IET Generation, Transmission & Distribution

Research Article

Two-stage stochastic demand response in smart grid considering random appliance usage patterns

ISSN 1751-8687
Received on 2nd February 2018
Revised 22nd June 2018
Accepted on 24th July 2018
E-First on 6th September 2018
doi: 10.1049/iet-gtd.2018.5943
www.ietdl.org

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Abstract: By effectively adjusting the appliance usage patterns of customers, demand response (DR) is expected to bring significant economic and environmental benefits to the future smart grid. Two kinds of appliances should be considered for DR, i.e. shiftable appliances such as dishwashers and laundry machines, and non-shiftable appliances such as lights and stoves. Although the shiftable appliances can be well controlled by energy management systems, the random usage patterns of non-shiftable appliances will result in uncertainties to electrical demands and thus, affect the efficiency and reliability of smart grid operation. A two-stage stochastic programming problem is formulated, for which the distribution system operation cost is minimised in the first stage, by considering various distribution system operation constraints. The scheduling of shiftable appliances is optimised in the second stage, by considering the random usage patterns of non-shiftable appliances. To reduce the computational complexity caused by a large number of home appliances in distribution systems, scenario reduction technique is applied to reduce the number of possible scenarios while still retaining the essential features of the original scenario set. Extensive simulations are performed to evaluate the proposed DR scheme in IEEE 33-bus and 119-bus test distribution systems based on real appliance usage pattern data.

Nomenclature

A_{NS}	non-shiftable appliances set
A_{SE}	shiftable EMS controlled appliances set
A_{SM}	non-shiftable price sensitive appliances set
C^m	household electrical cost
C^u	utility electrical operation cost
$G()$	price sensitive function
M	household set
$O()$	household occupied function
T	time set
W	market clear price
max	maximum value
min	minimum value
ψ	power operation to describe appliance property
a	appliance indices
c_{ω}	wholesale electricity market
m	household indices
t	time slot indices
E	energy used to describe appliance property
H	hour used to describe appliance property
I	power flow node current
L_{SR}	power loss between sending end and receiving end
N_{tap}	transformer tap operation
P	active power
Q	reactive power
V	power flow node voltage
τ	transformer tap-ratio
ξ	time of use probability profile
n_{R}	receiving end
n_{S}	sending end
x	decision variable of EMS controlled appliance
$\boldsymbol{y}$	admittance matrix
z	impedance matrix

1 Introduction

As a crucial component of the future smart grid, demand response (DR) can benefit from the two-way communications between electricity producers and customers and results in power quality improvement as well as power system operation cost reduction. On the other hand, the customers can adjust (or shift) the time of their appliance usage in response to different electricity pricing for energy bill savings. According to the U.S. residential energy consumption survey [1], the percentage of energy consumption at home by daily electrical appliances (e.g. lighting and air conditioning) is 34.6% in 2011, which is 1.44 times higher than that of 1993. DR is expected to play an important role in accommodating such a load increment in the near future.

DR, along with renewable energy sources and energy storage devices at the distribution system level, will have significant implications in the wholesale energy market. In [2], the authors distinguish various types of DR as market DR, while physical DR. Market DR focuses on the electrical pricing and the physical DR is more about the grid system management. DR programs can also be divided into time-based DR programs and incentive-based DR programs [3]. By these different classifications of DR, we can observe that the electricity market requires the customers to play an active role to improve the grid system, rather than pure receivers in the traditional market. Moreover, the close relationship between the customer and the electrical market helps enhance power quality, reduce peak period load demand, and enhance customer user experience. Therefore, customers can choose whether to shift electrical appliances to the low-price period or to their favourite hours.

In the literature, DR optimisation has been studied based on various pricing mechanisms, such as real-time pricing, day-ahead pricing, time-of-use pricing, and critical peak pricing [4, 5]. Several research works investigate the optimal design of price-based DR schemes by utility companies based on the prediction of customer load demand, in order to improve the efficiency of power system operation [6–9]. A recent research work pointed out that the consideration of uncertain load growth is critical for distribution network pricing [10], and a bidding strategy operation model of the virtual power plant has been formulated to make distributed energy resources more applicable and effective in electricity market [11]. Although there is existing research works on modelling the uncertainties of renewable power generation [12, 13], electric vehicle (EV) charging and discharging, and voltage regulation and inverter capacity [14], how to incorporate the random appliance usage patterns in the development of DR schemes in distribution system still requires extensive research. In order to address this issue, recent research works on DR investigate residential appliances with flexible service time period and power intensity, as well as day-ahead load forecast considering errors [15]. Also, a continuous decision-making process that allowed more flexibility of electricity customers is proposed in [16]. Probabilistic residential electrical load models are developed in some recent research works by considering the random operating conditions of each home appliance under uncertain human behaviour [17]. Yet, these works concentrated on the optimisation at the residential level, but how to utilise the probabilistic residential electrical load models and develop a stochastic DR scheme accordingly in the distribution system level is still an open issue.

In this paper, we proposed a two-stage stochastic programming scheme for DR in a smart grid. Different from the recent research works on the optimisation in distribution systems, which model each residential household by its total load, we establish detailed models of the usage patterns of each appliance in the household, as well as the customers' response to electrical price variation. Specifically, the operation cost minimisation of the distribution system is considered in the first stage by optimising electrical price, while the optimal scheduling of shiftable appliances is investigated in the second stages. The interaction between the two stages is established based on customers' response to electricity price, which can affect the usage patterns of various appliances. This work is important for the analysis of the impact of customers' uncertain behaviour in distribution systems with certain DR programs and for the utility companies seeking for optimal pricing schemes for the DR programs to indirectly affect customers' behaviour. The main contributions of this paper are threefold:

- A two-stage stochastic programming scheme is developed for DR in the smart grid, by considering the random appliance usage patterns of customers.
- In the first stage of the stochastic programming, a genetic algorithm is implemented to optimise the electricity price, by considering the responses of various types of appliances and non-linear distribution power flow.
- In the second stage of the stochastic programming, due to the existence of a large number of appliances with random usage patterns in each household, a modified scenario reduction technique is proposed to reduce the computational complexity of appliance scheduling optimisation.

This remaining of this paper is organised as follows. The related works are introduced in Section 2. In Section 3, the system model, including the models of both distribution system and household appliances, are introduced. The two-stage stochastic programming scheme, along with the genetic algorithm and scenario reduction technique are presented in Section 4. The simulation results are discussed in Section 5, followed by the concluding remarks in Section 6.

2 Related work

Random load demand has been studied in the literature as a part of uncertainties in power systems. A real-time interactive energy management scheme of microgrid was proposed by Marzband *et al.* [18], where various uncertainties including random load demand and renewable power generation are being considered. Also, the uncertainties of wind power generation and price elastic loads are investigated in [12] for security-constrained economic dispatch. Nevertheless, these research works study the random load demand based on data analysis and prediction, by assuming that the human behaviour is known in advance.

In practice, the knowledge of future human behaviour cannot be obtained accurately when the electrical price is released. To address this kind of uncertainty, two-stage approaches can be applied for stochastic programming [19]. Considerable efforts have been made in the past concerning applying two-stage approaches for DR [20–24]. In particular, a two-stage stochastic programming problem was formulated in [20], aiming at pursuing the optimal day-ahead power procurement with minimum costs and expected recourse cost, while considering the random actual power demand, renewable energy supply and storage. In [21], a two-stage operation scheme was introduced to reduce the uncertainty of the solar energy at the first stage, while maximising the total revenue of EV parking at the real-time operation in the second stage. Uncertainties such as renewable energy, power demand, and energy storage are considered in [22–24].

Although different kinds of uncertainties have been studied in power systems, all of the above-mentioned literature does not take into account human behaviour uncertainties in DR, or only considers the random appliance usage patterns in the household energy management system (EMS) instead of a distribution system. In this paper, we formulate a two-stage stochastic programming problem based on probabilistic residential electrical load models for DR in the smart grid. Besides, a genetic algorithm is implemented to solve the two-stage stochastic programming problem, in conjunction with a scenario reduction technique for computational complexity reduction.

3 System model

In this paper, we consider a typical residential distribution system and various types of appliances. The distribution system power flow model and household electrical appliance models are presented in the following.

3.1 Distribution system power flow model

We use a common branch model to characterise the transmission lines and transformers in an n-node distribution system, which consists of a standard π transmission line model and an ideal phase shifting transformer model. For a transformer with tap-ratio τ and phase shift angle θ, its turns ratio can be represented as $B = \tau e^{j\theta}$, while a transmission line can be modelled by letting $B = 1$.

Then, the complex current from the sending end (I_{n_S}) to the receiving end (I_{n_R}) of a branch can be expressed with branch admittance matrix and respective voltages V_{n_S} and V_{n_R}, given by

$$\begin{bmatrix} I_{n_\mathrm{S}} \\ I_{n_\mathrm{R}} \end{bmatrix} = \begin{bmatrix} \left(\dfrac{y}{2} + z\right)\dfrac{1}{B^2} & \dfrac{1}{B}z \\ \dfrac{1}{B}z & z + \dfrac{y}{2} \end{bmatrix} \begin{bmatrix} V_{n_\mathrm{S}} \\ V_{n_\mathrm{R}} \end{bmatrix}, \tag{1}$$

where the impedance z and the admittance y in the branch admittance matrix are elements between the sending end and receiving end. For an n-node distribution system, the complex nodal current injections from related node d to node b is $\boldsymbol{I}_b = \sum_{d=1}^{d=n} I_{bd}$. Then, the complex power flow can be calculated as a function of the complex nodal voltages, given by

$$P_b + jQ_b = V_b \boldsymbol{I}_b^* = V_b \boldsymbol{Y}_{bd}^* \boldsymbol{V}_d^*, \tag{2}$$

where P and Q refer to the active power and reactive power, respectively. $\boldsymbol{Y}_{bd}$ integrates all the impedance and admittance elements into a complex $n \times n$ admittance matrix. Once the active and reactive power consumed by all household appliances (which are random variables in nature due to the random appliance usage patterns) are realised, the voltage and related phase angle can be obtained based on power flow analysis.

For tap changing transformers (e.g. voltage regulators), the transformer tap-ratio can be calculated as [25]

$$\tau_t = 1 + b_{V,t}, \quad \forall t = 1, 2, \ldots, T, \tag{3}$$

where $b_{V,t}$ refers to the voltage regulator coefficient, based on the transformer turns ratio range. For a given period of time (T), the total number of transformer tap operations can be calculated as

$$N_{\text{tap}} = \sum_{t=1}^{T} |\tau_t - \tau_{t-1}| \,. \tag{4}$$

The branch power loss L_{SR} from node n_{S} to node n_{R} can be calculated based on bus voltages and branch parameters as

$$L_{\text{SR}} = |(V_{n_{\text{S}}}/B) - V_{n_{\text{R}}}|^2 / z \,. \tag{5}$$

As mentioned before, the voltage and phase angle can be acquired from power flow analysis. So, the system operation cost, which includes tap operation and power loss, can be calculated accordingly.

3.2 Residential load model

Two categories of household appliances are considered for DR programs. One includes the non-shiftable appliances which involve human participation, such as cooking and cleaning. The other consists of shiftable appliances such as washers and dryers, which can be controlled by an EMS with adjustable operation start times. A key feature of this research work is the consideration of the random usage patterns of appliances, which can significantly affect the optimisation of DR schemes.

3.2.1 Non-shiftable appliances: Recent research works have investigated how to quantify the randomness of human participated activities [26, 27]. In particular, the electric load profiles of individual appliances have been developed.

Based on these research works, we can introduce the appliance time-of-use (ToU) probability profile $\xi_{m,a,t}$, which represents the probability of operation of an electrical appliance $a \in A_{\text{SM}}$ in a household $m \in M_n$ during the time period $t \in T$.

The probability distribution profiles of human behaviour can be found in [28]. In this paper, we assume that the appliances related to one behaviour follow the same distribution (e.g. deep fryer and stove are related to the cooking behaviour). Fig. 1 shows several typical human behaviours in a common household with children. The probability distributions differ by household types. For more details, please refer to [28].

In practice, many factors can affect this ToU probability profile, among which the price sensitivity is a major factor.

Price sensitivity function $G(W)$ can be used to describe the sensitivity of human behaviours in response to different market clear prices (MCPs) W [29]. Affected by the price sensitivity, the new ToU probability profile can be calculated as follows:

$$\xi'_{m,a,t} = G(W_t) \cdot \xi_{m,a,t}, \quad \forall m \in M_n, \forall a \in A_{\text{SM}}, \forall t \in T, \tag{6}$$

where the MCP value applied to calculate the new ToU probability profile is given by

$$W_t = \frac{\alpha c_t}{c_t^{\max}}, \quad \forall t \in T \,. \tag{7}$$

Here α is a calibration scalar to adjust the MCP value. By applying different scalars, the effectiveness of the price sensitivity can be different.

Another common factor related to the random appliance usage patterns is whether the house is occupied or not. In other words, the appliances can only be operated when the house is not empty. Thus, a household occupation function can be applied to force the appliance turn-on probability to zero when the household is empty [17, 26]. Accordingly, a household occupation function with binary variables is introduced in this paper, given by

$$O(m) = \begin{cases} 1, & \text{if the house is occupied} \\ 0 & \text{otherwise} \end{cases} \quad \forall m \in M_n \,. \tag{8}$$

Consequently, the household occupation function affected ToU probability profile can be expressed as

$$\xi'_{m,a,t} = O(m) \cdot \xi_{m,a,t}, \quad \forall m \in M_n, \forall a \in A_{\text{SM}}, \forall t \in T \,. \tag{9}$$

The summation of the adjusted human activity probability profile $\xi'_{m,a,t}$ is still equal to 1. Therefore, a calibration equation can be introduced as follows:

$$\xi''_{m,a,t} = \frac{\beta \xi'_{m,a,t}}{\sum_t \xi'_{m,a,t}}, \quad \forall m \in M_n, \forall a \in A_{\text{SM}}, \forall t \in T \,. \tag{10}$$

The final ToU probability profile $\xi''_{m,a,t}$, which is affected by either the price sensitivity function or the occupation function, could be adjusted by the calibration scalar β to ensure the sum of probability profile equals to 1.

3.2.2 Shiftable appliances: Different from the non-shiftable appliances operated by a human which can cause uncertainty, the EMS controlled appliances can be shifted to the lower price period deterministically. For the various EMS controlled appliances, we proposed the following EMS appliance property matrix to describe each appliance

$$a \in A_{\text{SE}} := \{a\colon [H_a, H_{s_a}, H_{f_a}, E_a, \psi_a^{\max}, \psi_a^{\min}]\} \,. \tag{11}$$

Here, we denote the appliance operation duration by H_a, the operation starting time H_{s_a}, finishing time H_{f_a}, appliance total energy consumption E_a, and upper and lower bounds of power operation consumption $\psi_a^{\max}$ and $\psi_a^{\min}$, respectively. Based on this matrix, we can define appliances with different requirements, such as the appliances need to be operated during a specific period, with controllable or uncontrollable power consumption. Details of the EMS controlled appliance operation are discussed in Section 5.2.

Therefore, considering these two kinds of appliance and non-shiftable appliances without price sensitivity as base load ($a \in A_{\text{NS}}$), the power consumption of a single household $m \in M_n$ can be calculated as

$$P_m = \sum_a P_a, \{a \in A \,|\, A = A_{\text{SE}} \cup A_{\text{NS}} \cup A_{\text{SM}}\} \,. \tag{12}$$

The household reactive power for power flow computing can be achieved by using the power factor $\cos\theta_a$ of a specific appliance a, given by

$$Q_m = \sum_a P_a \left(\frac{1}{\cos^2\theta_a} - 1 \right)^{-1}, \quad \forall m \in M_n, \forall a \in A \,. \tag{13}$$

4 Problem formulation

In this paper, we consider the minimisation of distribution system operation cost based on DR. Therefore, we can define the operation cost as follows [In this work, the transmission investment and maintenance costs are not considered, since they are typically

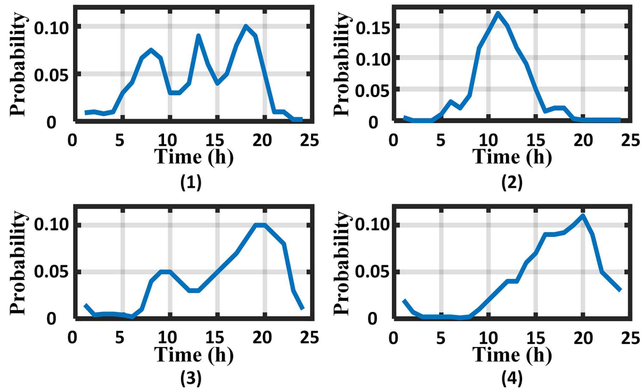

Fig. 1 *Time of use distribution profile with the following activities: (i) cooking; (ii) laundry; (iii) comfort and healthy; (iv) entertainment*

charged at a fixed rate and would not affect the optimisation results.]:

$$\text{Operation cost} = \text{Wholesale martket's electrical cost} \\ +\text{Power transmission cost}\,. \tag{14}$$

The electrical cost from wholesale market associated with the cost of energy procurement by retailers from the power pool, while the transmission cost reflects the power loss during transmission plus the cost associated with transformer wear and tear due to tap charging operation. Therefore, the objective function of the optimisation problem is formulated as

$$C^{u}(\boldsymbol{c}_w, P) = \sum_n \sum_m \sum_a \sum_t \boldsymbol{c}_w P_{n,m,a,t} + \left(\mu \sum_n \sum_t \boldsymbol{c}_w L_{n,t} + \nu N_{\text{tap}}\right), \\ \forall n \in N, \forall m \in M_{\text{NS}}, \forall a \in A, \forall t \in T\,. \tag{15}$$

where μ and ν are the weights given to the power transmission operation which depends on the level of priority, while $\boldsymbol{c}_w$ refers to the price of the wholesale electricity market, who offers the electricity to retailers. In this work, we consider that the electricity retailers purchase from the wholesale pool and decide the price for electricity customers. The optimised electrical price can help reduce power loss by indirectly affect the usage of price-sensitive appliances. Consequently, the residents would respond to the electrical price $\boldsymbol{c}$ presented by the utility company, and aspirate to reduce the electrical expenditure by arranging their behaviours related to non-shiftable appliances and shiftable appliances. Therefore, the household appliances electrical expenditure, that is, all the appliances $a \in A$ electrical cost C^m in a household $m \in M_n$ can be commonly expressed as

$$C^{m}(\boldsymbol{c}, P) = \sum_t \boldsymbol{c}\left(\sum_a P_{a,t}\right), \quad \forall a \in A, \quad \forall t \in T\,. \tag{16}$$

5 Two-stage stochastic programming

Considering the costs of two different parties in the distribution system, i.e. utility company and customers, the optimisation problem with random appliance usage patterns can be solved based on two-stage stochastic programming. The basic idea is that optimal decisions should be made based on available data, without a priori knowledge of future observations. The general formulation of a two-stage stochastic programming problem is given by [30]

$$\min \Bigg\{C^{u}(\boldsymbol{c}_w, P) = \sum_{n \in N} \sum_{m \in M_n} f_u\left(\boldsymbol{c}_w, \sum_{a \in A_{\text{SE}}} P_{a,t}\right) \\ + \sum_{n \in N} \sum_{m \in M_n} \mathbb{E}_{\varepsilon \in \mathscr{E} a \in A_{\text{SM}}}[g_u(\boldsymbol{c}_w, P_a(\varepsilon))] \\ + \left(\mu \sum_n \sum_t \boldsymbol{c}_w L_{n,t} + \nu N_{\text{tap}}\right)\Bigg\}, \tag{17}$$

where $f_u(\boldsymbol{c}_w, \sum_{a \in A_{\text{SE}}} P_a)$ refers to the shiftable appliance related cost of the utility company. Once the second stage realisation is achieved, it corresponds to a deterministic power consumption in the distribution system. Further, $g_u(\boldsymbol{c}_w, P_a(\varepsilon))$ is the non-shiftable appliance $a \in A_{\text{SM}}$ related cost, which can be calculated from the second stage problem

$$\min \{g_u(\boldsymbol{c}_w, P_a(\varepsilon)) | G_M(\boldsymbol{c}, \varepsilon) + O(m)P_a(\varepsilon) = h(\varepsilon)\}\,. \tag{18}$$

Here, ε refers to the appliance random turn-on scenarios, determined by the ToU probability profile $\xi_{m,a,t}$, which is related to the electrical price $\boldsymbol{c}$ presented by the utility company as well as the household occupation function $O(m)$. For the second stage, the electrical price $\boldsymbol{c}$ is determined before the realisation of the uncertain data ε. Once the realisation of ε becomes available, we can optimise the shiftable appliances by solving an optimisation problem.

In this paper, a genetic algorithm is applied in the first stage to seek for the optimal electrical price since the distributed power flow analysis is highly non-linear. A flowchart of the stage decomposition based genetic algorithm is shown in Fig. 2. Details of the techniques involved are discussed in the following subsections.

5.1 First stage optimisation

The general formulation of the proposed stochastic programming is shown in (17). Since the power loss L and transformer tap operation N_{tap} can be calculated based on power flow equations once all the appliance operations are settled, we can start with costs of EMS controlled shiftable appliances ($a \in A_{\text{SE}}$) and non-shiftable base load appliances ($a \in A_{\text{NS}}$). By considering all the appliances of these two categories in a household $m \in M_n$, we have

$$f_u\left(\boldsymbol{c}_w, \sum P_a\right) = \sum_{a \in A_{\text{SE}} \cup A_{\text{NS}}} \sum_{t \in T} \boldsymbol{c}_w P_{a,t}\,. \tag{19}$$

Further, we defined the stochastic appliance turn-on scenarios by $\varepsilon \in \mathscr{E}$. Therefore, for each scenario ε, the electrical cost of shiftable appliances with ToU probability profiles ($a \in A_{SM}$) in the household $m \in M_n$ can be formulated as

$$g_u(\boldsymbol{c}_w, P_m(\varepsilon)) = \sum_{t \in T} \boldsymbol{c}_w P_m(\varepsilon), \quad \forall \varepsilon \in \mathscr{E}\,. \tag{20}$$

Consequently, the electrical cost of a household by considering the random human behaviours can be formulated as

$$\mathbb{E}_{\varepsilon}[g_u(\boldsymbol{c}_w, P_m(\varepsilon))] = \sum_{\varepsilon} p_{\varepsilon} g_u(\boldsymbol{c}_w, P_m(\varepsilon)), \quad \forall \varepsilon \in \mathscr{E}\,. \tag{21}$$

By minimising the cost, the cost minimisation of distribution system operation can be achieved in the first stage. However, in (21), the total number of scenarios of appliance turn-on permutation in a household is given by

$$K_{\mathscr{E}} = (K_a)^{(K_{O(m)})}, \quad \forall a \in A_{\text{SM}}, \forall m \in M_n\,. \tag{22}$$

As the number of household appliances increase, the total number of scenarios increases exponentially. The situation becomes even worse if we model the ToU probability profile $\xi_{m,a,t}$ as random variables with continuous distributions. A common approach to reducing the scenario set to a manageable size is by using Monte Carlo simulations. Specifically, we can generate a set $\varepsilon^1, \varepsilon^2, \ldots, \varepsilon^K$ of K scenarios of the random vector $\boldsymbol{\varepsilon}$, which follow the same probability distribution. Furthermore, we assume that the samples are independent identically distributed. Therefore, the

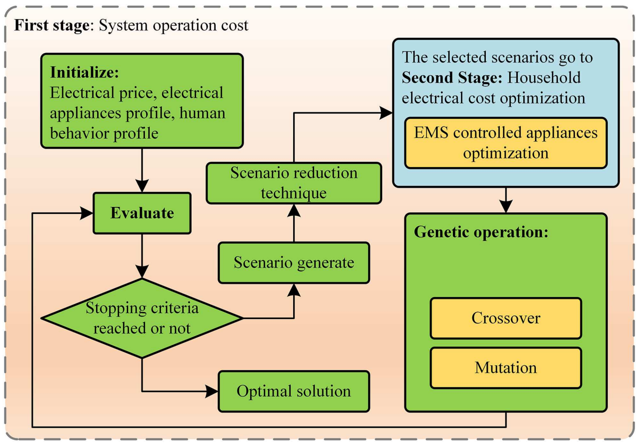

Fig. 2 *Flowchart of the stage decomposition based genetic algorithm*

approximated expectation $g_m(\boldsymbol{c}, P_m(\varepsilon))$ can be calculated based on an average over the samples, given by

$$\mathbb{E}_\varepsilon[g_u(\boldsymbol{c}, P_m(\varepsilon))] = \frac{1}{K}\sum_k g_u(\boldsymbol{c}, P_m(\varepsilon^k)), \quad \forall k \in K\,. \tag{23}$$

This formulation is also known as the Sample Average Approximation method [31]. The accuracy of the optimal result can be improved by increasing the number of samples (K). Therefore, the computational complexity of the algorithm could be excessive when considering a large distribution system. In order to address this issue, scenario reduction technique will be introduced in Section 5.3.

5.2 Second stage optimisation

The objective of second stage optimisation is to calculate an appliance schedule to minimise the electrical cost for each scenario $\varepsilon \in \mathscr{E}$, based on the electrical price $\boldsymbol{c}$ given by the first stage optimisation. Furthermore, once the operations of shiftable appliances with ToU probability profiles are realised, the shiftable appliances controlled by the EMS can be scheduled by solving a mixed-integer linear programming (MILP) problem. The general formulation of the second stage optimisation is introduced in (18) and (20), with the details given below:

$$\min_{\boldsymbol{x},\boldsymbol{r}} f_m(\boldsymbol{c}, P_a) + g_m(\boldsymbol{c}, P_m(\varepsilon))\,. \tag{24}$$

$$\text{s.t.} \sum_{a \in A_{\mathrm{SM}}} P_a + \sum_{a \in A_{\mathrm{SE}}} P_a + \sum_{a \in A_{\mathrm{NS}}} P_a \le P^{\max}, \tag{25}$$

$$G_M(\boldsymbol{c}, \varepsilon) + O(m)P_a(\varepsilon) = h(\varepsilon), \tag{26}$$

$$F(P_a(\boldsymbol{x}, \boldsymbol{r})) = b\,. \tag{27}$$

The value of $P^{\max}$ in the first constraint can be obtained by running the standard test system, according to certain voltage and loading constraints. The second constraint is used to simplify (6), (9) and (10) for the appliance $a \in A_{\mathrm{SM}}$. The last constraint refers to the EMS controlled appliances. The decision variables $(\boldsymbol{x}, \boldsymbol{r})$ for $a \in A_{\mathrm{SE}} \cup A_{\mathrm{NS}}$ are introduced as follows:

$$(\boldsymbol{x}, \boldsymbol{r}) = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_a, \boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_a], \tag{28}$$

$$\boldsymbol{x}_a = [x_{a,1}, x_{a,2}, x_{a,3}, \ldots, x_{a,t}], \tag{29}$$

$$\boldsymbol{r}_a = [r_{a,1}, r_{a,2}, r_{a,3}, \ldots, r_{a,t}], \tag{30}$$

where $\boldsymbol{x}_a$ is the energy consumption for each appliance $a \in A_{\mathrm{SE}} \cup A_{\mathrm{NS}}$, and each $\boldsymbol{x}_a$ is consumed within the T time slots. Also, $\boldsymbol{r}_a$ is the appliance operation status represented by binary variables, i.e. 1 and 0 for appliances turned on and off, respectively.

Based on the appliance property matrix (11), we can define several kinds of appliances: appliances with controllable power level such as light bulbs with controllable brightness, electric fans with controllable speeds; appliances with fixed power level such as battery chargers with fixed charging rates; and appliances need to operate at a specific period of day and so on. For the set of appliances operated by EMS ($a \in A_{\mathrm{SE}}$) with controllable power levels ψ_a, the properties can be described as follows:

$$\sum_{t = H_{s_a}}^{t = H_{f_a}} x_a^t = E_a, \quad \psi_a^{\min} \le x_a^t \le \psi_a^{\max}, \tag{31}$$

$$\sum_t s_a[t] = H_a, \quad H_{s_a} \le \{t \mid s_a[t] = 1\} \le H_{f_a}, \tag{32}$$

where $s_a[t] = 1$ refers to the appliance turn-on time. For the set of appliances operated by EMS ($a \in A_{\mathrm{SE}}$) with a fixed power level, it can be formulated as

$$\sum_{t = H_{s_a}}^{t = H_{f_a}} x_a^t = E_a, \quad \psi_a^{\min} = x_a^t = \psi_a^{\max}, \tag{33}$$

$$\sum_t s_a[t] = H_a, \quad H_{s_a} \le \{t \mid s_a[t] = 1\} \le H_{f_a}\,. \tag{34}$$

For an appliance $a \in A_{\mathrm{NS}}$, which refers to the non-shiftable appliance such as refrigerator, the parameters in the property matrix are all constant and cannot be optimised. Therefore, after the stochastic appliance scenarios are settled, MILP can be used for the EMS controlled appliance optimisation.

5.3 Scenario reduction for two-stage stochastic programming

In stochastic programming, the expectation of uncertainty related problem can be obtained by evaluating all possible scenarios, which usually results in an enormous scenario set. For the ease of implementation, we need to reduce the number of scenarios while still preserving the basic characteristics of the original scenario set. In other words, we seek a set of reduced scenarios to produce the optimal solution that can best approximate the solution of the original problem.

In this paper, the scenario reduction technique based on fast forward section is implemented [32], as shown in Algorithm 1. In this algorithm, $o(\varepsilon_k, \varepsilon_u)$ refers to the norm of ε_k and ε_u. This selection allows us to not only seek for the scenarios with the highest probability of occurrence but also concern the solution that is closest to the original optimal problem. In each step i, the closest scenario is selected. In general, more accurate results of scenario reduction can be obtained by increasing the number of steps.

Algorithm 1: Scenario reduction

for $i = 1$ **do**
  $o_{ku}^{[1]} = o(\varepsilon_k, \varepsilon_u), \quad k, u = 1, \cdots, \mathscr{E}$
  $z_u^{[1]} = \sum_{k=1, k \ne u}^{\mathscr{E}} p_k o_{ku}^{[1]}, \ u = 1, \cdots, \mathscr{E},$
  $u_1 \in \arg\min_{u \in \{1, \cdots, \mathscr{E}\}} z_u^{[1]}, \ J^{[1]} := \{1, \cdots, \mathscr{E}\} \setminus \{u_1\}$
end for
for $i = 2, 3, \ldots, \mathscr{E}$ **do**
  $o_{ku}^{[i]} = \min\left\{o_{ku}^{[i-1]}, o_{ku_{i-1}}^{[i-1]}\right\}, \quad k, u \in J^{[i-1]}$
  $z_u^{[i]} = \sum_{k \in J^{[i-1]} \setminus \{u\}}^{\mathscr{E}} p_k o_{ku}^{[i]}, \ u \in J^{[i-1]}$
  $u_i \in \arg\min_{u \in J^{[i-1]}} z_u^{[i]}, J^{[i]} := J^{[i-1]} \setminus \{u_i\}$
end for
for $i = n + 1$ **do**
  Redistribution by the minimum attained at:
  $\bar{q}_j = p_j + \sum_{i \in J_j} p_i, \quad$ for each $j \notin J$
end for

To further accelerate the scenario reduction process, we also combine the following method with the fast forward section. The key for this method is to transfer the appliance turn-on scenarios to the power consumption scenarios with the related probability. Specifically, the power consumption by a different power level $P(l)$ in a household $m \in M_n$ at a specific time $t \in T$ can be computed as

$$P_{m,t}(l) = \sum_a P_a(l), \quad \forall a \in A_{SM}, \ \forall l = 1, 2, \ldots, \tag{35}$$

where $\sum_a P_a(l)$ refers to the power consumption of power level $P_{m,t}(l)$ related appliance turn-on scenarios. For instance, if the power level is $P_{m,t}(l) = 50\,\mathrm{W}$, $\sum_a P_a(l)$ can be five light bulbs with 10 W rating, and two light bulbs with 25 W rating at the time $t \in T$. A power level is the total appliance turn-on scenarios

Table 1 Characteristics of typical household appliances

Appliance category and name	Average power consumption, W	Average operation duration, h	Power factor
Kitchen			
blender	175	0.2	0.73
coffee maker	900	0.4	1
deep fryer	1500	0.267	1
dishwasher	1300	0.667	0.99
food freezer	350	8	0.8
microwave oven	1500	0.333	0.9
range and oven	4000	0.833	1
toaster	1200	0.133	1
Laundry			
dryer	5000	0.933	0.99
iron	1000	0.4	1
washing machine	500	0.867	0.65
Entertainment			
computer (desktop)	250	8	0.8
computer (laptop)	30	8	0.8
laser printer	600	2	—
stereo	120	4	—
television	100	4.167	0.8
Comfort and health			
air conditioner	750	2.467	0.9
electric heating	1000	8.333	1
fan	120	0.2	0.87
lights	60	8	0.93
vacuum cleaner	800	0.333	0.9

without repetition. Consequently, the corresponding power consumption probability distribution at a time $t \in T$ can be calculated as

$$\zeta_P(l) = \Pi_a \xi_{a'} \cdot \xi_a, \quad \forall a \in A_{\text{SM}}, \tag{36}$$

where $\zeta_P(l)$ is the probability corresponding to the power level $P_m(l)$, a' refers to the turned-off appliances. The probability distribution ξ_a introduced in Section 3.2.1. Using this probability distribution profile, the turn on/off probability for a specific appliance in each time slot can be obtained. Therefore, the power consumption probability distribution profile in the household $m \in M_n$ can be calculated via (35) and (36). For each time period, there is a pool that contains the turn on/off operation scenarios for all appliances. For a specific scenario in the pool, there exists an optimal solution for the shiftable appliances controlled by EMS. Noted that the non-shiftable appliances are assumed to be operated once every day and non-interruptible. As the number of all the scenarios can be as large as 2^a, we define a redistributed power consumption probability distribution by d intervals, i.e. in the household $m \in M_n$ at a time $t \in T$, the redistributed power level $P_{m,t}(l_d)$ can be calculated as

$$P_{m,t}(l_d) = \frac{\max\{P_m(l)\}}{d}, \quad \forall a \in A_{\text{SM}}, \forall l_d = 1, 2, \ldots \tag{37}$$

and the corresponding probability is the sum of the probability in each interval d, given by

$$\zeta_P(l_d) = \sum_d \zeta_{P,t}(l), \quad \forall a \in A_{\text{SM}}. \tag{38}$$

Since this method allows us to reduce a large number of scenarios to d scenarios and obtain the corresponding probability $\zeta_P(l_d)$ of each scenario, the original probability distribution can be retained with proper value of d. By combining the sample average approximation and scenario reduction technique, the performance of our proposed algorithm can be improved significantly without sacrificing the accuracy.

5.4 Heuristic two-stage stochastic programming algorithm

L-shaped method [33] has been widely used to solve two-stage stochastic programming problems. However, for large-size problems, the study in [34] indicates that the evolutionary algorithm such as a genetic algorithm performs better in finding the optimal solutions than the L-shaped method. Besides, the genetic algorithm, as a common mature algorithm in evolutionary computing, has been widely used in DR problems [9]. Furthermore, the power flow analysis in our work is highly non-linear in nature. For the above reasons, a genetic algorithm is applied to solve the proposed problem instead of the L-shaped method.

The proposed stochastic two-stage programming scheme with a genetic algorithm is introduced as follows:

- **First stage:** System operation cost minimisation:

  i. Initialise: Generate the initial population electrical price $c_{t,i}^k$, where the subscript i refers to the ith individual in the iteration k. Then, input the household shiftable appliances $a \in A_{\text{NS}} \cup A_{\text{SE}}$ use pattern follow (11). Also, input non-shiftable appliances with probability usage pattern $a \in A_{\text{SM}}$, which is ToU probability profile $\xi_{m,a,t}$ in this paper. For a specific time slot t, the scenario pool for non-shiftable appliances is obtained from the probability profile, and the pool is utilised by the next process.
  ii. Evaluate: Use the initialled individuals $\boldsymbol{c}_i^k$ to apply the fitness function below that modified from (17) with second stage power consumption P_m. $C^{\max}$ is the maximum estimate value of system operation cost. This function helps transfer the minimisation problem to maximisation problem

  $$\text{fit} = C^{\max} - C^u(\boldsymbol{c}_w, P_m). \tag{39}$$

  iii. Scenario reduction: scenarios are exhausted and the most representative scenarios are selected via Algorithm 1 and functions (35)–(38). Only the selected scenarios are considered in the second stage optimisation.
- **Second stage:** Household electrical cost minimisation: For each scenario $\varepsilon \in \mathscr{E}$, the optimal power consumption P_a can be achieved by (27). The optimal shiftable appliances schedules are decided in this stage once the representative scenarios are selected in the first stage.
- **Genetic operation:** Electricity price optimisation:

  i. Select: The best-ranking individuals are preserved as parents to reproduce the offspring.
  ii. Breed: To generate a new generation population, a combination of crossover and mutation can be applied to give birth to offspring $c_{t,i}^{k+1}$.
  iii. Evaluate: Apply the offspring $c_{t,i}^{k+1}$ to the fitness function $C^u(c, P)$.

6 Case study

In this section, we evaluate the performance of the proposed DR scheme based on the IEEE 33-bus and 119-bus test distribution systems. The simulations are conducted on a Linux desktop with an Intel i7-4790 CPU at 3.60 GHz with 16 GB RAM. Several categories of typical household appliances are considered, with their characteristics shown in Table 1.

For each appliance, the average power consumption, average operation duration, and power factor can be obtained from [35, 36]. The appliances for entertainment activities are considered as non-shiftable and insensitive to price due to comfort reasons, while all other appliances are considered as price sensitive. In particular, the washing machine and dryer are assumed to be controlled by the

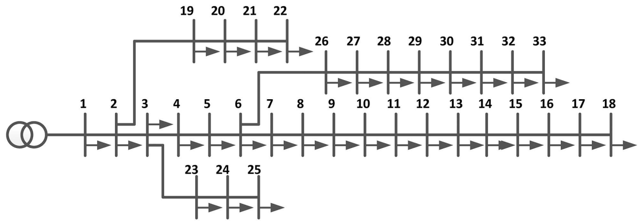

Fig. 3 *One-line diagram of the IEEE 33-bus test distribution system*

EMS in this case study. Note that the appliance list is expandable if more appliance usage data are available. In the case study, the time horizon is considered to be 24 h, with the duration of each time slot being 1 h. The wholesale market electrical pool price c_w is obtained from Alberta Electric System Operator (AESO) in April 2018 [37].

6.1 IEEE 33-buses test distribution system

The IEEE 33-bus test distribution system is used in the first case study. Due to the relatively small-scale of the distribution system with low computational complexity, we can evaluate our proposed DR scheme through extensive simulation runs under various system configurations. The one-line diagram of the system under study is shown in Fig. 3, where the detailed circuit data can be obtained from [38]. The system is operated at 12.66 kV, and the total real and reactive loads are 3715 kW and 2300 kVar, respectively. The voltage regulators can regulate system voltages in 32 steps with 0.625% for each step. The household type applied in this case study is the household with children, and the data related to human behaviour are collected from [28, 39].

In the simulation, the performance of the proposed two-stage stochastic programming scheme with scenario reduction technique is compared with that of the Monte Carlo simulation, which can be considered as the benchmark solution. Monte Carlo simulation method is widely used to generate random scenarios in stochastic programming [19, 40]. In this work, Monte Carlo simulation with repeated random sampling is applied to obtain the optimal results. These results still need to be sent to the genetic algorithm, and genetic operation (crossover and mutation) is used to generate a new population for the next iteration. A large number of Monte Carlo simulation runs can lead to better performance in terms of the DR outcome. However, the computational complexity can be prohibitive due to a large number of appliances in the distribution system. Furthermore, the results of both the proposed scheme and Monte Carlo simulation scheme, which are stochastic in nature, are also compared to that of traditional deterministic optimisation scheme, where only the expectations are used to model the uncertain factors in the simulation [19, 40]. Moreover, the mechanism of economic DR (EDR) introduced in [16] can be adapted for comparison.

A comparison of average fitness of different schemes is shown in Fig. 4*a*. Here, the modified objective function (39) that aims at finding the lowest cost for distribution system operation, is chosen as the fitness function for comparison. As we can see, the scenario reduction method shows a better convergence in the first 100 generations than other methods. Furthermore, the cost of EMS appliances $a \in A_{SE}$ is shown in Fig. 4*b*, with respect to the different percentage of controllable domestic appliances $a \in A_{SE} \cup A_{SM}$. In this simulation, after all the algorithms reach each their solutions, we use Monte Carlo simulation with random samples to simulate as the real scenario. Based on the EDR algorithm, the peak load is shifted to the off-peak period, which should lead to the most economical result. However, when the randomness of the appliance usage patterns is considered, the cost becomes higher than our proposed method, as shown in Figs. 4*b* and *c*. Also, Fig. 4*b* shows that our proposed method with scenario reduction technique has a better performance than that of Monte Carlo simulation and deterministic optimisation scheme, due to the consideration of random appliance usage patterns. Note that the cost based on Monte Carlo simulations with 1000 random samples is higher than that of our proposed scheme. The main reason is that Monte Carlo simulations choose scenarios randomly, while the scenario

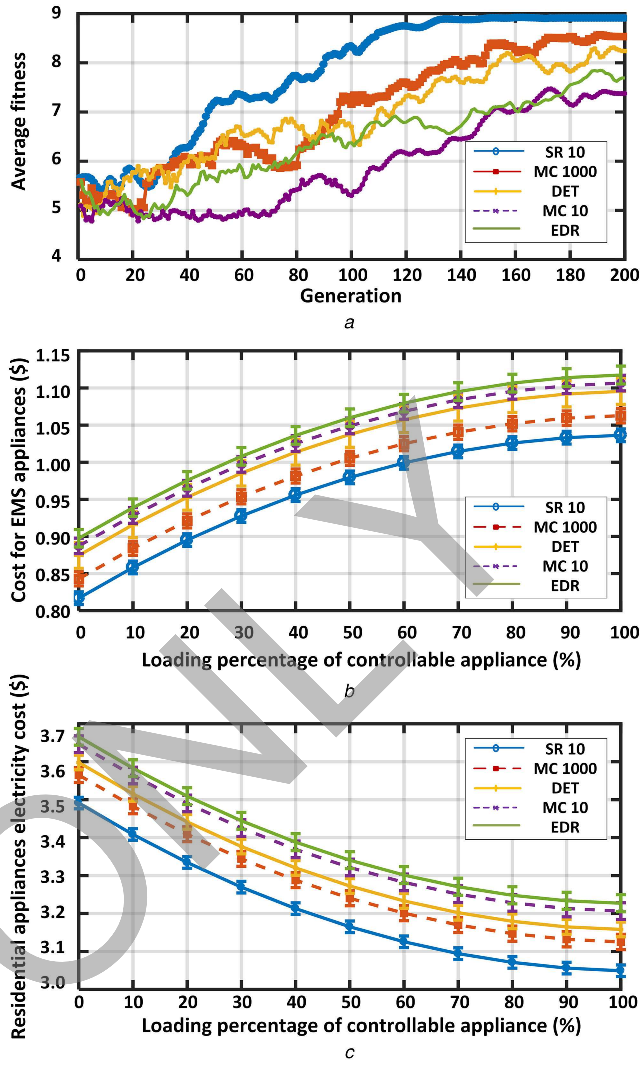

Fig. 4 ***(a)*** *Comparison of the average fitness,* ***(b)*** *Cost of shiftable appliances controlled by EMS,* ***(c)*** *Comparison of the pure electrical cost*

reduction technique used in this paper selects scenarios based on the probability of their occurrence. Since the probabilities for different human behaviours to occur are different, our proposed scenario reduction technique can effectively select the scenarios which may improve the outcomes of DR significantly.

From the utility companies' perspective, Fig. 4*c* shows the cost of electricity from the wholesale market for distribution system operation, which can be indirectly affected by the optimised electrical price. As we can see, the electrical cost is lower based on the proposed scheme. Besides, as the percentage of the domestic appliances increases, the electrical cost can be reduced significantly. In other words, the optimised electrical price can effectively reduce the distribution system operation cost while achieving reliable electrical grid operation. Note that in these figures, we allocate different loading percentages of shiftable appliances. Although the cases with close to 0% or 100% of domestic appliances may not happen in practice, these cases are still included in this case study to show the trend of the performance of different algorithms.

6.2 IEEE 119-buses test distribution system

A relatively large-scale case study is performed based on the IEEE 119-bus test distribution system to test the scalability and effectiveness of the proposed scheme, the system data can be found in [41]. As shown in Fig. 5, several types of households with various occupation function $O(m)$ and human behaviour ToU probability profiles are used. The test system operates at 11 kV with 22,709.7 kW and 17,041.1 kVar of real and reactive power

Table 2 Electrical price optimisation results

Method	SR10	MC10	MC1000	DET	EDR
user cost, $	5.0126	6.7451	5.2202	5.8152	6.8366
system operation cost, $	268,578	358,572	299,785	309,342	362,523
single individual average time, s	15.7467	8.8023	282.7716	2.6277	3.0424

demands, respectively. In this study, 30% of shiftable appliances are implemented.

Table 2 shows the performance of the proposed scheme (SR10), in comparison with the Monte Carlo simulations with 10 and 1000 random samples (MC10 and MC1000), respectively, the deterministic optimisation scheme (DET), and EDR. For all the simulation results, 200 generations are applied to the genetic algorithm for each method. Similar to the 33-buses test system, we still use Monte Carlo simulation with a random sample to test these algorithms as the real scenario. As we can see, the proposed scheme can optimise the electrical price more effectively, as reflected by the lower user cost and system operation cost. Also, the convergence of the proposed scheme is faster, since the average time for the calculation of every single individual is shorter. For the overall optimisation of the DR, the Monte Carlo simulation with 1000 random samples takes over 76 h to converge, while the scenario reduction technique can efficiently converge within 200 generations in about 4 h.

7 Conclusion

In this paper, a two-stage stochastic programming scheme is proposed for the purpose of optimal DR in the smart grid. A genetic algorithm is utilised to find the optimal electrical price under random appliance usage patterns, while a scenario reduction technique is embedded in the algorithm to reduce the computational complexity caused by a large number random scenarios of the household electrical appliance operation. Simulation results based on IEEE 33-bus and 119-bus test distribution systems indicate that our proposed scheme can provide better performance of DR in comparison with Monte Carlo simulations and deterministic optimisation. Also, the convergence of the proposed scheme is faster, which improves the efficiency of practical implementation of DR. Future research work involves the investigation of other types of uncertainties in DR, such as renewable energy sources with intermittent power generation and EVs with random driving cycles, and the development of an efficient stochastic programming scheme to optimise the DR process accordingly.

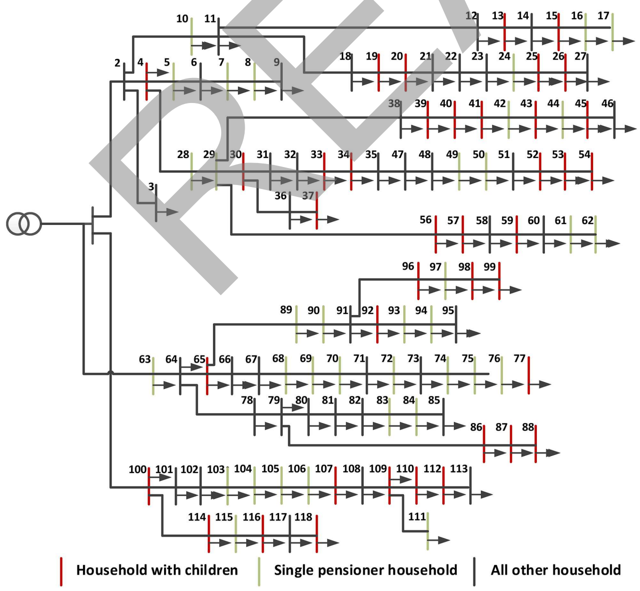

Fig. 5 *Illustration of the IEEE 119-bus test distribution system with various household types*

8 Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC). Yue Wang was sponsored by the China Scholarship Council (CSC) under grant no. 201406350066.

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