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THE STRUCTURE OF PERCEIVED RELATIONS AMONG PHYSICS CONCEPTS

by

© Heidi Kass

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled, "The Structure of Perceived Relations Among Physics Concepts," submitted by Heidi Kass in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

Supervisor

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Date August 11, 1969

ABSTRACT

The study was designed to investigate the applicability of multidimensional scaling techniques to the description of aspects of the structure of students' perceptions of subject-matter concepts encountered in the classroom.

More specifically, the investigation endeavored to ascertain the number and nature of the dimensions required to summarize the structure of perceived relations among selected mechanics concepts as derived from judgments of similarity with respect to their difficulty. Consistent individual viewpoints in the structuring of the given domain of concepts were sought and their relationships with mechanics achievement and various aptitude measures investigated.

The empirical data suggested that the group average perceptual space could be characterized in terms of either four or five dimensions. Two of the coordinates appeared to be interpretable as generalized continua while the remaining dimensions were characterized by clusters of concepts. The four viewpoint dimensions which emerged were not sufficiently different from one another to be considered distinct. No consistent significant differences among groups of subjects with high coefficients on different viewpoint dimensions emerged for any of the aptitude, achievement, or preference measures.

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TABLE OF CONTENTS

CHAPTER		PAGE
I.	THE PROBLEM	1
	Introduction	1
	A Method of Representing Viewpoints	3
	Some General Characteristics of Concepts	6
	A Basis for Making Judgments	9
	Purpose of the Study	11
	General Hypotheses	12
	Delimitations of the Study	17
	Definition of Terms	18
	Summary	20
II.	SURVEY OF THE LITERATURE	21
	Introduction	21
	Laboratory Studies of Concept Formation	22
	Concept Learning in School	26
	The Geneva View of Conceptual Development	33
	Cognitive Preference Studies in Science	35
	Formal and Empirical Structure in Mechanics	40
	Structure in Teaching	43
	Summary	46
III.	MULTIDIMENSIONAL SCALING	47
	Introduction	47
	The Basis for Multidimensional Scaling	47
	The Metric Structure of the Multidimensional Space	48
	Procedures for Obtaining Dissimilarity Estimates	53
	The Individual Differences Model for Multidimensional Scaling	56

CHAPTER PAGE

The "Idealized Individual" Concept 65

Studies Involving Individual Differences in Points of View 68

Nonmetric Multidimensional Scaling 71

Summary 80

IV. DESIGN OF THE INVESTIGATION: SUBJECTS, MATERIALS, AND PROCEDURES 81

The Population 81

The Paired Comparisons Task 82

The Achievement and Preference Measures: Development and Description. 85

Achievement Test 85

Cognitive Preference Test 86

The Aptitude Measures: Description 90

Differential Aptitude Tests. 91

Skemp's Tests of Reflective Thinking 94

Testing Procedure 97

V. ANALYSIS, RESULTS, AND INTERPRETATION 100

Dimensions of Viewpoint. 100

The Group Average Perceptual Space 105

The Viewpoint Representations. 132

The Achievement, Aptitude, and Preference Measures: Results 202

CHAPTER		PAGE
VI.	SUMMARY AND DISCUSSION	218
VII.	CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS	228
	Conclusions and Limitations.	228
	Implications for Science Education	231
	For Further Research	232
BIBLIOGRAPHY	. . . .	235
APPENDIX	. . . .	244
	Instructions for the Paired Comparisons Task	244
	Stimuli for the Paired Comparisons Task	245
	Physics 30 Achievement Test: Mechanics	247
	Cognitive Preference Test: Mechanics	254
	Skemp's Tests of Reflective Thinking	266
	Tables LXXII to XCIV	297

LIST OF TABLES

TABLE		PAGE
I.	Mean Intercorrelations for the DAT by Sex for Form L	92
II.	Distribution of Subjects, by Teacher, in Groups A, B, and C	102
III.	First Ten Eigenvalues, Groups A, B, and C	104
IV.	Stimulus-Pair Projections on First Principal Vector, Group A	107
V.	Stimulus-Pair Projections on First Principal Vector, Group B	108
VI.	Stimulus-Pair Projections on First Principal Vector, Group C	109
VII.	Minimum Stress for Group Average Configurations	110
VIII.	Final Unrotated Configuration of 20 Points in Four Dimensions, Group A Average	115
IX.	Final Unrotated Configuration of 20 Points in Five Dimensions, Group A Average	116
X.	Group B Rotated for Maximum Factor Vector Overlap with Group A	122
XI.	Cosines between Reference Axes: Group B Rotated for Maximum Factor Vector Overlap with Group A	124

TABLE PAGE

XII. Group C Rotated for Maximum Factor Vector Overlap with Group A 125

XIII. Cosines between Reference Axes: Group C Rotated for Maximum Factor Vector Overlap with Group A 127

XIV. Final Unrotated Configuration of 20 Points in Four Dimensions, Group B Average 129

XV. Cosines between Reference Axes: Group C Rotated for Maximum Factor Vector Overlap with Group B 130

XVI. Group C Rotated for Maximum Factor Vector Overlap with Group B 131

XVII. Number of Subjects Clustered on the Viewpoint Dimensions for Group A, B, and C 134

XVIII. Minimum Stress for Viewpoint Dimension II Configurations 136

XIX. Final Unrotated Configuration for 20 Points in Six Dimensions for High Positive Coefficients on Viewpoint Dimension II: Group A 140

XX. Final Unrotated Configuration for 20 Points in Six Dimensions for High Negative Coefficients on Viewpoint Dimension II: Group B 141

TABLE PAGE

XXI. Cosines between Reference Axes: Subgroup BII(-) Rotated for Maximum Factor Vector Overlap with Subgroup AII(+) 144

XXII. Subgroups BII(-) Rotated for Maximum Factor Fector Overlap with Subgroup AII(+) 145

XXIII. Cosines between Reference Axes: Subgroup BII(+) Rotated for Maximum Factor Vector Overlap with Subgroup AII(-) 146

XXIV. Subgroup BII(+) Rotated for Maximum Factor Vector Overlap with Subgroup AII(-) 147

XXV. Cosines between Reference Axes: Subgroup CII(+) Rotated for Maximum Factor Vector Overlap with Subgroup AII(+) 148

XXVI. Subgroup CII(+)Rotated for Maximum Factor Vector Overlap with Subgroup AII(+) 149

XXVII. Cosines between Reference Axes: Subgroup CII(-) Rotated for Maximum Factor Vector Overlap with Subgroup AII (-) 150

XXVIII. Subgroup CII(-) Rotated for Maximum Factor Vector Overlap with Subgroup AII(-) 151

XXIX. Cosines Between Reference Axes: Subgroup CII(+) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BII(-) 152

TABLE PAGE

XXX. Cosines between Reference Axes: Subgroup CII(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BII(+) 153

XXXI. Minimum Stress for Viewpoint Dimension III Configurations 155

XXXII. Final Unrotated Configuration for 20 Points in Four Dimensions for High Positive Coefficients on Viewpoint Dimension III: Group A 158

XXXIII. Final Unrotated Configuration of 20 Points in Four Dimensions for High Negative Coefficients on Viewpoint Dimension III: Group A 159

XXXIV. Cosines between Reference Axes: Subgroup BIII(+) Rotated for Maximum Factor Vector Overlap with Subgroup AIII(+) 161

XXXV. Subgroup BIII(+) Rotated for Maximum Factor Vector Overlap with Subgroup AIII(+) 163

XXXVI. Cosines between Reference Axes: Subgroup BIII(-) Rotated for Maximum Factor Vector Overlap with Subgroup AIII(-) 164

XXXVII. Subgroup BIII(-) Rotated for Maximum Factor Vector Overlap with Subgroup AIII(-) 165

TABLE PAGE

XXXVIII. Cosines between Reference Axes: Subgroup CIII(+) Rotated for Maximum Factor Vector Overlap with Subgroup AIII(+) 166

XXXIX. Subgroup CIII(+) Rotated for Maximum Factor Vector Overlap with Subgroup AIII(+) 167

XL. Cosines between Reference Axes: Subgroup CIII(-) Rotated for Maximum Factor Vector Overlap with Subgroup AIII(-) 168

XLI. Subgroup CIII(-) Rotated for Maximum Factor Vector Overlap with Subgroup AIII(-) 169

XLII. Cosines between Reference Axes: Subgroup CIII(+) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BIII(+) 170

XLIII. Cosines between Reference Axes: Subgroup CIII(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BIII(-) 171

XLIV. Minimum Stress for Viewpoint Dimension IV Configurations 173

XLV. Final Unrotated Configuration for 20 Points in Four Dimensions for High Positive Coefficients on Viewpoint Dimension IV: Group A 176

TABLE PAGE

XLVI. Final Unrotated Configuration of 20 Points in Four Dimensions for High Negative Coefficients on Viewpoint Dimension IV: Group A 177

XLVII. Cosines between Reference Axes: Subgroup BIV(-) Rotated for Maximum Factor Vector Overlap with Subgroup AIV(+) 180

XLVIII. Subgroup BIV(-) Rotated for Maximum Factor Vector Overlap with Subgroup AIV(+) 181

XLIX. Cosines between Reference Axes: Subgroup BIV(+) Rotated for Maximum Factor Vector Overlap with Subgroup AIV(-) 182

L. Subgroup BIV(+) Rotated for Maximum Factor Vector Overlap with Subgroup AIV(-) 183

LI. Cosines between Reference Axes: Subgroup CIV(-) Rotated for Maximum Factor Vector Overlap with Subgroup AIV(+) 185

LII. Subgroup CIV(-) Rotated for Maximum Factor Vector Overlap with Subgroup AIV(+) 186

LIII. Cosines between Reference Axes: Subgroup CIV(+) Rotated for Maximum Factor Vector Overlap with Subgroup AIV(-) 187

TABLE		PAGE

LIV. Subgroup CIV(+) Rotated for Maximum Factor Vector Overlap with Subgroup AIV(-) 188

LV. Cosines between Reference Axes: Subgroup CIV(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BIV(-) 189

LVI. Cosines between Reference Axes: Subgroup CIV(+) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BIV(+) 190

LVII. Minimum Stress for Viewpoint Dimension V Configurations 192

LVIII. Final Unrotated Configuration for 20 Points in Four Dimensions for High Positive Coefficients on Viewpoint Dimension V: Group A 195

LIX. Cosines between Reference Axes: Subgroup BV(-) Rotated for Maximum Factor Vector Overlap with Subgroup AV(+) 196

LX. Subgroup BV(-) Rotated for Maximum Factor Vector Overlap with Subgroup AV(+) 197

LXI. Cosines between Reference Axes: Subgroup CV(-) Rotated for Maximum Factor Vector Overlap with Subgroup AV(+) 198

TABLE PAGE

LXII. Cosines between Reference Axes: Subgroup CV(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BV(-) 199

LXIII. Cosines between Reference Axes: Subgroup CV(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BV(-) 200

LXIV. Intercorrelations among the Achievement and Aptitude Scores and Viewpoint Dimension Coefficients for Group A 204

LXV. Intercorrelations among the Achievement and Aptitude Scores and Viewpoint Dimension Coefficients for Group B 205

LXVI. Intercorrelations among the Achievement and Aptitude Scores and Viewpoint Dimension Coefficients for Group C 206

LXVII. Summary of Results for Stepwise Regression Analysis: Prediction of Viewpoint Dimension Coefficients from Eight Aptitude Measures 207

LXVIII. Intercorrelations among the Achievement and Aptitude Scores for Group A, B, and C Combined 211

TABLE PAGE

LXIX. Summary of One-way Analysis of Variance of Physics Test Scores for Subjects with High Coefficients on the Viewpoint Dimensions 213

LXX. Frequency of Selection as the Most Preferred Category on the Cognitive Preference Test 216

LXXI. Summary of One-way Analysis of Variance of Physics Test Scores for Subjects Grouped according to Most Frequently Chosen Category on the Cognitive Preference Test 217

LXXII. Individual Coefficients on Principal Vectors, Group A 297

LXXIII. Individual Coefficients on Principal Vectors, Group B 299

LXXIV. Individual Coefficients on Principal Vectors, Group C 301

LXXV. Individual Coefficients on Principal Factors, Group A 303

LXXVI. Individual Coefficients on Principal Factors, Group B 305

LXXVII. Individual Coefficients on Principal Factors, Group C 307

LXXVIII. Projection of Stimulus-Pairs on Principal Vectors (II) to (V), Group A 309

TABLE		PAGE

LXXIX. Projection of Stimulus-Pairs on Principal Vectors (II) to (V), Group B 314

LXXX. Projection of Stimulus-Pairs on Principal Vectors (II) to (V), Group C 319

LXXXI. Final Unrotated Configuration of 20 Points in Five Dimensions, Group B Average 324

LXXXII. Final Unrotated Configuration of 20 Points in Four Dimensions, Group C Average 325

LXXXIII. Final Unrotated Configuration of 20 Points in Five Dimensions, Group C Average 326

LXXXIV. Group A Individual Coefficients on Varimax Rotated Factors 327

LXXXV. Group B Individual Coefficients on Varimax Rotated Factors 329

LXXXVI. Group C Individual Coefficients on Varimax Rotated Factors 331

LXXXVII. Transformation Matrices for Varimax Rotation of Principal Factors II to V 333

LXXXVIII. Subgroup CII(+) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BII(-) 334

TABLE | PAGE

LXXXIX. Subgroup CII(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BII(+) 335

XC. Subgroup CIII(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BIII(-) 336

XCI. Subgroup CIII(+) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BIII(+) 337

XCII. Subgroup CIV(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BIV(-) 338

XCIII. Subgroup CIV(+) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BIV(+) 339

XCIV. Subgroup CV(-) Rotated for Maximum Factor Vector Overlap with Unrotated Configuration for Subgroup BV(-) 340

LIST OF FIGURES

FIGURE		PAGE
1.	Variation of Minimum Stress with Dimensionality for Group Average Configurations	111
2.	Variation of Minimum Stress with Dimensionality for Subgroups AII(+), BII(-), and CII(+)	137
3.	Variation of Minimum Stress with Dimensionality for Subgroups AII(-), BII(+), and CII(-)	138
4.	Variation of Minimum Stress with Dimensionality for Subgroups AIII(+), BIII(+), and CIII(+).	156
5.	Variation of Minimum Stress with Dimensionality for Subgroups AIII(-), BIII(-), and CIII(-).	157
6.	Variation of Minimum Stress with Dimensionality for Subgroups AIV(-), BIV(+), and CIV(+)	174
7.	Variation of Minimum Stress with Dimensionality for Subgroups AIV(+), BIV(-), and CIV(-)	175
8.	Variation of Minimum Stress with Dimensionality for Subgroups AV(+), BV(-), CV(-), and BV(+)	193

CHAPTER I

THE PROBLEM

I. INTRODUCTION

Science education at the secondary level is heavily dependent upon cognitive processes in which abstract concepts serve as guides to perception and action. Although terms such as "conceptual" are sometimes used in a broad sense to refer to types of cognitive processes frequently involving verbal expression, "conceptual learning" may be described more precisely as a change in the representational or symbolic capabilities of the learner.[1]

The capacity of the mind to become aware of and to manipulate its own concepts forms the basis of Piaget's formal operations,[2] an idea which has been extended in an educational context by Skemp.[3] Fundamental to Skemp's theory of mathematics learning is the notion of reflective intelligence. Reflective thought is regarded as a second-order mental system which is capable of manipulating the mental representations of one's sensory-motor system. Although mathematics

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[1]R. M. Gagné, "The Learning of Principles", Analyses of Concept Learning, H. J. Klausmeier and C. W. Harris, editors, (New York: Academic Press, 1966), p. 81.

[2]Barbel Inhelder and Jean Piaget, The Growth of Logical Thinking from Childhood to Adolescence (New York: Basic Books, Inc., 1958). English Edition.

[3]R. R. Skemp, "The Psychology of Learning and Teaching Mathematics", Study No. 1 on various aspects of teaching mathematics in secondary schools, Paris, UNESCO, 1962, (Mimeographed, Limited Distribution).

differs from the natural sciences in that new concepts are derived primarily from existing ones rather than from information obtained from investigation of the physical environment, Skemp's views might be of considerable relevance to the description of learning in highly structured and mathematical areas of science such as mechanics.

The lack of a theory of learning applicable to science has frequently been pointed out.[4,5,6] Yet before any theoretical formulation of how students go about learning science can be tested, much more needs to be known about the nature of the cognitive processes involved in this learning. Before specific courses of study and classroom activities can be evaluated, much more information about the cognitive functioning of the students and teachers who will be involved must be available. The conditions under which specific cognitive processes develop and the factors enhancing or impeding their functioning are far from explicit at present.

Consider a subject such as mechanics, or, for that matter, any one of a number of subjects. It does not appear unreasonable to suggest that the student perceives some concepts in the field as more essential or important than others, and given concepts as more or less closely related to specific other concepts. This would imply

---

[4] W. W. Cooley, "Challenges to the Improvement of Science Education Research", Science Education, 45: 383-387, 1961.

[5] R. W. Tyler, "Resources, Models and Theory in the Improvement of Research in Science Education", Journal of Research in Science Teaching, 5: 43-51, 1968.

[6] R. W. Tyler, "Analysis of Strengths and Weaknesses in Current Research in Science Education", Journal of Research in Science Teaching, 5: 52-61, 1968.

that the student has structured his perceptions of these concepts in some way. But what way? To what extent does this structuring reflect the logical organization of the subject itself, or the way the student was taught those particular concepts, or the characteristics of the student? To what extent do his perceptions differ from those of other students at the same stage in learning the subject, at different stages in learning the subject, and from those of the teacher? How does the structure of perceived relations among the concepts affect the student's performance in the subject? Additional questions of equal educational relevance might easily be asked. Before answers to such questions could be sought, means of isolating and representing aspects of this structuring must be available.

II. A METHOD OF REPRESENTING VIEWPOINTS

Merely asking the student to describe his structuring of perceived relations among selected concepts would complicate the matter by introducing all the ambiguities of expression and interpretation inherent in communication through language. Some alternative approach appears preferable.

A model of considerable potential utility employs the construct of psychological distance, widely used in the interpretation of psychophysical measurements, as its basis. The stimuli, in this case statements of concepts, are represented by points in a postulated psychological space. The distance between any two points in the space is a function of the degree of similarity of the two stimuli. In other words, two stimuli which are judged to be very "similar" can be considered as psychologically closer together than two stimuli which are judged to be "different". If the formal characteristics of the postulated psychological space correspond to those of some type of geometric space, e.g., Euclidean space, then mathematical techniques are available to obtain the dimensionality of the space defined by the distances. The dimensionality of the space may be regarded as corresponding to the number of different ways the stimuli are perceived to differ.

The possible existence of more than one point of view about the similarity of the stimuli must also be considered since it cannot be assumed that all judges possess equivalent cognitive structures. For example, if one group of judges was cognitively complex,[7,8] perceiving the concepts as differing in many ways, and another group of judges was cognitively simple, then any space which represents an average might distort the relationships by providing more dimensions than would characterize some of the judges and less than the number necessary to summarize the judgments for certain other of the judges.

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[7] James Bieri, "Cognitive Complexity - Simplicity and Predictive Behavior", Journal of Abnormal and Social Psychology, 51: 263-268, 1955.

[8] W. A. Scott, "Cognitive Complexity and Cognitive Flexibility", Sociometry, 25: 405-414, 1958.

Tucker and Messick[9] have developed a method whereby a separate multidimensional space is derived for each different viewpoint about stimulus similarity which is isolated. Each individual viewpoint could be considered as a vector in a multidimensional space of stimulus objects. The number of dimensions required to span this vector space is determined by principal components analysis, the resulting rotated factor loadings representing scale values for the various viewpoints.

The dimensions isolated reflect consistencies in the rating of pairs of stimuli and represent consistent individual viewpoints with respect to some specified attribute. Projections for pairs of stimuli on each rotated dimension of viewpoint yield measures of dissimilarity which can be analyzed according to the distance model[10,11] of multidimensional scaling.

Since each individual receives a loading on all viewpoint dimensions, correlations between the viewpoint dimensions and various measured characteristics of the judges may be computed. Correlations of viewpoint dimensions with measures of performance in the subject may also be obtained.

---

[9] L. R. Tucker and S. Messick, "An Individual Differences Model for Multidimensional Scaling", Psychometrika, 28: 333-367, 1963.

[10] W. S. Torgerson, Theory and Methods of Scaling, (New York: John Wiley & Sons, Inc., 1958).

[11] J. B. Kruskal, "Multidimensional Scaling by Optimizing Goodness to Fit to a Nonmetric Hypothesis", Psychometrika, 29: 1-27, 1964.

In terms of its primary purpose, the study being reported represents an attempt to apply the above model to the description of aspects of the structure of students' perceptions of selected concepts encountered in the classroom. The potential of such an approach in curriculum evaluation, particularly at the formative stages,[12] in the assessment of effects of different teaching methods, and for the study of individual differences in cognitive functioning rests on the possibility of obtaining descriptions of such a dimensionality as to be manageable and, more important, interpretable.

III. SOME GENERAL CHARACTERISTICS OF CONCEPTS

It seems advisable before proceeding any further to indicate briefly the sense in which the term "concept" is used in this study.

The existence of many different kinds of concept, as well as certain intrinsic characteristics of concepts, makes the definition of this term somewhat troublesome. Some concepts depend upon the isolation of a particular aspect (or set of aspects) of the stimuli which are exemplars of that concept. Others depend upon agreement of particular responses to the stimuli. There are also concepts which are based on systematic relations, in other words, which are constructs in general systems of relationships and rest on the

[12]Michael Scriven, "The Methodology of Evaluation" Perspectives of Curriculum Evaluation, No. 1 AERA Monograph Series on Curriculum Evaluation (Chicago: Rand McNally & Co., 1967), pp. 40-43.

inferential base specified by their systematic relationships.[13] It is in this sense that the term "concept" is most often used in science. It is also the sense in which the term is used here.

It should be pointed out that a concept and a "conception" are not the same thing. A conception may be regarded as "belonging to" a particular individual (although others may have similar conceptions) and will generally differ from time to time. A high school science student's conception of an atom is different from that held by a nuclear physicist and different again from what it was before he became aware of energy levels, orbitals, quantum numbers and the like.

In contrast to its conceptions, a concept may be regarded as being something impersonal. Although a concept is an abstract construction, this abstract quality should not obscure the point that as the conceptions which enter into its construction change, so will the concept. Although the percept - concept relation[14] will not be discussed at this time, in talking about the perceived relations among concepts one is, strictly speaking, referring to aspects of the subject's conceptions rather than to the concepts themselves.

A fundamental characteristic of a concept is that it has its existence in a body of rules. According to Kaplan,

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[13]J. J. Jenkins, "Meaningfulness and Concepts: Concepts and Meaningfulness" Analyses of Concept Learning, H. J. Klausmeier and C. W. Harris, editors (New York: Academic Press, 1966), pp. 68-69.

[14]Henry Margenau, The Nature of Physical Reality, (New York: McGraw-Hill Book Company Inc., 1950), p. 56.

> Since Kant, we have come to recognize every concept as a rule for judging or acting, a prescription for organizing the materials of experience so as to be able to go on about our business.[15]

Thus a concept can presumably be identified by means of testing procedures involving these rules in some fashion. It is, of course, common practice to assess a student's grasp of particular concepts, e.g., his conception of Newton's Second Law, by asking him to solve problems requiring the application of these concepts -- in this case, in problems involving the application of Newton's Second Law.

Two things might be noted. One cannot teach someone a concept in the same way as one can teach facts. One can only attempt to arrange for him to learn it. In confronting the student with the term, or symbol, or formula one is not necessarily communicating the concept since it is possible to have the word or symbol without having grasped the concept.[16] The extent to which the student understands the concept is demonstrated in how he uses it.

The second point is that the conceptual structure, that is, the concepts in their matrix of interrelatedness, available to the student at the particular time determines the extent to which the student can grasp the concepts in question, i.e., the meaningfulness of the particular concepts to the student. For instance, the expression ma=F might mean something to a high school physics student. It

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[15] Abraham Kaplan, The Conduct of Inquiry (San Francisco: Chandler Publishing Company, 1964), p. 46.

[16] Skemp, op. cit., p. 7.

is less likely that Newton's Second Law expressed in the form $m\frac{d^2x}{dt^2} = F(x)$ would have much meaning to such a student unless he were specifically familiar with force-functions and differential equations. Piaget refers to these available conceptual structures, or organizations of existing knowledge, as schemata[17] and Ausubel regards them as forming the prerequisite conceptual structure essential to the more abstract types of verbal learning so frequently encountered in the classroom.[18]

IV. A BASIS FOR MAKING JUDGMENTS

The procedure outlined in the previous section requires that judgments be made about the dissimilarity of stimuli. Now a concept, as such, is not a stimulus. Concepts are abstract constructions, varying in the extent of their removal from the sensory-empirical and in the complexity of their relations to other concepts. A statement of a concept, however, may act as a stimulus, its meaningfulness being determined by the conceptual structure of which it is a part, and may thus be responded to in some specified way.

Certain concepts in a subject may be communicated to the student in contexts which outline in some way their use in specified

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[17] J. H. Flavell, The Developmental Psychology of Jean Piaget (New York: D. Van Nostrand Company, Inc., 1963), pp. 52-57.

[18] D. P. Ausubel, The Psychology of Meaningful Verbal Learning (New York: Grune & Stratton, 1963).

types of situations. In dealing with various topics in physics, for example, the text-book writer and teacher delimit this range of application and therefore restrict the range of meaning which particular concepts may take. The student learns concepts in a particular context or set of contexts and it is in this framework that he can be expected to make judgments about specific concepts. To ask a student whether he regards the topic of freely falling bodies - in terms of the things he is expected to do with this area of concepts in his physics course - as more or less difficult than composition of forces - in the same context, i.e., his physics course, would provide a frame of reference which could act as a basis for making the judgment. On the other hand, to ask the student merely whether he regards "freely falling bodies" as being more or less difficult than "composition of forces" is essentially meaningless until some type of context is provided, either by the experimenter or by the student himself.

It might be expected that the student would be capable of rating the concepts directly on the basis of their similarity to each other. Some relative estimate of how "alike" or "different" the stimuli are perceived to be is sufficient to permit application of the scaling procedures. In such a case, however, the context in which the judgment was made would again be ambiguous and could very easily differ from one student to the next. If the object of the study involved the isolation of these frames of reference, then such an approach might be considered even if the possibility exists that a sizeable proportion of the students may have difficulty in understanding precisely what it is they are expected to do.

In this study it appears essential that the judgment be one that all the students could reasonably be expected to make and difference in difficulty between concepts seems to be something that students would encounter quite frequently at school. This is not to suggest that judgments about the dissimilarity of concepts with respect to other attributes, e.g., relative importance to the field could not be made readily, but it seems desirable for the purposes of this study to have a frame of reference which is based as much as possible on the experiences of the students themselves with the particular concepts to be rated.

The extent to which the structure of the subject or discipline itself determines the nature of the context will be considered later.

V. PURPOSE OF THE STUDY

Three apsects of the problem to be investigated have influenced the design of the study most extensively. The purpose of the study may be stated as follows:

1. To ascertain the number and nature of the dimensions required to summarize the structure of perceived relations among concepts in a subject area as derived from judgments of similarity with respect to their difficulty.

2. To investigate possible individual and group differences in conceptual structures with a view to isolating consistent individual viewpoints in the structuring of the given domain of concepts.

3. To describe relationships among the viewpoints for different groups of judges, and to relate these aspects of conceptual structure to measured characteristics and to the performance of the judges.

VI. GENERAL HYPOTHESES

The fundamental assumption upon which this study is based is that the construct of psychological distance applies to judgments involving relationships between concepts learned in the classroom. In its general form, this assumption appears difficult to test. However, when the additional assumption that these distances correspond to distances in Euclidean space is included, the resulting dimensionality of the perceived relations among concepts should provide some indication of the applicability of the model. This is the undertaking upon which the first two objectives focus.

The first objective is concerned primarily with mapping the stimulus domain - in other words, in accounting for the complexity inherent in the stimuli. The Kruskal - Shepard nonmetric scaling procedure,[19,20] in which a monotone relationship between the

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[19]Kruskal, op. cit., pp. 1-27.

[20]J. B. Kruskal, "Nonmetric Multidimensional Scaling: A Numerical Method", Psychometrika, 29: 115-129, 1964.

experimental dissimilarities and the distances in the configuration is sought by methods of numerical analysis appears most suitable since, in this approach, distributional assumptions with respect to the judgments are unnecessary.

The second objective is concerned primarily with perceiver differences. Since individual perceptions of stimuli as well as the stimuli themselves could vary in any number of ways, it seems reasonable to make provision for both types of variation in the investigation. A separate multidimensional representation of the perceived stimulus space would be obtained by means of the Kruskal - Shepard procedure for each viewpoint dimension which is isolated.

One aspect of the third objective of the study relates to the identification of measurable characteristics of the subjects which show indications of differentiating between their conceptual structures.

The complexity of an individual's conceptual structure may be regarded as a function of two variables:

1. Conceptual structures vary in the number of dimensions along which they are capable of ordering stimuli.

2. Conceptual mediating systems vary in the degree of superordination of schemata with which the perceived dimensions of information are organized.[21]

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[21] J. E. Sieber, "Problem Solving Behavior of Teachers as a Function of Conceptual Structure", Journal of Research in Science Teaching, 2: 64-68, 1964.

In addition to providing some theoretical basis for the first two objectives, these predictions lead to some general hypotheses regarding the types of variables which may account for some of the characteristics of the structures which are isolated.

Correlation of viewpoint loadings with extensively used measures of Verbal Reasoning, Numerical Ability, Abstract Reasoning, and Space Relations such as the above-named four subtests of the Differential Aptitude Tests (DAT) battery may serve as an indication of the degree of relationship of the viewpoint dimensions with such presently accepted intellectual abilities. Perhaps a better reason for including measures of the above four aptitudes is that they function as reasonably good predictors of achievement in high school science courses.[22]

> Hypothesis A: There is no significant relationship between the viewpoint dimensions isolated and any of the following subtests of the Differential Aptitude Tests, form L, battery: Verbal Reasoning, Numerical Ability, Abstract Reasoning, and Space Relations.

The apparent emphasis in high school physics upon the building of relatively abstract conceptual structures to aid in the perception and interpretation of natural phenomena seems consistent with Piaget's views on cognitive development. In adolescence, the student becomes

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[22] Norman Frederiksen, reviewing the DAT battery in The Fifth Mental Measurements Yearbook, O. K. Buros, editor (Highland Park, N. J.: Gryphon Press, 1959), p. 675.

capable of performing logical operations on verbal propositions. He is now able to go beyond those operations which are an immediate consequence of empirical reality and deal with all possible relations between ideas.[23] Consequently there is an increasing tendency for his generalizations to become second - order constructs which are derived from relationships between previously established concepts.

The relation between aspects of the conceptual structures which are isolated and Skemp's tests of reflective intelligence (Concept Formation, Reflective Action with Concepts, Operations Formation, and Reflective Action with Operations) which directly involve Piaget's formal operations would need to be studied before the consistency of this theoretical position with science learning and teaching could be assessed.

Hypothesis B: There is no significant relationship between any of the viewpoint dimensions isolated and any of the following of Skemp's reflective intelligence measures: Concept Formation, Reflective Action with Concepts, Operations Formation, and Reflective Action with Operations.

The relevance of Skemp's tests as predictors of achievement in high school physics courses has apparently not been investigated and forms the other aspect of the third objective of the study. There is some indication that reflective intelligence scores, as measured by Skemp's tests of Operations Formation and Reflective

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[23] Inhelder and Piaget, op. cit.

Action with Operations contribute significantly to the prediction of mathematics achievement as early as Grade VIII.[24]

Hypothesis C: The prediction of physics achievement scores is not significantly improved by adding to the differential Aptitude Tests, form L, battery any of Skemp's reflective intelligence measures.

Hypothesis D: There is no significant relationship between any of the viewpoint dimensions isolated and student performance as measured by a physics achievement test.

Currently there appears to be some interest in a measure of cognitive "style", namely the cognitive preference test developed by Heath to compare the cognitive preferences of students in the Physical Science Study Committee (PSSC) Course with those of students in a conventional physics course.[25]

The cognitive preferences chosen for comparison in this study are:

a. recall of factual material

b. mathematical application

c. experimental or practical application

d. principle or generalization

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[24] D. B. Harrison, "Reflective Intelligence and Mathematics Learning" (unpublished Doctoral Dissertation, The University of Alberta, Edmonton, 1967).

[25] R. W. Heath, "Curriculum, Cognition, and Educational Measurement", Educational and Psychological Measurement, 24: 239-253, 1964.

The above four are chosen because all are related to some extent to the goals of instruction in high school physics.

Hypothesis E: For subjects with high loadings in different viewpoint dimensions, there is no significant difference in the frequency of selection as the most preferred for the four cognitive preferences as measured by the *Cognitive Preference Test*.

Hypothesis F: There is no significant difference in performance as measured by a physics achievement test for subjects grouped according to the most frequently chosen category on the *Cognitive Preference Test*.

VII. DELIMITATIONS OF THE STUDY

The selection of concepts was confined to the mechanics section of grade twelve Physics 30 for the following reasons:

a. The concepts are generally well-defined in terms of their meaning and range of application.

b. The number of basic concepts encountered is relatively small.

c. The concepts are frequently high-level abstractions, e.g., point masses, instantaneous velocities.

d. The section is comparatively self-contained. The course does not assume any previous knowledge of mechanics on the part of the student.

e. The logical and mathematical structure of the concepts of mechanics and their interrelationships have been extensively scrutinized by scientists and philosophers of science.[26,27,28,29]

The selection of concepts from mechanics necessitates the choice of grade twelve Physics 30 students as subjects since it is in this course that the student first encounters classical mechanics as a formal discipline.

VIII. DEFINITION OF TERMS

Cognitive complexity is concerned with the differentiation of dimensions of judgment, and may be defined as the capacity of the mind to construe stimuli in a multidimensional way.

Concept. For the purposes of this study, a concept may be regarded as a construct in a system of relationships. It is an abstract construction associated with the usage of a term.

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[26] P. W. Bridgman, The Nature of Physical Theory (Princeton: Princeton University Press, 1936: New York: Wiley Science Editions, 1964), Ch. VI, VII.

[27] Margenau, op. cit., Ch. 9.

[28] Henry Margenau, "Is the Mathematical Explanation of Physical Data Unique?" in E. Nagel, P. Suppes, and A. Tarski (Eds.) Logic, Methodology and Philosophy of Science (Stanford, Calif.: Stanford University Press, 1962), pp. 348-355.

[29] Ernest Nagel, The Structure of Science (New York: Harcourt, Brace & World, Inc., 1961), Ch. 7-10.

Concept attainment. The extent to which a concept is understood or attained is demonstrated in its usage. Concept attainment is not assumed to occur always on an all-or-nothing basis.

Conceptual structure is designated as referring to the relations among component concepts rather than to the concepts themselves.

Reflective intelligence refers to the capacity of the mind to become aware of purely mental objects, to act on them in various ways, and to observe the results of these mental operations.

Schemata refers to organizations of existing knowledge which may be regarded as forming the prerequisite conceptual structure essential to the more abstract types of learning.

Structure refers to an organized system whose properties depend upon the interrelations of the various elements within the system. While constituting a relation among designated elements, the particular structure may itself form an element in some superordinate structure. Futhermore, the elements of any given structure may themselves be regarded as complex structures with their own elements and interrelations.

IX. SUMMARY

The considerations outlined to this point have led to the view that the construct of psychological distance might serve as a basis for representing viewpoints derived from judgments about relatively abstract and complex concepts such as the ones encountered by a grade twelve student in mechanics. A distinction is made between a concept in the sense that the term is most often used in science and science teaching, and a particular individual's conception of that concept.

The basis for differences among individuals is assumed to be reflected in organizations or structures which can be described and measured in terms of their dimensional characteristics.

CHAPTER II

SURVEY OF THE LITERATURE

I. INTRODUCTION

Comparatively little change with respect to the objectives of science teaching as set out in the educational literature is noted over the years.

Principles and generalizations of science are objectives.[1]

> It is one thing to be able to recite a neat statement covering a concept. It may be something else to be able to use the concept correctly in thinking, speaking, or writing about a relatively unfamiliar situation in which the concept properly plays a part.[2]

> From science courses, pupils should acquire a useful command of science concepts and principles. Science is more than a collection of isolated and assorted facts; to be meaningful and valuable, they must be woven into generalized concepts.[3]

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[1]National Society for the Study of Education, A Program for Teaching Science (The 31st Yearbook of National Society for the Study of Education, Chicago: The University of Chicago Press, 1932), p. 44.

[2]National Society for the Study of Education, Science Education in American Schools (The 46th Yearbook of the National Society for the Study of Education, Chicago: The University of Chicago Press, 1947), p. 27.

[3]National Society for the Study of Education, Rethinking Science Education (The 59th Yearbook of the National Society for the Study of Education, Chicago: The University of Chicago Press, 1960), p. 34

Criticisms which appear at first glance to be directed towards the objectives of science teaching may in fact be a censure of teaching methods which require too little conceptualizing and too much memorizing, with the consequent failure to realize the desired objectives.

If science teaching is to be oriented more towards conceptualizing knowledge, research on intellectual processes such as concept formation assumes particular relevance. Although the following comment by Watson and Cooley appeared in 1960, it remains timely.

> Without serious and continual study of the complex learning processes required in the sciences, we have no adequate basis on which to define, investigate, or appraise science - teaching methods.[4]

II. LABORATORY STUDIES OF CONCEPT FORMATION

Presumably educators should turn to laboratory studies conducted by psychologists for assistance in understanding concept attainment in the classroom. Although a large literature on concept formation has been developed by psychologists, its pertinence to the learning of concepts in school subjects, particularly at the secondary level, is not always obvious.

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[4]F. G. Watson and W. W. Cooley, "Needed Research in Science Education", Rethinking Science Education (The 59th Yearbook of the National Society for the Study of Education, Chicago: The University of Chicago Press, 1960), p. 300.

Carroll[5] notes a number of major differences between the conditions under which simple "concepts" are formed by subjects in psychological experiments and the teaching of concepts in school and questions whether there is any continuity in the processes involved between the generally non-verbal, inductive concept formation tasks used in laboratory experiments and the more verbal-explanatory kind of teaching encountered in school.

Before concept learning in school is discussed, it might be useful to elaborate on the view of a "concept" held by some psychologists and exemplified by the studies of Bruner and his associates.[6]

Despite differences in terminology, there appears to be considerable agreement among psychologists as to their meaning of the term "concept." According to Gagné:

> From the standpoint of the investigator of behavior, the notion of a concept as an "inferred process which enables the individual to classify objects" is both prominent and widely accepted.[7]

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[5] J. B. Carroll, "Words, Meanings and Concepts", Harvard Educational Review, 34: 178-202, 1964.

[6] J. S. Bruner, J. Goodnow, and G. A. Austin, A Study of Thinking (New York: John Wiley & Sons, Inc., 1956).

[7] R. M. Gagné, "The Learning of Principles," Analyses of Concept Learning, H. J. Klausmeier and C. W. Harris, editors, (New York: Academic Press, 1966), p. 83.

Concept attainment is therefore regarded as a process of categorization. The standard experimental procedure involves the presentation to the subject of a series of exemplars and non-exemplars of some arbitrary "concept". Generally the stimuli consist of visual material clearly differentiated with respect to dimensions such as shape, number, or color, e.g., a blue square and red triangle. The subject then makes guesses about other possible instances which are presented to him, often in some predetermined sequence. Various criterion measures, such as the number of tries required to correctly identify the concept as defined by the experimenter, may be employed.

The experimental work includes investigation of questions such as the relative importance of positive and negative instances[8] and relative difficulty in identifying disjunctive as opposed to conjunctive concepts.[9] Bruner's work in this area has been particularly influential on approaches to classroom concept learning because of his emphasis on strategies of concept attainment.[10]

Carroll summarizes the information about the concept attainment which has been obtained under the aforementioned experimental conditions in the following manner:

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[8]C. I. Hovland, "A Communication Analysis of Concept Learning," Psychological Review, 59: 461-472, 1952.

[9]Bruner, et al, op. cit., pp. 156-181.

[10]Ibid., pp. 81-155.

1. Concept attainment becomes more difficult as the number of relevant attributes increases, the number of values of attributes increases, and the salience of the attributes decreases.

2. Concept attainment becomes more difficult as the information load that must be handled by the subject in order to solve the concept increases, and as the information is increasingly carried by negative rather than positive instances.

3. Various strategies for handling the information load are possible and some are in the long run more successful than others.[11]

It might be noted that concept formation in the context of the experimental approach outlined to this point is frequently treated as synonymous with information processing.[12] This view is shared by some researchers in social psychology:

> A concept is a system of ordering that serves as the mediating linkage between the input side (stimuli) and the output side (response). In operating as a system of ordering, a concept may be viewed as a categorical schema, and intervening medium, or a program through which impinging stimuli are coded, passed, or evaluated on their way to response evocation.[13]

There are some indications that concept attainment and information processing need not be synonymous or, for that matter, unitary activities. Employing factor analysis and the conventional type of tasks - variation in the color, shape, size, etc. of the stimulus

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[11] Carroll, op. cit., p. 190.

[12] Hovland, op. cit., pp. 461-472.

[13] O. J. Harvey, D. E. Hunt, and H. M. Schroder, Conceptual Systems and Personality Organization (New York: John Wiley & Sons, Inc., 1961), p. 1.

figures, Lemke, Klausmeier and Harris[14] were able to identify two relatively distinct information processing factors in addition to three concept attainment factors. The correlations between the factors and the usual types of aptitude measures, e.g., verbal comprehension were found to be generally low.

Of course, the analysis of concept formation in terms of processes such as stimulus generalization of discrimination learning[15] does not in itself imply that continuity exists between such simpler forms of learning and concept attainment in the classroom.

III. CONCEPT LEARNING IN SCHOOL

A major difference between concept learning in the classroom and in the psychologist's laboratory lies in the nature of the concepts to be learned.[16] Rather than representing artificial and arbitrary combinations of attributes already familiar to the student, many of the concepts learned in school are legitimately "new." Carroll elaborates the distinction as follows:

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[14]E. A. Lemke, H. J. Klausmeier, and C. W. Harris, "Relation of Selected Cognitive Abilities to Concept Attainment and Information Processing", Journal of Educational Psychology, 58: 27-35, 1967.

[15]H. H. Kendler, "The Concept of the Concept", Categories of Human Learning, A. W. Melton, editor (New York: Academic Press, 1964), pp. 212-236.

[16]Carroll, loc. cit.

> New concepts learned in school depend on attributes which themselves represent difficult concepts. In more general terms, concepts learned in school often depend upon a network of related or prerequisite concepts. One cannot very well learn the concept of derivative, in the calculus, until one has mastered a rather elaborate structure of prerequisite concepts (e.g., slope, change of slope, algebraic function, etc.). Further, the attributes on which school-learned concepts depend are frequently verbal, depending on elements of meaning that cannot easily be represented in terms of simple sensory qualities as used in concept formation experiments.[17]

Many of the concepts in a subject are defined in terms of their relations with other concepts within the subject area rather than by the combined presence or absence of specific physical attributes. Part of the logical structure of mechanics, for instance, consists of such interrelationships among concepts. The attendant constraints are specified formally in the terms used, and learning mechanics becomes in part a matter of internalizing the relations. Thus the student is faced not only with learning a large number of new concepts, but with the additional problems of acquiring numerous unfamiliar words to attach to some of these concepts, e.g., torque, or to restricting the meaning of words with which he is familiar in other contexts to their scientific meaning, e.g., work. There are, of course, operationally defined quantities such as distance or time which are also essential to the structure of mechanics.

Whereas laboratory experiments in concept attainment are generally inductive, the teaching of concepts in the classroom

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[17] _Ibid._

generally incorporates deductive elements. The concept in question is frequently described in terms of previously established verbal abstractions. Carroll feels that

> Concept formation experimentation would be more relevant to school learning prpblems if it could give more attention to examining the role of verbalization and other deductive procedures in concept attainment.[18]

Underwood[19] has noted the similarity between the logical form of some types of concepts and certain verbal learning paradigms. Both verbal learning and concept learning are regarded as being concerned with implicit associative responses. Evidence such as that of Hunt and Hovland[20] that subjects find it easier to learn conjunctive and relational concepts than disjunctive concepts is related to such paradigms.

The importance of verbal associations in the learning of physics has been investigated by Johnson. In one experiment,[21] four groups of high school students gave free-association responses to eighteen mechanics concepts (words) and the number of times one of the

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[18] Carroll, op. cit., p. 191.

[19] B. J. Underwood, "Some Relationships between Concept Learning and Verbal Learning," Analyses of Concept Learning, H. J. Klausmeier and C. W. Harris, editors (New York: Academic Press, 1966), pp. 51-63.

[20] E. B. Hunt and C. I. Hovland, "Order of Consideration of Different Types of Concepts," Journal of Experimental Psychology, 59: 220-225, 1960.

[21] P. E. Johnson, "Associative Meaning of Concepts in Physics," Journal of Educational Psychology, 55: 84-88, 1964.

eighteen stimula words occurred as a response to another stimulus word was tabulated. From the outcome that students currently taking physics gave more such responses than students who had taken physics earlier, were planning to take physics, or were not planning to take physics, Johnson concluded that the extent to which students produce interrelated associations among mechanics concepts is a function of the extent of their involvement in physics.

In another experiment,[22] both a verbal association test and a problem solving test in mechanics were administered to two randomly equated groups of physics students. One group had the association test first, while the other group received the problem solving test first. In the association test the students were instructed to write the first physics word that came to mind rather than just the first word, as was the case in the previous experiment.

Johnson found that the students who had the problem test first gave significantly more stimulus words as responses than did the group who had the association test first. The students who had the problem test first also produced significantly more problem-relevant associations than the other group. Furthermore, students who gave a relatively large number of problem-relevant associations solved more problems than students who produced few such associations. It was also found that students who had the association test before the

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[22] P. E. Johnson, "Word Relatedness and Problem Solving in High School Physics", Journal of Educational Psychology, 56: 217-224, 1965.

problem test solved significantly more problems than the students who received the problem test first.

In the third study of the series,[23] students were administered a multiple-response verbal association test in which they were instructed to write down as many words as the key word brought to mind (in one minute). Students were also asked to rate each pair of key words with respect to their dissimiliarity using a seven-point scale. A tabulation of the frequency of occurrence of the words in the written instructional materials was also carried out.

Significantly more responses per stimulus word were given by high achievers than by low achievers and significantly more responses were elicited by the high-frequency words than by the low-frequency words. From this Johnson concluded that words which occurred frequently in the text were more meaningful for both high and low achievers than words which appeared more rarely and that high achievers found both kinds of words more meaningful.

Relationships consistent with physical equations appeared more frequently for high achievers and tended to occur early in the response hierarchies, particularly for the high achievers. The extent of the relationships between associative meaning and judged similarity across the word pairs varied both with the group and with the concepts themselves.

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[23]P. E. Johnson, "Some Psychological Aspects of Subject-Matter Structure", Journal of Educational Psychology, 58: 75-83, 1967.

In a science such as mechanics it is frequently possible to move overtly from one concept to another either by means of definitions or by substitution in equations. However, the ways in which a student gets from one concept to another implicitly are another matter. Johnson's suggestion that students may move from one concept to another by means of mediated associative equalities raises a number of interesting questions involving the extent to which associative meanings indicate which relationships are understood and the extent to which a student's problem solving behavior is influenced by the associative use of the relevant concepts.

One outcome emphasized by Johnson was the non-occurrence of certain relationships basic to mechanics. For example, force was found to be associated with energy, work, power, pressure, and weight but not as related in Newton's Second Law. It would be tempting to suggest that the associative meanings of terms in mechanics might in some way be influenced by the relative difficulty of the relationships involved.

Johnson's work was reviewed in some detail because the approach to investigating students' perceptions about concepts learned in school subjects might warrant further application.

The semantic differential type of test has also been investigated in relation to science achievement. Rothman[24] found that scores on a science-related semantic differential test function as small but

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[24] A. I. Rothman, "Responses to Science Concepts on a Semantic Differential Instrument and Achievement in Freshman Physics and Chemistry," Journal of Research in Science Teaching, 5: 168-173, 1967-1968.

significant predictors of achievement in freshman chemistry and physics.

Some work has been carried out in elementary school science on levels of understanding of certain concepts. In an approach somewhat similar to that of Gagné,[25] Pella and Carey[26] arranged sixteen selected concepts in a sequence designated to make the attainment of a desired terminal concept most probable and obtained a possible hierarchy in terms of their difficulty as indicated by scores earned by students.

Again at the elementary school level, Scott[27] has investigated the relation of inductive reasoning ability and cognitive style in categorization to science concept achievement. The results indicated that inductive reasoning was related to science concept attainment for ten-year old students but that for eleven-year old students, categorization style became the more influential of the two. In addition, an interaction effect emerged between age levels and sex categories.

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[25] R. M. Gagné, "The Acquisition of Knowledge," Psychological Review, 69: 355-365.

[26] M. O. Pella and R. L. Carey, "Levels of Maturity and Levels of Understanding for Selected Concepts of the Particle Nature of Matter," Journal of Research in Science Teaching, 5: 202-215, 1967-1968.

[27] N. C. Scott, Jr., "Science Concept Achievement and Cognitve Functions," Journal of Research in Science Teaching, 2: 7-16, 1964.

IV. THE GENEVA VIEW OF CONCEPTUAL DEVELOPMENT

The extensive and insightful inquiries of Piaget into how certain science concepts develop in young children offer numerous leads to further investigation. The volume prepared in collaboration with Inhelder[28] is particularly relevant to high school science.

Although it may be argued that studies conducted with young children are likely to be of most profit since their ideas tend to be relatively uncomplicated, there is at present little justification for the assumption that older students learn in the same way or with the same emphasis. As a matter of fact, the view that in the course of development a person's conceptual structure passes through stages characterized by qualitative differences is central to Piaget's psychology.

Intelligence is regarded as the possession of rules of transformation or operations which change as the person matures.

Some of the conflict with numerous American theorists appears a consequence of Piaget's assumption that the operations are logical structures which are <u>independent of language</u>.

In the analysis of the performance of subjects of various ages on a number of science experiments, Inhelder and Piaget[29] set out a view of cognitive growth up to and including propositional thinking, i.e., formal operations, in adolescence. A summary of Piaget's

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[28] Barbel Inhelder and Jean Piaget, <u>The Growth of Logical Thinking from Childhood to Adolescence</u>, (New York: Basic Books, Inc., 1958). English Edition.

[29] <u>Ibid</u>.

conception of the logical sturcture of formal operations and its implications for adolescent behavior appears in Flavell's book.[30]

Kagan[31] counters Piaget's view with the argument that it is not clearly apparent that the observed qualitative differences in performance necessarily derive from different logical structures. Different habits of perceptual analysis or different semantic structures could also account for the developmental difference in performance on the science experiments or on the more popular Piaget demonstrations such as conservation of volume.

Although the absence of any theoretical explanation which accounts for how or why a child passes from one stage of operations to another has been pointed out,[32] the work of Piaget has undeniably been suggestive of further lines of experimentation of psychology.[33,34,35]

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[30] J. H. Flavell, The <u>Developmental Psychology of Jean Piaget</u>, (New York: D. Van Nostrand Company, Inc., 1963), pp. 211-224.

[31] Jerome Kagan, "A Developmental Approach to Conceptual Growth," <u>Analyses of Concept Learning</u>, H. J. Klausmeier and C. W. Harris, editors, (New York: Academic Press, 1966), pp. 97-115.

[32] <u>Ibid</u>., p. 98.

[33] J. S. Bruner, R. R. Olver and P. M. Greenfield, <u>Studies in Cognitive Growth</u>, (New York: John Wiley & Sons, Inc., 1966).

[34] Jan Smedslund, "The Acquisition of Conservation of Substance and Weight in Children: II. External Reinforcement of Conservation of Weight and of the Operations of Addition and Subtraction" <u>Readings in the Psychology of Cognition</u>, R. C. Anderson and D. P. Ausubel, editors, (New York: Holt, Rinehart and Winston, Inc., 1965), pp. 581-601.

[35] Jan Smedslund, "The Acquisition of Conservation of Substance and Weight in Children: III. Extinction of Conservation of Weight Acquired 'Normally' and by Means of Empirical Controls on a Balance," <u>Readings in the Psychology of Cognition</u>, R. C. Anderson and D. P. Ausubel, editors, (New York: Holt, Rinehart and Winston, Inc., 1965), pp. 602-605.

Unfortunately, its implications for the learning of scientific concepts in school have, to date, remained relatively unexplored.

V. COGNITIVE PREFERENCE STUDIES IN SCIENCE

A number of studies attempting to identify curriculum-related differences in cognitive style have been conducted with variants of the "cognitive preference" type of test which Heath[36] used to compare the preferences exhibited by students taking the Physical Science Study Committee (PSSC) physics course with those of students taking conventional physics courses. The four preferences chosen by Heath for purposes of comparison were (a) memory for specific facts or terms (b) practical application (c) critical questioning of information and (d) identification of a fundamental principle.

Heath's study indicated that, on the average, the PSSC group exhibited stronger preference for critical questioning and identification of fundamental principle options and less preference for the practical application and memory options than was demonstrated by the students in conventional physics courses.

The difference between the PSSC and control group means was statistically significant for three of the four cognitive preference scales.

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[36] R. W. Heath, "Curriculum Cognition, and Educational Measurement," Educational and Psychological Measurement, 24: 239-253, 1964.

The difficulties in obtaining a clear analysis of the data due to the ipsative, or interrelated, nature of the scale scores were noted but not circumvented in Heath's study. An attempt to overcome some of the attendant problems was made by Schmedemann and LaShier[37] who used Heath's test with a scoring system based on the goals of PSSC physics in their study of the cognitive preferences and selected characteristics of teachers and the cognitive preferences of their students. No significant relationship was found to exist between teacher warmth, demand, and use of motivation and the cognitive preferences of their students. Similarly, no relationship between the cognitive preferences of the teachers and the cognitive preferences of their students emerged.

Atwood[38] prepared a cognitive preference examination with the same categories for chemistry. The items were based on the information in the first ten chapters of the Chemical Education Material Study (CHEM Study) course and the cognitive preference test was used only to classify students into groups. Although there was no significant difference in CHEM achievement among groups classified on the basis of a single preference score, students demonstrating a strong preference for the memory option in combination with a second

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[37]Gary Schmedmann and W. S. LaShier, Jr., "Cognitive Preferences of Students and Selected Characteristics of their PSSC Teachers", <u>Journal of Research in Science Teaching</u>, 5: 40, 1967-1968.

[38]R. K. Atwood, "A Cognitive Preference Examination Using Chemistry Content," <u>Journal of Research in Science Teaching</u>, 5: 34-35, 1967-1968.

cognitive preference tended to be at a disadvantage when compared to students showing other preferences.[39]

A cognitive preference test in chemistry was also prepared by Marks[40] for a study comparing the cognitive preferences of students taking the Chemical Bond Approach (CBA) chemistry with the cognitive preferences of students enrolled in conventional high school chemistry courses. Marks found that the CBA group indicated significantly stronger preference for the fundamental principle and critical questioning options.

The indications are that the reliabilities of the cognitive preference type of test tend to be somewhat low. Heath[41] reported K-R 20 reliabilities ranging from 0.37 for fundamental principles to 0.68 for memory for the PSSC group and from 0.31 for fundamental principles to 0.77 for memory for the control students. Test-retest stability coefficients reported by Atwood[42] ranged from 0.41 for practical application to 0.78 for critical questioning.

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[39] R. K. Atwood, "CHEM Study Achievement Among Groups Classified by Cognitive Preference Scores," Journal of Research in Science Teaching, 5: 154-159, 1967-1968.

[40] R. L. Marks, "CBA High School Chemistry and Concept Formation", Journal of Chemical Education, 44: 471-474, 1967.

[41] R. W. Heath, op. cit., p. 245.

[42] R. R. Atwood, "A Cognitive Preference Examination Using Chemistry Content", p. 35.

VI. COGNITIVE COMPLEXITY AND JUDGMENT

Concern with structural properties of cognition is becoming evident in areas of psychology such as contemporary personality theory. In addition to his discussion of specific structural properties postulated by various researchers, Scott[43] deals in considerable detail with the methodological considerations involved in assessing some of the structural concepts encountered in the more cognitive approaches to personality.

Cognitive complexity may be regarded as one such structural variable. Bieri[44] views cognitive complexity as indicative of the ways in which a person processes information primarily from his social environment. Cognitive complexity is defined in terms of dimensions (rather than categories) of judgment.[45] Analyses of degrees of differentiation in the conceptual structure of individuals are founded on the assumption that a person's perceptions of others are based on dimensional processes.

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[42] R. R. Atwood, "A Cognitive Preference Examination Using Chemistry Content", p. 35.

[43] W. A. Scott, "Conceptualizing and Measuring Structural Properties of Cognition", Motivation and Social Interaction, O. J. Harvey, editor, (New York: The Ronald Press Co., 1963), pp. 266-288.

[44] James Bieri, "Cognitive Complexity and Personality Development", Experience Structure and Adaptability, O. J. Harvey, editor, (New York: Springer Publishing Company, Inc., 1966), p. 15.

[45] Bieri, op. cit., p. 18.

Bieri regards the study of cognitive behavior as a twofold task, involving both analysis of the degree of differentiation of the individual's personal conceptual structure (i.e., the relative number of dimensions used to construe a particular domain of information) and the influence exerted upon the judgmental behavior by the stimulus conditions themselves (possibly in terms of the number of dimensions inherent in the stimuli). An overview of empirical research on the relation between cognitive complexity and social judgment which involves the discrimination of multidimensional stimuli is presented in Bieri, et. al.[46]

Bieri's concern with the stimulus conditions within which social perception and judgment take place is in contrast to approaches to cognitive behavior such as that of Piaget, where the focus appears to be primarily on the identification of stable and characteristic response modes which then serve as a basis for making inferences about structural properties of cognition.

The relatively pronounced emphasis found in fields such as perception and judgment on the role of stimulus variables in behavior might well be suggestive to educational researchers. Judgmental behavior is certainly not restricted to the area of social cognition.

It does not appear unreasonable to suggest that in solving a problem in mechanics, for instance, the student makes various

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[46] James Bieri, A. L. Atkins, Scott Briar, R. L. Leaman, Henry Miller, Tony Tripodi, Clinical and Social Judgment: The Discrimination of Behavioral Information (New York: John Wiley & Sons, Inc., 1966), pp. 182-206.

judgments about a given domain of concepts - perhaps about the relevance of certain relationships, how he is going to sequence them, and the like.

The influence exerted upon judgmental behavior in the science classroom by stimulus conditions confronting the student needs to be studied at a much more specific level than is attempted in the usual type of study dealing with something like "inquiry" as opposed to "conventional" teaching.[47] Gagné's[48] hierarchical approach to the acquisition of knowledge may be of relevance in this regard.

VII. FORMAL AND EMPIRICAL STRUCTURE IN MECHANICS

The concept "structure" appears in numerous contexts in various areas of knowledge. Consequently in dealing with such a generalized concept it often becomes desirable, if not essential, to specify the level of analysis to which one is referring. This consideration was pointed out when the term was first defined.

Qualitatively different kinds of structure which reflect different ways of looking at the particular domain under consideration may also exist. Thus, one may speak of the logical structure or the empirical structure of an area of science such as physics.

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[47] L. J. Cronbach, "The Logic of Experiments on Discovery," Learning by Discovery, L. S. Shulman and E. R. Keislar, editors, (Chicago: Rand McNally & Company, 1966), pp. 76-92.

[48] R. M. Gagné, The Conditions of Learning (New York: Holt, Rinehart & Winston , Inc., 1965).

In relation to empirical structure, one may wish to consider syntactical structure as distinct from substantive structure.[49]

Generally speaking, problems of the formal or logical structure of a discipline are concerned with setting up the discipline as an axiomatic of hypothetico deductive system. These may differ from problems of the empirical foundation of the discipline.[50]

For example, the logical status of Newton's axioms of motion and thus of classical mechanics is discussed, among others, by Nagel[51], Margenau[52], and Toulmin[53]. Only three points will receive brief mention, by way of suggesting considerations posed by problems in the logical structure of mechanics to the design and teaching of physics courses.

Although Newton's axioms of motion have been the object of critical appraisal and analysis since the time they were first formulated, there is still wide disagreement as to precisely what it is these axioms assert. This lack of consensus has resulted in

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[49] J. J. Schwab, "Structure of the Disciplines: Meanings and Significances", The Structure of Knowledge and the Curriculum, G. W. Ford and Lawrence Pugno, editors, (Chicago: Rand McNally & Co., 1964), pp. 1-30.

[50] Rudolf Carnap, The Logical Syntax of Language. (New York: Harcourt, Brace and Co., 1939), pp. 279-284, 322-323, 331-333.

[51] Ernest Nagel, The Structure of Science (New York: Harcourt, Brace & World, Inc., 1961), Ch. 7-10.

[52] Henry Margenau, The Nature of Physical Reality (New York: McGraw-Hill Book Company Inc., 1950), Ch. 9.

[53] Stephen Toulmin, The Philosophy of Science (New York: Harper & Row, Publishers, 1960), pp. 85-90.

numerous alternative interpretations of their status, particularly in relation to their intended empirical content.[54]

A consideration of only the mathematical form of fundamental equations is clearly insufficient to account for the distinctive features of Newtonian mechanics. That two laws have the same form does not in any way imply that one law may explain the other. It is well known that diverse theories in science may exhibit structures of relationships which are formally identical. Although this characteristic can assume considerable heuristic importance in conducting research, the axioms of motion, for instance, may not be regarded as the premises of a particular branch of science as a consequence of their mathematical form alone.[55]

The consideration of possibly the most direct relevance to the teaching of mechanics relates to the empirical content of the axioms of motion. <u>These axioms are not inductive generalizations from observed facts</u>.[56] It might therefore be desirable to point out to students that while Newton's axioms of motion serve as one schema, among others, for analyzing the motions of bodies, they need to be coupled with additional assumptions in order to be construed as possessing empirical content. Nagel summarizes the point as follows:

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[54]Nagel, *op. cit.*, p. 174.

[55]*Ibid.*, pp. 162-166.

[56]*Ibid.*, p. 200.

> . . . it is not possible to ascertain what empirical content if any, any one of the axioms of mechanics has, without reference to the other axioms and to the way the theory to which they belong as component parts is codified. It is the system of theoretical assumption taken as a whole that fixes the meanings of the terms occurring in them, and that determines whether a given sentence in a theory has the status of a convention or of a statement about matters of fact.[57]

As mentioned by Taylor, a search for logical or formal structure will not and is not intended to serve as a substitute for the search for empirical knowledge -- for the facts, concepts, and principles which constitute the subject matter of a discipline.

> A search for logical structure does not guarantee that it will lead to the discovery of empirical knowledge on which to base theory and practice. Moreover, logical structures cannot of themselves provide the total basis for all needed empirical knowledge. The formal structure may be able to serve as a means of tracing subtle implications of extant empirical knowledge, which in turn may allow for the extension of results obtained through empirical research. Nevertheless, formalogical structures do not necessarily replicate all existing empirical information or indeed all information that might be gained through observation and through learning.[58]

A given formal or logical structuring is itself, in effect, a hypothesis, subject to re-examination in terms of its "fit" to the current state of the discipline.

VIII. STRUCTURE IN TEACHING

In the past fifteen years the concept of structure in knowledge has received considerable emphasis in the reconstruction of

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[57] *Ibid.*, p. 202.

[58] P. A. Taylor, "The Mapping of Concepts," (Unpublished Doctoral Dissertation, The University of Illinois, Urbana, Ill., 1966) pp. 25-26.

courses in science, mathematics, history, and so forth. Subject matter may be organized on the premise that there exist in a discipline major principles or ideas which can be used to relate or subsume less inclusive ideas or informational data.[59] Bruner, who has been influential in the introduction of the notion of structure into educational thinking, remarks that

> . . . the structure of knowledge - its connectedness and the derivations that make one idea flow from another - is the proper emphasis in education. For it is structure, the great conceptual inventions that bring order to the congeries of disconnected observations, that gives meaning to what we learn and makes possible the opening up of new realms of experience.[60]

The major principles are to be used by the teacher to relate specific facts and examples, thus developing a cluster of specific concepts around a central theme.

The effectiveness of such an approach has been demonstrated by Ausubel[61], who found that the prior presentation of a general organizational statement about a topic increased students' acquisition and retention of more specific material presented subsequently.

Anderson[62] considers structure in teaching as involving both the nature of the subject matter and the events occurring in the

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[59] J. S. Bruner, The Process of Education (Cambridge: Harvard University Press, 1960).

[60] J. S. Bruner, On Knowing: Essays for the Left Hand (Cambridge: Harvard University Press, 1962), p. 120.

[61] D. P. Ausubel, "Use of Advance Organizers in the Learning and Retention of Meaningful Verbal Material", Journal of Educational Psychology, 51: 267-272, 1960.

[62] O. R. Anderson, "A Refined Definition of Structure in Teaching", Journal of Research in Science Teaching, 4: 289-291, 1966.

learning sequence. Structure is regarded essentially as the order in which information is presented to the learner. The purpose of the paper is to present a classification of various types of structure which, while conceptually distinct, are considered to be interrelated in the actual teaching process.

Anderson[63] examined the effects of different levels of structure on students' acquisition and retention of content in seventh grade biology and found significant differences in retention for different sequences.

Gagné's[64] "learning structures" or hierarchies of capabilities which are acquired as a subject is studied represent a somewhat different view of structure in teaching. In this approach, subject matter is subdivided into smaller and simpler units of student competency which may be regarded as hierarchial with respect to the types of learning involved. In contrast to the views of Bruner and Ausubel, Gagné's approach requires that the appropriate more specific prerequisite capabilities be mastered before the more advanced principle is attained.

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[63] O. R. Anderson, "The Strength and Order of Responses in a Sequence as Related to the Degree of Structure in Stimuli", Journal of Research in Science Teaching, 4: 192-198, 1966.

[64] R. Gagné, The Conditions of Learning (New York: Holt, Rinehart and Winston, Inc., 1965), pp. 172-203.

IX. SUMMARY

The survey of the literature related to theoretical aspects of the study which has been undertaken in this chapter attempted to elaborate on frequently somewhat diverse topics in educational discourse, namely concept formation, cognitive structure, and subject matter structure. However, all may be regarded as potential influences in determining students' perceptions of relations among subject matter concepts. In the selection of experimental studies to be reviewed, emphasis was placed on considerations which might relate to the teaching of a particular subject in the classroom.

The following chapter will consider literature of more direct relevance to the methodological aspects of the study.

CHAPTER III

MULTIDIMENSIONAL SCALING

I. INTRODUCTION

In this chapter attention is directed to the analytic procedures used to derive a structure of perceived relations from measures of psychological distance as obtained from estimates of pairwise similarities among a set of stimuli. Although illustrative studies will be cited to indicate the range of applicability of the techniques to empirical data, the major emphasis will be on methodological considerations.

II. THE BASIS FOR MULTIDIMENSIONAL SCALING

Multidimensional scaling methods deal with situations in which a set of stimuli may be considered to vary simultaneously with respect to several dimensions. The underlying notion involved is that a complex attribute may be represented by a postulated psychological space of dimensionality corresponding to the (frequently unknown) dimensionality of the attribute. The position of a stimulus as represented by a point in the space corresponds to the extent to which the stimulus possesses the attribute in question.

The distance between any two points in the underlying psychological space is postulated to be a function of the similarity of the corresponding stimuli. Two stimuli judged to be very similar can be regarded as being psychologically "closer" to each other than two stimuli which are rated as very different. The distance between stimulus points is related to their projections (scale values) on the axes of the space.

The typical problem of multidimensional scaling is twofold: to ascertain the minimum dimensionality of the set of stimulus points, and to determine the projections of the stimuli on each of the dimensions.

III. THE METRIC STRUCTURE OF THE MULTIDIMENSIONAL SPACE

A consideration of fundamental importance in any attempt to construct a spatial representation for a set of multidimensional stimuli concerns the question of just what metric is appropriate for the psychological space. A basic assumption of the multidimensional scaling model using the Young and Householder[1] theorems which has been developed by Richardson[2] and extended by Torgerson[3], Messick

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[1] G. Young and A. S. Householder, "Discussion of a Set of Points in Terms of Their Mutual Distances", Psychometrika, 3: 19-22, 1938.

[2] M. W. Richardson, "Multidimensional Psychophysics", Psychological Bulletin, 35: 659-660, 1938, (Abstract).

[3] W. S. Torgerson, "Multidimensional Scaling: I. Theory and Method", Psychometrika, 17: 401-419, 1952.

and Abelson[4], Shepard[5,6], and Tucker and Messick[7] is that the formal characteristics of the postulated psychological space are Euclidean in nature.

Multidimensional scaling procedures have been rather extensively applied to the domain of color perception and have generally been found to yield results, as in the studies reported by Indow and Kanazawa[8], Indow and Uchizono[9], Messick[10], Shepard,[11] and Torgerson[12], which may be regarded as an adequate fit to the data within the

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[4]S. Messick and R. P. Abelson, "The Additive Constant Problem in Multidimensional Scaling", Psychometrika, 21: 1-15, 1956.

[5]R. N. Shepard, "The Analysis of Proximities: Multidimensional Scaling with an Unknown Distance Function: I." Psychometrika, 27: 125-140, 1962.

[6]R. N. Shepard, "The Analysis of Proximities: Multidimensional Scaling with an Unknown Distance Function: II." Psychometrika, 27, 219-246, 1962.

[7]L. R. Tucker and S. Messick, "An Individual Differences Model for Multidimensional Scaling", Psychometrika, 28: 333-367, 1963.

[8]T. Indow and K. Kanazawa, "Multidimensional Mapping of Munsell Colors Varying in Hue, Chroma, and Value", Journal of Experimental Psychology, 59: 330-336, 1960.

[9]T. Indow and T. Uchizono, "Multidimensional Mapping of Munsell Colors Varying in Hue and Chroma", Journal of Experimental Psychology, 59: 321-329, 1960.

[10]S. J. Messick, "An Empirical Evaluation of Multidimensional Successive Intervals", Psychometrika, 21, 367-375, 1956.

[11]Shepard, "The Analysis of Proximities: Multidimensional Scaling with an Unknown Distance Function: II", pp. 230-237.

[12]W. S. Torgerson, Theory and Methods of Scaling (New York: John Wiley & Sons, Inc., 1958), pp. 247-297.

confines of the Euclidean metric. That this is not always the case, however, was demonstrated by Attneave[13] with sets of geometric figures. Attneave found that an additive space provided a closer fit to the data.

In Euclidean space, the distance between any two points equals the square root of the sum of squares of the differences in projections over all orthogonal axes of the space. Geometries have been developed for more general metric spaces which subsume Euclidean space as a special case. Perhaps of greatest immediate potential for multidimensional scaling applications is the class of Minkowski spaces implemented by Kruskal[14,15] in his scaling program.

For the class of Minkowski r-metrics, for any $r>1$, the r-distance between points $x = (x_1, \ldots, x_t)$ and $y = (y_1, \ldots, y_t)$ is defined as

$$d_r(x,y) = \left[\sum_{s=1}^{t} | x_s - y_s |^r \right]^{\frac{1}{r}} \tag{1}$$

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[13] F. Attneave, "Dimensions of Similarity", American Journal of Psychology, 63: 516-556, 1950.

[14] J. B. Kruskal, "Multidimensional Scaling by Optimizing Goodness of Fit to a Nonmetric Hypothesis", Psychometrika, 29: 1-27, 1964.

[15] J. B. Kruskal, "Nonmetric Multidimensional Scaling: A Numerical Method", Psychometrika, 29: 115-129, 1964.

If r=2, then d_r becomes the ordinary Euclidean formula. If r=1, then d_r becomes the "city-block" or Manhattan" metric used by Attneave[16] in which the distance between any two points equals the sum of the absolute differences of their projections on the axes of the space

$$d(x,y) = \sum_{s=1}^{t} \mid x_s - y_s \mid \tag{2}$$

Since the triangle inequality

$$d_r(x,z) \leq d_r(x,y) + d_r(y,z) \tag{3}$$

is satisfied, d_r is a genuine distance. (The inequality requiring that the direct distance between two points be less than or equal to the distance between them via a third point is necessary for any metric space.)

Of the class of Minkowski r metrics, the Euclidean metric is particularly useful because expressions for angle and distance are invariant under rotations of the coordinate system. In the more general case, only rigid rotations which transform coordinate axes into coordinate axes leave d_r unchanged. Thus while a configuration may be freely rotated within the Euclidean metric, this is not the case if more general distances are used.

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[16] Attneave, op. cit.

As already mentioned, in scaling stimuli which consisted of parallelograms and triangles varying in size and either brightness or shape, Attneave[17] found the "city block" metric to yield a more appropriate representation for data based on both similarity judgments and frequency of confusion. Shepard[18] conducted similar experiments using circles varying in size and in the angle of inclination of a radial line and obtained results incompatible with the Euclidean metric.

Kruskal[19] has reanalyzed Ekman's experimental data on color perception[20] (also reanalyzed by Shepard[21] in the Euclidean metric) for different values of r. A value of 2.5 for r was found to give the best fit, perhaps suggesting the possibility of a slight departure from Euclidean space for subjective estimates of distance between colors.

Torgerson's attempts to reconcile Attneave's results[22] have led him to suggest that perhaps a distinction should be made between

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[17] Attneave, _op_. _cit_.

[18] R. N. Shepard, "Attention and the Metric Structure of the Stimulus Space", _Journal of Mathematical Psychology_, 1: 54-87, 1964.

[19] Kruskal, "Multidimensional Scaling by Optimizing Goodness of Fit ot a Nonmetric Hypothesis", pp. 23-24.

[20] G. Ekman, "Dimensions of Color Vision", _Journal of Psychology_, 38: 467-474, 1954.

[21] Shepard, "The Analysis of Proximities ... II", pp. 235-237.

[22] Attneave, _op_. _cit_.

a multidimensional attribute and a set of stimuli which vary simultaneously with respect to several different attributes[23]. On the basis of a series of experiments using geometric figures varying in bipolar as well as simple physical attributes, Torgerson argues that

> - - - similarity is not a unitary concept. The major distinction is between similarity as a basic, perhaps perceptual, relation between instances of a multidimensional attribute and similarity as a derivative, cognitive relation between stimuli varying on several attributes. Similarity in the former case appears to have the properties of distance in Euclidean space. Similarity in the later case is complex, and is sensitive to all of the delicate problems of attitude and strategy involved in decision-making tasks in general. Here, degree of similarity is not an invariant relation between a pair of stimuli, but rather depends upon such things as stimulus context. The shape of the configuration and even its dimensionality varies with the set or strategy taken by the subject. And under some circumstances, the dimensions obtained turn out to be qualitative, class variables, rather than quantitative measures of degree.[24]

IV. PROCEDURES FOR OBTAINING DISSIMILARITY ESTIMATES

The initial stage of multidimensional scaling requires that some type of measurement procedure be used to obtain estimates of the pairwise dissimilarities among the set of stimuli being scaled. Subsequent procedures permit the stimuli to be represented as points in a (usually) Euclidean space of low dimensionality in such a way as to reproduce as closely as possible the corresponding pairwise dissimilarities.

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[23] Torgerson, <u>Theory and Methods of Scaling</u> pp. 254, 292.

[24] W. S. Torgerson, "Multidimensional Scaling of Similarity", <u>Psychometrika</u>, 30: 379-393, 1965, pp. 389-390.

The dissimilarities can be reproduced in two senses. The distances in Euclidean space may reproduce some actual numerical assessment of the dissimilarity of the stimulus pairs. This is the multidimensional scaling approach outlined in Torgerson[25]. The particular extension of interest here is that of Tucker and Messick[26] which yields separate multidimensional spaces for individuals with different viewpoints regarding the interrelationships within a set of stimuli.

An alternative approach in multidimensional scaling is concerned with only the order of the stimulus pairs. Shepard's papers on the analysis of proximities[27,28] set out the basic ideas which serve as a rationale for this type of scaling. Details of both approaches will be discussed in subsequent sections.

Numerical estimates perceived dissimilarity among the set of stimuli can be obtained by means of a number of experimental procedures, including applications of various generalizations of Thurstonian models.

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[25] Torgerson, Theory and Methods of Scaling, pp. 247-297.

[26] Tucker and Messick, op. cit.

[27] Shepard, "The Analysis of Proximities ... I".

[28] Shepard, "The Analysis of Proximities ... II".

Methods such as the multidimensional method of successive intervals[29,30] or the method of tetrads[31] may be regarded as direct extensions of unidimensional scaling methods. The method of triadic combination[32], the complete method of triads[33] and the method of multidimensional rank order[34] represent generalizations of condition C of the law of comparative judgment.[35]

Direct estimation techniques[36,37] or intrusion errors in identification learning[38] may also be used in some cases. Descriptions

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[29]Attneave, op. cit.

[30]Messick, op. cit.

[31]Torgerson, Theory and Methods of Scaling, pp. 261-262.

[32]M. W. Richardson, "Multidimensional Psychophysics", Psychological Bulletin, 35: 659-660, 1938 (Abstract).

[33]Torgerson, "Multidimensional Scaling: I. Theory and Method".

[34]F. L. Klingberg, "Studies in Measurement of the Relations between Sovereign States", Psychometrika, 6: 335-352, 1941.

[35]Torgerson, Theory and Methods of Scaling, pp. 159-204.

[36]Indow and Kanazawa, op. cit.

[37]Indow and Uchizono, op. cit.

[38]R. N. Shepard, "Stimulus and Response Generalization: Tests of a Model Relating Generalization to Distance in Psychological Space", Journal of Experimental Psychology, 55: 509-523, 1958.

of various measurement procedures appear in Messick[39], Shepard[40] and Torgerson[41].

The experimental procedure used in the collection of data for this study consisted of paired comparisons according to the multidimensional method of successive intervals. In this method the subject is required to arrange the n(n-1)/2 pairs of stimuli into categories according to the degree of similarity of the members of each pair. In practice, the subjects were required to rate all possible pairs of stimuli with respect to their dissimilarity in difficulty on a nine-point scale. The ratings were used directly as the estimates of inter-stimulus distances.

V. THE INDIVIDUAL DIFFERENCES MODEL FOR MULTIDIMENSIONAL SCALING

It appears at least possible that in some instances multidimensional scaling procedures which yield an average representation

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[39] S. J. Messick, "Some Recent Theoretical Developments in Multidimensional Scaling", Educational and Psychological Measurement, 16: 82-100, 1956.

[40] R. N. Shepard, "Similarity of Stimuli and Metric Properties of Behavioral Data", Psychological Scaling: Theory and Applications, H. Gulliksen and S. Messick, editors, (New York: John Wiley & Sons, Inc., 1960) pp. 33-43.

[41] Torgerson, Theory and Methods of Scaling, pp. 261-268.

of the structure of perceived relations among a set of stimuli may obscure consistent but different viewpoints about the stimulus interrelationships. The assumption that all raters perceive the stimuli in the same way may not always be tenable.

Tucker[42] has developed a vector model for paired comparisons capable of dealing with individual differences in the evaluation of stimuli with respect to some unidimensional attribute. The basic model involves the representation of an individual's scale value for a given stimulus by the scalar product between the individual's vector and the vector representing the stimulus.

In this vector model, the multidimensional space represents different individual viewpoints while the scale for each individual remains one-dimensional. In other multidimensional approaches, it is the stimuli which are considered to vary along different dimensions while all judges construe the space in essentially the same way.

Tucker and Messick[43] have combined the two approaches in a procedure which yields a separate representation of the stimulus

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[42] L. R. Tucker, "Intra-Individual and Inter-Individual Multidimensionality", Psychological Scaling: Theory and Applications, H. Guilliksen and S. Messick, editors, (New York: John Wiley & Sons, Inc., 1960), pp. 155-167.

[43] Tucker and Messick, op. cit.

space for each viewpoint about the interrelationships among the stimuli which may emerge. A mathematical outline of the individual differences model will now be presented.

It is assumed that ratio-scale estimates of distances or dissimilarities between stimuli are available for each individual. (The experimentally obtained estimates of dissimilarity may need to be scaled in order to meet this requirement.)

Let $x_{(jk)i}$ = estimate of interpoint distance between stimuli j and k by individual i

j,k = stimuli 1, 2, ..., n

(jk) = stimulus pairs $k>j$; there are $n(n-1)/2$ stimulus pairs

i,h = individuals 1, 2, ..., N

Since no missing data is permitted, there is one distance measure for each stimulus pair for every individual.

X = matrix of $x_{(jk)i}$ consisting of $n(n-1)/2$ rows for the stimulus pairs and N columns for the individuals

Once the matrix X of interpoint distances $x_{(jk)i}$ is compiled, questions can be raised with regard to the similarity of entries in the columns. If all individuals perceived the dissimilarities between stimuli the same way, the columns would be similar, i.e., the matrix of interpoint distances would be of rank 1. Any variation which may appear would be due to random dispersion or error of measurement. If there were two viewpoints about stimulus similarity, the columns of X could be arranged to form two sections. The columns in each section would be alike, but different from the columns in the other sections.

Rather than sorting columns of X, the actual method of analysis first searches for consistent covariation in the $x_{(jk)i}$ by factoring X into its principal components. If one factor satisfactorily accounts for the variance in X, then the corresponding average distances obtained from the factor loadings can be analyzed to yield a single multidimensional representation of the stimulus space. If more than one factor is required to account for the variance in X, then more than one set of distances between stimuli can be obtained. Each set of distances can in turn be analyzed, thus yielding several multidimensional spaces which reflect different points of view about the stimulus interrelationships.

Since X is not a symmetric, square matrix, the conventional factoring equations[44] are inapplicable. The procedure for factoring X is based on a theorem of Eckart and Young[45] for approximating one matrix by another matrix of lower rank.

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[44] H.H. Harman, Modern Factor Analysis (Chicago: University of Chicago Press, 1960).

[45] C. Eckart and G. Young, "The Approximation of One Matrix by Another of Lower Rank", Psychometrika, 1: 211-218, 1936.

As the number of stimulus pairs increases very quickly as the number of stimuli increases, Tucker and Messick[46] suggest that, if the number of individuals is moderately small, relationships between individuals rather than variables should be subject to analysis.

The basic matrix, designated P, is obtained as follows

$$P = X'X \tag{4}$$

P is an NxN matrix with sums of squares of measures for individuals as the diagonal elements and sums of cross products of measures between pairs of individuals as the off diagonal elements.

The procedure developed by Eckart and Young[47] yields a matrix $\hat{X}$ of lower rank than X which is a least-squares approximation to matrix X. The matrix $\hat{X}$ can be constructed from the _r_ largest characteristic roots and vectors of matrix X so as to approximate X to any desired extent. Thus

$$\hat{X}_r = U_r \Gamma_r W_r \tag{5}$$

is a matrix of rank r where

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46 Tucker and Messick, _op. cit._, pp. 337-338.

47 Eckart and Young, _op. cit._

U_r = n(n-1)/2 x r portion of an orthonormal matrix, i.e., $U_r^{-1} = U_r'$

Γ_r = rxr diagonal matrix of characteristic roots

W_r = rxN portion of an orthonormal matrix, i.e., $W_r^{-1} = W_r'$

It should be noted that the components are derived from a sums of squares and cross products matrix rather than from a matrix of intercovariances.

In contrast to X, P is a square, symmetric matrix and may thus be analyzed into principal components in the usual way.[48]

$$\hat{P}_r = \hat{X}_r'\hat{X}_r = W_r' \Gamma_r^2 W_r \qquad (6)$$

Γ_r^2 is a diagonal matrix containing the r largest characteristic roots of P and W_r contains the characteristic vectors of P. The elements of W_r' represent projections of points corresponding to people on unit length principal vectors of X (or of P).

U_r may now be obtained as follows,

$$U_r = XW_r' \Gamma_r^{-1} \qquad (7)$$

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[48]Harman, op. cit. Ch. 9.

since $U_r' U_r = I$ and $W_r W_r' = I$.

The elements of U_r represent projections of points which correspond to stimulus pairs on unit length principal vectors of X. Measures of distance between the stimuli are obtained from these stimulus pair projections when the latter are appropriately scaled and weighted. Each column of U_r will yield a set of distance measures which can be subsequently analyzed by multidimensional scaling methods.

As W_r is an rxN matrix, the coefficients both for individuals and for stimulus pairs are a function of the sample size N. (Both are scaled so that $WW' = I$). Thus, if two studies differed only in the number of individuals involved, with both sets of individuals consisting of random samples from the same population, the resulting numbers would not be comparable. W_r is rescaled into a matrix V:

$$V = K W_r \qquad (8)$$

so as to be independent of the number of individuals:

$$\frac{1}{N} V V' = I \qquad (9)$$

Substituting (8) into (9) and solving for K in terms of N leads to

$$K = N^{1/2} \qquad (10)$$

Thus,

$$V = N^{1/2} W_r \tag{11}$$

and

$$Y = U_r N^{-1/2} \tag{12}$$

in order to maintain the basic relationship of equation (5). From (7),

$$Y = X V' \Gamma_r^{-1} N^{-1} \tag{13}$$

The matrix V consists of scaled individual projections on the principal vectors while Y contains scaled stimulus pair projections on the principal vectors.

A factor matrix A of scaled projections for individuals on principal _factors_ can be obtained from the V matrix of scaled projections of individuals on the principal _vectors_ by weighting each vector by the square root of the corresponding latent root.

$$A = \Gamma_r V = N^{1/2} \Gamma_r W_r \tag{14}$$

Then,

$$\hat{X}_r = YA \tag{15}$$

Tucker and Messick point out that the Y and A matrices could be rotated to an orientation which may be more appropriate than the principal axes position. The situation is analogous to the rotation problem in factor analysis.

T_r is an rxr nonsingular transformation matrix which rotates the principal factors to some criterion:

$$B = TA \tag{16}$$

The inverse of this transformation is applied to Y to obtain the interstimulus distances.

$$Z = YT^{-1} \tag{17}$$

Now

$$\hat{X}_r = ZB \tag{18}$$

The matrix Z consists of scaled stimulus pair projections on the rotated axes. The n(n-1)/2 coefficients in each of the r columns of matrix Z represent distances between stimulus pairs in terms of a rotated viewpoint dimension. The r columns of matrix Z can be rearranged into r distinct nxn distance matrices and analyzed so as to yield r separate multidimensional spaces.

VI. THE "IDEALIZED INDIVIDUAL" CONCEPT

The factors or reference vectors may be construed as representing different viewpoints, real or hypothetical, with respect to interstimulus similarity. The entries in either matrix A or matrix B, which represent coordinates of points for individuals on unrotated and rotated axes of the space, respectively, may be plotted graphically. If specific individuals are of particular interest, perhaps because of their location with respect to other individuals in the factor space, a separate multidimensional space can be obtained for each such individual. Postmultiplication of matrix Z by the column vector of matrix B corresponding to the particular individual will yield the desired set of estimated interstimulus distances for that person. For i such individuals,

$$\hat{X}_i = ZB_i \qquad (19)$$

$\hat{X}_i$ is an n(n-1)/2 by i matrix containing distance estimates for the i selected individuals. B_i is an r by i matrix of individual coefficients on the viewpoint dimensions. The i columns of $\hat{X}_i$ each contain n(n-1)/2 measures of interstimulus distance which can be resolved to yield a multidimensional space for each individual.

It should be noted that the sets of interpoint distances in $\hat{X}_i$ are estimated on the basis of the r-dimensional viewpoint space.

thus eliminating some of the error variance in the original $x_{(dk)i}$ distance estimates.

It is also possible to generate hypothetical individuals by adding points in desired locations to the plots of the factor space for individuals. The location of these "idealized individuals"[49] may be determined in any number of ways. Points may be placed within clusters of points representing real individuals or at the extremities of the configuration of real points. Outside variables may also conceivably be used to determine the positions of the hypothesized individuals.

The coordinates for each point representing an idealized individual can then be read from the factor plots of matrix B and recorded in a column vector. The column vectors for g such idealized individuals can be assembled to form a matrix G and $\hat{X}_g$ computed as follows:

$$\hat{X}_g = ZG \tag{20}$$

$\hat{X}_g$ is an $n(n-1)/2$ by g matrix of estimated interstimulus distances for the g idealized individuals. G is an r by g matrix of coordinates for the idealized individuals on the rotated axes.

If points representing the g idealized individuals are inserted

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[49]Tucker and Messick, op. cit., pp. 314-343.

into the factor plots prior to rotation, the coordinates would be obtained with respect to the reference frame of matrix A. Then

$$X_g = YG_A \qquad (21)$$

G_A is an r by g matrix of coordinates for the idealized individuals on the unrotated factors.

The degree of relationship between the points of view of real individuals and each idealized viewpoint may be obtained by rotating the dimensions of the factor space of individuals to positions representing idealized individuals. In other words, a dimension is located for each idealized individual such that the loading on that dimension for the given idealized individual is unity and the loadings of other idealized individuals are zero. The projections of real individuals on each such dimension may be interpreted as indicative of the degree of relationship between the real individuals and that particular idealized viewpoint.

The idealized individual approach to the interpretation of viewpoint dimensions has elicited comment[50,51] on the grounds that ambiguities can occur in the meaning which may be ascribed to the points of view which are isolated.

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[50]John Ross, "A Remark on Tucker and Messick's 'Points of View' Analysis", Psychometrika, 31: 27-31, 1966.

[51]Norman Cliff, "The 'Idealized Individual' Interpretation of Individual Differences in Multidimensional Scaling", Psychometrika 33: 225-232, 1968.

VII. STUDIES INVOLVING INDIVIDUAL DIFFERENCES IN POINTS OF VIEW

A few representative studies will be cited to show the variety of areas in which there appear to be indications that more than one characteristic viewpoint about stimulus similarity may obtain. In each of these studies two or more viewpoint dimensions were found to be necessary in order to account for the points of view with respect to similarity judgments.

Studies dealing with individual differences in preference judgments are reported by Gulliksen[52] and by Tucker[53]. One of these is Tucker's study of dessert preferences[54] among students. In this study students were asked to rate several kinds of melons and berries with cream according to their preferences. Tucker's vector model for paired comparisons yielded two dimensions by preference: one reflecting a preference for melons and the other a preference for berries.

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[52] Harold Gulliksen, "The Structure of Individual Differences in Optimality Judgments," Human Judgments and Optimality, M.W. Shelly and G.L. Bryan, editors, (New York: John Wiley & Sons, Inc.,1964) pp. 72-84.

[53] L.R. Tucker, "Systematic Differences Between Individuals in Perceptual Judgments," Human Judgments and Optimality, M.W. Shelly and G.L. Bryan, editors, (New York: John Wiley & Sons, Inc. 1964), pp. 85-98.

[54] Tucker, "Intra-Individual and Inter-Individual Multidimensionality".

In the study by Tucker on goals of life[55], application of an adaptation of factor analysis led to the emergence of four dimensions, three of which were interpreted in terms of interest in service goals, religious goals, and power goals, respectively.

Helm and Tucker[56] have applied the individual differences model for multidimensional scaling to color perception. The subjects were ten individuals with normal color vision and four who were color-blind. The expectation that the color-blind subjects would form a separate group was borne out to some extent. In the three-dimensional representation which emerged, the subjects with normal color vision were found to lie in one plane and the color-blind subjects in a second plane.

The studies by Messick[57] and Tucker and Messick[58] of perceptions of prominent political figures represent another application of the individual differences model to the study of perceiver differences. The 39 subjects whose judgments were analyzed according to the individual differences model in the second study represented

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[55]L.R. Tucker, "Factor Analysis of Double Centered Score Matrices", Research Memorandum 56-3, (Princeton, N.J.: Educational Testing Service, 1956).

[56]C. Helm and L.R. Tucker, "Individual Differences in the Structure of Color Perception", American Journal of Psychology, 75: 437-444, 1962.

[57]Samuel Messick, "The Perceived Structure of Political Relationships", Sociometry, 24: 270-278, 1961.

[58]Tucker and Messick, op. cit., pp. 344-363.

a subgroup of the larger number of subjects in the first study and were chosen so as to be approximately equally divided among four categories: liberal Republican, conservative Republican, liberal Democrat, conservative Democrat. The subjects were asked to rate on a nine point scale the dissimilarity in political thinking of each of the 190 pairs of twenty selected political figures.

The analysis yielded three dimensions, each representing a separate multidimensional point of view. Idealized individuals located in terms of the unrotated factor space of individuals revealed spaces ranging from a one-dimensional representation with a marked evaluative component to a complex space of possibly six dimensions. A nonmathematical discussion of the above study appears in Jackson and Messick[59].

In Wiggins'[60] study the subjects were instructed to rate pairs of items in terms of their difference in social desirability. The axes of the factor space of individuals were rotated so as to have the idealized individuals load on only one dimension. The six rotated factors were interpreted as idealized individuals representing different perceptual structures or viewpoints with respect to social desirability.

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[59] D.N. Jackson and S. Messick, "Individual Differences in Social Perception", British Journal of Social and Clinical Psychology, 2: 1-10, 1963.

[60] Nancy Wiggins, "Individual Viewpoints of Social Desirability", Psychological Bulletin, 66: 68-77, 1966.

VIII. NONMETRIC MULTIDIMENSIONAL SCALING

Scales in which the relationships among interpoint distances are specified only by inequalities are generally termed nonmetric. In such situations, the information given about any two interpoint separations is only which is the larger, rather than how much larger.

Various types of nonmetric scales differ in the degree to which they approximate metric scales. If a sufficient number of nonmetric constraints are imposed, a nonmetric scale may begin to behave like a metric scale. In a simple ordinal scale there are comparatively few constraints with the result that the points on the scale can be moved about relatively freely without interchanging any two points. However, as the points are required to satisfy an increasing number of inequalities with respect to interpoint separations, the spacing becomes more and more constrained, until relatively minor perturbations of the points are sufficient to cause one or more of the inequalities to be violated. Hence, ordinal information on interpoint separations may imply a considerable amount of interval information about the location of the points. Conditions which need to obtain in order that metric information be realized from nonmetric data are discussed by Shepard[61].

The object of constructing a configuration of the n points representing n stimulus objects in such a way that the interpoint separations correspond in some sense to the experimentally obtained

[61] R.N. Shepard, "Metric Structures in Ordinal Data", Journal of Mathematical Psychology, 3: 287-315, 1966.

dissimilarity estimates has, as already mentioned, led to the development of two distinct avenues of approach to the problem.

The "nonmetric" approach, as originally set out by Shepard in his two papers on the analysis of proximities[62,63], has resulted in numerous computer based procedures for obtaining such configurations. In Shepard's original program the coordinates of a trial configuration were adjusted by means of an iterative procedure so as to make the rank order of the interpoint separations coincide more and more closely with the inverse of the rank order of the experimentally determined proximity measures. The only requirement is that the distances and proximity measures be monotonically related. No assumptions are made with regard to the specific form of the distance function required to convert the proximity measures to distances.

While retaining the goal of obtaining a monotone relationship between distances and the experimentally observed dissimilarities, Kruskal[64,65], has refined Shepard's approach by introducing a quantitative measure of departure from monotonicity which differs from that suggested by Shepard. Kruskal has proposed that a monotone regression of distance upon dissimilarity be performed and that the

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62 Shepard, "The Analysis of Proximities:...I".

63 Shepard, "The Analysis of Proximities:...II".

64 Kruskal, "Multidimensional Scaling by Optimizing Goodness of Fit to a Nonmetric Hypothesis".

65 Kruskal, "nonmetric Multidimensional Scaling: A Numerical Method".

residual variance, suitably normalized, serve as the measure of goodness of fit.

Let δ_{ij} be the experimentally obtained dissimilarity between stimuli i and j and let the n stimulus objects be represented by n points $x_1, \ldots, x_n$ in a t-dimensional space with interpoint distances d_{ij}. Then the <u>stress</u> of the configuration is defined as

$$S = \sqrt{\sum_{i<j} (d_{ij} - \hat{d}_{ij})^2 / \sum_{i<j} d_{ij}^2} \tag{22}$$

where $\hat{d}_{ij}$ are numbers which minimize S under the constraint that the $\hat{d}_{ij}$ have the same rank order as the δ_{ij}, i.e., $\hat{d}_{ij} \leq \hat{d}_{i'j'}$, whenever $\delta_{ij} \leq \delta_{i'j'}$. In short, the stress, S, is the square root of an appropriately normalized sum of squared deviations from the best-fitting monotonic sequence.

It is in effect hypothesized that the "true" dissimilarities, from which the observed dissimilarities differ only because of random variation, are the result of some unknown monotone distortion of the distances between the points in some "true" configuration. For a given t-dimensional space, the best-fitting configuration is the one which minimizes the stress. Kruskal regards a stress of 0.10 as "fair", 0.05 as "good" and 0.025 as "excellent"[66]. Zero stress is "perfect"

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[66]Kruskal, "Multidimensional Scaling by Optimizing Goodness of Fit to a Nonmetric Hypothesis", p.3.

in the sense that a perfect monotone relationship exists between the distances and the dissimilarities.

While invariant under any monotonic transformation of the dissimilarities, S varies continuously with changes in the coordinates of the configuration. Thus, instead of iterative adjustment of a set of trial values for the d_{ij}, the minimization is carried out by the method of gradients. Kruskal uses the negative gradient of S as the basis for his algorithm.

The numerical procedure requires that the number of dimensions, t, and the kind of metric (Euclidean or non-Euclidean) be specified. A single vector or point in nt-dimensional space is used to describe the entire configuration. The coordinates of point X_{ik} for $i = 1, \ldots, n$ and $k = 1, \ldots, t$ are the coordinates of the n points in t dimensions which constitute the configuration i.e.

$$(x_{11}, \ldots, x_{1t}, \ldots, x_{n1}, \ldots, x_{nt})$$

If the values of δ_{ij} are given, there exists a definite stress for any configuration:

$$S = S(x_{11}, \ldots, x_{1t}, \ldots, x_{n1}, \ldots, x_{nt})$$

The above function is minimized by taking the partial derivative with respect to each coordinate in turn.

An initial configuration is defined and subsequently improved on the basis of the direction in which S is decreasing most quickly. The negative gradient

$$\left(-\frac{\partial S}{\partial x_{11}}, \ldots, -\frac{\partial S}{\partial x_{1t}}, \ldots, -\frac{\partial S}{\partial x_{nt}} \right)$$

is determined and a suitable distance moved along it. The gradient is again determined and a move made. After a sufficient number of repetitions of the two steps a minimum value of S, indicated by all the partial derivations being zero, is obtained.

The procedure requires that the number of dimensions to be used be known. Since this is rarely the case in practice, Kruskal suggests that the computation be carried out in several dimensionalities and the minimum stress be plotted against the number of dimensions.[67]

Decisions as to which configuration to retain as the most appropriate representation of the data may contain some element of subjectivity since no statistical methods for testing the significance of results obtained from scaling procedures are available at present. According to Kruskal, interpretability should be considered along with an acceptable stress value in deciding on the number of dimensions to be retained.

Systematic investigation of the properties of the solutions

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[67]Ibid. p. 16.

generated by various nonmetric multidimensional scaling programs is only beginning. Green,[68] for instance, points out the growing need to use Monte Carlo runs and other empirical tests to determine further the properties of a variety of computer-oriented psychometric procedures.

The effect of the number of points on the accuracy of the solution has been explored by Shepard[69]. In order to evaluate the extent to which the results from the Kruskal program are influenced by the position from which the iterative process starts, Shepard compared the reconstructed configurations with two-dimensional "true" configurations for points ranging in number from 3 to 45. The results indicate that for a small number of points, say less than eight, the reconstruction tends to be relatively poor. As the number of points becomes larger, the accuracy of reconstruction increases until for fifteen or more points it becomes essentially perfect. This suggests that at least eight stimulus points are necessary if a unique best-fitting configuration is sought.

Klahr[70] has scaled randomly generated proximities for 6, 7, 8, 10, 12, and 16 points by means of the Kruskal program in order to

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[68] B.F. Green, Jr., "The Computer Revolution in Psychometrics". Psychometrika 31: 437-455, 1966.

[69] Shepard, "Metric Structures in Ordinal Data", pp. 296-299.

[70] David Klahr, "A Monte Carlo Investigation of the Statistical Significance of Kruskal's Nonmetric Scaling Procedure (Chicago: Graduate School of Business, University of Chicago, 1969, mimeographed). To appear in Psychometrika.

obtain estimates of the relative frequency with which unstructured data yields superficially "good" but entirely spurious solutions.

As in Shepard's study, the results were found to be extremely sensitive to the number of points when this is low. For instance, solutions with stress $\leq$ 0.05 were found in three dimensions 96 times out of 100 for six points, 74 times out of 100 for 7 points, and 33 times out of 100 for 8 points. Even for sixteen points, the average stress value on the basis of 50 random sets of data was found to be 0.130 in four dimensions and 0.096 in five dimensions. This suggests that what constitutes acceptable stress is strongly dependent upon the number of points involved. Klahr concludes that "if n is small, and if a low stress constitutes the only evidence of structure, then any results may be meaningless".[71]

However attractive the procedures for recovering metric structures from nonmetric data may appear, studies such as the two cited above suggest that problems may arise in their application and interpretation. In his discussion of some of the difficulties which may be encountered in practice, Torgerson[72] deals most extensively with problems inherent in the nature of similarity itself. Evidence is cited to the effect that similarity judgments do not necessarily remain invariant over changes in the composition of the set of stimuli. For geometric figures varying with respect to two simple physical

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[71]Ibid. p. 7.

[72]Torgerson, "Multidimensional Scaling of Similarity".

attributes, the rank order of the judgments was found to depend upon the composition of the set of stimuli, suggesting that stimulus context may affect the judgment rendered.

The influence of selective attention on the metric properties of the spatial representation has been investigated by Shepard[73]. The results suggest that when the stimuli vary with respect to dimensions which are perceptually distinct, the underlying metric changes as the focus of attention is shifted from one dimension to another. In such cases perhaps a separate multidimensional representation of each identifiable state of attention (or viewpoint?) might be sought.

Gregson[74] and Russell and Gregson[75] have applied Kruskal's method with various Minkowski r-values to the scaling of three-component taste mixtures. The data are better fitted by a representation in three-dimensional space with r equal to ten or six than in Euclidean space although the differences in minimum stress are not large. Gregson interprets a high r-value as suggesting that in such instances the largest component in the taste mixture may be

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[73]Shepard, "Attention and the Metric Structure of the Stimulus Space".

[74] R. A. Gregson, "Representation of Taste Mixture Cross-Modal Matching in a Minkowski r-metric", Australian Journal of Psychology 17: 195-204, 1965.

[75]P. N. Russell and R. A. Gregson, "A Comparison of Intermodal and Intramodal Methods in Multidimensional Scaling of Three-Component Tast Mixtures". Australian Journal of Psychology, 18: 244-254, 1966.

disproportionately influential in determining the similarity judgment. In any case, the results are not inconsistent with Shepard's finding that the metric seems to change with a shift in emphasis with regard to the characteristics of the stimuli to which the subject attends.

Although Kruskal's algorithm differs from that of Shepard, the solutions have been found in practice to converge to essentially the same configuration[76]. Conditions for the application of nonmetric multidimensional scaling were discussed in Shepard's original paper[77] on various applications of his technique. Difficulties were anticipated as the number of stimuli becomes very small. Shepard shows that for three points, the rank order only of the separations is insufficient to insure a unique solution.

Even if the number of points is adequately large, their actual spatial configuration may influence the determinacy of the solution. The rank order of the set may be insufficient to yield a determinate solution in the situation in which the points can be divided into two clusters in such a manner that the interpoint separations for all pairs of points within the same cluster are less than all the interpoint separations between clusters. The ratio of the between to within distances could in such a base be made arbitrarily large, with the result that the two clusters may be driven apart and collapsed internally

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[76]Shepard, "Metric Structures in Ordinal Data", p. 293.

[77]Shepard, "The Analysis of Proximities:...II", pp. 238-245.

into essentially one-dimensional configurations. Although the prevalence of such cases in practical situations has not been determined, the likelihood of encountering such configurations might be expected to decrease as the number of points increases.

IX. SUMMARY

The identification and description of consistent individual differences in judged similarity of a set of stimuli may be regarded as a two-stage process. The procedures which will be used to this end, and some of the considerations involved in the interpretation of their results, have been presented and discussed in this chapter.

CHAPTER IV

DESIGN OF THE INVESTIGATION: SUBJECTS, MATERIALS, AND PROCEDURES

I. THE POPULATION

After receiving permission from the Director of Research, Edmonton Public School System, to conduct the study in the Edmonton Public School System, the investigator approached the principals of Bonnie Doon Composite High School, Harry Ainlay Composite High School, and Queen Elizabeth Composite High School to request their assistance in its execution. The subjects selected to participate in this study were the students registered in Grade XII Physics 30 for the 1968-69 term at these three schools and their teachers. Eleven Physics 30 classes, with 353 students and seven teachers, were involved.

The subjects covered mechanics according to Stollberg, Hill, and Nygaard, [1] a new physics textbook introduced for the 1968-69 term. It was throught that some advantage might be gained from carrying out the study with this particular text in 1968-69 since Department of Education examination requirements for the new course would not as yet be revealed at the time the study was in progress. Consequently somewhat greater reliance on the part of the teachers on their own

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[1]R. Stollberg, F. F. Hill, and M. M. Nygaard, <u>Frontiers of Physics</u> (Don Mills, Ontario: Thomas Nelson & Sons (Canada) Limited, 1968). Canadian Edition.

judgment might be expected than would be the case if external examination requirements were known.

Because of the extensive testing involved in this study, and because an alternate Physics 30 course is available in some schools, random sampling of Physics 30 classes or students from within the Edmonton Public system was not regarded as feasible. For the purposes of this study, an indication of the applicability of multidimensional scaling to this type of problem is as relevant an outcome as are the actual representations which might emerge. Thus a well-defined population appeared preferable to an apparently randomly selected one for which all the desired measures would likely be unavailable. Since participation in some of the testing was voluntary, all students did not write all the tests. A subset of 180 subjects was chosen from the original population on the basis of the completeness of their set of scores.

II. THE PAIRED COMPARISONS TASK

The twenty concepts which constituted the set of stimuli to be scaled were selected from the mechanics section of Physics 30, namely Chapters 1, 2, 3, 4, 6, and 7 of Stollberg, _et. al_.[2] The choice was influenced by the relative emphasis which these topics receive in the text and by the time allotment for each chapter suggested in the

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[2]Stollberg, Hill and Nygaard, _op. cit_.

course outline.[3] The teachers appeared to be covering the material at roughly the same rate. All finished the mechanics section in time for the January examination.

According to the course outline, the four chapters to be dealt with most extensively were Chapter 1, Motion and Measurement; Chapter 2, Force and Motion; Chapter 3, Work and Energy; and Chapter 6, Forces in Equilibrium, with a recommended time allotment of sixteen class periods each. Four concepts were ultimately chosen from each of Chapter 1 and Chapter 6 and five concepts from each of Chapter 2 and Chapter 3. Although the suggested time to be spent on Chapter 4, Matter and Energy was twelve periods, only one concept was selected from this chapter because the bulk of it deals with topics such as atomic structure and transmutation which were not regarded as central to mechanics. One concept was also chosen from Chapter 7, Forces and Simple Machines since only part of the chapter is included in Physics 30 with a recommended time allotment of four class periods.

A preliminary list of twenty-four statements of concepts was drafted and submitted to a Physics 30 teacher for criticism. Four of the original statements of concepts were discarded and the remainder revised so as to yield the above distribution by chapter. The twenty statements of concepts which formed the set of stimuli to be scaled in this study appear in the Appendix.

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[3] Senior High School Curriculum Guide for Science (Province of Alberta, Department of Education, 1968) p. 33.

With two exceptions, the format in which the concepts were presented consisted of an underlined topic heading followed by a short elaboration in the form of a descriptive phrase, an example, a formula, or some combination of the three. The statement of one of the selected concepts which appears below serves as an illustration:

The resultant of concurrent forces: $\vec{R} = \Sigma\vec{F} = 0$ for equilibrium; the equilibrant is equal and opposite to the resultant

The 190 pairwise combinations of the twenty concepts were arranged in a random sequence and typewritten on 5 inch by 7 inch index cards. The deck of 95 index cards was arranged so that the first 95 concept pairs, numbered 1 to 95 at the upper left, appeared on one side and the pairs numbered 96 to 190 appeared on the other side. Forty such sets of cards were prepared, sufficient to provide each student in a class with a deck. A two-page answer sheet was devised, the first page with blanks numbered 1 to 95 and the second with blanks after the numbers 96 to 190.

The instructions, which appear in the Appendix called for the subject to rate the difference in difficulty between the two concepts on each numbered card on a 9-point scale and to place the rating in the correspondingly numbered space on the answer sheet.

1	2	3	4	5	6	7	8	9
very similar in difficulty							very different in difficulty	

The paired comparisons task was tried out in a pilot run by seven Physics 30 students and seven adults with background in physics. The indications were that the task was one which the students might reasonably be expected to carry out in one 42-minute class period. The paired comparisons task was performed by the eleven Physics 30 classes and their seven teachers during a class period in January, 1969.

III. THE ACHIEVEMENT AND PREFERENCE MEASURES: DEVELOPMENT AND DESCRIPTION

Achievement Test

A set of forty four-option multiple choice items based on subject matter included in the mechanics section of Physics 30 was prepared by one of the Physics 30 teachers who participated in the study. In formulating the items emphasis was placed on application of the principles of mechanics. Computational labor was reduced by stressing wherever reasonable the form of the solution (i.e., how it is set up) rather than the final numerical answer. A copy of this preliminary set of items was submitted to each of the other six teachers involved for criticism and improvement. Suggested revisions were discussed with the teachers and consensus as to their acceptability reached before the final draft of the test was duplicated.

The reliability of the test, as computed by means of the Kuder-Richardson 20 formula, was found to be 0.82 for the total population (N-352) and 0.84 for the sample (N=179) on which this study is based. The *Achievement Test* appears in the Appendix.

Cognitive Preference Test

The preliminary draft of the *Cognitive Preference Test* consisted of twenty-two items resembling a four-option multiple choice question in their general appearance. Each item presented some information or data dealing with mechanics, followed by four options, all of which were correct and related to the information given. The four options in each item were designed so that one of them reflected each of the following four categories or "preferences":

a. recall of factual material

b. mathematical application

c. experimental or practical application

d. principle or generalization

The instructions called for the subject to select the option he prefers most in conjunction with the introductory information.

The seven physics 30 students and two adults who wrote the pilot test expressed dissatisfaction with the above format for the items. There appeared to be general agreement that the nature of the options became sufficiently transparent once the first few items were completed to enable the subject to "slant" his responses to the remaining items to deliberately reflect any one of the four categories.

Revision of the format of the items consisted of using the four options in each item to construct two-option items. Each of the twenty-two items in the pilot test served as the basis for two such items. An introductory statement similar in content to the original was devised for the second "half" of the options. In order that the six combinations of the four categories appear an equal number of times, two additional sets of the four options were prepared and included to make a total of 48 two-option items.

Each combination of the four categories appeared four times in the first half and four times in the second half of the test. The revised items were also arranged so that each combination of the four categories appeared four times among the odd and four times among the even items. Thus, if an individual was perfectly consistent, he could choose a given type of option a maximum of twenty-four times in the course of the forty-eight choices. The total number of times one category was selected in preference to the remaining three categories was determined for each category for each subject. The following pair of items serves as an illustration:

Item 1

In circular motion, the acceleration vector is perpendicular to the velocity vector.

(a) A satellite launched horizontally with the right initial velocity will move at constant speed in a circular orbit around the earth. (experimental or practical application)

(b) A deflecting force of constant magnitude perpendicular to the motion makes a body move in circle with constant speed. (principle)

Item 25

The acceleration of an object traveling with uniform speed in a circle is directed inward toward the center of the circle.

(a) The magnitude of the acceleration is given by $a=v^2/r$ (factual material)
(b) The speed of a satellite moving in a circular orbit 400 km. above the earth is approximately 7.6×10^3 m/sec or 18,000 miles per hour (mathematical)

The ipsative, or interrelated, nature of the scale scores can present statistical problems. Since the score for each category cannot be regarded as independent of the scores for the other categories, the numbers obtained are not the result of measurement in the sense of being based on an interval scale. [4] The difficulty was overcome to some extent by retreating to the ordinal level of measurement. The relative frequencies of selection for each subject were converted to rank orders for purposes of further analysis.

The interdependence of the four scores for any given individual may also present problems in the calculation of an unambiguous index of reliability for each scale. However, an estimate of the agreement among the rankings of the four categories may be obtained since the two halves of the test, either odd/even or first half/second half contain the same number of each combination of categories.

The rank orders of the number of times each category was chosen in half the test were compared for each category for each individual. The comparison in terms of "agreement-disagreement" was

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[4] S. Siegel, Nonparametric Statistics for the Behavorial Sciences (New York: McGraw-Hill Book Co., 1956) pp. 18-34.

made for both the first half as opposed to the second half of the test and for the odd items as opposed to the even items. Because of the frequency of occurrence of tied ranks it was decided to regard the rank orders based on half the items as being in "agreement" if they matched to within ± 0.5. For instance, if the rank assigned on the basis of frequency of choice for a given category for a given subject is 2.0 for the first half and either 1.5, 2.0, or 2.5 for the second half of the test, then the two rank orders were considered to be in agreement. The ratio of the number of "agreements" to the total number of comparisons for a given category was regarded as an index of consistency of preference for that category.

The extent of agreement for the category relating to the recall of factual material was 0.43 for the first/second half of the test and 0.41 for the odd/even items, for an average of 0.42. For the category dealing with mathematical application, the proportion of agreement was 0.48 for the first/second half of the test and 0.42 for the odd/even items, for an average of 0.45. The agreement in the rankings for the experimental or practical application category was 0.43 in both cases. The agreement in preference rankings assigned to the category reflecting generalizations or principles was 0.36 for the first/second half of the test and 0.32 for the odd/even items, for an average of 0.34 i.e., approximately one-third of the subjects were consistent in the rank order of their relative frequency of choice for this type of option.

It is noted that the extent of the consistency of preference exhibited by the subjects on this <u>Cognitive Preference Test</u> is unsatisfactory. It is also noted that the same problem in relation to consistency or, if one prefers, "reliability," for this general type of test has emerged in the literature (cf. Chapter II, pp.38-39). Heath[5] has published K-R 20 reliabilities as low as 0.31 for options dealing with fundamental principles in physics, while Atwood[6] has reported a test-retest coefficient of 0.42 for a category dealing with practical application in chemistry.

Since it appears fairly obvious that estimates of consistency of preference exhibited by high school students for the types of categories on which cognitive preference tests in science have, to date, been based are far from impressive, it seems advisable to view the conclusions based on this measure with extreme caution. Subsequent analysis of the results of the Cognitive Preference Test will focus only on the most frequently chosen category.

IV. THE APTITUDE MEASURES: DESCRIPTION

One of the undertakings of this study involves investigation of some of the relationships of the viewpoint dimensions which may emerge with measures of aspects of the intellectual functioning of the subjects. The tests administered to the subjects with the view

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[5] R. W. Heath, "Curriculum, Cognition, and Educational Measurement, Educational and Psychological Measurement, 24:239-253, 1964, p. 245.

[6] R. K. Atwood, "A Cognitive Preference Examination Using Chemistry Content," Journal of Research in Science Teaching, 5:34-35, 1967-1968, p. 35.

of uncovering some of the relationships which might obtain are described in this section.

Differential Aptitude Tests

The Differential Aptitude Tests (DAT) are integrated battery of eight standardized tests designed to measure various abilities of students in grades eight through twelve primarily for purposes of educational and vocational guidance. The four subtests of the DAT battery selected for inclusion in this study are the Verbal Reasoning, Numerical Ability, Abstract Reasoning, and Space Relations subtests.

Considerable technical information is available on the battery as a whole[7] and reviews tend to be favorable.[8,9] A reservation seems necessary, however, with respect to the interdependence of the above four subtests. The intercorrelations of the four subtests, as reported in the DAT manual,[10] appear in Table I. The magnitude of these coefficients appears to suggest that the aptitudes measured by the four subtests may not be sufficiently different to be regarded as indicative of separate aspects of intellectual functioning.

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[7] G. K. Bennett, H. G. Seashore, and A. G. Wesman, Manual of the Differential Aptitude Tests, Fourth Edition, (New York: The Psychological Corporation, 1966).

[8] J. B. Carroll, Reviewing the DAT Battery in The Fifth Mental Measurements Yearbook, O, K. Buros, editor (Highland Park, N.J.: The Gryphon Press, 1959), pp. 672-673.

[9] R. E. Schutz, Reviewing the DAT Battery in The Sixth Mental Measurements Yearbook, O, K. Buros, editor (Highland Park, N. J.: The Gryphon Press, 1965, pp. 767-769.

[10] Bennett, Seashore, and Wesman, op. cit. p. 72.

TABLE I

MEAN INTERCORRELATIONS FOR THE DAT BY SEX
FOR FORM L*

Boys (N=913)		VR	NA	AR
Verbal Reasoning	(VR)			
Numerical Ability	(NA)	.70		
Abstract Reasoning	(AR)	.68	.66	
Space Relations	(SR)	.58	.53	.63
Girls (N=930)		**VR**	**NA**	**AR**
Verbal Reasoning	(VR)			
Numerical Ability	(NA)	.72		
Abstract Reasoning	(AR)	.68	.64	
Space Relations	(SR)	.58	.58	.67

*G. K. Bennett, H. G. Seashore, and A. G. Wesman, _Manual for the Differential Aptitude Tests_, Fourth Edition, (New York: The Psychological Corporation, 1966) p. 7-2.

1. Verbal Reasoning (VR) subtest of the DAT, Form L, battery. (Administration time: 30 minutes.) The VR test consists of fifty items in the form of verbal analogies. For the most part, the vocabulary is relatively straightforward and it is the complexity of the verbally phrased concepts which varies. A Spearman-Brown reliability coefficient of 0.94 for Grade X11 boys and 0.93 for Grade X11 girls has been reported for the VR test.[11] On the basis of the subjects of the present study, a K-R 20 reliability estimate of 0.84 was obtained for the VR test.

2. Numerical Ability (NA) subtest of the DAT, Form L, battery. (Administration time: 30 minutes.) The NA subtest consists of forty multiple choice items ranging from simple numerical skills to somewhat more complex computational problems. Each item includes a "none of these" option to discourage estimation of the answer. Spearman-Brown reliability coefficients of 0.92 for Grade XII boys and 0.91 for Grade XII girls are reported in the DAT manual.[12] A K-R 20 reliability of 0.79 was obtained for the subjects in the study being reported.

3. Abstract Reasoning (AR) subtest of the DAT, Form L, battery. (Administration time: 25 minutes.) The AR test consists of fifty items which involve the ability to perceive relationships in patterns of abstract figures. Designed as a non-verbal measure of

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[11] Ibid., p. 6-2

[12] Ibid.

[13] Ibid.

reasoning ability, each item requires the student to discover the operating principle in the sequence of diagrams and to designate the figure which should come next. The DAT manual reports Spearman-Brown reliability coefficients of 0.89 for Grade XII boys and 0.94 for Grade XII girls for the AR test.[13] A K-R 20 reliability coefficient of 0.81 was calculated on the basis of the subjects of this study.

4. Space Relations (SR) subtest of the DAT, Form L, battery. (Administration time: 25 minutes.) The SR test consists of sixty items of the "unfolded paper boxes" type. These require the subject to manipulate objects mentally in three-dimensional space both in terms of visualizing the object on the basis of a picture of a pattern and imagining how it would appear when rotated in various ways. Spearman-Brown reliability coefficients of 0.95 for Grade XII boys and 0.94 for Grade XII girls are reported in the DAT manual.[14] A K-R 20 coefficient of 0.91 was calculated for the SR test on the basis of the present sample.

Skemp's Tests of Reflective Thinking

1. (a) Skemp's Concept Formation (CFI) test. The test material consists of fourteen sets of geometric figures. In each set or problem, the subject is presented with three exemplars and three

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[13]Ibid.

[14]Ibid.

non-exemplars of a geometric concept and is required to identify three additional figures as exemplars or non-exemplars of the same concept. The CFI tests was designed to be administered immediately prior to Skemp's Reflective Action with Concepts test in order to familiarize subjects with the type of concept they would be expected to handle reflectively in the second test. Skemp[15] does not report reliability estimates for this test since it is expected that most of the subjects would answer most of the items correctly. A copy of the test appears in the Appendix.

(b) Skemp's Reflective Action with Concepts (CFII) test. In essence, the reflective process involved in this test is that of logical multiplication. The student is required to combine two geometric concepts to form a new concept which possesses both the properties of the original concepts. The format of the thirty-five sets of figures resembles that of the CFI test. Three exemplars of the double concept are followed by three non-exemplars having, respectively, one, the other, and neither of the class-properties of the exemplars. In order to classify the test figures as exemplars and non-exemplars of the double concept, the subject is presumed to exercise reflective intelligence in identifying the relevant properties by consciously separating and combining concepts. Skemp[16] reports a Spearman-Brown reliability coefficient of 0.76 for this test on

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[15] R. R. Skemp, "Reflective Intelligence and Mathematics," British Journal of Educational Psychology, 31:45-55, 1961 p. 53.

[16] Ibid.

the basis of 138 subjects. The K-R 20 estimate of reliability based on the present study was found to be 0.88 (N=170) for the CFII test.

2. (a) Skemp's Operations Formation (OFI) test (as modified by Harrison[17]). In this test, the subject is given an answer sheet and a demonstration sheet which presents three examples of each of ten operations such as 90° clockwise rotation or interchanging the numbers of the elements in two groups. Once two similar operations have been attempted on a practice sheet and explained, the subjects are asked to discover the operation which transforms the demonstration figure on the left of the arrow into that on the right and to carry out this operation, as identified on the basis of three such examples, on the three test figures. The subjects are requested to draw the result of the operation on each of the three specified test figures in a corresponding blank space on the answer sheet.

Skemp[18] has calculated a Spearman-Brown reliability coefficient of 0.94 for a somewhat longer original version of the OFI test. In the present study, a Spearman-Brown reliability estimate of 0.88 was obtained for the modified OFI test which appears in the Appendix.

The OFI test is designed to be administered immediately preceding Skemp's Reflective Action with Operations (OFII) test.
Since an understanding of the operations involved in the OFI test is

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[17] D. B. Harrison, "Reflective Intelligence and Mathematics Learning: (unpublished Doctoral Dissertation, The University of Alberta, Edmonton, 1967).

[18] Skemp, *op. cit.*, p. 53.

necessary in order that the subject be able to do the OFII test, it is intended that the answers to the OFI test be distributed to the subjects and any difficulties explained before the OFII test is administered.

(b) Skemp's Reflective Action with Operations (OFII) test (as modified by Harrison[19]). In the OFII test the subject is asked to indicate the results of carrying out operations in reverse, of combining two operations, and of simultaneously reversing and combining two operations, on test figures. Of the fifteen sets of figures constituting the test, there are five of each of the above type. To do this requires conscious awareness of the operations in question and of the possible results of such modifications and combinations.

Skemp[20] has calculated a Spearman-Brown reliability coefficient of 0.94 for his original test. On the basis of the present OFII test used in this study, the Spearman-Brown reliability coefficient was also found to be 0.94. A copy of the modified test appears in the Appendix.

V. TESTING PROCEDURE

As a consequence of constraints imposed by actual school operating procedures, the scheduling of some of the tests varied from school to school. What seemed to be basic requirements, however,

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[19] Harrison, *op. cit.*

[20] Skemp, *op. cit.*, p. 53.

were met in all three situations.

The scaling of the concepts and the administration of the *Achievement Test* and *Cognitive Preference Test* took place in January, 1969, when the mechanics section of the Physics 30 course was completed. In each instance, the paired comparisons task was carried out in a 42-minute class period which preceded by at least one week the *Achievement Test*. The seven teachers were asked to participate in the paired comparisons task.

The *Achievement Test* was administered as the initial one-hour section of the regularly scheduled January mid-term examination in Physics 30. At the end of one hour, the papers were collected and the students instructed to proceed with the remainder of the paper, which did not concern this study. It was thought that physics achievement scores obtained under such circumstances, in which the test "counts" as a substantial part of the student's physics grade, might be a better reflection of knowledge of mechanics than a period test taken to accommodate an outside investigator. The *Cognitive Preference Test* was administered to the students a few days after the *Achievement Test* was written, but before it was returned to them graded. These requirements appeared necessary in order to reduce any effect that a student's knowledge of his performance may have on his conceptions and performance.

Skemp's OFI and OFII tests were administered in December, 1968, and early January, 1969, to all eleven classes. In each instance, the two tests together required one period of class time. In two of the schools (7 classes), Skemp's CFI and CFII tests were also both administered in a class period which preceded the OFI and OFII tests. In the third school (4 classes), the CFI and CFII tests were written during a study period.

In two schools (7 classes), the four subtests of the DAT battery were written on a voluntary basis in a scheduled two and one-half hour block of time during January test week. In the third school (4 classes), the *Abstract Reasoning* subtest was written during regular class time and the other three DAT subtests in study periods during November and December, 1968. Since the recommended maximum time was allowed for each test in each instance, the variation in testing circumstances in the three schools occasioned by administrative differences was regarded as insufficient grounds for excluding these tests from the study.

CHAPTER V

ANALYSIS, RESULTS, AND INTERPRETATION

The computations involved in the analysis of the data were performed on the University of Alberta IBM 360/67 computer using programs from the computer program library of the Division of Educational Research Services.

I. DIMENSIONS OF VIEWPOINT

At the time the analysis was performed, the computer program based on the Tucker and Messick individual differences model was dimensioned for a maximum of 70 columns for the input matrix X. Therefore it seemed reasonable to divide the 180 subjects into three groups of 60. More important, it appeared necessary to analyze at least two sets of dissimilarity data for the purpose of cross-validating any results which might emerge.

The scaling procedures to be employed will always yield some type of solution, but in attributing meaning to aspects of this solution it appears desirable, if not essential, to compare the solutions for two or more statistically equivalent groups of subjects in order to get an idea of how well portions of the solution replicate. This is particularly important since there are at present no statistical tests for assessing the significance of the results. There seems little point in undertaking a detailed interpretation of an elaborate structure of perceived relations without the presentation of at least rudimentary indications about the extent to which one may expect to obtain a similar result with another sample drawn from the same population.

Upon being assigned an identification number coded by teacher and individual, the 180 students were each assigned to one of three groups, designated A, B, and C. Since students were selected as subjects on the basis of the completeness of their data, the number of subjects drawn from a given teacher's classes varied considerably. Consequently, random assignment of subjects to groups was accomplished separately for each teacher's students in order to insure that, as nearly as possible, an equal number of a particular teacher's students were included in each of the three groups. For instance, of the 24 students of teacher 600 who served as subjects in this study, eight were randomly assigned to each of Group A, B, and C. The number of students, by teacher, in each of the three groups appears in Table II. The seven teachers were also included in each of the three groups in order to obtain some indication of the extent to which their individual projections on the principal factors vary with the composition of the group of judges.

For each group, the 67 sets of dissimilarity ratings for the 190 pairs of 20 physics concepts were arrayed in the matrix X consisting of 190 rows for the stimulus pairs and 67 columns for the judges. Each entry $X_{(jk)i}$ was an integer from 1 to 9 representing the estimated difference in difficulty assigned to the stimulus pair (jk) by individual i.

TABLE II

DISTRIBUTION OF SUBJECTS, BY TEACHER, IN GROUPS A, B, AND C

TEACHER (ID)	GROUP A	GROUP B	GROUP C	TOTAL
100	10	10	10	30
200	6	6	6	18
300	5	5	4	14
400	8	8	8	24
500	8	8	8	24
600	8	8	8	24
700	15	15	16	46
TOTAL	60	60	60	180

The matrix P of sums of squares of measures for the individuals and sums of squares of cross products between individuals was then analyzed into principal components. As expected, the first latent root or eigenvalue was very large relative to the subsequent roots. More precisely, the first latent root accounted for 81.0 per cent of the sums of squares of matrix P for Group A, 80.2 per cent of the sums of squares of matrix P for Group B and 87.4 per cent of the sums of squares of matrix P for Group C. The next four roots also appeared to be somewhat larger than the remainder, with a slight break occurring between the fourth and fifth roots for Group A and Group B and between the third and fourth root for Group C. The four roots together accounted for four per cent of the sums of squares for each group. The remaining sixty-two eigenvalues trailed off in a regular manner to a negligible value. The first ten eigenvalues for each of the three groups are listed in Table III.

As a result, it was decided to characterize the structure of individual differences in perceived relations among the twenty physics concepts in terms of five dimensions. The diagonal matrix Γ_r of order rxr where r = 5 was formed from the five largest eigenvalues. The five corresponding principal vectors of matrix P were used to construct the matrix W_r of order rxN where r = 5 and N = 67 which was rescaled to form matrix V by multiplying each element by $\sqrt{67}$. The V matrices of individual coefficients on principal vectors for Group A, Group B, and Group C appear as Tables LXXII, LXXIII, and LXXIV, respectively, in the Appendix.

TABLE III

FIRST TEN EIGENVALUES

GROUPS A, B, AND C

	GROUP A	B	C
	227026	215869	216005
	3644	3766	3507
	2854	2862	2507
	2574	2468	2389
	2517	2410	2095
	2285	2096	2004
	2098	2025	1834
	2015	1905	1738
	1815	1724	1621
	1738	1704	1589
	⋮	⋮	⋮
TOTAL	280443	269249	265531
Per cent trace accounted for by first five roots	85.0	84.2	91.4

Matrix A of individual coefficients on principal factors was produced by weighting each row of V by the corresponding value of Γ_r. The A matrices for the three groups are also in the Appendix as Table LXXV, Table LXXVI, and Table LXXVII. The entries in the first column of each of these three matrices of factor loadings were found to be large and relatively uniform. The first property was expected since the first eigenvalue in each group was so very much larger than all the remaining ones and the second since the first factor loadings reflect the average rating assigned by the subjects over the 190 stimulus pairs.

The matrix U_r of projections of stimulus pairs on the unrotated principal vectors was computed according to equation (7) on page 61 and rescaled to form the matrix Y by multiplying each element by $1/\sqrt{67}$. The portions of the three Y matrices containing the stimulus pair projections on the principal vectors corresponding to the second, third, fourth, and fifth roots appear as Table LXXVIII, Table LXXIX, and Table LXXX in the Appendix. The stimulus pair projections on the first principal vector for each of Group A, B, and C were tabulated separately and will be considered in more detail in the next section.

II. THE GROUP AVERAGE PERCEPTUAL SPACE

Since it is the cross products which are subject to analysis, information in the means of the dissimilarity ratings is retained.

The means are in essence recovered in the first principal vector of U_r. This being the case, the first latent root of matrix P is expected to be large relative to the subsequent roots.

Although not precisely proportional to the mean dissimilarity ratings, the coefficients in the first unrotated vector of U_r are highly correlated with them and may therefore be viewed as distance measures for the "average individual" in the group. It should be noted that the distances and the associated spatial configuration are determined only to within a positive multiplicative constant.

Consequently, the 190 scaled stimulus pair projections on the first unrotated principal vector in Matrix Y were interpreted as distance measures and the group average perceptual space derived by means of the Kruskal program. The first column vector in the Y matrix (of order $n(n-1)/2 \times r$ where $n = 20$ and $r = 5$) for each group was rearranged to form a 20 by 20 distance matrix. The three matrices of stimulus pair projections on the principal vector for Group A, B, and C are displayed as Table IV, V, and VI, respectively. The parameter values used in the Kruskal program in the present study were: number of data points--20; maximum number of dimensions--9; minimum value to stop iteration--0.05; metric--Euclidean.

The minimum stress which was obtained for the group average configuration in a number of dimensions is reported in Table VII. Figure 1 also shows the dependence of minimum stress on dimension

TABLE IV

STIMULUS-PAIR PROJECTIONS ON FIRST PRINCIPAL VECTOR, GROUP A*

Stimulus	2	3	4	5	6	7	8	9	10	11
1	.0102									
2	.0092	.0100								
3	.0085	.0094	.0066							
4	.0088	.0099	.0080	.0090						
5	.0093	.0094	.0078	.0075	.0053					
6	.0111	.0076	.0083	.0097	.0098	.0096				
7	.0097	.0096	.0075	.0084	.0080	.0090	.0097			
8	.0094	.0054	.0089	.0082	.0110	.0096	.0075	.0098		
9	.0088	.0102	.0097	.0100	.0061	.0063	.0099	.0086	.0109	
10	.0080	.0094	.0082	.0092	.0067	.0069	.0107	.0082	.0113	.0044
11	.0092	.0088	.0080	.0088	.0077	.0074	.0093	.0074	.0099	.0074
12	.0097	.0092	.0075	.0088	.0076	.0072	.0097	.0072	.0085	.0077
13	.0083	.0088	.0077	.0091	.0084	.0091	.0093	.0075	.0089	.0085
14	.0099	.0099	.0086	.0083	.0100	.0093	.0094	.0087	.0087	.0098
15	.0072	.0101	.0096	.0090	.0100	.0093	.0092	.0090	.0098	.0097
16	.0071	.0097	.0086	.0079	.0086	.0090	.0106	.0090	.0103	.0083
17	.0078	.0102	.0096	.0077	.0081	.0087	.0103	.0090	.0112	.0091
18	.0100	.0087	.0089	.0092	.0098	.0098	.0085	.0084	.0097	.0093
19	.0097	.0103	.0080	.0090	.0098	.0097	.0092	.0092	.0093	.0079

Stimulus	12	13	14	15	16	17	18	19	20
11	.0075								
12	.0080	.0049							
13	.0083	.0074	.0066						
14	.0090	.0084	.0086	.0077					
15	.0099	.0098	.0101	.0098	.0093				
16	.0082	.0096	.0088	.0092	.0079	.0071			
17	.0082	.0084	.0090	.0088	.0084	.0076	.0070		
18	.0102	.0093	.0089	.0091	.0096	.0096	.0078	.0091	
19	.0087	.0079	.0086	.0087	.0094	.0082	.0086	.0087	.0090

*Principal diagonals omitted.

TABLE V

STIMULUS-PAIR PROJECTIONS ON FIRST
PRINCIPAL VECTOR, GROUP B*

Stimulus	Stimulus 2	3	4	5	6	7	8	9	10	11
1	.0102									
2	.0092	.0098								
3	.0098	.0091	.0066							
4	.0091	.0107	.0081	.0098						
5	.0082	.0099	.0077	.0093	.0103					
6	.0105	.0085	.0091	.0093	.0081	.0083				
7	.0092	.0094	.0065	.0077	.0108	.0098	.0084			
8	.0089	.0060	.0095	.0089	.0066	.0069	.0110	.0091		
9	.0080	.0116	.0095	.0100	.0076	.0075	.0098	.0087	.0116	
10	.0081	.0108	.0086	.0093	.0070	.0079	.0092	.0088	.0103	.0050
11	.0085	.0093	.0084	.0086	.0081	.0078	.0092	.0080	.0101	.0065
12	.0082	.0083	.0081	.0088	.0077	.0087	.0095	.0079	.0090	.0075
13	.0085	.0095	.0091	.0098	.0098	.0096	.0075	.0091	.0098	.0086
14	.0087	.0090	.0090	.0100	.0098	.0089	.0092	.0088	.0083	.0092
15	.0077	.0092	.0087	.0091	.0079	.0083	.0093	.0083	.0099	.0100
16	.0069	.0107	.0099	.0096	.0091	.0087	.0096	.0090	.0105	.0080
17	.0080	.0099	.0083	.0098	.0089	.0090	.0090	.0082	.0105	.0093
18	.0070	.0090	.0086	.0092	.0094	.0090	.0103	.0095	.0103	.0081
19	.0092	.0094	.0088	.0055	.0100	.0088	.0088	.0091	.0100	.0084

Stimulus	Stimulus 12	13	14	15	16	17	18	19	20
11	.0075								
12	.0078	.0047							
13	.0076	.0060	.0066						
14	.0103	.0097	.0081	.0086					
15	.0105	.0097	.0088	.0092	.0096				
16	.0078	.0087	.0091	.0088	.0091	.0078			
17	.0097	.0077	.0087	.0091	.0095	.0073	.0072		
18	.0086	.0089	.0096	.0088	.0097	.0081	.0075	.0074	
19	.0094	.0077	.0084	.0087	.0094	.0089	.0082	.0080	.0084

*Principal diagonals omitted.

TABLE VI

STIMULUS-PAIR PROJECTIONS ON FIRST
PRINCIPAL VECTOR, GROUP C*

Stimulus	2	3	4	5	6	7	8	9	10	11
1	.0099									
2	.0093	.0093								
3	.0091	.0085	.0077							
4	.0084	.0107	.0081	.0091						
5	.0088	.0098	.0078	.0090	.0060					
6	.0108	.0085	.0085	.0095	.0102	.0106				
7	.0086	.0088	.0082	.0068	.0090	.0091	.0097			
8	.0102	.0060	.0101	.0090	.0106	.0102	.0087	.0090		
9	.0081	.0108	.0089	.0101	.0069	.0069	.0106	.0107	.0118	
10	.0083	.0099	.0082	.0094	.0068	.0074	.0109	.0095	.0114	.0049
11	.0091	.0096	.0082	.0088	.0080	.0075	.0101	.0081	.0099	.0073
12	.0095	.0091	.0080	.0086	.0080	.0078	.0095	.0077	.0086	.0082
13	.0091	.0099	.0082	.0083	.0085	.0090	.0097	.0077	.0103	.0082
14	.0101	.0092	.0086	.0103	.0103	.0095	.0080	.0091	.0084	.0102
15	.0080	.0086	.0084	.0089	.0093	.0087	.0088	.0077	.0086	.0110
16	.0061	.0099	.0090	.0065	.0080	.0084	.0094	.0075	.0096	.0082
17	.0090	.0100	.0095	.0087	.0082	.0087	.0109	.0073	.0100	.0091
18	.0086	.0099	.0092	.0088	.0095	.0091	.0094	.0085	.0094	.0086
19	.0087	.0096	.0087	.0091	.0090	.0083	.0093	.0081	.0097	.0076

Stimulus	12	13	14	15	16	17	18	19	20
11	.0075								
12	.0075	.0056							
13	.0086	.0068	.0075						
14	.0090	.0098	.0094	.0087					
15	.0102	.0090	.0093	.0099	.0091				
16	.0082	.0081	.0088	.0081	.0089	.0071			
17	.0090	.0085	.0083	.0082	.0101	.0074	.0079		
18	.0083	.0085	.0085	.0091	.0098	.0081	.0075	.0075	
19	.0082	.0085	.0085	.0085	.0096	.0094	.0078	.0078	.0085

*Principal diagonals omitted.

TABLE VII

MINIMUM STRESS FOR GROUP AVERAGE CONFIGURATIONS

Dimensions	Group A	Group B	Group C
9	.050	.049	.046
8	.044	.050	.049
7	.048	.059	.049
6	.053	.081	.054
5	.068	.097	.068
4	.097	.135	.095
3	.143	.198	.135
2	.210	.272	.185

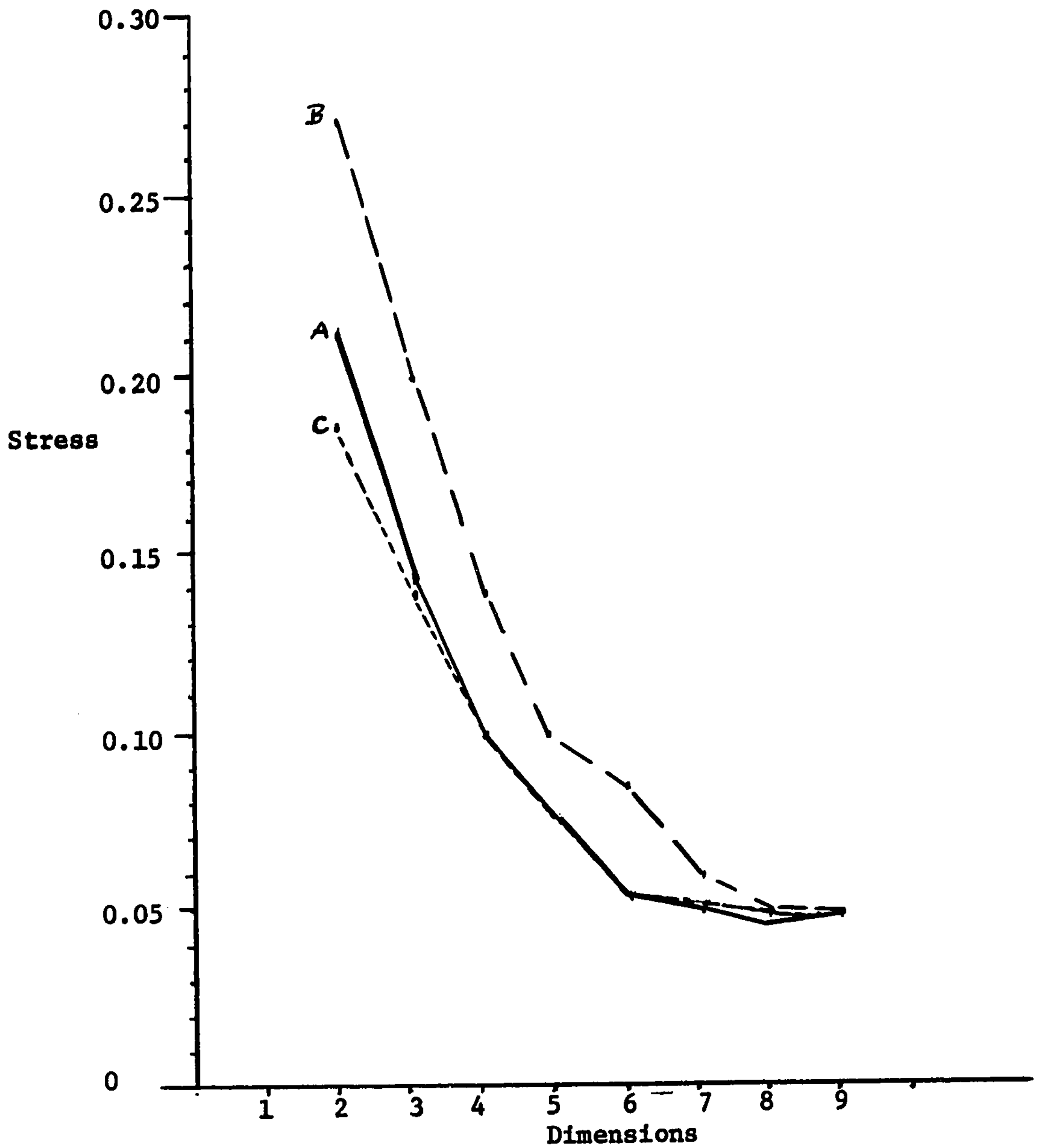

FIGURE 1

VARIATION OF MINIMUM STRESS WITH DIMENSIONALITY FOR GROUP AVERAGE CONFIGURATIONS for the three sets of data. The stress values appear generally well behaved in the sense of decreasing with increasing number of dimensions. The curves for Group A and Group C reflect the similarity of the minimum stress values obtained in various dimensions for the two groups. The discrepant nature of the minimum stress values for the best-fitting configurations obtained on the basis of Group B data in the same number of dimensions was immediately noted. The curve of minimum stress values for this group seems to be roughly parallel to the other two for configurations in two to seven dimensions.

Due to the absence of a distinct "elbow" in the plots, the figure does not clearly indicate the number of dimensions to be retained, but suggests that either the four or five dimensional representation might be appropriate. The stress for the best-fitting configuration in four dimensions was found to be 0.097 for Group A and 0.095 for Group C, both values regarded by Kruskal as "fair" in terms of departure from perfect fit. The minimum stress of 0.135 for the configuration of 20 points in 4 dimensions obtained for the Group B data is poor. As a matter of fact, Klahr has found (cf. Ch.III) that configurations based on random data for 16 points in four dimensions yielded an average minimum stress of 0.130. Although one might expect a slightly larger average minimum stress value for 20 points under the same conditions, the minimum stress value of 0.135 would not by itself provide much support for the existence of structure in the dissimilarity judgments which were rendered. In terms of Klahr's results, the stress values of the best-fitting configurations in four dimensions for the Group A and Group C data are considerably more convincing as evidence for structure.

The five-dimensional solutions reflect the same problem. The minimum stress value of 0.068 for both Group A and Group C is in the "fair to good" range in terms of Kruskal's evaluation of what constitutes an acceptable result. The discrepant Group B result in five dimensions again lies on the borderline in relation to Klahr's estimates of significance of minimum stress values. The minimum stress value of 0.097 obtained for the Group B data for 20 points in five dimensions is uncomfortably close to the average minimum stress of 0.096 reported by Klahr for solutions based on random data for 16 points in five dimensions.

As an aside, it might be mentioned that one-way analysis of variance for the three groups on nine variables, namely the Achievement Test, the four DAT subtests, and the four Skemp tests, failed to uncover any significant differences among the three groups at the <0.05 level. The analysis was undertaken in order to explore the possibility that the groups may in fact have differed in physics achievement or in terms of one or more of the aptitude measures which were administered to the subjects. Since the groups did not differ statistically on any of the nine variables, an outcome expected as a consequence of the random assignment of subjects to groups, the numerical results of the analysis of variance were not included in this report. Apparently the discrepant nature of the

Group B data insofar as the minimum stress is concerned requires explanation on other terms.

An acceptable stress value is not the only criterion for decisions with respect to the dimensionality of the spatial representation. The number of dimensions to be used in characterizing the structure of stimulus interrelationships may also depend upon the interpretability of the coordinates which are extracted. On this basis, both the four and five dimensional representations would seem to be appropriate. The tentative interpretation of at least three of the coordinates appears to be similar for both configurations. Although little additional structure was extracted from the five dimensional as opposed to the four dimensional solution, its retention might be warranted in terms of how well the configuration replicates across the three groups. The four-dimensional configuration based on the Group A data appears in Table VIII and the five dimensional representation in Table IX. The initial interpretation will be in terms of the Group A configurations, with subsequent extension to the results for the other two groups.

For both configurations, the first dimension seemed to be characterized by relatively high negative loadings for concepts dealing with motion and moderate positive loadings for concepts in which motion is generally not a primary focus of attention. The dimension appears to be somewhat better defined in the five-dimensional configuration.

The two most extreme negative loadings on the first coordinate were for stimuli 9 and 2, concepts dealing with centrifugal and

TABLE VIII

FINAL UNROTATED CONFIGURATION OF 20 POINTS IN FOUR DIMENSIONS

GROUP A AVERAGE

Points*	Dimension I	II	III	IV
1	.53	-.70	-.51	-.36
2	-.61	.38	-.96	.11
3	-.60	-.19	.49	-.11
4	-.50	-.81	.12	-.23
5	.12	.28	.30	-.83
6	.01	.40	.07	-.78
7	-.83	.40	-.71	.39
8	-.42	.26	.74	.11
9	-.92	-.05	-.71	.35
10	.52	.61	.11	-.52
11	.53	.42	.17	-.65
12	.08	.53	.46	-.19
13	-.20	.43	.49	-.16
14	-.19	.24	.64	.46
15	-.15	-.44	.47	.88
16	.48	-.83	-.60	.10
17	.58	-.68	-.19	.11
18	.46	-.88	.13	-.13
19	.25	.32	-.59	.89
20	.86	.33	.08	.54

*Mechanics concepts.

TABLE IX

FINAL UNROTATED CONFIGURATION OF 20 POINTS
IN FIVE DIMENSIONS

GROUP A AVERAGE

Points*	I	II	Dimension III	IV	V
1	.42	-.68	-.73	-.18	.00
2	-.62	.46	-.70	.05	-.56
3	-.51	-.26	.65	.04	-.01
4	-.40	-.82	.35	-.12	-.09
5	.04	.20	-.03	-.61	.67
6	-.08	.28	-.01	-.69	.49
7	-.57	.34	-.19	-.14	-.99
8	-.08	.17	.71	.44	.28
9	-.94	.03	-.45	.10	-.62
10	.43	.52	-.04	-.56	.36
11	.43	.39	-.17	-.45	.55
12	-.05	.51	.28	-.07	.49
13	-.30	.36	.35	.03	.42
14	-.29	.21	.11	.62	.40
15	-.25	-.26	-.30	.93	.34
16	.58	-.76	-.32	-.07	-.47
17	.64	-.55	-.30	.21	-.05
18	.68	-.69	.09	.07	.12
19	.46	.45	.01	.66	-.69
20	.40	.09	.71	-.25	-.64

*Mechanics concepts.

centripetal force and circular motion. Along this continuum, uniformly accelerated motion (stimulus 3) was regarded as less difficult than the above two closely related concepts, and projectile motion (stimulus 4) as less difficult than uniformly accelerated motion. Stimuli 16 to 19, which appear to form a cluster of moderately sized positive loadings, deal with topics in statics such as moments and composition and resolution of forces, with stimulus 17 (moments) seen as the most difficult. In the four dimensional configuration, the positive loadings for the friction-work-power concepts (stimuli 20, 10, and 11) became somewhat more prominent than the "statics" cluster but this was not seen as requiring a change in the interpretation accorded to this dimension since it still appeared plausible to suggest that one dimension used by the subjects to construe the relative difficulty of mechanics concepts involved the type of motion and the degree of explicitness of motion in the concepts. The high negative loading for Newton's Law of Gravity (stimulus 7) was not regarded as presenting a problem since this concept is used in conjunction with circular motion and centripetal and centrifugal force in numerous "satellite" problems in the course.

The difficulty continuum in terms of the types of motion seems to be reflected to some extent in the item difficulties of the questions on the physics <u>Achievement Test</u> which involve these topics. Although it is recognized that individual test items dealing with the same topic can vary considerably in difficulty, the two items involving circular motion and centrifugal and centripetal force, namely items 9 and 19 with item difficulties (fraction of correct responses) of 0.33 and 0.43, respectively, for an average of 0.38, seem to have been somewhat more poorly answered than the three items dealing with uniformly accelerated motion, namely questions 10, 24, and 25, with a mean item difficulty of 0.54. The fairly similar positive loadings of the four statics concepts (stimuli 16 to 19) appear reflected to some extent in the range of item difficulty for the six test questions involving these concepts: item 30--0.75, item 32--0.73, item 33--0.61, item 35--0.43, item 36--0.55, and item 39--0.81. The mean item difficulty as reflected in the fraction of correct responses for this cluster was found to be 0.66.

The second dimension in both configurations for the Group A data was found to be characterized by high negative loadings for stimuli 1, 4, 16, 17 and 18 and intermediate positive loadings for concepts 10, 11, 12. The five concepts with high negative loadings, namely velocity vectors, projectile motion, moments, and composition and resolution of forces involve vector quantities as the focus of attention while the positively loaded cluster of work, power, and potential energy does not reflect this focus. Consequently it was decided to characterize the second coordinate as a vector dimension. In addition to the item difficulties of the test questions dealing with statics presented above, question 8 on relative motion was found to have a difficulty of 0.894, yielding an average item difficulty of o.688 for the cluster. Five test questions involving the concepts in the work-power-potential energy cluster, namely questions 3,4,5,6, and 31 had a mean item difficulty of 0.69, perhaps suggesting that although students may include a vector dimension in the judgments of difficulty which are rendered, vectors as a concept may not present any more problems in relation to mechanics achievement than are encountered with a number of other concepts.

Although the remaining two or three dimensions were not as clearly defined as the first two, a number of fairly explicit clusters of concepts still appear to be present. A force-work-power cluster based on fairly high negative loadings for stimuli 5, 6, 10, and 11 appears on the fourth coordinate of the four dimensional configuration. What appears to be more or less the same cluster, namely force and work, is also reflected to some extent in the high negative loadings of stimuli 5, 6, and 10 on the fourth coordinate of the five dimensional representation.

The six test items which involve Newton's Second Law were found to have an average item difficulty of 0.52, with problems involving the absolute system of units reflecting a somewhat lower mean item difficulty in terms of the fraction of correct responses (0.60) than problems using the British engineering system of units (0.44). As expected, the loadings for the two stimuli involving Newton's Second Law are similar but not identical. The direction of the difference is not conclusively indicated on the basis of the Group A data alone.

From the standpoint of interpretability of the coordinates, the four dimensional representation may at this point seem preferable. Not only is the force-work-power cluster restricted to one dimension, but the third coordinate shows a relatively well-defined cluster of negative loadings along with a somewhat more diffuse group of medium-sized positive coefficients. The high negative loadings of concepts 2, 7, and 9, namely circular motion, Newton's Law of Gravity, and centrifugal and centripetal force are thought to reflect, as already mentioned, preoccupation with "satellite" type problems. The cluster emerges more explicitly in the third dimension than in the first, although the same possibility was presented as a rationalization for the clustering of Newton's Law of Gravity with concepts involving motion along the first coordinate. This particular group of concepts, reflected by high negative loadings for concepts 2, 7, and 9, also emerges on the fifth coordinate of the five dimensional representation.

Since it appears that similarly constituted clusters of concepts emerge in both the four dimensional and five dimensional representation, little further structure seems to be revealed as a consequence of including the additional dimension. Nevertheless, the extent to which the representation on the basis of Group A data is replicated in both the Group B and Group C data needs to be considered before either configuration is designated as preferable.

For the other groups of data, the four and five dimensional configurations were rotated orthogonally to maximum overlap with the corresponding Group A representation by means of the Kaiser[1] factor matching procedure. In this approach, the extent of the agreement between the representations for the rotated configuration and the reference configuration is presented as a matrix of cosines between the reference vectors for the two groups. The cosine of the angle between two vectors may be regarded as representing a correlation between the corresponding two variables in that it acts as a measure of the degree of relationship between them.

The unrotated five dimensional configuration for the Group B data and the unrotated four dimensional and five dimensional representations for the Group C data appear as Tables LXXXI, LXXXII, and LXXXIII in the Appendix.

Table X presents the four dimensional configuration for the Group B data which has been rotated to maximum overlap with the Group A four dimensional representation. The first coordinate, tentatively interpreted as a motion-statics continuum, appears well defined in the Group B representation. Again concepts involving motion are characterized by high negative loadings and the cluster of statics concepts by moderate to high positive loadings. The topic of circular motion appears again to be relegated to the extreme negative end of the dimension. The second coordinate, tentatively characterized as a vector dimension, appears to be somewhat more diffuse for the Group B data. Stimuli involving concepts dealing with vectors, e.g., 1, 3, 16, and 18 still exhibit negative, although

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[1]H. F. Kaiser, "Relating Factors between Studies Based upon Different Individuals," (Bureau of Educational Research, University of Illinois, 1960) mimeographed.

TABLE X

GROUP B ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH GROUP A

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.61	-.28	-.65	-.18
2	-.84	.12	-.79	.29
3	-.59	-.42	.58	-.29
4	-.65	-.30	.73	.41
5	-.09	-.21	-.05	-.92
6	-.44	.50	-.38	-.51
7	-.17	.75	-.27	.82
8	-.20	-.37	.94	-.05
9	-.82	-.02	-.62	-.46
10	.42	.43	.10	-.69
11	.41	.55	-.02	-.60
12	.22	.53	.52	-.13
13	.01	.47	.60	-.27
14	.22	.87	.10	.38
15	-.65	-.66	-.45	.53
16	.39	-.91	-.19	-.13
17	.92	-.19	-.20	.02
18	.64	-.50	.05	.46
19	.64	-.31	-.56	.42
20	-.03	-.08	.54	.86

*Mechanics concepts.

somewhat more variable, loadings. The work-power-energy cluster seems to be more pronounced than in the Group A representation, but this was not regarded as warranting a revision in the interpretation of the second dimension since the distinction along a vector/non-vector continuum appears to have been retained.

With respect to the third coordinate, high negative coefficients again appear for the two concepts involving circular motion, namely 2 and 9, although Newton's Law of Gravity is no longer regarded as exceptionally closely related to these two concepts. The force-work-power cluster on the fourth dimension again appears to be well defined.

The comparison matrix of cosines between the reference vectors for the Group A and Group B four dimensional configurations is presented in Table XI. If one regards a cosine of 0.800 or greater as indicative of a satisfactory degree of similarity between a given dimension as extracted from two sets of data based upon different individuals, the overall extent of relationship between the four dimensional configurations for the Group A data and Group B data seem to suggest that portions of the structure might be expected to replicate across samples drawn from the same population.

In assessing the degree of relationship between the dimensions extracted on the basis of Group A data and Group C data, the five dimensional representation appeared to yield a better match than was evident between the two four dimensional configurations. The five dimensional representation for the Group C data, rotated to maximum overlap with the Group A configuration, appears in Table XII.

TABLE XI

COSINES BETWEEN REFERENCE AXES: GROUP B
ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP
WITH GROUP A

(FOUR DIMENSIONS)

		Group A			
		I	II	III	IV
	I	.952	.239	-.119	-.152
GROUP	II	-.161	.892	-.033	.421
B	III	-.012	.226	.882	-.414
	IV	.261	-.310	.455	.793

TABLE XII

GROUP C ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH GROUP A

(FIVE DIMENSIONS)

Points*	Dimensions I	II	III	IV	V
1	.69	-.47	-.64	-.18	-.03
2	-.90	.03	-.03	-.20	-.85
3	-.58	-.25	-.18	.07	.58
4	-.38	-.79	.42	-.18	.02
5	.11	.04	-.15	-.57	.73
6	.11	.02	.66	-.45	.32
7	-.67	-.02	-.87	.59	-.39
8	-.24	-.29	-.73	-.07	.11
9	-.74	.23	.01	.13	-1.02
10	.46	.66	-.28	-.25	.56
11	.25	.72	-.24	-.23	.49
12	.02	.44	.51	-.30	.34
13	-.25	.43	.51	-.20	.12
14	-.06	.09	.65	.42	.54
15	-.55	.22	-.33	1.04	-.05
16	.05	-.67	-.26	-.05	-.65
17	.42	-.51	-.09	.13	.07
18	.71	-.29	.50	-.09	-.30
19	.66	.19	.02	-.21	-.66
20	.70	.27	.02	.51	.14

*Mechanics concepts.

The first coordinate characterized as a motion-statics continuum again appears to be fairly well defined. The concepts involving motion, namely stimuli 2, 3, 4, and 9, again exhibit moderately high negative loadings while the larger positive loadings seem to be associated with the statics concepts or with concepts in which the type of motion is generally not the primary focus of interest. As in the Group A data, Newton's Law of Gravity again appears as part of this cluster.

The second dimension was again interpreted as a vector continuum since the larger negative loadings were restricted to the concepts involving velocity vectors, projectile motion, and statics. As was apparent in the Group A representation, the positive loadings of comparable magnitude are confined to the work-power-energy cluster (stimuli 10, 11, 12, and 13).

The third coordinate of the five dimensional representation extracted on the basis of Group A data did not lend itself well to interpretation. According to the comparison matrix for the two five dimensional configurations presented in Table XIII, the fourth dimension of the Group C configuration resembles the third coordinate of the Group A representation more closely than does the third dimension of the Group C data. However, inspection of the two configurations shows that it is the fourth dimension in both cases which contains the high negative loadings for the two stimuli relating to Newton's Second Law and, relatively speaking, a very high positive loading for stimulus 15 (relativistic mass). The fifth coordinate again reflects, although somewhat weakly, the force-work-

TABLE XIII

COSINES BETWEEN REFERENCE AXES:
GROUP C ROTATED FOR MAXIMUM FACTOR VECTOR
OVERLAP WITH GROUP A

(FIVE DIMENSIONS)

		Group A I	II	III	IV	V
Group C	I	.871	.240	-.057	-.421	-.051
	II	.041	.806	.056	.502	.306
	III	-.419	.372	.354	-.722	.193
	IV	.182	-.115	.917	.220	-.252
	V	.176	-.374	.159	.022	.897

power cluster on the positive side and circular motion (stimuli 2 and 9), by means of somewhat larger loadings, on the negative side.

On the basis of the results to this point, some evidence of relationship between the Group B and Group C configurations might be expected. The four dimensional Group C representation was rotated to maximum overlap with the unrotated four dimensional Group B configuration. The unrotated Group B configuration in four dimensions appears in Table XIV, the matrix of cosines between the two sets of reference vectors in Table XV, and the rotated four dimensional Group C configuration in Table XVI. All four dimensions were found to match reasonably well. The interpretation to this point appeared satisfactory for the first, second, and fourth coordinate. The third dimension, although not readily interpretable, appeared in both instances to involve a cluster of stimuli with positive loadings which include the concepts of kinetic and potential energy.

To summarize briefly, the group average configuration of the dissimilarity estimates with respect to difficulty appears to be characterized by two fairly well-defined dimensions, one relating to motion and the other to the vector nature of some of the concepts. These two dimensions appear in all the four and five dimensional configurations, unrotated and rotated, for all three sets of data. The additional two or three dimensions which were extracted are somewhat less distinct. Essentially the same clusters of concepts, such as the force-work-power cluster which appears in each case on

TABLE XIV

FINAL UNROTATED CONFIGURATION OF 20 POINTS
IN FOUR DIMENSIONS

GROUP B AVERAGE

Points*	Dimension I	II	III	IV
1	.62	-.40	-.57	-.19
2	-.72	.39	-.78	-.39
3	-.69	-.42	.54	.01
4	-.84	-.01	.41	.58
5	.01	-.56	.30	-.71
6	-.17	.32	-.01	-.84
7	-.08	1.05	-.41	.25
8	-.39	-.35	.77	.45
9	-.64	-.06	-.35	-.85
10	.59	.03	.47	-.52
11	.61	.18	.35	-.55
12	.30	.37	.63	.03
13	.10	.29	.74	-.08
14	.34	.90	.13	.14
15	-.80	-.25	-.76	.25
16	.19	-.92	-.32	.20
17	.85	-.30	-.24	.23
18	.42	-.36	-.27	.71
19	.54	-.19	-.75	.35
20	-.24	.28	.11	.94

*Mechanics concepts.

TABLE XV

COSINES BETWEEN REFERENCE AXES: GROUP
C ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP
WITH GROUP B

(FOUR DIMENSIONS)

		Group B			
		I	II	III	IV
	I	.883	-.346	.310	.062
Group	II	.287	.932	.222	.014
C	III	-.307	-.101	.847	-.422
	IV	-.208	-.038	.371	.904

TABLE XVI

GROUP C ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH GROUP B

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.68	-.68	-.39	-.24
2	-1.25	-.11	-.39	-.08
3	-.28	.22	.04	-.71
4	-.59	-.66	.27	.01
5	.42	-.23	.51	-.63
6	.26	-.08	.52	-.63
7	-.49	.44	-1.12	-.33
8	-.37	-.29	.21	.60
9	-1.21	.00	-.59	.41
10	.91	.32	.40	-.41
11	.71	.39	.40	-.43
12	.19	.17	.76	.01
13	-.22	.28	.61	.00
14	-.02	.58	.58	.41
15	-.43	1.00	-.64	-.02
16	-.16	-.65	-.63	.24
17	.33	-.39	-.25	.07
18	.36	-.48	.22	.71
19	.50	-.26	-.31	.70
20	.65	.44	-.19	.34

*Mechanics concepts.

the last dimension extracted tend to recur, although not necessarily on the same dimension, for each group.

III. THE VIEWPOINT REPRESENTATIONS

The matrix A for each group could at this point be rotated to simple structure, producing the matrix B of individual coefficients on the five rotated axes. The matrix Z of stimulus pair projections on the rotated axes could then be formed by applying the inverse transformation to matrix Y and the set of distance measures in each column of Z analyzed by means of the Kruskal program to obtain the separate viewpoint representations.

The above approach would correspond to that outlined by Tucker and Messick. However, since the coefficients for all individuals on the first factor in matrix A are very large in relation the remaining coefficients, the result in the present case was that rotation of matrix A to the varimax criterion[2] obscured the signs of the individual coefficients on the other four viewpoint dimensions. Problems in interpretation were anticipated since all individuals now had positive coefficients on all rotated dimensions. Consequently it was decided to rotate viewpoint dimensions II to V for each group and to use the individual coefficients on the four rotated axes only as a basis for clustering the subjects. The perceptual space for a given cluster would be derived by application of the Kruskal program to the average

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[2]H.F. Kaiser, "The Varimax Criterion for Analytic Rotation in Factor Analysis," Psychometrika, 23: 187-200, 1958.

of the original dissimilarity ratings for the individuals in the cluster.

The corresponding portion of matrix A was rotated to the varimax criterion. The individual coefficients on the varimax rotated factors for Group A, B, and C appear as Table LXXXIV, LXXXV, and LXXXVI, respectively, in the Appendix. The three transformation matrices are presented in the Appendix in Table LXXXVII.

The coefficients for each individual on the four rotated dimensions were normalized to facilitate comparison and two clusters of individuals, one with high positive coefficients and one with high negative coefficients, were formed for each dimension for each of the three groups of data. If only individuals with normalized coefficients equal to or exceeding 0.80 in absolute value (accounting for 64 per cent or more of the sum of squares of the four coefficients) were considered, 99 of the 180 subjects in the three groups could be clustered on either the positive or negative side of one of the four viewpoint dimensions. If the criterion for inclusion in one of the two clusters for each dimension were reduced to a normalized coefficient equal to or exceeding 0.707 in absolute value (accounting for 50 percent or more of the sum of squares) on that particular dimension, 149 of the 180 subjects could be classified. Even though only 55 percent of the subjects could be clustered on this basis, it was decided to use the 0.80 value as a criterion since it was considered desirable, within practical limits, to restrict the composition of the clusters to individuals with a relatively large coefficient on the given viewpoint dimension. The number of subjects included in each cluster is presented in Table XVII.

TABLE XVII

NUMBER OF SUBJECTS CLUSTERED ON THE VIEWPOINT DIMENSIONS FOR GROUP A, B, AND C*

	Dimension II	III	IV	V	TOTAL
GROUP A	5+ 7-	8+ 1-	5+ 3-	6+ 0-	35
GROUP B	9+ 2-	5+ 5-	2+ 2-	3+ 6-	34
GROUP C	4+ 3-	3+ 4-	6+ 3-	0+ 7-	30
TOTAL	30	26	21	22	99

*Normalized individual coefficients $\geq \pm .080$.

The stress values for the best fitting configurations in a number of dimensions for the two clusters of subjects from each of Group A, B, and C on viewpoint dimension II are reported in Table XVIII. The viewpoint dimensions extracted from the three sets of data are labeled II, III, IV, and V in order to conform to the designations used in the complete V and A matrices which appear as Table LXXII to Table LXXVII in the Appendix. The first dimension in the V and A matrices, designated I, concerns the group average perceptual space considered in the previous section.

In the discussion which follows, the various subgroups will be designated by first, the capital letter A, B, or C, denoting the group of subjects from which the subgroup was drawn, followed by a Roman numeral from II to V denoting the viewpoint dimension on which the subgroup was clustered, and terminating with either (+) or (-) to indicated whether the individual coefficients of the members of the subgroup on that dimension are positive or negative.

The arrangement of the six subgroups into two sets of three in Table XVIII rests on results which will be presented shortly and is initiated here in order to preserve consistency with subsequent groupings. Figure 2 and Figure 3 show the variation of minimum stress with dimension for the viewpoint dimension II configurations. As was the case with the group average configurations, a clear indication of the number of dimensions to be used in the representations is lacking. However, the curves might be interpreted as suggesting that either six, five, or four dimensions could be appropriate. In order

TABLE XVIII

MINIMUM STRESS FOR VIEWPOINT
DIMENSION II CONFIGURATIONS

Dimensions	AII(+)	BII(-)	CII(+)
9	.050	.049	.049
8	.047	.048	.050
7	.049	.053	.063
6	.058	.070	.079
5	.072	.079	.103
4	.095	.109	.142
3	.129	.146	.190
2	.172	.216	.283
	AII(-)	BII(+)	CII(-)
9	.049	.048	.049
8	.048	.047	.049
7	.050	.050	.050
6	.052	.064	.059
5	.069	.085	.074
4	.090	.106	.094
3	.127	.146	.122
2	.164	.205	.176

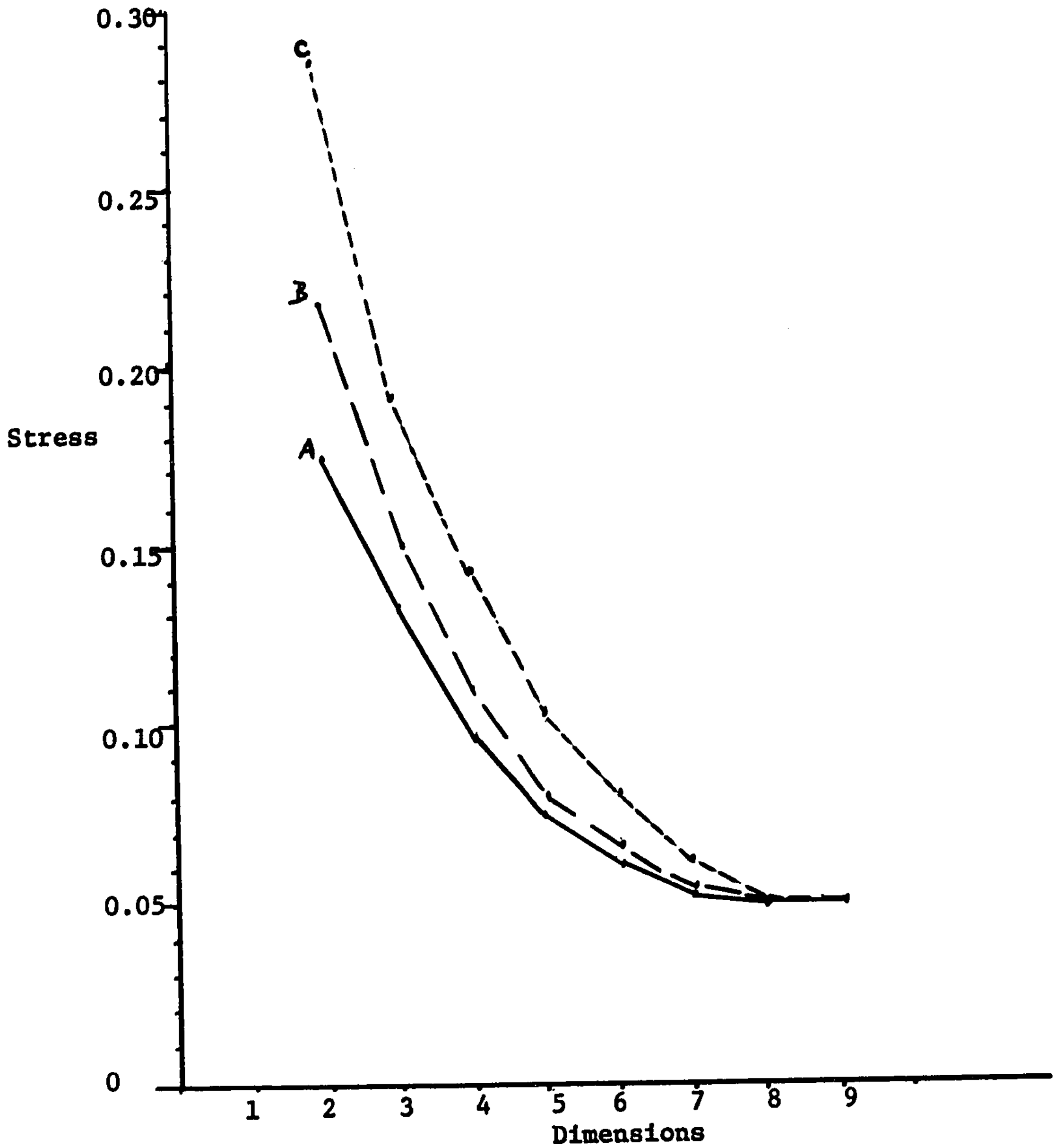

FIGURE 2

VARIATION OF MINIMUM STRESS WITH DIMENSIONALITY FOR SUBGROUPS AII(+), BII(-), AND CII(+)

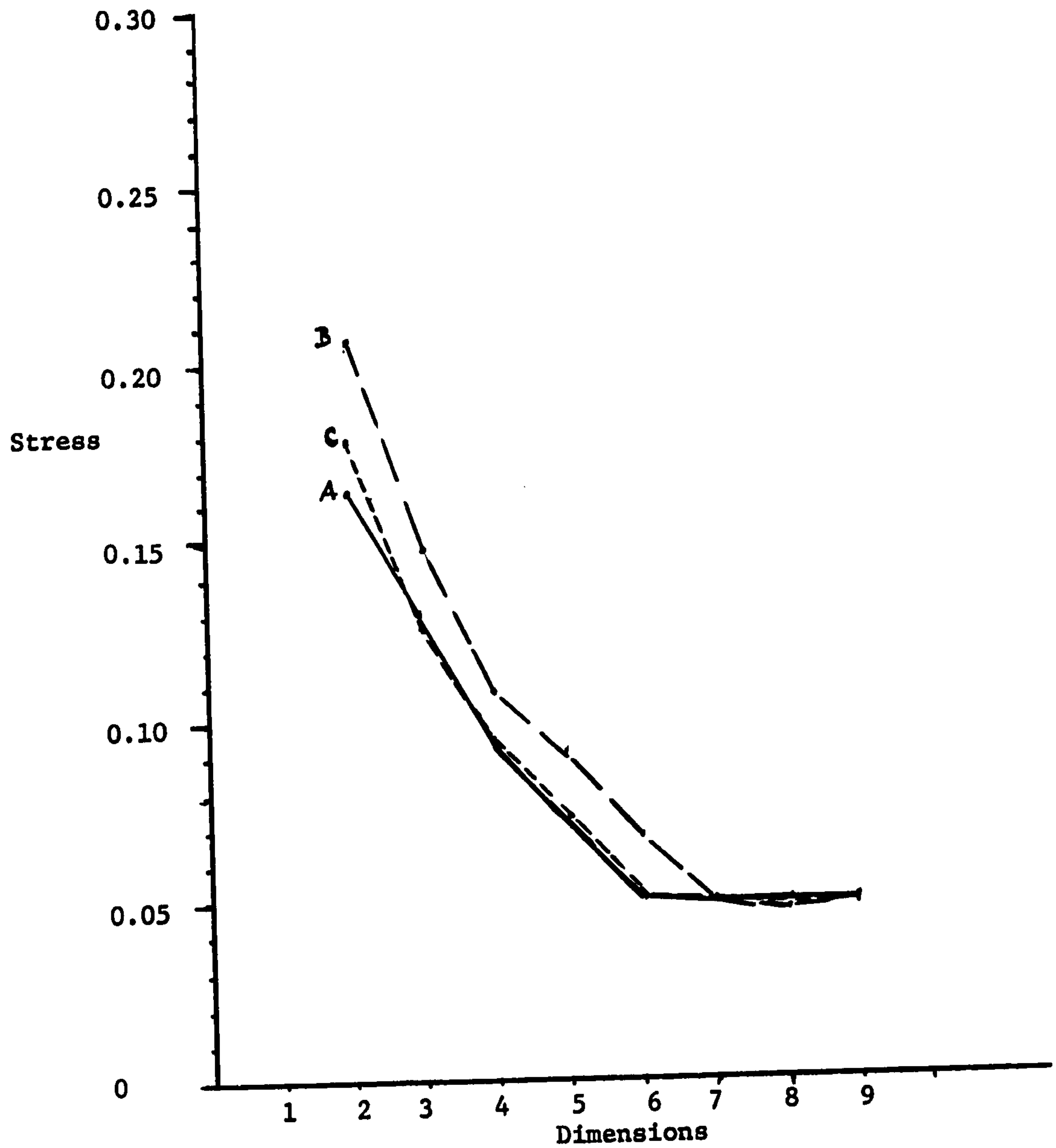

FIGURE 3

VARIATION OF MINIMUM STRESS WITH DIMENSIONALITY FOR SUBGROUPS AII(-), BII(+), AND CII(-)

to obtain further indication of the number of dimensions to be retained the best-fitting configurations for each subgroup in four, five, and six dimensions were matched for all possible combinations of subgroups in a given number of dimensions by the Kaiser procedure. The comparison matrices of cosines between reference axes were examined in order to ascertain the number of dimensions in which agreement among the configurations for the three groups, A, B, and C, appeared to be most extensive. As a result, it was decided to characterize viewpoint dimension II in terms of six coordinates. The average minimum stress of 0.064 for the six dimensional representations lies in the "fair to good" range in terms of Kruskal's evaluation of stress values. As before, the Group A structures of stimulus interrelationships will serve as the reference configurations.

The unrotated six dimensional configurations for subgroups AII(+) and AII(-) are presented as Tables XIX and XX, respectively. The structure of stimulus interrelationships for subjects with high negative coefficients on viewpoint dimension II appears to resemble the group average configurations discussed earlier somewhat more closely than does the configuration for the subgroup of subjects with high positive loadings. The statics-motion dimension emerges quite clearly on the first coordinate, perhaps encompassing a sufficient number of stimuli to warrant being characterized as a general statics-dynamics continuum. The dimension tentatively interpreted on a vector/non-vector basis for the group average configurations is reflected, although somewhat weakly, by the third coordinate of the AII(-) representation. A work-power-energy cluster involving

TABLE XIX

FINAL UNROTATED CONFIGURATION FOR 20 POINTS IN SIX DIMENSIONS FOR HIGH POSITIVE COEFFICIENTS ON VIEWPOINT DIMENSION II: GROUP A

	Dimension					
Points*	I	II	III	IV	V	VI
1	.75	-.08	-.36	.36	-.36	-.36
2	-1.05	.02	-.95	.07	-.09	.36
3	-.04	-.25	.85	-.32	.08	-.10
4	.66	.07	.30	.33	.03	.01
5	.50	-.06	.34	-.52	.41	-.22
6	-.07	-.78	.29	.33	.12	.20
7	-1.31	-.03	-.17	-.39	.33	-.34
8	.43	-.07	.43	.01	.56	.19
9	-1.27	-.20	-.55	-.38	-.18	-.05
10	.75	.43	-.14	-.17	.30	-.20
11	.52	.61	.10	.05	-.42	-.50
12	-.23	-.08	.29	.58	-.33	-.24
13	-.51	-.03	-.01	.53	.61	.09
14	.09	.19	-.32	.59	.16	.22
15	-.28	.53	.12	.05	-.26	.78
16	.20	-.38	-.15	-.33	-.27	.45
17	.32	.31	-.10	-.54	-.10	.48
18	.31	-.38	-.17	-.02	-.41	-.47
19	.36	-.40	-.18	-.02	.59	-.24
20	-.13	.57	.37	-.20	.36	-.07

*Mechanics concepts.

TABLE XX

FINAL UNROTATED CONFIGURATION FOR 20 POINTS IN SIX DIMENSIONS FOR HIGH NEGATIVE COEFFICIENTS ON VIEWPOINT DIMENSION II: GROUP A

	Dimension					
Points*	I	II	III	IV	V	VI
1	.35	-.54	-.39	-.26	.40	-.38
2	-.29	.26	-.29	-.46	-.24	-.57
3	-.44	-.49	-.19	-.20	-.66	-.26
4	-.46	-.12	-.33	.79	-.63	-.17
5	-.76	-.05	-.02	.14	.20	-.01
6	-.62	-.26	.01	-.59	.09	.10
7	-.44	.35	.58	.25	-.43	-.40
8	-.29	-.47	.38	-.20	-.17	.36
9	-.10	-.50	.35	.30	.08	-.33
10	-.13	-.31	.51	-.54	.25	-.27
11	-.40	-.20	.06	-.38	.42	-.36
12	-.20	.24	.44	-.19	.59	-.03
13	-.23	-.11	-.08	.36	.56	-.40
14	-.16	-.21	-.09	.26	.83	.32
15	.50	.18	.04	.15	.06	.97
16	.86	.25	.16	.08	-.60	.59
17	.95	.36	-.55	-.07	-.24	-.07
18	.89	-.09	-.29	.47	.04	.27
19	.42	.71	-.62	-.10	-.24	.66
20	.57	.98	.33	.11	-.32	.07

*Mechanics concepts.

stimuli 10 to 14 seems apparent on the fifth coordinate. The interpretation to the attributed to the remaining three dimensions is not clear.

In the configuration based on the AII(+) data, neither the statics-motion nor vector dimensions emerge clearly. The first coordinate is characterized by very high negative loadings for stimuli 2, 7, and 9, reflecting the circular motion cluster which appeared in the group average. The same cluster emerges somewhat more weakly on the third coordinate. The second dimension was tentatively characterized as a force-work-power continuum since one stimulus involving Newton's Second Law appears at the negative end and friction, work, and power at the positive end. A cluster of stimuli with fairly uniform positive loadings which relates to energy concepts is present on the fourth dimension.

The AII(-) dimensions appear somewhat more inclusive than the AII(+) dimensions in terms of the number of concepts clustered on various coordinates which may be regarded as related in the sense of reflecting aspects of the logical structure of mechanics. The discernible clusters on the AII(+) coordinates tend to encompass a smaller number of such concepts per cluster or dimension. On the basis of the two Group A interpretations, viewpoint dimension II might very tentatively be characterized as a factor involving the inclusiveness of the range of concepts perceived to be related.

The degree of relationship of the above two configurations with the representations based on the Group B and Group C data is again reflected in the comparison matrices of cosines. Table XXI presents the cosines between the reference axes for the subgroup BII(-) representation orthogonally rotated for maximum overlap with the subgroup AII(+) configuration. All six coordinates match fairly well. The rotated BII(-) configuration appears in Table XXII.

The situation now becomes constrained in that the AII(-) and BII(+) configurations need to match to a reasonable extent before viewpoint dimension II based on Group B data may be regarded as similar to viewpoint dimension II for the Group A data. Table XXIII of cosines between the reference axes when the BII(+) configuration is rotated for maximum overlap with the AII(-) configuration indicates that three of the six dimensions match to a fair extent. The rotated BII(+) representation appears in Table XXIV.

With reference to the Group C data, it was found that the CII(+) configuration matched satisfactorily with the AII(+) representation on only two of the six dimensions. The comparison matrix of cosines is presented in Table XXV and the rotated CII(+) configuration in Table XXVI. The extent of agreement between the CII(-) and AII(-) representations is also poor, with two of the six dimensions reflecting any sort of similarity. The matrix of cosines between reference axes is displayed in Table XXVII and the rotated CII(-) configuration in Table XXVIII.

The extent of the relationship between the Group B and Group C data is reflected in Table XXIX and Table XXX which present, respectively, the cosines between reference axes for the subgroup CII(+) configuration

TABLE XXI

COSINES BETWEEN REFERENCES AXES: SUBGROUP BII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AII(+)

(SIX DIMENSIONS)

		AII(+)					
		I	II	III	IV	V	VI
BII(-)	I	.979	-.113	.061	.106	-.112	-.037
	II	.095	.944	.286	-.105	-.076	.023
	III	-.045	-.204	.836	.043	.402	-.307
	IV	-.112	.052	.140	.928	-.162	.279
	V	.131	.169	-.320	.155	.891	.184
	VI	.030	-.151	.306	.301	.002	.890

TABLE XXII

SUBGROUP BII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AII(+)

(SIX DIMENSIONS)

Points*	Dimension I	II	III	IV	V	VI
1	.96	.22	-.48	-.25	.12	-.14
2	-.61	.01	-.69	.32	-.38	-.12
3	-.08	-.29	.94	-.20	.22	-.03
4	-.09	.21	.76	.46	-.13	-.03
5	.35	-.21	-.31	-.35	.95	-.16
6	.35	-.59	.00	-.34	-.56	.23
7	-.80	.67	-.13	-.26	.10	-.30
8	-.60	.10	.04	.45	-.31	-.31
9	-.36	-.71	.04	-.46	-.18	.48
10	.77	.07	-.29	-.11	-.59	-.25
11	.52	.74	.20	-.34	.09	.02
12	.27	-.36	.27	.78	.25	-.18
13	-.09	.15	-.04	.82	.24	.20
14	-.36	.16	-.21	.48	.40	.52
15	-.71	.03	-.15	-.37	.32	.28
16	.03	-.29	-.22	-.16	-.01	.85
17	.75	.66	.04	-.24	-.09	.00
18	-.68	-.38	-.42	.07	.01	-.44
19	.15	-.65	.30	-.03	.19	-.57
20	.24	.44	.35	-.28	-.64	-.05

*Mechanics concepts.

TABLE XXIII

COSINES BETWEEN REFERENCE AXES: SUBGROUP BII(+)
ROTATED FOR MAXIMUM FACTOR VECTOR
OVERLAP WITH SUBGROUP AII(-)

(SIX DIMENSIONS)

		AII(-)					
		I	II	III	IV	V	VI
BII(+)	I	.692	.147	.087	-.340	.414	.452
	II	-.179	.933	.279	.057	.072	-.105
	III	-.125	-.316	.902	-.140	.177	-.146
	IV	-.285	-.044	.148	.521	.028	.789
	V	-.599	.002	-.175	-.714	.142	.283
	VI	-.181	-.081	-.222	.283	.878	-.246

TABLE XXIV

SUBGROUP BII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AII(-)

(SIX DIMENSIONS)

	Dimension					
Points*	I	II	III	IV	V	VI
1	.79	-.35	.40	-.12	.16	.09
2	-.51	.67	-.21	.00	.09	-.44
3	-.30	-.43	.40	.01	-.33	-.37
4	-.47	-.10	.29	.45	-.47	-.10
5	-.81	.12	-.33	-.31	-.38	.14
6	-.32	-.66	-.61	.08	.17	-.27
7	.41	.26	.33	-.21	-.20	-.67
8	-.28	-.60	.21	.01	-.36	.73
9	-.21	-.20	-.49	.65	.40	-.42
10	-.29	-.19	.19	-.68	.14	-.14
11	-.40	-.08	.35	-.72	.36	-.08
12	-.27	-.13	.41	.31	.53	.04
13	-.41	.09	.37	.37	.41	.14
14	-.24	-.08	-.12	-.14	.63	.33
15	-.01	.20	.32	-.03	.87	-.23
16	.35	.62	.06	.18	-.55	.81
17	.73	.50	-.89	-.06	-.30	.14
18	.88	.00	-.50	-.22	-.60	-.11
19	.83	.11	-.43	-.10	.19	.54
20	.51	.25	.25	.54	-.76	-.11

*Mechanics concepts.

TABLE XXV

COSINES BETWEEN REFERENCE AXES: SUBGROUP CII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AII(+)

(SIX DIMENSIONS)

		AII(+)					
		I	II	III	IV	V	VI
CII(+)	I	.302	.100	-.010	.071	.917	-.231
	II	-.284	.739	-.166	.230	.126	.527
	III	-.556	.189	.729	-.056	.090	-.333
	IV	.212	-.007	.174	.926	-.184	-.182
	V	.551	.624	.119	-.274	-.314	-.345
	VI	.412	-.139	.629	-.077	.053	.637

TABLE XXVI

SUBGROUP CII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AII(+)

(SIX DIMENSIONS)

Points*	I	II	Dimension III	IV	V	VI
1	.63	-.61	-.51	.14	-.07	-.24
2	-.51	-.05	-.72	.42	-.12	.41
3	-.28	.06	.91	.00	-.12	-.07
4	.59	-.12	.26	.50	-.26	-.39
5	.47	-.04	-.30	-.99	-.10	-.03
6	.26	-.32	-.29	-.50	.37	.73
7	-.58	-.46	.42	-.46	.31	-.44
8	.02	-.28	.05	-.30	-.68	.28
9	-.68	.00	-.43	-.48	-.14	-.45
10	.34	.42	-.24	-.30	.83	-.01
11	.52	.78	.32	-.19	.14	.05
12	-.83	-.02	.05	.26	-.07	-.03
13	-.38	-.02	-.37	.61	.31	-.41
14	.40	.68	-.26	-.38	-.34	.27
15	.09	.18	.61	.18	.15	.76
16	.11	-.57	.81	.25	.13	.00
17	.18	.49	-.09	.06	.03	-.65
18	-.09	-.26	-.29	.27	-.82	-.44
19	.06	-.42	.00	.43	.65	.19
20	-.34	.57	.07	-.29	-.15	.52

*Mechanics concepts.

TABLE XXVII

COSINES BETWEEN REFERENCE AXES: SUBGROUP CII(-)
ROTATED FOR MAXIMUM FACTOR VECTOR
OVERLAP WITH SUBGROUP AII(-)

(SIX DIMENSIONS)

		AII(-)					
		I	II	III	IV	V	VI
CII(-)	I	.783	-.068	-.412	.092	.432	-.133
	II	.377	.262	.852	-.157	.196	-.026
	III	-.353	.373	-.138	-.324	.434	-.651
	IV	-.218	-.811	.256	.094	.443	-.152
	V	-.147	.067	-.132	-.488	.472	.704
	VI	-.228	.354	.048	.784	.408	.198

TABLE XXVIII

SUBGROUP CII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AII(-)

(SIX DIMENSIONS)

Point*	Dimension I	II	III	IV	V	VI
1	.88	-.05	-.39	.05	.28	.02
2	-.54	.02	.19	.60	-.79	-.40
3	-.44	.36	-.13	-.49	-.18	-.55
4	-.51	-.50	-.03	.72	-.43	.16
5	-.14	.10	-.56	-.56	.23	.44
6	-.71	-.37	.05	-.27	.39	.11
7	-.38	.38	.20	.06	-.98	.06
8	.08	-.49	.24	.37	-.79	.49
9	.14	-.15	.28	1.06	-.38	-.35
10	.18	-.36	.15	-.51	.45	-.08
11	.48	.17	.46	-.52	-.12	-.41
12	-.04	.26	-.38	.02	.69	.00
13	-.24	-.22	.20	-.16	.63	-.55
14	.08	.47	-.22	-.03	.56	.43
15	-.05	.17	.82	.92	-.34	.20
16	.23	-.14	.06	-.60	-.24	.28
17	.17	-.31	-.20	.05	.61	.57
18	.24	-.29	-.57	-.57	.07	-.23
19	.42	.50	-.45	.20	-.27	-.20
20	.15	.45	.31	-.31	.60	.01

*Mechanics concepts.

TABLE XXIX

COSINES BETWEEN REFERENCE AXES: SUBGROUP CII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR SUBGROUP BII(-)

(SIX DIMENSIONS)

		BII(-)					
		I	II	III	IV	V	VI
	I	.953	.077	.224	-.060	-.177	.027
	II	.008	.395	-.517	.462	-.560	.224
CII(+)	III	-.291	.359	.773	.022	-.432	.038
	IV	.075	.016	.266	.820	.488	.111
	V	.029	.838	-.108	-.258	.467	-.031
	VI	-.024	-.083	.049	-.208	.110	.967

TABLE XXX

COSINES BETWEEN REFERENCE AXES: SUBGROUP CII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR SUBGROUP BII(+)

(SIX DIMENSIONS)

		BII(+)					
		I	II	III	IV	V	VI
CII(-)	I	.741	-.582	-.153	-.030	-.285	.080
	II	.481	.272	.722	.227	.321	.139
	III	-.045	-.233	.106	-.796	.465	.288
	IV	-.406	-.670	.232	.468	.211	.262
	V	.222	.001	-.509	.259	.745	.265
	VI	.061	.290	-.362	.169	-.016	.868

rotated for maximum overlap with the unrotated BII(-) representation and the CII(-) configuration rotated for maximum overlap with the unrotated BII(+) representation. The CII(+) and BII(-) representations only match on one dimension.

On the basis of the above measures of relationship among the three sets of data, it appears obvious that characterization of viewpoint dimension II in terms of an "inclusiveness" factor must remain highly speculative.

The stress values for the best fitting configurations in a number of dimensions for the six subgroups on viewpoint dimension III are reported in Table XXXI. Figure 4 and Figure 5 also show the dependence of minimum stress of dimension for the viewpoint dimension III configurations. Again the Kaiser procedure was used to match the four, five, and six dimensional configurations for all possible combinations of subgroups in a given number of dimensions and the comparison matrices of cosines between the reference axes examined to ascertain the dimensionality in which agreement among the configurations for Group A, B, and C data appeared to be most extensive. As a consequence, viewpoint dimension III will be characterized in terms of four coordinates. The average stress of 0.118 for the best-fitting four dimensional representations is rather poor both in terms of Kruskal's evaluation and Klahr's estimates of significance.

The unrotated four dimensional configurations for subgroups AIII(+) and AIII(-) are set out in Tables XXXII and XXXIII, respectively. The structure of stimulus interrelationships for the subgroup characterized by high positive coefficients on viewpoint dimension III appears

TABLE XXXI

MINIMUM STRESS FOR VIEWPOINT DIMENSION III CONFIGURATIONS

Dimension	AIII(+)	BIII(+)	CIII(+)
9	.050	.049	.049
8	.049	.055	.050
7	.050	.065	.051
6	.060	.082	.066
5	.080	.100	.084
4	.111	.126	.122
3	.153	.167	.169
2	.230	.248	.256
	AIII(-)	BIII(-)	CIII(-)
9	.049	.050	.049
8	.049	.049	.050
7	.050	.055	.050
6	.058	.071	.064
5	.082	.090	.086
4	.116	.116	.115
3	.171	.171	.166
2	.260	.235	.240

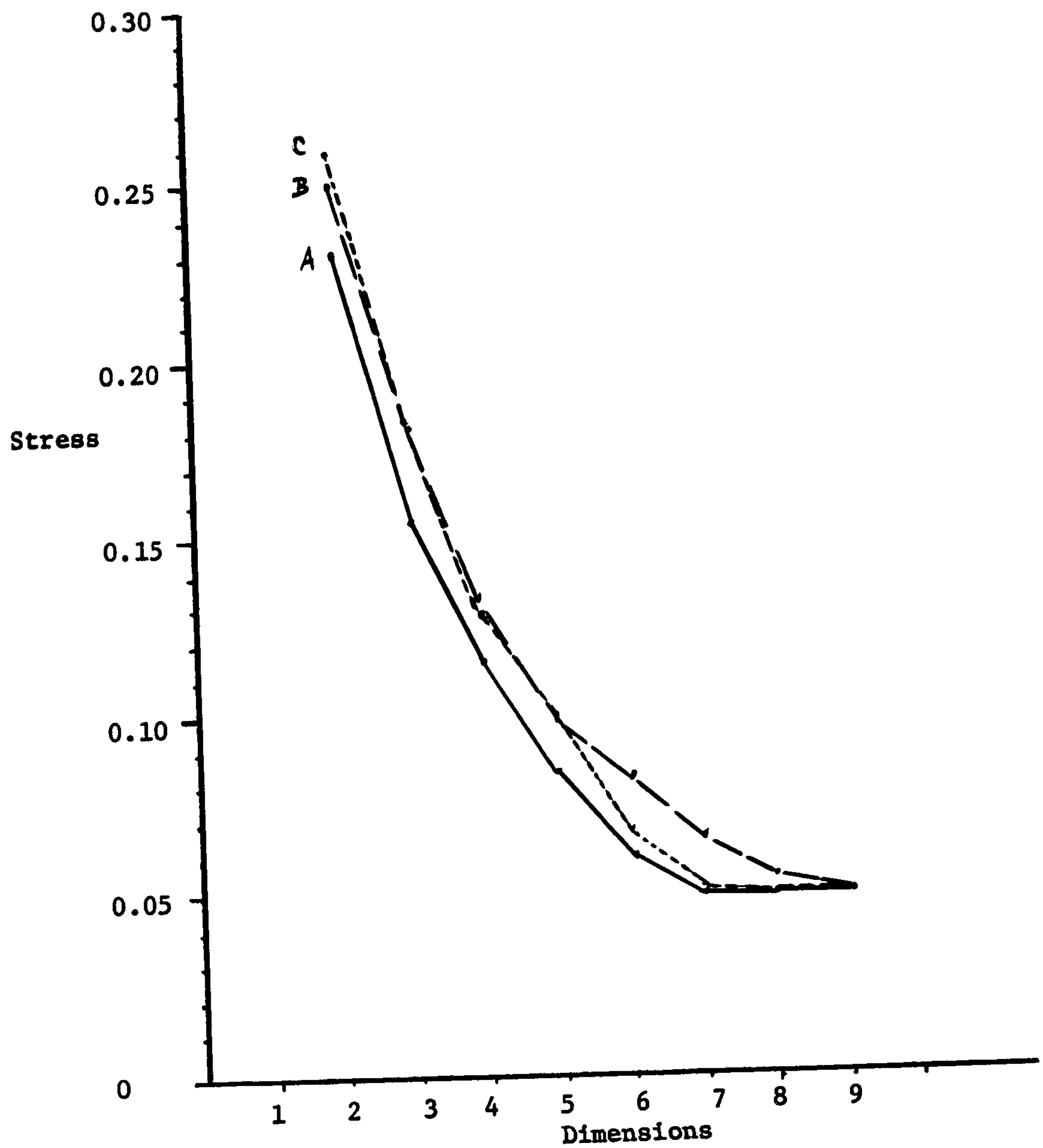

FIGURE 4

VARIATION OF MINIMUM STRESS WITH DIMENSIONALITY FOR SUBGROUPS AIII(+), BIII(+), AND CIII(+)

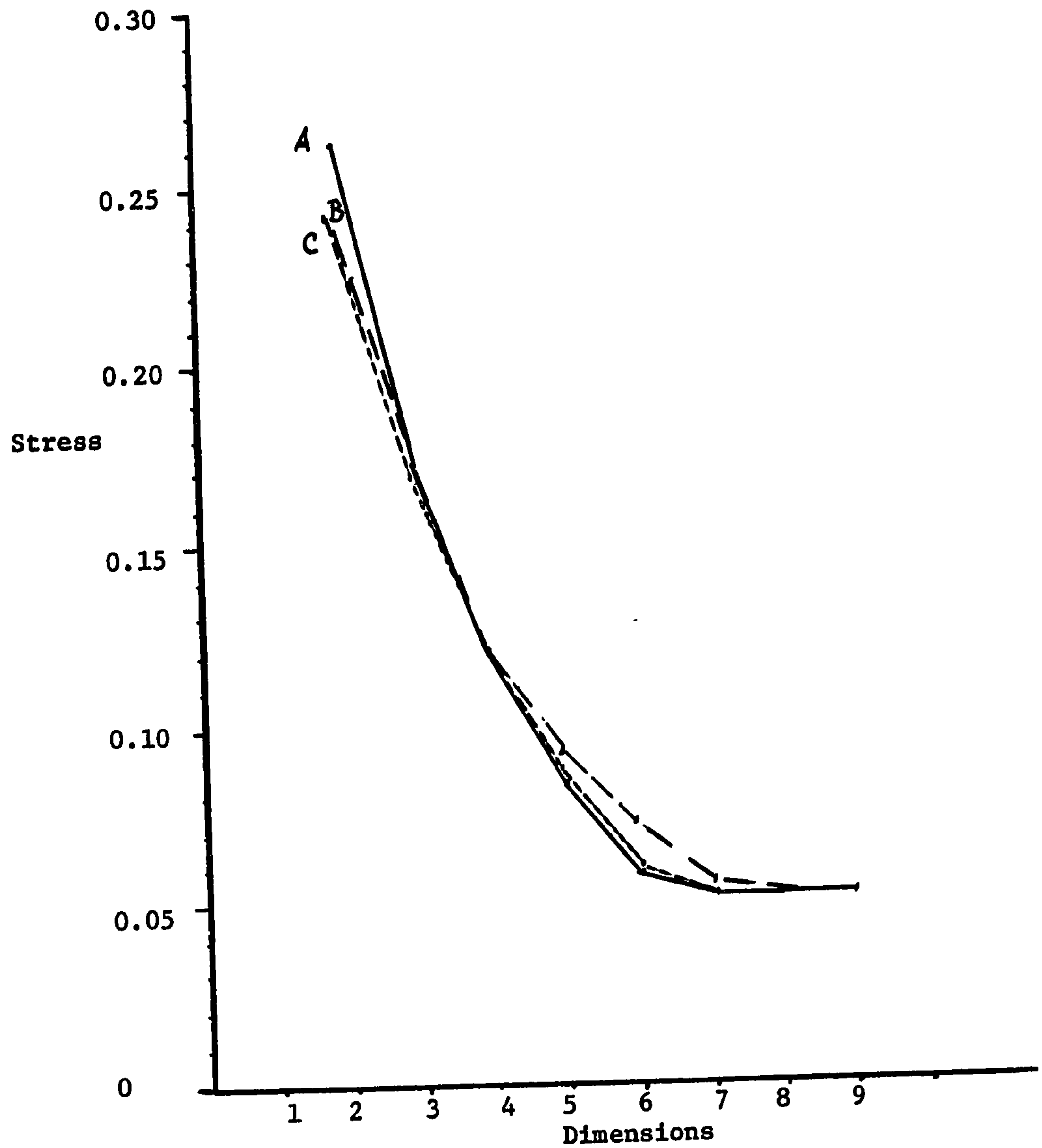

FIGURE 5

VARIATION OF MINIMUM STRESS WITH DEMENSIONALITY FOR SUBGROUPS AIII(-), BIII(-), AND CIII(-)

TABLE XXXII

FINAL UNROTATED CONFIGURATION OF 20 POINTS IN FOUR DIMENSIONS FOR HIGH POSITIVE COEFFICIENTS ON VIEWPOINT DIMENSION III: GROUP A

	Dimension			
Points*	I	II	III	IV
1	.05	.90	-.26	-.11
2	.14	-.61	-1.05	.34
3	-.60	-.72	.30	-.15
4	-.45	-.02	-.51	.72
5	.14	.35	.72	-.25
6	-.09	.02	.59	-.51
7	.10	-.89	-.13	-.76
8	1.08	.04	-.46	.18
9	-.49	-.55	-1.14	-.03
10	-.28	.42	.49	-.25
11	-.16	.72	.44	.30
12	.49	.24	.63	.20
13	-.15	-.14	.58	.58
14	.41	-.27	.21	.96
15	.48	-.52	.45	-.02
16	.27	.72	-.10	-.43
17	-.19	.53	-.05	.01
18	-.45	.50	-.15	-.49
19	.75	-.72	-.68	-.56
20	-1.04	.02	.12	.27

*Mechanics concepts.

TABLE XXXIII

FINAL UNROTATED CONFIGURATION OF 20 POINTS IN FOUR DIMENSIONS FOR HIGH NEGATIVE COEFFICIENTS ON VIEWPOINT DIMENSION III: GROUP A

	Dimension			
Points*	I	II	III	IV
1	.48	-.38	-.33	-.72
2	-.61	.27	.38	.49
3	.59	-.75	.38	.39
4	-.03	-.23	.25	.80
5	-.34	-.33	-.52	.66
6	.56	-.99	-.40	.16
7	-.55	.39	.16	-.80
8	-.52	.84	-.30	.26
9	-.69	-.21	.46	.12
10	.37	.45	.73	-.20
11	.92	.46	.04	.40
12	-.14	.17	.69	-.37
13	-.80	-.11	-.33	-.25
14	.54	.32	-.93	-.32
15	-.18	-.69	-.42	-.26
16	.06	.99	.01	-.36
17	.46	.26	-.06	.93
18	.19	-.27	.26	-.82
19	-.01	-.66	.73	.10
20	-.31	.49	-.81	-.21

*Mechanics concepts.

to resemble the group average configuration with respect to two coordinates. The second dimension of the AIII(+) configuration may be regarded as reflecting a statics-motion continuum in that stimuli 16, 17, and 18 (statics) as well as 10 and 11 (work and power) have moderate positive loadings while stimuli dealing with types of motion are characterized by negative coefficients. Both the force-work-power cluster and the grouping of stimuli 2 and 9 (circular motion) are apparent on the third coordinate. The nature of the first and fourth dimensions remains unclear.

Two of the coordinates of the AIII(-) representation also do not seem to lend themselves readily to interpretation. However, the cluster relating to circular motion which involves stimuli 2, 7, and 9 emerges grouped with kinetic energy on the first dimension and the third coordinate seems to be characterized by a force-energy cluster of moderate-sized negative loadings for stimuli involving Newton's Second Law, energy, and friction. The interpretation of the structures of relationships between the stimuli for the two subgroups of subjects was not sufficiently clear to permit any inferences regarding the nature of this viewpoint dimension.

Comparison matrices of cosines between the reference axes for the four dimensional representations again provide some indication of the extent of agreement among the configurations based on Group A, B, and C data. Table XXXIV presents the cosines between the reference axes for the BIII (+) configuration rotated for maximum overlap with the AIII (+) representation. The second coordinate, interpreted as a statics-motion continuum, matches fairly well.

TABLE XXXIV

COSINES BETWEEN REFERENCE AXES: SUBGROUP BIII(+)
ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP
WITH SUBGROUP AIII(+)

(FOUR DIMENSIONS)

		AIII(+)			
		I	II	III	IV
BIII(+)	I	-.278	.905	.123	-.299
	II	.310	.169	.765	.539
	III	.748	.364	-.524	.193
	IV	-.516	.143	-.360	.764

The rotated BIII(+) configuration appears in Table XXXV. The comparison matrix of cosines for the complementary pair of configurations, namely BIII(-) and AIII(-), set out in Table XXXVI indicates that these two representations match fairly closely on all four coordinates. The BIII(-) configuration rotated for maximum overlap with the AIII(-) configuration appears in Table XXXVII.

Turning to the Group C representations, it was found that the CIII(+) representation resembled the AIII(+) configuration on three of the four dimensions. The matrix of cosines between the reference axes for the AIII(+) and rotated CIII(+) representations is displayed in Table XXXVIII while the rotated CIII(+) configuration appears in Table XXXIX. The CIII(-) and AIII(-) representations appear similar with respect to two of the coordinates. Tables XL and XLI present, respectively, the cosines between the reference axes and the rotated CIII(-) configuration.

The degree of similarity of the Group B and Group C data is reflected in Tables XLII and XLIII which display, respectively, the cosines between the reference axes for the subgroup CIII(+) configuration rotated for maximum overlap with the unrotated BIII(+) representation, and the cosines between the reference axes for the CIII(-) configuration rotated for maximum overlap with the unrotated BIII(-) representation. In both cases, three of the four dimensions seem to match fairly well. The extent of the relationship between the Group B and Group C configurations was considered relatively more important for viewpoint dimension III than for the other viewpoint dimensions since the AIII(-) configuration is based on the dissimilarity estimates of only one individual.

TABLE XXXV

SUBGROUP BIII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIII(+)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.44	.64	-.56	.36
2	.26	-.62	-.76	.01
3	.30	-.89	.05	.12
4	-.29	-.71	.47	.57
5	.64	.04	.42	-.75
6	-.45	-.61	.61	-.29
7	.51	-.51	.31	-.77
8	.94	.21	.23	.07
9	-.02	-.73	-.49	.37
10	-.53	.51	.82	.18
11	.04	.52	.88	.00
12	-.15	.35	.46	.80
13	.22	.06	-.16	.82
14	-.34	.39	-.33	.79
15	-.72	-.54	-.42	-.41
16	.11	.12	-.81	-.31
17	-.02	.55	.55	-.25
18	.25	.35	-.54	-.64
19	-.47	.27	-.67	-.70
20	-.71	.60	-.06	.03

*Mechanics concepts.

TABLE XXXVI

COSINES BETWEEN REFERENCE AXES: SUBGROUP BIII(-)
ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP
WITH SUBGROUP AIII(-)

(FOUR DIMENSIONS)

		AIII(-)			
		I	II	III	IV
BIII(-)	I	.891	.251	.363	.106
	II	.212	.378	-.861	.266
	III	.327	-.889	-.247	.204
	IV	-.232	.057	.258	.936

TABLE XXXVII

SUBGROUP BIII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIII(-)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.60	.25	.46	-.43
2	-1.05	.26	.47	-.23
3	.35	-.76	.24	.37
4	-.61	.34	.17	1.28
5	-.03	.30	-.94	.27
6	.25	-.28	-.52	-.54
7	.03	.82	-.15	.10
8	-.43	-.87	.49	.20
9	-1.02	.42	-.03	-.12
10	.17	.02	.39	-.83
11	.26	.26	-.52	-.61
12	.01	-.72	-.50	.33
13	-.08	-.62	-.54	-.10
14	.12	-.05	-.80	.21
15	-.29	-.37	-.03	-.94
16	.87	.23	.09	.14
17	.65	.48	.31	-.11
18	.32	-.61	.65	.02
19	.45	.40	.82	.16
20	-.56	.47	-.07	.82

*Mechanics concepts.

TABLE XXXVIII

COSINES BETWEEN REFERENCE AXES: SUBGROUP CIII(+)
ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP
WITH SUBGROUP AIII(+)

(FOUR DIMENSIONS)

		AIII(+)			
		I	II	III	IV
CIII(+)	I	.064	.232	.886	-.395
	II	-.909	-.385	.157	-.023
	III	.368	-.750	.356	.419
	IV	-.183	.485	.250	.817

TABLE XXXIX

SUBGROUP CIII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIII(+)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.72	.63	-.32	-.75
2	.13	-.40	-.93	-.33
3	.70	-.66	-.10	-.24
4	-.71	.06	-.31	.46
5	-.13	-.02	.16	.78
6	-.58	-.35	.02	-.48
7	.45	-.27	-.22	-1.13
8	.44	.41	-.38	-.20
9	-.36	-.37	-1.10	.35
10	-.14	.07	.43	-.62
11	-.34	-.11	.74	-.03
12	.30	-.28	.38	.71
13	.27	.63	.39	.65
14	.59	.18	.57	-.12
15	.55	-.41	.97	.04
16	.24	.20	-.82	.78
17	-.88	-.05	.12	-.09
18	-.26	.98	.08	-.03
19	-.26	-.90	.26	.12
20	-.73	.65	.05	.13

*Mechanics concepts.

TABLE XL

COSINES BETWEEN REFERENCE AXES: SUBGROUP CIII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIII(-)

(FOUR DIMENSIONS)

		AIII(-)			
		I	II	III	IV
CIII(-)	I	.389	-.597	-.294	-.637
	II	.914	.278	.223	.194
	III	-.096	.565	.341	-.746
	IV	.066	.497	-.865	-.027

TABLE XLI

SUBGROUP CIII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIII(-)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	-.44	-.63	-.45	-.55
2	-.75	.25	.18	.34
3	-.08	.41	.78	.12
4	-.38	.18	-.56	.60
5	.66	-.68	-.41	.64
6	.70	-.82	-.38	.06
7	-.53	-.07	.65	.46
8	-.70	.53	-.36	.16
9	-.77	.07	.44	-.22
10	1.19	-.17	.50	-.47
11	1.00	-.39	-.09	-.40
12	.24	.63	-.35	-.86
13	.22	.37	-.69	-.39
14	.04	.97	.01	-.22
15	.15	-.13	.71	.71
16	-.25	.52	.18	.55
17	-.60	-.20	-.14	.16
18	.64	-.01	-.70	.40
18	-.23	-.65	.37	-.28
20	-.09	-.18	.31	-.81

*Mechanics concepts.

TABLE XLII

COSINES BETWEEN REFERENCE AXES: SUBGROUP CIII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR SUBGROUP BIII(+)

(FOUR DIMENSIONS)

		BIII(+)			
		I	II	III	IV
CIII(+)	I	.439	.869	-.182	-.139
	II	.373	-.388	-.825	-.172
	III	-.631	.289	-.524	.493
	IV	.519	-.105	.108	.841

TABLE XLIII

COSINES BETWEEN REFERENCE AXES: SUBGROUP CIII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR SUBGROUP BIII(-)

(FOUR DIMENSIONS)

		BIII(-)			
		I	II	III	IV
CIII(-)	I	.185	.864	-.102	-.457
	II	-.133	-.296	.580	-.746
	III	-.972	.223	-.054	.041
	IV	.053	.339	.806	.482

The CIII(-) configuration rotated for maximum overlap with the unrotated BIII(-) representation is reported in Table XC in the Appendix while the CIII(+) configuration rotated for maximum overlap with the unrotated BIII(+) representation appears as Table XCI. The first dimension of the CIII(-) configuration differs from that of the AIII(-) configuration in being characterized by relatively large positive loadings for concepts relating to force. The circular motion cluster appears on the third dimension and the force-work-power cluster on the fourth dimension.

Considering that the AIII(+) configuration is characterized by a statics-motion continuum on the first coordinate with the circular motion and force-work-power clusters on the third coordinate, the general properties of the structures of stimulus interrelationships for subjects with high positive coefficients and subjects with high negative coefficients on viewpoint dimension III appear to have a number of points of similarity. Thus even a tentative characterization of viewpoint dimension III would probably be pure speculation since the two interpretations appear to resemble each other and the group average configuration to an appreciable extent.

The minimum stress values for the two dimensional to nine dimensional representations for the six subgroups on viewpoint dimension IV are presented in Table XLIV. The variation of minimum stress with dimension for viewpoint dimension IV is shown in Figure 6 and Figure 7. Since the number of dimensions to be retained could not be clearly established by inspection of the plots, the comparison

TABLE XLIV

MINIMUM STRESS FOR VIEWPOINT DIMENSION IV CONFIGURATIONS

Dimensions	AIV(+)	BIV(-)	CIV(-)
9	.049	.050	.047
8	.049	.047	.049
7	.056	.049	.050
6	.064	.056	.060
5	.097	.079	.077
4	.124	.111	.109
3	.178	.168	.147
2	.272	.248	.238
	AIV(-)	BIV(+)	CIV(+)
9	.050	.050	.049
8	.047	.051	.049
7	.050	.056	.049
6	.056	.077	.062
5	.076	.101	.082
4	.099	.133	.097
3	.132	.198	.143
2	.192	.295	.201

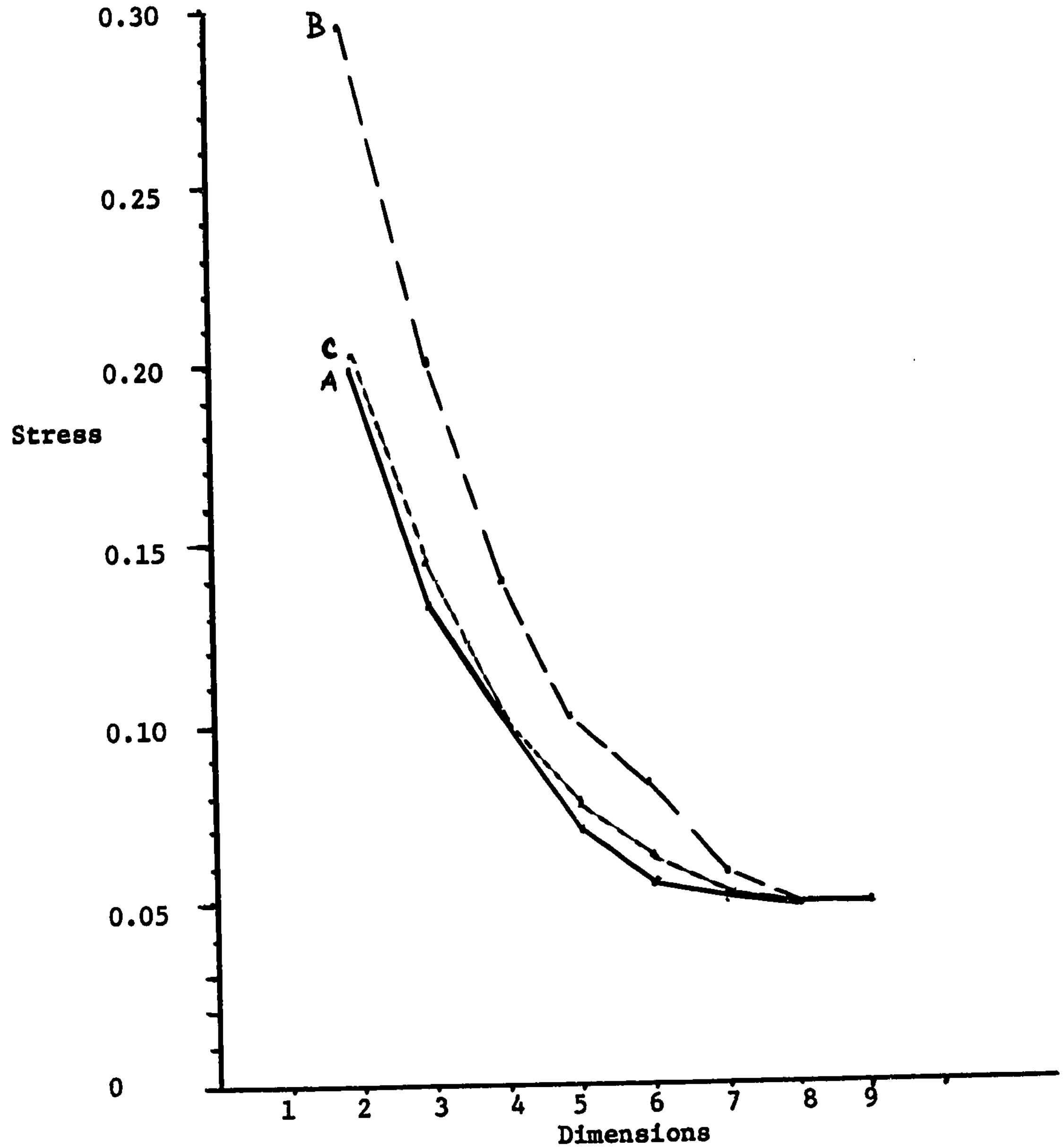

FIGURE 6

VARIATION OF MINIMUM STRESS WITH DIMENSIONALITY FOR SUBGROUPS AIV(-), BIV(+), AND CIV(+)

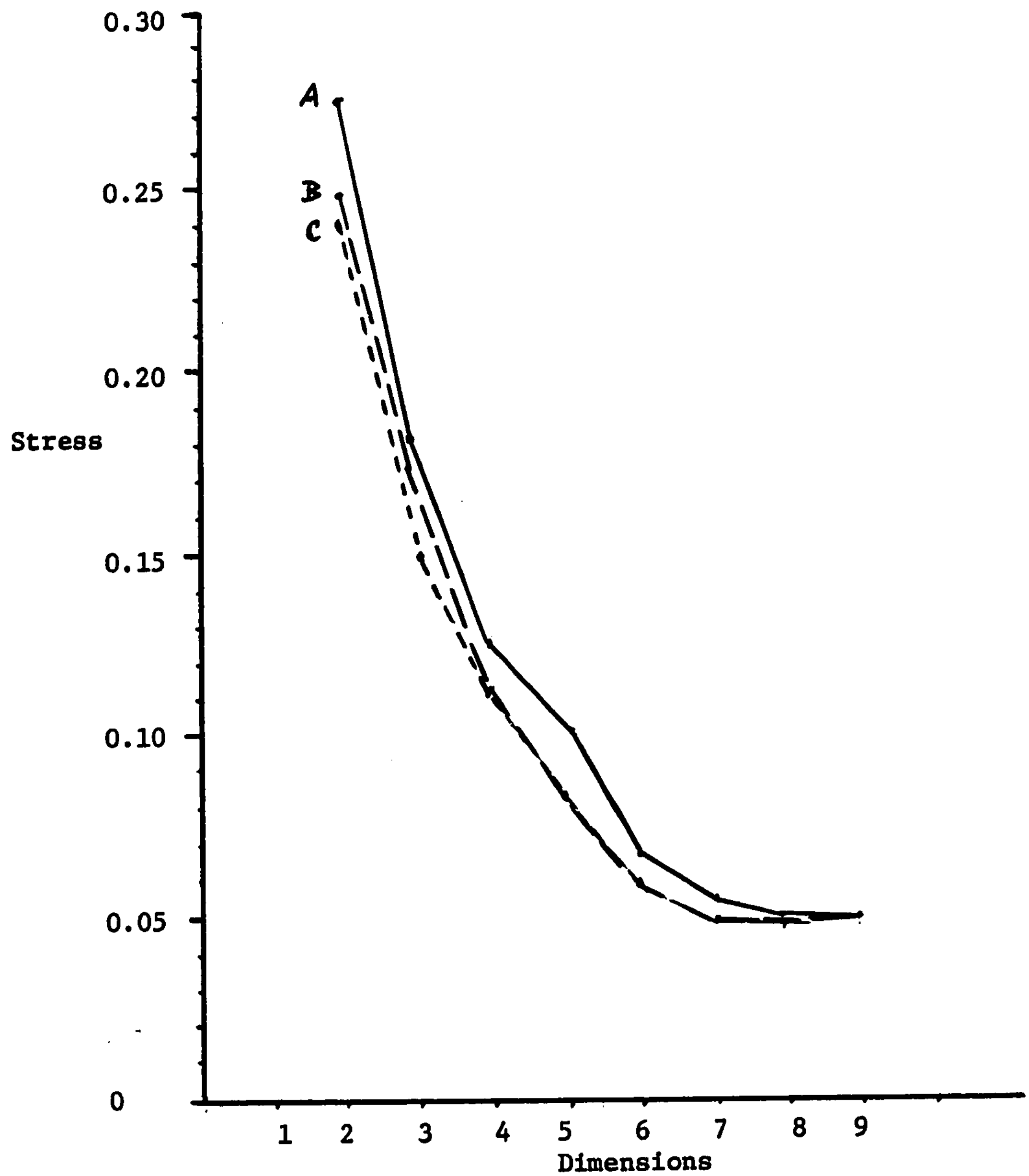

FIGURE 7

VARIATION OF MINIMUM STRESS WITH DIMENSIONALITY FOR SUBGROUPS AIV(+), BIV(-), AND CIV(-)

TABLE XLV

FINAL UNROTATED CONFIGURATION FOR 20 POINTS IN FOUR DIMENSIONS FOR HIGH POSITIVE COEFFICIENTS ON VIEWPOINT DIMENSION IV:

GROUP A

	Dimension			
Points*	I	II	III	IV
1	-.19	-.88	.21	.23
2	-.11	-.18	-.68	-.76
3	.62	.12	.78	.13
4	-.25	-.34	.60	.80
5	-.53	1.01	-.11	.10
6	.44	.90	-.26	.17
7	-.33	.14	-.20	-.80
8	.18	-.50	-.78	.31
9	-.81	-.51	.27	.11
10	.59	.40	-.53	-.46
11	.17	.70	-.22	-.45
12	-.33	-.49	.32	.55
13	-.04	-.04	-.57	.74
14	.26	-1.07	.02	-.24
15	-1.02	-.27	-.31	-.05
16	.24	-.02	.86	-.34
17	.37	.00	.79	-.34
18	.30	.45	-.30	.74
19	.93	-.13	-.29	-.32
20	-.47	.72	.42	-.12

*Mechanics concepts.

TABLE XLVI

FINAL UNROTATED CONFIGURATION OF 20 POINTS IN FOUR DIMENSIONS FOR HIGH NEGATIVE COEFFICIENTS ON VIEWPOINT DIMENSION IV: GROUP A

	Dimension			
Points*	I	II	III	IV
1	.38	-.39	-1.30	-.16
2	-.88	.27	-.17	-.40
3	-.65	.03	-.19	-.04
4	-.60	.10	-.11	.52
5	-.20	-.27	-.15	.65
6	-.45	-.32	-.08	.34
7	-.07	.97	.30	-.61
8	-.22	-.92	-.02	-.27
9	-.59	.31	-.06	-.80
10	.20	-.40	.69	.05
11	.22	.00	.91	.29
12	.44	-.18	.57	.30
13	.47	-.50	.38	-.11
14	-.07	-.18	.76	.30
15	.07	.46	.92	.41
16	.12	-.54	-1.22	-.41
17	.51	.26	-.81	.19
18	.98	.52	-.30	.44
19	-.31	.80	.01	.23
20	.65	-.02	-.13	-.92

*Mechanics concepts.

matrices of cosines between the reference axes were obtained for all possible combinations of subgroups in a given dimensionality for the four, five, and six dimensional configurations. Agreement among the Group A, B, and C representations appeared to be most extensive for the four dimensional configurations and viewpoint dimension IV will therefore be characterized in terms of four coordinates. The average minimum stress of 0.112, although somewhat below the average minimum stress of 0.130 obtained by Klahr for solutions based on random data for 16 points in four dimensions, is far from satisfactory.

The unrotated four dimensional configurations for subgroups AIV(+) and AIV(-) are reported in Tables XLV and XLVI, respectively. The structure of stimulus interrelationships for the subgroup with high positive coefficients on viewpoint dimension IV appears similar to the group average configuration with respect to the first two coordinates. The first dimension might be interpreted as reflecting a statics-motion continuum and a comparatively well defined force-work-power cluster is apparent on the second coordinate. The interpretation to be attributed to the other two dimensions remains obscure.

The AIV(-) configuration appears to lend itself a little better to interpretation since three of the coordinates resemble dimensions which tended to recur in the group average configurations. The first coordinate of the AIV(-) representation appears to reflect a statics-motion continuum while the third might be construed as a vector dimension. A cluster of concepts relating to circular motion is characterized by relatively high negative loadings for stimuli 2, 7, and 9 on the fourth coordinate. The clustering of stimuli 2 and 19, namely Newton's Law of Gravity and center of gravity, on the second dimension might perhaps be attributed to verbal association occasioned by the word "gravity." As appeared to be the case for viewpoint dimension III, viewpoint dimension IV seems to resemble to a considerable degree the group average structure of stimulus relationships discussed in Section II. The AIV(-) subgroup representation seems to lend itself particularly well to interpretation in terms of the group average designations.

The extent of agreement among the configurations derived from Group A, B, and C data is again reflected in the comparison matrices of cosines between the reference axes. Table XLVII presents the matrix of cosines between the reference axes for the BIV(-) configuration rotated for maximum overlap with the AIV(+) representation. The first coordinate, tentatively interpreted above in terms of a motion-statics continuum, appears to match fairly well. One other coordinate in the two configurations appears somewhat similar in the two representations. The rotated BIV(-) representation appears in Table XLVIII.

In relation to the complementary pair of configurations, Table XLIX of cosines between the reference axes for the BIV(+) configuration rotated for maximum overlap with the AIV(-) representation suggests that only the statics-motion coordinate is similar in the two structures. The rotated coefficients for the BIV(+) configuration are presented in Table L.

TABLE XLVII

COSINES BETWEEN REFERENCE AXES: SUBGROUP BIV(-)
ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP
WITH SUBGROUP AIV(+)

(FOUR DIMENSIONS)

		AIV(+)			
		I	II	III	IV
BIV(-)	I	.829	.273	-.424	.240
	II	-.204	-.189	-.812	-.512
	III	.081	.704	.235	-.666
	IV	.514	-.628	.324	-.487

TABLE XLVIII

SUBGROUP BIV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIV(+)

(FOUR DIMENSIONS)

	Dimension			
Points*	I	II	III	IV
1	.24	-.76	.00	.50
2	-.70	-.46	-.43	-.45
3	-.81	-.77	.31	-.05
4	-.04	-.81	.50	-.41
5	-.14	.30	-.75	.62
6	-.29	.52	-.05	-.10
7	-.30	.22	.62	-.81
8	-.86	.38	.01	.57
9	.28	.13	-.89	-.27
10	.79	.49	-.22	-.28
11	.03	-.07	-.97	-.43
12	-.02	.68	-.17	-.27
13	.76	-.13	-.07	-.40
14	.45	-.35	.19	-.70
15	-.60	-.32	.51	.72
16	.26	.09	1.11	.31
17	.25	.05	.10	1.07
18	.28	.65	.49	.61
19	.55	-.50	-.48	.30
20	-.12	.64	.19	-.52

*Mechanics concepts.

TABLE XLIX

COSINES BETWEEN REFERENCE AXES: SUBGROUP BIV(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIV(-)

(FOUR DIMENSIONS)

		AIV(-)			
		I	II	III	IV
BIV(+)	I	-.137	-.524	-.764	-.351
	II	-.025	.675	-.633	.378
	III	-.535	-.404	.033	.741
	IV	.834	-.325	-.123	.430

TABLE L

SUBGROUP BIV(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIV(-)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.69	-.70	-.38	-.45
2	.05	.87	-.51	.25
3	-.16	-.25	-.31	.87
4	-.43	-.21	.05	.89
5	-.86	.38	.04	-.05
6	-1.14	-.26	.25	-.54
7	-.06	-.15	1.07	-.10
8	.47	-.14	.45	.03
9	-.26	.83	.32	-.74
10	-.37	-.93	.18	.00
11	.31	-.25	.49	.79
12	.00	.62	.70	.35
13	.61	.13	.25	-.49
14	.74	-.14	-.48	.14
15	.20	-.40	.28	-.82
16	.06	-.72	-.27	.13
17	.29	.84	-.09	-.30
18	.55	.41	-.24	.60
19	-.18	.00	-.89	-.53
20	-.49	.07	-.92	-.04

*Mechanics concepts.

With reference to the Group C representations, the situation deteriorates still further. The comparison matrix of cosines for the CIV(-) configuration rotated for maximum overlap with the AIV(+) representation presented in Table LI suggests that only one dimension in the two configurations matches. Table LIII of cosines between the reference axes for the AIV(-) and rotated CIV(+) configurations indicates no discernible similarity in the two representations. The CIV(-) configuration rotated for maximum overlap with the AIV(+) representation is set out in Table LII and Table LIV shows the CIV(+) configuration rotated for maximum overlap with the AIV(-) representation.

The degree of similarity of the Group B and Group C configurations is indicated by Tables LV and LVI which display, respectively, the cosines between the reference axes for the subgroup CIV(-) configuration rotated for maximum overlap with the unrotated BIV(-) representation and the cosines between the references axes for the CIV(+) configuration rotated for maximum overlap with the unrotated BIV(+) representation.

The CIV(-) configuration is seen to match with the unrotated BIV(-) representation with respect to two dimensions while the CIV(+) and unrotated BIV(+) representations appear similar with respect to all four coordinates. The CIV(-) representation rotated for maximum overlap with the unrotated BIV(-) configuration appears in Table XCII in the Appendix while the CIV(+) configuration rotated for maximum overlap with the unrotated BIV(-) representation is set out in Table XCIII.

TABLE LI

COSINES BETWEEN REFERENCE AXES: SUBGROUP CIV(-)
ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP
WITH SUBGROUP AIV(+)

(FOUR DIMENSIONS)

		AIV(+)			
		I	II	III	IV
CIV(-)	I	.643	-.382	-.538	.389
	II	-.409	.560	-.607	.387
	III	.623	.719	.296	.084
	IV	-.173	-.154	.504	.832

TABLE LII

SUBGROUP CIV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIV(+)

(FOUR DIMENSIONS)

Points*	Dimensions I	II	III	IV
1	-.34	-.63	.54	-.11
2	-.59	.13	.18	-.69
3	-.15	.60	.94	-.31
4	-.60	-.62	.59	-.43
5	-.31	.58	-.54	-.23
6	-.46	.58	-.37	-.09
7	.63	-.32	-.04	-.77
8	-.15	-1.14	.19	-.27
9	.66	-.17	-.51	-.55
10	.09	.71	-.57	-.31
11	.37	.67	-.24	-.20
12	.67	.52	-.21	.22
13	.24	.80	.13	.28
14	-.11	.50	-.51	.54
15	-.63	.34	-.25	.77
16	.21	-.54	.87	.62
17	.55	-.45	.65	.04
18	.59	-.28	-.02	.70
19	-.12	-.82	-.69	.11
20	-.57	-.45	-.14	.67

*Mechanics concepts.

TABLE LIII

COSINES BETWEEN REFERENCE AXES: SUBGROUP CIV(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIV(-)

(FOUR DIMENSIONS)

		AIV(-)			
		I	II	III	IV
CIV(+)	I	.102	-.780	.504	.356
	II	-.265	.555	.462	.638
	III	.761	.088	-.383	.516
	IV	.583	.274	.621	-.446

TABLE LIV

SUBGROUP CIV(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AIV(-)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	-.38	-.77	-.25	-.42
2	-1.20	.39	.04	-.58
3	.56	.04	-.34	.10
4	.37	-.46	.60	-.31
5	-.54	.17	-.03	.90
6	.34	-.70	-.54	.38
7	.31	1.55	.17	-.75
8	.28	-.58	-.04	-.06
9	-.76	.26	-.44	-1.29
10	-.21	-.57	.48	.52
11	-.02	-.33	-.07	.84
12	.42	-.25	.17	.40
13	.35	-.15	.09	-.86
14	.24	.12	.63	-.07
15	-.53	.38	.08	-.25
16	.38	.30	-.89	.38
17	-.25	-.15	-.70	.21
18	.37	.50	.07	-.03
19	. 56	.40	.34	.64
20	-.26	-.16	.63	.25

*Mechanics concepts.

TABLE LV

COSINES BETWEEN REFERENCE AXES: SUBGROUP CIV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR SUBGROUP BIV(-)

(FOUR DIMENSIONS)

		BIV(-)			
		I	II	III	IV
CIV(-)	I	.871	-.363	.048	-.341
	II	-.145	.289	.793	-.529
	III	.279	.016	.572	.772
	IV	-.387	-.887	.234	-.013

TABLE LVI

COSINES BETWEEN REFERENCE AXES: SUBGROUP CIV(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR SUBGROUP BIV(+)

(FOUR DIMENSIONS)

		BIV(+)			
		I	II	III	IV
CIV(+)	I	.829	.042	.374	.414
	II	-.145	.860	.446	.202
	III	-.328	-.482	.812	-.025
	IV	-.429	.162	-.050	.887

As was the case for viewpoint dimension III, the similarity between the structure of stimulus interrrelationships for the subgroup with high positive loadings and the subgroup with high negative loadings and their resemblance to the group average representation precludes even a tentative characterization of the nature of viewpoint dimension IV.

In relation to viewpoint dimension V, no Group A individuals with normalized negative coefficients and Group C individuals with normalized positive coefficients equal to or greater than 0.80 were present in the sample. The minimum stress values for the four dimensions are presented in Table LVII. Figure 8 also shows how stress varies with dimension for the four sets of data. Since examination of the plots again yielded no clear indication of the number of dimensions in which the configurations might be characterized, the comparison matrices of cosines between the reference axes for the possible combinations of subgroups were obtained for the four, five, and six dimensional representations. Somewhat more agreement seemed evident among the four dimensional configurations for the AV(+), BV(-), and CV(-) subgroups than was apparent in either five or six dimensions or when the BV(+) subgroup replaced the BV(-) subgroup in the combination. Like viewpoint dimensions III and IV, viewpoint dimension V will therefore be characterized in terms of four coordinates. The average minimum stress of 0.106 is slightly better than the average stress value for the viewpoint dimension III or IV representations.

TABLE LVII

MINIMUM STRESS FOR VIEWPOINT
DIMENSION V CONFIGURATIONS

Dimensions	AV(+)	BV(-)	CV(-)
9	.049	.050	.049
8	.050	.053	.049
7	.049	.071	.050
6	.052	.083	.057
5	.067	.099	.072
4	.089	.125	.099
3	.117	.176	.153
2	.168	.253	.210
		BV(+)	
9		.048	
8		.050	
7		.050	
6		.060	
5		.082	
4		.111	
3		.156	
2		.232	

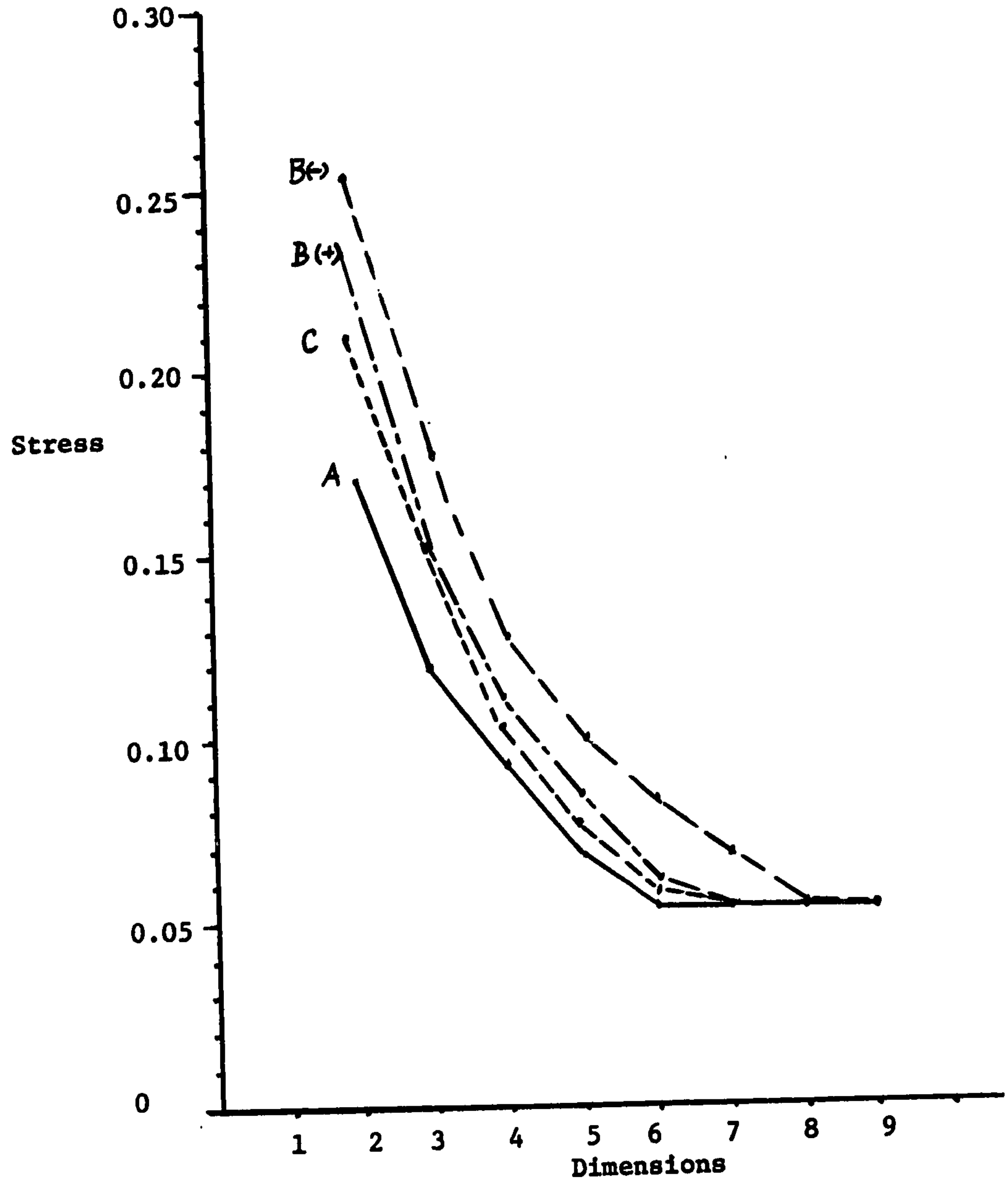

FIGURE 8

VARIATION OF MINIMUM STRESS WITH DIMENSION FOR SUBGROUPS AV(+), BV(-), CV(-), AND BV(+)

The final unrotated configuration for the Group A subgroup with high positive coefficients on viewpoint dimension V is presented in Table LVIII. The first coordinate is characterized by relatively high negative loadings for stimuli 2, 7 and 9, the cluster of concepts related to circular motion, as well as for stimulus 16 (composition of forces). A group of fairly high positive coefficients for concepts related to momentum and energy appears on the third coordinate while a force-work-power cluster with negative loadings seems fairly well defined on the fourth coordinate. An appropriate interpretation was not found for the second dimension.

Table LIX presents the comparison matrix of cosines between the reference axes for the subgroup BV(-) configuration rotated for maximum overlap with the AV(+) representation. Three of the four coordinates, more specifically the three which lent themselves somewhat to interpretation, appear related in the two configurations. The BV(-) representation rotated for maximum overlap with the subgroup AV(+) configuration appears in Table LX.

The cosines between the reference axes for the CV(-) configuration rotated for maximum overlap with the AV(+) representation are reported in Table LXI and the rotated CV(-) configuration in Table LXII. Two of the dimensions in the two representations appear to be somewhat similar in nature. Table LXIII of cosines between reference axes for the CV(-) configuration rotated for maximum overlap with the unrotated BV(-) representation indicates that one dimension might be regarded as matching in the two configurations. The corresponding CV(-) configuration appears as Table XCIV in the Appendix.

TABLE LVIII

FINAL UNROTATED CONFIGURATION FOR 20 POINTS IN FOUR DIMENSIONS FOR HIGH POSITIVE COEFFICIENTS ON VIEWPOINT DIMENSTION V:

GROUP A

Points*	Dimension I	II	III	IV
1	.40	-.44	-.32	-.91
2	-.56	.54	.22	.38
3	-.22	-.36	.28	.64
4	.42	-.36	-.23	.64
5	-.24	-.01	-.27	-1.08
6	-.09	-.22	-.99	-.46
7	-.93	.18	-.86	-.30
8	.36	-.10	.68	.36
9	-.65	.13	.30	.33
10	.62	.25	-.44	-.94
11	.32	.59	-.31	-.97
12	-.36	-.08	.83	-.13
13	-.47	-.21	.71	.16
14	.03	.15	.21	.75
15	.19	-.46	.22	.48
16	-.78	-.55	-.20	-.50
17	.85	-.66	-.05	.17
18	.61	.59	-.52	.46
19	.44	.39	.53	.38
20	.04	.60	.20	.54

*Mechanics concepts.

TABLE LIX

COSINES BETWEEN REFERENCE AXES: SUBGROUP BV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AV(+)

(FOUR DIMENSIONS)

		AV(+)			
		I	II	III	IV
BV(-)	I	-.186	-.573	-.017	-.798
	II	.878	.266	-.067	-.393
	III	-.211	.460	.809	-.298
	IV	.388	-.624	.583	.345

TABLE LX

SUBGROUP BV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AV(+)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.26	-.92	.34	-.08
2	-.07	.40	.75	.54
3	-.21	1.06	.45	-.05
4	-.19	-.67	-.14	.80
5	-.38	.17	-.79	-.14
6	-.62	.12	-.14	-.49
7	-.06	.08	-.76	-.61
8	-.02	.94	-.17	.49
9	-.01	-.54	-.52	.17
10	.27	.73	-.60	-.25
11	.70	.25	-.13	-.85
12	-.16	-.43	.73	-.37
13	.43	-.40	.61	-.50
14	1.09	-.30	-.43	.24
15	.38	-.54	.38	.75
16	-.84	-.48	-.04	-.08
17	.06	-.42	-.12	-.67
18	.51	.74	.32	-.09
19	-.67	.05	.65	.53
20	-.45	.16	-.39	.66

*Mechanics concepts.

TABLE LXI

COSINES BETWEEN REFERENCE AXES: SUBGROUP CV(-)
ROTATED FOR MAXIMUM FACTOR VECTOR
OVERLAP WITH SUBGROUP AV(+)

(FOUR DIMENSIONS)

		AV(+)			
		I	II	III	IV
CV(-)	I	.896	-.252	-.081	-.356
	II	.051	.729	.469	-.495
	III	-.101	-.557	.822	-.049
	IV	.429	.309	.309	.791

TABLE LXII

SUBGROUP CV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH SUBGROUP AV(+)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.72	-.37	-.24	-.45
2	-.38	.54	.89	-.01
3	-.73	-.95	.33	-.40
4	.86	-.10	.62	.42
5	.34	.51	-.45	-.79
6	-.34	.10	-.54	-.41
7	-.65	-.67	-.03	.66
8	.26	.17	-.20	.92
9	-.85	.30	.50	-.07
10	.86	-.03	.16	-.47
11	-.06	.25	-.15	-.84
12	.18	-.19	.51	.68
13	-.62	.60	-.09	.67
14	-.33	.30	.06	.96
15	-.33	-.75	.42	-.03
16	-.07	-.35	-.70	.02
17	.24	-.40	-.10	-.73
18	.32	.78	-.51	.11
19	.32	-.55	-.58	-.31
20	.26	.81	.11	.07

*Mechanics concepts.

TABLE LXIII

COSINES BETWEEN REFERENCE AXES: SUBGROUP CV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR SUBGROUP BV(-)

(FOUR DIMENSIONS)

		BV(-)			
		I	II	III	IV
CV(-)	I	-.159	-.878	-.207	-.401
	II	-.781	.278	.315	-.463
	III	-.258	-.387	.624	.628
	IV	-.547	.041	-.684	.481

The structures of perceived stimulus interrelationships for subgroups of subjects clustered on the various viewpoint dimensions appear, on the whole, to be quite similar, suggesting the possibility that the so-called viewpoint dimensions which emerged do not in fact reflect consistent individual differences in how relationships among the stimuli are construed. The discussion presented in Chapter VI will elaborate on this possibility.

It might be supposed that the manner in which the teacher himself structures information determines to some extent the environment which is provided for the development of conceptual structures in his students. However, an investigation of the teacher's perceptions of stimulus interrelationships on the conceptual structures of his students did not seem warranted for the present data for a number of reasons.

Since the teacher ratings were included in all three groups of data, one factor which emerged was the lack of consistency among the three groups with respect to the rotated viewpoint dimension on which the teacher received a high coefficient. Depending upon the group in question, anywhere from two to six of the teachers received normalized coefficients equal to or exceeding ± 0.80 on various dimensions. Inspection of both the unrotated and rotated individual coefficients indicated that the particular viewpoint on which the teacher received a high loading was not favored above the others by his students. Short of carrying out an extensive and most likely inconclusive series of factor matches for each teacher and his students, there appeared to be little that could be done by way of analysis for the present data. Since the structures of perceived relations for the various viewpoint dimensions tend to exhibit considerable similarity with respect to the interpretable coordinates, there appears little to be gained by undertaking the above analysis.

IV. THE ACHIEVEMENT, APTITUDE, AND PREFERENCE MEASURES: RESULTS

The magnitude of a given individual's coefficient on a viewpoint dimension may be regarded as indicative of the extent to which that individual's viewpoint about the stimulus interrelationships corresponds to the viewpoint dimension--in other words, his score on that particular dimension of viewpoint. Since each individual receives a score on each viewpoint dimension, these scores or coefficients may be correlated with various external measures in order to ascertain characteristics of the viewpoint dimensions. The aptitude measures selected for correlational purposes were four subtests from the Differential Aptitude Tests battery and Skemp's four tests of reflective intelligence.

Hypothesis A and Hypothesis B regarding the postulated relationships are at this point restated for reference:

> Hypothesis A: There is no significant relationship between the viewpoint dimensions isolated and any of the following subtests of the Differential Aptitude Tests, form L, battery: Verbal Reasoning, Numerical Ability, Abstract Reasoning, and Space Relations.

Hypothesis B: There is no significant relationship between any of the viewpoint dimensions isolated and any of the following of Skemp's reflective intelligence measures: Concept Formation, Reflective Action with Concepts, Operations Formation, and Reflective Action with Operations.

The Pearson product-moment correlations among the above aptitude measures and the viewpoint dimension coefficients for Group A, Group B, and Group C, are presented in Table LXIV, LXV, and LXVI, respectively. For a sample consisting of 60 observations, a mimimum coefficient of 0.25 is required in order that the correlation be considered significantly different from zero at the $p<0.5$ level. A summary of the results for the stepwise regression analysis appears as Table LXVII.

With respect to viewpoint dimension II, only the Spatial Relations subtest of the DAT battery was found to have a significant correlation ($r=0.27$) with the normalized viewpoint II coefficients for Group A data. Stepwise regression analysis with viewpoint dimension II as the criterion and the eight aptitude measures as predictors indicated that only the SR test functioned as a significant predictor, yielding a prediction equation of $0.015\ X_{SR} + 3.13$. The F value of 4.82 for the SR variable was significant at the $p<0.05$ level, with this predictor accounting for 7.7 per cent of the criterion variance. For Group B and Group C, none of the correlations of the eight aptitude measures with viewpoint dimension II attained significance at the 0.05 level. Thus, Hypothesis A was rejected at the 0.05 level only for the Spatial Relations subtest for Group A

TABLE LXIV

INTERCORRELATIONS AMONG THE ACHIEVEMENT AND APTITUDE SCORES AND VIEWPOINT DIMENSION COEFFICIENTS FOR GROUP A

	Physics	VR	NA	SR	AR	CFI	CFII	OFI	OFII	II	III	IV	V
Physics													
VR	.46												
NA	.59	.50											
SR	.32	.60	.38										
AR	.20	.63	.36	.38									
CFI	.15	.41	.27	.27	.58								
CFII	.21	.31	.35	.35	.35	.44							
OFI	.06	.46	.17	.31	.56	.35	.24						
OFII	.31	.54	.47	.44	.49	.38	.37	.42					
II	.28	.23	.10	.27	.06	-.04	-.04	.18	.08				
III	.09	.13	-.12	.18	.08	.08	.15	.04	.31	.17			
IV	-.14	.17	-.06	.16	.09	-.01	-.26	.20	.29	.17	.16		
V	-.16	.02	-.13	.06	-.03	-.01	-.14	.10	-.11	.06	-.40	-.02	
MEAN	22.8	38.7	35.3	40.4	42.2	39.6	76.9	26.3	25.0				
S.D.	7.5	7.3	3.8	9.8	5.1	3.3	13.0	3.1	8.1				

TABLE LXV

INTERCORRELATIONS AMONG THE ACHIEVEMENT AND APTITUDE SCORES AND VIEWPOINT DIMENSION COEFFICIENTS FOR GROUP B

	Physics	VR	NA	SR	AR	CFI	CFII	OFI	OFII	I	II	III	IV
Physics													
VR	.34												
NA	.25	.40											
SR	.26	.29	-.13										
AR	.14	.46	.27	.45									
CFI	.25	.51	.16	.31	.40								
CFII	.10	.26	.01	.27	.19	.54							
OFI	.14	.33	.09	.45	.59	.60	.36						
OFII	.36	.49	.26	.28	.28	.55	.23	.40					
II	.11	-.10	.03	-.14	-.08	-.24	-.10	-.20	-.24				
III	-.17	.07	-.10	.03	-.09	-.02	.04	-.06	.08	-.03			
IV	.13	.03	-.28	.31	-.05	.00	.09	-.11	.15	-.16	.07		
V	.25	.22	.13	.21	.07	.15	.16	.15	.23	-.04	-.02	.14	
MEAN	22.2	39.2	35.0	38.5	41.7	37.3	76.6	24.9	24.6				
S.D.	6.6	6.6	5.2	11.1	4.4	5.6	10.9	3.9	7.8				

TABLE LXVI

INTERCORRELATIONS AMONG THE ACHIEVEMENT AND APTITUDE SCORES AND VIEWPOINT DIMENSION COEFFICIENTS FOR GROUP C

	Physics	VR	NA	SR	AR	CFI	CFII	OFI	OFII	II	III	IV	V
Physics													
VR	.37												
NA	.55	.40											
SR	.09	.32	.19										
AR	.28	.38	.34	.43									
CFI	.19	.12	.18	.07	.16								
CFII	.06	.06	.02	.00	.18	.08							
OFI	.35	.25	.52	.27	.40	.08	.07						
OFII	.26	.32	.40	.42	.34	.36	-.08	.35					
II	.04	-.16	-.08	-.14	-.08	-.15	.02	-.05	-.23				
III	.09	.05	.20	.02	.11	.29	.04	.11	.29	-.24			
IV	-.02	.17	.16	.10	-.03	.16	.22	.09	.15	-.09	.30		
V	-.05	.08	.09	.13	.07	-.10	-.20	.10	.00	-.01	-.01	-.02	
MEAN	21.6	39.2	34.2	39.0	40.6	39.4	75.2	26.0	26.3				
S.D.	6.9	5.8	4.6	10.5	6.5	3.0	10.4	3.0	7.1				

TABLE LXVII

SUMMARY OF RESULTS FOR STEPWISE REGRESSION ANALYSIS:
PREDICTION OF VIEWPOINT DIMENSION COEFFICIENTS
FROM EIGHT APTITUDE MEASURES

	GROUP	Predictors	Per cent Variance Accounted for	F	p
Viewpoint	A	$-.015\ X_{SR} + 3.13$	7.7	4.82	<0.05
Dimension	B	(X_{OFII})	5.8	3.58	n.s.
II	C	(X_{OFII})	5.1	3.09	n.s.
Viewpoint	A	$-.044\ X_{NA} + .028\ X_{OFII}$	18.4	6.44	<0.01
Dimension	B	(X_{NA})	0.9	0.54	n.s.
III	C	$0.19\ X_{OFII}$	8.5	5.41	<0.05
Viewpoint	A	$-.016\ X_{CFII} + .028\ X_{OFII}$	24.1	9.04	<0.001
Dimension	B	$-.017\ X_{SR} - .034\ X_{OFII}$	17.4	5.99	<0.01
IV	C	(X_{CFII})	4.9	3.01	n.s.
Viewpoint	A	(X_{CFII})	1.9	1.1	n.s.
Dimension	B	(X_{OFII})	5.2	3.18	n.s.
V	C	(X_{CFII})	3.9	2.34	n.s.

viewpoint dimension II and retained for the remaining three DAT subtests for Group A and for all four DAT subtest for Groups B and C. Even for Group A, the relationship barely attained significance. Hypothesis B was retained for all three groups for the four Skemp tests.

Viewpoint dimension III was found to be significantly correlated with Skemp's Reflective Action with Operations test for both Group A ($r=0.31$) and Group C ($r=0.29$). Although the correlation with Skemp's Concept Formation test was also 0.29 for Group C, this variable did not function as a significant predictor when entered in order in the regression equation. However, the Numerical Ability subtest of the DAT ($r = -0.12$) was found to add significantly to the prediction for Group A. As with viewpoint dimension II, none of the eight aptitude measures attained significant correlation with viewpoint dimension III for Group B. However, the OFII variable with an F value of 5.41 ($p<0.05$) was found to account for 8.5 per cent of the variance for the Group C viewpoint dimension III. Gor Group A, the best weighted combination of $-0.044\ S_{NA} + 0.028\ S_{OFII}$ together accounted for 18.4 per cent of the criterion variance. The F value of 6.44 was significant at the $p<0.01$ level.

Consequently, both Hypothesis A and Hypothesis B were retained for all aptitude measures for Group B for viewpoint dimension III. Hypothesis B was rejected at the 0.05 level for Skemp's Reflective Action with Operations test for both Group A and Group B. Hypothesis A was rejected at the 0.05 level (F value for adding NA subtest=6.29, $p<0.05$) for the Numerical Ability subtest of the DAT battery for Group A. For the remaining measures, the hypotheses were retained.

With reference to Group A viewpoint dimension IV, both the Reflective Action with Operations test ($r=0.29$) and the Reflective Action with Concepts test ($r=0.26$) function as significant predictors of the viewpoint coefficients, the combination with regression weights $-0.016\ S_{CFII} + 0.028\ X_{OFII}$ accounting for 24.1 per cent of the criterion variance. The associated F value of 9.04 is significant at the $p<0.001$ level. For Group B viewpoint dimension IV, the Spatial Relations subtest ($r=0.31$) and the Operations Formation test ($r=-0.11$) together account for 17.4 per cent of the criterion variance. The F value of 5.99 for the combination is significant at the $p<0.01$ level and the best prediction equation is $0.017\ X_{NA} -0.034\ X_{OFI}$. For Group C, none of the predictor variables were found to correlate significantly with the criterion.

On the basis of the above results, Hypothesis B was rejected at the 0.001 level for Group A for Skemp's Reflective Action with Concepts and Reflective Action with Operations tests and retained for the other two tests. Hypothesis A was retained for all four tests. For Group B, Hypothesis A was rejected at the 0.01 level for the Spatial Relations subtest and Hypothesis B on the basis of the prediction equation for Skemp's Operations Formation test (F value for adding the OFI subtest=5.43, $p<0.05$). Hypothesis A and Hypothesis B were retained for the remaining DAT and Skemp tests, respectively. Both Hypothesis A and Hypothesis B were retained for all aptitude measures for Group C. For viewpoint dimension V, none of the intercorrelations with the predictor variables attained significance at the 0.05 level for any of the three groups.

It seems apparent that although a few intercorrelations among the viewpoint variables and aptitude measures attained statistical significance, very little in the way of consistent relationships across the groups was uncovered. Since Skemp's tests also involve geometric figures and are seen from Table LXVIII to exhibit correlations ranging from 0.21 to 0.37 with the Space Relations test, probably the most that could be said about the relationship between the viewpoint dimensions as a whole and the aptitude measures is that the ability to perceive spatial relationships appears to be related to a slight extent to some of these dimensions. The lack of any clear characterization of the viewpoint dimensions on the basis of the correlational data might be interpreted as further support for the contention that the four dimensions of viewpoint which were extracted in accordance with the individual differences model for multidimensional scaling are not distinct.

Turning now to the results relating to the physics achievement of the subjects, the two relevant hypotheses will first be restated:

Hypothesis C: The prediction of physics achievement scores is not significantly improved by adding to the Differential Aptitude Tests, form L, battery any of Skemp's reflective intelligence measures.

Hypothesis D: There is no significant relationship between any of the viewpoint dimensions isolated and student performance as measured by a physics achievement test.

Table LXVIII shows the intercorrelations of physics scores with the eight aptitude measures. For 180 subjects, a coefficient $r \geq 0.15$

TABLE LXVIII

INTERCORRELATIONS AMONG THE ACHIEVEMENT AND APTITUDE SCORES FOR GROUP A, B, C COMBINED

	Physics	VR	NA	SR	AR	CFI	CFII	OFI	OFII
Physcis									
VR	.39								
NA	.45	.41							
SR	.23	.40	.12						
AR	.22	.47	.32	.41					
CFI	.19	.37	.18	.23	.33				
CFII	.14	.22	.13	.21	.25	.37			
OFI	.18	.35	.23	.35	.49	.44	.23		
OFII	.31	.47	.35	.37	.35	.44	.19	.39	
MEAN	22.2	39.0	34.8	39.3	41.5	38.8	76.2	25.7	25.3
S.D.	7.0	6.6	4.5	10.5	5.4	4.3	11.5	3.4	7.7

is required before the correlation may be regarded as significantly different from zero at the $p<0.05$ level. All aptitude measures with the exception of Skemp's Reflective Action with Concepts test are seen to have significant correlations with the physics achievement test.

Stepwise regression analysis with the physics score as the criterion found the Numerical Ability and Verbal Reasoning subtests of the DAT battery to function as the only two significant predictors, accounting for 25.1 per cent of the criterion variance. The F value of 22.8 for the combination $0.26\ X_{VR} + 0.53\ X_{NA}$ was significant at the 0.001 level. Since the addition of Skemp's reflective intelligence tests to the DAT measures did not improve the prediction of physics achievement scores to a significant extent, Hypothesis C was retained.

The significance of the difference between the mean physics score for the two clusters of subjects on each viewpoint dimension was tested by means of the t test. None of the three t values for the two clusters of subjects on each of viewpoint dimension II, III, and IV ($t=.21$, 1.52, and 1.56, respectively) attained the 0.05 level of significance.

Since the two clusters of subjects for a given viewpoint dimension did not differ significantly with respect to physics achievement, the two subgroups for each viewpoint dimension were combined for one-way analysis of variance. The results summarized in Table LXIX indicate no significant difference in mean physics achievement among the four groups. However, stepwise regression analysis with coefficients for each of the four viewpoint dimensions in turn as the criterion and the physics test included as a predictor along with the eight aptitude

TABLE LXIX

SUMMARY OF ONE-WAY ANALYSIS OF VARIANCE OF PHYSICS TEST SCORES FOR SUBJECTS WITH HIGH COEFFICIENTS ON THE VIEWPOINT DIMENSIONS

Viewpoint dimension	Number	Mean	S.D.
II	30	21.5	5.6
III	26	21.2	7.0
IV	21	21.3	8.3
V	22	20.0	5.7
TOTAL	99	21.1	6.5

Homogeneity of variance test $\chi^2 = 4.73$ $p = 0.20$

ANALYSIS OF VARIANCE

Source	SS	MS	df	F	p
Between groups	29.9	10.0	3	0.23	n.s.
Error	4205.7	44.3	95		

measures indicated that the physics score functioned as a significant predictor in two instances. For Group A viewpoint dimension II, the physics test replaced the Spatial Relations subtest of the DAT battery as the one significant predictor, accounting for 8.1 per cent of the criterion variance. The F value of 5.09 was significant at the 0.05 level, with 0.020 $X_{Physics}$ as the best prediction equation. For Group B viewpoint dimension V, the physics test accounted for 6.5 per cent of the criterion variance. The F value of 4.0 was significant at the 0.05 level and the best prediction equation was again 0.020 $X_{Physics}$.

The results of the Cognitive Preference Test were equally disappointing. The two relevant hypotheses were:

> Hypothesis E. For subjects with high loadings on different viewpoint dimensions there is no significant difference in the frequency of selection as the most preferred for the four cognitive preferences as measured by the Cognitive Preference Test.
>
> Hypothesis F. There is no significant difference in performance as measured by a physics achievement test for subjects grouped according to the most frequently chosen category on the Cognitive Preference Test.

For the total sample, the mathematical application option was the most frequent choice by 45 per cent, the recall of factual material option by 23 per cent, the experimental application option by 18 per cent and the principle or generalization option by 14 per cent of the subjects. The results in terms of the most frequent choice for subjects with high loadings on different viewpoint dimensions are summarized in Table LXX. The significance of the difference in the frequency of

TABLE LXX

FREQUENCY OF SELECTION AS THE MOST PREFERRED CATEGORY ON THE COGNITIVE PREFERENCE TEST

		Cognitive Preference						
		1	2	3	4	Ties	No Score	TOTAL
Viewpoint dimension	I	5	2	10	6	6	1	30
	II	6	3	6	5	2	4	26
	III	2	4	7	5	1	2	21
	IV	8	2	6	5	1	0	22
	TOTAL	21	11	29	21	10	7	99
	Per cent	26	13	35	26			

selection of each of the four cognitive preferences as the most preferred for subjects with high coefficients on the four viewpoint dimensions was tested by means of the χ^2 test for independent samples. The χ^2 value of 6.57 with df=9 was not significant at the 0.05 level, necessitating the retention of Hypothesis E.

Hypothesis F was tested by grouping the subjects according to the most frequently chosen category and testing the significance of the differences between the group means by one-way analysis of variance. Table LXXI presents a summary of the results. No significant difference in mean physics achievement was found among the groups, necessitating the retention of Hypothesis F.

The low consistency exhibited in the extent of agreement in preference rankings assigned to a given category when two halves of the test are compared (0.34 to 0.45), coupled with the similarity of the structures of stimulus interrelationships for the various viewpoint dimensions, would tend to make the possibility of rejecting Hypothesis E somewhat unlikely. This low consistency in the selection of the preferences would also provide little justification for reliance being placed on any differences in achievement or aptitude which might emerge on the basis of the above groups.

TABLE LXXI

SUMMARY OF ONE-WAY ANALYSIS OF VARIANCE OF PHYSICS TEST SCORES FOR SUBJECTS GROUPED ACCORDING TO MOST FREQUENTLY CHOSEN CATEGORY ON THE COGNITIVE PREFERENCE TEST

Cognitive Preference	Number	Mean	S.D.
1. experimental in practical application	27	20.9	5.8
2. principles or generalizations	22	21.8	6.4
3. mathematical application	66	22.7	7.1
4. recall of facts	34	22.6	7.7
TOTAL	149	22.2	6.9

Homogeneity of variance test $\chi^2 = 2.57$ $p = 0.46$

ANALYSIS OF VARIANCE

Source	SS	MS	df	F	p
Between groups	76.7	25.6	3	0.53	n.s.
Error	6981.6	48.2	145		

CHAPTER VI

SUMMARY AND DISCUSSION

The primary undertaking of this study involved exploration of the appropriateness of multidimensional representation for perceived relations among stimuli such as subject-matter concepts with abstract cognitive characteristics as opposed to relatively simple perceptual attributes. The validity of the premise that interrelationships among such stimuli may be structured in a difficulty space with dimensional properties rests on the interpredictability and stability of the resulting configurations. The design of the study was most extensively influenced by the following three objectives:

1. To ascertain the number and nature of the dimensions required to summarize the structure of perceived relations among concepts in a subject area as derived from judgments of similarity with respect to their difficulty.

2. To investigate possible individual and group differences in conceptual structures with a view to isolating consistent individual viewpoints in the structuring of the given domain of concepts.

3. To describe relationships among the viewpoints for different groups of judges, and to relate these aspects of conceptual structure to measured characteristics and to the performance of the judges.

In January, 1969, eleven classes of grade twelve Physics 30 students and their seven teachers performed the paired comparisons task of rating 190 pairs of selected mechanics concepts in terms of the difference in difficulty of each pair. During November and December, 1968, and January, 1969, the students were administered a test battery consisting of a physics Achievement Test; a Cognitive Preference Test; the Verbal Reasoning Numerical Ability; Abstract Reasoning, and Space Relations subtests of the Differential Aptitude Tests battery; and Skemp's Concept Formation, Reflective Action with Concepts, Operations Formation, and Reflective Action with Operations tests.

The 180 subjects selected on the basis of the completeness of their set of scores were randomly assigned by teacher to one of three groups. The dissimilarity ratings for each group were analyzed by means of the individual differences model for multidimensional scaling to yield five viewpoint vectors. The stimulus pair projections on the first principal vector were analyzed by means of the Kruskal nonmetric scaling program to obtain, in accordance with the first objective restated above, a representation of the group average perceptual space.

The group average configuration was found to be characterized in terms of either four or five dimensions. Two of the coordinates, one interpreted as a statics-motion continuum and the other construed as reflecting the vector nature of some of the concepts, appeared in all three sets of data. The remaining two, or three, coordinates, while not interpretable in terms of generalized continua, were characterized by clusters of concepts which tended to recur, although not necessarily on the same dimension, in the configurations for the three groups. A cluster of force-work-power concepts was found on the last coordinate in each representation and a cluster of three concepts involving circular motion was also apparent in each configuration.

In relation to student performance on test items based on the concepts perceived as related on the various coordinates, circular motion emerged as the most poorly answered and statics as the best answered for the statics-motion dimension. (This could be because statics was the last topic covered before the judgments were rendered.) For the coordinate interpreted in terms of a vector/non-vector continuum, the test items relating to vector as opposed to non-vector type concepts were equally well answered. The clusters of concepts did not seem to reflect a trend for the particular coordinate on which they appeared.

In order to attain the second objective and the first part of the third objective, the mean dissimilarity ratings for subgroups of subjects selected on the basis of high normalized coefficients on each of the four varimax rotated viewpoint dimensions were analyzed to yield two representations, one for the subgroup with high positive loadings and the other for the subgroup with high negative loadings, for each viewpoint dimension. The configurations in three possible dimensionalities were compared for the three groups and the number of dimensions in which most agreement among the groups was evident retained as the preferred representation.

The structure of perceived stimulus interrelationships for one viewpoint dimension appeared best characterized in terms of six coordinates while the other three viewpoint dimensions were characterized in terms of four coordinates. On the whole, the solutions resembled the group average configuration with respect to the interpretation of individual coordinates. The motion-statics dimension and the coordinates characterized by the circular motion and force-work-power clusters of concepts tended to recur in most configurations. The viewpoint dimension characterized by six coordinates was tentatively interpreted in terms of the inclusiveness of the perceived interrelationships since the number of concepts which emerged as related on a given coordinate appeared to be considerably larger in the representation for the subgroup with high negative coefficients than for the subgroup selected on the basis of high positive coefficients. As highly tentative as it is, this interpretation is probably suspect since the other three viewpoint dimensions did not lend themselves to interpretation for the reason that the two structures presumably characterizing the extremes of the viewpoint continua were very much alike with respect to the interpretable coordinates.

With reference to the second part of the third objective, the results based on the coorelational data strongly suggest that the four so-called viewpoint dimensions are not distinct. This is not unexpected since the corresponding four eigenvalues together accounted for only four per cent of the total sums of squares in the sums of squares and cross products matrix. No significant differences among groups of subjects with high coefficients on different viewpoint dimensions emerged for any of the aptitude, achievement or preference measures. Although a few of the intercorrelations among the aptitude and preference measures and the viewpoint variables for the three groups attained statistical significance and some of these measures functioned as small but significant predictors of viewpoint dimension coefficients in isolated situations, very little in the way of consistent trends or relationships was apparent. One or the other of Skemp's reflective intelligence measures functioned as a significant predictor in five of the eight instances in which one or two variables did account for a significant amount of criterion variance. The Spatial Relations subtest of the DAT battery functioned as a significant predictor twice and the Numerical Ability subtest once. With respect to the aptitude measures, about all that could be said is that since Skemp's tests also involve geometric figures, the ability to perceive spatial and geometric relationships appears to be related to a slight extent to some of the viewpoint dimensions.

When the physics Achievement Test was included among the predictors, this measure emerged as a significant predictor for one of the viewpoint dimensions for each of two groups. However, it was not the same dimension in both cases.

The failure of any relationships exhibiting some degree of consistency across the three groups to emerge and the lack of any significant differences in the aptitude, achievement, and preference scores for subjects characterized by high coefficients on the four viewpoint dimensions was regarded as fairly strong support for the contention that the viewpoint dimensions which were isolated are not sufficiently different from one another to be considered distinct. Coupled with the similarity in the interpretable coordinates of the perceptual structures for the various subgroups and the very small per cent of trace accounted for by the corresponding roots, the conclusion that different structures of perceived stimulus interrelationships could not be isolated by grouping subjects according to their coefficients on the various viewpoint dimensions seems to be inescapable. It could very well be that any slight differences which might be apparent among the viewpoint dimensions are a consequence of differences in the way the response scale was being used by the subjects rather than in how the stimulus interrelationships were being construed.

However, variation in individual scale properties might not be the sole explanation for the comparatively slight differences in the structures of stimulus interrelationships which were encountered. It might well be that subjects do not on the whole construe stimuli characterized by a highly structured matrix of substantive interrelationships in more or less the same way in a difficulty space. Apparently Cliff[1] has encountered the same problem of similar structures for different viewpoint dimensions.

Substantial individual differences in perceived structures have been uncovered by Tucker and Messick[2] and Helm and Tucker[3] in their

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[1]Norman Cliff, "The 'Idealized Individual' Interpretation of Individual Differences in Multidimensional Scaling," Psychometrika, 33: 225-232, 1968.

[2]L. R. Tucker and S. Messick, "An Individual Differences Model for Multidimensional Scaling," Psychometrika, 28: 333-367, 1963.

[3]C. Helm and L. R. Tucker, "Individual Differences in the Structure of Color Perception," American Journal of Psychology, 74: 437-444, 1962.

studies of political judgment and color perception, respectively. However, in both instances a rather strong preselection variable was also operating. The 39 subjects in the political judgment study were selected from a set of 836 subjects on the basis of four desired combinations of responses to four questionnaire items. The preselection variable in the Helm and Tucker study, namely the physical factor of color-blindness, could hardly help but influence judgments related to color perception. On the other hand, Wiggins[4] preselection variable in the study on social desirability was found to be ineffectual. In any case, the failure to isolate distinct viewpoints with the Tucker and Messick model does not appear to be without precedent.

Another factor which may be operating is the nature of the substantive interrelationships among the stimuli themselves. When the stimuli are political figures or statements of behavior their interrelationships are free to exhibit considerable variability. However, with stimuli such as mechanics concepts, the substantive interrelationships among such stimuli are constrained by the structure of the discipline itself. Kinetic energy, for instance, is related to work in a very specific way regardless of how these concepts are construed in some judgment space. Consequently it seems reasonable to suppose that the actual scientific interrelationships which obtain between such stimuli influence any resultant structuring based on judgments of

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[4]Nancy Wiggins, "Individual Viewpoints of Social Desirability," Psychological Bulletin; 66: 68-77, 1966.

similarity with respect to some essentially subjective attribute such as perceived difficulty.

The logical and empirical structure of a discipline such as mechanics could well provide criteria for assessing the meaningfulness and relevance of any dimensional representation of its concepts derived on the basis of some attribute postulated as being shared by these concepts in the experience of the judges, i.e., their conceptions of these concepts. If no patterns consistent with the scientific structure emerge, then any ensuing multidimensional representations would very likely be useless. The primary finding of this study is that the fairly consistent structures of perceived relations based on judgments of dissimilarity with respect to difficulty which emerge for different samples drawn from the same population and which are reflected to some extent in representations derived for various subgroups within the sample are interpretable in terms of interrelationships among these concepts in the subject-matter. Evidence was obtained that clusters of concepts which are associated in the science of mechanics emerge from judgments with regard to pairs of such selected concepts presented to students in a random sequence.

In doing mechanics problems, some type of modification in associations among already familiar concepts may at times be required. What are generally regarded as "good" physics problems frequently require that familiar concepts be combined within the constraints of acceptable physics in some way which has not been previously encountered. Concepts frequently related in certain types of problems might be expected to appear as clusters on the coordinates of a difficulty space. Of the clusters which emerged, for instance, one of the most frequently occurring depicted the concepts of circular motion, Newton's Law of Gravity, and centrifugal and centripetal force as associated on some difficulty continuum. This is not in itself unexpected since problems dealing with projectiles in circular orbit about the earth are encountered in the course.

Useful associations, in the sense of being consistent with physics, would be of considerably more help to a student in doing physics problems than unproductive associations. One possible use for multidimensional scaling techniques in education could be to provide a way of isolating and representing these associations in some judgment space. For example, the concept of center of gravity was at times represented as related to the cluster of circular motion concepts considered above. Without indulging in too much speculation, it might be suggested that the word "gravity" could have something to do with this. One wonders if the association would still appear of the students had studied "center of mass" instead. Conversely, associations might appear which should be strengthened. One such instance involved the comparatively weak perceived relationship between kinetic energy and work. Of course, concepts might also be perceived as related in a difficulty space because common mathematical procedures are required in working with them. The vector dimension which emerged is a case in point.

Before such multidimensional representations could be used as a bench mark for studying the effects of changes in teaching practice, a number of factors which might complicate the issue need to be considered. The effect of the metric in which a representation is derived on the nature of the configuration would need to be established. Shepard[5] has found, for instance, that the nature of the underlying metric can change with different states of attention for the subjects.

The effects of changes in stimulus context on the stability of the ensuing representations would also need to be considered. Context in such cases refers to the composition of the set of stimuli to be scaled, not how the subject is presented. If some assurance is obtained that roughly the same perceived relations appear among given concepts when these are included in various sets of stimuli, then there would be some basis for looking at differences due to changes in extent and level of mathematical treatment, types of problems, the specific textbook, and so on.

The possibility of obtaining relatively stable and interpretable multidimensional group average representations for certain science concepts emerged as essentially the one potentially useful result of the investigation. Application of the individual differences model, however attractive this model may be conceptually, did not appear to provide any advantage over scaling methods based on the group average since clearly differing viewpoints with respect to the stimulus interrelationships for subgroups of individuals did not emerge.

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[5]R. N. Shepard, "Attention and the Metric Structure of the Stimulus Space," Journal of Mathematical Psychology, 1:54-87, 1964.

CHAPTER VII

CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS

I. CONCLUSIONS AND LIMITATIONS

The goal of attaining interpretable multidimensional representations of aspects of the structure of students' perceptions of concepts encountered in the classroom has been shown to be capable of realization to some degree in the area of mechanics. This in itself is, of course, no evidence that interpretable configurations might be obtained in other subjects, although such a possibility is perhaps somewhat greater in other relatively structured areas such as hydrostatics or electricity.

The group average perceptual space derived on the basis of dissimilarity judgments appeared to be relatively stable for samples drawn from the same population. Generalization of the interpretable portions of the structures to the entire population of Grade XII physics students who use Stollberg and Hill as a textbook should, however, be accompanied by caution since no evidence that the subjects constituted a representative sample of the total Grade XII physics population in the province was presented.

Although indications were obtained on the basis of the group average structure that multidimensional scaling techniques might be applicable in the domain of subject-matter concepts, nothing specific emerged to suggest that the full potential of the individual differences model for multidimensional scaling could be realized in such a context. Now the failure of distinct individual viewpoints with regard to stimulus interrelationships to emerge could well be because there were in fact no marked differences among the subjects with respect to these perceived interrelationships in the difficulty space. Be that as it may, there was still no empirical evidence to indicate that the special feature of the individual differences model, namely the capability of isolating separate perceptual structures for subgroups of individuals who differ with respect to their views regarding the stimulus interrelationships, would in fact isolate these presumed structures when applied to the domain of subject-matter concepts.

The possibility that any number of the other multidimensional scaling methods would have yielded equally interpretable results to describe the perceptions of the group as a whole should be considered. However, this possibility is not seen as detracting from the main positive finding of this study, namely that multidimensional representation of perceived relations among specific subject-matter concepts in some type of judgment space can lead to at least qualitatively interpretable results. The sense in which the magnitude of the coefficients on the coordinates of the multidimensional space reflects quantitative aspects of the given complex attribute serving as a basis for the judgments was not established in this study.

The degree of invariance of the dissimilarity judgments rendered across different stimulus samples and occasions also needs to be established. The results of this study do suggest, however, that a considerable degree of invariance across subjects might be expected to occur in some cases. Since the construct of psychological distance was postulated to exhibit the properties associated with distances in space, it becomes important to demonstrate that dissimilarity judgments in a subject-matter context do in fact behave like distances.

Even if the dissimilarity judgments are found to behave like distances, they may not necessarily behave like distances in an Euclidean space. Representations might be derived in other metrics in order to ascertain the nature of the most appropriate space. This might not be as straightforward as it first appears since there are indications that the characteristics of the space can vary with the state of attention or strategy of the subject.

Nevertheless, a few general conclusions might be ventured on the basis of the evidence gathered in this study. Emphasizing again the highly tentative and speculative nature of these conclusions, the three drawn on the basis of this study will now be stated:

1. Dissimilarity estimates among pairs of subject-matter concepts appear to lend themselves in certain cases to relatively interpretable multidimensional representation.

2. Judgments of relative difficulty with respect to subject-matter concepts do not seem to be unidimensional in nature. Students appear to be capable of construing such concepts along two or more difficulty dimensions.

3. For a given highly structured domain of subject-matter concepts, the nature of the difficulty space for subgroups of students seems to show little marked deviation from the group average representation.

Since no distinct viewpoint dimensions emerged, no generalized conclusions with respect to the findings from the correlational data appear justified.

II. IMPLICATIONS FOR SCIENCE EDUCATION

The potential of multidimensional scaling methods in research on science teaching remains to a large extent unexplored. If the reservations mentioned in the previous section with respect to their application to the scaling of stimuli which are abstract conceptual entities can be adequately met in a given domain of subject-matter concepts, an appropriate multidimensional representation could serve as an "initial configuration" for assessing the effects of a variety of variables. Changes in the organization of the subject and in specific teaching practices might be reflected in differences in the final configurations. Effects of variation in content might also show up. Specific characteristics of students such as their cognitive complexity might be reflected in such representations. In terms of classroom practice, the teacher might be interested in types of useful associations, relative to problem solving for instance, which could be strengthened and in the nature of nonproductive associations which might be weakened.

Differences in the structure of perceived relations in some other judgment space or subject area could be related to student performance and to various other variables. Curriculum workers might find information obtained from multidimensional representations useful in contemplating the nature and extent of revisions.

These few possibilities are mentioned only by way of illustrating the potential of this whole area of measurement techniques to research in science education. However, it cannot be emphasized too strongly that some of the objections to its application to conceptual stimuli are serious and need to be considered in any such attempts. There seems little point in deriving elaborate dimensional interpretations for stimulus interrelationships in cases where the attribute in question is clearly not dimensional in nature. However, with judicious and imaginative application, information useful to the classroom teacher might ultimately be obtained.

III. FOR FURTHER RESEARCH

A number of research possibilities of a more specific nature than the ones mentioned briefly in the previous section have suggested themselves in the course of this investigation. Some of these will be enumerated briefly.

1. The nature of the paired comparisons task might be modified and the study essentially replicated. A number of students indicated that they found the nine point scale somewhat difficult to handle and that they would have preferred a seven or even a five point scale. In addition, the list of concepts to be used as stimuli could be revised to exclude the one or two concepts, for example relativistic mass, which do not relate very well to other topics in the course.

2. Other multidimensional scaling models could be used to obtain the structure of perceived relations. It might be informative to compare the solutions obtained by various methods.

3. Effects of changes in the nature of the underlying metric on the properties of the solution could be assessed in a systematic fashion.

4. Multidimensional configurations for groups selected on the basis of some appropriate preselection variable might be obtained and compared. The individual differences model could be tried in such a situation.

5. A common set of concepts with, say, a seven point rating scale could be administered to subjects studying the PSSC course as well as to a group taking the conventional course and the ensuing multidimensional representations compared and related to various external measures of cognitive style or cognitive complexity.

6. The individual differences model might be used to scale perceptions of selected concepts in other areas of science, or perhaps statements reflecting various science activities or attitudes toward science.

7. A general set of concepts with which students would be expected to be familiar could be selected and the scaling be carried out at various grade levels to investigate the nature of changes in perception as the student learns science.

8. Variation in attitudes toward science or in perception of various types of science activities at different grade levels might also be investigated by means of multidimensional scaling methods.

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APPENDIX

INSTRUCTIONS FOR THE PAIRED COMPARISONS TASK

You have been presented with a deck of 95 cards, printed on both sides. Each card is numbered at the upper left and has on it two statements involving concepts with which you have worked in Physics 30.

Your task is to rate the DIFFERENCE IN DIFFICULTY between the two concepts on each card on a 9-point scale.

1	2	3	4	5	6	7	8	9
very similar in difficulty								very different in difficulty

If you regard the two physics concepts on the card as being very similar in difficulty (e.g., both very easy or both very difficult) in terms of what you are expected to do with them and how you are expected to apply them in your Physics 30 course, place a number at the lower end of the scale in the corresponding space on the Answer Sheet. For example, if you regard the two physics concepts on the card as being practically identical in difficulty, then you would rate their differences in difficulty as 1.

On the other hand, if you regard the two physics concepts on the card as being very different in difficulty, e.g., one very easy, the other very difficult, you would place a number at the upper end of the scale, e.g., 9, in the correspondingly numbered space on the Answer Sheet.

There are 190 pairs in all.

REMEMBER, you are estimating the DIFFERENCE in difficulty of the pair.

Work quickly and try to use the whole range of the scale.

STIMULI FOR THE PAIRED COMPARISONS TASK

MECHANICS CONCEPTS

1. Vector quantities: finding the vector sum or resultant of a group velocity vectors.

2. Circular motion: $a = v^2/r$, in which the direction of the acceleration is perpendicular to the direction of the velocity.

3. Uniformly accelerated motion involving gravitational acceleration, g; falling bodies and bodies thrown upward.

4. Projectile motion: the independence of the horizontal and vertical motions.

5. Newton's Second Law, F = ma; using the absolute system (mks and egs) of units, i.e., newtons, dynes.

6. Newton's Second Law, F = ma; using the British engineering system of units, i.e., slugs.

7. Newton's law of universal gravitation: $F = G\frac{m_1 m_2}{s^2}$.

8. Conservation of momentum: collisions and recoil velocities.

9. Centripetal and centrifugal forces: although equal in magnitude, these forces act on different objects; centripetal force = $\frac{mv^2}{r}$.

10. Work: the product of the component of a force in a particular direction and the distance.

11. Power: the rate of doing work, or the product of a force (or thrust) and velocity.

12. Potential energy: $E_p = mgh$, the work done to put an object into its position or condition.

13. Kinetic energy: $E_k = \frac{1}{2}mv^2$, the work done to accelerate an object from rest.

14. Conversion of potential energy to kinetic energy, e.g., falling objects.

15. The relation between the relativistic mass and the rest mass of an object.

16. <u>The resultant of concurrent forces</u>: $\vec{R} = \Sigma\vec{F} = 0$ for equilibrium; the equilibrant is equal and opposite to the resultant.

17. <u>Components of a force</u>: a single force may be broken into components.

18. <u>Moment of a force (torque)</u>: when parallel forces produce equilibrium, the sum of the clockwise moments equals the sum of the counterclockwise moments.

19. <u>Center of gravity</u>: the point of action of the resultant of the weights of the individual particles in the object.

20. <u>Friction</u>: the frictional force between two sliding surfaces is directly proportional to the normal force between them.

PHYSICS 30 ACHIEVEMENT TEST: MECHANICS

TIME: 1 hour

1. A unit for measuring work is

   (a) watt (b) newton (c) watt-sec (d) J/sec

2. Two boys are pulling on the same rope in opposite directions. If one boy exerts a force of 40 lb and the other a force of 60 lb, then the tension on the rope is

   (a) 40 lb (b) 60 lb (c) 100 lb (d) 20 lb

3. A boy is pulling a sled of mass 10 Kg with a force of 5.0N. If the force is applied at an angle of 30° to the horizontal then the effective force doing the work is

   (a) 5.0N sin 30° (b) 5.0N cos 30° (c) 5.0N tan 30°

   (d) 5.0 cos 30°

4. How much work is done if a horizontal force of 5.0N is used to push a box of mass 10 Kg along a level floor a distance of 2.0m?

   (a) 10J (b) 20J (c) 196J (d) 100J

5. A boy who weighs 160 lb, carries a 40 lb load up an incline ramp which is 12 feet long and is 4 feet high in 5 seconds. The useful power developed by the boy is

   (a) $\frac{200 \text{ lb} \times 12 \text{ ft}}{5 \text{ sec}}$ (b) $\frac{200 \text{ lb} \times 4 \text{ ft}}{5 \text{ sec}}$

   (c) $\frac{40 \text{ lb} \times 12 \text{ ft}}{5 \text{ sec}}$ (d) $\frac{40 \text{ lb} \times 4 \text{ ft}}{5 \text{ sec}}$

6. A load weighing 10 lb is raised to a position 20 feet above the ground level. If it is dropped, what is its potential energy relative to the ground at the instant it has fallen 15 feet.

   (a) 50 ft lb (b) 200 ft lb (c) 150 ft lb (d) 350 ft lb

7. A body whose weight is 10 lb is moving at 8 ft/sec. Its kinetic energy is

(a) $\frac{1}{2} \times 10 \text{ lb} \times 32 \text{ ft/sec}^2 \times \left(\frac{8 \text{ ft}}{\text{sec}}\right)^2$ (b) $10 \text{ lb} \times \left(\frac{8 \text{ ft}}{\text{sec}}\right)^2$

(c) $\frac{1}{2} \times \frac{10 \text{ lbs}}{32 \text{ ft/sec}^2} \left(\frac{8 \text{ ft}}{\text{sec}}\right)^2$ (d) $10 \text{ lb} \times 32 \text{ ft/sec}^2 \times 8 \text{ ft}$

8. You are on a train that is moving due south at 12 mi/hr. You walk across the floor of the train at right angles to the motion of the train at 5 mi/hr. What is your speed with respect to the earth?

(a) 17 mi/hr (b) 7 mi/hr (c) 8.5 mi/hr (d) 13 mi/hr

9. A car is travelling around a bend without changing its speed.

(a) Its acceleration is in the direction of its velocity.
(b) Its acceleration is perpendicular to its velocity.
(c) Its acceleration is away from the center of curvature.
(d) It has no acceleration.

10. A car travelling with an initial velocity of V_1 is uniformly accelerated in t seconds to a velocity of V_2. Its average speed during the interval of acceleration is

(a) $\frac{V_1 + V_2}{t}$ (b) $\frac{V_2 - V_1}{t}$ (c) $\frac{V_2 - V_1}{2}$ (d) $\frac{V_2 + V_1}{2}$

11. Which of the following is <u>NOT</u> a vector quantity.

(a) force (b) displacement (c) work (d) momentum

12. A single pulley is suspended from a fixed beam. A cord over the pulley has a mass of 100 kg on one end and a mass of 200 kg on the other. Neglecting the friction of the pulley and weight of the cord, the acceleration of masses is approximately

(a) 3.3 m/sec^2 (b) 9.8 m/sec^2 (c) 6.6 m/sec^2 (d) 4.9 m/sec^2

13. A cord over a single fixed pulley has two weights of 7 lb and 9 lb attached one to each end. If the masses are accelerating at 4 ft/sec^2 the tension on the cord is

(a) $16\ lb + \frac{16\ lb}{32\ ft/sec^2} \times 4\ ft/sec^2$

(b) $9\ lb + \frac{9\ lb}{32\ ft/sec^2} \times 4\ sec^2$

(c) $7\ lb - \frac{7\ lb}{32\ ft/sec^2} \times 4\ ft/sec^2$

(d) $7\ lb + \frac{7\ lb}{32\ ft/sec^2} \times 4\ ft/sec^2$

14. An unbalanced force of 1000 N causes a mass to be accelerated at 50.0 m/sec^2. How great is the mass?

(a) 1000 kg (b) 20 kg (c) 20 x 9.8 kg (d) 5.0×10^4 kg

15. If a man weighs 180 lb on the surface of the earth, what would he weigh on a planet whose mass is twice as great and whose radius is three times as great as that of the earth?

(a) 120 lb (b) 270 lb (c) 40 lb (d) 215 lb

16. Newton's Second Law of Motion may be expressed as

(a) $F = mgh$ (b) $F = Wa$ (c) $F = \frac{mV_2 - mV_1}{t}$ (d) $F = \frac{m(V_2 = V_1)}{2}$

17. A car of mass 1000 kg is travelling in a straight line at a speed of 10 m/sec. It is accelerated to 50 m/sec in 20 sec. Find its change in momentum.

(a) 1000 kg x 40 m/sec

(b) $\frac{1000\ kg \times 40\ m/sec}{20\ sec}$

(c) $\frac{1000\ kg \times 60\ m/sec}{2}$

(d) $\frac{1000\ kg \times 60\ m/sec}{20\ sec}$

18. A 100 g mass travelling horizontally in a straight frictionless groove at 200 cm/sec strikes a 400 g mass travelling at 100 cm/sec in the same direction. If upon collision the two masses stick together find the velocity of the combination of masses.

(a) 150 cm/sec (b) 120 cm/sec (c) 300 cm/sec (d) 0

19. A mass m is whirled on a weightless cord in a vertical circle of radius r at a velocity v. If no centripetal force is required when the mass is at the top of its curve, then the velocity of the mass is

(a) mgr (b) $\sqrt{mgr}$ (c) gr (d) $\sqrt{gr}$

20. What force is necessary to accelerate a 2000 lb car at 48 ft/sec^2?

(a) 3.0×10^3 lb (b) 9.6×10^4 lb

(c) 2.4×10^3 lb (d) 4.17×10^1 lb

21. A force of 1 lb will give an acceleration of 1 ft/sec^2 to

(a) 1 lb of mass (b) 1/32 lb of mass (c) 1 slug (d) 1/32 slug

22. The work done to accelerate a body from rest may be represented by

(a) ma (b) $\frac{1}{2}mv^2$ (c) mgh (d) $\frac{w}{g}a$

23. When a bullet is fired from a gun, the bullet has a greater capacity for doing work than the recoil of the gun because

(a) the momentum of the bullet is greater than that of the gun.
(b) the momentum of the bullet is equal to that of the gun.
(c) the kinetic energy of the bullet is greater than that of the gun.
(d) the kinetic energy of the bullet is equal to that of the gun.

24. A body starting from rest falls freely. How far does it fall in $\frac{1}{2}$ sec?

(a) 32 ft (b) 16 ft (c) 8 ft (d) 4 ft

25. A body starting from rest falls freely. How far does it fall during the <u>second</u> second?

(a) 64 ft (b) 48 ft (c) 32 ft (d) 8 ft

26. A force acting on a mass at rest accelerates it to half the speed of light. The mass of the body is increased by approximately

(a) 0.5% (b) 1.0% (c) 15% (d) 50%

27. The kinetic energy of a high speed particle is given by

(a) $\frac{1}{2}m_0v^2$ (b) mc^2 (c) $mc^2 - m_oc^2$ (d) $m\sqrt{1 - \frac{v}{c}^2}$

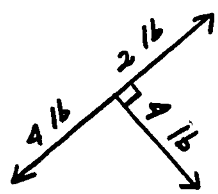
28. The mass of an electron

(a) decreases with increasing speed.
(b) is directly proportional to its speed.
(c) is independent of speed
(d) increases with increasing speed.

29. According to a conclusion of Einstein

(a) all energy has mass properties.
(b) moving particles apparently lose mass.
(c) momentum is not conserved in high speed collisions.
(d) a long stick moving rapidly sideways appears shortened.

30. An object is held in equilibrium by the action of four forces, three of which are shown in the diagram. Find the magnitude of the force not shown in the diagram.



(a) $\sqrt{20}$ lb (b) $\sqrt{8}$ lb (c) $\sqrt{10}$ lb (d) $\sqrt{12}$ lb

31. A dog sled plus the load weighs 1400 lb. The snow is hard packed and level but friction exerts a retarding force. If the coefficient of sliding friction is 0.05, how much work do the dogs do on the sled in pulling it 3000 ft?

(a) 1.4×10^5 ft lb (b) 2.1×10^5 ft lb

(c) 8.4×10^7 ft lb (d) 2.0×10^4 ft lb

32. What is the vertical component of a 100 N force acting at an angle of 30° with the horizontal?

(a) 100 N (b) 50 N (c) 86.6 N (d) 0

33. Two ropes tied to a load make an angle of 120° with each other. If each rope exerts a force of 500 N, the magnitude of the equilibrant is

(a) 1000 N (b) 500 N (c) 433 N (d) 250 N

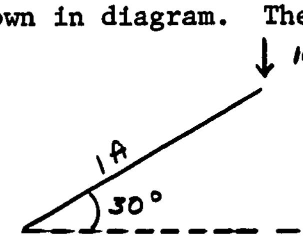
34. A ramp of length 13 ft has one end raised 5 ft. If a 1300 lb weight is placed on the ramp find the normal force. (Force exerted perpendicular to the ramp.)

(a) 1300 lb (b) 1200 lb (c) 650 lb (d) 1126 lb

35. A meter stick is balanced at the 50 cm mark. If a 20 gram mass is placed at the 100 cm mark, at what mark must a 50 gram mass be placed to keep the stick in a horizontal position?

(a) 10 cm (b) 20 cm (c) 30 cm (d) 40 cm

36. A force of 10 lb acts vertically downward on the end of a 1 ft crank as shown in diagram. The moment produce



(a) 10 ft lb (b) 8.66 ft lb (c) 5 ft lb (d) 4.33 ft lb

37. The coefficient of friction of stone on wood is 0.40. How much work must be done on a 200 kgf stone in moving it 15 m across a wooden floor?

(a) 3.0×10^3 Kgf m (b) 80 Kgf m

(c) 6.0×10^2 Kgf m (d) 1.2×10^3 Kgf m

38. The force of friction does <u>NOT</u> depend on

(a) the force pressing the two surfaces together.
(b) the nature of the surfaces involved.
(c) the area between the two surfaces.
(d) the kind of motion of the surfaces.

39. Find the magnitude of the resultant of the following three forces at a point 50 lb due East, 20 lb due West and 40 lb due South.

(a) 20 lb (b) 30 lb (c) 40 lb (d) 50 lb

40. A jet is rising with a uniform velocity of 600 mi/hr at 30° with the horizontal

(a) the sum of the vertical forces is zero and the sum of the horizontal forces is greater than zero.
(b) the sum of the vertical forces is zero and the sum of the horizontal forces is zero.
(c) the sum of the vertical forces is greater than zero and the sum of the horizontal forces is zero.
(d) the sum of the vertical forces is greater than zero and the sum of the horizontal forces is greater than zero.

COGNITIVE PREFERENCE TEST: MECHANICS

Instructions:

Each test item presents some information or data dealing with mechanics, followed by two options, both of which are correct and related to the information given.

YOUR TASK IS TO SELECT THE OPTION (a or b) WHICH YOU THINK CONSTITUTES THE BETTER OR PREFERABLE ENLARGEMENT OF THE INTRODUCTORY INFORMATION. In other words, select the option which, in your opinion, "goes better" with the introductory statement.

Please mark your choice on the separate Answer Sheet provided.

1. In circular motion, the acceleration vector is perpendicular to the velocity vector.

   a. A satellite launched horizontally with the right initial velocity will move at constant speed in a circular orbit around the earth

   b. A deflecting force of constant magnitude perpendicular to the motion makes a body move in a circle with constant speed

2. Potential energy is the available work or energy stored in a body on account of its position, condition, or state of stress.

   a. Potential energy is a relative quantity and must be measured with respect to some arbitrary reference level

   b. The potential energy of a raised weight is equal to the work done to raise it

3. Momentum is the product of mass and velocity.

   a. At a fire a stream of water moving horizontally delivers 100 kg of water per second at a speed of 5 meters per second as it hits the wall. If the water is assumed to lose all its momentum at the wall, the force exerted on the wall is 500 newtons

   b. Momentum is conserved when objects collide or cause each other to change velocity

4. The moment of a force about any point is equal to the sum of the moments of its components about the same point.

   a. The study of forces which act upon bodies without producing acceleration is known as statics

   b. Two boys sitting on a seesaw are able to make it oscillate at will

5. Newton's Second Law indicates that the mass of a body is everywhere constant.

   a. A newton is 10^5 dynes

   b. If a block of mass 5 kg resting on a level frictionless table is towed by a string of tension 10 newtons, its accelration is 2 m/sec^2

6. Whenever an object moves while in contact with another object, frictional forces oppose the relative motion.

   a. If it takes a horizontal push of 50 lb to move a 200 lb trunk across the floor, the coefficient of friction is 0.25

   b. The ratio of the force required for sliding one surface over another to the force which is pressing the surfaces together is the coefficient of friction

7. A set of components of a force is a set of forces whose resultant is the original force.

   a. The horizontal component of a vertical force is zero

   b. Pairs of component forces may be selected in any desired direction

8. Acceleration is the rate at which velocity changes with time.

   a. Unsupported bodies near the earth's surface have a downward accelration which may be regarded as constant

   b. When air resistance is negligible, a piece of paper and a brick will fall at the same rate

9. Work is done upon an object when a force causes the object to move in the direction of the force.

    a. The work done in a time t in pulling a body of weight w up a frictionless incline of angle θ with a speed v is wvt sin θ

    b. The work done by a force which is everywhere perpendicular to the path is zero

10. Quantities which have both magnitude and direction and which obey the parallelogram law of addition are called vector quantities.

    a. When two vectors are at right angles to each other, the magnitude of their vector sum or resultant can be found by the Pythagorean theorem

    b. An airplane, headed north, travels through the air at 250 km/hr. If a 50 km wind is blowing toward the east, the speed of the airplane relative to the ground is 255 km/hr

11. The lower the center of gravity of a body, the more difficult it is to tip the body over.

    a. A cylindrical piece of wood of height 6 in and diameter 4 in standing on one of its circular ends on a rough board can be tilted through 33°42' before it tips over

    b. If an object is supported only at its center of gravity, it remains in balance in any position

12. In mechanical systems at low speeds, there is frequently an interchange of kinetic and potential energies.

    a. The loss in potential energy of a falling object is equal to its gain in kinetic energy provided that forces other than gravity are absent

    b. The motion of a pendulum illustrates the transformations of potential and kinetic energy

13. Newton checked his law of gravitation by calculations upon the orbit of the moon.

  a. The force of attraction between two bodies varies directly as their masses and inversely as the square of the distance between their centers

  b. A mountain arising abruptly out of a flat plain has enough sidewise gravitational attraction to make a plumb bob on the plain hang perceptibly out of the vertical

14. Galileo studied the problem of acceleration by rolling balls down very smooth planes inclined at increasing angles.

  a. The acceleration of gravity is approximately 32 ft/sec^2 or 9.8 m/sec^2

  b. From his observations, Galileo derived the laws of motion regulating freely falling bodies

15. Work is the product of power and time.

  a. The horsepower as a unit of power was defined by James Watt in connection with his studies on the steam engine

  b. A truck climbing a steep mountain road slows down because there is a limit to the power it can develop

16. For equilibrium, the resultant of all the forces acting on a body must be zero.

  a. The above condition refers to forces <u>on</u> a body and to only one isolated body at a time

  b. A 150 lb tightrope walker stands at the center of a rope 100 ft in length. If the rope sags 10 ft at the center, the tension in the rope is 375 lb

17. Einstein's prediction of the equivalence of mass and energy is readily verifyable in nuclear reactors.

  a. In certain cases a particle may receive energy of the order of 10^{-12} joule as a result of the decrease in the mass of the reactants. This is measurable with a mass spectrometer

  b. If a body is accelerated to a speed approaching the speed of light, its mass increases appreciably

18. When work is done by a force acting on a mass in the direction of its motion, the kinetic energy of the mass is increased.

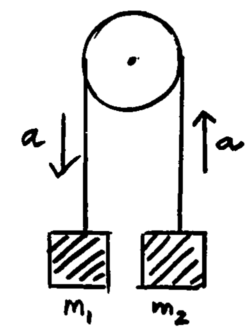
    a. The relation between the increase in kinetic energy of an object and the total work done on the object is known as the kinetic energy - work theorem

    b. A retarding force of 2.3×10^{-8} newtons is required to bring an electron traveling at 10^8 cm/sec to rest in a distance of 20 cm

19. Two masses m_1 and m_2 are connected by a light inelastic string which passes over a light fixed pulley as shown in the diagram:

    a. The system would accelerate in the direction indicated if m_1 was greater than m_2

    b. The acceleration of the system is given by $$a = \frac{m_1 - m_2}{m_1 + m_2} g$$



20. An object moving in a circular path exerts centrifugal force due to its inertia.

    a. A car rounding a curve at 60 mi/hr requires a centripetal force four times as great as that required in rounding the curve at 30 mi/hr

    b. If the string is suddenly cut, a heavy ball whirled on a horizontal surface by means of a string will travel in a straight line in the direction it was going when released.

21. Momentum is conserved when objects collide or cause each other to change velocity.

    a. Newton originally formulated his Second Law in terms of the time rate of change of momentum

    b. While momentum is conserved during any interaction, kinetic energy is not conserved if there is relative motion between the bodies interacting

22. The horizontal and vertical motions of a projectile are independent.

    a. Newton's Second Law is a vector law, valid in situations in which the net force acts at an angle to the velocity

    b. A flash photograph of two golf balls, one projected horizontally at the same time that the other is dropped would show that the two fall at the same rate

23. The weight force, the attraction between the earth and an object, is proportional to the mass of the object.

    a. A baseball pitcher needs to apply 5 lb of force in order to give a ball of mass 0.33 lb an acceleration of 480 ft/sec^2

    b. The mass of a body is a direct measure of its inertia

24. Unbalanced parallel forces cause rotation.

    a. The action of a lever is a simple illustration of moments

    b. A uniform bar 9.0 ft long and weighing 5.0 lb is supported by a fulcrum 3.0 ft from the left end. If a 12 lb load is hung from the left end, a downward force of 4.8 lb is required to hold the bar in equilibrium

25. The acceleration of an object traveling with uniform speed in a circle is directed inward toward the center of the circle.

    a. The magnitude of the acceleration is given by $a = v^2/r$

    b. The speed of a satellite moving in a circular orbit 400 km above the earth is approximately 7.6 x 10^3 m/sec or 18,000 mi/hr

26. The potential energy of a raised weight is its weight multiplied by the height through which it can fall.

    a. A 10 lb rock has 50 ft - lb of potential energy when it is 5 feet above the ground

    b. A wound up clock spring, a raised weight, a bent bow all possess potential energy

27. Conservation of momentum.

    a. If an explosive shell bursts in mid-air, the vector resultant of all the fragments is the same as the momentum of the whole shell if it had failed to explode

    b. The total momentum of a body or group of bodies can be changed only by the application of an unbalanced external force

28. For equilibrium the sum of the clockwise moments equals the sum of the counterclockwise moments.

    a. The above rule holds for any axis perpendicular to the plane of the forces

    b. If a meter stick weighing 120 g is supported by a fulcrum at the 60 cm mark, a 40 g weight must be suspended at the 90 cm mark in order to balance the stick

29. Unit force is that force which gives unit acceleration to unit mass.

    a. The application of an unbalanced force to a body gives the body an acceleration in the direction of the force

    b. For electrons and other atomic particles traveling with speeds approaching that of light, the acceleration is less than indicated by $F = ma$

30. The resisting force of sliding friction is usually less than that of starting friction.

    a. A block of wood rests on a level board. If one end of the board is slowly raised, the block will begin to slide when the angle with the horizontal reaches a certain value

    b. For many dry surfaces, the coefficient of friction does not vary greatly with the speed of the motion or the area of the surfaces of contact

31. A single force may be broken into components.

    a. When two strings supporting a weight are not equal in length, the steeper one has the greater tension

    b. If a boy weighing 40 lb sits in a swing and is pulled sideways with a horizontal force of 25 lb, the tension is each supporting rope is 23.6 lb

32. If the direction of the acceleration is parallel to the direction of the motion, only the speed of the object changes.

    a. The value of the acceleration of gravity is approximately 32 ft/sec^2 or 9.8 m/sec^2

    b. A ball thrown upward with an initial velocity of 75 ft/sec will reach a height of 87 ft in 2.5 seconds

33. The term work is restricted in physics to cases in which there is a force and a displacement along the line of the force.

    a. The work done under the action of gravity in moving an object from place A to place B does not depend upon the path by which the object was moved

    b. Work can be measured in ergs, joules, foot-pounds, or kilowatthours

34. Depending upon the angle between the two vectors, the magnitude of their resultant may be greater than, equal to, or less than either of them.

    a. If a point A moves with a velocity v_1 relative to a reference point 0, and if a second point B moves with a velocity v_2 relative to A, then the velocity of B relative to 0 is given by the vector sum of v_1 and v_2

    b. The composition of vectors finds useful application in problems of relative motion

35. The weight of an object may be regarded as being concentrated at its center of gravity.

    a. For uniform cubes, spheres, and rods, the center of gravity coincides with the geometrical center of symmetry

    b. The center of gravity is the point of action of the resultant of the weights of the individual particles in the object

36. When an object falls, its potential energy is converted to kinetic energy.

    a. Falling bodies represent a special case of the law of conservation of energy

    b. A 2 kg object falling from a height of 10 m into a box of sand exerts an average force of 65 newtons if it comes to rest at a distance of 3.0 cm beneath the surface of the sand

37. A spherical body attracts another body as though its mass were concentrated at its center.

    a. The numerical value of the gravitational constant, G, depends upon the units adopted for the measurement of force, mass, and distance

    b. Since the pull of the earth on one kilogram mass is 9.8 newtons and the radius of the earth is 6.37×10^6 m, the mass of the earth can be calculated to be 5.97×10^{24} kg

38. Galileo studied the problem of acceleration by rolling balls down very smooth planes inclined at increasing angles.

    a. The method was used to establish time intervals of sufficient length for him to be able to measure

    b. The acceleration of a body sliding down a smooth inclined plane which makes an angle of 30° with the horizontal is 16 ft/sec^2

39. The average power is the work performed divided by the time required for the performance.

    a. In energy transfers where work is involved, power is the rate of doing work

    b. If 10^{16} electrons strike the screen of a TV tube each second and each electron is accelerated through a voltage large enough to give it a speed of 10^8 cm/sec starting from rest, then 4.6×10^{-3} watts are expended in maintaining the beam

40. For equilibrium, the place at which a force is applied as well as its magnitude and direction must be considered.

    a. Forces whose lines of action intersect at a single point are termed concurrent

    b. A common trick is to prop open a door by placing a wedge of wood in the crack next to the hinge, usually ruining the hinge

41. According to Einstein, $\text{mass of a body in motion} = \text{mass at rest} + \dfrac{\text{kinetic energy of body}}{(\text{speed of light})^2}$

    a. The kinetic energy of a high speed particle is given by $E_k = mc^2 - m_oc^2$

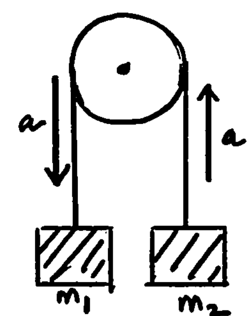
    b. It is incorrect to merely substitute the relativistic mass in the regular kinetic energy equation. The relationship must be derived from the beginning with mass variable

42. A body of mass m moving at velocity v has capacity for doing work to the extent $\frac{1}{2}mv^2$.

a. In a head-on collision between two billiard balls of masses m_1 and m_2, the kinetic energy lost by m_1 is equal to the amount gained by m_2 if m_1 and m_2 move the same distance during the collision

b. The increase in kinetic energy of a particle during an interval equals the total work done on the particle in that interval

43. Two masses m_1 and m_2 are connected by a slight inelastic string which passes over a light fixed pulley as shown in the diagram:



a. The pull of the string on the two bodies is equal

b. The system represents an application of Newton's Second Law

44. An object moving in a curved path is accelerated toward the center of rotation by centripetal force.

a. Objects moving in a curved path illustrate all three of Newton's laws of motion

b. For a satellite moving in a circular orbit around the earth, the centripetal force is the gravitational attraction of the earth

45. After any impact or collision, the total momentum is the same as the momentum before the event.

a. A moving billiard ball collides head on with a billiard ball of equal mass which is at rest. If the incident ball is stopped, the ball it hit will move off with the velocity of the incident ball

b. A moving railway car of mass M_1 and velocity v_1 couples with a standing car of mass M_2. The velocity of the two cars just after coupling is $\frac{M_1}{M_1 + M_2} v_1$

46. Projectile motion may be considered as the motion of a freely falling body which has a velocity component parallel to the surface of the earth.

    a. A bullet fired from a gun held horizontally with a muzzle velocity of 2500 ft/sec at a target 1000 ft away will drop 2.6 ft as it travels to the target

    b. The trajectory of a projectile is a parabola

47. The property of a body which requires that a force be exerted upon it to accelerate it is called the inertia of a body

    a. The same force will be required to accelerate an object within a satellite as would be required on earth

    b. When an object appears weightless to a man in an earth satellite, its mass is the same as when it was on earth

48. The moment of a force is a measure of its effectiveness in causing rotation.

    a. The product of the magnitude of a force and the perpendicular distance from its line of action to the center of rotation is known as the moment of the force

    b. In dealing with moments, it is assumed that the motio of the body under consideration is restricted to a plane

SKEMP'S TESTS OF REFLECTIVE THINKING

The following thirty pages present the four Skemp tests on reflective thinking in the form in which they were used in the present study. Although the format of the tests has been altered somewhat, the line drawings in each item have been faithfully reproduced.

Permission from Dr. R. R. Skemp to administer his tests was secured with the understanding that they were to be used only in the present study. If the reader should wish to use these tests or the originals, he should seek permission from:

Dr. R. R. Skemp,
Department of Psychology,
University of Manchester,
Manchester 13, England.

Skemp's doctoral dissertation is available on microfilm in the library of the University of Alberta.

NAME .... SCHOOL ....
Last First Middle

AGE .... Years GRADE .... BOY GIRL (Circle One) DATE .... Day Month Year

SK4: PART I

INSTRUCTIONS

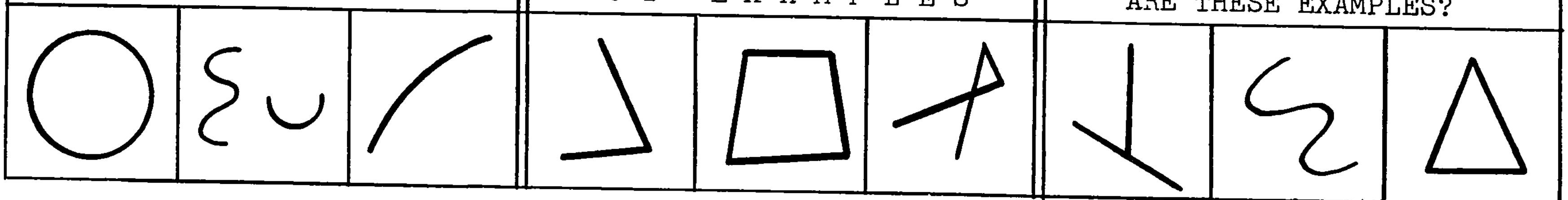
There are 15 rows of figures in PART I of this test. In each row of figures, the first three figures (marked "EXAMPLES") all have some property in common. In the second group of three (marked "NOT EXAMPLES") in the row, none of the figures has this property. Your problem is to decide whether or not each of the figures under the question "ARE THESE EXAMPLES?" has the property. If it has the property (i.e., if it is an example), circle YES under that figure. If it does not have the property, circle NO.

Look at the first row of figures at the top of the next page. Each of the figures under "EXAMPLES" is made of curved lines only. None of the figures under "NOT EXAMPLES" has this property. Look under the question "ARE THESE EXAMPLES?" Figure 1 is not made of curved lines, so you should circle NO for figure 1. Since figure 2 is curved, you should circle YES for figure 2. Figure 3 is not made of curved lines so you should circle NO for figure 3.

The second row of figures deals with another property. After deciding what the property is by looking at the figures under "EXAMPLES" and under "NOT EXAMPLES," answer the question "ARE THESE EXAMPLES?" by circling YES or NO for figure 4; then for figure 5; and then for figure 6. Then go on to the rest of the rows of figures in PART I of the test.

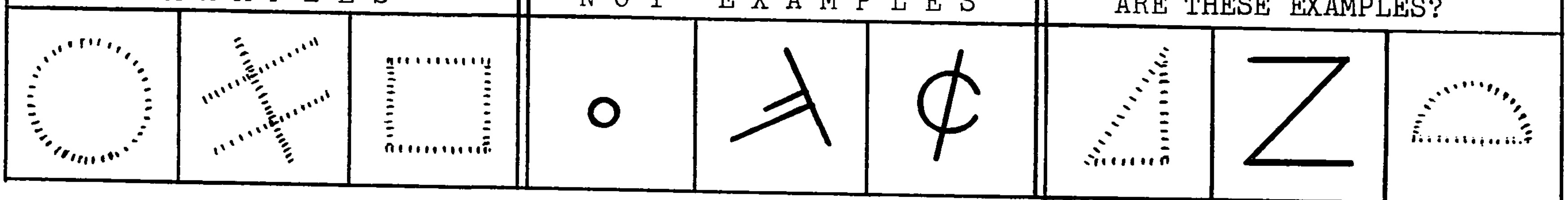
IMPORTANT: One, two, or all three of the figures under "ARE THESE EXAMPLES?" may have the property (i.e., may be examples).

E X A M P L E S	N O T E X A M P L E S	ARE THESE EXAMPLES?



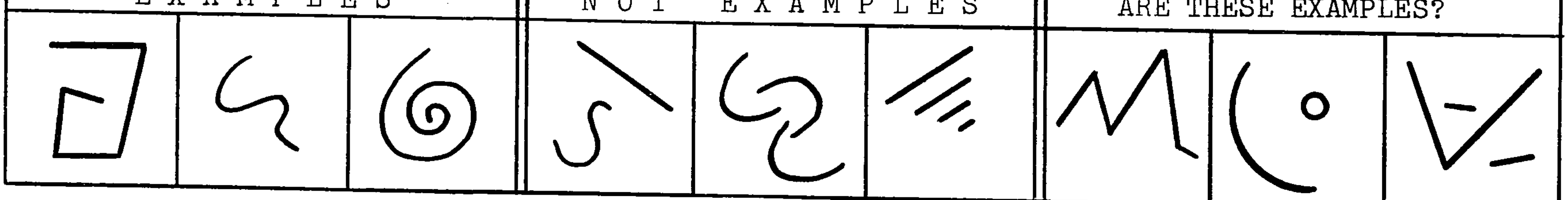
1. YES NO
2. YES NO
3. YES NO

E X A M P L E S	N O T E X A M P L E S	ARE THESE EXAMPLES?

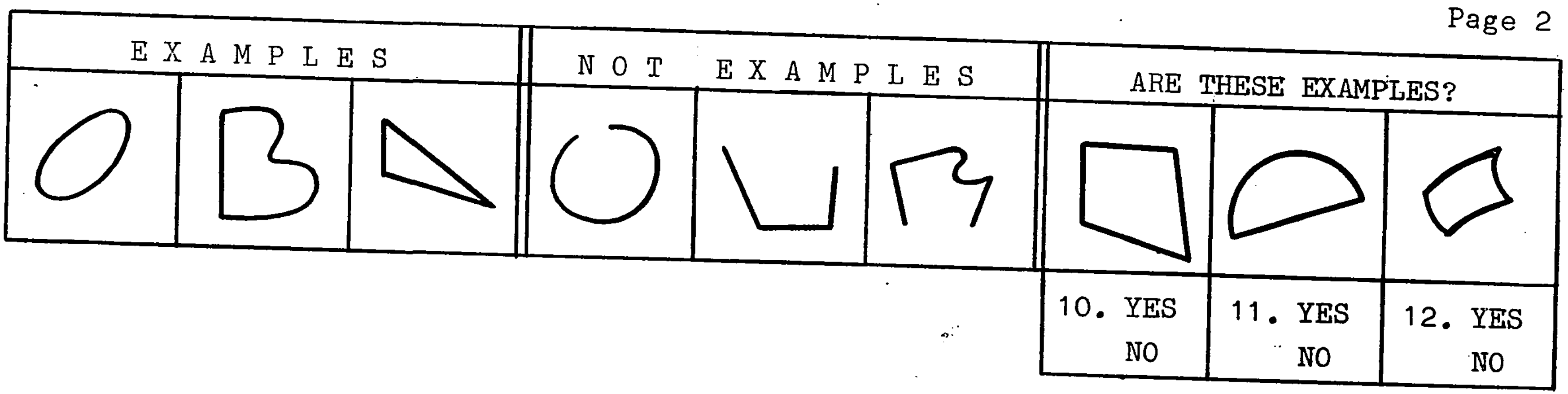


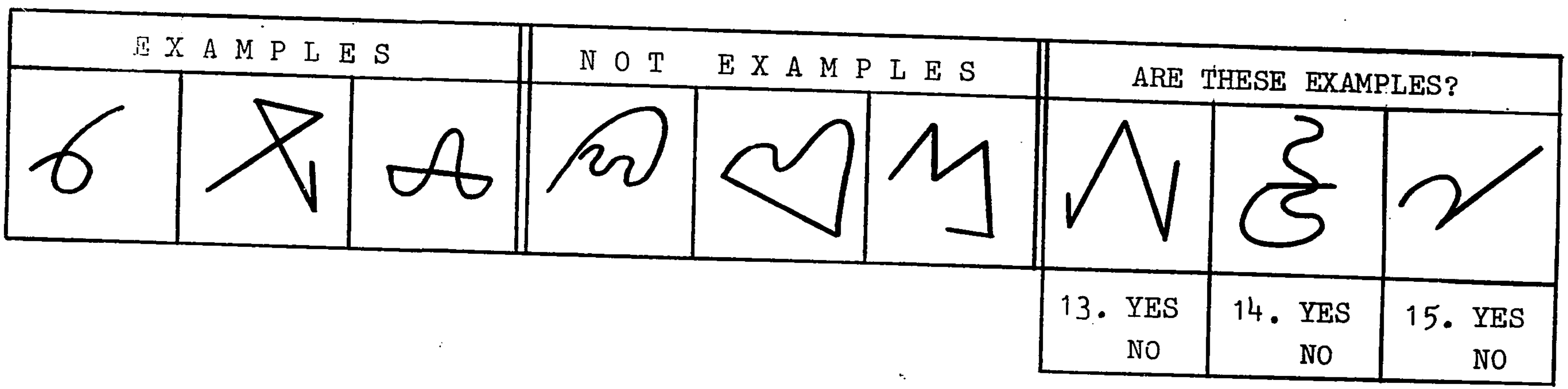
4. YES NO
5. YES NO
6. YES NO

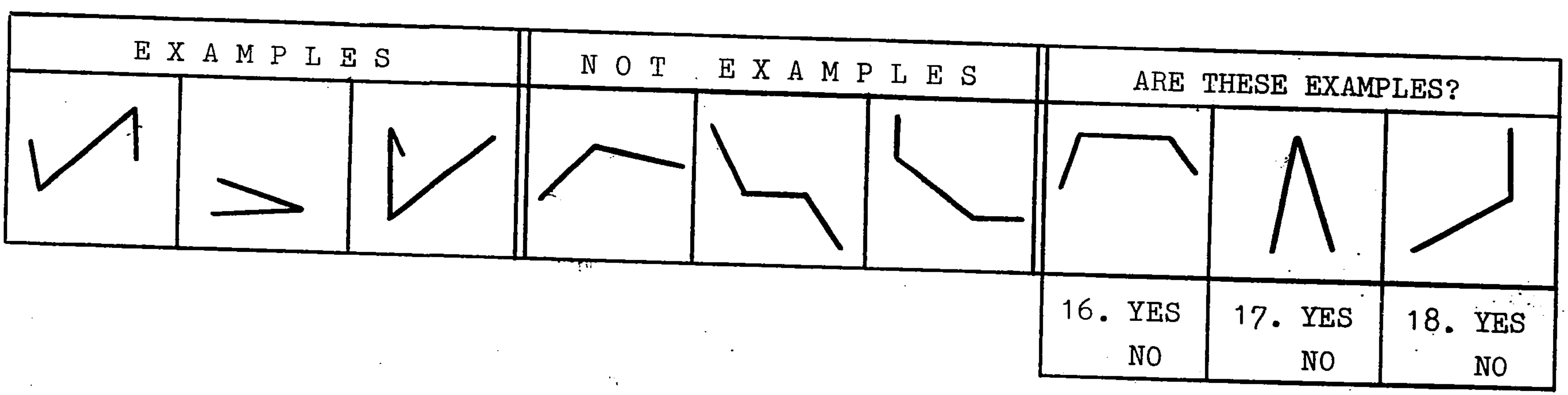
E X A M P L E S	N O T E X A M P L E S	ARE THESE EXAMPLES?

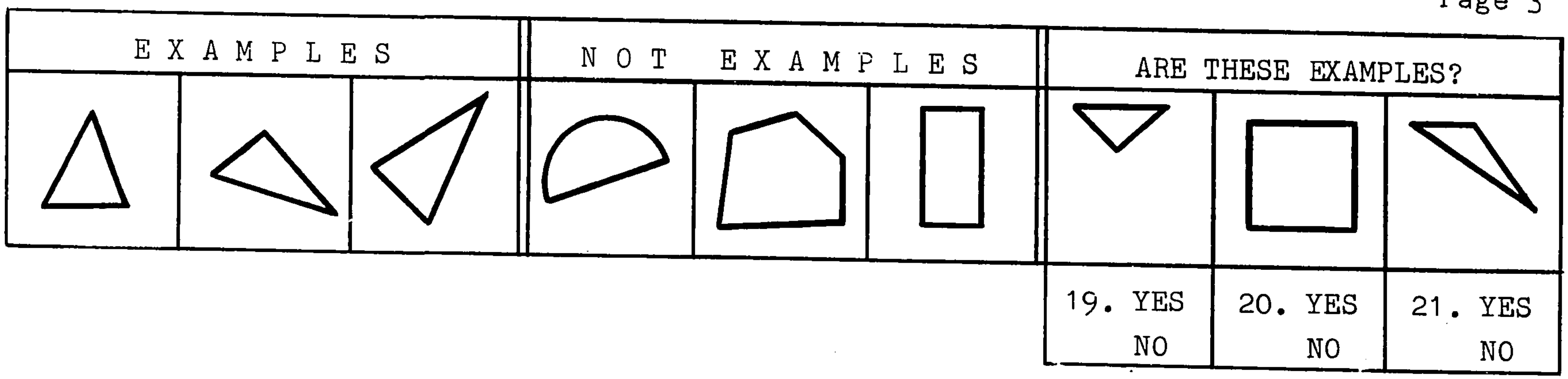


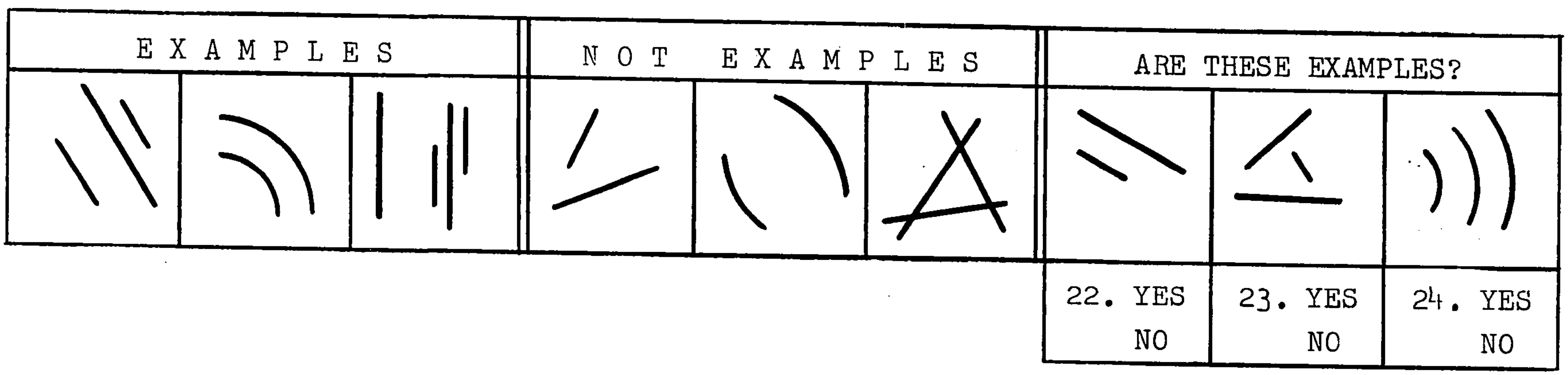
7. YES NO
8. YES NO
9. YES NO

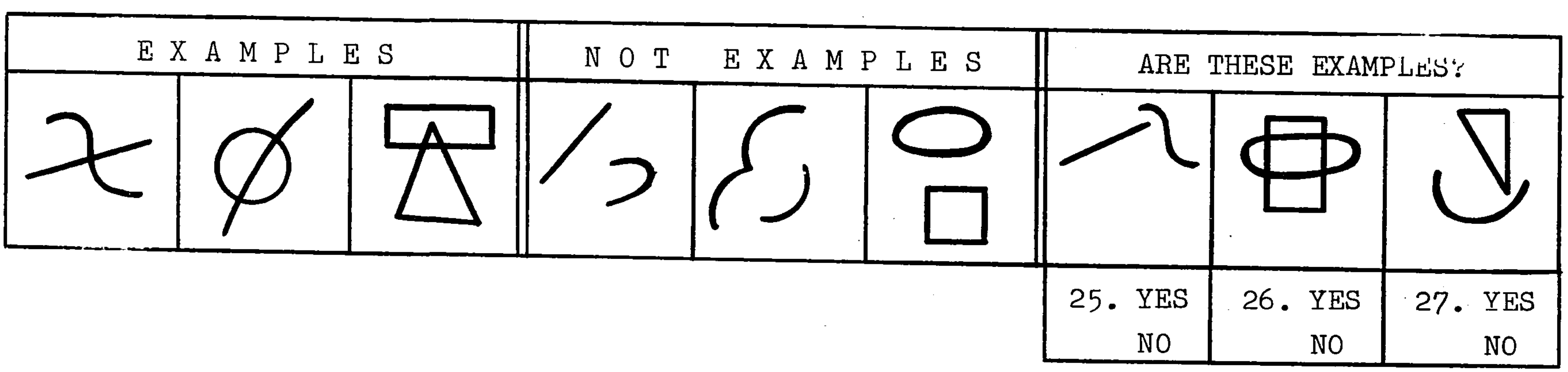

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
10. YES NO
11. YES NO
12. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
13. YES NO
14. YES NO
15. YES NO

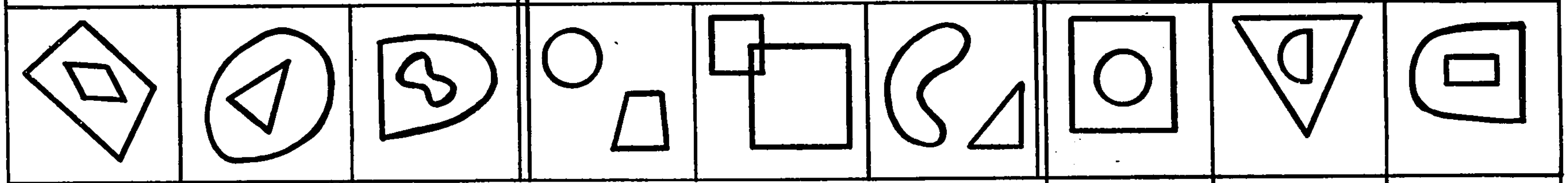

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
16. YES NO
17. YES NO
18. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
19. YES NO
20. YES NO
21. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
22. YES NO
23. YES NO
24. YES NO

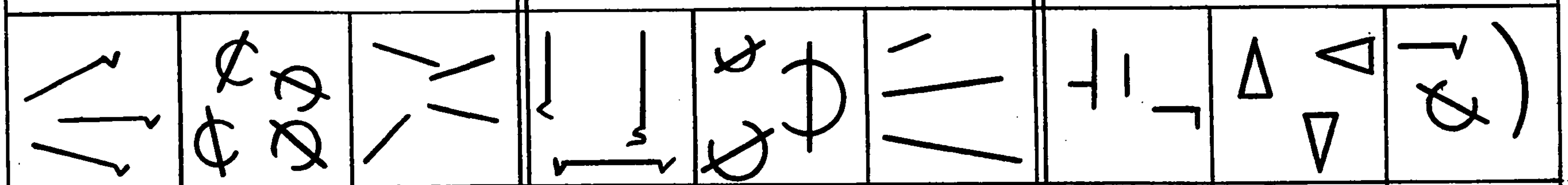

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
25. YES NO
26. YES NO
27. YES NO

E X A M P L E S | N O T E X A M P L E S | ARE THESE EXAMPLES?



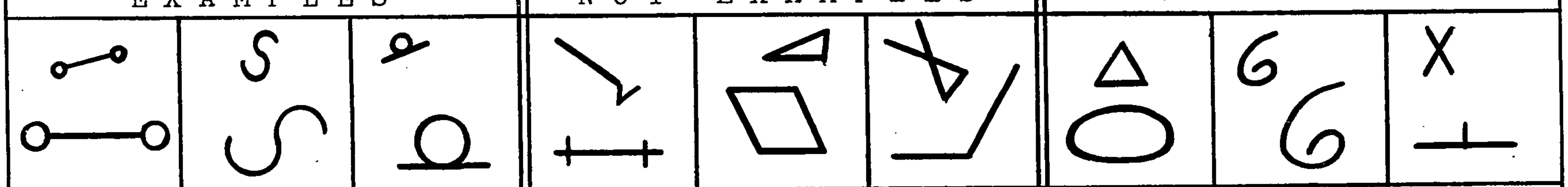
28. YES NO | 29. YES NO | 30. YES NO

E X A M P L E S | N O T E X A M P L E S | ARE THESE EXAMPLES?

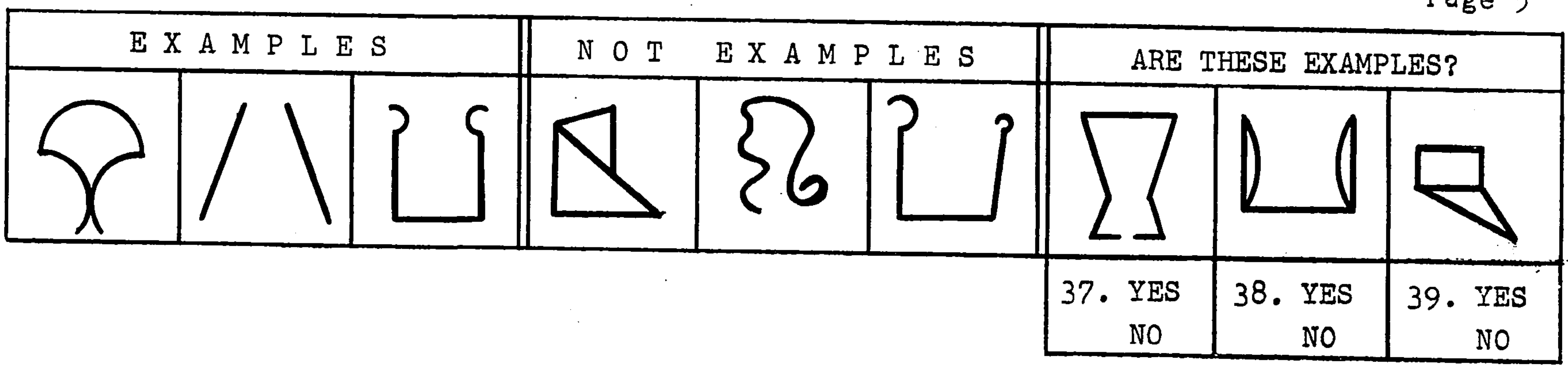


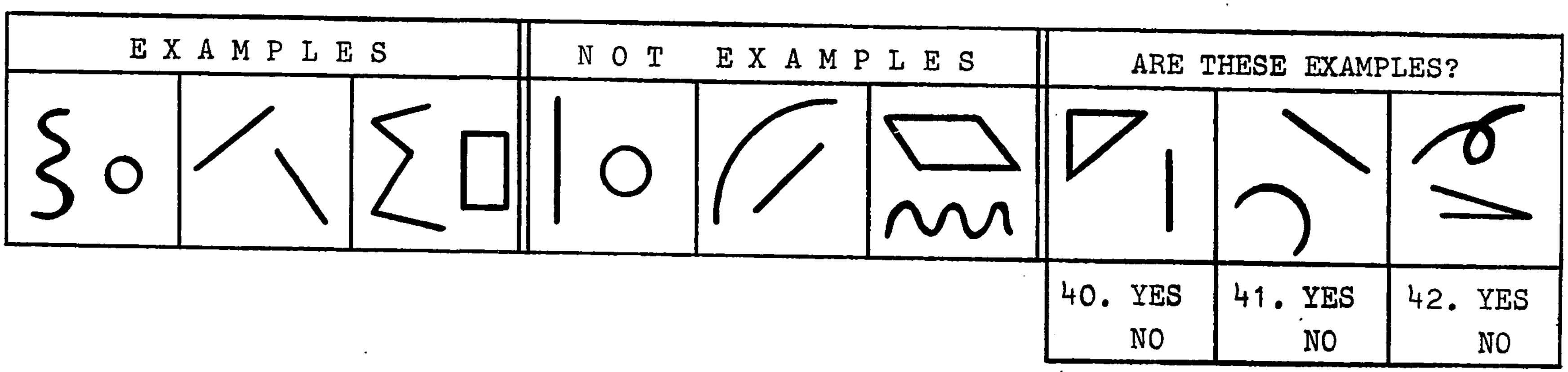
31. YES NO | 32. YES NO | 33. YES NO

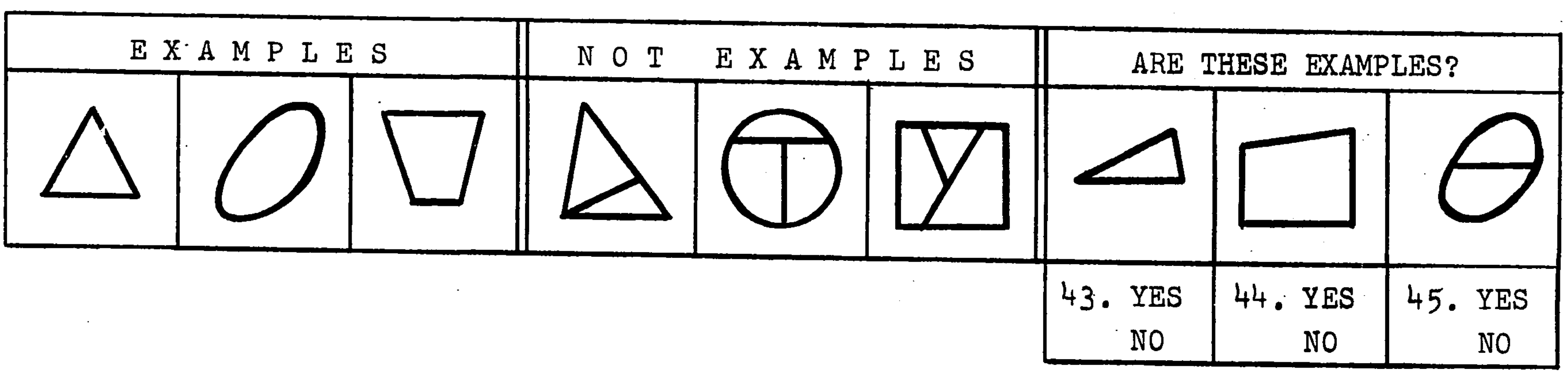
E X A M P L E S | N O T E X A M P L E S | ARE THESE EXAMPLES?



34. YES NO | 35. YES NO | 36. YES NO

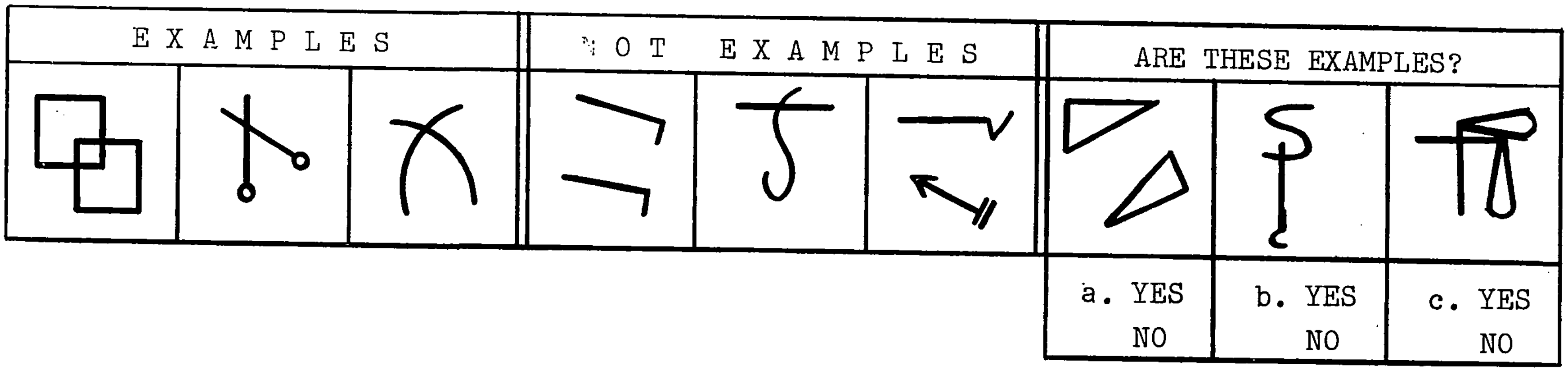

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
37. YES NO
38. YES NO
39. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
40. YES NO
41. YES NO
42. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
43. YES NO
44. YES NO
45. YES NO

SK4: PART II

PRACTICE PROBLEM



INSTRUCTIONS

In each of the problems in PART II, the figures marked "EXAMPLES" have two properties in common. The figures marked "NOT EXAMPLES" do not have both of these properties. They may have only one, or neither. To help you, in each problem (row of figures) the first "NOT EXAMPLE" has one of the properties, the second has the other, and the third has neither.

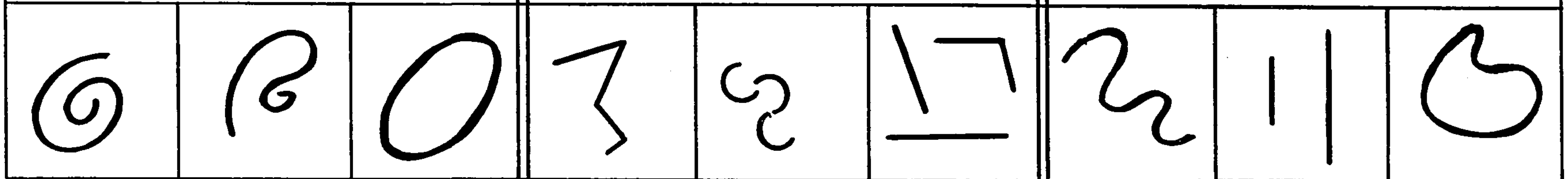
Your problem is to decide whether or not each of the figures under the question "ARE THESE EXAMPLES?" has both of the properties. If it has both properties, circle YES under that figure. If it does not have both properties, circle NO.

Look at the PRACTICE PROBLEM above. Each of the "EXAMPLES" is made up of two identical parts which intersect. In the first "NOT EXAMPLE," the parts are identical but do not intersect. In the second "NOT EXAMPLE," they intersect but are not identical. In the third "NOT EXAMPLE," they are neither identical nor intersecting.

For each of the figures under "ARE THESE EXAMPLES?", decide whether or not it has both properties. In the PRACTICE PROBLEM, figure a. has two identical parts but they do not intersect, so you should circle NO. Figure b. does have two intersecting parts but they are not identical, so circle NO under figure b. Figure c. does have two identical parts and they do intersect so circle YES under figure c.

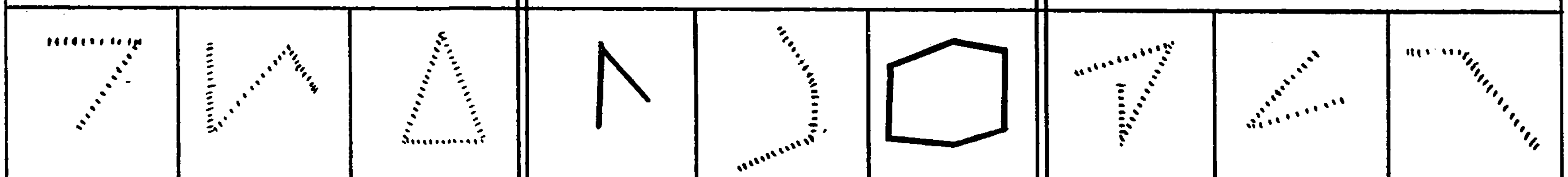
All of the problems in PART II are of the same kind as the PRACTICE PROBLEM, BUT one, two, or all three of the figures under "ARE THESE EXAMPLES?" may be examples

E X A M P L E S | N O T E X A M P L E S | ARE THESE EXAMPLES?



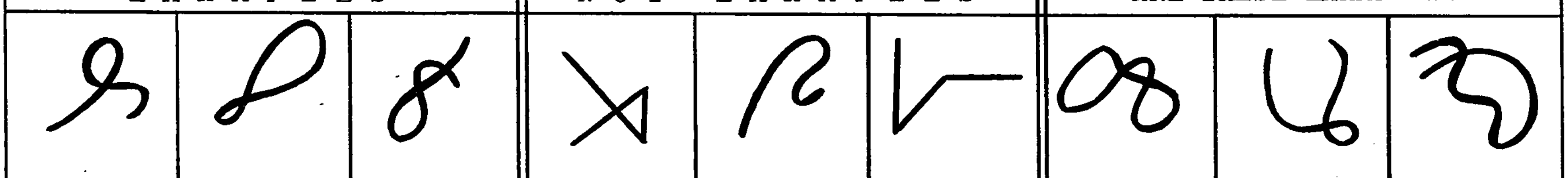
1. YES NO	2. YES NO	3. YES NO

E X A M P L E S | N O T E X A M P L E S | ARE THESE EXAMPLES?



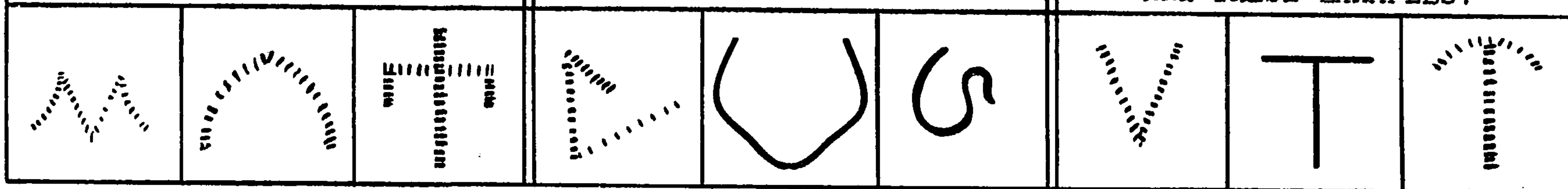
4. YES NO	5. YES NO	6. YES NO

E X A M P L E S | N O T E X A M P L E S | ARE THESE EXAMPLES?



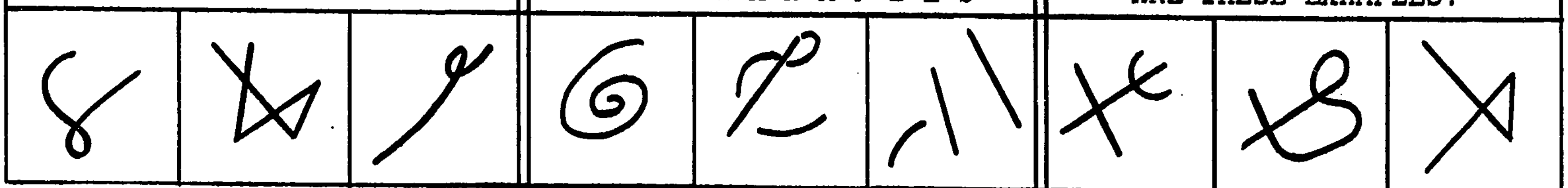
7. YES NO	8. YES NO	9. YES NO

EXAMPLES	NOT EXAMPLES	ARE THESE EXAMPLES?



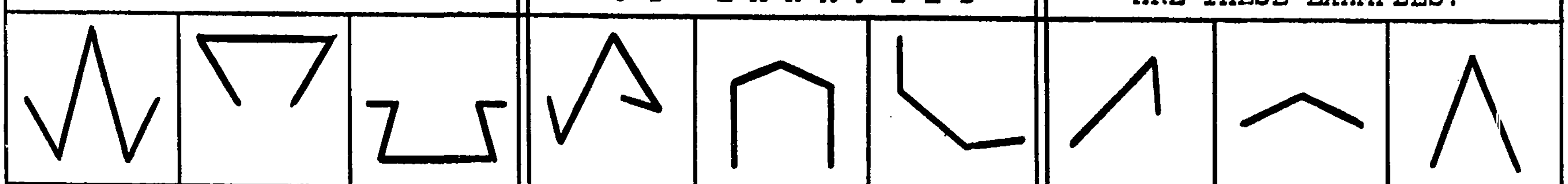
10. YES NO
11. YES NO
12. YES NO

EXAMPLES	NOT EXAMPLES	ARE THESE EXAMPLES?

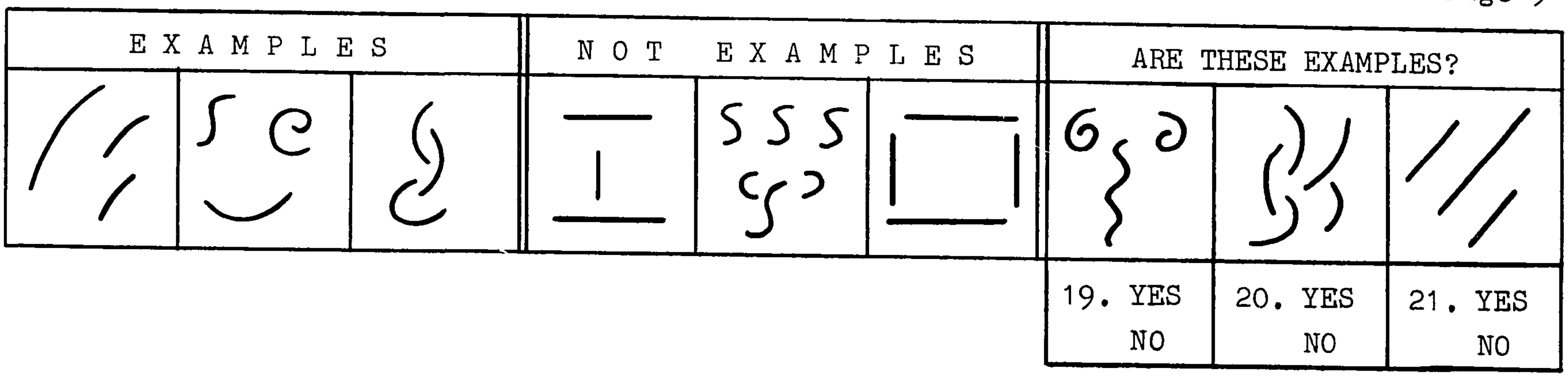


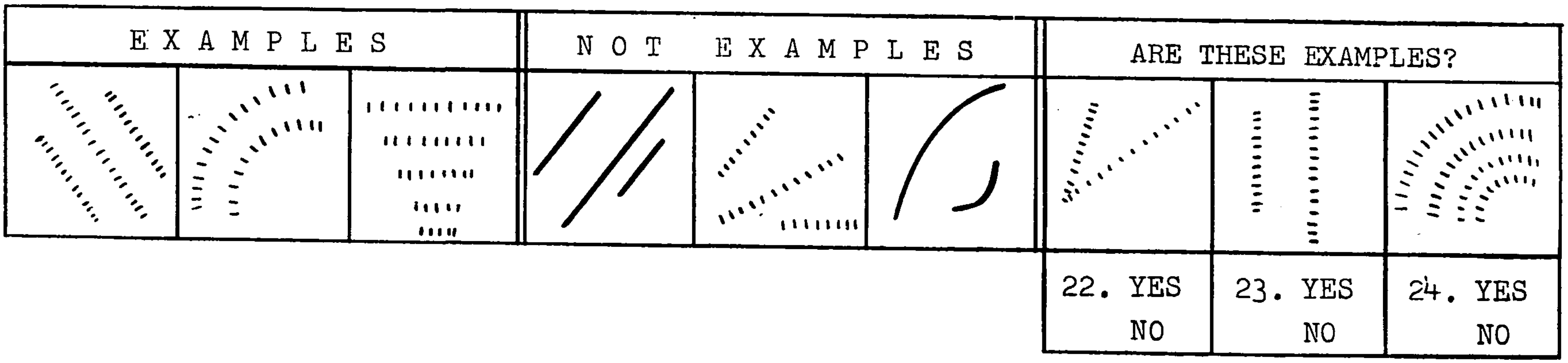
13. YES NO
14. YES NO
15. YES NO

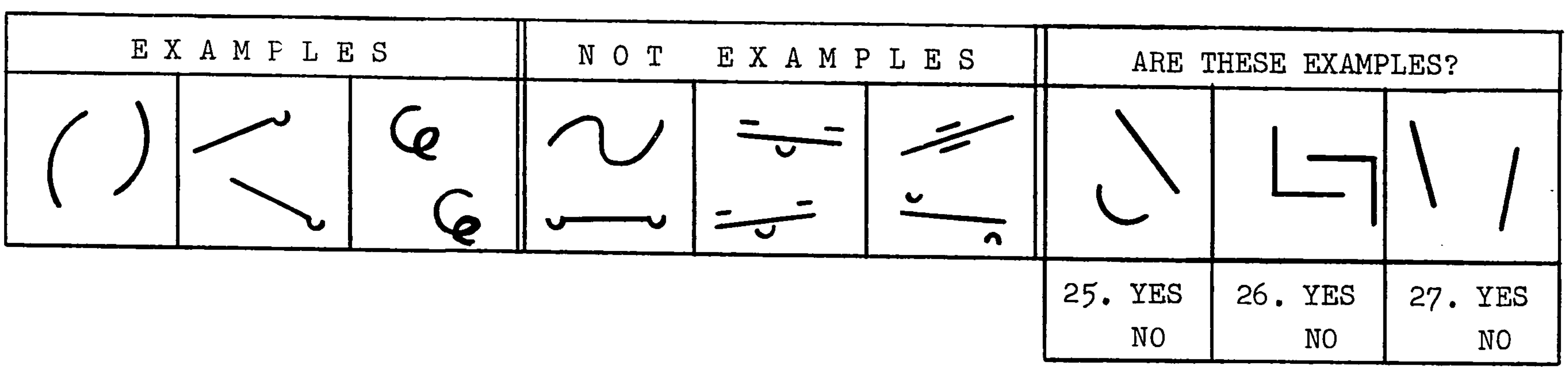
EXAMPLES	NOT EXAMPLES	ARE THESE EXAMPLES?

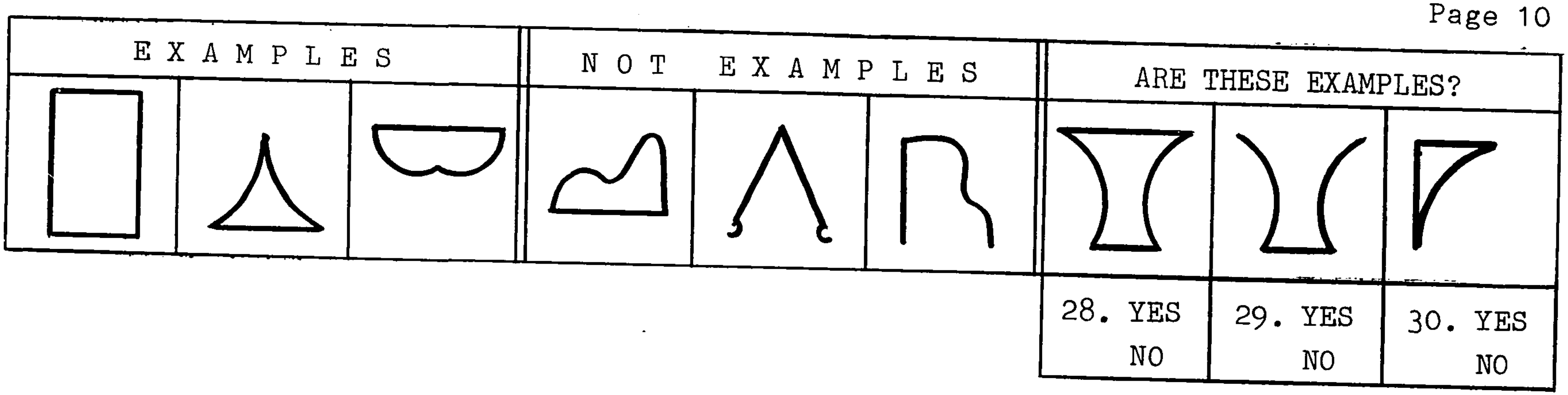


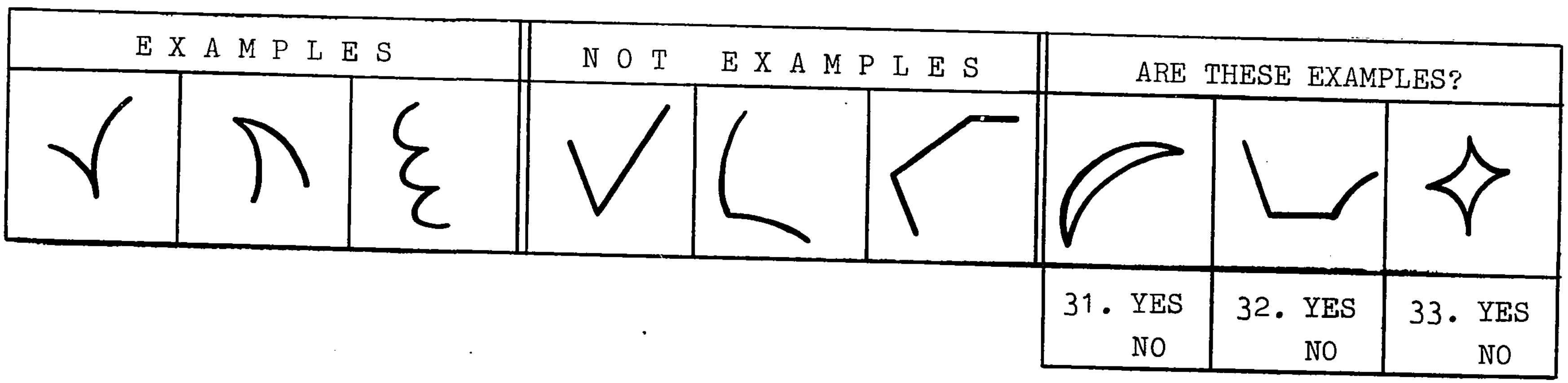
16. YES NO
17. YES NO
18. YES NO

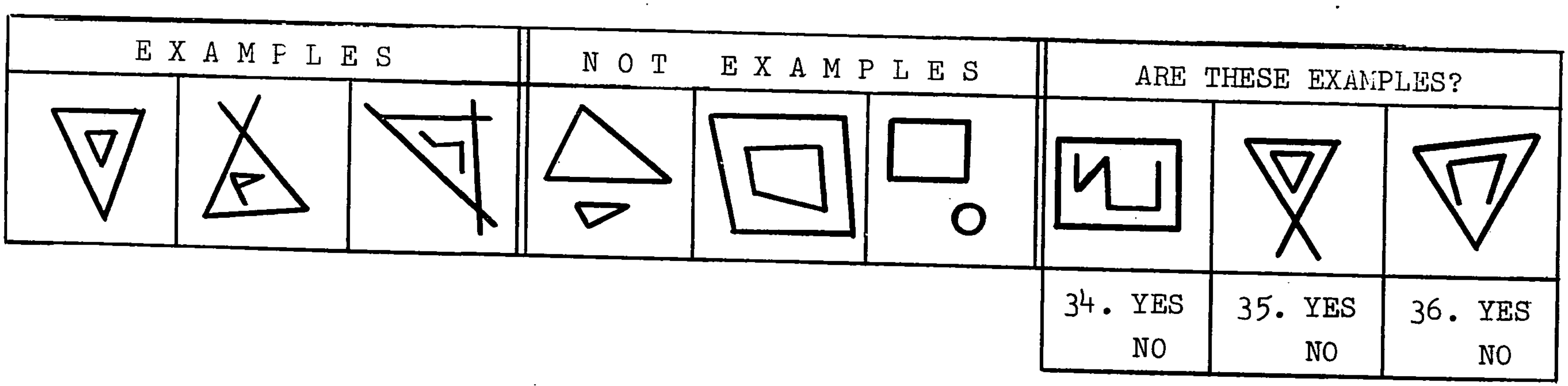

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
19. YES
NO
20. YES
NO
21. YES
NO

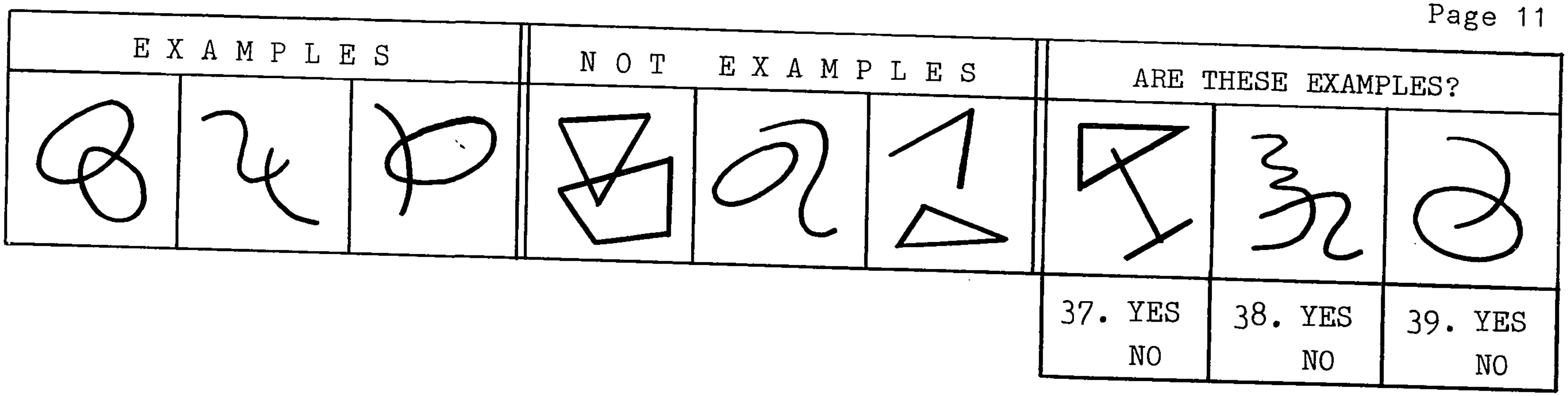

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
22. YES
NO
23. YES
NO
24. YES
NO

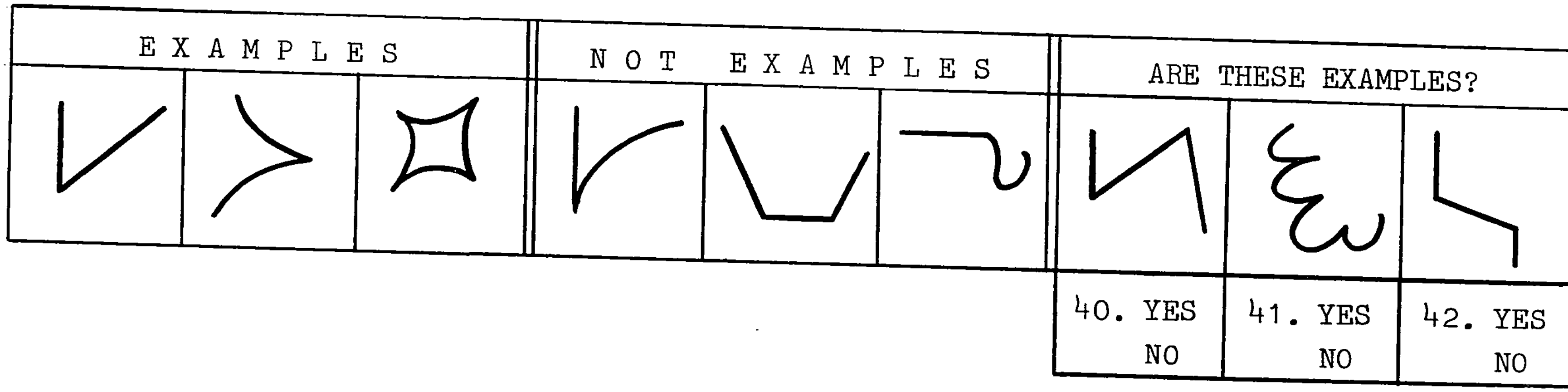

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
25. YES
NO
26. YES
NO
27. YES
NO

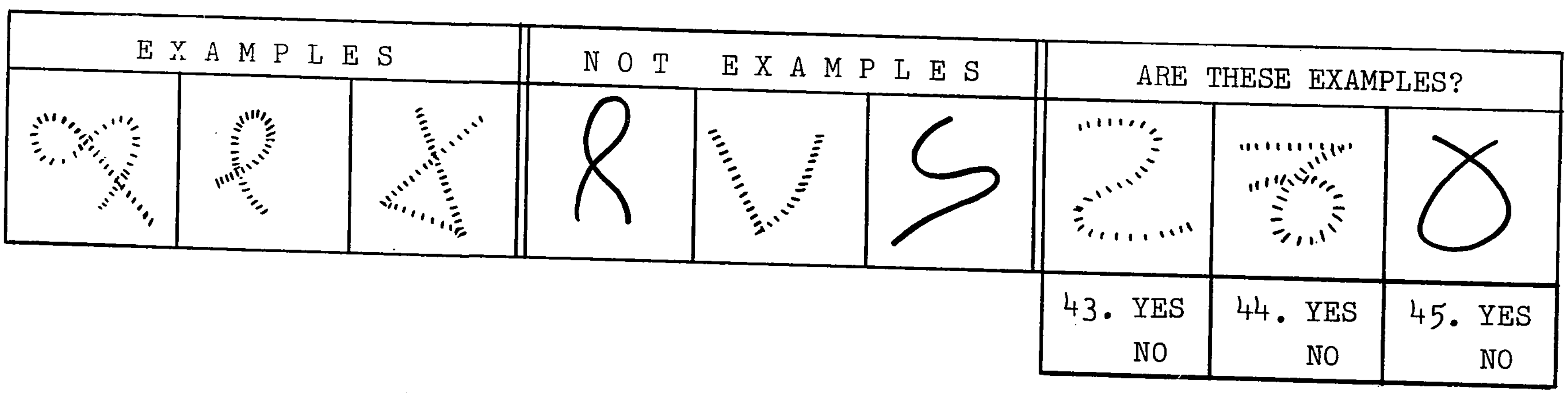

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
28. YES NO
29. YES NO
30. YES NO

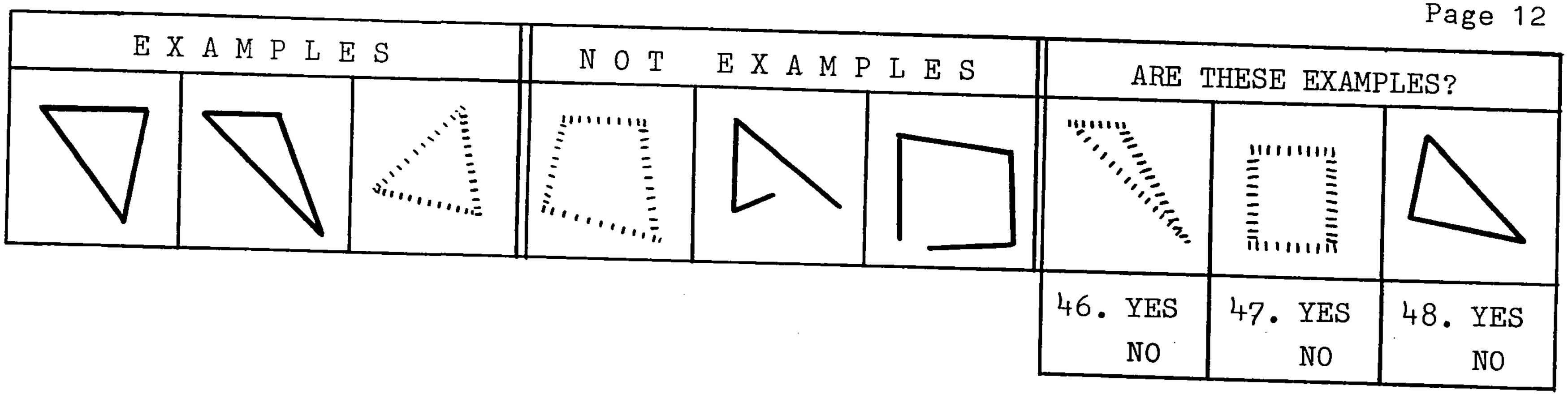

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
31. YES NO
32. YES NO
33. YES NO

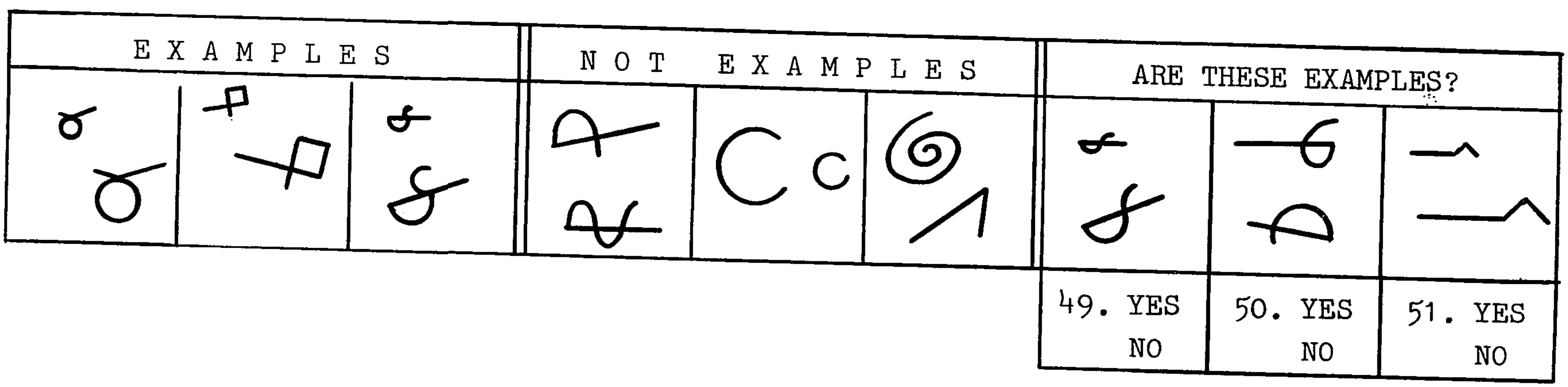

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
34. YES NO
35. YES NO
36. YES NO

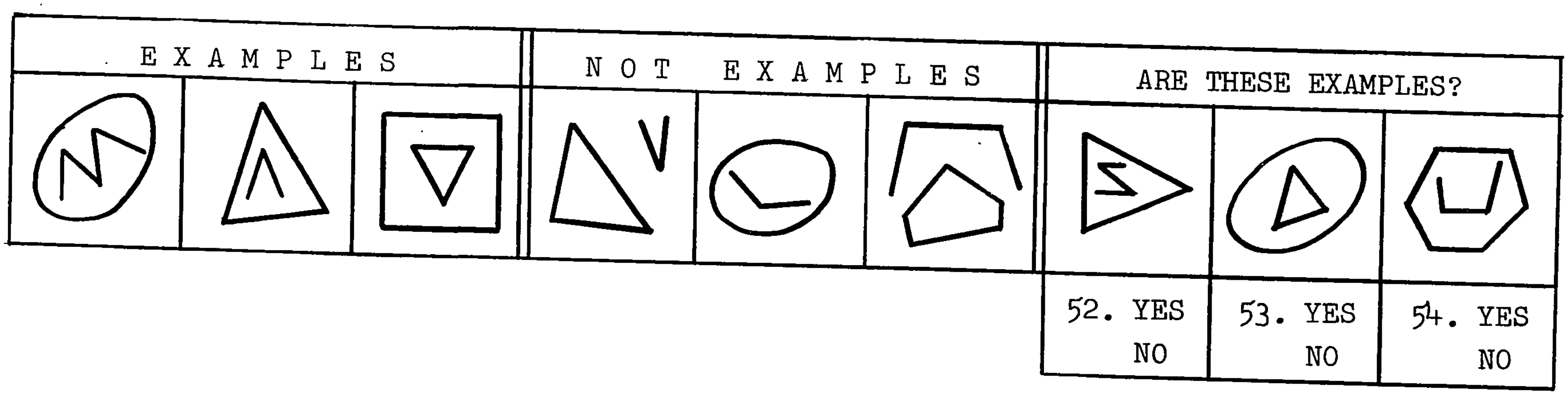

E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
37. YES NO
38. YES NO
39. YES NO

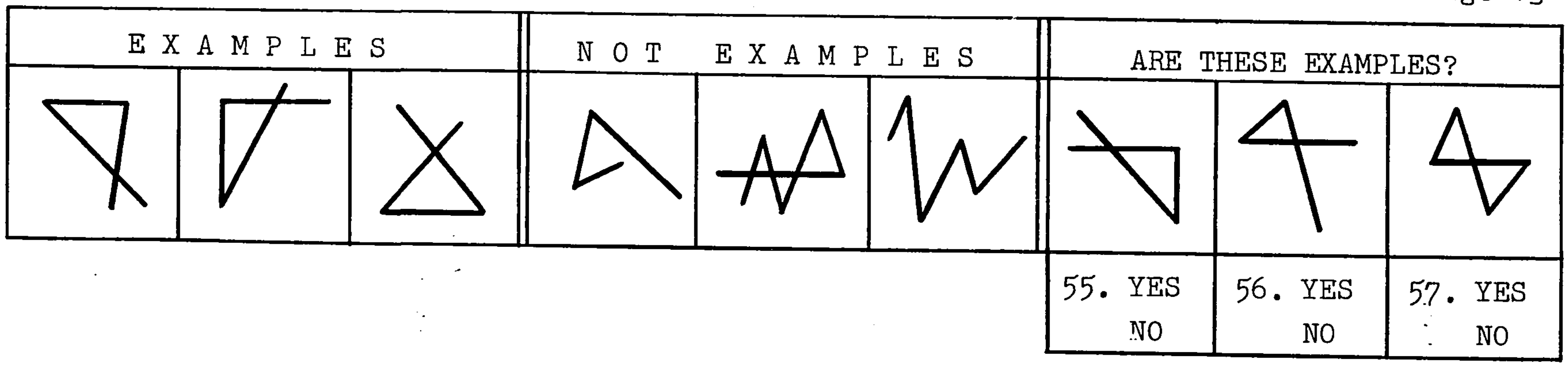

E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
40. YES NO
41. YES NO
42. YES NO

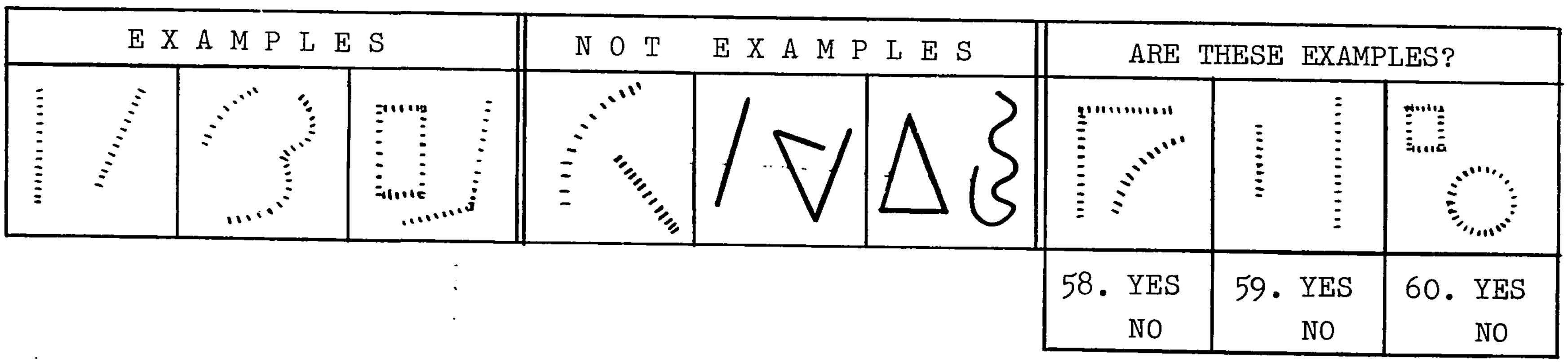

E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
43. YES NO
44. YES NO
45. YES NO

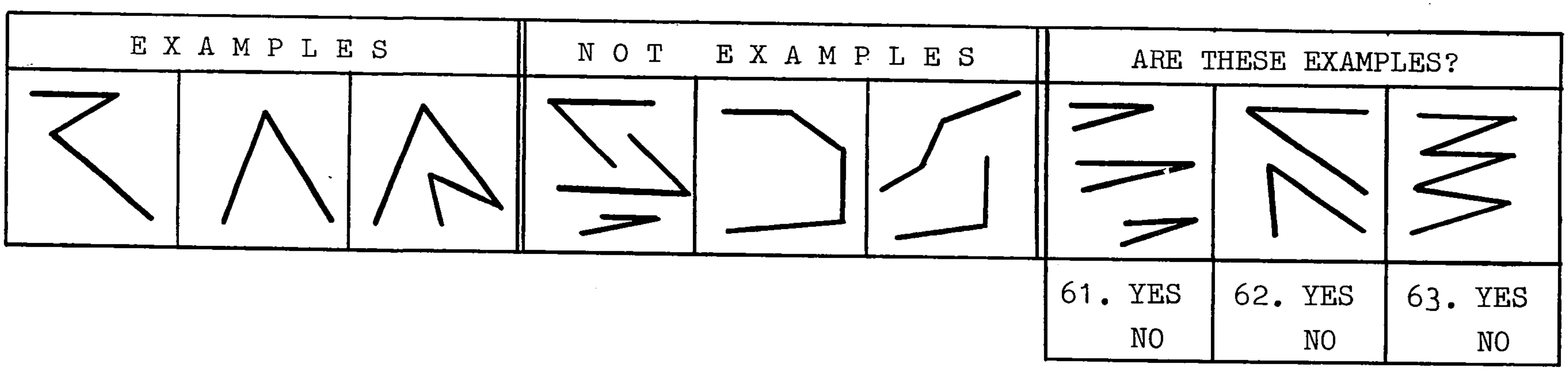

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
46. YES NO
47. YES NO
48. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
49. YES NO
50. YES NO
51. YES NO

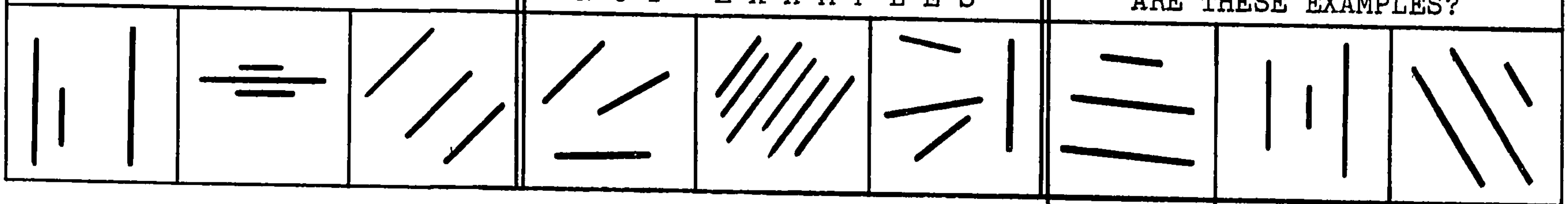

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
52. YES NO
53. YES NO
54. YES NO


E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
55. YES
NO
56. YES
NO
57. YES
NO


E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
58. YES
NO
59. YES
NO
60. YES
NO

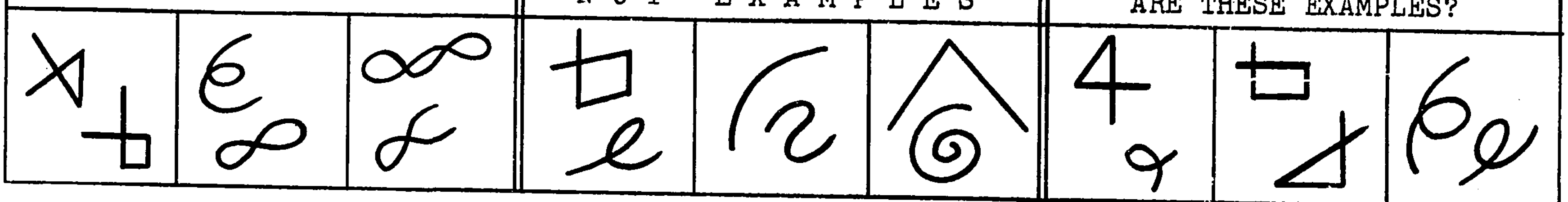

E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
61. YES
NO
62. YES
NO
63. YES
NO

EXAMPLES | NOT EXAMPLES | ARE THESE EXAMPLES?



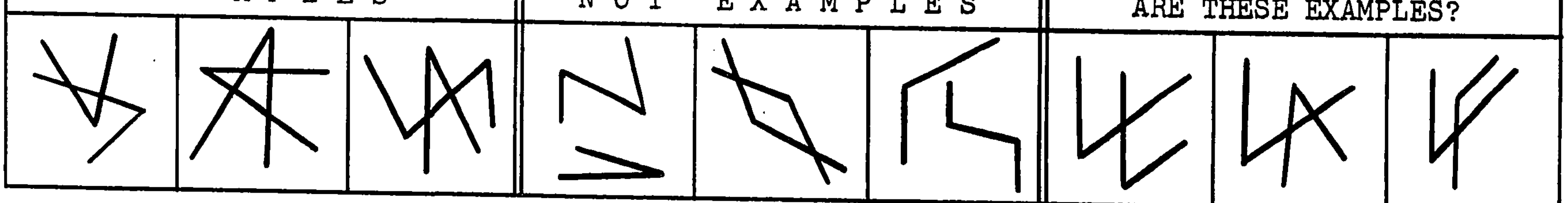
64. YES NO | 65. YES NO | 66. YES NO

EXAMPLES | NOT EXAMPLES | ARE THESE EXAMPLES?

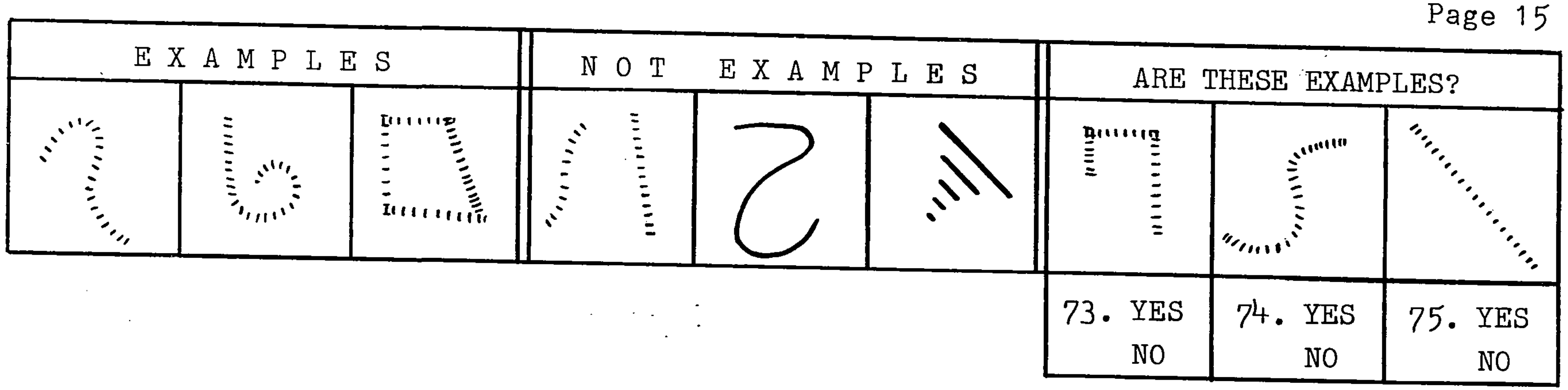


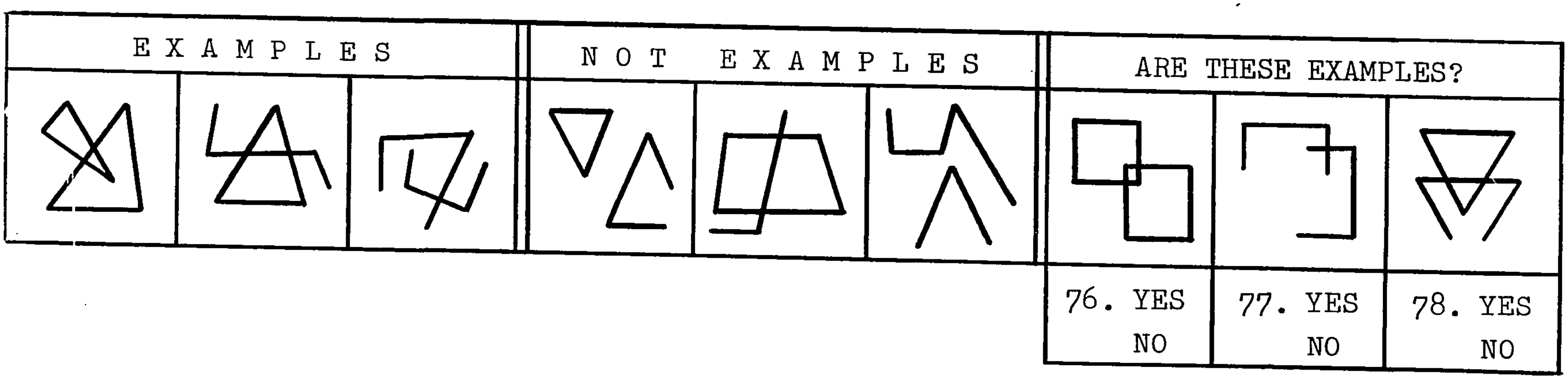
67. YES NO | 68. YES NO | 69. YES NO

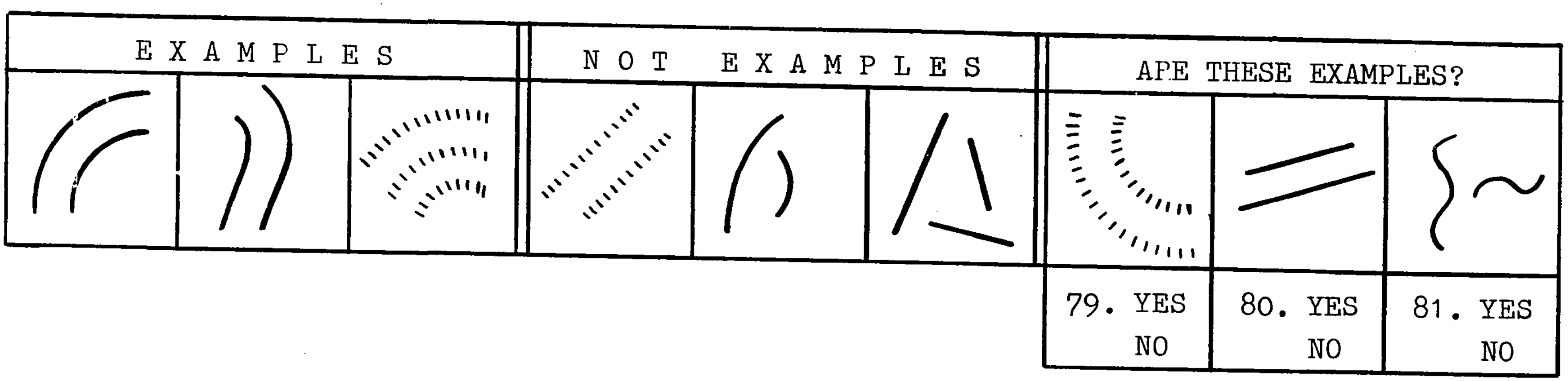
EXAMPLES | NOT EXAMPLES | ARE THESE EXAMPLES?



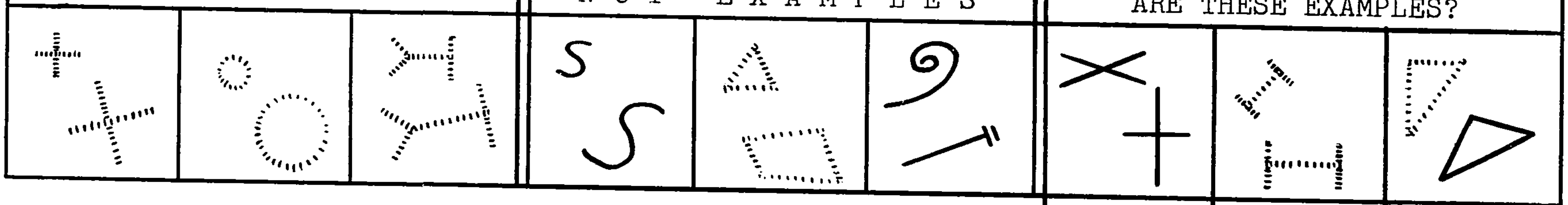
70. YES NO | 71. YES NO | 72. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
73. YES NO
74. YES NO
75. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
76. YES NO
77. YES NO
78. YES NO

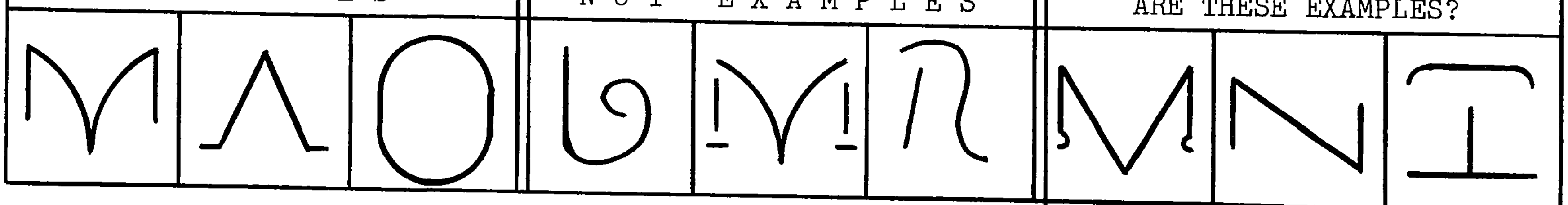

EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
79. YES NO
80. YES NO
81. YES NO

EXAMPLES | NOT EXAMPLES | ARE THESE EXAMPLES?



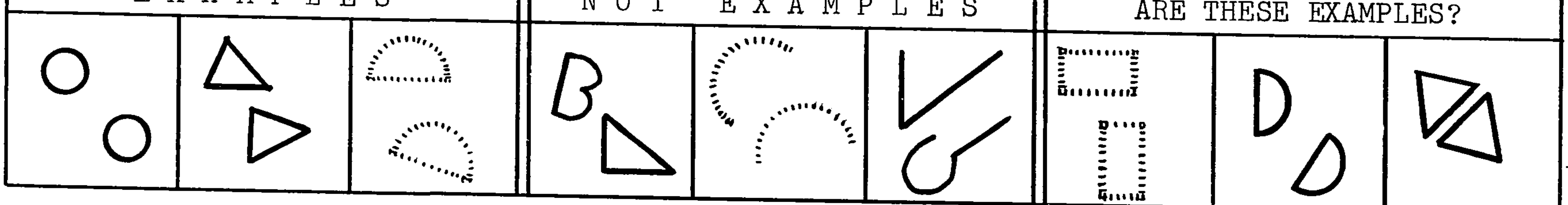
82. YES NO | 83. YES NO | 84. YES NO

EXAMPLES | NOT EXAMPLES | ARE THESE EXAMPLES?

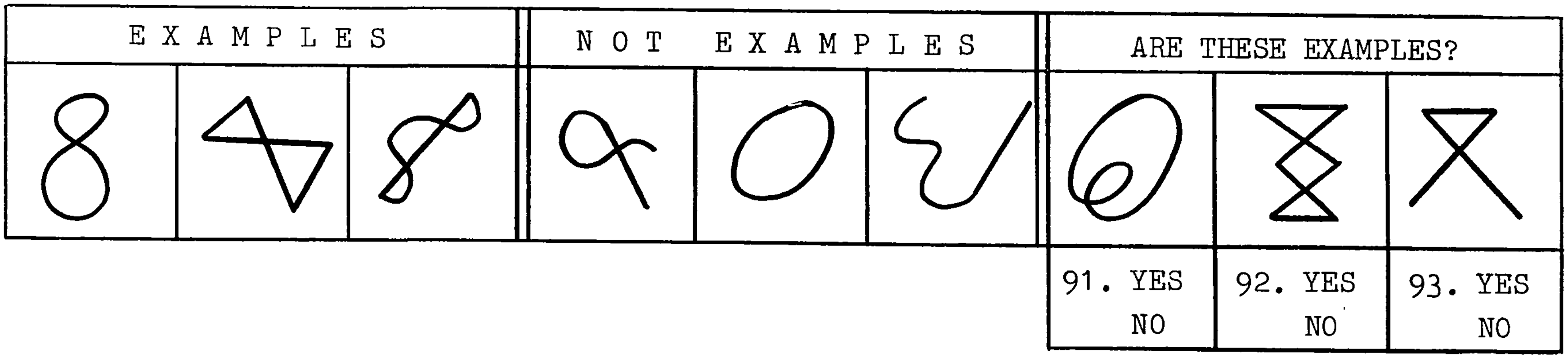


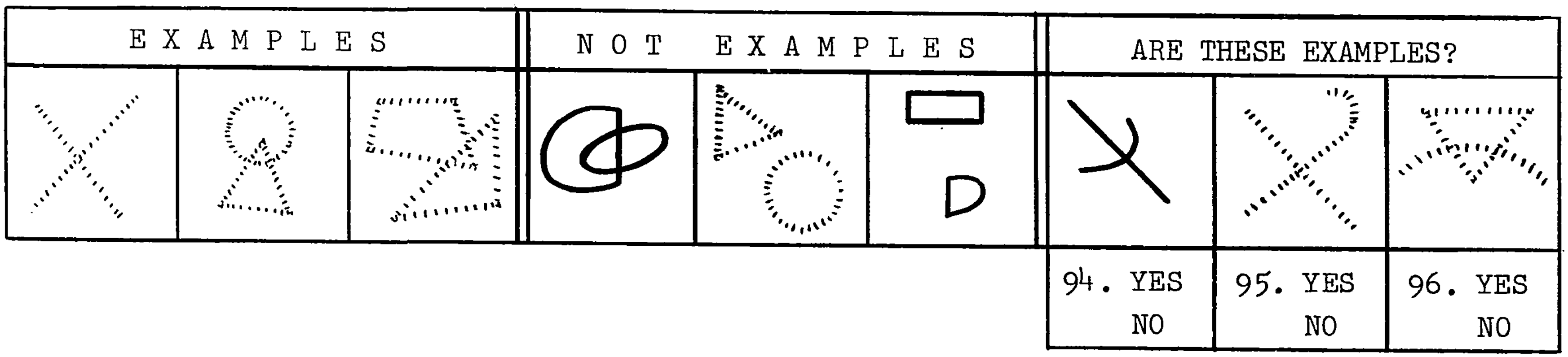
85. YES NO | 86. YES NO | 87. YES NO

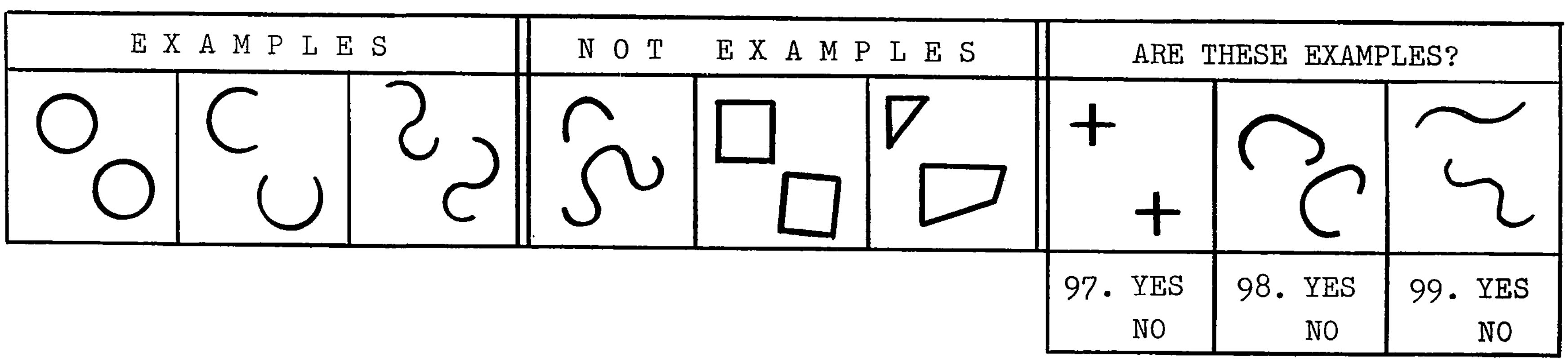
EXAMPLES | NOT EXAMPLES | ARE THESE EXAMPLES?

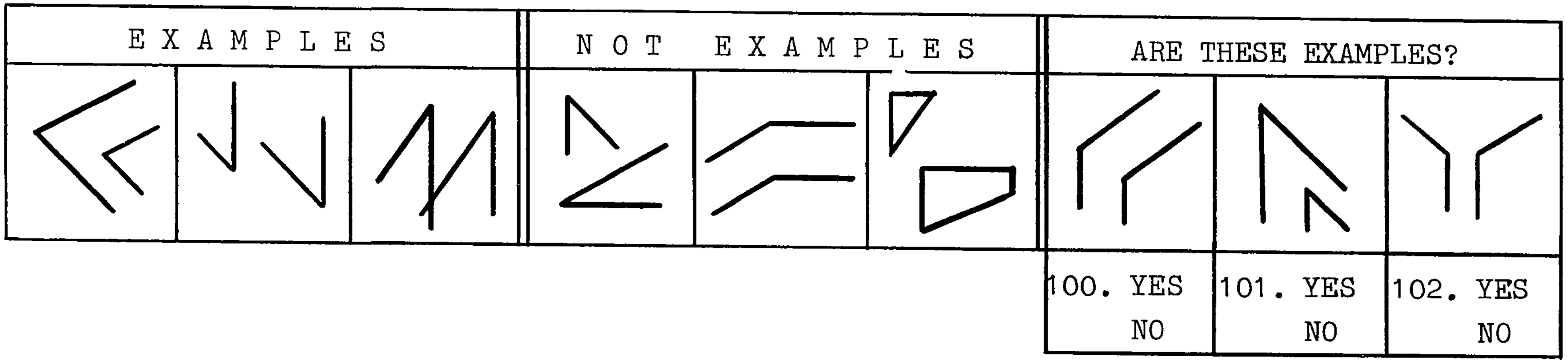


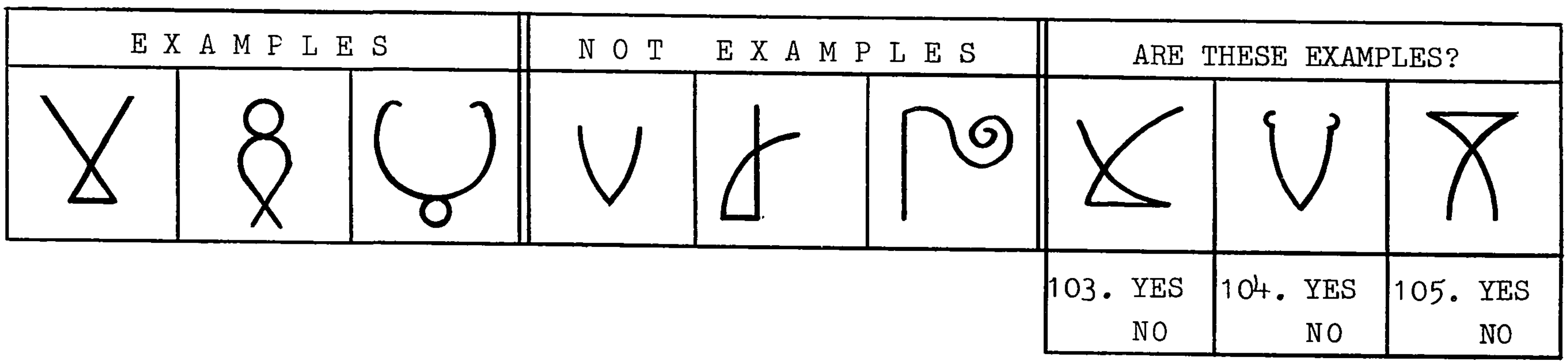
88. YES NO | 89. YES NO | 90. YES NO


E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
91. YES
NO
92. YES
NO
93. YES
NO


E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
94. YES
NO
95. YES
NO
96. YES
NO

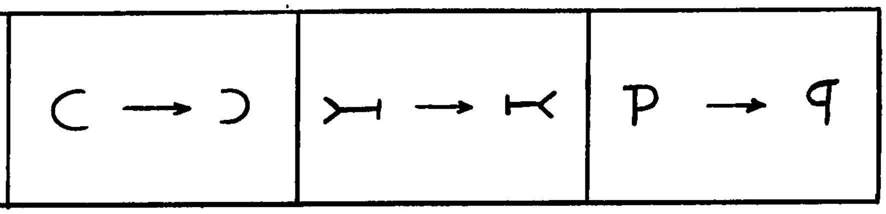

E X A M P L E S
N O T E X A M P L E S
ARE THESE EXAMPLES?
97. YES
NO
98. YES
NO
99. YES
NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
100. YES
NO
101. YES
NO
102. YES
NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
103. YES
NO
104. YES
NO
105. YES
NO

SK6: PRACTICE SHEET

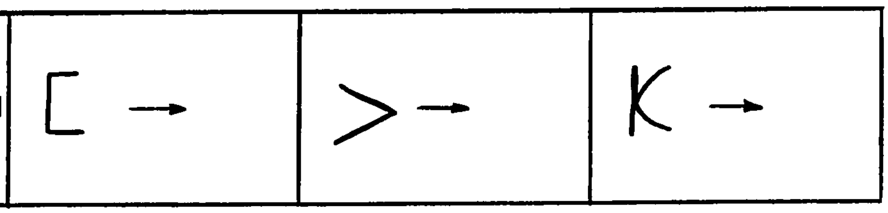
Operation 1



In the above figures, the one on the left of each pair has been changed to the one on the right by means of the same simple operation. In other words, the above figures give three examples of a particular operation. You have to find out what the operation is, and then do the same operation to some other figures.

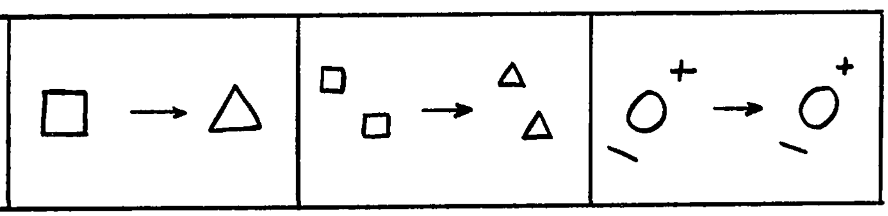
What is the operation? It is reversing from left to right. Do this on each of the figures below, and fill in the answers in the blank spaces. Check with the answers on the blackboard to make sure that you have understood.

Do Operation 1 on these.



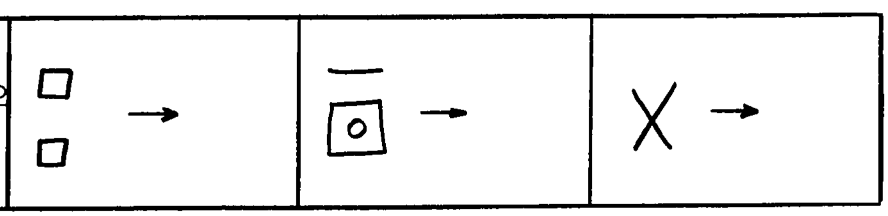
Here is a different operation:

Operation 2



When you have found out what it is, do it on the figures below. Check with the answers on the board.

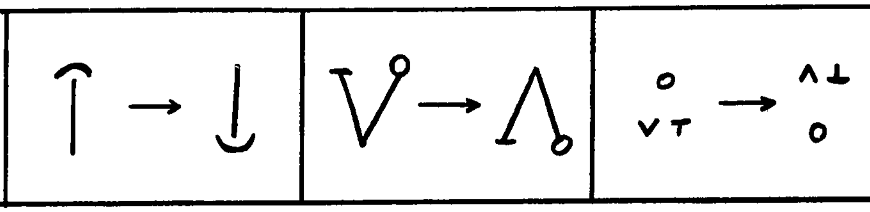
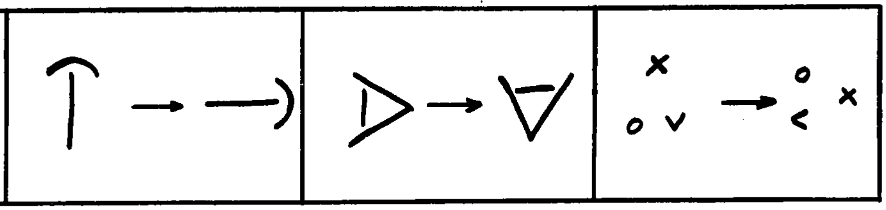
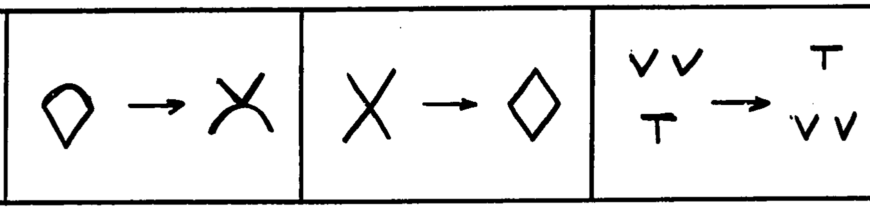
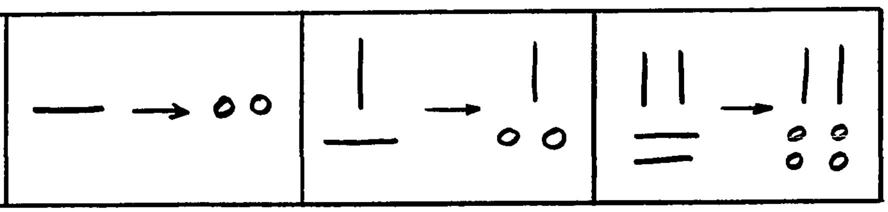
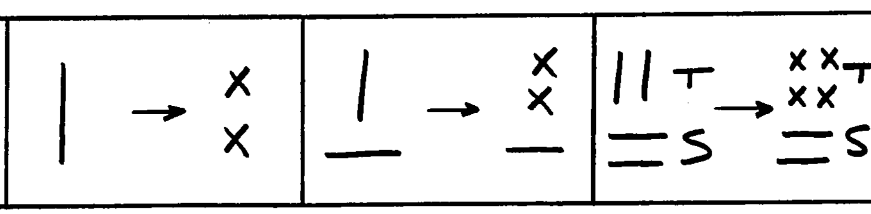
Do Operation 2 on these.



SK6: DEMONSTRATION SHEET

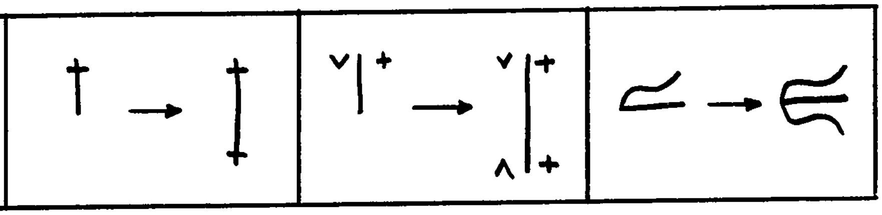
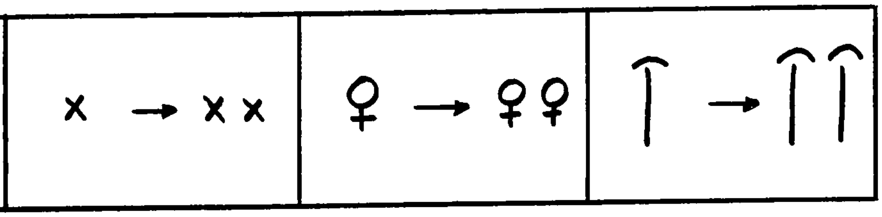
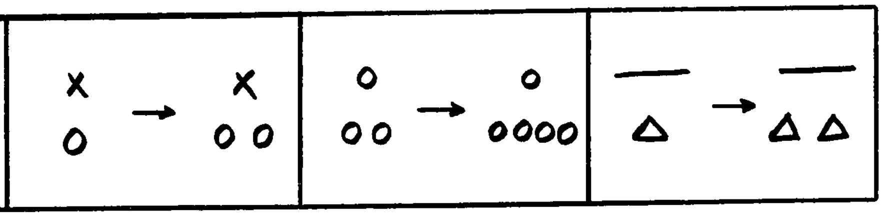
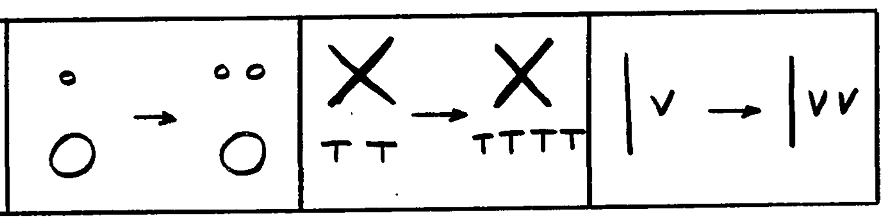
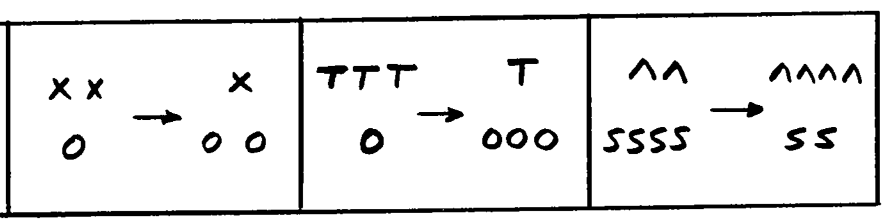
OPERATIONS A TO E

(OPERATIONS F TO J ARE ON THE NEXT PAGE)

Operation A	
Operation B	
Operation C	
Operation D	
Operation E	

SK6: DEMONSTRATION SHEET

OPERATIONS F TO J

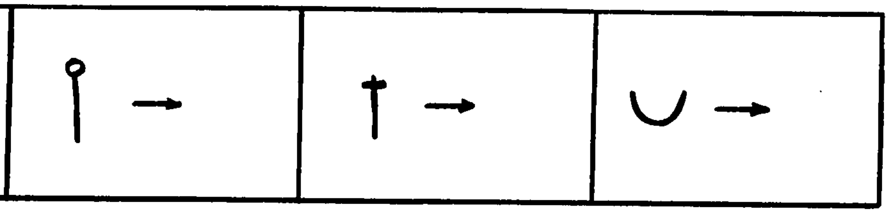
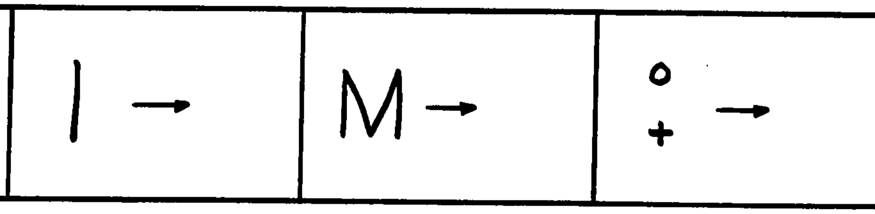
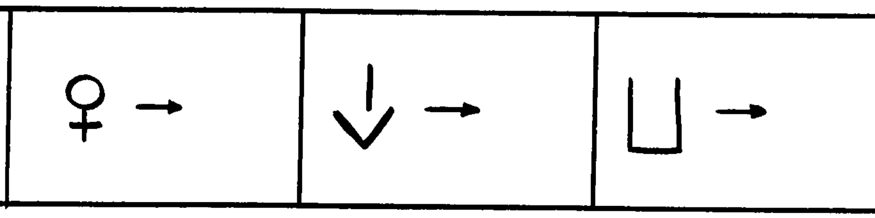
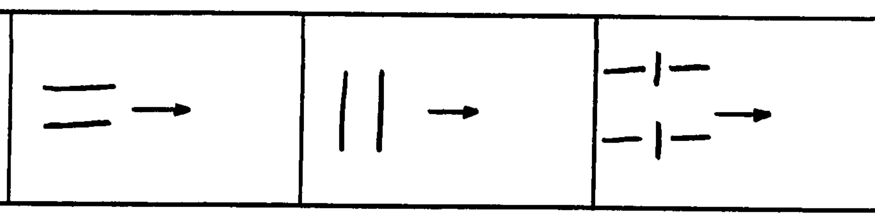
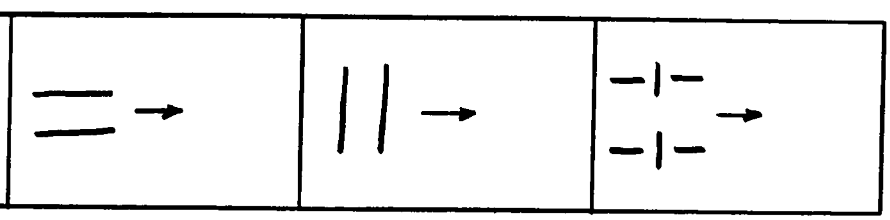
Operation F	
Operation G	
Operation H	
Operation I	
Operation J	

NAME .... SCHOOL ....
Last First Middle

AGE .... GRADE .... BOY GIRL DATE ....
Years (Circle One) Day Month Yr.

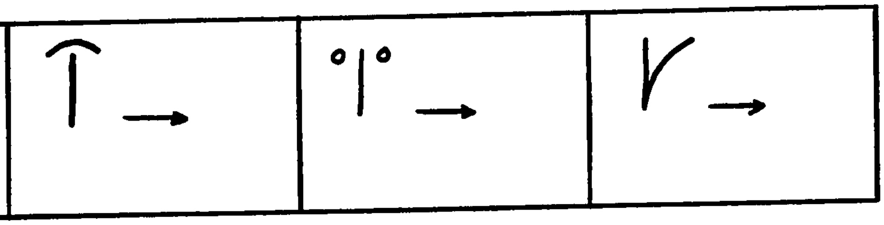
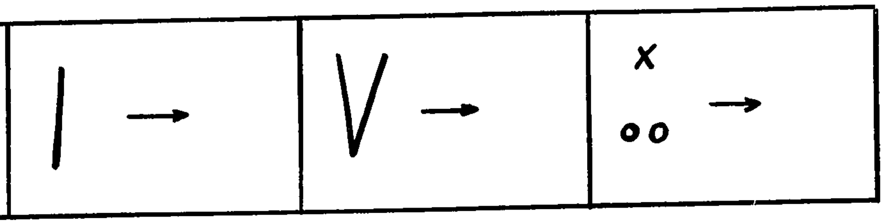
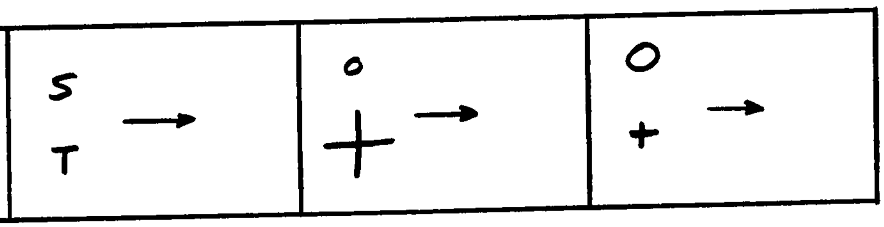
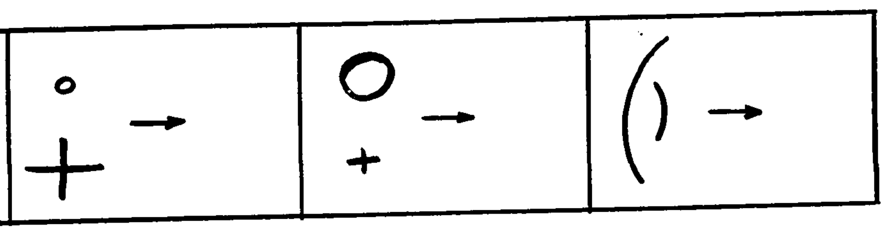
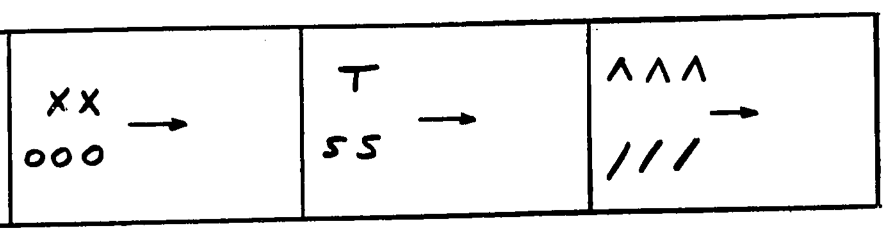
SK6: PART I

Find out the operations from the DEMONSTRATION SHEET, and fill in the answers in the blank spaces, just as you did on the PRACTICE SHEET.

Do Operation A on these.	
Do Operation B on these.	
Do Operation C on these.	
Do Operation D on these.	
Do Operation E on these.	

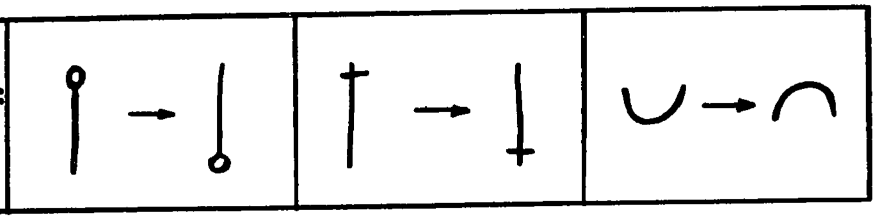
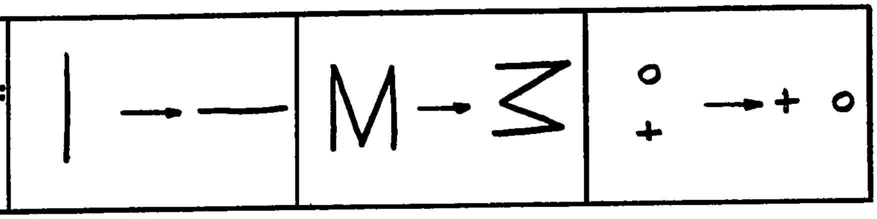
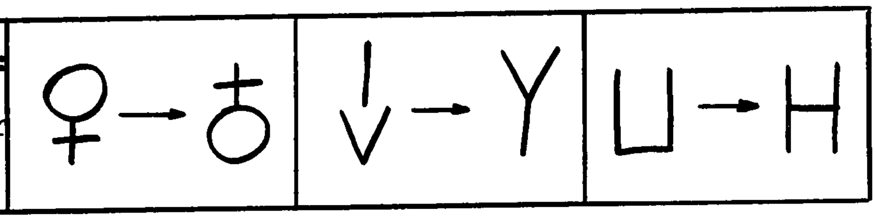
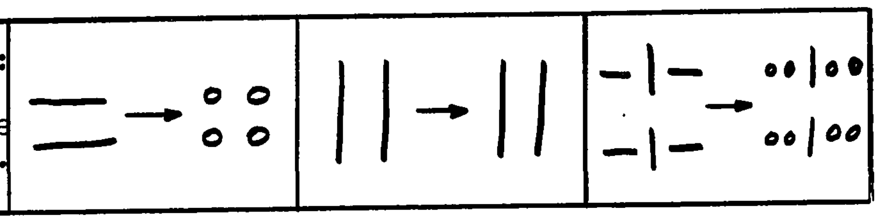
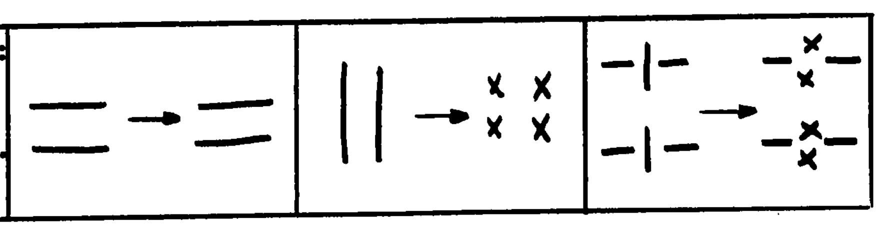
SK6: PART I

(CONTINUED)

Do Operation F on these.	
Do Operation G on these.	
Do Operation H on these.	
Do Operation I on these.	
Do Operation J on these.	

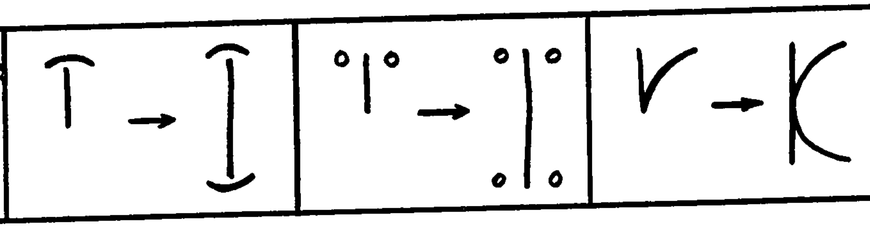
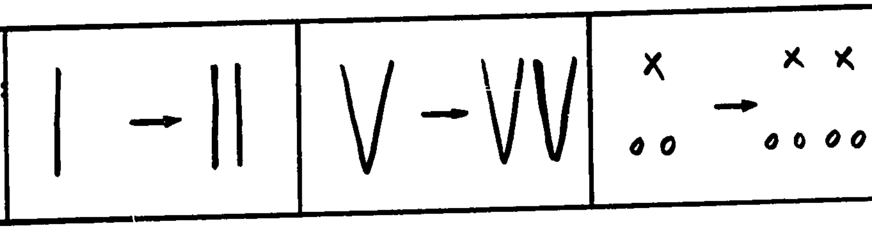
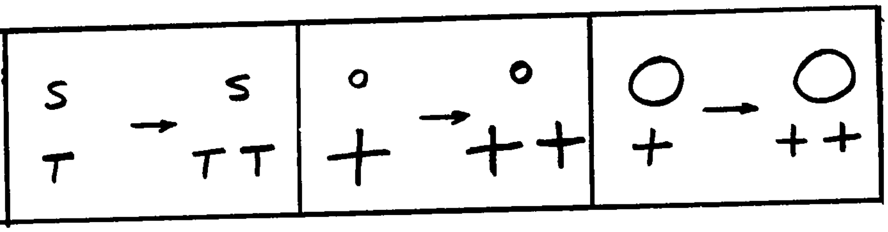
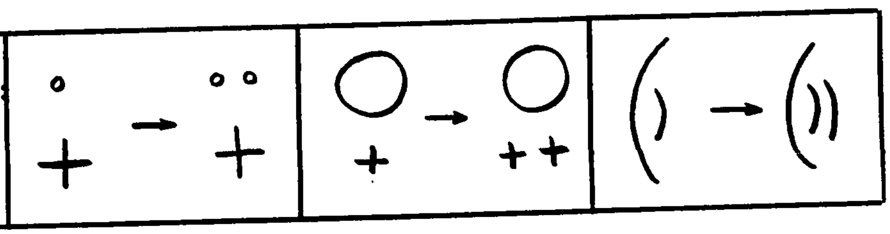
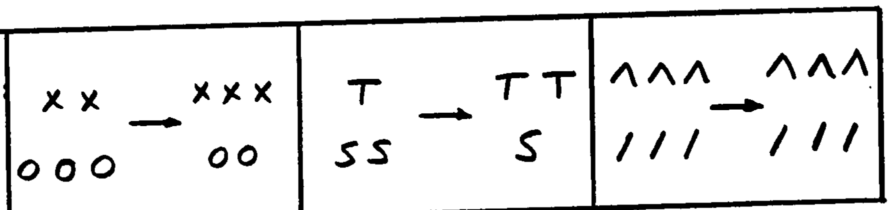
ANSWER SHEET FOR SK6, PART I

Here are the answers to the problems you did. Go through these carefully and put a tick in the right hand margin if you think that you got the whole line right. If you are not sure, ask for an explanation.

Operation A is: turn the other way up.	
Operation B is: rotate a quarter turn clockwise.	
Operation C is: interchange upper and lower parts.	
Operation D is: replace each horizontal line by two circles.	
Operation E is: replace each vertical line by two crosses.	

ANSWER SHEET FOR SK6, PART I

(CONTINUED)

Operation F is: add a symmetrical lower half.	
Operation G is: double everything.	
Operation H is: double the lower part.	
Operation I is: double the smaller part.	
Operation J is: interchange the numbers.	

NAME .... SCHOOL ....
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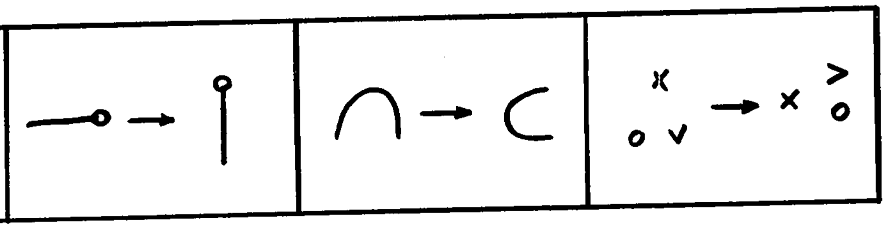
Years Day Month Yr.

SK6: PART II

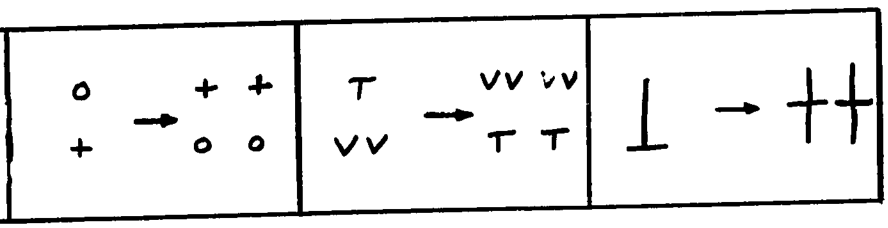
In PART II the problem is to combine the operations on the DEMONSTRATION SHEET, or to do them in reverse, or both. When combining operations, they are to be done in the order given (i.e., "Combine C and G" means "Do Operation C first and then do Operation G.")

Look at the examples given below and then carry out the operations indicated on the following three pages.

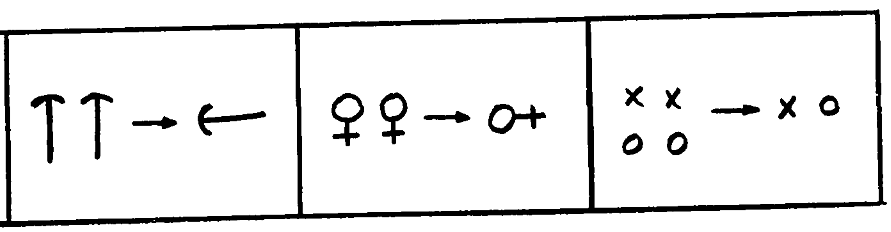
EXAMPLE:
Reverse B



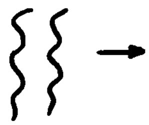
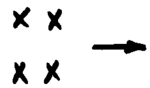
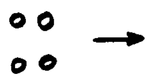
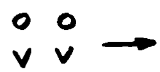
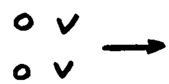
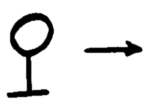
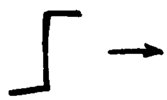
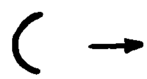
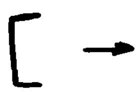
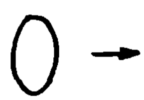
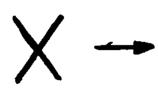
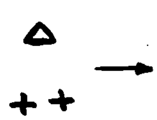
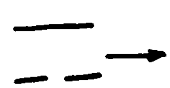
EXAMPLE:
Combine C & G



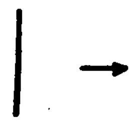
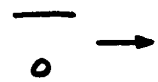
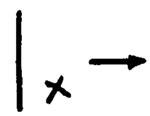
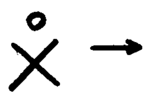
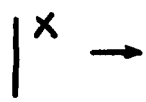
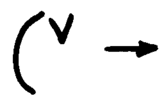
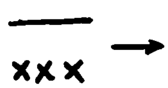
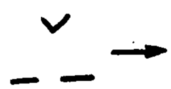
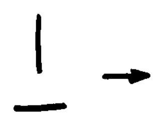
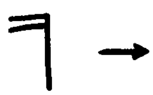
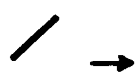
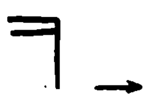
EXAMPLE:
Reverse and
Combine G & B



SK6: PART II

Reverse G			
Reverse D			
Reverse C			
Reverse F			
Reverse H			

SK6: PART II

Combine E & H			
Combine A & I			
Combine D & J			
Combine B & F			
Combine F & B			

SK6: PART II

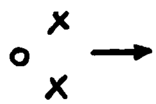
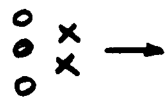
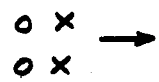
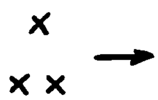
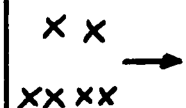
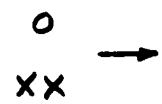
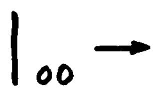
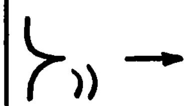
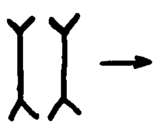
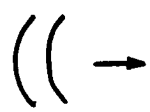
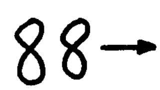
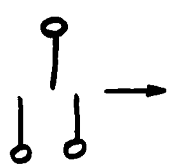
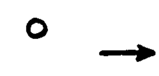
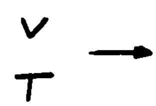
Reverse and Combine B & J			
Reverse and Combine H & E			
Reverse and Combine A & I			
Reverse and Combine F & G			
Reverse and Combine A & C			

TABLE LXXII

INDIVIDUAL COEFFICIENTS ON PRINCIPAL VECTORS
GROUP A

I.D.	I	II	III	IV	V
100	.925	1.266	-.167	-.001	-.281
200	.770	-.320	-.010	.404	.242
300	1.108	1.555	1.645	-.173	-1.743
400	.715	.906	.520	-1.073	-.540
500	.933	-.674	-.340	-.426	.136
600	.953	1.459	.063	.703	-.804
700	.431	-.156	.169	.125	-.101
168	1.135	.693	-.947	-.463	.998
131	.809	.355	-.051	.477	-.349
174	1.092	-.784	.216	1.292	1.204
165	1.049	-.649	-.570	-.466	.705
161	.991	-.415	.183	.764	-.294
155	.615	-.119	-.081	-.075	-.261
105	1.021	.420	-.505	-1.131	-.010
115	1.114	-.706	-.966	.424	-.557
103	1.088	.751	.173	.268	.122
171	.870	2.437	.841	1.057	-1.061
222	.906	.313	2.101	1.359	.329
229	1.097	1.710	-1.921	-2.148	.672
232	1.015	.933	-.508	-1.285	.181
210	.774	.344	-.718	-.017	-.092
211	1.072	1.967	-.247	-1.672	-1.793
311	1.047	-1.290	-.746	.550	-.726
317	1.068	-.269	.628	.123	.921
324	.973	.219	.154	.678	.336
302	.607	.784	-.270	-.149	-.517
309	1.212	-.790	.912	3.562	1.994
318	.751	1.287	.196	-.491	-.087
468	1.194	.277	.210	1.765	.882
470	.589	-.088	-.024	-.208	.097
421	1.072	-.309	.692	-.028	1.199
475	.565	.107	.067	.497	.430
465	.807	.722	.559	.191	.576
472	.769	-.603	-.019	.119	-.166
469	1.048	.225	.914	.705	1.183
417	1.128	.761	1.705	-.874	1.935
514	.989	.759	.078	-1.060	.809
531	1.107	-.282	-.619	.438	-.052

TABLE LXXII (continued)

529	.474	.149	.133	-.117	.253
510	1.212	.897	-.359	.251	.133
507	1.212	.561	-.818	-.472	-.142
575	1.129	-.669	-.040	.730	-.734
505	.994	1.478	-.394	-.572	.510
509	.957	.127	-.702	-.957	.536
681	1.035	.250	-.064	-.110	.273
604	1.244	.165	-.790	.025	.227
605	1.254	.507	.225	-.682	.481
687	1.531	-.069	-2.482	.944	-2.497
662	.956	.154	2.540	-.525	1.574
608	1.124	-.611	-.810	-2.590	.558
606	.984	.614	-.405	.448	-.369
619	.761	1.753	.115	.731	-.229
674	.914	-.245	.005	.449	-.124
628	1.124	.576	.816	-.403	-.634
670	1.221	-2.050	-.180	-1.051	1.400
684	1.502	-2.619	-.382	-.197	.756
607	1.065	.349	.372	.881	.097
624	.662	.410	-.123	-.263	.269
661	.684	.049	.029	.299	.134
723	1.148	-.483	-.876	.722	.358
716	.882	-.680	-1.263	2.664	-3.513
715	1.122	-2.595	-1.052	-1.215	-.787
708	1.387	-2.246	4.848	-1.998	-3.615
725	.838	-1.021	.065	.796	.121
728	.935	-.062	-.257	.617	.190
714	.882	-.765	-1.066	-.914	-.186
703	.684	-1.296	-.102	-.436	-.107

TABLE LXXIII

INDIVIDUAL COEFFICIENTS ON PRINCIPAL VECTORS
GROUP B

I.D.	I	II	III	IV	V
100	.949	-.026	.585	-.593	.546
200	.790	-.546	-.022	-.042	-.484
300	1.140	.557	2.180	-1.120	-.915
400	.739	.880	.598	.073	-.568
500	.956	-.911	-.481	.042	.414
600	.978	.072	.939	-.945	1.846
700	.442	-.119	-.076	.156	-.313
118	.889	-.969	-.775	-.552	1.038
123	.857	-.183	1.522	.057	.025
130	1.249	-1.189	1.616	.787	-1.884
169	.905	.541	.313	.418	.686
160	1.050	1.245	3.118	1.286	.862
128	.662	.274	.810	.034	-.160
120	1.048	.069	-1.355	.341	-1.198
113	1.116	.449	.537	1.160	.538
101	.891	-.364	-.364	.366	-.239
102	1.210	.994	1.037	.593	1.365
221	.701	-.318	.636	-.523	.898
214	.858	-.737	.513	.754	1.758
201	.957	.479	2.009	-.106	1.158
221	1.147	.523	-1.344	-.889	.816
208	1.049	-.904	-.843	.395	.224
310	.962	-1.237	-.120	.839	-.809
305	1.047	-1.490	-1.234	2.452	1.327
320	1.015	1.423	-.749	-1.849	2.210
316	1.329	3.639	.667	2.239	-1.041
319	1.220	1.130	-1.010	-2.112	1.945
327	.861	-.254	-.053	.678	-.136
463	.521	.136	-.450	-.366	.396
429	.958	-1.246	-1.221	-1.195	-.812
471	.699	-.221	-.405	-.140	.254
473	.797	-.031	1.017	.203	.028
427	1.111	.875	-1.274	-.855	-.754
460	.838	-.035	1.036	.365	.480
431	1.199	-.349	-.424	-.730	-.336
422	.937	.573	.030	.232	.300
562	.806	1.466	.292	.467	.253
559	.967	-.177	-.230	1.287	1.424

TABLE LXXIII (continued)

503	1.001	.588	-.469	-.833	-.079
530	.564	-.030	.006	-.245	-.304
516	.946	-.844	-1.449	-.466	-.304
579	.690	-.293	-.366	-.393	.560
567	1.109	-.544	.045	-.165	-1.097
555	.977	-.035	.843	-.280	-.301
663	1.046	1.531	-2.129	2.701	-2.853
669	.960	-.702	-.703	1.239	.992
685	1.187	3.932	-2.495	.125	.098
626	1.598	-.971	.100	.794	.806
679	1.300	-.185	.561	-.809	-.287
686	1.057	-.393	-.435	.584	-1.754
611	.942	-.596	-.708	-2.164	-1.660
660	1.172	-.002	.666	-.490	-.736
672	.726	-.590	-.627	.051	.212
673	1.209	-.348	.075	-.730	-.638
658	1.141	-.729	-1.476	.717	.802
625	1.105	-1.420	.645	.707	.223
652	1.191	-.276	-1.516	-.842	.276
614	.610	-.598	-.509	.337	.306
603	1.172	.188	-.126	.042	.428
719	1.263	.031	1.046	-2.902	-2.501
722	1.252	-.56]	.704	1.467	.440
733	.954	-.779	-.841	.817	1.124
720	1.013	.786	.723	-.404	-.803
731	1.048	-2.310	.216	1.325	-1.047
710	.464	-.169	-.040	-.262	.069
712	.890	-.180	-.069	-.538	-.496
701	.979	-.384	.287	.652	-.870

TABLE LXXIV

INDIVIDUAL COEFFICIENTS ON PRINCIPAL VECTORS
GROUP C

I.D.	I	II	III	IV	V
100	.947	1.035	-.536	-.352	1.039
200	.789	-.574	.275	-.278	.159
300	1.144	.688	2.165	-3.633	3.265
400	.738	1.255	.503	.537	-.663
500	.953	-.706	.148	.299	-.271
600	.979	1.235	-1.778	.825	1.639
700	.422	-.107	-.060	-.126	.177
125	1.121	-1.452	1.543	3.919	-.509
110	.725	1.201	-.431	.705	1.335
156	.874	.463	-.486	-.324	.830
173	1.186	-.677	1.193	-1.601	-.160
167	.463	-.898	.839	1.531	1.056
166	1.251	-.846	.318	.533	-.189
152	1.105	1.861	.034	-.011	-1.539
126	1.003	.648	-.723	-.198	-.613
112	1.113	4.592	2.757	-.231	-2.404
176	1.011	.556	-.773	.929	.890
216	1.156	.101	-.250	3.410	1.147
228	1.242	-1.141	1.029	.123	.881
231	1.034	-1.472	1.794	-1.542	-.601
220	.974	2.917	-.365	.416	.482
204	1.233	-.623	-.883	-.289	.213
331	1.063	-.025	.303	.230	-.208
330	1.154	-1.042	-1.399	-1.809	-.022
307	.911	-.789	-.785	.917	-.454
657	1.067	-.428	-1.048	-.511	-2.537
301	.952	1.246	-.545	-.357	1.408
304	1.104	-.193	-1.001	.759	.594
419	1.224	-.339	-.396	-.296	.575
451	1.248	-.216	-.935	-.343	-1.774
466	.382	-.371	.017	.040	-.115
480	1.027	-.635	.133	.664	.469
430	1.135	1.278	.336	.097	-.647
455	.918	.232	.643	.276	-1.017
432	1.166	-.553	1.555	-.701	-.136
461	.951	.297	-.806	-.098	.497
572	1.165	-.475	3.335	1.569	.205
524	.963	.805	.086	-.325	-.001

TABLE LXXIV (continued)

521	.746	-.087	.179	-.528	.327
504	.809	.807	-1.231	-.216	1.045
556	.850	-.425	-.105	-.191	-.009
561	.922	.540	-.421	.076	-.104
534	1.002	.933	-1.109	.379	-.492
570	.873	.843	-.434	-.131	.097
665	.868	-1.019	.711	.653	.210
612	1.054	-.995	-.804	.566	-.273
630	.735	-.313	-.090	.139	.534
651	.526	-.426	-.493	-.356	-.958
650	.674	.212	-.985	-.172	.395
664	.986	.486	1.660	-.404	.269
615	.533	.188	-.660	.304	-.547
617	1.193	.105	-.707	-1.515	-1.608
675	1.177	-.228	-.309	-.333	-.902
656	.750	.496	-.320	.659	-.053
666	.949	-.562	.421	-.310	.047
677	.906	.394	-.728	-.492	.780
678	1.176	-.380	-.766	-.262	-.808
635	1.095	-.125	.006	-.292	.838
616	.988	-.865	-.696	-.022	.915
729	1.261	-1.359	1.632	-.579	1.572
718	1.252	-.026	.376	.509	-.655
732	1.363	-.408	-1.101	-.361	.484
734	.733	-.094	1.281	1.273	.500
726	.807	-.765	-.406	.079	-.421
702	.886	-.600	.455	.194	-1.619
706	1.016	-.120	-.308	-.316	-.093
704	1.076	-1.252	-1.024	-1.147	-2.150

TABLE LXXV

INDIVIDUAL COEFFICIENTS ON PRINCIPAL FACTORS
GROUP A

I.D.	I	II	III	IV	V
100	440.7	76.4	-8.9	-0.1	-14.1
200	366.7	-19.3	-0.6	20.5	12.2
300	527.8	93.8	87.9	-8.8	-87.4
400	340.8	54.7	27.8	-54.4	-27.1
500	444.5	-40.7	-18.2	-21.6	6.8
600	454.3	88.1	3.3	35.7	-40.3
700	205.2	-9.4	9.0	6.4	-5.1
168	540.7	41.8	-50.6	-23.5	50.1
131	385.7	21.4	-2.7	24.2	-17.5
174	520.4	-47.3	11.5	65.5	60.4
165	499.9	-39.2	-30.5	-23.6	35.4
161	472.3	-25.0	9.8	38.8	-14.8
155	292.8	-7.2	-4.3	-3.8	-13.1
105	486.5	25.4	-27.0	-57.4	-0.5
115	530.6	-42.6	-51.6	21.5	-27.9
103	518.4	45.4	9.2	13.6	6.1
171	414.6	147.1	44.9	53.6	-53.2
222	431.8	18.9	112.2	68.9	16.5
229	522.6	103.2	-102.6	-109.0	33.7
232	483.5	-56.3	-27.2	-65.1	9.1
210	368.9	20.8	-38.4	-0.8	-4.6
211	510.6	118.7	-13.2	-84.8	-89.9
311	498.7	-77.9	-39.9	27.9	-36.4
317	508.9	-16.2	33.5	6.2	46.2
324	463.7	13.2	8.2	34.4	16.9
302	289.3	47.4	-14.4	-7.6	-25.9
309	577.3	-47.7	48.7	180.7	100.1
318	357.6	77.7	10.5	-24.9	-4.4
468	568.8	16.7	11.2	89.5	44.3
470	280.8	-5.3	-1.3	-10.5	4.9
421	510.5	-18.7	37.0	-1.4	60.2
475	269.4	6.4	3.6	25.2	21.6
465	384.3	43.6	29.9	9.7	28.9
472	366.5	-36.4	-1.0	6.0	-8.3
469	499.3	13.6	48.8	35.8	59.4
417	537.5	45.9	91.1	-44.4	97.0
514	471.5	45.8	4.2	-53.8	40.6

TABLE LXXV (continued)

531	527.4	-17.0	-33.0	22.2	-2.6
529	225.6	9.0	7.1	-5.9	12.7
510	577.6	54.2	-19.2	12.8	6.7
507	577.3	33.9	-43.7	-23.9	-7.1
575	538.1	-40.4	-2.1	37.0	-36.8
505	473.7	89.2	-21.1	-29.0	25.6
509	456.1	7.6	-37.5	-48.5	26.9
681	493.0	15.1	-3.4	-5.6	13.7
604	592.8	10.0	-42.2	1.3	11.4
605	597.6	30.6	12.0	-34.6	24.1
687	729.5	-4.2	-132.6	47.9	-125.2
662	455.6	9.3	135.6	-26.6	79.0
608	535.5	-36.9	-43.3	-131.4	27.8
606	468.7	37.1	-21.7	22.7	-18.5
619	362.5	105.8	6.1	37.1	-11.5
674	435.4	-14.9	0.2	22.8	-6.2
628	535.3	34.8	43.6	-20.4	-31.8
670	581.6	-123.7	-9.6	-53.3	70.2
684	715.9	-158.1	-20.4	-10.0	37.9
607	507.4	21.1	19.9	44.7	4.9
624	315.2	24.7	-6.6	-13.3	13.5
661	325.9	3.0	1.6	15.2	6.7
723	546.8	-29.2	-46.8	36.6	18.0
716	420.2	-41.0	-67.5	135.2	-176.3
715	543.4	-156.7	-56.2	-61.6	-39.5
708	660.8	-135.6	259.0	-101.4	-181.4
725	399.4	-61.6	3.5	40.4	6.1
728	445.6	-3.8	-13.7	31.3	9.5
714	420.4	-46.2	-56.9	-46.4	-9.4
703	325.8	-78.3	-5.4	-22.1	-5.4

TABLE LXXVI

INDIVIDUAL COEFFICIENTS ON PRINCIPAL FACTORS
GROUP B

I.D.	I	II	III	IV	V
100	440.8	-1.6	31.3	-29.5	26.8
200	367.2	-33.5	-1.2	-2.1	-23.8
300	529.6	34.2	116.6	-55.7	-44.9
400	343.3	54.0	32.0	3.7	-27.9
500	444.3	-55.9	-25.8	2.1	20.3
600	454.3	4.4	50.2	-46.9	90.6
700	205.4	-7.3	-4.1	7.7	-15.4
118	412.9	-59.5	-41.4	-27.4	50.9
123	398.2	-11.2	81.4	2.8	1.2
130	580.4	-73.0	86.4	39.1	-92.5
169	420.5	33.2	16.7	20.8	33.7
160	487.9	76.4	166.8	63.9	42.3
128	307.6	16.8	43.3	1.7	-7.9
120	486.8	4.2	-72.5	16.9	-58.8
113	518.4	27.6	28.7	57.6	26.4
101	413.9	-22.3	+19.5	18.2	-11.7
102	562.1	61.0	55.5	29.5	67.0
221	325.6	-19.5	34.0	-26.0	44.1
214	398.7	-45.2	27.4	37.5	86.3
201	444.7	29.4	107.5	-5.2	56.9
221	532.8	32.1	-71.9	-44.2	40.1
208	487.4	-55.5	-45.1	19.6	11.0
310	447.1	-75.9	-6.4	41.7	-39.7
305	486.3	-91.4	-66.0	121.8	65.2
320	471.7	87.3	-40.1	-91.9	108.5
316	617.5	223.3	35.7	111.2	-51.1
319	566.6	69.4	-54.0	-104.9	95.5
327	400.2	-15.6	-2.9	33.7	-6.7
463	242.2	8.4	-24.1	-18.2	19.5
429	445.0	-76.5	-65.3	-59.4	-39.9
471	324.6	-13.5	-21.7	-6.9	12.5
473	370.2	-1.9	54.4	10.1	1.4
427	516.0	53.7	-68.2	-42.5	-37.0
460	389.6	-2.1	55.4	18.1	23.6
431	556.9	-21.4	-22.7	-36.3	-16.5
422	435.3	35.2	1.6	11.5	14.7
562	374.4	89.9	15.6	23.2	12.4
559	449.4	-10.9	-12.3	63.2	69.9

TABLE LXXVI (continued)

503	465.1	36.1	-25.1	-41.4	-3.9
530	262.1	-1.8	0.3	-12.2	-14.9
516	439.7	-51.8	-77.5	-23.2	-14.9
579	320.5	-18.0	-19.6	-19.5	27.5
567	515.3	-33.4	2.4	-8.2	-53.9
555	453.8	-2.2	45.1	-13.9	-14.8
663	486.1	94.0	-113.9	134.2	-140.1
669	446.2	-43.1	-37.6	61.5	48.7
685	551.5	241.3	-133.5	6.2	-4.8
626	742.3	-59.6	5.3	39.4	39.6
679	604.1	-11.4	30.0	-40.2	-14.1
686	490.9	-24.1	23.3	29.0	-86.1
611	437.8	-36.6	-37.9	-107.5	-81.5
660	544.6	-0.2	35.6	-24.4	-36.1
672	337.4	-36.2	-33.6	2.4	10.4
673	561.9	-21.3	4.0	-36.3	-31.3
658	530.3	-44.7	-78.9	35.6	39.4
625	513.6	-87.2	34.5	35.1	11.0
652	553.4	-17.0	-81.1	-41.8	13.5
614	288.3	-36.7	-27.2	16.7	15.0
603	544.7	11.6	-6.7	2.1	21.0
719	586.7	1.9	56.0	-144.2	-122.8
722	581.6	-34.5	37.6	-72.9	21.6
733	443.3	-47.8	-45.0	40.6	55.2
720	470.8	48.2	38.7	-20.1	-39.4
731	486.8	-141.8	11.5	65.8	-51.4
710	215.4	-10.4	-2.2	-13.0	3.4
712	413.5	-11.1	-3.7	-26.7	-24.4
701	455.0	-23.6	15.3	32.4	-42.7

TABLE LXXVII

INDIVIDUAL COEFFICIENTS ON PRINCIPAL FACTORS
GROUP C

I.D.	I	II	III	IV	V
100	440.3	61.3	-26.8	-17.2	47.6
200	366.6	-34.0	13.8	-13.6	7.3
300	531.6	40.7	108.4	-177.6	149.5
400	343.1	74.3	25.1	26.2	-30.3
500	443.0	-41.8	7.4	14.6	-12.4
600	454.9	73.1	-89.0	40.4	75.0
700	205.5	-6.3	-3.0	-6.1	8.1
125	521.0	-86.0	77.3	191.6	-23.3
110	336.7	71.1	-21.6	34.5	61.1
156	406.1	27.4	-24.4	-15.8	38.0
173	551.3	-40.1	59.7	-78.3	-7.3
167	435.7	27.4	-45.0	41.0	70.1
166	581.6	-50.1	15.9	26.0	-8.7
152	513.6	110.2	1.7	-0.5	-70.5
126	466.0	38.4	-36.2	-9.7	-28.0
112	517.3	271.9	138.0	-11.3	-110.0
176	469.9	32.9	-38.7	45.4	40.7
216	537.2	6.0	-12.5	166.7	52.5
228	577.4	-67.6	51.5	6.0	40.3
231	480.7	-87.2	89.8	-75.4	-27.5
220	452.5	172.7	-18.3	20.5	22.1
204	573.3	-36.9	-44.2	-14.1	9.8
331	494.2	-1.5	15.2	11.2	-9.5
330	536.5	-61.7	-70.0	-88.4	-1.0
307	423.2	-46.7	-39.3	44.8	-20.8
657	495.9	-25.3	-52.5	-25.0	-116.2
301	442.3	73.8	-27.3	-17.5	64.5
304	513.1	-11.5	-50.1	37.1	27.2
419	569.0	-20.1	-19.8	-14.5	26.3
451	580.0	-12.8	-46.8	-16.8	-81.2
466	177.5	-22.0	0.8	2.0	-5.2
480	477.4	-37.6	6.6	32.5	21.5
430	527.5	75.7	16.8	4.7	-29.6
458	426.8	13.7	32.2	13.5	-46.6
432	542.1	-32.7	77.9	-34.3	-6.2
461	442.1	17.6	-40.4	-4.8	22.8
572	541.2	-28.1	167.0	76.7	9.4
524	447.5	-47.7	4.3	-15.9	-0.1

TABLE LXXVII (continued)

521	346.6	-5.1	9.0	-25.8	15.0
504	376.1	47.8	-61.6	-10.5	47.8
556	395.1	-25.2	-5.2	-9.4	-0.4
561	428.7	32.0	-21.1	3.7	-4.8
534	465.8	55.3	-55.5	18.5	-22.5
570	405.7	49.9	-27.3	-6.4	4.4
665	403.3	-60.3	35.6	31.9	9.6
612	489.9	-58.9	-40.3	27.7	-12.5
630	341.4	-18.5	-4.5	6.8	24.5
651	244.5	-25.2	-24.7	-17.4	-43.8
650	313.1	12.6	-49.3	-8.4	18.1
664	458.2	28.8	83.4	-19.8	12.3
615	247.7	11.1	-33.1	14.9	-25.0
617	554.0	6.2	-35.4	-74.1	-73.6
675	547.1	-13.5	-19.9	-16.3	-41.3
656	348.5	29.4	-16.0	32.2	-2.4
666	441.3	-33.3	21.1	-15.1	2.2
677	421.0	23.3	-36.5	-24.1	35.7
678	546.6	-22.5	-38.4	-12.8	-37.0
635	509.1	-7.4	0.3	-14.3	38.3
616	459.0	-51.3	-34.9	-1.1	41.9
729	586.1	-80.5	81.7	-28.3	72.0
718	581.7	-1.5	18.8	24.9	-30.0
732	633.5	-24.2	-55.1	-17.6	22.2
734	340.5	-5.5	64.1	62.2	22.9
726	375.1	-45.3	-20.3	3.9	-19.3
702	412.0	-35.5	22.8	9.5	-74.1
706	472.0	-7.1	-15.4	-15.4	-4.3
704	500.0	-74.1	-51.3	-56.1	-98.4

TABLE LXXVIII

PROJECTION OF STIMULUS-PAIRS ON PRINCIPAL VECTORS (II) TO (V)

GROUP A

Stimulus Pair	II	III	IV	V
1-2	.0071	-.0078	.0126	.0032
1-3	.0007	-.0044	.0148	.0042
1-4	-.0017	-.0122	.0041	.0008
1-5	-.0051	.0050	-.0039	-.0060
1-6	-.0037	.0053	.0022	-.0015
1-7	.0005	-.0004	.0153	-.0056
1-8	-.0054	.0001	.0098	-.0041
1-9	.0082	-.0160	.0047	-.0056
1-10	.0000	.0015	-.0009	-.0202
1-11	.0024	-.0118	.0074	-.0108
1-12	.0013	-.0073	.0221	-.0038
1-13	.0026	-.0014	.0142	-.0087
1-14	.0004	-.0133	.0158	-.0060
1-15	-.0047	-.0021	.0109	-.0095
1-16	-.0119	.0170	-.0095	.0048
1-17	-.0063	.0215	.0073	.0016
1-18	-.0051	-.0061	.0063	.0027
1-19	-.0111	-.0046	.0003	-.0084
1-20	-.0031	-.0035	.0142	-.0027
2-3	.0268	.0095	-.0067	.0100
2-4	.0029	.0153	.0009	.0046
2-5	.0130	.0038	-.0027	-.0071
2-6	.0146	.0015	.0017	.0082
2-7	.0014	-.0068	.0111	.0061
2-8	.0116	-.0055	-.0016	-.0009
2-9	-.0054	-.0032	-.0096	.0107
2-10	.0140	.0039	.0078	-.0170
2-11	.0189	-.0100	.0063	-.0024
2-12	.0107	-.0004	-.0099	-.0023
2-13	.0100	-.0076	.0019	-.0029
2-14	.0037	.0007	-.0102	-.0089
2-15	-.0048	.0009	-.0033	.0099
2-16	.0005	-.0073	-.0008	-.0095
2-17	.0077	.0033	-.0048	-.0063

TABLE LXXVIII (continued)

2-18	.0103	-.0056	-.0010	-.0068
2-19	.0052	.0088	-.0080	-.0024
2-20	.0011	.0044	-.0100	-.0146
3-4	.0031	-.0071	-.0090	.0086
3-5	-.0026	-.0022	.0058	.0207
3-6	.0045	-.0064	-.0015	.0162
3-7	.0017	.0082	.0060	-.0026
3-8	.0065	.0117	-.0066	.0044
3-9	.0176	.0085	-.0129	-.0060
3-10	.0005	.0091	-.0009	.0023
3-11	.0080	-.0062	.0110	.0088
3-12	-.0018	-.0132	-.0088	.0100
3-13	-.0031	-.0080	-.0101	.0033
3-14	.0018	-.0116	-.0030	.0082
3-15	-.0019	-.0028	-.0052	.0080
3-16	-.0135	-.0040	-.0038	-.0063
3-17	-.0070	-.0103	-.0016	.0052
3-18	-.0124	-.0046	-.0076	-.0029
3-19	-.0039	.0016	-.0120	-.0012
3-20	-.0069	-.0125	-.0089	.0039
4-5	.0057	-.0001	-.0082	.0130
4-6	.0022	.0068	-.0169	.0018
4-7	.0011	.0047	.0150	.0084
4-8	-.0047	.0038	-.0137	-.0012
4-9	.0065	-.0058	.0023	.0028
4-10	-.0008	.0033	-.0135	.0060
4-11	.0047	-.0068	-.0034	.0070
4-12	-.0060	-.0042	-.0096	-.0029
4-13	-.0018	.0006	-.0093	-.0064
4-14	.0019	.0167	-.0144	-.0034
4-15	.0001	-.0105	-.0132	.0109
4-16	-.0069	-.0026	.0042	-.0134
4-17	-.0078	.0030	-.0123	-.0046
4-18	.0044	-.0062	-.0001	-.0003
4-19	-.0019	-.0006	-.0195	.0184
4-20	-.0112	-.0095	-.0097	-.0129
5-6	.0041	.0036	.0039	.0062
5-7	.0117	-.0138	-.0044	-.0160
5-8	.0049	-.0044	.0040	.0220
5-9	.0242	-.0092	-.0011	.0042
5-10	-.0027	.0180	.0002	-.0035
5-11	.0017	.0112	.0003	-.0084

TABLE LXXVIII (continued)

5-12	-.0045	.0212	.0068	.0039
5-13	-.0005	.0066	.0065	.0021
5-14	.0025	-.0022	.0127	.0174
5-15	-.0028	.0100	-.0059	-.0027
5-16	-.0106	.0009	-.0030	-.0134
5-17	-.0151	.0087	-.0033	.0069
5-18	-.0180	-.0036	.0090	-.0084
5-19	-.0071	.0078	-.0146	.0126
5-20	-.0156	-.0048	-.0061	.0023
6-7	.0095	-.0070	.0029	-.0118
6-8	.0063	.0097	.0089	.0130
6-9	.0178	-.0083	-.0041	.0103
6-10	.0065	.0019	.0123	-.0027
6-11	.0069	-.0038	.0097	-.0011
6-12	-.0028	.0131	.0091	.0047
6-13	.0015	.0013	.0151	.0113
6-14	.0038	.0087	-.0017	.0130
6-15	.0018	.0133	.0071	.0034
6-16	-.0181	-.0050	-.0136	-.0042
6-17	-.0176	.0061	.0098	.0043
6-18	-.0088	-.0113	.0125	-.0001
6-19	-.0013	-.0130	-.0020	.0156
6-20	-.0107	.0040	.0031	-.0034
7-8	.0109	-.0089	.0024	.0093
7-9	-.0023	.0104	.0069	.0047
7-10	.0067	-.0006	-.0007	-.0178
7-11	.0092	-.0078	-.0014	-.0212
7-12	.0085	-.0065	.0100	-.0049
7-13	.0033	-.0055	-.0086	-.0117
7-14	.0047	-.0149	.0068	.0043
7-15	-.0145	.0011	.0025	.0034
7-16	.0016	-.0096	.0086	-.0028
7-17	-.0016	-.0011	.0121	-.0065
7-18	-.0027	.0092	.0070	.0014
7-19	-.0027	.0076	.0196	.0119
7-20	-.0036	.0041	-.0009	-.0050
8-9	.0116	.0087	.0081	-.0114
8-10	.0027	.0064	.0049	.0002
8-11	.0006	-.0059	.0079	.0113
8-12	.0050	.0033	-.0008	.0091
8-13	.0024	-.0074	-.0068	.0128
8-14	-.0057	-.0006	-.0080	.0052

TABLE LXXVIII (continued)

8-15	-.0048	-.0048	.0085	-.0082
8-16	-.0191	.0085	.0017	.0109
8-17	-.0107	.0030	-.0072	.0056
8-18	-.0030	-.0122	.0023	.0025
8-19	-.0122	-.0012	-.0133	-.0103
8-20	-.0050	-.0059	-.0158	-.0085
9-10	.0196	.0155	.0068	-.0049
9-11	.0240	.0045	.0094	.0023
9-12	.0176	-.0037	-.0147	-.0143
9-13	.0125	-.0003	-.0066	-.0168
9-14	.0198	-.0005	-.0038	-.0033
9-15	-.0017	-.0168	-.0050	.0006
9-16	-.0017	-.0237	-.0020	.0031
9-17	.0063	.0015	.0019	-.0081
9-18	.0110	.0073	.0058	-.0073
9-19	.0045	.0104	-.0074	.0028
9-20	.0085	-.0072	-.0107	.0026
10-11	-.0021	.0111	-.0044	-.0031
10-12	-.0039	.0184	.0017	.0119
10-13	.0010	.0133	.0014	.0090
10-14	-.0023	.0180	-.0029	.0107
10-15	.0052	.0101	-.0047	.0047
10-16	-.0118	.0067	.0023	-.0162
10-17	-.0144	.0068	.0120	-.0034
10-18	-.0192	-.0032	.0064	-.0123
10-19	-.0070	-.0187	-.0069	.0166
10-20	-.0115	-.0005	.0062	.0100
11-12	-.0052	.0168	.0052	.0116
11-13	.0053	.0110	.0145	-.0004
11-14	.0039	.0055	.0105	.0170
11-15	.0035	.0228	.0005	.0055
11-16	-.0126	.0048	.0015	-.0158
11-17	-.0104	-.0022	.0118	.0031
11-18	-.0188	-.0003	.0067	-.0094
11-19	-.0078	-.0096	-.0072	-.0031
11-20	-.0083	-.0155	.0106	.0055
12-13	.0061	.0010	-.0099	.0093
12-14	.0013	.0139	-.0040	-.0008
12-15	.0019	.0201	-.0110	-.0053
12-16	-.0118	-.0003	.0016	-.0150
12-17	-.0119	.0024	.0010	-.0014
12-18	-.0107	-.0004	.0048	.0224

TABLE LXXVIII (continued)

12-19	-.0023	-.0052	-.0237	.0020
12-20	-.0103	-.0133	-.0040	.0090
13-14	.0011	.0180	.0007	.0009
13-15	-.0031	.0056	-.0182	.0020
13-16	-.0120	-.0014	-.0020	-.0068
13-17	-.0050	-.0123	.0014	.0100
13-18	-.0059	-.0111	.0088	-.0083
13-19	-.0018	-.0046	-.0061	.0240
13-20	-.0121	.0088	-.0088	.0002
14-15	.0069	.0005	-.0165	.0139
14-16	-.0145	-.0159	.0088	.0033
14-17	-.0122	.0039	-.0000	.0001
14-18	-.0073	.0046	.0031	-.0048
14-19	-.0032	-.0020	-.0163	-.0083
14-20	-.0031	-.0086	-.0077	.0088
15-16	-.0050	.0034	.0090	-.0107
15-17	.0031	.0045	-.0144	-.0079
15-18	.0178	.0003	.0095	-.0017
15-19	.0137	.0096	-.0054	-.0025
15-20	.0021	-.0053	-.0077	-.0110
16-17	-.0013	.0022	.0192	-.0007
16-18	.0014	-.0012	.0139	.0027
16-19	.0012	.0076	.0038	-.0065
16-20	.0051	-.0043	.0110	.0046
17-18	-.0040	.0101	.0038	-.0033
17-19	.0126	-.0064	-.0061	.0035
17-20	-.0004	.0066	.0029	-.0008
18-19	.0004	-.0010	-.0068	-.0073
18-20	.0065	-.0004	-.0011	-.0046
19-20	.0062	-.0124	-.0109	.0023

TABLE LXXIX

PROJECTION OF STIMULUS-PAIRS ON PRINCIPAL VECTORS (II) TO (V)

GROUP B

Stimulus Pair	II	III	IV	V
1-2	.0149	.0044	.0018	.0085
1-3	.0043	-.0107	-.0093	-.0030
1-4	.0071	-.0089	-.0087	.0005
1-5	-.0191	-.0128	.0026	-.0013
1-6	-.0164	-.0102	.0049	-.0029
1-7	.0076	.0104	-.0064	-.0079
1-8	-.0024	-.0072	-.0110	.0061
1-9	.0142	-.0035	.0023	.0085
1-10	-.0157	.0038	.0020	-.0148
1-11	-.0007	.0032	-.0009	-.0126
1-12	.0011	-.0025	-.0067	.0030
1-13	.0005	.0110	.0141	.0126
1-14	-.0011	.0000	.0032	.0119
1-15	.0159	.0058	.0103	.0065
1-16	-.0112	-.0015	.0100	-.0124
1-17	-.0164	.0039	.0011	-.0000
1-18	-.0114	-.0027	-.0121	-.0037
1-19	-.0065	.0115	-.0093	-.0033
1-20	.0019	-.0063	-.0118	.0001
2-3	-.0026	.0193	-.0023	.0162
2-4	-.0109	.0195	-.0078	.0028
2-5	.0148	.0110	-.0007	.0021
2-6	.0162	.0096	-.0041	.0022
2-7	-.0088	-.0037	-.0166	-.0021
2-8	-.0023	-.0002	.0076	.0077
2-9	-.0080	.0033	-.0070	.0007
2-10	.0143	.0098	-.0051	-.0094
2-11	.0120	.0147	-.0037	.0009
2-12	.0032	.0056	.0082	.0119
2-13	-.0046	.0032	-.0080	.0109
2-14	.0037	.0089	-.0034	-.0006
2-15	.0002	.0048	-.0067	.0173
2-16	.0026	-.0195	-.0084	-.0028
2-17	.0130	.0009	-.0004	.0025

TABLE LXXIX (continued)

2-18	.0116	-.0046	.0061	.0171
2-19	.0197	.0008	.0109	.0129
2-20	.0051	.0042	.0130	.0052
3-4	-.0049	-.0065	-.0220	-.0021
3-5	.0095	-.0040	.0072	-.0121
3-6	.0038	-.0081	.0013	-.0051
3-7	-.0022	-.0024	.0056	.0002
3-8	-.0004	.0053	.0018	.0018
3-9	-.0010	.0083	-.0084	.0108
3-10	.0054	-.0022	-.0026	-.0199
3-11	.0078	-.0075	.0024	-.0156
3-12	.0066	-.0048	.0056	-.0123
3-13	-.0018	.0037	.0080	-.0041
3-14	.0053	-.0080	.0023	-.0105
3-15	.0030	-.0052	-.0143	-.0080
3-16	-.0049	.0018	-.0009	-.0040
3-17	.0001	-.0154	.0075	.0074
3-18	-.0072	-.0110	.0134	-.0056
3-19	.0040	-.0176	.0064	.0003
3-20	.0008	-.0060	.0007	.0036
4-5	.0131	-.0041	-.0041	-.0099
4-6	.0076	-.0040	-.0173	-.0088
4-7	-.0098	.0087	-.0071	.0083
4-8	-.0008	-.0102	-.0049	-.0103
4-9	-.0048	.0145	-.0040	.0018
4-10	.0100	-.0161	-.0067	-.0105
4-11	.0079	-.0115	.0017	-.0074
4-12	.0135	-.0067	-.0028	.0006
4-13	.0110	-.0019	-.0148	.0025
4-14	-.0081	.0105	.0067	.0053
4-15	.0066	-.0009	-.0019	.0105
4-16	.0052	-.0171	.0088	.0047
4-17	-.0018	-.0215	-.0040	.0133
4-18	.0088	-.0141	-.0023	.0020
4-19	-.0045	.0024	.0078	.0038
4-20	.0010	.0086	.0073	.0102
5-6	.0074	.0105	.0196	.0023
5-7	-.0001	-.0123	-.0037	.0015
5-8	.0157	.0082	-.0149	.0163
5-9	-.0042	.0054	-.0077	-.0061
5-10	-.0119	.0072	-.0080	-.0044
5-11	-.0075	.0004	-.0093	-.0030

TABLE LXXIX (continued)

5-12	-.0022	.0065	-.0018	-.0101
5-13	-.0061	-.0017	-.0117	-.0099
5-14	.0084	.0033	.0017	-.0213
5-15	-.0091	.0075	.0073	.0077
5-16	-.0134	-.0060	.0127	.0130
5-17	-.0046	.0017	.0114	.0042
5-18	-.0118	-.0016	-.0025	.0031
5-19	-.0040	-.0030	-.0025	.0043
5-20	.0023	.0033	.0157	.0003
6-7	.0005	-.0082	-.0052	.0079
6-8	.0120	.0008	-.0094	-.0020
6-9	-.0077	.0053	-.0119	-.0008
6-10	-.0076	.0081	-.0138	-.0041
6-11	-.0083	.0069	-.0113	-.0067
6-12	.0003	.0055	-.0012	.0031
6-13	-.0014	.0064	-.0088	-.0059
6-14	.0028	.0164	-.0040	-.0075
6-15	-.0146	.0051	-.0087	.0016
6-16	-.0158	-.0044	.0045	.0030
6-17	-.0165	.0017	.0059	.0116
6-18	-.0206	.0039	-.0016	.0070
6-19	-.0049	-.0001	.0023	.0153
6-20	-.0036	.0085	.0055	.0049
7-8	-.0132	-.0149	.0001	-.0007
7-9	.0072	.0132	.0192	-.0126
7-10	.0053	.0242	.0120	-.0018
7-11	-.0020	.0011	.0143	-.0120
7-12	.0012	.0026	.0097	-.0087
7-13	.0017	.0001	.0236	-.0049
7-14	-.0172	-.0055	.0077	.0055
7-15	.0002	-.0123	.0051	-.0010
7-16	.0069	.0076	.0109	-.0053
7-17	.0099	.0013	.0258	-.0012
7-18	.0158	-.0125	-.0036	-.0118
7-19	.0058	.0008	-.0037	-.0041
7-20	-.0056	.0147	-.0023	.0051
8-9	.0020	-.0105	-.0008	-.0080
8-10	.0125	-.0076	.0019	-.0079
8-11	.0051	-.0124	-.0023	-.0054
8-12	-.0018	-.0033	.0093	-.0010
8-13	.0011	-.0163	-.0036	-.0059
8-14	-.0102	-.0067	.0062	-.0160

TABLE LXXIX (continued)

8-15	-.0144	-.0021	.0000	-.0050
8-16	.0162	-.0123	.0002	.0041
8-17	-.0048	-.0190	-.0011	.0092
8-18	-.0042	-.0003	.0066	.0064
8-19	.0060	-.0143	.0014	-.0062
8-20	-.0109	.0002	-.0083	-.0055
9-10	.0012	.0160	-.0178	.0071
9-11	.0067	.0215	-.0023	.0161
9-12	.0107	.0127	-.0090	.0077
9-13	-.0039	.0012	-.0139	.0035
9-14	.0109	.0080	-.0082	.0255
9-15	-.0007	.0021	-.0010	.0042
9-16	.0093	-.0158	.0007	.0160
9-17	.0131	-.0003	.0031	.0014
9-18	.0051	-.0033	.0025	.0080
9-19	.0088	.0050	-.0047	.0166
9-20	.0003	.0149	-.0006	.0045
10-11	-.0032	-.0010	-.0082	.0007
10-12	-.0070	-.0033	-.0097	-.0005
10-13	.0048	.0106	-.0091	-.0039
10-14	-.0007	.0061	-.0194	-.0068
10-15	.0115	.0054	.0108	-.0205
10-16	-.0085	.0108	.0137	-.0125
10-17	-.0116	-.0018	-.0007	.0018
10-18	-.0106	.0071	.0051	-.0106
10-19	-.0174	-.0028	.0062	-.0033
10-20	-.0019	.0049	.0051	.0118
11-12	-.0099	.0043	-.0067	.0027
11-13	.0036	.0109	-.0050	-.0061
11-14	.0041	-.0021	-.0107	-.0024
11-15	.0120	.0117	-.0001	-.0266
11-16	-.0077	.0068	.0135	-.0084
11-17	-.0153	-.0098	.0169	.0116
11-18	-.0071	-.0074	.0155	-.0030
11-19	-.0162	-.0002	.0014	-.0029
11-20	-.0056	-.0134	-.0085	.0075
12-13	.0036	.0046	-.0048	-.0083
12-14	.0015	.0051	-.0075	-.0084
12-15	.0101	.0054	.0046	-.0158
12-16	-.0038	.0051	.0061	-.0010
12-17	-.0066	.0042	.0084	.0126
12-18	-.0140	-.0114	.0113	-.0076

TABLE LXXIX (continued)

12-19	-.0083	-.0005	-.0057	-.0036
12-20	-.0054	-.0166	-.0057	.0091
13-14	-.0054	.0059	-.0142	-.0078
13-15	.0011	-.0038	.0012	-.0234
13-16	-.0035	.0000	.0065	-.0034
13-17	-.0054	.01116	.0076	.0061
13-18	-.0102	-.0094	.0097	.0087
13-19	-.0079	.0033	.0061	.0036
13-20	.0054	-.0016	-.0006	.0095
14-15	.0037	.0056	.0053	-.0159
14-16	-.0131	.0042	.0062	.0125
14-17	.0008	-.0114	.0115	.0081
14-18	-.0063	-.0107	.0071	-.0120
14-19	-.0088	.0030	.0012	-.0016
14-20	-.0029	-.0107	-.0084	.0113
15-16	.0024	-.0088	.0020	.0048
15-17	.0065	-.0012	.0195	.0004
15-18	-.0004	-.0100	-.0006	.0096
15-19	.0147	-.0063	.0105	.0118
15-20	.0050	.0039	-.0049	-.0014
16-17	-.0130	.0154	.0085	-.0038
16-18	-.0079	-.0050	-.0168	.0006
16-19	-.0046	.0074	.0038	-.0053
16-20	.0183	-.0020	.0007	-.0009
17-18	-.0114	-.0012	-.0063	-.0089
17-19	-.0147	.0102	-.0016	-.0143
17-20	.0026	-.0117	-.0051	.0017
18-19	-.0037	-.0027	-.0158	-.0084
18-20	.0066	-.0072	-.0148	.0037
19-20	.0003	-.0140	-.0155	.0063

TABLE LXXX

PROJECTION OF STIMULUS-PAIRS ON PRINCIPAL VECTORS (II) TO (V)

GROUP C

Stimulus Pair	II	III	IV	V
1-2	.0114	-.0041	-.0089	-.0104
1-3	-.0035	-.0116	.0055	-.0046
1-4	.0019	.0017	-.0018	-.0012
1-5	-.0053	-.0068	.0152	.0041
1-6	-.0081	-.0038	.0104	.0017
1-7	.0129	-.0123	.0021	.0024
1-8	-.0014	-.0021	.0006	-.0077
1-9	.0056	-.0004	.0047	-.0167
1-10	-.0051	-.0005	.0074	.0191
1-11	-.0064	-.0009	.0073	.0118
1-12	-.0069	-.0062	.0092	.0015
1-13	-.0056	-.0138	.0112	-.0040
1-14	-.0136	-.0117	.0045	-.0058
1-15	.0002	.0035	.0084	-.0089
1-16	-.0007	.0008	-.0122	.0194
1-17	-.0039	-.0038	.0022	.0166
1-18	-.0164	.0088	-.0040	.0087
1-19	-.0058	-.0039	.0006	.0232
1-20	-.0009	-.0025	-.0001	.0157
2-3	.0022	-.0128	-.0035	.0188
2-4	.0028	-.0185	-.0049	.0135
2-5	.0139	-.0051	-.0054	.0023
2-6	.0104	-.0015	-.0133	.0005
2-7	-.0046	-.0098	.0147	.0032
2-8	.0074	-.0167	.0139	.0015
2-9	-.0045	-.0015	.0056	.0058
2-10	.0160	.0077	-.0032	.0037
2-11	.0119	.0060	-.0090	.0089
2-12	.0175	.0031	.0140	.0050
2-13	.0165	.0071	.0018	.0052
2-14	.0128	-.0012	.0044	-.0044
2-15	-.0050	-.0162	.0094	.0032
2-16	.0069	-.0122	.0097	-.0066
2-17	.0126	-.0009	.0094	-.0060

TABLE LXXX (continued)

2-18	.0145	.0044	-.0092	-.0040
2-19	.0169	-.0102	-.0026	.0012
2-20	.0143	.0065	.0002	.0025
3-4	.0036	.0014	.0041	-.0144
3-5	-.0069	-.0108	-.0082	-.0102
3-6	-.0044	-.0094	-.0001	-.0080
3-7	.0071	-.0029	-.0058	.0170
3-8	.0029	.0123	-.0003	-.0171
3-9	.0167	-.0105	.0005	.0089
3-10	-.0035	.0002	-.0156	-.0041
3-11	-.0079	.0016	-.0137	.0009
3-12	-.0090	-.0063	-.0078	-.0015
3-13	-.0116	-.0081	-.0068	-.0123
3-14	-.0005	-.0035	-.0092	-.0110
3-15	.0052	.0162	.0068	-.0106
3-16	-.0016	.0005	.0133	-.0014
3-17	.0030	.0027	.0142	-.0088
3-18	-.0094	.0004	.0081	-.0087
3-19	-.0035	-.0002	.0098	-.0061
3-20	-.0073	.0017	-.0082	.0007
4-5	.0024	.0105	.0017	-.0158
4-6	.0004	-.0059	-.0141	-.0055
4-7	-.0030	-.0097	.0021	.0059
4-8	-.0029	-.0076	-.0013	.0031
4-9	-.0023	-.0077	-.0012	.0054
4-10	-.0021	.0084	-.0003	-.0148
4-11	.0013	.0117	-.0009	-.0041
4-12	-.0021	.0107	.0107	-.0035
4-13	.0008	.0121	.0011	-.0084
4-14	.0090	.0113	.0023	-.0077
4-15	-.0136	.0030	.0002	.0032
4-16	.0063	.0003	.0057	-.0040
4-17	.0050	-.0072	-.0011	-.0110
4-18	.0045	.0045	.0083	-.0074
4-19	.0012	-.0090	.0033	-.0031
4-20	.0043	-.0020	-.0028	-.0137
5-6	-.0023	-.0171	-.0058	.0020
5-7	.0090	.0031	-.0033	-.0018
5-8	-.0049	.0055	-.0013	-.0141
5-9	.0137	.0029	-.0061	.0026
5-10	-.0069	-.0031	-.0126	.0075
5-11	.0011	-.0022	-.0088	.0090

TABLE LXXX (continued)

5-12	-.0161	-.0002	-.0210	-.0057
5-13	-.0127	-.0028	-.0073	-.0080
5-14	-.0154	-.0074	-.0129	-.0166
5-15	.0050	.0094	-.0035	-.0150
5-16	-.0104	.0092	.0087	.0117
5-17	-.0167	.0017	.0047	.0007
5-18	-.0120	.0008	.0083	.0031
5-19	-.0069	.0137	.0050	.0168
5-20	-.0133	.0038	-.0028	.0034
6-7	.0059	.0043	-.0068	.0126
6-8	-.0021	.0195	-.0025	-.0078
6-9	.0022	.0068	-.0086	.0002
6-10	-.0009	-.0096	-.0133	.0109
6-11	-.0040	-.0040	-.0121	.0065
6-12	-.0046	-.0104	-.0108	-.0110
6-13	-.0130	-.0084	-.0080	.0065
6-14	-.0108	-.0090	-.0089	-.0075
6-15	.0048	-.0037	-.0063	-.0030
6-16	-.0100	.0051	.0189	.0060
6-17	-.0092	.0042	.0063	.0074
6-18	-.0068	.0002	.0069	.0039
6-19	-.0049	.0088	.0151	.0013
6-20	-.0091	.0127	-.0011	.0060
7-8	.0044	-.0105	-.0017	.0065
7-9	-.0014	-.0111	.0094	.0032
7-10	.0081	.0057	-.0093	.0048
7-11	.0062	.0111	-.0023	.0118
7-12	.0096	.0063	-.0072	.0036
7-13	.0068	.0140	.0005	.0076
7-14	.0049	-.0004	.0081	.0016
7-15	-.0066	-.0059	.0161	.0083
7-16	.0069	-.0106	.0082	-.0011
7-17	.0174	-.0142	-.0003	-.0104
7-18	.0077	-.0085	.0140	-.0048
7-19	.0010	-.0125	.0077	-.0046
7-20	.0170	.0021	-.0054	-.0016
8-9	.0054	-.0137	.0049	.0083
8-10	-.0069	.0188	-.0100	-.0046
8-11	-.0053	.0213	-.0105	-.0066
8-12	.0001	.0166	.0005	.0067
8-13	-.0006	.0033	-.0012	-.0086
8-14	.0075	.0074	.0074	-.0004

TABLE LXXX (continued)

8-15	-.0097	-.0025	.0031	-.0194
8-16	-.0028	-.0045	.0028	.0073
8-17	.0016	-.0110	-.0034	-.0183
8-18	.0089	.0011	-.0024	.0020
8-19	-.0009	-.0009	-.0084	-.0036
8-20	.0038	.0028	-.0120	-.0013
9-10	.0120	-.0024	-.0205	.0110
9-11	.0129	.0019	-.0058	.0235
9-12	.0211	.0016	.0050	.0086
9-13	.0176	.0008	.0058	-.0002
9-14	.0149	-.0019	.0147	.0033
9-15	-.0088	-.0150	-.0006	.0037
9-16	.0096	-.0136	.0067	-.0099
9-17	.0149	-.0144	.0072	-.0091
9-18	.0147	-.0088	-.0161	-.0001
9-19	.0069	-.0174	-.0061	.0120
9-20	.0157	-.0011	.0014	.0074
10-11	-.0016	-.0108	.0016	-.0037
10-12	-.0137	-.0063	-.0122	-.0109
10-13	-.0136	-.0027	-.0150	-.0001
10-14	-.0071	-.0037	-.0225	-.0043
10-15	.0115	.0137	-.0102	-.0030
10-16	-.0099	.0070	.0060	.0206
10-17	-.0086	.0126	-.0057	.0148
10-18	-.0165	-.0180	-.0025	.0108
10-19	-.0063	-.0146	.0230	-.0025
10-20	-.0094	.0028	.0145	-.0095
11-12	-.0082	-.0105	-.0120	-.0086
11-13	-.0042	-.0026	-.0197	-.0036
11-14	-.0108	-.0076	-.0144	-.0064
11-15	.0077	.0238	-.0065	-.0012
11-16	-.0093	.0105	.0052	.0197
11-17	.0001	.0263	-.0010	.0034
11-18	-.0108	.0026	.0064	.0056
11-19	-.0079	.0093	.0175	.0036
11-20	-.0160	.0043	.0095	-.0088
12-13	-.0038	-.0092	.0022	-.0030
12-14	-.0092	.0050	-.0116	.0077
12-15	.0081	.0117	-.0138	-.0024
12-16	-.0039	.0109	.0122	.0091
12-17	-.0090	.0024	-.0061	-.0012
12-18	-.0122	.0029	-.0022	-.0115

TABLE LXXX (continued)

12-19	-.0047	.0040	.0099	-.0060
12-20	-.0091	.0025	.0144	-.0051
13-14	-.0034	-.0044	-.0073	.0090
13-15	.0075	.0122	-.0044	-.0053
13-16	-.0087	-.0008	.0046	.0012
13-17	-.0084	-.0056	.0106	-.0124
13-18	-.0037	-.0015	.0049	.0040
13-19	.0032	-.0029	.0096	-.0033
13-20	-.0069	.0040	.0039	.0091
14-15	.0042	.0121	-.0006	-.0119
14-16	-.0077	-.0115	.0133	-.0016
14-17	-.0095	-.0049	.0023	-.0069
14-18	-.0097	.0040	.0060	.0043
14-19	-.0110	.0025	-.0085	-.0037
14-20	-.0143	.0039	.0012	.0027
15-16	.0088	-.0101	.0151	-.0019
15-17	.0072	.0071	.0133	-.0159
15-18	.0090	.0041	-.0059	-.0120
15-19	-.0007	.0024	.0019	.0011
15-20	.0094	.0169	-.0057	-.0146
16-17	.0018	-.0114	-.0025	.0066
16-18	-.0093	-.0084	-.0110	.0080
16-19	-.0005	.0059	-.0102	.0095
16-20	-.0013	-.0053	-.0054	.0045
17-18	-.0128	.0035	-.0006	-.0082
17-19	.0051	.0068	.0113	.0101
17-20	-.0023	-.0066	-.0077	-.0002
18-19	-.0056	-.0038	-.0067	.0059
18-20	-.0005	-.0053	.0004	.0047
19-20	-.0051	-.0139	.0017	-.0012

TABLE LXXXI

FINAL UNROTATED CONFIGURATION OF 20 POINTS
IN FIVE DIMENSIONS

GROUP B AVERAGE

	Dimension				
Points*	I	II	III	IV	V
1	.71	-.24	-.46	.08	.32
2	-.86	-.03	-.82	-.08	-.07
3	-.38	-.22	.51	-.09	-.71
4	-.44	.32	.41	.31	-.78
5	-.27	-.52	.14	.61	.48
6	-.12	.33	.06	-.87	-.18
7	-.39	.90	-.49	.11	.25
8	-.37	-.45	.62	.43	-.35
9	-.70	-.18	-.45	-.68	.08
10	.54	.10	.55	-.41	.37
11	.31	.01	.35	-.46	.60
12	.14	.24	.65	.11	.33
13	-.19	-.03	.52	.35	.51
14	-.05	.57	.02	.42	.70
15	-.43	-.49	-.26	.79	.47
16	.32	-.79	-.35	-.10	-.42
17	.87	.02	-.30	-.17	-.19
18	.58	-.24	-.25	.37	-.55
19	.64	-.03	-.69	.20	-.27
20	.08	.74	.15	.31	-.59

*Mechanics concepts.

TABLE LXXXII

FINAL UNROTATED CONFIGURATION OF
20 POINTS IN FOUR DIMENSIONS

GROUP C AVERAGE

Points*	Dimensions I	II	III	IV
1	.70	-.53	-.37	-.48
2	-1.19	-.54	.10	.05
3	-.35	.12	.39	-.58
4	-.21	-.72	.47	.26
5	.57	.01	.59	-.46
6	.38	.11	.63	-.43
7	-.95	.01	-.70	-.63
8	-.12	-.32	.07	.71
9	-1.22	-.47	-.30	.40
10	.78	.64	.20	-.42
11	.58	.65	.26	-.40
12	.35	.39	.57	.24
13	-.10	.33	.55	.26
14	-.01	.66	.27	.57
15	-.92	.67	-.50	-.21
16	-.10	-.79	-.52	.05
17	.35	-.33	-.30	-.09
18	.60	-.29	-.17	.67
19	.48	-.16	-.69	.42
20	.39	.56	-.55	.08

*Mechanics concepts.

TABLE LXXXIII

FINAL UNROTATED CONFIGURATION OF 20 POINTS
IN FIVE DIMENSIONS

GROUP C AVERAGE

	Dimension				
Points*	I	II	III	IV	V
1	.59	-.47	-.55	-.46	.22
2	-.65	-.37	.35	-.02	-.94
3	-.61	-.02	.15	-.37	.49
4	-.47	-.72	.14	.36	.31
5	.32	-.03	.47	-.43	.63
6	.24	-.07	.59	-.45	.40
7	-.76	.08	-.54	-.69	-.58
8	-.21	-.29	-.20	.73	.05
9	-.59	-.10	.11	.13	-1.12
10	.65	.58	.25	-.44	.29
11	.47	.61	.34	-.43	.17
12	.21	.34	.62	.27	.22
13	-.06	.30	.62	.30	-.03
14	-.28	.49	.09	.53	.56
15	-.84	.64	-.57	-.19	-.25
16	.05	-.78	-.45	.00	-.36
17	.19	-.31	-.48	.07	.32
18	.57	-.31	-.22	.68	.04
19	.74	-.12	-.18	.24	-.55
20	.45	.55	-.53	.19	.16

*Mechanics concepts.

TABLE LXXXIV

GROUP A INDIVIDUAL COEFFICIENTS ON VARIMAX ROTATED FACTORS

I.D.	Viewpoint dimensions			
	II	III	IV	V
100	64.5	39.4	-0.6	-20.0
200	-10.2	-11.5	-1.7	26.5
300	117.0	-42.9	28.3	-89.0
400	33.8	17.7	27.3	-72.6
500	-48.7	1.1	-11.0	-2.6
600	100.6	13.7	-15.1	-11.4
700	-1.4	-15.2	-0.1	2.1
168	-2.2	84.5	9.1	12.0
131	33.1	-3.1	-15.0	5.6
174	-24.7	-28.6	20.1	92.1
165	-59.6	22.9	-1.1	14.1
161	3.6	-39.3	-16.9	24.4
155	-4.5	-5.3	-11.7	-8.5
105	-10.8	50.4	0.0	-44.8
115	-27.3	-8.6	-67.9	17.3
103	43.1	15.6	15.6	4.5
171	169.3	5.7	13.2	-22.5
222	64.9	-79.5	76.4	40.2
229	3.2	172.8	-4.7	-65.8
232	-83.7	14.9	-7.8	-31.0
210	9.9	33.9	-26.1	-0.2
211	82.6	60.7	-21.1	-136.3
311	-48.0	-40.5	-73.2	21.5
317	-17.3	-12.1	48.0	28.4
324	22.9	-3.1	10.1	32.8
302	39.9	25.3	-15.5	-26.6
309	22.8	-75.7	41.9	198.3
318	55.4	40.5	25.9	-37.4
468	43.0	-10.5	15.9	90.4
470	-10.8	3.8	3.8	-4.3
421	-26.3	-6.8	60.7	30.3
475	10.9	1.7	10.8	30.3
465	37.3	12.6	45.2	11.3
472	-24.6	-24.0	-14.4	6.5
469	19.4	-11.0	65.3	51.0
417	7.0	22.9	145.6	-2.9
514	1.6	57.6	50.8	-27.4
531	-9.9	3.6	-34.2	24.6

TABLE LXXXIV (continued)

529	2.3	7.7	16.2	-0.2
510	43.8	39.1	-2.5	7.6
507	9.8	51.1	-23.1	-21.0
575	-5.4	-48.6	-41.6	15.2
505	47.4	81.4	25.4	-19.9
509	-31.6	56.8	3.9	-17.2
681	4.9	18.0	10.4	1.5
604	-3.5	37.0	-21.4	13.2
605	4.5	30.8	38.3	-20.6
687	28.4	12.1	-185.8	-10.6
662	-0.1	-39.0	154.7	-0.9
608	-106.8	66.7	13.0	-72.5
606	41.5	17.3	-25.7	4.4
619	107.5	33.2	7.9	2.7
674	0.1	-18.6	-12.4	16.8
628	37.7	-16.8	23.1	-48.0
670	-147.9	-9.0	27.0	22.1
684	-148.8	-49.1	-18.5	45.3
607	40.0	-15.3	9.8	30.5
624	8.7	27.7	11.9	-5.9
661	7.6	-1.8	2.0	14.9
723	-22.3	10.0	-37.1	52.2
716	66.8	-101.9	-200.5	23.3
715	-155.1	-41.0	-78.9	-33.4
708	-47.5	-281.6	71.8	-204.8
725	-33.0	-45.8	-15.8	45.2
728	5.3	0.1	-12.6	33.0
714	-67.5	24.8	-43.4	-23.5
703	-73.2	-31.9	-16.5	-5.7

TABLE LXXXV

GROUP B INDIVIDUAL COEFFICIENTS ON VARIMAX ROTATED FACTORS

I.D.	Viewpoint dimensions			
	II	III	IV	V
100	-10.7	-14.7	47.3	-2.2
200	11.2	28.3	-1.7	-27.7
300	-120.8	23.6	64.1	-25.2
400	-65.3	0.0	-12.8	17.5
500	60.8	10.5	11.4	-16.6
600	0.0	-50.8	99.1	23.7
700	1.7	13.9	-11.6	-6.0
118	79.7	-25.0	29.6	-27.3
123	-37.7	41.6	58.3	14.7
130	-35.4	142.7	17.9	-30.7
169	-11.3	-16.5	10.7	49.3
160	-111.5	42.4	86.1	133.8
128	-38.9	13.8	19.0	12.7
120	15.0	5.0	-88.5	-30.5
113	-10.0	14.8	2.8	72.6
101	24.1	19.3	-18.9	-5.9
102	-34.4	-27.0	42.4	91.9
321	8.0	-10.6	63.0	2.2
214	61.3	13.5	66.1	57.9
201	-55.3	-1.2	95.5	59.1
221	28.8	-91.1	-21.0	-13.4
208	70.9	16.0	-13.5	-12.3
310	43.8	80.2	-16.3	-22.4
305	152.7	54.1	-24.8	70.4
320	-4.1	-166.7	37.2	15.8
316	-165.4	-21.7	-119.5	154.9
319	6.8	-163.4	31.6	-10.6
327	15.9	28.4	-13.0	14.2
463	13.2	-34.0	-1.7	-4.9
429	56.1	1.9	-18.0	-108.4
471	25.4	-11.7	-0.8	-8.8
473	-26.8	28.8	34.3	18.4
427	-22.2	-62.3	-67.9	-41.4
460	15.5	22.7	44.0	35.6
431	11.6	-9.1	-3.5	-48.3
422	-14.6	-19.6	-7.0	30.7
562	-56.9	-36.0	-21.3	63.6
559	59.6	0.1	9.1	74.9

TABLE LXXXV (continued)

503	-19.8	-50.3	-15.5	-22.3
530	-8.4	2.0	-2.7	-17.1
516	66.0	-9.7	-34.5	-61.7
579	31.2	-22.5	14.9	-11.8
567	-6.0	41.4	-13.3	-46.5
555	-33.9	20.2	28.4	-9.3
663	-32.0	41.1	-234.3	42.6
669	84.7	16.0	-7.9	43.9
685	-82.3	-181.9	-175.2	74.6
626	62.3	36.0	29.9	25.2
679	-24.6	4.5	31.5	-35.0
686	-30.6	80.9	-33.7	-27.6
611	-14.7	-13.1	-17.1	-142.5
660	-41.8	20.1	13.7	-28.8
672	47.9	1.6	-6.0	-14.7
673	-10.4	9.5	6.5	-50.3
658	99.5	-9.8	-30.6	11.2
625	48.7	75.2	44.9	2.2
652	54.3	-54.1	-26.1	-47.3
614	49.8	9.8	-4.4	-1.0
603	6.3	-18.4	2.2	15.5
719	-119.7	6.8	25.0	-154.9
722	-4.5	-14.7	76.0	-48.8
733	90.5	1.1	0.0	28.5
720	-75.6	-0.9	-3.7	-.67
731	76.3	142.1	2.3	-34.3
710	6.7	-3.9	8.7	-12.5
712	-7.5	2.3	-1.9	-37.1
701	-6.0	57.6	-17.0	-4.6

TABLE LXXXVI

GROUP C INDIVIDUAL COEFFICIENTS ON VARIMAX ROTATED FACTORS

I.D.	Viewpoint dimensions			
	II	III	IV	V
100	-27.5	18.6	72.2	26.8
200	27.4	-27.4	-8.8	1.0
300	-20.0	-191.5	137.0	107.0
400	-82.5	23.4	-15.0	13.6
500	30.0	-4.1	-34.5	-7.7
600	-5.1	102.3	91.8	40.0
700	9.2	-4.5	6.8	1.1
125	52.7	72.6	-190.6	78.9
110	-29.5	55.9	52.1	61.2
156	-2.5	9.6	52.0	15.2
173	9.5	-105.2	-7.1	-12.4
167	19.4	63.9	48.5	50.5
166	36.7	-3.2	-46.4	2.3
152	-123.6	28.1	4.2	-32.1
126	-33.8	25.6	23.8	-36.1
112	-321.4	-22.4	-19.2	32.9
176	1.5	65.3	29.4	34.3
216	28.2	131.8	-54.9	97.7
228	59.0	-45.5	-34.4	46.3
231	33.5	-133.3	-53.0	-19.9
220	-134.7	70.0	76.4	46.8
204	48.4	7.3	21.6	-27.2
331	-6.4	-1.3	-19.7	4.5
330	69.5	-37.8	60.6	-81.2
307	46.9	46.1	-33.4	-27.0
657	-8.6	9.9	-30.8	-128.0
301	-31.7	21.5	86.4	41.5
304	37.8	55.1	15.5	7.1
419	32.9	-4.1	24.5	1.0
451	-7.2	15.0	-14.8	-93.5
466	16.9	-4.6	-12.7	-6.8
480	40.8	9.9	-24.3	25.0
430	-82.1	13.0	2.0	2.0
458	-39.1	-5.5	-43.6	-11.0
432	0.5	-81.9	-37.2	15.7
461	5.6	25.6	42.4	-1.9
572	-18.4	-54.1	-134.1	115.7
524	39.2	-26.6	-11.8	-12.7

TABLE LXXXVI (continued)

521	6.1	-26.3	15.6	5.0
504	-4.8	41.7	81.5	9.5
556	22.8	-10.1	-2.0	-11.2
561	-23.0	24.1	18.6	-7.0
534	-38.8	62.7	27.7	-27.8
570	-33.7	24.8	39.1	-3.8
665	47.1	-14.0	-54.0	26.3
612	60.1	30.7	-23.7	-30.0
630	27.3	2.6	4.5	15.3
651	11.4	-3.1	-10.8	-56.7
650	10.6	27.2	44.5	-12.3
664	-46.6	-59.3	-14.5	49.3
615	-8.6	34.9	-0.1	-27.8
617	-27.4	-29.9	22.1	-100.3
675	0.8	-2.3	-7.9	-49.8
656	-19.8	41.4	0.6	7.7
666	22.5	-32.8	-14.2	0.4
677	3.4	10.0	60.0	3.5
678	16.1	9.3	-1.5	-56.2
635	20.5	-13.4	25.3	22.1
616	71.1	6.9	21.2	4.0
729	71.5	-93.7	-19.1	69.9
718	-14.7	7.0	-40.0	-3.8
732	45.4	14.5	40.6	-22.4
734	-2.3	4.5	-57.2	72.4
726	38.2	4.3	-19.3	-31.6
702	53.2	-17.6	8.9	64.3
706	8.3	-3.6	11.4	-18.2
704	38.6	-26.7	-23.7	-134.9

TABLE LXXXVII

TRANSFORMATION MATRICES FOR VARIMAX ROTATION OF PRINCIPAL FACTORS II TO V

GROUP A			
.813	.521	.191	-.176
.208	-.646	.714	-.175
.452	-.349	-.256	.780
-.302	.436	.624	.574
GROUP B			
-.651	-.542	-.332	.414
-.567	.424	.675	.205
.213	.531	-.376	.729
.457	-.494	.541	.505
GROUP C			
-.868	.255	.384	.184
-.304	-.620	-.516	.507
.055	.741	-.550	.381
.389	-.020	.533	.751

TABLE LXXXVIII

SUBGROUP CII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR SUBGROUP BII(-)

SIX DIMENSIONS

	Dimension					
Points*	I	II	III	VI	V	VI
1	.37	-.53	-.17	-.09	.76	-.29
2	-.18	-.45	-.60	.59	-.34	-.29
3	-.47	.27	.69	.01	-.13	.40
4	.13	.11	.33	.29	.82	.09
5	.06	.11	-.42	-1.02	.23	-.15
6	.29	-.50	-.57	-.56	-.22	.41
7	-.14	-.24	.79	-.49	-.45	-.26
8	-.67	-.18	-.19	-.29	.30	.11
9	-.34	.00	-.01	-.29	-.40	-.86
10	.88	.31	-.22	-.31	-.32	.02
11	.31	.85	-.13	-.17	.08	.41
12	-.42	-.14	.24	.42	-.53	-.25
13	.31	-.17	.19	.64	-.19	-.56
14	.02	.52	-.63	.47	.31	.18
15	-.05	.08	-.01	.17	-.24	.97
16	-.02	-.35	.78	.02	.20	.56
17	.28	.61	.14	.09	.21	-.43
18	-.56	-.14	.01	.28	.59	-.58
19	.57	-.56	.18	.28	-.15	.25
20	-.38	.39	-.41	-.05	-.53	.27

*Mechanics concept.

TABLE LXXXIX

SUBGROUP CII(-) ROTATED FOR MAXIMUM FACTOR VECTOR
OVERLAP WITH UNROTATED CONFIGURATION
FOR SUBGROUP BII(+)

SIX DIMENSIONS

Points*	Dimension					
	I	II	III	IV	V	IV
1	.80	-.45	-.17	.06	-.34	-.08
2	-.83	.49	.26	-.19	-.61	.27
3	-.31	.08	.17	-.83	.26	-.17
4	-.82	.12	-.22	.44	-.50	.25
5	.13	-.12	-.65	-.15	.47	-.45
6	-.56	-.28	-.15	.13	.67	.04
7	-.49	.90	.14	-.22	-.32	-.20
8	-.38	.36	.06	.73	-.61	-.30
9	-.23	.17	.33	.24	-.92	.65
10	.18	-.15	.26	.13	.45	-.20
11	.40	.00	.77	-.25	.10	-.31
12	.32	-.27	-.45	-.24	.33	.38
13	-.14	-.57	.34	-.20	.47	.40
14	.54	.11	-.52	-.01	.41	.21
15	-.16	.77	.39	.59	-.41	.64
16	.15	.02	.11	.14	.17	-.70
17	.31	-.38	-.50	.56	.28	.08
18	.12	-.64	-.14	-.34	.02	-.54
19	.43	.18	-.19	-.48	-.54	-.01
20	.55	.05	.17	-.11	.62	.21

*Mechanics concepts.

TABLE XC

SUBGROUP CIII(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR BIII(-)

(FOUR DIMENSIONS)

	Dimension			
Points*	I	II	III	IV
1	.29	-.84	-.47	.30
2	-.09	-.49	-.30	.66
3	-.57	-.64	-.21	-.13
4	.47	-.26	.32	.68
5	1.04	-.03	.31	-.53
6	.79	.46	.12	-.68
7	-.01	-.72	-.60	.19
8	-.12	-.14	.16	.93
9	-.41	-.13	-.67	.45
10	-.34	.23	.18	-1.31
11	.12	.54	.29	-.96
12	-.75	.69	.53	.07
13	-.15	.58	.64	.20
14	-.78	-.13	.55	.27
15	.18	-.89	-.25	-.41
16	-.13	-.70	.18	.36
17	-.26	-.03	-.34	.51
18	.59	.11	.81	-.19
19	.09	.20	-.77	-.21
20	-.50	.52	-.49	-.20

*Mechanics concepts.

TABLE XCI

SUBGROUP CIII(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR BIII(+)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	-.12	.41	1.01	-.62
2	-.68	-.60	.22	-.54
3	-.93	.32	-.06	-.17
4	.38	-.75	-.22	.24
5	.11	-.18	-.19	.76
6	.15	-.18	-.54	-.58
7	-.53	.38	.19	-1.07
8	-.13	.03	.71	-.18
9	-.41	-1.19	.02	-.01
10	.31	.46	-.19	-.51
11	.40	.40	-.59	.08
12	-.26	.22	-.24	.80
13	.38	.31	.39	.81
14	-.05	.81	.19	.13
15	-.29	1.01	-.45	.33
16	-.32	-.73	.61	.60
17	.58	-.35	-.55	-.21
18	.87	.02	.53	.02
19	-.37	-.05	-.90	.07
20	.92	-.34	.06	.05

*Mechanics concepts.

TABLE XCII

SUBGROUP CIV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR BIV(-)

(FOUR DIMENSIONS)

	Dimension			
Points*	I	II	III	IV
1	-.46	-.36	-.70	.01
2	-.61	.62	-.33	.09
3	-.68	.14	.09	.93
4	-.71	-.09	-.88	.02
5	-.08	.72	.44	-.25
6	-.30	.58	.48	-.23
7	.62	.35	-.67	.37
8	-.05	-.36	-1.11	-.31
9	.85	.43	-.32	.04
10	.26	.78	.52	.00
11	.34	.48	.48	.32
12	.60	.03	.60	.31
13	.02	.04	.77	.44
14	.08	.04	.82	-.38
15	-.48	-.21	.75	-.56
16	-.17	-1.13	-.26	.35
17	.23	-.61	-.47	.55
18	.55	-.77	.19	-.04
19	.36	-.15	-.44	-.91
20	-.36	-.55	.04	-.74

*Mechanics concepts.

TABLE XCIII

SUBGROUP CIV(+) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR BIV(+)

(FOUR DIMENSIONS)

Points*	Dimension I	II	III	IV
1	.67	-.43	-.58	-.08
2	-.01	.66	-1.10	-.53
3	-.29	-.48	.35	.06
4	.32	-.04	-.06	.84
5	.25	.54	.49	-.73
6	.45	-.77	.46	-.18
7	-1.60	.47	-.45	.29
8	.37	-.44	.09	.27
9	-.32	-.13	-1.51	-.30
10	.80	.31	.35	.11
11	.47	.02	.70	-.32
12	.18	-.10	.56	.25
13	-.18	-.39	-.56	.63
14	-.08	.36	.09	.57
15	-.17	.43	-.47	-.24
16	-.51	-.59	.48	-.59
17	.12	-.39	.02	-.67
18	-.57	.08	.17	.18
19	-.34	.30	.86	.24
20	.42	.57	.11	.21

*Mechanics concepts.

TABLE XCIV

SUBGROUP CV(-) ROTATED FOR MAXIMUM FACTOR VECTOR OVERLAP WITH UNROTATED CONFIGURATION FOR BV(-)

(FOUR DIMENSIONS)

	Dimension			
Points*	I	II	III	IV
1	.09	-.84	-.10	-.45
2	-.81	.53	.47	.27
3	.56	-.22	1.07	.46
4	-.73	-.65	-.41	.44
5	-.10	-.05	-.02	-1.09
6	.46	.27	.17	-.52
7	.66	.18	.09	.91
8	-.03	.16	-.88	.43
9	-.20	.74	.64	.26
10	-.45	-.76	-.06	-.46
11	-.02	.03	.44	-.77
12	-.32	-.15	-.25	.78
13	-.01	1.00	-.32	.33
14	-.06	.64	-.51	.67
15	.23	-.30	.60	.59
16	.74	-.10	-.21	-.16
17	.19	-.58	.41	-.46
18	-.18	.36	-.70	-.58
19	.60	-.60	-.08	-.34
20	-.63	.35	-.35	-.31

*Mechanics concepts.