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S. Djurfors, D. Netterville

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BUOYANT PLUME RISE IN NON-UNIFORM WIND CONDITIONS

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ABSTRACT

The conservation equations governing buoyant plume rise are solved for the case of non-uniform wind conditions. A simple power law is selected to represent the actual wind profile. Analytical solutions are presented both for uniformly stable and neutral atmospheric conditions. These solutions are shown to be of the same form as those obtained in the simpler uniform case but with the plume rise now depending explicitly on the wind speed shear, i.e., $z \propto t^{2/(3+\delta)}$.

A sensitivity analysis of the effects on plume rise of typical variation in wind shear and entrainment reveals that the two quantities have an almost equal effect therefore justifying the use of the present model. To simplify computations a "uniform wind" is introduced such that when used in conjunction with Briggs' equations the results become consistent with those of the present theory.

BUOYANT PLUME RISE IN NON-UNIFORM WIND CONDITIONS

Introduction

A common characteristic of recent plume rise models is their reliance on simple fluid mechanical principles. Thus the conservation equations of plume mass, plume potential temperature and plume vertical momentum are assumed to describe the essential features of buoyant chimney plume behavior. Closure is obtained by introducing an entrainment assumption in the mass flux equation which relates turbulent entrainment to local mean-flow conditions in the plume. For a bent-over plume only entrainment normal to the plume axis is of significance, i.e., entrainment is everywhere proportional to the vertical velocity of the plume. This implies that plume growth is essentially linear with height (Scorer 1958, Slawson and Csanady 1967, Bringfelt 1969, Briggs 1970 and Shwartz and Tulin 1971). Plume lapse photography and aerial plume sampling have provided evidence for this simple relationship at least during the initial plume phase where the vertical plume velocity is still sufficiently large (Bringfelt 1969, Briggs 1970). The conservation equations are generally solved for uniform wind conditions resulting in the now well-known "2/3 - laws." It is the authors' understanding that only one previous study has dealt analytically with the problem of non-uniform wind conditions, and then only for an unstratified atmosphere (Murthy 1970).

Theoretical Considerations for a Plume Rise Model in Non-Uniform Wind Conditions

In this paper the conservation equations are solved for the non-uniform wind case and solutions are presented for both stable and neutral atmospheres. It can probably be argued that in the case of tall stacks the already familiar solutions are quite adequate since wind speed shears are supposedly negligible at these heights (> 150 m). However, in our experience significant wind speed shears occur occasionally, primarily during very stable atmospheric conditions. Rather than complicating the problem by introducing a realistic but complex physical wind model, an attractive alternative to this model would be a simple power law which is known to well represent the wind profile at moderate heights. Thus, if U_s is the wind speed at the top of a chimney whose height above ground is h_s and z is the vertical distance coordinate originating at h_s, then the wind profile may be written as:

$$\frac{U}{U_s} = \left(1 + \frac{z}{h_s}\right)^{\gamma} . \tag{1}$$

Statistical values of γ may be generated for a particular location as a function of time of day or if wind profiles are obtained routinely better estimates of γ may be derived from the individual profiles. At the heights of interest it is reasonable to expect that $0 \leq \gamma \leq 1.0$; the value $\gamma = 0$ represents uniform flow, and $\gamma = 1/7$ is often reported as characteristic of a nearly neutral atmosphere.

The complete derivation of the conservation equations can be found in many recent publications on plume rise and may be considered as well known. Therefore the governing differential equations can simply be stated here in their conventional notation without further justification.

$$\frac{d}{dt}(UR^2) = 2RU\alpha|w| \tag{2}$$

$$\frac{d}{dt}(UR^2 g \frac{\theta'}{\theta_a}) = - N^2 UR^2 w \tag{3}$$

$$\frac{d}{dt}(UR^2 w) = UR^2 g \frac{\theta'}{\theta_a} \tag{4}$$

$$\frac{dz}{dt} = w \tag{5}$$

Eq. (5) is a kinematic relationship governing the bodily motion of the plume. The absolute sign on w in Eq. (2) is necessary because the plume entrains air whether it ascends or descends. It may be wise to review briefly the applicability of Eq.'s (2) to (4) inclusive. These conservation equations apply to buoyant bent-over plumes, requiring that the horizontal plume velocity essentially equals that of the ambient wind. Furthermore, they imply a cylindrical plume element having "top hat" potential temperature and vertical velocity profiles. The Boussinesq approximation is assumed valid and the physical properties of the plume are assumed to be those of the ambient air.

To solve the governing equations when the wind speed is given by the relationship (1), they are first conveniently transformed into z - coordinates and expanded. Thus:

$$\frac{dR}{dz} + \frac{R}{2U}\frac{dU}{dz} = \alpha \tag{2a}$$

$$\frac{d}{dz}\left(\frac{\theta'}{\theta_a}\right) + \frac{2\alpha}{R}\frac{\theta'}{\theta_a} = -\frac{N^2}{g} \tag{3a}$$

$$\frac{dw^2}{dz} + \frac{4\alpha}{R}w^2 = 2g\frac{\theta'}{\theta_a} \quad . \tag{4a}$$

Eq.'s (1) and (2a) may be combined and the resulting differential equation integrated to yield the expression for plume growth. Then for the initial conditions $R = R_s$ at $z = 0$ the solution is:

$$R = \frac{\alpha h_s}{1+\gamma/2}\frac{h}{h_s} + \left(R_s - \frac{\alpha h_s}{1+\gamma/2}\right)\left(\frac{h}{h_s}\right)^{-\gamma/2} \quad . \tag{6}$$

The quantity h is defined as the sum of the physical chimney height h_s and the height above the chimney z, the latter of which corresponds to the buoyant plume rise. Eq. (6) is rather too complicated to use directly in the subsequent development of the plume rise model. However as can be seen from Figure 1 Eq. (6) is very nearly linear in z and can in fact be adequately approximated by the relationship:

$$R = R_s + \frac{\alpha h_s}{1+\frac{2\gamma}{9}}\left(\frac{h}{h_s} - 1\right) = R_s + \alpha_* z \tag{7}$$

where $\alpha_* = \frac{\alpha}{1+\frac{2\gamma}{9}}$ is a rescaled entrainment constant. In Table 1 is shown the ratio of Eq. (7) and Eq. (6) for a range of values of h/h_s and γ. It is obvious that for the most part Eq. (7) is an excellent approximation to Eq. (6). Varying α and R_s/h_s over a broad range changes the ratios by only a few percent.

TABLE 1. $R_{approx.}/R_{exact}$

h/h_s	γ=0.	γ=.1	γ=.2	γ=.3	γ=.4	γ=.5	γ=.6	γ=.7	γ=.8	γ=.9	γ=1.
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	1.00	0.99	0.98	0.97	0.96	0.95	0.95	0.94	0.93	0.92	0.92
2.0	1.00	0.99	0.99	0.99	0.98	0.98	0.97	0.97	0.97	0.97	0.96
2.5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
3.0	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.02	1.02	1.02
3.5	1.00	1.00	1.01	1.01	1.02	1.02	1.02	1.03	1.03	1.04	1.04
4.0	1.00	1.01	1.01	1.02	1.02	1.03	1.03	1.04	1.05	1.05	1.06
4.5	1.00	1.01	1.01	1.02	1.03	1.04	1.04	1.05	1.06	1.07	1.07
5.0	1.00	1.01	1.02	1.03	1.03	1.04	1.05	1.06	1.07	1.08	1.08

Legend

$\alpha = .6$

$\frac{R_s}{h_s} = 0.022$

= represents error in excess of 5%

By displacing the point of origin a distance z_o beneath the chimney exit, Eq. (7) can be written in the form:

$$R = \alpha_* z \text{ for } z \geq z_o (= R_s/\alpha_*). \tag{8}$$

In view of Eq. (8), Eq. (3a) and therefore also Eq. (4a) can now be completely integrated. In the general case $N^2 \neq 0$ the excess plume temperature and the vertical motion of the plume are given by:

$$\frac{\theta'}{\theta_a} = S(\frac{z}{z_o}) + (G - S)\ (\frac{z}{z_o})^{-(2+\delta)} \tag{9}$$

$$\frac{w^2}{2} = \frac{g\ z_o}{3+\delta}\left\{\frac{S}{2}\left[(\frac{z}{z_o})^2 - (\frac{z}{z_o})^{-2(2+\delta)}\right] + (G - S)\left[(\frac{z}{z_o})^{-(1+\delta)} - (\frac{z}{z_o})^{-2(2+\delta)}\right]\right\} \tag{10}$$

where G is the initial value of θ'/θ_a, $S = -\frac{z_o N^2}{g\ (3+\delta)}$ is a nondimensional atmospheric stability parameter and $\delta = 4\gamma/9$. Furthermore a buoyancy dominated plume was assumed, i.e., one where $w = 0$ initially. The plume trajectory for any given atmospheric stratification can now be obtained from Eq.'s (5) and (10). In a uniformly stable atmosphere ($N > 0$) the solution is:

$$z = \left[\frac{2\ (3+\delta)\ z_o^{\delta}\ F_o}{U_s \alpha_*^{\ 2} N^2}\right]^{1/(3+\delta)} \left[\sin \frac{Nt}{2}\right]^{2/(3+\delta)} \tag{11}$$

where $F_o = U_s\ z_o^{\ 2}\ \alpha_*^{\ 2}\ g\ G$. The relationship between travel-time and downwind distance is given by $x = \int U\,dt$ + constant. Eq. (11) reduces to the famous "2/3-law" when $\delta = 0$, i.e., when the wind speed is constant with height. Eq. (11) possesses a global maximum when $t = \pi/N$, the magnitude of which is:

$$z_m = \left[\frac{2\ (3+\delta)\ z_o^{\ \delta}\ F_o}{U_s \alpha_*^{\ 2}\ N^2}\right]^{1/(3+\delta)}. \tag{12}$$

A final asymptotic rise is approached when $t \to \infty$. Thus:

$$\frac{z_f}{z_m} = 1 - \frac{1}{2(3+\delta)} \left(\frac{z_m}{z_o}\right)^{\delta} \frac{U_s}{U_m} \tag{13}$$

where U_m is determined from Eq. (1) at $z = z_m$. In deriving Eq. (13) Briggs' (1975) concept of the unaltered volume flux was used.

Finally, as $Nt \to 0$, Eq. (11) may be Taylor-expanded about the value zero to yield the result:

$$z = \left[\frac{(3+\delta)\, z_o^{\delta} F_o}{2\, U_s\, \alpha_*^2}\right]^{1/(3+\delta)} t^{2/(3+\delta)} \; . \tag{14}$$

Some typical plume trajectories are presented in Figure 2, in which the effect of wind exponent γ is seen to be significant.

<u>Sensitivity To Shear and Entrainment</u>

The question now arises as to the sensitivity of the predicted plume rise to changes in the observed values of γ and α. The sensitivities must be of the same magnitude in order to justify the use of the present theory, since it is more complex than the widely accepted Briggs' equations which require observations only of the entrainment constant α. The entrainment constant α is seldom obtained directly by measuring the plume radius. Usually its value is inferred by comparing the plume rise equation to observed trajectories, with the value of α being varied so as to obtain the best fit. Obviously this method will lump the effect of factors such as non-uniform winds into the observed variability of the entrainment constant. A physical mechanism which might explain the observed variability of α is not obvious, whereas the mechanisms which cause variability in wind shears are much better understood. It is the authors' belief that much of the observed variability of entrainment constant α can be ascribed to the hidden effect of variations in wind speed shear.

Sensitivity of a variable v is here defined as

$$S_v = 100 \frac{dv}{v} \approx 100 \frac{\Delta v}{v} \tag{15}$$

where Δv is the variability of v. Observed values of the entrainment constant α normally fall in the range 0.4 to 0.9, so that the variability is at most a factor of 2. Observed values of wind exponent γ usually fall in the range of .1 to .5, so that the variability can be as much as a factor of 5. The assumption is here made that in general,

$$\left.\frac{\Delta\gamma}{\Delta\alpha}\right|_{\text{observed}} = 2.5 \tag{16}$$

implying that a 25 % change in γ is observed as frequently as a 10% change in α.

Now,

$$dz = \frac{\partial z}{\partial \alpha} d\alpha + \frac{\partial z}{\partial \gamma} d\gamma \ , \tag{17}$$

and from equation (11) we obtain:

$$\frac{\partial z}{\partial \alpha} = - \frac{2+\delta}{3+\delta} \left(\frac{z}{\alpha}\right) \tag{18}$$

and

$$\frac{\partial z}{\partial \gamma} = \frac{4}{9} \frac{z}{3+\delta} \left[1 - \ell n\left(\frac{z}{z_o}\right)\right] \tag{19}$$

which are valid for both stable and neutral conditions. Combination of equations (15) through (19) yields

$$S_z = \frac{\delta}{3+\delta}\left[1 - \ln\left(\frac{z}{z_o}\right)\right] S_\gamma - \frac{2+\delta}{3+\delta} S_\alpha . \tag{20}$$

The first term on the right is the sensitivity of z to changes in γ, while the second term is the sensitivity to changes in α. The sensitivities can be approximated for small values of γ by

$$\left.\frac{S_z}{S_\gamma}\right|_\alpha \cong \frac{4\gamma}{27}\left[1 - \ln\left(\frac{z}{z_o}\right)\right] \tag{21}$$

and

$$\left.\frac{S_z}{S_\alpha}\right|_\gamma \cong -\frac{2}{3} \; . \tag{22}$$

These relations are plotted and compared in Figure 3 (the effect of assumption (16) has been included). It is apparent that S_z always depends linearly on S_α, with a 10% increase in α producing a 6.7% decrease in z no matter what the value of α or z might be. The dependence of S_z on S_γ is more complex since their ratio is a logarithmic function of z/z_o and depends linearly on the value of γ. Thus when $z/z_o = 10$ and $\gamma = 0.5$, a 25% increase in γ produces only a 2½% decrease in z. However, when $z/z_o = 100$ and $\gamma = 0.5$, a 25% increase in γ produces a 6.7% decrease in z. Therefore a change in γ has a cumulative effect which becomes most important when predictions indicate large values of plume rise.

It follows from the above that neither α nor γ is the predominant input variable. They have equally strong effects on the magnitude of plume rise and one should if possible, include both in the predictive equations.

The "Uniform Wind" Approximation

One disadvantage in applying the theory presented here is the necessity for an integral transformation between x and t. The equations are explicit in z and t, whereas the stack designer who might use these equations will normally want to work in z and x. This is somewhat difficult to do with the equations as given here, so a simpler approach is desirable. Such an approach could take the form of a "uniform wind" approximation.

For a uniform wind the velocity U_s at stack height should be used in the Briggs formulation. However, to account for vertical wind shear effects the present practice is to use a velocity U_B different from U_s. Usually the value of U_B chosen is the average velocity over the plume layer. Then for neutral conditions

$$z = \left(\frac{3}{2\alpha^2}\right)^{1/3} \frac{F_o^{1/3}}{U_B} x^{2/3} \tag{23}$$

and for stable conditions

$$z = \left(\frac{6F_o}{U_B \alpha^2 N^2}\right)^{1/3} \left[\sin\left(\frac{Nx}{2\,U_B}\right)\right]^{2/3} \tag{24}$$

where, for a power law wind profile,

$$U_B = U_{av} = \left[\frac{\left(\frac{h}{h_s}\right)^{1+\gamma} - 1}{\left(\frac{h}{h_s} - 1\right)\,(1+\gamma)}\right] U_s \,. \tag{25}$$

Recent comparisons (Slawson, 1977) of equation (23) with observed plume rise show an overestimation of z which can be corrected by using a velocity U_B somewhat greater than U_{av}. The disadvantages of this approach are that it requires an <u>a posteriori</u> adjustment of the plume rise equation in order to fit the observations, and the adjustment factor is usually different for each set of observations. A more straightforward method of adjusting equations (23) and (24) is to compare their predictions with those of the more exact theory presented here.

According to Briggs (1975) the final rise phase for neutral conditions occurs near

$$x_f = \begin{cases} 49\,F_o^{5/8}, & F_o \le 55\ m^4/s^3 \\ 119\,F_o^{2/5}, & F_o > 55\ m^4/s^3 \end{cases} \tag{26}$$

or at time $t_f = x_f/U_s$. Equating the final rise forms of (23) and (14) and solving for U_B, we obtain

$$\frac{U_B}{U_s} = \left[\frac{\frac{3}{2}\,t_f^{\,2}\alpha F_o}{U_s R_s^{\,3}\,(1+\frac{\delta}{3})^{3/\delta}\,(1+\frac{\delta}{2})^{(3/\delta)}\,(2+\delta)}\right]^{\frac{\delta}{3+\delta}} . \tag{27}$$

Similarly, for stable atmospheric conditions we have

$$\frac{U_B}{U_s} = \left[\frac{(6/N^2)\ \alpha F_o}{U_s R_s^{\,3}\,(1+\frac{\delta}{3})^{3/\delta}(1+\frac{\delta}{2})^{(3/\delta)}\,(2+\delta)}\right]^{\frac{\delta}{3+\delta}} \tag{28}$$

Comparison of equations (27) and (28) produces the single relationship

$$\frac{U_B}{U_s} = \left[\frac{A\alpha F_o}{U_s R_s^{\,3}\,(1+\frac{\delta}{3})^{3/\delta}\,(1+\frac{\delta}{2})^{(3/\delta)}\,(2+\delta)}\right]^{\frac{\delta}{3+\delta}} \tag{29}$$

where for a stable atmosphere

$$A = 6/N^2 \tag{30}$$

and for a neutral or slightly stable atmosphere

$$A = \frac{3}{2}\,t_f^{\,2} = \frac{3}{2}\left(\frac{x_f}{U_s}\right)^2 . \tag{31}$$

Equations (29) through (31) thus specify the uniform velocity U_B to be used in the Briggs' plume rise equations in order for that more simple formulation to predict final or maximum rises which are consistent with those from the more exact present theory. The error introduced by the "uniform wind" approximation is presented in Table 2. This conservative approximation to the actual plume rise is probably adequate for most applications.

TABLE 2. z("uniform wind")/z(shear)

t (sec)	NEUTRAL		STABLE	
	γ=.25	γ=.50	γ=.25	γ=.50
20	.93	.87	.96	.93
40	.94	.89	.98	.96
60	.95	.91	.99	.97
80	.96	.93	.99	.98
100	.96	.93	1.00	.99
120	.97	.94	1.00	1.00
140	.97	.95	1.00	1.00
160	.98	.95	1.00	1.00
180	.98	.96	1.00	1.00
200	.98	.96	1.00	1.00

Stack parameters same as Figure 2.

Comparison With Field Measurements

The verification of any plume rise theory eventually rests on comparison with data obtained from full-scale field measurements. The present theory has been applied to 21 time-mean stable plume trajectories observed at Ontario Hydro's 4000 MW Nanticoke Power Station (Djurfors, 1975). The results were inconclusive in that the observed scatter around the predicted plume trajectories was not reduced significantly below that found from comparison of observations with the commonly used uniform wind model. The difficulty lay in the fact that the Nanticoke study was not intended to provide data for comparison with a theory which requires the wind exponent γ as an input. Thus the values of γ which were available were subject to errors which were large enough to mask any improved predictive ability held by the present theory. It is therefore recommended that future full scale plume rise studies incorporate more accurate measurements of the vertical wind profile than are provided by single theodolite-pibal techniques. Double theodolite or tethersonde readings would be appropriate.

FIGURE I ILLUSTRATION OF PLUME GROWTH WITH HEIGHT

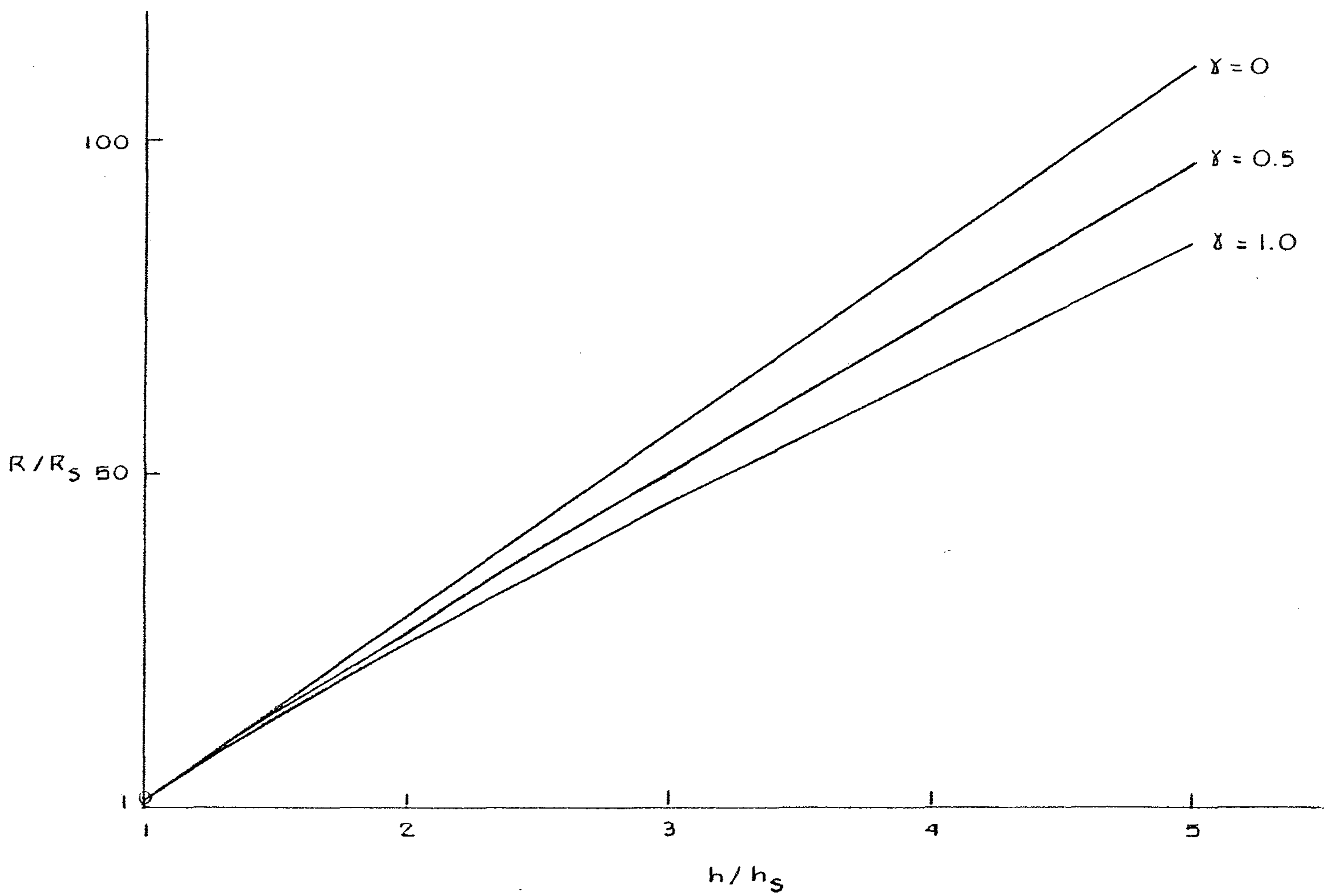
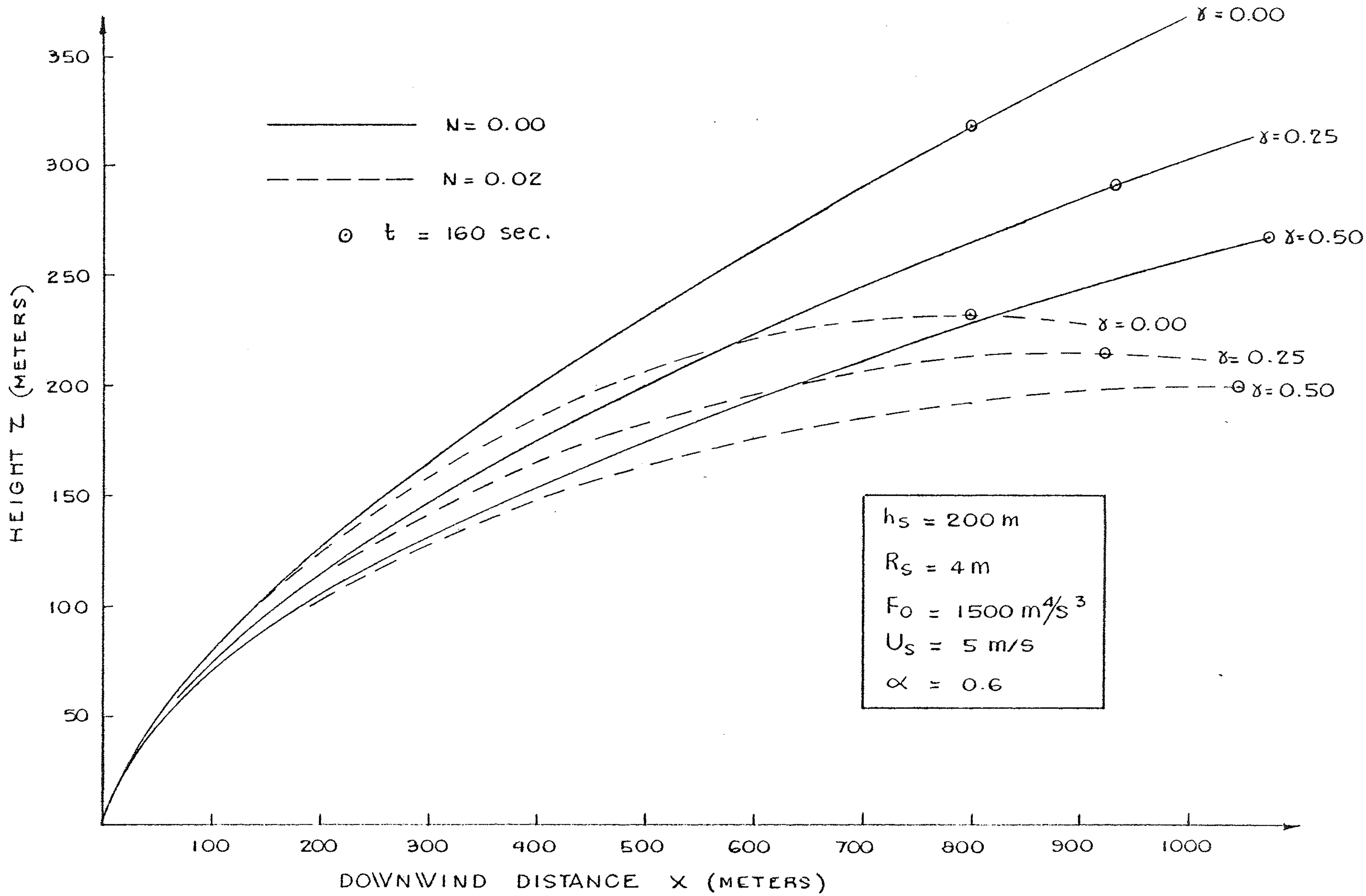


FIGURE 2 EFFECT OF WIND SHEAR ON PLUME RISE



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