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STOCHASTIC OPTIMIZATION OF WESTERN CANADIAN COAL PRODUCTION

by

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A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of Master of Science

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STOCHASTIC OPTIMIZATION OF WESTERN CANADIAN COAL PRODUCTION

Submitted by Alexandru Gheorghe Pop in partial fulfillment of the requirements for the degree of Master of Science in Mining Engineering.

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ABSTRACT

Western Canadian coal production faces significant production, haulage and marketing challenges from complex mining operations, long transportation distances to ports, competitive coal markets and price volatility. Thermal and metallurgical coal quality and quantity levels are presently managed using simplistic theories, and trial and error methods to meet consumer demands. This study uses stochastic modeling, linear and non-linear optimization techniques to formulate mathematical models of problems. The objective functions of the optimization models are derived from the profit maximization of an integrated mining system constrained by coal quality, quantity, demand and supply. The models are validated using data from Luscar-Sherritt and Fording mines. The results show that Luscar-Sherritt and Fording have profit expectations of $135.7M and $221.3M, at 50% probability of success, and $70M and $ 120M at zero failure probability. This unique study applies stochastic-optimization theories to develop comprehensive coal production, transport and marketing models for competitive business decisions.

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NOMENCLATURE

A_{ij}	Value per unit of metallurgical coal shipped from mine "i" to destination "j"
a_{im}	Ash content of metallurgical coal at mine "i"
a_{it}	Ash content of thermal coal at mine "i"
B_{ij}	Value per unit of thermal coal shipped from mine "i" to destination "j"
C_{asm}	Maximum ash content of metallurgical coal accepted on the market
C_{lasm}	Ash lower limit content of metallurgical coal input by the mining company
C_{ast}	Maximum ash content of thermal coal accepted on the market
C_{last}	Ash lower limit content of thermal coal input by the mining company
C_{em}	Minimum energy content of metallurgical coal accepted on the market
C_{et}	Minimum energy content of thermal coal accepted on the market
C_{fcm}	Minimum fixed carbon content of metallurgical coal accepted on the market
C_{fct}	Minimum fixed carbon content of thermal coal accepted on the market
C_{msm}	Maximum moisture content of metallurgical coal accepted on the market
C_{lmsm}	Moisture lower limit content of metallurgical coal set by the mining company
C_{mst}	Maximum moisture content of thermal coal accepted on the market
C_{lmst}	Moisture lower limit content of thermal coal input by the mining company
C_{sm}	Maximum sulfur content of metallurgical coal accepted on the market
C_{lsm}	Sulfur lower limit content of metallurgical coal input by the mining company
C_{st}	Maximum sulfur content of thermal coal accepted on the market
C_{lst}	Sulfur lower limit content of thermal coal input by the mining company
C_{vm}	Maximum volatile matter content of metallurgical coal accepted on the market
C_{vt}	Maximum volatile matter content of thermal coal accepted on the market
C^k	Available resources
d_{ij}	Distance from mine sites to destinations (km)
e_{im}	Energy content of metallurgical coal at mine "i"
e_{it}	Energy content of thermal coal at mine "i"
fc_{im}	Fixed carbon content of metallurgical coal at mine "i"
fc_{it}	Fixed carbon content of thermal coal at mine "i"
fc_{im}	Fixed carbon content of metallurgical coal at mine "i"

φ Function of random variables

k_{oi} Overhead cost per tonne of material handled at mine"i"

k_{mi} Unit cost of mining and processing of metallurgical coal at mine"i"

k_{pj} Port Charges at port "j"

k_s Unit cost per kilometer-tonne of coal hauled

k_{ti} Unit cost of mining and processing of thermal coal at mine "i"

M Total coal production from all the mines

M_i Maximum coal production capacity at mine "i"

m_{im} Moisture content of metallurgical coal at mine "i"

m_{it} Moisture content of thermal coal at mine "i"

MP_i Mining and processing cost at mine "i"

MP_T Total mining and processing cost

OVH_i Overhead cost at mine "i"

OVH_T Total overhead cost

PC_T Total port costs

P_j Maximum port capacity at destination "j"

P_T Profit function

p_m Price of metallurgical coal

p_t Price of thermal coal

R_j Maximum railway capacity from mines to destination "j"

Q_1 Total metallurgical coal production

Q_2 Total thermal coal production

Q_{im} Minimum metallurgical coal production requirement at mine "i"

Q_{it} Minimum capacity requirement of total coal production at mine "i"

Q_m Maximum amount of metallurgical coal that can be sold on the market annually

Q_{mc} Contracted metallurgical coal per year

Q_t Maximum amount of thermal coal that can be sold on the market annually

Q_{tc} Contracted thermal coal per year

s_{im} Sulfur content of metallurgical coal at mine "i"

s_{it} Sulfur content of thermal coal at mine "i"

TC_i Total cost at mine "i"

TC_T	Total cost
TR_{ij}	Transport cost from mine "i" to destination "j"
TR_T	Total transport cost
v_{im}	Volatile matter content of metallurgical coal at mine "i"
v_{it}	Volatile matter content of thermal coal at mine "i"
X_{ij}	Metallurgical coal produced at mine "i" and shipped to destination "j"
Y_{ij}	Thermal coal produced at mine "i" and shipped to destination "j"
λ^k	Lagrange multiplier
μ	Mean value
σ	Standard deviation

CHAPTER 1.0
INTRODUCTION

1.1 Background Information

Coal is the most abundant of the fossil fuels, and its reserves are also the most widely distributed within the earth crust. Estimates of the world's total recoverable reserves of coal in 1999, as reported by International Energy Outlook (IEO) are at about 1,0 billion tonnes [18]. The resulting ratio of coal reserves to production exceeds 220 years. Thus at current rates of production, coal reserves could last for another two centuries. The distribution of coal reserves around the world varies notably from that of oil and gas, in that significant reserves are found in the United States, Russia and other countries of the former Soviet Union (FSU) but not in the Middle East. The United States and the FSU each has roughly 25% of global coal reserves. China, Australia, India, Germany, and South Africa each has between 6 and 12 percent of world reserves as illustrated in Appendix 1.0 [18].

In spite of the fact that coal is expected to be displaced by natural gas for electrical and power generation in some parts of the world, only a slight drop in its share of total energy consumption is projected by 2020 [18]. In 1999, coal provided 22 percent of the world primary energy consumption and is projected to fall to 19 percent by 2020 [18]. World coal consumption has been in a period of generally slow growth since the late 1980s, a trend that is expected to continue. IEO projects some growth in coal demand between 1999 and 2020, at an average annual rate of 1.5 percent, but with considerable variation among regions [18]. Coal demand is expected to decline in Western Europe, Eastern Europe, and the FSU. Increases are expected in the United States, Japan, developing Asia (China, India, ASEAN countries), Brazil and Mexico. China and India are projected to account for 92 percent of the total expected increase in coal demand worldwide. Coal consumption is heavily concentrated in the electricity generation sector, and steel production. More than 55% of the coal consumed worldwide is used for electricity generation. Power generation accounts for virtually all the projected growth in coal consumption worldwide [18, 38].

Consumption of coking coal is projected to decline slightly in most regions of the world as a result of technological advances in steelmaking, increasing output from electric arc furnaces, and continuing replacement of steel by other materials in end-use applications [18, 38]. Quality and geological characteristics of coal deposits are other important parameters for coal reserves. Coal is a much more heterogeneous source of energy than oil and natural gas. Its quality varies significantly from region to region and even within various sections of a coal seam. For example, Australia, the United States, and Canada are endowed with substantial coal reserves that can be used to manufacture coke. Together, these three countries have supplied approximately 85 percent of globally traded coking coal during recent years (1985-2000) [18, 36, 38].

Canada has been a leading coal producer and exporter, particularly to the countries of the Pacific Rim. The coal mining industry is vital to Canada's economy. In 1999, the western provinces of Saskatchewan, Alberta and British Columbia produced 70 million tonnes of coal, which accounted for about 97% of the Canada's total coal production with a total market value of $1.4 billion [36]. Furthermore, Canada's coal mining industry provided about 7,145 direct jobs in 1999. Alberta, the largest coal-producing province in Canada, produced 34 million tonnes of coal in 1999 (47% of Canada's total production) and generated $430 million in total revenue. Similarly in 1999, British Columbia produced 24.2 million tonnes of coal (34% of Canada's total production) worth $805 million. The difference between the revenues comes from the fact that British Columbia produces mainly metallurgical coal and Alberta thermal coal. The remaining coal production came from Saskatchewan and Nova Scotia (up to 1999). Canada derives its energy sources mainly from coal, bitumen, natural gas and oil. About 16.5% of Canada's electricity is generated from coal, 59% from hydro, 13.5% from nuclear, 9% from natural gas, and less than 1.5% others (oil, solar, wind) (Flemming, U of A presentation 2001) [20, 30, 36,].

Geological reserve estimates indicate that coal constitutes about 67% of the total energy reserves of Canada against 25% for bitumen, 6% for natural gas and 3% for conventional oil [30]. A recent survey of the best US and Canadian power plants showed that the average costs of production and maintenance to be 1.4 cents/kilowatt-hour for natural gas, 1.35 cents/kilowatt-hour for nuclear and 1.02 cents/kilowatt-hour for coal [20]. Thus, Canada will continue to derive a major portion of its energy requirements from coal alone. Alberta exports all of its metallurgical coal but a significant portion of its thermal coal is used for domestic power generation. British Columbia exports all of its thermal and metallurgical coal. The coal exports from both provinces is hauled by Canadian National Railways (CNR) to the Neptune Bulk Terminals in the Vancouver area for export to Japan, South Korea, Taiwan, Brazil and Europe [9, 22, 36].

1.2 Fundamental Problem of Western Canadian Coal Production

Western Canadian coal mining companies use exclusively surface mining techniques to extract coal from its natural environment. The application of surface mining techniques depends on the amount of overburden to be removed in order to economically extract the coal. Surface mining methods can be classified into open pit, strip-mining, aqueous mining, placer mining and solution mining. In the prairie mines, strip-mining techniques are used to produce coal. The first step involves removing the topsoil and subsoil, which are either stockpiled for future reclamation or placed directly on previously mined and recountured land. Next the dragline digs the overburden and moves it a short distance to expose the coal. Shovel and loaders then excavate and load the exposed coal into trucks, which transport the coal to an adjacent preparation plant or electricity generation station. Once the coal has been removed, the overburden, subsoil and topsoil are replaced in their proper sequence, leveled and reclaimed, for agricultural use [9, 22].

At foothills and mountain mines, the open-pit mining method is used. After salvaging the surface soil suitable for reclamation, rock overburden is drilled and blasted. Loaders and shovels load the rock into trucks to be hauled into previously mined-out pits or dump areas. The exposed coal is extracted and loaded by shovels or loaders into haul trucks for delivery to preparation plants or power stations. At the preparation plant, further waste material is removed and the coal is sized for shipping. Coal is then loaded into railcars for delivery to customers. The mined-out pit is filled with previously mined rock overburden and the land is reclaimed for wildlife habitat [9, 22].

Two competing railways, Canadian National and Canadian Pacific, connect coal producers with west coast at Vancouver and Ridley, and with eastern Canada, both directly and through the port at Thunder Bay as illustrated in Figure 1.1. The principal rail routes for exports now run from southeast British Columbia and south-central Alberta to the Vancouver and Robert Bank terminals. Coal from Northeastern B.C and northern Alberta is carried by BC Rail on its dedicated line to the Ridley Island terminal, near Prince Rupert. The Canadian railways offer one of the lowest per-ton-km rates in the world. This advantage is offset by the long haul distances which, for eastbound coal movements, can be over 2300km. Westbound rail haulage averages around 1100 km. High relative freight costs mean that thermal coal exports are rarely competitive on a *fob* basis unless production costs are incremental to metallurgical coal output. Coal is transported in both high capacity aluminium and conventional steel rotary dump gondola unit trains. Train configuration can vary with total carrying capacity capable of exceeding 13 000 tonnes [9, 22, 38].

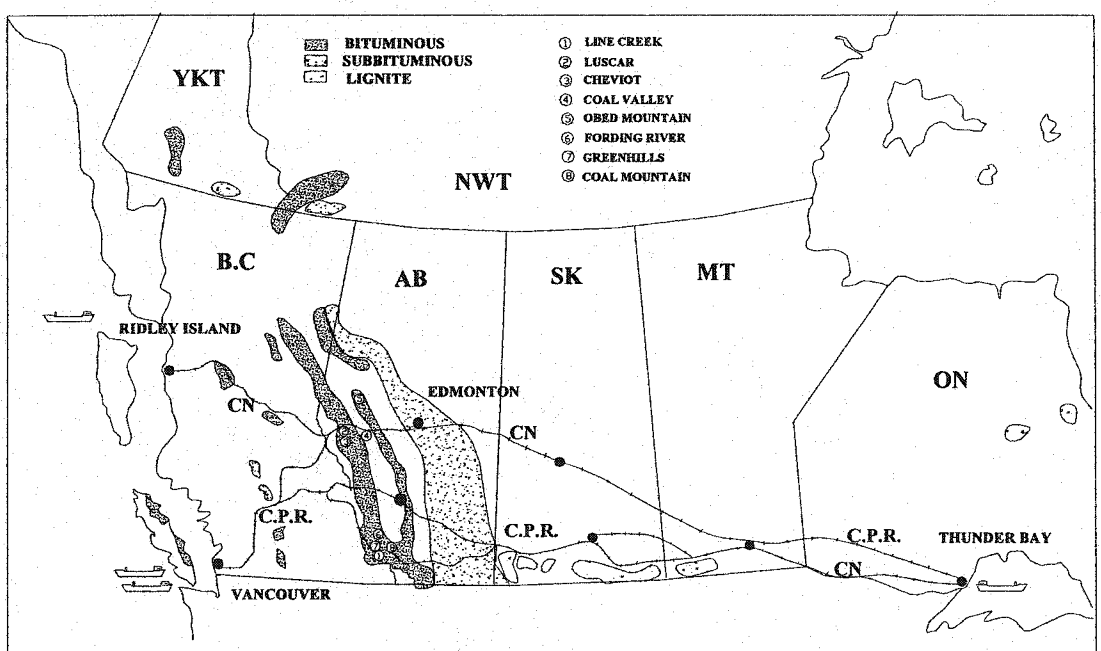


Figure 1.1 Coal Deposits, Mines, Ports, and Railways of Western Canada

Western Canada is estimated to contain some 190 billions tonnes of coal of all ranks from lignitic to anthracitic [18, 31, 36, 38]. The four ranks of coal are anthracite, bituminous, subbituminous and lignite. These are distinguished primarily by their percentage of fixed carbon and heating value. Certain bituminous coals have metallurgical or coking properties, which make them important in the production of steel. The rank of Western Canadian coal increases from east to west, with coals of similar rank occurring in broad belts parallel to the Rocky Mountains as illustrated in Figure 1.1. The quality issues for metallurgical and thermal coal include the amount of ash, sulfur, energy, moisture, fixed carbon, and volatile matter content. The main characteristic and advantage of Western Canadian coals over coal from other countries is the generally low sulfur content as illustrated in Appendix 2.0 of this report [22, 31, 38].

Canadian export coal producers struggled during the 1990s. Over-optimistic demand forecasts in the late 1970s led to the construction of high-cost mines and the upgrading, at great expense, of rail and port facilities. Since the mid 1980s, reduced Japanese demand for coking coal has left Western Canadian producers with serious over-capacity. Subsequent fluctuation in Asian steel production and unfavorable dollar exchange rate movements were exacerbated by Asian economic crisis of the late 1990s, which led to further substantial price cuts for coking coal [38]. With production and transport costs higher than exporting countries such as Australia and Indonesia, Canadian producers have had to bear severe profit margin reduction. Some of the mines have survived only through long-term contracts that have given *fob* prices well in excess of those received elsewhere for metallurgical coal. While operators have recently made some progress in cutting operating costs, Canada is still one of the countries most likely to be affected by reduced market share over the short-to medium-term. Increasing competition from lower-cost producers from Indonesia and Australia will continue to erode Canada's market base in the Far East. Higher shipping costs from western Canadian ports make market areas such as the Mediterranean or Western Europe equally unattractive [38].

The cost of mining and haulage of Western Canadian coal has become a major focal point when compared to the relatively inexpensive coal from Australia, and Indonesia as illustrated in Appendix 1.0. In open pit mine operations, material handling constitutes a significant portion of total cost. Studies by Caterpillar on global mining operations show that open pit mining cost can be broken down as 40% hauling, 25% supporting, 20% loading and 15% drilling and blasting [4, 6]. Thus, reducing the loading and hauling costs can significantly enhance the profitability of a mine. Coal transportation costs from mine sites to power plants, Vancouver Ports, Ridley Island and Thunder Bay range between 23% and 65%, while mining costs range between 35% and 77% of the total production and haulage costs. A good strategy for reducing production and transportation costs is the consolidation of the ownership and the creation of integrated mining systems. Using an integrated mining system some transportation and processing costs could be avoided by planning the production and coal products according to the coal qualities and distances from the mines to the final destinations. Capacity expansion to achieve economics of scale, advanced technologies and ingenious research must be undertaken to review, and assess areas for improvement and cost reduction. This research study uses linear and non-linear optimization and stochastic modeling techniques to formulate models and to examine areas for cost reduction, quality and capacity expansions in coal production and haulage.

1.3 Scope and Objectives of Study

In recent years, many coal companies have created joint ventures or publicly traded companies that have more than one coal mine in operation. Therefore, in increasingly competitive markets, companies have to redesign and simplify their production and management processes to make their operations more efficient and cost effective. This study presents a new approach for creating and optimizing an integrated production process. The models are based on the structure and composition of a typical Western Canadian Coal company like Fording Coal Ltd. and Luscar-Sherritt Int. Even though similar to these companies, the models are different to preserve the confidential information obtained from these mines. These coal mining companies have several mines geographically located at different regions as illustrated in Figure 1.1, but operate as an integrated single operation. While each mine has its own production schedule, centralized teams are responsible for carrying out many of the functions that are common to the entire operation. Under such a system, comprehensive blending of products using quantities of each type of product with specific quality levels from each mine takes place. The production occurs as a single, integrated process, which ends with assembled product stockpiles ready for shipping to customers. The final product is considered as a single desired product from integrated mining units.

The main objectives of this study are to (i) develop Linear Programming (LP) and Non-Linear Programming (NLP) optimization models for Western Canadian coal production and haulage systems; (ii) develop stochastic-optimization models for Western Canadian coal production and haulage systems; (iii) simulate these models to create a series of optima for coal production and haulage systems in Western Canada; (iv) provide detailed analysis of the system to examine the economics and risks associated with Western Canadian coal production and haulage. The scope of this work is limited to stochastic-optimization modeling and analysis of Western Canadian coal production using linear programming, Lagrange Parameterization, Monte Carlo and Latin hypercube techniques. Data and information from two different case studies, Luscar-Sherritt International and Fording Coal's operations, were used to validate the model. Different markets, mine capacities and coal quality scenarios were analyzed in the model validation process. The stochastic-optimization model validation demonstrates the power and usefulness of the model in finding an optimal solution and providing the management with a decision tool.

1.4 Research Methodology

Figure 1.2 illustrates the optimization model of coal extraction and transport. This model is divided into three main sections, including (i) production and haulage optimization model (PHOM); (ii) stochastic-optimization model (SOM); and (iii) the solution model. The main focus of the optimization process is to ensure that cost-effective production and haulage system are put in place to provide the contract requirements governing coal quantity and quality. The generalized optimization model comprises two main modules, the objective function and the underlying constraints developed in Chapter 3.0. The objective function is to maximize the total profits from the mines subject to the underlying constraints. The total profit is a function of the mining and processing costs, transportation, port charges and overhead costs and the revenues. The underlying field constraints are derived from railways, port and mine capacities; coal qualities and market supply and demand. The constraints are imposed to define the feasible region where the outputs ensure the attainment of the objective function.

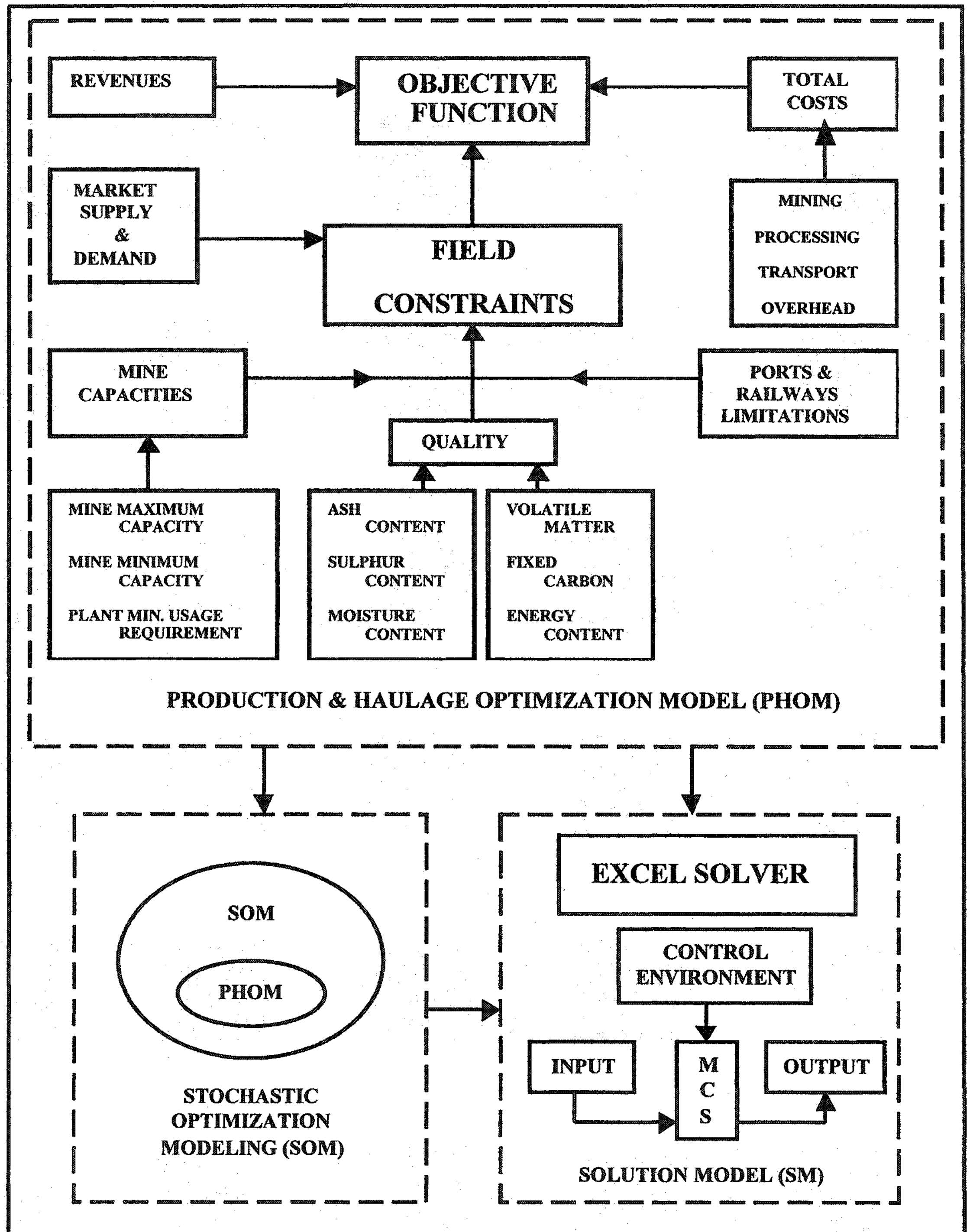


Figure 1.2 Stochastic-Optimization and Solution Models of Coal Production

To develop the objective function and the constraints, extensive and detailed data was collected on the geology and coal qualities; mine production and processing costs; mines, ports, and railway capacities; mine-port distances and freight costs; coal industry overview and markets. Solver software within Excel is used to solve both the LP and NLP models. Solver has a high processing speed and solves the problem in a mater of seconds and it also provides an output results with sensitivity analysis. Solver solution models are easily modified to analyze various options. The stochastic model is formulated by introducing the variable stochastic processes into the objective function in equation (3.7) in Chapter 3.0. The key objectives in the stochastic-optimization models are (i) maximize the expected value of the total profit; and (ii) minimize the variance associated with the total profit. The solution model comprises two main modules – the EXCEL SOLVER for solving the LP and the NLP models and the Monte Carlo simulation technique to provide solution to the stochastic model. Solver is linked with @RISK software to simulate a series of optima for the stochastic optimization model. The characterization of the risks associated with coal extraction and transport is based on the results of the stochastic simulation.

1.5 Scientific and Industrial Contributions

This is the first attempt in the application of LP and NLP optimization techniques to solve the problem associated with western Canadian coal production and transport to Vancouver Ridley Island and Thunder Bay ports. In increasing competitive coal markets, these techniques offer the management of coal companies' sophisticated tools for reducing costs, increasing efficiency and product quality. The results from the stochastic-optimization modeling provide coal companies with relevant data and information to make decisions under uncertainty and to assist in risk mitigation and control. The tools also provide relevant information on cost-composition and production levels from various satellite mines to create an appropriate product quality and quantity levels for market requirements. These tools will benefit Fording Coal Ltd., and Luscar-Sherritt International in managing coal haulage and transportation via CN and CP rails from various production sites to the shipping ports.

1.6 Report Structure

This report contains a thorough literature and past research initiatives review of linear programming and Lagrange parameterization optimizations techniques in Chapter 2.0. The generalized linear programming and Lagrange multipliers mathematical models are provided in Chapter 3.0. The computer model and two case studies are conducted in Chapter 4.0 to validate the mathematical models. Discussion and analysis of results is provided in Chapter 5.0 where different market, coal qualities and mine capacities scenarios were developed and the risk associated with the mines was analyzed. The conclusions and the recommendations regarding the report are provided in Chapter 6.0. The report also contains eight appendixes providing a world coal industry overview in Appendix 1.0; Canada's domestic coal market and industry overview in Appendix 2.0; an answer report and sensitivity report for Luscar-Sherritt case study in Appendix 3.0 and 4.0; an answer report and sensitivity report for Fording Coal case study in Appendix 5.0 and 6.0. Appendix 7.0 and 8.0 consist of tables and figures regarding the results of the base cases and several scenarios developed in Chapter 5.0.

CHAPTER 2.0
LITERATURE REVIEW

This chapter deals with the analytical focus of the literature underlying this study. It comprises the literature on linear and non-linear optimization and stochastic-optimization theories and their application for solving industrial problems.

2.1 Mineral Market Environment

Estimating mineral project revenue is a difficult and risky but necessary activity. Mining projects are particularly sensitive to projections of mineral prices, many of which are volatile. The unique nature of mineral markets, prices, and product specifications occupies a major role in mineral project evaluation. One of the key variables associated with annual production is the tonnage of ore produced. Annual ore production is derived from the mining schedule, which is a function of deposit characteristics, mining method and many other factors. The production from every mine has a unique analysis, which may significantly affect the product price in a contractual market environment. The price of contractual coal is greatly affected by sulfur and ash content and heating value [11, 14].

Mineral prices are ultimately determined by supply and demand like any other product. On the demand side, one encounters the fundamental uncertainty surrounding the general level of economic activity that will exist in some future period. The demand for minerals lags most other economic activity, as consumers either accumulate or work off inventories before concluding that changes in business conditions will be sustained long enough to warrant changes in their raw materials orders. On the supply side, mineral production curtailments or expansions are often not felt for several months in the market place due to the large amount of product in the "pipeline" en route to the market. Also, mines have a high level of fixed costs and often operate in remote locations, both of which increase the resistance to shutdowns and start-ups when economic condition change. Supply is also affected by new discoveries, new technology, and recycling. Many minerals are traded on world markets; therefore, in weighing supply and demand pressures the analyst must generally consider production and consumption in the entire free world along with any trade restrictions that might exist. In view of the above characteristics, the production process must be subjected to rigorous modeling and analysis for comprehensive decisions on product quality, quantity and periodic adjustments to meet contractual obligations [11, 14].

2.2 Fundamental Theories of Optimization Modeling

The concept of optimization is very important in the analysis of many complex decision or allocation problems. Using the optimization philosophy, one approaches a complex decision problem, involving the selection of values for a number of interrelated variables to quantify performance and measure the quality of decision. The objective is maximized (or minimized) subject to constraints that may limit the selection and values of decision variables. The objective function may be used to optimize profit, loss, speed or distance, expected return in risky investments, or social welfare in the context of government planning. Optimization may also provide a suitable framework for analysis. Many practical problems can be formulated as constrained optimization equations. These complex problems, for example, the detailed production policy of a giant corporation, the planning of a large government agency, or even the design of a complex device cannot be directly treated in its entirety. Thus, these problems must be decomposed into separate sub-problems with limiting constraints to restrict the solution space [17, 21, 26].

2.2.1 Linear Programming Optimization

A linear programming (LP) problem is characterized by linear functions of the unknowns with a set of constraints, which are also linear equalities or linear inequalities in the unknowns. LP problem can be studied both algebraically and geometrically. The two approaches are equivalent, but one or the other may be more convenient for answering a particular question about an LP problem. The algebraic point of view is based on creating standard forms from LP problems. The coefficient matrix of the constraints of the LP problem can be analyzed using tools of linear algebra. The geometric approach is based on the geometry of the feasible region, and uses ideas such as convexity to analyze the LP problem. It is less dependent on the particular way in which the constraints are written. The use of the geometrical approach makes many of the concepts in LP problems easy to understand, because they can be described in terms of intuitive notions, such as, moving along an edge of the feasible region. There is a direct correspondence between these two points of view. The generalized LP optimization algorithm is formulated as.

$$\text{Maximize} \quad f_i(x) = a_i{}^*x_i \tag{2.1}$$

$$\text{Subject to} \quad h_i(x) = 0, \qquad i = 1, 2, \ldots, m \tag{2.2}$$

$$g_j(x) \leq 0, \qquad j = 1, 2, \ldots, r \tag{2.3}$$

$$x \in S. \tag{2.4}$$

In this formulation, x is an n - dimensional vector of unknowns, $\{x = (x_1, x_2, \ldots, x_n)\}$, and $\{f_i, h_i, i = 1,2, \ldots, m\}$, and $\{g_j, j = 1, 2, \ldots, r\}$ are real valued functions of the variables$\{x_1, x_2, \ldots, x_n\}$. The set S is a subset of n-dimensional space. The function f is the objective function of the problem and the equations, inequalities, and set restrictions are constraints. One obvious measure of the complexity of an LP problem is its size, which is measured in terms of the number of unknown variables or the number of constraints. As might be expected, the size of problems that can be effectively solved has been increasing with advancing computing technology and with advancing theory. Today, with present computing capabilities, however, it is reasonable to distinguish three classes of problems: small scale problems having about five or less unknowns and constraints; intermediate-scale problems having from about five to a hundred variables; and large-scale problems having in the order of a thousand variables and constraints [21, 26].

The most important characteristic of a high-speed digital computer is its ability to perform repetitive operations efficiently. In order to exploit this basic characteristic, most algorithms designed to solve large optimization problems are iterative in nature. Typically, in solving a programming problem, an initial vector x_0 is selected and the algorithm generates an improved vector x_1. The process is repeated many times until the attainment of a better solution. Continuing in this fashion, a sequence of ever-improving points $\{x_0, x_1, ..., x_k, ...,\}$ is found that approaches a solution point x^*. For LP problems, the generated sequence is a finite length, reaching the solution point exactly after a finite (although initially unspecified) number of steps. For nonlinear programming (NLP) problems, the sequence generally does not ever reach the exact solution point, but converges toward this solution point. For NLP problems, the process is terminated when a point sufficiently close to the solution is obtained with a balance between the cost and the time for processing [17, 21, 26].

An LP problem is a mathematical formulation in which the objective function is linear in the unknowns with a set of constraints of linear equalities and/or linear inequalities. The exact form of these constraints may differ from one problem to another, but as shown below, any linear program can be transformed into the following standard form:

$$\text{Maximize} \quad c_1 x_1 + c_2 x_2 + \dots + c_n x_n \tag{2.5}$$

$$\begin{aligned} \text{Subject to} \quad & a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1 \\ & a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n = b_2 \\ & \cdot \\ & \cdot \\ & \cdot \\ & a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n = b_m \end{aligned} \tag{2.6}$$

$$\text{and} \quad x_1 \geq 0,\ x_2 \geq 0,\ \dots,\ x_n \geq 0, \tag{2.7}$$

The b_i 's, c_i 's and a_{ij} 's are fixed real constants, and the x_i 's are real numbers to be determined. It is always assumed that each equation has been multiplied by minus unity, if necessary, so that each $b_i \geq 0$. In more compact vector notation, the standard problem presented by equations (2.5) trough (2.7) could be written as in equations (2.8) and (2.9).

$$\text{Maximize} \quad cx \tag{2.8}$$

$$\text{Subject to} \quad Ax = b \quad \text{and } x \geq 0. \tag{2.9}$$

Here x is an n-dimensional column vector, c is an n-dimensional row vector, A is an $m*n$ matrix and b is an m-dimensional column vector. The vector inequality $x \geq 0$ means that each component of x is nonnegative. The basic solution of equation (2.7) can be found by considering the system of equalities

$$Ax = b \tag{2.10}$$

x is a vector, b an m-vector and A is an $m * n$ matrix. Suppose that from the n columns of A we select a set of m linearly independent columns (such a set exists if the rank of A is m). For notational simplicity assume that the first m columns of A are selected and denote the $m * n$ matrix determined by these columns by B. The matrix B is then nonsingular and the equation can be solved [17, 21, 26].

The simplex method is an iterative method for solving an LP problem written in standard form. It moves from one basic feasible solution to another. At each iteration the method tests to see if the current point is optimal. If it is not, the method selects a feasible direction along which the objective function improves, and moves to an adjacent basic feasible solution along that direction [21, 26]. Considering a general linear program, with the standard form given by equations (2.11), (2.12), (2.13), and assuming that the problem has n variables and m linearly independent equality constraints, the steps in the simplex method are as follows.

$$\text{maximize} \quad z = c^T x \tag{2.11}$$

$$\text{subject to} \quad Ax = b \tag{2.12}$$

$$x \geq 0 \tag{2.13}$$

If x is a basic feasible solution, with the variables ordered so that

$$x = \begin{pmatrix} x_B \\ x_N \end{pmatrix} \tag{2.14}$$

x_B is the vector of basic variables and x_N is the (currently zero) vector of non-basic variables. The objective function can be written as in equation (2.15)

$$z = c_B^T x_B + c_N^T x_N \tag{2.15}$$

The coefficients for the basic variables are in c_B and the coefficients for the non-basic variables are c_N. Similarly the constraints are written as

$$Bx_B + Nx_N = b. \tag{2.16}$$

The constraints can be rewritten as

$$x_B = B^{-1}b - B^{-1}Nx_N. \tag{2.17}$$

By varying the values of the non-basic variables one gets all possible solutions to $Ax = b$. If equation (2.17) is substituted in equation (2.15) the objective function z becomes (2.18).

$$z = c_B^T B^{-1}b + (c_N^T - c_B^T B^{-1}N)x_N \tag{2.18}$$

One can define $y = (c_B^T B^{-1})^T = B^{-T}c_B$, then z can be written as in equation (2.19).

$$z = y^T b + (c_N^T - y^T N)x_N \tag{2.19}$$

The formula in equation (2.19) is efficient computationally. The vector y is the vector of simplex multipliers and the current values of the basic variables and the objective are obtained by setting $x_N = 0$. The solution is given by the formulas in equation (2.20).

$$x_B = \hat{b} = B^{-1}b \qquad \text{and} \qquad z = \hat{z} + \hat{c}_N^T x_N \tag{2.20}$$

$\hat{z} = c_B^T B^{-1} b$ and $\hat{c}_j$ is the entry in the vector $\hat{c}_N^T \equiv (c_N^T - c_B^T B^{-1} N)$ corresponding to x_j. The coefficient $\hat{c}_j$ is called the reduced cost of x_j [17].

2.2.2 Non-Linear Programming

2.2.2.1 Lagrange Multiplier Method

The Lagrange multiplier method is commonly used to solve constrained problems. The general procedure of the Lagrange multiplier method is to combine the constraints into the objective function and then to take the derivatives of the objective function with respect to the variables. The generalized mathematical programming problem can be stated as:

$$\text{Maximize} \qquad f_i(x) = c_i * x_i \tag{2.21}$$

$$\text{Subject to} \qquad A(x) \le b, \qquad i = 1, 2, \ldots, m \tag{2.22}$$

$f_i(x)$ is twice continuously differentiable and A_{m*n} is a matrix of full rank. The Lagrange function is defined in equation (2.23)

$$L(x,\lambda) = f(x) - \lambda^T [A(x) - b] \tag{2.23}$$

First order necessary optimality condition: if x_* is a local maximizer of f over the set $\{x : Ax \le b\}$, then for some vector λ_* of Lagrange multipliers equations (2.24) through (2.27) have to be satisfied.

$$\nabla f(x_*) = A^T \lambda \tag{2.24}$$

$$\lambda_* \le 0 \tag{2.25}$$

$$\lambda_*^T (Ax_* - b) = 0 \tag{2.26}$$

$$Z^T \nabla^2 f(x_*) Z \qquad \text{-is positive definite} \qquad (2.27)$$

Z is a null-space matrix for the active constraints matrix at x_*. By solving the above system of equations one can find the stationary points, which are candidates for the optimal solution of the original problem. The second order optimality conditions are represented by equation (2.28).

$$Hf = \left(\frac{\partial f}{\partial x_i \partial x_j} \right)_{m*n} \qquad \text{-negative definite (for maximization problem)} \qquad (2.28)$$

H_f is the Hessian matrix of the first order derivatives [21, 26].

2.2.2.2 Generalized Lagrange Multiplier Method

Lagrange multipliers are introduced in a context of differentiable functions, and are used to produce constrained stationary points. Their validity or usefulness often appears to be connected with differentiation of the function to be optimized. But many typical operation research problems, however, involve discontinuous or non-differentiable functions, which must be optimized subject to constraints. Everett III (1963) has extended the application of the Lagrange multiplier method to optimization problems involving non-differentiable functions [8]. The procedure is first to identify those constraints which are to be handled by Lagrange multiplier method, next multiply each constraint by an undetermined Lagrange multiplier and then subtract the product from the original objective function. The use of Lagrange multipliers in this method constitutes a technique whose goal is maximization, rather than location of stationary points, of a function with constraints. Therefore, there are no restrictions such as continuity or differentiability on the function to be maximized. Everett (1963) defines the problem as a maximization of the payoff, subject to given constraints c^k, $k = 1 \ldots n$, on the resource, to find equations (2.29) and (2.30).

$$max_{x \in S} H(x) \qquad (2.29)$$

$$\text{subject to } C^k(x) \leq c^k, \text{ all } k. \qquad (2.30)$$

Everett (1963) has also presented and proved a theorem concerning the use of Lagrange multipliers, its meaning and implications [8]. The theoretical basis is defined by the theorem in equations (2.31) to (2.34).

1. λ^k, $k = 1, n$ are nonnegative real numbers, (2.31)
2. $x^* \in S$ maximizes the function (2.32)

$$H(x)-\sum_{k=1}^{n}\lambda^k C^k(x) \quad \text{- over all } x \in S, \tag{2.33}$$

3. x^* maximizes $H(x)$ over all those $x \in S$ such that $C^k \leq C^k(x)$ for all k.

The theorem states that for any choice of nonnegative λ^k, $k = 1, n$, if an unconstrained maximum of the new (Lagrangian) function (2.34) can be found (where x^*, say is a strategy which produces the maximum), then the solution to that unconstrained maximization problem are the amount of each resource expended in achieving the unconstrained solution.

$$L(x) = H(x)-\sum_{k=1}^{n}\lambda^k C^k(x) \tag{2.34}$$

Thus, if x^* produced the unconstrained maximum, and required resources $C^k(x)$, then x^* itself produces the greatest payoff which can be achieved without using more of any resource than x^* does [8]. A limitation of the Lagrange multiplier method arises from the fact that it does not guarantee that an answer can be found in every case. It simply asserts that if an answer can be found it will be indeed optimum. In general, different choices of the λ^k lead to different resource levels, and it may be necessary to adjust them by trial and error to achieve any given set of constraints stated in advance. The Lagrange multiplier method generates a mapping of the space of lambda vectors (λ^k, $k = 1, n$) into the space of constraint vectors (c^k, $k = 1, n$). There may be inaccessible regions (called gaps) consisting of constraints vectors that are not generated by any λ vectors. Optimum payoffs for constraints inside such inaccessible regions could be impossible to find by straightforward application of the Lagrange multiplier method, and must be sought by other means. The basic cause of an inaccessible region is non-concavity in the function of the optimum payoff vs. resource constraints. Each solution that can be obtained by Lagrange multipliers defines a bounding hyper-plane that gives an upper bound to the maximum payoff at all points [8].

2.3 Stochastic-Optimization Modeling

In most of the evaluation frequently performed in the mining industry, it has been assumed that input data are known with certainty. In reality, estimates of ore grade, mining costs, commodity price, etc., are subject to varying degrees of uncertainty due to the inability to predict the future with much precision. Consideration of time element cannot be ignored in many random phenomena observed in engineering and economic problems. To evaluate the statistical characteristics of these random phenomena, it is necessary to consider the concept of a family of random variables that is a function of time. Random phenomena whose characteristics can be determined by this concept are called stochastic processes. The general method by which the level and magnitude of risk associated with mining operations is determined is to establish probability distributions for the input variables rather than treating each of these parameters as being known with certainty. Several techniques are available to help managers analyze risk. Three of the most common are best-case/worst –case analysis, what-if analysis, and simulation. Of these methods, simulation is the most powerful. Simulation is a technique that measures and describes various characteristics of the bottom-line performance measure of a model when one or more values for the independent variables are uncertain. If any independent variables in a model are random variables, the dependent variable (z) also represents a random variable. The objective in simulation is to describe the distribution and characteristics of the possible values of the bottom-line performance measure z, given the possible values and behavior of the independent variables $\{x_1, x_2, \ldots, x_k\}$ [13, 14, 25, 27].

The idea behind simulation is similar to the notion of playing out many what-if scenarios. The difference is that the process of assigning values to the cell in the spreadsheet that represent random variables is automated so that: (i) the values are assigned in a nonbiased way, and (ii) the spreadsheet user is relieved of the burden for determining these values.

With simulation one repeatedly and randomly generates sample values for each uncertain input variable $\{x_1, x_2, ..., x_k\}$ in the model and then computes the resulting value of the bottom line performance measure (z). One can then use the sample values of z to estimate the true distribution and other characteristics of the performance measure z. For example, one can use the sample observations to construct a frequency distribution of the performance measure, to estimate the range of values over which the performance measure might vary. It can be also used to estimate its mean and variance, and to estimate the probability that the actual value of the performance measure will be greater than or less than a particular value. All these measures provide greater insight into the risk associated with a given decision than a single value calculated based on the expected values for the uncertain independent variables. A general approach which has been successfully applied to a variety of problems with uncertainty is to assign explicitly or implicitly, a probability distribution to the various unknown parameters [13, 25, 27, 33,].

A stochastic process is defined as follows: Given a set of random and continuous data x_i $(i = 1, n)$, a stochastic process will capture the uncertainty in the distribution of $f(x)$ at a level of confidence, α. If P_T is a function of many variables (technical, economic, safety and others) defined as in equation (2.35), x_i $(i = 1, n)$ are random and continuous, and $f(x)$ is the probability density function of the set of variables x_i $(i=1 ,n)$, then the expected value $E[P_T]$ and the variance $VAR[P_T]$ could be expressed as in equations (2.36) and (2.37).

$$P_T = \phi[x_i, i = 1, n] \tag{2.35}$$

$$E[P_T] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} \phi[x_i, i = 1, n] * f(x_i) dx_i [i = 1, n] \tag{2.36}$$

$$VAR[P_T] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} [\phi[x_i, i = 1, n] - E[P_T]]^2 * f(x_i) dx_i [i = 1, n] \tag{2.37}$$

Equations (2.36) and (2.37) are numerically solved using the variance propagation, Monte Carlo and Latin Hypercube simulation methods [12, 13, 25, 27, 33,]. Variance

Propagation generates value moments/propagation using Taylor Series Expansion. Monte Carlo technique is characterized by large number of iteration and generates one random value for each PDF for estimating P_T, while the Latin Hypercube technique uses Stratified sampling approach [12, 13, 15, 25].

2.4 Use of Past Research Initiatives

Everett III (1963) formulates and proves a theorem that Lagrange multipliers method, for optimization in the presence of constraints, is not limited to differentiable functions. The method can be applied to problems of maximizing an arbitrary real-valued objective function over any set *S*, subject to bounds of real valued functions defined on the same set. The generalized Lagrange multiplier method that Everett has created has a larger range of applicability to solve optimization problems. Moreover, this technique is particularly well suited for use with computers. A limitation of the Lagrange multiplier method arises from the fact that it does not guarantee that an answer can be found in every case. It simply asserts that, if an answer can be found, it will indeed be optimum. The Lagrange multiplier method for solving constrained maximum problems has considerably greater power than is generally realized. It is not limited to differentiable functions, but may often be applied in situations involving maximization of any type of function over any set of strategies, discrete or continuous, numerical or non-numerical, with constraints that can be represented as bounds on real valued functions over the same strategy set [8].

Barnes, King, and Johnson (1979) realizing the short-comings of the manual methods and the queuing theory models used for analyzing production estimations for the shovel-truck system considered stochastic simulation approach for open pit production modeling. The authors identified the limitation of the application of queuing theory to the problem of shovel-truck system as incomprehensive because it was in the development stage. Stochastic simulation modeling appeared to produce a more accurate estimate than the queuing theory techniques. The simulator was considerably more flexible, and it had the advantage that it did not assume steady state from the moment of start-up nor throughout the time of operation. However, the simulation modeling was also limited: (i) the time and manpower required to initially develop a simulator was significant; (ii) the required computer size was much larger than required for the queuing techniques; (iii) the computer run time required to produce the estimate using a stochastic simulator was considerably greater. The authors concluded that the application of queuing theory to the analysis of mine production systems appeared to hold a great deal of promise, although, some mathematical developments were required to make them practically useful [2].

Bott, and Badiozamani (1980) focus on developing a computer model to optimize the mine planning, sequencing, and blending of coal to meet quality compliance standards. The authors stated that in multiple seam operation where some of the individual seams or portion of seams will not meet the sulfur and BTU standards, it would be advantageous to blend such seams with other seams of higher quality. The blending of low and high quality coal will maximize the resource utilization. A computer program was developed to provide mine operators with the necessary means to determine the sequence of mining, which would maximize the usage of low quality coal. The model used optimization techniques to determine and report the rate of advance along each bench necessary to meet the emission standard. The model took into consideration factors such as pit geometry, maximum pre-stripping, maximum pit inventory, and minimum bench separation required for proper operation. The blending portion of the model used a linear programming algorithm, which determined how much coal from each mining block could be blended into each product, stored for later use, or disassociated. The main limitation of the model was that it did not take into consideration the transportation problem. The authors conclude that their Quality Compliance Model (QCM) is a method for blending coal from multiple pits to meet any product specifications. The model can be also used to determine how much coal can be recovered from a property and offers a fast and inexpensive method to evaluate a mining property [5].

Gershon (1982) applied linear programming approach to develop the mine scheduling optimization (MSO). The author states that while linear programming has been applied to mine scheduling before, the models have been either site-specific or they have only optimized one aspect of mine operations without regard to others. The (MSO) concept determined the optimal operation of a mine. The model took into consideration factors from mine to plant to market, accounted for mine-plant-market interfaces, optimized operations over the life of the mine, and developed long, intermediate, and short range planning. MSO was a large model made up of many smaller models, which independently addressed: the ultimate pit problem, blending problem, transportation problem, and production-scheduling problem. The author considered that if the sub-models were solved separately, the results of the various optimizations would conflict, creating a series of solutions, which would not be feasible to put into practice. The main achievement of MSO was that it addressed and solved the sub-models conflict problem. The system optimized the net present value of the profits over the life of the mine. The author concluded that the approach was applicable to a wide variety of mining operations [15].

Dagdelen and Johnson (1986) dealt with optimum pit mine production scheduling by Lagrangian Parameterization. They said that the need to plan and operate a mine with an optimum production schedule had been obvious for some time, but the methods used to obtain such a schedule had not been. In an attempt to determine a production schedule, which is optimum, a new algorithm was developed by the authors. The algorithm was based on the Lagrange theory concepts of mathematics and in practice was based on the parameterization concept of mining. In the authors' opinion, the problem in mine planning was to come up with an optimum mine production schedule, which maximized the NPV of a deposit while satisfying the following type of system constraints: mining capacity, milling capacity, mill feed grade, and slope limitation. To solve the Lagrangian optimization model the authors used the Generalized Lagrange Multiplier method developed by Everett III (1963). A two-period scheduling concept was used to find the yearly production schedule. The idea was to manipulate different period block values by subtracting a given amount until the right tonnage was scheduled in each period. The authors believed that through the use of techniques, such as the one presented by them, the mining industry would be more competitive and decision makers would be closer to making decisions, which would result in more profitable operations [7].

Zuo, Kuo, and McRoberts (1991) developed a series of mathematical programming models (LP) to optimize a large-scale agricultural production and distribution system. The task was to find ways to allocate various hybrids to production locations and to determine proper transportation between facilities and sales regions without compromising seed quality, work environment, facility capacities and markets demands. Different management decision options were studied to see their effects. The optimal routing scheme for hybrid production and distribution was to be identified. A baseline model was considered, in which the objective function was to minimize the sum of the production and transportation costs. The objective function was subject to maximum facility capacity and customer demand constraints, minimum capacity usage requirements, and either-or constraints that were regarded by manufacturers. To provide the management with decision options and comparison eight different models were constructed by altering some of the constraints of the objective function. A sensitivity analysis was conducted to study the effects of variation of right-hand-side constraint coefficients (facility capacity, transportation resource, and customer demand) and the objective function coefficients. The sensitivity analysis showed how sensitive the input data were, and provided the customer with confidence in the results obtained or gave a good reason to perform detailed study to obtain more accurate data and more reliable solutions. The limitation of the approach was that it did not guarantee that global optimal solutions would be obtained even though, very good sub-optimal solutions were obtained [45].

Yun, and Lu (1992) focused on optimization of transportation system in open-pit mines. The authors considered that in spite of the fact that much work had been done for the optimization in long range planning of open pit mines, transportation was still a weak link. They presented a new approach to optimize the transportation system in open pit mines by means of artificial intelligence, fuzzy sets, dynamic programming and CAD technique. It consisted of 4 phases: (i) initial selection of feasible alternatives, (i) alignment of access road, (iii) final decision of transportation system, and (iv) engineering drawing. These phases were correlated with each other and assembled as an integrated package. At first, the new approach obtained a set of feasible transportation alternatives after knowledge utilization and reasoning based on a fuzzy expert system. Secondly, the optimum alignment of access road was determined by means of dynamic programming. Then, the final decision was made by fuzzy judgment from the evaluation of all feasible alternatives with its road alignment. Finally, using CAD technique, a plan showing the structure of open-pit was drawn. The authors concluded that the approach had been successfully applied in China and it was proved that it could be used in other open-pit mines as well [42].

Zhao, and Kim (1992) applied the Lagrangian parameterization approach to mine production sequencing problems, and demonstrated the impossibility of obtaining the optimal mine production sequencing using this method. The main idea of the approach was to simplify the optimum production-sequencing problem into an ultimate pit limit design problem by applying the Langrangian relaxation. In this paper, the authors demonstrated the fact that the Lagrange approach fails to converge to an optimal solution under two conditions. These conditions include: 1) the existence of duplicated optimal solutions in a given sequence period for the targeted ore and waste tonnage requirements, and 2) advanced stripping is required beyond what is the necessary minimum stripping defined by the slope angles. These two situations could not be overcome by the relaxation of the error tolerances, neither by some methods to find the correct Lagrange multipliers. The authors believed that the non-convergence was not due to the general cases mentioned by Everett (1963), but it was due to the fact that relaxed problem was solved by an ultimate pit limit algorithm, which could not detect any of the optimal solutions. Therefore, the authors concluded that the proposed approach combining the ultimate pit limit algorithm could not identify any of them. They also concluded that, the approach would either pick up all the redundant optimal solution at once when the Lagrange multipliers were small or discarded all of them when Lagrange multipliers were big. The two ways of picking up the redundant optimal solution would result in non-convergence to the required ore and waste tonnages. Since the solution did not satisfy the tonnage constraints, the correct Lagrange multipliers would be further searched and the approach would go into a "dead" loop [44].

Mann and Wilke (1992) developed a computerized planning system, which combined the advantages of powerful graphic software with the optimizing strategy of linear programming. The system addressed mainly short-term mine planning and grade control, but special problems like the geometrical constraints, shovel moves and multiple period optimization have also been successfully considered in the model. The authors believed that the complex reality of an open pit could not be transformed into a linear model and they limited the application of the linear programming to the blending problem. Another limitation was the need of a complex algorithm to check the accessibility of the blocks. The authors used the mixed integer programming technique (MIPT) to handle the accessibility of the blocks. The disadvantage of the MIPT was that with increasing number of variables too many branches had to be calculated. Because of that complex system the LP selection had to be always interrupted if a block had been totally mined out and it should be checked whether another block had become accessible or not. The authors concluded that improvements could be made regarding synchronization with graphics system and integrating branch and bound techniques [23].

Jerez and Tivy (1992) described the progress made in the application of linear programming techniques to solve problems of economic dispatching, transportation and shipments of coal in Cyprus Minerals Company. They developed and evaluated numerous scenarios with respect to washing, routing and blending. The model allowed analysis of alternatives including washing higher quality coals with inherently higher yields and blending off the lower quality coals. A major improvement in the application of linear programming to coal mine planning provided by the authors was the introduction of time as a variable into the model. The model was also used to manage inventory levels and insure material balance. The authors believed that improvements could be made by a better modeling of the coal flow and by mine operator's participation in all the phases of data organization, model development and implementation [19].

Tan and Ramani (1992) explored the feasibility of linear programming (LP) and dynamic programming (DP) models to perform a scheduling ore and waste production in open pit mines. In the authors' opinion, the mathematical representation of the scheduling problem was to find an ore production curve below the upper bound function of ore production and a stripping ratio curve. The curve has to be within the feasible combination of ore and overburden that maximize the overall profit. In addition to the two major constraints, the feasible combination of ore and waste production and the limited ore production as a function of mine life, the authors considered various other contributing factors such as interest rate, equipment purchase cost and installation schedule, operating cost, and equipment idle cost. They applied the models to a case study and concluded that both the linear and dynamic models could be employed to find the optimal solution. The linear approach had more flexibility than the dynamic model and could also serve as a general decision support tool in planning open pit mine operation [35].

Muge, Santos, Vieira, and Cortez (1992) developed a dynamic programming model to address the mine planning and production scheduling. They applied the model to two undergrounds mines in South Portugal. The authors considered mining exploitation as a Markovian deterministic and discrete process consisting of *N* sequential steps. They used Bellman's principle of optimality to calculate the optimal exploitation policy. The methodology allowed the calculation, in *N* decision phases, of the sequence of block's combinatory, which minimized the total deviation according to constraints of production rates. They were encouraged by the results obtained and planned to extend the work and integrate the mineral processing models with the mining exploitation models [24].

Wilke, Fabian, and Oravec (1996) developed a practical application of production planning by means of operation research (OR) and knowledge-based techniques. The authors believed that the existing planning software based on LP and simulation predominantly failed in assisting the engineer to find the optimum mine plan over the total lifetime of a mine. The main objective of the research was to develop a knowledge–based system (KBS) to model the decision procedures of an optimizing algorithm in mine production planning. A multi-goal LP and simulation modules were developed for quantitative production planning of mining periods and the KBS technique was used to ensure practical design and combination of consecutive time steps. The consecutive time periods were considered as a chain of mutually dependent activities. Surpac software package was used for spatial modeling and 3D data representation. The authors considered that, by integration of existing knowledge into the KBS, substantial improvements in the field of planning could be achieved. KBS could be used to solve high levels of complex engineering problems. The authors suggested that improvements could be made in block-combination composition problems to guarantee an equable and acceptable mid-term production schedules [39].

Zhang, Li, Zhang, Li, Hou, and Li (1996) focused on developing a computer aided design system for long Range Planning in open cut coal mines to meet the demand of mining software applications. The system was based on mining oriented graph simulation method, which combined graph and network theory with systems simulation technique. The system comprised two subsystems, computer-aided design for coal deposit model and computer-aided design for surface mining model. A grid model was formed by estimating the geological parameters, and then a 3-D block model was developed for mining design. The mining model consisted of a number of sub-models for developing mining design functions. These functions were need to calculate mining volumes and coal quality by graph scanning, determine operating stripping ratio by interactive design, verify the annual coal production, delimit and develop the dumping site, optimize the material flow direction and volumes by LP, and surface mine production scheduling by interactive iteration. The system was applied to several coal mines in China and proved to be a powerful tool to describe the spatial development of the open pit. The authors concluded that their system was user-friendly, provided real-time representation of figure and graphs on the screen, and was built on flexible interactive functions. They believed that further improvements could be made on some problems as mining development system design, economic analysis, and evaluation of the project [43].

Achireko and Frimpong (1996) focused on open pit optimization using artificial neutral networks on conditionally simulated blocks. The authors considered that the algorithms used in the past failed to yield the truly optimized pit limits because they did not address the random field properties associated with ore grades, reserves and commodity prices.

Previous algorithms neglected the stochastic process of the state variables and they yielded sub-optimal pit limits. Therefore, they proposed a new algorithm, the modified conditional simulation/multilayer feedfoward neutral networks algorithm (MCS/MFNN), which would overcome those limitations. The random field properties of the ore reserves and grade were modeled by using a modified conditional simulation (MCS). Artificial neutral networks were used to classify the blocks into classes based on their values. The error back propagation algorithm (EBP) was used to optimize the pit limits by minimizing the outputs error under the pit wall slope constraint. The EBP is a training algorithm for the multilayer perceptron in which the error is back propagated and used to update the weights. The MCS/MFNN algorithm was one of the first algorithms that combine comprehensive orebody modeling and open pit optimization in random multivariable states. The MCS/MFNN algorithm was quicker, simpler and easier to understand compared to the Lerchs and Grossman (LG) algorithm. The authors concluded that for applications in the mining industry further work was required to develop a user-friendly 3D algorithm [1].

Vujic, Cirovic, and Radojevic (1996) proposed a mining system production planning by fuzzy linear modeling. The authors noted that mine production systems were characterized by uncertainty, subjectivity, impreciseness, instability and lack of data and linear or dynamic programming failed in cases of environmental uncertainty. The objective of the research is to develop a fuzzy linear programming (FLP) to solve the production optimization problem of a bauxite basin that includes five open pit mines. The authors used Beldman's and Zadeh's (1970) concept of fuzzy sets to model both the aim and limitations of the production systems. The authors believed that in fuzzy linear programming each of the solutions obtained is characterized by its affiliation level. Instead of maximizing the income, they introduced a concept of satisfactory income having the corresponding level of affiliation. The concept of satisfactory income was introduced and presented by a triangular fuzzy number (TNF). They concluded that the theory on fuzzy sets enabled a more real description of both production and processes systems [37].

Frimpong and Whiting (1996) developed a 3D numerical model of mine value stochastic processes using dynamic arbitrage theory. In the author's opinion to maximize a joint venture's value a stochastic process must be rigorously modeled and solved in a competitive market environment. The objective of the study was to provide a comprehensive evaluation methodology, which enables investors to model mineral venture's stochastic processes. The authors developed a 3D stochastic model of a copper mining venture by means of dynamic arbitrage theory. They also formulated a numerical model of the stochastic processes using finite difference approximations. The main achievement of the study was that the model enabled investor to analyze dynamic phases of the venture and provided the appropriate timing for investment decisions. Another achievement was that the study showed the amount and value of appropriate feasibility studies required to maximize the value of the mining venture [12].

Winkler (1996) developed a mixed integer linear programming (MILP) to optimize period fix costs in complex mine sequencing and scheduling problems. The author stated that fixed costs had a major influence on economic results and they could not be included in LP models. To address this problem the author developed a MILP as an extension to LP in the optimization of a mine-planning problem. The MILP was used to model fix cost aspects in the context of a quality-oriented plan for an underground hard rock coal mine. The main achievement of the study was that MILP enabled the users to split costs into variable and fix cost and capture the machine restrictions in the planning process. The main limitations of the model was that it did not take time into consideration. The author concluded that improvements could be made by extending the approach to mine planning into a multiple period model [40].

Frimpong, Asa, and Szymanski (1998) developed a multivariate optimized pit shells simulator, called MULSOP, for tactical mine planning. The main objective of the study were: to simulate the production targets from concurrent active mining faces in the multi-bench operation, simulate production sequence options, and select appropriate options that minimize total production cost and maximize operating profits. The multivariate production function was modeled and solved using geometrical, numerical and stochastic modeling techniques. Geometrical modeling was used to model the pit shell expansions and the interactions among successive shells. The geometrical models of the pit shells were simulated using the Latin hypercube techniques under different production-economic situations. Geometric formulations of the ellipsoidal approximations of the pit shells geometry were modeled to capture material displacement dynamics in an open pit operation. Stochastic and numerical modeling techniques were used to provide solutions to the time dependent geometric models in random multivariate states. Variance simulation (using VARSIM) was also used to provide analysts with sensitive stochastic variables for input data definition and tight production target tolerance. Detailed analysis of the production scheduling and costs results were carried out to yield desired results that satisfy the tactical plan objectives. The novelty of the MULSOPS simulator was the interface of the pit shells expansion and interaction modules in random multivariate fields. Practical implementation of the models would require careful attention to the deviations that may result from changes in the field and other parameters governing the production process in the pit layout. The authors recommended that, to maximize the present value of materials in the studied section of the pit, the periodic schedule and the cut sequence should follow the result of MULSOPS. It was also recommended by the authors that, before production begins, capabilities of MULSOPS be exploited to develop tactical mine plans for large-scale surface mining operations [10].

Frimpong, Whiting and Szymanski (1998) focused on stochastic-optimization annealing of an intelligent open pit mine design. The authors stated that many algorithms had been developed to optimize pit layouts in the past. These algorithms were limited in dealing with the stochastic processes governing ore reserves, commodity price and production cost. The algorithms did not take into account geological structures, such as faults and disturbed regions that may affect the pit layouts. Moreover, those algorithms did not allow incorporating mine operating strategies to maximize the net pit values. The main objective of the study was to develop an intelligent pit optimizer, IPOP, to apply IPOP to solve a pit optimization problem, and to compare the results from IPOP and the 2D Lerchs-Grossmann's algorithm. IPOP consists sub-models such as the modified conditional simulation (MCS), the multiplayer feedfoward neutral networks (MFNN), a pit search algorithm called PITSEARCH, and a stochastic-optimization annealing process. The algorithm was used to model ore grade, commodity price, and optimize open pit layouts. The model also provided mine planning engineers with estimates associated with the optimized pit value. The solution of the pit design and optimization problem was a snapshot of a dynamic uncertain system with a number of underlying stochastic variables. MCS was used to reduce the smoothing effects and gave better results than the kriging technique. LAS was used to generate a sequence of local averages of random field values while maintaining the internal consistency in the field data. The evolution and uncertainty associated with the commodity price were modeled using the standard Gauss-Wiener stochastic process. After the block partitioning was done, using Multilayer Feedfoward Neutral Network algorithm, the positive regions, the corresponding positive blocks, and their attributes were output to a file, which became an input file to PITSEARCH. After the search process, the sum of all the positive frustra in the optimized layout gives the net pit value, NPVALOPT. The WinNN package was used to train the neutral networks to recognize the block pattern of the pit layout. The authors suggested, that more work should be done to train the neutral networks to recognize geological structures that may affect pit layouts. Also, certain measures migth be incorporated, and studied thoroughly in the design phase before implementing the results [13].

The current MSc. Dissertation occurs in the context of the past researches attempts to solve similar optimization problems by using LP, NLP and stochastic techniques. Many of the past researches focused and solved mine design, production scheduling and blending optimization problems with excellent results. The LP and the NLP models developed were improved with each attempt and they addressed more and more complex problems. In increasing competitive markets and consolidation of ownership a new optimization problem faces the management of coal mining companies. This is the first attempt in the application of LP, NLP and stochastic optimization techniques to solve the problem associated with western Canadian coal production and haulage in the context of integrated mining systems. The models developed and presented in this report offer the management of coal companies' sophisticated tools for reducing costs, increasing efficiency and product quality. They also provide relevant information on cost composition, production levels and risks associated with various satellite mines. It can be concluded that this paper is a continuation of the past research, which addresses the new optimization problems that coal mining companies face in increasingly global competitive markets.

CHAPTER 3.0

COAL PRODUCTION HAULAGE AND OPTIMIZATION MODELING

The main objective of this chapter is to develop mathematical models for solving the coal production and haulage optimization problems facing the Western Canadian coal companies. These models are based on the LP and NLP optimization algorithms. The two different mathematical techniques will also be used to provide a basis for verifying the results from the models. The objective function of the LP algorithm approach is assumed to be linear and non-linear for the Lagrange Multiplier algorithm. The simplex method is used to solve the linear case and the Lagrange parameterization approach to solve the nonlinear case [21, 26].

3.1 LP Production Optimization Model

The first step in the process of developing an LP is to make sure that the following assumptions are met. These assumptions include (i) proportionality assumption: the contribution of each decision variable to the objective function and each constraint is proportional to its value; (ii) additivity assumption: the contribution of each decision variable is independent of the contribution of other decision variables; (iii) divisibility assumption: each variable may take fractional values, and (iv) certainty assumption: each coefficient is known with certainty [21.26].

3.1.1 Definition of Problem and Variables

Western Canadian Coal mining companies produce two final products with different market and contractual requirements coming from various mine-sites with specific chemical and physical characteristics. It is assumed that from each mine, a certain amount would be metallurgical coal expressed by the variables $\{X_{11}; X_{12}; \dots ; X_{1j}\}$ produced by Mine # 1 and hauled to Destination 1, 2, …, j; $\{X_{21}; X_{22}; \dots ; X_{2j}\}$ produced by Mine # 2 and hauled to Destination 1, 2, …, j; … ; and $\{X_{i1}; X_{i2}; \dots ; X_{ij}\}$ produced by Mine # i and hauled to Destination 1, 2, …, j; $\forall$ i from 1 to n and j from 1 to m. It is also assumed that from each mine a certain amount would be thermal coal expressed by the variables $\{Y_{11}; Y_{12}; \dots ; Y_{1j}\}$ produced by Mine # 1 and hauled to Destination 1, 2,

…, j; $\{Y_{21}; Y_{22}; \ldots ; Y_{2j}\}$ produced by Mine # 2 and hauled to Destination 1, 2, …, j; … ; and $\{Y_{i1}; Y_{i2}; \ldots ; Y_{ij}\}$ produced by Mine # i and hauled to Destination 1, 2, …, j; ∀ i from 1 to n and j from 1 to m. These production dynamics are illustrated in Figure 3.1.

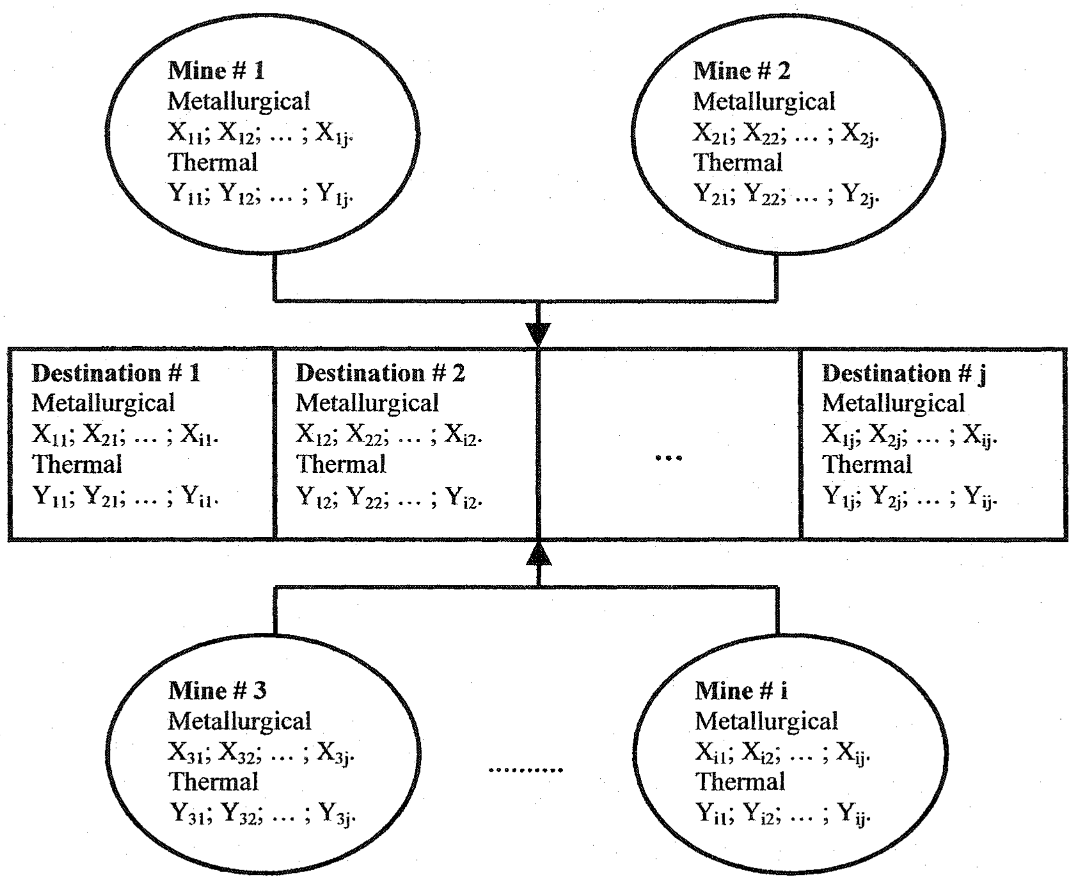


Fig. 3.1 Schematic Dynamic Diagram of Coal Production vs. Destination

3.1.2 The Objective Function Definition

The objective is to maximize the total profits of the company subject to the underlying constraints. The total profit is a function of the mining and processing, transportation, overhead costs, port charges and revenue. The cost of mining and processing the coal is given in equation (3.1). The transportation cost is determined by CNR and CPR per kilometer-tonne and is given in equation (3.2). Equations (3.3), (3.4) and (3.5) contain the respective overhead costs (OVH_T), port costs (PC_T) and the total cost (TC_T). The total revenue (REV_T) is a function of the revenues generated in the individual mines, which is expressed mathematically in equation (3.6). From equations (3.1) to (3.6), the objective function for this problem is formulated as in equation (3.7).

$$MP_T = \sum_{i=1}^{n} MP_i = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[k_{mi} * X_{ij} + k_{ti} * Y_{ij}\right] \tag{3.1}$$

$$TR_T = \sum_{i=1}^{n} \sum_{j=1}^{m} TR_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[\left(d_{ij} * X_{ij} + d_{ij} * Y_{ij}\right) * k_s\right] \tag{3.2}$$

$$OVH_T = \sum_{i=1}^{n} OVH_i = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[k_{oi} * \left(X_{ij} + Y_{ij}\right)\right] \tag{3.3}$$

$$PC_T = \sum_{j=1}^{m} PC_j = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[k_{pj} * \left(X_{ij} + Y_{ij}\right)\right] \tag{3.4}$$

$$TC_T = MP_T + TR_T + OVT_T + PC_T \tag{3.5}$$

$$REV_T = \sum_{i=1}^{n} REV_i = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[p_m * X_{ij} + p_t * Y_{ij}\right] \tag{3.6}$$

$$\text{Maximize} P_T = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[REV_{ij} - TC_{ij}\right] \tag{3.7}$$

The profit function is obtained by replacing all the coefficients in the profit function with their values in terms of X_{ij} and Y_{ij} variables; A_{ij} and B_{ij} known coefficients as illustrated in equation (3.8).

$$P_T = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(A_{ij} * X_{ij} + B_{ij} * Y_{ij}\right) \tag{3.8}$$

Equation (3.8) is subject to production, economic, logistic and quality constraints. These constraints include the total supply and demand, production capacities, ports, railways limitation and constraints and quality constraints in terms of ash, sulfur, moisture, energy, fixed carbon and volatile matter.

3.1.3 Definition of Constraints

The field constraints are derived from mine capacities and coal quality, demand and supply, respective port and railway limitations. The constraints are imposed to define the feasible region where the outputs ensure the attainment of the objective function. The constraints are formulated to capture the production, quality, supply and demand imposed on coal production. It is assumed that one company owns these mines and that production is constrained by contractual agreement requirements. The total metallurgical and thermal coal production is governed by equations (3.9) and (3.10). These equations represent the sum of the amounts of metallurgical and thermal coal produced by each mine.

$$Q_1 = \sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} \tag{3.9}$$

$$Q_2 = \sum_{i=1}^{n} \sum_{j=1}^{m} Y_{ij} \tag{3.10}$$

Equation (3.11) governs the total coal production of the company, which is equal to the metallurgical plus thermal coal production from all the mines.

$$M = Q_1 + Q_2 = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[X_{ij} + Y_{ij}\right] \tag{3.11}$$

The maximum mine capacities are controlled by equation (3.12). The sum of the variables X_i and Y_i representing the respective metallurgical and thermal coal produced by mine "i" is limited by the available resources M_i of that particular mine.

$$\sum_{j=1}^{m} (X_{ij} + Y_{ij}) \leq M_i \tag{3.12}$$

The respective market demand and supply for metallurgical and thermal coal, based on contractual arrangements and market absorption capability are provided in equations (3.13), (3.14), (3.15) and (3.16). Equation (3.13) constrains the sum of variables the X_{ij} to be less or equal to the maximum amount of metallurgical coal Q_m that could be sold on the market.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} \leq Q_m \tag{3.13}$$

The contractual metallurgical coal constraint is governed by equation (3.14). The sum of X_{ij} variables is larger than or equal to the company's contracted quantities Q_{mc}.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} \geq Q_{mc} \tag{3.14}$$

Equation (3.15) limits the total amount of thermal coal produced by the company to the maximum amount that could be absorbed by the overseas and domestic market Q_t.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} Y_{ij} \leq Q_t \tag{3.15}$$

The sum of Y_{ij} variables, which represent the company's thermal coal production, is equal to or larger than the quantity specified by the contract Q_{tc} as in equation (3.16).

$$\sum_{i=1}^{n} \sum_{j=1}^{m} Y_{ij} \geq Q_{tc} \tag{3.16}$$

A minimum capacity usage constraint is set to ensure that no metallurgical coal processing facility works below its economic efficiency level. The minimum capacity constraint refers to the metallurgical coal quantities and to the total coal production of a mine. The minimum capacity of a mine is determined from economic factors, equipment availability and utilization. These constraints are given by equations (3.17) and (3.18). Equation (3.17) sets the total coal production of mine "i" to be at least or larger than the economic efficiency level Q_{it}.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left[X_{ij} + Y_{ij}\right] \geq Q_{it} \qquad (3.17)$$

Equation (3.18) ensures that a steady metallurgical coal production occurs at each processing and upgrading facility. The sum of variables X_{ij} has to be greater than the minimum capacity usage requirement Q_{im}.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} \geq Q_{im} \qquad (3.18)$$

The respective port and railway limitations are governed by equations (3.19) and (3.20). According to these equations production at each particular mine could be limited by the quantity of coal that the ports and railways could handle.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij} + Y_{ij}\right) \leq P_j \qquad (3.19)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij} + Y_{ij}\right) \leq R_j \qquad (3.20)$$

The contractual agreements stipulate the quality of metallurgical and thermal coal production from these mines. The coal constraints require that the ash, sulfur, moisture, energy, fixed carbon and volatile matter contents are maintained above or below certain acceptable levels. These constraints have a different degree of importance from a production company's point of view. The energy, fixed carbon and volatile matter content in coal are physical characteristics that are practically very hard to improve. Ash, and moisture contents could be improved in the processing and upgrading facilities to meet the requirements. Ash and moisture content are derived from the coal formation, geology and dilution during excavation and transport operations. The relative ease of coal processing depends on the coal's percentage of ash and moisture content. Since coal characteristics are different for each coal deposit and even from one coal seam to another in the same deposit, the processing cost would be specific for each mine [3, 29, 30, 31]. At some mines it would be easier and cheaper to process and upgrade the coal than others. To reduce costs, coal-mining companies may decide to process and upgrade the coal at one or two of the mines and blend with the coal coming from other mines. The blending could be done at each of the final destination in blending stockpiles.

One of the risks associated with the blending processes is that the coal might end up having ash, sulfur and moisture content far below the maximum allowable requirements. If this happens the company will give away value from the mines. To avoid these situations a minimum percentage of ash, sulfur and moisture are imposed for each of the blending stockpile. Therefore, there will be an upper and a lower limit for each of the blending stockpiles regarding the ash, sulfur and moisture content. The mathematical equations that control coal quality elements are written as weighted averages of coal quantities and qualities. The equations that govern the upper and lower limit of ash content in metallurgical coal are provided in equations (3.21) and (3.22). The left hand side of these equations represents the quantity of ash existent in the metallurgical coal. The right hand side represents the maximum (3.21) respective minimum (3.22) ash quantity or content requirement.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(a_{im} * X_{ij}\right) \leq C_{asm} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij}\right) \tag{3.21}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m}(a_{im} * X_{ij}) \geq C_{lasm} * \sum_{i=1}^{n} \sum_{j=1}^{m}(X_{ij}) \quad (3.22)$$

Equations (3.23) and (3.24) control the upper and lower limit of the ash content in thermal coal. Similar to metallurgical coal these equations represent existent and required ash quantities.

$$\sum_{i=1}^{n} \sum_{j=1}^{m}(a_{it} * Y_{ij}) \leq C_{ast} * \sum_{i=1}^{n} \sum_{j=1}^{m}(Y_{ij}) \quad (3.23)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m}(a_{it} * Y_{ij}) \geq C_{last} * \sum_{i=1}^{n} \sum_{j=1}^{m}(Y_{ij}) \quad (3.24)$$

Sulfur content has a negative impact on both metallurgical and thermal coal. First, sulfur in the burning process ends up as sulfur dioxide, which is a poisonous gas and has a great environmental negative impact. Second, in the steel making process high levels of sulfur gives steel brittle characteristics. Western Canadian coal is characterized by low sulfur contents compared with coal from other countries [3, 31, 38]. Therefore, western Canadian coal-mining companies could take advantage of the low sulfur content and blend it with coal that has high sulfur content. The upper and lower limit of sulfur content in metallurgical coal are provided in equations (3.25) and (3.26). The market requires the upper limit while the mining companies impose the lower limit.

$$\sum_{i=1}^{n} \sum_{j=1}^{m}(s_{im} * X_{ij}) \leq C_{sm} * \sum_{i=1}^{n} \sum_{j=1}^{m}(X_{ij}) \quad (3.25)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m}(s_{m} * X_{ij}) \geq C_{lsm} * \sum_{i=1}^{n} \sum_{j=1}^{m}(X_{ij}) \quad (3.26)$$

The sulfur content in thermal coal is not a big issue for western Canadian coal producers [6, 7]. Equations (3.27) and (3.28) control the sulfur content requirements in thermal coal.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{it} * Y_{ij}\right) \leq C_{st} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) \tag{3.27}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{it} * Y_{ij}\right) \geq C_{lst}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) \tag{3.28}$$

Moisture is an important quality parameter for both metallurgical and thermal coal. To reduce the moisture content the coal has to be dried to meet the market requirements. Some coals are more difficult to dry out than others [31]. Therefore, there will be two boundaries regarding moisture content, the upper limit required by the market and the lower limit imposed by the producer. Equations (3.29) and (3.30) provide the upper and lower constraints equations of the moisture content in the metallurgical coal to be shipped to the markets.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{im} * X_{ij}\right) \leq C_{msm} * \sum_{i=1}^{n}\sum_{j=1}^{n}\left(X_{ij}\right) \tag{3.29}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{im} * X_{ij}\right) \geq C_{lmsm} * \sum_{i=1}^{n}\sum_{j}^{m}\left(X_{ij}\right) \tag{3.30}$$

Equations (3.31) and (3.32) govern the upper and lower limits of moisture content in thermal coal.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{it} * Y_{ij}\right) \leq C_{mst} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) \tag{3.31}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{it} * Y_{ij}\right) \geq C_{lmst} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) \tag{3.32}$$

The specific energy of coal is a quality parameter that is more important for thermal than metallurgical. Energy content has to be greater than or equal to the limits specified in the contracts or required by the market. Since western Canadian coal has usually large specific energy values, this parameter is not a big concern for the mining companies and do not impose upper limit energy content. Equation (3.33) and (3.34) provide the respective lower energy contents for metallurgical and thermal coal.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(e_{im} * X_{ij}\right) \geq C_{enm} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij}\right) \tag{3.33}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(e_{it} * Y_{ij}\right) \geq C_{ent} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(Y_{ij}\right) \tag{3.34}$$

Fixed carbon is a quality parameter that matters more for metallurgical than thermal coal. The higher the coal rank, the higher the fixed carbon content [3, 29, 30, 31]. Fixed carbon content has to be greater than or equal to certain values specified by the contract or required by the markets. Western Canadian coal mining companies do not impose an upper limit for this parameter; therefore, there will be only a lower boundary. Equation (3.35) provides the lower limit constraint of fixed carbon content in metallurgical coal and equation (3.36) governs the fixed carbon content in thermal coal.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(fc_{im} * X_{ij}\right) \geq C_{fcm} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij}\right) \tag{3.35}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(fc_{it} * Y_{ij}\right) \geq C_{fct} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(Y_{ij}\right) \tag{3.36}$$

Volatile matter content has a negative impact on coals. Volatile matter content has to be less than or equal to the markets requirements. Equation (3.37) and (3.38) provide the respective upper limit constraint equations of the volatile matter content in the metallurgical and thermal coal to be shipped to the customers.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(v_{im} * X_{ij}\right) \leq C_{vm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) \quad (3.37)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(v_{it} * Y_{ij}\right) \leq C_{vt} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) \quad (3.38)$$

The non-negativity constraints assume that none of the variables X_{ij}, Y_{ij} may take negative values. They are presented in equations (3.39) and (3.40).

$$X_{ij} \geq 0 \quad (3.39)$$

$$Y_{ij} \geq 0 \quad (3.40)$$

3.2. Non-Linear Programming (NLP) Production Optimization Model

The Lagrange Multipliers technique is applied to the same coal production and haulage optimization problem. The main purpose of this approach is to verify if the two different mathematic techniques provide similar optimal solutions. The objective function is to maximize the profit of the company obtained by shipping optimal amounts of coal from each mine to the final destinations while the blending requirements are met, as illustrated in equation (3.41)

$$\text{Maximize } P_T = \sum_{i=1}^{n}\sum_{j=1}^{m}\left(A_{ij} * X_{ij} + B_{ij} * Y_{ij}\right) \quad (3.41)$$

The main difference between the LP and the Lagrange parameterization algorithm is that instead of optimizing the profit function one has to build and optimize the Lagrange function. Everett III (1963) modified and simplified the Lagrange parameterization method and extended the applicability of the method to a larger range of optimization problems [8]. By applying Everett's method, the above problem would be solved when the Lagrange Function is optimized. The general form of Lagrange Function is given in equation (3.42) and the general form of the constraints in equation (3.43).

$$\text{Maximize}L(x) = P_T - \sum_{k=1}^{n} \lambda^k * \left[C^k\left(X_{ij}, Y_{ij}\right) - C^k\right] \tag{3.42}$$

$$C^k\left(X_{ij}, Y_{ij}\right) - C^k \leq 0 \tag{3.43}$$

λ^k, $k = 1, n$ are positive real numbers; and $C^k(X_{ij}, Y_{ij})$ are the optimized resources allocated by the model; C^k are the maximum available resources that could be allocated.

The main constraints of equation (3.41) include contractual, supply and demand, maximum and minimum capacities, ports and railways limitations and blending constraints in terms of ash, sulfur, moisture, energy, fixed carbon, and volatile matter. All the constraint equations for the Lagrange method are written in such a form that they have to be less than or equal to zero. The maximum capacities are controlled by equation (3.44) in which the sum of variables X_{ij} respective Y_{ij} representing the metallurgical and thermal coal could not exceed the available resources of each particular mine M_i.

$$\sum_{j=1}^{m} (X_{ij} + Y_{ij}) - M_i \leq 0 \tag{3.44}$$

Market supply and demand constraints are governed by equations (3.45) to (3.48). Equations (3.45) and (3.46) limit the variables X_{ij} respective Y_{ij} to the maximum amount of metallurgical and thermal coal that can be sold on the market.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} - Q_m \leq 0 \tag{3.45}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} Y_{ij} - Q_t \leq 0 \tag{3.46}$$

Equations (3.47) and (3.48) set the X_{ij} respective Y_{ij} variables to be larger than or equal to the company's contracted coal quantities.

$$Q_{mc} - \sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} \leq 0 \tag{3.47}$$

$$Q_{tc} - \sum_{i=1}^{n} \sum_{j=1}^{m} Y_{ij} \leq 0 \tag{3.48}$$

To prevent that no mine is producing under its minimum economic capacity a minimum usage capacity requirement is set for each mine. Equation (3.49) governs the mine minimum capacity usage requirements.

$$Q_{it} - \sum_{i=1}^{n} \sum_{j=1}^{m} \left[X_{ij} + Y_{ij}\right] \leq 0 \tag{3.49}$$

Equation (3.50) provides the minimum usage requirement constraint for the processing and upgrading facility of metallurgical coal.

$$Q_{im} - \sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} \leq 0 \tag{3.50}$$

Since the ports and railways handling capacities are limited, the sum of variables x_{ij} respective y_{ij} for each of the ports and railway route has to be less or equal to these capacities. The port limitations are controlled by equations (3.51) and the railway constraints by equation (3.52).

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij} + Y_{ij}\right) - P_j \leq 0 \tag{3.51}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij} + Y_{ij}\right) - R_j \leq 0 \tag{3.52}$$

From Section 3.1.3, coal quality requirements are specified by the contract. The coal quality requirement is one of the main concerns of coal mining companies since the coal processing and upgrading cost is a major driving factor. These costs could be reduced through an optimized blending process. It is assumed that the coal from different mines, owned by the same company, is blended in stockpiles at each final destination. The constraint equations that control coal quality are written as weighted averages of coal quantities and qualities. The difference between these weighted averages has to be always less than or equal to zero. The upper ash limit content is specified by the contract, and the mining company imposes the lower ash limit. The upper and lower metallurgical ash requirements are governed by equations (3.53) and (3.54).

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(a_{im} * X_{ij}\right) - C_{asm} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij}\right) \leq 0 \tag{3.53}$$

$$C_{lasm} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(X_{ij}\right) - \sum_{i=1}^{n} \sum_{j=1}^{m} \left(a_{im} * X_{ij}\right) \leq 0 \tag{3.54}$$

Equations (3.55) and (3.56) control the upper and the lower boundaries of the ash requirements for thermal coal.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \left(a_{it} * Y_{ij}\right) - C_{ast} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(Y_{ij}\right) \leq 0 \tag{3.55}$$

$$C_{last} * \sum_{i=1}^{n} \sum_{j=1}^{m} \left(Y_{ij}\right) - \sum_{i=1}^{n} \sum_{j=1}^{m} \left(a_{it}\right) \leq 0 \tag{3.56}$$

Sulfur content is an important coal quality parameter and has to be less than or equal to the amount specified by the contract. Western Canadian coal deposits are endowed with low sulfur content. Coal mining companies want to use this advantage and sell coal with higher sulfur content that would be normally accepted on the market by blending. Therefore they impose a lower limit for the sulfur content. The metallurgical sulfur constraints are given in equations (3.57) and (3.58).

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{im} * X_{ij}\right) - C_{sm} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) \leq 0 \qquad (3.57)$$

$$C_{lsm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{m} * X_{ij}\right) \leq 0 \qquad (3.58)$$

Equations (3.59) and (3.60) deal with the upper and lower limits of sulfur content in thermal coal.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{it} * Y_{ij}\right) - C_{st} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) \leq 0 \qquad (3.59)$$

$$C_{lst} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{it} * Y_{ij}\right) \leq 0 \qquad (3.60)$$

There are two constraints regarding moisture content: the upper limit content, which is required by the market and a lower limit imposed by the producer. Equations (3.61) and (3.62) govern the moisture content in metallurgical coal.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{im} * X_{ij}\right) - C_{msm} * \sum_{i=1}^{n}\sum_{j=1}^{n}\left(X_{ij}\right) \leq 0 \qquad (3.61)$$

$$C_{lmsm} * \sum_{i=1}^{n}\sum_{j}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{im} * X_{ij}\right) \leq 0 \qquad (3.62)$$

Equations (3.63) and (3.64) govern the upper and the lower boundaries of moisture content in thermal coal.

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{it} * Y_{ij}\right) - C_{mst} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) \leq 0 \qquad (3.63)$$

$$C_{lmst} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{it} * Y_{ij}\right) \leq 0 \qquad (3.64)$$

The specific energy content has to be greater than or equal to the limits specified in the contract. These constraints are not a big concern for Western Canadian coal companies since the coal has usually larger values than required and therefore do not impose an upper limit for energy content. The equations that govern these constraints regarding metallurgical and thermal coal are presented in equations (3.65) respective (3.66).

$$C_{enm} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(e_{im} * X_{ij}\right) \leq 0 \qquad (3.65)$$

$$C_{ent} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(e_{it} * Y_{ij}\right) \leq 0 \qquad (3.66)$$

Fixed carbon content has to be greater than or equal to the limits specified in the contract or required by the market. Similar to energy content, mining companies do not input an upper limit regarding fixed carbon. The lower boundary of fixed carbon in metallurgical and thermal coal, which is required by the markets, is governed by equations (3.67) respective (3.68).

$$C_{fcm} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(fc_{im} * X_{ij}\right) \leq 0 \qquad (3.67)$$

$$C_{fct} * \sum_{i=1}^{n} \sum_{j=1}^{m} (Y_{ij}) - \sum_{i=1}^{n} \sum_{j=1}^{m} (fc_{it} * Y_{ij}) \leq 0 \tag{3.68}$$

Volatile matter content has to be less than or equal to the requirements specified in the contract. There will be only an upper limit for volatile matter parameter since mining companies do not input a lower limit. Equations (3.69) and (3.70) control these limitations for metallurgical respective thermal coal.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (v_{im} * X_{ij}) - C_{vm} \sum_{i=1}^{n} \sum_{j=1}^{m} (X_{ij}) \leq 0 \tag{3.69}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (v_{it} * Y_{ij}) - C_{vt} * \sum_{i=1}^{n} \sum_{j=1}^{m} (Y_{ij}) \leq 0 \tag{3.70}$$

The non-negativity constraints are necessary to make sure that none of the X_{ij} and Y_{ij} variables take negative values. They are presented in equations (3.71) and (3.72).

$$X_{ij} \geq 0 \tag{3.71}$$

$$Y_{ij} \geq 0 \tag{3.72}$$

Introducing all the constraint equations (3.44) to (3.70) into the Lagrange function (3.42) the extended Lagrange function for the coal production and haulage and handling at the ports is obtained in equation (3.73).

$$\begin{aligned} L[X_{ij}, Y_{ij}] = A - [&B + C + D + E + F + G + H + I + J + K + \\ &L + M + N + O + P + R + S + T + U + V + W + X] \end{aligned} \tag{3.73}$$

$$A = \sum_{i=1}^{n} \sum_{j=1}^{m} (A_{ij} * X_{ij} + B_{ij} * Y_{ij}) \tag{3.74}$$

$$B = \lambda^1\left[\sum_{j=1}^{m} (X_{ij} + Y_{ij}) - M_i\right] + \lambda^2\left[\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij} - Q_m\right] + \lambda^3\left[\sum_{i=1}^{n}\sum_{j=1}^{m} Y_{ij} - Q_t\right] \tag{3.75}$$

$$C = \lambda^4\left[Q_{mc} - \sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}\right] + \lambda^5\left[Q_{tc} - \sum_{i=1}^{n}\sum_{j=1}^{m} Y_{ij}\right] \tag{3.76}$$

$$D = \lambda^6\left[Q_{it} - \sum_{i=1}^{n}\sum_{j=1}^{m}\left[X_{ij} + Y_{ij}\right]\right] + \lambda^7\left[Q_{im} - \sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}\right] \tag{3.78}$$

$$E = \lambda^8\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij} + Y_{ij}\right) - P_j\right] + \lambda^9\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij} + Y_{ij}\right) - R_j\right] \tag{3.79}$$

$$F = \lambda^{10}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(a_{im} * X_{ij}\right) - C_{asm} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right)\right] \tag{3.80}$$

$$G = \lambda^{11}\left[C_{lasm} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(a_{im} * X_{ij}\right)\right] \tag{3.81}$$

$$H = \lambda^{12}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(a_{it} * Y_{ij}\right) - C_{ast} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right)\right] \tag{3.82}$$

$$I = \lambda^{13}\left[C_{last} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(a_{it}\right)\right] \tag{3.83}$$

$$J = \lambda^{14}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{im} * X_{ij}\right) - C_{sm} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right)\right] \tag{3.84}$$

$$K = \lambda^{15}\left[C_{lsm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_m * X_{ij}\right)\right] \tag{3.85}$$

$$L = \lambda^{16}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{it} * Y_{ij}\right) - C_{st} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right)\right] \tag{3.86}$$

$$M = \lambda^{17}\left[C_{lst} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(s_{it} * Y_{ij}\right)\right] \tag{3.87}$$

$$N = \lambda^{18}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{im} * X_{ij}\right) - C_{msm} * \sum_{i=1}^{n}\sum_{j=1}^{n}\left(X_{ij}\right)\right] \tag{3.88}$$

$$O = \lambda^{19}\left[C_{lmsm} * \sum_{i=1}^{n}\sum_{j}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{im} * X_{ij}\right)\right] \tag{3.89}$$

$$P = \lambda^{20}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{it} * Y_{ij}\right) - C_{mst} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right)\right] \tag{3.90}$$

$$R = \lambda^{21}\left[C_{lmst} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(m_{it} * Y_{ij}\right)\right] \tag{3.91}$$

$$S = \lambda^{22}\left[C_{enm} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(e_{im} * X_{ij}\right)\right] \tag{3.92}$$

$$T = \lambda^{23}\left[C_{ent} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(e_{it} * Y_{ij}\right)\right] \tag{3.93}$$

$$U = \lambda^{24}\left[C_{fcm} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(fc_{im} * X_{ij}\right)\right] \tag{3.94}$$

$$V = \lambda^{25}\left[C_{fct} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right) - \sum_{i=1}^{n}\sum_{j=1}^{m}\left(fc_{it} * Y_{ij}\right)\right] \tag{3.95}$$

$$W = \lambda^{26}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(v_{im} * X_{ij}\right) - C_{vm}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(X_{ij}\right)\right] \quad (3.96)$$

$$X = \lambda^{27}\left[\sum_{i=1}^{n}\sum_{j=1}^{m}\left(v_{it} * Y_{ij}\right) - C_{vt} * \sum_{i=1}^{n}\sum_{j=1}^{m}\left(Y_{ij}\right)\right] \quad (3.97)$$

3.3 Risk Characterization

The objective function in equation (3.7) is a function of many continuous random variables, the most important of which are the coal quantity and quality, price and total costs, as illustrated in equation (3.98). These random variables create uncertainty in the projected optimized company profits. These uncertainties result in risks that must be managed to create competitive edge for companies.

$$P_T = \varphi\left(X_{ij}, Y_{ij}\right)_{i=1,n;\, j=1,m} \quad (3.98)$$

If x is continuous and f (x) is the probability density function of x, the objective function of the stochastic model therefore becomes a dual function. This dual function is to maximize the expected value, E [P_T], and minimize the variance, VAR [P_T], of the coal profit function in equation (3.99) and (3.100) respectively [12, 13, 25, 27, 33,].

$$\text{Maximize} E[P_T] = \int_{-\infty}^{\infty} \Lambda \int_{-\infty}^{\infty} \varphi(X_{ij}, Y_{ij}) * f(X_{ji}, Y_{ij}) dX_{ij} dY_{ij} \quad (3.99)$$

$$\text{Minimize} VAR[P_T] = \int_{-\infty}^{\infty} \Lambda \int_{-\infty}^{\infty} \left[\varphi\left(X_{ij}, Y_{ij}\right) - E\left(X_{ij}, Y_{ij}\right)\right]^2 * f(X_{ij}) X_{ij} * f(Y_{ij}) Y_{ij} \quad (3.100)$$

The Monte Carlo and Latin Hypercube simulation technique is used to estimate equations (3.99) and (3.100) numerically by simulating 10,000 iterations in one simulation run. The results are used to characterize the risk profile of the coal production and haulage system. To characterize the risk associated with the coal production and haulage costs, revenues, and profits various probabilistic distributions such as; normal, lognormal, truncated lognormal, uniform, and triangular can be used [13, 25, 27]. The lognormal distribution was found to fit best to the coal prices distribution, and normal and truncated lognormal distribution to the mining and processing and haulage costs. The probability density function (PDF) of the lognormal distribution is illustrated in equation (3.101). λ and ζ from equation (3.101) are defined in equations (3.102) and (3.103).

$$f(x) = \frac{1}{\varsigma x\sqrt{2\pi}} \exp\left[-\frac{1}{2} * \left(\frac{\ln x - \lambda}{\varsigma} \right)^2 \right] \tag{3.101}$$

$$\lambda = F[\ln x] \tag{3.102}$$

$$\varsigma = (VAR[\ln x])^{\frac{1}{2}} \tag{3.103}$$

The probability that the respective price of metallurgical and thermal coal (x_m), (x_t) will be within (a_1, b_1) and (a_2, b_2) is given by equations (3.104) and (3.105). Equations (3.106) and (3.107) present how λ and ζ are calculated for different values of the Coefficient of Variation (COV).

$$probability(a_1 \le x_m \le b_1) = \phi\left[\left(\frac{\ln b_1 - \lambda}{\varsigma}\right)\right] - \phi\left[\left(\frac{\ln a_1 - \lambda}{\varsigma}\right)\right] \tag{3.104}$$

$$probability(a_2 \le x_t \le b_2) = \phi\left[\left(\frac{\ln b_2 - \lambda}{\varsigma}\right)\right] - \phi\left[\left(\frac{\ln a_2 - \lambda}{\varsigma}\right)\right] \tag{3.105}$$

$$\lambda = \ln \mu - \frac{1}{2}\varsigma^2 \tag{3.106}$$

$$\varsigma^2 = \begin{cases} \ln\left(1+\frac{\sigma^2}{\mu^2}\right) & \forall \quad COV \geq 0.30 \\ \frac{\sigma^2}{\mu^2} & \forall \quad COV \leq 0.30 \end{cases} \tag{3.107}$$

COV is given in equation (3.108).

$$COV = \sigma/\mu \tag{3.108}$$

Equation (3.109) illustrates the PDF of the normal probability distribution. The probability that the mining and processing cost *(x)* will be within (a, b) is given by equation (3.110).

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{3.109}$$

$$probability(a \leq x \leq b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right) \tag{3.110}$$

Cumulative probability *F(x)* of a function f(x) on a set of random numbers ($a \leq x \leq b$) is illustrated in equation (3.111).

$$F(x) = \int_{-\infty}^{x} f(x)dx \tag{3.111}$$

Both the PDF and the cumulative probability distributions are used successfully to assess the risk associated with the western Canadian coal production and haulage.

3.4 Conclusions

The main objective of this chapter was to develop mathematical models for solving the coal production and haulage optimization problems facing the Western Canadian coal companies. The models were based on the LP and NLP optimization algorithms. The simplex method was used to solve the linear case and the Lagrange parameterization approach to solve the nonlinear case. The main purpose of using two approaches was to verify if the two different mathematical models would provide similar optimal solutions. The two different mathematical techniques will also be used to provide a basis for verifying the results from the models in Chapter 4.0. The objective functions are functions of many continuous random variables. These random variables created uncertainty in the projected optimized coal mining companies profits. A stochastic model was developed to analyze the risk associated with the coal mining companies, which will enable analysts to predict the associated short-and long-term risks.

CHAPTER 4.0

COMPUTER MODELING AND VALIDATION MODELS

In this chapter, the author discusses the computer models and the validation models used to analyze the case studies in this report. The complex mathematical equations derived in Chapter 3.0 are translated into computer models to facilitate the analyses. Computer modeling includes the LP, the NLP and the associated stochastic models. The logic flowcharts are also provided for the coal production process, metallurgical and thermal coal process, and stochastic sampling technique.

4.1 Optimization Flowcharts of Computer Models and Risk Simulation

The production and haulage optimization model is illustrated in Figure 4.1. The objective function is determined by the mine revenues and total costs subject to the field

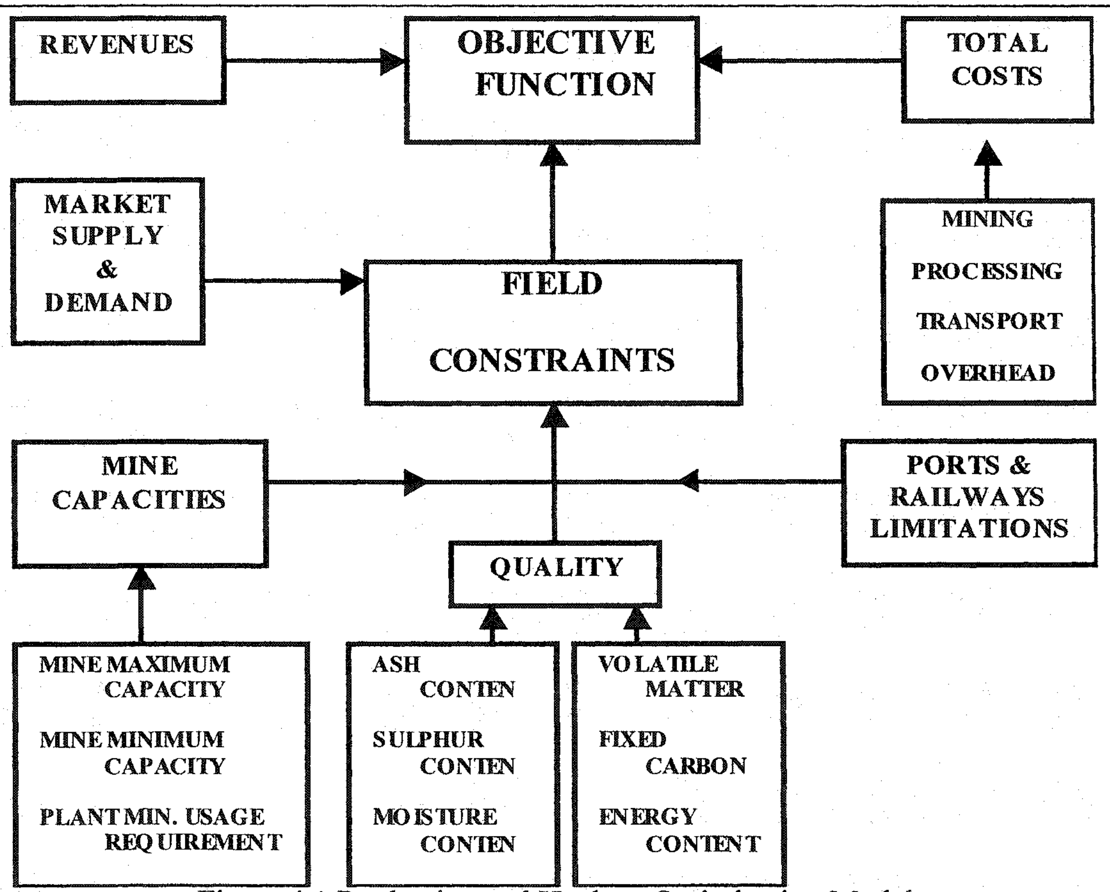
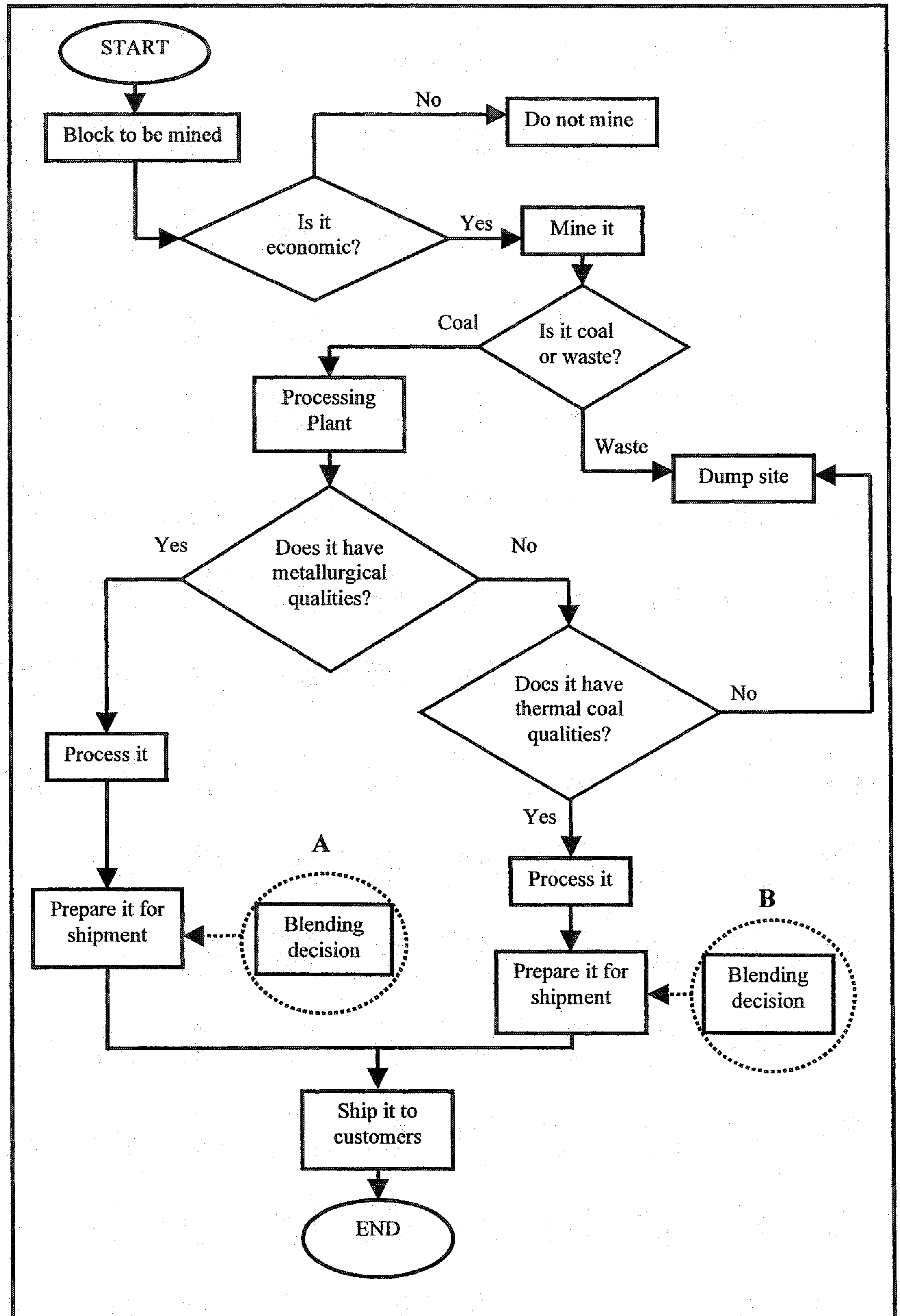


Figure 4.1 Production and Haulage Optimization Model

constraints. The revenue of each mine is given by the market value of the coal products multiplied by the quantity of coal produced and sold. Total cost of the company is calculated as a sum of mining, processing, transport, and overhead costs. The field constraints comprise a large range and variety of factors as market supply and demand, mine capacities, ports and railway limitations, and coal quality. All the above factors have an impact on the objective function. Therefore, the mathematical and the computer models have to be developed to capture and optimize the coal production and shipment from each particular mine to their final destinations. Solver, Excel add-in software, is used to solve the complex optimization problems in this study [Microsoft 1997].

Flow charts are also used to provide further understanding how the problem was solved in the optimization and simulation environments. The coal production process flowchart, as illustrated in Figure 4.2 is a chain of decisions and actions that have to be made in a coal operation. The first decision deals with extraction of mine blocks, from the coal deposit, based on economic analysis. If the block is mined, the next decision deals with its destination. The next decision is a two-part decision focusing on coal quality for both metallurgical and thermal coal. These processing of metallurgical and thermal coal have to analyze specific coal qualities as: ash, sulfur, energy, moisture, fixed carbon, and volatile matter for specific blending decisions. The metallurgical and thermal coal blending decision flowcharts are presented in Figure 4.3 and Figure 4.4. The blending decisions are made based on the content of ash, sulfur, moisture, fixed carbon, volatile matter and energy in the final product and the upper and lower limits of each of these contents accepted on the market or input by the producer. Since there are several final destinations, where coal is blended, blending constraints form the largest part of the mathematical and computer model.


START
Block to be mined
Is it economic?
No
Do not mine
Yes
Mine it
Is it coal or waste?
Coal
Processing Plant
Waste
Dump site
Does it have metallurgical qualities?
Yes
No
Does it have thermal coal qualities?
No
Process it
Yes
Process it
A
Prepare it for shipment
Blending decision
B
Prepare it for shipment
Blending decision
Ship it to customers
END

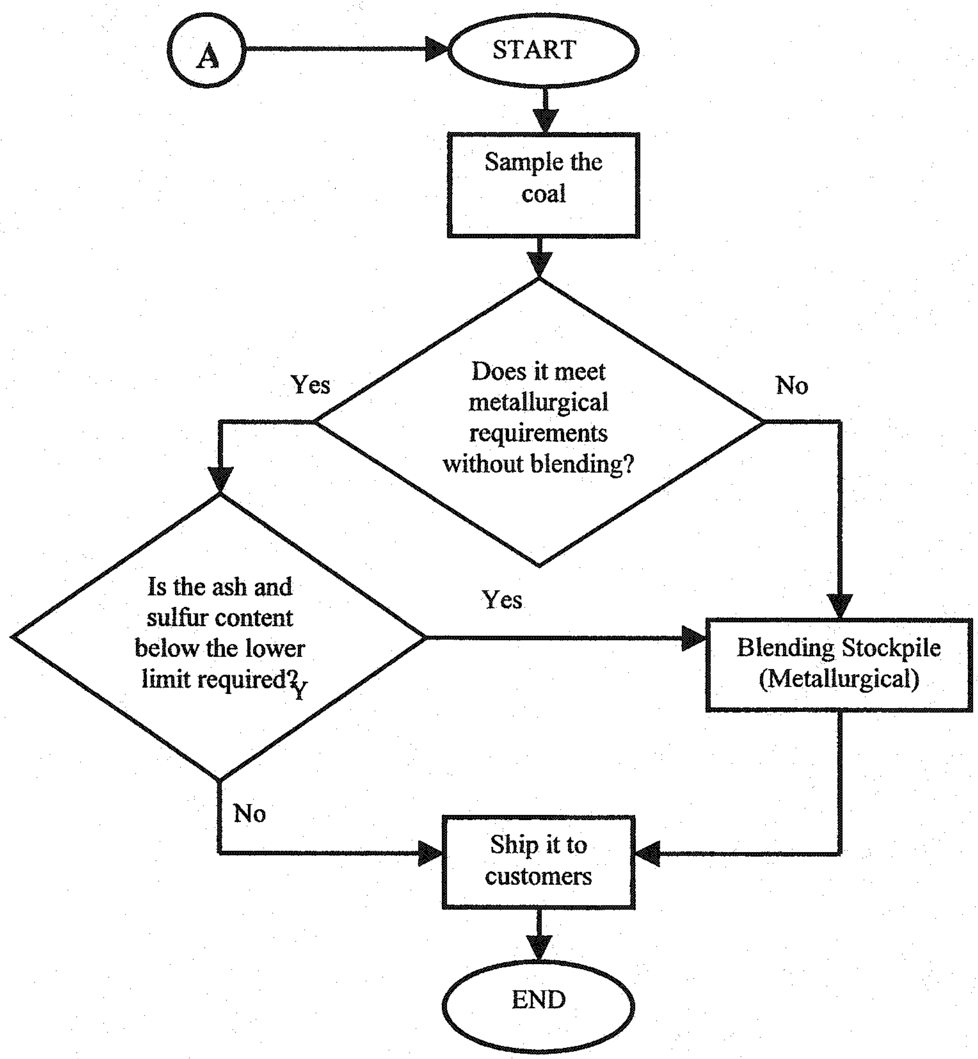


Figure 4.3 Flowchart for Metallurgical Coal Process

Solver uses The Simplex Method (TSM) to solve LP problems. The general algorithm of TSM is presented in Chapter 2.0 and a detailed LP algorithm for the problem addressed by this report is developed in Chapter 3.0 [8, 21, 26]. The TSM flowchart, which is illustrated in Figure 4.5, presents the logic of the computer model. The GLM method is applied to solve the same optimization problem. The flowchart of the GLM computer model is presented in Figure 4.6. The GLM's mathematical algorithm is presented in Chapter 2.0 and an expanded GLM algorithm to the problem solved in this report is presented in Chapter 3.0. The model is developed and solved in Excel-Solver with the non-linearity option [32, 41].

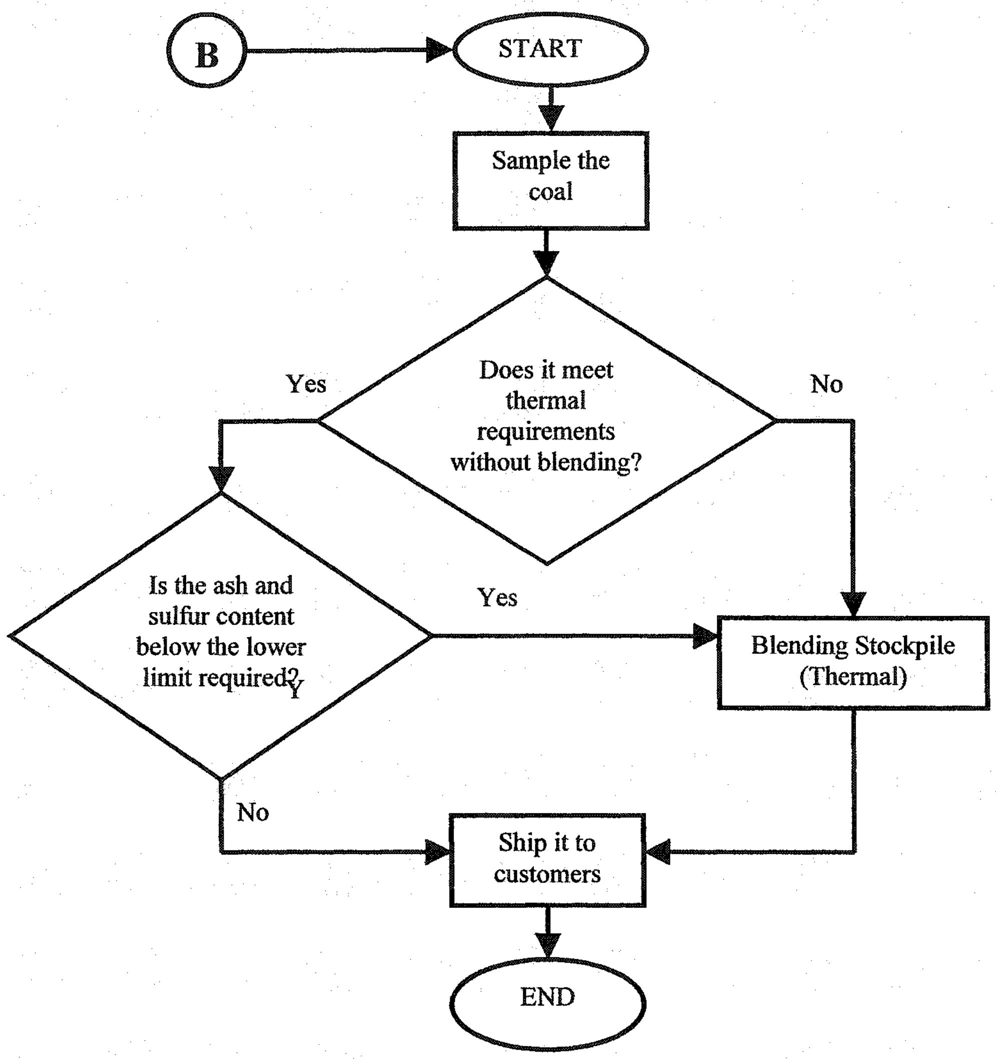


Figure 4.4 Flowchart for Thermal Coal Process

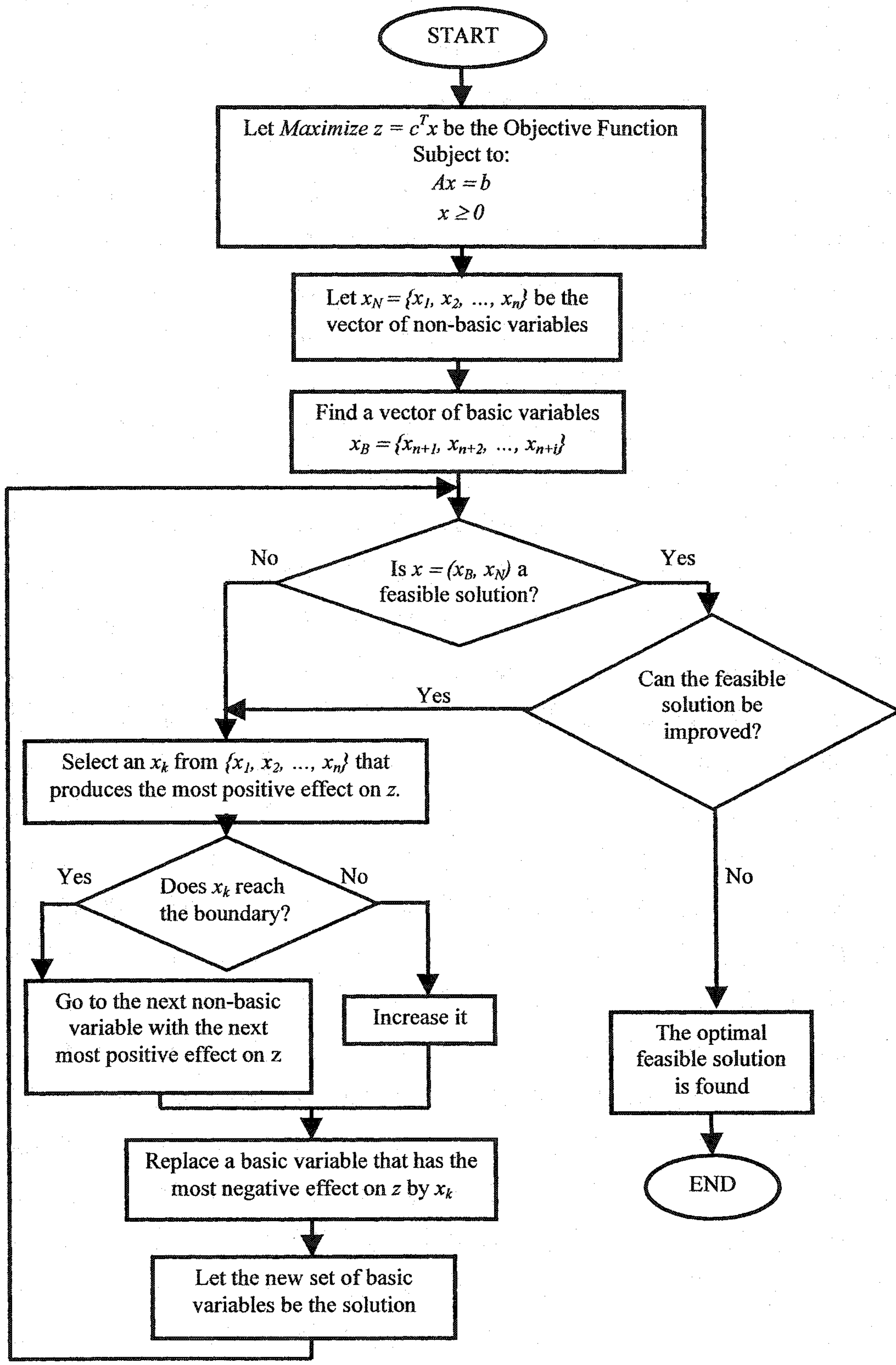


Figure 4.5 The Simplex Method Flowchart

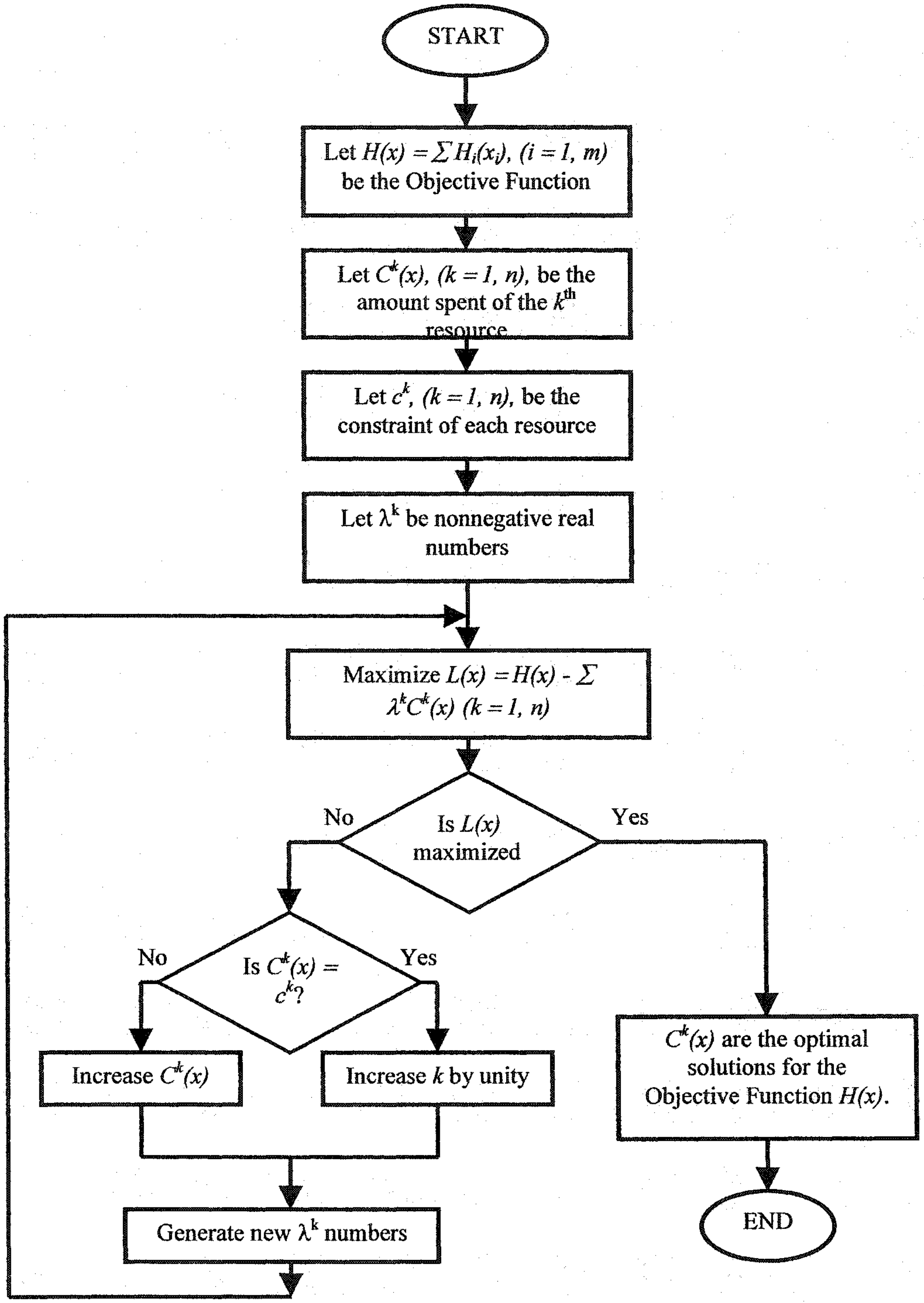


Figure 4.6 The Generalized Lagrange Multiplier Method Flowchart

Several mathematical techniques exist to solve LP and NLP problems involving almost any number of variables without visualizing or graphing their feasible regions. These techniques are now built into spreadsheet packages in a way that makes solving LP and NLP problems a fairly simple task. The main challenge is the correct formulation and communication of the problem to the Solver environment for accurate results. Solver provides three solution reports including an Answer, Sensitivity and Limits reports for detailed analysis and a basis for management [32, 41].

The design and solution of the stochastic models are carried out in the @RISK environment [Palisade, 1996]. Figure 4.7 illustrates the flowchart model for risk simulation using @Risk. The Monte Carlo and the Latin Hypercube techniques are used to provide multiple maps for the stochastic models in the @RISK environment as illustrated in Figure 4.8. The selected PDFs for the uncertain random variables are entered into the @Risk environment. The setup of the simulation model entails the selection of sampling type, standard recalculation, and outputs. The outputs of the function in the spreadsheet can be calculated using the expected value, Monte Carlo, and the true expected value. Random sample are generated from each PDFs, and used as inputs to the simulation model. Excel uses these random values to recalculate the functional value of the stochastic problem [25, 28, 32, 41].

The Latin Hypercube sampling technique is designed to accurately recreate the input distribution by using less iteration when sampling the distribution as compared with Monte Carlo sampling technique. Once the PDF is entered, it is converted to a cumulative distribution function (CDF). Then the input distribution is stratified, the cumulative curve is divided into equal intervals on the probability scale from 0 to 1. Random samples are drawn from each interval, which forces the recreation of the input probability distribution. The number of stratifications of the cumulative distribution is equivalent to the number of iterations selected for a simulation run. Once a sample is drawn from stratification, no other value is sampled from the same stratification [25].

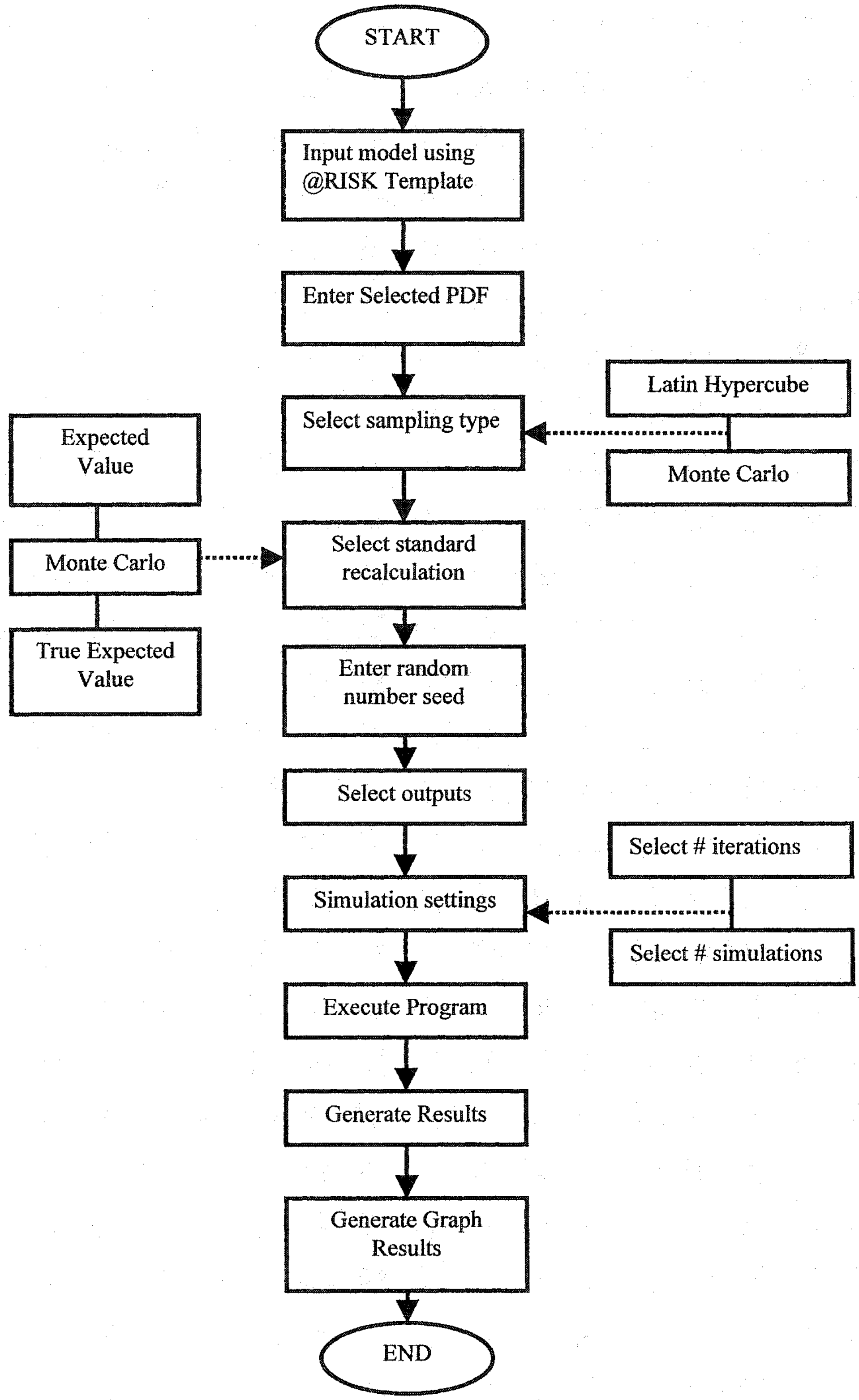


Figure 4.7 Flowchart Model for Risk Simulation Using @RISK

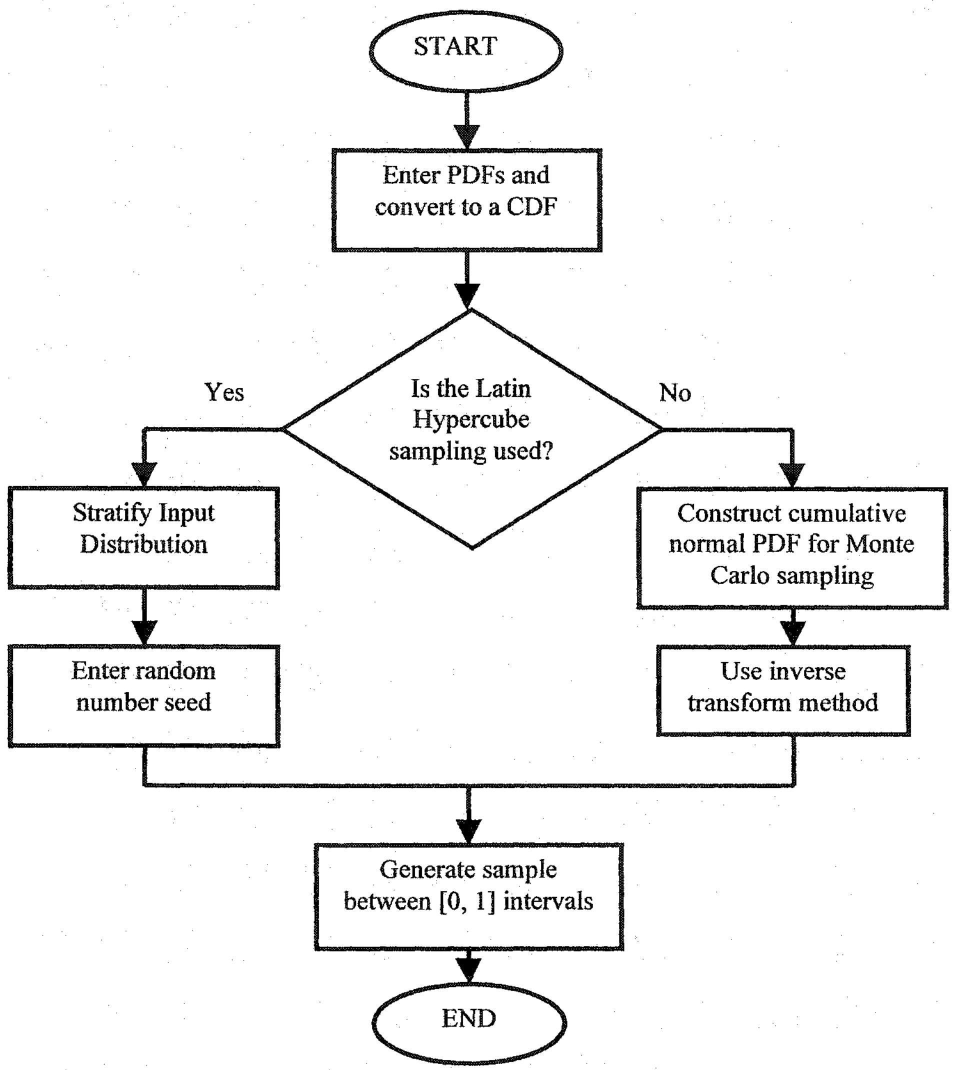


Figure 4.8 Flowchart for Simulation Model Sampling Techniques

The Monte Carlo sampling technique begins with the construction of a cumulative PDF and a cumulative normal PDF. An inverse transform methodology is used to transform the cumulative normal PDF into a cumulative uniform distribution to permit generating equal-probable sampling. A sample is selected along the probability interval from 0 to 1 by drawing a horizontal line from the cumulative uniform distribution curve across to the cumulative PDF. Selecting outputs ensures that only those cells are going to display simulation results [25, 28].

The simulations experiments are conducted with 10,000 iterations in one simulation run for each of the case study. The simulation model generates a range of graphical output results. An output distribution of each simulation random variable is provided, along with several other graphs, which include a frequency, and cumulative graphs of the objective function. The simulation results include summary statistics and data reports for both input and output variables. Generated statistics include minimum and maximum values, means, standard deviations, and percentiles [25, 28, 32, 41].

4.2 Formulation of the Case Studies Computer Models

In this section two case studies were conducted to validate the optimization and stochastic models in Chapter 3.0. The two largest coal mining companies in western Canada, Luscar-Sherritt International and Fording Coal, were selected to conduct the case studies. Each one of these two companies own several mines spread across the Mountain and Foothills regions of Western Canada. The mathematical models are designed to capture the production, transport and market structure used by these two companies. Some of the data used in the model validation had to be assumed since the coal mining companies, for confidential reasons, were somehow reluctant to release. However, these assumptions will not affect the model results but care must be taken in the results interpretation process since they could be different of what the mining companies would expect.

4.2.1 Case Study I: Luscar-Sherritt International.

Luscar-Sherritt International is a leading coal producer in Canada and one of the largest suppliers of coal in North America with its corporate offices located in Edmonton, Alberta. The company has proven reserves in excess of 800 [Mt], operates 11 coal mines in the provinces of British Columbia, Alberta and Saskatchewan producing sub-bituminous and bituminous coal. Luscar-Sherritt uses surface mining techniques such as strip mining and open pit mining methods for coal extraction. Luscar-Sherritt provides coal products to domestic utility customers in Alberta, Saskatchewan, and Ontario. The company also exports a large amount of metallurgical and thermal coal to countries on the Pacific Rim and Europe through the ports of Vancouver and Prince Rupert [22].

This study addresses the production and haulage optimization problem for the second category of Luscar's mines. Data is collected and analyzed from four mines and a project, owned by Luscar-Sherritt, for optimization in the study. These mines include Line Creek in southeast BC, Luscar, Coal Valley, Obed Mountain and the Cheviot project in Alberta. All these mines are linked to all the major ports by railway except Line Creek, which is connected to only Vancouver and Thunder Bay [22].

To optimize the coal production and haulage of these mines, LP and NLP models were developed. It is assumed that from each mine, a certain amount would be metallurgical coal expressed by the variables; x_{11} and x_{13} produced by Line Creek and transported to Vancouver and Thunder Bay; x_{21}, x_{22}, and x_{23} produced at Luscar and transported to Vancouver, Ridley Island and Thunder Bay; x_{31}, x_{32}, and x_{33} produced by Cheviot and transported to Vancouver, Ridley Island and Thunder Bay; x_{41}, x_{42}, and x_{43} produced by Coal Valley and transported to Vancouver, Ridley Island and Thunder Bay, and x_{51}, x_{52}, and x_{53} produced by Obed Mountain and transported to Vancouver, Ridley Island and Thunder Bay. It is also assumed that a certain amount would be thermal coal expressed by the variables; y_{11}, y_{13}, and y_{14} produced at Line Creek and transported to Vancouver, Thunder Bay and to TransAlta's power plants (Sundance and Keephills); y_{21}, y_{22}, y_{23}, and y_{24} produced at Luscar and transported to Vancouver, Ridley Island, Thunder Bay and to TransAlta's power plants; y_{31}, y_{32}, y_{33}, and y_{34} produced at Cheviot and transported to Vancouver, Ridley Island, Thunder Bay and to TransAlta's power plants; y_{41}, y_{42}, y_{43}, and y_{44} produced at Coal Valley and transported to Vancouver, Ridley Island, Thunder Bay and to TransAlta's power plants, and y_{51}, y_{52}, y_{53}, and y_{54} produced at Obed Mountain and transported to Vancouver, Ridley Island, Thunder Bay and to TransAlta's power plants. The dynamic of coal production and transport is presented in Figure 4.2.1. The total amount of metallurgical coal (Q1) produced by Luscar-Sherritt is governed by equation (4.1). Equation (4.2) governs the total quantity of thermal coal (Q2).

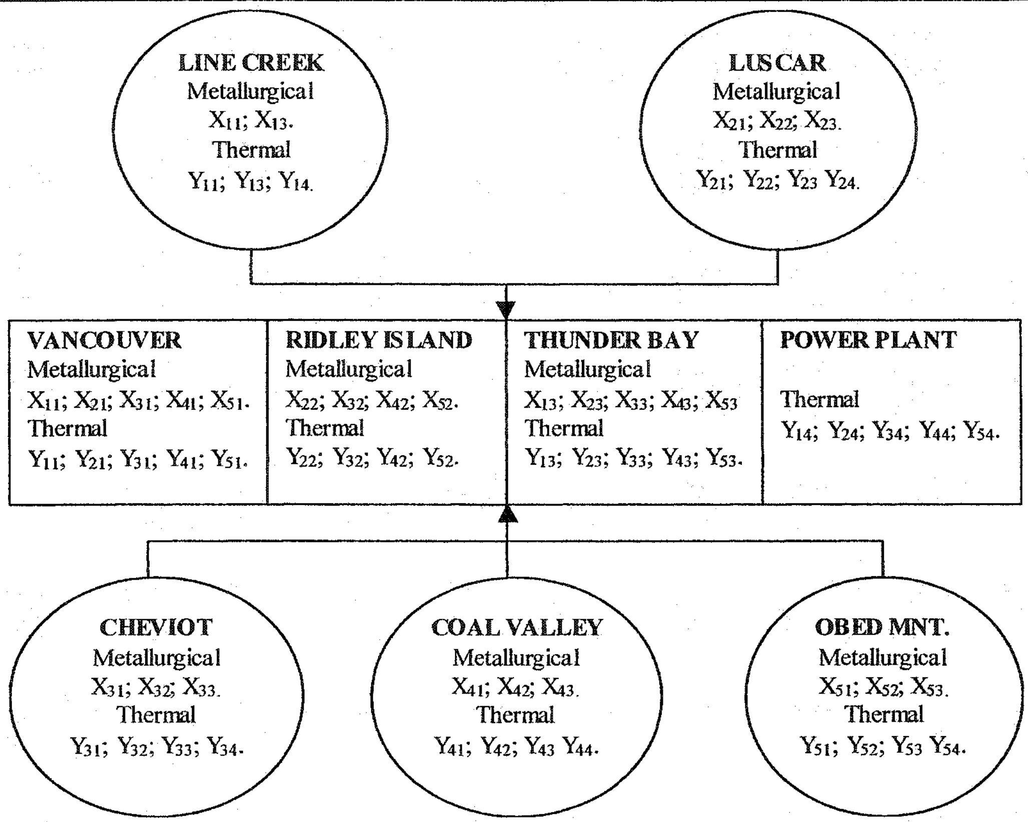


Figure 4.2.1 Schematic Diagram of Luscar Coal Production and Transports

$$Q_1 = X_{11} + X_{13} + X_{21} + X_{22} + X_{23} + X_{31} + X_{32} + X_{33} + X_{41} + X_{42} + X_{43} + X_{51} + X_{52} + X_{53} \quad (4.1)$$

$$Q_2 = Y_{11} + Y_{13} + Y_{14} + Y_{21} + Y_{22} + Y_{23} + Y_{24} + Y_{31} + Y_{32} + Y_{33} + Y_{34} + Y_{41} + Y_{42} + Y_{43} + Y_{44} + Y_{51} + Y_{52} + Y_{53} + Y_{54} \quad (4.2)$$

Equations (4.3) through (4.7) represent the total coal production at each particular mine. P_{LC}, P_L, P_C, P_{CV}, and P_{OB} represent total coal production at Line Creek, Luscar, Cheviot, Coal Valley, and respective Obed Mountain.

$$P_{LC} = X_{11} + X_{13} + Y_{11} + Y_{13} + Y_{14} \quad (4.3)$$

$$P_L = X_{21} + X_{22} + X_{23} + Y_{21} + Y_{22} + Y_{23} + Y_{24} \quad (4.4)$$

$$P_C = X_{31} + X_{32} + X_{33} + Y_{31} + Y_{32} + Y_{33} + Y_{34} \quad (4.5)$$

$$P_{CV} = X_{41} + X_{42} + X_{43} + Y_{41} + Y_{42} + Y_{43} + Y_{44} \quad (4.6)$$

$$P_{OB} = X_{51} + X_{52} + X_{53} + Y_{51} + Y_{52} + Y_{53} + Y_{54} \quad (4.7)$$

The profit of the company is calculated as difference between total revenue and total Cost as illustrated in equation (4.8).

$$P_t = R - T_C \quad (4.8)$$

The total revenue is a function of the revenues generated by each mine, which is expressed in equation (4.9). M_p represents the price of metallurgical coal, and T_p is the price of thermal coal. The average coal price is assumed to be 54.0 [$/t] for metallurgical coal and 37.5 [$/t] for thermal coal based on various sources [3, 16, 36].

$$R = Q_1 * M_P + Q_2 * T_P \quad (4.9)$$

The total operating cost of the company is expressed in equation (4.10), which contains the mining and processing cost of metallurgical coal (MP_m), mining and processing cost of thermal coal (MP_t), transportation cost (T_{rt}), port charges (P_c), and OVH_c overhead cost. The mining and processing costs are different for each mine, as illustrated in Table 4.2.1 [16].

$$T_C = MP_m * Q_1 + MP_t * Q_2 + (T_{rt} + P_c + OVH_c) * (Q_1 + Q_2) \quad (4.10)$$

Table 4.2.1 Luscar-Sherritt's Mining and Processing Costs

Mine	Production & Processing cost [$/t]	
	Metallurgical	Thermal
Line Creek	20.5	16.5
Luscar	20.0	16.0
Cheviot	20.5	16.5
Coal Valley	21.5	14.5
Obed Mnt.	21.0	14.0

The total railway transportation cost (T_{rt}) is governed by equation (4.11). T_{rt} is a function of the mine port distances (d_{ij}), railway fees (k_v, k_r, k_t, and k_p) and the respective quantities of coal transported on each route ($x_{11,...,}y_{54}$). k_v, k_r, k_t, and k_p represent the railway fees from the mine site to Vancouver, Ridley Island, and Thunder Bay ports and to TransAlta's power plants. The distances and fees are presented in Table 4.2.2 [9, 16, 22, 38].

$$T_{rt} = k_v * [d_{11} * (X_{11} + Y_{11}) + d_{21} * (X_{21} + Y_{21}) + d_{31} * (X_{31} + Y_{31}) + d_{41} * (X_{41} + Y_{41}) +$$
$$d_{51} * (X_{51} + Y_{51})] + k_r[d_{22} * (X_{22} + Y_{21}) + d_{32} * (X_{32} + Y_{32}) + d_{42} * (X_{42} + Y_{42}) +$$
$$d_{52} * (X_{52} + Y_{52})] + k_t * [d_{13} * (X_{13} + Y_{13}) + d_{23}(X_{23} + Y_{23}) + d_{33} * (X_{33} + Y_{33}) +$$
$$d_{43} * (X_{43} + Y_{43}) + d_{53} * (X_{53} + Y_{53})] + k_p * [d_{14} * Y_{14} + d_{24} * Y_{24} + d_{34} * Y_{34} +$$
$$d_{44} * Y_{44} + d_{54} * Y_{54}] \quad (4.11)$$

Table 4.2.2 Luscar-Sherritt's Mine Port Distances and Railway Charges

Mine / Destination	Vancouver		Ridley		Thunder Bay		Pw. Plant
	Distance [km]	Rail Fees [$/Km-t]	Distance [Km]	Rail Fees [$/Km-t]	Distance [Km]	Rail Fees [$/Km-t]	Distance [Km]
Line Creek	1141	0.013	-	-	2102	0.008	200
Luscar	1108	0.013	1404	0.012	2305	0.008	100
Cheviot	1128	0.013	1424	0.012	2325	0.008	100
Coal Valley	1093	0.013	1381	0.012	2282	0.008	50
Obed Mnt.	958	0.013	1257	0.012	2264	0.008	50

The total port cost is governed by equation (4.12) and is a function of charges at each individual port. V_{pc}, R_{pc}, TB_{pc}, and PW_c represent the charges at Vancouver, Ridley Island, and Thunder Bay ports and respective power plant. Port charges are illustrated in Table 4.2.3 [16].

$$P_c = V_{pc} * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51}) + R_{pc} * (X_{22} + X_{32} + X_{42} + X_{52} + Y_{22} + Y_{32} + Y_{42} + Y_{52}) + TB_{PC} * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53} + Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53}) + PW_C * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \quad (4.12)$$

Table 4.2.3 Luscar-Sherritt's Port and Power Plant Charges.

Mine	Port Costs [$/t]			Power Plant
	Vancouver	Ridley	Thunder Bay	Cost [$ / t]
Line Creek	3.75	-	3.50	1.50
Luscar	3.75	3.00	3.50	1.50
Cheviot	3.75	3.00	3.50	1.50
Coal Valley	3.75	3.00	3.50	1.50
Obed Mountain	3.75	3.00	3.50	1.50

The Overhead Cost is function of the individual fixed costs and the amount of coal produced at each mine and is governed by equation (4.13). M_1, M_2, M_3, M_4, and M_5 represent the fixed costs per tonne at Line Creek, Luscar, Cheviot, Coal Valley, and respective Obed Mountain. The overhead costs are presented in Table 4.2.4.

$$OVH_C = M_1 * (X_{11} + X_{13} + Y_{11} + Y_{13} + Y_{14}) + M_2 * (X_{21} + X_{22} + X_{23} + Y_{21} + Y_{22} + Y_{23} + Y_{24}) + M_3 * (X_{31} + X_{32} + X_{33} + Y_{31} + Y_{32} + Y_{33} + Y_{34}) + M_4 * (X_{41} + X_{42} + X_{43} + Y_{41} + Y_{42} + Y_{43} + Y_{44}) + M_5 * (X_{51} + X_{52} + X_{53} + Y_{51} + Y_{52} + Y_{53} + Y_{54}) \quad (4.13)$$

Table 4.2.4 Luscar-Sherritt's Overhead Costs by Mine

Mine	Overhead Cost [$ /t]
Line Creek	0.15
Luscar	0.11
Cheviot	0.12
Coal Valley	0.13
Obed Mnt.	0.11

By substituting equations (4.10), (4.11) (4.12) and (4.13) in (4.9) and (4.8) the profit function takes the form of equation (4.14).

$$P_t = Q_1 * M_p + Q_2 * T_p - MP_m * Q_1 + MP_t * Q_2 + (T_n + P_c + OVH_c) * (Q_1 + Q_2) \quad (4.14)$$

By replacing all the coefficients in the profit function with their values from Tables 4.2.1 through 4.2.4, results in a function of X_{ij} and Y_{ij} variables and some numerical values A_{ij} and B_{ij} as illustrated in equation (4.15).

$$P_t = A_{11} * X_{11} + A_{13} * X_{13} + ... + A_{53} * X_{53} + B_{11} * Y_{11} + ... + B_{54} * X_{54} \tag{4.15}$$

The profit function from equation (4.15) and all the specific costs, revenues, coal quantities, and other pertinent data are input in an Excel spreadsheet for calculation. The next step is to develop the optimization models into a spreadsheet. The optimization model consists of the objective function represented by equation (4.15) and the underlying field constraints, which include supply and demand, mine capacities, railway capacities, port limitations, and coal quality constraints.

4.2.1.1 LP Model

The objective function of the LP model is the profit function, presented in equation (4.15). The next step in the LP modeling is to develop the constraint equations that the objective function is subjected to. Market supply and demand is the main factor, which determines the quantities of coal that western Canadian coal mining companies produce. Coal mining companies usually conduct a careful analysis of the coal market and take their decisions accordingly. Most of the coal companies prefer long-term contracts. Recently, because of world coal production over-capacities and market's over-supply, the trend has shifted from long-term contracts to spot market and short-term contracts (See Appendix 1.0). This trend increases the risk borne by coal producers and it requires rigorous modeling and analysis of production capacities, quality and market strategies to stay competitive.

For the purpose of this study it is assumed that only a certain quantity of coal could be sold on the market and the company will limit its production to this quantity. For instance, it is assumed that Luscar-Sherritt's coal production, at the five mines taken into consideration in this study, would be limited to 8.5 [Mt/y] metallurgical coal and 9.0 [Mt/y] of thermal coal. This production is assumed to be distributed to the shipping ports and power plants as illustrated in Table 4.2.5, these figures are extracted from various sources and are close to the current production and transportation levels of Luscar-Sherritt [16, 22, 36]. The constraints that control the overseas market limitations are presented in equation (16) through (19). These equations will limit the coal quantities produced and shipped according to the market absorption capacity. Equation (4.16) and (4.17) governs the respective metallurgical and thermal coal transported to Vancouver ports (Neptune and West shore) for the overseas customers.

Table 4.2.5 Luscar-Sherritt's Market Capacity Limitation by Destination

Destination	Metallurgical [Mt/y]	Thermal [Mt/y]
Vancouver	≤ 5.5	≤ 2.0
Ridley Island	≤1.5	≤ 2.5
Thunder Bay	≤ 1.5	≤ 2.5
TransAlta's power plants	-	≤ 2.0
Total	≤ **8.5**	≤ **9.0**

$$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} \leq 5.5 \text{ [Mt/y]} \tag{4.16}$$

$$Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} \leq 2.0 \text{ [Mt/y]} \tag{4.17}$$

Equation (4.18) and (4.19) control the metallurgical and thermal coal transported to Ridley Island for customers in Japan, Korea.

$$X_{22} + X_{32} + X_{42} + X_{52} \leq 1.5 \text{ [Mt/y]} \tag{4.18}$$

$$Y_{22} + Y_{32} + Y_{42} + Y_{52} \leq 2.5 \text{ [Mt/y]} \tag{4.19}$$

The domestic market constraints (Alberta and Eastern Canadian provinces) are presented in equations (4.20) through (4.22). Equations (4.20) and (4.21) govern the respective metallurgical and thermal coal transported to Thunder Bay for eastern Canada customers. Equation (4.22) refers to domestic customer within Alberta (electricity and cement producers).

$$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} \leq 1.5 \text{ [Mt/y]} \quad (4.20)$$

$$Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} \leq 2.5 \text{ [Mt/y]} \quad (4.21)$$

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} \leq 2.0 \text{ [Mt/y]} \quad (4.22)$$

It is also assumed that Luscar-Sherritt has contracted certain amounts of coal, which are transported to Vancouver and Ridley Island for overseas consumers, and to Thunder Bay and TransAlta's power plants for domestic market. The total contracted coal quantities per year are assumed to be 5.0 [Mt/y] metallurgical coal and 7.5 [Mt/y] of thermal coal distributed to the shipping ports and power plants as illustrated in Table 4.2.6. The constraint equations that govern the contracted amounts of coal for overseas markets are presented in (4.23) through (4.26). These equations require the company to produce and ship the required coal quantities to customers. The contracted amounts of metallurgical and thermal coal shipped to Vancouver ports are controlled by equation (4.23) and respective (4.24).

Table 4.2.6 Luscar-Sherritt's Contracted Quantities of Coal by Destination

Destination	Metallurgical [Mt/y]	Thermal [Mt/y]
Vancouver	3.0	2.0
Ridley Island	1.0	2.0
Thunder Bay	1.0	2.0
TransAlta's power plants	-	1.5
Total	**5.0**	**7.5**

$$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} \geq 3.0 \text{ [Mt/y]} \quad (4.23)$$

$$Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} \geq 2.0 \text{ [Mt/y]} \quad (4.24)$$

Equations (4.25) and (4.26) force the amount of metallurgical and respective thermal coal to be larger than or equal to the contracted quantities for Ridley Island port.

$$X_{22} + X_{32} + X_{42} + X_{52} \geq 1.0 \text{ [Mt/y]} \quad (4.25)$$

$$Y_{22} + Y_{32} + Y_{42} + Y_{52} \geq 2.0 \text{ [Mt/y]} \quad (4.26)$$

The contracted coal quantities for domestic market are presented in equations (4.27) to (4.29). Equations (4.27) and (4.28) set the amounts of metallurgical and thermal coal transported to Thunder Bay to be greater than or equal to the contract specifications. Equation (4.29) controls the thermal coal quantities contracted with the power plants.

$$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} \geq 1.0 \text{ [Mt/y]} \quad (4.27)$$

$$Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} \geq 2.5 \text{ [Mt/y]} \quad (4.28)$$

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} \geq 1.5 \text{ [Mt/y]} \quad (4.29)$$

The maximum mine capacities is another important factor that constrains the coal quantities produced by Luscar-Sherritt. It is assumed that each mine has a limited production capacity and the constraint is captured into the model. Table 4.2.7 illustrates the assumed maximum coal capacities for each of the five mines, they were assumed based on the current level of production and on their assets capabilities. Equations (4.30) through (4.34) limit the available resources for the optimization model to the assumed maximum capacities of Line Creek, Luscar, Cheviot, Coal Valley, and Obed Mountain.

Table 4.2.7 Luscar-Sherritt's Mine Maximum Capacities

Mine	**Maximum Capacity [Mt/y]**
Line Creek	4.0
Luscar	3.0
Cheviot	4.0
Coal Valley	2.5
Obed Mnt.	2.0
Total	**15.5**

$$X_{11} + X_{13} + Y_{11} + Y_{13} + Y_{14} \leq 4.0 \text{ [Mt/y]} \tag{4.30}$$

$$X_{21} + X_{22} + X_{23} + Y_{21} + Y_{22} + Y_{23} + Y_{24} \leq 3.0 \text{ [Mt/y]} \tag{4.31}$$

$$X_{31} + X_{32} + X_{33} + Y_{31} + Y_{32} + Y_{33} + Y_{34} \leq 4.0 \text{ [Mt/y]} \tag{4.32}$$

$$X_{41} + X_{42} + X_{43} + Y_{41} + Y_{42} + Y_{43} + Y_{44} \leq 2.5 \text{ [Mt/y]} \tag{4.33}$$

$$X_{51} + X_{52} + X_{53} + Y_{51} + Y_{52} + Y_{53} + Y_{54} \leq 2.0 \text{ [Mt/y]} \tag{4.34}$$

The minimum capacity of a mine is determined mostly by its assets utilization. To guarantee that no producing and processing facility works below its economic efficiency capacity, a minimum capacity usage requirement is set for each mine. From the five mines only Line Creek, Luscar, and Cheviot have metallurgical coal processing and upgrading facilities, therefore a minimum metallurgical coal capacity is set for these particular mines [22]. There is no minimum production requirement of thermal coal for any specific mine. The minimum capacity requirements, for the five mines, are presented in Table 4.2.8. The minimum capacities are governed by equations (4.35) through (4.42). Equations (4.35) through (4.39) set the total coal production at the five mines to be larger than or equal to the minimum capacities in Table 4.2.8.

Table 4.2.8 Luscar-Sherritt's Mine Minimum Capacity Requirements

Mine	**Total [Mt/y]**	**Metallurgical [Mt/y]**
Line Creek	3.2	2.5
Luscar	2.5	2.0
Cheviot	3.2	2.5
Coal Valley	1.5	-
Obed Mnt.	1.5	-
Total	**11.9**	**7.0**

$$X_{11} + X_{13} + Y_{11} + Y_{13} + Y_{14} \geq 3.2 \text{ [Mt/y]} \tag{4.35}$$

$$X_{21} + X_{22} + X_{23} + Y_{21} + Y_{22} + Y_{23} + Y_{24} \geq 2.5 \text{ [Mt/y]} \tag{4.36}$$

$$X_{31} + X_{32} + X_{33} + Y_{31} + Y_{32} + Y_{33} + Y_{34} \geq 3.2 \text{ [Mt/y]} \tag{4.37}$$

$$X_{41} + X_{42} + X_{43} + Y_{41} + Y_{42} + Y_{43} + Y_{44} \geq 1.5 \text{ [Mt/y]} \tag{4.38}$$

$$X_{51} + X_{52} + X_{53} + Y_{51} + Y_{52} + Y_{53} + Y_{54} \geq 1.5 \text{ [Mt/y]} \tag{4.39}$$

Equations (4.40) to (4.42) set the metallurgical coal production at Line Creek, Luscar, and Cheviot to be greater than or equal to the minimum capacity requirements of the processing and upgrading facilities.

$$X_{11} + X_{13} \geq 2.5 \text{ [Mt/y]} \tag{4.40}$$

$$X_{21} + X_{22} + X_{23} \geq 2.0 \text{ [Mt/y]} \tag{4.41}$$

$$X_{31} + X_{32} + X_{33} \geq 2.5 \text{ [Mt/y]} \tag{4.42}$$

Coal production from the mines is transported by trains to ports or directly to customers. Luscar-Sherritt's mines are served by both Canadian National and Canadian Pacific railway companies. Railway-port distances average 1085.5 km to Vancouver 1366.5 to Ridley Island and over 2255.6 km to Thunder Bay on the western end of Lake Superior. Line Creek mine does not have any railway link with Ridley Island, therefore, it does not transport coal to this particular port [22]. The total coal railway capacity is assumed to be around 25.0 [Mt/y] from Sparwood to Vancouver, 15 [Mt/y] from Hinton-Grande Cache area to Vancouver, 10 [Mt/y] from Hinton-Grande Cache area to Ridley Island, and around 15.0 [Mt/y] to Thunder Bay. These assumptions were made based on the quantities of coal transported by a train unit and the time necessary for a complete cycle, they are proven to be real since in 1997 the level of production and transportations were very close to these values [3, 9, 22, 36].

Since there are other coal producers, served by the same railway companies, the railway capacities allocated to Luscar-Sherritt's mine are lower than the above figures as illustrated in Table 4.2.9.

Table 4.2.9 Total Railway Coal Capacities, Route vs. Customer

Mine - Destination Routes	**Total Railway Capacity [Mt/y]**	**Other Coal Producers [Mt/y]**	**Allocated to Luscar [Mt/y]**
Sparwood - Vancouver	25.0	19.5	5.5
Sparwood - Thunder Bay	15.0	4.0	11.0
Hinton/Grande-Cache - Vancouver	15.0	1.5	13.5
Hinton/Grande-Cache - Ridley	10.0	1.5	8.5
Hinton/Grande-Cache - Thunder Bay	15.0	2.0	13.0
Mines - Local Consumers	5.0	-	5.0

Total rail capacity from Sparwood area to Vancouver is less than 25.0 [Mt/y], of which 19.5 [Mt/y] are taken by Fording and Teck, the remaining capacity is 5.5 [Mt/y] [3, 36]. This limitation applies to Line Creek mine only. The amount of coal shipped by train from Line Creek to Vancouver is limited and controlled by equation (4.43).

$$X_{11} + Y_{11} \leq 5.5 \text{ [Mt/y]} \tag{4.43}$$

From Hinton-Grande Cache area to Vancouver ports the railway capacity is less than 15 [Mt/y] of which 1.5 [Mt/y] taken by Smoky River, and the remaining capacity of 13.5 [Mt/y] is used by Luscar-Sherritt [36]. This constraint refers to Luscar, Cheviot, Coal Valley, and Obed Mountain. The maximum coal quantity that could be produced by Luscar, Cheviot, Coal Valley and Obed Mountain and hauled to Vancouver is governed by equation (4.44).

$$X_{21} + X_{31} + X_{41} + X_{51} + Y_{21} + Y_{31} + Y_{41} + Y_{51} \leq 13.5 \text{ [Mt/y]} \tag{4.44}$$

From Hinton-Grande Cache to Ridley Island port, the coal railway capacity is less than 10 [Mt/y] of which 1.5 [Mt/y] taken by Teck, remaining capacity 8.5 [Mt/y] [36]. This constraint concerns Luscar, Cheviot, Coal Valley and Obed Mountain. Equation (4.45) governs this limitation.

$$X_{22} + X_{32} + X_{42} + X_{52} + Y_{22} + Y_{32} + Y_{42} + Y_{52} \leq 8.5 \text{ [Mt/y]} \quad (4.45)$$

From Sparwood area to Thunder Bay port, the coal railway capacity is less than 15.0 [Mt/y] of which 4.0 [Mt/y] are taken by Fording and Teck, and the remaining 11.0 [Mt/y] are used by Luscar-Sherritt [36]. This constraint concerns Line Creek only and is controlled by equation (4.46).

$$X_{13} + Y_{13} \leq 11.0 \text{ [Mt/y]} \quad (4.46)$$

From Hinton-Grand Cache to Thunder Bay port the coal railway capacity is less than 15 [Mt/y] of which 2.0 [Mt/y] are taken by Bienfait, and the remaining 13.0 [Mt/y] are used by Luscar-Sherritt [36]. This constraint applies to Luscar, Cheviot, Coal Valley and Obed Mountain. Equation (4.47) governs the limitation.

$$X_{23} + X_{33} + X_{43} + X_{53} + Y_{23} + Y_{33} + Y_{43} + Y_{53} \leq 13.0 \text{ [Mt/y]} \quad (4.47)$$

From the mine sites to power plants or domestic consumers within Alberta, the coal railway capacity is believed to be less than 5.0 [Mt/y]. Equation (4.48) controls this constraint.

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} \leq 5.0 \text{ [Mt/y]} \quad (4.48)$$

Port Capacities could limit the amounts of coal, which are sold overseas. As in the railway case, several coal producers and exporters use port facilities. Ports capacities are distributed to the users as illustrated in Table 4.2.10 [3, 9, 22, 36].

Table 4.2.10 Port Capacities vs. Customers Distribution

Port	Total Capacity [Mt/y]	Other Users [Mt/y]	For Luscar [Mt/y]
Vancouver	33.0	19.7	13.3
Ridley Island	16.0	2.0	14.0
Thunder Bay	10.0	4.0	6.0
Total	**59.0**	**25.7**	**33.3**

Vancouver ports have an annual coal handling capacity of 33.0 [Mt]. There are two main port coal facilities in Vancouver namely, Westshore with a capacity of 26.0 [Mt/y] and Neptune with a capacity of 7.0 [Mt/y]. From these capacities, 15.7 [Mt/y] are taken by Fording and 4.0 [Mt/y] by Teck, and the remaining 13.3 [Mt/y] are used by Luscar-Sherritt [7, 12]. Equation (4.49) governs the Vancouver port limitation

$$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} \leq 13.3\,[\text{Mt/y}] \tag{4.49}$$

Ridley Island port has 16.0 [Mt] annual capacity, of which 2.0 [Mt/y] is taken by Teck, and the remaining 14.0 [Mt/y] is used by Luscar-Sherritt [22, 36]. Equation (50) limits the coal shipped through Ridley Island port.

$$X_{22} + X_{32} + X_{42} + X_{52} + Y_{22} + Y_{32} + Y_{42} + Y_{52} \leq 14.0\,[\text{Mt/y}] \tag{4.50}$$

Thunder Bay has 10.0 [Mt] annual capacity, of which 2.0 [Mt/y] taken by Bienfait and 2.0 [Mt/y] by Fording and Teck, and the remaining 6.0 [Mt/y] is used by Luscar-Sherritt [22, 36].

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} \leq 2.0\ [\text{Mt/y}] \tag{4.51}$$

Blending constraints determine the quality of final coal products and the quantities of coal transported from each mine to the final destinations to meet the quality requirements. The quality of coal is determined by the percentage of ash, sulfur, fixed carbon, volatile matter, and moisture and by the content of energy. All these parameters must be within an acceptable range, which could be required by the market, set in the contracts or imposed by the producer. Table 4.2.11 and Table 4.2.12 present the existing clean coal physical and chemical characteristics at each mine and they also give the upper and lower limit of market requirements. All the values from these tables are assumed, based on geological data and ranges of coal quality requirements presented in Appendix 2.0 [3, 31].

Table 4.2.11 Metallurgical Coal Characteristics and Requirements

Metallurgical	Ash [%]	Sulfur [%]	Moisture [%]	Fixed C. [%]	Volatile M.[%]	MJ/kg
Requirements						
Upper Limit	8.5	0.40	8.0	58	22.6	30.2
Lower Limit	8.0	0.35	7.5	-	-	-
Existent (Clean Coal)						
Line Creek	7.5	0.35	8.2	60.0	21.0	32.0
Luscar	8.0	0.42	7.75	59.5	22.0	31.0
Cheviot	8.5	0.38	7.8	57.5	22.5	29.5
Coal Valley	9.0	0.43	8.5	57.0	23.0	29.0
Obed Mnt.	8.75	0.40	8.3	57.5	24.5	29.2

Table 4.2.12 Thermal Coal Characteristics and Requirements

Thermal	Ash [%]	Sulfur [%]	Moisture [%]	Fixed C. [%]	Volatile M.[%]	MJ/kg
Requirements						
Upper Limit	13.0	1.00	13.5	46.5	36.2	26.1
Lower Limit	12.5	0.75	13.0	-	-	-
Existent (Clean Coal)						
Line Creek	12.5	0.75	13.2	51.0	30.0	28.0
Luscar	13.1	0.78	13.75	49.0	35.0	27.5
Cheviot	13.8	0.85	13.8	48.5	37.0	26.0
Coal Valley	12.0	0.80	13.5	47.0	35.2	27.0
Obed Mnt.	12.75	0.90	13.3	46.5	36.0	26.5

From Figure 4.2.1, coal is shipped to four final destinations from five different mines. At each of the final destination, except power plants, the coal is stocked into two different stockpiles (metallurgical and thermal). Each of these stockpiles has to meet the required coal specifications. For each of these stockpiles, a set of blending constraints has to be developed and input into the optimization model to make sure that the coal would meet the quality specifications. Based on data from Tables 4.2.11 and 4.2.12, the blending constraints could be generated for each stockpile as weighted averages. The quality of metallurgical coal stockpiled in Vancouver is governed by equations (4.52) through (4.60). The content of ash in metallurgical coal will be controlled by equation (4.52) and (4.53). Equations (4.52) and (4.53) govern the respective upper and lower limits of the ash content. The ash content must be within 8% - 8.5%.

$$(0.075 * X_{11} + 0.08 * X_{21} + 0.085 * X_{31} + 0.09 * X_{41} + 0.0875X_{51}) -$$
$$0.085 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \leq 0 \quad (4.52)$$

$$(0.075 * X_{11} + 0.08 * X_{21} + 0.085 * X_{31} + 0.09 * X_{41} + 0.0875X_{51}) -$$
$$0.08 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \geq 0 \quad (4.53)$$

The sulfur content is determined by equations (4.54) and (4.55) and must be within 0.35% and 0.4%. Equations (4.54) and (4.55) control the respective upper and lower limits.

$$(0.0035 * X_{11} + 0.0042 * X_{21} + 0.0038 * X_{31} + 0.0043 * X_{41} + 0.004X_{51}) -$$
$$0.004 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \leq 0 \quad (4.54)$$

$$(0.0035 * X_{11} + 0.0042 * X_{21} + 0.0038 * X_{31} + 0.0043 * X_{41} + 0.004X_{51}) -$$
$$0.0035 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \geq 0 \quad (4.55)$$

The moisture content must be between 7.5% and 8.0%. The upper boundary of moisture content is governed by equation (4.56) and the lower boundary by equation (4.57).

$$(0.082 * X_{11} + 0.0775 * X_{21} + 0.078 * X_{31} + 0.085 * X_{41} + 0.083X_{51}) -$$
$$0.08 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \leq 0 \quad (4.56)$$

$$(0.082 * X_{11} + 0.0775 * X_{21} + 0.078 * X_{31} + 0.085 * X_{41} + 0.083X_{51}) -$$
$$0.075 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \geq 0 \quad (4.57)$$

The fixed carbon content for metallurgical coal is required to be larger than 58%. Equation (4.58) sets the content of fixed carbon to be higher than or equal to this value.

$$(0.60 * X_{11} + 0.595 * X_{21} + 0.78 * X_{31} + 0.57 * X_{41} + 0.575 * X_{51}) -$$
$$0.58 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \geq 0 \quad (4.58)$$

The volatile matter constraint is illustrated in equation (4.59). Volatile matter content must be less than or equal to 22.6%.

$$(0.21 * X_{11} + 0.22 * X_{21} + 0.225 * X_{31} + 0.23 * X_{41} + 0.245 * X_{51}) -$$
$$0.226 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \leq 0 \quad (4.59)$$

The energy constraint is presented in equation (4.60). The equation requires that the energy content is larger or equal to 30.2 [MJ/kg].

$$(0.32 * X_{11} + 0.31 * X_{21} + 0.295 * X_{31} + 0.29 * X_{41} + 0.292 * X_{51}) -$$
$$0.302 * (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \geq 0 \quad (4.60)$$

The quality of thermal coal stockpiled in Vancouver is controlled by equations (4.61) through (4.69). The constraint equations are based on Table 4.2.12, which illustrate the thermal coal quality requirements and the qualities of cleaned thermal coal produced by each mine. Equation (4.61) governs the upper limit of the ash content in thermal coal and equation (4.62) the lower limit. The content ash has to be between 12.5% and 13%.

$$(0.125 * Y_{11} + 0.131 * Y_{21} + 0.138 * Y_{31} + 0.12 * Y_{41} + 0.1275 * Y_{51}) -$$
$$0.13 * (Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51}) \leq 0 \quad (4.61)$$

$$(0.125 * Y_{11} + 0.131 * Y_{21} + 0.138 * Y_{31} + 0.12 * Y_{41} + 0.1275 * Y_{51}) -$$
$$0.125 * (Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51}) \geq 0 \quad (4.62)$$

The sulfur content of thermal coal must be within 0.75% and 0.1% limits. Equation (4.63) controls the upper limit and (4.64) the lower boundary.

$$(0.0075*Y_{11}+0.0078*Y_{21}+0.0085*Y_{31}+0.008*Y_{41}+0.009*Y_{51})-$$
$$0.01*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\leq 0 \quad (4.63)$$

$$(0.0075*Y_{11}+0.0078*Y_{21}+0.0085*Y_{31}+0.008*Y_{41}+0.009*Y_{51})-$$
$$0.0075*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\geq 0 \quad (4.64)$$

The moisture content is given by equations (4.65) and (4.66). It has to be between 13% and 13.5% boundaries. Equation (4.65) governs the upper boundary and (4.66) the lower boundary.

$$(0.132*Y_{11}+0.1375*Y_{21}+0.138*Y_{31}+0.135*Y_{41}+0.133*Y_{51})-$$
$$0.135*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\leq 0 \quad (4.65)$$

$$(0.132*Y_{11}+0.1375*Y_{21}+0.138*Y_{31}+0.135*Y_{41}+0.133*Y_{51})-$$
$$0.13*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\geq 0 \quad (4.66)$$

The fixed carbon constraint is governed by equation (4.67). It must be higher than or equal to 46.5 %.

$$(0.51*Y_{11}+0.49*Y_{21}+0.485*Y_{31}+0.47*Y_{41}+0.465*Y_{51})-0.465*($$
$$0.465*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\geq 0 \quad (4.67)$$

The volatile matter constraint is illustrated by equation (4.68). The content of volatile mater of thermal coal has to be lower than 36.2%.

$$(0.30*Y_{11}+0.35*Y_{21}+0.37*Y_{31}+0.352*Y_{41}+0.36*Y_{51})-$$
$$0.362*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\leq 0 \quad (4.68)$$

The energy content of thermal coal must be larger than or equal to 26.1 [MJ/kg]. Equation (4.69) sets the energy content of thermal coal stockpiled in Vancouver port to meet this requirement.

$$(0.28*Y_{11}+0.275*Y_{21}+0.26*Y_{31}+0.27*Y_{41}+0.265*Y_{51})-$$
$$0.261*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\geq 0 \qquad (4.69)$$

Ridley Island's blending stockpiles are based on the same data and have to meet the same quality requirements as Vancouver's blending stockpiles. The coal quality in the metallurgical blending stockpile is governed by equations (4.70) through (4.78). Equations (4.70) and (4.71) control the respective upper and lower limits of ash content.

$$(0.08*X_{22}+0.085*X_{32}+0.09*X_{42}+0.0875*X_{52})-$$
$$0.085*(X_{22}+X_{32}+X_{42}+X_{52})\leq 0 \qquad (4.70)$$

$$(0.08*X_{22}+0.085*X_{32}+0.09*X_{42}+0.0875*X_{52})-$$
$$0.08*(X_{22}+X_{32}+X_{42}+X_{52})\geq 0 \qquad (4.71)$$

The content of sulfur in metallurgical coal is based on equations (4.72) and (4.73), which define the upper and respective the lower limits of it.

$$(0.0042*X_{22}+0.0038*X_{32}+0.0043*X_{42}+0.004*X_{52})-$$
$$0.004*(X_{22}+X_{32}+X_{42}+X_{52})\leq 0 \qquad (4.72)$$

$$(0.0042*X_{22}+0.0038*X_{32}+0.0043*X_{42}+0.004*X_{52})-$$
$$0.0035*(X_{22}+X_{32}+X_{42}+X_{52})\geq 0 \qquad (4.73)$$

The moisture constraints are illustrated by equations (4.74) and (4.75) which control the respective upper and lower limits content.

$$(0.0775 * X_{22} + 0.078 * X_{32} + 0.085 * X_{42} + 0.083 * X_{52}) -$$
$$0.08 * (X_{22} + X_{32} + X_{42} + X_{52}) \leq 0 \quad (4.74)$$

$$(0.0775 * X_{22} + 0.078 * X_{32} + 0.085 * X_{42} + 0.083 * X_{52}) -$$
$$0.075 * (X_{22} + X_{32} + X_{42} + X_{52}) \geq 0 \quad (4.75)$$

The fixed carbon constraint is defined by equation (4.76), which sets it to be higher than or equal to 58%.

$$(0.595 * X_{22} + 0.575 * X_{32} + 0.57 * X_{42} + 0.575 * X_{52}) -$$
$$0.58 * (X_{22} + X_{32} + X_{42} + X_{52}) \geq 0 \quad (76)$$

The volatile matter constraint is governed by equation (4.77).

$$(0.22 * X_{22} + 0.225 * X_{32} + 0.23 * X_{42} + 0.245 * X_{52}) -$$
$$0.226 * (X_{22} + X_{32} + X_{42} + X_{52}) \leq 0 \quad (4.77)$$

The energy content of metallurgical coal in the Ridley Island's stockpile is controlled by equation (4.78).

$$(0.31 * X_{22} + 0.295 * X_{32} + 0.29 * X_{42} + 0.292 * X_{52}) -$$
$$0.302 * (X_{22} + X_{32} + X_{42} + X_{52}) \geq 0 \quad (4.78)$$

The quality of coal in the thermal blending stockpile in Ridley Island port is controlled by equations (4.79) through (4.87). The constraint equations are based on data in Table 4.2.12. Equations (4.79) and (4.80) define the respective upper and lower boundaries of ash content requirements.

$$(0.131 * Y_{22} + 0.138 * Y_{32} + 0.12 * Y_{42} + 0.1275 * Y_{52}) -$$

$$0.13*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\leq 0 \quad (4.79)$$

$$(0.131*Y_{22}+0.138*Y_{32}+0.12*Y_{42}+0.1275*Y_{52})-$$
$$0.125*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\geq 0 \quad (4.80)$$

The sulfur constraints are illustrated in equations (4.81) and (4.82), which govern the respective upper and lower limits of the sulfur content.

$$(0.0078*Y_{22}+0.0085*Y_{32}+0.008*Y_{42}+0.009*Y_{52})-$$
$$0.01*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\leq 0 \quad (4.81)$$

$$(0.0078*Y_{22}+0.0085*Y_{32}+0.008*Y_{42}+0.009*Y_{52})-$$
$$0.0075*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\geq 0 \quad (4.82)$$

The moisture content is controlled by equations (4.83) and (4.84), which define the respective upper and lower boundaries.

$$(0.1375*Y_{22}+0.138*Y_{32}+0.135*Y_{42}+0.133*Y_{52})-$$
$$0.135*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\leq 0 \quad (4.83)$$

$$(0.1375*Y_{22}+0.138*Y_{32}+0.135*Y_{42}+0.133*Y_{52})-$$
$$0.13*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\geq 0 \quad (4.84)$$

The fixed carbon constraint is presented in equation (4.85).

$$(0.49*Y_{22}+0.485*Y_{32}+0.47*Y_{42}+0.465*Y_{52})-$$
$$0.465*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\geq 0 \quad (4.85)$$

The volatile matter content is governed by equation (4.86)

$$(0.35*Y_{22}+0.37*Y_{32}+0.352*Y_{42}+0.36*Y_{52})-$$
$$0.362*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\leq 0 \quad (4.86)$$

The energy content of Ridley Island's thermal coal stockpile is defined by equation (4.87).

$$(0.275*Y_{22}+0.26*Y_{32}+0.27*Y_{42}+0.265*Y_{52})-$$
$$0.261*(Y_{22}+Y_{32}+Y_{42}+Y_{52})\geq 0 \quad (4.87)$$

Thunder Bay's metallurgical and thermal blending stockpiles follow the same mathematical models as those from Vancouver and Ridley Island. The quality of coal in the metallurgical stockpile is controlled by equations (4.88) to (4.96), which are based on data from Table 4.2.11. Equations (4.88) and (4.89) illustrate the ash constraints for Thunder Bay's metallurgical stockpile. They define the respective upper and lower boundaries of the ash content requirements.

$$(0.075*X_{13}+0.08*X_{23}+0.085*X_{33}+0.09*X_{43}+0.0875*X_{53})-$$
$$0.085*(X_{13}+X_{23}+X_{33}+X_{43}+X_{53})\leq 0 \quad (4.88)$$

$$(0.075*X_{13}+0.08*X_{23}+0.085*X_{33}+0.09*X_{43}+0.0875*X_{53})-$$
$$0.08*(X_{13}+X_{23}+X_{33}+X_{43}+X_{53})\geq 0 \quad (4.89)$$

The sulfur constraints are presented in equations (4.90) and (4.91), which control the respective upper and the lower limits of this parameter.

$$(0.0035*X_{13}+0.0042*X_{23}+0.0038*X_{33}+0.0043*X_{43}+0.004*X_{53})-$$
$$0.004*(X_{13}+X_{23}+X_{33}+X_{43}+X_{53})\leq 0 \quad (4.90)$$

$$(0.0035 * X_{13} + 0.0042 * X_{23} + 0.0038 * X_{33} + 0.0043 * X_{43} + 0.004 * X_{53}) -$$
$$0.0035 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \geq 0 \quad (4.91)$$

The upper and lower contents of moisture are governed by equation (4.92) and (4.93).

$$(0.082 * X_{13} + 0.0775 * X_{23} + 0.078 * X_{33} + 0.085 * X_{43} + 0.083 * X_{53}) -$$
$$0.08 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \leq 0 \quad (4.92)$$

$$(0.082 * X_{13} + 0.0775 * X_{23} + 0.078 * X_{33} + 0.085 * X_{43} + 0.083 * X_{53}) -$$
$$0.075 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \geq 0 \quad (4.93)$$

Equation (4.94) controls the content of fixed carbon.

$$(0.60 * X_{13} + 0.595 * X_{23} + 0.575 * X_{33} + 0.57 * X_{43} + 0.575 * X_{53}) -$$
$$0.58 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \geq 0 \quad (4.94)$$

The volatile matter constraint is illustrated in equation (4.95).

$$(0.21 * X_{13} + 0.22 * X_{23} + 0.225 * X_{33} + 0.23 * X_{43} + 0.245 * X_{53}) -$$
$$0.226 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \leq 0 \quad (4.95)$$

Equation (4.96) governs the content of energy in the metallurgical coal stockpiled in Thunder Bay.

$$(0.32 * X_{13} + 0.31 * X_{23} + 0.295 * X_{33} + 0.29 * X_{43} + 0.292 * X_{53}) -$$
$$0.302 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \geq 0 \quad (4.96)$$

The mathematical equations, which define thermal coal qualities for the Thunder Bay stockpile, are base on data presented in Table 4.2.12. These equations are illustrated in

(4.97) through (4.106). Equation (4.97) and (4.98) define the upper and lower limits of ash content.

$$(0.125*Y_{13}+0.131*Y_{23}+0.138*Y_{33}+0.12*Y_{43}+0.1275*Y_{53})-$$
$$0.13*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\leq 0 \quad (4.97)$$

$$(0.125*Y_{13}+0.131*Y_{23}+0.138*Y_{33}+0.12*Y_{43}+0.1275*Y_{53})-$$
$$0.125*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\geq 0 \quad (4.98)$$

The sulfur constraints are presented in equation (4.99) and (4.100), which define the respective upper and the lower limits of sulfur content.

$$(0.0075*Y_{13}+0.0078*Y_{23}+0.0085*Y_{33}+0.008*Y_{43}+0.009*Y_{53})-$$
$$0.01*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\leq 0 \quad (4.99)$$

$$(0.0075*Y_{13}+0.0078*Y_{23}+0.0085*Y_{33}+0.008*Y_{43}+0.009*Y_{53})-$$
$$0.0075*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\geq 0 \quad (4.100)$$

The moisture constraints, illustrated by equations (4.101) and (4.102), govern the content of this parameter in the Thunder Bay thermal coal stockpile.

$$(0.132*Y_{13}+0.1375*Y_{23}+0.138*Y_{33}+0.135*Y_{43}+0.133*Y_{53})-$$
$$0.135*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\leq 0 \quad (4.101)$$
$$(0.132*Y_{13}+0.1375*Y_{23}+0.138*Y_{33}+0.135*Y_{43}+0.133*Y_{53})-$$
$$0.13*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\geq 0 \quad (4.102)$$

The fixed carbon constraint is presented in equation (4.103).

$$(0.51 * Y_{13} + 0.49 * Y_{23} + 0.485 * Y_{33} + 0.47 * Y_{43} + 0.465 * Y_{53}) -$$
$$0.465 * (Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53}) \geq 0 \quad (4.103)$$

The volatile matter content is controlled by equation (4.104).

$$(0.30 * Y_{13} + 0.35 * Y_{23} + 0.37 * Y_{33} + 0.352 * Y_{43} + 0.36 * Y_{53}) -$$
$$0.362 * (Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53}) \leq 0 \quad (4.104)$$

The energy content of the thermal coal stockpiled in Thunder Bay has to be greater than or equal to 26.1 [MJ/kg]. Equation (104) governs this particular parameter.

$$(0.28 * Y_{13} + 0.275 * Y_{23} + 0.26 * Y_{33} + 0.27 * Y_{43} + 0.265 * Y_{53}) -$$
$$0.261 * (Y_{13} Y_{23} + Y_{33} + Y_{43} + Y_{53}) \geq 0 \quad (4.105)$$

TransAlta's power plant stockpile consists of only thermal coal. The coal shipped to the power plant has to meet certain quality requirements. In this study, the quality of coal shipped to the power plants is assumed to be similar to the thermal coal shipped abroad. Equations (4.106) through (4.115) govern the coal quality and are based on data from Table 4.2.12. The ash constraints, for the power plant coal, are presented in equation (4.106) and (4.107), which represent the upper and lower limit of ash content.

$$(0.125 * Y_{14} + 0.131 * Y_{24} + 0.138 * Y_{34} + 0.12 * Y_{44} + 0.1275 * Y_{54}) -$$
$$0.13 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \leq 0 \quad (4.106)$$

$$(0.125 * Y_{14} + 0.131 * Y_{24} + 0.138 * Y_{34} + 0.12 * Y_{44} + 0.1275 Y_{54}) -$$
$$0.125 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \geq 0 \quad (4.107)$$

Equations (4.108) and (4.109) control the upper and lower limit of sulfur content.

$$(0.0075 * Y_{14} + 0.0078 * Y_{24} + 0.0085 * Y_{34} + 0.008 * Y_{44} + 0.009 * Y_{54}) -$$
$$0.01 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \leq 0 \qquad (4.108)$$

$$(0.0075 * Y_{14} + 0.0078 * Y_{24} + 0.0085 * Y_{34} + 0.008 * Y_{44} + 0.009 * Y_{54}) -$$
$$0.0075 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \geq 0 \qquad (4.109)$$

The moisture constraints for the power plant's coal are illustrated in equations (4.110) and (4.111). They deal with the respective upper and lower moisture content boundaries.

$$(0.132 * Y_{14} + 0.1375 * Y_{24} + 0.138 * Y_{34} + 0.135 * Y_{44} + 0.133 * Y_{54}) -$$
$$0.135 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \leq 0 \qquad (4.110)$$

$$(0.132 * Y_{14} + 0.1375 * Y_{24} + 0.138 * Y_{34} + 0.135 * Y_{44} + 0.133 * Y_{54}) -$$
$$0.13 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \geq 0 \qquad (4.111)$$

The fixed carbon constraint is illustrated in equation (4.112).

$$(0.51 * Y_{14} + 0.49 * Y_{24} + 0.485 * Y_{34} + 0.47 * Y_{44} + 0.465 Y_{54}) -$$
$$0.465 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \geq 0 \qquad (4.112)$$

The volatile matter content is controlled by equation (4.113), which limits this parameter to 36.2%.

$$(0.30 * Y_{14} + 0.35 * Y_{24} + 0.37 * Y_{34} + 0.352 * Y_{44} + 0.36 Y_{54}) -$$
$$0.362 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \leq 0 \qquad (4.113)$$

The energy content in the coal shipped to the power plants has to be higher than or equal to 26.1 [MJ/kg]. Equation (4.114) defines and controls this limitation.

$$(0.28 * Y_{14} + 0.275 * Y_{24} + 0.26 * Y_{34} + 0.27 * Y_{44} + 0.265Y_{54}) -$$

$$0.261 * (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \geq 0 \tag{4.114}$$

The non-negativity constraints are imposed on all the variables of the objective function. Equations (4.115) and (4.116) govern the non-negativity constraints of Luscar-Sherritt LP optimization model.

$$X_{11}; X_{13}; X_{21}; X_{22}; X_{23}; X_{31} X_{32}; X_{33}; X_{41}; X_{42}; X_{51}; X_{52}; X_{53} \geq 0 \tag{4.115}$$

$$Y_{11}; Y_{13}; Y_{14}; Y_{21}; Y_{22}; Y_{23}; Y_{24}; Y_{31}; Y_{32}; Y_{33}; Y_{34}; Y_{41}; Y_{42}; Y_{43}; Y_{44}; Y_{51}; Y_{52}; Y_{53}; Y_{54} \geq 0 \tag{4.116}$$

The LP variables represent the quantities of metallurgical and thermal coal produced and hauled by each mine. The total number of variables is 33. The total number of constraints imposed on the objective function is 132. These constraints consist of 14 supply and demand, 13 minimum and maximum capacities, 9 port and railway limitations, 63 blending, and 33 non-negativity equations. Excel-Slover is used to maximize the objective function illustrated in equation (15) subjected to the 132 limits constraint equations.

4.2.1.2 NLP Model

The objective function for the NLP model is, as defined by Everett (1963) based on Lagrange theory [8]. The generalized Lagrangian model is the difference between the profit function, illustrated by equation (4.13), and the constraint functions multiplied by some real positive numbers arbitrarily chosen. The goal of this subsection is to develop the mathematical equations of the constraints. The constraint equations for the NLP model are basically the same as for the LP model. The only difference is that their mathematical formulations have to be always less than or equal to zero to be consistent with Lagrange and GLM theorems. The market constraints for Luscar-Sherritt coal production are based on data from Table 4.2.5. Equations (4.117) through (4.121) govern the markets capacity limitations. Equations (4.117) and (4.118) control the metallurgical and thermal coal quantities that could be sent annually to Vancouver ports.

$$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} - 5{,}500{,}000 \leq 0 \quad (4.117)$$

$$Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} - 2{,}000{,}000 \leq 0 \quad (4.118)$$

Equation (4.119) and (4.120) control the metallurgical and thermal coal transported to Ridley Island for customers in Japan and South Korea.

$$X_{22} + X_{32} + X_{42} + X_{52} - 1{,}500{,}000 \leq 0 \quad (4.119)$$

$$Y_{22} + Y_{32} + Y_{42} + Y_{52} - 2{,}5000{,}000 \leq 0 \quad (4.120)$$

Equations (4.121) and (4.122) govern the respective metallurgical and thermal coal transported to Thunder Bay for eastern Canada customers. Equation (4.123) refers to domestic customers within Alberta.

$$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} - 1{,}500{,}000 \leq 0 \quad (4.121)$$

$$Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} - 2{,}500{,}000 \leq 0 \quad (4.122)$$

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} - 2{,}000{,}000 \leq 0 \quad (4.123)$$

The total contracted coal quantities per year are assumed to be 5.0 [Mt/y] metallurgical coal and 7.5 [Mt/y] of thermal coal distributed to the shipping ports and power plants as illustrated in Table 4.2.6. The constraint equations that govern the contracted amounts of coal for overseas markets are presented in (4.124) through (4.127). These equations require that the company produces and transport the required coal quantities to customers. The contracted amounts of metallurgical and thermal coal shipped to Vancouver ports are controlled by equation (4.124) and respective (4.125).

$$3{,}000{,}000 - (X_{11} + X_{21} + X_{31} + X_{41} + X_{51}) \leq 0 \tag{4.124}$$

$$2{,}000{,}000 - (Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51}) \leq 0 \tag{4.125}$$

Equations (4.126) and (4.127) set the amount of metallurgical and respective thermal coal to be larger than or equal to the contracted quantities for Ridley Island port.

$$1{,}000{,}000 - (X_{22} + X_{32} + X_{42} + X_{52}) \leq 0 \tag{4.126}$$

$$2{,}000{,}000 - (Y_{22} + Y_{32} + Y_{42} + Y_{52}) \leq 0 \tag{4.127}$$

Equations (4.128) and (4.129) set the amounts of metallurgical and thermal coal transported to Thunder Bay to be greater than or equal to the contract specifications. Equation (4.130) controls the thermal coal quantities contracted within Alberta.

$$1{,}000{,}000 - (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \leq 0 \tag{4.128}$$

$$2{,}500{,}000 - (Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53}) \leq 0 \tag{4.129}$$

$$1{,}500{,}000 - (Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54}) \leq 0 \tag{4.130}$$

The maximum mine capacities are assumed to be those illustrated in Table 4.2.7. Equations (4.131) through (4.135) limit the available resources for the optimization model to the assumed maximum capacities of Line Creek, Luscar, Cheviot, Coal Valley, and respective Obed Mountain.

$$X_{11} + X_{13} + Y_{11} + Y_{13} + Y_{14} - 4{,}000{,}000 \leq 0 \tag{4.131}$$

$$X_{21}+X_{22}+X_{23}+Y_{21}+Y_{22}+Y_{23}+Y_{24}-3{,}000{,}000 \leq 0 \quad (4.132)$$

$$X_{31}+X_{32}+X_{33}+Y_{31}+Y_{32}+Y_{33}+Y_{34}-4{,}000{,}000 \leq 0 \quad (4.133)$$

$$X_{41}+X_{42}+X_{43}+Y_{41}+Y_{42}+Y_{43}+Y_{44}-2{,}500{,}000 \leq 0 \quad (4.134)$$

$$X_{51}+X_{52}+X_{53}+Y_{51}+Y_{52}+Y_{53}+Y_{54}-2{,}000{,}000 \leq 2.0 \quad (4.135)$$

Total minimum capacities of mines are governed by equations (4.136) through (4.140) and are based on the values illustrated in Table 4.2.8. They force the total coal production at the mines to be larger than or equal to the minimum capacities presented in this table.

$$3{,}200{,}000-(X_{11}+X_{13}+Y_{11}+Y_{13}+Y_{14}) \leq 0 \quad (4.136)$$

$$2{,}500{,}000-(X_{21}+X_{22}+X_{23}+Y_{21}+Y_{22}+Y_{23}+Y_{24}) \leq 0 \quad (4.137)$$

$$3{,}200{,}000-(X_{31}+X_{32}+X_{33}+Y_{31}+Y_{32}+Y_{33}+Y_{34}) \leq 0 \quad (4.138)$$

$$1{,}500{,}000-(X_{41}+X_{42}+X_{43}+Y_{41}+Y_{42}+Y_{43}+Y_{44}) \leq 0 \quad (4.139)$$

$$1{,}500{,}000-(X_{51}+X_{52}+X_{53}+Y_{51}+Y_{52}+Y_{53}+Y_{54}) \leq 0 \quad (4.140)$$

Equations (4.141) to (4.143) set the metallurgical coal production at Line Creek, Luscar, and Cheviot to be greater than or equal to the minimum capacity usage requirement of the processing and upgrading facilities presented in Table 4.2.8.

$$2{,}500{,}000-(X_{11}+X_{13}) \leq 0 \quad (4.141)$$

$$2{,}000{,}000-(X_{21}+X_{22}+X_{23}) \leq 0 \quad (4.142)$$

$$2{,}500{,}000 - (X_{31} + X_{32} + X_{33}) \le 0 \tag{4.143}$$

The total coal railway capacity is assumed to be around 25.0 [Mt/y] from Sparwood to Vancouver, 15 [Mt/y] from Hinton-Grand Cache area to Vancouver, 10 [Mt/y] from Hinton-Grand Cache area to Ridley Island, and around 15.0 [Mt/y] to Thunder Bay as illustrated in Table 4.2.9. The rail capacity from Sparwood area to Vancouver allocated to Luscar-Sherritt coal production is around 5.5 [Mt/y]. This limitation applies to Line Creek mine only and is controlled by equation (4.144)

$$X_{11} + Y_{11} - 5{,}500{,}000 \le 0 \tag{4.144}$$

From Hinton-Grand Cache area to Vancouver ports, the railway capacity is around 13.5 [Mt/y]. This constraint refers to Luscar, Cheviot, Coal Valley, and Obed Mountain is governed by equation (4.145) and limits the coal quantities shipped on this route.

$$X_{21} + X_{31} + X_{41} + X_{51} + Y_{21} + Y_{31} + Y_{41} + Y_{51} - 13{,}500{,}000 \le 0 \tag{4.145}$$

From Hinton-Grand Cache to Ridley Island port, the coal railway capacity is around 8.5 [Mt/y]. This constraint concerns Luscar, Cheviot, Coal Valley and Obed Mountain. Equation (4.146) governs this limitation.

$$X_{22} + X_{32} + X_{42} + X_{52} + Y_{22} + Y_{32} + Y_{42} + Y_{52} - 8{,}500{,}000 \le 0 \tag{4.146}$$

From Sparwood area to Thunder Bay port the coal railway capacity is around 11.0 [Mt/y]. This constraint concerns Line Creek only and is controlled by equation (4.147).

$$X_{13} + Y_{13} - 11{,}000{,}000 \le 0 \tag{4.147}$$

From Hinton-Grand Cache to Thunder Bay port the coal railway capacity is 13.0 [Mt/y]. This constraint applies to Luscar, Cheviot, Coal Valley and Obed Mountain. Equation (4.148) governs the limitation.

$$X_{23}+X_{33}+X_{43}+X_{53}+Y_{23}+Y_{33}+Y_{43}+Y_{53}-13{,}000{,}000\leq 0 \qquad (4.148)$$

From the mine sites to power plants or domestic consumers within Alberta, the coal railway capacity is believed to be less than 5.0 [Mt/y]. Equation (4.149) controls this constraint.

$$Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54}-5{,}000{,}000\leq 0 \qquad (4.149)$$

Several coal producers and exporters use port facilities. Ports capacities are distributed to the users as illustrated in Table 4.2.10. Vancouver ports have an annual coal handling capacity of 33.0 [Mt] out of which 13.3 [Mt/y] is allocated to Luscar-Sherritt. Equation (4.150) governs the Vancouver port limitation

$$X_{11}+X_{21}+X_{31}+X_{41}+X_{51}+Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51}-13{,}300{,}000\leq 0 \qquad (4.150)$$

Ridley Island port has 16.0 [Mt] annual capacity, out of which 14.0 [Mt/y] are allocated to Luscar-Sherritt. Equation (4.151) limits the coal shipped through Ridley Island port.

$$X_{22}+X_{32}+X_{42}+X_{52}+Y_{22}+Y_{32}+Y_{42}+Y_{52}-14{,}000{,}000\leq 0 \qquad (4.151)$$

Thunder Bay has 10.0 [Mt] annual capacity, of which 6.0 [Mt/y] is allocated to Luscar-Sherritt. Equation (4.152) defines this limitation.

$$Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54}-2{,}000{,}000\leq 0 \qquad (4.152)$$

Blending constraints are based on Table 4.2.11 and Table 4.2.12. These constraints define the existing clean coal physical and chemical characteristics for each mine and the market requirements. From Figure 4.2.1, coal is shipped to four final destinations from five different mines. At each final destination, except power plants, the coal is stocked into two different stockpiles (metallurgical and thermal). For each of these stockpiles, a set of blending constraints has to be developed and input into the optimization model to make sure that the coal would meet the quality specifications. The quality of metallurgical coal stockpiled in Vancouver is governed by equations (4.153) through (4.161). The upper limit (8.5%) of ash content in metallurgical coal is defined by equation (4.153). Equation (4.154) governs the lower limit (8.0%) of the ash content. The ash content is defined within 8% - 8.5%.

$$(0.075*X_{11}+0.08*X_{21}+0.085*X_{31}+0.09*X_{41}+0.0875*X_{51})-$$
$$0.085*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})\leq 0 \quad (4.153)$$

$$0.08*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})-(0.075*X_{11}+0.08*X_{21}+$$
$$0.085*X_{31}+0.09*X_{41}+0.0875*X_{51})\leq 0 \quad (4.154)$$

The sulfur content is defined by equations (4.155) and (4.156) and is limited within 0.35% and 0.4%. Equation (4.155) controls the upper limit while (4.156) the lower.

$$(0.0035*X_{11}+0.0042*X_{21}+0.0038*X_{31}+0.0043*X_{41}+$$
$$0.004*X_{51})-0.004*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})\leq 0 \quad (4.155)$$

$$0.0035*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})-(0.0035*X_{11}+0.0042*X_{21}+$$
$$0.0038*X_{31}+0.0042*X_{41}+0.004*X_{51})\leq 0 \quad (4.156)$$

The moisture content is set between 7.5% and 8.0%. The upper boundary of moisture content is governed by equation (4.157) and the lower boundary by equation (4.158).

$$(0.082*X_{11}+0.0775*X_{21}+0.078X_{31}+0.085*X_{41}+0.083X_{51})-$$
$$0.08*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})\leq 0 \quad (4.157)$$

$$0.075*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})-(0.082*X_{11}+0.0775*X_{21}+$$
$$0.078*X_{31}+0.085*X_{41}+0.083*X_{51})\leq 0 \tag{4.158}$$

The fixed carbon content for metallurgical coal is required to be larger than 58%. Equation (4.159) sets the content of fixed carbon to be higher than or equal to this value.

$$0.58*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})-(0.60*X_{11}+0.595*X_{21}+$$
$$0.575*X_{31}+0.57*X_{41}+0.575*X_{51})\leq 0 \tag{4.159}$$

The volatile matter content is illustrated in equation (4.160) and is set to be less than or equal to 22.6%.

$$(0.21*X_{11}+0.22*X_{21}+0.225X_{31}+0.23*X_{41}+0.245X_{51})-$$
$$0.226*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})\leq 0 \tag{4.160}$$

The energy constraint is presented in equation (4.161). The equation requires that the energy content is larger or equal to 30.2 [MJ/kg].

$$0.302*(X_{11}+X_{21}+X_{31}+X_{41}+X_{51})-(0.32*X_{11}+0.31*X_{21}+$$
$$+0.295*X_{31}+0.29*X_{41}+0.292*X_{51})\leq 0 \tag{4.161}$$

The quality of thermal coal stockpiled in Vancouver is controlled by equations (4.162) through (4.170). The constraint equations are based on Table 4.2.12, which illustrate the thermal coal quality requirements and the qualities of cleaned thermal coal produced by each mine. Equation (4.162) governs the upper limit of the ash content in thermal coal and equation (4.163) the lower limit. The ash content is set between 12.5% and 13%.

$$(0.125*Y_{11}+0.131*Y_{21}+0.138*Y_{31}+0.12*Y_{41}+0.1275*Y_{51})-$$
$$0.13*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\leq 0 \tag{4.162}$$

$$0.125*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})-(0.125*Y_{11}+0.131*Y_{21}+$$
$$0.138*Y_{31}+0.12*Y_{41}+0.1275*Y_{51})\leq 0 \quad (4.163)$$

The sulfur content of thermal coal is set within 0.75% and 0.1%. Equation (4.164) controls the upper limit and (4.165) the lower boundary.

$$(0.0075*Y_{11}+0.0078*Y_{21}+0.0085*Y_{31}+0.008*Y_{41}+0.009*Y_{51})-$$
$$0.01*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\leq 0 \quad (4.164)$$

$$0.0075*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})-(0.0075*Y_{11}+0.0078*Y_{21}+$$
$$0.0085*Y_{31}+0.008*Y_{41}+0.009*Y_{51})\leq 0 \quad (4.165)$$

The moisture content is governed by equations (4.166) and (4.167). It is set between 13% and 13.5% boundaries. Equation (4.166) governs the upper boundary and (4.167) the lower boundary.

$$(0.132*Y_{11}+0.1375*Y_{21}+0.138*Y_{31}+0.135Y_{41}+0.133Y_{51})-$$
$$0.135*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\leq 0 \quad (4.166)$$

$$0.13*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})-(0.132*Y_{11}+0.1375*Y_{21}+$$
$$0.138*Y_{31}+0.135*Y_{41}+0.133*Y_{51})\leq 0 \quad (4.167)$$

The fixed carbon constraint is governed by equation (4.168). It must be higher than or equal to 46.5 %.

$$0.465*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})-(0.51*Y_{11}+0.49*Y_{21}+$$
$$0.485*Y_{31}+0.47*Y_{41}+0.465*Y_{51})\leq 0 \quad (4.168)$$

The volatile matter constraint is illustrated by equation (4.169). The content of volatile matter of thermal coal must be lower than 36.2%.

$$(0.30*Y_{11}+0.35*Y_{21}+0.37*Y_{31}+0.352Y_{41}+0.36Y_{51})-$$
$$0.362*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})\leq 0 \quad (4.169)$$

The energy content of thermal coal has to be larger than or equal to 26.1 [MJ/kg]. Equation (4.170) sets the energy content of thermal coal stockpiled in Vancouver port to meet this requirement.

$$0.261*(Y_{11}+Y_{21}+Y_{31}+Y_{41}+Y_{51})-(0.28*Y_{11}+0.275*Y_{21}+$$
$$0.26*Y_{31}+0.27*Y_{41}+0.265*Y_{51})\leq 0 \quad (4.170)$$

Ridley Island's blending stockpiles are based on the same data and have to meet the same quality requirements as Vancouver's blending stockpiles. The coal quality in the metallurgical blending stockpile is governed by equations (4.171) through (4.179). Equation (4.171) controls the upper limit of ash content and (4.172) the lower limit.

$$(0.08*X_{22}+0.085*X_{32}+0.09*X_{42}+0.0875*X_{52})-$$
$$0.085*(X_{22}+X_{32}+X_{42}+X_{52})\leq 0 \quad (4.171)$$

$$0.08*(X_{22}+X_{32}+X_{42}+X_{52})-(0.08*X_{22}+0.085*X_{32}+$$
$$0.09*X_{42}+0.0875*X_{52})\leq 0 \quad (4.172)$$

The content of sulfur in metallurgical coal is based on equations (4.173) and (4.174), which define the respective upper and lower limits.

$$(0.0042*X_{22}+0.0038*X_{32}+0.0043*X_{42}+0.004*X_{52})-$$
$$0.004*(X_{22}+X_{32}+X_{42}+X_{52})\leq 0 \quad (4.173)$$

$$0.0035*(X_{22}+X_{32}+X_{42}+X_{52})-(0.0042*X_{22}+0.0038*X_{32}+$$
$$0.0043*X_{42}+0.004*X_{52})\leq 0 \tag{4.174}$$

The moisture constraints are illustrated by equations (4.175) and (4.176), which control the respective upper and lower limit content.

$$(0.0775*X_{22}+0.078*X_{32}+0.085*X_{42}+0.083*X_{52})-$$
$$0.08*(X_{22}+X_{32}+X_{42}+X_{52})\leq 0 \tag{4.175}$$

$$0.075*(X_{22}+X_{32}+X_{42}+X_{52})-(0.0775*X_{22}+0.078*X_{32}+$$
$$0.085*X_{42}+0.083*X_{52})\leq 0 \tag{4.176}$$

The fixed carbon constraint is defined by equation (4.177) and sets it to be higher than or equal to 58%.

$$0.58*(X_{22}+X_{32}+X_{42}+X_{52})-(0.595*X_{22}+0.575*X_{32}+$$
$$0.57*X_{42}+0.575*X_{52})\leq 0 \tag{4.177}$$

The volatile matter constraint is governed by equation (4.178).

$$(0.22*X_{22}+0.225*X_{32}+0.23*X_{42}+0.245*X_{52})-$$
$$0.226*(X_{22}+X_{32}+X_{42}+X_{52})\leq 0 \tag{4.178}$$

The energy content of metallurgical coal in the Ridley Island's stockpile is controlled by equation (4.179).

$$0.302*(X_{22}+X_{32}+X_{42}+X_{52})-(0.31*X_{22}+0.295*X_{32}+$$
$$0.29*X_{42}+0.292*X_{52})\leq 0 \tag{4.179}$$

The quality of coal in the thermal blending stockpile in Ridley Island port is controlled by equations (4.180) to (4.188). The constraint equations are based on data in Table 4.2.12. Equations (4.180) and (4.181) define the respective upper and lower boundaries of ash content requirements.

$$(0.131 * Y_{22} + 0.138 * Y_{32} + 0.12 * Y_{42} + 0.1275 * Y_{52}) -$$
$$0.13 * (Y_{22} + Y_{32} + Y_{42} + Y_{52}) \leq 0 \quad (4.180)$$

$$0.125 * (Y_{22} + Y_{32} + Y_{42} + Y_{52}) - (0.131 * Y_{22} + 0.138 * Y_{32} +$$
$$0.12 * Y_{42} + 0.1275 * Y_{52}) \leq 0 \quad (4.181)$$

The sulfur constraints are illustrated in equations (4.182) and (4.183), which govern the upper and lower limits of sulfur content.

$$(0.0078 * Y_{22} + 0.0085 * Y_{32} + 0.008 * Y_{42} + 0.009 * Y_{52}) -$$
$$0.01 * (Y_{22} + Y_{32} + Y_{42} + Y_{52}) \leq 0 \quad (4.182)$$

$$0.0075 * (Y_{22} + Y_{32} + Y_{42} + Y_{52}) - (0.0078 * Y_{22} + 0.0085 * Y_{32} +$$
$$0.008 * Y_{42} + 0.009 * Y_{52}) \leq 0 \quad (4.183)$$

The moisture content is controlled by equations (4.184) and (4.185), which define the upper and lower boundaries.

$$(0.1375 * Y_{22} + 0.138 * Y_{32} + 0.135 * Y_{42} + 0.133 * Y_{52}) -$$
$$0.135 * (Y_{22} + Y_{32} + Y_{42} + Y_{52}) \leq 0 \quad (4.184)$$

$$0.13 * (Y_{22} + Y_{32} + Y_{42} + Y_{52}) - (0.1375 * Y_{22} + 0.138 * Y_{32} + 0.135 * Y_{42} +$$
$$0.133 * Y_{52}) \leq 0 \quad (4.185)$$

The fixed carbon constraint is presented in equation (4.186).

$$0.465*(Y_{22}+Y_{32}+Y_{42}+Y_{52})-(0.49*Y_{22}+0.485*Y_{32}+0.47*Y_{42}+ 0.465*Y_{52})\leq 0 \qquad (4.186)$$

The volatile matter content is governed by equation (4.187)

$$(0.35*Y_{22}+0.37*Y_{32}+0.352*Y_{42}+0.36*Y_{52})-0.362*(Y_{22}+Y_{32}+ Y_{42}+Y_{52})\leq 0 \qquad (4.187)$$

The energy content of Ridley Island's thermal coal stockpile is defined by equation (4.188).

$$0.261*(Y_{22}+Y_{32}+Y_{42}+Y_{52})-(0.275*Y_{22}+0.26*Y_{32}+0.27*Y_{42}+ 0.265*Y_{52})\leq 0 \qquad (4.188)$$

Thunder Bay's metallurgical and thermal blending stockpiles follow the same mathematical models of Vancouver and Ridley Islands. The quality of coal in the metallurgical stockpile is controlled by equations (4.189) to (4.197), which are based on data from Table 4.2.11. Equations (4.189) and (4.190) illustrate the ash constraints for Thunder Bay's metallurgical stockpile. They define the upper and the lower boundaries of the ash content requirements.

$$(0.075*X_{13}+0.08*X_{23}+0.085*X_{33}+0.09*X_{43}+0.0875*X_{53})- 0.085*(X_{13}+X_{23}+X_{33}+X_{43}+X_{53})\leq 0 \qquad (4.189)$$

$$0.08*(X_{13}+X_{23}+X_{33}+X_{43}+X_{53})-(0.075*X_{13}+0.08*X_{23}+ 0.085*X_{33}+0.09*X_{43}+0.0875*X_{53})\leq 0 \qquad (4.190)$$

The sulfur constraints are presented in equations (4.191) and (4.192), which control the respective upper and lower limits of this parameter.

$$(0.0035 * X_{13} + 0.0042 * X_{23} + 0.0038 * X_{33} + 0.0043 * X_{43} + 0.004 * X_{53}) - 0.004 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \leq 0 \quad (4.191)$$

$$0.0035 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) - (0.0035 * X_{13} + 0.0042 * X_{23} + 0.0038 * X_{33} + 0.0043 * X_{43} + 0.004 * X_{53}) \leq 0 \quad (4.192)$$

The upper and lower contents of moisture are governed by equation (4.193) and (4.194).

$$(0.082 * X_{13} + 0.0775 * X_{23} + 0.078 * X_{33} + 0.085 * X_{43} + 0.083 * X_{53}) - 0.08 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \leq 0 \quad (4.193)$$

$$0.075 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) - (0.082 * X_{13} + 0.0775 X_{23} + 0.078 * X_{33} + 0.085 * X_{43} + 0.083 * X_{53}) \leq 0 \quad (4.194)$$

Equation (4.195) controls the content of fixed carbon.

$$0.58 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) - (0.60 * X_{13} + 0.595 X_{23} + 0.575 * X_{33} + 0.57 * X_{43} + 0.575 * X_{53}) \leq 0 \quad (4.195)$$

The volatile matter constraint is illustrated in equation (4.196).

$$(0.21 * X_{13} + 0.22 * X_{23} + 0.225 * X_{33} + 0.23 * X_{43} + 0.245 * X_{53}) - 0.226 * (X_{13} + X_{23} + X_{33} + X_{43} + X_{53}) \leq 0 \quad (4.196)$$

Equation (4.197) governs the content of energy in the metallurgical coal stockpiled in Thunder Bay.

$$0.302*(X_{13}+X_{23}+X_{33}+X_{43}+X_{53})-(0.32*X_{13}+0.31X_{23}+$$

$$0.295*X_{33}+0.29*X_{43}+0.292*X_{53})\leq 0 \qquad (4.197)$$

The mathematical equations, which define thermal coal qualities for the Thunder Bay stockpiles, are based on data in Table 4.2.12. These equations are illustrated in (4.198) through (4.206). Equations (4.198) and (4.199) define the respective upper and lower limits of ash content.

$$(0.125*Y_{13}+0.131*Y_{23}+0.138*Y_{33}+0.12*Y_{43}+0.1275*Y_{53})-$$

$$0.13*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\leq 0 \qquad (4.198)$$

$$0.125*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})+(0.125*Y_{13}+0.131*Y_{23}+$$

$$0.138*Y_{33}+0.12*Y_{43}+0.1275*Y_{53})\leq 0 \qquad (4.199)$$

The sulfur constraints are presented in equation (4.200) and (4.201) and define the respective upper and the lower limits of sulfur content.

$$(0.0075*Y_{13}+0.0078*Y_{23}+0.0085*Y_{33}+0.008*Y_{43}+0.009*Y_{53})-$$

$$0.01*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\leq 0 \qquad (4.200)$$

$$0.0075*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})+(0.0075*Y_{13}+0.0078*Y_{23}+$$

$$0.0085*Y_{33}+0.008*Y_{43}+0.009*Y_{53})\leq 0 \qquad (4.201)$$

The moisture constraints, illustrated by equations (4.202) and (4.203), govern the content of this parameter in the Thunder Bay thermal coal stockpile.

$$(0.132*Y_{13}+0.1375*Y_{23}+0.138*Y_{33}+0.135*Y_{43}+0.133*Y_{53})-$$

$$0.135*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\leq 0 \qquad (4.202)$$

$$0.13*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})+(0.132*Y_{13}+0.1375*Y_{23}+$$
$$0.138*Y_{33}+0.135*Y_{43}+0.133*Y_{53})\leq 0 \qquad (4.203)$$

The fixed carbon constraint is presented in equation (4.204).

$$0.465*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})+(0.51*Y_{13}+0.49*Y_{23}+$$
$$0.485*Y_{33}+0.47*Y_{43}+0.465*Y_{53})\leq 0 \qquad (4.204)$$

The volatile matter content is controlled by equation (4.205).

$$(0.30*Y_{13}+0.35*Y_{23}+0.37*Y_{33}+0.352*Y_{43}+0.36*Y_{53})-$$
$$0.362*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})\leq 0 \qquad (4.205)$$

The energy content of the thermal coal stockpiled in Thunder Bay has to be greater than or equal to 26.1 [My/kg]. Equation (4.206) governs this particular parameter.

$$0.261*(Y_{13}+Y_{23}+Y_{33}+Y_{43}+Y_{53})+(0.28*Y_{13}+0.275*Y_{23}+0.26*Y_{33}+$$
$$0.27*Y_{43}+0.265*Y_{53})\leq 0 \qquad (4.206)$$

TransAlta's power plants stockpile consists of only thermal coal. Equations (4.207) through (4.215) govern the coal quality and are based on data from Table 4.2.12. Ash constraints, for the power plant coal, are presented in equation (4.207) and (4.208), which represent the respective upper and lower limit of ash content.

$$(0.125*Y_{14}+0.131*Y_{24}+0.138*Y_{34}+0.12*Y_{44}+0.1275Y_{54})-$$
$$0.13*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})\leq 0 \qquad (4.207)$$

$$0.125*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})-(0.125*Y_{14}+0.131*Y_{24}+$$
$$0.138*Y_{34}+0.12*Y_{44}+0.1275*Y_{54})\leq 0 \qquad (4.208)$$

Equations (4.209) and (4.210) control the upper and lower limit of sulfur content.

$$(0.0075*Y_{14}+0.0078*Y_{24}+0.0085*Y_{34}+0.008*Y_{44}+0.009*Y_{54})-$$
$$0.01*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})\leq 0 \qquad (4.209)$$

$$0.0075*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})-(0.0075*Y_{14}+0.0078*Y_{24}+$$
$$0.0085*Y_{34}+0.008*Y_{44}+0.009*Y_{54})\leq 0 \qquad (4.210)$$

The moisture constraints for the power plant's coal are illustrated in equations (4.211) and (4.212). They deal with the upper and lower moisture content boundaries.

$$(0.132*Y_{14}+0.1375*Y_{24}+0.138*Y_{34}+0.135*Y_{44}+0.133*Y_{54})-$$
$$0.135*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})\leq 0 \qquad (4.211)$$

$$0.13*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})-(0.132*Y_{14}+0.1375*Y_{24}+$$
$$0.138*Y_{34}+0.135*Y_{44}+0.133*Y_{54})\leq 0 \qquad (4.212)$$

The fixed carbon constraint is illustrated in equation (4.213).

$$0.465*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})-(0.51*Y_{14}+0.49*Y_{24}+$$
$$0.485*Y_{34}+0.47*Y_{44}+0.465*Y_{54})\leq 0 \qquad (4.213)$$

The volatile matter content is controlled by equation (4.214), which limits this parameter to 36.2%.

$$(0.30*Y_{14}+0.35*Y_{24}+0.37*Y_{34}+0.352*Y_{44}+0.36Y_{54})-$$
$$0.362*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})\leq 0 \qquad (4.214)$$

The energy content in the coal transported to the power plants has to be higher than or equal to 26.1 [MJ/kg]. Equation (4.215) defines and controls this limitation.

$$0.261*(Y_{14}+Y_{24}+Y_{34}+Y_{44}+Y_{54})-(0.28*Y_{14}+0.275*Y_{24}+ 0.26*Y_{34}+0.27*Y_{44}+0.265*Y_{54})\leq 0 \quad (4.215)$$

Non-negativity constraints have to be imposed for the NLP optimization model as well. Equations (4.216) and (4.217) govern the non-negativity constraints.

$$X_{11};X_{13};X_{21};X_{22};X_{23};X_{31}X_{32};X_{33};X_{41};X_{42};X_{51};X_{52};X_{53}\geq 0 \quad (4.216)$$

$$Y_{11};Y_{13};Y_{14};Y_{21};Y_{22};Y_{23};Y_{24};Y_{31};Y_{32};Y_{33};Y_{34};Y_{41};Y_{42};Y_{43};Y_{44};Y_{51};Y_{52};Y_{53};Y_{54}\geq 0 \quad (4.217)$$

The objective function for the NLP model consists of the profit function (4.15) minus all the constraint functions multiplied by some real parameters. To simplify the Lagrange function, the Lagrange multipliers will be represented as λ^k ($k=1,99$) and the constraint functions by the constraint's equation numbers (4.117; 4.118; ... ; 4.214; 4.215). The NLP objective function is illustrated by equation (4.218). The equation is input into a spreadsheet and optimized using Excel-Solver with a non-linear option.

$$L_f=(4.15)-[\lambda^1*(4.117)+\lambda^2*(4.118)+...+\lambda^{99}*(4.215)] \quad (4.218)$$

(4.15) – represents the profit function, (4.117); (4.118); ... ;(4.214); (4.215) - represent the constraint functions.

4.2.2 Case Study II: Fording Coal Ltd.

Fording Coal Ltd. is Canada's largest export-coal producer and the world's second largest exporter of coking coal (after BHP Billiton) [9, 36]. It has the capacity to supply more than 20 million tonnes of metallurgical and thermal coal products to world and Canadian markets. The Company's economic reserves of high-quality coal include over a billion tonnes in proven and probable reserves. Fording coal operates mines for coking coal in British Columbia and for thermal coal in Alberta. Metallurgical coal accounts for about 85% of Fording's sales and its major customers include steelmaking companies from Japan, South Korea, Europe, South America and SUA. Thermal coal will be delivered to the Brooks power plant project, 180 km southeast of Calgary. The coal mines that Fording Coal operates are located in southeast of British Columbia and these include Fording River, Greenhills, and Coal Mountain. All these mines are linked by railway systems to Vancouver and Thunder Bay ports [9].

To optimize the coal production and haulage of Fording mines a LP and NLP were developed. It is assumed that from each mine a certain amount would be metallurgical coal expressed by the variables; X_{11} and X_{12} produced by Fording River and transported to Vancouver and Thunder Bay; X_{21} and X_{22} produced by Greenhills and transported to Vancouver and Thunder Bay; $X_{31}$and X_{32} produced by Coal Mountain and transported to Vancouver and Thunder Bay. It is also assumed that from each mine a certain amount would be thermal coal expressed by the variables; Y_{11}, Y_{12}, and Y_{13} produced by Fording River and transported to Vancouver, Thunder Bay and to Brooks' power plant; Y_{21}, Y_{22}, and Y_{23} produced by Greenhills and transported to Vancouver, Thunder Bay and to Brooks' power plant; Y_{31}, Y_{32}, and Y_{33} produced at Coal Mountain and transported to Vancouver, Thunder Bay and to Brooks' power plant. These production and transport dynamics are illustrated in Figure 4.2.2.1. The total amount of metallurgical coal (Q1) produced by Fording is governed by equation (4.219). Equation (4.220) governs the total quantity of thermal coal (Q2).

$$Q_1 = X_{11} + X_{12} + X_{21} + X_{22} + X_{31} + X_{32} \qquad (4.219)$$

$$Q_2 = Y_{11} + Y_{12} + Y_{13} + Y_{21} + Y_{22} + Y_{23} + Y_{31} + Y_{32} + Y_{33} \qquad (4.220)$$

Equations (4.221) through (4.222) illustrate the total coal production at each particular mine. P_{FR}, P_G, and P_{CM} represent total coal production at Fording River, Greenhills, and Coal Mountain.

$$P_{FR} = X_{11} + X_{12} + Y_{11} + Y_{12} + Y_{13} \tag{4.221}$$

$$P_G = X_{21} + X_{22} + Y_{21} + Y_{22} + Y_{23} \tag{4.222}$$

$$P_{CM} = X_{31} + X_{32} + Y_{31} + Y_{32} + Y_{33} \tag{4.223}$$

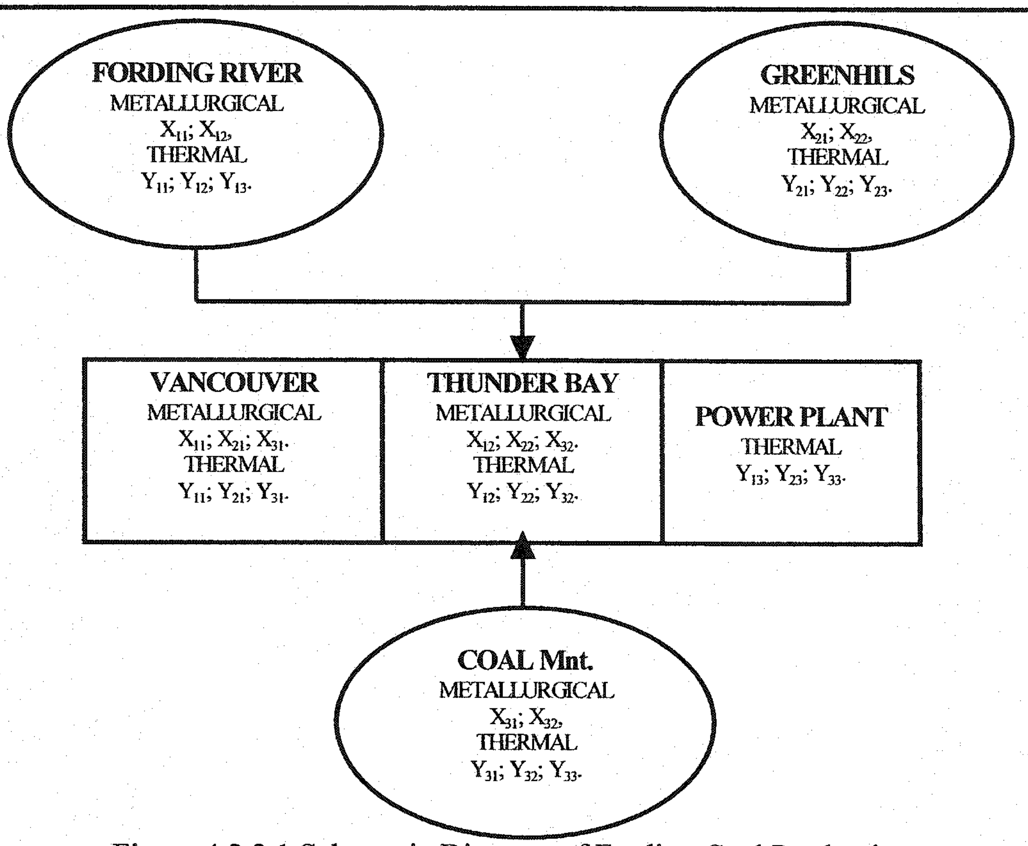


Figure 4.2.2.1 Schematic Diagram of Fording Coal Production

The profit of the company is calculated as difference between total revenue and total cost as illustrated in equation (4.224).

$$P_t = R - T_C \tag{4.224}$$

The total revenue is a function of the revenues generated by each mine, which is expressed in equation (4.225). M_p represents the price of metallurgical coal, and T_p is the price of thermal coal. The average coal price is assumed to be $54.0/tonne for metallurgical coal and $ 37.5/tonne for thermal coal based on various sources [3, 16, 36].

$$R = Q_1 * M_P + Q_2 * T_P \qquad (4.225)$$

The total cost of the company is expressed in equation (4.226), which contains the mining and processing cost of metallurgical coal (MP_m), mining and processing cost of thermal coal (MP_t), transportation cost (T_{rt}), port charges (P_c), and overhead cost (OVH_c). The mining and processing costs are different for each mine, as illustrated in Table 4.2.2.1 [16].

$$T_C = MP_m * Q_1 + MP_t * Q_2 + (T_{rt} + P_c + OVH_c) * (Q_1 + Q_2) \qquad (4.226)$$

Table 4.2.2.1 Fording Coal's Mining and Processing Costs

Mine	**Production and Processing Cost [$/t]**	
	Metallurgical	Thermal
Fording River	18.5	15.5
Greenhills	19	16
Coal Mountain	18.75	15.75

The total railway transportation cost (T_{rt}) is governed by equation (4.227). T_{rt} is a function of the mine port distances (d_{ij}), railway fees (k_v, k_t, and k_p) and the respective quantities of coal transported on each route ($x_{11,\ldots,}y_{33}$). k_v, k_t, and k_p represent the railway fees from the mine sites to Vancouver and Thunder Bay ports and to Brooks' power plant. The distances and fees are presented in Table 4.2.2.2 [9, 16, 22, 38,].

$$T_{rt} = k_v * [d_{11} * (X_{11} + Y_{11}) + d_{21} * (X_{21} + Y_{21}) + d_{31} * (X_{31} + Y_{31})] +$$
$$k_t * [d_{12} * (X_{12} + Y_{12}) + d_{22}(X_{22} + Y_{22}) + d_{32} * (X_{32} + Y_{32})] +$$
$$k_p * [d_{13} * Y_{13} + d_{23} * Y_{23} + d_{33} * Y_{33}] \qquad (4.227)$$

Table 4.2.2.2 Fording Coal's Mine-Port Distances and Railway Charges

Mine	Vancouver Dist. [Km]	Rail Cost [$/Km-t]	Thund. Bay Dist. [Km]	Rail Cost [$/Km-t]	Pw. Plant Dist. [Km]	Rail Cost [$/Km-t]
Fording River	1160	0.013	2125	0.008	175	0.05
Greenhills	1155	0.013	2120	0.008	170	0.05
Coal Mountain	1150	0.013	2080	0.008	150	0.05

Total port cost is governed by equation (4.228) and is a function of charges at each individual port. V_{pc}, TB_{pc}, and PW_c represent the charges at Vancouver and Thunder Bay ports and respective power plant. Port charges are illustrated in Table 4.2.2.3 [16, 38].

$$P_c = V_{pc} * (X_{11} + X_{21} + X_{31} + Y_{11} + Y_{21} + Y_{31}) + TB_{PC} * (X_{12} + X_{22} + X_{32} + Y_{12} + Y_{22} + Y_{32}) + PW_C * (Y_{13} + Y_{23} + Y_{33}) \quad (4.228)$$

Table 4.2.2.3 Fording Coal's Port and Power Plant Charges

Mine	Port Costs [$/t]		Power Plant
	Vancouver	Thunder Bay	Cost [$ / t]
Fording River	3.75	3.50	1.50
Greenhills	3.75	3.50	1.50
Coal Mountain	3.75	3.50	1.50

The Overhead Cost is function of the individual fixed costs and the amount of coal produced at each mine and is governed by equation (4.229). M_1, M_2, and M_3 represent the fixed costs per tonne at Fording River, Greenhills and respective Coal Mountain. The overhead costs are presented in Table 4.2.2.4.

$$OVH_C = M_1 * (X_{11} + X_{12} + Y_{11} + Y_{12} + Y_{13}) + M_2 * (X_{21} + X_{22} + Y_{21} + Y_{22} + Y_{23}) + M_3 * (X_{31} + X_{32} + Y_{31} + Y_{32} + Y_{33}) \quad (4.229)$$

Table 4.2.2.4 Fording Coal's Overhead Costs by Mine

Mine	Overhead Cost [$ /t]
Fording River	0.15
Greenhills	0.11
Coal Mountain	0.12

By substituting equations (4.226), (4.227) (4.228) and (4.229) in (4.224) and (4.225) the profit function takes the form of equation (4.230).

$$P_t = Q_1 * M_p + Q_2 * T_p - MP_m * Q_1 + MP_t * Q_2 + (T_n + P_c + OVH_c) * (Q_1 + Q_2) \qquad (4.230)$$

By replacing all the coefficients in the profit function with their values from Tables 4.2.2.1 through 4.2.2.4 results in a function in x_{ij} and y_{ij} variables and some A_{ij} and B_{ij} known coefficients as illustrated in equation (4.231).

$$P_t = A_{11} * X_{11} + A_{12} * X_{12} + ... + A_{32} * X_{32} + B_{11} * Y_{11} + ... + B_{33} * Y_{33} \qquad (4.231)$$

The profit function from equation (4.231) and all the specific costs, revenues, coal quantities, and other pertinent data are input in an Excel spreadsheet for calculation. The next step is to develop the optimization models (LP and NLP) into a spreadsheet. The optimization model consists the objective function represented by equation (4.231) and the underlying field constraints: supply and demand, mine capacities, railway capacities, port limitations, and coal quality.

4.2.2.1 LP Model

The objective function of the LP model is the profit function, presented in equation (4.231). The next step in the LP model is to develop the constraint equations that the objective function is subjected to. Market constraints for Fording coal production are based on data from Table 4.2.2.5, data, which is assumed, based on current sales and on the coal markets analyses presented in Appendix 1.0 and 2.0.

Table 4.2.2.5 Fording Coal's Market Limitations by Destination

Destination	Metallurgical [Mt/y]	Thermal [Mt/y]
Vancouver	≤ 12.5	≤ 2.5
Thunder Bay	≤ 1.25	≤ 1.0
Brooks power plant	-	≤ 1.25
Total	**≤ 13.75**	**≤ 4.75**

Equations (4.232) through (4.236) govern the markets capacity limitations. Equations (4.232) and (4.233) control the metallurgical and thermal coal quantities sold annually in Vancouver ports.

$$X_{11} + X_{21} + X_{31} \leq 12.5 \text{ [Mt/y]} \tag{4.232}$$

$$Y_{11} + Y_{21} + Y_{31} \leq 2.5 \text{ [Mt/y]} \tag{4.233}$$

Equations (4.234) and (4.235) limit the quantities of metallurgical and respective thermal coal transported annually to Thunder Bay to the market capacity of absorption.

$$X_{12} + X_{22} + X_{32} \leq 1.25 \text{ [Mt/y]} \tag{4.234}$$

$$Y_{12} + Y_{22} + Y_{32} \leq 1.0 \text{ [Mt/y]} \tag{4.235}$$

Equation (4.236) controls the limitation of thermal coal market within Alberta.

$$Y_{13} + Y_{23} + Y_{33} \leq 1.25 \text{ [Mt/y]} \tag{4.236}$$

It is assumed that Fording Coal has contracted out most of its coal production. The contracted coal quantities per year and destination are illustrated in Table 4.2.2.6.

Table 4.2.2.6 Fording Coal's Contractual Obligations by Destination

Destination	**Metallurgical [Mt/y]**	**Thermal [Mt/y]**
Vancouver	10.5	2.0
Thunder Bay	1.0	1.0
Brooks power plants	-	1.0
Total	**11.5**	**4.0**

Equations (4.237) through (4.241) control the contractual constraints, which are based on data from Table 4.2.2.6. Equations (4.237) and (4.238) set the amounts of metallurgical and thermal coal shipped to Vancouver to be larger than or equal to the quantities specified by the contract.

$$X_{11} + X_{21} + X_{31} \geq 10.5 \text{ [Mt/y]} \tag{4.237}$$

$$Y_{11} + Y_{21} + Y_{31} \geq 2.0 \text{ [Mt/y]} \tag{4.238}$$

Equations (4.239) and (4.240) control the quantities of metallurgical and thermal coal transported to Thunder Bay as contractual obligations.

$$X_{12} + X_{22} + X_{23} \geq 1.0 \text{ [Mt/y]} \tag{4.239}$$

$$Y_{12} + Y_{22} + Y_{23} \geq 1.0 \text{ [Mt/y]} \tag{4.240}$$

The annual quantity of contracted thermal coal transported to Brooks is governed by equation (4.241)

$$Y_{13} + Y_{23} + Y_{33} \geq 1.0 \text{ [Mt/y]} \tag{4.241}$$

It is assumed that Fording Coal's mine operations have their maximum annual production capacity as illustrated in Table 4.2.2.7. These assumptions were made based on mine's current level of production and their assets capabilities. The resources available for the optimization model will be limited to the values presented in this Table 4.2.2.7.

Table 4.2.2.7 Fording Coal's Mine Annual Maximum Capacities

Mine	**Maximum Capacity**
Fording River	10.5
Greenhills	5.5
Coal Mountain	3.5
Total	**19.5**

Equations (4.242), (4.243), and (4.244) illustrate the constraints that deal with maximum mine capacities at Fording River, Greenhills and Coal Mountain.

$$X_{11} + X_{12} + Y_{11} + Y_{12} + Y_{13} \leq 10.5 \text{ [Mt/y]} \quad (4.242)$$

$$X_{21} + X_{22} + Y_{21} + Y_{22} + Y_{23} \leq 5.5 \text{ [Mt/y]} \quad (4.243)$$

$$X_{31} + X_{32} + Y_{31} + Y_{32} + Y_{33} \leq 3.5 \text{ [Mt/y]} \quad (4.244)$$

Similar to Luscar-Sherritt case, to guarantee that no producing and processing facility works below its economic efficiency capacity, a minimum capacity usage requirement is set for each mine. The minimum capacity requirements, for the three mines, are presented in Table 4.2.2.8.

Table 4.2.2.8 Fording Coal's Mine Minimum Capacities Requirements

Mine	**Total [Mt/y]**	**Metallurgical [Mt/y]**
Fording River	9.0	7.5
Greenhills	4.0	3.2
Coal Mountain	2.0	1.85
Total	**15.0**	**12.55**

Equations (4.245) through (4.250) set the metallurgical and the total quantities of coal produced by each mine to be larger than or equal to the values presented in Table 4.2.2.8. Equation (4.245) and (4.246) control coal production at Fording River.

$$X_{11} + X_{12} + Y_{11} + Y_{12} + Y_{13} \geq 9.0 \text{ [Mt/y]} \quad (4.245)$$

$$X_{11} + X_{12} \geq 7.5 \text{ [Mt/y]} \quad (4.246)$$

Greenhills' minimum capacity requirements are governed by equations (4.247) and (4.248)

$$X_{21} + X_{22} + Y_{21} + Y_{22} + Y_{23} \geq 4.0 \text{ [Mt/y]} \tag{4.247}$$

$$X_{21} + X_{22} \geq 3.2 \text{ [Mt/y]} \tag{4.248}$$

Coal Mountain's total and metallurgical coal minimum capacity requirements are defined by equations (4.249) and (4.250).

$$X_{31} + X_{32} + Y_{31} + Y_{32} + Y_{33} \geq 2.0 \text{ [Mt/y]} \tag{4.249}$$

$$X_{31} + X_{32} \geq 1.85 \text{ [Mt/y]} \tag{4.250}$$

Similar to Luscar-Sherritt case, coal production from Fording Coal is hauled by trains to the final destinations. Fording Coal shares the railways capacity with other western Canadian coal producers. Total railway capacity from Sparwood area to Vancouver is assumed to be around 25.0 [Mt/y], out of which 7.0 [Mt/y] is taken by Luscar-Sherritt and Teck, and the remaining capacity of 19.0 [Mt/y] is allocated to Fording Coal [3, 36]. Equation (4.251) limits the amount of coal that Fording mines could produce and ship to Vancouver to 19.0 [Mt/y].

$$X_{12} + X_{21} + X_{31} + Y_{11} + Y_{21} + Y_{31} \leq 19.0 \text{ [Mt/y]} \tag{4.251}$$

From Sparwood area to Thunder Bay port, the coal railway capacity is assumed to be around 15.0 [Mt/y] of which 4.0 [Mt/y] is taken by Luscar-Sherritt and Teck, and the remaining 11.0 [Mt/y] are allocated to Fording Coal [3, 36]. This constraint is illustrated by equation (4.252).

$$X_{12} + X_{22} + X_{32} + Y_{12} + Y_{22} + Y_{32} \leq 11.0 \text{ [Mt/y]} \tag{4.252}$$

Port Capacities could limit the amounts of coal produced and sold by Fording Coal overseas. Vancouver ports have an annual coal handling capacity of 33.0 [Mt]. From these capacities, 7.5 [Mt/y] is taken by Luscar and 4.0 [Mt/y] by Teck, the remaining 21.5 [Mt/y] is allocated to Fording Coal [3, 36]. Equation (4.253) governs the Vancouver port limitation.

$$X_{11} + X_{21} + X_{31} + Y_{11} + Y_{21} + Y_{31} \leq 21.5 \text{ [Mt/y]} \tag{4.253}$$

Thunder Bay port has 10.0 [Mt] annual capacity, out of which 2.0 [Mt/y] are taken by Bienfait and 2.0 [Mt/y] by Luscar and Teck, the remaining 6.0 [Mt/y] is allocated Fording Coal [36]. This limitation is defined by equation (4.254).

$$X_{12} + X_{22} + X_{32} + Y_{12} + Y_{22} + Y_{32} \leq 6.0 \text{ [Mt/y]} \tag{4.254}$$

Blending constraints determine the quality of final coal products and the quantities of coal transported from each mine to the final destinations. The quality of coal is determined by the percentage of ash, sulfur, fixed carbon, volatile matter, and moisture and by the content of energy. All these parameters must be within an acceptable range, which could be required by the market, set in the contracts or imposed by the producer. It is assumed that the existing clean coal physical and chemical characteristics at each mine are as in Tables 4.2.2.9 and 4.2.2.10. All the values from these tables are assumed, based on geological data and ranges of coal quality requirements presented in Appendix 2.0 [3, 9, 31].

Table 4.2.2.9 Fording Mines' Metallurgical Coal Characteristics and Requirements

Metallurgical	Ash [%]	Sulfur [%]	Moisture [%]	Fixed C. [%]	Volatile M.[%]	MJ/kg
Upper Limit	8.5	0.4	8.0	58	22.6	30.2
Lower Limit	8.0	0.35	7.5	-	-	-
Existent (Clean Coal)						
Fording River	7.75	0.35	7.5	60.0	21.0	32.0
Greenhills	8.0	0.42	7.75	59.5	22.0	31.0
Coal Mountain	9.0	0.4	8.5	57.5	22.5	29.5

Table 4.2.2.10 Fording Mines' Thermal Coal Characteristics and Requirements

Thermal	Ash [%]	Sulfur [%]	Moisture [%]	Fixed C. [%]	Volatile M.[%]	MJ/kg
Upper Limit	13.0	1.0	13.5	46.5	36.2	26.1
Lower Limit	12.5	0.75	0.13	-	-	-
Existent (Clean Coal)						
Fording River	12.25	0.7	13.2	51.0	30.0	28.0
Greenhills	13.1	0.85	13.75	49.0	35.0	27.5
Coal Mountain	13.8	0.8	13.8	48.5	37.0	26.0

At each of the final destination, except the power plant, the coal is stocked into two different stockpiles (metallurgical and thermal). Each of these stockpiles has to meet the required coal specifications. For each of these stockpiles a set of blending constraints has to be developed and input into the optimization model to make sure that the coal would meet the quality specifications. Based on data from Tables 4.2.2.9 and 4.2.2.10, the blending constraints are generated as weighted averages for each stockpile. The quality of metallurgical coal stockpiled in Vancouver is governed by equations (4.255) through (4.263). The content of ash in metallurgical coal will be controlled by equation (4.255) and (4.256), which govern the respective upper (8.5%) and the lower limit (8.0%) of this parameter.

$$(0.0775 * X_{11} + 0.08 * X_{21} + 0.09 * X_{31}) - 0.085 * (X_{11} + X_{21} + X_{31}) \leq 0 \qquad (4.255)$$

$$(0.0775 * X_{11} + 0.08 * X_{21} + 0.09 * X_{31}) - 0.08 * (X_{11} + X_{21} + X_{31}) \geq 0 \qquad (4.256)$$

The sulfur content is determined by equations (4.257) and (4.258) and must be between 0.35% and 0.4%, which represent the upper and the lower limits.

$$(0.0035 * X_{11} + 0.0042 * X_{21} + 0.004 * X_{31}) - 0.004 * (X_{11} + X_{21} + X_{31}) \leq 0 \qquad (4.257)$$

$$(0.0035 * X_{11} + 0.0042 * X_{21} + 0.004 * X_{31}) - 0.0035 * (X_{11} + X_{21} + X_{31}) \geq 0 \qquad (4.258)$$

The upper boundary of moisture content (8.0%) is governed by equation (4.259) and the lower boundary (7.5%) by equation (4.260).

$$(0.075 * X_{11} + 0.0775 * X_{21} + 0.085 * X_{31}) - 0.08 * (X_{11} + X_{21} + X_{31}) \leq 0 \qquad (4.459)$$

$$(0.075 * X_{11} + 0.0775 * X_{21} + 0.085 * X_{31}) - 0.075 * (X_{11} + X_{21} + X_{31}) \geq 0 \qquad (4.460)$$

Equation (4.261) sets the content of fixed carbon to be higher than or equal to 58%.

$$(0.60 * X_{11} + 0.595 * X_{21} + 0.575 * X_{31}) - 0.58 * (X_{11} + X_{21} + X_{31}) \geq 0 \qquad (4.261)$$

The volatile matter content is illustrated in equation (4.262) and must be less than or equal to 22.6%.

$$(0.21 * X_{11} + 0.22 * X_{21} + 0.225 * X_{31}) - 0.226 * (X_{11} + X_{21} + X_{31}) \leq 0 \qquad (4.262)$$

Equation (4.263) requires that the energy content is larger or equal to 30.2 [MJ/kg].

$$(0.32 * X_{11} + 0.31 * X_{21} + 0.295 * X_{31}) - 0.302 * (X_{11} + X_{21} + X_{31}) \geq 0 \qquad (4.263)$$

The quality of thermal coal stockpiled in Vancouver is controlled by equations (4.264) through (4.272). The blending constraint equations are based on Table 5.3.10, which illustrate the thermal coal quality requirements and the qualities of cleaned thermal coal produced by each mine. Equations (4.264) and (4.265) govern the respective upper (13%) and the lower limit (12.5%) of ash content in thermal coal.

$$(0.1225 * Y_{11} + 0.131 * Y_{21} + 0.138 * Y_{31}) - 0.13 * (Y_{11} + Y_{21} + Y_{31}) \leq 0 \qquad (4.264)$$

$$(0.1225 * Y_{11} + 0.131 * Y_{21} + 0.138 * Y_{31}) - 0.125 * (Y_{11} + Y_{21} + Y_{31}) \geq 0 \qquad (4.265)$$

The sulfur content of thermal coal has to be within 0.75% and 0.1%. Equation (4.266) controls the upper limit and (4.267) the lower.

$$(0.007*Y_{11}+0.0085*Y_{21}+0.008*Y_{31})-0.01*(Y_{11}+Y_{21}+Y_{31})\leq 0 \quad (4.266)$$

$$(0.007*Y_{11}+0.0085*Y_{21}+0.008*Y_{31})-0.0075*(Y_{11}+Y_{21}+Y_{31})\geq 0 \quad (4.267)$$

Equations (4.268) and (4.269) govern the respective upper (13.5%) and the lower (13%) boundaries of moisture content.

$$(0.132*Y_{11}+0.1375*Y_{21}+0.138*Y_{31})-0.135*(Y_{11}+Y_{21}+Y_{31})\leq 0 \quad (4.268)$$

$$(0.132*Y_{11}+0.1375*Y_{21}+0.138*Y_{31})-0.13*(Y_{11}+Y_{21}+Y_{31})\geq 0 \quad (4.269)$$

The fixed carbon content of thermal coal is governed by equation (4.270) and must be higher than or equal to 46.5 %.

$$(0.51*Y_{11}+0.49*Y_{21}+0.485*Y_{31})-0.465*(Y_{11}+Y_{21}+Y_{31})\geq 0 \quad (4.270)$$

The volatile matter constraint is illustrated in equation (4.271). The content of volatile matter of thermal coal must be lower than 36.2%.

$$(0.30*Y_{11}+0.35*Y_{21}+0.37*Y_{31})-0.362*(Y_{11}+Y_{21}+Y_{31})\leq 0 \quad (4.271)$$

The energy content of thermal coal must be larger than or equal to 26.1 [MJ/kg]. Equation (4.272) sets the energy content of thermal coal stockpiled in Vancouver port to meet this requirement.

$$(0.28*Y_{11}+0.275*Y_{21}+0.26*Y_{31})-0.261*(Y_{11}+Y_{21}+Y_{31})\geq 0 \quad (4.272)$$

Thunder Bay metallurgical and thermal blending stockpiles have the same requirements as those of Vancouver's. The quality of coal in the metallurgical stockpile is controlled by equations (4.273) through (4.281), which are based on data from Table 4.2.2.9.

Equations (4.273) and (4.274) illustrate the ash constraints for Thunder Bay's metallurgical stockpile. They define the respective upper and lower boundaries of the ash content requirements.

$$(0.0775 * X_{12} + 0.08 * X_{22} + 0.09 * X_{32}) - 0.085 * (X_{12} + X_{22} + X_{32}) \leq 0 \tag{4.273}$$

$$(0.0775 * X_{12} + 0.08 * X_{22} + 0.09 * X_{32}) - 0.08 * (X_{12} + X_{22} + X_{32}) \geq 0 \tag{4.274}$$

The sulfur content is defined by equations (4.275) and (4.276), which control the respective upper and lower limits of this parameter.

$$(0.0035 * X_{12} + 0.0042 * X_{22} + 0.004 * X_{32}) - 0.004 * (X_{12} + X_{22} + X_{32}) \leq 0 \tag{4.275}$$

$$(0.0035 * X_{12} + 0.0042 * X_{22} + 0.004 * X_{32}) - 0.0035 * (X_{12} + X_{22} + X_{32}) \geq 0 \tag{4.276}$$

The upper and lower contents of moisture are governed by equation (4.277) and (4.278).

$$(0.075 * X_{12} + 0.0775 * X_{22} + 0.085 * X_{32}) - 0.08 * (X_{12} + X_{22} + X_{32}) \leq 0 \tag{4.277}$$

$$(0.075 * X_{12} + 0.0775 * X_{22} + 0.085 * X_{32}) - 0.0775 * (X_{12} + X_{22} + X_{32}) \geq 0 \tag{4.278}$$

Equation (4.279) controls the content of fixed carbon.

$$(0.60 * X_{12} + 0.595 * X_{22} + 0.575 * X_{32}) - 0.58 * (X_{12} + X_{22} + X_{32}) \geq 0 \tag{4.279}$$

The volatile matter constraint is illustrated in equation (4.280).

$$(0.21 * X_{12} + 0.22 * X_{22} + 0.22.5 * X_{32}) - 0.226 * (X_{12} + X_{22} + X_{32}) \leq 0 \tag{4.280}$$

Equation (4.281) governs the content of energy in the metallurgical coal, stockpiled in Thunder Bay port.

$$(0.32*X_{12}+0.31*X_{22}+0.295*X_{32})-0.302*(X_{12}+X_{22}+X_{32})\geq 0 \quad (4.281)$$

The mathematical equations, which define thermal coal qualities for the Thunder Bay stockpile, are based on data presented in Table 4.2.2.10. These equations are illustrated in (4.282) through (4.290). Equation (4.282) and (4.283) define the upper and lower limits of ash content.

$$(0.1225*Y_{12}+0.131*Y_{22}+0.138*Y_{32})-0.13*(Y_{12}+Y_{22}+Y_{32})\leq 0 \quad (4.282)$$

$$(0.1225*Y_{12}+0.131*Y_{22}+0.138*Y_{32})-0.125*(Y_{12}+Y_{22}+Y_{32})\geq 0 \quad (4.283)$$

The sulfur constraints are presented in equation (4.284) and (4.285), which define the respective upper and lower limits of sulfur content.

$$(0.007*Y_{12}+0.0085*Y_{22}+0.008*Y_{32})-0.01*(Y_{12}+Y_{22}+Y_{32})\leq 0 \quad (4.284)$$

$$(0.007*Y_{12}+0.0085*Y_{22}+0.008*Y_{32})-0.0075*(Y_{12}+Y_{22}+Y_{32})\geq 0 \quad (4.285)$$

Equations (4.286) and (4.287), govern the content of moisture in the Thunder Bay's thermal coal stockpile.

$$(0.132*Y_{12}+0.1375*Y_{22}+0.138*Y_{32})-0.135*(Y_{12}+Y_{22}+Y_{32})\leq 0 \quad (4.286)$$

$$(0.132*Y_{12}+0.1375*Y_{22}+0.138*Y_{32})-0.13*(Y_{12}+Y_{22}+Y_{32})\geq 0 \quad (4.287)$$

The fixed carbon content is controlled by equation (4.288).

$$(0.51*Y_{12}+0.49*Y_{22}+0.485*Y_{32})-0.465*(Y_{12}+Y_{22}+Y_{32})\geq 0 \quad (4.288)$$

Equation (4.289) defines volatile matter content.

$$(0.30*Y_{12}+0.35*Y_{22}+0.37*Y_{32})-0.362*(Y_{12}+Y_{22}+Y_{32})\leq 0 \quad (4.289)$$

The energy content of the thermal coal has to be greater than or equal to 26.1 [MJ/kg]. Equation (4.290) governs this particular parameter for the Thunder Bay's stockpile.

$$(0.28*Y_{12}+0.275*Y_{22}+0.26*Y_{32})-0.261*(Y_{12}+Y_{22}+Y_{32})\geq 0 \quad (4.290)$$

Brooks power plant stockpile consists of only thermal coal. The quality of coal shipped to the power plants is assumed to be similar to the thermal coal shipped abroad. Equations (4.291) through (4.299) govern the coal quality and are based on data from Table 4.2.2.10. The ash constraints, for the coal shipped to power plant, are presented in equation (4.291) and (4.292), which represent the upper and lower limit of ash content.

$$(0.1225*Y_{13}+0.131*Y_{23}+0.138*Y_{33})-0.13*(Y_{13}+Y_{23}+Y_{33})\leq 0 \quad (4.291)$$

$$(0.1225*Y_{13}+0.131*Y_{23}+0.138*Y_{33})-0.125*(Y_{13}+Y_{23}+Y_{33})\geq 0 \quad (4.292)$$

Equations (4.293) and (4.294) control the respective upper and lower limits of sulfur content.

$$(0.007*Y_{13}+0.0085*Y_{23}+0.008*Y_{33})-0.01*(Y_{13}+Y_{23}+Y_{33})\leq 0 \quad (4.293)$$

$$(0.007*Y_{13}+0.0085*Y_{23}+0.008*Y_{33})-0.0075*(Y_{13}+Y_{23}+Y_{33})\geq 0 \quad (4.294)$$

The moisture constraints for the power plants are illustrated in equations (4.295) and (4.296). They deal with the respective upper and lower boundaries of this parameter.

$$(0.132 * Y_{13} + 0.1375 * Y_{23} + 0.138 * Y_{33}) - 0.135 * (Y_{13} + Y_{23} + Y_{33}) \leq 0 \tag{4.295}$$

$$(0.132 * Y_{13} + 0.1375 * Y_{23} + 0.138 * Y_{33}) - 0.13 * (Y_{13} + Y_{23} + Y_{33}) \geq 0 \tag{4.296}$$

The fixed carbon content is governed by equation (4.297).

$$(0.51 * Y_{13} + 0.49 * Y_{23} + 0.485 * Y_{33}) - 0.465 * (Y_{13} + Y_{23} + Y_{33}) \geq 0 \tag{4.297}$$

The volatile matter content is controlled by equation (4.298).

$$(0.30 * Y_{13} + 0.35 * Y_{23} + 0.37 * Y_{33}) - 0.362 * (Y_{13} + Y_{23} + Y_{33}) \leq 0 \tag{4.298}$$

Equation (4.299) defines and controls the energy content requirement regarding coal transported to the power plant.

$$(0.28 * Y_{13} + 0.275 * Y_{23} + 0.26 * Y_{33}) - 0.261 * (Y_{13} + Y_{23} + Y_{33}) \geq 0 \tag{4.299}$$

Non-negativity constraints for Fording Coal optimization model are presented in equations (4.300) and (4.301) and are imposed on all the variables of the objective function.

$$X_{11}; X_{12}; X_{21}; X_{22}; X_{31}; X_{32} \geq 0 \tag{4.300}$$

$$Y_{11}; Y_{12}; Y_{13}; Y_{21}; Y_{22}; Y_{23}; Y_{31}; Y_{32}; Y_{33} \geq 0 \tag{4.301}$$

The LP variables represent the quantities of metallurgical and thermal coal produced and shipped by each mine. The total number of variables is 15. The total number of constraints imposed on the objective function is 83. These constraints consist of 10 supply and demand, 9 minimum and maximum capacities, 4 port and railway limitations, 45 blending, and 15 non-negativity equations. Excel-Solver is used to maximize the objective function illustrated in equation (4.231) subjected to the 83 limits constraint equations.

4.2.2.2 NLP Model

The objective function for the NLP model is, as defined by Everett (1963) based on Lagrange theory. The generalized Lagrangian model is the difference between the profit function illustrated be equation (4.231) and the constraint functions multiplied by some real numbers arbitrarily chosen. The goal of this subsection is to develop the mathematical equations of the constraints. The constraint equations for the NLP model are basically the same as for the LP model. The only difference is that their mathematical formulation have to be always less than or equal to zero to be consistent with Lagrange and GLM theorems. The market constraints for Fording Coal's coal production are based on data from Table 4.2.25. Equations (4.302) through (4.306) govern the markets capacity limitations. Equations (4.302) and (4.303) control the metallurgical and thermal coal quantities that could be sent annually to Vancouver ports.

$$X_{11} + X_{21} + X_{31} - 12{,}500{,}000 \leq 0 \tag{4.302}$$

$$Y_{11} + Y_{21} + Y_{31} - 2{,}500{,}000 \leq 0 \tag{4.303}$$

Equations (4.304) and (4.305) limit the quantities of metallurgical and thermal coal transported annually to Thunder Bay to the market capacity limits.

$$X_{12} + X_{22} + X_{32} - 1{,}250{,}000 \leq 0 \tag{4.304}$$

$$Y_{12} + Y_{22} + Y_{32} - 1{,}000{,}000 \leq 0 \tag{4.305}$$

Equation (4.306) controls the limitation of thermal coal transported to Brooks power plant.

$$Y_{13} + Y_{23} + Y_{33} - 1{,}250{,}000 \leq 0 \qquad (4.306)$$

It is assumed that Fording has contracted out most of its coal production. The contracted coal quantities per year and destination are illustrated in Table 4.2.2.6. Equations (4.307) through (4.311) control the contractual constraints based on data from this table. Equations (4.307) and (4.308) set the amounts of metallurgical and thermal coal shipped to Vancouver to be larger than or equal to the quantities specified by the contract.

$$10{,}500{,}000 - (X_{11} + X_{21} + X_{31}) \leq 0 \qquad (4.307)$$

$$2{,}000{,}000 - (Y_{11} + Y_{21} + Y_{31}) \leq 0 \qquad (4.308)$$

Equations (4.309) and (4.310) control the quantities of metallurgical and thermal coal transported to Thunder Bay as contractual obligations.

$$1{,}000{,}000 - (X_{12} + X_{22} + X_{23}) \leq 0 \qquad (4.309)$$

$$1{,}000{,}000 - (Y_{12} + Y_{22} + Y_{23}) \leq 0 \qquad (4.310)$$

Thermal coal shipped to Brooks is governed by equation (4.311)

$$1{,}000{,}000 - (Y_{13} + Y_{23} + Y_{33}) \leq 0 \qquad (4.311)$$

It is assumed that Fording Coal's mine operations have their maximum annual production capacity as illustrated in Table 4.2.2.7, and the resources available for the optimization model will be limited to those values. Equations (4.312), (4.313), and (4.314) illustrate the constraints that govern the maximum mine capacities at Fording River, Greenhills and respective Coal Mountain.

$$X_{11} + X_{12} + Y_{11} + Y_{12} + Y_{13} - 10{,}500{,}000 \leq 0 \qquad (4.312)$$

$$X_{21} + X_{22} + Y_{21} + Y_{22} + Y_{23} - 4{,}500{,}000 \le 0 \quad (4.313)$$

$$X_{31} + X_{32} + Y_{31} + Y_{32} + Y_{33} - 3{,}500{,}000 \le 0 \quad (4.314)$$

The minimum capacity usage requirement set for each mine is illustrated in Table 4.2.2.8. Equations (4.315) through (4.320) set the metallurgical and the total coal production at each mine to be larger than or equal to the values presented in this table. Equation (4.315) and (4.316) control coal production at Fording River.

$$9{,}000{,}000 - (X_{11} + X_{12} + Y_{11} + Y_{12} + Y_{13}) \le 0 \quad (4.315)$$

$$7{,}500{,}000 - (X_{11} + X_{12}) \le 0 \quad (4.316)$$

Greenhills' minimum capacity requirements are governed by equations (4.317) and (4.318)

$$4{,}000{,}000 - (X_{21} + X_{22} + Y_{21} + Y_{22} + Y_{23}) \le 0 \quad (4.317)$$

$$3{,}200{,}000 - (X_{21} + X_{22}) \le 0 \quad (4.318)$$

Coal Mountain's total and metallurgical coal minimum capacity requirements are defined by equations (4.319) and (4.320).

$$2{,}000{,}000 - (X_{31} + X_{32} + Y_{31} + Y_{32} + Y_{33}) \le 0 \quad (4.319)$$

$$1{,}850{,}000 - (X_{31} + X_{32}) \le 0 \quad (4.320)$$

Coal production from Fording Coal's mines is hauled by trains to the final destinations. As was mentioned in the LP model, Fording Coal shares the railways capacity with other western Canadian coal producers. Total railway capacity from Sparwood area to Vancouver allocated to Fording is assumed to be around 19.0 [Mt/y]. Equation (4.321) limits the amount of coal that Fording mines produce and ship to Vancouver to this quantity.

$$X_{12} + X_{21} + X_{31} + Y_{11} + Y_{21} + Y_{31} - 19{,}000{,}000 \leq 0 \quad (4.321)$$

From Sparwood area to Thunder Bay port, the coal railway capacity allocated to Fording is assumed to be around 11.0 [Mt/y]. This constraint is illustrated by equation (4.322).

$$X_{12} + X_{22} + X_{32} + Y_{12} + Y_{22} + Y_{32} - 11{,}000{,}000 \leq 0 \quad (4.322)$$

Port Capacities could limit the amounts of coal produced and sold by Fording Coal overseas. Vancouver ports have an annual coal handling capacity of 33.0 [Mt]. From this capacity around 11.5 [Mt/y] is taken by other coal producers, and the remaining 21.5 [Mt/y] is allocated to Fording Coal. Equation (4.323) governs the Vancouver port limitation.

$$X_{11} + X_{21} + X_{31} + Y_{11} + Y_{21} + Y_{31} - 21{,}500{,}000 \leq 0 \quad (4.323)$$

Thunder Bay port has 10.0 [Mt] annual capacity, of which around 4.0 [Mt/y] is taken by Luscar and Teck, and the remaining 6.0 [Mt/y] is allocated to Fording Coal. This limitation is defined by equation (4.324).

$$X_{12} + X_{22} + X_{32} + Y_{12} + Y_{22} + Y_{32} - 6{,}000{,}000 \leq 0 \quad (4.324)$$

Regarding the blending constraints, it is assumed that the existing clean coal physical and chemical characteristics at each mine are as illustrated in Table 4.2.2.9 and 4.2.2.10. At each final destination, except power plants, the coal is stocked into two different stockpiles (metallurgical and thermal). Each of these stockpiles has to meet the required coal specifications. For each of these stockpiles, a set of blending constraints has to be developed and input into the optimization model to make sure that the coal would meet the quality specifications. The quality of metallurgical coal stockpiled in Vancouver is governed by equations (4.325) through (4.333). The content of ash in metallurgical coal is controlled by equation (4.325) and (4.326), which govern the respective upper (8.5%) and lower limit (8.0%) of this parameter.

$$(0.0775 * X_{11} + 0.08 * X_{21} + 0.09 * X_{31}) - 0.085 * (X_{11} + X_{21} + X_{31}) \leq 0 \qquad (4.325)$$

$$0.08 * (X_{11} + X_{21} + X_{31}) - (0.0775 * X_{11} + 0.08 * X_{21} + 0.09 * X_{31}) \leq 0 \qquad (4.326)$$

The sulfur content is determined by equations (4.327) and (4.328) and must be within 0.35% and 0.4%.

$$(0.0035 * X_{11} + 0.0042 * X_{21} + 0.004 * X_{31}) - 0.004 * (X_{11} + X_{21} + X_{31}) \leq 0 \qquad (4.327)$$

$$0.0035 * (X_{11} + X_{21} + X_{31}) - (0.0035 * X_{11} + 0.0042 * X_{21} + 0.004 * X_{31}) \leq 0 \qquad (4.328)$$

The upper boundary of moisture content (8.0%) is governed by equation (4.329) and the lower boundary (7.5%) by equation (4.330).

$$(0.075 * X_{11} + 0.0775 * X_{21} + 0.085 * X_{31}) - 0.08 * (X_{11} + X_{21} + X_{31}) \leq 0 \qquad (4.329)$$

$$0.075 * (X_{11} + X_{21} + X_{31}) - (0.075 * X_{11} + 0.0775 * X_{21} + 0.085 * X_{31}) \leq 0 \qquad (4.330)$$

Equation (4.331) sets the content of fixed carbon to be higher than or equal to 58%.

$$0.58 * (X_{11} + X_{21} + X_{31}) - (0.60 * X_{11} + 0.595 * X_{21} + 0.575 * X_{31}) \leq 0 \qquad (4.331)$$

The volatile matter content is illustrated in equation (4.332) and must be less than or equal to 22.6%.

$$(0.21*X_{11}+0.22*X_{21}+0.225*X_{31})-0.226*(X_{11}+X_{21}+X_{31})\leq 0 \qquad (4.332)$$

Equation (4.333) sets the energy content to be larger or equal to 30.2 [MJ/kg].

$$0.302*(X_{11}+X_{21}+X_{31})-(0.32*X_{11}+0.31*X_{21}+0.295*X_{31})\leq 0 \qquad (4.333)$$

The quality of thermal coal stockpiled in Vancouver is controlled by equations (4.334) through (4.342). The blending constraint equations are based on Table 4.2.2.10, which illustrate the thermal coal quality requirements and the qualities of cleaned thermal coal produced by each mine. Equations (4.334) and (4.335) govern the respective upper (13%) and lower limit (12.5%) of ash content in thermal coal.

$$(0.1225*Y_{11}+0.131*Y_{21}+0.138*Y_{31})-0.13*(Y_{11}+Y_{21}+Y_{31})\leq 0 \qquad (4.334)$$

$$0.125*(Y_{11}+Y_{21}+Y_{31})-(0.1225*Y_{11}+0.131*Y_{21}+0.138*Y_{31})\leq 0 \qquad (4.335)$$

The sulfur content of thermal coal must be within 0.75% and 0.1%. Equation (4.336) controls the upper limit and (4.337) the lower.

$$(0.007*Y_{11}+0.0085*Y_{21}+0.008*Y_{31})-0.01*(Y_{11}+Y_{21}+Y_{31})\leq 0 \qquad (4.336)$$

$$0.0075*(Y_{11}+Y_{21}+Y_{31})-(0.007*Y_{11}+0.0085*Y_{21}+0.008*Y_{31})\leq 0 \qquad (4.337)$$

Equation (4.338) and (4.339) govern the respective upper (13.5%) and lower (13%) boundaries of moisture content.

$$(0.132*Y_{11}+0.1375*Y_{21}+0.138*Y_{31})-0.135*(Y_{11}+Y_{21}+Y_{31})\leq 0 \qquad (4.338)$$

$$0.13*(Y_{11}+Y_{21}+Y_{31})-(0.132*Y_{11}+0.1375*Y_{21}+0.138*Y_{31})\leq 0 \quad (4.339)$$

The fixed carbon content of thermal coal is governed by equation (4.340). It has to be higher than or equal to 46.5 %.

$$0.465*(Y_{11}+Y_{21}+Y_{31})-(0.51*Y_{11}+0.49*Y_{21}+0.485*Y_{31})\leq 0 \quad (4.340)$$

The volatile matter content is illustrated by equation (4.341) and must be lower than 36.2%.

$$(0.30*Y_{11}+0.35*Y_{21}+0.37*Y_{31})-0.362*(Y_{11}+Y_{21}+Y_{31})\leq 0 \quad (4.341)$$

The energy content of thermal coal must be larger than or equal to 26.1 [MJ/kg]. Equation (4.342) sets the energy content of thermal coal stockpiled in Vancouver port to meet this requirement.

$$0.261*(Y_{11}+Y_{21}+Y_{31})-(0.28*Y_{11}+0.275*Y_{21}+0.26*Y_{31})\leq 0 \quad (4.342)$$

Thunder Bay's metallurgical and thermal blending stockpiles have the same requirements as those from Vancouver. The quality of coal in the metallurgical stockpile is controlled by equations (4.343) through (4.351), which are based on data from Table 4.2.2.9. Equations (4.343) and (4.344) illustrate the ash constraints for Thunder Bay's metallurgical stockpile. They define the respective upper and lower boundaries of the ash content requirements.

$$(0.0775*X_{12}+0.08*X_{22}+0.09*X_{32})-0.085*(X_{12}+X_{22}+X_{32})\leq 0 \quad (4.343)$$

$$0.08*(X_{12}+X_{22}+X_{32})-(0.0775*X_{12}+0.08*X_{22}+0.09*X_{32})\leq 0 \quad (4.344)$$

The sulfur content is defined by equations (4.345) and (4.346), which control the respective upper and lower limits of this parameter.

$$(0.0035*X_{12}+0.0042*X_{22}+0.004*X_{32})-0.004*(X_{12}+X_{22}+X_{32})\leq 0 \qquad (4.345)$$

$$0.0035*(X_{12}+X_{22}+X_{32})-(0.0035*X_{12}+0.0042*X_{22}+0.004*X_{32})\leq 0 \qquad (4.346)$$

The upper and lower contents of moisture are governed by equation (4.347) and (4.348).

$$(0.075*X_{12}+0.0775*X_{22}+0.085*X_{32})-0.08*(X_{12}+X_{22}+X_{32})\leq 0 \qquad (4.347)$$

$$0.075*(X_{12}+X_{22}+X_{32})-(0.075*X_{12}+0.0775*X_{22}+0.085*X_{32})\leq 0 \qquad (4.348)$$

Equation (4.349) controls the content of fixed carbon.

$$0.58*(X_{12}+X_{22}+X_{32})-(0.60*X_{12}+0.595*X_{22}+0.575*X_{32})\leq 0 \qquad (4.349)$$

The volatile matter constraint is illustrated in equation (4.350).

$$(0.21*X_{12}+0.22*X_{22}+0.22.5*X_{32})-0.226*(X_{12}+X_{22}+X_{32})\leq 0 \qquad (4.350)$$

Equation (4.351) governs the content of energy in the metallurgical coal, stockpiled in Thunder Bay port.

$$0.302*(X_{12}+X_{22}+X_{32})-(0.32*X_{12}+0.31*X_{22}+0.295*X_{32})\leq 0 \qquad (4.351)$$

The mathematical equations, which define the thermal coal qualities for the Thunder Bay stockpile, are based on data presented in Table 4.2.2.10. These equations are illustrated in (4.352) through (4.360). Equations (4.352) and (4.353) define the respective upper and lower limits of ash content.

$$(0.1225 * Y_{12} + 0.131 * Y_{22} + 0.138 * Y_{32}) - 0.13 * (Y_{12} + Y_{22} + Y_{32}) \leq 0 \quad (4.352)$$

$$0.125 * (Y_{12} + Y_{22} + Y_{32}) - (0.1225 * Y_{12} + 0.131 * Y_{22} + 0.138 * Y_{32}) \leq 0 \quad (4.353)$$

The sulfur constraints are presented in equation (4.354) and (4.355), which define the respective upper and lower limits of sulfur content.

$$(0.007 * Y_{12} + 0.0085 * Y_{22} + 0.008 * Y_{32}) - 0.01 * (Y_{12} + Y_{22} + Y_{32}) \leq 0 \quad (4.354)$$

$$0.0075 * (Y_{12} + Y_{22} + Y_{32}) - (0.007 * Y_{12} + 0.0085 * Y_{22} + 0.008 * Y_{32}) \leq 0 \quad (4.355)$$

Equations (4.356) and (4.357), govern the content of moisture in the Thunder Bay thermal coal stockpile.

$$(0.132 * Y_{12} + 0.1375 * Y_{22} + 0.138 * Y_{32}) - 0.135 * (Y_{12} + Y_{22} + Y_{32}) \leq 0 \quad (4.356)$$

$$0.13 * (Y_{12} + Y_{22} + Y_{32}) - (0.132 * Y_{12} + 0.1375 * Y_{22} + 0.138 * Y_{32}) \leq 0 \quad (4.357)$$

The fixed carbon content is controlled by equation (4.358).

$$0.465 * (Y_{12} + Y_{22} + Y_{32}) - (0.51 * Y_{12} + 0.49 * Y_{22} + 0.485 * Y_{32}) \leq 0 \quad (4.358)$$

Equation (4.359) defines volatile matter content.

$$(0.30 * Y_{12} + 0.35 * Y_{22} + 0.37 * Y_{32}) - 0.362 * (Y_{12} + Y_{22} + Y_{32}) \leq 0 \quad (4.359)$$

The energy content of the thermal coal must be greater than or equal to 26.1 [MJ/kg]. Equation (4.360) governs this particular parameter for the Thunder Bay's stockpile.

$$0.261*(Y_{12}+Y_{22}+Y_{32})-(0.28*Y_{12}+0.275*Y_{22}+0.26*Y_{32})\leq 0 \quad (4.360)$$

Brooks power plant stockpile consists of only thermal coal. Equations (4.361) through (4.369) govern the coal quality and are based on data from Table 4.2.2.10. The ash constraints, for the coal shipped to power plants, are presented in equation (4.361) and (4.362), which represent the respective upper and lower limit of ash content.

$$(0.1225*Y_{13}+0.131*Y_{23}+0.138*Y_{33})-0.13*(Y_{13}+Y_{23}+Y_{33})\leq 0 \quad (4.361)$$

$$0.125*(Y_{13}+Y_{23}+Y_{33})-(0.1225*Y_{13}+0.131*Y_{23}+0.138*Y_{33})\leq 0 \quad (4.362)$$

Equations (4.363) and (4.364) control the upper and lower limit of sulfur content.

$$(0.007*Y_{13}+0.0085*Y_{23}+0.008*Y_{33})-0.01*(Y_{13}+Y_{23}+Y_{33})\leq 0 \quad (4.363)$$

$$0.0075*(Y_{13}+Y_{23}+Y_{33})-(0.007*Y_{13}+0.0085*Y_{23}+0.008*Y_{33})\leq 0 \quad (4.364)$$

The moisture constraints for the power plant are illustrated in equations (4.365) and (4.366).

$$(0.132*Y_{13}+0.1375*Y_{23}+0.138*Y_{33})-0.135*(Y_{13}+Y_{23}+Y_{33})\leq 0 \quad (4.365)$$

$$0.13*(Y_{13}+Y_{23}+Y_{33})-(0.132*Y_{13}+0.1375*Y_{23}+0.138*Y_{33})\leq 0 \quad (4.366)$$

The fixed carbon content is governed by equation (4.367)

$$0.465*(Y_{13}+Y_{23}+Y_{33})-(0.51*Y_{13}+0.49*Y_{23}+0.485*Y_{33})\leq 0 \quad (4.367)$$

The volatile matter content is controlled by equation (4.368).

$$(0.30*Y_{13}+0.35*Y_{23}+0.37*Y_{33})-0.362*(Y_{13}+Y_{23}+Y_{33})\leq 0 \qquad (4.368)$$

Equation (4.369) defines and controls the energy content requirement.

$$0.261*(Y_{13}+Y_{23}+Y_{33})-(0.28*Y_{13}+0.275*Y_{23}+0.26*Y_{33})\leq 0 \qquad (4.369)$$

The objective function of the NLP model consists of the profit function (4.231) minus all the constraint functions, defined above, multiplied by some real parameters. To simplify the objective function the Lagrange multipliers will be represented as λ^k ($k= 1,68$) and the constraint functions represented by their equation number (4.302) through (4.369). The NLP objective function is illustrated by equation (4.370). The equation is input into a spreadsheet and optimized using Excel-Solver with a non-linear option.

$$L_f=(4.231)-[\lambda^1*(4.302)+\lambda^2*(4.303)+...+\lambda^{68}*(4.369)] \qquad (4.370)$$

4.3 Experimental Design

4.3.1 Design Options

Many design options could be developed to solve the coal production and haulage optimization problem. In this research, a design option is defined as a technically-and economically feasible options that can be used to solve the problem. For instance, one of the basic design options could be the analysis of increasing and decreasing mining and processing costs at some of the mines correlated with the ash, and moisture contents. Market design options could be developed. For instance, increase or decrease in markets demand; shifts in markets demand (overseas versus domestic) and the volatility of the coal prices. Many variant options were conducted besides the base cases to analyze different economic and coal quality scenarios. The choice of one option over another largely depends on the cost structure and the economic uncertainty. Three variant options (the most relevant) for each base case have been considered as a sample space for management's decisions. Detailed discussion of the base options and the variant options are presented in Chapter 5.0.

4.3.2 Data Gathering and Processing

The source of the information about western Canadian coal production and haulage was mainly the Coal Association of Canada, BC Government, Luscar-Sherritt's and Fording Coal's websites. From these websites, information was extracted regarding the producing mines, the level of coal productions and capacities, haulage distances, main overseas and domestic customers, amounts of coal transported and sold at different destinations, coal prices and coal required qualities. The Fording Coal representatives confirmed that the information and data used in this study were close to the real data and it was reliable. The cleaned coal qualities in the models were assumed based on the ranges of coal qualities in Vancouver ports (See Appendix 2.0 Tables A.2.1 through A.2.6). The ports and railway capacities were extracted from the ports and rail company's websites and from the coal mining companies as well. The costs of mining and processing, railway fees, and port charges were collected during the meeting with the Fording Coal representatives in Calgary in January 2002 and from Hanam Canada Corporation website.

Other sources of important information were the International Energy Outlook website, Major Coalfields of the World, Alberta and British Columbia government websites and statistics, and geological conferences on western Canadian coal. Some of this data had to be filtered since sometimes it conflicted with different sources. The Bestfit software was used to characterize the stochastic process governing the random variables, such as coal price, mining and processing and haulage costs. No data collection is completely free of defects, however, the author believes that the data integrity is adequate for the purpose of the study. Besides, the Fording Coal representatives confirmed that the information and data used in this study are close to the real data and it is reliable.

4.3.3 Number of iteration

In this section, the required number of iterations, time, precision, and convergence for the LP and NLP validation models are discussed, as well as the number of iterations in a simulation run for the stochastic models. An experimental design is used to validate the required number of iterations and time for the LP and NLP models. The number of iterations was increased from 10 with 10 iterations increments. The time was increased from 2 seconds with 2 seconds increments. For Luscar-Sherritt models, it was found that 130 iterations and 10 seconds were required for the LP models and 100 iterations and 30 seconds for the NLP model. As a conclusion the NLP needs less iterations but more time to reach the optimal solution. Regarding the precision, tolerance, and convergence they are not a big concern for the LP but they are important for the NLP. The higher the precision and the smaller the convergence values, the more time Solver takes to reach a solution. Table 4.3.1 illustrates the recommended value of these parameters for the Luscar-Sherritt case study.

Table 4.3.1 Solver Parameters Recommended for Luscar-Sherritt's Case Study

	Iterations	Time [sec]	Precision	Tolerance [%]	Convergence
LP	130	10	0.5	1.0	0.5
NLP	100	30	0.0001	0.01	0.0001

For Fording Coal's LP and NLP models a similar experiment was conducted and it was found that 100 iterations and 5 seconds are required for the LP and 20 iterations and 20 seconds for the NLP model. Table 4.3.2 presents the recommended values of these parameters necessary to find and reach the optimal solution.

Table 4.3.2 Solver Parameters Recommended for Fording Coal Case Study

	Iterations	Time [sec]	Precision	Tolerance [%]	Convergence
LP	100	5	0.5	1%	0.5
NLP	20	20	0.0001	0.01	0.0001

An experiment was conducted to validate the required number of iterations for the stochastic model in a single simulation run. Many simulations runs can be carried out; however, the most important parameter is to ensure that one carries out a sufficient number of iterations. A new mean and standard deviation is calculated for the profit function, whenever a value is randomly sampled. Intervals of 100 iterations for a single run were carried out between 100 and 3000 and 1000 increments from 3000 to 10000, each time recording the mean and the standard deviation. The results are illustrated in Figure 4.3.1 and 4.3.2 for Luscar-Sherritt Case and Figure 4.3.3 and 4.3.4 for Fording Coal. It can be seen that for Luscar-Sherritt the mean values begin to stabilize between 1000 and 3000 and standard deviation values stabilize between 4000 and 6000 iterations. For Fording Coal the mean value begins to stabilize between 1300 and 2000 and the standard deviation stabilizes between 3000 and 5500 iterations. For the purpose of ensuring sufficient accuracy, 5000 iterations are adequate.

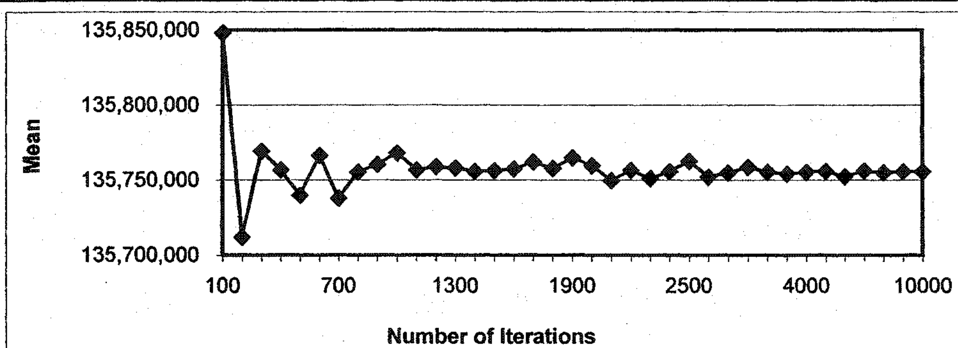


Figure 4.3.1 Mean Values vs. Numbers of Iterations: Luscar-Sherritt

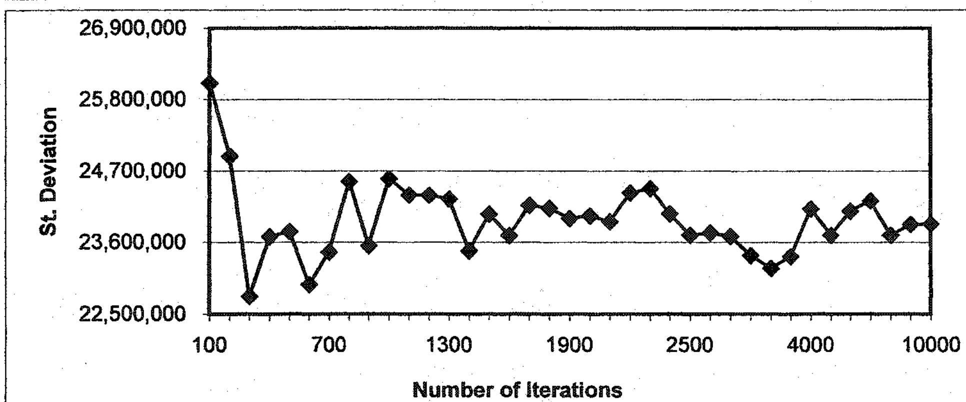


Figure 4.3.2 Standard Deviation vs. Numbers of Iterations: Luscar

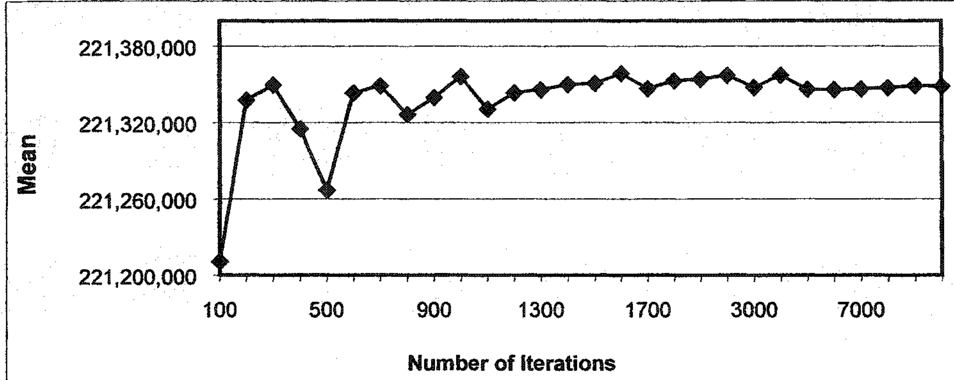


Figure 4.3.3 Mean Values vs. Number of Iterations Fording Coal

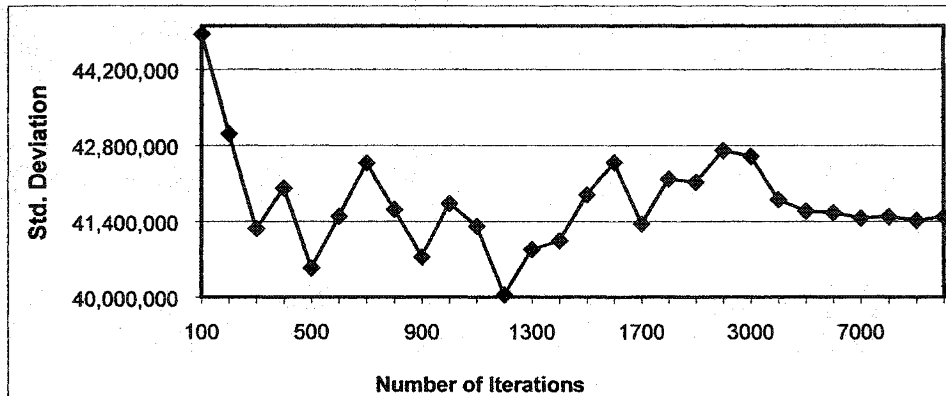


Figure 4.3.4 Standard Deviation vs. Number of Iterations Fording Coal

4.3.4 Experimental Setup and Experimentation

The experiment is carried out using a desktop personal computer (PC). The minimum requirements to run the experiment are a 386 DX-33 MHz with Microsoft Windows 97 or higher, with at least eight Megabytes of RAM. The average central processing time for a simulation run is less than two minutes for a Pentium (III) PC, compared to four minutes for the slower PC. The software used in this research study includes three add-in programs; Bestfit v.2.0, @RISK v.3.5, and Solver; and Excel v.7.0 spreadsheet. This computer software makes the use of the concepts in this study practical and helpful for the western Canadian coal companies at a relatively very low cost.

4.4 Conclusion

In this chapter the computer models were developed based on the complex mathematical equations derived in Chapter 3.0. Logic flowcharts for the coal production process, metallurgical and thermal coal process, TSM and GLM mathematic techniques, and stochastic sampling technique were developed to facilitate the analyses. Two case studies were conducted to validate the optimization models by developing an LP and an NLP for each case study. The computer models captured and optimized the coal production and haulage from each particular mine to their final destinations. Solver, Excel add-in software, was used to solve the complex optimization problems. The results of the base models and various optimization and risk analyses are provided in detail in the next chapter.

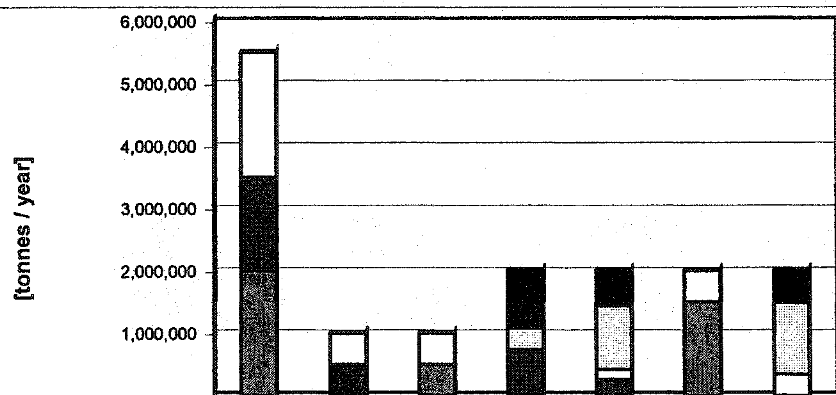
CHAPTER 5.0
DISCUSSION OF RESULTS

In this chapter, the results from the basic optimization models, various market and coal quality scenarios and sensitivity and risk analysis, are discussed in detail. The LP and NLP base models were developed and solved for each of the case studies, as presented in Chapter 4.0. To analyze various market and production scenarios, several changes were made in the objective function and constraint coefficients. The LP and NLP models were solved using Microsoft Solver-Excel 2000. Solver provides sensitivity and answer reports, which contain useful information about the problem (See Appendixes 3.0 through 6.0). These reports contain information regarding the variability of the right hand side of the constraints, highlight the binding constraints, provide the magnitude and the allowable increase and decrease margins for the constraints and the objective function coefficients. Based on this data, the management of coal mining companies could make decisions on how to improve the economic results of the company. The stochastic models provide the risks associated with the production and haulage of coal from mines to ports.

5.1 Analyses and Results of Luscar-Sherritt Optimization Models

5.1.1 Analysis of the Basic Optimization Model

The optimized profit function of the LP base case model for the Luscar-Sherritt operations is $160,626,790. The NLP model converged towards this value with a maximized profit of $160,626,756 ($34 less), providing practically identical optimal solutions for the objective function variables. The Lagrange multipliers for the NLP are provided in Table A.7.4 (Appendix 7.0). The optimized solutions for the objective function variables are illustrated in Figure 5.1.1 and Table A.7.1 (Appendix 7.0). This figure illustrates the level of coal production and haulage distribution for each mine. For this basic scenario, the markets demand was assumed to be relatively optimistic. The mine minimum and maximum capacities constraints were set to be 85% and 110% of the current level of production. The model's output suggests that Luscar-Sherritt should produce 7.5 [Mt] of metallurgical and 8.0 [Mt] of thermal coal annually. To ensure this level of production, all the mines should produce at their maximum capacity. The metallurgical coal production occurs at Line Creek, Cheviot, and Luscar and 5.50 [Mt / y] is hauled to Vancouver, 1.0 [Mt / y] to Ridley Island and 1.0 [Mt /y] to Thunder Bay. Thermal coal production occurs at all the mines at various levels and 2.0 [Mt / y] are hauled to each of the final destinations.



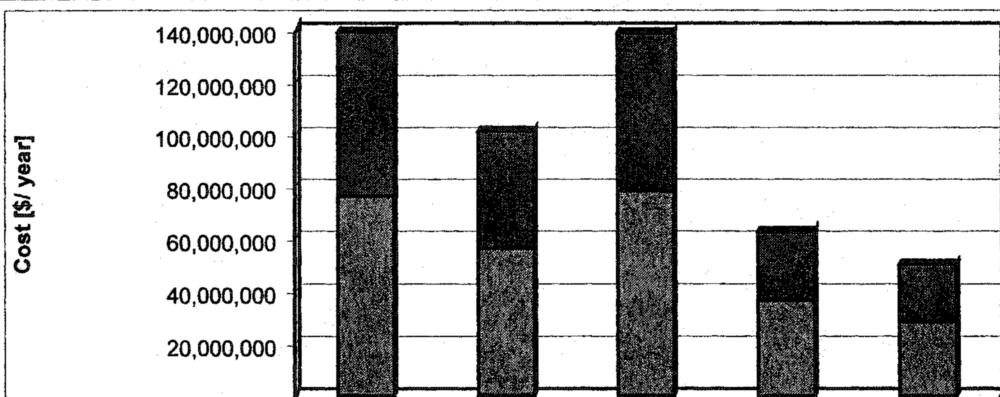
	Metallurgical			Thermal			
	Vancouver	Ridley	Thunder B	Vancouver	Ridley	Thunder B	Power Pla
Obed Mnt.	-	-	-	920,245	566,934	-	512,821
Coal Valley	-	-	(0)	343,558	1,011,143	-	1,145,299
Cheviot	2,000,000	500,000	500,000	-	158,120	500,000	341,880
Luscar	1,500,000	500,000	-	736,196	263,804	0	-
Line Creek	2,000,000	0	500,000	-	0	1,500,000	-

Figure 5.1.1 Luscar-Sherritt's Optimized Coal Production

No metallurgical or thermal coal is hauled from Line Creek to Ridley Island because there is no direct railway connection between them. Line Creek produces 2.5 [Mt /y] of metallurgical coal, of which 2.0 [Mt /y] is transported to Vancouver and 0.5 [Mt /y] to Thunder Bay. Line Creek also produces 1.5 [Mt /y] of thermal coal, which is entirely transported to Thunder Bay. Luscar produces 2.0 [Mt /y] of metallurgical coal, out of which 1.5 [Mt /y] is hauled to Vancouver and 0.5 [Mt /y] to Ridley Island, and 1.0 [Mt /y] of thermal coal of which approximately 0.74 [Mt /y] is transported to Vancouver and 0.26 [Mt /y] to Ridley Island. Cheviot should produce 3.0 [Mt /y] of metallurgical coal out of which 2.0 [Mt /y] should be transported to Vancouver, and 0.5 [Mt /y] to each Ridley Island and Thunder Bay. The quantity of thermal coal that Cheviot should produce is 1.0 [Mt /y] and 0.158 [Mt /y] should be hauled to Ridley Island, 0.5 [Mt /y] to Thunder

Bay, and 0.342 [Mt /y] to TransAlta's power plants. Coal Valley and Obed Mountain produce thermal coal only. Coal Valley produces 2.5 [Mt /y] of thermal coal and around 0.34 [Mt /y] is transported to Vancouver, 1.0 [Mt /y] to Ridley Island and 1.15 [Mt /y] to TransAlta's power plants. Obed Mountain produces 2.0 [Mt /y] and around 0.92 [Mt /y] is hauled to Vancouver, 0.56 [Mt /y] to Ridley Island and 0.51 [Mt /y] to TransAlta's power plants.

The mining and processing and haulage costs are the main factors that drive the total cost of a mine. Figure 5.1.2 illustrates the magnitude of these costs at each mine. Transportation costs represent 41% of the total cost at Line Creek, 40.2% at Luscar, 39.5% at Cheviot, 38% at Coal Valley, and 39.2% at Obed Mountain. Mining and processing costs represent 49.2% at Line Creek, 50% at Luscar, 51% at Cheviot, 52.6% at Coal Valley, and 49.8 at Obed Mountain.

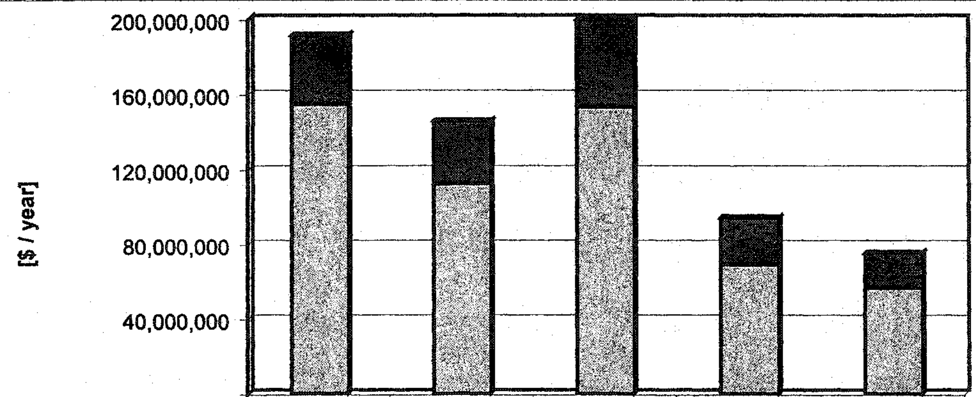


	Line Creek	Luscar	Cheviot	Coal Valley	Obed Mnt.
Transport [$]	63,298,000	45,078,736	60,883,350	26,219,471	22,063,652
Mining & Proc. [$]	76,000,000	56,000,000	78,000,000	36,250,000	28,000,000

Figure 5.1.2 Mining & Processing and Haulage Cost by Mine

The magnitude of the profits achieved and the total costs that occur at each mine are illustrated in Figure 5.1.3. The largest profit is achieved by Cheviot with 46.6 [M$ /y] followed by Line Creek with 36.8 [M$ /y], Luscar with 33.4 [M$ /y], Coal Valley with 24.9 [M$ /y], and Obed Mountain with 18.8 [M$ /y]. The total costs have a different distribution and Line Creek leads with 154.4 [M$ /y] followed by Cheviot with 152.8

[M$ /y], Luscar with 112.1 [M$ /y], Coal Valley with 68.8 [M$ /y], and Obed Mountain with 56.2 [M$ /y]. The total cost of Luscar-Sherit operations is 544.37 [M$ /y] with revenue of 705.0 [M$ /y]. These costs and profits occurred as an integrated process from the optimization model and they could be different if each mine optimizes the coal production and haulage without taking into consideration the other mines. Other relevant results regarding the coal quantities, costs, revenues and profits distribution by mine are illustrated in Table A.7.2 and A.7.3 (Appendix 7.0).



	Line Creek	Luscar	Cheviot	Coal Valley	Obed Mnt.
Profits [$]	36,852,000	33,414,117	46,649,470	24,915,809	18,795,395
Cost [$]	154,398,000	112,085,883	152,850,530	68,834,191	56,204,605

Figure 5.1.3 Luscar-Sheritt's Total Costs and Profits by Mine

The Sensitivity Report (Apendix 4.0) provides the objective function coefficients in dollars per tonne for each variable. Regarding metallurgical coal, the largest objective function coefficients are achieved by transporting metallurgical coal to Vancouver. They range from $14.41 to $16.68 per tonne from mine to mine. The objective function coefficients for transporting metallurgical coal to Ridley Island range from $12.789 to $14.806 per tonne. Transporting metallurgical coal to Thunder Bay gives a range of profits between $10.614 and $13.034 per tonne. Regarding thermal coal, the largest objective function coefficients are achieved by transporting thermal coal to the power plants. They range from $11.35 to $17.89 per tonne. Transporting thermal coal to Vancouver gives a range of profits from $2.267 to $7.186 per tonne. Ridley Island gives a range of profits from $0.792 to $5.306 per tonne. Transporting thermal coal to Thunder

Bay could be uneconomic for some of the mines and profitable for others but with a very low profitability margin. The objective function coefficients range from negative values of ($1.22) to positive values of $1.778. The model allocates coal to Thunder Bay only because it was constrained by the contract.

The feasible region for Luscar-Sherritt base case is defined by binding constraints of different type, such as, contractual, market capacities, mine's minimum and maximum capacities and blending constraints. The total number of the binding constraints is 26. None of the railway and port limitations are binding. There are 5 contractual binding constraints, which means that the model allocates only the minimum coal quantities, required by the contract, to these destinations. These binding constraints refer to Ridley Island and Thunder Bay for the metallurgical coal and to Vancouver, Ridley Island, and Thunder Bay for the thermal coal. The market supply binding constraints limit the quantities of metallurgical coal shipped to Vancouver to 5.5 [Mt/y], and thermal coal shipped to Power Plants and Vancouver port to 2.0 [Mt/y] each. All the maximum capacity constraints are binding, which means that all mines should produce at their maximum capacity. The minimum capacity usage requirement forces Line Creek and Luscar to produce metallurgical coal at least at their minimum economic capacity. The majority of the binding constraints consist of blending requirements. The lower limit of the ash constraint is binding for the metallurgical stockpiles in Vancouver and Thunder Bay, and for the thermal stockpiles at Ridley Island and power plants. The upper limit of the sulfur constraint is binding for the metallurgical coal stockpile in Ridley Island. The upper limit of the moisture content is binding for the metallurgical stockpile in Thunder Bay and for the thermal coal stockpiles in Vancouver, Thunder Bay and power plant. None of the other blending constraints are binding. However, some of them have very low margins and could become binding at very small changes of coal quality.

A sensitivity analysis on this base case was conducted to analyze the profit sensitivity to marginal changes in the determinant variables. The price of coal, the mining and processing costs of metallurgical and thermal coal, and transporting costs were varied in 10% increments from –30% to +30%. The greatest influence on the profit is the metallurgical coal price followed in order by haulage cost, metallurgical coal operating cost, thermal coal selling price and the least sensitive is thermal coal operating cost. The results of the sensitivity analysis are presented in Figure 5.1.4 and Table A.7.5 (Appendix 7.0).

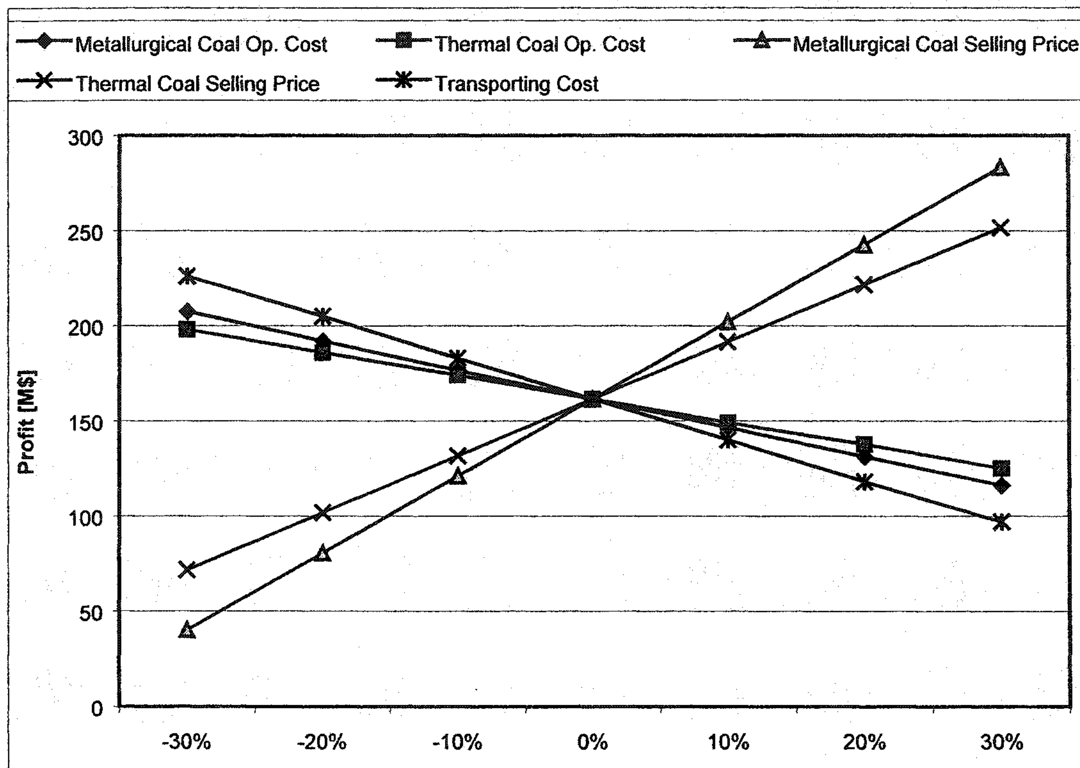


Figure 5.1.4 Profit vs. Costs and Selling Prices Sensitivity Analysis

It can be concluded that the model maximizes the profit by allocating all the available resources to produce metallurgical coal and haul it to Vancouver and to produce thermal coal for the power plant. Transporting metallurgical and thermal coal to Ridley Island is less profitable. Transporting thermal coal to Thunder Bay is the least profitable and it is uneconomic for some of the mines. From the binding constraints and from the magnitude of the objective function coefficients, different scenarios could be developed to improve the final output. The feasible region of any LP is defined by the binding constraints. Any changes in the coefficients of these constraints could result in a non-feasible solution, transform a binding into a non-binding constraint or transform a non-binding into a binding constraint. If a change in any of the objective function coefficients and the constraint coefficients occurs, the LP model should be run again to find out the new feasible optimal solution. To improve the output, some of the binding constraints could be relaxed or tightened and run the model again to find a new optimal feasible solution.

5.1.2 Changes in Market Condition Analysis

In an attempt to improve the solution provided by the base model, different market and coal quality scenarios were developed for thorough characterizations. The management of coal mining companies can use the LP and NLP models to make appropriate decisions, which respond to competitive market environment. In this study, a sensitivity analysis is run to study some changes in the market conditions and in the coal qualities, and their effect on the optimal solution. It must be understood that sensitivity analysis is conducted under limited access to the real information and some of the results could be different in real conditions. This analysis is rather a model validation and it is intended to demonstrate the power and usefulness of the LP model in finding an optimal solution and for management decisions.

Based on the world and domestic coal market analysis provided in Appendix 1.0 and 2.0, it is assumed that the overseas market demand decreases by 1.5 [Mt/y] metallurgical coal and by 1.0 [Mt/y] thermal coal, the domestic market demand increases by 0.5 [Mt/y] metallurgical and 0.5 [Mt/y] thermal coal, while the mine capacities and coal quality requirements stay the same. These changes will affect the coal distribution to the final destination. Therefore, it is assumed that Vancouver's metallurgical coal demand decreases from 5.5 [Mt/y] to 4.25 [Mt/y] and Ridley Island from 1.0 [Mt/y] to 0.75 [Mt/y]. Vancouver thermal coal demand decreases from 2.0 [Mt/y] to 1.5 [Mt/y] and Ridley Island from 2.5 [Mt/y] to 2.0 [Mt/y]. The contracted quantities of thermal coal will decrease from 2.0 [Mt/y] to 1.0 [Mt/y] for Vancouver, from 2.0 [Mt/y] to 1.5 [Mt/y] for Ridley Island and from 1.0 [Mt/y] to 0.5 [Mt/y] for the metallurgical coal to Ridley Island. Thunder Bay's metallurgical coal demand increases from 1.5 [Mt/y] to 2.0 [Mt/y]. Domestic thermal coal demand increases from 2.0 [Mt/y] to 2.25 [Mt/y] at the power plant and from 2.5 [Mt/y] to 2.75 [Mt/y] at Thunder Bay. These changes are illustrated in Table 5.1.1 and 5.1.2.

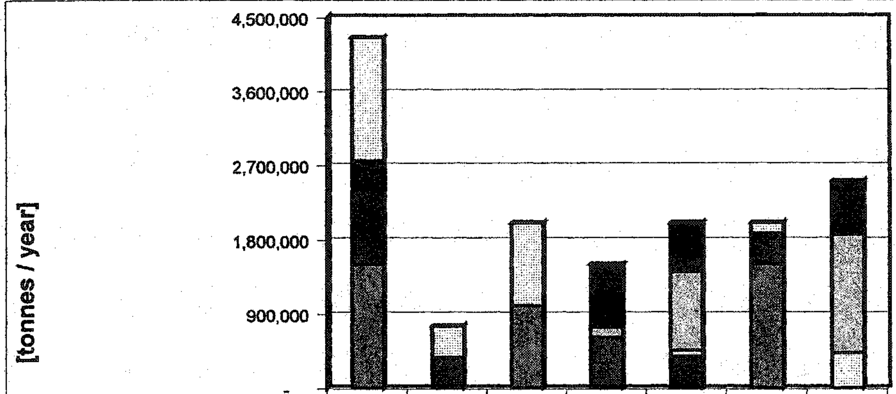
Table 5.1.1 Changes in Market Capacities

Destination	Metallurgical [Mt/y]		Thermal [Mt/y]	
	Before	Current	Before	Current
Vancouver	5.5	4.25	2.0	1.5
Ridley Island	1.0	0.75	2.5	2.0
Thunder Bay	1.5	2.0	2.5	2.75
Power Plant	-	-	2.0	2.5
Total	**8.0**	**7.0**	**9.0**	**8.75**

Table 5.1.2 Changes of Contracted Coal Quantities

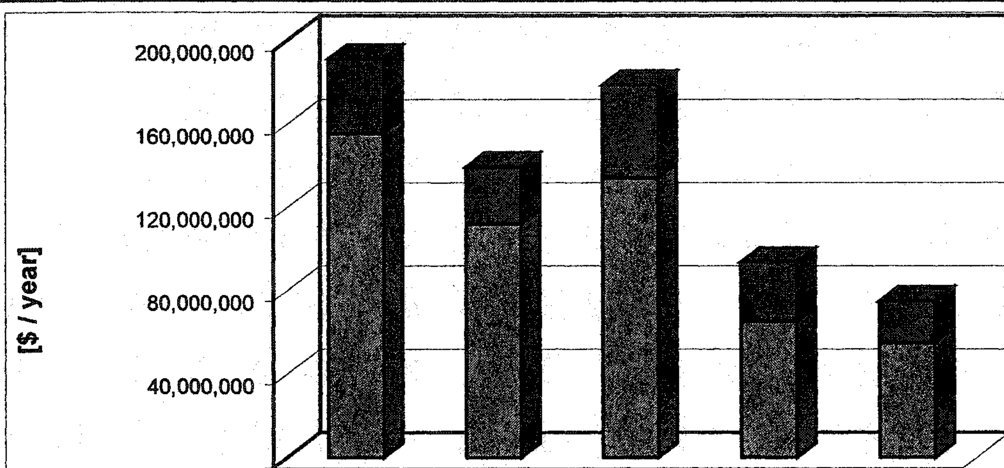
Destination	Metallurgical [Mt/y]		Thermal [Mt/y]	
	Before	Current	Before	Current
Vancouver	3.0	3.0	2.0	1.0
Ridley Island	1.0	0.5	2.0	1.5
Thunder Bay	1.0	1.0	2.0	2.0
Power Plant	-	-	1.5	1.5
Total	**5.0**	**4.5**	**7.5**	**6.0**

Management must rather respond to the model output to maximize profits by transporting metallurgical coal to Vancouver, Ridley Island and Thunder Bay and by hauling thermal coal to the power plant, Vancouver, and Ridley Island to the market's capacity of absorption. Allocate the rest of the resources to produce and haul thermal coal to Thunder Bay only to the limit required by the contract. To meet the market demand, all the mines produce at their maximum capacity. Since metallurgical coal demand dropped by 1.0 [Mt/y], the minimum capacity usage requirements for metallurgical coal should be relaxed, otherwise the model will not be able to find an optimal feasible solution. Using the LP model to find out which of these minimum capacities will be cost efficient to relax, it is recommended that this particular constraint be relaxed at Luscar mine from 2.0 to 1.65 [Mt/y]. This means that producing metallurgical coal at Luscar mine, in these market conditions, is the least profitable compared to Line Creek and Cheviot. Another conclusion that can be drawn is that selling metallurgical coal to the Eastern Canadian Provinces is more profitable than selling thermal coal to overseas customers. The maximized profit for this case is $156,764,264. The coal production and transportation distribution is illustrated in Figure 5.1.5 and Table A.7.6 (Appendix 7.0). It can be seen that all the changes in the coal production and distribution is almost proportional with the base case distribution. The only major change is that Luscar starts to transport thermal coal to Thunder Bay.



	Vancouve	Ridley	Thunder	Vancouve	Ridley	Thunder	Power Pl
	Metallurgical			Thermal			
Obed Mnt.	-	-	0	766,871	592,103	-	641,026
Coal Valley	-	-	-	119,632	948,744	-	1,431,62
Cheviot	1,500,00	375,000	1,000,00	-	72,650	125,000	427,350
Luscar	1,250,00	375,000	0	613,497	386,503	375,000	-
Line Creek	1,500,00	0	1,000,00	-	0	1,500,00	-

Figure 5.1.5 Coal Production Distribution in Market Changes



	Line Creek	Luscar	Cheviot	Coal Valley	Obed Mnt.
Profit [$]	35,985,500	27,310,764	44,763,838	28,583,778	20,120,384
Total Cost [$]	155,264,500	112,001,736	133,923,662	65,166,222	54,879,616

Figure 5.1.6 Profits and Total Costs in Market Changes Case

The Profits distribution changes compared to the base case as illustrated in Figure 5.1.6. The Cheviot project has still the largest profit but with its magnitude dropped by around 2.0 [M$ / y], followed by Line Creek whose profits dropped by 1.0 [M$ / y]. Coal Valley profits increase by 0.6 [M$ / y] and takes Luscar's place whose profit drops drastically by around 6.0 [M$ / y]. Last comes Obed Mountain with an increase in profit by around 1.5 [M$ / y]. It can be concluded that for this market scenario, profitability increases at the mines that produce thermal coal and decreases where they produce mainly metallurgical coal. The percentage of mining and processing and haulage costs compared with the total cost remains relatively the same except Coal Valley. Mining and processing cost share increases to 55.5 % and haulage drops to 35.3% of the total cost at this particular mine.

In the next scenarios, a drastic drop in the overseas coal demand is assumed, based on Appendix 1.0, while the domestic consumption remains stable. The overseas metallurgical coal demand drops by 2.5 [Mt/y] and thermal by 0.5 [Mt/y] to the contracted quantities (No coal could be sold on spot market). The coal haulage distribution is assumed to be as illustrated in Table 5.1.3.

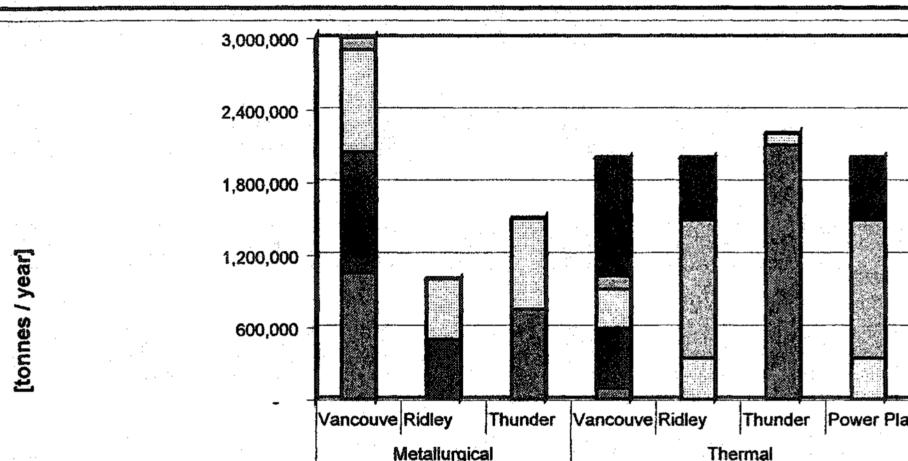
Table 5.1.3 Coal Transport Distribution in an Overseas Market Drop

Destination	**Metallurgical [Mt/y]**		**Thermal [Mt/y]**	
	Before	Current	Before	Current
Vancouver	5.5	3.0	2.0	2.0
Ridley Island	1.0	1.0	2.5	2.0
Thunder Bay	1.5	1.5	2.5	2.5
Power Plant	-	-	2.0	2.0
Total	**8.0**	**4.5**	**9.0**	**8.5**

To respond to these market changes, the minimum capacity production constraints, regarding metallurgical coal, should be relaxed before running the model. Otherwise they will be in conflict with the market absorption capacity constraints and the model will not find a feasible solution. The total minimum capacity usage requirements constraints are binding at Line Creek, Luscar, and Obed Mountain and are not binding at Cheviot and Coal Valley. To find out where production should be reduced to respond efficiently to these market changes, it is recommended that relaxation be considered for the total minimum capacity requirements. After relaxing both the total and the metallurgical coal minimum capacity usage requirements, a feasible optimal solution was found and a profit of $130,832,290. According to the model output, it was recommended that metallurgical coal production at Line Creek be reduced from 2.5 [Mt/y] to 0.75 [Mt/y] and increase from 2.5 [Mt/y] to 2.75 [Mt/y] at Cheviot. This drastic reduction in metallurgical coal production at Line Creek could affect the processing and upgrading facilities efficiency. By changing the minimum metallurgical coal production requirements, another level of production and feasible solution could be found. For instance, a level of metallurgical production of 1.8 [Mt/y] at Line Creek, 1.5 [Mt/y] at Luscar, and 2.1[Mt/y] at Cheviot provides an optimal feasible solution with a profit of $129,238,722 ($1.5 M less). Management could chose between these two or other options that fit within the strategic plan of the company. The new optimal minimum capacities are illustrated in Table 5.1.4, for the first option and the solution for coal production and haulage is illustrated in Figure 5.1.7 and Table A.7.7 (Appendix 7.0) for the second option.

Table 5.1.4 The New Minimum Capacities Usage Requirements

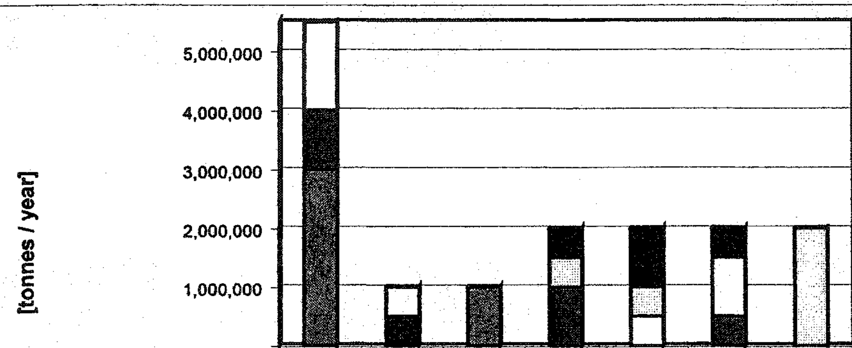
Mine	Total Production		Metallurgical	
	Before	Current	Before	Current
Line Creek	3.2	2.5	2.5	0.75
Luscar	2.5	2.0	2.0	2.0
Cheviot	3.2	3.2	2.5	2.75
Coal Valley	1.5	1.5	-	-
Obed Mnt.	1.5	1.0	-	-
Total	**11.9**	**10.2**	**7.0**	**5.5**



	Vancouve	Ridley	Thunder	Vancouve	Ridley	Thunder	Power Pla
Obed Mnt.	-	-	-	974,359	512,821	-	512,821
Coal Valley	100,000	-	0	109,402	1,145,29	-	1,145,29
Cheviot	850,000	500,000	750,000	324,573	341,880	91,667	341,880
Luscar	1,000,00	500,000	-	500,000	-	-	-
Line Creek	1,050,00	0	750,000	91,667	0	2,108,33	-

Figure 5.1.7 Coal Production in a Market Drop

Another scenario, in which the coal production and transport are set arbitrarily, could be taken into considerations. The purpose of this scenario is to determine the difference between an optimized and a non-optimized case. In this scenario, the blending constraints are not taken into consideration. Figure 5.1.8 and Table A.7.8 (Appendix 7.0) illustrate the coal production distribution and transport using the best estimates according to the magnitude of the profit function coefficients. The total coal production and distribution of the various mine versus destinations is the same as for the base case but the coal quantities produced and hauled from the mine sites are different.



	Vancouve	Ridley	Thunder	Vancouve	Ridley	Thunder	Power Pla
	Metallurgical			Thermal			
Obed Mnt.	-	-	-	500,000	1,000,00	500,000	-
Coal Valley	-	-	(0)	500,000	500,000	-	2,000,00
Cheviot	1,500,00	500,000	-	-	500,000	1,000,00	-
Luscar	1,000,00	500,000	-	1,000,00	-	500,000	-
Line Creek	3,000,00	0	1,000,00	-	0	-	-

Figure 5.1.8 Non-Optimized Coal Production

The profit for this case is $159,956,500. The difference between the optimized and the non-optimized profit is $ 0.7 M. Since the blending problem is not solved, it will cost the company a few millions to bring the coal to the required qualities at each of the mines.

5.1.3 Changes in Coal Quality

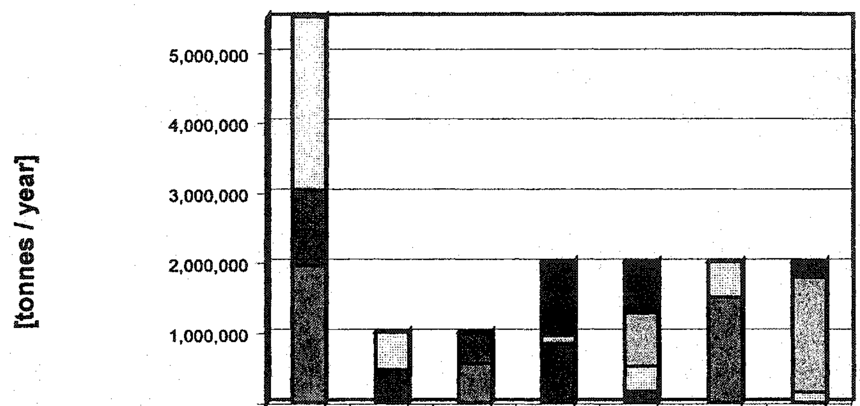
Coal deposits have different ash content (See Appendix 2.0). Some of the coals are easier and cheaper to wash than others. The ash content has generally two components; the inherit ash content depending on the coal petrography (hard to wash), and the ash coming from dilution during coal extraction process which is relatively easy to wash. For coals with high-inherent ash content there is an exponential interdependence between the cost of washing, coal losses, and the ash content during processing. The lower the ash content is wanted, the higher the washing cost and coal losses [31]. It is assumed that this is the case with Luscar-Sherritt mines. For instance, it is assumed that Cheviot mine has high-inherent ash content while Line Creek and Luscar have low inherent ash content. In these scenarios one can allow an increase in the ash content at Cheviot and blend the coal to get the required qualities. It is assumed that an increase in the Cheviot's cleaned coal ash content by 5 percent will give a reduction in the mining and processing costs of 5 percent for the metallurgical coal and a cost decrease of 3 percent for thermal coal. Cheviot's ash content will increase from 8.5% to 8.925% for metallurgical and from 13.8% to 14.49% for thermal coal. Mining and processing costs, for Cheviot, will decrease from $20.50 to $19.48 per tonne for metallurgical and from $16.50 to $16.01 per tonne for thermal coal. An optimized solution produced a slightly different coal production and transport distribution. The profit was increased by almost $ 4.0 millions from $160.60 millions, in the base case, to $164.40 millions. This improvement was limited to 5% percent increase in the ash content. Any further increase of the ash content in the coal produced at Cheviot is not possible without affecting the optimality of the solution. If it is increased the model will not be able to find a feasible solution. More improvements could only be made by decreasing the ash content at other mines. The decrease of the ash content in cleaned coal is recommended to be implemented at mines where it would be easier and cheaper to process.

At this stage most of the lower ash limit constraints are binding. These binding constraints force the model to consider transportation of coal over larger distances for blending purposes, and as a result cost increases. To overcome this problem the ash content could be increased at some of the mines, or another option is to decrease the lower limit of ash requirement. As it was mentioned before, by allowing an increase in the ash content, the processing costs could be reduced. Therefore, it is appropriate to increase the ash content at some mines. For instance by increasing the ash content in thermal coal from 12% to 13% at Obed Mountain and from 12% to 12.25% at Coal Valley, the profit will increase from $164.4M to $164.7M. No improvement could be made by increasing the ash content in thermal coal at Line Creek and Luscar. Improvements can be made by decreasing the ash content at Luscar but this means higher processing costs, and therefore, further analysis is needed before making a decision. A sensitivity analysis conducted on the ash content in the metallurgical coal at Line Creek suggested that improvements could be made by increasing the ash content from 7.5% to 8.2%, which is the optimal. These changes increased the optimized profit to $ 164.8M. Increasing the ash content at Luscar could make no improvements. The increase in profit resulted from the blending and haulage optimization only and no reduction in the processing costs, as a result of the increased ash content, was considered. The optimal ash content of cleaned coal, at each particular mine, obtained from the LP model are illustrated in Table 5.1.5.

Table 5.1.5 Luscar-Sherritt's Mines Optimized Ash Contents

Mine	Metallurgical [%]	Thermal [%]
Line Creek	8.15	12.50
Luscar	8.00	13.10
Cheviot	8.925	14.49
Coal Valley	9.00	12.25
Obed Mnt.	8.75	13.00

The moisture content of metallurgical coal at Cheviot can be increased from 7.8% to 7.95% without affecting the optimized results. Any further increase in the moisture content of metallurgical coal at Cheviot will have a negative impact on the profit. If the moisture content decreases from 8.2% to 8.0% at Line Creek the profit increases to $165.7M. No further improvements resulted if the moisture decreased below 8.0%. For thermal coal, only the moisture content at Line Creek can be increased from 13.2% to 13.3% without affecting the optimal solution. Any further increase in the moisture content will result in a profit decrease or the model will not find an optimal solution. Decreasing the moisture content, at any of the mines, could make improvement but this will increase processing costs. All the above model variation resulted in a profit increase from $160,626,790, in the base case, to $164,829,198. The new optimal coal production and transport distribution is illustrated in Figure 5.1.9 and Table A.7.9 (Appendix 7.0). In this scenario, the total revenue of the company is unchanged ($705.0 M) compared with the base case. The improvement resulted from decreasing the processing and haulage costs. For instance, the total cost of the company decreased form $544.3 M to $540.1 M.



	Metallurgical			Thermal			
	Vancouve	Ridley	Thunder B	Vancouve	Ridley	Thunder B	Power Pla
Obed Mnt.	-	-	-	1,048,951	728,166	-	222,883
Coal Valley	-	-	-	111,888	759,583	-	1,628,529
Cheviot	2,466,667	533,333	-	-	351,412	500,000	148,588
Luscar	1,088,889	466,667	444,444	839,161	160,839	-	-
Line Creek	1,944,444	0	555,556	0	0	1,500,000	-

Figure 5.1.9 Improved Coal and Transport Distribution

From the base case model and its scenarios, the most profitable option is to produce and sell metallurgical coal on the overseas market, followed by transporting coal to the power plant. The other best option is to transport metallurgical coal to the Eastern Canadian provinces, and the next option is to transport thermal coal to overseas customers. The last option is to ship thermal coal to Thunder Bay.

5.1.4. Luscar-Sherritt Economic Risk Analysis

In this part of the study, stochastic modeling and sensitivity analyses are conducted using the Bestfit and @Risk simulation packages [28, 32, 41]. Bestfit produces the best statistical distribution of the input variables for stochastic modeling and that for output results. These design and operating risks are captured in a quantitative model and simulated over an extended period using the Latin Hypercube and Monte Carlo simulation techniques with @RISK. The results of the simulation experiments enable analysts to predict the associated short-and long-term risks. The significance of the input variables and the definition of their stochastic processes are determined using the variance propagation and Bestfit algorithms. The Latin Hypercube simulation technique is used to simulate 10,000 iterations in one simulation run to ensure variance stability and accurate results.

The profitability of a coal mine is very sensitive to changes in the coal prices, mining and processing costs, and the haulage costs from the mine to customers. To analyze the risk associated with Luscar-Sherritt mines, the coal prices, mining, processing and transportation costs were input in the model as stochastic variables. The lognormal distribution is used to describe the stochastic behavior of coal prices. The lognormal distribution was chosen as a result of the analyses conducted on coal prices distribution using Bestfit software. The results are presented in Table A.7.10 (Appendix 7.0). The risk associated with the coal prices was analyzed and it is presented in Figure A.7.1 (Appendix 7.0). Mining and processing costs were assumed to have a truncated lognormal distribution with a coefficient of variation (COV) of 20% a minimum of 75% and a maximum of 125% of the expected value. The truncated lognormal distribution was selected to characterize the mining and processing costs behavior because of technical and economical reasons. For instance, high costs will drive the company into bankruptcy and very low costs are not technically possible. The characteristics that define the mining and processing costs probability distribution are presented in Table A.7.11 (Appendix7.0). Railway costs [$ /km-t], have a truncated lognormal distribution with a COV of 10%, a minimum and a maximum of 85% and respective 115% of the expected value. These assumptions were made based on the same reasoning as for the mining and processing cost. Railway costs are less volatile than mining and processing costs and the probability characteristics that governs them is presented in Table A.7.12 (Appendix 7.0).

The Latin Hypercube simulation technique was used to simulate 10,000 iterations in one simulation run with the total cost, revenue and profit by mine as output cells. Even though the experimental design showed that 5000 iterations would be adequate, 10000 iterations were simulated to increase the accuracy. The cost of timing is not of concern since it takes less than 2 minutes to run the simulation. The risk associated with the costs of each particular mine are presented in Figures 5.1.10 and 5.1.11. The expected total costs and the associated uncertainty for producing and transporting coal from Line Creek, Luscar and especially Obed Mountain are significantly higher. As a result further work must be carried out to reduce these uncertainties. The revenues from the various mines in Figure A.7.2 and A.7.3 (Appendix 7.0) show similar levels of uncertainty ranging between $50M and $235M. It is important that production and commodity prices be carefully modeled and analyzed to ensure revenue stability.

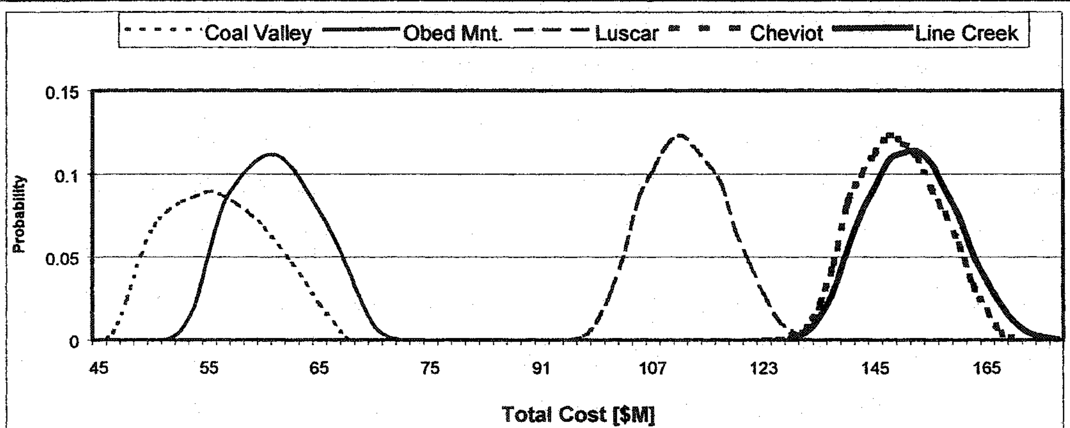


Figure 5.1.10 Risk Characterization of Mine's Total Costs

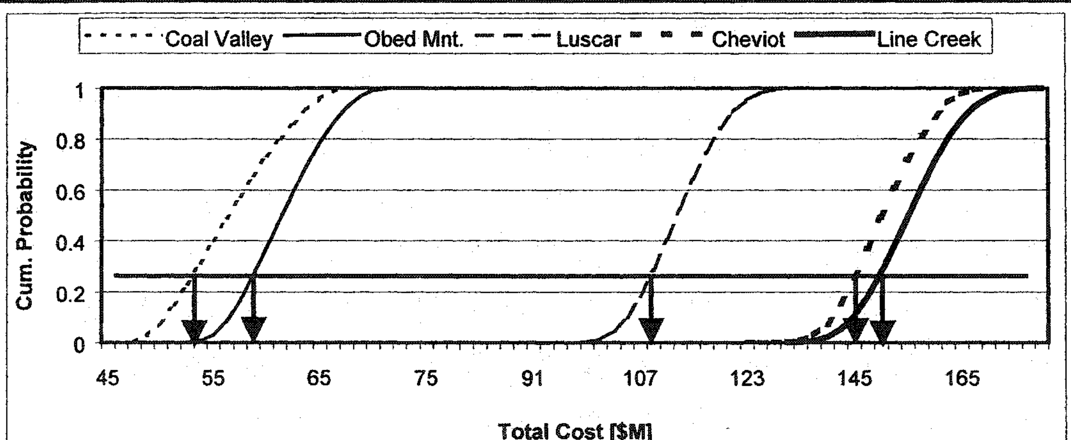


Figure 5.1.11 Cumulative Probability of Total Costs by Mine

Figures 5.1.12 and 5.1.13 show that the profits from Obed Mountain and Luscar have relatively high degree of uncertainty with lower expected profits. Obed Mountain and Line Creek have higher expected profits compared to that from the other mines. As a result, attention must be paid to these mines to reduce the level of uncertainties and to ensure company's profit stability. Figures 5.1.14 and 5.1.15 show that there is only 5% probability that Obed Mountain will have a loss, 10% probability that the profit will be below $1.57M, and 50 % risk that the profit will be lower than $10M. There is no risk of failure for Coal Valley; there is 10% probability that the profit will be below $22.3M and 50% that it will be lower than $32.7M. For Line Creek, Luscar and Cheviot the risk of having a negative return is lower than 1%. There is 10% probability that the profit will be below $13.4M for Line Creek, $18.4M for Cheviot and $13.9M for Luscar. Line Creek will have $29.7M, Cheviot $36.8M and Luscar $26.0M for a 50% risk.

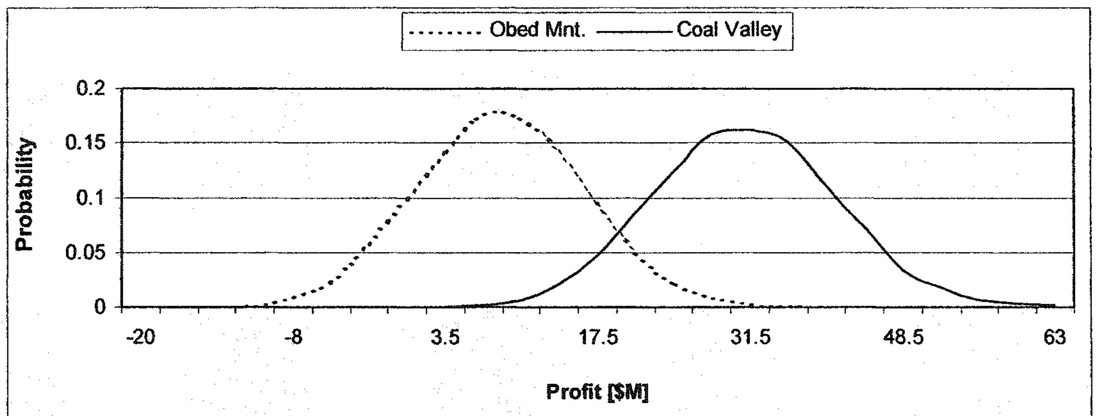


Figure 5.1.12 Risk Characterization of Mine's Profits

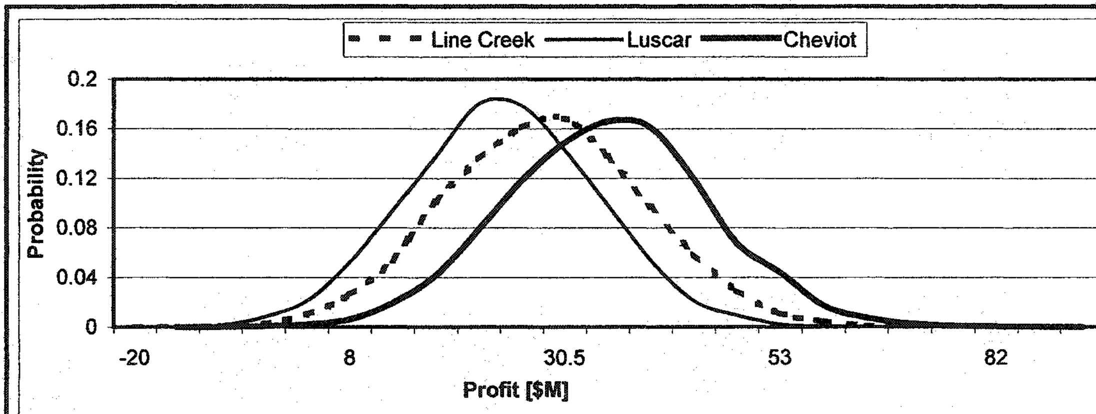


Figure 5.1.13 Risk Characterization of Mine's Profits

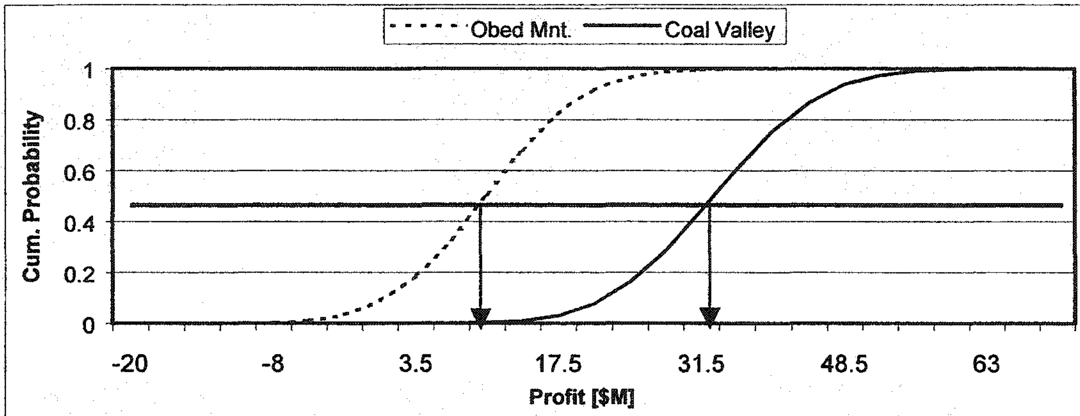


Figure 5.1.14 Probability of Success of Profits by Mine

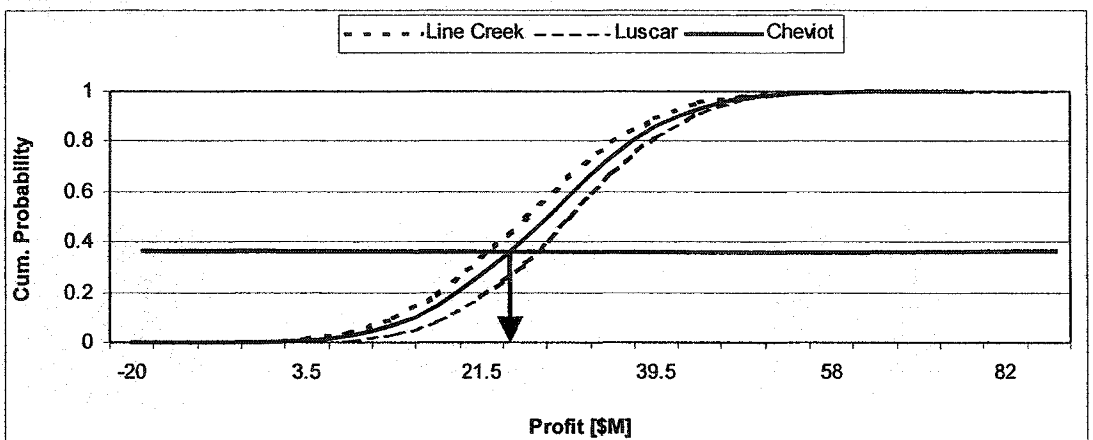


Figure 5.1.15 Probability of Success of Profits by Mine

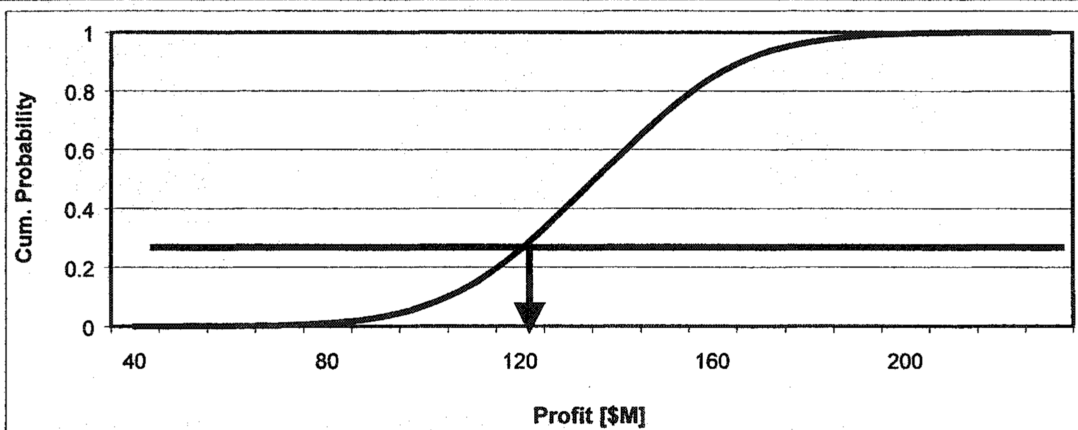


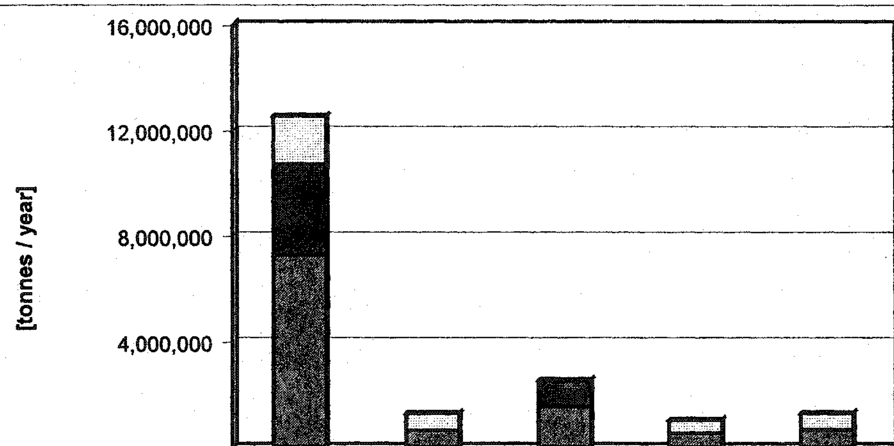
Figure 5.1.16 Company's Probability of Success at Different Profit Margins

Figures 5.1.16 and A.7.4 (Appendix 7.0) illustrate the risk characterization and the probability of success associated with Luscar-Sherritt operations. The above figures illustrate the probability of failure associated with the five mines. Mine managers can use this probability of failure concept to gauge the viability of projects in terms of risk and return. These figures show that overall the company is profitable under the current and anticipated technical, economic and uncertainty environments. The probability of failure is very low for all of the mines.

5.2 Analyses and Discussion of Fording Coal Optimization Models

5.2.1 Analysis of the Basic Optimization Model

An optimum profit of $247,290,805 was obtained by solving the LP base model. The NLP converged to exactly the same solution, which provides confidence in the results. The optimal coal production and transport distribution are illustrated in Figure 5.2.1 and Table A.8.1 (Appendix 8.0).

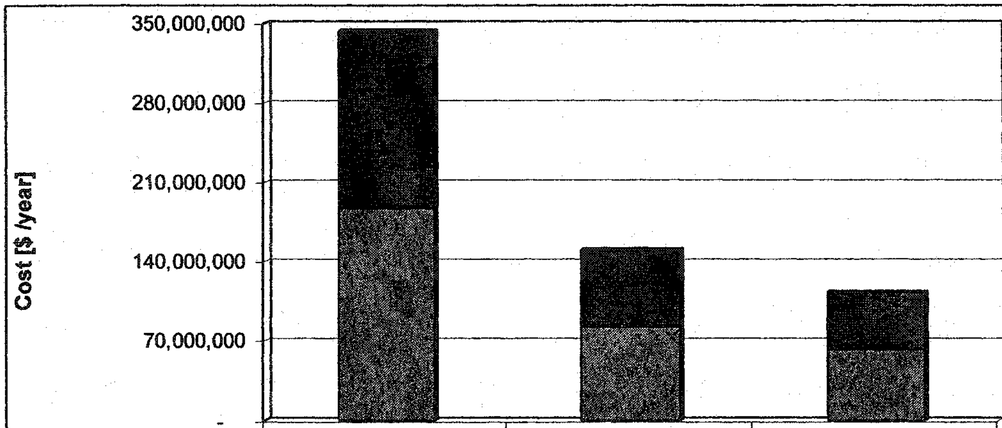


	Metallurgical		Thermal		
	Vancouver	Thunder Bay	Vancouver	Thunder Bay	Power Plant
Coal Mnt.	1,807,203	625,000	-	474,576	593,220
Greenhills	3,463,983	-	997,881	16,949	21,186
Fording River	7,228,814	625,000	1,502,119	508,475	635,593

Figure 5.2.1 Fording Coal's Production Distribution

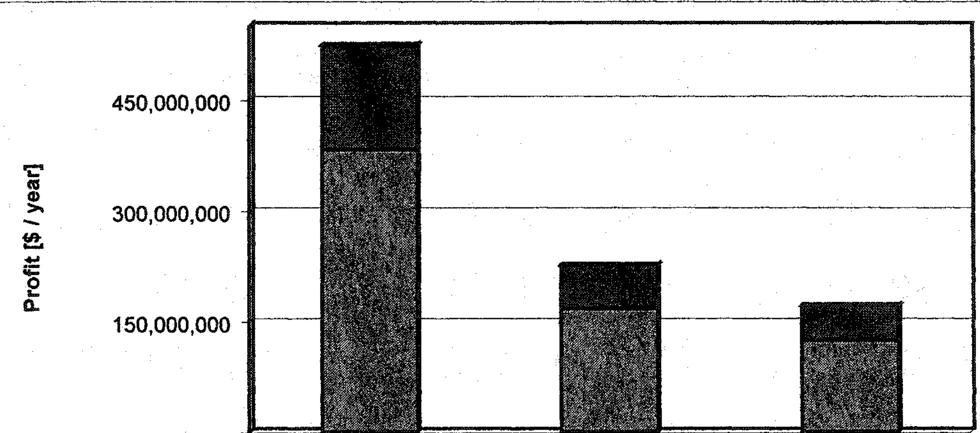
This figure illustrates the level of coal production and haulage distribution for each mine. According to the model's output Fording Coal should produce 13.75 [Mt] of metallurgical and 4.75 [Mt] of thermal coal annually. The metallurgical coal production occurs at all the mines and 12.50 [Mt / y] are hauled to Vancouver and 1.25 [Mt / y] to Thunder Bay. Thermal coal production also occurs at all the mines and 2.5 [Mt / y] are hauled to Vancouver, 1.0 [Mt / y] to Thunder Bay and 1.25 [Mt / y] to Brooks power plant. Fording River produces 7.85 [Mt / y] of metallurgical coal of which 7.23 [Mt / y] is hauled to Vancouver and 0.62 [Mt / y] to Thunder Bay. It also produces 2.65 [Mt / y] of thermal coal of which 1.50 [Mt / y] is hauled to Vancouver, 0.50 [Mt / y] to Thunder Bay, and 0.64 [Mt / y] to Brooks' power plant. Greenhills produces 3.46 [Mt / y] of metallurgical coal which is transported to Vancouver and 1.04 [Mt / y] of thermal coal of which 0.99 [Mt / y] is hauled to Vancouver, and approximately 0.02 [Mt / y] to each Thunder Bay and Brooks' power plant. Coal Mountain produces 2.43 [Mt / y] of metallurgical coal of which 1.80 [Mt / y] is hauled to Vancouver and 0.63 [Mt / y] to Thunder Bay. The total quantity of thermal coal produced at Coal Mountain is 1.07 [Mt / y] and 0.47 [Mt / y] is hauled to Thunder Bay and 0.59 [Mt / y] to Brooks' power plant.

The mining, processing and haulage costs are the main factors that drive the total cost of a mine. Figure 5.2.2 illustrates the magnitude of these costs at each mine. Transportation costs represent 41% of the total cost at Fording River, 40.3% at Greenhills, and 40.0% at Coal Mountain. Mining and processing costs represent 48.7% at Fording River, 49.2% at Greenhills, and 50.3% Coal Mountain. The magnitude of the profits achieved and the total costs that occur at each mine are illustrated in Figure 5.2.3. The largest profit is achieved by Fording with 141.3 [M$ /y] followed by Greenhills with 58.7 [M$ /y], and Coal Mountain with 47.3 [M$ /y]. The total cost has a similar distribution and Fording River has a cost of 523.3 [M$ /y] followed by Greenhills with 152.8 [M$ /y], and Coal Mountain with 171.4 [M$ /y]. The total cost of Fording Coal operations is 673.3 [M$ /y] with revenue of 920.6 [M$ /y]. These costs and profits occurred as if all the mines produced as a single mine, and they could be different if each mine optimizes the coal production and haulage without taking into consideration the other mines. Other relevant results regarding the coal distribution, cost, revenues and profits are illustrated in Table A.8.2 and A.8.3 (Appendix 8.0).



	Fording River	Greenhills	Coal Mtn.
Transport	156,492,966	67,462,436.4	49,763,792.4
Mining & Proc.	186,311,441	82,391,949	62,421,610

Figure 5.2.2 Fording Coal's Mining and Processing and Haulage Costs



	Fording River	Greenhills	Coal Mtn.
Profit	141,296,970	58,733,242	47,260,593
Total Cost	382,040,953	167,172,479	124,120,763

Figure 5.2.3 Fording Coal's Profits vs. Total Costs

The sensitivity report (Appendix 6.0) provides the objective function coefficients in [$/t] for each of the variables. The largest objective function coefficients are achieved by transporting metallurgical coal to Vancouver. They range from $16.125 to $16.52 per tonne. Transporting metallurgical coal to Thunder Bay gives a range of profits from $14.43 to $14.99 per tonne. Thermal coal hauled to Brooks' power plant gives a range of profits from $11.39 to $11.63 per tonne. Thermal coal hauled to Vancouver provides profits between \$2.625 and \$3.02 per tonne. Transporting thermal coal to Thunder Bay is barely profitable with a profit range between \$0.93 and \$1.49 per tonne.

The feasible region of Fording Coal LP model is defined by market demand and supply constraints, mine capacities, and blending constraints. None of the port and railway limitations is binding. The overall metallurgical and thermal coal production is limited by the overseas and domestic market capacity. All the market supply constraints are binding. Fording Coal's production capacity is larger than the market capacity. There is only one binding contractual constraint with regards to thermal coal shipped to Thunder Bay. The maximum mine capacity constraints are binding for Fording River and Coal Mountain but not for Greenhills. The output of the model suggests that Fording River and Coal Mountain should produce at their maximum capacity to increase profitability, while Greenhills should produce less because of the higher mining and processing costs. None of the minimum capacity usage requirements are binding.

The lower ash limit constraint is binding for metallurgical coal blending stockpile in Vancouver. The upper ash limit constraint, for thermal coal, is binding for the blending stockpiles at both Thunder Bay and power plant. The lower sulfur limit constraint is binding at Thunder Bay's thermal stockpile and at the power plant. The upper moisture limit constraint, for metallurgical coal stockpile, is binding at Thunder Bay. None of the other blending constraints are binding. Some of the blending constraints have very small margins and could become binding at the slightest change in coal quality. The upper limits of sulfur and moisture constraints have very small margin for the metallurgical and respective thermal stockpiles at Thunder Bay. The lower moisture limit constraint has a small margin at the power plant.

Sensitivity analysis on the base case model was conducted on the price of coal, the mining and processing costs of metallurgical and thermal coal, and transporting costs. The metallurgical coal price has the greatest impact on the profit, followed by the haulage cost, metallurgical coal operating cost, the price of thermal coal and the least sensitive is the thermal coal operating cost. The results of the sensitivity analysis are presented in Figure 5.2.4 and Table A.8.4 (Appendix 8.0)

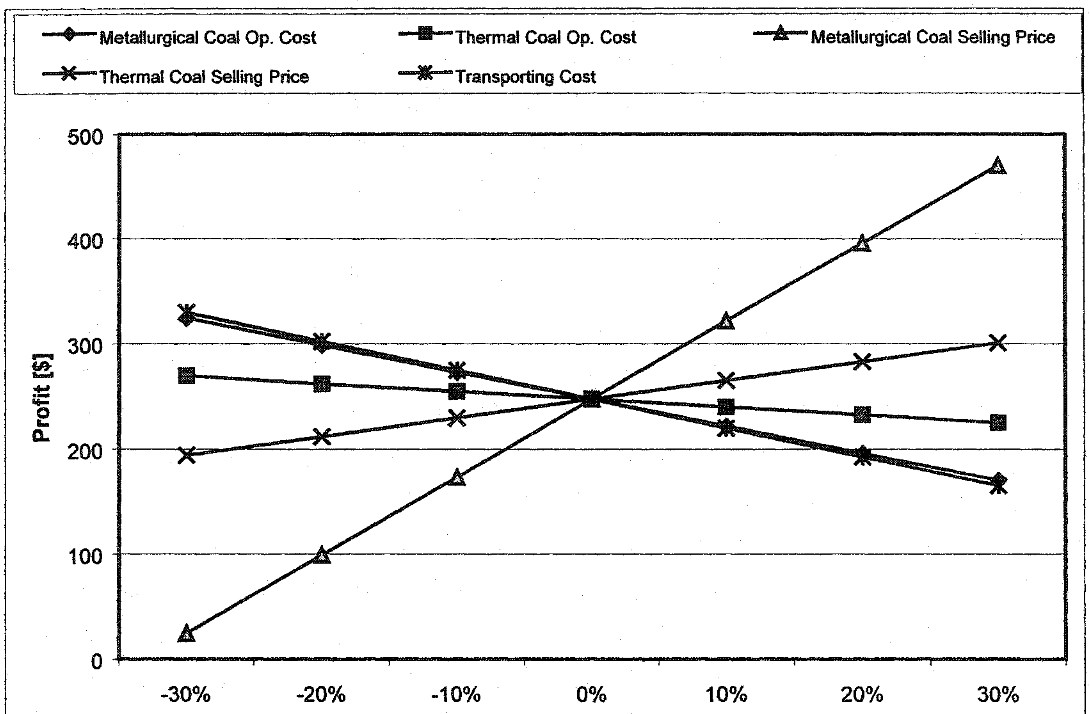


Figure 5.2.4 Profit vs. Coal Price and Costs Sensitivity Analysis

It can be concluded that the model maximizes the profit by allocating all the available resources to produce metallurgical coal and ship it to Vancouver and to produce thermal coal for the power plant. Transporting metallurgical and thermal coal to Thunder Bay is less profitable. From the binding constraints and from the magnitude of the objective function coefficients, different scenarios could be developed to improve the final output. Some of the binding constraints could be relaxed or tightened to find a new optimal feasible solution, which improves the output.

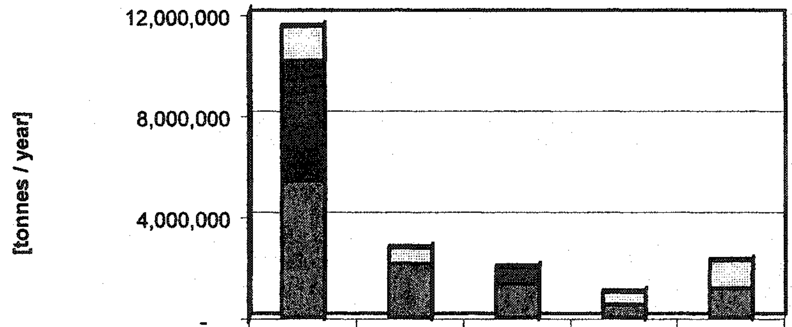
5.2.2 Changes in Market Condition Analysis

Different market and coal quality scenarios were developed in an attempt to improve the solution provided by the base model. A sensitivity analysis is run to study some changes in the market conditions and in the coal qualities. The effect of these changes on the optimal solution it is also analyzed. Based on the world coal market analysis provided in Appendixes 1.0 and 2.0, it is assumed that the overseas market demand decreases by 1.0 [Mt/y] metallurgical coal and increases by 0.5 [Mt/y] thermal coal. The domestic market demand increases by 1.5 [Mt/y] metallurgical, and by 1.0 [Mt/y] thermal coal at Thunder Bay and by 1.0 [Mt/y] at the power plants. Table 5.2.1 illustrates the changes in market conditions. The mine capacities and contractual requirements stay the same.

Table 5.2.1 Changes in Market Capacities

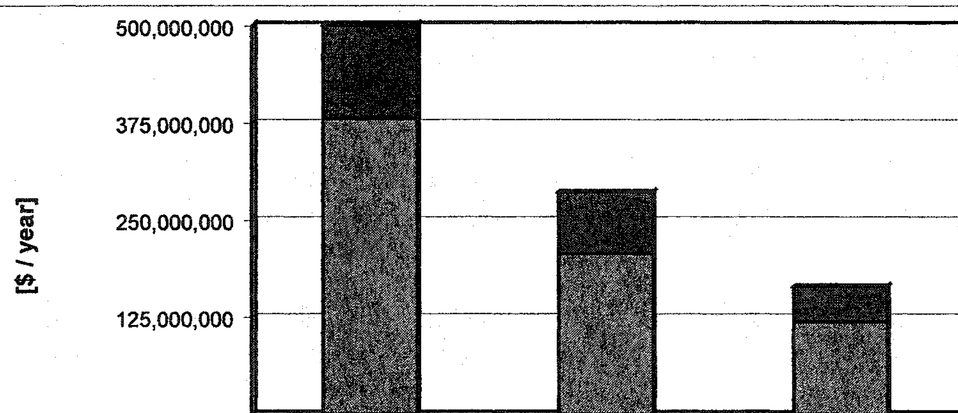
Destination	Metallurgical [Mt/y]		Thermal [Mt/y]	
	Before	Current	Before	Current
Vancouver	12.5	11.5	2.5	3.0
Thunder Bay	1.25	2.75	1.0	2.0
Power Plant	-	-	1.25	2.25
Total	**13.75**	**14.25**	**4.75**	**7.25**

The model responds by allocating all the resources available to produce and transport metallurgical coal to Vancouver and to Thunder Bay to the upper limit of the market capacity of absorption. Thermal coal is produced and transported to Vancouver only to satisfy the contractual requirements. Regarding the power plant, the model allocates thermal coal to the upper market limitation. This response indicates that transporting metallurgical coal to the Eastern Canadian Provinces is more profitable than selling thermal coal to overseas customers. In these scenarios, the coal demand exceeds the mines' total capacity. Therefore, all the mines produce coal at their maximum capacity. All three mines produce more metallurgical coal than their minimum capacity usage requirement. The maximized profit for this scenario is $263,272,770 with an increase of 15.9 [$M] compared with the base case. The coal production and shipments distribution is illustrated in Figure 5.2.5 and Table A.8.5 (Appendix 8.0).



	Metallurgical Vancouver	Metallurgical Thunder B	Thermal Vancouver	Thermal Thunder B	Thermal Power Pla
Coal Mnt.	1,344,350	613,277	-	474,576	1,067,797
Greenhills	4,778,249	-	666,667	16,949	38,136
Fording River	5,377,401	2,136,723	1,333,333	508,475	1,144,068

Figure 5.2.5 Coal Production Distribution: Market Changes



	Fording River	Greenhills	Coal Mtn.
Total Profits	138,549,299	79,249,386	45,474,085
Total Costs	379,183,751	205,841,716	118,076,763

Figure 5.2.6 Total Costs and Profits: Market Change Case

The profits and total costs distributions are similar to the base case but different in magnitude as illustrated in Figure 5.2.6. Fording River leads but with profits dropped by around 2.7 [M$ / y] followed by Greenhills with profits increased by 20.5 [M$ / y], and Coal Mountain with a decrease in profit by around 1.8 [M$ / y]. It can be concluded that for this market scenario, profitability increases at Greenhills because it produces more metallurgical coal than in the base case and transports it to Vancouver. Fording River and Coal Mountain profitability decreases because they transport more metallurgical and thermal coal to Thunder Bay to satisfy the market demand. The LP model optimizes the coal production and transport distribution as an integrated system and the increase that occurs at Greenhills compensates the decrease in profits at Fording River and Coal Mountain. The percentage of mining and processing and haulage costs compared with the total cost remains relatively the same as for the base case. Mining and processing cost share increases at Greenhills by 0.50 %. The transport cost increases at Fording River by 0.20 % and decreases by 0.33 % at Greenhills. The revenue of the company for this scenario is $966.3 M.

5.2.3 Changes in Coal Quality Analysis

It is assumed that the mining and processing cost at Coal Mountain mine will decrease by 5% if the ash content in cleaned coal increases by 5% in both metallurgical and thermal coal. The mining and processing cost of metallurgical coal changes from 18.75 to 17.81 [$/t] and the cost of thermal coal from 15.75 to 14.96 [$/t]. An increase of 5% in the ash content results in a change from 9.0% to 9.45%, for metallurgical coal, and from 13.8% to 14.49%, for thermal coal. Profit increases from $247.29 M to $250.17 M. A net increase of $2.88 M resulted, compared with the base model, just by blending the coal and avoiding some of the mining and processing costs.

It must be understood that any increase in the ash content, that does not affect the optimal solution, decreases the processing costs. Any decision to decrease the ash content at a particular mine must consider the corresponding increase in the processing costs. From a sensitivity analysis on the ash content in the metallurgical coal, it is realized that no improvements in the profit occur by increasing the ash content of metallurgical coal at Fording River without affecting the optimal solution. The ash content of metallurgical coal at Greenhills can be increased from 8.0% to 9.45% without affecting the optimal solution. By allowing this increase, the processing costs at Greenhills could be substantially reduced. If the ash content at Coal Mountain is increased, the profit decreases. If the ash content of metallurgical coal at Fording River and Coal Mountain decreases, the profit increases but an increase in the processing costs must be taken into account. No change in profit occurs by decreasing the ash content at Greenhills. Any increase in the ash content of thermal coal at Fording River, Greenhills, and Coal Mountain results in a profit decrease because the blending constraints force the coal to be transported over larger distances. The optimal ash contents from the model are illustrated in Table 5.2.2.

Table 5.2.2 Fording Coal's Optimized Ash Contents

Mine	Metallurgical [%]		Thermal [%]	
	Existent	Optimal	Existent	Optimal
Fording River	7.75	7.75	12.25	12.25
Greenhills	8.00	9.45	13.10	13.10
Coal Mtn.	9.00	9.45	13.80	14.49

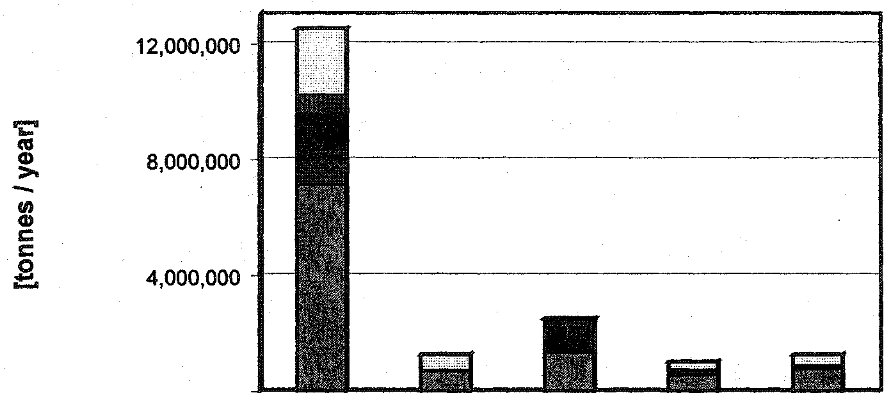
The moisture content of metallurgical coal at Fording River can be increased from 7.50 % to 7.60%, and from 7.75% to 8.55% at Greenhills without affecting the optimal solution. Any further increase results in a profit decrease. No improvement in profit can be made by increasing the moisture content at Coal Mountain. The moisture content of thermal coal can be increased from 13.75% to 13.85% at Greenhills and from 13.80% to 13.90% at Coal Mountain without affecting the optimal solution. Any further increase will reduce the profit. No increase in the moisture content of thermal coal can be made at Fording River without a negative effect on profit. The optimal moisture contents are illustrated in Table 5.2.3.

Table 5.2.3 Fording Coal's Optimized Moisture Contents

Mine	Metallurgical [%]		Thermal [%]	
	Existent	Optimal	Existent	Optimal
Fording River	7.50	7.60	13.20	12.25
Greenhills	7.75	8.55	13.75	13.10
Coal Mtn.	8.50	8.50	13.80	14.49

The lower sulfur limit constraints are binding at Thunder Bay's thermal stockpile and at the power plant. These constraints force the coal to be hauled through long distances for blending purposes. The lower sulfur limit requirement is imposed by the company and therefore, an analysis should be conducted to find its proper value. A sensitivity analysis conducted on this parameter suggests that for a decrease in the lower sulfur content limit in the thermal coal from 0.75% to 0.74% results in a profit increase from $250,177,700 to $ 250,194,913. No further increase results if the lower sulfur content is decreased below

0.74%. The optimal values for the ash, moisture and sulfur results in a profit increase from \$247,290,805 to \$250,194,913. The optimal solution for the coal production and transport distribution in the coal quality scenario is illustrated in Figure 5.2.7 and Table A.8.6 (Appendix 8.0).



	Metallurgical		Thermal		
	Vancouver	Thunder Ba	Vancouver	Thunder Ba	Power Plant
Coal Mnt.	2,291,537	551,471	-	270,035	386,958
Greenhills	3,092,228	-	1,153,846	170,732	83,194
Fording River	7,116,235	698,529	1,346,154	559,233	779,848

Figure 5.2.7 Fording Coal's Improved Coal Production

In this scenario, the total revenue of the company is unchanged (\$920.6 M) compared with the base case. The improvement resulted from decreasing the processing and haulage costs alone. For instance, the total cost of the company decreased form \$673.3 M to \$670.43 M.

5.2.4 Fording Economic Risk Characterization

To analyze the risk associated with Fording Coal mines, the coal prices, mining and processing costs and transportation costs were input in the model as stochastic variables. The lognormal distribution is used to describe the stochastic behavior of coal prices as presented in Appendix 7.0 Table A7.10. The risk associated with the coal prices was analyzed and it is presented in Figure A.7.1 Appendix 7.0. Mining and processing costs were assumed to have a Truncated Lognormal distribution with COV = 20%, a minimum of 75% and a maximum of 125% of the expected value. The truncated lognormal distribution is preferred to characterize the mining and processing costs, since it is not economically and technically feasible that these costs take higher or lower values than certain limits. The characteristics that define the mining and processing costs probability distribution are presented in Table A.8.7 (Appendix 8.0).

Railway Costs [$ /km-t] have a truncated lognormal distribution with COV = 10% a minimum and maximum of 85% and 115% of the expected value. Railway costs distributions follow the same logic as mining and processing costs with the difference that they have lower degree of uncertainty. Table A.8.8 (Appendix 8.0) presents the probability characteristics. The Latin Hypercube simulation technique was used to simulate 10,000 iterations in one simulation run with the total cost, revenue and profit by mine as output cells. The experimental design conducted, showed that 5000 iterations would be adequate. However, 10000 iterations were simulated to increase the accuracy since the cost of timing is not of concern. The probabilities and the risk associated with the total costs of Fording Coal's operations are presented in Figures 5.2.8, and 5.2.9.

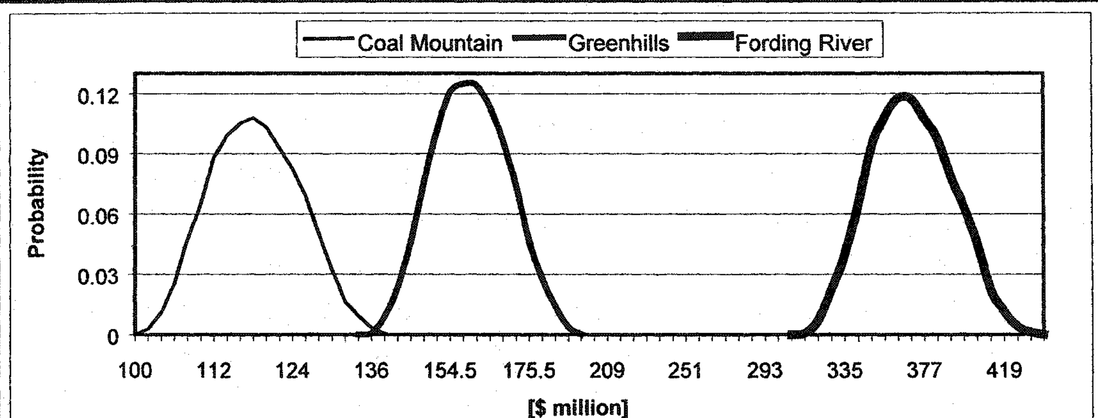


Figure 5.2.8 Risk Characterization of Fording Coal's Mines Total Cost

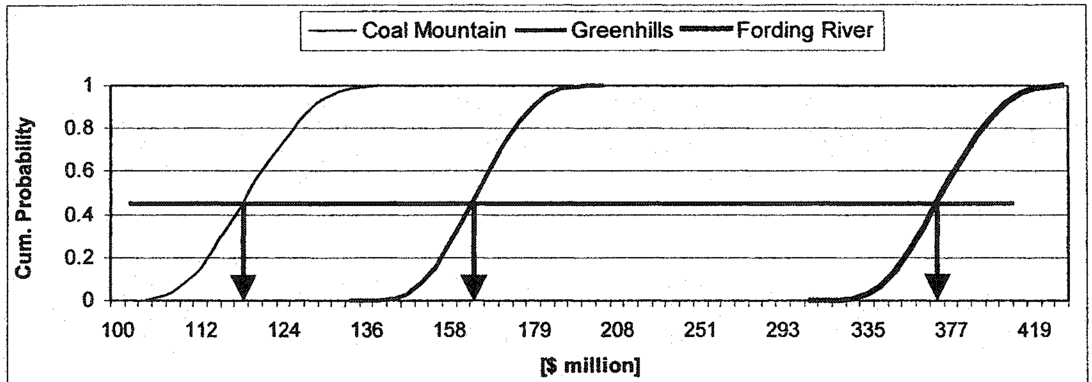


Figure 5.2.9 Cumulative Probability of Fording Coal's Mines Total Costs

The associated uncertainty for producing and transporting coal form Fording River and Coal Mountain is higher than for Greenhills. As a result further work must be carried out to reduce these uncertainties. The revenues of the company in Figures A.8.1 and A.8.2 show similar levels of uncertainty ranging between $120M and $620M. Greenhills mine has the highest uncertainty regarding its revenue. It is important that production and commodity prices be carefully modeled and analyzed to ensure revenue stability.

The company profit ranges from $78.9M to $375.0M, with an expected value of $221.3M as illustrated in Figures A.8.3 and A.8.4 in Appendix 8.0. Figures 5.2.10, 5.2.11 and 5.2.12 show the risk and probability of success associated with each particular mine. The profits from Greenhills and Fording River have relatively high degree of uncertainty. The expected profits at Coal Mountain is $44.4M, at Greenhills $50.6M, and $126.3M at Fording River. As a result, attention must be paid to these mines to reduce the level of uncertainties and to ensure company's profit stability.

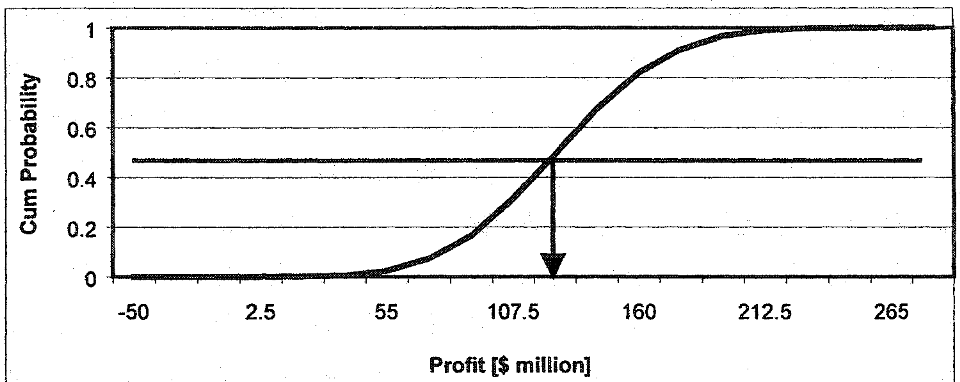


Figure 5.2.10 Probability of Success of Fording River's Profit

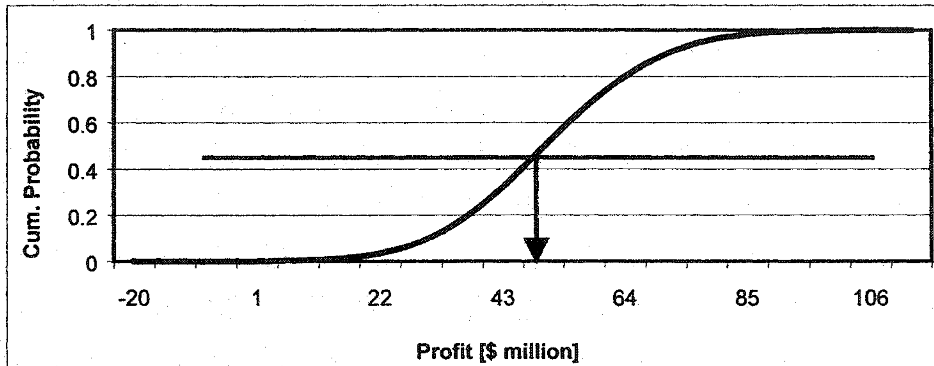


Figure 5.2.11 Probability of Success at Greenhills

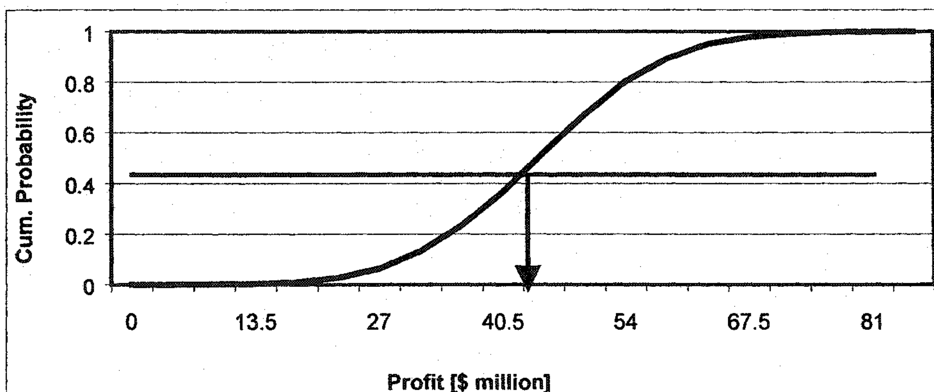


Figure 5.2.12 Probability of Success at Coal Mountain

The above figures illustrate the probability of failure or success associated with the mines. These figures show that overall, the company is profitable under the current and anticipated technical, economic and uncertainty environments. The probability of failure is very low for all of the mines. There is less than 0.5% probability of failure at Fording

River, 10% risk that the profit will be less than $79.3M and 50% that it will be less than $126.3M. There is less than 1% probability that Greenhills will have a loss, 10% probability that the profit will be below $29.9M, and 50 % risk that the profit will less than $51M. There is no risk of failure for Coal Mountain, there is 10% probability that the profit will be below $29.80M and 50% that it will be less than $44.4M. Figure 5.2.13 illustrates the risk associated with the total cost, revenue, and profit of the company.

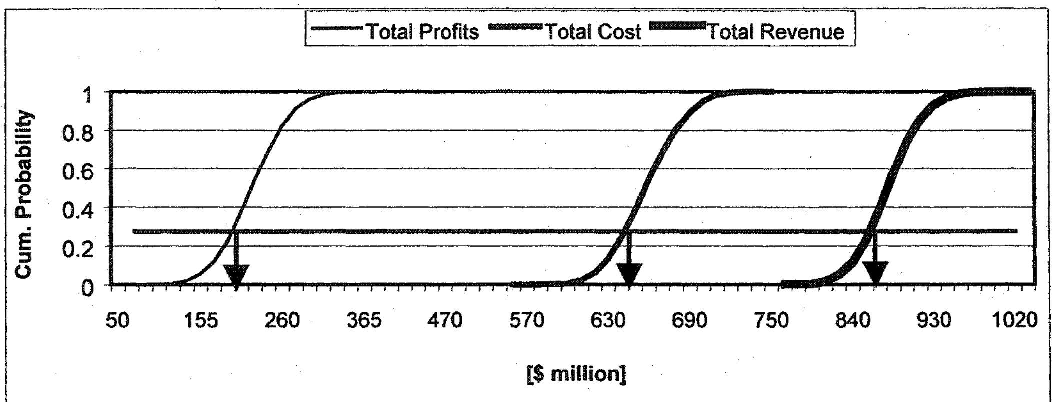


Figure 5.2.13 Cumulative Probability of Total Profits, Costs, and Revenues

Table A.8.9 (Appendix 8.0) illustrates the mean values, standard deviations, and the mean values plus/minus standard deviation for costs revenues and profits of Fording Coal's operations. Mine managers can use these probabilities of failure concept to gauge the viability of projects in terms of risk and return.

5.3 Conclusions

The results from the basic optimization models, various market and coal quality scenarios and sensitivity and risk analysis, were discussed in detail. LP models were developed and solved for each of these scenarios. To analyze various market and production scenarios, several changes were made in the objective function and constraint coefficients. Different market and coal quality scenarios were developed in an attempt to improve the solution provided by the base model, based on the world coal market analysis provided in Appendix 1.0 and Appendix 2.0. A sensitivity analysis was run to study some changes in the market conditions and in the coal qualities. The effects of these changes on the optimal solution are also analyzed in detail. The stochastic models provided the risk associated with the production and haulage of coal from mines to ports. These design and operating risks were captured in quantitative stochastic models and simulated over an extended period using the Latin Hypercube and Monte Carlo simulation techniques.

The results show that Luscar-Sherritt and Fording Coal have profit expectation of $135.7M and $221.3M, at 50% probability of success, and $70M and $ 120M at zero failure probability. The expected metallurgical coal production capacities at Luscar-Sherritt operations must be 2.5 [Mt/y] at Line Creek, 2.0 [Mt/y] at Luscar, 3.0 [Mt/y] at Cheviot, and zero at Coal Valley and Obed Mountain. The expected thermal coal production capacities at Luscar-Sherritt operations must be 1.5 [Mt/y] at Line Creek, 1.0 [Mt/y] at Luscar, 1.0 [Mt/y] at Cheviot, 2.5 [Mt/y] at Coal Valley, and 2.0 [Mt/y] at Obed Mountain. The expected metallurgical coal production at Fording Coal operations must be 7.85 [Mt/y] at Fording River, 3.46 [Mt/y] at Greenhills, and 2.43 [Mt/y] at Coal Mountain. The expected thermal coal production at Fording Coal operations must be 2.64 [Mt/y] at Fording River, 1.04 at Greenhills, and 1.07 [Mt/y] at Coal Mountain.

The models maximized the profit by allocating the available resources to produce metallurgical coal and haul it to Vancouver and to produce thermal coal for the power plants. In this regard, the production levels of metallurgical coal for Vancouver ports and the production level of thermal coal for the power plants were limited only by the market capacities. Transporting metallurgical and thermal coal to Ridley Island is less profitable and transporting thermal coal to Thunder Bay is the least profitable and even uneconomic for some of the mines. It must be understood that the optimization analysis was conducted under limited access to the real information and some of the results could be different in real conditions. However, Stochastic Optimization modeling provides coal companies with relevant information to make decisions under uncertainty and to assist in risk mitigation and control.

CHAPTER 6.0
CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

With production and transport costs higher than exporting countries such as Australia and Indonesia, the cost of mining and haulage of Western Canadian coal has become a major focal point. Thus, reducing hauling costs can significantly enhance the profitability of a mine. In increasingly competitive markets, companies have to redesign and simplify their production and management processes to make their operations more cost effective. Using integrated mining systems some transportation and processing costs can be avoided by planning the production and coal products according to the coal qualities and distances form the mines to the final destinations. This study presented a new approach for creating and optimizing an integrated production process. In this research study the author uses mathematical and computer models to examine areas for cost reduction, quality and capacity expansions in Western Canadian coal production and haulage. The objectives of this study were to:

(i) Develop LP and NLP optimization models for Western Canadian coal production and haulage systems;

(ii) Develop stochastic-optimization models for Western Canadian coal production and haulage systems;

(iii) Simulate these models to create a series of optima for coal production and haulage;

(iv) Provide detailed analysis of the system to examine the economics and risks associated with coal production and haulage.

The literature review dealt with the analytical focus of the literature underlying this study. It comprised the literature on linear and non-linear optimization and stochastic-optimization theories and their application for solving industrial problems. Many of the past researches focused and solved mine design, production scheduling and blending optimization problems with excellent results. The LP and the NLP models developed were improved with each attempt and they addressed more and more complex problems.

This study is a continuation of the past research, which addresses the new optimization problems that coal mining companies face in increasingly global competitive markets.

To address the Western Canadian coal production and transportation problem, the author of this study, developed and analyzed the following:

1. Detailed LP and NLP optimization models have been developed to address Western Canadian coal production, transport and marketing problem.
2. Stochastic models of the Western Canadian coal production and transport problem have been developed using the underlying random determinants.
3. The stochastic optimization models have been subjected under various paradigms to generate the economic-efficient operating curves for Western Canadian coal production.
4. Detailed analysis of the results has been conducted to examine the various strategies required for efficient production operations.

Two case studies have been conducted to validate the optimization models. The models were based on the structure and composition of a typical Western Canadian Coal company like Fording Coal Ltd. and Luscar-Sherritt Int. These coal mining companies have several mines geographically located at different regions, but could operate as an integrated single operation. Under such a system, comprehensive blending of products using quantities of each type of product with specific quality levels from each mine will take place. The production occurs as a single, integrated process, which ends with assembled product stockpiles ready for shipping to customers. The mathematical and computer models captured and optimized the coal production and haulage from each particular mine to the final destinations. In both case studies the LP and NLP models provided identical optimal solutions. From a detailed analysis of the LP and NLP results, the following conclusions are drawn:

1. The expected total coal production at Luscar-Sherritt mines is 7.5 [Mt/y] of metallurgical and 8.0 [Mt/y] of thermal coal distributed to the mines.

2. Line Creek should produce 1.94 [Mt/y] and 0.55 [Mt/y] of metallurgical coal and transport it to Vancouver and to Thunder Bay, and 1.5 [Mt/y] of thermal coal and transport it to Thunder Bay.
3. Luscar should produce 1.08 [Mt/y], 0.46 [Mt/y], and 0.44 [Mt/y] of metallurgical coal for Vancouver, Ridley Island and Thunder Bay ports and 0.84 [Mt/y] and 0.16 [Mt/y] of thermal coal for Vancouver and Ridley Island ports.
4. Cheviot should produce 2.46 [Mt/y] and 0.53 [Mt/y] of metallurgical coal for Vancouver and Ridley Island, and 0.35 [Mt/y], 0.50 [Mt/y], and 0.148 [Mt/y] of thermal coal and transport it to Ridley Island, Thunder Bay, and power plants.
5. Coal Valley should produce 0.11 [Mt/y], 0.76 [Mt/y], and 1.63 thermal coal and transport it to Vancouver, Ridley Island and power plants.
6. Obed Mountain should produce 1.05 [Mt/y], 0.73 [Mt/y] and 0.22 [Mt/y] of thermal coal for Vancouver, Ridley Island and Thunder Bay.
7. The expected coal production at Fording mines is 13.75 [Mt/y] of metallurgical and 4.75 [Mt/y] of thermal coal distributed to the mines.
8. Fording River should produce 7.11 [Mt/y] and 0.69 [Mt/y] of metallurgical coal for Vancouver and Thunder Bay, and 1.34 [Mt/y], 0.56 [Mt/y] and 0.77 [Mt/y] of thermal coal for Vancouver, Thunder Bay and power plants.
9. Greenhills should produce 3.09 [Mt/y] metallurgical coal and haul it to Vancouver, and 1.15 [Mt/y], 0.17 [Mt/y] and 0.08 [Mt/y] of thermal coal for Vancouver, Thunder Bay and power plants.
10. Coal Mountain should produce 2.21 [Mt/y] and 0.55 [Mt/y] metallurgical coal and transport it to Vancouver and Thunder Bay, and 0.27 [Mt/y] and 0.38 [Mt/y] of thermal coal for Thunder Bay and power plants.
11. The coal quality levels of metallurgical coal at Luscar-Sherritt's operations should be as follow: The percentage of ash content must be equal to 8.15; 8.0; 8.92; the percentage of sulfur must be less than 0.35; 0.42; 0.38, and the percentage of moisture must be equal to 8.20; 7.75; 7.95, respectively at Line Creek, Luscar and Cheviot. The percentage of fixed carbon must be larger than 60.0; 59.5; 57.5, the percentage of volatile matter must be less than 21.0; 22.0; 22.5, and the quantity

of energy in [MJ/kg] must be larger than 32.0; 31.0; and 29.5 respectively at Line Creek, Luscar and Cheviot.

12. The quality of thermal coal at Luscar-Sherritt's operations should be: The ash content [%] must be equal to 12.50; 13.10; 14.49; 12.25; and 13.0, the sulfur [%] must be less than 0.75; 0.78; 0.85; 0.80; and 0.90, the moisture [%] must be equal to 13.30; 13.75; 13.80; 13.50; and 13.30 respectively at Line Creek, Luscar, Cheviot, Coal Valley and Obed Mountain. The fixed carbon [%] must be larger than 51.0; 48.0; 48.5; 47.0; and 46.5, the volatile matter must be less than 30.0; 35.0; 37.0; 35.2; and 36.0, the energy [MJ/kg] must be higher than 28.0; 27.5; 26.0; 27.0; and 26.5, respectively at Line Creek, Luscar, Cheviot, Coal Valley and Obed Mountain.
13. The quality of metallurgical coal at Fording Coal's should be: the ash content [%] must be equal to 7.75; 9.45; and 9.45, the sulfur content [%] must be less than 0.35; 0.42; and 0.40, the moisture [%] must be equal to 7.60; 8.55; and 8.50, the fixed carbon [%] must be higher than 60.0; 59.5; and 57.5, the volatile matter [%] must be less than 21.0; 22.0; and 22.5, the energy must be higher than 32.0; 31.0; and 29.5 [MJ/kg], respectively at Fording River, Greenhills and Coal Mountain.
14. The quality of thermal coal at Fording Coal's operations should be; the ash content [%] must be equal to 12.5; 13.10; and 14.49, the sulfur content [%] must be less than 0.70; 0.85; and 0.80, the moisture [%] must be equal to 13.20; 13.85; and 13.90, the fixed carbon [%] must be higher than 51.0; 49.0; and 48.5, the volatile matter [%] must be less than 30.0; 35.0; and 37.0, the energy [MJ/kg] must be higher than 28.0; 27.5; and 26.0 respectively at Fording River, Greenhills and Coal Mountain.

The stochastic models provided the risk associated with the profits and the results showed that:

1. Luscar-Sherritt has profit expectation of $135.7M at 50% probability of success, and $70M at zero failure probability.
2. The failure probability is less than 1%; 1%; 1%; 0%; and 5%, there is 10% risk that the profits will be less than $13.4M; $13.9M; $18.4M; $22.3M; and $1.57M,

and 50% risk that the profits will be less than \$29.7M; \$26.0M; \$36.8M; \$32.7M; and \$9.84M, respectively at Line Creek, Luscar, Cheviot, Coal Valley and Obed

3. Fording Coal Ltd. has profit expectation of \$221.3M, at 50% probability of success, and \$ 120M at zero failure probability.
4. The failure probability is less than 0.5%; 1%; and 0%, there is 10% risk that the profits will be less than \$79.3M; \$29.9M; and \$29.8M, and 50% risk that the profits will be less than \$126.3M; \$51.0M; and \$44.0M respectively at Fording River, Greenhills and Coal Mountain.

6.2 Recommendations for Further Research

The author of this study wished to do a better job by expanding the models to the coal qualities in the coal deposits instead of the cleaned coal at the processing plants, expand the models to optimize a larger range of coal products and destinations, and analyze the mining and processing costs independently. The lack of time, resources, and the reluctance of mining companies to release confidential data and the information necessary limited the research to the actual optimization design. From the limitations of this research the following recommendations can be made:

1. Analyze the mining and the processing cost independently. The cleaned coal qualities have a larger impact on the processing costs than on mining costs. Therefore, analyzing them independently will give a better understanding and better decisional base to the management.
2. Expand the models to the qualities of coal in the deposits. Even though the model will be very complicated by taking the coal qualities from the coal deposits into the optimization model, it will create a fully integrated optimization process and will give the management a lager flexibility of where and how much to mine.
3. Expand the models to cover a larger range of coal products and destinations. The coal mining companies produce more than one quality of metallurgical and thermal coal. To address this problem, the model can be expanded by taking more variables into the model and develop the corresponding number of blending constraints for each of the coal product.
4. Develop a user-friendly interface of the models for industrial applications.

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APPENDIX 1.0
WORLD COAL INDUSTRY OVERVIEW

A.1.1 World Coal Recoverable Reserves

Coal is the most abundant of the fossil fuels, and its reserves are also the most widely distributed. Estimates of the worlds' total recoverable reserves of coal in 1999, as reported by I.E.O., are at about 1,089 billion tons. The resulting ratio of coal reserves to production exceeds 220 years, meaning that at current rates of production, coal reserves could last for another two centuries. The distribution of coal reserves around the world varies notably from that of oil and gas, in that significant reserves are found in the United States and the FSU but not in the Middle East. The United States and the FSU each have roughly 25% of global coal reserves. China, Australia, India, Germany, and South Africa each have between 6 and 12 percent of world reserves as illustrated in Figure A.1.1 [18].

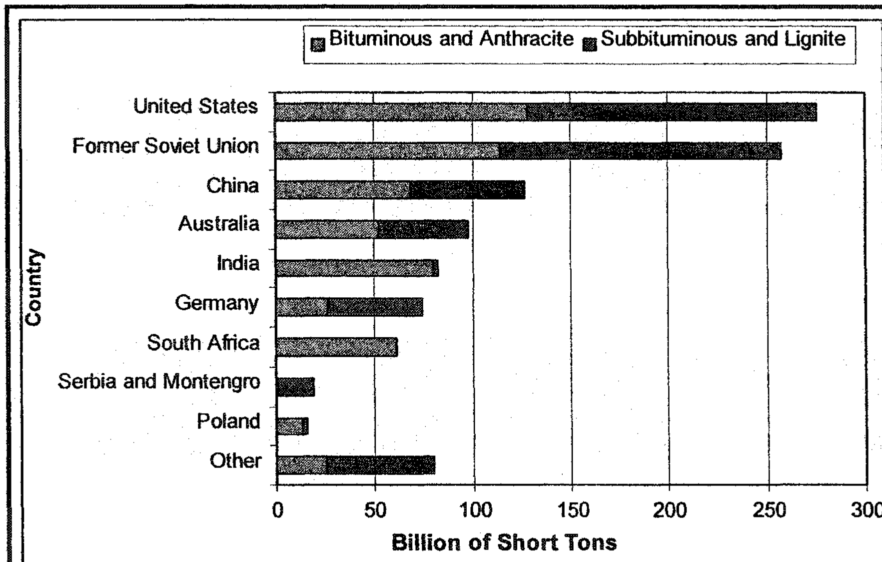


Figure A.1.1 Top Ten Coal Reseaves by Country

Quality and geological characteristics of coal deposits are other important parameters for coal reserves. Coal is a much more heterogeneous source of energy than oil and natural gas. Its quality varies significantly from one region to the next and even within an individual coal seam. For example, Australia, the United States, and Canada are endowed with substantial reserves of premium coals that can be used to manufacture coke. Together these three countries have supplied approximately 85 percent of globally traded coking coal during 1985-2000. Table A.1.1 presents the recoverable reserves (Bituminous and Subbituminous) by country and continent (Source I.E.O) [3, 18, 36, 38].

Table A.1.1 World Estimated Recoverable Coal by Countries (Million Short Tons)

Region Country	Recoverable Anthracite and Bituminous	Recoverable Lignite and Subbituminous	Total Recoverable Coal
North America			
Canada	4,970	4,535	9,505
Greenland	0	202	202
United States	127,748	147,824	275,572
Total	**132,718**	**152,561**	**285,279**
Central & South America			
Argentina	0	143	143
Bolivia	1	0	1
Brazil	0	13,173	13,173
Chile	34	1,286	1,320
Colombia	7,020	420	7,440
Ecuador	0	26	26
Peru	1,058	110	1,168
Venezuela	528	0	528
Total	**8,641**	**15,158**	**23,799**
Western Europe			
Austria	0	28	28
Croatia	7	36	43
France	105	23	128
Germany	26,455	47,399	73,854
Greece	0	3,168	3,168
Ireland	15	0	15
Italy	0	37	37
Netherlands	548	0	548
Norway	0	7	7
Portugal	3	36	39
Serbia and Montenegro	71	18,087	18,158
Slovenia	0	65	65
Spain	220	507	727
Sweden	0	1	1
Turkey	495	690	1,185
United Kingdom	1,102	551	1,653
Total	**29,021**	**70,635**	**99,656**
Eastern Europe & FSU			
Bulgaria	14	2,947	2,961
Czech Republic	2,880	3,929	6,809
Hungary	657	4,260	4,917
Kazakhstan	34,172	3,307	37,479
Kyrgystan	0	895	895
Poland	13,352	2,421	15,773
Romania	1	3,979	3,980

Russia	54,110	118,964	173,074
Slovakia	0	190	190
Ukraine	18,065	19,806	37,871
Uzbekistan	1,102	3,307	4,409
Total	**124,353**	**164,005**	**288,358**
Middle East			
Iran	213	0	213
Total	**213**	**0**	**213**
Africa			
Algeria	44	0	44
Botswana	4,754	0	4,754
Central African Republic	0	4	4
Congo	97	0	97
Egypt	0	24	24
Malawi	2	0	2
Morocco	6	0	6
Mozambique	265	0	265
Niger	77	0	77
Nigeria	23	168	191
South Africa	60,994	0	60,994
Swaziland	128	0	128
Tanzania	220	0	220
Zambia	-	61	61
Zimbabwe	809	0	809
Total	**67,419**	**257**	**67,676**
Far East and Oceania			
Afghanistan	73	0	73
Australia	52,139	47,510	99,649
Burma	2	0	2
China	68,564	57,651	126,215
India	80,174	2,205	82,379
Indonesia	849	0	849
Japan	865	0	865
Korea, North	331	331	662
Korea, South	90	0	90
Malaysia	4	0	4
Nepal	2	0	2
New Caledonia	2	0	2
New Zealand	32	597	629
Pakistan	0	3,228	3,228
Philippines	26	303	329
Taiwan	1	0	1
Thailand	0	2,205	2,205
Vietnam	165	0	165
Total	**203,319**	**114,030**	**317,349**
World Total	**565,684**	**516,646**	**1,088,602**

A.1.2 World Coal Consumption Overview

In spite of the fact that coal demand is expected to be displaced by natural gas in some parts of the world, only a slight drop in its share of total energy consumption is projected by 2020. In 1999,coal provided 22 percent of world primary energy consumption and is projected to fall to 19 percent by 2020. World coal consumption has been in a period of generally slow growth since the late 1980s, a trend that is expected to continue. The I.E.O. 2001 reference case projects growth in coal use between 1999 and 2020, at an average annual rate of 1.5 percent, but with considerable variation among regions. Coal use is expected to decline in Western Europe, Eastern Europe, and the F.S.U. Increases are expected in the United States, Japan, developing Asia (China, India, ASEAN countries), Brazil and Mexico. China and India are projected to account for 92 percent of the total expected increase in coal use worldwide [18, 38].

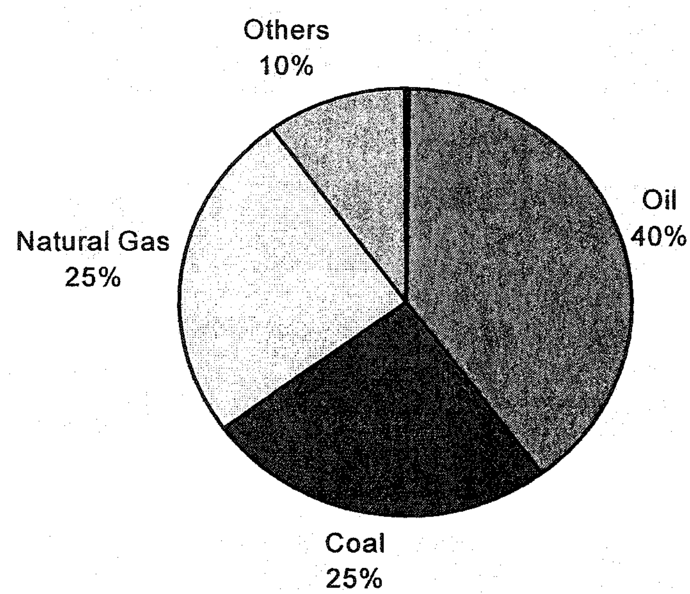


Figure A.1.2 World Primary Energy (Source BP Energy)

Coal consumption is heavily concentrated in the electricity generation sector, and significant amounts are also used for steel production. More than 55% of the coal consumed worldwide is used for electricity generation. Power generation accounts for virtually all the projected growth in coal consumption worldwide. Primary uses of coal are (i) 40% of the world's electrical energy (ii) 70% of steel production relies on coal as illustrated by Figure A.1.2 and A.1.3. Consumption of coking coal is projected to decline slightly in most regions of the world as a result of technological advances in steelmaking, increasing output from electric arc furnaces, and continuing replacement of steel by other materials in end-use applications [18, 38].

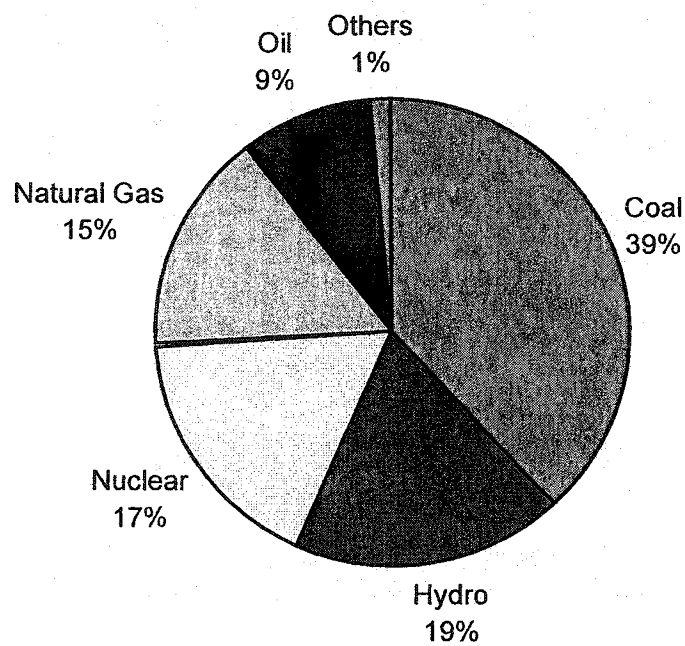


Figure A.1.3 World Electricity Generation 2000 (source BP)

A.1.2.1 Regional Coal Consumption

Asia. Large increases in coal consumption are projected for China and India based on outlook of strong economic growth for both countries. Much of their increased demand for energy will be met by coal particularly in the industrial and electricity sectors. China is the world's leading producer of both steel and pig iron (pig iron offers a more direct link to overall coal use). According to the I.E.O 2001 forecasts, China would account for

40 percent of world coal use in 2020. Energy consumption in India is also dominated by coal, and more than two-thirds of the coal consumed is used in the power sector, where most growth in coal demand is projected to occur. Coal use for electricity generation in India is projected to rise by 2.1 percent per year till 2020. South Korea is a significant coal user in both the power and steel industries. Coal consumption in South Korea is expected to increase from 1.4 quadrillion BTU in 1999 to 1.9 quadrillion Btu in 2020, accounting for more than 25 percent of the projected increase in developing Asia outside China and India [18].

Japan, which is the third largest coal user in Asia and the fifth largest globally, imports basically all the coal it consumes. Some coal is used for the country's steel production and is also used heavily in the Japanese power sector, accounting for about 16% of the energy used for electricity generation and 45% of the coal used in the country. Taiwan, Indonesia and Thailand are the next largest coal consumers in developing Asia. They are building and commissioning several large coal-fired power plants that will lead to rising coal use. They are also planning to develop significant new steel capacities in order to become net steel exporters. Most of the new capacities in the region will be in the form of electric arc furnaces although some integrated works are under construction and several other are planned. For instance the Vietnam Government has a master plan for the steel industry and high priority is being placed on developing the country's iron ore reserves. The Indian steel industry is expanding capacity and production the plan is to add 5 to 7 million tonnes of capacity by 2003. Because of India's shortage of scrap and high power prices, most new capacity will be blast furnace based or direct reduction fed electric arc furnaces [18].

Western Europe. Coal consumption in Western Europe has declined by almost 40% over last decade. Coal consumption is also expected to decline over the forecast period, but at a slower rate. All the countries have plans to restructure their coal industry and to reduce subsidies, which mean that they will become more dependent on imports for their coal needs since coal-consuming trend is projected to remain close to current levels [18].

Eastern Europe and FSU. Coal consumption in this region has fallen since 1988. In the future, total energy consumption in the EE/FSU is expected to rise, primarily as the result of increasing production and consumption of natural gas. Coal consumption in most of the E.E. countries is dominated by the use of low-Btu subbituminous coal and lignite produced from local reserves. The World Bank has approved loans to support restructuring of the coal industry by continuing to close unprofitable mines. Some of these countries as Ukraine, Bulgaria, Romania and Hungary, are becoming more and more dependent of coal imports for their needs. The general economic situation in E.E countries has been improving over the recent years and domestic steel consumption would be considerable higher, therefore, the need of coking coal will increase [18].

North America. Coal demand in North America is dominated by U.S. consumption. In 1999, the United States consumed 1,045 million tons, accounting for 93 percent of the regional total. By 2020 U.S. consumption is projected to rise to 1,297 million tons. The United States relies heavily on coal for electricity generation, a trend that continues in the forecast. Coal provided 51 percent of total U.S. electricity generation in 1999 and is projected to provide 44 percent in 2020. In Canada and Mexico, coal consumption is projected to rise from 77 million tons in 1999 to 93 million tons in 2020. On Mexico's Pacific coast, a newly completed import facility with a throughput of 10 million tons per year will supply CFE's Petacalco power plant and a nearby-integrated steel mill [18].

Africa. African coal production and consumption are concentrated heavily in South Africa. South Africa became the second largest coal exporter in 1999 when its export exceeded those from the United States. South Africa is also the world's largest producer of coal based synthetic liquid fuels. In 1998, about 15% of the coal consumed in South Africa was used to produce coal-based synthetic oil, which accounted for more than a quarter of liquid fuels consumed in South Africa. For Africa as a whole, coal consumption is projected to increase by 39 million tons between 1999 and 2020, primarily to meet increased demand for electricity. Most of the coal will be provided from domestic resources. South Africa's steel capacity is expected to expand assisted by low electricity costs, relative cheap labor, and abundance of row materials such as Ferro-chromium and iron ore, which will increase demand for metallurgical coal [18, 38].

Central and South America. Coal has not been an important source of energy in Central and South America, accounting for less than 5 percent of the region's total energy consumption in 1999. Brazil, with the eight largest steel industries worldwide in 1999, accounted for more than 66 percent of the region's coal demand, with Colombia, Chile, Argentina, and to a lesser extent Peru accounting for much of the remaining portion. The steel industry in Brazil accounts for more than half of the country's total coal consumption, relying on imports of coking coal to produce coke for use in blast furnaces. Although Brazil's steel production was fairly flat in the late 1990s, strong growth during the first part of 2000 was part of a broader industry trend. In the forecast, increased use of coal for making steel (both coking coal and coal for pulverized coal injection) makes up a large portion of the projected increase in Brazil's coal consumption. Coal could be competitive for power generation in those parts of Central America where pipeline natural gas and hydropower are not available [18].

Middle East. The Middle East, including Turkey, accounted for about 2 percent of global coal use in 1999. As a whole, the region relies heavily on oil and gas for its primary sources of energy. Still coal use is expected to grow in the region. In the I.E.O. 2001 reference case, coal consumption in the Middle East is projected to increase from 96 million tons in 1999 to 120 million tons by 2020, representing an average annual growth rate of 1.1 percent. Over the forecast period, coal consumption in Turkey (both lignite and hard coal) is expected to increase by 17 million tons, primarily to fuel additional coal-fired power generation. The start up of two new coal-fired generating units in Israel in 1999 and 2000 is expected to add approximately 3 million tons to Israel's total annual coal consumption. Table A.1.2 and Figures A.1.3 to A.1.6 illustrate the coal consumption and the forecasts from 1990 until 2020 (Source I.E.O) [10].

Table A.1.2 World Coal Consumption by Region: 1990-2020

	1990	1995	2000	2005	2010	2015	2020
Latin America	30	32	37	44	47	53	59
Africa	152	172	171	178	191	203	217
Middle East	6	8	10	12	13	15	16
FSU/Eeur	1372	934	919	918	887	848	805
Western Europe	958	674	679	670	676	681	686
North America	957	1013	1148	1209	1263	1319	1365
Industrial Asia	233	257	267	273	286	292	301
Developing Asia	190	244	247	273	298	324	355
India	242	312	387	444	499	537	581
China	1124	1489	1796	2176	2666	3374	4242

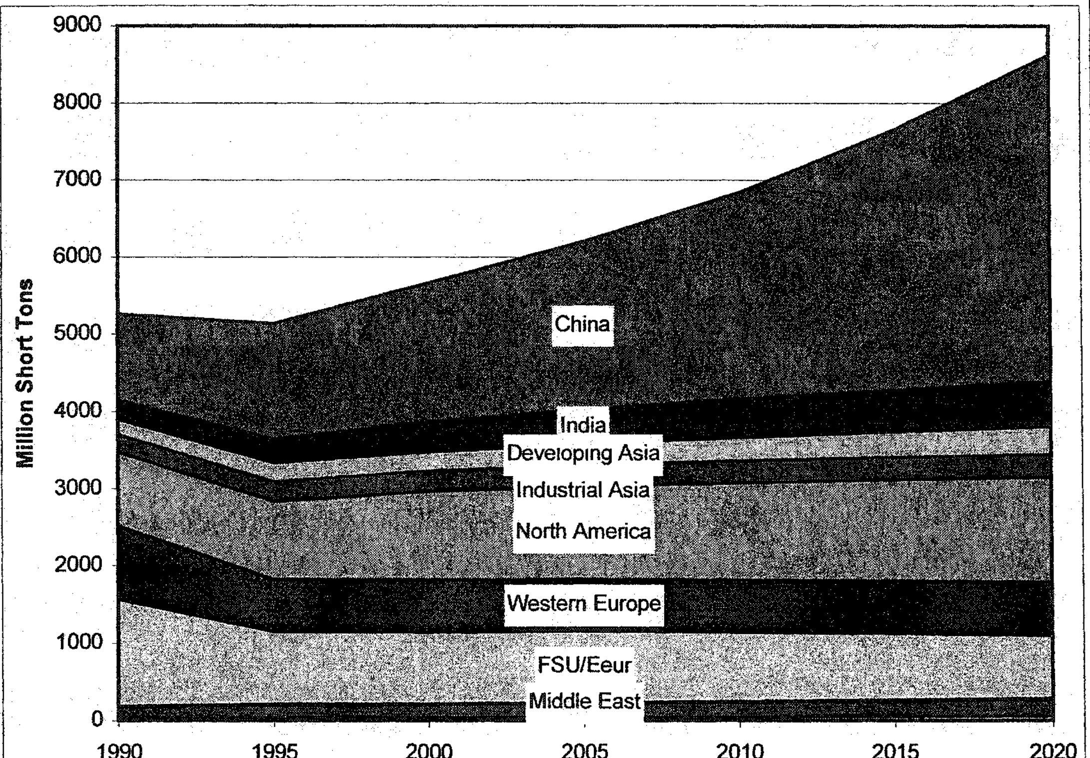


Figure A.1.4 World Consumption by Region (I.E.O)

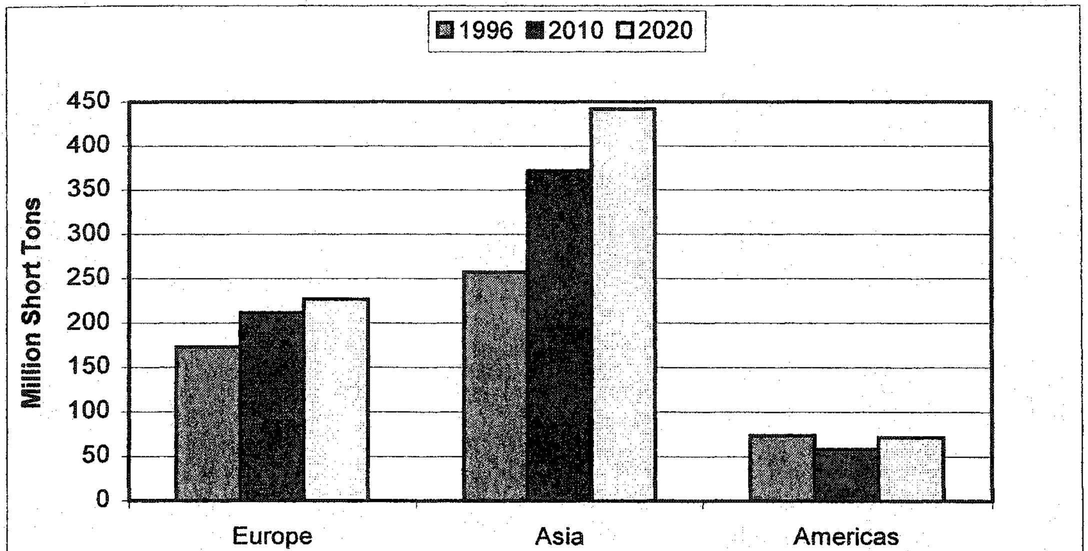
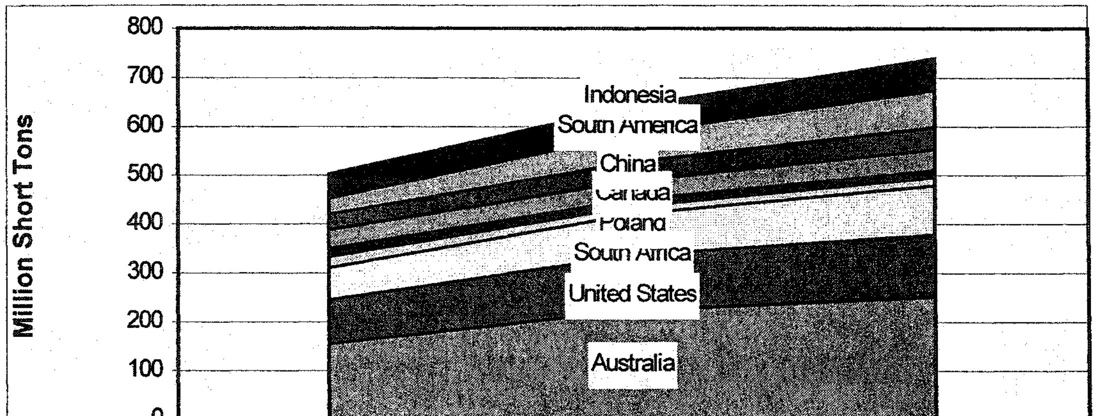


Figure A.1.5 World Coal Imports by Region (Source I.E.O)



	1996	2010	2020
Indonesia	51.9	56	66.6
South America	32.3	57.8	74.7
China	31	39.2	45.9
Canada	38.1	39.4	43.5
Poland	16.8	14.1	13.9
FSU	22.2	16	16.4
South Africa	65.6	82.8	100.1
United States	90.5	111.6	127.7

Figure A.1.6 World Coal Exports 1996 - 2020 (Source I.E.O.)

A.1.3 World Major Coal Producers and Exporters

Australia. Australia has a total of some 90,700 Mt of proven recoverable reserves, comprising 49,900 Mt of black coal and 40,800 Mt of brown coal and equivalent of about 8% of world hard coal and 15% of world lignite reserves. There is enough surface-mineable coal to maintain present and future production within a 20-year time frame, with additional underground resources available when surface-mineable coal is exhausted. The coals have high quality with a low sulphur content (less than 1%), low ash content, relatively low volatile mater and high specific energy. The industry has a very good structure and performance. The hard coal industry benefited substantially from initiatives such as the deregulation of the electricity supply sector and the railways leading to lower power and freight charges. In the brown coal sector, the sale of the State Electricity mining and generating assets to the private sector was undertaken on the basis of individual power station/mine units, such that each is now a wholly separate commercial entity. The industry benefits of good railway network and port facilities and the relatively short distances from mine sites to ports leading to low freight and port costs. Coal transport and handling costs are US$6.6-11.00/t [38].

The industry is taking advantage of the favorable unfaulted shallow coal deposits and the climatic condition to reduce production costs. Cash production cost FOB US$33.50/t compared with US$39.50/t in Canada and US$42.60 in the USA and is projecting a cash cost of US$27.90/t for hard coal and US$0.83/GJ for steam coal. Consolidation of ownership that happened between 1995 and 2000 is likely to continue. Such a scenario envisages the current 26 operations being merged into seven mines, each with a capacity of between 10 Mt/y and 25 Mt/y of coal, offering the potential annual output of around 120 Mt. AME Mineral Economics suggests the improvements that have occurred to Australia's competitive position in the coking coal market relative to Canada and the USA will enable the country to withstand the pressure caused by low prices [18, 38].

United States. USA has the largest coal reserves in the world accounting for 25% of the total world's recoverable reserves. The US recoverable reserves total 249,680 Mt of which 90,940 are classed as low-sulphur, 77,050 Mt are medium-sulphur and 81,680 Mt are high sulphur. Future developments will be driven more by environmental and commercial considerations than by the existence of in situ coal resources. Low sulphur coals will become more marketable, even if geographically less accessible to markets, at the expense of higher-sulphur coals in the Illinois basin and northern Appalachia. SUA is a large coal producer but most of its output is internally consumed for producing electricity. Coal provided 51% of total U.S. electricity generation in 1999 and is projected to provide 44% by 2020. With an output of 1014.5 Mt in 1998 and 998Mt in 1999, the United States retained its position as the world's second-largest producer and consumer of coal, behind China [38].

The USA is also a major coal exporter, with peak shipments of 102 Mt in 1981. Since then, exports have ranged between 64 Mt/y and 99 Mt/y, but in the late 1990s increased competition from lower-cost producers had a marked impact on traditional markets for US coal in Europe and the Far East. Conversely imports, mainly of steam coal, showed a significant rise in 1999 to 6.9 Mt, with the prospect of higher future imports into southern states. Since high production costs, United States saw a major reduction in its exports being displaced by South Africa as the world's second largest coal-exporting country. Coal imports to the United States are projected to increase from 9 million tons in 1999 to 20 million tons in 2020. Coal-fired power-plants in the southeast part of the country are expected to take most of the additional import tonnage, primarily as a substitute for higher priced coal from domestic producers. The forecast reflects projected declines in both minemouth coal prices and coal transportation rates [38].

All the US coal industry is in the private sector. Ownership consolidation in the industry had reached the stage that by the late 1990s the ten largest companies controlled nearly two-thirds of US production. Rail forms the principal means of transporting coal from mine to consumer. Competition and consolidation amongst the railway companies during 1990s helped to drive down costs, so permitting coal to be hauled over steadily increasing distances. Transport distances are long, especially from the coalfields of the western United States, averaging 2250 km Wyoming to Roberts bank or to ports in Washington state, and 1300 km from Uinta basin in Utah to the ports of Los Angeles and Long Beach in California. In an easterly direction, rail distances between the Appalachian basin and export ports on the eastern seaboard are considerably less, while companies producing coal from Illinois basin have the alternative outlets of ports on the Atlantic and Gulf coasts. The United States is unique amongst major producers in the extent and usage of its internal waterway system for bulk transport. The massive port construction programmed for coal export facilities that took place in the 1980s and 1990s has given USA the theoretical capacity to handle some 250 Mt/y of coal (IEA, 1999). Given the downward trend in exports, much of this capacity is likely to remain under-utilized [38].

Average mine-gate prices received by US coal producers fell consistently over the 1980s and 1990s. IEA (1999) reports that mine-gate production costs for export coking coal range between US$29.00 and US$42.80/t from the Appalachian coalfields, to which must be added up to US$18/t for the transport to the east cost ports or US$9/t for barging to the Gulf Coast. With handling and port charges this gives a fob cost of US$45-54/t, a figure that sits uncomfortably close to the average price of US$49.13/t received in 1998. Mine-gate costs from underground operations in the Uinta coalfield, which export through Californian ports, fell in the range US$16.20-20.30/t, giving fob costs of US$35.20-43.20/t. The overall scenario for US coal production to 2020 is for production to continue to increase. There is however, some divergence of views over the future for US exports. While the EIA forecasts a gradual increase in exports to perhaps 85 Mt/y by 2020, others are not so optimistic. The high cost base that characterizes US export coal relative to competitors such as Indonesia and Australian the Far East market and Columbia, South Africa in the European market, seems to be damaging US sales [38].

South Africa. South Africa has large hard coal reserves, 33,200 Mt. of high steam coal quality and relatively low producing costs. The uniformity of the coal deposits, relatively shallow and almost horizontally, permits the use of surface and underground mining of the same seams. In 1999, South Africa produced 248 million tons of coal, 30 % of which went to export exceeding U.S. and becoming the world's second largest coal exporter. South Africa is also the world's largest producer of coal based synthetic liquid fuels. In 1998, about 15 percent of the coal consumed was used to produce coal-based synthetic oil which in turn accounted for more than a quarter of all liquid fuels consumed in South Africa. Over 95% of South Africa coal reserves consist of bituminous coal, with 2 % consisting of anthracite and less than 2% comprising metallurgical coal [38].

The South African coal industry is based on resources found in 19 coalfields. All the mines are privately operated, with sales being made to domestic users such as the electricity generating utilities or into export markets. Exports are mainly of steam coal, with minor amounts of coking coal and anthracite. Principal customers are in Europe, Japan and other Pacific Rim countries. South African metallurgical coals are generally comparable to Australian coking coal after washing, although ash contents are usually higher (Falcon and Ham, 1988). Consolidation of ownership took place during the late 1990s. Coalfield development in South Africa has been undertaken in two largely separate directions. Coal mined as fuel for domestic electricity generation is often fed by conveyor to mine-mouth power stations, while export products are handled through a dedicated railway-port system. The principal route for coal exports is via the 580 km-long Spoornet railway from Witbank to Richards Bay. The line is fully electrified and has an annual capacity of some 60 Mt. Richard Bay is the largest single coal terminal in the world, capable of handling vessels of up to 190,000 dwt. In 1998, South Africa was the lowest-cost steam coal producer on a per-Gj basis, with an average fob cash cost of US$0.77GJ. This was equivalent cost of US$20.35/t, on which basis South Africa ranked second to Indonesia [38].

With the rail transport and export-handling infrastructure well established, South Africa producers are able to benefit from some of the lowest per-ton-kilometer freight costs of any of the major coal exporting countries. Faced with the prospect of lower steam coal prices, especially in the European market, continuing to fall as more production capacity comes on stream in Indonesia, Colombia, Australia and Venezuela, South African producers are unlikely to be able to commit investment to new greenfield mine projects for some time. Export production may actually decrease somewhat as more of the country's exports are sold on the spot market rather than at higher-price, longer-term contracts. Nonetheless, while short-to medium-term prospects are good, by 2010 the position may be different. With accessible resources unable to support production of adequate volumes of export coal of a sufficient quality unless prices are higher, it may be that South African exports will begin to tail off [38].

Indonesia. Indonesian coal resources have been estimated to total some 38,000Mt, comprising 21,460 Mt of lignite, 11,100 of subbituminous, 5200 Mt of bituminous and 130 Mt of anthracite (Sunardi, 1999). Thick seams that lie close to the surface, with widespread thermal metamorphism having increased the rank of the coal significantly, characterize deposits. In the last decade the output increased considerably and increased more than 40% in the last 5 years. In 2000 the output exceeded 70Mt with exports of over 50 Mt. The success of coal development in Indonesia has been dependent on the construction of deep-water facilities. No fewer than 6 ports have been constructed during the 1990s. The short distances to the sea ports and a variety of transport options for the new mine developments as truck haulage, barging down rivers to transshipment ports, or the construction of overland conveyors or railways are really advantages and lead to low FOB prices. AME estimated fob cash cost of US$0.79/GJ is the lowest in the world for steam coal. Indonesian coal production will continue to grow but the rate of increase in production will probably slow with exports as 70 Mt by 2005(Sunardi, 1999). Indonesia suffers from social disruptions that might result in continuing political uncertainty, which may affect the coal industry and coal exports [38].

China. China is the world's largest coal producer, the vast proportion of its output being used internally for industry and for electricity generation. China has still not made the impact on the coal export market that was anticipated in the mid-1980s. While there is undoubtedly significant potential for increased coal exports, rising internal demand coupled with continuing infrastructural problems- in particular transport-have meant that even those mines that were opened with the express intention of serving the export market have failed to fulfill their original targets. Despite the major economic progress being made in the country, China remains an enigma in terms of the reliability of available information about its coal industry. The complete range of coal is found, from lignite to anthracite. Bituminous coal is the most abundant, with good quality coking coal deposits. The country's proven reserves were estimated in 1994 to be 114,500 Mt, of which 13,5% comprises brown coal, 24% non-coking bituminous coal, 28% coking coal and 18.5% anthracite (Wang, 1999). It is considered that China's coal reserves to a depth of 150 m may be relatively small, with the bulk reserves available for the 20-year time frame covered held at depths of between 150 and 300 m. Only 7% of the country's reserves are considered surface–mineable, of which, 70% is lignite (Sanda, 1995) [38].

China's coal industry has undergone major restructuring during 1990s. The administrative structure now consists of three tiers; major state-owned mines, locally administered state mines and those operated on a township or village basis. In terms of mining technology, the state operations are the most heavily mechanized. The country has its own longwall equipment supply sector; although a number of recent mine developments have involved the installation of high-capacity Western systems. Productivities are extremely dependent on the size of the mine concerned and the level of sophisticated of the operation, ranging from less than 1 t/man-shift in small scale operations to more than 25 t/man shift on highly mechanized, high capacity faces. Around 75 longwall faces are equipped to produce more than 1 Mt/y, with the highest output achieved being 4.1 Mt/y. Some mines operating in thick seams use multi-pass longwall caving which offers better resource recovery at the cost of more complex technical and stability control problems. Mine safety has been of major concern for a number of years, with a reported total of nearly 7500 fatalities in mine accidents (mainly gas explosions). The property of many Chinese coals to spontaneous combustion also presents a significant hazard [38].

A network of rails serves China's coal industry lines, used to transport coal from the mines in the west-central region to the domestic users and export ports, located in the east. Overall, the situation has improved markedly since the early 1990s, such that the rail network no longer represents a major constraint to coal transport. Plans have been announced for a new rail line to link the coalfields in the Hilonjiang province (NE of China) with the Russian port of Voctochny, capable of handling 6-8 Mt/y of coal for export to the Pacific Rim. China's exports are largely routed through the ports of

Qinhuangdao and Shijiusuo. Qinhuangdao has been extensively modernized and now has an annual throughput capacity (imports/exports) of over 100 Mt/y. A high proportion of this capacity continues to be used for domestic transshipment, with coal being transported along the Chinese coast from north to south [38].

Much of the coal industry was loss-making for the 12 years prior to 1997, when the large state mine finally achieved profitability. However, the oversupply of coal into the Chinese market, within an environment in which coal prices have been freed from government control in 1994, resulted in prices falling to the extent that a considerable number of major producing companies were once again operating at loss in 1998. The coal industry lost some US$360 million in total during 1999, this being one factor in the government's reported decision to end state-run coal mining in the southeastern coastal province of Guangdong by the end of 2000 [38].

With forecasts predicting long-term growth in China's coal demand, rising to 2100 Mt/y by 2020 and 2600-3000 Mt/y by 2050 (Wang, 1999), the Government reportedly considering allowing widespread foreign investment in the coal sector. In March 2000, the government announced a relaxation of existing rules to permit foreign investors to hold a majority stake in coal industry projects for the first time. Government's small mine closure programmed, competition in Far Eastern markets for better-quality Australian and Indonesian steam coal make it improbable that China can achieve an increase of nearly 20 Mt/y in exports at current prices. While China continues to be a major coal producer and user, the environmental impact of its coal consumption will also remain of great concern. Aside from the wide spread installation of flue-gas desulphurization system on older power stations, one option for the Government will be to source limited supplies of better-quality coal from other countries. The Government has, in fact, already signaled its intention to replace domestic output in Guandong province with coal obtained from elsewhere in China, as well as from Australian and Indonesian sources [38].

India. India's hard coal proven reserves is estimated at 80,000 Mt and 2,205 Mt of Lignite. Since 1975, most coal production in the Country has been the responsibility of the nationalized organization, Coal India. Coal outputs have risen by almost 50% since the beginning of the 1990s. Production in 1999 totaled 353 Mt, comprising 330 Mt of hard coal and 23 Mt of lignite. Much of the increase in hard coal production has come from new opencast mines, which in 1999 accounted for nearly 79% of total output. As near-surface hard-coal resources are becoming increasingly scarce, or have been sterilized by the effects of poor mining practices in the past, future developments will have to be at greater depth. This will inevitably entail higher costs, and the need for more efficient mining techniques [38].

By world standards, India's coal industry remains extremely inefficient, with low-capacity mines, low productivity and substantial over manning with labor productivity of 0.5-0.6t/manshift in underground operations and 5t/manshift in opencast. India has a very poor infrastructure; the railway network is extensive, largely pre-dating the country's independence in 1948. India remains very short of coal washery capacity, leading to large amounts of waste rock being transported to consumers rather than being rejected close to the mine sites. Newer port facilities have been designed with the aim of importing coal rather than for exports and coastal trade. Although Coal India's production costs have remained relatively low US$10.25/t this is offset by freight cost which can add up to US$25/t (inefficient railway network and long distances to the coast) [38].

The industry is subsided by the government and the Coal Ministry decided not to provide further subsidize to some loss-making mines that are to be closed. Coal comprises an essential component of India's future energy policy. Throughout the 1990s, the coal industry has barely kept pace with the burgeoning demand for electricity, however, and most projections now forecast that before long there will be a significant gap between consumer demand and domestic coal production that can only be satisfied by increasing imports. Export potential for Indian coal during the next 20 years appears minimal. Indian coal is of inadequate quality to be able to compete on world markets. Perhaps of greater significance to world coal trade will be India's potential to become a coal importer, particularly if more thermal generation stations are constructed in coastal locations [38].

Poland. Its abundant reserves of coal provide a secure source of energy and foreign exchange, but heavy reliance on coal is also a major source of environmental pollution. While coal production is declining and will continue to decline over the coming years, it will remain a key energy source. Coal is by far the dominant fuel in Poland's economy, accounting for 95% of country's primary energy production. Poland's hard-coal recoverable reserves accounts for 13,352 Mt while the Lignite accounts for 2.241 Mt. Polish coal, though of high quality, has various geological features as changing seam thickness, gas and rock outburst, spontaneous combustion, highly faulted (short lives of longwall mining), reserves lying at depths of 300 or more that make it difficult to mine [38].

The coal industry has poor performance, low productivity, high production costs US$37.5/t (operate at loss) therefore is subsidized by the government (US$750 million deficit). There is a good railway network capable of serving both domestic and export, but the relatively long distance from the mines to the ports generate high freight costs. In spite of these conditions Poland is the world's ninth-largest coal exporter, which go primary to customers in Europe and former Soviet Union. Poland is undergoing a comprehensive restructuring program for the coal industry aimed at maximizing efficiency and paying off some of the industry debt. In 2000, Poland closed 22 coal mines and partially closed seven others. This reduced production by about 10.3 million metric tons, but the coal mining industry was profitable for the first time, and continued to be profitable into 2001[18, 38].

Ukraine. Ukraine has proven reserves of 18,065 Mt of bituminous coal and 19,806 Mt lignite and subbituminous coal. The coal deposits are characterized by adverse geology; many thin seams, intense faulting, high gas content, and average depth of mining 690m. Coal is of low quality with high ash and sulphur content and coking coal only present at depth. Between 1992 and 1998, Ukraine's coal production dropped 43%, from 146.8 Mmst to 83.3 Mmst, before rebounding to 90.8 Mmst in 1999. Ukraine exported approximately 2.5 Mmst of coal in 2000, some 9% higher than in 1999 but all of this coal went to Eastern Europe. However, even as Ukraine's coal production remained steady in 2000, demand for coal in Ukraine increased by 16.5 % in 2000 compared to 1999, when the country's net coal imports totaled 7 Mmst [18, 38].

Ukraine has nearly 200 coal mines, and remains heavily subsidized by the government. The country's coal industry continues to be plagued by labor strikes, hazardous working conditions, inefficiency and low productivity. Ukraine's coal industry has the world's highest death rate, over 300 deaths /year, mostly attributed to obsolete equipment and low safety standards. Average production costs of around US$55/t with mining costs of US$23/t and its limited port facilities on the Black Sea and high freight rates makes the export potential to appear minimal. Meanwhile, the industry's debt level has risen to more than $2 billion over 50% greater than the value of annual production and twice as much as its receivable debts. In the long run, however, rationalization of the Ukrainian coal industry is an economic imperative, and will likely include the closure of at least half the mines [38].

Russia. Russia was the leading coal producer of the former Soviet Union and its peak production of 416.5 Mt was achieved in 1988. Russia has huge coal reserves, 54,100 Mt of bituminous and 118,964 Mt of subbituminous of which those in eastern Siberia and the Russia Far East remain largely unexploited because of their remoteness and lack of infrastructure. Current hard coal production is won predominantly from numerous coalfields in European and Central Asian Russia. The Russian coal industry is in a continuing process of restructuring, by early 1999, privatized mines accounted for 22% of the country's coal production (ICR, 1999n). The Pechora basin, in the extreme northeast of European Russia, is a principal supplier of coking coal. Production is derived solely from highly mechanized underground operations. As in Ukraine, the Russian section of the Donetsk basin produces anthracite, coking coal and steam coal from operations that typically are deep, low-capacity and uneconomic. The Kuznetsk basin, (South-central Russia) is now the largest single producer in Russia. Coking and steam coal is derived from both surface and underground operations, and new mines are still being developed to replace uneconomic capacity in the coalfields elsewhere (MJ, 1999). Large-scale surface mining has been developed to provide power station fuel [38].

Privatization of Russia's coal industry will have a major effect on the geographical distribution of future mining. Already, the tendency is toward a concentration of capacity in the Kuzbass at the expense of the more difficult conditions and expensive mines in Donetsk and Pechora basins. It is therefore difficult to visualize major investment in regions other than the Kusbass and Kansk-Achinsc basins, and in other coalfields in the far east of Russia that can supply Pacific Rim export markets (MJ, 1999). Russia produced 232Mt of all types of coal in 1998, 30% lower than in 1990, although output recovered slightly in 1999 to 238.4 Mt, comprising 152.6 Mt of hard coal and 85.8 Mt of brown coal. Exports of 15.1 Mt of thermal coal and 1.6 Mt of coking coal were more than offset by imports (mainly from Kazakhstan) of 19.4 Mt of thermal coal and 5.6 Mt of coking coal [38].

Surface mining accounted for 62% of all output in 1998. In mid 1999, Russia had 159 mines in operation, of which 39 were scheduled for closure in 1999 and further 17 in 2000. However, some 42.5 Mt of annual capacity is currently under development, more than offsetting the 30 Mt that will be lost by the closures (MM, 1999). All the coalfields are well equipped with rail transport system that deliver coal to both domestic consumers and to Pacific Rim coast export ports. Dedicated coal-loading terminals have also been commissioned, that handle Kuzbass coking coal exports. One 3 Mt/y coal slurry pipeline is in use between Belov and Novosibirsk, reportedly giving 35-50% savings in transport cost (Mayshev, 1999). The average production costs across the industry of US$23/t in 1997, with an average selling price of US$25.20/t (ICR,1998a). Meanwhile, the large-scale strip mines in the Kansk-Achinsk basin currently achieve costs as low as US$2-3/t, and the target for new mines now being developed in the Kuzbass is a production cost of less than US$8/t (Malyshev, 1999) [38].

The other principal cost component for both export and domestic consumers is rail freight, given the huge distances that are involved from Kuzbass to both European Russia and the Far East ports. Costs from Kemerovo to St Petersburg fall in the range US$11.50 – 12.00/t, while the fob Pacific port prices in 1999 are quoted as being not more than US$25/t (Malyshev, 1999). The Government has projected an output of 245-290 Mt/y of coal by 2005 and plans to commission new mines with a capacity 110 Mt/y by 2010 (Yevtushenko, 1999). The coal industry in Russia is still beset by financial and structural problems that will take years to remediate. Its reliance on long-distance rail haulage to both domestic consumers and export ports is also a costly constraint. Nonetheless, the outlook is more positive than it was in the early 1990s, with the potential for foreign investment becoming more readily available for the development of new, low-cost mines. The use of high-voltage power transmission lines would assist in reducing the need for the physical transport of coal between producers and consumers, while the construction of long-distance coal slurry pipelines to carry coal both east and west from Siberia represents another prospective technology that could benefit Russian coal producers [38].

APPENDIX 2.0

CANADA'S COAL INDUSTRY AND DOMESTIC MARKET OVERVIEW

A2.1 Coal Deposits of Western Canada

A.2.1.1 Geological Overview

Western Canada is estimated to contain some 198 billions tons of coal of all ranks from lignitic to anthracitic. The coals, ranging in age from Jurassic to Tertiary, are widely distributed across this region and occur under widely diverse conditions. The vast coal-bearing area of western Canada extends from the lignite deposits of Saskatchewan into a sub-bituminous and bituminous coal region that covers about three quarters of the province of Alberta, and continues into northeast and southeast British Columbia. Three physiographic regions: Mountain, Foothills and Plains correspond to linear belts that parallel the Rocky Mountain Front, each exhibiting different coal rank and degrees of disturbance of the coal-bearing sequences as illustrated in Figure 1.1 (Chapter 1.0). From west to east, the Cordiliera or Mountain region extends southwards along the continental divide in British Columbia and Alberta and contains highly disturbed coal measures. The Foothills extend along the eastern margins of the Mountain region, comprising a zone of moderate disturbance, while the Plains region is generally flat-lying or gently inclined, with little or no disturbance to the coal seam [30, 31, 38].

Coal deposits formed in late Jurassic, Cretaceous and early Tertiary times. During middle to late Jurassic times, collision between the North American and Pacific crustal plates resulted in three major orogenic phases, each associated with the deposition of thick, wedge-shaped sedimentary sequences into the subsiding foredeeps on the eastern side of the rising mountains. The first (Columbian 1) orogeny extended from Jurassic to earliest Cretaceous times and resulted in the formation of up to 2700m of sediments containing the coal bearing Mines and Kootenay Groups. The second (Columbian 2), affecting most of what is now west-central Alberta and northeast British Columbia, is represented by the Bullhead, Fort St John and Luscar Groups of Lower to early Upper Cretaceous age. Following the formation of a major seaway between the artic and the Gulf of Mexico, the third (Laramide) orogeny in Upper Cretaceous to Tertiary times brought the coal-bearing

Horseshoe Canyon, Scollard and Paskapoo Formations of the Alberta plains and their equivalents in the foothills [31].

The main tectonic feature of the region, the Rocky Mountain Fold and Thrust Belt, formed during the Laramide Orogeny. Mountain-building, erosion and glaciations produced structurally complex coal deposits in mountainous terrain. Folding and faulting have caused shearing, repetition, thinning and thickening of the coal seams, which also exhibit a great variation in dip on both a local and regional scale. To the east, the Alberta Syncline developed in faulted contact along the southwestern margin of the Mountain and Foothill regions. Here the coal measures are generally undisturbed except for the effects of the near-surface glacial deformation [31].

The Mist Mountain Formation of the Kootenay Group forms potentially major economic coal deposits in the Mountain region of southwest Alberta and southeast British Columbia. The formation contains many of seams, 14 of which have been mined, with thicknesses ranging from 2 m in Alberta to over 15 m in British Columbia. The Bullhead, Fort St John and Luscar Groups occur in the Mountain and Foothills regions. The Gething Formation in the Bullhead Group varies from 120 m thick near Grand Cache with two, 2-5 m coal seams, to 1000 m thick in northeast British Columbia where it contains around 100 coal beds up to 4 m thick. The overlying Gates Formation of the Luscar Group contains seven tectonically thickened seams that reach 13 m in thickness, while the laterally equivalent formation within the Fort St John Group contains up to 11 seams with an aggregate thickness of 46 m. The Belly River, Edmonton, Wapiti and Saunders Group and the Brazeau Formation represent the third sedimentary wedge. Coal measures in the Belly River Group include the MacKay, Taber and Lethbridge Coal Zones, while the Coalspur Formation of the Saunders Group contains the Coal Valley Coal Zone, with 5-7 seams ranging from 1 m to 22 m thick. Its equivalent in the central Plains region, the Scollard Formation, contains the 60 m-thick Ardeley Coal Zone. The Wapiti Group in the northwest Plains and northern Foothills regions hosts, amongst others, the Paskapoo Formation, which contains the Obed Coal Zone. Comprising a 140 m-thick coal measures sequence, this is well-exposed in the Obed Mountain coalfield, where the seams range in thickness from 1 to 5 m [31, 38].

The rank of Western Canadian coal, as illustrated in Figure 1.1 (Chapter 1.0) and Table A.2.1, increases from east to west, with coals of similar rank occurring in broad belts parallel to the Rocky Mountains [22, 31]. In the Mountain region most of the coal is medium to low-volatile bituminous, with some areas of high-volatile bituminous or semi-anthracite. In the Foothills, the coal rank decreases to high-volatile bituminous A and the more widespread high-volatile bituminous C and sub-bituminous A, which extends into the southwest and northwest Plains region. The remainder of the Plains comprises sub-bituminous B and C coal which grades eastwards into lignite fields of Saskatchewan and North Dakota to the south. The Western Canadian coals generally have low sulphur content [3, 31, 38].

Coal rank is determined by various parameters, which are directly affected by petrografic composition. Petrography influences the coal-cleaning operation. From a petrografic point of view, coals are made of four coal types namely vitrain, clarain, durain, and fusain. Coal type is determined by the nature of the plant material from which the coal is formed and is an inherent characteristic of the coal. These coal types account also for the bright, soft, dull and hard properties. They have optical properties and reflectance varies significantly from vitrain to fusain. As rank increases, constituents of high reflectance become more abundant. Finely coal crushed material reacts more readily than coarser particles during carbonization. Vitrain, clarain, and fusain predominate in the finer fractions. Durain remains relatively coarse and may inhibit coke production. Fusain is a non-coking constituent is extremely soft and concentrates in the fines fraction. The coking properties of metallurgical coals can be controlled in the processing plant by different choices of separation gravity. Vitrain and clarain concentrate in the lighter fractions while durain and fusain in the heavier fractions [31].

Table A.2.1. Western Canadian Coal Qualities

Coalfield	Rank	Heating Value, [MJ/kg]	Moisture [%]	Volatiles [%]	Ash [%]	Sulphur [%]
Mountains region						
Flathead	HL	20~25	3~6	22	25~35	0.5
Crowsnest	HL	22~19	3~9	18~24	12~35	0.3~0.4
Elk Valley	HL	24~29	4~14	19~29	12~30	0.3~0.5
Cadomin-Luscar	HM	27~30	4~6	18~22	15~25	0.2~0.4
Smoky River	L	28~31	3~8	15~17	12~18	0.3~0.5
Peace River	MI	22~28	3~6	19~25	20~35	0.3~0.5
Foothills region						
Coalspur	H	19~29	10~14	24~28	25~35	0.2~0.4
McLeod River	H	27	11	35	11	0.3
Plain region						
East Brooks	S	20				
Brooks	S	19~21				
Blackfoot	S	19~20				
Sheerness	S	17~19	24~17	27~30	8~15	0.4~0.5
Drumheller	S	18~21				
Garden Plain	S	17				
Battle River	S	17~19	25~26	28~30	8~13	0.4~0.5
Tofield Dodds	S	19~20	19~25	28~30	7~9	0.4~0.5
Morinville	S	19~20	24~27	28~32	7~10	0.3
Ardley	S	15~20				
Alix	S	22	19	28	9	0.3
Wetaskwin	S	19~20	18~20	27~28	13~16	0.2~0.4
Wabamum	S	17~19	19~21	27~29	11~17	0.2~0.3
Meyerthorpe	S	21	20	27	11	
Obed Mountain	H	16~19	13~17	20~31	25~35	0.3~1.2

In Table A.2.1; H stands for high-volatile bituminous, HM for high-to medium-volatile bituminous, HL for high-to low-volatile bituminous, ML for medium-to low-volatile bituminous, and S for sub-bituminous.

A.2.1.2 Coal Resources

The current estimate of measured, indicated and inferred coal resources is 198,600 Mt, of which 90% lies in the three western provinces of Saskatchewan, Alberta and British Columbia. This total encompasses a resource base of 92,240 Mt of bituminous coal, 3455Mt of anthracite, 49,635 Mt of sub-bituminous and 53,270 Mt of lignite. Within the resource base, Canada's recoverable reserves comprise 3471Mt of bituminous coal, 871 Mt of sub-bituminous and 2236 Mt of lignite, totaling 6578 Mt. Of this, 1918 Mt are regarded as being coking coal and 4660 Mt thermal. Resources on a provincial basis are shown in Table A.2.2 Coals that occur in beds less than 450 mm thick, and beds that occur at depths below 600 m, are excluded from Western Canada's coal resources [38].

Table A.2.2 Coal Resources in Western Canada (Mourits, 2000)

Province	Type of Coal	Recoverable Reserves [Mt]	Total Resources [Mt]
Alberta			
	Bituminous	1,040	12,645
	Sub-bituminous	871	33,475
British Columbia			
	Bituminous	1,996	16,460
Saskatchewan			
	Lignite	1,670	7,595

A.2.1.3 Area of interest

Significant deposits in the Mountain region comprise the East Kootenay coal area in southeastern British Columbia, the Smoky River and Candomin-Luscar coalfields in west-central Alberta, and the Peace River coal area in northeast British Columbia. The Kootenay district, which encompasses the structurally separate Flathead, Crowsnest and Elk Valley coalfields, contains seams that have been tectonically thickened to provide substantial reserves. The Cadomin-Luscar coalfield contains one major seam up to 30 m thick, while the Smoky River coalfield contains 11 seams, of which three are mineable. Further north, the Peace River district contains up to six seams with large resources caused by the coal measures having been repeatedly brought to the surface by low-angle thrust faults, and locally thickened. Significant deposits in the Foothills region occur in northwest Alberta, where seams in the Coalspur and McLeod River coalfields have again been tectonically thickened, and contain large reserves. In the Plains region, Upper Cretaceous to Tertiary coal zones generally persist over large areas; these include the important Lethbridge, Carbon-Thomson, Drumheller and Obed Coal Zones, and the Ardeley Coal Zone, which contains vast surface reserves of subbituminous coal. Near-surface lignite resources in Saskatchewan are more than adequate to supply current and future needs [31, 38].

A.2.1.4 Coal Chemical and Physical Characteristics

This report is addressing only two of the coal deposits in Western Canada, the Kootenay Groups and Luscar Groups, which are the most important from an economic viewpoint. These are the groups where most of the coal mines are and where most of the metallurgical coal is produced. Coal chemical and physical characteristics are specific for each deposit. The rock coal physical and chemical characteristics of these deposits are presented in Table A.2.3 and the clean coal chemical and physical characteristics are presented in Table A.2.4 [3, 29, 30, 31, 38].

Table A.2.3 Rocky Mountain Belt Coalfields Rock Coal Specifications

Coalfield	Rank	Heating Value, [MJ/kg]	Moisture [%]	Volatiles [%]	Ash [%]	Sulfur [%]
Flathead	HL	20~25	3~6	22	25~35	0.5
Crowsnest	HL	22~19	3~9	18~24	12~35	0.3~0.4
Elk Valley	HL	24~29	4~14	19~29	12~30	0.3~0.5
Cadomin-Luscar	HM	27~30	4~6	18~22	15~25	0.2~0.4
Smoky River	L	28~31	3~8	15~17	12~18	0.3~0.5
Peace River	MI	22~28	3~6	19~25	20~35	0.3~0.5

Table A.2.4 East Kootenay Coalfields Cleaned Coal Specifications

Product Coal Dry Basis	Medium-Volatile	High-Volatile
Volatile matter (%)	21 - 28	32
Fixed Carbon (%)	64 - 69	62
Ash (%)	8 - 9.8	6.0
Sulfur (%)	0.3 - 0.7	0.4 - 0.8
Btu / lb	13,750 - 14,200	14,200 - 15,050
Mj / kg	32 - 33	33 - 35
Hardgrove index	> 80	> 60
Rmax (%)	1.1 - 1.35	0.8 - 1.1

A.2.1.5 Types of Coal Products and Requirements

The companies that mine coal in the Rocky Mountain Coalfields are Fording, Luscar-Sherritt, and Teck. These companies produce mainly metallurgical coal for overseas and domestic customers and thermal coal for electricity generation. Metallurgical coal is produced at different qualities required by the customers and can vary in a certain range. Metallurgical and thermal coal average quality, in Vancouver ports, is presented in Table A.2.5. The range of metallurgical coal specifications is presented in Table A.2.6 [3, 36, 38].

Table A.2.5 Metallurgical and Thermal Coal Qualities in Vancouver Ports

Requirements	Ash [%]	Sulphur [%]	Moisture [%]	Fixed C. [%]	Volatile M. [%]	MJ/kg
Metallurgical	8.5	0.4	8.0	58	22.6	30.2
Thermal	13	1.0	13.5	46.5	36.2	26.1

Table A.2.6 Range of Metallurgical Coal Characteristics

Volatile matter (%) dry	26.0	22.0
Ash (%) dry	9.0	6.0
Sulfur (%) dry	0.4	1.0
Moisture	8.0	5.0
Screen size (mm)	0 -30	0 - 30

A.2.2 Mines Production Capacities

There are 21 operating coal mines in Canada and most of them located in the Western Canadian provinces. Luscar-Sherritt International is Canada's leading coal producer and among the largest suppliers of coal in North America. Luscar operates 11 surface mines located in the provinces of British Columbia, Alberta and Saskatchewan. Luscar-Sherritt produces metallurgical and thermal bituminous coal for domestic and overseas markets and transports approximately 36 million tonnes of coal annually. The second largest coal producer but the largest metallurgical coal producer and exporter in Canada is Fording Coal. Fording Coal owns and operates 3 coal mines in South-east British Columbia and 2 in Alberta and produces over 20 million tonnes of coal annually. The next largest coal producer in Canada is Teck which operates 2 mines in British Columbia and ships approximately 5.5 million tonnes annually. The main Canadian coal mines and their saleable production are presented in Table A.2.7 [9, 16, 22, 36].

Table A.2.7 Saleable Productions of the Main Canadian Coal Mines

Mine	Million Tonnes
Bienfait	1.8
Boundary Dam	5.9
Bullmoose	1.4
Coal Mountain	2.3
Coal Valley	1.1
Elkview	4.1
Fording River	9.0
Genesee	3.5
Greenhills	4.4
Highvale	12.5
Line Creek	2.5
Luscar	2.6

New Brunswick Coal	0.23
Obed Mountain	1.5
Paintearth	2.8
Poplar River	3.5
Prince	0.866
Quinsam	N/a
Sheerness	3.6
Smoky River	0.7
Whitewood	2.0

A.2.3 Canada's Domestic Coal Market Overview

In 2000, Canada produced 69.1 million tonnes of coal. Nearly half, 31.7 million tonnes, was exported to the Pacific Rim, Europe and South America. Japan, Canada's largest coal customer, purchased 18.6 million tonnes of coal in 2000 of which 86 % (13.3 million tonnes) was metallurgical coal for steel making. Of the ten provinces, Ontario and Alberta consume by far the greatest amounts of coal consumed in Canada. In the same year, Ontario used 14.2 million tones, 25 % of Canada's coal use. Four other provinces use coal to generate electricity: Saskatchewan, Manitoba, Nova Scotia and New Brunswick. Although much of its production is used domestically, Canada imported 15 million tonnes of coal from United States in 2000 for industrial use in Ontario and Quebec and for electrical generation in Ontario and New Brunswick. Tables A.2.8 through A.2.12 and Figures A.2.1 and A.2.2 present some of the important statistical data regarding Coal mining industry in Canada [36].

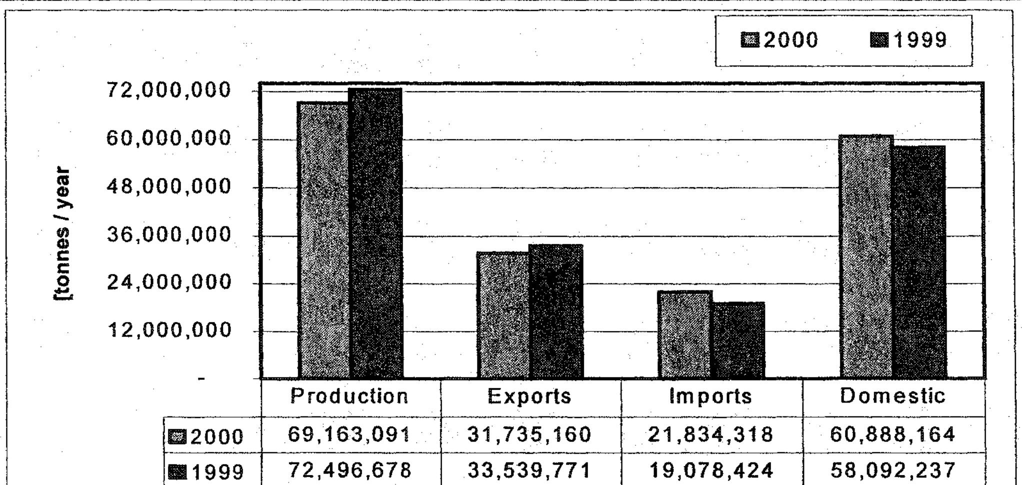


Figure A.2.1 Canadian Coal Production and Consumption 2000

Table A.2.8 Coal Production by Province 2000 (tonnes)

	Metallurgical	Thermal	Total
BC	24,230,573	1,450,762	25,681,335
Alberta	3,871,007	27,025,827	30,896,834
Saskatchewan	-	11,190,106	11,190,106
New Brunswick	-	229,290	229,290
Nova Scotia	-	1,165,526	1,165,526
Total	**28,101,580**	**41,061,511**	**69,163,091**

Table A.2.9 Coal Exports by Province 2000 (tonnes)

	Metallurgical	Thermal	Total
BC	24,038,367	1,138,517	25,176,884
Alberta	3,820,592	2,737,684	6,558,276
Total	27,858,959	3,876,201	31,735,160

Table A.2.10 Consumption of Imported Coal 2000 (tonnes)

Province	Bituminous	Anthracite	Sub-bituminous	Total
Alberta	-	5,990	-	5,990
Manitoba	-	-	537,149	537,149
Ontario	9,765,886	43,846	4,465,928	14,275,660
Quebec	408,243	396,277	-	804,520
New Brunswick	1,027,573	-	-	1,027,573
Nova Scotia	2,293,455	-	-	2,293,455
Newfoundland	-	74,161	-	74,161
Total	**13,495,157**	**520,274**	**5,003,077**	**19,018,508**

Table A.2.11 Domestic Consumption of Coal 2000 (tonnes)

Province	Steel Making	Electricity	Industrial/Domestic	Total
BC	-	-	348,557	348,557
Alberta	-	24,768,473	15,904	24,784,377
Saskatchewan	-	9,180,499	206,933	9,387,432
Manitoba	-	561,251	121,875	683,126
Ontario	4,264,600	15,209,353	718,644	20,192,597
Quebec	-	-	804,520	804,520
New Brunswick	-	1,242,277	-	1,242,277
Nova Scotia	-	3,321,714	49,403	3,371,117
Newfoundland	-	-	74,161	74,161
Total	**4,264,600**	**54,283,567**	**2,339,997**	**60,888,164**

Table A.2.12 Structure of Coal Industry Source: 1999 and 2000 Annual Reports

Company	Production Mt	Manpower	Assets M$	Revenues M$	Reserves Mt
Luscar	38.7	2,300	1,422	693	1,566
Fording	21.2	2,071	995	896	1,220
Teck	5.5	950	n/a	n/a	270

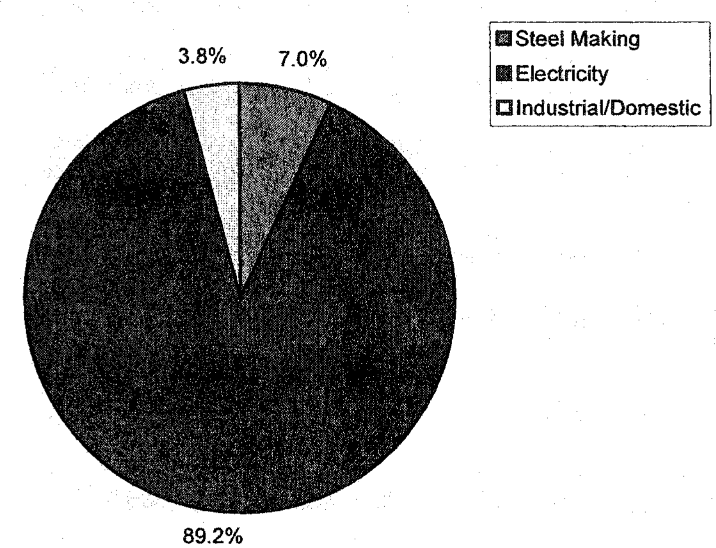


Figure A.2.2 Canada's Domestic Coal Consumption

In spite of the fact that Canada is producing more coal than is consumed domestically, Canada still imported almost 22 Mt of coal in 2000 and 19 Mt in 1999 from US, while only 1.5 Mt of coal was imported by the US from Canada in 2000. Canada seems to lose its Eastern Canada domestic market share against US. From the tables above it can be noticed that there is a big potential for coal in Eastern provinces of Canada especially Ontario. Ontario imported 9.7 Mt of bituminous coal, 4.5 Mt sub-bituminous coal representing a total of 14.2 Mt of coal from the US in 2000 [36].

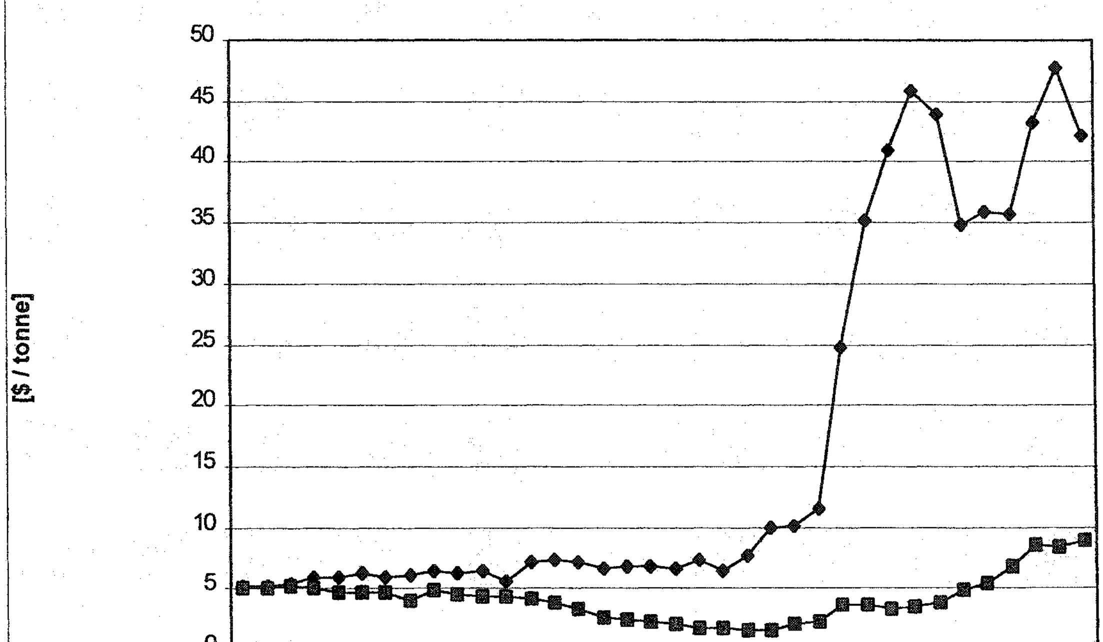
Canadian export coal producers struggled during the 1990s. Over-optimistic demand forecasts in the late 1970s led to the construction of high-cost mines and the upgrading, at great expense, of rail and port facilities. Since the mid 1980s reduced Japanese demand for coking coal has left Western Canadian producers with serious over-capacity. Subsequent fluctuation in Asian steel production and unfavorable dollar exchange rate movements were exacerbated by Asian economic crisis of the late 1990s, which led to further substantial price cuts for coking coal. With production and transport costs higher than exporters such as Australia and Indonesia, Canadian producers have had to bear brunt of these cuts [36, 38].

Consolidation of ownership has been marked during the 1990s and continues today with the take-over by Sheritt International of the country's largest producer Luscar. Luscar-Sheritt, Fording Coal and Teck Corp now control over 95% of Western Canadian production and 91% of exports. Production is almost exclusively from large surface operations. Geological conditions can be very difficult, particularly in northeastern British Columbia, where recoveries can be as low as around 20% of the in situ coal. Two competing railways, Canadian National and Canadian Pacific, connect producers with west coast at Vancouver, and with eastern Canada, both directly and through the port at Thunder Bay. The principal rail routes for exports now run from southeast British Columbia and south-central Alberta to the Vancouver and Robert Bank terminals. Coal from Northeastern B.C and northern Alberta is carried by BC Rail on its dedicated line to the Ridley Island terminal, near Prince Rupert. The Canadian railways offer one of the lowest per-ton-km rates in the world. This advantage is offset by the long haul distances which, for eastbound coal movements, can be over 2000km. Westbound rail haulage averages around 1100 km. High relative freight costs mean that steam coal exports are rarely competitive on a fob basis unless production costs are incremental to coking coal output. Cash costs fob port range from US$28.90/t to US$40.63/t for coking coal, and US$25.50-26.18/t for steam coal (ICR, 1999K). Mine-gate costs for 1999 range from US$16.30/t – US$26.50/t for metallurgical coal, to which must be added US$9.90-12.25 in rail costs and US$2.00-4.00 for port charges [38].

Canada is one of the countries most likely to be affected by reduced market share over the short-to medium-term. Increasing competition from lower-cost producers such as Indonesia and Australia will continue to erode its market share base in the Far East, with higher shipping costs from western Canadian ports making markets such as the Mediterranean or Western Europe equally unattractive. (MJ, 1998). With other countries already in a position to respond to continuing demand growth despite falling prices, the potential for Canada to increase its market share appears slim. With the loss of US coking coal capacity, Canada may inherit the role of being a swing supplier to the market (Whittington, 1999). Without the development of innovative concepts such as viable long-distance electricity transmission to supply eastern Canadian markets, western Canada's coal industry faces a difficult future (Downing, 1999) [38].

A.2.4 Coal Price History and Projected Forecasts

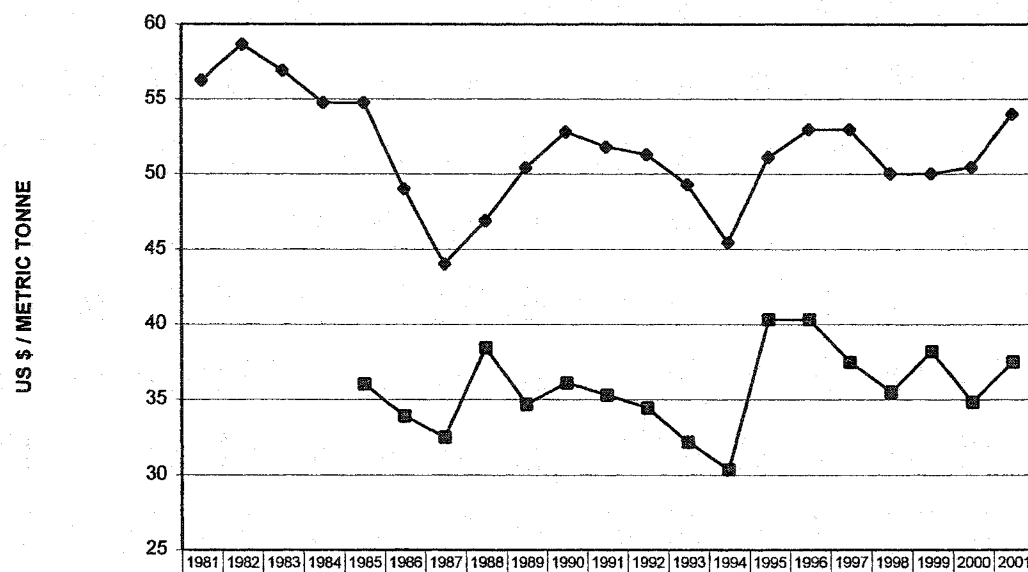
Coal market is mainly contractual. The contract usually protects both suppliers and buyers from large and speculative fluctuation. Little coal is sold on spot market and few companies survived selling on the spot market. Coal prices are mainly influenced by factor as world coal supply and demand, world steel demand, and oil and gas prices. Annual negotiations take place to settle benchmark prices between Australia and Japanese steel mils. Canada's Coal price history from 1949 to 1983 is presented in Figure A.2.3 (Source BC Government) [3].



	1949	1951	1954	1956	1959	1961	1963	1966	1968	1971	1973	1975	1978	1980	1983
BITUMINOUS CAN $	5.274	5.483	6.233	6.068	6.383	7.115	7.085	6.83	7.24	9.85	11.43	35.21	43.95	35.89	47.65
SUBBITUMINOUS CAN $	5.018	5.144	4.643	4.064	4.43	4.21	3.241	2.3	1.76	1.6	2.27	3.65	3.8	5.33	8.406

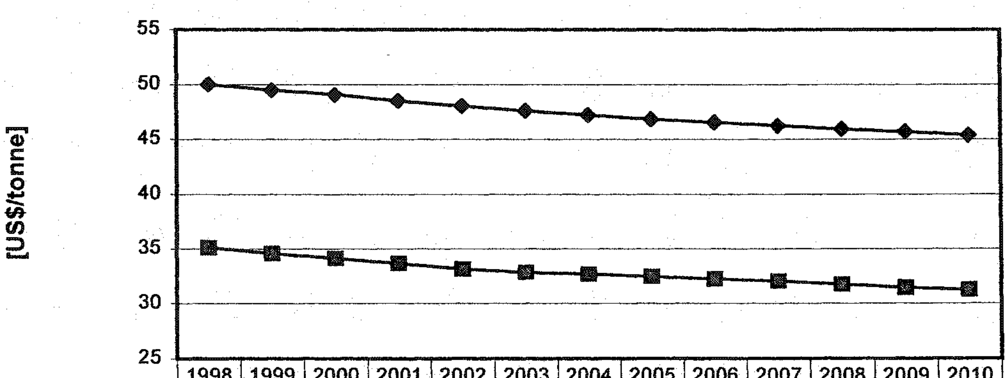
Figure A.2.3 Canada's Coal Price History (1949-1983) (BC Government)

Price of coal has been gradually decreasing over the past 20 years having small periods of fluctuation at about 6-7 seven years as illustrated in Figure A.2.4 (source BC Government). As a general conclusion the coal price is most likely to decrease over the next 20 years (See Appendix 1.0). Figure A.2.5 presents the USA government coal price forecast for the next 10 years. Because coal price has such a big impact on the economics of any new or old project, investors and producers must be careful and conservative in their managerial and economical policies [3, 18, 36].



	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
METALLURGICAL	56.3	58.7	56.9	54.8	54.8	49	44	46.9	50.4	52.8	51.8	51.3	49.3	45.5	51.1	53	53	50	50	50.4	54
THERMAL					36	33.9	32.5	38.4	34.7	36.1	35.3	34.5	32.2	30.3	40.3	40.3	37.5	35.5	38.2	34.8	37.5

Figure A2.4 Average Export Coal Price F.O.B. Vancouver Ports



	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Metallurgical	49.98	49.45	49.05	48.48	48.02	47.59	47.19	46.83	46.51	46.23	45.91	45.67	45.38
Thermal	35.1	34.57	34.11	33.65	33.13	32.82	32.67	32.45	32.21	32.03	31.71	31.43	31.24

Figure A.2.5 US Government Coal Price Forecast

A.2.5 Strategies to Compete in the Global Market in the Future

Coal future around the world is closely related to the economic growth and improved standard of living. Consumption of coal has two major components, coal for producing electricity and coal for producing steel. In all the forecasts the need of electricity and steel around the world is gradually increasing. Western Canada with its large high quality near-surface coal resources is still going to play a major role in world coal trade. In spite of certain advantages over other competitors Canadian coal producers have to deal with transportation constraints in order to maintain their competitiveness. The long haul distances from the mine site to the ports and mine sites to the eastern Canadian markets, leading to high freight costs, is a real concern. In analyzing the strategies that Western Canadian Coal Producers should follow, one has to take into account the two components of coal consumption, metallurgical and thermal. Most of the Canadian metallurgical coal is produced to be exported and most of the thermal coal produced is internally consumed [18, 36, 38].

A.2.5.1 Metallurgical Coal.

According to the forecasts Canada is one of the country most likely to be affected by increasing competition from lower-cost producers such as Indonesia and Australia [38]. Coal industry has to change to adapt to marketplace (domestic and export). Canadian metallurgical coal producers should focus their attention on new emerging steel producers as Brazil, India, Mexico and Turkey while maintaining their market share with classic steel producers as Japan and South Korea and Western Europe through long-term contracts. What seems to be occurring in the Asian coal markets is a shift away from contract purchases to the spot market which means that exporting countries will be under increased pressure to reduce mining costs in order to maintain current rates of return.

A good strategy of reducing producing and transportation costs is the consolidation of the ownership and the creation of integrated mining systems. In an integrated mining system several mines join and operate as a single mine. Using an integrated mining system some transportation and processing cost could be avoided by planning the production and coal products according to the coal qualities and distances form the mines to the final destinations.

Regain and conquer more of the Eastern Canadian market especially Ontario that consumed 20.1 Mt of coal in 2000 of which 14.2 Mt was imported from US. Expand Canada's market share in US. Canada imported almost 22 Mt of coal in 2000 and 19 Mt in 1999 from US, while only 1.5 Mt of coal was imported by the US from Canada in 2000 [7]. Strong pressure on contract prices negotiated between Canadian producers and their customers in Japan, South Korea, and elsewhere in the Far East. Improved productivity in surface mine operations by adopting new highly productive technologies. Better cooperation among Western Canadian coal producers within Coal Association of Canada and understand that before competing among themselves they have to compete with low-cost producers such as Indonesia and Australia.

A.2.5.2 Thermal Coal.

Thermal coal as a commodity seems to be more volatile than the metallurgical coal. The price of thermal coal is closely related with the oil and natural gas prices. The higher the oil and natural gas prices the better for the thermal coal producers. Some of the electricity generator has the capability to switch their power plants from oil-fired to coal-fired depending on costs. An additional factor contributing to coal import growth was higher oil prices, which led to substitution of coal-fired generation in some importing countries [18, 38]. New markets for thermal coal seem to appear in the southern states of US, Mexico and Brazil [18]. The Canadian thermal coal could become competitive for these new markets by lowering the production and transportation costs. Production costs could be decreased by improving productivity in surface mine operations and adopting new highly productive technologies.

The recently North American energy crisis proved that thermal coal has future as fuel for producing electricity. Since coal is abundant, low cost, secure and stable is preferred to other sources of energy in some of the Canadian provinces, US states and foreign countries. All forecasts project an increase in electricity demand in North America over the next 20 years and coal provides source of inexpensive fuel for electricity production. Therefore, Western Canadian coal producers should put pressure on the federal and provincial government to link the electric power grids of Alberta and the western US states where there is an increasing demand of electricity.

Consolidation of ownership in coal mining/electricity generation, individual power station/mine units, such that each would be a wholly separate commercial entity. Strong pressure on long-term contract prices negotiations between producers and customers. Development of innovative concepts such as viable long-distance electricity transmission to supply eastern Canadian markets (high-voltage power transmission lines). Construction of long-distance coal slurry pipelines to carry coal from Alberta or Saskatchewan to the Eastern Canadian provinces (See Russia case, 35-50% savings in transport costs).

A.2.5.3 Future Coal Opportunities.

The future of the coal industry lies in research into advancing clean coal technologies as Integrated Gasification Combined-Cycle, which can increase generating efficiencies by 20 to 30 percent and also reduce emission levels (See Western Europe) [10]. Production of hydrogen out of coal and production of electricity by burning hydrogen. Electricity can be produced with extremely high efficiency, 50% less CO_2, capture of all emission products. The cost of producing electricity from hydrogen is \$3.84/GJ when hydrogen is produced from coal, \$9.25/GJ for hydrogen produced from natural gas and \$16.38/GJ from oil (Flemming, U of A presentation 2001).

APPENDIX 3.0
LUSCAR-SHERRITT'S LP ANSWER REPORT

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$D$365		(4,500.00)	$D$365<=$F$365	Not Binding	4500
$D$372		-	$D$372<=$F$372	Binding	0
$D$378		-	$D$378<=$F$378	Binding	0
$D$386		7,500.00	$D$386>=$F$386	Not Binding	7,500.00
$D$390		77,500.00	$D$390>=$F$390	Not Binding	77,500.00
$D$395		(8,500.00)	$D$395<=$F$395	Not Binding	8500
$D$399		(89,000.00)	$D$399<=$F$399	Not Binding	89000
$D$404		5,500.00	$D$404>=$F$408	Not Binding	5,500.00
$D$415		(10,000.00)	$D$415<=$F$415	Not Binding	10000
$D$422		(3,316.24)	$D$422<=$F$422	Not Binding	3316.23932
$D$429		-	$D$429<=$F$429	Binding	0
$D$436		12,564.10	$D$436>=$F$436	Not Binding	12,564.10
$D$440		(9,743.59)	$D$440<=$F$440	Not Binding	9743.5897

					4
$D$80	X11+ X21+ X31+ X41+ X51 <=	5,500,000	$D$80<=$F$80	Binding	0
$D$82	Y11+ Y21+ Y31+Y41+ Y51 <=	2,000,000	$D$82<=$F$82	Binding	0
$D$86	X22+ X32+ X42+ X52 <=	1,000,000	$D$86<=$F$86	Binding	0
$D$88	Y22+ Y32+ Y42+Y52 <=	2,000,000	$D$88<=$F$88	Not Binding	500000
$D$93	X13+ X23+ X33+ X43+ X53 <=	1,000,000	$D$93<=$F$93	Not Binding	500000
$D$96	Y13+ Y23+ Y33 +Y43+ Y53 <=	2,000,000	$D$96<=$F$96	Not Binding	500000
$D$98	Y14+ Y24+ Y34+ Y44+ Y54 <=	2,000,000	$D$98<=$F$98	Binding	0
$E$155	X11 + Y11 <=	2,000,000	$E$155<=$G$155	Not Binding	3500000
$E$160	X21+ X31+ X41+ X51+ Y21+ Y31+ Y41+ Y51 <=	5,500,000	$E$160<=$G$160	Not Binding	8000000
$E$165	X22+ X32+ X42+ X52+ Y22+ Y32+ Y42+Y52 <=	3,000,000	$E$165<=$G$165	Not Binding	5500000
$E$170	X13 + Y13 <=	2,000,000	$E$170<=$G$170	Not Binding	9000000
$E$175	X23+ X33+ X43+ X53+ Y23+ Y33+ Y43+Y53 <=	1,000,000	$E$175<=$G$175	Not Binding	12000000
$D$211		-	$D$211>=$F$211	Binding	-
$D$217		500,000	$D$217>=$F$217	Not Binding	500,000
$D$224		165,000.0	$D$224>=$F$224	Not Binding	165,000.0
$D$230		177,300.61	$D$230>=$F$230	Not Binding	177,300.61
$D$237		2,375,000	$D$237>=$F$237	Not Binding	2,375,000
$D$243		1,000,000	$D$243>=$F$243	Not Binding	1,000,000
$D$280		250,000.00	$D$280>=$F$280	Not Binding	250,000.00
$D$286		-	$D$286>=$F$286	Binding	-
$D$293		50,000.00	$D$293>=$F$293	Not Binding	50,000.00
$D$299		159,323.32	$D$299>=$F$299	Not Binding	159,323.32
$D$306		275,000	$D$306>=$F$306	Not Binding	275,000
$D$312		1,000,000	$D$312>=$F$312	Not Binding	1,000,000
$D$349		-	$D$349>=$F$349	Binding	-
$D$355		23,650,000	$D$355>=$F$355	Not Binding	23,650,000
$D$362		15,000.00	$D$362>=$F$362	Not Binding	15,000.00
$D$368		50,000.00	$D$368>=$F$368	Not Binding	50,000.00
$D$375		500,000	$D$375>=$F$375	Not Binding	500,000
$D$381		700,000	$D$381>=$F$381	Not Binding	700,000
$D$418		-	$D$418>=$F$418	Binding	-
$D$425		168,376.07	$D$425>=$F$425	Not Binding	168,376.07
$D$432		1,000,000	$D$432>=$F$432	Not Binding	1,000,000

APPENDIX 4.0
LUSCAR-SHERRITT'S LP SENSITIVITY REPORT

Microsoft Excel 8.0 Sensitivity Report
Worksheet: [Luscar Scenario L.P..xls]
Report Created: 5/16/02 6:22:48 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$25	X11	2,000,000	-	14.767	2.153	1.716192
$D$25	X13	500,000	-	13.034	1.7161973	2.153
$E$25	Y11	0	-	2.267	1.6800123	1.451081
$G$25	Y13	1,500,000	-	0.534	2.3458027	1.680027
$H$25	Y14	-	(1)	11.35	1.4510658	1E+30
$B$27	X21	1,500,000	-	15.736	0.8580987	1.0765
$C$27	X22	500,000	-	14.042	1E+30	0.858096
$D$27	X23	-	(1)	11.95	1.0765	1E+30
$E$27	Y21	736,196	-	3.236	0.44311	0.093278
$F$27	Y22	263,804	-	1.542	0.0932778	0.106372
$G$27	Y23	-	(0)	-0.55	0.140001	1E+30
$H$27	Y24	-	(1)	14.89	0.6920855	1E+30
$B$29	X31	2,000,000	-	14.966	2.153	1.716192
$C$29	X32	500,000	-	13.292	1.1729013	0.858096
$D$29	X33	500,000	-	11.28	1.7161973	0.92557
$E$29	Y31	-	(0)	2.466	0.133908	1E+30
$F$29	Y32	158,120	-	0.792	0.1527284	0.13398
$G$29	Y33	500,000	-	-1.22	1.7161973	0.152729
$H$29	Y34	341,880	-	14.38	3.311408	0.993541
$B$31	X41	-	(3)	14.411	2.7166447	1E+30
$C$31	X42	-	(5)	12.798	5.3146727	1E+30
$D$31	X43	-	(2)	10.614	1.8511447	1E+30
$E$31	Y41	343,558	-	4.911	0.1834202	0.20919
$F$31	Y42	1,011,143	-	3.298	0.2091994	0.18342
$G$31	Y43	-	(1)	1.114	1.020546	1E+30
$H$31	Y44	1,145,299	-	17.37	1.4984708	1.32456
$B$33	X51	-	(4)	16.686	4.0087313	1E+30
$C$33	X52	-	(5)	14.806	5.3976327	1E+30
$D$33	X53	-	(5)	11.278	5.2424813	1E+30
$E$33	Y51	920,245	-	7.186	0.5186402	0.3318
$F$33	Y52	566,934	-	5.306	0.3318	0.518645
$G$33	Y53	-	(3)	1.778	3.327182	1E+30
$H$33	Y54	512,821	-	17.89	2.2076053	1.230244

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$55	X11+X21+X31+X41+X51	5,500,000	0	3000000	2500000	1E+30
$D$57	Y11+Y21+Y31+Y41+Y51	2,000,000	(12)	2000000	0	0
$D$61	X22+X32+X42+X52	1,000,000	-	1000000	0	1E+30
$D$63	Y22+Y32+Y42+Y52	2,000,000	(14)	2000000	500000	0
$D$67	X13+X23+X33+X43+X53	1,000,000	-	1000000	0	1E+30
$D$69	Y13+Y23+Y33+Y43+Y53	2,000,000	(15)	2000000	0	0
$D$73	Y14+Y24+Y34+Y44+Y54	2,000,000	-	1500000	500000	1E+30
$D$102	X11+X13+Y11+Y13+Y14	4,000,000	15	4000000	0	0
$D$104	X21+X22+X23+Y21+Y22+Y23+Y24	3,000,000	14	3000000	0	0
$D$106	X31+X32+X33+Y31+Y32+Y33+Y34	4,000,000	13	4000000	0	0
$D$108	X41+X42+X43+Y41+Y42+Y43+Y44	2,500,000	17	2500000	0	500000
$D$110	X51+X52+X53+Y51+Y52+Y53+Y54	2,000,000	20	2000000	0	500000
$E$179	Y14+ Y24+ Y34+Y44+ Y54	2,000,000	-	5000000	1E+30	3000000

$D$114	X11+X13+Y11+Y13+Y14	4,000,000	-	3200000	800000	1E+30
$D$116	X11+X13	2,500,000	(4)	2500000	0	0
$D$119	X21+X22+X23+Y21+Y22+Y23+Y24	3,000,000	-	2500000	500000	1E+30
$D$121	X21+X22+X23	2,000,000	(1)	2000000	0	0
$D$124	X31+X32+X33+Y31+Y32+Y33+Y34	4,000,000	-	3200000	800000	1E+30
$D$126	X31+X32+X33	3,000,000	-	2500000	500000	1E+30
$D$129	X41+X42+X43+Y41+Y42+Y43+Y44	2,500,000	-	1500000	1000000	1E+30
$D$132	X51+X52+X53+Y51+Y52+Y53+Y54	2,000,000	-	1500000	500000	1E+30
$E$139	X11+X21+X31+X41+X51+Y11+Y21+Y31+Y 41+Y51	7,500,000	-	13500000	1E+30	6000000
$E$143	X22+X32+X42+X52+Y22+Y32+Y42+Y52	3,000,000	-	14000000	1E+30	11000000
$E$147	X13+X23+X33+X43+X53+Y13+Y23+Y33+Y 43+Y53	3,000,000	-	6000000	1E+30	3000000
$D$408		28,000.00	-	0	28000	1E+30
$D$208		(27,500.00)	-	0	1E+30	27500
$D$214		(5,000.00)	-	0	1E+30	5000
$D$444		12,017.09	-	0	12017.094	1E+30
$D$221		(1,100.00)	-	0	1E+30	1100
$D$227		(3,226.99)	-	0	1E+30	3226.997
$D$234		(3,750.00)	-	0	1E+30	3750
$D$240		-	347.82	0	0	948.7179
$D$248		52,500.00	-	0	52500	1E+30
$D$252		20,122.70	-	0	20122.699	1E+30
$D$257		(43,000.00)	-	0	1E+30	43000
$D$261		(14,110.43)	-	0	1E+30	14110.42
$D$266		34,000.00	-	0	34000	1E+30
$D$270		17,079.75	-	0	17079.755	1E+30
$D$277		(2,500.00)	-	0	1E+30	2500
$D$283		(10,000.00)	-	0	1E+30	10000
$D$290		-	2,932	0	0	0
$D$296		(3,406.77)	-	0	1E+30	3406.766
$D$303		(2,250.00)	-	0	1E+30	2250
$D$309		-	404.98	0	0	948.7179
$D$317		5,000.00	-	0	5000	1E+30
$D$321		14,813.20	-	0	14813.198	1E+30
$D$326		(3,500.00)	-	0	1E+30	3500
$D$330		(13,145.98)	-	0	1E+30	13145.98
$D$335		500.00	-	0	500	1E+30
$D$339		14,903.15	-	0	14903.151	1E+30
$D$346		(5,000.00)	-	0	1E+30	5000
$D$352		(3,500.00)	-	0	1E+30	3500
$D$359		(350.00)	-	0	1E+30	350
$D$365		(4,500.00)	-	0	1E+30	4500
$D$372		-	-	0	1E+30	0
$D$378		-	-	0	1E+30	0
$D$386		7,500.00	-	0	7500	1E+30
$D$390		77,500.00	-	0	77500	1E+30
$D$395		(8,500.00)	-	0	1E+30	8500
$D$399		(89,000.00)	-	0	1E+30	89000
$D$404		5,500.00	-	0	5500	1E+30
$D$415		(10,000.00)	-	0	1E+30	10000
$D$422		(3,316.24)	-	0	1E+30	3316.239
$D$429		-	800.78	0	0	1615.384
$D$436		12,564.10	-	0	12564.103	1E+30
$D$440		(9,743.59)	-	0	1E+30	9743.58
$D$80	X11+ X21+ X31+ X41+ X51	5,500,000	3	5500000	0	500000
$D$82	Y11+ Y21+ Y31+Y41+ Y51	2,000,000	-	2000000	1E+30	0
$D$86	X22+ X32+ X42+ X52	1,000,000	0	1000000	0	0
$D$88	Y22+ Y32+ Y42+Y52	2,000,000	-	2500000	1E+30	500000
$D$93	X13+ X23+ X33+ X43+ X53	1,000,000	-	1500000	1E+30	500000
$D$96	Y13+ Y23+ Y33 +Y43+ Y53	2,000,000	-	2500000	1E+30	500000
$D$98	Y14+ Y24+ Y34+ Y44+ Y54	2,000,000	-	2000000	1E+30	0
$E$155	X11 + Y11	2,000,000	-	5500000	1E+30	3500000
$E$160	X21+ X31+ X41+ X51+ Y21+ Y31+ Y41+ Y51	5,500,000	-	13500000	1E+30	8000000

$E$165	X22+ X32+ X42+ X52+ Y22+ Y32+ Y42+Y52	3,000,000	-	8500000	1E+30	5500000
$E$170	X13 + Y13	2,000,000	-	11000000	1E+30	9000000
$E$175	X23+ X33+ X43+ X53+ Y23+ Y33+ Y43+Y53	1,000,000	-	13000000	1E+30	12000000
$D$211		-	(2)	0	0	0
$D$217		500,000	-	0	500000	1E+30
$D$224		165,000.0	-	0	165000	1E+30
$D$230		177,300.61	-	0	177300.61	1E+30
$D$237		2,375,000	-	0	2375000	1E+30
$D$243		1,000,000	-	0	1000000	1E+30
$D$280		250,000.00	-	0	250000	1E+30
$D$286		-	(0.20)	0	500000	311111.1
$D$293		50,000.00	-	0	50000	1E+30
$D$299		159,323.32	-	0	159323.32	1E+30
$D$306		275,000	-	0	275000	1E+30
$D$312		1,000,000	-	0	1000000	1E+30
$D$349		-	(4.34)	0	0	0
$D$355		23,650,000	-	0	23650000	1E+30
$D$362		15,000.00	-	0	15000	1E+30
$D$368		50,000.00	-	0	50000	1E+30
$D$375		500,000	-	0	500000	1E+30
$D$381		700,000	-	0	700000	1E+30
$D$418		-	(1)	0	462500	311111.1
$D$425		168,376.07	-	0	168376.07	1E+30
$D$432		1,000,000	-	0	1000000	1E+30

APPENDIX 5.0

FORDING COAL'S LP ANSWER REPORT

Microsoft Excel 8.0 Answer Report
Worksheet: [FORDING Scenario L.P..xls]
Report Created: 3/20/02 9:29:06 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
$B$29	Pf max = X31	$ -	$ 247,290,805

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$22	X11	-	7,228,814
$C$22	X12	-	625,000
$D$22	Y11	-	1,502,119
$E$22	Y12	-	508,475
$F$22	Y13	-	635,593
$B$24	X21	-	3,463,983
$C$24	X22	-	-
$D$24	Y21	-	997,881
$E$24	Y22	-	16,949
$F$24	Y23	-	21,186
$B$26	X31	-	1,807,203
$C$26	X32	-	625,000
$D$26	Y31	-	-
$E$26	Y32	-	474,576
$F$26	Y33	-	593,220

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$46	X11+X21+X31	12,500,000	$D$46>=$F$46	Not Binding	500,000
$D$48	Y11+Y21+Y31	2,500,000	$D$48>=$F$48	Not Binding	500,000
$D$52	X12+X22+X32	1,250,000	$D$52>=$F$52	Not Binding	250,000
$D$54	Y12+Y22+Y32	1,000,000	$D$54>=$F$54	Binding	-
$D$58	Y13+Y23+Y33	1,250,000	$D$58>=$F$58	Not Binding	250,000
$D$63	X11+X21+X31	12,500,000	$D$63<=$F$63	Binding	-
$D$65	Y11+Y21+Y31	2,500,000	$D$65<=$F$65	Binding	-
$D$68	X12+X22+X32	1,250,000	$D$68<=$F$68	Binding	-
$D$70	Y12+Y13+Y22+Y23+Y32+Y33	2,250,000	$D$70<=$F$70	Binding	-
$D$75	X11+X12+Y11+Y12+Y13	10,500,000	$D$75<=$F$75	Binding	-
$D$77	X21+X22+Y21+Y22+Y23	4,500,000	$D$77<=$F$77	Not Binding	1,000,000
$D$79	X31+X32+Y31+Y32+Y33	3,500,000	$D$79<=$F$79	Binding	-
$D$83	X11+X12+Y11+Y12+Y13	10,500,000	$D$83>=$F$83	Not Binding	1,500,000
$D$85	X11+X12	7,853,814	$D$85>=$F$85	Not Binding	353,814
$D$88	X21+X22+Y21+Y22+Y23	4,500,000	$D$88>=$F$88	Not Binding	500,000
$D$90	X21+X22	3,463,983	$D$90>=$F$90	Not Binding	463,983
$D$93	X31+X32+Y31+Y32+Y33	3,500,000	$D$93>=$F$93	Not Binding	1,500,000
$D$95	X31+X32	2,432,203	$D$95>=$F$95	Not Binding	582,203
$E$103	X11+X21+X31+Y11+Y21+Y31	15,000,000	$E$103<=$G$103	Not Binding	8,000,000
$E$107	X12+X22+X32+Y12+Y22+Y32	2,250,000	$E$107<=$G$107	Not Binding	3,750,000
$E$113	X11+X21+X31+Y11+Y21+Y31	15,000,000	$E$113<=$G$113	Not Binding	8,000,000
$E$117	X12+X22+X32+Y12+Y22+Y32	2,250,000	$E$117<=$G$117	Not Binding	8,750,000
$D$143		(62,500)	$D$143<=$F$143	Not Binding	62,500
$D$149		(10,268)	$D$149<=$F$149	Not Binding	10,268
$D$156		(2,922)	$D$156<=$F$156	Not Binding	2,922
$D$162		(6,003)	$D$162<=$F$162	Not Binding	6,003
$D$169		(35,768)	$D$169<=$F$169	Not Binding	35,768

$D$175	(2,012)	$D$175<=$F$175	Not Binding	2,012
$D$182	187,500	$D$182>=$F$182	Not Binding	187,500
$D$185	92,542	$D$185>=$F$185	Not Binding	92,542
$D$189	(138,252)	$D$189<=$F$189	Not Binding	138,252
$D$192	(105,106)	$D$192<=$F$192	Not Binding	105,106
$D$196	145,180	$D$196>=$F$196	Not Binding	145,180
$D$199	42,511	$D$199>=$F$199	Not Binding	42,511
$D$207	(1,563)	$D$207<=$F$207	Not Binding	1,563
$D$213	-	$D$213<=$F$213	Binding	-
$D$220	(313)	$D$220<=$F$220	Not Binding	313
$D$226	(2,500)	$D$226<=$F$226	Not Binding	2,500
$D$233	-	$D$233<=$F$233	Binding	-
$D$239	(59)	$D$239<=$F$239	Not Binding	59
$D$247	9,375	$D$247>=$F$247	Not Binding	9,375
$D$251	32,797	$D$251>=$F$251	Not Binding	32,797
$D$256	(10,625)	$D$256<=$F$256	Not Binding	10,625
$D$260	(27,932)	$D$260<=$F$260	Not Binding	27,932
$D$265	6,875	$D$265>=$F$265	Not Binding	6,875
$D$269	9,424	$D$269>=$F$269	Not Binding	9,424
$D$276	-	$D$276<=$F$276	Binding	-
$D$283	(3,125)	$D$283<=$F$283	Not Binding	3,125
$D$290	(74)	$D$290<=$F$290	Not Binding	74
$D$297	40,996	$D$297>=$F$297	Not Binding	40,996
$D$301	(34,915)	$D$301<=$F$301	Not Binding	34,915
$D$305	11,780	$D$305>=$F$305	Not Binding	11,780
$D$146	-	$D$146>=$F$146	Binding	-
$D$152	223,199	$D$152>=$F$152	Not Binding	223,199
$D$159	332,839	$D$159>=$F$159	Not Binding	332,839
$D$165	24,682	$D$165>=$F$165	Not Binding	24,682
$D$172	2,673,199	$D$172>=$F$172	Not Binding	2,673,199
$D$178	1,048,835	$D$178>=$F$178	Not Binding	1,048,835
$D$210	468,750	$D$210>=$F$210	Not Binding	468,750
$D$216	500,000	$D$216>=$F$216	Not Binding	500,000
$D$223	31,250	$D$223>=$F$223	Not Binding	31,250
$D$229	-	$D$229>=$F$229	Binding	-
$D$236	625,000	$D$236>=$F$236	Not Binding	625,000
$D$242	494,068	$D$242>=$F$242	Not Binding	494,068
$D$279	625,000	$D$279>=$F$279	Not Binding	625,000
$D$293	617,585	$D$293>=$F$293	Not Binding	617,585
$D$286	-	$D$286>=$F$286	Binding	-

APPENDIX 6.0

FORDING COAL'S LP SENSITIVITY REPORT

Microsoft Excel 8.0 Sensitivity Report
Worksheet: [FORDING Scenario L.P..xls]
Report Created: 3/20/02 9:29:09 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$22	X11	7,228,814	-	16.52	0.0575	0
$C$22	X12	625,000	-	14.85	0.23	0.11
$D$22	Y11	1,502,119	-	3.02	0	0.046
$E$22	Y12	508,475	-	1.35	0.74	0.33464
$F$22	Y13	635,593	-	11.6	0	0.95036
$B$24	X21	3,463,983	-	16.125	0	0.046
$C$24	X22	-	(0)	14.43	0.0825	1E+30
$D$24	Y21	997,881	-	2.625	0.046	0
$E$24	Y22	16,949	-	0.93	0	1.429E+14
$F$24	Y23	21,186	-	11.39	0	629.88
$B$26	X31	1,807,203	-	16.43	0.23	0
$C$26	X32	625,000	-	14.99	1E+30	0.23
$D$26	Y31	-	-	2.93	0	1E+30
$E$26	Y32	474,576	-	1.49	22.4957143	0
$F$26	Y33	593,220	-	12.63	1E+30	0.78265

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$46	X11+X21+X31	12,500,000	0	12000000	500000	1E+30
$D$48	Y11+Y21+Y31	2,500,000	-	2000000	500000	1E+30
$D$52	X12+X22+X32	1,250,000	-	1000000	250000	1E+30
$D$54	Y12+Y22+Y32	1,000,000	(11)	1000000	250000	1000000
$D$58	Y13+Y23+Y33	1,250,000	-	1000000	250000	1E+30
$D$63	X11+X21+X31	12,500,000	16	12500000	1000000	463983.05
$D$65	Y11+Y21+Y31	2,500,000	3	2500000	804661.017	246822.03
$D$68	X12+X22+X32	1,250,000	15	1250000	109698.682	185593.22
$D$70	Y12+Y13+Y22+Y23+Y32+Y33	2,250,000	12	2250000	118394.309	195535.71
$D$75	X11+X12+Y11+Y12+Y13	10,500,000	0	10500000	164548.023	365755.01
$D$77	X21+X22+Y21+Y22+Y23	4,500,000	-	5500000	1E+30	1000000
$D$79	X31+X32+Y31+Y32+Y33	3,500,000	0	3500000	91438.7519	41137.01
$D$83	X11+X12+Y11+Y12+Y13	10,500,000	-	9000000	1500000	1E+30
$D$85	X11+X12	7,853,814	-	7500000	353813.559	1E+30
$D$88	X21+X22+Y21+Y22+Y23	4,500,000	-	4000000	500000	1E+30
$D$90	X21+X22	3,463,983	-	3000000	463983.051	1E+30
$D$93	X31+X32+Y31+Y32+Y33	3,500,000	-	2000000	1500000	1E+30
$D$95	X31+X32	2,432,203	-	1850000	582203.39	1E+30
$E$103	X11+X21+X31+Y11+Y21+Y31	15,000,000	-	23000000	1E+30	8000000
$E$107	X12+X22+X32+Y12+Y22+Y32	2,250,000	-	6000000	1E+30	3750000
$E$113	X11+X21+X31+Y11+Y21+Y31	15,000,000	-	23000000	1E+30	8000000
$E$117	X12+X22+X32+Y12+Y22+Y32	2,250,000	-	11000000	1E+30	8750000
$D$143		(62,500)	-	0	1E+30	62500
$D$149		(10,268)	-	0	1E+30	10268.01
$D$156		(2,922)	-	0	1E+30	2921.61
$D$162		(6,003)	-	0	1E+30	6003.18
$D$169		(35,768)	-	0	1E+30	35768.01
$D$175		(2,012)	-	0	1E+30	2011.65
$D$182		187,500	-	0	187500	1E+30
$D$185		92,542	-	0	92542.3729	1E+30
$D$189		(138,252)	-	0	1E+30	138252.12

$D$192	(105,106)	-	0	1E+30	105105.93
$D$196	145,180	-	0	145180.085	1E+30
$D$199	42,511	-	0	42510.5932	1E+30
$D$207	(1,563)	-	0	1E+30	1562.5
$D$213	-	25.08	0	250	829.98
$D$220	(313)	-	0	1E+30	312.5
$D$226	(2,500)	-	0	1E+30	2500
$D$233	-	23.00	0	329.096045	731.51
$D$239	(59)	-	0	1E+30	59.32
$D$247	9,375	-	0	9375	1E+30
$D$251	32,797	-	0	32796.6102	1E+30
$D$256	(10,625)	-	0	1E+30	10625
$D$260	(27,932)	-	0	1E+30	27932.20
$D$265	6,875	-	0	6875	1E+30
$D$269	9,424	-	0	9423.72881	1E+30
$D$276	-	101.36	0	312.5	829.98
$D$283	(3,125)	-	0	1E+30	3125
$D$290	(74)	-	0	1E+30	74.15
$D$297	40,996	-	0	40995.7627	1E+30
$D$301	(34,915)	-	0	1E+30	34915.25
$D$305	11,780	-	0	11779.661	1E+30
$D$146	-	-	0	41137.0056	91438.75
$D$152	223,199	-	0	223199.153	1E+30
$D$159	332,839	-	0	332838.983	1E+30
$D$165	24,682	-	0	24682.2034	1E+30
$D$172	2,673,199	-	0	2673199.15	1E+30
$D$178	1,048,835	-	0	1048834.75	1E+30
$D$210	468,750	-	0	468750	1E+30
$D$216	500,000	-	0	500000	1E+30
$D$223	31,250	-	0	31250	1E+30
$D$229	-	(1.59)	0	2554.74453	1612.90
$D$236	625,000	-	0	625000	1E+30
$D$242	494,068	-	0	494067.797	1E+30
$D$279	625,000	-	0	625000	1E+30
$D$293	617,585	-	0	617584.746	1E+30
$D$286	-	(4.51)	0	3193.43066	2016.13

APPENDIX 7.0

Table A.7.1 Luscar-Sherritt's Optimized Coal Production and Shipments

Solution	Metallurgical			Thermal			
	Vancouver	Ridley	Thunder Bay	Vancouver	Ridley	Thunder Bay	Power Plant
Line Creek	X11	#	X13	Y11	#	Y13	Y14
	2,000,000	#	500,000	0	#	1,500,000	-
Luscar	X21	X22	X23	Y21	Y22	Y23	Y24
	1,500,000	500,000	0	736,196	263,804	-	-
Cheviot	X31	X32	X33	Y31	Y32	Y33	Y34
	2,000,000	500,000	500,000	-	158,120	500,000	341,880
Coal Valley	X41	X42	X43	Y41	Y42	Y43	Y44
	-	-	-	343,558	1,011,143	-	1,145,299
Obed Mnt.	X51	X52	X53	Y51	Y52	Y53	Y54
	-	-	0	920,245	566,934	-	512,821
TOTAL	**5,500,000**	**1,000,000**	**1,000,000**	**2,000,000**	**2,000,000**	**2,000,000**	**2,000,000**

Table A.7.2 Luscar-Sherritt's Optimized Costs by Mine

Mine	Mining & Proc.	Transport	Port & Plant	Overhead	Total
	Cost [$]	Cost [$]	Cost [$]	Cost [$]	Cost [$]
Line Creek	76,000,000	63,298,000	14,500,000	600,000	154,398,000
Luscar	56,000,000	45,078,736	10,677,147	330,000	112,085,883
Cheviot	78,000,000	60,883,350	13,487,179	480,000	152,850,530
Coal Valley	36,250,000	26,219,471	6,039,720	325,000	68,834,191
Obed Mnt.	28,000,000	22,063,652	5,920,953	220,000	56,204,605
Total	**274,250,000**	**217,543,210**	**50,625,000**	**1,955,000**	**544,373,210**

Table A.7.3 Optimized Revenues and Profits by Mine

Mine	Revenue	Profits
	[$]	[$]
Line Creek	191,250,000	36,852,000
Luscar	145,500,000	33,414,117
Cheviot	199,500,000	46,649,470
Coal Valley	93,750,000	24,915,809
Obed Mnt.	75,000,000	18,795,395
Total	**705,000,000**	**160,626,790**

Table A.7.4 Lagrange Multipliers for Luscar-Sherritt's Case

L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
1.139402	0	0	0	0	0	5896.82	0	0	0
L11	L12	L13	L14	L15	L16	L17	L18	L19	L20
25.36	879365	99.245	0	0	0	0	0	2569.4	0
L21	L22	L23	L24	L25	L26	L27	L28	L29	L30
686487.3	0	0	682647	1372974	686487.3	8237845	1.5E+07	4E+06	4801533
L31	L32	L33	L34	L35	L36	L37	L38	L39	L40
10987671	7551358	1E+07	1.6E+07	4118945	37744.53	0	6858.96	-346.9	1507.66
L41	L42	L43	L44	L45	L46	L47	L48	L49	L50
259139.6	4435.83	242894	5127.32	5.4E+07	0	1372959	72076.2	27812	58976.6
L51	L52	L53	L54	L55	L56	L57	L58	L59	L60
19578.89	46667.6	23489	3432.62	343258	13729.36	0	0	68648	4676.28
L61	L62	L63	L64	L65	L66	L67	L68	L69	L70

218898	3089.71	1E+07	0	1372326	6864.061	20317.4	4805.72	18051	687.888
L71	L72	L73	L74	L75	L76	L77	L78	L79	L80
20462.18	6864.76	0	4762.88	896679	476.13	20595.1	6174.09	69021	0
L81	L82	L83	L84	L85	L86	L87	L88	L89	L90
9954225	0	963467	10297.6	106291	11670.37	121912	7551.28	38369	13729.6
L91	L92	L93	L94	L95	L96	L97	L98	L99	#
0	4554.09	231098	0	1372953	17273.39	13406.3	16508.9	172.55	#

Table A.7.5 Sensitivity Analysis, Profit vs. Costs and Coal Prices

Change of Pf MAX (%)	28%	19%	9%	0%	-9%	-19%	-28%
Pf MAX [M$]	208	192	177	161	147	131	116
Change of Prod. Cost (%) Metallurgical	-30%	-20%	-10%	0%	10%	20%	30%
Change of Pf MAX (%)	23%	15%	8%	0%	-8%	-15%	-23%
Pf MAX	198	186	174	161	149	138	125
Change of Prod. Cost (%) Thermal	-30%	-20%	-10%	0%	10%	20%	30%
Change of Pf MAX (%)	-75%	-50%	-25%	0%	25%	50%	75%
Pf MAX [M$]	40	81	121	161	202	243	283
Change of Selling price Metallurgical.	37.8	43.2	48.6	54	59.4	64.8	70.2
Change of Selling price (%)	-30%	-20%	-10%	0%	10%	20%	30%
Change of Pf MAX (%)	-56%	-37%	-19%	0%	19%	37%	56%
Pf MAX [M$]	72	102	132	161	192	222	252
Change of Selling price Thermal.	26.25	30	33.75	37.5	41.25	45	48.75
Change of Selling price (%) I	-30%	-20%	-10%	0%	10%	20%	30%
Change of Pf MAX (%)	40%	27%	13%	0%	-13%	-27%	-40%
Pf MAX [M$]	226	205	183	161	140	118	97
Change of Transportation Cost (%)	-30%	-20%	-10%	0%	10%	20%	30%

Table A.7.6 Coal Production and Haulage Distribution in Changed Market Capacities

Solution	**Metallurgical**			**Thermal**			
	Vancouver	Ridley	Thunder Bay	Vancouver	Ridley	Thunder Bay	Power Plant
Line Creek	X11	#	X13	Y11	#	Y13	Y14
	1,500,000	#	1,000,000	-	#	1,500,000	-
Luscar	X21	X22	X23	Y21	Y22	Y23	Y24
	1,250,000	375,000	0	613,497	386,503	375,000	-
Cheviot	X31	X32	X33	Y31	Y32	Y33	Y34
	1,500,000	375,000	1,000,000	-	72,650	125,000	427,350
Coal Valley	X41	X42	X43	Y41	Y42	Y43	Y44
	-	-	-	119,632	948,744	-	1,431,624
Obed Mnt.	X51	X52	X53	Y51	Y52	Y53	Y54
	-	-	0	766,871	592,103	-	641,026
TOTAL	**4,250,000**	**750,000**	**2,000,000**	**1,500,000**	**2,000,000**	**2,000,000**	**2,500,000**

Table A.7.7 Solutions for the Dropping Metallurgical Coal Demand

Solution	Metallurgical			Thermal			
	Vancouver	Ridley	Thunder Bay	Vancouver	Ridley	Thunder Bay	Power Plant
Line Creek	X11	#	X13	Y11	#	Y13	Y14
	1,050,000	#	750,000	91,667	#	2,108,333	-
Luscar	X21	X22	X23	Y21	Y22	Y23	Y24
	1,000,000	500,000	-	500,000	-	-	-
Cheviot	X31	X32	X33	Y31	Y32	Y33	Y34
	850,000	500,000	750,000	324,573	341,880	91,667	341,880
Coal Valley	X41	X42	X43	Y41	Y42	Y43	Y44
	100,000	-	0	109,402	1,145,299	-	1,145,299
Obed Mnt.	X51	X52	X53	Y51	Y52	Y53	Y54
	-	-	-	974,359	512,821	-	512,821
TOTAL	3,000,000	1,000,000	1,500,000	2,000,000	2,000,000	2,200,000	2,000,000

Table A.7.8 Luscar-Sherritt's Non-optimized Coal Distribution

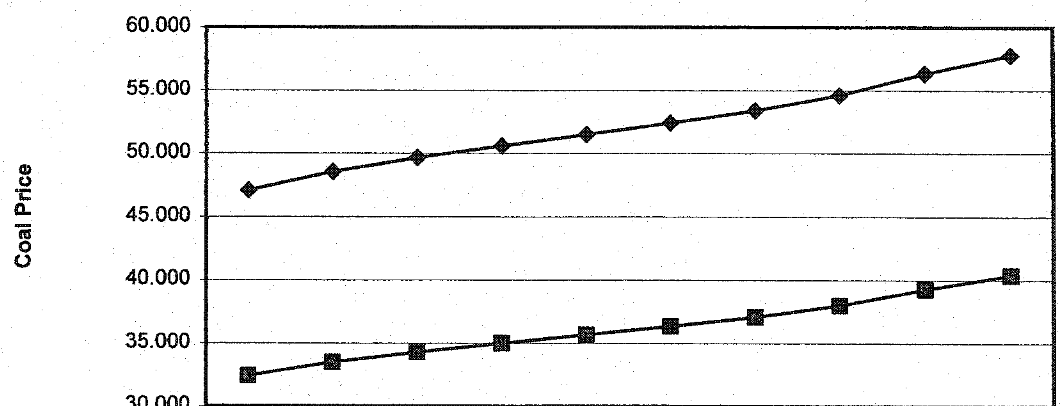
Solution	Metallurgical			Thermal			
	Vancouver	Ridley	Thunder Bay	Vancouver	Ridley	Thunder Bay	Power Plant
Line Creek	X11	#	X13	Y11	#	Y13	Y14
	3,000,000	#	1,000,000	-	#	-	-
Luscar	X21	X22	X23	Y21	Y22	Y23	Y24
	1,000,000	500,000	-	1,000,000	-	500,000	-
Cheviot	X31	X32	X33	Y31	Y32	Y33	Y34
	1,500,000	500,000	-	500,000	500,000	1,000,000	-
Coal Valley	X41	X42	X43	Y41	Y42	Y43	Y44
	-	-	-	-	500,000	-	2,000,000
Obed Mnt.	X51	X52	X53	Y51	Y52	Y53	Y54
	-	-	-	500,000	1,000,000	500,000	-
TOTAL	5,500,000	1,000,000	1,000,000	2,000,000	2,000,000	2,000,000	2,000,000

Table A.7.9 Luscar-Sherritt's Improved Coal Production and Transport Distribution

Solution	Metallurgical			Thermal			
	Vancouver	Ridley	Thunder Bay	Vancouver	Ridley	Thunder Bay	Power Plant
Line Creek	X11	#	X13	Y11	#	Y13	Y14
	1,944,444	#	555,556	0	#	1,500,000	-
Luscar	X21	X22	X23	Y21	Y22	Y23	Y24
	1,088,889	466,667	444,444	839,161	160,839	-	-
Cheviot	X31	X32	X33	Y31	Y32	Y33	Y34
	2,466,667	533,333	-	-	351,412	500,000	148,588
Coal Valley	X41	X42	X43	Y41	Y42	Y43	Y44
	-	-	-	111,888	759,583	-	1,628,529
Orbed Mnt.	X51	X52	X53	Y51	Y52	Y53	Y54
	-	-	-	1,048,951	728,166	-	222,883
TOTAL	5,500,000	1,000,000	1,000,000	2,000,000	2,000,000	2,000,000	2,000,000

Table A.7.10 Stochastic Characterization of Coal prices

	Metallurgical	Thermal
Mean	51.608	35.764
Median	51.230	35.462
Mode	51.482	35.663
Standard Deviation	3.616	2.691
Variance	13.076	7.242



	10%	20%	30%	40%	50%	60%	70%	80%	90%	95%
Metallurgical	47.066	48.537	49.627	50.577	51.482	52.403	53.406	54.605	56.312	57.762
Thermal	32.389	33.477	34.285	34.990	35.663	36.348	37.096	37.991	39.268	40.354

Figure A.7.1 Coal Price Risk Characterization

Table A.7.11 Mining and Processing Costs Probability Characteristics

Mine	Expected Value		Standard Deviation		Minimum		Maximum	
	Metallurgical	Thermal	Metallurgical	Thermal	Metallurgical	Thermal	Metallurgical	Thermal
Line Creek	20.5	16.5	4.1	3.3	15.375	12.375	25.625	20.625
Luscar	20	16	4	3.2	15	12	25	20
Cheviot	20.5	16.5	4.1	3.3	15.375	12.375	25.625	20.625
Coal Valley	21.5	14.5	4.3	2.9	16.125	10.875	26.875	18.125
Obed Mnt.	21	14	4.2	2.8	15.75	10.5	26.25	17.5

Table A.7.12 Railway Costs Probabilistic Distribution

	EPV				Standard Deviation			
Mine	Vancouver	Ridley	Thunder Bay	Power Plant	Vancouver	Ridley	Thunder Bay	Power Plant
Line Creek	0.013	#	0.008	0.04	0.0013	#	0.0008	0.004
Luscar	0.013	0.012	0.008	0.05	0.0013	0.0012	0.0008	0.005
Cheviot	0.013	0.012	0.008	0.05	0.0013	0.0012	0.0008	0.005
Coal Valley	0.013	0.012	0.008	0.08	0.0013	0.0012	0.0008	0.008
Obed Mnt.	0.013	0.012	0.008	0.08	0.0013	0.0012	0.0008	0.008
	Minimum				Maximum			
Mine	Vancouver	Ridley	Thunder Bay	Power Plant	Vancouver	Ridley	Thunder Bay	Power Plant
Line Creek	0.01105	#	0.0068	0.034	0.01495	#	0.0092	0.046
Luscar	0.01105	0.0102	0.0068	0.0425	0.01495	0.0138	0.0092	0.0575
Cheviot	0.01105	0.0102	0.0068	0.0425	0.01495	0.0138	0.0092	0.0575
Coal Valley	0.01105	0.0102	0.0068	0.068	0.01495	0.0138	0.0092	0.092
Obed Mnt.	0.01105	0.0102	0.0068	0.068	0.01495	0.0138	0.0092	0.092

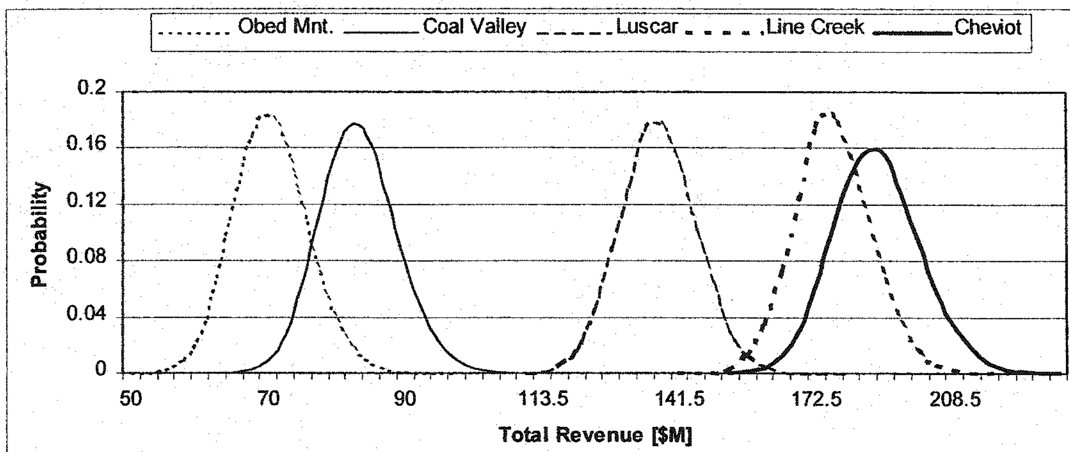


Figure A.7.2 Risk Characterization of Mine's Revenues

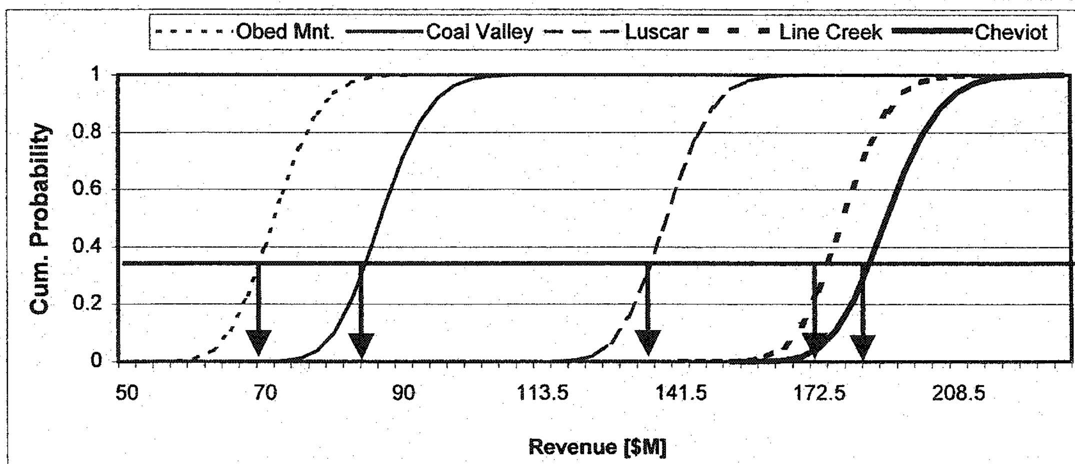


Figure A.7.3 Cumulative Probability of Revenues by Mine

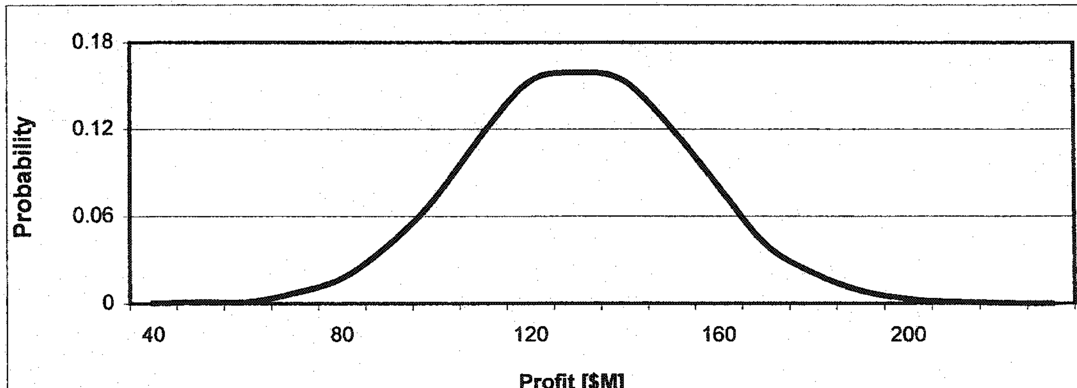


Figure A.7.4 Risk Characterization of Company's Total Profit

APENIDIX 8.0

Table A.8.1 Fording Coal's Optimized Coal Production and Transports

Solution	Metallurgical		Thermal			TOTAL
	Vancouver	Thunder Bay	Vancouver	Thunder Bay	Power Plant	
Fording River	X11	X12	Y11	Y12	Y13	
	7,228,814	625,000	1,502,119	508,475	635,593	**10,500,000**
Greenhills	X21	X22	Y21	Y22	Y23	
	3,463,983	-	997,881	16,949	21,186	**4,500,000**
Coal Mnt.	X31	X32	Y31	Y32	Y33	
	1,807,203	625,000	-	474,576	593,220	**3,500,000**
TOTAL	**12,500,000**	**1,250,000**	**2,500,000**	**1,000,000**	**1,250,000**	**18,500,000**

Table A.8.2 Optimized Costs by Mine

Mine	Mining & Proc. Cost [$]	Transport Cost [$]	Port & Plant Cost [$]	Overhead Cost [$]	Total Cost [$]
Fording River	186,311,441	156,492,966	37,661,547	1,575,000	382,040,953
Greenhills	82,391,949	67,462,436	16,823,093	495,000	167,172,479
Coal Mtn.	62,421,610	49,763,792	11,515,360	420,000	124,120,763
Total	**331,125,000**	**273,719,195**	**66,000,000**	**2,490,000**	**673,334,195**

Table A.8.3 Optimized Revenues and Profits by Mine

Mine	Revenue [$]	Profits [$]
Fording River	523,337,924	141,296,970
Greenhills	225,905,720	58,733,242
Coal Mtn.	171,381,356	47,260,593
Total	**920,625,000**	**247,290,805**

Table A.8.4 Sensitivity Analysis, Profit vs. Costs and Coal Prices

Change of Pf MAX (%)	31%	21%	10%	0%	-10%	-21%	-31%
Pf MAX [M$]	324	299	273	247	221	196	170
Change of Op. Cost (%) Metallurgical	-30%	-20%	-10%	0%	10%	20%	30%
Change of Pf MAX (%)	9%	6%	3%	0%	-3%	-6%	-9%
Pf MAX [M$]	269	262	255	247	240	232	225
Change of Op. Cost (%) Thermal	-30%	-20%	-10%	0%	10%	20%	30%
Change of Pf MAX (%)	-90%	-60%	-30%	0%	30%	60%	90%
Pf MAX [M$]	24	99	173	247	321	396	470
Change of Selling price Metallurgical	37.8	43.2	48.6	54	59.4	64.8	70.2
Change of Selling price (%)	-30%	-20%	-10%	0%	10%	20%	30%
Change of Pf MAX (%)	-22%	-14%	-7%	0%	7%	14%	22%
Pf MAX [M$]	194	212	229	247	265	283	301
Change of Selling price Thermal	26.25	30	33.75	37.5	41.25	45	48.75
Change of Selling price (%)	-30%	-20%	-10%	0%	10%	20%	30%
Change of Pf MAX (%)	33%	22%	11%	0%	-11%	-22%	-33%
Pf MAX [M$]	329	302	275	247	220	192	165
Change of Transporting Cost (%)	-30%	-20%	-10%	0%	10%	20%	30%

Table A.8.5 Coal Production and Haulage Distribution in Changed Market Capacities

Solution	Metallurgical		Thermal			TOTAL
	Vancouver	Thunder Bay	Vancouver	Thunder Bay	Power Plant	
Fording River	X11	X12	Y11	Y12	Y13	
	5,377,401	2,136,723	1,333,333	508,475	1,144,068	**10,500,000**
Greenhills	X21	X22	Y21	Y22	Y23	
	4,778,249	-	666,667	16,949	38,136	**5,500,000**
Coal Mnt.	X31	X32	Y31	Y32	Y33	
	1,344,350	613,277	-	474,576	1,067,797	**3,500,000**
TOTAL	**11,500,000**	**2,750,000**	**2,000,000**	**1,000,000**	**2,250,000**	**19,500,000**

Table A.8.6 Fording Coal's Improved Optimal Coal Production and Haulage Distribution

Solution	Metallurgical		Thermal			TOTAL
	Vancouver	Thunder Bay	Vancouver	Thunder Bay	Power Plant	
Fording River	X11	X12	Y11	Y12	Y13	
	7,116,235	698,529	1,346,154	559,233	779,848	**10,500,000**
Greenhills	X21	X22	Y21	Y22	Y23	
	3,092,228	-	1,153,846	170,732	83,194	**4,500,000**
Coal Mnt.	X31	X32	Y31	Y32	Y33	
	2,291,537	551,471	-	270,035	386,958	**3,500,000**
TOTAL	**12,500,000**	**1,250,000**	**2,500,000**	**1,000,000**	**1,250,000**	**18,500,000**

Table A.8.7 Fording Coal's Mining and Processing Costs Probability Characteristics

Mine	Expected Value		Standard Deviation		Minimum 75% of EPV		Maximum 125% of EPV	
	Metallurgical	Thermal	Metallurgical	Thermal	Metallurgical	Thermal	Metallurgical	Thermal
Fording River	18.5	15.5	3.7	3.1	13.875	11.625	23.125	19.375
Greenhills	19	16	3.8	3.2	14.25	12	23.75	20
Coal Mnt.	18.75	15.75	3.75	3.15	14.0625	11.8125	23.4375	19.6875

Table A.8.8 Fording Coal's Haulage Costs Probabilistic Characteristics

Mine / Destination	EPV			STD		
	Vancouver	Thunder Bay	Power Plant	Vancouver	Thunder Bay	Power Plant
Fording River	0.013	0.008	0.05	0.0013	0.0008	0.005
Greenhills	0.013	0.008	0.05	0.0013	0.0008	0.005
Coal Mtn.	0.013	0.008	0.05	0.0013	0.0008	0.005
	Minimum			**Maximum**		
	Vancouver	Thunder Bay	Power Plant	Vancouver	Thunder Bay	Power Plant
Fording River	0.01105	0.0068	0.0425	0.01495	0.0092	0.0575
Greenhills	0.01105	0.0068	0.0425	0.01495	0.0092	0.0575
Coal Mtn.	0.01105	0.0068	0.0425	0.01495	0.0092	0.0575

Table A.8.9 Stochastic Characterization of Fording Coal Costs, Revenue and Profits

Name	Mean	St.Dev	Mean-St. Dev.	Mean+St. Dev.
Total Mining & Processing Cost	325,204,500	22,280,650	302,923,850	347,485,150
Total Haulage Cost	264,446,800	11,212,270	253,234,530	275,659,070
Fording River Total Cost	373,613,600	21,864,050	351,749,550	395,477,650
Greenhills T. Cost	165,220,600	10,087,400	155,133,200	175,308,000
Coal M. T. Cost	119,307,000	6,693,044	112,613,956	126,000,044
Total Fording Cost	658,141,200	25,061,430	633,079,770	683,202,630
Fording River Revenue	499,597,600	29,342,330	470,255,270	528,939,930
Greenhills Revenue	215,821,900	12,807,760	203,014,140	228,629,660
Coal M. Revenue	163,710,000	9,280,184	154,429,816	172,990,184
Total Fording Revenue	879,489,500	33,588,900	845,900,600	913,078,400
Fording River Profits	126,344,000	36,701,100	89,642,900	163,045,100
Greenhills Profits	50,601,260	16,357,840	34,243,420	66,959,100
Coal Mountain Profits	44,403,000	11,427,500	32,975,500	55,830,500
Total Fording Profits	221,348,300	41,762,070	179,586,230	263,110,370

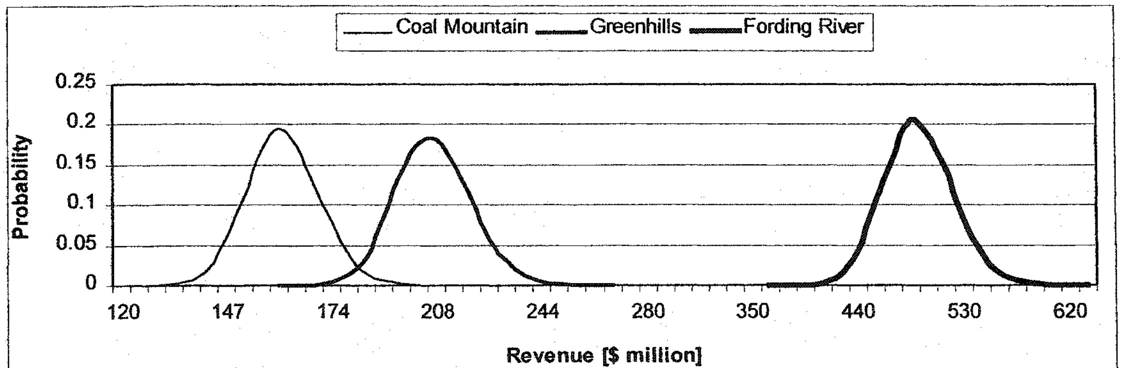


Figure A.8.1 Risk Characterization of Fording's Mines Revenue

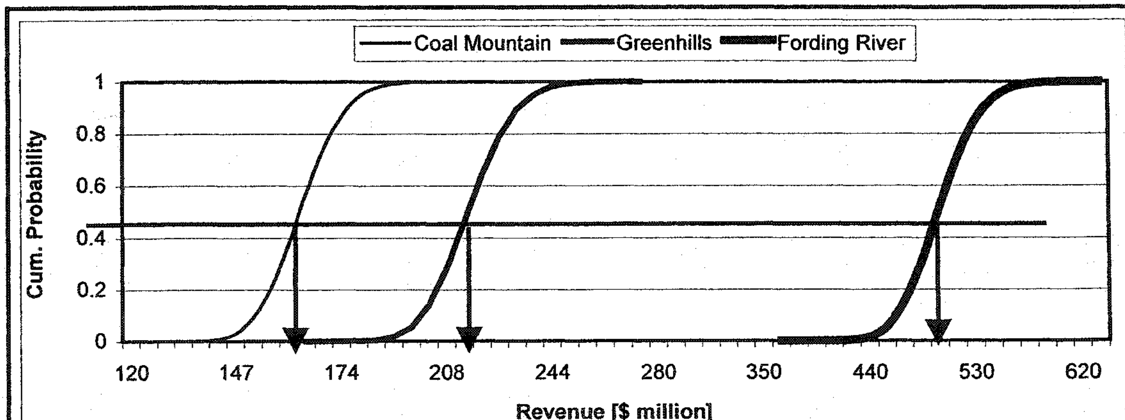


Figure A.8.2 Cumulative Probability of Fording Mines Revenue

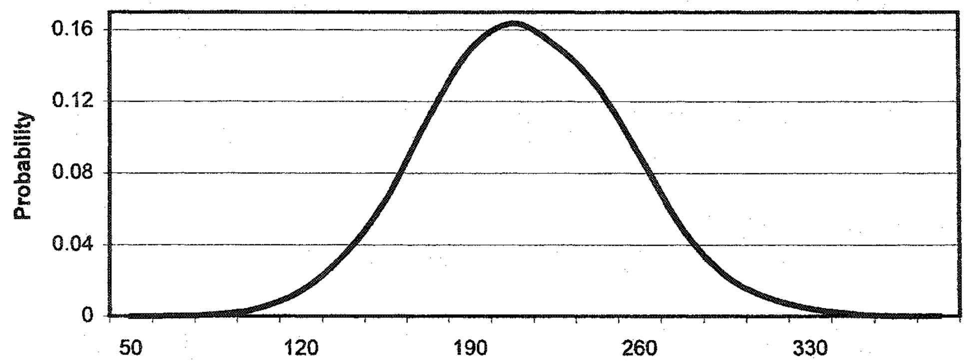


Figure A.8.3 Risk Characterization of Fording Coal Total Profit

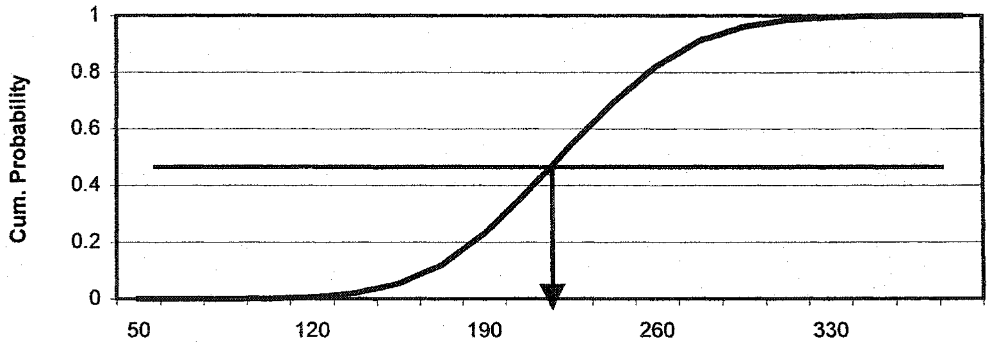


Figure A.8.4 Fording Coal's Probability of Success