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Being (Almost) a Mathematician:
Teacher Identity Formation in Post-Secondary Mathematics

by

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To my mother, Charla, and grandmothers, Mary and Margaret, for their courage, strength, and brilliance in trailblazing many paths ahead of me.

ABSTRACT

Within the field of mathematics teacher education, mathematics graduate students have recently become subjects of investigation. While research in this area tends to focus on future schoolteachers, little has been done to examine prospective university teachers of mathematics and their understanding of its teaching and learning. As a result, the experiences of mathematics graduate students and the development of their teaching practices are not well understood. Almost seventy-five percent of mathematics PhDs will become professors at post-secondary institutions dedicated to undergraduate education. Since much of their careers will be spent in the classroom, attending to the manner in which mathematics graduate students develop their teaching practices is important in understanding how they are shaped for their future profession.

The purpose of this research project was to uncover issues and difficulties that arise as mathematics graduate students develop their views of their possible future roles as university teachers of mathematics. Over a six-month period, conversations were held with six mathematics graduate students exploring their experiences of and perspectives on mathematics teaching. Using hermeneutic inquiry and thematic analysis, the conversations were analysed and interpreted with attention to themes and experiences that had the potential to influence the graduate students' ideas about and approaches to the task of teaching.

This dissertation also attends to notions of identity for mathematics graduate students, in particular their emerging identities as mathematicians and what being a mathematician in the world means to them, as well as their identities as future post-secondary teachers of mathematics. The structures and expectations

of behaviour within their department of mathematics had implications for how the participants formed their identities as mathematicians and mathematics teachers. Lave and Wenger’s notion of legitimate peripheral participation is explored with regard to the meta-themes that came through the analysis. These meta-themes are: replication – where university mathematics teacher identity and classroom practices became a process of replication; resignation – the research participants felt resigned to one particular way of being in mathematics and of mathematics teaching; and despondence – the participants were beginning to lose their excitement about becoming post-secondary teachers of mathematics.

Preface

Becoming a Professor of Mathematics

In the fall of 2008, after four years of doctoral studies in mathematics education, I began my search for an academic position in a university. I mostly looked in departments of mathematics because much of my graduate study and most of my teaching experiences were in such departments, and teaching university-level mathematics is still my wish for my career. What I found in most job postings was a persistent requirement that applicants have experience in teaching mathematics, even for positions at large research universities. There were few postings that did not require evidence of good teaching and, in many cases, they required a teaching dossier from the applicant.

I was invited to interviews at two very different universities. The first interview was in a department of mathematics at a small, public, undergraduate university. Along with submitting my teaching portfolio and letters of recommendation that spoke directly to my teaching experience, as part of the interview I was required to give two teaching demonstrations, including one to a group of future elementary school teachers and the other to the Mathematics Club, a group of undergraduate students working towards a bachelor degree in mathematics.

The second interview was in a department of mathematics at a large, public, research university. In this case, I was not asked to teach a class or interact with a group of students. I was only required to give an hour-long presentation on my doctoral research. The irony for me was that, in this particular department, there was a program that focused on college- and university-level teaching of mathematics. I wondered why did the hiring committee pay no attention to who and how I was as a teacher of mathematics? Why were my abilities as a post-secondary mathematics teacher seemingly not important, particularly in relation to a program with a supposed focus on teaching at the university level?

I was offered a position at the small, undergraduate university, which I accepted. Most of my work in this position will be focused on teaching mathematics to undergraduates. To support the transition into being a full-time faculty member in mathematics, the department supported my application to be a become teaching fellow in *Project NExT*, *New Experiences in Teaching* (Mathematics Association of America, 2009). This program has been established for new mathematics professors to help them learn not only the tasks of being a faculty member, but to also support them as they learn, often for the first time, about teaching mathematics to undergraduates.

As I think about my future in mathematics, I am compelled to look to my past in mathematics and how I came to this place of being a professor of mathematics. One experience that seems quite important in such a reflection was my discovery that teaching university-level mathematics was my passion, a dream job for me. In my desire to focus on my teaching as a mathematics graduate student, to learn and understand more about teaching and learning in post-secondary mathematics, I slowly became aware that furthering the discipline through teaching did not seem to represent real mathematical work in the eyes of departments of mathematics (see Chapters 1 and 2). I found that in order to explore the teaching and learning of mathematics, I had to leave mathematics and enroll in a department of mathematics education.

What I find most disconcerting about this was that my exploration of mathematics and its teaching and learning had to occur *outside* of a department of mathematics. Curiously enough, *because of* my attention to teaching and learning mathematics, I now find myself hired by a department of mathematics who expect their newest member to be educated in and enthusiastic about teaching mathematics. It seems that in the process of becoming a professor of mathematics who is knowledgeable about and interested in teaching mathematics, there have been interesting conflicts and contradictions in how or whether teaching mathematics is of any consequence in such a process.

Two years ago, as I prepared for my candidacy exam for the research in this dissertation, a friend asked me how I felt about things and whether I was nervous about the exam. After thinking about her question for a moment, I responded, "It's interesting becoming something that you've never been before" (Beisiegel, 2007). At that time, I was mostly thinking of what it meant to become a researcher in mathematics education. I had the sense that to become a researcher meant that I would spend many years continuing to learn about and understand research and about myself, that I would not instantly become a researcher, but that I would continue to grow into a way of being, of going about understanding the world.

I now find myself in quite a similar place, thinking about becoming an assistant professor of mathematics and a university teacher of mathematics when I begin my first academic, tenure-track appointment. September 15, 2009 marks my first official day as an assistant professor of mathematics. Yet, I do not want to feel that I am finished learning and understanding what it means to be a university teacher of mathematics. In becoming and imagining being an assistant professor of mathematics, I wonder what will it mean for who I am and how I want to be in mathematics. In *returning* to a department of mathematics, will I be able to express myself, my interests, and excitement about mathematics and its teaching to my students and my peers? I wonder if there will be a space (or whether I can make one) where I will be able to explore my understanding of mathematics teaching and learning with others. Or will I feel confined to a particular way of being in mathematics?

In these reflections and questions of my experiences and my soon-to-be role as a mathematics professor, there is a tension in what it means *to be* a mathematician and a professor of mathematics. There are also tensions in what one makes important on the path to becoming a mathematician and mathematics professor and what one might need to make important once having become a mathematician and mathematics professor. I have a strong desire to understand these tensions in my own and others' experiences. The research described in this dissertation, then, represents the beginning of that understanding of the process of and tensions in becoming a mathematician and a professor of mathematics.

Acknowledgements

I would like to thank my research participants for their willing and often courageous participation in this project, for answering my questions thoughtfully, and for helping me to find an understanding of what their experiences were like.

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To my peers and friends in mathematics education for the wonderful conversations and sharing of our learning and understanding. Thank you for being a sounding board for my ideas and for the fantastic collaborations.

To the first elementary education students I had the opportunity to teach – thank you for asking for more from your learning of mathematics. You opened my mind to a new way of thinking about teaching mathematics.

Finally, I would like to thank my grade 10 and 11 mathematics teachers for their patience with students and their passion for teaching. Without their joy for teaching mathematics, I would not have found my own joy.

TABLE OF CONTENTS

Chapter 1

You're Not Going to Let Those People Change You, Are You?

> Newcomers are caught in a dilemma. On the one hand, they need to engage in the existing practice, which has developed over time: to understand it, to participate in it, and to become full members of the community in which it exists. On the other hand, they have a stake in its development as they begin to establish their own identity in its future. (Lave & Wenger, 1991, p. 115)

Formative experiences of future professors have recently come to the fore as a topic worthy of exploration and research. For example, the purpose of the Carnegie Initiative on the Doctorate (Golde & Walker, 2006) in the United States was to gain an understanding of doctoral experiences in various disciplines and how future PhDs were prepared for their careers in academia and industry. As part of this work, the preparation for teaching that graduate students receive became a topic of interest, because it was recognized that doctoral programs do little to prepare future professors for their roles as post-secondary teachers (Golde & Walker, 2006; Prewitt, 2006).

With regard to mathematics in particular, Bass (2006) concluded that, "Apart from a minimally mentored apprenticeship, through teaching assistantships or graduate instructorships, scant professional development for the work of teaching has been provided to doctoral students in most mathematics departments" (p. 109). In an effort to inform teacher preparation programs in post-secondary mathematics and to contribute to the understanding of graduate school experiences in mathematics, this thesis describes an exploration of the graduate program experiences of six students in mathematics. In this project, I particularly focus on the mathematics graduate students' developing sense of themselves as future mathematicians and as post-secondary teachers of mathematics.

As the researcher of this phenomenon, I am not neutral in this arena. I have experienced graduate programs in two university departments of mathematics and encountered very different approaches to my preparation as a post-secondary teacher of mathematics. I have also taught college-level mathematics for ten years prior to undertaking this study. Thus, the stories that I share below come from my own experience as a graduate student and teacher in college and university departments of mathematics. To me, these experiences convey a great deal not only about the perceived insignificance of teaching mathematics, but also speak to the view that lecturing epitomizes post-secondary mathematics teaching. I do not supply these accounts only because they are important experiences in my own journey to find an enlivened and connected way of teaching mathematics. I offer them as examples of the experiences that graduate students may encounter as they learn about what it means to live the life of a university mathematics professor. In the remainder of this chapter, then, I document my personal history in mathematics graduate programs and as a college and university teacher of mathematics with particular reference to questions about the formation of my identity as a post-secondary teacher of mathematics.

Going to Graduate School

At the beginning of the last year of my undergraduate program in mathematics, I was invited to attend a National Science Foundation conference for undergraduate women in the sciences. The purpose of the conference was to encourage women to go onto graduate school. I had not considered graduate school as an option, but I was also unsure as to what to do with a bachelor's degree in mathematics. I had not thought that far ahead, nor had I chosen to study mathematics with a particular job in mind. Through talking about their experiences in graduate school, the speakers at the conference motivated me to consider graduate study. Among other suggestions made at the conference, I took some advice from other women in the sciences who recommended that I undertake some undergraduate research if I were interested in going to graduate school. After the conference, I asked one of my professors if I could work on a research problem with her. She agreed and for two quarters I worked toward understanding and programming a mathematical equation that described the diffusion of heat through different surfaces.

I found working with the research problem to be interesting and not too difficult, as it relied on the mathematics I had been learning in my courses, programming physical phenomena and creating three-dimensional pictures. The images I was able to create, play with, rotate, and transform allowed me to see an amazing part of mathematics that I had not yet encountered in my courses. It made me feel special to be doing work that none of my peers was doing, to be working one-on-one with and learning from my research professor, and to be going beyond what was expected of an undergraduate student. It felt like I had finally found a way of working in and with mathematics, one that would help me find a future in mathematics that taking courses did not provide. Even when some of my mathematical modeling failed to produce accurate results, my research professor reminded me that knowing what not to do was just as valuable as knowing what to do. As a result of these experiences, I decided to go to graduate school to become a research mathematician. While I did not have a particular research question in mind, it seemed that research was what I was preparing to do.

Throughout my undergraduate program, when asked whether I was going into mathematics to become a schoolteacher, I would respond, “Absolutely not!” Teaching mathematics as a potential career was out of the question. Many of the other young women who had also chosen mathematics for their undergraduate major did so because they were going to be teaching high-school mathematics. And while my stepfather was a professor of mathematics, I could not imagine myself being a mathematics professor. Nor could I imagine teaching mathematics in schools. I was adamant about this. I had negative ideas and impressions about what it meant to be a schoolteacher.

In unknowing support of my resistance to teaching, when speaking with my undergraduate mathematics professors about graduate school, teaching was never mentioned. Even though teaching was a large part of their work lives as university-based mathematicians and professors of mathematics, and the potential for teaching to be significant in my own future career, none of my professors asked me if I were interested in being a teacher as well as a researcher. Research was noticeably more important than teaching and, if I were interested in becoming a mathematician, research was what mattered. Teaching was secondary to research; an afterthought, if that. When I went to graduate school to study for a

PhD in mathematics, it was with the thought that I would carry out mathematical research.

Becoming a Mathematics Graduate Teaching Assistant

In the Department of Mathematics, in order to support the number of undergraduate courses that were offered, it was expected that the graduate students would teach during their graduate programs. As teaching assistants, graduate students would have their tuition deferred, earn a monthly stipend, and gain some experience of what was to come in their potential careers. Before fall semester courses began, all of the new graduate students were given a week-long training session for our duties as teaching assistants. We were taught about the software our students would be using, given instruction on marking papers, informed about the university honour code for academic dishonesty, and told of the expectations for undergraduate student behaviour. The mathematics graduate students were also required to present a mini-lecture to a few professors. If our mini-lectures were deemed successful by the professors, we were assigned a teaching mentor with whom we would work during our first semester. It was expected that we would regularly attend our mentor's class. Depending on the mentor, some graduate students would occasionally teach the mentor's class, and our mentors and the graduate teaching assistant coordinator evaluated our teaching. When our teaching was judged satisfactory, we were assigned our own calculus lecture sections the subsequent semester. The teaching assistant coordinator continued to hold meetings in order to discuss issues related to the course, which provided some support for the mathematics graduate students during our first experiences of teaching.

I had held off teaching as long as I could by opting to work on a research grant until its funding was exhausted. When the research grant was over, I was assigned my own classes to teach. Despite the mentoring process provided by the department, I was quite nervous about teaching – so nervous that even the tips of my fingers were sweating as I entered the classroom for the first time as an official instructor of mathematics. That first day was a very awkward experience. My voice broke many times as I talked to the class about the syllabus and the course requirements. I stuttered. I lost my place in my notes and my train of thought. My hands shook with nerves as I wrote on the board. My right leg trembled as it often does when I get nervous. Rings of sweat developed under my arms, and my shirt stuck to my back. I had never experienced forty sets of eyes looking at me, just looking at me – not with great enthusiasm or boredom, but just looking and listening to the introduction of the course and the explanation of what would be expected of them. Under their collective gaze, I felt uncomfortable and occasionally terrified.

Eventually, though, I began to review the rules of algebra and trigonometry we would need for calculus. My nerves faded as I waded into the familiar and comfortable territory of the mathematics that I knew. Within the mathematics I was safe. Once I was into the mathematics, I could work at the chalkboard and did not have to look at my students. Standing at the board, I found that the experience of talking about mathematics, although not necessarily talking to or with my students, was exhilarating. I had had many discussions about mathematics with my peers outside of the classroom, but I had never had the experience of sharing my knowledge with a classroom full of people.

More importantly, I had not yet had the chance to engage with mathematics in this way. Being at the chalkboard, talking about mathematics, was somehow different from the other ways I had worked with mathematics. Teaching a course meant one hour three days a week where I could talk about something that I loved. I got to talk about mathematics! When I left my class on that first day, despite the nerves, jitters and sweat, and my previous resistance to teaching, I knew deep within myself, despite not knowing why, that I had found what I wanted to do for the rest of my life.

My first year in graduate studies was a difficult one. Along with the demands of graduate school – taking courses, teaching courses, and doing research – I also experienced significant health issues that exhausted my energy to continue much longer in the program. I had discovered a greater fondness for teaching than research and had begun to feel uncertain about becoming a researcher. I knew that a master's degree in mathematics would be enough to secure a teaching job at a community college, so I consulted with my advisor about finishing a master's degree early and becoming an instructor. With his approval to change my program, I decided to leave the university with a master's degree in mathematics. I began teaching part-time at a community college immediately after graduation and soon after that I was hired to be a full-time instructor of mathematics.

Wanting to Be a Different Kind of Mathematics Instructor

I had not planned to be a mathematics teacher, but when I had my first official job as a college instructor, I wanted to be a good teacher. Having experienced six years of mathematics courses as a student, sitting through many lectures that were difficult to grasp and did not provide opportunities for understanding, my intention was to become a different kind of mathematics teacher. In this regard, I not only wanted mathematics to make sense to my students, but I also hoped that, through my approach to mathematics, they would find reasons within themselves for studying mathematics. I felt strongly that mathematics had given me good skills to think and reason, even just to be mindful, in and about many circumstances in life, and not just situations that were mathematical. I felt that attaining an understanding of mathematics could help my students in a similar fashion.

In several ways, I was different from conventional images of mathematics instructors and professors. Having been a student so recently myself, I could relate to and empathize with the busy lives of my students – working, going to school full-time, family pressures, and not being sure about what they wanted to do in life. I provided them with information on resources that would help them in their lives as students. I could talk about the world outside of the college setting – what movies I had seen, what adventures I had had, what was happening in the world. I would even talk about how I felt about mathematics – that I really enjoyed it and I think that was apparent in my teaching. I suppose I also felt a need to show my students that I was more than a mathematician. Or maybe that a mathematician could also be someone was aware of and participated in the happenings of the world.

My teaching, though, was not that far removed from the teaching I had experienced as a student. I lectured just as my professors had and I prepared my lectures directly from my textbooks, in a repetitive cycle of definition-theorem-proof-example. One mathematician and university professor of mathematics described a similar experience in her own teaching:

> Each class had a natural pattern: I introduced the topic, covered the blackboard with formulas and mathematical language, and worked a few problems. I asked a few questions and even elicited a few answers, though usually from the same three or four (male) students, and then I assigned homework. I was considered a successful teacher. In my course evaluations, students praised me for my enthusiasm, my organization, the clarity of my exposition, my knowledge of the material, and my accessibility. (Rogers, 1994, p. 385)

Similarly, I employed techniques that good teachers were supposed to use. I spoke and wrote clearly. I asked questions. At the beginning of class, I reviewed the material that had been covered in the preceding lecture. Essentially, though, my techniques were mostly a replication of what I thought were the good aspects of the mathematics classes I had had as a student. What I spoke of and wrote on the board was precisely the material in the textbook. The questions I asked were often yes-or-no questions or required particular, fixed answers. In reviewing the material, I was repeating the same mathematics from the day before, without an attempt to determine whether or not the students had understood it in the first place.

I had a few short-lived attempts to include group learning into my teaching, but such approaches felt awkward. I also had a strong sense that if a lecture were clear enough and detailed enough, that would be enough to cause my students to gain an understanding of mathematics, to see the wonder in mathematics that I saw, and embrace it. Despite my attempts to be different, I seemed to be firmly set in the tradition of lecture-based mathematics teaching that I had been exposed to for years. Often when I was at the board, my back to my students, going through the motions of a mathematics lecture, I could feel the boredom in the room. When I did turn to face my students, I could see their eyes glazed over, pointed in my direction, but not engaged by what I was I doing. I could sense a growing disinterest in something that I cared about deeply – mathematics. The way in which I presented mathematics to my students did little to encourage them to want to learn more. I had wanted to be an advocate for and to provide an inspiring example of mathematics, promoting it, enlivening it, and exciting others to study mathematics and further the discipline. Yet, in teaching this way, what was I doing to mathematics?

The personal exhilaration from talking about mathematics that I had discovered when I first began teaching started to fade. I would try to break up the monotony by leaving the mathematical topics of the course and turn to life happenings, sharing short stories about recent life experiences I had had, or even recounting stories about my own mathematics teachers and their quirky habits. I did not know how to make the *mathematics* interesting enough by itself to keep the students aware and involved for fifty minutes, so I would depart from the mathematics here and there to breathe some life into my classroom.

Eventually, I began to lose faith in the mathematics I was teaching to my students. When my students would ask, "When are we ever going to use this?", I no longer believed my own stock answer – "Even though you might not use this particular piece of mathematics in your daily life, it is helping you to build logical and critical thinking skills." That was my best answer, but I knew they were not going to use the mathematics I was teaching them, except for the few who might go on to study more mathematics or engineering. I sensed that my teaching was not helping them to build the critical thinking skills or understanding I alleged. I began to feel that a change in my approach to teaching was necessary for my students to obtain the understanding and appreciation of mathematics that I had always wanted for them.

Along with teaching at the community college, I often attended in-service sessions and conferences for mathematics instructors in an effort to learn more about teaching. Yet, the sessions I attended were conducted much the way my own classes were, in the manner that I was beginning to find constricting and confining. Rather than providing opportunities for exploring, experimenting, or discussion, the tasks that were given at the conferences told students exactly how to do mathematics, directed them to what number to plug in where, and pointed students precisely to what they needed to do. My filing cabinets became filled with tasks that I could use in the classroom, but essentially they promoted the same kind of teaching and learning that I was no longer interested in.

My colleagues, the other mathematics department members, taught in much the same way that I did and appeared to be satisfied with their teaching methods. My department chair and other faculty members also felt that there was nothing wrong with my teaching, and I could not find support for or an outlet to discuss my desire to change. Nevertheless, as I reflected on my teaching practice, it was similar to Rogers's (1994) experience: "It shocked me to realize how faithfully I was reproducing in my own classroom the structures which had so effectively silenced and disempowered me at that time (as a mathematics undergraduate)" (p. 385). Even with that realization, I felt as though I had to maintain a teaching style that was beginning to feel uninformed and inappropriate. I had no means to discover new methods or truly different ideas about my teaching practice. The information that claimed to provide a different perspective for university mathematics classes focused on the students and what they should be doing. There was little information or support for what *I* could or should be doing, nor was there an incentive for instructors and professors to change.

I became frustrated that my attempts to reach out for different approaches to teaching were met with more of the same approach to teaching that I was trying to shed. As my attempts to find alternative ways of teaching mathematics were unsuccessful, I resigned myself to a certain way of teaching mathematics, remaining entrenched in a lecture-based approach, and I started to feel that it would be unbearable to teach mathematics like this for thirty years. Similar to my students, I was becoming bored by my teaching and being an instructor was becoming just a job, not something that inspired me. Apathy was beginning to set in. When the time came for my husband and I to move to another city, while having to let go of something I was once passionate about, I was relieved that I would not have to continue teaching. I could move on to something else and leave behind the struggles and constraints I felt bound to in teaching mathematics.

Becoming a Graduate Teaching Assistant Again

During my time as a community college instructor, I was asked to teach a course on statistics. I had never learned statistics and when I was assigned to teach it I felt entirely out of my element and that I was just barely ahead of my students. I enjoyed the course material and wanted to know more about statistics in order to be a better teacher. So, after a move to a new city, I enrolled in graduate studies again in a Department of Mathematics and Statistics, but this time I took statistics courses rather than mathematics. From my previous experience as a graduate student, I knew that I would be expected to be a teaching assistant. Having had a few years of experience teaching, and having recently taken a break from teaching for a year, I was ready to be in the classroom again. Armed with the knowledge of the approaches to teaching I did *not* want to take, I hoped that I would have the opportunity to collaborate with other teaching assistants and instructors to try new methods, to learn how to teach in a different, more meaningful way.

My enthusiasm to be in the classroom evaporated when I was assigned to teach two statistics lab sections just one day before the labs were to begin. The course coordinator only told me which lab sections I would be teaching. I was not given information such as the web site address for the course, which textbook was required for the course, and what I should expect of the students. In contrast to my previous graduate program, I was not provided with any instruction or training on department policies, how I should interact with undergraduates, what my teaching should look like, or how I should mark the students' homework assignments. The lab sections that I was assigned required that I teach about a computer program the students would use to complete their homework assignments. I had to learn to use the software for the course on my own, using only the resources that would be given to my students. I was frustrated by the lack of oversight, training, and information that was given to me, but fortunately I had a few years of teaching experience to rely on.

On the day of the first lab, I went to the classroom and began by introducing myself and talking about what the assignments and due dates were. When the time came to make use of the software, I found it was not installed on any of the computers in the lab. I ended the session early because there was nothing left for the class to work on. When I went to the lab coordinator's office to let him know what had happened, I was told that it was my oversight that the class was assigned to the wrong room and that I should have checked on this before the labs started. And, because I had let my students leave early, I was also told that I would have to make up that time in an extra session. I left the coordinator's office knowing that, as far as my teaching was concerned, I would have to figure out everything on my own.

There were multiple lecture sections for the statistics course and dozens of lab sections. To make the testing uniform across the different lecture and lab sections, all of the students were required to take an on-line final lab exam that was created by the course coordinator. When all of the students' grades, including their lab assignment, mid-term, and final exam scores, were submitted to the course coordinator, he performed a statistical analysis that compared the grades of the students in different instructors' courses and teaching assistants' labs. My students had higher than average grades and I was subsequently told not only that I would have to mark their assignments more stringently, but that I should also not be as helpful as I had been to the students. In addition, the course coordinator sent out a department-wide email stating that some teaching assistants and instructors were providing the students with too much information. My interpretation of this email was that I was being told to change my approach to my students in such a way that I would satisfy the department standard for teaching, showing the students only how to calculate statistics, and not explaining why things were done in certain ways or how statistics were meaningful.

I began to observe the way other teaching assistants ran their lab sections. Often they provided a twenty-minute demonstration of how the software would run the necessary calculations for the particular assignment. Beyond that, the teaching assistants often sat at their desks, doing their own assignments or looking at the Internet. Their students would then have to come up to the teaching assistant if they needed help or had questions. My approach had been to talk with the students about the statistics, to explain why people would be interested in knowing a particular statistic, what a statistic meant, and how to interpret it. I recommended that students think critically about how statistics often benefited particular people or organizations. I wanted my students to be interested and motivated to learn and understand statistics. It was clear, though, that this way of working with students was not acceptable to the department, especially if my students earned better marks than others on their exams.

Without resources of support or avenues to discuss my teaching within the department, I became involved in the university-wide teaching program. Some of my fellow graduate students had taken courses in this program as well, but had found that they wanted to know more about teaching mathematics and statistics in particular. Some graduate students had approached different professors in the department who had been recognized for their excellent teaching, and asked these professors to teach them about teaching mathematics. The first session addressed how to teach calculus and was offered by a professor who had won local, national, and international teaching awards. I was happy that the department was going to be offering a session on teaching mathematics and I thought I could really learn something from this professor, someone who was considered to be an excellent university professor of mathematics.

The session was standing room only – dozens of mathematics graduate students who wanted to know more about teaching filled the room. The professor began the session by expressing his opinion that the first semester of mathematics should teach students only the procedures of calculus, providing them with the "drill and skill" so that calculus students could be technically proficient in their engineering courses. For those undergraduate students who wanted to know more, to know the reasons why the mathematics behaved the way it did, the conceptual piece would be taught during the second semester, but only to those who proved their interest by continuing to study mathematics. In saying that our teaching of calculus ought to be done in such a way, I heard him suggest that my teaching of mathematics should stay fixed in a lecture-based approach where I would provide examples of problems rather than aim for developing meaning for different methods.

After half an hour of this discussion, he proceeded to say, "How to teach Calculus? You need good notes." And that was it, really. He went on to say that having good notes was a key to his teaching. However, he did not indicate what good notes would look like or what they consisted of for him, or that good notes might take on different forms for different people. The suggestion I took from his talk was that good notes, and little else, were instrumental in helping this professor to teach mathematics, teaching that was considered worthy of many awards. It was unfortunate that this message was conveyed to a room full of future instructors and professors of mathematics, and that his talk was one of the very few explicit directions in this department that we would receive about how to approach our teaching.

It was becoming clear again that this particular life, a life in a department of mathematics, was not for me. In addition to observing my fellow teaching assistants, I also began to pay more attention to my own professors. They taught in the manner that the course coordinator and award-winning professor had suggested. There was little dialogue between the students and the professors, even outside of the classroom. If the professors had any passion for their work, it did not come through in their classrooms. More and more, I came to believe that to further the field of statistics or mathematics, my teaching needed to evolve in such a way that students would become interested in learning mathematics. To me, that evolution meant teaching in a way that would help them understand the concepts, where the concepts came from, and why they were important. I wanted my students to know more than just how to do things. I wanted to be an advocate, a steward, of mathematics and statistics. When I shared this ambition with my supervisor, though, her response was cold and disapproving, asking, “Why would you want to do that?” An exploration of my teaching did not feel possible in the department and I made changes to my program in order to leave as quickly as possible.

The insignificance of teaching and being knowledgeable about teaching became more apparent when the time came for me to move to a Faculty of Education to pursue a PhD in mathematics education. I decided to say goodbye to the undergraduate chair of the department of mathematics who had given me classes to teach. During our conversation, he made a few interesting and telling comments about the place of teaching in mathematics. One comment was, “You’re not going to let those people change you, are you?” I found it interesting that he would assume that by going into doctoral program in education I would change in some unacceptable way. When I asked him if he would hire me after I completed my PhD, his response was, “No. We would never hire anyone with a PhD in mathematics education.”

In this department, I had proven myself to be a capable statistician by earning an advanced degree in statistics. I had further proven myself to be a capable mathematics and statistics teacher, earning high teaching evaluations as well as a graduate student teaching award in this particular department. And yet to turn my attention to the education of future teachers of mathematics suddenly seemed to make me unacceptable within the realm of mathematics. My potential contributions as a future educator, my knowledge of students and their understanding of mathematics, would not be of value to mathematicians who, as seen in the previous sections, continued to recognize lecturing as the accepted way of teaching.

Reflections on Teaching Mathematics

In the three departments of mathematics that I have taught in, the status quo for teaching has arguably been the same. From the lowest to the highest levels of mathematics that I have learned or taught in, the form of teaching was predominately lecture-based, with teaching assistants, instructors, and professors most often reproducing the material from the textbook on a chalkboard and asking little more from the students than note taking. Innovative approaches to teaching meant the incorporation of graphing calculators at the community college and the use of computer software at the university. Sometimes alternative teaching methods meant lecturing during the first half of the class, then having students complete worksheets while sitting in groups. Calculus reform, the largest movement in university mathematics teaching, called for the use of applied problems and integration of software such as *Maple* or *Mathematica*, but did not call for changes to teaching styles.

Inattention to post-secondary mathematics teaching was apparent when I had the experience of working in a mathematics department that had received a grant to alter its entire mathematics curriculum so that mathematics would be presented by means of applied problems. After five years of using the new curriculum, it was found that student understanding and achievement had not improved relative to previous approaches to the curriculum. The grant had required a complete rewriting of the mathematics curriculum, but left teaching entirely untouched. Thus, while the entry points into mathematics were quite changed, the teaching was not. At a national conference for community college mathematics instructors, the researchers concluded that the instructors at the college had continued to teach mathematics in the same way they always had, thereby resulting in unchanged student performance.

In my experience, few of the previous or reform approaches to university mathematics seemed to help students to build a deeper understanding of mathematics. Many mathematicians, including myself, have often taught their students the *how to* of mathematics, but not the *why*. My experiences of spending the last fifteen years in departments of mathematics as a student, teaching assistant, and instructor of mathematics have been inspiring, difficult, thought-provoking, but also filled with struggle and dissatisfaction. For me, the most discouraging aspects of it, though, have been my own inability to change the way I teach, the lack of recognition for needed changes to teaching of post-secondary mathematics on a larger scale (see Chapter 2), and a refusal to accept or respect those who choose teaching as their focus and passion in post-secondary mathematics. While there is acknowledgment for those who further the field of mathematics through research, little is done in appreciation of those who further the field through their teaching.

As I look back, uncover, and rediscover my own experiences in mathematics, I question what it was about the experience in mathematics that precluded alternative approaches to teaching and learning mathematics for myself as well as other mathematics teachers. Why did alternative approaches feel so awkward? Why did I not persist until such new approaches felt more natural, especially when I had genuinely wanted to provide my students with more meaningful experiences? Through these reflections on my experiences in mathematics, many questions have surfaced. In particular, the following seem to be important in attending to experiences in post-secondary mathematics teaching and for exploring the life and work of being a mathematician and mathematics teacher:

Was there something in the presentation of mathematics that resulted in my teaching being disconnected from the way I experienced mathematics through my own learning and research? Was it the discipline itself that obliged me to employ lectures as the only way of presenting mathematics? Was there something within the lived experience of mathematics or of being in departments of mathematics that prevented me from making changes to my teaching practices? How does it come to pass that mathematics teaching, at the university level in this case, seems to replicate itself from class to class, teacher to teacher, and eventually graduate student to graduate student? What is our identity when we are in that space of mathematics teaching? Who are we supposed to be? Who are we allowed to be?

Reflections on the Mathematics Graduate Student Experience

I believe these reflections and questions necessitate a look at how I learned to teach, how I was prepared for my future profession as a post-secondary teacher of mathematics. What had I learned about teaching in the departments of mathematics where I had been a graduate student? What about those very different experiences – where in one department I was trained, mentored and supported and in the other I was left with almost no guidance or support – had an influence on how I learned to teach? How did these experiences have an influence on what I thought my role was, who I thought I needed to be and what I should do, as an instructor of mathematics?

Despite the considerable differences in approaches to preparing graduate students in these two programs, I cannot say that my teaching varied much between the two universities. At the first university, my teaching was described as good if it was similar, if not identical, to that of my teaching mentor, whose teaching was, quite arguably, rooted in a traditional, teacher-centered lecture format. While I was mentored in my teaching, it seemed that I was being guided to the form of teaching that was acceptable and customary in a community of mathematicians. Once it was confirmed that my teaching agreed sufficiently with particular expectations, I was given more courses to teach and generally left alone. At the second university, my teaching was mostly insignificant to those in charge of my progression through the statistics program and ignored almost entirely, except curiously when my students' performance was above average.

In becoming an instructor whose classroom practice was rooted in lecture-based methods, did the training and mentoring process at the first graduate program make a difference in how I went about teaching? Did it truly offer me anything more than the second graduate program? In both departments, it seemed that I was expected to reproduce and replicate the teaching methods that already existed, regardless of whether I was explicitly trained or not. With these reflections in mind, I have begun to ask the following questions:

In departments of mathematics, in the places where I learned to teach, what were the experiences that resulted in my adoption of traditional, lecture-based teaching methods, even though I had a strong desire not to? What would my teaching have been like had I been given the opportunity to engage in a dialogue with other mentors and other future mathematics teachers? If I had been given the opportunity to explore pedagogy, my own knowledge of mathematics, and alternate perspectives of mathematics teaching, would I have been able to embrace forms of teaching other than lectures?

I began to wonder about the possibilities if a graduate program were to go beyond the training and mentoring I received as a mathematics graduate student. What if mathematics graduate students were provided with a course on pedagogy as part of their programs? What if mathematics graduate students were involved in a reform-based project? Would projects such as these help mathematics graduate students to not only learn more about teaching, but also adopt classroom practices other than the lecture-based approaches they had been exposed to?

Few universities offer mathematics graduate teaching assistants opportunities or support to prepare for teaching. This has been recently been recognized by several researchers in mathematics education and they have taken steps to remedy the situation with various projects (e.g., Speer, 2001; DeFranco & McGivney-Burelle, 2001; Belnap, 2005). Speer (2001) worked with mathematics graduate students in the context of reform-based undergraduate calculus. In this project, the mathematics teaching assistants were expected to "teach in ways that differed substantially from their classroom experiences" (p. 1); in particular, the focus was on dialogue, problem solving, and exploring students' questions. Speer concluded that mathematics teaching assistants' beliefs had a significant impact on whether and how they would engage with students using reform-based methods. Further, their beliefs about mathematics and undergraduate students were enough to counteract the new methods they had learned in the reform-oriented program.

DeFranco and McGivney-Burelle (2001) developed a pedagogy course that was offered to graduate students in mathematics. In this course, the graduate students had the opportunity to address "issues surrounding pedagogy, epistemology, curriculum, and assessment" (p. 681). The teaching assistants were asked to make changes in their teaching based on what they had learned and discussed in the new course. The researchers concluded that while the graduate students developed new beliefs about teaching mathematics, "observations of the

TAs [teaching assistants] at the end of the mathematics pedagogy course revealed classroom instruction that was largely teacher-directed and involved very little, if any, student-student or student-teacher interaction" (p. 686).

Belnap (2005) studied mathematics graduate students in a department where they enrolled in year-long departmental teaching assistant training course. In his project, he explored whether the training course had an impact on the graduate students' views of teaching and teaching practices. Despite this course and Belnap's interaction with the graduate students, it was found that, while the course might have some influence on the graduate students' practices, there were other factors that could "support, constrain, and even counteract the impact of the training programs" (p. 11). Further, it was concluded that the training program had only a limited effect on mathematics graduate students' teaching.

In a few studies, then, despite learning about teaching and having the opportunity and support to explore other methods of classroom practice, mathematics graduate students held on to a lecture-based, teacher-centered method of teaching. To be sure, mathematics graduate students are busy with coursework, teaching, and research and often may not have the time to focus on such changes. Yet, some of these studies allowed the mathematics graduate students a great deal of time and support to explore and change their teaching.

Taking a closer look at the previous research, to simply insert a teacher education program into a person's routine and life experience without an understanding of that experience seems uninformed. As Brown (2001) stated, "There is not an unproblematic state of 'how things are' with straightforward implications for practice. Changing practice cannot normally rest solely on instantaneous substitution of techniques" (p. 210). This is seen in Speer's (2001) conclusion:

> On the surface, with some of these TAs everything seemed to be going well, but something about the nature of the interactions did not seem to make the most of the opportunities they had to help their students learn. Although I encountered this phenomenon several times over many semesters, I was not able to really articulate what the problem was. I also felt relatively ineffective in helping these TAs devise strategies for improving their practice. On the surface, they were doing everything "right," and I was at a loss to help them because my usual collection of intervention and consultation strategies just did not apply. (p. 5)

From such a statement, it seems as though many experiences and events in the lives of mathematics graduate student were not addressed or explored, but had significant influences on the graduate students and their teaching. It appears that an exploration of mathematics graduate students and their teaching from a different vantage point is needed.

The above studies concluded there are other issues at work in the lives of mathematics graduate students that are not yet fully understood. I believe that an understanding of life and lived experience is necessary to know how to proceed, to know how to move forward, to understand what types of teacher education, if any, might fit into the life of a mathematics graduate student. Britzman (2003) described future teachers as being "in a state of becoming" (p. 9); I believe that a more in-depth understanding of the lives of mathematics graduate students, of "becoming" a mathematician, is needed before teacher education programs can be developed and implemented, before a course can be set for altering their teaching.

The following questions represent my points of entry into the lives of graduate students in mathematics in their process of becoming mathematicians and post-secondary teachers of mathematics:

In thinking about the idea of graduate students becoming future academics, professors, researchers, teacher educators, or mathematicians, where do they look for who they are supposed to become? For indications of how they should be? Of how they attend to their work, students, their discipline, themselves? How might the graduate school experience shape the identity of these future mathematicians as teachers of mathematics? What has meaning for them in how they present themselves within their disciplines? What is it that mathematics graduate students interpret or understand their lives to be like in mathematics? What has meaning for them in their process of becoming? What might graduate students in mathematics interpret as having significance for who and how they should be as mathematicians and as university teachers of mathematics?
As these questions are not yet addressed but pointed to in previous research, I believe that a more thorough understanding of mathematics graduate students' lives and their processes of becoming mathematicians and future professors of mathematics is necessary to understand why the previous studies were not as effective as the researchers hoped. This dissertation, then, is an exploration of the lives of mathematics graduate students and is concerned with how mathematics graduate become mathematicians, the experiences they go through, and what has meaning for them as they become mathematicians and post-secondary teachers of mathematics.

Chapter 2

I Hate 'Teaching' […] I Love Lecturing

In the previous chapter, I attended to my experiences as a graduate student in mathematics and teaching mathematics in post-secondary institutions. I believe that many of these experiences and my thoughts about mathematics teaching are not limited to me. Thus, in this chapter I will address some of the literature that speaks to the importance of teaching at colleges and universities, with a focus on post-secondary mathematics teaching in particular. Within the discussion around the improvement of mathematics teaching, several concerns about the discipline of mathematics and its teaching are described.

In this chapter, I revisit the recent research that is concerned with mathematics graduate students and their teaching. I also describe some of the suggestions that graduate students tend to encounter within mathematics departments, from their professors, and from mathematicians who have published their views about mathematics teaching, suggestions that have the potential to shape graduate students' views of teaching and their roles as professors of mathematics. Finally, I turn the focus to the graduate students' experiences and understanding of mathematics and mathematics teaching that are explored through the research in this dissertation. Note that in the dissertation I use the terminology *post-secondary education* and *higher education* interchangeably to denote all vocational and community colleges, liberal arts colleges, and universities that represent the possible future employers of mathematics graduate students.

University Teaching – Past and Present

Kline (1977), Boyer (1990), and Kronman (2007) have each written about a long-past paradigm of post-secondary teaching by means of a historical description of the status of teaching in American universities. These authors share that, before the mid-nineteenth century, universities preferred that their teachers *not* conduct research as it interfered with their teaching. Historically, the most important objective of universities in North America was to educate and, in order to satisfy that purpose, professors were to focus on their teaching. Before the civil war in the United States, at American universities there were no distinct separations between disciplines. For example, at Yale University, professors were expected to teach all subjects to their students until their fourth year. According to Kronman (2007), at many institutions, administrators of the university also had a large role in teaching, and its president was often the highest-regarded teacher.

Through changes in industry, technology, and science, research began to take on a much larger role in the career of university professors. By the early twentieth century, it was recognized that teaching had begun to suffer under the new focus on research. An idea had come into North America from Germany, "that universities exist primarily to provide the space, books, and other resources that scholars need to engage in the work of producing new knowledge" (p. 59). Research became commonplace, where "only those who concentrated on a single discipline while ignoring all others could hope to add in a meaningful way" (p. 64). In order to manage the growth in knowledge, the disciplines were separated into discrete units. Teaching regressed to a transmission model where teachers provided information that students were only expected to remember and recall, not necessarily understand (Boyer, 1990; Kronman, 2007).

In his work *Scholarship Revisited: Priorities of the Professoriate*, Boyer (1990) provided a renewed vision of university teaching, "as a dynamic endeavor involving all the analogies, metaphors, and images that build bridges between the teacher's understanding and student's learning" (p. 24). The intention of Boyer's work was to advance the purpose of the Carnegie Foundation, an organization that has dedicated itself to the improvement of post-secondary teaching, and also to practices informed by teaching, for the last hundred years. Boyer described the importance of great teachers in students' learning and their growth as critical thinkers.

Several researchers have recently explained the many reasons why attention to and improvement of post-secondary teaching is critical. Austin (2002) described increasing pressure and accountability for better teaching from parents, employers, and legislators. Forest (2002) argued that it is through "teaching future generations of leaders that higher education serves its most important function in society" (p. 8). Huber (2005) put forth the claim that advancements in the world, such as new technologies, new kinds of learners, and changes to society, required renewed attention to teaching and learning. Boyer (1990) noted, "Almost all successful academics give credit to creative teachers. […] Without the teaching function, the continuity of knowledge will be broken and the store of human knowledge dangerously diminished" (p. 25).

Efforts to improve post-secondary teaching have now become global, with international journals, organizations, and colleges and universities now dedicated to research (Forest, 2002; Golde & Walker, 2006; Huber, 2005; Shulman, 2004). For example, at the University of Alberta, through the President's newest academic plan, teaching and learning are gaining more visibility through initiatives such as the Teaching and Learning Enhancement Fund. In the United States, the Carnegie Foundation has spent the past two decades exploring and researching the Scholarship of Teaching and Learning at post-secondary institutions. In the international community, research journals and conferences dedicated to university teaching have been established (e.g., *The Journal of University Teaching and Learning Practice* from Australia, *The Journal of Scholarship of Teaching and Learning* from the United States, and *Teaching and Learning in Higher Education* from Canada, among others).

Yet, Huber (2005) warns that while the "recent movement to broaden the idea of scholarship in the academy is beginning to bear fruit […] the question remains as to whether individual faculty, departments, and institutions will take the risks that embracing new work inevitably entails" (p. 2). Within the wider movement to improve teaching and learning at universities, some work has been done to understand and improve the teaching of mathematics specifically. At this point, I turn the focus to teaching in departments of mathematics, to explain why the improvement of teaching mathematics at the post-secondary level is necessary.

Teaching in Departments of Mathematics

Mathematics departments constitute one of the largest service departments within institutions of higher education, providing prerequisite courses for students in diverse disciplines such as engineering, physics, chemistry, business, medicine, psychology and education. As a result, the teaching of mathematics at the university level is quite important in undergraduate education and professors, instructors, and graduate teaching assistants in mathematics have a wide-reaching influence on the education of future researchers, teachers, and mathematicians (Golde & Walker, 2006). While there has been an increased focus on undergraduate mathematics education for the past twenty years and movements such as calculus reform initiatives have been put in place, the format and style of post-secondary mathematics teaching has remained problematic for undergraduates.

The National Science Foundation and the National Research Council each published reports on the status of undergraduate learning of mathematics during the 1990s. Some of their findings were:

> Lack of student-teacher dialogue, which was thought also to reflect faculty indifference. Classes were mainly one-way lectures.
>
> Evident poor preparation for lectures, indicating to students that faculty were disinterested in student learning. Students were particularly frustrated by faculty who seemed unable to explain their ideas sequentially or coherently.
>
> Students also wanted but typically did not find many illustrations, applications, and/or discussions of implications. Nevertheless, students did not believe there was anything dull about NS&E [natural sciences and engineering] class material, even though student interest in many classes began to flag when faculty failed to present material in a stimulating way. Many students made reference to the “monotone” voices and dry recitations of their instructors’ lecturing.

> Class tedium grew in instances where faculty were "over-focused" on getting students to memorize material.
>
> Students identified as worst practice reading or copying material straight from text books. Reports of this practice were common in every SME&T [sciences, mathematics, engineering, and technology] discipline and on every campus. (National Science Foundation, 1996, p. 37)

In Kyle's (1997) review of these reports, he concluded that, "undergraduate instruction in these areas is hindered by outmoded instructional techniques, a focus upon disciplines resulting in fragmentation, and curricular inertia" (p. 547).

The problems with the post-secondary teaching of mathematics do not reside solely in undergraduates' stated dissatisfaction with how they are being taught. The teaching methods used by university mathematics teachers contribute significantly to the drop-out rate in mathematics and the sciences, as found by Martin (2001): "Studies show that almost half of the students who decide to specialise in a science major switch to a non-science major soon after enrolment. […] Why the severe drop out? One factor seems to be poor pedagogy. Students who change to non-science subjects cite this as a factor for leaving" (p. 434). Further, when looking at the decline of interest among science majors, the largest drop was in mathematics. Again, one of the reasons cited was the form of teaching in mathematics classes (Seymour, 2000).

Besides having an influence on undergraduate science majors, post-secondary mathematics teaching has had a negative impact on future elementary and secondary teachers. Several mathematics education researchers and mathematicians have described the importance of how mathematics is taught to this particular group. Along with others, Kline (1977) described the importance of post-secondary mathematics teaching for pre-service elementary and secondary schoolteachers, as they tend to teach in the way they were taught as undergraduates. Selden and Selden (1993) found that the teaching of mathematics and the interpretation of how mathematics is or can be taught was powerful enough to negate alternative forms of teaching learned in the methods courses for future schoolteachers. Hodgson (2001) recognized that what pre-service teachers learn is deeply connected to how they learn it. Kessel and Ma (2001) found that the structure of university mathematics lessons had a large influence on pre-service teachers' perceptions of how mathematics could be taught. Moreover, within mathematics content courses previously formed ideas about mathematics were reinforced and new meanings were not created.

Shulman (2004) described the cycle that unchanging mathematics pedagogy produces:

> Whether we call ourselves professors of education or professors of mathematics, to the extent that in our classrooms day after day sit men and women who subsequently go out and teach youngsters, we are teacher educators. To the extent that they are likely to teach both *what* and *as* they have been taught, unlike any other students in your classes, the future teachers are, if you will, carriers. Whatever understandings or misunderstandings you infect them with, both about the content and regarding the pedagogy, they will carry to generations of young people whom they will subsequently teach, and who themselves will eventually appear at your doorstep. (p. 406; emphasis in original)

Hodgson (2001) also put across this view about the relationship between university mathematics teaching and school mathematics teaching with the statement, "The quality of students in university classrooms is conditioned to a large extent by the quality of the teachers they have had in school" (p. 502). Thus, the way in which mathematics is taught to pre-service teachers has considerable implications for students' learning and teaching of mathematics at all levels of their education.

While mathematicians have often viewed teaching mathematics to pre-service teachers as low-level work (Hodgson, 2001), professional organizations such as the American Mathematical Society (1994) have made public statements supporting the involvement of mathematicians in programs for pre-service teachers. In particular, their national policy statement has as one of its goals to "attain excellence at all levels of mathematics education, giving particular attention to the professional development of teachers" (para. 7). There is growing recognition that pre-service teachers must be exposed to "a different manner of teaching from the traditional practices that they themselves most often experienced and observed as learners," especially in their post-secondary level content courses such as mathematics (McGinnis, Watanabe & McDuffie, 2005, p. 408). Yet, it is unclear at this time whether college and university departments of mathematics recognize or acknowledge their role in mathematics education and whether they are heeding such advice and employing alternate pedagogical methods for future teachers.

Additionally, through their teaching, mathematicians also have a large role in the future of mathematics itself (Chan, 2006). As I continue through my career teaching mathematics, I find that I am genuinely concerned for the discipline of mathematics. While the teaching of mathematics in universities remains rooted in traditional, lecture-based approaches, I cannot help but think that not only are many gifted mathematics students finding themselves exasperated and alienated by this approach (Seymour, 2000), but also that negative images of mathematics are maintained in this way. I cannot forget the response of one of my most talented algebra students to my question "Have you thought about majoring in mathematics and being a mathematician?" His answer, stated without humor, was "I have never been so insulted." His reaction was representative of the feelings many people seem to have for mathematics. Yet, my own experience with and passion for teaching and learning mathematics tells me that mathematics is not as students perceive it, nor is it as mathematicians sometimes portray it.

As students' frustrations with mathematics teaching and learning continue, it is inevitable that not only will mathematics continue to fall out of favor, but I also fear that it will be, in a sense, taken away from mathematicians and made to be relevant solely through other disciplines. For instance, Steen (2001) described how other disciplines have recognized that "post-secondary mathematics is no longer a craft practiced only in the village of university mathematicians" (p. 310), and also that other disciplines are "setting up alternatives to university mathematics" (p. 309). Friedberg (2005) acknowledged that if professors of mathematics cannot effectively teach undergraduates, then "the pressure for others to do so in our place will increase" (p. 842). Indeed, in order to create more meaning in and deeper connections to mathematics for their own students, many disciplines have begun to teach mathematics and statistics within their own departments rather than having their students learn these subjects in traditional departments of mathematics and statistics.

While a great deal of mathematics had originally developed with connections to or for work in other disciplines, it seems that mathematics might soon find itself isolated, where much of the mathematics developed within departments of mathematics will be developed solely for mathematics' sake. Two mathematicians, Freudenthal (1973) and Steen (2001), have voiced their concern for mathematics. Respectively, they have stated, "A mathematician should never forget that mathematics is too important to frame its instruction to suit more or less the needs of future mathematicians" (p. 69) and "Mathematics is too important to be left to mathematicians" (p. 308). The notion that the teaching of mathematics is a central cause in the diminished appreciation of mathematics can be seen as explicit in the first statement, implicit in the second.

Another mathematician, Kline (1977), summarized this predicament of mathematics stemming from its teaching with the statement, "The greatest threat to the life of mathematics is posed by the mathematicians themselves, and their most important weapon is their poor pedagogy" (p. 5). Friedman (2005) recognized that "mathematics will do better if the next generation of mathematicians are excellent teachers" (p. 842). Chan (2006) remarked that mathematics is risking "turning our doctorate into a kind of esoteric priesthood for the few," if the field does not recognize the need for change and acceptance of teacher training programs (p. 125). Thus, several mathematicians are beginning to express a desire to improve teaching partly from a concern about the future of mathematics.

Considering the Future University Professor of Mathematics – Mathematics Graduate Students

> Graduate school is thus the common portal through which nearly all of America's college and university teachers now pass on their way to an academic career. It is the first stage of their professional lives, not just for a few devoted scholars but for all who choose a career in college or university teaching, at whatever level and whether or not they later engage in research themselves. It is in graduate school, therefore, that all but a few of America's college and university teachers are now introduced to the norms of the academic professor and where they first acquire an understanding of who possesses authority in the profession and why. As a result, our graduate schools, and the research universities that house them, exert an enormous influence on the values and expectations of young teachers. They are the nursery beds in which professional habits of most of our college and university teachers are formed, and the attitudes they acquire there are carried with them to every corner, and level, of American higher education. (Kronman, 2007, p. 92)

During their time as teaching assistants, mathematics graduate students "often have responsibility for teaching lower-division courses," including those for future teachers (Speer, Gutmann, & Murphy, 2005, p. 76). Further, most graduate students in mathematics either are or will become university teachers of mathematics with more than seventy percent of mathematics PhDs finding jobs at institutions focused mainly on undergraduate education (Chan 2006; Kirkman, Maxwell, & Rose, 2006; Kline, 1977). The National Science Foundation (1996) has recommended that, in order to improve undergraduate mathematics teaching, departments need to "provide opportunities for graduate students to learn about effective teaching strategies as part of their graduate programs" (p. 69). However, while the education of elementary and secondary teachers in teaching methods is viewed as essential, teacher training is not always acknowledged as required at the university level (Alsina, 2005; Golde & Walker, 2006; Herzig, 2002a).

In general, graduate students in many disciplines are not provided with opportunities to learn about pedagogical methods or theories, despite often taking on the role of teaching assistant (Golde & Walker, 2006; Forest, 2002). Because preparing to be an academic is often "rooted in a focus on developing expertise within the discipline," there is little interest in making that "expertise accessible to a diverse group of students" (Forest, 2002, p. 81). Boyer (1990) described the experience of graduate students as "a period of withdrawal – a time when many students are almost totally preoccupied with academic work and regulatory hurdle" (p. 69). Austin (2002) has argued that graduate teaching assistantships are often "structured more to serve institutional or faculty needs than to ensure a high quality learning experience for graduate students" (p. 95). Both Forest and Boyer have put forth the argument that teacher training for future professors must be considered and that graduate students must be provided with practicum experiences in teaching.

Within mathematics, Bass (2006) has noted that, "little attention is paid to preparing and supporting graduate teaching assistants" (p. 99). In the National Research Council (1991) and National Science Foundation (1992, 1996) studies cited previously, Kyle (1997) found "the current professional education and development of future SME&T [science, mathematics, engineering, and technology] faculty members places too little emphasis on teaching and teaching improvement" (p. 547). However, Herzig (2002b), echoing Shulman's statement in the previous section, described the important and far-reaching role that graduate students have in the teaching of mathematics:

> Indeed, since teachers' own classroom experiences shape their beliefs and knowledge about teaching and learning (Fennema and Franke, 1992; Thompson, 1992), these graduate students' educational experiences are likely to pass on to other students in a type of 'domino effect.' That is, if the survivors of this educational environment teach the way they were taught, and then the pre-service teachers they teach later teach the way they were taught, then it is critical, and alarming, to consider the effect of this model of graduate education on children learning mathematics in schools. (p. 205)

Despite the acknowledgement of graduate students' significant influence on the teaching of mathematics at many levels, this has not yet noticeably had an impact on the preparation graduate students receive for their current and future positions as post-secondary teachers of mathematics.

In following a new professor of mathematics in his first year in a department of mathematics, Gutmann's (2000) research provides an interesting argument for the preparation of future post-secondary mathematics teachers. Gutmann noted that during the interview process for a position as an assistant professor of mathematics, it was not the candidate's mathematical abilities that were questioned, but "the most important concern during the hiring process was that he would be an effective classroom teacher" (p. 103). Yet, despite the teaching experience hiring committees are looking for in new faculty members, the National Science Foundation (1992) found that "young faculty today are poorly prepared and lack adequate support to assume the full responsibilities of academic life. In large measure, young faculty are left to their own devices and therefore doomed to repeat the mistakes of their predecessors due to inadequate instructional preparation" (p. 24). This was observed in Guttman's research when it became evident that the new professor "had never taught a class in statistics at any level and said he does not know much about what an undergraduate studying statistics should learn" (p. 62).

Chan (2006) has claimed, "we must recognize the fact that most of our 'products' will not become professors in research universities, so we must train them in a way that better prepares them for a broad range of careers" (p. 125). However, few graduate programs in mathematics fully prepare their students for their future careers, nor do they require their students to take any courses on the subjects of pedagogy, learning theories, or classroom practice. As a result, many graduate students in post-secondary mathematics teach without any formal teacher training (Hodgson, 2001). Approaching graduate students while they are not yet instructors or professors is important as they have not yet been entirely enculturated into the paradigm of research that can be seen as detrimental for university professors' efforts to teach (Kline, 1977).

To compensate for the lack of preparation, the Mathematical Association of America (2009) has acknowledge the lack of preparation that mathematics graduate receive and have, as a result, created the program *Project NEXT* (*New Experiences in Teaching*), which is described as:

> A professional development program for new or recent Ph.D.s in the mathematical sciences. It addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, and participating in professional activities. It also provides the participants with a network of peers and mentors as they assume these responsibilities.

However, with limited resources this program unfortunately can only accept eighty of the over one thousand new PhDs in mathematics each year.

In the past several years, research concerning mathematics graduate students and their preparation as post-secondary teachers has begun. In a research project that involved mathematics graduate students in the context of calculus reform, when graduate students in mathematics could speak of teaching using reform-oriented terminology, they also reported rarely using the associated teaching methods and maintained a lecture style form of instruction (Speer, 2001). When mathematics graduate students were offered a course in pedagogy and teaching mathematics, it did not alter their teaching practices (Belnap, 2005; DeFranco & McGivney-Burelle, 2001). In another project, Golde and Walker (2006) found that changes to pedagogy were particularly difficult for mathematics doctoral students. Moreover, it has been concluded by researchers that positive attitudes and beliefs around teaching mathematics did not change graduate students' teaching practices (Belnap, 2005; Speer, 2001). As these studies have found that informing mathematics graduate students about different approaches to teaching, pedagogy, student learning, and curriculum reform did not change classroom practices, something remains to be explored.

While previous research reports that mathematics graduate students receive very little preparation for teaching, I would argue that they have essentially received years of tacit instruction in teaching mathematics through their experiences as students. Austin (2002) found that graduate students are "keen observers and listeners," gleaning information from their experiences to understand the emphasis they should place on their different tasks. As well, through their involvement in the routines of a department of mathematics, graduate students' views of the discipline and teaching are shaped (Austin, 2002; DeFranco & McGivney-Burelle, 2001). Further, graduate students in mathematics encounter many situations and structures that have the potential to be interpreted as having meaning and implications for how they should live their lives and convey their work as mathematicians (Austin, 2002).

In my review of the literature, it appears that in the lives of mathematics graduate students there exists a complex and intricate interplay among the structures that they encounter, their feelings about mathematics and themselves, and their ideas of their future role as mathematics instructors or professors. I believe it is the bearing that these experiences and structures have on mathematics graduate students' teaching practices that should be explored in order to gain some understanding of teacher preparation for future mathematicians might be better approached. In the following section, I describe and explore what I mean about the experiences, suggestions, and structures that mathematics graduate students live through and encounter, and what connotations these structures and contexts may have for their lives in mathematics and their teaching of mathematics.

Suggestions About Teaching Mathematics

> A graduate student, as part of his or her socialization into an academic discipline, will come into contact with two main categories of tacit knowledge. One of them is the knowledge that has grown out of long experience with the discipline. It is a practical, almost subconscious, knowledge or competence that the department elite fully master. The most important ingredient is the knowledge and command of the repertoire of scientific discourses. The other category of tacit knowledge is generated by the students themselves as they try to make sense of what they are

> experiencing in the graduate studies program. Like the former type, it is likely to be used as a guide for action. And for an understanding of what goes on in Academia they are both of great importance. (Gerholm, 1990, p. 271)

This description of the lives of graduate students is important in the following exploration of their experiences. Gerholm wrote of the categories of tacit knowledge that graduate students encounter and interpret as having meaning for their lives. An investigation of the word *tacit* shows that it means "understood or implied without being stated" (Soanes & Stevenson, 2005, p. 1794). In my own experience in graduate school, there were many such understandings conveyed without being openly expressed, suggestions that were hinted at often through implicit means. For the study of the experiences and literature surrounding mathematics graduate students' lives that now follows, I use the words *suggestion* – "the process of inducing a thought, sensation, or action in a receptive person without using persuasion and without giving rise to reflection in the recipient" (www.dictionary.com, 2008) and *intimation* – "to indicate or make known indirectly; hint; imply; suggest" (www.dictionary.com, 2008) to denote the tacit knowledge and understandings Gerholm refers to.

As "words and deeds, by their nature, 'reach out' to others; they 'speak across' generations, cultures, contexts and situations as a form of contribution to the universal voice which is man in the world" (Smith, 1983, p. 87), I will describe suggestions that are put forth regarding the status of teaching not only at institutions of higher education, but also specifically in post-secondary departments of mathematics. Graduate students in mathematics encounter many intimations about the form their lives should take and who they should become as they prepare to be mathematicians. Thus, within the discussion of graduate students' experiences, it is important to look at both the implicit and explicit suggestions communicated by departments of mathematics as well as by individual mathematicians. Among other things, post-secondary departments of mathematics represent future employers with expectations of behaviour and performance. And, since mathematicians are examples of the profession, the people who graduate students will emulate as they progress through their graduate studies and into their academic careers, their views of teaching are quite important in the development of graduate students as future professors. In the following subsections, then, I point to suggestions about teaching that are found in universities and communicated by mathematicians.

Suggestions about the importance of teaching at universities

Teaching at many universities and in many departments fails to be promoted or attain the status that many researchers and educators are now arguing for. Austin (2002) found that while graduate students are given teaching opportunities, "they are often not organized systematically to ensure growth or appropriate preparation" (p. 105). Prewitt (2006) found that even though some universities speak to the importance of teaching, "incentives," such as funding and grants, promote research and run counter to the efforts to focus on and improve post-secondary teaching (p. 23). Britzman (2003) also alluded to the university reward system, stating that "Trained as 'experts' in particular content areas but not in the production of their accompanying pedagogies, many [university professors] accept the view, instanced by the university reward system, that teaching is secondary to the 'real' work of scholarly research" (p. 55).

While some universities claim to acknowledge the importance of teaching through various programs, Shulman (2004) described how teaching remains a secondary concern for particular departments:

> Look, for instance, at the way the improvement of teaching is treated in most of our schools. Institutional support for teaching and its improvement tends to reside in a university-wide center for teaching and learning where most of the TAs are trained, and where faculty – regardless of department – can go for assistance in improving their practice. That's a perfectly reasonable idea. But notice the message it conveys – that teaching is generic, technical, and a matter of performance; that it's not part of the community that means so much to most faculty, the disciplinary, interdisciplinary, or professional community. It's something general you lay on top of what you really do as a scholar in a discipline. (p. 456)

Huber (2005), in her work with new academics who chose to focus on teaching to advance their discipline, found that teaching has remained a belittled activity on university campuses. In fact, each faculty member with whom she worked, all of whom had been labeled innovative educators, had been "told by caring and responsible mentors that they were undertaking a fool's errand in treating teaching so seriously" (p. xii). Despite promoting their discipline by working to improve undergraduate education in that discipline, one faculty member was begged by their mentor to engage in more "traditional" work. As Davis and Hersh (1981) stated, "such unorthodox and dubious adventures [a focus on teaching] would have seemed at best a foolish waste of precious time – at worst a disreputable dabbling with shady and suspect ventures such as psychology, sociology, or philosophy" (p. 2).

Suggestions about teaching mathematics from mathematicians

> Mathematicians in universities have great power to influence what is learned and how it is learned. (Burton, 2001, p. 589)

Several mathematicians have contributed their own ideas about the post-secondary teaching of mathematics through various publications, sometimes arguing for and sometimes belittling the importance of teaching. Mathematicians are significant figures in the lives of mathematics graduate students. As I mentioned in the discussion about pre-service elementary and secondary teachers, these future teachers tend to teach mathematics in the ways they have been taught, despite learning about alternative methods in their education programs. In a similar vein, as graduate students are the future teachers of post-secondary mathematics, it is the mathematicians with whom the graduate students interact who represent the models of teaching that graduate students will adopt (Herzig, 2002a). Not only is the manner of mathematicians' teaching of consequence for graduate students, but the suggestions they convey about teaching are significant as well.

Unfortunately, Kline (1977) described the belief amongst mathematicians that a focus on teaching "is a confession of failure as a researcher, a tacit admission of inability to compete in the arena of pure mathematics" (p. 240). Kline also noted that a graduate student will "undoubtedly lose the respect of his mathematics professors" should they choose to learn about teaching (p. 101). There is a feeling, then, among some mathematicians that successful teaching in mathematics represents failure as a mathematician. Moreover, there is the sense of what real teaching is in mathematics – upper-level undergraduate or graduate courses; certainly not courses for pre-service teachers. Kline described how one-hundred-level courses and courses for future teachers do not fit the “standards for real mathematics” (p. 308). This attitude is unfortunate, as Steen (2001) has pointed to the unique perspective that mathematicians could bring to courses for pre-service elementary and secondary teachers.

Individual mathematicians have put their stamp on the value of post-secondary teaching of mathematics. In *A Mathematicians Apology*, one of the first books written by a mathematician that relates somewhat to a mathematician’s role in the classroom, Hardy (1940) declared, “I hate ‘teaching’ […] I love lecturing” (p. 48). Krantz (2003), a mathematician who has been vocal about university teaching in mathematics, wrote the book entitled *A Mathematician’s Survival Guide: Graduate School and Early Career Development*. It is noteworthy that within his diagram of the “Steps to a Graduate Education” teaching and learning to teach are not included in the list of steps, which are summarized as taking courses and qualifying exams along with conducting research. Within this 222-page book about graduate studies and early career development, teaching is only mentioned a handful of times, indicating by its absence the perceived insignificance of teaching to a mathematics graduate student’s experience. When Krantz briefly addressed teaching in this guidebook, he stated “If the professor so chooses, s/he does not even have to entertain questions” (p. 48). In his advice to someone who cannot decide between teaching and research, Krantz implores them to not give up their research, explaining that teaching is easy to return to, while research is not.

Baumslag (2000), another mathematician, put forth his own ideas about mathematics teaching at the post-secondary level and research in mathematics education in the book *Fundamentals of Teaching Mathematics at the University Level*. He described the efforts and some reasons for improving university mathematics teaching in Europe:

> Governments are also convinced that the fault lies mainly with the teaching skills, or lack of them, at the university. Thus the United Kingdom has introduced an authority to assess teaching quality at universities, and, as mentioned in section 4.1, the Swedish government made it a point for the budget year 1997 that universities had to intensify their work on development of teaching methods and actively find new methods in teaching for both undergraduates and postgraduates. Lecturers had to be offered courses on pedagogy, and the university was required to account at the end of the year for the developments achieved and the number of their staff that had attended courses on pedagogy. There was thus clearly the suggestion that improved teaching is the way of making progress. (p. 49)

However, he described his opinion of the attempts to reform university mathematics teaching in the following way: "The problem is reminiscent of the alchemists' search for converting lead into gold; nobody knew whether it [reform of university mathematics teaching] was possible or economic, and much time and energy were frittered away" (p. 49). While this represents only his belief, it is unfortunate that this attitude is conveyed in a book with such an influential and important sounding title.

Stewart (2006), another mathematician who has put forth his own opinions about the importance of teaching post-secondary mathematics into the mainstream, recently wrote the book *Letters to a Young Mathematician*. In this book, through letters to a woman who is interested in becoming a mathematician, Stewart noted the most significant experiences and rites of passage in the life of a mathematics graduate student who becomes a professor. He acknowledged that new PhDs in mathematics are not prepared for teaching, but claimed that relying on their experiences as mathematics students would be sufficient. Regrettably, Stewart advised that teachers, "Stick to the main point, and try not to digress if doing so requires new ideas that are not on the syllabus" (p. 168), essentially promoting an adherence to traditional modes of teaching. In Gowers' (2006) book *The Princeton Companion to Mathematics*, five professional mathematicians offered guidance to young mathematicians. In eleven pages of advice, there is only one sentence about teaching: "Teaching should not be a burden, but a source of inspiration" (Bollobàs, 2006, p. 1004).

I believe it is also important to recognize the mathematics education research that has been organized at the university level by mathematicians in this discussion of the suggestions about post-secondary mathematics teaching. It is interesting to observe that articles and books related to mathematics education research at the university level do little to explicitly address teaching itself (Speer et al., 2005). Rather, these volumes (e.g., Selden & Selden, 1993; Selden, Dubinsky, Harel, & Hitt, 2003; Tall, 1991) focus a great deal on the learner rather than the teacher, with a large emphasis on the learner in calculus reform. To me, these research efforts imply that it is the student who must change and evolve, not the teacher. This lack of attention to the actions of the teacher can prevented me from exploring modes of teaching other than lecturing (see Chapter 1).

Suggestions About the Nature of Mathematics and About a Way of Being in Mathematics

One last aspect of mathematics graduate students' lives I wish to address is the texts or forms of mathematics that they experience and are expected to produce. I believe that these texts, along with the previously-mentioned suggestions conveyed by universities and mathematicians, compel mathematics graduate students and teachers to use fixed, inexpressive modes of writing and teaching when conveying their knowledge through mathematical research and when in the classroom. Moreover, the writing of mathematics has implications for one's identity and relationship with mathematics. Within this sub-section I also include an example of my own experiences as a student of mathematics in this regard.

Producing mathematics as a mathematician

As Burton (2004) discovered in her extensive interviews with seventy mathematicians, the work of a mathematician is often a creative and innovative process of exploration and discovery, of attempting new methods, of failure and success, of disappointment and joy. Yet, the mathematics that is presented in texts, journals, and in the classroom does not resemble or reveal the process involved in attaining the final results. Gone from the page and from the classroom are the people who discovered the mathematics, along with their joys, passions, and frustrations. This language of mathematics that is presented in public forums, which is ostensibly free from the human perspective that brought it about, has consequences for mathematicians and for teachers.

With regard to the person behind the mathematics, he or she is often not present in their own creation, as described by Davis and Hersh (1981): "His writing follows an unbreakable convention: to conceal any sign that the author or intended reader is a human being. It gives the impression that, from the stated definitions, the desired results follow infallibly by a purely mechanical procedure" (p. 36). The absence of the mathematician was also commented upon by Burton and Morgan (2000), where they explained that the mathematician's role becomes "subordinate to that of the mathematics itself" (p. 435).

Gadamer (1975) wrote, "the sheer fact that something is written down gives it special authority. […] The written word has the tangible quality of something that can be demonstrated and is like a proof" (p. 274), recognizing the influence of what is written. Morgan (1998) stated that presenting "processes as objects […] is part of the strength of mathematics, but at the same time, it increases the impersonal effect, strengthening the impression that it is these process-objects that are the active participants in mathematics rather than the human mathematicians" (p. 15). Thus, for the presentation of mathematics, it is not the discoverer who is important, but the discovery. Through the writing and language used in mathematics, mathematicians and their efforts are placed at the periphery. This has potentially detrimental effects on the mathematicians themselves, as Herzig (2002a) has noted that the view of mathematics as "an objective field of knowledge" has lead to a "cultural blindness to personal issues" (p. 10).

Burton (1998) noted the large discrepancy between the collaborative model of mathematicians and the transmission model of the classroom. Further, she wrote that mathematicians "speak in very different voices about the nature of the enterprise upon which they are engaged, making clear how integral it is to both their own person-ness and the nature of their professional interactions. These personal flavours are entirely lost in the 'objective' mathematics they, as teachers, thrust towards reluctant learners" (p. 140). Indeed, the writing and language used in mathematics leaves the teaching of mathematics without the inquiry, process, and characteristics of the people involved. Thus, in mathematical writing, the innovator and devotee of mathematics – the researcher, teacher, instructor, professor – becomes subordinate to the mathematics they are develop and present.

Producing mathematics as a student

Students at all levels experience constraints when they are learning mathematics. They encounter textbooks that most often provide one method for solving equations and are offered one form of a solution in the back of many textbooks. When teachers mark students' homework, often it is only the answer that is considered and not the process. Morgan (1998) stated that producing 'correct mathematics' was, "producing a correct sequence of symbols" where "the mathematical writer's task is merely to record the content without any need to pay separate attention to the form of the language in which it is recorded" (p. 12).

It is not only the methods of solutions offered, but also the language of mathematics texts themselves that seems to confine students to symbolic manipulation and prevents them from expressing themselves in other ways. The use of present tense or third-person passive language in mathematics texts suggests the "impersonal work of mathematical necessity rather than the accident of authentic discovery" (Netz, 2000, as cited by Pimm, 2005). Pimm (2005) depicted the language of mathematics problems as 'omniscient' and 'impersonal.' Morgan (1998) noted within mathematical writing was "a formal, impersonal style, including an absence of reference to human activity" (p. 11). Hersh (1979) alluded to a loss of meaning in the translation from "informal, intuitive theory to a formalized theory" (p. 389).

The impersonal language in mathematics has implications for graduate students as well. Jardine (1990/1998) wrote that, "The problem with the literalism and exactness of the discipline of mathematics is that it can live in a forgetfulness of the young and become (pedagogically speaking) tragically self-enamored and self-enclosed" (p. 67). As Burton (1998) explained, "Of course with each new generation of teachers who have not encountered the excitement and frustration and whose learning has always been dependent on a didactic and transmission-based model, there is no alternative experience for them to draw on in their own practices" (p. 140). Without opportunities during the mathematics graduate students' programs to learn how to teach other than directly out of their textbooks, the language and format of the texts takes on a large role in their teaching. When graduate students encounter no other model of teaching other than a transmission model, and with only texts to rely on, they have little to understand other than their teaching must be free from who they are as people and that they must attend only to the mathematical content of the course.

Exploring the Lived Experience of Graduate Students in Mathematics

> Insofar as the very character of the field of study is conditioned by social judgments about the norms of the profession, attainment of candidacy can be viewed as a form of social initiation into a group whose members have a vested interest in maintaining the norms of that group. And to the degree that that initiation requires prescribed social behaviors and beliefs, it can be argued that successful completion of a doctoral degree calls for the successful performance of a social role called 'graduate student.' (Tinto, 1993, p. 255)

I have described some of the messages and suggestions about teaching from universities, departments of mathematics, mathematicians, and the texts that graduate students encounter. These suggestions include the ways in which mathematicians should live, about how they represent their work, and the minimal and even insignificant role that mathematics teaching should have in the life of a mathematician. In reviewing previous studies with mathematics graduate students (e.g., Belnap, 2005; DeFranco & McGivney-Burelle, 2001; Speer, 2001), the relationship between mathematics graduate students and their teaching seems to have been significantly confounded by the suggestions put forth by mathematicians, departments of mathematics, and even by the very language of mathematics, which have somehow confined graduate students to one way of teaching. It appears that the suggestions graduate students encountered were powerful enough so that opportunities to enroll in teaching methods courses was not enough to alter their teaching practices.

What might have meaning in this context? What might be happening for the graduate students in the process of becoming mathematicians? The above quotation from Tinto (1993) suggests that, amongst other things, graduate students in mathematics undergo an initiation or manner of induction into being a member of a mathematics department. In this regard, Lave and Wenger (1991) have offered the term “legitimate peripheral participation” in relation to a community of practice to name one central process by which novices gain knowledge and understanding about the practices of a community. This concept is described more fully as learners “participate in communities of practitioners and that the mastery of knowledge and skill requires newcomers to move toward full participation in the sociocultural practices of the community” (Lave & Wenger, 1991, p. 29).

I believe that the suggestions that are put forth by mathematicians and departments point to the practices of the community and speak not only to what is important, but also to what is acceptable and sanctioned compared with what might be unacceptable. In particular, the noticeable omission about teaching being a salient part of the life activity of a mathematician, an omission that I pointed to earlier in citing the work of Krantz (2003) and Stewart (2006), is one that resonates throughout literature about becoming and being a mathematician. Through their initiation, then, in encountering such suggestions, the mathematics graduate students may be hearing that teaching is insignificant and, in order to become part of the community and to take on a larger role and more of an identity as a mathematician, they must not make their teaching important. Might this be the case for graduate students in mathematics?

Another area that this dissertation seeks to explore is to what extent being a mathematician is part of the identity of being a post-secondary teacher of mathematics, and to understand what such a relationship might be. Tinto's (1993) description of what graduate students encounter includes the concept of norms, which points to taking on a particular identity within a particular group. Within the framework of legitimate peripheral participant exist issues of identity where Lave and Wenger (1991) describe how "the development of identity is central to the careers of newcomers in communities of practice" where "learning and a sense of identity are inseparable" (p. 115). Again, the suggestions explored earlier in the chapter, in particular those I described from Stewart (2006) and Krantz (2003), speak to notions of community and identity, who mathematicians are, how they should be, and how newcomers can attend to this.

Britzman (2003) has explored issues of teacher identity in her book *Practice Makes Practice*. Unlike the schoolteachers with whom Britzman worked, mathematics graduate students do not explicitly learn about teaching, about becoming a teacher, but they are nonetheless in the process of becoming instructors or professors of mathematics. Yet, Britzman (2003) remarked that, "individuals feel encumbered in the institution of education and not recognized for who they hope to become (as opposed to who they might actually be). And this pressure for identity and identification seems to come from the outside" (p.

18). Further, she stated, “taking up of an identity means suppressing aspects of the self” (p. 27). Thus, within the framework of legitimate peripheral participation, and what it appears that mathematics graduate students are experiencing in becoming a part of a community and possibly leaving behind what is important to them, I believe that Britzman’s work around issues of teacher identity has much to offer to the exploration in this dissertation.

In her study of mathematics doctoral students leaving their programs before completing their degrees, Herzig (2002b) described departments of mathematics as a place in which a “doctoral student needs to do more than just learn the content of the mathematics taught in class, he needs to learn to participate in social and cultural practices” (p. 201), where “the mathematics community of practice imposes cultural practices with corresponding implicit expectations on students” (p. 178). I believe that the suggestions from individual mathematicians, the explicit and implicit directions from departments of mathematics, signify what the participants should and, at times, even *must* attend to as they become a part of this community of practice. I believe that aspects of legitimate peripheral participation describe the experiences I had as I learned more about what my role and practice should be as a future professor of mathematics. In so doing, in taking on what I believed was important and leaving behind what was not, I also began to develop my identity as a post-secondary teacher of mathematics with all of its implicit tensions and challenges. Similar to Britzman’s (2003) work, I focus on how “subjects produce and reproduce meaning and myths […] through their theories, practices, routines, discourses, contexts, and reflections on educational life, and how such meanings produce identities" (p. 37). I will return to issues of legitimate peripheral participation and identity in later chapters, as I address how these ideas capture the themes and challenges for the mathematics graduate students in this study.

Chapter 3
Hermeneutics as Restoring Life to Its Original Difficulty

In the first chapter, I provided an autobiographical account of my experiences as a graduate student in mathematics. I also described my difficulties with mathematics teaching – my initial resistance to teaching, my taking up of an indifferent and disengaged form of mathematics teaching and, eventually, a concern for and subsequent change to my teaching. The emerging attentiveness to my teaching included a realization that the way I taught mathematics, and the way I had been taught mathematics, was detached from how I felt about mathematics. From the beginning of my teaching career, I had wanted my students to experience the exploration, discovery, even exhilaration that I felt in and about mathematics. As I slowly reconnected my feelings for mathematics with my teaching, I began to question why I had taken up a form of teaching that I had disliked when I was a student in mathematics. What would cause me to teach mathematics in this way? Was it the discipline of mathematics itself that had brought me to this place? Through my professors' teaching and unspoken expectations during my graduate programs had I experienced and internalized a tacit form of training to take on a particular way of being in the classroom?

My experiences with teaching as a mathematics graduate student and as a post-secondary teacher of mathematics described in the first chapter are supported by what I have provided in the second chapter. In particular, along with sharing the reasons why mathematics teaching is important and a concern within larger movements to improve university teaching, I described the suggestions that mathematics graduate students experience as they learn to be and come to be mathematicians. I believe that as students of mathematics, graduate students encounter, interpret, and eventually come to embody a life in mathematics teaching that they observe in the behaviour of their professors and in the many suggestions they receive about a particular way of being in mathematics. These messages seem to be interpreted and internalized as the graduate students find themselves growing into their lives as mathematicians.

The life world that is conveyed through the teaching of mathematics appears to be far removed from the life world that is experienced in mathematics – at least in how I experienced mathematics, how I believe my peers experienced mathematics, and how others describe the lives of mathematicians (e.g., Burton, 2004; Davis & Hersh, 1983). Definitive and polished products replace the struggle, creativity, and exploration inherent in mathematical work. Classroom discourse is most often reduced to the replication of texts that do not convey the richness of the mathematical discovery that produces definitions, techniques, and theorems. The relationships and associations between topics and concepts are concealed by the linearity of texts. The innovative and exploratory behaviour of mathematicians is concealed by their static pedagogy, and their interests and passions often remain hidden from most of the world, kept behind closed office doors.

Within the first two chapters of this thesis is the emergence of a question, or questions, of what it means to live a life in mathematics as a graduate student in mathematics, as a future mathematician, and as a post-secondary teacher of mathematics. During this experience, what are the messages and suggestions that are encountered by graduate students? What meaning do these suggestions have for graduate students? How are these suggestions perceived, interpreted, and internalized? What might these suggestions imply for being in mathematics and for mathematics teaching? In undertaking this research study, my hope was to gain an understanding of the lives of mathematics graduate students, what they encounter, and the meaning that university mathematics teaching has for them.

While much of my research experience was previously based in quantitative methods, in the tradition of Heidegger and Gadamer who "sought to reclaim what they perceived had been lost through the use of empirical scientific explorations within the human realm" (Laverty, 2003, p. 11), I felt that my research and future endeavors as an educator ought to be based in a qualitative understanding of life and learning. Such an approach would lead to richer descriptions of experiences in mathematics and produce an understanding as to why teacher preparation has not yet been successful for graduate students in mathematics. Gadamer (1975) wrote, "Understanding begins […] when something addresses us. This is the first condition of hermeneutics" (p. 298). This chapter describes the theoretical framework of hermeneutics, a form of inquiry that I believe will help to shed light on what has addressed me – the lived experience of mathematics graduate students and their process of becoming mathematicians.

The title of this chapter comes from an article entitled *Reflections in education, hermeneutics and ambiguity: Hermeneutics as restoring life to its original difficulty* (Jardine, 1992). This title has significance for me as lecturing in university mathematics classes is often considered easy and straightforward. However, I believe that lecturing conceals the difficulty, challenges, debate, and human effort that exist in and are inherent in the creation and development of mathematics. Hermeneutics helps to trouble the notion of mathematics teaching as simple and reduced to the replication of texts, to remind us of the original difficulties in learning and understanding mathematics. In using hermeneutics, I hope to remind the community of the difficulties in the lives of graduate students that are not yet understood, but seem to have a significant influence on them as they make their way to becoming mathematicians.

Hermeneutics and Its Attentions

> *hermeneutic:* "interpretive," 1678, from Gk. *hermeneutikos* "interpreting," from *hermeneutes* "interpreter," from *hermeneuein* "to interpret," considered ultimately a derivative of Hermes, as the tutelary divinity of speech, writing, and eloquence. (www.etymonline.com, 2008)

> It distinguishes itself from other forms of inquiry by its essentially educational nature. That is to say, hermeneutic inquiry has as its goal to educe understanding, to bring forth the presuppositions in which we already live. Its task, therefore, is not to methodologically achieve a relationship to some matter and to secure understanding in such a method. Rather, its task is to collect the contours and textures of the life we are already living, a life that is not secured by the methods we can wield to render such a life our object. (Jardine, 1992, p. 116)

As my questions continue to find themselves concerned with the lived experiences of mathematics graduate students and their processes of becoming mathematicians and post-secondary teachers of mathematics, I am drawn to hermeneutics as a way to continue to uncover the questions about and to seek an understanding of these phenomena. I have found that hermeneutic inquiry is essential for this project, as “it is the interpretive study of the expressions and objectifications of lived experience in the attempt to determine the meaning embodied in them” (van Manen, 1997, p. 38). At the heart of hermeneutics is ontology, “the be-ing of being human” (Smith, 1983, p. 28). As I seek to understand what the experience is like in be-ing a mathematics graduate student and in be-coming a mathematician, this project is ontological rather than epistemological. Further, hermeneutics has as part of its focus “illuminating details and seemingly trivial aspects within experience that may be taken for granted in our lives, with a goal of creating meaning and achieving a sense of understanding” (Laverty, 2003, p. 7). As I hope to become familiar with the whole of the graduate students’ lives, hermeneutics provides a lens for exploring many aspects of their lives, including those that are seemingly insignificant.

Hermeneutics helps one to unearth the ways and the whys in which we understand life and lived existence, and how we can create and find meaning through experience, language, and social engagement (Brown, 2001; Gallagher, 1992; Jardine, 1992; Smith, 2006; Smits, 1997). Within mathematics graduate students’ experiences, their professors’ teaching, departmental expectations, and their teaching assistantships all have interpretive implications for what graduate students make important in their lives. As students’ knowledge of their future worlds develop, “as a consequence of their encounter with the department: semi-automatic, barely conscious interpretations of what teachers say and do”

(Gerholm, 1990, p. 264), hermeneutics opens up a space for understanding interpretations within these different encounters.

Hermeneutics aims to comprehend how knowledge of such phenomena "can be applied to our broader understanding of what it means to be human" (Smits, 1997, p. 281). As Crusius (1991) noted, hermeneutics takes an ontological approach to *Dasein* – "human being in the world" (p. 9), acknowledging "there is nothing to know and hence no problem of knowledge without beings" (p. 4). Smith (1991) recognized that hermeneutics is "*the* foundational practice of Being itself" and "interpretation is the means by which the nature of Being and human be-ing is disclosed" (p. 192). In the case of this project, the interpretations of the graduate students' lives will help to answer the questions: What does it mean to be a human being in mathematics? What does it mean to become and to be a mathematician in the world? As "being a member of a disciplinary community involves a sense of identity and personal commitment, a 'way of being in the world,' a matter of taking on 'a cultural frame that defines a great part of one's life'" (Geertz, 1983, cited in Becher & Trowler, 2001, p. 47), hermeneutics allows an exploration of what it means to be in mathematics.

Hermeneutic inquiry is concerned with the question we are attempting to ask, the question that "resists easy answers or solutions" (Smits, 1997, p. 281), but that "enables understanding to occur" (p. 285). Through hermeneutic inquiry, there is an awareness of the emerging question, where "to question something is to interrogate something from the heart of our existence, from the center of our being" (van Manen, 1997, p. 43). The questions that I am attempting to unearth resound in this project, taking on different forms as I look into my experiences as a graduate student in mathematics. It comes through the exploration of my own mathematics teaching and my desire to improve mathematics pedagogy at colleges and universities, my own experiences and struggles as a teacher, my feelings for mathematics, and from the hope that programs for mathematics graduate students can be better informed. There is still, however, the sense that the question is still taking shape, and will continue to develop through hermeneutic inquiry and the exploration of the participants' lives.

Hermeneutics is different from other forms of inquiry as it does not consist of any particular prescriptive methodology (Gadamer, 1975; Jardine, 1992; van Manen, 1997), nor does it belong to a domain of metaphysics or philosophy (Smith, 1991). It is described as a multidimensional mode of inquiry, which attends to interpretation, understanding, creating meaning, the role of language in interpretation and understanding, and the search for the true question within and underlying the inquiry (Brown, 2001; Smith, 1991). Thus, hermeneutics allows an attention not only to the experiences and questions that I have presented so far, but also allows those things to show themselves that have not yet emerged in my reflections and in my questions.

Davis (2004) offered a description of hermeneutics as "a mode of inquiry that is oriented by two intertwining questions: What is it that we believe? How did we come to think that way?" (p. 206). Smith (1983) stated that hermeneutics asks the question, "How is it, how has it come about, that I use these words or act in these ways?" (p. 28). Thus, within these questions, hermeneutics opens up a dialogue with the history of the phenomenon under consideration. It recognizes the 'historicality' in a way that we can begin to "understand the present only in the horizon of the past and future" (Smith, 1983, p. 41). This temporality of experience is seemingly absent both in mathematics and mathematics teaching, as concepts are provided without reference to their development in history, to their connectedness with what came before or after. Hermeneutics offers a historical perspective for the interpretation of experiences in mathematics and in mathematics teaching, a perspective that is disregarded in current programs for graduate students in mathematics.

Lastly, as Smith (1983) described, hermeneutics holds "particular import for questions surrounding the nature of human understanding, the meaning of interpretation, and the role of interpretation in life-world understanding" (p. 7). Hermeneutics profoundly resonates with my experience, the ideas I have put forth around various experiences, and the interpretations thereof as having an influence on graduate students' teaching methods. I believe that hermeneutics provides an entry point into and a way to approach graduate students in mathematics, what mathematics and its teaching means to them, and how the experiences they have serve to signify a life in mathematics.

Beyond this description of hermeneutics and the connections made between hermeneutics and the study of mathematics graduate students' lives, a further exploration of four matters hermeneutics attends to is needed in order to gain a better understanding of what hermeneutics offers this project, some of which I have briefly addressed in this section, such as questioning and language.

The following subsections take a deeper look into what I believe is important to the hermeneutic work in this research study. In particular, I focus on language, conversation, the priority of the question, and the hermeneutic circle.

Importance of language in hermeneutics

> Mind and thing, human beings and being dwell together in language. (Crusius, 1991, p. 37)

As "hermeneutics examines human understanding in general" and "all understanding is linguistic" (Gallagher, 1992, p. 7), language is fundamental to the work of hermeneutics. Laverty (2003) described hermeneutics as an "interpretive process that seeks to bring understanding and disclosure of phenomena though language" (p. 9). Language is the "method of human communication" (Soanes & Stevenson, 2005, p. 983) and has a "central role to play in understanding the world" (Gallagher, 1992, p. 5). As such, hermeneutics requires the researcher to "develop a deep attentiveness to language itself, to notice how one uses it and how others use it" (Smith, 1991, p. 199). Language is not necessarily viewed as an object, an "objective entity" to be studied, but rather the researcher is called to reflect on and interpret the way language is used (Gallagher, 1992, p. 6).

Within the hermeneutic focus on language, often an investigation into the etymological origins of words can "put us in touch with an original form of life where the terms still had living ties to lived experiences" (van Manen, 1997, p. 58). Smith (1991) wrote that in attending to language, in noticing "how one uses it and how others use it," etymology can help to see "what they [words] point to" (p.

199). In this chapter and throughout the remainder of the dissertation, I focus on particular words, their definitions and, in some cases, their etymological origins. In looking at the definitions and origins of words, I hope to gain an appreciation of the meaning of the words in the context of the study and an understanding of how the original meanings can help to make sense of the graduate students' construal of their circumstances. For example, in Chapter 8, I explore and compare the differences in the meanings of the words *teacher* and *professor* and how the different understandings of these words have implications and meanings in the context of the mathematics graduate students' perceptions of their future roles.

Smith (1991) also described how language "contains the story of who we are as people" (p. 199). For this project, I want to understand the story of graduate students in mathematics. This understanding will come through language and the interpretation of language. Smith (1983) stated that to understand "means to see into what is being spoken" (p. 69), and so a question for me now is 'where will this language come from?' – the language that will help me to understand the lives of the participants. In the next sub-section, I describe conversations as a source for language in this hermeneutic inquiry.

Conversation

> *conversation*: 1340, from O.Fr. conversation, from L. *conversationem* (nom. *conversatio*) "act of living with," prp. of *conversari* "to live with, keep company with," lit. "turn about with," from L. *com*- intens. prefix + *vertare*, freq. of *vertere* (see *versus*). Originally "having dealings with others," also "manner of conducting oneself in the world" specific sense of "talk" is 1580. (www.etymonline.com, 2008)

Hermeneutics is not a prescriptive form of inquiry, fixed to a particular mode of data collection. A great quality of hermeneutics is its recognition of the impossibility of establishing "a correct method for inquiry independently of what it is one is inquiring into. This is because *what* is being investigated itself holds part of the answer concerning *how* it should be investigated" (Smith, 1991, p. 198; Smith, 2006, p. 110). With this in mind, several researchers applying hermeneutics in their studies have used conversation as a departure from positivistic, deterministic forms of inquiry, and as a way, through dialogue, to uncover participants' interpretations and creation of meaning (Carson, 1986). Hermeneutics itself was considered by Gadamer to be the art of "conversation" (Smith, 2006, p. 108), and he considered conversation as fundamental to the process of understanding (Smits, 1997). Within the use of conversation lies the hermeneutic principle that the only way we can uncover meaning is through language (Smits, 1997). van Manen (1997) stated that, "The conversation has a hermeneutic thrust: it is oriented to sense-making and interpreting the notion that drives or stimulates the conversation" (p. 98). Through the non-linear paths of conversation, meaning is allowed to emerge through the participants' "efforts to discover what it is" (Carson, 1986, p. 79).

Conversation, by avoiding the prescriptive character of an interview, creates the "possibility of developing a community of cooperative investigation," thereby changing the relationship between researcher and participant (Carson, 1986, p. 83). Moreover, conversation allows a "self-forgetfulness as one gives oneself over to the conversation itself, so that the truth realized in the conversation is never the possession of any one of the speakers or camps, but rather is something all concerned realize they share together" (Smith, 1991, p. 198). Conversation further acts as a way of coming to the question or questions underlying the inquiry. Carson (1986) explained that there is, in fact, no separation between the conversation itself and the uncovering of the question, where each seems to emerge through a process of attending to the other.

Conversation has the potential to create an open and natural interchange of meaning and interpretation, as well as a discovery of the question underlying the research and the phenomenon being investigated. However, conversations are not inherently simple and the researcher must attend to several issues. The conversations that are recorded cannot be treated as a set of fixed data to be classified and reported. Rather, recorded conversations continue to be in conversation with the researcher, and the conversations and interpretations are never finished (Carson, 1986; Smith, 1991). As well, conversations, in and of themselves, will not necessarily cause participants to look for the meaning in the dialogue. The researcher must consciously introduce a "critical distance […] in order that what the language reveals may be placed into the open" (Carson, 1986, p. 81). In using conversation as a mode of research, I must develop the skill needed to keep conversation open and maintain a commitment "to a communal venture of discovering" (Carson, 1986, p. 82), which stands in contrast to misperceptions about mathematics that most discoveries are made in isolation.

Mathematics has been interpreted to be a rigid discourse (Jardine, 1990/1998), and I believe that to attempt to create meaning and understand interpretation through prescriptive methods of research or inquiry would reinforce participants' views that certain answers, or forms of answers, are expected. An inquiry into the graduate students' practices through conversation serves to create meaning, understanding, dialogue and community, allowing participants to gain new insights into their own practices (Carson, 1986), rather than attempting to incorporate someone else's. What has been observed in previous research was that conversation helped to "forge a reformed practice" by "helping to create spaces within educational institutions for thoughtful reflection oriented towards improving practice" (Carson, 1986, p. 84). With this in mind, I believe that conversation as a form of inquiry is what is important for gaining a deeper understanding of life in mathematics.

The hermeneutic priority of the question

> *question*: c.1300, from Anglo-Fr. *questiun*, O.Fr. *question* "legal inquest," from L. *quæstionem* (nom. *quæstio*) "a seeking, inquiry," from root of *quærere* (pp. *quæsitus*) "ask, seek" (see query). The verb is first recorded 1470, from O.Fr. *questionner* (13c.). (www.etymonline.com, 2008)

> The close relationship between questioning and understanding is what gives the hermeneutic experience its true dimension. (Gadamer, 1975, p. 367)

Another of hermeneutics' attentions is to questions and questioning as we attempt to understand and find meaning within a phenomenon (Smits, 1997). Gadamer wrote that questioning "is an essential aspect of the interpretive process as it helps make new horizons and understandings possible" (Laverty, 2003, p. 10), where questions open "possibilities of meaning, and thus what is meaningful passes into one's own thinking on the subject" (Gadamer, 1975, p. 375). A questioning stance suggests a “genuine desire to know” and asking questions “means to bring something into the open, to achieve a true openness” (Smith, 1983, p. 78). Davis (2004) described the hermeneutic question as “an entry point for excavation, not an arrow for answer seeking,” and a way of finding “how we arrived at our current place” (p. 25).

With the use of questions there is a significance in “revealing the questionability of what is being questioned” (Gadamer, 1975, p. 357), that the questioning within and of the lives of the graduate students is a noteworthy recognition that their experiences should be opened up and explored. Asking questions of the participants, of their experiences, and of the meaning found in the investigation thereof opens the study to new meaning and understanding. For the conversations described above, I will use questions not only as entry points into the dialogue but also as a way to look for meaning in the participants’ experiences, to not take their answers as endpoints, but rather as part of the process in finding the questions underlying the research. This form of questioning will not follow the characteristic form of an interview, as interviews often have the appearance of a question being answered, rather than a question being informed and evolving as one tries to find meaning.

There are limits to questions and questioning, however. In order to remain receptive to possibilities in questioning, the researcher must remain “open in such a way that in this abiding concern of our questioning we find ourselves deeply interested” (van Manen, 1997, p. 43). Carson (1986) also wrote of continuing to be open to the question and to make sure questions are “not cut off too early by rapidly formed opinions and conclusions" (p. 78). Gadamer (1975) asserted that the openness to questions is "not boundless. It is limited by the horizon of the question […] Posing a question implies openness but also limitation" (p. 357). He also recognized the person using the "the art of questioning and seeking truth […] comes off worse in the argument in the eyes of those listening to it" (p. 460). There is the potential, then, for the researcher to appear unskilled or uninformed in different phases of the inquiry. Moreover, questions always include "both negative and positive judgments" (Gadamer, 1975, p. 358), and so there is also the potential for the questioning of participants to be perceived as critical or judgmental. Further, beyond the understanding of what is being investigated, Smith (1983) alluded to the potential for alteration of the researcher in the questioning as "A question presses itself on us such that we can no longer avoid and persist in our accustomed opinion" (p. 78). Thus, for the researcher there are transformative possibilities in questioning.

Gadamer (1975) stated, "A person who wants to understand must question what lies behind what is said. He must understand it as an answer to a question. If we go back *behind* what is said, then we inevitably ask questions *beyond* what is said" (p. 363). Further he wrote "The art of questioning is that of being able to go on asking the questions, i.e., the art of thinking" (p. 330). In the preceding and proceeding sections and chapters, I have taken a questioning stance in the attempt to understand fully what I want to learn through this investigation, in questioning the participants, in questioning my own thinking and perceptions, and questioning the participants' responses and the dialogue. This questioning, though, is done with the knowledge that the hermeneutic question will continue beyond this study.

I feel an interesting sense of anticipation for the questions that will evolve in this inquiry. While "There is no such thing as a method of learning to ask questions, of learning to see what is questionable" (Gadamer, 1975, p. 359), the questions will, in a sense, represent uncharted territory in understanding the lives of the mathematics graduate students. I have a feeling of a newness, of discovery, that there is the potential of finding an understanding we do not have yet of this phenomenon because the questions are not defined and prescribed as they have been in other investigations. In allowing the questions to evolve as I begin to learn more about the participants and their experience, I believe the possibilities for understanding will go deeper than previous studies.

The hermeneutic circle

> The movement of understanding is constantly from the whole to the part and back to the whole. (Gadamer, 1975, p. 291)

Within hermeneutic inquiry, the hermeneutic circle is understood as a movement between individual parts and the whole as one tries to understand a phenomenon (Davis, 2004; Gadamer, 1975). More specifically, the hermeneutic circle:

> refers to the idea that one's understanding of the text as a whole is established by reference to the individual parts and one's understanding of each individual part by reference to the whole. Neither the whole text nor any individual part can be understood without reference to one another, and hence, it is a circle. (Ramberg & Gjesdal, 2009, para. 31)

Smith (1991) described the hermeneutic circle as “the interplay of the part and whole in the process of interpretation” (p. 190). Gadamer (1975) wrote that the hermeneutic circle is not “formal,” “subjective,” or “objective,” but that it “describes understanding as the interplay of the movement of tradition and the movement of the interpreter” (p. 293). The idea of a circle does not imply that we continuously go around trying to find meaning and that understanding never takes shape, but rather that it does so within particular contexts such as tradition and culture.

Brown (2001) wrote of the meaning of the hermeneutic circle for the researcher. In particular, he made a connection between explanation and understanding as the researcher works to find meaning:

> Whilst one’s understanding may become “fixed” in an explanation for the time being such fixity is always contingent. In choosing to act as if my explanation is correct, the world may resist my actions in a slightly unexpected way, giving rise to new understanding, resulting in a revised explanation, providing a new context for acting and so on. This circularity between explanation and understanding, termed the “hermeneutic circle” (p. 36)

Davis (1996) also expressed one significance the hermeneutic circle has for the researcher. He described how the investigator is ‘placed’ in the hermeneutic circle, where they are “constantly reading the particular against the general, the past against the present, and projected against one another, and open to the transformative demands of dedicated inquiry. In this way, the hermeneut is inevitably incorporated – embodied – in his or her research and complicit in the phenomenon under investigation” (pp. 24 - 25).

In this investigation, I focus on the parts of the graduate students’ lives (e.g. their coursework and teaching assistant duties) in finding an understanding of the whole (what life is like in becoming a mathematician), using the idea of a circle, of a movement between the parts and the whole, to find a richer understanding of what their lives are like. This exploration can be seen in Chapters 6, 7, and 8 as I focus on parts of their experiences and relate them back the whole of what it means to become a mathematician and professor of mathematics. I will also relate back the whole of becoming a mathematician and professor to the elements of the participants' lives in understanding how the whole and the various parts of their lives have meaning when placed up against each other.

Through this investigation of mathematics and hermeneutics, there is what I see as an interesting similarity between the process of coming to know mathematics and the process of understanding observed in the hermeneutic circle. Gadamer described the hermeneutic circle as a "circular movement of understanding that runs backward and forward along the text" (Gallager, 1992, p. 62), yet is never completed. Davis (1994) described a "back-and-forth movement between the particular and the general" as the hermeneutic circle (p. 20). Mathematics itself is a discourse that also deals with a back-and-forth movement between particular and general, with general theorems and particular cases, and general ways of solving individual occurrences. While this connection is not enough to claim an inherent bond between mathematics and hermeneutics, I find it to be an interesting characteristic shared by mathematics and hermeneutics.

Hermeneutics and Mathematics

> It [mathematics] is considered a serious and exact science, a strict discipline, and such images of seriousness, exactness and strictness often inform how it is taught and how it is understood. […] It has become inhuman, lacking humus, lacking any sense of direct presence in or relevance to our lives as they are actually lived. (Jardine, 1998, p. 53)

While it may seem unexpected to couple a hermeneutic study of human beings and their lived experience with what is most currently categorized as a science (i.e., mathematics), hermeneutics has interesting implications for a renewed understanding of mathematics and its teaching. In the development of hermeneutics, "we can see that it has almost always defined itself as an affirmative reaction against dominant, theological, epistemological, and metaphysical presuppositions deemed to foreclose and limit the possibilities of human understanding" (Smith, 1983, p. 29). Thus, hermeneutics also represents a way to disturb the perspective that mathematics is a fixed body of knowledge, free of context and experience, disconnected from the human life, interest, and passion that give rise to it. Hermeneutics provides a lens that departs from approaches that focus solely the acquisition on knowledge, that have been preoccupied "with intellectual and technical mastery of things" (Crusius, 1991, p. 18).

Mathematics often represents a drive for certainty, for classification and definition. Proof is often construed as the core of mathematics and is described as "evidence to establish the fact of (something)" (www.etymonline.com, 2008). There appears to be little ambiguity in mathematics. Yet, as Imre Lakatos (1976) illustrated in his book *Proofs and Refutations*, the process of mathematics is in developing definitions and formulating proofs, where ambiguities and contradictions indeed exist and are dealt with by inclusion or exclusion through dialogue within the community in order to aspire certainty, to have clear boundaries of which concepts belong where. Jardine (1992) wrote that, "hermeneutics is thus concerned with the ambiguous nature of life itself. It does not desire to render such ambiguity objectively presentable (as if the ambiguity of life were something to dispel, some 'error in the system' that needed correction) but rather to attend to it, to give it a voice" (p. 119). In hermeneutics' acknowledgement of the ambiguity in life, then, there is the potential to reveal the ambiguity in mathematics and a life in mathematics that Lakatos referred to.

Further, hermeneutics allows communication with the dogmatic traditions (Smith, 1991) of technical-scientific discourses such as mathematics, a contact that uncovers the "deep denial of desire found in that discourse […] unearthing the desire for finality, the desire for control" (Jardine, 1992, p. 118). Hermeneutics, in its effort to understand interpretation and create meaning within disciplines, does not do away with or undermine these disciplines. Rather, hermeneutic inquiry has become almost necessary in understanding the life of and the life within technical, inflexible discourses, which have failed "to deal single-handedly with the lived problems of modernity that makes interpretation or re-interpretation of contemporary paradigms and their institutional embodiments necessary" (Smith, 1991, p. 188).

For mathematics, where the mathematician is often distanced from their innovations, hermeneutics denies "the primacy of the subject-object dichotomy," which "treats all experience as if it were broken" (Crusius, 1991, p. 15, 16).

Further, “hermeneutics offers the ‘reflexivity’ required to see the falsity of the subjective-objective split” (Smits, 1997, p. 288) and provides a way to disrupt the split that is perceived to exist between subject and object. Indeed, hermeneutics acts to transform discourses, “to shake loose dogmatic notions of tradition to show how all traditions open up onto a broader world which can be engaged from within the language of one’s own space” (Smith, 1991, p. 195). Moreover:

> Hermeneutics demands of such disciplines and traditions that they tell us what they know about keeping the world open and enticing and alive and inviting. And to the extent that such disciplines and traditions can no longer serve this deeply pedagogical purpose, to that extent they are no longer telling, no longer helpful in our living, no longer true. (Jardine, 1990, p. 3)

Hermeneutics allows an opening to a dialogue that not only enriches the world through a lived understanding of the discipline, but also enriches it by allowing it to rediscover how it might keep “the world open and enticing and alive and inviting.”

In Chapter 2, I addressed the risks to mathematics that might be caused by pedagogy. In order to remain important to the world, to learners, and to other disciplines, I believe mathematics must let go of the “illusion of mastery,” which Crusius (1991) claimed to be “self-deceptive” and “horribly destructive” (p. 25). Within mathematics’ claim to truth, Davis (1994) recognized that truth “is not a static form which, after discovery or creation, takes on an autonomous existence; truth is always contingent, existing not in a single authority, but amid dynamic interaction and engagement” (p. 22). Thus, I claim that there is a need for mathematics to reconnect with its origins in our lived experience and our understanding. To this end, hermeneutics helps to “remind us that the space of human understanding is *within* the lived world of practice and human relationships" (Smits, 1997, p. 293). As mathematics consists of expressions, language, and traditions developed by humans as a way to understand the world (Brown, 2001), hermeneutics allows us to recognize that we are not only limited by traditions, but also enabled by them (Smits, 1997), permitting a two-way interchange to occur to create a new understanding, a new way of living in the world with mathematics, helping us to discover different circumstances for a new, enlivened pedagogy.

Conclusion

> If education involves understanding and interpretation; if formal educational practice is guided by the use of texts and commentary, reading and writing; if linguistic understanding and communication are essential to educational institutions; if educational experience is a temporal process involving fixed expressions of life and the transmission or critique of traditions; if, in effect, education is a human enterprise, then hermeneutics, which claims all of these as its subject matter, holds out the promise of providing a deeper understanding of the educational process. (Gallagher, 1992, p. 24)

With these definitions and pointings made by hermeneutic scholars, my genuine interest is to know how mathematics graduate students find themselves in the space of becoming mathematicians and what in their lives has meaning for their roles as future instructors or professors. Past studies with mathematics graduate students (e.g., Belnap, 2005; DeFranco & McGivney-Burelle, 2001; Speer, 2001), I believe, have failed to take into account what is at work in the lives of mathematics graduate students, such as departmental expectations for their behaviour, what meaning becoming a professor has for them in how they take up their various duties. It is these things that I wish to explore in order to understand the place that university mathematics teaching will have for graduate students. In this regard, hermeneutics offers a unique perspective as it "wants to recover the original difficulties of life, difficulties that are concealed in the technical-scientific reconstructions, concealed in the attempt to render human life objectively presentable" (Jardine, 1992, p. 118). Similar to Gallagher's (1992) questions of "to what extent are traditions (and various authority or power structures) necessarily assimilated or reproduced in understanding, thereby lending themselves to forces of domination" (p. 19), one of my questions is 'what has influence over the graduate students' views as they come to understand their roles?' I believe that hermeneutic inquiry affords a perspective that will bring understanding in these contexts and questions.

Hermeneutic inquiry into mathematics graduate students' understandings, experiences, and ideas about teaching will allow, even compel, a look at what is present in the structures of departments of mathematics that might cause these future professors of mathematics to adopt teaching methods that do not come from their own understanding. In this regard, Davis (2004) wrote:

> The promise of hermeneutics is not to unburden ourselves of this historical mass in a (modernist) quest to determine the one Truth; nor is its goal, through more profound understandings of the world, to control the future or to better manage the objects that surround us. Rather, the place of hermeneutics is to interrupt our unquestioned patterns of acting. (p. 20)

Further, "by questioning the terms, the traditions and the texts that shape our understandings," hermeneutics offers the possibility that we "might begin to think differently about ourselves and our situations" (Davis, 2004, p. 31). Along with gaining a new understanding, there is also the hope that the participants and myself might begin to think in and conceptualize different ways about our roles as university teachers of mathematics. In light of Smith's (1991) remark, "The aim of interpretation, it could be said, is not just another interpretation but human freedom" (p. 189), hermeneutics might afford the opportunity for the participants and myself to express ourselves for who we are and who we hope to be, not for how we are expected to be in mathematics.

Chapter 4
Situatedness and Context Out of Which the Speech Comes

This is a study of the lives and experiences of six graduate students in mathematics, who are in the process of becoming mathematicians and, most likely, future post-secondary teachers of mathematics. In this chapter, in an effort to understand the experiences of the participants, I provide a description of the department of mathematics itself, as it represents the space in which in the participants not only learn mathematics, but also learn about their possible future roles as instructors and professors of mathematics. The structure of the study will also be described along with the interactions I had with the participants. In particular, I discuss the format, number, and length of conversations I had with the participants. I introduce each of the mathematics graduate students and give details of their backgrounds and experiences in mathematics. I also reflect on the quality of the conversations with the research participants and my role as both an insider and an outsider in these interactions. As I have included my own experiences as a graduate student of mathematics in Chapter 1, I also discuss them as an additional resource for the study.

Beyond the location of the study and structure of the conversations are other sources that appear important for graduate students in mathematics. One aspect that is included in this study, as described in Chapter 2, are the suggestions about mathematics teaching and a life in mathematics put forth publicly by mathematicians. I have brought these suggestions into my investigation of the graduate students' lives in mathematics, so they also represent a resource for this project. The images of mathematicians that are portrayed in the media represent a further source of information that have the potential to influence the research participants and I have included these in Chapter 7 of this thesis. They are discussed in the final section of this chapter.

The Location for Research – A Department of Mathematics

In carrying out this research project, I hoped to gain a better understanding of the lives of mathematics graduate students whose experiences I expected would include the characteristics of the life described in Chapter 2. In particular, I wanted to involve mathematics graduate students who had had or would have the opportunity to be teaching assistants, to work with undergraduates likely in first-year calculus classes, and who had a variety of duties, which would also include a focus on research. I sought to work with graduate students who represented not only future instructors of mathematics, but also future professors of mathematics and future mathematicians. Accordingly, locating a doctorate-granting university was essential for this study. I approached mathematics graduate students at a public, doctorate-granting university in Canada for this research project. The department of mathematics at the university had almost fifty graduate students and about thirty full-time faculty members. Similar to many mathematics departments, there were two research groups in the department – the applied mathematics group and the pure mathematics group.

The department of mathematics and its faculty members' offices were located down a long, narrow hallway away from classrooms and the main pathways students would take to travel across campus. On several visits to the department during the two semesters of this study, I found that this area of the department was quiet with few people present and most of the office doors were closed. The hallway walls were partly covered with posters advertising upcoming mathematics conferences. Most professors had posted their course offerings and office hours on their door. A few professors also had comic strips related to mathematics posted on their door. Although there was small room for gatherings, there was not a substantial, common meeting place in the department for faculty members or graduate students.

Similar to many graduate programs, the department of mathematics guaranteed graduate students funding during their degree programs. The funding consisted of teaching assistant duties, a research assistantship provided by the student's supervisor, external funding such as a National Science and Engineering Research Council grant, or a combination thereof. Office space was also offered to all graduate students. I found the offices for mathematics graduate students were distributed among three connected buildings on campus, including the building that housed the department of mathematics. Some of their offices were located near faculty members' offices, while others were two buildings away. A number of the graduate students' offices consisted of partitioned areas with as many as eight desks that were assigned to specific graduate students. Other office space in a newer building had a more communal feel, with open areas of desk space available to students on a daily rather than an allocated semester basis. The newest building, where some mathematics graduate students were located, had designated meeting rooms with large tables and white boards where graduate students could meet to discuss homework and projects. I was surprised to find out that graduate students who did not have office space in this building were not allowed to use the meeting rooms without having a faculty sponsor associated with the particular research group in the building. To me that seemed to disconnect the graduate students from each other and, in a way, make some seem more important or valued than others.

Meeting with the Participants

Initially, I approached the department chair for permission to approach the graduate students for this study. With approval from the chair, I placed letters describing the study in all of the mathematics graduate students' mailboxes in the department of mathematics (see Appendix 1). One week later, I had not received a response to the letter, and I sent a follow-up email to the graduate students, again describing the study and inviting their participation (see Appendix 2). I thought that an email message would be easier for the graduate students to respond to. As a result, three graduate students responded to these invitations and agreed to participate in my project. These first three participants were all male and two of the three were doctoral students. Hoping for a wider sampling of graduate students, about one month later, I sent another follow-up email message to the graduate students inviting their participation in the project. Three more graduate students replied to this email expressing their interest in the study and subsequently agreed to participate. Of these three new participants, two were women, both of whom were master's students.

I then planned a series of meetings with the participants over a six-month period. The meetings were coordinated around their class schedules, teaching assistant duties, and breaks between semesters. In an effort to prevent their participation in the project from becoming burdensome, all of the meetings with the participants were held on the main campus, with the exception of two meetings at another campus of the university with one of the participants. My unfamiliarity with the campus and particulars in the lives of the graduate students, such as the layout of the department and the privacy their office space afforded, caused some issues for the first individual meetings with two participants on the main campus. During those meetings, it became clear that their office space and the public areas around their offices did not allow for sufficient privacy and anonymity.

After some exploring of the campus and some fine-tuning, a location was found that was convenient, comfortable, and familiar for the individual meetings with the participants. This location was relaxing and secure and it offered a space that I believe was "conducive to collaborative hermeneutic conversations" (van Manen, 1997, p. 99). However, this location was not suitable for the group meetings, and so a conference room in a building separate from the department of mathematics, but still on the central campus, was located. While the environment was not as familiar, the distance from the department and the privacy it afforded helped to reassure the participants it was safe to share their opinions and experiences.

I felt that it was important to have scheduled the meetings with the participants over a six-month period, as it afforded a look into their lives during significant events and allowed for various perspectives to emerge around critical happenings in their programs, such as the end of the semester, final or candidacy exams. The first individual meetings with the participants took place one month after the beginning of the academic year. The final individual meetings took place as the academic year was coming to an end. The time frame of the project allowed me to see how their ideas and thoughts changed during the research project as they encountered different experiences and structures, as they learned more in and about their graduate programs, and as they reflected on their participation in this research project.

In total, five meetings took place with the research participants. Three of the meetings were solely between the individual participant and the researcher. Two of the meetings included all of the research participants and myself. I first met with each participant twice individually, which helped to establish some familiarity with his or her background and experiences in mathematics. It also allowed me to identify similarities and contrasts among their experiences to explore in the subsequent group meetings. After these two individual meetings, the group meetings were held consecutively, with the second group meeting occurring one month after the first. At the end of the research project, one final individual meeting took place with each participant. For the most part, the individual meetings lasted between forty-five minutes and an hour and a half. The first group meeting lasted almost three hours. The second group meeting lasted just under two hours.

To promote reflection between and before the meetings, the participants were offered notes, ideas, and summaries of our meetings. As van Manen (1997) stated, allowing participants to reflect upon transcripts and notes helps the researcher in their "aim for as much interpretive insight as possible" (p. 99). In having the participants review and comment on my thoughts "both the researcher and the interviewee weigh the appropriateness of each theme by asking 'Is this what the experience is really like?'" (van Manen, 1997, p. 99). The process of reviewing the meetings, reflecting on the topics of the conversations, and summarizing what appeared to be most important in their experiences allowed me to be more informed about the participants and to prepare for the subsequent meetings. These reflections allowed entry points into each conversation. It is unclear whether the participants had time to review and reflect on the information I sent to them, as only one of them took up any of the thoughts of their own accord in later conversations. For the most part, they simply agreed that the summaries I forwarded to them were accurate descriptions of our conversations.

Questions and Conversations with the Participants

As I discussed in Chapter 3, I chose questions and conversations as a mode of inquiry through which to gain an understanding of the phenomenon investigated in this project. Throughout the conversations, I posed questions that were “not so much designed to gather information for other purposes but rather posed as a way of opening up the topic under consideration, or else of clarifying it as it unfolds such that there is a genuine understanding between the partners as the conversation proceeds” (Smith, 1983, p. 80). In the first meeting with each of the participants, I employed questioning to obtain specific information about the participants. For the remainder of the meetings, I used questions to explore their experiences more deeply and how they described aspects of their lives.

I prompted each conversation with particular starting points. The first conversation began with questions about who they were, where they came from, and what drew them to mathematics, among other questions (see Appendix 3). I initiated the second conversation with each individual participant by reflecting on the notes from the previous meeting. The entry points for the second meeting were different for each participant, depending on where the conversation had gone in the first talk and the matters I felt needed some clarification on for my own understanding. I did not want to focus entirely on their views about teaching and learning mathematics. With this in mind, in order to have a broader understanding of their experiences and perspectives, I asked questions that were not solely related to teaching and learning mathematics. For example, the first group conversation began by asking the participants about their ideas of mathematics. Then I also asked questions directly related to their teaching and learning experiences in mathematics (see Appendix 4). The second group conversation was prompted by reflecting on thoughts and some observations of similarities among the participants from the first group meeting, and I asked questions to once again to clarify my understanding of their perspectives. Finally, the last individual conversation with each participant was a reflection on their participation in the project, with final questions used to shed further light on their ideas and feelings.

Each participant granted permission to have the conversations tape-recorded. I then transcribed the recordings as precisely as possible, attending to their pauses, their “um”s, their laughter, and other features of the conversations. Bird (2005) stated that transcription is “a key phase of data analysis with interpretative qualitative methodology” (p. 227). Braun and Clark (2006) put forward the view that transcription is an “interpretative act, where meanings are created, rather than simply a mechanical act of putting spoken sounds on paper” (pp. 87 - 88). Having both the recordings and the transcripts allowed me to continue to hear and read the conversations throughout the research project. The recordings allowed me to revisit the “background noises, unnoticed vocal inflections and tonalities of speech, etc., all of which can be brought to bear in a more genuine hearing of what is being said” (Smith, 1983, p. 86). The recordings also brought me back to the physical space of the meetings, to become aware of previously unnoticed sounds in the conversations. The transcripts afforded a different perspective on the conversations; a way of seeing, as it were, the conversations unfold. The transcripts provided a visual perspective on how words and topics of conversations were connected, intertwined, picked up by those in the conversation, or left behind. They helped me to notice in different ways the duration of each participants’ attention to topics, the stuttering, and pauses in relation to the others, including myself.

The Participants

The following sub-sections are introductions to each participant in the study, to describe their education and their experiences. While the readers of this thesis will not have the opportunity to hear the voices of the participants and to see the ways in which they spoke and carried themselves, I believe it is important to include something of these details. As each participant revealed part of himself or herself in how they presented themselves, in the manner in which they spoke, I have included some descriptions of the participants' behaviours to help the reader to hear and understand what the participants had to say. I hope that, through the inclusion of these details, the reader will more fully understand the "situatedness and context out of which the speech comes" (Smith, 1983, p. 88) in the exploration of the participants' experiences in the later chapters.

Laverty (2003) has recommended having participants "who are diverse enough from one another to enhance the possibilities of rich and unique stories of the particular experience" (p. 18). The full group was quite diverse in certain aspects of their backgrounds: three were from Canada, one from Asia, one from Europe, and one from the United States; four were studying topics in pure mathematics and two were studying topics in applied mathematics; finally, they ranged from a first-semester master's degree student to a fourth-year doctoral student. All of the participants remained in the study for the duration. As a result, this study satisfied Laverty's (2003) recommendation for hermeneutic research that one have "participants who have lived experience that is the focus of the study, who are willing to talk about their experience" (p. 18). Note that pseudonyms are used for each participant.

Emily

At the time of the study, Emily was a Canadian, 23-year-old, first-year master's student. Emily had earned her bachelor's degree in mathematics and had chosen this particular university for her graduate studies because of the work of her supervisor. In describing her decision to study mathematics, she stated that she decided to study mathematics by process of elimination. To her, mathematics was always quite easy, she was always able to do it, and she always liked it. In comparison, Emily said that she felt out of her element in other subjects and not as good as she was in mathematics. Her parents encouraged her to take mathematics beyond what was required to graduate from high school, and in her final year of high school she traveled to a different city to take calculus. In an interesting contrast to her abilities in mathematics, though, she also described herself as "not particularly talented at math." She once said, "I kind of like it, but I'm not great at like the amazingly insightful things."

In coming to this university for graduate studies, she was living far away from home and was struggling somewhat with feeling isolated. While her roommate was a friend from the same area and a fellow graduate from her undergraduate university, her roommate was not in the Department of Mathematics. So aside from their common hometown, they shared little in their current graduate school experiences. Further, while her family supported her decision to study mathematics, they could no longer understand the mathematics she was studying. In our first two individual meetings, Emily spoke of feeling lonely and missing the mathematics cohort and department from her undergraduate experience. During the second semester of the research study, however, she had established friendships with other mathematics graduate students and her mood had noticeably changed. She smiled more and she was more at ease and confident compared with her previous feelings of tension and uncertainty.

At the time of the research study, Emily was taking two graduate mathematics courses each semester. She described the mathematics she was learning in graduate school as more difficult than she could have imagined. Because of her struggle to learn and understand, which was a new experience for her, she questioned how successful she would be in mathematics. Without actually knowing how her classmates were progressing, she often compared herself to them, and she described most of them as being much more talented than herself. Of the courses she was required to enroll in, some were related to her research interest and others were not, causing her to be uncertain about their value for what her research focus would eventually become. She spoke of wanting to complete her coursework, so that she could focus on research, on creating something of her own.

Emily’s teaching assistantship duties consisted of spending hours in a workshop setting (see Chapter 6), helping students individually with their homework problems, invigilating exams, and marking homework and exams.

During her first semester, she was assigned twelve hours of work per week. She found that this amount of work was too much when coupled with her own coursework and so she asked for half as many hours during her second semester. The department approved her request and she felt less overwhelmed during her second semester. In talking about teaching mathematics, she said that she was interested in being a schoolteacher but also said that maybe it was a “crazy idea.” When I asked her about this, she said that she was not sure if she knew enough yet to be a teacher. She envisioned good teachers as knowing everything about mathematics. Still uncertain about her own knowledge, she questioned whether she would be able to teach. In further discussions about teaching, though, she spoke of wanting to help students understand mathematics.

Because Emily had little experience in graduate school, she was initially apprehensive about participating in the study, worrying that she would not have much to contribute. Once I assured her that her perspective would be informative for the research study, she became more at ease. While her posture gave the impression that she was shy, she looked directly at me when we spoke and sometimes smiled. Her manner of speaking was slightly tense and uneasy, but she laughed often when sharing stories of her experience, although sometimes it was a nervous laugh. During our individual conversations, Emily was engaged and shared a lot about herself. In contrast, though, during the group conversations, she most often spoke only when she was asked questions directly. It was rare that she would freely contribute to the group conversation. Throughout our conversations, it appeared that Emily was trying to find her place in terms of how she fit in the department, what her future research might be, whether she might become a schoolteacher, and if a life in mathematics was really for her.

Chris

At the time of the study, Chris was a Canadian, 28-year-old, first-year PhD student in pure mathematics. He had earned a bachelor's degree and two master's degrees in mathematics before arriving at this university for his doctoral studies. He had recently completed his coursework and was beginning his doctoral research, but he had not yet completed his comprehensive exams or his candidacy exam. Similar to Emily, he had chosen to study at this particular university because of the research of his supervisor. However, after his first semester in the program, he was beginning to lose interest in his supervisor's line of work. Despite winning a research grant for research in his supervisor's area, by the time of our last meeting Chris had changed his supervisor and his research focus.

In talking about his past experiences, Chris described a negative encounter with mathematics as an elementary school student. After a bad experience in grade three, he said that he lost interest in mathematics for many years, doing only enough work to earn average or passing marks in his mathematics courses. He described an experience with a proof later in high school that was pivotal in changing his interest in and opinion about mathematics. He said that seeing a proof for the first time, in particular that the square root of the number two was irrational, helped him to finally "see what mathematics was all about."

Although he described this experience with a proof as the impetus for a new interest and enthusiasm in mathematics, he claimed that he chose mathematics because he was not smart enough for anything else. In particular, he shared his experience of mistakenly registering for the wrong mathematics course as a first-year undergraduate, signing up for an honours mathematics course rather than the general mathematics course for first-year, non-mathematics majors. He described this course as concerning proof rather than calculation, with a focus on understanding concepts. This course seemed to be the beginning of his good experiences with mathematics as an undergraduate and he said, “I loved the math courses. I never wanted to miss them. I always wanted to go.” He had expected to study physics and he did not intend to earn a degree in mathematics, but the experience of learning proofs in the honours mathematics course drew him to the subject and as a result he earned a degree in mathematics.

Chris described how his new experiences with research in graduate school were a struggle, in not knowing what a good problem would be and what direction he should take. What was most difficult for him in mathematical research was not knowing what to do next on a problem and feeling that he was wasting time. On the other hand, he mentioned that he could not imagine being happier doing anything else. He related the sense of fun he experienced when playing with mathematical ideas and trying different arguments to see where they would lead. His enthusiasm for mathematics became evident when during one of our meetings he drew on a piece of paper what the mathematics he was most interested in looked like. In his description of the concepts, he was instantly engaged and even excited to be sharing his topic with me, something he did not often get the chance to do with others outside the department of mathematics.

At the time of the study Chris's teaching assistantship duties were to spend some hours in the workshops, helping students individually with their homework assignments, teaching a one-hour tutorial session (see Chapter 6), and marking assignments and exams for a class. After earning two graduate degrees in mathematics, he was accustomed to teaching assistant duties, but also mentioned that he did not always agree with how issues such as marking and teaching were handled by the department and various professors. With regard to teaching, he spoke of how he prepared to lecture during the one-hour tutorials by preparing notes for himself. He described how he was happiest when explaining something to a student in a way that would help the student understand. He liked to share mathematics with students and wanted them to have the feeling of understanding that he experienced when working on mathematics.

During the first semester of the study, Chris's teaching assistant duties were located at a separate campus of the university. He was unaware as to how the decision had been made and expressed frustration at his placement. Chris said that he felt distant from the other mathematics graduate students and happenings of the department. Describing this campus as more of a commuter-student campus, he felt there was not a strong community among the students. With only a few mathematics faculty members and a handful of mathematics graduate students located on that campus, he felt isolated. During the second semester of the study, with the change in supervisor, his office and teaching assistant duties were relocated to the central campus of the university.

This circumstance had an impact on how he felt about his studies, as well as having an impact on how he communicated during the research project. In particular, in the first two meetings between Chris and myself, he was often withdrawn and quiet, seeming to lack confidence. He spoke of how he was unsure about how he conversed, whether the words he said came out properly and in the right order. However, I found that his speech was natural and flowed well without the missteps he was worried about. When he moved campuses and felt that he was more a part of the department, he became more confident and easy going, laughing more and did not appear to be as analytical and critical of himself. In both group meetings, he was often quiet and did not engage in the joking or banter that occurred between other participants. When he spoke, it was with purpose, to put forth something he felt quite strongly about and to introduce ideas that had not yet been discussed. While other participants sometimes laughed about the things they or others said in the group meetings, Chris's manner was sincere and composed.

John

At the time of the study, John was a 28-year-old, second-year PhD student in pure mathematics from the United States and he had earned a bachelor's degree and a master's degree in mathematics at universities there. For one year after his master's degree, he had worked as a Fulbright Scholar in an academy of science in Europe. He had already completed his coursework in the Department of Mathematics and, during the period of the research study, he successfully passed his candidacy exam. His supervisor was able to supply John with research funds and, as a result, John no longer had teaching assistantship duties in the department.

Unlike the other participants in the study, John was fairly accomplished in mathematics, having presented his research at a few conferences and he had published articles in mathematics journals. His master's thesis was also due to be published in a three-part article in a mathematics journal. As a doctoral student, John continued to do well-received work in mathematics. His supervisor was well known in a particular area of mathematics and was mentoring and promoting John in their common field of study. Interestingly, though, in contrast to his successes, when describing his decision to study mathematics, John stated, "I think, in the end, it's the only thing I'm really good at."

John began his undergraduate studies with a focus on chemistry, hoping to go into medicine. During this time, though, he took mathematics courses because he regularly earned high marks in those classes, which increased his grade-point average. At some point in his undergraduate experience, he took a course in number theory. It was during that course that he decided to study mathematics because he found he was able to understand theorems quickly, saying "I could just see it." He spoke of how he was drawn to mathematics because of the "system" aspect, that mathematics was a system that certain things would fit in certain ways. He observed that this systematic part of mathematics was not present in everyday life and he found mathematics to be an escape from an unsystematic world.

Unlike the other participants, John had already had opportunities to teach his own courses. In particular, in his master's program in the United States, graduate students were required to teach their own courses, such as first-year calculus or linear algebra. Also, at one point during his graduate experience in Canada, he had applied to the department to be a sessional lecturer in order to teach a number theory course. While he still had opportunities to teach in the department, during the time of the study he was mainly focused on his research project. In talking about preparing for the jobs that he would soon apply for, he felt that he had enough experience with teaching and would only teach again if it were what he called a "good course," something in pure mathematics such as analysis or possibly a course in his research area.

John spoke of the highs and lows in mathematics and said that most of the time the experience was very low. He said that he was happiest when working on his research, where the rewards of solving a problem made up for the lows one experienced when doing research. In contrast, he described teaching as easy, bringing with it little reward. He revealed that there were things in life that he valued more than mathematics, such as his marriage and a quality life. Despite his success and what seemed to be a promising career in mathematics, he said that he would give up mathematics if these valued parts of his life began to suffer. He spoke of wanting to ensure a semblance of a normal life and that sacrificing such a life to work in mathematics was "just not worth it."

With our common experiences of graduate study in mathematics in the United States, John and I were able to communicate fairly easily from the beginning of our meetings. John was confident about his work in mathematics. He was fairly secure about his future, talking about how his work had been well received. On a few occasions, however, he talked about how his area of pure mathematics was not appealing to the entire mathematics community. In general, he was somewhat serious about things, rarely laughing. During the group conversations, though, he became more spirited, playing off of the participants he knew and engaging in some friendly verbal sparring.

Sara

At the time of the study, Sara was a 24-year-old, first-year master's student from eastern Europe studying applied mathematics. She had earned a bachelor's degree in mathematics at this university and had chosen to stay for a master's degree. Because of the multilingual education in her home country and having spent two years in the United Kingdom in an international baccalaureate program, her English-speaking skills were excellent. In talking about her past experiences in mathematics, she spoke of how she was always good at mathematics. In grade five, a teacher noticed Sara's talent with mathematics and motivated Sara to start working on mathematics problems outside of school and to participate in mathematics competitions. For high school, she attended a school for gifted students and she continued to excel in mathematics. Unlike Emily, Chris, and John, Sara's choice to study mathematics at the university appeared to be a natural progression of her education.

At the time of the study, however, she was struggling with the choice as to whether she should continue to study mathematics. She was unsure if she would pursue a doctorate in mathematics, finding mathematical work to be restrictive for what one could do in the world. She spoke of wanting to be able to help as many people as possible and that doing specialized research in mathematics might not allow her to do so. A few times she wondered aloud about what level of education was necessary to help the world. At the end of the study, she was still unsure about pursuing a doctorate, but was beginning to feel that mathematics was something that she enjoyed and that she was becoming better at speaking the language of mathematics at this level.

During the time of the research study, she was taking two courses each semester as well as beginning her master's research and attending to her teaching assistant duties. It was clear that applied mathematics was one of her passions, but she was struggling to find a research topic that was inspiring to her. As part of her teaching assistant duties, Sara spent six hours each week in a workshop, helping students individually with their homework assignments. She would also mark papers and exams as part of her duties. Unlike the other research participants, she spent little time talking about her experiences in the workshops or marking papers.

Sara was energetic during all of our conversations. She passionately spoke her opinions and offered her beliefs. She was thoughtful and direct during our conversations, pausing before answering questions, taking time to think about how she felt. She smiled easily and often, looked directly at me and played with her hands when she was talking. During the individual conversations, she spoke a lot about wanting and needing to do things outside of her life as a graduate student in mathematics. She played guitar, spent time with friends, and was hoping to continue her learning outside of mathematics. However, she described how her coursework and other duties were requiring more of her time and energy, slowly causing her to give up her other interests.

Sara also spoke a great deal about her frustrations with the teaching in the department and about one unhappy experience in particular. She also shared her thoughts about her fellow graduate students and what she perceived to be their reluctance to confront various issues in the department, such as the teaching she observed in graduate-level courses. During the group conversations, she was not shy to contribute and solidly put forth her ideas about the topics of the conversation, even when her thoughts differed from others in the group. In doing this, it seemed as though she wanted to prompt new thoughts for the other participants. In contrast, though, she did not share her bad experience with the group, as she was apprehensive about others labeling her as a troublemaker.

Robert

At the time of the study, Robert was a 31-year-old, fourth-year PhD student in applied mathematics. He was born and raised in Hong Kong, and he and his family had moved to Canada when he was in high school. He had learned to speak English at an early age, which helped his transition into Canadian schools. He had earned a bachelor's degree and a master's degree in physics at a Canadian university. His doctoral studies were focused on an area in applied mathematics, and he described his shift to mathematics as "coming back to where I began." In describing his past experiences with mathematics, Robert said that he had always liked mathematics from a very young age, as early as he could remember. Because he was good at math, he always had an interest in it. However, when he chose his focus for his undergraduate studies, he found that he was more drawn to physics.

During his master's program in physics, he had teaching assistant duties and, as a result, he was somewhat familiar with the requirements of that type of work when he began his doctorate. Throughout his doctoral program in mathematics, he had teaching assistantship duties, which included some hours in a workshop helping students with assignments, leading one-hour tutorial sessions, invigilating exams, as well as marking exams and homework for various courses. At one time during his program, he had been assigned to as many as three one-hour tutorial sessions along with marking papers. He spoke of how tiring the experience was, leaving him little energy to focus on his research, which was the work he would be judged by in graduate school. During the time of the study, he applied for a doctoral research grant, which provided him with a different source of funding. This allowed him to forego his teaching assistantship in order to focus on his dissertation during the final semester of his doctorate.

In our individual conversations, Robert frequently gave his views about the quality of elementary and high school education in North America. Because of his experience in schools in both Hong Kong and Canada, he concluded that the quality of education in Hong Kong was much better than in Canada. He had strong opinions about the status of education in high schools, and had decided that university-level mathematics was suffering because students were ill prepared in high school.

During the individual conversations, Robert candidly shared stories about himself and his experiences. He would pause for reflection before answering a question or putting forth his own ideas to the topic of conversation. Often, when thinking about his next comments, he would look to the ceiling when mulling over what to say, suggesting a thoughtful disposition. He offered his opinions freely and recognized them as coming from a particular experience. During the group meetings, Robert was often quiet, mostly speaking only when he was directly asked a question. When he did contribute to the conversation, he was articulate, self-assured, and introspective, and his statements frequently caused the group to pause and reflect.

One thing of note about Robert was that throughout the study, during individual meetings and in the group meetings, he was reluctant to identify himself as a mathematician. He viewed the work that he was doing as a lot of physics and not necessarily mathematics. Moreover, he claimed that he was not doing math, but rather was someone who "used mathematics as a tool to do some science." He also alluded to pure mathematics as being real mathematics, whereas applied mathematics did not have the rigour necessary to be considered true mathematics. Being one of only two applied mathematicians among my research participants, he did not have the connection with the pure mathematicians in the group. Because of his lower opinion of his own work, he appeared to hold back his opinions on some topics, possibly feeling that he did not have the knowledge necessary to contribute to certain parts of the conversations.

Steven

At the time of the study, Steven was a Canadian 23-year-old, second-year master's student in pure mathematics. He had earned a bachelor's degree in mathematics and had chosen to study at this university because of the work of his supervisor. His research interests were in an up-and-coming area of mathematics and he was excited about his work, but unsure as to whether the field of mathematicians would recognize it. He was uncertain if he would continue to pursue a doctorate. If he did choose to continue his studies, he mentioned that he would return to central Canada in order to complete them there.

Steven described how he had never had positive experiences with mathematics until he was in university. He shared stories of debating with his teachers in junior high school about mathematical concepts and being sent to the principal's office for speaking up about his ideas in mathematics. When I asked him about why he chose to go into mathematics, Steven responded by saying that he went into mathematics because he was not good at science and that not being good at science pigeonholed him into mathematics. He said that it was not a conscious decision to go into mathematics, but that there was “literally nothing else” he could do.

He spoke of his positive experiences in mathematics as an undergraduate student, relating stories of his professors. He credited the institution he attended for the passion he found for mathematics. In contrast, though, he described how he went to that university by a curious accident. In particular, when listing his choices for universities for undergraduate study, he had listed his first few choices and could not think of a final one to include. To find one more, he randomly pointed to a list of universities, finding his finger aimed at that particular one. In the end, that final choice was where he earned his bachelor’s degree. Consequently, he saw his choice to study mathematics as an “accident” that came about through the random selection of a university, which happened to have a mathematics department that inspired his interest.

At the time of the study, being a second-year master’s student, Steven had thought that he no longer had course requirements, but his supervisor advised him to take another course. He had already begun the research for his thesis, but he did not talk about how much he had yet accomplished. Along with one final course and his research, Steven also had teaching assistantship duties, which included some hours in a workshop setting, helping students individually, invigilating and marking exams, and marking homework assignments. He often shared his struggles with his teaching assistant duties, particularly his construal of undergraduates’ behaviour.

Steven's manner of speaking was very direct and intense. He would often talk about his frustrations with the undergraduates and with departmental policies. And while he had only his own experiences upon which to rely, he often complained about the state of education in general. At the time of the study, Steven was frustrated with mathematics, with the department of mathematics in particular, and with what he perceived as the poor quality of the undergraduate students. Steven had very strong opinions about the quality of undergraduate students, blaming their high school experiences for their inability to do and understand university mathematics. The sharing of his views was not limited to our individual conversations, as he often inserted these ideas into the group dialogue, looking for affirmation from his peers. During the group conversations, he was the most vocal participant in sharing his opinions, often putting forth somewhat controversial ideas to spur the other participants either into agreement with him or into a debate.

One thing of note about Steven was that he often followed up his statements with a question such as "You know?" or "Right?", as though he was looking for corroboration or approval of his opinions. While coming across as brash by stating contentious ideas, the questions with which he would end his statements seemed to reveal some anxiety, need of approval, or a lack of confidence. Another interesting observation about Steven was that when talking about the master's degree he was working toward, he described it as a "fake degree." When I asked him about this, he stated it was because his thesis would not contain any proofs, which he believed were the sign of a truly mathematical thesis.

My experience

As I have introduced this study through my own encounters as a mathematics graduate student, I will continue to bring in and reflect upon those experiences. Although I was not explicitly a participant of this project, there were points in this research study where my past experience resonated or contrasted with that of the participants. At times, this was observed in the transcripts of the conversations, in the ways I contributed to the conversation, and at other times in the ways I reflected on the connections or dissimilarities of my experience compared to the participants' in the follow-up analysis of the transcripts. My experiences, interpretations of, and feelings about being a mathematics graduate student are included as part of the experiences that inform the study.

Finding Common Ground – My Roles as an Insider and an Outsider

My background in mathematics afforded me positions as an insider in the research project. Having twice experienced the role of being a mathematics graduate student, I was able to relate to the experiences of the research participants, particularly in regard to the workload of their courses, their teaching assistant duties, and their various interests in mathematics. During my master's program in mathematics, I decided to take courses in both pure and applied mathematics, in order to provide me with a broad background in mathematics.

Thus, despite their different areas of study in mathematics, I was able to understand somewhat each of their fields of study and research projects. My familiarity with mathematics allowed me to understand and discuss their projects with them, which I believe helped the participants to feel comfortable talking to me. Because I had knowledge of and could even understand the mathematics they were studying, this helped a great deal in establishing genuine, and fairly deep conversations. My background in mathematics provided me a sense of legitimacy among the participants and I think they were excited to know that I (that someone outside of their cohort in mathematics) understood the descriptions of the mathematics they were studying and was interested in what they were doing. I believe that my background in and familiarity with graduate work in mathematics helped to create "an environment of safety and trust, that needs to be established at the outset and maintained through the project" (Laverty, 2003, p. 19).

With this role as an insider and the relevance of my own experiences to the dialogue, I had to be careful that my own experiences did not overwhelm the conversations. For myself, there was excitement and even relief in being understood by others in the interesting experience of being a mathematics graduate student and all that that particular life entails. At times, I was drawn into sharing my own stories to the point of needing to remind myself that "the conversation is a place of listening" (Davis, 1994, p. 27).

In contrast to feeling like an insider with the research participants, as the researcher, I was also an outsider to their environment. For the most part, I could understand and empathize with the participants, but I also brought with me into the research ideas, knowledge, and experiences that made me an outsider and these were sometimes in tension with the participants' views. In particular, my understanding of research in mathematics education and the knowledge of the influence that university mathematics teaching has at all levels of education came into tension, for example, with the participants' ideas that the problems with undergraduates' learning rested solely in their pre-tertiary education. In these moments, where their opinions conflicted with what I knew of the research, I had to hold back my frustration and be careful to not contradict their views. My purpose was not to change their opinions, but rather to understand what about their experiences had an influence on their teaching and how they came to feel the way they did. To that end, I wanted the participants to feel comfortable expressing their feelings fully. In order to create that environment, instead of challenging them directly, I continued to ask them to clarify their views. At some points, though, because some participants' views seemed extreme, this was quite difficult for me to do.

My own struggles to learn mathematics in high school and university also seemed to conflict with some of the participants' views. For example, my troubles in learning mathematics allowed me to relate to undergraduates who needed extra help with mathematics and to understand that not all students learned mathematics in the same way and in the same time frame as others. Further, from my experience working thirty hours per week as an undergraduate, I knew that some students were in the same position and often were, by necessity, finishing their homework at the last minute. I understood that this last-minute rush could be due to their schedules and not their degree of motivation. Some of the participants in the study, however, interpreted the behaviour of the undergraduates as symbolic of a lack of motivation and interest. There seemed to be very little effort on the part of the participants to understand what the undergraduates might be going through. This was frustrating to me because, regardless of circumstance, I felt that the undergraduates should not have been judged, nor should the help they received been conditioned on when they completed their assignments.

My views that university mathematics teaching could take on forms other than lecturing also came into conflict with some of the participants' views of the professor's role. Once I started to learn about mathematics education, students' understanding of mathematics, and my role in students' learning as a post-secondary mathematics teacher, I found that I could make changes in my teaching practices that were beneficial to my students and my own understanding of mathematics. In contrast, though, the participants held on to their view that they would be professors, not teachers, and so lecturing represented what they would do in the classroom. In these moments, I found it difficult to hold back, and not say to them "But it could be different. This is how it could be different." While I wanted them to experience changes in their opinions and in their classroom practices, I knew that it was something that they would most likely have to realize on their own. It was not something I could teach them or help them come to understand in the little time we had together, nor was it the purpose of the study to show them alternate ways of teaching mathematics.

Further Reflection on the Conversations

Entering into each conversation, I had some expectation of where it might go based on the entry points. However, I frequently found myself surprised by what became the topic of conversation, or how the conversation would bend and move in different ways, sometimes away from where I thought or hoped it might go, to a new, unexpected place of understanding and unearthing different perspectives. As Carson (1986) described it, "Unlike writing, conversation does not have a linear logic. Conversation has an appearance of 'discursus' – of a running from place to place" (p. 79). The conversations did run to unforeseen places, allowing me to hear new perspectives and opinions about the experience of graduate study in mathematics. Because the directions of the conversations were unexpected, it allowed the dialogue to be a fresh and original exploration not only for the research participants, but also for myself.

The conversation and questioning required a different focus in my role as the researcher in the dialogue. Carson (1986) stated:

> Hermeneutical reflection requires that a critical distance be taken in order that what the language reveals may be placed into the open. This can be accomplished by imposing a formal dialectic of question and answer. […] However, the dialectic is not a part of the natural structure of conversation and it must, therefore, be consciously introduced by the researcher as a second layer providing the needed critical moment. (p. 81)

I had to continually attend to my responsibility to the dialogue, to "find what to say and what to ask in the midst of the dialogue itself" (Crusius, 1991, p. 38). This attention was difficult to maintain as an insider to the research project because at times I found myself very much involved in the relating of experiences. In those moments, I had to remind myself of my role as the researcher, how I needed to pay attention to the dialogue, and know “what to ask in the midst of the dialogue itself.”

The use of group dialogue contributed greatly to the research project, where the conversations among members of the same department contributed in the effort to find an understanding of what life was like for them. Using only individual conversations would not have provided the same insights as the group meetings, as “dialogue proposes to work with and through personalities and opinions” (Crusius, 1991, p. 37). The research participants had varied personalities and opinions and using the group meetings allowed some of those differences to fade away as the group worked to find common understanding, to arrive “at shared understandings” (Davis, 1994, p. 27).

In listening to the conversations, there were moments where we related to each other and understood each other in such a way that we finished each others' sentences and completed each others' thoughts, representing a “fusion of horizons,” where “we come to understand the viewpoints of the other participants, and as all viewpoints are modified and enlarged by each other” (Crusius, 1991, p. 95). Often, in the conversations there were “mm-hmm”s and “yes”s heard from other participants while one person spoke, which Gadamer viewed as symbolic of the participants staying together (Smith, 1983). There was no talk “at cross purposes or with the intention of scoring points” in either the individual or group conversations (Smith, 1983, p. 79). Rather, when participants disagreed with one another, they were not confrontational, nor did they make moves to disprove one another. Rather, they patiently described their experience and how they did not see the particular issue in the same way.

Further, the group conversations allowed me to bring “contradictions to the attention of the participants” (Carson, 1986, p. 81) to the group as a whole, which allowed a more in-depth dialogue and exploration of the contradictions that had arisen for particular individuals and within the group. For example, eventually the more extreme views about undergraduates faded away as the group continued to mull over their opinions in the conversation, and to explore whether the more extreme views were valid. On the whole, the group meetings allowed me to continue to find the place of their experiences and understanding of experience, not as an isolated opinion, but as a shared meaning, of commonality, of what their lives as mathematics graduate students were like. The number of conversations appeared to be a good amount, meeting “a point of saturation” (Laverty, 2003, p. 18) where the participants continued to be pleased to contribute to the project. Yet, at the final individual meetings, a sense of relief came through, that their role was completed and they would now have fewer meetings in general.

Other Resources for Understanding a Life in Mathematics

In addition to the investigation of the lives of the participants who volunteered for this project, I have also relied upon other sources of information for this study. In Chapter 2, I wrote about the suggestions proposed by mathematicians regarding mathematics teaching at universities. At times in our conversations, the participants referred to famous mathematicians such as Terence

Tao or Ian Stewart, and physicist Richard Feynman, among others. They looked to these scientists as exemplars of experts in the field, to be attended to for indications of how they should be in mathematics. In my exploration of the conversations, I have included further evidence of the influence mathematicians might have by looking at some mathematicians' publications and what they have to say for living a life in mathematics. In the excerpts of the conversations discussed in Chapters 6, 7, and 8, I will return to these professional mathematicians and the potential meaning they have for the participants' experiences and for the understanding of university teaching of mathematics.

Some of the participants also looked to images of mathematicians that are depicted in the media and how those portrayals sometimes had an impact on how they themselves were viewed. These images represent a public face of mathematics, what others see when mathematicians are presented in the media. As another source of information for the study, I have looked to some of these descriptions and images of real-life mathematicians. Because there is limited information or research on mathematicians' identities, I have also looked to motion pictures as a resource of how mathematicians are construed and how these depictions might have an impact on the research participants. Such images are explored in Chapters 7 and 8.

Chapter 5

A Means to Get at the Notion We Are Addressing

> *explore:* c.1450 (implied in explorator), "to investigate, examine," from L. *explorare* "investigate, search out," said to be originally a hunter's term meaning "set up a loud cry," from *ex*- "out" + *plorare* "to cry." But second element also explained as "to make to flow," from *pluere* "to flow." Meaning "to go to a country or place in quest of discoveries" is first attested c.1616. (www.etymonline.com, 2008)
>
> *explore:* travel through (an unfamiliar area) in order to learn about it (Soanes & Stevenson, 2005, p. 610)

When reading other dissertations and research papers, I noticed that the language commonly used at this point in various projects was *data analysis*. I had originally named this chapter using the same terminology, but something about these words troubled me. I researched the meaning of the words *data* and *analysis* and what I found did not resonate with the approach I wanted to take in this research project. The word *analysis* had some definitions that were congruent with how I wanted to think about the conversations. However, other meanings of *analysis*, such as "the separating of any material or abstract entity into its constituent elements" (www.dictionary.com, 2009), gave me the sense that this word had connotations that were too rigid and pre-determined. In thinking of other language that would more fully describe my work in this project, the word *explore*, as seen above, and the sense that it was a "traveling through in order to learn about" seemed to provide an openness for the discovery of unknown and unexpected things that might come out of the conversations. Using the word *explore* gave me the sense that I would not have to follow a fixed direction when reading the conversations, but rather I could move in many directions and follow different paths "in quest of discoveries" when attending to the graduate students' spoken experiences.

The word *data* was often affiliated with statistics and numerical data, and did not seem suitable as a description for the conversations I had with the participants. Having worked with data and the analysis of data in mathematics and statistics, I felt that using the word *data* in the description of the different approach I would take in this project would not be appropriate. As opposed to referring to the conversations as data, I chose to refer to them as what they are – conversations. The word *data* seemed fixed, as though the transcripts of the conversations represented what the participants said, implying a sense of past tense that they no longer had anything to say. In comparison, though, the word *conversation* allowed me to continue to be *in conversation* with them ("I am in conversation" rather than "I have data or I collected data"), that the transcripts represented what the participants are saying, where a continued sense of present tense helped me to know that the recordings and transcripts of the conversations still had something to say to me.

With this language in mind and the idea of wanting to remain open to what the conversations had to reveal, in this chapter I describe thematic analysis, which is the approach I used to explore the conversations. While the word *analysis* is used in the name, I describe how the process of thematic analysis allows the researcher to be receptive to the discovery of new ideas and themes. Along with the description of this approach, I discuss the nature of themes and explain the choices I made as a researcher making use of this approach. I also describe how I believe thematic analysis resonates with the work of hermeneutics. Finally, I address the themes that I developed through reading and hearing the conversations with the participants.

Finding and Understanding Hermeneutics

> A person trying to understand something will not resign himself from the start to relying on his own accidental fore-meanings, ignoring as consistently as possible the actual meaning of the text until the latter becomes so persistently audible that it breaks through what the interpreter imagines it to be. Rather, a person trying to understand a text is prepared for it to tell him something. That is why a hermeneutically trained consciousness must be, from the start, sensitive to the text's alterity. But this kind of sensitivity involves neither 'neutrality' with respect to content nor the extinction of one's self, but the foregrounding and appropriation of one's own fore-meanings and prejudices. The important thing is to be aware of one's own bias, so that the text can present itself against all its otherness and thus assert its own truth against one's own fore-meanings. (Gadamer, 1975, pp. 271 - 272)

Before moving forward into the exploration of the conversations in the following chapters, I am compelled to convey my struggles to connect hermeneutically with the transcripts. While in the previous chapter I reflected on the quality of the conversations with the research participants, this manner of engagement was not easily transferred when reviewing the transcripts of the conversations. As seen in the quotation from Gadamer above, he understood the potential for someone to be distracted by his or her previous experiences. The distractions proved difficult for me, and I feel it would be disingenuous to not describe this part of the research process. As van Manen (1997) recommended:

> It is better to make explicit our understandings, beliefs, biases, assumptions, presuppositions, and theories. We try to come to terms with our assumptions, not in order to forget them again, but rather to hold them deliberately at bay and even to turn this knowledge against itself, as it were, thereby exposing its shallow or concealing character. (p. 47)

Further, Laverty (2003) wrote that for hermeneutic research:

> The biases and assumptions of the researcher are not bracketed or set aside, but rather are embedded and essential to interpretive process. The researcher is called, on an ongoing basis, to give considerable thought to their own experience and to explicitly claim the ways in which their position or experience relates to the issues being researched. (p. 17)

Smith (1991) wrote that “any study carried on in the name of hermeneutics should provide a report of the researcher’s transformations undergone in the process of the inquiry” (p. 198). Here, then, I describe how my experiences as a graduate student in mathematics became obstacles for me when I attempted to engage with and interpret the transcripts of the conversations. In doing this, not only do I “come to terms with my assumptions,” but also this process will help me to “explicitly claim the ways in which my position relates to the issues.”

One of the questions I hoped to answer with this research project was ‘what is it about the graduate school experience in mathematics that prevents future mathematicians from engaging in and embracing their roles as teachers of university mathematics?’ As I described in Chapters 1 and 2, my own experiences as a mathematics graduate student and in learning to teach mathematics were not unproblematic. The questions that motivated the research came directly from my experiences and I had certain ideas in mind as to what the answers to the questions might be. As a result, rather than respecting the participants’ voices and experiences and giving them a space to be explored, at first I looked to their experiences to validate my own answers to this question.

For the first few readings of the transcripts, I focused on the topics that I expected, rather than those things that were unexpected, that might have been different from my experience. In this regard, I singled out the participants’ experiences that coincided with my own as way to validate my own frustrations and disappointments with the experience of being a graduate student in mathematics. I had the initial sense that I had answered all of my questions and simply needed to write about my questions and the answers I heard. Yet, when I attempted to write, I could not. Sitting in front of my computer every day for some time, my body resisted my computer, my fingers not wanting to touch the keyboard, almost as though my computer and I were opposing magnets. After days of staring at my computer or sitting with pencil and paper in hand, waiting for the sense of having answered my questions to translate into a written dissertation, I wrote a note to myself: "I'm not writing because I am not ready to write." I started to feel that there was something almost negligent, indifferent, and unwise in the sense that I was done, that I had answered the questions I had entered the research asking. It felt incomplete and not truthful. This required an uncovering of my own motivations and experiences.

In reflecting on the struggle to write, I suppose I wanted to prove that I had not continued in mathematics because there was something wrong within the space of being in mathematics and not because there was something wrong or unacceptable in who I was and in not earning a PhD in mathematics. I realized that I needed, through my research, to justify my own reasons for leaving mathematics, and these reasons needed to be about mathematics, not me. I also wanted to illustrate that the negative experiences of being a graduate student in mathematics were universal and to argue that there was something wrong in the process of educating future professors of mathematics.

Like Husserl, I seemed to be "dominated by the one-sidedness" (Laverty, 2003, p. 14) I was criticizing, possibly reaching out for an almost mathematical certainty in attending to the questions of the research project. In approaching the transcripts with particular questions *and* particular answers in mind, it was almost as though I framed the question and my approach to the transcripts so that the result would be the very answers I wanted. In an interesting way, I found this approach to be similar to the form of engagement and questioning that had occurred in my mathematics classrooms, where I would ask questions in such a way that the students would *have* to give me the particular answer I was looking for. The ways in which I engaged students foreclosed on what they might have to offer the conversation in the classroom. For this research project, it occurred to me that I was interpreting the conversations in a similar fashion, taking an approach that would compel a particular answer – mine.

In hermeneutic inquiry, there are several issues that must be attended to by the researcher. In particular, what was required of me was "an *openness* to my prejudice, not only to see clearly the way that my self-understanding emerges from a set of particular conditions, but also to see how my identity opens out onto the horizon of Other identities" (Smith, 2006, p. 111). My journey to understand my experiences as a mathematics graduate student was not free from my own opinions, false perceptions, and frustrations. Within this recognition of the potential for prejudice, hermeneutics held a responsibility for me, "a taking of responsibility for myself as an integral part of other things, other people, and accepting the fact that my self-understanding must change as my interpretations are shown by the Other to be wrong or in need of revision" (Smith, 2006, p. 109). I had to become aware of the ways in which I came to the place of understanding the graduate student experience, including the things I have rejected, which might be relevant to another's experience. I could not hold on to the prejudices that would prevent me from acknowledging that someone else's path may include those things that I could not accept for myself.

As Gadamer (1975) stated, understanding requires "the fundamental suspension or our prejudices" (p. 298). Gradually it became clear to me that I had to let go of my inclination to be right, the desire to prove to myself that I was correct in my feelings about the world of mathematics, my prejudices about mathematics. In fact, what I found was that I could not write about or interact with the transcribed conversations until I could hold these feelings at a distance, recognizing them as significant in my own experience, but not helpful for clearly hearing what others had to say about their own lives in mathematics. In this regard, Gadamer (1975) wrote:

> We are always affected, in hope and fear, by what is nearest to us, and hence we approach the testimony of the past under its influence. Thus it is constantly necessary to guard against overhastily assimilating the past to our own expectations of meaning. Only then can we listen to tradition in a way that permits it to make its own meaning heard. (p. 304)

To respect the participants' experiences, I had to learn to hold to one side the sense that my experience in mathematics was necessarily representative of others' experiences. I began to ask myself the question "how do I know that I experience things in the same way as does someone else?" (van Manen, 1997, p. xii). When looking at the transcripts, I had concluded that I already understood my research participants, which had the potential to "reflect condescension" (Smith, 2006, p.

106). But I did not want to condescend to my research participants and their perspectives, their aspirations, and their lives in mathematics. Nor did I want to undercut and belittle the connections and the understanding that had come through in our conversations. It was clear that my first attempt to write did not "care for their [the participants'] integrity, humanity, and struggles" (Britzman, 2003, p. 35).

van Manen (1997) stated "we must dislodge and confront our unexamined assumptions" (p. xii). Yet, Gadamer (1975) reminded us that the hermeneutic inquirer "cannot separate in advance the productive prejudices that enable understanding from the prejudices that hinder it and lead to misunderstanding. Rather, this separation must take place in the process of understanding itself" (p. 295). Thus, through the processes of transcribing, listening, understanding, and interpreting, I began to let go of my once-insider, now-outsider critique of mathematics and mathematicians. I needed and wanted to reconnect with my love and passion for learning mathematics, to what drew me to mathematics, and to remember what had inspired me as a graduate student in mathematics. Through this process, I recognized that I had two very different perspectives of being in mathematics as harmful to one's spirit and, in contrast, mathematics as thoughtful, creative, and deeply connected to lived experience. I began to realize that one perspective was not mutually exclusive of the other, that they could co-exist, even within myself, and that I did not have to subscribe solely to one perspective or the other.

Eventually, I found a new patient thoughtfulness about my own experience, and a new attention to more than just what I was hoping to find. There were amazing moments of transition when it finally became less important to prove correct any hypothesis I might have. It became more important to create an openness for understanding what I was hearing in the graduate students' voices. Listening to and *hearing* the participants' voices and their differences from me – they did not have the distanced feeling of disappointment that I had been carrying with me – helped me reconsider my opinions and that which I thought would be true for other mathematics graduate students.

Thematic Analysis

> *theme*: c.1300, from O.Fr. *tesme* (13c., with silent *-s-*), from L. *thema* "a subject, thesis," from Gk. *thema* "a proposition, subject, deposit," lit. "something set down," from root of *tithenai* "put down, place," from PIE base **dhe-* "to put, to do" (see *doom*). (www.etymonline.com, 2008)
>
> *theme:* the subject of a talk, piece of writing, exhibition, etc.; an idea that recurs in or pervades a work of art or literature (Soanes & Stevenson, 2005, p. 1828)
>
> *analysis*: c.1581, "resolution of anything complex into simple elements" (opposite of *synthesis*), from M.L. *analysis,* from Gk. *analysis* "a breaking up," from *analyein* "unloose," from *ana-* "up, throughout" + *lysis* "a loosening" (see *lose*). Psychological sense is from 1890. Phrase *in the final (or last) analysis* (1844), translates Fr. *en dernière analyse.* (www.etymonline.com, 2008)

While listening to the recorded conversations and transcribing the dialogue, I noticed that there were similarities among the experiences the participants described and in the language that they used. At times, I had a strong sense that I had previously heard and typed something comparable to what I was writing down at that moment. There was a noticeable consistency around some notions or, in other words, an obvious attention on the part of the participants to certain topics or points of view, and so my attention was drawn to these. In the notes that I took while listening to the conversations over the six-month period of our meetings, I also noticed a resemblance in what they chose to speak about. These similarities were not limited to broad categories of their lives, such as how they each had to attend to their teaching assistantship duties or their graduate-level coursework. It was also opinions and perspectives about various aspects of their experiences that appeared to be in common.

I began to put down in words the ideas, experiences, and opinions that resonated among the participants, that appeared to be the subject of conversations. Some of these points were consistently the topic of conversation while others were surprising to me and, as a result, caught my attention. Eventually, my notes grew and evolved into coherent groupings that I describe here as themes, which are defined above as "an idea that recurs in or pervades a work of art or literature." Thus, rather than only focus on my feelings or impressions about what commonalities existed in the data, I looked to thematic analysis (Braun & Clarke, 2006; van Manen, 1997) as a thoughtful and conscious way to listen for and unearth themes that I found to echo among the participants.

Thematic analysis begins "when the analyst begins to notice, and look for, patterns of meaning and issues of potential interest in the data" (Braun & Clarke, 2006, p. 86) and is characterized as "a method for identifying, analysing and reporting patterns (themes) within data. It minimally organizes and describes your data set in (rich) detail. However, it frequently goes further than this, and interprets various aspects of the research topic" (p. 79). van Manen (1997) described themes as "a means to get at the notion we are addressing" and as providing "control and order to our research and our writing" (p. 79). Further, van Manen stated, "Theme is the form of capturing the phenomenon one tries to understand" (p. 87). In these researchers' descriptions of theme and thematic analysis, I found the process that I was beginning to use. Consequently, I followed their guidelines in the development of themes.

Braun and Clarke (2006) described the decisions one must make in thematic analysis and also make explicit when reporting research. They first explained the choice between an inductive and deductive approach to coding and categorizing data. I chose the inductive approach to thematic analysis as it is a process that does not try to "fit into a pre-existing coding frame […] the themes identified may bear little relation to the specific questions that were asked of the participants […] the themes identified are strongly linked to the data" (p. 83). In contrast, a deductive approach required the researcher to look for data that answers a "quite specific research question" (p. 84). With this comparison, the inductive approach could contribute to hermeneutic work in that it would allow an openness to what surfaced when reviewing the conversations. Further, in not attending to a particular question, an inductive approach to themes would be helpful to a hermeneutic approach to the question, allowing the questions of the research to reveal themselves and evolve further.

A second choice Braun and Clarke (2006) described is the "level at which themes are to be identified" (p. 84), in particular, a latent or a semantic approach.

The semantic approach looks at "the explicit or surface meanings of the data, and the analyst is not looking *beyond* what a participant has said" (p. 84). In comparison, Braun and Clarke (2006) acknowledged that the latent approach, of "identifying or examining the *underlying* ideas, assumptions, and conceptualizations – and ideologies," involves "interpretive work" (p. 84). As the work of hermeneutics involves interpretation and going beyond the semantics of the conversations in order to find meaning, I chose to take a latent approach to exploring the conversations and extracting and naming themes.

Beyond these choices, Braun and Clarke (2006) list six phases that researchers follow when using thematic analysis. The first phase is the process of transcribing, which they describe as needing to be "a 'verbatim' account of all verbal (and sometimes non-verbal – e.g., coughs) utterances" (p. 88). For this study, as I transcribed the conversations, I included the periods of silence, laughter, and the "um"s and "hmm"s of the participants (see Appendix 5 for a portion of an individual transcript and Appendix 6 for a portion of a group conversation transcript). I was attentive to the participants' unfinished thoughts and what they emphasized, typing out half sentences even when they switched to an entirely different topic in mid-thought. Each participant had peculiarities when speaking, one saying "Mm-hmm, mm-hmm" before most thoughts, another asking the question "Right?" after most statements of opinion. It was important to include these as they allowed me to hear the participants through the written transcripts, to see what was typed on paper as representing the specificity of each person. It helped me to recognize the typed transcripts as belonging to each participant because their particular ways of conversing were fully set down.

In their description of the second phase of thematic analysis, Braun and Clarke (2006) explained the use of coding schemes to categorize data. However, van Manen (1997) described coding schemes as mechanical processes of counting the frequency of various concepts, rather than attending to “a process of insightful invention, discovery or disclosure” where “grasping and formulating a thematic understanding is not a rule-bound process but a free act of ‘seeing’ meaning” (p. 79). Thus, rather than a numerical coding or counting occurrences of particular topics of the conversations, I categorized issues in the dialogue into broader groupings and then refined those categories as I continued to listen to the recordings and read the transcripts. For example, if a participant’s statement was related to their teaching assistantship duties, it was first placed into that broad category.

With regard to the second phase, Braun and Clarke (2006) stated that the researcher “may initially identify the codes, then match them with data extracts that demonstrate that code” (p. 89). They described the third phase as “sorting the codes into potential themes, and collating all the relevant coded data extracts within the identified themes” (p. 89). In carrying out the second and third phases, I colour-coded pieces of the conversations that appeared to fit into broad categories, such as teaching or graduate-level coursework. I then extracted these pieces from the larger conversations and placed them under headings denoting the categories I had specified. In reviewing the categories I had named, I asked the question “What does this category speak of?” Further, of the quotations that I pulled out of the data, I asked the question “What is this statement speaking of?” Each statement was again listened to, thought about, and grouped into more refined categories, such as the participants’ experiences in the workshop situations or experiences as a learner.

van Manen (1997) has described three approaches for “uncovering or isolating thematic aspects of a phenomenon in some text” (p. 93). In carrying out Braun and Clarke’s (2006) second and third phases in the aforementioned ways, I was following and combining two of van Manen’s recommendations. The first suggestion was to take a “selective or highlighting” approach where one asks “What statement(s) or phrase(s) seem particularly essential or revealing about the phenomenon or experience being described?” where the statements are then “circled, underlined, or highlighted” (p. 93). When categorizing the data, I looked to highlight those statements that seemed to speak to particular categories. The second suggestion was to take a “detailed or line-by-line approach” where “we look at every single sentence or sentence cluster” (p. 93). For all of the categories, I took this approach to each statement that was placed there, and asked what they might say about the particular category or theme.

The fourth phase of thematic analysis recommended by Braun and Clarke (2006) is a two-fold review of the defined themes. The first stage of this review process occurs at the “level of the coded data extracts” where “it will become evident that some candidate themes are not really themes (e.g., if there are not enough data to support them), while others may collapse into each other (e.g., two apparently separate themes might form one theme)" (p. 91). In reviewing the names of the themes and what they consisted of, I began to bring together those that appeared to address similar issues while other themes waned in their importance, and others began to appear more significant through continued reading of the dialogue. The second stage of this review process addressed the relation of each theme to the entire collection of conversations. At this stage, I reconsidered each theme and whether it was an appropriate piece on which to focus within the entirety of the conversations.

The fifth phase of thematic analysis is concerned with defining and refining themes, meaning "identifying the 'essence' of what each theme is about, as well as the themes overall, and determining what aspect of the data each theme captures" (Braun & Clarke, 2006, p. 92). At this stage, I reconsidered the names I had given to each theme. Rereading the statements under each theme and thinking about what they spoke of allowed me to reconsider and, in some cases, rewrite the labels I had used for particular themes. The sixth and final phase of thematic analysis required that I write the research in such a way that one "provides sufficient evidence of the themes within the data – i.e., enough data extracts to demonstrate the prevalence of the theme" (Braun & Clarke, 2006, p. 93). The following chapters are an expression of my work related to the fifth and sixth phases of thematic analysis. The writing in the following chapters presents the revised themes, the participants' statements within each them, and the interpretive, hermeneutic exploration of the themes and conversations.

Thematic Analysis and Hermeneutic Inquiry

While I have discussed the resonance of some aspects of thematic analysis with hermeneutic inquiry, I would like to explore further the reasons for using this approach to the conversations within hermeneutic inquiry. From a more pragmatic perspective, Braun and Clarke (2006) remind the researcher that unlike grounded theory and other methods, thematic analysis is “not wedded to any pre-existing theoretical framework, and therefore it can be used within different theoretical frameworks” (p. 81). Further, thematic analysis is flexible, and “has the potential to provide a rich and detailed, yet complex, account of data” (p. 78). The stages of thematic analysis are in accord with Laverty’s (2003) description of a hermeneutic project where “the multiple stages of interpretation allow patterns to emerge, the discussion of how interpretations arise from the data, and the interpretive process itself are seen as critical” (p. 23). Beyond this, though, are deeper connections to hermeneutics.

Hermeneutics is described as a multidimensional mode of inquiry, which attends to interpretation, understanding, the creation of meaning, the role of language in interpretation and understanding, and the search for the true question within and underlying the inquiry. My choice of a latent approach to thematic analysis supports the interpretive work of hermeneutics as this methodology recognizes that “the development of the themes themselves involves interpretative work” (Braun & Clarke, 2006, p. 84). Hermeneutic inquiry is concerned with the questions we are attempting to ask and allows the questions to emerge and evolve.

In this regard, Braun and Clarke (2006) remark that thematic analysis allows the research question to evolve through the process of coding and evaluating themes.

Hermeneutic inquiry also attends to a movement between the part and whole. Thematic analysis resonates here as well as it "involves a constant moving back and forward between the entire data set, the coded extracts of data that you are analysing, and the analysis of the data that you are producing" (Braun & Clarke, 2006, p. 86). Thematic analysis represents a recursive rather than linear process, an approach that I utilized in the conversations with the participants and is also a feature of hermeneutics. Lastly, hermeneutics is concerned with meaning and understanding. In this research project I am attempting to make sense of the lives of graduate students in mathematics. In order to do so, I used conversations, which were instrumental for hermeneutics and thematic analysis, since "the collaborative quality of the conversation lends itself especially well to the task of reflecting on the themes of the notion or phenomenon under study" (van Manen, 1997, p. 98).

Reflecting on the Themes

> The initial meaning emerges only because he is reading the text with particular expectations in regard to a certain meaning. Working out this fore-projection, which is constantly revised in terms of what emerges as he penetrates into the meaning, is understanding what is there. (Gadamer, 1975, p. 269)

As I described in the first section of this chapter, I struggled to find an openness, a readiness to hear and accept what was said in the conversations that went beyond my already-understood experience, and to also hear what was said beyond the topic of conversation, to allow the participants' voices to speak to me.

This struggle came and went as I proceeded to read the transcripts and name themes I heard in the conversations. Here, I would like to explain a bit further the process I went about, and what in my experience caused me to hear or see the initial themes.

What were the themes that I first identified as having relevance within the conversations with the participants? In my first few readings of the transcripts, I had in mind some themes that had already occurred to me and that I took note of in the midst of holding the conversations. These topics mostly resonated with my own experiences in mathematics and so were somewhat expected. One theme was labeled "identity in mathematics" and I extracted participants' statements that were related to this. For example, some statements described how the participants did not fully engage with others outside of mathematics because being in mathematics somehow made them different from others. This was something that was very much wrapped up in my own experiences. I tended to shy away from revealing my work in mathematics to others, which was a common response to people who claimed that mathematics was something they had always disliked. As the participants' experiences resonated with my own, it felt important to include this theme in trying to understand their lives in mathematics.

Some of the other early themes that I paid attention to represented subjects that were the topic of conversation for long periods of time and they seemed to be most relevant. One of these themes came from their statements about elementary and secondary education. Several of the participants spoke at great length about the problems with pre-tertiary education. As this made up a significant portion of the dialogue, it became one of the themes. However, as I will describe later, Braun and Clarke (2006) state that the length of a topic in a conversation should not immediately imply its importance in thematic analysis.

In the first section of this chapter, I also wrote about an early sense of being done with the project. One of the questions that I hoped to answer through this research was 'what experiences in the lives of graduate students have the potential to prevent them from engaging in learning how to teach?' In one theme that I named "What life is like," I collected those statements that reflected the participants' struggles with time, coursework, and being a graduate student that had the potential to interfere with a graduate students' teaching. At first, this list of collected statements seemed to answer that question fully. In the little time the participants have and with the minimal department focus on teaching, in the hours they must spend attending to their own learning of mathematics and engaging in research, it was clear that graduate students simply had very little time to embrace and explore their teaching. However, this theme was not the only answer to this question, as I illustrate in the following chapters.

There were some surprises, though, when reading the transcripts. In particular, I was taken aback by the participants' construal of undergraduates' behaviour. To some of the participants, any undergraduate's struggle with mathematics signified a lack of motivation or desire to learn mathematics. This was a surprise because, based on my own experiences as a student in university, I could empathize with others' labours to learn mathematics. I had struggled in my undergraduate mathematics courses and had worked diligently to find ways to understand mathematics. When I encountered undergraduates having similar experiences, I could understand their difficulties and shared my perspectives with them. It was surprising to me that the participants would take meaning from and make judgments about the undergraduates' actions.

After the first few passes through the transcripts, I had collected six themes and I believed that I had enough to write my dissertation. But, as I stated previously, I could not write and I began to reflect on this. Many of the themes I had named resonated in my own experience and I had collected them, in part, to validate that experience. In looking at the pieces of conversation that I had extracted for some of these initial themes, I saw my own voice coming through in them, in what *I* had said in the dialogue, in how my experience had, at times, directed the conversations. At this stage, I went through the thematic analysis process I described above, using the literature and guidance of hermeneutics to learn to put these themes aside and read the transcripts again with a new openness and thoughtfulness about the participants' own experiences.

On the next several readings of the transcripts, I did not ignore the themes I had already defined, but put them to one side in order to hear new experiences and ideas. I began to see the participants as separate from myself, as people who might not have had the same experiences as me during their graduate programs in mathematics. Rather, I took the standpoint of being other and began to put "myself in someone else's shoes" in order to "become aware of the otherness […] by putting ourselves in his position," which allowed "a higher universality that overcomes not only our own particularity but also that of the other" (Gadamer, 1975, p. 304). New ideas and expressions came through in the graduate students' experiences which asserted their "own truth against [my] own fore-meanings" (Gadamer, 1975, p. 272) as I followed the process that Braun and Clarke (2006) and van Manen (1997) recommended for thematic analysis.

After a new look at the conversations, I reflected on all the themes I had named, rereading them and evaluating whether they spoke only to my experience or whether they had relevance in understanding the participants' lives in mathematics or both. For the most part, I discovered that the themes, even those that were named in the first readings of the transcripts, were appropriate and significant to the graduate students' experience. I also found that some of the first and second set of themes spoke to similar topics and could be combined.

While I have not named all of the themes here, they will be named and discussed in the following chapters. The next three chapters will explore themes that I identified when listening to the conversations and reading the transcripts. These chapters will attend to the questions that Braun and Clarke (2006) describe as an integral part of thematic analysis, which are also central in hermeneutic inquiry. These questions are:

> What does this theme mean?
> What are the implications of this theme?
> What conditions are likely to have given rise to it?
> Why do people talk about this thing in a particular way (as opposed to other ways)?
> What is the overall story the different themes reveal about the topic? (p. 94)

As Smith (1983) has stated "The truly hermeneutic imagination does not abandon itself directly to the tangibility of words and appearances, or to the fixed determinateness of the meant, but is able to reflect that which brings to fullness what lies silent" (p. 91). This perspective will also be reflected in the analysis of the themes.

Exploring the Conversations

In the following chapters, I will include portions of the transcripts as they relate to the themes I explore in this dissertation. After each of the excerpts is a reflective examination of each theme and of the "ways in which language is used, an awareness of life as an interpretive experience, and an interest in human meaning and how we make sense of our lives" (Laverty, 2003, p. 22). Chapter 6 explores an overarching theme I have named "what life is like." This chapter addresses the various tasks the participants had to attend to and the structures they encountered in their lives as mathematics graduate students. As such, Chapter 6 represents a broader context necessary for understanding Chapters 7 and 8. Chapter 7 deals with a theme of being almost a mathematician in the world, as the graduate students are in an in-between space of being a student on their way to possibly becoming a mathematician. Chapter 8 speaks to the experiences and the spoken and unspoken messages the participants go through as they begin to explore and form their identities as post-secondary teachers of mathematics.

One note to the reader is that the ellipsis seen in the excerpts in the following chapters, (…), is symbolic of a participant trailing off, not completing their thought, or changing their thought in mid-sentence. Note also that, "The retelling of another's story is always a partial telling, bound not only by one's perspective but also by the exigencies of what can and cannot be told. The narratives of lived experience – the story, or what is told, and the discourse, or what it is that structures how a story is told – are always selective, partial, and in tension" (Britizman, 2003, p. 35). What you will read in the following chapters, then, is my interpretation and understanding of the lives of the six mathematics graduate students in this study.

Chapter 6
I Have to Cut Off the Rest of the World

The participants spoke at length of the work of their daily lives – how they came to be teaching assistants for particular courses, what their duties were, and what was involved in their graduate studies. As hermeneutics "has to do with interpreting – making sense of, bringing to intelligibility and understanding – the meaning of human destiny as it reveals itself in the occurrences of daily life" (Smith, 1983, p. 28), this chapter comprises an exploration of an overarching theme described as "what life is like," an investigation of what the research participants experienced in their day-to-day tasks.

In his work with future schoolteachers of mathematics, Brown (2001) claimed that their "perspectives, it is suggested, are imbued with culturally derived or institutionally imposed structures, present both in the words used by inhabitants and in the physical space they occupy" (p. 3). Further, in his study following a first-year assistant professor of mathematics, Gutmann (2000) wrote about how the "formal structures (the promotion and tenure process, yearly reviews, student evaluations, observations by senior faculty) and informal structures (casual conversations, spontaneous 'thank you's,' enrollment by students in future courses) each have a part to play in passing on local values about teaching to newcomers" (pp. 4 - 5). With this in mind, in this chapter, the word *structure* has multiple meanings for the exploration of the conversations. Below are some of the different definitions of the word *structure*:

> Mode of building, construction, or organization; arrangement of parts, elements, or constituents

> Anything composed of parts arranged together in some way
>
> From sociology: the system or complex beliefs held by members of a social group; the system or complex of beliefs held by members of a social group
>
> The way in which parts are arranged or put together to form a whole (www.dictionary.com, 2008)

In light of these definitions, I focus on the structures the participants encountered in their daily lives as graduate students in mathematics – the structures that existed in their coursework, research, and teaching assistantship duties, as well as the structures that were found in both the spoken and unspoken expectations in the department.

Gadamer (1975) wrote, "We are always situated within traditions, and this is no objectifying process – i.e., we do not conceive of what tradition says as something other, something alien. It is always part of us" (p. 283). The structures of the participants' coursework and teaching assistant duties represent such traditions and also represent what they must pass through on their way to completing their degrees. Further, these structures characterize what mathematics graduate students encounter in their everyday work.

I believe it is important to look at "that which has been sanctioned by tradition and custom," those things that are nameless, but have an influence where our "being is marked by the fact that the authority of what has been handed down to us – and not just what is clearly grounded – always has power over our attitudes and behavior" (Gadamer, 1975, p. 281). Within the mathematics graduate students' lives there is an authority handed down that exists within the structures they encounter, that they must attend to, and that has meaning for their becoming mathematicians. In this chapter, I explore the ways in which the participants interpreted and made meaning of these traditions and structures in their day-to-day experiences as graduate students and as future mathematicians.

Structures of Undergraduate Courses and Teaching Assistantships

> Your instructor [mathematics graduate teaching assistant]: has always been good at mathematics; has never taught before; has never selected and graded homework assignments; is unfamiliar with the content and the pace of the course; has never before written, let alone graded, an exam; and has less than a week of training, a good portion of which involved learning university policies, departmental policies, and administrative procedures. (Belnap, 2005, p. 13)

The Department of Mathematics is considered partly to be a service department for other disciplines in the university as each year it offers first- and second-year mathematics courses for dozens of non-mathematics subject majors in engineering, business, education, science, and the social sciences. Many of these courses, such as first-year calculus, are often scheduled in such a way that hundreds of students enroll in a particular lecture section, and smaller groups of students register in one of the accompanying one hour per week tutorial sessions. Unlike the courses that are offered to non-mathematics students, the upper-level courses that are required for mathematics majors have far fewer students enrolled. This is a common structure for departments of mathematics in large universities.

To support the configuration of the one- and two-hundred-level courses that have large lecture sections, where undergraduate students generally have little or no contact with the professor, the Department of Mathematics employs graduate students to help the undergraduates in these courses in various ways. The teaching assistantship duties that a graduate student might be assigned to are the following:

Tutorials for a particular course – The teaching assistant works under the faculty member teaching the lecture section of the course. The teaching assistant leads one-hour tutorial sessions, holds office hours for students, and invigilates and grades exams. On some course syllabi found on the department website, tutorials are described as an opportunity to hear a different explanation of the concepts covered during the lecture. Other syllabi describe tutorials as a time to receive help on homework assignments. The department offers tutorials for a wide variety of courses, from one- to three-hundred level courses, including introductory mathematics, calculus, numerical analysis, and the history of mathematics, among others. The assignment to a tutorial section is typically given to more advanced graduate students.

Workshops for one- and two-hundred-level courses – The teaching assistant works under the workshop coordinator, someone who supervises several graduate students in the workshop. Each teaching assistant has regularly assigned hours during which they help students with homework problems. In addition to their hours in the workshop, the teaching assistants grade homework assignments, and invigilate and grade exams for the courses that utilize the workshop. Workshops are offered mostly for lower-level courses, such as linear algebra, calculus, and introductory mathematics. The workshops are intended to give undergraduate students assistance with assignments, test preparation, as well as assistance with mathematical concepts. This teaching assignment is most often given to new graduate students.

Marking papers for courses – The teaching assistant works under the faculty member for the course or under the supervision of the workshop coordinator. The teaching assistant marks homework assignments and exams and has no direct contact with students. The teaching assistants are provided with direction as to which exercises should be marked according to particular criteria. According to the department website, graduate students with poor English skills are assigned to this duty.

Before each semester, the mathematics graduate students are asked to rank their preferred choices for teaching assistantship duties. The department then makes arrangements for staffing the workshops and tutorials for the undergraduate courses according to the graduate students' requests and schedules. The typical workload for graduate teaching assistants is described as approximately two hundred hours over the fourteen-week semester. The graduate students in this department are not required to teach courses with full responsibility before completing their degrees. Should they wish to teach a course with full responsibility, they must apply to the department as a sessional lecturer. Even though the mathematics graduate students were required to interact with undergraduates, mark papers, and serve as instructors, the participants of this study were not offered specific guidance in how to work with undergraduates in the workshop settings or in the tutorials.

Typically, the graduate students were given a teaching assignment that translated to a twelve hours per week assistantship, where they spent five to eight hours each week in the workshop and the remainder of their twelve hours was spent marking papers. However, the department expectation was that the graduate students should be prepared to help the undergraduates with homework problems. As a result, the graduate students sometimes spent several hours beyond their allotted twelve to ensure they knew how to solve the problems that had been assigned to the undergraduates. Additionally, during mid-term and final exams, the graduate students were expected to spend extra time marking exams. This often took between one and two full days of work.

Mathematics graduate teaching assistants in the workshops and tutorials

The rooms in which the workshops were held were located along the main walkway on campus. This caused the area around the rooms and the rooms themselves to go from being fairly quiet, while students were in classes, to being filled with the noise of hundreds of students walking by as they made their way to their next class. The workshop rooms were converted classrooms with clusters of desks where the undergraduate students could work. The workshop rooms accommodated approximately thirty to forty students at a time. As the hallway outside the workshops experienced times of quiet and times of commotion, so did the workshops themselves, depending on the due date of an assignment or whether an exam would soon take place. When the workshop was filled with students needing help, the space was crowded and noisy. On a few visits to campus during the time of the research study, wide fluctuations in the use of the workshops were noted – sometimes teaching assistants sat in an empty workshop waiting to help students, other times the teaching assistants appeared to be overwhelmed by the number of students waiting to receive help.

The mathematics graduate students expressed a genuine interest in working with undergraduates. Several of them described the best moments in their graduate programs as those when they were able to help someone understand a mathematical concept. Emily described an event of helping a student not only solve, but also understand a particular problem:

> There was this question about a baseball diamond today in the pre-calculus workshop, just trying to find the distance between the pitcher's mound and third base. And it's a great question, you know? And there was a guy there who clearly was just, "I don't understand what a baseball diamond is." So I drew a picture of it – "here's the base, and you run around, and what we're looking for." I drew the triangle and this is what we're looking for. And he says, "Okay, so if I do this and I do this and I do this I can get it, right?" And I said, "Yeah." And he said, "Cool." And I'm like, "Yeah." I gave him a thumbs up.

What came through in this description of helping a student was not only Emily's enthusiasm about mathematics itself and that there are great questions in mathematics, but also about helping students learn mathematics. There was a sense of excitement that came through Emily's voice in this story of relating mathematics to a student. In order to help the student understand the problem, she had taken the time to draw a picture. To me, what came though her account of this interaction was a sense of camaraderie that developed between her and the undergraduate student and she spoke of giving the student a thumbs-up, a sign of approval and connectedness. She felt excited and happy when the student made progress in his understanding.

The ways in which the workshops were arranged and utilized, though, did not often allow for the encounters the participants hoped to have with undergraduates, like Emily's experience illustrated above. After sharing the situation of helping the student with the baseball diamond problem, Emily went on to describe further what she frequently noticed in the workshop setting:

> So often when they come to you, they, like it's pretty demanding of, well, there's so many of them that it's, I try so hard not to give the answers away, but so often you're basically one step away and it's nice to see them do a few steps on their own. And to be able to like see that process, which I don't find the workshop is very conducive to that because the second they're able to be independent, they move away because there's someone else in line. Right? Like you don't get to see that.

In her follow-up statement, Emily observed that the workshop situations were often quite busy and the interactions with undergraduates too brief to have more in-depth discussions about mathematics.

Why would it be important for Emily to go beyond providing answers to the students in the workshops? Her desire was to not solely provide answers, but to see the students work through the processes in the mathematics they were assigned. I saw Emily's interest in over-seeing how the students worked through their problems as two-fold. First, what came across was an interesting protectiveness towards the students she helped in the workshop setting, wanting to see them work through the processes, in seeing the students connect to and understand the mathematics, and the enjoyment she felt in that moment. Second, based on her previous story of helping a student, it appeared that she had hoped to connect with the students through mathematics. However, she also voiced the pressure she felt from the number of students waiting for help, and sensed that the undergraduates needed to move way from her once they received assistance on one particular step of the problem. The structure of the workshop setting and the number of students waiting for help seemed to make Emily feel that she could not offer these connections to all of the students.

Robert voiced feelings similar to Emily's, particularly about the number of students who required assistance with their homework assignments:

> I don't like to teach the workshop in the sense that you don't really teach things. You're just kind of being a problem solver. You know, people have a particular problem on the assignment. They come in, they ask you and they go. They sit down in the workshop. They do their homework and if they have a problem they come and ask you and go back and work on it. And so I find that, a lot of TAs at the end of the day, because they're being asked the same question for the thirtieth or the fortieth time that week, that at the end of the day, they are just so tired of the question that they would just tell anyone who comes in and asks that question basically how to do it. And I kind of found that because it happened to me, too.

Here Robert described the toll that helping dozens of students with a particular problem had on how he and other teaching assistants interacted with the students, which caused them to resort to telling students how to solve the problem because they were too tired to do anything else.

To me, what came across in both Robert's and Emily's descriptions of the workshop setting and how they were able to work with the undergraduates is a difference between *helping students understand the mathematics* versus showing students *how to do the mathematics*. While Robert and Emily were not explicit about the differences in these two ways of helping students, what I heard in their voices is that the latter was not their preferred way of working with students.

Showing students *how to do* mathematics seemed to be viewed as an unsatisfactory interaction with students. Robert said, “I don’t like to teach the workshop in the sense that you don’t really teach things. You’re just kind of being a problem solver,” pointing to a perceived difference in the types of interactions with students.

What would it mean to “just be a problem solver”? In what the participants described, I heard that there was something wanting in just being a problem solver. As seen in Emily’s earlier statement, there was an interest and desire in connecting with others about mathematics and in helping students understand. Yet, as a problem solver, are those connections with students necessarily absent? Or are the connections just felt as being absent from the interactions that are seen as “problem solving”? It may be that the undergraduates indeed learned and understood the mathematics in those problem-solving moments, but my participants did not seem to feel or know that might be the case. Why would these problem-solving encounters be seemingly disheartening to the participants? Why did it appear that deeper connections with the undergraduates through mathematics were important? How do these feelings compare with how they viewed their future roles as professors of mathematics? These questions are addressed in Chapters 7 and 8 in the context of the participants’ views of themselves as potential future mathematicians and future professors.

Along with discussing the bearing the workshop situations had on the ways in which they interacted with the undergraduates, the participants also described the exhaustion they felt after working in labs, running tutoring sections, and marking papers. Robert spoke about his most recent experience running three consecutive tutorial sessions for an undergraduate numerical analysis course:

> It takes up a lot of time. And it takes up a lot of energy, too. I mean, this semester I was doing three one-hour tutorials in the morning for the numerical analysis class. It was a lot of time. By the end of the third tutorial, I was completely run out of gas because standing up and talking for almost three hours nonstop is really, really hard.

In running the tutorial sessions, Robert felt the amount of time and energy the physical act of teaching required, finding that he was worn out at the end of three hours. John spoke about being in the workshop for up to eight hours each week in addition to the marking he was required to do, and commented that all of the teaching assistant work from week to week was tiring. While most of the participants expressed their feelings about the workshop setting for undergraduate courses in our individual meetings, they also found their fatigue to be a common experience:

> Steven: It's exhausting. It would almost be better if we could teach. At least we could do it in sort of our own like … Like I'd prefer to teach forty of them than sit in the workshop and essentially teach all of them.
>
> Emily: One at a time…
>
> John: Yeah.
>
> Steven: I think it's honestly the one-on-one aspect of the teaching that's more exhausting than anything else.
>
> John: And it's not necessarily training to be a teacher. It's training you to be a tutor.

What was conveyed here was a frustration that their time in the workshop was demanding and exhausting, but also ineffective. In communicating their burden of having to help each student individually, together they considered the structure of helping undergraduates one-on-one to be unproductive compared to what occurs in a classroom setting. John's final comment speaks of an understanding that the workshop setting was not an example of teaching, nor was it a helpful experience for knowing or learning how to teach.

The dissatisfaction and exhaustion within the structure of the workshop setting was common among the graduate students. Beyond these particular feelings, within their comments of how things could be improved, there was a sense of disappointment in how things transpired. The graduate students did not have opportunities to observe the undergraduate students' progress and their understanding of concepts develop over time, and so the act of tutoring in the workshop situation became an unrewarding and tiring experience. Emily described how her work became "how fast can you turn them over." Rather than being able to provide the undergraduate students with a more in-depth learning experience, when there were many students waiting for help, it became "a lot faster to plug and chug." So, despite the participants' desire to help the undergraduates understand the material, and knowing the reward that they experienced in doing so, my participants felt the pressure to solve the problems for the students rather than taking the time to help the students understand the processes and find the solutions in the mathematics.

I noticed a curious paradox in the language and descriptions the participants used to describe what they did in the workshops and tutorials. They talked about the workshop situation as tutoring not teaching, but they seemed to have a desire to offer teaching moments. The group dialogue presented an interesting contrast to the views expressed individually by Emily and Robert.

What had previously come across was a sense that working one-on-one with students held possibilities for dialogue, connection, and understanding around mathematics. Yet, in the group Robert spoke of his role in the tutorials as “standing up and talking for almost three hours nonstop.” So, while there seemed to be a desire among some of the participants to have opportunities to share mathematics and help students understand, there seemed to be some conflict in their views of what they should offer the students. In particular, the participants appeared to want to interact with students, to converse about mathematics, and to watch the undergraduates grow in their understanding. In spite of this, at times when the participants were in a role where they could interact with the undergraduates in the ways they wanted, they reverted to a tutoring or lecturing stance.

Did the workshop and tutorial settings contradict their views of what teaching is, of how they wanted to work with students? Or, rather, did the structure of the workshop impose a view of how they should, or even must, work with students? How and why would their desires or interests in helping undergraduates be overridden by the structures of their work environment? These issues are explored further in this chapter, as well as in Chapters 7 and 8. In particular, in this chapter I continue to address the structures of the participants’ lives that appeared to have an influence on how they went about their various duties. In Chapter 7, I explore the participants’ need for connection with others about mathematics, and in Chapter 8, I further discuss the participants’ views of their work as future professors of mathematics.

Tacit and unknown structures of graduate teaching assistant work

> We are always looking for something, something made significant by the explicit and tacit rules of the game, for what counts in some particular inquiry and context. (Crusius, 1991, p. 15)

As the mathematics graduate students began their work with undergraduates in the workshop setting, aside from the number of hours they were expected to spend in the workshop, they were given little guidance as to what assistance they should offer undergraduates. While little was stated in a formal policy with regard to the graduate students' roles and what was expected of them in how they interacted with undergraduates, a structure did exist and eventually became explicit when the graduate students deviated from the unspoken department expectations for how they could and should interact with undergraduates.

Emily described her struggle to know all the topics and problems that students in the workshops might ask her about. She did not have time to learn all of the solutions to the homework problems. Rather than strain to solve a problem while being watched by an undergraduate student, she thought that admitting she did not know how to solve the problem and sending the student on to another teaching assistant who knew and understood the problem would speed along the process along, and get the student help as quickly as possible. However, when it came to the attention of the department that another mathematics graduate student had admitted their lack of knowledge on a particular problem, an email was sent to all graduate students telling them that they could not say “I don’t know” to the undergraduates. Emily remarked:

> I feel so bad about it, you know, and you’re not allowed to say you don’t know. Like that’s one of the rules. Isn’t it better to admit you don’t know and say, “Go find somebody else”?

Here Emily voiced some distress about the recently stated rule of not being able to say that she did not know something; in the workshop setting, not knowing how to solve a problem had become unacceptable. To be sure, Emily felt pressure to know everything and did not want to stumble in front of others. Yet, her distress (“I feel so bad about it”) seemed more wrapped up in her concern for the undergraduate students, in getting them help sooner rather than later. She has felt at ease in admitting that she did not know how to solve a problem, but now felt a new pressure that it was unacceptable to do so.

John spoke of helping an undergraduate who was frustrated with his [the undergraduate’s] mathematics class:

> A student [an undergraduate] said, “I just don’t understand how it works. I do see why some things are a certain way” or they’re [the undergraduate] kind of fed up with the course a little bit and I might have the same complaints. I mean, but there’s nothing I can do about it, so it’s, you have no control. You can’t really work outside of a certain box.

John articulated a tacit understanding that he could not “work outside of a certain box,” even when he agreed with the undergraduate’s opinions of the class. As a teaching assistant for a course, he felt he had “no control” and he could do nothing about it.

It is interesting to compare these two experiences. While Emily, a first-year master’s student, could see a way of helping students by sending them to another teaching assistant, John, who was a fourth-year doctoral student, felt that he had no options to help the student, to connect with them, or provide them with something other than what was offered by the professor for the course. Is this difference of perspectives, of the possibilities that one can offer to an undergraduate, a function of the years in the department, of experiencing the unspoken and spoken expectations of behaviour? Why would a first-year graduate student, with less mathematical knowledge and teaching experience, appear to feel more empowered to offer an undergraduate options for learning mathematics than a fourth-year graduate student? John's feeling of having no control over or no say in what he could offer the student in this situation to me speaks of the weight graduate students are under to behave in particular ways in mathematics. In contrast, Emily had not experienced the years in the department and the expectations of behaviour that John had, but it nevertheless seemed she was beginning to feel the pressure to alter her way of being in mathematics and with undergraduates.

While the expectations for teaching assistant conduct in the workshop setting slowly became explicit over time, with regard to their marking duties, the graduate students were given clear and precise directions about which problems to mark and how to grade them. Chris described his experiences with this particular duty as a teaching assistant:

> It's been, it's very frustrating when I'm marking one question and, I mean, like I can see all the other questions they've done and it's just complete crap and I'm only supposed to mark one question. So it's hard and I just don't have time to go through everything and make comments.

His directions were to only mark one or two questions on the homework assignment, even though the students were required to do more problems. To Chris, it seemed negligent to not correct more problems, especially when he could see that students were doing other problems incorrectly. Like Emily, Chris's expressed desire to go about things in a particular way resided in a concern for the students. He wanted students to understand the mathematics and felt that they would not learn if most of their homework problems went uncorrected. Yet, he felt obliged to follow the instructions given by the professor of the course. In light of this, though, Chris admitted to secretly marking more problems when he had the time, revealing his belief that this was an important way to help undergraduates in their learning even if the department did not recognize it.

The duty of marking assignments and exams demanded a great deal of the participants' time and energy. Emily reported that she often spent up to five or six hours on the weekends marking papers and then found herself too tired to focus on her own coursework. Steven described the experience of marking mid-term and final exams, where graduate students were needed to mark hundreds of exams. These times during the semester often required up to ten extra hours of work in one day. Most often, each graduate student was assigned one problem to mark on each of the exams. Steven expressed the heavy load and exhaustion this work brought: "It's just tough when you're on your two thousandth paper of the day. I just want to get the hell out of there."

As the mathematics graduate students moved further into their programs and gained more experience, they were often given greater responsibility in their teaching assistantship duties. The more advanced duties of the doctoral students included holding tutorial sessions, which meant that they might lecture on particular topics or solve problems for one hour with a group of students. They also had the responsibility to create exams for those sessions. While it seemed that spoken and unspoken rules existed for their previous duties, for these higher up tasks there was much that was simply unknown, with no spoken or unspoken directions for how the graduate students were supposed go about things.

For example, Chris spoke about how he might structure his exams, balancing challenging questions with easy or procedural questions. Wanting to include problems of varying difficulty, he was unsure as to how many problems of each type he could have on an exam. Robert also spoke of his uncertainty about making exams for students, of not knowing how to write tests. He was not sure what would be a good or a bad problem. And, more importantly, he questioned what a test would or should be testing – knowledge and understanding or procedural skill and which was more important to test. Robert also wondered how long a test should be. He spoke of a fellow graduate student who would have Robert take the test the other graduate student had developed for his students. In order to estimate how long it might take undergraduate students to complete the exam, the graduate student would multiply Robert's time by three. This was not a direction given by the department, but rather a rule of thumb that had been shared amongst the graduate students. In my own experience as a teaching assistant, we often multiplied our test taking times by six, twice the amount of time allowed by Robert's friend.

To me what is revealed here was that without guidance for their tasks in teaching, testing, and working with students, the more advanced graduate students were left to create meaning and strategies among themselves and sometimes on their own. They were unsure as to whether their ideas, ways of teaching, and ways of working with students were satisfactory, and there appeared to be no feedback loop that would either help them learn about, support, or change their approaches. In that system, they were left with many unknowns and a sense of arbitrariness for their duties.

Often, the directions from the department and the directions the graduate students wanted to pursue were contrary to each other. John described how his views often were not in step with the work that was expected of him: “And if it’s different, it’s really hard to kind of go along with what’s happening.” Moreover, the graduate students had ideas of how things might be improved, not only for their own work, but also for the ways in which they helped the undergraduates. Yet, it did not appear that the graduates either had a voice or felt that they could express their own ideas within the structures for working with undergraduates. In the midst of the departmental spoken and unspoken expectations for their behaviour, they were not able to express or explore what they felt was best for the undergraduates’ learning of mathematics.

To me, an outcome of their teaching assistant experiences seemed to be a sense of powerlessness to help undergraduates, a feeling that came from being hindered or impeded by the department in their efforts to help students in the ways the graduate students thought would be best. This powerlessness or ineffectualness also seemed to stem from the lack of a space or an environment in which to express themselves, who they wanted to be as teaching assistants for undergraduates, and what was important to them in this role. There seemed to be a bending or molding of the graduate students into particular ways of being – to conform to the ways of the department. The participants' interests in and even passions for helping students learn were muted and sometimes silenced by the structures of their teaching assistant work and the expectations for certain behaviour that came from the department.

If experiences such as these represented the participants' first interactions with undergraduates, and these experiences might continue to occur for as many as two to four years while they are in their graduate programs, what might happen to their views of teaching, to those ideas that came through in their reactions to the happenings in and rules of the department? How might their aspirations for what they want for the students (what I wanted for my own students, as expressed in Chapter 1) be influenced by the years they spend within these structures? Who will they be and how they will interact with students as they enter their careers as professors of mathematics? And will their interests in and passions for helping undergraduates persist?

Structures of Coursework and Research

> The life of a full-time graduate student: in addition to spending time preparing for class, teaching, preparing/grading assignments and exams, holding office hours, attending course meetings, and tutoring, [sic] he will be: taking nine units of difficult core graduate mathematics courses; doing his homework; preparing for his own exams; and studying for the written qualifying exams, which will determine whether he will

be allowed to continue on in graduate school. (Belnap, 2005, p. 13)

Aside from their teaching assistantship duties, the research participants were required to take their own courses. Typically, the coursework in this department consisted of four to six courses for a master's degree, and six to eight courses for a PhD. Master's students tend to complete their coursework in their first year, spending one to two years afterwards working on their research projects. The courses were often quite demanding of the students' time and mental energy. Depending on whether or not they earned their master's degree at this university, doctoral students often took two years to complete their coursework and spent two to three years on their research.

The graduate students in this department were also expected to be working on the preliminary stages their research in their first year. This consisted of reading articles outside of their coursework and meeting with their supervisor to discuss ideas and directions for their research. The amount of time spent with their supervisors varied greatly, from a few meetings each semester to as many as three meetings each week. Graduate students in the applied mathematics doctoral program were not required to take comprehensive exams, whereas pure mathematics doctoral students were, which represented a substantial workload. In addition to their coursework, both applied and pure mathematics doctoral students had to pass a candidacy exam based on their proposed research topic. The following subsections explore the participants' experiences in the traditions and structures of their coursework and research.

Structure and traditions of graduate-level coursework

The research participants were at varying stages in their coursework. Emily and Sara were just beginning the courses for their master's degrees, Steven had only one course left to take, and Chris, Robert, and John had completed their courses. Consequently, Emily and Sara had the most to say about their coursework, as the doctoral participants had passed that phase of their programs. Sara had completed her bachelor's degree at the site of this research project, but Emily was entirely new to the environment and she contrasted her undergraduate experience to what she was now going through:

> Oh, I worked very hard in my undergrad. Like they worked us hard, but somehow it's even harder here. Like I always say, it can't get any harder. There's no way. But it always does.
>
> I've been more discouraged about it in this first term than I've ever been before and I think it's just because it's that much harder. And I'm also in a new place and I don't really know anyone yet, so …
>
> And it's different. Like it's a different type of trying to figure it out. And I haven't quite done it, figured it out, I don't think. I'm hoping that by next term maybe I'll do better. But, I don't know.
>
> I haven't quite figured out how to like, how it works, yet. You know? Like where you need to be spending the time and the effort because there're these days that are so much more stressful than any of the undergrad days. Then there are these days which are kind of like what are you doing when you're so burnt out that you can't because you've been working so hard, that you can't even make yourself look at it. Well, you can, but it doesn't, it's not good work at that point and … I don't know. It's like waves. It comes in waves, big crashing waves.

What I heard in Emily's voice was the same culture shock I experienced in my first year of graduate studies, almost a feeling of being knocked from my foundation, from what I knew about mathematics and what I knew about myself as a student. What is it about studying mathematics in graduate school that makes it feels so different from one's undergraduate experience? The number of courses in graduate school tends to be fewer than what students take during their undergraduate programs. And I do not believe that the upper-level, undergraduate mathematics courses are much easier than graduate-level courses. It is hard to pinpoint exactly what is different – the coursework, the level of learning, the amount of homework.

If we revisit Belnap's (2003) description of a mathematics graduate student's life – "in addition to spending time preparing for class, teaching, preparing/grading assignments and exams, holding office hours, attending course meetings, and tutoring, he will be: taking nine units of difficult core graduate mathematics courses; doing his homework; preparing for his own exams; and studying for the written qualifying exams" (p. 13) – we see that graduate student life as a whole can be significantly more complicated than what an undergraduate experiences. Thus, it may not be only the coursework that makes life difficult, but the combination of the new duties that graduate students must attend to.

Beyond the level of difficulty they experienced in their coursework, the participants talked about the amount of work in their courses and the time the work required. Sara spoke a great deal about this, describing how she and fellow graduate students were often "down" because:

> We don't feel that we have enough time to understand things really in depth. You know, really get to the bottom of things and understand what is divergence.
>
> When I do all these other things – teaching, going to class, doing homework assignments – I split my time way too much and I cannot really go in depth.

Sara spoke several times of going “in depth,” signifying an importance for her in understanding mathematics at a deeper level. What came through in her voice was a passion, a profound desire, to not only know more, but to gain an understanding of mathematics. This sense came through when she spoke of having to push herself in order to complete her work:

> I learned what I had to. But it’s not the best way of learning because you don’t remember stuff. You realize you can push yourself far. You can do, it seems, almost anything if you have to. But, apart from that, I don’t know how much you truly learn.

The language Sara used here is interesting to me – talking of learning what she “had to,” but that pushing herself to do things did not necessarily mean that she had “truly” learned.

Did she learn or did she not? Or is there an in-between space of learning? What might her language mean about her learning of mathematics? In her statement, there was a difference to her in the way she learned things. I believe the first sentence “I learned what I had to” represented solely a knowing of mathematics and that by pushing herself she came to know a great deal. But what she desired was to “truly learn,” which corresponded to the in-depth understanding she spoke of previously. If I compare the meaning of *learn* with the meaning of *understand – learn:* to acquire knowledge of or skill in by study, instruction, or experience, to become informed of or acquainted with; ascertain; to memorize; *understand:* to perceive the meaning of; grasp the idea of; comprehend; to be thoroughly familiar with; apprehend clearly the character, nature, or subtleties; to grasp the significance, implications, or importance of (www.dictionary.com, 2008) – I believe this is the difference in experiences that is suggested by Sara's statement. This distinction came through again when she spoke of wanting to see and understand the "big picture" of the mathematics she was learning, described below.

Sara expressed how the perceptions of her learning (i.e., understanding concepts) versus not truly learning (i.e., only knowing how to solve problems) were beginning to have an effect on her:

> It's like you start questioning everything – why am I doing this right now, like why am I spending two days on this thing, is it going to make any difference? Really now, if you think about it. If we sit down and we talk about this, does it make a difference? It made me feel good because I got pretty pictures in the end, but I mean this is a silly example, but often there'll be some things where you wonder how is this ever going to become useful? Why am I doing this?

While the accomplishment of doing the problem successfully and "getting pretty pictures" made her "feel good," what came through in her voice was not only a desire, but also a need for more than that. She began to question the usefulness of her coursework, twice asking if it was going to make any difference – is doing a mathematics problem going to make a difference and, if she were to speak with other mathematicians about the problem, would it make a difference?

What would such a difference have looked like or felt like to Sara? The discrepancy between how she wanted to learn and understand and what her coursework required her to do had caused her to reflect on an academic life in mathematics:

> Would I just be publishing papers, making slight improvements on an already-existing theory and not really making a difference? And there's all these other things that I would like. I'd like to work for the UN. I don't know, so we'll see…

Sara’s interests went beyond mathematics to helping the world in some way, such as working for the United Nations, helping to create technology that would better the lives of people in third-world countries, or finding ways to help the environment. She hoped to use mathematics in those efforts and she wanted to see a big picture of how the mathematics she was required to know might be applied in some way to help the world. Yet, her experiences in mathematics caused her to see it as a “narrow path,” where “now there’s just a small group of people listening to you.” Sara’s experience was similar to Stage and Maple’s (1996) findings where mathematics graduate students “described a growing frustration with the seeming lack of connection of mathematics with the world surrounding them” (p. 32).

The sense I have from the graduate students is that they had a genuine desire to learn and understand the material that they were being taught. In fact, they were quite passionate about and interested in their own learning and had their own ideas of how they might learn best. However, the organization of their time, imposed by various structures and traditions of the department, along with the amount of work required in their courses, did not allow for the level of understanding they hoped for. They were left without a sense of purpose about their homework assignments. Either they were unclear about what learning they should expect from their assignments or the purpose was not made explicit, and, for most of them, their coursework became an exercise in pushing equations around or solving problems that had already been solved. Similar to the graduate students that Stage and Maple (1996) interviewed, the research participants felt that their coursework was "an endless series of puzzles that could be solved if enough time or effort was invested. Sometimes solving problems did not seem to lead to the learning of mathematics but merely represented the results of that investment of extensive time and effort" (p. 32). Without understanding the meaning of continuing to do homework assignments at the graduate level, they did not see what they might be learning and how they might be growing in their knowledge of mathematics.

(Lack of) Structure and traditions of research

For Emily, Sara, and Steven, the participants who were still taking courses, there was a sense that doing the mathematics of their coursework did not represent authentic mathematical work. They spoke of creating their own mathematics as the experience they were now hoping for, and that their time would be better spent working on their own rather than going to classes. Emily, in her first semester of graduate studies, expressed her desire to be done with classes and working on something else. Compared with the certainty of the expectations of coursework, though, research presented a lack of structure, a new set of unknowns, and uncertainty about what direction to go. At the time of the research study, Emily had started her first steps into exploring what her research project might be:

> So I have articles to read, you know, and there's no one breathing over, looking over my shoulder going "Have you read that yet?" So I open them up, I get through about a page and a half and it's like, "Well, this is really boring." And I think I need to start emailing her [supervisor], and say, "Okay, we need to like do something" because she's given me a list of probably about twenty

> articles to look through and I think I don't know which ones I'm supposed to do. So I don't know whether that's bad on her, bad on me, bad on what, like, or it's just not working.
>
> Um, I'm very interested in getting a project. I really want a project to look into. Like right now I'm just, like she keeps telling me, "Oh, it's early, don't worry about it. It's early. You don't need to have a project yet." But I'm very much like, I just feel so all over the place, you know, like I read this article, but then the next one I read is completely unrelated and the project I might do might be completely unrelated to both of them. And I know that I'm supposed to be becoming more mathematically balanced perhaps by doing this, but …

Emily, who had yet to choose a research topic, spoke about spending time reading articles that did not appear to be related to each other. Without having specific guidelines for what to make important, she found that she often gave up quickly when attempting to move forward with her research. Emily wanted to be finished taking courses, to move on to what she saw as the real learning that would happen in research. On the other hand, though, she either did not know how to learn from the articles or may not have realized that she could learn from the articles. Yet, what drew my attention to her statement was when she said, "I'm *very interested* in getting a project. I *really* want a project to like look into" (emphasis added). Her language was quite resolute and determined here. What Emily might have been experiencing was not only the desire to be done with coursework, but also the impending pressure of research and publishing that graduate students and new PhDs face, where "they are likely to feel that their professional lives, however fulfilling in other ways, have been of a lesser sort than those of scholars who have contributed something new to their fields" (Kronman, 2007, p. 94).

In comparison, though, Chris, a doctoral student, reminisced about his coursework and how that experience was easier or more familiar than his research project. He had recently completed his coursework and was beginning his research. He spoke of knowing what he had needed to do in order to be successful in his coursework, but he also shared uncertainties about doing research:

> I do well in my courses when I take courses and I work really hard on them. I guess because I, I'm trying to do research and I just find that I get stuck too easily. It's hard. I guess it's hard to do research. When I do my coursework I have a problem and I know how to approach something, right, because I'm in the course. So I guess it's reassuring to know that something is, to know what I'm supposed to be proving is true.
>
> I guess in research I get nervous that, or I worry too much that maybe what I'm doing is wrong. Or just maybe, maybe I do okay because I do well in my courses and I can solve pretty hard problems that they give, that they assign. But, I mean those exercises are supposed to be do-able.
> I guess I worry that, you know, maybe, maybe it's [the research problem] not do-able, maybe this problem is impossible, or maybe it's too easy or maybe … but I think in the end I just don't know, so then it makes me not work hard enough, I guess.

Chris felt confident in working on assignments because he knew the problems would always have solutions and he felt that he could solve the problems with enough work. Research brought unknowns that seemed to shake his confidence, particularly in not knowing whether his research problem would have a solution and how much of his time might be wasted on false starts. He described how the known structure of homework assignments prompted him to work hard towards a solution, whereas the new and unfamiliar structure of research caused him to avoid working on his project.

The distinction Chris made between a problem in a course and a problem in research is interesting to me. What came through in his statement was that he knew that he could work very hard and be successful, and had done so in his courses. He had solved very difficult problems. But something I observed, which

Chris did not seem to acknowledge, is that, at some point in time, the problems he had solved in his courses were questions that did not have solutions and someone had worked to find those solutions, without the knowledge that a solution existed. I wanted to understand why this disconnect might exist for him. What Chris might be experiencing here is what I wrote about in Chapter 2:

> Gone from the page is the person who discovered the mathematics, along with their joys, passions, and frustrations. This language of mathematics that is presented in public forums is free from the human perspective that brought it about and conceals the missteps that occurred on the path to its creation. With regard to the person behind the mathematics, they and their struggles and efforts are often not present in their own creation. As Davis and Hersh (1982) describe the mathematician: "His writing follows an unbreakable convention: to conceal any sign that the author or intended reader is a human being. It gives the impression that, from the stated definitions, the desired results follow infallibly by a purely mechanical procedure." (p. 36)

The mathematics that Chris had seen over the years did not include the difficulty, the missteps, and the hard work that went into solving the problems. He had always known that problems could be solved. I believe the understanding that came through observing and experiencing mathematics in such ways is what brought Chris to say, with some surprise and disbelief, "It's hard. I guess it's hard to do research."

The space of doing research was complicated for both new and experienced graduate students, where new graduate students wanted to move into research because that was what they saw as important in becoming a mathematician, while more experienced graduate students longed for the relative certainty of coursework. Within the structure of doing assignments came the reassurance that Chris would be able to solve problems, which was something that Emily and Sara both found to be disheartening about their coursework. If they had to continue to work problems that had already been solved, were they truly learning anything? Were they doing the perceived authentic work of mathematicians? While Chris had completed the part of his graduate program where he worked on assignments, he longed to be back within that structure. In comparison, Emily and Sara, who had not yet finished their coursework and had yet to begin their research projects, no longer saw the value of doing assignments, with working in certain ways.

More About Their Lives

Aside from the parts of their lives that I have included above and what they discussed about learning mathematics, being teaching assistants, and their ideas about research, several of the participants described what their lives were like in graduate school in poignant and somewhat dispiriting ways. Emily, as a first-year master's student, talked about her graduate school experience as being quite often scary. Sara, also a first year master's student, spoke at length about how life in mathematics was not a life that would allow her to better the world in the ways she hoped to. She saw the life of a mathematician, of working on particular research problems in mathematics, as isolated and contributing to the world in minute ways at best. In confirmation of Sara's intuition about life in mathematics, Stewart (2006), a lifelong mathematician, acknowledged that "much of a mathematician's work is solitary, even lonely" (p. 123).

Interestingly, Emily, Sara, and Steven, all master's students, were unsure and hesitant as to whether they would continue in mathematics. While they all planned to earn their master's degrees in mathematics, with which they could explore other options outside of mathematics, they felt earning a PhD in mathematics would restrict them to a certain life. Their hesitation about continuing in mathematics echoed my own experience in a department of mathematics, where none of the seven students in my master's program cohort remained in the program to earn their doctorate. What I observed, and what I believe my peers observed as well, was not only a life of isolation, but also a life where we would not be able to explore our roles as teachers in the ways we wanted. In that particular department, there seemed to be very little enjoyment or happiness about being a mathematician and what that meant for being in the world.

Herzig (2002a) wrote about the emotional side of mathematical work that graduate students encounter. In particular, she described the experience as disheartening and depressing. John, a fourth-year graduate student who had recently passed his qualifying exams, described a similar experience of going through a doctoral program in mathematics:

> I guess if … as far as if I would suggest to someone if they, if they can live with the kind of rollercoaster of reward, they can probably do it. I mean, there's just so many low points that you have to be able to deal with. Most of the people that I've seen that can't hack it, they just can't get over the low points.

He spoke of the lows being incredibly low and about how a line of work that one might spend a great deal of time on could eventually be of no value for their research. In contrast to John talking about the lows of mathematical work, Stewart (2006) alluded to the "highs" of research with the statement, "You need to be something like an addict for this feeling to provide sufficient recompense for the all the work" (p. 123). Stewart recognized the hard work that is involved in achieving such a "high," and disclosed his own uncertainty as to whether or not the reward is enough. To John, even for those who could sustain the rollercoaster ride that he described, he was unsure that they would be successful, saying "they can probably do it," conveying a sense that even if someone could adapt to the lows, there might not be enough to hold a person to mathematics.

Sara also revealed the emotional nature of mathematical work Herzig (2002a) wrote about by describing how being a graduate student in mathematics had influenced her:

> I felt like, that I guess the system or something just drew the love out, all the energy because it was just fed fast without any appreciation for the beauty of it or for the usefulness. I guess this is how much you need to learn in three or four years, so let's just put it into your brain and you'll be done with it. So, it lost the beauty there.

To Sara, mathematics had been a beautiful thing, but through learning by being "fed fast" and "putting things into her brain," it had lost its beauty and meaning.

A price to be paid

Herzig's (2002a) work investigated the reasons for the large attrition rate of doctoral students from mathematics. What she found was that the graduate students described "portraits of isolation and lack of social interaction, expectations that involved extensive time commitments, and few interests outside of mathematics. These former students reported how isolating graduate study had been" (p. 35). Emily shared similar thoughts about how she viewed a life in mathematics:

> It's not really the academic part that really intimidates me. It's that I wasn't sure that I wanted that life [in mathematics]. And I'm not really sure I want that life yet. You know? Like it's kind of isolated to a certain extent. But we'll see. I don't think it would be a terrible life, I think.

Emily revealed her uncertainty about continuing in mathematics. She drew an interesting line between two sides of the life of a professor of mathematics. To her, there was the academic part, by which she was not intimidated, but also the mathematical part, which she viewed as being isolating. Emily's feelings of isolation were represented in Stage and Maple's (1996) study of mathematics graduate students, where one student said, "Getting your PhD in mathematics means spending your life in a closet with a light and a desk… which is another way of saying isolation" (p. 33).

Emily's statement, "I don't think it would be a terrible life," presents what I see as a lower bound for what life in mathematics might be like, that if it were only as bad as "terrible" then it would be okay. But to enter a life where there is an expectation or perception that life could be terrible is to me quite disheartening, and it signifies a surrendering or resignation of a life that might be exciting, wonderful, and connected to the world. While Emily presented a lower bound of how life, at a minimum, could still be bearable, she did not share what an upper bound might be like for a life in mathematics.

Sara spoke of the sacrifices she needed to make in order to have the kind of success or understanding she hoped for:

> If you take the time to really understand everything, to understand every term and understand how it relates to reality, then you will have to give up the million other things you want to do, like play guitar, etcetera. In order for me to feel like I'm really learning something in depth, I feel like I have to kind of cut off the rest of

> the world and just do that, do math because we are required to learn a lot.

As I explored earlier, Sara had hopes for how she would learn mathematics – she wanted an in-depth understanding of the material. Here, though, I took notice of what that kind of learning required under the weight of the amount she needed to learn. In order to understand mathematics in the ways she wanted, she had "to give up a million other things" and "to kind of cut off the rest of the world." Yet, she had previously expressed how she wanted to connect the mathematics she was learning to the world, to find how mathematics could help the world. It appeared that she thought she could not have it both ways – she needed to attend to the mathematics or the world, but not both.

Herzig (2002b) found that for doctoral students in mathematics there was guilt "over having other commitments in their lives, and the difficulties of balancing a life outside of mathematics with the demands of the program. Some students who were close to completing their degrees said that they could have been done more quickly if they had been more focused on their work, but that it would not have been worth the price to them" (p. 186). This was heard in what John had to say about the choices he felt he needed to make between being a successful, famous mathematician and having quality of life:

> Yeah, I mean it's, it's poor quality of life, right? You don't do math, at least I don't do math, for the sole reason of doing math. I mean I enjoy it a lot, you know. It's something that I like, but, I mean, if it starts to hurt my life, I'm going to quit. Like, you know, it's like there's … I guess it depends on what your priorities are. I mean I know mathematicians who do math seemingly twenty-three hours a day, you know, and, I don't know, they go on coffee breaks the other hour and they never sleep …

John's description of the mathematician who does mathematics "seemingly twenty-three hours a day" is not entirely an exaggeration. In one department of mathematics where I was a graduate student, there was a professor who spent weeks at a time in his office, only leaving to teach a class, microwave his food, and use the restroom. This is an extreme example, but I hope that it conveys a sense that John's perception is somewhat based on reality. There are indeed mathematicians who appear to work twenty-three hours a day.

Beyond that perception, though, is John's declaration that he would quit mathematics if it started to hurt his life. Why would he feel that mathematics might begin to hurt his life? Does he feel that those mathematicians who work twenty-three hours a day are harming in their lives and that is a life that he does not want? Does mathematics require a surrendering of what is important in one's life? Is there an in-between space where one can maintain a healthy concentration on mathematics that does not impair one's quality of life? It appeared that John's view of his quality of life went beyond his achievements and work in mathematics. His sense that he would have to give up mathematics if he could not have the quality of life he wanted revealed John's feelings that we would have to make a choice between the two. Interestingly, the sense of having to make a choice was also found in Stage and Maple's (1996) work where a mathematics graduate student said, "If, for whatever reason, I thought that my marriage was suffering because of this PhD program, I would quit right now. […] I never thought I would say 'If I had to make a decision between my husband and my child, or this PhD, I'll give up. Just like that.'" (p. 35). John similarly spoke of his own marriage and how he would choose it over a life in mathematics.

In Emily, Sara, and John's voices I heard that there is a price to be paid for a life in mathematics. For each of them, that price meant slightly different things. But for all of them, though, it seemed to mean a choice between mathematics and the life they wanted or hoped for. As Kronman (2007) wrote, "Whatever the discipline, graduate students are taught to accept the limits of specialization and to see these as the price that must be paid for the powers and opportunities it affords" (p. 94). Thus, it appeared there was a significant cost of being in mathematics for some of the research participants in this study.

Conclusion

The structures that the mathematics graduate students encountered had implications for how they were able to move forward in graduate school as learners, as teachers, and as new members in the community of mathematicians. Within the structure of their coursework, their teaching assistantship duties, and their forays into research, the participants had to surrender their own ideas about their learning and the ways in which they hoped to interact with students. These structures seemed to produce within the graduate students an emotional response that affected them significantly in many ways, on deeper levels, with regard to their identities and how they felt in the world, which I explore further in Chapters 7 and 8.

One hope for this project was to find understanding that would help the inquiry concerning why education programs for mathematics graduate students had failed to instill hoped for changes in future university teachers of mathematics (e.g., Belnap, 2005; DeFranco & McGivney-Burelle, 2001; Speer, 2001). From the structures explored in this chapter, it appears that the whole of the mathematics graduate students' lives prevented a focus on teaching. Thus, if a program were to be introduced to the group, the structures, time, and encouragement necessary to support learning about teaching do not currently exist.

Chapter 7
Small Talk Becomes a Problem

> Mathematics is a cruel profession. Solving a mathematical problem is for most mathematicians an arduous and lengthy process, which may take years, even a lifetime. The final conquest of the truth comes, if ever, inevitably tinged with disillusion, soured by the realization of the ultimate irrelevance of all intellectual endeavor. For Stan Ulam, this process took place instantaneously, unremittingly, day and night, as a condition of his being in the world. (Rota, 1986, p. 240, speaking about a fellow mathematician,)

Burton (2004) stated, “There is a myth associated with mathematicians that they are born, not made, and that their life histories are necessarily particular to mathematics. […] The myth of born not made is alive and well.” (p. 34). Often there is a perception that mathematicians are cut from the same cloth. Yet, the participants' varying backgrounds in and experiences with mathematics conflict with this perception. It seemed that even with their diverse histories, their wide-ranging physical appearances, and their interests outside of mathematics, through their experiences in becoming mathematicians, the graduate students were being knitted or woven into a fabric in which individual pieces became almost indistinguishable from the others simply by being identified as a mathematician.

Are mathematicians different from other people, other human beings in the world? Does the choice to study mathematics shape someone to be unlike others, so that they no longer feel comfortable revealing that part of themselves? With the perspective that “We do not and cannot exist as an isolated, individual mind or consciousness. We belong to a society and culture in the sense of unquestioning interiorization of its norms and ways long before we have the capacity to reflect and criticize” (Crusius, 1991, p. 15), in this chapter I would like to take a deeper look into the lives of these future mathematicians and how a life in mathematics has meaning for their identities in a way that allows reflection and questioning of their lives as future mathematicians.

Before progressing, it is important for me to make a distinction between being in mathematics and being mathematical. While I recognize that most, if not all, people are mathematical and engage in activities that are mathematical, I believe that there is an experience particular to individuals who describe themselves as mathematicians and who, through various actions, behaviour, and work, belong to a group of mathematicians. I am putting forward a notion here, and I will explore it in this chapter, that being a mathematician is a particular and unique experience of identity. The mathematical knowledge of the participants is not surveyed here, nor are their mathematical abilities. Rather, I will investigate the experience of *being in mathematics as a future mathematician* – how that particular name or label, together with the experience of how mathematicians go about being in the world, sets them apart from others. Therefore, in the following sections, I use the words *being in mathematics* to describe the research participants as they live in a world of mathematics, how they are in the process of becoming mathematicians, and what that might mean for their for lives, for who and how they are in the world.

Strange In and To the World

> Some of you may have met mathematicians and wondered how they got that way. (Thomas Andrew Lehrer, www.absoluteastronomy.com/topics/Mathematician, 2009)

One theme that came through in the conversations was the sense that being a mathematician meant being somehow different in and to the world. Being a mathematician was a complicated and conflicted space of being set apart and unusual, of belonging, to some extent, to a distanced group of brainy eccentrics who, despite their mathematical abilities, were unable to function normally in the world. The participants' sense of difference was sometimes welcomed and held onto, while at other times it was felt as a burden. Gutmann (2000) observed a similar sentiment among the members of a department of mathematics, noting, "the mathematics faculty labor under a burden of being mathematicians" (p. 86), while at the same time they enjoy the stigma.

To the participants in the study, being a mathematician was sometimes experienced as something that made them better, smarter, and set apart from others in a superior sense. At other times, though, some of the participants were pained by the otherness they felt and distanced themselves from mathematics and their particular interests in the discipline. In the conversations, I noticed a significant oscillation in how they felt – what I would describe as a pulling towards and pushing back between their feelings of the safe and familiar space of group membership of mathematicians and the space of an outcast, an eccentric, of being other. Where might these feelings of the otherness of being a mathematician come from? Did the confluence of these feelings play a part in how the participants felt about being in the world? Did these feelings come into play in how, whether, and when they revealed themselves as mathematicians?

Mathematicians in the media

In this chapter, I focus on being a mathematician, on what it means to be a mathematician in the world. However, not much literature exists that explores the being of mathematicians, as Burton (2004) found that there "have been few investigations into 'who is a mathematician?' other than someone who does mathematics" (p. 33). Rather, much of the literature that concerns mathematicians focuses on a more epistemological perspective in how mathematicians think about and come to know mathematics and particular mathematical concepts (e.g., Burton, 2004; Byers, 2007; Nardi, 2007). The literature that does address who mathematicians are, who and how they are in the world, often present stereotypical images of mathematicians. In investigating what it means to be a mathematician, I saw most resources coming from stereotypes in the literature and the media.

With this in mind, what does it look like to be a mathematician in the world? Are mathematicians portrayed differently from other people? A common view of a mathematician is "an unkempt, glasses-wearing, balding middle-aged white male" (Higginson, 2006, p. 134). Beyond stereotypes such as this, however, are more somber examples of who others think mathematicians are. Observe some of the mathematicians who have been taken up as symbolic of those in mathematics. Theodore Kaczynski, the Unabomber, was a mathematician. When he was apprehended for his crimes, his career in mathematics was mentioned often, even though he had resigned his position as a mathematics professor almost ten years earlier. His behaviour was described as typical of a mathematician – brilliant but withdrawn, isolated and embittered. Andrew Wiles, another mathematician, became a focus of attention in the world when he proved Fermat's last theorem. In the *Nova* documentary *The Proof* (Lynch & Singh, 1997), he was portrayed as odd and almost absurd for spending seven years of his life working in isolation on a proof only a few people in the world would understand.

The motion pictures that depict mathematicians also do little to encourage positive attitudes about them. A fairly harsh example is the movie *Pi* (Watson & Aronofsky, 1998), which is about a brilliant mathematician and computer scientist who is paranoid and spends time only with a fellow mathematician. The movie concludes with the mathematician causing himself debilitating harm, so that he can no longer physically and mentally function in society, forcing a complete isolation from the world. The movie *A Beautiful Mind* (Grazer & Howard, 2001), about real-life mathematician John Nash, who developed Nobel Prize winning theories about game theory in economics, focused on his battles with schizophrenia.

Contemporary portrayals of mathematicians depart to some extent from the stereotype of a white, privileged, middle-aged man, as though to say "one should not assume that mathematical ability comes only in certain preconceived personifications" (Higginson, 2006, p. 138). For example, in the movie *Good Will Hunting* (Bender & Van Sant, 1997), we are introduced to a young, white man of lower socio-economic status who was exceptionally talented in mathematics despite not having had a formal education. In the play and movie *Proof* (Hart & Madden, 2005), the brilliant mathematician this time was a white woman, the daughter of a famous mathematician. However, despite the addition of these new faces in the mathematical world, the image of the troubled mathematician remains. The young man in *Good Will Hunting* was unsettled, lonely, and prone to violence, while the young woman in *Proof* feared that the mental illness that haunted her father would lay claim to her as well. In these examples, despite the somewhat varied physical appearances and circumstances of the mathematically gifted, what is inferred is a notion of mathematicians as mentally imbalanced and unstable.

The weight of the negative portrayal of mathematicians was felt by at least one of the research participants. John had alluded to being perceived as strange or even crazy by others and I asked for some clarification of this:

> Mary: One thing you mentioned briefly in our first conversation that when you tell people that you're in math, you're kind of seen as nutty. Like you said math is kind of seen as being nutty …
>
> John: Yeah.
>
> Mary: I was just wondering if you could explain that a little more. Like where that …
>
> John: Hmm.
>
> Mary: Do you feel like when you introduce yourself as a math person, like what kind of response do you get?
>
> John: Well, a lot of response, of course, is, "Oh, I hate math" or "I could never do math" or "Wow, I can't believe anyone does that" or, you know, or they … or you get the stereotype, you know, of the TV math personalities, or in movies, you know. And I think this is why they think you're a little weird. Because, because, it's just we've been typecast …
>
> Mary: Yep.
>
> John: … and maybe some of that is, I think some of it is true a little bit, at least the social side.
>
> Mary: How so?
>
> John: I mean, well there's a lot of mathematicians which, which have social problems.
>
> Mary: Yeah.
>
> John: I mean and I think it's a higher percentage than just your normal population and those are the ones that stand out.

> Mary: … have you seen that a lot?
>
> John: Yeah. Sure.
>
> Mary: How would you describe, is there a generality?
>
> John: I think people just… they [mathematicians] have hard times holding a conversation. Small talk becomes a problem. Just people seem awkward when you're standing there talking to them. And I don't know if, if they do math, math has appealed to them because they don't have to deal with people as much or if it's the other way around – you don't deal with people a lot so, and you're kind of in your own little world a lot and that's different from the real world and maybe it's hard to transition. And I think maybe that's a part of it, too. I mean I find myself, it's harder sometimes, I have a harder time with some social interaction as things go on. But I don't know if it's, and that could just be getting older and, yeah, I think that could just be getting older, too.

John spoke of how the stereotypes of mathematicians seemed to influence others' perceptions of him: "And I think this is why they think you're a little weird." In explaining why this might be the case, he at first attributed it to being "typecast," but then acknowledged that the typecasting might be somewhat true to reality. His own perceptions of mathematicians are interesting here – that they have social problems more often than others, that they have "hard times holding conversations" and that they are "kind of in their own little world." In the last piece of the dialogue, he wondered aloud whether this same behaviour he saw in himself was because he was in mathematics. He then sidestepped being in mathematics as a reason, attributing his own difficulties in social interactions to getting older.

In this portion of dialogue, John at first shared his self-identification with the group of mathematicians when he said "*we've* been typecast." While the conversation progressed, though, I noticed that he began to distance himself by talking about mathematicians as a group separate from himself, no longer using the word *we* to describe how mathematicians are construed and instead talking about *them* in such a way that conveyed mathematicians as other, as separate from himself. John's change in the use of pronouns is noteworthy, as his dialogue changes from *they* to *you* to *I*, which to me signified a wavering in being in the group membership of mathematicians. While he seemed to bristle at being associated with the typecasting and stereotypes of mathematicians, he also recognized a tendency for mathematicians in general and for him in particular to be socially awkward, but he moved to keep his behaviour and that of mathematicians unrelated. However, it could also be a truth about himself he was unable or unwilling to accept.

Tinto (1993) stated, "as the very character of the field of study is conditioned by social judgments about the norms of the profession," becoming a mathematician "can be viewed as a form of social initiation into a group whose members have a vested interest in maintaining the norms of the group" (p. 255). With this perspective in mind, I began to think of some questions when reading John's last statement. Does mathematics appeal to those who are socially awkward because it involves or requires working alone? Or do people working in mathematics become awkward because mathematical work is a seemingly solitary endeavor? In other words, do the norms or stereotypes determine the mathematician? Or does the mathematician who, in having a "vested interest in maintaining the norms of the group," participate in the upkeep of the stereotypes or norms by behaving in particular ways?

Herzig (2002a) concluded that to be successful in his or her PhD program "a doctoral student needs to adopt the identity of a mathematician" (p. 40). Here, John saw similarities between his behaviour and that of particular norm of the group of mathematicians, revealing a possible investment in preserving the stereotype by "adopting the identity of a mathematician." The distance he created between himself and the perceived norm or stereotype disclosed a pulling towards and pushing away of being a mathematician and what that might mean for who he is or who he may have to become. Aside from feeling set apart by others because of stereotypes, there were other ways the participants felt different in the world, some of which are addressed in the following section.

Being different, being other, because of being in mathematics

> The conclusion of most people seems to be that anybody who moves in such an intellectual realm must be equally removed from life in other areas – social, moral, ethical. (Brooks, 2009, p. 1, relating the experience of how mathematicians are perceived)

Some of the familiar responses to letting others know that my career is in mathematics and mathematics teaching are, "I was always bad at math" or, more severe, "I always hated mathematics." These responses were also common for the participants in the study – introducing oneself as being in mathematics to an outsider to mathematics and at once becoming framed as very different from them. In admitting to your choice of mathematics, you become unusual, one of the few people who not only understood mathematics, but also continued to study mathematics even when you could have chosen to do something else.

Higginson (2006) gave a sense for what one experiences when revealing oneself as a mathematician: "There is, of course, something uncomfortably familiar about the baleful looks perennially cast at the subject, its institutional purveyors and, most certainly, its perpetrators" (p. 127). In my experience, in the moment of telling someone I am in mathematics, that I am a mathematician, there is an alteration in the feeling between me and the other person, in the connection that might have been possible between us. Often the response is a look of surprise or sometimes disdain. I begin to feel different, strange, and even sometimes abnormal, as though somehow I represent the mathematics teachers, exercises, and concepts that the person disliked in school. As quickly as conversations begin, it seems as though there is very little to talk about, very little to share. Despite outward appearances, when revealing yourself as a mathematician almost at once you become something other, different, strange.

The participants spoke of their feelings of otherness, of feeling different, around people who were not in mathematics. At times, they attributed these feelings of being different in the world to a perceived social acceptance of mathematical illiteracy. I first encountered such feelings in a conversation with Chris:

> Chris: This is not my observation but, you know, it's like if you, if you're a writer and you tell someone that you're a writer, they don't say "Oh, I can't read, I can't write."
>
> Mary: Right.
>
> Chris: "I'm a bad speller" – you wouldn't say that to someone.
>
> Mary: No.
>
> Chris: There's some sort of pride when you can't do math, I guess.

This perspective was also heard in a group conversation:

> Robert: Yeah, and it seems like, you know, that's one thing. I think I forgot to mention it to you last time is that it seems like the inability of doing math is kind of … what is the right word? Regularized now?
>
> Mary: Yeah, it's socially acceptable.
>
> Robert: It's socially acceptable, where someone who cannot read, cannot write, who are illiterate, they will feel ashamed. So, I think to a large extent, it's social attitudes. And so math ability it seems like you, I guess, for lack of a better word, well, you can do math, you're kind of like a freak. I may be exaggerating, but …
>
> Steven: No, I agree. I'm a freak. No, somebody at [the university] gave a talk about mathematical illiteracy versus actual illiteracy and she's like,

> yeah, exactly what you said – if you're illiterate, you're like an outcast to society. But if you're mathematically illiterate, people like sympathize with you and they're going to give you a hug. It's okay. We're all equally illiterate.

The participants described what they observed in others as pride in the inability to do mathematics, a social acceptance of mathematical illiteracy, and construed what that meant for who they are, as people who are in mathematics. They made a comparison between illiteracy and mathematical illiteracy, conveying their views that these should be on equal footing. To me this contrast exposed their sense of the importance of mathematics, their pride in being mathematical, and served as an argument that who they are is valuable and useful. Yet, being in mathematics made them different from a perceived large group of people who are mathematically illiterate, so much so that they described themselves as strange and Steven, by staking a claim to the label of "freak," revealed a willingness to be seen and characterized in such a way. Still, I heard a sense of unfairness and even a grievance that came across in these parts of the conversation, as though the participants and mathematics had not been given fair consideration by those outside of it.

There was also an assumption by the participants that other disciplines would not encounter the same resistance from others that mathematics would, as though, in this regard, mathematics resided in a space separate from other disciplines. At various times in the conversations, though, the participants talked about how they had resisted and disliked other disciplines. This resistance was true for me as well. As an undergraduate studying mathematics, I argued to several of my writing teachers that, as a mathematician, I should not be required to take writing courses and I very much resisted what I was supposed to learn. Some of the participants railed against what they viewed as the triviality of other disciplines, such as biology, or they wondered why they should have been required to take courses in history. In this respect, I observed a noteworthy forgetfulness of their own resistance to other disciplines that helped them to keep mathematics and their place in it as an experience separate from other disciplines.

The participants expressed more of their feelings of how being in mathematics made them different in the world, in the ways they thought about things, and the ways in which they were compelled to interact with others:

> Sara: You're [a mathematician] definitely-detail oriented, but then I mean, I don't know. I have conversations with all sorts of people and often I'm the one who's very logical and who will find a flaw in your argument and then I'll ask you "But what about that and that's wrong." Or, you know, detail-oriented, but then other people are, have a lot more just creative random ideas about life, the world, while I'm here, stuck in my own little thing, you know, very narrow mathematically that I cannot really share with other people.
>
> Emily: Yeah.
>
> Sara: So, you're good at seeing this one thing that you're doing and that's great. And that does give you some qualities that you can use, like detail-oriented logical thinking, but you do lose a lot of other kinds of things.
>
> Robert: Well, yeah. Actually I can relate …
>
> Sara: And it depends. How much time do you hang out with math people or other kind of people …
>
> Robert: That's true.
>
> Sara: … because if you don't hang out much then you probably just don't know.
>
> (Laughter in the group.)
>
> Robert: Yeah. I was, I was …
>
> Mary: That you're hanging out with people who aren't mathematicians, then …
>
> Sara: Exactly, yeah. I mean we're all ignorant in different ways but it comes from where I stand so …
>
> Robert: Well, one person … One time I remember that my wife told me that, well, what she observed in terms of how I talked to other people and

how they react to me when I was talking to them. She said there was some people who doesn't like the way that I talk and some people would react really positively. And she said the difference, she thinks, is that I'm very good talking to people who are, who are patient, because I tend to, maybe I didn't know this myself, but I tend to talk in very, very logical manner, like very detail-oriented and logical. And so if I talk to someone who is patient, they start to think "Wow, this guy really knows what he's talking about because his whole argument seems so …"

Mary: Well-structured.

Robert: Seamless, well-structured. And she said people who don't react to me well or maybe not that enthusiastically, if I can put it that way, are people who just don't have patience for that …

Mary: I think that, too. Before I explain something to somebody, you know, I think about it – what's the most logical order to explain this? What must come before something else? In how I explain this. I find that I do that, too, and I probably drive people crazy like, "Would you just get to the point?"

Emily: Everyone gets so mad at me because …

Sara: That happens to me, too.

Emily: … like "What is the point to this?" Like "I'll get to the point of this just …"

Mary: But you have to know all these things before I get to the point.

Robert: If you don't get through one through nine you won't get to ten.

Mary: Right.

Robert: I can't just give you ten. It's not logical.

Mary: Right.

Emily: It's like there's a point to this, but here's why.

Mary: Yeah, I find that, I wouldn't say that I'm obsessive about having to explain things in a particular order, but I kind of laugh at myself when I feel my mind doing that, you know?

Sara: I do that all the time.

In this part of the dialogue, the research participants and myself found common ground in our ways of communicating that felt rooted in our being in mathematics. We related to each other in how we structured our discussions in a particular manner, following a logical, step-by-step procedure we felt compelled to adhere to or subconsciously followed. What came through as we shared this common way of being in the world was the feeling of otherness, of communicating in a way that made sense to us, yet was frustrating and bewildering to those who were outside of this mathematical part of our world. During this part of the conversation, there was a sense of relief in being understood by the other participants in the conversation. There was an openness among us, all mathematicians, where we could safely reveal, share, and relate these parts of ourselves, maybe in ways we had never done before. Similar to her feelings that were explored in Chapter 6, Sara again expressed her view that being and communicating in a particular mathematical way caused her to be “stuck in her own little thing,” narrowing her possibilities for making connections with others.

Reminiscent of John sidestepping the issue of having his behaviour attributed to his being in mathematics, which was described in the previous section, there again was a reluctance to make a direct, explicit connection between who we are and the ways in which mathematicians are in the world. While it is not explicit in the dialogue, the ways in which we related to each other revealed that being in mathematics was an undercurrent. There was an underlying understanding that mathematics was the reason for our ways of communicating, where the logical structure of mathematics spread over the surface of interacting with others, an inevitable part of who we were are as mathematicians, of how we went about being in the world. However, there was also a tempering of what this might mean for who we were. There were indeed particularities that we felt about ourselves, our behaviour, and what others observed about us, but we refrained from explicitly attributing our behaviour to mathematics. Moreover, in claiming that our manner of communicating was not “obsessive,” there was a distancing of our behaviour from the stereotypical acts of mathematicians. In our remarks, we were connecting to each other through the one thing we shared, mathematics, yet we continued to distance ourselves from what that might mean and what implications it might have for who we were in the world.

Remaining Other

In the previous sections, I have illustrated the ways in which the participants felt different in and to the world. These feelings translated in interesting ways in how they went about being in the world. In particular, what I heard was a reluctance to share themselves and their identities as mathematicians with others. For most of the participants, it was clear that they felt most safe in a community of fellow mathematicians where they were safe to be themselves and interact in ways that felt most natural to them without feeling different. These themes are investigated in the following subsections.

Reluctance to share themselves

> "He [a mathematician] finds it difficult to establish meaningful conversation with that large portion of humanity that has never heard of [his research topic in mathematics, the effect of which they describe as] creating grave difficulties for him." (Davis & Hersh, 1981, p. 35)

> How could we as mathematicians prove to a skeptical outsider that our theorems have meaning in the world outside our own fraternity? (Davis & Hersh, 1981, p. 44)

Most of the participants shared their reluctance to tell others outside of mathematics that they were mathematicians. For some, there was a sense of dread over being asked the question "What is it that you do?", which some of the participants and I responded to by creating cover stories:

> Mary: Well, I wonder if it's, I mean if you say you're good at math what kind of response do you get from people?
>
> Steven: I don't actually. I have a joke, I tell my friends, I tell everyone I'm in business. Because as soon as you tell someone you're in business, they stop. They're like, "Oh, okay, cool." No more questions because it's obvious what you're going to do in business.
>
> Mary: Yeah. I used to say to my husband, "You know, I'm going to start telling people I'm in waste management," because I'm so tired of people saying, "Oh, yeah" or rolling, or having this you know these bugged out eyes when I tell them I'm a math teacher and, you know, that look of either pity or horror.
>
> Steven: Yeah, like they're still afraid of you.
>
> Mary: You know when you say, "Oh, I'm in waste management," they'd be like "Great."
>
> Steven: So I usually say I'm a grad student and hope for no follow-up questions.
>
> Mary: Yeah, yeah.
>
> Steven: That works like thirty percent of the time. Well, I don't mind, like, being a mathematician. I'm proud of it. I walk in the math department everyday and go like, "I'm a mathematician." Like that, that's pretty awesome. Not many people are capable of doing this job. I don't really care that society doesn't necessarily respect us. I know I have job security. I know I'll be fine.

This portion of dialogue disclosed a pushing away and pulling back into being in mathematics. At first, in a pushing away of mathematics being a part of us, there was the description of our alternate personas, what we told people to order to avoid the reality of what the response could be to our being in mathematics. These alternate occupations helped to avoid the appearance of being other, of being unknown and not understood, and to avoid the look of dread sometimes encountered. It was also an evasion of the feeling of being feared by others, something that both Steven and I had experienced. Further, it seemed to be a departure from the group membership of mathematicians, a move that we made in order to be accepted into the flow of conversation with others, rather than be set apart. At the end, though, there was a pulling back into mathematics, a pride in being a mathematician that was accompanied by a pushing away of society. However, in this piece of dialogue, Steven connected his pride of being in mathematics to job security, to being better than others, rather than a security within himself or an assuredness about being in mathematics.

In the second group meeting, the feelings of otherness and a reluctance to reveal themselves as mathematicians were shared by a few of the participants:

> Emily: And then when you get, but like I think like the "Yeah" reaction comes from when you choose to continue in math, you know because everyone feels like it's a terrible people thing and they're like, "Well, you're good at it. It's cool, but why would you continue to put yourself through that," you know? Like that's I think where the "ah" factor, like why I don't understand. You must be this weird person, you know. I think that's where it is and like you don't get that until the higher levels and I think that's when you get the awkward reaction.
>
> Sara: I didn't have … maybe I didn't have too much interaction with normal people lately, but …
>
> Steven: Why is that?
>
> Sara: But I think, um, from what I remember as an undergrad often people would just think like, "Oh, that's cool," like you're smart, like what you said. And so then you're in a good place. They don't think that you're weird. They just think that you're smart and maybe they are going to start feeling slightly inferior because you're doing something they don't understand. And you make it better by just saying, "It's not really that hard. You must have had a bad teacher" or something funny, and then it's fine. Then you're at the same level.
>
> John: But it's not necessarily the people at the university you ever have to worry about having this discussion with …
>
> Sara: Eh, that's a good point.
>
> John: Right?
>
> Steven: It's parents.
>
> John: Well, it's parents or it's people who are outside of the university …
>
> Sara: Exactly.
>
> John: You're sitting on a plane, someone asks you what you do and you have to decide if you really want to have this conversation, right? Do you

> really want to know that their uncle is an accountant? I think that's when it's awkward myself.
>
> Sara: No, I could see that.
>
> John: I don't feel awkward about it in any other sense, except when you're describing what you do to someone you just don't want to have that conversation with.
>
> Emily: We can talk about so many other things …
>
> John: Yeah, yeah.
>
> Emily: … like why do we have to talk about this? This is going to be a really big deal, like …
>
> John: Yeah.

While at first the participants talked about how they became strange or different when they began to study mathematics in graduate school, what seems present in this portion of dialogue is a consternation of having conversations during which they might encounter a resistance to their being in mathematics. Beyond this, the participants expressed frustration with either an unwelcome relating of someone else's experience or with describing their particular interest in mathematics to someone whom they thought would inevitably not understand. The former encounter was experienced as burdensome, as though the ways in which others attempted to relate to mathematics represented misunderstandings of what mathematicians do rather than a desire others might have to make a connection with them. The latter encounter was experienced as having the potential to bring more misapprehensions, to be "a really big deal." Thus, there was a burden expressed here in the participants feeling unable to reveal themselves because of the anticipated responses from others.

The complicated space of being in mathematics was again revealed in a pulling towards and pushing away of mathematics and also of people outside of mathematics. In the statement "be on the same level," I heard the distancing from mathematics and the pulling toward others in the participants' need to downplay being in mathematics. In the two previous pieces of dialogue was also a "distancing oneself from a general populous that is unable to understand the mathematics that it uses" (Gutmann, 2000, p. 101). There was a sense among the participants that people would not understand the mathematics that they do, but this also seemed to imply that the participants themselves would be misunderstood as well, which was something they hoped to sidestep by creating the cover stories discussed previously. Yet, in the attempt to "be on the same level" and in the statement "we can talk about so many other things," there was both the desire to relate to people outside of mathematics and to show to others that the participants were defined by more than just mathematics. In contrast, though, there was recognition that they felt most comfortable among their academic peers, in a space where they would not have to explain themselves.

A Reflective Interlude

> We need to stop hiding our delight at a well-formed graph; our admiration for a clean, crisp proof, our realization that higher mathematics is one of the finest accomplishments of the human spirit. It's time to stand on our desktops and shout to the world: "Say it out loud, I like math and I'm proud!" (Brooks, 2009, p. 1)

Thus far, I have explored the participants' experiences with others outside of mathematics, in how the participants chose to interact and present themselves. They felt a sense of difference by being in mathematics that came through in their voices, in what they disclosed in the dialogue. The participants seemed to endure a weight of apprehension and tension of how others would view them when and if they revealed their mathematician selves. Being in mathematics and being mathematically literate and expressing themselves through logically structured communications further set them apart from others. At times, the participants' difference was celebrated and they enjoyed being special, being other, while at other times in their voices came through a sense that there was more to them than mathematics. Further, in developing cover stories for their identities and in distancing themselves in various ways from mathematics, the participants revealed a desire to belong, to blend in, to be able to ease into a connection with others without resistance to who they were.

There were perplexing and distressing words that the graduate students used to label themselves and others who are in mathematics. They described mathematicians as being feared. On occasion, some of the participants set themselves apart with labels such as "freak" or referred to mathematicians as "terrible" people. The claiming of these labels, coupled with the participants' distancing of themselves from mathematics, speaks to the complicated and difficult space that they live in – that of a smart, even brilliant, person, but odd for being so, revered and reviled, stereotyped yet misunderstood. It is interesting to think of how the participants classified or described mathematicians and how such labels might take a toll on them. With such impressions of mathematicians and responses to who they are, it is understandable that they would be reluctant to reveal to others that they will be mathematicians.

Within this problematic space, though, all of the participants shared their passions and joy in doing mathematics. To love what you do, but to be afraid to admit to it, is both interesting and confounding to me. Despite the participants' enthusiasm for their work, they shied away from sharing it with others, apprehensive of what the response might be, afraid of becoming different in that moment of revealing themselves. What would it mean for the participants to, as Brooks (2009) calls on mathematicians to do, "stand on our desktops and shout to the world: 'Say it out loud, I like math and I'm proud'"? Would this announcement close them into the world of mathematics, never again to be a part of the community outside mathematics? Would making such a declaration efface the other parts of their identities, relegating them to the stereotype of mathematicians they claimed not to resemble? Could the participants find self-assuredness, a stable footing both in the world of mathematics and the world outside of it, where they would not feel that they had to continue to oscillate, pulling towards and pushing away from being in mathematics?

As I continue to explore the lives of mathematics graduate students, how and who they are in the world, my focus now turns toward the participants and how and who they are in mathematics itself, rather than in the context of the world outside of mathematics. I will again point to the pulling towards and pushing away from mathematics that I observed in the conversations with the participants. The lived experience of the complicated spaces of being in mathematics will once more be heard in their voices as they describe the difficulty and trepidation up against the enjoyment and satisfaction of being in mathematics.

Clinging to, safety in, and familiarity with the community of mathematicians

In being different from others outside of mathematics, the participants' interest in mathematics made them part of a group of mathematicians. In this community, the participants found familiarity, safety, and camaraderie:

Emily: I have like … the stereotypical mathematician is not me or you, right? Like it's the old, white-haired guy that can't talk about anything but math, right? Who stays in his office …

Mary: Forgets to zip his zipper.

Emily: Yeah, that guy… I feel so happy because like finally I've got other mathematicians to be around and now that I do, it's like I don't really have a particular desire to see the other friends I made. Like it's like, I'll go out and see them, but it's kind of like I like my other friends more, you know? We can talk about math, we can talk about school, we can talk about you know what so-and-so and so-and-so did. It's just like … we're all a terrible little clique is what we are, but …

Mary: But that, I mean, what you're studying is such a big part of your life. And so to have friends that you can relate to on that really big part of your life …

Emily: Yeah, no, like I've had this argument with my roommate all the time, though, whether or not I use math to define myself. And like I don't … It's like what do I say? "I'm Emily, I'm in math, I'm from […]," you know? It's pretty much, you know, like it's as surface as it gets, but it, that's pretty much what I do, whereas like […] in public policy, she's like "I'm doing my master's, I'm from […]," you know, and she doesn't really mention the public policy. Like people will go on and ask about it and sometimes I hide it and sometimes I go through phases where I hide it because I don't, it drives me crazy, you know, because you get this reaction. And other times it's like, "No, this is who I am. Go away," you know? And I like to think that it doesn't define me. But to a large extent it probably does. It's a huge aspect of my life.

Mary: For sure.

Emily: Like I spend the majority of my waking hours and too many of my sleeping hours with math in my mind. You know? Like it's just … It's a huge aspect of me.

While Emily at first distanced herself from the stereotypical mathematician, she also recognized how important being a part of the mathematics community was to her happiness. The stereotypical mathematician, whom she viewed as not representative of her, could only talk about math. Then again, though, Emily wanted to spend time only with people with whom she could share mathematics, which revealed her comfort being in the world of mathematics. Among that group of people, there was a shared experience and a shared language that only mathematicians could understand

(Burton, 1999). Emily expressed her happiness in being a part of this community and a sense of belonging once she became friends with fellow mathematics graduate students.

In this dialogue, there was a contrariness that exposed a pushing away and pulling toward mathematics, as though to say "I am not that person – the stereotypical mathematician. I am different." In contrast, though, Emily recognized that the space within mathematics, with other mathematicians, was the place that she felt safest and could most be herself. In light of this, Emily's back-and-forth thoughts about her identity are interesting. While she felt compelled to introduce herself to others as being in mathematics, she distanced herself from mathematics by claiming that it did not define her. Slowly, she acknowledged that mathematics was a "huge part" of her and essential to who she was in the world. I observed this again in another conversation with Emily:

> A friend of mine who's not in math, like, I was telling him how I don't know how to bring it up, you know. It's like one of those things, you know. Like people can label each other so easily, right? And so, it's like, if you, you do something a little more normal ... but I try to avoid it and he's like, "Because you're in math, it doesn't define you." I say, "I know this, but not everyone else does," you know. And he just doesn't see that. Again, it's not like political science, you know. Like, you could talk, you could talk to people, the politics, you know, and they understand that. But, yeah, it just doesn't, they don't understand. Like, it doesn't define who you are. Like, sometimes it kills conversation.

In this statement, Emily spoke of another friend who told her that being in mathematics did not define her, but she saw mathematics as different from other disciplines. Emily perceived other disciplines, such as political science or public health, as allowing students or scholars in those disciplines to be able communicate with others freely, to be accepted in the world. Yet, to her, mathematics was different from these other disciplines. Her experience and her perceptions of experience caused her to feel different and unusual, so much so that she feared people would not continue to talk with her when she declared mathematics as part of herself. What is more, from these two portions of dialogue, it seemed that Emily was not able to leave out being in mathematics from how she presented herself to others. Mathematics was such a part of her that rather than omit it, even temporarily, from how she introduced herself as a way to connect with others, she held onto to it tightly, unable to let it go.

Why was mathematics inseparable from Emily's idea of who she is? Why did she feel that her friends' disciplines did not define them, but that mathematics defined her? She spoke of how being in fields other than mathematics allowed her friends to easily relate to others, to be able to connect over things that everyone might understand. I saw Emily's delineation as revealing a perspective that one cannot connect with or relate to others through mathematics. But it was not just anyone with whom she could not relate, as she had recently found happiness in meeting fellow mathematics graduate students and had been able to connect with them. It was those who are outside of mathematics with whom she felt she could not relate, while there was safety in being with other mathematicians.

Uncertainty About Being in Mathematics

No one drifts into being a mathematician. (Stewart, 2006, p. 16)

In contrast to their feelings of safety and familiarity with their peers and fellow mathematicians, the participants distanced themselves from their choice to be in mathematics. In interesting ways, a few of the participants shared their stories of how, by a series of accidents, they came to study mathematics. Other participants described how mathematics was, in a sense, the field of last resort because they were not good at anything else. The participants' descriptions of how they came to study mathematics are explored in the following sections.

A distancing of their choice

Beyond the reluctance to reveal their mathematician selves to others, most of the research participants claimed that their choice to study mathematics came about either by interesting accidents or because they were not good at anything else. For example, Chris began his undergraduate studies in physics and happened upon an interest in mathematics by registering in the wrong course. Emily and Steven both commented that they were not good at writing, art, or the other sciences and so, as a result, their only choice was to study mathematics:

> Emily: I haven't really taken a break, like you know, high school right into… I went to university and did math there by process of elimination basically and have really quite taken to it and at one point along the way I got hired and given money to do research. And I thought that was great. And so they're [her professors] like, "If you think that's great, go to grad school." And here I am on the other side of the country now. So, yeah, I just graduated in May so …
>
> Mary: So when you say you chose mathematics by a process of elimination, could you explain that?
>
> Emily: Oh, well just like when you're a high school kid and you have no idea what's going on.
>
> Mary: Mm-hmm.
>
> Emily: Like I was good at science in general and so at university they ask you to choose your major before you even start, which is …
>
> Mary: Oh, before you start?
>
> Emily: Yeah. So I had to choose and like they said over and over again you can change as soon as you get there if you want, so I looked at the options and I didn't have any clue about rocks and so geology was out and

> I really didn't like chemistry. I have a real thing against that. And then it was like physics. I hadn't taken physics in two years. For biology, I was just kind of like everyone takes biology and I was just kind of like, "I guess I'll try math." I'd always liked math. I took a calculus class. I call it extracurricular. It wasn't really, there was, like I kind of come from a very small high school. My high school is very small. And so there was one calculus class for the entire county and so I had to go Wednesdays and Thursdays from four to six to learn calculus in another city, which is like a twenty-minute drive away. And so I did that. I didn't have any pre-calculus background. It was the first time that I was really like challenged by it to any extent. And it was hard, but I got through it and I figured if I could do that, I should be able to do a math degree. And looking back, they're not the same, but …

In describing the process of elimination she followed, Emily talked about mathematics almost as a last resort for her undergraduate studies, again revealing a hesitation or reluctance to embrace mathematics fully. While she went beyond what her high school required her to do in mathematics, taking more courses than necessary and commuting to another city to do so, she continued to hold mathematics at arm's length, never admitting to a intentional choice to study mathematics. Yet, to me, her past behaviour of taking the extracurricular calculus disclosed her interest in mathematics and unveiled a reality that mathematics may have been a conscious and deliberate choice. Although, in light of her previous comments of feeling different being in mathematics, she may have held on to her claim of mathematics as a last resort to avoid being seen as different, other, or strange.

Steven discussed his decision to study mathematics in ways similar to Emily – as a process of elimination, rather than a choice made with enthusiasm and ownership:

> Mary: And you went to university knowing you were going to study math?
>
> Steven: Yeah.

> Mary: … but you didn't …
>
> Steven: Well, because I sucked at science …
>
> Mary: Mm-hmm.
>
> Steven: … like I just couldn't get behind the idea of chemistry. To me, it's a very unelegant, unbeautiful beast.
>
> Steven: Like I don't think it was really a conscious decision for me to go into mathematics. I just happened to be good at it. Like, you could ask me the same question if I was good at English and I'd go into writing. So, it's like, it's fine. It's my calling, but I don't think… There's literally like nothing else I could do. Like this is my skill. Like I have to do it.
>
> Mary: Okay.
>
> Steven: Like it's everyone's responsibility to like figure out what they're good at and then pursue it. Anything else would be a mistake.
>
> Mary: So you don't think you're good at anything else?
>
> Steven: No.
>
> Mary: No?
>
> Steven: Well, I know I'm not good at anything else. I'm not a, I'm not good at writing, I'm not a musician, like, I can't draw.
>
> (Later in the conversation.)
>
> Steven: Yeah. That's, so I don't, I think it's sort of a weird question because I don't know if I chose it. It chose me. … Nothing else was appealing. It seemed to be like, you know, I knew not a lot of people were good at it. It seemed like the easiest road to take.

What spoke to me in Steven's description of his choice to study mathematics was how he held mathematics at a distance. While he said he was "not being able to get behind the idea of chemistry," it appeared that he was also not fully able to get behind mathematics either, saying that he *had* to do it, rather than declaring that he wanted to do it, claiming that mathematics chose him, rather than him choosing mathematics. When he said "it's fine" about his choice of mathematics with a tone of resignation or disappointment in his voice, that signaled to me an acceptance of something that was not truly okay. He spoke of mathematics as a calling, something that he had to do, but he did not claim that it was something he freely chose to do. However, he alluded to his skill in mathematics, which revealed some ownership in his choice.

Chris said his decision to study mathematics was due to a mistake he made during course registration in his first year of undergraduate studies. Unlike Emily and Steven, who knew upon entering university that mathematics would be their focus, Chris began his undergraduate studies with the intention of earning a degree in physics:

> And it was… I had … I guess maybe one, one thing… a funny story I like to tell is that I was actually just too stupid to not be a math major because I was supposed to take a regular, generic first year math course. But at… so at my university there was, you could take Calc I, Calc II, which were generic, or there was a full year for honours students. I didn't understand that I wasn't an honours student at that point so I guess I might as well take the full year – why register in two classes, I might as well register in one.

As with Steven and Emily, in Chris's speech there was no declaration of mathematics as a purposeful decision or choice. Chris repeated this story in our third meeting, holding on to it as his reason for studying mathematics. To describe this experience as "a story that he tells people" is interesting – a developed, remembered, and held-onto account that signified to me his way of holding mathematics at a distance. At this point in his education, though, he had earned a bachelor's degree and two master's degrees in mathematics, but he continued to maintain the story that his choice was by accident. One last remark that I found interesting was Chris's description of being "too stupid to not major in mathematics." To me this statement appeared to represent a downplaying of his intelligence and a move to distance himself from the stereotype of mathematicians.

John, arguably the most published and established participant, also claimed that mathematics was the only thing he was good at:

> Mary: My next question is what drew you to mathematics?
>
> John: Wow… I think in the end it's the only thing I'm really good at.
>
> Mary: Really?
>
> John: Yeah.
>
> Mary: You've got to be, well…
>
> John: I mean I enjoy it …
>
> Mary: Mm-hmm.
>
> John: … sure. I definitely enjoy it, but …
>
> Mary: Mm-hmm.
>
> John: … as far as what I was actually good at …
>
> Mary: Mm-hmm.
>
> John: So I didn't start off with a math major. I started out in chemistry, pre-med for chemistry.
>
> Mary: Oh, wow.
>
> John: And I was doing fine in chemistry. I just didn't really enjoy it. Like it wasn't that interesting. Whereas I saw the value in education and the value and need for, for mathematics. And I'd always kept doing it because it's an easy, an easy A, you know. I could always do it so …

As with Chris, John did not begin his undergraduate studies focusing on mathematics, but he had continued to take courses in mathematics during his pre-medicine program to elevate his grade point average. At a certain point in his program, he realized that he did not want to be a medical doctor and saw a "need for mathematics." Having already taken many courses in mathematics, he was able to easily change his degree program. John did not describe a deliberate choice to earn a degree in mathematics, but rather he seemed to describe mathematics as a default after deciding against medicine. It is interesting to me, though, that while John alluded to studying mathematics as a default, he would see a need for mathematicians rather than a need for doctors, and so he made the switch to mathematics. Similar to Emily, John continued to take mathematics, even when it was unnecessary, revealing more of an interest in mathematics than it seemed he would like to declare.

Like many of the other participants, Robert was hesitant to claim mathematics as his ambition or purpose. However, he went further in avoiding a connection to mathematics than the other participants:

> It's technically not strictly something that I … want to do in the sense that I was actually not a mathematician… Let me put it this way – it seems more as, instead of saying that I'm doing mathematics, I would probably classify myself as someone who uses mathematics … That probably fits my own personal research and reflects a little bit more of my own ability in mathematics better than, you know … So I guess to answer your question is probably in the line of I feel that mathematics is a … very powerful tool that I use more than I, it's a little bit more than this is the subject that I'm studying. Yeah, that's probably my feeling.

Robert had earned his bachelor's and master's degrees in physics and was enrolled in the doctoral program in applied mathematics. Even though he was a PhD candidate in mathematics, doing the work of a mathematics graduate student and mathematician, he was reluctant to label himself as such. He sometimes wondered about the validity of his experience and perspective in this research project because he did not feel that he was a mathematician.

The participants' statements about mathematics are fascinating to me. Why did the participants not refer to mathematics as their preference, as something they were interested in? What would it mean for the participants if mathematics had been their choice rather than an accident or simply a default, a forced choice at best? What would an intentional decision to study mathematics say about them? In describing their experiences as accidents or claiming that mathematics was their only option, what were the participants saying about themselves? Why would they continue to hold mathematics at a distance, to push it away? I see this distancing, holding mathematics at arm's length, as another way for them to show that they are still acceptable within society and that they do not conform to the stereotype that people associate with mathematicians.

Beyond their hesitation to acknowledge mathematics as their choice, I found something else interesting within the participants' descriptions of the specific topics in mathematics that they studied. Not only were they reluctant to acknowledge their choice to study mathematics, but, with regard to the particulars of the mathematics they were interested in, they often had negative things to say about their particular focus of study. For instance, the participants who were working in pure mathematics felt that the mathematics they were working on was not valued or interesting.

Steven, when speaking about his master's research, had referred to his degree as "fake," and in our final meeting I asked him for clarification of what he meant by this:

> Steven: Oh (laughs) … Yeah, no, so I don't think I've, in terms of mathematics, like I'm not in a pure form of mathematics. Like this was a deliberate decision to remove myself more from the pure mathematicians to do something that would be more practical.
>
> Mary: Okay.
>
> Steven: Just because like, you know, it's one thing to be an artist, it's another thing to starve along with them.
>
> Mary: Mm-hmm.
>
> Steven: So, yeah, I don't think this is a pure math degree at all. Like this is more, like what I'm doing is more appropriate, well, it's really this odd, new cohesion between mathematics and computer science. There's no name for it yet. So I would say, you know, this is a fake degree in computer science as well as in mathematics because I just have no place.

Steven had made moves to do work in mathematics that would depart from what he alluded to as the impractical work of pure mathematics. As a result he saw his new line of work as not having a place in mathematics.

John went further than Steven by inferring that his area of pure mathematics was unappealing:

> John: Maybe if I worked in a more appealing area.
> Mary: More appealing area such as?
> John: Such as applied math.
> Mary: Oh, so then you must be in pure math?
> John: I'm in pure math.
> Mary: Okay.
> John: I'm in number theory.
> Mary: Okay.
> John: So analytic number theory even.

John expressed that his specific interest in mathematics was not only uninteresting to the world, but also in the world of mathematics. If this were true, what did it say about him to be working in this area? What would it mean for someone to work not only in a discipline that most others do not understand, but also where your own interest is described as unappealing to those who are in your discipline? What would that mean for who you are and how you go about being in the world – to be one of possibly a few dozen people in the world to work in something that is so disliked? What would this kind of isolation mean – a world with few coworkers, colleagues, or people with whom to collaborate, commiserate, and relate?

Not being a superstar

> One of the most striking statements of the doctrine of the individual in mathematics was put forward in an article by Alfred Adler […]: "Each generation has its few great mathematicians, and mathematics would not even notice the absence of the others. They are useful as teachers, and their research harms no one, but it is of no importance at all. A mathematician is great or he is nothing." (Davis & Hersh, 1981, P. 61)

For some of the participants, I heard a note of sad resignation that they would not be a superstar or a winner of the Fields Medal, the highest recognition for research in mathematics. Only being in their mid-twenties and early thirties, but not having yet been singled out for their work in mathematics, they seemed to acquiesce in a role of less brilliant, not as special, not as important as others within the world of mathematics. Some of them spoke of their talents as a reason, while others described their unwillingness to work as hard as they needed to in order to become a superstar. Chris mentioned this first:

> I certainly don't think I'm dumber than other students, but I just don't know if it's a … you know, I think you need to be a superstar researcher. I think you need to be able to work pretty much all the time on research. I realize now that I'm not going to be, you know, the superstar or win the Fields Medal.

Emily expressed feelings similar to Chris:

> Emily: I kind of know how to work. I'm not particularly talented at math. I kind of like it, but like I'm not great at like the amazingly insightful things. I don't grasp things immediately. Like I'm fairly quick, but I'm not a superstar by any extent, you know. I'm like, "I don't know if I have what it takes to do this."
>
> Emily: Yeah, like I know I'm not, you meet some very brilliant people when you're studying math, right? Like you're just like, "Whoa, and you understand this so much more than I do." And like I'm well aware that I'm not one of them, you know? Like I'm good and if I work really hard …
>
> Mary: But you're in the same program that they are …
>
> Emily: Yes, but like I'm very good if I work really hard, I can do quite good work …

> Mary: Mm-hmm.
>
> Emily: But it takes an extra something to do something spectacular, you know? And I'm quite comfortable now with not being the spectacular one. I'm someone that works really hard and usually does pretty well and like I'm okay with that and but I don't want them to think I'm this really spectacular person then I, and then I disappoint them. I have a good transcript, but it's like, it's not because, like I'm not going to be the Fields medal winner. Like I know that I'm not going to be like, like someone that solves one of the Millennium problems. I'm not going to do that. Maybe I'm not shooting high enough. It's just, like, you kind of know where your strengths are and, anyway, like I don't have that drive I guess to do… Like I'm quite good and when you put pressure and deadlines on me I can, I can perform quite well. I'm not the person that can remember everything the first time I see it.

For Emily, even though she had been accepted into the same graduate program as the superstars she perceived to be in her program, it appeared that she had decided that she was not one of them. In what she said about superstars, there appeared to be some assumptions about others' behaviour and what it took to be spectacular. There was an image she had of what it would require to be a stand out – including hard work, but also being able to remember things the first time they were seen. Based on this impression and what she saw as her own way of being in mathematics, she was resigned to not become a superstar.

John had similar conclusions about his place in mathematics, but, while Emily alluded to an inability to work in ways that superstars did, John seemed to decide that he was not willing to let go of other parts of himself in order to be a superstar:

> I've come to the conclusion, and I'm okay with it now, that I will never be a mathematician who works at Princeton, you know? Like I'm just not willing to spend my time like that.
>
> I'm not going to give up something like my marriage to do math, and I'm not going to give up, say, being able to ski once a week to do math because I know I won't enjoy it, right? Like if I gave up skiing or the

other things I've become used to, it's like I… it's not worth it.

There seemed to be an assumption John made that brilliant mathematical work was done in a particular way by talented people. Yet, despite also being accepted into the very department in which they saw the gifted students, the participants seemed reluctant to recognize their own abilities. There was an interpretation of others' behaviour that made them feel different, other, not good enough, or, in John's case, even hesitant about wanting be the superstar because of the perceived costs. Not only did the participants express their feelings of otherness, of being strange to the world, but they also seemed to find themselves in a space of being different in their particular mathematics worlds.

Conclusion

What does mathematics mean for being in the world? What experiences come into play as future mathematicians interpret their worlds and their place in it? Being in mathematics is a profound (deep, weighty, intense) and complicated part of who the participants are and how they are in the world. The paradox of pulling toward mathematics and at the same time pushing it away resounded throughout the conversations. This was observed in how Emily alternated between hiding and revealing mathematics as part of herself to others, and acknowledging that mathematics was a large part of who she is. It was heard in how Chris, John, Steven, and Emily, while being graduate students in mathematics, and thus having a made a choice to be in mathematics, held on to their descriptions of "not being good at anything else" or becoming mathematicians by accident.

Even though they attempted to distance themselves from mathematics in various ways, mathematics had left an indelible imprint on who they are. The participants expressed a sensitivity about being in mathematics and could not seem to separate themselves from mathematics and the awkwardness they felt, not only in the world, but also in the world of mathematics. In light of the stereotypes described earlier, how would they appear to the world if they were to lay claim to their choices to study mathematics? It seems as though holding on to their various genesis stories prevented them from having to feel the possible rejection by others of their being in mathematics.

What might this mean for a mathematician's teaching? Do mathematicians remove their passions and interests from their teaching because they do not want to further expose themselves as being different? If the participants were to share their true passion for mathematics with their students, what would that reveal about them? The participants felt different enough just by being in mathematics, but then to be passionate and excited about it as well – what would that mean for how they are seen by others? There is safety in teaching a particular way, in presenting mathematics on a chalkboard and not interacting with the students. John talked about math as an escape from the world and I spoke about a certain way of teaching (lecturing) as being safe (Chapter 1). Is it safer, is there less resistance to one's mathematical self, in becoming a conduit for the material, where, in keeping one's passions and interests hidden, one can protect his or her self from further judgment, from being seen as even more strange?

Or are classroom identities formed less by what and how one thinks he or she should be and more by what others think they should be? What are the expectations for the behaviour of mathematicians and post-secondary teachers of mathematics? Is teaching more influenced by what others expect? Such questions remind me of one of my student's evaluation of my teaching. He or she wrote, "Easy to understand instructions and a teacher with a pulse. That's all that matters! Good teacher." Is that *all* that matters in mathematics teaching? What if I had been something beyond this, beyond being just a teacher with a pulse? What would that have looked like and how would it have been perceived? Could I have been something other than this? If I were to become a teacher who opened up possibilities for their students, who enlivened mathematics, who revealed the creativity and dialogue inherent in mathematics, how would I be received in the community of mathematicians? With these questions in mind, the following chapter is an investigation of themes around teaching mathematics and the implications of being a mathematician might have for teaching post-secondary mathematics.

Chapter 8

How Many Ways Can You Skin a Calculus Class?

In this chapter, I explore the participants' views of their roles as university teachers of mathematics. As with the previous chapters, there are many questions regarding their views about their current positions as teaching assistants and their potential futures as mathematics professors. In particular, I focus on what had meaning for the participants as they began to or continued to put together ideas of how they would approach teaching. Beyond the stereotypes of college and university mathematics professors, were there experiences that the participants interpreted to have meaning for who and how they would be as university teachers of mathematics? What of their identities as mathematicians was held onto as they began to interact with students? What of their individual identities and interests in mathematics came through in their interactions with students?

Interpretations of Mathematics Teaching and Mathematics Teachers

> I hate 'teaching' and have had to do very little […] I love lecturing, and have lectured a great deal to extremely able classes; and I have always had plenty of leisure for the researches which have been the one and great permanent happiness of my life. (Hardy, 1940, p. 48)

In the previous chapter, I presented a look at the stereotypes of mathematicians put forth in various contexts and I explored the influence those stereotypes might have for my participants. Do similar stereotypes of university mathematics teaching and mathematics teachers also exist? How are mathematics teachers depicted and described? Are there particular ways of being in the classroom that are characteristic of university professors of mathematics? To revisit the film *Good Will Hunting* (Bender & Van Sant, 1997), the professor of mathematics depicted in the film finishes his lesson with a chalkboard filled with formulas and definitions. At the point we see him in the movie, he is speaking to the students and in his final remark he asks the students if they have any questions. Rather than wait for his students to answer, the professor answers his own question by saying "If so, you can ask Tom," referring to his teaching assistant. The professor then leaves the room. I believe that this portrayal is illustrative of many university mathematics professors in that the professor has given a lecture and there is little, if any, interaction with the students.

At least one of the participants had a sense of how professors of mathematics are portrayed in films. In particular, Emily said:

> I have like … the stereotypical mathematician is not me or you, right? Like it's, you know, it's the old, white-haired guy that can't talk about anything but math, right? Who stays in his office… And, you know, people will bow down around him and give him whatever he wants and they don't really ask a whole lot of him, right? Like in the movies they demand they [professors] teach a class, they're terrible teachers and they go too fast and they understand it too well.

Emily had an image of mathematics teachers not only fitting a certain physical profile, but also a certain way of teaching, of hurrying through the course material, of being "terrible" teachers. Her portrayal of a mathematician seemed to put the mathematician on a pedestal, with a revered status where "people will bow down around him and give him whatever he wants," despite their classroom practice.

Some of the language Emily used to describe the professor's circumstances was quite interesting to me. She mentioned that mathematics professors were "demanded" to teach. Yet, what is a role of a professor? Is teaching not a part of their profession, part of their work of being an academic? She also spoke of professors knowing mathematics "too well," as though their in-depth knowledge was a hindrance to their ability to teach well. Where might Emily's ideas about mathematics professors and their roles come from? I explore these questions further in later sections of this chapter.

Mathematicians have written about their views of the professor's role in the classroom. Models for mathematics professors similar to the one seen in *Good Will Hunting* (Bender & Van Sant, 1997) exist in the literature and were described in Chapter 2. In particular, Krantz (2003), in his instructions to mathematics graduate students, wrote, "If the professor so chooses, s/he does not even have to entertain questions" (p. 48). Stewart (2006) advised that mathematics graduate students and professors "Stick to the main point, and try not to digress if doing so requires new ideas that are not on the syllabus" (p. 168), revealing a view that university mathematics teaching should follow a certain structure. Further, Higginson (2006) recounted Polya's description of a "traditional mathematics professor" as someone who "writes *a*, he [*sic*] says *b*, he means *c*, but it should be *d*" (p. 133), giving the impression that the traditional mathematics professor is unmindful of his classroom practice.

Hardy (1940), a twentieth-century mathematician, in his declaration of love for lecturing and dislike of teaching, made a distinction between these two acts and in doing so communicated a common attitude about mathematics teaching at the university level. With this distinction made, I began to think of a question – do professors of mathematics teach or do they lecture? Are these unalike acts? In my investigation of the differences between these words, I observed some dissimilarity. The etymology of *lecture* is the "action of reading, that which is read," whereas the etymology of the word *teach* is "to show, point out, to give instruction" (www.etymonline.com, 2009). The definitions of *lecture* and *teach* also revealed some differences in these two acts, where to give a *lecture* is "to read a speech to an audience or class" and to *teach* is "to impart knowledge of or skill in" (Soanes & Stevenson, 2005, p. 997 and p. 1809, respectively). Lecturing is a reading or a speech, in contrast to teaching, which is also defined as a sharing or imparting of one's knowledge. In later sections, I explore this difference within the context of the participants' views of their roles as mathematics professors. In particular, how might this delineation come into play as the participants began to think of their future classrooms and how they might engage their own students?

Beyond the paradigm for mathematics professors, I wondered if there was an experience characteristic of what occurs in university mathematics classrooms? In Davis and Hersh's (1981) quotation below, an ordinary mathematics class is portrayed as programmatic and predictable:

> In an ordinary mathematics class, the program is fairly clear cut. We have problems to solve, or a method of calculation to explain, or a theorem to prove. The main work to be done will be in writing, usually on the blackboard. If the problems are solved, the theorems proved, or the calculations completed, then the teacher and class know that they have completed the daily task […] one knew where one was supposed to be going; one also knew that the main thing was what you wrote down. As to spoken words, either from the class or from the teacher, they were important insofar as they helped to communicate the import of what was written. (p. 3)

To me, what comes through as central to the time spent in class is the importance of the mathematics, the theorems, the calculations, the problems; when these are presented, proved, or solved, then it can be concluded that a mathematics class has successfully taken place. The class is unidirectional and focused on the mathematics the teacher must attend to. What is missing from this description is an interaction of students and teacher, of conversation, a back-and-forth communication of ideas and learning.

With these stereotypes of mathematics professors and the image of a mathematics classroom in my mind, I began to wonder what have the participants experienced in their university mathematics courses? Are there images that they have taken to have meaning for how they will be when they enter their own classrooms as professors? How did the participants describe what occurs in a university mathematics classroom? Based upon the stereotypes and general descriptions of mathematics professors and classrooms discussed previously, in the following exploration I will make an assumption that university mathematics classes consist of a lecture, a reading of the course material, rather than a dialogue that occurs between students and teacher. In this regard, the participants expressed a similar interpretation:

> Steven: I can tell you the structure. I think it's just usually – it's like definition, theory, example. Example, definition, theory, over and over and over again.
>
> John: And it's an instructor saying something without much student interaction.
>
> Emily: Very little, yeah.
>
> Sara: Which they don't mind.
>
> Steven: It's sort of worse than less, it's …

John: But it's efficient.

Steven: You barely have their attention.

In this group dialogue, the participants related what they perceived as the typical structure in mathematics classrooms – a recycled pattern of definitions, theorems, and examples. They had experienced this structure as students of mathematics and they recognized the role that it played in not engaging students. John further described how this model had an influence on students by saying "A lecture is easy for a student really, too, you know. They do homework. They take their tests. That's it." Here it seemed that John acknowledged that the structure of mathematics classes does not challenge students, stating "That's it" in terms of what is required of students, which he claimed was nothing more than doing homework and taking tests.

The participants continued to share their interpretations of the occurrences in mathematics classes. In particular, they agreed that the lecture structure is a one-way street, where professors relate information and students sit passively, receiving this information:

> Steven: Well, I think people in math expect when they go to class you will be like, there's like no, there's no requirements for you to show up to that class. Like, you're just going to have information flung at you.
>
> John: It's a one-way street now.
>
> Steven: It's a one-way, whereas in other classes they're expected to like read a journal or a short story and like, then when they go to that class they're going to discuss …
>
> Sara: Discuss things.
>
> Steven: How come? … That is the difference. That's never going to happen in math. And if there's any expectation of a math teacher it's that, that, it's just like you're expected to like come and like take up all this information …

Steven went further than the other participants in declaring that students and teachers were used to and now expected this particular routine for mathematics classes. At one point, though, he questioned the format of mathematics classes and their lack of discussion and interaction, asking “How come?”, wondering why and whether it had to be that way. After a short pause, he seemed resigned to the structure of mathematics classroom, inferring that a departure from lecturing, such as having a discussion, would “never happen in math.”

Beyond this part of the discussion, John and Robert both expressed how the structure of mathematics courses had a bearing on teachers. Robert spoke of his experience teaching three sections of a tutorial: “You are actually in essence teaching about the same thing three times… They are very different individual sections, but you pretty much do the same thing.” Despite the varied character or makeup of the students in his classrooms, Robert presented the same material in much the same way to each section. Having not received support or encouragement for other ways of being in the classroom, John said, “We’re not encouraged to ask open-ended questions in mathematics.” He seemed bound to continue lecturing in the courses he would teach.

Steven’s description of the passive role of students – “you’re just going to have information flung at you” – stood out for me. To me, his statement conveyed a sense of mistreatment in a classroom where something, in this case mathematical concepts, were thrown at students. It revealed a strong sense of discomfort in a classroom where students are consigned to sit and listen to mathematics. Have such perceptions of their own learning experiences in mathematics had an influence on the participants? What meaning might be taken from these feelings of sitting there, “having information flung at them”? In this regard, John related his observations of how lecturing has an effect on students: “When you’re in a lecture you don’t think, right? I mean you follow in some respects, but your own intuition doesn’t come out. There’s really just a one-way flow of information. And people don’t learn that way, I think.” To John, hearing a lecture also seemed to represent a certain harshness – a suppression of one’s thinking, intuition, and learning.

In the previous piece of group dialogue, Steven put forth an interesting observation – that the ways in which mathematics is presented to students would not occur in some other disciplines. From this observation, a question came to my mind – does a distinction exist between the structures of mathematics classes and the classes of other disciplines? John spoke of the differences he perceived in mathematics lectures compared to what might happen in the classrooms of other disciplines:

> You don’t go to a language lecture, right? They would never do this. It would never work. And music is the same way, right? You don’t go to a music lecture. You go to a practice room and you have lessons with your teacher and he makes you play.

Based on their experiences as students, the participants perceived and described a difference between what occurs in mathematics classrooms and the classrooms of other disciplines. While they did not necessarily agree with this difference, that mathematics should not be presented in ways similar to music and language, they were unable to move beyond the paradigm of university mathematics classes as lectures and to see how different approaches might suit mathematics.

The participants' inability to envision a way of teaching of mathematics other than lecturing caused me to think of another question – why might a lecture, a "reading" of the course material, seem more reasonable for mathematics than other subjects? Why might it seem acceptable to a professor of mathematics to communicate with students in such a way? I had studied and taught mathematics for years. It had not yet occurred to me that maybe this way of presenting mathematics might make sense within people's perceptions of mathematics. Burton (2004) found that, "A singular view of mathematics also controls the ways in which it is presented to students" (p. 21), which was a perspective that seemed to be true for the participants:

> Steven: Within whatever our truth system is, but I think that's as close to absolute truth that you could probably get in mathematics. In no other subject can you make a statement and say, "Well, this is right." Right? There's no, you can't have an opinion, you can't have anything except the fact that "Yeah, this is true."
>
> John: Yeah. True. It's… Yeah. It doesn't have the subjectivity that most other things would have.
>
> Steven: Well, most every other thing has subjectivity. Mathematics would be the only subject where you can take out subjectivity. Right?

Steven and John saw mathematics as embodying an objective system of truth and their point of view resonates with Burton's (2004) finding that "mathematicians are presented in the literature as engaged in searching for The Truth of which, by definition, there can be only one" (p. 21). With this idea in mind, if there is only one truth in mathematics, could there be more than one way to present the truth, some way other than a simple reading of the truth? How might someone present the truth in a different way to students? To some of the participants, mathematics was seen as a topic free from opinion and subjectivity, therefore reducing the possibilities for engaging with students.

John went further in his description of how mathematics is presented in classrooms:

> As you develop as a mathematician, you see that there are right ways to go about things and there are… Well, maybe not right and wrong, but there are better and easier ways to go about things and there are harder ways to go about things. And so I guess part of teaching is having that come out in a classroom context. It seems like what we teach is kind of the, how do you say, it's always the best possible situation.

John provided the perspective that mathematical work consists of finding "better and easier" ways of doing things, which then get passed on in the classroom. What seemed to underlie this idea was that there exists one best way, the best possible situation to present mathematics to students. With this view of "best," of one way that is best to present mathematics, a lecture or a reading of mathematics might be seen by some as the most efficient way to communicate mathematics to students. Such a perspective seems to resonate with the views of Krantz (2003) and Stewart (2006) discussed earlier; in particular, that one need not entertain questions or one should not digress from the syllabus because lecturing is the most effective way to teach mathematics.

I believe there is an important question to be asked here – the "best" way for whom in particular? The word *best* signifies that one technique is better than any other, which is a judgment that can only be determined by people, in this case by mathematicians. However, there seems to be an incongruity in what John says that he is unaware of. On one hand, he spoke of presenting the *best* case, and he connected this idea to his understanding that mathematics is objective and that an impartial way of going about and communicating mathematics exists. He did not recognize that something that is considered to be the *best* was rendered so through a subjective determination. This revealed a subject-object split, a view that the *best* is determined by the perceived objectivity of mathematics, rather than the subjectivity of mathematicians. This standpoint will be explored further in terms of what it means for professors of mathematics in the following section.

With the images of mathematics professors and classrooms in mind, what meaning might be taken from the stereotypes of mathematics classrooms and mathematics professors? How might such an archetypal image of university mathematics classroom as lecture come into play as the research participants began to think about themselves as professors of mathematics? Britzman (2003) described the influence of such stereotypes:

> These images displace the collective concerns of real teachers with measures of individual behaviour based upon […] notions of a unitary-contradictory identity […] The persistency of such stereotypes, however, does more to caricature the opinions and hopes of a community. […] Stereotypes engender a static and hence repressed notion of identity as something already out there, a stability that can be assumed. Here, identity is expressed as a final destination rather than a place of departure. (p. 29)

I heard the participants' concerns, such as the hope for their students' understanding that was discussed in Chapter 6, fall by the wayside as they expressed their interpretations of what mathematics teaching is and who mathematics professors are supposed to be. The images of these seemed to "displace the collective concerns" of the research participants and replace them with a "static and repressed notion of identity."

With stereotypes of mathematics classrooms and professors carrying the weight of a unitary identity, an ideal of a final destination for how mathematics professors *should* be in the classroom, and the idea that a way of being can be assumed, I began to wonder – does it matter *who* the mathematics professor is? Chris stated, "I don't really care who teaches calculus and who doesn't," and so to him the *who* did not seem to be of importance. This perspective came through for Robert as well:

> There's a certain rigor involved that how, you could teach a little bit better, but I don't know how much variety you can actually put in. How much different with professor A, different from professor B, … I mean for the standard undergraduate classes anyway. I mean for a graduate course, you can use a different textbook, a different approach, but then even in that case, I think the difference is subtle.

In referring to professor A and professor B, the image that Robert conveyed was one where the professor did not have much bearing on what might occur in the classroom. In the language of professor A and professor B, I heard an interchangeability between professors, as though their identities might be so alike or the differences so insignificant that it would not matter who was in the classroom. This resonates with Jardine's (2006) view that in mathematics there exists a "mood of detached inevitability: anyone could be here in my place and things would proceed identically" (p. 187). Robert went on to describe the textbooks that are the standards for certain courses:

> The first time you teach a course, the first book you use is Churchill. Churchill is probably the standard text that you do, right? You may not use Churchill, but that's probably one of the reference books that will be on your desk when you teach that, when you teach the first complex analysis class. You probably have a Churchill sitting there and to teach maybe real hardcore analysis you probably have a Rudin sitting somewhere.

To Robert, it was as though the text for the course became the salient identity and that things would "proceed identically" in courses such as complex analysis, regardless of the professor.

What I interpreted from the participants' descriptions of classrooms was that the professor becomes subordinate to the mathematics. This was somewhat evident in John's view that what is presented in classrooms is the "best-case scenario." There is no flexibility in presenting what is perceived as the best and so, regardless of the professor, the same mathematics will be presented in the classroom. I make an argument here that the "best case" is perceived to be that which is given in textbooks and in curriculum guides. Jardine (1990/1998) pointed to the difficulties of such interpretations of mathematics in the classroom, the text, and the curriculum:

> The difficulty with the mathematics curriculum is that it appears to not be conversant with anything outside of itself. It appears self-closed, complete, detached – it doesn't *speak*, it will not *answer*. (p. 62)

This resounded in Steven's statement "you can't have an opinion, you can't have anything except the fact that 'yeah, this is true.'" In this light, how might a mathematics professor have a voice beyond what is the truth, what is "best," and beyond what is "self-closed, complete, detached"? How could a student have a voice? There seems to be no dialogue between the professor and the mathematics curriculum, and the mathematics comes to overshadow the professor. Mathematics, in this view, seems to take precedence over the mathematician and his or her knowledge, experience, and subjectivity.

The significance of the images of the university mathematics classroom, the stereotypes of mathematics professors, and perceptions of mathematics as embodying truth were heard in the participants' voices, in how they viewed the arrangement of professor and mathematics in the classroom. If mathematics is absolute truth, if it is fixed, if the curriculum represents what is best, then what imprint can a professor give to the classroom or leave with students? What influence can he or she have over what is and how it is presented to students? What place can their identities have in this situation? Such influences, issues, and questions are explored further in the participants' lived experiences as teaching assistants for first-year calculus courses.

Teaching Calculus Gives Us This Very Rigid Direction

At most universities, including the site for this project, first-year calculus courses represent the core initial encounters mathematics graduate students will have working with undergraduate students in an instructional capacity. The research participants' experiences with first-year calculus courses included helping students one-on-one in the workshop setting, marking homework assignments and exams, and leading one-hour tutorial sessions, all of which were described in Chapter 6. Within our conversations, the subject of calculus became a noticeable concern for the participants. In their first experiences taking on some of the work of professors, the participants felt the weight of perceptions of mathematics, both their own and their students', an awareness of a defined and fixed curriculum, and the complexities of working with students.

In a conversation with Robert, I heard the authority of a perceived fixed curriculum for calculus:

> I mean first-year calculus you have to teach differentiation. You have to teach, you have the concept of continuity. I mean there's, there's not much freedom involved. I mean it's pretty standard.
>
> Right? At the end it depends on, you know, the bigger room maybe for biological science students who are maybe not taught the formal definition of the limit. You can get away with that. They may not be able to understand what, you know, to understand probably, you probably need some more real analysis anyway. And so, you can argue maybe that you don't really need to teach it in a first-year calculus class if the people learning mathematics, if they have to do it all over again in an analysis class anyway, in the first analysis class anyways, so you just choose to skip that.
>
> But that doesn't actually affect a lot of things that you do in the calculus class. You can definitely teach a calculus class without the formal definition of the limit. You can do that. The formal definition of the limit without a doubt is very confusing for first-year students.
>
> So, you can skip that and, and it's fine, but besides that, I don't know what else you could put in for what you take off. I mean you may probably do a little more, you know, basic stuff if the class background is really weak. And then choose not to go as deep in the textbook, whatever textbook you choose there's … as, as you may want, but there's no freedom in talking about it, you know what I mean.
>
> It's pretty standard stuff, you know.

Robert used the word *standard* twice when talking about what happens in first- and second-year mathematics courses. What does it mean for something to be standard? The etymology of the word *standard* includes "authoritative or recognized exemplar of quality or correctness" and "rule, principle, or means of judgment" (www.etymonline.com, 2009). In Robert's description of what might occur in these courses, he pointed to a particular way of how things are done, to a perceived "exemplar of quality or correctness." With this perception, then, there is little room for the professor to move, to do things other than present the material. This resonated in a question he put forth: "How many ways can you skin a calculus class?"

Robert's ideas for how a professor might go about things differently in the classroom were strongly connected to the mathematics. To him, the personal imprint a professor could put on the classroom consisted solely of the mathematics that could be omitted depending on whether the students were capable of the mathematics. Aside from describing the mathematics of the course, though, Robert did not talk about how else the professor might be different. In light of this, Robert twice saying that there is "no freedom" is telling. What would freedom mean? Some synonyms for *freedom* are autonomy, lack of restrictions, independence, self-determination, choice, and free will. Yet, to Robert the mathematics professor seemed to represent Britzman's unitary identity, an identity in which freedom is subordinate to the structure of mathematics and the curriculum to be presented.

John also spoke of his perceptions of the ways in which calculus is presented:

> It's easy to keep teaching calculus like this. We've done it forever. We know exactly what we have to do. Almost everyone does it the same way. I mean even by the time you have your PhD, you've probably been teaching calculus three or four times. You've taken it. You've TA'd for it. I mean, you know the problems. You know the classic examples. You almost don't even need a book. You can just walk up there and start teaching.

There was the sense in what John said that there is nothing left for the professor to understand, know, or learn about teaching calculus, and there are no other directions to go. Teaching calculus is simple, rote, and effortless. It seems that the years of seeing calculus presented in a standard way have left an indelible imprint that there is but one way to offer calculus to students.

Brown (2001) discussed the phenomenon of how “Certain styles of signification become naturalized in the sense that they become culturally conventional in a way that seems to be entirely neutral […] Mathematics becomes locked into certain conventional symbolizations which then dominate thinking about it” (p. 27). I heard this in Robert’s description of how he would give a calculus lecture: “You know, giving a calculus lecture is easy because, OK, today I’m going to do this and I just, I could use a transparency or whatever, just do examples and then fifty minutes later, the class is over.” It was also heard in John’s statement “You know the problems. You know the classic examples.” There is an interpretation that all one must do in a calculus lecture is use transparencies, do particular examples and problems, and then the class will be over. Robert and John both said this in a matter-of-fact manner, as though it was natural, neutral, and conventional to approach teaching calculus in such a way.

Britzman (2003) wrote about how inflexible perceptions of curricula can influence teachers: “When knowledge is reduced to rigid directives that demand little else from the knower than acquiescence, knowers are bereft of their capacity to intervene in the world, and knowledge is expressed as static and immutable” (p. 46). In the context of this study, it seems that the presentation of mathematics was tied to “rigid directives” and that the participants, these future professors of mathematics, were unable to see any other way to be or any other way of teaching mathematics. Aside from teaching, Sara’s questions about whether being in mathematics would allow her to “intervene in the world” were explored in

Chapter 6. To me, her questions came from the "rigid directives that demanded little else" from her and Sara wanted more from life than this.

Britzman (2003) described what she called 'routinization' where "the repetition of activity desensitizes us and undermines our critical capacity to transform it into something more than going through the motions" (p. 50). I heard the effect of routinization in Steven's statement below:

> I don't know. It's like people, it's like we're both [the professor and the students] going through the motions. Maybe it's like the students are going through the motions by coming to class, the professor's going through motions of teaching it. It's like we're all going through the motions. And maybe that's the point, then, of why be enthusiastic because we're just going through the motions.

I noticed a tone of resignation in Steven's voice, what I perceived as almost a hopelessness, in thinking of the routinization of what occurs in mathematics classes. Emily echoed this, too, saying, "I like the 'going through the motions' idea. It's like, I'm coming here, he [the professor] doesn't want to be here, I don't want to be here, no one wants to be here."

The weight of routinization was also heard in the participants' comments about calculus. I observed it in Steven's contribution to the discussion about calculus. While John and Robert spoke of the perceived ease and straightforwardness in teaching calculus, I noticed in Steven's voice the limitation of what he was able to offer to undergraduate students within the standard approaches to calculus:

> No one really enjoys calculus. Like it's an interesting exercise in pushing around equations, but it wouldn't say it's… I think there's much more interesting things you can teach these students at that level, right? Is all mathematics required to learn calculus? So teaching calculus gives us this very rigid direction, right?

Steven expressed further dissatisfaction with calculus, not only its place in the undergraduate mathematics curriculum, but also with the direction calculus seemed to require.

Eventually, the participants departed from calculus and began to share the topics they felt were important for students to learn. They spoke of wanting to include number theory, statistics, counting, and probability, among other topics, in the mathematics curriculum that students in other disciplines were required to take. They spoke of creating a survey course that would present multiple views of mathematics, which would, in their minds, promote deeper understanding and appreciation of mathematics. Their desire to include other kinds of mathematics did not stem solely from their own particular interests, but came from their wish for others to have a better sense of mathematics. However, to revisit Britzman (2003): “When knowledge is reduced to rigid directives that demand little else from the knower than acquiescence, knowers are bereft of their capacity to intervene in the world” (p. 46). With similar feelings about the mathematics curriculum and its “rigid directives,” the participants expressed how they would have very little power to influence or change what mathematics and how mathematics might be presented to undergraduates, even once they became professors. The participants conveyed a sense of resignation, an acquiescence, as to what they could do in their roles as graduate students and future professors, with one participant going so far as to say, “No one is ever going to listen to me.”

Taking It Personally

Aside from their interpretations of mathematics, mathematics professors, and perceptions of how mathematics is to be presented, their daily lives as teaching assistants held particular importance in how the participants viewed the possibilities for their futures as professors of mathematics. While I had expected that their professors' behaviour would carry the most weight in how the participants thought about their teaching, I was surprised to hear how profoundly the participants experienced the undergraduates' behaviour.

It was in the workshops that the mathematics graduate students had the most opportunity to interact with undergraduates. They worked as many as eight hours each week in the calculus or algebra workshops, helping undergraduate students individually with homework problems. The few minutes the participants spent with individual students appeared to have a bearing on the ways in which the participants engaged with the students, and the participants expressed a sensitivity to how the undergraduates approached them and asked for help on assignments.

When describing her experiences with the different approaches the undergraduates took in order to receive help on their assignments, Emily's willingness to assist students appeared to be conditioned upon the ways in which the students initiated conversations with her:

> Like there's never any "Hi, how are you today?" It's just kind of like "How do you do this?" Well, there's a couple of kids that are really, really good, you know. They're like "How are you doing today? Do you have a long day again?" "Yeah, I have a long day." And then they're like "Okay, sorry I have another question." And I'm like "Don't worry about it. That's what I'm here for." You

> know? Like I appreciate that. I kind of get a little annoyed when they're just like "How do you do this?"

Emily mentioned this again, going further in interpreting the students' actions as having meaning for how important the undergraduates made their learning and whether they were interested in mathematics:

> You're in the workshop and it's pretty rare, maybe once every month that you'll get a kid coming up and being like "I really don't understand this concept. Like I'm losing it here and here. What's going on?" And then you can kind of like, and it's not like one of those questions, rather than a "So question sixteen's really bothering me." You know? "I can't do question sixteen. Can you do it for me?" And when you're in the workshop, like them talking to you is just a means to an end, you know? It's just nothing other than, you know, they get stuck. They've tried for, you know, maybe thirty seconds on the question. They can't get it and they come and they talk to you and they want the answer because their deadline is in two hours to pass it in. They're not interested in the real math yet. You can't really blame a pre-calculus kid because the majority of those are just trying to fill up math credit hours. I don't know why they're there. They're really… some of them really struggle. Some of them really get it, but they don't really need to come to the workshop. And so often it's just like they don't really want to talk to you about why something's the case. They don't want to discuss all the things about, it's just like "Okay, you've given me enough. I can see how to do it now." And they go, they take your page and they go and they finish it.

I observed an interesting construal of the undergraduates' behaviour in Emily's description. In particular, she seemed to assume that, based on their approach to her, the students had little interest in mathematics. With this in mind, if their approach was to ask for help rather than engage with Emily as person, she attributed their struggle to solve a problem to a lack of motivation and interest in mathematics. This was surprising to me. As an undergraduate, I worked as many as thirty hours a week to put myself through school. Sometimes I did not have as much time as I needed to understand mathematics, but it did not mean that I was not interested in learning or in mathematics. I found Emily's interpretations to be a bit unfair, but her interpretations of the undergraduates may have deeper meaning.

I heard disappointment in her voice about how the undergraduates did not engage with her as a person, but only spoke with her "as a means to an end," in order to complete their homework. She seemed to think that the students saw her only as a vessel of mathematics. I noticed a sense of rejection in what Emily said about her interactions with undergraduates, as though she had hoped that they would express some interest in her as well as needing help with mathematics. This also came through in Emily's hopes for connecting with students in Chapter 6. Here, however, if any teaching assistant would do when helping the students, then it as though who Emily is as a person and all that she is going through ("Yeah, I have a long day") did not matter. Emily appeared to become invisible behind the mathematics she was expected to help students with. It was her content knowledge that mattered, not her personality or identity, and feeling this way in her interactions with students seemed to have a downbeat impact on how she engaged with them.

Chris had similar interpretations of the undergraduates' behaviour and pointed to a difference in his approaches to mathematics versus the undergraduates' approached. He repeated these views a few times during our second conversation and again during our third conversation:

> I feel most of them [undergraduates], they just want the answers to their assignment. I'm kind of bad that if I get a problem and I want to work on it, I kind of want to work through it rather than just give them the hints. So it's probably something I have to work on. But

> yeah it is a bit disconcerting when you can kind of tell that they don't really understand, they don't care. They don't care what they learn. They just want to know how to, they want to memorize the formula. They expect it to be a one-second question.
>
> I guess that's been a bit disappointing. I mean I don't know. It's just maybe but I think none of the people here are majoring in math therefore they don't really have, they may like it, but they don't really care. … Or they don't, they don't know that they could do well if they wanted. I don't know. I think there's just a lot of motivation issues.
>
> So it's frustrating when they don't try to, to figure out what, what needs to be done. They just, they just try to do it by rote even though they don't really understand what they're doing. That's frustrating.

Chris expressed disappointment in how the undergraduates approached him for help. He construed the undergraduates' behaviour to mean that they were not concerned about mathematics. The comparison of his way of being in mathematics with the undergraduates' ways of being came across as disapproving, and in saying that "they don't try to figure out what needs to be done," Chris appeared to put them down for not understanding an unspoken way of being in mathematics.

Robert echoed Emily and Chris in how he made sense of the undergraduates' ways of asking for help:

> I sort of, almost never ran into the problem of someone who actually wanted to come in and say "I want to know more about this," you know, a little bit more about the math of this because a lot of them are just engineering students. They just want, pretty much they just want the answer. Yeah. How do you solve this? Okay, this is the answer. Done.

Based on the actions of the undergraduates, Robert had also made an assumption that their motives and concerns stemmed from wanting to know how to solve problems rather than a desire to understand the material. His assumption prevented any further engagement he might have had with them. Because he thought he had not met a student who asked for more explanation, he felt that students did not want to know more beyond the particular question.

John also seemed to take the perceived lack of interest on the part of the undergraduates as a slight, but he went further by saying that undergraduates who did not display certain behaviours should not be allowed to pass their courses:

> And a really unmotivated student like that shouldn't pass. Like we shouldn't have a system where you can just come to class and not care and pass. Right? Because then we're saying well, you know, the average student is unmotivated, so it's … But it would be nice if they would learn to teach themselves. I would think that's what, maybe not necessarily just in math, but that's what a bachelor's degree should do is it should give you an educated start and to be able to teach yourself about things.

To John, being in mathematics involved not just learning content; it was also about emulating a certain kind of behaviour. He moved to distance himself from interactions with or responsibility for students by remarking that, "it would be nice if they would learn to teach themselves."

Steven's comments about the undergraduates provided another look into how perceptions of undergraduates influenced the emotions of the participants:

> How long can you really, it's disheartening if you have maybe three students who are interested and then like three hundred who just hate it, right, so it's good to have optimism. I'm still young, but let's see how long it lasts. I've seen it just completely smothered. I can feel it now. It's just easier for me to not care about it. It's too exhausting to teach the students who don't want to learn. And what's the point? They just end up hating you anyway…
>
> Apparently this is what happens to grad students. Because when I came here I was very enthusiastic about teaching and everyone was making fun of me. And they said "Oh, by next year, you're going to hate the students." And they were right. It's, well, it's completely disappointing, but …

This impact of his interpretations of the undergraduates went a bit further than the other participants, as Steven talked about losing his optimism and how he was beginning to hate the students. In this regard, I found Steven's transition in dialogue from the students hating him to him hating the students to be interesting. Why did he make this transition? Might it represent a move to deflect a sense of grief that came from feeling hated by the students he had once hoped to inspire? What would it mean for him to be hated by his students? It is possible that his life as a professor would be more bearable if such feelings were turned in the other direction.

The perceived indifference that was felt coming from the undergraduates translated into reticence on the part of the graduate students. The help the graduate students provided was conditional, whereby those undergraduate students who appeared to have an interest in mathematics, who emulated a way of being similar to the group of mathematicians, were deserving of in-depth help, of becoming a part of the group. Otherwise, the undergraduates were given only what the graduate student believed they wanted – the solution to the problem, rather than a learning experience where their understanding might be improved. Herzig (2002a) found a similar perspective in a department of mathematics:

> Overall, rather than defining instruction as an avenue to teach students to be mathematicians, these faculty described instruction as an avenue for students to improve themselves. The approach removes most of the responsibility of teaching and learning from the faculty, and instead places that responsibility almost entirely in the hands of the students, requiring students to develop a certain degree of competence <u>before</u> they can interact with the faculty in meaningful ways. (p. 189)

The participants often perceived the undergraduates as not knowing what it might take to be successful in mathematics. Further, they expressed certain expectations of behaviour and competence that determined whether they would engage in meaningful ways with the undergraduates. The participants' assumption that the undergraduates would not take the time to learn beyond being told how to solve a problem is an unfortunate assessment. The participants expressed frustration that many of the undergraduates were only *doing* the mathematics, rather than *attempting to understand* the mathematics. And, yet, instead of motivating a conversation that might compel the understanding the participants hoped for the undergraduates, they declined to provide educative moments to the very students who seemed only to do the mathematics.

I found it interesting that the participants would perceive such a great deal of information about the undergraduates in so little time and that it would be taken personally. It seemed as though the participants had handed some influence over to the undergraduates, a form of influence over how the participants perceived their own relevance or importance. Steven's comment "Like it's hard for me to like wrap my head around, you know, I've just become a bad teacher because … I have to, I'm convinced that it's the students … that I'm teaching" speaks to this influence, revealing a view that the undergraduates' behaviour affected him in such ways that he had begun teaching differently.

It appeared that the mathematics graduate students had taken on a passive role not only to the curriculum, but also to the undergraduates, to what they perceived the undergraduates as wanting from them. Why would the participants have been so influenced by the behaviour of the undergraduates that they helped in the workshops? The participants had a unique perspective of learning and understanding mathematics that could help them to bridge the divide between themselves and the undergraduates. However, they refrained from relying on their experiences and their own ways of learning and understanding mathematics. Why did they not engage this side of themselves when working with undergraduates? I believe that this is partly due to department expectations, the sheer number of students needing help, and in the participants' reluctance to share themselves with others. But also I believe this is due to their perceptions of their roles as professors, which are described in the following section.

I Am Professor – Hear Me Not Teach

Up to now, I have intentionally chosen to mainly use the word *professor* rather than *teacher* to describe the participants and those who present mathematics to university students. I have done so because the participants referred to themselves as professors rather than teachers, some of them holding on tightly to this classification. More than claiming this particular identity for themselves, the participants also delineated the differences between the two. Steven and John spoke explicitly of being professors rather than teachers:

> Steven: But this is why, this is the first thing we need to get across is that professors and teachers are two completely different things. They're professors.
>
> John: I think they [undergraduates] view their professors as teachers, right? This is why they're here – to be taught. They think that it's not their responsibility. It's the professor's responsibility to teach them.

I found the participants' focus on this difference worthy of note and I began to question it. If a person is called or named *professor*, then is what they do in the classroom necessarily *professing*? When one describes what a professor does when they are in the classroom, most often the word used is *teach*. What is frequently heard is "I am *teaching* algebra this semester," not "I am *professing* algebra this year." I observed an interesting co-mingling in the language the participants used to describe the classroom role of a professor. I wondered if there were there distinct, formal characterizations of teacher and professor.

> *Teacher*: One who teaches or instructs; one whose business or occupation is to instruct others; an instructor; a tutor.
>
> *Professor*: One who professes, or publicly teaches, any science or branch of learning; especially, an officer in a university, college, or other seminary, whose business it is to read lectures, or instruct students, in a particular branch of learning; as a professor of theology, of botany, of mathematics, or of political economy. (www.dictionary.reference.com, 2009)

I observed some overlap between these two as the words *teach* and *instruct* are both used to describe part of what each of these occupations entails. What does it mean to teach then?

> *Teach*
> 1. impart knowledge or instruct (someone) as to how to do something; give information about or instruction in a (subject or skill)
> 2. cause (someone) to learn or understand something by example or experience encourage someone to accept (something) as a fact or a principle make (someone) less inclined to do something (Soanes & Stevenson, 2005, p. 1809)

I wanted to investigate how these definitions and distinctions came into play for my participants. Did the participants envision professors as teaching? If not, what was the role they expected the professor to fulfill?

> Steven: I never really saw them as teachers. I never saw them as teachers. I always knew there was a line between teachers and professors. I always felt that professors are more there to guide you in self-learning rather than, like, it's not their responsibility to

> make you understand. They're just presenting you with the material and then you have to take that material …
>
> John: At the university level the professor should just be there to guide the students. I mean, they have books. They should be motivated to work on their own. You know, if they really want to do well.
>
> Robert: I agree with John that it is difficult to ask a professor to teach, to be like that in a first-year calculus class.

John and Steven put forth their ideas that professors are not teachers and are thus not responsible for students' understanding. With that, they asserted that students must teach themselves. With regard to how teaching is defined, these participants were in agreement with the first definition, but did not accept the second. Robert's statement is interesting in two ways. Similar to Emily's earlier assertion that professors are demanded to teach, here Robert said "it is difficult to ask a professor to teach." Again, I wondered why the participants had a view of professors as not teaching. If they are not teaching, then what is it that professors do in the classroom? Second, Robert said that it would be difficult "to be like that in a first-year calculus class," again suggesting a conditional interaction with students that depended on whether the undergraduate students met certain benchmarks for behaviour or accomplishment.

The participants discussed the role of the professor later in the conversation:

> John: If you go to school to be educated well, sure, you want presentable material and I think it's the professor's job to present the material in a coherent way. But I think it's the student's job to make their own motivation.
>
> Steven: But if it's their job to do anything, if you're paying them to do anything, it's to get correct guidance, right? Because it's impossible to study mathematics. You're paying them to tell you

> what direction an area of mathematics that you just studied and what's the best way of doing it, right? But this idea that they [the professors] need to explain it enough or well enough in order for us to understand it is absurd.
>
> Sara: I think the professor should still do a very good job of presenting the material in a coherent way.
>
> John: It's not the responsibility of the professor to motivate the students. I think it is their responsibility to kind of motivate the subject matter.

In several parts of the dialogue, the participants described what they saw as the professor's job – to motivate the material, to present material coherently, but they stopped short at saying that part of a professor's work was to help students understand the mathematics. In this regard, they saw this task as entirely the students' function.

Why would these distinctions be true and important for the participants? One thing that came to mind was that, as graduate students, the participants were required to work very hard in their programs to learn mathematics, and they spoke of how they often needed to teach themselves. Through this experience, it is possible that they developed the impression that students need to work in such ways and to not rely on their professors. When they described their own learning in graduate school, the participants spoke of how they could only depend upon themselves, alluding to a notion that their professors were not explicitly teaching them. Another idea is that they may possibly have a fear of the unknown. With the exception of John, the participants had not formally taught their own courses, nor had they been given guidance in how to do so. Thus, they might have been anxious about the responsibility of teaching, of how they might go about helping students understand mathematics, students that they had not, as of yet, established good impressions about.

Another interpretation I have of the participants' views as to how calculus should be taught and what the professor's role is may stem from their impression that professors are experts and must relay their expertise through lecturing, through a mastered reading of the material. A summary of these possible interpretations is illustrated by Britzman (2003):

> For example, in university settings, the lecture format is typically employed to dispense knowledge, and examinations are the chief means for exchanging knowledge for credits leading to credentials. Here, knowledge inevitably appears as self-referential, something transmitted to students who have no voice in determining its relevancies and who have gained no insight into the struggles of selection or their own power to interpret. This form of presentation bestows both knowledge and the teacher representing it with an immutable quality of certainty, efficacy, and authority. (p. 56)

All of these – lecturing, students as powerless, teacher as expert – seemed to come into play as the participants brought together their experiences and formulated ideas as to how they should be as professors. I interpreted these perspectives, though, as contradictory to their desire to help students understand, which was described in Chapter 6. This paradox resonates with DeFranco and McGivney-Burrelle's (2001) findings that, "although the TAs [mathematics teaching assistants] indicated a new understanding of how students learn mathematics, this belief seemed to be held peripherally and in conflict with their views about the role of teachers (i.e., to deliver information or as the central authority figure in class" (p. 687).

The More Things Stay the Same, the More They Stay the Same

The participants had already begun to develop their own ideas of how they would prepare and teach their own courses. Without exposure to alternative models of university mathematics teaching, they were left to rely on their own histories and professors as exemplars for how they *should be* in the classroom. Steven explained how he would look to the good things he observed in his own professors for ideas of how he would be in the classroom:

> I guess, you know, you always sort of want to copy the good things about a professor. You know, I want to act like them. I guess they're good role models in that sense, but I don't know.

He did not specifically describe what he thought were the good things he saw in his professors, but his view of a whom mathematician is was tied tightly to his view that he would be a professor not a teacher.

Robert described a possible scenario a first-time university professor might follow when preparing for a particular course:

> I think a lot of times what happens is that maybe the first time a teacher teaches a course, and what happens is your supervisor is nice enough. He or she taught that course before. They lend you the set of books they have. They say, "Oh, you can use my books" and then you probably pick up a couple texts, besides the textbook, you probably pick up some relevant texts and look through them. And then think "Okay, I'm going to teach this" and then look at the notes and look at the book and maybe "Oh, I'll do it this way this time" not following one set of things completely, but you sort of take the ingredients a little bit from here, a little bit from there, sort of suit what you want in a sense that it suit what the class is doing. But I would have to say that even to do that, there's a lot of times that's what happens and in that situation I don't think there's much, the teaching that actually comes out at the end of the day is not much different from if you just take your supervisor's notes and you just do it in class. I don't think it's really that different.

Even though Robert saw the potential for developing one's own ideas for a class by pulling together multiple resources, he felt that doing this would not have an outcome that would differ, even slightly, from the situation where a new professor would follow his or her supervisor's notes entirely. I heard in Robert's statements that the professor's identity and his or her ideas did little to alter what might occur in the classroom.

How did the participants think about preparing for their own classes? Chris spoke of writing good, thorough notes to himself for his tutorial sections. His idea of preparing for teaching echoed the award-winning professor described in Chapter 2 who told mathematics graduate students that the key to good teaching was to make good notes. What do notes have to do with what one does in the classroom? How do notes have meaning for how one goes about teaching? Robert stated his own ideas about notes: "If you prepare well, write down good notes, the worst you could do is just copy everything on the blackboard and try to explain your way along. Maybe your information may not be very clear, but that's probably the worst you could do." It seems as though notes represented a guide for what was to be copied onto the blackboard, a reading and rewriting of the mathematics the students would be expected to learn, a replication not only of the text, but also of other professors' approaches to their classes.

Robert spoke of "the worst thing you could do," which consisted of copying of notes onto the blackboard. The act of copying notes onto the blackboard is what is most typical in a mathematics lecture. It is interesting, then, that Robert described this common way of being in the classroom as "the worst thing you can do." Similar to Emily's description of a lower bound of what a life in mathematics might be, it seems that Robert was creating an image of what the lower bound might be for the teaching of mathematics. He later spoke of how none of the students would complain if the copying of notes were done in a coherent way. To me there appeared to be a tacit acceptance of how mathematics professors are and what they do in the classroom and an understanding that the "the worst you could do" would continue to be accepted.

Sara offered an interesting experience she had with some of her fellow graduate students, her peers who seemed to be complicit with a professor's lecturing when it came to filling out teacher evaluation forms at the end of the semester:

> A few of the students, they knew I was pretty vocal about me disliking what was happening in class. And they asked me, "So you filled out those evaluations? You know, how did it go?" And I said "I just filled it out in a fair and polite way, but I said what I thought, as you know. You know what I think." And they said "Oh, okay, I just tried, I gave him like A's and B's", which are the best things you can give to a prof, "because I didn't think there was any need to be mean." And I said "It's not about being mean," or it's, well, if you don't give them feedback, then there's no way they can change. So I'm guessing that these students who are letting them go easily probably wouldn't care as much when they, if they ever become teachers because they'll think "Oh well, it's just small and informal and students will still give me good evaluations so as long as I do a mediocre job, it's okay." So, if they're not very vocal about what's happening now, they probably won't put in the effort that I imagine one should if they were to become a teacher in the future.

An acceptance of the worst one can do came through in Sara's description of the professor and her peers' evaluation of the professor. Sara also spoke of one of her favourite professors, whom she described as an inspired teacher:

> … so he's been kind of on, he's says he's kind of on the blacklist because he speaks up too much about these issues [teaching] and a lot of people are like "Whatever. Forget about it. Come on, relax."

Similar to the complacency of the graduate students to accept a professor's teaching, Sara spoke of how the professors took the same attitude about teaching – that poor teaching should be accepted so that their own diminished focus on teaching would still be within acceptable limits.

John had spoken of his enthusiasm for the R.L. Moore Method (Jones, 1977), a teaching method by which students discover mathematics through developing their own proofs rather than receiving them from professors during lectures. He hoped to eventually employ this method, but then resigned himself to the thought that he would not be able to:

> It's almost like the people who could change things are so entrenched in the way they do things already, kind of the senior professors or the senior lecturers … the chairs of departments. I mean it's, you can't as a chair of a department, I, I can't imagine you saying "You know, I think you guys should teach the discovery method this year." You know? "Our new policy is that we're not gonna have lectures." Or, "In these five courses we're not going to have lectures and we're going to see what happens." I don't know that people are ready to take those chances.

To me, John was speaking of being resigned to a certain way of being in mathematics and that there were risks in breaking away, that university faculty weren't "ready to take those chances."

What seemed to repeat in each of these stories was a sense of conformity and of reproduction or replication when becoming a university professor of mathematics. Robert shared his ideas of how one becomes like their professors and inferred that the worst one could do was acceptable. Sara related a story where graduate students did not poorly evaluate a professor because they could then teach in similar ways without reproach. Sara's second story of her professor also reveals a sense of conformity – that even a good teacher is told to "forget about it," to forget about their passion for teaching. Britzman (2003) explained what such conformity can do:

> Conformity, however, speaks to something more than the uniformity of thought and the standardization of activity. As a measure for being, conformity diminishes the prospects of something other than what has been previously established. In this sense, the forces of conformity are suppressive. […] In other words, conformity, in its adherence to the dictates of social convention, privileges routinized behavior over critical action. Its centripetal force pulls toward reproducing the status quo in behavior as it mediates our subjective capacity to intervene in the world. (p. 46)

Britzman's thought that "conformity diminishes the prospects of something other than what has been previously established" came through in Chris's wondering about his own teaching: "I always have these pictures in my head of when I teach it's going to be different and I'm sure everyone does. And I'm sure it won't be different." Chris and the other participants seemed to recognize what Britzman has observed in future schoolteachers – "The problem of acting in an inherited tradition, while at the same time trying to establish one's authority […] often finds student teachers embodying the very traditions they hoped to change" (p. 41).

Conclusion

In the previous chapter, I heard a pulling towards and pushing away from a mathematician's identity in the participant's voices – an interesting coupling of a reluctance to be, yet safety in being a mathematician. However, when it came to their identities in the university classroom, it appeared that they felt the weight of many things such as the stereotypes of mathematics teachers, their perspectives of the role of professors and students, the view that what is taught in classrooms is the best-case scenario, and so on. In holding onto the idea that they were being or becoming professors, not teachers, and what they could do in that role, I heard what Britzman (2003) described as the "startling idea that taking up of an identity means suppressing aspects of the self" (p. 27).

She also wrote of how "teachers' classroom appearance, sustained by school structure and serving as a basis for cultural myths, represses teachers' subjectivity: they are subsumed by predictability and hence immune to changing circumstances and incapable of interventions" (p. 30). I heard something similar when I asked the participants how they might be as professors of mathematics, what they might change once they became professors, and they seemed to be unable to consider the question, as though a space did not exist in which to consider such things.

Chapter 9
Becoming a Mathematician

For the six graduate students in mathematics who volunteered for this study, what did it mean to become a mathematician and a professor of mathematics? How did they understand their roles as mathematics teaching assistants and possibly future professors of mathematics? How did they experience and make meaning of the various suggestions they encountered about teaching? What was important for success in their programs? What did the participants interpret as having meaning for who and how they should be as mathematicians and as professors of mathematics?

In this chapter, I look at meta-themes that come out of the explorations in Chapters 6, 7, and 8 and discuss how these meta-themes resonate with Lave and Wenger's (1991) idea of legitimate peripheral participation and Britzman's (2003) work on identity in relation to teaching. I then describe Caputo's (1987) notion of repetition as a way to understand how the participants began to find hope, possibility, and even themselves through the conversations. In light of what I have learned in this exploration of the mathematics graduate students' lives, I also address the idea that graduate students in mathematics are in the process of becoming mathematicians and not professors of mathematics. Finally, because of the complicated notion of *teaching* post-secondary mathematics found in this dissertation, I close this chapter with questions to departments of mathematics, questions I believe need to be understood before moving forward with teacher preparation plans for graduate students in mathematics.

Replication

> *replication*: make an exact copy of; reproduce; repeat (a scientific experiment or trial) to obtain consistent results (Soanes & Stevenson, 2004, p. 1493)

> The larger community of practitioners reproduces itself through the formation of apprentices. (Hanks, 1991, p. 16).

One meta-theme that came out of many of the conversations is that of replication in mathematics teaching and of mathematicians. The structures of the participants' programs, their teaching assistant work, and the suggestions that were put forth by the department either through direct communication or the lack of it seemed to point to a particular, sanctioned way of being and becoming a mathematician, a way of being which implied that teaching was unimportant and determined solely by what had been observed in other professors' classrooms. As I continued to read and hear how the participants began to make certain tasks more important than others through what they were and were not allowed to do as newcomers in the department, it seemed that they were being primed for a particular way of being.

The idea of legitimate peripheral participation offers an interesting lens through which to interpret, understand, and describe what is happening in this context. Lave and Wenger (1991) wrote that, "Communities of practice have histories and developmental cycles, and reproduce themselves in such a way that the transformation of newcomers into old-timers becomes remarkably integral to the practice" (p. 122). Further, Lave and Wenger (1991) claimed that, "even in cases where a fixed doctrine is transmitted, the ability of the community of practice to reproduce itself through the training process derives not from the doctrine, but from the maintenance of certain modes of coparticipation in which it is embedded" (p. 16).

In a similar vein, framed by the idea of legitimate peripheral participation, through their process of becoming mathematicians, the participants in this study seemed to move from a peripheral position to a slightly more central standing in the community as their identities became closer to that of a mathematician. As they learned of the relative importance or unimportance of different aspects in the life of a mathematician, they grew into the community of mathematicians and reproduced the community. This transition was not overt, nor was it explicitly stated anywhere. The participants did not report a public statement or even an acknowledgement that they had to abandon their own ideas about teaching, that they should no longer consider teaching important and, by maintaining "certain modes of coparticipation," they would move toward a more central position in the department. Rather, it seemed that the set-up, the structure of the department, the behaviours that were legitimate, and the progression to becoming a mathematician rendered it so.

The notion of legitimate peripheral participation also seems to describe the suggestions and messages that were experienced explicitly and implicitly by the participants and how the participants interpreted and internalized those suggestions as they proceeded through their programs. I address this in the following subsections, which relate to some of the participants' experiences that speak to their peripheral location in the department, to post-secondary teaching of mathematics as replication or reproduction, and, finally, to moving towards taking on the identity of a mathematician.

Peripheral experiences and (il-)legitimate behaviour in mathematics

There were some instances in the participants' experience that I believe are important in understanding how they moved from a peripheral position as a newcomer into a more experienced place of doctoral candidate and future mathematician. In this regard, the comparison of the perspectives of John, a fourth-year doctoral student, and Emily, a first-year master's student is again significant. When Emily spoke of her teaching, she was excited about the possibilities of teaching mathematics and she took pleasure in assisting undergraduates. Her interest in helping students understand mathematics was fresh and relatively unencumbered by the pressures of the department. In contrast, John spoke of how he could not work "outside of a certain box" and appeared to no longer have an interest in teaching. I believe this comparison of graduate students on the extremes of the continuum of experience in the department speaks to the "transformation of newcomers into old-timers" (p. 121) and how "an extended period of legitimate peripherality provides learners with opportunities to make the culture of practice theirs" (p. 95). In the final year of his doctoral program, John had taken on a more uniform identity of mathematician and spoke in such a way that revealed that certain behaviour was not allowed or acceptable for that identity.

There were other experiences that also appeared to resonate with the notion of legitimate peripheral participation. In particular, Sara had observed her fellow graduate students accept the poor teaching of one of their professors by refusing to give a negative evaluation. She spoke of how her peers were not vocal about the ways they were being taught and she interpreted their behaviour as an excuse to take on poor teaching in their future roles as professors. I believe that Sara saw her classmates as engaging in the "maintenance of certain modes of coparticipation" (Lave & Wenger, 1991, p. 16), as though their peripheral position would change to more central position provided they "maintained" this form of teaching. Steven's comment that the "idea that they [professors] have to explain it enough or well enough in order for us to understand it is absurd" also revealed the notion of legitimate behaviour for professors, which did not include making certain that students understand mathematics.

Two other experiences seem to be captured well by legitimate peripheral participation. First, Emily's experience of being told that she could not say "I don't know" to an undergraduate represents a disclosure that certain forms of behaviour were considered illegitimate within the department of mathematics. In her peripheral role as a teaching assistant, she had to abide by this prohibition in order to become more central on her way to becoming a mathematician. Second, Steven spoke of wanting to create a presentation for the undergraduates, yet was denied the opportunity to do so, as though this was not a legitimate pursuit. Similar to Emily, being a teaching assistant meant Steven's role was peripheral to the interests of the department, and he had to let go of his ideas for helping undergraduates in their understanding of mathematics because such work was not legitimate.

It is also interesting to observe how the notion of legitimate peripheral participation makes the participants' reactions to and interactions with the undergraduates more comprehensible. As I described in Chapter 8, the participants offered more in-depth learning experiences to those students who behaved in ways that were deemed sufficiently mathematical or displayed behaviour which demonstrated that the students treated mathematics as important. In contrast, students who did not exhibit such mathematical behaviour were not offered these opportunities. In this regard, the participants had interesting things to say about the undergraduates. Emily said, "They're not interested in the real math yet," while John remarked that "a really unmotivated student like that shouldn't pass the class."

In this interesting turn, where it became the graduate students who had expectations of behaviour for undergraduates, Herzig's (2002a) findings are helpful: "students develop a certain degree of competence <u>before</u> they can interact with the faculty in meaningful ways" (p. 189, emphasis in original). It appeared that the participants were operating in a similar manner with the undergraduates, where the participants "interacted in meaningful ways" with students who demonstrated legitimate behaviour, as though, along with the legitimate ways of becoming and being a mathematician, there were also legitimate ways of being a student in mathematics.

Post-secondary mathematics teaching as replication

> The problem of acting in an inherited context […] often finds student teachers embodying the very traditions they hoped to change. (Britzman, 2003, p. 41)

Similar to Lave and Wenger's (1991) idea that communities "reproduce themselves" (p. 121), the post-secondary teaching of mathematics, as viewed by the participants, appeared to be a practice of replication, a reproducing of others' teaching and the material in mathematics textbooks. I observed this in a few of the participants' comments. Specifically, Steven spoke of the structure of all mathematics courses as "definition, theory, example." This interpretation resonates with Rogers's (1994) description of her teaching mentioned in Chapter 1. She wrote how "Each class had a natural pattern: I introduced the topic, covered the blackboard with formulas and mathematical language, and worked a few problems" (p. 385), finding that her teaching was quite similar to that of her own teachers. Certainly, my own teaching was also modeled on what I had seen for years as a student – a mathematical four-step of definition, theorem, proof, example.

The understanding of mathematics teaching as involving replication and reproduction resonated in several of the other participants' comments. Robert asked the interesting question "How many ways can you skin a calculus class?" and John said "It's easy to keep teaching calculus like this. We've done it forever. We know exactly what we have to do." To me, their comments revealed that replication would be the only way to teach calculus because, in their minds, there was only one way to teach it. To revisit John's comment "We teach the best possible situation," which I addressed in Chapter 8, the idea of "best" denotes one way of teaching. Because other models of post-secondary mathematics teaching were not present in their world, it is as though this "best" type of teaching represented the legitimate behaviour of a mathematics professor, where teaching practices needed to become an exercise of replicating their own teachers in order to be acceptable within the community of mathematicians.

Taking on the identity of mathematician

> *identity*: c. 1570, from M.Fr. identité (14c.), from L.L. (5c.) *identitatem* (nom. *identitas*) "sameness," from ident-, comb. form of L. idem (neut.) "the same" (see identical); abstracted from *identidem* "over and over," from phrase idem et idem (www.etymonline.com, 2009)

> A [mathematics] doctoral student needs to adopt the identity of a mathematician. (Herzig, 2002a, p. 40)

In this thesis, I posed several questions about the experiences of mathematics graduate students and what might have meaning for them in their process of becoming mathematicians, future professors, or future post-secondary teachers of mathematics. The notion of becoming a mathematician, of taking on a certain character or qualities, speaks to identity. Within this meta-theme of replication or reproduction, I believe that the notion of identity is quite important as the exploration of a mathematician's identity and the identity of a professor of mathematics became significant to the participants' experiences, as described in Chapters 7 and 8, respectively.

Reflecting on the etymology of the word *identity* in this context is interesting in that it derives from a word that means "the same" and "over and over." Thus, using the word *identity* resonates with this idea of replication as the participants appeared to be undergoing a process so that they would be quite similar to, if not identical with, each other in becoming post-secondary teachers of mathematics. Lave and Wenger (1991) remarked about identity: “We have claimed that the development of identity is central to the careers of newcomers in communities of practice, and thus fundamental to the concept of legitimate peripheral participation” (p. 115). Again, legitimate peripheral participation provides a way to interpret and describe the participants’ experiences in the formation of their identities as mathematicians and post-secondary teachers of mathematics.

In Chapter 7, I explored issues of identity in being a mathematician. From several directions, such as films, literature, and the participants’ experiences, there was a pointing to a particular view of whom a mathematician is; in particular, a person who is isolated, and who works, behaves, and communicates in specific ways. Several participants spoke of an image of a mathematician and vacillated in their perspectives of themselves as being similar to this persona. The participants sometimes pushed this particular identity away, denying any resemblance, while at other times they acknowledged their tendency to behave in ways similar to the stereotype of a mathematician. They seemed to struggle somewhat with this image, as though it represented who they would be perceived to be, even if not necessarily who they would become. I feel that their resistance to this image came from not wanting to wholly take on this particular identity, of feeling that they had to give up their non-mathematical activities because such extra-mathematical conduct was not legitimate for a mathematician.

In contrast, however, the participants also seemed to find some comfort in how they recognized their own behaviour in the image of a mathematician. In this regard, they sometimes felt strange in and to the world and so finding an identity with which to relate was quite important and reassuring. Again, though, taking on the identity of mathematician appeared to be problematic and troubling at times, as that identity clashed with who and how they wanted to be in the world, when they felt their interests were not legitimate to that identity. This is similar Herzig's (2002a) conclusion that many of the doctoral students who left mathematics described their departure as strongly related to the incongruity between who they were expected to be and who they wanted to become, that being a mathematician precluded certain ways of being.

In Chapter 8, the participants' views of their future possible roles as instructors and professors of mathematics were explored. Again, the idea of replication or reproduction seems to describe what was happening here, as the participants looked to their professors for how and who they should become. Herzig (2002a) found that the mathematicians with whom graduate students interacted represented the models of teaching that graduate students would adopt. This was true in my study as well, as several of the participants noted that they would copy (replicate) their professors, with Robert going so far as to say that a new professor might just rely on someone else's lecture notes because the teaching would not be that different from professor to professor, as if there were a unitary, shared practice.

In replicating their professors' classroom practices, it appeared that the participants were attending to what was legitimate in the department of mathematics, in the behaviour of being a mathematician, and to who they would be as professors of mathematics. It looked as if the participants were marching on a singular path to a singular identity (what Britzman, 2003, referred to in her own research as a "unitary identity") that seemed to come through the participants' strong emphasis on being professors, not teachers, where their perception of the role of a professor spoke to a particular image of what they might be in the future.

The participants did not see their professors as teachers, as teaching. With that perspective in mind, what did the participants learn from their professors about being teachers or teaching, especially when they looked to their professors as models for how they should be in the classroom? Lave and Wenger's (1991) work provides an interesting insight here. They stated, "If masters don't teach, they embody practice at its fullest in the community of practice. […] Identities of mastery, in all their complications, are there to be assumed" (p. 85). The participants' views of who they would become as professors were fixed on this identity, which echoes with what Britzman (2003) found in her work, where identity is seen as a "final destination rather than a place of departure" (p. 29). In this study, the instructor or professor represented what the participants will become, with few possibilities for moving beyond that role as they continued to attend to what would maintain their legitimacy and help them become more central within the department.

The ideas of the replication of mathematicians, the reproduction of the community, and the participants' need to focus on what was legitimate are interesting in revisiting Speer's (2001) findings in her work with mathematics graduate students. She wrote:

> On the surface, with some of these TAs everything seemed to be going well, but something about the nature of the interactions did not seem to make the most of the opportunities they had to help their students learn. Although I encountered this phenomenon several times over many semesters, I was not able to really articulate what the problem was. I also felt relatively ineffective in helping these TAs devise strategies for improving their practice. On the surface, they were doing everything "right," and I was at a loss to help them because my usual collection of intervention and consultation strategies just did not apply. (p. 5)

I believe Speer encountered an unspoken and not yet understood resistance to changes in the mathematics graduate students' teaching because such attention to teaching was not part of the legitimate behaviour the graduate students believed they needed to engage in. Further, it may be that the notions of unitary identity of a mathematician and the perspective of professors versus teachers caused the graduate students to not engage with their students. Without the full support of the department, and without models for different ways of interacting with undergraduate or graduate students, the mathematics graduate students would continue to emulate their own professors and attend solely to what was recognized by the department, even though they hoped to be different.

Resignation

> *resign*: accept something that cannot be avoided; surrender oneself to another's guidance (Soanes & Stevenson, 2004, p. 1498)

> What happens when the authoritarian ways of conceptualizing experience bump up against the wishes one has for experience? (Britzman, 2003, p. 18)

To respond to Britzman's question above in regard to the participants of this study, as they encountered different suggestions about their current and future roles, another meta-theme that came through in the exploration of the conversations was that of resignation, of being resigned to the "authoritarian ways" of the department of mathematics. This feeling surfaced as the participants spoke of how they viewed teaching, both in their current situations as students themselves and for their future practices. The sense of resignation also came out in their views of how successful they would be as mathematicians.

With regard to his current role as a graduate student Steven said, "You can't have an opinion, you can't have anything except the fact that 'yeah, this is true.'" Here, it seemed that Steven was resigned to a passive position with respect to his own learning, and that he must accept facts rather than engage in a different form of learning. Further, when speaking about the possibilities for his future teaching practice and, in particular, about the use of discussion in a mathematics classroom, Steven said, "that's never going to happen in math," a statement that expressed a resigned view that there are no alternative possibilities for what occurs mathematics classrooms. Concerning his own observations of the ways in which the undergraduates were being taught by professors, John remarked "I might have the same complaints, but there's nothing I can do about it," signaling a resignation to being unable to change the way mathematics courses are taught or structured.

When Chris spoke of his future role as a mathematics professor, he expressed an interest in being different from his own teachers, but then followed this interest with "I don't think I will be," revealing a resignation to a particular way of teaching in mathematics. Some of the participants had resigned themselves to the idea that they would be average or ordinary mathematicians. Both Chris and Emily spoke of how they would never win the Fields Medal or be a superstar in mathematics, while John said that he "will never be the mathematician who works at Princeton." Finally, with regard to a life in mathematics, John seemed resigned to a "poor quality of life."

The resignation that the participants seemed to feel is interesting up against the framework of legitimate peripheral participation. Again, there was a sense of legitimacy or rather illegitimacy that came through in how they spoke of their teaching and how they saw their current and future success as mathematicians. With regard to their teaching practices, there was a feeling that incorporating opinions or discussions into mathematics classes or the notion of being different from one's professors represented unauthorized or illegitimate practices or behaviours in mathematics. Further, I interpreted the participants' views of their future roles as professors, and not being superstars, as a continued form of peripherality, where their resignation seemed to reveal that they would not be admitted to the higher echelons of mathematicians.

Despondence

> *despondence*: 1676, from L. *despondere* "to give up, lose, lose heart, resign" (especially in phrase *animam despondere,* lit. "give up one's soul"), from the sense of a promise to give something away, from *de-* "away" + *spondere* "to promise" (see *spondee*). A step above *despair* (www.etymonline.com, 2009)

The last meta-theme I would like to address here is that of despondence, which, as seen above, means "to give up, lose heart, resign." I believe that legitimate peripheral participation is again relevant to the participants' feelings of despondency and disappointment, because in attending to what was deemed legitimate, they often had to relinquish things that they felt were important to them. In being peripheral, they realized how little their roles and actions as teaching assistants and mathematics graduate students made a difference within the department of mathematics. As teaching assistants, they had no voice, even in the matters in which they were involved, such as the workshops and tutorials. Their various encounters with departmental structures often brought about emotional and disheartening responses.

Other researchers have conveyed such experiences as well. Herzig (2002a) wrote about the emotional side of mathematical work that graduate students encounter. In particular, she described the experience of their graduate studies as discouraging and depressing. What she found was that the graduate students described "portraits of isolation and lack of social interaction, expectations that involved extensive time commitments, and few interests outside of mathematics. These former students reported how isolating graduate study had been" (p. 35). Bass (2006) confirmed the situation that mathematics graduate students face, depicting graduate education in mathematics as "being too isolating and competitive" (p. 98). Kline (1977) painted a grim picture for new mathematics PhDs: "They have just emerged from indoctrination in the purity of mathematics and from the dark recesses of some specialty they have pursued for two or three years. The doctorate conferred upon them is not the certification of a teacher but the official stamp of cultural deprivation" (p. 74). The life of a mathematics graduate student appears to be a period of attending almost entirely to learning and expanding a rigorous language within a seemingly constrained culture where little of their own identities is valued or expressed.

I found that the participants in this study had similar reactions to and descriptions of their experience, all of which revealed despondency. For example, Sara said, "You start questioning everything – why am I doing this right now?" In questioning her experience of becoming a mathematician, she asked, "Is it going to make any difference?" I believe her question speaks to a sense of losing heart, of giving up. For much of her time in the project, Sara talked about how she did not think she would continue in mathematics once she had completed her master's degree, sadly saying, "I guess the system or something just drew the love out. I feel that I have to cut off the rest of the world."

Steven also expressed feelings of despondency in relation to various experiences. As a graduate student still enrolled in courses, he talked about how "we're both [professor and students] going through the motions. And maybe that's the point, then, of why be enthusiastic because we're just going through the motions." As a teaching assistant, he spoke about losing his optimism as a teacher, saying, "I've seen it just completely smothered here" and that his experience in helping students had been "completely disappointing." During the study, Steven frequently spoke of his disappointment with regard to what he was able to do. As with Sara, he had lost heart in what he was doing in mathematics and was no longer sure that he would continue to study mathematics beyond his master's degree.

Replication, Resignation, Despondency

Through their experiences as graduate students, the participants have observed that post-secondary mathematics teaching takes on a particular form and that mathematicians seem to take on particular identities. As graduate teaching assistants, they encountered rules and structures that did not allow or support them to diverge from a particular form of interaction with undergraduates. Their interpretations of mathematics curricula as emblematic of mathematics, or as the best-case scenario, also seemed to bind them to a specific way of presenting mathematics. It appeared that the participants were not just learning mathematics, but also how to be in mathematics.

As the graduate students moved to the next step on the ladder toward the completion of their programs, none of the steps seemed to address how they interacted with students or what their teaching practices might be like. In other words, similar to Krantz's (2003) diagram of progression through a graduate program in mathematics that I referred to in Chapter 2, the itinerary of progress through the department did not explicitly address their teaching at all, other than to suggest that it take on a particular form. It is interesting to note that as the three doctoral students got closer to earning their degrees and formally being mathematicians, teaching became less important as their supervisors and the department worked to find them other sources funding so that they would not have to teach, as though, as they became more central to the community of mathematicians, teaching itself was no longer seen as a legitimate task.

The resignation and despondency I have described here seemed to develop from the participants having to give up their hopes and expectations in order to be considered legitimate within the department. But in attending to what was legitimate in the department, there seemed to be a considerable cost. What they had to produce as mathematicians was, in a way, how they seemed to present themselves as mathematicians and as mathematics teachers – restricted and disconnected from the creative and active processes that inspired mathematics, including the mathematics they were most interested in and passionate about.

Repetition – Seeing a Way Forward

Being able to express their opinions, concerns, and frustrations and hearing those of the other participants seemed not only to bring about a sense of relief, but also a significant realization that there were plural perspectives about learning and teaching mathematics, even amongst their peers. This new awareness helped the participants to understand that they were not alone in their frustrations, and that, beyond their own experiences, there were other possibilities for how to learn and teach mathematics. Along with the relief came a new sense of openness to others' ideas and to other possibilities for being in mathematics.

The conversations provided the research participants with an otherwise-absent forum to talk about their experiences in their graduate programs and how they were affected by them, as well as identifying the aspects of their programs they were struggling with. Robert reflected upon the lack of discussion in the department, saying, "It's a very rare opportunity that you talk about these sort of things in a little bit more formal setting." Sara stated, "I really enjoyed participating in the study because I got to think about some more questions in depth that I maybe wouldn't have on my own." With regard to his participation in the study and having a place to voice his opinion, Steven said, "I did feel like no one was listening to me. You offered me a place to tell these things to." While Emily often worked closely with her classmates, there was little time to discuss what being in mathematics meant to her and how she experienced it in the world. The conversations provided her with a different opportunity to interact with her peers and talk about being in mathematics. She said, "I just like talking about being in math. And being with other people that feel the same way about, you know, teaching and trying to figure that out and, or, like being in classes still, and just the general reaction that you get being a math major, and commiserating with people on it."

Both Chris and Robert remarked how the conversations also prompted them to think more consciously about things they often held in the back of their minds. Chris said, "I've been thinking about things kind of in a background level I think, but not so focused, or as focused as when I'm talking to you. So it's been interesting to be able to frame my feelings more concretely than I would have otherwise." Robert spoke of how participating in the project made him "think about a lot of different things where the idea was sort of always there, it's just I wasn't really aware of it until maybe now that I'm involved in this project and I think about this sort of thing." Rather than keeping their thoughts and ideas in the background where they might not have had to chance to explore them, having the space to voice their concerns and talk about teaching mathematics allowed them to think and talk about their ideas more explicitly.

Mason (2001) wrote, "Exposing one's principles-beliefs-theories makes them available to questioning, critique and modification, whereas when they remain embedded below the surface of behaviour, they are not amenable to modification" (p. 72). In this regard, the conversations were transformative for Steven. In hearing that some of the participants disagreed with his opinions, Steven discovered that he had "really demonized the students." In listening to others' views about their teaching and undergraduate students, it was helpful for him to find that not all of the participants shared his opinion. He said, "I think some people [participants] are still more optimistic about them [undergraduates] and like saying that we should come up with more creative, you know, approaches for teaching them. So, in a way, it sort of undid some of the bitterness and resentment I have toward some of those students." Steven had expected the other participants to feel as he did. In hearing otherwise, it seemed that he was able recognize how negative he had become. I got the sense from Steven that through the process of dialogue and hearing others' views, he was able to take a new look at his opinions, whether they were true to his experience, and whether they were helpful in realizing what he wanted to be and do as a mathematician. In this respect he said, "So it was nice to get together and see, to have people disagree with me, and agree possibly. It was nice to see some sort of variation in expectations and experiences."

In seeing the "variation of expectations and experiences" among the participants, an openness to other possibilities seemed to develop. Carson (1986) wrote:

> In the final analysis, the practice of conducting conversations with participants is in itself a form of action which helps forge a reformed practice. By engaging in conversation, researchers are helping to create spaces within educational institutions for thoughtful reflection oriented towards improving practice. (p. 84)

John remarked that hearing the other participants' experiences and opinions opened his mind to others' experiences and opinions. In particular, it was surprising for him to learn that some of his peers preferred lectures while he did not. It was not hearing that lectures were the choice for some students in their learning that was important for John, but rather it was realizing that others experienced things differently from him that was remarkable. To hear this helped John to realize that teaching and learning mathematics did not have to rest solely in one mode of communication.

With this understanding of what their participation meant for the participants, in realizing that they were not alone and had similar concerns, and that other future mathematicians had opinions that were distinct from theirs, in relating to each other and in differing from one another, there seemed to be a sense of new possibilities, that they were not limited to a particular way of being in the world and in mathematics. While the participants would most likely continue to struggle with the happenings and suggestions of the department and its mathematicians, it seemed that in the relief and understanding they had realized through the dialogue, they sensed they could hold on to more of themselves than they otherwise thought.

In contrast to the meta-theme of replication that I described earlier in this chapter, Caputo's (1987) notion of repetition provides a different and more hopeful way of looking at the participants' processes of becoming mathematicians and post-secondary teachers of mathematics:

> Repetition is always an originary operation by which Dasein [human being in the world] opens up possibilities latent in the tradition, bringing forth something new. […] In repetition/retrieval Dasein is productive of what it repeats; it does not simply go over old ground. The self produces itself by repetition. In repetition Dasein discloses its own Being and that of the historical situation in which it belongs, that of its generation, *for the first time*. Repetition is a first, a breakthrough, a retrieval which pushes forward, which opens what was previously closed, liberates what was previously held in check. Repetition is a new beginning which aims at the possible. (Caputo, 1987, p. 90, emphasis in original)

Caputo's description of repetition illustrates that it is more than a replication of what has taken place in the past, but rather repetition brings forth new possibilities for Dasein, for being, where repetition "is not a matter of making actual again what has been previously actualized." It is not, then, "what one ordinarily means, in English, by repetition – the simple reduplication of a previous act" (p. 90), but instead repetition "means to produce something, not to reproduce a prior presence" (p. 15). Further, Caputo also wrote "Repetition aims at not the *actual* but the *possible*" (p. 91).

This idea of repetition also has meaning for identity, as Caputo (1987) stated, "Repetition says that actuality must be continually produced, brought forth anew, again and again. Identity must be established, produced. Identity, as Derrida would say, is an effect of repetition" (p. 17). In this regard, in providing a new way of thinking of how one establishes his-self or her-self within a particular circumstance, Caputo also wrote:

> Repetition is the power of the individual to forge his personality out of the chaos of events, in the midst of the flux, the power to create an identity in the face of the incessant "dispersal" of the self, of the dissipating effects of the flux. There is always a "remainder" no matter how much is subtracted from the individual by the taxing business of everyday existence. Repetition is the exacting task of constituting the self as a self. (p. 21)

Caputo's explanation of repetition shows that while the participants in this study are undergoing the "chaos of events" and are "in the midst of the flux," there is always something left of themselves, that "there is always a remainder no matter how much is subtracted from the individual." In thinking of what the study offered the participants, and the sense of possibility that emerged in their reflections on the study, it was clear to me that despite the pressures, time constraints, and expectations they were under in their process of becoming mathematicians, there were "remainders" for each of them. There were various signposts which signified that the participants were different from those they were supposed to replicate, where it seemed the participants were subtly and gradually "carving out an identity for themselves" (Caputo, 1987, p. 30).

Jardine (2006) gives further insight to the potential for possibilities for the mathematics graduate students in how he described the generative possibilities within mathematics itself. He wrote, "Each new example, each new interweaving thread or fiber, reopens the 'kind' to new permutations and possibilities, and each new permutation has a cascade effect, rattling through each instantiation, giving it new relations" (p. 194). To me, each of the participants represents a new "example" or "permutation" in how they came to mathematics from different situations, with different interests and passions. I believe that as they found that they could hold on to more of themselves, they will each have a “cascade effect, rattling through each instantiation.”

Understanding and finding hopefulness in my own experience

In undertaking this project, I wanted to understand my past experiences as a graduate student in mathematics, my process of becoming a mathematician and a post-secondary teacher of mathematics. Similar to the participants in this study, I had not had the time or a forum to make sense of what I was going through during my graduate studies or as a post-secondary teacher of mathematics. For many years, my own teaching had troubled me. I had had a strong desire to be different, yet was seemingly constrained to lecture-based teaching. To revisit Rogers’s (1994) experience: “It shocked me to realize how faithfully I was reproducing in my own classroom the structures which had so effectively silenced and disempowered me at that time (as a mathematics undergraduate)” (p. 385). As a learner, I loved working with, exploring, and discovering mathematics. Why did that energy and enthusiasm not show itself when I taught mathematics? What caused me to teach in such a way? Why did there seem to be a large gulf between *how I wanted to be* and *who I was* when it came to my mathematics teaching practice? Why did I not or could I not engage in alternatives even when I had a strong and enthusiastic desire to do so? After earning my degrees and having a few years of teaching experience and the knowledge of whom I did not want to be, how did I not have the wherewithal to depart from such a way of teaching?

These questions about my teaching seemed to compel an investigation into how or whether I was prepared to be a post-secondary teacher of mathematics, to understand what had been important for me and to the different departments of mathematics in that process. In beginning to reflect on my experiences and the discrepancy in how I was prepared for my teaching assistant duties at the universities I attended, initially I thought that the first experience had better prepared me for teaching. On later consideration, though, as I described in Chapter 1, the manner in which I taught at that first university was not far removed from the way I had been taught. While I was provided with information about various policies, I was given little guidance on the possibilities of how and who I could be as a post-secondary teacher of mathematics. I was deemed suitable to teach my own classes when my teaching practice closely resembled my teacher mentor's practice.

In looking back at my experiences in mathematics departments, it seemed that I was looking for something, some other way of being that felt true to who I wanted to be as a post-secondary teacher of mathematics. When I left a department of mathematics after not finding this, or at least not finding support in my longing and my search for it, sometimes it was with frustration, at other times with anger. Why did it seem that I could not be myself? Why was it not a worthy enterprise to further the field by teaching? Why did I have to find myself, my passions for mathematics teaching, who I wanted to be in mathematics *outside of a department of mathematics*?

My frustrations, my feelings of rejection – that I did not have what it took to be in mathematics – and a sense that I still wanted to be accepted into mathematics, eventually transformed into a desire to be vindicated by others' experiences. As I described in Chapter 5, at first I looked to the participants to validate my feelings about mathematics, and that the problems, the faults, and the discomforts I had felt were due to mathematics and not me. In looking back now at my experiences and at the desire to be justified in my feelings, these reveal to me my own complicated space of being in mathematics. I had done a similar pushing away and pulling towards mathematics that I had noticed in the research participants. I loved mathematics and I wanted to belong in that community, and at times I simply declared myself as belonging to the community of mathematicians. But at other times, I felt relief in not belonging, and I distanced myself, coming up with a cover story that I would use when talking to others.

When I was able to back away from my own experiences and the need to have my feelings and interpretations of my experiences confirmed by others, this project offered me an opportunity to learn what the experience of being a graduate student in mathematics was like for others. Through this research project, I have learned that my struggles and frustrations being in mathematics were not only about me and who I am, but that other mathematics graduate students felt similar pressures and limitations. In reviewing some of the literature and learning of the other research projects with mathematics graduate students, I came to realize that the pulls, contradictions, dilemmas, and difficulties exist for others and not just for me.

In taking on this project, I hoped to gain a better understanding of the lives of mathematics graduate students, to become aware of the particulars of their lives and what had meaning for them in their process of becoming mathematicians and post-secondary mathematics teachers. The use of hermeneutic inquiry provided a unique perspective for the research as:

> Hermeneutic inquiry has as its goal to educe understanding, to bring forth the presuppositions in which we already live. Its task, therefore, is not to methodologically achieve a relationship to some matter and to secure understanding in such a method. Rather, its task is to collect the contours and textures of the life we are already living, a life that is not secured by the methods we can wield to render such a life our object. (Jardine, 1992, p. 116)

I believe that what lies in this project represents an understanding of the contours of the mathematics graduate students' lives and helps to understand what such a life is like.

Aside from offering the possibility of a new understanding of this phenomenon, hermeneutics offered me something as well. Smith (1991) wrote, "Hermeneutics is about finding ourselves, which, also, curiously enough, is about losing ourselves" (p. 201). At times in this project, I did feel lost – lost in the sense that I had to temporarily *lose* (to loosen, divide, cut apart, untie, separate; www.etymonline.com, 2009) what I thought I understood about myself and my experience in order to discover how one goes about a hermeneutic project, as well as to find a new understanding of my life in mathematics. In attending to things hermeneutically, in losing what I thought I understood, I unearthed my experiences from a perspective of understanding rather than judgment, in a way that helped me to make sense of what I had faced in departments of mathematics and how I had interpreted and reacted to those events.

Further, this hermeneutic project has also offered me a new opportunity to see "how we might make sense of our lives in such a way that life can go on" (Smith, 1991, p. 200). In finding an understanding of who I was and now am in mathematics, and who I hope to be as an assistant professor of mathematics, Caputo's (1987) notion of repetition offers me a new awareness of the possibilities that did exist in my previous experiences and I hope will come forth in my future career:

> By virtue of repetition the individual is able to press forward, not toward a sheer novelty, which is wholly discontinuous with the past, but into the being which he himself is. By repetition the individual becomes himself, circling back to the being which he has been all along. (Caputo, 1987, p. 12)

Here, in thinking of who I want to be as a teacher of mathematics, I feel that I am "circling back to the being" which I have been all along, and that I can hold on to the part of myself that wants to make teaching central to who I am and how I live as a teacher of mathematics.

Further, with Caputo's idea of repetition, I can now reflect on my graduate programs in mathematics and see that there were differences within my experience, the "remainders" that signified that who I was as a mathematician and a post-secondary teacher of mathematics was not bound to what I discussed previously as replication. In particular, I will not forget the professor's comment after I completed my master's presentation. At one point during my presentation, in both nervousness and enthusiasm, I said aloud to the audience, "Cool math." When all was said and done, a mathematics professor whom I did not formally know said to me, "You're who we need in mathematics." Maybe he was right.

Educating Graduate Students in Mathematics to be Mathematicians

> *mathematician*: an expert or student of mathematics (Soanes & Stevenson, 2004, p. 1083)
>
> *professor*: a university academic of the highest rank; a university teacher (Soanes & Stevenson, 2004, p. 1405)

Throughout this dissertation, I have written that the research participants are in the process of simultaneously becoming both mathematicians *and* professors of mathematics. I grouped these two terms together based on the knowledge that as many as seventy-five percent of new PhDs in mathematics (Kirkman et al., 2006) will find employment at colleges and universities where their primary role will be to teach mathematics to undergraduates. Yet, when I look again at what mathematics graduate students *do* in their programs, at what they must attend to, I question whether they are actually being prepared to be both a mathematician *and* a post-secondary teacher of mathematics. What, in fact, are their programs educating them for, as well as directly and indirectly preparing them for?

When I reflect on the definition of mathematician above, given my experiences with post-secondary mathematics and the conversations with the graduate students in my study, it becomes clear that through their master's and doctoral programs, graduate students in mathematics are on a path to becoming mathematicians. They learn about the discipline of mathematics through their coursework and they learn about mathematical research through undertaking their theses and dissertations. To be successful in their programs, they must become skilled in mathematics by mastering their coursework by earning high marks and they must undertake a research project.

In contrast to the mathematics they must become skilled in, mathematics graduate students are not required to demonstrate competence in teaching, in how they work with students, how they present material to a class of students, or how they assess students' learning. There is often no point in their programs where they are evaluated on their teaching, even though teaching might be a very important part of their future careers. Mathematics graduate students seldom learn explicitly about what it means to be an instructor or professor of mathematics, and they are left to create meaning and develop proficiency amongst themselves. In other words, to be successful, to earn a graduate degree in mathematics, they must become proficient in high-level mathematics and little, if any, attention is paid to their development as instructors or professors, to who they are or will be as teachers of mathematics.

The guides that have been published by mathematicians support this idea that graduate students in mathematics are being prepared solely to be mathematicians. Krantz (2003) did not include teaching or preparing to teach in his list of the "Steps to a Graduate Education" in mathematics. Stewart (2006), in writing about the rites of passage in the life of a mathematics graduate student, attended almost entirely to learning mathematics and doing research. Further, in Gower's (2006) book *The Princeton Companion to Mathematics*, of the five professional mathematicians who offered guidance to a young mathematician, only one addressed teaching in a very brief and vague statement: "Teaching should not be a burden, but a source of inspiration" (Bollobàs, 2006, p. 1004).

From my exploration of the conversations in the previous chapters, though, not only did the participants have significant ideas about teaching and their identities as both mathematicians and post-secondary teachers of mathematics, but there also existed many tensions between these two aspects of their current and future selves. I believe that the exploration in the section titled "I am Professor – Hear me not teach" in Chapter 8 sheds light on some of these tensions. In that section, I investigated the differences between the definitions of *teacher* and *professor* because some of the participants were adamant that they would be professors and not teachers. As one participant said, "It is difficult to ask a professor to teach."

In the tensions between what it means to be a professor and what it means to teach, there is a question – does being a professor exclude being a teacher? Or does there exist a conflict in the understanding of what it means to be a professor? To revisit the meaning of *professor* explored in Chapter 8, teaching is included as one of the duties:

> *Professor*: One who professes, or publicly teaches, any science or branch of learning; especially, an officer in a university, college, or other seminary, whose business it is to read lectures, or instruct students, in a particular branch of learning; as a professor of theology, of botany, of mathematics, or of political economy. (www.dictionary.reference.com, 2009)

In their descriptions of a mathematics professor, the participants said that the professor's role was to "present the material clearly" and "to motivate the subject matter." They described the difference between teachers and professors in the following way: that teachers help students to understand, but professors present material clearly to students most often by lecturing and it is the students' responsibility to find understanding on their own. One participant even said, "The idea that they [professors] need to explain it well enough for us to understand is absurd." In the above definition, teaching is referred to, but lecturing and instructing are also listed. Up against the definition of professor, then, the participants have accurately described some of what professors are characterized as doing in the classroom. Yet, there still exists a question about what it means to teach mathematics and why the participants did not want to be described as teachers.

In thinking about this comparison, the tensions may not solely rest in the differences between teacher and professor that the participants are pointing to here. It seems that beyond such a comparison exists another – of mathematician and mathematics professor – that is creating a discord not only for the participants, but also for the discipline of mathematics. Graduate students in mathematics are on a path to becoming mathematicians. They are not being prepared to be professors of mathematics. The participants' ideas of their future selves and the professor's role, their possible future profession, is seen to rest in becoming a mathematician, in being knowledgeable in mathematics with little foresight of what else will be expected of them beyond their education. Such a view, that one is a mathematician, as seen in Chapters 7 and 8, seems to preclude other ways of being in the world, of expressing oneself and one's passions, and fails to convey that, as a mathematician, one can connect with students in educative moments and create possibilities, interest, and help students understand mathematics. As noted by Kronman (2007), Kline (1977), and Boyer (1990), in the history of universities and professors in North America, it seems that something has indeed been lost on the road from teacher to researcher in a discipline.

To End With Another Question

In this dissertation, I wanted to step back from recent research (e.g., Belnap, 2005; DeFranco & McGivney-Burelle, 2001; Speer, 2001) and gain an understanding of what might exist in the experiences of mathematics graduate students that could interfere with their taking up methods of classroom practice that are different from lecturing. I explored their day-to-day experiences as learners, as new teaching assistants and as new researchers. I asked questions about their experiences and interpretations in mathematics and found that that there are significant conflicts and tensions in being in mathematics, not only in what it means for one's self in the world, but also concerning being a mathematician and professor of mathematics.

In most graduate programs, mathematics graduate students are often assigned teaching assistant duties while they work to become knowledgeable in mathematics. So there exists the illusion that they are being prepared to be both mathematicians and professors of mathematics. However, Lave and Wenger (1991) described how newcomers "need to engage in the existing practice, which has developed over time: to understand it, to participate in it, and to become full members of the community in which it exists" (p. 115). As the participants attended to the existing practices of the department, there were suggestions and rites of passage which revealed that only one of these, becoming a mathematician, really counts.

As seen in the participants' and my own experiences, it appears that there were repercussions in taking on alternate methods of working with undergraduate students, in engaging in dialogue and inquiry into teaching and learning in mathematics, that might diverge from what is considered legitimate in the department. In the communities of departments of mathematics, the existing practices seem to focus solely on the practice of developing mathematical proficiency. As the participants moved closer to being mathematicians, as their peripheral position became more central in the department, teaching became insignificant to their successful completion of their degrees. Consequently, their degrees would certify them as sufficiently mathematical, but would not certify them as being satisfactorily competent to be a professor, a teacher, of university mathematics.

After all that I have learned from this study, I am left with a question that I believe needs to be asked and understood within departments of mathematics in order to know whether and how to move forward in preparing mathematics graduate students for their possible futures as professors. I found that the notion of teaching mathematics was complex, troublesome, and not well understood by the participants, as it had become synonymous with lecturing, and as they refused to be labeled *teacher*, but then referred to what they did as teaching. While calls to improve post-secondary mathematics teaching have been made (e.g., Alsina, 2005; Bass, 2006; Chan, 2006), I suggest that *before* the question of how mathematics graduate students might be prepared for teaching is addressed, an understanding of what it means to *teach* post-secondary mathematics is needed. The questions I am left with are thus posed to the departments of mathematics that are charged with educating mathematics graduate students – Why does the notion of *teaching* appear to be so disconcerting in post-secondary mathematics? What does it mean, what does it look like, *to teach* college- and university-level mathematics?

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Appendix 1: Information Letter to Participants

This letter is an invitation to you to take part in a research project, *Conversations with Mathematics Graduate Students: Understanding in Mathematics and Teaching*, to be conducted by Mary Beisiegel as part of the requirements for a doctoral degree. The purpose of this research is to determine the experiences with mathematics and mathematics teaching that graduate students have, and how those experiences have meaning for graduate students' future careers as instructors or professors of mathematics. Gaining an understanding of how mathematics graduate students encounter and interpret various experiences in mathematics has implications for their education as teachers, and will inform teacher education for graduate students. The results from this study will be presented in the doctoral dissertation, and may be used in conference presentations, journal articles, and other writing such as book chapters. The data produced out of this research will be dealt with according to the University of Alberta Standards for the Protection of Human Research Participants, which can be found at the website:
http://www.uofaweb.ualberta.ca/GFCPOLICYMANUAL/content.cfm?ID_page=37738.

Participation in this study will include between four and six meetings with the researcher, all of which will occur away from the Department of Mathematics, all of which will be audio recorded for analysis. The first meeting will be an introductory conversation to introduce the participant and the researcher. This meeting will address details of the participants such as the year they are in their program, the focus of their studies in mathematics, what drew them to mathematics, to determine whether they have taught mathematics, what their experiences in mathematics teaching have been, and what they intend to do once they have completed their degrees in mathematics. This first conversation should take between 1 and 2 hours and should occur during October 2007.

After this conversation, I will analyse the tape recordings, listening in particular for thoughts, images, and impressions that arise about experiences with mathematics teaching. The analysis will also include tone of voice, tone of the conversation, utterances, silences, and gestures observed during the conversation. A summary of the individual conversations will be sent to each participant separately via email five days before the second conversation. The participant will be asked to reflect upon these themes in advance of this second conversation between the participant and researcher. Also, the themes will be used as a starting point into this second conversation about mathematics teaching and the participants' life in mathematics. The second conversation should take between 1 and 2 hours.

Again, the recordings from the individual conversations will by analysed, with attention paid to dialogue that has implications for the participants' teaching and ideas about teaching. The themes that emerge from all of the individual conversations will be summarized and sent to all participants via email five days before a third conversation. This third meeting will be a group conversation, motivated by the themes about mathematics and teaching that come out of the second conversations. This third conversation should take between 1 ½ and 3 hours.

A fourth meeting with each participant will occur during the second week of January 2008 and will be centred on the analysis of the group conversation, which will be provided to the participants no less than five days before the meeting. Each of these individual meetings should take between 1 and 2 hours. A fifth meeting, which will be a group meeting with all research participants will be based upon the analysis of the previous group meeting. It is expected that this meeting will take between 1 and 3 hours.

A final conversation with each participant will take place during March 2008, for which the participant will be provided with themes from all previous conversations five days in advance of the meeting. This final meeting will provide an opportunity for the participant and researcher to reflect upon the conversations that have occurred, what issues the conversations raised for the participants, their teaching and their lives in mathematics. This conversation should take between 1 and 2 hours.

As is noted in the research process described above, participants will have opportunities after each meeting to review my reflections, interpretations, and summaries. Feedback on my interpretations is welcome. Participants will also be able to review transcripts of their individual conversations as well as transcripts from the group conversation.

Participants in this project have a right to privacy, anonymity, and confidentiality. As the researcher I will protect participants' rights by using pseudonyms. Participants will be given notice of the importance of maintaining this right to privacy for all participants, and will be asked that personal information that is shared during the group conversation not be discussed with others outside of the group. Data will be shared with my supervisor, Dr. Elaine Simmt, who will also protect the participants' privacy and confidentiality by meeting the terms of the University of Alberta Standards for the Protection of Human Research Participants. Tape recordings, transcripts, and other data that comes out of the research will be protected for a minimum of five years after the completion of the research.

Participants also have the right to remove themselves at any time during the study. Should this be the case, you may request that any data that has been collected concerning you and contributions you made to the group conversation will be destroyed and thus not used in any writing that comes out of this project. If you elect to withdraw from the study, you can contact my supervisor, Dr. Elaine Simmt, or me.

I hope that you will choose to participate in the study. I believe this research presents a good opportunity to engage in conversations about mathematics and teaching with your colleagues and may contribute to your ideas about teaching mathematics.

If you have any questions or concerns, please contact:

- Mary Beisiegel, researcher for this study, at mdb5@ualberta.ca
- Dr. Elaine Simmt, Department Chair of Secondary Education at University of Alberta and supervisor for this project at elaine.simmt@ualberta.ca or 780-492-0753
- Dr. Susan Barker, Graduate Coordinate for the Department of Secondary Education at University of Alberta, at susan.barker@ualberta.ca or 780-492-5415.

Thank you for your cooperation.

Sincerely, Mary Beisiegel

The plan for this study has been reviewed for its adherence to ethical guidelines and approved by the Faculties of Education, Extension and Augustana Research Ethics Board (EEA REB) at the University of Alberta. For questions regarding participant rights and ethical conduct of research, contact the Chair of the EEA REB at (780) 492 – 3751.

CONSENT FORM

Please sign the form below to indicate your willingness to take part in the study *Conversations with Mathematics Graduate Students: Understanding in Teaching and Learning*, described on the previous pages.

I, ________________________________, give my informed consent to participant in the research study *Conversations with Mathematics Graduate Students: Understanding in Teaching and Learning*, conducted by Mary Beisiegel.

I hereby agree to:

- Have individual and group interviews and conversations audio recorded, transcribed, and analysed for themes related to mathematics, interpretations of mathematics and its teaching, and teaching and learning related to mathematics
- Allow my statements and contributions from the interviews and conversations to be analysed and presented in the dissertation, research articles, conference presentations, and teaching workshops presented by Mary Beisiegel
- Protect other research participants' identities

I understand that I am under no obligation to participate in this study and that I can withdraw from the study after which any information or data directly related to me as an individual will be removed from the study and dissemination of results.

____
(print name)

____
(sign name)

____
(date)

The plan for this study has been reviewed for its adherence to ethical guidelines and approved by the Faculties of Education, Extension, and Augustana Research Ethics Board (EEA REB) at the University of Alberta. For questions regarding participant rights and ethical conduct of research, contact the Chair of the EEA REB at (780) 492 – 3751.

Two copies of this form are provided for you. One should be returned to the researcher and you should keep the other copy for your records.

Appendix 2: Follow-Up Letter to Potential Participants

Dear Mathematics Graduate Students,

I am writing to you to follow up on the letter I put in your mailboxes last week. I am a graduate student at the University of Alberta. I am currently working on a research project in mathematics education. In particular, my research revolves around interviews and conversations with mathematics graduate students about their experiences in teaching and learning mathematics. My research project would require between four and six meetings between October and April (about one meeting per month), where each interview/conversation would last between 1 ½ and 2 ½ hours. Students at all levels of graduate experience and various backgrounds are welcome to participate.

I hope that you will choose to participate in the study. I believe this research presents a good opportunity to engage in conversations about mathematics and teaching with your colleagues and may contribute to your ideas about teaching mathematics. My hope is that my research project will inform teacher education for graduate students, in an effort to help them better prepare for their future careers as professors of mathematics.

If you would like to participate, please email me at mdb5@ualberta.ca.

Best Regards,
Mary Beisiegel BSc, MSc
PhD Candidate – Mathematics Education
University of Alberta

Appendix 3: Questions From the First Individual Meetings with Participants

Name:

Age:

Year in Program:

Focus in mathematics:

What drew them to mathematics:

To determine whether they have taught mathematics:

What their experiences in mathematics teaching have been (student/teacher):

What they intend to do once they have completed their degrees in mathematics:

Ideas about mathematics teaching:

Appendix 4: Questions from the First Group Meeting

Question 1:
The first question we're going to talk about is 'What is math?' question. I'm just going to read you a few quotes from mathematicians about what mathematics is. So Hersh says "Mathematics is a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context." Ian Stewart says "Math is a product of human minds but not bendable to human will. Exploring it is like a new tract of country; you may not know what is around the next bend in the river, but you don't get to choose. You can only wait and find out." And he also says, "Mathematics is the shared social construct created by people who are aware of certain opportunities, and we call those people mathematicians."

These are some interesting ideas about mathematics from mathematicians. And I'd like you to think about those things. Do you feel like there's anything missing in these ideas about mathematics? How would you expand on that, or … ?

Question 2:
We're just going to switch gears a little bit and think about the classroom in particular. And say you're walking across campus and you pass a classroom where you can see teachers and students. So you look in this particular classroom, can't really see what's on the board, but you know it's a math classroom. What's going on in there? Is there, is there someway that you just know that you've walked by a math classroom versus another kind of classroom?
So we started this part of the conversation by me saying 'So, you've walked by this classroom and you know it's a math classroom based on what you've observed.' So, just for fun, say you've walked by a classroom and you see what the class is doing, teacher and students, and you know it's not a math class. What do you think you'd be seeing? You'd say that's definitely not a math class.

Question 3:
So if we have the expectation that students, once students get to university level, they should be self-reliant. They should be making efforts to learn things on their own and understand things on their own. What does that mean for the professor? What is the professor's role if we have the expectation that that's what student should be doing?

Question 4:
Ian Stewart wrote a book called 'Letters to a Young Mathematician.' In it is a series of letters to a young woman named Meg and she's considering mathematics. You don't see her letters, but his letters are a response to hers. She's exploring mathematics as a possible major as an undergrad and he responds to her a couple times through that. And then getting in to grad school, there are some letters he writes to her. And then becoming a professor, there are letters that he

writes her, and so on. In one of the letters his response to her is “You asked me whether you would have to give up your sense of beauty to study mathematics, whether everything would become just numbers and equations.” In your choice of being in mathematics, do you feel you’ve had to give up anything, I mean, that you loved in your life or that, do you feel like you’ve had to give? Meg’s worried that she’ll have to give up her sense of beauty. Do you feel like that’s true for you?

Question 5:
In a letter to Meg in response to her being nervous about teaching Ian Stewart says “You’ve been in classrooms all your life. You’ve observed several dozen teachers at length and you have strong opinions about what makes a course good or bad. All of this is preparation.” With that quote in mind, my study is about how graduate education or graduate study shapes the future mathematician for teaching. And, so, what kind of thoughts do you have about that? What would you say about this – this idea that your education here or other places where you’ve earned degrees in mathematics is shaping you for teaching?

Appendix 5: Five Page Sample of a Transcript from a One-on-one Conversation

MARY: How old are you?

CHRIS: 28.

MARY: 28, and, um, are you studying master's? PhD?

CHRIS: PhD.

MARY: Okay. What year in the program?

CHRIS: Just started my second year.

MARY: How's it going?

CHRIS: Uh, overall?

MARY: Yeah.

CHRIS: It's okay. I've finished all my coursework, which is good.

MARY: Wow, in your first year.

CHRIS: Yeah, the research I'm trying to… so my official supervisor… I have two supervisors, co-supervisors now, but my main supervisor does applied stuff and I like more pure, so…

MARY: Oh, okay.

CHRIS: So I'm trying to push my way to the, to the actual more theoretical.

MARY: Oh, okay. What in particular?

CHRIS: Um, so he wants me to look at, uh, so my general area is discrete math.

MARY: Okay.

CHRIS: And, we, my main supervisor does optimization, kind of thing, so…

MARY: Mm hmmm.

CHRIS: You know what I mean? (apprehensively).

MARY: Mm hmm. I have a master's degree in math also. It's been about nine years, so, but…

CHRIS: So he does combinatorial optimization and I like more theory, graph theory, but I'm looking at at geometry problems now.

MARY: Cool. I took graph theory as a graduate student.

CHRIS: Yeah, okay. Where'd you go?

MARY: Um, I'm, uh, more an applied mathematician than a pure mathematician. I went to xxxx.

CHRIS: Oh, really?

MARY: So, um, and went there thinking I was going to get a PhD, but stopped with a master's.

CHRIS: Okay.

MARY: I was really homesick, because I'm originally from xxxx so xxxx was opposite side of the country. Umm. So, graph theory. And, umm, so most, was most of your coursework in pure, like real analysis?

CHRIS: Yeah, I had real analysis and some discrete math courses.

MARY: Mm hmm, okay.

CHRIS: Yeah. And I, I did a master's at xxxx, which is a very specialization, (inaudible) discrete math (inaudible) in that area.

MARY: Okay.

CHRIS: Combinatorics.

MARY: Now, do you think you focused on, um, discrete math at xxxx because that was at xxxx, or was that something you were interested in?

CHRIS: You mean, why did I go to xxxx?

MARY: Mm hmm.

CHRIS: Uhh, well, it was something I was, I mean, it has a very strong reputation for that area.

MARY: Okay.

CHRIS: And it. They have a special department just for that area.

MARY: Oh, okay.

CHRIS: It's unique in the country and maybe even the world, I'm not sure. Uh, but, I mean, I got interested in it at xxxx, which is where I did my undergraduate degree.

MARY: Okay.

CHRIS: And, so I went there for my master's for that reason. Also, my dad lives in the same city, so that was one reason, because I'd never really lived near him. Uh, and my girlfriend at the time was in that area.

MARY: Okay.

CHRIS: That's why I went there.

MARY: Is your dad in mathematics? Or your mother? Or?

CHRIS: No, neither of them. Neither at (went to) university.

MARY: Um, do you remember, um, any experience that really drew you to mathematics, or is there something about mathematics that drew you into it?

CHRIS: Uh, in, throughout my entire life?

MARY: Uh-huh.

CHRIS: Uh, well, I, I guess, so, uh, I found in my k – 12, uh, education that there wasn't, it was more about arithmetic and competition and stuff and so I was, when I was, I lived, I lived in xxxx and I hadn't, hadn't been exposed to proofs or anything and I remember on one exam I was supposed to prove the Pythagorean Theorem using the dot product and I, I managed to solve it and I was really proud. I had had just never seen proofs and I had never really understood what math was about, I guess.

MARY: Mm hmm.

CHRIS: I mean, even then it was vague. This proof idea was vague. It wasn't until later that I, I knew what a proof was and how to do 'em

MARY: Mm hmm.

CHRIS: But I think just seeing, being able to derive something, or, maybe a connection between two things that, I, were kind of separate in my, my head at the time.

MARY: Mm hmm.

CHRIS: It was kind of inspiring for me, I guess.

MARY: Yeah. Does it, so, um, that kind of in-, that insp-… Did you, did that knowingly inspire you to study more math? Or just kind of?

CHRIS: Uh, I just thought "Wow, that's really cool." I was really proud of myself, I guess, which, which made me want… I mean, I always liked math, math, but I would never… I had a… I have a weird, kind of, I was really great at it, like, in grade one and two and then I kind of… I was told I had a bad… Later on, I was told I my teacher in grade 3 was really bad and then after that I was kind of, I wasn't struggling, but I never really cared…

MARY: Yeah.

CHRIS: … much about homework or anything. But it was only at grade xxxx and then more in university that I got more into, into math. So, I'm sorry, what was your question again?

MARY: Oh, just, um, like, uh, well, we were talking about your, um, that experience with the proof, and then…

CHRIS: Yeah.

MARY: What was my question?

Mumbling, laughing

CHRIS: You're recording.

MARY: Sorry. I can listen to it later, but, um, no, it's really interesting to hear your experience. I think a lot of people assume that those who go on to study mathematics have just always been really great at it there entire lives.

CHRIS: Yeah. Yeah.

MARY: My experience, um, math was a breeze until I got to grade seven.

CHRIS: Yeah.

MARY: Pre-algebra and people started talking about x and I was like "What? What are you talking about?"

CHRIS: Yeah, it was confusing… yeah.

MARY: And seven, eight, and nine are all a blur. And I passed my classes. I really can't remember why or how. Um. But then grade 10, I was in second year algebra and I had a great teacher and in grade 11 …

CHRIS: Yeah. I think I was more of a slacker. I mean, in grade ten I had, my final mark was in the 50's.

MARY: Wow.

CHRIS: And then, I got, I mean, in grade xxxx I had in the low 80's and then in university I just kind of…

MARY: Took off.

CHRIS: Yeah. Cuz I think it was again the idea of the proof and the logic of it was really…

MARY: Mm hmm.

CHRIS: … what I liked about it.

MARY: So, more of the algebra, um, well, algebra and arithmetic are tools for proof.

CHRIS: Yeah.

MARY: They didn't quite give you the …

CHRIS: Yeah, it was just, yeah. I always thought I liked phys, I would have liked, I would have been a physicist, when I was, when I was younger.

MARY: Mm hmm.

CHRIS: Then I found out I didn't, uh, so physics and math kind of work differently. So physics you have, you look at a principle, you want to try to find the basic principle, right? You see the effect and you want to try to find the basic principle, but I never understood, I guess what I liked about math was that you start with the basic principles and then you go up, so I guess, I guess I liked the certainty of the logic, there was no question. Physics I didn't like well, why is, I didn't understand what these vibrate, this resonance, and what these kind of concepts were. I like math where it's just, uh, just so you knew, you started somewhere and you knew where you were going. There was no question ever of…

MARY: Mm hmm.

CHRIS: Of what you were doing was wrong, I guess…

MARY: Yeah, yeah. And, um, I think for me, there's a sense of exploration there, right?

CHRIS: Yeah.

MARY: Like you can start with something and …

CHRIS: Yeah, yeah, yeah.

MARY: Well, let's try it over here, let's try with this, let's try it with that, whereas I think the physics, the physics experience …

CHRIS: Yeah, you're forced to, yeah…

MARY: You're locked into something…

CHRIS: Yeah, yeah…

MARY: Whereas mathematics, well what the heck, you can try this, try that…

CHRIS: Yeah.

MARY: Yeah. Um, so you were saying your grade 3 teacher wasn't too inspiring. Did you, did you have any teachers in math that …

CHRIS: … were inspiring?

MARY: Yeah.

CHRIS: In like, in like elementary school?

MARY: Or in k through 12 or undergrad in mathematics that…

CHRIS: Um, no, not particularly.

MARY: No?

CHRIS: No.

Laughter

CHRIS: I had, you know, some good, some bad, but no one that, that really I could point back to and say …

MARY: Mm hmm.

CHRIS: … that they're the one that made me go into math. Like …

MARY: Mm hmm. So that, that's really interesting, um, so the, that grade xxxx experience of the proof was like that first real fire that, you know?

CHRIS: Yeah.

MARY: That's really interesting.

CHRIS: Yeah, yeah.

MARY: So in, at that time in xxxx once you finished grade xxxx, you went into university?

CHRIS: Yeah. Basically grade xxxx was more or less what the first year of university courses would be like here, you know...

MARY: Mm hmm.

CHRIS: So, I mean I took calculus and linear algebra. Well, not linear algebra, but vectors and geometry and stuff like that.

MARY: And did you find that you were more drawn to your pure discrete courses than …?

CHRIS: Uh, well, not right away. Actually, the, so there's a, a bit of an ironic story, I guess. So, in grade xxxx they offered three different math courses – calculus, algebra, and discrete math. And I had, I guess, I guess even in [the grade before] I must have known that I liked math enough at least enough to, that I had to register for all three of them, but then the guidance counselor said you should not take all three because, because I had not had great marks before, so it was like okay you need calculus and algebra as the main two that'll get you into university, so I dropped the discrete math one. So I didn't know what discrete math wasn't until third year of university when I took my first course.

MARY: Oh, interesting. Yeah.

CHRIS: I always liked calculus with, I mean, I like the analysis kind of stuff.

MARY: Yeah. Um, uh, I, the discrete math course I had I was completely baffled by.

CHRIS: Yeah?

MARY: You know, I loved linear algebra, just totally dug it and wish I could have taken more, and, um, real analysis, whoo, proofs were really hard for me. I really really struggled with proofs. So, for me, the applied mathematics where I got to program ode's and pde's and watch their behavior and play around with that…

CHRIS: Yeah.

MARY: … and, um, that's what, you know, really got me going. Uh, it's interesting to contrast our stories because for me, um, I was never spectacular at math by any means, but … And I went to college right after high school and actually flunked out, got kicked out because of bad grades. And then when I went back to school a few years later I really wasn't sure what I wanted to do I just knew I needed to stop having horrible jobs because I didn't have a degree and, um, my grade 10 and 11 math teachers, while they weren't, I wouldn't say they were innovative, spectacular teachers, but who they were as people, they were really incredible people and they seemed to really like, they really liked what they did, and I thought, you know, I think I'll study math. And you know, I started with, I had to go back to the basics like algebra, trig, calculus and, um, it's funny how we make our choices because when I got, I start, I took, I spent a year at community college and then when I switched to university and had to choose – the university I went to, I went to xxxx and they had six choices for math majors. So, actuarial …

CHRIS: Oh, okay.

MARY: Um, here was a comprehensive for future high schoolteachers. Um, applied, pure and I can't remember what else. And I actually chose applied, um, in my very first year because I needed the fewest amount of credits. And it just lucked out that that's what I liked the most, right? Because I still had to take real analysis and graph theory and abstract algebra and, um, but then like I took continuum mechanics and numerical analysis and stuff and just loved it, loved that kind of math. So, but it is funny how, you know, the choices that seem kind of random that actually work out for us, so. Um, yeah that's interesting. How did you, um, enjoy your undergrad experience in math?

CHRIS: Oh, I loved it. Yeah.

MARY: Mm hmmm.

CHRIS: Um, in fact, initially I was, I was accepted into computer math, which was, I guess, halfway between computer science and math. And I found that I hated the computer science courses, which was what I thought I'd like. But I loved the math courses. I never wanted to miss them. I always wanted to go.

MARY: Mm hmm.

CHRIS: So then I just figured… I made the switch.

MARY: So, that sense that had of, like, that you had of not wanting to miss your math classes

CHRIS: Yeah.

MARY: Was it really the math that was drawing you in? Like

CHRIS: Yeah.

MARY: You were just really drawn in by what you saw?

CHRIS: Yeah. I guess because I understood it.

MARY: Mm hmm.

CHRIS: And it was… I had … I guess maybe one, one thing… a funny story I like to tell is that I was actually just too stupid to not be a math major because I was supposed to take a regular, generic first year math course. But at… so at my university there was, you could take calc I, calc II, which were generic, or there was a full year for honors students. I didn't understand that I wasn't an honors student at that point so I guess I might as well take the full year – why register in two classes, I might as well register in one.

MARY: Right.

CHRIS: And so it was, it was much more … I find here… Well, I guess we'll probably get to it later … I find, I find in the generic courses that they teach here, it's just more of packing in, you know, how to, how to compute integrals, derivatives.

MARY: Mm hmm.

CHRIS: And I guess, I guess that I did have a very good teacher in that first year. He was, he talked more about theory, he talked more, you know, it was more of a discussion about concepts, infinities and theories. It was less how do you, how do you compute this derivative.

MARY: Less procedural.

CHRIS: Yeah, like, I don't think, I don't remember … I mean, most of my actual calculus I remember is from high school.

Appendix 6: Five Page Sample of a Transcript from a Group Conversation

MARY: It'll be interesting to see how many of you know each other. Well, I knew that you two knew each other (Steven and Chris) already.

STEVEN: We know John as well, yeah.

MARY: Oh, John. Okay.

ELAINE: (Laughing) Yeah, that's always interesting. It's going to be confidential… oh, just wait a minute…

STEVEN: Confidential, except we all know each other.

MARY: Yeah. Chris, are you taking classes? Steven, you're not taking classes? You are? In?

STEVEN: (can't understand…) I thought I'd finished my coursework.

MARY: I thought you were done taking courses.

STEVEN: Yeah, so did I.

MARY: How did that happen?

STEVEN: My supervisor says take this course. And I said "Okay."

MARY: Yeah, yeah. You're working on research (directed to Chris). How is your problem coming along?

CHRIS: Oh, the one I had. I switched supervisors, so …

MARY: Oh, did you? Oh, okay.

CHRIS: Yeah.

MARY: Although, that campus is pretty nice. It's pretty small.

CHRIS: It's okay. (Can't understand). It's isolated, too. Like there are not too many graduate students. Like I know Steven like not through math at all. We know each other randomly through a common friend.

STEVEN: Although the common friend was in math.

CHRIS: Well, yeah, but…

MARY: Um, well, Elaine, um, this is Robert…

ELAINE: Hi Robert.

SARA: Sara.

MARY: … and Sara. And John.

ELAINE: Hi John.

MARY: This is my supervisor, Elaine Simmt, who has come out for this meeting. So, um, we're just waiting for one more person, for Emily. But, um, I guess we can kind of get started with introductions. Well, why don't we grab a bite to eat if you'd like.

MARY: So it seems like most of you know each other already? More or less.

?: Oh yeah.

?: Oh yeah.

ELAINE: You do? You all recognize each other? Well, of course, you're all in the same department.

ROBERT: We're all in the same department here, so...

MARY: So …

STEVEN: There goes confidentiality.

MARY: You know there are about xxxx graduate students, which seems kind of small for a math department.

SARA: Yeah.

JOHN: But the actual proportion that come to school is like half of that.

SARA: And, as well, we all work in our own little parts of the campus.

STEVEN: That's small?

MARY: Okay, so do any of you share offices with each other or… ?

STEVEN: A little. He comes and bothers me all the time.

CHRIS: I'm trying not to do that, though.

MARY: So are you…? Now you're in xxxx, right? (To John).

JOHN: Robert and I are.

MARY: You're both in xxxx (to John and Robert) and you'll be in xxxx (to Chris)?

STEVEN: No, he's trying to move into xxxx with me.

MARY: Oh, okay.

STEVEN: That's where all the cool people are.

MARY: So, just briefly, why don't we just make a little introduction, so Elaine can get to know you better and if you want to start.

ELAINE: Sure. Um, okay, well my name's Elaine, obviously, and, um, my area is mathematics education at University of Alberta and, so, I supervise doctoral students as part of my work. And a year ago, I guess, I went to Namibia with one of Mary's peers and it happened that she, uh, we secured a grant for her to go back to Namibia to do her data collection. And as part of that I went back for a week while she was collecting data and I realized how valuable that was to actually participate in the data collection. I was kind of the research assistant. So, that's what I am today – the research assistant. So I've sort of made a personal commitment that I want to participate at least for a very short piece of time in data collection, because then it helps me understand better when they're trying to work through ideas. So, um, Vancouver is much closer, much easier to do than Namibia, although Namibia was quite nice. Um, so, yeah, I have a colleague that I did some work with for the last two days at UBC then caught up with Mary yesterday, so this is good for me. Well, thanks for making it so that everyone could come at one time.

CHRIS: I'm Chris.

Laughter

CHRIS: You all know me. I'm working on my PhD in math and uh…

ELAINE: And you're from xxxx?

CHRIS: Yeah, I'm from xxxx.

MARY: And you did a master's in math before you came to xxxx?

Chris: Yeah.

STEVEN: I'm Steven. I'm also in math. I'm also from xxxx. Most of us know each other.

JOHN: And you're doing what kind of degree?

STEVEN: A fake one. I'm doing computer algebra.

MARY: And this is the second year of your master's?

STEVEN: Yes.

ROBERT: So, I'm Robert. I'm working in applied math. It's my fourth year in PhD and it's my hope and expectation that I'll probably finish this year. Yeah. Before that, I was, I actually did my master's degree in Physics, wasn't here, so it's kind of, kind of a change period for me. And then, and then, when I get to time (?) I realized it wasn't that much of a change in career, pretty much do the same thing. There you go.

SARA: Okay, well my name is Sara. I'm a first year master's student in applied math. Um, I did my undergraduate degree here and I've been in Canada for five years. I'm originally from xxxx, southeastern Europe. And I work on a problem in material science. And so.

JOHN: John. Um, I work in number theory. Uh, I'm a second year PhD, but hopefully pretty close to the end. Um, I'm American, from xxxx. And I came here with a master's already.

MARY: I'm Mary. I'm a PhD student in math education. Um, looking at math education at post-secondary level. So, working with grad students in math as future teachers and talking to them about teaching and learning in math. So, I emailed you some questions on Friday, um, to think about for this weekend, for today's meeting. And, so, the first question we're going to talk about is 'What is math?' question and I'm just going to read you a few quotes from mathematicians about what mathematics is.

Um, so Hersh says "Mathematics is a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context."

Ian Stewart, I don't know how many of you know of Ian Stewart, he says "Math is a product of human minds but not bendable to human will. Exploring it is like a new tract of country; you may not know what is around the next bend in the river, but you don't get to choose. You can only wait and find out."

And I he also says, "Mathematics is the shared social construct created by people who are aware of certain opportunities, and we call those people mathematicians."

Some interesting ideas about mathematics from mathematicians. And, um, I'd like you to, um, think about those things and if you want me to repeat them, uh, I would definitely do so. Um, but, do you feel like there's anything missing in these ideas about mathematics? How would you expand on that, or … ?

JOHN: Could you read it one more time?

MARY: Mm-hmm.

SARA: Sounded pretty vague to me because many things can be defined in that way.

MARY: Okay.

SARA: You could put different words in there besides math and it would work.

MARY: Okay, okay.

SARA: But…

MARY: So, just for John, um, Hersh says "Mathematics is a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context."

JOHN: And who is this?

MARY: Reuben Hersh.

JOHN: And he is? I, I don't know…

MARY: He's a mathematician. He worked with another person, Davis, and they wrote a book in 1984 called The Mathematical Experience.

JOHN: So, he's in math education?

MARY: He's a mathematician. He's mathematician …

JOHN: Pure mathematician? Applied mathematician?

MARY: Um, I believe he's an applied mathematician. So, um, yeah. And then Ian Stewart has a similar statement "Math is a shared social construct created by people who are aware of certain opportunities, a product of human minds, but not bendable to human will." So, Sara, you were saying that lots of things could be defined in this way. So, so, what would you add to these definitions that would make it, would make people know that if the word mathematics was taken out of these descriptions, that they would know that it was mathematics you were describing.

SARA: Um, difficult question, um. I guess first one point the only thing that it seemed like it singled math out a bit was the last statement "not bendable to human will," um, because a lot of other things are bendable to human will, but the math is based on a certain set of rules. And, unlike, … yes, but are those rules free from human will? I'm not sure about that.

MARY: John, you were nodding when Sara was talking, so what …?

JOHN: Well, yeah, I think it's true. I think if you take out "bendable to human will" you could call it language, you could call it history. I mean there are so many things that are just a shared social construct. I mean academia in general. Um. I mean it seems kind of vague.

STEVEN: Yeah, but how come none of them uses the term 'truth'? Which is probably the most definitive word you could talk about in mathematics.

SARA: True.

JOHN: No, I don't know that that's true. I don't think all math is truth at all. I mean I think it's…

STEVEN: It's the study of absolute truth.

SARA: Absolute truth.

JOHN: Well, no, it's not.

STEVEN: All mathematics is absolutely true. That's the way we set it up. That's the whole point of mathematics. It's the whole bendable to human will …

JOHN: It's true based on a certain set of axioms.

SARA: Yeah.

JOHN: Real truth is independent of axioms, I would say. So, I mean, it's all in context, right? I mean you set up axioms whether you're an applied mathematician or not and then you see what you can do with those axioms, right?

STEVEN: Yeah, but we're sort of, you know, on top of physical law. Like physicists can make observations and be wrong or be accurate. All of our observations about our consequences according to our axioms have to be true inherently.

JOHN: Well, that's true. But, I mean does truth …

SARA: And they're relative to the axioms. They can't be absolute – they depend on the axioms.

STEVEN: Well, okay, you have to start somewhere, but starting at a set of axioms. This could be a good definition of math – the study of logical consequences to axioms, right? But all of these things have to be true, right?

JOHN: Within our set.

STEVEN: Within whatever our truth system is, but I think that's as close to absolute truth that you could probably get in mathematics. In no other subject can you make a statement and say 'well, this is right.' Right? There's no, you can't have an opinion, you can't have anything except the fact that 'yeah, this is true.'

SARA: Yeah. True. It's… Yeah. It doesn't have the subjectivity that most other things would have.

STEVEN: Well, most every other thing has subjectivity. Mathematics would be the only subject where you can take out subjectivity. Right?

CHRIS: I mean, Goedel's imcompleteness theorem showed us that no matter how, you know, no matter how good your axioms are there's going to be things that are true that can't be proved.

STEVEN: Well, Goedel showed that you can't ever have a perfect system which can prove everything or disprove everything.

CHRIS: So, that's what John is saying – truth is independent of the axioms.

STEVEN: So, truth is in that little place where…

SARA: What little place?

STEVEN: that's outside...

MARY: So, so, with what you're talking about in mind, so how would you expand these definitions that they've given, like, so it's "the shared social construct created by people who are aware of certain opportunities" or "a human activity, a social phenomenon." What, how, what would you add on to that?

STEVEN: Well, I don't think it's a human activity.

MARY: No?

STEVEN: Well, it sort of touches on did we discover mathematics or invent it, right? I mean no one can really ever answer that.

JOHN: These are both very discover-oriented.

STEVEN: Yeah, these are both very like…

JOHN: I mean, sorry, the other one…

STEVEN: Invented.

JOHN: Yeah. It even said, the second guy, the calculus book guy…

MARY: Um, Hersh?

-discussion about Ian Stewart versus James Stewart…

JOHN: Ian Stewart? The Scottish guy. Yeah, yeah.

STEVEN: The one that does the algebra book. Get your story straight.

JOHN: The book writer. He wrote Letters to a Mathematician.

MARY: Yeah.

ELAINE: Yeah.

JOHN: He even said something about constructivism, math as a construct.

MARY: "A shared social construct created by people who are aware of certain opportunities." So, it's created? It doesn't exist already?

STEVEN: I think you need to pen the words 'truth' and 'language' into all of those definitions before you can start getting any rich language about mathematics, right?

JOHN: I think of mathematics as more of a language of logic. I mean you set up axioms and you, and then you see what you can do with them. It's a lot more linguistic, I think.

SARA: Yeah, it is.

JOHN: At least on the pure side.

SARA: I was going to add, actually, it's linguistic, um, and the grammar rules are very strict. But if you get, onto, into some areas of applied math, it becomes hand-wavey, so that's where you start throwing in slang or don't really look at, don't check that it satisfies the rules that we suppose. But the best version of it is when it truly satisfies all the rules. Unlike in a human language when you don't always do that.

MARY: Robert, what are you thinking? You've been a little quiet.

ROBERT: Oh, no. Been thinking about the human language part. Because I, I think… human language is kind of a convention because it's fluid, it changes. For example, English is different 500 years ago from what it is now. But I don't think, for example, mathematics, well, we know more, but it's basically the same.

SARA: But it is different, like changing notation or changing the words we use. Some are archaic, some is more common.

ROBERT: Yeah, but there are some people who try to impose rigor in language

SARA: Yeah, but as you speak, you follow rules all the time. Unless you start talking slang, then you (laughs) well, even then you follow rules, but not ones that are commonly accepted.

STEVEN: Well, you had this great analogy this morning comparing English to Mathematics.

JOHN: Yeah, I forgot it.

MARY: Do you remember?

STEVEN: Theories of poetry and….

JOHN: Well no, yeah. So you write a paper, each paper is either, like we even have an idea of a standard in mathematics like we do in language, right? You read a paper and you say "This is beautiful" or you read a paper and you say, "Well, it's communicating."

STEVEN: Well, even more important than that…

JOHN: Right? Maybe it's ugly, you know?

MARY: Come on in. (Emily comes in after her morning class.) Um, so you were saying?

JOHN: Yeah, yeah. Some things are pretty and some things are ugly and some things communicate, some things don't, so I mean it's …

STEVEN: Yeah, but I mean even within the notion of a proof, right, not in a paper, we can talk about proofs being beautiful or ugly, which is even more remarkable.