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Long-Term Mine Planning in Presence of Grade Uncertainty

by

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To

My wonderful parents, Nasrin and Mohammad,

my lovely sister, Mahsa,

and my beautiful wife, Elmira.

ABSTRACT

Open-pit mining is widely used to extract natural resources. Low cut-off grades and large operations can make open-pit mining profitable. An important challenge is to determine the optimum production schedule. Usually the goal is to maximize the net present value of the project while delivering ore to the plant at full capacity. The best production plan would require complete knowledge of the orebody and all other engineering and economic parameters. An estimated block model is often used to determine the production schedule. Uncertainty is inevitable with widely spaced drill holes. The open-pit production schedule based on estimated models may be suboptimal and affected dramatically by grade uncertainty. The research documented herein develops, implements and verifies four mixed integer optimization frameworks for long-term production scheduling in the presence of grade uncertainty. The main contributions of this research are (1) consideration of cost of grade uncertainty to influence the production plan, (2) accounting for the linear and nonlinear effects of the grade uncertainty on the long-term mine planning, (3) development of a mixed integer linear programming model that maximizes NPV and minimizes the cost of the grade uncertainty by considering a stockpile, and finally (4) implementation of a quadratic optimization model accounts for grade uncertainty in the long-term production plan.

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Behrang Koushavand, January 2014

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LIST OF ABBREVIATIONS

COU	Cost of grade Uncertainty
DCF	Discounted Cash Flow
DCOU	Discounted Cost of grade Uncertainty.
DTP	Deviation from Target Production
DEBV	Discounted Economic Block Value
EBV	Economic Block Value
GSLIB	Geostatistical Software Library
LTPP	Long-Term Production Plan
LP	Linear Programming
m	Meters
MD	Million Dollars
MIP	Mixed Integer Programming (linear or non linear)
MIPGAP	Relative MIP Gap Tolerance
MIQP	Mixed Integer Quadratic Programming
MILP	Mixed Integer Linear Programming
MT	Million Tonne
NPV	Net Present Value
OK	Ordinary Kriging
SGS	Sequential Gaussian Simulation
SIS	Sequential Indicator Simulation

LIST OF NOMENCLATURE

a	Range of the variogram
$a(t,i)\in\{0,1\}$	A binary integer variable indicates whether block i has been extracted until period t
$\mathbf{A}$	The coefficient matrix of linear constraints of the MILP model in general form
$\mathbf{b}_L$	Lower limit of the linear constraints
$\mathbf{b}_U$	Upper limit of the linear constraints
$\mathbf{c}$	Coefficient of linear objective function in the general form of a MILP model
$C(i)$	Set of immediate blocks on top of block i that are needed to be extracted before start extraction of the block i
$\mathrm{C}_{up}(t;l)$	Cost of under production for realization l in period t
C_p	Cost o processing for a unit tonnage of ore
C_m	Cost of mining for a unit tonnage of ore
$C_p(t)$	Cost of processing in period t
$C_m(t)$	Cost of mining in period t per tonne
$c_{up}(t)$	Discounted cost of the under production per tonne in period t
$c_{op}(t)$	Cost of over production in period t per tonne
$c_{op}(t+1)$	Cost of over production in period t+1 per tonne
$C_{RH}(t)$	Cost of the re-handling of ore in the stockpile in period t per tonne
$c_{RH}(t)$	Discounted re-handling cost in period t per tonne
$\hat{c}_{op,RH}(t)$	Adjusted cost of unit tonne of over production ore in present of stockpile in period t
$\overline{c}_{op}(t)$	Average cost of over production in period t over all L realizations
$\overline{c}_{up}(t)$	Average cost of under production in period t over all L realizations
C_0	Nugget effect in the variogram model

C	Sill of the variogram model
$C_{\text{UserDefined}}$	User-defined upper limit for the variance of the NPV
DEBV(i)	Discounted economic block value of block i .
$DCF(t)$	Discounted cash flow for each period t that is calculated by an estimation block model
$DTP(t)$	Deviation from target production in period t
$DTP(t;l)$	Deviation from target production for realization number l
EBV(i)	Economic Block Value of block i
$E\{v(t;i)\}$	Expected value or the average of discounted ore value for block i in period t
$\overline{g}(i)$	Average grade of block i calculated from all L realizations
g	Grade of ore
g_{cut}	Cut-off grade
$g(i)$	Estimated grade value of the block i
$g_l(t)$	Allowable lower limit of the head grade in period t
$g_u(t)$	Allowable upper limit of the head grade in period t
$\overline{g}(t)$	Average input grade to the mill in period t.
$i \in \{1 \cdots N\}$	Index of the block at the block model
IR	Interest rate of the project or the discounting rate
$j \in \{1 \cdots N\}$	Index of the block at the block model
K	Total number of precedence relationships between all the mining cuts
$K_{\text{UserDefined}}$	User-defined lower limit the NPV of the project
L	Total number of conditional realization
$\lambda_{\text{UserDefined}}$	User-define value to control the trade-off between the maximization of the NPV and the minimization of the variance of the NPV.
M	Total number of mining cuts inside the final pit
$m_l(t)$	Lower limit for mining capacity in period t
$m_u(t)$	Upper limit for mining capacity in period t
m	Total number of linear constraint in the MILP model in the general form
N	Total number of blocks inside the final pit

$NPV(l)$	NPV of realization l
NPV_{es}	NPV of the project that is calculated from an estimation block model such as OK
P	Price of commodity per tonne
$p_l(t)$	Lower limit (target production) for the designed processing plant in period t
$p_u(t)$	Upper limit (target production) for the designed processing plant in period t
$P(t)$	Commodity price per tonne in period t
$q(i;t)$	Cost of mining of block i in period t
$q(t;i;l)$	Cost of mining of block i in period t at realization l
R_m	Mining recovery
$R_m(t)$	Mining recovery in period t
R_p	Processing recovery
$R_p(t)$	Processing recovery in period t
$S_u(t)$	Upper limit tonnage of the stockpile in each period and this constraint is applied to each realization.
t	Period in year.
T	Total number of period or the mine life
T_p	Total number of production years
$T(i)$	Tonnage of the block i
$T_{up}(t;l)$	Tonnage of under produced ore for realization l in each period t
$T_{op}(t;l)$	Tonnage of over produced ore for realization l in each period t
$T_t(t)$	Tonnage of the target production in period t
$v(i;t)$	Ore value of block i in period t
$v(t;i;l)$	Ore value of block i in period t at realization l
$Var\{NPV\}$	Variance of NPV calculated from all conditional realizations

$Var\{g(i)\}$	Or $\sigma_g^2(i)$; Variance of grade for the block i calculated from all conditional realizations
$w(t;i)$	Binary decision variable indicates if block i is extracted in period t or not
X	Coordinate of the blocks in x-axes
$\mathbf{x}$	Vector of the decision variables of the MILP model including binary and continues variable
$\mathbf{x}_L$	Lower limit of the decision variables for a general form of MILP
$\mathbf{x}_U$	Upper limit of the decision variables for a general form of MILP
$y(t;i)$	Continues decision variables indicates the portion of extraction for block i in period t
Y	Coordinate of the blocks in y-axes
$z(t;i)$	Continues decision variables indicates the portion of processing for block i in period t
Z	Coordinate of the blocks in z-axes

Chapter 1 Introduction

Mine planning is reviewed in Section 1.1. The challenge of mine planning in presence of grade uncertainty is presented in Section 1.2. The objective of this thesis is described in Section 1.3. Section 1.4 describes additional background and the relevant literature. The algorithms used to determine optimal pit limits are explained first; then the methods of long-term mine planning with and without uncertainty are reviewed. Shortcomings of the currently available mine planning methods in presence of grade uncertainty are described to motivate the research. Finally, Section 1.6 presents an outline of the thesis.

1.1 Introduction

Mine planning defines the source, destination and sequence of extraction of ore and waste over the mine life. The result of mine planning is a production schedule that defines the tonnage of ore and waste and the input grade to the plant in each period of time. This production schedule has a significant influence on the economics of the mine. Improving production scheduling is essential as the mining industry considers more marginal resources. The natural complexity of mineral deposits makes mine planning more difficult. Moreover, the production schedule must follow physical constraints and meet the target capacity of the processing plant.

The typical mine life is usually between 20 to 30 years. There are often three time ranges for production scheduling: long-term, medium-term and short-term. Long-term will be for the full life of mine. This period is broken into several medium-term periods between 1 and 5 years. Medium-term schedules provide more detailed information that allows for a more accurate design of ore extraction. The short-term schedule is also broken down into short weekly or even daily plans for detailed scheduling (Osanloo et al., 2008).

Optimization algorithms are starting to be used in mine planning to maximize the overall profit of the project and minimize deviation from target production. In traditional long-term mine planning a geological block model is used as the main input to maximize the net present value (NPV) of the project. The geological block model is a quantitative definition of the available resource. Data from drillholes are used to construct the block model using geostatistical techniques.

Uncertainty in the block model is inevitable with relatively widely spaced drillholes. The optimality of the open-pit production schedule will be affected by this uncertainty. Recent research initiatives have attempted to consider the effect of grade uncertainty on production schedules. Another major challenge in open-pit production scheduling is the size of the optimization problem. The mathematical programming formulation of realistic long-term open-pit production schedules often exceeds the capacity of current hardware and optimization software.

In this thesis, the geological uncertainty is taken into account in the long-term production scheduling. The main focus of this thesis is on long-term mine planning in presence of grade uncertainty. Therefore, instead of using only one block model, a number of simulated realizations, that are representative of grade uncertainty, are used in the optimization process. In addition to maximization of the NPV of the project, a second objective is to minimize the negative impact of grade uncertainty.

1.2 Statement of the Problem

Numerical modeling is a robust method to quantify geological complexity (Hustrulid and Kuchta, 1995). A limited number of samples are collected and used to build numerical models. These samples are considered to be at the point scale. There may be some additional secondary information such as geophysical data at a larger scale.

A geological block model is obtained by dividing the deposit into a three-dimensional grid at the block scale. The block model may have tens of millions of blocks sometimes referred to as grid nodes or cells. The recommended size for each block is about 1/3 the anticipated blast-hole spacing and the bench height for short-term mine planning (Leuangthong et al., 2004). The block dimensions for long-term mine planning are selected according to the exploration drilling pattern, geology of the ore-body, mine equipment and anticipated operating conditions. The size of the blocks used in long-term mine planning is a function of the selective mining unit (SMU). Leuangthong et al. (2004) are defined as the SMU size, that is, “the block model size that would correctly predict the tonnes of ore, tonnes of waste, and diluted head grade that the mill will receive with the anticipated grade control practice”. This SMU block size is relatively large with the vertical size equal to the bench height and the horizontal size between 10 to 20m.

The basic problem in mine planning is to find a sequence of ore and waste blocks to mine so that the NPV of the operation is maximized. The production schedule is subject to a variety of constraints including overall pit slopes, mining, milling, and refining capacities, blending requirements, minimum mining width and the number of active mining benches in each production period. A cut-off grade is often defined and used to distinguish between ore and waste blocks based on some economical parameters such as price of commodity and the costs of processing and mining and technical parameters such as processing techniques that are going to be used.

There are two kinds of geological uncertainties that are numerically modeled using geostatistical simulation: rock type uncertainty and grade uncertainty inside each rock type. The main focus of this thesis is on grade uncertainty and how that uncertainty is handled in mine planning.

As shown in Figure 1-1, in traditional mine planning methods, a grade is assigned to each block using estimation techniques. Kriging (Deutsch and Journel, 1998; Goovaerts, 1997) is a common estimation method. however, kriging does not capture uncertainty and it creates estimates that are too smooth leading to systematically biased reserve estimates (McLennan and Deutsch, 2004).

Geostatistical simulation algorithms help quantify and assess grade uncertainty. The generated realizations represent plausible geological outcomes and quantify the uncertainty. The problem presented in this thesis involves scheduling of N different ore and waste blocks within the final pit outline over T different periods of extraction in the presence of grade uncertainty.

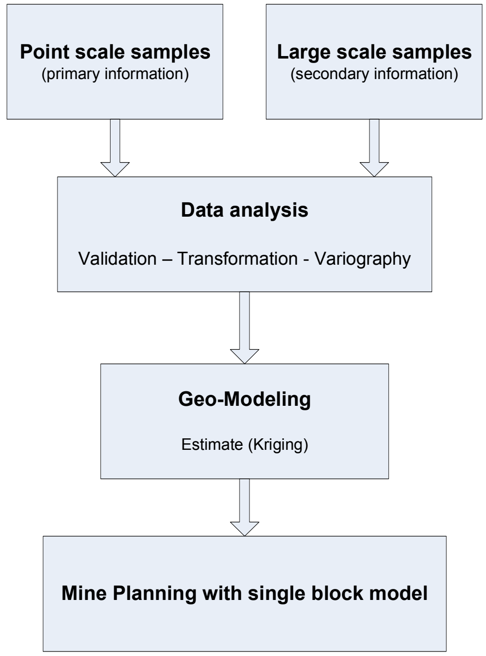


Figure 1-1: Traditional mine planning stages.

1.3 Objectives of the Study

The main scope of this thesis relates to long-term open-pit extraction scheduling using optimization methods and geological realizations. The objective is to account the grade uncertainty in the optimization model such that the expected NPV is maximized and the risk of grade uncertainty is minimized. A new term called the cost of grade uncertainty is introduced. The simulated realizations are used to calculate expected NPV, expected deviation from target production at each time period, the cost of the grade uncertainty and the variance of NPV. These parameters are used to assess the uncertainty and the quality of the generated schedule in the presence of grade uncertainty.

The basic assumption is that operating projects will be risk averse, that is, among options with the same expected profit, the one with the least uncertainty is preferred. Higher uncertainty may be chosen if the expected profit is sufficiently higher. The challenging question is how to quantify and choose the trade-off point between NPV and the risk incurred by grade uncertainty.

The goal of this research is to develop, implement and verify a theoretical framework based on optimization methods to address the long-term production plan (LTPP) problem in the presence of grade uncertainty. The generated schedule will be less sensitive to the grade uncertainty in early years of production. Additional information obtained from infill drill holes and blast holes will reduce the uncertainty in later years.

1.4 Background and Literature Review

Surface mining is a method of extraction where the operations are open to the surface during the mine life (Askari-Nasab, 2006). Non-valuable surface material that covers the deposit (called overburden) as well as waste rock within the deposit must be removed to gain access to the mineralized ore.

The main goal of a mining operation is to extract and process the ore with minimum cost and maximum profit. To achieve this goal, it is required to have and follow a schedule of extraction considering a number of economic and technical parameters including:

- Input block model of grades for the entire deposit.
- A cut-off grade that distinguishes ore blocks from waste blocks (Osanloo et al., 2008).
- Mining capacity: the nominal rate limit that a mine can extract and haul ore and waste from the mine.
- Processing capacity: the nominal rate limit of the processing plant where ore is upgraded to the final product.
- Type of excavators and hauling system that controls the mining selectivity and minimum mining width.
- Wall slopes: the angle of the walls in an open-pit mine which is determined based on rock mechanics, geological characteristics and some safety factors.
- The preferred direction of mining.
- Interest rate or discount factor to calculate the present value and costs in a long-term project.

A surface mine is designed by considering these parameters. The selection of these parameters and the schedule of extraction of ore and waste material are complex engineering decisions that have economic significance (Askari-Nasab, 2006).

Whittle (1989) defined open-pit mine planning as “specifying the sequence of blocks extraction from the mine to give the highest NPV, subject to a variety of production, grade blending and pit slope constraints”. The goal is to find an optimal schedule that satisfies all technical and physical constraints. One of the first steps in mine planning is to find the ultimate pit limit. The heuristic floating cone technique (Pana and Davey, 1965) is sometimes used. The method is simple and fast but it may not find the optimum solution for the final pit (Huttagosol and Cameron, 1992). Lerchs and Grossman (1965) presented two methods to determine ultimate pit limits: (1) a 2D algorithm based on dynamic programming and (2) a 3D algorithm that uses graph theory. The graph theory approach is widely accepted and used throughout the mining industry. Other methods have been presented such as maximum network flow algorithm (Johnson, 1968) and transportation algorithm (Huttagosol and Cameron, 1992). Within the ultimate pit, push-backs or stages are designed. Often, elevated selling prices are used to find the initial pit. These push-backs are used as a large scale guide for the detailed production schedule.

There are two general classes of methods to solve the LTPP problem: (1) deterministic methods that assume the input values and parameters are known and fixed, and (2) uncertainty based methods that consider some input parameters as uncertain.

1.4.1 Deterministic Approach for LTPP

In medium-term and short-term production scheduling, the main goal is to find an optimum plant feed schedule (Chanda and Dagdelen, 1995; Dowd and Elvan, 1987; Huang, 1993; Zhang et al., 1993). In long-term production planning, the goal is to find the sequence of extraction such that the future cash flow is maximized subject to economic, technical and environmental constraints (Askari-Nasab and Awuah-Offei, 2009). In long-term production planning, it is common to consider yearly production volumes. An interest rate is used to calculate discounted economical values such as revenue and mining cost of each block at any given period. Askari-Nasab and Awuah-Offei (2009) reported three general categories of long-term mine planning methods: (1) heuristic methods, (2) application of artificial intelligence techniques and (3) operations research (OR) methods. Some of these algorithms are embedded into commercial software packages. XPAC AutoScheduler (Runge Limited, 1996-2009) is a commercial software that uses the heuristic method proposed by Gershon (1987). In this method, an upward cone close to the shape of the pit is generated for each block. A factor called positional weight is calculated for each block based on whether it is more desirable to be extracted in the present time or not. These weights are used to determine the removal sequence. Steps of this approach are as follows:

1. Determine the set of blocks currently available for mining,
2. Calculate a positional weight for each of those blocks,
3. Use positional weight to decide which blocks to be mined and return to step 1.

Whittle (Gemcom Software International, 1998-2008) and NPV Scheduler (Datamine Corporate Limited, 2008) are popular software packages which are also based on heuristic algorithms. They apply the parametric analysis method introduced by Lerchs and Grossmann (1965) to generate a series of nested pits using different revenue factors. The nested pits do not guarantee the global optimum solution. This may cause financial losses (Askari-Nasab and Awuah-Offei, 2009).

There are artificial intelligence techniques applied to the mine scheduling problem. Tolwinski and Underwood (1996) proposed a method that is a combination of dynamic programming, stochastic optimization and artificial intelligence with heuristic rules. This method finds the ultimate pit and generates the production schedule concurrently. Denby et al. (1996) proposed a method based on genetic algorithm and simulated annealing. This method also generates the ultimate pit limit and production schedule in the same time. Askari-Nasab (2006) developed an intelligent agent-based theoretical framework for open-pit mine planning. It has a component that simulates push-backs and the intelligent agent learns the optimal push-back using a reinforcement learning method. There is no guarantee that the results will be close to the theoretical optimum solution.

Operations Research (OR) methods have been employed in mine production scheduling by a variety of authors. Linear programming (LP), integer programming (IP) and mixed integer linear programming (MILP) are commonly used in literature for long-term mine planning. These methods are exact since they are used to solve convex models. An optimization model is called convex if the solution can be proven to be the global optimum solution.

Johnson (1968) used an LP model to maximize NPV of a mining project. His model considers discounted values of revenues and costs, different processing types and dynamic cut-off grade. To solve this LP model, a large multi-period model should be decomposed into sub-model. Each sub-model considers only one period at a time. Although the model generates optimum results for each period individually, the results are not optimum taken all together. Also, the precedence of block extraction is not satisfied. This causes some percentage of the overlaying blocks be suspended in air (Figure 1-2).

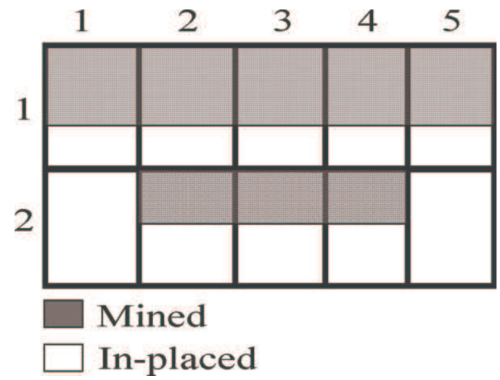


Figure 1-2: Problem of partial block mining of Jonson's (1968) model (Osanloo et al., 2008).

Gershon (1983) modified Johnson's LP model to a general MILP model. A set of binary variables was considered to satisfy the precedence of block extraction. The main disadvantage of this model is that it is intractable for a real size mine planning project because there are too many binary variables. MILP models can handle multiple ore processing options and multiple grades. Although the global solution exists and it is unique, there is a gap between the numerical results and the theoretical solution. Over time, several authors tried to make MILP models tractable for real size mine planning. The first attempts were made by using the Lagrangian relaxation approach. Dagdelen and Johnson (1986) decomposed a multi-period problem into smaller single-period problems. Mining capacity and processing capacity constraints are relaxed into the objective function with Lagrange multipliers. In some cases, the multipliers cannot be adjusted to get a feasible solution. Therefore, this method may not converge to an optimum solution (Osanloo et al., 2008). Akaike and Dagdelen (1999) presented a 4D-network relaxation method. A dynamic cut-off grade method is considered and the method can include a stockpile.

Caccetta and Hill (2003) presented a customized branch and cut algorithm to speed up the convergence of the MILP model. Some other authors tried to use aggregation methods to reduce the number of variables. Ramazan and Dimitrakopoulos (2004) proposed a method where the waste blocks are considered as real variables to reduce the size of the problem. Ramazan et al. (2007; 2005) employed a fundamental tree algorithm to reduce the number of decision variables. They showed a case study for 38,457 blocks. They used Whittle software (Gemcom Software International, 1998-2008) to design 4 push-backs and within each push-back the fundamental tree method was applied to aggregate. The 5512 fundamental tress are used in a mixed integer programming to generate a schedule for 8 periods. According to Osanloo et al. (2008) the disadvantages of this method are: (1) the large number of trees and number of periods that cannot be handled by current commercial MILP solvers, (2) the optimality of the final solution is highly dependent on how the push-backs are designed, (3) the MILP problem may need to be solved several times to identify the fundamental trees and (4) the method is complex and not very popular.

Boland et al. (2009) used a different approach. They defined two separate real variables for the portion of mining and processing. They used aggregated blocks to determine the order of extraction and the processing decision variables were applied at the block level. In their case study, the 96821 blocks were aggregated to 125 units and the problem was solved for 25 periods. Considering only 125 units reduces the degrees of freedom of the optimization. Therefore, the generated schedule would not be optimum solution comparing using all 96821 blocks. Boland et al. (2009) did not present much information about the aggregation technique that they have used.

Askari-Nasab and Awuah-Offei (2009) also reviewed a commercial software package called MineMax (1998-2009) that generates schedules by solving a MILP optimization problem using ILOG CPLEX (1987-2009) solver. This software uses the parametric analysis technique presented by Lerchs and Grossmann (1965) to divide the deposit into several nested pits or pit shells. Then, for each pit shell the MILP is solved. The number of variables is reduced by only considering the blocks in the pit shell. The result will not be the global optimum solution because the pit shells are already defined. Sliding windows is another method which uses sub-problems to solve the MILP problem on a period by period basis.

Askari-Nasab et al. (2010; 2011) presented the objective functions of the LP formulations that maximize the NPV of the mining operation. Their model was generalized form of an earlier model presented by Caccetta and Hill (2003) that is widely accepted. Their model is also an MILP optimization problem. TOMLAB (Holmström, 1989-2011) was employed to solve the proposed optimization problem.

1.4.2 Uncertainty Based Approaches for the LTPP Problem

Dimitrakopoulos (1998) has classified the uncertainties involved in mine planning as: (1) orebody model and in situ grade uncertainty and material type distribution; (2) technical mining specification uncertainty such as extraction capacities and slope consideration and (3) economic uncertainties including capital and operating costs.

A solution to the LTPP problem is affected by uncertainties related to the input parameters. Osanloo et al. (2008) mentions several authors who consider the uncertainty of ore grade in the LTPP problem. They report that grade uncertainty may cause some shortfalls at the designed production and discrepancies between planning expectations and actual production, especially in early years of production.

Since the1960's, different authors have tried to solve the LTPP problem with both deterministic and uncertainty based approaches. There are few works on uncertainty based approaches. There are some papers that show the effect of grade uncertainty and consequently the value of each block in production schedules

Vallee (2000) reported that 60% of the mines surveyed had 70% less production than designed capacity in the early years. Rossi and Parker (1994) reported shortfalls against predictions of mine production in later stages of production.

Dimitrakopoulos et al. (2001) show that there are substantial conceptual and economic differences between risk based frameworks and traditional approaches. Dowd (1994) and Ravenscroft (1992) used stochastic orebody models sequentially in traditional optimization methods. Dowd (1994) proposed a framework for risk integration in surface mine planning. Ravenscroft (1992) discussed risk analysis in mine production scheduling. He used simulated ore bodies to show the impact of grade uncertainty on production scheduling. The geostatistical simulation techniques are used to generate realizations that are representative of the grade uncertainty of ore body. Each realization is an alternative image of the ore body. Any deterministic method can be used to solve the LTPP problem using the realizations one at a time. He concluded that conventional mathematical programming models cannot accommodate quantified risk.

Godoy and Dimitrakopoulos (2003) and Leite and Dimitrakopoulos (2007) presented a risk inclusive LTPP approach based on simulated annealing. A multi-stage heuristic framework is presented to generate a final schedule, which considers geological uncertainty so as to minimize the risk of deviations from production targets. A basic input to this framework is a set of realizations. They report significant improvement on NPV in presence of uncertainty. This method has the following disadvantages: (1) it does not consider grade blending, (2) it does not control the risk distribution for the production target, (3) the optimality of this method cannot be guaranteed and (4) simulated annealing requires tuning parameters that make it difficult to apply.

Figure 1-3 shows the stages of the stochastic production schedule framework presented by Leite and Dimitrakopoulos (2007) . For each realization, an optimum schedule is generated and a single schedule is generated based on all schedules such that deviation from target production is minimized.

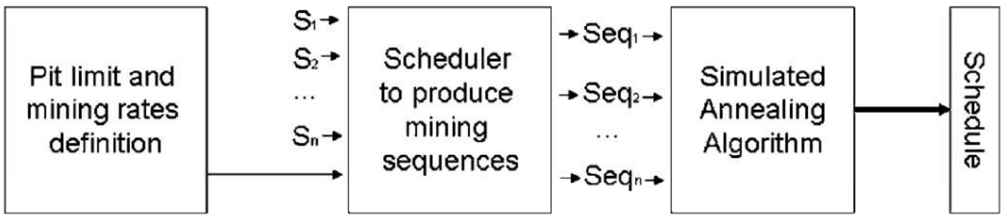


Figure 1-3: Three stages of mine production scheduling process (Leite & Dimitrakopoulos, 2007).

Dimitrakopoulos and Ramazan (2004) proposed a probabilistic method for long-term mine planning based on linear programming. This method uses probabilities of being above or below a cut-off to account for uncertainty. An LP model is used to minimize the deviation from target production. This method does not directly and explicitly account for orebody uncertainty and also does not maximize NPV.

Boland (2008) in an unpublished paper which is available through the internet presented a new stochastic based linear model for long-term mine production in presence of grade uncertainty. The uncertainty depends on the material mined in earlier periods.

Dimitrakopoulos and Ramazan (2008) presented a stochastic integer programming (SIP) model to generate the optimal production schedule using multiple realizations as input. There is a penalty function that is the cost of deviation from the target production. The function is calculated from a geological risk discount rate (GDR) that is discounted unit cost of deviation from a target production. They use linear programming to maximize NPV minus penalty costs. They concluded that the generated production schedule is the optimum solution that can produce the maximum achievable discounted total value from the project, given the available orebody uncertainty described through a set of stochastically simulated orebody models. The proposed scheduling approach considers multiple simulated orebody models without increasing the required number of binary variables and thus computational complexity. In their model, it is not clear how to define the GDR parameter. Adding constraints increases the complexity and CPU time to solve the optimization. The short-term production schedule is not taken into account. It is not dynamic and flexible to new information that is acquired during the mine life.

Koushavand and Askari-Nasab (2009) showed the impact of grade uncertainty on the production plan. The research was based on a case study on an oil sand deposit in Canada. The details of this research are presented in Section 4.1. The idea was to transfer grade uncertainty to the mine production schedule. Two different methodologies are used to assess the impact of grade uncertainty on output parameters of mine production scheduling such as NPV, ore tonnage, head grade, stripping ratio, amount of final production and annual target production.

Table 1-1 summarizes the uncertainty based approaches to LTPP.

Table 1-1: Summary of uncertainty based algorithms to solve LTPP problems.

Type of model	Author	Year	Solution	Advantages	Disadvantages
Risk analysis using deterministic algorithm	Rovenscroft	1992	Conditional simulation technique and Deterministic LTPP algorithm	Shows the impact of grade uncertainty on LTPP.	Cannot quantify the risk of a project. Does not give the optimal solution in presence of grade uncertainty.
Risk analysis using dynamic programming	Dowd	1994	Conditional simulation and DP	Quantify risk associated in a project.	Does not give any criteria to accept or reject the risk. Does not produce optimal solution in the presence of grade uncertainty.
Linear programming	Dimitrakopoulos and Ramazan	2003	Linear goal programming	Generates the schedule that reduces the risk at early production stages. Considers equipment mobility and block access in production planning.	Does not generate maximum NPV in presence of grade uncertainty.
Meta-heuristic	Gody and Dimitrakopoulos	2003	Conditional simulation and Simulated annealing	Integrates ore body uncertainty, waste management and economic and mining consideration to generate optimal mining rates. Produces a single optimum production planning in presence of uncertainty.	Implementation is complicated. The optimality of this method cannot be guaranteed. Does not consider equipment access. Grade uncertainty is not incorporated explicitly in the production planning process.
Mixed integer programming	Ramazan and Dimitrakopoulos	2004	Mixed integer programming	Maximizes NPV explicitly with the consideration of equipment mobility and block access.	Cannot implement on large deposit. Grade uncertainty has not been used directly.
Meta-heuristic	Leite and Dimitrakopoulos	2007	Conditional simulation and Simulated annealing	Integrates ore body uncertainty, waste management and economic and mining consideration to generate optimal mining rates. Produces a single optimum production planning in presence of uncertainty. Minimizes deviation from target production that is caused by grade uncertainty.	Cannot be implemented on large deposit. The optimality of this method cannot be guaranteed. Simulated annealing technique is very complex and needs lots of parameters. It is very slow and time consuming.
Stochastic mixed integer programming	Dimitrakopoulos and Ramazan	2008	Linear stochastic programming	Maximizes NPV. Uses grade uncertainty directly. Produces a single optimum production planning for a geological risk discount rate (GDR) value.	Cannot be implemented on large deposit. GDR parameter is not very clear. Short-term production schedule issue and blast data set are not taken into account.
Risk analysis	Koushavand, Askari-Nasab and Deutsch	2009	Conditional simulation technique	Shows the impact of grade uncertainty on LTPP.	Quantifies the risk of a project Does not guarantee optimal solution in presence of grade uncertainty.

1.4.3 Application of Clustering in Mine Planning

In the model presented by Askari-Nasab et al. (2010; 2011), there are $3 \times TN$ decision variables where T is the number of periods and N is the number of blocks. The number of binary variables is TN and the rest of the variables indicate portions of the blocks that should be extracted and processed in each period. For example, with a block model of 20,000 blocks and 20 years of mine life, there would be 1,200,000 variables where 400,000 are binary. Solving such a big model is not tractable with current commercial mixed integer programming solvers. Tabesh and Askari-Nasab (2011) tried to solve this problem by clustering the blocks in order to reduce the number of variables. Using grade aggregation methods, similar blocks are summarized to a group and are dealt with as one variable which will be extracted in the same period. Each group of blocks is called a mining cut. Figure 1-4 shows a schematic plan view of mining cuts. The mining cuts are distinguished with different colors. In this example, 226 blocks in a mining level are aggregated into 13 mining cuts. The goal was to create clusters with 17 blocks at each group.

Figure 1-4. Schematic plan view of aggregated blocks into 13 mining-cuts on a mining bench; each mining cut is identified by a number (Askari-Nasab and Awuah-Offei, 2009)

Clustering is a process to assign entities into groups called clusters. The objective is to maximize the similairy between entities inside the groups and maximize the dissimilarity between entites in different goups. Clustering is a nonlinear programming (NP) model which Gonzalez (1982) proved to be NP-hard. Instead of solving this NP-hard problem, there are some non-exact algorithms that have been developted by several authers. There are two main clustering algorithms: (1) hard clustering, where each entity blongs to a group or not and (2) soft clustering or fuzzy clustering, where each entity belongs to each group with a certain degree.

The different clustering algorithms can be organized as follows: (1) Hierarchical clustering, (2) partitional clustering and (3) overlapping clustering.

Only hierarchical and partitional clustering can be used in mine planning because all blocks must belong to a single cluster. Tabesh and Askari-Nasab (2011) reviewed different clustering algorithms and presented a new method for clustering. Hierarchical clustering methods generate better results but they are computationally expensive (Feng et al., 2010). A well-known partitional clustering method is the k-mean algorithm. It is an iterative algorithm that tries to find better partitions at each loop. Recently some authors tried the k-mean algorithm (Bagirov, 2008; Chang et al., 2009; Chung and Lin, 2006; Niknam and Amiri, 2010; Niknam et al., 2010; Zalik, 2008). Jain (2010) has reviewed the k-mean method in more detail.

The advantages of clustering are: (1) it decreases the number of variables in optimization stage and therefore increases the speed of the algorithm, (2) it reduces the gap factor and (3) because any real surface mine is not extracted in block scale resolution, applying clustering method to define mining cuts in optimization models is more accurate.

1.5 Contributions

There is a need for a well-established solution for long-term mine planning in presence of uncertainty. Heuristic methods and artificial intelligence techniques are not exact methods in which the global optimum is not guaranteed. There are subjective decisions involved with these methods that may lead to different solutions. Obtaining an exact solution based on MILP methods is computationally very expensive. Even with current super computers/computer clusters, it is intractable to solve a real size mine planning problem with more than 10,000 blocks and 10 periods. The suggested methods for reducing number of variables leads to suboptimal solutions.

The main contribution of this research is the integration of geostatistical simulation methods with mixed integer programming in the context of mine planning. The generated schedule is near optimum. Some detailed contributions are:

- Several methods are presented with different objective functions: (1) the expected value of NPV is maximized and the expected deviation from target production is minimized considering grade realizations with and without a stockpile, (2) the expected value of NPV is minimized and the variance of the block is penalized to provide a smooth ore production with less fluctuations in input ore tonnage to the mill in early years of production and (3) the mean-variance method is adapted to the long-term mine planning such that the expected NPV is maximized while the variance of the NPV is minimized. The objective functions of the first two methods are a mixed integer linear programming and the last method is a mixed integer quadratic programming.
- A new term called 'cost of grade uncertainty' is introduced and investigated. It is used as a guideline to determine the optimum trade-off factor in the optimization problem.
- The effect of grade uncertainty, the cost of grade uncertainty and how to generate a near optimum schedule in presence of grade uncertainty are also investigated.
- The relationship between mining and processing capacities and the cost of grade uncertainty is shown.

The proposed methods and the developed software can be applied for real size industrial applications.

1.6 Thesis Outline

The main theoretical developments of the thesis are presented in Chapter 2. First, the MILP model without grade uncertainty is presented. The effect of grade uncertainty in mine planning is investigated. A formulation is developed to estimate the possible cost of unexpected shortfalls from the target production that is caused by grade uncertainty. Two new linear programming models are presented to use the grade uncertainty explicitly in scheduling. The complete definition of the methods and the variables are presented in this chapter. In the first model the cost of grade uncertainty is used in the optimization stage. The NPV is maximized while the cost of grade uncertainty is minimized. In the second model a stockpile is mathematically considered in the objective function. A new linear programming model is presented in presence of grade uncertainty and a stockpile. Also, in this chapter a nonlinear programming model is presented. The mean-variance method is applied in long-term mine planning to minimize the effect of grade uncertainty on the NPV of the project. The expected value of NPV is maximized while the variance of NPV is minimized.

Chapter 3 explains some implementation details, the steps and details that have to be taken to use block models in mine planning. FORTRAN programs and MATLAB codes are presented to implement developed models. The clustering methods also are presented in this chapter.

Chapter 4 verifies the models and provides a case study. First, a schedule is generated using Whittle software. An ordinary kriging block model is used for this purpose. The traditional MILP method and all proposed optimization models are applied. For each model verification and uncertainty assessment procedure are implemented. This procedure includes quantifying the cost of grade uncertainty, calculating deviation from target production, evaluating the variance of NPV and creating proper graphs. Finally, using the aforementioned steps performances of different techniques are compared in this chapter. It is shown that grade uncertainty can cause shortfalls and sub-optimal production plans. By taking grade uncertainty into account and using a stockpile the negative effects of grade uncertainty on the production plan can be reduced significantly.

Chapter 5 presents a comprehensive sensitivity analysis on the input parameters of the models such as the number of mining cuts, the mining and processing capacity and the cost of grade uncertainty.

Chapter 6 presents conclusions and final remarks. Also limitations of the proposed methods are discussed in this chapter.

Chapter 2 Theoretical Framework

The MILP framework without grade uncertainty is initially explained and formulated in Section 2.2. Then, the effect of grade uncertainty is considered in Section 2.3. The concept of the cost of grade uncertainty is proposed in Section 2.4 and equations are presented to estimate the cost of grade uncertainty in a mine production schedule with and without considering a stockpile. A new optimization model for generating the long-term production schedule in presence of grade uncertainty is presented in Section 2.5. This model simultaneously considers NPV maximization and minimization of the cost of grade uncertainty. Two new variables are introduced to control the overproduction and underproduction in each period for each realization. In Section 2.6 the model is expanded to implement a stockpile. The idea is that any overproduction that occurs in some realizations in any period is transferred to the next periods. Section 2.7 presents a quadratic optimization based on a portfolio optimization method using mean-variance approach from economics. The idea is to maximize the expected return and minimize its variance. The method is adapted for multi-period long-term mine production scheduling in presence of grade uncertainty. The effect of grade uncertainty is minimized not only in the target production but also in the input head grade to the mill. The Hessian matrix is positive definite and therefore it is a convex mixed integer quadratic optimization problem and the unique optimum solution exists. In Section 2.8, a block aggregation method is presented to reduce the size of the optimization problem. Section 2.9 presents a discussion on the adaptive approach to the long-term mine planning problem and the combination of the anticipative and adaptive models. Finally, the chapter is summarized in Section 2.10.

2.1 Introduction

The first step in any mining project is to gather information from the orebody. Diamond core drilling is mostly used (Hustrulid and Kuchta, 1995) for this purpose. The core provides a continuous string of geologic information. The drillholes are subdivided into rock-types. Inside each rock-type there are a number of continues grade variables. A 3D characterization of the orebody in the form of a block model is constructed from the available drillholes. Geostatistical methods are widely used for block model generation. There are many methods to calculate the block estimates (Deutsch and Journel, 1998;

Deutsch et al., 2002; Goovaerts, 1997; Journel and Huijbregts, 1981). Among these techniques the most popular methods are Kriging and Simulation. The generated block model is the main input to long-term production planning (LTPP). The main goal of LTPP is to maximize the NPV of the project. There are constraints such as the annual mining limit and processing capacity. The optimal production schedule specifies the sequence of extraction of ore and waste material from the mine such that all constraints are satisfied and the NPV is maximized. In order to solve this optimization problem, the objective function and all constraints are required to be formulated in a mathematical form. A number of decision variables are defined for each block. There are T binary decision variables for each block where T is the mine life. These variables indicate whether or not the block is extracted in each period; 1 indicates that the block has been extracted and 0 indicates that the block has not been extracted yet. In these models the entire block should be extracted in one period. In some other models, a fraction of a block could be mined in each period. Nevertheless, binary variables are still required to track if a block has been extracted to control the precedence of block extractions. Therefore, the optimization problem is to find the order of extraction of the blocks such that the NPV is maximized and the constraints are satisfied. The mathematical form of such optimization problem can be written as Eq. (2.1):

$$Max \quad NPV \tag{2.1}$$

Such that the following constraints are satisfied:

- Grade blending: the average input grade to the mill needs to be within specified limits due to the processing limitations and the plant design. Upper and lower constraints are specified for each element that is processed in each period.
- Processing capacity: there is a specified annual capacity for the processing plant. The generated schedule should provide sufficient ore which should be between specified upper and lower processing capacities per period.
- Mining capacity: there is a maximum mining capacity based on the excavation method and equipment that are used in the mine.
- Mining precedence: the production schedule needs to be feasible. This means that if a block is planned to be extracted in period t, all the blocks above should have already been extracted or they are going to be extracted in the same period. This rule is enforced in the optimization model with mining precedence constraints. A

set of constraints is constructed to ensure that a block is not extracted until all the blocks located directly above are extracted.

In the next section, a mixed integer linear programming model is developed using the aforementioned framework for the case where no uncertainty is involved in the LTPP problem. This model is later used for uncertainty based methods.

2.2 MIPL Formulation for Deterministic Approach (Model #1)

The MILP model developed by Askari-Nasab et al. (2010; 2011) is described in detail. This model is generalized from an earlier model presented by Caccetta and Hill (2003).

For each block, a parameter called the Economic Block Value (EBV) is calculated. The EBV depends on the value of the block and the costs incurred during mining and processing stages. The mining cost of a block is a function of the distance between its location and its final destination. The EBV of a block is the revenue generated by selling the final product less all the costs involved in extracting and processing the block. Because the long-term production plan is a multi-period optimization problem and blocks are extracted in different periods, a discount rate is applied to calculate the present value of the EBV, revenue and the costs. Therefore, the Discounted Economic Block Value (DEBV) is calculated using Eq. (2.2):

$$\text{DEBV} = \text{discounted revenue} - \text{discounted costs} \tag{2.2}$$

The discounted revenue is the present value of the ore minus the cost of processing. The value of the ore is the amount of money generated by selling the final product of the plant. Therefore for a block *i*, if there is only one valuable commodity, the ore value can be calculated by Eq. (2.3)

$$\text{Ore Value} = (\text{Tonnage of block}) \times \text{Grade} \times \text{Price} \tag{2.3}$$

The discounted revenue of block *i* in period *t* is denoted by $v(i;t)$ and calculated as Eq. (2.4).

$$\begin{aligned} v(i;t) &= \frac{\text{Ore Value of block } i}{(1+IR)^t} - \frac{\text{Processing Cost of block } i}{(1+IR)^t} \\ &\text{or} \\ v(i;t) &= T(i) \times R_m(i) \times \left[g(i) \times R_p(i) \times \frac{P}{(1+IR)^t} - \frac{C_p(i)}{(1+IR)^t} \right] \end{aligned} \tag{2.4}$$

And the discounted cost of block *i* in period *t*, $q(i;t)$, can be written as Eq. (2.5):

$$
\begin{aligned}
&q(i;t) = \frac{\text{Mining Cost of block i}}{(1+IR)^t} \\
&or \\
&q(i;t) = T(i) \times \frac{C_m(i)}{(1+IR)^t}
\end{aligned}
\tag{2.5}
$$

Where

- i is the identification number of the block.
- t is the period number.
- $T(i)$ is the tonnage of block i.
- $C_p(i)$ and $C_m(i)$ are the cost of processing and mining of block i, respectively.
- $R_p(i)$ and $R_m(i)$ are the recovery of processing and mining of block i, respectively.
- $g(i)$ is the estimated grade value of block i.
- IR is the interest rate of the project or the discounting rate.
- P is the selling price per tonne of the final product.

There are some considerations:

1. Eq. (2.4) gives the discounted revenue of a single product. In general, there are several valuable products and the overall revenue is the summation of all discounted revenues from each element. Also, because in the processing plant there is a certain cost to remove the contaminants, the processing cost should include this cost. A multivariate geostatistical modeling technique is recommended to model both valuable properties as well as contaminants. $v(i;t)$ is the total discounted revenue of block i by considering all the valuable elements, the processing cost and the removal cost of the contaminants from final product.
2. The tonnage of a block that is calculated by Eq. (2.6) is a function of density of the blocks, $\rho(i)$, and the volume of the block, V. Usually the density of a block

depends on the rock-type and can be modeled as a continuous variable using geostatistical techniques.

$$T(i) = V \times \rho(i) \tag{2.6}$$

3. Based on the processing method, the required degree of liberation may be different for low and high grades. Also, the amount of required chemical reagents may vary. Therefore, the processing cost depends on the grades of the block. Moreover, the processing recovery factor is not constant value for all input grades. Usually these factors are modeled as a function of the rock-type and the grades of the blocks. Therefore, the grade uncertainty would influence on the recovery factors and processing cost, and it can be qualified by the realizations.

4. Most of the time a cut-off grade (g_{cut}) is used to determine whether a block is ore or waste. If the estimated grade of a block is less than cut-off grade, the block is considered as waste. The ore value of such a block is zero.

Therefore, the EBV of block *i* can be shown as Eq. (2.7) by assuming the cut-off grade:

$$EBV(i) = \begin{cases} T(i) \times R_m(i) \times \left[g(i) \times R_p(i) \times P - C_p(i) \right] - T(i) \times C_m(i) & if\ g(i) \geq g_{cut} \\ -T(i) \times C_m(i) & if\ g(i) < g_{cut} \end{cases} \tag{2.7}$$

The DEBV is given by Eq. (2.8).

$$DEBV(i;t) \begin{cases} v(i;t) - q(i;t) & if\ g(i) \geq g_{cut} \\ -q(i;t) & if\ g(i) \geq g_{cut} \end{cases} \tag{2.8}$$

The NPV of the project is discounted revenue minus discounted costs of all blocks summed over all periods. The objective function for LTPP is to maximize NPV. There are different methods to define a linear optimization problem that maximizes the NPV. As described before, one method is to assign an integer (binary) variable $w(t;i)$ for block *i* in period *t* to determine whether the block is extracted or not. The general form of this model is presented in Eq. (2.9):

$$\text{Max} \quad NPV = \sum_{t=1}^{T} \sum_{i=1}^{N} DEBV \times w(t;i) \tag{2.9}$$

- *T* is the number of time periods or the mine life.

- N is the total number of blocks.

There are $T \times N$ binary decision variables. Since all the decision variables are binary, this optimization problem is called integer linear programming (ILP).

In the second model, decision variables $y(t;i)$ are defined as the extraction portion of block i in period t. However, binary variables $a(t;i)$ are still required to satisfy the precedence of the block extraction. These binary variables appear in the constraints of the optimization problem. The second objective function is presented in Eq. (2.10):

$$Max\ NPV = Max \sum_{i=1}^{N} \sum_{t=1}^{T} EBV(t;i) \times y(t;i) \tag{2.10}$$

This is a linear model with continues decision variables, $y(t;i)$, which can take any number between 0 to 1. Binary variables only control the precedence of the block extraction. This model is a mixed integer linear programming (MILP) problem.

The cost of mining and processing always are a function of grade of the block. In the earlier versions of long term optimization models the cut-off grade has been considered as a constant number and it has been decided before the optimization problem. Therefore, the destination of the blocks are decided by a static cut-off grade, and in the optimization process only the period of the extraction has been determined (and the portion of the extraction). Such a model has been shown in Eq. (2.9) where the decision variables are binary and Eq. (2.10) with continues decision variables that indicate the proportion of the block extraction. There is another approach which is called dynamic cut-off grade. In this approach, there is another decision variable that determines the portion of the block that should be processed. By using two different decision variables for extraction and processing of each block, the optimizer decides whether the block is processed or it should be sent to the waste dump. Therefore, the cut-off grade is implemented in the optimization process. In other words, because two separate variables are defined for extraction and processing, it is possible to generate a schedule that may send low quality ore blocks located on upper benches to waste dump (or, more likely to a low grade stockpile), in order to gain access to high quality ore blocks in the lower levels. This produces more cash flow in early periods of the project and increases the total profit of the project. So, in this model, the cut-off grade is dynamic through the mine life. This concept is modeled in the third model.

In the third model, there are two decision variables for each block i in each period t: (1) $y(t;i)$ is the portion of block i to be extracted in period t and (2) $z(t;i)$ is the portion of block i to be processed (if it is ore) in period t. So, in this model, the cut-off grade is dynamic through the mine life. This model is called Model #1 from this point on.

The mathematical form of optimal mining schedule with separate decision variables for mining and processing is presented in Eq. (2.11):

$$\text{Max} \quad NPV = \sum_{t=1}^{T}\sum_{i=1}^{N}\left[v(t;i)\times z(t;i) - q(t;i)\times y(t;i)\right] \tag{2.11}$$

As mentioned before, any feasible production plan should satisfy technical and environmental constraints. These constraints are:

- Grade blending constraints: These inequalities ensure that the head grade of estimated block model is within the desired range in each period.

$$\begin{cases} \sum_{i=1}^{N}\left[T_o(i)\times\left(g(i)-g_u(t)\right)\times z(t;i)\right]\le 0 \\ \sum_{i=1}^{N}\left[T_o(i)\times\left(g_l(t)-g(i)\right)\times z(t;i)\right]\le 0 \end{cases} \qquad \forall\, t=1,2,\ldots,T \tag{2.12}$$

  - $g_l(t)$ and $g_u(t)$ are the allowable lower and upper limits of the head grade in period t. A feasible solution is not guaranteed for any range of input head grade. So, these two parameters should be chosen such that the optimization problem is feasible. Therefore these two constrains should be within the range of values in the block model. There are separate constraints for each element of interest and any contaminants in each period. Therefore, there are two equations (upper bound and lower bound) per element per period. For consistency, the lower limit constraint has been multiplied by a negative sign. Therefore, both constraints are enforced to be less than zero.

- Processing capacity constraint: these inequalities ensure that the total ore processed in each period is within the acceptable range of the processing plant capacity. There are two equations, (upper and lower limits) per period.

$$p_l(t) \le \sum_{i=1}^{N} \left[T_o(i) \times z(t;i) \right] \le p_u(t) \qquad \forall t = 1,2,\ldots,T \tag{2.13}$$

- $p_l(t)$ and $p_u(t)$ are the lower limit and upper limit (target production) for the designed processing plant in period *t*, respectively.

- Mining Capacity constraint: these inequalities ensure that the total tonnage of material mined (ore, waste, overburden, and undefined waste) in each period is within the acceptable range of mining equipment capacity in that period. There are two equations (upper bound and lower bound) per period.

$$m_l(t) \le \sum_{i=1}^{N} \left[T_{total}(i) \times y(t;i) \right] \le m_u(t) \qquad \forall t = 1,2,\ldots,T \tag{2.14}$$

- $m_l(t)$ and $m_u(t)$ are lower and upper limits for the mining capacity in period *t*.

- These inequalities ensure that the amount of ore of any block processed in any given period is less than or equal to the amount of rock extracted in the considered period.

$$z(t;i) \le y(t;i) \qquad \forall t = 1,2,\ldots,T \;,\; i = 1,2,\ldots,N \tag{2.15}$$

- The constraints that specify the block extraction precedence: These equations control the relationship of block extraction precedence.

$$a(t;i) - \sum_{u=1}^{t} y(u;j) \le 0 \qquad \forall t = 1,2,\ldots,T \;,\; i = 1,2,\ldots,N,\; j = 1,2,\ldots,C(i) \tag{2.16}$$

$$\sum_{u=1}^{t} y(u;i) - a(t;i) \le 0 \qquad \forall t = 1,2,\ldots,T \;,\; i = 1,2,\ldots,N \tag{2.17}$$

$$a(t;i) - a(t+1;i) \le 0 \qquad \forall t = 1,2,\ldots,T-1 \;,\; i = 1,2,\ldots,N \tag{2.18}$$

- $a(t,i)$ is a binary integer variable. It equals one if the extraction of block *i* has started before or in period *t* (otherwise it is zero).
- $C(i)$ is the set of blocks that are needed to be extracted before extraction of block *i*.

- j is the index for set of the blocks, $C(i)$, that need to be extracted prior to the extraction of block i.
- This model only needs the set of immediate predecessor blocks on top of each block to model the order of block extraction. This is presented by set C(M) in Eq. (2.16). Figure 2-1 shows the 1-9 and 1-5 immediate blocks above each block that should be considered for precedence constraints.

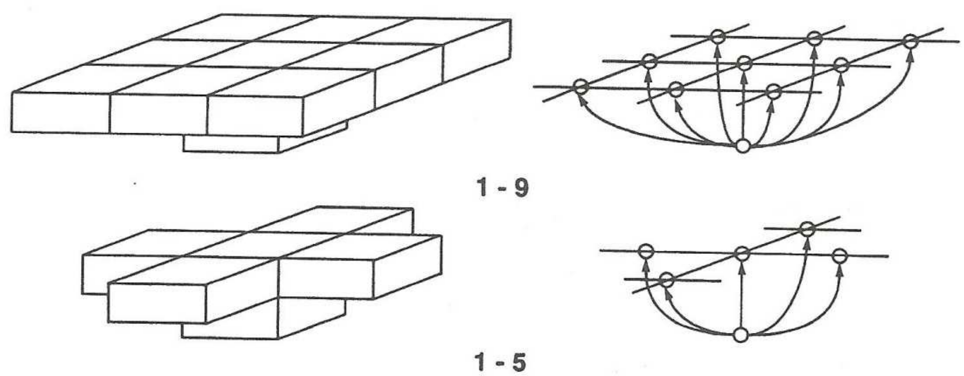


Figure 2-1. Representation of the 1-5 and 1-9 block constraints(Hustrulid and Kuchta, 1995).

- Reserve constraints: these constraints enforce that all portions of the blocks inside the ultimate pit limit to be extracted until the end of the mine life.

$$\sum_{t=1}^{T} y(t;i) = 1 \qquad \forall\ i = 1,2,\ldots,N \qquad (2.19)$$

All the ore blocks in the model have to be mined. This is based on the assumption that a fixed final pit limit is used. In this case, the mining capacity should be large enough to extract all the material inside the pit during the mine life. Otherwise the optimization will be infeasible when the Eq. (2.20) is applied. In the case that the optimization is not forced to extract all the blocks inside the ultimate pit limit, Eq. (2.19) should be replaced by Eq. (2.20). This constraint forces the sum of the extraction portions of a block to be less than or equal to one.

$$\sum_{i=1}^{t} y(t;i) \leq 1 \qquad \forall\ i = 1,2,\ldots,N \qquad (2.20)$$

The optimization model presented in Eqs. (2.11) to (2.20) is a mixed integer linear programming (MILP) problem with linear constraints. There are two types of variables: (1) continuous values $y(t;i)$ and $z(t;i)$ and (2) the binary variables $a(t;i)$ that indicate whether a block has been extracted or not in each period; 0 indicates that the bock has not been extracted until period *t* and 1 otherwise.

Wolsey (1998) categorizes the MILP as an NP-hard problem. Branch-and-cut methods that are widely used to solve MILP problems are exact algorithms consisting of a combination of cutting plane algorithm with a branch-and-bound algorithm (Mitchell, 1999). These methods solve the MILP problem by solving a sequence of linear programming relaxations of the MILP problem. Cutting plane methods improve the relaxations of the problem by close approximations of the MILP. The mixed integer programming problems are usually not efficiently solved by the cutting plane approach. A branching technique is used that results in the branch-and-cut approach.

2.3 Effect of Grade Uncertainty in Mine Planning Optimization

In this section, the effect of the grade uncertainty on the long-term mine production plan is studied. Grade uncertainty is modeled by generating multiple realizations using geostatistical simulation techniques. Usually, the average grade of a block is used to determine whether a block should be processed or not. The average grade of a block is calculated as the arithmetic mean of all simulated values. In this section, the effect of grade uncertainty and cut-off grade on the mine planning is investigated.

The cut-off grade is a critical threshold. Any block with a grade above this limit is considered as ore and has an economical value. Any material below the cut-off grade is considered to be waste with no economic value; however, there is a mining cost associated with that block. Lane (1988) presented the fundamentals of cut-off grade calculation. There are two theoretical cut-off grades: (1) marginal and (2) break-even. The marginal cut-off grade is the critical grade threshold where the value of the material is equal to the cost of mining. Therefore, marginal cut-off grade is the point that the EBV is equal the negative value of the cost of mining or $EBV(g_M) = -C_m$.

The break-even cut-off grade is the critical grade value that the net value of the block is zero. This means that any block above this limit has positive net value. This can be denoted by: $EBV(g_{BE}) = 0$. In other words, the break-even cut-off grade is the point that the revenue of a bock cancels out all the costs including cost of mining and processing.

On the other hand, operational cut-off grade (g_{OP}) is applied in most of the industrial projects. This value is mostly imposed by the limitations of the processing methods and it is higher than the break-even cut-off grade. Therefore, the EBV of a block above operational cut-off grade is always a positive value: $EBV\left(g_{OP}\right) \geq 0$.

Figure 2-2 (a) to (c) shows the cases where the marginal cut-off grade (g_M), break-even (g_{BE}) and operational (g_{OP}) cut-off grades are applied respectively to calculate the EBV for a single block. Figure 2-2 (a) shows the case that the marginal cut-off grade (g_M) is applied. Any block for which the average grade is lower than the marginal cut-off grade is not processed and it is sent to the waste dump. On the other hand, if the grade of a block is higher than g_M and lower than g_{BE} , still it generates negative EBV. However, processing these blocks reduces the negative impact of the mining cost. Therefore, it is economical to process these blocks. Figure 2-2 (b) shows the case that break-even cut-off grade is applied. All the blocks above g_{BE} have positive EBV values. Figure 2-2 (c) shows the case that operational cut-off grade is applied. Usually the operational cut-off grade is higher or equal to the break-even cut-off grade.

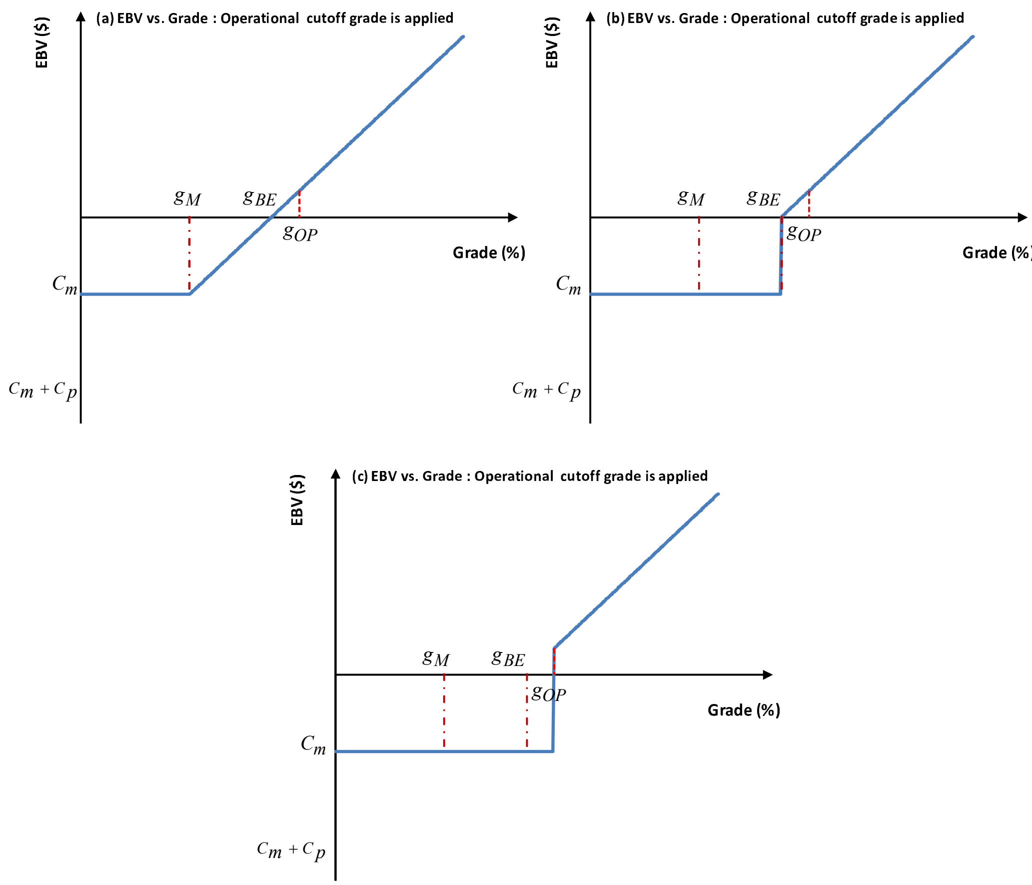


Figure 2-2. EBV of unit tonnage vs. input grade for an ore block with different cut-off grade (a) $g_{cut} = g_M$,(b) $g_{cut} = g_{BE}$ and (c) $g_{cut} = g_{OP}$

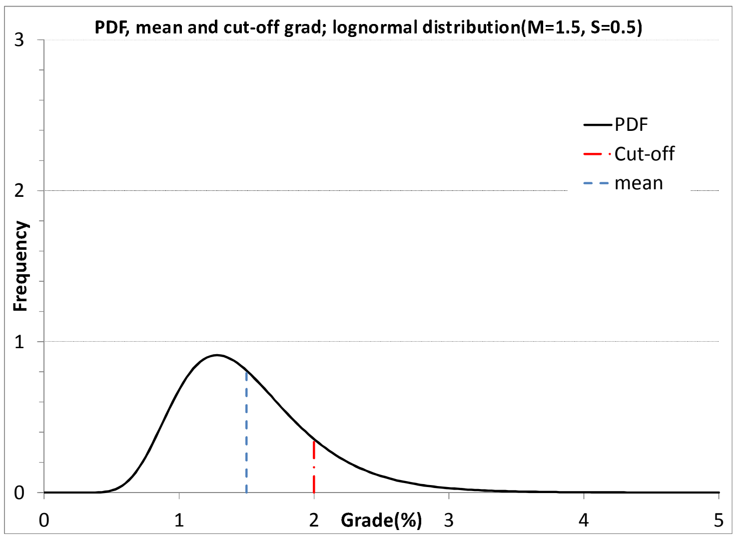
The grade uncertainty can affect the production plan in two different ways: (1) the tonnage of ore sent to the mill could vary and (2) the average EBV value over all realizations could be different than that calculated from the average grade. In this section each of these factors is studied.

2.3.1 The Effect of Grade Uncertainty on the Input Tonnage

There is uncertainty in the grade of each block. The local uncertainty in each block is described by L realizations. To show the effect of grade uncertainty on the tonnage of ore, four synthetic blocks are simulated based on a lognormal distribution. The following conclusions are drawn:

1. The first case (Figure 2-3): The mean of the block is 1.5% and the standard deviation is 0.5%. The distribution of this block is also shown in Figure 2-3. 85.3% of the realizations are below the cut-off grade. The average grade of the

block is also less than cut-off grade and in traditional LTPP methods is considered as a waste block. However, with the probability of 14.7%, the grade of this block can be above the cut-off grade and it should be processed. Therefore, with the probability of 14.7 % this block can produce an unexpected ore tonnage which is not planned in the traditional long-term production schedule. In this thesis, this type of ore is referred to as overproduced ore. The EBV of this block can be calculated by Eq. (2.7) for each realization. Therefore, the expected value of EBV and the variance of EBV can be calculated for this block.



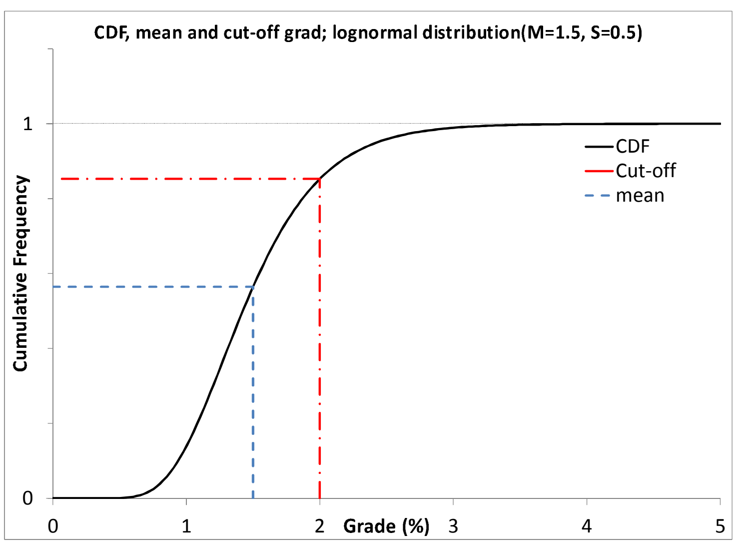


Figure 2-3. PDF (top) and CDF (bottom) for case 3, not all n realizations and mean (dashed blue line) are less than cut-off grade (dashed red line).

2. The second case (Figure 2-4): The mean and the standard deviation for this block respectively are 2.5% and 0.5%. The average grade of the block is above the cut-off grade and the grade of the 85.3% the realizations is above the cut-off. Therefore in deterministic mine production plans this block is assumed an ore block and it is scheduled to be processed during mine life. However, there is a probability of 14.7% that the grade of this block is below the cut-off grade and it should not be processed. Therefore, it may cause the shortfall from target production. Same as the previous case, the expected value of EBV and the variance of EBV is calculated for this block using the Eq. (2.7).

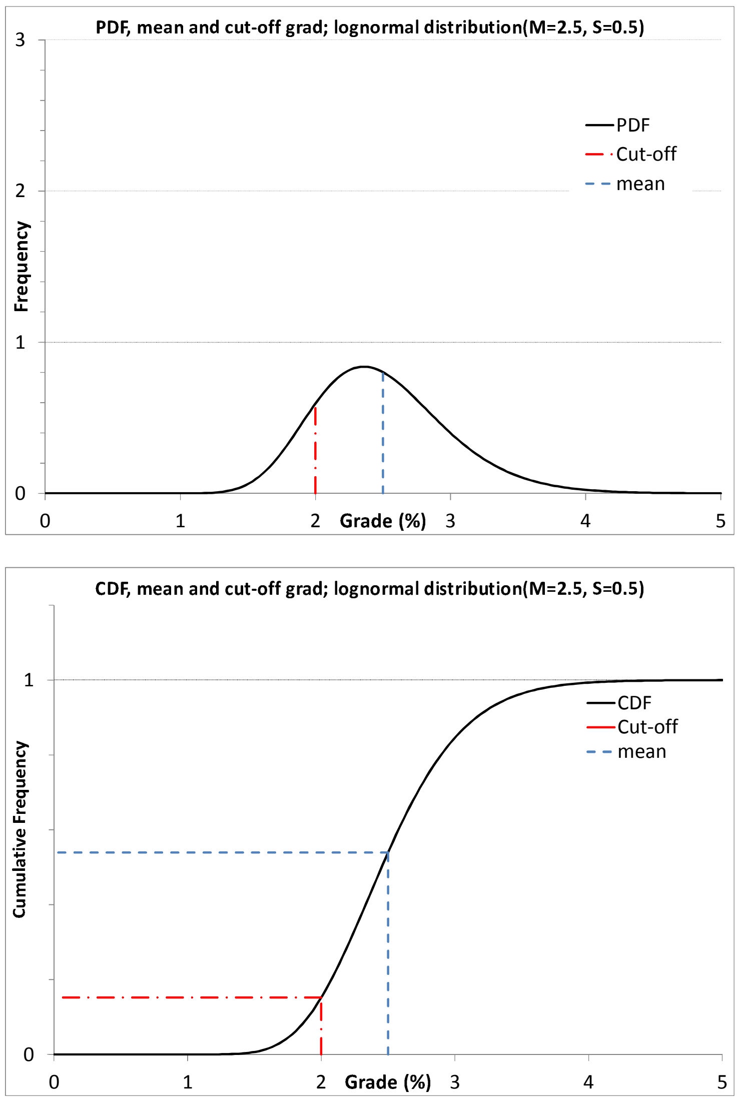
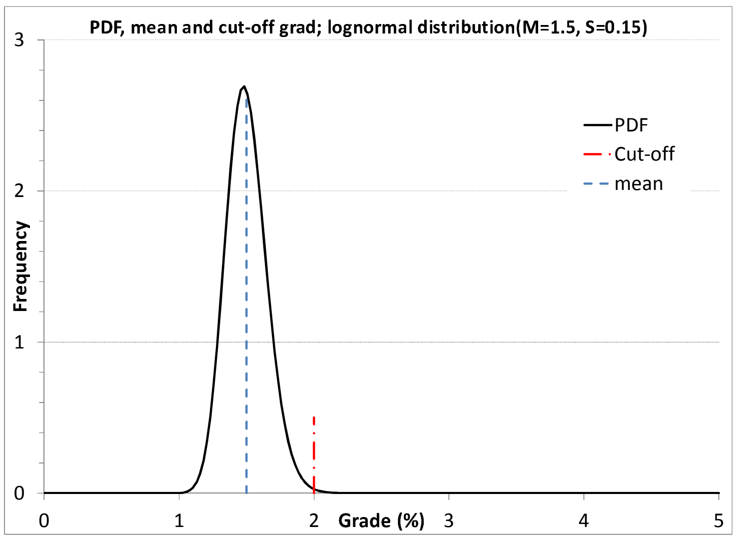


Figure 2-4. PDF (top) and CDF (bottom) for case 3, not all n realizations and mean (dashed blue line) are higher than cut-off grade (dashed red line).

In real life, the number of realizations is much lower than what we used here in this synthetic case. Therefore, two more cases may happen when the number of realizations is limited. For these two cases 100 realizations are generated:

3. Third case (Figure 2-5): The average grade of the block is 1.5% and the standard deviation is 0.15%. All of the realizations are below the cut-off grade. Therefore, in all realizations this block is a waste block and there is no grade uncertainty associated with this block when the number of realization is low. The EBV of the block in all realizations equals to the negative cost of the mining. So, the variance of the EBV is zero.



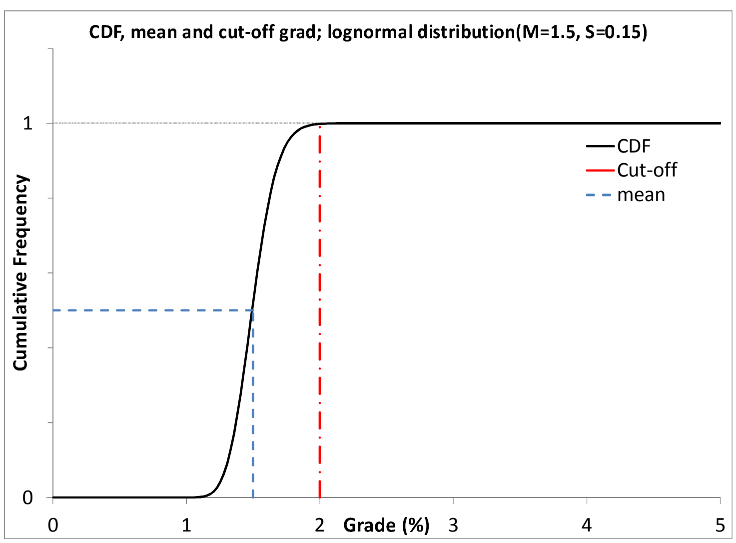
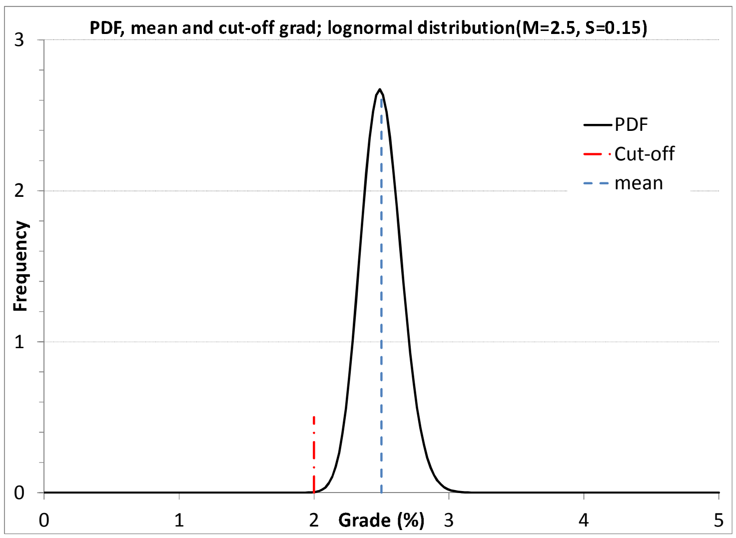


Figure 2-5. PDF (top) and CDF (bottom) for case 1, all n realizations and mean (dashed blue line) are less than cut-off grade (dashed red line).

4. The Fourth case (Figure 2-6): The average grade of this block is 2.5% and the standard deviation is 0.15%. All of the L realizations are above the cut-off grade. Therefore, this block will be processed in all realizations. There is no uncertainty for the tonnage of ore for this block. However, the EBV of the block which is related to the grade of the block has a different value for each realization. So, the expected value and the variance of EBV can be calculated for this block. The variance of EBV is not zero for this case.



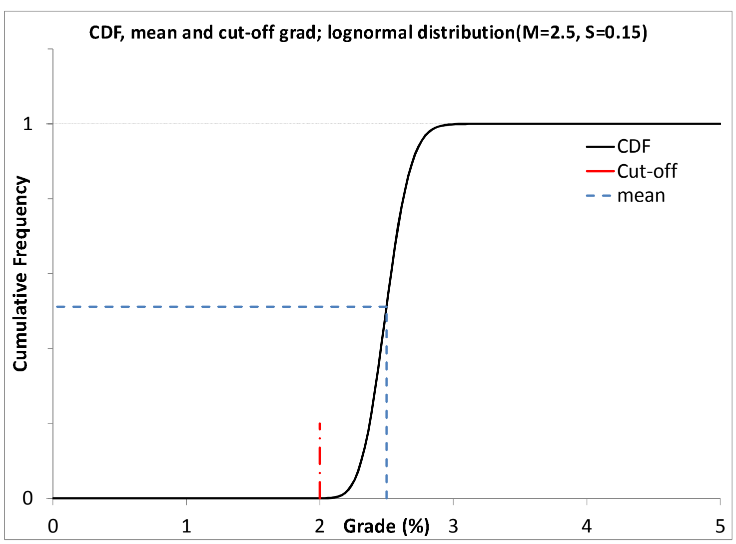


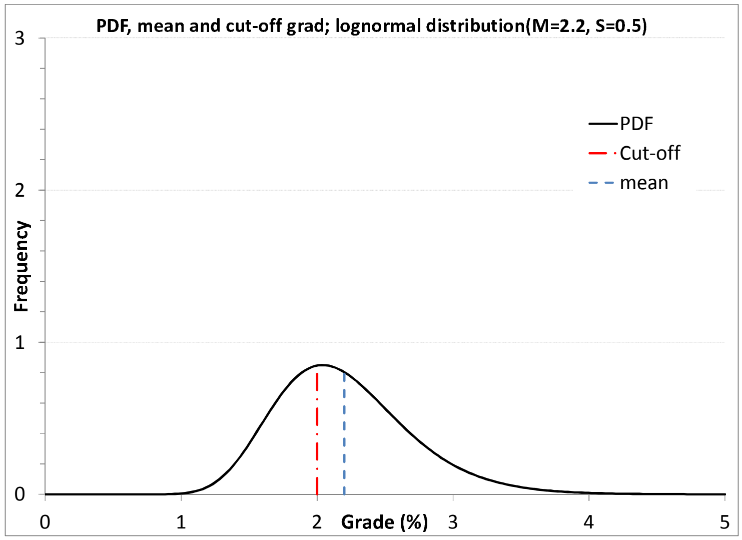
Figure 2-6. PDF (top) and CDF (bottom) for case 2, all L realizations and mean (dashed blue line) are higher than cut-off grade (dashed red line).

Grade uncertainty influences the ore tonnage uncertainty and the input ore sent to the mill at different periods.

2.3.2 The Effect of Grade Uncertainty on the Average EBV of Blocks

The EBV of each block is highly dependent to the grade of the blocks in several ways: (1) simulated grade is used directly at EBV calculation, (2) the ore tonnage of the block which is controlled by simulated grade and the cut-off grade, (3) the processing cost and (4) recovery factor. In this section, considering a synthetic case, the effect of grade uncertainty on the average EBV value over all realizations is studied. A block with lognormal distribution of grade with a mean and standard deviation of 2.2% and 0.5% respectively is simulated 10,000 times (Figure 2-7). The cut-off grade is assumed to be 2%. Therefore, this block is considered as an ore block because the average grade is above the cut-off grade. Eq. (2.7) is used to calculate EBV of this block per tonne for all 10,000 realizations where $R_p = 100\%$, $\text{Price} = 1\$$, $C_p = 0.5\$ / tonne$ and $C_m = 1.5\$ / tonne$. The histogram of EBV is shown in Figure 2-8. In 3744 cases over 10,000 generated values (37.44%), the block is decided to be waste because the simulated grade is less than the cut-off grade. In 62.56% of the cases, the simulated grade is above the cut off-grade and the block is decided to be ore (gray columns in the Figure 2-8). The average EBV is -\$0.26 which is less than zero. This means that even for a block with an average grade above the cut-off grade, the average EBV from simulations may be less than zero, so it is not economic to be processed. This is due to the nonlinearity of the EBV calculation. Therefore, with the methods that maximize NPV based on one single block model (that can be the kriging or average of realizations), this block is assumed as ore and will be processed. This phenomenon has direct impact on the NPV of each realization.

The grade uncertainty has two main effects on the LTPP: (1) it may cause deviations from target production. This can be calculated from the simulated realizations by taking into account the grade variation and the input cut-off grade; and (2) the grade uncertainty directly has effect on the EBV calculation and consequently on the NPV of the project. In this thesis, the new term of “the cost of grade uncertainty” is proposed to quantify the first effect. In this thesis, two LTPP models are proposed to maximize the NPV and minimize the expected value of the cost of grade uncertainty. The second effect of grade uncertainty is also measured by the variance of NPV. In addition, a third LTPP model is proposed which tries to maximize the expected mean of NPV and minimize the variance of NPV by taking all the realization into account directly at optimization procedure.



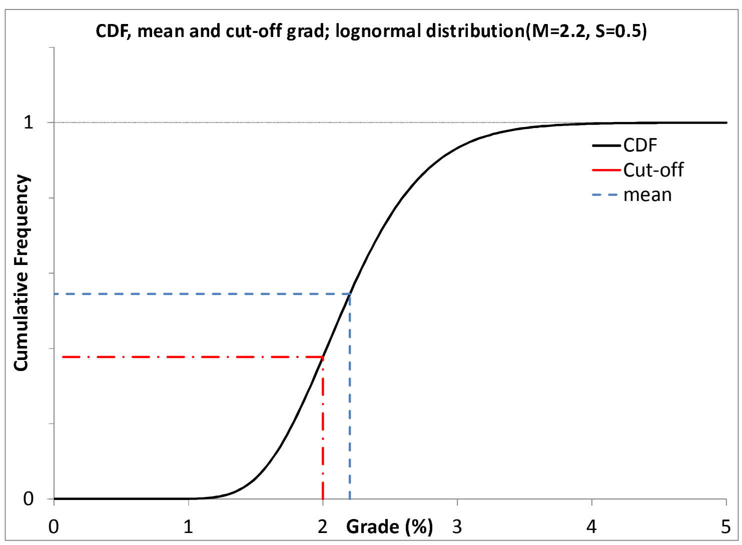


Figure 2-7. PDF (top) and CDF (bottom) for a synthetic case to calculate expected value of EBV per tonne of ore.

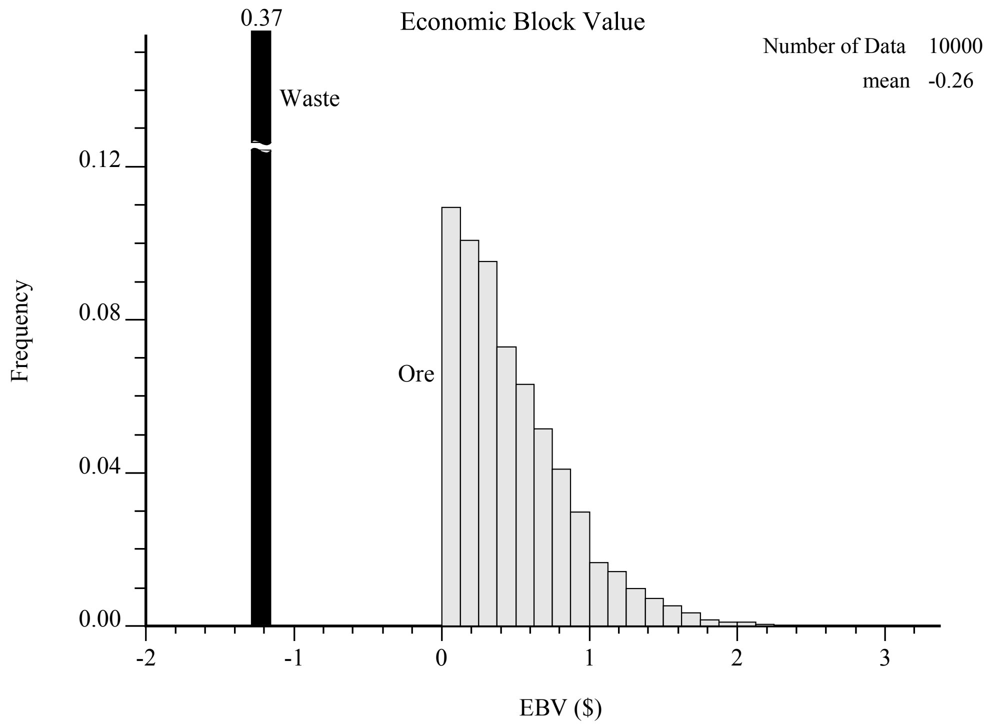


Figure 2-8. Histogram of EBV for a block with lognormal distribution, mean=2.2, Std. dev=0.5 and cut-off grade=2.

2.4 Cost of Grade Uncertainty

In this section, a new term called "cost of grade uncertainty" is introduced. Cost of grade uncertainty is a quantitative parameter that estimates the cost in long-term production plans due to of grade uncertainty. In traditional mine planning methods, in order to generate a long-term production schedule, it is required to predict the grade of blocks and use these predictions to maximize the NPV of the project. This prediction may lead to miss-classification of ore and waste blocks. Consequently, during the extraction, it may cause an unexpected ore production or a shortfall from target production. Generally, the cost of grade uncertainty is caused by two main reasons:

1. Cost of underproduction: where the mine has to react quickly to make up for an unexpected shortfall.

2. Cost of overproduction: unexpected extra available ore in mine leads to sub-optimal use of resources and/or imposes a cost for stockpiling.

Both of these two costs are calculated using the realizations generated by geostatistical simulation algorithms. The realizations are used to assess the uncertainty and the average cost of it.

The cost of underproduction is the loss of revenue due to the shortfall from target production at the processing plant which causes the processing plant to operate sub-optimally. The idea is that revenue will be lost if the plant is not running at full capacity. Although in real cases, mines make short-term arrangements to prevent this type of revenue loss, these extra arrangements incur a significant cost. A mathematical equation to calculate the cost of underproduction is proposed as:

$$\begin{aligned}\text{Cost of under production} &= \text{Tonnage of shortfall} \times \\ &\quad \left(\text{Average revenue per tonne - Processing cost per tonne}\right)\end{aligned}$$

or

$$\text{Cost of under production} = \text{Tonnage of shortfall} \times \text{Cost of under production per tonne}$$

This equation is used to calculate the discounted cost of underproduction for realization l in period t:

$$\begin{aligned}\mathrm{C}_{up}\left(t;l\right) &= T_{up}(t;l) \times \left[\overline{g}\left(t\right) \times \frac{P}{(1+IR)^t} - \frac{C_p}{(1+IR)^t}\right] \\ &= T_{up}(t;l) \times c_{up}\left(t\right)\end{aligned} \tag{2.21}$$

Where

- $\mathrm{C}_{up}\left(t;l\right)$ is the cost of underproduction using realization l.
- $T_{up}(t;l)$ is the tonnage of underproduction using realization l.
- $\overline{g}\left(t\right)$ is the average input grade to the mill in period t.
- $c_{up}\left(t\right)$ is the discounted cost of underproduction per tonne in period t and it is calculated by Eq.(2.22) as below:

$$c_{up}\left(t\right) = \overline{g}\left(t\right) \times \frac{P}{(1+IR)^t} - \frac{C_p}{(1+IR)^t} \tag{2.22}$$

The same concept is applied for the cost of overproduction. It is assumed that there is no stockpile available to store extra unplanned ore produced due to grade uncertainty. Also, in a hypothetical case, the processing plant is not able to handle extra ore and it is decided to send the extra ore to the waste dump. This assumption is revised in the Section (2.6) where a stockpile is considered in the calculation of cost of overproduction. For the case with no stockpile, the hypothetical cost of overproduction is presented in Eq. (2.23)

$$
\begin{aligned}
\mathrm{C}_{op}\left(t;l\right) &= T_{op}(t;l) \times \left[\overline{g}\left(t\right) \times \frac{P}{(1+IR)^t} - \frac{C_p}{(1+IR)^t} \right] \\
&= T_{op}(t;l) \times c_{op}\left(t\right)
\end{aligned}
\qquad (2.23)
$$

Where

- $T_{op}(t;l)$ is the tonnage of overproduced ore in period *t* and realization *l*.
- $c_{op}\left(t\right)$ is the cost of overproduction per tonne which is calculated by Eq. (2.24)

$$
c_{op}\left(t\right) = \overline{g}\left(t\right) \times \frac{P}{(1+IR)^t} - \frac{C_p}{(1+IR)^t} \qquad (2.24)
$$

In this case, the cost of overproduction and underproduction per tonne are equal: $c_{up}\left(t\right) = c_{op}\left(t\right)$. Figure 2-9a shows the discounted cost of grade uncertainty at different periods which is a symmetric penalty function for this specific hypothetical case.

The more accurate and realistic case is that the cost of overproduction is less than the cost of underproduction because over production will not be wasted. The mining plan will be modified or the extra ore will be stockpiled. The new assumption here is that any extra ore is sent to the stockpile and it is processed in the next periods. There are different components involved in the cost of overproduction in presence of a stockpile. When a mine defers the processing of the extra ore to the next periods, this processed ore will have less value due to discounting. The discounting factor also applies to the processing costs. A cost of stockpiling should be considered as well. As a result, the cost of overproduction is summarized as:

$$
\text{Discounted cost of over production} = \text{Extra ore tonnage} \times \begin{Bmatrix} \text{Loss of value of ore due to processing in next period} \\ +\text{Cost of stockpiling and rehandling} \end{Bmatrix}
$$

or

$$
= \text{Extra ore tonnage} \times \begin{Bmatrix} \text{Revenue for a tonne of ore in period t - Revenue for a tonne of ore in period t+1} \\ +\text{Cost of stockpiling and rehandling} \end{Bmatrix} \qquad (2.25)
$$

Eq. (2.26) is proposed to calculate the discounted cost of overproduction in presence of a stockpile:

$$C_{op}\left(t;l\right)=T_{op}(t;l)\times\left\{\underbrace{\left(\frac{\overline{g}\left(t\right)\times P}{(1+IR)^{t}}-\frac{\overline{g}\left(t\right)\times P}{(1+IR)^{t+1}}\right)}_{\text{the lost of the value of ore}}+\underbrace{\left(\frac{C_p}{(1+IR)^{t+1}}-\frac{C_p}{(1+IR)^{t}}\right)}_{\text{the difference of processing costs}}+\underbrace{\frac{C_{RH}}{(1+IR)^{t}}}_{\substack{\text{rehandling}\\ \text{cost}}}\right\} \tag{2.26}$$

This equation can be simplified by the following assumptions:

$$\begin{aligned} c_{op}\left(t\right)&=\overline{g}\left(t\right)\times\frac{P}{(1+IR)^{t}}-\frac{C_p}{(1+IR)^{t}}\\ c_{op}\left(t+1\right)&=\overline{g}\left(t\right)\times\frac{P}{(1+IR)^{t+1}}-\frac{C_p}{(1+IR)^{t+1}}\\ c_{RH}\left(t\right)&=\frac{C_{RH}}{(1+IR)^{t}}\end{aligned} \tag{2.27}$$

Where

- $c_{op}\left(t\right)$ is the cost of overproduction per tonne in period t
- $c_{op}\left(t+1\right)$ is the cost of overproduction per tonne in period t+1
- $C_{RH}\left(t\right)$ is the re-handling cost of stockpile per tonne in period t.

Eq. (2.26) is simplified as Eq.(2.28):

$$\begin{aligned} C_{op}\left(t;l\right)&=T_{op}(t;l)\times\left\{c_{op}\left(t\right)-c_{op}\left(t+1\right)+c_{RH}\left(t\right)\right\}\\ C_{op}\left(t;l\right)&=T_{op}(t;l)\times\hat{c}_{op,RH}\left(t\right)\end{aligned} \tag{2.28}$$

Where

- $c_{RH}\left(t\right)$ is the discounted re-handling cost per tonne in period t.
- $\hat{c}_{op,RH}\left(t\right)$ is the adjusted cost per tonne of overproduction in presence of stockpile in period t.

The cost of overproduction presented in Eq. (2.26) is an approximation because the mine may adapt dynamically to divert mining capacity to other locations to deal with the extra ore. However, extra unexpected ore makes the LTPP to be sub-optimal and this can be quantified and implemented in the new LTPP models to reach the optimal solution in presence of the grade uncertainty. Any overproduced ore that has been transferred to the stockpile will be processed in the following periods whenever a shortfall happens. Therefore, the cost of overproduction is only related to losing value of ore due to processing in next periods plus re-handling and stockpiling costs. Figure 2-9b shows the penalty function for the second assumption, where the discounted cost of overproduction at different periods is less than underproduction. This figure shows that the slope of the linear penalty function for underproduction is more than overproduction in any period.

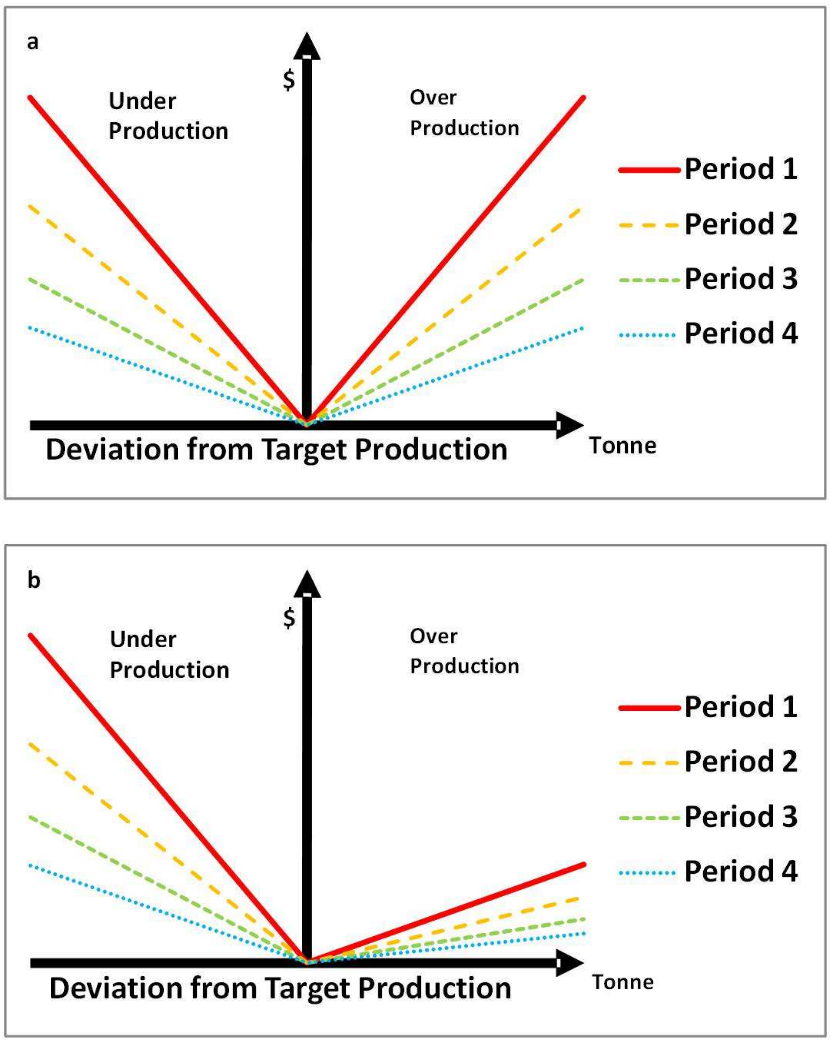


Figure 2-9. Penalty function for over and under production at different periods based on a discounting factor; a: for the case that there is no stockpile and b: there is a stockpile.

The cost of over and under production are calculated based on realizations. $C_{up}(t;l)$ and $C_{op}(t;l)$ are calculated for each realization in each period. The average discounted cost of grade uncertainty in period *t* over all *L* realizations is presented in Eq. (2.29) :

$$\begin{aligned} \bar{C}_u(t) &= \frac{1}{L}\sum_{l=1}^{L}\left[C_{up}(t;l)+C_{op}(t;l)\right] \\ \bar{C}_u(t) &= \frac{1}{L}\sum_{l=1}^{L}\left[T_{up}(t;l)\times c_{up}(t)+T_{op}(t;l)\times c_{op}(t)\right] \end{aligned} \tag{2.29}$$

The Discounted Cost of grade Uncertainty (DCOU) is calculated by Eq. (2.30) :

$$DCOU=\sum_{t=1}^{T-1}\bar{C}_u(t) \tag{2.30}$$

This gives a single value for the discounted cost of grade uncertainty over all periods and all realizations. It can be used to compare different schedules. It gives a quantitative measurement for the effect of the grade uncertainty on the long-term production plan.

The cost of grade uncertainty is calculated over all periods except the final period. Any ore that is left for the final period will be processed and will not exceed the target production; and any shortfall in the final period is not relevant to the grade uncertainty.

2.5 Grade Uncertainty Based LP Model Without Stockpile (Model #2)

A Mixed Integer Linear Programming model for optimizing the long-term production scheduling in open-pit mines is developed with an objective function that maximizes the total NPV of the project under a managed grade risk profile. For the starting point, the MILP model that is developed by Askari-Nasab and Awuah-Offei (2009) is used. This model is the generalized form of an earlier model presented by Caccetta and Hill (Caccetta and Hill, 2003) that is widely accepted.

Two main assumptions are made to model this optimization problem:

1. There is no stockpile considered in the optimization model. However a post-processing procedure is applied for each realization to remove possible extra ore to the stockpile to be processed at next periods.

2. Long-term scheduling is a dynamic process. This means that it changes during the mine life. There are many situations that may occur at the operational level

that change the extraction schedule, such as misclassification of ore and waste that is determined by taking extra information (it is called as information effect), failures of equipment, price changes and changes to processing and mining costs. In addition, during each period, the generated schedule is updated with new information such as blast-hole data and new exploration drill holes. Therefore, no long-term production schedule is exactly followed from the first year until the end of the mine life. However, the goal here is to find a long-term schedule using all useful information in a way that the probability of over and under production is minimized for early years of production. This will reduce the mine reaction to unexpected shortfalls and/or surpluses. The optimization should be run again with new information to find the new optimum schedule as the mine life proceeds. By using the new methods that are proposed here, negative effects of the grade uncertainty is reduced for the next production periods.

The main idea of the proposed method is to generate a schedule that postpones the extraction of uncertain blocks with lower grade to later years when there will be new information and less uncertainty. The discounted cost of grade uncertainty that is presented in previous section is added to the objective function. The schedule that is generated using the proposed method poses less risk in the early years of production. However, there is always a risk that the generated schedule may not meet the target production. These probabilities can be calculated using the realizations.

Two new variables are defined, $T_{op}(t;l)$ and $T_{up}(t;l)$, to represent the amount of overproduction and underproduction for realization l in period t. Each of these variables is multiplied by the discounted cost for over and under production, $c_{op}\left(t\right)$ and $c_{up}\left(t\right)$. These are the discounted penalty dollar values per tonne for probable over and under production. It is important to determine reasonable values for $c_{op}\left(t\right)$ and $c_{up}\left(t\right)$. Section 5.2 provides two methods to determine these parameters.

The main objective of this model is to maximize the NPV that is calculated by an estimated block model and to minimize the cost of grade uncertainty that is calculated by conditional realizations:

$$\begin{cases} \text{Max. NPV of estimation block model} \\ \text{Min. The cost of uncertainty} \end{cases} \tag{2.31}$$

These objectives can be combined as one optimization objective shown in Eq. (2.32):

$$\text{Max. } \{\text{NPV - The cost of uncertainty}\} \tag{2.32}$$

This can be formulated as Eq. (2.33)

$$\begin{aligned} & Max\{NPV - DCoU\} \\ & = Max \sum_{t=1}^{T} \{DCF(t) - \bar{C}_u(t)\} \end{aligned} \tag{2.33}$$

Where

- $DCF(t)$ is the discounted cash-flow for period t that is calculated by an estimation block model and it can be defined as Eq. (2.34).

$$DCF(t) = \sum_{\text{i}=1}^{\text{N}} \left[v(t;i) \times z(t;i) - q(t;i) \times y(t;i) \right] \tag{2.34}$$

The mining cost does not depend on the grade of the blocks and has the same value for all realizations. In this model, the NPV is calculated based on the estimated block model. The revenue calculated from the estimated block model is denoted by $v(t;i)$. The conditional realizations are used to penalize the cost of over and under production.

The mathematical form of Model #2 is presented in Eq.(2.35). This model generates the optimal production schedule in presence of grade uncertainty based on the concept that is presented in Eq.(2.31).

$$Max \sum_{t=1}^{T} \left[\begin{array}{l} \underbrace{\sum_{i=1}^{N} \left[v(t;i) \times z(t;l) - q(t;i) \times y(t;i) \right]}_{DCF_{es}(t)} \\ \underbrace{-\frac{1}{L} \sum_{l=1}^{L} \left[c_{up}(t) \times T_{up}(t;l) + c_{op}(t) \times T_{op}(t;l) \right]}_{\bar{C}_u(t)} \end{array} \right] \tag{2.35}$$

Subject to:

- Eq. (2.12) to Eq. (2.20). These constraints are applied to the estimated block model and are required for maximization of the NPV of the project based on the average block model.

- Tonnage of over and under production variables that are controlled with these two constraints per period and per realization:

$$\begin{cases} \sum_{i=1}^{N} \left\{ -T_o\left(i;l\right) \times z\left(t;i\right) - T_{up}(t;l) \right\} \le -p_u(t) \\ \sum_{i=1}^{N} \left\{ T_o\left(i;l\right) \times z\left(t;i\right) - T_{op}(t;l) \right\} \le p_u(t) \end{cases} \quad \forall\, t = 1,2,\ldots,T \;\;,\;\; l = 1,2,\ldots,L \quad (2.36)$$

  These constraints control two new variables: $T_{op}(t;l)$ and $T_{up}(t;l)$. $T_{op}(t;l)$ and $T_{up}(t;l)$ are decision variables and determined by optimization process. Both of these variables are present in the objective function (Eq. (2.35)). $T_{op}(t;l)$ is used to calculate $C_{op}(t;l)$ and $T_{up}(t;l)$ is included in $C_{up}(t;l)$. Because they have a negative impact on the objective function, the optimizer tries to assign the lowest positive values to these variables. However, constraints presented in Eq. (2.36) enforce these variables to get lower limit values. $T_{op}(t;l)$ is the tonnage of overproduction for realization l in period t and $T_{up}(t;l)$ is the tonnage of underproduction in period t for realization l.

The amount of processed ore and the amount of mined material are controlled by two separate continuous variables rather than binary integer variables. The NPV of the estimation block model can be calculated using Eq. (2.37):

$$NPV_{es} = \sum_{t=1}^{T} \sum_{i=1}^{N} \left[v\left(t;i\right) \times z\left(t;l\right) - q\left(t;i\right) \times y\left(t;i\right) \right] \quad (2.37)$$

Where

- NPV_{es} is the NPV of the project that is calculated from an estimation block model such as kriging.

This optimization model is linear programming (LP). There are binary variables to control the precedence of the block extractions. Therefore, MILP methods are used to solve the model.

The advantage of this model is that it generates a smooth production plan. NPV_{es} is also used to show the estimated NPV of the project that is calculated by the estimation model.

The cost of grade uncertainty is applied to the objective function such that all conditional realizations are explicitly used in the optimization stage.

This model has two disadvantages:

- The stockpile is not considered at the optimization model.
- A symmetric penalty function is applied to this model (Figure 2-9a) which is not realistic. The cost of underproduction is more than overproduction because the overproduced ore can be saved in a stockpile to be used in next periods.

In the next section, the stockpile has been embedded at the optimization procedure and an asymmetrical penalty function is used for the cost of under and over productions.

2.6 Grade Uncertainty Based LP Model With a rule-based Stockpile (Model #3)

The stockpile considered in this model is used to store surplus ore. The extra ore can be used in circumstances where the plant cannot be fed at full capacity, such as a failure of the extraction and hauling system, or when there is a grade blending problem with input material to the mill. Any possible overproduced ore will be processed in later years, so, the penalty value defined in Eq. (2.23) for overproduction will be less in the presence of a stockpile. For this case, the overproduced ore is calculated by Eq. (2.26). Therefore, any plausible overproduced ore from a realization is kept in a stockpile and it is used in the next periods.

As shown in Section 2.1, the reasons for the costs of overproduction for this case are:

- The cost of re-handling material from a stockpile.
- The loss of discounted value of ore transferred to the next period.

To calculate the cost of overproduction in presence of a stockpile, the cost of overproduction for each period is deducted from the cost of overproduction in the next period. This means that for each period, the penalty value is only the loss of the discounted value of ore that is transferred to the next period. Meanwhile, any re-handling cost is added to the cost of overproduction in each period. Therefore, a new optimization model is developed based on Eq. (2.32) for the long-term mine planning in the presence of grade uncertainty and a stockpile. The new model is presented in Eq. (2.38):

$$Max \sum_{t=1}^{T} \left[\underbrace{\sum_{i=1}^{N} \left[v(t;i) \times z(t;l) - q(t;i) \times y(t;i) \right]}_{DCF_{es}(t)} \underbrace{- \frac{1}{L} \sum_{l=1}^{L} \left[T_{op}(t;l) \times \hat{c}_{op,RH}(t) + T_{up}(t;l) \times c_{up}(t) \right]}_{\bar{C}_u(t)} \right] \tag{2.38}$$

Subject to:

- Eq. (2.12) to Eq. (2.20). The estimated block model is used for these constraints.
- Modified version of Eq. (2.36) to control stockpile variables:

$$\begin{cases} \sum_{i=1}^{N} \left[-T_o(i;l) \times z(t;i) \right] - \left(T_{op}(t-1;l) + T_{up}(t;l) \right) \leq -P_u(t) \\ \sum_{i=1}^{N} \left[T_o(i;l) \times z(t;i) \right] + \left(T_{op}(t-1;l) - T_{op}(t;l) \right) \leq P_u(t) \end{cases} \quad \forall t = 1,2,\ldots,T \ , \ l = 1,2,\ldots,L \tag{2.39}$$

- Upper and lower limits for stockpile in each period:

$$S_l(t) \leq T_{op}(t;l) \leq S_u(t) \qquad \forall t = 1,2,\ldots,T \ , \ l = 1,2,\ldots,L \tag{2.40}$$

Where

- $S_u(t)$ and $S_l(t)$ are the upper and lower limits of the stockpile tonnage in each period. These constraints are applied in each realization. So, there are two constraints per period and per realization.

The number of decision variables and binary variables are the same as the previous model (Eq. (2.35)). This model is also a linear programming optimization problem with mixed integer variables.

There are two main differences between this model and the previous one in which the stockpile was not considered. The first difference is that the cost of overproduction for this model is less than the previous one ($c_{op}(t)$ vs. $\hat{c}_{op,RH}(t)$). This was shown in Figure 2-9. Because the tonnage of overproduction, $T_{op}(t;l)$, is used in the next period, it has less effect on optimization process than $T_{up}(t;l)$. Therefore, the tonnage of underproduction is penalized more in this model. This difference is applied on the second part of objective function. The second difference is hidden in the constraints that controls the variables $T_{op}(t;l)$ and $T_{up}(t;l)$ in Eq. (2.39). This equation contains two constraints that control the over and under production variables. The upper constraint, $\sum_{n=1}^{N}\left\{-T_o\left(i;l\right)\times z\left(t;i\right)-\left(T_{op}(t-1;l)+T_{up}(t;l)\right)\right\}\leq -P_u\left(t\right)$, controls the possible underproduction of realization l in period t. If there is any overproduced ore from previous year in the stockpile, $T_{op}\left(t-1;l\right)$, it is transferred to the current year. This is the reason that the overproduced tonnage in period t-1 is added to the under-produced tonnage in period t. The lower constraint, $\sum_{n=1}^{N}\left\{T_o\left(i;l\right)\times z\left(t;i\right)-\left(T_{op}(t-1;l)+T_{op}(t;l)\right)\right\}\leq P_u\left(t\right)$, also controls the possible overproduced ore for realization l in period t, $T_{op}\left(t;l\right)$, by adding the possible overproduction of previous year that has been transferred from stockpile, $T_{op}(t-1;l)$.

The concept of a stockpile is applied by using overproduced ore from previous period. Therefore, if there is a shortfall in current year, the model penalizes only the difference between overproduction from previous year and the shortfall of the current year. If the overproduction from last year is equal to or more than underproduction of current period, there will be no penalty for the current period. Another scenario is that both previous period and current period have overproduction. In this case, the overall surplus ore will accumulate for the current period.

In Model #3 a rule-based stockpile is used in the optimization process. The stockpile considered in this model is used to minimize the division from target production and is not used as part of NPV maximization. Therefore, the treatment of the stockpile in this approach is not a full optimization approach, that is, mining cannot be accelerated to mine high grade where low grade ore is stockpiled for later treatment. The stockpile is treated in a rule-based manner where surplus ore is saved and drawn down in time periods where there is inadequate ore supply. Although the stockpile is not being optimized, this approach provides a mechanism to account for over and under production more realistically. In Model #3 it has been tried to overcome some limitations of Model #2 in which the costs of over and under productions are assumed to be the same. To solve this problem, stockpile is used in Model #3. The soft constraints on the processing capacity allow the optimizer to produce extra ore in some periods, store it in the stockpile and use it in different periods to reduce the possibility of shortfall. Although this can increase the expected value of the NPV of the project, the stockpile is not part of the NPV maximization.

2.7 A Quadratic Optimization Method for the Mean-Variance Approach (Model #4)

As shown in Section 2.3, grade uncertainty affects not only the tonnage of ore, but also the average input grade to the mill. The NPV of the project that is calculated by combining the input grade and the input tonnage to the mill will have a different value for each realization. Both Model #2 and Model #3 assume that the grade uncertainty affects only the input tonnage of the ore to the mill. A more robust method to minimize the effect of grade uncertainty is to minimize the variance of NPV while the expected value of NPV is maximized. It is called mean-variance method.

The portfolio optimization by mean-variance approach is well known in Economic Science. It is mainly referred to as modern portfolio theory (MPT) and its creator has been awarded by Nobel memorial prize in 1990. Harry Markowitz (1952) introduced the MPT for risk based methods in portfolio optimization. The main idea of MPT is to consider a weighted average combination of assets' return and calculate expected value and variance of the return. The portfolio optimization with mean-variance approach is a quadratic optimization problem that tries to maximize the expected return and minimize the standard deviation. The decision variables are the weights or the portions of each asset.

In this section, a modified version of mean-variance method is adapted to long-term mine planning in presence of grade uncertainty. For each realization, EBV of each block is calculated. The average and variance of discounted EBV in each period for each block can be calculated. The main idea of the mean-variance method is to find the portion of each block to be extracted and processed, such that the average NPV calculated from all realizations is maximized and the variance of NPV is minimized simultaneously.

$$\begin{cases} \text{Maximize} \begin{pmatrix} \text{expected discounted} \\ \text{profit from} \\ \text{mining operation} \end{pmatrix} \\ and \\ \text{Minimize} \begin{pmatrix} \text{expected discounted} \\ \text{variance of profit} \end{pmatrix} \end{cases} \tag{2.41}$$

To model this optimization problem, the NPV is required for each realization. Eq. (2.42) is used to calculate $NPV(l)$.

$$NPV(l) = \sum_{i=1}^{N} \sum_{t=1}^{T} \left[v(t;i;l) \times z(t;i) - q(t;i;l) \times y(t;i) \right] \tag{2.42}$$

Where

- $v(t;i;l)$ is the discounted ore value of block *i* in period *t* in realization *l*.
- $q(t;i;l)$ is the discounted mining cost of block *i* in period *t* in realization *l*. $q(t;i;l)$ has the same value for all realizations and it can be calculated by Eqs. (2.43) and (2.5).

$$q(t;i;l) = q(t;i) \quad \forall l \tag{2.43}$$

Therefore using all realization, the expected NPV is calculated by Eq. (2.44):

$$\mathrm{E}\{NPV\} = \overline{NPV} = \sum_{n=1}^{N} \sum_{t=1}^{T} \left[E\{v(t;i)\} \times z(t;i) - q(t;i) \times y(t;i) \right] \tag{2.44}$$

Where

- $E\{v(t;i)\}$ is the expected value or the average of discounted ore value for block *i* in period *t* that is calculated by considering all realizations

The variance of NPV is also calculated as Eq. (2.45)

$$\begin{aligned} Var\{NPV\} = \sigma^2_{NPV} &= \sum_{i=1}^{N} \sum_{t=1}^{T} \left(z(t;i) \right)^2 \times Var\{v(t;i)\} \\ &= \sum_{i=1}^{N} \sum_{t=1}^{T} \left[\sigma_v^2(t;i) \times z^2(t;i) \right] \end{aligned} \tag{2.45}$$

Three solution options are suggested for the mean-variance approach:

1- Maximize expected NPV with restricted variance: in this model, a linear objective function (expected NPV) is maximized such that a nonlinear constraint (variance or standard deviation of NPV) does not exceed a critical user-defined value.

   The general form for this method is shown in Eq. (2.46)

$$\begin{aligned} &Max\,\mathrm{E}\left\{NPV\right\} \\ &s.t: \\ &\sigma_{NPV}^{2} \le C_{\mathrm{UserDefined}} \end{aligned} \tag{2.46}$$

   $C_{\mathrm{UserDefined}}$ is a user-defined upper limit for the variance of NPV. The model maximizes the NPV such that the variance does not exceed $C_{\mathrm{UserDefined}}$. The optimization model consists of a linear objective function with linear and quadratic constraints.

2- Minimize the variance or standard deviation of NPV with given minimum expected NPV: in this model, a quadratic optimization is minimized such that a minimum user-defined expected NPV is satisfied. The general form of the second type of optimization is shown in Eq. (2.47)

$$\begin{aligned} &Min\,\sigma_{NPV}^{2} \\ &s.t: \\ &\mathrm{E}\left\{NPV\right\} \ge K_{\mathrm{UserDefined}} \end{aligned} \tag{2.47}$$

   $K_{\mathrm{UserDefined}}$ is an input parameters that controls the minimum NPV. In this model, instead of maximization of NPV, the variance of NPV is minimized such that NPV is not less than a user-defined value called $K_{\mathrm{UserDefined}}$. To use this solution method, user needs to make good estimation of expected NPV of the project. The optimization model consists of a quadratic objective function with linear constraints.

3- Maximize the weighted differences of expected NPV and its variance: in this model, a quadratic function is maximized. The user-defined weight is a critical value that controls the trade-off between maximizing the expected NPV and minimizing its variance. Eq. (2.48) shows the general form of the third type.

$$Max\,\mathrm{E}\left(NPV\right)-\lambda_{\text{UserDefined}}.\sigma_{NPV} \tag{2.48}$$

$\lambda_{\text{UserDefined}}$ is a user-define value to control the trade-off between the maximization of NPV and the minimization of variance of NPV. This method is a more general way to optimize the dual objective functions. The weight can be any number from zero to infinity. Zero weight leads to a linear optimization without minimization of the variance of NPV. By increasing $\lambda_{\text{UserDefined}}$, both the expected NPV and the variance of the NPV decrease. User needs to have a clear understanding of the behavior of the Lambda parameters by doing a sensitivity analysis with different values.

All three methods have user-defined parameters. These parameters control the trade-off between the expected NPV and variance of NPV.

There are some considerations for these three methods:

- There is no known relationship between C, K and λ.
- Above a critical value of K in the second model (Eq. (2.47)), the optimization problem is infeasible and no solution exists. This means that, it is not possible to generate a schedule with NPV above this critical value. This critical value is the NPV that achieved by Model #1 presented in Section 2.1. It is the result of maximization of the NPV without any uncertainty.
- All of the three methods are nonlinear mixed integer optimization problems. The integer (binary) variables control the precedence of the block extraction. Types two (Eq. (2.47)) and three (Eq. (2.48)) have a quadratic objective function with linear constraints. The Hessian matrix is positive definite for both of them, so, the optimization model is always convex. There are well-developed algorithms to solve these kinds of optimization problems and the optimality of the solution is unique and guaranteed. Kozlov et al (1980) claimed that a convex quadratic optimization can be solved in polynomial time. In the first solution method (Eq. (2.46)), the objective function is linear, however, there are nonlinear constraints. This kind of optimization problems is mainly categorized as hard-NP and only numerical methods are available that may not be practically efficient. The iteration methods mainly are used to solve these types of optimization problems.

Based on the above comments, the third model is chosen because it is more robust with less implementation details. The first model presented early in this chapter has been used as the basis for this model. The general form of the objective function for mean-variance approach is presented in Eq. (2.49).

$$Max\left\{\sum_{n=1}^{N}\sum_{t=1}^{T}\left[\overline{v}(t;i)\times z\left(t;i\right)-q(t;i)\times y(t;i)\right]-\lambda\times\sum_{n=1}^{N}\sum_{t=1}^{T}\left[\sigma_{v}^{2}\left(t;i\right)\times z^{2}\left(t;i\right)\right]\right\} \tag{2.49}$$

Subject to the constraints in Eq. (2.12) to Eq. (2.20).

λ is the user-defined parameter that controls the trade-off between expected NPV maximization and minimization of variance of NPV. High λ values generate a schedule with lower expected NPV and also lower variance. Therefore, in a very conservative case, a large λ value should be used. On the other hand, in a case that the project is flexible with grade uncertainty, lower λ values should be preferred. Section 5.4 presents a sensitivity analysis to study the effect of Lambda.

The final optimization model presented in Eq. (2.49) is a Mixed Integer Quadratic Programming (MIQP) problem with linear constraints.

Wolsey (1998) categorizes the MIQP as NP-Hard problems. Volkovich et al. (1987) have done a detailed survey about the quadratic integer programming. Based on a literature review by Axehill (2005), most commonly used methods to solve MIQP problems are :

- Cutting plane methods
- Decompositions methods
- Logic-based methods
- Branch-and-bound (B&B) methods

Fletcher and Leyffer (1998) present a branch-and-bound (B&B) method for MIQP problems. They compare the result with generalized benders decompositions, outer approximation and LP/QP based branch-and-bound methods. They conclude that the B&B method is the superior method for solving MIQP problems. The B&B algorithm introduced by Land and Doig (1960) is a general method to solve discrete optimization problems.

2.8 Mining Cuts

One of the well-known methods in operations research for optimization of large problems is aggregation of similar variables in groups and assign a single new variable for all aggregated variables. In this method, the size of optimization reduces and the optimization problem can be solved in less CPU time. This has two main advantages:

- It decreases the number of variables, so, the optimization stage can be tractable with current computer hardware and software.
- It creates a schedule that is not spatially scattered. Using the high resolution block scale in optimization stage (even if it was tractable with current software and hardware) generates scattered schedules and there would be gaps between blocks extracted in the same period. Also in real mine operations, the production schedule is not at a block scale. The movement of huge excavators is minimized due to cost. Therefore, in real life the extraction unit is mining cut which contains several blocks instead of an individual block.

A MATLAB Fuzzy c-means clustering algorithm ('fcm') is used as the clustering method. The clustering step should be done for each level of elevation separately. Blocks at different levels should not be aggregated in one cluster. The c-means clustering algorithm creates clusters based on the number of clusters and the input properties as similarity factors. The data that are used in this thesis as similarity factors are:

- X, Y coordinates of the blocks: it is important that the blocks within each cluster are not spatially separated. The coordinates of the blocks are the main input properties for the clustering algorithm.
- Binary indicator that shows a block is ore or waste: it is 1 if the block is ore and 2 if the block is waste. It prevents mixing the ore and waste blocks in one mining cut.
- The average grade of the blocks: this helps the algorithm to cluster blocks with similar grades.
- The grade variances of the blocks: Because the main goal in this thesis is to minimize the risk of grade uncertainty, it is important to not aggregate highly uncertain blocks with lowly uncertain blocks. Therefore, the grade variances of blocks which are good measurements of the uncertainty of the blocks are used here.

By applying MATLAB fcm function for each branch, N blocks are aggregated into M mining cuts. For each mining cut the following parameters are calculated and stored in a MATLAB "mat" file:

- Center coordinate: X, Y and Z coordinate of the mining cut.

- Total tonnage, EBV, ore value, mining cost, ore tonnage and waste tonnage of the mining cut by adding up the values of the blocks inside each mining cut.
- Average grade of the mining cut which is calculated by taking a volumetrically average of the blocks.
- Precedence of mining cuts is stored in an array for each mining cut. This is the list of the indexes of the mining cuts that are needed to be extracted before getting access to the specific mining cut. First for each block inside a mining cut, the list of all nine blocks (Figure 2-1) that are located at the immediate top of the block is generated. Then, indexes of the mining cuts that these blocks belong to are determined and listed in an array.

Using clustered blocks in optimization of LTPP may lead to a sub-optimal solution. This is the disadvantage of using mining cuts instead of blocks in the optimization problem. Therefore, it is important to determine the correct number of cuts and use an efficient method to aggregate blocks such that the optimality of the objective function is not affected that much, the generated schedule is feasible and also the optimization process is tractable for industrial scale projects. A sensitivity analysis is required to determine the optimal clusters. In Section 5.1, the results of such a sensitivity analysis on the number of cuts and its effects on objective function are presented.

2.9 Discussion

LTPP in presence of grade uncertainty is discussed in this chapter. The input grade is an uncertain parameter and this uncertainty can be modeled by geostatistical simulation methods. Multiple and equal probable scenarios are used in LTPP optimization models to consider the uncertainty in the optimization process. In mine planning, there are always integer variables involved in the objective function. Therefore, mine planning in presence of grade uncertainty is considered as a mixed integer programming (MIP). Birge and Louveaus (1997) have discussed different methods to solve the optimization problems in presence of uncertainty. The main approaches that exist for these types of optimization problems are: anticipative models, adaptive models, anticipation and adaptation (recourse) models and chance constraints.

To explain the differences between each of these methods consider a situation that a decision has to be made in an uncertain world where the uncertainty is described by the random vector x. A prudent plan should anticipate the possible future realizations.

In anticipative models, the uncertainty is not changed over the optimization process and the decisions are independent from uncertainty. Anticipative models can be adapted to the long-term mine planning problems, because when a planner generates a long-term production schedule for the mine life, the grade uncertainty of the blocks are modeled by a finite number of realizations.

In the adaptive models, the decision is dependent on the uncertainty of the input parameters. The observation related to the uncertainty becomes available before the decision is made. Therefore, such an optimization model takes place in a learning environment. In this approach, the observation provides partial information about the uncertainty related variables. The anticipative model is not suitable for long-term mine planning problems, "because block grades will not be known in time, or by waiting, as required for the purpose of optimisation" (Dimitrakopoulos and Ramazan, 2008). The adaptive models can be suitable for short-term mine planning.

The recourse models are the combination of anticipative and adaptive models. The problem is solved by taking all future realizations (anticipation) and thus can be adapted by taking recourse decisions. For example, a long-term mine planning model, generated by maximizing the NPV and minimizing the deviation from target production, is an anticipative model. In future, whenever new information is gathered such as new infill drill holes, blast drill holes or the average grade of extracted blocks, new optimum decisions are made by solving the new model.

The fourth approach is the chance constraints that are not suitable for long-term mine planning due to the unrealistic assumptions such as normality of grade distributions in mining blocks (Dimitrakopoulos and Ramazan, 2008).

All methods presented in this thesis are anticipative models. They can be used as adaptive models anytime that new information is available.

The NPV of Models #2, 3 and 4 will be less than Model #1. In Model #1, NPV of the ordinary kriging is maximized without considering any grade uncertainty. However, in all other models presented in this chapter, there is an extra term in the objective function that is related to the grade uncertainty. It is either the cost of grade uncertainty or the variance of the NPV. Therefore, NPV of the OK block model that is gained from production schedule of any of these methods will be less than Model #1. This can be summarized in three main reasons:

1. Model #1 maximizes NPV directly without considering any other limitations such as minimizing the cost of grade uncertainty. Therefore, the feasibility region for any other models would be smaller and the solution will be sub-optimal relative to the maximum NPV solution that is achieved by Model #1.
2. The objective function of Model #2, 3 or 4 is quite different. It has an extra term that is the minimization of the discounted cost of grade uncertainty and/or the variance of the NPV. It is obvious that an optimum solution for one objective function is not optimal for the others. It is concluded that the objective function of any of these models (e.g. Model #2) has the maximum value with its own solution. For example, the solution of Model #1 is not optimum with objective function Model #2.
3. There is a trade-off between NPV maximization and minimization of the negative effect of uncertainty. By reducing the effect of grade uncertainty at the production plan, it is expected that the NPV of the project may be slightly decreased.

Model #1 to 3 are mixed integer linear programming (MILP) problems and Model is mixed integer quadratic optimization problems (MIQP) where the Hessian matrix is positive definite. Therefore, the solution for any of these models is the global optimum solution. The numerical methods, algorithms and available software to solve these kinds of optimization problems are very well developed. However, a user-defined factor called the gap tolerance is required as the termination criterion of the algorithm. This parameter also shows how different the current solution is from the real achievable optimum solution and it shows how good the current answer is. This is one of the main advantages of using operations research methods to find the optimal mine production schedule. There is no guarantee with heuristic methods that the final solution is close to the global optimum solution. Also, there is no parameter to show the goodness of the solution.

2.10 Summary

An optimization model based on the deterministic approach was presented in Section 2.2 for mine planning problem. The effect of grade uncertainty on the input tonnage and the NPV of the project were shown by studying a single block model with different local grade distributions. The concept of cut-off grade and its effect on the EBV and input tonnage in presence of grade uncertainty was presented in Section 2.3.

The cost of grade uncertainty was presented in Section 2.4. The cost of overproduction and the cost of underproduction were taken into account in the proposed models. Equations were presented for both cases. The more realistic method to calculate the cost of grade uncertainty was considered by assuming a stockpile. The equations for this case also have been provided in Section 2.4.

Three main methodologies are proposed to consider grade uncertainty in long-term production scheduling in Sections 2.5 to 2.7. The first method uses the cost of grade uncertainty and tries to minimize this cost and maximize net present value of the project simultaneously. This method does not consider any stockpiles. Two variables control the over and under production of ore in each period for each realization. A linear optimization model is developed to generate an optimum solution for ore production problem. The main goal in this method is to feed the plant with less deviation in early years of production. By deferring highly uncertain blocks to the later years, a schedule with lower deviation from target production is generated.

In the second method a stockpile is also used in the optimization process. The presence of a stockpile has two main effects on the model: (1) the extra ore can be processed at the next period, therefore, the cost of overproduction is much less than the cost of underproduction; and (2) the extra ore from the previous period can reduce or eliminate the cost of underproduction of the current period.

The final method is based on the mean-variance approach. It is more robust than other two methods. Since the grade uncertainty causes the fluctuation of the NPV, the expected value of NPV and the variance of NPV are used directly in the objective function. With this approach, the optimization model maximizes the expected value of NPV and minimizes the variance of NPV.

Finally, two main anticipative and adaptive methods to solve LTPP in presence of grade uncertainty are discussed. It is briefly explained why adaptive models are not suitable for long-term mine planning. The best solution for LTPP will be obtained by recourse models. In this method, a schedule is generated using all available information such as generated realizations for uncertain parameters. The recursive scheme can be generated by using all new information that will be available during the mine life.

Chapter 3 Implementation Details

In this chapter, implementation details of the proposed methods are discussed. In Section 3.1, the steps to generate an open-pit production schedule are presented. Section 3.2 explains how to use geostatistics to generate a geologically representative block model. Section 3.3 shows how to import that block model into the Whittle program in order to generate the optimum final pit limit by using 3D LG method. In Section 3.4, all realizations are imported into MATLAB. The implementation details for each of mine schedule optimization methods are described in Section 3.5. Finally, a summary of the chapter is presented in Section 3.6.

3.1 Introduction

In previous chapter, four optimization models are presented. Three are mixed integer linear programming (MILP) problems and one is a mixed integer quadratic programming (MIQP) problem. There are different software packages to handle MILP and MIQP problems. One approach is to utilize MATLAB (MathWorks Inc., 2011) and the optimization toolbox called TOMLAB/CPLEX (Holmström, 1989-2011). MATLAB is a numerical computing software package that contains a high level programming language with an interactive environment for numerical computation and visualization. MATLAB has many built-in functions, procedures and toolboxes for data analysis and visualization. Therefore, MATLAB is a convenient environment to experiment and test new mathematical algorithms and workflows. However, the computational time for a MATLAB code may be longer than the codes that are written and compiled in traditional programming languages such as FORTRAN and C/C++.

A commercial optimization package called TOMLAB/CPLEX is available to plug into the MATLAB environment. This package has some very powerful optimization engines and algorithms that can be used for different purposes. The important optimization engine called CPLEX (ILOG Inc, 2007) has been integrated with TOMLAB. CPLEX has been developed since 1988 and it is a very powerful solver for large scale LP, MILP, QP and MIQP problems.

TOMLAB has a standard form for MILP and MIQP problems. The required matrices and vectors are easily prepared for input to the TOMLAB solvers.

The general workflow for numerical implementation of the proposed methods that are theoretically discussed in the previous chapter is as follow:

1. Generate multiple conditional realizations for grade of the blocks to capture uncertainty using geostatistical methods. The GSLIB programs are used in this step (Deutsch and Journel, 1998).

2. Generate ultimate pit limit by 3D LG method using the average block model or an estimated block model from Ordinary Kriging (OK). The Whittle software is employed for this purpose (Gemcom Software International, 1998-2008).

3. For each realization filter all of the blocks outside of the final pit limit and save the ones inside to an ASCI file. This custom written program also applies the cut-off grade to the input realizations and calculates the EBV.

4. Import all the ASCI output files into MATLAB and run the MATLAB subroutines step by step to generate the standard format for TOMLAB/CPLEX solver.

5. Run the TOMLAB/CPLEX solver.

6. Post-process the results to create summary tables and plots, and export the final solution as an ASCI file format that can be used in other software.

A computer with 8 CPUs and 20 GB of ram has been used with Windows 7 Professional. The processor is an Intel(R) Core(TM) i7 CPU 930 @ 2.80GHz, 2801 MHz, 4 Core(s), 8 Logical Processor(s). All the methods are solved by TOMLAB/CPLEX solver.

In this chapter, all the important implementation steps are described. Additional details of some steps, input parameters and subroutines are presented in an Appendix. Figure 3-1 shows the steps that are described in this section.

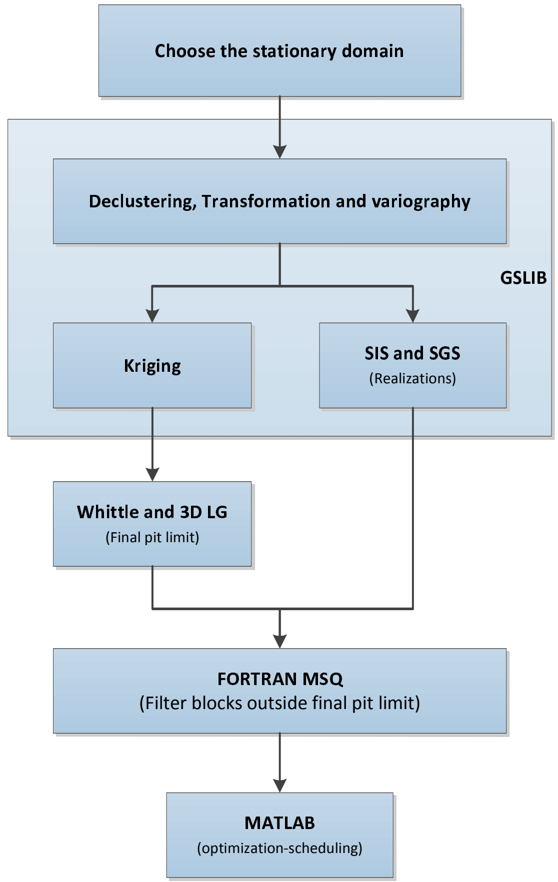


Figure 3-1. Summary of the steps to generate optimum mine schedule with grade uncertainty

3.2 Geostatistical Modeling

GSLIB (Deutsch and Journel, 1998) is used for geostatistical modeling. The first step for a geostatistical study is to define stationary subsets of the data and the deposit. For each of these different subsets, geostatistical modeling is performed separately. This step can be done by defining different rock-types of different depositional or digenetic alteration style. Geological knowledge has a critical role in this step. Usually, the geologist defines the subsets based on drillhole data. Geophysical data may also be used to supplement drillhole data. For each stationary subset the following steps are implemented to generate a geological block model:

1. The first step is to determine the representative distribution of each grade variable. Due to preferential sampling from high grade zones, usually distributions of the variables are not representative of the entire area. Therefore, a declustering algorithm has to be done to adjust the distributions. The `declus`

program in GSLIB is used for this purpose. This program generates numerical weights that indicate the effect of each sample in a representative distribution.

2. The next step is to perform a multivariate statistical analysis to determine the correlation of the multivariate data. For example, Copper is more likely to have high positive correlation with Molybdenum in porphyry deposits. In this step, the goal is to find all other properties that have either positive or negative relationship with the property of interest. `Scatplt`, `scatnscores`, `corrmat_plot` etc. are used for this purpose.

3. The next step is to transform the data to Gaussian units. For univariate data, `nscore` and for multivariate data `sctrans` is used. In both cases, the output data has a standard normal distribution. The `sctrans approach` removes any correlation between the variables using the Stepwise Conditional Transformation (Leuangthong, 2003) method. Therefore, each of transformed variables is standard normal distribution and not correlated to any other variable. So, they can be modeled separately.

4. The experimental variograms are calculated with normalized data by using `gamv`. `vmodel` is used to fit and calculate the variogram model in different directions. `vargplt` plots the experimental variograms and fitted variogram models.

5. The next stage is to generate the rock-type model for the chosen grid definition. There are two main approaches to generate a rock-type model: estimation and simulation. `ik3d` applies indicator kriging to estimate the probability distribution of a discrete variable such as rock-type. Indicator variograms are required as an input for `ik3d`. The most probable rock-type is chosen for each block or grid node. Another method to create a rock-type model is to use sequential indicator simulation (SIS) algorithm. `sisim` is used for this purpose. This algorithm generates multiple realizations of the rock-types.

6. The final step is to generate rock-property models such as grade, density, etc. for each rock-type, and back-transform data to the original unit. `kt3d` estimates continuous variables. This program provides a variety of kriging algorithms such as simple, ordinary and universal kriging. The OK method does not require any transformation and it should be applied to the data in original unit. In order to

generate conditional realizations that are representative of the grade uncertainty in the area of interest, `sgsim` is employed. This program is based on Sequential Gaussian Simulation (SGS) algorithm.

The generated block model needs to be checked and verified. The most important statistics that should be reproduced are the histogram and variogram. The realizations should reproduce the input histogram and variogram model within acceptable statistical fluctuations. `Gam` is used to calculate directional variograms from a simulated grid. `Vargplt` plots the calculated variograms and the input variogram model. There should be a good match between the input variogram and the average variogram calculated from all realizations. `Histplotsim` is also used to plot cumulative distribution function (CDF) of each realization and the reference distribution (input data) in one graph. Some additional post processing procedures are done to check the model such as calculating E-Type mean and variance using `postsim`. E-Type mean of all realizations should be close to the kriging estimation.

3.3 Final Pit Limit Design Using 3D LG

The next step is to determine the final pit limit using 3D LG algorithm. An estimation block model such as ordinary kriging is used to generate the final pit limit. This block model is imported into Whittle (Gemcom Software International, 1998-2008) where 3D LG algorithm is employed to determine the optimum final pit limit. A series of nested pits or push-backs are generated by different revenue factors. A long-term production plan is generated by push-backs and all required input parameters. The msq file (The mining sequence file) is exported from Whittle. This file contains the extraction portion of the blocks in each period and their destinations. A program called `MSQ90` was written to extract all the blocks inside the final pit and remove the blocks outside. The final block model is saved in an ASCII format for each realization. These files are used as the input in the optimization stage. `MSQ90` also can be used to read the production schedule generated by Whittle and assess the effect of grade uncertainty on a production schedule using the input realizations. The idea is that by following the production schedules with each realization the input ore tonnage to the plant and other parameters such as input head grade, NPV etc. are recalculated. This program needs to be run separately for each realization. The second output of this program is a summary file that includes ore tonnage, waste tonnage, average input grade, etc. for each period.

The parameter file and detail information on this program are found in an appendix.

3.4 Data Preparation for the Optimization Stage

The mathematical optimization models that are proposed in the previous chapter, are coded in MATLAB (MathWorks Inc., 2011) version 2009. The TOMLAB/CPLEX solver (Holmström, 1989-2011) is used for optimization.

There are five preliminary steps to be taken before starting the optimization. These steps include importing the input block models and parameters, creating the precedence matrix, clustering the blocks, creating the mining cuts and adding the simulation realizations to each mining cut. For each step, there is a folder that contains the functions and procedures that should be used. The output file of each step is transferred to the next step. Each step is described in detail in an appendix; however, a brief summary of these steps is as follows:

- Step 1: Import data to MATLAB. In this step, each of output files that are generated by `MSQ90`, are transferred to a separate MATLAB ".mat" file.
- Step 2: Create adjacency matrix. An estimation block model such as OK, which has been imported to MATLAB in previous step, is used to generate the adjacency matrix. The precedence of block extraction is defined in this step.
- Step 3: Create clusters or mining cuts. The output file of previous step is used here to generate the clusters or mining cuts.
- Step 4: In this step, all the '.mat' files that are generated in first step are merged with the output file from step 3. For each mining cut, total tonnage, EBV, ore value, mining cost, ore tonnage, waste tonnage and average grade are calculated for each realization. All simulated numbers for each of these parameters are stored in separate vectors and one ".mat" file is created. This file has a cell array in which each cell contains information on each mining cut.
- Step 5: Input user parameters that are required for the optimization models #1, 2, 3 and 4. At this step, the final input parameters such as the minimum and maximum mining and processing capacities for each period, number of periods or mine life in years, number of pre-striping years, number of simulation realizations, the discounted cost of over and under production in each period ($c_{up}(t)$ and $c_{op}(t)$) which are used in models #2 and #3 and Lambda (λ) which is used in Model #4 are added to the generated ".mat" file.

The final production of these steps is a MATLAB ".mat" file that contains all input parameters required for the proposed models. In the next section the implementation details of each model is presented.

3.5 MILP Formulation Implementation

The CPLEX solver starts with relaxing the LP model. During this procedure, all integer variables are relaxed by being changed to real variables and the LP model is solved. Then, CPLEX uses the branch-and-cut search algorithm to reach a feasible integer solution. Branch-and-cut is a combination of branch-and-bound and cutting plane methods (Horst and Tuy, 2003; Wolsey, 1998).

There is an important termination criterion that is set by the user. It is called the MIP gap (MIPGAP) and it is the absolute tolerance of the gap between the best integer objective and the objective of the best node remaining in the branch-and-bound algorithm. This parameter instructs CPLEX to stop as soon as it has found a feasible integer solution proved to be within the MIPGAP limit.

TOMLAB uses a general form as an input for all optimization problems. The general form for an MILP problem is stated by Equations (3.1) to (3.3).

$$\min f(x) = \mathbf{c}'\mathbf{x} \tag{3.1}$$

Such that:

$$\mathbf{x}_L \leq \mathbf{x} \leq \mathbf{x}_U \tag{3.2}$$

$$\mathbf{b}_L \leq \mathbf{A}\mathbf{x} \leq \mathbf{b}_U \tag{3.3}$$

Where

- $\mathbf{c}$ is the coefficient of linear objective function in the MILP model; a vector of $n \times 1$.
- n is the total number of decision variables.
- $\mathbf{x}$ is the vector of the decision variables of the MILP model including binary and continues variable; vector of $n \times 1$.
- $\mathbf{x}_L$ and $\mathbf{x}_U$ define the lower and upper limit of the decision variables respectively; vectors of $n \times 1$.

- $\mathbf{A}$ is the coefficient matrix of linear constraints of the MILP model; a matrix of $m \times n$.
- m is the total number of linear constraints in the MILP model.
- $\mathbf{b}_L$ and $\mathbf{b}_U$ define the lower and upper bounds of the linear constraints; vectors of $m \times 1$. Any equality constraint can be added to the model by setting equal values for upper and lower boundaries for the respective elements of vectors $\mathbf{b}_L$ and $\mathbf{b}_U$.

In this section, the implementation details of Model #1 are presented first followed by the other models which are updated versions of Model #1. This includes all the matrices of the objective functions and constraints that are required to build each model in order to solve by TOMLAB. The goal is to create the models in the standard format shown in Eqs. (3.1) to (3.3). All implementations are at the in mining cut level. The assumption is that *N* blocks are clustered in *M* mining cuts. The mine life is denoted by *T* and *L* is used for total number of realizations.

3.5.1 Implementation of Model #1: MILP Without Considering Grade Uncertainty

In order to solve Model #1 (MILP model), the objective function presented in Eq. (2.11) and all constraints formulated in Eqs. (2.12) to (2.20) are reformatted in matrices and vectors. The matrices or vectors for each of these equations are shown below:

- $\mathbf{x}$ vector: The vector of decision variables. For each mining cut there are binary decision variables $a(t;i) \in \{0,1\}$ that are used to control the precedence of the blocks. $a(t;i)$ equals to 1 if mining cut *i* has been extracted before period *t* and equals to 0 if it has not been extracted yet. Also there are two continuous decision variables, $y(t;i)$ and $z(t;i)$ $\in \{0...1\}$. $y(t;i)$ is the portion of extraction and $z(t;i)$ is the portion of processing of mining cut *i* in period *t*. $\mathbf{x}$ is a vector that includes all decision variables. The size of the $\mathbf{x}$ vector is $3TM \times 1$ which means that $\mathbf{x}$ is a vector with $3TM$ of rows and 1 column as shown below:

$$\mathbf{x} = \begin{bmatrix} [a(t;i)]_{TM\times 1} \\ [z(t;i)]_{TM\times 1} \\ [y(t;i)]_{TM\times 1} \end{bmatrix}_{3TM\times 1} \tag{3.4}$$

The structure for each of these variables is also shown in Eq. (3.5) :

$$\begin{bmatrix} \begin{bmatrix} \text{mining cut 1} \\ \text{mining cut 2} \\ \vdots \\ \text{mining cut M} \end{bmatrix} \text{at period 1} \\ \begin{bmatrix} \text{mining cut 1} \\ \text{mining cut 2} \\ \vdots \\ \text{mining cut M} \end{bmatrix} \text{at period 2} \\ \vdots \\ \begin{bmatrix} \text{mining cut 1} \\ \text{mining cut 2} \\ \vdots \\ \text{mining cut M} \end{bmatrix} \text{at period T} \end{bmatrix} \tag{3.5}$$

All three decision variables are between zero and 1 because the binary variables, $a(t;i)$, are zero or one and the other two variables, $z(t;i)$ and $y(t;i)$, are continuous variables between 0 and 1. Therefore, the upper limit and lower limit for $\mathbf{x}$ that are called $\mathbf{x}_U$ and $\mathbf{x}_L$ are respectively defined as below:

$$\begin{aligned} \mathbf{x}_U &= [1]_{3TM\times 1} \\ \mathbf{x}_L &= [0]_{3TM\times 1} \end{aligned} \tag{3.6}$$

Where $[1]_{3TM\times 1}$ is a vertical vector with the size of $3TM \times 1$. The lower limit, $[0]_{3TM\times 1}$, is a zero vertical vector with all elements equal to zero.

- Objective function: Eq. (2.11).

  The objective function maximizes NPV of the mining operation. However, as shown in Eq. (3.1), the general form of the objective function is a minimization. Therefore, the objective function coefficients of Model #1 and all other models that are presented in previous chapter should be multiplied by a negative sign to change a maximization of NPV to the minimization of –NPV:

$$-NPV = \mathbf{c}'\mathbf{x} \tag{3.7}$$

The coefficients of objective function are stored in a vector called $\mathbf{c}$. It is a column vector with size of $3TM \times 1$. Each row of this vector is the coefficient for relevant column of the decision variables. The binary variables do not have any coefficient in the objective function, so, the first TN rows of $\mathbf{c}$ vector are zero. These zero values remove the binary variables from the objective function. The second part of this vector is $v(t;i)$ values or the discounted revenue of block i in period t. These values are the coefficients of $z(t;i)$. These values occupy TN rows of the $\mathbf{c}$ vector ($(TN+1)\cdots(2TN)$). The last part is $q(t;i)$ values or the discounted cost of mining of block i in period t ($(2TN+1)\cdots(3TN)$). Both $v(t;i)$ and $q(t;i)$ are positive values, therefore, as discussed before, to change the maximization to the minimization problem, only $v(t;i)$ are multiplied by a negative sign. The final form of $\mathbf{c}$ vector:

$$\mathbf{c} = \begin{bmatrix} [0]_{TM\times 1} \\ [-v(t;i)]_{TM\times 1} \\ [q(t;i)]_{TM\times 1} \end{bmatrix}_{3TM\times 1} \tag{3.8}$$

- Grade blending constraints: Eq. (2.12).

  These constraints enforce the average input grade to the mill to be in the acceptable range in each period. The standard form of this constraint is indicated as Eq.(3.9):

$$[-\infty] < \mathbf{A}_{grade}\mathbf{x} \le [0] \tag{3.9}$$

Coefficients of this constraint are stored in a matrix called $\mathbf{A}_{grade}$. As shown in Eq. (2.12), $\mathbf{A}_{grade}$ should be multiplied by $z(t;i)$. To eliminate other variables from this constraint, coefficients of other decision variables are set to zero. Matrix $\mathbf{A}_{grade}$ has $3TM$ columns in which each column is related to the same row of decision variables in vector $\mathbf{x}$. There are upper and lower limits for each period; therefore, the total number of grade blending constraints is $2T$. Each constraint occupies a row in $\mathbf{A}_{grade}$; so, there are $2T$ rows for this matrix. The general form of matrix $\mathbf{A}_{grade}$ is shown in Eq. (3.10). The upper limit of this constraint is a zero vector called $\mathbf{b}_{U\_grade}$. There is no lower limit for these constraints. Therefore, $\mathbf{b}_{L\_grade}$ is a vector with very large negative number as shown in Eq. (3.11).

$$\mathbf{A}_{grade} = \begin{bmatrix} [0]_{T\times TM} & \left[T_o(i)\times\left(g(i)-g_u(t)\right)\right]_{T\times TM} & [0]_{T\times TM} \\ [0]_{T\times TM} & \left[T_o(i)\times\left(g_l(t)-g(i)\right)\right]_{T\times TM} & [0]_{T\times TM} \end{bmatrix}_{2T\times 3TM} \quad (3.10)$$

$$\begin{aligned} \mathbf{b}_{U\_grade} &= [0]_{2T\times 1} \\ \mathbf{b}_{L\_grade} &= [-\inf]_{2T\times 1} \end{aligned} \quad (3.11)$$

Where

- $[0]_{T\times TM}$ is the zero matrix with T rows and TM columns.
- $[0]_{2T\times 1}$ is the zero vector with $2T$ rows and 1 column.

- Processing capacity constraints: Eq. (2.13)

  These constraints control the input ore tonnage to the mill in each period. The general form is presented in Eq. (3.12):

$$\mathbf{b}_{L\_PC} < \mathbf{A}_{processing}\mathbf{x} \le \mathbf{b}_{U\_PC} \quad (3.12)$$

There are two constraints per period to control the upper and lower limits of input tonnage. The matrix of coefficients of constraints is called $\mathbf{A}_{processing}$ which is shown in Eq. (3.13). The upper and lower limit vectors of the constraints are called $\mathbf{b}_{U\_PC}$ and $\mathbf{b}_{L\_PC}$ which are shown in Eq. (3.14).

$$\mathbf{A}_{processing} = \left[[0]_{T\times TM} \quad [T_o(i)]_{T\times TM} \quad [0]_{T\times TM} \right]_{T\times 3TM} \quad (3.13)$$

$$\begin{aligned} \mathbf{b}_{U\_PC} &= [p_u(t)]_{T\times 1} \\ \mathbf{b}_{L\_PC} &= [p_l(t)]_{T\times 1} \end{aligned} \quad (3.14)$$

- Mining capacity constraints: Eq. (2.14).

Same as the processing capacity constraints, there are $2T$ number of constraints which occupy one row of the coefficient matrix called $\mathbf{A}_{Mining}$. The general form is:

$$\mathbf{b}_{L\_MC} < \mathbf{A}_{Mining}\mathbf{x} \leq \mathbf{b}_{U\_MC} \tag{3.15}$$

The coefficients are multiplied by the decision variables that indicate the portion of extraction, $y(t;i)$. The coefficients related to $a(t;i)$ and $z(t;i)$ must be zero. The coefficient matrix and the upper and lower limit vectors are shown in Eqs. (3.16) and (3.17).

$$\mathbf{A}_{Mining} = \left[[0]_{T\times TM} \quad [0]_{T\times TM} \quad \left[T_{Total}(i)\right]_{T\times TM} \right]_{T\times 3TM} \tag{3.16}$$

$$\begin{aligned} \mathbf{b}_{U\_MC} &= \left[m_u(t)\right]_{T\times 1} \\ \mathbf{b}_{L\_MC} &= \left[m_l(t)\right]_{T\times 1} \end{aligned} \tag{3.17}$$

- Ore-mining constraints: Eq. (2.15).

  These constraints enforce the portion of processing to be equal or smaller than the portion of extraction at the same period for each mining cut. Eq. (2.15) is modified as:

$$z(t;i) - y(t;i) \leq 0 \tag{3.18}$$

Therefore, the upper limit of the constraint equals to zero. The general form for this constraint is presented in Eq. (3.19)

$$-\infty < \mathbf{A}_{Ore\_Mining}\mathbf{x} \leq [0] \tag{3.19}$$

There will be TM constraints. The coefficient matrix and upper and lower limit vectors are shown in Eq. (3.21).

$$\mathbf{A}_{Ore\_Mining} = \left[[0]_{TM\times TM} \quad I_{TM\times TM} \quad -I_{TM\times TM} \right]_{TM\times 3TM} \tag{3.20}$$

$$\begin{aligned} \mathbf{b}_{U\_OM} &= [0]_{TM\times 1} \\ \mathbf{b}_{L\_OM} &= [-\inf]_{TM\times 1} \end{aligned} \tag{3.21}$$

Where $I_{TM\times TM}$ is the identity matrix with size of $TM \times TM$.

- Extraction precedence constraints: Eqs. (2.16) to (2.18).

  The general form of these constraints is shown in Eq. (3.22):

$$[-\infty] < \mathbf{A}_{precedence}\mathbf{x} \le [0] \tag{3.22}$$

There are three constraints for each mining cut. These constraints enforce all immediate mining cuts above mining cut i to be extracted completely before ahead with i. Binary decision variables, $a(t;i)$, are used for this purpose. $a(t;i)$ is 1 if mining cut i is completely extracted before period t; otherwise it is zero.

  - The first Eq. (2.16) enforce $a(t;i)$ to be zero if any of the mining cuts located above mining cut i is not extracted completely until period t. If all the precedent mining cuts are extracted completely until period t, the decision variable $a(t;i)$ can be either 0 or 1. The coefficient of this constraint is shown in Eq. (3.23):

$$\Big[\big[1(t;i)\big]_{TMK\times TM} \quad [0]_{TMK\times TM} \quad \big[-1(u;j)\big]_{TMK\times TM}\Big]_{TMK\times 3TM} \tag{3.23}$$

    Where

    - K is the total number of precedence relationships between all mining cuts.
    - $\big[1(t;i)\big]_{TMK\times TM}$ is the coefficient matrix for $a(t;i)$. In this vector, all the elements for mining cut i (i=1..M) and j (j=1..$C(i)$) in period t (t=1..T) are one; other elements are zero.
    - $[0]_{TMK\times TM}$ is a zero matrix and it is the coefficient matrix of $z(t;i)$.
    - $\big[-1(u;j)\big]_{TM\times TM}$ is the coefficient matrix for $y(u;j)$ that is related to mining cut j in period u. For mining cut i, there are $T\times C(i)$ rows and each row is a separate constraint related to mining cut i in period t; there are $C(i)$ number of precedent mining cuts each mining cut i. For any i and j th mining cut in period 1 to t, the element of this matrix are -1.

- The second constraint, Eq. (2.17), enforces binary variable $a(t;i)$ to be 0 if mining cut *i* has not been extracted until period *t*; otherwise it can be either 0 or 1. The coefficient of this constraint is shown in Eq. (3.24):

$$\left[-I_{TM\times TM} \quad [0]_{TM\times TM} \quad \left[adj(u;i)\right]_{TM\times TM} \right]_{TM\times 3TM} \tag{3.24}$$

  Where

  - $\left[adj(u;i)\right]_{TM\times TM}$ is the adjacency matrix with size of $TM \times TM$. The element representing mining cut *i* in period *t* is 1 if mining cut *u* should be extracted before; otherwise it is zero.

- The third constraint, Eq.(2.18), ensures that if a mining cut has been extracted in period *t*, all $a(t;i)$ to $a(T;i)$ are 1. There is a constraint for mining cut *i*, in period *t*. These constraints are shown in matrix form as Eq. (3.25).

$$\left[\left[1(t;i),-1(t;i+1)\right]_{TM\times TM} \quad [0]_{TM\times TM} \quad [0]_{TM\times TM} \right]_{TM\times 3TM} \tag{3.25}$$

The complete precedence constraint coefficient matrix called $\mathbf{A}_{precedence}$ is summarized as below:

$$\mathbf{A}_{precedence} = \begin{bmatrix} \left[\left[1(t;i)\right]_{TMK\times TM} \quad [0]_{TMK\times TM} \quad \left[-1(u;j)\right]_{TMK\times TM} \right] \\ \left[-I_{TM\times TM} \quad [0]_{TM\times TM} \quad \left[1(u;i)\right]_{TM\times TM} \right] \\ \left[\left[1(t;i),-1(t;i+1)\right]_{TM\times TM} \quad [0]_{TM\times TM} \quad [0]_{TM\times TM} \right] \end{bmatrix}_{(TM(K+2))\times 3TM} \tag{3.26}$$

The upper and lower limit vectors are also shown as below:

$$\begin{aligned} \mathbf{b}_{U\_precidence} &= [0]_{(TM(K+2))\times 1} \\ \mathbf{b}_{L\_precidence} &= [-\inf]_{(TM(K+2))\times 1} \end{aligned} \tag{3.27}$$

- Reserve constraint: Eq. (2.19) or Eq. (2.20).

  There are two versions for this type of constraints: The first type, Eq. (2.19), ensures all the blocks inside the final pit limit are extracted. The summation of extraction portions of any mining cut is one. It is an equality constraint and upper limit and lower limit of the constraint should be set to one.

The assumption here is that the final pit limit that is determined by 3D LG method is the optimum limit. Therefore, all the blocks inside the final pit should be extracted. The second type, if for any reason one does not agree with this assumption and decides to let optimizer determine the final pit limit inside the ultimate pit limit, the equality constraint is not required anymore. However, it is required that summation of extraction portions of a single mining cut not to exceed 1.

$$\begin{aligned}&[1] < \mathbf{A}_{Reserve}\mathbf{x} \leq [1]\\&or\\&[-\infty] < \mathbf{A}_{Reserve}\mathbf{x} \leq [1]\end{aligned} \tag{3.28}$$

The coefficient matrix for both cases is the same and it is called $\mathbf{A}_{Reserve}$ that is shown in Eq. (3.29). The upper limit vector of these constraints is shown in Eq. (3.30). The lower limit depends on which case is considered. If the ultimate pit limit determined by 3D LG is decided to be the final pit limit, an equality constraint can be satisfied by setting the lower limit equal to the upper limit as shown in Eq. (3.31). Otherwise, the lower limit is not required anymore and it can be eliminated by setting to the negative infinity (Eq. (3.32)).

$$\mathbf{A}_{Reserve} = \Big[[0]_{M\times TM} \quad [0]_{M\times TM} \quad \big[1(1\ldots T;i)\big]_{M\times TM} \Big]_{M\times 3TM} \tag{3.29}$$

$$\mathbf{b}_{U\_reserve} = [1]_{M\times 1} \tag{3.30}$$

$$\mathbf{b}_{L\_reserve} = [1]_{M\times 1} \tag{3.31}$$

$$\mathbf{b}_{L\_reserve} = [-\inf]_{M\times 1} \tag{3.32}$$

- Where $\big[1(1\ldots T;i)\big]_{M\times TM}$ is a matrix that has M rows (one row for each mining cut). For mining cut *i,* all the columns that are related to $y(1;i), y(1;i),\ldots, y(T;i)$ are set to 1; all other elements are zero.

The general form of the constraints can be rewritten as one matrix and two upper and lower limit constraints as Eq. (3.33):

$$\mathbf{b}_L \leq \mathbf{A}\mathbf{x} \leq \mathbf{b}_U \tag{3.33}$$

Where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{grade} \\ \mathbf{A}_{processing} \\ \mathbf{A}_{Mining} \\ \mathbf{A}_{Ore\_Mining} \\ \mathbf{A}_{precedence} \\ \mathbf{A}_{Reserve} \end{bmatrix}_{(4T+TM(K+3)+M)\times 3TM}$$

$$\mathbf{b}_L = \begin{bmatrix} \mathbf{b}_{L\_grade} \\ \mathbf{b}_{L\_processing} \\ \mathbf{b}_{L\_Mining} \\ \mathbf{b}_{L\_Ore\_Mining} \\ \mathbf{b}_{L\_precedence} \\ \mathbf{b}_{L\_reserve} \end{bmatrix}_{(4T+TM(K+3)+M)\times 1}$$

$$\mathbf{b}_U = \begin{bmatrix} \mathbf{b}_{U\_grade} \\ \mathbf{b}_{U\_processing} \\ \mathbf{b}_{U\_Mining} \\ \mathbf{b}_{U\_Ore\_Mining} \\ \mathbf{b}_{U\_precedence} \\ \mathbf{b}_{U\_reserve} \end{bmatrix}_{(4T+TM(K+3)+M)\times 1} \tag{3.34}$$

3.5.2 Implementation of Model #2: MILP Considering Grade Uncertainty Without Stockpile

In this section, the implementation of Model #2 is presented. The constraints are same as Model #1 as shown in Eqs. (2.12) to (2.20) plus one extra constraint to control the tonnage of over and under production ore that is shown in Eq. (2.36). To implement this model, implementation of Model #1 has been updated as below:

- $\mathbf{x}$ vector: is same as the previous model as shown in Eq. (3.4); except there are additional variables that store the tonnage of over and under production. Therefore $\mathbf{x}$ vector is updated as Eq. (3.35) as below:

$$\mathbf{x} = \begin{bmatrix} [a(t;i)]_{TM\times 1} \\ [z(t;i)]_{TM\times 1} \\ [y(t;i)]_{TM\times 1} \\ [T_{op}(t;l)]_{TL\times 1} \\ [T_{up}(t;l)]_{TL\times 1} \end{bmatrix}_{(3TM+2TL)\times 1} \tag{3.35}$$

Where $T_{op}(t;l)$ and $T_{up}(t;l)$ are tonnage of over and under production in period t for realization l. The size of $\mathbf{x}$ vector increases to $3TM + 2TL$. The structure of each of the vectors $\left[T_{up}(t;l)\right]_{TL\times 1}$ and $\left[T_{op}(t;l)\right]_{TL\times 1}$ are shown in Eq. (3.36).

$$\begin{bmatrix} \begin{bmatrix} \text{realization 1} \\ \text{realization 2} \\ \vdots \\ \text{realization L} \end{bmatrix} \text{at period 1} \\ \begin{bmatrix} \text{realization 1} \\ \text{realization 2} \\ \vdots \\ \text{realization L} \end{bmatrix} \text{at period 2} \\ \vdots \\ \begin{bmatrix} \text{realization 1} \\ \text{realization 2} \\ \vdots \\ \text{realization L} \end{bmatrix} \text{at period T} \end{bmatrix} \tag{3.36}$$

The upper limit of the $\mathbf{x}$ vector which is defined by vector $\mathbf{x}_U$ is required to be modified as well. Since new variables $T_{up}(t;l)$ and $T_{op}(t;l)$ are the tonnage of under and over production, they can take any positive number. The lower limit vector, $\mathbf{x}_L$, is still 0:

$$\begin{aligned} \mathbf{x}_U &= \begin{bmatrix} [1]_{3TM\times 1} \\ [\inf]_{2TL\times 1} \end{bmatrix}_{(3TM+2TL)\times 1} \\ \mathbf{x}_L &= [0]_{(3TM+2TL)\times 1} \end{aligned} \tag{3.37}$$

- Objective function: Eq. (2.35).

The tonnage of over and under production is multiplied by the cost of over and under production per tonne. Therefore, the objective function coefficient vector, $\mathbf{c}$, is modified in order to add these extra terms in Model #2. The new vector $\mathbf{c}$ is shown in Eq. (3.38):

$$\mathbf{c} = \begin{bmatrix} [0]_{TM\times 1} \\ [-v(t;i)]_{TM\times 1} \\ [q(t;i)]_{TM\times 1} \\ [\bar{c}_{op}(t)]_{TL\times 1} \\ [\bar{c}_{up}(t)]_{TL\times 1} \end{bmatrix}_{(3TM+2TL)\times 1} \tag{3.38}$$

Where $\bar{c}_{op}(t)$ and $\bar{c}_{up}(t)$ are the average cost of over and under production per tonne, respectively. They are simply calculated by dividing $c_{up}(t)$ and $c_{op}(t)$ by the total number of conditional realizations *L*. $\bar{c}_{up}(t)$ and $\bar{c}_{op}(t)$ are equal in all realizations in period *t*.

$$\begin{cases} \bar{c}_{op}(t) = \dfrac{c_{op}(t)}{L} \\ \bar{c}_{up}(t) = \dfrac{c_{up}(t)}{L} \end{cases} \qquad \forall\, l = 1,2,\ldots,L \tag{3.39}$$

- Grade blending constraint: Eq. (2.12).

  Because $T_{up}(t;l)$ and $T_{op}(t;l)$ do not have any effects on the input grade to the mill, the coefficients related to these variables are set to zero in $\mathbf{A}_{grade}$. The modified version of this matrix is shown in Eq. (3.40):

$$\mathbf{A}_{grade} = \begin{bmatrix} [0]_{T\times TM} & [T_o(i)\times(g(i)-g_u(t))]_{T\times TM} & [0]_{T\times TM} & [0]_{T\times 2TL} \\ [0]_{T\times TM} & [T_o(i)\times(g_l(t)-g(i))]_{T\times TM} & [0]_{T\times TM} & [0]_{T\times 2TL} \end{bmatrix}_{2T\times(3TM+2TL)} \tag{3.40}$$

  The upper and lower limit vectors for this constraint are same as Model #1 as shown in Eq. (3.11).

- Processing capacity constraint: Eq. (2.13).

  Same as grade blending constraint, $\mathbf{A}_{processing}$ is modified. The new variables do not have any contribution in processing capacity constraints as well. Therefore, the coefficients related to these variables are zero.

$$\mathbf{A}_{processing} = \begin{bmatrix} [0]_{T\times TM} & [T_o(i)]_{T\times TM} & [0]_{T\times TM} & [0]_{T\times 2TL} \end{bmatrix}_{T\times(3TM+2TL)} \tag{3.41}$$

The upper and lower limit vectors are same as Model #1 that are shown in Eq. (3.14).

- Mining capacity constraint: Eq. (2.14).

  $\mathbf{A}_{Mining}$ is also updated as below:

$$\mathbf{A}_{Mining} = \left[[0]_{T\times TM} \quad [0]_{T\times TM} \quad \left[T_{Total}(i)\right]_{T\times TM} \quad [0]_{T\times 2TL} \right]_{T\times(3TM+2TL)} \tag{3.42}$$

  The upper and lower limit vectors are same as Model #1 that are shown in Eq. (3.17).

- Ore-mining constraint: Eq. (2.15).

$$\mathbf{A}_{Ore\_Mining} = \left[[0]_{TM\times TM} \quad I_{TM\times TM} \quad I_{TM\times TM} \quad [0]_{TM\times 2TL} \right]_{TM\times(3TM+2TL)} \tag{3.43}$$

  The upper limit vectors are same as Model #1 that are shown in Eq.(3.21).

- Extraction precedence constraints: Eqs. (2.16) to (2.18).

  For these constraints, the coefficient matrix, $\mathbf{A}_{precedence}$, is calculated by modifying Eq. (3.26). The result is shown in Eq.(3.44). The upper and lower limits are exactly same as Eq. (3.27).

$$\mathbf{A}_{precedence} = \begin{bmatrix} \left[[1(t;i)]_{TMK\times TM} \quad [0]_{TMK\times TM} \quad [-1(u;j)]_{TMK\times TM} \quad [0]_{TMK\times 2TL} \right] \\ \left[-I_{TMK\times TM} \quad [0]_{TMK\times TM} \quad [1(u;i)]_{TMK\times TM} \quad [0]_{TMK\times 2TL} \right] \\ \left[[1(t;i),-1(t;i+1)]_{TMK\times TM} \quad [0]_{TMK\times TM} \quad [0]_{TMK\times TM} \quad [0]_{TMK\times 2TL} \right] \end{bmatrix}_{(TM(K+2))\times(3TM+2TL)} \tag{3.44}$$

- Reserve constraint: Eq. (2.19) or Eq. (2.20).

  Same as the previous constraints, the coefficient matrix, $\mathbf{A}_{Reserve}$, that has been presented in Eq. (3.29) is updated as Eq.(3.45).

$$\mathbf{A}_{Reserve} = \left[[0]_{M\times TM} \quad [0]_{M\times TM} \quad [1(1\ldots T;i)]_{M\times TM} \quad [0]_{M\times 2TL} \right]_{M\times(3TM+2TL)} \tag{3.45}$$

  The upper limit is equal to Eq. (3.30) and lower limit is equal to Eq. (3.31) or Eq. (3.32) .

- Over and under production constraints: Eq. (2.36).

  These constraints assign the tonnage of over and under production for realization l in period t. If there is a surplus ore at realization l in period t, $T_{op}(t;l)$ is a positive number and equals to the tonnage of overproduction and $T_{up}(t;l)$ is zero. If there is a shortfall from target production at realization l in period t, $T_{up}(t;l)$ equals to the tonnage of underproduction and $T_{op}(t;l)$ is zero. The matrix for the coefficients of these constraints is called $\mathbf{A}_{OU\_Prod}$ which is shown in Eq. (3.46). The upper and lower limit vectors of the constraints are called $\mathbf{b}_{U\_OU}$ and $\mathbf{b}_{L\_UO}$. They are shown in Eq.(3.47):

$$\mathbf{A}_{OU\_Prod} = \begin{bmatrix} [0]_{T\times TM} & [T_o(i)]_{T\times TM} & [0]_{T\times TM} & [-T_{op}(i;l)]_{T\times TL} & [0]_{T\times TL} \\ [0]_{T\times TM} & [-T_o(i)]_{T\times TM} & [0]_{T\times TM} & [0]_{T\times TL} & [-T_{up}(i;l)]_{T\times TL} \end{bmatrix}_{2T\times(3TM+2TL)} \tag{3.46}$$

$$\begin{aligned} \mathbf{b}_{U\_OU} &= \begin{bmatrix} [p_u(t)]_{T\times 1} \\ [-p_u(t)]_{T\times 1} \end{bmatrix}_{2T\times 1} \\ \mathbf{b}_{L\_UO} &= [-\inf]_{2T\times 1} \end{aligned} \tag{3.47}$$

The general form of the constraints are shown in Eq. (3.48). All of the coefficient matrices are collected in one matrix called $\mathbf{A}$. The upper and lower limits for the constrains are gathered in two vectors called $\mathbf{b}_U$ and $\mathbf{b}_L$:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{grade} \\ \mathbf{A}_{processing} \\ \mathbf{A}_{OU\_Prod} \\ \mathbf{A}_{Mining} \\ \mathbf{A}_{Ore\_Mining} \\ \mathbf{A}_{precedence} \\ \mathbf{A}_{Reserve} \end{bmatrix}_{(4T+2TL+TM(K+3)+M)\times 3TM}$$
$$\mathbf{b}_U = \begin{bmatrix} \mathbf{b}_{U\_grade} \\ \mathbf{b}_{U\_processing} \\ \mathbf{b}_{U\_OU} \\ \mathbf{b}_{U\_Mining} \\ \mathbf{b}_{U\_Ore\_Mining} \\ \mathbf{b}_{U\_precedence} \\ \mathbf{b}_{U\_reserve} \end{bmatrix}_{(4T+2TL+TM(K+3)+M)\times 1}, \mathbf{b}_L = \begin{bmatrix} \mathbf{b}_{L\_grade} \\ \mathbf{b}_{L\_processing} \\ \mathbf{b}_{L\_OU} \\ \mathbf{b}_{L\_Mining} \\ \mathbf{b}_{L\_Ore\_Mining} \\ \mathbf{b}_{L\_precedence} \\ \mathbf{b}_{L\_reserve} \end{bmatrix}_{(4T+2TL+TM(K+3)+M)\times 1} \tag{3.48}$$

3.5.3 Implementation of Model #3: MILP Considering Grade Uncertainty With Stockpile

The main idea with this model is that any overproduced ore is transferred to the next periods. Therefore, Model #2 is modified. The cost of overproduction is reduced since the only cost of overproduction is the loss of ore value due to the selling the commodity at the latter years. This loss is calculated by the difference of the discounted ore value in two periods. Therefore, two main changes are needed to implement Model #3. The implementation details for this model are presented as below:

- $\mathbf{x}$ vector: same as Eq. (3.35).
- Objective function: Eq. (2.38).

  The format of the objective function is same as Model #2 that is shown in Eq. (3.38). However, the cost of overproduction is different and it is shown in Eq. (3.49) and Eq. (2.28).

$$\overline{c}_{op}\left(t,l\right)=\frac{\hat{c}_{op,RH}\left(t,l\right)}{L} \qquad \forall l=1,2,\ldots,L \tag{3.49}$$

- Grade blending constraint: Eq. (2.12).

  It is same as Model #2 as shown in Eq. (3.40).
- Processing capacity constraint: Eq. (2.13).

  It is same as Model #2 as presented in Eq. (3.41).
- Ore-mining constraint Eq. (2.15).

  It is same as Model #2 as indicated in Eq. (3.43).
- Extraction precedence constraints: Eqs. (2.16) to (2.18).

  They are same as Model #2 as shown in Eq. (3.44)
- Reserve constraint: Eq. (2.19) or Eq. (2.20).

  Both cases are same as Model #2 as presented in Eq. (3.45)
- Over and underproduction constraints: Eq. (2.21) and Eq. (2.23).

  These constraints are needed to be modified in order to account for the overproduced ore from the previous period. Eq. (3.46) is modified as below to Eq. (3.50). The upper and lower limits of this constraint are also same as Model #2 as demonstrated in Eq. (3.47).

$$\mathbf{A}_{OU\_Prod} = \begin{bmatrix} [0]_{T\times TM} & [T_o(i)]_{T\times TM} & [0]_{T\times TM} & [T_{op}(t-1;l), -T_{op}(i;l)]_{T\times TL} & [0]_{T\times TL} \\ [0]_{T\times TM} & [-T_o(i)]_{T\times TM} & [0]_{T\times TM} & [T_{op}(t-1;l)]_{T\times TL} & [-T_{up}(i;l)]_{T\times TL} \end{bmatrix}_{2T\times(3TM+2TL)} \tag{3.50}$$

- Stockpile capacity constraint: Eq. (2.40).

  The stockpile limit constraints are consistent of *TL* constraints. The general form of this constraint is shown as below:

$$\mathbf{b}_{L\_Stockpile} < \mathbf{A}_{Stockpile}\mathbf{x} \le \mathbf{b}_{U\_Stockpile} \tag{3.51}$$

  For each realization in each period the overproduced ore is controlled by these constraints. The coefficient matrix called $\mathbf{A}_{Stockpile}$ is shown in Eq. (3.52).

$$\mathbf{A}_{Stockpile} = \begin{bmatrix} [0]_{TL\times TM} & [0]_{TL\times TM} & [0]_{TL\times TM} & [T_{op}(i;l)]_{TL\times TL} & [0]_{TL\times TL} \end{bmatrix}_{TL\times(3TM+2TL)} \tag{3.52}$$

  The upper and lower limits are shows in Eq. (3.53).

$$\begin{aligned} \mathbf{b}_{U\_Stockpile} &= \left[S_u(t)\right]_{TL\times 1} \\ \mathbf{b}_{L\_Stockpile} &= \left[S_l(t)\right]_{TL\times 1} \end{aligned} \tag{3.53}$$

  $\left[S_l(t)\right]_{TL\times 1}$ and $\left[S_u(t)\right]_{TL\times 1}$ are vertical vectors with size of *TL*.

The general form of the constraints are shown in Eq. (3.54). All of the coefficient matrices are collected in one matrix, $\mathbf{A}$, and the upper and lower limits for the constrains are gathered in two vectors called $\mathbf{b}_U$ and $\mathbf{b}_L$:

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}_{grade}\\ \mathbf{A}_{processing}\\ \mathbf{A}_{OU\_Prod}\\ \mathbf{A}_{Stockpile}\\ \mathbf{A}_{Mining}\\ \mathbf{A}_{Ore\_Mining}\\ \mathbf{A}_{precedence}\\ \mathbf{A}_{Reserve}\end{bmatrix}_{(4T+3TL+TM(K+3)+M)\times 3TM}$$

$$\mathbf{b}_{U}=\begin{bmatrix}\mathbf{b}_{U\_grade}\\ \mathbf{b}_{U\_processing}\\ \mathbf{b}_{U\_OU}\\ \mathbf{b}_{U\_Stockpile}\\ \mathbf{b}_{U\_Mining}\\ \mathbf{b}_{U\_Ore\_Mining}\\ \mathbf{b}_{U\_precedence}\\ \mathbf{b}_{U\_reserve}\end{bmatrix}_{(4T+2TL+TM(K+3)+M)\times 1},\mathbf{b}_{L}=\begin{bmatrix}\mathbf{b}_{L\_grade}\\ \mathbf{b}_{L\_processing}\\ \mathbf{b}_{L\_OU}\\ \mathbf{b}_{L\_Stockpile}\\ \mathbf{b}_{L\_Mining}\\ \mathbf{b}_{L\_Ore\_Mining}\\ \mathbf{b}_{L\_precedence}\\ \mathbf{b}_{L\_reserve}\end{bmatrix}_{(4T+2TL+TM(K+3)+M)\times 1} \tag{3.54}$$

3.5.4 Implementation of Model #4: MIQP for Mean-Variance Approach

In this section, the mixed integer quadratic programming that has been presented in Eq. (2.49) is transformed into the standard form of TOMLAB. Model #2 is used and modified. The general form of MIQP problems can be written as below:

$$Max\ \frac{\mathbf{1}}{\mathbf{2}}\mathbf{x}'\mathbf{Q}\mathbf{x}+\mathbf{c}'\mathbf{x} \tag{3.55}$$

Subject to:

$$\mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \tag{3.56}$$

$$\mathbf{b}_L \le \mathbf{A}\mathbf{x} \le \mathbf{b}_U \tag{3.57}$$

Where

- $\mathbf{x}$, $\mathbf{c}$, $\mathbf{x}_L$, $\mathbf{x}_U$, $\mathbf{A}$, $\mathbf{b}_L$, $\mathbf{b}_U$ and $\mathbf{A}$ are same as the general form of MILP, and they have been described in Model #2. The implementation details for each of them have been shown in Section 3.5.1.
- $\mathbf{Q}$: The quadratic part of the objective function.

  This is a diagonal matrix with the size of $3TM \times 3TM$. $\mathbf{Q}$ is the covariance matrix of all parameters that are used in NPV calculation in each realization. Since only variance of ore value is considered in the objective function presented in Eq.(2.49), the coefficients for $z(t;i)$ that are $\sigma_v^2\left(t;i\right)$ must only be included in $\mathbf{Q}$ matrix. $\mathbf{Q}$ is a diagonal matrix in which each diagonal elements is the variance of a variable, and other elements are zero. This matrix can be calculated by eq. (3.58) as below:

$$\mathbf{Q} = \begin{bmatrix} [0]_{TM\times TM} & [0]_{TM\times TM} & [0]_{TM\times TM} \\ [0]_{TM\times TM} & \left[\sigma_v^2\left(t;i\right)\right]_{TM\times TM} & [0]_{TM\times TM} \\ [0]_{TM\times TM} & [0]_{TM\times TM} & [0]_{TM\times TM} \end{bmatrix}_{3TM\times 3TM} \tag{3.58}$$

Where $\left[\sigma_v^2\left(t;i\right)\right]_{TM\times TM}$ is also the diagonal matrix in which each diagonal element is the variance of the ore value of mining cut *i* in period *t*.

$\mathbf{Q}$ is a "strictly diagonally dominant matrix". It means that the magnitude of the diagonal entry in a row is larger than or equal to the sum of the magnitudes of all the other (non-diagonal) entries in that row. It can be proved that these type of matrices are positive semi-definite. Also $\mathbf{Q}$ is the covariance matrix of NPV in presence of grade uncertainty. It is a well known fact that all covariance matrices are positive definite. Any quadratic optimization problem with a positive semi-definite Hessian matrix is convex. Therefore, there is always a unique solution for this optimization problem which is also the global optimum solution.

3.6 Summary

In this chapter, the programs and steps that are required to generate geostatistical model by GSLIB software were described. The estimation model is imported to Whittle to generate the final pit limit that is required for optimization stage. A program called `MSQ90` was presented. This program is used to assess the grade uncertainty for the production plan, remove the blocks outside the final pit, apply the cut-off grade to the blocks and calculate the economic parameters for each block and realization. The generated files from `MSQ90` are used as an input to the clustering. The clustering technique and the implementation details were presented. Finally, in this chapter, the implementation details for each of four proposed models have been showed. All the generated matrices and vectors are in standard input format for TOMLAB solver.

Chapter 4 Case Study

This chapter focuses on the application of the approaches that are presented in Chapter 2. A data set from an oil sands deposit has been used for this purpose. In Section 4.1, summary statistics of the data are presented. In this section, a geostatistical workflow is followed to generate a block model. Ordinary kriging is used to estimate the bitumen grade and sequential Gaussian simulation is used to simulate 50 realizations. In Section 4.3, the Whittle software is employed to generate the ultimate pit limit and the long-term production plan using the estimated values. The simulated bitumen grade values are used to assess the effect of grade uncertainty on the long-term production schedule. Section 4.5 shows the schedule generated by Model #1 based on the estimated values. Section 4.6 presents the results of Model #2. In this model the realizations are used to penalize input ore tonnage fluctuations into the plant that are caused by grade uncertainty. A symmetric penalty function has been used for this model. In Section 4.7, an optimum production schedule has been generated using Model #3 with an asymmetrical penalty function. In this model the stockpile is modeled as part of the optimization stage. In each section, the generated schedules are compared to each other. Finally, in Section 4.8 a summary of the chapter and some conclusions are presented.

4.1 Practical regulations of oil sands mining in Alberta

The Energy Resources Conservation Board (ERCB) has been succeeded by the Alberta Energy Regulator (AER). The Alberta Energy Regulator is a regulatory body with a mandate to provide for the efficient, safe, orderly, and environmentally responsible development of Alberta's energy resources. The following information is extracted from directive 082. (http://www.aer.ca/). Here is some information that is required for a long term mine planning project for oil sand deposits in Alberta.

4.1.1 Bitumen Recovery

The in situ oil sands cut-off grade is defined as the minimum bitumen content of the oil sands that would be classified as ore. It has been set at 7 weight per cent bitumen. The minimum mining thickness (mining selectivity) is defined as the minimum thickness of ore that can be separated from waste or waste that can be separated from ore. It has been set at 3 meters (m). Processing plant recovery is a variable factor based on the average bitumen content of the as-mined ore. The factor is determined as follows:

- If the average bitumen content of the as-mined ore is 11 weight per cent bitumen or greater, the recovery factor is 90 weight per cent.
- If the average bitumen content of the as-mined ore is less than 11 weight per cent bitumen, recovery is determined by the following equation, where x is the average weight per cent bitumen content of the as-mined ore:
  $\text{Recovery} = -202.7 + 54.1(x) - 2.5(x)$

4.1.2 Drilling Density Requirements

For all areas subject to development within the first ten years and for a 1 kilometer (km) buffer around these areas, the maximum spacing between drillholes meeting the drilling data quality requirements must be 350 m as determined by triangulation. If any one side of the triangle is greater than 350 m, the ERCB will determine if additional drilling is required.

For all other areas subject to development after the first ten years and for a 1 km buffer around these areas, the maximum spacing between drillholes meeting the drilling data quality requirements must be 700 m as determined by triangulation. If any one side of the triangle is greater than 700 m, the ERCB will determine if additional drilling is required.

4.2 Input Data

The input data set belongs to an oil sand deposit in Fort McMurray, Alberta, Canada. The only rock property that is used in this study is mass percent of the bitumen. There are 210 vertical boreholes in this data set. The location of the boreholes is shown in Figure 4-1. The horizontal and vertical scales are in meters. This figure also shows the boundary of the ore body. This polygon is chosen manually based on an approximately a constant distance from the boreholes. All the blocks outside this polygon are considered as waste.

The histogram and cumulative density function (CDF) of the data are shown in Figure 4-2. The data set has 7887 samples of the mass percent bitumen with a mean and standard deviation of 8.73 and 4.76 m%, respectively. The P10, P50 (median) and P90 of the data, respectively are: 1.9, 9.2 and 14.7. Also, from the CDF curve 68% of the data are above 6 m% of bitumen. This is the operational cut-off grade that is used in this case study. This is 1 weight per cent less than the cut-off grade that has been defined by AER.

Also, the recovery factor in the case study is assumed to be a constant number for all input grades. This has been explained in Section 2.2. On the other hand, stockpiles are not popular in oil sand open pit mines. Due to the huge size of oil sand operations, it is almost impractical to have a stockpile and keep the ore in the stockpile. However, the presented case study in this chapter has tried to show the application and results of Model #1 and #2 without stockpile and Model #3 in which the stockpile is considered.

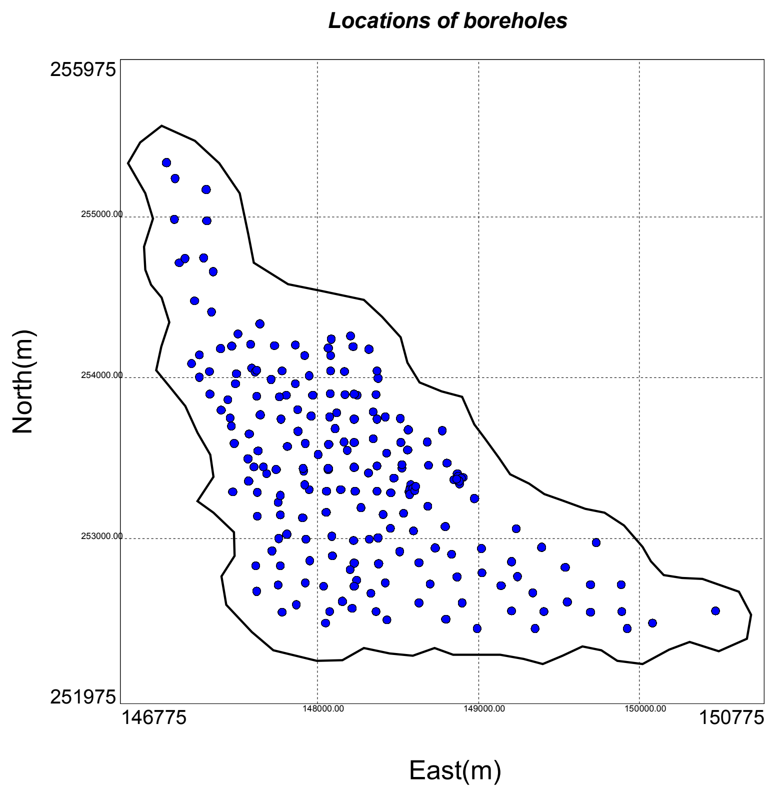


Figure 4-1: location map of boreholes and the boundary of the ore body in meters.

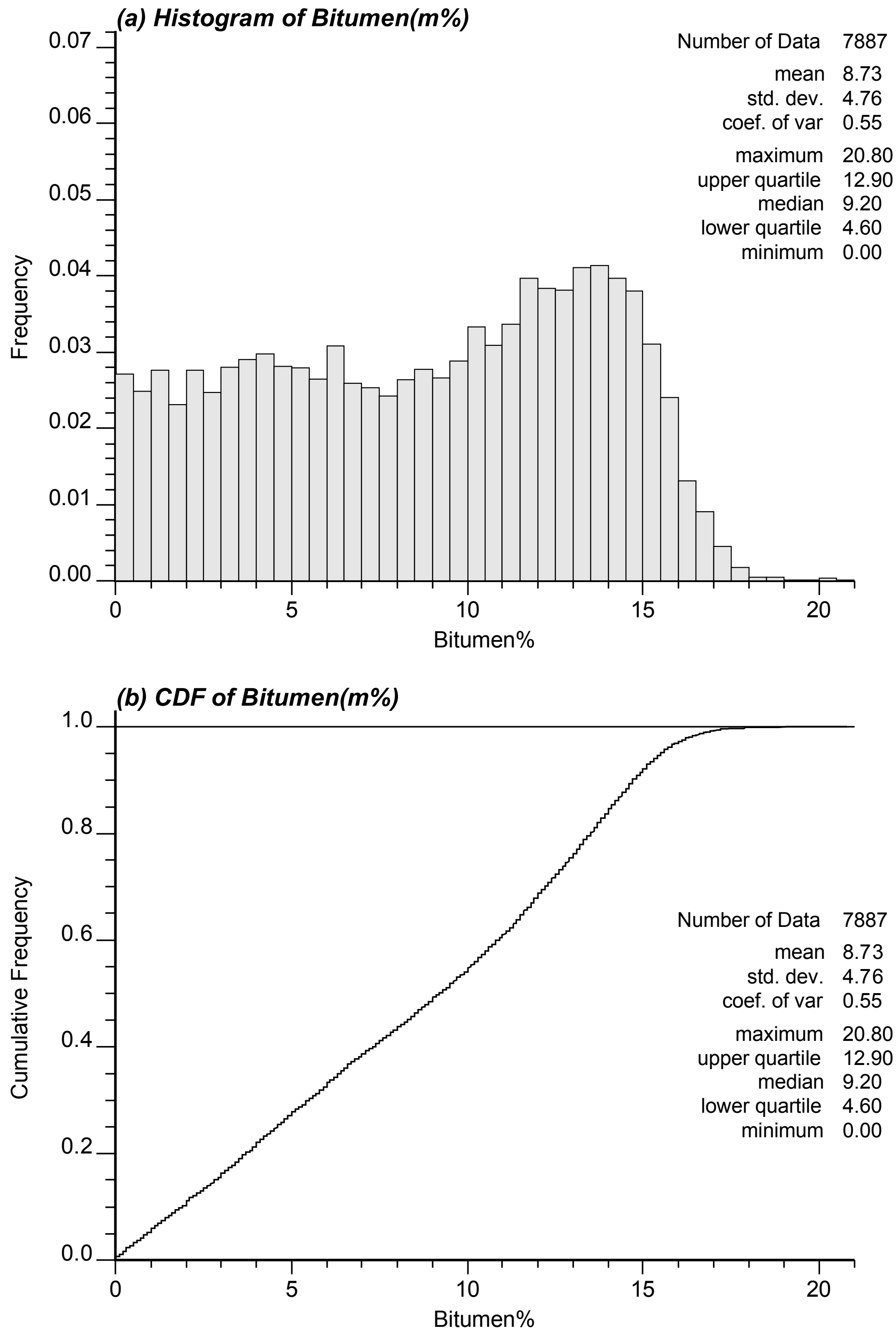


Figure 4-2: (a) Histogram and (b) the CDF of bitumen grade (m%).

As described in Section 3.2, the next step is to find the representative distribution of the bitumen by reducing the effect of clustered samples. The cell declustering technique is used for this purpose. Since the clustered wells have been drilled in high grade zones, the cell declustering method is used to correct the mean of the data. Figure 4-3 shows a cross-plot of cell size versus the declustered mean. From this graph, the optimum cell size is chosen as 45m. From the location map of the boreholes in Figure 4-1, it is concluded that the cell de-clustering is affected by vertical clustered samples. Figure 4-4 shows the vertical locations of the samples at each borehole. It is clear that the distance between the samples is not regular and in some areas the density of the samples is less than other boreholes. In order to check the results of cell-declustering method, another technique called Global Kriging method declustering method is applied. In this method the cumulative sum of the weights that each data location receives is calculated. These cumulative weight sums can be used as declustering weights as those data located in a densely sampled area will not receive as much weight as those data located in sparsely sampled areas. The average of the bitumen grade with this method is reduced to 7.27 m%. This is close to the cell declustering method that is performed for this data set and proves that the data has vertical declustering.

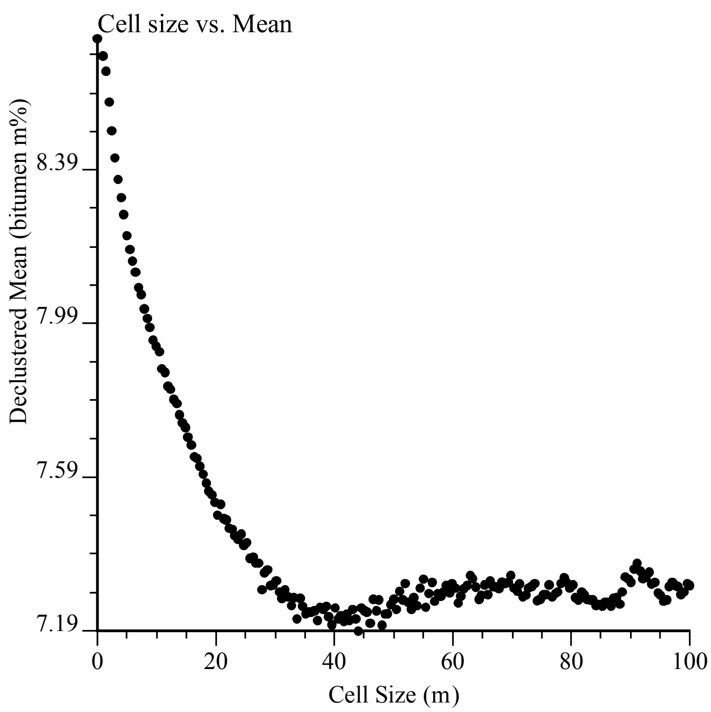


Figure 4-3: Scatter-plot of cell sizes versus declustered mean

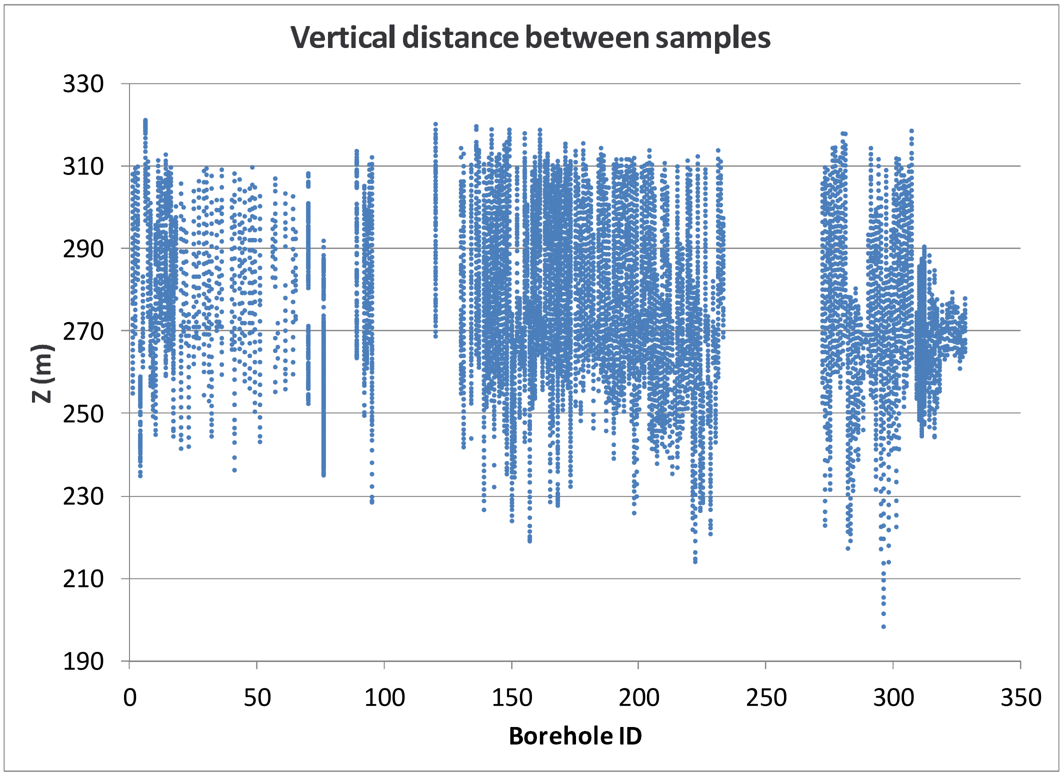


Figure 4-4: The vertical locations of the samples at each borehole

Figure 4-5 shows the histogram and CDF of the bitumen grade using the declustering weights. The representative distribution of the data has a mean and standard division of 7.19 and 5.00 mass percent respectively. Also P10, P50 and P90 of the data respectively are: 0.70, 6.60 and 14.2 m%. The grade of the 64.0 percent of declustered data is above 6.00 m%.

Both histograms (Figure 4-2 and Figure 4-5) clearly show a bimodal distribution. This also can also be seen in Figure 4-6. This graph shows the normal probability plot of the bitumen grade using declustering weights. The two populations could be separated into two domains; however during this thesis, the modeling techniques are aimed at the grade uncertainty on the production plan. Therefore, all of the data are considered in one domain.

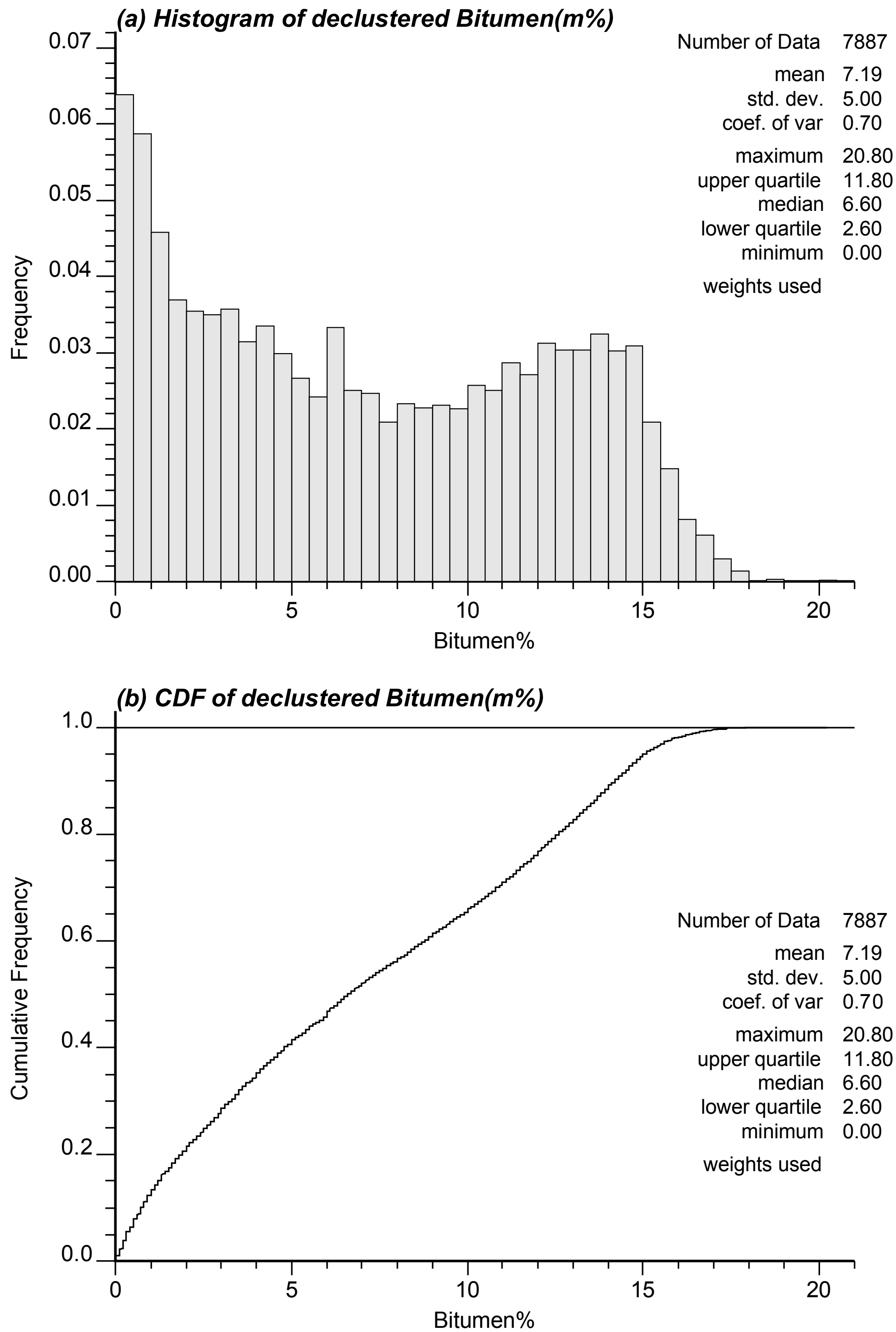


Figure 4-5: (a) Histogram and (b) the CDF of bitumen with declustered weights

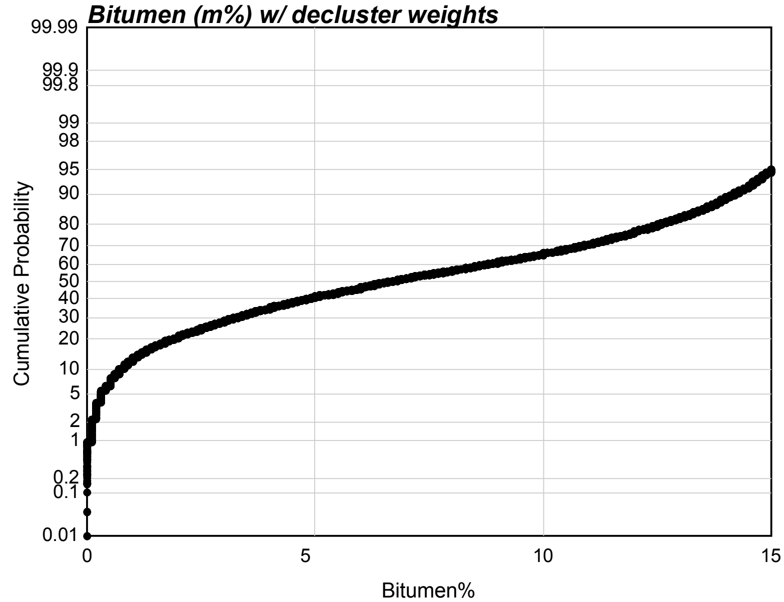


Figure 4-6: Probability plot of bitumen using declustered weights.

Directional experimental variograms are calculated and a theoretical variogram model is fitted. The azimuths of the major and minor directions were found to be 50 and 140 degrees. Figure 4-7 shows the experimental and the fitted variogram models in major (Figure 4-7a), minor (Figure 4-7b) and vertical (Figure 4-7c) directions. The variogram model consists of two nested spherical models. The general equation for an isotropic spherical variogram is given by Eq. (4.1).

$$\gamma(h)=\begin{cases}0 & if\ h=0\\ C_0+C\times\left(\frac{3}{2}\frac{h}{a}-\frac{1}{2}\frac{h^3}{a^3}\right) & if\ h\leq a\\ C_0+C & if\ h>a\end{cases} \tag{4.1}$$

Where

- C_0 is the nugget effect.
- C is the sill of the variogram model.
- a is the range of the variogram.

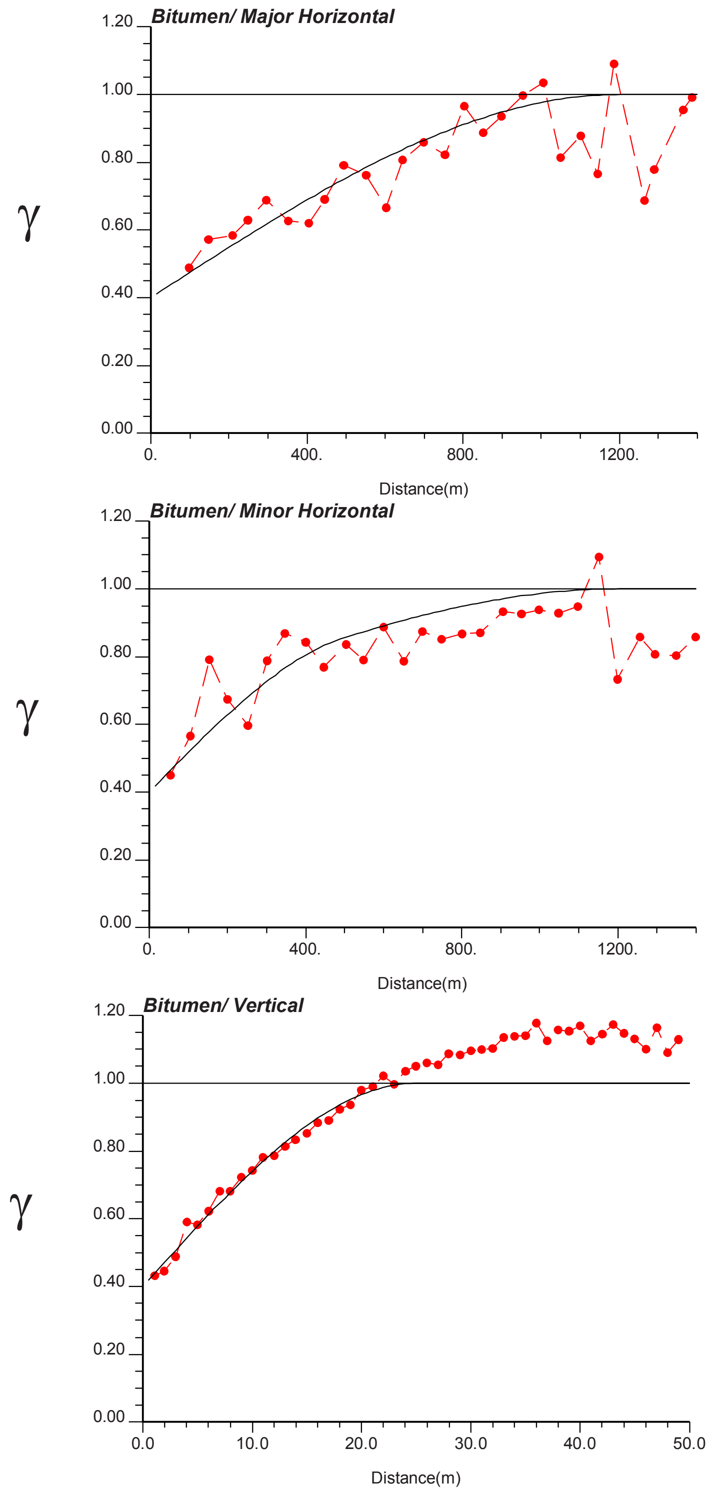


Figure 4-7: Experimental directional variograms (dots) and the fitted variogram models (solid lines), distance units in meter

Table 4-1 shows the parameters for the variogram model that were used for bitumen modeling.

Table 4-1: Variogram model for bitumen

		Spherical				Spherical			
Bitumen	Nugget Effect	Sill	Range N50E	Range N40W	Range Vert.	Sill	Range N50E	Range N40W	Range Vert.
	0.4	0.25	1200m	500m	25m	0.35	1200m	1200m	25m

The next step is to define a regular grid that will be used for estimation and simulation. The parameters that are required to define a grid are: the distance between each grid nodes in the X, Y and Z direction, the number of grid nodes in the X, Y and Z directions and the coordinates of the first grid node. As discussed in Section 1.2, the size of the block should be chosen carefully. Using small blocks increases the size of the model and the overall number of blocks in the model. Having too many blocks increases the computational time for the modeling and optimization process. Also, for a relatively homogeneous oil sand deposit, a larger block size can be used. On the other hand, extremely large block sizes will smooth out the grade fluctuations and therefore the effect of grade uncertainty will be diminished. Therefore, considering all of these parameters, the size of the blocks is chosen to be 50 by 50 by 10 meters. The height of each level or bench is considered as the height of one block which is 10 meters. To estimate (or simulate) at the block level, the scale difference of the input data and the blocks should be taken into account: The input data are at a point scale and the calculated histogram and variograms are at this scale. Therefore, the correct way to create a geomodel at a block scale is to build a high resolution grid and then scale it up to the block size. For this case study each block was discretized into 2 points in length, 2 point in width and 5 points in height. Therefore, inside each block there are 20 point scale simulation or estimation values. The grid definition of the high resolution grid for point scale modeling is given in Table 4-2. There are 6624000 nodes in this grid. The block scale grid is also given in Table 4-3. There are 89600 blocks in this grid.

Table 4-2: High resolution grid definition for point scale modeling.

Direction	Number of nodes	Center coordinates of first node	Grid spacing
Easting	240	146000	25.0m
Northing	240	251000	25.0m
Elevation	115	190	2.0m

Table 4-3: Block scale grid definition.

Direction	Number of nodes	Center coordinates of first node	Grid spacing
Easting	80	146800	50.0m
Northing	80	252000	50.0m
Elevation	14	220	10.0m

The next step is to create the necessary bounding surfaces. There are three surfaces in this model: the surface of the ground, the top of the ore body and the base of the ore body. Each of these surfaces is created based on the borehole information. The ore body is limited between the top and base surfaces and inside the polygon defined earlier (Figure 4-1). There are three types of blocks in this block model: (1) potential ore blocks that are inside the ore body, (2) waste blocks that are outside the orebody and (3) the air blocks. Figure 4-8 shows a plan view at 290m and two perpendicular cross sections at 252750m looking north and 148250m looking east. The grey area is the waste rocks, the yellow blocks are ore and the transparent area is above the ground surface.

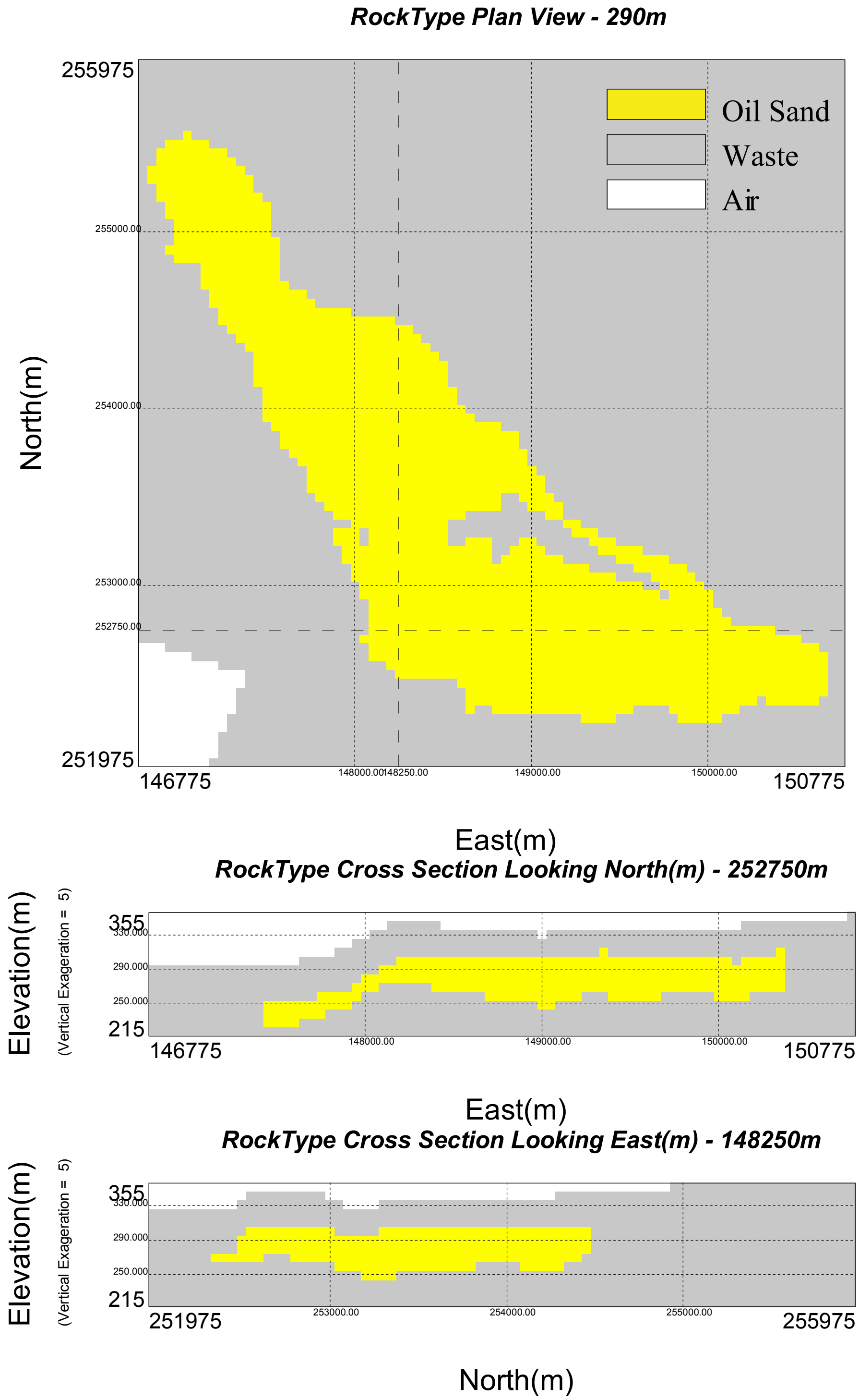


Figure 4-8: Plan view, cross section looking north and east of the ore body and waste blocks

The next step is to estimate the bitumen grade. Ordinary Kriging (OK) is used to estimate the bitumen grade in original units (without normal score transform) at each block location with a 2*2*5 block discretization. A minimum of 30 and maximum of 40 samples were used to estimate each node. The search ellipsoid is defined as two times the variogram range in each direction.

Fifty realizations of the bitumen grade are generated using Sequential Gaussian Simulation (SGS) at the high resolution grid at the point scale. The declustered weights are used in SGS to transform the data to Gaussian space and back-transform to original units. The main difference between simulation and estimation is that the estimated model does not reproduce the input histogram and variogram. Figure 4-9 shows the variogram reproduction in three major, minor and vertical directions from the realizations. Figure 4-10 shows the histogram reproduction in Gaussian units and original units. Since the histogram and variogram are reproduced quite reasonably, the generated realizations are considered representative of the grade uncertainty. Finally the block averaging step is applied to get the simulated values at block scale.

Kriging with a limited search is conditionally biased (Isaaks, 2005) and “there is no conditional bias of simulation when the simulation results are used correctly” (McLennan and Deutsch, 2004). The grade-tonnage curve is a good tool to understand the predicted values. Figure 4-11 shows the grade tonnage curve of simulation realization (dashed lines), OK (bold solid line) and E-type mean (bold dashed line) all at the block scale. The E-type mean is simply the average of the simulated values in each block. The E-type mean is slightly different than kriging because kriging is applied in original units; theoretically they would be the same in Gaussian units (Journel and Huijbregts, 1981). For this case study, the large search radii (12000m in horizontal and 250m in vertical) and up 40 conditional data has been applied for OK algorithm to reduce the conditional biasness of the OK. Using large search routines reduces the uncertainty and increases the smoothness of the OK block model. On the other hand, the uncertainty of the data is taken into account in optimization process for long term production plan.

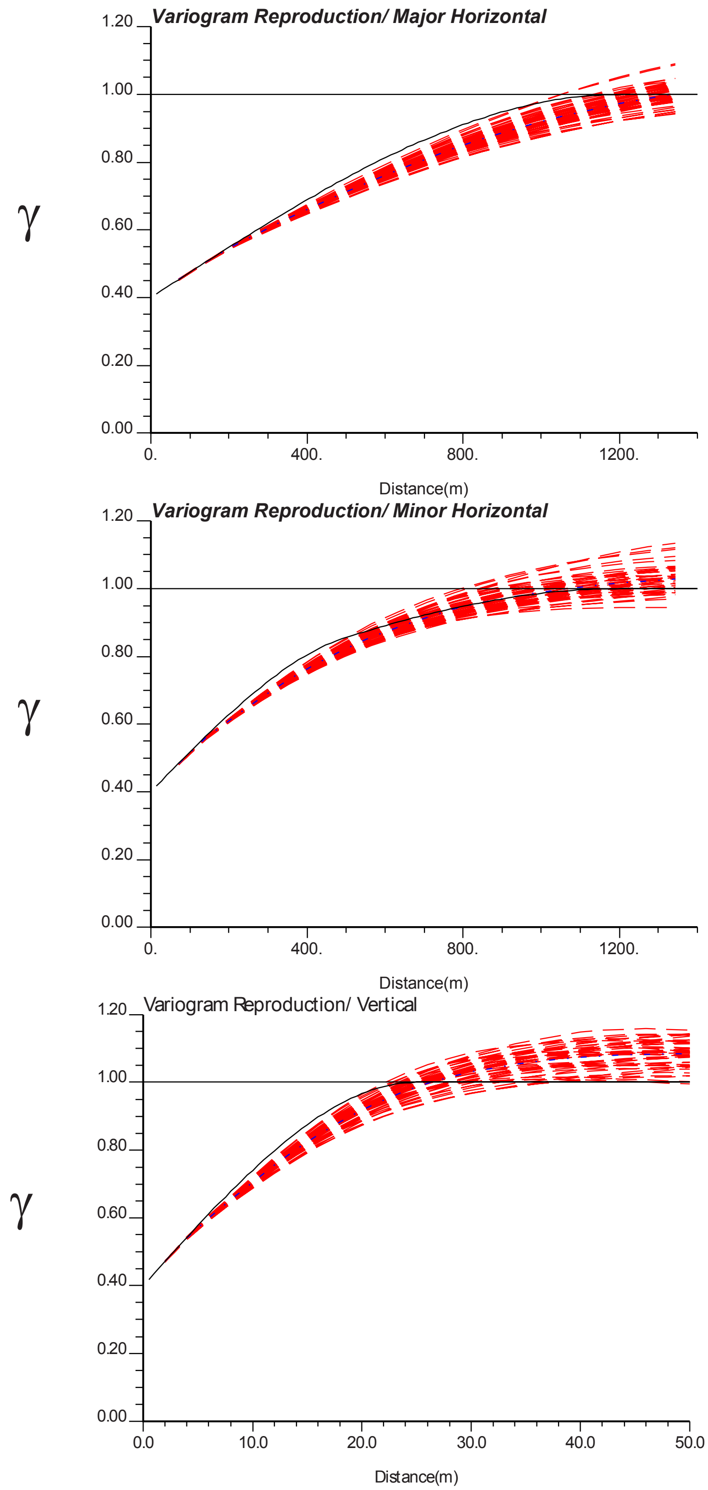
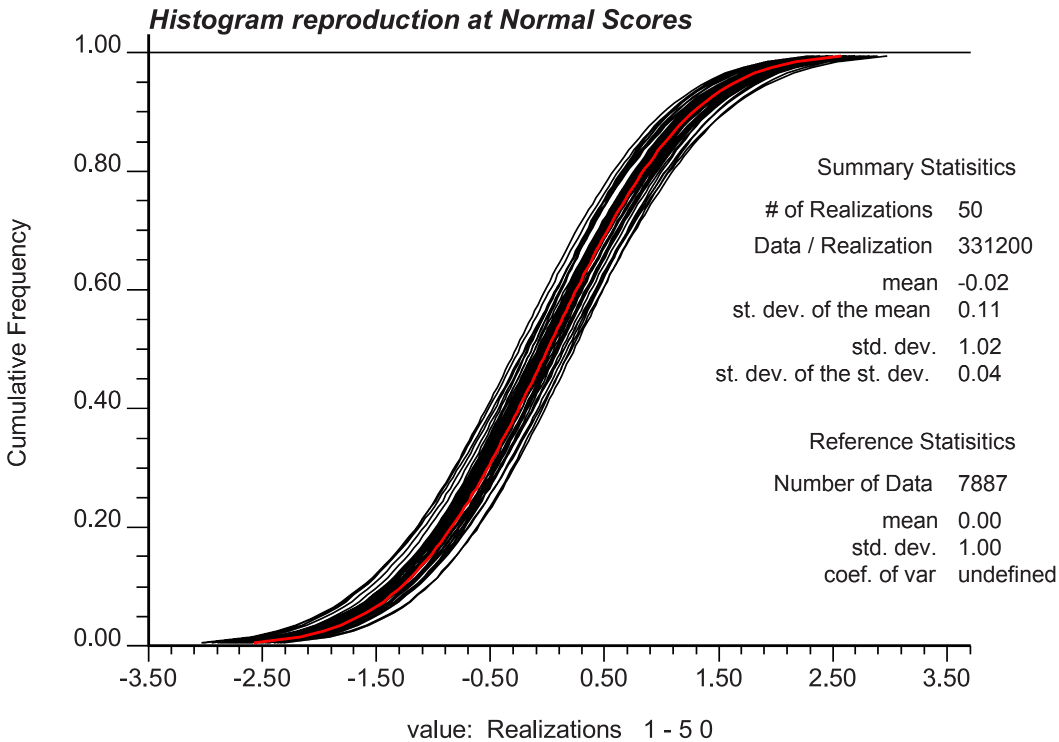


Figure 4-9: Variogram reproduction at Gaussian units of conditional simulation realizations (red dash lines), the reference variogram model (solid black lines) and the average variogram from realization (dashed blue line).



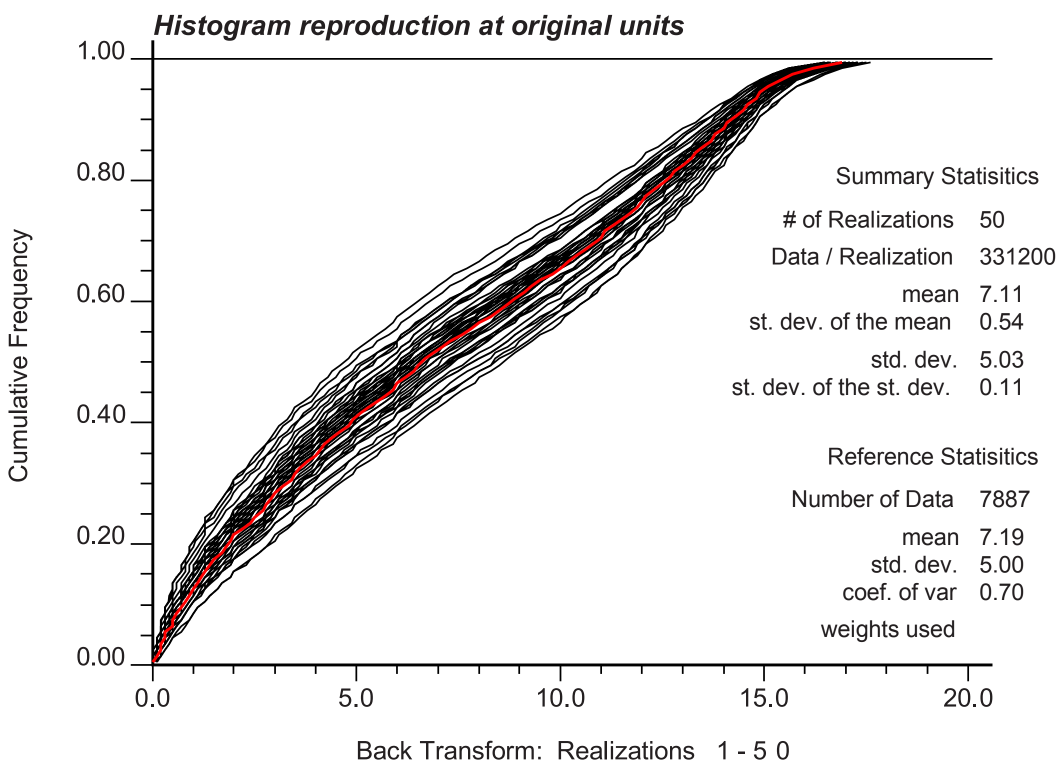


Figure 4-10: Histogram reproduction of simulation realizations (dashed lines) and histogram of original data (bold line) at Gaussian unit (top) and original unit (bottom)

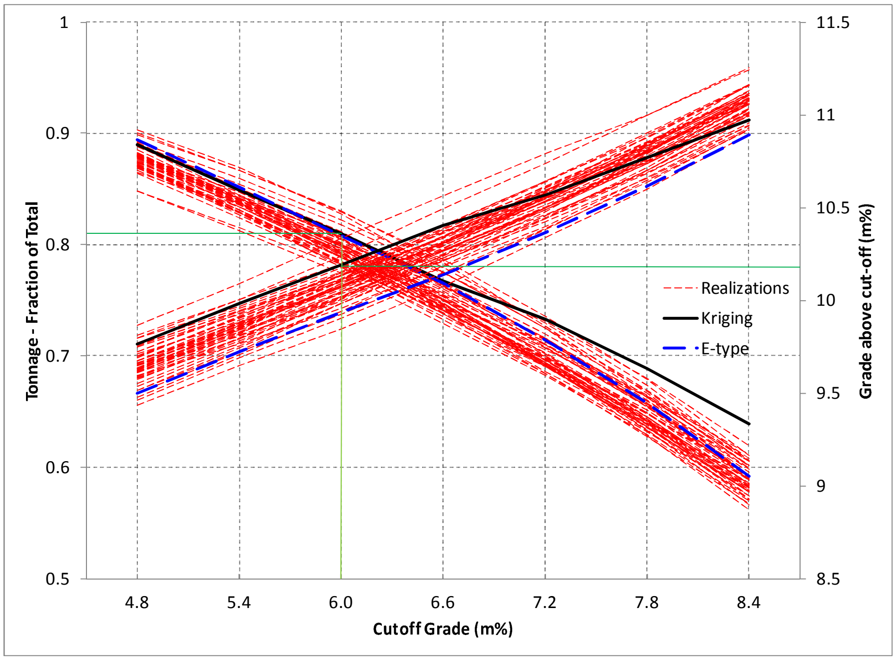


Figure 4-11: Grade tonnage curve of simulation realizations, OK, and E-type mean at block scale

All of the simulated and estimated blocks outside the area of interest are clipped and removed from the model. The same plan view and cross sections that are shown in Figure 4-8 are shown in Figure 4-12 for the kriged bitumen grade. Figure 4-13 and Figure 4-14 show the same maps and cross sections for the E-type mean and E-type variance, respectively. The E-type variance shows the areas that are uncertain. The uncertainty at the center of the domain is lower than the edges where there are fewer drillholes. Realization number 26 is shown. Figure 4-15 shows the plan view and the cross sections.

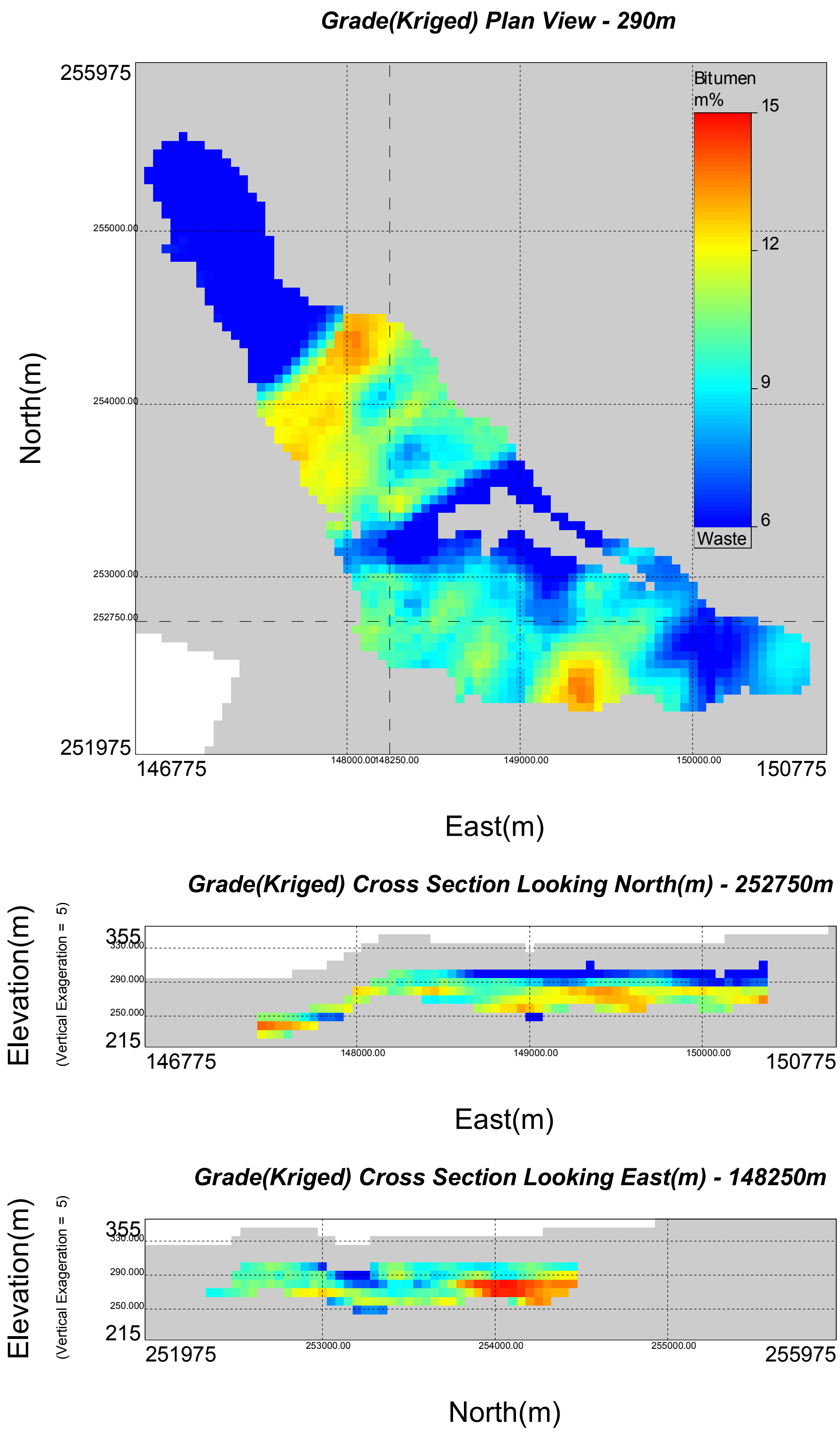


Figure 4-12: Plan view, cross section looking north and east of estimated Bitumen grade (m%) using OK method.

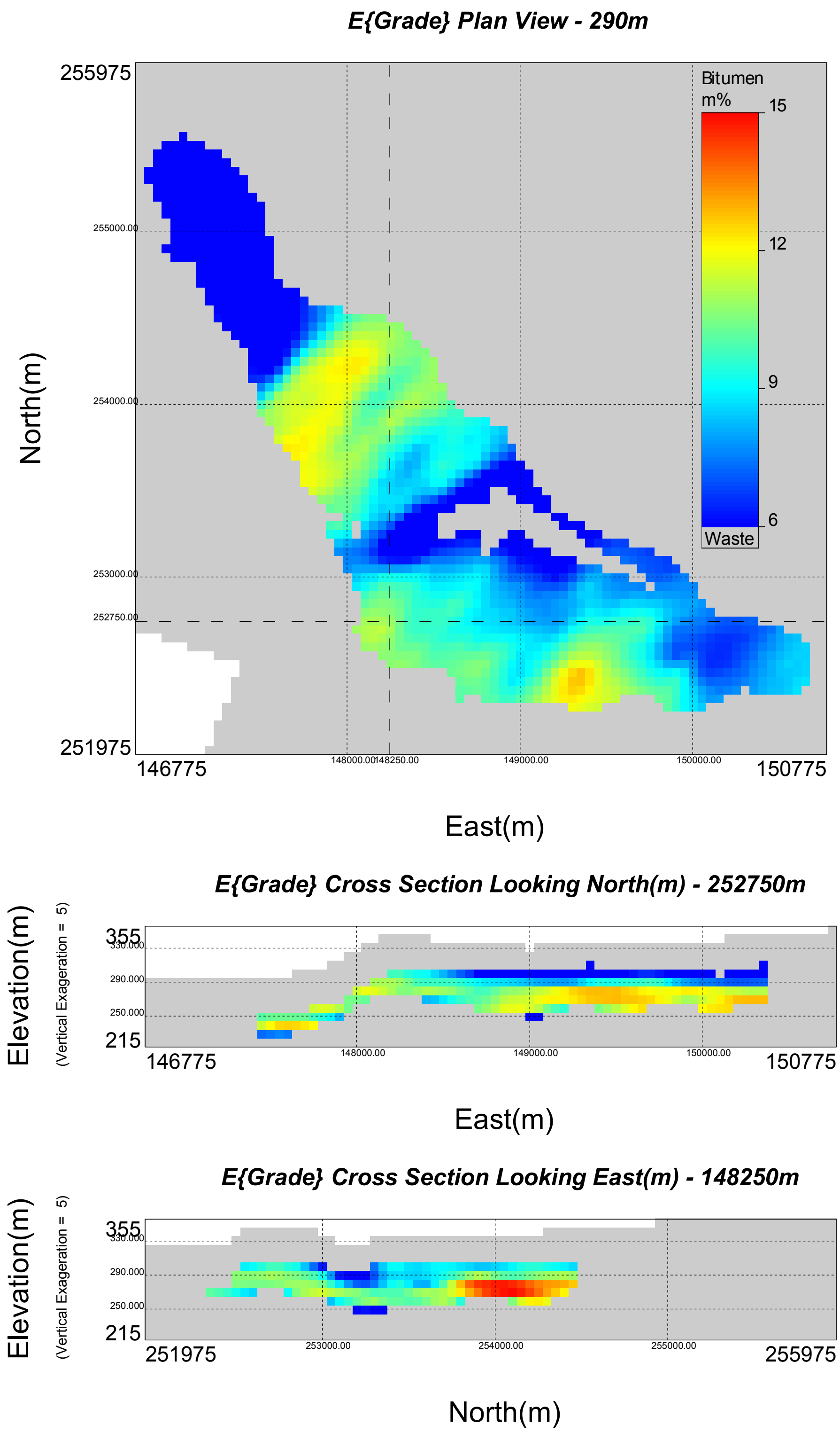


Figure 4-13: Plan view, cross section looking north and east of E-Type mean of Bitumen grade (m%) using 50 conditional realizations.

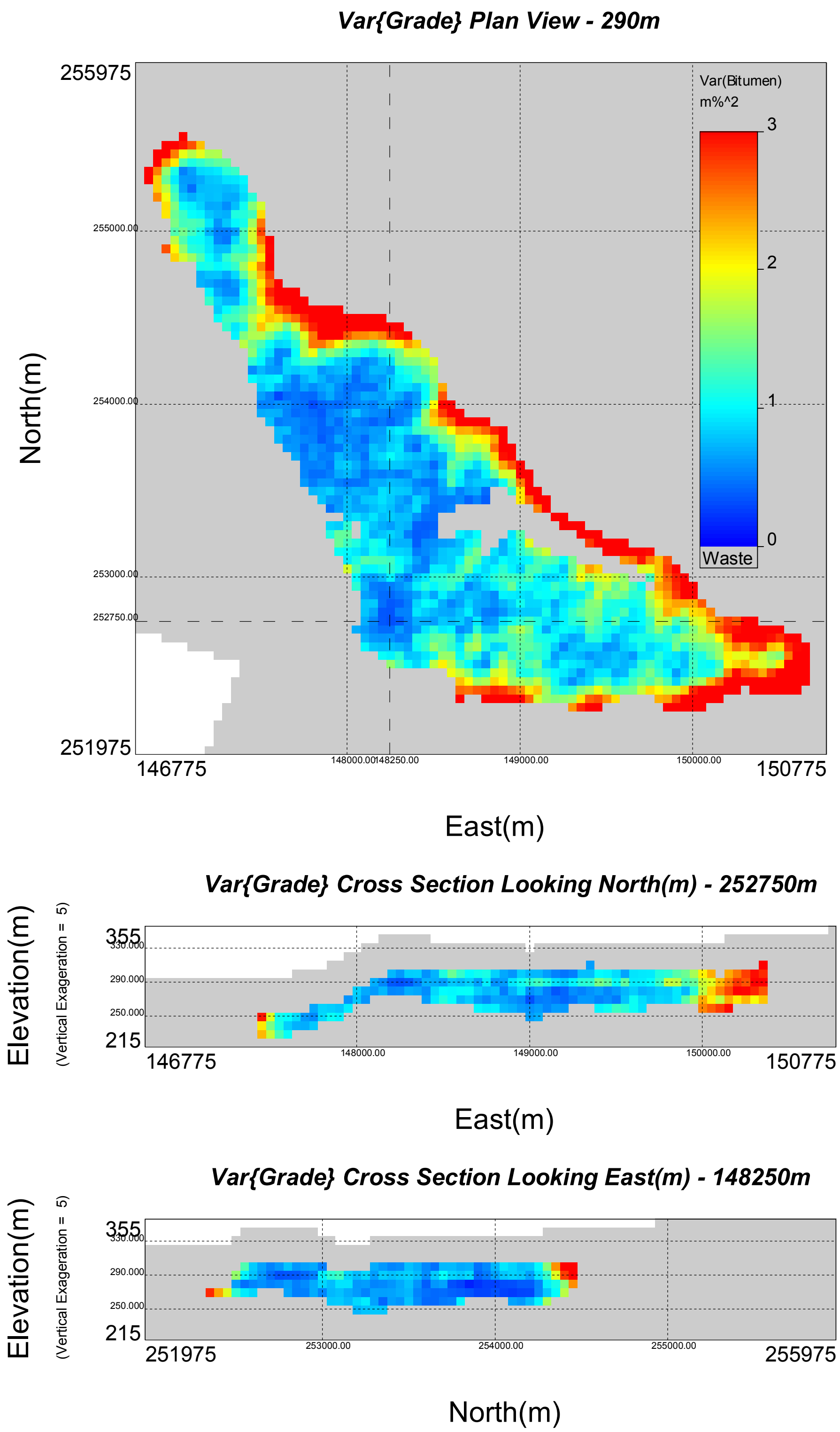


Figure 4-14: Plan view, cross section looking north and east of E-Type variance of Bitumen grade (m%^2) using 50 conditional realizations.

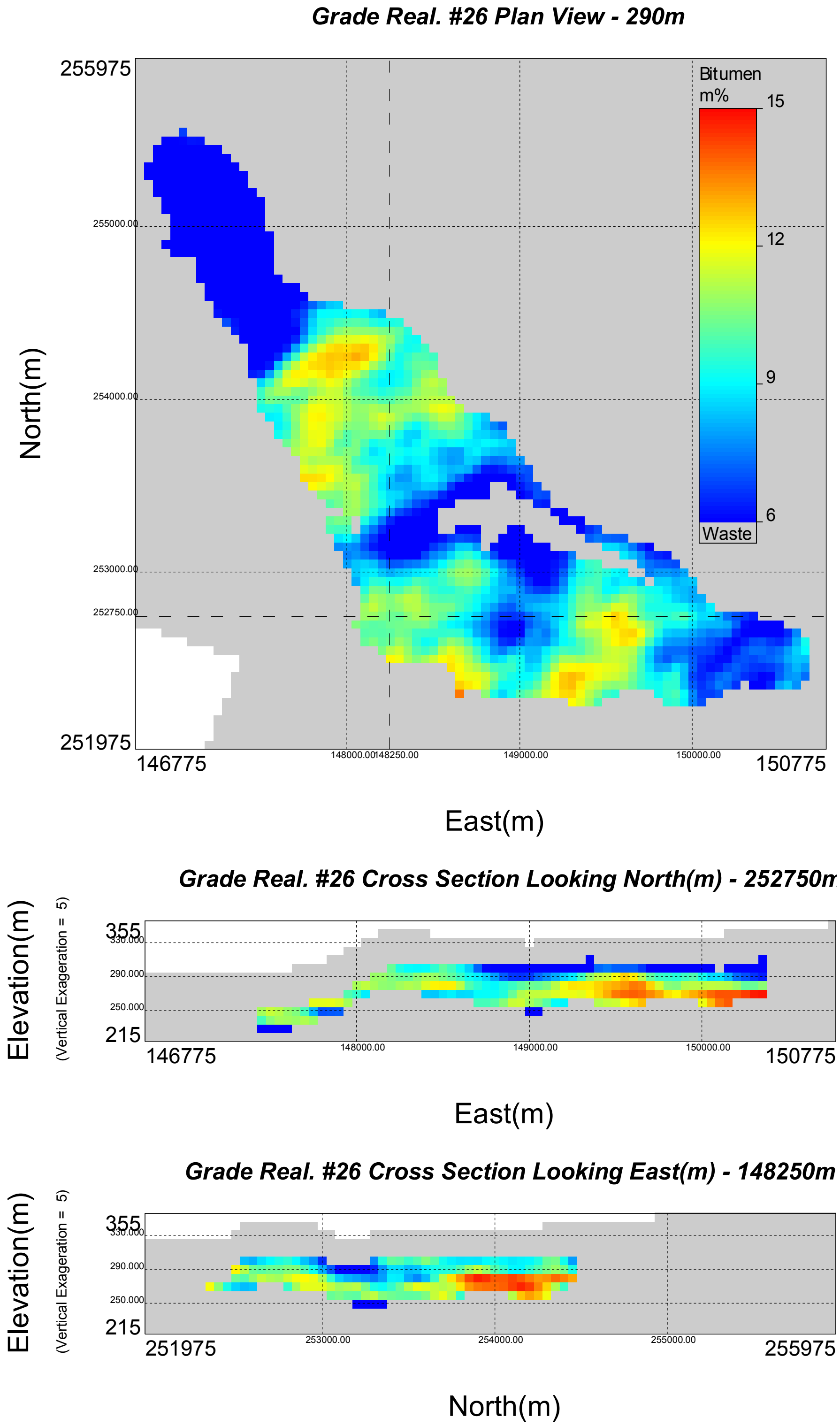


Figure 4-15: Plan view, cross section looking north and east of simulated Bitumen grade (m%) realization number 26.

The economic block value was calculated for the OK values and realizations. See Eq. (2.7). The E-type mean and variance of the EBV are also calculated.

The economic parameters that are required for this calculation are based on the Syncrude's costs in CAN$/bbl of sweet blend for the third quarter of 2008 (Jaremko 2009). These parameters are summarized in Table 4-4.

Table 4-4: The parameters that are required to calculate EBV

Arcanum	Description	Value
P	Selling price ($/tonne)	281.25
C_m	Mining Cost ($/tonne)	4.6
C_p	Processing Cost ($/tonne)	0.5025
R_m	Mining recovery factor	0.88
R_p	Processing recovery factor	0.95
g_{cut}	Cut-off grade (%m)	6
T_o	Tonnage of ore blocks (tonne)	54000
T_w	Tonnage of waste blocks(tonne)	52500

The price of oil is considered 44.7 $/bbl and the density of the bitumen is approximately 1 $tonne/m^3$ at 20 degree Celsius. 1 cubic meter is 6.2895 bbl. Therefore, the price of 1 tonne of bitumen is calculated as $281.25.

The density of ore blocks and waste block are assumed 2.16 $tonne/m^3$ 2.1 $tonne/m^3$, respectively. Figure 4-16 shows a plan view and two cross sections of the EBV for the OK values. The expected value (mean) and the variance of the EBV also are calculated for each block. Figure 4-17 also shows the expected value of EBV for each block at the same maps and Figure 4-18 shows the variance of EBV.

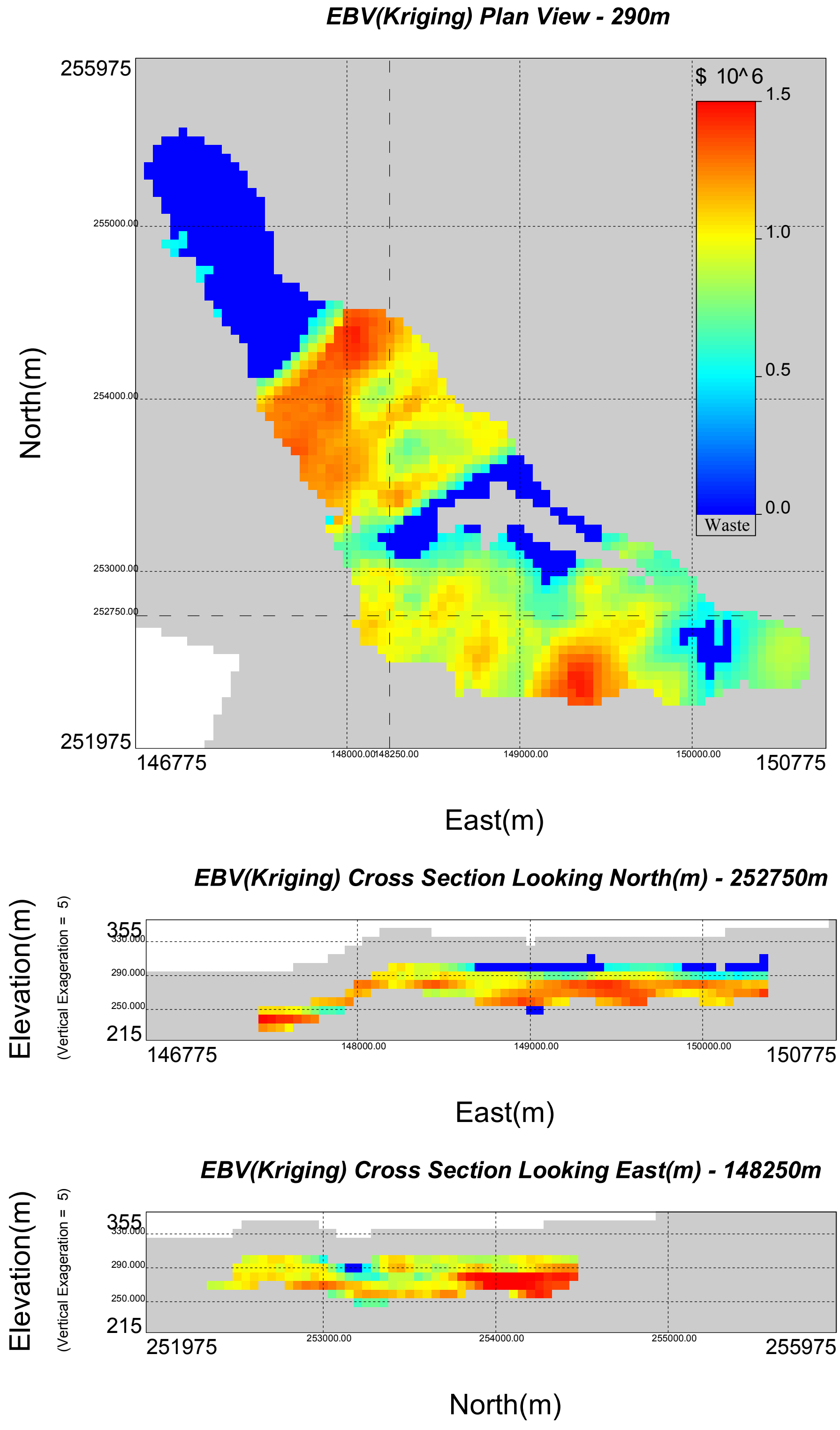


Figure 4-16: Plan view, cross section looking north and east of EBV($) using kriged bitument grade (m%).

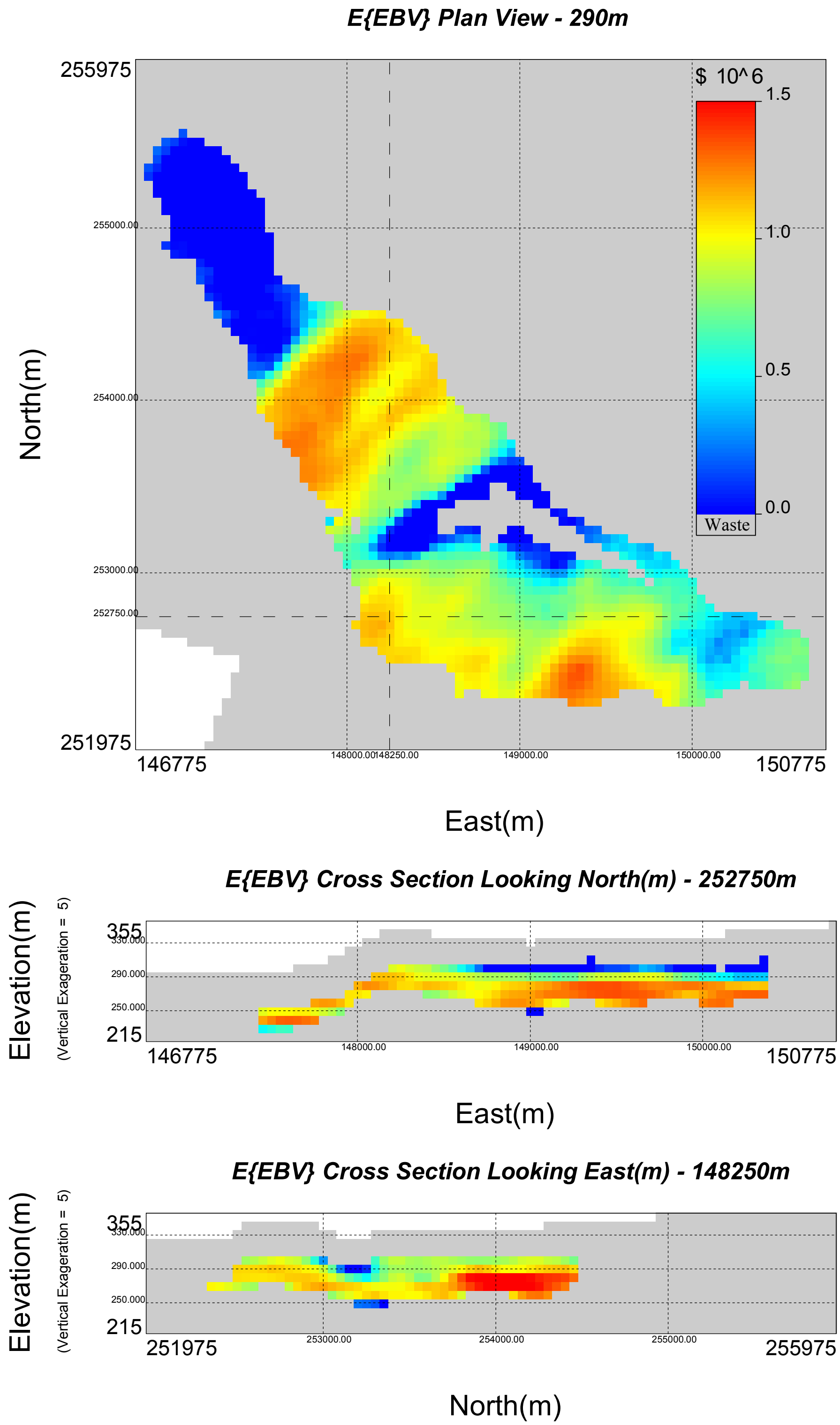
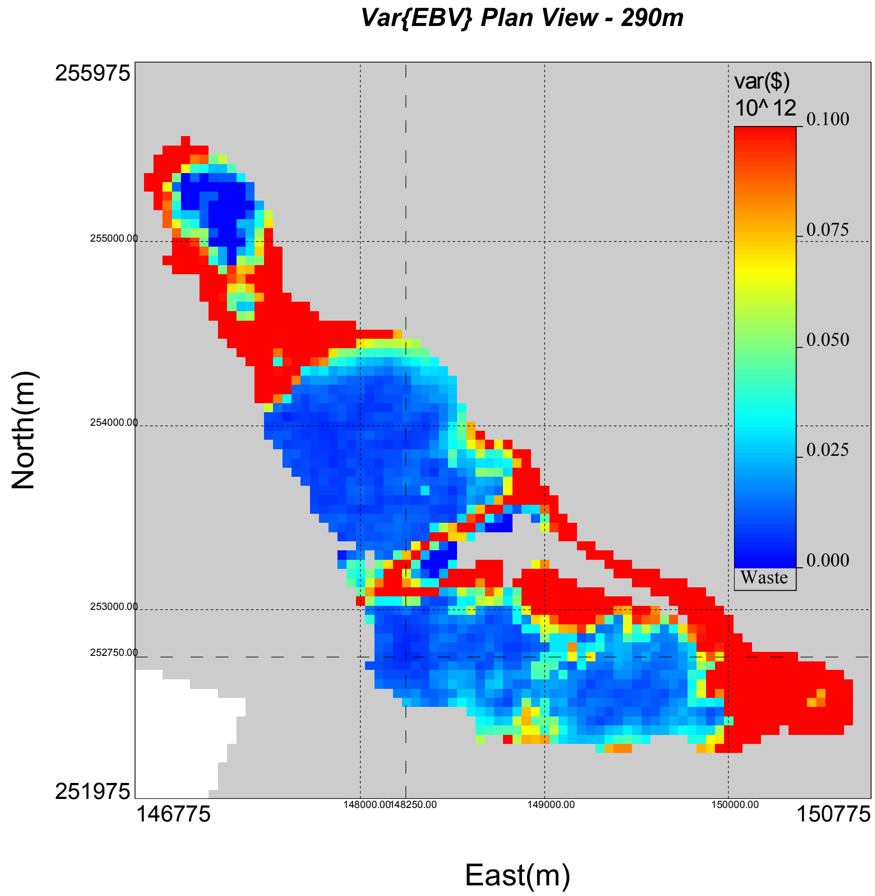
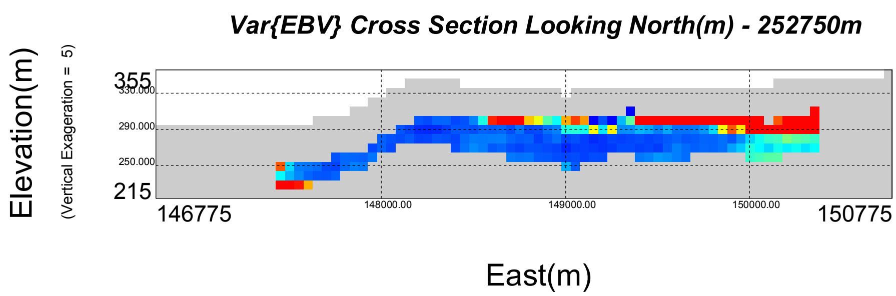


Figure 4-17: Plan view, cross section looking north and east of E-Type mean of EBV($) using 50 conditional realizations of simulated bitumen grade (m%).





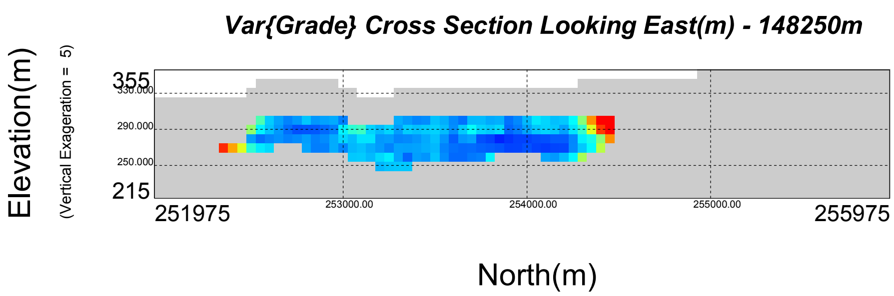


Figure 4-18: Plan view, cross section looking north and east of E-Type varaince of EBV($) using 50 conditional realizations of simulated bitumen grade (m%).

4.3 Determine the Ultimate Pit Limit

The next stage is to determine the ultimate pit limits with the ordinary kriging block model. The same economic parameters as presented in Table 4-4 are used here. The overall slope of the pit is assumed to be 20 degrees. 3D LG algorithm is applied to generate 33 pit-shells by using revenue factors ranging from 0.1 to 2.5. Some revenue factors generate the same pit shell. Table 4-5 shows the error of the pit slope for the ultimate pit limit. The values are in the acceptable range. Usually the average slope error should not exceed 1 to 2 degrees. Table 4-6 shows the 14 pit-shell generated by different revenue factors. By increasing the revenue factor, the selling price increases and therefore the size of the pit and the strip ratio increase. The revenue factors less than one create smaller pit-shells aimed to extract more ore blocks with higher grade. Lower revenue factor create more conservative pit limits where the price of ore is predicted to be lower than the current price. The base case pit-shell is generated with a revenue factor equal to one (pit shell number 14).

Table 4-7 show the number of the blocks inside the final pit. The cut off grade is not applied to the values presented in this table. There are 653.6 million tonnes of ore and waste inside the ultimate pit limit. There are 297.4 million tonnes of waste. By applying cut off grade of 6 mass percent to the estimated bitumen values, the ore tonnage is calculated to be 282.4 million tonnes with an average grade of 10.31 m%. The waste tonnage is calculated to be 371.17 million tonnes. The estimated total tonnage of bitumen is 29.11 million tonnes. Considering the overall recovery factor of 83.6% (from Table 4-4), the total estimated recoverable bitumen is 8.2 billion dollars.

Figure 4-19 shows the ultimate pit limit in plan view and two cross sections at 290m, 25275m and 148250m, respectively. The final pit does not cover all the ore blocks; some ore blocks in the Northwest of the area are not inside the final pit limit.

Table 4-5: Errors of generated pit slope by LG method

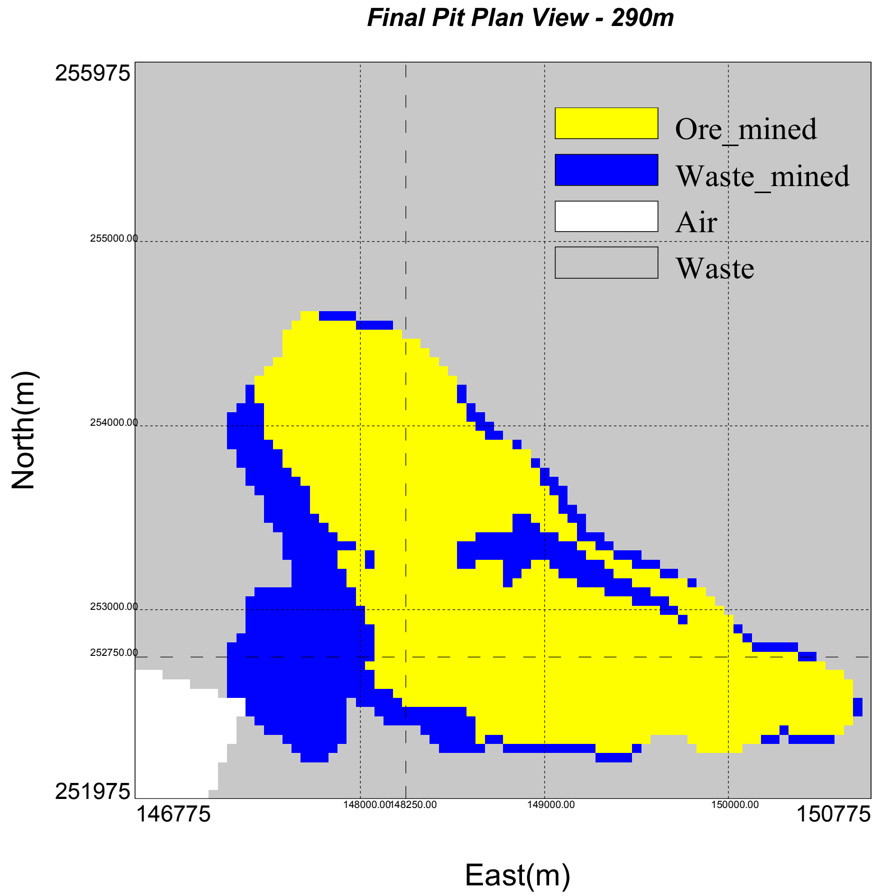
Description	Value
Minimum slope error	0.1 degrees
Average slope error	0.8 degrees
Maximum slope error	1.1 degrees

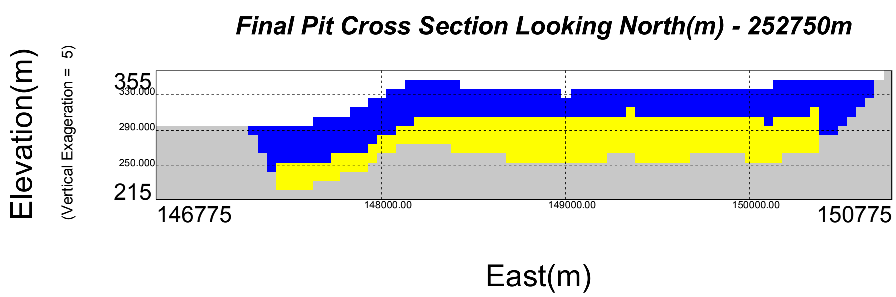
Table 4-6: Generated pit shells using different revenue factors using OK block model

Pit	Rev. Factor	Rock (M tonne)	Ore (M tonne)	Strip Ratio	Max Bench	Min Bench	Bitumen (M tonne)	Bitumen Grade(m%)
1	0.35	137.30	71.37	0.92	18	7	7.95	11.14
2	0.4	373.83	190.73	0.96	18	7	19.88	10.42
3	0.45	440.18	219.54	1.01	18	6	22.70	10.34
4	0.5	458.05	226.26	1.02	18	6	23.37	10.33
5	0.55	505.06	241.68	1.09	18	5	24.96	10.33
6	0.6	576.25	263.24	1.19	18	5	27.15	10.31
7	0.65	607.98	272.22	1.23	18	5	28.05	10.3
8	0.7	619.51	275.09	1.25	18	5	28.35	10.31
9	0.75	630.11	277.59	1.27	18	5	28.61	10.31
10	0.8	640.61	279.90	1.29	18	5	28.85	10.31
11	0.85	646.19	281.03	1.3	18	5	28.97	10.31
12	0.9	647.86	281.36	1.3	18	5	29.00	10.31
13	0.95	652.23	282.21	1.31	18	5	29.08	10.31
14	1	653.61	282.44	1.31	18	5	29.11	10.31
15	1.05	656.81	283.06	1.32	18	5	29.16	10.3
16	1.1	726.12	296.17	1.45	18	5	30.32	10.24
17	1.15 - 1.2	731.27	297.06	1.46	18	5	30.40	10.23
18	1.25	733.90	297.54	1.47	18	5	30.44	10.23
19	1.3	737.64	298.20	1.47	18	5	30.49	10.22
20	1.35	739.31	298.48	1.48	18	5	30.51	10.22

Table 4-7: Number of blocks and tonnages for ore and waste rock-types

Block Type	Number of blocks	Tonnage (M tonne)
Ore	6625	356.2
Waste	7987	297.4
Total	14612	653.6





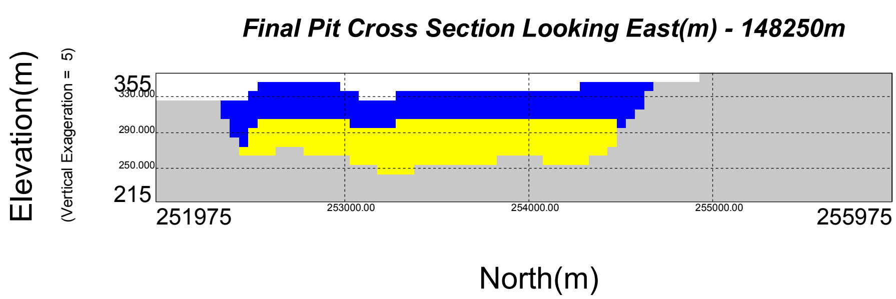


Figure 4-19: Plan view, cross section looking north and east of ultimate pit limit with over bourden (blue blocks), ore body (yellow blocks) and surrounding waste blocks outside the pit limit (gray blocks)

4.3.1 Uncertainty Assessment of Final Pit Limit Using Simulation Realizations

The effect of grade uncertainty is now studied on the size of the final pit limit. There are two main options: apply the 3D LG algorithm on each realization to generate different ultimate pit limits for every realization. In the second method the simulated realizations are assessed in the ultimate pit generated by kriging. The total tonnage (ore+waste) would be the same for all realizations in the second method. Koushavand and Askari-Nasab (2009) presented these two methods for the same case study presented here. The results for the second method are reported here. Figure 4-20 to Figure 4-23 show the histogram and the box-plot of the average simulated grade, total ore tonnage, strip ratio and the total tonnage of bitumen inside the final pit. The cut-off grade is applied for all realizations such that all blocks below the 6 m% of the bitumen grade are considered as waste blocks.

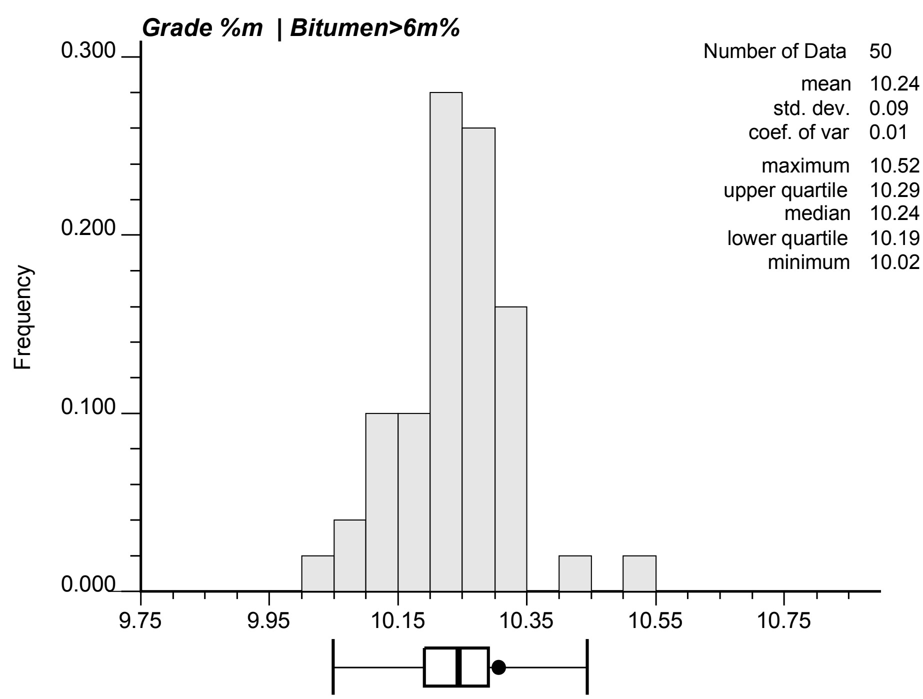


Figure 4-20: Histogram and box-plot of bitumen grade inside the final pit that was generated by OK. OK result is marked by solid circle. Cut-off grade has been applied to the all realizations.

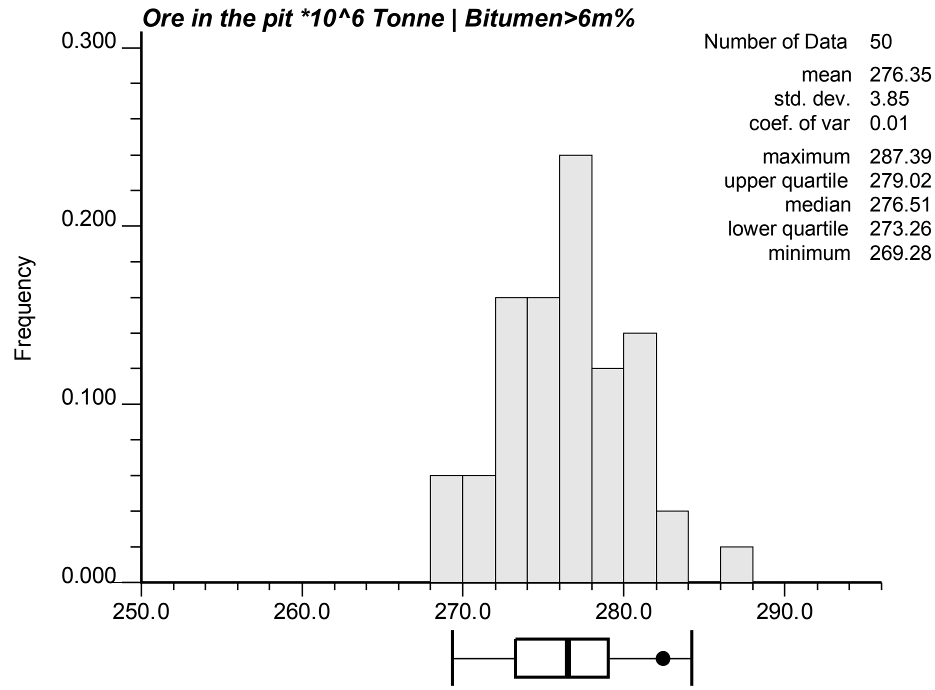


Figure 4-21: Histogram and box-plots of ore tonnage inside the final pit. OK result is marked by solid circle. Cut-off grade has been applied to OK values and all realizations.

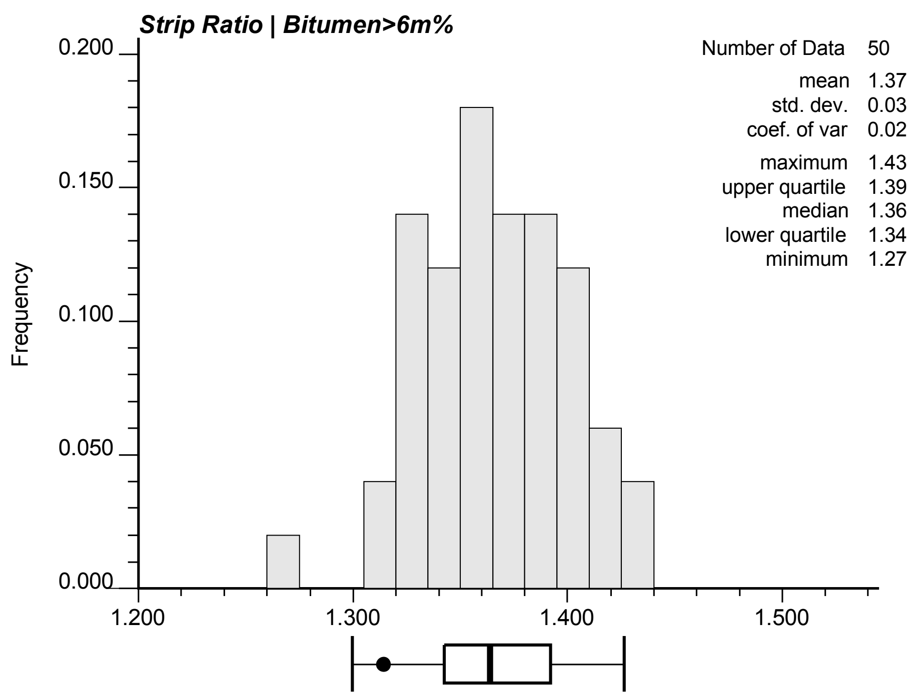


Figure 4-22: Histograms and box-plots of overall stripping ratio inside the final pit. OK result is marked by solid circle. Cut-off grade has been applied to OK values and all realizations.

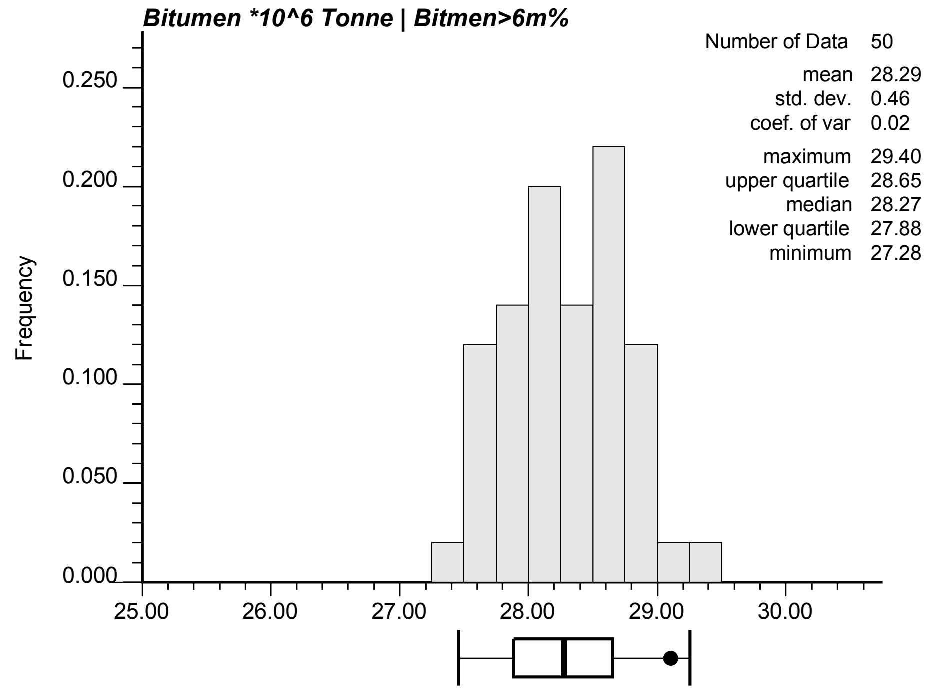


Figure 4-23: Histograms and box-plots of tonnage of bitumen inside the final pit. OK result is marked by solid circle. Cut-off grade has been applied to OK values and all realizations.

Table 4-8 shows the percentile rank of realizations based on the producible bitumen inside the final pit. The smoothing of kriging is clearly shown in this table. The producible bitumen calculated by ordinary kriging is more than 97 percent of the realizations (only 2 realizations produce more ore than OK). The histograms and box plots shown in Figure 4-20 to Figure 4-23 suggest that the OK is significantly biased from realizations. The reason for this biasness is that the final pit limit has been derived from OK block model. Therefore, the results are optimum for the OK block model and not for the realizations. The final pit covers all the ore blocks in OK block model only. This is a good example to motivate the future research to take into account grade uncertainty in pit limit optimization. The E-type mean is much closer to the median of the distribution. There are 40 percent of the realizations that produce more ore than E-type mean. Also, Table 4-9 shows the summary statistics of the final pit from realizations for ore tonnage, strip ratio and bitumen in place.

The NPV calculated with the OK block model will be highest because of the smoothing effect and overestimation of ore tonnes.

Table 4-8: The producible bitumen and rank of realizations

Realization	Bitumen tonnage (MT)	Percentile
4	27.28	1
27	27.62	10
5	27.88	24
15	28.27	50
47	28.65	75
7	28.81	90
10	29.40	99
OK	29.11	97
E-type Mean	28.50	61

Table 4-9: Summary of statistics for the ultimate pit limit

Statistics	Ore tonnage (MT)	STRO	Bitumen tonnage (MT)
Mean	276.35	1.37	28.29
Std. dev	3.89	0.03	0.47
Min	269.28	1.27	27.28
Quartile 1	273.31	1.34	27.90
Median	276.51	1.36	28.27
Quartile 2	278.91	1.39	28.65
Max	287.39	1.43	29.40

4.4 Production Scheduling With Whittle

The kriged model is used to generate a production schedule. The same economical parameters presented in Table 4-4 are used. No stockpile and capital costs are considered. Additional parameters are shown in Table 4-10.

Table 4-10: Mine planning input parameters

Parameter	Value
Mining dilution fraction	1.0
Discount factor per period	10%
Mining Limit	67.5 M Tonne
Processing Limit	36 M Tonne
Pre-stripping mining	2 years
Mine life	10 years

Milawa NPV is used as the scheduling algorithm. All configurations of 3 push backs up to 9 push-backs are checked with “Full” calculation mode. In this mode, the program searches all the possible combinations of the pushbacks and reports the case with higher

NPV. Among all configurations, three pushbacks case generates reasonable result such that by increasing the number of push-backs, the NPV is not improved significantly. The pit-shells number 1, 5 and 14 are chosen as the push-backs by the algorithm. In all the scenarios that are generated by Whittle, the mine life is 11 years. This is because the processing capacity is very tight for Whittle to be able to process all the ore tonnage inside the final pit. On the other hand, the generate schedules by Model #1 to #4 have 10 years of mine life. Therefore, to keep consistency with other production schedules and to compare the result, the mining and processing capacity at the final period (10) is increased to 87MT and 40 MT respectively.

4.4.1 Verification of the Generated Production Schedule

Table 4-11 shows the summary of the schedule for each period. The NPV of the project is 2387 million dollars. This schedule is valid based on the input parameters because there are two pre-striping periods and also the mining and processing capacity constraints are satisfied in all periods.

The total tonnage of processed ore is 281 MT. The comparison of this number with the total of 282.4 million tonnes of ore inside the final pit limit shows that there are 1MT of rejected ore. Each block is flagged with: (i) the extraction period (ii) the portion of the extraction and (iii) the destination of the block. Any ore block that is sent to the waste dump is called rejected ore. The total average head grade is 10.28 m%.

Figure 4-24 shows the schedule generated by Whittle. The mill is fed at full capacity in periods 3 to 8. There is shortfall in period 9 while 40 MT of ore is processed in the period 10.

Figure 4-25 shows the plan view and two cross section looking north and east for the extraction periods of the blocks. The three push-backs that control the extraction strategy can clearly be observed in this figure. The extraction of the blocks is continuous and there is no gap effect. The production schedule can be easily followed without any extra equipment movement during each period.

Table 4-11: Summary of Production schedule in each period for OK block model; the schedule generated by Whittle with OK block model.

Period	Input Ore MT	Waste MT	Mined MT	SR	Grade M%	stockpile MT	stockpile grade M%	CDCF MD
1	0.0	67.5	67.50	Inf.	0.00	0.0	0.00	-282.28
2	0.0	67.5	67.50	Inf.	0.00	0.0	0.00	-538.88
3	36.0	25.8	61.80	0.72	10.35	0.0	0.00	-17.96
4	36.0	31.1	67.10	0.86	11.38	0.0	0.00	506.47
5	36.0	31.5	67.50	0.87	9.17	0.0	0.00	850.19
6	36.0	17.0	53.00	0.47	10.01	0.0	0.00	1,245.88
7	36.0	11.1	47.10	0.31	10.71	0.0	0.00	1,654.02
8	36.0	31.5	67.50	0.88	10.85	0.0	0.00	1,987.45
9	25.2	42.3	67.50	1.68	9.67	0.0	0.00	2,126.13
10	40.0	47.0	87.00	1.18	10.22	0.0	0.00	2,387.24
Total	**281.2**	**372.3**	**653.5**	**1.37**	**10.28**	**0.0**	**0.00**	**2,387.24**

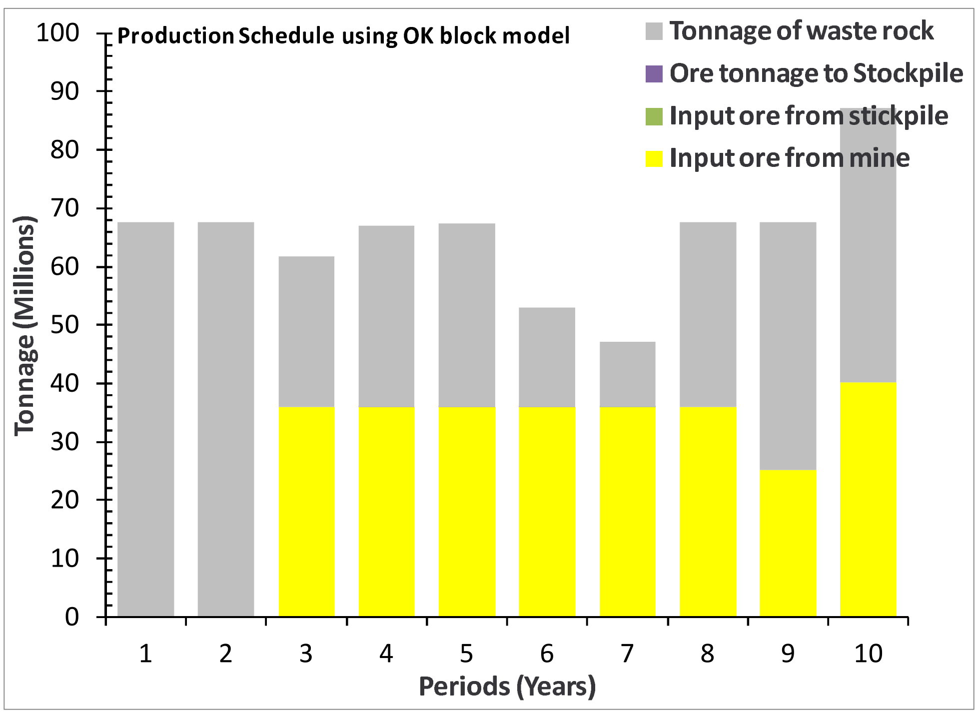


Figure 4-24: Production schedule generated by Whittle with OK block model.

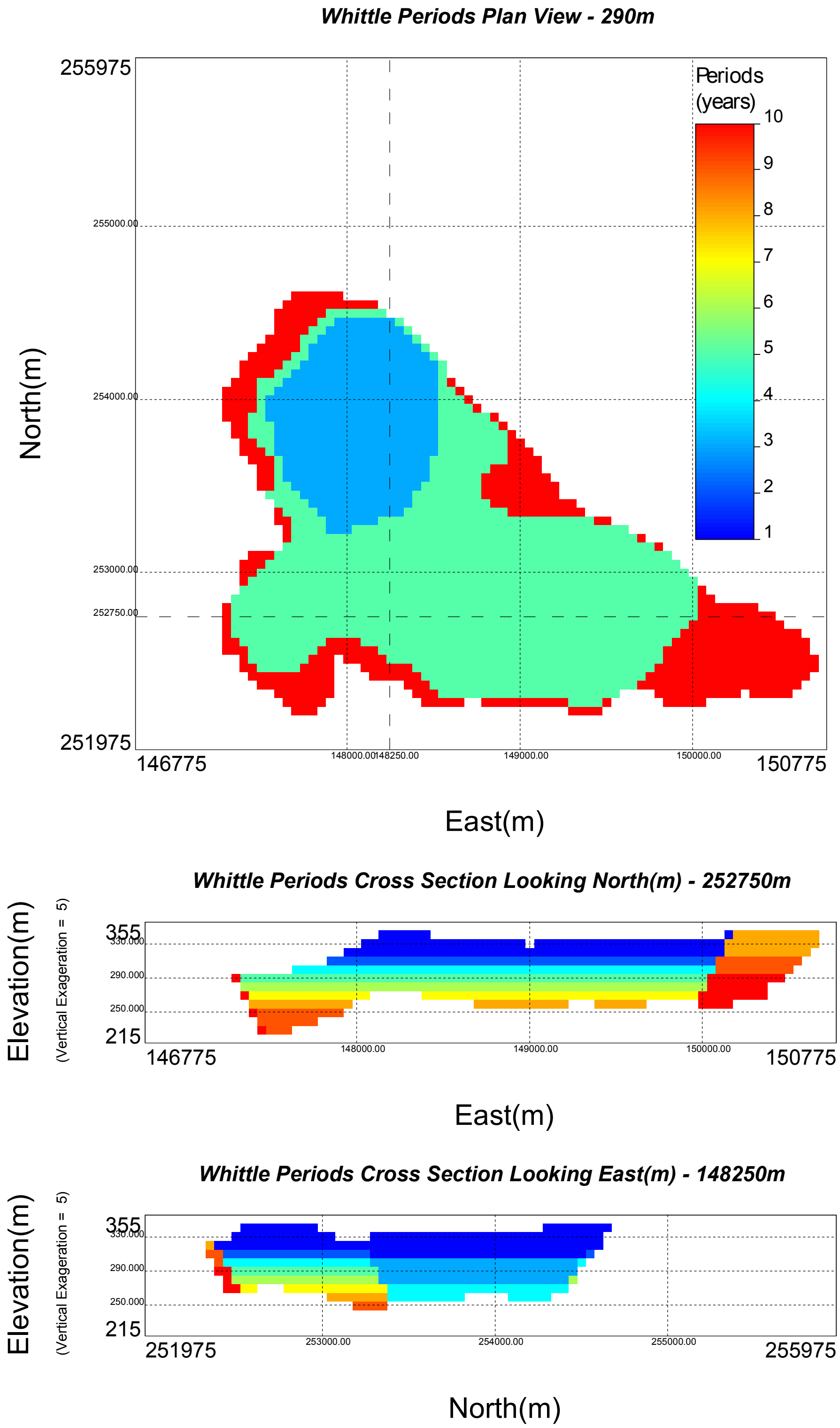


Figure 4-25: Extraction periods of production schedule generated by Whittle with OK block model.

4.4.2 Uncertainty Assessment of the Whittle Generated Production Schedule

By following the single schedule for all realizations the uncertainty in the production schedule can be measured. The period of extraction for each block and the portion of the extraction are specified by the production schedule, but the destination of the blocks will be different for the simulated realizations. The assumption here is that the destination of the blocks is decided based on the simulated grade value of each block and the cut-off grade. There will be two contradictions:

1. The block is estimated above 6% and is processed in the schedule; however the simulated value is lower than the cut-off grade. These kinds of blocks are sent to the waste dump for that specific realization. This can cause shortfall from target production.
2. The block is estimated as waste and the simulation value is above the cut-off grade. This block is sent to the mill and processing plant. The extra ore tonnage of such blocks in a realization and a specific period can exceed the target production and generate overproduced ore for that realization.

The overproduced ore is transferred to a stockpile to be used in later periods. It is unrealistic to dispose the extra ore. Ore that is transferred to the stockpile will be available for subsequent periods if there is any shortfall from target production.

Figure 4-26 shows 6 graphs with different aspects of the generated schedule in each period. The black bold line is the result of OK in each period and dashed red lines are the results of the realizations that are followed by the same generated schedule. Figure 4-26a shows the average grade of the mined ore. The average grades of ore for all realizations are roughly between 8 to 10 m%. Figure 4-26b shows the ore tonnage that is mined in each period. Extra ore is generated only in periods 6, 7 and 8 with very few realizations and the tonnage of extra ore for these periods is very low. Figure 4-26c and Figure 4-26d show the average input grade of ore and the tonnage of ore to the mill. Any extra ore that generated with this schedule has transferred to the next earliest year that shortfall happens. The average input grade is updated by the tonnage of ore from the stockpile. Figure 4-26f shows the cumulative discounted cash flow (CDCF). Only one realization (#10) has higher NPV than OK. This is fully expected because of the smoothing of kriging and the fact that the cut-off is less than the average.

Table 4-12 shows the discounted cost of grade uncertainty before and after post-processing calculated by Eq. (2.30). The discounted cost of grade uncertainty is calculated before and after post-processing as 218.67 and 214.76 million dollars, respectively. The cost of grade uncertainty is reduced by 1.8% after post-processing the production schedule.

Figure 4-27 shows the box-plot of the input tonnage of the mined ore. The target production is marked by the horizontal black line. The second vertical axis at the right shows the percentage of the average deviation from the target production. The yellow transparent columns show the average deviation from target production for each period. The numbers attached to each column also show the average deviation from target production. The deviation from the target is the absolute value of the difference of the actual value from the target value divided by the target value:

$$\text{deviation from targert}=\frac{\left|\text{Actual value - Target}\right|}{\text{Target}} \tag{4.2}$$

The deviation from target production in period t is calculated by taking the average values from all the realization. This is called the deviation from target production in period t and it is shown by $DTP(t)$ in Eq. (4.3) :

$$\begin{aligned} &DTP(t)=\frac{1}{L}\sum_{l=1}^{L}DTP(t;l) \\ &Where \\ &DTP(t;l)=\frac{\left|T_o(t;l)-T_t(t)\right|}{T_t(t)} \end{aligned} \tag{4.3}$$

Where

- $DTP(t;l)$ is the deviation from target production for realization number l.
- $T_t(t)$ is the target production in period t. $T_t(t)$ for this case study is zero in pre-striping periods (1 and 2), and 36MT for periods 3 to 9.

The deviation from target production for each period is shown in Figure 4-28. This graph shows the results after post-processing. Figure 4-27 and Figure 4-28 are not very different from each other because this production schedule does not produce a significant tonnage of extra ore from the realizations. There is a shortfall from target production in the period 9.

The box-plots in these figures also show the variation of the input feed to the mill from all realizations. Each box-plot shows the minimum and maximum values in the tails. The limits of the box itself show the 25 and 75 percentile of the values or first and third quartile. The middle line also shows the second quartile or the median of the data.

The average of deviation from target production is shown by $\overline{DTP}$ and it is simply the average of $DTP(t)$ at all production periods except the last period and it is calculated by Eq.(4.4) as below:

$$\overline{DTP} = \frac{1}{T_p - 1} \sum_{t=1}^{T_p - 1} DTP(t) \tag{4.4}$$

Where T_p is the total number of production years. For this case it is 8 years.

$\overline{DTP}$ is another variable that can be used to compare the performance of the production schedule in the present of grade uncertainty.

Table 4-13 shows the percentage of the average deviation from target production calculated form period 3 to 9 before and after post-processing. These numbers are used to compare the Whittle generated schedule with other methods that are presented in this thesis. As expected, the post-processing stage reduces the effect of the grade uncertainty and the average of the deviation from target production is reduced.

Table 4-14 shows the summary statistics for production schedule that is generated by Whittle after post-processing. Since there are some ore tonnage left at the end of period 10, the statistics for ore tonnage, strip ratio and tonnage of bitumen are less than the values that are reported in Table 4-9. Also, because the OK block model has more ore tonnage than almost all of the realizations (Table 4-8), the NPV of the OK block model is higher than most of the realizations.

Figure 4-29 shows the histogram of the mined ore tonnage at different periods. The solid black columns (e.g. in periods 7 and 8) indicates that the frequency of that particular range of data exceeds the vertical axes upper limit which is 0.4. The box-plot is plotted for each histogram at the bottom of the horizontal axis. The vertical dashed line shows the processing limit of 36MT per year (3 to 9). There are some realizations where the tonnage of mined ore is more than the processing limit in time periods 6, 7 and 8. After post-processing, all overproduced ore is removed and added to the next period that shortfall happens.

Figure 4-30 shows the histogram and box-plot of the tonnage of processed ore for the realizations and ordinary kriging. Post-processing the schedule helps to reduce the effect of grade uncertainty because any overproduced ore can be used in subsequent periods. This can be observed by comparing the histogram of periods 6, 7 and 8 from Figure 4-29 and Figure 4-30. This effect can be easily detected from these histograms even though the tonnage of overproduced ore that is transferred to the stockpile is very low.

4.4.3 Conclusion

The Whittle software is a very well-known mine planning software package. It has many tools to create long-term production plans. The push-back design algorithm increases the feasibility of the generated production schedule. However there are some shortcomings:

- There is no control for the mine life. The user needs to change the mining and processing limit to create a schedule with desired mine life.
- Whittle uses a heuristic algorithm to maximize the NPV; however the program does not give any measurement of the optimality of the solution.

The generated schedule from Whittle is not necessarily the optimum solution.

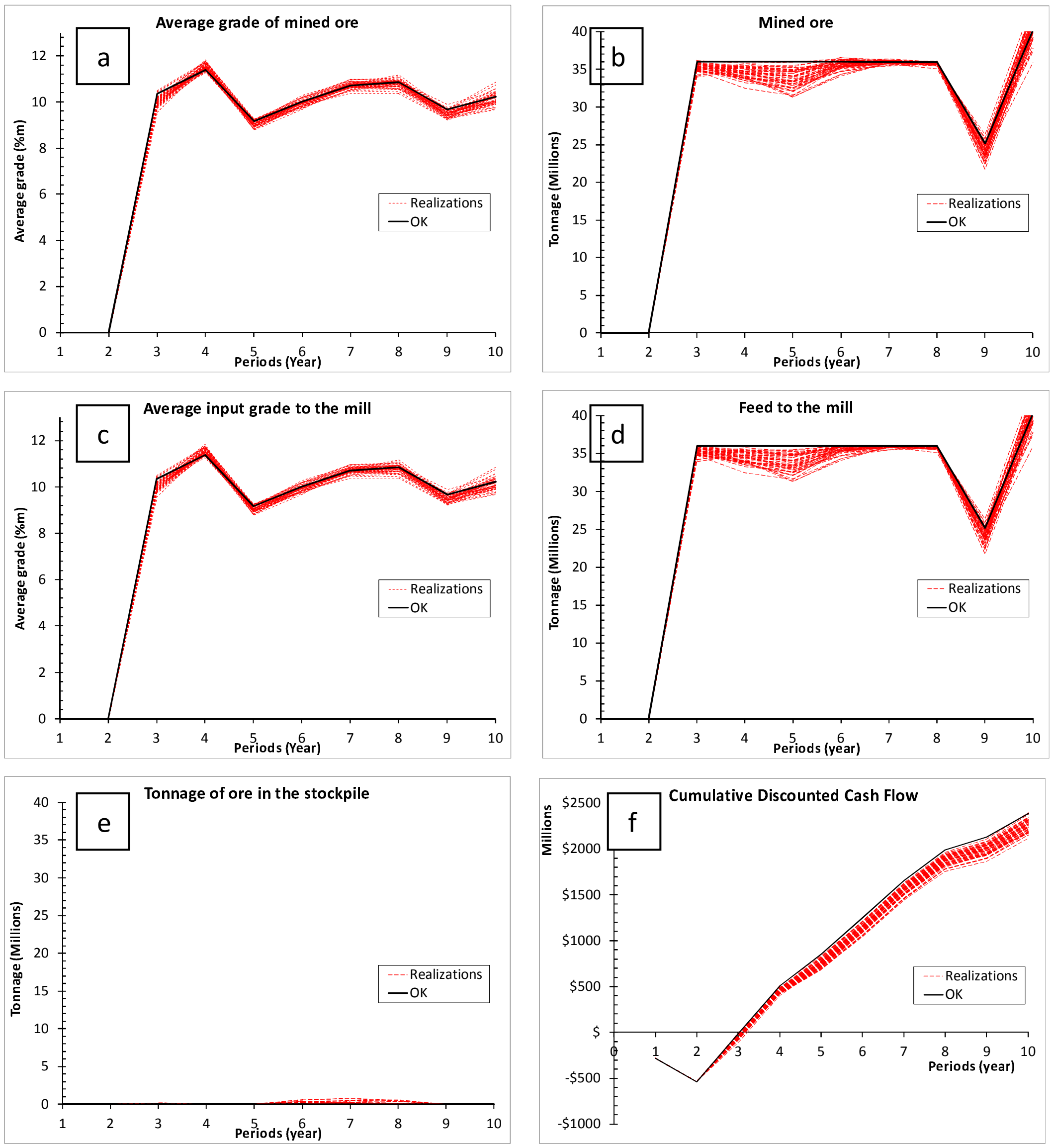


Figure 4-26: a: average grade of mined ore, b: tonnage of mine ore, c: average grade of input ore to the mill before post processing, d: tonnage of ore input to the mill after post processing, e: tonnage of ore at the stockpile after post processing and f: CDCF after post processing stage; the schedule generated by Whittle with OK block model.

Table 4-12: DCOU with and without stockpile for the Whittle generated schedule

	Discounted cost of grade uncertainty (MD)
Before post processing	218.67
After post processing	214.76

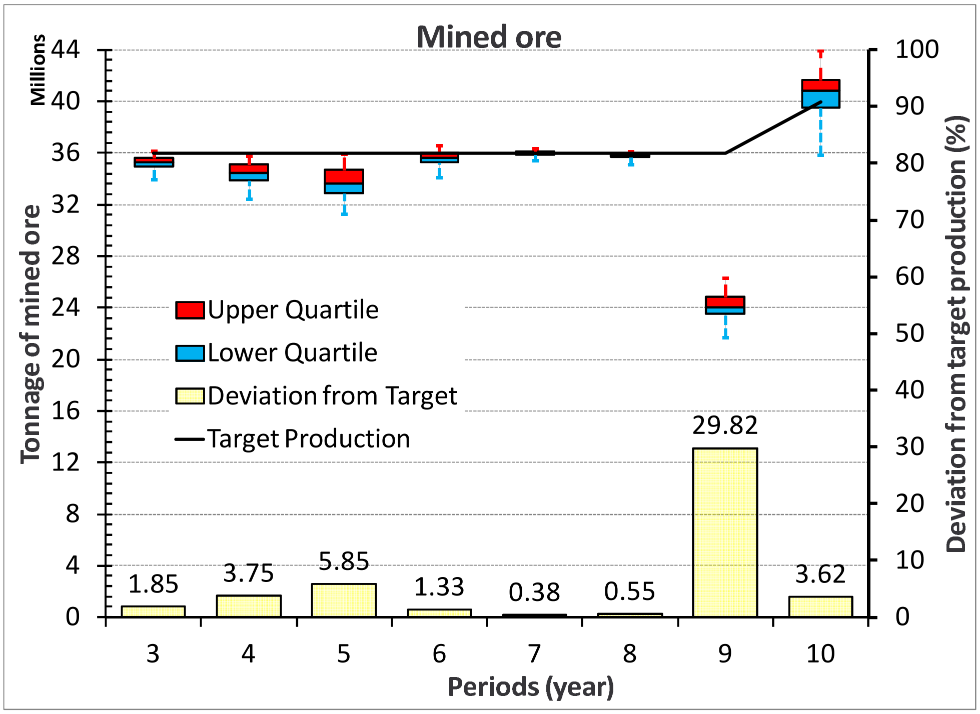


Figure 4-27: box-plot and deviation from mine ore for each period (before post-processing); the schedule generated by Whittle with OK block model.

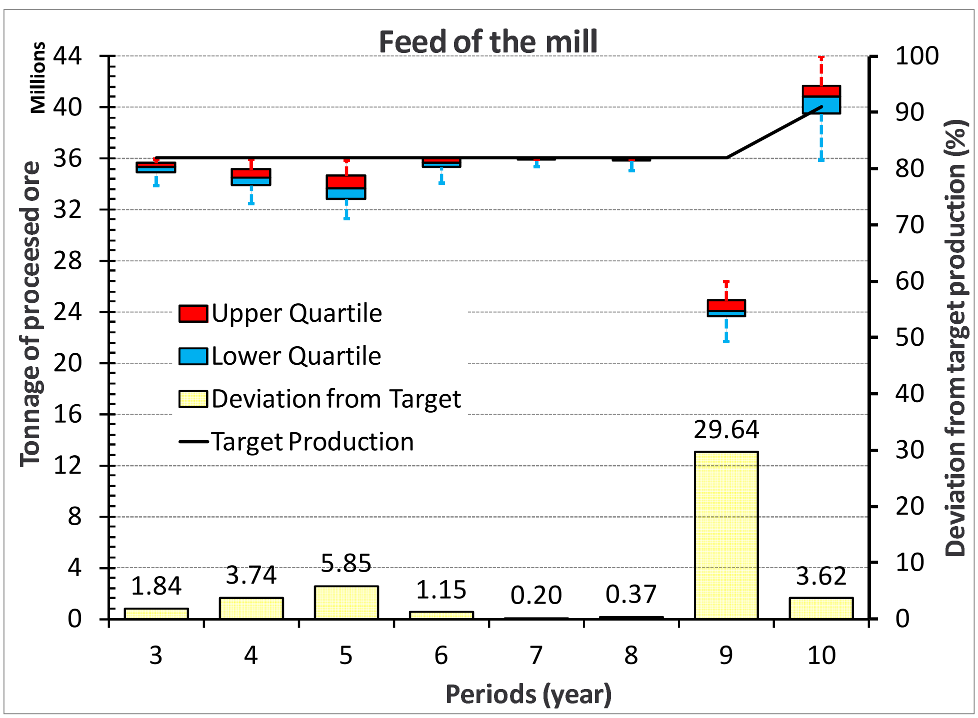


Figure 4-28: Feed of the plant and the box-plot of the deviation from target production for each period (after post-processing); the schedule generated by Whittle with OK block model.

Table 4-13: $\overline{DTP}$ with and without stockpile for the Whittle generated schedule

	Average deviation from target production ($\overline{DTP}$)
Before post processing (No stockpile)	6.90%
After post processing (With stockpile)	6.79%

Table 4-14: Summary of statistics for the production schedule after post processing stage; the schedule generated by Whittle with OK block model

Statistics	Ore tonnage (MT)	STRO	Bitumen tonnage (MT)	Average m%	NPV (MD)	DCOU (MD)
Mean	275.48	1.37	28.23	10.25	2256.32	214.76
Std. dev	3.74	0.03	0.46	0.09	64.14	43.96
Min	268.84	1.29	27.24	10.04	2113.15	107.02
Quartile 1	272.75	1.35	27.87	10.20	2205.85	183.66
Median	275.83	1.37	28.21	10.26	2255.90	211.61
Quartile 2	278.02	1.40	28.60	10.30	2310.99	247.76
Max	285.87	1.43	29.27	10.53	2394.84	290.40
OK	281.36	1.32	29.03	10.32	2387.24	N/A

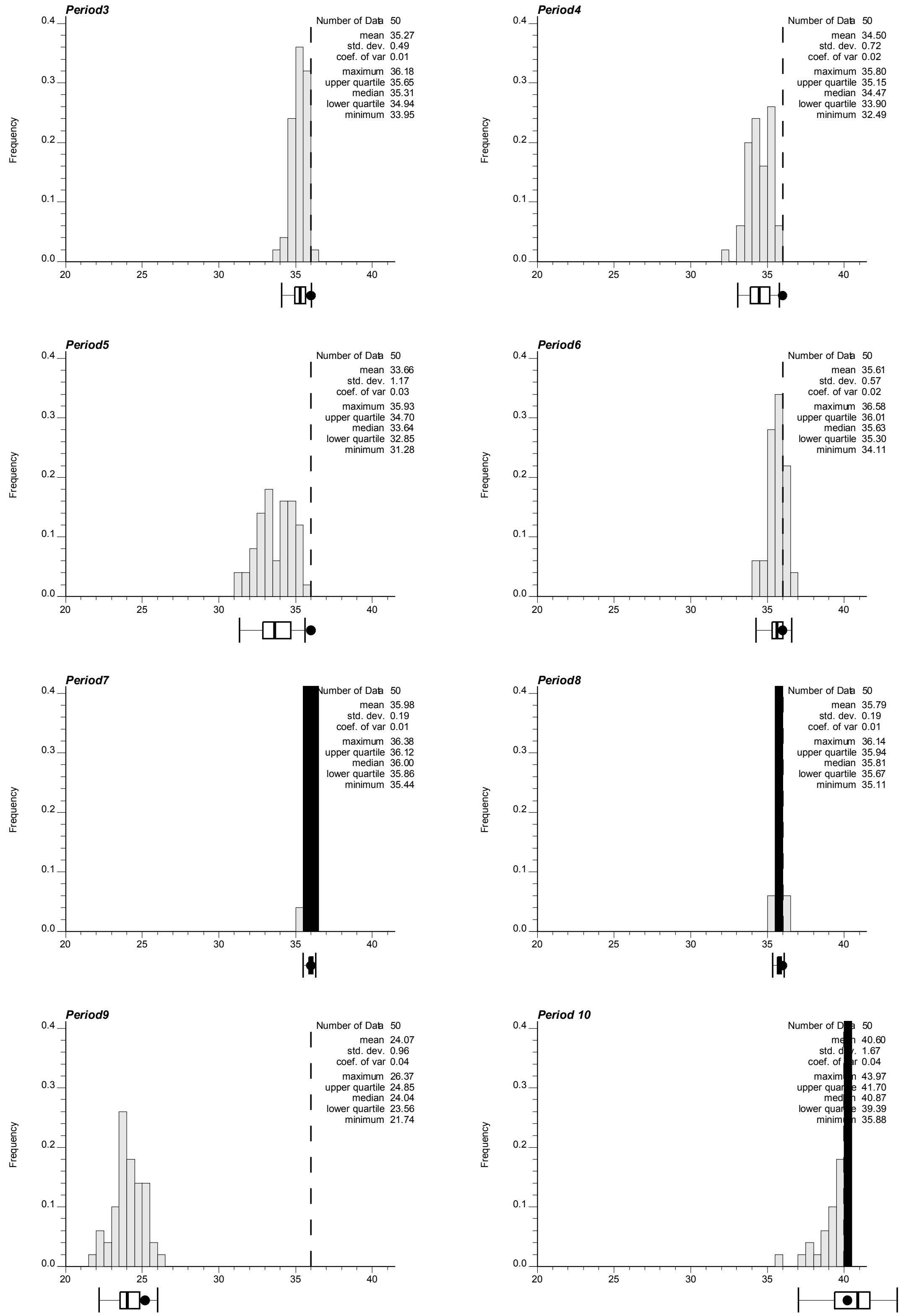


Figure 4-29: Histograms of mined ore tonnage. The schedule is generated with Whittle using OK block model. OK result is marked by solid circle and dash line indicates target production.

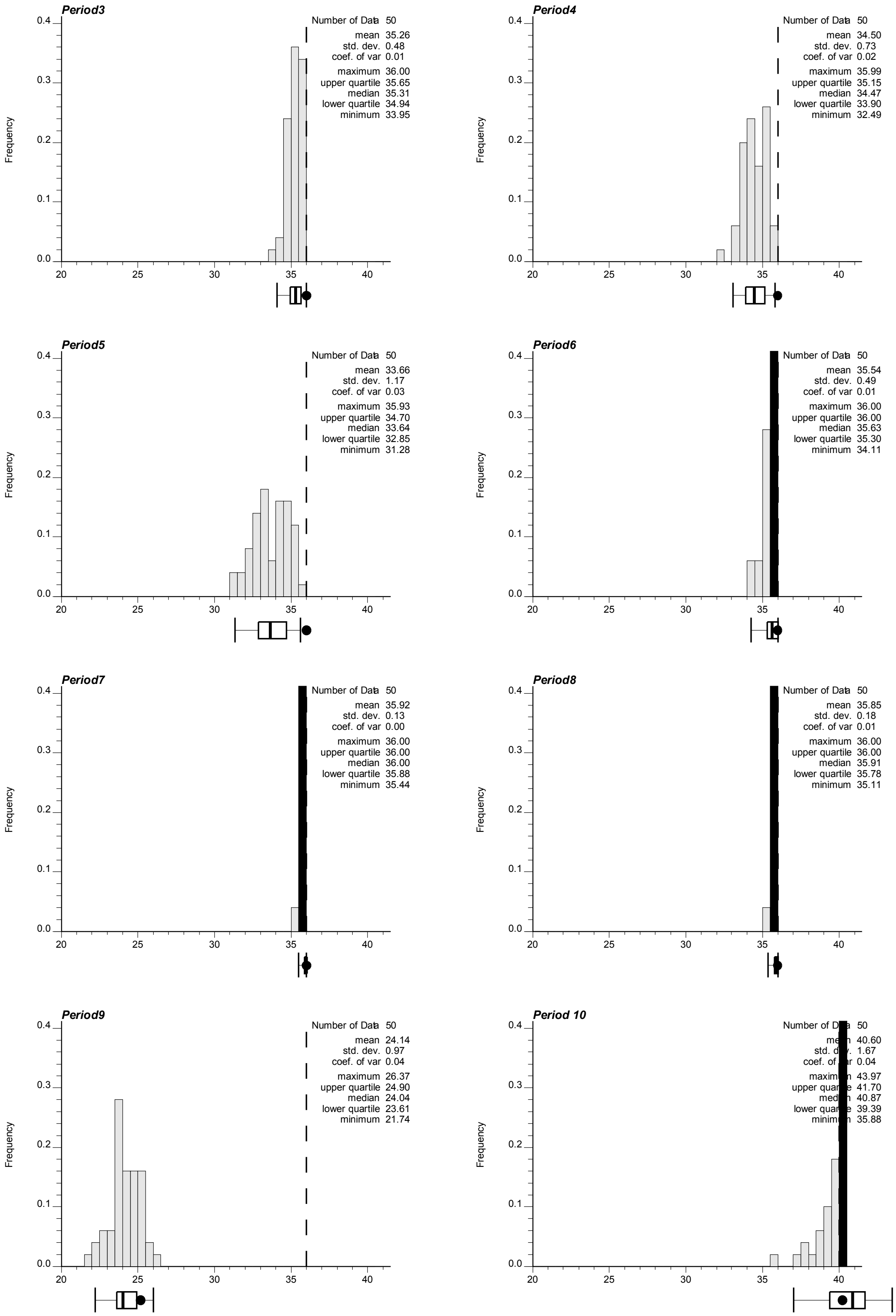


Figure 4-30: Histograms of input ore tonnage to the mill. The schedule is generated with Whittle using OK block model and post pressed by assuming presence of stockpile. OK result is marked by solid circle and dash line indicates target production.

4.5 Production Scheduling With Model #1

The MILP approach described in Section 2.1 is coded in MATLAB (MathWorks Inc., 2011) environment. Model #1 maximizes the NPV of an input block model. This model is formulated in Eq. (2.11). For this case study, the ordinary kriging block model is used. The output of the 3D LG that was used in Whittle was imported to MATLAB. All the steps are described in the previous chapter. The 14612 blocks inside the final pit are aggregate into 1834 mining cuts by using MATLAB's c-mean clustering function. First the number of mining cuts for each level is calculated and then for each level Fuzzy C-mean clustering technique is applied to aggregate based on similar grades, X and Y coordinates. Total tonnage, EBV, ore value, mining cost, ore tonnage and waste tonnage of the mining cut is calculated by adding up the values of the blocks inside each mining cut. The average grade of the mining cut is calculated by taking a volumetrically average of the blocks for each realization. Figure 4-31 shows the histogram of average grade of mining cuts calculated from OK block model. There are 973 waste mining cut which is about 53% of the all mining cuts. Figure 4-32 shows the histogram of ore mining cuts. The average grade of ore mining cuts in OK block model is about 10 m%. The Histplotsim is used to calculate and plot the CDF of average grade of mining cuts for all realizations shown in Figure 4-33. In this graph the waste mining cuts are filtered. OK block model is indicated by red line and the realizations by black lines. The small deviation of the OK from the realizations is due to the smoothing effect of the OK.

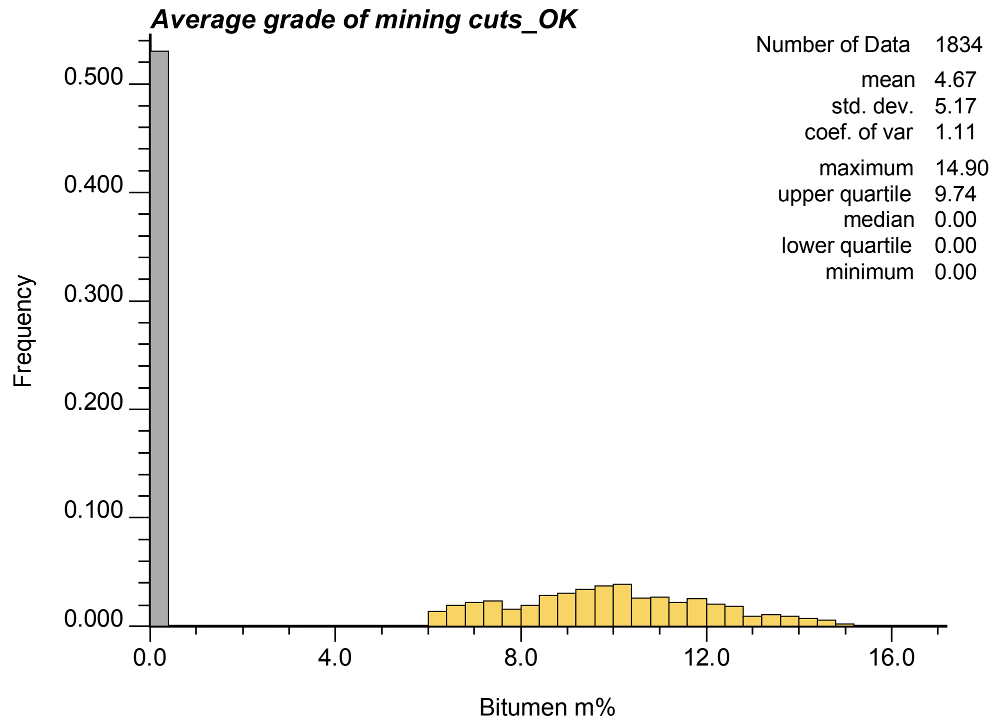


Figure 4-31: Histogram of average grade of ore and waste mining cuts calculated from OK block model; yellow bars indicate ore and the gray bar indicates waste mining cuts

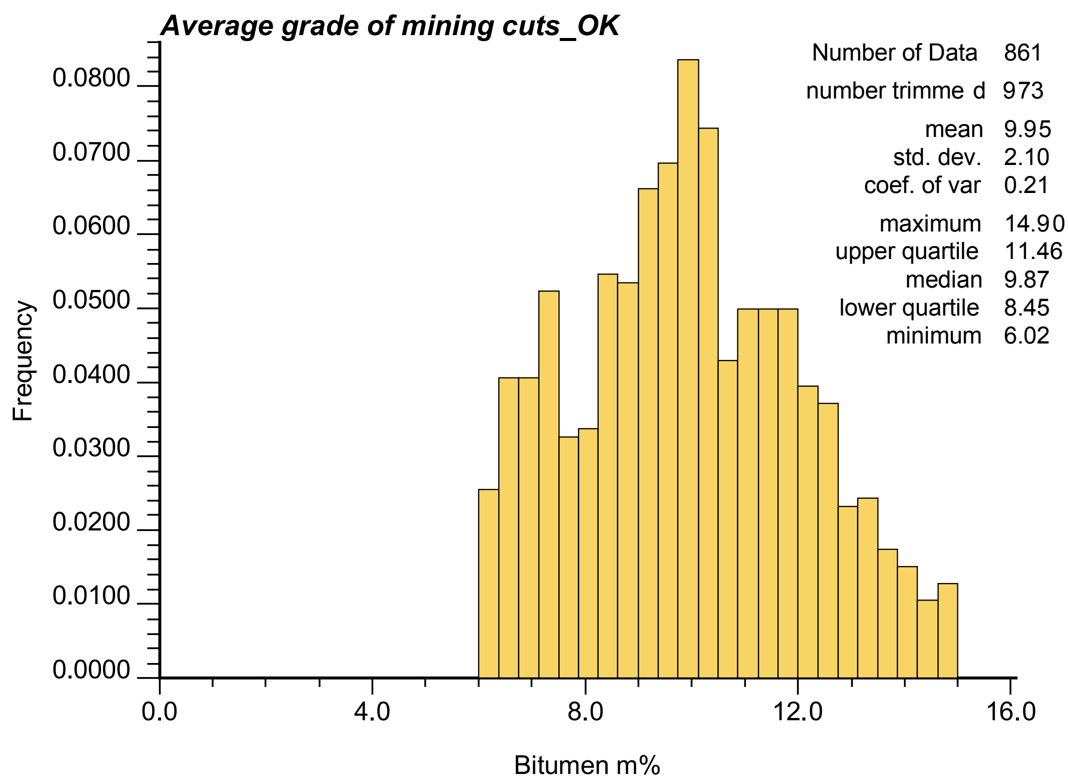


Figure 4-32: Histogram of average grade of ore mining cuts calculated from OK block model

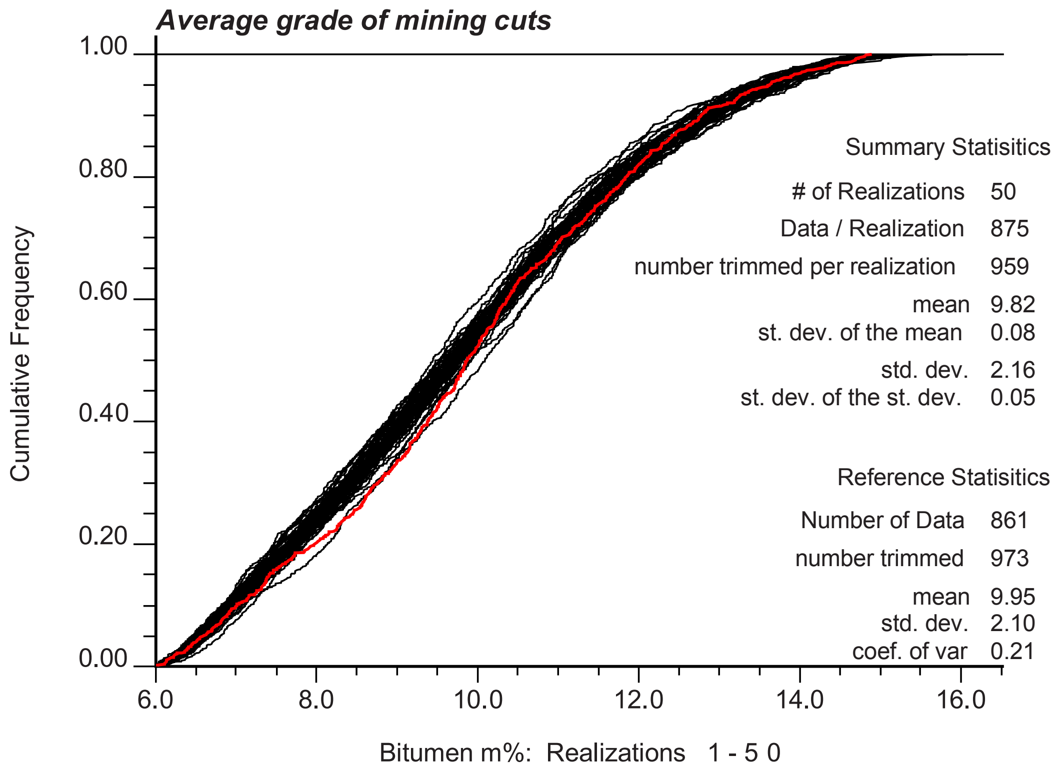


Figure 4-33: CDF of average grade of ore mining cuts; simulation realizations (black lines) and OK (red line)

The same strategy, input parameters, costs and limits that were used in the previous section to generate the schedule with Whittle are employed in this section as well. Two years of pre-stripping is assumed to provide enough operating space and ore availability. The target production is set to 36 million tonnes of ore per year with a mining capacity of 67.5 million tonnes per year. The mine life is 10 years. Table 4-10 summaries all these input parameters.

Implementation details for Model #1 are presented in Section 3.5.1. Vector $\mathbf{x}$ contains the decision variables: $z(t;i)$, $y(t;i)$ and $a(t;i)$. The total number of variables is 55020 which is three times of number of mining-cuts multiplied by the number of periods. There are 18340 binary variables and 36680 continuous variables in this model.

Vector $\mathbf{c}$ which stores the coefficient values for the objective function contains 55020 elements. Matrix $\mathbf{A}$ contains the coefficients of the constraints. The number of columns is equal to the number of variables which is 55020. The rows of this matrix contain the following constraints:

- Zero constraint for input head grade (Eq. (2.12)). For this case study no constraint is considered for input head grade.
- 20 constraints (two for each period) for processing capacity; the lower limits are set to zero. These constraints are formulated in Eq. (2.13) and implemented in Eqs. (3.13) and (3.14).
- 20 for mining capacity; two (upper and lower) limits for each period. The lower limits are set to zero. These constraints are formulated in Eq. (2.14) and implemented in Eqs. (3.16) and (3.17).
- 18340 ore-mining constraints based on Eq.(2.15), which enforce the portion of the block that is extraction in each period to be greater than the portion of processing. These constraints are implemented in Eq. (3.20) and (3.21).
- There are 126626 constraints that control the precedence of block extraction based on the Eqs. (2.16), (2.17) and (2.18). The implementations for these constraints are shown in Eqs. (3.26) and (3.27).
- There are 3668 constraints that force all the blocks inside the final pit to be extracted. This constraint is so called reserve constraint (Eq. (2.19)). The equality constraints are split into two inequality constraints with equal lower and upper limits. Therefore the total number of reserve constrains equals to two times of number of mining cuts. The implementations are based on Eq. (3.29), (3.30) and (3.31).

There are 146840 constraints based on the constraints that are models in Eq. (2.12) to (2.19). Therefore matrix **A** has 146840 rows and 55020 columns.

The upper limit and lower limit of the variables are also kept in the $\mathbf{x}_U$ and $\mathbf{x}_L$. Each of these vectors has 55020 rows. Therefore there are 110040 constraints (two for each variable) to control the upper and lower limit of the variables. The lower and upper limits for all decision variables are respectively zero and one.

Table 4-15 summaries the size of each of these matrices and vectors. The total number of constrains is 403720. However not all of these constraints are active. There are many inactive constraints in $\mathbf{b}_U$. The total number of active constraints equals to 258714.

There are some input parameters that are required by the CPLEX solver. These parameters are:

- MIPGAP: is set to 0.1% or 0.001. The MIPGAP is the gap of optimization.

- EPMRK: is set to 0.001. EPMRK species the amount by which an integer variable can be different from an integer and still be considered feasible.
- EPOPT: is set to 0.00001. It is called optimality tolerance. This parameter governs how closely CPLEX must approach the theoretically optimal solution. This parameter is used in LP relaxation stage.
- MIPEMPHASIS: is set to 1. This parameter emphasizes feasibility over optimality. It is very important that the generated answer to be feasible and satisfy all the constraints.
- MIL probe level: is set to 0. Determines the amount of probing on variables to be performed before MIP branching. Higher settings perform more probing. Probing can be very powerful but very time consuming at the start. Setting the parameter to values above the default of 0 (automatic) can result in dramatic reductions or dramatic increases in solution time, depending on the model. Probing can dramatically improve performance, although it may also consume significant amounts of time.
- DIVE-TYPE (MIP dive strategy): is set to 2. The MIP traversal strategy occasionally performs probing dives, where it looks ahead at both children nodes before deciding which node to choose. The default (automatic) setting lets CPLEX choose when to perform a probing dive, if the value is set to1 then CPLEX will never perform probing dives; 2: always to probe, and 3: spend more time exploring potential solutions that are similar to the current incumbent. The recommended number for DIVE-TYPE in mixed integer programming is 2.
- CLIQUES (MIP cliques indicator): is set to 0. This parameter determines whether or not clique cuts should be generated for the problem. Setting the value to 0, the default, indicates that the attempt to generate cliques should continue only if it seems to be helping.

CPLEX is a parallel solver and it can be run in as many CPUs that are available in the system. For this case study 7 CPUs are assigned to CPLEX solver.

Table 4-16 shows CPU time and the real time in seconds. The real time is the elapsed real runtime that CPLEX takes to solve the optimization problem and terminates based on the MIPGAP parameter. It takes 11723 seconds or 3 hours and 15 minutes to solve this problem. The CPU time is the sum of the all seconds that any of these CPUs was busy with the solving the problem. It is 58915 seconds or 16 hours and 22 minutes. This means that if the system had 1 CPU, the real time would be close to this number. On the other hand, by having 7 CPUs, the real time is not decreased by the same factor. The real time is about 5 times less than the CPU time. There are two reasons for this: (1): all algorithms inside the CPLEX solver are not parallelized. There is also wait-time as some subroutines wait to get results from other subroutines.

Table 4-15: The dimensions of the matrices for MILP with OK block model (Model #1)

Matrix	$\mathbf{x}$	$\mathbf{c}$	$\mathbf{A}$	$\mathbf{b}_U$ and $\mathbf{b}_L$	$\mathbf{x}_U$ and $\mathbf{x}_L$
row	55020	55020	146840	146840	55020
column	1	1	55020	1	1

4.5.1 Verification of the Schedule Generated by Model #1

Table 4-17 summarizes the production schedule in each period. The values are based on the OK block model. The generated model satisfied all the input constraints. The input ore to the mill in first two periods are zero due to the pre-striping. The annually input ore tonnage to the mill does not exceed the processing limit of 36 MT in each period. The annual mining limit which is the sum of ore and waste tonnage in each period does not exceed the total tonnage of 67.5 MT in each period. All the precedence relationships are satisfied. The NPV of the project is 2461 million dollars. This is 3% higher than the NPV that has been reported by Whittle. The total processed ore tonnage after 10 years is 282.5 MT that is equal to the total ore tonnage inside the final pit. Therefore there is no rejected ore tonnage in this schedule. The total average of the input head grade is 10.3 m%. The mill has been feed with full capacity at all the periods except the final period. The shortfall in the final period is due to the less remained ore in this period. This model shows a significant improvement at feeding ore to the plant with full capacity compared to the Whittle generated schedule.

The cross sections and plan view of the generated schedule are presented in Figure 4-35. These are the same cross sections and plan view that has been presented for Whittle results in Figure 4-25. There are no push-backs for this model therefore the generated schedule is quite different from Whittle. However the schedule is still feasible and no gap effect is detected.

Table 4-16: The performance of MILP for the Model #1

Parameter	CPU time seconds	Real time seconds	Final Gap percentage
value	58,915	11,723 or	0.01%

Table 4-17: Summary of Production schedule in each period for OK block model (Model #1).

Period	Input Ore MT	Waste MT	Mined MT	SR	Grade M%	stockpile MT	stockpile grade M%	CDCF MD
1	0.0	46.2	46.20	Inf.	0.00	0.0	0.00	-193.18
2	0.0	67.5	67.50	Inf.	0.00	0.0	0.00	-449.79
3	36.0	31.5	67.50	0.88	11.47	0.0	0.00	132.02
4	36.0	31.5	67.50	0.88	11.31	0.0	0.00	650.56
5	36.0	31.5	67.50	0.88	10.17	0.0	0.00	1053.99
6	36.0	31.5	67.50	0.88	9.85	0.0	0.00	1403.09
7	36.0	31.5	67.50	0.88	9.59	0.0	0.00	1707.77
8	36.0	31.5	67.50	0.88	10.05	0.0	0.00	2005.40
9	35.1	32.4	67.50	0.92	10.27	0.0	0.00	2274.61
10	31.4	36.1	67.50	1.15	9.66	0.0	0.00	2460.98
Total	**282.5**	**371.2**	**653.6**	**1.31**	**10.3**	**0.0**	**0.00**	**2460.98**

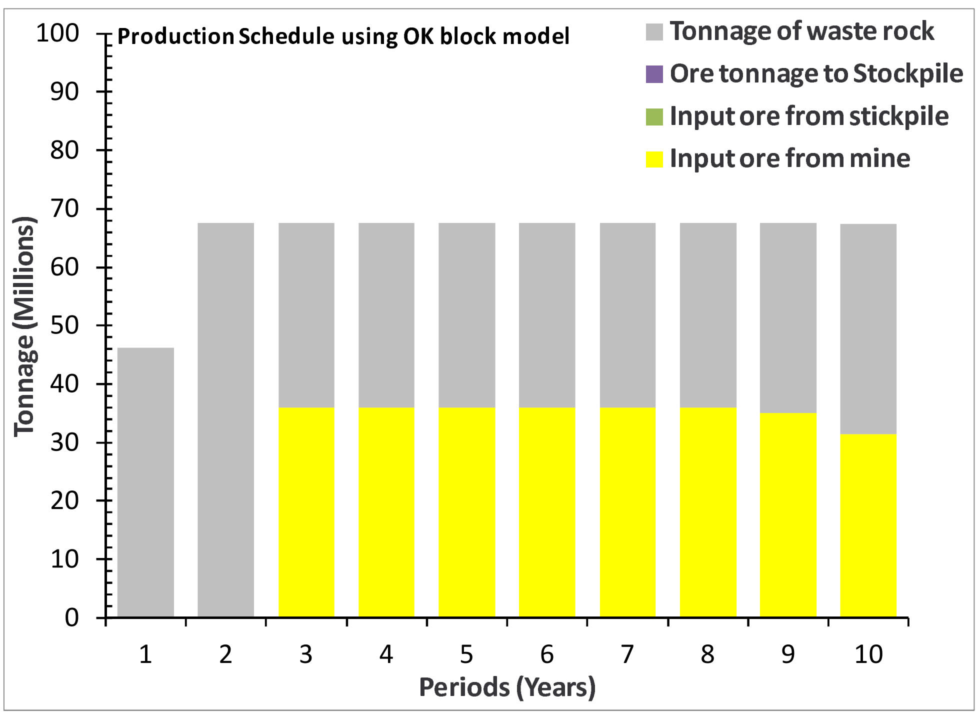


Figure 4-34: Production schedule generated by MILP with OK block model (Model #1)

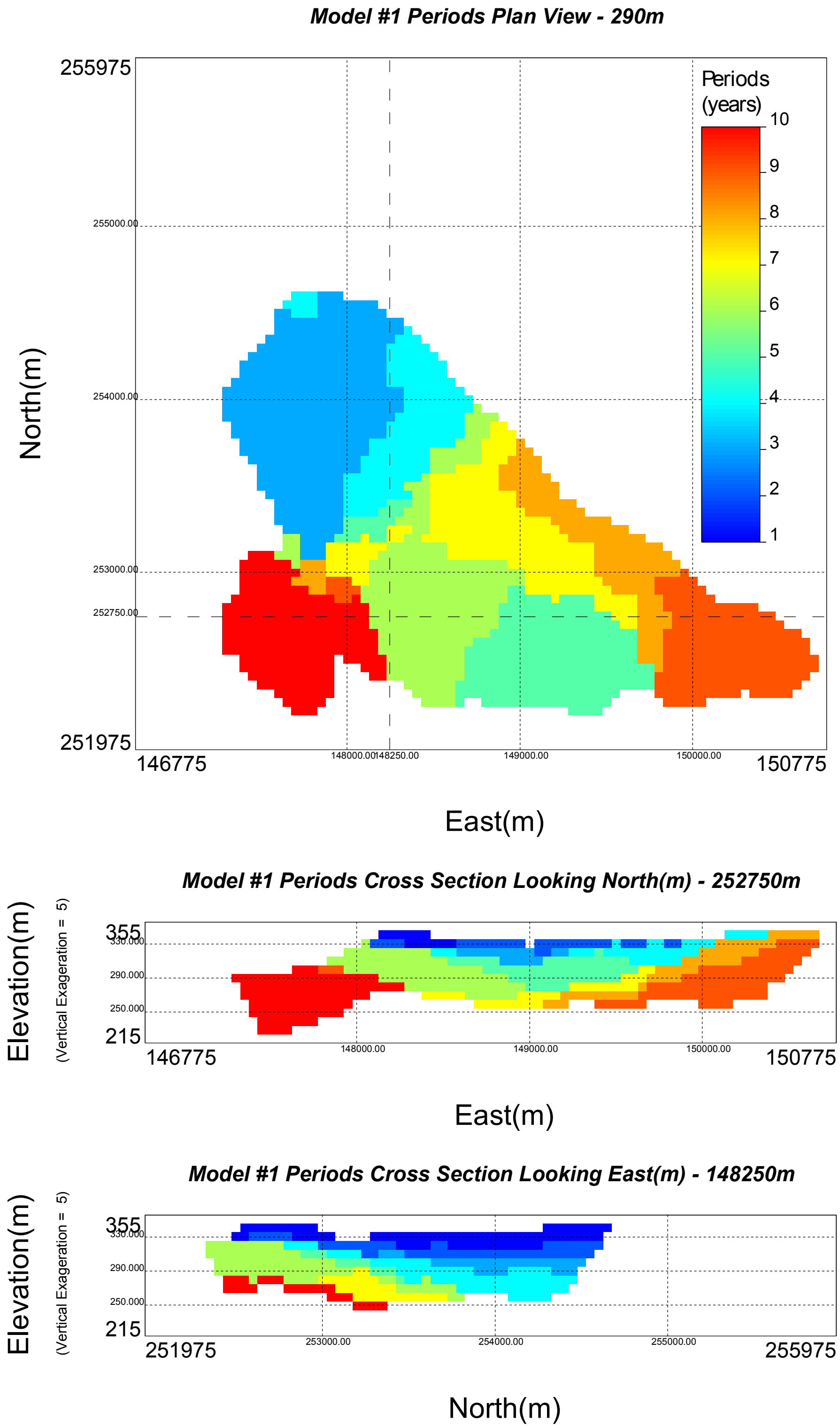


Figure 4-35: Extraction periods of production schedule generated by MILP with OK block model (Model #1).

4.5.2 Uncertainty Assessment of the Schedule Generated by Model #1

The performance of the generated schedule in presence of grade uncertainty is studied. Everything inside the final pit has been removed by this model, so the total statistics such as total tonnage of ore, strip ratio and total tonnage of bitumen are the same as Table 4-9 and Figure 4-20 to Figure 4-23. The detailed uncertainty assessment of the final pit limit has been studied in Section 4.3.1.

To assess the effect of grade uncertainty on the production schedule the same procedure as Section 4.4.2 is followed here. None of the realizations is used in the objective function and the constraints. The schedule may generate extra ore that exceeds the annual feed limit of the plant. Also, in the pre-striping periods it is possible that extra ore is produced. The post processing step is applied to make the schedule feasible at all realizations. The same assumption as taken in Section 4.4.2 is applied here to transfer any extra ore to the next periods and NPV and other parameters are recalculated for each of the realizations.

Figure 4-36 contains the same 6 graphs that are presented with Whittle. The orders of the graphs are kept the same. Therefore in Figure 4-36a and Figure 4-36b the average grade of mined ore and the tonnage of mined ore are presented, respectively. These two graphs are before post-processing. In most of the realizations there is some extra ore that is mined in first two periods that the processing plant is not working due to the pre-striping. Therefore the extra ore of periods 1 and 2 has been transferred to the period 3. Figure 4-36c and Figure 4-36d show the post processing results for input average ore and tonnage to the mill respectively. Figure 4-36e shows the tonnage of ore in the stockpile for each realization. Figure 4-36f also shows the discounted case flow over the mine life. Unlike Whittle, Model #1 does not have a significant shortfall in the periods 9 and 10. The generated schedule with this model has higher NPV and fewer shortfalls compare to the Whittle.

Table 4-20 also presents summary statistics for this model. The ordinary kriged block model has the largest NPV over all realizations because the optimization model has been applied to this block model. The average and the standard deviation of NPV that is calculated from all realizations are 2334 and 62.82 million dollars, respectively. Table 4-18 shows the effect of post-processing stage for this model. The cost of grade uncertainty before and after post-processing is 118 million dollars and 81 million dollars, respectively. The stockpile can reduce the cost of grade uncertainty up to 31.2%.

Figure 4-37 and Figure 4-38 show the box-plot of the mined ore and the input ore to the mill that is calculated from the realizations. The post processing stage removes overproduced ore and transfers the extra ore to the next period if there is any shortfall from target. Therefore the deviations from target production are reduced.

The schedule created by Model #1 has less deviation from the target production than Whittle. By comparing Figure 4-38 with Figure 4-28 there is a significant reduction in the deviation from target production in the first three years of production. The main reason is that the extra ore that is generated in period 3 is used to reduce the deviation from target in later years.

Table 4-19 shows $\overline{DTP}$ values before and after post-processing. Both of these two values are less than the equivalent values from Whittle.

By comparing the NPV of this model and Whittle (Table 4-25 vs. Table 4-20) it can be concluded that the average NPV increases by 8.6% while the average cost of grade uncertainty is decreased by 26.5%. Also the standard deviation of the NPV has been decreased from 63.7 to 62.8 million dollars.

Figure 4-39 shows the histograms of mined ore in different periods. Period 3 has the highest average of overproduced ore among all other periods. This extra ore can be use in later years whenever a shortfall happens. The shortfalls at the final period (period #10) is due to the less amount of ore remained to this period and does not taken in to account in the calculation of the average cost of grade uncertainty.

The histograms of processed ore in each period are showed in Figure 4-40. These are post-processed results. The post-processing by assuming a presence of a stockpile has a significant impact on the production schedule.

4.5.3 Conclusion

The Model #1 has been applied to the OK bock model and the uncertainty has been assessed by following the same schedule for each of the realizations. The NPV of the OK block model is maximized by considering hard constraints on the mining and processing limits. It is a feasible production plan and there is not significant gap effect in the periods. The NPV is significantly higher that Whittle. The generated schedule provides a smooth input ore to the mill in each period. However the post-processing stage has a huge impact on the uncertainty of the input tonnage to the mill. The average cost of the uncertainty of this model is less than the Whittle generated schedule. The variance of the NPV also is less than the Whittle method.

The disadvantage of the model is that:

- The realizations and grade uncertainty are not taken into account in this model at the optimization stage.
- The stockpile has not been used directly in the model.

The generated schedule is sensitive to grade uncertainty and the post-processing stage. Therefore, the production schedule that does not consider the grade uncertainty may not be the optimum solution. In the next section, Model #2 is used to take the realizations into account at LTPP problem.

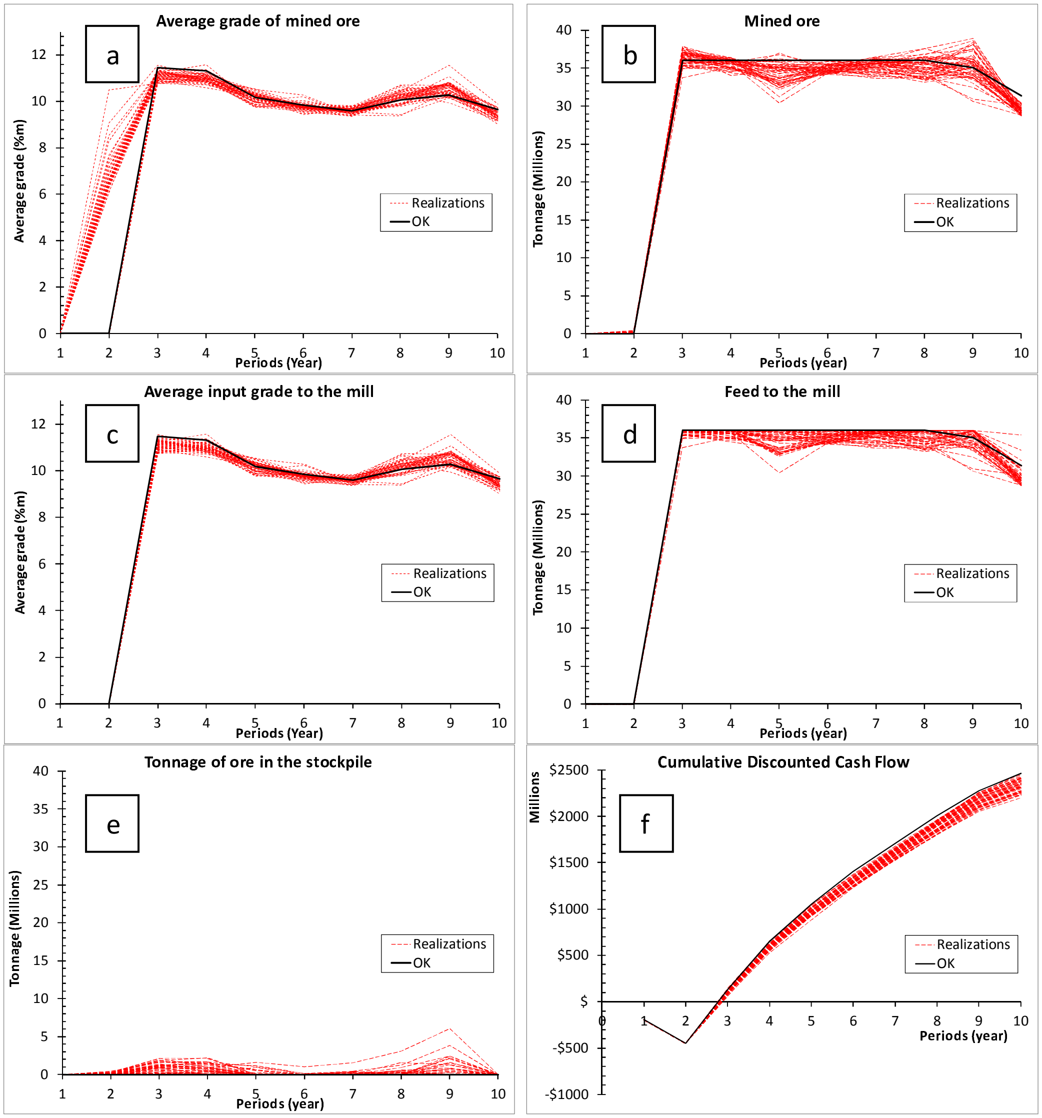


Figure 4-36: a: average grade of mined ore, b: tonnage of mine ore, c: average grade of input ore to the mill before post processing, d: tonnage of ore input to the mill after post processing, e: tonnage of ore at the stockpile after post processing and f: CDCF after post processing stage; the schedule generated by MILP with OK block model (Model #1).

Table 4-18: DCOU with and without stockpile for the schedule generated by Model #1

	Discounted cost of grade uncertainty (MD)
Without post processing (No stockpile)	117.97
After post processing (With stockpile)	81.15

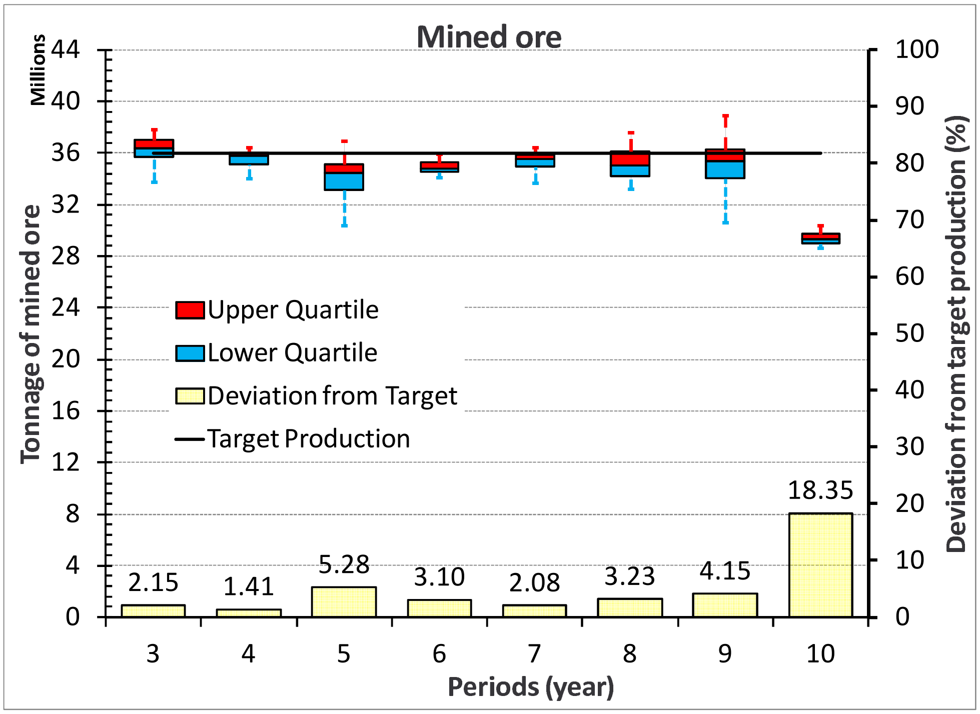


Figure 4-37: Box-plot and deviation from target production for each period; the schedule generated by MILP with OK block model (Model #1).

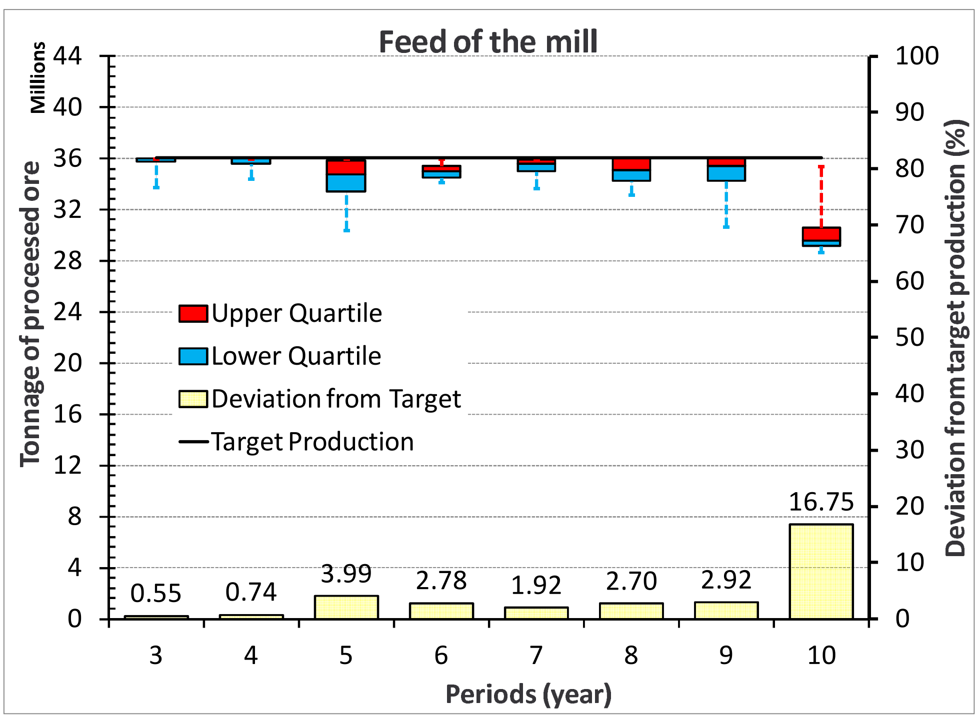


Figure 4-38: Feed of the plant and the box-plot of the deviation from target production for each period; the schedule generated by MILP with OK block model (Model #1).

Table 4-19: $\overline{DTP}$ with and without stockpile the schedule generated by MILP with OK block model (Model #1).

	Average deviation from target production ($\overline{DTP}$)
Without post processing (No stockpile)	3.06%
After post processing (With stockpile)	2.23%

Table 4-20: Summary of statistics for the production schedule after post processing stage; the schedule generated by MILP with OK block model (Model #1).

Statistics	Average grade (m%)	NPV (MD)	DCOU (MD)
Mean	10.24	2334.41	81.15
Std. dev	0.09	62.82	45.29
Min	10.02	2201.39	0.00
Quartile 1	10.19	2283.42	44.13
Median	10.24	2331.40	77.61
Quartile 2	10.29	2386.67	116.49
Max	10.52	2458.97	192.14
OK	10.31	2460.98	N/A

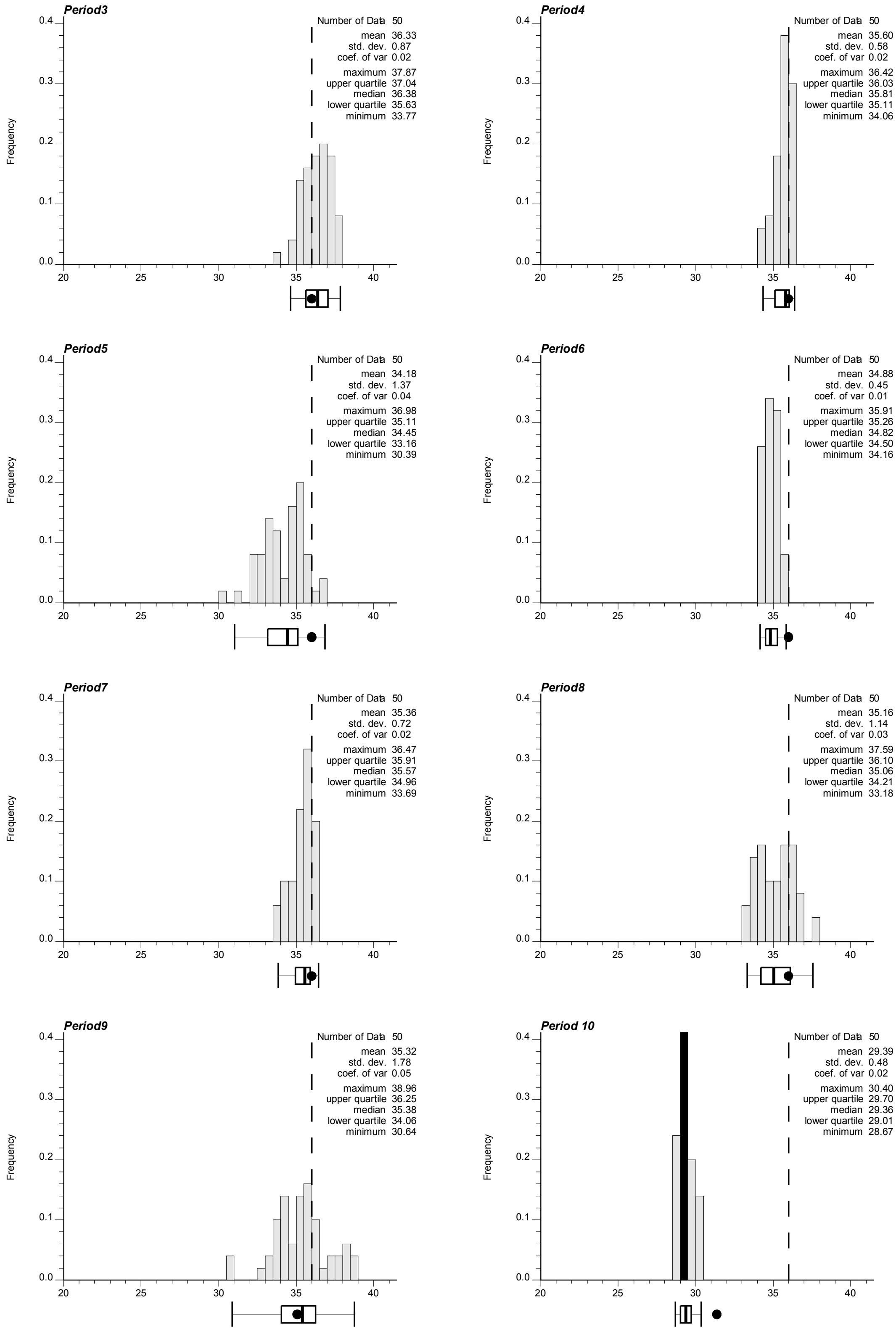


Figure 4-39: Histograms of tonnage of mined ore in different periods. The schedule is generated with MILP using OK block model (Model #1). OK result is marked by solid circle and dash line indicates 36 MT of target production.

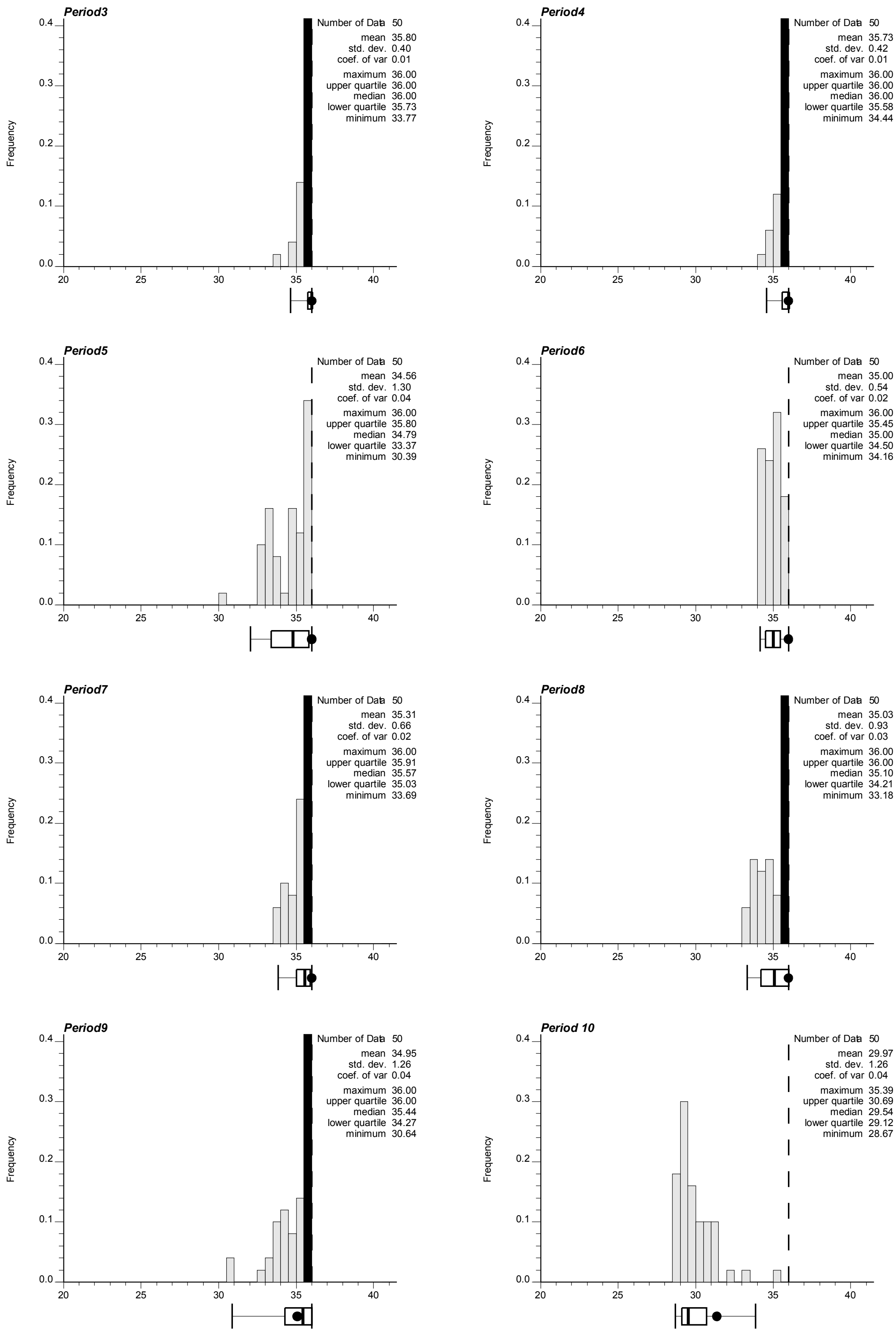


Figure 4-40: Histograms of input ore tonnage to the mill. The schedule is generated with MILP (Model #1) using OK block model and post pressed by assuming presence of stockpile. OK result is marked by solid circle and dash line indicates 36 MT of target production.

4.6 Production Scheduling With Model #2

As discussed in Section 4.5.3, the effective way to consider the grade uncertainty is to implement the realizations in the optimization model. Model #2 (Eq. (2.35)), that has been shown in Section 2.5 uses the realizations to optimize over and under production in each period. Generally, the average cost of grade uncertainty has been minimized while the NPV of the OK block model is maximized. The unit discounted cost of over and under production per tonne ($c_{up}(t)$ and $c_{op}(t)$) is an input parameter for this model. Model #2 can be run with two different modes:

- With a symmetric penalty function (Figure 2-9a): In this model the cost of overproduction is equal to the underproduction, $c_{up}(t)=c_{op}(t)$. Symmetrical penalty function should be used when there is no stockpile available and production plan cannot handle the possible extra ore tonnage effectively.
- With an asymmetric penalty function (Figure 2-9b): This case is recommended when there is a stockpile in the mine and any probable overproduced ore can be sent to the stockpile to be used in the next periods. Therefore the cost of overproduction is lower than the cost of underproduction.

In this section the results of the model are presented with the symmetric penalty function. Other parameters such as the mining and processing limits, the discounting rate and all other economical parameters are kept exactly the same as previous cases (Whittle and Model #1).

Model #2 has more decision variables compared to Model #1. There are 2 new continuous variables $T_{up}(t;l)$ and $T_{op}(t;l)$ for each realization in each period (totally 1000). These variables contain the tonnage of over and under production at reach period and each realization. Therefore the total number of variables increases to 56020. There are also two more constraints per period per realization that are showed in Eq. (2.36). Therefore Model #2 has 1000 extra constraint.

Table 4-21 shows the sizes of the matrices and vectors that are imported to the CIPLEX.

MIPGAP is set 0.5% or 0.005. Other parameters of CPLEX are the same as Model #1.

Table 4-21: The dimensions of the matrices for MILP with OK block model and realizations (Model #2)

Matrix	$\mathbf{x}$	$\mathbf{c}$	$\mathbf{A}$	$\mathbf{b}_U$ and $\mathbf{b}_L$	$\mathbf{x}_U$ and $\mathbf{x}_L$
row	56020	56020	147840	147840	56020
column	1	1	56020	1	1

The penalty function is the discounted cost of under and overproduction ore in each period. $c_{up}(t)$ and $c_{op}(t)$ that were introduced in Chapter 2 are used in this model for the discounted cost of under and overproduction per unit ore tonnage in period t. Eqs. (2.22) and (2.24) calculate these two values. It is assumed that the average grade of the input grade to the mill is about 11m%. Other parameters are given at Table 4-4. So, the cost of the under and overproduction in period zero ($c_{up}(0)$ and $c_{op}(0)$) equals 25 \$/tonne. The average cost of under and overproduction ($\overline{c}_{up}(t)$ and $\overline{c}_{op}(t)$), which are calculated by Eq. (3.39), in period zero are 0.5 \$/tonne. Table 4-22 shows the $\overline{c}_{up}(t)$ (on top) and $\overline{c}_{op}(t)$ (on bottom) values over the time periods for one tonne of ore. The symmetric function has been considered for this case. The only difference is in period 10 where the average discounted cost of underproduction is zero. The reason has been given in Section 2.4.

Table 4-23 shows the performance of this model. The run time to solve this problem was 183600 seconds or 2 days 3 hours 1 minute and 0 seconds. The CPU time was 1272500 seconds or if the problem has been tried to solve in one CPU it would take 14 days 17 hours 28 minutes and 20 seconds. The run time in parallel mode is reduced by 6.9 times. This shows that by using more CPUs the speed of the algorithm can be increased significantly. However this model is 20 times slower than Model #1.

Table 4-22: Discounted penalty value of over and under production over the periods.

Periods	0	1	2	3	4	5	6	7	8	9	10
$\overline{c}_{up}(t)$	0.5	0.455	0.413	0.376	0.342	0.310	0.282	0.257	0.233	0.212	0.000
$\overline{c}_{op}(t)$	0.5	0.455	0.413	0.376	0.342	0.310	0.282	0.257	0.233	0.212	0.193

4.6.1 Verification of the Schedule Generated by Model #2

Table 4-24 shows the summary of the Model #2 with symmetric penalty function. Figure 4-41 also shows the tonnage of ore, waste and stockpiled ore in different periods. The annual processing capacity has been satisfied in all periods. The mining capacity is satisfied in all periods too. The same as Model #1, there is no rejected ore in this case and all the ore blocks inside the final pit have been processed. The NPV of this model is 2454 million dollars. There is no significant difference in the NPV of this model and Model #1.

Figure 4-42 shows the plan view and two cross sections. The plan view and the cross sections are not very different from the previous model shown in Figure 4-35. The big difference is in early periods. This model tries to defer the uncertain blocks to later years. The discounted cost of grade uncertainty has less effect on the objective function at the later years. Therefore, in the early years of the mine life less uncertain blocks are extracted.

Table 4-23. The performance of MILP for the Model #2

Parameter	CPU time seconds	Real time seconds	Final Gap percentage
value	1,272,500	183,660	0.5%

Table 4-24: Summary of Production schedule in each period for OK block model.

Period	Input Ore MT	Waste MT	Mined MT	SR	Grade M%	stockpile MT	stockpile grade M%	CDCF MD
1	0.00	46.40	46.40	Inf.	0.00	0.0	0.00	-194.03
2	0.00	67.50	67.50	Inf.	0.00	0.0	0.00	-450.64
3	36.00	31.50	67.50	0.88	11.11	0.0	0.00	105.08
4	36.00	31.50	67.50	0.88	11.24	0.0	0.00	619.03
5	36.00	31.50	67.50	0.88	10.43	0.0	0.00	1038.05
6	36.00	31.50	67.50	0.88	9.98	0.0	0.00	1394.61
7	36.00	31.50	67.50	0.88	9.56	0.0	0.00	1697.95
8	36.00	31.50	67.50	0.88	10.03	0.0	0.00	1994.55
9	35.12	32.38	67.50	0.92	10.34	0.0	0.00	2266.71
10	31.33	35.89	67.22	1.15	9.68	0.0	0.00	2453.88
Total	**282.5**	**371.2**	**653.6**	**1.31**	**10.31**	**0.0**	**0.00**	**2453.88**

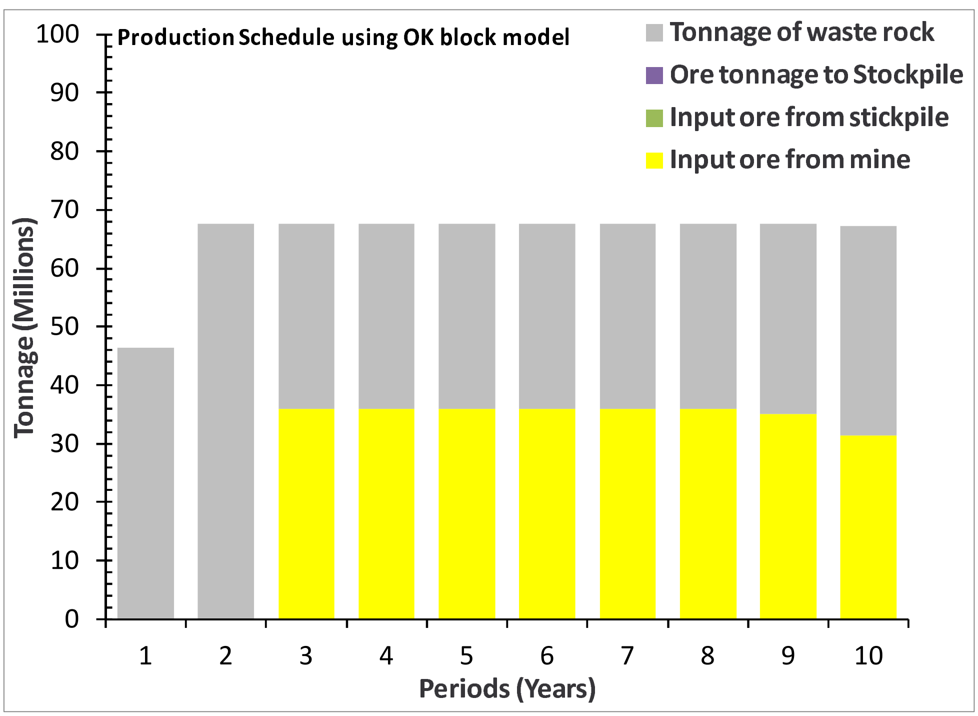


Figure 4-41. Production schedule generated by MILP with OK block model and realizations (Model #2) with symmetrical penalty function

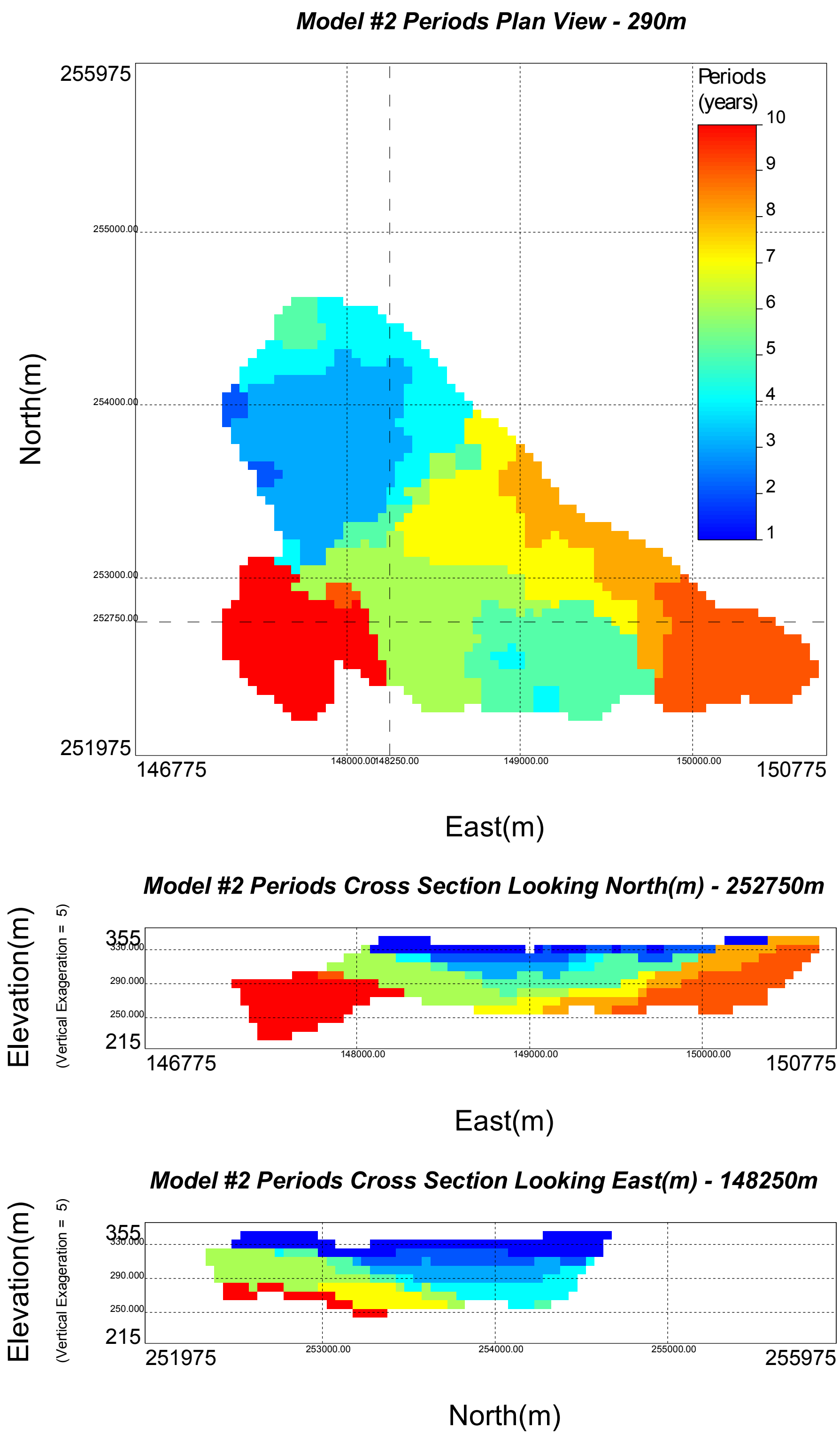


Figure 4-42: Extraction periods of production schedule generated by MILP using OK block model and realization with symmetric penalty function (Model #2).

4.6.2 Uncertainty Assessment of the Schedule Generated by Model #2

Figure 4-43 shows the same graphs as Figure 4-26 for Whittle and Figure 4-36 for Model #1. The discounted cost of grade uncertainty for the schedule generated with Model #2 with symmetric penalty function is shows in Table 4-25. The values of DCOU before and after post-processed are 89.4 MD and 80.5 MD, respectively. The following conclusions are drawn from this table:

- The DCOU of Model #2 before post-processing is 89.4 where the relevant value form previous case with Model #1 is 118. The cost of grade uncertainty is reduced by 23%.
- The post-processing stage does not make a big difference in DCOU in Model #2. The DCOU after post-processing is 80.5 and it is reduced only about 10 % after prost-processing the schedule.

Therefore Model #2 is more robust than Model #1 in the presence of grade uncertainty. The requirement for a stockpile can be reduced by using realizations in the optimization stage. The amount of over and under production of ore is reduced significantly. On the other hand because the stockpile is not implemented in the objective function explicitly, the post-processing of the schedule does not have a big impact and it does not reduce the cost of grade uncertainty significantly.

Figure 4-44 shows the box-plot and the deviations from target production of the mined ore in different periods. The same graph has been presented in Figure 4-37 for Model #1. The Model #2 generates less deviation from target production compared to Model #1.

Figure 4-45 shows the box-plots and deviations from target production for each period after post-processing. The values of deviations from target production at all periods are slightly less than the same values from Figure 4-44. Also, this graph is not very different from Figure 4-38 that is generated from post-processed schedule of Model #1. The overproduced ore is less for this case therefore the presence of a stockpile does not have a big effect on reducing the cost of grade uncertainty.

Table 4-26 shows the average deviation from target production. $\overline{DTP}$ before and after post-processing is 3.06% and 2.23% respectively. As it is expected, both of these two values are less than the equivalent values from Whittle and Model #1.

Table 4-27 summarizes the statistics of Model #2. The DCOU for realization #10 is 0, which means that this schedule does not generate any shortfall for realization #10.
Figure 4-46 shows the histograms of tonnage of mined ore. This model has less standard deviation in periods 3, 5, 6 and 9 compared to the Model #1.

Figure 4-47 shows the input ore tonnage to the mill after post processing the results. By post-processing the schedule, the uncertainty of input ore tonnage is reduced slightly in all periods. Since the stockpile is not implemented in the optimization, the post-processing does not have a big impact.

4.6.3 Conclusion

The results of Model #2 showed with a symmetric penalty function were more robust than Model #1. Therefore, the presence of a stockpile is essential to reduce the cost of grade uncertainty. However the presence of a stockpile does not have much effect on Model #2.

The stockpile has not been implemented in Model #2. On the other hand, the hard constraint on the processing limit that was applied to the OK block model prevents the optimization to create extra ore in early periods. If there is a stockpile in the mine, this extra ore can be used to fill the shortfall in later years. The next case shows the application of Model #3, where a stockpile is modeled directly in the optimization.

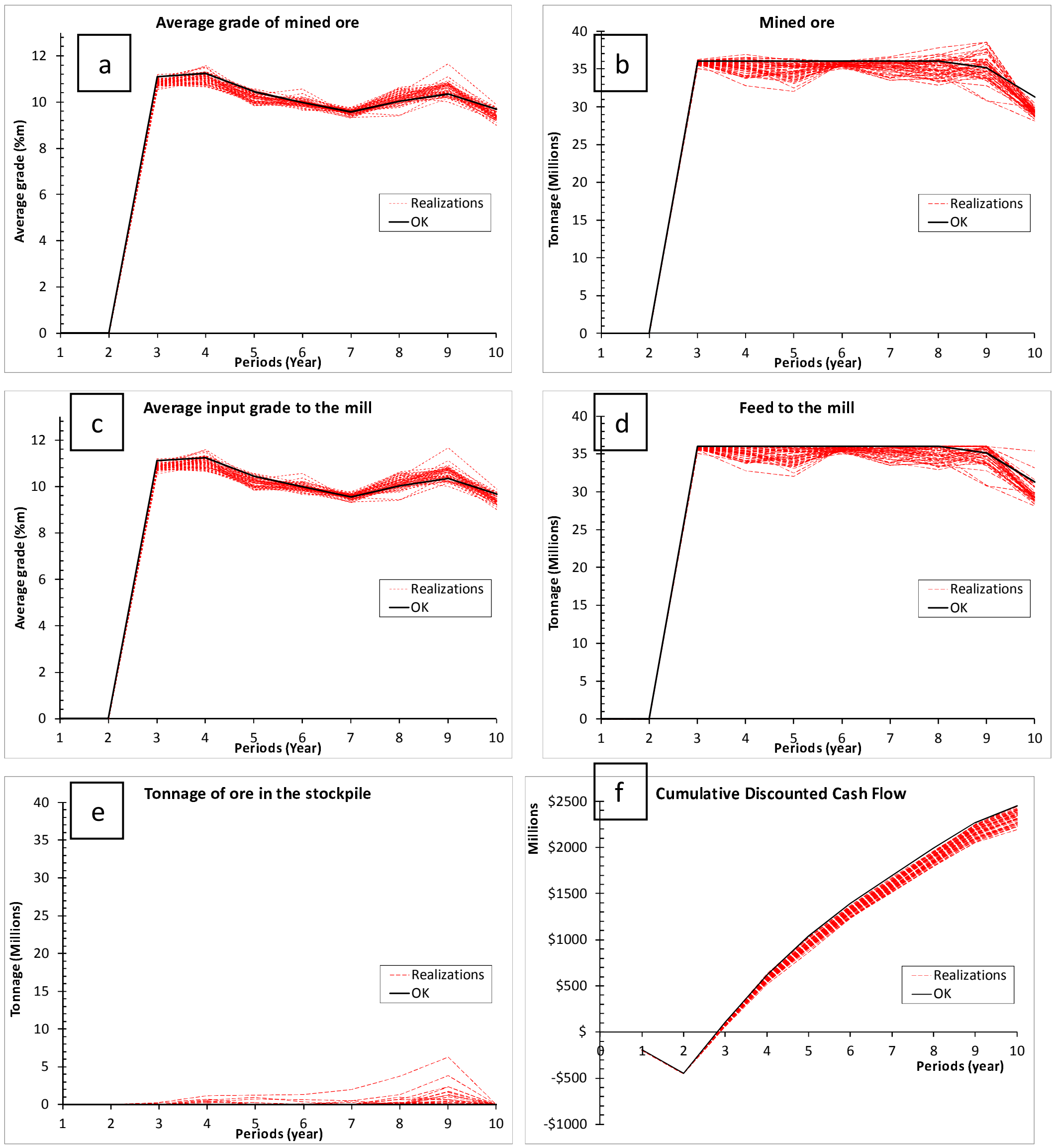


Figure 4-43: a: average grade of mined ore, b: tonnage of mined ore, c: average grade of input ore to the mill before post processing, d: tonnage of ore input to the mill after post processing, e: tonnage of ore at the stockpile after post processing and f: CDCF after post processing stage; the schedule generated by MILP with OK block model and realizations with symmetrical penalty function (Model #2).

Table 4-25: DCOU with and without stockpile for the schedule generated with Model #2 with a symmetric penalty function

	Discounted cost of grade uncertainty (MD)
Without post processing (No stockpile)	89.39
After post processing (With stockpile)	80.46

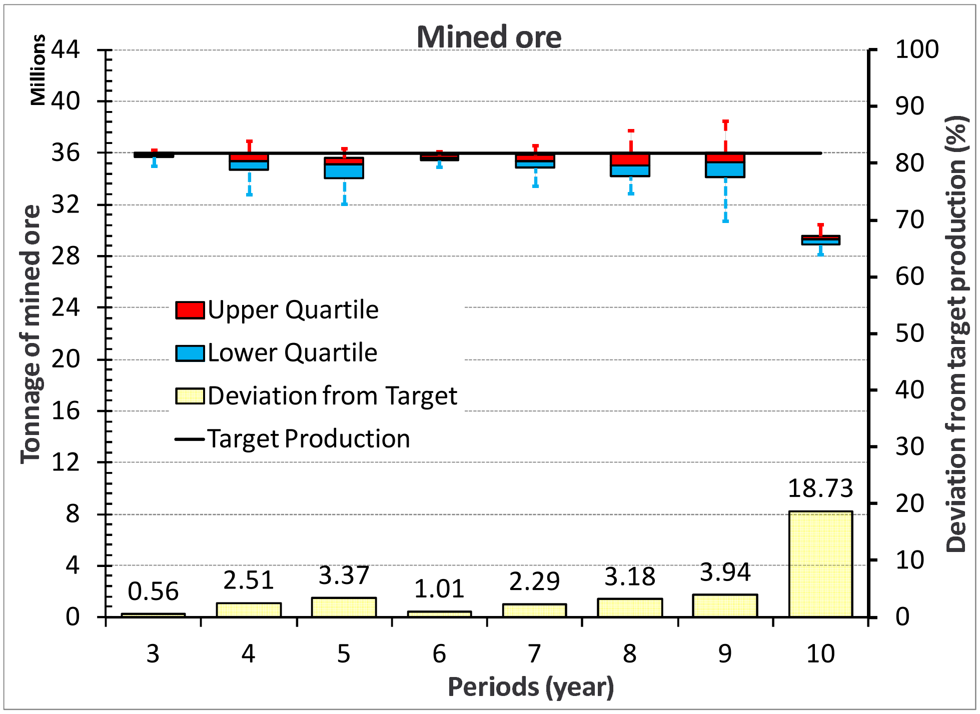


Figure 4-44: Box-plot and deviation from target production for each period; the schedule generated by Model #2 with symmetrical penalty function.

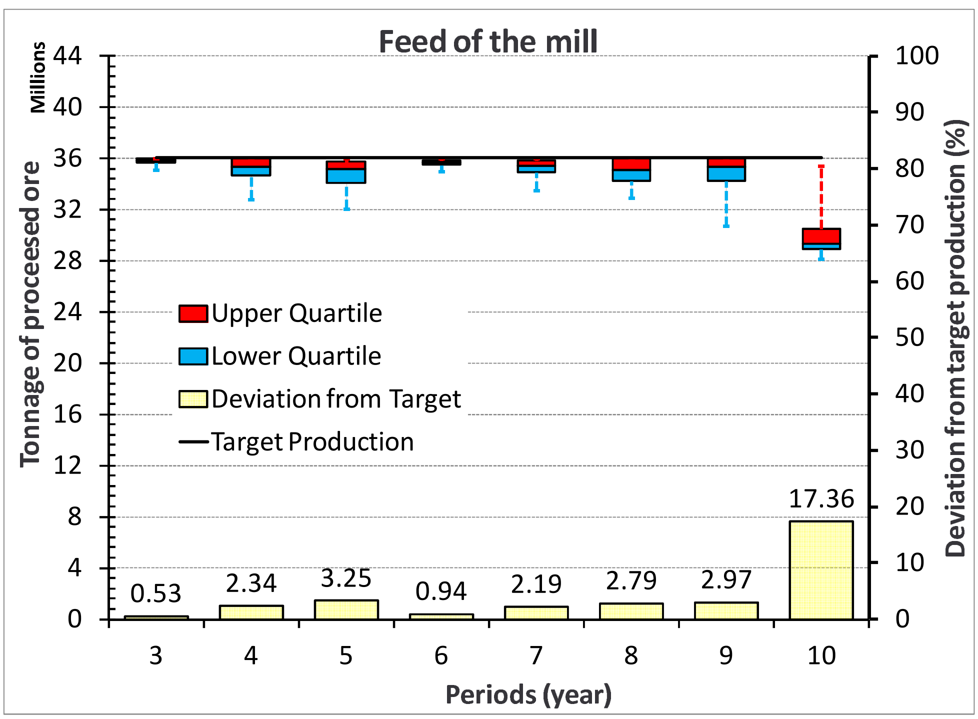


Figure 4-45: Feed of the plant and the box-plot of the deviation from target production for each period; the schedule generated by MILP with OK block model and realizations with symmetrical penalty function (Model #2).

Table 4-26: $\overline{DTP}$ with and without stockpile the schedule generated by MILP with OK block model (Model #2).

	Average deviation from target production ($\overline{DTP}$)
Without post processing (No stockpile)	2.41%
After post processing (With stockpile)	2.14%

Table 4-27: Summary of statistics for the production schedule after post processing stage.

Statistics	Average grade (m%)	NPV (MD)	DCOU (MD)
Mean	10.24	2329.06	80.46
Std. dev	0.09	63.10	47.16
Min	10.02	2195.53	0.00
Quartile 1	10.19	2276.80	41.48
Median	10.24	2324.25	80.11
Quartile 2	10.29	2380.74	115.85
Max	10.52	2456.52	203.18
OK	10.31	2453.88	N/A

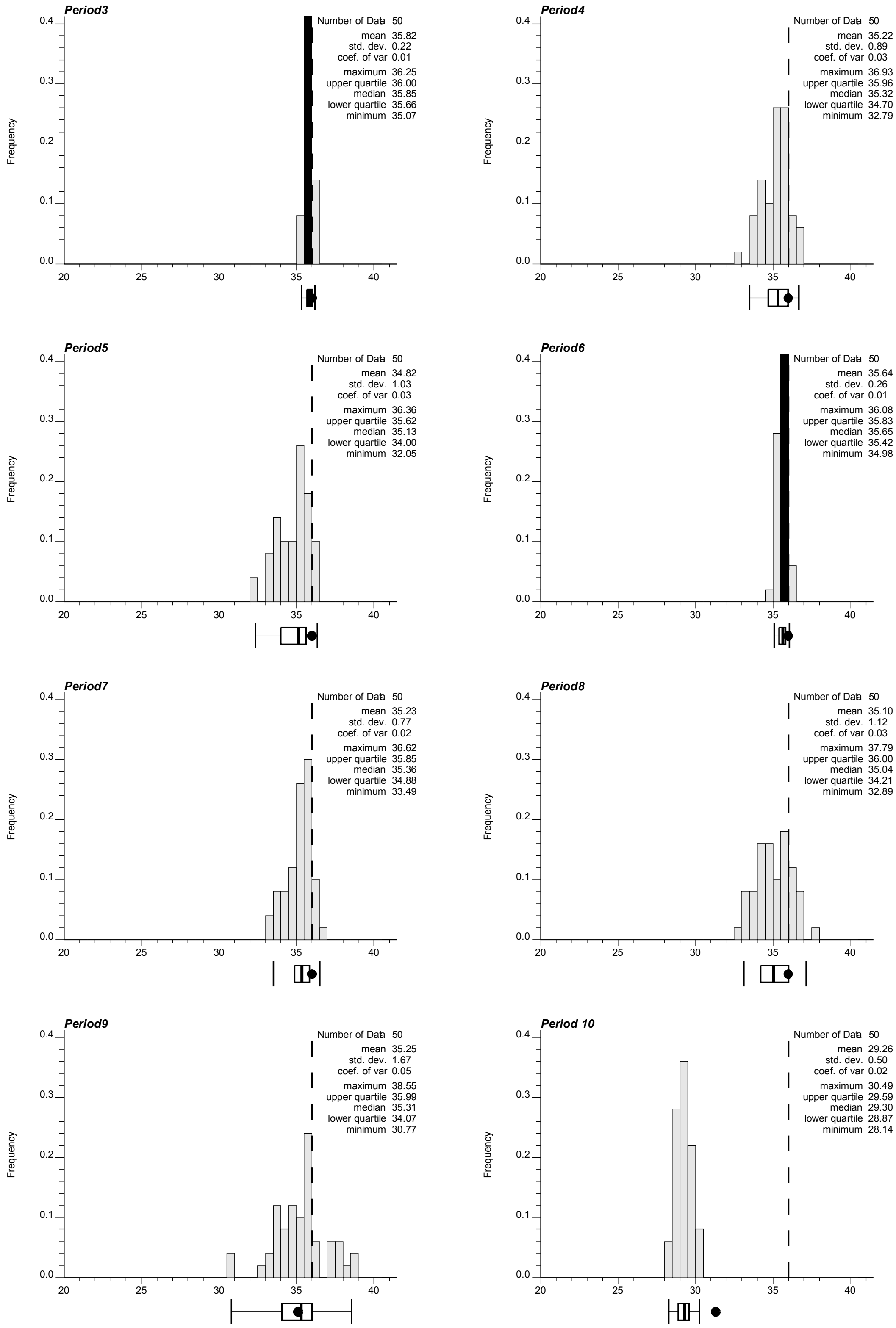


Figure 4-46: Histograms of tonnage of mined ore in different periods. The schedule is generated with Model #2 with symmetrical penalty function. OK result is marked by solid circle and dash line indicates 36 MT of target production.

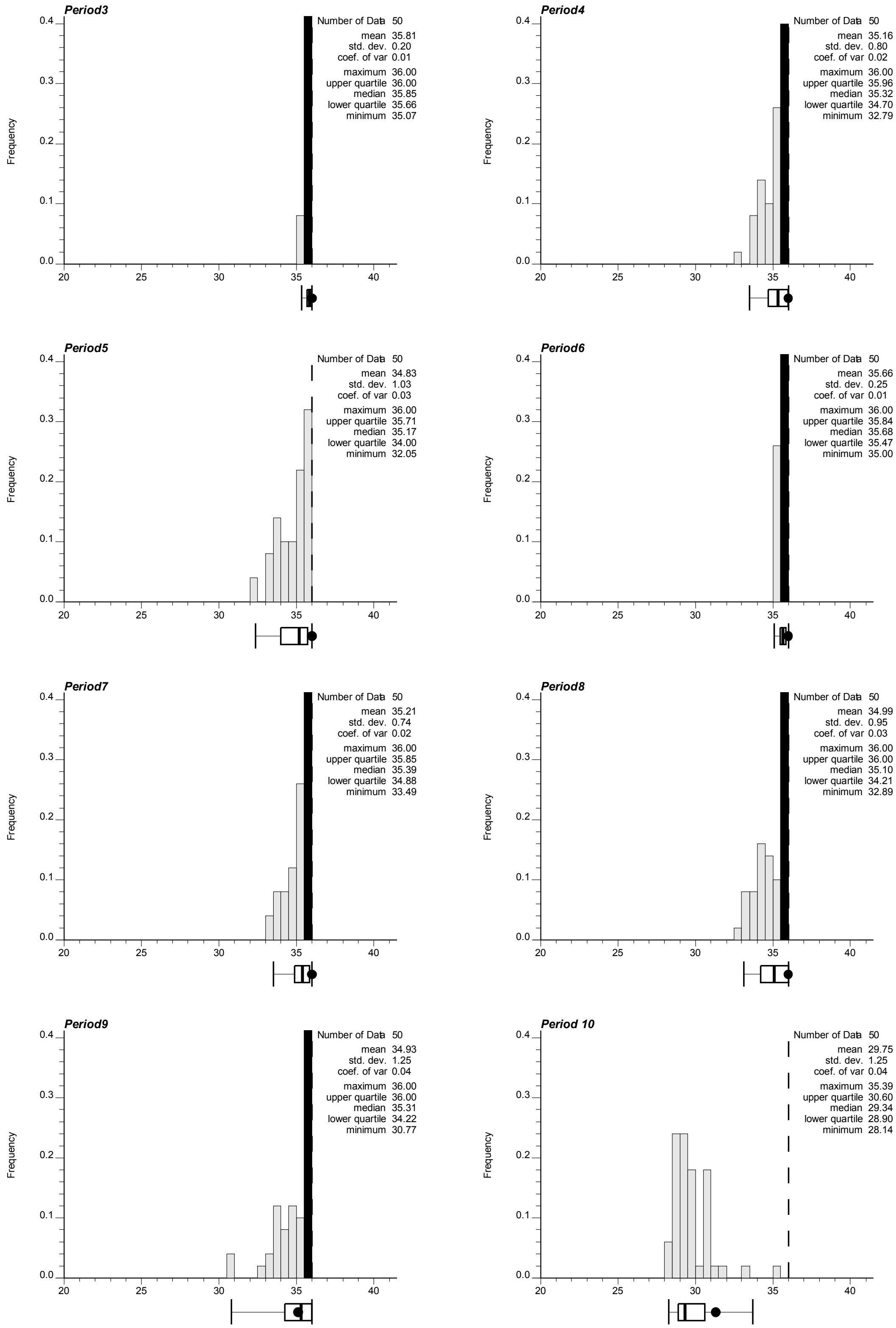


Figure 4-47: Histograms of input ore tonnage to the mill. The schedule is generated with Model #2 with symmetrical penalty function and post pressed by assuming presence of stockpile. OK result is marked by solid circle and dash line indicates 36 MT of target production.

4.7 Production Scheduling With Model #3

In real life projects, the cost of over-production is not equal to the cost of under-production. There are always good ways to deal with any extra unpredicted ore, such as stockpiling. The extra ore can be used in the later periods if for some reason the mill cannot be fed. Model #3 has a stockpile concept in the optimization stage. The same as Model #2, the idea here is to penalize over and under production of ore such that the NPV of the OK block model is maximized and the deviations from target production is minimized. There are three differences between Model #2 and #3:

1. The cost of underproduction is reduced by the additional value of extra ore available from previous periods. On the other hand the tonnage of underproduction is re-adjusted by the extra ore from previous periods.
2. The tonnage of overproduction in each period equals to the extra ore available from current period plus any extra ore remaining from previous period.

These two conditions are embedded in the constraints that controls the under and overproduced ore in Eq. (2.21) and Eq. (2.23). The implementation is shown in Section 3.5.3.

3. The hard constraint on the OK is replaced with the same constraints that controls the over and under production. Therefore with this model, it is expected that the OK block model also generates overproduction and shortfalls before post-processing stage. It is also expected that the schedule after post-processing with OK block model should be smooth and feed the plant with full capacity. The OK block model is used to maximize NPV, so it will have the largest NPV.

The upper limit of the stockpile is set to 40 million Tonne. The Lower limit is set to zero. The stockpile capacity constraints are shown in Eq. (2.40). The implementations are shown in Eq. (3.52) and (3.53) in Section 3.5.3.

With this model, there are two new variables in each period: tonnage of over and under production of OK block model. Therefore there are 20 more continuous variables than Model #2 which increases the total number of variables to 56040. The number of constraints however is the same as Model #2. The hard constraints on the upper and lower limit for the processing that is applied to the OK block model in previous models (1 and 2) is replaced with the same number of constraints that controls the over and under production constraints for the OK block model (Table 4-28).

Table 4-28: The dimensions of the matrices for MILP with OK block model and realizations (Model #3)

Matrix	$\mathbf{x}$	$\mathbf{c}$	$\mathbf{A}$	$\mathbf{b}_U$ and $\mathbf{b}_L$	$\mathbf{x}_U$ and $\mathbf{x}_L$
row	56040	56040	147840	147840	56040
column	1	1	56040	1	1

The asymmetric penalty function that is used for this model is shown in Table 4-29. The new cost of over-production is calculated by Eq. (2.28). $\hat{c}_{op,RH}(t)$ is the difference of the average cost of over-production in periods t and t+1 calculated at previous section. The average cost of under-production is the same as the previous section.

Table 4-29: Discounted penalty value of over and under production over the periods.

Periods	0	1	2	3	4	5	6	7	8	9	10
$\overline{c}_{up}(t)$	0.5	0.455	0.413	0.376	0.342	0.310	0.282	0.257	0.233	0.212	0.000
$\overline{c}_{op}(t)$	0.5	0.041	0.038	0.034	0.031	0.028	0.026	0.023	0.021	0.019	0.193

Table 4-30 shows the run times for this case. The CPU time is 16 hours, 12 minutes and 24 seconds. The real time that elapsed for this case was 3 hours and 35 minutes with 7 CPUs. The performance of the model is increased 4.5 times with 7 CPUs. This model is much faster than Model #2: The CPU time and real times are14 times and 11 times less in Model #3.

4.7.1 Verification of the Schedule Generated by Model #3

Table 4-31 shows the summary of the production schedule in each period after the post-processing stage. The input ore tonnage to the mill satisfies the processing capacity constraints (first column) at all time periods. Also, the mining capacity is satisfied at all periods because the total tonnage of mined material (third column) does not exceed the mining capacity. One of the important aspects of this model is that during the two pre-striping periods the schedule extracts 9.54 and 27.5 million tonnes of ore, respectively. This ore is transferred to the stockpile. At the beginning of the third period, 37.1 million tonnes of ore is stockpiled. The tonnage of ore that has been reclaimed from the stockpile during period 3 and 4 are 2.66 million 19.4 million. Nevertheless, the upper limit of the stockpile is satisfied and total tonnage of stockpiled ore does not exceed the upper limit of 40 million tonne.

The average grade in the stockpile in different periods is 10.52 m% to 11.03 m%. These values are very close to the assumption of 11m% that was used to calculate the cost of over and under production in Section 4.6. Therefore, there is no need to rerun the optimization with adjusted costs.

The NPV of the OK block model is 2460 million dollars. It is very close to the previous models. The stockpile does not increase the NPV of the model, but, it reduces the effect of grade uncertainty and the potential cost of grade uncertainty.

Figure 4-48 also shows the annual distribution of the input ore, tonnage of ore in stockpile, mined waste rock and input ore tonnage from stockpile to the mill over the mine life. The same as Table 4-31, these values are related to the OK block model. In periods 1 and 2 the large amount of ore is mined and transferred to the stockpile. Most of these extra ore are used in periods 3, 4 and 5. Periods 6, 7 and 8 also generate a small amount of extra ore that is transferred to the stockpile. The other portion of the ore in the stockpile is used in period 9 to feed the plant with full capacity and the remained ore in the stockpile is processed in period 10. This graph shows that in all production periods except final period the mill is fed in full capacity the same as all previous models. Also the same as previous models all ore tonnage inside the final pit is processed therefore there is no rejected ore for this model as well. The same as Model #1 and #2 the uncertainty of the total ore, average grade and bitumen are the same as Section 4.3.1.

Figure 4-49 shows plan view and cross sections for this model. There are big differences between the new schedules with other methods. This schedule digs deep early on to the ore zone rather than simply extracting over burden. The cross section looking east (bottom graph) clearly shows extraction of blocks deeps in first periods in the north part of the model area. This can be seen from plan view. As with all previous models, the southwest part of the model is extracted at the end.

Table 4-30: The performance of MILP for the Model #3

Parameter	CPU time seconds	Real time seconds	Final Gap percentage
value	58344	12891	0.56%

Table 4-31: Summary of Production schedule in each period for OK block model.

Period	Input Ore MT	Waste MT	Grade M%	stockpile MT	stockpile grade M%	CDCF MD
1	0.00	47.76	0.00	9.54	10.52	-199.72
2	0.00	39.84	0.00	37.07	11.39	-351.18
3	36.00	34.16	11.50	34.41	11.39	213.93
4	36.00	50.94	10.60	14.97	11.39	626.83
5	36.00	40.63	10.52	5.84	11.39	1030.51
6	36.00	30.37	9.76	6.97	11.13	1380.24
7	36.00	31.23	9.57	7.24	11.07	1685.78
8	36.00	31.15	9.99	7.58	11.03	1984.16
9	36.00	32.93	10.67	6.15	11.03	2258.61
10	30.40	32.20	9.74	0.00	0.00	2460.00
Total	282.4	371.2	10.27	0.0	0.00	2460.00

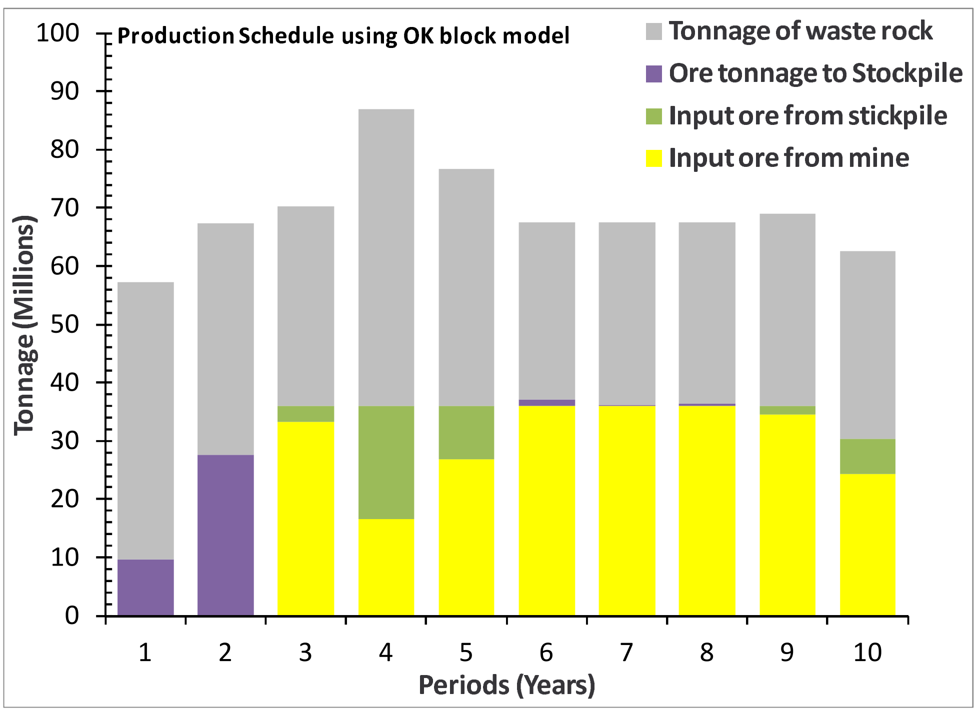


Figure 4-48: Production schedule generated by MILP and realizations and a stockpile (Model #3) with asymmetrical penalty function

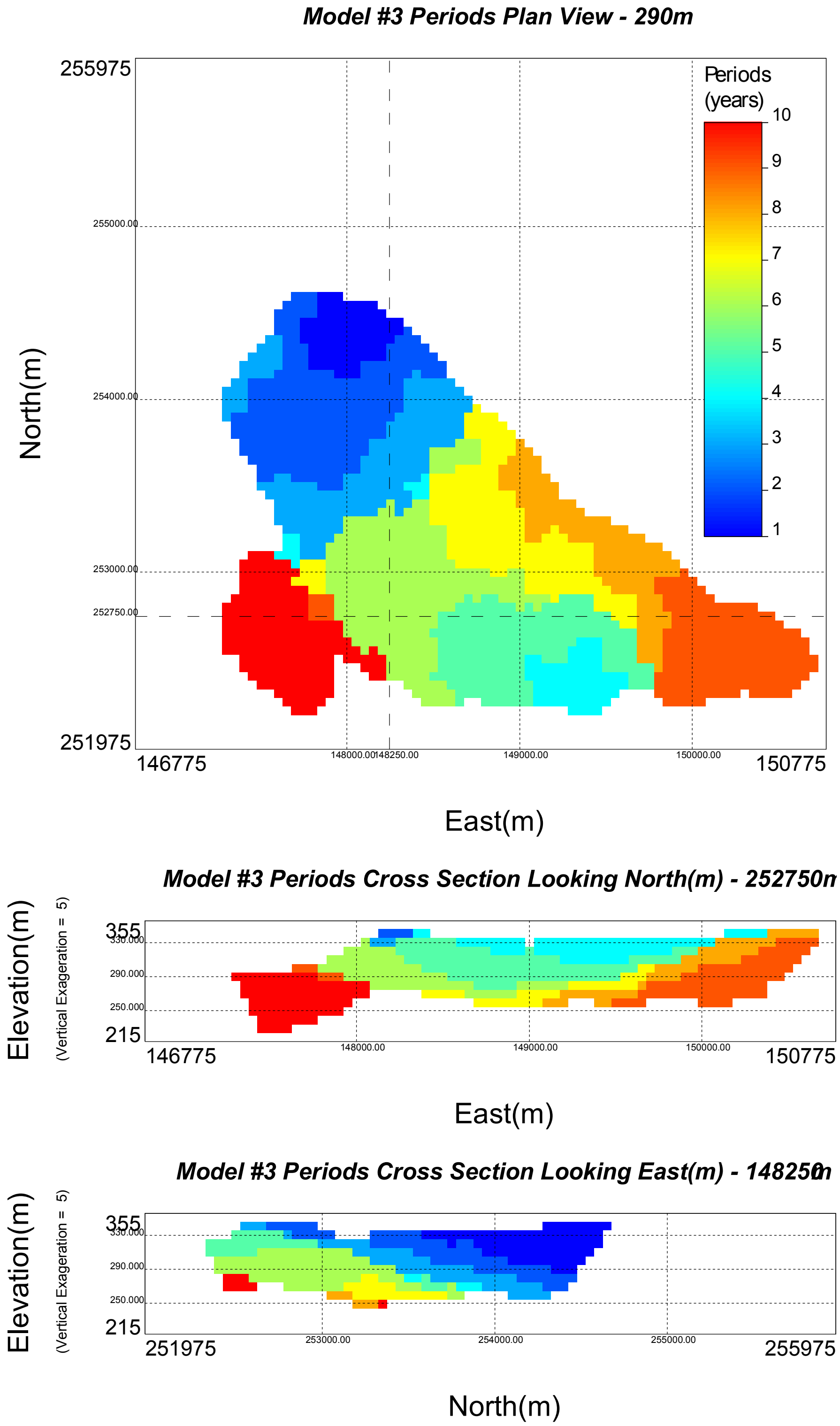


Figure 4-49: Extraction periods of production schedule generated by MILP using OK block model and realization and by considering a stockpile with un-symmetric penalty function (Model #3).

4.7.2 Uncertainty Assessment of the Schedule Generated by Model #3

The same procedure as previous models is followed for this model as well. The generated schedule is followed for all the realizations and post-processed. Figure 4-50 shows the response of the generated schedule for each of the realizations. Figure 4-50a and Figure 4-50b show the mined ore and the average extracted ore in each period. These graphs show the results before post-processing. All of the realizations generated ore in the pre-striping periods (1 and 2) is transferred to the stockpile. There is a big gap in period 4 between the mined ore and the target production. This gap is filled by the ore tonnage from stockpile. The results after the post processing are shown in Figure 4-50c and Figure 4-50d. All the extra ore that is mined in the first and second periods are transferred to the later periods such that in periods 3 and 4 all of the realizations feed the plant with full capacity. Figure 4-50e shows the tonnage of ore inside the stockpile in each period for each realization. The maximum amount of ore is stockpiled in period 2. Finally the cumulative discounted cash flow of the realizations and OK block model are shown in Figure 4-50.

The discounted cost of grade uncertainty before and after post-processing is shown in Table 4-32. After post-processing the extra ore tonnage and transfer to later years the DCOU is dropped to 26.1 MD. This is the lowest DCOU among all other models. It is 8.38 times or 193 million dollars less than Whittle generated schedule and 3.7 times or 70.8 million dollars less than Model #2. There are 13 out of 50 realizations with zero DCOU. This means that with this production shortfall will not happen with 13 realizations. In other words there is a 26% chance that no shortfall happens during the mine life with this production schedule.

Figure 4-51 and Figure 4-52 show the box-plot and deviations from target production in each period before and after post processing the schedule. The first two periods (pre-striping periods) are not shown in these graphs. After post-processing the schedule (see Figure 4-52), none of the realization creates a shortfall in first two production periods (periods 4 and 5). Therefore, the deviation from target production in period 3 and 4 is zero. Also the periods 5 and 6 have very small shortfall from target production such that the deviations from target production for periods 5 and 6 respectively are 0.18% and 0.06%. The deviation from target production increases toward the end of the mine life. This uncertainty can be reduced by new data that are collected during mine life.

Table 4-33 indicates the average deviation from target production before and after post processing that are calculated between periods 3 to 9. The $\overline{DTP}$ after post-processing the schedule drops to 0.84%. This is the lowest $\overline{DTP}$ value compare to the previous models.

Table 4-34 shows the summary statistics for all realizations after post-processing. Another interesting point of this schedule is that the average NPV of this model is increased and the standard deviation of the NPV is decreased significantly. One should note that none of these two parameters are in the objective function therefore these values can be different in other cases. Having low deviation from target at the early years of the production can increase the average NPV and reduce the variance of the NPV significantly in most cases.

Figure 4-53 shows the histogram of mined ore tonnage in each period except periods 1 and 2. These histograms are before the post-processing stage. Most of the shortfalls are compensated by the extra ore available in the stockpile.

Figure 4-54 illustrates the histograms of total input ore tonnage to the mill in each period. These histograms are after post processing the schedule. All of the realizations feed the plant with full capacity in periods 3 and 4. Therefore the histograms are one column at 36MT. There are small deviations in periods 5 and 6.

4.7.3 Conclusion

The concept of a stockpile used in Model #3 helps to reduce the shortfalls and deviations from target production. The impact on the cost of grade uncertainty is significant. This model is superior to the previous models because the stockpile is considered in the optimization process. The asymmetric penalty function, where the cost of underproduction is higher than cost of overproduction. However the presence of a stockpile allows optimization to extract extra ore at early stages of the mine life. These extra ore reduce the chance of short falls at later years.

With this method the average of NPV can be increased and the variance of the NPV can also be decreased.

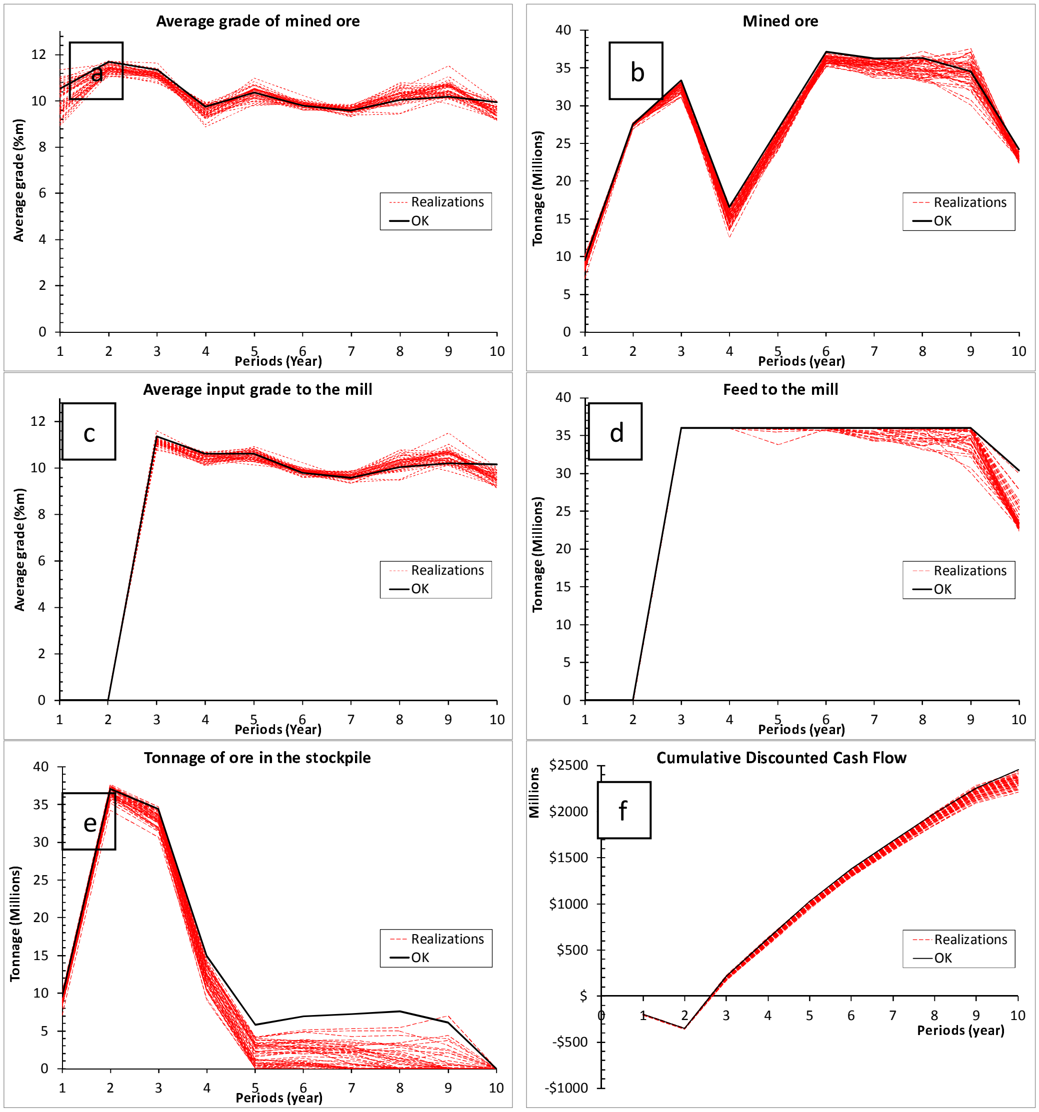


Figure 4-50 a: average grade of mined ore before post processing, b: tonnage of mine ore before post processing, c: average grade of input ore to the mill after post processing, d: tonnage of ore input to the mill after post processing, d: tonnage of ore at the stockpile after post processing and f: CDCF after post processing stage,

Table 4-32: DCOU with and without stockpile for the schedule generated by Model #3

	Discounted cost of grade uncertainty (MD)
Without post processing (No stockpile)	1561.98
After post processing (With stockpile)	26.11

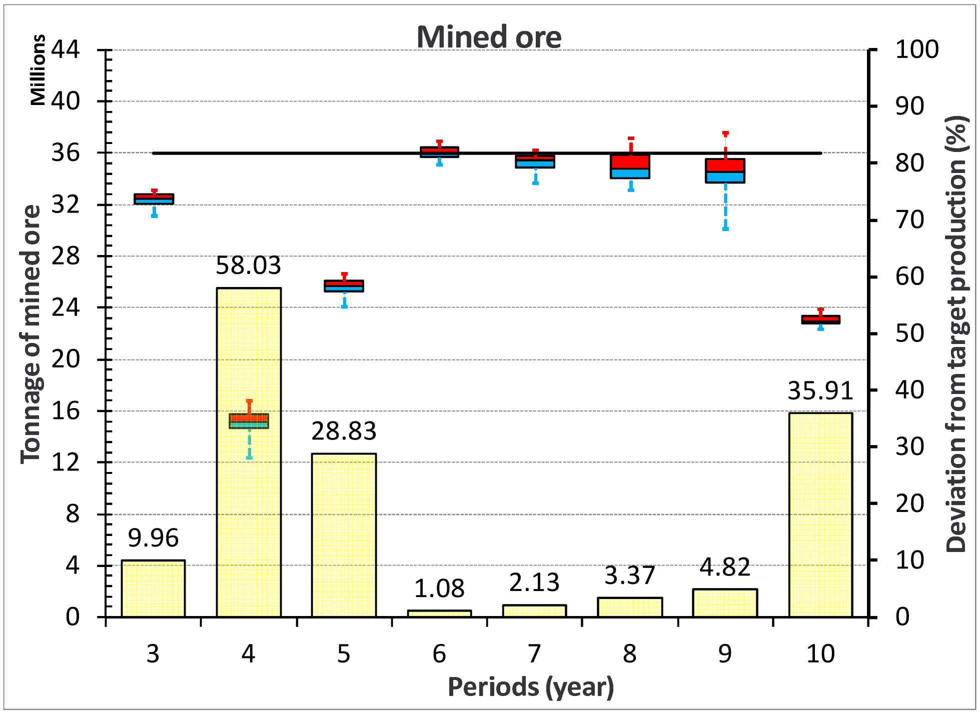


Figure 4-51: Box-plot and deviation from target production for each period; the schedule generated by Model #3 with asymmetrical penalty function with stockpile.

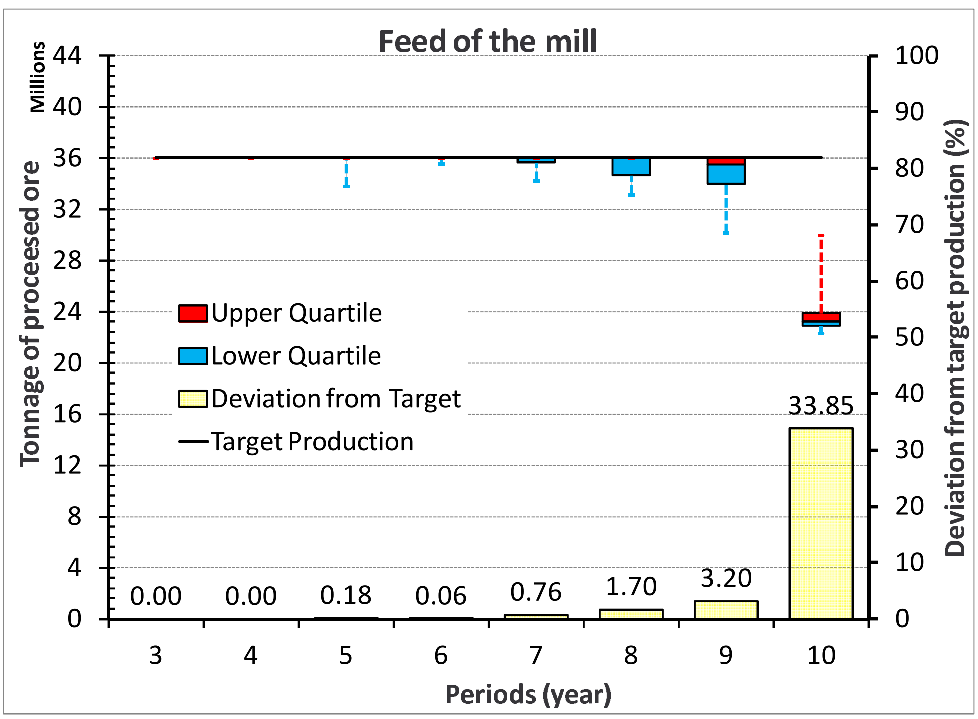


Figure 4-52: Feed of the plant and the box-plot of the deviation from target production for each period; the schedule generated by MILP with OK block model and realizations with asymmetrical penalty function with stockpile (Model #3).

Table 4-33: $\overline{DTP}$ with and without stockpile the schedule generated by MILP with OK block model (Model #3).

	Average deviation from target production ($\overline{DTP}$)
Without post processing (No stockpile)	15.46%
After post processing (With stockpile)	0.84%

Table 4-34: Summary of statistics for the production schedule after post processing stage.

Statistics	Average grade m%	NPV MD	DCOU MD
Mean	10.27	2336.01	26.11
Std. dev	0.09	55.34	30.08
Min	10.06	2211.27	0.00
Quartile 1	10.24	2291.79	0.00
Median	10.28	2334.51	14.85
Quartile 2	10.32	2378.72	45.30
Max	10.56	2441.89	99.65
OK	10.31	2460.00	N/A

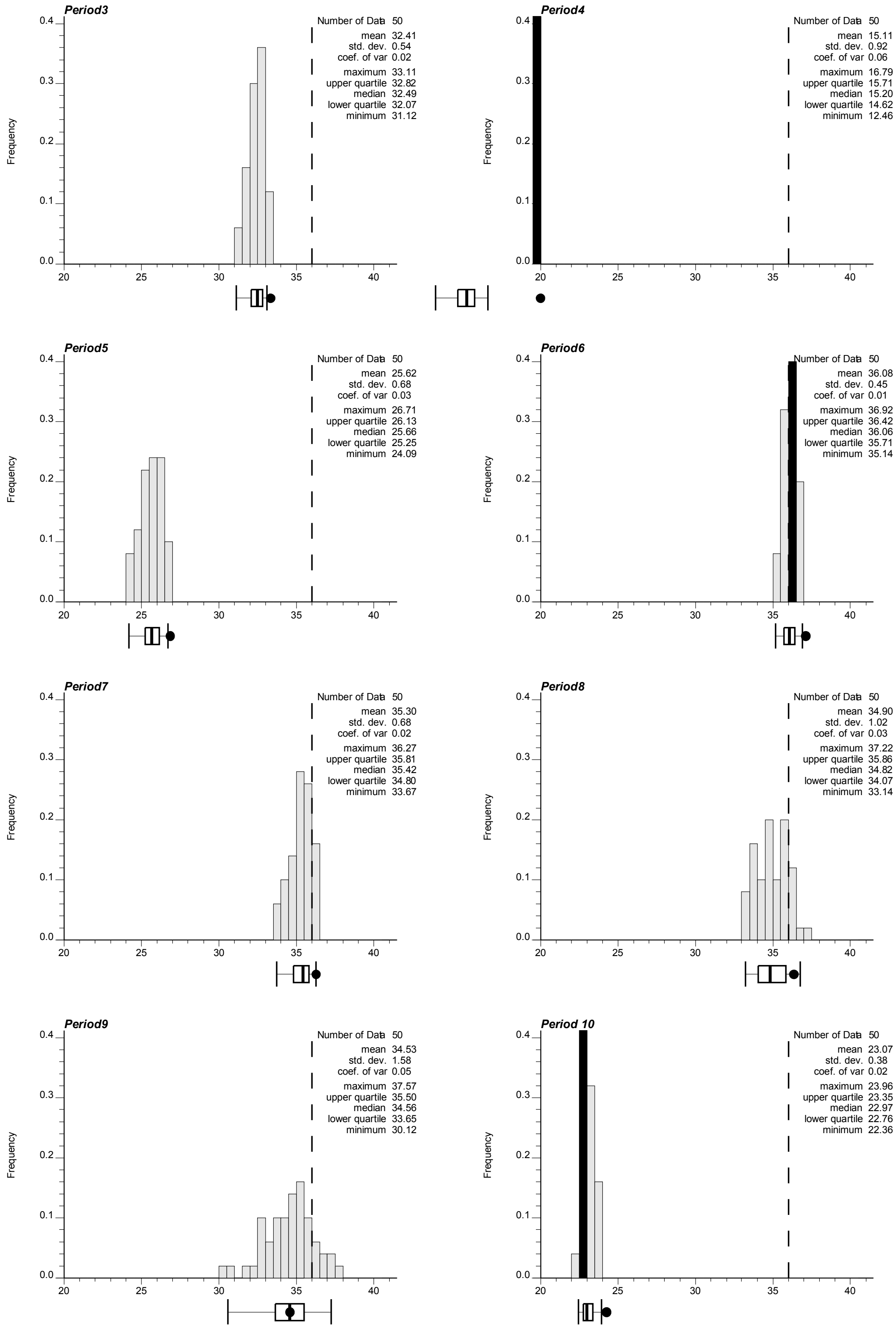


Figure 4-53: Histograms of tonnage of mined ore in different periods. The schedule is generated with Model #3 with asymmetrical penalty function. OK result is marked by solid circle and dash line indicates 36 MT of target production.

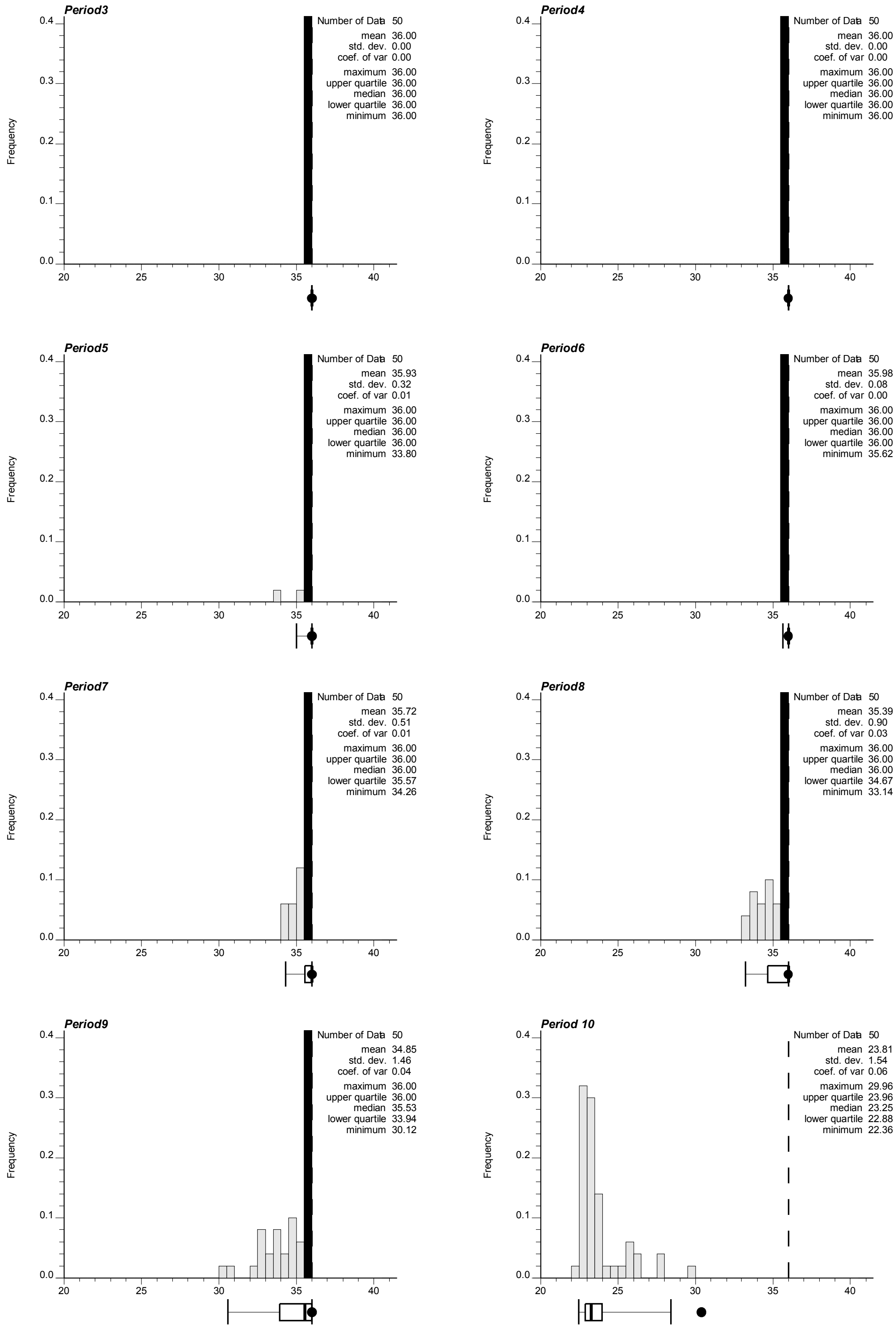


Figure 4-54: Histograms of input ore tonnage to the mill. The schedule is generated with MILP and realization and a stockpile with un-symmetric penalty function (Model #3). OK result is marked by solid circle and dash line indicates 36 MT of target production.

4.8 Summary

In this chapter, an oil sands data set from Alberta has been used. The Whittle software has been employed to generate a final pit limit using the 3D LG method. This program created a production schedule based on three push-backs to maximize the NPV of the OK block model. The simulated realizations are used to measure the effect of grade uncertainty on the production plan. Two new quantities are introduced to quantify the effect of grade uncertainty: (1) the average deviation from target production and (2) the average discounted cost of grade uncertainty. The optimization algorithms called Model #1, #2 and #3 have been used in this chapter to create the production schedules. The results of each of these models are compared with the previous model.

Table 4-35 shows the summery of statistics for the different methods. The values in this table have been used in Figure 4-55 to Figure 4-57. Table 4-36 and Table 4-37 show the percentages of change for NPV and DCOU. In both of these tables, the first column presents the base method. As shown in Table 4-36, generally the percentage changes of the NPV in proposed optimization methods (1 to 3) are very small. The biggest change is 0.3% reduction of NPV in Model #2 relative to Model #1. However, as shown in Table 4-36, there is a 78% reduction in the cost of grade uncertainty in Model #3 relative to Model #1.

The effect of grade uncertainty is quantified by the concept that has been introduced in this thesis. The effect of grade uncertainty highly depends on the nature of the deposit. Oil sand deposits are huge homogenous deposits. The number of boreholes in the case study is quite reasonable and the density of the boreholes at most of the area is higher that the AER requirement. Therefore, the grade uncertainty has relatively small effect on the production plan. This can be investigated from Table 4-35 and Table 4-36. Model #1 is the deterministic model based on OK block model. The discounted cost of grade uncertainty before post processing for this model is 118 MD while the NPV of the project is 2461MD. The effect of grade uncertainty is about 4.8% of the NPV of the project. However, this percentage will change for different deposits. Smaller deposits such as gold, copper, etc. with higher grade uncertainty, lower number of boreholes and naturally more variable elements will have significantly higher cost of grade uncertainty. Nevertheless, the effect of grade uncertainty is reduced for the presented case study. The percentage of the cost of grade uncertainty is about 3.6% and 1% of the NPV for Model #2 and #3, respectively. The NPV of the project does not change significantly from

Model #1 to #3; however, the cost of grade uncertainty is reduced by 25% and 78% by using Model #2 and #3.

Table 4-35. Summary of statistics for the different methods.

Method	NPV of Kriging MD	E{NPV} MD	Std.dev{NPV} MD	DCOU MD
Whittle	2387	2256	64	219
Model #1	2461	2334	63	118
Model #2	2454	2329	63	89
Model #3	2460	2336	55	26

Table 4-36. Percentage change of NPV for different methods.

	Whittle	Model #1	Model #2	Model #3
Whittle	+0.0%	-3.0%	-2.7%	-3.0%
Model #1	+3.1%	+0.0%	+0.3%	+0.0%
Model #2	+2.8%	-0.3%	+0.0%	-0.2%
Model #3	+3.0%	-0.0%	+0.2%	+0.0%

Table 4-37. Percentage change of DCOU for different methods.

	Whittle	Model #1	Model #2	Model #3
Whittle	+0%	+86%	+146%	+742%
Model #1	-46%	+0%	+33%	+354%
Model #2	-59%	-25%	+0%	+242%
Model #3	-88%	-78%	-71%	+0%

Figure 4-55 compares all four case studies. This graph shows that there are substantial differences between NPVs resulted from Whittle and other MILP-based methods. On the other hand, NPV of the three optimization-based methods are quite close. However, NPV of Model #2 is lower than Model #1 and #3. The reasons have been explained in this chapter.

Figure 4-56 shows the standard deviation of the NPV that is calculated using all 50 realizations. Different production plans are generated using different methods that are presented in this chapter. The standard deviation of Model #3 is lower than all other three methods. The production plan generated by Whittle has more variation in NPV.

The discounted cost of grade uncertainty for all methods is shown in Figure 4-57. As discussed before, Model #3 has the lowest value among all other methods because the stockpile is used in this optimization model.

Figure 4-58 shows the runtime for each method in logarithmic scale. Whittle takes about 24 seconds to create a production schedule. The 3D LG also takes 27 seconds. Whittle is very fast compared to all optimization-based methods. Model #3 is faster than other two optimization-based methods.

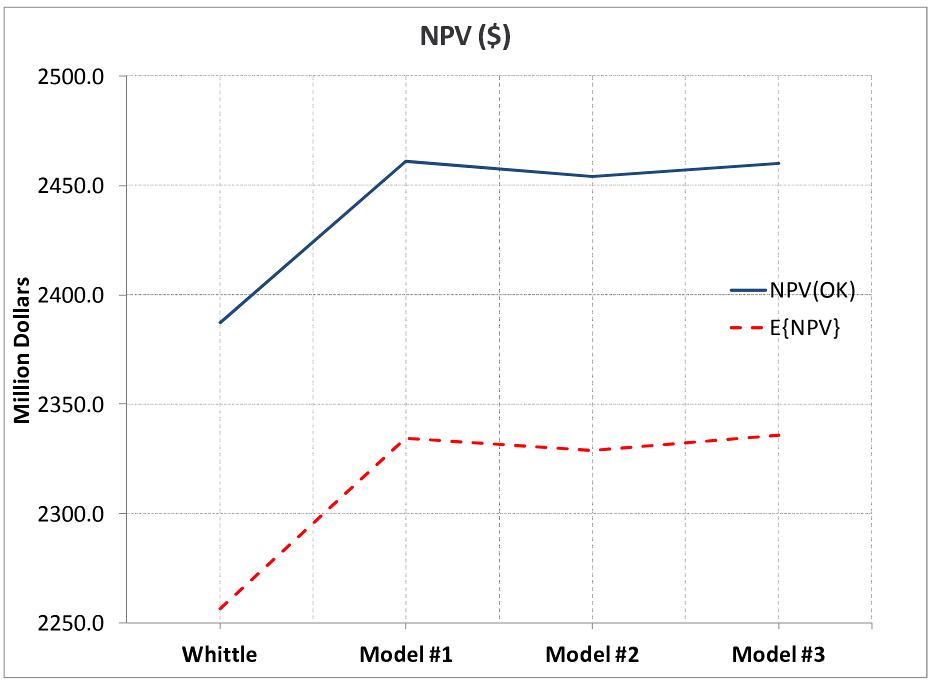


Figure 4-55: NPV of the kriging (bold blue line) and the expected value of the NPV calculated from realizations for each of the production schedules generated by Whittle and models #1, #2 and #3.

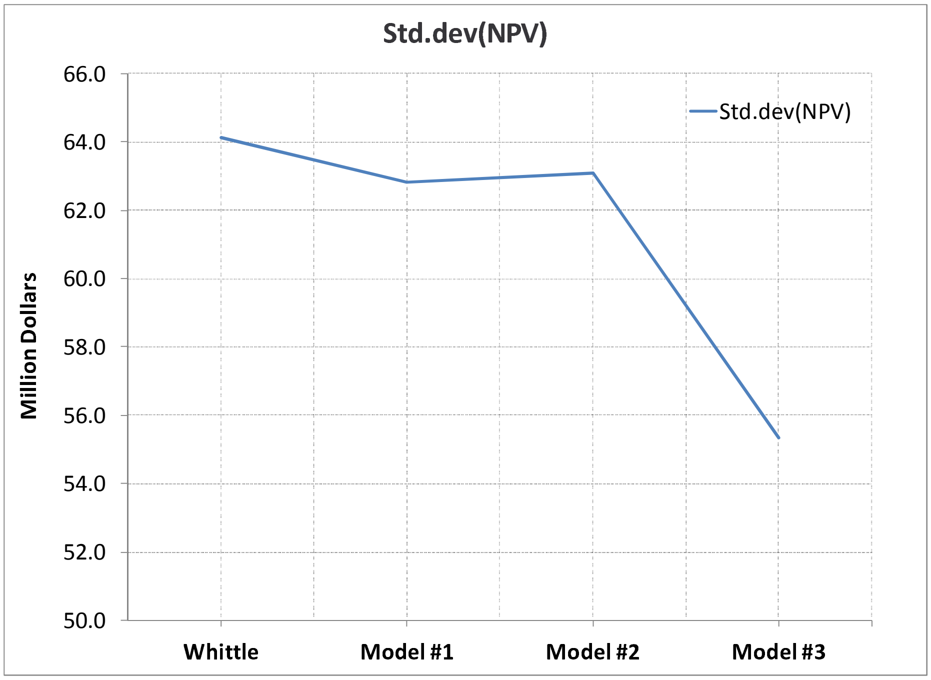


Figure 4-56: The standard deviation of the NPV calculated from realizations for each of the production schedules generated by Whittle and models #1, #2 and #3.

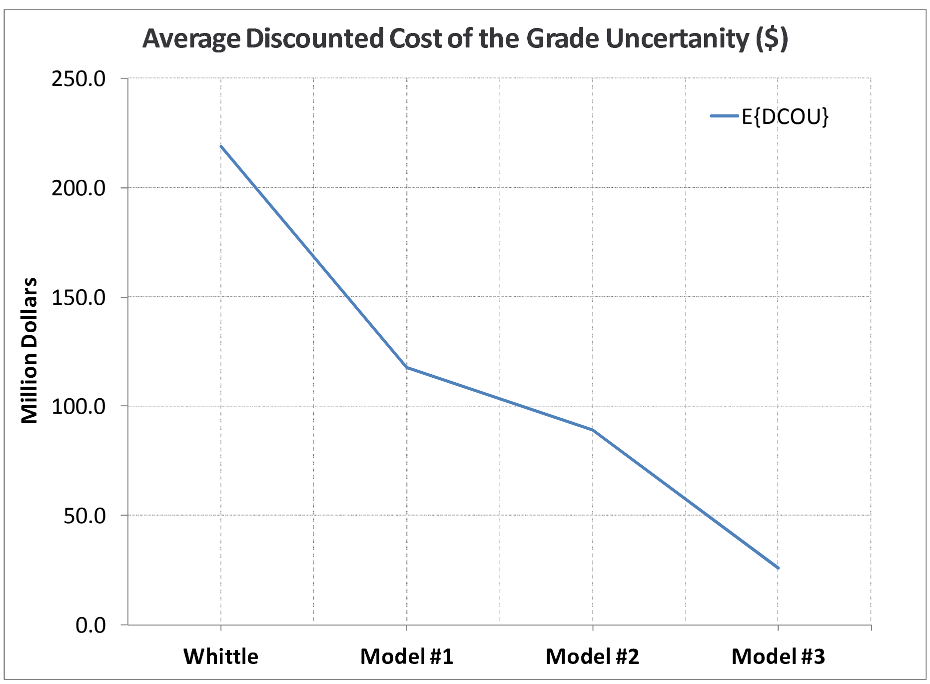


Figure 4-57: The average discounted cost of the grade uncertainty calculated from realizations for each of the production schedules generated by Whittle and models #1, #2 and #3.

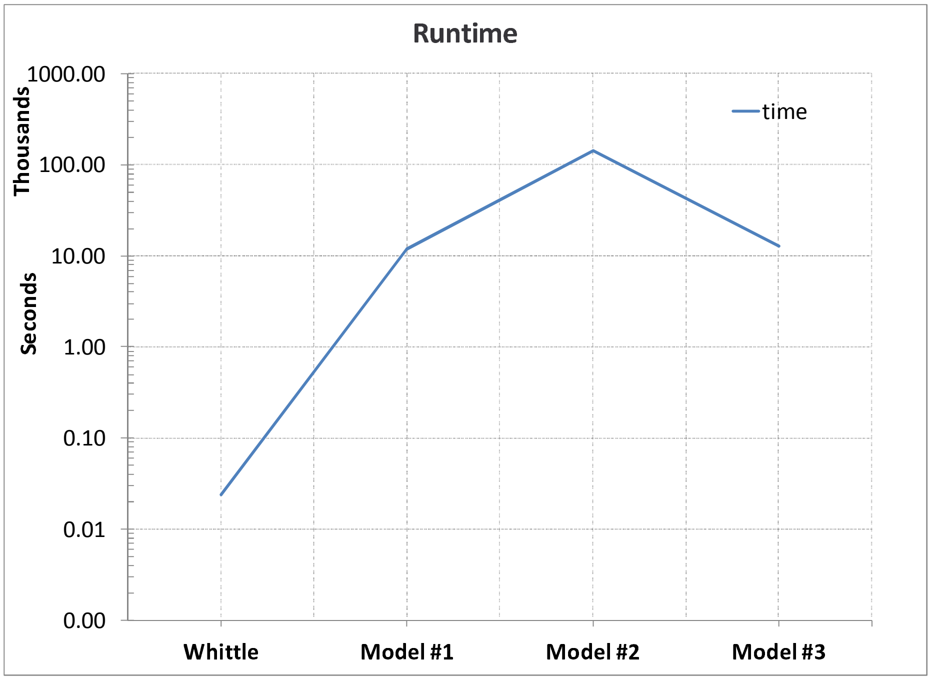


Figure 4-58: Runtime values for each of the production schedules algorithms generated by Whittle and models #1, #2 and #3.

Chapter 5 Sensitivity Analysis

In this chapter, a sensitivity analysis is performed on important parameters. First, a sensitivity analysis on the clustering methods and the number of cuts is presented in Section 5.1. The input parameters that are used in Sections 2.5 and 2.6 are studied in Section 5.2. Two practical methods are presented to estimate the optimal values for the costs of over and under production. Section 5.3 presents a methodology to determine mining capacity based on the cost of grade uncertainty. Also In this section, the effect of grade uncertainty is studied with different processing capacities. In Section 5.4, sensitivity analysis is applied on the lambda parameter needed for the mean-variance method in Eq. (2.49). Finally, the chapter summary is in Section 5.5.

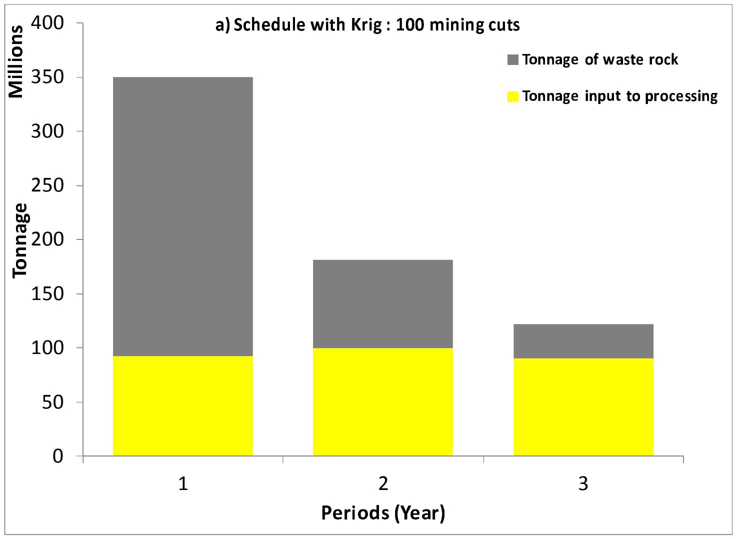
5.1 Sensitivity Analysis on Clustering and Number of Cuts

To investigate the sensitivity of the project to the number of clusters or mining cuts, different numbers of mining cuts are generated. Model #1 presented in Eq. (2.11) is used with a kriged block model. Although this model does not consider any grade uncertainty in the optimization stage, it is used because: (1) it is much faster than considering grade uncertainty and (2) the variance of each block has already been used as an input parameter in creating the clusters. The absolute MIPGAP tolerance is set to 0.0001 or 0.01%. To make the algorithm faster, the mine life is set to 3 years with no pre-striping, and very high mining and processing capacities are assumed (350 MT for mining capacity and 100MT for processing capacity). The optimization is run with 100, 500, 1000, 2000, 3000 and 4000 mining cuts with 8 CPUs. Figure 5-1 shows the schedules generated for 100 (top figure) and 2000 (bottom figure) mining cuts. Yellow bars are the amount of ore that is sent to the processing plant. The annual processing capacity limit is satisfied at all three periods of mine life. Grey bars indicate the waste tonnage. The sum of the yellow and grey bars shows the total amount of material (ore and waste) that is removed from the mine. The mining capacity limit is also satisfied during the mine life. It is clear from these graphs that the production schedule with more mining cuts feeds the plant at full capacity over the mine life. The schedule generated with 100 mining cuts has a 7.7 MT shortfall in the first period and the plant was not fully fed (92.3 MT). Also, with 2000 mining cuts, in order to maximize the NPV, less waste material has been removed in the first period (158 MT vs. 285 MT). This increases the NPV of the project by deferring the removal of waste material. Also, with smaller mining cuts, the schedule can reach to the high grade zone faster with less waste mining.

Figure 5-2 shows clusters generated with 100 mining cuts in plan view of 310 m and two cross sections of 252750 m looking north and 14825 m looking east. Figure 5-3 shows the generated schedule and each block's extraction period in the same plan view and cross sections. Figure 5-4 and Figure 5-5 show the plan view and cross sections of mining cuts and extraction period of each block with 2000 mining cuts. It is clear that by increasing the number of clusters, the size of each cluster decreases. These two cases have different production schedules (compare Figure 5-3 to Figure 5-5).

The NPV of each case has been calculated using Eq. (2.37). Figure 5-6 shows the changes in NPV versus the number of cuts. By increasing the number of cuts, the optimization generates higher NPV because of more flexibility in decision variables. However, after a point the NPV does not increase significantly because the clustering method used here aggregates blocks with similar grades and similar EBVs. Therefore, the NPV from 2000 mining cuts can be considered as a good estimate of the maximum achievable NPV of the project. 2000 mining cuts have been used for the case study presented in Chapter 4.

Figure 5-7 shows elapsed real time for solving the optimization problem. To generate an optimum schedule with 2000 mining cuts elapsed real time is about 300 seconds. This number increases dramatically for larger numbers of cuts. For 5000 cuts, 4000 seconds was required to reach the 0.01% gap. The relationship between elapsed real time and CPU time is shown in Figure 5-8. This graph shows that with the same CPU specifications and using only one core, the run time of the optimization would be 5 times more than using all 8 cores.



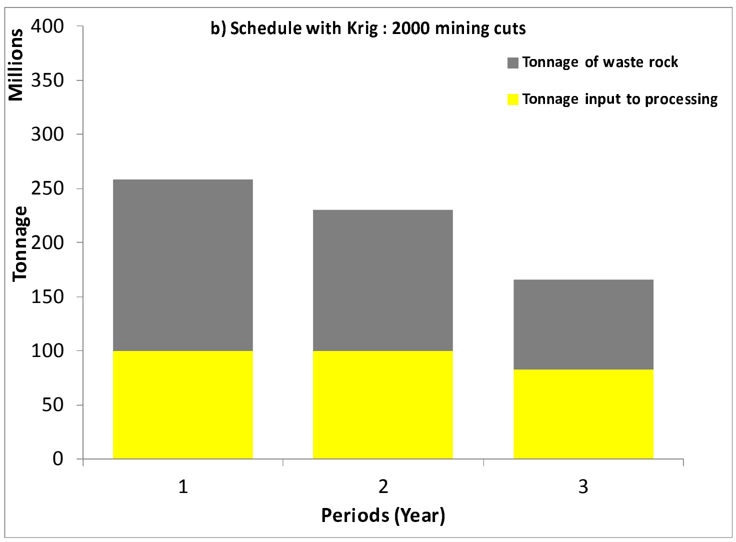
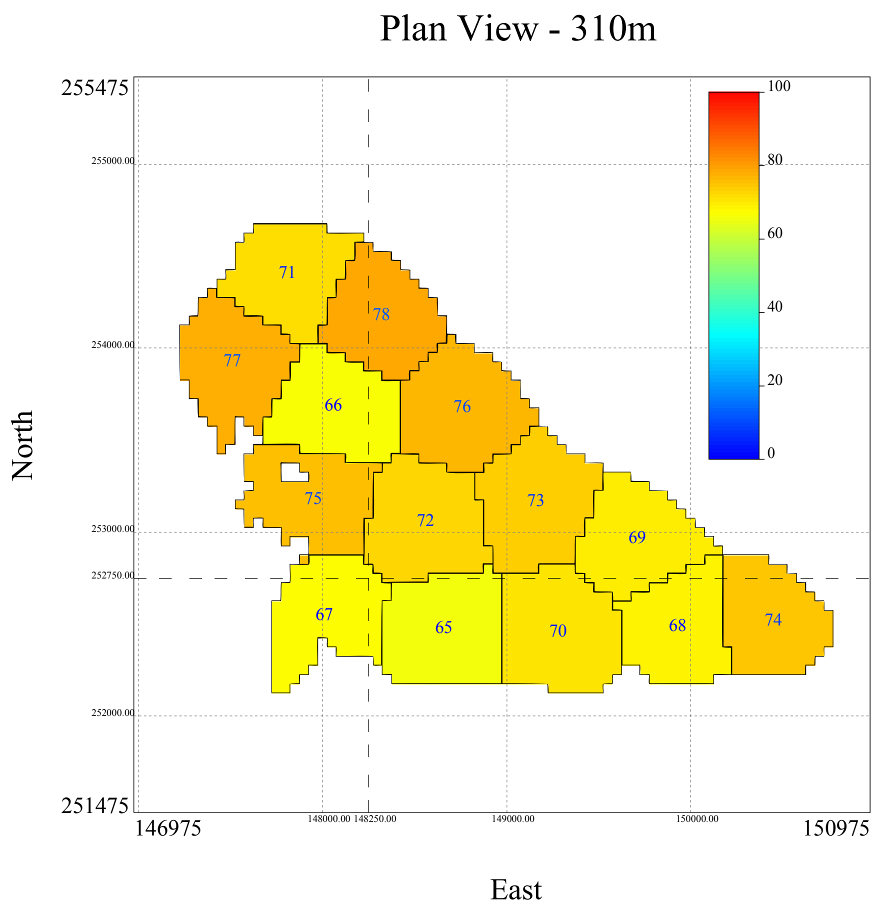
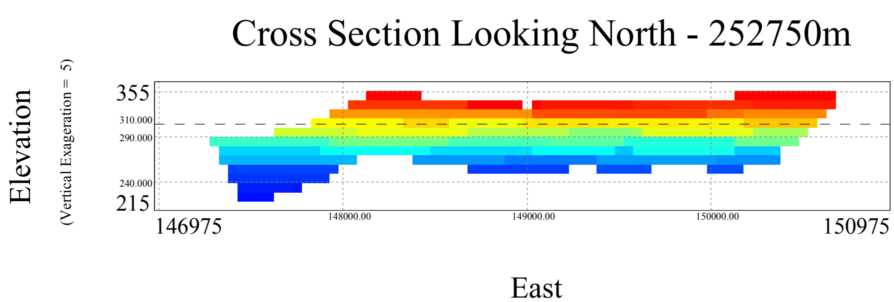


Figure 5-1. Schedules generated by a) 100 mining cuts (top) and b) 2000 mining cuts (bottom).





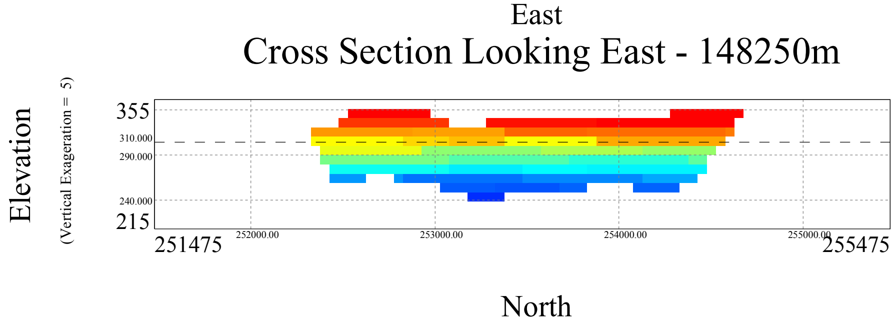


Figure 5-2. Plan view and cross sections for 100 generated mining cuts.

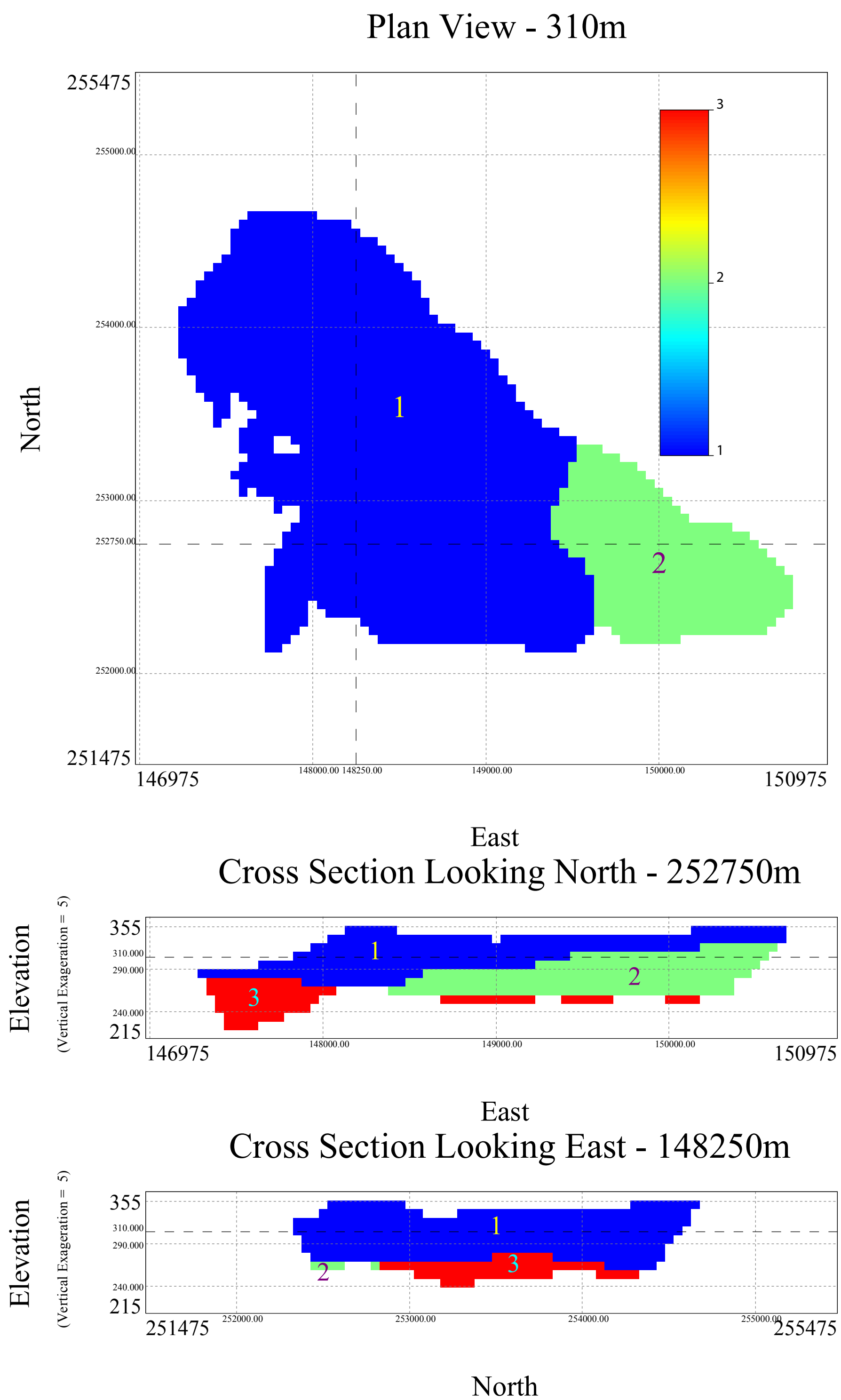


Figure 5-3. Plan view and cross sections of extraction period for each block with 100 mining cuts.

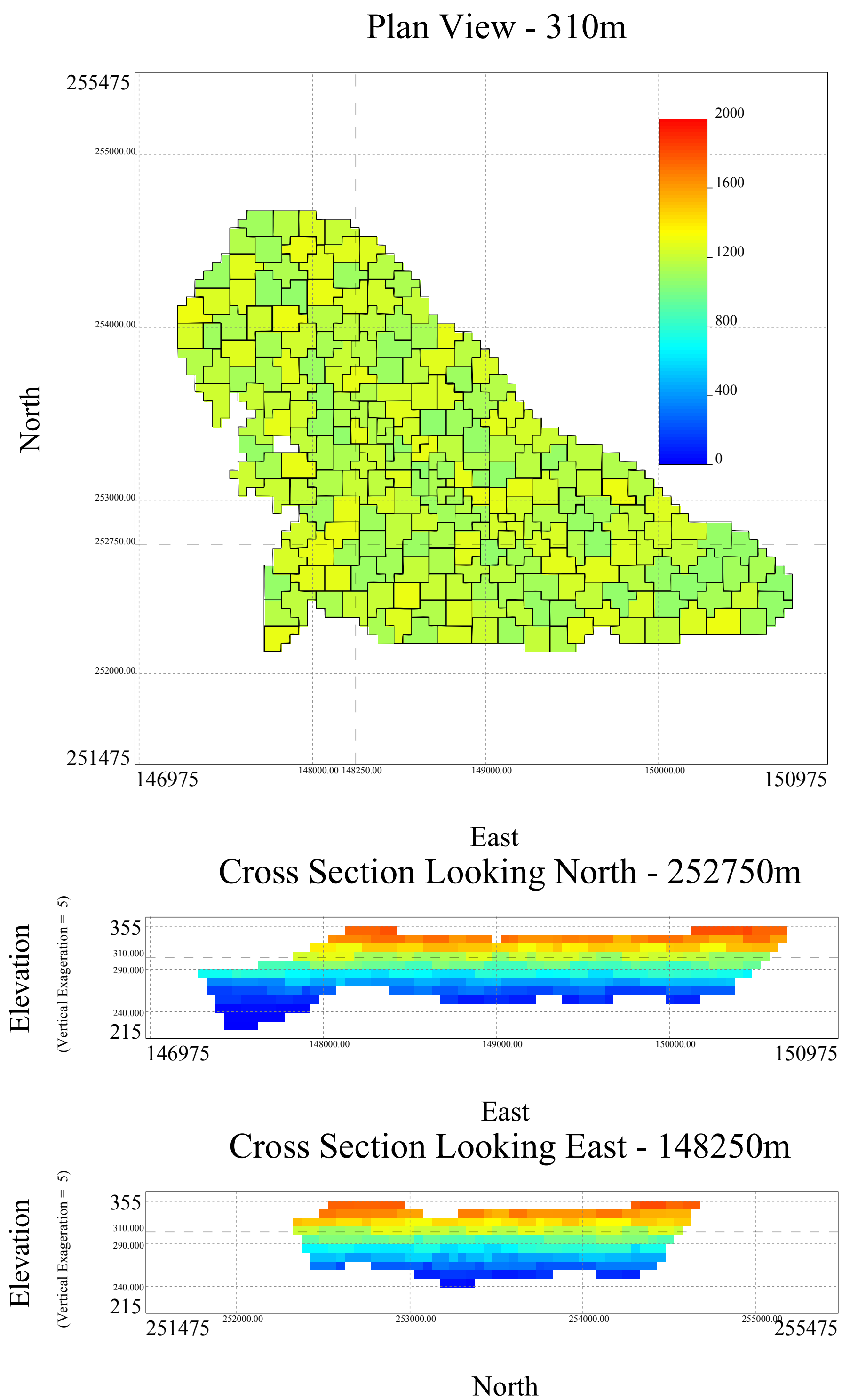


Figure 5-4. Plan view and cross sections for 2000 generated mining cuts.

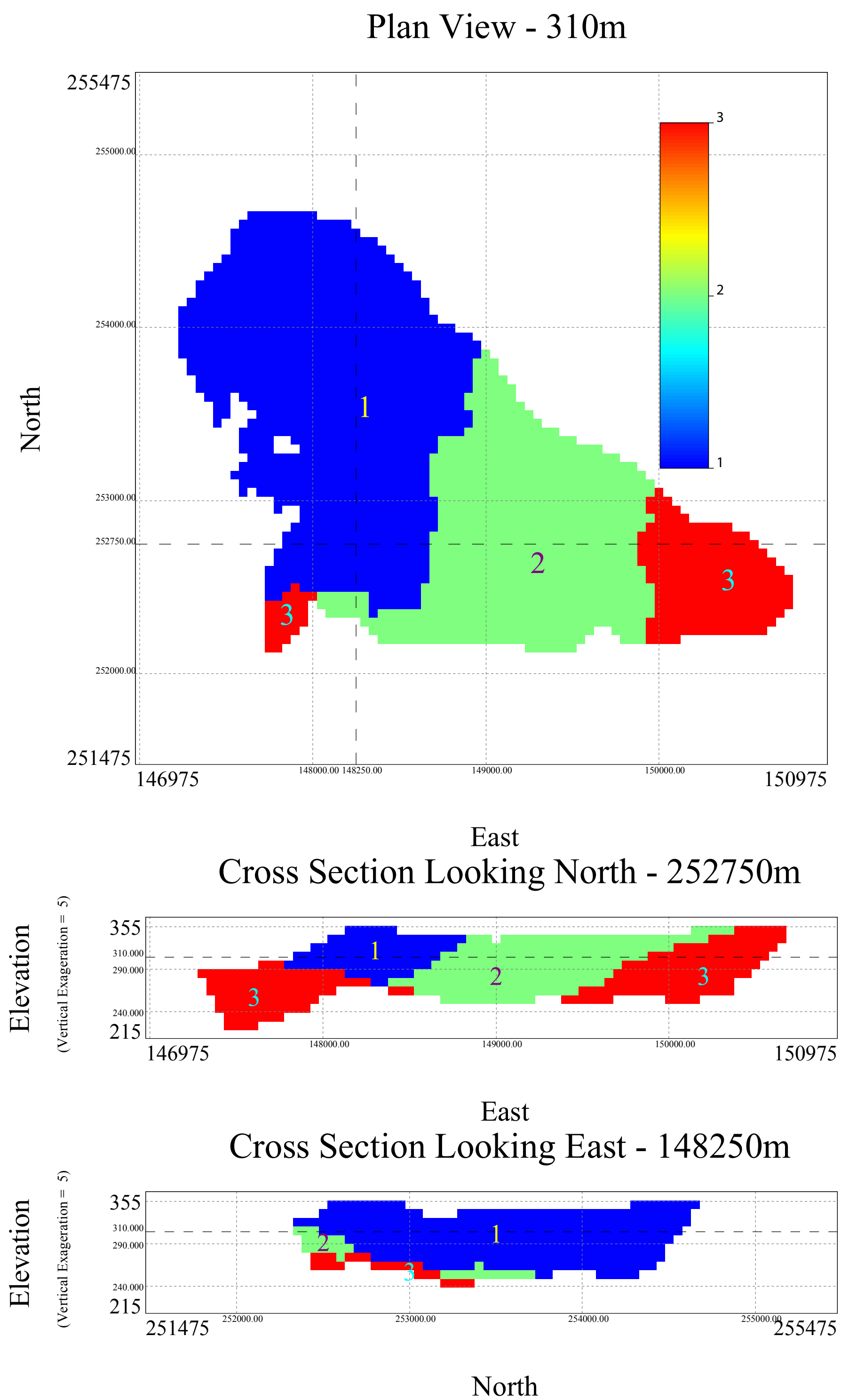


Figure 5-5. Plan view and cross sections of extraction period for each block with 2000 mining cuts.

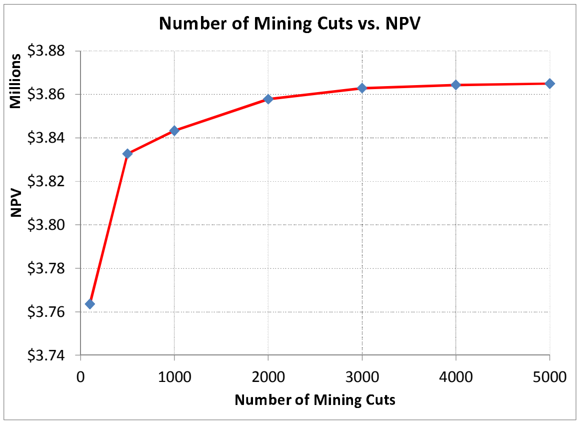


Figure 5-6. NPV versus the number of mining cuts for the case that the mine life is 3 years.

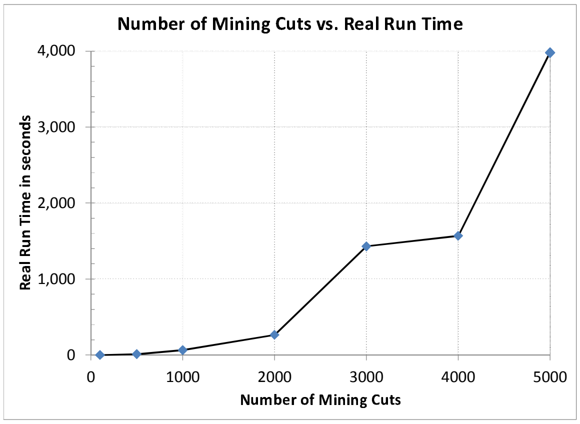


Figure 5-7. Number of mining cuts versus real run time of the optimization stage in seconds; Mine life is 3 years.

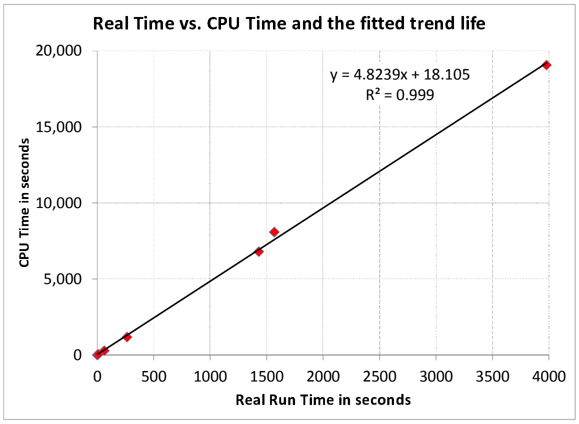


Figure 5-8. Correlation between CPU time and real run time for a case that the optimization has been run in a computer with 8 CPUs; Mine life is 3 years.

In order to compare the results with 10 year mine life that was used in case studies, the same configurations as the previous section are used. Model #1 is run with 100, 500, 1000, 2000 and 3000 mining cuts. Figure 5-9 shows the number of mining cuts versus the NPV of the project with 10 years of mine life. As it is shown in Figure 5-6, 2000 mining cut can be used for this case as the sufficient number of mining cuts. Using more mining cuts does not have significant improvement on the NPV maximization. The runtime for 10 years of mine life is much longer. Figure 5-10 shows the real run time for the case of 10 years mine life with different number of mining cuts. A polynomial function has been used to capture the trend which has been shown with dashed line in the graph. High R^2 value shows that the number of mining cuts has polynomial effect on the run time of the optimization. From this line it can be predict that the run time for the case with 5000 mining cuts is about 90,000 seconds or 25 hours with 8 CPUs. The correlation between CPU time and real run time also has been shown in Figure 5-11. Running the code with 8 CPUs with 10 years of mine life decreases 4.3 times of the real run time, which is very close to what has been shown in Figure 5-8 with 3 year of mine life.

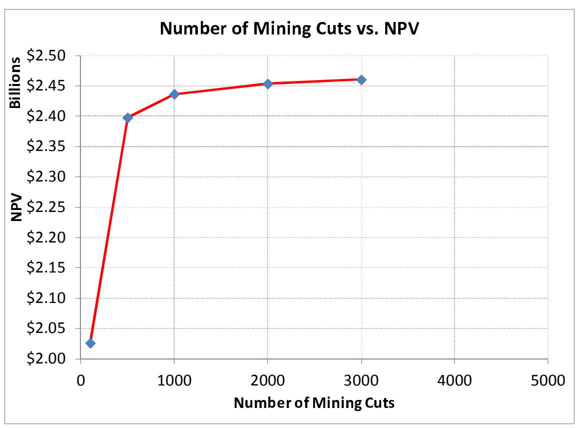


Figure 5-9. NPV versus the number of mining cuts for the case that the mine life is 10 years.

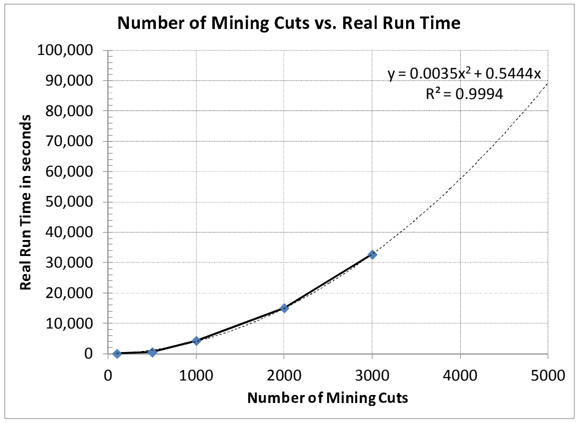


Figure 5-10. Number of mining cuts versus real run time of the optimization stage in seconds; Mine life is 10 years.

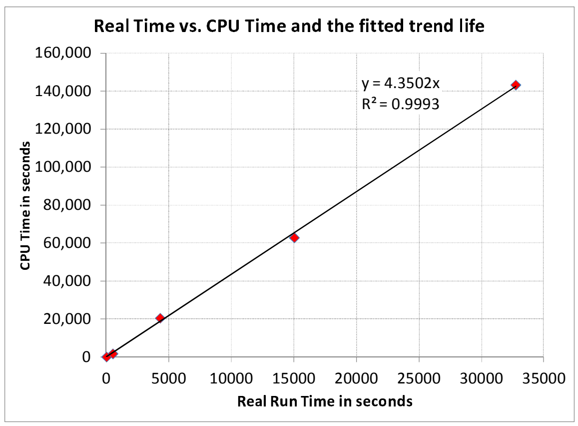


Figure 5-11. Correlation between CPU time and real run time for a case that the optimization has been run in a computer with 8 CPUs; Mine life is 10 years.

5.2 Determining Optimal Values for Costs of Under and Over Productions

In this section, two methods are presented to estimate the costs of over and under productions that balance the trade-off between NPV and risk associated with grade uncertainty; they are used in Models #2 and 3. As discussed before, the cost of overproduction is less than underproduction, because there are always ways to deal with unexpected extra ore such as stockpiling, blending with low grade material or adjusting the mine plan.

In the proposed objective functions, Eq. (2.35) or Eq. (2.38), there are two input parameters that control the uncertainty part of the optimization problem: (1) $c_{up}(t)$ and (2) $c_{op}(t)$ or $\hat{c}_{op,RH}(t)$. Dimitrakopoulos and Ramazan (2008) call these parameters the discounted unit cost of deviation from target production. They do not suggest how to calculate or estimate these parameters; choosing these parameters are subjective. A high geological risk discount or GDR rate means that low uncertainty blocks are going to be extracted in the early years of production if at all possible. The generated schedule may be quite conservative with lower NPV. In this research, only one discount factor is used in the objective functions. In this section, two different techniques are presented to calculate and calibrate these factors.

- Deterministic method: The factors as shown in Eq. (2.21) and Eq. (2.23) can be calculated as:

$$c_{up}(t) = c_{op}(t) = \bar{g}(t) \times R_p(t) \times \frac{P(t)}{(1+IR)^t} - \frac{C_p(t)}{(1+IR)^t}$$
$$\hat{c}_{op,RH}(t) = c_{op}(t) - c_{op}(t+1) + c_{RH}(t) \tag{4.5}$$

  This requires a prior knowledge of the average input grade to the mill for each realization and the average grade of the stockpile in each period. This cannot be calculated exactly until the optimization finds the solution. Therefore, the user needs to have a rough estimate, run the optimization and iterate if necessary. Our observation shows that the optimization process is robust with respect to small changes in the $c_{up}(t)$ and $c_{op}(t)$ or $\hat{c}_{op,RH}(t)$ values.

- Numerical method: In this method, the same cost is applied for over and under production $c(t) = c_{up}(t) = c_{op}(t)$. $c(t)$ is the discounted cost of not meeting the target production. In this method, the optimization algorithm is run with different $\bar{C}_u$ values that are introduced in Eq. (2.29). For any value of *c*, the values of NPV_{es} and $DCOU$ in Eqs. (2.37) and (2.30) are calculated. The difference between NPV_{es} and $DCOU$ is also calculated as:

$$Delta = NPV_{es} - DCOU \tag{4.6}$$

  The $\bar{C}_u$ value is the trade-off parameter between the maximization of NPV and the minimization of the cost of grade uncertainty. When it is zero, the optimization problem turns to the NPV maximization such as Model #1. By increasing $\bar{C}_u$, the NPV and the cost of grade uncertainty are decreased. However the slope of declination for NPV and DCOU are not the same. With lower $\bar{C}_u$, the DCOU is reduced more than what we lose at NPV of the project and after some point, higher $\bar{C}_u$ cannot reduce the DCOU more than NPV reduction. Therefore the Delta reaches to the maximum value where the difference between NPV and DCOU is maximized. Therefore the optimum $\bar{C}_u$ is where Delta reaches to its maximum value.

Both of these two methods converge to the same value with small differences. However a sensitivity analysis is required for detail investigation. For this purpose the three year mine life that has been introduced in previous section is used with Model #2. Table 5-1 shows the NPV, Delta and the $\bar{C}_u$ values that are used for each case. Figure 5-12 shows the NPV values with different $\bar{C}_u$. This graph and the values in Table 5-1 suggest that a new schedule is generated is not generated for any changes of $\bar{C}_u$. For example the generated schedule with $\bar{C}_u$ equals to 0.5, 0.6 and 0.7 are identical. And the reduction rate of the NPV with small $\bar{C}_u$ is lower than with higher $\bar{C}_u$. For example by changing $\bar{C}_u$ from zero to 0.7, the NPV is dropped 3 million dollars. Figure 5-13 shows the DCOU values with the same $\bar{C}_u$ values. The DCOU is 133 million dollars when $\bar{C}_u$ is zero. This values is dropped to 75 million dollars when $\bar{C}_u$ is 0.5 dollars per tonne. From Figure 5-14, it can be conclude that the Delta is maximized when $\bar{C}_u$ is 0.9 dollars per tonne. This value is little bit different from what it has been suggested from deterministic method, 0.5 dollars per tonne, and has been used in case study in the previous chapter.

Table 5-1. The NPV and Delta values at different $\overline{C}_u$ values with 3 year of mine life case.

$\overline{C}_u$	NPV	DCOU	Delta
0	3.86E+09	1.33E+08	3.72E+09
0.1	3.86E+09	8.12E+07	3.78E+09
0.2	3.86E+09	8.10E+07	3.78E+09
0.3	3.86E+09	8.11E+07	3.78E+09
0.4	3.86E+09	7.95E+07	3.78E+09
0.5	3.85E+09	7.50E+07	3.78E+09
0.6	3.85E+09	7.50E+07	3.78E+09
0.7	3.85E+09	7.50E+07	3.78E+09
0.8	3.85E+09	5.89E+07	3.79E+09
0.9	3.84E+09	4.67E+07	3.80E+09
1	3.84E+09	4.60E+07	3.80E+09
1.2	3.84E+09	4.57E+07	3.80E+09
1.4	3.84E+09	4.57E+07	3.80E+09
1.6	3.84E+09	4.42E+07	3.79E+09
1.8	3.83E+09	4.38E+07	3.79E+09
2	3.83E+09	4.37E+07	3.79E+09
2.5	3.83E+09	4.23E+07	3.78E+09
3	3.83E+09	4.22E+07	3.78E+09
3.5	3.83E+09	4.22E+07	3.78E+09
4	3.82E+09	4.10E+07	3.78E+09
5	3.81E+09	3.73E+07	3.77E+09
10	3.78E+09	2.40E+07	3.75E+09

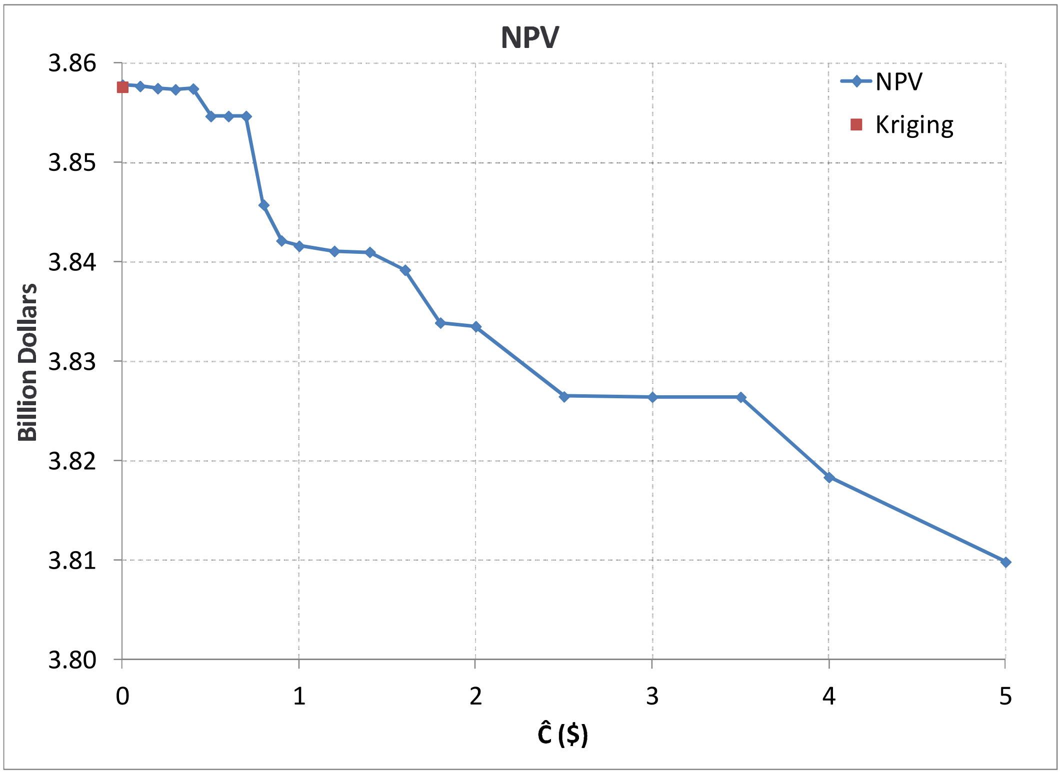


Figure 5-12. The NPV vs. $\overline{C}_u$ with 3 year of mine life case using Model #2.

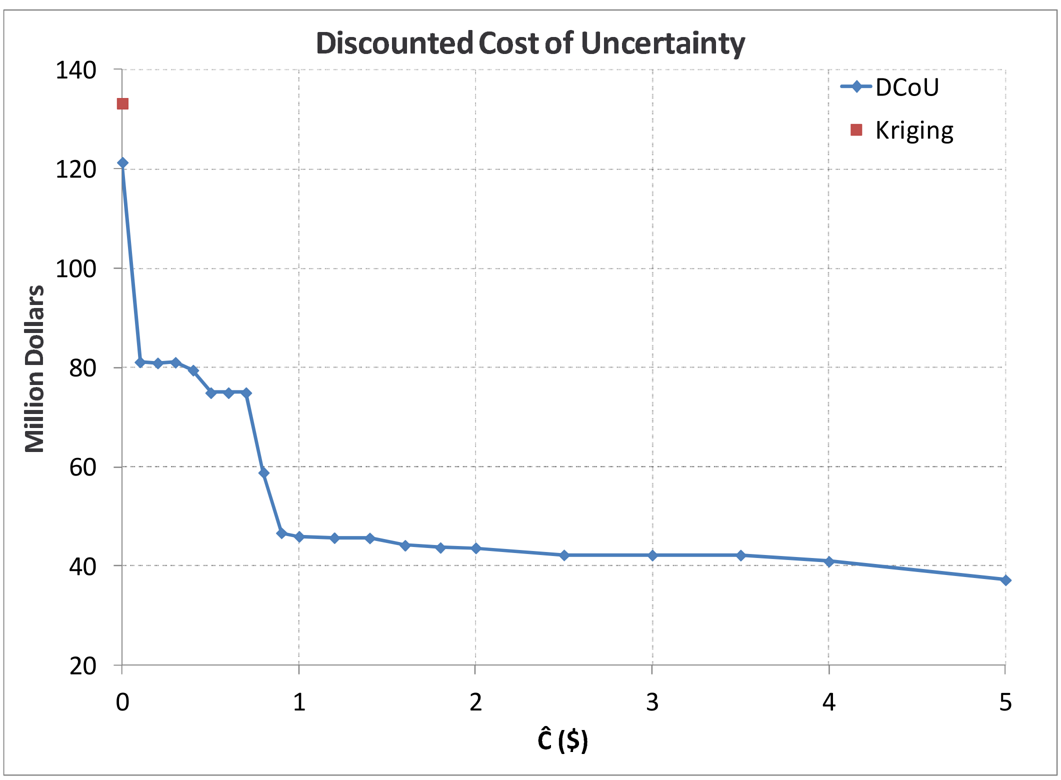


Figure 5-13. The DCOU vs. $\overline{C}_u$ with 3 year of mine life case using Model #2.

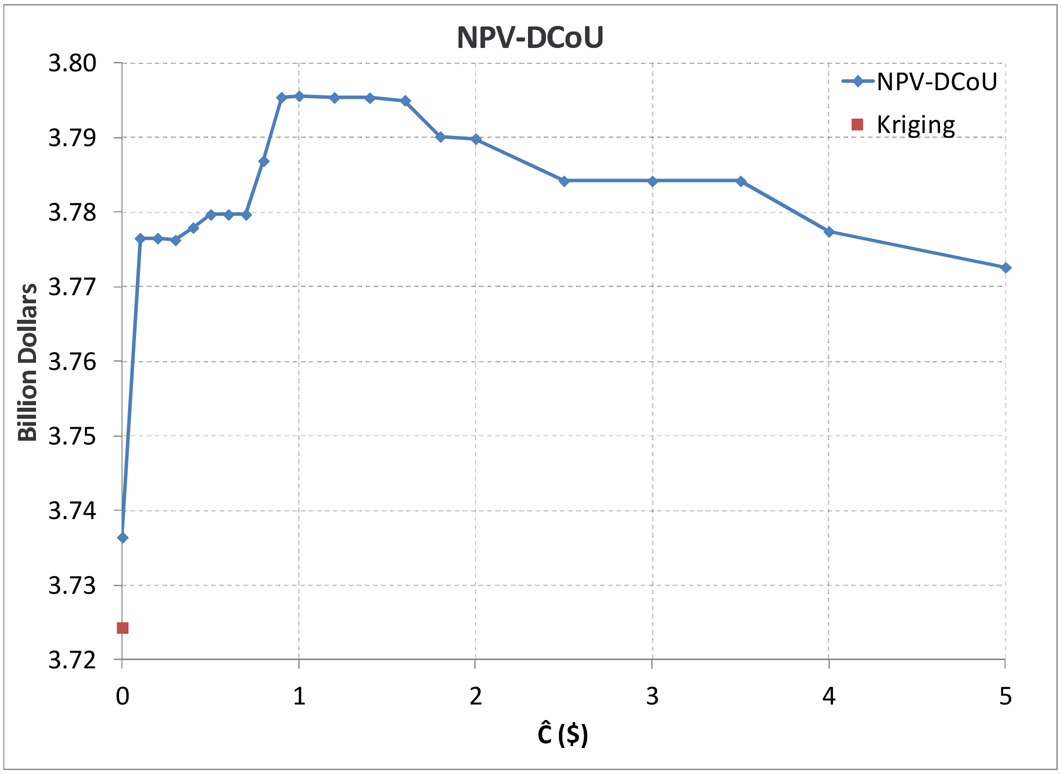


Figure 5-14. The Delta vs. $\bar{C}_u$ with 3 year of mine life case using Model #2.

5.3 Cost of grade uncertainty Versus Mining and Processing Capacities

One of the most important input parameters in an open-pit mine scheduling process is the capacity of mining. Usually, in traditional methods, where no uncertainty is considered, the mine planner starts with an initial mining capacity and tries to find a reasonable schedule. The schedule should be smooth during the mine life and it should also be feasible. The plant should be fed at full capacity during the mine life, except in the last period; and the annual tonnage of mined material should not exceed the mining capacity. The mine planner increases the mining capacity if the assumed value is not able to feed the plant at full capacity.

In this section, a new technique to find the optimum mining and processing limits is presented by considering the cost of grade uncertainty. For this case, Model #2 is used. The objective function presented in Eq. (2.33) allows the mine planner to consider grade uncertainty. The objective function maximizes NPV as well as minimizes the cost of grade uncertainty. By changing mining and processing limits, different optimum solutions are generated. The cost of grade uncertainty is calculated for each case. Figure 5-15 shows the relationship between cost of grade uncertainty and mining and processing capacities. Both mining and processing capacities are increased 10 times with 5 and 10 unit intervals, respectively. Therefore, the optimization has been run 100 times. Each line shows a different scenario where the processing capacity is constant and the mining capacity changes. The vertical axis is the cost of the uncertainty. The following points can be concluded from this graph:

- For a chosen processing capacity (each line), by increasing the mining capacity, the cost of grade uncertainty reduces until a certain point.
- With a larger processing capacity, it is required to set a much larger mining capacity in order to get the minimum cost of grade uncertainty. This can be well understood by comparing two processing capacities of 95 and 140. In these two lines, the mining capacities resulting in minimum costs of uncertainty are 290 and 350, respectively.
- Larger mines with high processing and mining capacities have higher cost of grade uncertainty.

A higher mining capacity reduces the cost of grade uncertainty because there is flexibility to makeup an unexpected shortfall. Also, any possible extra ore can be handled easily by stockpiling. As shown in this graph, as the mining capacity increases, the cost of grade uncertainty decreases. This decrease is not significant after a certain amount of mining capacity. For each processing capacity, there is an optimum mining capacity. After this point, the cost of grade uncertainty does not decrease further by increasing the mining capacity. The mining capacity estimated this way is higher than the capacity found by the traditional method with an estimated model. The best practice to get the optimum value of mining limit is to increase the mining capacity and calculate the cost of grade uncertainty for each case. The optimum mining capacity is the capacity that the higher values (than that capacity) do not change the cost of grade uncertainty, because after some point the processing capacity limits the total ore that is mined and higher mining capacity does not have any impact on the schedule. However if the model #3 is used, the stockpile capacity is the factor that confine the cost of grade uncertainty and by increasing the mining capacity after a critical value the cost of grade uncertainty cannot be decreased. However if the stockpile capacity is high enough, the cost of grade uncertainty can be reach to zero with higher mining capacity.

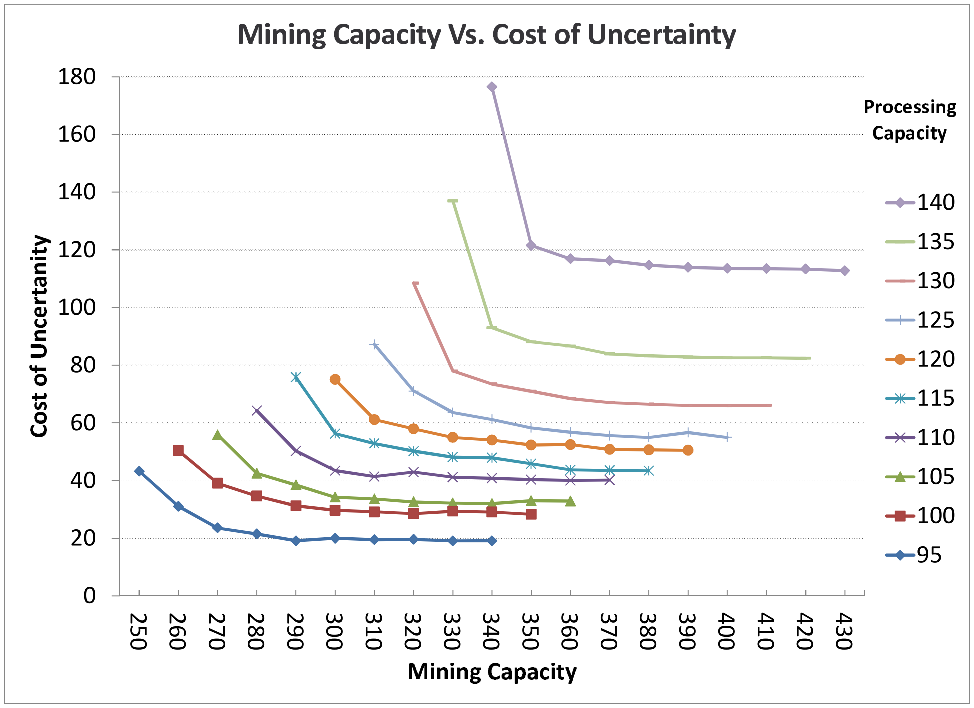


Figure 5-15: The cost of grade uncertainty versus different mining and processing capacities in a synthetic case.

5.4 Sensitivity Analysis on Lambda

In this section the sensitivity analysis on lambda in the mean-variance method (Model #4) is presented (Eq. (2.49)). Different Lambda values are used with 100 mining cuts and mine life is 3 years. Figure 5-16 to Figure 5-18 show the different Lambda values versus the average NPV, variance of NPV and total tonnage of mined material (Ore + Waste). In each pair of graphs, the graph at the top shows the changes in the variable of interest when Lambda changes between 0 and 0.65. To show details on the small Lambda values, the same graph is shown with a smaller range of Lambda: 0 to 0.5. Figure 5-16 shows the effect of Lambda on average NPV. With higher Lambda values, the average NPV is reduced. Figure 5-17 shows that higher Lambda values reduce the variance of NPV. Because the reserve constraint is not considered in this case, with higher Lambda values, the size of the final pit is smaller. Figure 5-18 shows that when a large Lambda value is chosen, the average tonnage of mined material is less and the final pit is smaller. The size of final pit is reduced dramatically for Lambda 0.2 and 0.5.

Figure 5-19 and Figure 5-20 show a plan view and two cross sections for the cases where Lambda is 0.10 and 0.65. The schedules are quite different. From cross section looking north, it is clear that some blocks are not extracted when Lambda is 0.65.

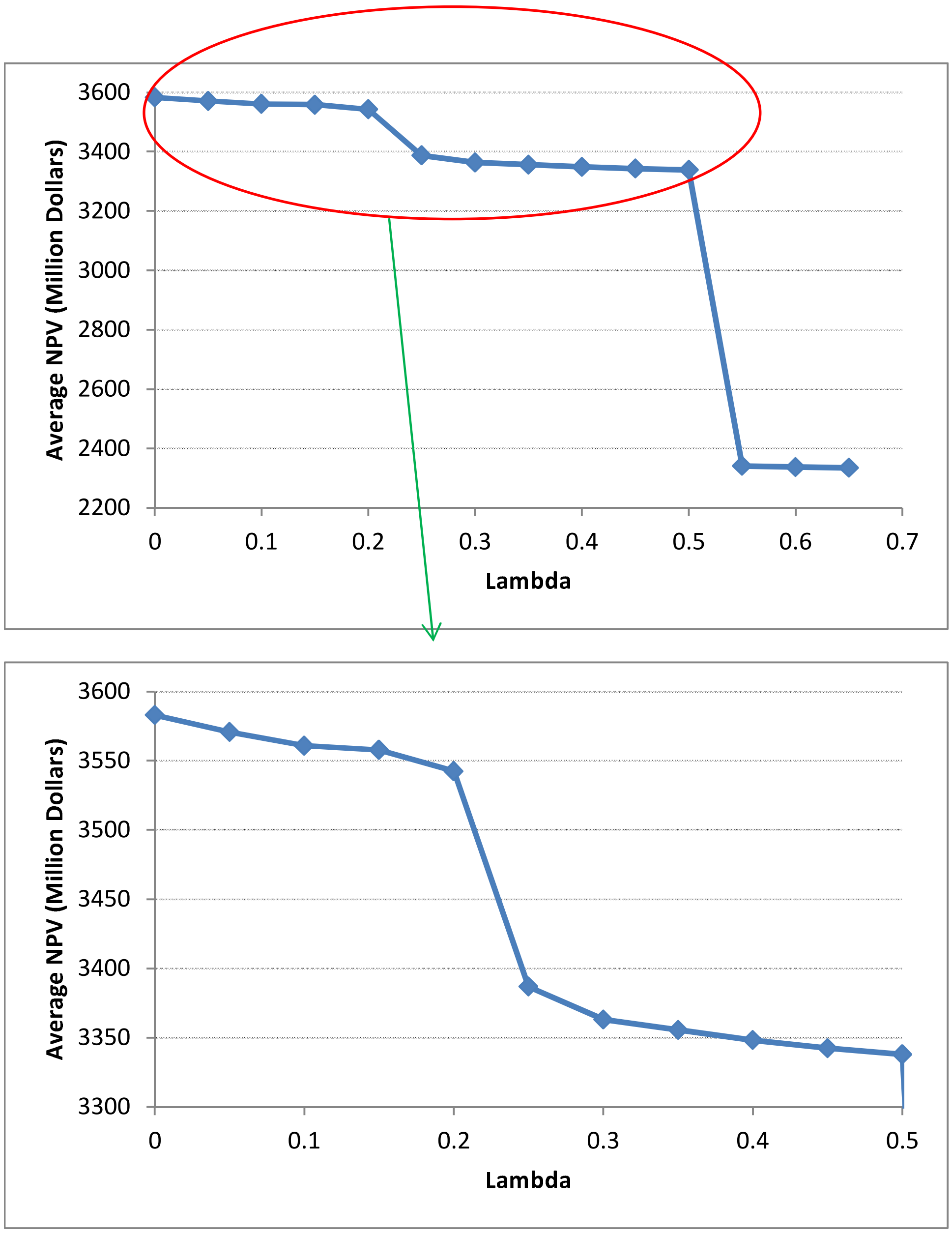


Figure 5-16. Lambda factor versus average NPV.

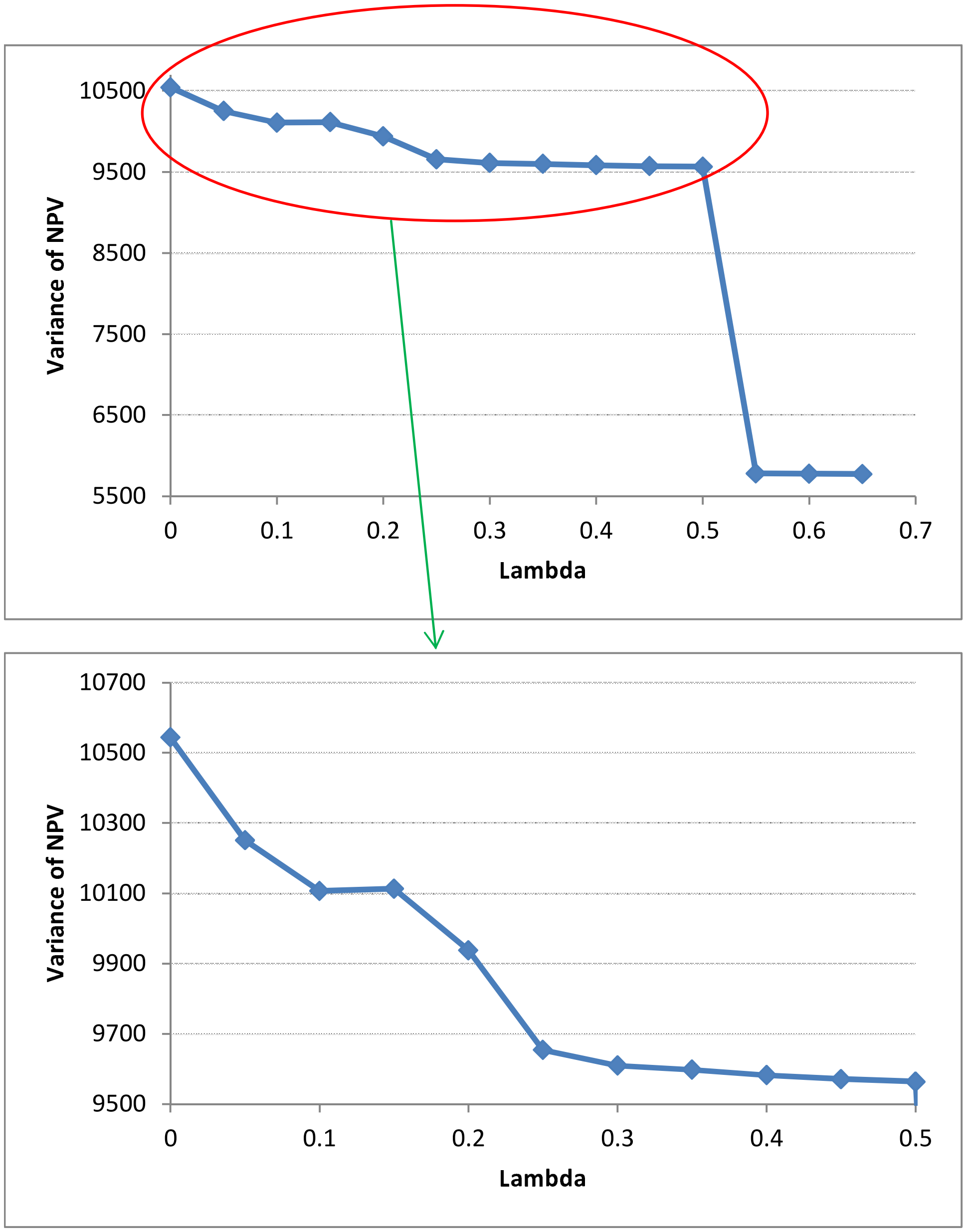


Figure 5-17. Lambda factor versus variance of NPV.

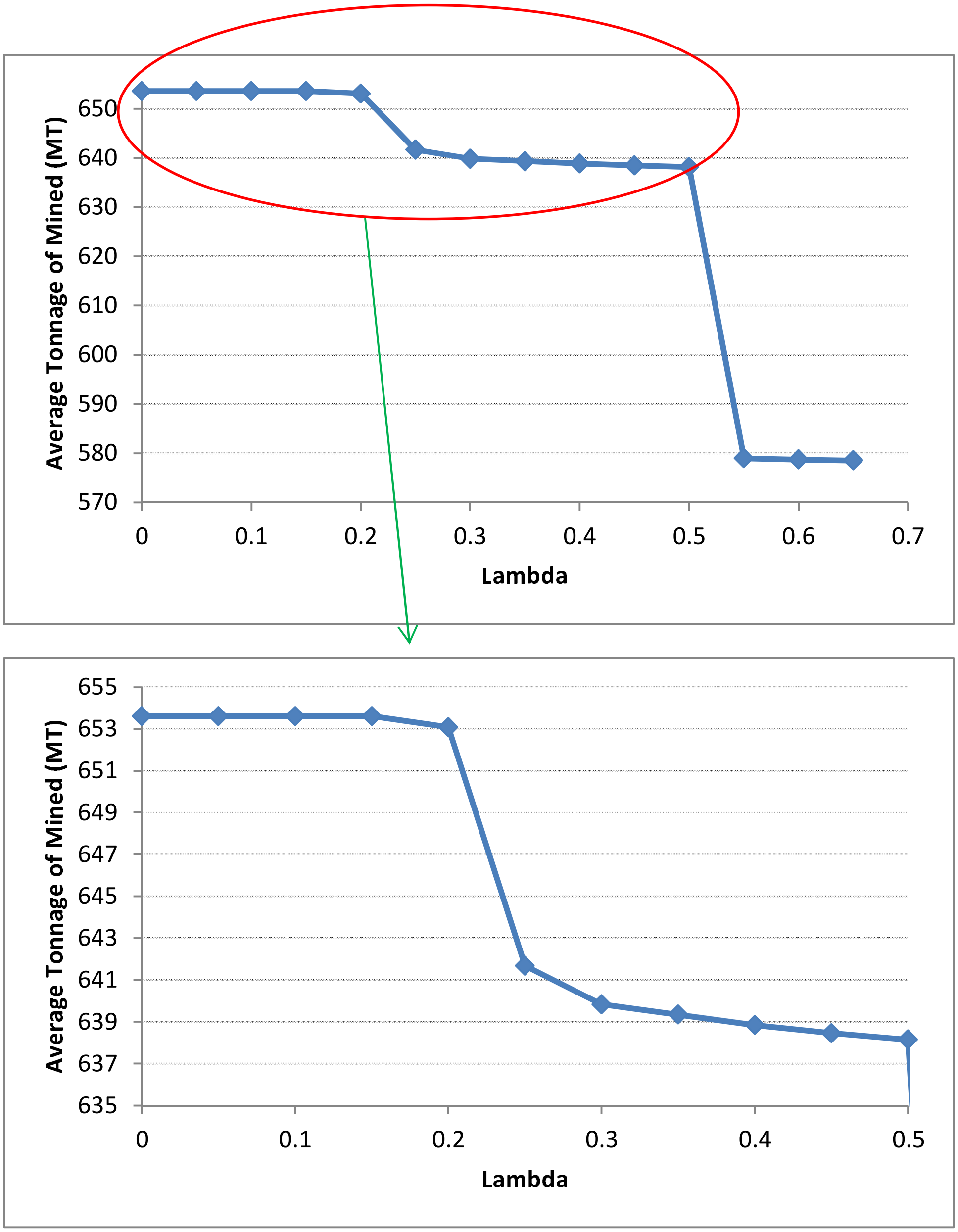


Figure 5-18. Lambda factor versus average tonnage of mined material (ore + waste).

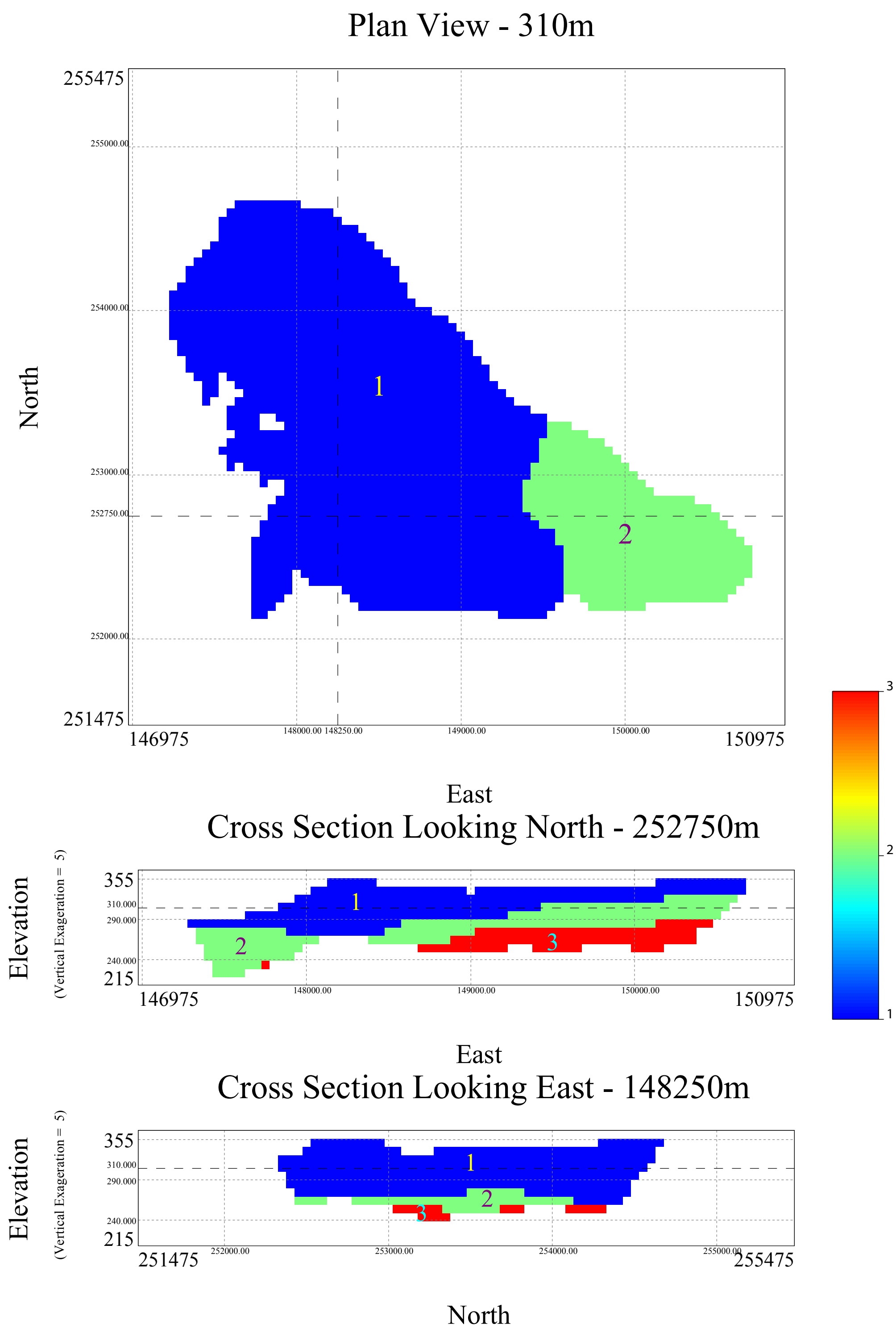


Figure 5-19. Plan view and two cross sections of the schedule generated by mean-variance approach using 100 cuts and Lambda=0.1.

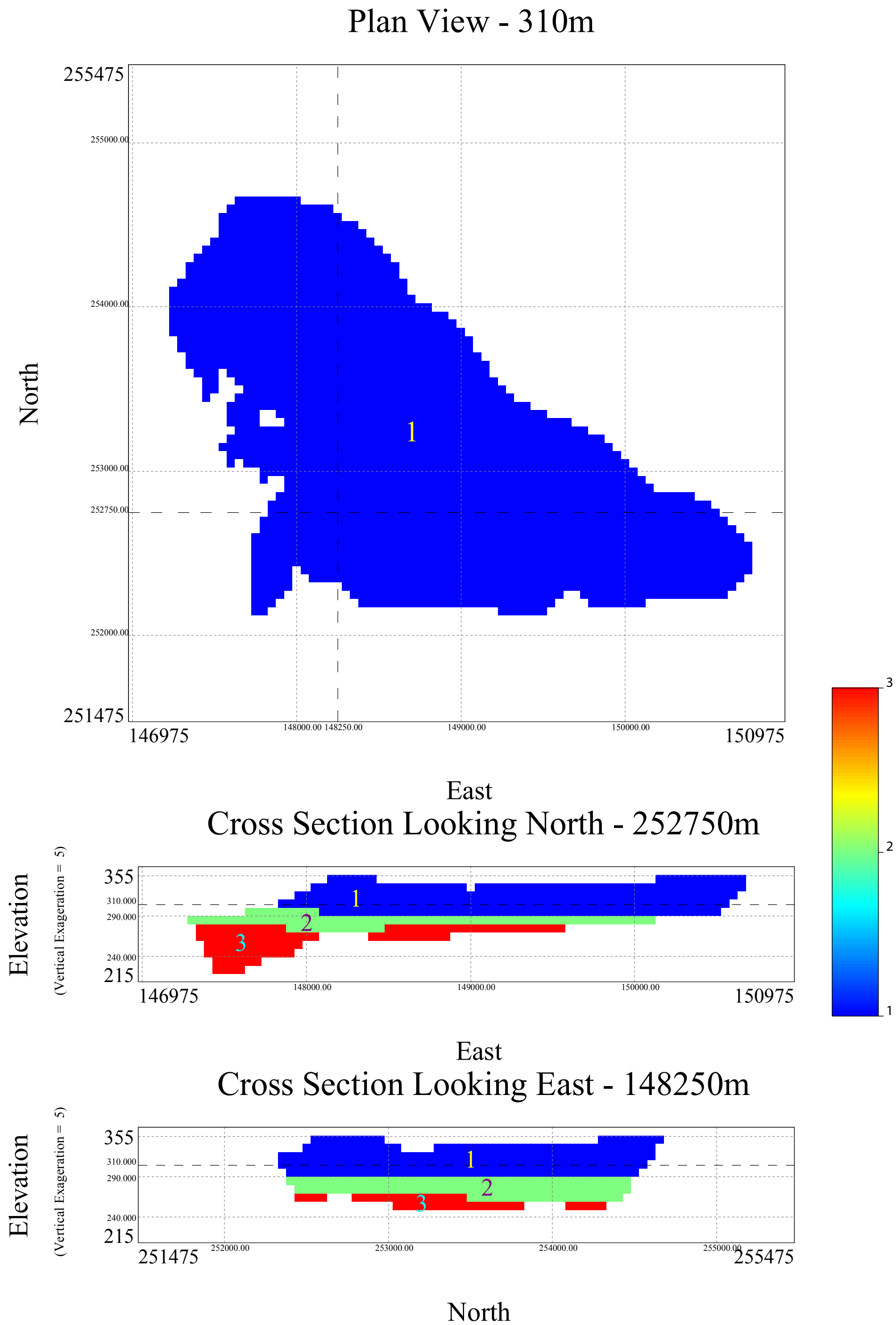


Figure 5-20. Plan view and two cross sections of the schedule generated by mean-variance approach using 100 cuts and Lambda=0.65.

5.5 MIPGAP versus RunTime

MIPGAP is the absolute tolerance of the gap between the best integer objective and the objective of the best node remaining in the branch-and-bound algorithm. This is an important termination criterion that is set by user. This parameter instructs CPLEX to stop as soon as it has found a feasible integer solution proved to be within the MIPGAP limit. MIPGAP shows the goodness of the optimal solution and it indicates how far we are from the theoretical solution. CPLEX reduces the GAP of current answer from theoretical answer by creating new branches and exploring new nodes in the tree of answers in two ways: (1) finding new better answers with higher objective function and (2) getting better estimation of the theoretical answer. Choosing higher MIPGAP decreases the runtime in two ways: (1) the algorithm needs to create less branches and (2) every time that CPLEX creates a new branch, it eliminates the branch for which the gap is less than the target MIPGAP. Therefore, it reaches the required MIPGAP much faster. There is always a tradeoff between the goodness of the optimum solution and the runtime of the optimization problem. As shown in Figure 5-21, by increasing the runtime, the gap between red and blue lines decreases until the gap of 0.1% is reached after 14 hours of runtime. As shown in this graph, after around 5 hours of running, the current solution does not improve significantly (blue line) and only the theoretical answer (red line) is decreased to reach the required GAP. Therefore, by setting the gap to higher values the runtime can decrease significantly.

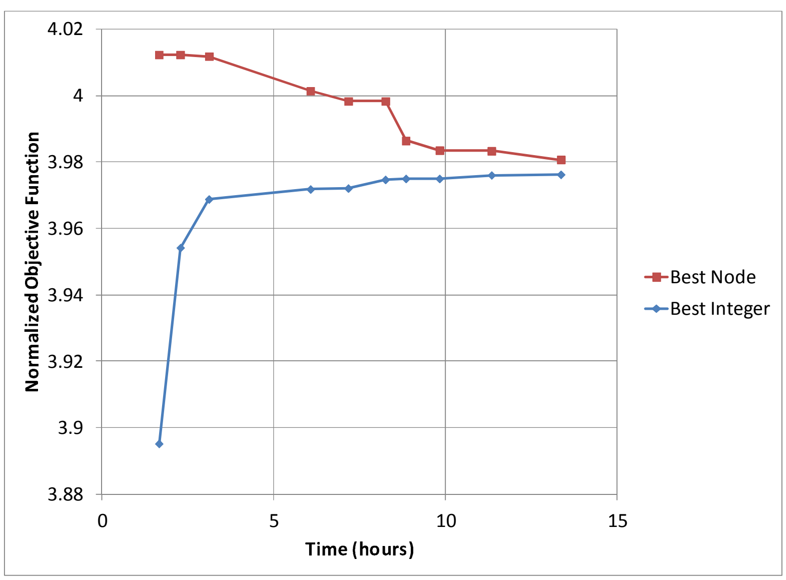


Figure 5-21: NPV of the project in current best answer (best integer) and the theoretical NPV at current node (Best Node) vs. the runtime

5.6 Summary

A sensitivity analysis on the number of cuts, the cost of grade uncertainty, the Lambda value and MIPGAP has been considered.

The clustering method is an important step for all optimization-based production planning algorithms. The size of the optimization problem is directly related to the number of blocks or cuts that exist in the model. The number of continuous and binary decision variables is related to the number of cuts and the number of periods in the project. Typically, for long-term production plan the mine life is consider between 10 to 30 years. A large numbers of blocks increase the complexity of the problem. On the other hand, by clustering similar blocks and assigning a single variable for each cluster, the size of the problem decreases. The side effect of clustering blocks to the mining cuts is that the maximum achievable NPV of the project is reduced due to the sub-optimality of the problem compared to using all blocks in the optimization.

It is almost impossible to solve the optimization with all blocks. Using mining cuts is more practical in that the long-term production plan is kept at an appropriate scale and the movement of the equipment during a single period is minimized.

The NPV of the project increases by increasing the total number of mining cuts. However, after a point there is no significant improvement in the NPV. An important issue is to choose the clustering algorithm and the input parameters for the similarity parameters between the blocks.

The other important input parameters that have been investigated in this chapter are the trade-off parameters. These parameters are required to be selected by user: the *c* factor for Model #2 and #3 and the Lambda factor for Model #4. The main conclusion here is that production plans are not very sensitive to this parameter. It is suggested to use a range of *c* factor within acceptable limits in order to find the optimal values for this parameter. On the other hand, Model #4 is very sensitive to the Lambda value. This model aims to minimize the variance of NPV. The variance of the NPV is highly correlated to the NPV itself. Therefore, in most of the cases, by increasing Lambda, the optimizer finds a solution with lower NPV. On the other hand, in order to reduce the variance of NPV, the optimizer reduces the size of the pit and total processed ore; which is not a good solution. The recommendation here is to use a reserve constraint to force the optimizer to process all the blocks inside the final pit. A sensitivity analysis is also required to determine the optimum Lambda value which has been shown in this chapter as well.

Finally, the behavior of the cost of grade uncertainty has been studied in different mining and processing limits. Increasing the mining capacity reduces the cost of grade uncertainty. An optimum ratio of processing capacity to mining capacity can be determined by using the concept of the cost of the uncertainty.

Chapter 6 Concluding Remarks

A summary of the research is presented in Section 6.1. Conclusions and applications are discussed in Section 6.2. The contributions of this dissertation are summarized in Section 6.3. Finally, recommendations for future work are presented in Section 6.5.

6.1 Summary of Research

Open-pit mining is the most widely used mining technique. Open-pit mining has high capital cost and the average grade of the ore-body may be relatively low. An important problem is to determine the optimum production schedule. Recently, the operations research methods are used to generate the optimal long-term production plan. Usually the goal is to maximize the net present value of the project in long-term and feed the plant at full capacity in the short-term. To determine the optimum production plan it is required to have a complete knowledge of the value of ore and the cost of mining and processing of the blocks, and this is not possible until extraction of all blocks. In traditional methods the estimated block values are used as the main input to determine the optimum production schedule.

Currently, commercial software tools are available to determine production schedules. One of these software tools is Whittle which works based on some heuristic methods. This software creates the production schedule at the block level and provides the destination of each block and the portion of the extraction in each period. Although this software does not generate the optimum solution that maximizes NPV of the project, it is widely accepted as a good tool in the industry to generate long-term production plans. This software uses the 3D LG algorithm to find the final pit limit and push-backs.

The demand to apply the operations research-based algorithms in open-pit mining is increasing. The main reason for this is the recent improvements in the optimization methods and in the computer power that allows solving large scale optimization problems more efficiently. The advantage of the operations research-based algorithms that are discussed in Chapter 1 can be summarized as:

- Operations research-based algorithms such as linear programming generate the best achievable solutions. The optimality of the solutions from heuristic-based algorithms is not guaranteed.

- There is a gap parameter that measures the goodness of the solution. This parameter is used as the stop criterion of the algorithm. It shows the gap between the best integer objective and the objective of the best node remaining.
- The proposed algorithms are much more flexible than currently available commercial software such as Whittle. It is easy to add constraints such as directional mining and to optimize based on multiple valuable elements and multiple processing plants.

Many recent techniques are based on linear programming techniques without considering the uncertainty in the input block model. However, the uncertainty in the input block model may cause shortfalls from target production.

There are very limited numbers of uncertainty-based methods. The major shortcomings of the current uncertainty-based methods in the literature for long-term mine planning are:

- Some of these methods are only based on local uncertainty. These methods don't accept jointly simulated realizations as an input. The main input for these types of methods is the variance of each individual block. This is not a correct way to transfer the geological uncertainty into the production plan. The best practice is to use simulated realizations. These values are generated based on the fact that geological properties are spatially correlated and the values of the blocks are dependent on the nearby blocks.
- Although the computer power and the optimization methods have improved significantly, it is not possible to use all blocks in the optimization directly due to the size of the problem.

A methodology has been developed to transfer grade uncertainty into the mine production plan. A program has been developed to assess the effect of grade uncertainty on the production plan. A theoretical framework for the long-term production planning problem in presence of grade uncertainty was developed, implemented and verified by a case study. Three different models were developed based on a deterministic model. The deterministic model was presented by Askari-Nasab et al. (2010; 2011), which was referred as Model #1. Models, #2 and #3 are mixed integer linear programming problems and the last model, #4, is a quadratic optimization problem. The simulation realizations are directly used in the proposed models.

MATLAB (MathWorks Inc., 2011) environment was used to implement the codes. A standard form of the MILP problem that can be used by TOMLAB/CPLEX (Holmström, 1989-2011) optimization toolbox has been developed. The TOMLAB Optimization Environment is a modeling platform for solving applied optimization problems in MATLAB. This toolbox contains a large number of different solvers. CPLEX (ILOG Inc, 2007) was used as the main solver engine to find the solution for the proposed MILP and MIQP models. The implementation details for each model and the vectors and matrices that are required to build the models in standard format were presented in Chapter 3.

An oil-sand deposit was used to apply models #1, #2, #3 and #4. A comprehensive geological analysis and the geostatistical simulation steps were presented in Chapter 4. The ordinary kriging block model and 50 realizations were generated in high resolution and up-scaled into blocks. Whittle was employed to generate the final pit using the 3D LG algorithm and to estimate a production schedule based on OK block model. The realizations were used to assess the effect of grade uncertainty on the final pit limit and on the production schedule generated by Whittle. The deterministic method to generate the production schedule using the mixed integer linear optimization approach was applied with aggregated mining cuts. Later on in Chapter 5, a sensitivity study has been done to confirm the number of mining cuts. Model #1 was used to optimize the NPV of the kriging without considering the grade uncertainty. Model #2 was applied to this data set with symmetrical penalty function. All Realizations are used in the objective function to reduce the cost of grade uncertainty. Although the NPV of Model #2 was very close to the NPV of Model #1, it reduced the cost of grade uncertainty. Finally, Model #3 was applied. In this model, the stockpile is considered directly in the objective function. Therefore, although the NPV of the project was not changed significantly compared to model #1 and #2, the cost of grade uncertainty was reduced by 68% (Table 4-37). Also, the variance of NPV was reduced compared to other methods. Furthermore, the run time of Model #3 is much less than Model #2.

Finally, the sensitivity analysis on the input parameters was presented in Chapter 5. It was shown that the optimum number of mining cuts can be determined. A deterministic and a numerical method were presented to determine the optimum penalty values for over and under productions. It was shown that the generated schedule by either Model #2 or #3 is not very sensitive to the penalty values of over and under productions. On the other hand, the production schedule created by Model #4 is sensitive to Lambda parameter. If a high value is selected for this parameter, the optimizer can reduce the variance of NPV by reducing NPV itself. Therefore, a lower Lambda parameter has to be chosen. A sensitivity analysis is required to obtain the optimum values for all of the input parameters. Finally, the application of the cost of grade uncertainty in finding the optimum mining and processing capacities or the ratio of mining to processing capacity was shown in Chapter 5.

Table 6-1 summarizes the proposed optimization models and the main advantages and disadvantages of each model.

Table 6-1. Summary of the proposed methods, advantages and disadvantages.

Method	Solution	Advantage	Disadvantage
Model #1 **(MILP)**	Deterministic approach: NPV of the estimate block model is maximized.	The generated schedule for the estimate block model has the highest NPV among all methods.	The geological uncertainty is not taken into account.
Model #2 **(MILP)**	Uncertainty based approach: NPV of the estimate block model is maximized and the COU is minimized.	A symmetrical and asymmetrical penalty functions are applied to minimize the COU. The results are optimum when there is no stockpile available.	No stockpile is considered. The over produced ore in any period is not used to reduce the underproduction during following periods.
Model #3 **(MILP)**	Uncertainty based approach: NPV of the estimate block model is maximized and the COU is minimized.	Stockpile is modeled explicitly and used in optimization stage.	The average grade of the ore inside the stockpile is given by user. A recursive scheme may need to rerun the model to adjust the assumed grade values.
Model #4 **(MIQP)**	Uncertainty based approach: The expected mean of the NPV is maximized while the variance of NPV is minimized.	No need for estimate block model. It only uses realizations. The nonlinearity effect of grade uncertainty on NPV is taken into account.	Due to the nature of the quadratic optimization, the size of the problem is larger than other methods. Therefore it is very slow and difficult to handle large problems.

6.2 Conclusion

- All proposed models have two separate decision variables for the portion of extraction and the portion of processing. This is very useful when there is no certain value for the cut-off grade. In this case, the optimization procedure decides which blocks should be processed and which blocks should be sent to the waste dump. Therefore, in the case that there is a cut-off grade to distinguish ore and waste blocks and also there is a stockpile to store extra ore, there is no need to define two separate variables. All of the ore blocks are going to be processed or sent to the stockpile for being processed at later years. Therefore, only one continuous decision variable was used for each period in Model #3. This variable is the extraction portion of the blocks. The destination of each block (or mining cut) is decided based on the average grade of that block.
- The grade uncertainty has linear and nonlinear effects on the mining project. The linear effect is applied to the input ore tonnage to the mill. This effect is due to the existence of a cut-off grade. If in a realization a block gets less simulated value than the cut-off grade, it is considered as a waste block in that realization. In this way, the grade uncertainty is transferred to the ore tonnage uncertainty.
- The other effect of grade uncertainty is on the ore value. Different simulated grades generate different ore values and this affects the NPV. Therefore, if the simulated grade of a block in all realizations is less than the cut-off grade, this block does not have any effects on the ore tonnage or NPV. In general, the grade uncertainty does not have linear effect on the NPV.
- Models #2 and #3 account for the grade uncertainty and reduce the effect of grade uncertainty in long term production plan. Model #4 accounts for both the linear and the nonlinear effect of the grade uncertainty by minimizing the variance of NPV.
- The cost of grade uncertainty can be reduced by using Model #2 and #3. However, there is always a trade-off between the maximization of the NPV and minimization of the negative effect of grade uncertainty. By reducing the effect of grade uncertainty, it is expected the NPV of the project to be reduced too. However, as it is shown in Table 4-36, the percentages of reduction for proposed models are not significant.

- To solve MIQP, new variables are defined inside the solver. Therefore, Model #4 is slower than any other proposed model. The other models are recommended. If the grade uncertainty significantly changes the results, then Model #4 should be applied. A good initial point such as the results of Model #1 can increase the speed of this model.
- The long-term mine plan is established using all available information. The production plan will be updated as new information becomes available during the mine life. The new information is obtained from new infill and blast holes. Therefore, the best practice is to re-run the optimization algorithm based on all available data to generate the optimum schedule as the mine life evolves.

6.3 Summary of Contributions

This research has studied the effect of grade uncertainty on the long-term production planning problem. The goal was to investigate the negative effect of the grade uncertainty, propose a robust algorithm and develop tools to incorporate grade uncertainty in the long-term production planning. The grade uncertainty is modeled by realizations that are generated by geostatistical simulation algorithms such as sequential Gaussian simulation. This dissertation is an effort to develop long-term production planning algorithms in presence of grade uncertainty. The proposed methodologies offer the following significant improvements over existing methods in the context of long-term mine planning in presence of grade uncertainty.

6.3.1 The Effect of Grade Uncertainty on Long-term Production Planning

The first contribution of this research is that it provides a body of knowledge on the effect of grade uncertainty on the LTPP. It has been shown that grade uncertainty has linear and quadratic effects on NPV of the project. The linear effect is on the input ore tonnage to the mill and can cause shortfalls from target production or surplus unexpected ore. Both of these effects make the production plan sub-optimal if the grade uncertainty is not considered in the optimization process. The nonlinear (quadratic) effect of the grade uncertainty occurs on the NPV of the project. A methodology has been presented to transfer the grade uncertainty into the production plan.

6.3.2 The Cost of grade uncertainty

A new term called the cost of grade uncertainty has been introduced. It gives a good measurement of the potential cost of the grade uncertainty on the long-term production schedule. It is based on the linear effect of the grade uncertainty on the input ore tonnage into the plant. The cost of grade uncertainty can be used to compare different production schedules in presence of grade uncertainty. Also, it has been used in the proposed models. The goal was to incorporate the cost of grade uncertainty in the optimization stage.

6.3.3 Two MILP Models for LTPP (Model #2 and #3)

In this research, two different MILP models were presented and implemented in MATLAB environment to generate the optimum LTPP in presence of grade uncertainty. Both of these models have dual objective functions to maximize the NPV and minimize the cost of grade uncertainty. The difference between these two models is that in Model #3, the stockpile has been considered in the objective function while in Model #2 it is assumed that no stockpile is available. As it is summarized in Table 4-37, by using Model #3 the cost of grade uncertainty is reduced up to 68% compared to Model #1 for the case study that has been presented in Chapter 4.

6.3.4 MIQP Model for LTPP (Model #4)

A mixed integer quadratic optimization model has been proposed in this thesis. The idea is to maximize the expected return and minimize the variance of the result that are calculated directly from realizations. Therefore, there is no need for an estimation block model in Model #4.

6.4 Assumptions and Limitations

The following assumptions and limitations are considered for the proposed methods in this thesis:

- The generated realizations and the number of realizations are assumed to be enough to represent the geological uncertainty.
- The uncertainty on the rock-type, the density of the rock, the mining cost and the price of the commodity are not taken into account in any of the models.

- The proposed models are adapted for single-element deposit. The production plan for a multi-element deposit needs a multivariate geostatistical model as an input. However, the proposed models can be adapted for a general case with multi-element deposit.
- All models are designed for a single-process open pit mine. It is required extra effort to adapt the models for a complex multi-process open pit mines.
- For all the models, push-backs are not considered. However, this can be implemented by adding extra precedence constraints for the blocks that are in different push-backs.
- There is no constraint to apply a preferable directional mining in none of the proposed models. However, same as push-backs, by adding extra precedence constraints, this feature can be added to each of the models.
- Long term mine planning is a recursive procedure such that at the end of each period or any time that is required, the optimization models should be run. However, the goal here is to use as much information as possible to reduce the effect of grade uncertainty on the production plan. For this purpose, the extraction of low grade high uncertain blocks is deferred to the later years or stockpile is used. All methods presented in this thesis are anticipative models. The best practice would be to use them as adaptive models anytime that new information is available.
- In Model #3, in order to model the stockpile and keep the optimization model linear, the average grade of stockpile for each realization at each period is assumed to be an input parameter. As it was shown in the case study, this assumption is checked and retuned in a recursive scheme, so that the assumed values are close to the actual numbers. However, the grade blending procedure always smoothes out the fluctuations in average grade of stockpile. Therefore, the average grade of the stockpile is close to the average grade of the block model when the cut-off grade is applied.
- In Model #3, the stockpile is treated in a rule-based manner where surplus ore is saved and drawn down in time periods where there is inadequate ore supply. Although the stockpile is not being optimized, this approach provides a mechanism to account for over and under production more realistically.
- Model #4 is a quadratic optimization problem. A positive definite quadratic optimization problem always can be solved in a polynomial time. However, the

size of Model #4 is much bigger than the other models considering the same mine life and the same number of mining cuts. Because it takes too much time to solve this model, it is not a practical model for real size problems with current software and hardware capacity.

- Clustering is required to reduce the size of the problem.

Aforementioned limitations derive some recommendations for the future research which are presented in the following section.

6.5 Recommendations for Future Research

Although the LTPP models developed in this thesis have provided pioneering efforts to effectively use the grade uncertainty on the production plans, there is still a need to continue investigation on using simulation realizations in the mine planning context.

The following recommendations could be considered:

- Push-backs are very well known extraction strategies that are used for many years in open-pit mine design. However, there is a debate that they are not required anymore because of increased usage of operations research algorithms in mine planning. Push-backs could be implemented in any of models by adding extra constraints.
- Directional mining can be implemented in all of the models. New sets of blocks are added to each block precedence list. These new block sets enforce the mining to be in a specific direction.
- The long-term production planning optimization models at the block scale are intractable. There is still a significant requirement to aggregate blocks into mining cuts. A more efficient clustering technique in which grade uncertainty is considered exclusively is recommended for future research.
- Rock-type uncertainty is not considered in this thesis. Realizations for rock-types; then, inside each rock type, grade realizations should be generated. For future work, it is recommended to explicitly use the rock-type in the optimization process.
- The recovery factor and the cost of processing should depend on the input grade.
- As shown in different chapters, the NPV of the project is not a linear function of the grade uncertainty. The quadratic optimization model developed in this thesis

is the first attempt to model nonlinear behavior of the grade uncertainty. The next step in LTPP problems is to investigate the efficiency of MIQP models and compare them to MILP models.

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Appendix

MSQ90 FORTRAN Program

A FORTRAN 90 program called MSQ90 is developed to assess the uncertainty of the pit limit and production schedule generated by Whittle. Whittle uses a file format called mining sequence file (msq) to report the portion of the extraction and the destination for each block. Only the blocks inside the final pit are reported in this file. Also some parameters of the blocks such as grade, EBV, cost of mining, and cost of processing are calculated based on the input block model that imported to the whittle. Based on this, the final pit and the schedule are generated. To assess the uncertainty of the final pit or the production schedule that is usually generated by OK block model, it is required to read this msq file and replace the economical parameters of the OK block model with the realizations. The MSQ90 program reads the Whittle msq (mining sequence) file and a conditional realization (in GSLIB grid format) and recalculates all the economical parameters for each block. The extraction and the portion of the extraction of the blocks are followed based on what is reported in msq file. However, the destination of a block may be different when the values are replaced by the realizations. Based on the grade uncertainty of the blocks and the operational cut-off grade two different situations may happen that MSQ90 program changes the destination of the block:

1. In the msq file, a block is waste (and is seeing sent to the waste dump), but is has higher simulated grade than the cut-off grade in the realization: this block is considered as an ore block and is sent to the processing plant. This kind of blocks may cause overproduced ore.
2. The block is ore in the msq file but the simulated grade is less than the cut-off grade: The program changes the destination of this block from processing to the waste dump. This type of blocks cause the shortfall from target production and it is the source of the cost of underproduction.

The general steps of MSQ90 program includes:

1. Read the parameter file that contains all the input parameters.

2. Read the msq file. This file contains the index of each block, the period of the extraction, the portion of the extraction and the destination of the block if it is an ore block.
3. Read the realization file in GSLIB grid format.
4. Apply the cut-off grade to the simulated value. If the simulated value is below the cut-off grade, the block is considered as a waste block. If the simulated value is above the cut-off grade the block is considered as an ore block.
5. Recalculate the tonnage of ore, EBV, ore value, and waste cost.
6. Calculate the total tonnage of ore and waste EBV discounted cash flow for each period.
7. Report the results in the two different output files

The MSQ90 program is based on GSLIB codes. All the parameters are stored in the parameter file. A sample parameter file for the program is shown in Figure APX 1.

```
                  Parameters for MSQ
                  ***********************

START OF PARAMETERS:
1 : krig.msq                             -MSQ file
2 : 1                                    -number of rock-type that are ore
3 : ORE                                  -name of the rock-type 1
4 : sim_1.out                            -file with blocks
5 : 1                                    -   column for variable
6 : 120   146000.0     50                    -nx, xmn, xsiz
7 : 120   251000.0     50                    -ny, ymn, ysiz
8 : 23    190.0        10                    -nz, zmn, zsiz
9 : 4.6                                    -Mining Reference Cost
10: 0.88                                   -Mining Recovery
11: 0.5025                                 -Processing Costs
12: 0.95                                   -Processing Recovery
13: 281.25                                 -Selling price ($ per %mass)
14: 6                                      -Cut off grade
15: 0.1                                    -Interest Rate
16: msq.csv                                -file with output
17: msq.dbg                                -file with Debug
```

Figure APX 1: parameter file for the MSQ90 program

Line 1 is the location and the file name of the input msq file. This file is in the standard format of the Whittle mining sequence file format. It is an ASCII format. Figure APX 2 shows a part of an msq file. There are two types of blocks in an msq file: (1) air blocks and (2) non-air blocks. There is one line per air block in the msq file. It is highlighted by grey color in Figure APX 2. The non-air blocks may be ore or waste. A waste block is highlighted by blue and an ore block is highlighted by yellow. The first line for any type of block contains: ix, iy and iz (indices of the block that are same as GSLIB), number of parcel, MCAF (Mining Cost Adjustment Factor), PCAF (Processing Cost Adjustment Factor), BlockTonnage, Period, BlockFraction, PushbackNum. If the block is air, there will not be any value for BlockFraction and PushbackNum. If the block is not air, there will be a second line which contains: ix, iy and iz, name of the block (ore, waste, etc), ParcelTonnes, and the name of the processing plant. If a block is being extracted in different periods, there are these two lines for each portion that is extracted at each period. Therefore, in an msq file, a block can be reported several times in different locations of the file.

```
1,1,9,0,1.000,1.000,0,1,1.0000
66,23,15,1,1.000,1.000,25000,8,1.0000,3
66,23,15,WAST,25000,0,-np-
40,48,8,1,1.000,1.000,54000,9,0.0006,1
40,48,8,ORE,54000,545007.188,UPGR
```

Figure APX 2: Part of a msq file

Line 2 is the number of ore rock-type.

Line 3 specifies the names of the ore rock-types.

Line 4 specifies the name and the location of the file of the grade realization. Each realization should be stored in a separate file.

Line 5 is the column in the realization file that contains the grade of the blocks.

Line 6 to 8 is the grid definition of the realization.

Line 9 is the mining cost per tonne.

Line 10 is the mining recovery fraction.

Line 11 is the processing cost.

Line 12 is the processing recovery fraction.

Line 13 is the selling price ($ per %mass).

Line 14 is the cut-off grade.

Line 15 is the interest rate.

Line 16 is the name of output file. This file contains the tonnages of input ore, mined ore, the average grade, etc for each period.

Line 17 is the name of the debug file. It reports one line for each block with different block properties such as EBV and period that the block is started to be extracted and finish the extraction.

MATLAB codes

MATLAB codes are categorized in different folders:

- Step01_Read: A MATLAB function called `ReadData.m` reads all block models that are saved in separate files using MSQ90, and transfers them to MATLAB data files. Therefore, there will be a separate output file for each block model. The output file is a MATLAB data file called '*.mat'. The user also should enter the following parameters as well:
    - Path: it is the location where all the MSQ90 output files are saved.
    - GSLIB grid definition that is used to generate the block model: nx,ny,nz, xmin,ymin,zmin, xsize,ysize and zsize.
    - numBlocks which is the number of blocks inside the final pit limit.

  In each output MATLAB file, there is a cell array called Blocks that contains numBlocks cells. Each cell contains the information of each block such as x, y, z, ix, iy, iz, tonnage, ore tonnage, waste tonnage, grade, ore value, mining cost, EBV and rock-type.
    - Cut-off: the cut-off grade of the project.
    - NumOfPeriods: the number of periods.
    - interestRate: the interest rate or the discount factor of the project.
    - mcMax and mcMin: the maximum and minimum mining capacity in each period.
    - pcMax and pcMin: the maximum and minimum processing capacity in each period. If there is a pre-striping stage, it should be defined here by forcing the pcMax to be zero in the pre-striping periods.
    - oreGradeMin and oreGradeMax: the maximum and minimum of average input grade to the mill in each period.
    - numOfRockType: the number of rock-types that are going to be extracted
    - rockTypesMined: the array of 1 to numOfRockType that indicates the rock codes that will be extracted.

- Step02_Adjacency_Matrix: In this step the adjacency matrix is generated. There are a main function called '`adjacency_matrix.m`' and 4 subroutines: '`arcs_graph.m`', '`blocks_above.m`', '`blocks_indices.m`', '`find_precedent_blocks.m`'. These codes have been mostly written by Dr. Hooman Askari Nasab and some minor modifications have been made to fix some bugs. The main function reads a file called "`blocks.mat`" as an input. It is recommended that the same block model that is used to generate ultimate pit limit to be used in this step. For example, if ultimate pit limit is generated from kriging block model, it is a good idea to copy the kriging mat file from previous step to this folder and rename it as '`blocks.mat`'. The output file will be '`inputToMILPBlocks.mat`'. This file contains same properties of input block model '`blocks.mat`' and the adjacency matrix:
    - blocksAbove: cell array from 1 to numBlocks. For each block or cell there is an array that shows the indices of blocks above that cell or block.
- Step03_Clustering: the output file of previous step is used here to generate the clusters. One block model is used to generate clusters. In this code, the blocks in each branch or level are used to create the mining cuts inside each bench. '`clustering_mining_cuts.m`' is the main function and '`blocks_on_top_of_cuts.m`', '`blocks2cuts.m`' and '`cut_clustering.m`' are the subroutines. The output file is '`inputToMILPcuts.mat`'.
- Step04_addSims: This folder contains two MATLAB files, the main one is '`AddSims.m`' and '`blocks2cuts.m`' is a subroutine. In this step, the output file of previous step is used. For each mining cut, the economical values are recalculated using simulated grade values from all realizations. At the end, the output file called '`inputToMILPcuts2.mat`' contains all realizations.

In this step, one single file is generated. This file has a cell array that each cell contains information of each mining cut for each realization such as tonnage, ore tonnage, waste tonnage, grade, ore value, mining cost and EBV. Finally one of the two MATLAB codes called '`params.m`' or '`params_Stockpile.m`' in case that there is a stockpile needs to be run. These codes read the '`inputToMILPCuts2.mat`' and add some final input parameters such as minimum and maximum mining and processing capacity for each period, number of periods or mine life in years, number of pre-striping years, number of simulation realizations and the discounted cost of over and under production in each period ($c_{up}(t)$ and $c_{op}(t)$) . These values can be equal specially when there is no stockpile. For the case that there is a stockpile, the overproduction of each period is calculated as Eq.(2.26).

Figure 2-9 shows the over and under production costs at different periods.

Also in the mean-variance approach, there is one input parameters called Lambda (λ). Lambda is set in '`params.m`' in case that the mean-variance is the interested approach.

The output file name for both of '`params.m`' and '`params_Stockpile.m`' is '`inputToMILPcuts3.mat`'. This file is the only input file for all the optimization steps that are described below.

The next step is use the '`inputToMILPcuts3.mat`' to generate schedule with different methods. For each method there is a folder. As before the main code called '`main.m`' is the one should be run. Other files are the sub-routines and are required to run the code.

Here is the description for each method:

- MILP_Cuts: this folder contains the optimization main codes and subroutines that are witten for generating schedule without considering grade uncertainty. Basically, it is based on Eqs.(2.11) to (2.19). It is a mixed integer linear programming (MILP) in cut level. The input file is the output file of '`params.m`'
- MILP_Cuts_Sims: This folder has MATLAB codes for generating schedules based on Eqs.(2.35) to (2.36) and its constrains described in section 2.6. This model is a MILP problem too. The input file is also the output file of '`params.m`'
- MILP_Cuts_Sims _Stockpile: In this folder, there are codes that generate the schedule based on Eqs. (2.38) and its constrains described in section 2.6. The input file is the out file of '`params_Stockpile.m`'.
- MIQP_Cuts_Sims: to generate a schedule based on mean-variance approach presented in section 2.7 the codes inside this folder is required. Three different methods are presented in section 3.5. But the last solution was coded. Therefore

the schedule generated by the codes inside this folder is based on Eq.(2.49) and its constraints Eq. (2.12) to Eq. (2.20).