THE UNIVERSITY OF CALGARY

AN ANALYSIS OF THE COMPARATIVE EFFECTIVENESS OF TWO MATHEMATICS PROGRAMS FOR LOW ACHIEVERS IN GRADE TEN

A Dissertation

Submitted to the Faculty of Graduate Studies

In Partial Fulfilment of the Requirements for the Degree

Master of Education

Division of Education Curriculum and Instruction

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by

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Calgary, Alberta

August, 1966

THE UNIVERSITY OF CALGARY

FACULTY OF GRADUATE STUDIES

The undersigned hereby certify that they have read and recommended to the Faculty of Graduate Studies for acceptance, a thesis entitled AN ANALYSIS OF THE COMPARATIVE EFFECTIVENESS OF TWO MATHEMATICS PROGRAMS FOR LOW ACHIEVERS IN GRADE TEN, submitted by Ivo Joseph Rodrigues, in partial fulfilment of the requirements for the degree of Master of Education.

Professor .....

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ABSTRACT

This study attempts to investigate the relative effectiveness of "modern" and "traditional" mathematics programs for low achievers in grade ten. The experiment was conducted in Calgary, Alberta, during the 1964-65 school year.

Using as criterion variables, (1) a modified form of the 1964 Alberta Departmental Grade Nine Mathematics Test, (2) the California Achievement Test in Mathematics, (Advanced), Form W, and (3) an Attitude Toward Mathematics Scale, that had been developed by the investigator, the study seeks answers to two main questions: In comparison to the "traditional" mathematics programs, does the "new" approach to mathematics for the low achiever show any significant evidence of: (1) better achievement in mathematics as measured by a conventional mathematics test and a standardized achievement test in mathematics?, and (2) better attitude toward mathematics as measured by the attitude toward mathematics scale?

The general approach to the statistical treatment of the data gathered was that of the multiple linear regression. This technique subsumes the analyses of variance and covariance.

The analysis shows that, (1) students studying the new materials achieved significantly better on the conventional mathematics test than students studying the traditional materials, (2) students studying the new materials achieved significantly better on the standardized achievement test in mathematics than students studying the traditional materials,

and (3) students studying the new materials scored significantly higher on the attitude toward mathematics scale than students studying the traditional materials.

In summary, the overall comparison between "modern" and "traditional" mathematics programs for low achievers seems to indicate that students studying the new materials show better achievement and display better attitude toward mathematics than students studying the traditional materials.

ACKNOWLEDGMENTS

The bound thesis is seldom, if ever, the result of one person's effort.

The writer wishes to acknowledge his gratitude for the constructive suggestions and guidance and for the continuous assistance and support received from his thesis supervisor, Dr. S. A. Lindstedt.

Grateful thanks are also extended to Messrs. David P. Flathman and Fred Pysh for their assistance in the statistical analysis.

Grateful acknowledgments are also made to the Examinations Branch of the Alberta Department of Education for making available the records of the 1964 Departmental Grade Nine Mathematics Examination.

Thanks are also extended to the superintendents of the Calgary Public and Separate School Boards, for permission to conduct the testing program, and to the principals and teachers of the participating schools for the assistance rendered in the testing process.

Finally, the writer wishes to express his thanks to his wife, Roma, and to his children, Hyacinth and Ivan, for their support and patience throughout the preparation of this report.

ABBREVIATIONS USED

MOD refers to the Treatment Group--students in the "modern" mathematics classes.

TRA refers to the Control Group--students in the "traditional" mathematics classes.

RS refers to the 1964 Alberta Departmental Grade Nine Mathematics Examination raw score.

MGN refers to the Modified Grade Nine Mathematics Test.

SMT refers to the California Standardized Achievement Test in Mathematics (Advanced), Form W.

SVT refers to the Iowa Standardized Vocabulary Test.

AMS refers to the special Attitude Toward Mathematics Scale.

DEFINITIONS OF TERMS USED

Modern or New Mathematics. Throughout the report of this investigation, the term "modern mathematics" or "new mathematics" shall be interpreted as meaning a carefully articulated sequence of mathematics, with emphasis upon concepts and structure. The most important changes in the "new mathematics" have been in the style in which content has been formulated and presented. Logic continues to be an important element in "modern mathematics," and therefore great tribute is no longer paid to the 'self-evident' truth but greater emphasis is given to the exploration of systems based on postulates.

Traditional Mathematics. The term "traditional mathematics" shall be interpreted as meaning the sequence of mathematics which, until the recent revolution in mathematics, was the accepted science of space and numbers. Most textbooks designed for the traditional curriculum place much emphasis on recipes for manipulating symbols.

Low Achiever. The term "low achiever" shall be interpreted to refer to those students who obtained a letter grading below achievement 'B' in the 1964 Alberta Departmental Grade Nine Mathematics Examination.

Significant. Wherever this adjective is used the implication is "statistically significant" at the .05 level.

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SETTING FOR THE STUDY

I. THE NATURE OF THE PROBLEM

In recent years the content of the mathematics program in the secondary school has been subjected to frequent criticism. Critics and supporters alike have expressed dissatisfaction with the results of traditional teaching of school mathematics. Kline points out that evidence supporting the above dissatisfaction is unmistakable; in his words, "The vast majority of the students who take mathematics dislike it."[1] Critics of the traditional approach to the teaching of mathematics believe that the main cause of its failure is that it emphasizes rote-learning and computational skills.[2]

The evident need for careful research in this area, designed to make mathematics programs more meaningful, has led groups of mathematicians, teachers of mathematics and psychologists to become active in a movement carrying the force of a revolution. The purpose of this movement has been to reform the old high school mathematics curriculum

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[1]Morris Kline, "Reform in the Mathematics Curriculum," The Education Digest, Vol. XXIV, No. 6, (February, 1959), p. 36.

[2]E. G. Begle, "Revolution in Mathematics," Overview, (July, 1960), p. 97.

and to install a program which is logically structured and which stresses understanding of concepts, ideas and mathematical thought. This long-overdue revision to the mathematics curriculum probably began in 1952 when the Carnegie Corporation and the University of Illinois jointly supported a curriculum study in mathematics. This study produced a number of textbooks which emphasize the structure of mathematics and the 'discovery theory' of learning. By 1958, other groups at the University of Maryland, the University of Minnesota, Ball State Teachers College, Southern Illinois University and Boston College were forming new programs. Finally, the Commission on Mathematics of the College Entrance Examination Board and the Curriculum Committee of the National Council of Teachers of Mathematics formulated reports that presented detailed instructions for improved mathematics curricula for secondary schools. Since 1958, the National Science Foundation has supplied in excess of five million dollars to the program of the School Mathematics Study Group for the improvement of mathematics training of youth at all levels from Kindergarten through Grade twelve.

In this great surge of activity, a great deal of energy has been expended in an effort to provide new courses for the college preparatory student, but very little attention has been devoted to the needs of other types of students, especially the so-called low achiever. The general consensus of opinion among some mathematicians has been that the role of school mathematics is to develop the kind of mathematical literacy that is needed by the average citizen in this era of

automation and nuclear energy;[3] however, the initial efforts in the direction of reform in the mathematics curriculum would infer that the only students worth cultivating are those who might enter the college preparatory pattern.

It is only recently that a new approach to teaching the low achiever is beginning to take hold.[4] Despite the attitude of some school administrators and teachers, who tend to regard "modern" mathematics programs as primarily for high ability students, findings of classroom experimentation with "modern" mathematics materials indicate that such programs have been found effective in increasing a low achiever's grade level in mathematics.[5] Teachers engaged in such experimental studies have reported that some of the students, who had considered mathematics dull and uninteresting, came to life after exposure to the new materials.[6] Commenting on the results of the aforesaid studies, Rosenbloom suggests that low ability students gain even more, in comparison with their achievement in conventional courses, than do high ability students.[7]

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[3]Viewpoints by 65 Mathematicians, "The Mathematics Curriculum of the High School," The Mathematics Teacher, Vol. 55, (March, 1962), pp. 191-193.

[4]Preliminary Report of the Conference on the Low Achiever in Mathematics, (Washington, D.C.: NCTM, March, 1964).

[5]K. E. Easterday, "An Experiment with Low Achievers in Arithmetic," The Mathematics Teacher, Vol. 62, (November, 1964), pp. 462-468.

[6]Comments from teachers, The Revolution in School Mathematics, (NCTM publication, 1961), p. 32.

[7]Paul C. Rosenbloom, "Mathematics K-14," Educational Leadership, (March, 1962), pp. 359-363.

There is a challenge here to determine the extent to which mathematics for the low achiever should be the same as that offered in the college preparatory sequence. The plausible answer to this question lies in extensive experimentation and carefully conducted research in this area.

Having seen the need for such experimentation, the Calgary Public School Board adopted a "modern" mathematics textbook[8] on an experimental basis and the program was offered to a pilot group consisting of over three hundred and fifty low achievers in the two vocational high schools in the city of Calgary, Alberta.

This study attempts to evaluate the relative effectiveness of the "modern" program, as developed by the text used by the pilot group, as compared with the "traditional" programs offered to students of similar ability.

II. SIGNIFICANCE OF THE PROBLEM

A low achiever has been, and still is, classified as "non-academically minded" based on the unproved premise that he is a person incapable of attempting college preparatory courses. The Preliminary Report of the Conference on the Low Achiever in Mathematics describes such a student as, "That student, who by teacher estimates, achievement

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[8]Marie S. Wilcox, and John E. Yarnelle, Mathematics A Modern Approach, (Addison-Wesley Pub. Co. Inc., 1963).

tests, or by whatever means the school uses for marking, grouping, or promotion, ranks below the 30th percentile of the student population in achievement in mathematics."[9] Within this "slow-learner" group may be included youngsters who appear to try hard and progress little by accepted standards; those who try hardly at all and yet consistently meet some sort of minimal standard; and those who are present--and of whom not much else can be said. This is hardly a pure culture, and it is unlikely that a single program will produce the desired changes in its varied patterns. Yet, these individuals are assigned the common label, "slow learner," and accordingly are assigned to a general mathematics class, where they repeat the same program that has frustrated and repelled them in earlier grades. The net effect of this treatment is that it perpetuates their retardation in mathematics. During the school year 1964-65 the two school systems in Calgary, Alberta, accounted for one thousand one hundred and seven students who may be classified in this group.

In light of this problem, facing the educational authorities, it seems reasonable to suggest that there appears to be a need to assess the present status of the low-academic student. Children differ markedly in many respects; some differences are easily observed, for example, height, weight, complexion, color of hair and eyes, and various motor skills; however, the more subtle differences of intelligence and

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[9] Preliminary Report of the Conference on the Low Achiever in Mathematics, op. cit., p.2.

personality are less apparent and perhaps less understood. Review of the related research and literature indicates that the slow learner is faced with a future of unemployment because of failure to develop his potential abilities. Thomas W. Mahan argues that, "These youngsters demonstrate in other areas competence similar to those we attempt to encourage. Perhaps the 'slowness' lies in the teaching process rather than in the learner."[10] The findings of a recent study by Robey and Cody show that low-academic students performed at a level below the average-academic student in relation to study skills as well as achievement.[11] "These two findings suggest that the methodology of preparing lessons should be considered by individuals who work with low achievers."[12] The lack of ability to organize and to do thinking of an abstract nature appear to point out that:

> The slow learner might have failed to learn because he lacked the insight into the essential elements of organizing subject matter into meaningful patterns of learning. It appears that a unique curriculum should be devised for the slow learner, one which will place central emphasis on his slower development in study skills as well as achievement.[13]

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[10]Thomas W. Mahan, Jr., "The Slow Learner: Fact or Excuse," The Education Digest, (November, 1965), p. 15.

[11]Dale Lewis Robey, and John J. Cody, "A Differential Diagnosis of Low and Average Academic Ninth Grade Male Students," The Journal of Experimental Education, Vol. 34, No. 4 (Summer, 1966), pp. 38-43.

[12]Ibid., p. 42

[13]Ibid., p. 42

The new approach in the teaching of mathematics recognizes that low innate ability may not be the only reason for low achievement in mathematics. Inherent in this new approach is the assumption that the low achiever is probably capable of doing better if he is adequately motivated. The "modern" mathematics textbook,[14] used by the pilot group in this study, attempts to do just this--it seeks to arouse and maintain the student's interest in mathematics and provides him with opportunities to make genuine mathematical discoveries at his level of knowledge and performance.

The specific function of this study is to investigate whether or not the "modern" mathematics approach, as presented to the low achiever, has served the intended purpose--which is to help him to achieve better in mathematics.

III. STATEMENT OF THE PROBLEM

The particular problems that this study considers for investigation are:

(1) Does the new approach to mathematics produce any significant evidence of responses which can be identified as related to this particular approach and which are different from those responses promoted by a traditional approach to mathematics? The specific responses considered

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[14]Marie S. Wilcox, and John E. Yarnelle, op. cit.

are:

(a) How do students studying the new materials compare with students studying the traditional materials with respect to achievement as measured by a conventional mathematics test?

This study attempts to measure the mathematical achievement by the use of a modified form of the 1964 Alberta Departmental Grade Nine Mathematics Examination.

(b) Is there any evidence to indicate that students are better able to transfer the knowledge of concepts learned, in the solving of mathematical problems that call for logical reasoning and discovery?

This study attempts to measure the performance in mathematics by the use of a standardized California Progressive Achievement Test in Mathematics.

(2) Does the new approach to mathematics produce any significant evidence of better attitudes toward mathematics which can be identified as related to this particular approach and which are different from those attitudes generated by a traditional approach to mathematics? This study attempts to measure the attitude toward mathematics by the use of a special Attitude Toward Mathematics Scale.

In summary, the problems that this study attempts to investigate are:

In comparison to the "traditional" mathematics program, does the "new" approach to mathematics show any significant evidence of (1) better achievement in mathematics, (2) better attitude toward

mathematics?

CHAPTER II

SURVEY OF RELATED LITERATURE

I. REVIEW OF CURRENT ATTEMPTS TO IMPROVE MATHEMATICS PROGRAMS FOR LOW ACHIEVERS

There has probably never been a time in the long history of mathematics education when the curriculum has received more attention than at present. The vigorous movement under way, to reexamine and modify the teaching of mathematics, has been prompted by the feeling that traditional mathematics curriculum does not serve the functional need of children for the life they are to live as adults.

Initial attempts in this area were directed toward improving the content of mathematics programs designed for the academic student. The low achiever received little attention and almost no funds in the current revolution were expended on his behalf; it is only recently that a dramatic turn in attention, to the student at the other end of the academic achievement scale, has taken place.

As mentioned in the previous chapter, in March 1964, a conference on the low achiever in mathematics was held to consider the problem of improving the mathematics education of low achievers and to seek solutions and to propose actions to improve the existing mathematics programs. The conference participants have recommended

sets of guidelines for administrative provisions[15] and for teaching the low achiever in mathematics.[16] Guidelines were also suggested for the preparation of instructional materials for the low achiever in mathematics.[17] In summary, the conferees agreed that, for the effective instruction of low achievers, good organization, competent and sympathetic teachers, and suitable materials were needed.

The conferees reviewed the reports on recent projects for low achievers in mathematics[18] and were unanimous in their opinion that the experiments in teaching the new mathematics programs indicate that "all pupils can learn more mathematics than we previously thought."

II. REVIEW OF RESEARCH ON THE EVALUATION OF NEW MATERIALS

Time has not permitted extensive research on the impact of modern programs in mathematics for the low achievers. Therefore, the literature relating to comparative studies is still very limited.

Teachers engaged in experimental studies with the "modern" mathematics materials have reported that some of the students, who had

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[15] Preliminary Report of the Conference on the Low Achiever in Mathematics, op. cit., pp. 5-9.

[16] Ibid. pp. 12-13.

[17] Ibid. pp. 10-11.

[18] Ibid. p. 16.

considered "traditional" mathematics dull and uninteresting, came to life after exposure to the new materials and performed remarkably well. These students, who had gone unnoticed before, became interested in mathematics because "it gave them quick access to a kind of intellectual adventure that was enticing and satisfying."[19]

James Miller reported a classroom project in which he used a modern program of arithmetic with sixth-grade children; these pupils were below grade level in achievement. Miller placed more stress upon meaningful presentation of subject matter than on routine drill exercises. The Stanford Achievement Test was given as a pre-test and repeated as a post-test. He observed that gains on the arithmetic computation section of the test were remarkable in spite of less time spent on routine drill exercises. On the basis of the results, Miller suggested that there is good reason to believe that modern mathematics programs are effective in developing mathematical skills in computation.[20]

Kenneth E. Easterday reported a study designed to determine if "modern" mathematics could be taught effectively to a group of low achievers.[21] The group was composed of thirty-seven eighth grade students and forty-one seventh grade students. Forty-five students

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[19]The Revolution in School Mathematics, op. cit., p. 32.

[20]James Miller, "Children Score Higher in Computation Despite Less Time for Drill," Mathematics Forum, (Fall, 1961), p. 3

[21]Kenneth E. Easterday, op. cit., pp. 462-468.

in this group had a history of either academic difficulties, or behavior problems, or psychological problems prior to the beginning of the experiment.

Because the children in the sample indicated definite difficulties in both fundamentals and reasoning, a program was devised to attempt to strengthen both areas. Traditional mathematics was used to strengthen the fundamentals or computation in mathematics. Modern mathematics was used in an effort to strengthen both the reasoning and fundamentals. The modern and the traditional materials were used together in about equal portions.

Easterday observed that motivation was not a basic problem for most of the group. Many found personal and academic success in arithmetic for the first time. Once they met success, many of the discipline problems decreased. He concluded that:

(1) "Modern" mathematics can be blended with "traditional" material into a successful subject-matter program.

(2) Such a program may be effective in increasing a low achiever's grade level in mathematics up to as much as three years.

(3) Such a program may be effective in increasing a student's level of arithmetic reasoning up to three years, and competency in fundamentals up to as much as four years.[22]

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[22]Kenneth E. Easterday, op. cit., p. 468.

CHAPTER III

THE EXPERIMENTAL DESIGN AND STATISTICAL PROCEDURES

I. THE NATURE OF THE SAMPLE

During the 1964-65 school year the Calgary Public School Board introduced, on an experimental basis, a modern mathematics program in two vocational high schools in Calgary. The course of studies was called Mathematics Fifteen. The book adopted for use in this course was Mathematics A Modern Approach.[23] Three hundred and forty-seven low achievers, at grade ten level, were enrolled in this program.

This study was designed to test the relative effectiveness of the "modern" mathematics program developed by the textbook mentioned above, compared with a "traditional" mathematics program offered to low achievers in the other high schools in Calgary. A sample from the Mathematics Fifteen group, chosen for the study, was called the 'Treatment' group.

The low achievers, in grade ten, in the other high schools in Calgary were enrolled in a course of studies called Mathematics Eleven. The authorized textbook used in this course was Canadian Business Mathematics.[24] This textbook follows the traditional approach. The

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[23]Marie S. Wilcox, and John Yarnell, op. cit.

[24]Catherine Lund, Canadian Business Mathematics, (McGraw-Hill Co. of Canada, 1962).

sample from the Mathematics Eleven group chosen for the study was called the 'Control' group.

All students in the Mathematics Fifteen group were tested, but Mathematics Eleven classes were chosen by random selection from the high schools referred to above. The subjects in the 'Treatment' group and the 'Control' group in the study were those who wrote the Alberta Departmental Grade Nine Examination in 1964; consequently, a certain number of students, who wrote the tests, could not be considered in the sample either because they previously attended school in another province or country, or because they were registered in grade eleven.

II. TESTS AND INSTRUMENTS

(1) Pretest: June 1964, Alberta Departmental Grade Nine Mathematics Examination.

(2) A modified form of the June 1964, Alberta Departmental Grade Nine Mathematics Examination.

(3) The California Achievement Test in Mathematics (Advanced), Form W.

(4) The Iowa Standardized Vocabulary Test.

(5) A special Attitude Toward Mathematics Scale.

(6) A Teacher Questionnaire.

Purpose of the Tests

(1) The pretest score data was used as a covariate and also as a predictor of the criterion measure.

(2) The modified form of June 1964, Alberta Departmental Grade Nine Examination was used as a criterion to investigate Question 1.

(3) The California Achievement Test in Mathematics was used as a criterion variable to investigate Question 2.

(4) The Iowa Standardized Vocabulary Test was used as a covariate and also as a predictor of the criterion measure.

(5) The special Attitude Toward Mathematics Scale was used as a criterion variable to investigate Question 3.

(6) The Teacher Questionnaire had a twofold purpose: to obtain information on the characteristics of the teachers in the two groups, and to rank teachers on the basis of their attitude toward the two mathematics programs.

Validity and Reliability of the Tests

(1) The validity and reliability of the June 1964, Alberta Departmental Grade Nine Mathematics Examination scores was not available from the Alberta Department of Education. However, on the basis of previous studies,[25] one may conclude that the validity and reliability of the grade nine mathematics examinations are acceptable.

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[25]D. B. Black, "The Prediction of University Freshman Success using Grade Nine Departmental Examination Scores," The Alberta Journal of Educational Research, Vol. V, No. 4, (December, 1959), p. 234.

(2) An attempt was made to determine the face and content validity of the Modified Grade Nine Mathematics Test by obtaining the judgment of the participating teachers with respect to the relevance of the questions contained in the test. A copy of the test was distributed to each teacher in the study; he was asked to assess the relevance of each question on a five point scale. Only those questions that were unanimously rated 1, 2, or 3 by the teacher group were included in the test. Reference to Table I indicates that the correlation between the Modified Grade Nine Test scores and the Standardized Achievement Test scores is .47.

The reliability of this test was measured by the 'split-half' method.[26] The coefficient[27] was found to be .73.

(3) The California Achievement tests have been developed over a period of years and through four editions. They have been administered to over 100,000 students, and the authors claim that their validity is high.[28]

The test is divided into two parts--mathematics reasoning and mathematics fundamentals. The reliability coefficient of the mathematics reasoning subtest is .91 with a standard error of measurement of .73. The mathematics fundamentals subtest has a reliability coefficient of .93 with a standard error of measurement of .68. The coefficient of

[26] George A. Ferguson, Statistical Analysis in Psychology and Education, (McGraw-Hill Book Company, Inc., 1959), p. 279.

[27] Ibid., pp. 92-93.

[28] California Achievement Test Manual, (Form W,1957), pp. 8-9.

TABLE I

CORRELATION MATRIX OF VARIABLES

(N = 522)

VARIABLE	MOD	RS	MGN	SMT	SVT	AMS
Treatment Group	1.0000	-.1948	.0143	.0572	-.0628	.2586
Grade Nine Raw Score		1.0000	.6659	.5286	.1821	.1862
Modified Grade Nine Test			1.0000	.4700	.1860	.2224
Standardized Achievement Test in Mathematics				1.0000	.2787	.3084
Standardized Vocabulary Test					1.0000	.0829
Attitude Toward Mathematics Scale						1.0000

reliability of the entire mathematics test is .93 with a standard error of measurement of .66.[29]

(4) The Iowa Standardized Vocabulary Test was tested for reliability using 11,911 students.[30] The coefficient of reliability of the test is .92 with a standard error of measurement of 2.88.[31]

(5) An attempt was made to determine the validity of the special Attitude Toward Mathematics Scale by obtaining a critical analysis of the test's specifications and content. The judges involved were some of the mathematicians and psychologists at the Universities of Alberta and Calgary.

The reliability of this test was measured by applying a scalogram analysis suggested by W. H. Goodenough.[32] The coefficient of reproducibility, which is an approximate reliability coefficient, was found to be .704.

Construction of Tests

(1) As mentioned earlier, the content of the Modified Grade Nine Test consisted of those questions from the June 1964, Alberta Departmental Grade Nine Mathematics Examination, which the participating teachers considered valid for testing.

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[29] California Achievement Test Manual, (Form W, 1957), p.8.

[30] L. Siegel, "Review of Iowa Tests of Basic Skill," Journal of Counsel Psychology, (Fall, 1957), p. 252.

[31] Ibid., p. 252.

[32] Allen L. Edwards, Techniques of Attitude Scale Construction, (New York: Appleton Century Crofts, 1957), pp. 184-188.

(2) In preparing the special Attitude Toward Mathematics Scale, the investigator attempted to elicit student responses to the following:

(a) The way he felt about mathematics the previous year.

(b) The way he feels about mathematics at present.

(c) His level of understanding of the present mathematics.

(d) His progress in mathematics this year as compared with that in grade nine.

(e) His reaction to the amount of home study the present mathematics program requires of him.

(f) The impact of the present mathematics in the way he predicts his performance in the final examination.

(g) The extent of his keenness in wanting to take a mathematics course in grade eleven.

(h) The value he attaches to mathematics.

Care was taken to ensure that all questions compared the present mathematics program to the grade nine mathematics. To avoid ambiguous responses, each question had only two response categories--more favorable and less favorable.

(3) In preparing the Teacher Attitude Toward Mathematics Scale, care was taken to devise a method of determining the attitude of the teacher toward the mathematics course he was teaching by obtaining his evaluation of the course.

The investigator attempted to elicit teacher responses to the

following:

(a) Teacher's preference for teaching mathematics.

(b) How he felt about teaching that specific mathematics course in comparison to the other mathematics courses.

(c) His reaction to teaching mathematics to low achievers.

(d) The amount of lesson preparation involved in teaching that specific course compared to the preparation required for other mathematics courses he might have taught.

(e) His assessment of the appropriateness of the content of that specific mathematics course for low achievers.

(f) His thoughts on the amount of revision desirable in the course content.

(g) His assessment of student progress.

(h) His impressions on the student attitudes toward the mathematics program.

The investigator attempted to assess the attitude toward mathematics of the teachers because it seems logical to believe that the teacher's attitude toward the course he teaches reflects upon the achievement of his students. Speaking of teacher attitude, Rosenbloom writes:

> The teacher's attitude is a more important factor than his formal preparation in his effectiveness with the new courses. If he is flexible, willing to try new things and exercises critical judgement, he is likely to do well. If he is rigid, resistant to change or uncritically for or against the new courses he is likely to do poorly. Where the teacher has used the new mathematics materials because of pressure from administrators or parents, the results

have not been as favorable.[33]

III. ADMINISTRATION OF TESTS AND COLLECTION OF DATA

In April 1965, the Modified Grade Nine Test was administered to all students in the classes chosen for the study. Three hundred and eight eligible students in the Treatment group and two hundred and seventy-seven eligible students in the Control group wrote this test.

In June 1965, the following tests were administered, again to all students in the classes chosen for the study: (1) The California Achievement Test in Mathematics (Advanced), Form W; (2) The Iowa Standardized Vocabulary Test; and (3) The special Attitude Toward Mathematics Scale.

Complete data was obtained for two hundred and eight-two subjects in the Treatment group and two hundred and forty subjects in the Control group. Incomplete data resulted in some students having failed to write any one of the tests; the data of the above four tests are reported in Table II.

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[33] Rosenbloom, op. cit., p. 361.

TABLE II

DESCRIPTIVE STATISTICS OF THE ACHIEVEMENT TESTS

AND ATTITUDE SCALE

(N = 522)

TEST	ITEMS (N)	MEAN	S.D.	RELIABILITY
MGN	65	25.3275	10.0596	.73
SMT	140	93.5076	16.5042	.93
SVT	48	35.6896	5.7830	.92
AMS	8	5.7528	1.6301	.704

IV. THE STATISTICAL TECHNIQUE

The general approach to the statistical treatment of the data was that of the Multiple Linear Regression.[34] This technique subsumes the analysis of variance and the analysis of covariance.

The linear regression analysis conducted, used in each case a specific combination of variables as predictors and a single variable as the criterion. Two of the variables in the study are categorical (class membership--MOD or TRA) and the others are continuous variables (RS, MGN, SMT, SVT, and AMS).

The essential problem in this research was to determine whether or not members of the two groups (MOD and TRA) differ significantly on a selected criterion variable, controlling statistically for initial differences that may exist on a number of other variables. In reference to a categorical variable, a subject who was in the 'Treatment' group was assigned a score of one (1), and a subject who was in the 'Control' group was assigned a score of zero (0).

The 'full regression model' consisted of a criterion variable and several predictor variables, the computer program solving for the best-weighted linear combination of these predictors (least error sum of squares). The squared-multiple-correlation (R_1^2), of the full model

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[34]Robert A. Bottenberg, and Joe H. Ward, Jr., Applied Multiple Linear Regression, (6570th Personnel Research Laboratory Aerospace Medical Division Air Force Systems Command, Lackland Air Force Base, Texas, 1963), pp. 22-48.

is a measure of the efficiency of the prediction, i.e. it indicates how much of the criterion variance can be accounted for by the best-weighted set of predictor variables.

The investigator then attempted to determine the extent to which the knowledge of group membership improved the predictive efficiency. This was achieved by using a 'restricted model', which excluded knowledge of group membership.

The difference between the squared-multiple-correlation (R_1^2) of the full model and the resulting squared-multiple-correlation of the restricted model (R_2^2) is evaluated by using the following F-ratio:

$$\frac{(R_1^2 - R_2^2) / df_1}{(1 - R_1^2) / df_2}$$

where:

R_1^2 = the squared-multiple-correlation of the full model.

R_2^2 = the squared-multiple-correlation of the restricted model.

df_1 = numerator degrees of freedom--the difference between the number of linearly independent variables, including the constant, in the full and restricted model.

df_2 = denominator degrees of freedom--the number of subjects minus the number of independent variables in the full model minus one.

In summary, using the multiple linear regression technique, to test any hypothesis, a full and a restricted model are needed. The R^2 of the full and restricted models are compared, to evaluate the unique contribution of the particular variable being investigated.

The IBM-1620 computer at the University of Calgary was used extensively in the analyses of the data.

CHAPTER IV

THE STATISTICAL ANALYSIS OF DATA COLLECTED

I. FINDINGS FROM THE STATISTICAL ANALYSES

As mentioned in the previous chapter, the general approach to the statistical treatment of the data was that of multiple linear regression as described by Bottenberg and Ward.[35] This method permits the simultaneous inclusion of categorical and continuous data in a linear system which predicts a continuous criterion. The question asked is whether the inclusion of the predictor under consideration adds significantly to the prediction system. In essence, this is the same question that the analyses of variance and covariance are required to answer.

In this chapter, each of the questions asked is stated immediately before the presentation of the results.

Question 1

Using as a criterion a Modified Grade Nine Mathematics Test, will there be a difference in achievement between the MOD and the TRA students?

The first question which the multiple linear regression analysis attempts to answer is: What proportion of the variance in the achievement scores on MGN can be accounted for by the knowledge of MOD, RS, and SVT

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[35] Robert A. Bottenberg, and Joe H. Ward, op. cit., pp. 22-48.

scores?

This question is answered by carrying out comparisons among a full model and a restricted model. The full model is based on a linear combination of all predictor variables. The restricted models drop one predictor variable at a time.[36]

The results of the analysis, including the regression weights, are reported in Table III.

Interpretation

The squared-multiple-correlation (R^2) indicates how much of the criterion variance can be accounted for by the best-weighted set of predictor variables. For the full model, $R^2 = .4699$. This indicates that 46.99 per cent of variance in MGN scores is accounted for by knowledge of MOD, RS, and SVT scores. For the first restricted model, $R^2 = .4478$. This indicates that 44.78 per cent of variance in MGN scores is accounted for by the knowledge of RS and SVT scores. For the second restricted model, $R^2 = .0353$. This indicates that 3.53 per cent of variance in MGN scores is accounted for by knowledge of MOD and SVT scores. For the third restricted model, $R^2 = .4650$. This indicates that 46.50 per cent of variance in MGN scores is accounted for by knowledge of MOD and RS.

Conclusion

Since the R^2 value for the second restricted model is

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[36] Ibid. pp. 30-31.

TABLE III

SUMMARY OF ANALYSIS OF MULTIPLE LINEAR REGRESSION TESTS (1)

QUESTION	MODEL	CRITERION	REGRESSION WEIGHTS			CONSTANT	R^2
			MOD	RS	SVT		
1	Full	MGN	3.064	.339	.123	1.201	.4699
1	Restricted (1)	MGN		.325	.117	3.866	.4478
1	Restricted (2)	MGN	.528		.327	13.388	.0353
1	Restricted (3)	MGN	3.023	.345		5.298	.4650
2	Full	SMT	5.724	.429	.553	47.821	.3423
2	Restricted (1)	SMT		.403	.538	52.828	.3139
2	Restricted (2)	SMT	2.486		.809	63.296	.0833
2	Restricted (3)	SMT	5.516	.457		66.166	.3061
3	Full	AMS	1.009	.019	.017	3.607	.1285
3	Restricted (1)	AMS		.014	.014	4.484	.0372
3	Restricted (2)	AMS	.866		.028	4.283	.0768
3	Restricted (3)	AMS	1.003	.019		4.156	.1251

significantly less than the R^2 value for the full model, it is concluded that RS is a good predictor of mathematics achievement measured by student performance in the Modified Grade Nine Test.

The second question which the multiple linear regression analyses attempts to answer is: How does one evaluate the unique contribution of the MGN score?

This question is answered by obtaining the F-ratio by using the formula:

$$\frac{(R_1^2 - R_2^2) \ / \ df_1}{(1 - R_1^2) \ / \ df_2}$$

The results of the analyses are reported in Table IV.

Interpretation

With reference to Table IV it can be seen that, using as a criterion the MGN scores:

(1) Subjects in the MOD group scored signfficantly better than subjects in the TRA group.

(2) The RS is a significant predictor of MGN achievement.

Conclusion

In terms of the full model, the analysis shows that achievement scores of subjects in the MOD group have a weight three times that of subjects in TRA group, and that subjects in the MOD group achieved significantly better on the Modified Grade Nine Test than subjects in the TRA group, controlling for initial differences on grade nine raw

TABLE IV

SUMMARY OF ANALYSIS OF MULTIPLE LINEAR REGRESSION TESTS (2)

QUESTION	CRITERION	PREDICTOR	F	df	R_1^2	R_2^2	Exact Prob.*
1	MGN	MOD	21.649	1/518	.4699	.4478	0.00000
1	MGN	RS	424.736	1/518	.4699	.0353	0.00000
1	MGN	SVT	4.801	1/518	.4699	.4650	0.02885
2	SMT	MOD	22.413	1/518	.3423	.3139	0.00000
2	SMT	RS	204.024	1/518	.3423	.0833	0.00000
2	SMT	SVT	28.504	1/518	.3423	.3061	0.00000
3	AMS	MOD	54.294	1/518	.1285	.0372	0.00000
3	AMS	RS	30.761	1/518	.1285	.0768	0.00000
3	AMS	SVT	2.024	1/518	.1285	.1251	.15530

*Frequencies for significance at .05 level (df = 1/518)

score, and standardized vocabulary test.

Question 2

Using as a criterion a Standardized Mathematics Test, will there be a difference in performance between the MOD group and the TRA group?

The first question to be answered is: Does the knowledge of MOD, RS, and SVT scores help us to predict the achievement on SMT?

The results of the analyses, using one full model and three restricted models, are reported in Table III,page 29.

Interpretation

For the full model, R^2 = .3423. This indicates that 34.23 per cent of variance in SMT scores is accounted for by knowledge of MOD, RS, and SVT scores. For the first restricted model, R^2 = .3139. This indicates that 31.39 per cent of variance in SMT scores is accounted for by knowledge of RS, and SVT scores. For the second restricted model, R^2 = .0833. This indicates that 8.33 per cent of variance in SMT scores is accounted for by knowledge of MOD and SVT scores. For the third restricted model, R^2 = .3061. This indicates that 30.61 per cent of variance in SMT scores is accounted for by knowledge of MOD and RS.

Conclusion

Since the R^2 value for the second restricted model is significantly less than the R^2 value for the full model, it is concluded that RS is a good predictor of mathematics achievement measured by student performance in the standardized mathematics test.

The second question to be answered is: How does one evaluate the unique contribution of the SMT score?

The results of the analyses are reported in Table IV, page 31.

Interpretation

With reference to Table IV, page 31, it can be seen that, using as a criterion the SMT scores:

(1) Subjects in the MOD group scored significantly better than subjects in the TRA group.

(2) The RS is a significant predictor of MGN achievement.

Conclusion

In terms of the full model, the analysis shows that, achievement scores of subjects in the MOD group have a weight five times that of subjects in the TRA group, and that subjects in the MOD group achieved significantly better on the California Achievement Test in Mathematics than subjects in the TRA group, controlling for initial differences on grade nine raw score, and standardized vocabulary test.

Question 3

Using as a criterion an Attitude Toward Mathematics Scale, will there be a difference in attitude between the MOD group and the TRA group?

The first question to be answered is: Does the knowledge of MOD, RS, and SVT help us to predict student attitude toward mathematics?

The results of the analyses, using one full model and three

restricted models, are reported in Table III, page 29.

Interpretations

For the full model, R^2 = .1285. This indicates that 12.85 per cent of the variance in AMS scores is accounted for by knowledge of MOD, RS, and SVT scores. For the first restricted model, R^2 = .0372. This indicates that 3.72 per cent of the variance in AMS scores is accounted for by knowledge of RS, and SVT scores. For the second restricted model, R^2 = .0768. This indicates that 7.68 per cent of the variance in AMS scores is accounted for by knowledge of MOD and SVT scores. For the third restricted model, R^2 = .1251. This indicates that 12.51 per cent of the variance in AMS scores is accounted for by knowledge of MOD and RS.

The second question to be answered is: How does one evaluate the unique contribution of the AMS score?

The results of the analyses are reported in Table IV, page 31.

With reference to Table IV it can be seen that, using as a criterion the AMS scores:

(1) Subjects in the MOD group indicated a significantly different attitude toward mathematics than the subjects in the TRA group.

(2) The RS is a significant predictor of the AMS score.

(3) The SVT score is not a significant predictor of the AMS score.

TABLE V

SUMMARY OF ANALYSIS OF GROUP MEANS AND

STANDARD DEVIATIONS

	GROUP	N	RS	MGN	SMT	SVT	AMS
Mean	MOD	282	49.54	25.98	93.59	35.35	6.13
S.D.	MOD	282	18.35	9.17	15.65	5.56	1.33
Mean	TRA	240	56.99	24.95	91.66	35.52	5.27
S.D.	TRA	240	16.03	8.17	13.49	5.48	1.56

Conclusion

In terms of the full model, the analysis shows that, scores of the MOD group have a weight slightly higher than one compared to the weight of scores of the TRA group, and that subjects in the MOD group achieved significantly better on the Attitude Toward Mathematics Scale than subjects in the TRA group, controlling for initial differences on grade nine raw score, and standardized vocabulary test.

II. SUMMARY OF TEACHER QUESTIONNAIRE FINDINGS

In June 1965, the teachers involved in the experiment were asked to complete a questionnaire. This questionnaire was comprised of three parts: Part A was designed to obtain information on the characteristics of the teachers in the two groups; Part B was designed to obtain teacher evaluations of the MOD and TRA programs; and Part C was left blank for teachers' comments.

A summary of the information gathered from the responses to Part A of the questionnaire is reported in Table VI. The data collected did not lend itself to rigorous statistical analysis because the teacher sample was not large enough. However, a subjective analysis of the information collected provides interesting contrasts. For example, the

TABLE VI

SUMMARY OF TEACHER CHARACTERISTICS

(N: MOD = 7; TRA = 10)

CHARACTERISTIC	MOD	TRA
Number of teachers having teacher preparation of three years or more	6	10
Number of teachers having teacher preparation of two years or less	1	0
Average number of university mathematics courses each teacher had taken	2	2
Average number of years of teaching experience per teacher	4	9
Average number of years each teacher had previously taught MOD or TRA program	1	2
Average teaching load in minutes per week spent in actual classroom instruction	1323	1107
Average time spent in teaching each class involved in the experiment in minutes per week	207	211
Average amount of time each teacher was released from classroom instruction or supervision in minutes per week	210	241
Average number of students in each class involved in the experiment	33	30

teachers in the TRA group, on the average, were academically better qualified, had more years of teaching experience, had a smaller teaching load, and had more time released from classroom instruction or supervision than the teachers in the MOD group. Despite the obvious advantages in favor of the TRA group, the students in the MOD group performed significantly better on the two achievement tests.

The investigator used the teacher evaluations of the two mathematics programs to elicit teacher attitude toward the course he taught. The teachers were then ranked according to the teacher's attitude toward the mathematics course he taught. The class means and standard deviations along with ranks of five highest ranked teachers and five lowest ranked teachers are reported in Table VII, page 39, and Table VIII, page 40.

Examination of teacher comments in Part C of the questionnaire led the investigator to the following observations:

(1) Only one teacher in the MOD group had a University course in modern mathematics.

(2) Two teachers in the MOD group and seven teachers in the TRA group stated that mathematics was not their preferred subject.

(3) Four teachers in the TRA group complained that teaching low achievers was not satisfying. Teachers in the MOD group made no comment on teaching low achievers.

(4) Eight teachers in the TRA group and two teachers in the MOD group remarked that many of their students made little progress because they seemed "frustrated" with school.

TABLE VII

MEANS AND STANDARD DEVIATIONS OF

CLASSES TAUGHT BY FIVE HIGHEST RANKED TEACHERS

	CLASS	N	MGN	SMT	SVT	AMS	Teacher*
Mean	MOD	29	25.03	95.55	33.66	6.52	1
S.D.	MOD	29	8.98	12.37	4.82	0.97	1
Mean	MOD	27	22.89	96.11	34.89	5.59	1
S.D.	MOD	27	10.02	15.35	7.55	1.99	1
Mean	MOD	36	22.42	97.06	36.86	6.42	2
S.D.	MOD	36	10.77	19.95	5.02	1.32	2
Mean	MOD	30	27.83	101.53	34.77	6.17	2
S.D.	MOD	30	13.10	16.15	6.11	1.29	2
Mean	TRA	28	31.92	98.75	36.25	4.46	3
S.D.	TRA	28	8.25	15.09	4.83	1.99	3
Mean	TRA	29	28.23	92.91	37.14	4.09	3
S.D.	TRA	29	7.84	12.51	5.11	1.81	3
Mean	MOD	31	21.05	97.80	35.05	5.70	4
S.D.	MOD	31	5.97	13.08	5.71	1.76	4
Mean	MOD	29	29.31	97.50	36.04	6.54	4
S.D.	MOD	29	9.13	21.59	5.99	1.18	4
Mean	TRA	28	30.14	97.76	38.00	5.86	5
S.D.	TRA	28	8.64	11.97	6.16	1.46	5

*Teachers are ranked relative to their attitude toward the mathematics course they taught.

TABLE VIII

MEANS AND STANDARD DEVIATIONS OF

CLASSES TAUGHT BY FIVE LOWEST RANKED TEACHERS

	CLASS	N	MGN	SMT	SVT	AMS	Teacher*
Mean	MOD	23	24.81	85.81	35.88	5.81	13
S.D.	MOD	23	7.99	11.68	5.05	1.13	13
Mean	TRA	29	17.11	97.07	36.93	5.18	14
S.D.	TRA	29	5.94	13.52	6.08	1.52	14
Mean	TRA	15	22.56	80.22	31.22	5.22	15
S.D.	TRA	15	5.19	20.55	6.29	1.75	15
Mean	TRA	27	20.18	90.73	30.09	4.82	16
S.D.	TRA	27	6.31	10.01	5.37	2.25	16
Mean	TRA	36	15.65	74.88	37.00	4.73	17
S.D.	TRA	36	7.95	14.53	5.23	1.53	17

*Teachers are ranked relative to their attitude toward the mathematics course they taught.

CHAPTER V

SUMMARY, LIMITATIONS AND IMPLICATIONS OF THE STUDY

I. PURPOSE OF THE STUDY

The purpose of this study was to gather experimental evidence to determine the relative effectiveness of "modern" and "traditional" mathematics programs for low achievers. The subjects in this study were low achievers at grade ten level studying in Calgary. The students in the Treatment group (MOD) studied a "modern" mathematics program while those in the Control group (TRA) were taught "traditional" mathematics. All students involved in the experiment had studied traditional mathematics in grade nine.

The experiment was so designed that data could be gathered and analyzed to shed light on the following questions:

(1) How do low achievers studying the new materials compare with low achievers studying the traditional materials with respect to achievement as measured by a grade nine conventional mathematics test and a standardized achievement test in mathematics?

(2) How do low achievers studying the new materials compare with low achievers studying the traditional materials with respect to attitude toward mathematics, as measured by a special attitude toward mathematics scale?

II. THE NATURE OF THE EXPERIMENT

This study was conducted during the 1964-65 school year. Twenty-three classes of low achievers in mathematics and seventeen teachers were involved in the experiment. The Treatment group accounted for twelve classes and seven teachers, and the Control group had eleven classes and ten teachers.

In April, 1965, the Modified Grade Nine Mathematics Test (MGN) was administered to all students involved in the experiment. In June, 1965, the California Achievement Test in Mathematics (SMT), and the Iowa Vocabulary Test (SVT) were administered along with the Attitude Toward Mathematics Scale (AMS) that had been developed by the investigator.

The general approach to the statistical treatment of the data collected was that of the multiple linear regression technique.[37]

In addition to the statistical findings, the responses to the teacher questionnaire supplied information about teacher characteristics and teacher attitude toward mathematics.[38]

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[37] Bottenberg and Ward, op. cit., pp. 22-48.

[38] Chapter IV, pp. 36-40.

III. FINDINGS AND INTERPRETATION OF DATA

The multiple linear regression technique, used to analyse the data, involved the following procedure: A full model was constructed for each criterion variable using RS and SVT scores (covariates) and MOD as predictor variables, to predict the total variance in the criterion scores. Restricted models were then constructed, by dropping one predictor at a time, to determine the predictive efficiency of the combination of variables. The difference of R^2 of the full model and the R^2 of each restricted model was evaluated by obtaining the F-ratio in each case.

The following questions were asked:

Question 1

> Using as a criterion the Modified Grade Nine Mathematics Test, will there be a difference in achievement between the MOD and the TRA students?[39]

The analysis shows that students in the MOD group achieved scores which were, in a statistical sense, significantly better than those of students in the TRA group, controlling statistically for initial differences on grade nine raw score and vocabulary test. The analysis also shows that grade nine raw score is a good predictor of achievement in mathematics.

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[39]Chapter IV, p. 27.

Question 2

> Using as a criterion, the Standardized Mathematics Test, will there be a difference in performance between the MOD group and the TRA group?[40]

The analysis shows that students in the MOD group achieved scores which were, in a statistical sense, significantly better than those of students in the TRA group, controlling statistically for initial differences on grade nine raw score and vocabulary test. The analysis also shows that grade nine raw score is a good predictor of performance in mathematics.

Question 3

> Using as a criterion, the Attitude Toward Mathematics Scale, will there be a difference in attitude between the MOD group and the TRA group?[41]

The analysis shows that students in the MOD group had scores which were, in a statistical sense, significantly higher than those of students in the TRA group, controlling statistically for initial differences on grade nine raw score, and vocabulary test. On the basis of this test there is some indication that students in the MOD group now regard mathematics in a more favorable light.

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[40] Chapter IV, p. 32.

[41] Chapter IV, p. 33

Examination of teacher responses to the questionnaire, led the investigator to make the following observations:

(1) The teachers in the TRA group had, on the average, more years of teacher preparation than the teachers in the MOD group.

(2) Considered as groups, the teachers in the TRA group had slightly over twice as many years of teaching experience as those possessed by teachers in the MOD group.

(3) On the average, the teachers involved in the experiment were not markedly different from one another with respect to university mathematics courses taken, and the number of years each teacher had previously taught the MOD or the TRA program.

(4) The groups did not differ markedly with respect to classroom time spent in mathematics instruction.

(5) The groups also did not differ markedly with respect to the number of students in each class.

(6) On the average, teachers in the TRA group had a smaller teaching load and had more time released from classroom instruction or supervision, than teachers in the MOD group.

(7) Two teachers in the MOD group indicated preference for teaching traditional mathematics.

(8) All teachers in the MOD group felt that the "modern" mathematics program demanded more preparation time.

(9) Teachers in the MOD group were divided on the issue of the appropriateness of the "modern" mathematics program for low achievers.

(10) Only one teacher in the MOD group had taken a University course in modern mathematics.

(11) Two teachers in the MOD group and seven teachers in the TRA group stated that mathematics was not their preferred subject.

(12) Four teachers in the TRA group complained that teaching low achievers was not satisfying.

(13) Eight teachers in the TRA group and two teachers in the MOD group remarked that many of their students made little progress because they appeared "frustrated" with school.

IV. LIMITATIONS

Every effort was made to provide for a study that was both experimentally sound and statistically accurate. However, the investigator freely admits that there are some obvious shortcomings:

(1) The sample in the study cannot be considered as representative of the school population because of the limitations imposed on the selection of the Treatment group--this group was not a random sampling from the student population studying "modern" mathematics.

(2) A second limitation may well lie in the construction of the Attitude Toward Mathematics Scale. Since the instrument is used as a criterion variable, the investigator strongly urges the reader to carefully scrutinize the items in the test (included in Appendix C), to see

if they do appear to elicit student attitude toward mathematics.

(3) The reliability of the measuring instruments, particularly the Attitude Toward Mathematics Scale, should ideally have been better. This lack of precision, of course, increases error variance with resulting loss of power in tests of significance.

(4) Another limitation may lie in the construct validity of the Modified Grade Nine Mathematics Test, in view of the fact that items to be tested were made known to the teachers--some teachers might have drilled their students in the same questions.

(5) As another instance of limitation, one might question why the Attitude Toward Mathematics Scale was not administered at the beginning of the experiment. If the initial scores on the attitude scale were known, these would enable the investigator to detect significant differences in attitude at the beginning of the experiment. Knowledge of initial scores on the attitude scale could also be used to determine if attitude toward mathematics is a predictor of achievement in mathematics.

(6) Still another limitation lies in the small sample of teachers. If the teacher group was large enough, the data gathered on the Teacher Questionnaire would be amenable to statistical analysis to determine the relationship between teacher attitude toward mathematics and student performance in mathematics.

V. IMPLICATIONS FOR FURTHER RESEARCH

This study has some apparent implications for further research and experimentation with new and different teaching techniques for low achievers in mathematics. Very little is known about methods which are most effective with low achievers. Persons engaged in providing courses for the slow learner would like to have answers to questions such as: How do they learn best? What is an appropriate curriculum for them? What mathematical competences do they need? What are the characteristics of teachers who work most effectively with them? The answers to these and similar questions may be found in the results of classroom experimentation and research.

Very few, if any, follow-up studies have been made of slow learners. With this in mind, the investigator proposes the following recommendations:

(1) Similar studies should be conducted to investigate the effectiveness of modern educational technology. Multisensory aids often add a dimension to provide insight for the low achiever that he does not get from chalkboard or the printed page. For example, the overhead projector furnishes avenues to capture the attention and curiosity of the learner.

(2) Similar studies should also be conducted to investigate the effectiveness of programmed instructional materials.

(3) The experiment reported herein should be replicated using different experimental materials and at other grade levels. Appropriate

measuring instruments should be designed and constructed to determine the degree to which the new materials accomplish their purposes with slow learners.

(4) While a study such as the one described in this report serves the purpose of providing overall comparisons between "modern" and "traditional" mathematics programs, there is a need for studies that concentrate on specific aspects of mathematics. For example, a detailed study of a low achiever's facility with basic arithmetic skills or facility in solving mathematical problems, would do much to improve the techniques in the teaching of slow learners.

(5) A crucial consideration in an effective program for low achievers seems one of locating teachers willing and competent to teach slow learners. Studies should be made to determine what teacher characteristics are most efficacious in dealing with slow learners.

VI. IMPLICATIONS FOR EDUCATIONAL PRACTICE

Secondary schools might take account of the findings which have been presented in this study and analyse their curriculum in relation to the needs of their low achievers.

It is hoped that more studies, relating to mathematics curriculum and instruction for low achievers, will be conducted at different grade levels, and that findings from such studies will provide guidelines for the preparation of appropriate teaching materials and for more effective instruction of the so-called low achiever.

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APPENDIX A

LETTER TO PARTICIPATING TEACHERS REQUESTING EVALUATION OF THE 1964 ALBERTA GRADE IX MATHEMATICS EXAMINATION

Dear Teacher,

I shall be grateful for the favor of sparing some of your time to assess the validity of the attached 1964 Alberta Grade IX Mathematics examination paper, as a suitable test to be administered in April, 1965, to students presently taking Mathematics 11/15.

Please indicate your opinion of each question with a rating on a five-point scale. A separate sheet is provided for this purpose.

Please use the rating scale as indicated below:

1. This material has been covered completely.
2. Most of this material has been covered.
3. This material has been covered partly.
4. Very little of this material has been covered.
5. This material was not covered.

I shall deem it a favour if you will kindly consider this as very urgent. I trust you will be able to return your evaluation of the questions at your earliest convenience.

Thank you.

Sincerely yours,

Ivo Rodrigues

EVALUATION SHEET

Question Number	1	2	3	4	5
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					

Question Number	1	2	3	4	5
18					
19					
20					
21					
22					
23					
24					
25					
26					
27					
28					
29					
30					
31					
32					
33					
34					

Please place a check (✓) in the proper square against the Number of the question, to indicate your rating of the question.

APPENDIX B

THE MODIFIED GRADE NINE MATHEMATICS TEST

INSTRUCTIONS

* Do not spend too much time on any one question. Read the questions carefully.
* For the first 9 questions in Section "A", five solutions are given for each question, only ONE of which is correct. Use the reverse of the sheet to work out the answers and CIRCLE the NUMBER of the correct answer with a pencil or pen.
* For questions 10 to 12, work out the answers on the reverse of the sheet and write your answers on the blank spaces provided.
* For questions 13 to 16 in Section "B", all work should be shown in the space provided for the solution.
* You will be allowed ONE HOUR to complete this paper.

SECTION A

With a pencil or pen, circle the number of the correct answer in each of the following 12 questions. If you are not sure of the right answer, make the best choice you can. An example is:

The product of 3 and 12 is — 1. 15 2. 123 3. 36 4. 9 5. 4

Since the correct answer is 36, the number 3 has been circled.

1. 0.25% of 432 is

(1) 1. 108 2. 1.08 3. 0.108 4. 10,800 5. 1080

2. Which of these is an equation?

(2) 1. $3x + 4y$ 2. $\frac{3x}{2} = 8$ 3. $2x + 3y - 5$ 4. $x + x + 2x$ 5. $4(x)(y)$

3. $\frac{a}{b} + \frac{b}{c} =$

(3) 1. $\frac{ac + b^2}{bc}$ 2. $\frac{a}{c}$ 3. $\frac{ab}{bc}$ 4. $\frac{a + b}{b + c}$ 5. $\frac{a + b}{bc}$

4. In a triangle one angle is 54°. Which of the following could form the other two angles?

(4) 1. 80°, 110° 2. 36°, 54° 3. 36°, 144° 4. 23°, 31° 5. 68°, 58°

5. The radius of the circular top of a tower is 4 2/3 yards. ($\pi = \frac{22}{7}$) The area of the circular top is

(5) 1. 44 sq.ft. 2. $14\frac{2}{3}$ sq.ft. 3. $68\frac{4}{9}$ sq.ft. 4. 616 sq. ft. 5. $229\frac{1}{3}$ sq.ft.

6. The L.C.M. of 5,8, and 12, is

(6) 1. 12 2. 40 3. 120 4. 60 5. 80

7. $\dfrac{\frac{4}{5} - \frac{3}{4}}{\frac{5}{6} + \frac{3}{8}} =$

(7) 1. $\frac{6}{145}$ 2. $\frac{6}{55}$ 3. $1\frac{3}{4}$ 4. $\frac{19}{20}$ 5. $\frac{3}{16}$

8. Divide the difference of 36 and 27 into their product. (8) 1. $22\frac{10}{11}$ 2. 18 3. 36 4. 108 5. $29\frac{5}{11}$

9. The face of an insurance policy on a house is $ 6500 and the rate is 54¢ per $100 for 3 years. What is the 3-year premium? (9) 1. $ 105.30 2. $ 87.75 3. $ 35.10 4. $ 11.70 5. $ 351.00

10. Work out the answers to these questions on the back of this sheet and then place your answers on the lines provided.

(a) Simplify: $8x - 9y + 3y - 6x + 7 + 3x - y - 9$ . . . . . .

(b) The perimeter of a triangle is 14n inches and two of its sides are 3n inches and 5n inches respectively. Find the third side. . . . . .

(c) Simplify: $5 + 3 \times 8 - 6 \div 2 =$ . . . . . .

(d) How many square yards in 108 sq. ft.? . . . . . .

(e) $(1.03)^2 =$ . . . . . .

(f) $5\frac{1}{3} + 6\frac{2}{5} - 4\frac{1}{2} =$ . . . . . .

(g) Simplify: $6a(3a + 2b + 4) =$ . . . . . .

(h) The H.C.F. of 12 and 16 is . . . . . .

11. Place your answer in their simplest form on the lines provided.

(a) $(-9m) + (+7m) =$ . . . . . .

(b) $(-11k) - (+3k) =$ . . . . . .

(c) $\frac{(4a)(-3b)(-2x)}{-6ax} =$ . . . . . .

(d) $5x^2 - 2x + 5x - 11x^2 =$ . . . . . .

(e) $(x + y)(x - y) =$ . . . . . .

(f) 15 is 45% of what number? . . . . . .

(g) 36 is what per cent of 180? . . . . . .

(h) On a commission of 2½% how much does an agent receive for selling 6 dozen cases of dried fruit at $ 22.80 a case? . . . . . .

12. Place your answer on the line provided.

(a) Solve the formula: $N = a + bc^2$ for c . . . . . .

(b) $144a^4b^2$ = ...

(c) $(2\frac{1}{4}x)^2$ = ...

(d) 43 and 26 are approximate numbers. Find their product and round it off to the required number of significant digits. . . . . .

(e) The coefficient of x in $4x^2y$ is . . . . . .

(f) What is the interest on $800x for 20 months at 5y%? . . . . . .

SECTION B

13. Solve the following equations and check your answers by substitution. All your work must be shown in the space provided.

(a) $11m + 3 = 29 - 2m$ Check Here

(b) $0.3x + 5 = 0.5x + 9$ Check Here

(c) $\frac{3t}{5} + 2 = \frac{2t}{3} - 2$ Check Here

14. Tom purchased a transistor radio which was advertised at 20% off the regular price. He paid cash and was allowed another discount of 5%. If the original price was $ 72.50, how much did he pay for the transistor?

15. In order to pay the college fees for his son Jim, Mr. Smith borrowed $ 250 on a note, agreeing to pay interest at 4% per annum. After 9 months he paid back the loan in full. What amount did he pay?

16. The price of a television set is $ 350 cash or $ 50 down and $ 315 in 10 months. If a purchaser makes his payments on the second plan what rate of interest does he pay?

APPENDIX C

THE ATTITUDE TOWARD MATHEMATICS SCALE

INSTRUCTIONS

* Here is a list of questions intended to help you recognize the cause or causes which may be affecting your present performance in mathematics.
* This is not an intelligence test, nor are there any right or wrong answers.
* It is hoped that these questions will assist you in evaluating your past performance in mathematics. It may also serve as a guide for the decision you may be required to make regarding the choice of a course in mathematics next year.
* READ each question CAREFULLY.
* After each question you will find two statements. Decide which of the two statements best reflects your opinion. In recording your response, DRAW a CIRCLE around the NUMBER of the statement you agree with.
* Example:

In which part of Alberta do you live?

1. In Edmonton
(2) In Calgary

(A) When you were in grade nine, how did you feel about mathematics as compared with the other courses on your school program?

1. I liked mathematics.
2. I disliked mathematics.

(B) How has what you learned in the present mathematics course compared with what you learned in the grade nine mathematics course?

1. I have learned more mathematics this year.
2. I think I learned more mathematics in grade nine.

(C) How does the understanding of mathematics you have gained this year compare with that of the mathematics courses you have previously taken?

1. I have understood the things we have taken in the course this year better than in previous mathematics courses.
2. I have more difficulty understanding the things that are taught in the mathematics course this year.

(D) Try to recall how you felt last year at this time, as regards the way you expected to do in the June examination. How do you think you will do this year in the June examination?

1. I think my performance in the June examination this year will be better than that of last June.
2. I am afraid I will do worse this year.

(E) How have you felt about the amount of study work that was required in the present mathematics course as compared with the work that was required in the grade nine mathematics course?

1. I need to devote less time to mathematics this year.
2. I have to work longer at mathematics this year.

(F) If you had complete freedom in making out your grade eleven program, how do you feel about taking another mathematics course?

1. I will choose to take another mathematics course.
2. I will <u>not</u> choose another mathematics course.

(G) What are your impressions of your present mathematics course as compared with the grade nine mathematics course?

1. I prefer the kind of mathematics I am studying this year.
2. I think I liked the grade nine mathematics better than the kind of mathematics I am studying this year.

(H) What are your views on the usefulness of learning mathematics at high school?

1. I feel that knowledge of high school mathematics is essential for an average person today.
2. I don't believe that one needs high school mathematics to be a success in life.

APPENDIX D

THE TEACHER QUESTIONNAIRE PREPARED BY THE INVESTIGATOR

PART A

The purpose of these questions is to assist the investigator to describe teachers teaching the experimental group (Math. 15) and those teaching the control group (Math. 11). You are assured that the information supplied in this questionnaire will not be used to rate individual teachers nor will the names of participating teachers appear in the description of the study. The information collected will be referred to as, "Teacher Group Characteristics."

The investigator solicits your cooperation in supplying the requested information.

Teacher Preparation

Directions: Please indicate your 'teacher preparation' by drawing a circle around the number of the item that applies to you.

1. One year of teacher training.
2. Two years of teacher training.
3. Three years of teacher training.
4. One degree (specify) . . . . . .
5. Two degrees (specify) . . . . . .
6. List the University Mathematics courses you have taken:
   . . . . . .
   . . . . . .
7. List other mathematics courses you have taken, such as 'inservice.'
   . . . . . .
   . . . . . .

Teaching Experience

1. Years of teaching experience . . . . . .
2. Number of years taught Mathematics 11/15 . . . .

Teaching Load

1. List the subjects you are now teaching:
   . . . .
   . . . .
   . . . .
2. Teaching load in minutes per week spent in actual classroom instruction . . . .
3. Time released from classroom instruction or supervision in minutes per week . . . .
4. Time spent in teaching each Mathematics 11/15 class in minutes per week . . . .
5. Average size of Mathematics 11/15 class . . . .

PART B*

The following is a list of questions intended to elicit your evaluation of the Mathematics 15 program. After each question you will find two statements. Decide which of the two statements best reflects your opinion. In recording your response, draw a circle around the number of the statement you agree with. In all cases, by "traditional" mathematics courses we are thinking of Math. 11 or Math. 12.

(a) Is mathematics your preferred subject to teach?

1. Yes 2. No

(b) How do you feel about teaching 'modern' mathematics as compared to 'traditional' mathematics?

1. I prefer teaching 'modern' mathematics to 'traditional' mathematics.
2. I would rather teach 'traditional' mathematics.

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*This part of the Questionnaire was answered by teachers in the Treatment group.

(c) How have you felt regarding teaching mathematics to this ability group?

1. I have liked the challenge of teaching low achievers.
2. These students are a problem to teach.

(d) What have you found regarding the preparation involved in the teaching of 'modern' mathematics as compared to teaching 'traditional' mathematics?

1. 'Modern' mathematics calls for more preparation time.
2. Less preparation time is involved in teaching 'modern' mathematics.

(e) What is your belief concerning the appropriateness of the content of the Mathematics 15 course for the students enrolled in the program?

1. Low achievers can grasp the concepts of 'modern' mathematics satisfactorily.
2. These students find the concepts of 'modern' mathematics very difficult to grasp.

(f) Do you feel that the Mathematics 15 course content needs to be modified or revised?

1. It should be extensively modified.
2. The course needs little or no revision.

(g) How do you feel about the progress of the students in these classes in comparison with the progress they would likely have made in 'traditional' mathematics classes, such as Mathematics 11 or Mathematics 12?

1. They appear to have gained more from the 'modern' approach to mathematics.
2. They would probably have done better in Mathematics 11 or Mathematics 12.

(h) What is the most prevalent student attitude toward 'modern' mathematics?

1. They like 'modern' mathematics.
2. They are not enthused with 'modern' mathematics.

PART B**

The following is a list of questions intended to elicit your evaluation of the Mathematics 11 program. After each question you will find two statements. Decide which of the two statements best reflects your opinion. In recording your response, draw a circle around the number of the statement you agree with.

(a) Is mathematics your preferred subject to teach?

1. Yes 2. No

(b) How do you feel about teaching Mathematics 11 as compared to other non-matric mathematics courses, such as Mathematics 12?

1. I prefer teaching Mathematics 11.
2. I would rather teach Mathematics 12.

(c) How have you felt regarding teaching mathematics to this ability group?

1. I have liked the challenge of teaching low achievers.
2. These students are a problem to teach.

(d) What have you found regarding the preparation involved in the teaching of Mathematics 11 as compared with that in other mathematics courses you have taught?

1. Mathematics 11 calls for more preparation time.
2. Less preparation time is involved in teaching Mathematics 11.

(e) What is your belief concerning the appropriateness of the content of the Mathematics 11 course for the students enrolled in the program?

1. The content of the Mathematics 11 course is adequate for the needs of the low achiever.
2. Mathematics 11 course content does not appeal to the low achiever.

**This part of the Questionnaire was answered by the teachers in the Control group.

(f) Do you feel that the Mathematics 11 course content needs to be modified or revised?

1. It should be extensively modified.
2. The course needs little or no revision.

(g) How do you feel about the progress of the students in these classes in comparison with the progress they would likely have made in other non-matric mathematics courses, such as Mathematics 12?

1. They appear to have gained more from the Mathematics 11 program.
2. They would probably have done better in Mathematics 12.

(h) What is the most prevalent student attitude toward Mathematics 11?

1. They like Mathematics 11.
2. They are not enthused with Mathematics 11.