Understanding Collective Conversations in a Mathematics Professional Learning Network

by

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Abstract

Online professional learning communities have become prominent in teachers' professional development in recent years (Beach & Willows, 2014; Borba & Llinares, 2012; Dash, de Kramer, O'Dwyer, Masters, & Russell, 2012; Trust, 2012). As a new form of them (Trust, 2016), professional learning networks (PLNs) have the potential to make teachers' professional learning more "participatory, grassroots and supportive" (Carpenter & Krutka, 2015, p. 708) and make it possibile for teachers to access important resources that they could not afford or even access in the local communities (Dede, Breit, Ketelhut, McCloskey, & Whitehouse, 2005). It is not surprising, then, that a growing number of mathematics teachers have participated in PLNs to extend their professional learning. Yet, what their conversation structures look like in PLNs and what could emerge from their conversations in relation to mathematics-for-teaching remains unknown. This study addresses this gap by investigating the collective conversations in a PLN to understand its affordances.

This research used interpretive inquiry as the methodology and complexity thinking as the theoretical framework. One PLN was targeted to collect the archived data — blog posts and comments — from which four blog posts and their comments were selected as illustrative examples. Several data analysis techniques and conceptual frameworks including recursive dynamics, the features of fractal images, thematic analysis, mathematics-for-teaching, and necessary conditions for complex systems were adopted in this study.

The results presented the diverse conversation structures through conversation weaving and conversation expanding as well as the multiple types of knowing emergent from the conversations including: *mathematics-for-teaching*, *beliefs about teaching*, *social relationships*, *blog resources*, and *recounting experiences*. The knowing of *mathematics-for-teaching* was enacted in the moments of mathematics teachers' participation in the conversations. The other four types of knowing (i.e., *beliefs about teaching*, *social relationships*, *blog resources*, and *recounting experiences*) were implicated with the emergence of *mathematics-for-teaching*, the teachers' participation in the PLN, and the evolvement of the PLN itself. However, they have not yet been explored in the predominant research on teachers' disciplinary knowledge of mathematics.

The study helps me to better understand mathematics teachers' professional learning through their participation in the professional learning networks. It also contributes to the rapidly growing literature on teachers' professional learning, particularly in online learning communities (Dash et al., 2012). Additionally, it offers a valuable reference for reviewing online and even conventional teacher professional development. Looking forward, the study will inform further exploration of the nature of the relatively new form of teacher professional learning when we come to realize the affordances of our digitally connected world and the intricacies of teachers' professional growth as indicated by Brooks and Gibson (2012).

Preface

This research is an original study by Xiong Wang. Parts of the abstract have been published in X. Wang, 2018, “Toward an understanding of mathematics teachers’ participation in professional learning networks,” In J. Holm, S. Mathieu-Soucy, & S. Oesterle (eds.), Proceedings of the 2017 Annual Meeting of the Canadian Mathematics Education Study Group (pp. 291-292), CMESG. Parts of chapters 1, 2, 4 and 5 have been published in X. Wang, 2018, “Mathematics for teaching in a professional learning network: Complexity inquiry,” In H. Braund, B. Lester, S. MacGregor, and J. McConnel (Eds.), The Rosa Bruno-Jofré Symposium Proceedings (Vol 12) (pp.15-31). ON: Queen’s University.

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1. My Way to PLNs

In a math education class on May 26, 2015, my professor asked the participants to create a word problem using the expression "540÷40." It seemed very basic. I started to calculate the expression of "540÷40" and I quickly obtained the answer: 13.5. With the correct answer, I attempted to figure out what kind of unit, such as money, people, food, or days, could reasonably correspond with 13.5. It was not easy but I stayed focused and kept others from perceiving my fretfulness. Eventually, I created a problem with the context "the days spent on doing a project," with which I was quite familiar: "if one person completes a project in 540 days, then in how many days can 40 persons complete the same project?" The unit "day" exactly matched with 13.5 without any rounding off since 0.5 made sense for calculating a day and could be rationally expressed as a half day.

During the posting session, my colleagues provided plenty of problems with very different answers. For example, a primary school teacher produced this problem: "A teacher is grouping 540 students into classrooms. If each classroom could not accommodate more than 40 students, then at least how many classrooms should the teacher prepare for?" She presented us with the answer — 14 classrooms, with 13.5 rounded up; for practical purposes, it is unreasonable to have 0.5 of a classroom. A college teacher made another problem: "A pharmacist is equally allocating 540 tablets into 40 boxes; how many tablets are put in one box?" She offered the key with 13 tablets, with 13.5 rounded down, as it typically did not make practical sense to set 0.5 tablets in a box. A university teacher demonstrated the third problem: "if there is $540 to be evenly distributed among 40 members in a club, how much could each of them get?" He gave the reasonable answer: 13 dollars and 50 cents. Lastly, I was also invited to present my problem. Was it surprising to see that the basic expression "540÷40" yielded more than one answer, which depended on the units used?

This was my first time that I had experienced knowing that was not delivered by professors (teachers) or through textbooks, but which emerged from the classroom discussions as collective learning. Encouragingly, I also contributed my knowing to the whole class as a collective, just like other people did. In this case, I was acutely aware of these aspects: the uniqueness of myself as a human subject, the importance of my individual contribution to the whole class as a collective, and the power of collective learning. These collective-based learning experiences, which further motivated me to reflect on the projects I had done about teachers' online professional learning and on the courses I had taken for my own professional learning, helped me to shape and outline my research.

It was almost ten years ago that I explored e-technology for the first time in my research career. In February 2008, as a research fellow at the National Institute of Education, Singapore, I was invited to join a project about building an online learning community for mathematics teachers. Honestly, I did not think highly of this project at first, because I held slightly biased beliefs about e-technology in teachers' or students' learning. I treated the research on the application of the technology to mathematics education as non-core and therefore, non-significant, for I knew from my experiences that the core research areas had usually been identified as mathematics curriculum, students' mathematics learning, mathematics teaching, and conventional professional development for mathematics teachers.

In the project, all the team members struggled to design an online community intended for those Singaporean teachers who were too busy with their routine work to participate. After several rounds of discussions, arguments, and debates about the project, we finally came up with a design strategy — meeting teachers' needs — to attract these teachers' participation in the online community (Wang & Fang, 2010). However, because we did not know exactly what those teachers needed, we decided to presume and predict what they might need (Wang & Fang, 2010).

During the design process, we assumed the characteristics of the community as what we had prepared for a conventional professional development program. Unfortunately, upon the design completion, I was assigned to join another project — Lesson Study — so I did not know how the teachers used the online community. I also did not expect that such a community would exert a great impact on teachers' professional learning given that it was built upon the researchers' assumptions rather than the teachers' real needs.

In February 2010 when the lesson study project was completed, I returned to Shanghai, China to continue my teaching and research in a university. One day, I read in a journal an article describing an influential blog community run by teachers themselves. It reminded me of the community we had painstakingly designed to get the Singaporean teachers' attention. I began to think about what attracted so many teachers to join the blog community and started investigating the contents of mathematics teachers' blogs within that community. Adopting Ball, Thames, and Phelps' (2008) framework to categorize the contents, I found that the blogs dealt with Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) (Wang, 2015). The results assured me that such content had the potential to attract mathematics teachers to participate in the community. Nevertheless, I was quite certain that, in contrast to the community we designed in Singapore, this community was not differentiated from the others in the literature. Regretfully, at that time, I had not been able to explore how such an online community as a new field could also work for teachers' professional learning.

It was not until September 2014 when I started my PhD program at the University of Alberta and learnt about complex systems and their application to curriculum and teacher education, particularly mathematics teacher education, that I inferred that the notion of open space from complex systems might be one of the rationales behind the community function. To ascertain the nature of open space in an online community, I commenced searching and analyzing the related literature. I came to realize that the term "professional learning networks (PLNs)" exactly described the blog community that I found, in contrast to the previously designed one in Singapore. Because I wanted to know about mathematics teachers' participation in PLNs, I began to inquire about what they did and what emerged from their doings in that open space. Fortunately, my inquiry was inspired and directed by my learning experiences of doing a *winter count*.

The *winter count*, pictorial historical records made by Native Americans in North America, was set as a part of the pedagogy in my course, *Advanced Research Seminar in Secondary Education* (Fall, 2015). Having prepared myself as an individual learner, the pedagogy offered me a special opportunity of learning through participating in class discussions and interactions.

Students taking the course were required to produce a weekly symbol (for their own winter counts) to represent the ideas, feelings, or stories that emerged from the sharing of individual responses to the weekly readings and the follow-up class discussions that inspired them. Creating each symbol was actually a process of producing my own knowledge. I, however, did not yield that knowledge by myself; it emerged from collective contributions.

For example, in my winter count, the idea of *leaving more space for students' exploration* originated from a classmate's weekly response, which involved leaving more negative space for imagination in her painting. The negative space was compared to thinking space for school students' exploration in our classroom discussions, which inspired me to reflect on students' mathematics learning. From the perspective of traditional mathematics learning, we did not set enough space for school students to explore mathematics; instead, they were allowed to spend too much time on memorizing and practicing the facts, procedures, and formulas (Orton, 2004) that are often taken as the "true" mathematical knowledge. Figure1-1 shows a symbol I created to represent the idea *leaving more space for students' exploration.*

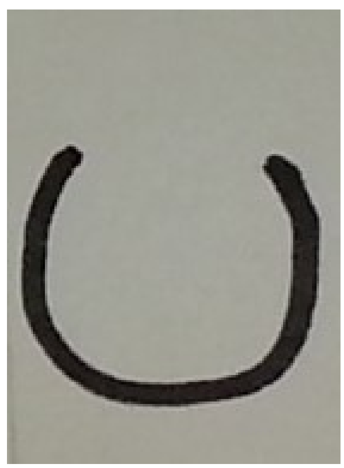


Figure 1-1. The symbol of *leaving more space for students' exploration.*

Towards the end of the semester, based on participants' individual sharing, classroom discussions, and my own reflections, I had produced a total of 12 symbols (see Figure1-2) representing the corresponding 12 ideas. The symbols were eventually pasted on the strips of a lantern (see Figure 1-3).

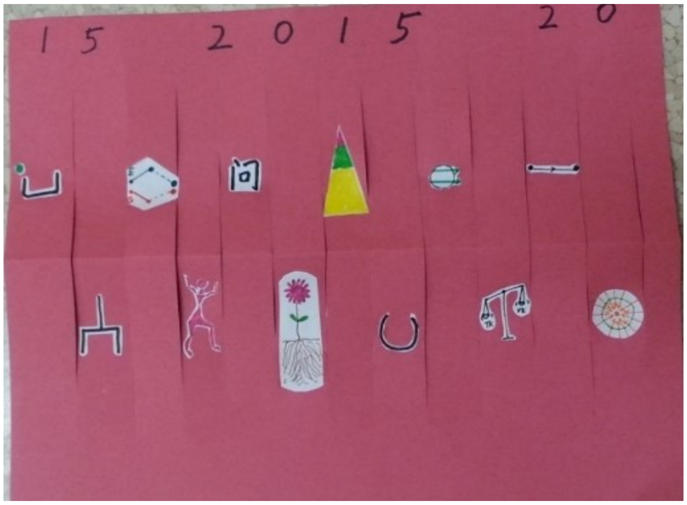


Figure 1-2. The symbols of my winter count.

Figure 1-3. The winter count lantern.

Learning occurs in “nonlinear patterns, emergent, divergent, and convergent” (Smitherman, 2004, p. 15). My experiences of building a winter count enabled me to understand that knowledge emerged from my participation in the class rather than it being transferred from professors or via authorized documents (Osberg & Biesta, 2008). This highlights the “dynamic and collective aspect of teacher knowledge” (Charalambous & Pitta-Pantazi, 2015, p. 32) in a “space of emergence” (Osberg & Biesta, 2008, p. 326). These experiences also encouraged me to further explore mathematics teachers’ learning through their participation in PLNs.

Integrating my winter count learning with my knowing about complex systems drove me to be attentive to mathematics teachers' participation in PLNs. In recent years, online professional learning communities have already significantly influenced teachers' professional development (Beach & Willows, 2014; Borba & Llinares, 2012; Dash, de Kramer, O'Dwyer, Masters, & Russell, 2012; Trust, 2012). For example, providing face-to-face quality professional development was conventionally viewed as difficult for all Alberta's math teachers because they were distributed widely across the province of Alberta in Canada. Similarly, to offer such kind of professional development was also challenging for all of Shanghai's math teachers because they were so densely located in a large number of schools throughout Shanghai in China. However, with the advancement of Internet technology and its widespread use,[1] online professional learning communities could be a viable alternative to onsite workshops (Francis-Poscente & Jacobsen, 2013) for teachers.

PLNs, a relatively new form of online professional learning communities (Trust, 2016), have the potential to make teachers' professional learning more "participatory, grassroots and supportive" (Carpenter & Krutka, 2015, p. 708) in their own ways. It is also possible for teachers to access important resources that they could not otherwise afford or access in the local communities (Dede, Breit, Ketelhut, McCloskey, & Whitehouse, 2005).

Mathematics teachers participating in PLNs might not be aiming to obtain prescribed knowledge but, rather, to share their interests, seek help or support, review others' viewpoints, or respond to others' needs. Similar to what I had experienced in the winter count, their individual efforts eventually shaped collective contributions, which may have surpassed any individual contribution on its own (Davis & Renert, 2014). In turn, collective contributions potentially advance individual participants' knowledge and knowing (Leikin, 2007). Based on the properties of complex systems (see details in Chapter 3: "Complex Systems"), a PLN could be viewed as a complex system consistent with the concept of "collective learner" (Davis & Renert, 2014, p. 31). Thus, it can be

[1] As of December, 2017, 54.6% of the population in China use the Internet (https://www.internetworldstats.com/asia.htm#cn), and 89.9% in Canada (https://www.internetworldstats.com/america.htm#ca).

regarded not only as "a collection of learners but as a collective learner" (Davis & Renert, 2014, p. 32).

Though more and more mathematics teachers are engaging in PLNs, the related research does not focus on how participants interact and what could emerge from their interactions. This gap will be brought to light after my review of the relevant literature in Chapter 2: "Teacher Online Professional Development." This study addressed the research gap by interpreting mathematics teachers' participation in a PLN.

According to Gadamer's (1990) notion about a genuine conversation, the interactions between or among the participants with a certain topic in a PLN could be looked upon as a down-to-earth or genuine conversation. Gadamer notes that the genuine conversation "has a spirit of its own, and that the language in which it is conducted bears its own truth within it — i.e., that it allows something to 'emerge' which henceforth exists" (p. 383). Similarly, a PLN is open to anyone and the conversation topics within are not pre-existent but arise from the conversations that follow their natural flow, and which allow "a mathematical world" and more to be "brought forth" (Gordon Calvert, 2001, p. 142). The interactions in question correspond to Gadamer's (1990) notion of how a genuine conversation takes "its own twists" and reaches "its own conclusion" (p. 383). Thus, I specified my research questions as:

- *what did the structures of the conversations among the participants look like? and*
- *what could emerge from the conversations in relation to mathematics-for-teaching*?

The questions engaged me in the field of interpretive inquiry, which I used as the methodology for the research process (see Figure 1-4), and they were further illustrated methodologically in Chapter 4: "Interpretive Inquiry."

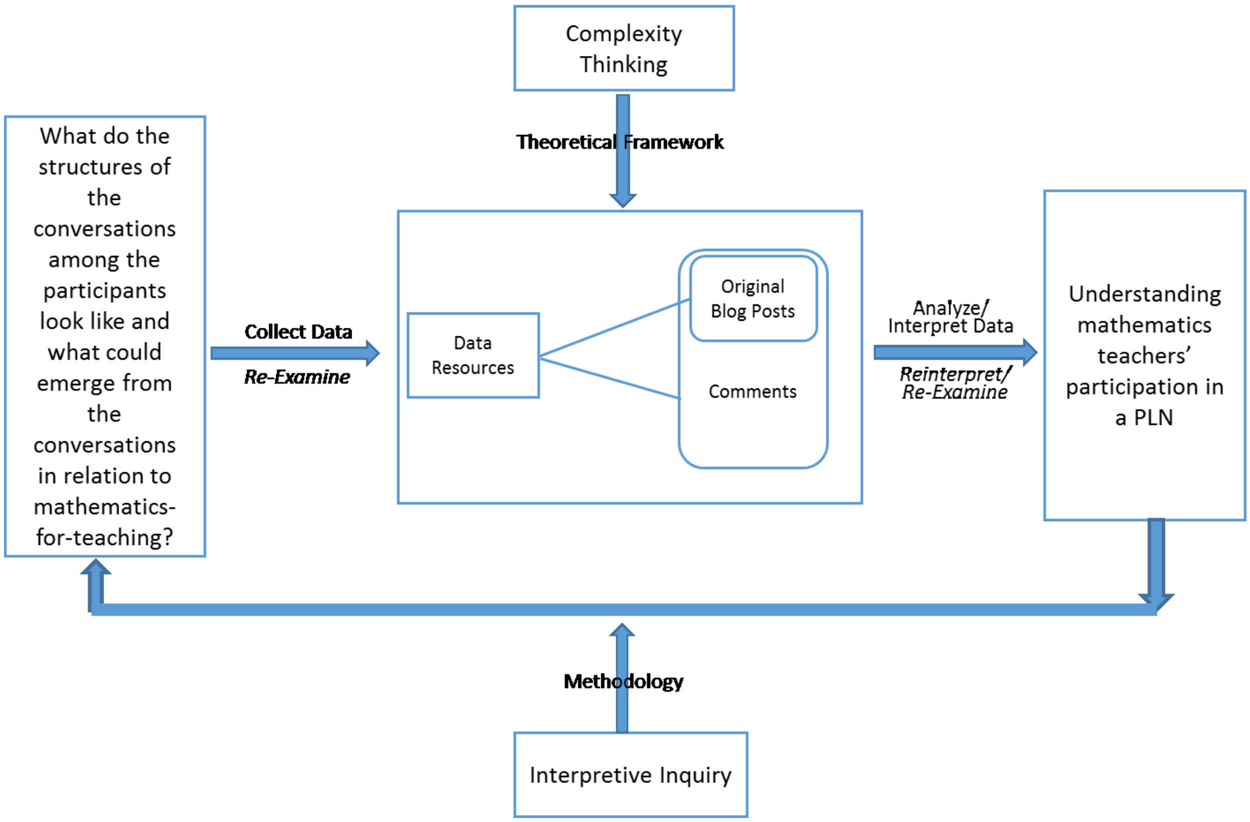


Figure 1-4. The diagram of the inquiry.

For this research, the original blog posts and their related comments could represent mathematics teachers' participation in a PLN, because they are regarded as the basic participatory actions in an online community (Wang & Yu, 2012). As such, I collected the archived documents of these actions in a targeted PLN as my data. However, these two kinds of data were not considered to separate teachers' participation into two different actions but to provide more possibilities to illustrate their engagement in a PLN. For example, some people might hold that blogging presents a solitary action of participation in a PLN and they preferred to share their original blog posts as a form of involvement. Others, however, might enjoy commenting on the original blog posts or other related comments involved in the interactive discussions in the PLN. Also, there were those who were interested simultaneously in sharing their original blog posts and engaging in the related discussions. As a result, participants' original blog posts and comments could appropriately provide an overview of teachers' participation in a PLN.

It should also be noted that for this research lurking or "zero posting" (Walker, Redmond, & Lengyel, 2010, p. 156) was not viewed as a type of participation even if it was taken as an "integral part of any online community" (Ridings, Gefen, & Arinze as cited in Walker et al., 2010, p. 157), because it does not have an observable or explicit participatory nature (Walker et al., 2010).

The data collection and analysis were detailed in Chapter 4: "Interpretive Inquiry" and the results were elaborated in Chapter 5: "The Conversations and the Emergence of Knowing in the Illustrative Examples."

The aim of this research was to understand mathematics teachers' participation by examining the conversations within through the analysis on the conversation structures and the emergence of knowing. The results, which helped me to understand mathematics teachers' professional learning through their participation in the targeted PLN, also contribute to the rapidly increasing literature on teachers' professional learning in online learning communities in particular (Dash et al., 2012). They also offer a reference for reviewing teachers' online professional learning and for reflecting on conventional professional development, the latter of which has been criticized for not meeting teachers' needs. Learning through participation in a PLN, however, is totally based on their needs and/or interests (see Chapter 2). Looking forward, it may also be fruitful for researchers to further explore the nature of the relatively new form of teacher professional learning when we come to realize the affordances of our digitally connected world and the intricacies of teachers' professional growth as indicated by Brooks and Gibson (2012).

This dissertation organized according to the following structure (Figure 1-5): Chapter 1 introduces my inquiry into the field of PLNs; Chapter 2 reviews the literature related to PLNs and reveals the research gap for this study; Chapter 3 describes complexity thinking as the theoretical framework; Chapter 4 discusses interpretive inquiry theoretically and applies it to a PLN practically; Chapter 5 presents the results from the analysis of four selected examples; and Chapter 6 interprets the implications of this study and concludes the reflections on the whole study.

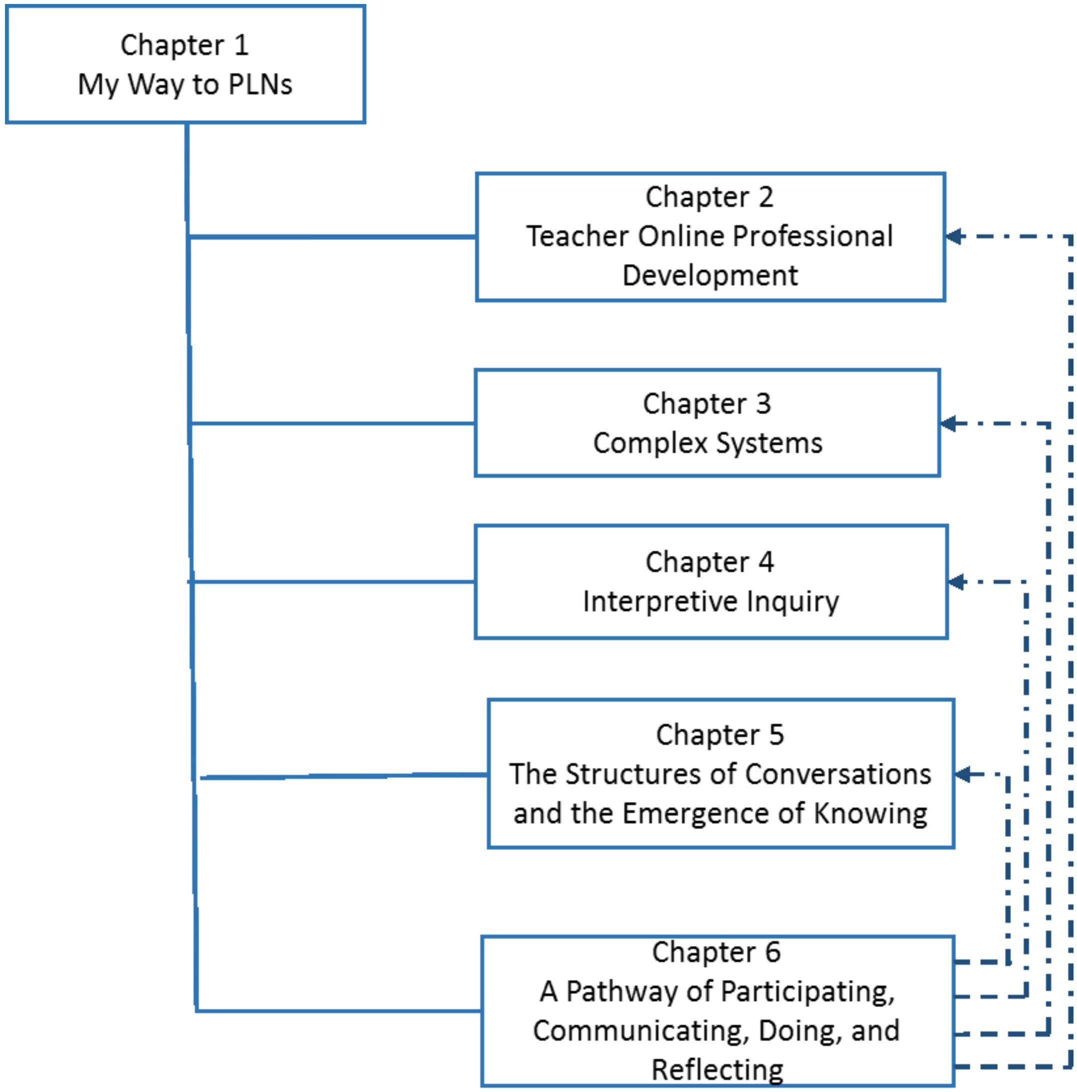


Figure 1-5. The structure of this dissertation.

2. Teacher Online Professional Development

This chapter reviews the literature on teacher online professional development (PD) and professional learning networks (PLNs). The rapid development of online PD renders necessary related research to focus on the following aspects: how to design, develop, and sustain online PD; what might influence it; and how to evaluate online professional learning. As a new type of professional learning, PLNs break away from the conventional views on PD and allow teachers to actively share their perspectives or viewpoints in their own way so as to meet their professional needs without predetermination of outcomes. To date, researchers have begun exploring a number of the issues related to PLNs, including:

- purposes of using PLNs,
- impacts of PLNs,
- analysis of PLNs,
- critical aspects of PLNs,
- challenges of participation in PLNs, and
- the concerns of anonymity.

Nevertheless, they have not paid much attention to what the structures of the conversations among the participants look like in PLNs and what could emerge from these conversations.

2.1 Literature Searching and Categorizing

I took several considerations into account in searching and categorizing the literature. I used key words such as *online learning community*, *mathematics teachers*, *professional development*, *Facebook*, *Twitter*, *Blog*, *professional learning network*, and *community of practice* to search the related literature. I narrowed down the publication years used for literature searching to the recent two decades because online professional learning/development prevailed during that period.

One hundred and twenty-four items were targeted to provide a holistic picture and detailed portrayal about PLNs, including: 2 books, 104 journal articles, 13 dissertations, 2 reports, 1 book chapter, and 2 conference papers from databases such as EBSCOhost, ProQuest, ERIC, ProQuest Dissertations and Theses Global, and Google Scholar. I sorted these into three categories: teachers' professional development (PD), online teachers' PD, and professional learning networks (PLNs). These categories were connected to each other: professional learning networks (PLNs) was nested within online teachers' PD, which was further nested within the teachers' PD.

The following section reviews the targeted literature and presents the background and rationale for this study.

2.2 Teacher Professional Development

Teacher professional development (PD) has been considered as a necessary approach and (in some contexts) a policy solution to promote teacher quality (Dash et al., 2012). Conventional professional development is defined by Guskey (2000) as "those processes and activities designed to enhance the professional knowledge, skills, and attitudes of educators so that they might, in turn, improve the learning of students" (p. 16). Successful PD is suggested to "be responsive to the teachers' needs and experience, tailored, and personalized" (Francis-Poscente & Jacobsen, 2013, p. 321).

However, conventional PD has received criticism in the literature, including that it may not offer what teachers actually need for their teaching practices (Wilson & Berne as cited in Marrero, Woodruff, Schuster, & Riccio, 2010); that it often occurs offsite, after school, or on holidays; that teachers usually have few choices, or sometimes no choice, in participation type and timing (Francis-Poscente & Jacobsen, 2013); and even worse, that it could provide little support for quality PD in many teachers' school environments (Darling-Hammond & McLaughlin, 2011). Accordingly, for the purpose of acquiring quality PD, a number of teachers and teacher educators are resorting to online sources and approaches (Marrero et al., 2010).

2.3 Teacher Online Professional Development

Generally, teachers' engagement in online professional development is called online PD and this has resulted from the evolution of web-based technology in education (Borko, Whitcomb, & Liston, 2009). Online PD is viewed as a potential way of affording rich, multivariate, and significant professional learning opportunities for teachers (Darling-Hammond & McLaughlin, 2011) because of its multiple advantages. For example, Gray (2004) points out that it could reduce isolation and create conversations about practice-based problems while Johnson (2001) identifies its advantages as breaking the formal boundaries of time or space and equalizing communication practices. Online PD therefore provides a possible way to provide vast numbers of teachers with opportunities to access quality professional development (Beach & Willows, 2014; Ginsburg, Gray, & Levin, 2004).

The research on online PD focused mainly on the following themes:

- design strategies,
- strategies for community development,
- critical factors influencing community maintenance,
- evaluations, and
- potential and realized challenges.

2.3.1 Design strategies

The question of how to design online PD was explored by several researchers. Ostashewski, Moisey, and Reid (2011) developed a teacher professional "courselet" based on constructivist principles to provide opportunities for teachers to engage in ongoing online professional development. Similarly, Cady and Rearden (2009) implanted mathematics contents and pedagogy courses in their online PD programme to promote teachers' professional learning. And Chinnappan (2006) used productive pedagogies as a design framework for mathematics teachers' online professional learning. In addition, much research focused specifically on the learning task design for online PD, including: developing multimedia video cases for the online professional learning community (Boling, 2007; Fang, 2010), designing qualified learning tasks (Francis-Poscente & Jacobsen, 2013; Maor, 2003), and embedding artifacts (e.g., animation) (Chieu, Herbst, & Weiss, 2011).

2.3.2 Strategies for developing online learning community

Researchers revealed several strategies for developing online learning communities. For example, Macdonald and Hills (2005) piloted a reflective method through online networks to support teachers' professional learning. This method first uses a log to structure teachers' reflections and then shares these reflections with other teachers through an online conference. Facilitation has also been recognized as a critical strategy in building and developing online learning communities (Gunawardena et al., 2006; Frady, 2012).

Online video case discussions are also viewed as an important strategy of improving teachers' online professional learning (Liu, 2012). In particular, researchers discovered and adopted the following strategies of stimulating online discussion: involving a case teacher in video case discussion (Koc, Peker, & Osmanoglu, 2009); positioning facilitators as both facilitator and co-participant (Lu & Jeng, 2006); and providing prompts for online discussion (McGraw, Lynch, Koc, Budak, & Brown, 2007).

2.3.3 Critical factors

Some critical factors were examined with respect to their influences on the development of online learning communities. A sense of community has been identified as the critical factor affecting members' participations and social interactions in online learning communities (Riverin & Stacey, 2008; Tsai, 2012; Visnovska, 2010). It is viewed as "a feeling that members have of belonging, a feeling that members matter to one another and to the group, and shared faith that members' needs will be met through their commitment to be together" (McMillan & Chavis as cited in Tsai, 2012, p. 272). The more members experience a sense of community, the more the community increases information flow, provides supports, and presents opportunities for collaboration and satisfaction (Job-Sluder & Barab, 2004; Scott, 2004).

Online interactions are viewed as an important factor affecting participants' learning process and their satisfaction with respect to online professional learning (Holmes, Signer, & MacLeod, 2010). It often takes the forms of asynchronous forum discussions, instant chats, paper or artifact uploads, and email. It is possible that participants are able to develop collaborative relationships in the communities that could promote their learning through the clustering of peer-to-peer interactions.

Teachers' prior content knowledge is described as a possible factor affecting their satisfaction with the learning in online learning programs. Owston, Sinclair, and Wideman (2008), for example, assess a blended learning program, which combined face-to-face with online learning and its influence on teachers' attitudes and knowledge. They found that teachers with relatively weaker subject-matter knowledge benefit less from the learning program. They suggest that online learning program developers who cannot obtain immediate feedback from participants in online learning environments attend more closely to teachers' subject area backgrounds because they may otherwise take a long time to realize what the teachers' weaknesses may be.

Finally, teachers' perceptions of more knowledgeable participants (e.g., university faculty members) are considered to be an adverse factor that could affect their communications. For example, Kale, Brush, Bryant, and Saye (2011) examine teachers' online participations and the depth of their messages in a professional learning program by analyzing the relations among the messages. They realize that teachers are inclined to post more messages to those participants assumed to have "more expert knowledge" (p. 509), such as the university faculty members involved. This practice results in the dominance of university faculty members over the entire communications.

2.3.4 Evaluations of online PD

Researchers conducted evaluations to examine the impact of online PD on the development of teachers' professional knowledge. Such evaluations included:

- teachers' construction of subject knowledge and pedagogical content knowledge (Burgess & Mayes, 2008; Cady & Rearden, 2009);
- teachers' perceptions about topics such as the merit and demerit of online learning spaces (Moore-Russo, Wilsey, Grabowski, & Bampton, 2015); and
- teachers' interactions such as more social interactions in peer-led discussions than in facilitator-led ones (Lalli & Feger, 2005).

Multiple evaluation forms are presented in the research, including:

- the development of assessment items such as pre-surveys and post-surveys on mathematics teachers' pedagogical content knowledge and teaching practices (Dash et al., 2012);
- the adoption of diagnostic tests, such as Diagnostic Teacher Assessments in Mathematics and Science (DTAMS), to explore teachers' mathematics knowledge for teaching (Cady & Rearden, 2009);
- the adoption of the National Council of Teachers of Mathematics (NCTM) standards — "problem solving, reasoning and proof, communication, connections, and representation" — to explore mathematics teachers'

understanding of student learning (Osmanoglu, Koc, & Isikasal, 2013, p. 1298); and

- the identification of a four-level assessment framework to test the effectiveness of the teacher professional learning model; the frame includes teachers' perceptions, the acquirement and the application of new knowledge and skills, and organisational support and change (Owston et al., 2008).

2.3.5 Maintaining sustainability

How to maintain the sustainability of online PD was a big concern for developing online PD. Primary factors that affect the sustainability of online PD include:

- low levels of trust and social affiliation,
- performance anxiety,
- lack of time, and
- failure to see the relevance of online interactions in the context of practitioners' needs (Thang et al., 2010; Riverin & Stacey, 2008).

Besides these factors, researchers also identified the quality of the shared information as an additional factor of influence upon learners' participation based on the hypothesis that "the greater the perceived usefulness of the knowledge-sharing system, the greater a user's participation in knowledge-sharing" (Sharratt & Usoro 2003, p. 190). However, as Sharratt and Usoro (2003) note, simply sharing information and knowledge does not always generate new ideas and produce new knowledge.

2.3.6 Learning space

Francis-Poscente and colleague (Francis-Poscente, 2009; Francis-Poscente & Jacobsen, 2013) launched an online learning program by borrowing a format of a face-to-face program for teacher's professional learning. Different from other online learning programs, this program attempts to provide a learning space — a synchronous online environment — for mathematics teachers to play with mathematics without geographic barriers rather than to "mandate a rote, linear, sequential, procedural form of learning" (Francis-Poscente, 2009, p. 27). A hermeneutic inquiry presents the journey of designing the online learning space, inviting the participants, fostering mathematical play, enjoying the play, and perceiving the potential of the online learning space for teacher professional learning. The researchers suggests that mathematical problems with emphasis on concepts and making connections and with less requirements on writing and more possibilities for drawing symbols can facilitate online interactions and conversations; that the online environment can overcome geographic barriers and invite dispersed individuals to collectively construct knowledge for mathematics problem solving as well as mathematics learning and teaching (Francis-Poscente, 2009; Francis-Poscente & Jacobsen, 2013).

My review of the research related to online PD programs found that teachers in these programs are mostly treated as "objects" (Osberg & Biesta, 2008, p. 323) of predicted transferring or transmitting systems. This means that the validated strategies of the design, development, evaluation, and application of various online PD programs mainly aim to facilitate and evaluate teachers' mastery of content and pedagogical content knowledge. Therefore, most of the online PD programs by their nature are alternative forms of achieving predetermined knowledge.

Certainly, this is not the case for Francis-Poscente's (2009) program, which aims to engage participants in discovering mathematical beauty as learners. This is consistent with Davis and Renert's (2014) notion of teachers' learning. For them, teachers' learning should not be viewed as a means to master a domain of mathematics but as a mode of enacting mathematics-for-teaching in different situations. Such a mode could be achieved through the platforms of PLNs, which are still relatively new environments for professional learning.

2.4 Professional Learning Networks

PLNs, as "system[s] of interpersonal connections and resources" (Trust, 2012, p. 133), have extended teachers' formal professional development to informal professional learning. Research indicates that conventional views on professional learning could not interpret teachers' actions in PLNs (Trust, 2015) because PLNs have dramatically changed the way teachers access professional learning (Trust, 2012). While online networks enable diverse viewpoints to be activated and shared among participants (Wenger, White, & Smith, 2009; Ebner, 2009) to meet teachers' professional learning needs, related research has only just begun to explore teachers' use of social networks for their professional learning.

2.4.1 The purposes of educators' using PLNs

Educators have different purposes for using PLNs, as specified in the research. Ross, Maninger, LaPrairie, and Sulliva (2015), for example, demonstrate that educators often use Twitter to collaborate, network, and engage in their professional learning. Adjapong, Emdin, and Levy (2018) hold that participants from all over the world share resources and information as well as their cultural perspectives on education. Forbes, (2017) in reviewing the literature, summarizes that PLNs in teacher education are for the purposes of content production and sharing, understanding content, and building collaborative connections with others. And Hur and Brush (2009), who examine teachers' motivation for participation in self-generated online communities through interviews, identify five reasons for participation: emotional support, academic or teaching issue related help, interaction with others, idea exploration, and a sense of camaraderie. Achieving these purposes through participation in the PLN could enhance participants' professional learning.

2.4.2 The impacts of PLNs

The research shows that PLNs have various impacts on teachers' professional learning and teaching practices. Levenberg and Caspi (2010), for example, indicate that teachers view informal learning as more meaningful than formal learning after comparing teachers' perceptions[2] of their learning in formal (face-to-face) and informal (online) settings. Further, they reveal that this is because fuzzy boundaries online can make more space and opportunities for online interactions, which can turn the informal learning environments into appropriate and secure learning places.

Moser (2012), exploring online collaborations among novice teachers and the positive impacts of conversations on their teaching practice, finds that collaborations can

- promote novice teachers' reflections,
- engage them in exploring new teaching approaches, and
- encourage them to adopt resources.

Parrish (2016) examines the impact of a PLN as an online community on teachers' selection and implementation of cognitively demanding tasks. Performing a

[2] The author mainly uses "the conception one holds of what learning is" to reveal the teachers' perceptions of their learning (Levenberg & Caspi, 2010, p. 324).

qualitative analysis on content developed by the PLN and teacher interviews, he concludes that the online community could provide solid support for teachers selecting and implementing tasks with high cognitive demands. Parrish (2017) also examines the mentoring emergent in the PLN. Through a thematic analysis of the contents of the blog posts in the PLN, he finds that participants can gain and/or seek mentoring from the PLN for their teaching, including “advice for teaching specific students, advice for goals or issues in teaching, suggestions for teaching or resources, transparency in planning mathematics instruction, and request for mentoring” (p. 120).

Some researchers touched upon teachers’ professional learning and teaching practice. Duncan-Howell (2010), who investigated participants’ professional learning in PLNs, reveals that professional development and classroom/student needs are the main reason for maintaining their participation therein. Noble, McQuillan, and Liteenberg-Tobias (2016) explore how PLNs impact teachers’ professional growth. By interviewing Twitter participants and observing their online posts, they find four key areas of professional growth: creating a supportive network, enhancing confidence in teaching, reflecting on teaching practice, and making changes in teaching practice. Trust, Carpenter, and Krutka (2018) attend to the impact of PLNs on the professional learning of instructional leaders (e.g., principals, superintendents, librarians, specialists, coaches, and facilitators). They report that PLNs enhance instructional leaders’ professional learning such as finding new ideas, approaches, or resources for teaching, developing new professional knowledge and/or skills, learning about leadership, realizing the value of community, and shaping learning disposition. Carpenter and Morrison (2018) discuss that social media platforms such as Twitter offer pre-service teachers opportunities to build professional networks, translate theory into practice, and access various mentors.

In my study, the impact of the investigated PLN on teachers’ professional learning and teaching practice was beyond my research purpose. However, some insights emerged from teachers’ interactions in the PLN, such as teachers adopting ideas from the PLN to improve their teaching practice.

2.4.3 Participants’ experiences and actions in PLNs

Researchers also attended to participants’ experiences and actions in the PLNs. Some explored participants’ experiences of participating in PLNs while others attended to specific actions of teachers in the PLN. Using an online survey, Trust, Krutka, and Carpenter (2016) investigated teachers' conceptions of PLNs and the effects of these on participants' teaching and learning and on their students' learning. Having conducted a thematic analysis, they identified that participation in PLNs supports participants' various affective, social, cognitive, and identity needs, which emerged from their teaching practice and professional learning.

Colwell and Hutchison (2018) attended to teachers' experiences of participating in a PLN. They note that teachers view posting professional ideas or resources to Twitter as a complex process, that they have skeptical concerns about participating in the PLN, and that they value the benefits of access to multiple resources. Larsen and Parrish's (2019) exploration of participants' experiences engaging in a PLN is grounded on Luehmann's (2008) framework of community building activities. They note that participants predominately regard the PLN as a space of sharing and acquiring resources and that they value these resources because of their inspiration, relevance, and reliability which are established by participants over time.

While participants' experiences in PLNs are not the concern of my study, insights from my study related to PLNs supporting participants' cognitive, social, and affective needs are reflected in participants' online interactions. These insights will be elaborated on in chapters 5 and 6.

Researchers also provide analysis tools or frameworks for exploring participants' experiences in PLNs. Krutka, Carpenter, and Trust (2016) offer a model related to these key elements of viewing participants' experiences in PLNs:

- engaging with PLNs,
- discovering through PLNs,
- experimenting teaching practice by learning from PLNs,
- reflecting on teaching practices for improvement, and
- sharing knowledge, skills, and resources.

Krutka, Carpenter, and Trust (2017) further propose a framework for reflecting and enriching participants' experiences in PLNs. The framework includes three dimensions — people, spaces, and tools — and three analysis orientations — identification, reflection, and intention. The orientations direct the analysis in each dimension. Such a framework can serve as a tool for participants to review their experiences in PLNs and plan for the future.

Researchers also closely attend to the participants' actions in PLN(s). Larsen and colleague (Larsen, 2016; Larsen & Lijedahl, 2017) focused on the interactions among participants in a PLN. Larsen (2016) examined a discussion among participants about mathematical abstraction based on negotiation of meaning and concludes that blogging and tweeting can support the continuing of discussion and allow the negotiation of meaning to occur. Using the perspective of necessary conditions for complex emergence, Larsen and Lijedahl (2017) find that the interactive productivity from the discussions among participants partially relies upon redundancy of sources and diversity of mathematical ideas.

Kontorovich (2016), who explored how explanations are developed in an online forum, constructed an individual and collective explanation space. In the individual space, he describes the explanations from individual participants. In the collective space, he presents the explanations constructed with the participants' collective efforts. In my study, such kinds of interactions and collective work also occurred, and this will be elaborated on in Chapter 5 in particular.

2.4.4 The critical aspects of PLNs

Various critical aspects of PLNs aroused researchers' attention. Some explored characteristics of participants and of PLNs while others attended to critical factors that impact members' participation. Uses online surveys, Fucoloro (2012) explored the characteristics of participants who seek out informal online professional learning. The surveys indicate the facts that the average age of participants is 43, their teaching levels are from Pre–K to the 5th grade, and most respondents are classroom teachers from suburban schools. She also presents an interesting result — that the increase in teachers' age leads to a decrease in the number of participants who adopt social media for their professional learning. Holmes, Preston, Shaw, and Buchanan (2013) investigated the characteristics of effective professional learning through Twitter. They identify the characteristics of effective professional learning as:

- being sustained over time,
- meeting learners' practical needs,

- being collaborative,
- involving knowledge sharing, and
- endowing learners with a certain degree of control power and ownership.

In this sense, the use of Twitter may be viewed as a way to potentially access effective professional learning. Elias (2012), who examined the essential aspects of PLNs, demonstrates that the interactions through PLNs approximate face-to-face communities of practice. The social and informal nature of PLNs supports the relevant, timely, and contextualized learning necessary for learners' professional growth.

Researchers have also explored the crucial factors that impact members' participation in PLNs. Generally, Sie et al. (2013) identify seven factors that play a pivotal role in participants' engagement in PLNs: "sharing, motivation, perceived value of the network, feedback, personal learning, trust and support, and peer characteristics and peer value" (p. 59). Smith Risser and Bottoms (2014) examined the role of individual participation in the whole network and found that commenting on others' postings is one possible way to gain a status in PLNs. However, they claim that commenting only is insufficient to make a status transition because what an initial post contributes to the community also plays a crucial role in transforming participants' status. For instance, they argue that if a peripheral blogger is recognized by participants in a PLN, the blogger's status could change from "Newbie" to "Celebrity" (p. 446) more quickly than those who only comment on the post. Ranieri, Manca, and Fini (2012) investigated the impact of group types (e.g., generic group-sharing experiences related to school in general and thematic group sharing of a school project or discussion theme) upon group membership. They found that the members of generic groups take knowledge sharing as a means of gaining status while those in thematic groups emphasize the emotional expressions and personal experiences of sharing in order to satisfy their needs of community belonging rather than of knowledge sharing.

2.4.5 The challenges of PLNs

Some research concerned the kinds challenges PLNs faced. Brown and Munger (2010) found that it is not easy to engage participants in a deep discussion through a virtual network because the discussion is affected by external influences (e.g., technology functioning), personal conditions (e.g., self-confidence, sense of responsibility, being a lifelong learner), and social situations (e.g., positive relationships among members) (Baran & Cagiltay; 2010). For example, some teachers receiving feedback from online discussions may feel motivated to respond to others who asked for advice or assistance (Baran & Cagiltay, 2010; Hew & Hara, 2007); if not, they may not be encouraged to join the discussions, which could affect the discussion depth.

Hew and Hara (2007) identify several barriers that hinder teachers from sharing knowledge. For instance, some teachers' lack of related knowledge and time prevent them from responding to the online discussions. Others are not inclined to share their knowledge because they fear being misunderstood and judged unjustly in a virtual environment devoid of verbal and visual cues. Moreover, some researchers indicate that the difficulty with technology, particularly the challenge of processing massive amounts of information (Baran & Cagiltay, 2010; Kear, 2011; Wang, 2008), could also decrease the quality of online discussions (Baran & Cagiltay, 2010).

While my research was not intended to dig out the kinds of challenges mathematics teachers faced in the targeted PLN, certain challenges demonstrated in the literature (e.g., lack of self-confidence and time) also appeared in my study, as revealed by participants' sharing in the conversations in the PLN.

2.4.6 Anonymity and privacy protection

How to protect one's privacy is also a big concern for the participants when they choose to use social networking technologies (Bristol, 2010; Lagu, Kaufman, Asch, & Armstrong, 2008). Hur and Brush (2009) note that teachers participating in online discussions favour anonymity in a virtual world: the anonymity encourages them to boldly share with others without fear of being judged or criticized. The researchers further explain that participants are able to express their needs in online environments, which they cannot share in their physical world where they might be viewed as incompetent if they express problems or seek advice from others. Admittedly, anonymity protects not only the teachers who create blog posts but also others involved in the discussions. For instance, in Hur and Brush's (2009) study, one participant noted that anonymity enabled her to share and air her views about teaching experiences connected to a specific colleague without hurting anyone at her school.

Individuals may either stop using social networks or be reluctant to use them again once they perceive or fear a loss of privacy associated with these networks. For instance, Andergassen, Behringer, Finlay, Gorra, and Moore (2009) find that many students in universities choose not to blog in informal contexts, even if they are offered appropriate facilities, because they were concerned about “the loss of privacy through blogging” (p. 211).

My study considered the significant role of anonymity and privacy protection in mathematics teachers’ participation in the investigated PLN. Even if the PLN was a public one, care was taken to avoid privacy breaches during my data analysis and reporting of the findings. I elaborate further on anonymity and privacy protection in the section “Ethical Considerations” in Chapter 4: “Interpretive Inquiry.”

As the literature review shows, although some studies related to teachers’ professional learning in online environments, only a few of them touched upon teachers’ participation in PLNs. There is (generally speaking) a lack of studies of PLNs, as Trust et al. (2016) suggest. The review also revealed that, although more and more mathematics teachers had participated in PLNs (Francis-Poscente & Jacobsen, 2013), few studies were undertaken on what the structures of the conversations among the participants look like and what could emerge from the conversations in PLNs. This study aims to bridge the research gap by investigating the conversations among participants in a targeted PLN.

3. Complex Systems

This chapter describes complexity thinking, which was used as the theoretical framework for this study. It begins with an introduction to the properties of, necessary conditions for, and fractal geometry of complex systems, then reveals the rationale for taking complexity thinking as the theoretical framework. Finally, the chapter inquires into the impact of complexity thinking on mathematics learning and mathematics for teaching. This impact mattered for my interpretations on mathematics teachers' participation in the investigated PLN.

Complex systems are described as "large numbers of relatively simple entities [that] organize themselves, without the benefit of any central controller, into a collective whole that creates patterns, uses information, and in some cases, evolves and learns" (Mitchell, 2009, p. 4). Davis and Sumara (2012) suggest that complexity thinking — or "complexity-oriented research" (p. 30) — has shifted its emphases in recent decades. For example, Davis and Simmt (2016) specify that complexity thinking has undergone three phases: *complexity theory*, *complexity science*, and *complexity research*.

In the first phase, research focuses on the identification of complex systems such as by their common properties. In the second phase, research attends to the analysis of "similar roots, structures, and consequences" (p. 418) of systems, for example the recursive dynamics of their development and growth. And in the third phase, research attends to "[the] matters of scale-free connective networks" (p. 469) and their ways of enabling learning systems.

These three phases are presented in this study. They are also used to identify a PLN as a complex system (see next section: "The Common Properties of Complex Systems"), analyze the emergence of knowing from the conversations among participants (e.g., mathematics-for-teaching and other types of knowing; see Chapter 5), and present the evolving features involved in the emergence of knowing through networked images (Chapter 5). As such, this study relies on complexity thinking as its general theoretical framework.

3.1 The Common Properties of Complex Systems

Mitchell (2009) argues that the simple entities or parts of a complex system are irreducibly woven. At an abstract level, complex systems have three properties in common:

a) complex behaviours from the collective actions of an immense number of parts;

b) information processing through internal and external settings; and

c) adaptation through behavioural changes (i.e., learning or evolutionary processes).

Mitchell (2009) illustrates those properties with a number of examples, such as "insect colonies, immune systems, brains, and economies" (p. 4). In particular, she defines a complex system as "a system in which large networks of components with no central control and simple rules of operation give rise to complex collective behavior, sophisticated information processing, and adaptation via learning or evolution" (p. 13).

Mitchell's (2009) definition of complex systems and her identification of their common properties led me to propose that a professional learning network (PLN) is a complex system. This proposal is based on the following four aspects. First, a PLN is developed by participants who try to "ask advice, offer opinions, and engage in deep discussion" (Flanigan, 2011, para. 1) with other teachers. In a PLN, participants readily create the discussion topics that might relate to "lesson plans, teaching strategies, and student work, as well as collaboration across grade levels and departments" (Flanigan, 2011, para. 1), and which could be switched freely from one to another without any intention of control but with simple, certain rules of operation to follow. For example, participants who use blogs in a PLN are not to post content that "promotes or condones violence against individuals or groups," as clearly stated in the Blogger website (https://www.blogger.com/content.g?hl=en).

Second, a PLN allows collective learning to occur. Participants can interact with others by dealing with a concept, an idea, a problem, or an issue derived from their interests or from the online discussion forum in PLNs. From their online interactions might emerge some new and/or related concepts, ideas, problems, or issues, which could possibly enhance participants' understanding of the previously held concept, idea, problem, or issue.

Third, posts or their comments under discussion in a PLN always carry ideas or queries that can potentially engage participants to communicate with each other to generate the professional knowledge/knowing for mathematics teaching.

Fourth, participants' posts and interactions (comments) are the determinants of the survival and development of a PLN. Within a PLN, it is essential for the participants' needs to be met, their sharing to be responded to, and their voices to be heard. This could motivate them to participate in a PLN more voluntarily. The more contributions participants make to a PLN, the better they could advance its development. Therefore, a PLN must "learn" how to survive and thrive. In short, based on the above said features of a PLN and the definition and common properties of complex systems, it is reasonable to see a PLN as a complex system.

In addition to the definition and properties of complex systems, Davis and Simmt's (2003) research on the necessary conditions for complex systems will also facilitate my understanding of them and their applications to a PLN.

3.2 Necessary Conditions for Complex Systems

Davis and Simmt (2003) propose five necessary conditions for the emergence and maintenance of complex systems. These are: "internal diversity, redundancy, decentralized control, organized randomness, and neighbour interactions" (p. 147).

Internal diversity within a complex system acts as a source that potentially facilitates reacting to emergent circumstances. While it is impossible to specify in advance the necessary variations for intelligent reactions, it is necessary to ensure the presence of diversity. Davis and Simmt (2003) emphasize that "diversity can't be assigned or legislated, it must be assumed — and it must be flexible" (p. 149). For example, in a PLN Math Teachers Circle,[3] participants from different walks of life (e.g., students, teacher educators, mathematicians, math teachers, designers, technicians, etc.) bring in multiple and diverse viewpoints or experiences. This creates the internal diversity within the PLN.

Redundancy refers to "duplication and excesses" (p. 150) of features of agents with respect to particular emergent events in a particular complex system. Generally

[3] The PLN uses Twitter for "building professional learning communities of mathematicians and teachers." Retrieved May 8, 2019 from https://twitter.com/MathTeachCircle/followers?lang=en

speaking, it is the “similarity” (p. 150) among agents and their backgrounds or learning purposes that plays an essential role in a transition from “a collection of me to a collective of us” (p. 150). As Davis and Simmt (2003) point out, whether or not a system can maintain its coherence heavily depends upon the redundancy demonstrated among agents because this redundancy enables their interactions.

Thus, similarity among participants (agents) is necessary for such interactions and could be embodied in different ways. Take the PLN Math Teachers Circle as an example again: participants expected to promote the development of their professional knowledge in relation to mathematics, and they had similar learning purposes with a mutually intelligible language and common interests necessary for provoking and promoting their interactions.

Decentralized control refers to how a complex system organizes itself without a central controller. This means that the sustenance of a system is achieved through shared projects rather than by planning or management. For instance, participants in a PLN access a flexible online learning environment in their own way and make contributions as they desire, rather than follow preplanned or predetermined programs. Thus, their participation shapes and develops the PLN.

Organized randomness implies a structural condition that balances redundancy and diversity of agents. It is rule-bound within complex systems. However, the rule only demarcates “the boundaries of activities” (Davis & Simmt, 2003, p. 154) of agents rather than limits the possibilities of what might be generated from the activities. Within the systems, the structures are intended to delicately balance both “sufficient organization to orient agents’ actions and sufficient randomness to allow for flexible and varied responses” (Davis & Simmt, 2003, p. 155). This implies that every agent does neither “the same thing” nor “their own thing” but, rather, participates in “a joint project” (Davis & Simmt, 2003, p. 155). In a PLN, for instance, participants are able to discuss whether teaching is an art or a science, review the problem structures of the textbooks, construct the teaching of rational functions, solve the problems related to chord lengths, and so forth.

Neighbour interactions signify the interactions of agents within a complex system. Davis and Simmt (2003) underline that “neighbours” in mathematical classrooms, groups or communities are not “physical bodies or social groupings” (p. 156) but “ideas, hunches, queries, and other manners of representation” (p. 156) that “bump” against each other. To be more specific, the interactions that occur among the agents are “ideas, metaphors, and words” (p. 156). Without interactions, classrooms could not be conceptualized as complex systems. It is particularly true for a PLN that, because participants do not physically present themselves, their interactions are totally based on the ideas or viewpoints from their posts or comments.

This study applied the above five necessary conditions to the examination of the environment of the investigated PLN and is elaborated on in chapters 4 and 6.

3.3 Fractal Geometry

Researchers Davis and Sumara (2000) suggest fractal geometry, an alternative way of understanding the patterns underlying complex systems, calls for an alternative perspective with which to view curriculum and teacher learning. This also offers a new perspective to visualize teacher professional learning in PLNs. Therefore, it is necessary to briefly introduce fractal geometry and fractal curriculum beforehand.

Fractal geometry presents an alternative discourse about knowing and knowledge and challenges the pervasive assumption that complex phenomena are reducible to “root causes” or “basic components” (Davis & Sumara, 2000, p. 825). A fractal can be produced through the reiterative process of “establishing a rule, applying that rule to generate a result, reapplying the rule to that initial result, and continuously reapplying the rule to results” (Davis, 2005, p. 124). The process can be visualized as a growing tree image (Figure 3-1). In each generation of the image, two branches are grafted onto each limb that was produced in the previous generation. Such a process presents an iterative procedure: “at any particular level of computation, the new input is the output from the previous level” (Davis & Sumara, 2000, p. 827). The infinitely regressive, nested, and implicated fractal images challenge the fundamental, structural, and hierarchical images that are embodied in Euclid’s geometry (Davis & Sumara, 2000). The fractal images also help to structure curriculum differently from the dominant Euclidean curriculum.

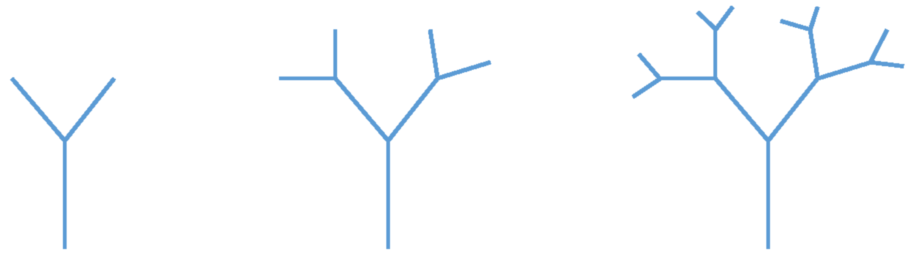


Figure 3-1. The first, second, and third iterations of a simple fractal tree (adapted from http://davis.wpi.edu/~matt/courses/fractals/trees.html).

Euclidean curriculum adopts a linear or spiral approach characterized as "bit-at-a-time" or accumulative instruction. Its structure would be problematic if learners encounter "messier, less delineated situations" and need to extend or apply their understandings to these situations (Davis & Sumara, 2000, p. 840). The fractal curriculum employs a nonlinear way characterized as "all-at-once or interpretive" (Davis & Sumara, 2000, p. 840). It is for opening a space to "talk about events simultaneously" and to consider "the importance of false starts, surprise turns and even-mounting complexity" rather than "steady progressions" advancing to optimality (p. 841).

In addition, a fractal curriculum also provides an alternative perspective to look at teacher learning. Learning now occurs not through "direct transmission from experts to learners" (Doll, 2012, p. 25) in a "sequential form" (Smitherman, 2004, p. 24) but through a "space of emergence" (Osberg & Biesta, 2008, p.326) in a "nonlinear manner" (Doll, 2012, p. 25). Thus, learners working in a fractal curriculum will not be asked only to reproduce or repeat knowledge but to work with others to generate knowledge (Davis et al., 1996). The curriculum could be generated by "linking pedagogical goals with the unpredictable behavior of learners" (Simitherman, 2004, p. 10). In Doll's (2012) words, it is "an emerging one within an ongoing process that actually catalyzes itself via interactions within the system or network" (p. 25).

3.4 Mathematics Learning and Teacher Professional Learning

The research on complex systems came to be recognized as a movement in the middle of the twentieth century (see Davis & Renert, 2014; Davis & Simmt, 2016). However, it is only in the last two decades that researchers and practitioners in the mathematics education community have started to pay close attention to complex systems. Some researchers pay attention to collective understanding in mathematics learning

(Davis & Simmt, 2016; Towers, Martin, & Heater, 2013) and compare learning to "coherence maintaining" (Davis & Renert, 2014, p. 28).

Davis and Simmt (2016) describe mathematics learning in the following way: "there is individual knowledge made public, there are a bumping up of ideas, there emerges a shared project, and there is collective action on a task that produces the emergent learner, and products attributed to the group" (p. 478). They argue that collective behaviour or learning is "neither the individual nor the group but the emergent product of actions and interactions" (2016, p. 477). Further, Davis and Renert (2014) identify some dramatic transformations to the attributes of mathematics learning. They discuss three related aspects:

- logical and analogical learning,
- surface and deep learning, and
- individual and collective learners.

Logical learning tends to structure the modern mathematics curricula under such assumption as "humans [are] logical creatures" (p. 28) just as we have taken it for granted that learning mathematics is to develop students' logical thinking. However, "knowledge" and "learning" are viewed from the perspective of complex systems as establishing "ever-more complex webs of connection" (p. 28). But most of these connections are not strictly logical — they are often analogical. Thus, Davis and Renert (2014) suggest that structuring mathematics learning must recognize "humans' penchant for analogical thought" and "humans' potential for logical thought" (p. 29).

Educators (Carpenter & Lehrer, 1999; Hiebert et al., 1997; Stylianides & Stylianides, 2007) also call for students to learn in a deep rather than superficial (or rote) way. Undoubtedly, many mathematics educators attempt to assist students in building connections among mathematics ideas, finding core arguments, hooking new knowledge to prior knowledge, and linking mathematics knowledge to daily life (Davis & Renert, 2014). On the other hand, it is also true that there is a disconnection between "such noble goals and the institutional structures" (Davis & Renert, 2014, p. 30) that result in students learning in a mechanical way. Thus, to take mathematics as "a warm and rich source of possible meanings and action[s]" (p. 31) enables students to reach a conceptual appreciation of the rationale behind a memorized procedure or fact.

Finally, collective learners are viewed as "coherence-maintaining, self-changing system[s]" from the perspective of complex systems (Davis & Renert, 2014, p. 30). For instance, the classroom collective could be regarded as both a collection of learners and a collective learner. In the classroom, students could work together to create a collective interpretation that goes beyond any individual interpretation. Davis and Renert (2014) therefore suggest that "the possibility for the individual learner and the collective learner [could] and should amplify one another" (p. 32). Indeed, it is important to underscore the collective dynamics of individuals instead of adding more structure to develop each individual's mathematics competence.

A collective understanding of mathematics learning helps me illustrate teacher collective learning in PLNs. The PLN in question is proposed as a learner (in complexity terms), since it demonstrates "collective, participatory engagements" (Davis & Renert, 2013, p. 33). The PLN can afford "instant access to information and connections to thousands of individuals with an array of expertise [in math teaching]" and can transform "professional development and learning opportunities for teachers" (Trust, 2012, p. 133) rather than transmit the static product derived from Euclidean architectures to teachers (Davis & Sumara, 2000).

For example, to know more about rational functions, a blogger created a post in which the blogger invited participants to think more about the concepts. In response to it, some of the participants actively offered interpretations about the concepts from various perspectives and experiences. Under the circumstance, the conversations between the blogger and the participants about rational functions enabled these aspects of the concepts to emerge: the multiple realizations, the rationale of graphing, the connections of different representations, the assessments, and the unsettling inquiries (e.g., the history of rational function and the impact of technology upon the understanding of the concepts).

Understanding those emergent aspects of the concepts from the conversations was unique and could not be found from any kind of authorized documents or research literature. The "all-at-once or interpretive" (Davis & Sumara, 2000, p. 840) understanding provides a different way of knowing the emergent sensibilities and recognises "the context and the immediate" (p. 843) as well as the individuals. The individual contributions are the demonstrations of individual “internal understanding” (Davis & Sumara, 2000, p. 831).

Interactions among the participants can be considered as collective learning that enables multiple aspects of rational functions to be explored and connected as a web. This means that when the interactions occur, the exploration of multiple aspects of the concepts and the web of the concepts are viewed as “understanding” (Davis & Sumara, 2000, p. 831). Therefore, it is through the interactions of the group that knowing occurs, which “open[s] up a range of new possibilities” (Davis & Sumara, 2000, p. 830).

Emergent understanding in a learning group is not the aggregation of individual understanding. Rather, “the individual is embedded in the collective” (Davis & Sumara, 2000, p. 832). In the example of rational functions, the understanding of both the multiple aspects and the web of the concepts is considered collective construction. However, individual contributions are not simply added to the collective understanding but actually function as a trigger, an element, a context, or a disposition to the collective understanding. Learning in online PLNs shifts the professional learning diagram from delivery-based modes towards participatory and inquiry-based ones (Brooks & Gibson, 2012; Laferrière, Lamon, & Chan, 2006) and from teachers as recipients towards teachers as (co) producers of professional and scientific knowledge (Kieran, Krainer, & Shaughnessy, 2013).

3.5 Mathematics-for-Teaching

Extensive research has been conducted on teachers’ disciplinary knowledge of mathematics for decades. Here, I focus only on the research from a small group of highly cited scholars who loosely form subsets of this field.

3.5.1 The character of mathematics-for-teaching

Ma’s (1999) work presents the character of teachers’ disciplinary knowledge of mathematics through understanding teachers’ mathematics content and their teaching practice. She describes the particular character of that knowledge by using “profound understanding of fundamental mathematics” (PUFM). However, after reviewing the term “fundamental,” which has been interpreted as “foundational, primary, and elementary” in Ma’s (1999, p. 116) work, Davis and Renert (2013) criticize the term based on complexity thinking as suggesting “a closed set of insights and understanding” (p. 247).

Additionally, they argue that teachers' disciplinary knowledge of mathematics cannot be considered as a defined and well-connected set of basics, but as "a sophisticated and largely enactive mix of familiarity with various realizations of mathematical concepts and awareness of the complex processes through which mathematics is produced" (p. 247). Accordingly, Davis and Renert (2013) develop the notion for teachers' disciplinary knowledge of mathematics as "profound understanding of emergent mathematics" (p. 247), which highlights "emergent" as different from "fundamental" in the sense of vast not limited, intricate not straightforward, and evolving not static. In this case, mathematics-for-teaching (M_4T) is taken as a term to mark the distinct character of teachers' disciplinary knowledge of mathematics (Davis & Renert, 2013; Davis & Simmt, 2006). M_4T is defined as:

> A way of being with mathematics knowledge that enables a teacher to structure learning situations, interpret student actions mindfully, and respond flexibly, in ways that enable learners to extend understandings and expand the range of their interpretive possibilities through access to powerful connections and appropriate practice. (Davis & Renert, 2014, p. 4)

3.5.2 The categories of mathematical knowledge for teaching

Ball and colleagues propose what has become known as, in the mathematics education community, domains of knowledge of mathematics for teaching (KMT) (e.g., Ball & Bass, 2009; Ball et al., 2008). The domains are split into two categories: Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), which are mapped as neighboring domains (Figure 3-2).

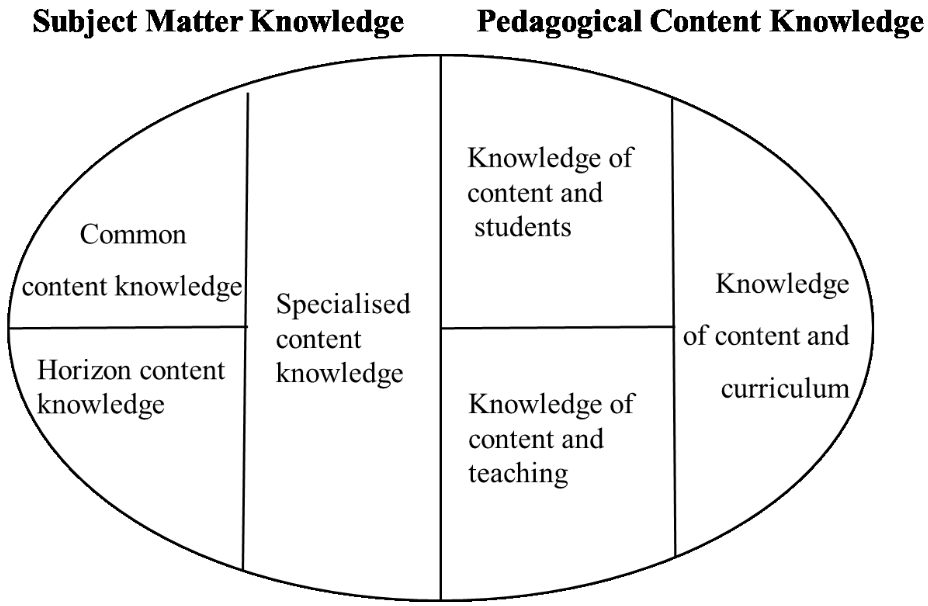


Figure 3-2. Domains of Knowledge of Mathematics for Teaching *(adapted from Ball et al., 2008, p. 403).*

The above structure of KMT distinguishes Subject Matter Knowledge (SMK) from Pedagogical Content Knowledge (PCK). SMK embraces three domains: Common Content Knowledge, Specialized Content Knowledge, and Horizon Content Knowledge. Common Content Knowledge is defined as “the mathematical knowledge and skill used in settings other than teaching” (Ball et al., 2008, p. 399). In other words, while teachers are capable of working out the assignments they assign to their students, the required knowledge and skill of the assignments is not special to teaching but is applicable to a variety of settings.

Specialized Content Knowledge is defined as “the mathematical knowledge and skill unique to teaching” (Ball et al., 2008, p. 400). Ball et al. (2008) further elaborates that teachers must work with mathematics in “decompressed or unpacked form” (p. 400), which is not necessary in settings other than teaching. Horizon Content Knowledge is regarded as “an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball et al., 2008, p. 403). This sort of knowledge could help teachers set the mathematical foundation for students’ later mathematics learning.

PCK includes the domains of Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum. Knowledge of Content and Students combines “knowing about students and knowing about mathematics” (Ball et al., 2008, p. 401). Within this domain, mathematical tasks of teaching require the teacher to integrate specific mathematical understanding with understanding of students and their mathematical thinking.

Knowledge of Content and Teaching incorporates “knowing about teaching and knowing about mathematics” (p. 401). Within this domain, mathematical tasks of teaching also require teachers to integrate specific mathematical understanding with pedagogical considerations that could impact student learning. Knowledge of Content and Curriculum mainly relates to Shulman’s (1986) curriculum knowledge, which is represented by designed programs, instructional materials, and indications/ contraindications for curriculum usage. Within this domain, mathematical tasks of teaching require the combination of knowing about curriculum and knowing about mathematics.

In collaboration with practicing teachers, Davis and Simmt (2006) generate a different image (see Figure 3-3) from the neighbouring domains (Figure 3-3) to highlight the intertwining categories of mathematics-for-teaching: mathematical objects, curriculum structures, classroom collectivity, and subjective understanding. Mathematical objects are relevant to concepts. Emergent from their interactions, teachers' ideas/thoughts about a concept do not simply add to what they have known, but integrate and are integrated into what they have established. The integrations are regarded as conceptual blends, which allow teachers to approach the web of interconnections of a concept. In my study, mathematical objects are also relevant to mathematical problems.

Curriculum structures highlight the presentation and elaboration of concepts across the grades in the curriculum. In my study, they also relate to the problem structures of textbooks, the prerequisites for concept learning, and the various levels of mathematics involved in problem solving. Classroom collectivity attends to the contexts in which teachers are engaged in the collective production of new possible interpretations. Classroom communities or learning groups are collective learners.

Subjective understanding, a prominent topic in mathematics education research in the past decades, focuses on individual understanding. In my study, it relates to student thinking and student conceptual understanding or misunderstanding.

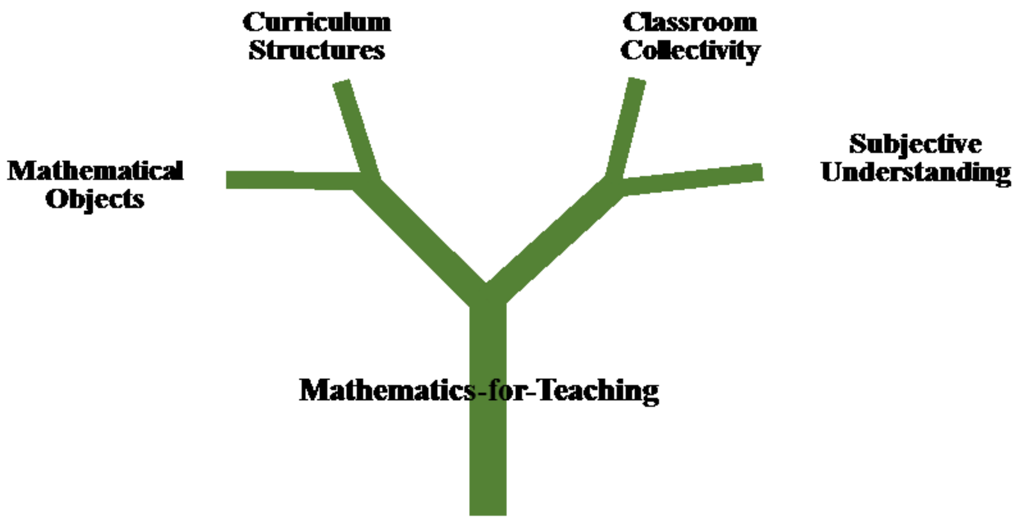


Figure 3-4. Perceived relationships among some aspects of teachers' mathematics-for-teaching *(adapted from Davis & Simmt, 2006, p. 298).*

Davis and Renert (2014) present the distinction between the categories of KMT and of M4T. The distinctions lie in that M4T signals the differentiation of *knowledge* from *knowing*. Mathematical objects and curriculum structures are described as relatively stable knowledge because it evolves "at a pace and a scale" (Davis & Renert, 2014, p. 90) while classroom collectivity and subjective understanding are characterized as "volatile and unstable" knowing (Davis & Renert, 2014, p. 90). However, KMT emphasizes that all the content domains are regarded as knowledge, or in Shulman's (1987) words, "the knowledge base" (p. 8).

Overall, my study adopted mathematics-for-teaching as a model to analyze what emerged from the collective interactions in the investigated PLN. The full details are presented in Chapter 4: "Interpretive Inquiry" and Chapter 5: "The Conversations and the Emergence of Knowing in the Illustrative Examples."

4. Interpretive Inquiry

This chapter elaborates on interpretive inquiry theoretically and practically and, in particular, on how this methodology was embodied in a PLN through data collection and analysis. It also addresses ethical considerations and clarifications for the research.

4.1 Interpretive Inquiry as a Methodology

My theoretical understanding of interpretive inquiry came mainly from Ellis' research (e.g., Ellis, 1998a; Ellis, 1998b; Ellis, 2006; Ellis, Janjic-Watrich, Macris, & Marvnowski, 2011) and her course, *Interpretive Inquiry* (Winter, 2016). Learning from her ideas helped me understand the nature of interpretive inquiry and informed me how to work through my research with interpretive inquiry as a methodology.

Interpretivists see research as "an eminently practical and moral activity" (Smith, 1992, p. 100) to the effect that interpretive inquiry dispenses with long-standing positivism as what is understood as rational is no longer confined to what is understood as scientific (Roth, 1987; Turner as cited in Smith, 1992). In addition, interpretivists do not believe that there is a bottom line or foundation upon which knowledge is constructed, and neither is there a privileged approach or position by which we can understand the targeted social phenomena (Smith, 1992). In other words, Smith (1992) suggests that interpretive inquiry interprets the interpretations that people display through their actions or interactions with social phenomena. This position is further demonstrated by Ellis (1998a) who believes that "there is no reality 'out there,' no meaning or knowledge waiting to be disclosed to the 'mind's eye,' until the act of understanding brings it into being" (p. 7).

I found myself searching for a correct method or a right way to follow after I decided to use interpretive inquiry for understanding mathematics teachers' participation in a PLN. I wanted to achieve "valid results" or do "accurate interpretations" which, as Ellis (1998a) indicates, are "part of the legacy of the positivist and post-positivist traditions" (p. 7). Derived from the natural sciences, the research paradigm of positivism and post-positivism emphasize the "efforts to verify (positivism) or falsify (post-positivism) a priori hypotheses" (Guba & Lincoln, 1994, p. 106).

Positivists objectively observe what happens in the world. However, the scientific method still leaves people in uncertainty and confusion (Guba & Lincoln, 1994). Thus, interpretivists move from "a natural sciences preoccupation with *explaining* to a humanities interests in *understanding*" (Ellis, 1998a, p. 7), which suggests that people have come to realize that it is more plausible to interpret social phenomena based on the interactions between the researcher and the observed phenomena rather than on observations in an objective approach.

4.1.1 The goals of interpretive inquiry

Interpretive inquiry is conducted to develop an understanding that is "more informed and sophisticated" than the previously held understanding (Guba & Lincoln, 1994, p. 112). In that sense, I used interpretive inquiry to understand participants' actions/interactions more intensely, carefully, and self-consciously when I felt the ambiguity in their meanings or reasoning. Packer and Addison (1989) discern that the vagueness or preliminary understanding of events embodies a particular concern and caring, which invites a possible reading, an initial access, a preoccupied stance or perspective (a fore-structure) to open up the field for inquiry.

My preliminary concerns when I initiated this study were about what mathematics teachers did and what could emerge from their doings in PLNs. These two questions, on which few studies have focused, invited me to step into the research field of PLNs. Thus, I began to review the related literature, reflect on my own experiences of encounters with PLNs, and prepare to inquire into a targeted PLN. When I began my inquiry into the field of the targeted PLN, my concerns directed me to attend to participants' doings (e.g., blogging and/or commenting); when I went through their texts in blogs, I realized that the very nature of their doings was through interactions. This realization led me to further inquire about what the interactions among mathematics teachers looked like and what could emerge from their interactions in the PLN. These inquiries led me to my actual research questions as specified in Chapter 1.

4.1.2 The way to interpretive inquiry

I wondered what interpretive inquiry would look like with respect to my study. Smith (1992) claims that interpretivists take self-inquiry and self-reflection as a doable way to proceed with the inquiry and that interpretations could vary from setting to setting.

In other words, as Smith (2006) suggests, no presumption of correctness can be followed in advance of my inquiry. Therefore, how to proceed with my inquiry was not a predetermined procedure but an ongoing process (Peshkin, 2000).

4.2 Finding the Path to Inquiry

Ellis (1998b) also provides suggestions on how to find a possible path to my inquiry. Starting with interpretive inquiry is daunting for novice researchers since there is no orientation toward or clear-cut destination to it. Considering this situation, Ellis (1998b) uses her experiences of doing interpretive inquiry projects to illustrate how to make a possible path. She calls for "mak[ing] the path by walking it" (p. 16), with the walking starting from the entry question.

4.2.1 The entry question

Ellis (1998b) suggests starting with entry questions that embody researchers' "openness, humility, and genuine engagement" (p. 18). These questions cannot be for abstract debate or from the perspective of certain positions on issues. Rather, the questions have to be real ones that engage the inquirers in exploring what they care about. And as for the questions, Ellis (1998b) emphasizes that the inquirers know neither their answers nor how to get the answers, which is helpful for the inquirers' position on the issues.

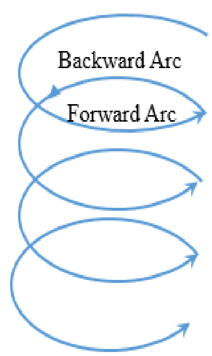
In the early stage of approaching PLNs, I was curious about the questions of "what mathematics teachers do in PLNs" and "what could emerge from their doings in PLNs?" These questions came naturally when I noticed more and more teachers were participating in PLNs. Having reviewed the related literature, I found that the questions were not yet attended to by the mathematics education research community. Accordingly, setting them as my original research subject matter in the first draft of my candidacy paper outline, I started to re-read the literature and browse PLNs to approach the original questions. However, I had to face the following challenges: how to continue the inquiry, where I should go, and whether or not I could find the answers to the questions. Fortunately, Ellis' (1998b) viewpoint enabled me to realize that these were my entry questions, that they were to engage me in advancing my inquiry, and that a possible way of advancing my inquiry was to enter spiral loops and/or a hermeneutic circle as an ongoing conversation with the phenomena of mathematics teacher participation in PLNs.

4.2.2 An unfolding spiral

Ellis (1998b) visualizes the interpretive inquiry process as "a series of loops in a spiral" (p. 19) (see Table 4-1). Her model demonstrates that each loop might express a separate but coherent exploration, which may include one effort of "data collection and interpretation" (p. 19). When an inquiry goes through a series of spiral loops, each loop might illustrate a separate attempt to get closer to what one cares about and wants to understand. Once inquirers enter into a loop with questions, confusion, or curiousness, what they explore in that loop might further direct or reframe more questions, confusion or curiousness for the next loop.

Table 4-1

The unfolding spiral (adapted from Ellis, 1998b, p. 20)

Visualized Spiral	Descriptions
	"Each loop may represent a separate 'data collection and analysis' activity or a return to a constant set of data with, however, a different question" (p. 20). "The question for each new loop has been influenced by what was uncovered in the inquiry represented by the previous loop" (p. 20).

Since what one might learn from one loop is emergent, Ellis (1998b) points out that it is possible to have a dramatic turning point after the loop, such as replacing one concern by another, because one's understanding of the question and/or its related contexts change. However, what will be explored in the next loop cannot be predetermined.

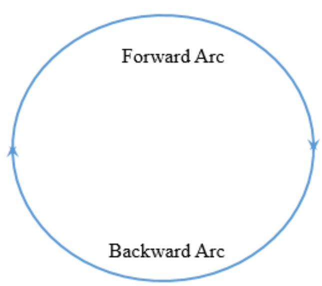
Looking back at the whole process of my encounter with PLNs in the projects about teachers' online professional learning as indicated in Chapter 1, my understanding of PLNs was always intertwined with wondering and re-interpretation (further understanding). This shaped the unfolding spiral loops.

4.2.3 The forward and backward arcs

Alternately, Ellis (1998b) realizes that one might make consecutive attempts to reinterpret one set of data in one loop rather than move onto the next one after the first effort of data collection and interpretation. This implies that unfolding spiral loops do not always occur. However, some studies might only present a single loop with only one set of data. With regard to these studies, Ellis (1998b) suggests that researchers might make several repeated loops to reinterpret the same data each time according to the reframed questions from what they learned from the previous set of exploration. Therefore, the whole inquiry process is constantly "cycling in questioning and understanding" (Mayers, 2001, p. 12), which is illustrated by "the forward and backward arcs of the hermeneutic circle" (Ellis, 1998b, p. 26) (see Table 4-2).

Table 4-2

The hermeneutic circle (adapted from Ellis, 1998b, p. 27)

Visualized Circle	Descriptions
	**Forward Arc**: Entails making sense of a research participant, situation, or a set of data by drawing on one's fore-structure, which is the current product of one's autobiography (belief, value, interests, interpretive framework) and one's relationship with the question or problem. (p. 27) **Backward Arc**: Entails endeavoring to see what went unseen in the initial interpretation resulting from projection. The data are re-examined for contradictions, gaps, omissions, or confirmations of the initial interpretation. Alternate interpretive frameworks are purposefully searched for and 'tried on.' (p. 27)

In a forward arc, researchers draw on their fore-structure and pre-understanding to "make sense of what is encountered" (Ellis et al., 2011, p. 12), while in a backward arc, they will re-examine the previous interpretation for "contradictions, gaps, or material not adequately explained by the [previous] interpretation" (p. 12). The backward arc aims to develop "the most adequate interpretation" (p. 12) so that all that is uncovered is interpreted or understood clearly enough.

Considering the role of the hermeneutic circle in interpretive inquiry, Ellis (1998b) poses that what one understands is based only on what one already knows; hence, the hermeneutic circle may be viewed as "tautological" (p. 29). But "a circularity of understanding is essential" (Packer & Addison as cited in Ellis, 1998b, p. 29). In my study, the circulatory understanding brought about different layers of interpretations of math teacher participation in the targeted PLN.

Researchers quite often combine unfolding spiral loops with a single-loop or a hermeneutic circle to achieve the part–whole relationship (Ellis, 1998b). Similarly, in this study, I blended the spiral loops with the hermeneutic circle to shape the path of my inquiry, including targeting a PLN, attending to the preferred data, collecting the data, analyzing the data, and so on.

4.2.4 Uncovering

It could not be predicted what emerges from each loop or exploration. To put it differently, some findings might be unexpectedly obtained from one loop or exploration. In Ellis' (1998b) model, these unexpected findings or dimensions are called *uncoverings*. For instance, when I discovered that the nature of participants' doings in a PLN was their interactions, I commenced my inquiry into what the interactions among mathematics teachers looked like and what could emerge from their interactions in the PLN. Unexpectedly, after my preliminary examination of their interactions, I ascertained that varied structures and abundant communications on mathematics teaching occurred within them.

Thus, unexpected findings led me to review the original questions and ask myself about what the structures of the conversations among mathematics teachers looked like and what could emerge from their conversations in relation to mathematics-for-teaching. Later, those two questions became my new research subject matter, which I addressed by my exploration of the structures of the interactions and the emergent knowing from selected illustrative examples (see Chapter 5). The answers to these questions, of course, were not mapped directly to my original questions, but they revealed the essential aspects of my original inquiry.

Another example of *uncoverings* appeared during the process of selecting examples from one targeted PLN. I expected to find multiple examples within the PLN to illustrate math teachers' participation. At the beginning of selecting the examples, I presented the diversity of the examples mainly through the different topics (e.g., concept understanding, problem solving, or learning tasks) involved in my chosen examples. Unexpectedly, however, after analyzing the first selected example, I found recursions of interactions occurring within it, and thus used them as a new dimension to select other examples for the further exploration of diversity.

As indicated by Ellis (1998b), these uncoverings helped me better understand the nature of my research questions or data and to frame the next loop(s) or circle(s) towards the very inquiry even though they did not directly result in solutions or answers to the inquiry.

4.3 Concerns Over Interpretive Inquiry

Evaluating an interpretive account is a persisting concern with interpretive inquiry. Issues about validation have consistently dominated perspectives about evaluation because "the positivist model of natural sciences" (Packer & Addison, 1989, p. 276) has been uncritically applied to the field of evaluation. However, evaluation in interpretive inquiry does not follow the predominant evaluation path. An interpretive account is not used to work out "validated knowledge or timeless truth" (Packer & Addison, 1989, p. 279) but "possibilities that have become apparent in a preliminary, dim understanding" (p. 277) of social phenomena. Accordingly, it is not considered a conjecture or guess at all, and neither should we treat evaluation as "testing a hypothesis" (Packer & Addison, 1989, p. 278).

Rather, evaluating an interpretive account is to advance "our concerns" about the phenomena we study (Packer & Addison, 1989, p. 279). Real concerns always start interpretation, and our engagement does not lie in pure truth (Packer & Addison, 1989). Therefore, an attempt to evaluate an interpretive account is often considered in light of whether "it reveals a solution to the difficulty that motivated the inquiry" (p. 29) or "our concern has been advanced" (Ellis, 1998b, p. 30) rather than whether or not the interpretation is validated.

As for the interpretation process, Ellis (1998a) expresses a concern that researchers, in particular novices, might hesitate to discuss the interpretation process at their early stage of research because their own interpretations might change over time. Indeed, it is known that our standpoints, pre-concepts, pre-understandings, or prejudices might confine what we can see at any given time and place. However, these dominant viewpoints or prejudices might change gradually when we are in contact or have a dialogue with the texts we care about. This might be able to "transform one's initial interpretation or understanding and gain new insight" (Ellis, Hetherington, Lovell, McConaghy, & Viczko, 2013, p. 491). Since language and interpretations are interconnected, no final or fixed language can express the changes and the interpretations.

In my study, the later data analysis always brought me some new perspectives with which to review the earlier data; this resulted in a re-understanding of the previously collected data. For instance, the perspectives resultant from my earlier data analysis for the first selected example have changed many times because my subsequent analyses for the other examples provided me with new perspectives to re-analyze the earlier data.

4.4 Interpretive Inquiry and Complex Systems

As mentioned in the previous section "Finding the Path to Inquiry," there is no prescribed procedure in interpretive inquiry to follow. This is because the path to the inquiry emerges from the process of data collection, data understanding, and data analysis. In particular, what emerges, or what one learns from the interpretive process of a spiral loop and/or a hermeneutic circle, will direct the exploration of the next loop and/or circle. This process could be appropriately characterized as "adaption via learning or evolution" (Mitchell, 2009, p. 13) in complex systems.

Furthermore, both interpretive inquiry and complex systems underscore the part–whole relationship. In particular, they lay much more emphasis upon the interplay of part and whole. Ellis (1998b) underlines that "to understand a part, one must understand the whole, and to understand the whole, one must understand the individual part" (p. 16). Mitchell (2009) also argues that parts are irreducibly woven into a system and that "the whole is more than the sum of its parts" (p. x). In short, interpretive inquiry was a methodology suitable for my study in terms of its purpose and theoretical framework — complexity thinking.

4.5 Interpretive Inquiry in a PLN

This section elaborates on the application of my methodology to this study as well as on data collection and analysis. Data collection involved a process of selecting rich and diverse data while data analysis was a process represented by interpretive cycles, with several particular analysis emphases:

- recursive dynamics,
- thematic analysis,
- the model of mathematics-for-teaching, and
- necessary conditions for complex systems.

Collecting the data simultaneously involved analyzing the data during the building up of the criteria for data selection since interpretive inquiry establishes no clear-cut boundary between data collection and data analysis.

4.5.1 Participants

It should be noted that participants in this study were referred to as "bloggers" and/or "commenters of blog posts"[4] (abbreviated to posts) in the targeted PLN. Here, a blogger was defined as someone who wrote the content of one or more posts for a blog, while a commenter was defined as someone who made a comment on a blogger's post.

4.5.2 Targeting a professional learning network

A PLN is theoretically championed as an anytime and anywhere option for flexible professional pursuits (Stanford-Bowers, 2008). It allows teachers to freely choose what they want to do within and could help to customize their professional learning. For example, teachers can participate in a PLN based on their individual interests and needs during the day or the night, at home or school or in any other place; they can also upload and download related files, post on interested topics to invite other teachers' ideas or comments on related topics, and they can also work on the problems that might arise from their students' learning in the classroom, parents' feedback on their children's

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[4] Blog Posts, "informal diary-style text entries", constitute a blog as "a discussion or informational website published on the World Wide Web" (para. 1). Retrieved May 5, 2018, from https://en.wikipedia.org/wiki/Blog

learning, colleagues' suggestions on certain issues, or the books/articles they enjoyed reading. In short, teachers can personalize their own professional learning through a PLN at any time and in any place. These possibilities are represented in Figure 4-1.

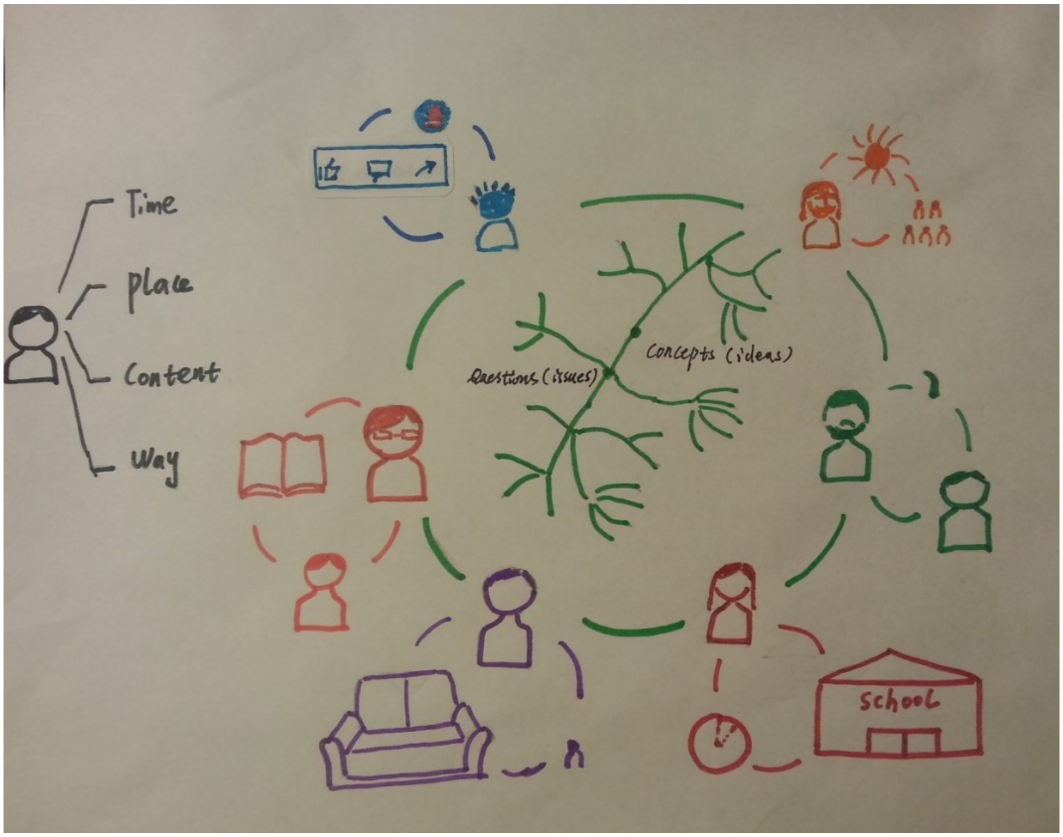

Figure 4-1. The diagram of mathematics teachers' professional learning in a PLN.

Professional learning within a PLN involves collective work, which is indicated by the branches within the main circle in Figure 4-1. According to their interests, individuals can work together with their counterparts on a concept, an idea, a problem, or an issue, which is discussed in the open space of a PLN. From the individuals' discussions with other participants could emerge completely new or related concepts, ideas, problems, or issues, which in turn might advance their understanding of the previously-held concepts, ideas, problems, or issues, or elicit other discussion topics.

This study targeted a particular PLN — *mathtwitterblogosphere* — that focuses on mathematics teaching and learning. Before I made that decision, there were three reasons to be considered. First, the targeted PLN presented the theoretical features of a

PLN illustrated in the diagram of Figure 4-1; second, it followed the specific characteristics of PLNs: "participation is open to anyone; conversations are held in public; participation is voluntary; topics of conversation are created by participants themselves; and participants are free to come and go" (Schwier & Seaton, 2013, p. 3); and third, I presumed it would be able to provide "rich data" (Morrow, 2005, p. 255) for my research based on my experiences of doing research on teacher professional learning.

The PLN website consisted of four modules: the first was intended for participants, the second offered reasons to participate, the third explained how to participate in the blog, and the fourth presented an advice module. The first module displayed the videos created by participants to introduce themselves and presented a participant location map available to anyone. The map (my last visit was on July 18, 2018) showed the majority (88%) of participants were from North America, 6% from Europe, 4% from Asia, and 2% from Oceania. The second module showed several activities that participants did together, such as using visual patterns to stimulate discussion about mathematics; it also listed some interesting posts to illustrate the grounded goodness that blogs were able to offer. The third module elaborated on how to join the PLN and categorized Twitter and blogs into two groups: *Academic* and *Interest*. These were publicly accessible to anyone. The last module provided some suggestions/examples for tweeting, blogging, or protecting privacy. These modules offered a useful reference for the people who expected to participate in the PLN.

4.5.3 Data attention

The blogs of the PLN. Twitter and blogs were the core parts of the PLN. Following either/both of them was the main path to participation in the PLN. In this study, I focused only on the analysis of blogs. I did so for two reasons: a) not enough time was set for me to deal with both types of data simultaneously and b) I assumed the blogs had more space for participants to explore ideas than Twitter, which allows a limited number of characters for each entry.

In addition, I centered only on the analysis of blogs under the category Academic because I held that academic-based mathematics far outweighed interest-based mathematics in relation to mathematics teaching and learning. This type of blog was therefore prioritized as my major data source.

Posts and their comments. Blogs consist of two major archived document types: posts and their comments. Posts are taken as being of "shared interest and importance to [participants'] teaching and students' learning" (Restivo, 2012, p. 43) and are logged as recorded history. They are frequently followed by comments. Comments are often regarded as "a great way to exchange ideas, thoughts, or opinions about what people feel for a particular topic or a blog post" (RankWatch, n.d., para 2) and can be metaphorized as "fuel" for blogs (RankWatch, n.d., para 3). They are the elements of conversations between bloggers and commenters.

Posts and their attached comments do not operate independently within a blog because they are connected with other internet websites through at least four kinds of pathways. First, blogs often have built-in links that connect to other websites such as other blogs and to Twitter through blogroll, Twitter ID linkage, and other followers. This means that each post and its comments are associated with blog networks.

Second, commenters also automatically bring in the hyperlinks to their commented blogs or Twitter once they post their comments. That is because once published, their comments will be hyperlinked automatically to their own blog websites or Twitter IDs. To illustrate this, I created an example of graphing rational functions (Figure 4-2). In this example, when the commenters, A. Frank and C. Liu, provided their comments following the post, their names were automatically hyperlinked to their social networks such as blog or Twitter (their names were underlined in Figure 4-2). This meant that the commenters' networks were associated with the original post through their comments.

Third, a pingback [5](e.g., How to introduce rational functions in Figure 4-2) as a type of "remote" comment (BESTWEBSOFT, 2015, para. 2) can link one post to another site such as another blog or a tweet. In the current example, the pingback How to introduce rational functions (Figure 4-2) linked the post to another post, "How to Introduce Rational Functions,*"* which was called the pingback blog post (shortened as

[5] A pingback is "a special type of comment that's created when you link to another blog post, as long as the other blog is set to accept pingbacks" (para. 1). Retrieved April 24, 2018, from https://en.support.wordpress.com/comments/pingbacks/

pingback post). Thus, such pingback activated the post to connect with other cyberspaces, which were nodes of a huge network.

And finally, the hyperlinks or websites embedded in the post (e.g., "Ann has some colorful diagrams at her site" [Figure 4-2]) or in the comments (e.g., link http://blog.anniefrank.com [Figure 4-2]) can also relate the post and its comments to other cyberspaces, which in turn can strengthen the ideas from the post and its comments.

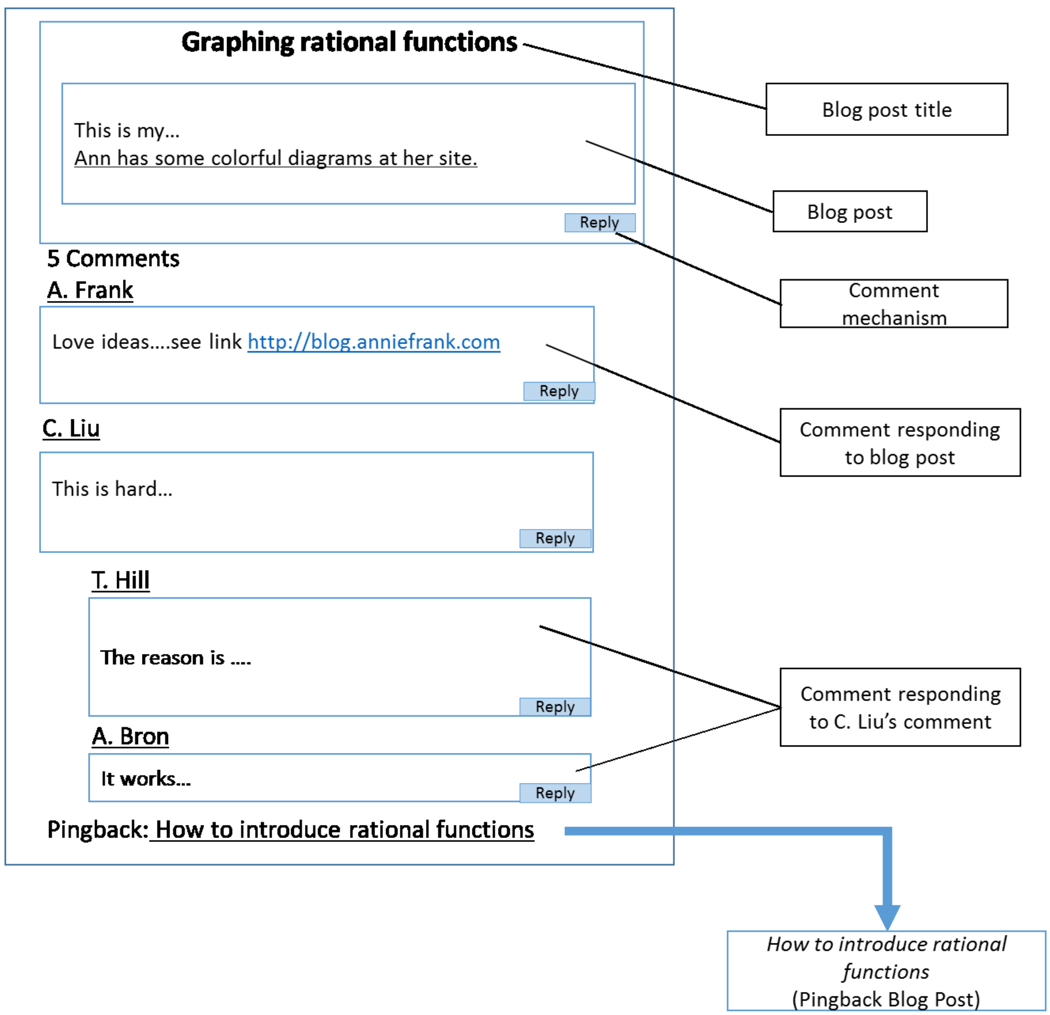

Figure 4-2. A diagram of the post of graphing rational functions and its comments.

The above description showed how posts and their comments were associated with other sites in multiple ways. The first two pathways of associations (i.e., the blogs' built-in links and the automatic hyperlinks of commenters) were not addressed in this study because the study attended to the interactions directly from posts and their comments.

4.5.4 Data collection

It was impossible to make an exhaustive collection of all the posts and their comments from the targeted PLN because they were dynamic, changing all the time. It was also unlikely to analyze all the collected posts and comments because time and space were limited for this study. Therefore, I chose some illustrative examples from them for further analysis. For the purpose of the research, I selected examples with diverse and rich conversations and established the selection criteria for them. However, I could not anticipate what types of and how many examples would form the subject matter because selecting them was also considered a part of data analysis in interpretive inquiry. Thus, the criteria, though not absolute, were true, that is, not pre-determined.

A large post pool. The data collection commenced with building up a large pool of posts. The pool accommodated posts and their comments transcribed directly from blogs in the PLN. A post and its comments were first transcribed and then saved together as a unique file. The existent posts were mined and about four hundred files were transcribed from blogs and stored in the pool.

Upon establishing the large post pool, I classified the posts into 11 categories based on the explored topics of their contents (Table 4-3). These topic categories, though not exhaustive, presented a “big picture” about what mathematics teachers explore in the PLN and provided a framework from which I was able to strategically select the illustrative examples for my data analysis.

Table 4-3

Emergent topic categories and their meanings (derived from the contents of the posts)

Topic Categories	**Meanings**
Concept understanding	Designing learning activities or highlighting the implicit ideas for better understanding of concepts
Teacher beliefs	Focusing on values, ideas, or viewpoints that teachers hold in their teaching, or experiences generated from their practices, which could provide references for themselves and others
Curriculum	Focusing on teaching materials from textbooks or theoretical reflections on curriculum
Problem solving	Solving problems or discussing how to teach problem solving
Teaching practice	Sharing general teaching plans or strategies, or specific learning activities or experiences without explicit purpose for facilitating concept understanding or problem solving
Student learning	Attending to strategies of engaging students, learning through errors, or focusing on student learning difficulties
Assessment	Grading, providing feedback, or reviewing student learning
Parental involvement	Inviting parents to visit the classroom, or reflecting on experiences with parent volunteers, or tackling parent complaints
Professional development	Introducing professional development programs, providing suggestions or references for others' professional learning, and self-reflecting on teaching practices for professional growth
Life	Focusing on teachers' life out of classroom teaching
Technology	Reviewing or experiencing technology in math teaching

Criteria for selecting illustrative examples. Two criteria were used for selecting illustrative examples from the large post pool. One was about the topics involved in the examples. It was set so that the illustrative examples should be selected from different topic categories to ensure their diversity. The other was about the depth of the conversations involved in the examples. The conversations among participants were revealed by the comments following the original posts. In other words, the richness of comments could uncover the depth of the conversations. Here, the word "richness" had double meanings: high quantity and high quality of the comments. The former referred to the number of comments following up an original post while the latter referred to the contents of the comments centring on ideational interactions or arguments rather than "gibberish" comments seemingly made for fun or advertisement. Thus, to explore high quantity and quality comments was the other criteria of selecting the illustrative examples.

Selection process. Selecting the illustrative examples involved the following four steps.

Step 1: Rename each file in the large post pool by using the total comment number as well as the file's abbreviated title. For instance, I renamed the first file (Figure 4-3) under the topic category *problem solving* as *6 IC*, with 6 referring to 6 comments that followed the post and with IC referring to the abbreviation of the post title to avoid disclosure of the traceable clues to the post.

6 IC	2017/1/19 14:09	Microsoft Word ...	213 KB
6 SP	2017/1/17 22:39	Microsoft Word ...	140 KB
8 MPSPMC	2017/1/12 14:17	Microsoft Word ...	45 KB
8 PPG	2017/1/17 22:47	Microsoft Word ...	129 KB
8 PWS	2017/1/15 22:14	Microsoft Word ...	129 KB
8 SSP	2017/1/17 22:44	Microsoft Word ...	61 KB
9 BB	2017/1/16 9:40	Microsoft Word ...	2,933 KB
9 DTN	2017/1/12 13:55	Microsoft Word ...	3,302 KB
9 FYPM	2017/1/17 22:10	Microsoft Word ...	218 KB
9 PMR	2017/1/15 22:49	Microsoft Word ...	4,668 KB
9 PSVS	2017/1/18 22:48	Microsoft Word ...	43 KB
9 UCK	2017/1/19 0:02	Microsoft Word ...	372 KB
10 ASG	2017/1/17 22:07	Microsoft Word ...	426 KB
10 IQ	2017/1/18 22:34	Microsoft Word ...	455 KB
10 MMM7S	2017/1/18 23:44	Microsoft Word ...	41 KB
10 OCP	2017/1/22 22:06	Microsoft Word ...	765 KB
10 S	2017/1/18 23:42	Microsoft Word ...	35 KB
10 UMPEGK	2017/1/16 9:40	Microsoft Word ...	343 KB
10 WAQQ	2017/1/18 23:40	Microsoft Word ...	39 KB
10 WDIMBG	2017/1/19 13:27	Microsoft Word ...	111 KB
10 WDWYDKWD	2017/1/12 16:48	Microsoft Word ...	498 KB
11 AGPS	2017/1/18 23:54	Microsoft Word ...	86 KB
11 ASET	2017/1/18 22:35	Microsoft Word ...	78 KB

Figure 4-3. Partial file list of the topic category of problem solving.

Step 2: Build a small post pool by selecting the posts according to density of comments. I chose the bottom half of the files in the new list under each topic category to build a small blog pool in which the posts were still folded under the same topic categories as had been done in the large one. Thus, the small post pool included posts that had a high quantity of comments.

Step 3: Create a core post pool by selecting potential examples. The selection was based on my personal preference and prejudice rather than on an established standard. I classified the contents of the posts into 11 topic categories (see Table 4-3):

- concept understanding,
- teacher beliefs,
- curriculum,
- problem solving,
- teaching practice,

- student learning,
- parental involvement,
- professional development,
- life,
- assessment, and
- technology.

I preferred the examples to be selected from the first six of eleven topic categories for this pool, because these six categories mattered more to me in relation to mathematics teaching and learning than the last five categories. Thus, I started with the file with the highest number of comments within each category. Then, I went through a determining process about the file: if the file involved plenty of ideational interactions rather than gibberish, I took it as a potential example; if not, I turned to the next files and selected the one with richer ideational interactions. After reviewing all the files, I eventually determined six files as the potential examples selected from six topic categories.

Step 4: Choose the illustrative examples by analyzing the structures of the conversations and the presented conversation topics in the cases from the core post pool. I prioritized the first example of *teaching improvement* from the topic category *teacher beliefs* because its related conversations and arguments fascinated me. Within the arguments, the participants were less interested in responding to the post than to its comments. The conversations were weaved recursively, hence I explored such conversations as recursions (Chapter 5). The topics emergent from the conversations were very close to the topic of the post.

I then selected the second example, *textbook presentations of the Handshake Problem* from *curriculum,* because the participants were more interested in responding to the post than to its comments. This made this example different from the first one, but its conversations were close to the inquiry of the post, which was similar to the first example.

When I came to select the third example, I hoped to see a different kind of conversation structure or conversation topic presentation from the first two examples. When I analyzed the example *introduction of rational functions* from *concept understanding,* I found it included a new type of comment pingback. This kind of comment extended the conversations dramatically and its conversations went beyond the topic of the post (see details in Chapter 5). Thus, *introduction of rational functions* was selected as the third example.

For the fourth example, I chose *solving problems about chord lengths* from *problem solving,* because the pingbacks within transformed the conversation topic from problem solving into other topics. Such transformations had not ever occurred from the previous three selected examples. I did not select the remaining two potential examples as illustrative ones, because they did not show obvious differences in both the conversation structures and conversation topic presentation from the selected four examples.

The four selected examples met the selection criteria. They sufficed to show the diversity and richness of the conversations occurring in the PLN even if their selection was not absolute.

4.5.5 Data analysis

Interpretive paths. I commenced the data analysis as soon as I collected the first post because collecting guided me to make further inquiry possible, as Pattern and Williams (2002) suggest. To put it differently, I engaged simultaneously in the processes of data selection, collection, and analysis.

Specifically, data analysis and collection were interwoven in an unfolded spiral and the hermeneutic circle (Figure 4-4). The unfolded spiral in Figure 4-4 shows the interplay between the data collection and the data analysis. Each of the four selected examples might go through one or more loops, and all the sets of example collection and analysis were eventually taken as the forward arc (the left side of the diagram circle) of the whole hermeneutic circle (the diagram circle) and examined through the backward arc (the right side of the diagram circle) as a whole to see the relationship between the whole and the part of data collection and analysis.

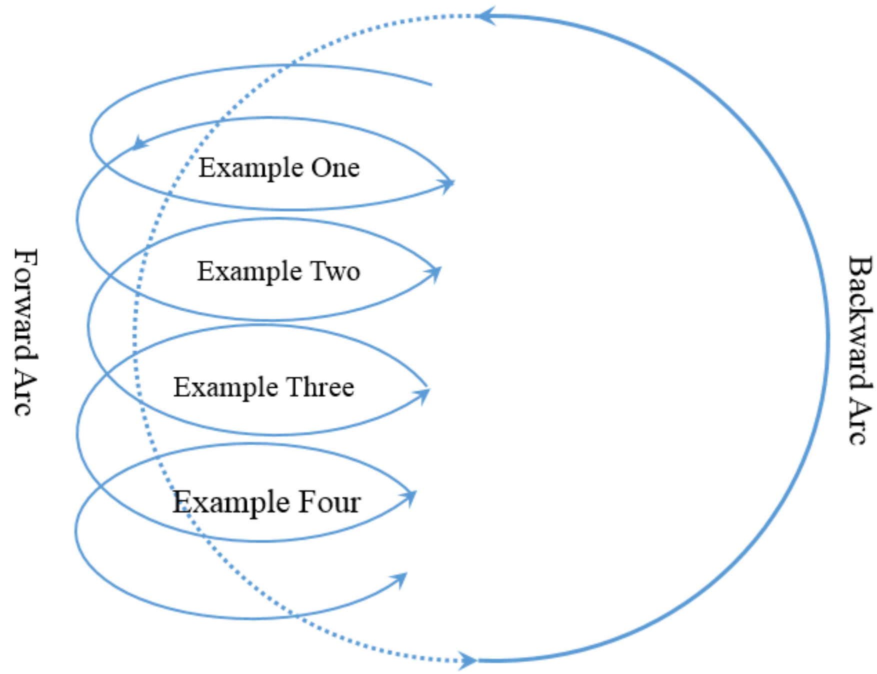

Figure 4-4. A diagram showing the process of data collection and analysis.

In addition, writing was used as a way of engaging my thinking in terms of its significant role in interpretive inquiry. I wrote out whatever emerged from the data collection and analysis and rewrote it over and over again as a part of my reflection to uncover what "insights and connections emerged from the very process of the writing itself" (Ellis, 1998a, p. 6).

The analysis of examples. The analysis of each example ran through several interpretive cycles (Figure 4-5) derived from Ellis' (1998b) hermeneutic circle. I began by summarizing the posts and their comments to handle the very informative conversations. If necessary, I went back and forth to the original data from time to time to double check the relevant details. The back-and-forth process was represented by a double arrow in Figure 4-5.

With the data in hand, I started the analysis. First, I mapped the conversations for a better view of what they looked like. Then, I inquired about what could emerge from them. This inquiry directed me to analyze the data by using thematic analysis and the model of mathematics-for-teaching, which resulted in the emergence of the multiple types of knowing. Next, wondering how the conversations were weaved and extended to underpin the emergence of knowing, I examined the recursions and the fractal-like conversation extensions, which resulted in the diverse structures of the conversations.

However, it still remained unclear what roles the conversation weaving and the conversation extending were playing in underpinning the emergence of knowing. Thus, I explored the connections between the conversation structures and the emergence of knowing and revealed the recursions' intensification and the conversation extension's transformation of the emergence of knowing. After that, I inquired into the roles the individual contributions and the conversations were playing in the emergence of knowing. This inquiry motivated me to re-examine the data from the perspective of interactions between the individual contributions, the collective conversations, and the emergence of knowing. The inquiry process, to date, is not closed but open for future revision. This is represented by the dotted line in Figure 4-5.

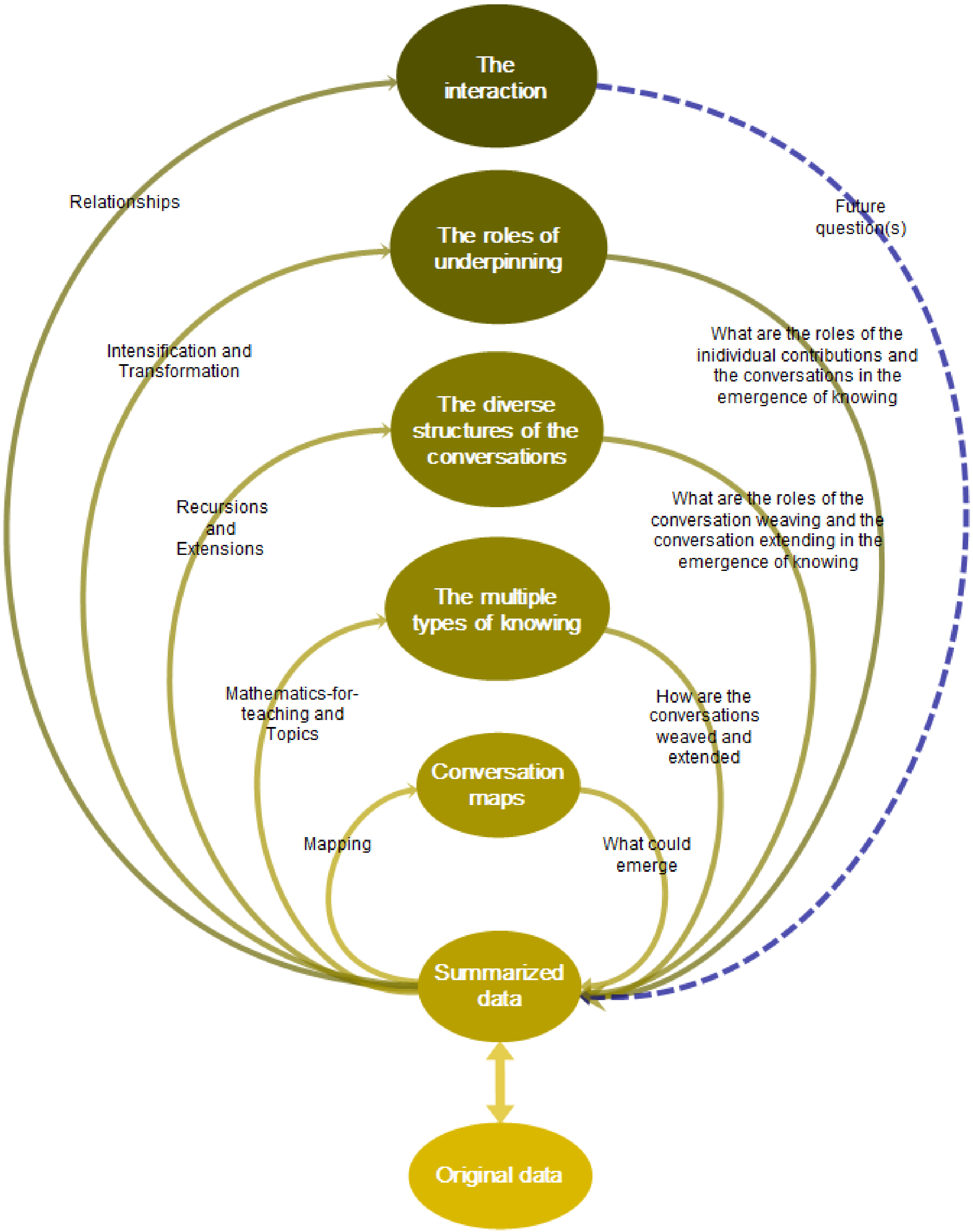

Figure 4-5. The analysis processes.

Analysis techniques and conceptual frameworks. In this study, I adopted certain analysis techniques and conceptual frameworks (see Table 4-4) to provide “enough illustrative material” (Ellis, 1998b, p. 32) for the entire process of data collection and analysis:

a) I applied thematic analysis and used the model of mathematics-for-teaching to investigate the emergence of knowing from the conversations;

b) I used recursive dynamics and fractal images to illustrate the structures of the conversations among the participants; and

c) I attended to necessary conditions for complex systems to understand the environment of the PLN for the emergence of knowing and the interactions among participants.

Details about these techniques and conceptual frameworks follow.

Table 4-4

An overview of techniques and conceptual frameworks

Data	**Purpose**		**Analysis Techniques and Conceptual Frameworks**
Posts and Comments	The Emergence of Knowing	The emergent topics	Thematic analysis
		The collective knowing	The model of mathematics-for-teaching
	The Structure of the Conversations	Weaving	Recursive dynamics
		Extending	Fractal images
	The Environment of the PLN		The necessary conditions for complex systems

Thematic analysis. To better understand the conversations that occurred in the PLN, I applied thematic analysis (Guest, MacQueen, & Namey, 2011) to analyze both the topics emerging from the conversations and the topics that mathematics teachers shared in their posts. I considered that categorizing these topics would reveal tacit information for enriching my understanding of mathematics teachers' participation in the PLN rather than provide the truth or fact of their interests or interactions.

The model of mathematics-for-teaching. I deemed the online conversations to be a knowing generating and an evolving process. I elaborate further on this in Chapter 5. The model of mathematics-for-teaching had potential to help me perceive the emergent knowing from the conversations in relation to mathematics-for-teaching. Thus, the tree model of mathematics-for-teaching (Davis & Simmt, 2006) was used as a framework to examine the emergent knowing from the conversations.

Recursive dynamics. In addition to the consideration of mathematics-for-teaching, this study also probed into the situations which underlined the ideas or thoughts that ran through the conversations. The situations could be examined through the responses of some commenters to other comments. Imagine the conversations flowing from the post like water from a river, but sometimes vortexes might occur where the ideas become twisted because some comments work together on these ideas through their mutual criticism, questioning, or refinement. In addition, the situations of twisted ideas looked like the entangled dynamics (Figure 4-6) of complex systems, which presents a "recursive process within cycles of development and growth" (Davis & Simmt, 2016, p. 418). As such, the entangled dynamics were used to account for the recursive procession occurring in the conversations.

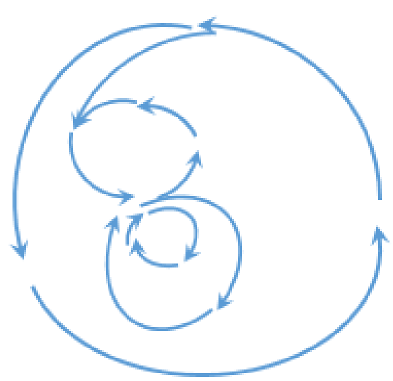

Figure 4-6. Entangled dynamics *(adapted from Davis & Simmt, 2016, p. 469).*

Fractal images. The conversations were dramatically extended by the hyperlinks and the pingbacks in the PLN. Fractal images (see Chapter 3) were used to understand the conversation extensions since the hyperlinks and the pingbacks involved in the conversations extended the conversations in “an unending, reiterative process” (Davis, 2005, p. 124). They were helpful not only in understanding how the conversations were extended but also in revealing one of the features of learning in the PLN: the infinite possibilities of meeting the needs and/or interests of participants.

Necessary conditions for complex systems. Regarding the online discussions, the interactions among participants and the knowing/knowledge emerging therein were not guaranteed — nor did they occur in a fixed way. Rather, they were heavily dependent on the environments in which participants positioned themselves. The environments of the PLN were examined based on the five necessary conditions for complex systems (see details in Chapter 3: “Complex Systems”) from Davis and Simmt (2003), including:

- internal diversity,
- redundancy,
- decentralized control,
- organized randomness, and
- neighbour interactions.

These conditions were employed to describe the environments of the PLN as well as its features.

4.6 Ethical Considerations

The data collected encompassed online posts and their comments. According to the guidelines of Tri-Council Policy Statement of Canada (TCPS, 2010), Association of Internet Researchers (AoIR, 2012), and British Psychological Society (BPS, 2013), the study was characterized as Internet Research (IR) or Internet-Mediated Research (IMR). Thus, the critical matter was whether the targeted PLN was a public or private space. According to TCPS (2010), a PLN is a public space if its information is viewed as “accessible in the public domain” and if there is “no reasonable expectation of privacy” (p. 18). In my study, the targeted PLN was considered a public space because it was accessible to anyone. In Trust’s (2015) words, it was a site “open to anyone who is interested in sharing math resources, adding to the collective knowledge of the field of math, and connecting with other math educators and experts" (p. 74).

Generally speaking, as Holmes (2009) indicates, most IR or IMR contains minimal risks or threats to individual participants; the PLN in question is not the exception. She does, however, reveal two potential risks: the breach of individual confidentiality and the damaged welfare of discussion groups. To address these issues, I explored the ethical considerations in relation to the participants and the whole PLN involved in the data collection and analysis to minimize the potential risks to them.

First, the data collected in this study were not used to analyze individual situations such as individual knowledge, ideas, thoughts, or viewpoints but the conversations among participants and the emergent ideas from the conversations in relation to mathematics-for-teaching. Therefore, they did not, do not, and will not pose any kinds of risks to the breach of individual privacy.

Second, according to the guideline from BPS (2013), the traceable quotes from the online conversations were not directly adopted but paraphrased and/or summarized in my data analysis to avoid any possibility of compromising individuals' anonymity and confidentiality.

Finally, I used gender neutral elements in the writing to further protect individual privacy. The gender-neutral pronoun "ze" (Table 4-5) in lieu of "she or he" referred to an individual participant (e.g., blogger or commenter) with or without "gendered characteristics" (University of Wisconsin-Madison's Writing Center, n.d., para.1).

Table 4-5

Pronoun ze (adapted from University of Wisconsin-Madison's Writing Center, 2018, para. 3)

Pronoun	Subject	Object	Possessive Determiner	Possessive Pronoun	Reflexive Pronoun
Ze	Ze	Hir	Hir	Hirs	Hirself

My examination of the environment of the PLN aimed to understand how the environment allowed ideational interactions and the emergence of knowing from these interactions. Such understanding was possible to help people to better perceive the affordances of the PLN and to promote the application of the PLN to math teachers' professional learning. This could benefit rather than damage the welfare of the PLN.

4.7 Coding Comments

For convenience of writing up the data analysis and its results, I used numbers to code all the individual comments. Specifically, each comment was assigned a number representing its sequential occurrence among all the comments. If a comment replied to the post, it was coded as an integer based on its sequence with respect to all comments responding to the same post. Take the post of graphing rational function (Figure 4-2) as an example again. C. Liu's comment (Figure 4-7) was coded as "Comment 2" since it was the second comment responding to the post. The ones standing at this level were referred to as the first level comments.

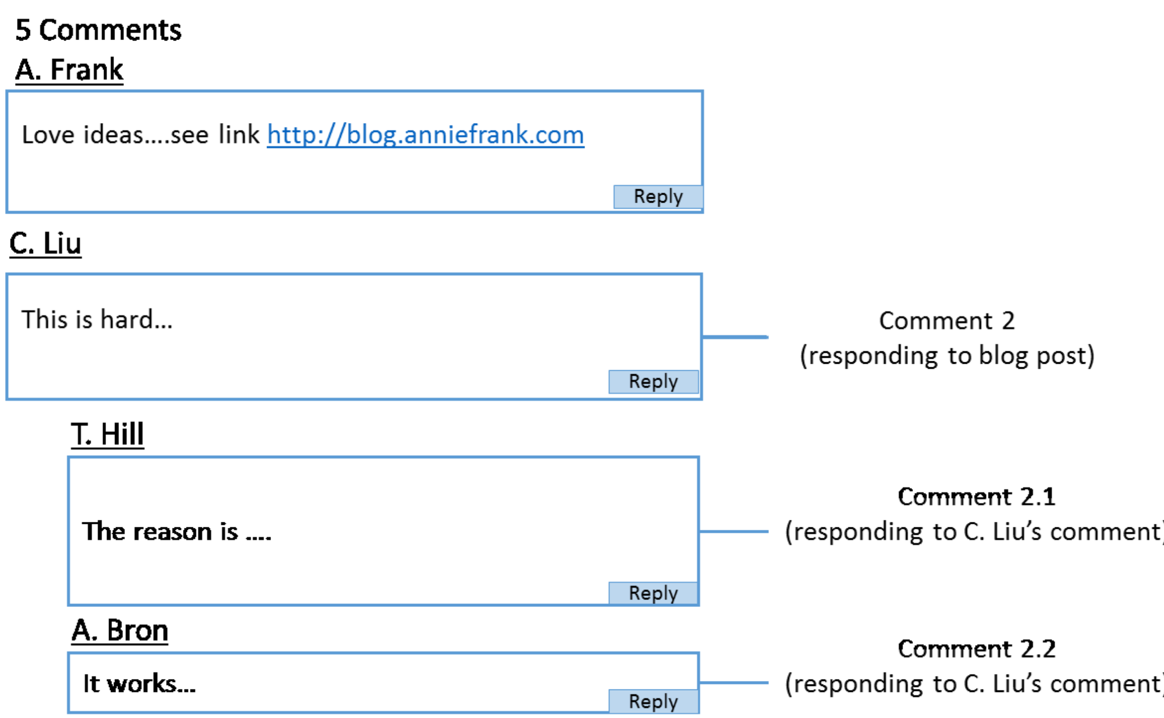

Figure 4-7. Partial diagram of the post of graphing rational functions and its comments (adapted from Figure 4-2).

If responding to another comment instead of the post, a comment (e.g., A. Bron's in Figure 4-7) was marked with a decimal number within which the integer part expressed the number of the responded comment and the fractional part represented its sequence in all the responses to that comment. For example, A. Bron's comment was designated as "Comment 2.2," indicating its response and ranking was second among all responses to Comment 2. All the comments at this level like A. Bron's were regarded as second-level comments. Certainly, if the second-level comments were further reviewed, it is possible that a third level may appear (e.g., Comment 2.2.1).

4.8 Limitations

There were certain limitations in this study. Data collection was limited because of my restricted time. It was impossible for me to transcribe and analyze all the archived documents even if they were available across the scheduled duration of this dissertation writing. Therefore, I spent my time and energy primarily on the selected examples only. In addition, the ways of selecting the examples directly impacted my interpretations because different focuses could result in varied understandings. To address the limitation of data collection, I familiarized myself with the contexts and the contents of the PLN and undertook a preliminary analysis on the texts within before selecting the examples to ensure that the selected data was as diverse and rich as possible.

Further, my language might limit my interpretations because language is critical for interpretive inquiry from "the perspective of understanding my data" (Mayers, 2001, p. 16) and for the writing of my understanding or interpretations. It is through language, as Mayers (2001) highlights, that I am able to "take you, convince you" as well as "run the risk of losing you" (p. 16). Thus, to address the limitation of my language, I tried to be "vigilant and gentle, careful and powerful" (p. 16) in the interactions with the data and the interpretations.

In brief, my understanding of interpretive inquiry directed me to find the path to my inquiry. The path included how I selected the analysis examples from the targeted PLN and how I analyzed them. In fact, the path to my inquiry was not straight but looped in backward and forward processes within which the data collection and the data analysis were not separate but interwoven. The results from the data collection and the data analysis are elaborated on in the next chapter.

5. The Structures of Conversations and the Emergence of Knowing

This chapter presents the results from the data analysis about the structures of the conversations and the emergence of knowing (Figure 5-1). It begins with the introduction of four selected examples followed by the interpretation of the emergence of knowing and the structures of the conversations. The emergence of knowing is interpreted through a two-layered analysis. First, on the emergent topics and second, on the collective knowing. Here, it is presented before the structures of the conversations because first understanding the contents of the conversations used for interpreting the emergence of knowing contribute to better understanding the structures of the conversations. Then, the structures of the conversations are analysed through two dimensions: recursions and fractal-like conversation extensions.

The ensuing sections describe the connections between the structures of conversations and the emergence of knowing. The two dimensions — recursions and fractal-like conversation extensions — are associated with each other through the recursions' intensification and the conversation extension's transformation of the emergence of knowing. The results show the diverse conversation structures and the multiple types of knowing inclusive of *mathematics-for-teaching*, *beliefs about teaching, blog resources, documenting experience*, and *social relationships*. The uncovered structures of the conversations provide the necessary underpinnings for understanding the emergence of the knowing. This chapter is organized as indicated in the following diagram (Figure 5-2):

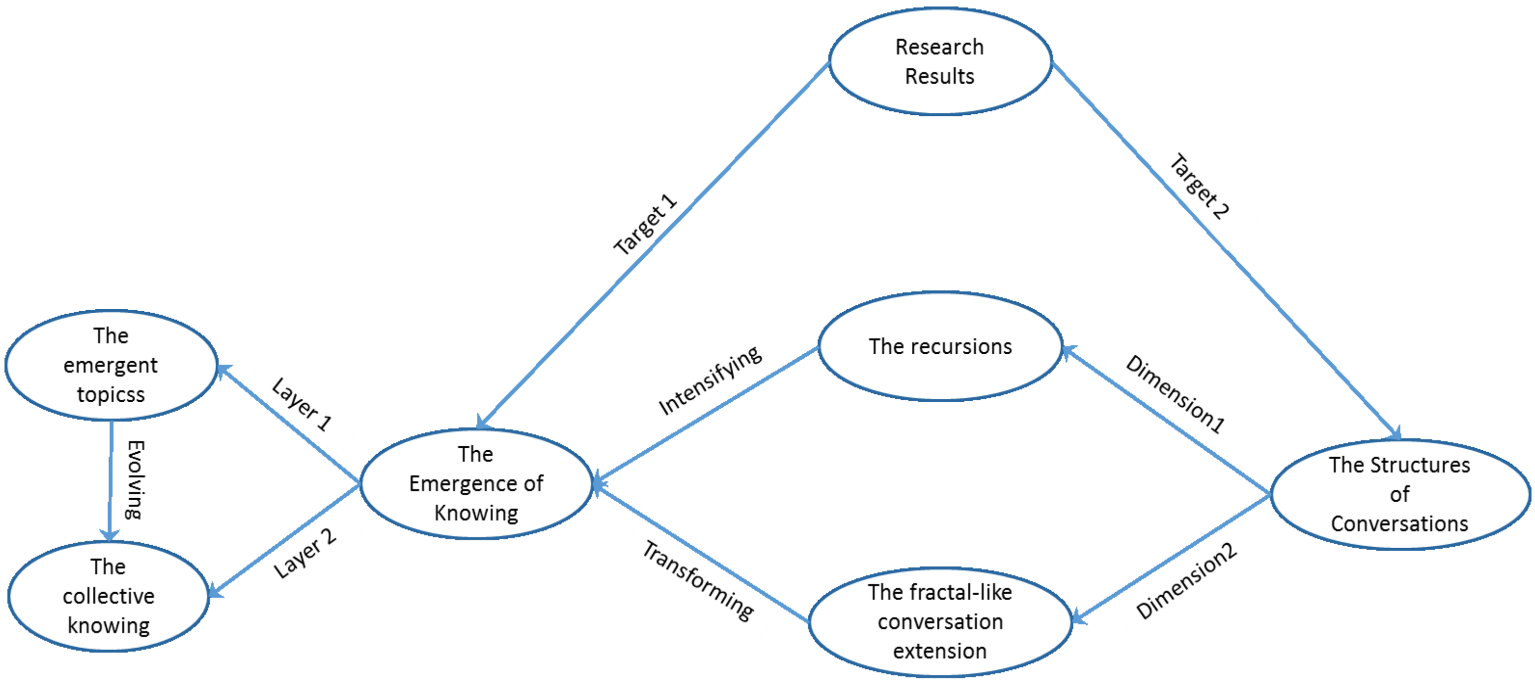

Figure 5-1. The diagram showing the presentation of the results from the analysis.

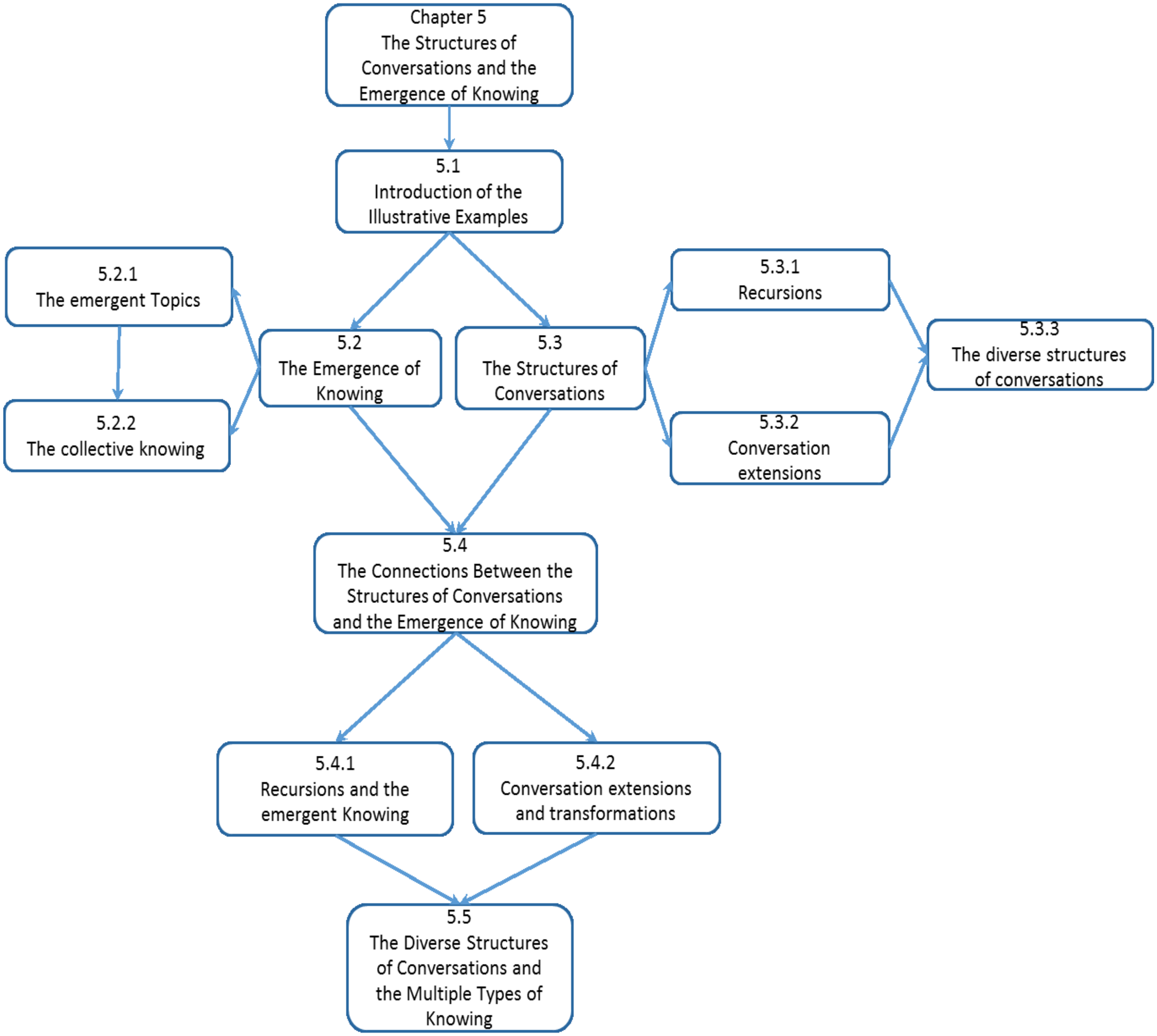

Figure 5-2. The structure of Chapter 5.

5.1 Introduction of the Illustrative Examples

The four illustrative examples are concerned about 1) teaching improvement, 2) textbook presentations, 3) introducing rational functions, and 4) solving problems about chord lengths. Each of them was named after its content. For instance, the example dealing with the application of writing improvement to teaching improvement was titled *Teaching Improvement*; the example with the presentations of the Handshake Problem from two textbooks was titled *Textbook Presentations of the Handshake Problem*; the example with introducing the concepts of Rational Functions in a graphical way to facilitate students' conceptual understanding was named *Introduction of Rational Functions*; and the one about solving the problem(s) of the chord lengths was called *Solving Problems About Chord Lengths*. All the posts and their comments in those illustrative examples are mapped as conversation maps (Figure 5-3, 5-4, 5-5, and 5-6) in the following sections.

5.1.1 Teaching improvement

In the original post (585 words) of this example, the author of the post, hereafter called the blogger, described a scenario in which a novelist attempted to use research results about writing to improve hir own writing. The scenario stirred up the blogger's curiosity about whether or not the novelist's strategies could be applicable to teaching. Ze also brought up two points to consider in the post: a) the difference between research relevant to practice and research on that practice and b) teaching as an art or as a science and its related meaning. Then ze invited participants to join the discussion on the post.

Following the original post were 31 comments (3488 words), out of which 16 (red nodes in the Figure 5-3) responded directly to the post, and 15 (yellow nodes in the Figure 5-3) to its comments. Of 31 comments, 28 (3402 words) were analyzed, because one comment (i.e., Comment 8) was removed by its commenter and two comments (i.e., Comments 15 and 16) were gibberish.

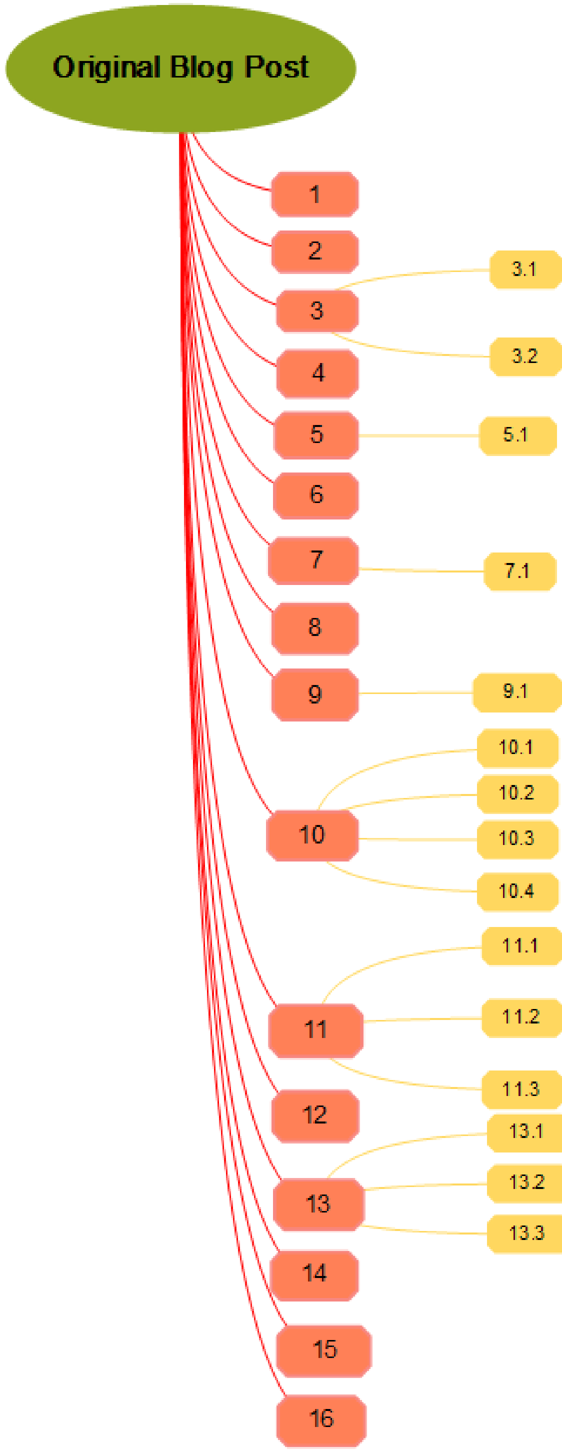

Figure 5-3. The conversation map of *Teaching Improvement* (Note: Code n represents Comment n and hereafter the same applies to the following pictures).

5.1.2 Textbook presentations of the Handshake Problem

In this example, the blogger of the original post introduced the Handshake Problem: *If each person at a party has to shake hands with everyone else, then how many handshakes take place?* Ze believed that if participants shared about how they used the problem, their sharing could offer interesting perspectives to this well-known problem.

Ze wrote a post of 337 words and added a 4-page textbook photocopy presentation about the problem from two textbooks (T1 and T2), then invited participants to think about the different geometric presentations. Specifically, the presentation of T1 used scaffolding to find a rule (function) to predict the number of handshakes at a party. The presentation of the example from T2 was about looking for the similarity between patterns that are found in the Handshake Problem and the maximum chords given points on a circle problem. Furthermore, ze shared hir own observations about supports for the modeling process, the connections between the handshake and some geometrical problems (such as chords), and the structures of the Handshakes Problem in the textbooks. Finally, ze revealed that these two presentations of the Handshake Problem were based on two assumptions: T1 assumed that it was essential for students to learn how to reason through an explicit modelling process; and T2 assumed that it was essential for students to shape their reasoning capabilities on their own.

Eleven comments (1,320 words) followed the post, which was mapped as a spraying image (Figure 5-4). Of 11 comments, 8 responded directly to the original post (red nodes) and one to another comment (yellow node), and one comment (i.e., Comment 9) was removed by its commenter.

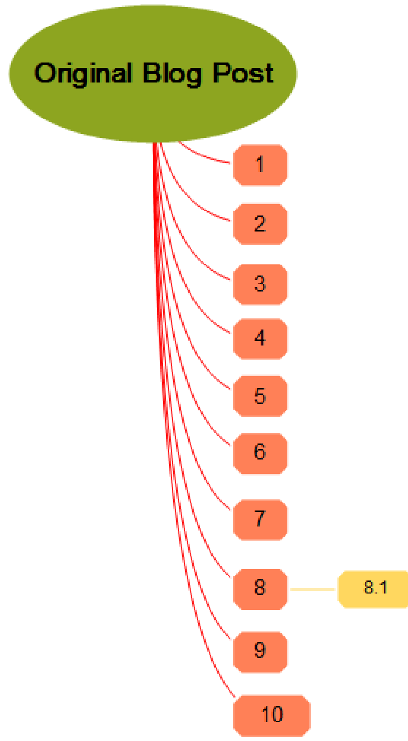

Figure 5-4. The conversation map of *Textbook Presentations of the Handshake Problem.*

5.1.3 Introduction of rational functions

In this example, the original post (1282 words and 13 pages of worksheets) contained three main sections: introducing a graphical approach to introduce rational functions (RF) and their rationale, reviewing teaching practice of graphical approaches in the classroom, and inviting people to further think about the graphical approach. At the beginning, the blogger of the post revealed that RF was quite often introduced by using procedures such as starting with a complex function, then asking questions about x-intercept(s), y-intercept(s), and vertical asymptotes, and finally working towards procedural understanding of RF. To avoid the procedural way of introducing RF, ze turned to the graphical approach of introducing RF so as to provide space for students to build up RF.

In teaching practice, the blogger first guided hir students to discuss the holes and the vertical asymptotes of RF. Then ze graphically underscored the significance of sign analysis. Using sign analysis enabled students to understand the particular graphical features indicated by the special places such as vertical asymptotes, holes, and x-intercepts. Finally, ze posted some “why” questions to facilitate participants’ thinking about the graphical approach.

Followed by the original post were 43 comments (2005 words), out of which 30 (red nodes in the Figure 5-5), including 6 pingbacks, responded directly to the post, and 13 (yellow nodes in the Figure 6-4) to the other comments. Interestingly, at my last visit to this PLN on July 18, 2018, I found that the post and its first comment were posted on May 31, 2013 while its latest comment was on April 8, 2018. This suggests that, even if they were created nearly five years ago, they still attracted participants’ attention and stirred up further conversations.

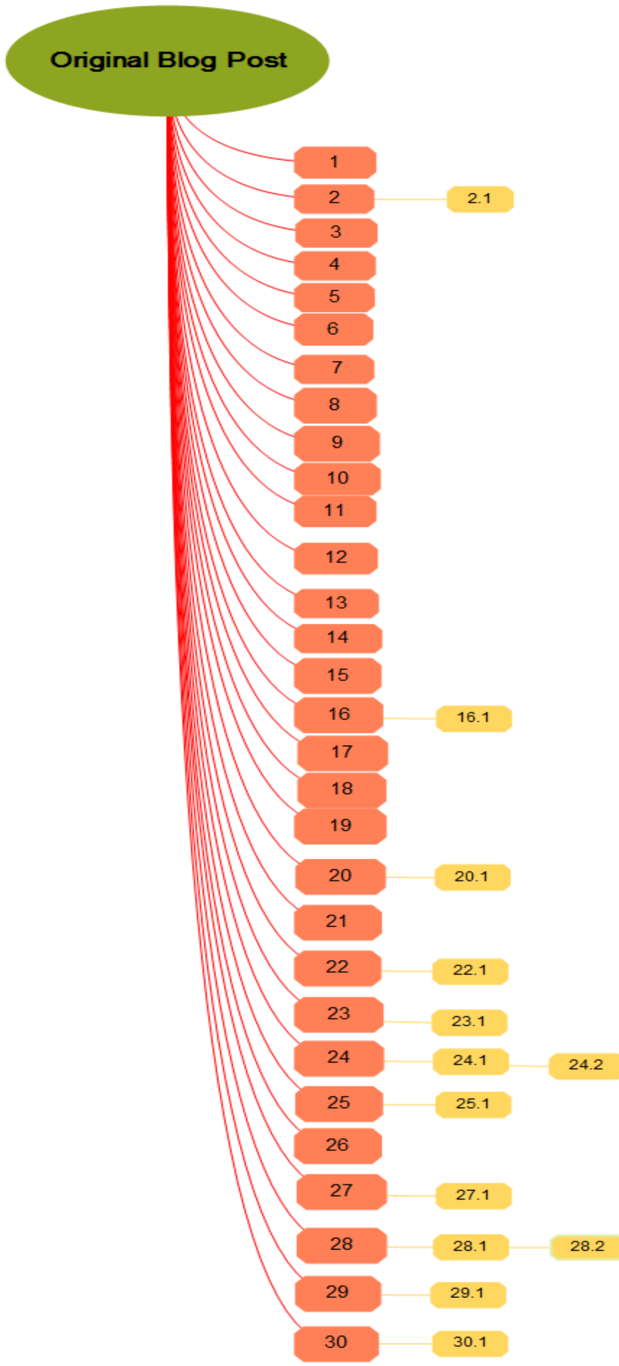

Figure 5-5. The conversation map of *Introduction of Rational Functions*.

5.1.4 Solving problems about chord lengths

In this example, the original post (318 words) briefly introduced an original problem related to chords: *Space n points evenly on a unit circle $x^2 + y^2 = 1$. If we draw chords from $(1, 0)$ to the other points, then what is the product of these chord lengths?* The blogger of the post presented a conjecture about the problem but could not prove it. Accordingly, ze invited participants to solve this problem and design its related lessons for pre-calculus students. In addition, ze posted an extension problem of the original problem for those who might be interested in it: *If we stretch the unit circle into $5x^2 + y^2 = 5$ and scale all the chords, then what is the product of these chord lengths?*

After this, the blogger hyperlinked two posts from the other two bloggers who tried to solve the original problem. However, only one of them was valid. I named it the "embedded post."

Following the original post were 37 comments (4203 words). Of them, 23 (red nodes in the Figure 5-6), including 6 pingbacks, replied directly to the post, and 4 (yellow nodes in the Figure 5-6) to the other comments. However, two of these pingbacks were not available and four of them had links to another four posts (total 5927 words).

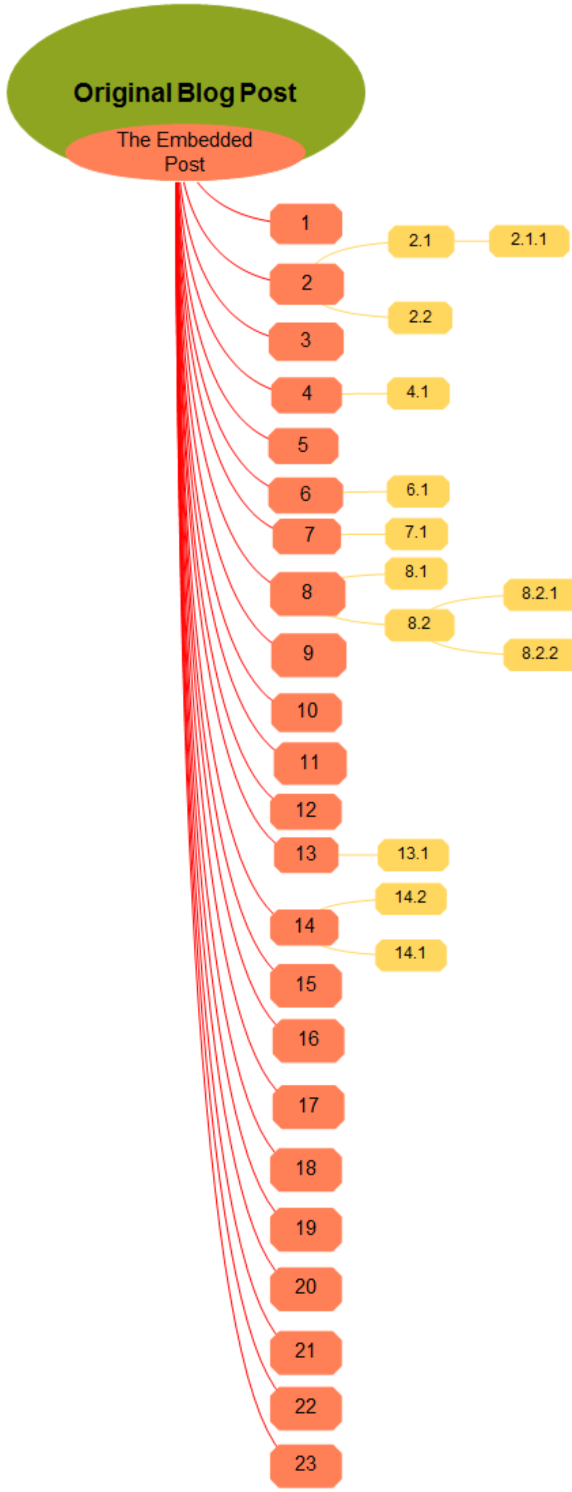

Figure 5-6. The conversation map of *Solving Problems About Chord Lengths*.

5.2 The Emergence of Knowing

This section begins with the related analysis on the emergent topics from the conversations, then interprets the collective knowing that evolved from the emergent topics, and finally describes the nature of the collective knowing.

5.2.1 The emergent topics

From the conversations among the participants emerged multiple conversation topics connected to the four examples. As stated previously, the four examples are:

- *Teaching Improvement,*
- *Textbook Presentations of the Handshake Problem,*
- *Introduction of Rational Functions,* and
- *Solving Problems About Chord Lengths*.

With the numerous topics in each example, the conversation map (Figure 5-3, Figure 5-4, Figure 5-5, and Figure 5-6) were coded into a corresponding topical conversation map on which the comments were coloured to reflect emergent topics (Figure 5-7, 5-9, 5-11, 5-13). In some cases, a few comments (e.g., Comment 3 or 4 in Figure 5-7) were coded with two colors because they straddled more than one topic. Nevertheless, the majority of comments in each example served a single topic. The topical conversation maps could illustrate the emergent topics but could not highlight their dynamic and evolving characters. Therefore, I sought a better way of presenting the topics.

I used a tree-like image because of its potential to prompt attention toward "a growing and evolving form" (Davis, Sumara, & Luce-Kapler, 2008, p. 3), to signify "a deep connectivity" in "describing a phenomenon as emergent" (Davis & Renert, 2014, p. 45) and "the vibrancy, the sufficiency, the contingency, the evolving character of knowing" (Evernden, 1993, cited by Davis et al., 2008, p. 3). It seems that the tree image has more possibility for illustrating emergent topics compared with the topical conversation map. Based on the visual metaphor of the tree image of "complex emergence" (Davis & Renert, 2014, p. 45), the emergent topics in each example were re-portrayed as branching images (Figure 5-8, 5-10, 5-12, and 5-14) to demonstrate their dynamic and evolving features when possibility of conversations expanded.

The topics emergent from teaching improvement. From this example emerged four topics (Figure 5-7 and 5-8):

- *the role of research in teaching*,
- *teaching as an art or a science*,
- *the analogy between writing and teaching*, and
- *the applicability of the traditions of improvement in crafts*.

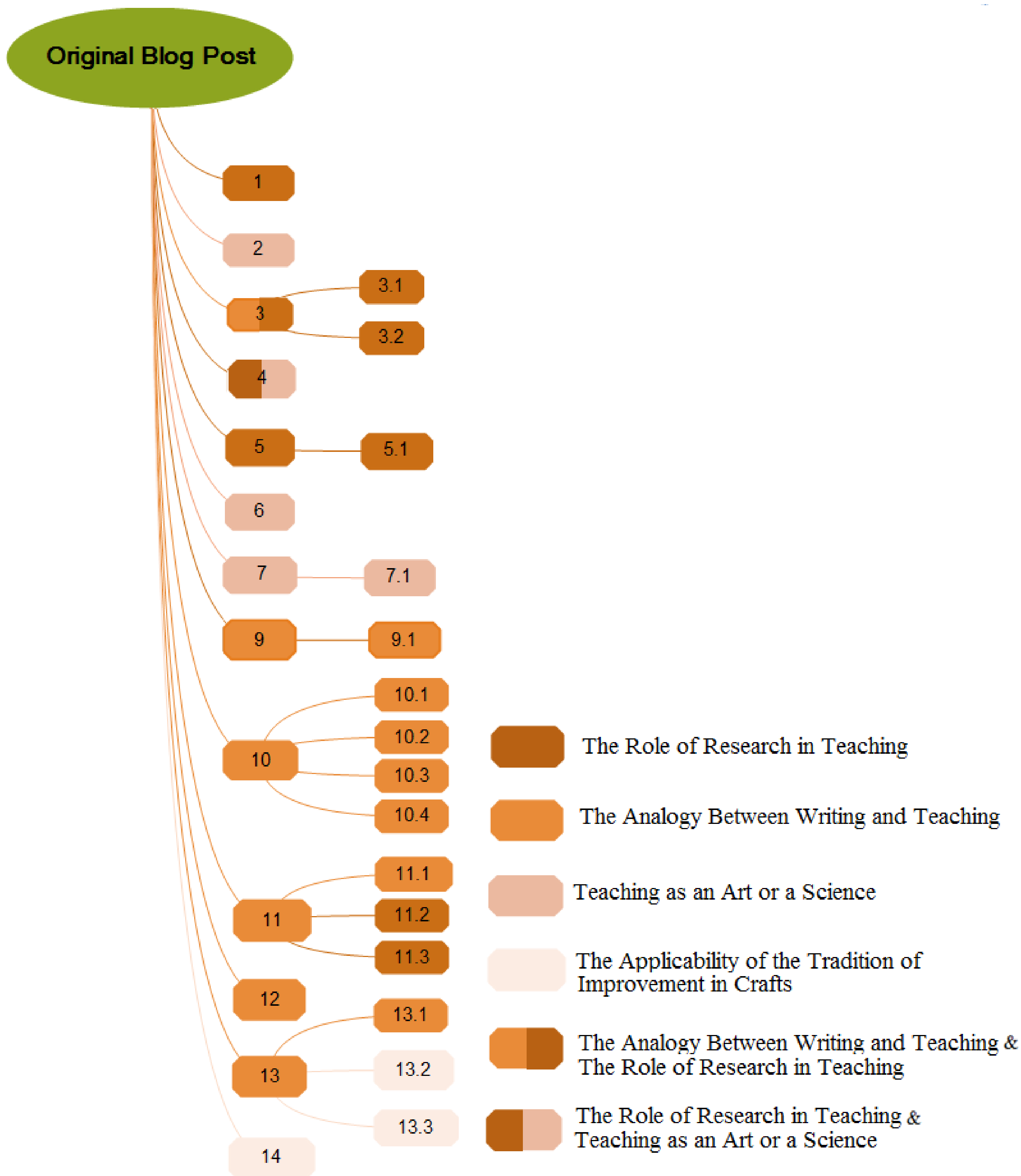

Figure 5-7. The topical conversation map of *Teaching Improvement.*

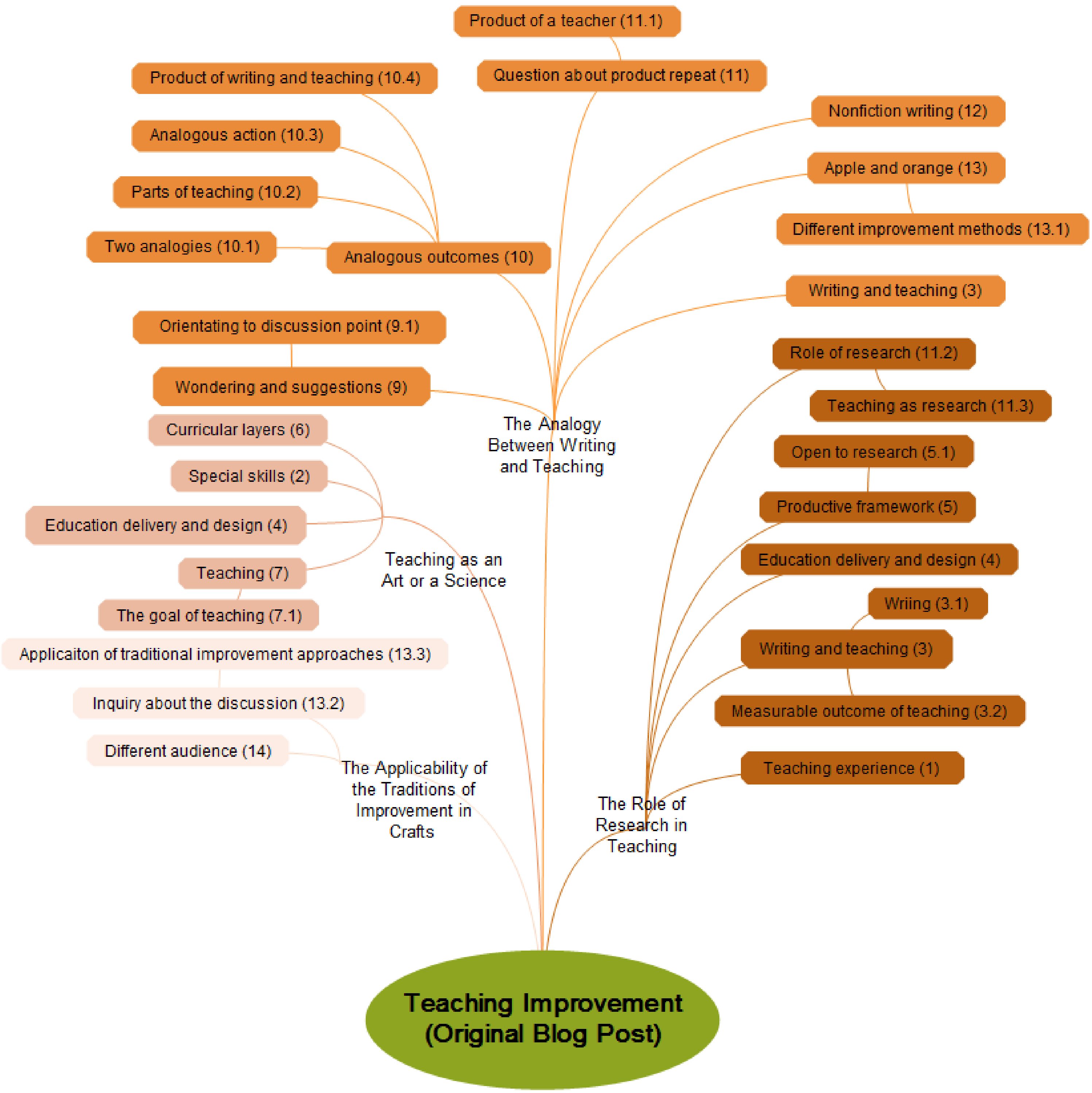

Figure 5-8. The clusters of conversation topics of *Teaching Improvement.*

The role of research in teaching. While the conversations in this example paid much attention to *the role of research in teaching*, no unanimous viewpoints were expressed on it. Some comments made it clear that research could add value to teaching but teaching still depended upon teachers' identification of what their students needed. For instance, in Comment 1 it was argued that a teacher's concerns were necessarily about how to interact with a group of students, analyze the students' thinking, and figure out what could work for the students in a particular situation, but that these concerns could not be attainable from research. In contrast, a few other comments implied that research could play an active role in teaching. For instance, in Comments 3 and 4 it was revealed that empirical studies could make math teaching more scientific than it was and that related research could provide direction for designing teaching tasks.

Other comments aimed to argue that it was impossible to think of the role of research in teaching from one single perspective. For instance, in Comment 5 it was argued that research was able to play a generally influential role in teaching because it could make teachers sensitive to the teaching conditions under which they could get a better understanding of what teaching might be. However, the so-called experimental studies did not contribute much to teachers' daily work because they were set to uncover the generalized truths about the complex world through randomized control, while teaching always proceeded within particular conditions or contexts.

The analogy between writing and teaching. A key point from the original post was about *the analogy between writing and teaching;* this was criticized in numerous comments. For instance, in Comment 3, it was assumed that writing a novel was remarkably distinct from teaching: writing was held to be an act of self-expression without measurable outcomes while teaching embodied the elements of self-expression but with measurable outcomes. In Comment 12, it was argued that teaching was more analogous to persuasive and non-fictional writing, such as in a health education pamphlet. In Comment 13, it was claimed that it was not appropriate to compare artists with teachers because doing art was completely different from teaching art, which required a full set of special skills.

Teaching as an art or a science. The conversations also tended to clarify *teaching as an art or a science*. In some comments, it was made clear that teaching was not a science but an art. For instance, Comment 2 was an argument that effective teaching as a great art depended upon skilled craftsmanship, which would differ from teacher to teacher. Comment 6 was an expression that teaching was done by people's own instincts rather than research and that designing interesting tasks for a textbook was not guided by research but by people's thinking. On the contrary, in a few other comments, it was claimed that teaching was not an art but a science. For instance, Comment 7 expressed the opinion that teaching was a science defined as the study of learning. In other comments, it was argued that it was impossible to determine teaching as either a science or an art because teaching required multiple skills (e.g., running a classroom) as well as being informed by research (Comment 4).

The applicability of the traditions of improvement in crafts. The conversations eventually reached the point about *the applicability of improvement traditions in crafts*, particularly in the traditions of writing and teaching, which the post tried to target. For instance, in Comment 13.2, the commenter inquired about what the discussion was really referring to. In Comment 13.3, the blogger clarified that the analogy between writing and teaching was not the focus of the inquiry, and ze hoped that the discussion could explore the general applicability of the traditions of improvement in crafts and unveil the reason why the traditions of improvement in the arts were not suitable to teaching but to writing in particular.

In Comment 14, the commenter suggested a possible reason that the difference in the improvement of teaching and writing might result from the various types of audiences. Ze also contended that an artist could judge the impact of hir artwork solely by audiences' engagement while a teacher could not do the same because teaching was not simply about engaging students' interest or cooperation but also required measurement of the experience that a teacher provided for the students. Therefore, the way of improving writing could not be assumed to be appropriate to teaching.

The topics emergent from textbook presentations of the Handshake Problem.
From this example, I extracted four topics (Figures 5-9 and 5-10):

- *problem structures,*
- *student learning*,
- *classroom conversations*, and
- *mathematical problems*.

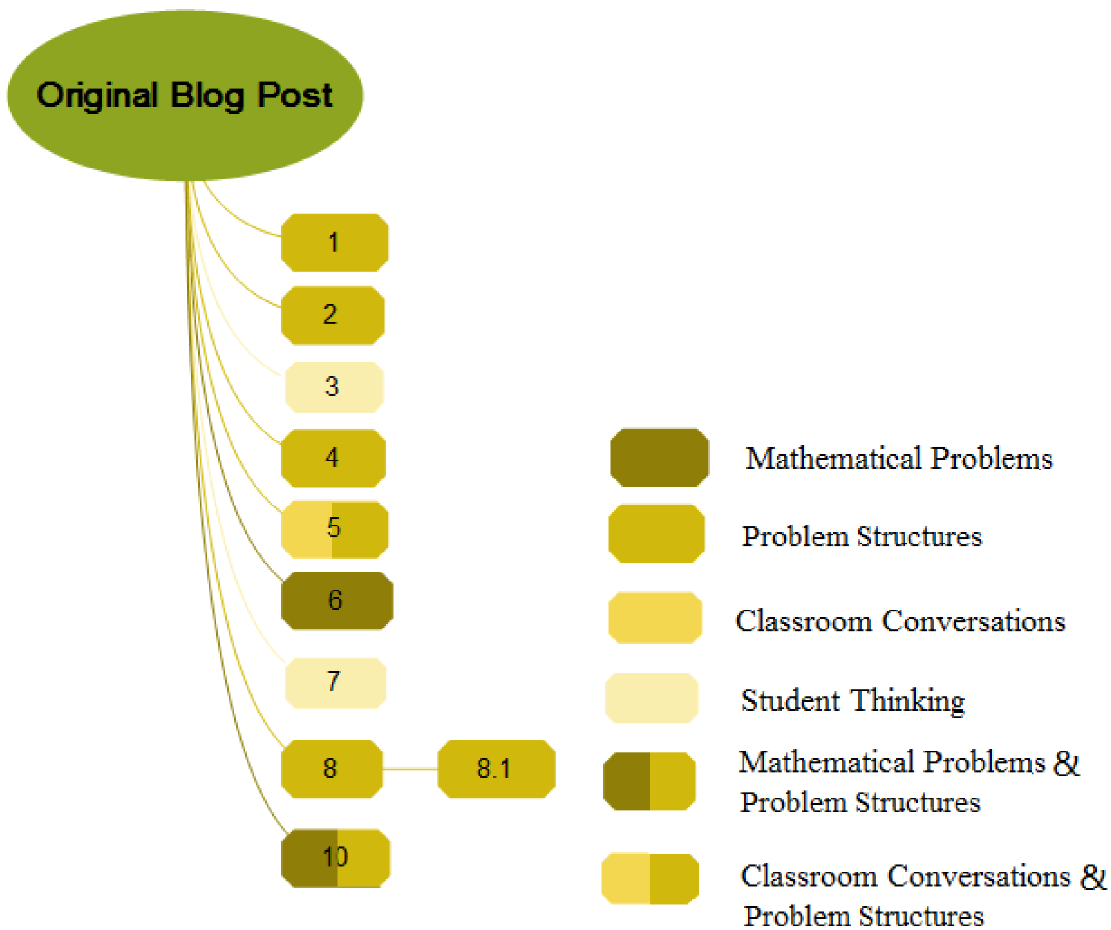

Figure 5-9. The topical conversation map of *Textbook Presentations of the Handshake Problem.*

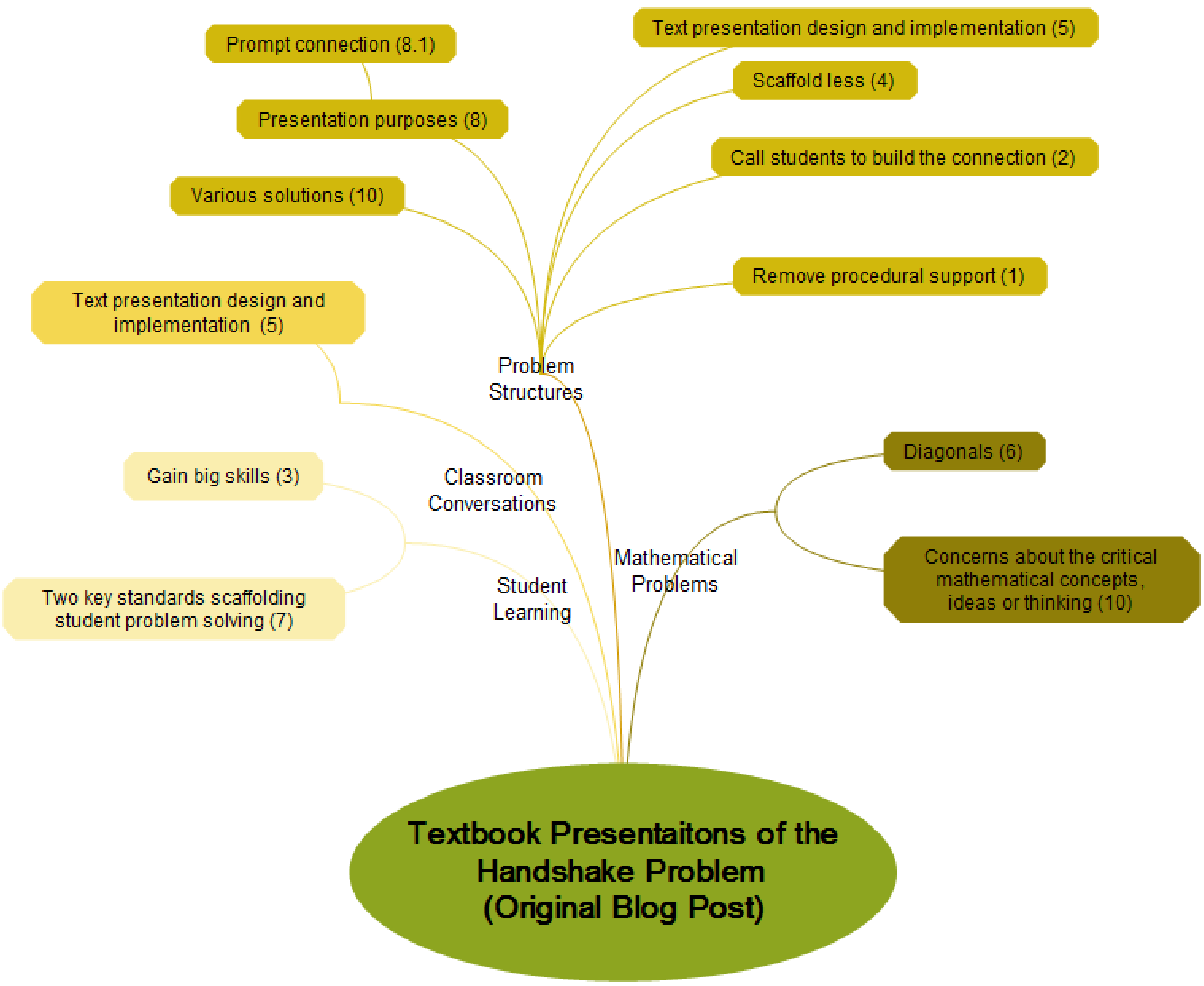

Figure 5-10. The clusters of conversation topics of *Textbook Presentations of the Handshake Problem.*

Problem structures. The conversations on the original post mainly focused on *problem structures* of the Handshake Problem from T1 and T2. They also included suggestions and expressed confusions related to problem structures in the textbooks. For instance, several actions on problem structures were suggested in Comment 1, including removing procedural support, building the connection between the handshake and the diagonal problems, and providing more guidance for teachers. In Comment 2 there was a call for an alternative way of prompting students to establish the association between the handshake and the diagonal problems. In Comment 4 it was suggested that there be less scaffolding for student thinking in the problem structures. And in Comment 8, confusion related to the gap between the problems was indicated, which then was dispelled by Comment 8.1.

Student learning. In addition to the major concern about *problem structures*, the conversations also referred to how to support *student learning*. In Comment 3, for example, how to help students obtain thinking skills through the presentations was considered. In Comment 7, tools to support students' thinking when they struggled with problems were demonstrated.

Classroom conversations. The topic of *classroom conversations* arose when participants considered providing support for student learning. For instance, the commenter of Comment 5 shared hir practical experiences of providing less support for students' thinking skills. Meanwhile, ze and hir colleagues designed their own geometry text and allowed the classroom conversations to flow in a natural way or without any control to achieve the target of their text design.

Mathematical problems. Attention to *mathematical problems* was observed when the conversations referenced the diagonal problem directly. In Comment 6 was released a diagonal problem from another textbook. This diagonal problem was very similar to the ones from T1 and T2 but without the Handshake Problem and other problems as prompts. And in Comment 10, it was argued that problems could offer mathematics learning a wide range of solutions, critical concepts, ideas, or thinking. These comments suggested that the problems in the textbooks needed a clear purpose. As the commenter mentioned in Comment 8, the questions in T1 could lead students to experience inductive reasoning from a specific case to a generalized situation.

The topics emergent from introduction of rational functions. Six topics emerged in this example from the comments on the post (Figure 5-11 and 5-12):

- *the graphical approach,*
- *teaching of rational functions,*
- *student learning,*
- *classroom environment,*
- *blog resources,* and
- *social relationships.*

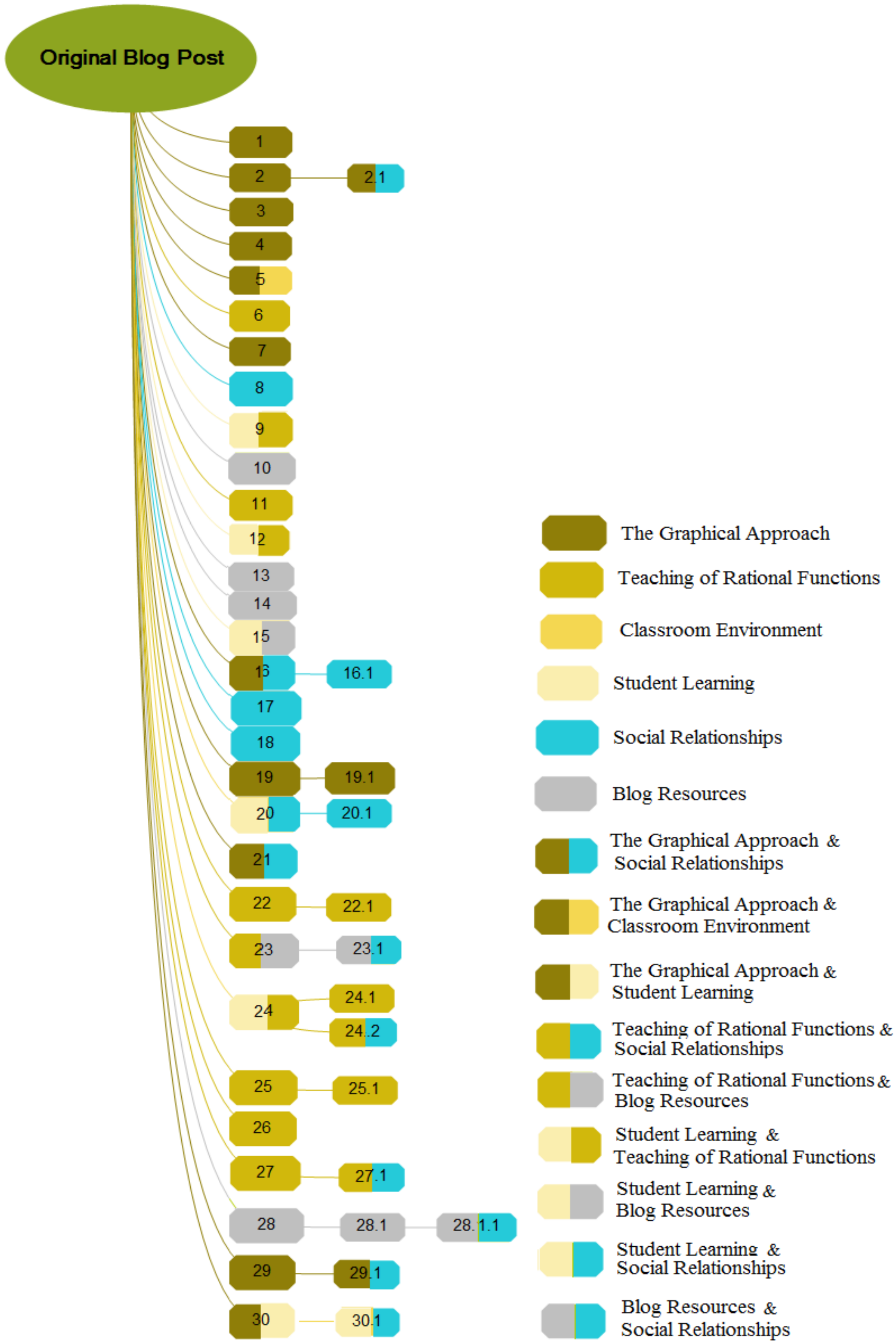

Figure 5-11. The topical conversation map of *Introduction of Rational Functions.*

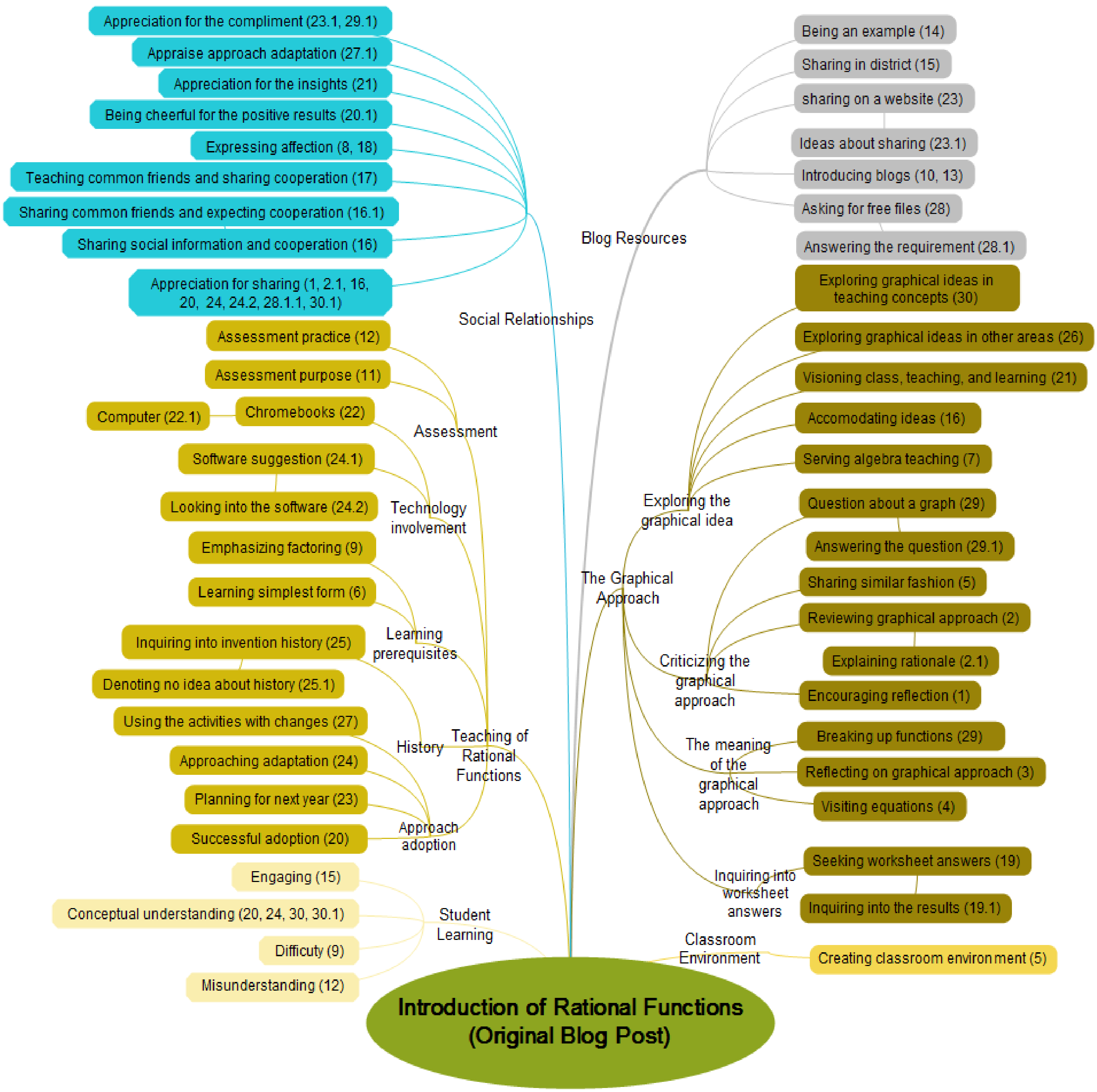

Figure 5-12. The clusters of conversation topics of *Introduction of Rational Functions.*

The graphical approach. This topic was observed from the discussion about the introduction of rational functions in the post. It was intended to help students build up the concepts of rational functions. The graphical approach of introducing rational fractions aroused participants' interest as it generated several discussion topics such as exploring the graphical idea, criticizing the graphical approach, understanding the meaning of the approach, and inquiring into worksheet answers.

For example, some participants explored the graphical idea involved in the approach and applied the idea to other mathematics content, such as polynomial and rational inequalities (e.g., Comment 26). Others dug out the meaning of the graphical approach, such as breaking up the functions (e.g., Comment 3), triggering analysis on what happened around the special places (values) (e.g., Comment 4), and transforming a rational function into the multiplication of fractions (e.g., Comment 29). However, a few participants criticized that it was difficult to handle the graph without equations or numbers (e.g., Comments 2 and 5). Indeed, some participants requested the answers to the attached working sheets about the graphical approach from the blogger (e.g., Comments 19 and 19.1) because there were no attached answers to these working sheets.

The above-mentioned discussion topics directly related to the graphical approach proposed in the post. Thus, proposing a particular approach was like a "seed" in the emergence of those topics. Simply put, the topic of *the graphical approach* grew up from the "seed" of the introduction of graphical approach, which could help participants to better understand the approach and its related rationale and activities.

Teaching of rational functions. Introducing rational functions (RF) was merely regarded as one aspect of teaching RF. Five aspects of teaching RF were also taken up for consideration: learning prerequisites, approach adoption, assessment, technology involvement, and history. Among the related comments, the prerequisites for students' learning RF at an early age were suggested. For instance, it was suggested that if students had learnt negative numbers and decimals, and know graphing point by point, they could begin learning RF from its simplest form in Grade 7 (Comment 6). It was recommended students learn factoring and polynomials in advance because factoring RF is often required before graphing (Comment 9). They were also suggestions for children to learn the simplest rational function and to do its graph at an early stage when learning RF (Comments 6 and 9).

The assessment of RF was initiated by a participant who presented an approach for assessing students' learning of RF. The approach required students to create RF that could satisfy specific subset criteria (Comment 11). The assessment approach caught the blogger's attention. The blogger followed up and posted a pingback (Comment 12) about the assessment of RF in which an assessment sample was attached with the explanation of the assessment purpose.

Some participants tried the graphical approach in their own rational function teaching and reported successful results (e.g., Comments 24 and 27). Others claimed that they would use the approach in their teaching practice in the future (e.g., Comments 23 and 26).

The use of technology aroused the participants' attention, as technology might provide a visual aid for graphing. It was triggered when one participant wondered, what if technology (e.g., Chromebooks) was involved in teaching RF (Comment 22)? The blogger replied that ze rarely used computers in hir teaching and that ze believed that graphing by hand was important for students to visualize their understanding of the concepts (Comment 22.1). But ze did not reject the idea of trying software that featured the advantage of graphing (Comments 24.1 and 24.2). Nevertheless, whether or not the technology usage could benefit the teaching of RF is still unknown in the conversations.

Comment 25 was an inquiry into the history of RF. Through Comment 25.1, the blogger replied that ze had no idea about that.

The topic of *teaching of rational functions* extended the conversations from introducing RF to teaching RF with various prerequisites and alternatives. Some related topics, even though they did not result in deep discussion among participants, added confusion or uncertainty to the comments. For instance, it was left unknown who invented RF and what technology usage would bring about for the teaching of RF.

Student learning. During the discussion about the graphical approach, comments also referred to students' understanding of RF.[6] On one hand, some comments revealed students' understanding of RF. For instance, in Comment 15 it was recognized that the activities with a visual scaffold (noted in the original post) enabled the students' engagement in conceptual understanding of RF. In Comments 20, 24, and 30, the effectiveness of applying the approach from the original post to teaching practice was exposed. On the other hand, some other comments disclosed students' difficulty in or misunderstanding about RF. For instance, Comment 9 noted that students had difficulty in understanding the relationship between the linear factors and their graphs. Their misunderstanding about RF could be uncovered through assessment, as suggested in Comment 12.

Classroom environment. The classroom environment received only a small amount of attention in the conversations. This topic appeared only in Comment 5. Its commenter shared an innovative way of approaching education with the emphasis upon creating a learning environment in which students were motivated to figure out their own learning instead of accepting the so-called correct approach.

Blog resources. The topic of *blog resources* in this example referred to sharing the original post as a resource within larger communities. From the conversations, there appeared two kinds of blog sharing: pingback sharing and community sharing. Participants employed pingbacks to share the post on other blogs or websites. For instance, the commenter of Comment 10 (pingback) linked the post as a resource for improving teaching practice. The commenter of Comment 13 (pingback) took the post as

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[6] *Student learning* mainly refers to students' understanding of RF in this example. In some comments, when commenters discussed other topics, they also mentioned student learning. For example, the commenter of Comment 5 improved the classroom learning environment to motivate student learning. The commenter of Comment 13 reviewed the post as an example of stimulating student learning. In these comments, the commenters emphasize the creation of the classroom learning environment and the trait of the post as a lesson for introducing RF: motivating student learning as the purpose of creating a classroom learning environment and the trait of the post as a lesson example. However, when I categorized these comments, I grouped them into other topics such as *classroom environment* and *blog resources* because of their emphases rather than under the topic of *student learning*. This does not mean the other topics (e.g., *classroom environment*) exclude *student learning*. Actually, they embrace *student learning*. And *student learning* is nested in those topics. Such understanding is consistent with Davis and Renert's (2014) notion of "nested phenomena" in mathematics-for-teaching in which the four aspects of mathematics-for-teaching are nested rather than "neighboring regions" (p. 92).

an example of activating students to be the owners of learning by providing them with exploring opportunities. And the commenter of Comment 14 (pingback) embedded the post into another post as an example detailing the classroom activities during the online sharing.

Other participants suggested that the post be shared in larger communities with more audiences. For example, the commenter of Comment 15 showed hir willingness to share the post within a district. The commenter of Comment 23 brought forward the idea about sharing the post on another website — TeachersPayTeachers. In response to Comment 23, the blogger commented that although ze had many reasons for not sharing hir work on that website, ze loved to be a member of a big math teacher community where resources, ideas, and suggestions are freely shared. And ze suggested that such community should be named TeachersSupportingTeachers. Ze also posted two websites for teachers to access free resources. In addition, the commenter of Comment 28 hoped to access free shared files from the post instead of the ones to be paid through an online account. And in Comment 28.1, the blogger replied to Comment 28 that those Microsoft Word files were available in the post and were free for anyone to download and use.

To summarize, participants were willing to share the original post and its related conversations with larger audiences via web links or presentations in larger communities. As such, the post and its conversations were considered as resources to configure another topic of the conversations — *blog resources* — that was helpful for teachers' professional learning.

Social Relationships. This topic arose from two situations: participants exchanging their social information and expressing their appreciation or positive feelings for the posts and replies of others. For instance, a participant described how ze and hir colleague loved a particular blogger's ideas shared in hir blogs and later became the fans of the blogger (Comment 16). They exchanged social information and came to realize that they might have some common friends (Comments 16.1 and 17). They also expressed their appreciation for the blogger's personality (e.g., openness to sharing), academic work (e.g., teaching approaches/ideas/actions), and social information (e.g., common friends). These social information exchanges and expressions of appreciation could help the participants and the blogger establish social ties.

Some participants also expressed their affection for the post (e.g., Comments 8 and 18) or their cheerful feelings for the positive results from using the graphical approach in their classrooms (e.g., Comment 20.1). They also appreciated the sharing of free documents (e.g., Comment 28.1.1) and the successful results after using the graphical approach (e.g., Comments 27.1 and 30.1). Their appreciation, affection, and cheerful feelings for others' contributions could make other participants feel that their sharing is appreciated, loved, and valued. This could help participants form close social ties (Algoe, Haidt, & Gable, 2008), including facilitating the initiation of new relationships, orientating to existing relationships, and maintaining or strengthening these relationships (Algoe, 2012).

Not limited to the above two situations, social relationships could also be constructed in another context: when the commenters responded to the original post or its comments without any intention to exchange social information with or express their appreciation to bloggers or other commenters. This is called broad-sense social relationships, and it commonly occur in blogs. Such social relationships presented one of the attributes of blogs, "interaction possibility" (Tan, 2009, p. 199), an attractor for participants to blog or comment on posts (Lu & Lee, 2012), an important contributor to the value of blogs (Du & Wagner, 2006). However, this was not explored in this study because of my focus on the posts and their direct comments.

The topics emergent from solving problems about chord lengths. Four topics emerged from the comments following the original post in this example (Figure 5-13 and 5-14):

- *problem solving*,
- *teaching of problem solving*,
- *blog resources,* and
- *recounting experiences.*

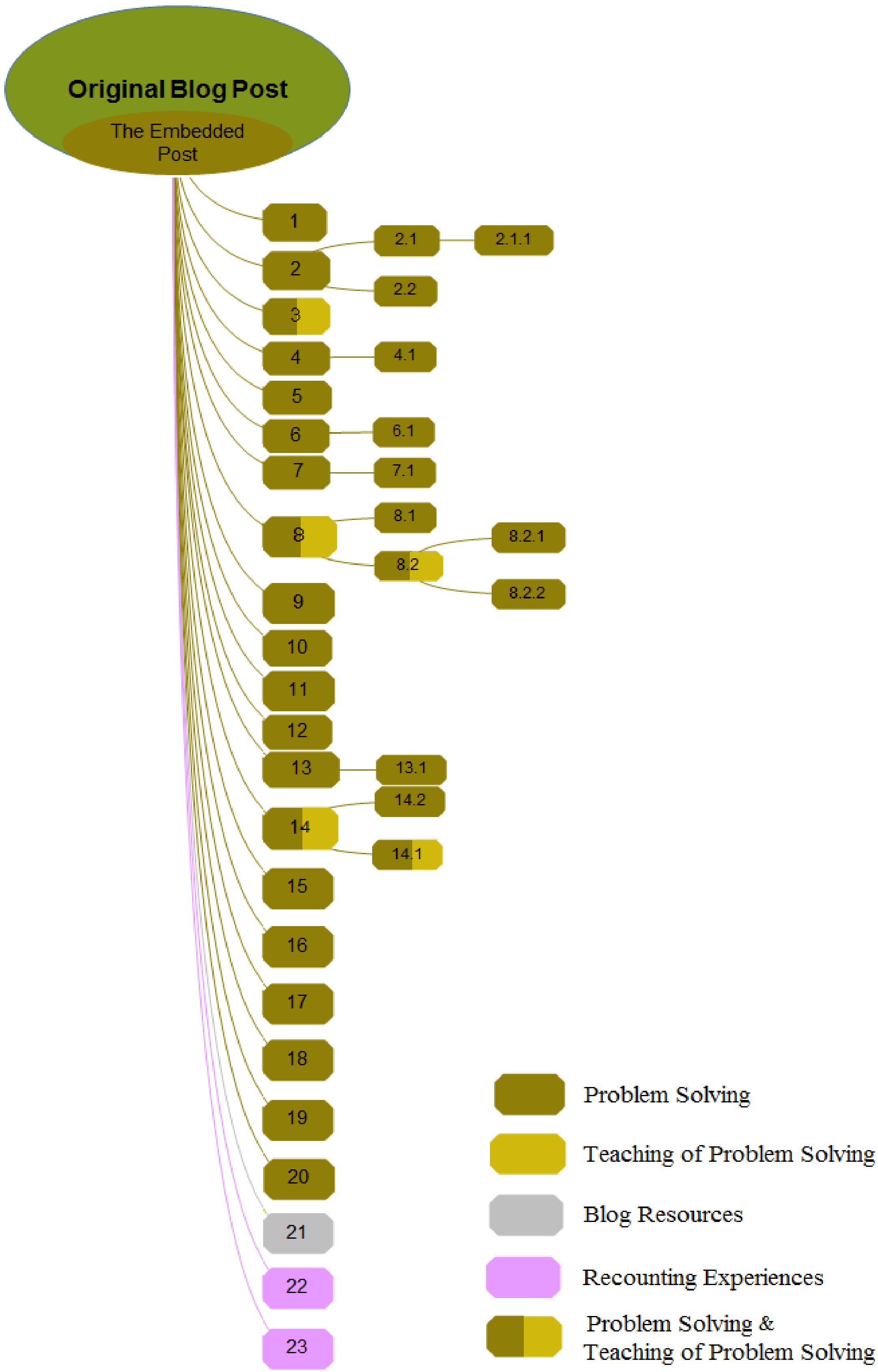

Figure 5-13. The topic conversation map of *Solving Problems About Chord Lengths.*

Among the four topics, the most prominent was *problem solving,* from which arose four approaches: *using trigonometry*, *applying complex numbers*, *drawing diagrams*, and *making conjectures*. The aforementioned four emergent topics, including the four approaches to problem solving, were presented in the bifurcating form (Figure 5-14).

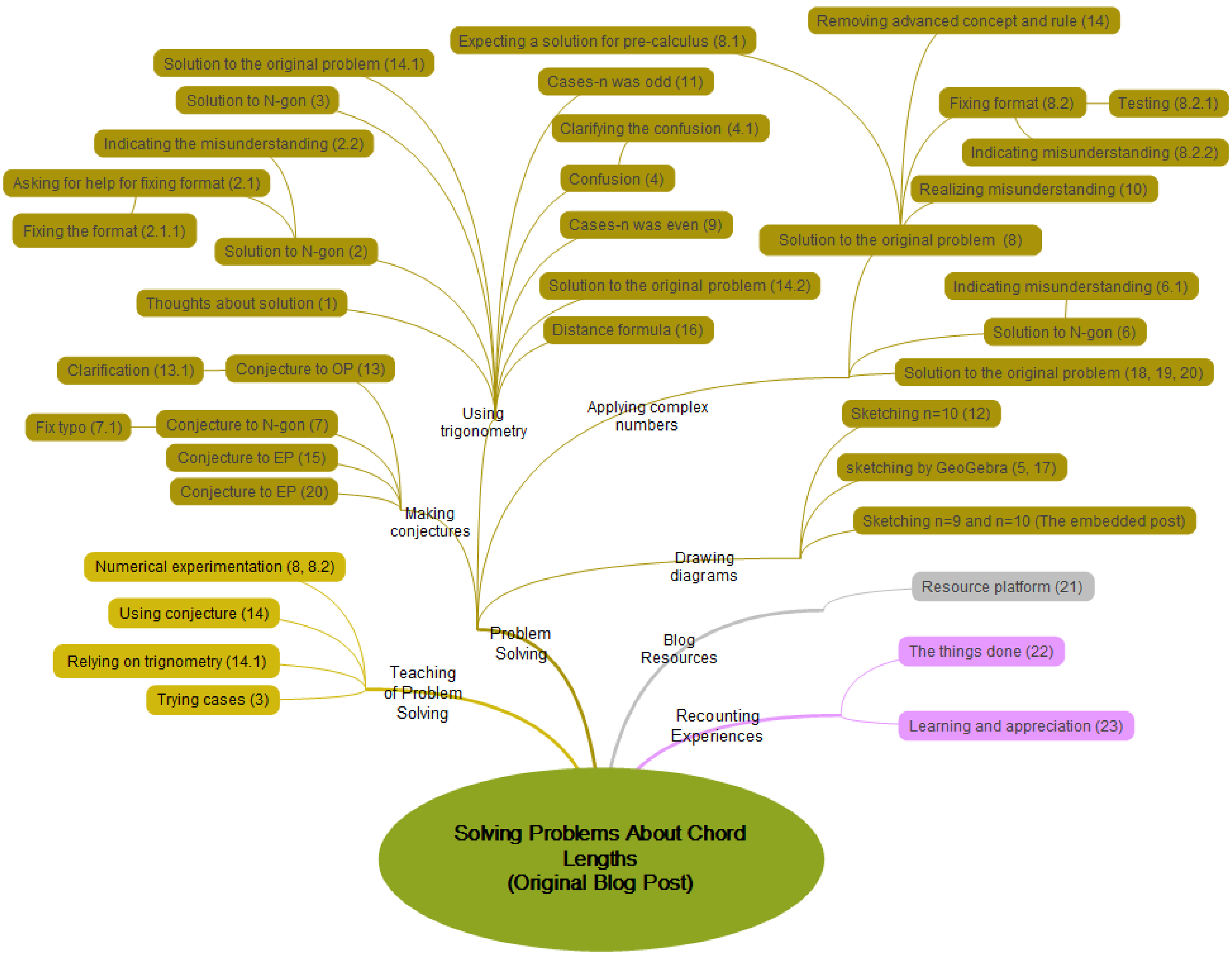

Figure 5-14. The clusters of conversation topics of *Solving Problems About Chord Lengths.*

Problem solving: Using trigonometry. Most comments focused on using trigonometry to solve the problem(s). In some comments, an attempt was made to break the original problem up into two situations: even cases and odd cases. For instance, by using basic trigonometry knowledge, the even cases were worked out in Comment 9, the odd cases in Comment 11, and the two types of cases were figured out in Comment 14.1. However, using trigonometry to deal with the odd and/or the even cases could not easily lead to the conjectured result *n* because it needed a complicated simplifying process.

In several comments, trigonometry was used to find the length of the chords and the product. For instance, in Comment 1, using the sine rule to create an isosceles triangle with two radii and a chord in a circle was proposed to find the chord lengths. In Comment 14.2, another approach was adopted: using a distance formula with trigonometry to work out the product. Nevertheless, the approach involved a complicated simplification and the simplification was not completed.

Trigonometry was also tentatively applied to solving the original problem due to a misunderstanding that the chords were connected from one point to the next and their lengths were equal. In other words, if the length of one chord were known and raised to the n^{th} power, the problem would have been solved (Comments 2 and 3). The misunderstanding was then clarified in Comment 2.2 and 4.1. Overall, trigonometry was used to find a route to solve the original problem but did not lead to the anticipated result because the related approach involved a complicated simplifying process.

Problem solving: Applying complex numbers. Complex numbers were used in some comments to solve the original problem, thereby resulting in complete solutions to the original problem. For instance, in Comment 8.2, using complex numbers and roots of unity to find the lengths of the chords was offered as a precise solution to the original problem. That solution was later refined in Comments 14, 18, 19, and 20 by using polynomial expansions. Thus, the refined solution could better serve the pre-calculus students.

Problem solving: Making conjectures. Some comments attended to the proof of the conjectures for the original problem and the extension problem. The conjectures, in fact, set an orientation for the problem solving, particularly for solving the extension problem. For instance, in Comment 15, the conjecture (C1) for the ellipse $5x^2 + y^2 = 5$

in the extension problem was worked out through a GeoGebra applet based on the cases from *n*=2 to *n*=10. In Comment 20, a further effort was made to prove C1, but in vain. Unexpectedly, however, a more general conjecture (C2) for a general ellipse $(4a+1)x^2+y^2=4a+1$ was worked out. C2 could not be proved mathematically yet but worked in cases such as the unity circle (a=0), the ellipse of $5x^2+y^2=5$ in the extension problem (a=1), and both an integer *a* and non-integer *a*. Accordingly, participants, particularly the creator of the extension problem, were invited to work on the further proof of C2.

Problem solving: drawing diagrams. Drawing diagrams for specific cases was presented in some comments. For instance, in both Comments 5 and 17 (pingbacks) GeoGebra was used to diagram the cases *n*=9 and *n*=10 for the original problem. In Comment 12, a geometric approach illustrated by a diagram (*n*=10) was demonstrated. The approach provided a clear view of the constructed right-angled triangles with hypotenuses from (0, 1) to (-1, 0) and with the chords as legs. Within the right-angled triangles, using the length of the hypotenuses (constant 2) and the angles (available) could help find the lengths of the chords.

The four types of approaches to the previously presented problem(s) resulted in multiple ways of solving the problem(s) and contributed to cooperative problem solving. *Drawing diagrams* could help people work on the cases, which could lead to conjecture(s). The major concern of solving the problem(s) centered on how to prove the conjectures. Using the elementary mathematics of trigonometry was potential to solve the problem(s) but did not lead to a complete solution because of the complicated simplification process. Nevertheless, compared with *making conjectures*, it actually boosted the problem solving even though its related simplification was not completed. *Applying complex numbers* could avoid the complicated simplification process and produce a complete solution to the original problem, which resulted in the advancement of problem solving. Overall, the numerous approaches effectively worked together towards problem solving.

Teaching of problem solving. The multiple approaches for solving the original problem involved different levels of mathematics. As suggested in Comments 8.2 and 14, the methods of d*rawing diagrams* and *making conjectures* could be regarded as the starting point for directing students to probe into OP and come up with a conjecture. Drawing diagrams with cases could provide students with opportunities for knowing sequences and indexes (e.g., Comment 3), and using the conjecture could motivate students to learn the topics of complex numbers (e.g., Comments 8 or 8.2) and the roots of unity (e.g., Comment 14).

Certainly, these two methods were rather rudimentary in comparison with the other two methods — *using trigonometry* and *applying complex numbers.* When compared with *applying complex numbers*, the method of *using trigonometry* was more highly recommended by participants because the problems were originally set for pre-calculus students who were not ready to handle advanced mathematics (e.g., Comment 14.1). Thus, the different levels of mathematics should be taken into account when the multiple approaches were integrated into the related teaching.

Blog resources. The topic of *blog resources* was mainly demonstrated in this example by curating blogs and sharing them as resources. More specifically, Comment 21 (pingback) was linked to a post that curated numerous posts on resources about knowing mathematics. In the post, Comment 20 — which contained the refined solution to the original problem — was curated as one resource. This means that participants were able to revisit Comment 20 via Comment 21. In this case, Comment 21 could be regarded as a resource-sharing platform where the updated problem solving (e.g., Comment 20) was available.

Recounting experiences. The topic of *recounting experiences* mainly referred to the participants' recounting of what they had done in their work and/or life, as well as in the online learning environment. Their experiences were related to the original post but went much beyond that. For instance, the commenter of Comment 22 (pingback) recounted what ze had done during the time that ze did not blog. Hir doings included working on the conjecture of the extension problem and sharing hir favorite blogs. These blogs were linked to interesting sites, such as a policy webpage, and to inspiring videos and journals. These sites, from which the emergent topics went beyond problem solving and even the domains of mathematics, mathematics education, and education, were likely to bring in a wealth of information as well as open new doors for participants to explore what they might be interested in. The commenter of Comment 23 recounted hir experiences of visiting the PLN. Ze appreciated the contributions the participants made to this community from which ze acknowledged that ze had benefited considerably, and ze decided to revisit it often.

5.2.2 The collective knowing

The aforementioned topics from the four examples were neither independent of one another nor pre-existent to the conversations but evolved from the collective conversations related to the following contexts:

- *Teaching Improvement*,
- *Textbook Presentations of the Handshake Problem*,
- *Introduction of Rational Functions*, and
- *Solving Problems About Chord Lengths*.

Now, my concern is about how to present the dynamic, evolving, and contextual nature of those emergent topics. The concept of collective knowing is proposed to address this concern.

The concept of collective knowing. Davis and colleagues assert that knowledge has been "popularly characterized as an object" (Davis et al., 2008, p. 57) and suggest that "knowing has slowly but steadily been taken up in place of knowledge" by educational scholars (Davis & Renert, 2014, p. 23). They posit knowing "as a complex process of co-evolution" (Davis et al., 2008, p. 57), reminding people of "the dynamic characters of both knowers and knowledge" and the "contextual and embedded" features of knowing (Davis & Renert, 2014, p. 23), implying that "personal cognition, collective knowledge, and social interaction are tightly interrelated" (Davis et al., 2008, p. 57).

Karlsen and Larrea (2016) present the concept of collective knowing as a different mode of knowledge construction from the construction of "explicit, analytical knowledge, or theoretical knowledge within epistemic communities" (Karlsen & Larrea, 2014, cited by Karlsen & Larrea, 2016, p. 77). They view collective knowing as a "shared understanding that [participatory] actors create through dialogue in action" (p. 76) and as "a construction of understanding that happens within a social environment" (p. 77) that is being constructed "through dialogue between different actors" (p. 83).

The emergent topics from the conversations are featured as dynamic, evolving, and contextual. These features are consistent with the features of knowing. The emergent topics are perhaps better described as "the emergent knowing" since the word "topic"[7] does not comprehensively present those features. I use "collective knowing" to refer to all types of emergent knowing when I elaborate on them as a whole at a broader level for explicitly highlighting the interrelationships among those types of knowing. This does not imply, however, that "the emergent knowing" is not collective. The emergent knowing is collective in nature because it is a process of "agents adapting to and affecting one another and their dynamic circumstances" (Davis et al., 2008, p. 57).

The multiple types of knowing. Across the four examples emerged the multiple types of knowing:

- *mathematics-for-teaching,*
- *beliefs about teaching,*
- *blog resources,*
- *recounting experiences,* and
- *social relationships*.

Mathematics-for-teaching. The emergent collective knowing from the four selected examples, if examined from the perspective of mathematics-for-teaching shown in Figure 4-7, embodied the four aspects described by Davis and Simmt (2006): subjective understanding, classroom collectivity, curriculum structures, and mathematical objects.

First, subjective understanding was embodied by *student learning* in two contexts: *Textbook Presentations of the Handshake Problem* and *Introduction of Rational Functions.* In the example of *Textbook Presentations of the Handshake Problem*, the knowing of *student learning* was relevant to the considerations from the participants about developing student thinking with the supportive tools that could help build up the individuals' thinking skills or problem-solving skills. These considerations were intended for developing students' subjective understanding when they solved problems such as the Handshake Problem. In the example of *Introduction of Rational Functions*, *student*

[7] The word "topic" comes from the Greek "topos" — a place. From the later 15th century and derived from the Latin "topica," it literally means a matter concerning commonplaces. In Webster's New Collegiate Dictionary, it is used to describe "the subject of a discourse or of a section of a discourse" (1981, p. 1222) or, in Google dictionary, as "a matter dealt with in a text, discourse, or conversation" (Topic, n.d.).

learning was exemplified in comments related to teacher knowing about students' understanding of RF, including their difficulties and misunderstandings, their conceptual understanding, and their graphical performance through visual scaffolding.

Second, classroom collectivity was presented in two situations: *classroom conversations* and *classroom environment*. In the example of *Teaching Improvement*, the knowing of *classroom conversations* arose from the interactions among the participants when they discussed how to support student thinking by allowing the classroom conversations to flow naturally without any control. The conversations would be pivotal for facilitating student thinking because they enable the occurrence of "unanticipated possibilities," which is taken as the essential idea about classroom collectivity (Davis & Simmt, 2006, p. 311). In the example of *Introduction of Rational Functions*, the knowing of *classroom environment* was intended to create a learning environment within which students could design their own learning.

Third, curriculum structures of *mathematics-for-teaching* were exposed by the knowing of *problem structures*, *teaching of rational functions*, and *teaching of problem solving* in three contexts: *Textbook Presentations of the Handshake Problem, Introduction of Rational Functions, and Solving Problems About Chord Lengths* respectively. In the example *Textbook Presentations of the Handshake Problem*, the knowing of *problem structures* mainly highlighted how to improve the structures of the problems in the textbooks to build the connections between the handshake and the diagonal problems. In developing the related mathematical concept of *diagonal*, building the connections by going through procedural steps from the handshake to the diagonal problems in the textbooks was a linear rather than a "recursive elaboration" (Davis & Simmt, 2006, p. 308). Thus, the structures of these two problems were accordingly criticized by the participants, who then suggested leaving more chances for students to construct those connections in their own way. The suggestions supported "the critical mathematical competency of generalization-making" on which the curriculum structures rest (Davis & Simmt, 2006, p. 308).

In the example *Introduction of Rational Functions*, the curriculum structures were considered both the broad considerations for the teaching of RF and the prerequisites for learning and understanding RF in the knowing of *teaching of rational functions*. In the knowing, several aspects related to the teaching of RF were taken into account. Examples include technology involvement, historical knowledge, and assessment, as well as some prerequisites for learning and understanding RF at an early age, such as the mastery of negative numbers, graphing points, converting fractions into decimals, and factoring a polynomial.

In the example *Solving Problems About Chord Lengths*, the multiple approaches were explored by participants to solve the problem(s). As revealed in the knowing of *teaching of problem solving,* using the approaches involved various levels of mathematics. The methods of *drawing diagrams* and *making conjectures* could not solve the problem(s) directly, but orientate the problem solving. They were rudimentary, however, in comparison with the other two methods of *using trigonometry* and *applying complex numbers,* which attempted to prove the made conjectures. When compared with *using trigonometry*, the method of *applying complex numbers* involved more advanced knowledge, such as complex numbers and the roots of unity. Thus, when exploring curriculum structures (planning or design), the integration of the approaches with the various levels of mathematics should be taken into account for related teaching.

Fourth, mathematical objects of *mathematics-for-teaching* were strongly embodied in three contexts: *Textbook Presentations of the Handshake Problem, Introduction of Rational Functions, and Solving Problems About Chord Lengths*. In the example of *Textbook Presentations of the Handshake Problem*, the knowing of *mathematical problems* was relevant to the role of the Handshake Problem in diagonal learning, the carrying of mathematical problems such as a wide range of solutions and critical mathematical concepts/ideas, and the purposes of textbook presentations for student mathematics learning. Such knowing started with the presentations of the textbook and stretched into the role of mathematical problems (e.g., the Handshake Problem) in geometry learning and to the implications and purposes of mathematical problems. These relevant explorations gave shape to “the web of interconnections” (Davis & Simmt, 2006, p. 301) that constituted one of the mathematical objects — *mathematical problems*.

In the example *Introduction of Rational Functions*, mathematical objects were mainly demonstrated in the knowing of *the graphical approach*. The graphical approach could break up the functions, stimulate more analysis on what happened around the special places (values), and transform a rational function into the multiplication of fractions.

In the example *Solving Problems About Chord Lengths*, the knowing of *problem solving* presented mathematical objects through numerous approaches of solving the original problem. The approaches included *drawing diagrams*, *making conjectures*, *using trigonometry*, and *applying complex numbers*. As suggested in the comments, *drawing diagrams* with cases could be regarded as the starting point for probing into the original problem such as leading to the conjecture(s). For that reason, *making conjectures* could be considered a higher level of problem solving.

Furthermore, the major concern about solving the problem centered on how to prove the stated conjecture(s). Using the elementary mathematics — trigonometry — was potential to prove the conjecture(s), but it required a complicated simplification of the intermediate results and the simplification was never completed. Nevertheless, even though incomplete, it actually boosted the problem solving in comparison to *making conjectures*. On the contrary, however, *applying complex numbers* could avoid the complicated simplification process, prove the conjecture(s), and produce complete solutions to the original problem, thereby resulting in the advancement of the problem solving. Overall, all four approaches worked effectively as a whole towards solving the problem. Thus, the problem and its solutions could be viewed as one of the mathematical objects related to the problem solving.

Beliefs about teaching. The knowing of *beliefs about teaching* emergent from the context of *Teaching Improvement* revealed many facets of the collective's beliefs about teaching, such as teaching as an art or a science, the role of research in teaching improvement, and the uniqueness of teaching. Different beliefs made teaching different. For instance, when teaching was viewed as an art, its practice could be informed by a teacher's intuition; when teaching was regarded as a science, its practice could be informed by the suggestions or results from research or others' empirical experiences.

In addition, beliefs could exert a determinant influence upon teaching practices (Tan, 2001; Thomas, 2013). For instance, in the knowing of *teaching of rational functions*, some participants firmly believed that the blogger's empirical experiences about the graphical approach were valuable references for them to improve their teaching of rational functions, and thus they decided to apply hir approach to their own teaching to improve their students' learning of rational functions.

Blog resources. The knowing of *blog resources* was shaped from two contexts: *Introduction of Rational Functions* and *Solving Problems About Chord Lengths*. In the context of *Introduction of Rational Functions*, the knowing of *blog resources* showed that sharing the original post as a resource in other cyberspaces could stimulate more conversations on the post and the related blog. This could transform the conversation topics. Meanwhile, the sharing could also open up more spaces for the post and its related blog such as within larger local or virtual communities. This could contribute to the spread of traffic and visitors from other blogs to the post and blog (Nasr & Ariffin, 2008). As a result, sharing blogs could excite more related conversations than before.

In the context of *Solving Problems About Chord Lengths*, the knowing of *blog resources* demonstrated how to take blogs as resources through curating. The curation categorized the posts for knowing mathematics into several groups: popular mathematics, investigation/computation, probability/statistics/data, proof, and mathematics history/mathematicians' biographies. In this case, the knowing of *blog resources* embraced a great deal of resources about mathematics for participants to explore.

Blogging itself has been regarded as a useful approach to sharing resources/knowledge (Chai, Das, & Rao, 2011; Deng & Yuen, 2011; Nasr & Ariffin, 2008; Loving, Schroeder, Kang, Shimek, & Herbert, 2007). Taking blogs as resources through purposeful sharing or curation could facilitate the sharing of resources or knowledge and satisfy participants' needs since sharing and receiving resources has been perceived as one benefit of PLNs for teacher professional learning (Colwell & Hutchison, 2018; Larsen & Parrish, 2019).

Recounting experiences. Recounting experiences appeared in the example of *Solving Problems About Chord Lengths*. Not only did it transform the conversation topics but also it resonated among the participants. In fact, *recounting experiences,* which was not confined to this example, was a typical way of blogging. For instance, the post *Introduction of Rational Functions* recounted the blogger's doings and experiences of hir introduction of RF. Recounting experiences represents one of the features of blogs as a convenient platform for sharing experiences (Deng & Yuen, 2011) and creates positive effects on the participants' critical reflection (Deng & Yuen, 2011; Yang, 2009).

Social relationships. Engaging in the PLN enabled the participants to build *social relationships* that were explicitly emergent in the example *Introduction of Rational Functions*. In this example, *social relationships* could be established among participants through exchanging their social information (e.g., sharing of personal social information) and expressing their appreciations for others' posts or comments. In addition, they could be developed once participants post their comments or link posts to other cyberspaces because social interactions with interlinks came along with the actions of commenting or linking posts or comments to other cyberspaces. This was noted previously in Chapter 4.

More importantly, interlinks have been defined as one of the features of PLNs (Wenger, Trayner, & de Laat, 2011). In general, *social relationships* could be achieved through explicit efforts from participants to exchange social information or express appreciation along with use of the social interlinks inherent in the PLN. Sharing could hold participants together, smooth their interactions, and make them feel comfortable with one another.

Additionally, it is conceivable that *social relationships* can be regarded as the critical elements of online community development (Chang, 2011; Sie et al., 2013) by motivating the participants to continue participating in the community (Chang, 2011; Dholakia, Bagozzi, & Pearo, 2004; Kim, 2000). The strength of community participation would "in turn help [the] virtual community to sustain its development" (Chang, 2011, p. 1).

Overall, the emergent knowing from the four selected examples consisted of the knowing of *mathematics-for-teaching,* as well as four other types of knowing: *beliefs about teaching*, *blog resources, recounting experiences,* and *social relationships*. These four types of knowing, though not directly connected to mathematics teaching, were essential to:

- the emergence of the knowing of *mathematics-for-teaching*,
- teachers' teaching practices,
- teachers' participation in the PLN, and
- the development of the PLN.

The multiple types of knowing evolving as a whole. Along with the knowing of *mathematics-for-teaching* came four other types of emergent knowing: *beliefs about teaching, blog resources*, *recounting experiences*, and *social relationships*. These five kinds of knowing were associated with each other and evolved as a whole. This is diagramed as a network image (Figure 5-15) based on the notion of networks from Capra (1996):

> We must visualize the web of life as living systems (networks) interacting in network fashion with other systems (networks). For example, we can picture an ecosystem schematically as a network with a few nodes. Each node represents an organism, which means that each node, when magnified, appears itself as a network. Each node in the new network may present an organ, which in turn will appear as a network when magnified, and so on. (p. 35)

Thus, the multiple types of knowing were further interpreted as a broader level of collective knowing.

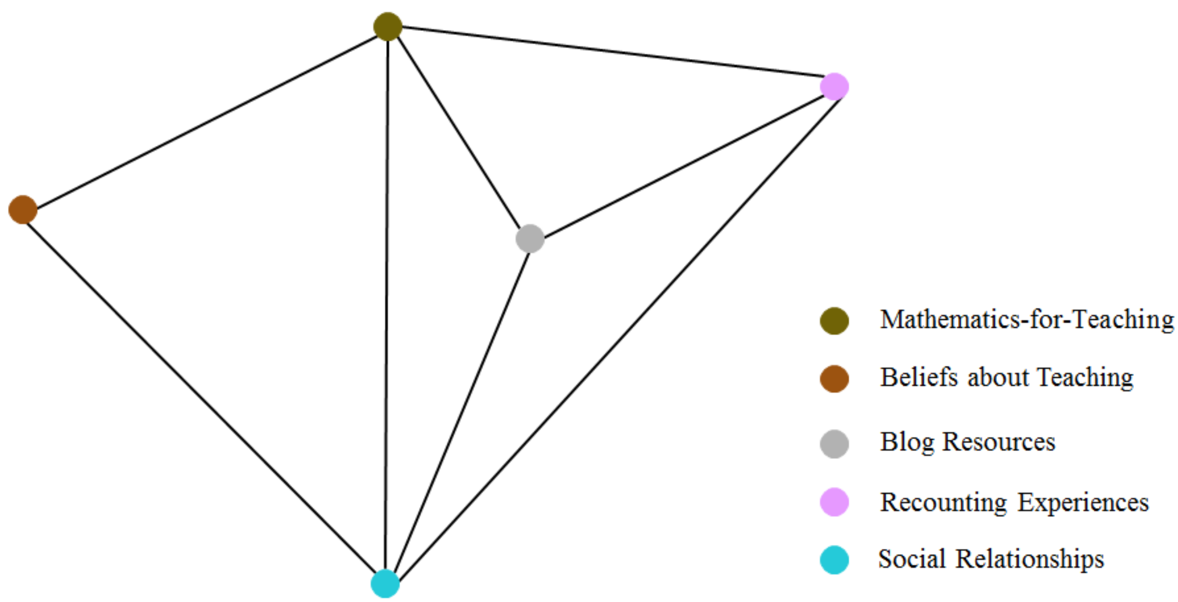

Figure 5-15. The emergent knowing.

The knowing of *mathematics-for-teaching* was the focus of the conversations in the illustrative examples, as revealed in the previous section. Specifically, the knowing in the PLN involved understanding student learning, creating classroom environment, structuring mathematics content or problems for teaching, and revealing mathematical approaches for building up concepts and solving problems. It lays emphasis on what the math teachers know for their teaching and for their professional learning; this has been explored by Davis, Simmt, and colleagues (e.g., Davis & Simmt, 2003, 2006, 2016; Davis & Renert, 2014).

The knowing of *beliefs about teaching* revealed how teachers thought of their teaching and student learning. For instance, teachers might believe that mathematics teaching could be improved on the basis of the ideas/concepts/approaches from research or the empirical experiences of others. Such knowing would not only theoretically impact the knowing of *mathematics-for-teaching*, but it also practically influenced the participants' teaching in the PLN. For instance, they applied the ideas from the post to their own teaching for supporting students' learning of RF or took the post as an example of motivating student learning in *Introduction of Rational Functions*. These two kinds of knowing were associated with each other.

The other two types of knowing, *blog resources* and *recounting experiences,* were considered essential to the emergence of *mathematics-for-teaching*. Related to the knowing of *mathematics-for-teaching,* the posts have been regarded by the participants as resources and experiences to be shared. Recounting these resources and experiences through blogging or commenting could shape and/or intensify the emergence of knowing, such as *mathematics-for-teaching.* This is because these resources and experiences revealed *mathematics-for-teaching*. For instance, the resources curated in *blog resources* were about knowing mathematics, and a problem-solving process embedded in *recounting experiences* contributed to the problem solving in *Solving Problems About Chord Lengths*. Additionally, the two types of knowing also could bring more audiences and more conversations to intensify the knowing. As the example of *Solving Problems About Chord Lengths* showed, the knowing of *recounting experience* embraced the knowing of *blog resources* since participants not only recounted their working and learning experiences but also recommended for others the blog resources they explored by themselves. Thus, these five types of knowing — *blog resources*, *recounting experiences*, *mathematics-for-teaching*, and *beliefs about teaching* — were interrelated with each other.

The knowing of *social relationships* was embedded in the conversations thereby enabling the participative environment to be comfortable for participants. More importantly, this type of knowing made the collective conversations possible because it helped participants achieve social ties, which are considered to be the "channels through which [ideas], information and resources can flow" (Tsai & Ghoshal as cited in Chai et al., 2011, p. 315). In other words, they made the emergence of knowing possible. Therefore, the knowing of *social relationships* was associated with the other four types of knowing.

All five types of knowing demonstrated the possibilities of participants' co-construction of knowing in the PLN. The network image (Figure 5-15) of collective knowing stands as a whole with the connections running through the emergent knowing. From the perspective of the living networks (Capra, 1996), the nodes representing the multiple types of knowing are considered to be the organisms. This means that when magnified, a node appears as a network within which the nodes representing another layer of emergent knowing represents the organisms. For instance, the dark yellow node for the knowing of *mathematics-for-teaching,* if magnified, was a network covering the knowing of:

- *student learning,*
- *classroom conversations or environment,*
- *teaching of rational functions or teaching of problem solving,* and
- *mathematical problems, the graphical approach,* or *problem solving*.

The visualized connections between *mathematics-for-teaching* and the other types of knowing provide a different picture about mathematics-for-teaching or teachers' disciplinary knowledge of mathematics from the well-known theories such as Ball et al.'s (2008) knowledge of mathematics for teaching and Davis and Renert's (2014) mathematics-for-teaching. *Mathematics-for-teaching* did not stand alone but was implicated with *beliefs about teaching*, *blog resources*, *recounting experiences*, and *social relationships*. These emerged as a whole.

5.3 The Structures of Conversations

The above analysis places heavy emphasis upon the emergence of knowing from the collective conversations. Even if the emergence of knowing was unveiled, however, the conversation structures were not yet clear. They are uncovered in the following subsections through two dimensions: conversation weaving and conversation extending, with the former revealed by the analysis on the recursions and the latter by the conversation extensions. The analysis of these dimensions demonstrates the diverse conversational structures.

5.3.1 Recursions

How the comments were weaved in the conversations mattered for the structures of the conversations. The comment threads manifested how ideas were developed collectively in the conversations. This could be particularly elaborated by the responses of participants to other people's comments based on the recursive dynamics (Chapter 4).

Defining recursions. In this study, two ways of participants' reviewing or responding to others' comments were observed: technical reply and semantical reply. One commenter could reply to another comment through the reply button under that comment. This is referred to as a technical post. For instance, in the example of *Teaching Improvement*, the commenter of Comment 7.1 replied to Comment 7 through the reply button and questioned the definition of teaching offered in the comment. In contrast a semantical reply involves one commenter providing feedback to another comment without posting directly to the comment that triggered the response. Such comments may appear as new comments to the original post or to comments on other comments but not technically connected to them.

For instance, one commenter posted Comment 4 by using the reply button under the post. This was a technical post in relation to the original post but semantical in relation to Comment 2: Comment 4 disagreed with the argument in Comment 2 that teaching was an art not a science. To make the response clear that it was in response to Comment 2, the information "@ Comment 2" was written at the beginning of Comment 4. Comment 6 continued that particular conversation thread with a refutation of the statement of Comment 4. Comment 6 was also a technical reply to the post and a sematic reply to Comment 4 as revealed by a direct quote from Comment 4. These three comments (Comment 2, 4, and 6) are specified as an example of a *onefold loop with three nodes* (Figure 5-24).

Overall, one commenter could either respond to another comment a) technically by using the reply button under that comment, or b) semantically by referencing its content. In my study, I tried to interpret the responsive relationships semantically rather than technically, even though there were cases when participants responded technically and semantically to the same comments.

Diagramming the dynamic interactions involved in the responsive relationships was done through the creation of semantic loops. Take the relationship between Comments 7 and 7.1 as an example again. If the commenter of Comment 7.1 responded to Comment 7, the responsive relationship could be presented as a flow chart (see Figure 5-16).

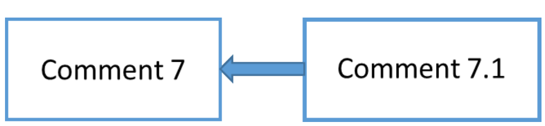

Figure 5-16. The diagram of Comments 7 and 7.1.

The flow chart fully displays the responsive relationship, but it fails to express the implied meaning involved in the interactions, which was underlined in my interpretations. For that reason, based on the connotations in the comments, I imaged the recursive dynamics (Figure 4-6) as a “feedback loop” (Capra, 1996, p. 56; Davis et al., 2008, p. 77) (Figure 5-17), which is described by Capra (1996) as “a circular arrangement of causally connected elements” (p. 56).

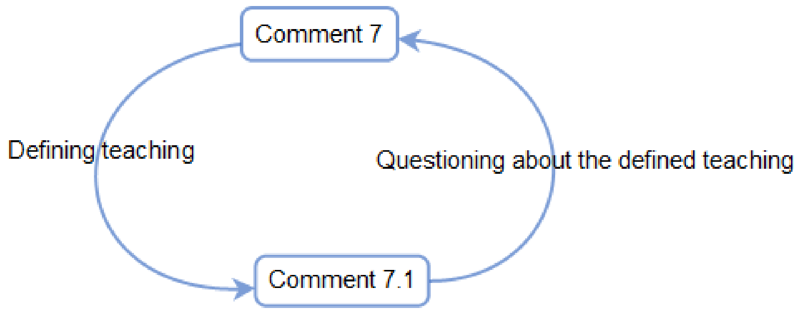

Figure 5-17. The semantic loop between Comments 7 and 7.1 of *Teaching Improvement.*

The feedback loop has both nodes and oriented arcs that illustrate how key ideas emerged from the comments. The loop is called a semantic loop. Generally speaking, when the commenter of *a comment* (e.g., Comment 7) brings new viewpoints or perspectives (e.g., defining teaching) to the conversation, the viewpoints or perspectives are (re)examined by the commenter of *another comment* (e.g., Comment 7.1) according to hir knowing, experiences, and beliefs. Thus, the semantic loop between the comments (e.g., Comment 7 and Comment 7.1) could result in a disagreement (e.g., questioning about the defined teaching), a further question, a new argument, and/or beyond. Even though it was done collectively, this process could be regarded as a recursion according to Doll’s (2008) framing of recursion as “a looping back to what one has already seen/done to ‘look back and see, yet again, for the first time’ ” (p. 9).

Multiple types of recursions. In the conversations from the four selected examples, multiple types of recursions were embedded:

- onefold loop with two nodes,
- onefold loop with three nodes,
- implicative loops,
- nested loops, and
- reflective loops.

These recursions are respectively presented with recursion maps, which were reshaped from the conversation maps (Figure 5-3, Figure 5-4, Figure 5-5, and Figure 5-6) presented in Section 5.1. Within the recursion maps, the comments coded as red nodes refer to the ones responding to the original posts while the comments coded as yellow nodes refer to the ones responding semantically to other comments. There were cases where the commenters of some comments responded technically to the post but semantically to others. For example, the following recursion map for the context of *Teaching Improvement* (Figures 5-3 and 5-18) shows that the commenter of the yellow-coloured Comment 14 replied technically to the original post but semantically to Comment 13.2.

The recursion map for the example of *Teaching Improvement* (Figure 5-18) demonstrates that almost all the comments were involved in a diverse set of feedback loops.

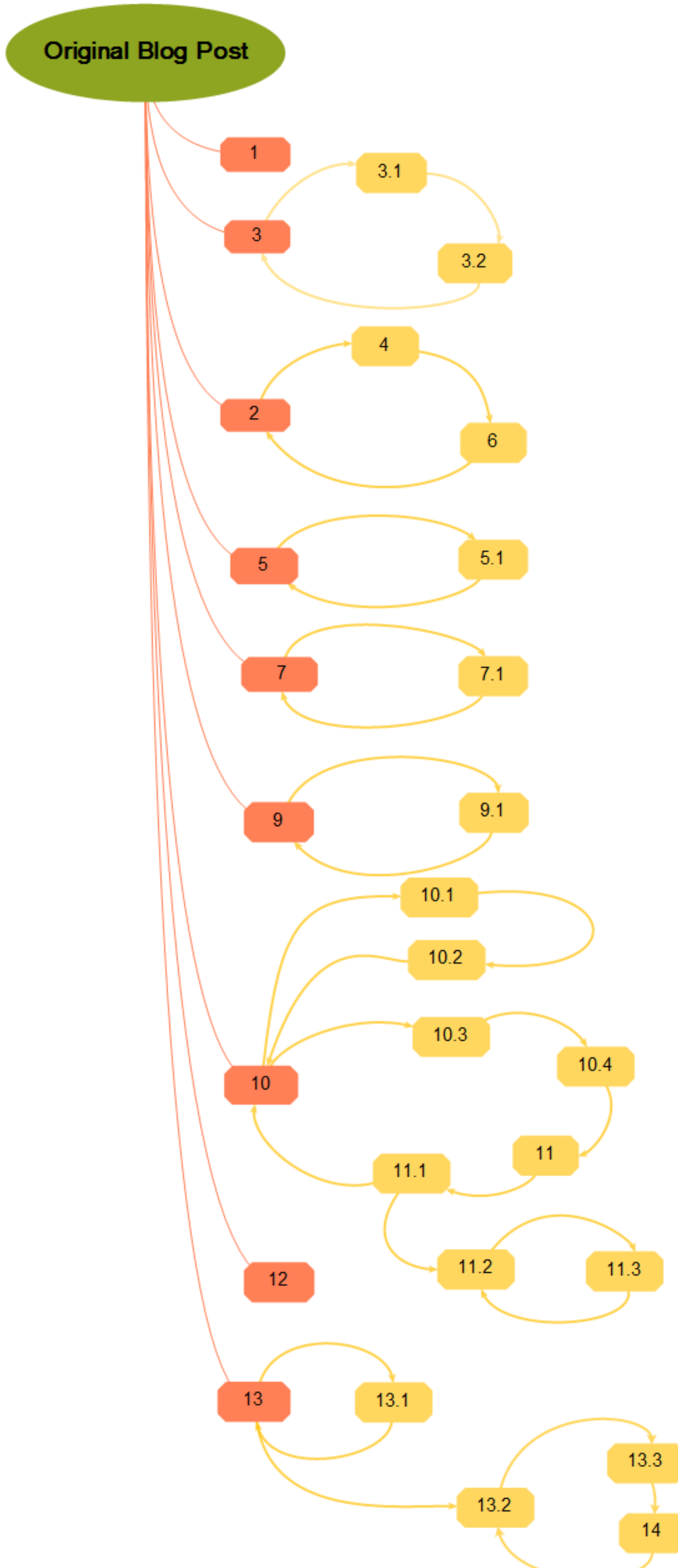

Figure 5-18. The recursion map of *Teaching Improvement.*

The recursion map for the example of *Textbook Presentations of the Handshake Problem* shows only two semantic loops (Figure 5-19) characteristic of a onefold loop with two nodes, which is detailed in the upcoming section "*Onefold loop with two nodes*."

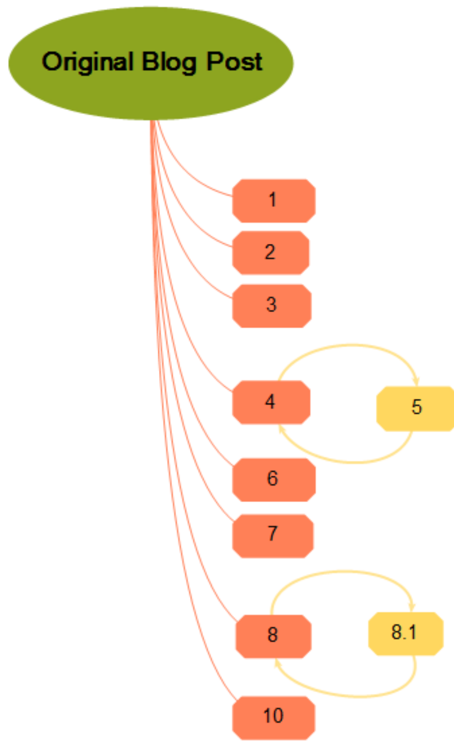

Figure 5-19. The recursion map of *Textbook Presentations of the Handshake Problem.*

The recursion map for the example *Introduction of Rational Functions* (Figure 5-20) indicates that the majority of the comments were involved in the semantic loops. However, the loops are mainly onefold with two nodes (see the details in the upcoming section "*Onefold loop with two nodes*"). They took shape during the conversations when the comments involved posting and answering question, increasing and clarifying confusion, sharing and appreciating what was shared, and exchanging social information.

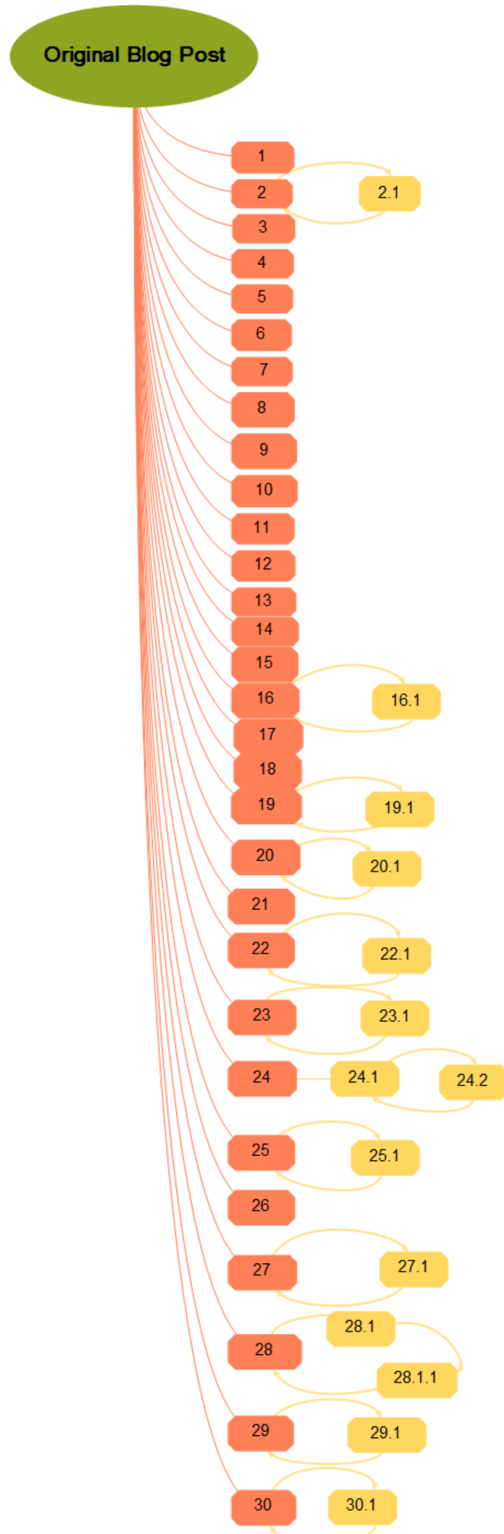

Figure 5-20. The recursion map of *Introduction of Rational Functions.*

Finally, the recursion map for the example *Solving Problems About Chord Lengths* (Figure 5-21) illustrates that the majority of the comments were attached with complicated semantic loops.

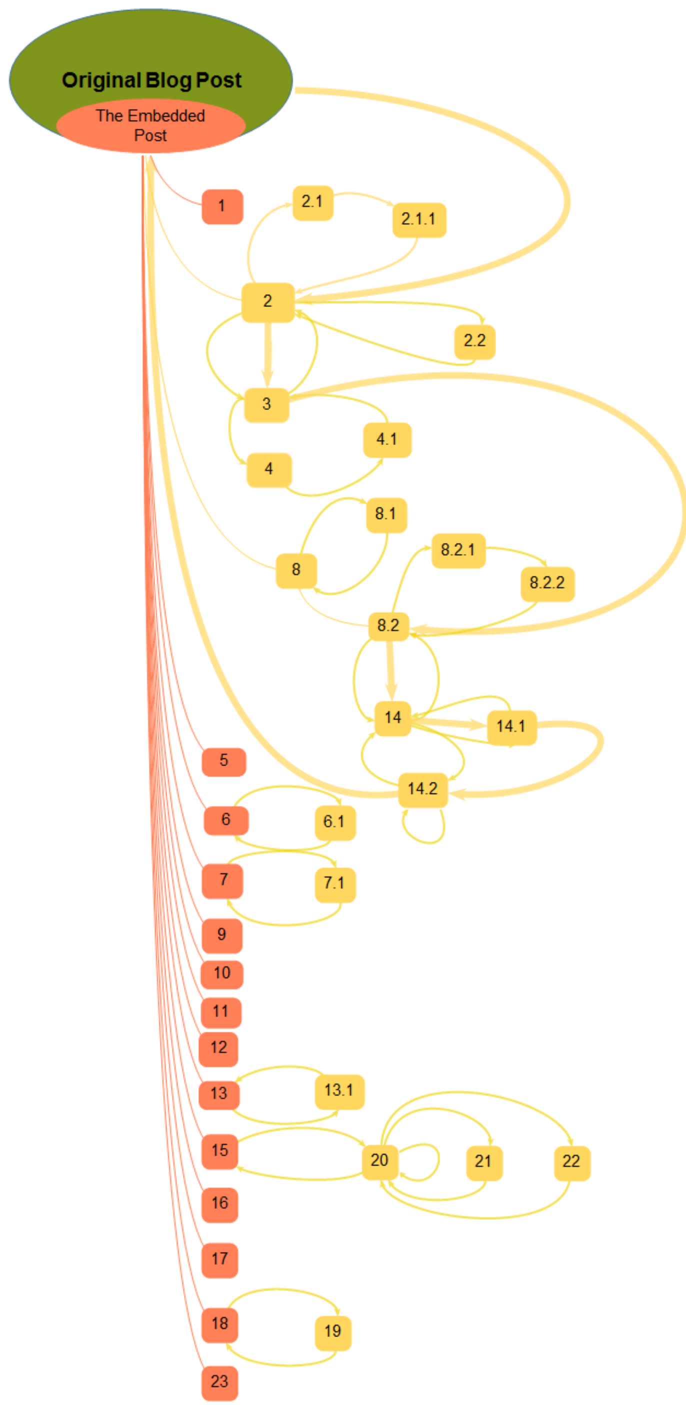

Figure 5-21. The recursion map of *Solving Problems About Chord Lengths.*

Overall, from the recursive maps, multiple types of recursions are manifested in those four examples. The recursions are divided into five main categories:

- *onefold loop with two nodes*,
- *onefold loop with three nodes*,
- *implicative loops*,
- *nested loops*, and
- *reflective loop*.

Onefold loop with two nodes. This type of loop occurs between two cooperative comments. Examples appeared in all four blogs, particularly in: *Textbook Presentations of the Handshake Problem* and *Introduction of Rational Functions*. The loop emerges in this way: when the viewpoints or definitions are provided in one comment, they are criticized or favored in the other; or when a question is put forward or confusion is expressed in a comment, it is answered or clarified in another comment.

A good example for such a loop was the one that occurred between Comment 5 and Comment 5.1 (Figure 5-22) from the example of *Teaching Improvement.* The commenter of Comment 5 confirmed a role for research in informing teaching and clarified that although some sorts of research could not support the teachers' daily teaching, others could. Subsequently, that viewpoint was criticized by the commenter of Comment 5.1, who argued that teaching was an art rather than a science and hence independent of research, though a teacher could learn much from it.

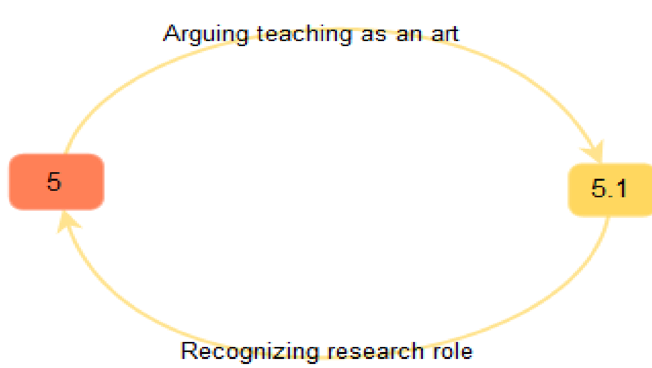

Figure 5-22. The semantic loop between Comments 5 and 5.1 from the example of *Teaching Improvement.*

The loop that appeared between Comment 4 and Comment 5 from *Textbook Presentations of the Handshake Problem* is another example of such a loop. The commenter of Comment 4 suggested that a textbook should leave more space for students to build the connections between the Handshake Problem and the diagonal problem. The commenter of Comment 5 agreed by exemplifying hir own teaching experiences, including working with colleagues to design their own geometry text, allowing classroom conversations to flow naturally, and offering students the opportunity of approaching the problem authentically.

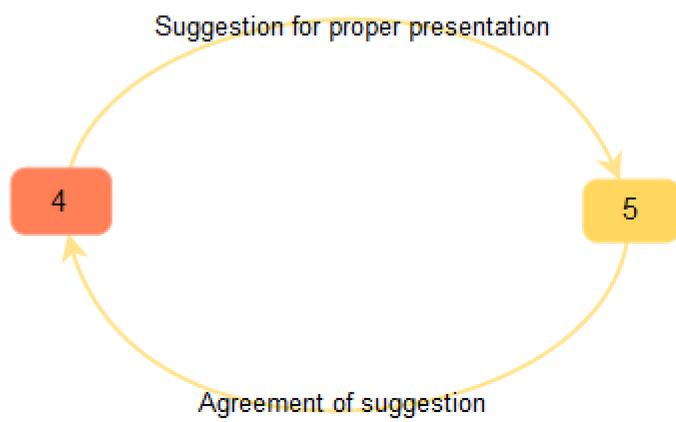

Figure 5-23. The semantic loop between Comment 4 and 5 from the example of *Textbook Presentations of the Handshake Problem.*

Onefold loop with three nodes. This type of loop occurs among three interrelated comments and is exemplified in the blog of *Teaching Improvement.* A onefold loop with three nodes appears when a chain reaction is triggered: when a viewpoint or definition was offered in one comment, it was criticized in another one and, further, the criticism was reviewed in the other; or when a question or confusion was raised or caused in one comment, it was answered or cleared up in another one and, further, the answer or clarification was reviewed or criticized in the other.

For instance, a onefold loop occurred among Comments 2, 4, and 6 from the example of *Teaching Improvement*, with the commenters arguing about whether or not teaching was an art (Figure 5-24). The commenter of Comment 2 made explicit that teaching was not a science but an art because it always requires skills; that statement was doubted by the commenter of Comment 4 who believed that teaching generally contained two aspects of delivery and design — both of which depended on skills and suggestions from research — thereby arguing that the claim in Comment 2 was far from convincing.

However, the argument in Comment 4 was refuted by the commenter of Comment 6 because ze believed that, in addition to those two aspects, teaching also included two more: learning and testing. Furthermore, according to hir own experiences, the commenter of Comment 6 considered that even though there was some influential research in teaching, teaching was not informed by research as much as hir own instincts, thereby still insisting that teaching was an art and not a science.

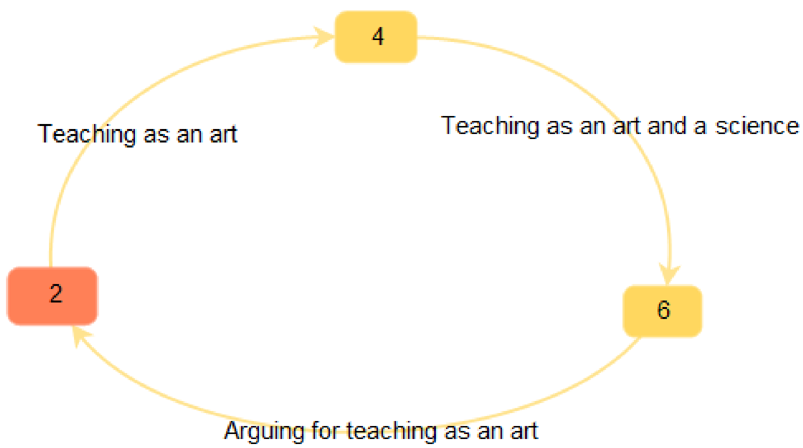

Figure 5-24. The semantic loop among Comments 2, 4, and 6 from the example *Teaching Improvement.*

Implicative loops. These types of loops illustrate how some loops are implicated with other loops through common comments. That means that one comment straddled more than one loop. For instance, Comment 10 from *Teaching Improvement* was involved in two implicative loops. One loop occurred among Comments 10, 10.1, and 10.2 (Figure 5-25). In Comment 10 there was a call for analogies for writing that are applicable to teaching; then in Comment 10.1 two analogies were made about the desired outcomes of writing. Further, in Comment 10.2 the two analogies were reviewed, one of which was criticized for its unreasonableness.

Figure 5-25. The semantic loops among Comments 10, 10.1, and 10.2 and among Comments 10, 10.3, 10.4, 11, and 11.1 from the example of *Teaching Improvement.*

Another loop appears among Comments 10, 10.3, 10.4, 11, and 11.1 (Figure 5-25). To respond to Comment 10, the commenter of Comment 10.3 made two more analogies regarding the repetition of writing product and teaching product: writing the same popular novel again (writing) and using the same interesting activity again (teaching). Ze further inquired about why it was ridiculous to use product repetition in writing rather than in teaching; the commenter of Comment 10.4 replied to the inquiry and explained the reason by defining the products of teaching and writing. The commenter of Comment 11 questioned the definition of teaching product alone; the commenter of Comment 11.1 then reflected upon that definition and began to refine it.

The two loops present the conversational manner: a question about the analogy between writing and teaching is posted in a comment (Comment 10), then the answers to it are offered, reviewed, and/or refined respectively in two loops (Comments 10.1 and 10.2; Comments 10.3, 10.4, 11, and 11.1). Such engagement could deepen participants' and/or audiences' understanding about the analogy between writing and teaching.

Comment 11.1 is also implicated with the loop between Comments 11.2 and 11.3. In Comment 11.1, the refined definition of teaching product triggered the commenter of Comment 11.2 to ponder how research could play a role in teaching. In response to Comment 11.2, the commenter of Comment 11.3 argued that research could provide helpful insights or suggestions for students' learning. Thus, the loop between Comments 11.2 and 11.3 explores the role of research in teaching through posing and answering questions.

Nested loops. These types of loops present the embedded relationships between or among loops. Such relationships can occur when the loops are hyperlinked to each other or when one loop embraces the other or several others. The nested loops appear in the example of *Solving Problems About Chord Lengths.*

For instance, Comment 20 was a pingback linking to a post about a partial solution to EP that could be revisited through Comments 21 and 22 (Figure 5-26), respectively; Comment 21 was also a pingback connecting to a resource sharing platform to which Comment 20 was hyperlinked. Thus, Comment 20 could be revisited through Comment 21 if a participant chose to do so, and between these two comments appeared one loop. Like Comment 21, Comment 22 was also a pingback linking to a post that contained experience sharing, to which Comment 20 was hyperlinked too. Thus, Comment 20 could also be revisited through Comment 22, and between them occurred the other loop.

The aforementioned two loops proceed towards the same comment, Comment 20, in the contexts of resource sharing and experience sharing. The hyperlinks of that comment to resource sharing and experience sharing enabled the loop between Comments 21 and 20 to be embedded in the loop between Comments 20 and 22.

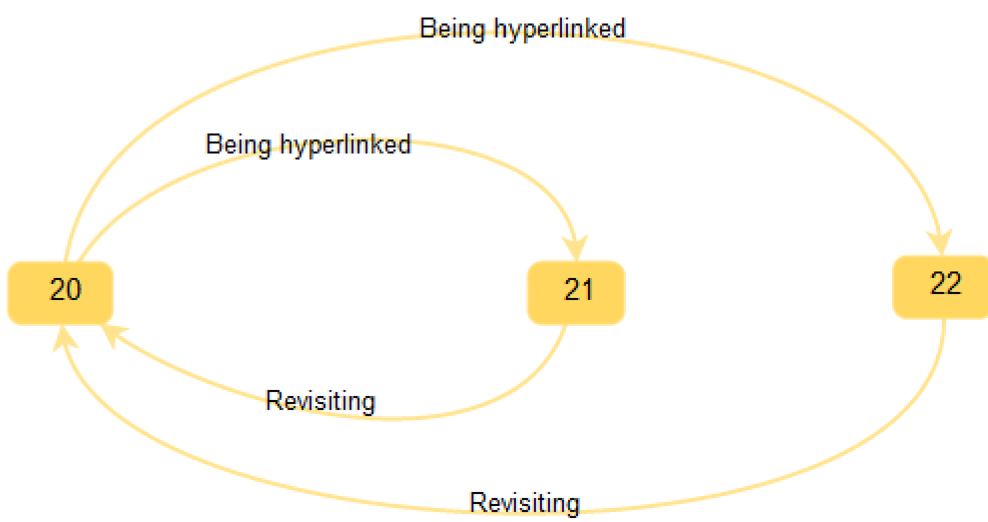

Figure 5-26. The semantic loops of revisiting Comment 20 from the example *Solving Problems About Chord Lengths.*

Another example of nested loops comes from the efforts made by the participants to figure out an applicable solution to the original problem (OP), which was set for pre-calculus students. These efforts are demonstrated as a loop across the comments, which alternatively referenced the solutions using trigonometry or complex numbers (Figure 5-27).

Early efforts among the participants used trigonometry to find a solution (e.g., Comments 2 and 3). At first, they thought of it as a potential approach to solve the original problem, but they soon found it difficult to obtain the final result because of its complicated simplifying process. Then, they turned to another approach — applying complex numbers (e.g., Comment 8.2) — but it involved advanced knowledge which were viewed as unsuitable for pre-calculus classes. Thus, some other participants (e.g., Comment 14) tried to use polynomial expansions to refine that solution; others (e.g., Comments 14.1 and 14.2) attempted to completely withdraw the advanced knowledge from the solution of Comment 14 and resorted to re-using trigonometry in a different way from the early efforts of some participants. Re-using trigonometry was regarded as the proper approach for pre-calculus students to solve OP, but the approach failed again in the complicated simplification.

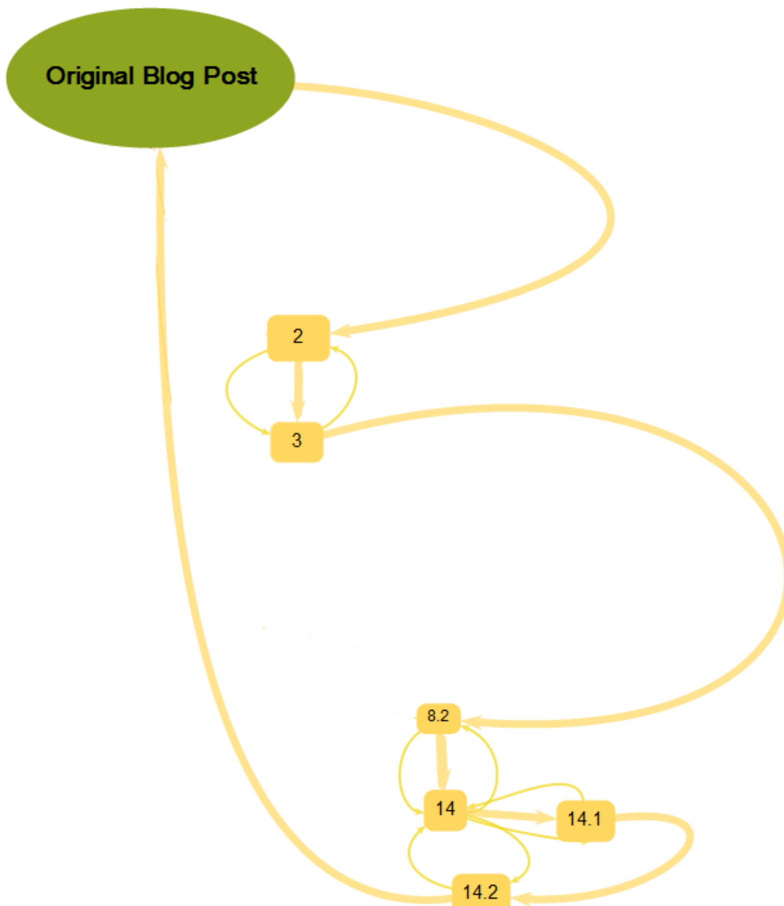

Figure 5-27. The loop of building up solutions among the comments in the recursion map for the example *Solving Problems About Chord Lengths.*

To better view the alternate route of finding the solutions, the loop (Figure 5-27) is re-diagramed into the following big blue routed loop (Figure 5-28). The big loop demonstrates that some small (grey) routed loops are enveloped by broader nodes, such as the loop between Comments 2 and 3 testifying a potential solution, or the one between Comments 8.2 and 14 refining a complete solution, or the reflective loop within

Comment 14.2 finding a solution by re-using trigonometry. It also displays that across the nodes appeared other small grey routed loops, such as the loops between Comments 14 and 14.1 and between Comments 14 and 14.2, which tried to refine the complete solution with advanced knowledge. This big loop seems to go through several small ones and take them as its integral parts.

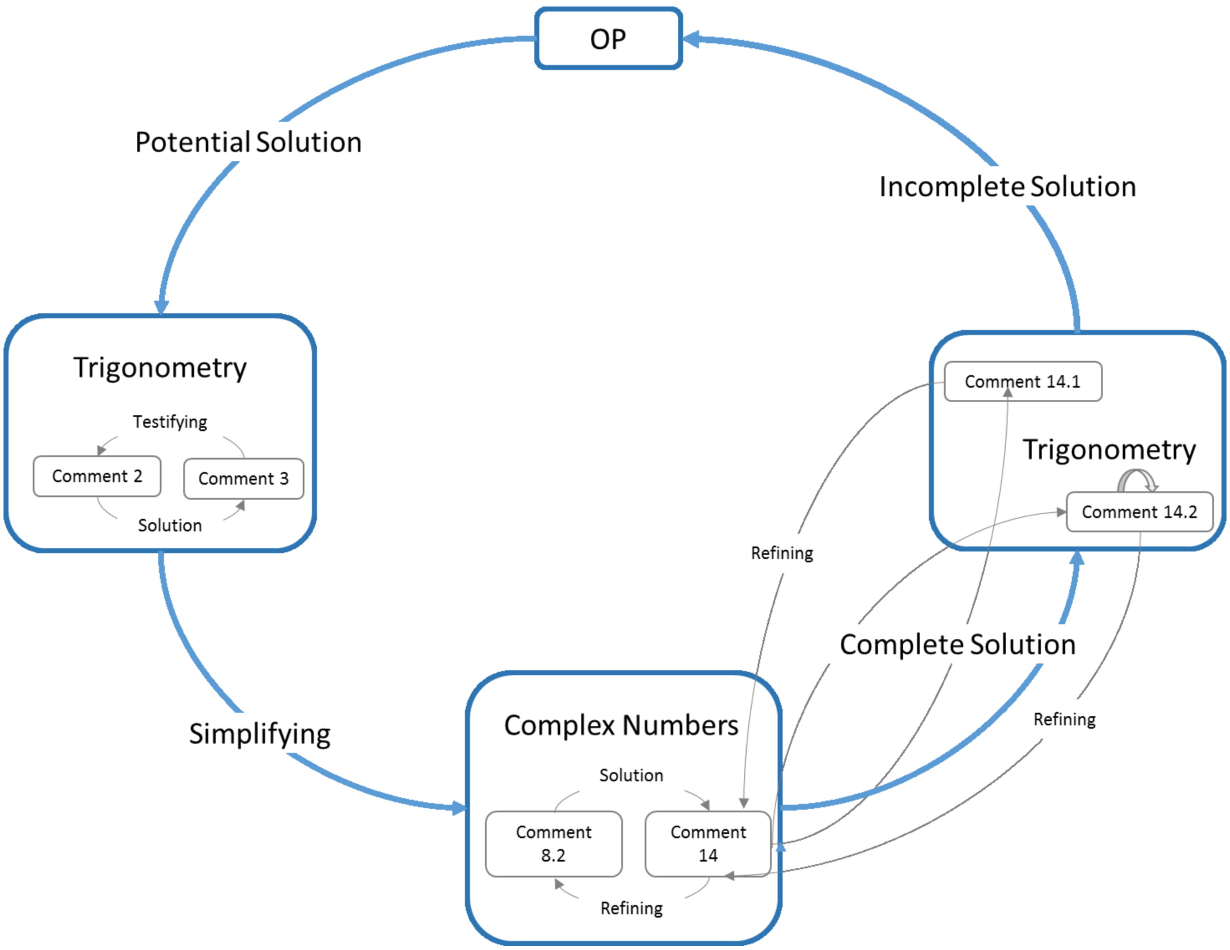

Figure 5-28. The semantic loop of building up solutions among the comments from the example *Solving Problems About Chord Lengths.*

Reflective loops. These types of loops are different from the above ones referred to as individual loops, which presented a detailed thinking process including self-reflection and self-refinement about a solution to or suggestion about a problem, or about an issue from the posts or conversations. They involve a self-looping-back process, which could be considered as recursions. They appear in the example of *Solving Problems*

About Chord Lengths. From the emergent thinking processes of solving OP and the extension problem (EP), reflective loops occur within Comments 14.2 and 20, respectively.

Specifically, the commenter of Comment 14.2 presented a process of finding a solution to OP (Figure 5-29). Ze made the first try to enumerate six cases (from n=1 to n=6) for the search of the pattern of the product by creating special triangles, but ze failed. Then, ze turned hir attention to another approach of the distance formulas that could yield the product, but it involved a complicated simplifying process: first using the trigonometric identities and then splitting the situations when n was an even and an odd number. Unfortunately, the approach did not work out the final result. Thus, those two approaches fell into abeyance. Nevertheless, it was the case that the second approach simplified the process of computing the product even if no complete solution occurred.

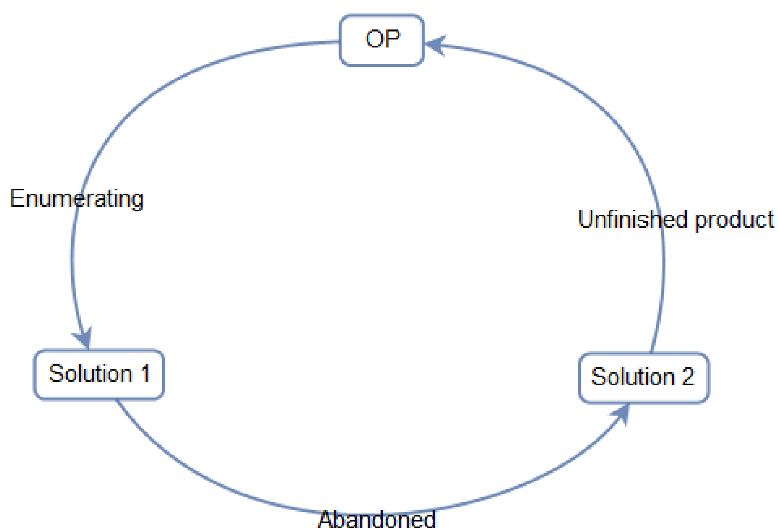

Figure 5-29. The semantic loop of improving solution within Comment 14.2 from the example *Solving Problems About Chord Lengths.*

For another example, the commenter of Comment 20 contributed to a partial solution to the extension problem (Figure 5-30). Ze started to work on the original conjecture (C1) for EP from Comment 15, which had been advanced by hir friend through their email interactions. On the basis of C1, hir friend found a new conjecture (C2) for a general problem (GP) when expanding EP into a more general problem — EP was assumed to be a case of GP. This meant that if C2 for GP was proved to be correct, so was C1 for EP. In that case, C2 could be understood as facilitating the problem solving of EP. On the basis of C2, the commenter of Comment 20 tried to work out some forms for C2. Ze discovered another new conjecture (C3) for GP when deploying the forms. In other words, if correct, C3 could be used to prove both C1 and C2 to be true. In this sense, C3 could be construed to advance the problem solving of EP.

Based on C1, the loop within Comment 20 began working on EP and produced C2. If true, C2 could be used to prove C1 to be correct. Then, based on C2, the loop worked on GP and yielded C3. If true, C3 could be used to prove both C1 and C2 to be right.

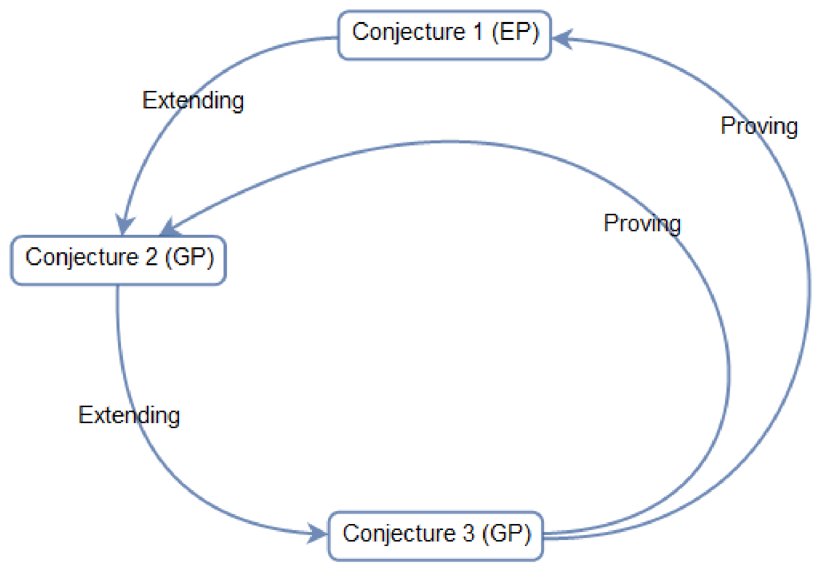

Figure 5-30. The semantic loop of Comment 20 from the example *Solving Problems About Chord Lengths.*

The multiple types of loops were embedded in the conversations in the four examples. For example, onefold loops with two nodes mainly appeared in the examples *Textbook Presentations of the Handshake Problem* and *Introduction of Rational Functions*; however, in addition to onefold loops with two nodes, more kinds of recursions existed in the other two examples: onefold loops with three nodes and implicative loops occurred in the examples *Teaching Improvement* and *Solving Problems About Chord Lengths,* while reflective loops and nested loops occurred in the example *Solving Problems About Chord Lengths.*

Further, more comments were involved in the recursions in the examples *Teaching Improvement* and *Solving Problems About Chord Lengths* than in the other two. For instance, the recursion maps showed that 27 out of 28 comments (96%) were involved in the recursions in the example *Teaching Improvement* (Figure 5-18), 27 out of 37 (73%) in *Solving Problems About Chord Lengths* (Figure 5-21), 25 out of 43 (58%) in *Introduction of Rational Functions* (Figure 5-20), and 2 out of 11 (18%) in *Textbook Presentations of the Handshake Problem* (Figure 5-19). In brief, the types of recursions and the number of comments involved herein made explicit that the conversations were more recursive in the examples *Teaching Improvement* and *Solving Problems About Chord Lengths* than in the other two.

5.3.2 Conversation extensions

Web linkages, hyperlinks, and pingbacks were all used in the comments and/or posts to extend the conversations. These are classified as conversation extensions. They mainly occurred in the examples *Introduction of Rational Functions* and *Solving Problems About Chord Lengths.*

In the example *Introduction of Rational Functions,* web linkages were embedded in the comments to introduce a similar post (Comment 2), a new school learning environment (Comment 5), the free resources for teachers (Comment 23.1), and an alternative approach of introducing RF (Comment 6). In the example *Solving Problems About Chord Lengths,* hyperlinks were implanted in the post and the comments to update the work on the problem solving by diagraming the problem(s) (an embedded post) and illustrating a solution to the original problem (Comment 19). Those linkages and hyperlinks not only produced or extended conversations related to the posts but also facilitated participants and/or audiences gaining a better understanding of the viewpoints expressed in the comments.

A pingback is a special type of comment which links the original post to other posts, and which plays multiple roles in the conversations, such as bringing the conversations into broader contexts, generating new conversational points, and transforming the conversation topics. In the example *Introduction of Rational Functions*, the original post was pingbacked as:

- a teaching resource for RF in another post (Comment 7) and on a resources repository website for teachers (Comment 13);
- an example of moving away from the procedural understanding of RF in a post (Comment 10) and of sharing blogs about teaching practice with details (Comment 14);
- the background knowledge of an extended conversational topic of RF — *assessment* — in another post (Comment 12); and/or
- a reference for graphing rational functions in another post (Comment 26).

These comments as pingbacks extended the conversations related to the original post to such broader contexts as gathering resources for teachers, highlighting conceptual understanding, assessing RF, and using lines as a graphical way of learning algebra. In the example of *Solving Problems About Chord Lengths*, the pingbacks transformed the conversation topic on problem solving. For instance, Comment 22 (pingback) brought in 9 further comments and 17 hyperlinks connecting to other blogs, videos, web pages of governments, journals, and so forth, which completely went beyond and transformed the conversation topic of solving problems about chord lengths.

The conversation extensions are depicted as the images of stretched branches (Figure 5-31 and 5-32). With the original post (green node) considered as an "initial seed" (Smitherman, 2005, p. 158), the pingbacks or web links attached to the comments (blue nodes) functioned as the new "seeds" (purple nodes), which yielded further comments (yellow nodes) (e.g., Comments p16 and p16.1, Figure 5-31) or hyperlinks (yellow nodes) (e.g., Comments e5 and HBP30, Figure 5-32). For instance, in the example *Introduction of Rational Functions*, Blog Post 4 (purple node) as a pingback post spawned 45 further comments (5733 words) (see the yellow nodes on the right side of Figure 5-31); and in *Solving Problems About Chord Lengths*, Blog Post 3 (purple node) as a pingback post linked to 30 hyperlinks (see the yellow nodes on the right side of Figure 5-32). In addition, these derived comments and hyperlinks would act as other new "seeds" and continue to produce more new topics, ideas, thoughts, or perspectives. In the example *Solving Problems About Chord Lengths*, the hyperlink 13 (HBP 13) of Blog Post 4 (Figure 5-32) brought participants to a new cyberspace, within which they were able to further explore a teacher's thoughts about life, science, and religion posted using the formats of water painting and handwriting. Those explorations were far beyond the inquiry of the post.

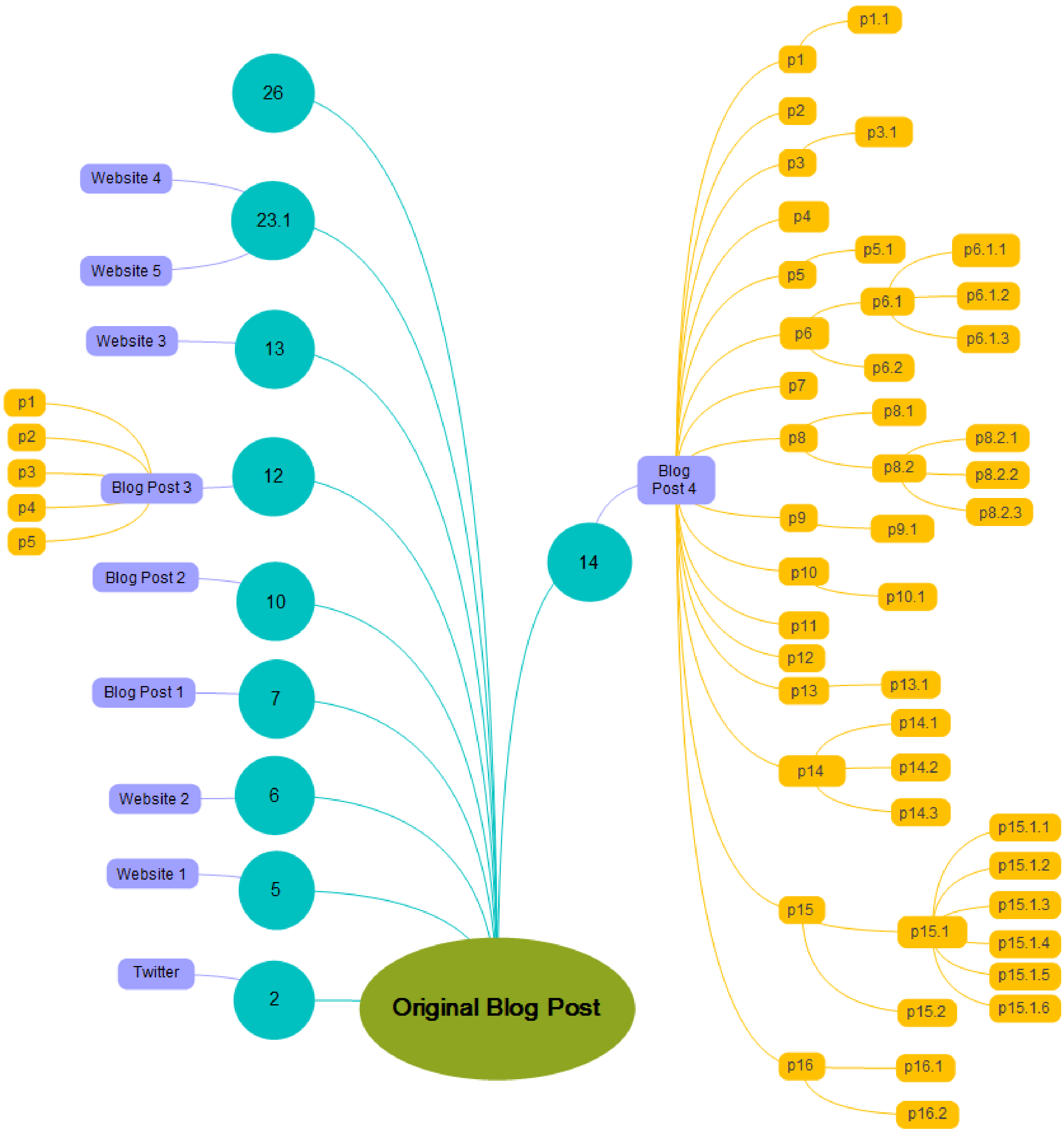

Figure 5-31. The conversation extension in the example *Introduction of Rational Functions* (Note: Number n represents Comment n; Code pn represents Comment n, which follows a pingback as a comment of the original post; Twitter and websites 1–5 are embedded in the related comments; Blog Posts 1–5 are pingbacks of the original post).

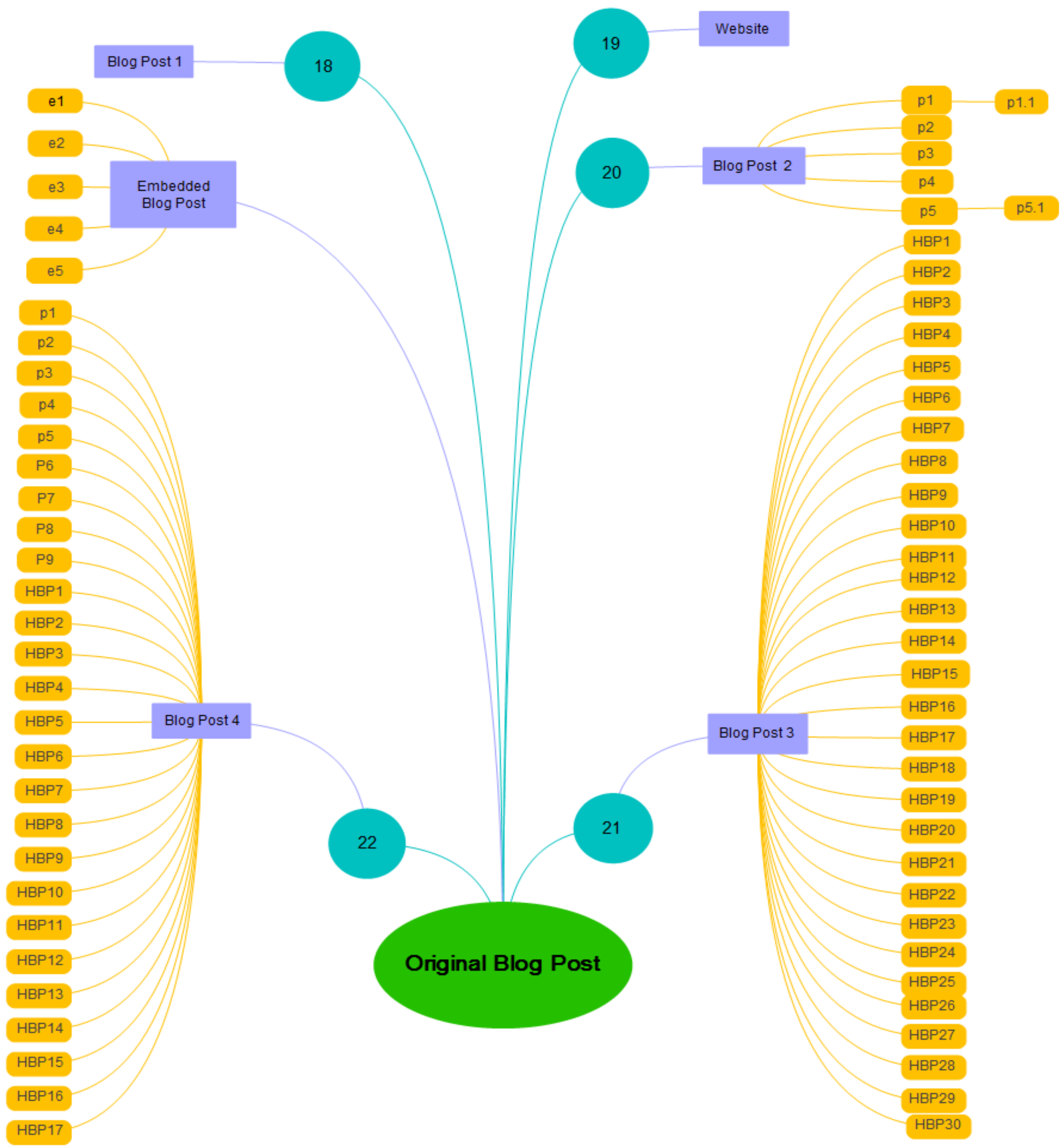

Figure 5-32. The conversation extension in the example *Solving Problems About Chord Lengths* (Note: Number n represents Comment n; Code pn represents Comment n following a pingback as a comment of the original post; Code en represents Comment n following the embedded post in the original post; Code HBPn represents the post n hyperlinked in the pingback. The website is embedded in the comments on the original post. Blog Posts 1–4 are pingbacks of the original post).

Theoretically, the process of "seeding" could go on forever without end. Such kind of extensions present in a fractal-like manner as a continuous, iterative process. In fact, the above branching images are not exhaustive. At the top end of their branches, all the potential "seeds" could still grow outwards and direct people to further explore the online learning environment. For instance, the hyperlinks or the hyperlinked blog in Comments 22 and 21 (Figure 5-32) allowed people to directly link to other cyberspaces within which the focal topics extended beyond both the topic of the problem solving in question and even the domain of mathematics education.

5.3.3 The diverse structures of conversations

The structures of the conversations are presented through conversation weaving and conversation extending. The analysis on the recursions indicates that the conversations weaved differently in the four examples. As described in the previous subsections, the conversations in the examples *Teaching Improvement* and *Solving Problems About Chord Lengths* were found to be much more recursive than those in the examples *Textbook Presentations of the Handshake Problem* and *Introduction of Rational Functions.* The recursions occurring in the first two examples are mainly presented in the context of reviewing or providing feedback based on criticism, refinement, or refutation of the proposed viewpoints or solutions.

Recursions in the last two examples are chiefly expressed through confusion clarifications, the sharing of appreciation, or social information exchange. Moreover, the first two examples possess nearly all forms of semantic loops (onefold loops with two nodes, onefold loops with three nodes, implicative loops, nested loops, and reflective loops) while the last two have only one type of loop — onefold loop with two nodes.

The analysis on the conversation extensions shows that the conversations extend variously in all the examples. For instance, the conversations in the examples *Teaching Improvement* and *Textbook Presentations of the Handshake Problem* are primarily extended by regular comments (i.e., the comments were posted by the commenters who wrote the comments through the function of "post a comment" or "reply") while the ones in the examples *Introduction of Rational Functions* and *Solving Problems About Chord Lengths* are more dramatically extended by hyperlinks and/or pingbacks in a fractal-like manner than by regular comments. Those hyperlinks and/or pingbacks bring in heavyloaded content because their linkage to other blog posts generally accommodated much more content than regular comments. They also attracted more potential audiences to the posts because they open more cyberspaces for the posts.

Overall, the conversations from the selected four examples are characterized by the diversity of the structures. And more importantly, they were shown to produce multiple types of knowing.

5.4 The Connections Between the Structures of Conversations and the Emergence of Knowing

The emergence of knowing and the conversation structures are not separate but interdependent. My main concern is about the roles the conversation weaving and conversation extending played in the emergence of knowing. Re-examining the conversations resulted in observing the different kinds of roles they play in underpinning the emergence of knowing: recursions intensify the emergent knowing and conversation extensions transform the emergent knowing.

5.4.1 Recursions and the emergent knowing

The recursions occurring from the comments essentially intensify the evolution of the emergent knowing when participants worked on discussion points or ideas. Take Comments 2, 4, and 6 in the context of *Teaching Improvement* (see details in "*Onefold loop with three nodes*," subsection 5.3.1) as an example again; the loop between Comments 2, 4, and 6 illustrates the dynamics of the arguments about whether or not teaching was an art. The arguments could strengthen the viewpoints about teaching as an art or a science and shape the knowing of *teaching as an art or a science*. To fully express the recursions in relation to the emergent knowing, the recursion maps in Figure 5-18, 5-19, 5-20, and 5-21 were reshaped into the topical ones.

The recursions touched upon different types of emergent knowing from the example of *Teaching Improvement* (Figure 5-33), but in particular one of them, the knowing of *the analogy between writing and teaching,* received more attention than the other kinds. Reviewing the recursions helped me to identify different kinds of knowing emergent from each loop. For instance, the loop between Comments 5 and 5.1 focused on the research role that pertains to the knowing of *the role of research in teaching*; the loop between Comments 7 and 7.1 illustrates the attempt to understand the concept of teaching, which could support the knowing of *teaching as an art or a science* ; and the loops between Comments 9 and 9.1, between Comments 13 and 13.1, among Comments 10, 10.1 and 10.2, and among Comments 10, 10.3, 10.4, 11, and 11.1 illustrate the aim to understand the analogy between writing and teaching, which could directly enhance the knowing of *the analogy between writing and teaching*.

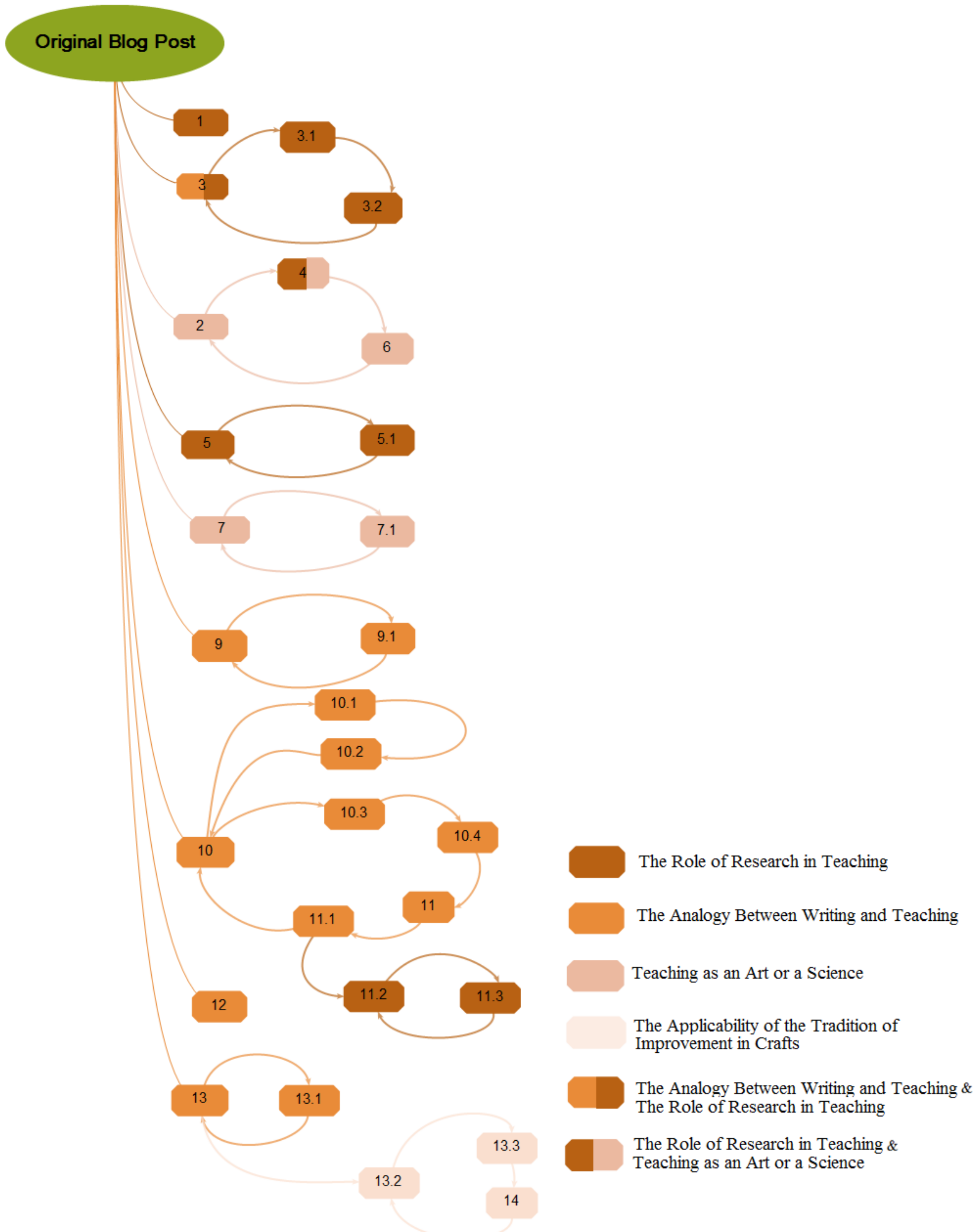

Figure 5-33. The topical recursion map in the example *Teaching Improvement.*

The recursions in the example *Textbook Presentations of the Handshake Problem* (Figure 5-34) focused only on the knowing of *problem structures.* Figure 5-34 shows that the two loops between Comments 4 and 5 and between Comments 8 and 8.1 clarified the confusion about the gap in problem structures and strengthened the ideas about leaving space for students to explore. The recursions were considered part of the knowing of *problem structures.*

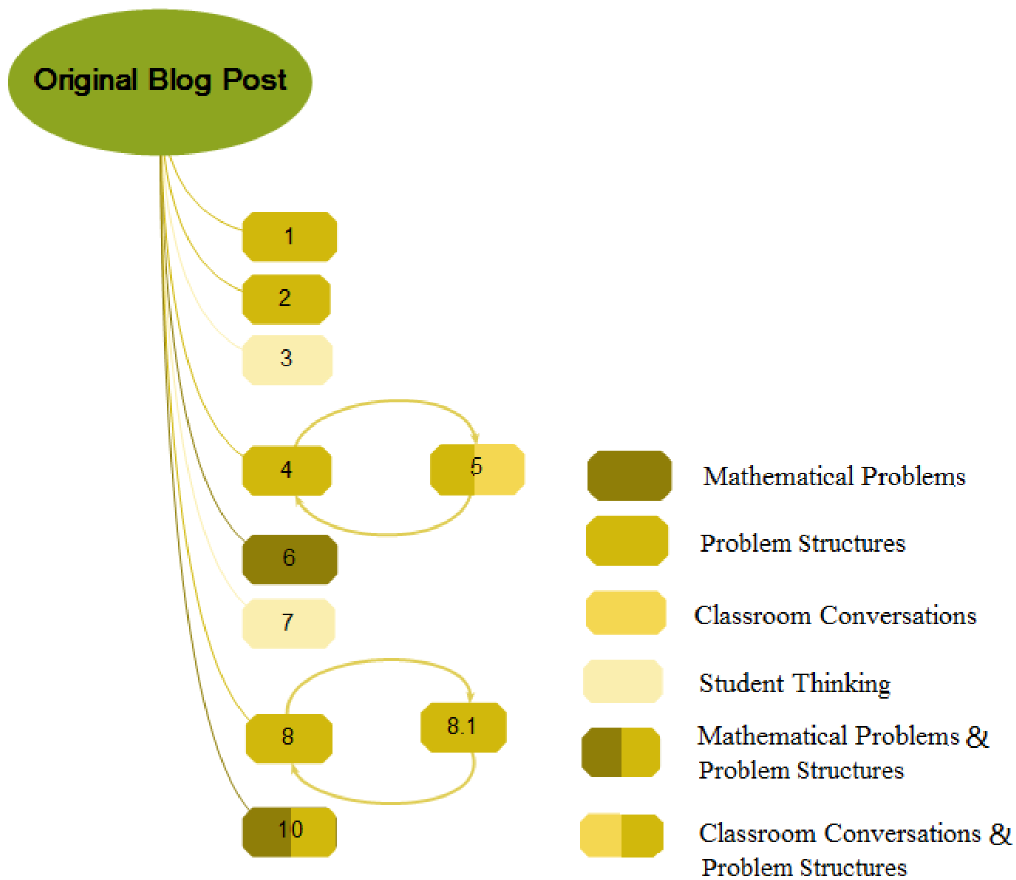

Figure 5-34. The topical recursion map in the example *Textbook Presentations of the Handshake Problem.*

The recursions in the example *Introduction of Rational Functions* (Figure 5-35) carry the diverse knowing that emerged from the conversations. For instance, the loop between Comment 2 and Comment 2.1 concentrated on clarifying the implied confusion about the graphical approach from the blog post. The clarification could intensify the knowing of *the graphical approach.*

The loop between Comments 27 and 27.1 was about adopting the ideas from the blog post to teach rational functions. This could strengthen the knowing of *teaching of rational functions.* The loop between Comments 16 and 16.1 shared social information about common friends that could help build their *social relationships*. The loop between Comments 30 and 30.1 illustrated that students' understanding of rational functions became deeper when they used the graphical approach in their learning. This was relevant to the knowing of *student learning.* The loop between Comments 23 and 23.1 revolved around sharing the blog in teacher communities. This discussion was related to the knowing of *blog resources*. In brief, the recursions underpinned the diverse knowing.

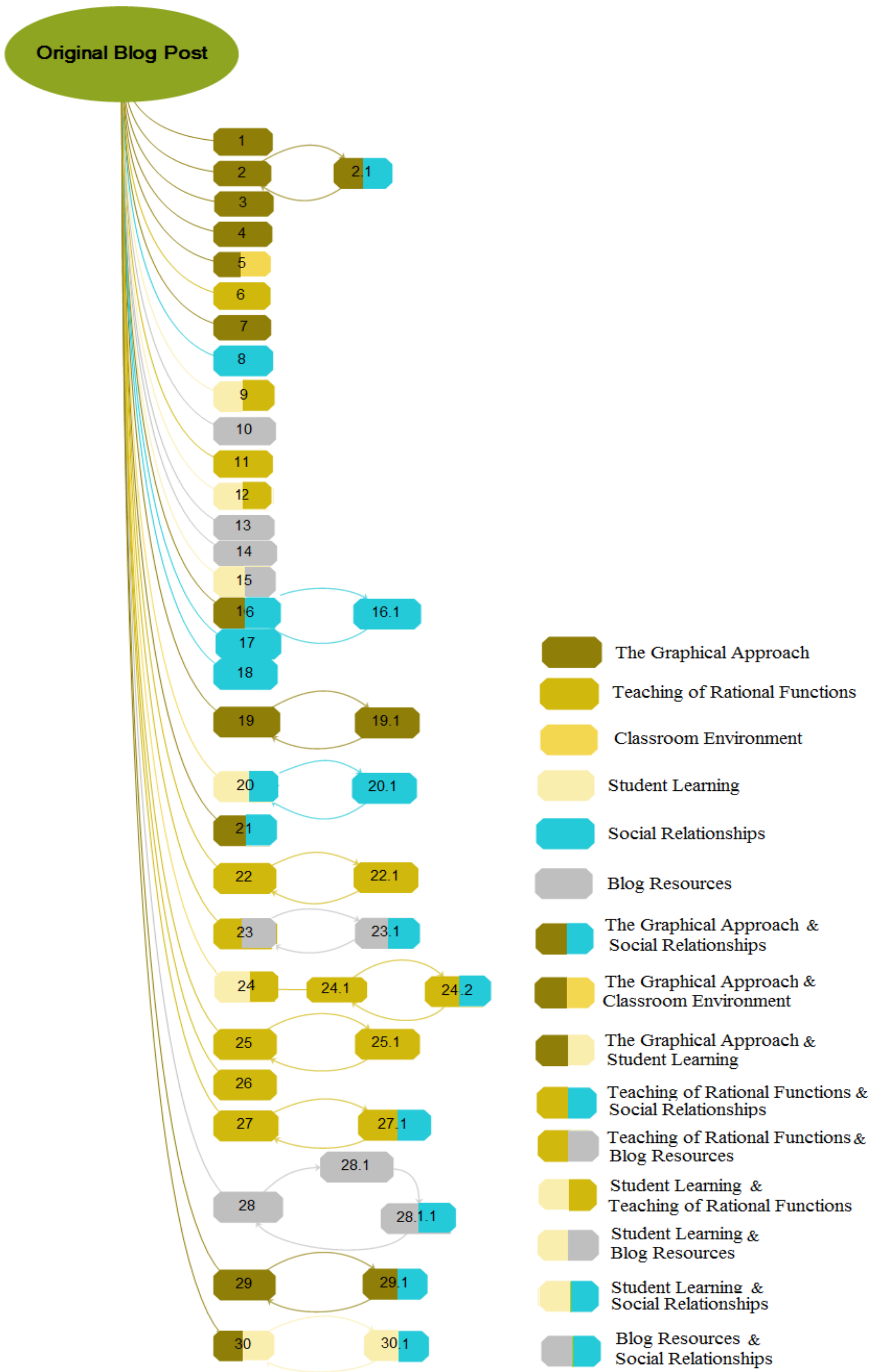

Figure 5-35. The topical recursion map in the example *Introduction of Rational Functions.*

The recursions from the example *Solving Problems About Chord Lengths* (Figure 5-36) illustrate a variety of types of emergent knowing. However, the majority attempted to advance problem solving through several strategies:

- uncovering misunderstanding (e.g., the loop among Comments 3, 4 and 4.1);
- testifying about solutions (e.g., the loop among Comments 8.2, 8.2.1 and 8.2.2);
- finding solutions (e.g., the reflective loop within Comment 14.2);
- refining solutions (e.g., the loop among Comments 8.2, 14, and 14.1); and
- revisiting the posts (e.g., the nested loop related to Comments 20, 21 and 22).

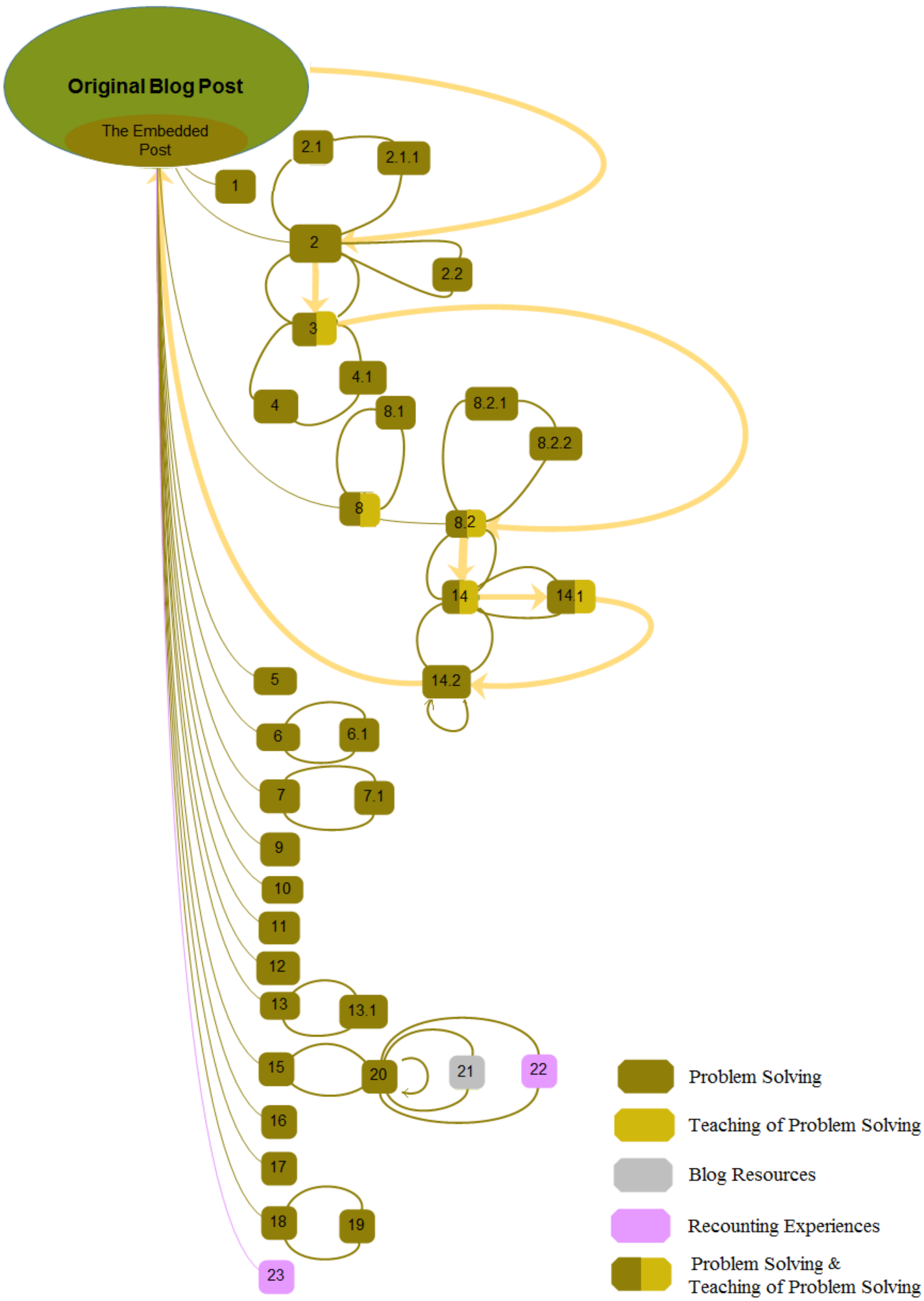

Figure 5-36. The topical recursion map in the example *Solving Problems About Chord Lengths*.

The above recursions demonstrated how comments wove together to carry on the conversations about the essential aspects related to the emergent knowing. Through recursive loops, participants worked together to:

- review the presented viewpoints,
- question the made statements,
- criticize the clarified standpoints,
- refute the established arguments, and
- improve the crafted definitions.

Thus, looping back with the reviews, questions, criticisms, refutations, and improvements could deepen the conversations on the particular knowing and emphasize the essential aspects related to the inquiries from the posts in the four examples.

5.4.2 Conversation extensions and transformations

The conversation extensions elaborated on in subsection 5.3.2 showed that hyperlinks and/or pingbacks played essential roles in extending the conversations. The hyperlinks and/or pingbacks could change the conversation topics as the hyperlink 13 (HBP 13) of Blog Post 4 (Figure 5-32) did. The changes[8] in conversation topics brought by the hyperlinks or pingbacks could be described as transformations.

As mentioned in the previous subsection, "Conversation Extensions" (5.3.2), the pingbacks could breed different conversation topics from those relevant to the original posts and transform the latter into other topics. For instance, in the example *Introduction of Rational Functions,* the pingback Comment 14 was linked to a post in which a celebrity math teacher and hir thoughts about mathematics teaching were reviewed. But the review resulted in controversial conversations about the teacher's identity and viewpoints (i.e., 46 three-layer comments of Blog 4 in Figure 5-31). In fact, the disputed conversations had nothing to do with the topic of the original post at all, hence there was a transformation of its topic into another one.

Hyperlinks also dramatically transformed the conversation topics. In the example *Solving Problems About Chord Lengths,* Comment 21 was hyperlinked to many posts

[8] Change is used to define transformation in multiple situations such as "composition or structure; outward form or appearance; or character or condition" (p. 1231) in Webster's New Collegiate Dictionary (1981).

related to education, life and, more broadly, society. In one of the hyperlinked posts, a heated discussion was initiated about whether or not children should be allowed to use finger counting. In another hyperlinked post, a personal perspective on the unfairness associated with female mathematicians in the family was expressed in terms of the unfair division of household labor and childcare. These hyperlinks could enrich participants' professional learning and broaden their horizons about teaching, learning, and even life; hence they are regarded as the transformations of the inquiry into solving the problems. Certainly, the hyperlinks were not limited to the posts and their comments since they were associated with other cyberspaces in multiple ways (see details in the subsection, "Posts and Their Comments" (4.5.3).

Pingbacks and hyperlinks make possible continuous navigation from one webpage to another. They served as doors for the whole conversations enabling the participants to enter new cyberspaces, which might bring them further into another new world. This process could go on forever. For instance, in the context of *Solving Problems About Chord Lengths,* an embedded hyperlink in Comment 22 directed the participants to a blog with plenty of posts. These posts mainly discussed the topics of pedagogy, students, society, politics, and life, all of which went completely beyond the topics of the original post and its related comments. When the posts were browsed by the participants, their related comments could be viewed or their commenters' related social networks — blog, Twitter, Facebook, or Instagram, to which their names (usernames) were hyperlinked at the top of the comments — could be visited. Moreover, blogrolls[9] embedded in the blogs could be unendingly explored by the participants because these are hyperlinked to other new cyberspaces within which more topics are displayed than in the original blogs. It was also observed that the transformations of the original posts and infinite possibilities of exploration within the new cyberspaces also occurred.

In brief, the embedded pingbacks and/or hyperlinks in the comments could extend the conversations as well as transform the conversation topics. The transformations could occur at any point of the online conversations because those linkages (e.g., pingbacks or hyperlinks) were the prominent features of social networks (Medaglia, Rose, Nyvang, &

[9] A blogroll is a list (hyperlinked) of other blogs or websites that a blogger endorses, commonly references, or is affiliated with. A blogroll is generally found on one of the blog's side columns. https://www.techopedia.com/definition/4822/blogroll

Sæbø, 2009). They could be considered a trigger point or relay station of conversations that is able to introduce the original topic into other ones. In this regard, I believe that the transformations of conversation topics could have yielded more various types of knowing if I were to have explored more conversations directed by the linkages.

5.5 The Diverse Structures of Conversations and the Multiple Types of Knowing

The above data analysis of the selected four illustrative examples resulted in the identification of structures of conversations among the participants and the emergence of knowing from the conversations in the PLN. To be more specific, the analysis on the recursions and the conversation extensions revealed the diversity of conversation structures, while the analysis on the emergent topics and the collective knowing did the emergence of the multiple types of knowing from the conversations: *mathematics-for-teaching*, *beliefs about teaching*, *blog resources*, *recounting experiences*, and *social relationships*.

Mathematics-for-teaching was embodied in multiple contexts. For instance, in the example *Textbook Presentations of the Handshake Problem*, all the aspects of mathematics-for-teaching were presented in the conversations about how to structure the Handshake Problem in the textbook; in the example *Introduction of Rational Functions*, mathematics-for-teaching was comprehensively immersed in the conversations about the concepts of rational functions and their introduction; and in the example *Solving Problems About Chord Lengths*, two aspects of mathematics-for-teaching (i.e., mathematical objects and curriculum structures) were demonstrated in the considerations of solving the problem(s). The emergent mathematics-for-teaching from the various situations could be significant for a teacher's professional growth as it is regarded as what "the teaching community (needs to) know" (Davis & Renert, 2013, p. 263). However, the other four types of knowing — *beliefs about teaching*, *blog resources*, *recounting experiences*, and *social relationships* — though not directly connected to mathematics teaching, were essential to the emergence of mathematics-for-teaching. They were also crucial to the teacher's teaching practices, the teacher's participation in the PLN, and the PLN's development.

A reflection on the recursions, the conversation extensions, the collective knowing, and the transformations motivated me to understand the roles that the individual contributions and the conversations played in the emergence of collective knowing. The individual contributions literally refer to what the individual participants post in the PLN. They include:

a) the arguments about teaching and the analogy between writing and teaching;
b) the suggestions, expectations, visions, criticisms, experiences, and comparisons related to textbook presentations;
c) the thoughts about the approaches of graphing and building up the concepts;
d) the alternative ways of introducing the concepts; and
e) the explorations of the solutions to the targeted problems.

These experiences, suggestions, viewpoints, sharing, critiques, and explorations from the individual comments could intensify the participants' and/or audiences' understanding of topics within the blogs. Meanwhile, they constituted the collective conversations that made possible the emergence of collective knowing.

The body of collective knowing was shaped through considering the individual contributions as elements of the collective. The posts and their comments were combined into the body, and in turn, the integrated body provided a larger living environment for them. For instance, Comment 8 from the example *Textbook Presentations of the Handshake Problem* expressed a concerned about the textbook presentation purposes. The concern was integrated into the knowing of *problem structures* with the ideas from other comments, such as the suggestions for the support for students' thinking in Comments 1, 2, and 4, and the shared visions and experiences in Comments 5 and 10. In this way, the concern was turned into an integral element of the knowing of *problem structures*, and in turn the knowing provided a wider living environment for Comment 8 because it encompassed other concerns as well as suggestions, critiques, and notions related to problem structures in other comments.

Another case came from the topic of adopting the graphical approach in the example *Introduction of Rational Functions*. Originating from the blending of six individual comments and coupling with other topics (i.e., exploring the graphical idea, understanding the meaning of the approach, and inquiring into worksheets answers), the topic of criticizing the graphical approach was incorporated into the knowing of *the graphical approach*. Thus, the individual comments became the elements of the knowing, which covered a range of viewpoints, suggestions, and confusions relevant to the graphical approach of introducing rational functions.

Evidently, based on their attached concepts, ideas or notions, the individual comments/contributions were considered the elements of collective knowing when they were blended together. They endowed the collective knowing with specific connotations in contexts. In addition, the emergent collective knowing embraced the individual comments/contributions as nodes and brought them into a network within which a larger picture or background was available with respect to their related original inquiries.

The full analysis on the conversations uncovered the emergence of the collective knowing from the conversation weaving. In fact, the collective knowing is "not available to any [participants] prior to the engagement, but that depends entirely on the combined knowledge of all participants" (Davis & Renert, 2014, p. 63); its components are not isolated but are evolving as a whole body, which directs the conversations to explore the inquiry as well as to "infuse the collective with a richness of interpretation" (Davis & Renert, 2014, p. 85). In addition, the transformations also occurred from the collective knowing. The transformations changed the topic of the conversations by linking the original inquiries to other topics.

Moreover, the recursions presented how the posts and their comments interacted as a whole and how the comments interwove collectively to strengthen the knowing appearing from the conversations. Therefore, it is evident that the recursions in the whole conversations shaped the collective knowing and facilitated the discussion subjects entering into the critical elements of the multiple types of knowing.

The pingbacks and/or hyperlinks involved in the conversation extensions also had the potential to transform the conversation topics. They linked the original posts to other posts from which arose the new topics. Therefore, they were able to transform the related topics attached to the original posts into other ones and produce the various types of knowing. On the whole, the analysis allowed me to see that the collective knowing presented a holistic picture for the conversations within which the individual comments were interconnected and integrated into the different elements of the collective knowing.

6. A Pathway for Participating, Communicating, Doing and Reflecting

This chapter returns me to my research questions,

- *what did the structures of conversations among the participants in a PLN look like?* and
- *what could emerge from the conversations in relation to mathematics-for-teaching*?

My exploration of the results in a more conceptual way includes recounting the affordances of the PLN for teacher professional learning, examining the environment of the PLN for emergence of knowing, discussing the implications for teacher professional learning, illustrating the contributions of this study, and reflecting on the research results.

6.1 Affordances of the PLN for Teacher Professional Learning

The PLN afforded much more than I expected for teacher professional learning. In general, the affordances can be elaborated on with respect to three dimensions: the individual dimension of self-expression and self-reflection; the collective dimension of social interaction and cognitive interaction; and the provocative dimension of boosting the individual and collective affordances. The first two dimensions were derived from Deng and Yuen's work (2009; 2011) and the third emerged from this study.

Self-reflection is regarded as a trait of effective professional learning (Lapointe-McEwan, Deluca, & Klinger, 2017; Quatroche, Bauserman, & Nellis, 2014) and as a tool for professional learning (McAleer & Bangert, 2011). Researchers have found that blogs are a valuable platform for people to project their own expression (self-expression) and reflection (self-reflection) (Brescia & Miller, 2006; Deng & Yuen, 2011) and that PLNs can promote participants' reflections on teaching practice (Moser, 2012; Noble et al., 2016). Participating in PLNs provides opportunities for teachers to reflect on their teaching experiences and ideas (thoughts), express their feelings, and seek or offer support (Trust et al., 2016; Hur & Brush, 2009).

In my study, the posts in the examples reflected teachers' thoughts about teaching improvement, textbook presentations of the Handshake Problem, and teaching experiences with respect to the introduction of rational functions. Teachers also sought help for solving problems about chord lengths and for resources for teaching and beyond. Additionally, self-expression, particularly regarding the expression of feelings, is considered integral to the learning process but is often underestimated in the formal learning environment (Boud & Walker, 1998; Deng & Yuen, 2011). The PLN is a place where participants can voice their feelings regardless of their learning, teaching, or life situation. For instance, in my study the participants expressed negative feelings about their lack of confidence in sharing their teaching experiences and cheerful or positive feelings about the successful results from others' application of their teaching approaches.

The social nature of blogs or PLN(s) would not let individual self-expression or self-reflection stand alone in the online space. Luehmann and Tinellli (2008) claim that self-expression and self-reflection through blogging are "conversational in nature" (cited by Deng & Yuen, 2011, p. 449) and will be affected by the audiences. In this regard, Deng and Yuen (2011) find that "blogging [is] not just about keeping account of personal events, but reaching out and updating others on what had happened…seeking social connections and support as well" (p. 449). In the PLN I studied, participants voiced self-expression and self-reflection not only for themselves but also for others. For instance, the blogger of *Introduction of Rational Functions* explicitly invited other participants to join hir in reflecting on hir teaching of rational functions, and their feedback on hir post dramatically influenced the blogger's further engagement in the conversations, such as sharing further on the rationales behind hir teaching approaches and linking hir other post about the assessment of rational functions to the related conversations.

Another distinct advantage of the PLN is that participants' interactions construct the collective conversations through their posting of comments. In the PLN, the bloggers and the commenters are aware of "how others might be engaged in productive collectivity" (Davis & Simmt, 2006, p. 309). Making conversations or communicating with other teachers has been regarded as a crucial characteristic of teachers' effective professional learning (Patahuddin, 2013), the "important avenues" toward their professional growth (Bangert, 2011, p. 106), and the best service for their learning (Dewey as cited in McAleer & Bangert, 2011).

The interactions among participants are dissected by Deng and Yuen (2011), based on Gilbert and Moore's (1998) idea, into two types of presence: the social and the cognitive. Both are observed in my study. For instance, participants sought to build up social relationships when they exchanged social information in the conversations; related interactions emphasized the social presence. However, most of the interactions in the examples were represented by the participants' exchanging of ideas, thoughts, or viewpoints related to the conversational topics, such as criticizing the analogy between teaching and writing, reviewing different kinds of textbook presentations, understanding the graphic approach to introducing rational functions, and providing thoughts or solutions to problem solving. These interactions addressed the cognitive presence.

In this sense, the cognitive presence was in association with two types of emergent knowing: *mathematics-for-teaching* and *beliefs about teaching*. In other words, participants could know mathematics-for-teaching and shape their beliefs about teaching through participation in the PLN. On the whole, the collective interactions, particularly the cognitive presence, were emergent as salient affordances of the PLN for teacher professional learning. This is not surprising since the PLN is dominated by the discussion of mathematics for teaching and the choice of the blog posts used for this study. Even though with less emphasis on the social presence, the affordances of the social and cognitive presences are essential to meet the participants' social and cognitive needs. In another word, as Trust et al. (2016) suggest, the participatory learning in PLNs can support participants' various social and cognitive needs.

The provocative dimension speaks to the other noticeable advantage of PLNs in relation to two types of knowing: *blog resources* and *recounting experiences*. They did not directly touch upon the individual and the collective affordances, but they opened more spaces and brought more audiences or resources for self-reflection and collective interactions. Therefore, these two types of knowing played the role of boosting the individual and the collective affordances, and even the development of the PLN. On one hand, sharing blogs as resources in larger communities or in other cyberspaces could draw much more attention from audiences to the related posts. The more attention participants paid to the posts, the more ideas they could bring into the related conversations, which enabled the bloggers and the participants to reflect more on their posts, generate more types of knowing, and deepen the emergent knowing. In addition, curating blogs as resources could strengthen the knowledge sharing in the PLN and offer more opportunities for the participants to satisfy their own needs and support others' needs. Certainly, it could also attract more potential participants to join the PLN and benefit its development.

On the other hand, as shown in the example of *Solving Problems About Chord Lengths,* recounting experiences could provide the collective conversations with more resources and reflections when the participants shared their doings with others. It could also promote the cognitive presence once it touched upon the related conversation topic(s) or professional learning resources. And more significantly, recounting experiences could even transform the conversation topics, upgrade the emergent knowing, and possibly benefit the development of the online learning community (Loving et al., 2007).

Recounting experiences could also promote social presence because such sharing reached out to others. Sometimes, it purposely responded to others' concerns. For instance, one participant's friend was concerned about what ze had been doing during hir non-blogging time. In response to this expression of care, the participant recounted what ze had been doing. In this context, actions such as recounting experiences reinforced the existing friendships. Furthermore, recounting experiences could possibly encourage other participants to share their doings or trigger resonation among them. This could help build richer and wider social relationships among the participants.

Sharing and receiving resources has been regarded as a prominent benefit to participants of PLNs (Colwell & Hutchison, 2018; Larsen & Parrish, 2019). However, in my study, *blog resources* and *recounting experiences* are beyond the role of resources, because they could enhance the individual and the collective affordances, and even the development of the PLN, as elaborated above.

In the PLN, the participants could reflect on their individual teaching or learning experiences as well as express their feelings or emotions. Their self-reflection and self-expression represented the individual dimension of math teacher participation. Since the PLN was social in nature, the individual posts reached out both to arrest the attention of other participants (i.e., bloggers and commenters) and to initiate their interactions. The interactions could shift their participation from the individual to the collective dimension.

Unexpectedly, not only did the social presence (e.g., *social relationships*) and the cognitive presence (e.g., *mathematics-for-teaching* or *beliefs about teaching*) emerge from the interactions in this study, but also two types of knowing — *blog resources* and *recounting experiences*. These types of knowing could facilitate individual reflection and expression as well as social and cognitive presence. Thus, these types of knowing played a positive role in boosting the individual and the collective affordances of the PLN. Their role represented the provocative dimension, which fueled the affordances from the individual and the collective dimension. Thus, the affordances of the PLN were interrelated and functioned as a whole to provide support for individual needs, collective knowing, and the development of the PLN.

6.2 The Environment of the PLN for Emergent Knowing

In a PLN, there is no guarantee that ideational interactions and the emergence of knowing will occur. Even if they occur, neither would occur in a deterministic way. Their occurrence is heavily dependent upon the environment of a PLN within which the participants position themselves. In my study, multiple types of knowing emerged out of the blogs in the PLN. Within the PLN, the individuals were not passive receivers of knowledge shared by others but were active co-constructors of the knowing for their learning — particularly mathematics-for-teaching. Their contributions were not "discrete or isolated components" but "interacting elements within the evolving system" of knowing (Davis & Renert, 2014, p. 61).

But what kind of environment did the PLN have? To answer this, I examined the environment of the PLN from the perspective of the five necessary conditions for complex systems according to Davis and Simmt (2003): *internal diversity*, *redundancy*, *decentralized control*, *organized randomness*, and *neighbour interactions*.

Internal diversity of the PLN is demonstrated through the various participant backgrounds and the numerous discussion topics. Participants in the PLN are situated in different countries, including the United States of America, Canada, and Jordan. They also teach different levels of mathematics — primary, high school, or college/university levels. They explore multiple mathematical content related to algebra, geometry, calculus, statistics, art or craft in math, or games or gamification in math, as well as variant aspects of mathematics education such as special education, interdisciplinary work, modelling, project/rich tasks, or technology. Thus, the internal diversity of the PLN enabled various conversations to occur, because they could scale up “the range and contours of possible response” (Davis & Sumara, 2008, p. 39).

Redundancy of the PLN is related to the common interests of the participants. The majority of participants in the PLN are mathematics teachers. A few of them are mathematics teacher educators or mathematics education researchers who are interested in mathematics teaching and/or learning. Thus, they have common interests in mathematics teaching and/or learning.

Decentralized control of the PLN is demonstrated by its self-development and self-maintenance. The PLN is developed and sustained by the participants in their own way and viewed as non-centralized and grassroots. It welcomes creativity from any entries and is developed well in its own sustained way.

Organized randomness of the PLN is relevant to free-rule participation. There are no established rules or “right” ways of participation; different viewpoints, ideas, or experiences are welcome, in Davis and Renert’s (2014) words, to “infuse the collective with a richness of interpretation” (p. 85). The conversations in the PLN usually start with inquiries from the initial posts and then flow without the explicit control of someone or something. The analysis of the four illustrative examples revealed that the transformations of conversation topics occurred unexpectedly in the PLN. Thus, this well exemplifies the PLN’s “openness to randomness” that allows for “the emergence of unanticipated possibilities” (Newell, 2004, p. 12)

Neighbour interactions in the PLN is about ideational interactions. Neighbours are not considered “physical bodies or social groupings” but “ideas, hunches, queries, and other manners” of participants (Davis & Simmt, 2003, p. 156). In the PLN, participants are not physically present; their interactions are primarily initiated by their ideas, viewpoints, values, questions, or suggestions implied in their posts.

In brief, these five conditions were essential for the emergence of knowing. They admitted “an open, participatory mode of attendance” (Davis & Renert, 2014, p. 87) and were accountable for “an openness to emergent possibility” (Davis & Renert, 2014, p. 48). They also provided learning space and possibility for the emergence of knowing without any desired objects by unsettling participants' doings in order to keep their ways open and call them into presence (Osberg & Biesta, 2008).

6.3 The Implications for Teacher Professional Learning

Conventional teacher professional learning is often largely stereotyped as transmitting predetermined knowledge to the effect that teachers are expected to "sit, listen, maybe try it on Monday" (Francis-Poscente & Jacobsen, 2013, p. 321). It is claimed that this established professional learning does not satisfy teachers' needs for their teaching practices very well (Corcoran, 1995; Wilson & Berne as cited in Marrero et al., 2010). Moreover, as mentioned in Chapter 2, its type, content, and time are often assumed to be "pre-determined" (Osberg & Biesta, 2008, p. 314), not allowing teachers to have choices for their participation in professional learning (Francis-Poscente & Jacobsen, 2013).

As far as teacher professional learning is concerned, participating, communicating, doing, and reflecting are considered the "learnable participatory disposition" (Davis & Renert, 2013, p. 247). Participating and communicating highlight the priority role of teachers themselves in their professional learning. For example, the participants in the PLN could determine their learning topics (e.g., teacher beliefs, curriculum, concept understanding, or problem solving) and their communicative approaches (e.g., blogging or commenting) according to their own needs.

Doing and reflecting tend to encourage teachers to learn from their practices rather than only from the authorities. For instance, the participants reflected on their teaching practice with respect to introducing rational functions and the problem-solving process on chord lengths in the PLN. The whole process of doing and reflecting was enacted mathematics-for-teaching. More importantly, as illustrated by the recursions in the illustrative examples, the individual contributions were not "simply 'piled onto' what ha[d] already been established, but incorporated and were incorporated into existing ideas" (Davis & Simmt, 2006, p. 301).

Accordingly, it was reasonable for me to consider that the PLN could function as a pathway of learning through participating, communicating, doing, and reflecting. I strongly believe that the PLN, like concept study (Davis & Renert, 2014), was an open space for enacting mathematics-for-teaching or for "enacting emergent evolutionary possibilities in mathematics pedagogy" (p. 48). The emergent knowing evolved from the contexts (e.g., *Teaching Improvement*, *Textbook Presentations of the Handshake Problem*, *Introduction of Rational Functions*, *Solving Problems About Chord Lengths*) within which the bloggers and the commenters were engaged in "productive collectivity" (Davis & Simmt, 2006, p. 309). Thus, the PLN, which featured diverse participants and collective structures, could advance mathematics-for-teaching.

6.4 Contributions of This Study

6.4.1 Theoretical relevance

The research goal was achieved by addressing a gap in the literature. In my study, the original goal was to unveil the phenomena of an online PLN — what the structures of conversations looked like and what could emerge from the conversations in relation to mathematics-for-teaching. The gap was addressed by interpreting the conversations and the emergence of knowing based on the theoretical framework of complexity thinking and the methodology of interpretive inquiry.

The major results in this study were identifying the emergent knowing from the conversations that occurred in the PLN: *mathematics-for-teaching*, *beliefs about teaching, blog resources*, *recounting experience*, and *social relationships*. Those results motivated me to reflect upon the theories of teachers' disciplinary knowledge, as well as the affordances of the PLN. First, *mathematics-for-teaching* was embodied in the contexts of *Textbook Presentations of the Handshake Problem*, *Introduction of Rational Functions*, and *Solving Problems About Chord Lengths*. This means that mathematics-for-teaching could be brought forth by teachers' participation in the PLN and it could be enacted in the context of their participation.

Second, the knowing of *blog resources* and of *recounting experiences* could be used to create curriculum and even become learning materials. However, they do not serve as "tools of the trade" for teaching as program materials do in Shulman's curriculum knowledge (Shulman, 1987, p. 8). This was because those two types of knowing were emergent and not pre-specified as something that must be grasped.

Third, these knowings about *beliefs about teaching* and *social relationships* are not yet considered in the dominant research of teachers' disciplinary knowledge, particularly for mathematics. For instance, they have not yet been addressed in Shulman's

(1986) pedagogical content knowledge, Ball and colleagues' (Ball et al., 2008) knowledge of mathematics for teaching, Ma's (1999) profound understanding of fundamental mathematics, and Davis and Renert's (2014) profound understanding of emergent mathematics. But in this PLN, those two types of knowing were indispensable for the emergence of mathematics-for-teaching, the teachers' participation, and the evolvement of the PLN in itself. Accordingly, it is imperative to view such kinds of knowing from a systematic and dynamic perspective as elements of teachers' disciplinary knowledge, particularly of mathematics.

Finally, the study proposed a three-dimensional perspective (individual, collective, and provocative) for the affordances of the PLN. The perspective is a breakthrough with respect to the conventional two-dimensional coordinate one that has been applied to constructivist learning (Jonassen, Davidson, Collins, Cambell, & Haag, 1995) and learning in educational blogs (Deng & Yuen, 2011). It embodied the systematic and dynamic way of viewing the affordances of the PLN as the knowing in the provocative dimension, which could boost the knowing in the individual and the collective dimensions through opening more spaces and bringing more attention or resources for knowing. The knowing in the individual dimension gave shape to and integrated into the knowing in the collective dimension.

6.4.2 Practical relevance

Davis and Renert (2014) reveal that concept study can be viewed as one of the means to enact mathematics-for-teaching. The present study suggests that in addition to concept study, participation in a PLN also could enact mathematics-for-teaching and, accordingly, be taken as another means to do so. Indeed, participation in the PLN could achieve effective professional learning. Five characteristics of effective professional learning — *instructive*, *reflective*, *active*, *collaborative*, and *substantive* (Quatroche et al., 2014) — were demonstrated by teachers' participation in the PLN.

First, teachers' participation in the PLN was considered *instructive* because their participation had an impact on their teaching — some participants reported adopting the ideas from the posts in their teaching. Second, their participation and sharing were viewed as *reflective* in so far as they explicitly shared their reflections. Third, their participation in the PLN was regarded as *active* because they engaged themselves in self-expression about and self-reflection on their teaching practice, curriculum, feelings, or school life, as well as in social interactions with other teachers. Fourth, their active participation could foster their *collaboration*. For example, participants from the same school worked together to reference teaching approaches from a particular blogger and prepare their lessons because both of them appreciated the blogger's teaching approaches. Finally, participation in the PLN is *substantive* in the following ways: knowing mathematics-for-teaching, exploring the extensive topics in the PLN (e.g., 11 emergent topic categories in Subsection 4.5.4), accessing the blogs or joining the conversations with no time limit (e.g., participants continue to visit a blog post even after five years), and enhancing professional growth (e.g., learning through participation).

6.5 Reflections on Research Results

As a study of interpretive inquiry, I conclude my dissertation with reflections on the research results: the research results themselves, the methodology of interpretive inquiry, the participation in the PLN, the affordances of the PLN for teacher professional learning, and the future vision.

6.5.1 Reflection on the analysis results

The analysis results addressed the research questions,

- *what did the structures of conversations among the participants in a PLN look like?* and
- *what could emerge from the conversations in relation to mathematics-for-teaching*?

The analysis of the recursions, the conversation extensions, the emergent topics, and the collective knowing appearing in the four selected examples uncovered the diversity of conversation structures and the emergence of multiple types of knowing from the conversations including: *mathematics-for-teaching* and the other four types of knowing: *beliefs about teaching, blog resources*, *recounting experiences*, and *social relationships*. These types of knowing arose from the conversations generated by participants spontaneously and unpredictably.

6.5.2 Reflection on the methodology

The methodology of interpretive inquiry allows researchers to find their own paths to the inquiries. My path included:

- improving my understanding of the PLN as a complex system,
- selecting four illustrative examples,
- analyzing each selected example, and
- examining the learning environment of the PLN.

To understand the PLN as a complex system, I underwent a process of reviewing the theories about complex systems and the literature about research on PLNs. My selection of the four examples involved an unfolding spiral through which the selection criteria were set up and the examples were selected through layers of refinement.

To analyze the examples, I went through an unfolding process — first analyzing the individual examples, then going back to reflect on the previous data analysis, then reviewing or making improvements on or fine tuning the ever-changing method and process I used as well as the ever-changing interpretations for the analysis results, and finally, setting up the analysis framework for all the examples by analyzing, reviewing, and refining each example backward and forward. Throughout the whole back and forth process, the previous and the later data analysis influenced each other. In addition, I examined the learning environment of the PLN for understanding the allowance of the emergence of knowing in the PLN.

6.5.3 Reflection on the participation in the PLN

Participation in the PLN shifted the mode of professional learning for the participants from passively receiving predetermined knowledge to actively generating knowing in their learning. There was no assumed significant knowledge out there for them to learn. The knowing was emergent from their participation. Within the PLN, math teachers were able to decide on participative topics, approaches, and durations according to their own needs.

First, plenty of resources and topics were available to them in the form of very detailed learning activities, challenging mathematics problems, specific textbook presentations, inspiring thoughts about teaching issues, and critical reviews about educational thoughts/theories. Undoubtedly, the teachers had space to create their own resources and topics based on personal interests. Second, they were able to choose many ways to participate in the PLN including sharing, commenting, conversing, and reflecting. Third, they were also able to determine the duration of their participation in conversations at their convenience.

6.5.4 Reflection on the affordance of the PLN

The PLN provided an open space for participants to explore in their own ways. They had space to satisfy individual needs and interests, such as reflecting on their individual learning and/or teaching experiences, expressing their feelings, and sharing or searching for resources. Certainly, the openness and sociality of the PLN meant individual doings with regard to reflections, expressions, or resources were available to others. This initiated collective learning from which different kinds of knowing emerged. For instance, through the collective learning in the PLN, participants were able to know mathematics-for-teaching, argue about teachers' beliefs about teaching, build up social relationships, and share blogs and experiences as resources.

In addition, the individual doings and the collective learning interacted with each other in the open space. The collective learning brought more conversations on the topics of individual doings. This encouraged the individuals to do more and share more. The more doings or sharing done by individuals, the more collective learning occurred. There are reasons to believe that such interplay is conducive to the development of the PLN and, in turn, the well-being of the PLN will nourish the individual doings and the collective learning.

6.5.5 Reflection on the contributions of this study

This study contributed to several research areas relevant to teacher professional learning through PLNs, teachers' disciplinary knowledge of mathematics, and the affordances of a PLN. First, this study contributed to the rapidly growing literature on teacher professional learning through PLNs. It could help people better understand the structures of conversations among participants and the emergence of knowing from conversations within a PLN.

Second, the study contributed to the theorization on teachers' disciplinary knowledge of mathematics. Its results uncovered the five types of knowing emergent from the conversations in the PLN. As an open learning site, a PLN makes it possible to observe how learning occurs (Bates, Phalen, & Morgan, 2018) and allows different types of knowing to present explicitly. Except for the knowing of *mathematics-for-teaching,* the other four types of knowing inclusive of *beliefs about teaching*, *blog resources*, *recounting experiences*, and *social relationships* have not yet been addressed in the dominant research on teachers' disciplinary knowledge of mathematics. However, they were implicated with the emergence of *mathematics-for-teaching*, the teacher's engagement in the PLN, and even the sustainability and development of the PLN itself. Therefore, the other four types of knowing are proposed to be elements of the teachers' disciplinary knowledge of mathematics from the systematic and dynamic perspective.

Third, the study proposed a three-dimensional perspective for understanding the affordances of the PLN, including individual, collective, and provocative dimensions. More importantly, the affordances allowed individual reflection and expression, collective knowing, and the self-maintenance and development of the PLN to occur in a systematic way. Specifically, the knowing of *mathematics-for-teaching* did not emerge alone, but in the company of the other four types of knowing. All five types of knowing worked together as a system, which offers a systematic perspective for understanding the emergence of knowing in the collective activity of participants in an online PLN. The PLN may be seen as a system for teacher knowing.

6.6 Future Vision

I have so much to tell as I try to wrap up what I learned through this study. I remember moments of being so touched by what I was reading from the bloggers that I almost forgot that I was doing data collection; I remember the uncertainty about what I had observed from the first example analysis; I remember the strength of conviction in what I had obtained from the analysis of the four examples; I remember the irrepressible feelings when I tried to explore teachers' participation beyond the examples; I remember the exhilaration when I found unexpected results; and I remember sense of inner peace when I felt I was able to answer my research questions. All these experiences shaped my beliefs about the value of a PLN intended for mathematics teachers' professional learning.

I am fully convinced that further exploration of this PLN will strengthen what I have found from the four illustrative examples and unveil more about PLNs as a system for teacher knowing. Therefore, in order to offer a wider picture for mathematics teachers' participation in PLNs, my future research plan is to explore more examples from this PLN and others. This may help me perceive other types of knowing, which might have not yet been identified in the various research studies already conducted on teachers' disciplinary knowledge of mathematics, and inspire me to know more about the learning space for teachers to learn by themselves in PLNs.

To continue the research on PLNs will be my major future academic plan. Meanwhile, using PLN(s) will be a learning module in my future teaching and my own professional learning because I strongly believe that participation in PLNs is an effective way to facilitate mathematics teachers' professional learning. From the bottom of my heart, I live this belief and it will live in my research, in my teaching, and even in my own professional learning.

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