University of Alberta

Application of performance measures

to mergers and acquisitions

by

Anna Evstafyeva

A thesis submitted to the Faculty of Graduate Studies and Research

in partial fulfillment of the requirements for the degree of

Master of Science

in

Mathematical Finance

Department of Mathematical and Statistical Sciences

Fall 2009

Edmonton, Alberta

Examining Committee

B. Schmuland (Committee Chair and Examiner), Math and Stat Sciences

A. Melnikov (Supervisor), Math and Stat Sciences

V. Yaskin, Math and Stat Sciences

C. Scepesvari, Computing Science

ABSTRACT

The main part of this thesis was devoted to applying performance measures in the analysis of mergers and acquisitions. Performance measures take into account the profitability of a company and the risk associated to that company. To analyze potential post-merger synergies one can combine the pre-merger financial results of two companies and calculate the value of an applicable performance measure. If the value exceeds that of the bidder by itself the merger is said to generate positive synergies. The thesis demonstrates how the calculations can be performed as well as advantages and disadvantages of various performance measures.

Most performance measures involve company's returns. It was shown that only quarterly return is a reliable source of information about company's performance. To deal with quarterly data annualization method for all applicable ratios was developed. Several examples were created to demonstrate advantages and disadvantages of annualized applicable performance measures. Three real-world examples of recent acquisitions in Canada and USA were given and analyzed .

Contents

List of Tables

Chapter 1

Introduction

This thesis is devoted to an application of performance measures to mergers and acquisitions (M&A). A performance measure is a rating system which is used in evaluation of mutual fund or stock risks and returns. However, their application to mergers and acquisitions has not been studied thoroughly yet.

A merger occurs when two companies combine to form a single company. A merger is very similar to an acquisition or takeover, except that in the case of a merger existing stockholders of both companies involved retain a shared interest in the new corporation. By contrast, in an acquisition one company purchases a bulk of a second company's stock, creating an uneven balance of ownership in the new combined company.

Usually the reason for M&A is that acquiring firms seek improvement of financial performance. For example, combined company can often reduce its fixed costs by removing duplicate departments or operations, lowering the costs of the company relative to the same revenue stream, thus increasing profit margins or a profitable company can buy a loss maker to use the target's loss as their advantage by reducing their tax liability or by merging with major competitors, a company can come to dominate the market they compete in. Thus, there are lots of way how to increase company profitability through M&A. On the other hand, every company carries it's own risks, so reducing these risks by using M&A also should be considered when a company is choosing another company for acquisition.

The simplest performance measure, which counts for both: profitability and risk, is Sharpe ratio. Sharpe ratio is usually applied to stock valuation and portfolio theory. However, we can think about two companies as about two stocks, which together form a portfolio (a combined company) and we can use Sharpe ratio to evaluate the effectiveness of merger these to companies. A large number of different performance measures like Sharpe ratio have been developed in literature. The main goal of this work is to describe what measures can be applied for analysis of M&A.

Most performance measures involve such variable as company's return. There are two main approaches to measuring company's returns. One is to measure company's stock returns and another is to measure company's real quarterly returns from its financial statements. This thesis is limited to company's real quarterly returns for the following reasons:

- Most companies involved in merger activity is private companies not a public company, so they just don't have stocks. "Sixty to 70 percent of U.S. acquisitions - even more in Europe and Asia - are private," (see Capron (2008))

- Stock is usually much more volatile than real quarterly returns because it reflects available public information including rumours about possible mergers and acquisitions. Acquisition is a complex transaction, and it is possible that new information would be released over time which would cause stock unpredictably to drop down or to jump up.

- "There are a number of problematic issues concerning the use of share price data ... including the inefficiency of stock market." (see Manson, Stark and Thomas (1994))

- "Gains from merger could arise from a variety of sources, such as operating synergies, tax savings, transfer from employees or other stakeholders, or increased monopoly rents... Stock prices studies are unable to provide evidence on the source of any merger related gains."(see Healy, Palepu and Ruback (1992))

- Acquiring firm attempts to increase its share price before merger because a) existing shareholders prefer a higher price in order to minimize earning dilution; b) a stock issue dilutes voting power and control of existing shareholders (particularly managers-shareholders); c)the higher the value of the acquiring firm's stock, the lower the cost of acquiring the target firm (see Erickson and Wang (1999)).

Thus, because of these reasons and to avoid problems associated with market anticipation of the gain from a takeover, only company's quarterly returns are considered to be a reliable source of information about company's performance. In this case we should define what company's quarterly returns are. There are two different approaches for evaluating company's quarterly returns.

First of all, profitability can be describe in four different ways as pre-tax (or post-tax) profit divided by assets (or sales) (see Mueller(1980)). This approach was very common in 70's and 80's in US and UK. For example, in UK profitability is defined as pre-tax (or post-tax) profit divided by average net asset (see Singh (1971), Meeks (1977), Cosh, Hushes & Singh (1980) and Holl & Pickering (1988)). In US it was said that one can understand an accounting-based return as a ratio of net income to total assets or as a ratio of operating income to sales(see Lev & Mandelker (1972)) and also the ratio of operating income to asset was used (see Ravenscraft and Scherer (1987)).

However, accounting rate of return (ARR), which is usually defined as net profit before interest expenses and tax (EBIT) divided by (depreciated) net assets, is not the best approach because of inconsistent accounting methods for different companies and because of opportunities for earning management (see Appleyard (1980)). It was also found that "earnings management is implemented through a composite strategy of accounting policy choices" before merger (see Perry and Williams (1994)). Managers of acquiring companies often manipulate last quarter income and try to increase company share price pre-merger in order reduce the cost of buying the target, but managers of target companies don't (see Erickson and Wang (1999)). So, to mitigate the impact of the financing of acquisition and the method of accounting for transactions, the second approach, based on operating cash flow, was developed.

The choice of cash flow as a numerator instead of net income was adopted in many studies. For example, in US operating cash flow (before interest expense and income from short-term investments) deflated by the market value of asset was used (see Healy, Palepu and Ruback (1992)). It was argued that cash flows represent the actual economic benefits generated by the asset and that such cash flow return is unaffected by the choice of financing. Also pre-tax cash flow normalized by market value of asset was used (see Anand and Singh (1997)). The similar measure of cash flow from operations standardized not by market value of asset but by sales (revenues) was used (see Clark and Ofek (1994)). Also ratio of operating cash flow (operating income before depreciation minus interest minus taxes minus change in noncash working capital) to sales was calculated (see Harford (1999)). In UK it was agreed that the best measure of operating performance is the ratio of operating cash flow to the market value Manson, Stark & Thomas (1994)) and operating cash flow to operating asset was prefered sometimes (see Steve & Robin (2003)).

As a result, it was propose to use even five various measures of cash flow and show that cash flows provide better forecast of future cash flow, than do earnings numbers based on accrual accounting (see Bowen, Burgstahler and Daley (1986)). "Many financial analysts regard operating cash flow as a better gauge of corporate financial performance than net income, since it is less subject to distortion from differing accounting practices" (see Skala (1991)). "A growing number of portfolio managers and analysts insist that cash flow is a more meaningful measure of a company's value than reported earnings" (see Dreyfus (1988)).

Thus, it is up to a researcher what approach to choose in calculating company quarterly return. In all examples in this thesis quarterly return is just a number which can be understood according to one of these approaches as operating income to total sale (or total asset) or operating cash flow to total sale (or total asset). It is not a goal of this work to study this question in details. The purpose of this work is to study what performance measures can be applied to M&A using company's quarterly returns, which can be calculated in any way described above.

The structure of the remaining paper is follows. Section 2 describes performance measures, which can be applied to M&A. Section 3 discusses annualization methods for company's quarterly returns data and calculated all measures as annual values (corresponding expamles are given in Appendix). Section 4 provides 1) some examples to illustrate how performance measures can be applied to M&A and what performance measure is a better estimation of a future combined company's return (accounting for market condition as well); 2) provides some real-world examples of mergers and acquisitions, where all suitable performance measures are calculated and corresponding conclusions are given. Section 5 describes performance measures, which are difficult to apply to M&A if company's quarterly returns are used instead of company's stock returns. Finally, Section 6 provides a summary and conclusions together with suggestions for further research.

Chapter 2

On performance measurements and some performance measures.

The following performance measures are considered to be applicable for mergers and acquisitions if real quarterly returns are used instead of stock returns.

In this chapter mean (E(R)) and standard deviation ($\sigma = \sqrt{\mathrm{VAR}(R)}$) are defined as usual in statistics:

$$\mathrm{E}(R) = \frac{1}{T}\sum_{t=1}^{T} R_t,$$
$$\sigma = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_t - \mathrm{E}(R))^2},$$

where R is a company's return and T is number of returns.

2.1 Sharpe ratio and Information ratio.

Sharpe ratio or reward-to-variability ratio was first introduced by Sharpe (1975) as a measure of the excess return per unit of risk in an investment asset.

$$\text{Sharpe ratio}_A = \frac{\mathrm{E}(R_A) - R_f}{\sigma_A} = \frac{\mathrm{E}(R_A - R_f)}{\sqrt{\mathrm{var}(R_A)}},$$

here R_A is a return on a security A, R_f is a risk free rate of return and σ_A is a standard deviation of a security A return.

In finance, a portfolio is an appropriate mix of or collection of investments held by an institution or a private individual. Portfolio return and standard variance can be calculated as

$$\mathrm{E}(R_p) = \omega_A \mathrm{E}(R_A) + \omega_B \mathrm{E}(R_B),$$
$$\sigma_p = \sqrt{\omega_A^2 \sigma_A^2 + \omega_B^2 \sigma_B^2 + 2\rho_{AB}\omega_A\omega_B\sigma_A\sigma_B},$$

where ω_A and ω_B are weights of assets A and B in the portfolio, σ_A and σ_B are standard deviations of each asset return and ρ_{AB} is the correlation coefficient of assets A and B (see Dowd (2000)).

In this way, Sharpe ratio of a portfolio of two securities is equal to

$$\text{Sharpe ratio}_p = \frac{\mathrm{E}(R_p) - R_f}{\sigma_p} = \frac{\mathrm{E}(R_p - R_f)}{\sqrt{\mathrm{var}(R_p)}}.$$

Note, that to estimate all these parameters it is assumed that returns or excess returns have identical, independent, normal distribution over time, which is a common assumption in the context of basic performance measures (see Grinblatt and Titman (1989)).

The generalization of Sharpe ratio is the Information ratio (see Goodwin (1998)). The only difference is that return on benchmark portfolio is used instead of risk-free rate and standard deviation of a portfolio excess return is used instead of standard deviation of a portfolio return:

$$\text{Information ratio}_p = \frac{\mathrm{E}(R_p - R_b)}{\sqrt{\mathrm{var}(R_p - R_b)}},$$

here R_b is the return on a benchmark portfolio.

Note, that if R_b is a constant risk free return throughout the period, then

$$\sqrt{\mathrm{var}(R_p - R_b)} = \sqrt{\mathrm{var}(R_p - R_f)} = \sqrt{\mathrm{var}(R_p)} = \sigma_p,$$

and Information ratio becomes Sharpe ratio.

Please see Examples 4-6 for numerical illustration of application of the Sharpe ratio to mergers and acquisitions case.

2.2 Sortino ratio.

Sortino ratio is a modification of the Sharpe ratio and was introduced by Sortino and Meer (1991). Sharpe ratio equally penalizes both upside and downside volatility. Sortino ratio assumes that only downside volatility is bad from the investor's point of view and proposes to penalizes only those returns which less then a target, such as the risk free rate for example.

Downside deviation is a square root of a lower partial moment with n=2, which can be defined as

$$LPM_{nA} = \frac{1}{T}\sum_{t=1}^{T} \max(0, L - R_A)^n,$$

where R_A is a return on a security A and L is a minimal acceptable return or threshold (see Harlow (1991)). In this way, Sortino ratio of a security A is equal to

$$\text{Sortino ratio}_A = \frac{\mathrm{E}(R_A) - L}{\sqrt{LPM_{2A}}}.$$

Since for portfolio of two securities A, B with weight (ω_A, ω_B) and correlation coefficient ρ_{AB}

$$\mathrm{E}(R_p) = \omega_A \mathrm{E}(R_A) + \omega_B \mathrm{E}(R_B),$$

$$LPM_{2p} = \omega_A^2 LPM_{2A} + \omega_B^2 LPM_{2B} + 2\rho_{AB}\omega_A\omega_B\sqrt{LPM_{2A}}\sqrt{LPM_{2B}},$$

then the Sortino ratio of a portfolio is equal to

$$\text{Sortino ratio}_p = \frac{\mathrm{E}(R_p) - L}{\sqrt{LPM_{2p}}}.$$

Thus, this ratio is the actual rate of return in excess of the investor's target rate of return, per unit of downside risk.

Please, see Example 1 for numerical illustration of application of the Sortino ratio to mergers and acquisitions case and for comparison it with Sharpe ratio.

2.3 Omega ratio.

Shadwick and Keating (2002) introduced a new measure, called Omega ratio, which takes into account the returns below and above a given loss threshold. More precisely, Omega ratio is defined as the probability weighted ratio of gains to losses relative to a return threshold. For any investor, returns below her loss threshold are considered as losses and returns above as gains. The exact mathematical definition is given by:

$$\text{Omega ratio}_A = \frac{\int_L^b (1 - F(x))dx}{\int_a^L (F(x))dx}$$

where $F(.)$ is the cumulative distribution function of the asset A returns defined on the interval (a, b), with respect to the probability distribution P and L is the return threshold selected by the investor. Thus, investor should always prefer the portfolio with the highest value of Omega for a given return threshold.

Note, that Omega ratio can be considered as the ratio of the prices of a call option to a put option written on A with strike price L and both evaluated under the historical probability P (see Kazemi et al (2004)). Ti was shown, that Omega ratio for a security A can be written as

$$\text{Omega ratio}_A = \frac{\mathrm{E}(R_A - L)^+}{\mathrm{E}(L - R_A)^+} = \frac{HPM_{1A}}{LPM_{1A}} = \frac{\mathrm{E}(R_A) - L}{LPM_{1A}} - 1,$$

where R_A is a return on a security A. If T is a number of returns, then higher partial moment (HPM) and lower partial moment (LPM) are correspondently equal to

$$HPM_{nA} = \frac{1}{T}\sum_{t=1}^{T} \max(0, R_A - L)^n,$$
$$LPM_{nA} = \frac{1}{T}\sum_{t=1}^{T} \max(0, L - R_A)^n$$

(see Harlow (1991)).

Since for portfolio of two securities A and B with weight (ω_A, ω_B)

$$HPM_{1p} = \omega_A HPM_{1A} + \omega_B HPM_{1B},$$
$$LPM_{1p} = \omega_A LPM_{1A} + \omega_B LPM_{1B},$$

then the Omega ratio of a portfolio is equal to

$$\text{Omega ratio}_p = \frac{HPM_{1p}}{LPM_{1p}} = \frac{\mathrm{E}(R_p) - L}{LPM_{1p}} - 1,$$

Additionally, one can note, that in the special case, when the threshold L is zero, the Omega ratio becomes Gain-Loss ratio described by Bernardo and Ledoit (2000).

Gain-loss ratio is the ratio of the expectation of the positive part of the returns divided by the expectation of the negative part. A Gain-Loss ratio of 1 implies that the investment is fairly priced and a Gain-Loss ratio above one implies the existence of the an attractive investment opportunity.

Please, see Example 1 for numerical illustration of application of the Omega ratio to mergers and acquisitions case and for comparison it with Sharpe ratio.

2.4 Kappa$_3$ ratio.

Kaplan and Knowles (2004) introduced the generalization of Omega and Sortino ratio, called Kappa$_n$ ratio, which use lower partial moment as well. More precisely, Kappa$_n$ ratio is defined as

$$\text{Kappa}_n\text{ratio} = \frac{\mathrm{E}(R_A) - L}{\sqrt[n]{LPM_{nA}}},$$

where R_A is a return on a security A and L is a threshold. If T is a number of returns, then a lower partial moment is equal to

$$LPM_{nA} = \frac{1}{T}\sum_{t=1}^{T}\max(0, L - R_A)^n$$

(see Harlow (1991)).

It was shown that

$$\begin{aligned}
\text{Omega ratio}_A &= \text{Kappa}_1 \text{ ratio} + 1, \\
\text{Sortino ratio}_A &= \text{Kappa}_2 \text{ ratio},
\end{aligned}$$

and proposed to use Kappa$_3$ ratio:

$$\text{Kappa}_3 \text{ ratio}_A = \frac{\mathrm{E}(R_A) - L}{\sqrt[3]{LPM_{3A}}}$$

(see Kaplan and Knowles (2004)).

Since for portfolio of two securities A, B with weight (ω_A, ω_B) and correlation coefficient ρ_{AB}

$$\begin{aligned}
\mathrm{E}(R_p) &= \omega_A \mathrm{E}(R_A) + \omega_B \mathrm{E}(R_B), \\
LPM_{3p} &= \omega_A^3 LPM_{3A} + \omega_B^3 LPM_{3B} \\
&\quad + 3\omega_A^2\omega_B\rho_{AB}\left(\sqrt[3]{LPM_{3A}}\right)^2\left(\sqrt[3]{LPM_{3B}}\right) \\
&\quad + 3\omega_A\omega_B^2\rho_{AB}\left(\sqrt[3]{LPM_{3A}}\right)\left(\sqrt[3]{LPM_{3B}}\right)^2,
\end{aligned}$$

then the Kappa$_3$ ratio of a portfolio is equal to

$$\text{Kappa}_3 \text{ ratio}_p = \frac{\mathrm{E}(R_p) - L}{\sqrt[3]{LPM_{3p}}}.$$

However, they noted that they "are not aware of any generally applicable rule for choosing the "correct" Kappa variant for a given purpose".

Please, see Example 1 for numerical illustration of application of the Kappa 3 ratio to mergers and acquisitions case and for comparison it with Sharpe ratio.

2.5 The Upside potential ratio.

Another ratio, which was introduced by Sortino et al. (1999), is the Upside potential ratio. The idea is to upgrade Sortino ratio by replacing the excess return with the upside potential or higher partial moment (HPM) with n=1. It is defined as follows:

$$HPM_{nA} = \frac{1}{T}\sum_{t=1}^{T} \max(0, R_A - L)^n,$$

where R_A is a return on a security A and L is a minimal acceptable return or threshold. In this way, Upside potential ratio of a security A is equal to

$$\text{Upside potential ratio}_A = \frac{HPM_{1A}}{\sqrt{LPM_{2A}}}$$

Since for portfolio of two securities A, B with weight (ω_A, ω_B) and correlation coefficient ρ_{AB}

$$\begin{aligned} \mathrm{E}(R_p) =\ & \omega_A \mathrm{E}(R_A) + \omega_B \mathrm{E}(R_B), \\ LPM_{2p} = & \omega_A^2 LPM_{2A} + \omega_B^2 LPM_{2B} + 2\rho_{AB}\omega_A\omega_B\sqrt{LPM_{2A}}\sqrt{LPM_{2B}}, \end{aligned}$$

then the Upside potential ratio of a portfolio is equal to

$$\text{Upside potential ratio}_p = \frac{HPM_{1p}}{\sqrt{LPM_{2p}}}.$$

Thus, this ratio considers upside potential over downside deviation with minimum acceptance return instead of risk free rate. This ratio allows to choose strategies with growth that is as stable as possible for a given minimum return.

Please, see Example 2 for numerical illustration of application of the Upside potential ratio to mergers and acquisitions case and for comparison it with Sharpe ratio.

2.6 Israelsen Ratio.

Another modification of Sharpe ratio was introduced by Israelsen (2003 and 2005). Usually, Sharpe ratio works well in normal market, but it is not an appropriate measure in ex post periods with average negative market excess returns. For bear market Israelsen proposed to multiply the mean excess return by its standard deviation. For normal market he agrees to use Sharpe ratio. Thus

$$\text{Israelsen ratio}_A = \begin{cases} \left[\mathrm{E}(R_A) - R_f\right]/\sigma_A & \text{if } \mathrm{E}(R_A) - R_f > 0, \\ \left[\mathrm{E}(R_A) - R_f\right] \cdot \sigma_A & \text{if } \mathrm{E}(R_A) - R_f < 0, \end{cases}$$

here R_A is a return on a security A, R_f is a risk free rate of return and σ_A is a standard deviation of a security A return.

Since for portfolio of two securities A, B with weight (ω_A, ω_B) and correlation coefficient ρ_{AB}

$$\begin{aligned} \mathrm{E}(R_p) &= \omega_A \mathrm{E}(R_A) + \omega_B \mathrm{E}(R_B), \\ \sigma_p &= \sqrt{\omega_A^2 \sigma_A^2 + \omega_B^2 \sigma_B^2 + 2\rho_{AB}\omega_A\omega_B\sigma_A\sigma_B}, \end{aligned}$$

then the Israelsen ratio for portfolio of two securities is equal to

$$\text{Israelsen ratio}_p = \begin{cases} \left[\mathrm{E}(R_p) - R_f\right]/\sigma_p & \text{if } \mathrm{E}(R_p) - R_f > 0, \\ \left[\mathrm{E}(R_p) - R_f\right] \cdot \sigma_p & \text{if } \mathrm{E}(R_p) - R_f < 0. \end{cases}$$

Intuitively it is clear that if two assets have the same negative excess return, but different deviation, the portfolio with smaller deviation should be preferred. However, the portfolio with bigger deviation gets the bigger Sharpe ratio since excess returns are negative. Thus, the right way to deal with bear market according to Israelsen is to multiply portfolio excess return by its standard deviation.

Please, see Examples 3 for numerical illustration of application of the Israelsen ratio to mergers and acquisitions case and for comparison it with Sharpe ratio.

2.7 Ferruz-Sarto Ratio.

Alternative ratio for bear market was introduced by Ferruz and Sarto (2004). Their idea is to use relative return premium of the security A instead of excess return, i.e.

$$\text{Ferruz-Sarto Ratio}_A = \frac{\mathrm{E}(R_A)/\mathrm{E}(R_f)}{\sigma_A},$$

where R_A is a return on a security A, R_f is a risk free rate of return and σ_A is a standard deviation of a security A return.

Since for portfolio of two securities A, B with weight (ω_A, ω_B) and correlation coefficient ρ_{AB}

$$\mathrm{E}(R_p) = \omega_A \mathrm{E}(R_A) + \omega_B \mathrm{E}(R_B),$$
$$\sigma_p = \sqrt{\omega_A^2 \sigma_A^2 + \omega_B^2 \sigma_B^2 + 2\rho_{AB}\omega_A\omega_B\sigma_A\sigma_B},$$

then the Ferruz-Sarto ratio for portfolio of two securities is equal to

$$\text{Ferruz-Sarto Ratio}_p = \frac{\mathrm{E}(R_p)/\mathrm{E}(R_f)}{\sigma_p}.$$

They point out that "in dynamic studies that take the evolution of portfolio performance over time into account, the return on risk-free assets, considered initially as a constant, becomes a significant and volatile variable. As a result, it is more appropriate to treat the return on risk free assets in the same manner as the total risk inherent in the portfolios analysed."

Please, see Examples 3 for numerical illustration of application of the Ferruz-Sarto ratio to mergers and acquisitions case and for comparison it with Sharpe ratio.

Chapter 3

Annualization.

The most common data for a company is its quarterly returns. Since the benchmark return such as risk-free rate is usually annualized, we need to annualize company's quarterly data as well. There are several ways to do it (see Goodwin (1998)). We will describe three of them.

3.1 Arithmetic mean excess return.

The most common practice to annualize data for company is to multiply quarterly return by 4 and quarterly standard deviation of return by $\sqrt{4}$. Thus, applying this idea to all ratios, which are suitable for mergers and acquisitions, we are getting the following list of annualized ratios.

We assume that risk-free rate and minimal acceptable return (L) are compounded annually and define any variable with symbol $\sim$ as a quarterly compounded variable. Also T is a number of quarterly returns. Since it is most common approach for annualizing quarterly data, it is used it in almost all examples.

3.1.1 Sharpe ratio and Information ratio.

The annualized Sharpe ratio is equal to

$$\text{Sharpe ratio}_p^1 = \frac{4\mathrm{E}(\tilde{R}_p) - R_f}{\sqrt{4} \cdot \tilde{\sigma}_p}.$$

Assuming quarterly return on benchmark, the annualized Information ratio will double:

$$\text{Information ratio}_p^1 = \frac{4\mathrm{E}(\tilde{R}_p) - 4\mathrm{E}(\tilde{R}_b)}{\sqrt{4} \cdot \sqrt{\mathrm{var}(\tilde{R}_p - \tilde{R}_b)}} = 2 \cdot \frac{\mathrm{E}(\tilde{R}_p) - \mathrm{E}(\tilde{R}_b)}{\sqrt{\mathrm{var}(\tilde{R}_p - \tilde{R}_b)}}.$$

3.1.2 Sortino ratio.

The annualized Sortino ratio is equal to

$$\text{Sortino ratio}_p = \frac{4\mathrm{E}(\tilde{R}_p) - L}{\sqrt{4} \cdot \sqrt{LPM_{2p}}},$$

where

$$LPM_{2p} = \omega_A^2 LPM_{2A} + \omega_B^2 LPM_{2B} + 2\rho_{AB}\omega_A\omega_B\sqrt{LPM_{2A}}\sqrt{LPM_{2B}},$$

$$LPM_{2A} = \frac{1}{T}\sum_{t=1}^{T} \max(0, \frac{L}{4} - \tilde{R}_A)^2.$$

3.1.3 Omega ratio.

The annualized Omega ratio will stay the same:

$$\text{Omega ratio}_p = \frac{4 \cdot HPM_{1p}}{4 \cdot LPM_{1p}} = \frac{HPM_{1p}}{LPM_{1p}} = \frac{4 \cdot \mathrm{E}(\tilde{R}_p) - L}{4 \cdot LPM_{1p}} + 1,$$

where

$$HPM_{1p} = \omega_A HPM_{1A} + \omega_B HPM_{1B},$$

$$LPM_{1p} = \omega_A LPM_{1A} + \omega_B LPM_{1B},$$

$$HPM_{1A} = \frac{1}{T}\sum_{t=1}^{T}\max(0, \tilde{R}_A - \frac{L}{4})^1,$$
$$LPM_{1A} = \frac{1}{T}\sum_{t=1}^{T}\max(0, \frac{L}{4} - \tilde{R}_A)^1.$$

3.1.4 Kappa$_3$ ratio.

The annualized Kappa$_3$ ratio is equal to

$$\text{Kappa }_3 \text{ ratio}_p = \frac{4 \cdot \mathrm{E}(\tilde{R}_p) - L}{\sqrt[3]{4 \cdot LPM_{3p}}},$$

where

$$\begin{aligned} LPM_{3p} = {} & \omega_A^3 LPM_{3A} + \omega_B^3 LPM_{3B} \\ & + 3\omega_A^2\omega_B\rho_{AB}\left(\sqrt[3]{LPM_{3A}}\right)^2\left(\sqrt[3]{LPM_{3B}}\right) \\ & + 3\omega_A\omega_B^2\rho_{AB}\left(\sqrt[3]{LPM_{3A}}\right)\left(\sqrt[3]{LPM_{3B}}\right)^2, \end{aligned}$$

with

$$LPM_{3A} = \frac{1}{T}\sum_{t=1}^{T}\max(0, \frac{L}{4} - \tilde{R}_A)^3.$$

3.1.5 The Upside potential ratio.

The annualized Upside potential ratio will double:

$$\text{Upside potential ratio}_p = \frac{4 \cdot HPM_{1p}}{\sqrt{4 \cdot LPM_{2p}}} = 2 \cdot \frac{HPM_{1p}}{\sqrt{LPM_{2p}}},$$

where

$$HPM_{1p} = \omega_A HPM_{1A} + \omega_B HPM_{1B},$$
$$LPM_{2p} = \omega_A^2 LPM_{2A} + \omega_B^2 LPM_{2B} + 2\rho_{AB}\omega_A\omega_B\sqrt{LPM_{2A}}\sqrt{LPM_{2B}}.$$

with

$$HPM_{1A} = \frac{1}{T}\sum_{t=1}^{T}\max(0, \tilde{R}_A - \frac{L}{4})^1,$$

$$LPM_{2A} = \frac{1}{T}\sum_{t=1}^{T}\max(0, \frac{L}{4} - \tilde{R}_A)^2.$$

3.1.6 Israelsen Ratio.

The annualized Israelsen ratio is equal to

$$\text{Israelsen ratio}_p = \begin{cases} \left[4\mathrm{E}(\tilde{R}_p) - R_f\right]/\sqrt{4}\tilde{\sigma}_p & \text{if } 4\mathrm{E}(\tilde{R}_p) - R_f > 0, \\ \left[4\mathrm{E}(\tilde{R}_p) - R_f\right]\cdot\sqrt{4}\tilde{\sigma}_p & \text{if } 4\mathrm{E}(\tilde{R}_p) - R_f < 0. \end{cases}$$

3.1.7 Ferruz-Sarto Ratio.

The annualized Ferruz-Sarto ratio will double:

$$\text{Ferruz-Sarto ratio}_p = \frac{4\mathrm{E}(\tilde{R}_p)/\mathrm{E}(R_f)}{\sqrt{4}\tilde{\sigma}_p} = 2\cdot\frac{\mathrm{E}(\tilde{R}_p)/\mathrm{E}(R_f)}{\tilde{\sigma}_p}.$$

3.2 Geometric mean excess return.

It is probably more preferable to use geometric mean excess return instead of arithmetic mean excess return. One can find geometric mean excess return solving

$$\mathrm{E}(R_p - R_b) = \left(\prod_{t=1}^{T}\frac{1+\tilde{R}_p}{1+\tilde{R}_b}\right)^{4/T} - 1.$$

However, standard deviation of excess return usually is calculated in the same way as in arithmetic mean excess return, i.e.

$$\text{Information ratio}_p^2 = \frac{1}{\sqrt{4}\tilde{\sigma}_p}\left[\left(\prod_{t=1}^{T}\frac{1+\tilde{R}_p}{1+\tilde{R}_b}\right)^{4/T} - 1\right].$$

For Sharpe ratio return on benchmark is risk-free rate. If risk-free rate is a constant and is compounded annually the mean excess return is

$$\mathrm{E}(R_p) - R_f = \left(\frac{1}{(1+R_f)} \prod_{t=1}^{T} (1+\tilde{R}_p)^{4/T} \right) - 1.$$

In this way, Sharpe ratio is

$$\text{Sharpe ratio}_p^2 = \frac{1}{\sqrt{4}\tilde{\sigma}_p} \left[\left(\frac{1}{(1+R_f)} \prod_{t=1}^{T} (1+\tilde{R}_p)^{4/T} \right) - 1 \right].$$

Since this approach is not very common, because sometimes it is very complicated to apply it, it will not be used for other ratios in this work.

3.3 Frequency-converted data.

The best method, which provides the exact ratio that would be calculated if return were observed only annually, is frequency-converted method. We can combine 4 quarterly returns of a portfolio to one annual return like this

$$R_p^{\text{year 1}} = (1+\tilde{R}_p^1)(1+\tilde{R}_p^2)(1+\tilde{R}_p^3)(1+\tilde{R}_p^4) - 1,$$

where $\tilde{R}_p^j$ is return of portfolio in quarter j.

Next we need to calculate annual return on benchmark separately

$$R_b^{\text{year 1}} = (1+\tilde{R}_b^1)(1+\tilde{R}_b^2)(1+\tilde{R}_b^3)(1+\tilde{R}_b^4) - 1.$$

If we continue to do it assuming that N is a number of years (i.e. number of quarterly returns (T) divided by 4), we can derive the general formula for mean excess return, standard deviation and Information ratio:

$$
\begin{aligned}
\mathrm{E}(R_p - R_b) &= \frac{1}{N}\sum_{j=1}^{N}\left(R_p^{\text{year j}} - R_b^{\text{year j}}\right), \\
\sqrt{\mathrm{var}(R_p - R_b)} &= \sqrt{\frac{1}{N-1}\sum_{j=1}^{N}\left(R_p^{\text{year j}} - R_b^{\text{year j}}\right)^2}, \\
\text{Information ratio}_p^3 &= \frac{\mathrm{E}(R_p - R_b)}{\sqrt{\mathrm{var}(R_p - R_b)}}.
\end{aligned}
$$

Applying the same idea to Sharpe ratio and keeping in mind that risk-free rate is compounded annually and is a constant, we get

$$
\begin{aligned}
\mathrm{E}(R_p) - R_f &= \frac{1}{N}\sum_{j=1}^{N}\left(R_p^{\text{year j}}\right) - R_f, \\
\sqrt{\mathrm{var}(R_p)} &= \sqrt{\frac{1}{N-1}\sum_{j=1}^{N}\left(R_p^{\text{year j}}\right)^2}, \\
\text{Sharpe ratio}_p^3 &= \frac{\mathrm{E}(R_p) - R_f}{\sqrt{\mathrm{var}(R_p)}}.
\end{aligned}
$$

The obvious disadvantage of this approach is that from T numbers of company quarterly returns (where T is not a big number, 28 for example), you get only N=T/4=7 yearly returns to analyse. Also if you want to recalculate Sharpe ratio every quarter to see how well company is doing, you have no easy way to do it.

Chapter 4

Examples and Case Studies.

4.1 Examples.

Only first annualization approach is commonly used because arithmetic averages are the easiest to calculate, that is why in almost all examples this method was used. Please, see Examples 1-3 for numerical illustration of application of all ratios to mergers and acquisitions and for comparison between these ratios.

However, for Sharpe ratio it is very interesting to compare results of three annualization methods described in the previous section. Please, see Example 4-6 in Appendix for numerical illustration of application these three methods to Sharpe ratio calculation and for comparison between them.

All data in examples were created using special software as normally distributed data.

4.1.1 Sortino ratio, Omega ratio, Kappa$_3$ ratio in comparison with Sharpe ratio.

From the previous chapter you saw that Sortino ratio, Omega ratio, Kappa$_3$ ratio are very similar to Sharpe ratio. They use the same numerator assuming that minimal acceptable return (L) is equal to riskless rate (R_f), but substitute different low partial moments for standard deviation in denominator:

$$
\begin{aligned}
\text{Sharpe ratio}_A &= \frac{4 \cdot \mathrm{E}(\tilde{R}_A) - R_f}{\sqrt{4} \cdot \sqrt{\mathrm{var}(\tilde{R}_A)}}, \\
\text{Sortino ratio}_A &= \frac{4 \cdot \mathrm{E}(\tilde{R}_A) - L}{\sqrt{4} \cdot \sqrt{LPM_{2A}}}, \\
\text{Omega ratio}_A &= \frac{4 \cdot \mathrm{E}(\tilde{R}_A) - L}{4 \cdot LPM_{1A}} + 1, \\
\text{Kappa }_3 \text{ ratio}_A &= \frac{4 \cdot \mathrm{E}(\tilde{R}_A) - L}{\sqrt[3]{4 \cdot LPM_{3A}}},
\end{aligned}
$$

where

$$
\begin{aligned}
LPM_{nA} &= \frac{1}{T} \sum_{t=1}^{T} \max(0, \frac{L}{4} - \tilde{R}_{At})^n, \\
\mathrm{var}(\tilde{R}_A) &= \frac{1}{T-1} \sum_{t=1}^{T} (\tilde{R}_{At} - \mathrm{E}(\tilde{R}_A))^2.
\end{aligned}
$$

Now let's understand in what situation Sortino ratio, Omega ratio, Kappa_3 ratio are better estimations of company's performance when Sharpe ratio. Consider two companies X and Y: their quarterly returns have the same average, skewness X is almost 0 and skewness Y is substantially negative.

When skewness of data is substantially negative, this means that some quarterly returns are outliers and far away less than the average. From economic point of view this indicates that company Y sometime has big losses. Thus, if you are looking for a target for merger of course company X is more preferable than company Y since the averages are the same, but "big losses" of company Y even sometimes sounds very unpromising.

What will we get by calculating Sharpe ratio? Since in denominator of Sharpe ratio the standard derivation is used, then this standard deviation will catch only about 68% of the sample (i.e. 68% of quarterly returns of companies X and Y). Of course, it will not catch outliers ("big losses" of company Y) and will give us the smaller denominator for company Y since most data will be concentrated near average on the right hand side. Thus, company Y is getting bigger Sharpe ratio than company Y.

What will we get by calculating Sortino ratio, Omega ratio and Kappa_3 ratio? Since in denominator of these ratios the lower partial moments are used, then these moments will catch only quarterly returns which less than average. For company X the denominator would be about the half of standard deviation. However, for company Y, which sometimes has data outliers, this would be much more than the half of standard deviation. Thus, if skewness of company Y is negative enough so, that low partial moments become much more than half of variation, then Sortino ratio, Omega ratio and Kappa_3 ratio will give us bigger number for company X than company Y.

Let's illustrate this simple explanation with numerical Example 1.

Example 1. Company AAA, a \$4 billion firm located in Calgary, wants to expand within their industry through merger. Their preliminary scan of candidates yielded two firms. The first, BBB, is a \$4 billion company. The second, CCC, is also a \$4 billion company. In order to prepare a more thorough analysis of the acquisition candidate, AAA collected information about the long-term pattern of returns of all two firms. The quarterly time series of total returns for the last few years for each firm plus the AAA are shown in Table 4.1.

To figure out who AAA should pick up as a target for merger we need to understand which combined company AAA+BBB or AAA+CCC is more profitable and less risky. To do so we will calculate expected Sharpe ratio, Sortino ratio, Omega ratio, Kappa_3 ratio of these firms assuming, that minimal acceptable return is equal to riskless rate of 15% compounded annually.

Solution of Example 1. We calculate historical quarterly mean, standard deviation, third, second and first lower partial moments for each firm and correlation among three series, based upon quarterly data. Assuming the weight on AAA is 0.5, the weight on BBB is 0.5 and the weight on CCC is also 0.5, we find the expected quarterly return and standard deviation of the new combined firms AAA+BBB and AAA+CCC and also first, second

Table 4.1: Quarterly total returns of each company for Example 1.

Quarter	AAA	BBB	CCC
2000.1	0.130473	0.194567	0.123
2000.2	0.049805	0.100248	0.195649
2000.3	0.072087	0.085047	0.221497
2000.4	-0.00302	0.212819	0.205222
2001.1	0.324495	0.057727	-0.10575
2001.2	0.15546	0.168392	0.151827
2001.3	0.057778	-0.03445	0.181054
2001.4	0.177308	0.102843	0.092422
2002.1	0.07421	0.297607	0.151915
2002.2	-0.01584	-0.00645	0.107772
2002.3	0.131349	0.097234	-0.171
2002.4	0.232838	0.063335	0.103154
2003.1	0.098856	-0.04934	0.179318
2003.2	0.070508	0.052942	0.157812
2003.3	0.177321	0.323227	0.200817
2003.4	-0.02208	0.104044	0.111085
2004.1	-0.15291	0.126053	-0.01722
2004.2	0.183839	0.146254	0.181389
2004.3	0.115958	0.087738	-0.01814
2004.4	0.173991	-0.01331	0.113696
2005.1	0.052907	0.213481	-0.02315
2005.2	-0.07237	0.12952	-0.04802
2005.3	-0.03101	0.062172	0.103259
2005.4	0.07828	-0.00346	0.136787
2006.1	0.198191	0.152489	0.000465
2006.2	-0.00362	0.113507	0.149059
2006.3	0.123852	-0.14508	0.165615
2006.4	0.047839	0.228307	0.163461

and third lower partial moments of the new firms according to the formulas described above.

As a result for combined companies expected Sharpe ratio, Sortino ratio, Omega ratio and Kappa_3 ratio are summarized below:

	AAA+BBB	AAA+CCC
Expected Sharpe ratio	1.600	1.634
Expected Sortino ratio	3.623	3.245
Expected Omega ratio	4.070	3.597
Expected Kappa_3 ratio	3.560	3.334

On one hand AAA alone has expected Sharpe ratio of 0.973. A merger with BBB and with CCC will increase this ratio to 1.600 and 1.634. Thus, both mergers are profitable, but if all else equal merger with CCC according to Sharpe ratio is preferable.

On another hand AAA has expected Sortino ratio of 2.077, Omega ratio of 3.443 and Kappa_3 ratio of 1.805. A merger with BBB and with CCC will increase these ratios to 3.623 and 3.245, 4.070 and 3.597, 3.560 and 3.334. Thus, both mergers are profitable, but if all else equal merger with BBB according to Sortino ratio, Omega ratio and Kappa_3 ratio is preferable.

Thus, the results are opposite: according to Sharpe ratio company AAA should choose company CCC as a target for acquisition, according to Sortino ratio, Omega ratio and Kappa_3 ratio company AAA should choose BBB as a target for acquisition. The better choice for acquisition from business point of view of course is company BBB since given the same average returns company CCC tend to have big losses. Thus, if quarterly returns are negatively skewed it is better to use one of these three ratios than Sharpe ratio.

4.1.2 Upside potential ratio in comparison with Sharpe ratio.

From the previous chapter you saw that Upside potential ratio is designed as Sharpe ratio. However, it uses low partial moment in denominator instead of standard deviation and substitutes high partial moment for average access return in numerator:

$$\text{Sharpe ratio}_A = \frac{4 \cdot \mathrm{E}(\tilde{R}_A) - R_f}{\sqrt{4} \cdot \sqrt{\mathrm{var}(\tilde{R}_A)}},$$
$$\text{Upside potential ratio}_A = \frac{4 \cdot HPM_{1A}}{\sqrt{4 \cdot LPM_{2A}}},$$

where

$$HPM_{1A} = \frac{1}{T}\sum_{t=1}^{T} \max(0, \tilde{R}_A - \frac{L}{4}),$$
$$LPM_{2A} = \frac{1}{T}\sum_{t=1}^{T} \max(0, \frac{L}{4} - \tilde{R}_A)^2,$$
$$\mathrm{var}(\tilde{R}_A) = \frac{1}{T-1}\sum_{t=1}^{T} (\tilde{R}_{At} - \mathrm{E}(\tilde{R}_A))^2.$$

As was mentioned in previous example if quarterly returns are negatively skewed it is better to use low partial moment in denominator than standard deviation. However, not only this make Upside potential ratio a better measure of company performance than Sharpe ratio. Let's consider two companies X and Y: their quarterly returns have the same average, skewness X is almost 0 and skewness Y is substantially positive.

When skewness of data is substantially positive, this means that some quarterly returns are outliers and far away more than the average. From economic point of view this indicates that company Y sometime has big gains. Thus, if you are looking for a target for merger of course company Y is more preferable than company X since the averages are the same, but "big gains" of company Y even sometimes sounds very promising.

What will we get by calculating Sharpe ratio? Since in numerator of the ratio is average excess return and for both companies X and Y the averages are the same, then the numerator of Sharpe ratio for both companies is the same. Thus, there is no difference between these companies according to Sharpe ratio.

What will we get by calculating Upside potential ratio? Since in numerator of the ratio is high partial moment, then for company X the numerator will be about the half of average excess return because of symmetry of distribution. However, for company Y, which has positive data outliers, the high partial moment will be more than half of average excess return. Thus, if skewness of company Y is positive enough so, that high partial moments becomes much more than half of average excess return, then Upside potential ratio will give us bigger number for company Y than company X. Let's illustrate this simple explanation with numerical Example 2.

Example 2. Company AAA, a \$4 billion firm located in Wonderland, wants to expand within their industry through merger. Their preliminary scan of candidates yielded two firms. The first, BBB, is a \$4 billion company. The second, CCC, is also a \$4 billion company. In order to prepare a more thorough analysis of the acquisition candidate, AAA collected information about the long-term pattern of returns of all two firms. The quarterly time series of total returns for the last few years for each firm plus the AAA are shown in Table 4.2.

To figure out who AAA should pick up as a target for merger we need to understand which combined company AAA+BBB or AAA+CCC is more profitable and less risky. To do so we will calculate expected Sharpe ratio and Upside potential ratio of these firms assuming, that minimal acceptable return is equal to riskless rate of 10% compounded annually.

Solution of Example 2. We calculate historical quarterly mean, standard deviation, the second lower partial moment(downside deviation) and the first higher partial moment (upside potential) for each firm and correlation among

Table 4.2: Quarterly total returns of each company for Example 2.

Quarter	AAA	BBB	CCC
2000.1	0.130473	0.09053	0.042371
2000.2	0.049805	0.147587	0.218673
2000.3	0.072087	0.126617	0.122524
2000.4	-0.00302	0.199911	0.142039
2001.1	0.324495	0.147808	0.032115
2001.2	0.15546	0.086237	0.030944
2001.3	0.057778	0.21364	0.175281
2001.4	0.177308	0.120551	0.136306
2002.1	0.07421	0.158105	0.131987
2002.2	-0.01584	0.089328	0.054021
2002.3	0.131349	0.032243	0.097938
2002.4	0.232838	-0.00435	0.1285
2003.1	0.098856	0.085218	-0.00866
2003.2	0.070508	0.137518	0.191061
2003.3	0.177321	0.104526	0.206198
2003.4	-0.02208	0.001088	0.063117
2004.1	-0.15291	0.127026	0.042882
2004.2	0.183839	0.076535	0.254741
2004.3	0.115958	0.106556	0.463713
2004.4	0.173991	0.153284	0.024647
2005.1	0.052907	0.048649	0.23362
2005.2	-0.07237	0.042739	0.010985
2005.3	-0.03101	0.064242	0.15854
2005.4	0.07828	0.075799	0.064183
2006.1	0.198191	0.073751	0.060486
2006.2	-0.00362	0.135012	0.111644
2006.3	0.123852	0.06004	0.072393
2006.4	0.047839	0.134727	0.078385

three series, based upon quarterly data. Assuming the weight on AAA is 0.2, the weight on BBB is 0.8 and the weight on CCC is also 0.8, we find the expected quarterly return and standard deviation of the new combined firms AAA+BBB and AAA+CCC and also and the second lower partial moments and the first higher partial moment of the new firms according to the formulas described above.

As a result for combined companies expected Sharpe ratio and Upside potential ratio are summarized below:

	AAA+BBB	AAA+CCC
Expected Sharpe ratio	3.188	2.127
Expected Upside potential ratio	15.749	16.897

On one hand AAA alone has expected Sharpe ratio of 1.221. A merger with BBB and with CCC will increase this ratio to 3.188 and 2.127. Thus, both mergers are profitable, but if all else equal merger with BBB according to Sharpe ratio is preferable.

On another hand AAA has expected Upside potential ratio of 3.732. A merger with BBB and with CCC will increase these ratios to 15.749 and 16.897. Thus, both mergers are profitable, but if all else equal merger with CCC according to Upside potential ratio is preferable.

Thus, the results are opposite: according to Sharpe ratio company AAA should choose company BBB as a target for acquisition, according to Upside potential ratio company AAA should choose CCC as a target for acquisition. The better choice for acquisition from business point of view of course is company CCC since given the almost same average returns company CCC tend to have big gains. Thus, if quarterly returns are positively skewed it is better to use Upside potential ratio than Sharpe ratio.

As you can see, Upside potential ratio is the only ratio, which takes into account not only downside risk, but also the upside potential of a company.

4.1.3 Israelsen ratio and Ferruz-Sarto ratio in comparison with Sharpe ratio.

From the previous chapter you saw that Israelsen ratio and Ferruz-Sarto ratio use average quarterly returns and standard deviation as well as Sharpe ratio:

$$\text{Sharpe ratio}_A = \frac{4 \cdot \mathrm{E}(\tilde{R}_A) - R_f}{\sqrt{4} \cdot \sqrt{\mathrm{var}(\tilde{R}_A)}},$$

$$\text{Ferruz-Sarto ratio}_A = \frac{4\mathrm{E}(\tilde{R}_A)/\mathrm{E}(R_f)}{\sqrt{4}\tilde{\sigma}_A},$$

$$\text{Israelsen ratio}_A = \begin{cases} \left[4\mathrm{E}(\tilde{R}_A) - R_f\right] / \sqrt{4}\tilde{\sigma}_A \text{ if } 4\mathrm{E}(\tilde{R}_p) - R_f > 0 \\ \left[4\mathrm{E}(\tilde{R}_A) - R_f\right] \cdot \sqrt{4}\tilde{\sigma}_A \text{ if } 4\mathrm{E}(\tilde{R}_A) - R_f < 0 \end{cases}$$

Both ratios were designed to solve the following problem of Sharpe ratio. Let's say we have two companies X and Y with the same average quarterly returns which less than risk free rate, then numerator of Sharpe ratio for both companies is the same and is negative. Now let's assume that company X is much more risky, than company Y, and then standard deviation of company X is much bigger than standard deviation of company Y. Thus, Sharpe ratio of company X will be bigger that Sharpe ratio of company Y and according to Sharpe ratio one should prefer company X as a target for acquisition. This is not right, because both companies make the same profit, but company X is more risky. This situation appears when market is falling down and many companies start to make less money than risk free rate or even when their average return becomes negative.

Israelsen ratio corrects this mistake by letting standard deviation be in numerator instead denominator in case of negative excess returns. This leads to bigger ratio for company Y as it supposed to be. Ferruz-Sarto ratio avoids negative value by dividing average quarterly return by risk free rate, however it will not correct the mistake if average quarterly return not only less than risk free rate, but also less than 0. In this case Ferruz-Sarto ratio gives ridicules answer as well as Sharpe ratio for the same reason.

Let's illustrate this simple explanation with numerical Example 3

Example 3. Company AAA, a \$ 4 billion firm located in Toronto, wants to expand within their industry through merger. Their preliminary scan of candidates yielded two firms. The first, BBB, is a \$1 billion company. The second, CCC, is also a \$1 billion company. In order to prepare a more thorough analysis of the acquisition candidate, AAA collected information about the long-term pattern of returns of all two firms. The quarterly time series of total returns for the last few years for each firm plus the AAA are shown in Table 4.3.

To figure out who AAA should pick up as a target for merger we need to understand which combined company AAA+BBB or AAA+CCC is more profitable and less risky. To do so we will calculate expected Sharpe ratio Israelsen ratio and Ferruz-Sarto ratio of these firms assuming, that riskless rate is 5% compounded annually.

Solution of Example 3. We calculate historical quarterly mean, standard deviation for each firm and correlation among three series, based upon quarterly data. Assuming the weight on AAA is 0.8, the weight on BBB is 0.2 and the weight on CCC is also 0.2, we find the expected quarterly return and standard deviation of the new combined firms AAA+BBB and AAA+CCC according to the formulas described above.

As a result, expected Sharpe ratio, Israelsen ratio and Ferruz-Sarto ratio are summarized below:

	AAA+BBB	AAA+CCC
Expected Sharpe ratio	-0.057	-0.044
Expected Israelsen ratio	-0.135	-0.172
Expected Ferruz-Sarto ratio	-0.484	-0.378

On one hand, AAA alone has an expected Sharpe ratio of (-0.045) and Ferruz-Sarto ratio of (-0.384). A merger with BBB will decrease this ratio to

Table 4.3: Quarterly total returns of each company for Example 3.

Quarter	AAA	BBB	CCC
2000.1	-0.692304	1.18506	4.9049
2000.2	-0.382942	0.33062	-5.641771
2000.3	0.09786	-0.03709	2.59107
2000.4	-0.038824	-0.504763	-3.776911
2001.1	-0.644825	-0.092759	2.30142
2001.2	0.16728	0.20083	-1.452975
2001.3	0.60122	-0.2364	-1.683909
2001.4	-1.105647	0.28495	-2.917469
2002.1	-0.296209	-0.292041	4.80952
2002.2	0.46407	0.97343	7.54398
2002.3	0.61386	-0.62245	-3.154525
2002.4	1.79844	-0.134503	-0.093248
2003.1	0.06074	-0.556165	1.11819
2003.2	0.64722	-0.108913	5.59352
2003.3	1.22611	-0.086408	-3.281975
2003.4	-0.874273	0.00577	0.86601
2004.1	1.53291	1.04488	-3.490502
2004.2	0.53255	0.1783	3.69672
2004.3	-0.521431	-0.95086	-0.310528
2004.4	0.78777	-0.508129	-2.358486
2005.1	-2.502664	0.46475	2.15234
2005.2	-0.329463	-0.702484	2.29454
2005.3	0.44039	0.33171	-2.425912
2005.4	-1.244399	-0.031919	-5.08795
2006.1	-1.448381	-0.295632	0.32678
2006.2	0.55246	0.84243	-3.377154
2006.3	-0.757409	-0.018679	-0.830793
2006.4	1.05711	-0.93744	1.4156

(-0.057) and (-0.484) correspondently. A merger with CCC will increase this ratio to (-0.044) and to (-0.378) correspondently. None of the merger will give the possibility to achieve a return in excess of treasure bills, but, according to Sharpe ratio and Ferruz-Sarto ratio, company CCC should be preferred.

On another hand, AAA alone has an expected Israelsen ratio of (-0.167). A merger with BBB will increase this ratio to (-0.135). A merger with CCC will decrease this ratio to (-0.172). Thus, only merger with BBB is profitable and should be preferred.

The results are opposite: according to Sharpe ratio and Ferruz-Sarto ratio company AAA should choose company CCC as a target for acquisition, according to Israelsen ratio company AAA should choose BBB as a target for acquisition. The better choice for acquisition from business point of view of course is company BBB since given the same negative average returns company CCC is 6 times more risky than BBB because its standard deviation is 6 times bigger. Thus, the best performance measure, which let us make the right decision about choice of company for merger in case of bear market, is Israelsen ratio.

4.2 M&A: Case Studies with performance measures.

In this section let's consider several real-world examples of acquisitions in Canada and US, which were completed recently. Information about quarterly and annual financial statements for every company and company's profile were found in the following internet resources: EDGAR (www.sec.gov), Mergent Online (www.mergentonline.com), HighBeam Research (www.highbeam.com). The technical information about acquisitions was drown from Financial Post (www.financialpost.com). Risk free rate as a rate of 1 year Treasury bills and exchange rates can be found in Bank of Canada website (www.bankofcanada.ca). All historical share price was found on Canadian Financial Markets Research Centre (CHASS) website using University of Alberta access and on Investcom website (www.investcom.com).

As described in Introduction, there are many ways of defining company returns. We use a couple of simplest returns R_1 and R_2, which we can get from companies quarterly and annual reports:

$$1)\ R_1 = \frac{Net\ cash\ provided\ by\ (used\ in)\ operating\ activities}{Revenue}$$

$$2)\ R_2 = \frac{Net\ cash\ provided\ by\ (used\ in)\ operating\ activities}{Total\ Assets}$$

Using these returns we calculate all performance measures suitable for mergers and acquisitions desribed in previous sections and compare results.

4.2.1 Aur Resources Inc. and Teck Cominco Ltd.

Teck Cominco, located in Vancouver, is engaged in mining and related activities including exploration, development, processing, smelting and refining. Co.'s major products are zinc, copper, and metallurgical coal. Co. also produces precious metals, lead, molybdenum, electrical power, fertilizers and various specialty metals. Co. also owned an interest in certain oil sands leases and has a partnership interest in an oil sands development project.

Aur Resources, located in Toronto, is a mining company engaged in the acquisition, exploration, development and mining of mineral properties. Co. has the Andacollo and Quebrada Blanca producing mining operations in Chile, the Duck Pond mining operation in Canada which commenced production in Jan. 2007, as well as the Andacollo primary copper-gold deposit (the "Andacollo Hypogene Project" in Chile) which is under development.

On the 3rd of July 2007, Teck Cominco Limited agreed to acquire Aur Resources Inc. for Cdn$41 or 0.8749 Teck class B subordinate voting shares and Cdn$0.0001 in cash per Aur common share held in a transaction valued at Cdn$4.1 billion.The offer was subject to pro ration and acceptance by 66.66% of Aur shareholders on a fully diluted basis. Teck Cominco would pay a maximum of Cdn$3.1 billion in cash and issue a maximum of 22,000,000 class B subordinate voting shares. The board of directors of Aur Resources unanimously supported the bid and agreed to pay a break fee of Cdn$140,000,000. Transaction was complited on the 1th of January 2008. Last quarter before the announcement ended on 30th of June 2007.

At that moment, Teck Cominco Limited had 419,300,000 shares outstanding with an average price of $49.355 per share, thus market capitalization of this company was 20,694.6M. At the same moment, Aur Resources Inc. had 98,672,000 shares outstanding with an average price of $31.675 per share, thus market capitalization of this company was 3,125.4M. This means, that weights, which we need for calculating performance measures, are $(\omega_1, \omega_2) =$

$(0.869, 0.131)$.

In Table 4.4 and in Table 4.5 you can see values for Total Assets from Balance Sheets, values for Revenues and Net earnings from Net Income statements and values for Cash provided by (used by) operating activities from Cash Flow Statements for Teck Cominco Ltd. and for Aur Resources Inc. Note, that value for the forth quarter every year is calculated as a difference between annual number and sum of previous 3 quarters.

Table 4.6 and Table 4.7 contain risk-free rate and two types of returns R_1 and R_2 calculated according to the formulas described above using values from Table 4.4 and from Table 4.5 correspondently.

Let's now calculte all performance measures applying the same technique as in Section 4.1 using returns R_1 and R_2 from Tables 4.6 and 4.7.

Performance measures calculated based on R_1 returns			
Ratios	Teck Cominco	Combined company	Aur Resources
Sharpe ratio$_1$	2.993	3.424	5.572
Sharpe ratio$_2$	3.991	4.785	10.104
Sharpe ratio$_3$	1.387	1.531	1.760
Sortino ratio	65.434	83.604	undefined
Omega ratio	157.905	201.476	undefined
Kappa$_3$ ratio	48.887	62.462	undefined
Upside-Potential ratio	65.851	84.021	undefined
Israelsen ratio	2.993	3.424	5.572
Ferruz-Sarto ratio	94.285	107.531	172.926

Performance measures calculated based on R_2 returns			
Ratios	Teck Cominco	Combined company	Aur Resources
Sharpe ratio$_1$	2.099	2.378	3.309
Sharpe ratio$_2$	2.154	2.459	3.569
Sharpe ratio$_3$	1.127	1.234	1.418
Sortino ratio	23.082	30.651	undefined
Omega ratio	33.547	44.218	undefined
Kappa$_3$ ratio	18.732	24.874	undefined
Upside-Potential ratio	23.791	31.360	undefined
Israelsen ratio	2.099	2.378	3.309
Ferruz-Sarto ratio	82.095	90.280	113.975

Conclusion from Section 4.2.1. As you can see from the resulting table all performance measures for Teck Cominco are less than numbers for combined company if one uses R_1 or R_2 returns for company evaluation. This means that this acquisition for Teck Cominco was profitable and Teck Cominco's managers made the right choice choosing Aur Resources as a target.

For Aur Resources all performance measures except Sortino ratio, Omega ratio and Kappa$_3$ ratio are bigger than numbers for combined company if one uses R_1 or R_2 returns for company evaluation. This means that this acquisition for Aur Resources was not profitable, however, Aur Resources got $4,100M in cash from Teck Cominco. Sortino ratio, Omega ratio and Kappa$_3$ ratio for Aur Resources are undefined, because denominator of these ratios turn to be zero in this case.

Table 4.4: Total Assets, Revenues, Net earnings and Cash provided by (used by) operating activities for Teck Cominco Ltd. (in thousands).

Date	Total assets	Revenues	Net earnings	Cash
06/30/2007	10,595,000	1,561,000	485,000	193,000
03/31/2007	11,129,000	1,340,000	360,000	152,000
12/31/2006	11,447,000	2,088,000	866,000	1,182,000
09/30/2006	9,896,000	1,632,000	504,000	752,000
06/30/2006	9,454,000	1,546,000	613,000	600,000
03/31/2006	8,928,000	1,273,000	448,000	371,000
12/31/2005	8,809,000	1,343,000	510,000	648,000
09/30/2005	8,042,000	1,150,000	405,000	383,000
06/30/2005	6,518,000	994,000	225,000	307,000
03/31/2005	6,293,000	928,000	205,000	309,000
12/31/2004	6,059,000	893,000	285,000	390,000
09/30/2004	5,854,000	978,000	120,000	262,000
06/30/2004	5,793,000	835,000	116,000	283,000
03/31/2004	5,588,000	722,000	96,000	181,000
12/31/2003	5,267,000	744,000	107,000	215,000
09/30/2003	5,238,000	590,000	19,000	79,000
06/30/2003	4,844,000	502,000	12,000	79,000
03/31/2003	5,113,000	574,000	11,000	27,000
12/31/2002	4,958,000	625,000	15,000	199,000
09/30/2002	4,940,000	540,000	5,000	-15,000
06/30/2002	4,809,000	521,000	8,000	31,000
03/31/2002	5,105,000	501,000	2,000	37,000
12/31/2001	5,153,000	527,000	6,000	13,000

Table 4.5: Total Assets, Revenues, Net earnings and Cash provided by (used by) operating activities for Aur Resources Inc.(in thousands)

Date	Total assets	Revenues	Net earnings	Cash
06/30/2007	1,221,597	214,366	84,125	85,550
03/31/2007	1,204,259	156,412	57,820	79,011
12/31/2006	1,148,215	181,390	68,487	134,013
09/30/2006	1,049,261	202,346	84,572	142,919
06/30/2006	920,755	219,442	93,904	151,687
03/31/2006	815,946	135,104	46,755	76,005
12/31/2005	753,381	132,699	41,513	89,300
09/30/2005	701,103	108,097	31,018	64,333
06/30/2005	648,620	109,636	36,486	52,339
03/31/2005	604,109	96,513	33,260	43,807
12/31/2004	574,653	97,884	33,538	57,131
09/30/2004	532,703	80,139	20,729	31,561
06/30/2004	510,522	70,031	16,034	34,130
03/31/2004	491,985	88,826	26,634	40,538
12/31/2003	447,160	61,433	7,353	21,746
09/30/2003	427,712	50,652	2,827	11,238
06/30/2003	421,531	50,252	985	15,377
03/31/2003	456,981	53,264	-305	11,861
12/31/2002	452,245	54,341	1,915	14,809
09/30/2002	454,201	46,301	2,882	16,200
06/30/2002	450,745	48,414	2,622	16,500
03/31/2002	461,296	46,566	2,639	9,561
12/31/2001	471,674	56,008	5,368	26,597

Table 4.6: Risk-free rate and two types of returns, calculated for Teck Cominco Ltd.

Date	Risk-free rate	R_1	R_2
06/30/2007	0.0473	0.12364	0.01822
03/31/2007	0.0418	0.11343	0.01366
12/31/2006	0.0415	0.56609	0.10326
09/30/2006	0.041	0.46078	0.07599
06/30/2006	0.0458	0.38810	0.06347
03/31/2006	0.0406	0.29144	0.04155
12/31/2005	0.0387	0.48250	0.07356
09/30/2005	0.0324	0.33304	0.04762
06/30/2005	0.0274	0.30885	0.04710
03/31/2005	0.0304	0.33297	0.04910
12/31/2004	0.0276	0.43673	0.06437
09/30/2004	0.0286	0.26789	0.04476
06/30/2004	0.0261	0.33892	0.04885
03/31/2004	0.02	0.25069	0.03239
12/31/2003	0.0262	0.28898	0.04082
09/30/2003	0.0263	0.13390	0.01508
06/30/2003	0.0282	0.15737	0.01631
03/31/2003	0.0363	0.04704	0.00528
12/31/2002	0.0291	0.31840	0.04014
09/30/2002	0.0312	-0.02778	-0.00304
06/30/2002	0.032	0.05950	0.00645
03/31/2002	0.0342	0.07385	0.00725
12/31/2001	0.022	0.02467	0.00252

Table 4.7: Risk-free rate and two types of returns, calculated for Aur Resources Inc.

Date	Risk-free rate	R_1	R_2
06/30/2007	0.0473	0.39908	0.07003
03/31/2007	0.0418	0.50515	0.06561
12/31/2006	0.0415	0.73881	0.11671
09/30/2006	0.041	0.70631	0.13621
06/30/2006	0.0458	0.69124	0.16474
03/31/2006	0.0406	0.56257	0.09315
12/31/2005	0.0387	0.67295	0.11853
09/30/2005	0.0324	0.59514	0.09176
06/30/2005	0.0274	0.47739	0.08069
03/31/2005	0.0304	0.45390	0.07252
12/31/2004	0.0276	0.58366	0.09942
09/30/2004	0.0286	0.39383	0.05925
06/30/2004	0.0261	0.48736	0.06685
03/31/2004	0.02	0.45638	0.08240
12/31/2003	0.0262	0.35398	0.04863
09/30/2003	0.0263	0.22187	0.02627
06/30/2003	0.0282	0.30600	0.03648
03/31/2003	0.0363	0.22268	0.02596
12/31/2002	0.0291	0.27252	0.03275
09/30/2002	0.0312	0.34988	0.03567
06/30/2002	0.032	0.34081	0.03661
03/31/2002	0.0342	0.20532	0.02073
12/31/2001	0.022	0.47488	0.05639

4.2.2 Emergis Inc. and Telus Corp.

TELUS, located in Vancouver, is a telecommunications company in Canada. Co. provides a wide range of wireline and wireless telecommunications products and services including data, Internet Protocol (IP), voice, video and entertainment services. Co.'s main business segments consist of TELUS Mobility and TELUS Communications. TELUS Mobility is a national facilities-based wireless provider. Through this segment, Co. provides national digital wireless voice, Push To TalkNULL (PTTNULL), data and Internet services across Canada. TELUS Communications is an incumbent local exchange carrier (ILEC) offering local, long distance, data, Internet and other services to consumers and businesses in Canada.

Emergis, located in Longueuil, is primarily engaged in provision of health and finance solutions services, through developing and managing solutions that automate transactions and the secure exchange of information to increase the process efficiency and quality of service of its customers. Through its subsidiaries, Co. is engaged in provision of health information software solutions services to health care providers; provision of payment solutions services in U.S.; and provision of tax filing and payment solutions services. Co. operates two business segments: Health and Finance. Co. operates predominantly in Canada and U.S.

On 29th of November 2007, TELUS Corporation agreed to acquire Emergis Inc., a provider of electronic health care and financial services solutions, for $763,000,000 cash or $8.25 per share. The agreement included a $15,000,000 break fee payable by Emergis. Transaction was complited on the 18th of January 2008. Last quarter before the announcement ended on 30th of September 2007.

At that moment, Telus Corp. had 327,431,260 shares outstanding with an average price of $57.85 per share, thus market capitalization of this company was 18,942M. At the same moment, Emergis Inc. had 90,117,951 shares outstanding with an average price of \$7.03 per share, thus market capitalization of this company was 633.5M. This means, that weights, which we need for calculating performance measures, are $(\omega_1, \omega_2) = (0.968, 0.032)$.

In Table 4.8 and in Table 4.9 you can see values for Total Assets from Balance Sheets, values for Revenues and Net earnings from Net Income statements and values for Cash provided by (used by) operating activities from Cash Flow Statements for Telus Corp. and for Emergis Inc. Note, that value for the forth quarter every year is calculated as a difference between annual number and sum of previous 3 quarters.

Table 4.10 and Table 4.11 contain risk-free rate and two types of returns R_1 and R_2 calculated according to the formulas described above using values from Table 4.8 and from Table 4.9 correspondently.

Let's now calculte all performance measures applying the same technique as in Section 4.1 using returns R_1 and R_2 from Tables 4.10 and 4.11.

Performance measures calculated based on R_1 returns			
Ratios	Telus	Combined company	Emergis
Sharpe ratio$_1$	2.993	3.424	5.572
Sharpe ratio$_2$	3.991	4.785	10.104
Sharpe ratio$_3$	1.387	1.531	1.760
Sortino ratio	65.434	83.604	undefined
Omega ratio	157.905	201.476	undefined
Kappa$_3$ ratio	48.887	62.462	undefined
Upside-Potential ratio	65.851	84.021	undefined
Israelsen ratio	2.993	3.424	5.572
Ferruz-Sarto ratio	94.285	107.531	172.926

Performance measures calculated based on R_2 returns			
Ratios	Telus	Combined company	Emergis
Sharpe ratio_1	2.099	2.378	3.309
Sharpe ratio_2	2.154	2.459	3.569
Sharpe ratio_3	1.127	1.234	1.418
Sortino ratio	23.082	30.651	undefined
Omega ratio	33.547	44.218	undefined
Kappa_3 ratio	18.732	24.874	undefined
Upside-Potential ratio	23.791	31.360	undefined
Israelsen ratio	2.099	2.378	3.309
Ferruz-Sarto ratio	82.095	90.280	113.975

Conclusion from Section 4.2.2. As you can see from the resulting table all performance measures for Telus are less than numbers for combined company if one uses R_1 or R_2 returns for company evaluation. This means that this acquisition for Telus was profitable and Telus's managers made the right choice choosing Emergis as a target.

For Emergis all performance measures except Sortino ratio, Omega ratio and Kappa_3 ratio are bigger than numbers for combined company if one uses R_1 or R_2 returns for company evaluation. This means that this acquisition for Emergis was not profitable, however, Emergis got \$763M in cash from Telus. Sortino ratio, Omega ratio and Kappa_3 ratio for Emergis are undefined, because denominator of these ratios turn to be zero in this case.

Table 4.8: Total Assets, Revenues, Net earnings and Cash provided by (used by) operating activities for Telus Corp. (in thousands).

Date	Total assets	Revenues	Net earnings	Cash
09/30/2007	16,697,300	2,309,900	409,900	831,800
06/30/2007	16,567,800	2,228,100	253,100	1,061,900
03/31/2007	17,403,400	2,205,600	194,800	460,600
12/31/2006	16,508,200	2,254,600	236,200	747,200
09/30/2006	16,454,500	2,210,700	319,600	570,400
06/30/2006	16,007,200	2,135,200	356,600	813,000
03/31/2006	16,017,800	2,080,500	210,100	673,100
12/31/2005	16,222,300	2,086,700	78,500	805,000
09/30/2005	18,050,100	2,062,800	190,100	693,500
06/30/2005	17,993,900	2,018,500	189,500	687,700
03/31/2005	18,136,500	1,974,700	242,200	728,400
12/31/2004	17,838,000	1,964,900	135,600	613,800
09/30/2004	17,643,300	1,946,900	156,600	847,200
06/30/2004	17,589,300	1,865,600	172,300	489,000
03/31/2004	17,563,500	1,803,800	101,300	588,100
12/31/2003	17,477,500	1,825,600	49,600	423,900
09/30/2003	17,459,900	1,806,200	115,900	849,700
06/30/2003	17,622,900	1,773,300	74,800	470,700
03/31/2003	17,806,900	1,740,900	91,200	399,700
12/31/2002	18,219,800	1,794,400	-139,200	367,800
09/30/2002	18,255,700	1,766,300	-107,400	804,300
06/30/2002	18,444,400	1,748,000	18,400	281,200
03/31/2002	18,482,200	1,698,000	-800	288,700

Table 4.9: Total Assets, Revenues, Net earnings and Cash provided by (used by) operating activities for Emergis Inc.

Date	Total assets	Revenues	Net earnings	Cash
09/30/2007	291,300,000	48,100,000	6,600,000	4,300,000
06/30/2007	278,500,000	46,300,000	6,900,000	11,700,000
03/31/2007	271,800,000	45,700,000	5,400,000	-3,000,000
12/31/2006	280,000,000	46,200,000	9,700,000	10,500,000
09/30/2006	268,700,000	43,300,000	5,200,000	2,100,000
06/30/2006	264,600,000	40,200,000	10,200,000	9,200,000
03/31/2006	259,000,000	40,300,000	3,700,000	-5,300,000
12/31/2005	271,300,000	38,900,000	6,900,000	11,600,000
09/30/2005	274,000,000	40,100,000	-1,200,000	-300,000
06/30/2005	309,700,000	40,700,000	11,500,000	1,300,000
03/31/2005	319,300,000	39,300,000	-5,700,000	-30,200,000
12/31/2004	352,000,000	37,400,000	-22,900,000	3,300,000
09/30/2004	399,300,000	48,300,000	-1,500,000	5,600,000
06/30/2004	420,200,000	62,500,000	-37,200,000	-16,200,000
03/31/2004	633,600,000	70,300,000	-100,000	-13,100,000
12/31/2003	640,700,000	-49,600,000	-113,700,000	-19,300,000
09/30/2003	757,300,000	117,200,000	6,200,000	55,500,000
06/30/2003	722,700,000	124,000,000	5,900,000	6,100,000
03/31/2003	766,500,000	124,100,000	4,800,000	12,200,000
12/31/2002	813,200,000	130,900,000	8,600,000	13,600,000
09/30/2002	851,100,000	135,100,000	4,800,000	21,500,000
06/30/2002	797,200,000	141,900,000	-95,800,000	9,200,000
03/31/2002	1,071,500,000	132,000,000	-27,900,000	-29,100,000

Table 4.10: Risk-free rate and two types of returns, calculated for Telus Corp.

Date	Risk-free rate	R_1	R_2
09/30/2007	0.0429	0.36010	0.04982
06/30/2007	0.0473	0.47659	0.06409
03/31/2007	0.0418	0.20883	0.02647
12/31/2006	0.0415	0.33141	0.04526
09/30/2006	0.041	0.25802	0.03467
06/30/2006	0.0458	0.38076	0.05079
03/31/2006	0.0406	0.32353	0.04202
12/31/2005	0.0387	0.38578	0.04962
09/30/2005	0.0324	0.33619	0.03842
06/30/2005	0.0274	0.34070	0.03822
03/31/2005	0.0304	0.36887	0.04016
12/31/2004	0.0276	0.31238	0.03441
09/30/2004	0.0286	0.43515	0.04802
06/30/2004	0.0261	0.26211	0.02780
03/31/2004	0.02	0.32603	0.03348
12/31/2003	0.0262	0.23220	0.02425
09/30/2003	0.0263	0.47044	0.04867
06/30/2003	0.0282	0.26544	0.02671
03/31/2003	0.0363	0.22959	0.02245
12/31/2002	0.0291	0.20497	0.02019
09/30/2002	0.0312	0.45536	0.04406
06/30/2002	0.032	0.16087	0.01525
03/31/2002	0.0342	0.17002	0.01562

Table 4.11: Risk-free rate and two types of returns, calculated for Emergis Inc.

Date	Risk-free rate	R_1	R_2
09/30/2007	0.0429	0.08940	0.01476
06/30/2007	0.0473	0.25270	0.04201
03/31/2007	0.0418	-0.06565	-0.01104
12/31/2006	0.0415	0.22727	0.03750
09/30/2006	0.041	0.04850	0.00782
06/30/2006	0.0458	0.22886	0.03477
03/31/2006	0.0406	-0.13151	-0.02046
12/31/2005	0.0387	0.29820	0.04276
09/30/2005	0.0324	-0.00748	-0.00109
06/30/2005	0.0274	0.03194	0.00420
03/31/2005	0.0304	-0.76845	-0.09458
12/31/2004	0.0276	0.08824	0.00938
09/30/2004	0.0286	0.11594	0.01402
06/30/2004	0.0261	-0.25920	-0.03855
03/31/2004	0.02	-0.18634	-0.02068
12/31/2003	0.0262	0.38911	-0.03012
09/30/2003	0.0263	0.47355	0.07329
06/30/2003	0.0282	0.04919	0.00844
03/31/2003	0.0363	0.09831	0.01592
12/31/2002	0.0291	0.10390	0.01672
09/30/2002	0.0312	0.15914	0.02526
06/30/2002	0.032	0.06483	0.01154
03/31/2002	0.0342	-0.22045	-0.02716

4.2.3 Cognos Inc. and IBM Corp.

International Business Machines, located in Armonk, New York, is a worldwide information technology company which primarily provides a variety of business products and services through the utilization of information technology. Co.'s primary operations comprise a Global Technology Services segment, which primarily reflects Internet Technology infrastructure services and business process services; a Global Business Services segment, which primarily reflects professional services and application outsourcing services; a Software segment, which consists primarily of middleware and operating systems software; a Systems and Technology that provides business applications; and a Global Financing segment.

Cognos, located in Ottawa, is primarily engaged in business intelligence and performance management software applications. Co.'s applications help organizations plan, understand, and manage financial and operational performance. Co.'s applications achieve this by supporting effective decision-making at all levels of the organization through the consistent reporting and analysis of data derived from various sources, and enabling Co.'s customers to understand and monitor current performance while planning future business strategies. Co.'s integrated applications components are supported by software services for administration, deployment, integration, and extraction, transformation, and loading.

On the 12th of November 2007 IBM Corporation agreed to acquire Cognos Incorporated for US$58 cash per share for a total price of about US$4.9 billion. The board of directors of Cognos unanimously recommended that shareholders vote in favour of the acquisition. Transaction was complited on the 31th of January 2008. Last quarter before the announcement ended on 30th of September 2007 for IBM Corp. and on 31th of August 2007 for Cognos Inc.

At the moment, IBM Corp. had 1,377,960,000 shares outstanding with an average price of $US117.8 per share, thus market capitalization of this company was US$162,323.688M. The exchange rate between CAD$ and US$ at that moment was 1US=0.9963CAD, so the market capitalization of this company was CAD$161,723.09M. At about the same time, Cognos Inc. had 83,213,180 shares outstanding with an average price of CAD$42.45 per share, thus market capitalization of this company was CAD$3532.4M. This means, that weights, which we need for calculating performance measures, are $(\omega_1, \omega_2) = (0.979, 0.021)$.

In Table 4.12 and in Table 4.13 you can see values for Total Assets from Balance Sheets, values for Revenues and Net earnings from Net Income statements and values for Cash provided by (used by) operating activities from Cash Flow Statements for IBM Corp. and for Cognos Inc. Note, that value for the forth quarter every year is calculated as a difference between annual number and sum of previous 3 quarters.

Table 4.14 and Table 4.15 contain risk-free rate and two types of returns R_1 and R_2 calculated according to the formulas described above using values from Table 4.12 and from Table 4.13 correspondently.

Let's now calculte all performance measures applying the same technique as in Section 4.1 using R_1 and R_2 from Tables 4.14 and 4.15.

Performance measures calculated based on R_1 returns			
Ratios	IBM Corp.	Combined company	Cognos Inc.
Sharpe ratio$_1$	9.849	10.597	2.882
Sharpe ratio$_2$	13.798	14.866	4.034
Sharpe ratio$_3$	38.914	45.965	3.475
Sortino ratio	undefined	2,244.278	54.813
Omega ratio	undefined	5,611.694	138.033
Kappa$_3$ ratio	undefined	1,653.598	40.387
Upside-Potential ratio	undefined	2,244.678	55.213
Israelsen ratio	9.849	10.597	2.882
Ferruz-Sarto ratio	313.286	337.011	91.006

Performance measures calculated based on R_2 returns			
Ratios	IBM Corp.	Combined company	Cognos Inc.
Sharpe ratio$_1$	6.753	7.195	2.418
Sharpe ratio$_2$	7.849	8.364	2.798
Sharpe ratio$_3$	19.876	21.721	4.065
Sortino ratio	undefined	1,142.142	29.299
Omega ratio	undefined	2,380.615	62.044
Kappa$_3$ ratio	undefined	851.653	21.847
Upside-Potential ratio	undefined	1,142.622	29.779
Israelsen ratio	6.753	7.195	2.418
Ferruz-Sarto ratio	260.832	277.596	89.610

Conclusion from Section 4.2.3. As you can see from the resulting table all performance measures for IBM except Sortino ratio, Omega ratio and Kappa$_3$ ratio are less than numbers for combined company if one uses R_1 or R_2 returns for company evaluation. This means that this acquisition for IBM was profitable and IBM's managers made the right choice choosing Cognos as a target. Sortino ratio, Omega ratio and Kappa$_3$ ratio for IBM are undefined, because denominator of these ratios turn to be zero in this case.

For Cognos all performance measures are less than numbers for combined company if one uses R_1 or R_2 returns for company evaluation. This means that this acquisition for Cognos was profitable, and Cognos got US$4,900M in cash from IBM.

Since for both companies acquisition was profitable, this is the pure example of synergy effect.

Table 4.12: Total Assets, Revenues, Net earnings and Cash provided by (used by) operating activities for IBM Corp. (in millions).

Date	Total assets	Revenues	Net earnings	Cash
09/30/2007	108,609	24,119	2,361	4,484
06/30/2007	102,548	23,772	2,260	3,443
03/31/2007	101,619	22,029	1,844	3,016
12/31/2006	103,234	26,258	3,540	5,334
09/30/2006	104,155	22,617	2,222	4,008
06/30/2006	103,377	21,890	2,022	2,567
03/31/2006	102,468	20,659	1,708	3,110
12/31/2005	105,748	24,427	3,187	5,420
09/30/2005	101,009	21,529	1,516	4,281
06/30/2005	103,388	22,270	1,829	3,133
03/31/2005	104,899	22,908	1,402	2,080
12/31/2004	109,183	27,461	3,040	4,050
09/30/2004	100,676	23,429	1,800	3,876
06/30/2004	99,582	23,153	1,988	3,819
03/31/2004	101,825	22,250	1,602	3,661
12/31/2003	104,457	25,913	2,706	4,752
09/30/2003	97,190	21,522	1,785	3,844
06/30/2003	96,938	21,631	1,705	3,735
03/31/2003	95,720	20,065	1,387	2,238
12/31/2002	96,484	23,163	1,018	4,373
09/30/2002	83,956	19,821	1,313	3,575
06/30/2002	84,211	19,651	56	3,185
03/31/2002	83,056	18,551	1,192	2,655
12/31/2001	88,313	22,826	2,333	4,832
09/30/2001	85,094	20,428	1,595	4,046

Table 4.13: Total Assets, Revenues, Net earnings and Cash provided by (used by) operating activities for Cognos Inc. (in thousands)

Date	Total assets	Revenues	Net earnings	Cash
08/31/2007	991,227	252,367	26,545	22,327
05/31/2007	1,206,397	236,654	22,386	29,881
02/28/2007	1,292,761	284,535	60,856	118,781
11/30/2006	1,140,309	247,799	16,543	23,456
08/31/2006	1,143,012	229,890	23,760	16,121
05/31/2006	1,131,459	217,040	14,538	72,640
02/28/2006	1,145,050	253,129	43,990	92,695
11/30/2005	990,095	212,254	28,268	8,764
08/31/2005	987,208	212,042	28,719	28,812
05/31/2005	978,898	200,075	23,825	-5,333
02/28/2005	1,063,967	256,326	54,335	97,640
11/30/2004	923,969	210,366	34,545	38,324
08/31/2004	836,346	185,220	27,599	29,414
05/31/2004	803,261	173,619	20,125	33,089
02/29/2004	827,471	202,146	46,100	80,477
11/30/2003	722,405	172,227	24,248	29,200
08/31/2003	674,275	158,181	18,158	21,711
05/31/2003	653,568	150,563	12,391	10,541
02/28/2003	658,551	163,728	29,565	55,727
11/30/2002	530,481	138,074	19,929	15,329
08/31/2002	509,430	129,104	13,739	3,041
05/31/2002	514,592	120,130	9,911	30,428
02/28/2002	522,152	142,792	10,120	44,152
11/30/2001	469,305	124,181	13,286	22,183
08/31/2001	462,808	116,313	7,104	28,646

Table 4.14: Risk-free rate and two types of returns, calculated for IBM Corp.

Date	Risk-free rate	R_1	R_2
09/30/2007	0.0429	0.18591	0.04129
06/30/2007	0.0473	0.14483	0.03357
03/31/2007	0.0418	0.13691	0.02968
12/31/2006	0.0415	0.20314	0.05167
09/30/2006	0.041	0.17721	0.03848
06/30/2006	0.0458	0.11727	0.02483
03/31/2006	0.0406	0.15054	0.03035
12/31/2005	0.0387	0.22189	0.05125
09/30/2005	0.0324	0.19885	0.04238
06/30/2005	0.0274	0.14068	0.03030
03/31/2005	0.0304	0.09080	0.01983
12/31/2004	0.0276	0.14748	0.03709
09/30/2004	0.0286	0.16544	0.03850
06/30/2004	0.0261	0.16495	0.03835
03/31/2004	0.02	0.16454	0.03595
12/31/2003	0.0262	0.18338	0.04549
09/30/2003	0.0263	0.17861	0.03955
06/30/2003	0.0282	0.17267	0.03853
03/31/2003	0.0363	0.11154	0.02338
12/31/2002	0.0291	0.18879	0.04532
09/30/2002	0.0312	0.18036	0.04258
06/30/2002	0.032	0.16208	0.03782
03/31/2002	0.0342	0.14312	0.03197
12/31/2001	0.022	0.21169	0.05471
09/30/2001	0.0297	0.19806	0.04755

Table 4.15: Risk-free rate and two types of returns, calculated for Cognos Inc.

Date	Risk-free rate	R_1	R_2
08/31/2007	0.0429	0.08847	0.02252
05/31/2007	0.0473	0.12626	0.02477
02/28/2007	0.0418	0.41746	0.09188
11/30/2006	0.0415	0.09466	0.02057
08/31/2006	0.041	0.07012	0.01410
05/31/2006	0.0458	0.33468	0.06420
02/28/2006	0.0406	0.36620	0.08095
11/30/2005	0.0387	0.04129	0.00885
08/31/2005	0.0324	0.13588	0.02919
05/31/2005	0.0274	-0.02666	-0.00545
02/28/2005	0.0304	0.38092	0.09177
11/30/2004	0.0276	0.18218	0.04148
08/31/2004	0.0286	0.15881	0.03517
05/31/2004	0.0261	0.19058	0.04119
02/29/2004	0.02	0.39811	0.09726
11/30/2003	0.0262	0.16954	0.04042
08/31/2003	0.0263	0.13725	0.03220
05/31/2003	0.0282	0.07001	0.01613
02/28/2003	0.0363	0.34036	0.08462
11/30/2002	0.0291	0.11102	0.02890
08/31/2002	0.0312	0.02355	0.00597
05/31/2002	0.032	0.25329	0.05913
02/28/2002	0.0342	0.30920	0.08456
11/30/2001	0.022	0.17863	0.04727
08/31/2001	0.0297	0.24628	0.06190

Chapter 5

On some other performance measurements and discussion about their applicability to M&A.

The following performance measures are considered to be difficult to apply for mergers and acquisitions at least if real quarterly returns are used instead of stock returns.

The reason is that almost all performance measures from this list are based on some parameters from the Capital Asset Pricing Model (CAPM) or are based on daily returns and were developed for more frequent data as quarterly returns. Of course, one can estimate these parameters for real company's quarterly returns. However, in this case, one needs to assume that CAPM works not only for stock, but for quarterly returns as well. It is very strong assumption, which should be studied very carefully. That is why, this measures considered to be difficult to apply to mergers and acquisitions.

5.1 Treynor ratio.

One of measurements of the returns earned in excess of that which could have been earned on a riskless investment (per each unit of market risk assumed) is called the Treynor ratio after Treynor (1965). Unlike Sharpe ratio, Treynor ratio uses systematic risk instead of total risk. The higher the Treynor ratio, the better the performance under analysis.

$$\text{Treynor ratio} = \frac{(R_p) - R_f}{\beta_p},$$

where R_p is a return on a portfolio, R_f is a risk free rate of return and β is a portfolio beta from the Capital Asset Pricing Model (CAPM).

Only in case, when the portfolios under consideration are sub-portfolios of a broader, fully diversified portfolio, ranking system based on the Treynor ratio is useful. Otherwise, portfolios with identical systematic risk, but different total risk, will be rated the same. However, the portfolio with a higher total risk is less diversified and therefore has a higher unsystematic risk, which is not priced in the market.

5.2 Jensen's Alpha Measure.

A measure, that is used to determine the excess return of a portfolio of securities over the portfolio's theoretical expected return, is Jensen's alpha after Jensen (1968). The portfolio's theoretical return is predicted by a market model, most commonly the Capital Asset Pricing Model (CAPM).

$$\text{Jensen's Alpha} = \alpha_p = R_p - (R_f + \beta_p(R_m - R_f)),$$

where R_p is a return on a portfolio, R_f is a risk free rate of return, R_m is a market return and β is a portfolio beta.

The CAPM return is supposed to be 'risk adjusted', which means it takes account of the relative riskiness of the asset. After all, riskier assets will have higher expected returns than less risky assets. If an asset's return is even higher than the risk adjusted return, that asset is said to have "positive alpha" or "excess returns". Investors are constantly seeking investments that have higher alpha.

5.3 Appraisal Ratio.

Treynor and Black (1973) create a new ratio, called Appraisal ratio:

$$\text{Appraisal ratio} = \frac{\alpha_p}{\sigma(e_p)},$$

where α_p is alpha of the portfolio and $\sigma(e_p)$ is its nonsystematic risk.

Note, that nonsystematic risk could, in theory, be eliminated by diversification. Thus, the ratio measures how far one has to depart from perfect diversification to obtain a given level of expected independent return. The higher the ratio, the better the portfolio's performance.

5.4 Modigliani ratio (M^2).

M^2 was developed by Modigliani and Modigliani (1997). It facilitates interpretation of the reward-to-variability measure and equates the volatility of the managed portfolio with the market by creating a hypothetical portfolio (p^*) made up of T-bills and the managed portfolio. If the volatility of the managed portfolio less than the market, leverage is used (borrow money and invest the proceeds in the portfolio) and the hypothetical portfolio is then compared to the market.

Thus, you need to create risk-adjusted portfolio p^* (RAP), which is combination of portfolio p and risk free asset: $p^* = \omega p + (1-\omega)R_f$ in such a way that $\sigma_m = \sigma_{p^*} = \omega\sigma_p$. In this way, return on this portfolio is

$$R_{p^*} = \omega R_p + (1 - \omega)R_f,$$

which should be compared with return on market over the same period of time:

$$M^2 = R_{p^*} - R_m.$$

5.5 Muralidhar ratio (M^3).

Arun Muralighar (2000) argued, that M^2 measure is not correct, since it does not capture correlation risk of the portfolio's returns with the benchmark returns. Thus, the investor may reject a portfolio in favor of another purely on the basis of RAP, but if the correlations are different, the comparison is not fair. He proposed to use M^3 measure to solve this three-dimensional problem: the comparison of returns, standard deviations and correlations.

Let CAP be the correlation-adjusted portfolio and ω_1 , ω_2, $1 - \omega_1 - \omega_2$ be the proportions invested in the manager or mutual fund A, benchmark and riskless asset. The return of a CAP is equal then

$$R_{CAP} = \omega_1 R_A + \omega_2 R_b + (1 - \omega_1 - \omega_2)R_f,$$

where R_A is manager's or mutual fund's return, R_b is a benchmark return, R_f is riskless asset's return.

The optimization problem is to hold such proportion, that the optimal portfolio has 1) the highest CAP's return within the target tracking error, and 2) the same standard deviation as the benchmark. Revoke, that in finance, tracking error is a measure of how closely a portfolio follows the index to which it is benchmarked. It measures the standard deviation of the difference between the portfolio and index returns. Thus, the solution is

$$\omega_1 = \frac{\sigma_b}{\sigma_A} \cdot \sqrt{\frac{1-\rho_{T,b}^2}{1-\rho_{A,b}^2}},$$
$$\omega_2 = \rho_{T,b} - \omega_1 \frac{\sigma_A}{\sigma_b} \rho_{A,b},$$

where $\rho_{T,b} = 1 - \mathrm{TE}(target)^2/2\sigma_b^2$ with TE standing for tracking error and $\rho_{A,b}$ is the correlation between A and benchmark.

Notice, that if the correlations are not important, then $\omega_1 = \sigma_b/\sigma_A$, which is M^2 leverage measure, and $\omega_2 = 0$.

5.6 Scholz ratio.

Let's first describe a single factor model:

$$\widehat{R}_A = \alpha_A + \beta_A \widehat{R}_m + \varepsilon_A,$$
$$\widehat{R}_A = R_A - R_f.$$

where α_A is Jensen's alpha (stock's excess return), β_A (the beta coefficient) is the sensitivity of the asset returns to market returns, ε_A is the component of return due to unexpected firm-specific events (unsystematic or firm specific risk), R_A is the expected return on the capital asset A and R_f is the risk-free rate of interest such as interest arising from government bonds.

In this way, Sharpe ratio can be rewritten according to a single factor model as

$$\text{Sharpe ratio}_A = \frac{\alpha_A + \beta_A \mathrm{E}(\widehat{R}_m)}{\sqrt{\beta_A^2 \cdot s_m^2 + s_{\varepsilon(A)}^2}}$$

where α_A ,β_A ,$s_{\varepsilon(A)}^2$ is asset-specific characteristic, but $\mathrm{E}(\widehat{R}_m)$ and s_m^2 (mean and variance of the market excess return) should be estimated. Sharpe ratio is commonly calculated based on relatively short-term evaluation periods of 3-5 years. Long-data often do not exist, especially for new assets or funds and additional these characteristics can change in the long run.

Henrdrik Scholz (2006) proposed to eliminate the random market climate impact on the Sharpe ratio by using much longer evaluation periods of 20 years, but only for market parameters to avoid market climate bias. Thus, in combination with asset-specific characteristics (α_A ,β_A and $s^2_{\varepsilon(A)}$) it gives the normalized Sharpe ratio of an asset for an 'average' market period and therefore, it is not affected by random market climate.

5.7 Sterling Ratio.

Sterling ratio as described for example by Kestner (1996) is

$$\text{Sterling ratio} = \frac{R_A}{(MDD_A^{year1} + MDD_A^{year2} + MDD_A^{year3})/3 + 10\%},$$

where R_A is average annual return over the past three years and MDD (maximum drawdown) up to time T is the maximum of the drawdowns over the history of the variable, i.e.

$$MDD_A^{(0,T)} = \max_{\tau \in (0,T)} \left[\max_{t \in (0,\tau)} (A(t) - A(\tau)) \right].$$

A higher Sterling ratio is generally better because it means that the investments are receiving a higher return relative to risk.

5.8 Calmar ratio.

Calmar Ratio is a performance measurement used to evaluate Commodity Trading Advisors and hedge funds. It was created by Young (1991). Young owned California Managed Accounts, a firm in Santa Ynez, California, which managed client funds and published the newsletter CMA Reports. The name of his ratio "Calmar" is an acronym of his company's name and its newsletter: CALifornia Managed Accounts Reports.

The Calmar ratio uses a slightly modified Sterling ratio and it is calculated on a monthly basis, instead of the Sterling ratio's yearly basis.

$$\text{Calmar ratio}_A = \frac{R_A^{(0,T)}}{MDD_A^{(0,T)}},$$

where R_A is a return of security A over time T (T=12 in case of monthly returns) and MDD (maximum drawdown) up to time T is the maximum of the drawdowns over the history of the variable (security A), i.e.

$$MDD_A^{(0,T)} = \max_{\tau \in (0,T)} \left[\max_{t \in (0,\tau)} (A(t) - A(\tau)) \right].$$

Young states that "the Calmar ratio changes gradually and serves to smooth out the overachievement and underachievement periods of a CTA's performance more readily than either the Sterling or Sharpe ratios."

5.9 Burke Ratio.

Burke (1994) introduces new modification of Sharpe ratio, which "penalizes a trader's upside variability (desirable) along with downside variability (the dreaded drawdown)." Remember, that drawdown at any time τ for security A is defined as

$$DD_A(\tau) = \max(0, \max_{t \in (0,\tau)} (A(t) - A(\tau))).$$

So, he proposed to use the square root of the sum of the squares of each monthly percentage drawdown, i.e.

$$\text{Burke ratio}_A = \frac{R_A - R_f}{\sqrt{\sum_{\tau=0}^{T} DD_A(\tau)}},$$

where R_A is return on security A, R_f is riskless return and T=12 in case of monthly returns.

5.10 VaR.

VaR is a measure of potential loss from an unlikely, adverse event in a everyday normal market environment (see Duffie and Pan (1997)). VaR is denominated in units of a currency, e.g., dollars. Given some confidence level $\alpha \in (0,1)$ the VaR of the security A at the confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is not larger than $(1-\alpha)$:

$$\mathrm{VaR}_A^{\alpha} = \inf \{l \in R : P(L_A > 1) \leq (1-\alpha)\}$$

or in case of normally distributed returns

$$\mathrm{VaR}_A^{\alpha} = -(R_A + z_{\alpha} \cdot \sigma_A)$$

where R_A is return on security A, σ_A is its standard deviation and z_{α} denotes the α-quantile of the standard normal distribution.

For example, we are 99% certain that we will not lose more then 1 million dollars in the next 10 days. Thus, 1 million is the 10-days VaR for 99% confidence level. For most financial business with active trading portfolios the standard time horizon is one day and the standard probabilities are 99% or 95%. Value at risk is merely a benchmark for relative judgment, such as the risk of one portfolio relative to another.

However, many securities experience a higher frequency of extreme outcomes, than is predicted by the normal distribution. In this case VaR understates the risk of large losses. Additionally, there is a statistical warning, that the past is not necessarily a guide to the future and historical data yield poor predictions about future outcomes, if the process generating rates of returns changes due to alterations in the underlying economic situation. Thus, under extreme economic conditions (a natural disaster, a currency crisis) historical relationships may fall apart, so instead of making VaR control or VaR reduction the central concern of risk management, it is far more important to worry about what happens when losses exceed VaR.

5.11 Excess return on VaR.

One of the first measures based on VaR is Excess return on VaR (see Dowd (2000)):

$$\text{Excess return on VaR}_A = \frac{R_A - R_f}{\text{VaR}_A},$$

where R_A is return on security A and R_f is riskless return. So, it is basically a Sharpe Ratio using Value-At-Risk instead of volatility as the risk measure.

5.12 Conditional Sharpe ratio.

One can define Conditional VaR of a security A for a given probability $(1-\alpha)$ as

$$\text{CVaR}_A = \text{E}(-R_A \mid R_A \leq \text{VaR}_A).$$

While the VaR focuses only on the frequency of extreme events, CVaR focuses on both the frequency and size of losses in case of extreme events. Another advantage of Conditional VaR is that it satisfies certain plausible axioms (see Artzner et al., 1999). Thus, according to Agarwal and Naik (2004), Conditional Sharpe ratio is defined as

$$\text{Conditional Sharpe ratio}_A = \frac{R_A - R_f}{\text{CVaR}_A},$$

where R_A is return on security A and R_f is riskless return.

So, "since hedge funds exhibit significant left-tail risk using the traditional mean-variance framework substantially underestimates the tail losses and this underestimation is most severe for portfolios with low volatility" (see Agarwal and Naik (2004)).

5.13 Modified Sharpe ratio.

Another risk measure to use in hedge fund performance measurement, which is based on VaR was developed by Gregoriou and Gueyie (2003). They used Cornish-Fisher expansion to include skewdness and kurtosis in computing Modified VaR, which was developed by Favre and Galeano (2002). So, Modified VaR is calculated as

$$\mathrm{MVaR}_A = -\left[R_A + \sigma_A\left(z_\alpha + (z_\alpha^2 - 1)\frac{S_A}{6} + (z_\alpha^3 - 3z_\alpha)\frac{K_A}{24} - (2z_\alpha^3 - 5z_\alpha)\frac{S_A^2}{36}\right)\right]$$

where R_A is return on security A, σ_A is standard deviation, z_α denotes the α-quantile of the standard normal distribution, S_A is skewness and K_A is excess kurtosis for security A.

In this way the Modified Sharpe ratio is

$$\text{Modified Sharpe ratio}_A = \frac{R_A - R_f}{\mathrm{MVaR}_A},$$

where R_f is riskless return.

Thus, using not only the first two moments of a distribution, namely mean and standard deviation, but also taking in consideration the third and the forth moments of a distribution, skewness and kurtosis, allows investors to obtain a more accurate picture without any bias.

Chapter 6

Summary and conclusions.

The thesis was devoted to application of performance measures to analysis of mergers and acquisitions. Performance measures usually take into account profitability and risk and are applied to stock valuation and portfolio theory. However, we can think about two companies as about two stocks, which together form a portfolio (a combined company) and we can use these performance measures to evaluate the effectiveness of merger of these companies. In other word using performance measures we can say if two companies together create a synergy effect.

Most performance measures involve company's returns. There are two main approaches to measuring them: stock's returns and real quarterly returns from financial statements. It was shown that only quarterly return is a reliable source of information about company's performance. After a well supported discussion of different kinds of returns, two quantities were chosen and used in this thesis: "Operating net cash over Revenue" and "Operating net cash over Total Assets".

Twenty major performance measures were described. Only seven of them can be calculated using company quarterly returns derived from financial statements. Other thirteen performance measures are difficult to calculate because they are either based on some parameters from the Capital Asset Pricing Model (CAPM) or daily returns, i.e. they were developed for more frequent data. It is very unclear if CAPM works for company's quarterly returns as well as for stock's returns and this should be studied carefully the future work. That is why these thirteen performance measures were deemed "inapplicable" and were described in the last chapter.

To deal with quarterly data annualization method for all applicable ratios was developed. This allows to calculate annual ratios from quarterly returns and annual risk free rates. Three methods were described and an example was created to demonstrate how they work for Sharpe ratio calculation. All three methods resulted in identical ranking; however, it is not clear if different methods of annualization always lead to the same results. That is why one method, arithmetic mean excess return, was chosen as the simplest one and was used in all other examples and case studies in this thesis.

Several examples were created to demonstrate advantages and disadvantages of annualized applicable performance measures. In every example we consider three companies: AAA (acquirer), BBB(target) and CCC(target). Manager from company AAA needs to choose a target for acquisition (BBB or CCC). Using quarterly returns of all three companies he can calculate a performance measure of combined companies (AAA+BBB and AAA+CCC). In this way the manager would choose BBB as a target for merger if combined company AAA+BBB has bigger performance measure than AAA+CCC. Additionally, if a performance measure of combined companies (AAA+BBB and AAA+CCC) is less than a performance measure of company AAA, then it is not worth to acquire neither BBB nor CCC.

The first choice of performance measure which could be applied to M&A is well-known Sharpe ratio. Sharpe ratio or reward-to-variability ratio is

$$\text{Sharpe ratio} = \frac{\mathrm{E}(R) - R_f}{\sigma},$$

here E(R) is an average return of a company, R_f is a risk free rate of return and σ is a standard deviation of a company's returns. However, different ratios capture different features of company's and market behavior. Thus, there exist situations when Sharpe ratio gives misleading information forcing to choose wrong target for merger.

It was found, that if company's quarterly returns are negatively skewed then the Sharpe ratio does not work properly. Negatively skewed data usually means that the company tends to have big losses making it a bad target for acquisition. The Sharpe ratio doesn't account for this effect, but Sortino ratio, Omega ratio and Kappa_3 ratio give the right result. All three ratios provide the same ranking for companies that tend to have big losses.

If company's quarterly returns are positively skewed and leptokurtic, the Sharpe ratio again does not work properly. In this case, the company tends to have big gains making it a good target for acquisition. The Sharpe ratio doesn't capture this, but the Upside potential ratio does. If company's quarterly returns are negatively skewed (the company tends to have big losses), the Upside potential ratio captures that effect as well and gives correct results. The formula for Upside potential ratio is

$$\text{Upside potential ratio} = \frac{\frac{1}{T}\sum_{t=1}^{T}\max(0, R - L)}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\max(0, L - R)^2}},$$

where R is a return of a company and L is a minimal acceptable return or threshold and T is a number of returns. Thus, the Upside potential ratio is the only one among the ratios studies in this work that takes into account not only downside risk, but also the upside potential of a company.

Finally, it was found that in case of a bear market when company's quarterly returns are lower than risk free rate, the Sharpe ratio gives wrong rating simply because company's expected excess return becomes negative. Two ratios that can fix this problem are Israelsen ratio and Ferruz-Sarto ratio. However, the disadvantage of Ferruz-Sarto ratio is that it gives wrong answers for negative returns. The formula for Israelsen ratio is

$$\text{Israelsen ratio} = \begin{cases} \left[\mathrm{E}(R) - R_f\right]/\sigma & \text{if } \mathrm{E}(R) - R_f > 0, \\ \left[\mathrm{E}(R) - R_f\right] \cdot \sigma & \text{if } \mathrm{E}(R) - R_f < 0, \end{cases}$$

here E(R) is an average return of a company, R_f is a risk free rate of return and σ is a standard deviation of a company's returns. Thus, the best performance measure, which lets one make the right decision about the target in case of bear market, is Israelsen ratio.

Three real-world examples of recent acquisitions in Canada and USA were given: Aur Resources (target) and Teck Cominco (acquirer), Emergis (target) and Telus (acquirer), Cognos (target) and International Business Machines (acquirer). For all three cases all applicable performance measures were calculated using the technique developed in this thesis.

In all three cases according to all calculated ratios the combined company is more profitable and less risky then acquirer. This means that acquirer made the right decision choosing the particular target for acquisition. On the other hand we found that the combined company is less profitable and more risky than the target in two cases. Thus, the acquisition from target point of view is not leading to any profit; however, target company gets a big cash prize from acquirer. Finally, in the last real-world example two companies together create positive synergy since according to all calculated ratios acquisition was profitable for both companies.

Appendix A

Sharpe ratio with different annualization methods.

Example 4 (**Sharpe ratio with the first annualization method.**). Company AAA, a \$4 billion firm located in Edmonton, wants to expand within their industry through merger. Their preliminary scan of candidates yielded two firms. The first, BBB, is a \$1 billion company. The second, CCC, is also a \$1 billion company. In order to prepare a more thorough analysis of the acquisition candidate, AAA collected information about the long-term pattern of returns of all two firms. The quarterly time series of total returns for the last few years for each firm plus the AAA are shown in Table A.1.

To figure out who AAA should pick up as a target for merger we need to understand which combined company AAA+BBB or AAA+CCC is more profitable and less risky. To do so we will calculate expected Sharpe ratio of these firms assuming, that minimal acceptable return is equal to riskless rate of 5% compounded annually.

Solution of Example 4. Let's use arithmetic mean excess return method in this example.

First we need to calculate the historical quarterly mean and standard deviation for each firm:

Table A.1: Quarterly total returns of each company for Example 4.

Quarter	AAA	BBB	CCC
2000.1	0.016557	0.007543	0.059277
2000.2	0.064433	0.02247	0.048597
2000.3	0.018049	0.006852	0.062662
2000.4	0.024344	0.031064	0.057061
2001.1	0.045369	0.011778	0.047062
2001.2	0.022144	0.028809	0.043896
2001.3	0.012475	0.027889	0.043387
2001.4	0.038219	0.025297	0.032208
2002.1	0.031894	0.00739	0.06796
2002.2	0.042062	0.0209	0.034095
2002.3	0.031964	0.016457	0.054668
2002.4	0.053523	0.030041	0.048487
2003.1	0.054002	0.011701	0.064172
2003.2	0.036693	0.029125	0.043641
2003.3	0.040284	0.013279	0.049076
2003.4	0.033045	0.004224	0.061252
2004.1	0.030244	0.026591	0.062164
2004.2	0.041506	0.006842	0.052677
2004.3	0.030039	0.018696	0.070089
2004.4	0.041861	0.020135	0.055931
2005.1	0.057139	0.034415	0.037456
2005.2	0.046625	0.031559	0.047398
2005.3	0.037314	0.027411	0.040012
2005.4	0.026592	0.022173	0.066659
2006.1	0.033078	0.014949	0.043669
2006.2	0.009844	0.013994	0.047655
2006.3	0.03816	0.013616	0.049792
2006.4	0.045486	0.010384	0.05664

	AAA	BBB	CCC
Quarterly mean ($E(\tilde{R})$)	0.036	0.019	0.052
Quarterly SD ($\tilde{\sigma}$)	0.013	0.009	0.010

Second we calculate correlation coefficient ρ among three series, based upon quarterly data:

AAA	1.000		
BBB	0.168	1.000	
CCC	-0.221	-0.484	1.000
	AAA	BBB	CCC

Now let's say AAA mergers with BBB and AAA mergers with CCC. Assuming the weight on AAA is 0.8, the weight on BBB is 0.2 and the weight on CCC is also 0.2, the new expected return of the new firm is

$$\mathrm{E}(\tilde{R}_p) = \omega_A \mathrm{E}(\tilde{R}_A) + \omega_B \mathrm{E}(\tilde{R}_B).$$

Using the formula for standard deviation of a portfolio we calculate the combined standard deviation of a new firm:

$$\tilde{\sigma}_p = \sqrt{\omega_A^2 \tilde{\sigma}_A^2 + \omega_B^2 \tilde{\sigma}_B^2 + 2\rho_{AB}\omega_A\omega_B\tilde{\sigma}_A\tilde{\sigma}_B}.$$

Finally, the expected Sharpe ratio of the new firm is

$$\text{Sharpe ratio}_p^1 = \frac{4 \cdot \mathrm{E}(\tilde{R}_p) - R_f}{\sqrt{4} \cdot \tilde{\sigma}_p}.$$

As a result for combined companies expected quarterly returns, quarterly standard deviations and annual Sharpe ratios, calculated according to the first method of annualization are summarized below:

	AAA+BBB	AAA+CCC
Quarterly return of the new firm	0.032	0.036
Quarterly SD of the new firm	0.011	0.010
Annual Sharpe ratio$_p^1$	3.602	5.102

To understand which merger is more profitable we have use the expected Sharpe ratio calculated above. AAA alone has an expected Sharpe ratio of 3.505. A merger with BBB will increase this ratio to 3.602. A merger with CCC also will increase this ratio to 5.102. Thus both mergers increase the probability of exceeding T-Bills, but if all else are equal merger with CCC is preferable.

Example 5 (**Sharpe ratio with the second annualization method.**). Let's use data from Example 4 (Table A.1), but instead of using arithmetic mean excess return annualization method, we will use geometric mean excess return annualization method.

Solution of Example 5. First we need to calculate quarterly returns of the combined companies AAA+BBB and AAA+CCC (see Table A.2) according to the formula:

$$\tilde{R}_p = \omega_A \tilde{R}_A + \omega_B \tilde{R}_B.$$

Now we need to calculate geometrical mean excess return for every combined company AAA+BBB and AAA+CCC using formula

$$\mathrm{E}(R_p) - R_f = \left(\frac{1}{(1+R_f)} \prod_{t=1}^{T} (1+\tilde{R}_p)^{4/T} \right) - 1.$$

Thus, Sharpe ratio calculated according to the second annualization method is

$$\text{Sharpe ratio}_p^2 = \frac{1}{\sqrt{4}\tilde{\sigma}_p} \left[\left(\frac{1}{(1+R_f)} \prod_{t=1}^{T} (1+\tilde{R}_p)^{4/T} \right) - 1 \right].$$

As a result for combined companies expected quarterly standard deviations (calculated in previous example), annual excess returns and annular Sharpe ratios, calculated according to the second method of annualization, are summarized below:

Table A.2: Quarterly total returns of combined companies for Example 5.

Quarter	AAA	AAA+BBB	AAA+CCC
2000.1	0.016557	0.016557	0.025101
2000.2	0.064433	0.064433	0.061265
2000.3	0.018049	0.018049	0.026971
2000.4	0.024344	0.024344	0.030887
2001.1	0.045369	0.045369	0.045707
2001.2	0.022144	0.022144	0.026494
2001.3	0.012475	0.012475	0.018657
2001.4	0.038219	0.038219	0.037017
2002.1	0.031894	0.031894	0.039108
2002.2	0.042062	0.042062	0.040469
2002.3	0.031964	0.031964	0.036505
2002.4	0.053523	0.053523	0.052516
2003.1	0.054002	0.054002	0.056036
2003.2	0.036693	0.036693	0.038083
2003.3	0.040284	0.040284	0.042043
2003.4	0.033045	0.033045	0.038686
2004.1	0.030244	0.030244	0.036628
2004.2	0.041506	0.041506	0.04374
2004.3	0.030039	0.030039	0.038049
2004.4	0.041861	0.041861	0.044675
2005.1	0.057139	0.057139	0.053202
2005.2	0.046625	0.046625	0.046779
2005.3	0.037314	0.037314	0.037853
2005.4	0.026592	0.026592	0.034606
2006.1	0.033078	0.033078	0.035196
2006.2	0.009844	0.009844	0.017406
2006.3	0.03816	0.03816	0.040486
2006.4	0.045486	0.045486	0.047717

	AAA+BBB	AAA+CCC
Annual geometric mean excess return	0.082	0.110
Quarterly SD of the new firm	0.011	0.010
Annual Sharpe ratio$_p^2$	3.698	5.278

To understand which merger is more profitable we have use the expected Sharpe ratio calculated above. AAA alone has an expected Sharpe ratio of 3.607. A merger with BBB will increase this ratio to 3.698. A merger with CCC also will increase this ratio to 5.278. Thus, both mergers increase the probability of exceeding T-Bills, but if all else equal merger with CCC is preferable.

Example 6 (**Sharpe ratio with the third annualization method.**). Let's use data from Example 4 (see Table A.1), but instead of using arithmetic mean excess return method or geometric mean excess return method, we will use frequency-converted data method of annualization.

Solution of Example 6. First we need to calculate annual returns of each company AAA, BBB and CCC according to the formula:

$$R_A^{\text{year 1}} = (1 + \tilde{R}_A^1)(1 + \tilde{R}_A^2)(1 + \tilde{R}_A^3)(1 + \tilde{R}_A^4) - 1.$$

Please see Table A.3 for resulting annual total returns for each company:

Table A.3: Annual total returns of each company for Example 6.

Year	AAA	BBB	CCC
2000	0.128402	0.069462	0.247708
2001	0.123194	0.097024	0.177178
2002	0.169062	0.076773	0.221221
2003	0.174256	0.059448	0.236483
2004	0.151503	0.074141	0.263402
2005	0.178233	0.120617	0.205439
2006	0.132321	0.053997	0.212862

Second we calculate annual mean and standard deviation of every company according to the formulas:

$$\mathrm{E}(R_A) = \frac{1}{N}\sum_{j=1}^{N}\left(R_A^{\text{year j}}\right),$$

$$\sqrt{\mathrm{var}(R_A)} = \sqrt{\frac{1}{N-1}\sum_{j=1}^{N}\left(R_A^{\text{year j}}\right)^2}.$$

	AAA	BBB	CCC
Annular mean	0.151	0.079	0.223
Annular SD	0.023	0.023	0.029

Now let's say AAA mergers with BBB and AAA mergers with CCC. Assuming the weight on AAA is 0.8, the weight on BBB is 0.2 and the weight on CCC is also 0.2, the new expected return of the new firm is

$$\mathrm{E}(R_p) = \omega_A \mathrm{E}(R_A) + \omega_B \mathrm{E}(R_B).$$

Using the formula for standard deviation of a portfolio we calculate the combined standard deviation of a new firm:

$$\sigma_p = \sqrt{\omega_A^2\sigma_A^2 + \omega_B^2\sigma_B^2 + 2\rho_{AB}\omega_A\omega_B\sigma_A\sigma_B}.$$

Remember, that correlation coefficient ρ we already calculated before.

Thus, the Sharpe ratio of combined companies AAA+BBB and AAA+CCC, calucated according to the third annualization method is

$$\text{Sharpe ratio}_p^3 = \frac{\mathrm{E}(R_p) - R_f}{\sigma_p}.$$

As a result for combined companies expected annual returns, standard deviations and Sharpe ratios, calculated according to the third method of annualization, are summarized below:

	AAA+BBB	AAA+CCC
Annual return of the new firm	0.137	0.165
Annual SD of the new firm	0.020	0.018
Annual Sharpe ratio$_p^3$	4.350	6.343

To understand which merger is more profitable we have use the expected Sharpe ratio calculated above. AAA alone has an expected Sharpe ratio of 4.345. A merger with BBB will increase this ratio to 4.350. A merger with CCC also will increase this ratio to 6.343. Thus, both mergers increase the probability of exceeding T-Bills, but if all else equal merger with CCC is preferable.

Conclusion from Section A.0.1. Please, see summary table below with Sharpe ratios, calculated using different annualization methods in Examples 4-6 for comparison.

	AAA	AAA+BBB	AAA+CCC
Expected Sharpe ratio$_p^1$	3.505	3.602	5.102
Expected Sharpe ratio$_p^2$	3.607	3.698	5.278
Expected Sharpe ratio$_p^3$	4.345	4.350	6.343

As you can see, different annualization methods lead us to the same conclusion, that merger AAA with BBB and AAA with CCC are profitable, but if all else equal, then merger with CCC is preferable. However, it is not clear if different methods of annualization always lead to the same result. That is why it is better to choose one method and use it for different ratios. In all following examples the first method, arithmetic mean excess return method, was used.

Bibliography

[1] Agarwal, V. and Naik, N.Y. (2004) 'Risk and portfolio decisions involving hedge funds', *Review of Financial Studies*, 17 (1), pp. 6398.

[2] Appleyard, A. R. (1980) 'Takeovers: accounting policy, financial policy and the case against accounting measures of performance', *Journal of Business Finance and Accounting*, 7(4), pp. 541554.

[3] Artzner, P., Delbaen, F., Eber, J.-M. and Heath, D. (1999) 'Coherent measures of risk', *Mathematical Finance*, 9 (3), pp. 203228.

[4] Bernardo, A.E. and Ledoit, O. (2000) 'Gain, Loss and Asset Pricing', *The Journal of Political Economy*, 108(1), pp. 144-173.

[5] Bowen, R., Burgstahler, D. and Daley, L. (1986) 'Evidence on the relationship between earnings and various measures of cash flow', *The Accounting Review*, 61(4), pp. 713725.

[6] Burke, G. (1994) 'A Sharper Sharpe ratio', *Futures*, Vol. 23(3), p. 56.

[7] Burt, S. and Limmack, R. (2003) 'The operating performance of companies involved in acquisitions in the U.K. retailing sector, 1977-1992', *Advances in Mergers and Acquisitions*, 2, pp. 147-176.

[8] Capron, L. (2008) 'The private M&A.: does the 'private firm' discount exist?, *Chief Executive (U.S.)*, 236(Sept-Oct), pp. 62-67.

[9] Clark, K. and Ofek, E. (1994) 'Mergers as a means of restructuring distressed firms: an empirical investigation', *Journal of Financial and Quantitative Analysis*, 29(4), pp. 541565.

[10] Cosh, A., Hughes, A. and Singh, H. (1980) 'The causes and effects of takeovers in the U.K.: An empirical investigation for the late 1960s at the micro-economic level', in: D. C. Mueller (Ed.), *Determinants and Effects of Mergers*, Cambridge, Mass.: Oelgeschlager, Gunn and Hain.

[11] Dowd, K. (2000) 'Adjusting for risk: An improved Sharpe ratio', *International Review of Economics and Finance*, 9(3), pp. 209-222.

[12] Dreyfus, P (1988) 'Go with the (cash) flow', *Institutional Investor*, August, pp. 129-132.

[13] Duffie, D. and Pan, J. (1997) 'An Overview of Value at Risk', *The Journal of derivatives*, 4(3).

[14] Erickson, M. and Wang, S-W. (1999) 'Earnings management by acquiring firms in stock for stock mergers', *Journal of Accounting and Economics*, 27(2), pp. 149176.

[15] Favre, L. and Galeano, J.A. (2002) 'Mean-Modifies Value-at-Risk with Hendge Funds', T*he Journal of Alternative Investments*, 5(2), pp. 21-25.

[16] Ferruz, L. and Sarto, J.L. (2004) 'An Analysis of Spanish Investment Fund Performance: Some Consideration Concerning Sharpe's Ratio', *Omega-The International Journal of Management Science*, 32, pp. 273-284.

[17] Goodwin, T.H. (1998) 'The Information Ratio', *Financial Analyst Journal*, 54(4), pp. 34-43

[18] Gregoriou, G.N., Gueyie, J.-P. (2003) 'Risk-adjusted performance of funds of hedge funds using a modified Sharpe ratio', *Journal of Wealth Management*, 6 (Winter), pp. 7783.

[19] Grinblatt, M. and Titman, S. (1989) 'Portfolio Performance Evaluation', *The Review of Financial Studies*, 2(3), pp. 393-421.

[20] Harford, J. (1999) 'Corporate cash reserves and acquisitions', *Journal of Finance*, 54(4), pp. 19691997.

[21] Harlow, W. Van (1991) 'Asset Allocation in a Downside Risk Framework', *Financial Analysts Journal*, 47(5).

[22] Healy, P. M., Palepu, K. G. and Ruback, R. S. (1992). 'Does corporate performance improve after mergers?', *Journal of Financial Economics*, 31(2), pp. 135175.

[23] oll, P. and Pickering, J. F. (1988) 'Determinants and effects of actual, abandoned and contested mergers', *Managerial and Decision Economics*, 9(1), pp. 119.

[24] Israelsen, C.L. (2003) 'Sharpening the Sharpe Ratio', *Financial Planning*, 33(1), pp. 49-51

[25] Israelsen, C.L. (2005) 'A Refinement to the Sharpe Ratio and Informational Ratio', *Journal of Asset Management*, 5(6), pp. 423-427

[26] Jensen, Michael. (May, 1968) 'The Performance of Mutual Funds in the Period 1945-1964,' *Journal of Finance*. Reprinted in *Investment Management: Some Readings*, J. Lorie and R. Brealey, Editors (Praeger Publishers, 1972).

[27] Kaplan, P.D., Knowles, J.A. (2004) 'Kappa: A Generalized Downside Risk-Adjusted Performance Measure', *Morningstar Associates and York Hedge Fund Strategies*, January 2004.

[28] Kazemi, H., Schneeweis, T. and Gupta, R. (2004) 'Omega as performance measure', *Journal of performance measurement*, Spring.

[29] Kestner, L.N.(1996) 'Getting a handle on true performance', *Futures*, 25(1), pp. 4446.

[30] Meeks, G. (1977) 'Disappointing marriage: a study of the gains from mergers', *Cambridge University Press*, Occasional Paper 51.

[31] Modigliani, F. and Modigliani, L. (1997) 'Risk-Adjusted Performance', *Journal of Portfolio Management*, 23(2), pp. 45-54.

[32] Mueller, D. C. (1980) 'The U.S. 196272. In: D. C. Mueller (Ed.)', *Determinants and Effects of Mergers*, Cambridge, Massachusetts, Oelgeschlager, Gunn and Hain, pp. 271298.

[33] Muralidhar, A. (2000) 'Risk-adjusted Performance: The Correlation Correction', *Financial Analysts Journal*, 56(5), p.63.

[34] Perry S.E. and Williams T.H. (1994) 'Earning management preceding management buyout offer', *Journal of Accounting and Economics*, 18, pp. 157-179.

[35] Ravenscraft, D. and Scherer, F. M. (1987) 'Mergers, sell-offs, and economic efficiency', Brookings Institution, Washington.

[36] Scholz, H. (2007) 'Refinements to the Sharpe ratio: Comparing alternatives for bear markets', *Journal of Asset Management*, 7(5), pp 347-357.

[37] Shadwick, W. F., Keating, C. (2002) 'A universal performance measure', *Journal of Performance Measurement*, 6(3), pp. 59-84.

[38] Sharpe, W.F. (1975) 'Adjusting for Risk in Portfolio Performance Measurement', *Journal of Portfolio Management*, 1(2), pp. 29-34.

[39] Singh, A. (1971) 'Takeovers', Cambridge University Press.

[40] Skala, M. (1991) 'U.S. industry's cash flow maintains uptrend', *Chemical Week*, 148(18), pp. 28.

[41] Sortino, F.A. and van der Meer, R. (1991) 'Downside risk', *Journal of Portfolio Management*, 18, pp. 27-31

[42] Sortino, F.A., van der Meer, R. and Plantinga, A. (1999) 'The Dutch triangle', *Journal of Portfolio Management*, 26 (Fall), pp. 5058.

[43] Treynor, J. (1965) 'How to rate management of investment funds', *Harvard Business Review*, 43 (1), pp. 63-75.

[44] Treynor, J. and Black F. (1973) 'How to use security analysis to improve portfolio selection', *Journal of Business*, 46(1).

[45] Young, T. W. (1991) 'Calmar Ratio: A Smoother Tool', *Futures magazine*, 20(1), p. 40.

```
 (\../)
=(';')=
((")("))
```