**ENACTIVE HERMENEUTICS AS AN INTERPRETIVE FRAMEWORK IN THE MATHEMATICS CLASSROOM**

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**Introduction**

Some time ago, as a secondary classroom mathematics teacher, I was fortunate to teach an unconventional math class in which we constructed 14’ Prospector canoes. We would often begin standing around th complex set of diagrams and instructions taped to our shop wall. We would discuss the day’s work, how we would approach it, and make explicit the mathematics underlying the work. The groups of students who took “canoe math,” as we came to call it, would be typically characterized as “low-achievers” in the traditional mathematics classroom, but I was always struck by the depth, however informal, of their mathematical intuitions and understandings. One student I worked with took the class for both grade ten and eleven math credits. He was an intelligent but reluctant student. In our first iteration of the class, the excitement of offering such a novel program was compounded by my own inexperience and lack of qualifications in the wood shop. In short, we all struggled. But one moment with this student stands out in particular. In building our canoes, we had to first stitch together the twenty planks that would form the canoe’s hull. This was a complex exercise in spatial reasoning: first we had to organize the planks in a two-dimensional plane (i.e., on the floor of the shop), stitch them together with wire, then jostle them into a three-dimensional canoe shape. It involved bending and contorting all of these planks in different ways in different places. In one particular class, nine or ten of us were doing just this. This student had not engaged in the class thus far, and he stood off to the side by himself, watching the process. We tried and failed to get the canoe into shape a couple of times and frustration had begun to mount. On our third try to get the canoe into shape, this particular student called out: “It’s not going to work!”

 When I looked over to the student, he was making a motion with his hand as if to signify a curved surface. He was looking at a particular spot of the canoe and making that gesture. Another student asked him what he was talking about and he came over to us, first working in one spot, then another, saying things like, “it has to be like this,” until the canoe seemed to pop into shape. The canoe program ran for five semesters, and I was consistently taken aback by the work we did together. Our best work was marked by a degree of cooperation and conviviality that I had never experienced, in the classroom or otherwise. And while I have returned to that work often in reflection, one event stands out to me in particular each time: the sweeping motion of that student’s hand. To be sure, one could argue that the student simply lacked some of the mathematical vocabulary for describing the understanding he embodied through gesture. But I argue there was something ineffable in that sweeping hand gesture, both in terms of *how* this student understood and what that disclosed of what was possible. Risser (2019) evoked *the silence of the word that speaks along with the spoken*. This student did utter a few words—*it’s got to be like this*—but he was mostly silent, moving his hand through the air over and over again as if testing the sharpness of scythe’s blade with his fingertips. He was silent, but silence is not simply the absence of speech or the limit of understanding (Risser, 2019, p. 2). Rather, it is a disclosure: silence can reveal in both what is said and what is not said. It pervades and sustains speech. It was through this experience that I became interested in the body’s silent expression of mathematical understanding.

 It is this provocation that has led me to my current work. In this essay, I offer *enactive hermeneutics* as a framework for interpreting classroom experience. This approach combines the principles of enactivism, in which cognition is viewed as a complex biological phenomenon, with the ideas of carnal hermeneutics (Kearney & Treanor, 2015), in which the body is understood as both interpretable and interpretive. In the next section, I draw extensively on the work of Varela, Thompson, and Rosch (1991) and Thompson (2007), as well as enactivist work in mathematics education such as Towers and Martin (2015) and Proulx (2015), to highlight the enactive foundations of this framework. In a following section, I propose carnal hermeneutics as a source of insight for an enactive approach. The carnal turn in hermeneutics “transcends the traditional dualism between rational understanding and embodied sensibility” (Kearney & Treanor, 2015, p. 2). One significant feature of an enactive approach is the importance of understanding and interpreting the lived experience of the body. I argue that carnal hermeneutics develops this emphasis on the lived body by merging insights from philosophical hermeneutics with a “reengagement with both our human senses (the medium of lived flesh) and the ‘flesh of the world’” (Kearney & Treanor, 2015, p. 3). A subsequent section describes an enactive hermeneutic reading of an experience I had with students in a secondary calculus classroom, which was part of a larger study on students’ embodied experiences of spatial reasoning. Rather than provide a definitive account of how enactive hermeneutics can be applied in educational research, this section is intended as a provocation, an opening up of the topic to question. I conclude by suggesting some implications for mathematics educators and researchers.

**Enactivism**

 Enactivism is one of a group of theories that push back on cognitivist notions of perception and view cognition as an embodied phenomenon. Perception in cognitivism is representational. That is, perception is a process of representing external stimuli in the mind. Although there are a variety of approaches and currents of thought within the field, enactivism at its most fundamental level consists in two ideas: “(1) perception consists in perceptually guided action and (2) cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided” (Varela et al., 1991, p. 173). Within this framework coalesce several distinct but related notions, including the importance of understanding lived experience, autonomy and emergence, and skillful know-how (Varela et al., 1991; Thompson, 2007; Gallagher, 2017). In this section, I sketch an outline of enactivist thinking and draw out the implications of the ideas listed above for teaching and learning mathematics.

 Enactivism is a theory of embodied cognition, which posits that perception is grounded in the body and experience, a “product of the neural capacities of our brains, the nature of our bodies, our evolution, our environment, and our long social and cultural history” (Lakoff & Nunez, 2000, p. 9). More specifically, rather than focus on the perceiver as the unit of analysis, enactivism takes the “explanatory unit of cognition (perception, action, etc.) [to be] not just the brain, or even two (or more) brains in the case of social cognition, but dynamic relations between organism and environment, or between two or more organisms” (Gallagher, 2017, p. 11). In enactivist thought, perception and action are co-determined and coordinated. Consider a hypothetical example of a mathematics classroom: a student sits at a desk, looking forward to the front of a rectangular classroom, a body alongside other similarly oriented bodies. The students might be focusing on the abstract symbols on the board, say of a quadratic equation. From an enactivist perspective, perception is perceptually guided action. A student is not a detached observer responding to external stimuli on the board, but rather sustains and is sustained by an environment that is a “set of features which can perceptually guide [the student’s] ongoing activities” (Ward et al., 2017, p. 367).

 As Casey (2015) noted, effect a change in the place—say by instead exploring quadratics through land- and place-based learning—and you will elicit a change in the body. This insight that perception is perceptually guided action is also found in Merleau-Ponty’s theory of perception as an embodied phenomenon. “To move one’s body is to aim at the things through it,” he wrote, “or to allow one’s body to respond to their solicitation, which is exerted upon the body without representation” (Merleau-Ponty, 2012, p. 140). Merleau-Ponty thus provides insight into the lived experience of the body, which is a significant feature of an enactive approach. As Thompson (2007) noted, an integral feature of enactivism is “explicat[ing] selfhood and subjectivity from the ground up by accounting for the autonomy proper to living and cognitive beings” (p.14). But enactivism expands on Merleau-Ponty in important ways, in particular, I argue, in a classroom context. In addition to bringing to bear on the lived experience of the body insights gleaned from cognitive science over the last several decades, enactivism also speaks to how understanding might emerge out of the interactions between bodies and the environment.

 In addition to the importance of a phenomenological understanding of lived experience, there are several other features of an enactive approach pertinent to the math classroom. Enactivism is first and foremost a theory grounded in biological processes. In *The Tree of Knowledge* (1987), Maturana and Varela defined *autonomy* a feature of the systems that support biological activity, which entails the capacity for self-determination. An autonomous system “can specify its own laws, what is proper to it,” and a living being’s organization, which is comprised of autonomous systems such as the nervous system, “is such that [its] only product is [itself], with no separation between produce and product” (Maturana & Varela, 1987, pp. 48-49). Interactions between these systems give rise to patterns of behaviours; that is, “patterns that enable action to be perceptually guided” (Varela et al., 1991, p. 173), for example, emerge from these interactions. Through this coupling of the body with other bodies and our environment, we come to experience the world as “a steady flow of skillful activity in response to one’s sense of the situation” (Dreyfus, 2002, p. 378). Gallagher (2017) described this state as one in which “I understand the world in terms of pre-reflective, pragmatic, action-oriented use, rather than in reflective terms of an intellectual or overly cognitive attitude of conceptual contemplation or scientific observation” (p. 49). Enactivism sees phenomenological analysis as a means of better understanding this state of being. It is important to stress that autonomy, emergence, and skillful know-how are not strictly biological phenomena. In fact, that is what makes enactivism such a fecund theoretical space for making sense of the classroom context: the unit of analysis can be chosen to study autonomy and emergence on varied scales.

 The enactivist framework has proven a rich source of insight in mathematics education (e.g., Maheux & Proulx, 2015; Towers & Martin, 2015). The classroom experience I describe in this paper occurred during a study exploring students’ embodied experiences in the math classroom, and in the study, I had adopted an enactivist framework to attend to that phenomenon. Towers and Martin (2015) noted that enactivism “posits that bodily actions (gestures, facial expressions, utterances, etc.) are intimately part of the cognitive process, not simply expressions of an already-generated, brain-based cognition” (pp. 249-250). Additionally, Gallagher suggested that spatial and geometric concepts in mathematics offer opportunities for an enactive approach:

Just as we can understand the movement possibilities of our bodies as tracing out the physical affordances to be found in particular environments, the principles and rules of operations of geometry and mathematics can be understood as cultural affordances that allow us to solve problems, to communicate at abstract levels, to model knowledge, to construct stone walls and skyscrapers, and thereby transform our environments; all of which also loop back into bodily process and reshape our lives and minds. (Gallagher, 2017, p. 212)

A key insight here is that collective understanding does not transpire between vatted brains, but involves the body and the environment, and the aesthetic, hedonic, and affective nature of perceptive action. Enactivism views the body “as an adaptively autonomous and sense-making system,” one in which “cognition and the world are interdependently originated” (Varela et al., 1991/2016, p. xxvi).

 As I noted above, there are other theories of embodied cognition. Gallagher (2018) described the “4E’s” of cognition: embodied, enactive, extended, and ecological (p.8). This list is often extended to include notions of embedded, empathic, and affective cognition. All of these forms of cognition can be interpreted as repudiations of classical cognitivism that in some way give primacy to the relationship between bodies and environments in understanding cognition. And within that list, as Gallagher (2017) noted, there is “no consensus on what weight to give the concept of embodiment” (p. 26). As a researcher in the mathematics classroom, the enactive approach is a compelling framing of how mathematical understanding is developed because of its emphasis on the interactions between an organism and its environment. Rather than view the mind as a computer, in which external stimuli are inputs, cognition in enactivism is a situated and emergent phenomenon. As Proulx (2015) noted of viewing mathematical problem solving through an enactive lens, “the problems that we encounter and the questions that we ask are thus as much a part of us as they are a part of the environment: they emerge from our interaction with it” (p. 159). Enactivism thus views understanding in the mathematics classroom as an emergent phenomenon brought forth in response to a particular place populated by particular bodies.

**Carnal Hermeneutics**

 It is worth returning to the point made by Gallagher (2017), mentioned above, regarding the varying role of the body in theories of embodied cognition: each must speak to the degree to which cognition is an embodied phenomenon. But within that lies a second question: in what ways is the body an interpretive phenomenon? Carnal hermeneutics (Kearney & Treanor, 2015) offers insight into this second question. Hermeneutics is a philosophy of understanding, or as Schwandt (2004) has called it, a *poetics of inquiry*. Clearly our bodies—and understanding our lived experiences of them—are critical to an enactive approach. So, if enactivism already gives primacy to the role of the body in understanding the mind, what need is there for drawing on a *carnal* turn in hermeneutics? Treanor (2015) identified three main reasons for taking the carnal turn: we live (and try to live well) bodily, we live with (and through) the bodies of others, and the body is interpretive (pp. 58-59). Indeed, with respect to this last point, Treanor (2015) identified the need to make “explicit *both* the inescapable nature of our hermeneutic condition *and* its ineluctable carnal character” (p. 59, emphasis in original). I argue that because our bodies are not only sites of interpretation, but are themselves interpretive, an enactive approach can benefit from taking a carnal turn toward describing and interpreting our lived experience.

 In this work, I offer that one means of doing this is through Merleau-Ponty’s notion of *flesh*. Through this, an enactive hermeneutics can attend to both the interpretive and embodied aspects of our condition. Kearney (2015) wrote that our flesh is “the cradle of both perception and the word” (p. 38). But our bodies do not stop at our own physical flesh. Rather, our bodies serve “as a precondition for the existence of all other sensations—visual, acoustic, olfactory, gustatory—which participate in it” (Kearney, 2015, p. 28). Merleau-Ponty (1968) described this as a reversibility of the senses: in touching we are touched, in seeing we are seen. As Moran (2015) noted, this was an insight made by others, including Husserl. But Merleau-Ponty expands on this notion of flesh to include reversibility between ostensibly disparate senses:

There is a circle of the touched and the touching, the touched takes hold of the touching; there is a circle of the visible and the seeing, the seeing is not without visible existence; there is even an inscription of the touching in the visible, of the seeing in the tangible—and the converse (Merleau-Ponty, 1968, p. 143)

Moreover, our understanding and experience of the world is not only mediated by our bodies but sustained by it and the bodies of others. Carnal hermeneutics addresses the “surplus of meaning arising from our carnal embodiment, its role in our experience and understanding, and its engagement with the wider world” (Kearney & Treanor, 2015, p. 1). It recognizes that while we are indeed languaged beings, the linguistic turn in the philosophical hermeneutics of Gadamer and Ricoeur emphasized the “temporality of understanding” at the expense of the “spatiality of the flesh” (Kearney & Treanor, 2015, p. 17). A carnal approach to hermeneutics posits that sensation is interpretation. Kearney (2015) noted that there are three connotations of sense: sensation, as when we smell the spring air or touch the bark of a tree; meaning, as when we make sense of a math problem; and orientation, which refers to “how we orient ourselves in space and time, how we move towards or away from, fore and aft, hither and tither” (p. 16). This is not restricted to how we orient ourselves in three-dimensional, bodily space, but also to the mathematical spaces that are derived from that bodily space. Carnal hermeneutics points to that liminal gap in the very distinction between bodily and mathematical spaces—and “where there is a gap there is a surplus: something more to be understood” (Casey, 2015, p. 73).

 The body has a long tradition in philosophy. As Kearney (2015) noted, Aristotle identified touch in his *De Anima* as a “discriminating sense” (p. 19). To be tactile, for Aristotle, was to be “exposed to otherness across gaps, to negotiate sensitively *between* other embodied beings, to respond to solicitations, to orient oneself” (Kearney, 2015, p. 19, emphasis in original). In short, touch—and carnal hermeneutics argues to include all the other senses as well—is interpretive. Carnal hermeneutics draws on the phenomenology of Merleau-Ponty to make sense of this provocative statement, in particular his notions of flesh, reversibility, and diacritical perception. For Merleau-Ponty, flesh is the “pre-existing, pre-reflective chiasm which allows the mutual insertion of the world between the folds of my body and my body between the folds of the world” (Kearney, 2015, p. 38). In essence, it is that through which we mediate all experience as embodied beings. Flesh is the means by which we bring forth a world, and in doing so “we are constantly prefiguring, refiguring, and configuring our experience” (Kearney, 2015, p. 21). This notion of the flesh evokes the Aristotelian idea of tact as a sensitivity between embodied beings, discussed above. As Kearney (2015) wrote, “In order to make flesh part of the world (*mondanéiser*) one needs to be not just oneself but oneself as another—a self with others” (p. 52). Flesh sustains a quality of reversibility, the idea, described originally in Husserl’s example of the left hand touching the right hand, that in touching we are also touched. Merleau-Ponty took this a step further: reversibility is multitudinous, a property of all the senses—in hearing we are heard, in seeing we are seen. Indeed, Merleau-Ponty argued, language is sustained by this same property: in speaking, we are spoken. Merleau-Ponty claimed reversibility as a “fundamental phenomenon…which sustains both the mute perception and the speech and which manifests itself by an almost carnal existence of the idea, as well as a sublimation of the flesh” (Merleau-Ponty, 1968, p. 155).

 For Merleau-Ponty, “to have a body capable of expressive articulation or action and to have a phonetic system capable of constructing signs, is the same thing” (Merleau-Ponty, cited in Kearney, 2015, p. 43). Rather than resort to correlationism or Cartesian dualism, a diacritical perception acknowledges that language and sense emerge out of tension created by the “encroachment of things on things” Kearney, 2015, p. 43). In a manner that evokes the enactivist perspective discussed in the previous section, a student does not simply make or receive meaning, but rather is enmeshed in meaning as a “diacritical play of visible and invisible, tangible and intangible, an embodied vigilance capable of signaling and resuscitating ‘full being’ (*létre total*) on the basis of a fragment” (Kearney, 2015, p. 44).

 Inherent in an enactivist framework is the idea that cognition is embodied. In this work I present the case here that to be embodied is also to be interpretive. Carnal hermeneutics “returns us to thinking about the lived body but with all the resources of phenomenology and diacritical hermeneutics” (Casey, 2015, p. 59). Visualizing functional relationships in the mathematics classroom, as I discuss in the vignette below, is not necessarily the most obviously embodied act, but I argue that it is indeed carnal all the way down, mediated by flesh. As such, it is also an interpretive phenomenon that occurs between bodies, including students, teachers, and the environment. Kearney (2015) mused that if the patron saint of hermeneutics is Hermes, carnal hermeneutics must also include Argos, the dog of Odysseus who recognized his disguised master with “canine flair” (p. 18). But if we are to accept that perception and language are diacritical, then we must also accept that that primitive savvy is also the purview of Hermes:

The work of Hermes is everywhere—from the inner capillaries of our heart to the nerve endings of our fingers—sounding and coding, ciphering and signifying through skin and flesh. Sometimes this work of mediating conceals itself diaphanously, as Aristotle notes, in which case Hermes proves hermetic. Other times, it serves to transmit between deep and surface messages, translating between inner wounds and outer scars, between secrets and signs, in which case Hermes assumes his hermeneutic task and enlists us in the act of deciphering flesh. (Kearney, 2015, p. 26)

**Visualization in the Mathematics Classroom: An Enactive Hermeneutic Account**

 What is to be gained by drawing enactivism and carnal hermeneutics together? Of course, enactivism is arguably already hermeneutic. Thompson (2007) noted that Varela in fact designated his early enactivist ideas as a “hermeneutic approach” (p. 444). Gallagher, Martinez, and Gastelum (2017) have called for an “enactivist hermeneutics” on grounds that “interpretation is always *in* and *of* a hermeneutical *situation* where ‘situation’ includes not only the horizonal limits of interpretation, but the interpreter him- or herself, understood as an agent, and the affordances (physical, pragmatic, social and cultural) that are relative to that agent” (p. 91, emphasis in original). In an analysis of how we interact with technology, Friis (2020) called enactivism an “organically oriented hermeneutics” that can trace its lineage to hermeneutic philosophers such as Paul Ricouer and Mark Johnson (para. 8). I concur, but an enactivist approach entails understanding lived experience, which is not only biological, but social and cultural as well. Dierckxsens (2018), in discussing Ricouer’s relevance to enactivism in particular, spoke to this when he wrote that enactivism “finds its roots in hermeneutics and phenomenology and explains cognition, in general terms, as the interaction between the lived, *interpreting body* [emphasis added] and the surrounding world” (p. 42). In this section, I approach a classroom experience I had with a group of secondary calculus students from an enactive hermeneutic perspective. This is not meant to be an exhaustive account; rather, my intent is to open up the topic to possibility and question. By drawing on enactivism and carnal hermeneutics, what might we able to say about our embodied experiences in the classroom?

*Study Context*

 The data discussed in this paper were collected as part of a larger study in a grade 12 International Baccalaureate (IB) calculus class at high school in western Canada. The study attended to the following question: *How can we better understand students’ embodied experiences of spatial visualization in the mathematics classroom?* This cohort of participants (N=16), the first in the study, engaged in a series of lessons as part of regular course programming. I designed and taught the lessons, which were oriented around eliciting bodied responses (e.g., gesture) and spatial approaches (e.g., visualization) to topics in mathematics that are often taken up in a non-spatial way in the secondary mathematics classroom. Each lesson utilized spatial visualization exercises to investigate an algebraic reasoning task. For example, the lesson in which these data were generated began with warm-up visualizations focused on conic sections[[1]](#footnote-1), then moved to a paper folding task in which students physically constructed a parabola (Hull, 2013). After investigating the properties of the folded parabola and deriving its equation, participants attempted to perform the construction in Geogebra, a digital geometry environment. As a culminating task, participants completed a final conic visualization.

*Bringing Forth Data*

 Students participated in a set of three visualization activities at the beginning of the lesson and a final visualization task at the end of the lesson. Figure 1 provides a transcript of one of the four conic visualization exercises from the recording of the lesson and an example of the intended imagery constructed in Geogebra by the author.

**Figure 1**

*The Third Visualization Exercise: A Hyperbola*



After each visualization exercise, participants were given the following prompt for written reflection: *To the best of your ability, describe your experience of visualizing.* Several induced themes, which I include in parentheses below, emerged early on from an hermeneutic reading of the reflection transcripts. These included the extent to which participants saw themselves as visual learners (*Identity*), how participants incorporated collaboration, online tools, and physical manipulatives into their visualizations (*Affordances*), the role their bodies played in visualization (*Dis/Embodiment*), and the ephemeral qualities of their mental imagery (*Straying Visualizations*). It is crucial to stress these are not themes in the sense that they generalize about the participants or topics. Approaching the data hermeneutically, “it can be helpful to identify themes in transcript data in order to organize the material and relate statements from participants to emerging interpretations” (Moules et al., 2015, p. 119). Rather than indicating generalizations of this group or similarities between its members, I identified these themes as a preliminary means of divergence, of opening the topic up to question in all its complexity. Freeman (2014) succinctly characterized this kind of work as one in which the “multiple particularities along with their multiple competing meaning frames are reconsidered and examined in light of the human problem under investigation” (p. 829). And, from an enactive perspective, this multiplicity explicitly entails the particulars of each participant situated in their environment.[[2]](#footnote-2)

*Visualization*

 In what follows, I focus on the third and fourth preliminary themes that emerged out of my analysis, which I denote as *Dis/Embodiment* and *Straying Visualizations*, respectively. First, I think it necessary to foreground what I mean by visualization, and more generally, spatial reasoning. Spatial reasoning is a broad set of skills and capabilities for engaging with objects in space. Mix and Cheng (2012), drawing mostly on research in cognitive psychology, list five categories of spatial reasoning: disembedding, spatial visualization, mental rotation, spatial perception, and perspective taking (p. 200). With an eye toward how spatial reasoning is enacted in the mathematics classroom, Davis et al. (2015) argued that spatial reasoning is an emergent phenomenon comprised of “locating, orienting, decomposing/recomposing, shifting dimensions, balancing, diagramming, symmetrizing, navigating, transforming, comparing, scaling, feeling, and visualizing,” (p. 140). Visualization features prominently in these conceptualizations of spatial reasoning and is the focus of the present study, but it, too, is subject to varied definitions. Arcavi (2003) offers what is perhaps the most widely cited definition of spatial visualization in the mathematics education literature, writing that it is “the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings” (p. 217).

 As a former secondary mathematics teacher and mathematics education researcher, these conceptions of spatial and visual thinking resonate with my own experiences working alongside students in the classroom, but I approached visualization in this study in a particular way. From an enactive perspective, visualization is not simply something that takes place in the mind or on paper, but rather is an embodied and sensual experience. Although the mind’s eye is a commonly invoked metaphor for visualization, it is a potentially misleading one. As Thompson (2007) wrote, a “phenomenal mental image is not a phenomenal picture in the mind’s eye, nor indeed is it any kind of static image or depiction” (p. 297). An enactive approach sees metal imagery as imagined perceptual experiences influenced by the way our senses and bodies enfold and involve us in the world. Visualization is “not a disembodied exercise of abstract sensorimotor skill, but rather a genuine sensorimotor act” (Thompson, 2007, p. 299). It is this notion of visualization that I bring to bear on the experiences I had with participants in this study in the mathematics classroom.

*Data Analysis and Interpretation*

 The data I describe and interpret here are from written reflections conducted in class as part of a lesson oriented around making connections between algebraic and geometric thinking. Beginning directly after the lesson, I read through the reflections several times. On a third reading, I annotated a printed copy to identify ideas that provoked me in some way. On a subsequent reading, I began to compile a formal list of topics and themes, as described above. Finally, I transcribed the hand-written reflections and colour-coded relevant text for each of the identified themes, which involved several re-readings. From the first reading, I had been struck by how polarizing the experience had been for the participants. They fell into one of two groups: those who explicitly said visualizing was “easy” and those who said it was “difficult”. But it was not until I had transcribed the reflections and was colour-coding according to my list of themes that a salient difference struck me.

 Consider the following excerpts from participant CR’s reflection:

I tried to visualize a piece of paper in which the circle was on the whole time, and tried to keep everything I had added so far in view and in focus. I also tried to play out how the movement would work in real time (i.e., the ball rolling up and down the right side of the outside of the larger circle) when I did this I found it easier to visualize. [M]any of the visualizations would stray out of focus while listening to the different limitations and instructions and sometimes lead to them straying off of the path they were meant to be on. (Participant CR)

It was a single word that had caught my attention: *straying*. I immediately thought of Ernst Mach’s well-known sketch of his own visual field (for an in-depth discussion, see Thompson, 2007). The image is drawn from Mach’s perspective, sitting in a chair with his right eye closed, and the entire monocular field is in crisp focus: his bookshelf to the left, his left hand resting on his lap, even the left side of his nose. It is a provocative drawing, but as Thompson noted, “readily available phenomenological evidence shows that our visual experience is not like this depiction” (Thompson, 2007, p. 280). In reading the words on this page, for example, notice how your own peripheral view of the page seems to fall away into the background. It *strays*. Again, per Thompson (2007), Mach’s “drawing is thus a representation that abstracts and combines the contents of many attentional phases of visual experience” (pp. 281-282). This kind of depiction arguably is not amenable to “play[ing] out how the movement would work in real time,” as CR reported attempting to do during the visualization exercise in the excerpt above. Mach’s drawing is a static composition, not an image subject to possible perceptual experience. As Gallagher et al. (2017) noted, this sort of geometrization of space “is not true to our embodied experiences of space and … [it is] a kind of de-temporalization of space that falsifies it” (p. 84). But in my work in the mathematics classroom, students often seem to feel their mental images should resemble Mach’s drawing. This provoked a troubling question: if, as I assumed in my theoretical approach to the study, that imagination was an embodied, sensorimotor act, then why did my participants’ reflections seem to depict visualization as a static, depictive, and seemingly disembodied experience?

 In a well-known study of visualization in the secondary mathematics classroom, Presmeg (1986) opened with Francis Galton’s admonition that “an over-readiness to perceive clear mental pictures is antagonistic to the acquirement of highly generalised and abstract thought” (p. 42). Galton, a member of the English aristocracy, observed an ostensible hierarchy in thinking, with the “thinkers” from his peerage engaging in more refined analytical thought (i.e., textual), and the “doers” from stations below his own thinking in pictures (Ferguson, 1994). In fact, the gulf between thinker and doer was not only a function of the mind’s eye, but the role of the body itself: “mechanicians, engineers, and architects,” Galton wrote, “possess the faculty of seeing mental images with remarkable clearness and precision….inventing their machines as they walk, and see[ing] them in height, breadth, and depth as real objects, and…in action” (Galton, cited in Ferguson, 1994, p. 44). Galton was clearly mistaken in subordinating the image to the word, and Presmeg (1986) noted the potential importance of being able to see “clear mental pictures” for engaging in so abstract a subject as mathematics (p. 42).

 However, in my experience as a mathematics educator, it is exactly this idea of “clear mental pictures” that students tend to find difficult and frustrating. In the present study, participants other than CR spoke to their struggle to compile clear images.

When doing these visualizations I struggled to see what was going on. Initially, it was pretty straightforward and simple, however as more layers/instructions were added I struggled to visualize it. (Participant LH)

As time went along and the more we added to the circle, the harder it became to visualize. (Participant BP2)

Keeping the visualized images in my head while adding more and more details was especially hard. (Participant EW)

The visualizations of these participants became increasingly difficult with additional details and instructions, but they were not explicit about what that experience was like for them, like CR was in describing the straying images. With this in mind, I returned to notes I had made on two other participants, AQ and SB. These participants’ reflections had stood out to me in my first reading because they were the only two that made explicit reference to their bodies (e.g., eye movement, gesture). Revisiting their writings with this notion of straying visualization in mind yielded further insight. For example, consider participant AQ’s reflection:

I felt like if my eyes, with my eyes closed, moved around when I was visualizing and if I looked away from a specific spot in my mind, [the visualization] would fade until I brought my attention back to it. i.e., if I was manipulating the right half of the circle with another circle, the left half would fade to black until I looked back at it. (Participant AQ)

Rather than try to fix a set of de-temporalized objects as a static image in the mind’s eye, AQ’s engagement seemed to be explicitly sensorimotor. This imaginary experience is more akin to our perceptual experience. Take Merleau-Ponty’s (2012) example of viewing a neighbour’s house. The front of the house is available to you visually—you see the colourful front door, gables over the upstairs windows, and so on—but the visual scene is not equivalent to a picture of the house in which every aspect is in focus simultaneously. Rather, when you look at the colourful front door, your visual perception of the gabled windows falls away. You also perceive parts of the house you cannot see from your vantage point. This perception is anticipatory and a function of your particular form of embodiment: as you walk around the house, aspects of the house previously unavailable to perceptual experience yield themselves to your visual field and are reconciled with your anticipations of them. This is just what AQ appears to do in visualizing. When AQ moves their eyes, it is to a different *spot in the mind*. Rather than try to see the imagined objects all at once, AQ “close[s] off the landscape and open[s] up the object” (Merleau-Ponty, 2012, p. 70).

 Another participant, SB, described bringing forth the object in a different way, but perhaps to similar effect. Rather than move around the imagined object to different spots in the mind, as AQ had described, SB enacted the object’s movement with their body in external space:

When the instructions were given that required the object to move, I noticed that I used my hands to mimic the movements in the air, both stretching and moving my pointer finger to follow the path of the point. This helped me visualize the transformations and understand the resulting shape. (Participant SB)

SB’s mental imagery is not simply a depiction in the mind, but emerges from her bodied, situated experience. Notably in the context of this discussion, SB does not explicitly mention a straying or fading visualization, but consider how she describes her gestures in the context of the visualization exercise. Her index finger both moves with the point and stretches as the moving circle enlarges to its limit. It is as if her gesture temporarily fixes her visualization and forestalls its transience. In this sense, SB’s bodily movement is perceptive, and from an enactive hermeneutic perspective, interpretive. Merleau-Ponty (2012) described this sort of engagement with the world as an “original intentionality” in which our world does “not exist around us as a system of objects whose synthesis we perform, but rather as an open ensemble of things toward which we project ourselves” (p. 407).

 Perhaps it is in this example that the benefit of drawing together enactivism and carnal hermeneutics is most salient. Consider SB’s experience in light of Kearney’s (2015) notion of an “encroachment of things on things” (p. 43). SB’s visualization emerges from this overlapping of imagination and gesture, just like the geometer who, Merleau-Ponty wrote, “only knows the relations that he is interested in by tracing them out—at least virtually—with his body” (Merleau-Ponty, 2012, p. 406). This hints at the concept of reversibility, discussed above. Indeed, for Merleau-Ponty, no one sense subjugates the others, even in the act of visualizing. Rather, the body’s senses are intertwined, are mediated by flesh: “we must habituate ourselves,” Merleau-Ponty (1968) wrote, “to think that every visible is cut out in the tangible, every tactile being in some manner promised to visibility, and there is encroachment … between the tangible and the visible” (p. 134). And one must also consider the overlapping between bodies, as the instructions borne by my voice and my own visualizations and gestures intertwined with those of the participants. In discussing reversibility, Moran (2015) wrote that the “seer forms part of the visible and is in communication with it” (p. 230). So too with the other senses, which, not surprisingly, is an idea amenable to an enactive approach to interpreting how understanding unfolds in the mathematics classroom.

**Discussion**

 I provide above an interpretation of a classroom experience from an enactive hermeneutic perspective. This approach draws on the principles of enactivism—including the priority given to a full account of the lived experience of the body, the situated and embodied character of our experience of the world, and the complex, emergent nature of how that world is enacted—as well as the ideas of carnal hermeneutics, in which interpretive bodies are seen as enmeshed in a world of sense and meaning. Carnal hermeneutics offers, if not a strict methodology, a set of principles and sensibilities for interpreting phenomena in the mathematics classroom through the theoretical lens of enactivism. The impetus for the overarching study was better understanding the embodied experiences of spatial reasoning in the mathematics classroom from an enactive perspective, but the approach described here was initiated by a *gap*, to paraphrase Treanor (2015). How can I speak to the embodied experiences of my students without confronting the ways in which bodies, including mine, enfold each other and the world in the classroom? And what of the ways the body’s senses imbue each other? The way participant SB reached out with their index finger to touch the world was not confined to any one carnal sense, nor was it strictly languaged. Rather, the student’s perception was diacritical, which entails the “crossing of sensation and language” (Kearney, 2015, p. 42). An enactive hermeneutic approach brings these carnal ideas to bear on understanding classroom experience from an enactivist perspective.

 The kind of classroom experience I discuss in this paper involved a unique set of challenges. Visualization is acknowledged as an important facet of spatial reasoning, yet it is a phenomenon that resists rigid definition. In a commentary in a special issue of ZDM focused on visualization as an epistemological tool, Presmeg (2014) pointed out that the papers collected in the issue generally adhere to the definition of visualization provided by Arcavi (2003), stated above, but she argued that a “finer grain of research” is still needed for teaching and learning visualization in the classroom (p. 155). For example, Presmeg noted that only one of the papers in this relatively recent issue attended to the role of gesture in visualization, despite this being a significant avenue of psychological research. Moreover, as I describe in this paper, visualization is experienced differently by different people, and arguable differently by the same individual in different contexts. Imagery can stray and remerge; it can be chroegraphed with the movements of an eye or a fingertip. An enactive hermeneutic approach to understanding classroom experience can help teachers and researchers better understand how students experience mathematics as embodied and situated.

**Concluding Remarks**

 In this paper, I offer an enactive hermeneutic approach for interpreting students’ experiences of mathematics in the classroom. Enactivism is a theory of embodied cognition in which cognition emerges from interactions between organisms and the environment. Although the body plays a central role in enactive thought, for example through its emphasis on skillful know-how and the importance of understanding how we experience the world in an embodied way, carnal hermeneutics suggests that the body is itself a site of interpretation. The body is both interpretable and interpretive. An enactive hermeneutics can aid researchers and educators in making sense of how mathematical understanding is enacted in the classroom.

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1. These visualization tasks are based on a series of stop motion films created by mathematician J. L. Nicolet and inspired by a similar exercise the author took part in at the 2018 meeting of the Canadian Mathematics Educators Study Group (see Chorney, Sinclair, & Coles, 2018). [↑](#footnote-ref-1)
2. This final point requires elaboration. Working from within an enactive hermeneutic framework, my interest is in interpreting students’ e*mbodied* experiences in the mathematics classroom. To capture this embodiment (e.g., gesture, bodily orientations to the environment, etc.), one of the primary methods of data generation in this study is through video. However, the lessons described in this paper were conducted virtually through Zoom due to Covid-19, which limited what could be captured by video. As such, much of my interpretation is drawn from written reflections and informed by dialogic experiences with participants throughout the lesson. [↑](#footnote-ref-2)