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Tool Wear Function Identification And Tool Replacement/Adjustment Decision Making

by

© Dian Xiaodan Wang

A thesis

submitted to the Faculty of Graduate Studies and Research

in partial fulfilment of the requirements for the degree of

Master of Science

in

Engineering Management

Department of Mechanical Engineering

Edmonton, Alberta

Fall, 1994

ISBN 0-315-95130-3

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Name of author : Dian Xiaodan Wang

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Degree : Master of Science

Year of this degree granted : Fall, 1994

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Date: Oct. 7, 1994

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled:

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submitted by **Dian Xiaodan Wang** in partial fulfilment of the requirements for the degree of Master of Science.

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Date: Oct. 7, 1994

Abstract

This thesis addresses on-line and off-line tool management in a manufacturing environment. The objective is to minimize the unit time production cost considering tool replacement/adjustment cost and the economic quality loss.

A systematic approach is developed for on-line tool management when the prior information on the tool wear trend is not accurate due to working condition changes. Monitoring with the EWMA control chart can detect when the actual tool wear process deviates from the expected tool wear function. A method is proposed to identify new tool wear function under changed working conditions. This method integrates the least square estimation and the prior information available before a new tool is used. With an updated tool wear function available, the optimal tool replacement/adjustment decision can be made on-line to minimize the unit time production cost. Therefore, the optimal decision of tool adjustment and replacement time can be obtained specifically for every new tool, without extensive prior data collection or controlled experiments.

Off-line decision making for tool replacement/adjustment is also studied. Under identical working conditions, With known tool wear function, the optimal tool replacement decision is developed. Under similar but varied working conditions, if on-line tool wear monitoring is not applicable, a robust optimal tool replacement time model is proposed for optimal decision making to reduce the average production cost. Regarding in-process tool adjustment, the optimal decision of adjustment times are analyzed. While an analytical solution is impossible due to the complexity of the optimization model, several heuristic tool adjustment options are discussed and compared with the solutions from the numerical search method.

Acknowledgements

I wish to thank my supervisor, Dr. M. J. Zuo, for his guidance, patience, and help in the preparation of this thesis.

I wish to thank my examiners, Dr. S. M. AbouRizk and Dr. M. Rao, for their time of reading and examining this thesis.

I also wish to express my deep gratitude to my husband, Kent, for his understanding, encouragement, and advice, to my baby son, Andrew, who has shared my pressure and added sunshine to my life.

Contents

List of Tables

List of Figures

Notation

a_0	initial tool setting at the beginning of each replacement (adjustment) cycle
a_t	initial tool setting offset; $a_t = a_0 - T$
b_i	the parameters of $R(t)$
$\bar{b}_i$	the mean value of b_i when b_i is uncertain
C	unit time production cost over replacement cycle
C_a	tool adjustment cost, including mainly production time loss cost which is smaller than that in a tool replacement situation
C_r	replacement cost, including tool cost, set up cost and production time loss cost
J	number of adjustment and replacement times ($J - 1$ adjustments and 1 replacement)
k	cost coefficient in the symmetric Taguchi quality loss function
K	weighting matrix, a diagonal matrix with elements of k_is
k_c	coefficient of control limits, a design variable of EWMA control chart
k_i	weighting factor for parameter $\hat{b}_i$'s deviation from the prior value
K_s	weighting matrix on θ_s in $\hat{\theta}$ solution
K_p	weighting matrix on θ_p in $\hat{\theta}$ solution
L_t	economic quality loss at time t
Q	tool replacement time without adjustment
$R(t)$	average tool wear at time t, a n-th order polynomial model reflecting a normal tool wear pattern
$\hat{R}(t)$	estimated $R(t)$
$R_p(t)$	prior information of $R(t)$
t	the working time of a new tool since installed
T	target value for the part dimension
T_j	duration between consecutive tool adjustments; $j = 1, 2, \ldots J$
U_t	the expected value of X_t; $U_t = a_0 + R(t)$
X_t	measurement of the part at time t, $X_t = U_t + \xi$
X_{Rt}	residual variable $X_{Rt} = X_t - [-a_t + R_p(t)]$
z_t	EWMA variable of X_{Rt}
θ	the parameter vector of $R(t)$. When $R(t) = b_1 t + b_2 t^2 + b_3 t^3$, $\theta = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$
θ_p	the parameter vector of $R_p(t)$
θ_s	the estimation of θ by using the least square method
$\hat{\theta}$	the estimation of θ by using the proposed method
$\Delta\theta_s$	estimation error by using the least square method

$\Delta\theta_p$	the inaccuracy of the prior information
$\Delta\theta$	estimation error by using the proposed identification method
$\overline{\sigma}^2$	the mean value of σ^2 for different processes when b_i is uncertain
σ_i^2	the variance of b_i when b_i is uncertain
σ_{ij}	the covariance of b_i and b_j,($i \neq j$)
λ	forgetting factor, a design variable of EWMA control chart
Φ	time state matrix consisting of sampling times
τ	the working time of a tool since its last adjustment
ξ	random variable with mean zero and standard variation σ $X_t = T - a_o + R(t) + \xi = -a_t + R(t) + \xi$

Chapter 1

Introduction

Facing increasing global competition, manufacturers are pursuing the most effective ways to improve product quality, as well as reduce production cost. Quality related research work is appearing in the literature at a fast rate to address various quality topics in manufacturing. One of the most concerned issues is tool management, since cutting tools play a major role in producing quality products economically in a manufacturing establishment. For example, "tool wear" is inevitable in a machining process. The dimension of the final work piece depends on the tool wear status. To ensure the workpiece quality, the tool has to be adjusted or replaced at certain time intervals. However, the adjustment and replacement costs have to be taken into consideration. The benefits resulting from better tool management include improved product quality, reduced machine down time and above all, lower production cost.

According to Taguchi, the economic quality loss L_t due to the deviation of a part's dimension from its target value is a quadratic function:

$$L_t = \begin{cases} k_{q1}(X_t - T)^2, & X_t < T \\ k_{q2}(X_t - T)^2, & X_t \geq T, \end{cases} \tag{1.1}$$

where X_t is the measurement of the part at time t, T is the target value for the part dimension, k_{q1} and k_{q2} are cost coefficients determined from the requirements of the customers [1, 2]. When $k_{q1} = k_{q2}$, the quality loss function is said to be symmetric.

As a major factor affecting product quality, tool wear is a complex phenomenon, involving a variety of wear mechanism, such as diffusion, abrasion, fatigue, attrition and plastic deformation. The dominant wear mechanism and the rate of wear are subject to temperature, stress, and other factors. Since the underlying physics of tool wear is very complicated, tool wear model $R(t)$ is usually fitted from empirical tool wear data. When the working condition remains the same during a tool life, the tool wear amount usually follows a non-decreasing, non-linear trend, consisting of three distinct periods: initial wear period, normal wear period, and accelerated wear period [3]. The average tool wear can usually be expressed as an m-th order polynomial tool wear function $R(t)$. This tool wear function can be fitted from the tool wear data obtained either from the controlled experiments or from historical tool wear data. While on-line tool wear monitoring using sensors is not widely available in industry, the finished part dimension measurement can be collected easily in the process. The average tool wear trend $R(t)$ can be reflected by the finished part dimension changes.

As an integration of three technical papers, this thesis addresses on-line and off-line tool management, including the optimal tool replacement/adjustment decision making based on the non-linear tool wear function. The objective is to minimize the unit time production cost considering tool replacement/adjustment cost and the economic quality loss.

To make optimal decisions for tool adjustment and replacement, the tool wear function $R(t)$ plays an essential role. The parameters of $R(t)$ will change when working conditions change [3, 4, 5]. To apply an existing tool wear function, the machining process must be under identical working conditions. In a practical situation, especially in a flexible manufacturing environment, working condition variations are very common. With application of group technology, each tool may be responsible for certain operations in which simiiar workpiece material and machining conditions are involved. However, tool property variations and working condition changes still exist. For example, cutting speed and cutting depth could be varied; workpiece material and geometry could be slightly different from batch to batch; tools themselves, could have similar but different properties from one to another; even the tool installations could be different from one operator to another. Under these known or unknown working condition changes, tool wear curves could be different even for a single tool [3, 6]. The parameters of $R(t)$ are uncertain to some degree. Due to working condition changes, the prior information on the tool wear function $R(t)$ obtained from experience may not be accurate for a new tool. The optimal result obtained according to a specific $R(t)$ function can not guarantee the real optimal solution under a changed working condition. However, it is either impossible or uneconomical to obtain the exact $R(t)$ functions under every different working condition.

In Chapter 2, a systematic approach is developed for on-line tool management when the prior information about the tool wear trend is not accurate due to the change of working conditions. On-line monitoring with the Exponential Weighted Moving Average(EWMA) control chart can detect when the actual tool wear process deviates from the expected tool wear function. A new identification method is proposed to identify new tool wear function under the changed working condition. This proposed method integrates the least square estimation and the prior information available before the new tool is used. In particular, the features of this proposed method are analyzed compared to the least square estimation and the prior information. With updated tool wear function available by applying this method, the optimal tool replacement/adjustment decision can be made on-line to minimize the unit time production cost. Therefore, the optimal decision of tool adjustment and replacement time can be obtained specifically for every new tool, without extensive prior data collection or controlled experiments.

Chapter 3 studies off-line decision making for tool replacement. Under identical working conditions, with a known tool wear function, the optimal tool replacement decision has been studied by Drezner and Wesolowsky [7], Jeang and Yang [8], etc. Their research results are reviewed in Chapter 3. This chapter also analyzes the research result claimed by Jean and Yang and shows that the tool wear model $R(t)$ by Jeang and Yang is too general to have a unique optimal solution (a_0^*, Q^*). The sufficient and necessary conditions for the optimal tool replacement decision is developed. On the other hand, under similar but varied working conditions, the parameters of the tool wear function are uncertain. If on-line tool wear function identification is not available, a robust optimal tool replacement model is proposed for optimal off-line decision making to reduce the average production cost.

Chapter 4 discusses the off-line optimal decision making for the in-process tool adjustment. In-process tool adjustment can reduce the dimension deviation of a workpiece from the target value caused by tool wear. With in-process adjustment, the tool could be replaced less frequently with improved workpiece quality. Usually, in-process adjustment can be performed at much less cost than tool replacement. Compared to performing tool replacement only, in-process tool adjustment could save not only the tool cost but also the production down time for the tool installation procedures. In this chapter, the optimal decision of adjustment times is analyzed based on a known tool wear function $R(t)$. An optimization model is developed. While analytical solution is impossible due to the complexity of this model, several heuristic tool adjustment options are discussed and compared with the solution by the numerical search method.

Finally, Chapter 5 provides a summary of this thesis and the suggestions of further research direction in the optimal tool management area.

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Chapter 2

On-Line Tool Management With Tool Wear Function Identification

2.1 Introduction

In a machining process, tool wear is a common phenomenon which affects the final product quality. Tool wear results in part dimension deviations from its target value. According to Taguchi quality loss function, the economic loss L_t can be expressed as:

$$L_t = k(X_t - T)^2 \tag{2.1}$$

where X_t is the measurement of the part at time t, T is the target value for the part dimension, and k is the cost coefficient determined by the requirements of the customers [1]. A good tool management strategy including tool adjustment and replacement is essential to product quality improvement and production cost minimization.

In terms of tool replacement, some research results have reported to minimize the expected unit time cost, based on the replacement cost and economic quality loss

[0] A version of this chapter will be submitted to the International Journal of Production Research

[2, 3, 4]. Further studies along this line will be presented in Chapter 3. However, most of these studies on tool replacement are based on the assumption that the average tool wear function versus time is known for a new tool. The validity of such an assumption in industry is limited.

A tool wear process usually follows a non-decreasing, non-linear trend, consisting of three distinct periods: initial wear period, normal wear period, and accelerated wear period [5]. When the working condition remains the same during a tool life, the average tool wear can usually be expressed as a polynomial function $R(t)$. This tool wear function can be fitted from the tool wear data obtained from either controlled experiment or historical data. While on-line tool wear monitoring using sensors may be few available in industry, finished part dimension measurement can be collected easily in a machining process. The average tool wear trend $R(t)$ can be reflected by the dimension change in the finished parts. The parameters of $R(t)$ will change when working conditions change [5, 6, 7]. The working conditions mentioned here include tool material, cutting speed, temperature, workpiece material and geometry, coolant condition, etc. To apply an existing tool wear function, the machining process must be under identical working conditions. Since the working condition often changes in a practical manufacturing environment, especially in an FMS environment [8, 6], it is either impossible or uneconomical to obtain the exact $R(t)$ functions under different working conditions.

Serval papers have addressed stochastic tool wear process adjustment without the knowledge of tool wear function $R(t)$. Quesenberry developed a two-part compensator to minimize the expected mean squared error of the measurement from the target value [9]. The method emphasizes the quality aspect and ignores the tool adjustment cost and replacement cost. It also assumes that the tool wear process follows a rough linear trend and the adjustments are done at the same time interval. Galante and Lombardo studied tool replacement with adaptive control for a generalized process using an unlimited number of tools [4]. The result would be useful if the tool wear process is a an unpredictable stochastic process. However, when working condition change is not dramatic, tool wear usually follows a predictable non-decreasing trend [3, 5], especially in a mass production environment or in a Flexible Manufacturing System (FMS) with application of group technology. Even though change could happen in the process, the decision of next adjustment time is still related to the previous adjustment time and the number of adjustments. The algorithm provided by Galante and Lombardo will not give the real optimal on-line tool adjustment/replacement strategy in a common tool wear process. Sanjanwala et. al. [10] proposed a feedback system with an on-line pneumatic sensor and an actuating mechanism to move the tool to compensate the tool wear. This method requires extensive control system implementation and initial investment, which may not be widely applicable in industry.

There are four typical situations when a new tool is to be used:

1. The exact tool wear function is available and the working condition for the new tool is well under control. This situation is rare in practice and there are many research results available, including Chapter 3 and 4 in this thesis for the optimal tool replacement/adjustment time determination.

2. The only information available is how the same type tool performed in similar working conditions. The available tool wear function may not suit the upcoming process very well.

3. The exact tool wear function for the upcoming process has been obtained. However, there is a possibility that the working conditions for the process will change unexpectedly.

4. There is no information related to the upcoming tool wear process at all.

This chapter discuss Situation 2 and 3 described above. A systematic approach is proposed for the on-line tool management, including process monitoring, tool wear function identification, and replacement/adjustment decision making.

To make an optimal tool placement/adjustment decision for a working tool, we need to know or estimate the tool wear progress not only in the past, but also in the future. The tool wear history can be obtained by measurement of the finished parts. To predict tool wear in the future, an on-line tool wear function identification method with the prior information is necessary. Whenever the tool wear function has been updated, the new tool adjustment decision can be made on-line to minimize unit production cost. Therefore, the best tool management can always be achieved for every single tool, without extensive prior data collection or controlled experiments. This is the goal of this chapter. Specifically, at first, the tool wear process is monitored with the Exponential Weighted Moving Average (EWMA) control chart. The control chart can tell whether the average tool wear has deviated from the existing tool wear function, i.e., out of control. Whenever the process is out of control, an effective identification algorithm is used to update the tool wear function. This algorithm is based on the prior information on this function and the latest available tool wear measurement data. Then an updated on-line adjustment strategy can be developed correspondingly.

In this chapter, the development of the identification algorithm is discussed with details. It has to be quick to respond to process parameter changes and insensitive to the process noise. Since the tool wear function during a tool life follows a non-linear, non-decreasing trend which can be described as an m-th order polynomial model $R(t)$. In this chapter, we will use the 3rd order model.

2.2 Tool Wear Process Monitoring with EWMA Control Chart

The part dimension measurement data has a non-decreasing trend $R(t)$ along time. Before a new tool is used, assume we have the prior information available for the wear of the new tool, i.e., $R_p(t)$. However, we do not know whether it will represent the average tool wear for the coming process perfectly. That is why the monitoring of tool wear process becomes necessary. During the tool wear process, even though the new tool wear function has been updated by the on-line identification algorithm, we still need the on-line tool wear monitoring to detect whether the actual tool wear process deviates again from what is expected.

Together with Shewart X-bar control chart, the EWMA chart is widely used in statistic process control (SPC) to detect process mean shift from the original value. Although the mean of a tool wear process data is not constant, a simple conversion can make SPC control possible. We know that:

$$X_t = -a_t + R(t) + \xi, \tag{2.2}$$

To verify whether $R_p(t)$ is the mean function of tool wear data, we can examine the residual variable:

$$X_{Rt} = X_t - R_p(t) = R(t) - R_p(t) + \xi \tag{2.3}$$

When the prior information is precise, i.e. $R_p(t) = R(t)$, the residual variable X_{Rt} ξ, with mean zero and stand deviation σ. Otherwise, the mean of residual variable will shift form zero with the trend of $R(t) - R_p(t)$.

The EWMA chart can be used to monitor residual variables. Compared to the Shewart X-bar chart, it performs better in detecting small process shifts [11]. It is also easier to set up and operate than the cumulative-sum control chart [11]. Distinguishing with Shewart control chart, the EWMA is insensitive to the normality assumption. It is therefore an ideal control chart for use with a small number of observations [11], as in tool wear process.

To apply EWMA chart here, we calculate the EWMA variable z_t based on the on-line measurement residuals:

$$z_t = \lambda X_{Rt} + z_{t-1} \tag{2.4}$$

where $0 < \lambda \leq 1$ is a constant and the starting value $z_0 = 0$. The variance of z_t is $\sigma_{z_t}^2 = \sigma^2 \frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2t}]$ [11]. The upper and lower control limits are:

$$\begin{aligned} UCL_t &= k_e \sigma_{z_t} \\ UCL_t &= -k_e \sigma_{z_t} \end{aligned}$$

When $R_p(t) = R(t)$, the process is in control. If $k_e = 3$, z_t will stay inside the control limits with a probability of 97.3% under the normal distribution assumption. When $R_p(t) \neq R(t)$, the process is out of control and z_t will be out of control limits after several observations.

Figures 2.1–2.3 show tool wear process monitoring by EWMA control chart under different situations. We have assumed the prior information $R_p(t) = 55t - 12t^2 + t^3$ on the upcoming process. The estimated average tool wear process is depicted with symbol 'o's. The actual part dimension measurements are shown as '*'s. These actual measurements are simulated with $X_t = R(t) + \xi$, where random variable ξ has a variance of $\sigma^2 = 16$.

- Figure 2.1 shows the case when the prior information $R_p(t)$ exactly represents the tool wear process mean, i.e. $R_p(t) = R(t)$. The EWMA chart indicates that the process is in control, because z_t stays inside the control limits during the whole process.

- Figure 2.2 shows the case when the prior information does not reflect the real process mean when $R(t) = 60t - 12t^2 + t^3$. The EWMA chart indicates that the process is out of control after the first few observation points.

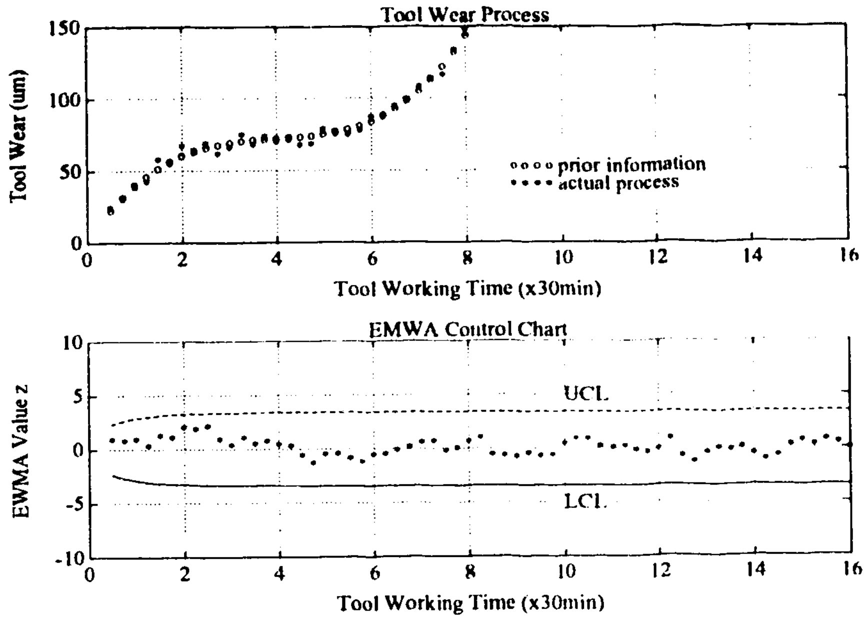

Figure 2.1:Process in control

- Figure 2.3 shows the case when the prior information fits the process very well at the beginning. However, at time $t = 4$, due to working condition changes, $R(t)$ changes to $R(t) = 60t - 12t^2 + t^3$. $R_p(t)$ is no longer precise when $t > 4$. The EWMA chart indicates that process is out of control after several points are observed after $R(t)$ change.

The average run length (ARL) to detect an out of control state is a control chart criterion. It reflects how many observations on average that the control chart will need to detect the process shift. The ARL to detect a tool wear process depends not only on values of λ, k_e, but also on the magnitude of mean shift $R(t) - R_p(t)$. An extensive study of optimal selection of λ and k_e has been reported [12, 13]. The best λ, k_e combination to detect an out of control state as soon as possible is found in these studies assuming that the magnitude of mean shift in the process is constant after the shift happened. These results are not applicable here since $R(t) - R_p(t)$

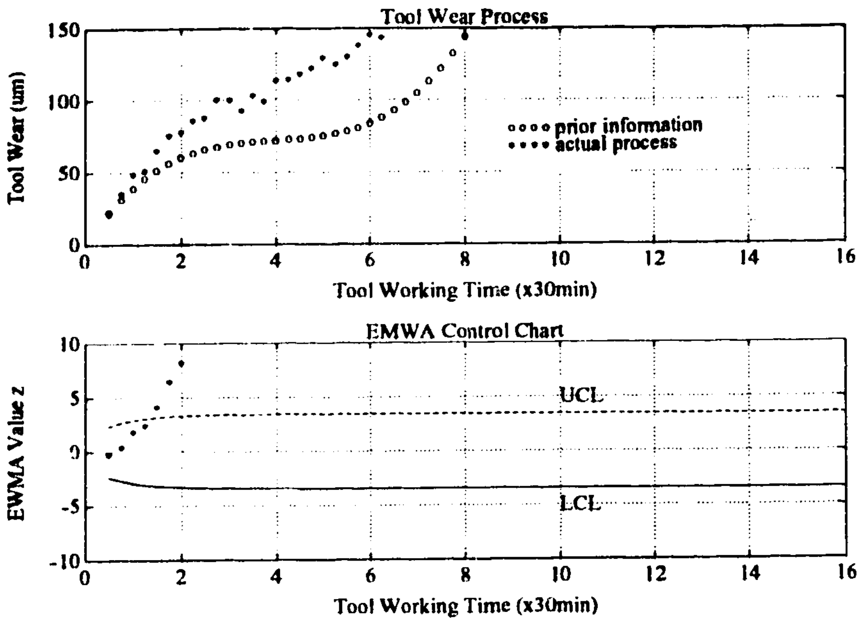

Figure 2.2:Process deviation from the beginning

changes with time. Therefore, the selection of λ and k_e values for the tool wear process monitoring needs further study. We have used $k_e = 3, \lambda = 0.15$ in this chapter, since as a good rule of thumb, $k_\epsilon = 3$ and $0.05 \leq \lambda \leq 0.25$ generally work very well in practice [11].

2.3 Tool Wear Function Identification With The Prior Information

An identification algorithm is critical to ensure effective on-line tool adjustment decision making. Whenever tool wear process monitoring indicates that the process is out of control, tool wear function identification becomes necessary to predict new tool wear trend. The new parameter θ of tool wear function $R(t)$ has to be estimated based on the part measurements and prior information. Since tool

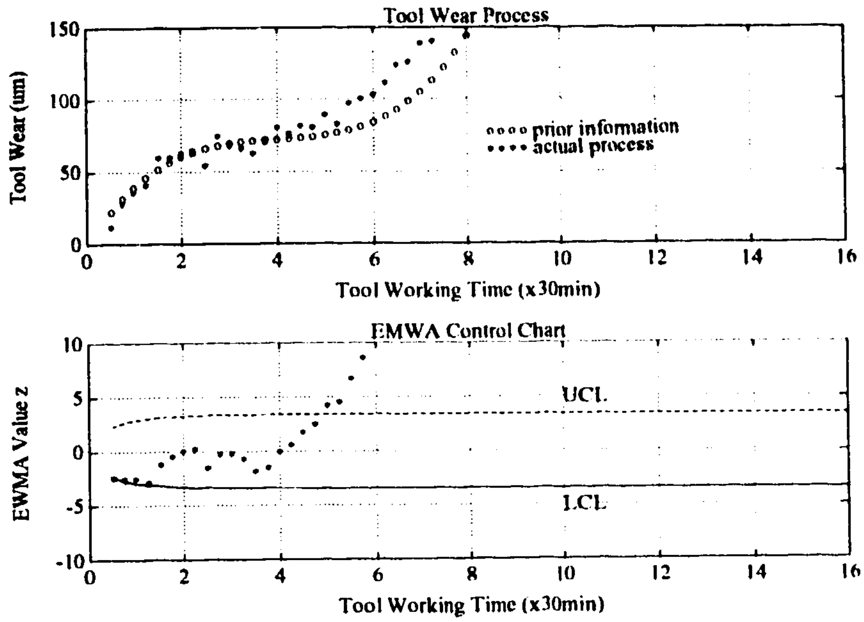

Figure 2.3:Process deviation from $t = 4$

adjustment/replacement decisions will be made on-line based on $\hat{\theta}$, a wide variation of $\hat{\theta}$ will result in a wide decision variation which may confuse the operator, while a slow $\hat{\theta}$ converge speed will not be able to provide correct adjustment/replacement decision in time. The identification algorithm has to be insensitive to the process noise, stable, and converging fast to the new parameters. Then the optimal decision of the next tool adjustment or replacement time can be made in time and effectively.

Based on the measured data, a curve fitting by the least square method can be performed to identify the parameters of a changed process. This method is widely used in the adaptive control of a dynamic process. However, it is not very effective for the tool wear process when noise exists in the process, because the sampling interval can't be sufficiently small due to the fact that the finished part dimension measurements can only be taken part by part. The number of data points X_t is limited. Since process noise exists, the least square method will result in a wide variation of $\hat{\theta}$ during the first several identification steps. It takes a long time for $\hat{\theta}$ to reach a stable state, especially for $\hat{b}_2$ and $\hat{b}_3$. For example, if the tool wear process can be expressed as $R(t) = b_1 t + b_2 t^2 + b_3 t^3$ with noise, where $b_1 = 60, b_2 = -12, b_3 = 1$. Based on the sampling data points, by using the least square method, the parameter estimation $\hat{\theta}$ can be made at every sampling time interval. To evaluate the accuracy of the identification result $\hat{\theta}$, the normalized parameter estimations are plotted in Figure 2.4a, where $\hat{b}_1/b_1, \hat{b}_2/b_2$, and $\hat{b}_3/b_3$ are shown as by solid line, dashed line, and dotted line correspondingly. The closer the normalized value $\hat{b}_i/b_i$s to 1, the better the estimated $\hat{b}_i$s. We can see that the estimated parameters change dramatically except $\hat{b1}$. They require long time to become stable. Obviously, the least square method is not very effective for on-line identification of the tool wear process.

In this section, a new identification algorithm is developed which outperforms the least square method considerably. The new algorithm is based on the combination of the least square method and the prior information of the process. It assumes that the prior information is close to the true tool wear function rather than dramatically different. It aims at making the best estimation of the parameters of the tool wear function $R(t)$, in terms of minimizing not only the sum of the squared errors between the part measurements and the estimations, but also the difference between the estimated $\hat{\theta}$ and prior information. The mathematical descriptions follow.

2.3.1 The model of tool wear function

The parameters of tool wear function $R(t)$ are related to the working condition. Assume that under working condition 1, the average tool wear is $R_p(t) = b_{p1}t + b_{p1}t^2 + b_{p1}t^3$, and under working condition 2, $R(t) = b_1 t + b_2 t^2 + b_3 t^3$. If working condition changes from 1 to 2 during the tool life at time t_{mc}, the actual tool wear amount trend will smoothly changes from $R_1(t)$ to $R_2(t)$:

$$
\begin{aligned}
R_a(t) &= \begin{cases} R_p(t) & \text{when } t < t_{mc} \\ R(t) - R(t_{mc}) + R_p(t_{mc}) & \text{when } t \geq t_{mc} \end{cases} \\
&= \begin{cases} b_{p1}t + b_{p2}t^2 + b_{p3}t^3 & \text{when } t < t_{mc} \\ b_1(t - t_{mc}) + b_2(t^2 - t_{mc}^2) + b_3(t^3 - t_{mc}^3) + R_{pmc} & \text{when } t \geq t_{mc} \end{cases}
\end{aligned}
$$

where $R_{pmc} = R_p(t_{mc})$

We know the parameters of $R_p(t)$ from the prior information before applying a new tool. The prior information is accurate only before time t_{mc}. When $t_{mc} = 0$, the actual tool wear trend $R_a(t) = R(t)$ does not have any adjoint part with the prior information $R_p(t)$. When $t_{mc} > 0$, the actual tool wear trend $R_a(t)$ starts to deviate from the prior information at time t_{mc}.

The EWMA control chart discussed in Section 2.2 can be used to detect when the prior information is not accurate, i.e. t_{mc}. Then the identification algorithm will identify the parameters of the new trend, i.e. $R(t)$.

2.3.2 Objective function for the identification algorithm

The finished part dimension measurement and estimation can be expressed as:

$$
\begin{aligned}
X_t &= T - a_0 + R_a(t) + \xi \\
\hat{X}_t &= T - a_0 + \hat{R}_a(t) \\
&= T - a_0 + \hat{R}(t) - \hat{R}(t_{mc}) + R_{pmc} \text{ when } \quad t \geq t_{mc}
\end{aligned}
$$

Based on the on-line part dimension measurement and the prior information available, the proposed algorithm aims at minimizing the sum of two terms, OBJ_1 and OBJ_2:

- OBJ_1—the sum of square errors between the part measurement X_t and estimated value $\hat{X}_t$.

- OBJ_2—the square error between the prior information parameters θ_p and $\hat{\theta}$.

At the current time t_{mi}, (when $t_{mi} \geq t_{mc}$), the objective function is:

$$\begin{aligned} OBJ &= OBJ_1 + OBJ_2 = \Sigma_{i=mc}^{mi}(X_{ti} - \hat{X}_{ti})^2 + \Sigma_{i=1}^{3} k_i(\theta_p - \hat{\theta})^2 \\ &= \Sigma_{i=mc}^{mi}\{X_{ti} - [T - a_0 + R_{pmc} + \hat{b}_1(t_i - t_{mc}) + \hat{b}_2(t_i^2 - t_{mc}^2) + \hat{b}_3(t_i^3 - t_{mc}^3)]\}^2 \\ &\quad + \Sigma_{i=1}^{3} k_i(\hat{b}_i - b_{pi})^2 \end{aligned}$$

Or, equivalently,

$$OBJ = OBJ_1 + OBJ_2 = (Y - \Phi\hat{\theta})^T(Y - \Phi\hat{\theta}) + (\hat{\theta} - \theta_p)^T K(\hat{\theta} - \theta_p) \tag{2.5}$$

Where,

$$Y = \Phi\theta + \xi \tag{2.6}$$

$$Y = \begin{bmatrix} X_{t_{mc}} - (T - a_0 + R_{pmc}) \\ X_{t_{mc+1}} - (T - a_0 + R_{pmc}) \\ \vdots \\ X_{t_{mi}} - (T - a_0 + R_{pmc}) \end{bmatrix}, \quad \xi = \begin{bmatrix} \xi_{mc} \\ \xi_{mc+1} \\ \vdots \\ \xi_{mi} \end{bmatrix}, \tag{2.7}$$

$$\Phi = \begin{bmatrix} t_i - t_{mc} & t_i^2 - t_{mc}^2 & t_i^3 - t_{mc}^3 \\ t_i - t_{mc+1} & t_i^2 - t_{mc+1}^2 & t_i^3 - t_{mc+1}^3 \\ \vdots & \vdots & \vdots \\ t_i - t_{mi} & t_i^2 - t_{mi}^2 & t_i^3 - t_{mi}^3 \end{bmatrix} \tag{2.8}$$

$$\theta = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \quad \hat{\theta} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix}, \quad \theta_p = \begin{bmatrix} b_{p1} \\ b_{p2} \\ b_{p3} \end{bmatrix}, \tag{2.9}$$

$$K = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}. \tag{2.10}$$

The best estimation of parameter vector $\hat{\theta}$ can be obtained by minimizing objective function (2.5). Note that when $K = 0$, (i.e. $k_1 = k_2 = k_3 = 0$), OBJ is equivalent to the objective function of the least square method.

2.3.3 Analytical solution of the best estimation

Given objective function (2.5), the necessary condition of the optimal solution is that the first derivative function equals to zero:

$$\begin{aligned}
\frac{\partial(OBJ)}{\partial\hat{\theta}} &= [-\Phi^T(Y-\Phi\theta)]^T - (Y-\Phi\theta)^T\Phi + [K(\hat{\theta}-\theta_p)]^T + (\hat{\theta}-\theta_p)K^T \\
&= 2[-\Phi^T(Y-\Phi^T\theta)]^T + 2[K(\hat{\theta}-\theta_p)]^T = 0 \\
&\Rightarrow -\Phi^T Y + \Phi^T\Phi\hat{\theta} + K\hat{\theta} - K\theta_p = 0 \\
&\Rightarrow \hat{\theta} = (\Phi^T\Phi + K)^{-1}(\Phi^T Y + K\theta_p).
\end{aligned} \tag{2.11}$$

When $(\Phi^T\Phi+K)$ in Equation (2.11) is non-singular, the solution (2.11) is unique [14].

Recall equation (2.8), Φ is a matrix with dimension of $(mi - mc + 1) \times 3$, where mi is the current time point and mc is the time point when the identification begins. The number of rows of Φ equals the number of data points used in the identification calculation. Here we assume $(mi - mc + 1) \geq 3$. The special structure of Φ matrix makes it is full column ranked matrix, i.e., its rank is 3. To prove this, by arbitrarily selecting 3 rows from Φ, we have:

$$\Phi_3 = \begin{bmatrix} t_{i1} & t_{i1}^2 & t_{i1}^3 \\ t_{i2} & t_{i2}^2 & t_{i2}^3 \\ t_{i3} & t_{i3}^2 & t_{i3}^3 \end{bmatrix} \tag{2.12}$$

The determinant of this matrix is:

$$det(\Phi_3) = t_{i1}t_{i2}t_{i3}(t_{i2} - t_{i1})(t_{i3} - t_{i1})(t_{i3} - t_{i2}) \tag{2.13}$$

Since $t_{i1} \neq t_{i2} \neq t_{i3}$ and $t > 0$, $det(\Phi_3) \neq 0$. Therefore, Φ has a column rank of 3 and $\Phi^T\Phi$ is a full ranked 3×3 matrix. All elements in $\Phi^T\Phi$ are positive.

Considering that the weighting matrix K (2.10) is a full ranked diagonal matrix with positive elements, it is reasonable that $\Phi^T\Phi + K$ is full ranked, i.e. non-singular. The solution (2.11) is unique to minimize the objective function OBJ (2.5).

2.3.4 Selection of weighting factors in K

From equation(2.11), the best estimation of θ is dependent on the weighting matrix K.

$$\text{When } K = 0, \Rightarrow \hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T Y, \tag{2.14}$$

which is the solution of the least square method.

$$\text{When } K \rightarrow \infty, \Rightarrow \hat{\theta} = \theta_p, \tag{2.15}$$

which is the prior information.

The selection of the weighting values in K is very important to ensure the effectiveness of this algorithm. The values of K can be selected according to different criteria for different purposes. However, the basic requirement is to avoid either least square estimation or the prior information to dominate the result.

In order to see the relationship between the two factors determining $\hat{\theta}$, namely, the least square estimation, and the prior information, we rewrite equation (2.11) as:

$$\begin{aligned}
\hat{\theta} &= (\Phi^T\Phi + K)^{-1}(\Phi^T Y + K\theta_p) \\
&= (\Phi^T\Phi + K)^{-1}(\Phi^T\Phi)(\Phi^T\Phi)^{-1}\Phi^T Y + (\Phi^T\Phi + K)^{-1}K\theta_p \\
&= K_s\theta_s + K_p\theta_p
\end{aligned} \tag{2.16}$$

where θ_s is the least square solution as equation (2.14) and θ_p is the prior information. Their corresponding weighting matrix are:

$$K_s = (\Phi^T\Phi + K)^{-1}(\Phi^T\Phi) \quad (2.17)$$

$$K_p = (\Phi^T\Phi + K)^{-1}K \quad (2.18)$$

In order to avoid either θ_s or θ_p to dominate the $\hat{\theta}$, K_s and K_p must be comparable, i.e. K and $\Phi^T\Phi$ must be comparable. In this chapter, we choose k_i values to be the same as the eigenvalues of $\Phi^T\Phi$:

$$k_1 = \Lambda(1),\ k_2 = \Lambda(2),\ k_3 = \Lambda(3),$$

$$\Rightarrow \|K\|_2 = \|\Phi^T\Phi\|_2 \quad (2.19)$$

where $\Lambda(i), i = 1, 2, 3$ are eigenvalues of matrix $\Phi^T\Phi$. Under this condition, mathematically,

- the 2-norms of both the weighting matrix K and the matrix $\Phi^T\Phi$ are equal,
- the weighting matrix K is similar to the matrix $\Phi^T\Phi$, and
- the determinants of K and $\Phi^T\Phi$ are equal.

Therefore, the prior information is comparable to the estimation by the least square method.

2.3.5 Simulations

The advantages of this new identification algorithm can be demonstrated by the following simulations.

With the prior information $R_p = 50t - 12t^2 + t^3$ available, four new tool wear processes with noise ($\sigma^2 = 16$) need to be identified:

1. From the beginning, b_1 is different from the prior information, where $b_1 = 60$.

2. From time $t_{mc} = 4$, b_1 is different from the prior information, where $b_1 = 60$.

3. From the beginning, three parameters are all different from the prior information, where $b_1 = 60, b_2 = -14, b_3 = 1.2$.

4. From time $t_{mc} = 4$, three parameters are all different from the prior information, where $b_1 = 60, b_2 = -14, b_3 = 1.2$.

The simulation results for the above four cases are shown in Figures 2.4–2.7 correspondingly. In these figures, the normalized parameter identified by the least square are shown in (a). The results by the proposed method are shown in (b). Compared to the least square method, we can see that proposed method greatly reduces convergence time and parameter variations. For further illustration, the predictions of future tool wear trend at every two sampling time intervals are also calculated. The first five tool wear trends predicted based on the least square estimation θ_s are shown in (c). And the corresponding first five tool wear trends estimated based on the proposed method estimation $\hat{\theta}$ are shown in (d). In (c) and (d), symbol '*'s stand for the actual tool wear process and symbol 'o's stand for the prediction based on the prior information θ_p, while other five lines stand for on-line predictions based on the identified parameters. From the figures, we can see that:

- the prior information can not provide a good estimation for these four tool wear processes.

- the least square method is extremely unreliable during the early several identification steps. The prediction of tool wear trend can be much worse than the prior information. It is slow to obtain a good estimation.

- the proposed method provides much better estimation compared to the least square method at the same sampling interval. It is much quicker to approach to the good estimation. In the four simulation cases, the fourth prediction

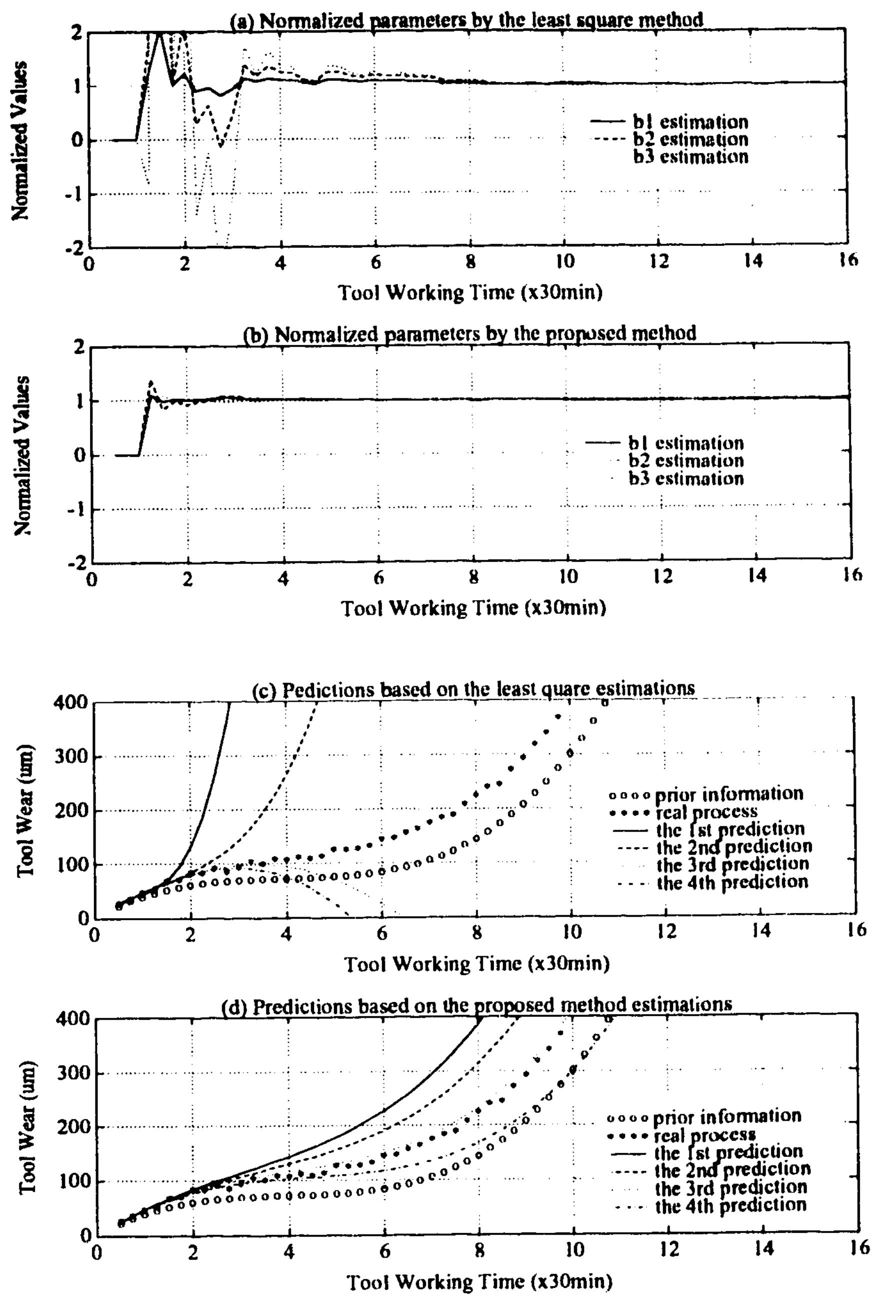

Figure 2.4: Comparison of the least square method and the proposed method when b_1 deviates from the prior information from the beginning

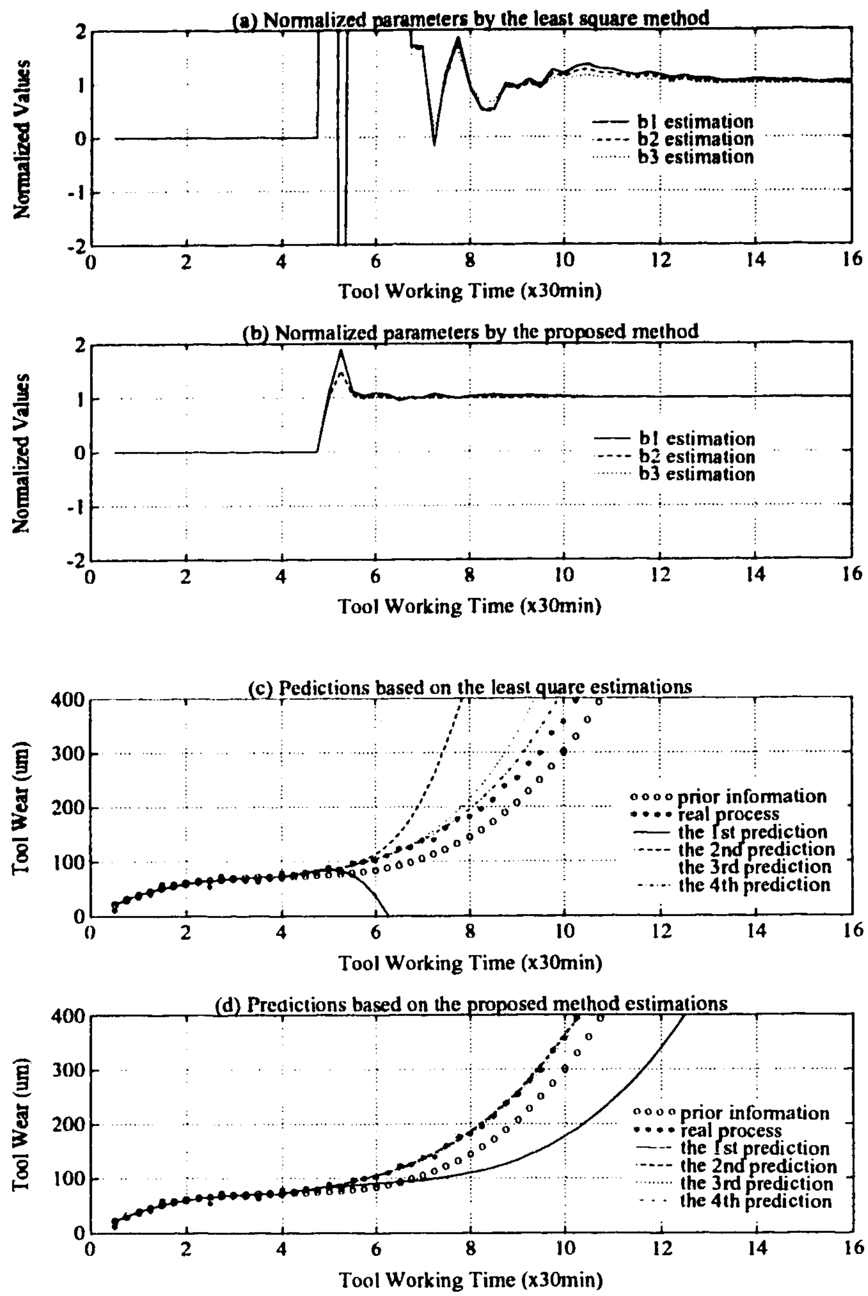

Figure 2.5: Comparison of the least square method and the proposed method when b_1 deviates from the prior information from $t = 4$

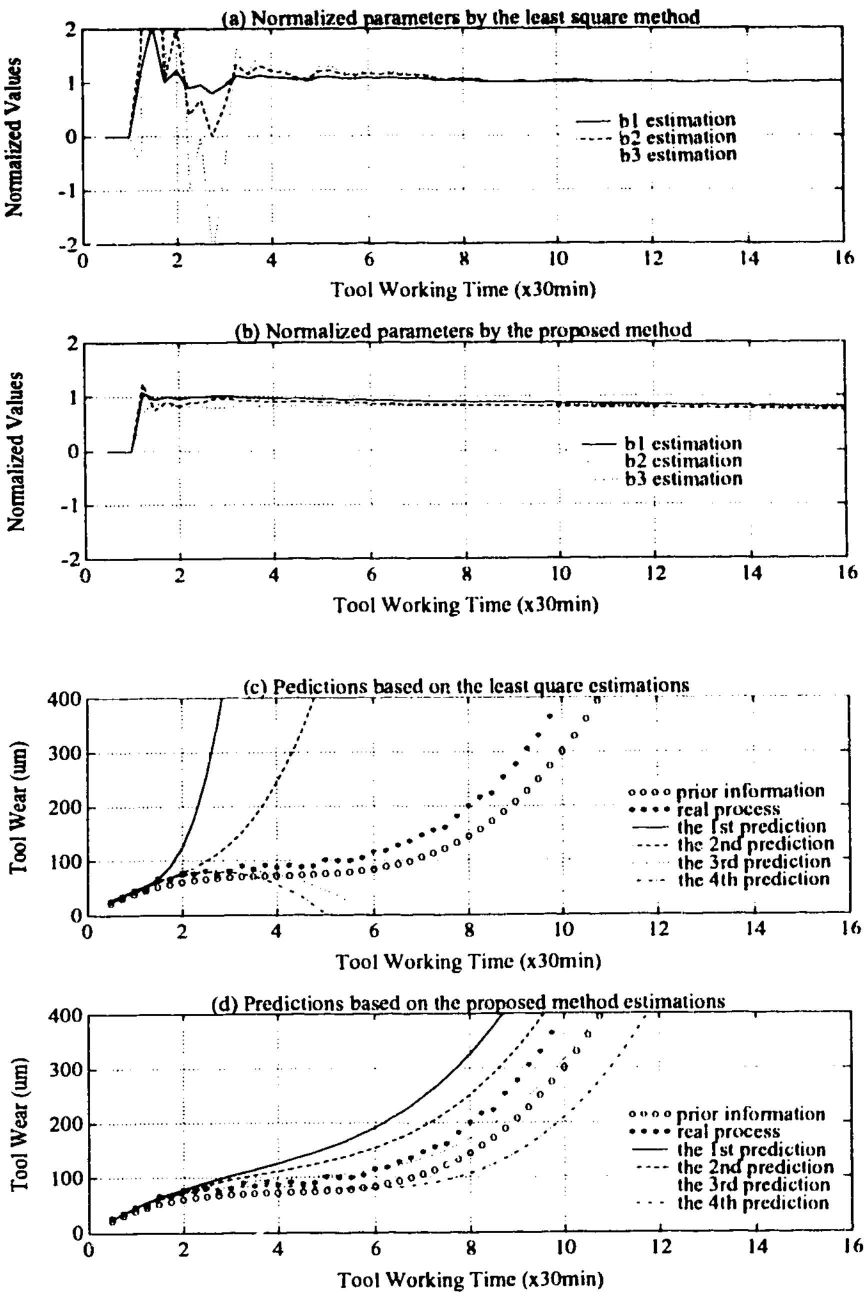

Figure 2.6: Comparison of the least square method and the proposed method when b_1, b_2, b_3 deviate from the prior information from the beginning

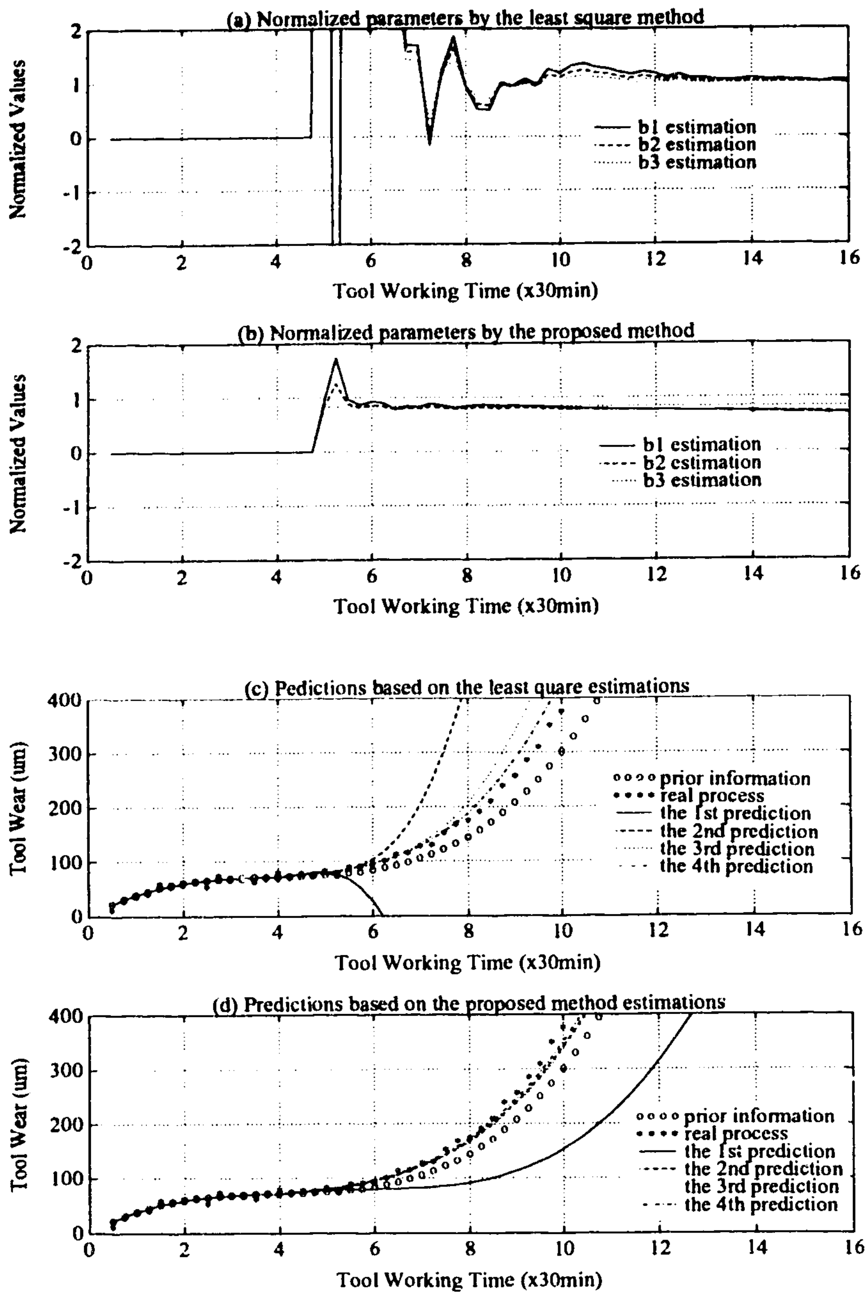

Figure 2.7: Comparison of the least square method and the proposed method when b_1, b_2, b_3 deviate from the prior information from $t = 4$

based on $\hat{\theta}$ is much closer to the actual process than the fourth prediction based on θ_s. It is also a much better prediction than that provided by the prior information.

2.3.6 Investigations of the proposed method

The performance of the proposed method can be shown by the following investigations:

- The least square method is sensitive to noise during the early several identification steps. It is already shown by the simulations (Figures 2.4 2.7). Mathematically, the difference between the estimated parameter θ_s and the true value θ can be developed. From equation (2.11) with $K = 0$, we known the least square estimation:

$$\theta_s = \hat{\theta}|_{K=0} = (\Phi^T\Phi)^{-1}\Phi^T Y \tag{2.20}$$

Replace Y by equation (2.6), then:

$$\theta_s = \theta + (\Phi^T\Phi)^{-1}\Phi^T\xi$$
$$\Rightarrow \quad \Delta\theta_s = \hat{\theta} - \theta = (\Phi^T\Phi)^{-1}\Phi^T\xi \tag{2.21}$$

We can see that the estimation errors of parameters are independent of parameters themselves and only relative to the process noise and sampling time. To illustrate the features of the estimation errors, assume that samples are taken every 0.25 time unit and ξ are random noise with mean zero and variance $\sigma^2 = 16$. Figure 2.8(a) shows the estimation errors along time when identification starts from time 1 and Figure 2.9(a) shows the estimation errors along time with identification starts from time 5. Obviously, the errors are significant during the early several identification steps and slowly approach zero.

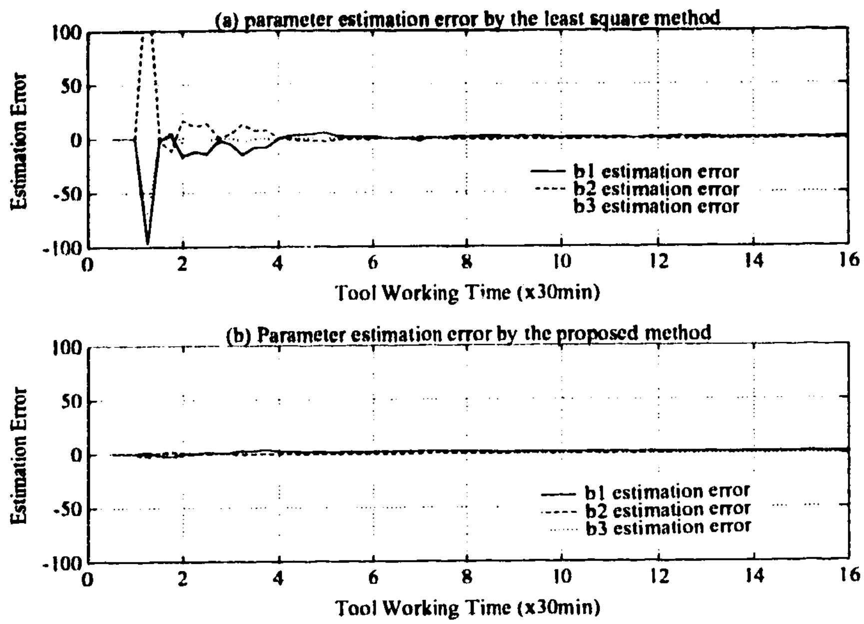

Figure 2.8: Comparison of estimation errors when identification starting from $t = 1$

- The proposed method greatly improves the performance of the traditional least square methods by taking advantage of the prior information, under the condition that the prior information is close to the parameters of the actual process. Compared to the prior information, this method provides better estimation by utilizing the newest information of process when working condition changes.

Assuming that the inaccuracy of the prior information is $\Delta\theta_p = \theta_p - \theta$, the estimation error by the proposed method can be developed. In equation (2.11), replacing θ_p with $\theta + \Delta\theta_p$, and replacing Y with equation (2.6), we obtain that:

$$\hat{\theta} = (\Phi^T\Phi + K)^{-1}K\Delta\theta_p + \theta + (\Phi^T\Phi + K)^{-1}\Phi^T\xi$$

$$\Rightarrow \quad \Delta\theta = \hat{\theta} - \theta = (\Phi^T\Phi + K)^{-1}(K\Delta\theta_p + \Phi^T\xi) \qquad (2.22)$$

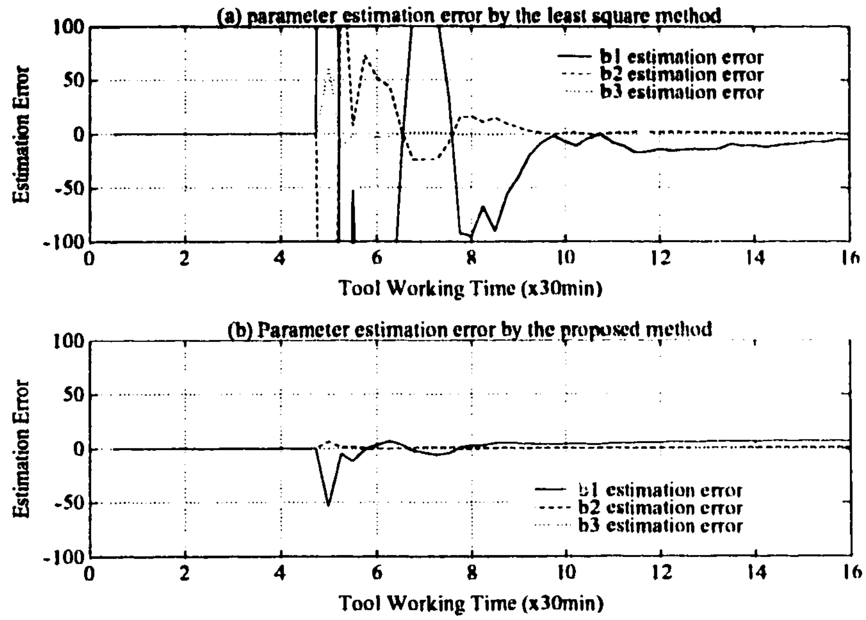

Figure 2.9: Comparison of estimation errors when identification starting from $t = 5$

Given weighting matrix K, the estimation error is independent of the process parameter but dependent on the accuracy of the prior information.

When the prior information is very close to the true process parameters, i.e. $\Delta\theta_p \approx 0$, then

$$\Delta\theta = \hat{\theta} - \theta \approx (\Phi^T\Phi + K)^{-1}\Phi^T\xi \qquad (2.23)$$

Compared to Equation (2.21), we can see that estimation errors are reduced due to the effects of K. That is why the proposed method can improve the performance of the traditional least square method.

On the other hand, if the process noise is not significant, i.e. $\xi \approx 0$, then

$$\Delta\theta = \hat{\theta} - \theta \approx (\Phi^T\Phi + K)^{-1}K * \Delta\theta_p \qquad (2.24)$$

Compared to the prior information error, the estimation is improved due to the effects of $\Phi^T\Phi$.

To illustrate Equation (2.22), with the prior information error $\Delta\theta = [5\ \ 1\ \ 0.1]^T$ for three parameters, the estimation errors are shown in Figure 2.8(b) and Figure 2.9(b) respectively, under the same sampling condition as Figure 2.8(a) and Figure 2.9(a). By comparison, we can see that the estimation errors from the proposed method are much smaller than that from least square method during the early several identification steps.

Therefore, by taking advantage of the prior information which is usually available, the proposed method makes tool wear process identification feasible for on-line decision making.

- The proposed method takes both advantages of the least square estimation and the prior information. The selection of K is important to the performance of the proposed method, as revealed by Equation (2.22). In this chapter, we choose K elements equal to the eigenvalues of $\Phi^T\Phi$. It is a good option of the tool wear process identification. Nevertheless, it is not the only option. The fine tuning can be done for different requirements.

  From the equations (2.16), (2.17), and (2.18), we can see that parameter estimation $\hat{\theta}$ by the proposed method is a complicated combination of the least square estimation θ_s and the prior information θ_p. The weighting matrix K determines the combination pattern. Since the matrix K_s in (2.17) and matrix K_p in (2.18) are not diagonal, we have selected to use their eigenvalues to roughly evaluate the weighting on the corresponding parameters in θ_s and θ_p. The results are shown in Figure 2.10, when we choose the elements of diagonal matrix K to be equal to the eigenvalue of $\Phi^T\Phi$ as discussed in Section 2.3.4. Figure 2.10 shows that this selection put a heavy weighting on b_1 estimated by the least square method, a heavy weighting on b_3 in the prior information, and average weighting on b_2 by the least square and the prior information.

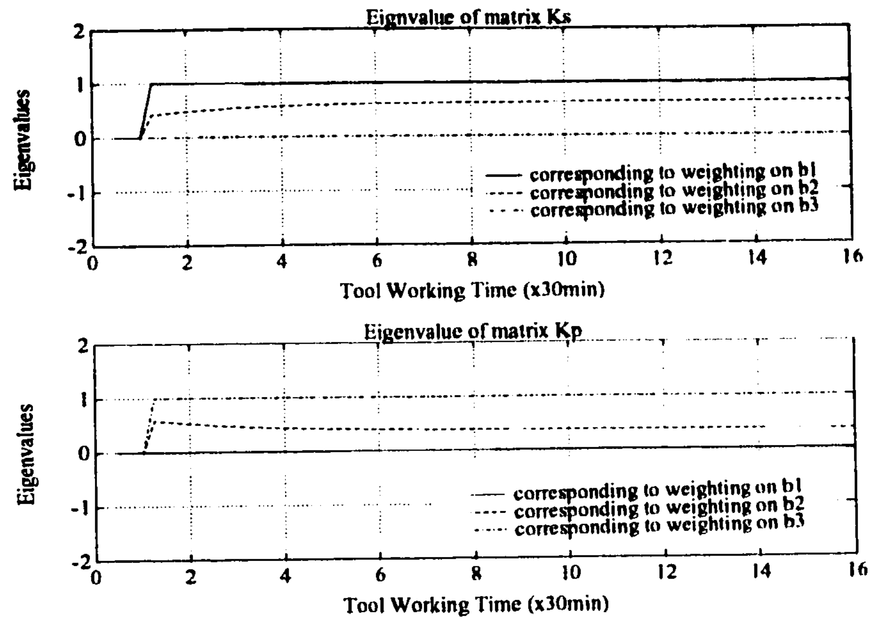

Figure 2.10:Comparison of eigenvalues of matrices K_s and K_p

In a tool wear function, the highest order term usually reflects the accelerated wear period rather than the beginning period. The parameter b_3 usually can't be decided precisely by the data points in the beginning period with the least square estimation. With a heavy weighting on b_3 in the prior information, the proposed method compensates this disadvantages of the least square method, hence providing better prediction of tool wear trend for on-line tool replacement/adjustment decision making.

- A heuristic method can be inferred based on the above analysis: the on-line identification can be done by a weighted sum of the least square estimation and the prior information. However, the selection of the weighting greatly depends on the experience. The complexity of calculation remains the same as the proposed method.

2.4 On-line Tool Replacement/Adjustment Decision Making

With the known non-liner tool wear function, the optimal tool replacement and adjustment time are studied by Jean and Yong [3], and discussed in the following two chapters. These results can be used for on-line decision making with little revision. Whenever the estimation of the tool wear function $\hat{R}(t)$ has been updated by the identification algorithm, the decisions, including adjustment and replacement time, will be updated correspondingly. The objective here is to minimize the expected cost per unit of time during every single tool's life. The objective function can be expressed as:

$$\begin{aligned} \text{Minimize } C &= \frac{C_r + (J-1)C_a + \sum_{j=1}^{J} L_j}{\sum_{j=1}^{J} T_j} \\ \text{subject to} \quad & T_j > 0, \quad , \; j = 1, 2, \ldots, J \end{aligned} \tag{2.25}$$

Where J is the total number of adjustments in a tool life. The J-th adjustment is considered as replacement. Adjustment time T_js are decision variables. L_j is quality loss in adjustment intervals. From the $(j-1)$-th to the j-th adjustment, the expected quality loss can be expressed as:

$$L_j = \int_0^{T_j} k[a_\tau + \hat{R}(\tau + \sum_{i=1}^{j-1} T_i) - \hat{R}(\sum_{i=1}^{j-1} T_i)]^2 d\tau. \tag{2.26}$$

When $J = 1$, only replacement will be made on each tool.

Starting at the beginning, the optimal decision of $T_j, j = 1, 2, \ldots, J$ can be decided according to the prior information $R_p(t)$. Then updated decision can be obtained for every updated $\hat{R}(t)$. Whenever current time $t = \Sigma_{s=1}^{j} T_s$, adjustment or replacement takes place. When $t > T_s$, $(1 < s < J)$, s adjustment have been already made and the decision variables reduce to $T_{s+1}, \ldots, T_J$. Chapter 4 will discuss this optimization problem. It can be solved either by numerical search or by heuristic methods. A combination approach can also be taken to avoid a local minimum solution obtained by using the numerical search method only. That is to use heuristic methods to provide the initial values for the numerical search. The details are omitted here.

Figure 2.11 and Figure 2.12 show how the replacement ($J = 1$) decision is made on-line, without the exact tool wear information before applying the new tool. Figure 2.11 shows the situation when the prior information does not reflect the actual tool wear process. Figure 2.12 shows the situation when the process starts to deviate from the prior information from $t_{mc} = 2.5$, due to unexpected working condition change. In both figures, (a) shows the difference between the actual tool wear and the prior information. (b) shows the on-line process monitoring chart. Whenever the process is out of control, the on-line process identification begins. The normalized identification results are shown in (c). Then, the replacement time decisions according to the identification results are shown in (d). We can see the decision updating along the time. Finally, the decision converges to a fixed optimal time before this time point. If current time exceeds this optimal time point, the replacement time will be the current time, which means the replacement has to be taken immediately.

In both cases, we assume that $C_r = \$270, k = 0.006, a_t = 0$. According to the prior information, the optimal replacement time will be $T = 8$. In the case shown in Figure 2.11, replacing the tool at $t = 8$ will result unit time cost of \$160.2, while the updated decision $T = 6.5$ give an unit time cost of \$145.8. In the case of Figure 2.12, replacing the tool at $t = 8$ will result unit time cost of \$133.5, while the updated decision $T = 7.25$ give an unit time cost of \$124.8.

Obviously, we can achieve considerable cost savings by using the proposed approach for the on-line tool wear process management.

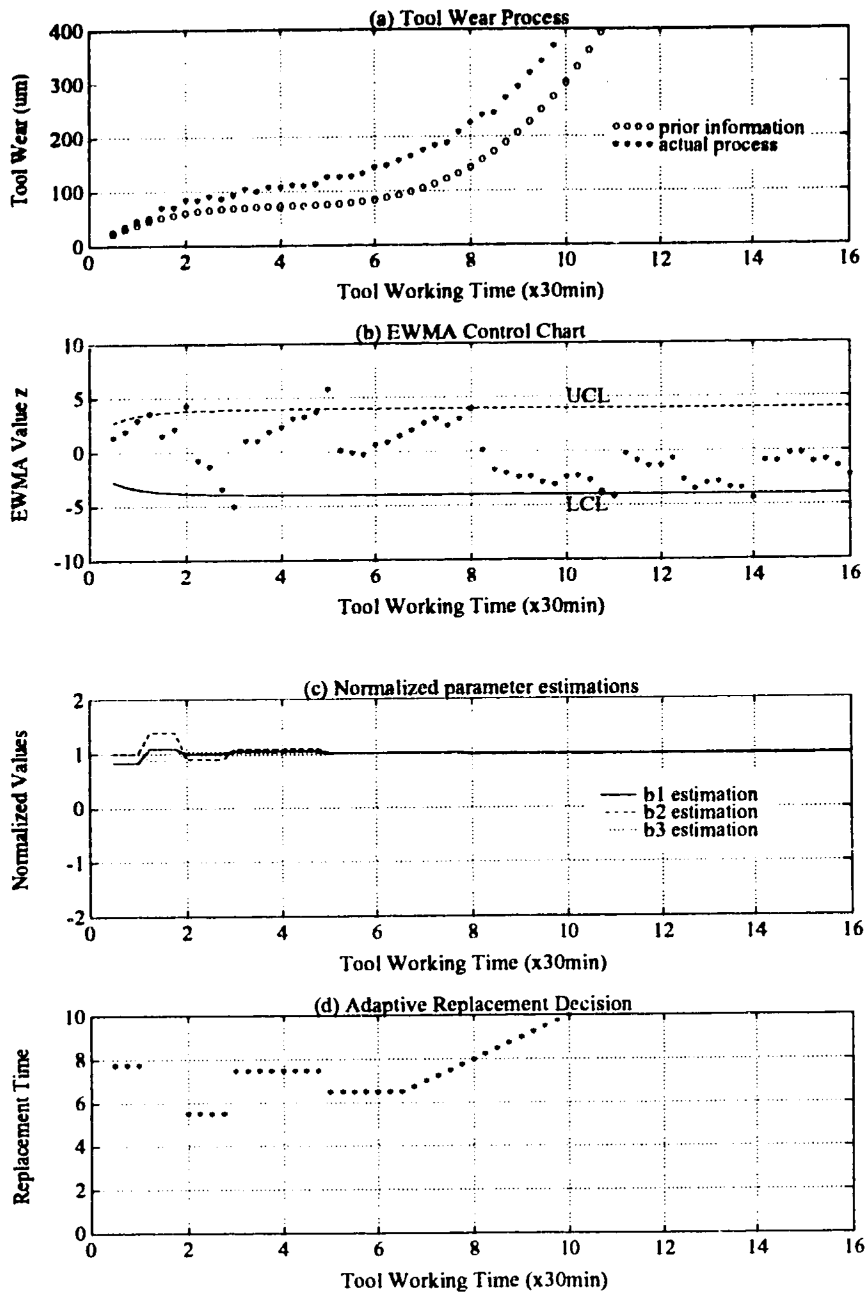

Figure 2.11: Simulation of on-line tool replacement decision making when b_1 deviates from the prior information from the beginning

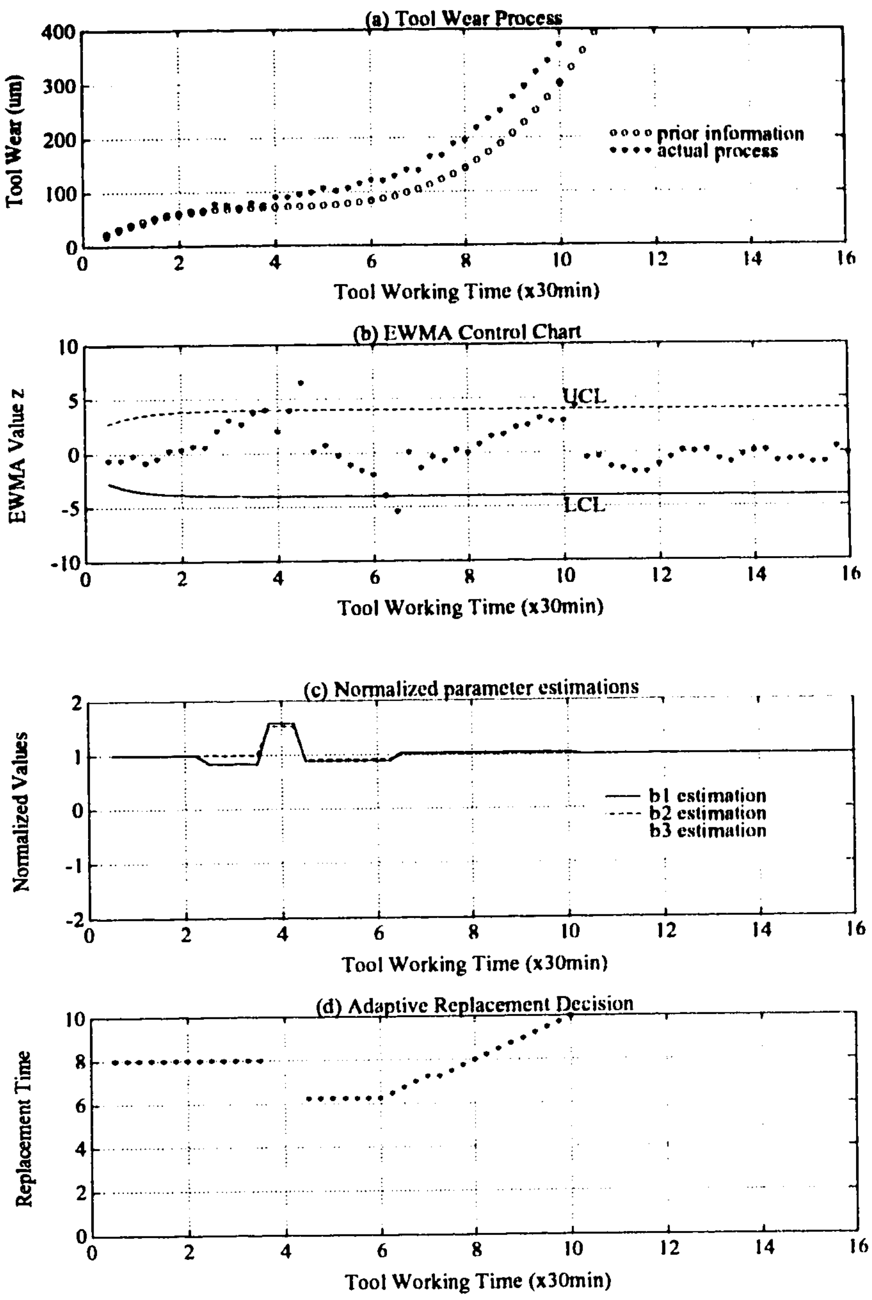

Figure 2.12: Simulation of on-line tool replacement decision making when b_1 deviates from the prior information from $t = 4$

2.5 Conclusion

This chapter provides a systematic approach for on-line tool wear process management when the prior information about the tool wear trend is not accurate due to working condition change. The objective is to minimize the unit time production cost. On-line monitoring by the EWMA control chart can detect when the actual tool wear process deviates from the original value. The tool wear process identification with the proposed method with prior information can identify new tool wear function under the changed working condition. In particular, the performance of this proposed method is investigated. The advantages include faster convergence and insensitivity to the process noise. Within a few identification steps, the parameter estimation error of the proposed method is much less than that of the least square method or the prior information. These features are very important to the on-line decision making. Finally, on-line optimal tool replacement/adjustment decision can be made based on the updated tool wear function. Therefore, the optimal decision of tool adjustment and replacement time can be obtained specifically for every single tool, without the extensive prior data collection or controlled experiments. Considerable cost saving can be achieved. The implementation of this approach does not need the extensive hardware support except dimension measurement of the finished part and a personal computer.

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Chapter 3

Off-line Tool Replacement Decision Making

3.1 Introduction

This chapter studies off-line optimal replacement decision making for the tool wear process, where on-line monitoring and decision making are not applicable. The objective is to minimize the unit time production cost considering both tool replacement cost and product quality loss.

Drezner and Wesolowsky [1] reported a procedure to determine the optimal replacement time in a tool wear process. They assumed that the average tool wear trend is linear, which greatly simplified the tool wear model. Jeang and Yang [2] generalized the model by Drezner and Wesolowsky by assuming that tool wear follows a general non-decreasing function $R(t)$ rather than a linear function. They claimed that they had developed an optimal tool replacement strategy consisting of an optimal initial tool setting a_0^* and an optimal cycle time Q^*, when the tool wear

[0] A version of this chapter was published in the Proceedings of the 2nd Industrial Engineering Research Conference, May 26-27, 1993, pp 350-354.

function $R(t)$ is known.

Tool wear model $R(t)$ is affected by tool properties, working conditions and their interactions. As discussed in Chapter 1, in a practical situation, especially in a flexible manufacturing environment, working condition changes are very common. Due to these known or unknown working condition changes, tool wear curves will be different even for the same type tool [3, 4]. Zhou et al. [5] put forward an optimization model based on the tool life distribution and a time index to decide the optimal tool replacement time when using a single tool to process different parts. The limitations of their study are: i) All expected tool life under possible working conditions are needed; ii) The quality loss existing along with tool wear has not been considered. In a varied but stable production system like routine working situations of an FMS, without on-line monitoring and identification of tool wear function, a robust optimal replacement strategy is needed to deal with the uncertainty of the tool wear function for an overall cost-effective tool management.

This chapter firstly analyzes the work by Jean and Yang, then shows that the tool wear model $R(t)$ by Jeang and Yang is too general to have a unique optimal solution (Q^*, a_0^*). The necessary and sufficient conditions for the unique optimal replacement time are developed. It is under the assumption that the tools are used under the identical working conditions, and $R(t)$ can be obtained from a controlled experiment. Furthermore, an optimization model is developed to obtain a robust optimal tool replacement strategy when the tool wear function is uncertain.

3.2 Optimal Tool Replacement with Known Tool Wear Function

A tool replacement decision discussed here includes the selection of an initial tool setting a_0 and a tool replacement cycle time Q. Associated with each decision is the corresponding economic quality loss during the replacement cycle and the corresponding tool replacement cost. According to Taguchi quality loss function, the economic quality loss L_t due to the deviation of a part's dimension from its target value is:

$$L_t = k(X_t - T)^2. \tag{3.1}$$

With an initial setting a_0 and a replacement cycle time Q, the expected loss over a cycle Q is:

$$L(Q, a_0) = \int_0^Q k[\sigma^2 + (a_0 + R(t) - T)^2]dt. \tag{3.2}$$

The expected cost per unit of time is:

$$C(Q, a_0) = \frac{L(Q, a_0) + C_r}{Q}. \tag{3.3}$$

A decision (Q, a_0) is optimal if it minimizes the unit time expected cost expressed in Equation (3.3).

The tool wear function $R(t)$ can be obtained under controlled experiments. With assumptions (1) the tool always works under identical machining conditions, (2) there is no in-process adjustment, and (3) tool failure does not occur within the planning horizon, a necessary condition for a_0^* and Q^* to be the optimal replacement decision is developed by Jeang and Young [2]:

$$a_0^* = T - \frac{\int_0^{Q^*} R(t)dt}{Q^*}, \tag{3.4}$$

$$Q^* = \frac{k \int_0^{Q^*} \left[R(t) - \frac{\int_0^{Q^*} R(s)ds}{Q^*}\right]^2 dt + C_r}{k\left[R(Q^*) - \frac{\int_0^{Q^*} R(t)dt}{Q^*}\right]^2}. \tag{3.5}$$

Jeang and Yang believed that equations (3.4) and (3.5) were both sufficient and necessary conditions for (Q^*, a_0^*) to be optimal and an optimal tool replacement decision could be obtained by solving these equations. However, we have discovered that without additional restrictions, even though $R(t)$ is non-decreasing, these two equations cannot guarantee the sufficiency for (Q^*, a_0^*) to be optimal. Equations

(3.4) and (3.5) may have more than one solution. An example is shown in Figure 3.1. In Figure 3.1, although R(t) is non-decreasing, the expected unit time cost curve has three stationary points which all satisfy equations (3.4) and (3.5). One of these three points is actually a maximum cost point.

Theorem 1 *A solution (Q^*, a_0^*) obtained from equations (3.4) and (3.5) is a local minimum if the following condition is satisfied:*

$$Q^* \frac{dR(t)}{dt} \Big|_{t=Q^*} + \frac{\int_0^{Q^*} R(t)dt}{Q^*} - R(Q^*) > 0 \tag{3.6}$$

Proof of Theorem 1:

Since (Q^*, a_0^*) is a stationary point of the objective function $C(Q, a_0)$, it is a local minimum point if the Hessian matrix at this point is positive definite. The Hessian matrix of $C(Q, a_0)$ at (Q^*, a_0^*) is:

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2 C}{\partial Q^2} & \frac{\partial^2 C}{\partial a_0 \partial Q} \\ \frac{\partial^2 C}{\partial Q \partial a_0} & \frac{\partial^2 C}{\partial a_0^2} \end{pmatrix}. \tag{3.7}$$

Since $\frac{\partial C(Q, a_0)}{\partial Q} = 0$ at (Q^*, a_0^*), we have:

$$\frac{\partial L(Q, a_0)}{\partial Q} = \frac{L(Q, a_0) + C_r}{Q}. \tag{3.8}$$

$$\begin{aligned} h_{11} &= \frac{\partial^2 C(Q, a_0)}{\partial Q^2} = \frac{\partial}{\partial Q} \left(\frac{\partial C(Q, a_0)}{\partial Q} \right) \\ &= \frac{\partial}{\partial Q} \left(\frac{1}{Q} \frac{\partial L(Q, a_0)}{\partial Q} - \frac{L(Q, a_0) + C_r}{Q^2} \right) \\ &= \frac{1}{Q} \frac{\partial^2 L(Q, a_0)}{\partial Q^2} + \frac{2}{Q^3} [L(Q, a_0) + C_r] - \frac{2}{Q^2} \frac{\partial L(Q, a_0)}{\partial Q} \\ &= \frac{1}{Q} \frac{\partial^2 L(Q, a_0)}{\partial Q^2} + \frac{2}{Q^3} [L(Q, a_0) + C_r] - \frac{2}{Q^3} [L(Q, a_0) + C_r] \\ &= \frac{1}{Q} \frac{\partial^2 L(Q, a_0)}{\partial Q^2} \\ &= 2k[a_0 - T + R(Q)]r(Q) \end{aligned} \tag{3.9}$$

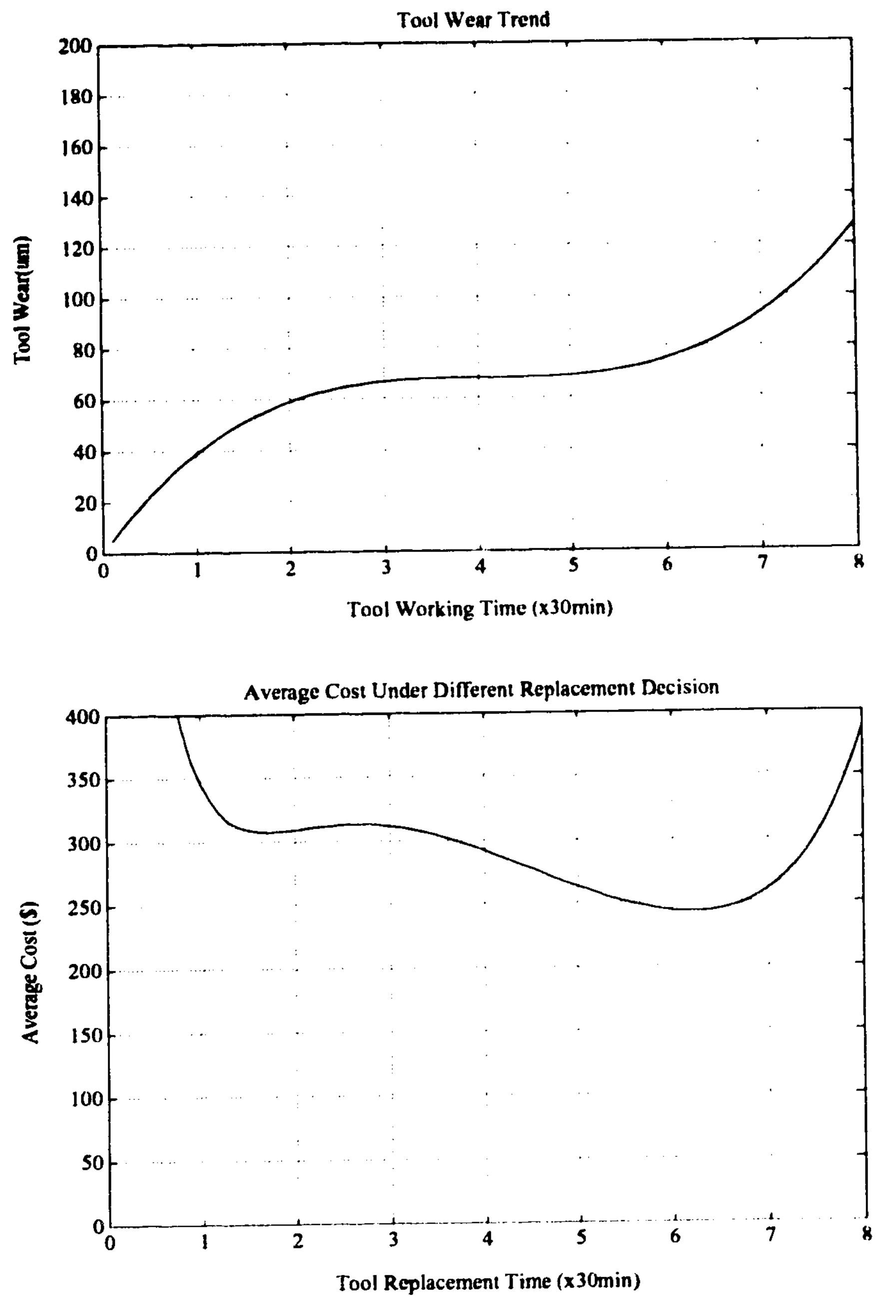

Figure 3.1: **A case showing two local minimum points, when** $R(t) = t^3 - 12.47t^2 + 50t, k = 0.6, C_r = 270$

h_{21}, h_{12}, and h_{22} are as developed by Jeang and Yang [2], namely,

$$h_{21} = h_{12} = \frac{2k}{Q}[a_0 - T + R(Q)], \tag{3.10}$$

$$h_{22} = 2k. \tag{3.11}$$

$$\begin{aligned} det(\mathbf{H}) &= h_{11}h_{22} - h_{21}h_{12} = \frac{4k^2}{Q}[a_0 - T + R(Q)]r(Q) - \frac{4k^2}{Q^2}[a_0 - T + R(Q)]^2 \\ &= \frac{4k^2}{Q^2}[a_0 - T + R(Q)][r(Q)Q - (a_0 - T + R(Q))] \end{aligned} \tag{3.12}$$

Since $[a_0^* - T + R(Q^*)] > 0$ [2], for $det(\mathbf{H}) > 0$ at (Q^*, a_0^*), the following condition must be satisfied:

$$Q^* r(Q^*) > a_0^* - T + R(Q^*) = R(Q^*) - \frac{\int_0^{Q^*} R(t)dt}{Q^*}. \tag{3.13}$$

Therefore, Equation (3.6) must be satisfied for a solution of equations (3.4) and (3.5), (Q^*, a_0^*), to be a local minimum. (QED)

If $R(t)$ is convex, $\frac{dR(t)}{dt}\mid_{t=Q} Q - R(Q) > 0$, and Equation (3.6) is automatically satisfied. Then, equations (3.4) and (3.5) have a unique minimum solution. However, a practical tool wear curve is usually not convex.

Another special case is to set a_0 at a fixed value. Under this condition, a unique optimal tool replacement decision Q^* can be obtained as stated in the following theorem.

Theorem 2 *If $R(t)$ is increasing and a_0 is fixed, the necessary and sufficient condition for Q^* to be a unique optimal solution of $C(Q, a_0)$ is:*

$$k[a_0 - T + R(Q^*)]^2 = \frac{1}{Q^*}\left\{\int_0^{Q^*} k[a_0 - T + R(t)]^2 dt + C_r\right\} \tag{3.14}$$

Proof of Theorem 2:

From $\frac{dC(Q)}{dQ} = 0$, Equation (3.14) can be obtained immediately, hence verifying the necessity.

The sufficiency for Q^* to be a minimum point is that $\frac{d^2C(Q)}{dQ^2} > 0$ at Q^*.

$$\begin{aligned}
\frac{d^2C(Q)}{dQ^2} &= \frac{d}{dQ}\frac{dC(Q)}{dQ} \\
&= \frac{d}{dQ}\{-\frac{k}{Q^2}\int_0^Q [a_0 - T + R(t)]^2 + \frac{k}{Q}[a_0 - T + R(t)]^2 - \frac{C_r}{Q^2}\} \\
&= \frac{2k}{Q^3}\int_0^Q [a_0 - T + R(t)]^2 dt + \frac{2k}{Q}[a_0 - T + R(t)]r(Q) \\
&\quad -\frac{2k}{Q^2}[a_0 - T + R(t)]^2 + \frac{2C_r}{Q^3} \\
&= \frac{1}{Q^2}(\frac{2k}{Q}\int_0^Q [a_0 - T + R(t)]^2 dt - 2k[a_0 - T + R(t)]^2 + \frac{2C_r}{Q}) \\
&\quad +\frac{2k}{Q}[a_0 - T + R(t)]r(Q)
\end{aligned} \tag{3.15}$$

At Q^*, recall Equation (3.14), we know:

$$\frac{d^2C(Q)}{dQ^2}|_{Q=Q^*} = \frac{2k}{Q^*}[a_0 - T + R(Q^*)]r(Q^*) \tag{3.16}$$

Since $R(t)$ is increasing, $r(Q^*) > 0$. We can show that $[a_0 - T + R(Q^*)]$ must be greater than 0 from the following. If $[a_0 - T + R(Q^*)] \leq 0$, $[a_0 - T + R(t)]^2$ would be decreasing during the interval $0 < t < Q^*$, and:

$$\frac{\int_0^{Q^*}[a_0 - T + R(t)]^2 dt}{Q^*} > [a_0 - T + R(Q^*)]^2 \tag{3.17}$$

$$\frac{k\int_0^{Q^*}[a_0 - T + R(t)]^2 dt + C_r}{Q^*} > k[a_0 - T + R(Q^*)]^2 \tag{3.18}$$

(3.19)

Equation (3.18) would violate Equation (3.14). Since (3.14) must be satisfied, we must have $[a_0 - T + R(Q^*)] > 0$. Then, from Equation (3.16), we know:

$$\frac{d^2C(Q)}{dQ^2}|_{Q=Q^*} > 0 \tag{3.20}$$

Hence verifying the sufficiency for Q^* to be a minimum point.

Furthermore, since Q^* which satisfies Equation (3.14) is always minimal and the objective function is differentiable for all $Q > 0$, it is impossible to have more than one local minimum points without a local maximum point. Q^* is unique. (QED)

3.3 Off-line Robust Optimal Tool Replacement with Uncertain Tool Wear

Section 3.2 discussed the optimal replacement decision when tool wear function $R(t)$ is known. The results obtained are rather restrictive since they need the assumption that the tools are used under the identical working conditions. However, in most practical situations, especially in a flexible manufacturing system, changes of working conditions are inevitable. Some of these changes could be unknown. Due to the changes in working conditions, the parameters of $R(t)$ are uncertain to some degree. An optimal decision based on a specific $R(t)$ model may not be the real optimal under the changing working conditions. In the working situations of an FMS, a robust optimal replacement strategy is needed to deal with the uncertainty of the tool wear curve for an overall cost-effective tool management, if on-line tool wear function monitoring and identification are not available.

3.3.1 Tool Wear Function With Uncertainty

As shown in Figure 3.2, the typical tool wear process is random but with a non-decreasing trend, consisting of three distinct periods: initial wear period, normal wear period, and accelerated wear period. Without loss of generality, an m-th order polynomial model $R(t)$ can be used to represent the average tool wear during a tool's life:

$$R(t) = b_1 t + b_2 t^2 + \cdots + b_m t^m = \sum_{i=1}^{m} b_i t^i. \tag{3.21}$$

This $R(t)$ can be fitted from past tool wear data, assuming that tool wear is zero when the tool starts working. With n measurements of parts produced during a tool life cycle, we have:

$$\begin{bmatrix} X_1-(T-a_0) \\ X_2-(T-a_0) \\ \vdots \\ X_n-(T-a_0) \end{bmatrix} = \begin{bmatrix} t_1 & t_1^2 & \cdots & t_1^m \\ t_2 & t_2^2 & \cdots & t_2^m \\ \vdots & \vdots & \ddots & \vdots \\ t_n & t_n^2 & \cdots & t_n^m \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} + \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_n \end{bmatrix} \quad (3.22)$$

where X_i is the measurement at time t_i, $i = 1, 2, \ldots, n$.

Or simply:

$$Y = \Phi\theta + \xi \quad (3.23)$$

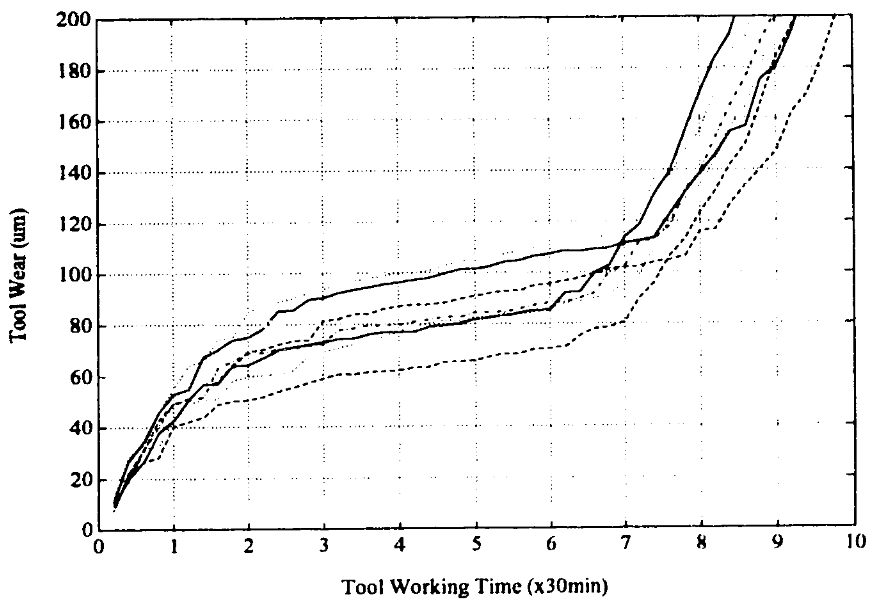

Figure 3.2:The simulated tool wear data in slightly different processes

By the least square method, the estimation of θ:

$$\hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T Y \quad (3.24)$$

Due to working condition changes, the estimated $b_i\,(i = 1, 2, \ldots, m)$ could be different for different sets of measurements. b_i's are random variables which can be related to each other to some degree. The detailed multivariate probability distribution of b_i's may be quite complicated and very hard to know. However, in a stable system where one type of tool is used under similar working conditions, the mean value, standard deviation and covariance of b_i's can be obtained from N trials of measurements and fitted curves:

$$\bar{b}_i = \frac{\sum_{l=1}^{N} b_{i,l}}{N} \tag{3.25}$$

$$\sigma_i^2 = \frac{\sum_{l=1}^{N} (b_{i,l} - \bar{b}_i)^2}{N} \tag{3.26}$$

$$\sigma_{ij} = \frac{\sum_{l=1}^{N} (b_{i,l} - \bar{b}_i)(b_{j,l} - \bar{b}_j)}{N}, \quad i, j = 1, 2, \ldots, m \text{ and } i \neq j \tag{3.27}$$

where $b_{i,l}$ is the estimation of b_i value from the l-th curve fitting.

Therefore, although the parameters of average tool wear function $R(t)$ is uncertain, their statistical characteristics can be obtained.

3.3.2 The Objective Function

Since $R(t)$ is uncertain, optimal tool setting and replacement strategy according to one specific $R(t)$ may not be optimal for another production trial. If $b_i\,(i = 1, 2, \ldots, m)$ changes stochastically due to working condition changes, optimal replacement points will change correspondingly. From a long run point of view, an integrated objective function model should be developed. The expected unit time cost must consider the uncertainty of $R(t)$ so that a robust tool replacement strategy can be developed.

According to Equation (3.3), under the identical working conditions, the expected unit cost is:

$$
\begin{aligned}
C(Q,a_0) &= \frac{L(Q,a_0)+C_r}{Q} \\
&= \frac{\int_0^Q k[\sigma^2+(a_0+R(t)-T)^2]dt + C_r}{Q} \\
&= k\sigma^2 + \frac{C_r}{Q} + \frac{k}{Q}\int_0^Q (a_0+\sum_{i=1}^m b_i t^i - T)^2 dt \qquad (3.28)
\end{aligned}
$$

One may simply use the mean value $\bar{b}_i$ to replace b_i in the original objective function $C(Q,a_0)$. $C(Q,a_0)\mid_{b_i=\bar{b}_i}$ can be minimized according to Section 3.2. However, the expected value of C when b_i's are uncertain is:

$$
\begin{aligned}
E(C) &= E\left\{k\sigma^2 + \frac{C_r}{Q} + \frac{k}{Q}\int_0^Q (a_0+\sum_{i=1}^m b_i t^i - T)^2 dt\right\} \\
&= k\overline{\sigma}^2 + \frac{C_r}{Q} + \frac{k}{Q}\int_0^Q E[(a_0-T+\sum_{i=1}^m b_i t^i)^2]dt
\end{aligned}
$$

Let $a_0 - T = b_0$, then:

$$
\begin{aligned}
E[(a_0-T+\sum_{i=1}^m b_i t^i)^2] &= E[(\sum_{i=0}^m b_i t^i)^2] \\
&= E^2[\sum_{i=0}^m b_i t^i] + Var[\sum_{i=0}^m b_i t^i] \\
&= (\sum_{i=0}^m \bar{b}_i t^i)^2 + \sum_{i=0}^m \sigma_i^2 t^{2i} + 2\sum_{i=0}^m \sum_{j=i+1}^m \sigma_{ij} t^{i+j} \\
&\quad \text{note that } \sigma_0^2 = 0,\ \sigma_{0j} = 0, (j = 1,2,\ldots,m) \\
&= (a_0 - T + \sum_{i=1}^m \bar{b}_i t^i)^2 + \sum_{i=1}^m \sigma_i^2 t^{2i} + 2\sum_{i=1}^m \sum_{j=i+1}^m \sigma_{ij} t^{i+j} \qquad (3.29)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
E(C) &= k\overline{\sigma}^2 + \frac{C_r}{Q} + \frac{k}{Q}\int_0^Q (a_0+\sum_{i=1}^m \bar{b}_i t^i - T)^2 dt \\
&\quad + \frac{k}{Q}\int_0^Q (\sum_{i=1}^m \sigma_i^2 t^{2i} + 2\sum_{i=1}^m \sum_{j=i+1}^m \sigma_{ij} t^{i+j})dt \\
&= C(Q,a_0)\mid_{b_i=\bar{b}_i} + \frac{k}{Q}\int_0^Q (\sum_{i=1}^m \sigma_i^2 t^{2i} + 2\sum_{i=1}^m \sum_{j=i+1}^m \sigma_{ij} t^{i+j})dt. \qquad (3.30)
\end{aligned}
$$

This is our new objective function.

3.3.3 Optimal Strategy

Based on the our new objective function (3.30), the optimal tool replacement strategy can be developed.

Theorem 3 *When b_i, $i = 1, 2, \ldots, m$ are uncertain, the necessary condition for (Q^*, a_0^*) to be a local minimum is:*

$$\sum_{i=1}^{m} \frac{2i}{2i+1}\left(\frac{i^2}{(i+1)^2}\bar{b}_i^2 + \sigma_i^2\right)Q^{*^{2i+1}}$$
$$+2\sum_{i=1}^{m}\sum_{i=j+1}^{m} \frac{i+j}{i+j+1}\left(\frac{ij}{(i+1)(j+1)}\bar{b}_i\bar{b}_j + \sigma_{ij}\right)Q^{*^{i+j+1}} = \frac{C_r}{k} \quad (3.31)$$
$$a_0^* = T - \sum_{i=1}^{m} \frac{\bar{b}_i}{i+1} Q^{*^i} \quad (3.32)$$

Proof of Theorem 3:

For (Q^*, a_0^*) to be an optimal solution of $E[C]$, it is necessary that:

$$\frac{\partial E[C]}{\partial a_0} = 0$$
$$\text{That is } a_0 = T - \sum_{i=1}^{m} \frac{\bar{b}_i}{i+1} Q^{*^i} \quad (3.33)$$

Substitute equation (3.33) into objective function, equation (3.30), then:

$$\frac{dE[C]}{dQ} = \frac{d}{dQ}\{k\sigma^2 + \frac{C_r}{Q} + \frac{k}{Q}\int_0^Q \left(\sum_{i=1}^{m} \frac{i}{i+1}\bar{b}_i t^i\right)^2 dt$$
$$+\frac{k}{Q}\int_0^Q \left(\sum_{i=1}^{m}\sigma_i^2 t^{2i} + 2\sum_{i=1}^{m}\sum_{j=i+1}^{m}\sigma_{ij}t^{i+j}\right)dt\}$$
$$= -\frac{C_r}{Q^2} - \frac{k}{Q^2}\int_0^Q \{\sum_{i=1}^{m}[\frac{i}{i+1}\bar{b}_i^2 + \sigma_i^2]t^{2i} + 2\sum_{i=1}^{m}\sum_{j=i+1}^{m}[\frac{ij}{(i+1)(j+1)}\bar{b}_i\bar{b}_j + \sigma_{ij}]t^{i+j}\}dt$$
$$\frac{+\frac{k}{Q}\{\sum_{i=1}^{m}[\frac{i}{i+1}\bar{b}_i^2 + \sigma_i^2]Q^{2i} + 2\sum_{i=1}^{m}\sum_{j=i+1}^{m}[\frac{ij}{(i+1)(j+1)}}{line b_i\bar{b}_j + \sigma_{ij}]Q^{i+j}\}}$$
$$= -\frac{C_r}{Q^2} - \frac{k}{Q^2}\{\sum_{i=1}^{m}\frac{2i}{2i+1}[\frac{i^2}{(i+1)^2}\bar{b}_i^2 + \sigma^2]Q^{2i+1}$$

$$+2\sum_{i=1}^{m}\sum_{i=j+1}^{m}\frac{i+j}{i+j+1}[\frac{ij}{(i+1)(j+1)}\bar{b}_i\bar{b}_j+\sigma_{ij}]Q^{i+j+1}\}$$

(3.34)

Let $\frac{dE[C]}{dQ}|_{Q=Q^*}=0$, that is:

$$\sum_{i=1}^{m}\frac{2i}{2i+1}[\frac{i^2}{(i+1)^2}\bar{b}_i^2+\sigma^2]Q^{*^{2i+1}}$$

$$+2\sum_{i=1}^{m}\sum_{i=j+1}^{m}\frac{i+j}{i+j+1}[\frac{ij}{(i+1)(j+1)}\bar{b}_i\bar{b}_j+\sigma_{ij}]Q^{*^{i+j+1}}=\frac{c_r}{k} \quad (3.35)$$

$$a_0^*=T-\sum_{i=1}^{m}\frac{\bar{b}_i}{i+1}Q^{*^i} \quad (3.36)$$

(QED)

Equation (3.31) is an $(2m+1)$-th order polynomial equation. It can be solved for Q^* by an analytical method to find all $(2m+1)$ roots and real roots. In Matlab [6], the Equation (3.31) can be solved by the command 'roots'. Then with known Q^*, the a_0^* can be obtained from equation (3.32) directly. The global optimal solution can be obtained by simply comparing objective function values corresponding to real roots and selecting the minimum.

3.3.4 Discussion and Simulations

In this section, an robust optimal tool replacement strategy has been proposed to deal with tool wear process under similar but uncertain working conditions, when on-line tool wear monitoring and identification are not available. Instead of taking the conventional approach that simply averages tool wear function parameters by allocating all variations to a single noise term, we use a tool wear function with the uncertain parameters to approximate working condition changes. It is closer to the practical situation. The tool wear function is a mathematic model to describe the tool wear progress in a single tool life. The tool wear amount in any period of tool life depends on current working conditions, as well as existing tool wear status and history [4]. No matter what kind of function is used, the tool wear model is directly related to the underlying wear progress mechanism. The parameters of the tool wear function should be fitted by the tool wear data in a single tool life.

The parameters of the tool wear function will change when working conditions change. A few research results reported that cutting forces and tool temperature are the major reasons causing tool wear. For a type of cutting tool, the change pattern of cutting forces and tool temperature during tool life could be different from one tool to another due to slightly different installation, changed workpiece geometry and material, different production batch size, or, varied cutting speed and cutting depth, etc. Tool temperature is also dependent primarily on the length of cutting cycles and the length of cooling intervals between cycles. Temperatures are generally lower in interrupted cutting than in continuous cutting under the same conditions [7]. Therefore, the parameters of fitted tool wear function will be different from one tool to the other. There are random components associated with each parameter. The statistical characteristics of the parameter changes can be obtained from equations (3.25) through (3.27).

Compared to the conventional approach, the proposed approach can provided a better optimization result. By this approach, the objective function (3.30) contains terms related to σ_i^2 and σ_{ij}, $i, j = 1, 2, 3$, which are caused by parameter variations. This term can be minimized by the optimization solution. However, with the conventional approach, the tool wear variations caused by parameter changes are allocated to the single, uncontrollable noise term σ^2 in the objective function (3.28). The optimization solution can not be adjusted according to the effects of parameter noise while the proposed method does. Therefore, the optimization result by the conventional method will be worse than the proposed method. The following simulation will show the difference.

For simplicity, we use the 3-rd order polynomial model to fit the empirical tool wear data here. Firstly, 20 sets random tool wear data are generated by using Matlab which are similar to the tool wear process tested by Shiraishi [8] and Ravindra et al. [9]. These data can be generated by a polynomial function with random changed coefficients plus a white noise term. Shown in Figure 3.2a, each trial of data has a certain trend but with random components. They represent the tool wear data during a tool life under similar but different working conditions. The fitted $R(t)$ curves for these processes are shown in Figure 3.3. The corresponding parameters and residuals are shown in Table 3.1. For each process, The unit time production cost curves are shown in Figure 3.4. In a practical situation, due to the changes of working conditions, each curve has a probability to show up even though the specific distribution is unknown. These curves show different minimum points. A robust replacement strategy is to make the best trade-off between these points.

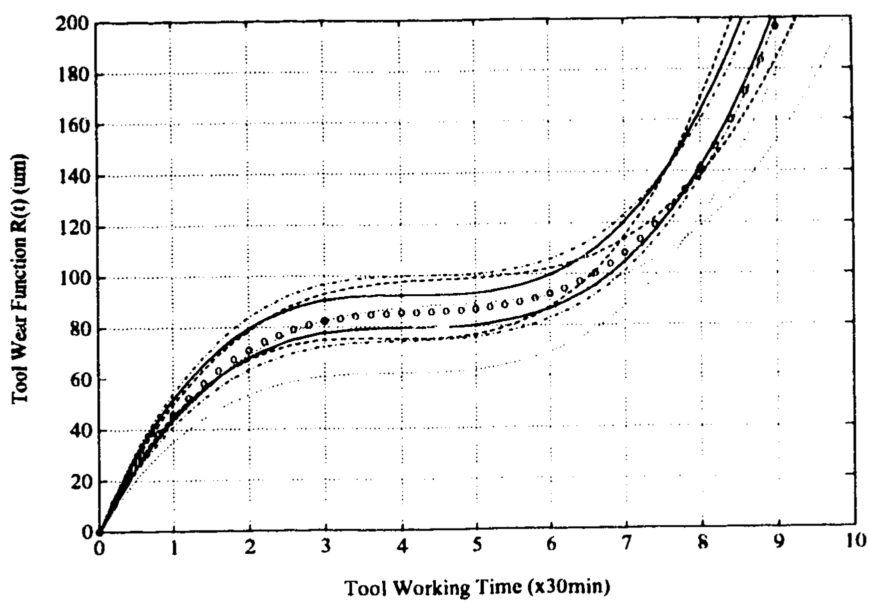

Figure 3.3:The fitted tool wear curves

From the parameters of the 20 different curves, the covariance matrix of b_1, b_2, b_3 can be obtained as Table 3.2:

Table 3.1:The Parameters and Residuals of Fitted Tool Wear Curves

20 Curves	b_1	b_2	b_3	σ
1	42.7190	-10.1719	0.8326	23.6905
2	60.2522	-14.2397	1.1302	13.1186
3	54.2524	-14.5976	1.3191	13.6722
4	64.6501	-13.9753	1.0149	11.3031
5	39.8398	-8.84640	0.6826	16.1532
6	59.1858	-13.4729	1.0313	6.87052
⋮	⋮	⋮	⋮	⋮
20	61.3036	-14.9659	1.2134	12.5708
Means of 20	56.1338	-13.0607	1.0319	11.7799
Conventional	b_1	b_2	b_3	σ
o	56.1338	-13.0607	1.0319	536.8806

Table 3.2:Covariance Matrix of b_i, $i = 1, 2, 3$

σ_{ij}	$i = 1$	$i = 2$	$i = 3$
j=1	67.7973	-14.691	0.9995
j=2	-14.691	1.6849	-0.455
j=3	0.9995	-0.455	0.0503

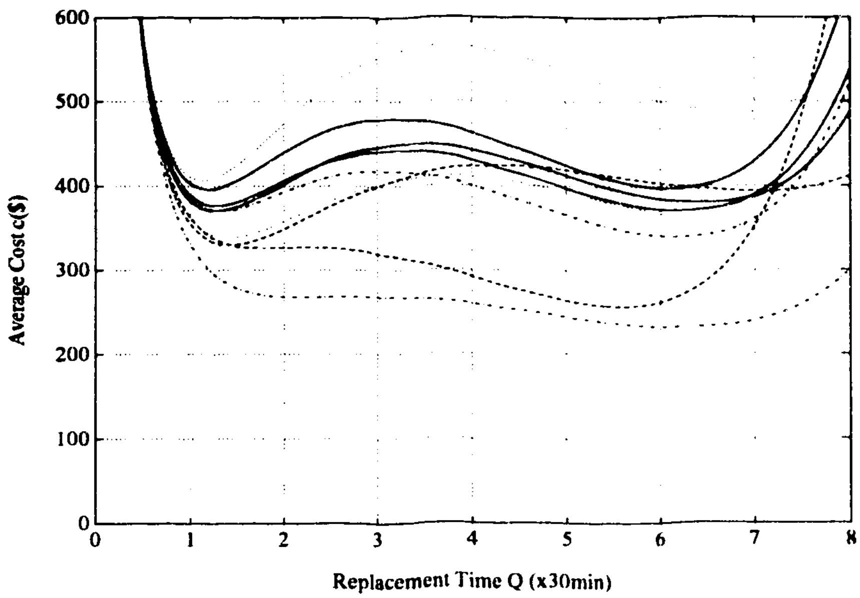

Figure 3.4:The unit time cost curves for each process

By using the proposed method, in Figure 3.5, the objective function is shown as the curve marked with '*'. The minimum point $Q^* = 1.35$ on this curve is the optimal replacement point. Applying this replacement policy under 20 different tool wear process will result in different unit costs with an average of \$362.64.

By the conventional approach, these 20 trials of data are used to fit a single tool wear curve marked with 'o' in Figure 3.3. Shown in Table 3.1, the parameters of this curve are the same with the mean values of corresponding parameters of 20 different curves. According to this tool wear curve, in Figure 3.5, the unit time cost function is shown as the curve marked with 'o'. Simply minimizing it will give a replacement point $Q_o = 6.25$. Applying this replacement policy under 20 different tool wear process will result in different unit cost with an average of \$423.55.

By Comparison, we can see that replacing tools at Q^* results in a lower production cost.

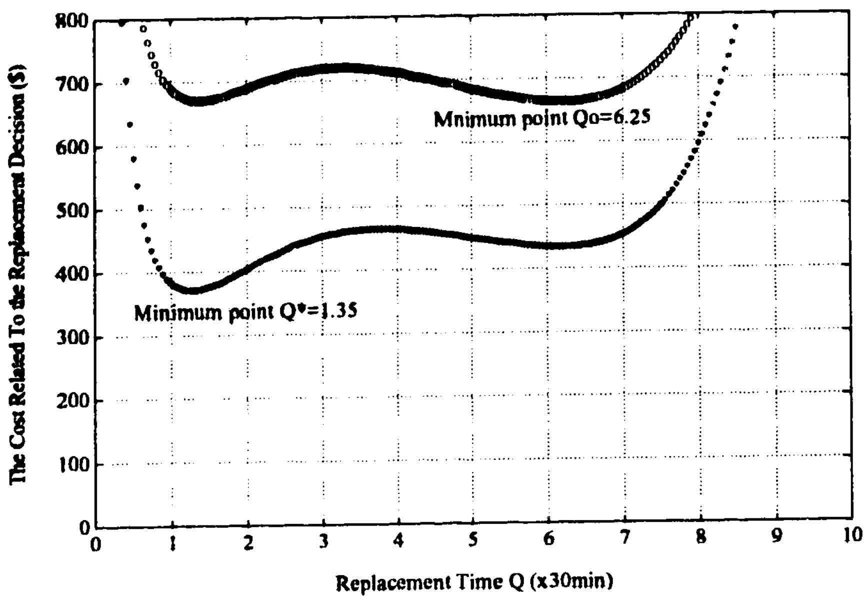

Figure 3.5:The comparison of results by two different approaches

3.4 Conclusion

This chapter studied the optimal tool replacement strategy, based on non-decreasing tool wear functions fitted from empirical tool wear data. When the tool is working under an identification working condition with a known tool wear function, the optimal tool replacement strategy is analyzed to correct the result obtained by Jeang and Young [2]. When the tool wear model is uncertain due to working condition changes, and on-line tool wear identification is not applicable, a new objective function model and a robust optimal replacement strategy have been presented. Compared to the conventional approach which simply averages the parameters of the tool wear function by allocating all parameter variations to a single noise term, the proposed method provides better solution to minimize the average unit production cost. The results developed in this chapter can be used in the off-line decision making in the production planning stage.

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Chapter 4

Off-line Tool Adjustment Decision Making

4.1 Introduction

This Chapter discusses off-line decision making for tool adjustment in manufacturing. In-process tool adjustment can reduce the dimension deviation of a workpiece from the target value caused by tool wear. With in-process adjustment, the tool could be replaced less frequently with the improved workpiece quality. Usually, in-process adjustment can be performed at much less cost than tool replacement. Compared to tool replacement, tool adjustment could save not only the tool cost but also the production down time for the tool installation procedures. Therefore, the ultimate benefit from a good tool adjustment strategy is the reduction of the unit time cost including quality loss.

Considering both tool replacement cost and product quality loss, a few researchers have reported optimal tool replacement strategies minimizing expected

[0] A version of this chapter was published in the Proceedings of the 3rd Industrial Engineering Research Conference, May 18-19, 1994, pp7-12.

unit time cost [1, 2, 3]. A complete review of these works was presented in Chapter 3. However, few papers addressed optimal tool adjustment in a tool wear process. Regarding quality control in a manufacturing process, most papers devote either to the design of Statistical Process Control (SPC) charts or to the analysis of process capability index, without considering the predictable tool wear pattern [4, 5]. Quesenberry [6] provides an SPC approach for quality improvement under linear tool wear, which aims at minimizing the expected mean square error of the part measurements from the target value. It does not consider either the unit production cost or the process adjustment cost. Galante and Lombardo [2] reports a study of tool replacement with adaptive control for a generalized process using an unlimited number of tools. They assume that the process is a non-stationary non-periodic stochastic process, without information regarding the marginal cost beforehand. The result is applicable to optimal tool adjustment if the tool wear process is a similar stochastic process. However, tool wear usually follows a predictable non-decreasing trend [7, 3]. The algorithm provided by Galante and Lombardo will not give a real optimal tool adjustment strategy in a tool wear process.

In this chapter, we will discuss the off-line decision making for tool adjustment and replacement in a non-decreasing tool wear process. We will use a non-decreasing function $R(t)$ tc fit the empirical tool wear data and use Taguchi quadratic quality loss function to estimate the economic loss caused by tool wear.

The economic quality loss L_t due to the deviation of a part's dimension from its target value can be expressed as:

$$L_t = k(X_t - T)^2 \tag{4.1}$$

where X_t is the measurement of the part at time t, T is the target value for the part dimension, and k is cost coefficients determined by the requirements of the customers [8].

4.2 Formulation Of Optimal Tool Adjustment Model

A tool adjustment decision includes the selection of the number of adjustment/replacement times J and the corresponding adjustment time intervals T_j ($j = 1, 2, \ldots, J$). The J-th adjustment is considered to be the replacement of the tool. In mass production, the tool wear information $R(t)$ can be obtained either from controlled experiments or from past tool wear data. An optimal tool adjustment scheme can be determined off-line in the production planning stage. The implementation of the optimal tool adjustment scheme in production does not require additional on-line hardware.

In developing optimal tool adjustment model, we assume that (1) the tool always works under identical machining conditions, (2) tool failure does not occur within the planning horizon, and (3) the tool is always adjusted to the fixed initial setting offset a_t. Then, from the $(j-1)$-th to the j-th adjustment, for a duration of T_j, the expected quality loss can be expressed as:

$$L_j = \int_0^{T_j} k[a_t + R(t + \sum_{i=1}^{j-1} T_i) - R(\sum_{i=1}^{j-1} T_i)]^2 dt. \tag{4.2}$$

The expected cost per unit of time during a single tool's life is:

$$\begin{aligned} C &= \frac{C_r + (J-1)C_a + \sum_{j=1}^{J} L_j}{\sum_{j=1}^{J} T_j} \\ &= \frac{C_r + (J-1)C_a}{\sum_{j=1}^{J} T_j} + \\ &\quad \frac{\sum_{j=1}^{J} \int_0^{T_j} k[a_t + R(t + \sum_{i=1}^{j-1} T_i) - R(\sum_{i=1}^{j-1} T_i)]^2 dt}{\sum_{j=1}^{J} T_j}. \end{aligned} \tag{4.3}$$

This objective function is subject to constraints $T_j > 0$ for $j = 1, 2, \ldots, J$

With the decision variables J and T_j's, this is a sophisticated multivariable optimization problem. It is impossible to solve it analytically. To simplify the decision process, we take the following two-stage optimization approach in this chapter.

- Stage one: fix the J values at $1, 2, \ldots$, respectively and find the corresponding optimal adjustment time intervals T_j's under each J value.
- Stage two: choose the J value with the minimum unit cost.

Since stage two is easy to perform, we will concentrate on stage one in the following sections.

4.3 Optimal Tool Adjustment Strategy With Fixed J

Even if J is fixed, the objective function (4.3) is still a multivariable function which is hard to optimize. For example, with $J = 3$, the necessary conditions for T_j's to be an optimal solution should be:

$$\frac{\partial C}{\partial T_3} = \frac{k[a_t + R(T_1 + T_2 + T_3) - R(T_1 + T_2)]^2 - C}{T_1 + T_2 + T_3} = 0$$

$$\Rightarrow k[a_t + R(T_1 + T_2 + T_3) - R(T_1 + T_2)]^2 = C \tag{4.4}$$

$$\begin{aligned} \frac{\partial C}{\partial T_2} &= \frac{k[a_t + R(T_1 + T_2) - R(T_1)]^2 - C}{T_1 + T_2 + T_3} - \\ &\quad \frac{\frac{\partial C}{\partial T_2} \int_0^{T_3} k[a_t + R(T_1 + T_2 + t) - R(T_1 + T_2)]^2 dt}{T_1 + T_2 + T_3} = 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow\ & k[a_t + R(T_1 + T_2) - R(T_1)]^2 = C - \\ & \frac{\partial C}{\partial T_2} \int_0^{T_3} k[a_t + R(T_1 + T_2 + t) - R(T_1 + T_2)]^2 dt \end{aligned} \tag{4.5}$$

$$
\begin{aligned}
\frac{\partial C}{\partial T_1} &= \frac{k[a_t + R(T_1)]^2 - C}{T_1 + T_2 + T_3} - \\
&\quad \frac{\frac{\partial C}{\partial T_1}\int_0^{T_2} k[a_t + R(T_1 + t) - R(T_1)]^2 dt}{T_1 + T_2 + T_3} - \\
&\quad \frac{\frac{\partial C}{\partial T_1}\int_0^{T_3} k[a_t + R(T_1 + T_2 + t) - R(T_1 + T_2)]^2 dt}{T_1 + T_2 + T_3} = 0
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\ & k[a_t + R(T_1 + T_2) - R(T_1)]^2 = C - \\
& \frac{\partial C}{\partial T_1}\int_0^{T_2} k[a_t + R(T_1 + t) - R(T_1)]^2 dt - \\
& \frac{\partial C}{\partial T_1}\int_0^{T_3} k[a_t + R(T_1 + T_2 + t) - R(T_1 + T_2)]^2 dt
\end{aligned}
\tag{4.6}
$$

One special case occurs when $R(t)$ is linear. With a linear tool wear trend, a unique optimal solution can be obtained analytically. When $R(t)$ is linear,

$$R(\sum_{i=1}^{j} T_i + t) - R(\sum_{i=1}^{j} T_i) = R(t) \tag{4.7}$$

We can observe from equations (4.4), (4.5), and (4.6) that:

$$
\begin{aligned}
& \frac{\partial C}{\partial T_j} = \frac{k[a_t + T_j]^2 - C}{T_1 + T_2 + T_3} = 0 \\
\Rightarrow\ & k[a_t + R(T_j)]^2 = C, \quad j = 1, 2, \ldots, J
\end{aligned}
\tag{4.8}
$$

Consider the second order derivatives at a stationary point:

$$
\begin{aligned}
\frac{\partial^2 C}{\partial T_j^2} &= -\frac{k[a_t + R(T_j]^2 - C}{(T_1 + T_2 + T_3)^2} + \frac{2k[a_t + R(T_j)]r(T_j) + \frac{\partial C}{\partial T_j}}{T_1 + T_2 + T_3} \\
&= \frac{2k[a_t + R(T_j)]r(T_j)}{T_1 + T_2 + T_3} \\
\frac{\partial^2 C}{\partial T_i \partial T_j} &= -\frac{k[a_t + R(T_j)]^2 - C}{(T_1 + T_2 + T_3)^2} + \frac{\frac{\partial C}{\partial T_j}}{T_1 + T_2 + T_3} = 0
\end{aligned}
$$

The Hessian matrix is:

$$
\begin{aligned}
\mathbf{H} &= \begin{pmatrix} \frac{\partial^2 C}{\partial T_1^2} & \frac{\partial^2 C}{\partial T_1 \partial T_2} & \cdots & \frac{\partial^2 C}{\partial T_1 \partial T_J} \\ \frac{\partial^2 C}{\partial T_2 \partial T_1} & \frac{\partial^2 C}{\partial T_2^2} & \cdots & \frac{\partial^2 C}{\partial T_2 \partial T_J} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 C}{\partial T_J \partial T_1} & \frac{\partial^2 C}{\partial T_J \partial T_2} & \cdots & \frac{\partial^2 C}{\partial T_J^2} \end{pmatrix} \\
det(\mathbf{H}) &= \prod_{j=1}^{J} \frac{2kr(T_j)}{\sum_{j=1}^{J} T_j} > 0
\end{aligned}
$$

Therefore, the stationary point which satisfies the equation group (4.8) will be a minimum point. The unique optimal solution $T_1 = T_2 = \ldots = T_J = T^*$ can be found by solving the equation group (4.8):

$$
k[a_t + R(T^*)]^2 = C = \frac{C_r + (J-1)C_a + J\int_0^{T^*} k[a_t + R(t)]^2 dt}{JT^*} \tag{4.9}
$$

Equation (4.9) provides an optimal tool adjustment strategy when the average tool wear is linear. However, a tool wear process usually follows a non-decreasing, non-linear trend, consisting of three distinct periods: initial wear period, normal wear period, and accelerated wear period (Figure 4.1). Under such a condition, the original multivariable optimization problem can only be solved by numerical search algorithms.

Numerical search is often used when a mathematical model is too complicated to be solved analytically. However, a potential risk of using a numerical search method is that the result obtained may be a local optimum rather than a global optimum. A numerical algorithm is usually quite sensitive to the initial values and search step length. The next section describes a few heuristics for solving the optimization problem.

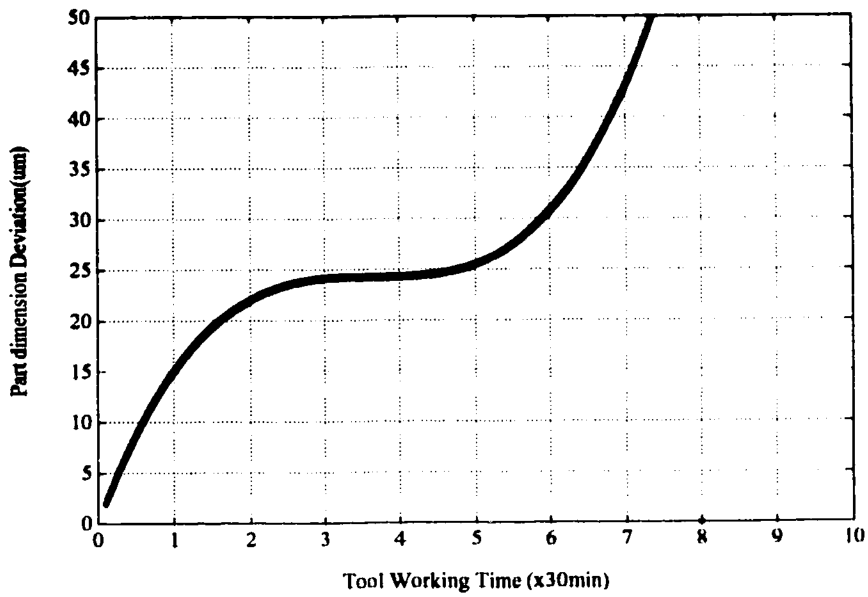

Figure 4.1:Non-linear tool wear trend

4.4 Discussion Of A Few Heuristic Tool Adjustment Methods

Since the original multivariable optimization problem is hard to solve, one idea is to simplify it to a single variable optimization problem with fixed J. We have listed three heuristic methods below:

1. Scheduled equal time interval adjustment: That is to force $T_1 = T_2 = \ldots = T_J$ (Figure 4.2). This method is easy to manage in production. However, it may not give good optimization results unless $R(t)$ is linear. As shown in Figure 4.2, the adjustment policy resulted from this method is obviously unreasonable since the tool wear amount during the second adjustment cycle is much smaller than that during the first adjustment cycle.

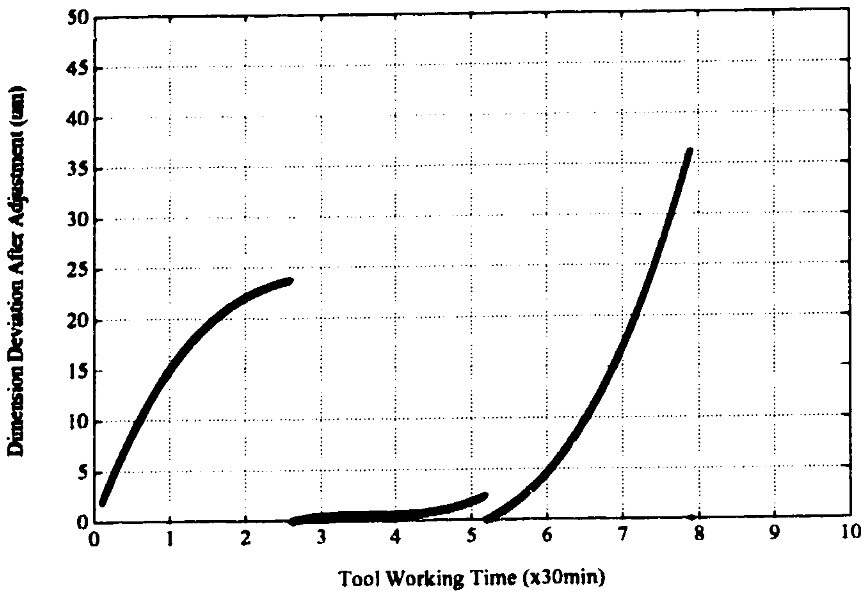

Figure 4.2:Scheduled equal time interval adjustment

2. Scheduled equal tool wear amount adjustment: The tool will be adjusted whenever the average tool wear amount reaches a certain level (Figure 4.3). That is to choose adjustment time interval T_j according to $T_j = R^{-1}(\frac{jR(\sum_{i=1}^{J} T_i)}{J})$. With the consideration of adjustment cost and replacement cost, this method also complies with the basic idea of SPC, which adjusts the process whenever the deviations of the workpiece dimension exceeds a control chart limit.

3. Cumulative interval optimization method: This method considers one additional tool adjustment cycle at a time, starting with cycle 1. When adjustment cycle j is considered $(1 \leq j < J)$, the only decision variable is T_j. The solutions T_i^* and L_i^* $(1 \leq i < j)$ obtained for previous adjustment cycles are treated as constants. The objective function to be minimized is the unit time

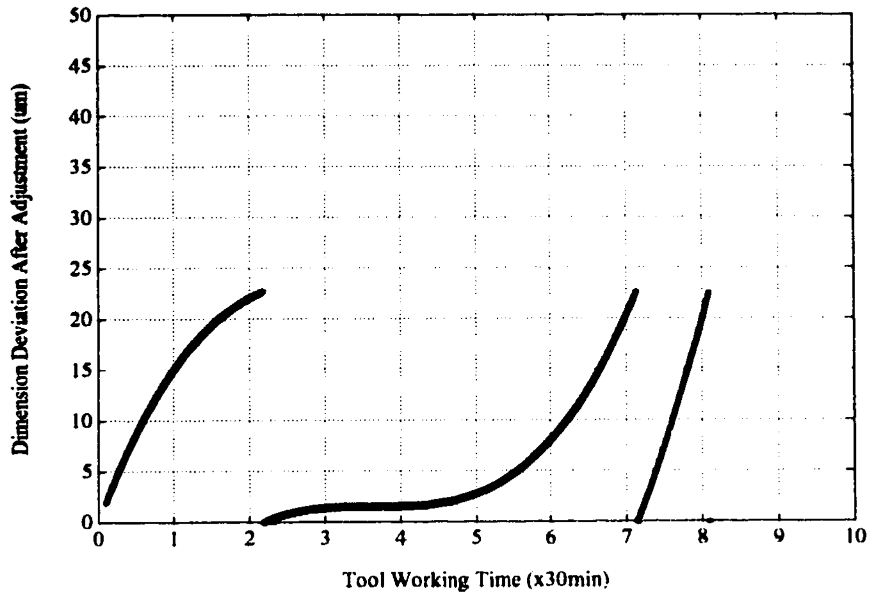

Figure 4.3:Scheduled equal tool wear amount adjustment

cost for the first j cycles, i.e.,

$$C_j = \frac{\sum_{i=1}^{j-1}(C_a + L_i^*) + C_a + L_j}{\sum_{i=1}^{j-1} T_i^* + T_j}.$$

When adjustment cycle J is considered ($j = J$), one of the J C_a's in function C_j is replaced by C_r. See Figure 4.4 for a set of results obtained with this method. Solving the optimization problem for each adjustment cycle can be done by applying the optimal tool replacement strategy developed in Chapter 3 with a little modification to include past information. This method can be used for on-line decision making when the past adjustments have been performed and further tool wear information is uncertain. The details are omitted here.

For comparison purposes, we use an optimization program 'constr.m' provided in Matlab [9] to perform the numerical search for the original multivariable opti-

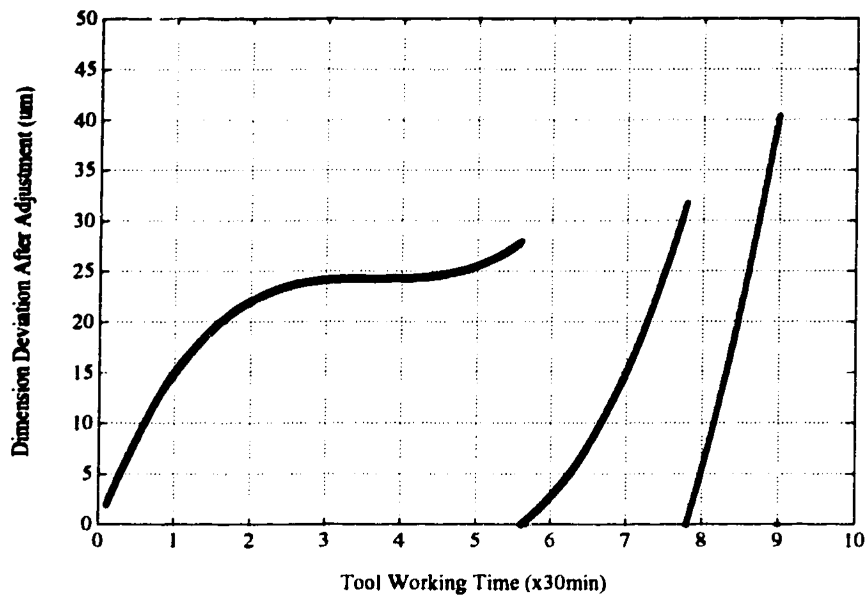

Figure 4.4:Cumulative interval optimization adjustment

mization problem. The algorithm is based on a Sequential Quadratic Programming (SQP) method. An estimation of the Hessian matrix of the Lagrangian function is updated at each iteration by using a gradient based line search method. The termination tolerance for the objective value is 0.0001. Two simulated cases with the specified tool wear functions are used to compare the results obtained from those three heuristic methods and the numerical search method (Table 4.1 and Table 4.2).

From the tables we can observe the following:

- $J = 1$, which represents the case of replacement without adjustment, results in high unit costs in both cases.
- When J increases beyond a certain level, the objective function value increases along with J.

- The numerical search method usually provides better results than the heuristic methods. However, it can not guarantee the best results all the time. In Table 4.2, when $J = 4$, the numerical method provides an adjustment policy with an objective function value 72.9, which is worse than 72.3967, obtained from heuristic method 2. With the solution generated with heuristic method 2 as a new starting point, the numerical search method finds a better solution with an objective function value 70.9.

- Among the three heuristic methods, Method 2 provides an adjustment policy with the lowest cost in most cases.

- In the case of Table 4.1, we should choose the strategy with $J = 3$, i.e., adjusting the tool at time intervals $T_1 = 0.96$ and $T_2 = 2.46$ and replacing the tool at the third time interval $T_3 = 7.26$, with the minimum unit time cost of 79.1.

- In the case of Table 4.2, we should choose the strategy with $J = 2$, i.e., adjusting the tool at the first time interval $T_1 = 1.79$ and replacing the tool at the second time interval $T_2 = 7.47$, with the minimum unit time cost of 57.6.

4.5 Conclusion

In-process adjustment of cutting tools is very important in manufacturing toward production cost reduction and quality improvement. For off-line decision making in the production planning stage, when tool wear can be represented by a fitted tool wear function, this chapter develops an optimal tool adjustment and replacement model to minimize unit time production cost. The formulated multivariable optimization problem can be solved by a numerical search method. However, a numerical search is sensitive to the initial value and search step length. It may not provide the global optimal solution. Additionally, as J increases, the number of decision variables increases, hence increasing the complexity of the search algorithm. It is not a good method for industry use.

Several simplified single variable heuristic tool adjustment methods are presented and compared with the numerical search method. Among these heuristic methods, the scheduled equal tool wear amount adjustment method seems to provide the best results. It is a single variable optimization problem which can be solved easily in practice. With this method, a simple algorithm code can be incorporated into any production management software in industry.

Table 4.1: The Comparison of Heuristic Method Solutions and Optimal Search Solution When $R(t) = 50t - 12.47t^2 + t^3$, $C_a = 100$, $C_r = 270$, $k = 0.06$

J = 1	T_1	T_2	T_3	T_4	ObJ
All Methods	2.3				227.7000
J = 2	T_1	T_2	T_3	T_4	ObJ
Method 1	2.95	5.90			133.7459
Method 2	1.67	7.50			87.3723
Method 3	1.40	6.70			84.8000
Num. Search	1.67	6.95			84.7000
J = 3	T_1	T_2	T_3	T_4	ObJ
Method 1	2.33	4.66	7.00		107.1061
Method 2	0.80	2.73	7.20		80.0800
Method 3	1.40	6.50	7.9		89.1756
Num. Search	0.96	2.46	7.26		79.1000
J = 4	T_1	T_2	T_3	T_4	ObJ
Method 1	1.90	3.80	5.70	7.60	103.1113
Method 2	0.78	2.50	7.12	8.00	85.6232
Method 3	1.40	6.50	7.80	8.70	95.2337
Num. Search	0.93	2.37	7.01	8.06	84.8000

Table 4.2: The Comparison of Heuristic Method Solutions and Optimal Search Solution When $R(t) = 20t - 5.4772t^2 + 0.5t^3$, $C_a = 100$, $C_r = 270$, $k = 0.06$

J = 1	T_1	T_2	T_3	T_4	ObJ
All Methods	6.50				74.1500
J = 2	T_1	T_2	T_3	T_4	ObJ
Method 1	3.80	7.60			64.5000
Method 2	2.09	7.10			58.7683
Method 3	5.60	8.00			71.6633
Num. Search	1.79	7.47			57.6000
J = 3	T_1	T_2	T_3	T_4	ObJ
Method 1	2.60	5.20	7.90		71.0700
Method 2	2.19	7.14	8.10		65.1371
Method 3	5.60	7.80	9.00		76.9600
Num. Search	1.72	7.22	8.45		63.9000
J = 4	T_1	T_2	T_3	T_4	ObJ
Method 1	2.10	4.20	6.30	8.40	78.9625
Method 2	2.31	7.18	8.14	8.80	72.3967
Method 3	5.60	7.80	8.90	9.80	83.0370
Num. Search	0.97	2.67	6.69	7.88	72.9000

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Chapter 5

Conclusions

This thesis integrates several studies in the optimal tool management area, including tool wear monitoring, tool wear function identification, on-line decision making, and off-line decision making for tool replacement/adjustment in a machining process. The objective of these studies is to minimize the unit time production cost considering both economic quality loss and replacement/adjustment cost.

A systematic approach is proposed for on-line tool wear process management when the prior information about the tool wear trend is not accurate due to working condition changes. In addition to on-line monitoring by using EWMA control chart, a new identification method is proposed to identify the new tool wear function parameters under the changed working condition. We have demonstrated that, this method is insensitive to the process noise, and faster to converge than the least square method. Secondly, the parameter estimation error during the identification beginning period by the proposed method is much less than that by the least square method or by the prior information. These features are very important for effective on-line decision making. With updated tool wear function available, the optimal tool replacement/adjustment decision can be made on-line, specifically for every single tool, without extensive prior data collection or controlled experiments. Therefore, considerable cost saving can be achieved. The implementation of this approach does not need extensive hardware support except dimension measurements of the finished parts and a personal computer.

Off-line tool replacement decision making is also studied in this thesis. When the tool works under identical working conditions with a known tool wear function, the optimal tool replacement strategy is developed. When the tool working under varied but similar working conditions, and the on-line identification is not applicable, a new objective function model and a robust optimal replacement strategy is proposed. Compared to the conventional approach which simply averages the parameters of the tool wear function under different working conditions, the proposed approach provides better tool replacement solution to minimize the average unit time production cost.

Off-line decision making for in-process adjustment of a cutting tool is analyzed in this thesis. The objective function is developed with a know tool wear function. Since an analytical solution is impossible for this formulated multivariable optimization problem, a numerical search method can be used to search the optimal solution. However, the numerical solution is sensitive to the initial value and the search step length, and may not give the global optimal solution. Several simplified single variable heuristic tool adjustment methods are presented and compared with the numerical search method. Among these heuristic methods, the scheduled equal tool wear amount adjustment method seems to provide the best results. It is a single variable optimization problem which can be solved easily for industry application. Furthermore, this heuristic method can also be used to provided the initial value for the numerical search algorithm to obtain a better solution.

Along the line of this thesis, further studies can be done in the tool management area. A short list follows:

- How to design the best EWMA control chart specifically for on-line tool wear monitoring.

- For the implementation of the proposed tool wear function identification algorithm, how to select the best weighting factors k_is according to different confidence levels on the prior information.

- Investigation of better heuristic method for in-process tool adjustment decision making.

This thesis studies tool management based on a non-linear tool wear function obtained from prior information, on-line identification, or controlled experiment. The tool has to work under similar working conditions. Another research direction is to incorporate the most critical working conditions into the tool wear function, such as cutting speed, cutting force, tool temperature, etc., hence providing a more clear relationship between the tool wear and those physical parameters. The tool wear will then be more predictable even under different working conditions and the optimal tool management can be achieved correspondingly.