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DISTANCE METRICS IN SIMILARITY JUDGMENTS OF MULTIDIMENSIONAL STIMULI

by

© ANDREW KINLOCH CLARK

A THESIS

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IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE

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THE UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES AND RESEARCH

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Distance Metrics in Similarity Judgments of Multidimensional Stimuli," submitted by Andrew Kinloch Clark in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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Abstract

The spatial or geometric model in multidimensional scaling which relates perceived distances to psychological space was studied for university students and Grade VII pupils.

Similarity judgments of sets of multidimensional stimuli of varying degrees of complexity were obtained by a modified version of the method of multiple ratios. The similarity judgments were analyzed by nonmetric multi-dimensional scaling procedures and by direct analysis of distance estimates.

The Minkowski r-metric for the group data on the bidimensional set of stimuli was close to a value of one, or city block metric, for both groups of subjects. On the four-dimensional set of stimuli the values of the r-metric were close to two and three for child and adult group respectively. The individual data for both groups of subjects showed a wide range of distance metrics on both sets of stimuli.

It was concluded that variations in distance metrics between subjects of different ages was not independent of the number and type of dimension incorporated in the set of multidimensional stimuli.

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Chapter 1

I. Introduction

Many investigators have attested to the importance of multidimensional scaling techniques in the research on cognition, perception, and related processes. Isaac (1970) has stated that similarity judgments and multidimensional configurations are indices of perceptual structure. It has been argued by Cliff and Young (1968) that when an individual makes similarity judgments about a set of stimuli, he has a psychological map of the stimuli, and multidimensional scaling enables the researcher to determine this map. Fenker and Brown (1969) made use of multidimensional scaling techniques in their study of conceptual space and cognitive efficiency. Beals, Krantz and Tversky (1968), while criticizing the manner in which multidimensional scaling techniques are sometimes applied, recognized the value of these techniques in research.

> Similarity between stimuli such as words, forms, or colors, contains important information about the way the stimuli are perceived and coded. Hence similarity judgments provide a valuable tool in the study of perception and cognition (p. 127).

In like manner Hake (1966) and Hake and Rodwan (1967) strongly endorsed the use of multidimensional scaling in research in perception.

Some reservations. Although the ready availability of programs and computing facilities has resulted in what Zinnes has described as an avalanche of articles (1969) using multidimensional scaling techniques, there has been comparatively little research of a critical nature into the techniques. In a major paper on the theory underlying multidimensional scaling, Beals, Krantz and Tversky (1968) show that there are certain basic assumptions which must be satisfied in multidimensional scaling. They advocate empirical investigation of the extent to which these assumptions are actually being met by the present techniques. Similarly, although Behrman and his colleagues have conducted a series of studies on the application of multidimensional scaling to define perceptual space, they raised the question of the assumption that spatial models are appropriate tools for the analysis of perceptual judgments (Behrman and Brown, 1968).

The Distance Metric Problem

Multidimensional scaling, in general, involves two distinct operations: the determination of distances between the stimuli, and the determination of the dimensionality and the projections of the stimuli on those dimensions in psychological space. The use of similarity judgments is a well-established and dependable method for obtaining perceived distances between stimuli. The spatial or geometric model which relates the perceived distances to the psychological space has been the cause of some controversy. The controversy centers around the particular distance metric which characterizes the spatial model: Euclidean, city block, or dominance. When a subject makes a similarity judgment of multidimensional stimuli, how does the overall judgment of similarity depend on differences on the dimensional components? The problem may be considered from two viewpoints: (a) which distance metric model provides the best fit to the similarity judgments, or (b) which distance metric are subjects using when they make similarity judgments? Probably both viewpoints have some validity in certain cases; a Kruskal (1969) analysis at various distance metrics of stimuli of unknown dimensionality might be more relevant to the data model viewpoint, whereas an analysis of distance settings as in Hyman and Well (1967) might be considered to be concerned with the behavior of the subject.

Distance Metrics

Regardless of the particular distance metric used, any distance function is characterized by certain properties (Beckenbach and Bellman, 1961):

1. Translation invariance: the distance between two points depends only on the differences x_1-x_2 and y_1-y_2 of their coordinates.
2. Symmetry: the distance from point A to point B equals the distance from B to A.
3. Triangular inequality: the distance from A to C is not less than the sum of the distances from A to B and B to C.
4. Positivity: the distance between any two points is nonnegative.
5. Homogeneity: if A has coordinates (x, y) and B has

coordinates (*a*x, *a*y) then the distance from the origin to B equals the product of *a* and the distance from the origin to A.

Euclidean metric. The Euclidean distance metric proposes that the perceived distance between two multidimensional stimuli is described by the Pythagorean distance theorem. That is, for two dimensions, the distance between two stimuli is the square root of the sum of the squares of the differences of the projections on the two dimensions. The Euclidean metric has, in addition to the five distance properties noted above, the property of *rotation invariance*; the distance between two points remains unchanged with angular rotation of the configuration about the origin.

City block metric. The city block metric states that the perceived distance between the multidimensional stimuli is obtained by adding the perceived distances on the component dimensions. The city block metric implies that the subject be able to analyze dimensional differences separately. This metric does not allow for rotation of the configuration.

Dominance metric. The dominance metric proposes that only one dimension is considered in the perceived distance between multidimensional stimuli. Thus for two dimensions, the distance between two stimuli would be described entirely in terms of the differences in projection on one or other of the dimensions, but not both. That is, the subject is attentive to only one aspect of difference, and he ignores any others in estimating the similarity between stimuli. Like the city block metric and unlike the Euclidean metric, the dominance does not have the property of rotation invariance.

Minkowski r-metrics. A class of distance metrics which contains the above-noted alternatives is the class of Minkowski r-metrics. The basic equation for the distance between two points is:

$$d(i,j) = \left[\sum_{m}^{p} \left(\left| a_{im} - a_{jm} \right| \right)^{r} \right]^{1/r} \qquad r \geqslant 1$$

where $a_{im} - a_{jm}$ is the difference between stimuli j and i on dimension m; p is the number of dimensions. When the Minkowski r value (M_r) is one, the formula gives the city block distance; when M_r is two the distance is the familiar Pythagorean formula for Euclidean distance; when M_r is infinity the formula gives the distance according to the dominance metric. It should be noted that intervening values of M_r between one and infinity correspond to distance metrics intermediate to the three well-defined alternatives.

A graphical representation is given at Figure 1 of some unit circles (locus of points one unit from the origin) for Minkowski r-metrics from one to infinity. Since the distance from the origin to any point on a curve equals one it is evident that on curve (1) this can only be achieved by adding the projections on X and Y, that is according to the city block metric. In curve (2) a point is on a curve whose formula is the locus of a circle: $X^2 + Y^2 = 1$, which gives

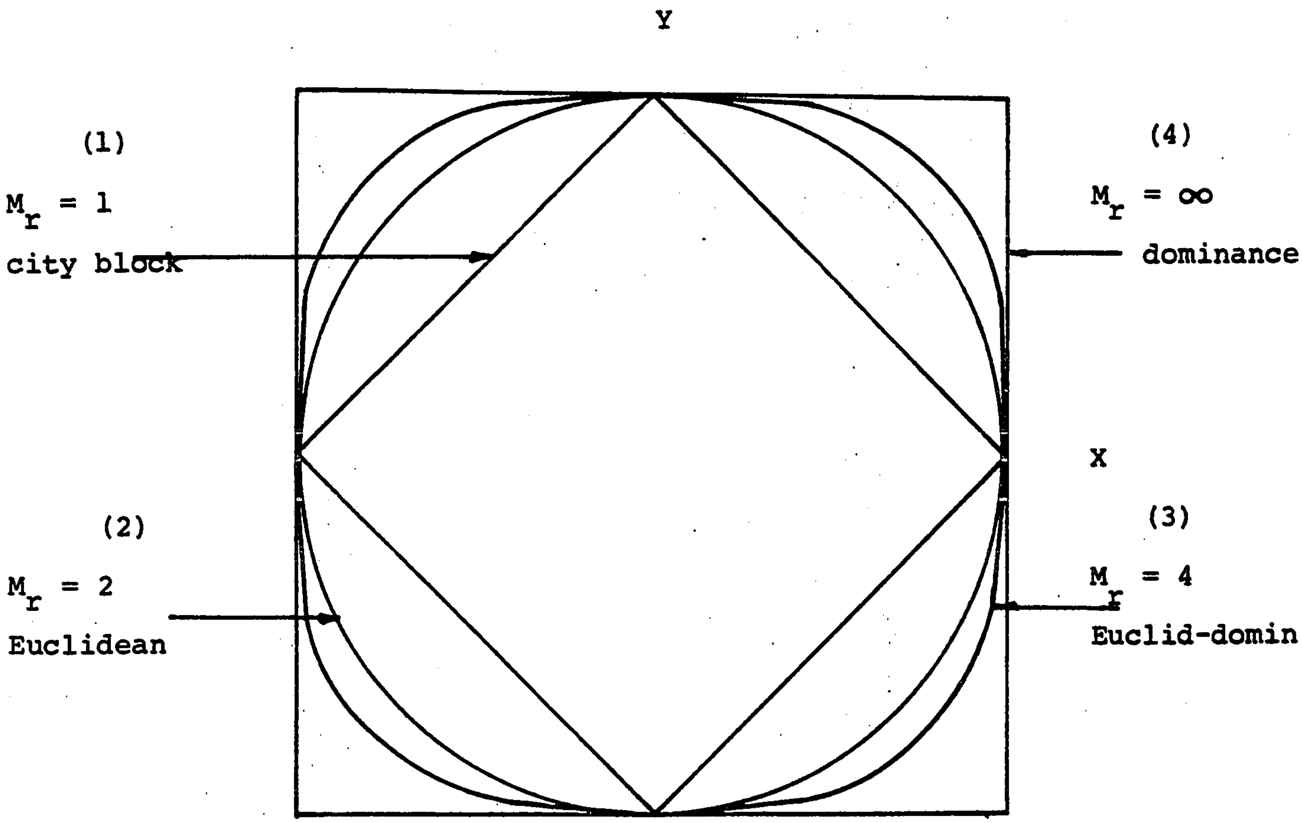


Figure 1. Unit circles at different Minkowski r-metrics.

Euclidean distance. In curve (3) the locus of the curve is between the circle and the square, indicating that the M_r value is between two and infinity, and that the metric is between Euclidean and dominance. In curve (4) the M_r value is infinity; only one dimension can contribute to a locus value of one on the curve.

Most of the discussion and controversy concerning the choice of distance metrics for various kinds of data has emphasized the choice between the city block and Euclidean models, but there is recent empirical evidence that the dominance metric may be applicable in some cases.

Some Implications of the Distance Metric Problem

For multidimensional scaling in general. In addition to the previously noted applications of multidimensional scaling in research into cognitive and perceptual processes, there have also been applications in such diverse fields as personality traits (Walters & Jackson, 1966); facial expressions (Abelson & Sermat, 1962); taste mixtures (Gregson, 1965); Morsecode symbols (Rothkopf, 1957); automobiles (Green *et al.*, 1969) and female body types (Wiggen, 1968). In view of the widespread applications of multidimensional scaling, it would appear highly desirable that the matter of the appropriate spatial model be clarified. It may well be that no great loss of scientific accuracy may be suffered by the assumption of an Euclidean metric. However, if the results of some studies can be more meaningfully interpretated by an alternate distance metric, then provision for such analysis should be provided. In short, an Euclidean metric should not be assumed in MDS analysis without some justification being given for the assumption.

For psychological research. In one of the few systematic investigations into the metric structure of the stimulus space, Shepard (1964) pointed out some direct implications of the appropriate distance metric for learning theory and discrimination problems. He proposed that inconsistencies in results of identification and classification learning studies was probably caused by a change in the distance metric corresponding to differences in the stimulus characteristics. In the problem of the determination of the maximum number of discriminably different stimuli in a given domain, for example, discriminable colors, the estimate would vary considerably with the metric structure of the psychological space.

Interpretation of Distance Metrics

Several explanations have been put forward to explain the different spatial models reported in research. These explanations are of three main types: erroneous data analysis, the nature of the multidimensional stimulus, and the attention state of the subject.

Hake (1966), after reanalysis of Attneave's (1950) experimental data, concluded that the data did not indicate a city block metric, as reported by Attneave, but that the results were equally consistent with the Euclidean metric. In like fashion Cross (1965) reexamined two studies on multidimensional stimulus generalization which had reported a

Euclidean metric (Buller, 1963; Jones, 1962). Cross believed that the data actually indicated a city block model rather than an Euclidean model.

Several authors have suggested that the spatial model which best describes the similarity judgments is the result of the type of multidimensional stimulus employed in the study. Torgerson (1958) first suggested that where the dimensions of the multidimensional stimulus were perceptually distinct, as in form, then the city block metric would be appropriate; where the stimuli were homogeneous in nature, as in the color domain, the Euclidean metric would be apparent. Shepard (1964) has named these types of stimuli 'analyzable' and 'unanalyzable' respectively. A similar idea is expressed by Handel (1967) who describes stimuli which vary along geometric dimensions as having a 'dimensional' structure, compared to stimuli which vary along color dimensions, which have a 'distance' structure. When a subject makes a similarity judgment of analyzable stimuli, he is presumed to note that the two stimuli differ by a certain number of units of shape, say, and by a certain number of units of size, with his similarity judgment being the sum of the dimensional differences. With unanalyzable or homogeneous stimuli, such as those differing only on value and chroma, his similarity judgment is related to the dimensional differences by the Pythagorean or Euclidean distance measure.

A theory which also considers the nature of the stimulus but differs in its explanation of how this causes different distance metrics has been put forward by Shepard (1964 and Micko and Fischer (1970). This theory states that the variation of spatial models is the result of the shift of the subject's attention state from one dimension to another in multidimensional stimuli. This shift in attention state is most pronounced if the dimensions are perceptually distinct.

Purpose of the Study

As the review of the literature will indicate, previous research concerning spatial models has utilized either color or geometric stimuli. The resultant findings have been influenced by this fact; as Hyman and Well (1967) observed:

> Unfortunately, the evidence for and against the Euclidean spatial model is completely confounded by the type of stimulus objects and the type of approach employed (p. 233).

Since the nature of the stimuli employed appears to bias the results in a particular direction, it would appear desirable to provide the experimental stimuli set with analyzable (form) and unanalyzable (color) dimensions. The stability of the manifested metric structure will be investigated by the use of two sets of stimuli, one set in which the form dimensions and color dimensions are correlated, resulting in an effective dimensionality of two, and another set in which the form dimensions and the color dimensions will be uncorrelated, resulting in a dimensionality of four.

In view of the educational implications of the use of similarity judgments in the investigation of perceptual and cognitive processes, there is a surprising lack of research into the relationship of the age variable and similarity judgments. An exploration of this relationship at this time appears necessary and valuable.

The purpose of the study is to investigate the following questions:

A. Is there any difference in the spatial models derived from similarity judgments of multidimensional stimuli made by adults and children?

B. Is there any difference in the spatial models derived from similarity judgments of individuals within the adult group and the child group?

C. How stable are the spatial models in each group as the configuration of the multidimensional stimulus sets is increased in complexity?

The study also proposes to explore some secondary questions concerned with the methodology of multidimensional scaling:

How effective is the Kruskal (1964, 1969) scaling program in determining the most appropriate value of the distance metric in a particular configuration?

How successful is the modified method of multiple ratios (Indow & Uchizono, 1960; Hyman & Well, 1967) in obtaining reliable similarity judgment data?

To a certain degree these questions are fundamental; if the data collection and data analysis procedures are unsatisfactory any conclusions concerning spatial models must be considered suspect.

Definition of Terms

Multidimensional stimuli: Stimuli which vary with respect to more than one attribute or dimension. The determination of the number of dimensions and the projections of the stimuli in each dimension are the objectives of multidimensional scaling. It is noted that the recovered multidimensional configuration based on subjects' similarity judgments need not correspond to the original physical configuration of the stimulus set.

Psychological distance: the perceived distance between two stimuli, obtained when an individual makes a similarity judgment. It is distinguished from two other distance measures:

1. Physical distance: the distance between stimuli derived from the physical coordinates of the stimuli on the dimensions, according to some distance metric.
2. Configuration distance: the distance between stimuli derived from the coordinates in the configuration which is the solution produced by the multidimensional scaling methods.

Psychological space: the multidimensional perceptual system in which stimuli are represented by points whose interpoint distance is psychological distance. The dimensions of psychological space need not correspond to the physical dimensions of the stimuli.

Distance metric: the particular distance function or formula by which psychological distance is specified; the possible metrics range from city block, through Euclidean to dominance.

Spatial model: the geometric model which specifies the characteristics of the psychological space. It is classified by the distance metric utilized.

Multidimensional scaling: the determination, given a set of interpoint distances, of the dimensionality of the multidimensional space in which the points are located and the projections on the dimensions of the points.

Attribute: a particular kind of property of a stimulus. For example, color is a multidimensional attribute and length is a undimensional attribute.

Dimension: a particular characteristic of an attribute. Thus length is a dimension as well as an attribute, whereas value and chroma are dimensions of the attribute color.

Complexity: for the present study refers to the number of independent dimensions in the stimulus set, not to any property of an individual stimulus. Thus the same stimulus could be part of two stimulus sets of vastly different complexity.

Chapter 2

Review of the Literature

The following review is classified under three main areas of concern: the development of multidimensional scaling, both metric and non metric, and some relevant issues thereto; the research bearing on one or other of the alternative distance metrics; and research studies involving similarity judgments and the variables of age, complexity and individual differences.

Multidimensional Scaling (MDS)

As stated earlier, there are two steps involved in MDS: the determination of the interpoint (or interstimulus) distances, and the determination of the dimensionality of the multidimensional space and the projections of the points on those dimensions. Measures of interpoint distance have been obtained from sources as diverse as correlation matrices (Guttman, 1966), confusion matrices (Hake, 1966), and measures of latency (Egeth, 1967). However, one of the most frequently employed methods is the use of similarity (or dissimilarity) judgments; indeed Shepard (1964) equates similarity and psychological distance, and Dember (1960) has defined similarity as the distance between points on an attribute. The data on similarity judgments may be obtained in various ways; indirect, as in the triadic methods of Richardson (1938) and Torgerson (1958), and direct, such as by rating stimuli on a scale of similarity (Abelson and Sermat, 1962) or by physical manipulation of the stimuli (Indow and Uchizono, 1960).

Metric MDS. The metric MDS procedures pioneered by Richardson and refined by Torgerson are still in widespread use in many areas of behavioral research. In the method of triads devised by Richardson (1938) to collect his data the subject is presented with three stimuli and asked to decide which two are most alike and which two are most different. In Torgerson's "complete method of triads" (1958) each triad is presented three times. On each presentation the subject is asked to select the stimulus from the other two stimuli which is most like the standard. By application of a procedure based on a generalization of Condition C of the law of comparative judgment (Torgerson, 1958, p. 165) a matrix of interpoint comparative distances is obtained. As absolute distances are required for the second stage of MDS, an additive constant must be determined and applied to yield a matrix of interpoint absolute distances.

To solve the problem of the dimensionality of the space and the projections on those dimensions Richardson made use of three theorems provided by Young and Householder (1938):

1. If a matrix B_i is positive semidefinite, the distances between the stimuli may be considered as distances between points lying in a real, Euclidean space, with origin at point i.

2. The rank of any positive semidefinite matrix B_i is equal to the dimensionality of the set of points.
3. If the rank of matrix B_i (where $B_i = AA'$) is equal to r, where r = (n-1), then matrix A is an (n-1) x r rectangular matrix whose elements are the projections of the points on r orthogonal axes with origin at point i of the r-dimensional, real, Euclidean space. (This theorem was later modified by Torgerson, who placed the origin at the centroid of all the points).

It is noted that no provision is made for other than an Euclidean space; if the latent roots of the matrix B_i are negative an imaginary space is implied.

Nonmetric MDS. Although manual methods of nonmetric MDS are feasible (Guttman, 1966; Coombs, 1964), the main thrust in nonmetric MDS dates from what Guttman (1967, p. 81) calls 'the computer breakthrough.' This was largely based on Shepard's papers on proximity analysis (1962). In brief, Shepard devised a procedure which utilized the constraints imposed by the rank ordering of interpoint distances to reconstruct the configuration. The success of the procedure was indicated by the degree of monotonicity in the relationship between the experimental similarities and the reconstructed distances.

Although Kruskal (1964, p. 2) states that he provided a 'solid logical foundation' for Shepard's technique in practice derived configurations using his MDSCAL program are almost identical to Shepard's. Kruskal's main contributions appear to be:

1. He supplied a quantitative measure, which he called the 'stress,' of the monotone criterion. The program strives to obtain a minimum value of stress.
2. Whereas Shepard's procedure assumed an Euclidean distance metric for the psychological space, Kruskal's procedure made provision for differing values of the Minkowski r-metrics, with Euclidean ($M_r = 2$) as one of the options.

Some of the proposed variations on the Kruskal-Shepard theme are merely programs which claim to obtain similar results in a more efficient manner, such as the TORSCA program (Young and Torgerson, 1967). The approach of Guttman-Lingoes and of McGee, however, appear to differ in some major respects from the Kruskal-Shepard techniques.

Of the many programs originated by Guttman and Lingoes, those known as smallest space analysis (SSA) are most similar to the Kruskal program (Guttman, 1967; Lingoes, 1968). A major difference in the Guttman-Lingoes program is that some of the information from the obtained similarity judgments is used in the initial configuration on which the computer program operates. This feature has been proposed as the reason for the less frequent occurrence of local minima problems in the Guttman-Lingoes program (Young and Applebaum, 1968; Spaeth and Guthery, 1969); the local minimum problem will be taken up in Chapter III.

There are several ostensible differences between

McGee's 'elastic' MDS and Kruskal's. McGee (1966) allows for greater error in similarity judgments, which indicates relatively greater separation of stimuli. It is this feature which contributes the term 'elastic' to the title of the method. Instead of the stress criterion McGee has a "Work" criterion which has a chi-square distribution. Despite these apparent improvements comparison of McGee and Kruskal solutions for the same input data (McGee, 1968) indicates that the derived configurations are almost indistinguishable.

Research On Distance Metrics

Research involving appropriate distance metrics may be divided into studies supporting one or other of city block, Euclidean or dominance metrics, plus comparative studies designed to investigate the conditions under which the alternative distance metrics might occur.

Favoring city block metric. Although his work was theoretical rather than empirical, Landahl (1946) was apparently the first to propose the city block distance metric in the judgment of similarity. Landahl, a mathematical biophysicist, explained similarity judgments in terms of the neural system. His theory was adapted by Attneave (1950) when he reported the first finding of the city block metric in an empirical study. Attneave's stimuli consisted of: Parallelograms varying in size, angularity and hue (one stimulus only); squares varying in area and reflectance; triangles varying in area and angularity. Although this single paper established the city block metric as an alternative to the Euclidean metric in MDS, it has been criticized in several respects. The geometric nature of the stimuli has been cited as a predisposing factor in the results (Torgerson, 1958; Shepard, 1964). The stimulus pattern has also been questioned as tending to emphasize the given dimensions (Torgerson, 1958; Hyman and Well, 1967). Finally, the data analysis has been strongly criticized by Hake (1966) as being nonconclusive.

Surprisingly, in view of the interest aroused by Attneave's study, there was no further empirical investigation on distance metrics until Shepard's work (1964), although a similar approach was being studied in research into multidimensional stimulus generalization. Shepard's stimuli consisted of a set of circles of varying diameters, each circle with a radius drawn in of varying inclination. Shepard concluded that the results indicated a distance metric between the city block and Euclidean metrics.

The city block metric also received support from a study by Cross (1965) concerned with generalization of multidimensional stimuli. In addition to criticizing previous studies with Euclidean findings, such as that of Jones (1962), Cross presented results which favored the city block metric as the most accurate predictor of the generalization surface of multidimensional stimuli. (This generalization surface corresponds to the unit circles of Figure 1).

Favoring Euclidean Metric. As noted earlier, the experiments of Richardson and Torgerson were based on analytic procedures which made no provision for a spatial model other than a Euclidean.

In a series of MDS studies carried out by Brown and colleagues (Behrman and Brown, 1968; Brown and Andrews, 1968; Brown and Brumaghim, 1968) the general conclusion was that the Euclidean metric provided a better fit to the data than the city block metric. The stimuli used were four sided randomly generated polygons. Employing similar stimuli, Stenson (1968) came to the same conclusion. A feature of these studies is that they appear to contradict the previously-noted popular theory that geometric stimuli will result in a city block metric.

From a study of stimulus generalization at various dimensionalities, Jones (1962) concluded that the Euclidean distance metric was a more successful predictor of the multi-dimensional generalization surface than was the city block metric. The stimuli used by Jones consisted of: lines varying in length and inclination from the horizontal, and Munsell color chips varying in hue, value and chroma. It should be noted, however, that Cross (1965) reanalyzed Jones' data and concluded that the city block metric was in fact the most appropriate.

Favoring dominance metric. As values of the Minkowski r-metric exceed two the metric approaches the dominance metric. The first theoretical proposal of the dominance metric appears to be that of Lashley (1942), who proposed that when any complex of stimuli arouses nervous activity, that activity is immediately organized, and certain stimulus properties become dominant for reaction while others become ineffective.

The first report of a dominance metric in a MDS study is credited to Gregson (1965). Using multidimensional stimuli consisting of solutions containing three taste components, Gregson found that the stress values were at a minimum at a M_r value of 10. For a perfect dominance metric the M_r value would have to be infinity, but it is apparent from Figure 1 that the obtained value of ten would yield a unit circle almost square in configuration, indicating that one of the taste components was responsible for almost the entire perceived similarity between the stimuli.

Comparative studies. In contrast to the somewhat partisan nature of much of the cited literature, two studies exist which were designed to investigate the conditions under which the alternative metrics might occur.

Handel (1967) compared classification and similarity judgments for different types of multidimensional stimuli. In general, in similarity judgments he found that the city block metric was the best fit for geometric stimuli and the Euclidean metric was the best for color stimuli.

Hyman and Well (1967) conducted the most comprehensive investigation to date of the occurrence of distance metrics in similarity judgments. Their study will be reviewed in some detail as the present study is in some respects a continuation of their research.

In their collection of data Hyman and Well used a modified version of Indow's method of multiple ratios (Indow and Uchizono, 1960) in which the subject adjusts the physical distance between stimuli to correspond to their perceived similarity. In the first part of their study Hyman and Well replicated the Torgerson, Attneave, and Shepard studies cited above, duplicating not only the stimuli dimensions but the original configurations as well. In the second part of the study they investigated the configuration effect by using the same configuration of stimuli, that of Shepard, for all three sets of stimuli. Again, in general, the results of the experiments were in agreement with the opinion that the Euclidean metric was most appropriate for color stimuli and the city block metric was most appropriate for geometric stimuli.

A useful feature of Hyman and Well's study was their investigation of individual differences in spatial models. They found that, although the group mean distances for the color stimuli indicated an Euclidean metric, the metric for individual S's ranged from dominance to city block. Similarly, the distance metrics for individual S's on the two sets of geometric stimuli showed a wide range of metrics, whereas the mean metrics were city block for the Attneave stimuli and between city block and Euclidean for the Shepard stimuli.

Similarity Judgments and Age

The conceptual preferences of children of varying ages when they were asked to group stimuli varying in form, color, and size into similar pairs were studied by Kagan and Lemkin (1961). The results showed that both sexes made their judgments on the basis of similarity of form, color, and size dimensions, in that order. There was no age difference for boys, but older girls were less likely than younger girls to use color as a preferred dimension. It is noted that the range of ages was from three to eight; a wider age span might have procuded different results.

Ginsburg and Gamlin (1967) studied the effects of instructions and class contrast on the similarity judgments of children (5-6 yrs) and adolescents (12-18 yrs). They found that the age variable made little difference to the similarity judgments of both groups. They did find that the kind of instructions given to subjects affected the nature of the similarity judgments at both age levels. They concluded that investigators must be cautious about comparing scaling studies unless the instructions given to subjects are identical.

In a relatively rare instance of a MDS study involving a comparison of the similarity judgments of 7-10-13 year olds and adults, von Wright and Niemala (1966) studied the dimensions of moral judgments using as stimuli short strip cartoons. They used Kruskal's program for the analysis of the similarity judgments; no mention is made of analysis under different distance metrics, so it is assumed that an Euclidean spatial model was used. Distinct differences were found between the configurations of the various age levels, with the youngest subjects paying particular attention to the seriousness of the damage resulting from actions, and the adult subjects to the motives involved in the actions.

Individual Differences in MDS

In conjunction with the increasing application of MDS techniques in the study of cognitive and perceptual processes has come a corresponding interest in the problem of how individual differences are handled in MDS. An analysis of group mean similarity judgments will provide certain information on the group psychological space, but in many cases analysis of the group data may hide wide variations in the similarity judgments of individuals or subgroups. Some approaches to the problem of individual differences include: individual analyses, the Tucker and Messick technique (1963), and the nonmetric procedure of McGee (1968).

Individual Analyses. At first glance it might appear that the problem of individual differences would be readily solved by subjecting the interpoint distance matrix of each subject to individual analysis. Unfortunately, the cost of a large number of individual analyses by a program such as Kruskal's would quickly become prohibitive. A further deterrent to individual analysis is that the error factor in individual solutions tends to be greater (as evidenced by larger stress values in the Kruskal solutions) than for group or subgroup solutions.

In their treatment of individual differences Hyman and Well (1967) did not do a MDS analysis of each subject's similarity judgments. Because of the nature of their data-gathering instrument they were able to analyze directly the distance settings of each subject to determine the relevant spatial model.

The Tucker-Messick technique. Tucker and Messick have provided a technique (1963) for identifying 'points of view' of subgroups and of determining separate multidimensional perceptual spaces for each point of view. The procedure is based on a matrix reduction technique of Eckart and Young (1936) applied to the matrix of similarity judgments. The Tucker-Messick technique has been applied to the analysis of individual differences in color perception (Helm and Tucker, 1962), social perception (Jackson and Messick, 1963), trait inference (Walters and Jackson, 1966), perception of visual form (Silver, Landis and Messick, 1966) and the judgment of facial expressions (Cliff and Young, 1968).

In conjunction with its widespread applications, the Tucker-Messick technique has been the object of some widespread criticism. Ross (1966) criticized the points of view concept as being illogical, an opinion disputed by Cliff (1968). Zinnes (1969) cites the Tucker-Messick procedure as an example of a new model being put into circulation before the properties are fully determined, and Gollob (1968) claimed that only the interaction variance should be analyzed and not the total matrix of raw similarity judgments.

McGee's technique. McGee (1968) proposed a nonmetric individual differences model based on his 'elastic' MDS technique (McGee, 1966). In this model four different kinds of analysis mey be made of N sets of similarity judgments. The alternative analyses are dependent on whether or not the same monotone transformation is applied to the sets of similarity judgments and whether or not each set is allowed to have its own configuration. The case where the monotone transformation and configuration are the same would correspond to an analysis of the group mean similarity judgments; the case where monotone transformation and configuration are different for each set of similarity judgments would correspond to individual analyses. A useful feature of McGee's technique is the provision of an index of correspondence between the obtained configurations.

Similarity Judgments and Stimulus Complexity

Miller (1956) has examined in entertaining fashion an interesting paradox: the human ability to discriminate between thousands of stimuli, such as faces or colors, coupled with an apparent limitation on the number of dimensions utilized in any discrimination task. Fenker and Brown (1969) have suggested that there is a geometric analog to Miller's 'magic number seven' in MDS results. Fenker and Brown found that when a subject scaled a set of sixteen random polygons under fifteen different task conditions the number of obtained dimensions in individual tasks ranged between one and three; the total number of linearly independent dimensions produced over all tasks was ten. They concluded that the psychological space resulting from a single MDS study was not the whole story--that subjects were capable of perceiving more dimensions than those uncovered by a single task.

In a series of studies by Brown and colleagues (Behrman and Brown, 1968; Brown and Andrews, 1968; Brown and Brumaghim, 1968) it was found that no matter how many dimensions were incorporated in a set of geometric stimuli, only three dimensions were obtained by MDS analyses.

An approach related to the above studies on stimuli complexity is that of Schroder, Driver and Streufert (1967) in their volume on human information processing. They have used MDS techniques to examine the integrative complexity of the cognitive structure in a given stimulus domain as a measure of the personality trait of abstractness. Integrative complexity was determined not only by the number of dimensions perceived but how they were combined in the similarity judgment. This combination feature has a certain resemblance to the distance metric although Schroder _et al_. do not discuss it in those terms.

Discussion

From the foregoing review of literature it may be seen that some of the opposing arguments concerning the various distance metrics have been based on generalization and classification studies, rather than similarity studies, e.g., Cross (1965), Handel (1967). In Cross' study, subjects were trained to discriminate stimuli along one or other of two dimensions; then stimuli varying in both dimensions were presented and the frequency and latency of response were noted. In classification tasks such as that used by Handel, subjects were required to split sets of multidimensional stimuli into discrete groups. While it is true that the findings from these different tasks may well be applicable to the similarity judgment task, such application must be cautiously approached. For example, in the Handel study it was found that some subjects were using different distance metrics in their classification and similarity judgments of the <u>same</u> multidimensional stimuli.

The review of research has shown that there is lack of agreement on the occurrence and nature of the distance metric, and a scarcity of research relevant to the importance of age, complexity of stimuli, and individual differences in MDS studies based on similarity judgments. It is hoped that the present study will make some contribution in these areas.

Since some of the concerns being investigated in the present study have not been specifically attended to previously in the literature, predictions on the results of the study must be tentative. With respect to the question of whether age has any effect on the distance metric, if the complexity theory of Schroder <u>et al</u>. (1967) is relevant here then there should be more dominance metrics in the younger group for the two-dimensional stimuli. This may not apply for the four-dimensional stimuli; if the above-noted dimensional limitation of three is applicable, the distance metric for each group may depend on which dimensions are recovered by the MDS prodedure. If the attribute preferences indicated by the Kagan and Lemkin (1961) study are still applicable at the 12 year old level the form dimensions may be favored over the color dimensions, which should result in a distance metric closer to the city block than to the Euclidean metric for the complex stimuli set. The Hyman and Well (1967) results indicate that even where there may be little difference in the mean distance metric there can be wide variation in the distance metrics of individual subjects.

Chapter 3

Design of the Study

General Approach

As stated earlier, the three main questions which the study was designed to investigate were as follows:

A. Is there any difference in the spatial models derived from similarity judgments of multidimensional stimuli made by adults and children?

B. Is there any difference in the spatial models derived from similarity judgments of individuals within the adult group and the child group?

C. How stable are the spatial models in each group as the configuation of the multidimensional stimuli is increased in complexity?

The first question was investigated by analyzing the similarity judgments in the form of distance estimates by both computer-based and direct methods of analysis. The group solutions for both adult and child groups on two sets of stimuli were compared with respect to dimensionality and distance metric.

The second question was studied by analyzing in similar fashion the individual solutions within each group to determine their dimensionality and distance metric, and how the dimensionality and distance metrics of individual solutions differs between the two groups of subjects.

The third question was investigated by examination of the difference in dimensionality and distance metric for groups and individuals as the configuration of the multi-dimensional stimuli increased from two-dimensions to four dimensions.

Subjects

Two groups of subjects were studied. Group One was made up of undergraduate students in Educational Psychology and Vocational Education programs at the University of Alberta. Group Two was Grade VII students of approximately twelve years of age attending a junior high school of the Edmonton Public School System.

The initial number of subjects in each group from whom similarity judgment data were obtained was approximately thirty. After initial analysis of the similarity judgments of all subjects only the data of the twenty-four subjects of highest reliability in each group were retained for further comparative analysis.

Stimuli

Since much of the literature on distance metrics has involved the nature of the multidimensional stimulus, the selection of the stimuli required particular care. The required stimuli should possess multi-attribute aspects (such as form) as well as multi-dimensional aspects (such as color). It was also required that the sets of stimuli be of different complexity.

Set One. The stimuli consisted of ten rectangles of standard Munsell color stock of 5.0 RP hue, varying on four dimensions. Two dimensions were of color, namely value and chroma, and two were of form, namely size and rectangularity. The color dimensions were highly correlated, as were the form dimensions. For example, from Table I it may be seen that the stimulus coordinates ranged from low values on dimensions I and II (value and chroma) for stimuli 4 and 5 to high values on the same dimensions for stimuli 9 and 10. Similarly, for dimensions III and IV (area and rectangularity) coordinates ranged from high area and low rectangularity values for stimuli 3 and 7 to low area and high rectangularity for stimuli 2 and 8. The net effect of the correlated dimensions is to reduce the effective dimensionality of Stimulus Set One to only two: form and color, as in the two-dimensional configuration shown in Figure 2.

Set Two. Set Two also consisted of ten stimuli varying on four dimensions. The dimensions are labelled as Set One, but the value and chroma dimensions, and the area and rectangularity dimensions, have little or no correlation, as indicated in Figure 3.

The difference in complexity between the two configurations is further indicated by a comparison of Figure 3 with Figure 4, which gives the location of the Set One stimuli on the four dimensions.

Apparatus

The method used to obtain similarity judgments from

TABLE I

Physical Coordinates of Stimulus Sets One and Two

Stimulus	Color		Form	
	I Value	II Chroma	III Area	IV Rect. (Horiz.=Vert.)
<u>Set One</u>				
5	4	2	.75	1.8
4	4	2	1.25	1.4
3	5	4	1.5	1.2
2	5	4	.5	2.0
1	5	4	1.0	1.6
6	6	6	1.0	1.6
7	6	6	1.5	1.2
8	6	6	.5	2.0
9	7	8	.75	1.8
10	7	8	1.25	1.4
<u>Set Two</u>				
5	4	2	.75	1.2
4	3	4	1.0	1.4
3	5	4	1.25	1.2
2	6	2	.75	1.6
1	7	4	.5	1.6
10	6	6	.5	1.6
9	3	8	1.0	1.8
8	5	8	1.5	1.6
7	7	8	.75	2.0
6	6	10	1.25	2.0

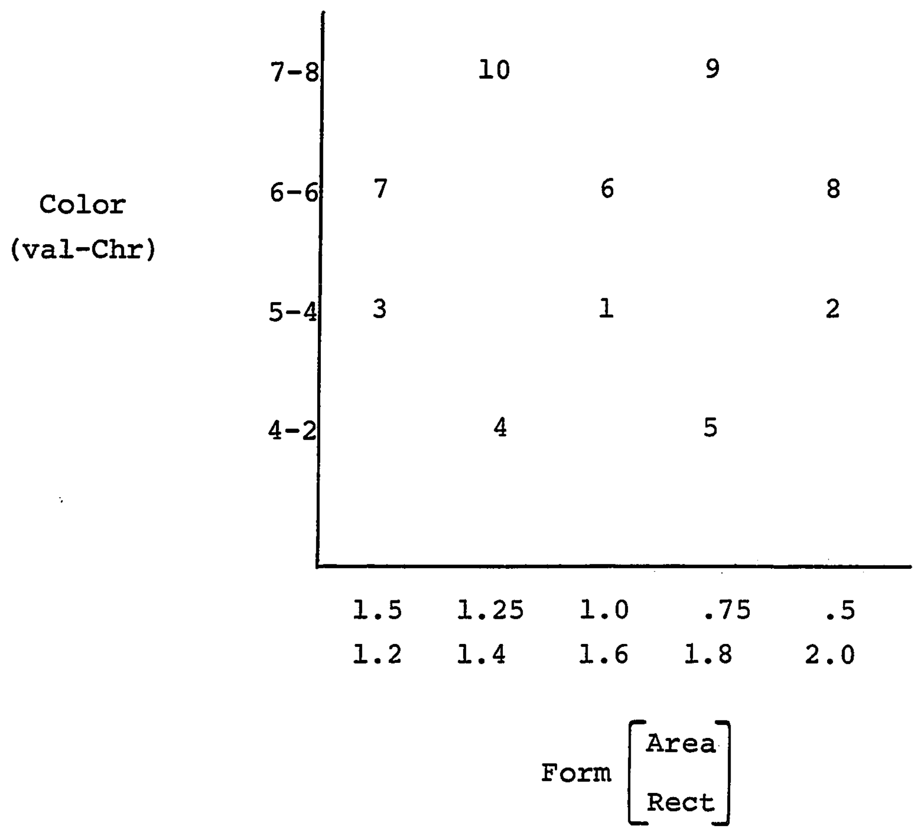


Figure 2. 2-Dim. Configuration of Stimulus Set One.

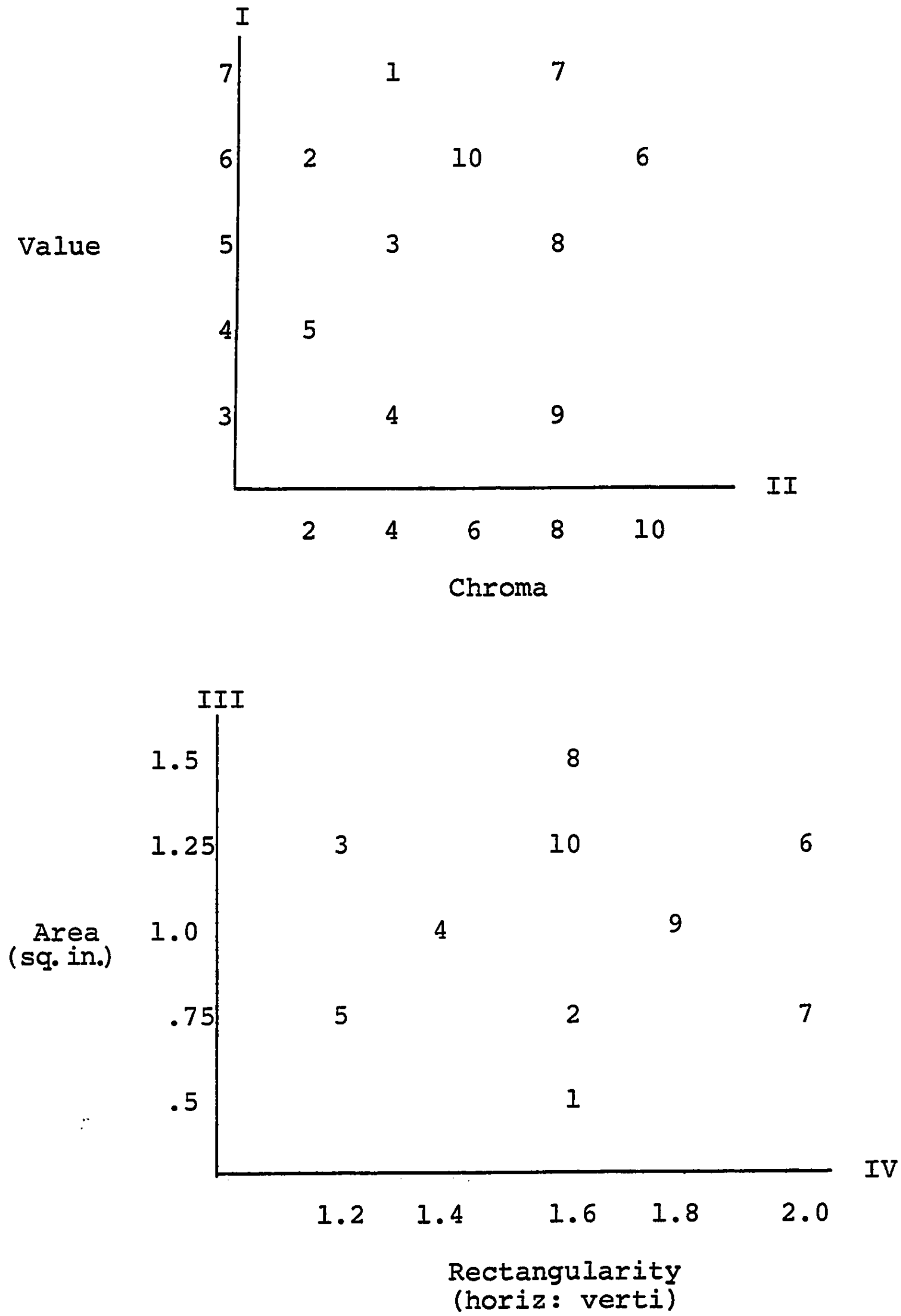


Figure 3. 4-Dim. Configurations of Stimulus Set Two.

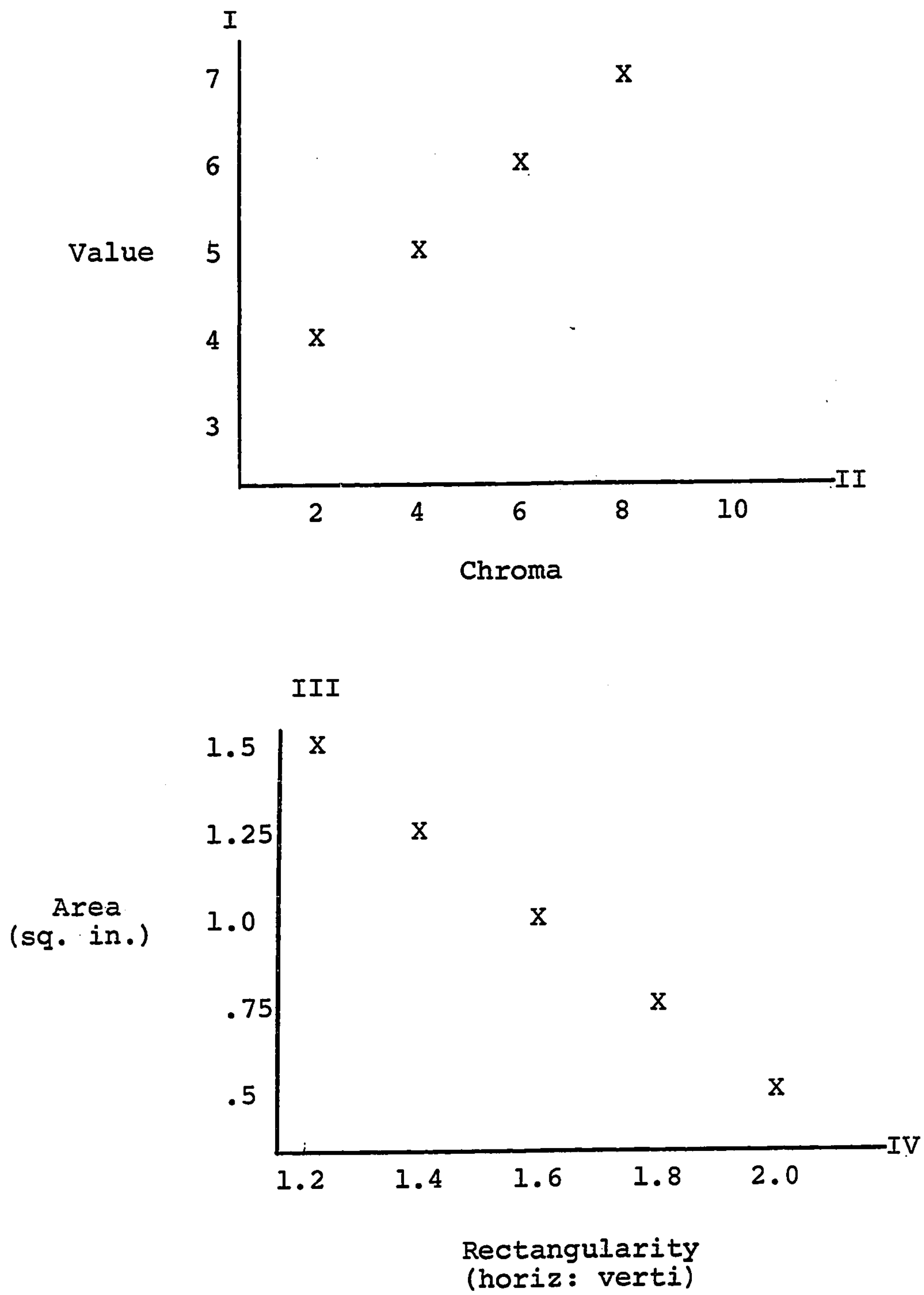


Figure 4. 4-Dim. Configurations of Stimulus Set One.

subjects was a modified version of Indow's Method of Multiple Ratios (Indow and Uchizono, 1960). The modifications will be more clearly apparent if this method is understood. Indow, who used a 45cm X 95cm board for his experiment, described the method as follows:

> On the gray board, a standard Color i (45cm. long, 1.0 cm. wide) was placed along the edge of the left side (45 cm.). Parallel to the 95-cm.-long side, 21 rows were marked by gray threads on the board, and a Color j was placed in each row at its left end. That means, 21 colors made a column next to the standard Color i on its right side. The same one as the standard was included among the 21 colors. The S was told that this duplicate of the standard would always be present and that he should place it right beside the standard. Then S was required to move, one by one, each of the other 20 colors to the right in the row according to the difference he observed between Color i and Color j. For the purpose of furnishing S with a unit for moving the colors, a pair of gray cards, either N 5.0 and N 5.5 or N 5.0 and N 4.5 were placed on the top of the board. The interval between the cards (.7 X 1.0 cm. each) was fixed at 3 cm. Should the difference between this pair of grays be represented by this distance (center to center), how far should the Color j be moved to the right to represent the difference between the Colors i and j? This was what S was instructed to do (with full knowledge that distances would be measured from center to center in the strips), hence the method may be called the multiple-ratio judgment (p. 323).

The major deviation from Indow's procedure is that the stimuli were ranked on a single slot on the board rather than each stimulus having its own slot. It is noted that in their modified version of Indow's procedure, Hyman and Well (1967) retained the individual slots but instead of providing a standard for comparison on each slot they provided only one standard in one of the middle slots. A further variation in the Hyman study is the omission of a unit of comparison. Indow provided this unit by the presence on the board of two stimuli whose physical distance represented the difference in similarity between the two stimuli. The Hyman study gives no explanation of the omission of this feature, which is retained in the present study because of the necessity to provide subjects with a common unit of distance.

The board used in the present study was painted white, rectangular in shape, size 16 in. x 40 in., and appeared to the subject as shown in Figure 5.

Procedure

Each subject attended two experimental sessions, at the first of which Stimulus Set One was presented, followed approximately a week later by the second session, at which Stimulus Set Two was presented.

Unfortunately, it was not possible to standardize the experimental conditions to the extent of having both groups provide data under identical environmental conditions. However, in each case data were collected in a quiet room free from interruptions. The lighting for each group was similar--overhead fluorescent.

The board was set at an angle on a table with the subject seated comfortably in front of the board. The remainder of the procedure will be clarified by reference to Figure 6, which gives the instructions read by the experimenter to each subject.

Subjects were also informed that three minutes was usually sufficient time for each trial, but that they could have an extra minute if necessary. With ten stimuli, ten trials per session were required, with the time per session

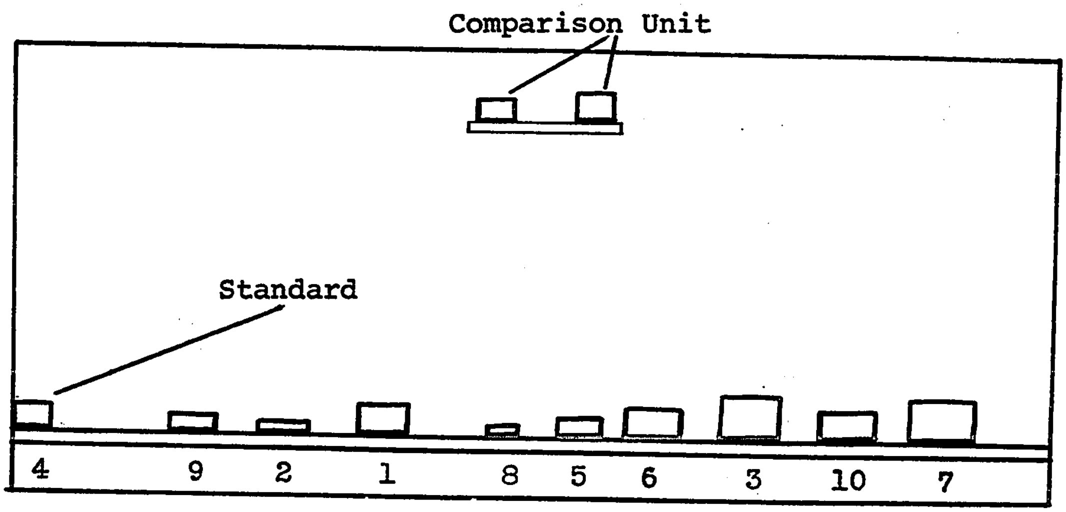


Figure 5. Experimental Apparatus.

(Note - raw data for this trial consist of nine distances from stimulus No.4 to each of the other stimuli)

Instructions to Subjects

--You see before you a white board with a number of pieces of card on the lower ledge (INDICATE) and two pieces on the upper ledge (INDICATE). No two pieces are identical.

--This card (INDICATE) is the standard for the first trial.

--Your task is to move the other nine cards so that the distance between the standard and any other card will represent the amount of similarity or likeness between them, just as the distance between the two cards on the upper ledge (INDICATE) is an indication of how similar or alike they are.

--When you are finished the nine cards will be lined up along the bottom ledge (with the white edge down), the card you think is most like the standard will be closest to the standard and the one you think is least like the standard will be furthest from the standard.

--You may rearrange the cards as often as you wish till you are satisfied that the distances of the nine cards from the standard represent their similarity or likeness to the standard.

--Are there any questions?

Figure 6. Instructions Read to Subjects

ranging from 30 to 50 minutes.

When the subject had arranged the stimuli to his satisfaction he stepped back and the experimenter measured the linear distance from the center of the standard to the centers of each of the other nine stimuli. To save time during the experimental session these values were not written but were recorded on a portable tape recorder. When the distances had been recorded the stimuli were then turned over and the numbers on the back of each stimuli were also recorded.

The order in which the stimuli appeared at the left of the board as the standard was determined by reference to a table of random numbers. These numbers were recorded on data sheets bearing the name of the subject (see Appendix B). When the distances and stimuli numbers for Trial One had been recorded the experimenter referred to this sheet and placed the second number as standard, and so on. At the conclusion of the day's experimental sessions the distances and stimuli numbers were transcribed to each subject's data sheet. Each data sheet thus contained nine distances from each standard, or two estimates of each of the 45 interpoint distances, as well as the rank order of similarity of the other nine stimuli to each standard.

Analysis of Data

Form of Raw Data

An experimental session with a subject provided two estimates of the 45 interpoint distances for the ten stimuli in each set. It should be noted that if the Kruskal MDSCAL-5 (Kruskal and Carmone, 1969) program were the only method of data analysis, only the rank order of the interpoint distances would have been required. However, the actual distance values were also required for an additional check on the distance metric, as explained below.

The initial analysis for each subject provided the correlation coefficient between the 45 interpoint distances with the stimulus as standard and the 45 interpoint distances with the other stimulus as standard; and the mean of the two estimates of each interpoint distance. The array of 45 mean values was then used for further analysis by the Kruskal MDSCAL program.

Reliability of Similarity Judgments

The correlation between the similarity judgments (in the form of distance estimates) of the same pairs of stimuli supplied an indication of the consistency of each subject's data. For a subject to have perfect consistency he should give the same distance setting for stimuli i and j when i is the standard as when j is the standard. Thus the correlation between the distances was regarded as an approximate reliability coefficient. The 24 subjects in each age group with the highest correlation coefficients were retained for further data analysis.

Kruskal's Nonmetric Multidimensional Scaling (NMMDS)

Although there are numerous versions of computer-based NMMDS methods, the version of concern for the present study was that based on Kruskal's work (1964a, b), the current title of the computer program being MDSCAL-5 (Kruskal and Carmone, 1969). The general features of NMMDS have already been described in Chapter II, but since the study relies in large part on Kruskal's procedures these will now be considered in some detail in terms of: the basic ideas involved, the 'stress' measure, and some problems which have become apparent in the application of NMMDS methods.

Basic principles. In brief, given the rank order of the inter-stimulus dissimilarities the procedure strives to obtain a configuration of specified dimensionality and distance metric, such that there is a monotone relationship between the dissimilarities and the configuration distances. The steps in this process will now be considered more closely.

The procedure starts with a set of experimental data values (which are similarity or dissimilarity estimates of some sort), which are usually arranged in a matrix format. From a configuration containing a similar number of points the corresponding inter-point distances are calculated according to a particular distance function. It is noted that in the standard option of the program this configuration is arbitrary, a feature which has resulted in some criticism, as discussed below.

A monotone regression is then performed of the configuration distances on the data values. This procedure is summarized as follows: the configuration distances (D) are arranged in the same order as the dissimilarities, the set of D values is then partitioned into smallest blocks so that the averages of the D values in the block are non-decreasing with the dissimilarities. These average values ($\hat{D}$) are then compared to the configuration distances in the following expression:

$$S = \sqrt{\frac{\Sigma\ (D - \hat{D})^2}{\Sigma\ (D - \bar{D})^2}}$$

where D is configuration distance, $\hat{D}$ is regression function value, and $\bar{D}$ is the mean configuration distance. This quantity, called 'Stress' by Kruskal, is an indication of how well the obtained configuration fits the original data. In the case of an arbitrary configuration this would obviously be a poor fit at first. An iterative procedure called the 'method of steepest descent' is used to move the points of the configuration around so as to minimize the value of the above expression.

The method of steepest descent assumes that a configuration on n points in t dimensions is considered as a point or vector in nt - dimensional space. A small step is made in the direction in which the stress is decreasing most quickly. This direction is known as the negative gradient and is obtained by calculating the partial derivatives of the S function. After arriving at a better point (better because value of S is lower) the gradient is again determined, and so on until S no longer decreases, or when the partial derivatives are zero.

The stress measure. Typical results of a Kruskal analysis report the stress values at specified dimensions and distance metrics, and give the coordinates for each configuration. One of the most crucial issues in NMMDS is the determination from the stress pattern of the correct dimensionality of the recovered configuration. Kruskal (1964a) suggests that a graph be plotted of stress versus dimensions; a noticeable 'elbow' in the curve, meaning that stress is not decreasing with increase of dimensionality, is taken as an indication of correct dimensionality. Two other criteria proposed by Kruskal are: interpretability of the obtained coordinates, and an estimation of the statistical error in the data, which is usually not available.

The expression for stress given above was not that of Kruskal's 1964 paper, which had as denominator the sum of the squared distances only. The revised expression is called Formula 2 (Kruskal and Carmone, 1969), and is now considered the standard formula.

Kruskal's descriptive evaluation of various stress values for both formulae is as follows:

Formula 2		Formula 1
.40%	Poor	.20%
.20%	Fair	.10%
.10%	Good	.05%
.05%	Excellent	.025%
.00%	Perfect	.00%

'Perfect' means that there is a perfectly monotonic relationship between the dissimilarities and the distances in the configuration obtained. It should be pointed out that while the stress value indicates the goodness-of-fit of the configuration to the data values it gives little or no indication of how well the configuration corresponds to the original configuration of stimuli on which the dissimilarities were made.

As noted previously, an important feature of this program for the present study is that it allows for solutions using other distance metrics than Euclidean, or M_r values other than two. Consequently, the interstimulus distances from each subject were analyzed at M_r values from one to five. The M_r value at which stress is at a minimum indicates the distance metric which best fits the similarity judgments.

NMMDS--Some problems. Although the Kruskal program has had widespread acceptance and use, several aspects of the technique have received critical attention: goodness-of-fit criterion, significance of the obtained solution, dimensionality and local minima.

In a Monte Carlo study, Sherman and Young (1968) studied the behavior of the stress variable in relation to the number of stimuli and the amount of error in the data. They found that with constant error, the mean stress value increased with the number of stimuli. Similar findings were reported by Stenson and Knoll (1969). The criticism based on these findings is that a measure of goodness-of-fit in NMMDS should be indicative of the amount of error in the data and not of the size of the data matrix.

The most common approach in the study of the significance of NMMDS solutions has also been by Monte Carlo methods using artificial data (Shepard, 1966; Klahr, 1969; Wagenaar and Padmos, 1971). Shepard found that with ten or more sets of stimuli consisting of random coordinates the configuration could be reproduced with a correlation of .99 between the distances in the initial configuration and the distances in the recovered configuration, or almost perfect recovery. Klahr studied the behavior of the stress values with changes in dimensionality and number of random coordinates. Klahr's results also showed that a minimum of ten stimuli were required if any conclusions were to be drawn about the evidence of structure in a configuration. For example, analysis of 100 sets of eight random stimuli in three dimensions produced 33 'good' stress values, and eight 'excellent' stress values; the same analysis with ten instead of eight stimuli produced no stress values in these two categories. Wagenaar and Padmos conducted an extensive study similar to Klahr's and have suggested critical values of stress for various combinations of stimuli and dimensionality (1971).

There have been numerous studies bearing on the appropriate dimensionality of an obtained solution, (Rankin _et al_., 1970; Stenson, 1968; Green _et al_., 1969) but since the dimensionality of the stimulus configuration in the present study is known these studies will not be reviewed.

The local minima question is whether a minimum stress value is an overall minimum as desired, or whether it is a local minimum. In one study Spaeth and Guthery (1969) found that over half of the trials on the Kruskal program landed at a local minimum instead of at an overall minimum. Isaac (1970) also studied the problem and believed that he greatly minimized the possibility of local minima by restarting the program in each of several dimensions. Young and Applebaum (1968) claimed that the key to the local minima problem lay in the initial configuration. They suggested that the arbitrary starting configuration of Kruskal's MDSCAL program would be more susceptible to local minima problems than the starting configuration of Guttman and Lingoes SSA technique (1967), which utilizes the original similarities data.

Guttman's goodness-of-fit measure is similar to Kruskal's but his regression value ($\hat{D}$) is made equal to the value of the configuration distance (D) whose rank number in the distance values is equal to the rank number of the dissimilarity in the data array.

Once the Kruskal program was selected the stress measure must also be accepted, although the above findings should be considered in its evaluation. The use of ten stimuli should increase the chances that the stress values will be indicative of the structure of the stimuli configuration. As noted, the dimensionality of the stimuli configurations was known; the focus of attention was on how they were recovered. In the analysis of mean interpoint distances, the local minimum problem was overcome by starting from three different trial configurations. For the data of individual subjects, analyses were made under alternate configurations only if there were anomalous stress values indicating local minima solutions.

Direct Distance Analysis

To provide an independent check on the distance metric, a direct distance analysis of the Stimulus Set One data was carried out as in the Hyman and Well study (1967). Only the data from Stimulus Set One were analyzed in this way because only Stimulus Set One contained the necessary combination of unidimensional and bidimensional judgments.

For the purpose of clarifying the direct distance analysis the two-dimensional configuration of Stimulus Set One is reproduced as Figure 7. Within this configuration it is apparent that similarity judgments, or distance estimates, will be either unidimensional or bidimensional. For example, in Triangle 4-5-10 the distance estimates between stimuli 4 and 5, and between stimuli 4 and 10 will be unidimensional, since stimuli 4 and 5 have the same color coordinate and

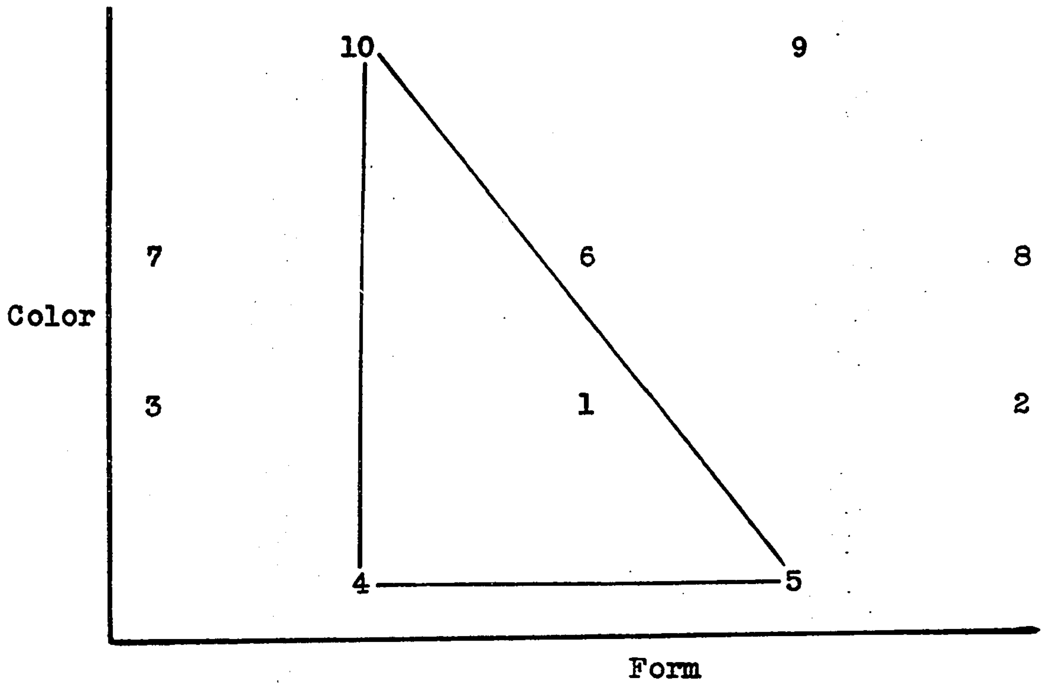


Figure 7. Direct distance analysis triangle example

stimuli 4 and 10 have the same form coordinate. The hypotenuse of triangle 4-5-10 will be a bidimensional distance estimate between stimuli 5 and 10 because these two stimuli differ on their color and form coordinates. The sixteen triangles with these characteristics are as follows:

4-5- 9	2-3-7	3-6-7	2-6-8
4-5-10	2-3-8	3-7-8	2-7-8
5-9-10	1-3-6	1-2-6	1-6-7
4-9-10	1-3-7	1-2-8	1-6-8

During the initial analysis of similarities data for Stimulus Set One the distance estimates of the above 16 triangles were examined in the following manner: (triangle 4-5-10 retained as illustration). The estimated distance of the hypotenuse 5-10 is compared to two values.

Value 1 = estimated distance 4-5 plus estimated distance 4-10

i.e., (4-5) + (4-10)

Value 2 = $[(4-5)^2 + (4-10)^2]^{1/2}$

That is, Value 1 would be the distance by the city block metric and Value 2 would be the distance by the Euclidean and Pythagorean metric.

The ratios of the hypotenuse estimates to Values 1 and 2 were compared. Whichever ratio is closer to unity is an indication of which distance metric provides a best fit to the similarity judgments. Further interpretation of this direct distance analysis will be given later under data analysis.

Chapter 4

Results and Discussion

The raw data for the study consisted of distance estimates representing similarity judgments of multidimensional stimuli. From this raw data the following results are presented and discussed: reliability of subjects' similarity judgments; analysis of group data in terms of dimensionality and goodness-of-fit of recovered configurations; analysis of distance metrics in group data; analysis of individual patterns of similarity judgments; and analysis of effects of stimulus complexity.

Reliability of Judgments

The first essential information required from the data was an indication of the reliability or consistency of the similarity judgments made by subjects. A measure of this consistency is simply the correlation between the distance estimates of a stimulus-pair with each stimulus in turn as the standard. That is, how does the distance from stimulus *i* as standard to stimulus *j* compare to the distance from stimulus *j* as standard to stimulus *i*? The correlation coefficients for both groups on both sets of stimuli are given in Table II. Three subjects in each group whose correlation coefficients were less than .24 (.05 significance level for Pearson r with n of 45) on stimulus set one were assumed to have misunderstood the task and were excluded from subsequent

TABLE II

Consistency of Judgments (Pearson 'r') of Adults and Children on Stimuli Sets One and Two

	Adult Group		Child Group	
	St. Set I	St. Set II	St. Set I	St. Set II
	739	613	*098	
	485	411	546	530
	*167		420	392
	242	259	*144	
	584	688	381	706
	452	332	495	464
	444	609	584	449
	274	523	563	732
	785	490	555	486
	525	566	260	531
	303	566	411	516
	426	604	705	790
	*167		*170	
	565	378	470	654
	735	609	289	625
	313	341	419	452
	504	436	549	594
	741	662	547	546
	825	577	499	451
	761	591	863	449
	434	500	441	635
	478	487	380	521
	531	513	578	578
	*184		332	519
	527	565	452	544
	422	567	652	705
	330	630	642	706
Means =	661	754	763	818

*not included in mean calculations

data analyses.

As expected, there was great variation in consistency of judgments, with the coefficients for stimulus set one ranging from near zero to a high of .825 in the adult group and .863 in the child group. Unexpectedly, the consistency of the child group was greater than the adult group on both sets of stimuli. Although the coefficient for means for the adult group increased from .661 on stimulus set one to .754 on stimulus set two it is still lower than that of the child group on stimulus set one.

Another unexpected finding was the greater correlation between means for stimulus set two than for stimulus set one. It should be noted that these means are not means of individual correlation coefficients but rather correlations between the means of the estimated distances for each group over all stimulus pairs. It was anticipated that because of the greater complexity in set two subjects might have more difficulty in making consistent judgments. In fact, the complexity of stimulus set two appears to have made the stimuli more discriminable. Referring to Figure 3, it is seen that in stimulus set one, two or more stimuli share each coordinate level of color and form, whereas in stimulus set two (Figure 4), no two stimuli share identical coordinates of value and chroma or area and rectangularity. During the experimental sessions it was observed that subjects spent less time deliberating on their choice in set two than in set one. It is also possible that there was a practice effect.

from the stimulus set one session.

As noted, the correlations for the child group were higher for each stimulus set than the correlations for the adult group. The variability within the child group was also less than that of the adult group. From Table II it is seen that six adult subjects, Nos. 2, 3, 5, 14, 19 and 20 had correlations on both stimulus sets of .50 or less, compared to four in the child group, Nos 2, 4, 13 and 16. In the adult group the correlations of 14 subjects varied by more than .10 on the two stimulus sets: seven were higher on stimulus set two and seven were lower. In the child group the correlations of twelve subjects varied by more than .10: ten were higher and only two were lower on set two.

In a similar check on group mean reliability Hyman and Well (1967) found coefficients of .78 and .80 for stimuli differing on color dimensions and coefficients of .89 and .90 for stimuli differing on form dimensions. Since the stimuli in the present study varied on both color and form dimensions no direct comparison is possible, although it may be inferred from the correlations of the group means that the group mean similarity judgments were reliable. It is also evident, however, that some individuals in each group were not able to perform the task consistently.

Group Data--Validity Analysis

The consistency of subjects' judgments indicated that most of the subjects could reliably follow the manipulations required by the task, but the reliability of the results is no guarantee of their validity. It is necessary to consider the question: does the experimental procedure result in similarity judgments, in the form of distance estimates, which may be processed by appropriate multidimensional scaling methods to produce a meaningful configuration which represents the original configuration of experimental stimuli? This question should be answerable in the light of the results of the analysis of the group data.

Criteria for dimensionality. Before presenting the results of the MDSCAL-5 analysis of the group mean similarity judgments it is necessary to explain the criteria to be applied in determination of the dimensionality of the derived solutions. Kruskal (1964) suggested several criteria: elbows in the curve of stress vs dimension; interpretability of the coordinates; and the amount of error in the raw data. The latter criterion will not be considered for the group data, but it will be relevant for analysis of individual configurations.

Stimulus Set One. The stress values resulting from analysis by Kruskal's (1969) MDSCAL-5 program are given in Table III. The 'Arbitrary,' 'Random 3' and 'Random 5' headings refer to the starting configurations used to initiate analysis in the MDSCAL program. The importance of this variation in starting configuration is evident from a study of Table III. At the $M_r = 1$ level, the child group reveals a stress of .641 at the arbitrary two-dimensional solution, a value which is obviously a local minimum when compared to the corresponding values of .113 and .119 at the other two starting configurations. Similarly, for adults the stress

TABLE III

Stress Values for Group Mean Similarity Judgments
Stimulus Set One

DIMS	Mr: 1	2	3	4	5
Adult-Arbitrary					
4	076	156	106	149	144
3	099	189	157	105	144
2	117	317	233	192	170
1	774	774	544	544	544
Random 3					
4	103	136	058	156	117
3	101	189	119	130	075
2	391*	317	233	192	170
1	774	507	544	511	506
Random 5					
4	079	134	078	144	067
3	036	189	120	096	164
2	107	317	233	192	602*
1	511	506	544	767	544
CHILD-Arbitrary					
4	101	151	110	082	063
3	087	223	163	134	079
2	641*	279	204	169	150
1	718	636	687	581	581
Random 3					
4	091	196	035	075	079
3	130	224	242	136	146
2	113	279	559*	169	150
1	718	744	697	581	581
Random 5					
4	080	134	088	359*	080
3	097	523*	143	569*	105
2	119	289	204	169	150
1	718	718	687	581	581

Decimals omitted
* Local minimum indicated.

at the two-dimensional Random 3 solution of .391 appears to be a local minimum value compared to the Random 5 and Arbitrary values of .107 and .117. Further obvious local minima are indicated by an asterisk in Table III. If the MDSCAL-5 analysis had been carried out at only one of the three starting configurations there could be an apparent difference between the two groups, attributable to a solution based on one or more local minima.

Inspection of the stress pattern of Table III, disregarding local minima, reveals in many instances, a considerable drop in stress value between dimensions one and two, with further small reductions in stress as dimensionality is reduced from two to four. There are also several instances where this elbow effect is weak or missing, for example the adult stress values at $M_r = 2$ show a sizeable drop between two and three dimensions, therefore further corroboration of dimensionality is necessary. Investigation of the interpretability of the coordinates also suggested two-dimensional solutions for both groups.

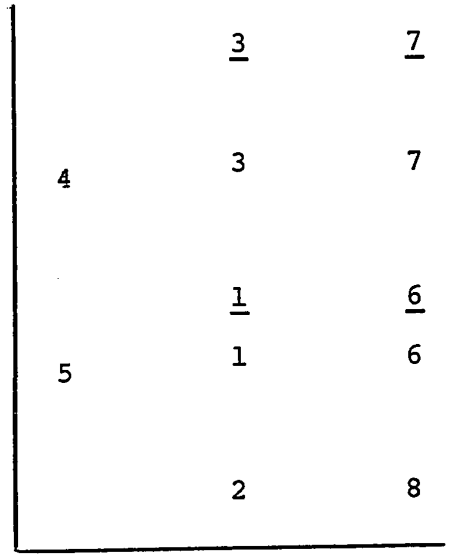
While the analysis of the group data appeared satisfactory in terms of stress pattern and dimensionality, the obtained solutions were examined to determine the degree of fit of the reconstructed configurations to the original configuration of stimuli. The two-dimensional solution at which stress was at a minimum in each group was plotted. In each case this was at the $M_r = 1$ value, the stress value for the adult group being .107 and for the child group being .113.

These configurations are given in Figures 8 and 9 for adult and child groups respectively. On comparing the derived configurations to the original in Figure 2 it is evident that there is a close resemblance for both groups.

It is noted that the steps on the color dimension are not equal to the steps on the form dimension as arbitrarily depicted in Figure 2, but as reconstructed are approximately twice as great, resulting in the elongated appearance of the configurations in Figures 8 and 9, compared to that of Figure 2.

Stimulus Set Two. The results of the Kruskal MDSCAL-5 analysis of group mean distance estimates of stimulus set two is given in Table IV. Interestingly, inspection of the stress values at different starting configurations indicate fewer local minima values than for the stimulus set one analysis. The only major discrepancies are the Random 5 configuration stress values of .627 and .757 at M_r = 3 in the child group data, values which are obviously incompatible with the stress pattern in the remainder of the table. A less serious discrepancy is indicated by the Random 3 stress value of .108 at M_r = 5 in the child group. While a stress value of .108 would normally suggest a good solution, the four-dimensional stress value should be equal to or lower than the three-dimensional value.

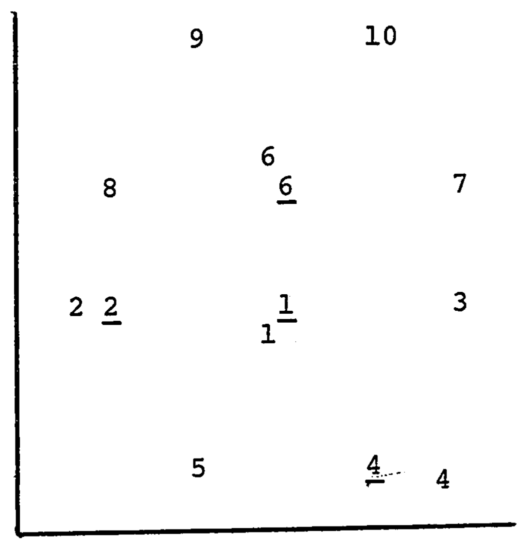
Examination of the stress pattern for both groups in Table IV suggests that a three-dimensional solution is satisfactory. It should be noted that the two-dimensional stress values in Tables III and IV are in the same range. However,



Stress = .107
Mr = 1

Numerals in italics indicate positions of stimuli not identical to original configuration.

Figure 8 Derived configuration, stimulus set one, adult group.



Stress = .113
Mr = 1

Numerals in italics indicate positions of stimuli not identical to original configuration

Figure 9 Derived configuration, stimulus set one, child group.

TABLE IV

Stress Values for Group Mean Similarity Judgments
Stimulus Set Two

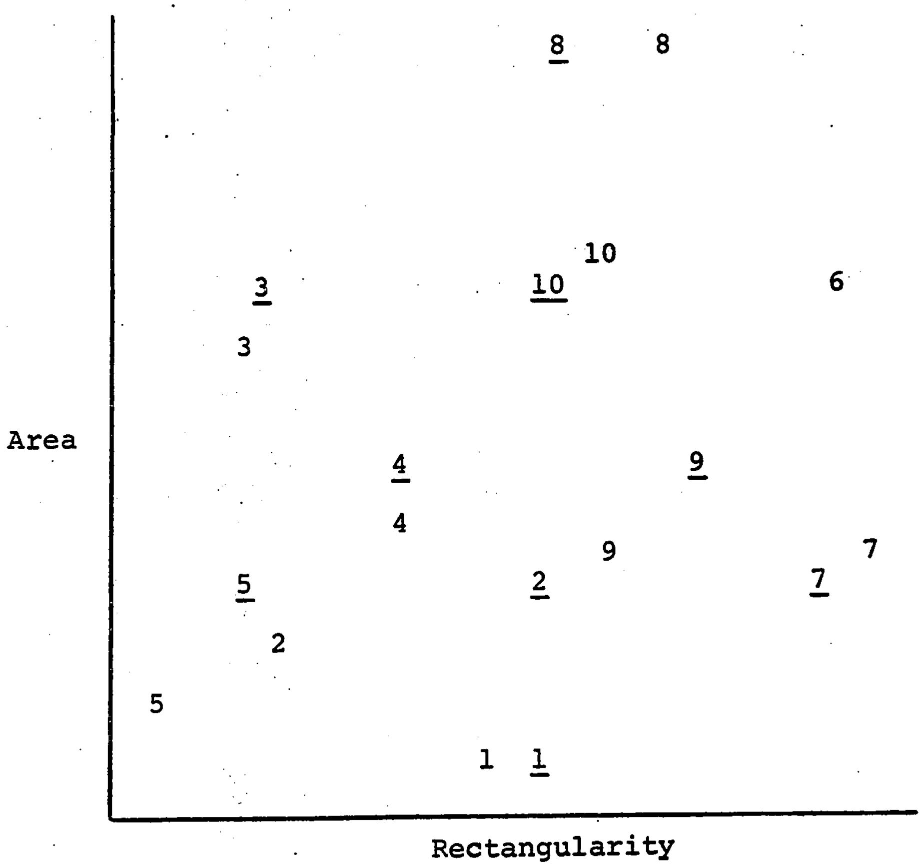
DIMS	Mr: 1	2	3	4	5
ADULT-Arbitrary					
4	102	018	016	018	022
3	098	044	029	034	036
2	294	195	213	219	224
1	558	598	592	588	588
Random 3					
4	088	029	035	043	046
3	115	044	036	076	078
2	259	195	177	176	224
1	588	625	821	820	614
Random 5					
4	021	018	016	018	022
3	054	044	029	034	036
2	249	195	214	176	228
1	558	653	637	558	588
CHILD-Arbitrary					
4	050	021	017	034	037
3	127	024	044	058	088
2	386	228	237	260	264
1	572	582	574	582	574
Random 3					
4	048	024	010	033	108*
3	143	024	045	050	080
2	250	229	205	202	199
1	564	564	564	582	582
Random 5					
4	054	023	627*	032	021
3	169	024	757*	060	082
2	204	228	204	200	263
1	564	572	564	582	574

Decimals omitted
* Local minimum indicated.

on Table IV there is almost invariably a further sizeable decrease in stress from two to three dimensions. Further confirmation of the proper dimensionality is obtained from an examination of the interpretability of the coordinates of the obtained solutions, which also indicated a three-dimensional solution for the analysis.

Stimulus set two had been constructed to a four-dimensional configuration. The three-dimensional solutions for both groups implied that subjects were making similarity judgments without considering all of the dimensions. Since much of the relevant research in this area has stressed the importance of the relationship of the type of dimensionality to the distance metric solution it is necessary to determine which dimensions were being recovered.

The three-dimensional solutions for each group in which stress was at a minimum are given in Figures 10 and 11 for adults and children respectively. In each case a two-dimensional solution is plotted and the third dimension coordinates are listed. The solutions are presented in this manner for easier comparison to the original configurations as shown in Figure 3. As with the set one reconstructed configurations the stimuli in italics indicate the location of the stimuli in the original configuration, where this is different from the obtained solutions. Although there are minor differences between the two configurations their similarity is evident. In each case the largest deviation from the original configuration (area and rectangularity) is

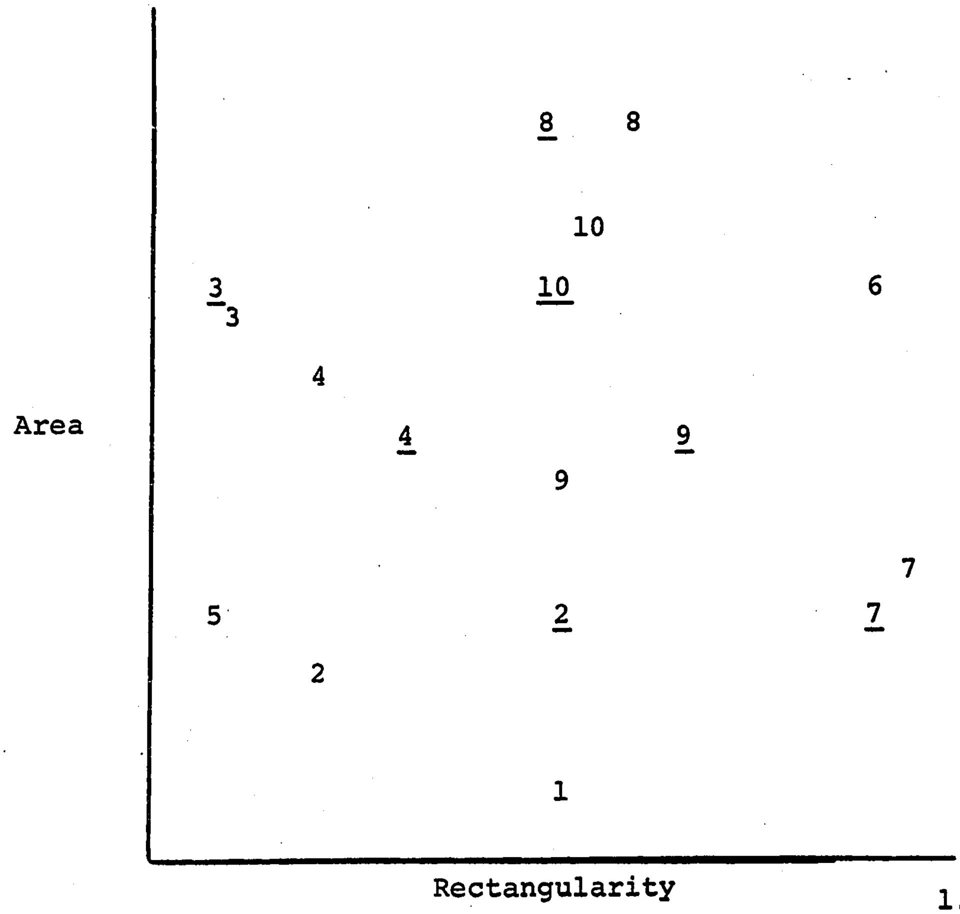


III	
1.	.764
2.	.358
3.	.123
4.	.938
5.	.631
6.	.188
7.	.604
8.	.090
9.	.993
10.	.525

Stress: .029

$M_r = 3$

Figure 10. Three-dimensional configuration, stimulus set two, adult.



III	
1.	.816
2.	.559
3.	.364
4.	1.021
5.	.546
6.	.206
7.	.719
8.	.064
9.	.797
10.	.364

Stress: .024

$M_r = 2$

Figure 11. Three-dimensional configuration, stimulus set two, child.

the location of stimulus No. 2. Examination of the stimulus revealed no discrepancy in its physical dimensions. From Figure 3 it may be seen that Stimulus No. 2 and Stimulus No. 5 differ on only two dimensions, those of value and rectangularity, thus they may have been perceived as being closer than they actually were in physical space.

Despite the meaningful two-dimensional plots shown in Figures 10 and 11, the third dimension must also be meaningful for a successful three-dimensional scaling solution. The rank order of the third dimension coordinates for adult and child groups is given in Table V, with the rank order of the stimuli on the value dimension in stimulus set two (from Table I and Figure 3) for comparison. The rank order correlations between these rankings are .98 for the adult group and .94 for the child group.

Comment. Initial analysis of the similarity judgments data for both groups of subjects suggested that meaningful solutions can be obtained by the experimental procedure employed in the study. There was a wide variability in the consistency of individuals' similarity judgments, but the group mean similarity judgments were found to have satisfactory reliability, after the exclusion of the three worst performers in each group. The validity of the procedure was evidenced by a high degree of success in the reproduction of the stimulus set one configuration. It was found that only three dimensions could be meaningfully extracted from the stimulus set two data. The question as

TABLE V

Comparison of Value Dimension in Stimulus Set Two With Third Dimension in MDS Solution for Adult and Child Groups

Value Rank	Dimension Stimuli	Adult Dim III	Child Dim III
1	4	9	4
1	9	4	9
2	5	5	5
3	3	8	3
3	8	3	10
4	6	2	8
4	2	10	6
4	10	6	2
5	7	7	7
5	1	1	1

to whether this finding is the result of the scaling technique used or of the particular combination of dimensions incorporated in the stimulus configuration is difficult to answer at this point. It is apparent, however, that the scaling solutions are sufficiently interpretable that the scaling technique may be considered successful, allowing attention to be directed to the question of the distance metrics.

Group Data--Distance Metric Analysis

The results of the Kruskal MDSCAL-5 analyses have been considered to date in investigating the group parameters and the fit of the derived configurations to the original configurations of both sets of stimuli. The involvement of the distance metric with the stress pattern is now considered.

Stimulus Set One

MDSCAL-5 analysis. From the data in Table III curves of stress vs distance metric are plotted in Figure 12. The anomalous values indicating local minima solutions were not considered in plotting these curves, which were based on the best obtained solution at each M_r value. Thus the best stress value at $M_r = 1$ is .107 for the adult group and .113 for the child group.

There are two noteworthy features to Figure 12: the close similarity between the two curves and the apparent maximum stress value at $M_r = 2$ (Euclidean distance metric). This is contrary to the expectation that there would be a minimum stress value at a certain distance metric. The stress value of .11 for both groups is of course the lowest value reported, and may actually be the minimum stress in question. Further clarification of these group findings may be possible after analysis of the direct distance data and of the individual data.

Direct distance analysis. The particular configuration of stimulus set one made possible a direct estimation of the distance metric involved in the similarity judgments made by each subject. The required data for the direct analysis of the adult and child groups are given in Tables VI and VII. For each of the sixteen right-angled triangles

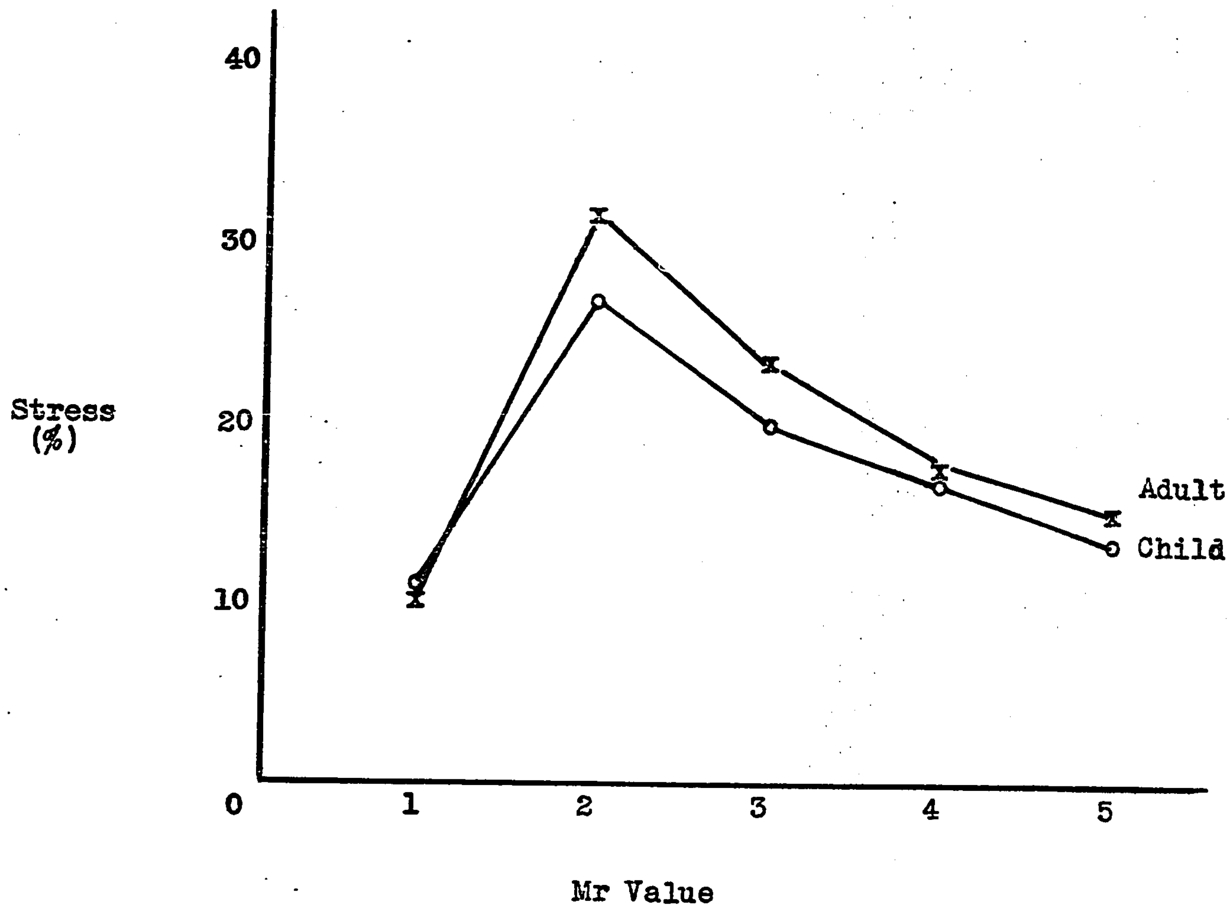


Figure 12. Stress vs distance metrics for both groups, stimulus set one

TABLE VI

Direct Distance Measures for right-Angle Triangles in Stimulus Set One Configuration-Adults

Triangle	Distance Estimates			Dist 1	Dist 2	Ratio 1	Ratio 2
	Hypot.	Side 1	Side 2	(C.B.)	(Eucl)	(C.B.)	(Eucl)
4-9-5	25.3	6.9	19.7	26.6	20.9	0.95	1.21
5-10-4	25.0	6.9	19.5	26.4	20.7	0.95	1.21
5-10-9	25.0	19.7	6.1	25.8	20.6	0.97	1.21
4-9-10	25.3	19.5	6.1	25.6	20.4	0.99	1.24
2-7-3	19.3	11.9	8.3	20.2	14.5	0.95	1.33
3-8-2	18.3	11.9	9.0	20.8	14.9	0.88	1.23
3-6-1	15.1	8.0	8.2	16.2	11.5	0.93	1.32
1-7-3	16.2	8.0	8.3	16.3	11.6	0.99	1.40
3-6-7	15.1	8.3	9.0	17.3	12.3	0.87	1.23
3-8-7	18.3	8.3	13.2	21.5	15.6	0.85	1.17
2-6-1	14.3	6.8	8.2	15.0	10.6	0.96	1.35
1-8-2	14.3	6.8	9.0	15.8	11.2	0.91	1.27
2-6-8	14.3	9.0	8.5	17.4	12.3	0.82	1.16
2-7-8	19.3	9.0	13.2	22.2	16.0	0.87	1.21
1-7-6	16.2	8.2	9.0	17.2	12.2	0.94	1.33
1-8-6	14.3	8.2	8.5	16.7	11.8	0.86	1.21
					Means =	0.91	1.24

TABLE VII

Direct Distance Measures for Right-Angled Triangles In Stimulus Set One Configuration-Children

Triangle	Distance Estimates			Dist 1	Dist 2	Ratio 1	Ratio 2
4-9-5	21.5	7.6	16.7	24.3	18.3	.89	1.17
5-10-4	20.6	7.6	17.0	24.6	18.6	.84	1.10
5-10-9	20.6	16.7	8.0	24.7	18.5	.83	1.11
4-9-10	21.5	17.0	8.0	25.0	18.8	.86	1.14
2-7-3	18.8	13.0	6.0	20.0	14.8	.94	1.28
3-8-2	17.8	13.0	7.9	20.9	15.2	.85	1.17
3-6-1	15.4	9.2	8.0	17.2	12.2	.89	1.26
1-7-3	15.2	9.2	6.9	16.2	11.5	.94	1.32
3-6-7	15.4	6.9	9.1	16.1	11.5	.96	1.34
3-8-7	17.8	6.9	13.3	20.3	15.0	.88	1.19
2-6-1	13.6	8.5	8.0	16.5	11.7	.82	1.16
1-8-2	14.3	8.5	7.9	16.4	11.6	.87	1.23
2-6-8	13.6	7.9	8.0	15.8	11.2	.86	1.21
2-7-8	18.8	7.9	13.3	21.2	15.5	.89	1.21
1-7-6	15.2	8.0	9.1	17.1	12.1	.89	1.25
1-8-6	14.3	8.0	8.0	15.9	11.3	.90	1.27
					Means =	.87	1.20

in the stimulus set one configuration the estimated distances of the sides are listed in columns 2, 3, and 4. In column 5 is the value of the hypotenuse calculated in accordance with the city block metric. In the example given previously in Figure 7 of triangle 4-5-9 the estimated lengths of the undimensional sides were 6.9 and 10.7 for the adult group (top line Table VI). Addition of these values gives 26.6 as the city block distance and the square root of the sum of their squares gives 20.9 as the Euclidean distance in column 6. Ratio 1 is the ratio of the estimated hypotenuse 4-9 to distance 1 and Ratio 2 is the ratio of the estimated hypotenuse to distance 2. For the triangle 4-5-9 the ratio values of .95 and 1.21 indicate that the distance metric which best describes the triangle is much closer to the city block than to the Euclidean.

No computations for Dominance metric are necessary as this distance is merely the larger of the two estimated unidimensional distances and is readily achieved by inspection. For example, in triangle 4-5-9 a hypotenuse estimate of 19.7 would indicate a dominance metric. It is noted that a Ratio 2 value of less than 1.0 is an indication that the M_r value is greater than two and that a Ratio 1 value of greater than one is an indication of a violation of the triangular inequality.

The values of Ratio 1 in Table IV ranged from a low of .81 to a high of .99, with a mean Ratio 1 of .91, while the Ratio 2 values ranged from a low of .97 to high of 1.35, with a mean of 1.24. On the basis of the mean ratios it may be concluded that for the adult group as a whole the city block distance metric is the combinatorial rule which best describes the distance estimates.

The corresponding data for the child group, given in Table VII show a range of values of Ratio 1 from .73 to a high of .96, with a mean of .87; the Ratio 2 values ranged from .94 to a high of 1.34, with a mean of 1.20. Although the city block ratio is again closer to unity, it is evident that the distance metric applicable to the child group data is closer to the Euclidean than in the adult group.

The results of the direct distance analysis would appear to confirm that the minimum stresses at the M_r value of 1, as depicted in Figure 12, are a generally correct indication of the distance metric, although the child group data are less positive. They do not, however, afford any explanation for the gradual reduction of the stress curves in Figure 12, after the peak at $M_r = 2$.

Stimulus Set Two

The plot of the mean stress values against distance metrics for both groups on stimulus set two is given in Figure 13. The values for the curves were obtained from the minimum stress values at each M_r value in Table IV. The curves in Figure 13 are markedly different from the Stimulus set one curves in Figure 12. The shape of the curves of both groups in Figure 13 shows a definite minimum stress, compared to the reverse situation in Figure 12. Further, the variation

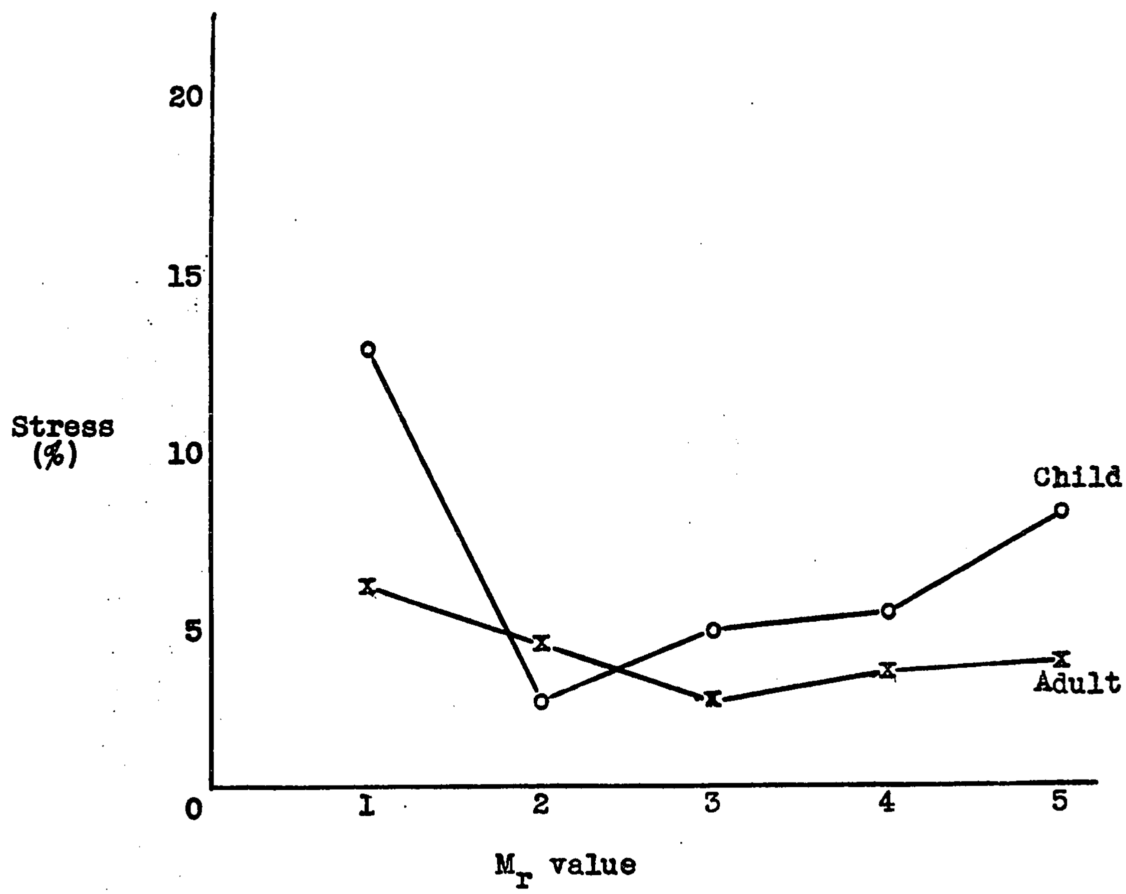


Figure 13. Stress vs distance metrics for both groups, stimulus set two

between the two curves on the stimulus set two data is greater than for the set one data; the adult curve is markedly flatter and the minima are at different levels. The curves in Figure 13 suggest a stress minimum at a M_r value of two for the child group and at three for the adult group.

Comment. Analysis of the mean similarity judgments for both groups of subjects revealed no major differences between the two groups in their distance estimates of stimulus set one. Both the Kruskal MDSCAL-5 analysis and the analysis of direct distance settings indicated that the city block metric (M_r = 1) was the most appropriate, with a tendency for the child group metric to be closer to the Euclidean metric than was the adult group. The MDSCAL-5 analysis of the group mean similarity judgments of stimulus set two showed that, while both groups reproduced similar three-dimensional configurations, the stress for the adult data was at a M_r value of three, whereas for the child group the minimum stress was at M_r value of two.

Individual Data

Stimulus Set One

MDSCAL-5 analysis. The stress values at various distance metrics for the individual subjects of both groups are given in Appendix A. On comparing these values to the group results in Table III the extreme deviations are immediately apparent. Many of the individual results are similar to the group data but the range of values is considerable. In addition to the subjects whose stress patterns showed reasonable stress values there were a sizeable number, such as numbers 1, 6, 7, 14, 22, and 24 in the adult group and numbers 5, 8, 9, 11, 16, 19, 20, 21 and 24 in the child group whose two-dimensional stress values were .25 or higher.

The individual stress patterns allow isolation of individuals who only attend to one dimension when making their similarity judgments. In this category numbers 5, 8 and 15 in the adult group and number 17 in the child group had stress values for all three dimensions close to .01, indicating almost perfect fit of the derived configurations to the similarity judgments. It should be noted that stress patterns of this nature, when plotted, give a configuration of points in a straight line, and tightly clustered in two or locations.

The stress values in Appendix A illustrate the previously noted disadvantage of individual analyses of similarity judgment data, namely the large error factor compared to group data. Consequently, determination of dimensionality and distance metric for each subject is less clear-cut than for the group data considered earlier. As before, the two main criteria for dimensionality of individual solutions are stress pattern and interpretability of coordinates. In the case of poor stress values, interpretability becomes the important criterion. The dimensionality and distance metric at which stress was at a minimum are summarized in Table VIII. Since

TABLE VIII

Dimensionality and Distance Metrics for Individuals of Adult and Child Groups, Stimulus Set One

Subject	Adult Dimensionality	Mr	Child Dimensionality	Mr
1	2	5	2	1
2	3	2	3	3
3	3	1	2	4
4	2	5	1	
5	1		3	5
6	3	5	2	1
7	2	4	2	1
8	1		3	5
9	2	5	2	1
10	2	5	1	
11	2	1	3	1
12	3	3	1	
13	1		1	
14	3	4	2	5
15	1		2	5
16	1		3	3
17	1		1	
18	1		2	4
19	1		2	1
20	2	1	2	1
21	2	2	3	5
22	3	4	1	
23	2	1	2	1
24	2	1	2	5

a feature of unidimensional solutions is invariably little or no variation of stress with distance metric no M_r value is reported in these cases.

Examination of Table VIII reveals several one dimensional solutions in addition to the examples already cited whose unidimensionality was obvious from the stress pattern. Although the stress values at one dimension for some of these subjects is only fair, for example numbers 13 and 19 in the adult group and numbers 10 and 12 in the child group, their coordinates clearly indicated a one dimensional solution. Of the eight unidimensional solutions in the adult group six were on the color dimension and two were on the form dimension. Of the six such solutions in the child group three were on the color dimension and three were on the form dimension.

In addition to the eight one-dimensional solutions in the adult group, there were ten two-dimensional and six three-dimensional solutions, and in the child group there were twelve two dimensional solutions, three three-dimensional and three whose stress values at the three-dimensional level were greater than 20 per cent. It is suggested that only the one and two-dimensional solutions in each group should be considered feasible, as the third dimension in a three-dimensional solution, even if the stress value is satisfactory, is uninterpretable.

<u>Direct Distance Analysis</u>. The direct distance ratios for individual subjects of both groups are listed in Table IX. The Ratio 1 and Ratio 2 values for each subject are the means of the ratios for each of the 16 triangles in the stimulus

TABLE IX

Mean Values of Direct Distance Ratios for Individual Subjects-Adult and Child

Subject	Adult		Child	
	Ratio 1(CB)	Ratio 2(Euc)	Ratio 1(CB)	Ratio 2(Euc)
1	1.11	1.36	.92	1.18
2	.90	1.11	.98	1.29
3	.88	1.11	.85	1.06
4	.91	1.10	.85	.96
5	.69	.82	.79	.98
6	1.25	1.60	1.01	1.30
7	1.02	1.24	.86	.99
8	.90	1.08	1.03	1.31
9	.99	1.22	1.07	1.38
10	.94	1.16	.98	1.19
11	1.00	1.29	1.31	1.70
12	.99	1.27	.74	.90
13	.98	1.16	.87	.99
14	.85	1.11	1.09	1.39
15	.92	1.03	.95	1.15
16	1.00	1.23	.72	.89
17	.74	.92	.75	.87
18	.78	.93	1.10	1.43
19	.69	.82	1.01	1.25
20	1.23	1.52	1.03	1.39
21	.73	.88	1.05	1.35
22	1.32	1.71	.87	.97
23	1.04	1.33	1.00	1.23
24	.90	1.22	1.16	1.48

set one configuration. On comparing the values in Table IX to the group mean values in Tables VI and VII the wide variation in the individual data is again noticeable: the Ratio 1 values for adults ranging from .69 to 1.32 and for children from .79 to 1.31. The Ratio 2 values ranging from .82 to 1.71 in the adult group and from .87 to 1.70 in the child group. As with other tabulations, the child group shows consistently less variation than the adult group. If a Ratio 1 value of greater than 1.10 be considered a violation of the triangular inequality, four adult subjects would qualify: numbers 1, 6, 20 and 22; only two child subjects, numbers 11 and 24 had such values. Referring to Table II it is noted that these violations are not because of low reliability; the coefficients for these adult subjects ranging from .739 to .444 and for the two child subjects the coefficients were .642 and .470.

As stated previously, a Ratio 2 value of less than 1.00 indicates that the distance metric is greater than two (Euclidean) and is tending towards dominance metric. On comparing Table VIII and Table IX for the adult group it is seen that of the eight lowest values of Ratio 2, six have one-dimensional configurations. Similarly, for the child group, five of the six subjects with one-dimensional solutions had Ratio 2 values of less than 1.00. These findings may be considered to confirm to some extent the dominance metrics indicated by the direct distance analysis. It is pointed out that there is rather poor correspondence between the distance metrics for individual subjects in Tables VIII and IX.

Possibly the distance metrics indicated by Table IX are more accurate in the light of the above-noted analysis of the dominance metrics.

<u>Stimulus Set Two</u>

The stress patterns for individuals of adult and child groups is given in the latter half of Appendix A, with a summary of the dimensionality and distance metric for each subject in Table X. In determining dimensionality the interpretability of coordinates is again of major concern. In the analysis of set one data, solutions of greater than two dimensions were considered uninterpretable. In the analysis of individual data on set two, since the stimuli were constructed to a dimensionality of four, solutions of three or four dimensions were possible. When the solution coordinates were not interpretable the stress pattern was evaluated in terms of significance levels for stress suggested by Wagenaar and Padmos (1971). Their discussion referred to an earlier stress formula, which supplied values approximately half as great as the formula used in the present study (Kruskal and Carmone, 1969). Consequently, the values suggested have been doubled for use in the present application. For ten points (or stimuli) the maximum stress which can be accepted at a significance level of $\alpha = .05$ is .14 in three dimensions and .06 in four dimensions. Three and four dimensional values which meet these levels are indicated by an asterisk in Table X. These indications must be treated tentatively, since a poor two-dimensional solution, if taken to four

TABLE X

Dimensionality and Distance Metrics for Individuals of Adult and Child Groups, Stimulus Set Two

Subject	Adult		Child	
	Dim.	Mr	Dim.	Mr
1	3	2	4	4
2	3*	4	2	4
3	3*	5	3*	4
4	3*	5	4*	2
5	1		4*	5
6	2	2	2	5
7	2	4	3*	4
8	3*	3	4	4
9	2	3	2	3
10	2	5	3*	5
11	3*	3	3*	5
12	3*	3	3*	5
13	4*	1	3*	5
14	4*	1	2	2
15	4	4	3	4
16	2	2	4	1
17	3*	3	4	3
18	2	1	3*	3
19	4	4	4	5
20	4*	5	4	4
21	3*	4	4*	3
22	4	1	4*	3
23	3*	3	2	4
24	2	2	2	3

* indicates significant stress at .05 level

dimensions, might appear to be a good four-dimensional solution. Also many of the values excluded by the application of the significant criteria might appear to be 'Fair' or 'Good' by the descriptive scale stated earlier (p. 40).

It is noteworthy that only subject, number 5 in the adult group, had one-dimensional stress values below .20. Examination of the coordinates confirmed that the solution was unidimensional for this subject who also had a unidimensional solution of set one. In the adult group there were seven two-dimensional solutions, including several, such as numbers 6 and 7, in which the stress values were poor but the configuration was clearly two-dimensional. The child group contained six two-dimensional solutions.

Comparison between Table X and the corresponding group results in Table IV again points out the great variability of the individual data. The group data revealed successful three-dimensional solutions for both groups of subjects, whereas, in those cases on the set two data where the dimensionality could be determined with some certainty, few three-dimensional solutions were found.

Complexity of Stimuli Configuration

In a sense some aspects of the complexity question have already become evident from the preceding analyses of group and individual data. No further data will be presented, but some relationships will be considered.

Group Data

Analysis of the group mean distance estimates by the Kruskal MDSCAL-5 multidimensional scaling program and by direct analysis of the distance settings indicates that for both groups there was a change in the distance metric from set one to set two, as summarized in Table XI. As previously noted, the mean ratios from which the direct distance values in Table XI were obtained were actually much closer to the city block metric than to the Euclidean metric, with the adult being closer.

TABLE XI

Variation of M_r Value (Approx) With Change in Stimulus Complexity, Group Data

	MDSCAL-5		Direct Distance
	Set One	Set Two	Set One
Adult	1	3	1-2
Child	1	2	1-2

The greater variability of the adult group data at each analysis is again seen in the comparison in Table XI; the M_r value for the adult group has moved from one to three, while that of the child group, which was already closer to the Euclidean than the adult group was, is now firmly Euclidean according to the MDSCAL-5 analysis of the set two data.

Individual Data

Although the stress patterns resulting from the MDSCAL-5 analysis of group data were relatively specific, the above analysis of the individual data indicates that conclusions concerning the influence of complexity on the data of individuals must be considered tentative, particularly in the case of the set two data.

Referring to the data on dimensionality and distance metrics listed for both groups in Tables VIII and X, one obvious difference between the two sets of results is the virtual elimination of the unidimensional solution in the set two data. Of the eight adult and six child subjects with one-dimensional solutions on set one only one persisted on set two.

Comparison of the individuals in each group with respect to effect of complexity on distance metric is greatly hampered by the variability of the individual data. Of the 18 interpretable solutions by adult individuals on set one, eight were unidimensional, and as stated the distance metric was not evident from the stress values. Of the remaining ten subjects on the adult group only four, numbers 7, 9, 10 and 24 had interpretable configurations on set two. The corresponding data for the child group are: six unidimensional solutions, twelve two-dimensional, and of these twelve only five subjects, numbers 6, 9, 14, 23 and 24 with interpretable configurations on set two. Since the number of comparable subjects in each group is so small further comparison appears of limited value.

Chapter 5

Summary and Conclusions

Summary

Similarity judgments of multidimensional stimuli of varying degrees of complexity were obtained from subjects of different age levels. One group of subjects consisted of twenty-four university students and the other were twenty-four Grade VII public school students. A bi-dimensional set of ten stimuli varied along the dimensions of color and form; the other set varied along the color dimensions of value and chroma and the form dimensions of size and rectangularity.

The experimental procedure required each subject to move the stimuli on a continuum such that the linear distance between a standard and each of the other nine stimuli represented their similarity to the standard. Each session thus provided two estimates of each of the forty-five inter-stimulus distances. These distance estimates were taken as a quantitative representation of the psychological distance between the stimuli.

An estimate of the reliability of the similarity judgments made by each subject was obtained by correlating the two sets of forty-five estimates of inter-stimulus distance.

The validity of the experimental procedure was evaluated through an analysis of the group mean distance estimates by the Kruskal MDSCAL-5 computer program. This provided a description of the group parameters in terms of how the obtained solutions related to the distance estimates, the dimensionality of the obtained solutions, and the fit of the obtained solutions to the original configurations of stimuli.

The distance metric for each group solution was determined by examination for minimum stress values at distance metrics from one to five. An additional measure of distance metric was also obtained from an analysis of the distance settings in the stimulus set one data. Similar analyses were made for the individual subjects in each group.

Conclusions

A. Is there any difference in the spatial models derived from similarity judgments of multidimensional stimuli made by adults and children?

The results of this study indicate little or no difference between the adult and child groups in terms of the distance metric which specifies the psychological space of bi-dimensional stimuli varying on color and form. Analysis of the group mean similarity judgments of the bi-dimensional stimuli by the Kruskal MDSCAL-5 multidimensional scaling program indicated stress was at a minimum at a Minkowski r-metric (M_r value) of one for both groups. Further analysis of the distance estimates also pointed to a M_r value of one for the bi-dimensional stimuli by both groups, although the child group result was closer to the Euclidean metric than that of the adult group.

A difference was revealed between adult and child groups on the distance metrics which characterized their similarity judgments of multidimensional stimuli varying on two dimensions of color and two of form. The MDSCAL-5 analysis of the child group mean similarity judgments indicated a best solution at the M_r value of two, or Euclidean metric, while that of the adult group data showed a best solution at the M_r value of three, or between Euclidean and dominance metric. It would appear from the results of the study that, as suggested by Torgerson (1958) and Shepard (1964) the distance metric problem is not independent of the nature of the multidimensional stimulus. If only one or other of the two sets of multidimensional stimuli had been used for data collection, the group results would have shown no difference or a difference of one in the distance metrics of the two groups.

The solutions based on group mean similarity judgments of stimulus set one showed little difference between the two groups, but some confirmation was suggested of the Kagan and Lemkin findings (1961) on children's conceptual preferences. Although six of the eight adult unidimensional configurations were on the color dimension only three of the six configurations in the child group were on the color dimension.

B. Is there any difference in the spatial models derived from similarity judgments of individuals within the adult group and the child group?

Analysis of the similarity judgments of the twenty-four subjects in each group showed there was considerable variation within each group. The MDSCAL-5 analysis of the individual data was affected by the apparently large error factor to which individual data of this type is prone. The analyses did reveal those individuals in each group whose results were so poor that they probably misunderstood the task. The analyses also isolated other individuals whose solutions showed that they were making unidimensional judgments.

Analysis of the direct distance settings of the bidimensional stimuli for the individual subjects in each group also revealed the wide variation within each group. Although the MDSCAL-5 analyses of individual data were perhaps inconclusive, they were confirmed to some extent by the direct distance analysis in that the unidimensional stress patterns corresponded well with the dominance metric values indicated in the distance analysis. Possibly the direct distance data for individuals has greater validity than the MDSCAL-5 data, because the ratios in which the individual data are presented are the means of sixteen ratios, and thus the random errors are reduced.

Although the group data on set one for both groups of subjects indicated a city block distance metric, the data for individual subjects ranged from triangular inequality violation to city block metric to Euclidean metric to dominance metric. The composition of these metrics was approximately similar for both groups, but the child group showed less extremes of variation than the adult group; they were higher in consistency on both sets of stimuli, their range of distance ratios was less than the adult range, and they had fewer instances of violation of triangular inequality. It had been anticipated that the child group would do less well than the adult group, but on nearly all criteria their data were more consistent. A possible reason for this superiority could be the naivete, experimental and otherwise, of the younger subjects. It was noted during the experimental sessions that the child subjects generally listened attentively as the procedure was explained and applied themselves diligently to the task at hand. The adults, however, were more apt to question the intent of the study, perhaps suspecting some kind of projective instrument.

The findings of the present study are in general agreement with those of Hyman and Well (1967) who showed that the distance metric based on group solutions did not describe the individual distance metrics with much fidelity. It may be concluded that group solutions may not contribute greatly to the analysis of individual perceptual behavior.

C. How stable are the spatial models in each group as the configuration of the multidimensional stimulus sets is increased in complexity?

The greatest effect of the change in complexity between the two sets of stimuli was in the distance metric for the adult group, which changed from near city block (M_r of one) on set one to Euclidean plus (M_r of three) on set two. The metric for the child group, which was already slightly closer to the Euclidean metric on set one, changed to Euclidean on set two. The increase of stimulus complexity had the effect on both groups of reducing the number of unidimensional judgments, with only one subject on the adult group having a one-dimensional solution on the second stimulus set. It would appear, as noted above, that the number and type of dimension must be considered in any study of distance metrics.

The individual results did not support the prediction based on the complexity theory of Schroder et al. (1967) that there would be a greater number of dominance metrics in the younger group on the two-dimensional stimulus set.

Although the distance metrics for the adult and child groups differed on the four-dimensional set of stimuli, only three dimensions, two of form and one of color, were recovered by the group solutions. This indicates some support for the dimensional limitations on similarity judgments, as proposed by Miller (1956), Fenker and Brown (1969) and others.

The present study has enabled investigation of two separate aspects of scaling methodology: the collection of similarities data by the modified method of multiple ratios, and the multidimensional scaling analysis of similarities data by the Kruskal MDSCAL-5 computer program.

The satisfactory correlation coefficients for both groups of subjects on both sets of stimuli showed that the experimental procedure produced sufficient reliable group data. It could also be concluded from the individual coefficients that the range varied from very satisfactory to completely unsatisfactory. Thus for the collection of group data the method appears to provide consistent data, but for individual data a wide variability in consistency may be expected.

A more important feature of the data collection procedure is the nature of the data which it provides. The group similarity judgments of the bi-dimensional stimuli could be processed to yield bi-dimensional configurations which were close reproductions of the original stimulus configurations. The reproduction of the group configurations for the four-dimensional stimuli was less successful; only three dimensions could be meaningfully extracted from the mean similarity judgments of each group. As noted above, this finding is in accord with much of the previous research in this area (Miller, 1956, Fenker and Brown, 1969). Since the dimensionality of the stimuli appears to be a factor in the study of the distance metric problem, some means of obtaining data which will lead to the recovery of all relevent dimensions would be preferable.

The Kruskal MDSCAL-5 program achieved better solutions in the analysis of group data than of individual data, although there were also some anomalous features in the group data analysis. The solutions on stimulus set one for both groups showed a minimum stress at the M_r value of one, rising to a maximum at $M_r = 2$, and decreasing gradually thereafter. The direct distance analysis confirmed the mimimum at $M_r = 1$, which appears to rule out the possibility of an alternate good solution at metrics greater than two. For the group solutions of stimulus set two the stress analysis identified stress minima for adult and child groups at M_r values of three and two respectively. Unfortunately, in the case of the stimulus set two data, no alternative measure of the distance metric was available.

Although the dimensionality and distance metric of some individual solutions were satisfactorily obtained by the MDSCAL-5 program, in many other instances this was not the case, particularly with stimulus set two data. The MDSCAL-5 analyses were able to show those subjects with extremely poor configurations, which did not always correspond with poor consistency of similarity judgments. The program was also successful in revealing those individuals who made unidimensional judgments.

The data analysis of the present study was facilitated by the nature of the multidimensional stimuli, which were constructed to known dimensions of color and form. This in turn allowed configurations and coordinates to be evaluated for goodness of recovery or interpretability. If the stimuli were of unknown dimensionality, such as the names of politicians or other personalities, on which subjects were required to make similarity judgments, solutions would be more difficult to evaluate. Thus in a table of stress values at different dimensionality and distance metrics, if the configuration coordinates were not easily interpretable, reliance would have to be placed on the stress pattern as a criterion. As the tables of group and individual stress values indicate, an empirical confusion might be possible between distance metric and dimensionality.

Much of the above discussion suggests the need for further experimentation in the areas of: characteristics of the subjects involved, the nature of the multidimensional stimuli, the data collection process, and methods of data analysis.

Selection of subjects of various age levels different from those in the present study could result in different findings. Similarity judgments from very young children would require a different method of data collection, but these methods are available. Further variation could involve subjects from differing cultures or occupational fields.

The nature of the multidimensional stimulus requires further research, particularly with respect to the dimensionality, the kind of dimensions and the particular mix of dimensions involved in a stimulus set. The city block metric for the bidimensional stimuli in the present study is in general accord with the view that this metric is appropriate for stimuli whose dimensions are perceptually distinct. How would the findings for the second set be affected if they were tri-dimensional, perhaps two of color and one of form, or two of form and one of color? It is true that the group analyses extracted only three dimensions, but it is not clear what effect the fourth dimension had on the data; possibly it contributed to the error in the solutions.

As noted earlier, there are numerous methods of collecting similarity judgments data. Further research might be directed to testing which methods provide similarity judgment data which are most amenable to distance metric analysis. The method used in the present study could be modified; the original method of Indow (1960) or the version of Hyman and Well (1967) might be more effective. Possibly presenting the stimuli in pairs instead of as a set might change the manner in which they are perceived by the subject.

The methods by which similarity judgments may be analyzed are also numerous. Research could be done on computer programs other than MDSCAL-5 to determine their success in delineating the distance metrics which characterize the similarity judgments. Further research on the MDSCAL-5 program itself would be highly desirable. A Monte Carlo study is required of the behavior of the stress variable when multiple configurations of random points are analyzed at different dimensions and distance metrics to determine what reliance can be placed on an apparent stress minimum at a particular distance metric.

Further attention should also be given to the development of alternative measures of distance metric, such as the direct distance analyses of the bi-dimensional data. Such measures should be amenable to non-computer manipulation and would provide a valuable check on experimental results.

Research such as that suggested above, as well as the present study, is designed to delimit the extent and conditions under which distance metrics may be identified. A more fundamental question concerns their importance. Some consequences were discussed in Chapter One, but further research is necessary into the implications. Given that certain solutions are better at a distance metric other than Euclidean, what is the actual difference in interpretability between the two solutions? It may well be that in many cases the improvement does not warrant the extra labor.

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Appendix A

Appendix A

Stress Values of Individual Solutions
Stimulus Set One-Adult

Sub No	Dim	$M_{r=1}$	2	3	4	5
1	3	171	179	122	154	146
	2	443	390	367	271	253
	1	496	496	651	495	495
2	3	290	101	155	134	161
	2	307	183	190	167	203
	1	323	500	323	323	323
3	3	010	016	011	026	203
	2	259	237	241	232	208
	1	413	677	735	510	416
4	3	142	061	136	097	114
	2	188	188	149	131	123
	1	440	454	501	440	441
5	3	010	010	009	010	009
	2	007	010	008	008	010
	1	004	005	005	006	008
6	3	305	258	188	145	121
	2	663	566	377	379	372
	1	686	770	701	765	766
7	3	398	206	151	216	230
	2	436	340	383	352	415
	1	549	532	532	544	558
8	3	010	009	010	011	013
	2	027	009	008	009	010
	1	010	008	009	006	007
9	3	046	148	089	087	084
	2	201	177	151	141	139
	1	438	456	441	440	440
10	3	186	190	163	073	083
	2	284	330	310	205	197
	1	411	470	411	423	423
11	3	165	089	092	095	091
	2	181	205	196	212	229
	1	303	303	303	303	303

Sub No	Dim	$M_{r=1}$	2	3	4	5
12	3	219	185	099	119	122
	2	386	240	225	221	219
	1	410	443	410	410	410
13	3	094	080	066	100	097
	2	170	165	168	151	147
	1	231	239	239	239	239
14	3	211	096	051	042	043
	2	247	318	267	281	278
	1	648	586	509	639	510
15	3	010	010	010	009	010
	2	010	009	010	010	009
	1	010	010	005	006	008
16	3	176	044	039	045	044
	2	182	073	094	080	183
	1	191	191	191	191	191
17	3	054	039	050	047	066
	2	054	057	055	050	111
	1	124	123	131	123	131
18	3	118	083	101	099	109
	2	148	107	125	117	113
	1	154	154	154	154	154
19	3	129	014	027	030	137
	2	143	061	009	009	138
	1	188	188	189	189	188
20	3	190	136	100	104	107
	2	193	304	337	241	336
	1	763	425	439	439	439
21	3	140	095	071	106	147
	2	132	113	130	158	164
	1	167	167	167	167	167
22	3	266	216	181	174	186
	2	374	428	405	418	492
	1	670	655	715	723	672
23	3	094	128	101	066	063
	2	101	191	171	209	206
	1	751	271	271	271	271
24	3	3200	206	151	182	201
	2	259	327	298	286	279
	1	767	485	485	485	485

Sub No	Dim	$M_{r=1}$	2	3	4	5
Stimulus Set One--Child						
1	3	131	131	110	094	102
	2	134	211	173	153	142
	1	549	679	559	549	558
2	3	155	139	133	180	232
	2	217	205	204	213	*530
	1	426	448	427	655	409
3	3	200	119	084	061	234
	2	273	199	180	171	171
	1	386	375	375	375	375
4	3	017	016	035	037	042
	2	007	012	010	010	009
	1	075	*461	075	075	075
5	3	159	164	166	155	147
	2	261	367	288	277	300
	1	449	449	546	467	467
6	3	120	064	126	091	149
	2	107	206	201	187	242
	1	263	263	289	263	263
7	3	100	071	056	040	048
	2	097	137	115	105	101
	1	*533	305	305	316	305
8	3	522	361	405	396	296
	2	533	420	506	520	523
	1	611	624	624	611	611
9	3	253	176	160	154	165
	2	276	343	280	275	284
	1	417	417	417	417	417
10	3	098	112	085	123	089
	2	110	164	119	177	143
	1	177	177	177	177	177
11	3	162	319	252	234	216
	2	667	481	382	363	381
	1	723	694	618	618	694
12	3	111	100	105	085	078
	2	206	174	194	166	163
	1	237	238	279	237	*532

Sub No	Dim	$M_{r=1}$	2	3	4	5
13	3	103	064	064	069	089
	2	112	105	105	099	099
	1	141	204	141	141	141
14	3	128	055	054	073	062
	2	150	151	157	237	135
	1	801	416	420	418	415
15	3	014	034	032	015	013
	2	208	139	118	107	102
	1	341	515	515	555	341
16	3	299	295	253	313	292
	2	382	333	311	355	367
	1	416	416	416	419	416
17	3	009	010	009	010	010
	2	009	009	010	010	010
	1	009	009	005	008	001
18	3	228	127	128	114	131
	2	266	250	270	228	274
	1	745	337	337	337	337
19	3	244	139	082	078	151
	2	308	372	330	496	336
	1	483	599	483	483	483
20	3	130	231	163	136	221
	2	297	392	359	337	334
	1	711	810	809	809	810
21	3	412	281	359	344	258
	2	549	510	473	575	422
	1	667	665	692	639	639
22	3	151	053	051	045	041
	2	220	138	144	135	130
	1	243	243	243	243	243
23	3	064	114	155	101	087
	2	166	175	139	201	202
	1	707	210	210	210	210
24	3	145	225	149	122	135
	2	435	303	283	277	275
	1	477	477	477	477	477

Sub No	Dim	$M_{r=1}$	2	3	4	5
Stimulus Set Two-Adult						
1	4	047	083	077	073	063
	3	184	127	145	129	130
	2	275	218	225	257	218
	1	300	270	271	274	270
2	4	067	093	029	044	040
	3	155	147	134	131	148
	2	393	361	450	469	498
	1	704	662	571	563	560
3	4	163	119	091	080	113
	3	225	194	186	161	135
	2	278	275	270	303	253
	1	400	401	401	400	400
4	4	101	055	047	040	104
	3	112	095	079	055	049
	2	242	225	223	221	241
	1	265	265	265	265	265
5	4	155	051	019	017	012
	3	137	107	074	049	056
	2	141	123	122	127	131
	1	183	183	183	183	183
6	4	210	068	029	045	026
	3	148	122	087	101	106
	2	563	305	306	307	318
	1	681	653	740	626	626
7	4	097	060	030	035	053
	3	183	182	163	343	197
	2	342	319	472	186	185
	1	602	675	628	517	516
8	4	174	070	089	062	107
	3	200	166	123	191	185
	2	333	306	291	313	348
	1	576	647	625	670	576
9	4	126	098	108	110	105
	3	307	227	198	188	223
	2	381	271	266	272	382
	1	631	729	576	625	650
10	4	098	010	010	010	012
	3	078	074	050	036	032
	2	226	227	220	149	134
	1	489	597	603	605	666

Sub No	Dim	$M_{r=1}$	2	3	4	5
11	4	162	026	015	077	131
	3	154	092	079	082	093
	2	253	193	195	210	208
	1	410	558	559	410	410
12	4	067	067	049	041	058
	3	171	134	109	118	118
	2	216	271	223	299	327
	1	426	426	426	426	578
13	4	095	170	117	106	102
	3	324	214	173	215	285
	2	328	304	337	338	295
	1	424	424	427	463	458
14	4	078	173	158	115	109
	3	490	471	453	409	398
	2	561	495	585	540	558
	1	728	777	755	777	786
15	4	258	280	174	148	209
	3	492	411	389	440	426
	2	523	492	477	506	500
	1	596	599	596	599	599
16	4	036	060	025	026	026
	3	141	112	124	119	173
	2	318	242	285	282	255
	1	589	589	589	714	715
17	4	092	063	044	050	079
	3	116	072	058	084	263
	2	466	550	273	445	320
	1	633	655	701	654	657
18	4	097	041	040	065	060
	3	162	096	083	114	158
	2	187	225	198	190	188
	1	537	530	502	503	507
19	4	105	175	115	077	146
	3	418	291	241	262	419
	2	434	404	386	429	647
	1	626	558	748	545	783
20	4	189	109	057	051	044
	3	218	231	257	211	239
	2	347	309	318	373	322
	1	806	796	538	508	667

Sub No	Dim	$M_{r=1}$	2	3	4	5
21	4	260	151	080	079	133
	3	217	213	167	136	138
	2	326	316	286	263	283
	1	391	391	391	391	391
22	4	103	130	113	113	115
	3	439	283	234	197	251
	2	390	351	370	348	330
	1	496	544	539	496	577
23	4	208	090	148	182	090
	3	274	256	140	141	254
	2	378	310	251	281	279
	1	390	813	390	390	390
24	4	125	070	048	039	048
	3	148	112	088	125	093
	2	232	154	378	160	198
	1	549	584	538	538	549

Stimulus Set Two--Child

1	4	113	080	052	031	047
	3	373	168	174	179	302
	2	321	304	317	323	302
	1	748	449	449	448	629
2	4	128	067	009	069	031
	3	294	133	096	093	099
	2	333	278	250	245	333
	1	635	583	583	583	583
3	4	151	102	082	065	091
	3	258	176	130	117	095
	2	297	235	196	193	195
	1	325	325	325	325	325
4	4	062	049	071	088	080
	3	289	211	198	234	250
	2	393	376	365	444	303
	1	504	538	538	504	504
5	4	220	142	066	064	050
	3	153	174	187	152	155
	2	198	241	262	216	293
	1	326	326	326	326	326

Sub No	Dim	$M_{r=1}$	2	3	4	5
6	4	050	045	044	045	085
	3	159	093	117	106	182
	2	265	*511	*588	239	235
	1	691	650	661	709	709
7	4	290	109	074	069	086
	3	373	180	129	104	213
	2	397	383	392	380	373
	1	527	724	516	535	528
8	4	244	154	110	076	080
	3	442	329	237	334	330
	2	525	597	411	406	401
	1	672	619	601	618	609
9	4	129	086	136	126	132
	3	189	151	161	161	142
	2	257	265	246	252	255
	1	400	459	407	407	407
10	4	129	064	034	105	072
	3	147	125	122	122	099
	2	187	221	204	195	190
	1	706	523	516	566	523
11	4	164	036	074	048	082
	3	282	139	115	126	111
	2	299	327	287	270	261
	1	551	551	540	548	548
12	4	082	039	035	026	037
	3	144	220	448	125	106
	2	253	247	*561	240	236
	1	471	360	361	360	360
13	4	066	045	070	082	087
	3	184	103	096	116	067
	2	286	190	262	270	287
	1	313	308	308	314	313
14	4	179	075	083	106	196
	3	235	126	136	158	145
	2	467	279	398	388	370
	1	674	649	632	645	645
15	4	045	078	087	061	051
	3	137	144	109	101	126
	2	234	*520	217	222	223
	1	650	575	650	658	681

Sub No	Dim	$M_{r=1}$	2	3	4	5
16	4	099	146	136	155	172
	3	456	257	234	220	211
	2	483	514	351	349	456
	1	690	582	667	684	683
17	4	187	269	170	247	270
	3	444	366	308	280	266
	2	491	504	509	599	496
	1	650	694	721	686	685
18	4	175	126	074	072	064
	3	266	167	125	134	127
	2	352	*621	344	329	299
	1	601	755	654	656	655
19	4	167	139	108	096	094
	3	310	216	210	200	199
	2	405	375	368	381	379
	1	701	702	723	722	723
20	4	191	170	115	097	115
	3	273	246	290	246	249
	2	454	444	436	438	443
	1	712	731	715	730	707
21	4	204	110	041	097	080
	3	170	178	224	124	292
	2	248	333	311	237	293
	1	534	425	425	411	425
22	4	101	080	062	064	178
	3	440	318	249	201	185
	2	361	379	362	342	330
	1	718	497	495	495	495
23	4	115	034	028	011	014
	3	194	100	098	100	127
	2	206	149	131	127	128
	1	448	463	463	463	463
24	4	191	021	025	067	052
	3	243	112	087	093	082
	2	318	259	255	292	257
	1	475	475	507	475	475

*Indicate local minimum value.

Appendix B

Appendix B--Data Sheet

Subject: . . . . Stimulus Set: . . . .

<u>Trial No.</u>

1. Dist / 0 / / / / / / / / / /
   Stim / 7

2. Dist / 0 / / / / / / / / / /
   Stim / 10

3. Dist / 0 / / / / / / / / / /
   Stim / 6

4. Dist / 0 / / / / / / / / / /
   Stim / 1

5. Dist / 0 / / / / / / / / / /
   Stim / 9

6. Dist / 0 / / / / / / / / / /
   Stim / 8

7. Dist / 0 / / / / / / / / / /
   Stim / 4

8. Dist / 0 / / / / / / / / / /
   Stim / 2

9. Dist / 0 / / / / / / / / / /
   Stim / 3

10. Dist / 0 / / / / / / / / / /
    Stim / 5