Manipulability of Teleoperated Surgical Robots with Application in Design of Master/Slave Manipulators

Ali Torabi, Mohsen Khadem, Kourosh Zareinia, Garnette Roy Sutherland, Mahdi Tavakoli

***Abstract*—Teleoperated surgical robots can significantly improve the performance of minimally invasive surgeries. The performance of a master-slave robotic system depends significantly on the capability of its master device to appropriately interface the user with the slave robot. However, master robots currently used in the clinic present several drawbacks such as the mismatch between the slave and master workspaces and the inability to intuitively transfer the slave robot's dexterity and joint limits to the user. In this paper, the "teleoperation manipulability index (TMI)" is introduced as a quantifiable measure of the combined master-slave system manipulability. We also demonstrate the application of the TMI in the design of master-slave robotic systems. By employing the proposed manipulability index, we are able to modify the design of a commercially available master robot that 1) enhances surgeon's control over force/velocity of a surgical robot, 2) minimizes the master robot's footprint, 3) optimizes the surgeons' control effort, and 4) avoids singularities and joint limits of the master and slave robots. A simulation study is performed to validate the performance of the modified master-slave robotic system.**

I. Introduction

Surgical procedures are being transformed by robots entering the operating rooms. Robots are enhancing surgical techniques and expanding surgeons' capabilities. Master-slave teleoperation is a common and effective mean of providing an intuitive user interface for controlling surgical robots. The slave robot, in the context of surgery, is the manipulator that performs the surgery within the patient's body while the master is a user interface that allows the surgeons to control the slave. This approach allows the surgeon to benefit from robotic advantages such as motion scaling and tremor reduction while retaining direct control over the robot motions to ensure the safety of the procedure.

Teleoperated surgical systems such as the da Vinci®(Intuitive Surgical, Sunnyvale, CA, USA), neuroArm® [1], and Raven Surgical Robot [2] have successfully solved many of the problems encountered in minimally invasive surgeries, such as loss of dexterity [3]. The enhanced dexterity offered by teleoperated surgical robots can significantly improve the performance of minimally invasive surgeries [4]. However, master robots currently used in the clinic present several drawbacks such as the mismatch between slave and master workspaces and the inability to intuitively transfer the slave robot's dexterity and joint limits to the user.

The first step in improving the kinematic dissimilarity and workspace mismatch in a teleoperated robotic systems is to define and estimate a measure that quantifies the teleoperation system's manipulability. In this paper, we propose the teleoperation manipulability index (TMI) as a quantifiable measure of kinematic similarity between the master and slave robot. Such a quantifiable measure of dexterity can be used for analysis and comparison of designs of master-slave robotic systems. Also, this allows for considering dexterity in motion planning and control of complex surgical tasks such as suturing or navigation in the presence of anatomical obstacles.

Manipulability of robots was first introduced in [5], [6]. Manipulability describes how a manipulator can freely apply forces and torques or move in arbitrary directions, and quantifies the ability to perform an action quickly and skilfully [7]. Manipulability analysis consists of describing directions in the task or joint space of a robot with the best ratio between some measure of effort in joint space (e.g., joint torque) and a measure of performance in task space (e.g., position accuracy). Yoshikawa introduced the manipulability index [6] as a quality index for a single manipulator, which describes the characteristics of feasible motions in the Cartesian space corresponding to unit joint velocity vectors. He defined a quality measure based on analysis of the manipulability ellipsoid (ME). ME is a volume/surface in the Cartesian velocity space, which is mapped from the unit sphere in the joint velocity space by a Jacobian transformation [6].

Manipulability analysis has been widely used for analysis of motion of multiple cooperating robots [8]–[11]. Lee [8] defined a dual-arm ME as the maximum volume ellipsoid determined by the intersection between the two single-arm MEs. This is because the required cooperation between the two arms imposes additional kinematic constraints to the manipulability of individual arms. Chiacchio *et al.* [9] extended the concept of ME to the multi-arm case independent of the number of arms involved in the cooperation by regarding the system of multiple arms as a closed-chain system. Chiacchio *et al.* also introduced the concept of force manipulability for fully actuated robotic chains by a duality argument, considering the principal directions for force and velocity MEs are the same while the lengths of axes are inversely proportional to each other [9]. Bicchi *et al.* [10] extended the kinematic ME problem to general cooperating arms, with arbitrary number

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of joints per arm. Melchiorri [11] applied similar tools to address force manipulability in cooperative robots with active and passive joints [12].

Researchers have studied the application of manipulability index in design and control of surgical robots. Konietschke *et al.* [13] and Li *et al.* [14] used the manipulability index to optimize the design of single robots. Maddahi *et al.* [15] showed the correlation between manipulability of a master robot and the performance of a teleoperated surgical system emulating a micro-neurosurgical task defined in terms of actuators efforts and distances travelled by the slave end-effector.

Most studies of manipulability of teleoperated systems only consider the manipulability of one robot (master or slave) [13]–[16]. By using the manipulability index of one robot as the design criterion, the solution to the design space search would result in a robot with very large links, such that joints angle deviate as little as possible from the isotropic pose while still reaching the target in the workspace. However, long links reduce the flexural stiffness of the manipulator and increase inertia and the robot's footprint. The robots used in the operation room work in a limited workspace and must have a small footprint with maximum rigidity and stability.

The aim of this research is to develop a manipulability index for quantifying the dexterity of surgical master-slave systems. We also demonstrate the application of manipulability in the design of master-slave robotic systems. We demonstrate that by modifying a commercially available master robot using the proposed manipulability index, we are able to enhance the surgeon's control over force/velocity of the surgical robot, minimizes the master robot's footprint, optimizes the surgeons' control effort, and avoid singularities of the master and slave robots.

The rest of the paper is organized as follows: In Section II, an overview of manipulability index for a single robot is presented. In Section III, manipulability of teleoperated systems is discussed. Application of the TMI in the design of master-slave robotic systems and simulation results to validate the performance of such designs are presented in Section IV. Concluding remarks appear in Section V.

II. Manipulability for a Single Robot

For a robotic manipulator, the Jacobian matrix provides a transformation from the velocity of the end-effector in Cartesian space to the actuated joint velocities as the

$$\dot{x} = J\dot{q} \tag{1}$$

where $\dot{q}$ is an n-dimensional vector that represents a set of actuated joint rates, $\dot{x}$ is an m-dimensional output velocity vector of the end-effector, and J is the $m \times n$ Jacobian matrix. The Jacobian J defines the mapping from R^n to R^m. The unit sphere in R^n can be mapped into an ellipsoid in R^m through J as shown bellow:

$$\begin{aligned} \|\dot{q}\|^2 = \dot{q}^T\dot{q} &= \dot{x}^T(J^\dagger)^T(J^\dagger)\dot{x} \\ &= \dot{x}^T(JJ^T)^{-1}\dot{x} \end{aligned} \tag{2}$$

here the superscript "$\dagger$" indicates the pseudo-inverse of a matrix, $J^\dagger = J^T(JJ^T)^{-1}$. The ellipsoid in R^m is called the manipulability ellipsoid (ME), and describes the versility of moving in the task space. The ME is a surface/volume that helps to visualize the feasible directions of velocity at the end-effector of a robot. This ellipsoid can be spanned using the singular values of the Jacobian matrix, which can be calculated using the singular value decomposition (SVD) [17]. As J is $m \times n$, there exist orthogonal matrices $U \in R^{m\times m}$ and $V \in R^{n\times n}$ such that

$$J = U\Sigma V^T \tag{3}$$

where $U = [u_1 \cdots u_m]$ is an $m \times m$ unitary matrix, Σ is an $m \times n$ rectangular diagonal matrix in which the diagonal entries ($\sigma_i, i = 1 \cdots m$) are known as the singular values of J with $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_m$, and $V = [v_1^T \cdots v_n^T]$ is an $n \times n$ unitary matrix

Now, the manipulability index can be defined based on the ME. Proportional to the volume of the ME spanned by singular values of J, the manipulability index can be defined as [6]

$$\mu = \sqrt{det(JJ^T)} = \sigma_1\sigma_2\cdots\sigma_m. \tag{4}$$

μ is the manipulability index at one point in the robot's workspace. To define a global manipulability index, one can use

$$GM = \frac{\int_W \mu dW}{\int_W dW} \tag{5}$$

where W is the workspace of the robot.

In addition to the manipulability index, the isotropy of the robotic arm is also important [5]. It is a measure of how well the mechanism can move in all directions, i.e., directional uniformity. Assuming that the surgical motion demands are uniform with respect to the robot in the surgical site, a good isotropy score would indicate that the load on the motors of each joint would be similar. The isotropy index has been introduced as the inverse of the condition number of the Jacobian, i.e., relation of the smallest to the largest singular value

$$\frac{1}{\kappa} = \frac{1}{\|J\|\|J^{-1}\|} = \frac{\sigma_m}{\sigma_1}. \tag{6}$$

Using (6) the global isotropy index or commonly called global conditioning index GC can be defined as

$$GC = \frac{\int_W (\frac{1}{\kappa})dW}{\int_W dW} \tag{7}$$

III. Manipulability of Teleoperated Surgical Systems

In this section, we modify the manipulability index to extend the definition of manipulability to teleoperated master-slave surgical systems. In teleoperation, we want the user to feel as if he/she is directly interacting with the slave's environment. This requires matching of the positions and forces on both the slave and the master side. It is assumed that the master robot and the slave robot follow each other position perfectly. Thus, it can be assumed that the end-effector of the master and the end-effector of the slave are physically attached together, similar to two cooperative robots manipulating a mass-less point object with tight grasps. This prompts the observation that the teleoperation manipulability is the volume of the intersection between the MEs for the individual arms, where the intersection of the two MEs is subject to the constraints imposed by the teleoperation system.

A. Teleoperation manipulability ellipsoid

Let us assume that the teleoperation task is defined in the slave robot workspace and the ME of the master robot is transformed to the task frame, i.e., slave robot frame. Using (2), the MEs for the master robot and the slave robot can be found as

$$\dot{x}^T(HJ_MJ_M^TH^T)^{-1}\dot{x} = 1 \tag{8a}$$

$$\dot{x}^T(J_SJ_S^T)^{-1}\dot{x} = 1 \tag{8b}$$

where J_M is the Jacobian of the master robot, J_S is the Jacobian of the slave robot, and H is the transformation matrix which transforms the Jacobian of the master robot to the task frame. Following the approach first presented in [8], the combined ME of two arms is the largest ellipsoid that can be fitted into the intersection of the ME of the master robot defined in (8a) and the ME of the slave robot given in (8b). To find the largest ME in the intersection of the two given MEs in (8a) and (8b), we first assume that the principal axes of the intersecting ellipsoid coincide with the principal axes of the master ME. Knowing the intersection points of master and slave MEs we find the intersecting ME. Next, it is assumed that the principal axes of the intersecting ellipsoid coincide with those of the slave robot ME and the intersecting ME is calculated again. Finally, the ME that has the largest volume between the two calculated MEs is selected as the TME. This strategy is discussed in the following.

An ellipsoid can be specified by a set of principal axes $\sigma_i u_i$ where $u_i, i = 1, \ldots, m$ denote the orthonormal vectors specifying the orientation of the principal axes, and $\sigma_i, i = 1, \ldots, m$ represent the length of the corresponding principal axes. The principal axes of the intersecting ellipsoid $\sigma_i^I u_i^I$ can be obtained assuming they coincide with the principal axes of the master ME as

$$_1\sigma_i^I\, _1u_i{}^I = \begin{cases} \eta_i^1 u_i{}^M, & \text{if } \eta_i^1 < \sigma_i^M \\ \sigma_i^M u_i{}^M, & \text{if } \eta_i^1 \geq \sigma_i^M \end{cases} \tag{9}$$

where $\sigma_i^M u_i^M, i = 1, \ldots, m$ are the set of principal axes of the master robot ME, and $\eta_i^1 u_i{}^M, i = 1, \ldots, m$ represents the intersecting points between the principal axes of master robot ME in (8a) and the boundary of slave robot ME in (8b). $\eta_i^1, i = 1, \ldots, m$ can be obtained using equation (8b). Considering $\eta_i^1 u_i{}^1$ lies on the slave robot ellipsoid, we have

$$(\eta_i^1 u_i{}^M)^T(J_SJ_S^T)^{-1}(\eta_i^1 u_i{}^M) = 1 \tag{10}$$

and thus

$$\eta_i^1 = [(u_i{}^M)^T(J_SJ_S^T)^{-1}u_i{}^M]^{-1/2} \tag{11}$$

Now, we assume that the principal axes of the intersecting ellipsoid coincide with those of slave robot ME. This results in a different representation for the intersecting ellipsoid:

$$_2\sigma_i^I\, _2u_i{}^I = \begin{cases} \eta_i^2 u_i{}^S, & \text{if } \eta_i^2 < \sigma_i^S \\ \sigma_i^S u_i{}^S, & \text{if } \eta_i^2 \geq \sigma_i^S \end{cases} \tag{12}$$

here, $\eta_i^2 u_i{}^S$ represents the intersecting point between the principal axes of slave robot ME and the boundary of master robot ME. Following the same method used in the derivation of η_i^1, η_i^2 can be obtained by

$$\eta_i^2 = [(u_i{}^S)^T(HJ_MJ_M^TH^T)^{-1}u_i{}^S]^{-1/2}. \tag{13}$$

B. Teleoperation manipulability index

The TMI (λ) is the largest ME between ellipsoids defined in (9) and (12).

$$\lambda = max\{\prod_{i=1}^{m} {}_1\sigma_i^I\, _1u_i{}^I, \prod_{i=1}^{m} {}_2\sigma_i^I\, _2u_i{}^I\} \tag{14}$$

Now we can define the global teleoperation manipulability index (GM_T) to determine the overall conditioning of the manipulability index of the teleoperation system across the slave workspace W rather than at each point therein:

$$GM_T = \frac{\int_W \lambda dW}{\int_W dW}. \tag{15}$$

Larger values of GM_T correspond to better manipulability of the teleoperation system.

We note that beside the singular values and manipulability index, joint limits have a major impact on the end-effector's dexterity in the workspace. In order to consider the effects of mechanical constraints of the manipulator, we deployed the joint-limit constrained Jacobian J^q [16]. The constrained Jacobian J^q is formed by penalizing the columns of Jacobian individually using

$$J_i^q = P_i^q J_i \tag{16}$$

where J_i is the i^{th} column of the robot Jacobian. P_i^q is the joint-wise penalization function given by

$$P_i^q = \frac{1 - exp(\frac{-4k_q(q_i - q_{i,min})(q_{i,max} - q_i)}{(q_{i,max} - q_{i,min})^2})}{1 - exp(-k_q)} \tag{17}$$

where the coefficient "4" and the denominator "$1 - exp(-k_q)$" in equation (17) are needed to normalize the penalization term such that P_i^q spans the interval $[0, 1]$. At the joint-limits, P_i^q becomes zero. In the neutral position, $\frac{q_{i,max}+q_{i,min}}{2}$, P_i^q becomes one. The scaling coefficient k_q specifies the functional shape in between these points. Using this penalty function, the individual columns of J are penalized when the ith joint value q_i approaches the limits $q_{i,min}$ or $q_{i,max}$.

Penalization of the Jacobian for calculating the manipulability index was first addressed by Tsai *et al.* [9]. However, unlike the global penalization approach used in [18], the individual columns of J are penalized. Now, by substituting the constrained Jacobian J^q in (4), (6), (11), and (13) for J, we can calculate the manipulability and isotropy indices for a single robot and the TMI considering the robot mechanical constraints.

Considering conservation of energy and neglecting the potential terms, a measure of master and slave robots' joints kinetic energy, $\|\dot{q}\|^2$, can be directly related to the human users effort while manipulating the robot, and the effort needed to move the slave's joints. Assuming the master robot is manipulated to move the slave robot's end-effector in its task space at a velocity $\dot{x}$, $\|\dot{q}\|^2$ can be calculated as follows

$$\begin{aligned}\|\dot{q}\|^2 = \dot{q}^T\dot{q} &= \dot{x}^T(J_T^\dagger)^T(J_T^\dagger)\dot{x} \\ &= \dot{x}^T(J_TJ_T^T)^{-1}\dot{x}\end{aligned} \tag{18}$$

where

$$(J_TJ_T^T) = U_T\Lambda_TU_T^{-1} \tag{19}$$

in which $U_T = [u_1^I, \cdots, u_m^I]$ and $\Lambda_T = diag(\sigma_1^I, \cdots, \sigma_m^I)$ are set of principal axes of the teleoperation system's ME.

In the next section, we will use GM_T as a design criterion to modify the master robot of a teleoperated system. Simulations are performed to demonstrate the benefits of using the TMI such as reducing the surgeon's control effort while manipulating the slave robot's end-effector.

IV. Manipulability as a Design Criterion

A slave robot will not perform according to its full potential if paired with a master robot with lower dexterity and manipulability. There are several commercial master manipulators designed and developed to operate in conjunction with slave robots, offering advantages in terms of generality and ease of use. However, their lack of kinematic similarity to a given slave robot presents several disadvantages such as reduced overall manipulability and dexterity as discussed below.

The control of a master-slave system can be based on force control, position control or a combination of both. To improve the control accuracy for teleoperated robotic systems in all these cases, one must improve the manipulability of the system. There are two ways to do this.

Most studies consider only the global manipulability index, GM_M, and global conditioning index, GC_M, of the master robot [13]–[16]. Following this approach, the optimal design for the master robot can be obtained by

$$\underset{D}{\text{maximize}}\{C_1 := K_1GC_M + K_2GM_M\} \tag{20}$$

where D is the set of parameters to be optimized, and K_1 and K_2 are appropriate scaling factors. By using (20), the kinematics of the master robot is optimized by maximizing the master robot manipulability and isotropy. In this optimization, the kinematic performance of the teleoperated system is enhanced by enlarging the master robot's links length which obviously faces practical limitation. In fact, such a solution would result in a robot with large links, small flexural stiffness, and big footprint. The robots used in the operating room work in a limited workspace and must have a small footprint with maximum rigidity and stability.

Instead of the above, we propose an approach that considers the global condition index GC_M of the master robot and the global manipulability index GM_T of the teleoperated system as design criteria. This way, in addition to maximizing GC_M for the master robot, which enhances surgeon's control over force/velocities, kinematic compatibility between the master and slave robot is also considered. The goal is to design a master robot for a given slave robot with maximum possible manipulability while maintaining a small footprint. By considering the kinematics of the slave robot, the optimal design for the master robot can be obtained by

$$\underset{D}{\text{maximize}}\{C_2 := K_1GC_M + K_2GM_T\}. \tag{21}$$

By using (21), the kinematics of the master robot is optimized for the best kinematic similarity to the slave robot.

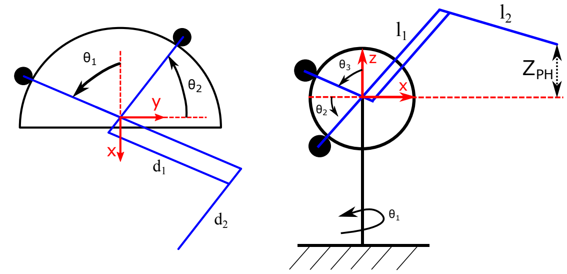

Fig. 1. (Left) Top view of Rehabilitation Robot. (Right) Side view of PHANToM 1.5A.

As a case study, the PHANToM 1.5A robot (Geomagic Inc., Morrisville, NC, USA), which provides position measurement and force feedback at its end point in three translational DOFs is used as the master robot, and a 2-DOF planar upper-limb rehabilitation robot and haptic device (Quanser Inc., Markham, ON, Canada) is used as the slave robot. The schematic diagram of rehabilitation robot and PHANToM robot are shown in Fig. 1. We propose to use C_2 as a quantitative measure for optimal selection of the PHANToM robot's placement and other kinematic parameters.

The Jacobians of the rehabilitation and the PHANToM robots in their base frames are

$$J_{RE} = \begin{bmatrix} -d_1s_1, & d_2c_2 \\ d_1c_1, & d_2s_2 \end{bmatrix} \tag{22a}$$

$$J_{PH} = \begin{bmatrix} -s_1(l_1c_2 + l_2s_3), & -l_1c_1s_2, & l_2c_1c_3 \\ c_1(l_1c_2 + l_2s_3), & -l_1s_1s_2, & l_2c_3s_1 \\ 0, & l_1c_2, & l_2s_3 \end{bmatrix} \tag{22b}$$

where $s_i = sin(\theta_i)$, $c_i = cos(\theta_i)$, $i = 1, 2, 3$.

The Jacobian of the Master robot needs to be transformed to the slave coordinate frame. The transformed Jacobian of the master robot is

$$J_M^{2D} = \begin{bmatrix} c_z & -s_z & 0 \\ s_z & c_z & 0 \end{bmatrix} J_{PH} \tag{23}$$

where $s_z = sin(\theta_z)$, $c_z = cos(\theta_z)$, and θ_z is the orientation of the master robot with respect to the center of the workspace of the slave robot. θ_z is shown in Fig. 2.

For the master robot, there are three parameters to be optimized, i.e. the last link's length (l_2), the orientation of the master robot (θ_z), and the level, Z_{PH}, of the plane in which the master robot works as a 2D robot. This means $D = \{l_2, \theta_z, Z_{ph}\}$.

In our simulations, first, the master robot parameters are optimized without considering the slave robot kinematic. The

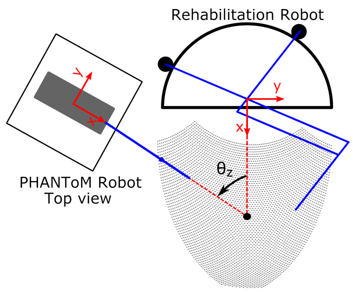

Fig. 2. Orientation of the PHANToM robot with respect to the rehabilitation robot, θ_z

TABLE I
OPTIMIZATION RESULTS, $D = \{l_2, \theta_z, Z_{ph}\}$

	C_1 as the cost function	C_2 as the cost function
l_2 (m)	0.365	0.269
Z_{PH} (m)	-0.2162	-0.065
θ_z (deg)	-63.5	0
GM_T	0.0549	0.0590
GC_M	0.7785	0.7315
GM_M	0.1023	0.0728

cost function for this optimization is C_1 given in (20). The constraints for the optimization are

$$0.165 < l_2 < 0.365, -\frac{\pi}{2} < \theta_z < \frac{\pi}{2}. \quad (24)$$

Next, the cost function is selected such that the kinematics of the slave robot is also considered. For this optimization, the cost function is C_2 given in (21). The same constraints given in (24) are used. In the first round of simulations, it is assumed that the master orientation, θ_z, is the only optimization variable. The teleoperation system manipulability can be optimized by rotation of the Master robot around the center of the slave workspace. For this case, $l_2 = 0.165m$ and $Z_{PH} = 0.0539m$ are constants. θ_z equals to 89° and θ_z equals to 2° are obtained from maximizing the C_1 and C_2 cost functions, respectively. The optimization result shows that if the orientation of the master robot changes from 89° to 2°, the GM_T varies from 0.0378 to 0.0437 while GM_M for the phantom robot changes from 0.0433 to 0.0428. The MEs at six points of the workspace for the master robot, slave robot, and the teleoperated system are depicted in Fig. 3.

In the second round of optimization, l_2, θ_z, and Z_{PH} are all considered as optimization variables. The optimized variables are summarized in Table I. As it can be seen, the optimization for C_1 cost function results in a bigger link length l_2. It can also be noticed that the global manipulability index GM_M and the global conditioning index GC_M of the master robot are higher for C_1 optimization. However, the global manipulability index of the teleoperated system, GM_T, decreases for the C_1 optimization.

Fig. 4 shows the MEs at six points of the workspace for the master robot, slave robot, and the teleoperated system. The ME of the master robot for C_2 optimization is smaller than that from C_1 optimization. However, the principal axes of the ME of the master robot are aligned with principal axes of the ME of the slave robot, which results in an overall large ME for the

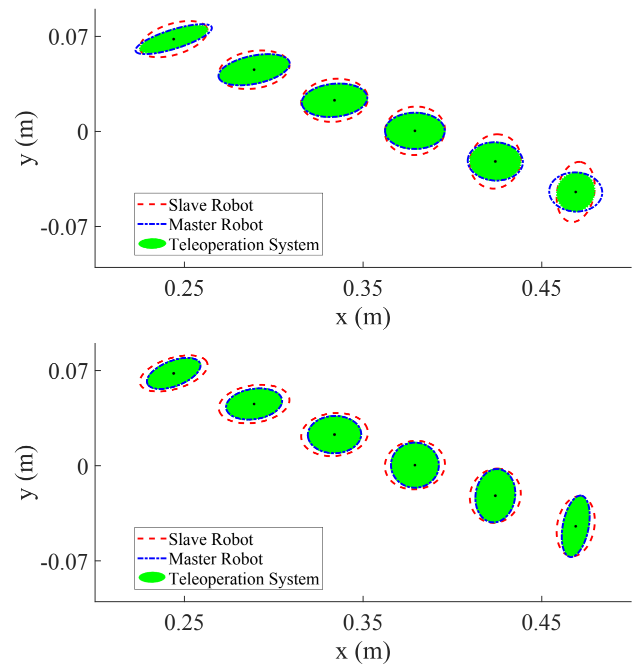

Fig. 3. (Top) $\theta_z = 89°$, as a result of maximizing the C_1 cost function. (Bottom) $\theta_z = 2°$, as a result of maximizing the C_2 cost function. $D = \{\theta_z\}$

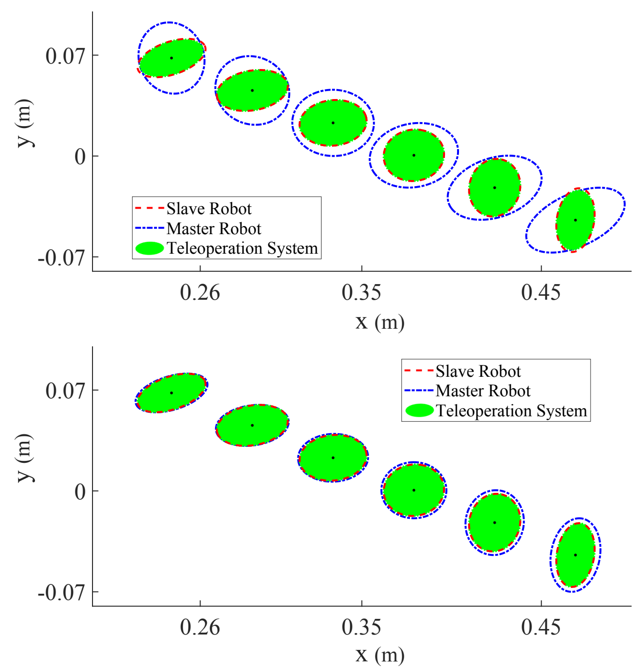

Fig. 4. (Top) Results of C_1 optimization. (Bottom) Results of C_2 optimization. $D = \{l_2, \theta_z, Z_{ph}\}$

teleoperation system. The calculated TMI from optimization using C_1 and C_2 is depicted in Fig. 5

From the ellipsoid obtained in the simulations, one can calculate a measure of master and slave robots' joints kinetic energy, defined as $\|\dot{q}\|^2$ by using (18). Fig. 6 shows the teleoperation system's joint energy for various simulation scenarios discussed above in a task of moving an object from point A (0.424 m,-0.1 m) to point B (0.424 m, 0.1 m) at a constant speed of $\dot{y} = 0.01m/s$ in the slave robot frame.

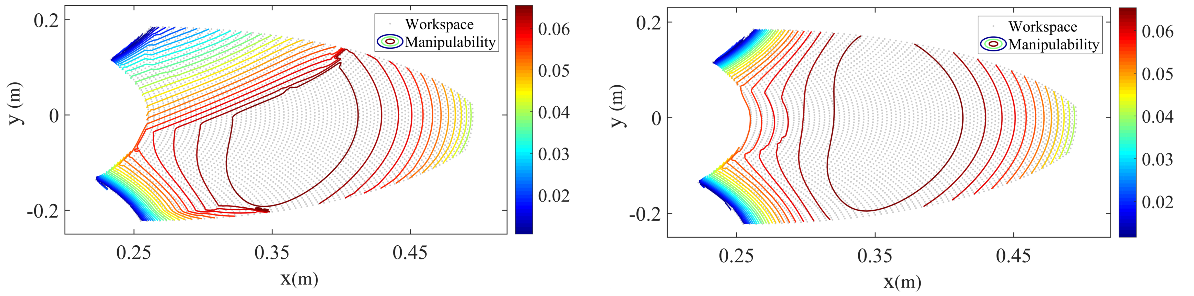

Fig. 5. (Left) The TMI as a result of maximizing the C_1 cost function. (Right) The TMI as a result of maximizing the C_2 cost function. $D = \{l_2, \theta_z, Z_{ph}\}$

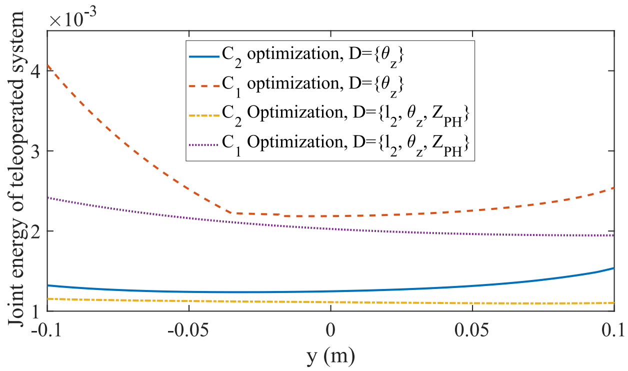

Fig. 6. A comparison of user's effort for different optimization approaches.

It can be seen in Fig. 6 that the teleoperation system's joint energy and consequently the user's input energy for performing a task in the slave robot's task space for cases where C_2 is optimized is much smaller.

V. Concluding Remarks

In this paper, the manipulability for a surgical master-slave robotic system is defined. We demonstrate the application of manipulability in the design of master-slave robotic systems. It is shown that by modifying the design of a commercially available master robot using the proposed manipulability criterion, we are able to enhance the surgeon's control over force/velocity of the surgical robot by increasing the conditioning index, minimizing the master robot's footprint via minimizing its link length, optimizing the surgeons control effort via minimizing the required input energy for moving the slave's end-effector, and avoiding singularities of the master and slave robots. In the future, we will perform experiments to validate the proposed manipulability criterion and its application in designing master-slave robotic systems. Here, the manipulability index of the teleoperation system was only used to modify the design of the master robot. In the future, we will employ the same index to modify the design of the slave robot. We will also use the manipulability criterion in motion planning of surgical tasks to find optimal trajectories that enhances manipulability and minimizes surgeon's control effort.

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