University of Alberta

Controller Performance Assessment of Time Variant Processes

by

Folake Bolanle Olaleye ©

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of **Master of Science**

in

Process Control

Department of Chemical and Materials Engineering

Edmonton, Alberta

Fall 2002

0-612-81454-8

University of Alberta

Library Release Form

Name of Author: Folake Bolanle Olaleye

Title of Thesis: Controller Performance Assessment of Time Variant Processes

Degree: Master of Science

Year this Degree Granted: 2002

F Olaleye ....

Folake Bolanle Olaleye
CME 536
University of Alberta
Edmonton, Alberta
Canada, T6G 2G6

Date: SEPTEMBER 30, 2002

University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Controller Performance Assessment of Time Variant Processes** submitted by Folake Bolanle Olaleye in partial fulfillment of the requirements for the degree of **Master of Science** in Process Control.

. . . .
Biao Huang (supervisor)

. . . .
Scott Meadows

. . . .
Bob Koch

Date: . . Sept. 30, 2002 . .

Abstract

The objective of this thesis is to develop techniques for control loop performance assessment of linear time variant (LTV) processes. Two classes of problems are considered according to different assumptions.

The first class, which deals with a general LTV performance assessment problem, assumes both time-variant process (including disturbances) and time-variant controller. The difficulty in handling of LTV operators is the non-commutativity involved in manipulation of these transfer functions. Therefore, normal multiplication or division, which is non-commutative is applied in deriving the LTV minimum variance control (MVC) benchmark. This methodology is extended to performance assessment of LTV feedforward/feedback control schemes.

The second one, which is more specific, assumes time-variant disturbance dynamics, but time-invariant controller and process. The general time-varying MVC benchmark is too demanding and inappropriate if the controller to be evaluated is time-invariant, as is the case for most industrial non-adaptive controllers. Therefore, alternative time-invariant performance benchmarks that are more suitable for time-variant processes under time-invariant control are studied.

Acknowledgements

I would like to express my profound gratitude to my supervisor, Dr. Biao Huang. His ingenuity, experience, support and encouragement have been an invaluable asset during the entire course of my master's program. He has built in me a new level of confidence by inspiring me to persevere in challenging circumstances. I would really like to thank him for his promptness, gentleness, kindness and great ideas, which have been instrumental to making appropriate decisions on several occasions.

I am greatly indebted to Dr. Shah and Dr. Huang for the knowledge they have imparted on me in the advanced process control and system identification courses, which have been useful tools for my research. I am grateful to Dr. Meadows whose experience and kindness helped me in carrying out my TA duties. All the Profs who instructed me deserve appreciation. I would also like to thank Edgar Tamayo and members of the Upgrading Unit in Syncrude Canada Ltd. for their invaluable guidance and support.

It is a privilege to be a member of the Computer Process Control (CPC) group of the University of Alberta. I would like to give my sincere thanks to my past and present student colleagues: Giti, Vikas, Arun, Raghu, Harigopal, Ramesh, Zhengang, Amar, Vinay, my dear sister and friend, Sade, Xin, Monjur, Salim, Fangwei, Liqian, Lisheng, and others for the stimulating environment. Special thanks to Giti, my former office partner for a continued friendship and encouragement, and to Fangwei, thanks for the times of discussion. I wish to thank the faculty and staff of Dept. of Chemical Eng. for the help and resources provided. Thanks to friends outside the CPC group; Emmanuel, Kayode, Niyi, Lekan, Sam, Saheed, Jummy and others for the good wishes and prayers.

I wish to express my whole-hearted appreciation and deepest gratitude to my wonderful and loving parents for their invaluable guidance, unwavering support, constant encouragement, unceasing prayers and for bestowing their unconditional and affectionate love on me. They are a source of inspiration and best parents ever! My heartfelt thanks go to my dearest sister, Folasade and her family and to my darling brothers, Olayemi and Olayimika for their love, prayers and encouragement. I also want to specially express my immense thanks to my sweetheart, Oluwagbemiga for showering me with love, affection & prayers, and for being there for me always. You're all the best!

I give all my thanks and endless gratitude to the Lord God Almighty whose everlasting and unfailing love has abounded towards me. Without his grace, this work would never have been possible. He only has been my very present help in the times of need. The Lord has been my defence; and my God is the rock of my refuge. Thank you God for all my loved ones and for everything.

This thesis is dedicated to God the Father, God the Son and God the Holy Spirit
My Help in Ages past, my Hope for ever

Contents

List of Tables

List of Figures

Chapter 1

Introduction

There has been an increasing interest in the research area of control loop performance assessment in the past decade. It has become attractive to industries because the information on performance measure of control loops gives control engineers insight into potentials of improving control system performance and/or finding problem in the existing controllers. Continuous performance assessment of process operations allows timely detection of performance degradation in the control loops and routine maintenance of such loops at optimal settings can result in huge monetary savings for a typical chemical complex.

Harris (1989) has reported how minimum variance or the "feedback control invariant" term of an LTI SISO process can be estimated from closed loop process output data, and used as a benchmark for control loop performance assessment. Desborough and Harris (1992) further proposed the use of least squares regression in the estimation of minimum variance control and performance index. Stanfelj et al (1993) presented the use of autocorrelation and cross correlation functions in the diagnosis of feedback/feedforward control loop performance. Eriksson and Isaksson (1994) proposed some modified criteria and monitoring tools for control loop performance assessment. Huang et al (1995) presented the on-line control performance monitoring of MIMO processes using filtering and correlation analysis while Martin et al (1996) presented an overview of multivariate statistical process control and its nonlinear extension for process performance monitoring. Tyler and Morari (1996) reported the performance monitoring of control systems using likelihood methods while Lynch and Dumont (1996) reported the use of Laguerre network to model the closed loop system in order to estimate the minimum variance control for controller performance monitoring. Isaksson (1996) defined a set of alternative indices for PID controller performance assessment. Vishnubhotla (1997) reported the use of spectral analysis of routine operating data and time-domain techniques for control loop performance assessment. Kendra and Cinar (1997) also presented controller performance assessment by frequency domain techniques. Li and Evans (1997) presented the minimum variance control of linear time-varying discrete-time systems. There has also been an extension of controller performance assessment techniques of univariate systems to multivariate systems by Harris et al. (1996), Huang (1997), Huang and Shah (1998) and Huang et al. (2000). Qin (1998) presented a review and assessment of control performance monitoring techniques. There has also been a review of performance monitoring and assessment techniques by Harris et al. (1999), refinery-wide control loop performance assessment by Thornhill et al. (1999), performance assessment of PI controllers based on setpoint response data by Swanda and Seborg (1999) and performance assessment of processes with abrupt changes of disturbances by Huang (1999). Horch and Isaksson (1999) proposed a modification of Harris' index for performance assessment of control loops, Forsman (1999) presented a holistic perspective of control performance monitoring and Cinar and Undey (1999) presented multivariable statistical process monitoring techniques for both continuous and batch processes. Some of the most recent work on control loop performance assessment include: control performance monitoring by Vaught and Tippett (2001), feedforward plus feedback controller performance assessment of MIMO systems by Huang et al. (2000), process controller performance monitoring and assessment by Hugo (2001). Ko and Edgar (2000, 2001) developed a procedure which does not require any knowledge of the interactor matrix for the estimation of minimum variance performance bounds in multivariable feedback control systems. Bezergianni and Georgakis (2000) proposed the use of a relative variance index for controller performance assessment. Wan and Huang (2002) have reported the robust performance assessment of feedback control systems. Huang and Jeng (2002) presented the monitoring and assessment of controller performance for single loop control system with controller of general structure or with PI/PID structure. Huang (2002) complemented the work of Li and Evans and proposed a more general method of performance assessment of LTV feedback processes.

If the control objective is to minimize process variance, minimum variance control can be used as the benchmark against which control loop performance is evaluated.

Minimum variance control is a bound on achievable performance against which performance of other controllers can be compared. It requires minimum effort to estimate (routine closed-loop operating data plus a priori knowledge of time delays). This benchmark might not be recommended for practical implementation due to excessive control action, poor robustness, poor process invertibility and other physical constraints on the process. However, as a benchmark, it indicates how good the controller performance is compared to the minimum variance control and it gives how much potential there is to further improve the controller performance (Huang, 1997). If the controller performance is inadequate but is already close to the minimum variance control, then further controller tuning would not be helpful. Substantial improvement is possible only by changing the system structure such as addition of feedforward control, reduction of dead time and another possibility might be reduction of disturbance by introducing inventory between units or through process modifications. However, if the controller indicates a poor performance, then the control system performance could be improved with the current control structure with simple retuning.

Although, there exist various performance assessment methods for time invariant processes, there are few results available for time-variant processes. However, in practice, most processes have certain degree of time varying behavior and this has brought about a need to develop performance assessment methods for time varying processes, which could be under adaptive or time-invariant control. Goodwin and Sin (1984) and many other researchers have studied these time-varying properties in the "Adaptive Filtering Prediction and Control" literature but it has not been well addressed in control loop performance assessment. Li and Evans (1997) presented the minimum variance control of linear time varying systems. The potential difficulty in the handling of LTV operators is seen in the non-commutativity involved in manipulation of these transfer functions. This property is important and has to be taken into consideration in the analysis of performance assessment of LTV systems.

Abrupt change is any change in the parameters of the model that occurs either instantaneously or very fast with respect to the sampling period of the measurement (Basseville and Nikiforov, 1993). It has been observed that abrupt changes of disturbances are often encountered in many chemical processes and they result in time varying dynamics or non-stationary time series of these processes. Therefore, if the controller is time invariant, as is the case for most industrial non-adaptive controllers, the time varying minimum variance control benchmark is clearly too demanding and is not appropriate. Also, the traditional performance assessment technique for time-invariant controllers may yield erroneous results when there is an abrupt change in the disturbance dynamics in the control loop. This is because a minimum variance control benchmark estimated from a set of data when only stationary disturbance affects the process could give a poor performance indication in regulating a new disturbance that has a different dynamics. Huang (1999) has reported that without considering time-variant characteristics, the classical performance assessment results may be incomplete or can be misinterpreted. It is therefore necessary to obtain an alternative time-invariant performance benchmark that is more suitable for time-variant processes under time invariant control.

The performance of control schemes is often enhanced by including feedforward elements. This is usually achieved in two ways; a feedforward variable could be measured and used in the control scheme, or the potential benefit of implementing feed-forward control can be estimated. Box and Jenkins (1976), and Sternad and Stoderstrom (1988) have discussed the design of minimum variance feedforward and feedback controller. Stanfelj et al. (1993) presented a hierarchical method for monitoring and diagnosing the cause of poor performance of feedforward/feedback control systems using autocorrelation and cross correlation functions. Huang (1997) and Huang et al. (2000) have extended methods for performance assessment of multivariate feedback control systems to performance assessment of multivariate LTI feedback plus feedforward control systems using minimum variance control as the benchmark. Huang (1999, 2002) and some other researchers have developed performance assessment techniques for LTV feedback control loops. Due to non-stationary time series often observed in performance assessment of control loops, it is also important to develop performance assessment techniques for LTV feedforward/feedback control loops. The estimation of the time-variant lower bound of variance for each of the controller in the control scheme allows for the performance of the individual controllers to be assessed from time series analysis of closed-loop routine operating data.

1.1 Scope and outline of this thesis

The purpose of this work is to discuss and study the control loop performance assessment of time variant processes in the field of chemical engineering. The methods and algorithms in this thesis have been applied to industrial case study on sulphur recovery process at Syncrude Canada Ltd. The structure of the thesis is as follows:

Chapter 2 discusses the control loop performance assessment of time-variant SISO processes. It has been found that there is a potential difficulty in the handling of LTV operators, and this is seen in the non-commutativity involved in manipulation of these transfer functions. A general algorithm for performance assessment of LTV loops is developed in this chapter. However, the time-variant minimum variance control discussed in this chapter is found to be suitable for time-variant controllers but will clearly be too demanding on time-invariant controllers. This is the focus in chapter 3.

Chapter 3 and chapter 4 deal with developing an alternative time-invariant performance benchmark that is more suitable for time-variant processes under time invariant control. In these two chapters, the discussion is limited to time variant disturbance models. Chapter 3 discusses the benchmark that is useful when minimization of a particular type of disturbances amongst other forms of disturbance dynamics affecting the process is of the only interest while chapter 4 presents the optimization technique that can be used to obtain time-invariant minimum variance control benchmark that can "optimize" overall performance of these time-variant processes.

In Chapter 5, the performance assessment methodology developed in chapter 2 is extended to feedforward/feedback control loop performance assessment of Linear Time Variant (LTV) MISO processes. If the controller performance is inadequate but is already close to the minimum variance control, then further controller tuning would not be helpful, and substantial improvement is possible only by changing the system structure such as addition of feedforward control. Therefore, the LTV benchmark of feedforward plus feedback control is discussed and illustrated by a simulation example to demonstrate the feasibility of the algorithm.

1.2 Contributions of this thesis

The contributions of this thesis include:

- Generalization of a technique for control loop performance assessment of linear time variant (LTV) processes, which assumes time variant process (including disturbances) and time variant controller. It is an efficient algorithm suitable for performance assessment of LTV controllers and particularly for that of adaptive control.

- Generalization of alternative time-invariant performance benchmark that is more suitable for time-variant processes under time invariant control using time series analysis on closed-loop routine operating data and/or optimization techniques.

- New development of the performance assessment methodology for feedforward plus feedback control loops.

- Extensive case studies of the developed algorithms in simulated examples including chemical process examples.

- Actual applications of the developed algorithm in industrial processes.

Chapter 2

Feedback Controller Performance Assessment of Time-Variant Processes

Abstract

This chapter discusses the theoretical extension and a practical application of Linear Time Variant (LTV) minimum variance control as a benchmark for control loop performance assessment of time-variant processes. This time-variant minimum variance control, which is found to be suitable for time-variant controllers or processes, is referred to as the type-A benchmark. The proposed performance monitoring method is illustrated through a simulated stirred tank reactor and applied to a case study on a Sulphur Recovery Unit that is under adaptive control in Syncrude Canada Ltd.

The main contributions of the chapter include: (1) generalization of LTV control loop performance assessment technique by deriving expressions of the LTV minimum variance term and actual variance term, and subsequent calculation of LTV performance index, which have not been achieved in previous work; (2) a detailed industrial case study to illustrate the applicability of the LTV control performance assessment techniques in practice.

2.1 Introduction

Automatic process control has been widely used in process industries to achieve objectives which vary from maintaining safe process operations to process optimization. Industrial processes include control loops whose number varies from a single loop in simple processes to thousands in large integrated plants. Routine maintenance of such loops at optimal settings can result in huge monetary savings for a typical chemical complex. Since these loops are maintained and serviced occasionally, it is important that the control loop performance is efficiently monitored and controller is retuned if necessary. Continuous performance assessment of process operations allows timely detection of performance degradation in the control loops. However, assessment of control loops should not disturb routine operations of the processes. That is, performance monitoring should be non-invasive or at least should be done under closed loop conditions (Huang, 1997). It is also required that the performance assessment algorithm should be simple and non-complex, and should require minimal process knowledge (Horch, 2000).

Performance assessment of control loops is often measured with respect to response of a process to step change in set point (servo performance) or to load disturbance variable (regulatory performance). Performance characteristics such as integral of the absolute value of the error (IAE), settling time, overshoot, damping e.t.c. are calculated and often used for monitoring purposes. This is a simple and useful method when experiments or set point changes can be made periodically on each control loop (Stanfelj et al, 1993). But continuously operating processes are subject to numerous disturbances that make the controlled variable behave as a random time series. The mean square error (MSE) or variance of the process variable is commonly used as the measure for control loop performance. The variance (or standard deviation) is used for monitoring because of its direct relationship to process performance and profit (Bozenhardt and Dybeck, 1986, Marlin et al, 1987, Stanfelj et al, 1993).

Harris (1989) has found that a "feedback control invariant" or controller independent term of an LTI SISO process can be estimated from closed loop process output data, and this term represents the process output under minimum variance control. The minimum variance term is used as the benchmark against which control loop performance is evaluated. Minimum variance control is a bound on achievable performance against which performance of other controllers can be compared. It requires minimum effort to estimate (routine closed-loop operating data plus a priori knowledge of time delays) and it is the best possible control in the sense that no other controller can provide a lower output variance. This benchmark might not be recommended for practical implementation due to excessive control action, poor robustness, poor process invertibility and other physical constraints on the process. However, as a benchmark, it indicates how good the controller performance is compared to the minimum variance control and it gives how much potential there is to further improve the controller performance (Huang, 1997). If the controller performance is inadequate but is already close to the minimum variance control, then further controller tuning would not be helpful. Substantial improvement is possible only by changing the system structure such as addition of feedforward control, reduction of dead time, and another possibility might be reduction of disturbance by introducing inventory between units or through process modifications. However, if the controller indicates a poor performance, then the control system performance could be improved with the current control structure with simple retuning.

Significant progress has been made in assessment of time invariant processes or to time series that can be made stationary by some simple transformation. However, non-stationary time series are often observed in performance assessment of control loops due to varying process dynamics, change of disturbance models, and non-linearity of actuators and sensors e.t.c. Although, there exist various performance assessment methods for time invariant processes, there are few results available for time-variant processes. In practice, most processes have certain degree of time varying behavior and this has brought about a need to develop performance assessment methods for time varying processes. The most intuitive extension of performance assessment technique from Linear Time Invariant (LTI) processes to Linear Time Variant (LTV) processes is through the recursive estimation technique, also referred to as "sequential parameter estimation" or "adaptive control algorithm" (Ljung and Soderstrom, 1983). The recursive identification algorithm (for LTV assessment) uses information from past observations recursively by focusing on the most recent data and discounting remote past measurements exponentially. Several recursive algorithms have been proposed to estimate control loop performance in the presence of non-stationary characteristics in the data (for example, Huang and Shah 1999). It is found that any recursive time series algorithm can be used to estimate the LTV ARMA model for performance assessment of LTV processes. However, the potential difficulty in the handling of LTV operators is seen in the non-commutativity of the multiplication and/or division of these transfer functions. This is illustrated in the multiplication of two LTV polynomials, $u(q^{-1},t)$ and $v(q^{-1},t)$ in the backshift operator q^{-1}:

$$\begin{aligned} u(q^{-1},t) &= u_0(t) + u_1(t)q^{-1} + \ldots + u_n(t)q^{-n} \\ v(q^{-1},t) &= v_0(t) + v_1(t)q^{-1} + \ldots + v_m(t)q^{-m} \end{aligned} \tag{2.1}$$

The multiplication of $u(q^{-1},t)$ and $v(q^{-1},t)$ is given by

$$\begin{aligned} u(q^{-1},t)v(q^{-1},t) &= \sum_{i=0}^{n} \sum_{j=0}^{m} u_i(t)q^{-i}v_j(t)q^{-j} \\ &= \sum_{i=0}^{n} \sum_{j=0}^{m} u_i(t)v_j(t-i)q^{-(i+j)} \end{aligned} \tag{2.2}$$

The multiplication of $v(q^{-1},t)$ and $u(q^{-1},t)$ is given by

$$\begin{aligned} v(q^{-1},t)u(q^{-1},t) &= \sum_{j=0}^{m} \sum_{i=0}^{n} v_j(t)q^{-j}u_i(t)q^{-i} \\ &= \sum_{j=0}^{m} \sum_{i=0}^{n} v_j(t)u_i(t-j)q^{-(i+j)} \end{aligned} \tag{2.3}$$

Hence, $u(q^{-1},t)v(q^{-1},t) \neq v(q^{-1},t)u(q^{-1},t)$

The multiplication of $u(q^{-1},t)$ and $v(q^{-1},t)$ in eqn. (2.2) and eqn. (2.3) is referred to as normal multiplication of the LTV polynomials. Normal multiplication is therefore said to be non-commutative and it is seen that this type of multiplication causes time delay in the LTV operators. This result shows that care has to be taken to the non-commutativity involved in manipulation of LTV transfer functions. This property of LTV polynomials is important and has to be taken into consideration in calculating the minimum variance term when the LTV ARMA model, for example, is transferred to an LTV MA model.

However, unlike normal multiplication, pointwise multiplication does not cause any time delay in the multiplication and/or division of the LTV polynomials as is illustrated in the following:

For two LTV polynomials, $u'(q^{-1},t)$ and $v'(q^{-1},t)$ in the backshift operator

$$\begin{aligned} u'(q^{-1},t) &= u_0'(t) + u_1'(t)q^{-1} + \ldots + u_n'(t)q^{-n} \\ v'(q^{-1},t) &= v_0'(t) + v_1'(t)q^{-1} + \ldots + v_m'(t)q^{-m} \end{aligned} \tag{2.4}$$

The multiplication of $u'(q^{-1},t)$ and $v'(q^{-1},t)$ is given by

$$\begin{aligned} u'(q^{-1},t)v'(q^{-1},t) &= \sum_{i=0}^{n} \sum_{j=0}^{m} u_i'(t)q^{-i}v_j'(t)q^{-j} \\ &= \sum_{i=0}^{n} \sum_{j=0}^{m} u_i'(t)v_j'(t)q^{-(i+j)} \end{aligned} \tag{2.5}$$

The multiplication of $v'(q^{-1},t)$ and $u'(q^{-1},t)$ is given by

$$\begin{aligned} v'(q^{-1},t)u'(q^{-1},t) &= \sum_{j=0}^{m} \sum_{i=0}^{n} v_j'(t)q^{-j}u_i'(t)q^{-i} \\ &= \sum_{j=0}^{m} \sum_{i=0}^{n} v_j'(t)u_i'(t)q^{-(i+j)} \end{aligned} \tag{2.6}$$

From eqn. (2.5) and eqn. (2.6), it can be seen that for pointwise multiplication, $u'(q^{-1},t)v'(q^{-1},t) = v'(q^{-1},t)u'(q^{-1},t)$. Thus, pointwise multiplication is said to be commutative.

It should be noted that pointwise multiplication may yield erroneous results if the plant or disturbance dynamics has relatively fast parameter change (as would be seen in the simulation example). Therefore, normal multiplication, which is non-commutative, is recommended when handling LTV operators.

2.2 Control Loop Performance Assessment of LTV Processes

Consider the LTV SISO process shown in Figure 2.1:

$$y_t = q^{-d}\tilde{T}(q^{-1},t)u_t + N(q^{-1},t)a_t \tag{2.7}$$

The time-delay, d is considered to be constant; $\tilde{T}(q^{-1},t)$ is the delay-free LTV plant transfer function; $N(q^{-1},t)$ is the LTV disturbance transfer function; a_t is a white noise sequence with zero mean and time-variant variance, $\sigma_a^2(t)$.

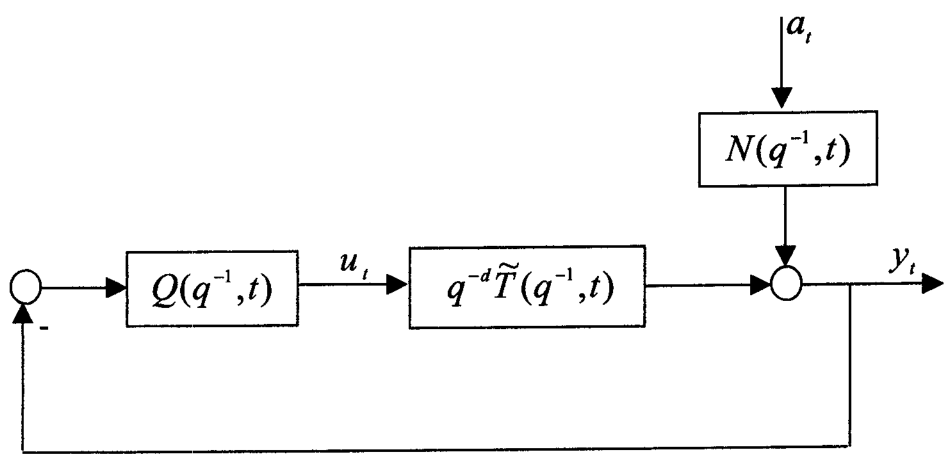


Figure 2.1: Schematic of time-variant SISO process under feedback control

By applying time-series analysis to the routine operating data, the LTV closed loop SISO response can be expressed as an ARMA model

$$A_{cl}(q^{-1},t)y_t = C_{cl}(q^{-1},t)a_t \tag{2.8}$$

The LTV ARMA model can be transferred to the LTV MA model and obtained as

$$y_t = (f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + \ldots + f_{d-1}(t)q^{-(d-1)} + f_d(t)q^{-d} + \ldots)a_t \qquad (2.9)$$

It has been shown in Huang (1997) that the closed-loop response under time-variant minimum variance control constitutes the first d-terms of the MA model

$$y_t\big|_{mv} = (f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + \ldots + f_{d-1}(t)q^{-(d-1)})a_t \qquad (2.10)$$

The LTV minimum variance can be calculated as

$$\sigma_{mv}^2(t) = (f_0^2(t) + f_1^2(t) + \ldots + f_{d-1}^2(t))\sigma_a^2(t) \qquad (2.11)$$

Note that in calculating the minimum variance term for LTV processes, the non-commutativity associated with LTV operators should be taken into account in transferring the LTV ARMA model to LTV MA model by using normal multiplication rather than pointwise multiplication or division.

The above procedure can be used to calculate the time variant minimum variance term for any model structure with any order. The complexity is, however, increased quickly with the increase of the model order. In the following sections, we shall focus on the derivation of the minimum variance terms for two special, yet most frequently used model structure and order, namely AR(4) and ARMA(2,2).

2.2.1 Calculation of the LTV minimum variance for AR model

Let us consider that the process output, y_t is represented by an LTV AR model of order 4, which is a default choice in most applications (Ljung, 1999) and in MATLAB System Identification toolbox:

$$A_{cl}(q^{-1},t)y_t = a_t \qquad (2.12)$$

where

$$y_t = \left(f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + f_3(t)q^{-3} + \ldots\right)a_t \tag{2.13}$$

$$A_{cl}(q^{-1},t) = 1 + \alpha_1(t)q^{-1} + \alpha_2(t)q^{-2} + \alpha_3(t)q^{-3} + \alpha_4(t)q^{-4} \tag{2.14}$$

Substituting eqn. (2.13) in eqn. (2.12) yields

$$A_{cl}(q^{-1},t)\left(f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + f_3(t)q^{-3} + \ldots\right)a_t = a_t \tag{2.15}$$

The LTV impulse response coefficients are obtained by equating coefficients on the right and left hand sides of eqn. (2.15):

$$\begin{cases} f_0(t) = 1 \\ f_1(t) = -\alpha_1(t)f_0(t-1) \\ f_2(t) = -\alpha_1(t)f_1(t-1) - \alpha_2(t)f_0(t-2) \\ f_3(t) = -\alpha_1(t)f_2(t-1) - \alpha_2(t)f_1(t-2) - \alpha_3(t)f_0(t-3) \\ f_k(t) = -\alpha_1(t)f_{k-1}(t-1) - \alpha_2(t)f_{k-2}(t-2) - \alpha_3(t)f_{k-3}(t-3) - \\ \qquad \alpha_4(t)f_{k-4}(t-4) \quad k>3 \end{cases} \tag{2.16}$$

It follows from eqn. (2.16) that

$$\begin{cases} f_0(t) = 1 \\ f_1(t) = -\alpha_1(t) \\ f_2(t) = \alpha_1(t)\alpha_1(t-1) - \alpha_2(t) \\ f_3(t) = \alpha_1(t)\alpha_2(t-1) - \alpha_1(t)\alpha_1(t-1)\alpha_1(t-2) + \alpha_1(t-2)\alpha_2(t) - \alpha_3(t) \\ \vdots \end{cases} \tag{2.17}$$

That is,

$$y_t = (1 - \alpha_1(t)q^{-1} + (\alpha_1(t)\alpha_1(t-1) - \alpha_2(t))q^{-2} + \\ (\alpha_1(t)\alpha_2(t-1) - \alpha_1(t)\alpha_1(t-1)\alpha_1(t-2) + \alpha_1(t-2)\alpha_2(t) - \alpha_3(t))q^{-3} + \ldots)a_t \tag{2.18}$$

However, direct long division of eqn. (2.12) gives

$$y_t = (1 - \alpha_1(t)q^{-1} + (\alpha_1^{\ 2}(t) - \alpha_2(t))q^{-2} + (2\alpha_1(t)\alpha_2(t) - \alpha_1^{\ 3}(t) - \alpha_3(t))q^{-3} + \ldots)a_t \tag{2.19}$$

From eqn. (2.19), it can be seen that unlike the result obtained with normal multiplication or division of LTV polynomials, pointwise multiplication does not cause any time delay in the coefficients of the moving average model.

As an example, from eqn. (2.16), the appropriate minimum variance for the LTV process with a time delay of 3 can be calculated as:

$$\begin{aligned}\sigma_{mv}^2(t) &= (1 + f_1^{\,2}(t) + f_2^{\,2}(t))\sigma_a^2(t) \\ &= (1 + \alpha_1^2(t) + (\alpha_1(t)\alpha_1(t-1) - \alpha_2(t))^2)\sigma_a^2(t)\end{aligned} \tag{2.20}$$

2.2.2 Calculation of the actual time-variant variance for AR model

To calculate a time-variant performance index, the actual time-variant variance has to be calculated. An algorithm based on the LTV AR(4) model is discussed in this section. The LTV AR model in eqn. (2.12) can be expressed as

$$y_t + \alpha_1(t)y_{t-1} + \alpha_2(t)y_{t-2} + \alpha_3(t)y_{t-3} + \alpha_4(t)y_{t-4} = a_t \tag{2.21}$$

The $\alpha_i(t)'s$ represent the parameters while a_t is the white noise sequence. Both sides of eqn. (2.21) are multiplied by $y_t, y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4}$ respectively and the expectations are taken to obtain:

$$\begin{aligned}&\gamma_0 + \alpha_1(t)\gamma_1 + \alpha_2(t)\gamma_2 + \alpha_3(t)\gamma_3 + \alpha_4(t)\gamma_4 = \sigma_a^2(t) \\ &\gamma_1 + \alpha_1(t)\gamma_0 + \alpha_2(t)\gamma_1 + \alpha_3(t)\gamma_2 + \alpha_4(t)\gamma_3 = 0 \\ &\gamma_2 + \alpha_1(t)\gamma_1 + \alpha_2(t)\gamma_0 + \alpha_3(t)\gamma_1 + \alpha_4(t)\gamma_2 = 0 \\ &\gamma_3 + \alpha_1(t)\gamma_2 + \alpha_2(t)\gamma_1 + \alpha_3(t)\gamma_0 + \alpha_4(t)\gamma_1 = 0 \\ &\gamma_4 + \alpha_1(t)\gamma_3 + \alpha_2(t)\gamma_2 + \alpha_3(t)\gamma_1 + \alpha_4(t)\gamma_0 = 0\end{aligned} \tag{2.22}$$

The γ_i's represent the time-variant auto covariance of the process variable, y with lag 'i', and $\sigma_a^2(t)$ is the variance of the white noise (or shock). The equations are re-organized in matrix format and solved to obtain the required time-variant process variance, σ_y^2 which is denoted by γ_0.

$$\underbrace{\begin{bmatrix} 1 & \alpha_1(t) & \alpha_2(t) & \alpha_3(t) & \alpha_4(t) \\ \alpha_1(t) & 1+\alpha_2(t) & \alpha_3(t) & \alpha_4(t) & 0 \\ \alpha_2(t) & \alpha_1(t)+\alpha_3(t) & 1+\alpha_4(t) & 0 & 0 \\ \alpha_3(t) & \alpha_2(t)+\alpha_4(t) & \alpha_1(t) & 1 & 0 \\ \alpha_4(t) & \alpha_3(t) & \alpha_2(t) & \alpha_1(t) & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{C} \sigma_a^2(t) \quad (2.23)$$

From eqn. (2.23), the γ_i's in vector "B" are estimated by

$$B = A^{-1} C \, \sigma_a^2(t) \quad (2.24)$$

The 1st element of the vector "B" represents the process variance, σ_y^2. The control loop performance can be determined by comparing the minimum variance with the process variance. Note that σ_y^2 is also time variant and eqn. (2.24) calculates the variance at time instant, t.

2.2.3 Calculation of the time-variant minimum variance for ARMA Model

A more general representation of a time series is the ARMA model. It is known that ARMA(2,2) is a typical representation of time series process and most physical process can be well fitted by this model. The calculation of the time variant minimum variance and actual variance for an ARMA model is more involving than the AR model. The calculation of the time-variant minimum variance for this model is illustrated next.

$$A_{cl}(q^{-1},t) y_t = C_{cl}(q^{-1},t) a_t \quad (2.25)$$

where

$$y_t = \left(f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + f_3(t)q^{-3} + \ldots\right)a_t \tag{2.26}$$

$$\begin{aligned} A_{cl}(q^{-1},t) &= 1 + \alpha_1(t)q^{-1} + \alpha_2(t)q^{-2} \\ C_{cl}(q^{-1},t) &= 1 + c_1(t)q^{-1} + c_2(t)q^{-2} \end{aligned} \tag{2.27}$$

Substituting eqn. (2.26) into eqn. (2.25) yields

$$A_{cl}(q^{-1},t)\left(f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + f_3(t)q^{-3} + \ldots\right)a_t = C_{cl}(q^{-1},t)a_t \tag{2.28}$$

The LTV impulse response coefficients are obtained by equating coefficients on the right and left hand sides of eqn. (2.28):

$$\begin{cases} f_0(t) = 1 \\ f_1(t) = c_1(t) - \alpha_1(t)f_0(t-1) \\ f_2(t) = c_2(t) - \alpha_1(t)f_1(t-1) - \alpha_2(t)f_0(t-2) \\ f_k(t) = -\alpha_1(t)f_{k-1}(t-1) - \alpha_2(t)f_{k-2}(t-2) \qquad k > 2 \end{cases} \tag{2.29}$$

It follows from eqn. (2.29) that

$$\begin{cases} f_0(t) = 1 \\ f_1(t) = c_1(t) - \alpha_1(t) \\ f_2(t) = c_2(t) - \alpha_1(t)c_1(t-1) + \alpha_1(t)\alpha_1(t-1) - \alpha_2(t) \\ f_3(t) = -\alpha_1(t)c_2(t-1) + \alpha_1(t)\alpha_2(t-1) + \alpha_1(t)\alpha_1(t-1)c_1(t-2) - \\ \qquad \alpha_1(t)\alpha_1(t-1)\alpha_1(t-2) - \alpha_2(t)c_1(t-2) + \alpha_2(t)\alpha_1(t-2) \\ \vdots \end{cases} \tag{2.30}$$

That is,

$$y_t = (1 + (c_1(t) - \alpha_1(t))q^{-1} + (c_2(t) - \alpha_1(t)c_1(t-1) + \alpha_1(t)\alpha_1(t-1) - \alpha_2(t))q^{-2} + \ldots)a_t \tag{2.31}$$

However, direct long division gives a different result

$$\begin{aligned} y_t = & (1+(c_1(t)-\alpha_1(t))q^{-1}+(c_2(t)-\alpha_1(t)c_1(t)+\alpha_1(t)^2-\alpha_2(t))q^{-2}+ \\ & (-\alpha_1(t)c_2(t)+2\alpha_1(t)\alpha_2(t)+\alpha_1(t)^2c_1(t)-\alpha_1(t)^3-\alpha_2(t)c_1(t))q^{-3}+\ldots)a_t \end{aligned} \tag{2.32}$$

As an example, from eqn. (2.29), the appropriate minimum variance for a time delay of 3 can be calculated as:

$$\sigma_{mv}^2(t)=\left(1+(c_1(t)-\alpha_1(t))^2+(c_2(t)-\alpha_1(t)c_1(t-1)+\alpha_1(t)\alpha_1(t-1)-\alpha_2(t))^2\right)\sigma_a^2 \tag{2.33}$$

2.2.4 Calculation of the actual time-variant output variance for ARMA Model

The LTV ARMA (2,2) model can further be expressed as

$$y_t+\alpha_1(t)y_{t-1}+\alpha_2(t)y_{t-2}=a_t+c_1(t)a_{t-1}+c_2(t)a_{t-2} \tag{2.34}$$

Both sides of eqn. (2.34) are multiplied by y_t, y_{t-1}, y_{t-2} respectively and the expectations are taken to obtain:

$$\begin{aligned} \gamma_0+\alpha_1(t)\gamma_1+\alpha_2(t)\gamma_2 = & \ (1-\alpha_1(t)c_1(t)+c_1^{\,2}(t)+c_2^{\,2}(t)-\alpha_2(t)c_2(t) \\ & -\alpha_1(t)c_1(t)c_2(t)+\alpha_1^{\,2}(t)c_2(t))\sigma_a^2(t) \\ \gamma_1+\alpha_1(t)\gamma_0+\alpha_2(t)\gamma_1 = & \ (c_1(t)+c_1(t)c_2(t)-\alpha_1(t)c_2(t))\sigma_a^2(t) \\ \gamma_2+\alpha_1(t)\gamma_1+\alpha_2(t)\gamma_0 = & \ c_2(t)\sigma_a^2(t) \end{aligned} \tag{2.35}$$

In a similar format with the LTV AR model, the equations are re-organized and solved to obtain the required time-variant process variance, σ_y^2 which is denoted by γ_0.

$$
\underbrace{\begin{bmatrix} 1 & \alpha_1(t) & \alpha_2(t) \\ \alpha_1(t) & 1+\alpha_2(t) & 0 \\ \alpha_2(t) & \alpha_1(t) & 1 \end{bmatrix}}_{A}
\underbrace{\begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix}}_{B}
=
\underbrace{\begin{bmatrix} 1-\alpha_1(t)c_1(t)+c_1^2(t)+c_2^2(t)-\alpha_2(t)c_2(t) \\ \quad -\alpha_1(t)c_1(t)c_2(t)+\alpha_1^2(t)c_2(t) \\ c_1(t)+c_1(t)c_2(t)-\alpha_1(t)c_2(t) \\ c_2(t) \end{bmatrix}}_{C}
\sigma_a^2(t) \tag{2.36}
$$

The actual variance γ_0 is solved from eqn. (2.36) and the control loop performance is determined by comparing the minimum variance with the process variance.

So far, we have been able to show how to estimate the control loop performance for LTV AR and LTV ARMA models. From this analysis, it can be seen that the proposed methodology for performance assessment of LTV processes can be obtained from routine operating data and the results can be extended to higher-order models following the same procedure if necessary.

2.3 Evaluation via Simulation

In this section, we consider a stirred tank heater shown in Figure 2.2. The objective is to raise the temperature of the inlet stream to a desired value. The stirred tank heater is an example of mixing vessel, which is heated by a jacket surrounding the vessel. A mixing vessel may serve as a chemical reactor, where two or more components are reacted under certain conditions to produce one or more products. The reaction often occurs at a certain temperature to achieve the desired yield. In this example, saturated steam is the heat transfer fluid that is circulated through the jacket to heat the fluid in the tank. The assumptions made in writing the dynamic modeling equations to find the tank temperature include:

- The volume and liquids have constant density and heat capacity
- Perfect mixing is assumed in both the tank and jacket
- The temperature of the saturated steam is constant throughout the jacket

- The flowrate of the saturated steam is time varying, and this causes the heat transfer coefficient U to be time varying.
- The tank inlet flowrate F_i, tank outlet flowrate F, jacket flowrate F_j, tank inlet temperature T_i, and jacket inlet temperature T_{ji}, vary with time.

Neglecting the work done by the impeller, energy balance around the tank is used to obtain the modeling equation given by

$$\frac{dT}{dt} = \frac{F}{V}(T_i - T) + \frac{Q}{V\rho C_p} \tag{2.37}$$

where the rate of heat transfer from the jacket to the tank, Q is given by

$$Q = UA(T_j - T) \tag{2.38}$$

T is the tank temperature, F is volumetric flowrate, ρ is the density, C_p is the heat capacity, U is the overall heat transfer coefficient and A is the area for heat transfer. The subscripts i, j and ji denote inlet, jacket and jacket inlet respectively.

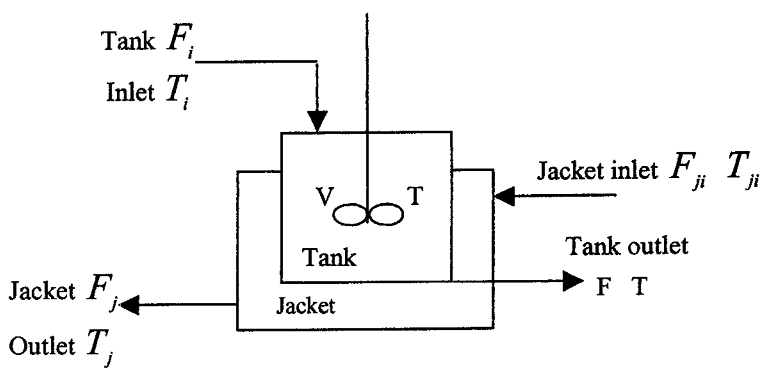


Figure 2.2: Jacket stirred tank heater

In linearizing the non-linear model in eqn. (2.37), it is assumed that the tank outlet flow rate, F, and the tank temperature, T are manipulated and controlled variables respectively. The overall heat transfer coefficient U is time varying and T_i is considered as the disturbance affecting the system. The steady state is obtained by solving the dynamic equation for $dT/dt = 0$. The steady state values of the system variables and some parameters for the process are given in Bequette (1998).

$$F_s = 1.0\, ft^3/\min \quad \rho C_p = 61.3\, Btu/°F\, ft^3 \quad V = 10 ft^3 \quad T_{is} = 50°F \quad T_s = 125°F$$
$$F_{js} = 1.5 ft^3/\min \quad \rho_j C_{pj} = 61.3 Btu/°F ft^3 \quad V_j = 1 ft^3 \quad T_{jis} = 200°F \quad T_{js} = 150°F$$

Linearizing and applying Laplace transform yields

$$\tilde{T}(s) = \frac{1/V(T_{is} - T_s)}{(s + F_s/V + UA/\rho V C_p)}\tilde{F}(s) + \frac{F_s/V}{(s + F_s/V + UA/\rho V C_p)}\tilde{T}_i(s) \quad (2.39)$$

Assume the temperature measurement has a time delay of 4, eqn. (2.39) yields

$$\tilde{T}(s) = \frac{-7.5e^{-4s}}{s + (0.1 + 0.00163\,\bar{u})}\tilde{F}(s) + \frac{0.1}{s + (0.1 + 0.00163\,\bar{u})}\tilde{T}_i(s) \quad (2.40)$$

where $\bar{u} = UA$

Eqn. (2.40) can be written in the general form

$$y(s) = \frac{ke^{-ds}}{\tau s + 1}u(s) + \frac{K}{\tau s + 1}D_o(s) \quad (2.41)$$

A continuous-time transfer function with the following form:

$$y(s) = \frac{k}{\tau s + 1}x(s) \quad (2.42)$$

can be discretized as

$$y_n = e^{(-T_s/\tau)}y_{n-1} + (1 - e^{(-T_s/\tau)})\,k\,x_{n-1} \quad (2.43)$$

Thus, eqn. (2.40) can be expressed as

$$\tilde{T}(s)=\frac{-7.5e^{-4s}/(0.1+0.00163\;\bar{u})}{\left(\dfrac{1}{0.1\;+\;0.00163\;\bar{u}}\right)s\;+\;1}\tilde{F}(s)\;+\;\frac{0.1/(0.1+0.00163\;\bar{u})}{\left(\dfrac{1}{0.1\;+\;0.00163\;\bar{u}}\right)s\;+\;1}\tilde{T}_i(s) \tag{2.44}$$

and with a sampling time of 1 unit, eqn. (2.44) can be discretized as

$$T(t)=q^{-4}\frac{\dfrac{1}{0.1+0.00163\bar{u}}7.5\left(e^{-(0.1+0.00163\bar{u})}-1\right)}{1-e^{-(0.1+0.00163\bar{u})}q^{-1}}F(t)+\frac{\dfrac{0.1}{0.1+0.00163\bar{u}}\left(1-e^{-(0.1+0.00163\bar{u})}\right)}{1-e^{-(0.1+0.00163\bar{u})}q^{-1}}T_i(t) \tag{2.45}$$

Let's consider that UA is time varying and is given by $UA=183.9(1+0.5\sin(t/x))$, where x is a variable oscillation period. Thus, eqn. (2.45) can be written as

$$T(t)=q^{-4}\frac{(5.03e^{-0.15\sin(t/x)}-7.5)/(0.4+0.15\sin(t/x))}{1-0.67e^{-(0.15\sin(t/x))}q^{-1}}F(t)\;+\;\frac{(0.1-0.067e^{-0.15\sin(t/x)})/(0.4+0.15\sin(t/x))}{1-0.67e^{-(0.15\sin(t/x))}q^{-1}}T_i(t) \tag{2.46}$$

From eqn. (2.46), it can be seen that both the process model and the disturbance model are time-variant. In this example, it is assumed that the disturbance has three different time-variant dynamics from relatively slow parameter change to relatively fast parameter change. This time varying nature is induced by the time varying steam flow rate for example. This illustration is chosen to compare the performance monitoring methodology (which takes non-commutativity associated with LTV transfer functions into account) with the conventional performance assessment algorithm. That is, the minimum variance term is calculated using normal multiplication (non-commutative) and pointwise multiplication (commutative) and the difference between the two methods are compared for each of the disturbance dynamic.

Assuming $T_i(t)$ is random white noise disturbance representing the driving force of the unmeasured disturbances, then, the process model in eqn. (2.46) can further be expressed as

$$y_t = q^{-4}\frac{\phi(t)}{1-\delta(t)q^{-1}}u_t + \frac{\upsilon(t)}{1-\delta(t)q^{-1}}a_t \tag{2.47}$$

where y_t is the process variable and u_t is the manipulated variable. The time variant process and disturbance dynamics are given by

$$\begin{cases} \phi(t) = (5.03e^{-0.15\sin(t/x)} - 7.5)\big/(0.4 + 0.15\sin(t/x)) \\ \upsilon(t) = (0.1 - 0.067e^{-0.15\sin(t/x)})\big/(0.4 + 0.15\sin(t/x)) \\ \delta(t) = 0.67e^{-0.15\sin(t/x)} \end{cases} \tag{2.48}$$

A PI controller is used to control the process and is given by

$$Q(q^{-1}) = \frac{-0.05 + 0.045q^{-1}}{1-q^{-1}} \tag{2.49}$$

Three cases of time-varying dynamics are being considered in ascending order of increasing parameter-varying rate:

$$\begin{aligned} &case\,1:\ x = 10 \\ &case\,2:\ x = 1 \\ &case\,3:\ x = 0.5 \end{aligned} \tag{2.50}$$

The simulation results in Figure 2.3 show a comparison of the difference between normal multiplication (solid line) and pointwise multiplication (dotted line). It can be observed that the difference between the minimum variance terms calculated using the normal multiplication and pointwise multiplication increases as the parameter-varying rate increases from the top to the bottom subplot. This result shows that it is important to use normal multiplication rather than point multiplication in the estimation of minimum variance term for time varying processes.

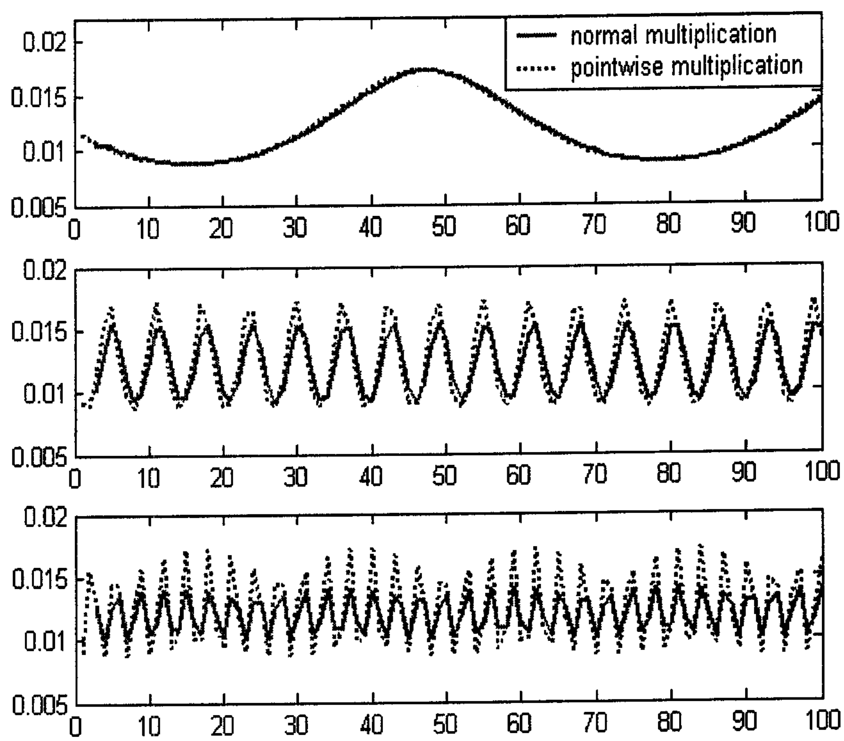


Figure 2.3: Comparison of time-variant minimum variance term using normal multiplication and pointwise multiplication

2.4 Case Study on Adaptive Control of Sulphur Recovery Unit

2.4.1 Process description

The proposed performance assessment methodology is applied to monitor the control loop performance of an adaptive controller in a Sulphur Recovery Unit (SRU) in Syncrude Canada Ltd.

The purpose of the SRU is to extract elemental sulphur from the Hydrogen Sulphide (H_2S) component of the acid gas stream obtained as by-products of plant operations.

A simplified schematic of the SRU is shown in Figure 2.4. (modified from "Adaptive Control of Sulphur Recovery Units, www.brainwave.com library").

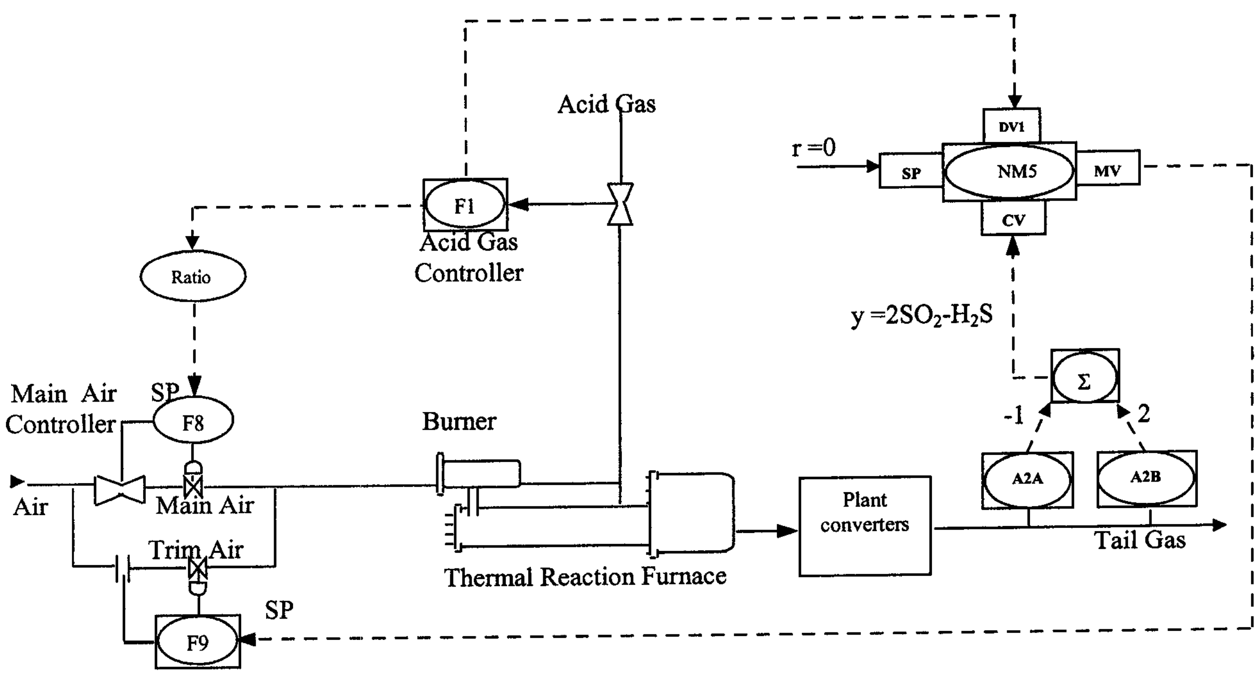


Figure 2.4: Schematic of Sulphur Recovery Unit

The SRU consists of a Claus process, which involves vapor phase oxidation of H_2S. The conventional Claus plant consists of thermal conversion unit and three stages of catalytic conversion in series. The description of a conventional Claus Sulphur Recovery plant is shown in Figure 2.5.

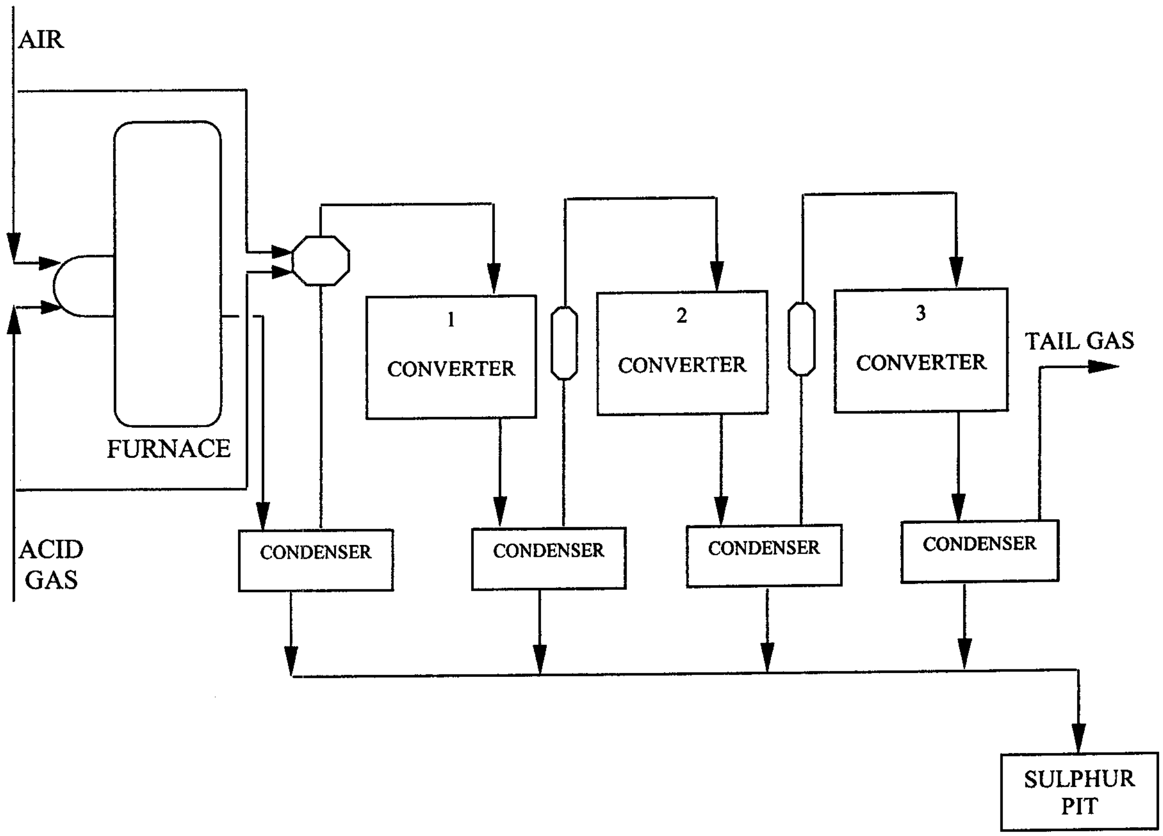


Figure 2.5: Claus Sulphur Recovery Plant

The acid gas feed stream which enters the thermal reaction furnace consists of H_2S, CO_2, hydrocarbons and traces of other substances. In the furnace, O_2 is added under controlled conditions to react with H_2S to form SO_2 under highly exothermic reactions. The complex reactions in the furnace can be simplified by the following reactions:

$$
\begin{aligned}
&H_2S + 3/2\, O_2 \Leftrightarrow SO_2 + H_2O + Heat \\
&2H_2S + O_2 \Leftrightarrow 2S + 2H_2O + Heat
\end{aligned}
\qquad (2.51)
$$

The amount of excess oxygen at the thermal stage is regulated such that 1/3 of the H_2S is converted to SO_2 in the furnace.

The hot gases exiting the furnace are passed through a waste heat boiler where they give off heat and are cooled. At the high temperatures of the reaction furnace and while cooling in the waste heat boiler, H_2S and SO_2 react to form sulphur vapors which are passed into a condenser.

$$2H_2S + SO_2 \Leftrightarrow 2H_2O + 3S + Heat \tag{2.52}$$

At this stage, approximately 67% of the conversion of the total H_2S to sulphur takes place. The remaining gases are then passed through stages of in-line re-heaters, three catalytic converters and three condensers to achieve the balance of the conversion to sulphur. To drive the catalytic reaction as close to completion as possible, the ratio 2:1 of H_2S to SO_2 obtained in the reactor should be maintained in the tail gas exiting the last converter. The control configuration is such that the gross air flow-rate is set by the main air controller F8, which adjusts the flow of combustion air to the reaction furnace thus providing feedforward control of air to the acid gas ratio. The trim air controller, F9 uses a gas analyzer (comprising of A2A, the analyzer that measures SO_2 and A2B, the analyzer for H_2S) to determine the ratio of H_2S to SO_2 in the tail gas and performs feedback control of this ratio by adjusting the amount of airflow to the reaction furnace.

In addition to the main H_2S conversion reactions, secondary reactions occur which result in low sulphur yield; hydrocarbons and oxygen compounds in the feed react with sulphur to form carbon disulphide (CS_2) and carbonyl sulphide (COS). To minimize these reactions, the temperature of the gas entering the first catalytic converter is maintained so that the converter outlet temperature is above 320°C. At the 320°C outlet temperature, 98% of the CS_2 and COS is expected to decompose. The sulphur produced in each converter is removed before passing through subsequent conversion stages to maximize H_2S conversion. Thus, process gases from each conversion stage are cooled to condense the sulphur vapor to liquid, and removed from the gas.

2.4.1.1 The adaptive controller

The adaptive controller, NM5 is used to control the tail gas ratio effectively by maintaining the appropriate set point for the trim air controller in order to achieve optimal performance. The adaptive controller updates its process model automatically and continuously as required to maintain optimal control of the process. This controller increases the unit efficiency because it is able to handle the long time-delay inherent in the process and it can handle the small variations in the acid gas flow/composition (i.e. feed changes). The adaptive nature of the controller makes the control loop time-variant. The effect of this time-variant nature will appear as non-stationarity of the closed-loop data. Therefore, the proposed time variant performance assessment algorithm is used to estimate control loop performance in the presence of non-stationary characteristics in the data. It involves a systematic solution where the non-commutativity problem associated with time variant processes is taken into consideration.

2.4.2 Data analysis

The time delay of the process including a zero-order-hold from a priori analysis is known to be no less or approximately 2 minutes. The sampling interval is one minute and a sample size of 2076 data points collected over a three-day period is used in this analysis. The data is assumed to contain a representative sample of normal process operations. The plot of the operating data (mean centered output) for the adaptive controller is shown in Figure 2.6.

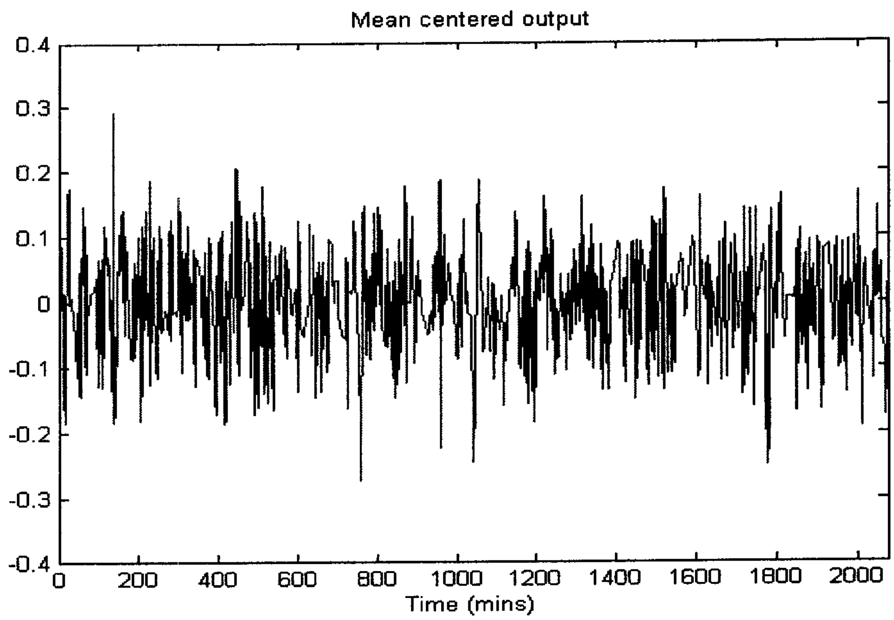


Figure 2.6: Plot of the operating data for the adaptive controller in the Sulphur Recovery Unit (SRU)

In addition to the minimum variance control benchmark used for performance assessment, other indicators of control loop performance are also considered and these include the closed-loop impulse response and autocorrelation function (ACF) of the output error.

An impulse response function curve represents the closed-loop impulse response between the whitened disturbance sequence and the process output and it is a direct measure of how well the controller is performing in rejecting disturbances or tracking set-point changes (Huang et al., 1997). The autocorrelation function (ACF) of the output error is an approximate measure of how close the existing controller is to minimum variance condition. The minimum variance control performance has been achieved if the autocorrelation function decays to zero beyond 'd-1' lags where d is the delay of the process. The rate at which the autocorrelation tends to zero beyond 'd-1' lags indicates the closeness of the existing controller to the minimum variance condition.

It is worthwhile to point out that the impulse response curve or autocorrelation function only applies to time invariant processes. However, both control loop performance measures are straightforward to calculate using process data and are therefore used as an initial, approximate estimation of performance by considering the control loop as time invariant. After the preliminary tests, the proposed time-variant performance assessment methodology is used to estimate the performance index of the time-variant control loop.

The estimation of the time-variant impulse response coefficients for the AR model (in eqn. (2.18) and eqn. (2.19)) and ARMA model (in eqn. (2.31) and eqn. (2.32)) shows that the first two terms of the moving average operators using pointwise and normal multiplication are the same. It has been discussed that the minimum variance term constitutes the first "d" terms of the moving average model. Hence, there will be no difference in the minimum variance term for a time delay of 2 for the traditional assessment method and LTV assessment method. However, the exact time delay is not known and time delay may also be time varying. The more effective method to account for such time delay uncertainty is to use extended horizon prediction method (Harris et al., 1999), that is, we need to calculate performance index as a function of time delay. With this consideration, the difference between normal multiplication and pointwise multiplication will show up. In this case study, we shall also compare the difference between the LTV assessment methodology (normal multiplication) and the traditional assessment method (pointwise multiplication) over a range of time delays.

Figure 2.7 shows the results for the autocorrelation test where a rather smooth autocorrelation plot is observed. Although it has not achieved minimum variance control performance, the response is relatively fast (almost settles down in about four samples). The test is only an indication of opportunity to further improve performance because the minimum variance control is typically not achievable in practice. In fact, it can be seen that the adaptive controller has achieved a close-to-optimal performance. It is observed that the closed-loop impulse response in Figure 2.8 shows a similar slow but smooth decay to zero. This result confirms a good performance of the adaptive controller. The initial analysis using the impulse response curve and autocorrelation function shows that the adaptive controller performance is good and the control loop is well tuned most of the time.

Further analysis is done on the LTV process by calculating the time-variant performance index using the proposed performance assessment algorithm. The plot of the performance index estimated over the three-day time period is given in Figure 2.9. Also, a moving window size of 100 (with an overlap of 99 data points) and window size of 50 (with an overlap of 49 data points) with conventional pointwise multiplication approach were used in assessing the performance of the control loop, and the results are compared with the LTV performance assessment algorithm.

Figure 2.9 shows the performance measure of the control loop using minimum variance control benchmark. By visualizing this plot, it can be seen that the controller performance is good and exhibits performance close to minimum variance control most of the time. The controller has a performance index that is greater than 0.7 approximately 60% of the sampled time.

Figure 2.10 shows a comparison of the estimated performance indices using the LTV performance assessment method and the moving window-based method (a conventional method). Figure 2.10(a) shows that although the performance indices of the two methods follow the same trend, but different values are obtained for most parts of the data. Figure 2.10(b) clearly reveals a greater difference between the LTV assessment method and moving window-based method when the window size was decreased from 100 to 50 data points. Hence, window-based method is unlikely to produce correct estimates of the performance index of a control loop with time-variant disturbance and, or process dynamics. This result shows that it is important to use the proposed LTV performance assessment algorithm in assessing the performance measure for time varying processes. Since it is known that there is a fundamental incorrectness in pointwise multiplication for LTV process, it is always better to use normal multiplication in the analysis of variance for time-variant processes. Furthermore, the plots clearly reveal time-varying behavior of the process and appropriate tool for performance assessment must therefore be time-variant based. It can be seen that the parameter-varying rate is slow and the difference between normal multiplication and pointwise multiplication may not be great as will be seen in the following.

The comparisons between the LTV assessment methodology (normal multiplication) and the traditional recursive assessment method (pointwise multiplication) for time delays of 4 and 6 are shown in Figure 2.11. The top subfigure 2.11(a) is a plot of the performance index calculated for the adaptive controller using the normal multiplication and pointwise multiplication for an assumed time delay of 4. Figure 2.11(b) shows the calculated performance index using the two performance assessment methods for an assumed larger time delay of 6. The plots reveal that there is a difference between the LTV assessment methodology and the traditional assessment for a time delay greater than 2 and the difference increases as the time delay becomes larger. Therefore, it is important to use normal multiplication in the calculation of the minimum variance term for an LTV process since the time varying nature of process data is frequently observed in practice and the time delay could be any value for different process operations.

Figure 2.12 shows the corresponding plots for the relative difference between the performance indices using normal multiplication (NM) and pointwise multiplication (PM) over the range of time delays discussed in the last section. The formula for calculating the relative difference (RD) is given as follows:

$$RD = \frac{|NM\ \ result\ - PM\ \ result\ |}{|PM\ \ result\ |} \tag{2.53}$$

It can be seen that the relative difference between the performance indices using normal multiplication and pointwise multiplication increases with the increase in time delay and the difference can be up to 40%. This result further shows that it is important to take non-commutativity into account in the manipulation of LTV operators.

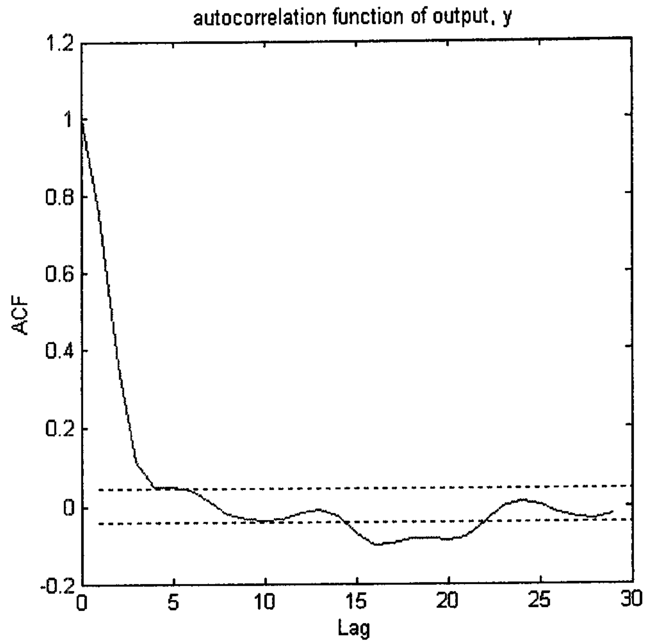


Figure 2.7: Autocorrelation Function

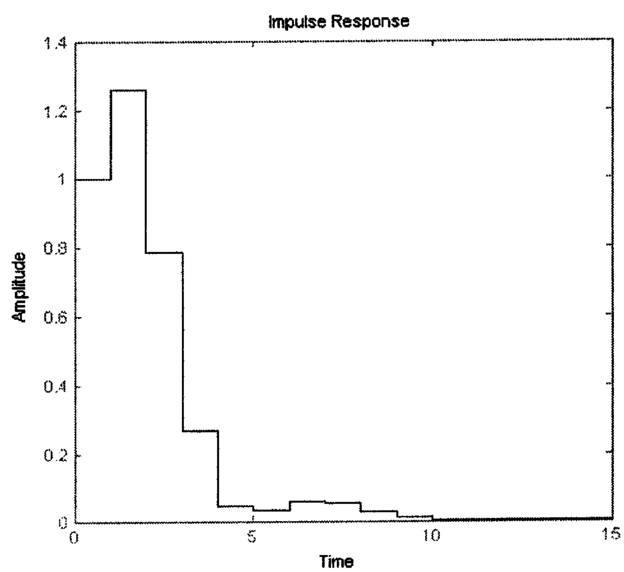


Figure 2.8: Estimated impulse response

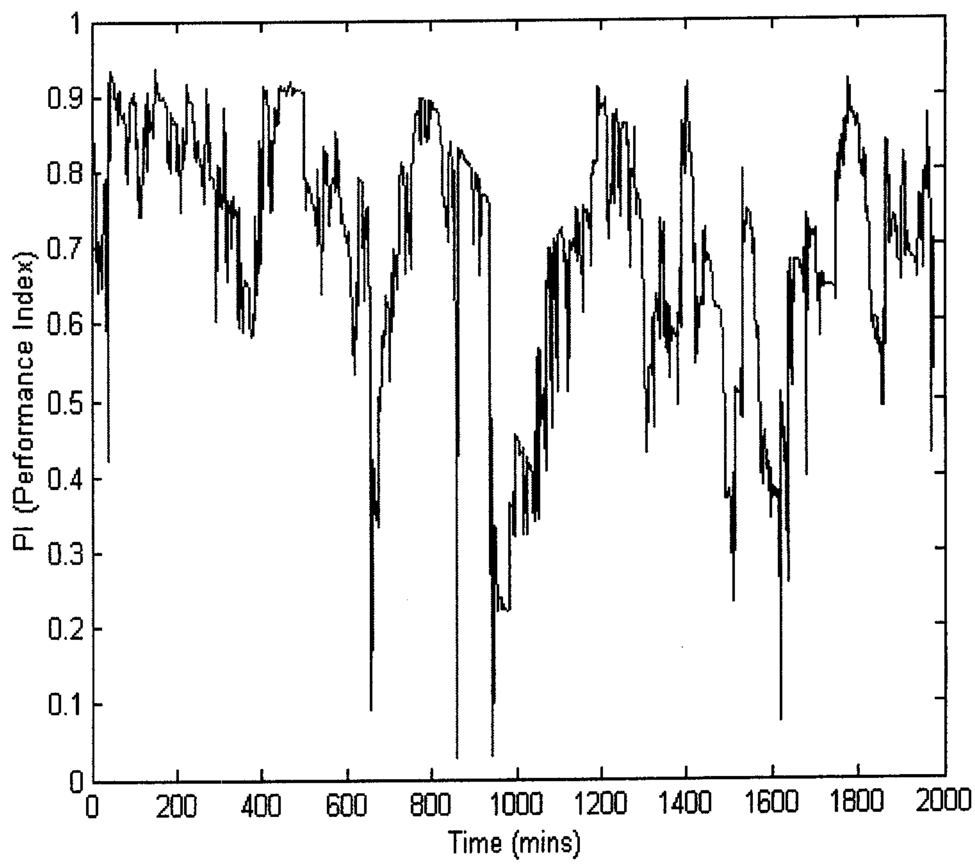


Figure 2.9: Estimation of performance index for the adaptive controller in the Sulphur Recovery Unit

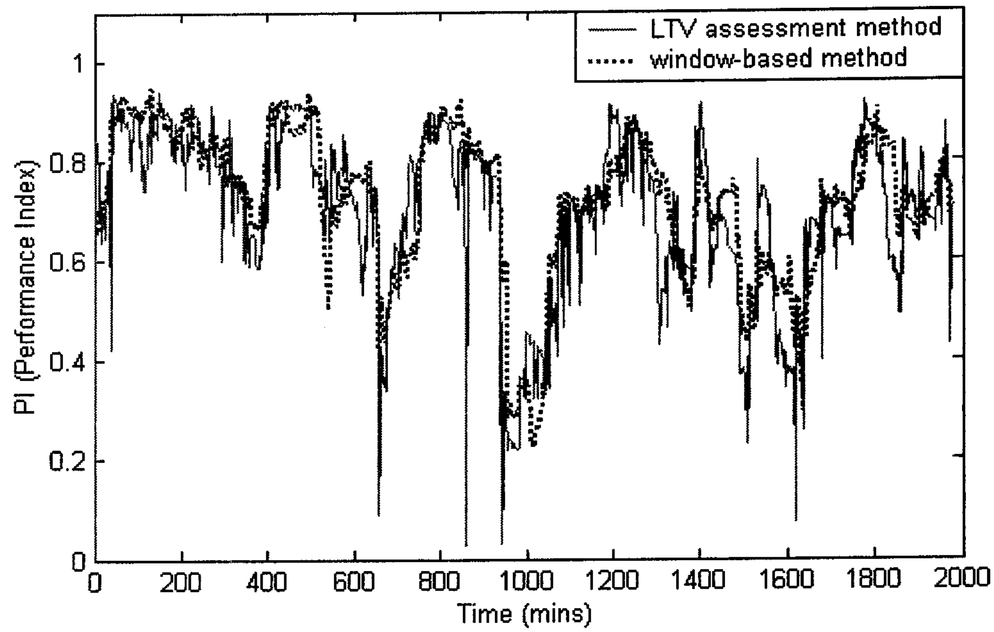


Figure 2.10(a): (window size of 100 data points)

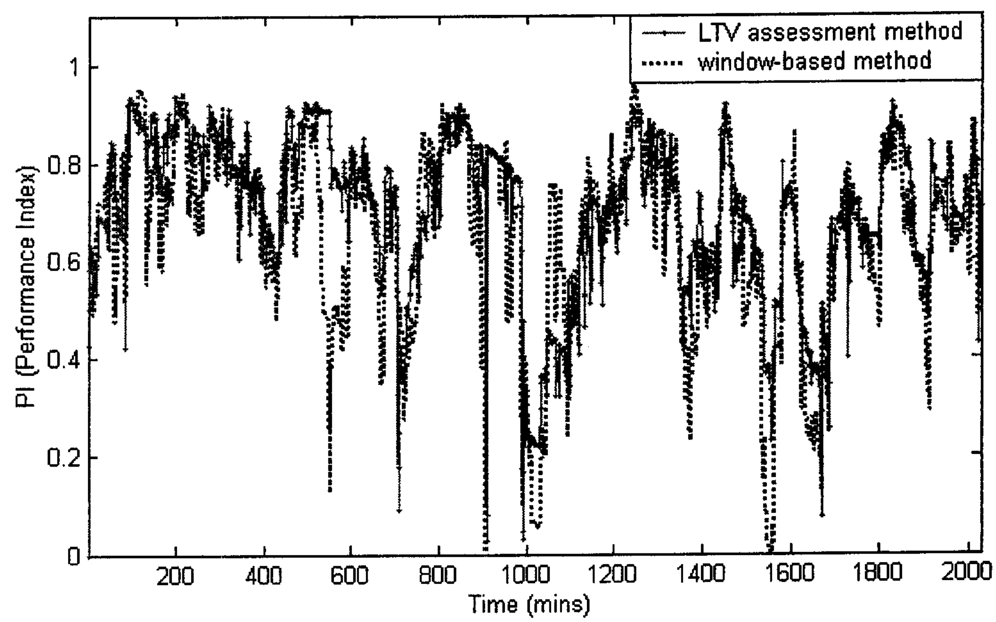


Figure 2.10(b): (window size of 50 data points)

Figure 2.10: Comparison of performance index using LTV assessment methodology and window-based method

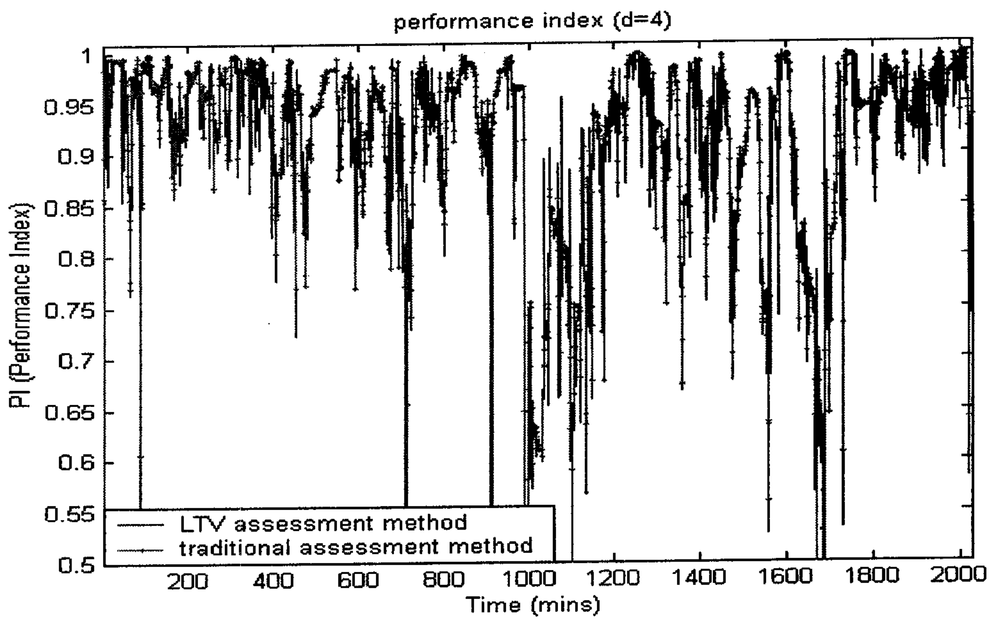


Figure 2.11(a)

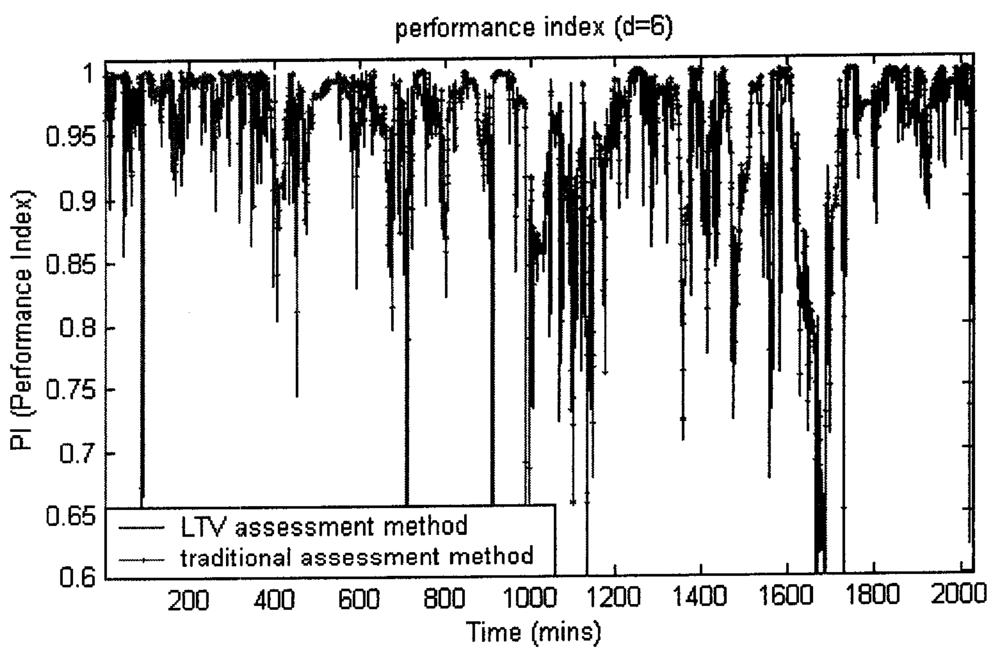


Figure 2.11(b)

Figure 2.11: Estimation of performance index as a function of time-delay

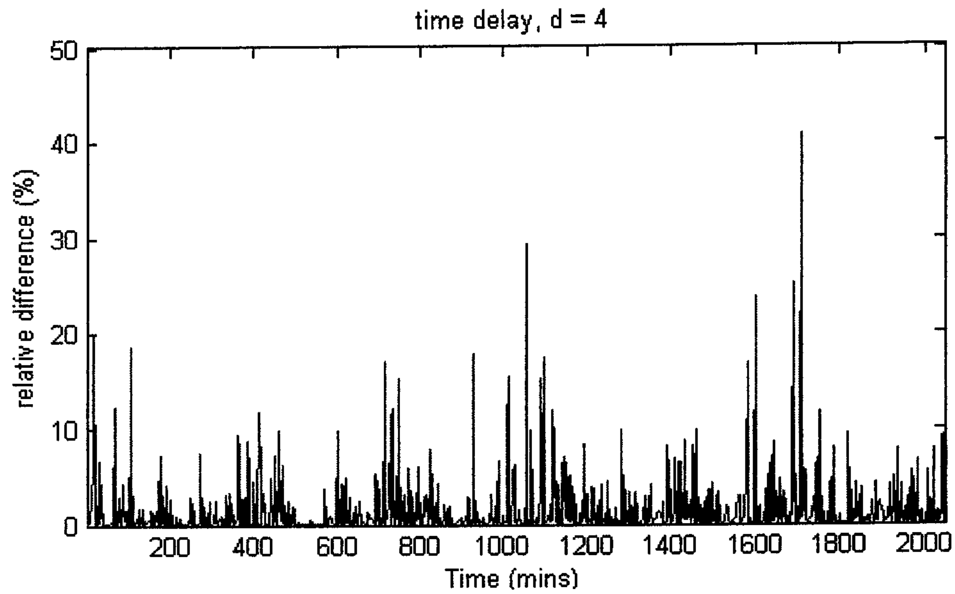


Figure 2.12(a)

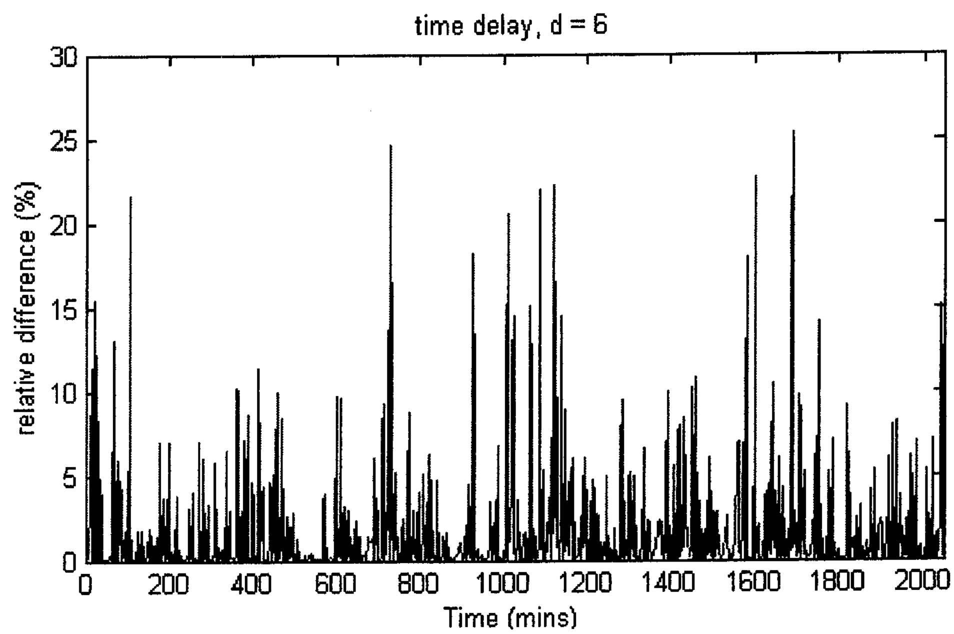


Figure 2.12(b)

Figure 2.12: Relative difference between the performance indices using normal multiplication and point multiplication

2.5 Conclusion

The technique for evaluating the type-A benchmark for control loop performance assessment of time-variant processes under time-variant or adaptive control has been discussed in this chapter. The method provides a way to monitor control loop performance of time-variant processes by taking the non-commutativity associated with LTV systems into account in calculating both minimum variance term and the process variance. The proposed performance monitoring method has been illustrated through a simulated example and demonstrated by an industrial application.

Chapter 3

Performance Assessment of Processes with Abrupt Changes of Disturbances Dynamics – Type-B Benchmark

Abstract

This chapter discusses the issue of performance assessment of time-variant processes due to abrupt changes of disturbances dynamics. It has been shown in the last chapter that type-A benchmark is time varying minimum variance control and is suitable for time-variant controllers or process or disturbance with the assumption that the controller can be time variant. If the controller is time invariant, as is the case for most industrial non-adaptive controllers, the type-A benchmark is clearly too demanding and is not appropriate. This results in a need to develop an alternative time-invariant performance benchmark that is more suitable for time-variant processes under time invariant control. In this chapter and the following one, we will limit our discussion to the time variant disturbance dynamics or models. This class of performance assessment problem and its solution is discussed and illustrated through a simulated example and an industrial application.

The main contributions of this chapter include (1) establishment of a general computation framework for a new control benchmark applicable to any change of disturbance dynamics; previous work in the literature has been limited to the assumption that only two different disturbance dynamics affect the process; and (2) a detailed industrial application of the proposed method.

3.1 Introduction

Abrupt change is any change in the parameters of the model that occurs either instantaneously or very fast with respect to the sampling period of the measurement (Basseville and Nikiforov, 1993). Abrupt changes involve changes with large or small magnitude but the magnitudes usually remain fairly constant for a time period before and after the abrupt change.

Most studies on control loop performance assessment have been focused on stationary time series or time-invariant processes, or to time series that can be represented as a time-invariant linear function of present and past values of an independent "white noise" process. Harris (1989) proposed that the minimum variance control or best possible control could be estimated by applying time series analysis to routine operating data. Desborough and Harris (1992) further proposed the use of least squares regression in the estimation of minimum variance control and performance index. Stanfelj et al (1993) presented the use of autocorrelation and cross correlation functions in the diagnosis of feedforward/feedback control loop performance. Lynch and Dumont (1996) reported the use of Laguerre network to model the closed loop system in order to estimate the minimum variance control for controller performance monitoring. Li and Evans (1997) presented the minimum variance control of linear time varying systems. There has also been an extension of controller performance assessment techniques to multivariate systems by Harris et al. (1996), Huang and Shah (1998) and Huang et al. (2000). Some of the recent work on control loop performance assessment includes a review of performance monitoring and assessment techniques by Harris et al. (1999), refinery-wide control loop performance assessment by Thornhill et al. (1999), and robust performance assessment of feedback control systems by Wan and Huang (2002). Huang (2002) complemented the work of Li and Evans and proposed a more general method of performance assessment of LTV feedback processes.

Although, there exist various performance assessment methods for time-invariant processes, there are few results available for time-variant processes. But most processes have certain degree of time varying behavior in nature. Goodwin and Sin (1984) have studied these time-varying properties in the "Adaptive Filtering Prediction and Control" literature but it has not been well addressed in control loop performance assessment. However, it has been observed that abrupt changes of disturbance dynamics are often encountered in many chemical processes due to, for example, the grade change of feed, change of downstream demands e.t.c. and they result in time varying dynamics or non-stationary time series of these processes. It is therefore necessary to obtain performance assessment benchmarks that take these abrupt changes of disturbance into account.

Traditional performance assessment technique for time-invariant controllers may yield erroneous results when there is an abrupt change in the disturbance dynamics in the control loop. This is because a minimum variance control benchmark estimated from a set of data when only stationary disturbance affects the process could give a poor performance in regulating a new disturbance that has a different dynamics. Therefore, without considering time-variant characteristics, the classical performance assessment results may be incomplete or can be misinterpreted (Huang, 1999).

3.2 Effect of Abrupt Changes of Disturbance Dynamics on the Performance of Time-Invariant Minimum Variance Control

In terms of controller performance assessment, we are facing a practical dilemma. It has been observed that abrupt changes of disturbances dynamics are often encountered in industrial processes but most of the industrial controllers are time-invariant. Although performance assessment of time varying processes has been studied by Huang (2002) and generalized in the previous chapter known as type-A benchmark, this benchmark assumes that the controller of concern is time varying or adaptive. Since most industrial controllers are time-invariant, performance assessment of such time-variant processes under time-invariant control is of practical interest. In this section, we would show how varying disturbance dynamics affects performance of time-invariant control and we shall develop a time-invariant control benchmark to evaluate controller performance of processes with abrupt change(s) of disturbance dynamics.

Let us consider the LTV SISO process shown in Figure 2.1:

$$y_t = q^{-d}\widetilde{T}(q^{-1},t)u_t + N(q^{-1},t)a_t \tag{3.1}$$

The closed-loop response under any time-variant feedback control $u_t = -Q(q^{-1},t)y_t$ can be written as

$$y_t = F(q^{-1},t)a_t + \ell(q^{-1},t)a_{t-d} \tag{3.2}$$

where

$$\begin{aligned}\ell(q^{-1},t) &= R(q^{-1},t) - \frac{\widetilde{T}(q^{-1},t-d)Q(q^{-1},t-d)N(q^{-1},t-d)}{1+Q(q^{-1},t-d)q^{-d}\widetilde{T}(q^{-1},t-d)} \\ &= R(q^{-1},t) - \frac{N(q^{-1},t-d)}{Q^{-1}(q^{-1},t-d)\widetilde{T}^{-1}(q^{-1},t-d)+q^{-d}}\end{aligned} \tag{3.3}$$

$F(q^{-1},t)$ and $R(q^{-1},t)$ are solved from the Diophantine identity:

$$N(q^{-1},t) = \underbrace{f_o(t) + f_1(t)q^{-1} + \ldots + f_{d-1}(t)q^{-d+1}}_{F(q^{-1},t)} + R(q^{-1},t)q^{-d} \tag{3.4}$$

The closed-loop response under time-variant minimum variance control is given by

$$y_t\big|_{mv} = (f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + \ldots + f_{d-1}(t)q^{-(d-1)})a_t \tag{3.5}$$

The time-variant minimum variance response can be estimated from time series analysis of closed-loop data. This time-variant minimum variance control benchmark, which has been discussed in chapter 2, is referred to as Type-A benchmark. For time-invariant processes, the type-A benchmark is the time-invariant minimum variance control. For processes with abrupt changes of disturbance dynamics, the type-A benchmark is piecewise time-invariant minimum variance control (Huang, 1999).

It follows from eqn. (3.3) that for a time-invariant process transfer function $q^{-d}T(q^{-1})$ and a time-invariant control law $Q(q^{-1})$, the closed-loop response can be written as

$$y_t = F(q^{-1},t)a_t + \left[R(q^{-1},t) - \frac{N(q^{-1},t-d)}{Q^{-1}(q^{-1})\widetilde{T}^{-1}(q^{-1}) + q^{-d}} \right] q^{-d} a_t \tag{3.6}$$

Let us assume that there are three different disturbance dynamics affecting the process with the first change in disturbance dynamics occurring at $t = \theta$, and the second abrupt change occurs at $t = 2\theta$:

$$N(q^{-1},t) = \begin{cases} N_1(q^{-1}) = F_1(q^{-1}) + q^{-d}R_1(q^{-1}) & t < \theta \\ N_2(q^{-1}) = F_2(q^{-1}) + q^{-d}R_2(q^{-1}) & \theta \le t < 2\theta \\ N_3(q^{-1}) = F_3(q^{-1}) + q^{-d}R_3(q^{-1}) & t \ge 2\theta \end{cases} \tag{3.7}$$

For the first section of data sampled at time $t < \theta$, eqn. (3.6) can be written as

$$y_t^{(1)} = F_1(q^{-1})a_t + \left[R_1(q^{-1}) - \frac{N_1(q^{-1})}{Q^{-1}(q^{-1})\widetilde{T}^{-1}(q^{-1}) + q^{-d}} \right] q^{-d} a_t \tag{3.8}$$

The closed-loop response in eqn. (3.8) is time-invariant and the corresponding time-invariant minimum variance control law is calculated when the right hand side of the equation is equated to zero to obtain

$$Q_{mv}^{(1)}(q^{-1}) = \widetilde{T}^{-1}(q^{-1})R_1(q^{-1})F_1^{-1}(q^{-1}) \tag{3.9}$$

Substituting eqn. (3.9) into eqn. (3.8) yields

$$y_t^{(1)}\big|_{mv} = F_1(q^{-1})a_t \tag{3.10}$$

$F_1(q^{-1})a_t$ represents the process output under minimum variance control for the first section of data.

If the minimum variance control $Q_{mv}^{(1)}(q^{-1})$ is chosen as the benchmark, the performance of this benchmark for the second section of data $\theta \le t < 2\theta$ can be estimated. From eqn. (3.6), the closed-loop response to the second disturbance dynamics can be written as

$$y_t^{(2)} = F_2(q^{-1})a_t + \left[R_2(q^{-1}) - \frac{N_2(q^{-1})}{Q^{-1}(q^{-1})\widetilde{T}^{-1}(q^{-1}) + q^{-d}} \right] q^{-d} a_t \tag{3.11}$$

The true minimum variance control $Q_{mv}^{(2)}(q^{-1})$ for this data set can be calculated by equating the right hand side of eqn. (3.11) to zero and this gives

$$Q_{mv}^{(2)}(q^{-1}) = \widetilde{T}^{-1}(q^{-1})R_2(q^{-1})F_2^{-1}(q^{-1}) \tag{3.12}$$

The process output under this true minimum variance control can be obtained by substituting eqn. (3.12) into eqn. (3.11) and this yields

$$y_t^{(2)}\big|_{mv} = F_2(q^{-1})a_t \tag{3.13}$$

However, if the minimum variance control of the first section is used, i.e. $Q_{mv}^{(1)}(q^{-1})$ is used to control the section, we would naturally want to know what the response of the second section will be.

Hence, substituting the control law $Q_{mv}^{(1)}(q^{-1})$ into eqn. (3.11) yields

$$
\begin{aligned}
y_t^{(2)} &= F_2(q^{-1})a_t + \left[R_2(q^{-1}) - R_1(q^{-1})\frac{N_2(q^{-1})}{N_1(q^{-1})} \right] q^{-d} a_t \\
&= \left[F_2(q^{-1}) + q^{-d} R_2(q^{-1})\right] a_t - q^{-d}\frac{N_2(q^{-1})}{N_1(q^{-1})} R_1(q^{-1}) a_t \\
&= N_2(q^{-1}) a_t - q^{-d}\frac{N_2(q^{-1})}{N_1(q^{-1})} R_1(q^{-1}) a_t \\
&= \frac{N_2(q^{-1})}{N_1(q^{-1})}\left[N_1(q^{-1}) - q^{-d} R_1(q^{-1})\right] a_t \\
&= \frac{N_2(q^{-1})}{N_1(q^{-1})} F_1(q^{-1}) a_t
\end{aligned}
\tag{3.14}
$$

Eqn. (3.14) can be expressed in additive terms as

$$
y_t^{(2)} = F_2(q^{-1})a_t + \left(\left(\frac{F_1(q^{-1})N_2(q^{-1})}{F_2(q^{-1})N_1(q^{-1})} - 1\right) F_2(q^{-1}) \right) a_{t-d} \tag{3.15}
$$

This is the response of the second section when the minimum variance control of the first section is applied.

We can also estimate the performance of this benchmark when applied to the third section of data $t \geq 2\theta$. From eqn. (3.6), the closed-loop response to the third disturbance dynamics can be written as

$$
y_t^{(3)} = F_3(q^{-1}, t)a_t + \left[R_3(q^{-1}) - \frac{N_3(q^{-1})}{Q^{-1}(q^{-1})\widetilde{T}^{-1}(q^{-1}) + q^{-d}} \right] q^{-d} a_t \tag{3.16}
$$

The true minimum variance control $Q_{mv}^{(3)}(q^{-1})$ for this data set is calculated by equating the right hand side of eqn. (3.16) to zero:

$$
Q_{mv}^{(3)}(q^{-1}) = \widetilde{T}^{-1}(q^{-1}) R_3(q^{-1}) F_3^{-1}(q^{-1}) \tag{3.17}
$$

The process output under this true minimum variance control can be obtained by substituting eqn. (3.17) into eqn. (3.16):

$$y_t^{(3)}\Big|_{mv} = F_3(q^{-1})a_t \tag{3.18}$$

With the benchmark control law $Q_{mv}^{(1)}(q^{-1})$, eqn. (3.16) yields

$$\begin{aligned}
y_t^{(3)} &= F_3(q^{-1})a_t + \left[R_3(q^{-1}) - R_1(q^{-1})\frac{N_3(q^{-1})}{N_1(q^{-1})}\right]q^{-d}a_t \\
&= \left[F_3(q^{-1}) + q^{-d}R_3(q^{-1})\right]a_t - q^{-d}\frac{N_3(q^{-1})}{N_1(q^{-1})}R_1(q^{-1})a_t \\
&= N_3(q^{-1})a_t - q^{-d}\frac{N_3(q^{-1})}{N_1(q^{-1})}R_1(q^{-1})a_t \\
&= \frac{N_3(q^{-1})}{N_1(q^{-1})}\left[N_1(q^{-1}) - q^{-d}R_1(q^{-1})\right]a_t \\
&= \frac{N_3(q^{-1})}{N_1(q^{-1})}F_1(q^{-1})a_t
\end{aligned} \tag{3.19}$$

Similarly, eqn. (3.19) can be expressed as

$$y_t^{(3)} = F_3(q^{-1})a_t + \left(\left(\frac{F_1(q^{-1})N_3(q^{-1})}{F_3(q^{-1})N_1(q^{-1})} - 1\right)F_3(q^{-1}) \right)a_{t-d} \tag{3.20}$$

The expressions in eqn. (3.10), eqn. (3.14) and eqn. (3.19) represent the performance of the time-invariant minimum variance control for the three disturbance dynamics occurring in the process. It should be noted that the result obtained could be extended to a more general case (which shall be discussed next) where more than three disturbance dynamics may affect the process.

Let us consider a general case of a process affected by "$i = 1, 2, 3, \ldots, n$" disturbance dynamics. Assuming an abrupt change of disturbance occurs at each data section, then the data Y_t could be segregated into "n" piecewise stationary sections $Y_1, Y_2, Y_3, \ldots Y_n$. If the minimum variance control $Q_{mv}^{(j)}(q^{-1})$ of the "j^{th}" section of data $(1 \leq j \leq n)$ is chosen as the benchmark, then the closed-loop response of this time-invariant benchmark control law to the "i^{th}" disturbance dynamics $(i \neq j)$ can be written as:

$$
\begin{aligned}
y_t^{(i)} &= F_i(q^{-1})a_t + \left[R_i(q^{-1}) - R_j(q^{-1})\frac{N_i(q^{-1})}{N_j(q^{-1})} \right] q^{-d} a_t \\
&= \left[F_i(q^{-1}) + q^{-d} R_i(q^{-1})\right] a_t - q^{-d} \frac{N_i(q^{-1})}{N_j(q^{-1})} R_j(q^{-1}) a_t \\
&= N_i(q^{-1}) a_t - q^{-d} \frac{N_i(q^{-1})}{N_j(q^{-1})} R_j(q^{-1}) a_t \\
&= \frac{N_i(q^{-1})}{N_j(q^{-1})} \left[N_j(q^{-1}) - q^{-d} R_j(q^{-1})\right] a_t \\
&= \frac{N_i(q^{-1})}{N_j(q^{-1})} F_j(q^{-1}) a_t
\end{aligned}
\qquad (3.21)
$$

Eqn. (3.21) implies that the difference of performance between the true minimum variance control $Q_{mv}^{(i)}(q^{-1})$ and the benchmark minimum variance control $Q_{mv}^{(j)}(q^{-1})$ applied to the current i^{th} disturbance is given by a multiplicative factor that depends only on the disturbance dynamics.

The difference of performance between the true minimum variance control $Q_{mv}^{(i)}(q^{-1})$ and the benchmark minimum variance control $Q_{mv}^{(j)}(q^{-1})$ applied to the current i^{th} disturbance can be further expressed in additive terms:

$$
y_t^{(i)} = F_i(q^{-1})a_t + \left(\left(\frac{F_j(q^{-1})N_i(q^{-1})}{F_i(q^{-1})N_j(q^{-1})} - 1\right) F_i(q^{-1}) \right) a_{t-d} \qquad (3.22)
$$

3.3 Performance Assessment with Type-B benchmark

As has been discussed above, type-A benchmark, which is time varying minimum variance control, is not a suitable benchmark for performance assessment of most industrial controllers that are time-invariant. But it has been discussed that abrupt changes of disturbances dynamics are often encountered in many chemical processes and they result in time varying dynamics of these processes. Therefore, it is important to obtain suitable time-invariant control benchmark for performance assessment of these processes with time-variant disturbance dynamics. This leads to the discussion of type-B benchmark, which is time-invariant minimum variance control.

The type-B benchmark uses minimum variance control of the data section that is the most representative of the process operation as a benchmark to evaluate controller performance over the entire time period. The rationale is that since the controller is time invariant, the benchmark controller can only be time invariant.

For instance, consider a time-invariant process transfer function $q^{-d}\widetilde{T}(q^{-1})$ and a time invariant control law $G_c(q^{-1})$. In this example, we have a set of data $Y = \{y_0, y_1, y_2, \ldots y_n\}$ sampled from $0 \le t \le n$. It is assumed that the process is affected by two different disturbance dynamics $N_1(q^{-1})$ and $N_2(q^{-1})$ with the change of disturbance dynamics occurring at $t = \theta$, where $0 < \theta < n$. Hence, data is segregated into 2 sections and for the first section of disturbance, we have

$$\begin{cases} Y_1 = \{y_0, y_1, \ldots, y_{\theta-1}\} \\ N_1(q^{-1}) = F_1(q^{-1}) + q^{-d}R_1(q^{-1}) \qquad 0 \le t < \theta \end{cases} \tag{3.23}$$

For the second section of disturbances, we have

$$\begin{cases} Y_2 = \{y_\theta, y_{\theta+1}, \ldots\ldots y_n\} \\ N_2(q^{-1}) = F_2(q^{-1}) + q^{-d}R_2(q^{-1}) \qquad \theta \le t \le n \end{cases} \tag{3.24}$$

The minimum variance or lower bound of variance for each data section could be estimated by applying any time-invariant control loop performance assessment algorithm (Harris, 1989; Desborough and Harris, 1992; Huang et al., 1997). These piecewise lower bounds can be achieved simultaneously under time variant minimum variance control (the type-A benchmark). However, a time-invariant controller can possibly achieve only one of the lower bounds. For type-B benchmark, the minimum variance control of one representative section of the data could be used as a benchmark to evaluate performance of the entire process but note that the chosen minimum variance control benchmark is optimal for that data section only. Since the disturbance dynamics is time-variant, it is important to check the suitability of this benchmark by estimating the possible response when it is applied to other sections of data (Huang, 1999). This should be carried out because the minimum variance control of one data section might not be appropriate for other sections of the data set especially when there is a significant change in the disturbance dynamics.

In this example, it is assumed that the minimum variance control of the first section of data $Y_1 = \{y_0, y_1, ...y_{\theta-1}\}$ is used as the benchmark without loss of generality.

From Figure 2.1, the closed loop transfer function for the first section of data can be written as

$$H_{cl}^{(1)}(q^{-1}) = \frac{N_1(q^{-1})}{1 + q^{-d}\widetilde{T}(q^{-1})G_c(q^{-1})} \tag{3.25}$$

Applying time series analysis and using pointwise multiplication and/or division, the closed loop transfer function for the first section can be expressed as the sum of impulse response coefficients:

$$Y_t^{(1)}(q^{-1}) = H_{cl}^{(1)}(q^{-1})a_t = (f_0^{(1)} + f_1^{(1)}q^{-1} + ... + f_{d-1}^{(1)}q^{-(d-1)} + q^{-d}G_R^{(1)}(q^{-1}))a_t \tag{3.26}$$

The minimum variance term is:

$$F_1(q^{-1}) = f_0^{(1)} + f_1^{(1)}q^{-1} + \dots + f_{d-1}^{(1)}q^{-(d-1)} \tag{3.27}$$

Similarly, applying time series analysis to the second section of data yields the closed loop transfer function

$$H_{cl}^{(2)}(q^{-1}) = \frac{N_2(q^{-1})}{1 + q^{-d}\widetilde{T}(q^{-1})G_c(q^{-1})} \tag{3.28}$$

Using long division, we can obtain

$$Y_t^{(2)}(q^{-1}) = H_{cl}^{(2)}(q^{-1})a_t = (f_0^{(2)} + f_1^{(2)}q^{-1} + \dots + f_{d-1}^{(2)}q^{-(d-1)} + q^{-d}G_R^{(2)}(q^{-1}))a_t \tag{3.29}$$

and the minimum variance term for the second section is:

$$F_2(q^{-1}) = f_0^{(2)} + f_1^{(2)}q^{-1} + \dots + f_{d-1}^{(2)}q^{-(d-1)} \tag{3.30}$$

From eqn. (3.25) and eqn. (3.28),

$$\frac{N_2(q^{-1})}{N_1(q^{-1})} = \frac{H_{cl}^{(2)}(q^{-1})}{H_{cl}^{(1)}(q^{-1})} \tag{3.31}$$

Eqn. (3.31) can be estimated from time-series analysis of the closed-loop operating data. Substituting eqns. (3.27) and (3.31) into eqn. (3.14) or (3.15) gives the closed loop response of the type-B benchmark control when it is applied to the second section of data.

Although the analysis has been carried out for two different disturbance dynamics, this result can also be extended to the case where there are more than two disturbance dynamics affecting the process. The illustration for the more general case is summarized and given next.

For a general case, consider a time-invariant process transfer function $q^{-d}\widetilde{T}(q^{-1})$ with a time invariant control law $G_c(q^{-1})$. It is assumed that the process is affected by "n" different disturbance dynamics $N_1(q^{-1}), N_2(q^{-1}), N_3(q^{-1}), ..., N_n(q^{-1})$. The sampled data is segregated into "n" sections and each disturbance dynamic is given by the diophantine identity:

$$N_i(q^{-1}) = F_i(q^{-1}) + q^{-d}R_i(q^{-1}) \qquad i = 1, 2, 3, ..., n \tag{3.32}$$

Typically, for type-B benchmark, the minimum variance control of one representative section of the data is used as a benchmark to evaluate performance of the entire process. Since the disturbance dynamics is time-variant, it is important to check the suitability of this benchmark by estimating the possible response when it is applied to other sections of data (Huang, 1999). This should be carried out because the minimum variance control of one data section might not be appropriate for other sections of the data set especially when there is a significant change in the disturbance dynamics.

It is assumed that the "j^{th}" section of data $(1 \le j \le n)$ is the most representative of process operations and the minimum variance control of this section of data is used as the benchmark for performance evaluation of the entire process. The closed loop transfer function of the "j^{th}" section of data can be written as:

$$H_{cl}^{(j)}(q^{-1}) = \frac{N_j(q^{-1})}{1 + q^{-d}\widetilde{T}(q^{-1})G_c(q^{-1})} \tag{3.33}$$

and the minimum variance term is given by

$$F_j(q^{-1}) = f_0^{(j)} + f_1^{(j)}q^{-1} + ... + f_{d-1}^{(j)}q^{-(d-1)} \tag{3.34}$$

The closed loop transfer function for the i^{th} section of data $(i \ne j)$ can be written as

$$H_{cl}^{(i)}(q^{-1}) = \frac{N_i(q^{-1})}{1 + q^{-d}\tilde{T}(q^{-1})G_c(q^{-1})} \tag{3.35}$$

and the corresponding minimum variance term is given by:

$$F_i(q^{-1}) = f_0^{(i)} + f_1^{(i)}q^{-1} + \ldots + f_{d-1}^{(i)}q^{-(d-1)} \tag{3.36}$$

From eqn. (3.33) and eqn. (3.35),

$$\frac{N_i(q^{-1})}{N_j(q^{-1})} = \frac{H_{cl}^{(i)}(q^{-1})}{H_{cl}^{(j)}(q^{-1})} \tag{3.37}$$

Substituting eqn. (3.34) and eqn. (3.37) in eqn. (3.21) gives the closed loop response when the benchmark control is applied to the i^{th} section of data, and can be written in a general form as

$$y_t^{(i)} = \frac{H_{cl}^{(i)}(q^{-1})}{H_{cl}^{(j)}(q^{-1})}(f_0^{(j)} + f_1^{(j)}q^{-1} + \ldots f_{d-1}^{(j)}q^{-(d-1)})a_t \tag{3.38}$$

Note that the basic assumption that has been made in the analysis of type-B benchmark is that only changes of disturbance dynamics affects the process operation.

3.4 Evaluation via Simulation Example

Huang (1999) showed that the controversial result obtained by Eriksson and Isaksson (1994) in their simulation example is due to the change of disturbance dynamics rather than inadequacy of the time-invariant minimum variance control benchmark. Huang (1999) demonstrated the validity of the minimum variance control benchmark by considering a case where there is an abrupt change of disturbance to random walk dynamics in the second data section. This simulation compliments the contribution by

Huang (1999), and further shows that we can extend the result to a more general case where we have more than two disturbance dynamics affecting the process.

The process is a discrete-time model given by

$$q^{-d}T(q^{-1}) = q^{-4}\frac{0.33}{1-0.67q^{-1}} \tag{3.39}$$

The process is controlled by a Dahlin controller, which is given by

$$Q(q^{-1}) = \frac{0.7-0.47q^{-1}}{0.33-0.10q^{-1}-0.23q^{-4}} \tag{3.40}$$

We assume that the time-variant disturbance transfer function is

$$N(q^{-1}) = \frac{1-\beta q^{-1}}{1-\lambda q^{-1}} \tag{3.41}$$

where λ and β can assume values from 0 to 1. Here, we'll be considering a total of 7000 data points with four different types of disturbance dynamics in the simulation. The disturbance dynamics are given as follows:

$$N_1(q^{-1}) = \frac{1-0.4q^{-1}}{1-0.67q^{-1}} \qquad t < 2501 \tag{3.42}$$

$$N_2(q^{-1}) = \frac{1-0.4q^{-1}}{1-q^{-1}} \qquad 2501 \le t < 3501 \tag{3.43}$$

$$N_3(q^{-1}) = \frac{1-0.4q^{-1}}{1-0.87q^{-1}} \qquad 3501 \le t < 5001 \tag{3.44}$$

$$N_4(q^{-1}) = \frac{1}{1-0.67q^{-1}} \qquad 5001 \le t \le 7000 \qquad (3.45)$$

From eqn. (3.42), the theoretical minimum variance term for the first section of data is

$$F_1(q^{-1}) = 1 + 0.27q^{-1} + 0.18q^{-2} + 0.12q^{-3}$$

and its variance is $\sigma^2_{mv,1} = 1.12\sigma^2_a$

From eqn. (3.43), the minimum variance term for the second section of data is calculated as

$$F_2(q^{-1}) = 1 + 0.6q^{-1} + 0.6q^{-2} + 0.6q^{-3}$$

and its variance is $\sigma^2_{mv,2} = 2.08\sigma^2_a$

From eqn. (3.44), the calculated minimum variance term for the third section of data is

$$F_3(q^{-1}) = 1 + 0.47q^{-1} + 0.41q^{-2} + 0.36q^{-3}$$

and its variance is $\sigma^2_{mv,3} = 1.51\sigma^2_a$

From eqn. (3.45), the theoretical minimum variance term for the fourth section of data is calculated as

$$F_4(q^{-1}) = 1 + 0.67q^{-1} + 0.45q^{-2} + 0.30q^{-3}$$

and its variance is $\sigma^2_{mv,4} = 1.74\sigma^2_a$

A plot of the four disturbance dynamics affecting the process is shown in Figure 3.1.

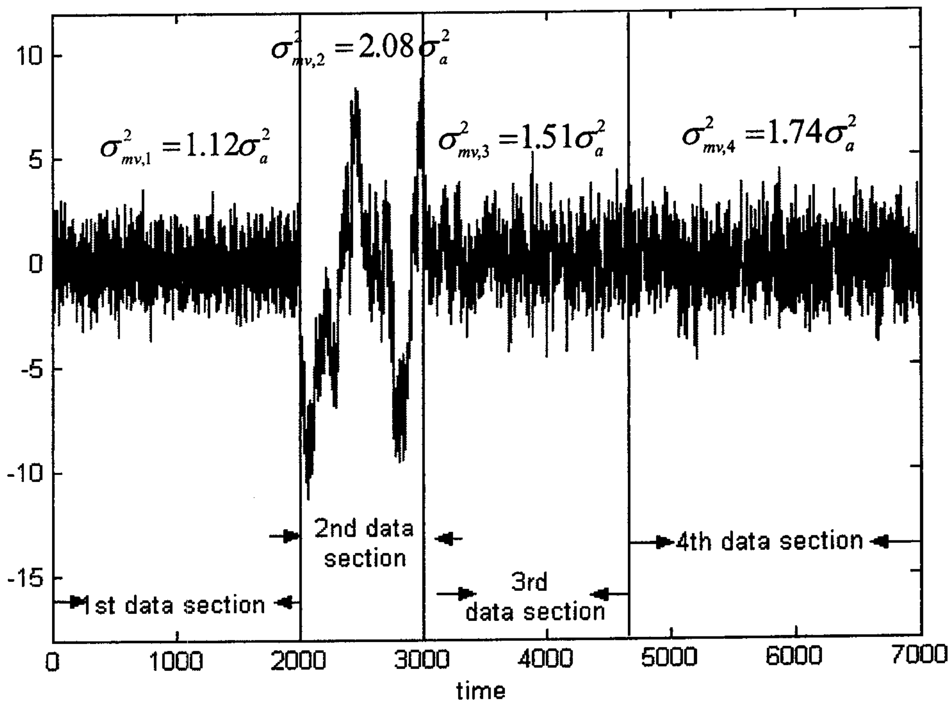


Figure 3.1: Plot of the sections of data showing the four disturbance dynamics affecting the process

A time-invariant controller cannot achieve the four minimum variance control benchmarks $\sigma^2_{mv,1}, \sigma^2_{mv,2}, \sigma^2_{mv,3}, \sigma^2_{mv,4}$ simultaneously. For type-B benchmark, only one of them is used as the benchmark depending on which section of disturbance is most representative. It is assumed that the disturbance dynamics in the first section of data is most representative, such that $\sigma^2_{mv,1} = 1.12\sigma^2_a$ is used as the benchmark for performance assessment but this value is only valid for the first section. The minimum variance control will give different variances for the remaining sections. Therefore, it is imperative to check if this benchmark control is suitable for the other sections of data after each abrupt change of disturbance.

From eqn. (3.21) the closed loop response of the chosen benchmark control to other sections of data is given by

$$y_t^{(i)} = \frac{N_i(q^{-1})}{N_{ref}(q^{-1})} F_{ref}(q^{-1}) a_t \tag{3.46}$$

where the subscript, "ref" represents the section of data that is used as the benchmark for performance assessment and "i" denotes the other sections of data.

Thus, the closed-loop response of the benchmark control $Q_{mv}^{(1)}(q^{-1})$ to the 2^{nd}, 3^{rd} and 4^{th} sections of data can be expressed as

$$y_t^{(2)} = \frac{1-0.67q^{-1}}{1-q^{-1}}(1+0.27q^{-1}+0.18q^{-2}+0.12q^{-3}) \qquad 2501 \le t < 3501 \tag{3.47}$$

$$y_t^{(3)} = \frac{1-0.67q^{-1}}{1-0.87q^{-1}}(1+0.27q^{-1}+0.18q^{-2}+0.12q^{-3}) \qquad 3501 \le t < 5001 \tag{3.48}$$

$$y_t^{(4)} = \frac{1}{1-0.4q^{-1}}(1+0.27q^{-1}+0.18q^{-2}+0.12q^{-3}) \qquad 5001 \le t \le 7000 \tag{3.49}$$

Although the minimum variance control of the first section of disturbances is a suitable benchmark for the disturbance dynamics in the 3rd and 4th data sections, it can be seen that a pole $q = 1$ appears in the denominator of the closed loop transfer function for the second data section which has a disturbance model with random walk dynamics. This results in a drifting process response, which is clearly unacceptable. Table 3.1 shows that the performance index calculated for the 2nd section of data is much greater than one and this further confirms that a very poor performance has been achieved by this benchmark control in this data section. Once the first section benchmark is found to be unsuitable, one has to search an alternative time invariant benchmark, which will be discussed in the next chapter.

Table 3.1: Type-B benchmark (with 1st section of the data as benchmark control)

data section, (i)	time (mins)	$\sigma^2_{mv(1,i)}$	$\sigma^2_y(t)$	$\eta(t)$
1st	1-2500	1.1203	1.6985	0.6596
2nd	2501-3500	14.7299	2.0911	7.0442
3rd	3501-5000	1.7290	1.9149	0.9029
4th	5001-7000	1.7581	2.6432	0.6651

Now, as an illustrative example, let's consider choosing the minimum variance control of the second section as a benchmark. If the minimum variance control of the second data section is used as the benchmark to evaluate controller performance over the entire time period, we need to check the suitability of this benchmark control when applied to other sections of data.

From eqn. (3.46), the closed-loop response of the benchmark control $Q^{(2)}_{mv}(q^{-1})$ to the 1^{st}, 3^{rd} and 4^{th} sections of data can be expressed as

$$y_t^{(1)} = \frac{1-q^{-1}}{1-0.67q^{-1}}(1+0.6q^{-1}+0.6q^{-2}+0.6q^{-3}) \qquad t < 2501 \qquad (3.50)$$

$$y_t^{(3)} = \frac{1-q^{-1}}{1-0.87q^{-1}}(1+0.6q^{-1}+0.6q^{-2}+0.6q^{-3}) \qquad 3501 \le t < 5001 \qquad (3.51)$$

$$y_t^{(4)} = \frac{1-q^{-1}}{1-1.07q^{-1}+0.268q^{-2}}(1+0.6q^{-1}+0.6q^{-2}+0.6q^{-3}) \qquad 5001 \le t \le 7000 \qquad (3.52)$$

From eqn. (3.50) to eqn. (3.52), it can be seen that the minimum variance control of the second section of disturbance is a suitable benchmark for the disturbance dynamics in the 1st, 3rd and 4th data sections. However, there is an integral action in the minimum variance controller as it can regulate the random walk disturbances. This integral action will inflate the achievable variance.

Table 3.2 reveals the inflation in variance due to the integrator of the benchmark control of the second section of data when applied to other sections of data. This results in high performance indices obtained in all the data sections. Therefore, according to this minimum variance control benchmark, one can see that the actual controller is very close to the benchmark in all sections. In other words, the existing controller is optimum among the class of controllers that have an integrator in regulating all four different disturbance dynamics.

It should be noted that if the minimum variance control of either the 3rd or 4th data section is chosen as a benchmark to evaluate performance of the entire process, the closed-loop response to the second section of data will also have a pole $q = 1$ appearing in the denominator of the closed loop transfer function and this results in an unstable response which is not acceptable as we have seen in the first scenario.

From this simulation example, it can be seen that for a time-varying process with at least one of its disturbance models having random walk dynamics, only the minimum variance control for the sections of data with random walk dynamics can be used as benchmark control to evaluate controller performance over the entire time period. However, this/these data section(s) might not be representative of the process operation as is required by type-B benchmark. Therefore, type-B benchmark might not always be appropriate and another benchmark has to be considered which would be suitable for regulatory performance of different types of disturbance dynamics within a process. This will be the focus in the next chapter.

Table 3.2: Type-B benchmark (with 2nd section of the data as benchmark control)

data section, (i)	time (mins)	$\sigma^2_{mv(2,i)}$	$\sigma^2_y(t)$	$\eta(t)$
1st	1-2500	1.6087	1.6985	0.9471
2nd	2501-3500	2.0800	2.0911	0.9947
3rd	3501-5000	1.8618	1.9149	0.9723
4th	5001-7000	2.5622	2.6432	0.9694

3.5 Industrial Application

The time-invariant type-B benchmark is used to assess the control loop performance in a Sulphur Recovery Unit (SRU) in Syncrude Canada Ltd. The general description and the control objective of this unit have been discussed in the last chapter. However, the output data of the PID controller that was used before the installation of the adaptive controller in the sulphur recovery unit is considered in this case study. Type-B benchmark is applicable to this data set since the actual control is time invariant.

A total of 740 data points are used for this case study and the data is assumed to contain a representative sample of normal process operations. The results obtained from the analysis of the control loop performance are applicable to the time over which the data was collected. The sampling interval for the data is 1 minute and the time delay of the process including zero-order-hold is estimated to be no less than 2. A time delay 2 will be used in the following discussions. The plot of the operating data is shown in Figure 3.2.

It is clear from Figure 3.2 that a major disturbance was affecting the process for a time period and the process and/or disturbance dynamics is clearly time varying. It is believed that the time-varying nature of the process data is due to time-variant disturbance dynamics. It is also believed the significant variation (nonstationary behavior) of data is not due to control tuning or instrumentation problem as the loop response was fairly stationary for most of other time period.

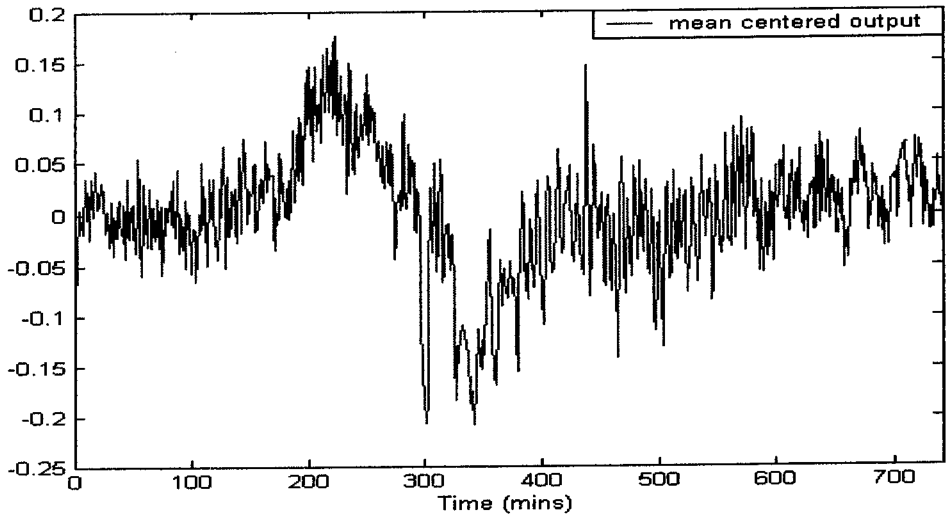


Figure 3.2: Plot of the operating data for the PID controller

The data set is divided into three sections according to the observed data trend as shown in Figure 3.3. The first section represents the data sampled from time 0 to 179 minutes, the second section is between 180 minutes and 400 minutes while the third data set is sampled from 401 minutes to 740 minutes.

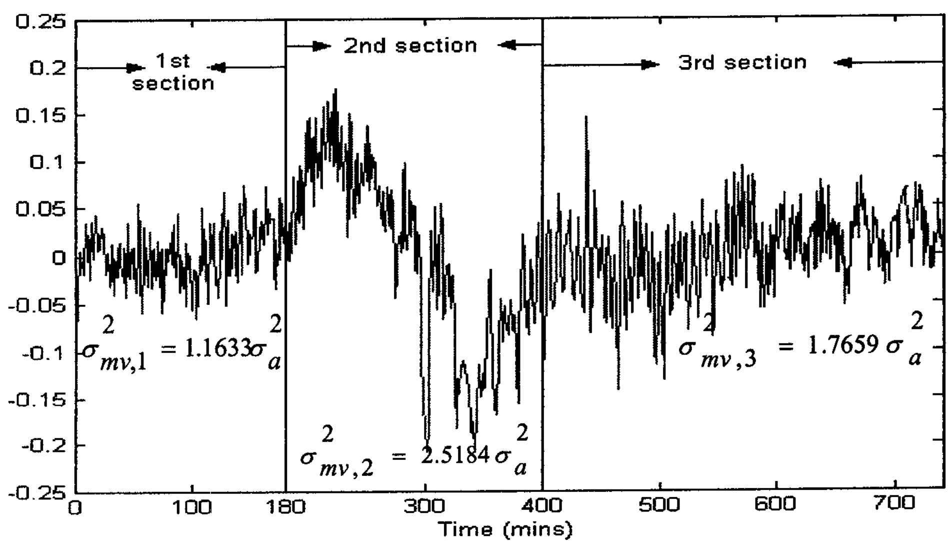


Figure 3.3: Data segmentation

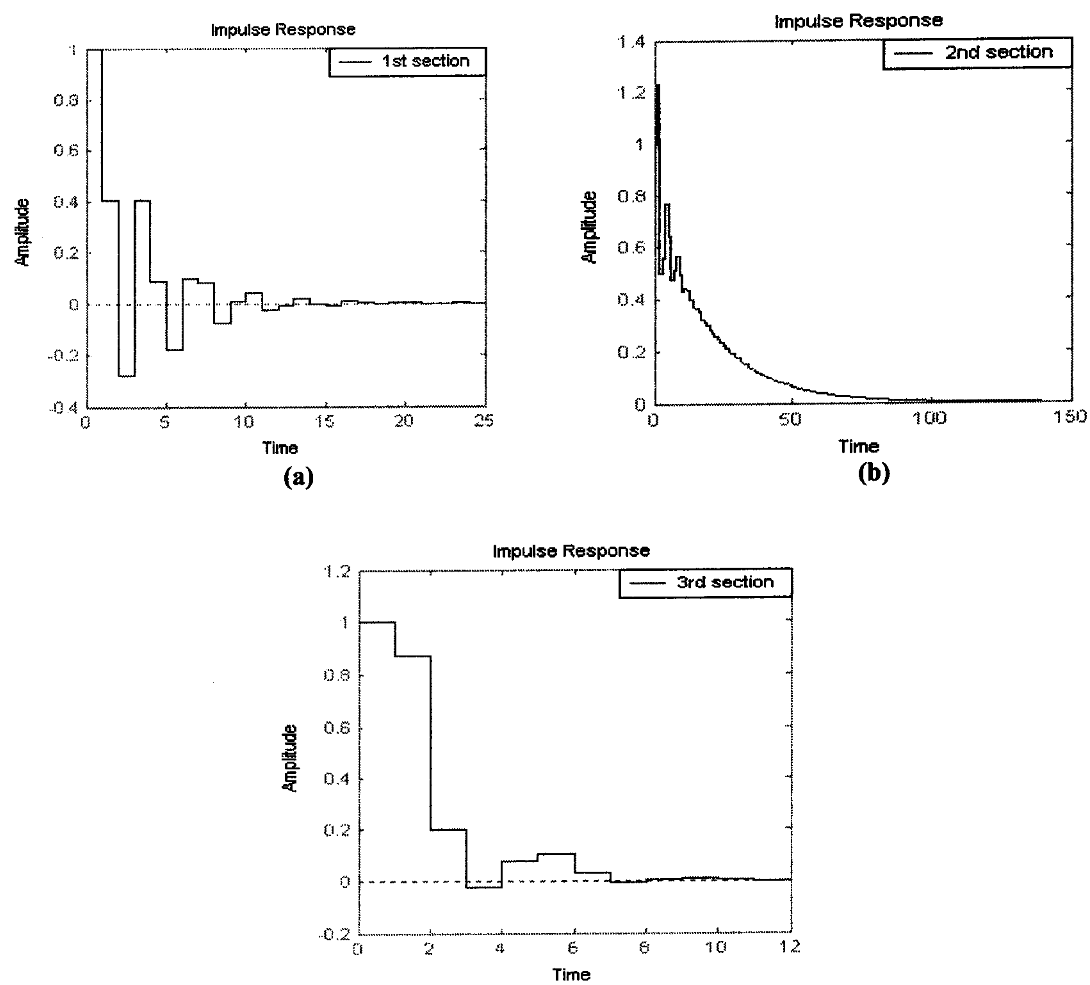
Let the 3rd section of data be the "routine" disturbance affecting the process and thus the minimum variance controller of this section should be chosen as the benchmark. Applying time-invariant performance assessment algorithm to each section of data yields the minimum variance for corresponding sections as shown in Fig 3.3. These minimum variances can only be achieved by (piece-wise constant) time variant minimum variance control. However, for the time-invariant type-B benchmark, only one of the lower bounds can be possibly achieved. Therefore, for this example, the minimum variance control of the 3rd section of data is considered as benchmark control and its closed-loop response to other sections of data are calculated and the results are presented in Table 3.3. The closed-loop impulse response of the benchmark control to the three data sections are shown in Figure 3.4

Table 3.3 shows that the performance index of the 1st section is greater than one. This indicates that the benchmark control has a relatively poor performance in regulating the 1st section of disturbance, and in fact this benchmark controller is poorer than the existing controller in regulating the disturbance occurring in the first section.

From Figure 3.4a, it can be seen that a settling time of 12 minutes is achieved by the benchmark control to the 1st section of disturbance while Figure 3.4b shows that the settling time to the disturbance dynamics in the 2nd section of data is 70 minutes. The settling time of the minimum variance control benchmark to the disturbance dynamics in the 3rd section of data is 7 minutes. Though stable responses have been obtained in all the data sections, the long settling time of 70 minutes to the abnormal disturbance in the second section of data is not acceptable. It is required that the closed-loop settling time when significant upset occurs should be approximately 5 minutes, which is the same as the open-loop settling time. Therefore, it is necessary to obtain another time-invariant benchmark that would give suitable regulatory performance for all the disturbance dynamics occurring in a process, which will be discussed in the next chapter.

Table 3.3: Type-B benchmark (with 3rd section of the data as benchmark control)

data section, (i)	time (mins)	$\sigma_{mv(3,i)}$	$\sigma^2_{mv(3,i)}$	$\sigma_y(t)$	$\sigma^2_y(t)$	$\eta(t)$
1st	0-179	1.2132	1.4718	1.1446	1.3102	1.1234
2nd	180-400	2.7220	7.4095	3.1733	10.0696	0.7358
3rd	401-740	1.3289	1.7659	1.3510	1.8251	0.9676



(c)

Figure 3.4: Impulse response for the three data sections with 3rd data section as the benchmark control

3.6 Conclusion

Type-B benchmark has been discussed in this chapter. For processes with piecewise constant disturbance models, type-B benchmark refers to a minimum variance control that corresponds to one representative disturbance model of user's choice. An example has been given to show that type-B benchmark is useful when minimization of a particular type of disturbances amongst other forms of disturbance dynamics affecting the process is of the only interest. In using type-B benchmark, it should be verified if it is suitable for other sections of data when abrupt changes of disturbance occur. If type-B benchmark is not appropriate, there is a need to obtain another form of time-invariant benchmark that will be suitable for the various types of disturbance dynamics occurring within the process.

Chapter 4

Performance Assessment of Processes with Abrupt Changes of Disturbances – Type-C benchmark

Abstract

The objective of this chapter is to develop another time-invariant benchmark that is able to regulate different disturbance dynamics within a given process. In Chapter 3, it has been shown that minimum variance control of one section of data may not be suitable for other sections of data if disturbance dynamics changes significantly. Thus, we need to discuss an alternative time-invariant minimum variance control benchmark that can optimize overall performance of these time-variant processes. It is shown that this performance benchmark that is referred to as the type-C benchmark may be found from routine operating data through some time series analysis and optimization technique. The proposed performance assessment method is illustrated by a simulated example, and a case study on an industrial process is presented.

The main contributions of this chapter include (1) a systematic solution of (optimal) type-C benchmark; previous solution of type-C benchmark is based on a simulation method; (2) establishment of a general computation framework applicable to any change of disturbance dynamics; previous work is limited to the assumption that only two different disturbance dynamics affect the process; (3) a detailed industrial case study.

4.1 Introduction

It has been shown that minimum variance control of one data section may not be appropriate for the whole time period of the process operation in the presence of time-variant disturbance dynamics. For such time-variant processes to be controlled by time-invariant controllers, it is important to obtain or search for a suitable time-invariant control benchmark that can optimize their overall performance while only routine operating data can be used. A section of data or disturbance may be more representative of process operation and a focus on the control of this section of disturbances is possibly most important. Other forms of disturbances within the process are said to be transient and it is not required to minimize the variances of these sections of data provided they could be controlled within some predefined performance targets or time limits (Huang, 1999).

Type-C benchmark is characterized by a controller that minimizes the variance of a most representative section of the disturbance subject to some predefined regulatory performance of other sections of disturbances within the process. That is, these abnormal disturbances or major upsets, which are typically transient, should be settled down along some user-defined reference trajectory or funnel. This is beneficial because it ensures that a specified performance of the controlled variable is achieved in such data sections (Qin and Badgewell, 1996). The desired closed-loop response to these sections of disturbances could be first order or a higher order with a pre-defined regulatory performance but it leaves out some free parameters that could be used to search for a time invariant controller that minimizes the variance of the most representative section of the disturbance (Huang, 1999). In general, there is a control performance requirement such as the settling time or time constant to regulate the abnormal disturbances of the process and it is required to find a time-invariant controller that can minimize the representative disturbance subject to the performance requirement of the sections of data with the significant upsets. Once the "constrained" benchmark control response is known, the control loop performance assessment can be carried out by comparing the benchmark control with the existing process output. This defines the Type-C benchmark.

It should be noted that like the type-B benchmark, a basic assumption that is also made with type-C benchmark is that process subjects to change of disturbance dynamics only.

4.2 The Optimization Problem - Type-C Benchmark

This section discusses the optimization problem involved in searching for a controller that minimizes the variance of the most representative section of the disturbance subject to some predefined performance requirement in regulating other major but transient upsets within the process. This constitutes an extension to the recent contribution by Huang (1999) in which the time-invariant control benchmark was obtained via an ad hoc method rather than through dynamic optimization methods.

For an illustration, we consider a process $q^{-d}\widetilde{T}(q^{-1})$ which is subject to two disturbance dynamics, $N_1(q^{-1})$ and $N_2(q^{-1})$. The change of disturbance dynamics takes place at $t=\theta$. It is assumed that the first disturbance dynamics is more representative section of the disturbance while the second section of disturbance corresponds to the significant but transient upset affecting the process. For type-C benchmark, it is required that the closed-loop response to this section of data is settled along some user-defined trajectory. Thus, the closed-loop response to the second section of disturbance can be written in a general form:

$$y_t^{(2)} = (\underbrace{f_0^{(2)} + f_1^{(2)}q^{-1} + \ldots + f_{d-1}^{(2)}q^{-(d-1)}}_{F_2(q^{-1})} + q^{-d}L_R(q^{-1}))a_t \tag{4.1}$$

where

$$F_2(q^{-1}) = f_0^{(2)} + f_1^{(2)}q^{-1} + \ldots + f_{d-1}^{(2)}q^{-(d-1)} \tag{4.2}$$

$L_R(q^{-1})$ is a stable and proper transfer function that depends on the feedback controller. However, $L_R(q^{-1})$ is not exclusively determined but it is defined in such a way that it leaves some free parameters that can be used to find a controller that optimizes the regulatory performance of the first section of disturbance. Here, let us consider the desired closed-loop response to the second section of disturbance is first order with time constant, τ. The transfer function $L_R(q^{-1})$ may be written as

$$L_R(q^{-1}) = \frac{\psi}{1 - \xi q^{-1}} \tag{4.3}$$

From eqn. (4.3), is left as the free parameter to be determined. ξ can be obtained from the desired time constant, τ via

$$\xi = \exp(-T_s / \tau) \tag{4.4}$$

where T_s is the sampling period of the data.

In general, $y_t^{(2)}$ in eqn. (4.1) is an achievable process response. We can find a time-invariant controller $Q_2(q^{-1})$ that will give the closed-loop response

$$y_t^{(2)} = (F_2(q^{-1}) + q^{-d} L_R(q^{-1}))a_t = \frac{N_2(q^{-1})}{1 + q^{-d}\widetilde{T}(q^{-1})Q_2(q^{-1})} a_t \tag{4.5}$$

Similarly, we can write the closed-loop response for this controller $Q_2(q^{-1})$ to the first section of disturbance:

$$y_t^{(1)} = \frac{N_1(q^{-1})}{1 + q^{-d}\widetilde{T}(q^{-1})Q_2(q^{-1})} a_t \tag{4.6}$$

From eqn. (4.5) and eqn. (4.6), we have

$$y_t^{(1)} = \frac{N_1(q^{-1})}{N_2(q^{-1})}\left(F_2(q^{-1}) + q^{-d} L_R(q^{-1})\right)a_t \tag{4.7}$$

The objective is to minimize the variance of the first section of data set $y_t^{(1)}$ with the time-invariant controller $Q_2(q^{-1})$ subject to a predefined performance of the second section of disturbance. Thus, we need to find the free parameter in the transfer function $L_R(q^{-1})$ that will minimize the variance of $y_t^{(1)}$. This is solved as an optimization problem using the Lyapunov equation solver and unconstrained non-linear minimization method:

$$\mathrm{var}(y_t^{(1)}) = \mathrm{var}\left(\frac{N_1(q^{-1})}{N_2(q^{-1})} \left(F_2(q^{-1}) + q^{-d} L_R(q^{-1})\right) a_t \right) \tag{4.8}$$

$$\psi = \arg_\psi \min\left(\mathrm{var}(y_t^{(1)})\right) \tag{4.9}$$

4.2.1 Solution of the optimization problem

The approach used in solving the optimization problem is to convert the transfer function model in eqn. (4.7) to a state space description:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{4.10}$$

where the matrix A in eqn. (4.10) is stable.

The variance of $y_t^{(1)}$ is obtained by calculating the H_2 norm of the discrete transfer function:

$$G(z) = C(ZI - A)^{-1} B + D = G(q^{-1}) \tag{4.11}$$

The H_2 norm represents the 2-norm of a stable, strictly proper system matrix. A discrete time system is said to be stable when the magnitude of all its poles are less than one.

The H_2 norm of the system is given by

$$\left\|G(q^{-1})\right\|_2^2 = trace(D^T D + B^T L_o(\psi)B) \tag{4.12}$$

where L_o is the observability gramian.

The H_2 norm of a system follows from the solution to the Lyapunov equation

$$AL_o(\psi)A^T - L_o(\psi) + C^T(\psi)C(\psi) = 0 \tag{4.13}$$

$L_o(\psi) = dlyap(A, C^T(\psi)C(\psi))$ in MATLAB environment solves the discrete Lyapunov equation. $L_o(\psi)$ is returned as a function of the free parameter ψ.

Once $L_o(\psi)$ is obtained, the optimal value of the free parameter ψ is calculated via the unconstrained non-linear minimization method "fminsearch" command in MATLAB. This optimization method makes use of the Nelder-Mead simplex (direct search) method to evaluate the free parameter ψ by minimizing the H_2 norm i.e.

$$\psi_{\min} =_{\min} (D^T D + B^T L_o(\psi)B) \tag{4.14}$$

Once the transfer function $L_R(q^{-1})$ is completely determined, the control loop response of the benchmark control $Q_2(q^{-1})$ to the disturbances occurring in both sections of data can be calculated from eqn. (4.1) and eqn. (4.7). It has been discussed that $F_2(q^{-1})$ in eqn. (4.2) can be estimated from time series analysis of routine operating data and

$$\frac{N_1(q^{-1})}{N_2(q^{-1})} = \frac{H_{cl}^{(1)}(q^{-1})}{H_{cl}^{(2)}(q^{-1})} \tag{4.15}$$

can also be estimated from closed-loop operating data. Therefore, both eqns. (4.1) and (4.7) can be estimated from routine operating data plus predefined performance requirements for regulating the major but transient upsets. Once the benchmark control response is obtained, the performance measure of the control loop can be determined by comparing the benchmark control response with the existing process output.

Up to now, we have only considered the process subject to two different disturbance dynamics. However, if there are more than two sections of varying disturbance dynamics, then the suitability of type-C benchmark to other sections of data has to be determined.

In general, for a process $q^{-d}\widetilde{T}(q^{-1})$ that is subject to "n" different disturbance dynamics $N_1(q^{-1}), N_2(q^{-1}), N_3(q^{-1}), \ldots, N_n(q^{-1})$, the process data can be segregated into piecewise stationary sections Y_i, $(i = 1, 2, 3, \ldots, n)$. If the j^{th} disturbance dynamics is the most representative section of the disturbance while the k^{th} section of disturbance corresponds to the significant but transient upset affecting the process, the objective is to minimize the variance of the j^{th} section of the data set, subject to a pre-specified performance for regulating the abnormal disturbance occurring in Y_k.

From eqn. (4.7), the closed-loop response of the benchmark control $Q_k(q^{-1})$ to the representative disturbance occurring in Y_j can be calculated as

$$y_t^{(j)} = \frac{N_j(q^{-1})}{N_k(q^{-1})}\left(F_k(q^{-1}) + \frac{\psi}{1-\xi\, q^{-1}} q^{-d}\right)a_t \qquad (4.16)$$

and from eqn. (4.1), the benchmark performance to the abnormal disturbance occurring in the k^{th} section of data can be written in a general form as

$$y_t^{(k)} = (F_k(q^{-1}) + \frac{\psi}{1-\xi q^{-1}} q^{-d}) a_t \tag{4.17}$$

where

$$F_k(q^{-1}) = f_0^{(k)} + f_1^{(k)} q^{-1} + \ldots + f_{d-1}^{(k)} q^{-(d-1)} \tag{4.18}$$

We can also write the closed-loop response for the controller $Q_k(q^{-1})$ to the i^{th} section of disturbance as

$$y_t^{(i)} = \frac{N_i(q^{-1})}{1 + q^{-d}\widetilde{T}(q^{-1}) Q_k(q^{-1})} a_t \qquad i \neq j, \ i \neq k \tag{4.19}$$

For the time-invariant controller $Q_k(q^{-1})$, the closed-loop response to the k^{th} section of disturbance can be further expressed as

$$y_t^{(k)} = (F_k(q^{-1}) + q^{-d} \frac{\psi}{1-\xi q^{-1}}) a_t = \frac{N_k(q^{-1})}{1 + q^{-d}\widetilde{T}(q^{-1}) Q_k(q^{-1})} a_t \tag{4.20}$$

From eqn. (4.19) and eqn. (4.20), the closed-loop response of the controller $Q_k(q^{-1})$ to the disturbance occurring in the i^{th} section of data can be obtained as

$$y_t^{(i)} = \frac{N_i(q^{-1})}{N_k(q^{-1})} \left(F_k(q^{-1}) + \frac{\psi}{1-\xi q^{-1}} q^{-d} \right) a_t \qquad i \neq k \tag{4.21}$$

Therefore, in addition to obtaining the benchmark performance to the representative disturbance and the major upset occurring in the process, we can also check the suitability of the type-C benchmark when it is applied to the entire duration of the process.

4.3 Evaluation via Simulation Example and An Industrial Application

4.3.1 Simulation example

Huang (1999) gave an example to demonstrate the optimization problem involved in type-C benchmark. In his simulation, an estimated solution was found by direct observation of curvature rather than the use of dynamic optimization methods. However, this simulation shows that the type-C benchmark may be found from routine operating data through some time series analysis and optimization techniques.

The process transfer function is given by

$$q^{-d}\tilde{T}(q^{-1}) = q^{-4}\frac{0.33}{1-0.67q^{-1}} \tag{4.22}$$

A Dahlin controller is used to control the process and it is given by

$$Q(q^{-1}) = \frac{0.7-0.47q^{-1}}{0.33-0.10q^{-1}-0.23q^{-4}} \tag{4.23}$$

In this example, a total of 4500 data points are considered and it is assumed that the process is affected by three different disturbance dynamics with an abrupt change of disturbance occurring at the 2001^{st} data point and 3001^{st} data point. The disturbance dynamics are given by:

$$N_1(q^{-1}) = \frac{1-0.4q^{-1}}{1-0.67q^{-1}} \qquad 1 \le t < 2001 \tag{4.24}$$

$$N_2(q^{-1}) = \frac{1-0.4q^{-1}}{1-q^{-1}} \qquad 2001 \le t < 3001 \tag{4.25}$$

$$N_3(q^{-1}) = \frac{1-0.4q^{-1}}{1-0.87q^{-1}} \qquad 3001 \le t \le 4500 \tag{4.26}$$

A plot of the disturbance dynamics is shown in Figure 4.1.

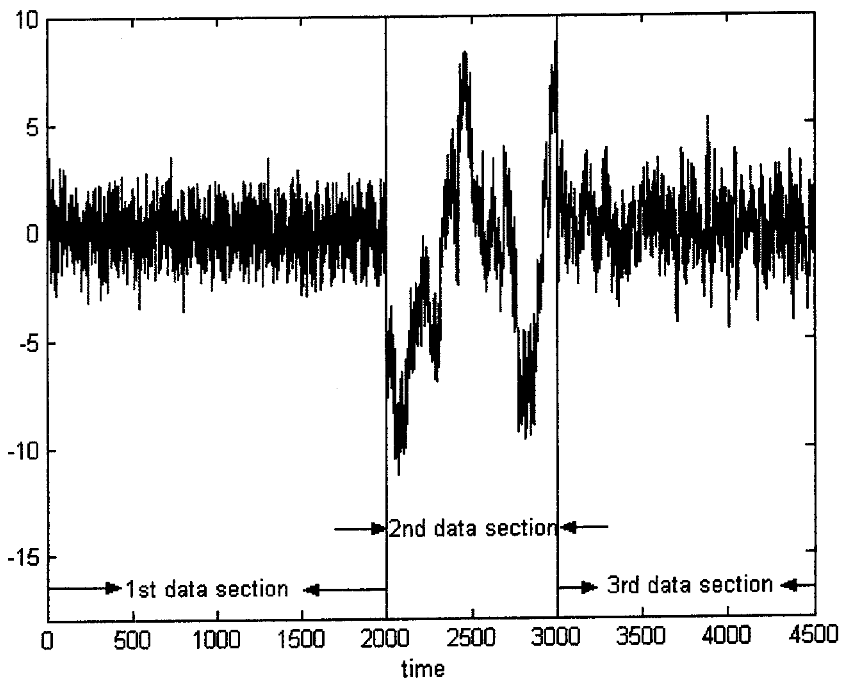


Figure 4.1: Plot of the three different disturbance dynamics affecting the process

For the first 2000 data points, the minimum variance term can be calculated as

$$F_1(q^{-1}) = 1 + 0.27q^{-1} + 0.18q^{-2} + 0.12q^{-3} \tag{4.27}$$

The minimum variance term for the second section of data after the first abrupt change of disturbance can be calculated as

$$F_2(q^{-1}) = 1 + 0.6q^{-1} + 0.6q^{-2} + 0.6q^{-3} \tag{4.28}$$

and the minimum variance term for the third disturbance dynamics can be calculated as:

$$F_3(q^{-1}) = 1 + 0.47q^{-1} + 0.41q^{-2} + 0.36q^{-3} \tag{4.29}$$

With type-C benchmark, the more representative disturbance that requires high-performance control is determined. The section of disturbance that constitutes the abnormal disturbance is considered to be typically transient and it is to be controlled within some predefined trajectory or funnel. For example, let us assume that the disturbance appearing in the time $0 \le t \le 2000$ is the most representative section of data. The disturbance dynamics in the second section of data is considered to be the abnormal disturbance. The type-C benchmark minimizes the variance of the first section of data subject to some predefined performance in regulating the second section of disturbance. This benchmark control is applied to the third section of data to check its suitability.

It follows from eqn. (4.1) that the closed-loop response to the second section of disturbance can be written as

$$y_t^{(2)} = (1 + 0.6q^{-1} + 0.6q^{-2} + 0.6q^{-3} + L_R(q^{-1})q^{-4})a_t \tag{4.30}$$

Let us consider that the closed-loop response to the second section of the disturbances be first order with time constant τ. Thus, the transfer function $L_R(q^{-1})$ may be expressed as

$$L_R(q^{-1}) = \frac{\psi}{1 - \xi\, q^{-1}} \tag{4.31}$$

where is the free parameter, and

$$\xi = \exp(-T_s / \tau) \tag{4.32}$$

From eqn. (4.16), the closed-loop response to the first section of data can be written as

$$y_t^{(1)} = \frac{N_1(q^{-1})}{N_2(q^{-1})}(1 + 0.6q^{-1} + 0.6q^{-2} + 0.6q^{-3} + \frac{\psi}{1-\xi\, q^{-1}} q^{-4})a_t$$

$$= \frac{1-q^{-1}}{1-0.67q^{-1}}(1 + 0.6q^{-1} + 0.6q^{-2} + 0.6q^{-3} + \frac{\psi}{1-\xi\, q^{-1}} q^{-4})a_t \qquad (4.33)$$

The free parameter can be solved via the optimization problem

$$= \arg_{\psi} \min\left(\operatorname{var}\left(\frac{1-q^{-1}}{1-067q^{-1}}(1 + 0.6q^{-1} + 0.6q^{-2} + 0.6q^{-3} + \frac{\psi}{1-\xi\, q^{-1}} q^{-4})a_t \right)\right) \qquad (4.34)$$

The general methodology that is used for the solution of the optimization problem is given in section 4.2.1.

Having obtained the closed loop response to the *1*st and 2^{nd} sections of data, the closed loop response of the benchmark control $Q_2(q^{-1})$ to the 3^{rd} data section is calculated from eqn. (4.21) and is given by

$$y_t^{(3)} = \frac{N_3(q^{-1})}{N_2(q^{-1})}(1 + 0.6q^{-1} + 0.6q^{-2} + 0.6q^{-3} + \frac{\psi}{1-\xi\, q^{-1}} q^{-4})a_t$$

$$= \frac{1-q^{-1}}{1-0.87q^{-1}}(1 + 0.6q^{-1} + 0.6q^{-2} + 0.6q^{-3} + \frac{\psi}{1-\xi\, q^{-1}} q^{-4})a_t \qquad (4.35)$$

The optimal value of is calculated for different τ and the corresponding benchmark variance $\min(\operatorname{var}(y_t^{(1)}))$ is calculated. The variance of the closed-loop response of the chosen benchmark control to the second and third sections of disturbances $var\ (y_t^{(2)})$ and $var\ (y_t^{(3)})$ are also calculated and the results are summarized in Table 4.1.

Let us consider, for an example, that the specified settling time to the abnormal disturbance in the middle section of data is 15 minutes. It can be seen from eqn. (4.30), eqn. (4.33) and eqn. (4.35) that stable responses have been obtained for all the sections of data. The closed-loop responses of the three sections of data to the benchmark control are given in Figure 4.2. Figure 4.2(a) shows that the settling time to the 1st section of disturbance is 18 minutes while Figure 4.4(b) gives the settling time to the abnormal disturbance to be 15 minutes. Figure 4.4(c) shows that the settling time of the type-C benchmark control to the 3rd section of disturbance is 30 minutes. This result shows that the benchmark control has achieved the closed-loop settling time that has been specified for the regulation of the abnormal disturbance occurring in the second section of data while minimizing the variance of the most representative section of the disturbance in the process operation. Thus, type-C benchmark is a suitable time-invariant control benchmark that can "optimize" overall performance of time-variant processes in the presence of significant changes in disturbance dynamics.

Table 4.1: Benchmark variance and optimal values of ψ for user-specified ξ (simulation example)

τ	ξ	ψ_{opt}	$\min(var(y_t^{(1)}))$	$var(y_t^{(2)})$	$var(y_t^{(3)})$
0	0	0.2594	1.5281	2.1473	1.8532
1.5	0.5134	0.3926	1.4228	2.2893	1.8382
3.0	0.7165	0.4453	1.3427	2.4875	1.8225

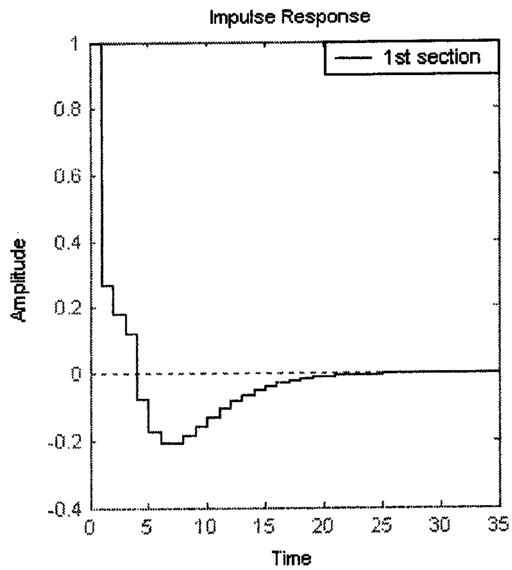


Figure 4.2(a)

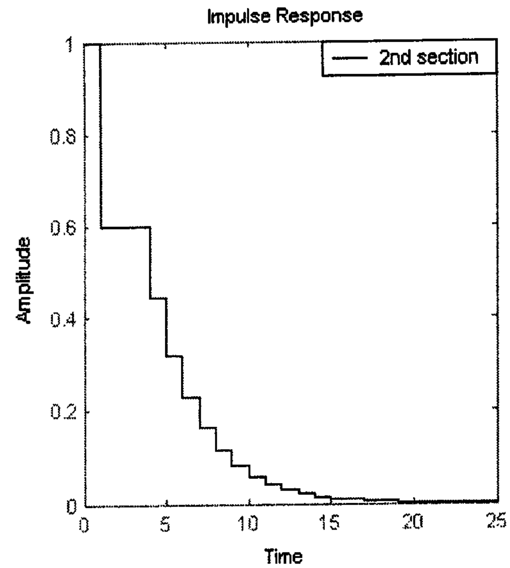


Figure 4.2(b)

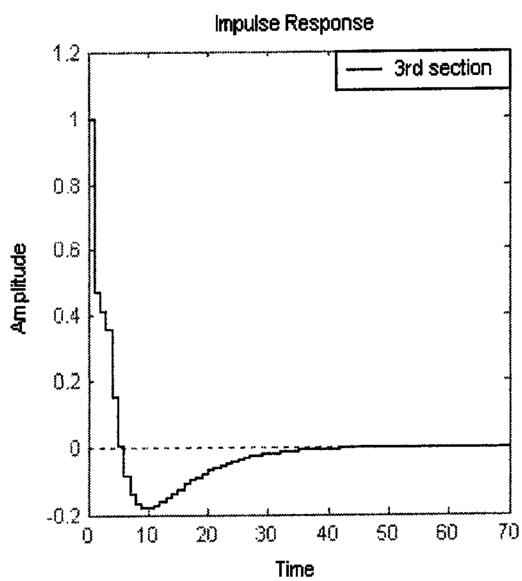


Figure 4.2(c)

Figure 4.2: Impulse response for the three data sections with type-C benchmark (simulation example)

4.3.2 Industrial application

For this case study, the time-invariant type-C benchmark is applied to monitor the control loop performance of the Sulphur Recovery Unit (SRU) described in chapter 2. The output data of the PID controller that was considered in chapter 3 is also used in this case study. The data segmentation, which is according to the trend observed in the process data, is shown in Figure 4.3.

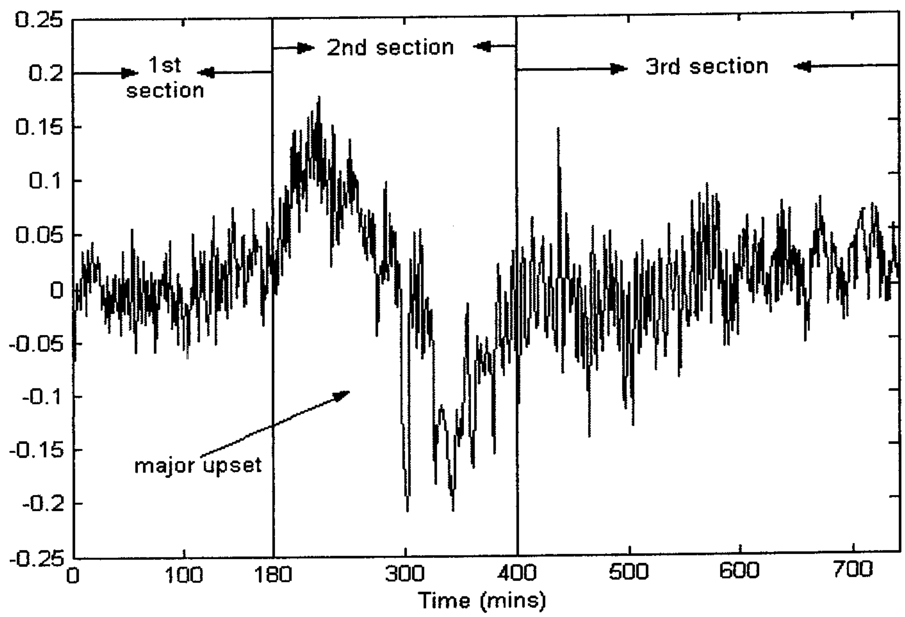


Figure 4.3: Data segmentation showing abnormal disturbance in the 2nd section of data

The disturbance dynamics in the 1st and 3rd sections of data are stable disturbances and for this example, the 3rd section of disturbance is assumed to be more representative of the process operation. A significant change is observed at the sample time 180 minutes and the disturbance that occurs from sampling time 180 minutes to 400 minutes is treated as a abnormal disturbance occurring within the process. The type-C benchmark is applied by minimizing the variance of the 3rd section of disturbance subject to some pre-defined performance in regulating the disturbance occurring in the 2nd section of data. That is, it is required that the abnormal disturbance be settled down along some user defined trajectory or time limits. In this example, the time constant of the open-loop system is approximately 1.0 minute. Let us consider that the desired closed-loop settling time when significant upset occurs be 5 minutes. The benchmark control is applied to the 1st section to check its suitability.

The results of performance assessment using type-C benchmark are presented in Table 4.2. Figure 4.4 shows the closed-loop impulse responses for the three data sections and it can be seen that the benchmark control gives stable responses throughout the entire process. Figure 4.4(a) shows that the settling time to the 1st section of disturbance is 5 minutes. It can be seen from Figure 4.4(b) that the settling time to the abnormal disturbance is 5 minutes. Figure 4.4(c) shows that the settling time of the benchmark control to the 3rd section of disturbance is 11 minutes. This result indicates that the type-C benchmark has achieved the desired regulatory performance which requires that the closed-loop settling time when significant upset occurs (in the 2nd data section) should be same as the open-loop settling time of 5 minutes. Table 4.2 shows that the variance of the "routine" disturbance in the 3rd section of data has also been minimized subject to the "constraint" in regulating the performance of the significant upset in the middle section of the data. This can be seen in the high performance index of 0.9262 obtained in the 3rd section of data. From Table 4.2, the performance index of the 1st section, which is greater than one, is an indication that the actual controller has a better performance than the benchmark control in regulating the 1st section of disturbance. The low performance index obtained in the 2nd data section shows that a significant improvement could be achieved by re-tuning the PID controller without affecting the performance specifications for regulating the abnormal disturbance. Therefore, in this example, the type-C benchmark is found to be suitable for all the sections of data and performance specification for regulating the abnormal disturbance has been achieved.

Table 4.2: Benchmark variance and values of ψ for user-specified ξ or (τ)

	$\tau = 1.0$ $\xi = 0.3679$ $\psi_{opt} = 0.3759$			
data section, (i)	time (mins)	$\sigma^2_{mv(3,i)}$	$\sigma^2_y(t)$	$\eta(t)$
1st	1-179	1.5039	1.3138	1.1447
2nd	180-400	2.8137	11.7941	0.2386
3rd	401-740	1.9631	2.1195	0.9262

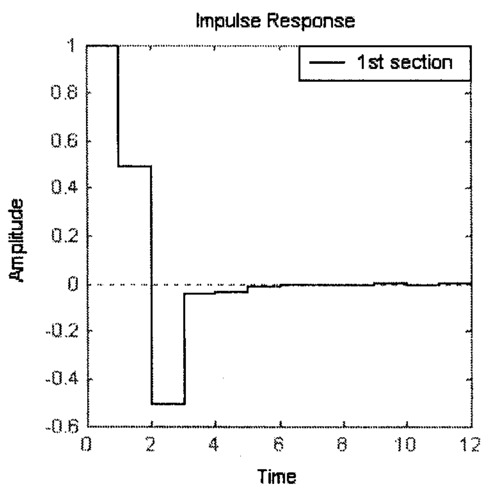


Figure 4.4(a)

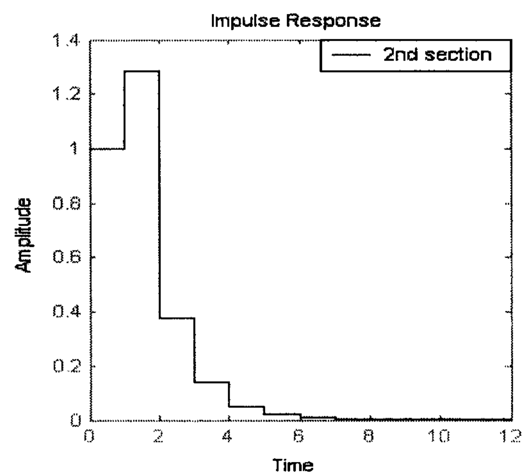


Figure 4.4(b)

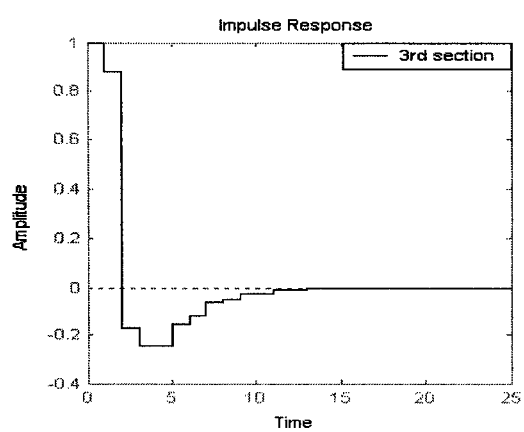


Figure 4.4(c)

Figure 4.4: Impulse response for the three data sections with type-C benchmark

4.4 Conclusion

Control-loop performance assessment of time-variant processes using type-C benchmark has been discussed. Type-C benchmark is a time-invariant minimum variance control that can minimize one disturbance dynamics subject to some pre-specified performance requirement on another disturbance dynamics. The results have shown that if type-B benchmark is not appropriate for processes with significant change in disturbance dynamics, then type-C benchmark can be used as an alternative performance benchmark. The results also provide guidelines for using routine operating data through time series analysis and optimization techniques to obtain suitable time-invariant control benchmark that can "optimize" overall performance of time-variant processes.

Chapter 5

Feedforward and Feedback Controller Performance Assessment of LTV systems

Abstract

In this chapter, the feedforward/feedback control loop performance assessment of Linear Time Variant (LTV) control systems is discussed. The time varying minimum variance feedfoward plus feedback control is used as the benchmark for performance assessment of the time-variant control loop and the non-commutativity associated with manipulation of LTV transfer functions is taken into consideration in the analysis of feedforward plus feedback control systems. The feedforward/feedback minimum variance control benchmark of the control loop can be obtained using routine operating data and time-series analysis. The proposed performance methodology is illustrated through a simulated stirred tank reactor and applied to a case study on a feedforward plus feedback control scheme in the sulphur recovery process under adaptive control, which is clearly an LTV control system. The study on LTV feedforward and feedback control performance assessment has not been done in the literature and this chapter constitutes a completely new contribution in this area.

5.1 Introduction

Performance assessment of LTV feedback control loops has been discussed in the last three chapters. This includes the use of time-invariant and time-variant minimum variance control as the benchmark to evaluate performance of the control loop and it has been seen that the proposed performance assessment methodology is an efficient and a convenient technique for monitoring industrial processes. Although minimum variance control is not desirable for practical implementation due to its poor robustness to modeling errors in addition to other physical constraints on the process, it provides a lower bound on the process variance that serves as an appropriate benchmark against which the control loop performance can be evaluated.

The performance measure of control loops gives control engineers insight into potentials of improving control system performance. If the controller performance does not satisfy requirement but is already close to the minimum variance control, then further tuning of the controller will not be useful (Huang et al., 2000). Substantial improvement is possible only by changing the control strategy such as addition of feedforward control, reducing dead time and/or reducing the disturbance in its source. The performance of control schemes is often enhanced by including feedforward elements and this is usually considered in two ways; a feedforward variable could be measured and used in the control scheme, or the potential benefit of implementing feed-forward control can be estimated. The latter is achieved by estimating the potential variance reduction for a prospective feedforward variable, which is not already incorporated in an existing control scheme.

Studies and discussions on the design of minimum variance feedback-only controllers can be found in Astrom (1970), Box and Jenkins (1976) and other references. Box and Jenkins (1976), and Sternad and Stoderstrom (1988) have also discussed the design of minimum variance feedforward and feedback controller. Stanfelj et al. (1993) presented a hierarchical method for monitoring and diagnosing the cause of poor performance of feedforward/feedback control systems using autocorrelation and cross correlation functions. Huang (1997) and Huang et al. (2000) have extended methods for performance assessment of multivariate feedback control systems to performance assessment of multivariate LTI feedback plus feedforward control systems using minimum variance control as the benchmark. Li and Evans (1997) and Huang (1999, 2002) have developed minimum variance control and performance assessment techniques for LTV feedback control loops respectively. This chapter is an extension of this method to LTV feedforward plus feedback control systems.

In this study, the analysis for the multiple-input single-output (MISO) feedforward and feedback control scheme is presented. The estimation of the lower bound of variance for each of the controller in the control scheme allows for the performance of the individual controllers to be assessed and it is shown that this can be obtained from the closed-loop routine operating data.

5.2 Analysis of LTV Feedforward and Feedback Control Systems

A review of the performance assessment of LTI processes with feedforward and feedback control scheme is presented as a preface to the analysis for LTV processes. Desborough and Harris (1992), Huang (1997), Kadali et al (1999), and Huang et al (2000) have reported that the closed-loop response to both unmeasured and measured disturbances can be modeled as:

$$y_t(q^{-1}) = G_a(q^{-1})a_t + \sum_{i=1}^{n} G_{i,b}(q^{-1})b_{i,t} \tag{5.1}$$

where n is the number of measured disturbance variables. y_t is the measured process variable and G_a and $G_{i,b}$ are proper and rational transfer functions. a_t, which is white noise, is the driving force for the unmeasured disturbances while $b_{i,t}$ represents the white noise sequence which is the driving force for the measured disturbance(s). The driving force $b_{i,t}$ can be generated by applying time series analysis to the feedforward variable $D_{i,t}$:

$$D_{i,t}(q^{-1}) = G_{i,m}(q^{-1})b_{i,t} \tag{5.2}$$

In this analysis, it is considered that there is no cross-correlation between the unmeasured disturbances and the measured disturbances. Therefore, for these time-invariant systems, eqn. (5.1) can be represented as a sum of impulse response coefficients

$$y_t = \left(f_0^{(a)} a_t + \ldots + f_{d-1}^{(a)} a_{t-(d-1)}\right) + \left(f_d^{(a)} a_{t-d} + \ldots\right) + \sum_{i=1}^{n}\left(f_0^{(b_i)} b_{i,t} + \ldots + f_{d-1}^{(b_i)} b_{i,t-(d-1)}\right) + \left(f_d^{(b_i)} b_{i,t-d} + \ldots\right) \tag{5.3}$$

where d represents the time delay of the process including zero-order-hold. Eqn. (5.3) can be further expressed as the sum of the contribution of the measured and unmeasured disturbances to the actual process variance:

$$y_t = \underbrace{e_t^u + \hat{e}_t^u}_{y_t^u} + \sum_{i=1}^{n} \underbrace{\left(e_t^{i,m} + \hat{e}_t^{i,m}\right)}_{y_t^{i,m}} \tag{5.4}$$

y_t^u is the contribution of unmeasured disturbances to the process variance while $\sum_{i=1}^{n} y_t^{i,m}$ is the contribution of measured disturbances to the process variance, and

$$\begin{aligned} e_t^u &= \left(f_0^{(a)} a_t + \ldots + f_{d-1}^{(a)} a_{t-(d-1)}\right) \\ \hat{e}_t^u &= \left(f_d^{(a)} a_{t-d} + f_{d+1}^{a} a_{t-(d+1)} + \ldots\right) \end{aligned} \tag{5.5}$$

$$\begin{aligned} e_t^{i,m} &= \left(f_0^{(b_i)} b_{i,t} + \ldots + f_{d-1}^{(b_i)} b_{i,t-(d-1)}\right) \\ \hat{e}_t^{i,m} &= \left(f_d^{(b_i)} b_{i,t-d} + f_{d+1}^{(b_i)} b_{i,t-(d+1)} + \ldots\right) \end{aligned} \tag{5.6}$$

The output under minimum variance feedback and feedforward can be expressed as:

$$y_t^{mv} = e_t^u + \sum_{i=1}^{n} e_t^{i,m} \tag{5.7}$$

The minimum variance feedforward and feedback control is calculated and used as the benchmark to evaluate the performance of feedforward and feedback controllers respectively. $\hat{e}_t^u$ represents the inflation to e_t^u due to non-optimal feedback control to the unmeasured disturbances and $\hat{e}_t^{i,m}$ is the inflation to $e_t^{i,m}$ due to non-optimal feedforward/feedback control of the measured disturbances. For a detailed discussion readers are referred to Huang (1997).

However, due to the fact that most processes have certain degree of time varying behavior, it is imperative to develop performance assessment methods for time-variant feedforward plus feedback control loops. For this class of time-variant systems, care has to be taken to the non-commutativity associated with manipulations of LTV transfer functions. An illustration has been given in chapter 2 (section 2.1) to show that the multiplication or division of two LTV polynomials, $u(q^{-1},t)$ and $v(q^{-1},t)$ is non-commutative, that is, $u(q^{-1},t)v(q^{-1},t) \neq v(q^{-1},t)u(q^{-1},t)$. This is important and has to be taken into consideration in calculating the minimum variance term when an LTV ARMA model is transferred to an LTV MA model. It is seen that pointwise multiplication may yield erroneous results especially when the parameter change for the plant or disturbance dynamics is relatively fast. Therefore, normal multiplication, which is non-commutative is recommended when handling LTV operators and is used in the analysis of variance for LTV feed-forward and feedback control systems.

In this chapter, a basic property of the backshift operator q^{-1} when it is multiplied by an LTV transfer function, $\lambda(q^{-1},t)$, for example, will be applied. See Huang (1999, 2002):

$$\begin{aligned} q^{-n}\lambda(q^{-1},t) &= \lambda(q^{-1},t-n)q^{-n} \\ q^{n}\lambda(q^{-1},t) &= \lambda(q^{-1},t+n)q^{n} \end{aligned} \tag{5.8}$$

Consider the LTV multiple-input single-output (MISO) process in Figure 5.1:

$$y_t = q^{-d}\widetilde{T}(q^{-1},t)u_t + N_a(q^{-1},t)a_t + N_b(q^{-1},t)D_t \tag{5.9}$$

y_t is the measured process variable, u_t is the manipulated variable and $\tilde{T} = K^{-1}(q^{-1},t)J(q^{-1},t)$ is the delay-free plant transfer function. $N_a = M^{-1}(q^{-1},t)L(q^{-1},t)$ represents the time-variant transfer function of the unmeasured disturbances while a_t is the "driving force" for realization of the unmeasured disturbances. $N_b D_t = X^{-1}(q^{-1},t)P(q^{-1},t)D_t$ is the effect of the measured feedforward variable, D_t, on the process variable. Note that $\tilde{T}, N_a, N_b$ given above are also known as LTV Box-Jenkins model, which are the most general linear dynamic models.

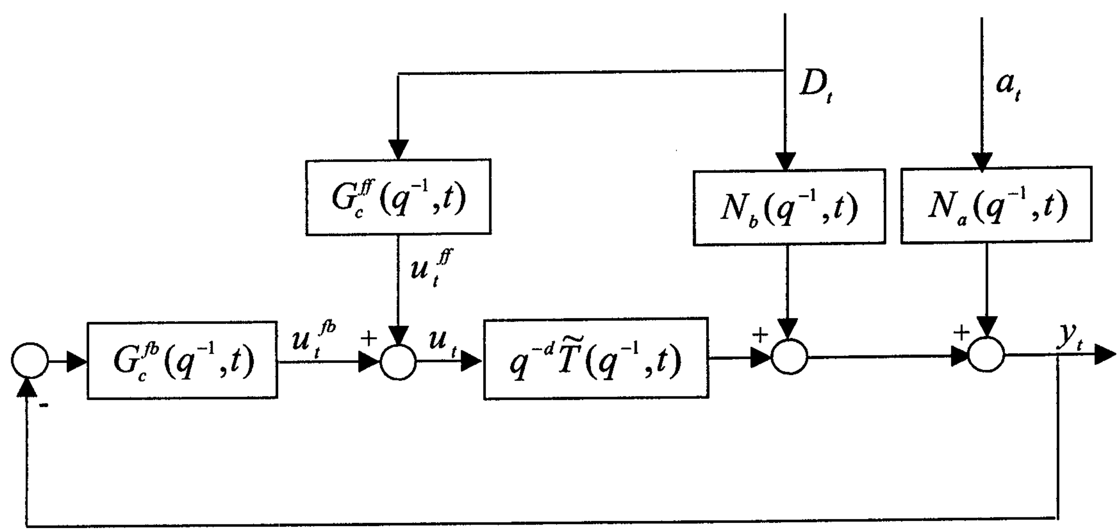


Figure 5.1: Schematic of time-variant MISO process under feedforward plus feedback control

For the LTV MISO process in eqn. (5.9), the closed loop response under LTV feedforward/feedback control can be represented by an LTV ARMAX model as will be seen in the following Lemma:

Lemma: The closed loop representation of an LTV Box-Jenkins model shown in Fig. 5.1 under feedback and feedforward control can be written as an LTV ARMAX model.

Proof:

The manipulated variable for the feedforward/feedback control system is the sum of the outputs from the feedback controller and the feedforward controller:

$$u_t = u_t^{fb} + u_t^{ff} \tag{5.10}$$

where $u_t^{fb} = -G_c^{fb}(q^{-1},t)y_t$ and $u_t^{ff} = G_c^{ff}(q^{-1},t)D_t$

Let us consider that the time-variant feedback and feedforward controllers are given by:

$$\begin{cases} G_c^{fb} = Z^{-1}(q^{-1},t)W(q^{-1},t) \\ G_c^{ff} = S^{-1}(q^{-1},t)N(q^{-1},t) \end{cases} \tag{5.11}$$

Substituting eqn. (5.10) and eqn. (5.11) in eqn. (5.9) yields

$$\begin{aligned} y_t = {} & [1+q^{-d}\widetilde{T}(q^{-1},t)G_c^{fb}(q^{-1},t)]^{-1}N_a(q^{-1},t)a_t + \\ & [1+q^{-d}\widetilde{T}(q^{-1},t)G_c^{fb}(q^{-1},t)]^{-1}(q^{-d}\widetilde{T}(q^{-1},t)G_c^{ff}(q^{-1},t)+N_b(q^{-1},t))D_t \end{aligned}$$

$$\begin{aligned} = {} & [1+q^{-d}K^{-1}(q^{-1},t)J(q^{-1},t)Z^{-1}(q^{-1},t)W(q^{-1},t)]^{-1}M^{-1}(q^{-1},t)L(q^{-1},t)a_t + \\ & ([1+q^{-d}K^{-1}(q^{-1},t)J(q^{-1},t)Z^{-1}(q^{-1},t)W(q^{-1},t)]^{-1}) \\ & (q^{-d}K^{-1}(q^{-1},t)J(q^{-1},t)S^{-1}(q^{-1},t)N(q^{-1},t)+X^{-1}(q^{-1},t)P(q^{-1},t))D_t \end{aligned}$$

$$\begin{aligned} = {} & [1+K^{-1}(q^{-1},t-d)J(q^{-1},t-d)Z^{-1}(q^{-1},t-d)W(q^{-1},t-d)q^{-d}]^{-1} \\ & (M^{-1}(q^{-1},t)L(q^{-1},t))a_t + \\ & [1+K^{-1}(q^{-1},t-d)J(q^{-1},t-d)Z^{-1}(q^{-1},t-d)W(q^{-1},t-d)q^{-d}]^{-1} \\ & (K^{-1}(q^{-1},t-d)J(q^{-1},t-d)S^{-1}(q^{-1},t-d)N(q^{-1},t-d)q^{-d}+X^{-1}(q^{-1},t)P(q^{-1},t))D_t \end{aligned} \tag{5.12}$$

From Li and Evans (1997) and Huang (1999,2002), pseudocommutation equations can be written such that:

$$\begin{cases} J(q^{-1},t)Z^{-1}(q^{-1},t) = \widetilde{Z}^{-1}(q^{-1},t)\widetilde{J}(q^{-1},t) \\ J(q^{-1},t)S^{-1}(q^{-1},t) = \widetilde{S}^{-1}(q^{-1},t)\widetilde{J}(q^{-1},t) \end{cases} \tag{5.13}$$

Substituting eqn. (5.13) into eqn. (5.12) yields

$$\begin{aligned} y_t = {} & ([1 + K^{-1}(q^{-1},t-d)\widetilde{Z}^{-1}(q^{-1},t-d)\widetilde{J}(q^{-1},t-d)W(q^{-1},t-d)q^{-d}]^{-1}) \\ & (M^{-1}(q^{-1},t)L(q^{-1},t))a_t + \\ & ([1 + K^{-1}(q^{-1},t-d)\widetilde{Z}^{-1}(q^{-1},t-d)\widetilde{J}(q^{-1},t-d)W(q^{-1},t-d)q^{-d}]^{-1}) \\ & \quad (K^{-1}(q^{-1},t-d)\widetilde{S}^{-1}(q^{-1},t-d)\widetilde{J}(q^{-1},t-d)N(q^{-1},t-d)q^{-d} + \\ & \qquad X^{-1}(q^{-1},t)P(q^{-1},t))D_t \end{aligned} \tag{5.14}$$

Let $\begin{cases} E^{-1}(q^{-1},t-d) = K^{-1}(q^{-1},t-d)\widetilde{Z}^{-1}(q^{-1},t-d) \\ G^{-1}(q^{-1},t-d) = K^{-1}(q^{-1},t-d)\widetilde{S}^{-1}(q^{-1},t-d) \end{cases}$ (5.15)

Substituting eqn. (5.15) into eqn. (5.14) yields

$$\begin{aligned} & M(q^{-1},t)X(q^{-1},t)G(q^{-1},t-d)[1+E^{-1}(q^{-1},t-d)\widetilde{J}(q^{-1},t-d)W(q^{-1},t-d)q^{-d}]y_t = \\ & \quad M(q^{-1},t)(X(q^{-1},t)\widetilde{J}(q^{-1},t-d)N(q^{-1},t-d)q^{-d} + G(q^{-1},t-d)P(q^{-1},t))D_t + \\ & \qquad G(q^{-1},t-d)X(q^{-1},t)L(q^{-1},t)a_t \end{aligned} \tag{5.16}$$

Let $\overline{M}(q^{-1},t) = M(q^{-1},t)X(q^{-1},t)$ (5.17)

We can also write the pseudocommutation equation:

$$\overline{M}(q^{-1},t)G(q^{-1},t-d)E^{-1}(q^{-1},t-d) = \widetilde{E}^{-1}(q^{-1},t-d)\widetilde{G}(q^{-1},t-d)\widetilde{\overline{M}}(q^{-1},t) \tag{5.18}$$

Substituting eqn. (5.18) into eqn. (5.16) yields

$$[\overline{M}(q^{-1},t)G(q^{-1},t-d)+\widetilde{E}^{-1}(q^{-1},t-d)\widetilde{G}(q^{-1},t-d)\widetilde{\widetilde{M}}(q^{-1},t)\widetilde{J}(q^{-1},t-d)W(q^{-1},t-d)q^{-d}]y_t = (\overline{M}(q^{-1},t)\widetilde{J}(q^{-1},t-d)N(q^{-1},t-d)q^{-d}+M(q^{-1},t)G(q^{-1},t-d)P(q^{-1},t))D_t + G(q^{-1},t-d)X(q^{-1},t)L(q^{-1},t)a_t \qquad (5.19)$$

Multiplying eqn. (5.19) by $\widetilde{E}(q^{-1},t-d)$ yields

$$[\widetilde{E}(q^{-1},t-d)\overline{M}(q^{-1},t)G(q^{-1},t-d)+\widetilde{G}(q^{-1},t-d)\widetilde{\widetilde{M}}(q^{-1},t)\widetilde{J}(q^{-1},t-d)W(q^{-1},t-d)q^{-d}]y_t = \widetilde{E}(q^{-1},t-d)(\overline{M}(q^{-1},t)\widetilde{J}(q^{-1},t-d)N(q^{-1},t-d)q^{-d}+M(q^{-1},t)G(q^{-1},t-d)P(q^{-1},t))D_t + \widetilde{E}(q^{-1},t-d)G(q^{-1},t-d)X(q^{-1},t)L(q^{-1},t)a_t \qquad (5.20)$$

Eqn. (5.20) can be expressed as

$$A_{cl}(q^{-1},t)y_t = B_{cl}(q^{-1},t)D_t + C_{cl}(q^{-1},t)a_t \qquad (5.21)$$

Eqn. (5.21) has the form of an LTV ARMAX model, where $A_{cl}(q^{-1},t)$, $B_{cl}(q^{-1},t)$ and $C_{cl}(q^{-1},t)$ are polynomials in the backshift operator q^{-1}. Therefore, for a time-variant feedforward plus feedback control loop, the closed-loop response to unmeasured and measured disturbances can be expressed as an LTV ARMAX model.

End of Proof

A general form of the closed loop response can be written for cases in which there are more than one measured disturbances and this is given by:

$$A_{cl}(q^{-1},t)y_t = \sum_{i=1}^{n} B_{cl}^{(i)}(q^{-1},t)D_{i,t} + C_{cl}(q^{-1},t)a_t \qquad (5.22)$$

where

$$\begin{aligned} A_{cl}(q^{-1},t) &= 1+\alpha_1(t)q^{-1}+\alpha_2(t)q^{-2}+\ldots+\alpha_k(t)q^{-k}+\ldots \\ B_{cl}^{(i)}(q^{-1},t) &= b_1^{(i)}(t)q^{-t_d^{(i)}}+b_2^{(i)}(t)q^{-(t_d^{(i)}+1)}+\ldots+b_k^{(i)}(t)q^{-(t_d^{(i)}+k-1)}+\ldots \\ C_{cl}(q^{-1},t) &= 1+c_1(t)q^{-1}+c_2(t)q^{-2}+\ldots+c_k(t)q^{-k}+\ldots \end{aligned} \qquad (5.23)$$

n is the number of measured disturbance variables. y_t is the time-varying process variable, $D_{i,t}$ is the measured disturbance for the feedforward variable, "i" and, t_d is the time delay it takes for a change in $D_{i,t}$ to affect the process variable. a_t is white noise sequence with zero mean and time-variant variance $\sigma_a^2(t)$.

From eqn. (5.22), the closed-loop response of the process variable to the measured and unmeasured disturbances can further be written as

$$y_t = \sum_{i=1}^{n} \underbrace{A_{cl}^{-1}(q^{-1},t)B_{cl}^{(i)}(q^{-1},t)D_{i,t}}_{y_{t,i}^{(m)}} + \underbrace{A_{cl}^{-1}(q^{-1},t)C_{cl}(q^{-1},t)a_t}_{y_t^{(u)}} \tag{5.24}$$

$$y_t = \sum_{i=1}^{n} y_{t,i}^{(m)} + y_t^{(u)} \tag{5.25}$$

$y_{t,i}^{(m)}$ represents the closed-loop response to the i^{th} measured disturbance while $y_t^{(u)}$ represents the closed-loop response to the unmeasured disturbance, and

$$y_{t,i}^{(m)} = A_{cl}^{-1}(q^{-1},t)B_{cl}^{(i)}(q^{-1},t)D_{i,t} \tag{5.26}$$

$$y_t^{(u)} = A_{cl}^{-1}(q^{-1},t)C_{cl}(q^{-1},t)a_t \tag{5.27}$$

Time series analysis can be applied to each measured disturbance, $D_{i,t}$ to obtain an LTV ARMA model

$$\bar{\partial}^{(i)}(q^{-1},t)D_{i,t} = \bar{\eta}^{(i)}(q^{-1},t)b_{i,t} \tag{5.28}$$

$b_{i,t}$ is the driving force for the measured disturbance $D_{i,t}$. The analysis presented in this chapter considers the multiple-input single-output (MISO) processes in which there is no cross-correlation between the unmeasured and measured disturbances.

Thus, the driving force $b_{i,t}$ is independent of a_t, which is the driving force for the unmeasured disturbance.

5.2.1 Analysis of minimum variance for feedback control

For the analysis of the LTV feedforward and feedback control scheme, ARMAX $(2, [nb_1\ nb_2\ ...\ nb_n], 2,\ [t_d^{(1)}\ t_d^{(2)}\ ...\ t_d^{(n)}])$ model that is representative of, and applicable to most physical processes is considered. Here, $nb_1 = nb_2 = ... = nb_n = 2$. $t_d^{(i)}$ is the time delay it takes for a change in each feedforward variable to affect the process variable. This time series modeling will yield an estimation of the model

$$A_{cl}(q^{-1},t)y_t = \sum_{i=1}^{n} B_{cl}^{(i)}(q^{-1},t)D_{i,t} + C_{cl}(q^{-1},t)a_t$$

The model can then be transferred to the form of (5.24). The effect of measured and unmeasured disturbances can be calculated from eqns. (5.26) and (5.27). In this section, we shall discuss the calculation of the minimum variance term for the unmeasured disturbance.

For the unmeasured disturbance, we have

$$\begin{aligned} A_{cl}(q^{-1},t) &= 1 + \alpha_1(t)q^{-1} + \alpha_2(t)q^{-2} \\ C_{cl}(q^{-1},t) &= 1 + c_1(t)q^{-1} + c_2(t)q^{-2} \end{aligned} \tag{5.29}$$

Eqn. (5.27) can be written in the impulse response form:

$$y_t^{(u)} = \left(f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + f_3(t)q^{-3} + ...\right)a_t \tag{5.30}$$

Substituting eqn. (5.30) into eqn. (5.27) yields

$$A_{cl}(q^{-1},t)\left(f_0(t) + f_1(t)q^{-1} + f_2(t)q^{-2} + f_3(t)q^{-3} + ...\right)a_t = C_{cl}(q^{-1},t)a_t \tag{5.31}$$

The time-variant impulse response coefficients are estimated by equating the coefficients on the right and left hand sides of eqn. (5.31):

$$\begin{cases} f_0(t) = 1 \\ f_1(t) = c_1(t) - \alpha_1(t) f_0(t-1) \\ f_2(t) = c_2(t) - \alpha_1(t) f_1(t-1) - \alpha_2(t) f_0(t-2) \\ f_k(t) = -\alpha_1(t) f_{k-1}(t-1) - \alpha_2(t) f_{k-2}(t-2) \qquad k > 2 \end{cases} \tag{5.32}$$

and from eqn. (5.32),

$$\begin{cases} f_0(t) = 1 \\ f_1(t) = c_1(t) - \alpha_1(t) \\ f_2(t) = c_2(t) - \alpha_1(t)c_1(t-1) + \alpha_1(t)\alpha_1(t-1) - \alpha_2(t) \\ f_3(t) = -\alpha_1(t)c_2(t-1) + \alpha_1(t)\alpha_2(t-1) + \alpha_1(t)\alpha_1(t-1)c_1(t-2) - \\ \qquad \alpha_1(t)\alpha_1(t-1)\alpha_1(t-2) - \alpha_2(t)c_1(t-2) + \alpha_2(t)\alpha_1(t-2) \\ \vdots \end{cases} \tag{5.33}$$

Therefore, the process output can be expressed as

$$y_t^{(u)} = (1 + (c_1(t) - \alpha_1(t))q^{-1} + (c_2(t) - \alpha_1(t)c_1(t-1) + \alpha_1(t)\alpha_1(t-1) - \alpha_2(t))q^{-2} + \dots)a_t \tag{5.34}$$

The corresponding minimum variance (for the feedback control) for the LTV process with a time delay of 3 as an example, can be calculated:

$$\begin{aligned} \sigma_{mv}^2(t) &= \left(1 + f_1^2(t) + f_2^2(t)\right)\sigma_a^2(t) \\ \sigma_{mv}^2(t) &= \left(1 + (c_1(t) - \alpha_1(t))^2 + (c_2(t) - \alpha_1(t)c_1(t-1) + \alpha_1(t)\alpha_1(t-1) - \alpha_2(t))^2\right)\sigma_a^2(t) \end{aligned} \tag{5.35}$$

However, direct long division gives

$$\begin{aligned} y_t^{(u)} = (1 + (c_1(t) - \alpha_1(t))q^{-1} + (c_2(t) - \alpha_1(t)c_1(t) + \alpha_1(t)^2 - \alpha_2(t))q^{-2} + \\ (-\alpha_1(t)c_2(t) + 2\alpha_1(t)\alpha_2(t) + \alpha_1(t)^2 c_1(t) + \alpha_1(t)^3 - \alpha_2(t)c_1(t))q^{-3} + \dots)a_t \end{aligned} \tag{5.36}$$

and the corresponding LTI minimum variance is given by

$$\sigma_{mv}^2(t) = \left(1 + (c_1(t) - \alpha_1(t))^2 + (c_2(t) - \alpha_1(t)c_1(t) + \alpha_1(t)^2 - \alpha_2(t))^2\right)\sigma_a^2(t) \quad (5.37)$$

which is not correct. Therefore, it is important to consider non-commutativity in handling an LTV process.

5.2.2 Analysis of minimum variance for feedforward control

For each feedforward variable $D_{i,t}$, ARMA (2,2) model is considered for the time series analysis since this model is generally applicable to most physical processes. However, the result can be extended to a higher order time series model by following the same procedure.

From eqn. (5.28), $\overline{\partial}^{(i)}(q^{-1},t)$ and $\overline{\eta}^{(i)}(q^{-1},t)$ can be written as

$$\begin{aligned}\overline{\partial}^{(i)}(q^{-1},t) &= 1 + \partial_1^{(i)}(t)q^{-1} + \partial_2^{(i)}(t)q^{-2} \\ \overline{\eta}^{(i)}(q^{-1},t) &= 1 + \eta_1^{(i)}(t)q^{-1} + \eta_2^{(i)}(t)q^{-2}\end{aligned} \quad (5.38)$$

The measured disturbance can be expressed as an impulse response form:

$$D_{i,t} = \left(f_0^{m_i}(t) + f_1^{m_i}(t)q^{-1} + f_2^{m_i}(t)q^{-2} + \ldots\right)b_{i,t} \quad (5.39)$$

Substituting eqn. (5.38) and eqn. (5.39) in eqn. (5.28) yields

$$\begin{aligned}(1 + \partial_1^{(i)}(t)q^{-1} + \partial_2^{(i)}(t)q^{-2})\left(f_0^{m_i}(t) + f_1^{m_i}(t)q^{-1} + f_2^{m_i}(t)q^{-2} + \ldots\right)b_{i,t} \\ = (1 + \eta_1^{(i)}(t)q^{-1} + \eta_2^{(i)}(t)q^{-2})b_{i,t}\end{aligned} \quad (5.40)$$

The time-variant impulse response coefficients, $f_o^{m_i}, f_1^{m_i}, \ldots, f_n^{m_i}$ are calculated from eqn. (5.40) and are given by

$$\begin{cases} f_0^{m_i}(t) = 1 \\ f_1^{m_i}(t) = \eta_1^{(i)}(t) - \partial_1^{(i)}(t) f_0^{m_i}(t-1) \\ f_2^{m_i}(t) = \eta_2^{(i)}(t) - \partial_1^{(i)}(t) f_1^{m_i}(t-1) - \partial_2^{(i)}(t) f_0^{m_i}(t-2) \\ f_3^{m_i}(t) = -\partial_1^{(i)}(t) f_2^{m_i}(t-1) - \partial_2^{(i)}(t) f_1^{m_i}(t-2) \\ \vdots \\ f_k^{m_i}(t) = -\partial_1^{(i)}(t) f_{k-1}^{m_i}(t-1) - \partial_2^{(i)}(t) f_{k-2}^{m_i}(t-2) \qquad k \geq 3 \end{cases} \tag{5.41}$$

It follows from eqn. (5.41) that

$$\begin{cases} f_0^{m_i}(t) = 1 \\ f_1^{m_i}(t) = \eta_1^{(i)}(t) - \partial_1^{(i)}(t) \\ f_2^{m_i}(t) = \eta_2^{(i)}(t) - \partial_1^{(i)}(t)\eta_1^{(i)}(t-1) + \partial_1^{(i)}(t)\partial_1^{(i)}(t-1) - \partial_2^{(i)}(t) \\ f_3^{m_i}(t) = -\partial_1^{(i)}(t)\eta_2^{(i)}(t-1) + \partial_1^{(i)}(t)\partial_1^{(i)}(t-1)\eta_1^{(i)}(t-2) + \partial_1^{(i)}(t)\partial_2^{(i)}(t-1) \\ \qquad - \partial_1^{(i)}(t)\partial_1^{(i)}(t-1)\partial_1^{(i)}(t-2) - \partial_2^{(i)}(t)\eta_1^{(i)}(t-2) + \partial_2^{(i)}(t)\partial_1^{(i)}(t-2) \\ \vdots \end{cases} \tag{5.42}$$

However, direct long division yields

$$\begin{aligned} y_t = (1 &+ (\eta_1^{(i)}(t) - \partial_1^{(i)}(t)]q^{-1} + [\eta_2^{(i)}(t) - \partial_1^{(i)}(t)\eta_1^{(i)}(t) + \partial_1^{(i)}(t)^2 - \partial_2^{(i)}(t))q^{-2} + \\ &(-\partial_1^{(i)}(t)\eta_2^{(i)}(t) + 2\partial_1^{(i)}(t)\partial_2^{(i)}(t) + \partial_1^{(i)}(t)^2\eta_1^{(i)}(t) + \partial_1^{(i)}(t)^3 - \partial_2^{(i)}(t)\eta_1^{(i)}(t))q^{-3} + \ldots)\, b_{i,t} \end{aligned} \tag{5.43}$$

which is, once again, not correct.

Eqn. (5.26) can be written in the impulse response form as

$$y_{t,i}^{(m)} = \left(f_0^{(i)}(t) + f_1^{(i)}(t)q^{-1} + f_2^{(i)}(t)q^{-2} + \ldots\right)b_{i,t} \tag{5.44}$$

Substituting eqn. (5.23), eqn. (5.39) and eqn. (5.44) into eqn. (5.26) yields

$$(1+\alpha_1(t)q^{-1}+\alpha_2(t)q^{-2})\left(f_0^{(i)}(t)+f_1^{(i)}(t)q^{-1}+f_2^{(i)}(t)q^{-2}+\ldots\right)b_{i,t}=$$
$$(b_1^{(i)}(t)q^{-t_d^{(i)}}+b_2^{(i)}(t)q^{-(t_d^{(i)}+1)})\left(f_0^{m_i}(t)+f_1^{m_i}(t)q^{-1}+f_2^{m_i}(t)q^{-2}+\ldots\right)b_{i,t} \quad (5.45)$$

The first $t_d^{(i)}$ coefficients of the moving average (MA) model are zero for $t_d^{(i)}>0$, and the remaining impulse response coefficients are obtained by equating the terms on the left and right hand sides. That is,

$$\begin{cases} f_0^{(i)}(t)=0 \\ \vdots \\ f_{t_d^{(i)}-1}^{(i)}(t)=0 \\ f_{t_d^{(i)}}^{(i)}(t)=b_1^{(i)}(t)f_0^{m_i}(t-t_d^{(i)}) \\ f_{t_d^{(i)}+1}^{(i)}(t)=b_1^{(i)}(t)f_1^{m_i}(t-t_d^{(i)})+b_2^{(i)}(t)f_0^{m_i}(t-t_d^{(i)}-1)-\alpha_1(t)f_{t_d^{(i)}}^{(i)}(t-1) \\ \qquad -\alpha_2(t)f_{t_d^{(i)}-1}^{(i)}(t-2) \\ \vdots \\ f_k^{(i)}(t)=b_1^{(i)}(t)f_{k-t_d^{(i)}}^{m_i}(t-t_d^{(i)})+b_2^{(i)}(t)f_{k-t_d^{(i)}-1}^{m_i}(t-t_d^{(i)}-1)-\alpha_1(t)f_{k-1}^{(i)}(t-1) \\ \qquad -\alpha_2(t)f_{k-2}^{(i)}(t-2) \qquad k>t_d^{(i)} \end{cases} \quad (5.46)$$

Eqn. (5.46) shows that if the delay of the feedforward variable $D_{i,t}$ is greater than or equal to the process delay, then the contribution of the measured disturbance to the minimum variance of the feedforward plus feedback control will be equal to zero.

Assume that the measured disturbances are mutually uncorrelated, the minimum variance of each measured disturbance can be calculated from eqn. (5.46) as

$$\sigma_{mv}^{2\ (i)}(t)=\left(f_0^{(i)2}(t)+f_1^{(i)2}(t)+\ldots+f_{d-1}^{(i)2}(t)\right)\sigma_{b,i}^2(t) \quad (5.47)$$

where d is the time delay of the process including the delay due to zero-order-hold device.

Let us consider that the time delay it takes a change in $D_{i,t}$ to affect the process variable is 2, then $t_d^{(i)} = 2$. The corresponding minimum variance of each measured disturbance for the LTV process with a time delay of 3 can be calculated as

$$\begin{aligned}\sigma_{mv}^{2\ (i)}(t) &= \left(f_0^{(i)}(t)^2 + f_1^{(i)}(t)^2 + f_2^{(i)}(t)^2\right)\sigma_{b,i}^2(t) \\ &= (b_1^{(i)}(t) f_0^{m_i}(t - t_d^{(i)}))^2 \sigma_{b,i}^2(t)\end{aligned} \tag{5.48}$$

It follows from eqn. (5.41) that

$$\sigma_{mv}^{2\ (i)}(t) = b_1^{(i)}(t)^2 \sigma_{b,i}^2(t) \tag{5.49}$$

One can evaluate the performance measure of the control loop by comparing the total contribution of the minimum variance terms from the measured and unmeasured disturbances with the actual process variance. However, the above procedure needs the assumption that the shock of the measured disturbances, $b_{i,t}$'s, are mutually independent. This assumption can be satisfied if the measured disturbance, $D_{i,t}$'s are mutually independent or if there is only one measured disturbance. Otherwise, we have to use a more tedious multivariable approach, which is discussed next.

5.2.3 Performance assessment of feedforward plus feedback LTV systems with correlated disturbances

This section addresses the more general case that accounts for cross correlation amongst the measured disturbances, $D_{i,t}$.

Let $\underline{D}_t$ be the vector of "n" feedforward variables which can be represented as

$$\underline{D}_t = [D_{1,t} \quad D_{2,t} \quad \ldots \quad D_{n,t}]^T \tag{5.50}$$

and $\underline{D}_t$ can be modeled by

$$\widehat{\partial}(q^{-1},t)\underline{D}_t = \widehat{\eta}(q^{-1},t)B_t \tag{5.51}$$

B_t is the vector of driving force $b_{i,t}$ for the measured disturbances, which can be expressed as

$$B_t = \left[b_{1,t} \;\; b_{2,t} \ldots b_{n,t}\right]^T \tag{5.52}$$

$\underline{D}_t$ can be expressed in impulse response form as

$$\underline{D}_t = F_0^{(D)}(t)B_t + F_1^{(D)}(t)B_{t-1} + \ldots F_{d-1}^{(D)}(t)B_{t-d+1} + \ldots \tag{5.53}$$

Consider eqn. (5.51) to be a vector ARMA(2, 2) process

$$\text{i.e.} \begin{cases} \widehat{\partial}(q^{-1},t) = I + \overline{\partial}_1(t)q^{-1} + \overline{\partial}_2(t)q^{-2} \\ \widehat{\eta}(q^{-1},t) = I + \overline{\eta}_1(t)q^{-1} + \overline{\eta}_2(t)q^{-2} \end{cases} \tag{5.54}$$

I is "n x n" identity matrix. $\overline{\partial}_1$ is "n x n" matrix that consists of the autoregressive coefficients of the ARMA model in eqn. (5.51) which corresponds to q^{-1} while $\overline{\partial}_2$ is "n x n" matrix that consists of the autoregressive coefficients corresponding to q^{-2}. Similarly, $\overline{\eta}_1$ is "n x n" matrix that consists of the moving average coefficients of the ARMA model in eqn. (5.51) which corresponds to q^{-1} while $\overline{\eta}_2$ is "n x n" matrix that consists of the moving average coefficients corresponding to q^{-2}.

(Note that ARMA(2, 2) is also the typical representation of time series in practice).
Substituting eqn. (5.53) and eqn. (5.54) into eqn. (5.51) yields

$$(I+\bar{\partial}_1(t)q^{-1}+\bar{\partial}_2(t)q^{-2})(F_0^{(D)}(t)B_t+F_1^{(D)}(t)q^{-1}+\ldots+F_{d-1}^{(D)}(t)q^{-d+1}+\ldots)B_t = (I+\bar{\eta}_1(t)q^{-1}+\bar{\eta}_2(t)q^{-2})B_t \tag{5.55}$$

The coefficients, $F_0^{(D)}(t), F_1^{(D)}(t), \ldots, F_k^{(D)}(t)$ are calculated from eqn. (5.55) and are given by

$$\begin{cases} F_0^{(D)}(t)=I \\ F_1^{(D)}(t)=\bar{\eta}_1(t)-\bar{\partial}_1(t)F_0^{(D)}(t-1) \\ F_2^{(D)}(t)=\bar{\eta}_2(t)-\bar{\partial}_1(t)F_1^{(D)}(t-1)-\bar{\partial}_2(t)F_0^{(D)}(t-2) \\ F_3^{(D)}(t)=-\bar{\partial}_1(t)F_2^{(D)}(t-1)-\bar{\partial}_2(t)F_1^{(D)}(t-2) \\ \vdots \\ F_k^{(D)}(t)=-\bar{\partial}_1(t)F_{k-1}^{(D)}(t-1)-\bar{\partial}_2(t)F_{k-2}^{(D)}(t-2) \qquad k\geq 3 \end{cases} \tag{5.56}$$

It follows from eqn. (5.56) that

$$\begin{cases} F_0^{(D)}(t)=I \\ F_1^{(D)}(t)=\bar{\eta}_1(t)-\bar{\partial}_1(t) \\ F_2^{(D)}(t)=\bar{\eta}_2(t)-\bar{\partial}_1(t)\bar{\eta}_1(t-1)+\bar{\partial}_1(t)\bar{\partial}_1(t-1)-\bar{\partial}_2(t) \\ F_3^{(D)}(t)=-\bar{\partial}_1(t)\bar{\eta}_2(t-1)+\bar{\partial}_1(t)\bar{\partial}_1(t-1)\bar{\eta}_1(t-2)+\bar{\partial}_1(t)\bar{\partial}_2(t-1) \\ \qquad -\bar{\partial}_1(t)\bar{\partial}_1(t-1)\bar{\partial}_1(t-2)-\bar{\partial}_2(t)\bar{\eta}_1(t-2)+\bar{\partial}_2(t)\bar{\partial}_1(t-2) \\ \vdots \end{cases} \tag{5.57}$$

Eqn. (5.22) can be expressed as

$$A_{cl}(q^{-1},t)y_t=B_{cl}^{(1)}(q^{-1},t)D_{1,t}+B_{cl}^{(2)}(q^{-1},t)D_{2,t}+\ldots+B_{cl}^{(n)}(q^{-1},t)D_{n,t}+C_{cl}(q^{-1},t)a_t \tag{5.58}$$

Eqn. (5.58) can be re-arranged to give

$$A_{cl}(q^{-1},t)y_t=[B_{cl}^{(1)}(q^{-1},t)\ B_{cl}^{(2)}(q^{-1},t)\ \ldots\ B_{cl}^{(n)}(q^{-1},t)]\begin{bmatrix} D_{1,t} \\ D_{2,t} \\ \vdots \\ D_{n,t} \end{bmatrix}+C_{cl}(q^{-1},t)a_t \tag{5.59}$$

Therefore

$$y_t = \underbrace{A_{cl}^{-1}(q^{-1},t)[B_{cl}^{(1)}(q^{-1},t)\ \ B_{cl}^{(2)}(q^{-1},t)\ \ldots\ B_{cl}^{(n)}(q^{-1},t)]\underline{D}_t}_{y_t^{(m)}} + \underbrace{A_{cl}^{-1}(q^{-1},t)C_{cl}(q^{-1},t)a_t}_{y_t^{(u)}} \qquad (5.60)$$

Eqn. (5.60) can be expressed as

$$y_t = y_t^{(m)} + y_t^{(u)} \qquad (5.61)$$

where

$$y_t^{(m)} = A_{cl}^{-1}(q^{-1},t)[B_{cl}^{(1)}(q^{-1},t)\ \ B_{cl}^{(2)}(q^{-1},t)\ \ldots\ B_{cl}^{(n)}(q^{-1},t)][D_{1,t}\ D_{2,t}\ \ldots D_{n,t}]^T \qquad (5.62)$$

That is

$$A_{cl}(q^{-1},t)y_t^{(m)} = [B_{cl}^{(1)}(q^{-1},t)\ \ B_{cl}^{(2)}(q^{-1},t)\ \ldots\ B_{cl}^{(n)}(q^{-1},t)][D_{1,t}\ \ D_{2,t}\ \ldots D_{n,t}]^T \qquad (5.63)$$

Substituting eqn. (5.23) into eqn. (5.63) yields

$$\begin{aligned}&(1+a_1(t)q^{-1}+a_2(t)q^{-2})y_t^{(m)} = \\ &[\,b_1^{(1)}(t)q^{-t_d^{(1)}} + b_2^{(1)}(t)q^{-(t_d^{(1)}+1)}\ \ \ldots\ \ b_1^{(n)}(t)q^{-t_d^{(n)}} + b_2^{(n)}(t)q^{-(t_d^{(n)}+1)}\,]\underline{D}_t\end{aligned} \qquad (5.64)$$

where $t_d^{(i)}$ is the time delay it takes for a change in each feedforward variable $D_{i,t}$ to affect the process variable.

$$\text{Let}\begin{cases}\theta = maximum\,(t_d^{(i)}) - minimum\,(t_d^{(i)}) + 2\\ and\\ \theta_1 = minimum\,(t_d^{(i)}) \qquad\qquad (for\ i = 1, 2, \ldots n)\end{cases} \qquad (5.65)$$

Then, eqn. (5.64) can be re-arranged and expressed as

$$(1+a_1(t)q^{-1}+a_2(t)q^{-2})y_t^{(m)} = (B_1^{(cl)}(t)q^{-\theta_1} + B_2^{(cl)}(t)q^{-(\theta_1+1)} + \ldots + B_\theta^{(cl)}(t)q^{-(\theta_1+\theta-1)})\underline{D}_t \quad (5.66)$$

where θ represents the total number of terms on the right hand side of eqn. (5.66). $B_1^{(cl)}(t)$ is a "$1 \times n$" row vector that consists of coefficients on the right hand side (r.h.s) of eqn. (5.64) which corresponds to $q^{-\theta_1}$. Similarly, $B_2^{(cl)}(t)$ is a "$1 \times n$" row vector that consists of coefficients on the r.h.s of eqn. (5.64) which corresponds to $q^{-(\theta_1+1)}$ until $B_\theta^{(cl)}(t)$ which is also a "$1 \times n$" row vector that consists of coefficients on the r.h.s of eqn. (5.64) corresponding to $q^{-(\theta_1+\theta+1)}$.

$y_t^{(m)}$ can be expressed in impulse response form as

$$y_t^{(m)} = F_0^{(m)}(t)B_t + F_1^{(m)}(t)B_{t-1} + \ldots + F_{d-1}^{(m)}(t)B_{t-d+1} + \ldots \quad (5.67)$$

Substituting eqn. (5.53) and eqn. (5.67) into eqn. (5.66) yields

$$(1+a_1(t)q^{-1}+a_2(t)q^{-2})(F_0^{(m)}(t)B_t + F_1^{(m)}(t)B_{t-1} + \ldots F_{d-1}^{(m)}(t)B_{t-d+1} + \ldots) =$$
$$(B_1^{(cl)}(t)q^{-\theta_1} + B_2^{(cl)}(t)q^{-(\theta_1+1)} + \ldots + B_\theta^{(cl)}(t)q^{-(\theta_1+\theta-1)})(F_0^{(D)}(t) + F_1^{(D)}(t)q^{-1} + F_2^{(D)}(t)q^{-2} + \ldots)B_t$$
(5.68)

The impulse response coefficients (which are "$1 \times n$" row vectors) are obtained by equating the terms on the left and right hand sides. However, it should be noted that if $\theta_1 > 0$, then, the first θ_1 coefficient(s) of the moving average (MA) model is(are) "$1 \times n$" zero row vector(s). That is,

$$
\begin{cases}
F_0^{(m)}(t) = [0\ 0 \ldots] \\
\vdots \\
F_{\theta_1-1}^{(m)}(t) = [0\ 0 \ldots] \\
F_{\theta_1}^{(m)}(t) = B_1^{(cl)}(t) F_0^{(D)}(t-\theta_1) \\
F_{\theta_1+1}^{(m)}(t) = -a_1(t) F_{\theta_1}^{(m)}(t-1) - a_2(t) F_{\theta_1-1}^{(m)}(t-2) + B_2^{(cl)}(t) F_0^{(D)}(t-\theta_1-1) + \\
\qquad B_1^{(cl)}(t) F_1^{(D)}(t-\theta_1) \\
\\
F_k^{(m)}(t) = -a_1(t) F_{k-1}^{(m)}(t-1) - a_2(t) F_{k-2}^{(m)}(t-2) + B_{\theta-j}^{(cl)}(t) F_{k-\theta_1-\theta+j+1}^{(D)}(t-\theta_1-\theta+j+1) \\
\qquad (for\ j = 0,1,2,\ldots,\theta-1),\ k > \theta_1
\end{cases}
\tag{5.69}
$$

The minimum Feedforward & Feedback variance can be calculated from eqn. (5.69):

$$
\sigma_{mv}^{2\ (m)}(t) = F_0^{(m)}(t) \Sigma_B F_0^{(m)^T}(t) + F_1^{(m)}(t) \Sigma_B F_1^{(m)^T}(t) + \ldots + F_{d-1}^{(m)}(t) \Sigma_B F_{d-1}^{(m)^T}(t) \tag{5.70}
$$

where

$$
\Sigma_B = \text{cov}\begin{bmatrix} b_{1,t} \\ b_{2,t} \\ \vdots \\ b_{n,t} \end{bmatrix} \tag{5.71}
$$

and d is the time delay of the process including the delay due to zero-order-hold device.

5.3 Simulation

5.3.1 Simulation example with one measured disturbance

For the same stirred tank reactor in chapter 2, the performance of the control loop is re-assessed by including a feedforward control variable. Let us consider that the tank inlet temperature T_i, is the measured feedforward variable and the time delay it takes for this measured disturbance to affect the tank temperature is 1. A deterministic step-type disturbance is the unmeasured disturbance added to the system and the white noise sequence, a_t is the driving force for the unmeasured disturbance.

Therefore, the process output, $T(t)$ can be expressed as:

$$T(t) = q^{-4} \frac{(5.03e^{-0.15\sin(t/x)} - 7.5) \big/ (0.4 + 0.15\sin(t/x))}{1 - 0.67e^{-(0.15\sin(t/x))} q^{-1}} F(t) \;+\; \frac{0.1}{1 - q^{-1}} a_t + \ldots$$

$$q^{-1} \frac{(0.1 - 0.067e^{-0.15\sin(t/x)}) \big/ (0.4 + 0.15\sin(t/x))}{1 - 0.67e^{-(0.15\sin(t/x))} q^{-1}} T_i(t)$$

(5.71)

Assume that the measured disturbance, T_i can be represented as

$$T_i(t) = \frac{1}{1 - 0.9q^{-1}} b_t \tag{5.72}$$

where b_t is white noise representing the driving force for realization of the measured disturbance, which is independent of a_t.

The process model can be expressed as

$$y_t = q^{-4} \frac{\phi(t)}{1 - \delta(t)q^{-1}} u_t + q^{-1} \left(\frac{\upsilon(t)}{1 - \delta(t)q^{-1}} \right) \left(\frac{1}{1 - 0.9q^{-1}} \right) b_t + \frac{0.1}{1 - q^{-1}} a_t \tag{5.73}$$

where the time variant process and disturbance dynamics are given by

$$\begin{cases} \phi(t) = (5.03e^{-0.15\sin(t/x)} - 7.5) \Big/ (0.4 + 0.15\sin(t/x)) \\ \upsilon(t) = (0.1 - 0.067e^{-0.15\sin(t/x)}) \Big/ (0.4 + 0.15\sin(t/x)) \\ \delta(t) = 0.67e^{-0.15\sin(t/x)} \end{cases} \tag{5.74}$$

Eqn. (5.71) shows that both the process model and the disturbance model are time-variant. Let us consider that the disturbance has three different time-variant dynamics in ascending order of increasing parameter-varying rate, from relatively slow parameter change to relatively fast parameter change:

$$\begin{aligned} &case\ 1: \\ &\upsilon(t) = (0.1 - 0.067e^{-0.15\sin(t/10)}) \Big/ (0.4 + 0.15\sin(t/10)) \\ &\delta(t) = 0.67e^{-0.15\sin(t/10)} \end{aligned} \tag{5.75}$$

$$\begin{aligned} &case\ 2: \\ &\upsilon(t) = (0.1 - 0.067e^{-0.15\sin(t)}) \Big/ (0.4 + 0.15\sin(t)) \\ &\delta(t) = 0.67e^{-0.15\sin(t)} \end{aligned} \tag{5.76}$$

$$\begin{aligned} &case\ 3: \\ &\upsilon(t) = (0.1 - 0.067e^{-0.15\sin(2t)}) \Big/ (0.4 + 0.15\sin(2t)) \\ &\delta(t) = 0.67e^{-0.15\sin(2t)} \end{aligned} \tag{5.77}$$

This example compares the feedforward/feedback minimum variance control (which takes non-commutativity associated with LTV transfer functions into account) with the conventional minimum variance control benchmark that is calculated using point multiplication. This is carried out by calculating the feedforward/feedback minimum variance term using normal multiplication (non-commutative) and pointwise multiplication (commutative), and comparing the difference between the two methods.

The simulation results in Figure 5.2 show a comparison of the difference between normal multiplication (solid line) and pointwise multiplication (dotted line). It can be seen that the difference between the minimum variance terms calculated using the normal multiplication and pointwise multiplication increases as the parameter-varying rate increases from the top to the bottom subplot. This result shows the importance of using normal multiplication rather than pointwise multiplication in the estimation of the LTV feedforward/feedback minimum variance term for time varying processes.

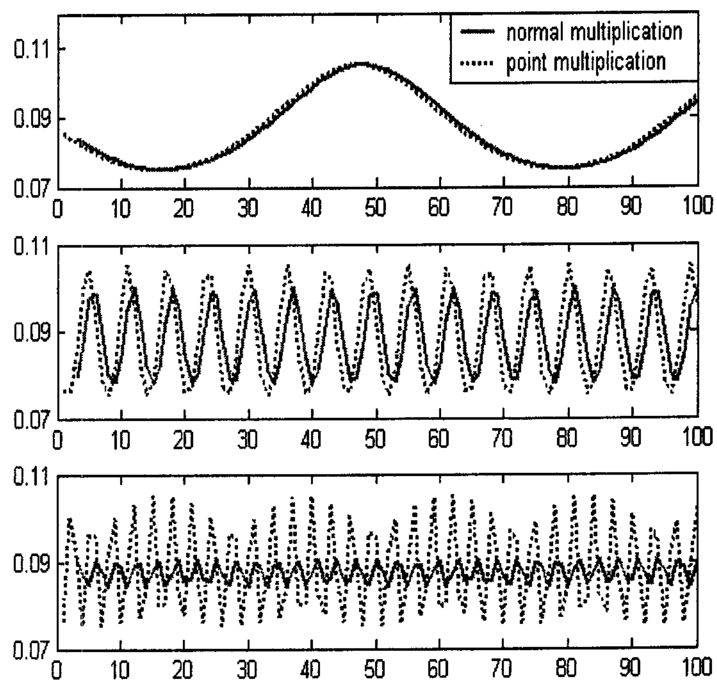


Figure 5.2: Comparison of time-variant feedforward/feedback minimum variance term using normal multiplication and pointwise multiplication

5.3.2 Simulation example with correlated disturbances

For the same example in section 5.3.1, let us consider that the ambient temperature, T_a is also measured and included as a feedforward variable in the feedforward plus feedback control scheme. The time delay it takes for this measured disturbance to affect the tank temperature is 3. The measured ambient temperature is considered to be correlated with the measured tank inlet temperature, T_i as will be seen in the analysis:

The process variable, $T(t)$ can be expressed as:

$$T(t) = q^{-4} \frac{(5.03e^{-0.15\sin(t/x)} - 7.5) \big/ (0.4 + 0.15\sin(t/x))}{1 - 0.67e^{-(0.15\sin(t/x))} q^{-1}} F(t) \; + q^{-3} \frac{0.02}{1 - 0.8q^{-1}} \mathrm{T_a} + \ldots$$

$$q^{-1} \frac{(0.1 - 0.067e^{-0.15\sin(t/x)}) \big/ (0.4 + 0.15\sin(t/x))}{1 - 0.67e^{-(0.15\sin(t/x))} q^{-1}} T_i(t) + \frac{0.1}{1 - q^{-1}} \mathrm{a_t} \tag{5.78}$$

The measured disturbance, T_i can be expressed as:

$$T_i(t) = \frac{1}{1 - 0.9q^{-1}} b_{1,t} \tag{5.79}$$

where $b_{1,t}$ is white noise representing the driving force for realization of the measured disturbance, T_i

The ambient temperature is given by:

$$T_a = \frac{1}{1 - 0.9q^{-1}} b_{2,t} \tag{5.80}$$

where $b_{2,t}$ is white noise which is correlated to $b_{1,t}$ by:

$$b_{2,t} = 0.5\, b_{1,t} + 0.5\, e \tag{5.81}$$

"e" in eqn. (5.81) represents random noise.

Hence, the process model can be expressed as

$$y_t = q^{-4}\frac{\phi(t)}{1-\delta(t)q^{-1}}u_t + q^{-1}\left(\frac{\upsilon(t)}{1-\delta(t)q^{-1}}\right)\left(\frac{1}{1-0.9q^{-1}}\right)b_{1,t} + q^{-3}\left(\frac{0.02}{1-0.8q^{-1}}\right)\left(\frac{1}{1-0.9q^{-1}}\right)b_{2,t} + \ldots \frac{0.1}{1-q^{-1}}a_t \tag{5.82}$$

where the time variant process and disturbance dynamics are given by eqn. (5.74).

In this example we also consider that the disturbance has three different time-variant dynamics in ascending order of increasing parameter-varying rate, from relatively slow parameter change to relatively fast parameter change as given by eqn. (5.75) to eqn. (5.77).

The feedforward/feedback minimum variance control benchmark is calculated using the developed LTV algorithm for correlated disturbances (which takes non-commutativity associated with LTV transfer functions into account) and compared with the conventional minimum variance control benchmark that is calculated using pointwise multiplication (which is non-commutative).

The simulation results for the correlated disturbances in Figure 5.3 show a comparison of the difference between the developed algorithm and the conventional method. It can be seen that the difference between the two methods increases as the parameter-varying rate increases from the top to the bottom subplot. This result further shows that it is important to consider non-commutativity associated with the manipulation of LTV operators in calculating the LTV feedforward/feedback minimum variance control benchmark to evaluate the performance of LTV feedforward/feedback control systems.

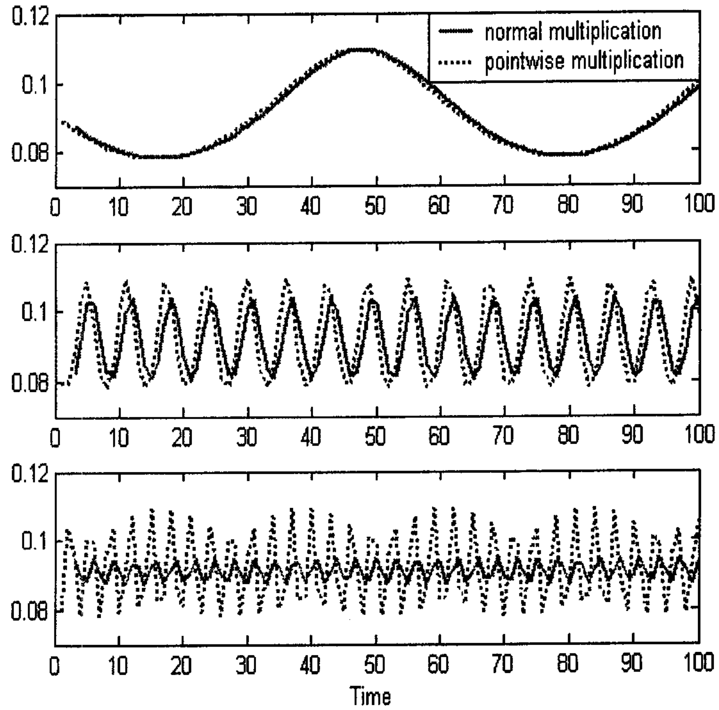


Figure 5.3: Comparison of time-variant feedforward/feedback minimum variance term using normal multiplication and pointwise multiplication for correlated disturbances

5.4 Case Study on An Industrial Process

The present case study is concerned with the feedforward/feedback control loop performance assessment of sulphur recovery process under adaptive control that is an LTV control system. One feedforward variable is considered in this analysis with the time delay of $t_d = 1$. The sulphur recovery process description is given in chapter 2 (section 2.4.1) and the schematic of the process is shown in Figure 2.4.

The data set has a sample size of 314 data points taken over a five-hour period with sampling interval of one minute. The time delay of the process is approximately no less than 2 minutes including the delay due to zero-order-hold device and is taken as 2 in this case study. The operating data is shown in Figure 5.4 while Figure 5.5 is the plot of the measured disturbance.

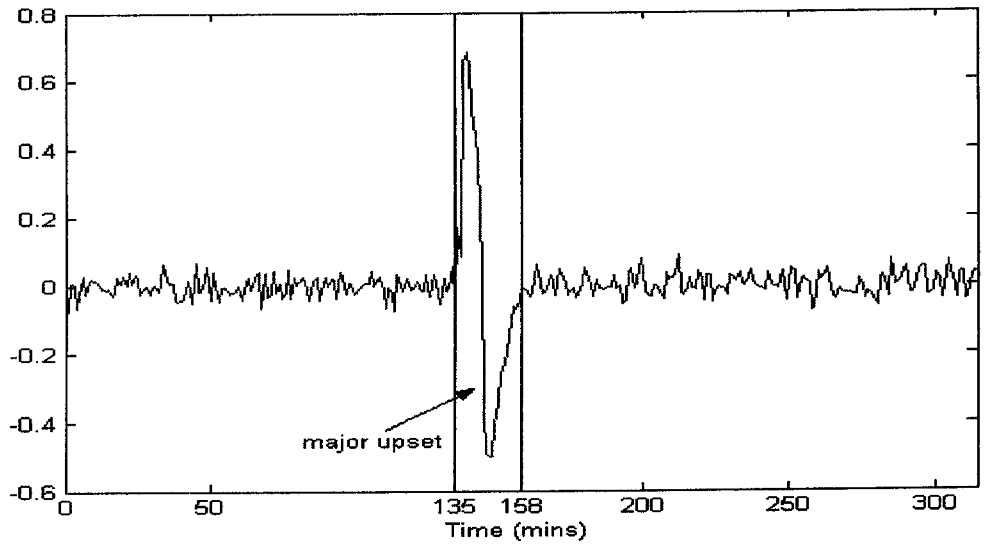


Figure 5.4: Plot of the operating data for the adaptive controller

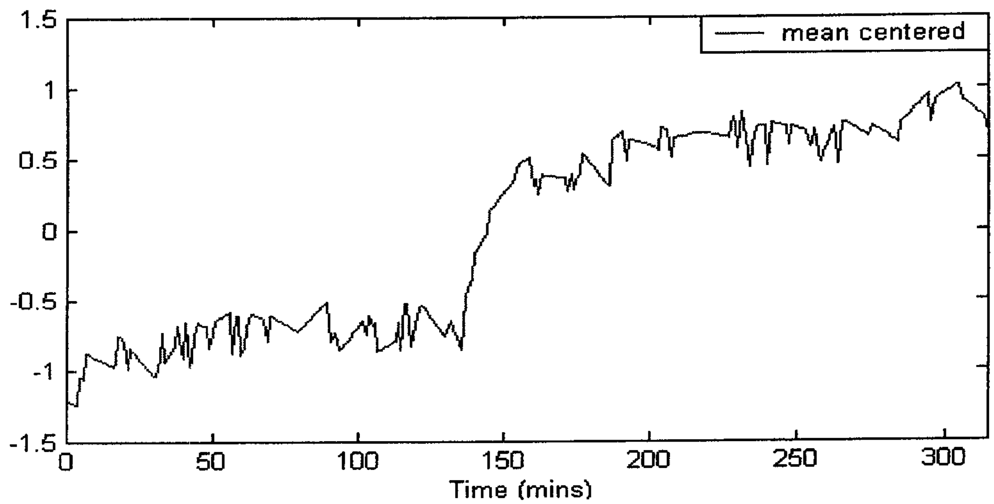


Figure 5.5: Plot of the measured disturbance, $D_{1,t}$

The data set used in this example clearly reveals that there is a significant disturbance occurring in certain limited time period (approx. 8% time) within the given data and the process/disturbance dynamics is clearly time varying.

The analysis carried out here compares the relative difference between the feedforward/feedback minimum variance term using normal multiplication (non-commutative) and pointwise multiplication (commutative).

Figure 5.6 shows the relative difference between the minimum variance terms using normal multiplication (NM) and pointwise multiplication (PM). One can see that the relative difference between the LTV performance method and the traditional recursive method can be up to 50%. The result shows that it is important to take non-commutativity into account in the manipulation of LTV operators of feedforward/feedback control loops. Thus, it is recommended that normal multiplication rather than pointwise multiplication is used when one deals with time varying processes.

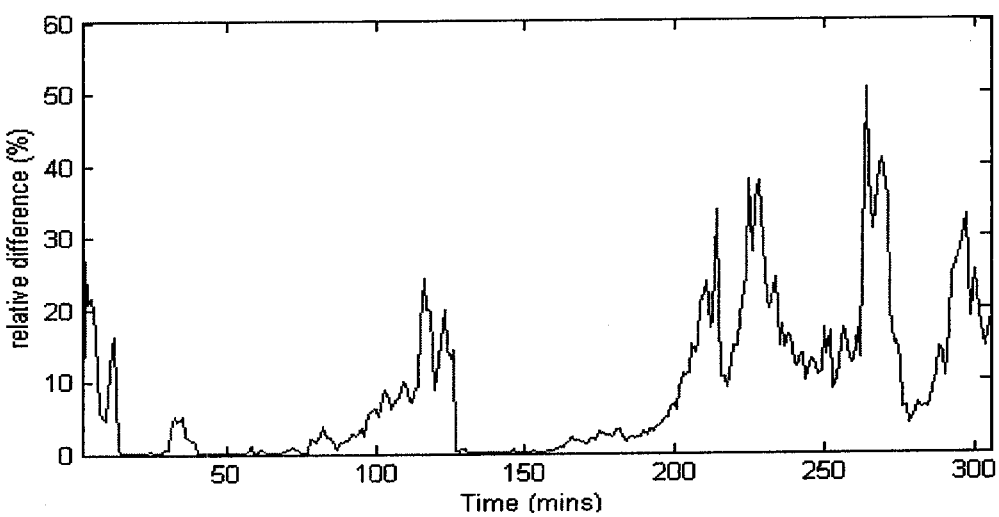


Figure 5.6:Relative difference between LTV feedforward/feedback minimum variance terms using normal multiplication and point multiplication

5.5 Conclusion

The LTV feedforward/feedback minimum variance control benchmark for time-variant processes has been derived in this chapter. This result is a significant extension of the recent contribution by Huang (2002) from performance assessment techniques for LTV feedback control loops to LTV feedforward/feedback control systems. The proposed method provides a way to calculate the minimum variance control benchmark from routine operating data and has been illustrated through a simulated example and an industrial case study.

Chapter 6

Conclusions

This work deals with the development of techniques for control loop performance assessment of linear time variant (LTV) univariate processes, including LTV feedback and/or feedforward control loops. The algorithms have been illustrated through simulation examples as well as case study on a sulphur recovery process.

Chapter 2 deals with the development of a computational algorithm for control loop performance assessment of linear time variant (LTV) processes, which assumes time variant process (including disturbances) and time variant controller. The methodology is found to be appropriate for performance assessment of LTV control such as adaptive control and has been used to evaluate the performance of a sulphur recovery process that is under adaptive control. However, the time-variant minimum variance control, which is found to be suitable for time-variant controllers, will clearly be too demanding on time-invariant controllers and this is the focus in chapter 3.

Alternative time-invariant performance benchmarks that are more suitable for time-variant processes under time invariant control have been developed in chapter 3 and chapter 4, with a limited discussion to time variant disturbance models. The benchmark developed in chapter 3 is useful for minimization of a particular type of disturbance of interest amongst other forms of disturbance dynamics affecting the process. However, it is found that this benchmark might not always be appropriate. Therefore, another benchmark that would be suitable for regulatory performance of different types of disturbance dynamics within a process is presented in chapter 4. This performance benchmark, which may be found from routine operating data through some time series analysis and optimization technique has been applied to the performance assessment of a PID controller in a sulphur recovery process.

The performance assessment methodology developed in chapter 2 is extended to feedforward/feedback control loop performance assessment of Linear Time Variant (LTV) MISO processes in chapter 5. The LTV benchmark developed for feedforward plus feedback control schemes is presented and the feasibility of the algorithm is illustrated by a simulated stirred tank reactor as well as a case study on sulphur recovery process.

As has been discussed by many researchers and practicing engineers, routine control loop monitoring is gaining increasing attention. New control loop performance assessment technologies will permit automated and repeated monitoring of the design, tuning and upgrading of the control loops. Poor design, tuning or upgrading of the control loops will be detected, and continuous performance monitoring will indicate which loops should be retuned or which loops have not been effectively upgraded when changes in the disturbances, in the process or in the controller occur. The results in this thesis show that the use of the correct methodology for continuous performance assessment of process operations allows timely detection of unwanted control loop variability. This can give control engineers insight into focusing control re-tuning and maintenance efforts on such control loops with poor performance.

6.1 The Contributions of this Thesis

The main contributions of this work include:

1. Establishment of a general computation framework for a new time-invariant control benchmark applicable to any change of disturbance dynamics, for time-variant processes under time invariant control.

2. A systematic solution of (optimal) time-invariant minimum variance control benchmark that can "optimize" overall performance of time-variant processes under time invariant control has been established.

3. Development of the performance assessment methodology for LTV feedforward plus feedback control loops.

4. Evaluation of the proposed algorithms/methodology using simulation examples.

5. A detailed industrial case study to illustrate the applicability of a generalized control performance assessment technique for linear time variant (LTV) processes is presented.

In this thesis, the algorithms are coded in MATLAB and used for the simulations and industrial applications.

6.2 Recommendations For Future Work

It has been discussed that there is an increasing interest in the research area of control loop performance assessment. Continuous performance assessment of time-variant processes is important since most processes have certain degree of time varying behavior and this has brought about a need to develop performance assessment methods for time varying processes. Although this thesis has presented some of the fundamental techniques, but there are some issues on performance assessment which have not been addressed. Therefore, some recommendations for future work are listed:

1. It is important to develop a method for calculation of the actual, time-variant process variance for feedforward plus feedback control loops.

2. It will be interesting to obtain a general performance assessment algorithm for processes with time-varying time-delay.

3. It would be worthwhile to develop a performance assessment methodology for time-variant processes, which would incorporate fault detection, as well as recommendations for controller tuning.

4. It will be certainly of interest to incorporate detection of abrupt change algorithms into the algorithms developed from this thesis.

5. Extension of this work to multivariable process is interesting but challenging.

Bibliography

Astrom, K.J. (1970). Introduction to Stochastic Control Theory. Academic Press, NewYork.

Basseville, M. and Nikiforov, I. (1993). Detection of Abrupt Changes. Prentice Hall, Englewood Cliffs, New Jersey.

Bequette, B.W. (1998). Process Dynamics, Modeling, Analysis and Simulation. Prentice Hall, New Jersey.

Bezergianni, S. and Georgakis, C. (2000). Controller Performance Assessment based on Mnimum and Open-loop Output Variance. Control Engineering Practice, **8**(7), 791- 797.

Box, G.E.P. and Jenkins, G.M. (1976). Time Series Analysis: Forecasting and Control. Holden-Day.

Bozenhardt, H. and Dybeck, M. (1986). Estimating Savings from Upgrading Process Control. Chemical Eng. Science, **93**(3), 99-102.

Cinar, A. and Undey, C. (1999). Statistical Process and Controller performance Monitoring. Proceedings of the American Control Conference, **4**, 2625-2639.

Desborough, L and Harris, T. (1992). Performance Assessment Measures for Univariate Feedback Control. Canadian Journal of Chemical Engineering, **70**(6), 1186-1197.

Desborough, L and Harris, T. (1992). Performance Assessment Measures for Univariate Feedforward/Feedback Control. Canadian Journal of Chemical Engineering, **71**(4), 605-616.

Eriksson, P.G. and Isaksson, A.J. (1994). Some Aspects of Control Loop Performance Monitoring. Proceedings of the 3rd IEEE conference on Control Applications, 1029-1034.

Forsman, K. (1999). Control Performance Monitoring-A Holistic Perspective. Process Control News for the Pulp and Paper Industries, **19**(9), 3-4.

Goodwin, G.C. and Sin, K.S. (1984). Adaptive Filtering Prediction and Control. Prentice Hall, Englewood Cliffs, New Jersey.

Harris, T. (1989). Assessment of Closed Loop Performance. Can. J. Chem. Eng., **67**, 856-861.

Harris, T., Boudreau, F. and MacGregor, J. (1996). Performance Assessment of Multivariable Feedback Controllers. Automatica, **32**(11), 1505-1518.

Harris, T.J., Seppala, C.T., and Desborough, L.D. (1999). Review of Performance Monitoring and Assessment Techniques for Univariate and Multivariate Control Systems. Journal of Process Control, **9**(1), 1-17.

Horch, A. and Isaksson, A.J. (1999). Modified Index for Control Performance Assessment. Journal of Process Control, **9**(6), 475-483.

Horch, A. (2000). Condition Monitoring of Control Loops. PhD Thesis, Royal institute of Technology, Sweden.

Huang, B. (1997). Multivariate Statistical Methods for Control Loop Performance Assessment. PhD Thesis, University of Alberta, Canada.

Huang, B. (1999). Performance Assessment of Processes with Abrupt Changes of Disturbances. Can. J. Chem. Eng., **77**(5), 1044-1054.

Huang, B. (2002). Minimum Variance Control and Performance Assessment of Time-Variant Processes. Journal of Process Control, **12**, 707-719.

Huang, B. and Shah, S.L. (1998). Practical Issues in Multivariable Feedback Control Performance Assessment, Journal of Process Control, **8**, 421-430.

Huang, B. and Shah, S.L. (1999). Control Loop Performance Assessment: Theory and Applications, Springer Verlag, London.

Huang, B.; Shah, S.L.; and Kwok, E.K. (1995). On-line Control Performance Monitoring of MIMO Processes. Proceedings of the American Control Conference, Seattle, Washington, 1250-1254.

Huang, B., Shah, S.L., and Kwok, E.K. (1996). How Good is Your Controller? Application of Control Loop Performance Assessment Techniques to MIMO Processes. Proc. 13th IFAC World Congress, M, 229-234.

Huang, B., Shah, S.L., and Kwok, E.K. (1997). Good, Bad or Optimal? Performance Assessment of Multivariable Processes. Automatica, **33**(6). 1175-1183.

Huang. B., Shah. S.L., and Miller, R. (2000). Feedforward plus Feedback controller Performance Assessment of MIMO Syystems. IEEE T. CONTR. SYS, **8**(3), 580-587.

Huang, H.P. and Jeng, J.C. (2002). Monitoring and Assessment of Control Performance for Single Loop Systems. Ind. Eng. Chem. Res., **41**(5), 1297-1309.

Hugo, A. (2001). Process Controller Performance Assessment. Hydrocarbon Process, **80** (4), 85-90.

Isaksson, A.J. (1996). PID Controller Performance Assessment. Proceedings of the 1996 Control Systems Conference, Halifax, 163-169.

Kadali, R., Huang, B., and Tamayo, C. (1999). A Case Study on Performance Analysis and Trouble Shooting of an Industrial Model Predictive Control System. Proceedings of the American Control Conference, San-Diego, California, 642-646.

Kendra, S.J. and Cinar, A. (1997). Controller Performance Assessment by Frequency Domain Techniques. Journal of Process Control, **7**(3), 181-194.

Ko, B.S.; Edgar, T.F. (2000). Performance Assessment of Multivariable Feedback Control Systems. Proceedings of the American Control Conference, **6**, 4373-4377.

Ko, B.S.; Edgar, T.F. (2001). Performance Assessment of Multivariable Feedback Control Systems, Automatica, **37**(6), 899-905.

Li, Z. and Evans, R.J. (1997). Minimum-Variance Control of Linear Time-Varying Systems. Automatica, **33**(8), 1531-1537.

Ljung, L. (1999). System Identification - Theory For the User. PTR Prentice Hall, New Jersey.

Ljung, L. and Soderstrom, T. Theory and Practice of Recursive Identification. MIT Press, England.

Lynch, C.B. and Dumont, G.A. (1996). Control Loop Performance Monitoring. IEEE Transactions on Control Systems Technology, **4**(2), 185-192.

Marlin, T, Barton, G., and Brisk, M. (1987). Advanced Process Control Applications. ISA, Research Triangle Park.

Martin, E.B., Morris, A.J., and Zhang, J. (1996). Process Performance Monitoring Using Multivariate Statistical Process Control. IEEE Proceedings on Control Theory and Applications, **143**(2), 132-144.

Qin, S.J. (1998). Control Performance Monitoring - A Review and Assessment. Computers and Chemical Engineering, **23**(2), 173-186.

Qin, S. and Badgewell, T. (1996). An Overview of Industrial Model Predictive Control Technology. Proceedings of CPC-V, Tahoe City, California.

Stanfelj, N., Marlin, T., and MacGregor, J. (1993). Monitoring and Diagnosing Process Control Performance: The Single Loop Case. Industrial and Engineering Chemistry Research, **32**(2), 301-314.

Sternad, M. and Soderstrom, T. (1988). LQG-optimal Feedforward Regulators. Automatica, **24**(4), 557-561.

Swanda, A.P. and Seborg, D.E. (1999). Controller Performance Assessment Based on Setpoint Response Data. Proceedings of the American Control Conference, **6**, 3863-3867.

Thornhill, N.F.; Oettinger, M.; and Fedenczuk, P. (1999). Refinery-wide Control Loop Performance Assessment. Journal of Process Control, **9**(2), 109-124.

Tyler, M.L. and Morari, M. (1996). Performance Monitoring of Control Systems Using Likelihood Methods. Automatica, **32**(8), 1145-1162.

Vishnubhotla, A. (1997). Frequency and Time-domain Techniques for Control Loop Performance Assessment. MSc Thesis, University of Alberta, Canada, 105 pages.

Vaught, R. and Tippett, J. (2001). Control Performance Monitoring: Shaman or Saviour?. Pulp and Paper Canada, **102**(9), 26-29.

Wan, S. and Huang, B. (2002). Robust Performance Assessment of Feedback Control Systems. Automatica, **38**(1), 33-46.