UNIVERSITY OF ALBERTA

PROMOTING MEANINGFUL TALK IN THE MATHEMATICS CLASSROOM

by

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ABSTRACT

The purpose of this study was to investigate a variety of instructional emphases designed to promote student ability to articulate mathematical knowledge. As part of this research, a unit of mathematics focusing on the meaningful interpretation of concepts was developed, and this unit was implemented using three different teaching approaches.

The research sought to determine if there were any differential effects on student ability to talk meaningfully about mathematics as a result of the type of instruction they received. A major part of the study was an investigation of the implementation of the unit by the cooperating teachers for the purpose of identifying those features of the classroom interactions that might have contributed to improvements in student talk.

Three grade eight mathematics teachers and three classes participated in the study. The three instructional approaches were identified as conventional teacher-centered, teacher-led discussion, and cooperative learning, and each teacher was responsible for a single approach. Data were collected through in-class observations, written tests, interviews, and small group tasks.

Results indicated that an emphasis on meaning tended to lead to an increase in teacher talk during class time, but not necessarily to a similar increase in student talk. Each of the teachers reported difficulty in implementing the unit due to the amount of information prescribed in the unit outlines. Other difficulties arising from the shift in instructional emphasis were also noted.

In the assessment of student talk at the unit's end, most students demonstrated an improvement in their abilities to identify meaningful

contexts for mathematical concepts. Students who were given greater opportunity to talk during class time demonstrated increased ability to articulate mathematical knowledge in the post-implementation assessments.

Based on these results, it was surmised that the factors most essential to the promotion of meaningful talk among students are the provision of opportunities to discuss concepts (both with the teacher and with fellow students), the example set by the teacher, the clear articulation of expectations on the part of the teacher, and an instructional emphasis on the meaningful interpretation of concepts.

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A. Introduction

Language -- spoken and written -- permeates every aspect of school life. For the student, the school is a place of listening, reading, writing, and occasionally speaking. Language forms the center of instruction, and a significant focus of educational research is on the careful selection of the language used to embody concepts so that they might be made accessible to learners.

But language may occupy an even more fundamental place in the learning process. According to psycholinguists such as Vygotsky and Piaget, language and thought are inextricably linked; and the simple act of talking about one's perceptions and interpretations of situations might dramatically affect the learning (that is, the development of the conceptualizations) that occurs. Thus language is seen as serving two purposes in the educational context: as a basis of communication and as a means of exploring and consolidating one's knowledge.

One would assume that, given the tremendous importance of language, students would be invited to articulate their thoughts and knowledge as a matter of course in the learning process. But such is not the case. Consistently it has been shown that students contribute little to the content of lessons, and what they do offer rarely extends beyond the regurgitation of factual information (see, for example, Barnes, 1986).

Nowhere is this more true than in mathematics class. The conciseness of mathematical expression, the use of familiar terms in unfamiliar situations, the uniqueness of the symbol system, the abstract nature of the concepts, the frequent decontextualization of ideas, and the algorithmic focus of lessons all contribute to render mathematics a subject which is simply not talked about in a consistently meaningful way. By the time students have reached the junior high school, most discussion of mathematics is reduced to simple restatements of definitions and algorithms, and the predominant perception is that mathematics is a set of rules designed to be used in specific situations (Hiebert, 1984).

While it may be argued that there is a role for such instrumentalism (see Resnick and Ford, 1981), many educators consider this perception of mathematics to be inappropriate. A number of recommendations have been put forward in an effort to ameliorate the situation. From the constructivists come two suggestions: that mathematics can be made more "meaningful" to learners if concepts are embedded in familiar contexts (Blais, 1988); and that educators must become more aware of the significance of personal interpretations of mathematical ideas (von Glaserfeld, 1987). To these ends, "talk" -- encouraging learners to manipulate concepts through some type of discourse as they attempt to make sense of mathematical ideas -- has recently emerged as a promising focus.

B. Background

The use of talk in the mathematics classroom is a topic which has received increasing attention over the past decade (Pirie et al., 1989). The reasons identified in support of this emphasis range from the notion that organized discussion will foster social skills among students

(Johnson et al., 1986) to the contention that such an approach will encourage pupils to take greater responsibility for their own learning (Hoyles, 1985).

Most reasons focus on the use of discussion as a tool for assisting learners as they construct knowledge (eg. Resnick, 1987; NCTM, 1989). The amount written on the importance of student talk in the development of understanding seems to have paralleled the increasing acceptance of constructivism among mathematics education researchers. In spite of this interest, however, there has been little or no empirical evidence to indicate that discussion is effective to this end (Pirie and Schwarzenberger, 1988).

Others view student talk as a valuable source of insight into student knowledge and conceptualizations. Pirie and Schwarzenberger, for example, viewed the ability to talk purposefully about mathematics as an important indicator of student understanding. It is thought that information gathered from student talk can be used to develop learning experiences better suited to each child's needs, previously acquired knowledge, and current perceptions. The extent to which student talk can be used to gather this sort of information, however, is also largely unknown.

The question of greatest interest in the area relates to the role played by talk in the enhancement of student understanding. The investigation of the issue has begun, but is hampered by the fact that encouraging students to express their knowledge and perceptions has not been widely implemented as a primary instructional emphasis in the mathematics classroom. There is, in fact, a conspicuous absence of research into techniques designed to promote talk. Lacking this, investigators are unable to develop an adequate research base for examining the essential qualities of student talk and assessing the relationship between talking about mathematics learning about mathematics.

C. Statement of the Problem

A commonly noted difficulty encountered by researchers in studying student talk about mathematics is that comments made by students often do not reflect the full extent of their understandings (Resnick, 1987), as children may lack the needed symbol systems or vocabulary, or are unable to articulate intuitive notions. Such observations are consistent with the contention that the ability to express one's knowledge -- particularly of mathematics -- does not occur naturally (Gatherer, 1977), leading one to question how the ability to talk about mathematical conceptualizations might be developed. Demonstrating that it is possible to encourage such skills and investigating instructional approaches designed to promote talk would represent important steps in the process of determining the relationship between articulation and mathematical understanding.

In structuring classroom environments conducive to talk, an important consideration is the identification and presentation of contexts which allow students to meaningfully relate mathematical concepts to their current stores of knowledge and experiences. Attempting to promote talk without providing situations familiar to students might be seen as somewhat counterproductive.

Another consideration is development and implementation of specific classroom activities to encourage talk. A cursory glance of the literature surrounding the issue presents a number of possibilities, including classroom discussions of mathematical issues, story-writing to illustrate concepts, and small group tasks to investigate mathematical issues.

The research question that emerges from this analysis is: What sorts of classroom experiences are most effective in developing student ability to talk meaningfully about mathematics? This question involves both the investigation of a variety of instructional emphases and the assessment of student ability to articulate their mathematical knowledge.

D. Definition of Terms

The key term in this analysis is "meaningful talk". Its definition is based in part on the work of Pirie et al. (1989) who defined "discussion" as talk among students on a mathematical topic in which the pupils are genuinely interacting (i.e. Piagetian ego-centric speech is omitted) and each participant is contributing. The current research is not primarily concerned with discussion -- that is the interactive version of talk -- but with the talk or articulation itself. Talk thus includes any verbal expression of mathematical knowledge, whether directed to the teacher, to the student, or to self.

The word "meaningful" is used to give an indication of the type of talk which is considered appropriate. Meaningful talk is defined according to the extent to which the speaker is able to relate a given concept to appropriate interpretations, which might include associated mathematical concepts, illustrative applications (or "contexts"), and analogous situations. The ability of the speaker to clearly articulate the relevance of a meaningful interpretation is also a feature of meaningful talk.

"Classroom experiences" refer to all the learning situations presented to the student within the classroom. In this research, the term is used primarily to refer to those experiences which are intended to improve abilities to talk about mathematics. These include the teacher-led discussions and the cooperative learning groups, in addition to the emphasis on the meaning of mathematical concepts.

"Mathematics" refers to the topics and concepts represented by the objectives outlined in the Junior High Mathematics: Teacher Resource Manual (Alberta Education, 1988) for grade eight.

E. Delimitations Imposed on the Research Setting

The research was conducted in cooperation with three grade eight mathematics teachers, each of whom took responsibility for a single unique approach to teaching a single unit. Classes were presented with the same topics of instruction. The choice, sequencing, and pacing of topics were based on the outline presented in the current curriculum guide (Alberta Education, 1988) and the textbook used by each of the teachers.

The teaching approaches used in this investigation were developed around a meaning emphasis and can be described as a conventional teacher-centered approach, a teacher-centered approach with an emphasis on classroom discussion, and a student-centered approach focusing on small group tasks.

It must be emphasized that the focus of this study -- one's ability to articulate mathematical knowledge -- is not viewed as an end, but as a means to an end. The eventual goal remains to develop learning opportunities which will foster student understanding of mathematics.

F. Significance of the Study

This investigation has both theoretical and practical significance to the field of mathematics education. As for theoretical considerations, the study contributes to the general understanding of the use of talk as a goal of instruction in the learning of mathematics. This is accomplished through the development of an interpretive framework for the analysis of talk and through the inclusion of meaningfulness as a primary focus in both promoting and analyzing student expression.

On the practical side, this study includes the development and analysis of three instructional approaches which might be of use to teachers wishing to encourage talk in their own classrooms.

G. Outline of the Report

The present chapter is an outline and preview of the study. Chapter II consists of a review of the literature relating to the research. Chapter III contains a description of the design of the study, and Chapters IV and V describe the development of the testing instruments and the three teaching approaches. Chapter VI consists of the results and analysis of the data gathered, while Chapter VII consists of a summary of the investigation and a discussion of possible conclusions and limitations, together with recommendations for further research.

Chapter II - Review of Related Literature

A. Introduction

The present study is an investigation of some of the instructional approaches which might be used to encourage eighth grade students to articulate their mathematical knowledge. All of these approaches are structured around an emphasis on the mathematical meaning of concepts and processes.

The study focuses on two issues: meaning and talk. This chapter thus begins with a brief look at meaning, including an analysis of the theoretical and psychological bases for such an emphasis. Included as part of this analysis is a discussion of constructivism, the theoretical perspective taken in the research. While it is not the purpose of this chapter to describe how a meaning emphasis can be brought into the classroom, some details on the topic will be outlined.

The theoretical base for the use of talk as an important mechanism in the learning process is then presented, followed by a more specific analysis of the use of talk in the mathematics classroom. Theoretical issues, research, and practical considerations are reviewed.

In addition, a few prominent research methods will be presented relating to the study of talk in the educational setting.

B. The Meaning Emphasis in Mathematics Education

Pimm (1987) has suggested that the construction of meaning rather than the question of rigor is the central problem currently facing mathematics education.

The term "meaning" has been interpreted in several ways. Resnick and Ford (1981) report that early attempts to make mathematics learning meaningful focused on embedding arithmetic skills and concepts in practical exercises with everyday life settings in an effort to reduce the risk of having students view mathematics as a set of unrelated facts and procedures. Later, from mathematicians, came the suggestion that mathematics would become more meaningful if students were taught the structures of mathematics. Resnick and Ford suggest that this emphasis was based on the notion that children could intuitively appreciate the concepts that underlie mathematical procedures. Alternatively, Schwab (1962) suggests that the emphasis may have emerged from the notion that learning the structure of mathematics is tantamount to learning the discipline.

Among today's mathematics education researchers, the term meaning is generally used to describe the variety of interpretations of a concept available to a learner. A concept is meaningful insofar as the person is able to think about and discuss is in a number of ways (Mick, 1987).

An emphasis on teaching for meaning has accompanied the emergence of constructivism as a dominant influence in mathematics education research. Two key underlying notions of this perspective, according to von Glaserfeld (1987), are that learners actively construct knowledge and that an individual's prior knowledge plays a critical role in her/his learning and performance. In this context, mathematical knowledge is not viewed as a store of retrievable facts, but as a product of reflection whereby the learner actively "makes sense of" or comes to understand new concepts.

Because learning is not viewed as a process of passive absorption and imitation, the responsibility for the active incorporation of information into one's own store of knowledge must fall on the student. The key instructional problem from the constructivist perspective is not one of teaching additional information, but one of helping students see connections between pieces of information they already possess. That is, learning involves the explicit construction of links between symbol procedures and intuitive understandings. Resnick (1987) suggests that current mathematics education "does not adequately engage students' interpretive and meaning-construction capacities" (p. 14).

Blais (1988) states that, from a constructivist point of view, meaningfulness is dependent on the number and type of associations between a concept and related concepts in the learner's cognitive structure, and he includes the use of manipulative materials, models, applications, and other mathematical concepts as possible bases for establishing these associations. Consistently, researchers have emphasized that a variety of such interpretations is important because, as Alibert (1988) puts it, "knowledge is all the more solid when it has been constituted and applied in more than one conceptual setting" (p. 31).

Hiebert (1984) argues that mathematics educators must acknowledge that children enter the classroom with substantial prior learning, rather than focusing on the formal manipulation of symbols and ignoring the intuitive knowledge of students. To this end, he suggests that verbal problems should not be viewed "as applications of learned symbolic procedures, but rather as referents for initial symbolic representation" (p. 508). Hiebert is recommending an instructional sequence which seems to presume that the teacher is able to identify learning contexts that lie within students' experiences and that are meaningful to them.

The educational consequences of a constructivist perspective are not minor. The teacher's task includes the selection of meaningful interpretations of concepts and the presentation of these interpretations in a manner which assists students in making appropriate associations to previously acquired knowledge. That is, the teacher must not only engage the students' sense-making abilities, but must somehow continually assess student conceptualizations in an effort to ensure that the sense being made is not fraught with misconceptions or missing details. From this perspective emerge two of the fundamental reasons for the promotion of student talk in the classroom: as a source of insight into their understandings and interpretations of the concepts under study, and as a means of determining the nature and extent of their existing knowledge.

C. The Talking-Learning Connection

Historically, talk in the educational setting has been viewed as valuable for quite a different reason than the ones presented above: that thought and language are inexorably linked, and speaking is thus closely tied to the processes involved in learning. Gatherer (1977) points out that the "relationship of language and thought has occupied the minds of scholars for centuries, and the growth of psychology and linguistics has produced a massive literature on the subject" (p. 61).

Vygotsky (1962) suggests that the acquisition of complex concepts seems to be inextricably linked with language: "the birth of a new concept is invariable foreshadowed by a more or less strained or extended use of old linguistic material; the concept does not attain to individual and independent life until it has found a distinctive linguistic embodiment" (p. 74).

A number of reasons can be identified for encouraging students to talk about what they are learning, ranging from the need (or desire) to share knowledge with another person to a means of self-analysis. As a form of communication in the mathematics context, Hoyles (1985) contends that talk forces the speaker to identify those aspects of a mathematical situation that are important for meaning and to describe how those aspects relate to one another, adding that "language facilitates reflection and internal regulation" (p. 206). As for the more egocentric use of talk, Austin and Howson (1979) describe how many professional mathematicians will talk to colleagues in a one-sided manner (i.e. the identity and attentiveness of the colleague would appear to be of little consequence). They also report that expert readers will start making subvocal movements when presented with difficult material, indicating the importance of even silent articulation in the interpretation of new information, and underscoring the connection between speaking and thinking. To this end, Pimm (1987) suggests that "articulating can aid the process of reflection by affording better access to thought itself" (p. 125). In a similar vein, Head and Sutton (1985) state that most of our thought constructs have a strong language component: "The resolution of a problem in understanding things usually involves adopting a particular way of talking about them.... Words form centers for the crystallization of ideas" (p. 92).

Skemp (1987) cites a number of benefits to the use of discussion in the learning of mathematics:

> The mere act of communicating our ideas seems to help clarify them, for, in so doing, we have to attach them to words (or other symbols), which makes them more conscious....
>
> Discussion also stimulates new ideas. One factor is simply the pooling of ideas, so that the ideas of each become available to all....
>
> The cross-fertilization of ideas is another benefit which comes from discussion. Listening to someone else (or reading what they have written) may spark off new ideas in us which were not communicated to us by the other, but which we would not have had without their communication. (pp. 88-89)

Skemp also points out that discussion involves the interrelating of ideas: "the expansions of our own schemas to enable us to assimilate [new] ideas, and the explanation of out ideas to enable [others] to assimilate our ideas into their schemas" (p. 88).

This notion is of course based on the work of Piaget (1963) who described two processes -- assimilation and accommodation -- used by the learner when incorporating new knowledge into her/his cognitive structure. In many cases, knowledge can be readily assimilated and Skemp seems to be suggesting that discussion can be used as a catalyst ... process. Other times, learners must accommodate, or reorganize ...nitive structure, to incorporate new knowledge. It seems ...ble to suggest that the manipulation of concepts through discussion, allowing one to analyze and contrast the new information with existing knowledge, would also facilitate the process of accommodation. Claxton (1984) supports this notion in stating that the purpose of discussion is to explore the implications of current understandings, to "build a trial bridge, and expose flaws in its design or suggest neater ones" (p. 107).

Talk has also been identified as a means to encourage the abstraction of ideas. Dienes (1963) suggests that having students talk about their perceptions causes them to generalize their learning away from concrete materials and situations. Dialogue may thus be a powerful means of creating links between formal and informal mathematics knowledge (Putnam, 1988) -- that is, a means of making the mathematics more meaningful to students.

Bruner (1975) views language as an instrument of thought, arguing that language is necessary for reflecting upon and evaluating one's own thinking processes. Gatherer (1977) takes the work of Bruner a step further in suggesting that

> 'speaking one's thoughts' is in a real sense the very act of thinking.... Speech is therefore the primary instrument of thought and there is a process of 'talking one's way through' ideas which is essential to learning. (p. 64)

He later states that learning to convey information and ideas is not a natural acquisition, and that teachers "should attempt to teach the language skills essential to [their] own subject" (p. 65).

D. Using Talk in the Mathematics Classroom

Over the past decade, the movement toward an increased use of discussion in the mathematics classroom has been gaining momentum. To this end, a frequently cited reference comes from the very influential British Cockcroft Report (1982):

> The ability to 'say what you mean and mean what you say' should be one of the outcomes of good mathematics teaching.... Pupils need the explicit help, which can only be given by extended discussion, to establish these relationships (between different mathematical topics) even pupils whose mathematics attainment is high do not easily do this themselves. (Section 246, p. 72)

In spite of this, as Mitchell (circa 1984) points out, the process of talking about mathematics is generally ignored by teachers as an aid to learning. He offers two reasons for this situation: mathematics is not seen as lending itself to discussion, and the role played by talk in the learning of mathematics is yet to be clearly identified.

Beyond those benefits suggested in the previous section for the promotion of talk in the mathematics classroom -- in addition to its potential use as a source of insight into student background knowledge and as a mechanism for monitoring student conceptualizations -- there are a number of advantages to discussion which are not strictly related to enhancing mathematical understanding.

Foremost among these relates to the student's perception of mathematics as a discipline. Using a teaching approach which required college-level students to participate in debates in addition to traditional lectures, Alibert (1988) noted significant improvements in students' perception of the role of mathematics in their lives. Similarly, Sigurdson and Olson (1989) reported that students involved in situations which invite increased participation view mathematics as being "more fun." Such comments are in welcome contrast to the oft-noted observation that students feel either alienated and bored with mathematics, or helpless and dependent on teachers (Hoyles, 1985).

Sullivan and Clarke (1989) add that encouraging classroom discourse (in this case, through questioning techniques) can help the teacher to tailor instruction to the broad range of abilities and needs found in a typical classroom. Further, Hoyles (1985) notes that using discussion "suggests a recognition of the significance of the socio-cultural setting in the development of a child's mathematical knowledge" (p. 205). Other benefits are described by Johnson et al. (1986) in their promotion of the cooperative learning paradigm. Increased interaction between students may result in improved social, communication (listening and speaking), and critical thinking skills.

A number of specific techniques have been proposed in the implementation of talk as a classroom tool. These can be broadly classified as Teacher-Directed Approaches, Inter-Student Talk; and Student Reflection. A common theme among these suggestions is that there is a need to engage the students' sense-making capacities through the provision of meaningful activities (see Lester, 1989) -- in essence, providing them with topics that are sufficiently familiar (meaningful) to encourage discussion.

Teacher-Directed Approaches

Most writers who are concerned with teacher-student communication in the mathematics classroom have as their focus the use of specific questioning techniques to encourage student reflection and to elicit student thought. Heddens (1986), for example, recommends the use of Bloom's taxonomy to guide students from concrete representations to abstract concepts. Sullivan and Clarke (1989) describe "good questions" as having more than a single solution (so the focus is on the process rather than on the solution), as involving situations that are meaningful and significant, as allowing students to work productively with minimal assistance, and as permitting extension and reflection upon completion of set tasks. Mitchell (c. 1984) describes the use of "interpretive discussion" which involves fairly lengthy, open-ended questions. Teacher talk is reduced to a minimum, and students are required to present opinions, defend interpretations, and analyze the views of others.

Lampert (1990) reports on an actual teaching project designed to affect discourse within the classroom. Using an action research approach, Lampert sought not only to promote classroom dicussion through investigations of mathematical "problems" which allow students to suggest, test, and defend mathematical hypotheses, but to "change students' ideas about what it means to do mathematics" (p. 58) in the process. Her report, which focuses on the classroom interaction that occurred during a brief series of lectures on exponentiation, includes a number of recommendations for promoting student participation, including criteria for selection of problems and the importance of modeling by the teacher. In her concluding remarks, Lampert claims that there was "convincing evidence that [the] students learned to do mathematics in a way that is consistent with the disciplinary discourse" (p. 58). However, she does not attempt to describe the nature of the changes in student ability to articulate mathematical knowledge, nor does she identify specific features of the instructional process which may have led to the noted improvements.

Peck et al. (1989) recommend the use of brief interviews in addition to pencil-and-paper tests, suggesting that interviews can yield useful information about children's understandings of mathematical concepts. Such an approach shifts the focus of instruction from the acquisition of skills to the acquisition of conceptual ideas and understandings. They add that students quickly discover the importance of searching for meanings when they are evaluated with respect to their understandings as well as their skill at producing answers.

A frequently noted difficulty with the methods mentioned above is that each places a heavy demand on teacher and classroom time, and yet relatively little opportunity is provided for each student to participate. As a result, numerous approaches that focus on written expression of knowledge rather than verbal talk have also been put forward. Clearly, there are some significant differences between written and verbal expression, including the time one is given to consider statements and the level of interaction with the audience, but the two forms of expression share some key features. Gatherer (1977), in describing the contribution of Wittgenstein, points out that the "verbal teasing out of one's thought which is characterized by 'thinking aloud' can also be achieved by expressive writing" (p. 65) -- a technique which sees little use outside of the English classroom. He further notes that the ability to write well on mathematical matters arises from an ability to "think well" in the language of mathematics, adding that "expressive writing should therefore become a normal practice within the subject; for this promotes good thinking in the subject" (p. 65).

Keith (1988) discusses the use of "explorative writing" wherein students investigate their knowledge about a topic by writing out what they know. Such exercises, she contends, can help expose and identify learning problems. A number of specific activities are provided in her article, including student journals, qualitative exam items, and concept mapping. Lester (1989) suggests that students should be invited to create their own story problems. Such an approach would result in familiar settings, and the problems would be stated in familiar language. Not only would it make evident the range of student experience, it would require students to search for associations between classroom mathematics and the everyday world.

Inter-Student Talk

Research indicates that the teacher does most of the talking with few students responding in most classrooms, and that improvements in mathematics instruction will depend largely on changes in organization, with more use made of manipulative materials and small group discussions (Austin and Howson, 1979). The intent here is to provide the students with increased opportunity to interpret mathematical concepts and greater responsibility for their own learning.

Significant attention has been focused on the use of small-group discussions as a classroom mechanism, with an increasing regard for the cooperative learning paradigm (as recommended by Johnson et al., 1986). A common caution regarding the use of discussion is that its effectiveness depends on a number of conditions relating to the structure of the task, the age and motivation of the children, and social relations within the group (Hoyles, 1985). In particular, it is noted that the social skills required for effective group functioning must be taught and encouraged by the teacher.

Alibert (1988) describes the use of "scientific debate" in which college-level students generate mathematical conjectures, discuss their validity, and then prove or disprove them formally. He states that this method gives rise to a clarification of individual student conceptualizations. This approach is consistent with Hoyle's (1985) emphasis on conflict as a source of discussion. Conflict (i.e. presenting students with possibly contradictory views) is seen as an effective method of developing an awareness that mathematical ideas are open to a variety of interpretations, and that meaning can only be shared after careful elaboration of one's own interpretation of the given concept.

In recent years, mathematics education has seen a movement towards the increased use of manipulative materials, especially at the secondary level. While many suggest that their purpose in mathematics instruction is "to help students understand (and remember) concepts" (Alberta Education, p. 221), another main argument for their use is that they provide students with concrete referents and thus facilitate discussion. Hoyles (1985) also claims that the use of the computer (specifically, Logo language) can encourage mathematical discussion. Similar arguments have been put forward for the use of applications, projects, and mathematical games by those encouraging the use of talk.

Student Reflection

It seems that little has been written on means of encouraging students to reflect on knowledge. Perhaps this is because we are, as yet, unable to motivate most learners to the point where they will independently contemplate the nature of their conceptualizations. Those techniques which have been suggested include a major emphasis on individual accountability. This is certainly the case with, for example, student journals, which tend to become a component of a teacher's evaluation scheme.

Many of the reasons that have been mentioned in support of meaningful talk (eg. as a means of increasing student interest, enjoyment, and appreciation of mathematics) may provide the motivation for such reflection. Certainly, student reflection can easily be seen to be the ultimate goal in a mathematics classroom, and may be best thought of as the indirect product of the sorts of activities recommended rather than the direct result of instruction. Clearly, there is a sizable gap between encouraging reflective thought and actual reflection among students.

Recent theoretical work in the area of metacognition may be seen as fitting in with this category. According to Costa (1984):

> Metacognition is our ability to plan a strategy for producing what information is needed, to be conscious of our own steps and strategies during the act of problem solving, and to reflect on and evaluate the productivity of our own thinking. (p. 57)

Costa outlines a number of strategies for enhancing metacognition, including journal-keeping, modeling, and paraphrasing student ideas.

E. Research on the Use of Talk in Mathematics

Pirie and Schwarzenberger (1988) point out that, while the notion that discussion is an aid to understanding has an intuitive appeal, the evidence to support the hypothesis is scant. They were able to find little or no published psychological work relating to the learning of mathematics (or, indeed, to general learning theory) which could substantiate the claim that understanding is enhanced by discussion. In a subsequent paper, Pirie et al. (1989) stated that, while there is an abundance of literature on the workings of cooperative groups, none of the research appears to focus on the connection between the qualities of discussion and student understanding.

Such conclusions are consistent with those of Austin and Howson (1979) who conducted a meta-analysis of the research that has been done in the area of language and mathematics education. Their purpose was to indicate topics for investigation and to draw attention to those areas in which research has begun. They noted, for example, that few writers in mathematics education research have derived much inspiration from research in linguistics. Among other relevant findings reported were that the premature introduction of formal language can be mathematically unproductive, and the level of vocabulary used in mathematics class is generally higher than that of other subject areas. This is particularly true of mathematics textbooks and, to a lesser extent, of the instruction based on those texts.

Austin and Howson recognized the need to work towards a theoretical framework for a study of classroom language. They also recommended that any study of the role of language in the development of mathematical concepts make a clear distinction between the language of the learner (eg. how a child's language development is related to the learning of mathematics), the language of the teacher (especially as it differs from the language of the learner), and the language of mathematics (i.e. its similarities and differences from vernaculars). They stated that any in-depth study must take into account such disciplines as sociology, psychology, and anthropology.

Pirie et al. (1989) reported on a longitudinal study of the issue of whether or not discussion in a mathematics classroom is an aid to understanding. They categorized talk into three broad groups: talk related to mathematics, incoherent verbal exchanges, and social chat. They chose to discard interactions with the teacher (which were considered to be adjuncts to exposition) as well as egocentric speech (i.e. verbalized thinking which does not involve interaction). The research methodology was based on an ethnographic, non-interventionist approach.

Analyses of tapes and transcripts have shown that few instances of talk fitted their definition, which included the criterion that students must make appropriate use of mathematical terminology. The findings were inconclusive, largely because classroom talk observed tended to be extremely imprecise. For example, they noted that students frequently resorted to conversation using "this", "that" and physical actions rather than making use of mathematical terms when talking about of manipulative materials which were intended to facilitate purposeful discussion. Consequently, the researchers have suggested that their definition of purposeful discussion may have been too restrictive; in that many instances of student talk were omitted, not because they were non-mathematical, but because the students did not make use of the prescribed vocabulary.

Hoyles (1985) described the implications of inter-student discussion within the mathematics classroom and presented several hypotheses as to the role of discussion in promoting mathematical understanding. Unlike Pirie et al., Hoyles included egocentric speech (i.e. reflective, non-interactive talk) as an aspect of discussion.

Hoyles interpreted her observations of pairs of children engaged in discussions of Logo-based problems as evidence that genuine discussion among pupils about mathematics is possible. Unfortunately, although identified as a primary reason for the use of discussion, no elaboration on its effects on student understanding was presented in the article, nor was a concise definition of discussion presented.

Despite these shortcomings, Hoyles did offer a definition of mathematical understanding from a social interaction perspective, recommending that it include the ability to apply a mathematical concept appropriately, communicate it effectively, and use it to defend or challenge the views of others. The definition reflects the notion that understanding is closely linked to one's ability to articulate knowledge.

F. The Study of Talk in Education

Pirie et al. (1989), as well as Austin and Howson (1979), have identified the need to work towards an interpretive framework for the study of talk in the mathematics classroom. To this end, numerous approaches for the investigation of the talk that occurs in educational settings have been developed. To the extent that some of these techniques have influenced this research, a few are briefly described below.

Barnes (1986) and Flanders (1970) have developed two unique descriptive systems used in the study of verbal interactions between teachers and students. Driven by the tenet that students should be encouraged to participate and to draw on previously-acquired knowledge, Barnes developed a system which focuses on two aspects of the interaction: students' participation (the amount and type) and teachers' questions (which were categorized as factual, reasoning, open, and social). Based on the analysis of lesson transcripts, Barnes noted that student participation is very low, they ask few questions, and the teacher rarely builds on their contributions. He also noted in his cross-subject analysis that there was a predominance of factual over reasoning questions, and entirely open-ended questions were not at all common.

Flanders developed a system comprising ten categories, seven for "teacher talk" (accepting feelings, encouraging, using student ideas, questioning, lecturing, giving directions, and criticizing), two for "pupil talk" (response and initiation), and one for "silence or confusion". This "live" analysis (i.e. not based on recordings or transcripts) consisted of a series of symbols recorded at three-second intervals which indicated what was happening at any moment in the lesson. Flanders' primary interest was in who controlled the topic and the extent to which student responses were used by the teacher, and he was not attempting a more detailed examination of the interactive use of language in his analysis. Among his findings was his "Two-Thirds Rule": typically, approximately two-thirds of classroom time is spent talking, two-thirds of the talk is by the teacher, and two-thirds of the teacher talk is "initiation" (lecturing, directing, or criticizing).

G. Summary

The emphasis on meaning in mathematics learning has accompanied the emergence of constructivism as an important influence in the field. The argument for meaning is based on the notion that learners invariably "make sense" of mathematical concepts by relating them to existing knowledge, although not always appropriately. Emphasizing appropriate meaningful interpretations would thus provide the learner with a more effective means of developing associations between concepts.

The reasons for encouraging student talk as part of an instructional approach in mathematics is supported for a number of reasons, including:

1. student talk provides insight into their background knowledge,
2. student talk provides a mechanism for monitoring student conceptualizations,
3. talking seems to be closely linked to thought and the development of conceptualizations,
4. the use of talk encourages students to take greater responsibility for learning and may positively affect attitudes towards mathematics,
5. using talk implies a recognition of the socio-cultural setting of learning.

Suggested techniques for implementing talk in the classroom include various teacher directed-approaches (for example, questioning techniques, interviews, expressive writing assignments), inter-student discussions (small group work, debates), and encouraging student reflection.

While the notion that talk affects understanding has an intuitive appeal, there is little evidence to support the hypothesis.

Chapter III - Design of the Study

A. Introduction

This comparative study was conducted at the grade eight level with the cooperation of three teachers, each of whom used a particular instructional approach in presenting the integers unit to one of their classes. This chapter consists of a detailed look at the research issues and brief descriptions of how these issues were investigated. The chapter includes descriptions of how the unit and sample were chosen, how the in-class observations were structured, and how the various assessments were conducted. The subsequent chapters provide a description of the instruments used and a more detailed overview of the development of the unit of study.

B. Research Issues

What sorts of classroom experiences are most effective in developing student ability to talk meaningfully about mathematics?

This question suggests two issues for investigation: the development and implementation of techniques to promote meaningful talk in the classroom, and the differential effects of the various instructional approaches on student ability to talk meaningfully about mathematics.

The approaches selected for investigation were a conventional teacher-centered model, a teacher-led discussion model, and a cooperative learning model, each of which focused on the meaningful interpretation of mathematical concepts. (More detailed descriptions of the approaches are included in Chapter V.) These approaches were selected because of the range of opportunities they provide in regard to the quantity of student talk and the quality of student talk. For example, the small group emphasis of the cooperative learning model allows for more student input than the teacher-led discussion approach, which places a greater emphasis on student talk than conventional approaches. As for the quality of student talk, with the classroom discussion approach the teacher is able to model meaningful talk and to monitor student articulation in class. In contrast, monitoring student talk is much more difficult in the small group settings of the cooperative learning approach when several students are able to speak at the same time. Further, with the conventional approach, the clarity of student talk is generally not a matter of central concern.

In assessing the development and implementation of these approaches, the following research questions were developed.

1. Do the lessons-as-taught reflect the implicit and explicit intentions presented in the unit outlines?

2. What is the effect of each of the approaches of the amount and type of talk that occurs in the classroom?

3. What difficulties arise when implementing each of these units?

For the investigation of the differential effects of the varied approaches on student ability to talk meaningfully about mathematics, three additional research questions were developed.

4. Does an emphasis on the meaningful interpretation of mathematical concepts in instruction assist students to talk more meaningfully about mathematics?

5. Does an increase in the opportunities students have to express their mathematical knowledge lead to improvements in their abilities to talk meaningfully about mathematics?

6. Does an emphasis on the quality of talk (i.e. clear articulation of one's knowledge), as is possible in the teacher-led discussion approach, promote student ability to talk meaningfully about mathematics?

C. The Choice of Integers as the Topic of Study

The integers unit was chosen for three main reasons: it is a relatively new topic to grade eight students, there is a variety of meaningful interpretations which can be readily applied to the concepts, and as a matter of convenience.

Integers are introduced only briefly as part of the grade seven mathematics course. Thus, in comparison to most of the other concepts which comprise the grade eight mathematics course, this unit explores relatively new material, making it reasonable to assume that student background knowledge in each of the three classes would be comparable, and possibly resulting in greater interest and fewer deeply-rooted misconceptions. Further, as most students are initially unaware of the algorithms used to perform the basic operations on integers, they may be less subject to the temptation of simply regurgitating and applying the "rule" and more amenable to examining the models and applications associated with the concepts at hand. These factors make it possible to structure the unit in a way that focuses both on the meaning of integers and on the fundamental meanings of the operations of addition, subtraction, multiplication, and division.

Perhaps the most important reason behind choosing the integers unit arises from the fact that the topic lends itself to a variety of readily accessible meaningful interpretations, including manipulative materials, models, diagrams, and a wide range of applications, some of which are briefly described in Chapter V.

The final reason for choosing the topic of integers, that of convenience, arose because the research was conducted near the end of the school year when the selection of remaining topics is somewhat limited. For each of the cooperating teachers, the integers unit was scheduled to begin in mid-April, making it ideally situated for the purposes of this study. The recommended duration of the unit of approximately three weeks (according to the Teacher Resource Manual, Alberta Education, 1988) was also appropriate to the scope of this research.

D. The Sample

Each of the three teachers volunteered to participate in the study and was contacted informally several months prior to the implementation of the integers unit. The assignments of teaching approaches were made on the basis of these preliminary discussions. In addition, information regarding school population, surrounding neighborhoods, and teacher background was gathered in order to establish comparability.

To further establish the comparability of the groups, each teacher and class was observed on three occasions prior to the implementation of the unit. During these observations, information was gathered on the formating of lessons, instructional emphases, and patterns of interaction in the classroom.

An analysis of the talk occurring during each lesson, based on the research framework developed by Flanders (1970), was also conducted during these observations. In this analysis, classroom activity was classified as either teacher talk, student talk, or silence/confusion at three-second intervals throughout the 40-minute period. The results were then converted to percentages of total class time, and these scores were averaged over the three observed lessons. Although the primary purpose for these observations was to establish comparability of classroom experiences across the groups of students, some of the data gathered served as a basis of comparison for the investigation of the effects of the prescribed integers unit on classroom procedures.

To establish comparability of the students in each group, three sources of information were used: scores from recently administered (February, 1990) Canadian Tests of Basic Skills (CTBS), the pre-implementation Achievement Examination, and the pre-implementation Explaining Test. The Achievement Examination and Explaining Test (as described in Chapter IV) were administered to each of the participating classes during a single 40-minute time block in the week preceding the implementation of the integers unit.

E. The Implementation

Each of the three teachers implemented a clearly-defined approach in presenting their units. These approaches, which are described in greater depth in Chapter V, are identified as:

Approach A: Conventional Meaning Approach,

Approach B: Teacher-Led Discussion Approach,

Approach C: Cooperative Learning Approach.

In order to ensure an accurate interpretation of the intents and methods presented in each of the unit outlines, a series of in-service sessions, in the form of informal discussions between the participating teacher and the researcher, were conducted prior to and during the implementation. In addition, four lessons for each of the approaches were observed (Lessons 2, 4, 6, and 9) in order to provide a basis for immediate feedback and to assess how closely the unit outlines were followed.

While conducting these observations, the researcher summarized and prepared an outline of the lesson as taught. This outline, which included details on the topics presented and the lesson format, was later compared to the lesson plan presented in the unit guide. To assess the effects of the prescribed lessons on classroom talk, the observed lessons were audio-taped and analyzed according to the proportion of teacher talk, student-to-teacher talk, and silence/ student-to-student talk/off-task behavior in the same manner as the pre-implementation observations (although, unlike those earlier analyses, these were not "live"). An examination of student talk during each lesson was also undertaken through the transcription of student verbal contributions during class time. In the case of the cooperative learning approach, a group was chosen randomly during each observation and recorded for a similar analysis.

F. Post-Implementation Assessments

The post-implementation assessments were designed to investigate the differences in student ability to talk meaningfully about integer concepts as a result of the varied instructional approaches. These assessments consisted of four parts: an Achievement Examination, an Explaining Test, Interviews, and Small Group Tasks (as described in Chapter IV). The written tests were completed at the end of the units, and the other assessments were conducted within one or two days of the written tests.

The Achievement Examination was included as a source of validation of the curricular appropriateness of the unit as it was presented to students. The results were not intended to be used as part of any subsequent analysis.

The Explaining Test served as a basis for a gross comparison of the students in the three classes. To minimize bias on the part of the researcher, the response sheets from all of the classes were combined and randomly arranged prior to scoring. Names of the students were also hidden (by folding the pages) in an effort to effectively render the grading anonymous. To determine the nature of any apparent differences in student ability to express mathematical knowledge in writing, an analysis of their responses (in terms of the range of meaningful interpretations identified and the clarity of articulation) was undertaken.

A more in-depth comparison of student talk was afforded by the Interviews and Small Group Tasks. These were administered to six members from each class who were chosen according to the following criteria:

1. three were male and three were female; and

2. all of the students selected fell within the second and third quartiles on each of the pre-implementation assessments: the Achievement Examination, the Explaining Test, and the CTBS scores.

The purpose of selecting students according to these criteria was to ensure that the candidates had similar profiles, thus permitting a more reasonable comparison of the results of the Interviews and Small Group Tasks. The selection included equal numbers of males and females because adolescent females have, on average, superior verbal abilities (see Maccoby and Jacklin, 1974). It would thus be inappropriate to compare verbalization skills without regard to the gender of the speaker. The selection was further restricted to students from the second and third quartiles as measured by the pre-implementation assessments on the basis of the findings of a recent study reported by Sigurdson and Olson (1989) in which it was demonstrated that an emphasis on meaning in instruction has the greatest impact on achievement for students in the average to above average range.

These assessments were evaluated according to a framework outlined in Chapter IV which yields scores for the content and the clarity of expression of the responses. Average scores for each item in these assessments were then produced using the individual and small group scores for each group of six students. These averages were compared to determine if any differences between group performances emerged. Wherever they occurred, an analysis of the differences in student responses (i.e. in the range of meanings presented and/or in articulation skills) was undertaken.

Chapter IV - Design and Preparation of the Assessment Tools

A. Introduction

Six separate tests were developed and administered as part of this study: the pre- and post-implementation Achievement Examinations, the pre- and post-implementation Explaining Tests, and the post-implementation Interviews and Small Group Tasks. This chapter presents the rationale behind each, and briefly describes how they were constructed and pilot tested. The chapter also includes a description of the Dimensions of Meaningful Talk used to evaluate student talk.

B. The Achievement Examinations

Pre-Test

The purpose of this test was to provide an overview of the average mathematical performance of students, and thus establish some measure of comparability for each of the classes participating in the research.

This examination consists of thirty multiple choice items drawn from previously administered examinations (unit tests and year-end examinations) for which some data was available regarding the difficulty and validity of each question. It is intended to assess student knowledge of the grade eight mathematics program as outlined in the Teacher Resource Manual (Alberta Education, 1988), and the number of questions on each topic is proportional to the number of hours recommended for that topic in the current curriculum guide.

The test has a 22-minute time limit so that it can be completed in the same 40-minute class period as the pre-implementation Explaining

Test. In this study the pre-test was administered by the cooperating teacher and scored by the researcher.

Post-Test

This 40-minute (one class period) test consists of forty-five multiple choice items and is intended to assess the levels of student mastery of the curriculum objectives associated with Integers, the topic of the unit of study, as outlined in the Teacher Resource Manual (Alberta Education, 1988).

Pilot Testing

The pre-implementation Achievement Examination was administered to three separate grade eight classes, and the post-implementation Achievement Examination was completed by two different grade nine classes. Responses to the multiple choice Achievement Examinations were computer-scored and analyzed. Those questions on which more than 90% or fewer than 10% of the students chose the correct response were deleted, resulting in the removal of five items from each of the examinations. Responses to the multiple choice items that were not chosen (or rarely chosen) by students were modified in an effort to present alternatives that reflect "typical" student errors.

C. Dimensions of Meaningful Talk

The "Dimensions of Meaningful Talk" is a framework used to examine student expression of mathematical knowledge. As no system had been developed to assess the quality of meaningful mathematical talk, it was necessary to construct such a framework in order to conduct this research. A first step was to identify the salient features of such talk, and these were broadly defined as the content (relating to the substance of the talk) and the presentation (relating to the clear articulation of the content).

Content is further defined as consisting of a meaningfulness component and a completeness component, as represented by the ordinal scales A1 and A2 in Figure 1. Presentation is analyzed according to the level of vocabulary (referring to the appropriate and consistent use of mathematical terms), logical sequencing (referring to the logical flow of statements), and fluency (referring to the portion of response time spent in silence). The ordinal scales B1, B2, and B3 in Figure 1 provide a clearer description of how these qualities are used in the analysis of student talk. In evaluating student talk, the entire utterance is analyzed (i.e. answers are not broken into smaller units). Scores on each of the individual components are added to produce three summary scores: content (A1 + A2); presentation (B1 + B2 + B3); and an aggregate index.

It must be noted that the framework has been developed to examine extended vocalizations and/or writings in the analysis of meaningful talk, as many of the components require that the speaker/writer presents more than a single statement. Noteworthy also is that fact that the description of meaningful talk offered by the components listed above is incomplete. Such qualities as authority, confidence, and enthusiasm, among others, might also be considered in a more comprehensive analysis.

FIGURE 1: **The Dimensions of Meaningful Talk**

A. Content (relating to the substance of student talk)

A1: **Meaningfulness**

0 - No response is given, or the talk is not related to the assigned task.

1 - The talk is on the assigned task, but no appropriate meaningful interpretations are brought to bear on the task (eg. a rote response dealing with the use of an algorithm is provided).

2 - A single meaningful interpretation is brought to bear on the assigned task.

3 - Two or more interpretations are brought to the assigned task.

4 - A novel and unique interpretation (i.e. one that has not been covered in class) is brought to the assigned task.

A2: **Completeness**

0 - The student is unable to elaborate on a meaningful interpretation.

1 - Some salient details are omitted.

2 - Some minor details are omitted.

3 - The student is able to provide a complete response.

B. Presentation (relating to the ability of the student to clearly articulate an appropriate response)

B1: **Level of Vocabulary**

0 - The vocabulary is entirely non-mathematical, or use of mathematical terms is entirely inappropriate.

1 - The vocabulary includes some mathematical terms, but only those which were presented in the statement of the problem.

2 - The vocabulary include some mathematical terms other than those presented in the problem statement, but their use is limited and/or sometimes inappropriate.

3 - Mathematical terms are used appropriately and regularly.

B2: **Logical Sequencing**

0 - The response consists of a single statement, or thoughts are disjointed and there is no apparent flow to the talk. (The talk consists of a series of disjointed statements.)

1 - There is some sense of logical flow, but some statements do not necessarily follow from prior statements.

2 - The student is able to provide a logically-sequenced explanation.

B3: **Fluency**

0 - More than 50% of the time spent responding is in silence or incoherent utterance.

1 - There is some hesitation. Between 25% and 50% of the time spent responding is in silence or incoherent utterance.

2 - The response is fluent. Less than 25% of the time spent responding is in silence or incoherent utterance.

D. The Explaining Tests

Rationale

These written tests were developed as a means of gross comparison of the students in the three classes in terms of their ability to talk meaningfully about mathematics. Although it is recognized that there is a significant difference between verbal and written expression, and that a written test is limited insofar as it does not permit the active face-to-face interaction that is possible in the interview setting, the written test has the distinct advantage of allowing the researcher to assess all the students without consuming an excessive amount of class time.

Pre-Test

The first Explaining Test consists of three items which were selected according to the criteria that each must be based on concepts that are studied at the grade eight level and that there are a number of interpretations that can be identified when responding. Student answers are scored according to the Dimensions of Meaningful Talk (with the necessary exclusion of the Fluency component).

In the study, each student was given a copy of the question sheet (as shown in Figure 2) and a piece of foolscap. They were also informed that scores would be determined according to the following criteria:

a) the correctness of the answer,
b) the clarity of explanation,
c) proper grammar,
d) the variety of responses presented.

Teachers were asked to underscore the importance of identifying a variety of examples. The purpose of the emphasis on presentation indicated in this set of instructions was to encourage students to present more elaborate responses.

This test had a 12-minute time limit and was written immediately after the completion of the pre-implementation Achievement Examination.

Post-Test

This Explaining Test consists of four items, as presented in Figure 3, which are to be completed in a recommended time span of 20 minutes. The questions were selected according to the same criteria as the pre-test items. This test was structured and administered in much the same manner as the pre-implementation Explaining Test, and was also evaluated according to the Dimensions of Meaningful Talk.

Pilot Testing

As with the Achievement Examinations, the pre-implementation Explaining Test was completed by three grade eight classes, and the post-intervention Explaining Test was written by two grade nine classes. Earlier, a preliminary version of the Dimensions of Meaningful Talk was developed and was used as a basis for initial evaluations of student responses. The actual evaluation process consisted of reviewing all student responses and then altering the Dimensions of Meaningful Talk in an effort to develop a framework that provided a clearer and more accurate description of the sorts of responses provided by students. Their answers were analyzed several times in this process.

Similarly, the phrasing of the questions was also altered in an effort to provide a clearer indication of the type of responses desired, These modifications were based both on written student responses and on the questions that they asked while writing the tests.

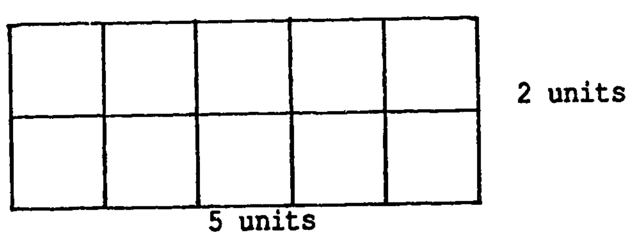
FIGURE 2: The Pre-Implementation Explaining Test

Part 2 - Explaining Test

Instructions: Answer each question with <u>complete sentences</u>. <u>Use examples</u> in your explanations. Write your answers on the paper provided.

Your responses will be graded according to:
a) your choice of example(s), and
b) how clearly you express your answer.

1. One way to explain "multiplication" is to talk about the area of a rectangle. For example, 2 x 5 is equal to the number of squares in a rectangle that is 2 units by 5 units.



What is another way of explaining "multiplication"?

2. To answer a question, a student used the following equation:

$$5 - (0.65 + 0.10) = 4.25$$

Write a real life problem that would use the same equation.

3. The following graph was in a Social Studies book, but the publishers forgot to print the labels. What might the graph represent? (Tell what information is missing and what the graph might mean.)

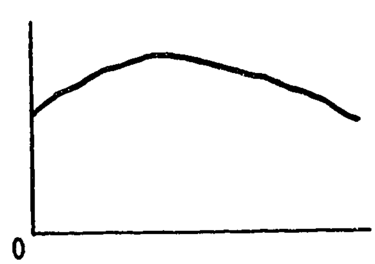


FIGURE 3: The Post-Implementation Explaining Test

JUNIOR HIGH MATHEMATICS
GRADE 8 INTEGERS TEST

Instructions: Answer each question with complete sentences. Use examples in your explanations. Write your answers on the paper provided.

Your responses will be graded according to:
a) your choice of example(s), and
b) how clearly you express your answer.

1. Where might we use "positive numbers" and "negative numbers"? For each answer, give an example to show how the numbers are used.

2. To answer a question, a student used the following equation:

$$(-6) + (+2) = -4$$

Write a real life problem that would use the same equation.

3. How would you prove to a friend that the following statement is true?

When you add two negative numbers together you get another negative number.

4. How would you prove to a friend that the following statement is true?

When you multiply two negative numbers together, you get a positive number.

E. Interviews and Small Group Tasks

Interviews

The individually-administered interviews consisted of four items which were similar to those found in the Explaining Tests. These were presented to the student both verbally and in an abridged written form, as shown in Figure 4. Verbal responses were audio-recorded, transcribed, and assessed according to the Dimensions of Meaningful Talk. As part of this analysis, responses were timed, and the portion of time spent in silence or incoherent utterance was determined for the purpose of assessing the fluency component. Prompts used during the interviews were limited to re-reading questions, to reminding students to provide examples, and to requesting further information for specific interpretations.

Small Group Tasks

The Small Group Task consists of a single item for which groups of three students (mixed gender and representing, to the extent possible, a range of abilities) were required to first discuss the task for 15 minutes and then present a group response to the researcher. The researcher was not present during the group discussion. The Small Group Task as presented to the students is shown in Figure 5.

Both the discussion and the presentation were recorded; and each was analyzed in the same manner as the Interview items, although group scores rather than individual tallies were assigned. This resulted in two sets of scores for each group: (A) one based on the 15-minute small group discussion and (B) a second based on the presentation of the group response to the researcher.

FIGURE 4: The Interview Items

Question as Read to Student by Researcher	Question as Presented to Student in Written Form
Instructions: Listen carefully to each question. It will be repeated as many times as you wish. Answer the question in complete sentences, giving as many examples as you can and speaking as clearly as possible. Try to think about your answers before you start responding. You do not have to answer immediately.	GRADE 8 INTEGERS
1. Pretend that I'm a person who has never heard of an "integer." It's your job to explain to me what an integer is and where integers are used. What would you tell me?	1. What is an INTEGER?
2. What does "zero" mean?	2. What does ZERO mean?
3. To answer a question, a student used this equation: (+3) x (-8) = -24. Make up a real-life problem that would use the same equation.	3. (+3) x (-8) = -24
4. a) When a positive number is added to another positive number, the answer is always positive. For example: (+2) + (+8) = +10.	4.a) (+2) + (+8) = +10
b) And the answer when you add two negative numbers is always negative. For example: (-6) + (-7) = -13.	4.b) (-6) + (-7) = -13
c) But when you add a positive number and a negative number together, sometimes the answer is positive, sometimes the answer is negative, and sometimes it's zero. Give an example of each case, and explain why this happens using examples.	4.c) +ve + -ve = { +ve; -ve; or zero.

Figure 5: The Small Group Task

Small Group Task

You are a teacher and have to prepare a 15-minute lesson on the addition of integers for a group of grade 7's who have never done the topic before. Describe what you would say in the lesson, including the examples you might use and some of the questions that might be asked.

You have 15 minutes to plan the lesson. At the end of the time, you will present your plan. Each person must participate in the presentation.

Pilot Testing

Interviews were conducted with eight students (four in grade eight and four in grade nine), and the Small Group Task was completed by two groups of grade nine students. Their responses were audio-recorded and transcribed, and then analyzed and scored according to the Dimensions of Meaningful Talk which had, by that point, already undergone considerable revision. The results of the timed analysis of student responses served as a basis for the modification of the Fluency component of the framework.

Informal interviews with the students were conducted following the administration of the tests. On the basis of information gleaned from these interviews, some questions were altered to make them clearer and it was decided that students should be given a written version of the questions in addition to the oral version provided by the researcher.

Chapter V - Unit Development

A. Introduction

The purposes of this chapter are to identify some of the key features of the unit design and to delineate some of the differences in emphasis among the three approaches. Because the mathematical content of the three units is identical, only one unit plan has been included in this report (the Cooperative Learning Approach in Appendix 1). Appendix 2 includes copies of the first lesson from each of the three unit plans as a basis of comparison of the outlines as provided to the cooperating teachers.

B. A Unit Overview

The learning objectives for this unit are based on those outlined in the Teacher Resource Manual (Alberta Education, 1988) for the topic of integers at the grade 8 level. The structure of the unit is derived from the sequence of concepts presented in the textbook used by each of the teachers, Journeys in Mathematics 8 (Connelly et al., 1987).

The unit begins with a review of the definitions of the terms "integer", "positive", "negative", and "zero", along with a look at a variety of situations in which integers are used. It is followed by a lesson on the comparison of integers. For each of these topics, an emphasis is placed on identifying familiar terms and applications associated with each concept. (For example, "zero" might be associated with the freezing point of water, sea level, par, or a neutral electrical charge.)

The bulk of the unit focuses on the basic operations with integers. In each case, students are to be provided with experiences with manipulative materials (bingo chips), diagrams (in the form of a worksheet based on the use of the bingo chips), models (number lines and thermometer scales), applications, efficient algorithms, and the calculator. The same process is followed in developing each of the algorithms: concepts are introduced at the intuitive level through the presentation and discussion of appropriate meaningful interpretations, first of the operation (eg. subtraction was presented as the opposite of addition, as "taking away", and as the distance between two points on a number line) and then of the operation on integers; several examples and exercises are done prior to the discussion of a rule; and through an analysis of emerging patterns, algorithms are developed. Drill on the use of these algorithms is purposely de-emphasized in the hope that students will not become totally reliant on a rule at the expense of the learning contexts provided. In place of this sort of practice, a heavy emphasis is placed on "real life" applications of integers, particularly for the topics of addition and subtraction.

C. Lesson Plan Formats

The structure of each lesson is based loosely on that recommended by Good et al. (1983), as follows:

A. Daily Review (first 8 to 10 minutes);
   1) review of concepts
   2) homework
   3) mental computation exercises

B. Development (from 15 to 30 minutes);
   (including important points, topics of discussion, relevant questions, and other classroom activities)

C. Seatwork (and/or Small Group Work);
(to be completed and reviewed during class time)

D. Homework Assignment.

The standardized format was chosen for two reasons. First, as will be noted in the results chapter, it reflected the lesson structure already used by each of the teachers, thus minimizing potential difficulties associated with the implementation of the new instructional emphases. Second, it allowed the researcher to structure three units which were identical in content and pacing, but clearly different in terms of the teaching approach.

Further, it must be emphasized that this format was strictly a guideline and the teachers were not expected to rigidly adhere to the timeline provided. In the case of the Cooperative Learning Approach, for example, considerable flexibility was required to accommodate an appropriate amount of time for small group work (usually 15 to 20 minutes per class).

D. Differences in Unit Structure Among the Groups

The purpose in developing three approaches to a single unit was to provide a range of opportunities for student talk -- from straightforward presentations of meaningful interpretations to exercises which require thoughtful and involved discussion.

Each teacher was provided with a manual consisting of a rationale and a brief overview of the prescribed approach, outlines for the twelve lessons, and master copies for each of the handouts required in the units (see Appendix 1). In each case, the introduction of the specific concepts followed the same sequence; and the pacing of topics, to the extent possible, was held consistent across the groups. Further, the information to be presented to the students was the same, and students in each group were assigned the same textbook exercises and worksheets.

The primary differences in the unit outlines provided to the teachers are in the manner in which each of the lesson plans are presented; in particular, in the sorts of opportunities students are given to talk about the concepts under study. In the case of Approach A (Direct Meaning Approach), each lesson plan consists of little more than a list of the meaningful interpretations to be introduced and the assignments to be completed. The manner in which these are presented is left to the discretion of the teacher, with the intention that instruction be similar to conventional teacher-centered approaches. (In order to reduce the risk having this teacher duplicate the Discussion Approach which was implemented by Teacher B, the Direct Meaning Approach Teacher was not informed as to the true purpose of the study. Instead, she was told that the focus of the research was on a variety of issues associated with the meaning emphasis in mathematics instruction.)

The lesson plans for Approach B (Teacher-Led Discussion Approach) contain the same information, supplemented with some suggestions as to how meaningful talk among students might be promoted through the course of each lesson. These recommendations take the form of possible questions and student activities. Instruction is to remain primarily teacher-centered, with an increased emphasis on student input.

The manual for Approach C (Cooperative Learning Approach) consists of the same basic information as that of Approaches A and B, plus some suggestions as to how group work might be used effectively in each lesson according to the cooperative learning model as set out by Johnson et al. (1986). Possible means of increasing student accountability during group work are also provided in the lesson plan.

These differences can be illustrated through a quick comparison of the first lesson in each of the units. (A copy of each of these lessons is provided in Appendix 2). The Direct Meaning Approach (A) lesson consists of a straightforward outline of the information to be presented (eg. a list of some situations in which integers are used). The lesson for the Teacher-Led Discussion Approach (B) contains this same outline, as well as some questions which might be used to spark discussion (eg. "What does positive seven mean in this case?") and guidelines for presentation (eg. "Stress the appropriate use of terms ...", "Model answers ..."). The Cooperative Learning (C) lesson plan provides the same guidelines for presentation, but outlines a small-group activity in place of the suggestions to promote discussion. Similar differences can be found in most of the other lessons.

Each guide also includes a mid-unit quiz (which was graded by the teachers). None of the other assessment tools are included in the guide, but they were provided to the teachers upon completion of the unit.

Chapter VI - Results

A. Introduction

This chapter provides a summary of the data collected through the course of the study, including information on the classroom observations conducted prior to and during the implementation, and results of pre- and post-implementation assessments.

B. Establishing the Comparability of the Three Classroom Settings

Each of the participating teachers had more than ten years mathematics teaching experience at the junior high level. At the time of the research, the teaching assignment of each included three sections of grade eight mathematics (in addition to grade nine mathematics, health and/or religion).

The schools were located in predominantly middle class neighborhoods. Two of the schools offered kindergarten through grade nine and the third was strictly junior high (although situated adjacent to an elementary school). For mathematics instruction, all class periods were 40 minutes in duration. Two schools (A and B) offered five classes of mathematics each week, while School C offered six. Class sizes of the Groups A, B, and C, were 23, 24, and 27, respectively. Table 1 provides a summary of comparison of the teachers and schools.

The lesson formating analysis portion of the pre-implementation observations indicated that the teachers structured their class time in similar manners. Each began by reviewing homework and presenting a brief lesson, and ended the class with seatwork and/or an assignment, as summarized in Table 2.

TABLE 1: A Comparison of the Participating Teachers and Their Schools

	Teacher A	Teacher B	Teacher C
Educational Background	4-year B.Ed., Math Major	4-year B.Ed., Math Major	4-year B.Ed., Math Major
Experience	12 years (all at junior high level)	25 years (most at junior high level)	11 years (all but 1 a at j.h. level)
Current Assignment	Math 8, 9; Religion (3 sections of Math 8)	Math 7, 8, 9; Health (3 sections of Math 8)	Math 8, 9; Religion; Health (3 sections cf Math 8
	School A	**School B**	**School C**
Grades	7 to 9	K to 9	K to 9
School Population	360 students	680 students (350 j.h.s.)	550 students (230 j.h.s.)
Number of Sections of Grade Eight	5	5	3
Neighborhood (as described by the teacher)	Middle Class (large ESL population)	Middle Class (large ESL population)	Lower Middle - Middle Class
	Group A	**Group B**	**Group C**
Duration of Math Class	40 minutes	40 minutes	40 minutes
Frequency of Math Class	5 classes/wk (not daily)	daily	6 classes/wk
Class Size (Males/Females)	23 (14/9)	24 (10/14)	27 (13/14)
Textbook	Journeys in Mathematics 8	Journeys in Mathematics 8	Journeys in Mathematics 8

During the observations, the lessons as taught were outlined by the researcher and the time spent on each section of the lesson was recorded. This was done over three lessons, and the results were averaged to produce the data presented in the second half of Table 2. It must be noted that it was not possible to observe lessons in which the same topics were being presented. Consequently, the information presented in Table 2 cannot be used as a basis for a strong comparison. It is useful, however, as a source of verification of the teachers' own descriptions of their lesson formats; and it does offer further evidence that the mathematical experiences of the students in the three classes were similar.

TABLE 2: Pre-Implementation - Lesson Formats

	TEACHER ESTIMATIONS		
	Teacher A	Teacher B	Teacher C
Homework Check	10 minutes	10 minutes	10 minutes (with review)
Lesson	15 minutes	15 minutes	10-15 minutes
Notes taken during lesson	usually	rarely	occasionally
Seatwork	10-15 minutes	10-15 minutes	15-20 minutes
Homework (average per night)	15 minutes	20 minutes	20 minutes
	AVERAGE ACTUAL TIMES DURING OBSERVED LESSONS (Based on the observation of three lessons)		
	Teacher A	Teacher B	Teacher C
Homework Review	8 minutes	8 minutes	11 minutes
Lesson	11 minutes	12 minutes	7 minutes
Seatwork	18 minutes	15 minutes	16 minutes

The results of the analysis of the portion of classroom time taken up by teacher talk, student talk, and silence/confusion indicates that in each of the classrooms, from 42% to 49% of class time was occupied by teacher talk; and 45% to 52% of class time was spent in silence or confusion. This means that student talk accounted for 4% to 8% of class time, on average, during the observed lessons. These data are summarized in Table 3. Again it must be noted that the analysis is based on lessons which covered different topics, and a detailed comparison is thus inappropriate. However, these data indicate that the use of class time, as well as the opportunities for student talk, were similar.

TABLE 3: Pre-Implementation - Analysis of Classroom Talk
(Based on the observation of three lessons)

	Class A	Class B	Class C
Teacher Talk	44 %	49 %	42 %
Student Talk	4 %	6 %	8 %
Silence/ Confusion	52 %	45 %	50 %
TOTALS	100 %	100 %	100 %

C. Assessing the Implementation of the Three Instructional Approaches

The purposes of the classroom observations conducted during the implementation were to assess the manner in which the prescribed approaches were implemented by the teachers and to gather information on the effects of the emphasis on meaningful talk on classroom procedures.

The observations consisted of three components: an analysis of the formating of the lessons as taught, an analysis of the type of talk occurring in the classroom, and an examination of the meaningful interpretations presented by each teacher. Each of these was contrasted with the implicit and explicit intentions contained in the unit plans provided.

Lesson Formats

In the guidelines presented in the unit plans, it was suggested that the introductory portion of each lesson should take 8 to 10 minutes and the actual lesson 15 to 30 minutes (depending on the topic and the approach), with any remaining time set aside for seatwork. During the observed lessons, the actual times for each portion of the lessons tended to vary considerably from these recommendations, particularly in the case of Teacher B (Teacher-Led Discussion Approach). The introductory portion of her lessons lasted, on average, nearly 20 minutes, forcing her to reduce the time for development to just over 12 minutes, and leaving less than 8 minutes for seatwork. This appeared to be in part a consequence of her efforts to encourage students to articulate their knowledge during the time set aside to review the concepts covered in the previous lesson, causing her to use what might be considered an exorbitant amount of time during the preliminary portions of the lesson.

On the average, Teacher A was fairly successful in adhering to the guidelines, but at the expense of omitting certain exercises (for example, the "Time Zones" worksheet and some of the mental computations).

Teacher C completed the introductory portion in just under 11 minutes, on average, but was forced to present the lesson in a brief 11 minutes in order to allow sufficient time for small group work. While she reported that this meant she had to "rush through the lesson," she also noted that any detrimental effects were minimized by the fact that students had the opportunity to discuss the concepts with their peers.

Table 4 presents the results of the lesson format analysis. Each lesson has been divided into three sections, as noted in the leftmost column of the chart; and the time spent over the four observed lessons is given as a percentage of the total lesson time. The bracketed figures indicate the average time spent in a 40-minute period. The final column indicates the times recommended in the unit manuals.

TABLE 4: During Implementation - Lesson Formats
(Based on the observation of Lessons 2, 4, 6, and 9)

	DIRECT-MEANING (Approach A)	TEACHER-LED DISCUSSION (Approach B)	COOPERATIVE LEARNING (Approach C)	Recommended
Introduction/ Review/ Mental Computation	29.9 % (12:00)	50.8 % (20:00)	26.8 % (10:40)	20-25 % (8-10:00)
Lesson	37.2 % (14:50)	30.3 % (12:10)	27.7 % (11:10)	40-75 % (15-30:00)
Seatwork (including time to correct it)	32.9 % (13:10)	18.9 % (7:30)	45.4 % (18:10)	(Any time remaining)
			(Small Group Work 31.2 % (12:30))	

Classroom Talk

As for the analysis of classroom talk, a large increase was noted in the amount of time Teachers A and B spent talking -- up to over 26 minutes, or from 65% to 70% of classroom time, on average. This represents a marked increase over the pre-implementation levels of less than 50% in each case. Teacher C also talked more, but used less than 50% of class time.

Of interest also is the fact that Teachers B and C spent more than twice the amount of time on classroom management (an average of 2 minutes) in comparison to Teacher A. In the case of Teacher B this seems to have been due to the fact that students appeared to become quite restless and increasingly inattentive during the lengthy teacher-dominated introduction. For Teacher C, extra management time was required to re-arrange and re-focus students for group work.

In comparison to teacher talk, student talk (that is, the students' contributions to teacher-student discourse) was scant, accounting for only 3 minutes of class time in Approach B and slightly less in the others. For the Teacher-Led Discussion Approach, these 3 minutes represent substantially less time than was anticipated during the design of the approach. However, although the quantity of student talk appeared to be at the same level as pre-implementation levels, the type of pupil contributions during the lesson were quite different, as will be discussed later.

A summary of the results of the analysis of talk in the classroom is contained in Table 5. The categories used in this table, as indicated in the leftmost column, are "Teacher Talk - Lesson" (when the teacher was talking specifically about the concepts which formed part of the lesson), "Teacher Talk - Management" (which includes all other instances of teacher talk), "Student-to-Teacher Talk" (which includes only those utterances focusing on mathematics), and "Silence/Student-to-Student Talk/Off-Task Behavior" (which covers everything else, including the time taken for seatwork and small group activities). As with the previous table, the numbers indicate the time spent in each category over the four lessons expressed as a percent of the total lesson time. Bracketed figures indicate the average time spent in each 40-minute period. This analysis is based on audio-recordings of the lessons, which made it possible to validate the results by repeating the analysis. This was done for four of the twelve observed lessons, and the results varied by less than 3% in all cases.

TABLE 5: During Implementation - Analysis of Talk
(Based on the observation of Lessons 2, 4, 6, and 9)

	Class A	Class B	Class C
Teacher Talk (On Lesson)	62.0 % (24:50)	65.8 % (26:20)	42.8 % (17:10)
Teacher Talk (Management)	2.0 % (0:50)	4.8 % (2:00)	4.8 % (2:00)
Student-to-Teacher Talk	6.7 % (2:40)	7.3 % (3:00)	5.3 % (2:10)
Silence/Student-to-Student Talk/ Off-Task Behavior	29.3 % (11:40)	22.3 % (9:00)	47.3 %* (19:00)

* This includes the time set aside for small group work, during which there was considerably more student talk than is indicated.

A further analysis of student responses in each lesson yields the data presented in Table 6, which indicate the lengths of student utterances (including both their responses and questions) during the "Student Talk" portions of the observed lessons. These data suggest that students in Class B offered nearly twice as many "long answers" in comparison to students in Class A, and 30% more than students in Class C, so the student contributions in Group B were much more directed toward "explaining" than in the other classrooms. The fact remains, however, that their contributions accounted for fewer than three minutes of class time, on average.

TABLE 6: During Implementation - Analysis of Student Utterances (Based on the observation of Lessons 2, 4, 6, and 9)

	Class A	Class B	Class C
Average Number of Single Word Responses	14	26	12
Average Number of Single Number Responses	45	26	46
Average Number of Utterances Longer than a Single Word	7	12	9
Average Number of Words in an Extended Utterance	9	9	7

Of perhaps equal importance were the differences in teacher expectations as articulated to students. For example, Teacher B, in contrast to the others, regularly alerted the students to the importance of "being able to explain what you know about math" and made it clear that part of their unit grade would be based on their responses to the sorts of questions they were being asked. She also posed nearly three times the number of "long-answer" questions (averaging approximately 20 per lesson) in comparison to the other teachers, although she tended to cut off students answers, accept incomplete (i.e. one-word) responses, or provide responses herself when answers were not immediately forth-coming. To a large extent, this accounts for the fact that there were twice as many single word responses in Class B as in the other classes.

An effort was also made to analyze student utterances according to the Dimensions of Meaningful Talk, but no notable differences emerged. Possible reasons for this include:

- the talk tended to focus on single interpretations, rather than a variety of possibilities;
- the teacher, in general, requested only a small amount of information on the topic at hand, rather than encouraging the student to provide a broader explanation (as is the case in the Explaining Tests);
- the utterances tended to be brief and few remarks would have been comprehensible if presented outside the context of the lesson.

For these reasons, the use of the Dimensions of Meaningful Talk for the analysis of student talk during class time was deemed inappropriate. The data from this analysis are thus not included in this report.

Lesson Content and General Comments

With regard to the presentation of meaningful interpretations during the lessons, each of the teachers adhered to the topics and concepts as outlined. In each of the observed lessons, all of the meaningful interpretations were presented, although not always as intended by the researcher. For example, Teacher A chose not to use thermometer scales or time zones when discussing addition, but introduced them for subtraction. Further, with Group A, the students were paired to work with bingo chips for addition and subtraction (rather than working individually); and, although bingo chips were used during the presentation of multiplication and division, students did not actually manipulate the chips themselves for those concepts.

Teacher B made a concerted effort to include all the prescribed meaningful interpretations -- so much so that she ended up talking for more than 70% of each class, on average (as noted above). As with Teacher A, it seemed that one of the immediate effects of the increased emphasis on meaning was a dramatic increase in the amount of teacher talk. In both cases, this increase was largely attributable to the introduction of the broad range of contexts which simply demanded more explanation. Attempts to encourage discussion in Approach B, as already mentioned, required even more time to ask and respond (or review the response) to explaining-type questions.

Perhaps the greatest difficulty encountered by Teacher B was in developing effective discussion-promoting questions. It appears that the Teacher-Led Discussion Approach requires a great deal of planning and foresight, and may have represented an instructional emphasis which was simply too different from this teacher's normal approach.

In terms of lesson formating, Teacher C managed to implement her approach more effectively than the other teachers. This meant that she talked less, and students were provided with, on average, 10 to 15 minutes each day to discuss the concepts and the assignment. Typically, there was not a great deal of what would be classified as meaningful talk in the small group settings (most of the time was spent working individually, occasionally comparing answers to exercises), and the discussion that did occur generally consisted of short phrases and single words, and often what Pirie et al. would refer to as "incoherent utterances."

D. A Basis for the Comparison of Scores on the Assessments of Meaningful Talk

The next sections of the chapter present the results of the assessments of the three participating groups of students. Prior to undertaking this sort of analysis, it is necessary to explore the basis on which the comparisons are made and to discuss the issue of what constitutes a "large difference" in student performance on the Explaining Tests, Interviews, and Small Group Tasks.

Two types of comparison are used in this analysis: a pre/post contrast (that is, differences between scores on the pre- and post-implementation assessments) and inter-group comparisons (that is, differences between the averaged scores of each group on each item of the assessments). In each case, an attempt is made to describe differences in performance in terms of the range of interpretations identified (Content) and the clarity with which responses were articulated (Presentation).

Because the ordinal scale used with the Dimensions of Meaningful Talk is quite narrow, a wide range in scores on the assessments is not possible. In the analysis which follows, the term "large difference" will be used to describe a disparity of at least one point on any single component (either Content of Presentation) in averaged student scores.

No statistical analyses were conducted on the data gathered. Reasons for this include that the sample was very small (reducing the likelihood of any sort of statistical significance) and that there were too many uncontrollable factors in the research. Details such as differences in classroom routine and variations in the personalities of the students involved could have greatly affected the results.

E. Gross Comparison of the Groups

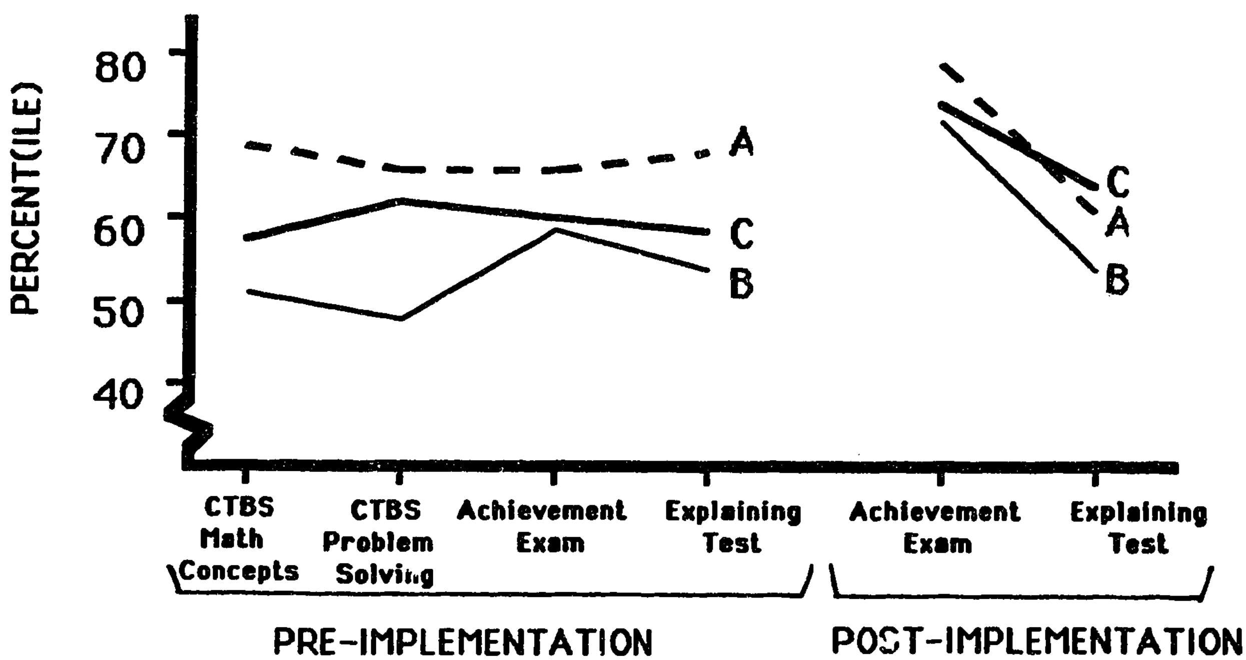
This section consists of a summary of the results of the pre- and post-implementation Achievement Examination and Explaining Tests. A summary of the class averages on each of these assessments is provided in Table 7 and interpreted graphically in Figure 6. Scores on the Achievement Examination were converted to percents and then averaged. For the Explaining Tests, both the average raw score and a percent score for each class are included. The table also presents the average percentile rankings of each of the classes on the Mathematics Concepts and Problem Solving portions of the Canadian Test of Basic Skills.

It must be noted that the raw scores on the Explaining Test cannot be considered to be "out of" some total. The "percent" scores provided were produced by multiplying each of the raw score averages by the same factor in order to produce a mean of 60. This was done in an effort to simplify the process of comparing the Explaining Test scores with the results of the other assessments.

TABLE 7: Results of the Gross Assessments (Class Averages)

	PRE-IMPLEMENTATION ASSESSMENT SCORES Class A	Class B	Class C
CTBS Math Concepts	69.0 %-ile	53.0 %-ile	57.7 %-ile
CTBS Problem Solving	66.6 %-ile	48.2 %-ile	62.1 %-ile
Achievement Examination	67 %	59 %	60 %
Explaining Test Percent Score (Raw Score)	67.7 % (10.4)	53.4 % (8.2)	58.6 % (9.0)
	POST-IMPLEMENTATION ASSESSMENT SCORES Class A	**Class B**	**Class C**
Achievement Examination	79 %	72 %	74 %
Explaining Test Percent Score (Raw Score)	61.1 % (13.6)	54.0 % (12.0)	64.8 % (13.9)

FIGURE 6: Profiles of the Three Classes



Pre-Implementation Results

The primary purpose for the pre-implementation assessments was to establish the comparability of the students in the three participating groups.

The data indicate that, on average, the students in Group A scored higher on all the assessments, followed by students in Group C, and then Group B students. That this result occurred for all four of the assessments, as illustrated by Figure 6, is an indication that the results of the pre-implementation Achievement Examination and Explaining Test are reliable. The fact that students in School A are streamed according to mathematical achievement (so that there were few low-achieving students in Group A) likely contributed to the relatively high averages in that group.

Of greatest interest among the data are the results of the Explaining Test. An in-depth analysis of these results is presented in a later section of this chapter in conjunction with the results of the post-implementation Explaining Test.

The pre-implementation data indicate that there were wide discrepancies between the performances of the classes, and that the comparability of the groups cannot be established on this basis. However, this result was deemed inconsequential for the purposes of the reported research, as the bulk of the analysis was to be based on individual assessments administered to only six students from each class. These students were chosen according to their individual scores on the pre-implementation assessments, allowing the selection of more comparable sub-groups. Profiles of the students selected are included later in this chapter.

Post-Implementation Results

The class averages on the post-implementation Achievement Examination for Groups A, B, and C, were 79%, 72%, and 74%, respectively, indicating that the three approaches were instructionally appropriate. Just as with all the pre-implementation assessments, the Group A average was highest and the Group B average lowest, demonstrating that the Achievement Examination was valid. These results were not used as part of any further analysis.

On the Integers Explaining Test, however, the pattern varied, with Group C attaining the highest average, followed by Group A. Group B remained third.

There were some discrepancies in the administration of the Explaining Test which may have confounded the results. Teachers B and C gave the test at the beginning of the class, as recommended in the guidelines provided. Teacher A had students write the test during the last half of the class, after reviewing the Achievement Exam (thus providing a review of many of the meaningful contexts). Further, students in Group A were told that the results of this test would form part of their report card marks, unlike the members of the other groups who were informed that their responses were not part of their evaluation. Consequently, both the assistance provided and the importance placed on the test were substantially different for Group A. The results are still useful, however, as a basis of comparison for Groups B and C.

Analysis of Student Responses to the Explaining Test

The following is a brief item-by-item analysis of student responses to the Explaining Tests. Sample answers are included immediately after the analysis to illustrate the sorts of responses given and the evaluation scheme used.

Among the notable differences between classes, the average scores on question 1 (on possible interpretations of multiplication) ranged from 2.8 for Group B to 3.9 and 4.1 for Groups C and A respectively. Closer analysis reveals this difference is almost entirely due to the range of meanings which were identified -- that is, on average, students in Group B tended to offer only one meaningful context for the given situations or they failed to elaborate on the examples that were identified. Discussions with each of the teachers after the administration of this test indicated that students in Groups A and C may have benefited from prompting by the teacher who felt their students "needed some help to get started." Even so, these students rarely identified more than two meaningful contexts for any single question.

The responses to question 3 (on interpreting an unlabeled graph) were also varied, but in this instance members of Group C demonstrated greater difficulty in identifying possible meaningful interpretations. What emerged to be consistent across the groups were the scores on the Presentation component (use of vocabulary and logical sequencing of responses). That is, while the each group showed strengths in different areas with regard the range of meaningful interpretations they were able to identify, the ability to express their mathematical knowledge in writing was comparable.

As for the post-implementation Explaining Test, students in Group A were most apt to identify examples dealing with money (bank balances, stocks, gains and losses) or sports (especially hockey and golf). Students in Group B tended to focus on sports (particularly golf, but they also mentioned hockey, baseball, basketball, football, and billiards). Group C students identified temperature almost twice as frequently as members of the other groups. Money applications were also very popular among Group C members. Scores on the Content component of the Dimensions of Meaningful Talk were thus fairly similar across the groups, and slightly higher than the scores earned on the pre-implementation test. Although the scores indicate only a slight difference, students were consistently able to identify a wide variety of interpretations, providing three to four examples on average. In contrast, students rarely identified more than one or two interpretations on the pre-implementation test.

Scores on the Presentation component, however, tended to be fairly low, as they were on the pre-implementation test. This indicates once again that students, in general, had difficulty describing the associations between the meaningful interpretations identified and the mathematical concept in question. A graphical interpretation of these scores is presented in Figures 10 to 12 at the end of this chapter.

In response to the first question of the Explaining Test ("Where might we use 'positive numbers' and 'negative numbers'? For each answer, give an example to show how the numbers are used."), the members of each group generated comparable lists of integer applications, although Group A students generally offered more thorough descriptions of how the integers are used, and thus received higher scores in the Presentation component (use of vocabulary and logical sequencing). Figure 7 presents two "typical" responses.

The results for each of the classes were very similar on the second question ("To answer a question, a student used the following equation:

(-6) + (+2) = -4. Write a real life problem that would use the same equation."), with money and temperature being the most popular contexts. As with the first question, the widest range of "real-life" situations came from Group A. Notably, few students identified more than one meaningful context in their solution to this question. Figure 8 presents a few responses.

On the third question ("How would you prove to a friend that the following statement is true: When you add two negative numbers together you get another negative number."), members of Groups A and B generally scored very poorly, largely because more than half the students simply presented the addition algorithm -- in essence stating that the sum of two negatives is negative because that is the rule. In Group C, bingo chips proved to be the most popular means of explanation, followed by money. Two members of Group C answered this question by describing the relationship between addition and subtraction. Figure 9 presents two sample responses.

In Groups B and C, most students were unable to meaningfully respond to the fourth question ("How would you prove to a friend that the following statement is true: When you multiply two negative numbers together, you get a positive number."), with almost half not even attempting a response. Most answers focused on a restatement of the algorithm. In contrast, over half of the students in Group A described the "Good Guy/Bad Guy" scenario (as presented in lesson 9, see Appendix 1). This result has been discounted, however, because of the brief review provided to these students prior to writing the test.

FIGURE 7. Integers Explaining Test - Sample Responses to Question 1

QUESTION:
Where might we use "positive numbers" and "negative numbers"? For each answer, give an example to show how the numbers are used.

STUDENT 1.A
In bank acounts (sic) amounts, where negative numbers signify overdrawn amounts, is an example of where we might use positive and negative numbers. Some other examples are golf scores, where negative is below par, zero is par, and positive is above par. Maps also use integers for latitude and longitude, where 30 degrees south would mean negative 30. Temperatures use them, -18 degrees C is 18 degrees below freezing.

CONTENT	**[5]**	
A1. Meaningfulness	<3>:	More than two meaningful contexts are identified.
A2. Completeness	<2>:	Minor details (eg. the meanings of "zero" and of positive values) are omitted.
PRESENTATION	**[4]**	
B1. Vocabulary	<2>:	Mathematical terms ("integer") other than those presented in the question are used, but their use is limited.
B2. Sequencing	<2>:	The student has provided a logically-sequenced explanation.
AGGREGATE INDEX	**[9]**	

STUDENT 1.B
We could use them in money to show the gain or loss of it. Or in golf for a way of making points. Temperature is another good way of using positive and negative numbers. The stock market in the exchange of money, the different kinds of money and value. And the value of the company's (sic).

CONTENT	**[3]**	
A1. Meaningfulness	<3>:	More than two meaningful contexts are identified.
A2. Completeness	<0>:	Examples lack elaboration. It is not shown how positive and negative numbers are used.
PRESENTATION	**[1]**	
B1. Vocabulary	<1>:	"Positive" and "negative," which were stated in the question, are used.
B2. Sequencing	<0>:	This is a series of disjointed statements.
AGGREGATE INDEX	**[4]**	

FIGURE 8. Integers Explaining Test - Sample Responses to Question 2

QUESTION:
To answer a question, a student used the following equation:

$$(-6) + (+2) = -4.$$

Write a real life problem that would use the same equation.

STUDENT 2.A
The temperature was -6 degrees C. It went up 2 degrees C. What is the temperature now?

CONTENT [5]
A1. Meaningfulness <2>: One meaningful context is identified.
A2. Completeness <3>: The question is complete.

PRESENTATION [2]
B1. Vocabulary <0>: No mathematical terms are used.
B2. Sequencing <2>: The question, though brief, contains all the pertinent information and it is sequenced in a logical manner.

AGGREGATE INDEX [7]

STUDENT 2.B
I had 6 apple (sic) and I gave 2 away. So I now have 4 apples left.

CONTENT [1]
A1. Meaningfulness <1>: The response is on task, but not deemed meaningful.
A2. Completeness <0>: It seems possible to "fit" this situation to the given equation by letting each apple represent a value of -1. The student did not do this, nor is there any clear indication that this was the intent.

PRESENTATION [0]
B1. Vocabulary <0>: No mathematical terms are used.
B2. Sequencing N/A: The logical flow cannot be assessed if the response is incorrect.

AGGREGATE INDEX [1]

FIGURE 9. Integers Explaining Test - Sample Responses to Question 3

QUESTION:
How would you prove to a friend that the following statement is true: When you add two negative numbers together you get another negative number.

STUDENT 3.A
If you owed 4 dollars to the bank and then you borrowed another 4 dollars from the bank, you'd still owe money, but a little more.

CONTENT	**[3]**	
A1. Meaningfulness	<2>:	One meaningful context is identified.
A2. Completeness	<1>:	The context is not well-developed. It assumes the reader is aware that a debt is to be represented by a negative. The answer may have been clearer if the final debt had been mentioned.
PRESENTATION	**[1]**	
B1. Vocabulary	<0>:	No mathematical terms are used.
B2. Sequencing	<1>:	The statements are logically ordered, but the conclusion is not logically supported because of the missing information.
AGGREGATE INDEX	**[4]**	

STUDENT 3.B
I would get some stones and say they stand for negatives. I would then put some stones in a pile and count them. Then put three or four more stones in the pile. This would show that a negative plus a negative equals a negative.

CONTENT	**[5]**	
A1. Meaningfulness	<2>:	One meaningful context is identified.
A2. Completeness	<3>:	The explanation is brief, but thorough. It is clear that the writer has chosen something to represent negative, and has demonstrated that the combining of two groups of negatives results in a negative.
PRESENTATION	**[3]**	
B1. Vocabulary	<1>:	The mathematical terms that are used were provided in the statement of the question.
B2. Sequencing	<2>:	The flow is logical.
AGGREGATE INDEX	**[8]**	

F. In-Depth Comparison of the Effects of the Three Approaches

Student Profiles

The Interview and Small Group Task portions of the final assessment were administered to six students from each group who were chosen on the basis of their performances on the pre-implementation assessments. For the purpose of demonstrating the comparability of the students selected, Tables 8 to 10 provide a profile of each of these students as well as a "Summary Profile" of the each group's average score.

To the extent possible, the selection of these students was based on the comparability of their scores to the scores of their counterparts from the other classes (i.e. Female 1 in Group A has a similar profile to Females 1 from Groups B and C, etc.). However, the data presented in the following tables indicate some notable differences between the three groups of students, with each demonstrating particular strengths and weaknesses. These differences were deemed to be inconsequential for the purposes of this research.

TABLE 8: Profiles of the Six Students Chosen from Group A

	Females			Males			
	1	2	3	1	2	3	Average
CTBS Math Concepts (Percentile)	61	72	81	61	77	77	71.5
CTBS Problem Solving (Percentile)	45	57	91	57	77	54	63.5
Achievement Pre-Test (Percent)	40	53	70	53	60	77	58.8
Explaining Pre-Test (Percent Score)	52	85	85	52	46	95	68.5

TABLE 9: Profiles of the Six Students Chosen from Group B

	Females 1	Females 2	Females 3	Males 1	Males 2	Males 3	Average
CTBS Math Concepts (Percentile)	28	61	72	43	50	81	55.8
CTBS Problem Solving (Percentile)	39	50	57	36	70	50	50.3
Achievement Pre-Test (Percent)	57	57	70	50	70	80	64.0
Explaining Pre-Test (Percent Score)	46	72	59	33	52	78	56.7

TABLE 10: Profiles of the Six Students Chosen from Group C

	Females 1	Females 2	Females 3	Males 1	Males 2	Males 3	Average
CTBS Math Concepts (Percentile)	33	33	61	23	65	77	48.7
CTBS Problem Solving (Percentile)	50	50	77	65	65	54	60.2
Achievement Pre-Test (Percent)	57	63	70	53	67	77	64.5
Explaining Pre-Test (Percent Score)	39	72	72	59	46	65	58.8

Interviews and Small Group Tasks

In terms of the Dimensions of Meaningful Talk, the scores attained by students on these assessments tended to be fairly consistent within the groups. That is, within-group differences were, in general, less pronounced than the differences that were noted between groups.

However, as is illustrated by Figure 12, there were large variations in the results on the Interview items. Notably, the responses to question 3 (which dealt with the multiplication of integers) resulted in much lower scores in all three groups. Even so, the between-group differences on items 1, 2, and 4 of the Interview are large in comparison to these variations. This aberration is thus deemed to be inconsequential and likely attributable to the complexity of the concept of multiplication.

An analysis of student responses to the Interview items and Small Group Tasks indicates that, as with the gross assessments, consistent patterns emerged in the individual assessments. In general, students were able to identify a wide range of appropriate meaningful interpretations in response to most questions. As on the Explaining Test, students from Group A continued to focus on money and sports; students from Group B identified sporting contexts most often; and students from Group C spoke most frequently of temperature and money, rarely identifying any sport. The students from Groups B and C mentioned mathematical models (eg. number lines and bingo chips) as meaningful interpretations four times more often than students from Group A.

In general, the six Group C students provided the widest range of meaningful interpretations during the Interviews and Small Group Tasks, followed by Group B, and then Group A. (Note that this ranking is different from that of the pre-assessments in which Group A consistently scored highest, followed by Group C and Group B.) This result is illustrated by Figure 10 which presents the average scores of the students on the Content component of each assessment on an item-by-item basis.

Of perhaps greatest significance is the fact that the scores on the Presentation component (vocabulary, sequencing, and fluency) for the students from Groups B and C were substantially higher than their Presentation scores in the Explaining Tests, yet a similar increase was not noted among the students from Group A. In other words, students in Groups B and C, who had greater opportunity to practice their talking skills during class time, demonstrated superior ability to talk in the interview setting. This result is illustrated by Figure 11.

It must be noted that the increase in Presentation scores between the gross assessments and the individual assessments is in part attributable to the inclusion of the Fluency component in the scoring of the verbal responses. Interestingly, Group A students did not do well on this aspect of talk, whereas students from the other groups were able to express their responses with much less hesitation and fore-thought.

The differences noted above were even more pronounced in the Small Group Tasks, where the results from Groups A and B were quite similar but substantially lower that the scores attained by the students from Group C. This is illustrated by Figures 10 to 12 which present profiles of the averaged performances of the students on the two Explaining Tests (pre- and post-), the Interviews, and the Small Group Tasks.

In response to the Small Group Tasks, all groups developed teacher-centered lessons on the assigned topic. Groups A and B students prepared very direct lessons, each of which included a few examples, the "rule", and some drill-type questions. Although Group C lessons followed a similar structure, the explanations provided were much more detailed and more clearly articulated. Further, in addition to drill-type exercises, both of the Group C lesson plans included questions which required learners to provide explanations. The following extracts from their responses illustrate this point.

> I would use bingo chips... and ask the children how to cancel out positive and negative numbers. (Group C-1)

> I'll ask them questions like "positive two plus negative four equals what?"... and then I'll make them give examples to prove their answers. (Group C-2)

Figure 10: Results of the Assessments of Meaningful Talk-
Profile of the Content Component

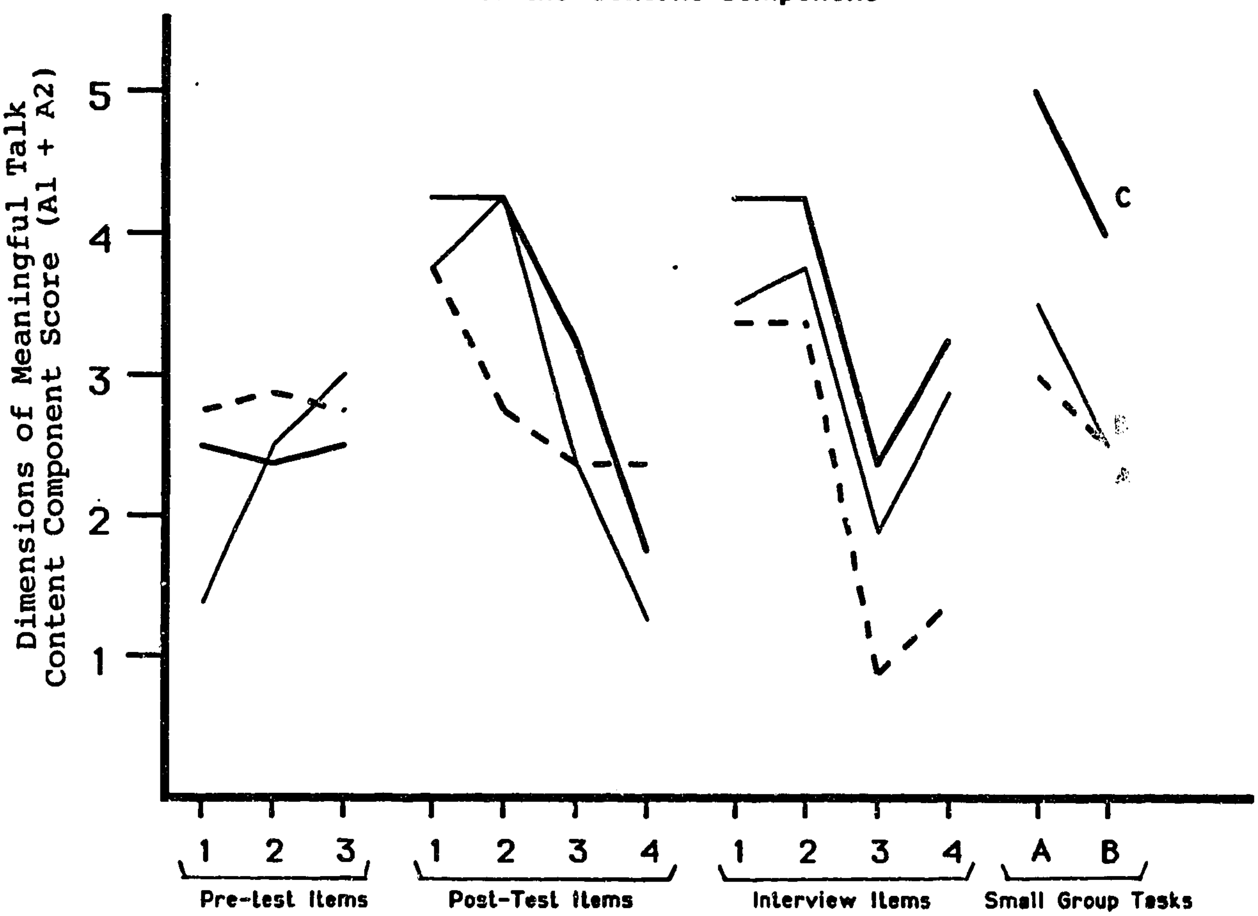


Figure 11: Results of the Assessments of Meaningful Talk-
Profile of the Presentation Component

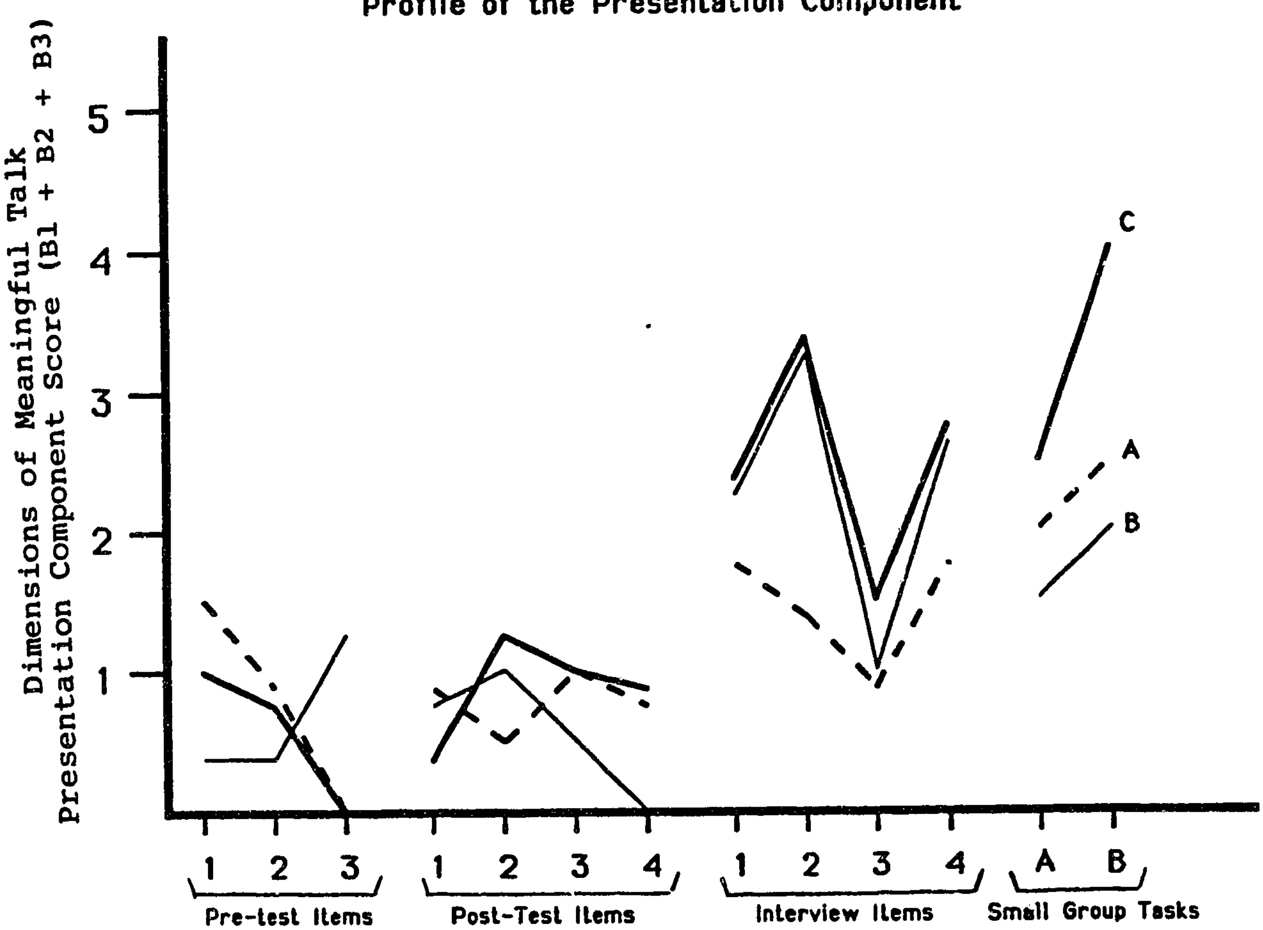
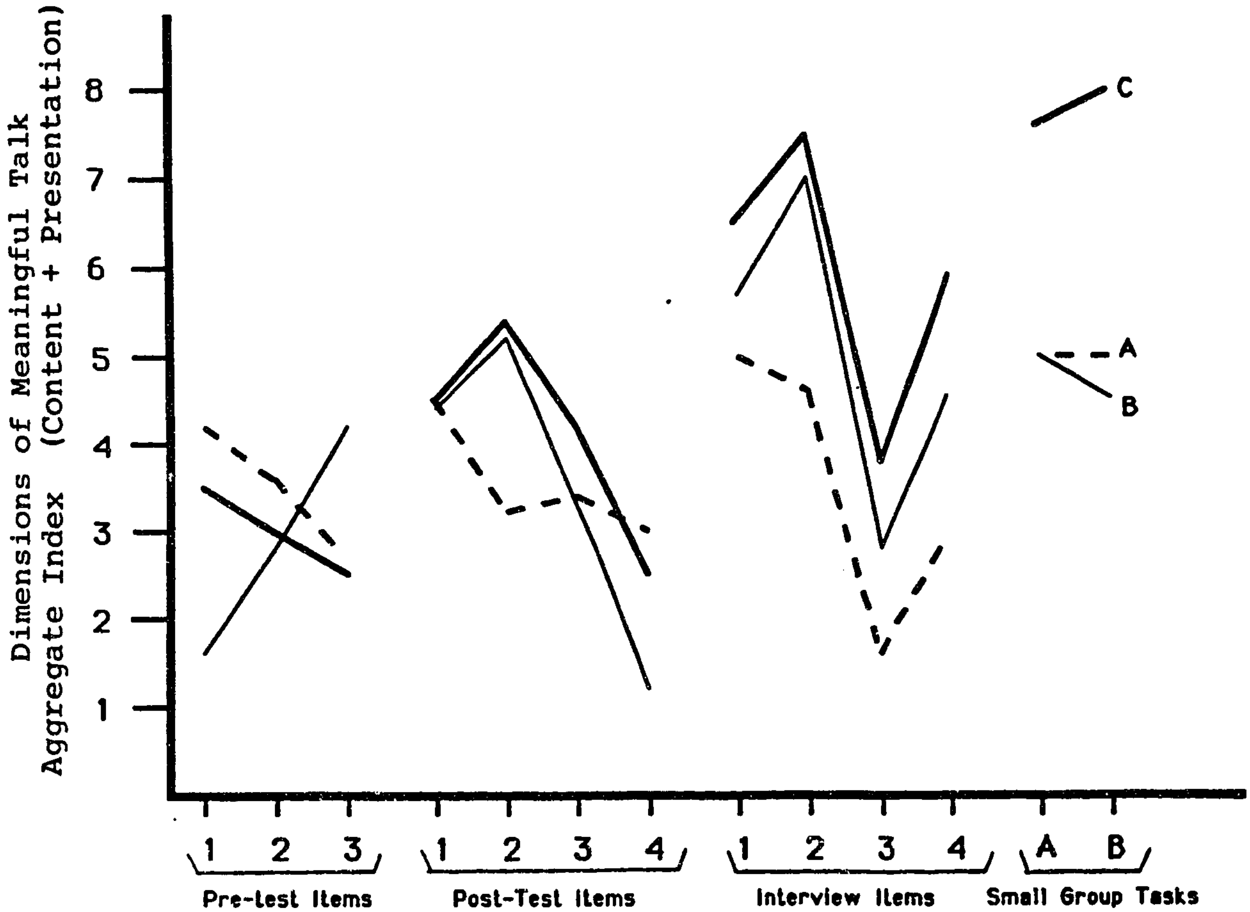


Figure 12: Results of the Assessments of Meaningful Talk-
Profile of Total Scores



Chapter VII
Interpretations, Discussion, and Further Study

A. Introduction

A common observation among teachers and researchers in mathematics education is that children tend to have inappropriate conceptions of the nature and purpose of mathematics. For many students, the discipline is not seen as a coherent scientific subject, but as a collection of propositions to be used in a narrowly defined realm of specific applications. Some students are of the opinion that there are correct rules to be followed in every mathematical situation, and these rules are often memorized as isolated procedures rather than being associated with related knowledge -- that is, they lack meaning for students.

A number of reasons have been put forward to explain such perceptions, but perhaps the critical factor arises from the observation that school mathematics has little in common with the kinds of problems children face in out-of-school settings. The situations are different, and the terms used for even "realistic" questions in mathematics textbooks bear little resemblance to the everyday language of students. A proposed solution is to embed concepts in meaningful contexts in an effort to narrow the "gap" separating student knowledge of classroom mathematics and their real-world "intuitive" understandings.

Many of those supporting an emphasis on meaning also advocate the use of talk as an instructional emphasis, based on the increasingly popular notions that thought and language are inextricably linked and one's conceptualizations are fundamentally affected by the contexts in which information is presented. Further, one's talk offers insights into those conceptualizations, and these insights can be useful in the structuring of learning experiences. Some educators also cautiously suggest that if students were able to learn mathematics within a meaningful context, they might come to recognize the central importance of the discipline in their lives, leading to improved attitudes and increased motivation to learn.

B. The Study

The purpose of this study was to investigate a variety of instructional emphases to determine the sorts of classroom experiences that are most effective in developing students' abilities to talk meaningfully about mathematics. This was done by developing a unit of instruction that focused on the meaningful interpretations of the concepts introduced, and presenting this unit in three different manners: a conventional teacher-centered approach, a teacher-led discussion approach, and a small group learning approach.

Three grade eight mathematics classes participated in the study. Each teacher implemented one of the approaches for the integers unit over a period of three to four weeks, during which time a number of the lessons were observed and analyzed. When the unit was complete, student ability to talk meaningfully about mathematical knowledge was assessed through a series of written tests, interviews, and small group tasks.

The analysis of the data obtained in this study resulted in the following interpretations and implications.

C. Interpretations and Implications

Student Talk

An emphasis on the meaningful interpretation of mathematical concepts in instruction appears to lead to an improved ability on the part of students to identify applications for the concepts under study, as indicated by the fact that students were able to cite a greater number of meaningful contexts in the post-assessments in comparison to the pre-assessments. There was also a general improvement in the quality of responses, indicating progress in student ability to talk meaningfully about mathematics. While it might be argued that the perceived improvements are attributable to more concise perceptions of expectations, it is clear that students were able to express their knowledge in writing more effectively at the end of the unit.

It also appears that the instructional bias towards certain contexts affects the interpretations that students tend to use. A dramatic example of this can be seen by contrasting the examples cited by students in Groups B and C in the post-assessments. Group B members identified sporting applications most often, whereas Group C students focused on money and temperature, and rarely mentioned any sport. These observations parallel the in-class emphases noted during the implementation. Similarly, Group A students identified mathematical models (eg. number lines and bingo chips) much less often than their counterparts in the other groups, consistent with Teacher A's lack of emphasis on these contexts.

Increased opportunities for students to express their mathematical knowledge also appears to affect their abilities to talk meaningfully about mathematics, as indicated by the results of the Interviews. Students from Groups B and C, who were encouraged to talk about the concepts during the unit either through teacher-led discussion or in small cooperative groups, consistently identified and were able to elaborate upon a wider range of meaningful interpretations than the students from Group A, as evidenced by the fact that the averages on both the Content and Presentation components of their talk were consistently higher.

This result is of particular interest because, as noted in the previous chapter, while there were increased opportunities for students to talk in Classes B and C, these opportunities were not fully exploited. This suggests that the process of making students aware that they are expected to be able to "explain what they know" (as was done by Teacher B, and to a lesser extent by Teacher C) may be as critical a factor in promoting talk as the increased opportunity to practice. A third factor affecting student talk may have been the example set by the teacher. Unlike Teachers B and C, Teacher A was unaware as to the true purpose of the study. It is thus not unreasonable to suggest that Teachers B and C were more cognizant of their own examples and took care (consciously or unconsciously) to model clear and precise mathematical talk.

Interestingly, an emphasis on the quality (i.e. clear articulation) of one's talk, an important feature of the Teacher-Led Discussion Approach, did not appear to lead to the same level of improvement in student talk as did an emphasis on quantity, as was possible in the small group settings of the Cooperative Learning Approach. This is indicated by the results of the Small Group Tasks, where Group C students scored much higher than members of Group B, while their scores on the Interviews were comparable. Other possibilities are that Group C students may have developed the skills necessary to work together in a small group setting as a result of the cooperative learning focus or may have been more familiar with the type of task assigned as a result of those experiences. Whatever the reason, it appears that the learning opportunities afforded by the Cooperative Learning Approach led to a substantial improvement in student ability to discuss mathematical knowledge with one another. The implication would be that, in promoting student ability to talk, an important consideration is the provision of opportunities for students to discuss mathematical concepts with one another.

Promoting Meaningful Talk

Much of the value of this study is in providing insight into the processes of structuring learning opportunities in ways that encourage talk. It must be noted at this point that the study was quite limited in its scope with regard to the variety of techniques that might be implemented, and thus the implications for teaching are similarly limited. (A summary of other approaches that might have been taken is included in the literature review.) From the above analysis, it appears that the meaning emphasis, the model provided by the teacher, the clear articulation of expectations, and the provision of opportunities for student talk (to one another and to the teacher) are the critical factors in promoting meaningful talk.

While it is difficult to rank these factors in order of significance on the basis of the limited research reported, it is surmised that the provision of opportunities for students to talk to one another about mathematical concepts is of upmost importance. As noted above, such provision appears to influence student ability to articulate mathematical knowledge. Of perhaps even greater importance, there is some evidence to indicate that providing opportunities for student discussion may have affected the way they view the learning of mathematics. For example, unlike their counterparts in the other groups, students from the Cooperative Learning Class (Group C) included explanation-type questions as part of their lesson plans in the Small Group Tasks. Their responses suggest a developing perception that mathematical concepts can and should be explained -- or talked about -- meaningfully.

Several difficulties arose in efforts to promote meaningful talk. For example, as noted in the cases of Teachers A and B, one of the immediate consequences of an increased emphasis on meaning seems to be a dramatic increase in the amount of teacher talk, especially when attempting to encourage discussion. That this situation need not occur was demonstrated by Teacher C, who maintained the same levels of teacher talk during the integers unit as were noted in the pre-implementation observations, leaving much of the "sorting out of details" to the students as part of their small group work. However, she too reported that it was difficult to fit all the recommended content into the limited time. Perhaps the range of meaningful contexts outlined in the unit guides was too broad, and teachers might be well-advised to limit themselves to a few choices that can be effectively used throughout the unit.

To this end, the mathematical models of the bingo chips and the number lines appear to be particularly meaningful to students, although the applications of temperature, golf, and money were the most popular among students in the post-assessments. While each of these models and applications can be used to illustrate (that is, make more meaningful) all of the basic operations on integers, it was noted by the cooperating teachers that bingo chips and monetary applications were most easily adapted for this purpose. They would thus appear to have the greatest potential for use within the unit. It must also be noted that while the fore-mentioned models reduce the addition of integers to an almost trivial exercise, the task of using them to illustrate and explain the other basic operations is not so easily established. Each of the teachers in this study suggested that more class time -- preferably dedicated strictly to the use of the models, as in a lab-type setting -- would likely be more effective. Unfortunately, the time required to implement these activities, when considered over all the possible models that might be introduced through the year, may be excessive.

On the basis of the research conducted, it is a fairly simple task to select those interpretations of concepts which seemed to be most meaningful to students. Such was not the case during the design of the unit, and the issues of which meanings are significant for young people and how those topics are best brought into the classroom is one that is not easily addressed. Clearly, meaning is not in the lesson content but in what students do with the content, and mathematics cannot be considered apart from its appropriation and transformation by students.

Further to the meaning issue, while evaluating the post-assessments it was interesting to note the number of students who believe that stating an algorithm is tantamount to providing a meaningful explanation. Many students apparently see the rule as being self-explanatory, or as an unquestionable fact, and therefore an adequate explanation for any question on the topic. While it is beyond the purpose of this paper to speculate as to the origin of such a notion, it is important for teachers to recognize this tendency.

Dimensions of Meaningful Talk

Because the framework used to analyze and interpret student utterances formed a key underpinning of this study, a few comments on that scheme bear mentioning.

A limiting factor on the use of the Dimensions of Meaningful Talk is that it was developed to analyze extended vocalizations (i.e. longer than a few sentences) by students, and these represent a relatively rare phenomenon in a junior high school mathematics classroom. (This was also noted by Pirie et al., 1989.) Unfortunately, the Presentation component becomes almost meaningless when examining brief statements, as does the Completeness (A2) portion of the Content component.

Even with the seemingly straightforward Meaningfulness (A1) portion of the Content component, difficulties often arose when attempting to determine if a context identified by a student was actually significant to him or her, or if the student simply recalled it being discussed in class. Many students demonstrated great difficulty in elaborating on examples even when prompted during interviews, forcing one to infer the existence of personal significance and underscoring the limited scope of the term "meaningful" as defined in this research. This leads to a question as to how one might more effectively assess students' personal interpretations of mathematical concepts. An answer appears to be dependent on a willingness to spend considerably more time interviewing and observing individual students, observing groups, using "think aloud" protocols, and analyzing a collection of information rather than focusing on the "snapshot" afforded by a brief interview. This, in turn, reflects one of the fundamental reasons for promoting student talk: to provide insight into student knowledge and conceptualizations.

The need to develop a framework for the analysis of talk within the classroom setting was also identified by Pirie et al. (1989). They too suggested that talk cannot be removed from its context. With regard to the "meaningful talk" described in this study, it thus appears that the problem must be tackled from two perspectives: the improvement of talk-promoting techniques and the development of a more appropriate analytical framework.

Male/Female Differences

On the issue of gender related differences in verbalization skills, the contention of Maccoby and Jacklin (1974) that adolescent females have superior verbal abilities is unsupported by this research. Possible reasons for this include that the sample was extremely small, the amount of data collected was limited, and the selection of the students for the Interviews was not random. Because an effort was made to choose students whose mathematical abilities were in the average range, the profiles of the students, male and female, were quite similar.

D. Further Discussion

A factor which may have had a significant effect on the outcome of this investigation, but which did not form a central part of the analysis, is the teacher variable. The three approaches were designed to allow for individual differences in classroom style between participating teachers, and these differences may have played a key role in determining the final outcomes. To illustrate, Teacher B's usual method, as observed prior to the implementation, tended to be very direct and included little student input. The Teacher-Led Discussion Approach she was asked to implement may have thus represented a radical shift, and one to which she had considerable difficulty adapting. Certainly her tendency to respond to her own questions and her unwillingness to wait for student responses bear this out. Similarly, Teacher A admitted to seeing little benefit in the use of manipulative materials, this was reflected in her under-emphasis on the use of bingo chips and number lines during the unit.

Encouraging students to make conjectures and to personalize knowledge in a discipline that is generally characterized as fixating on a single correct solution requires a fundamental change in the way mathematics is approached in the classroom. The ability of the teacher to establish an environment that supports the risks and the trust required for students to express their knowledge would also seem to be a critical element in promoting classroom talk. Fortunately, this appeared to be a particular strength of the teachers who participated in this research.

By way of teacher reaction to the meaning emphasis of the units, each stated that they enjoyed the focus and felt that the students did as well. However, each also noted that there was some difficulty with the speed at which the unit progressed. Unfortunately, there was no attempt to determine student opinions on the meaning emphasis, so the conjecture that it leads to increased interest and greater awareness of the relevance and usefulness of mathematics in one's life was not explored in this research.

Other factors which were given little consideration in this research were the differences that may have existed among the students. No attempt was made to assess such traits as motivation, willingness to cooperate with a relative stranger, ability to adapt to a new emphasis in instruction, or personal perception of mathematics. Clearly, these factors would have a significant influence on a student's reaction to this sort of investigation and on the manner in which s/he is able to talk about mathematics. In particular, if, as Hiebert (1984) and others suggest, students tend to view mathematics as a set of rules to be applied in a narrowly defined realm, a general resistance or impatience might emerge among learners who are suddenly expected to discuss their knowledge in a meaningful way. Indeed, this was observed in at least one case when Group B students reproved their teacher for dragging them through a meaningful explanation of subtraction (using bingo chips) rather than simply stating "the rule" in the first place.

Additionally, the duration of this study, at three to four weeks, might be considered to have been very brief. That differences were apparent over this short period suggests that designing mathematics instruction around talk might result in tremendous differences over extended spans of time. This notion is supported by Lampert (1990) who claims that, at the end of a year-long study, "students learned to do mathematics in a way that is congruent with a disciplinary discourse" (p. 58).

E. Further Research

The results of this study are not generalizable beyond the population of grade eight mathematics students on the topic of integers. The results, however, suggest several related areas for possible research.

For the identified population and topic, the ability to talk meaningfully about mathematics is a skill that can be developed, and some practical recommendations to this end have been presented. A suggestion for further research would be to implement similar approaches at other levels, for different topics, and over varying lengths of time to determine if this observation is generally true. Part of that research might focus on other instructional approaches (eg. expressive writing, debates, etc.) designed to encourage talk. Subsequent investigations might focus on implementation of these suggestions over extended periods, further refining approaches and research methodologies.

Another research focus might be to examine the affective outcomes of the use of talk as an instructional emphasis. A review of the current research on the issue reveals this to be a topic of key concern.

There is also a need to refine the techniques of evaluating meaningful talk. The opportunity to do so would emerge from the research proposed above. A major focus of this area of study would be to determine the extent to which utterances on mathematical topics actually reveal student conceptualizations and background knowledge.

Such developments would set the stage for what might be considered the most important issue in the study of talk: establishing the connection between student talk and understanding -- how one influences the other. Clearly, it cannot be accomplished until talk has been widely implemented as an important instructional emphasis.

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APPENDIX 1

COOPERATIVE LEARNING APPROACH

UNIT OUTLINE

Grade 8 Mathematics
INTEGERS

I. Overview

The unit begins with a review of the concept (definition) of the term "integer", followed by a lesson on the comparison of integers. These topics are supplemented with a discussion of the meanings of the term "zero".

The bulk of the unit focuses on basic operations with integers. Students are to have experiences with manipulative materials (BINGO chips), models (number lines), efficient algorithms, and the calculator.

The unit will also include graphing integers and order of operations, both of which lie outside the bounds of the specified curriculum objectives.

A major emphasis is placed on the meaningful interpretation of concepts. A lesser, but related, focus is placed on applications and problem solving. All situations and applications have been chosen on the basis of their potential to promote meaningful discussion among students. (i.e. There is an assumption that each interpretation is already be meaningful to students, or can easily be made so. It is also assumed that the students enter the unit with very little formal knowledge of integers.)

II. Meaningful Interpretations

Manipulative Materials: BINGO Chips

These will be used to represent integers and to illustrate one of the meanings of "zero" (i.e. as the balance of positive and negative units, or neutrality). The chips will also be used for developing the concepts of addition, subtraction, multiplication, and division.

Models: Number Lines (and Thermometer Scales)

These will be used to represent integers. The primary, though not exclusive, use of number lines will be in the lessons on comparison, addition, and subtraction of integers. The number line will also be used to illustrate another interpretation of "zero" (that is, as an arbitrarily chosen position).

Applications

The following will be used not only to illustrate the use of integers, but to bring out and develop intuitive notions relating to positive and negative numbers.

TEMPERATURE (vertical number line)
ELECTRIC CHARGE (as a real-world analog of BINGO chips)

ALTITUDE RELATIVE MOTION LATITUDE & LONGITUDE	Location and Movement
PROFIT/LOSS BANK BALANCES STOCK MARKET INDICES	Money
LAUNCH COUNTDOWNS TIME ZONES YEARS (A.D/B.C.)	Time
+/- HOCKEY STATISTICS GOLF SCORES GAINS/LOSSES IN FOOTBALL	Sports

Developing Algorithms

All algorithms will be developed through the following sequence:

> Concepts will be introduced at the intuitive level through the presentation and discussion of meaningful interpretations.

> Numerous examples and exercises will be done prior to the discussion of an algorithm.

> Through the analysis of emerging patterns, algorithms will be developed. Use of these algorithms will be purposely de-emphasized while an ability to describe the processes followed (through reference to some sort of meaningful context) is emphasized.

Basic Operations

It is important to note that this unit affords an opportunity to review the meaning of the basic operations. There is also an opportunity to investigate properties such as the identity and commutativity. This will be done on an informal level. The following provides an outline of some of the interpretations that will be brought into discussions.

Addition - as the combining of smaller groups into a larger group
- as movement or change (eg. along a number line)

Subtraction - as the opposite of addition
- as the removal of objects from a group
- as the distance between two positions between two positions/values

Multiplication - as repeated addition

Division - as the opposite of multiplication
- as the number of objects in each subgroup when a larger group is divided
- as the number of subgroups when a larger group is divided

III. Lesson Plan Format (loosely based on Missouri Mathematics Project)

A. Daily Review (first 8 - 10 minutes)
1) review of concepts
2) homework
3) mental computations

B. Development (from 20 to 30 minutes)
1) teacher activities (important points, topics of discussion, relevant questions, etc.)
2) small group work
3) teacher-led summary discussion

C. Assignment (All textbook references are to Journeys in Mathematics 8; worksheet references are to Journeys in Mathematics 8: Teaching Aids and Journeys in Mathematics 9: Teaching Aids.)

D. Positive Interdependence
This will contain brief descriptions of how interdependence and accountability can be worked into the lesson.

IV. Unit Outline

Day	Topic	Meaningful Interpretations
1	Definition and Use of Integers; What does "ZERO" mean?	Temperature, Altitude, Profit/Loss, Electric Charge, Launch Countdowns, +/- Hockey Stats, Years(A.D./B.C.), Motion, Bank Balances
2	Comparing and Ordering Integers	Temperature, Altitude, Profit/Loss Time Zones, Number Lines
3	Graphing Integers	Standard (x,y) Coordinates, Locations on a World Map (W & S are negative, N & E are positive)
4	Adding Integers (Part 1)	BINGO Chips, Temperature Changes, Altitude Changes, Profits and Losses
5	Adding Integers (Part 2)	Number Lines, Time Zones, Golf Scores, Bank Balances, Years, Relative Motion

6	Subtracting Integers (Part 1)	BINGO Chips, Golf Scores
7	Subtracting Integers (Part 2)	Number Lines, Temperature Changes, Years, Bank Balances
8	Practice/Review and QUIZ	
9	Multiplying Integers	BINGO Chips, Oilers/Flames/Wins/ Losses, Drill, Number Patterns
10	Dividing Integers	BINGO Chips, As an extension of the multiplication rule
11	Order of Operations	
12	Calculator and Review	
13	Testing	Achievement
14	Testing	Written Explanation

V. Choosing Groups

Students will be divided into heterogeneous groupings according to sex and achievement. Each group of 3 will have at least one male and one female, and a person from the each category of achievement: low, medium and high. Once set, the groups will remain unchanged through the duration of the unit.

An effort will be made to separate close friends in order to expose students to other classmates.

Similar guidelines to the above will be followed when assigning students to expert groups for JIGSAW activities.

Lesson 1: Definition and Use of Integers; The ZERO Concept

Daily Review

1. Review: none
2. Homework: none
3. Mental Computation: none

Development (T = teacher-led; S = done in small groups)

T: definitions of INTEGER, POSITIVE, NEGATIVE and ZERO (as an integer that is neither positive nor negative)

T: situations in which integers might be used. (Through a group discussion, elicit as many of the following situations as possible from the students.)

TEMPERATURE	(0 = freezing point of water; + = warmer)
ALTITUDE	(0 = sea level; + = above or higher)
PROFIT/LOSS	(0 = neither gain nor loss)
BANK BALANCES	(0 = no money; - = in debt)
GOLF SCORES	(0 = par)
ELECTRIC CHARGE	(0 = neutral or no charge)
COUNTDOWNS	(0 = take-off time; - = pre-launch)
MOTION	(0 = no movement or the starting point)
+/- HOCKEY STATS	(0 = same # of goals scored by both teams when player X was on the ice)

T: Working Together: p. 275, qq. 1-3

S: For each example above, tell what a value of "zero" means, what a positive value means, and what a negative value means. Make a list of all the words that stand for zero. (A central purpose here is to point out that "zero" doesn't always mean "nothing". Sometimes it's an arbitrary value, chosen for convenience.)

S: Make a list of words that might be translated as positive and negative. (Begin to build a chart that can be added to as the unit progresses.)

POSITIVE	NEGATIVE
increase/rise	decrease/fall
above/over	below/under
gain/profit	loss
in the black	in the red
up	down

S: p. 275, qq. 1-4

T: Sum up with a class discussion, ensuring that everyone has an appropriate list and the correct responses to the seatwork. Stress the proper use of terms and expressing answers in complete sentences. Model answers. Have students rephrase and/or repeat responses.

Homework

- p. 275, qq. 5-8 and Keeping Sharp

Positive Interdependence

For questions 1-4 on page 275 (which are to be done in class), one person's paper from each group will be randomly chosen and scored. All group members will receive the same score.

Lesson 2: Comparing and Ordering Integers

Daily Review

1. Review: Definition of "integer", "positive", "negative", "zero"
2. Homework: p. 275, qq. 5-8 (on transparency)
3. Mental Computation: (Oral)
   What integer is suggested by each statement?

12 m below sea level	writing a cheque of $32.00
37 s before take-off	a raise of 5.8%
a gain of 4 points	a golf score of par
8 steps backward	the year 149 B.C.

Development

T: Ask which number in each of the following pairs is greater. Have students justify their responses with a meaningful example. (eg. +2 > -1 because a profit of $2 is better than a loss of $1.) If no one answers meaningfully, provide an example or two using temperatures or altitude.

+3 or +5   +2 or -1   -3 or -5   -3 or 0

T: Using a number line (-5 to +5), discuss how integers compare with each other and how this is related to their positions on the number line.

Have students draw a number line and graph each of the following (see p. 276 of the text):

a) all integers > +3
b) all integers > -5
c) a integers < +2
d) all integers > -5 and < +4

Establish a set of rules linking number line position and comparative value. (For example, +4 > +3 because +4 is to the right of +3. Therefore, "to the right of" means "greater than".)

T: Repeat with a vertical number line (and/or temperature scale). (In this case, "greater" translates as "above" or "warmer".)

T: Distribute handout of world time zones. (NOTE: It would probably be helpful to have a globe on hand when explaining this concept.) Point out that the Prime Meridian runs through Greenwich, England. Everyone east of the p. M. sets their clock ahead of English time, and everybody west sets their clocks behind English time. (Since we're in the -7 time zone, we are 7 hours behind London.) Discuss the meaning of "greater" and "less" in this context (i.e. greater means "ahead of").

S: Have students repeat this type of analysis with several meaningful situations. In each case, have them provide an example which illustrates the "greater" or "less" concept. (For example, if using years, 1 A.D. came after 2 B.C. because +1 > -2, so "came after" is the same as "greater" in this case.) Compile a short list of phrases that mean the same as "greater" (eg. over, warmer, to the right, more, came after) and "less".

Potential situations include temperature, altitude, electric charge, golf scores, years, profit/loss.

S: p. 277, qq. 1-4, TRY THIS; Time Zones Worksheet

T: Review student answers and word lists. Through discussion, establish a list of rules that can be used to quickly compare a pair of integers. For example:
Positive numbers are always greater than zero.
Positive numbers are always greater than negative numbers.
Etc.

Homework

- p. 277, qq. 5-8

Positive Interdependence

During the follow-up discussion, students will be chosen at random to provide and explain answers. The group will get 1 point for a correct answer, and two points if the explanation is given in clear, concise sentences. Points are recorded as "bonus points" for the next assignment.

Lesson 3: Graphing Integers

Daily Review

1. Review: Ordering Integers (some quick rules, each with a meaningful example)
2. Homework: p. 277, qq. 5-8 (on transparency)
3. Mental Computation: (On board)
    a) Choose the larger number in each case.
        +6, +3   -4, +3   -5, -3
        -2, -1   -5, 0   0, +8
    b) Arrange each list in order.
        -3, +7, -4, -7, +1
        -1, +13, +11, +8, -6, -11
        -5.5, -7.8, -9.2, -5.2, -7.4

Development

Background: A graph is really a pair of number lines placed at right angles. This divides the graph into four areas, called quadrants. (Show how the quadrants are numbered with a sketch.)

History: This type of graph, the Cartesian Coordinate Plane, is named in honor of its inventor, Rene Descartes (France, 1600's). He was in jail one day (as a result of outspoken views on religion) watching a fly buzz around on the other side of the bars. He realized that he could determine the precise location of the fly by identifying the nearest vertical and horizontal bars.

Definitions: quadrant; X-axis; Y-axis; origin; coordinates; ordered pair

World Map (on transparency): One of the earliest uses of Cartesian Coordinates was on a flattened map of the world. When the map is drawn this way, it is cut into quarters by the Prime Meridian and the Equator. This forms four quadrants, which early explorers referred to as the "Four Corners of the World". (To help familiarize students with the map, identify the location of a few cities. For the sake of consistency, be sure to give the E/W coordinate first.)

Small Group Task: Do the Locating Earthquakes handout and answer the following questions:

Which directions (i.e. N, S, E, W) would be considered positive and negative in this case?
What does zero mean in this case? (i.e. 0 degrees E/W is the p. M.; 0 degrees N/S is the equator.)
Why do we bother with these coordinates? (To make locating things a simpler task.)
What are the coordinates of Edmonton on the map? (NOTE: For the sake of consistency, be sure to identify the E/W coordinate first.) What is on the opposite side of the world from Edmonton?

Homework

- p. 279, qq. 1-6

Positive Interdependence

Distribute only one map per group, which will be turned in to be graded (and photocopied so that each group member will end up with their own copy).

Choose a homework assignment randomly (one from each group), and assign a group mark.

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Lesson 4: Adding Integers (Part I)

Daily Review

1. Review: Graphing (ordered pairs; quadrants; origin; axes; etc.)
2. Homework: World Map (completed version on transparency. You may want to digress by discussion the significance of the pattern that emerges.)
   p. 279, qq. 1-6 (on transparency)
3. Mental Computation: (Oral, with the world map on the overhead.)
   1. Identify the coordinates of (a) Johannesburg; (b) Tokyo; and (c) Inuvik.
   2. Name a city that is (a) 150 degrees left of the Y-axis; (b) at 60 degrees latitude; and (c) in the fourth quadrant.
   3. Which directions on the world map (i.e. N,S,E,W) are considered positive.

Development

- BINGO CHIPS (SEE Journeys in Mathematics 8: F Teacher's Resource Manual, p. 190)

  Divide the class into their groups for the demonstration. Each group will need about 15 chips of each color. Begin by establishing a color code. (The text uses red for negative and black for positive.)

  Demonstrate that an equal number of oppositely colored chips is equivalent to zero since they cancel each other out. You can then demonstrate some addition statements (eg. (-2) + (-4) = (-6), and (+2) + (-1) = (+1)). Have the students work out a few in their groups. (There are some questions on p. 280 in the WORKING TOGETHER that are appropriate.) When going over responses, have individual students explain the solution process.

- NUMBER LINES (Again, the Teacher Resource Manual covers this nicely on p. 190.)

  Make the students responsible for this part of the lesson. The textbook covers it nicely (p. 280). Distribute CHECKER MATH - ADDITION, NUMBER LINES, and THERMOMETERS worksheets, and give the homework assignment.

Homework

- Worksheet: Checker Math - Addition
- p. 281, qq. 1-6 (NOTE: Question #3 is to be done with thermometers, not on number lines as the text recommends.)

Positive Interdependence

- Provide each group with a single set of chips.

- At the end of the worktime, randomly chose a single person from each group to come up and demonstrate an addition statement on the overhead with BINGO chips and/or on the chalkboard with a number line or thermometer. Offer BONUS MARKS (+1 for a correct answer, +2 for a clearly explained response) towards the next written assignment. (In order to keep things moving. place a 1 minute time limit on each answer. To keep things as fair as possible, avoid correcting a student's explanation until everyone is done.)

- One student's homework (p. 280) from each group will be randomly chosen and a group grade assigned.

Lesson 5: Addition (Part II)

Daily Review

1. Review: Adding integers with BINGO chips
   Adding integers on a number line
2. Homework: Worksheet: CHECKER MATH - ADDITION, qq. 4-6; p. 281, qq. 4-6
3. Mental Computation: (On board. Allow students to use BINGO chips or the number line, if desired.)

   (NOTE: If you write these on the board in the same format, it will be set up nicely for use in the lesson.)

(+3) + (+4)	(-1) + (-7)	(+2) + (-1)	(+2) + (-9)
(+2) + (+6)	(-3) + (-8)	(-4) + (+8)	(-6) + (+5)

Development

T: Looking at patterns to establish some shortcut rules: Use the Mental Computation questions to determine if there are any patterns emerging. The aim here is to produce a set of efficient rules that can be used to add integers. (NOTE: This is one of the objectives outlined in the curriculum guide, but I suggest that you present it as just another option to use when adding integers.)

A recommended tack is to ask students if they notice any patterns in the above questions. If they don't, more direct questioning (eg. "What is happening here when two positive numbers are being added together?" "Will the answer to a question like this always be positive?").

The rules that are established will probably be something like: "When adding two integers whose signs are the same, add the values and keep the signs. When the signs are not the same, subtract the values and keep the sign of the larger value."

S: Have students work on the assignment in groups. Encourage them to BINGO chips so that the skill is reinforced.

Homework

- p. 280, qq. 7-9
- Worksheet: ADDITION QUESTIONS

Positive Interdependence

- Students are to turn in a single worksheet. Each student is to participate in answering the questions so there must be 3 different styles of writing on the assignment (i.e. two questions per student). Students are expected to help one another and to check each other's work, but must not do so until an answer is written out. (This will help to prevent one student from dictating an answer to another.)

Lesson 6: Subtracting (Part I)

Daily Review

1. Review: the algorithm for adding integers
2. Homework: worksheet: ADDITION QUESTIONS
3. Mental Computation: (oral)
   Write out an equation and solve for each of the following.
   a) The temperature wa -8 degrees Celsius, but rose 4 degrees.
   b) Hawaii is 4 time zones behind us. What time is it in Hawaii right now?
   c) Sunspot activity occurs every 11 years. It occurred in 24 B.C. When was the next occurrence?
   d) Sarah had a golf score of 3 under par until she double bogeyed. What was her final score?

Development

- Subtraction with BINGO chips (This is covered quite nicely in the Teacher Resource Manual.)

  Ask students for the meaning of "subtraction". The answers will likely be along the lines of "to take away", although other interpretations are possible (as will be seen in Lesson 7).

  Begin with an example with matching signs: eg. (-5) - (-3).
  It's fairly easy to establish that the answer will be negative using BINGO chips and the notion of "subtract means to take away".

  Move to a question with different signs: eg. (-5) - (+3).
  This isn't so obvious because there are no positive chips to take from a pile of 5 negatives. Have students suggest ways of overcoming this problem. (It can be done by adding three +/- pairs -- that is, 3 "zero's" -- so that there are 8 negatives and 3 positives. NOTE that it's important to emphasize that the

number represented by this configuration is still -5. When +3 is removed, the result is -8.)

Set up several examples and have students come up and explain the process.

Assign the worksheet: CHECKER MATH - SUBTRACTION. Allow the students to work together with members of their own group if they desire.

- Using the answers students generate from the worksheet exercises, look to see if any patterns emerge.

Compare the results from subtraction and related addition questions: eg.

(-3) - (-5) = (+2) These give the same answer.
(-3) + (+5) = (+2)

Do several examples to establish the rule:
"To subtract an integer, add its opposite."

Homework

- worksheet: CHECKER MATH - SUBTRACTION
- p. 283, qq. 1-7

Positive Interdependence

- This is a teacher centered lesson.

Lesson 7: Subtracting (Part II)

Daily Review

1. Review: Subtracting Integers with BINGO Chips
   The Algorithm for Subtracting Integers
2. Homework: p. 283, qq. 1-7
3. Mental Computation: (On board. Allow BINGO chips if desired.)

(+8) - (-2)	(+11) - (-13)	(-8) - (-3)
(+5) - (+9)	(-2) - (+8)	(-14) - (-18)

Development

T: BACKGROUND - Subtraction on the Number Line

Another interpretation of subtraction (in contrast to "taking away" or "removing") is the difference between two quantities, or the distance between two points. This is suggested by the symbol "-" which is used in contexts such as "the week of May 3 - 10" and "Do questions 3 - 9." In these cases, the dash (-) refers to everything between the numbers.

Such is the case of subtraction on the number line. In the case of (-5) - (+3), it can be interpreted as "How many spaces are there between the positions of -5 and +3 on the number line. The answer is, of course, 8, although this lacks a sign.

In a subtraction statement, the first term is where you finish, and the second term tells you where you started. This can be illustrated by the following example:

> It was +3 degrees C when I arrived at school, and -5 degrees when I went home. What was the temperature change?

Answer: (-5) - (+3) = -8, so the temperature fell 8 degrees.
[finish - start = change]

On the number line, this example appears as follows:

```
          finish                                       start
            <-------------------------------------------o
<--+----+----+----+----+----+----+----+----+----+----+-->
  -6   -5   -4   -3   -2   -1    0   +1   +2   +3   +4
```

It may be helpful to lead students through a few examples similar to the ones found on the worksheet. (NOTE: THIS WORKSHEET IS NOT MEANT TO BE DISTRIBUTED!) Encourage students to describe the process of determining the response when using the number line.

S: Do the JIGSAW, described below.

T: Collect the sheets from the expert groups. (Each person should turn in a single piece of paper with a single question that is solved in two ways -- with a number line and with an integer number sentence.)

Positive Interdependence

This lesson is developed around the JIGSAW concept. Each member of each group needs to be labeled as either A, B, C. The students are then re-grouped so that EXPERT GROUPS (of A's, B's, or C's) are formed. Expert Groups should be have no more than 4 members. (This may take a fair bit of advance preparation to this. Expert Groups should, ideally, be heterogeneous.)

In this lesson, the Expert Groups will be assigned a topic, as follows:

A: MONEY (bank balances; profits/losses);
B: SCALES (temperature; altitude);
C: TIME (launch countdowns, years).

The expert group's task is to determine how integers are used in one of the assigned contexts, and to develop a subtraction question which can be interpreted using a number line. (It is recommended that you put on a stipulation that each question must involve at least one negative integer.) The group must solve the question using a number line and using an integer number sentence. (This should take about 10 - 15 minutes.) Warn students that this will be handed in at the end of the lesson.

Each member of the expert group must then return to their home group and present the question. The other 2 members are to solve it using both a number line and a number sentence. (This part of the lesson should take about 15 minutes.) To increase accountability, tell them that 2 of the questions produced in class will appear on an upcoming quiz.

Lesson 8: Review and Quiz

Daily Review

1. Homework: p. 284, qq. 1-12
   As you go over this, make sure that meaningful interpretations are brought up every now and then. Have students provide real life examples that can be used to illustrate some of the concepts.

Development

QUIZ: Part 1 is fairly straightforward. Part 2 will likely require some special instructions. (This part is included to give students some practice with the type of questions that will be asked in the explaining test at the end of the unit.) Emphasize that marks will be awarded on the basis of the correctness of the answer and on <u>how clearly the answer is expressed</u>. The second component is actually the more important one.

Homework

- none

Positive Interdependence

Each group will have the opportunity to earn bonus marks through the following procedure:

Home group averages for the previous unit <u>and</u> for this quiz will be calculated. The group that shows the greatest improvement will receive a bonus of +3 marks on the quiz. All other groups showing an improvement will get a bonus of +1. A group that shows no improvement or that goes down will get no bonus points.

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Lesson 9: Multiplying

Daily Review

1. Review: Return quiz
2. Homework: none
3. Mental Computation: (Oral. Allow students to use BINGO chips and/or number lines if desired.)

Add:	(+5) + (-4)	(-8) + (-1)	(-5) + (+9)
Subtract:	(+5) - (-4)	(-8) - (-1)	(-5) - (+9)

Development

- Hockey can be used to illustrate the result of multiplying two signed numbers. Begin by establishing whether the following items would be considered to be positive or negative.

Oilers (+)	Winning (+)	Happy (+)
Flames (-)	Losing (-)	Sad (-)

  Combining these we get:

  Oilers Winning --> Happy <u>or</u> (+) x (+) = (+);
  Oilers Losing --> Sad <u>or</u> (+) x (-) = (-);
  Flames Winning --> Sad <u>or</u> (-) x (+) = (-);
  Flames Losing --> Happy <u>or</u> (-) x (-) = (+).

  There's no need to immediately tie this into some multiplication examples. It can be left on the board for later reference.

- The Meaning of Multiplication: Recall that multiplication is really repeated addition. Thus, 3 x 5 can be restated as 5 + 5 + 5. This can be demonstrated with BINGO chips and on the number line.

  This means that (+3) x (+2) can be thought of as +3 groups of +2, or (+2) + (+2) + (+2) = (+6). This can be demonstrated with BINGO chips, and it can quickly be established that the product of two positives is a positive.

Similarly, (+3) x (-2) can be seen as +3 groups of -2, or (-2) + (-2) + (-2) = (-6). Demonstrating this and a few other examples will make it apparent that the product of a positive and a negative is a negative.

It can be shown that (-3) x (+2) produces the same answer as (+2) x (-3), since multiplication is commutative. One sequence to follow in establishing this is: (-3) x (+2) is the same as (+2) x (-3), which means you have +2 groups of -3, or (-3) + (-3) = -6.

Another way is to interpret (-3) x (+2) as (the opposite of +3) x (+2). That is, the you can simply replace the term "-3" with "the opposite of +3". The sequence would appear as follows:

(-3) x (+2) = (the opposite of +3) x (+2)
= the opposite of (+3) x (+2)
= the opposite of +6
= -6.

The product of a negative and a positive is thus a negative.

This argument becomes relevant when looking at the product of 2 negatives.

(-3) x (-2) = (the opposite of +3) x (-2)
= the opposite of (+3) x (-2)
= the opposite of -6
= +6.

The product of a two negatives is a positive. (This would be a good place to tie in the "rules" established in the "hockey game" business.)

- The worksheet (CHECKER MATH - MULTIPLICATION) can be given at this point. You may want to work through the examples in question 1 together and then assign question 2.

- Near the end of class, you can point out that there is yet another way to establish the multiplication rules, as outlined in the text using number patterns.

Homework

- p. 287, qq. 1-10

Positive Interdependence

This is a teacher-led lesson and students will be working on the homework assignment independently. At the start of the next class, have them get into their groups and check their responses. If there is any disagreement on an answer, members must discuss it until everyone is in agreement. All group members are then to sign and turn in a single assignment for a group grade.

Lesson 10: Dividing

Daily Review

1. Review: Meaning of Multiplication; Multiplying Integers with BINGO Chips; Quick Rules for Multiplying Integers
2. Homework: p. 287, qq. 1-10 (Home groups check each others homework and then turn in a single signed copy.)
3. Mental Computation: (Oral)

(+11) x (+2)	(+8) x 7	(-8) x (+2)
(+7) x (-8)	(-4) x (-3)	(+9) x (-9)

Development

Division: Possible Interpretations (Ask students what division means or what you're doing when you divide.)

- the opposite of multiplication (which is can be thought of as putting together groups of objects into a larger group);
- splitting a group of objects into smaller groups;

- Two possible ways of looking at division (and specifically, the quotient) are illustrated by the following example:

  8 / 2 can be interpreted as:

  a) take 8 objects and split them into 2 groups. How large is each group?

  (o o o o) (o o o o)

  b) take 8 objects and split them into groups of 2. How many groups do you get?

  (o o) (o o) (o o) (o o)

- The first interpretation (a) is the one that will be used in this lesson. So, in the dividing sentence: 6 / 3 = 2, the first term (6) is the number of objects you start with, the second (3) is the number of groups you want to create, and the quotient (2) is the number of objects in each group.

Dividing Integers

- Dividing by a positive integer

  eg. (+12) / (+4) means +12 objects split into +4 groups. How many will there be in each group? (This is easily demonstrated with chips.) [ESTABLISH THAT (+) / (+) = (+).] Once this has been demonstrated, it should be reasonable to expect that some students will be able to explain their way through the next example.

eg. (-10) / (+5) means -10 objects (i.e. 10 red chips) divided into +5 groups. How many in each group? [ESTABLISH THAT (-) / (+) = (-).]

- Dividing by a negative integer
  eg. (+6) / (-3) means +6 objects divided into -3 groups, which makes little sense. We can split them into +3 groups, however, but that produces an opposite answer. So:

  (+6) / (-3) = the opposite of (+6) / (+3)
  = the opposite of (+2)
  = -2 [ESTABLISH THAT (+) / (-) = (-).]

  eg. (-8) / (-2) means -8 objects split into -2 groups. Again, this makes little sense, so:

  (-8) / (-2) = the opposite of (-8) / (+2)
  = the opposite of (-4)
  = +4 [ESTABLISH THAT (-) / (-) = (+).]

- Note that the division rules are the same as the multiplication rules, which makes sense since division and multiplication are so closely related.

- Split the class into their groups and assign the worksheet: CHECKER MATH - DIVISION. When reviewing answers, have individual students explain the process involved.

Homework

- worksheet: CHECKER MATH - DIVISION
- p. 289, qq. 1-9

Positive Interdependence

Grade the homework assignment in the same way as the multiplication assignment was graded. If there is time, allow students to work together on the assignment.

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Lesson 11: Order of Operations

Daily Review

1. Review: Quick Rules for Multiplication
   Quick Rules for Division
2. Homework: p. 289, qq. 1-9
3. Mental Computation: (On board)

a) 7 x 4 - 3	c) 12 / 6 + 3 - 1	e) 5 + 3 x 2
b) 8 / 2 + 6 x 1	d) 5 x (8 + 1) - 10	f) 8 - 2 x 3 + 1

Development

T: The mental computation exercises should provide a smooth lead-in for this lesson. Review the rules for order of operations and point out that the same rules apply for integers.

The textbook sets out the lesson well. No additional meaningful interpretations will be brought in, so the lesson will be handled much as it is described in the resource manual. The main purpose here is to provide some practice for all of the basic operations.

Provide a few examples (eg. "Working Together" on page 290), and have students describe the processes followed, emphasizing the importance of properly naming numbers and operations.

S: Send students into their groups to do the groupwork assignment, which is due prior to the end of class.

Homework

- p. 291, qq. 3, 6 (Groupwork Assignment)
- p. 291, qq. 1, 2, 4, 5

Positive Interdependence

Each group will be turning in a single worksheet for questions 3 and 6 on page 291. For each question, the original number statement must be written out, and then solved step-by-step. As students are working out the answers, the sheet is passed from person-to-person, each doing the next step in the process.

You can therefore expect three different types of writing on the answer page, and the style should change from line to line.

Students are allowed to check each other's work after a line is written, but are not permitted to say anything while someone else is writing. (Hopefully this will reduce dictation.)

Lesson 12: Calculator and Review

Daily Review

1. Review: none
2. Homework: p. 291, qq. 1-6 (on transparency)
3. Mental Computation: (Oral)

(-6) + (-8)	-2 - 11	-8 x (-4)	-15 / 3
8 + (-8)	(-2) - (-20)	-4 x -4 x -4	-45 / (-9)

Development

T: Calculator: The is well-laid out in the the textbook (p. 285). The aim here is to draw students' attention to the SIGN CHANGE (+/-) KEY and its use. A quick description of how it is used and a few practice questions should be adequate.

S: Chapter Checkup: The unit is complete. As a final review exercise, the three classes will be doing questions 1 through 8 o page 303 of the textbook.

Seatwork/Homework

- p. 303, qq. 1-8

Positive Interdependence

A single student's work will be randomly chosen at the start of the next class and a group grade assigned.

APPENDIX 2

THE FIRST LESSON OF EACH OF THE THREE INSTRUCTIONAL APPROACHES

CONVENTIONAL TEACHER-CENTERED MODEL
(APPROACH A)

Lesson 1: Definition and Use of Integers
The ZERO Concept

Daily Review

1. Review: none
2. Homework: none
3. Mental Computation: none

Development

- definitions of INTEGER, POSITIVE, NEGATIVE and ZERO (as an integer that is neither positive nor negative)

- situations in which integers might be used. For each instance, present the meaning of "zero", of positive terms, and of negative terms. (A central purpose here is to point out that "zero" doesn't always mean "nothing". Sometimes it's an arbitrary value, chosen for convenience.)

TEMPERATURE	(0 = freezing point of water; + = warmer)
ALTITUDE	(0 = sea level; + = above or higher)
PROFIT/LOSS	(0 = neither gain nor loss)
BANK BALANCES	(0 = no money; - = in debt)
GOLF SCORES	(0 = par)
ELECTRIC CHARGE	(0 = neutral or no charge)
COUNTDOWNS	(0 = take-off time; - = pre-launch)
MOTION	(0 = no movement or the starting point)
+/- HOCKEY STATS	(0 = same # of goals scored by both teams when player X was on the ice)

- words that can be translated as positive and negative (Begin to build lists that can be added to as the unit progresses.)

POSITIVE	NEGATIVE
increase/rise	decrease/fall
above/over	below/under
gain/profit	loss
in the black	in the red
up	down

- Working Together: p. 275, qq. 1-3

Seatwork

- p. 275, qq. 1-4 (Take this up before the end of class.)

Homework

- p. 275, qq. 5-8 and Keeping Sharp

TEACHER-LED DISCUSSION MODEL
(APPROACH B)

Lesson 1: Definition and Use of Integers
The ZERO Concept

Daily Review

1. Review: none
2. Homework: none
3. Mental Computation: none

Development

- definitions of INTEGER, POSITIVE, NEGATIVE and ZERO (as an integer that is neither positive nor negative)

- situations in which integers might be used. For each instance, discuss the meaning of "zero", of positive terms, and of negative terms. (A central purpose here is to point out that "zero" doesn't always mean "nothing". Sometimes it's an arbitrary value, chosen for convenience.)

TEMPERATURE	(0 = freezing point of water; + = warmer)
ALTITUDE	(0 = sea level; + = above or higher)
PROFIT/LOSS	(0 = neither gain nor loss)
BANK BALANCES	(0 = no money; - = in debt)
GOLF SCORES	(0 = par)
ELECTRIC CHARGE	(0 = neutral or no charge)
COUNTDOWNS	(0 = take-off time; - = pre-launch)
MOTION	(0 = no movement or the starting point)
+/- HOCKEY STATS	(0 = same # of goals scored by both teams when player X was on the ice)

Elicit as many of the above situations as possible from the students. Have them elaborate on their answers through questions such as: What does zero mean in this case? What does positive seven mean here? What would it mean if a value were below zero?

Stress the appropriate use of terms and expressing answers in complete sentences. Model answers. Have students rephrase and/or repeat responses.

- words that can be translated as positive and negative (Begin to build lists that can be added to as the unit progresses. This portion may rise naturally from the preceding discussion.)

POSITIVE	NEGATIVE
increase/rise	decrease/fall
above/over	below/under
gain/profit	loss
in the black	in the red
up	down

- Working Together: p. 275, qq. 1-3

Seatwork

- p. 275, qq. 1-4 (Take this up before the end of class.)

Homework

- p. 275, qq. 5-8 and Keeping Sharp

COOPERATIVE LEARNING MODEL
(APPROACH C)

Lesson 1: Definition and Use of Integers;
The ZERO Concept

Daily Review

1. Review: none
2. Homework: none
3. Mental Computation: none

Development (T = teacher-led; S = done in small groups)

T: definitions of INTEGER, POSITIVE, NEGATIVE and ZERO (as an integer that is neither positive nor negative)

T: situations in which integers might be used. (Through a group discussion, elicit as many of the following situations as possible from the students.)

TEMPERATURE	(0 = freezing point of water; + = warmer)
ALTITUDE	(0 = sea level; + = above or higher)
PROFIT/LOSS	(0 = neither gain nor loss)
BANK BALANCES	(0 = no money; - = in debt)
GOLF SCORES	(0 = par)
ELECTRIC CHARGE	(0 = neutral or no charge)
COUNTDOWNS	(0 = take-off time; - = pre-launch)
MOTION	(0 = no movement or the starting point)
+/- HOCKEY STATS	(0 = same # of goals scored by both teams when player X was on the ice)

T: Working Together: p. 275, qq. 1-3

S: For each example above, tell what a value of "zero" means, what a positive value means, and what a negative value means. Make a list of all the words that stand for zero. (A central purpose here is to point out that "zero" doesn't always mean "nothing". Sometimes it's an arbitrary value, chosen for convenience.)

S: Make a list of words that might be translated as positive and negative. (Begin to build a chart that can be added to as the unit progresses.)

POSITIVE	NEGATIVE
increase/rise	decrease/fall
above/over	below/under

gain/profit	loss
in the black	in the red
up	down

S: p. 275, qq. 1-4

T: Sum up with a class discussion, ensuring that everyone has an appropriate list and the correct responses to the seatwork. Stress the proper use of terms and expressing answers in complete sentences. Model answers. Have students rephrase and/or repeat responses.

Homework

- p. 275, qq. 5-8 and Keeping Sharp

Positive Interdependence

For questions 1-4 on page 275 (which are to be done in class), one person's paper from each group will be randomly chosen and scored. All group members will receive the same score.
