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# Finite Larmor radius effects in stimulated Raman scattering

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Stimulated Raman scattering from a magnetized plasma has been examined using the set of Vlasov-Maxwell equations. The incident laser light, propagating in the form of an extraordinary mode, decays into a scattered extraordinary mode together with electron-Bernstein waves. For sufficiently strong magnetic fields and high electron temperatures, the growth rates of these waves become comparable to the field-free values. At relatively low plasma densities the magnetic field can result in scattered light at  $\omega_0/2$  (where  $\omega_0$  is the laser frequency), a frequency associated with radiation emitted from the quarter-critical density region of unmagnetized plasmas. We have compared the results from the kinetic theory analysis with observations from numerical experiments using a 13-D electromagnetic particle code and found them to be in good agreement.

## **I. INTRODUCTION**

The presence of large self-generated magnetic fields in nonuniformly irradiated target plasmas is well established.<sup>1,2</sup> The fact that these fields can assume their peak values in the underdense corona close to the quarter-critical density<sup>2,3</sup> has led naturally to some interest as to whether fields of several megagauss can influence processes occurring there to any measurable degree.<sup>4-6</sup> In particular Barr, Boyd, Gardner, and Rankin<sup>7</sup> examined the effects of strong magnetic fields on the Raman and two-plasmon decay instabilities and showed that the maximum growth rates of both instabilities are increased on account of the field. For the Raman instability the field-induced shift in frequency from  $\omega_0/2$  was such that the ordinary thermal red shift<sup>8</sup> was converted to a blue shift. Recent experimental observations<sup>9,10</sup> of stimulated Raman scattering have shown clear evidence of structure in the  $\omega_0/2$  scattered light spectrum, and in particular, in many instances a well-defined double-peaked spectrum. Various mechanisms have been proposed to explain this feature in the spectra including, for example, linear mode conversion of  $2\omega_p$  plasmons<sup>9,10</sup> as well as the magnetic field effects discussed in our earlier paper.<sup>7</sup> However, the question of the origin of the observed structure is largely unresolved. In this article we look again at the effects of strong magnetic fields on Raman scattered light and, in particular, generalize the earlier theory which was valid only in the fluid limit, to examine the influence of finite Larmor radius effects on the Raman instability.

#### II. MODEL

We present results of an analysis of stimulated Raman scattering from a uniform magnetized plasma based on the Vlasov-Maxwell equations. The nonlinear dispersion relation describing Raman decay is valid for arbitrary magnetic field strengths and for pump powers and electron temperatures up to the relativistic threshold. In contrast to our previous study,<sup>7</sup> we have not included electromagnetic effects in the dispersion relation that describes the plasma waves. Using an electrostatic approximation for the Bernstein waves restricts the validity of our results to densities well below quarter-critical, thus ensuring that the phase velocity of the upper-hybrid waves is less than the speed of light. Although this means that we do not determine accurately the fieldinduced frequency shifts, the instability growth rates are not likely to be affected significantly by this approximation.<sup>7</sup> We have also carried out a study of Raman scattering levels using a 1<sub>2</sub>-D particle code and compared the results of these numerical experiments with the growth rates predicted theoretically.

The analysis, though straightforward, is very cumbersome and is not presented here.<sup>11</sup> The dispersion relation is solved numerically as described previously.<sup>7</sup> The laser light is assumed to be incident at right angles to a uniform magnetic field  $\mathbf{B}_c$  so that it propagates as an extraordinary wave with wavenumber  $\mathbf{k}_0$  and wave magnetic field  $\mathbf{B}_0$  with  $\mathbf{k}_0 \perp \mathbf{B}_c || \mathbf{B}_0$ . Raman decay produces a scattered extraordinary wave together with an electron Bernstein mode.

#### **III. DISCUSSION**

In Figs. 1 and 2 we plot the Raman growth rate  $\gamma$  versus the wavenumbers k of the electrostatic modes excited. The numbers on the curves denote the frequencies of the electrostatic waves (normalized to  $\omega_0$ ) at the various positions of



FIG. 1. Growth rate  $\gamma$  as a function of k for decay into Bernstein waves n = 1,2,3. The numbers on the curves indicate the frequencies of the excited Bernstein waves at the position of maximum growth:  $\omega_p = 0.4\omega_0$ ,  $V_0/c = V_r/c = 0.1$ ,  $\Omega/\omega_0 = 0.16$  (solid line), 0.17 (broken line).



FIG. 2. Growth rate  $\gamma$  as a function of k for decay into Bernstein waves n = 1,2,3. The numbers on the curves indicate the frequencies of the excited Bernstein waves at the position of maximum growth:  $\omega_p = 0.4\omega_0$ ,  $V_0/c = 0.1$ ,  $V_e/c = 0.06$ ,  $\Omega/\omega_0 = 0.16$  (solid line), 0.17 (long dashed line), 0.18 (short dashed line).

maximum growth (cf. Barr, Boyd, Gardner, and Rankin<sup>7</sup>). For the values chosen for the plasma frequency,  $\omega_p$ , electron thermal velocity  $V_e$ , and imposed magnetic field, only lower-order Bernstein modes  $n\Omega < \omega < (n+1)\Omega$  with n = 1,2,3are excited;  $\Omega$  is the electron-cyclotron frequency. Note that there are two positions in k space at which curves with the same value of n occur. These correspond to forward- (ck / $\omega_0 < 0.9$ ) and backward- (ck / $\omega_0 > 0.9$ ) propagating scattered light waves; c is the speed of light. It is of course well known from fluid theory<sup>7</sup> that the growth rate is largest for backscatter and this is clearly true in this case as well. The reflection points of the scattered extraordinary waves occur when their respective frequencies approach the right-hand cutoff frequency:  $\omega_R = \frac{1}{2} \left[ \Omega + (\Omega^2 + 4\omega_p^2)^{1/2} \right] - \omega_p$  for small values of  $\Omega$ . At the reflection points of these waves k  $\rightarrow k_0$  (~0.9  $w_0/c$ ) and forward- and backward-propagating waves begin to merge. Note that this occurs in the case of the n = 3 mode in Fig. 1. The turning points for the other two modes lie at higher densities. Note too that the effect of increasing the magnetic field is simply to move individual modes nearer to their reflection points. Thus the presence of strong fields repositions the instabilities at lower densities. The growth rates of the n = 2,3 modes in Fig. 1 are by comparison much larger than that for n = 1. At smaller values of  $\Omega$  ( $\Omega/\omega_0 < 0.15$ ), the n = 2.3 growth curves merge and become indistinguishable; hence large fields are needed in order for growth to occur in the upper frequency band.

The  $2\Omega < \omega < 3\Omega$  cyclotron harmonic band contains the upper-hybrid frequency, and hence the n = 2 mode can be interpreted as the corresponding fluid mode with finite Larmor radius corrections. This is apparent in Fig. 2 where a lower electron temperature has been used. The growth rates of the n = 1 and n = 3 Bernstein waves are much reduced (the n = 1 branch is too small to be plotted). By contrast, growth in the  $2\Omega < \omega < 3\Omega$  cyclotron harmonic band has increased. Thus the n = 2 mode is the only mode that persists in the limit of low Larmor radius. For the sake of comparison note that the field-free growth rate is given approximately by  $\gamma = \frac{1}{2}k\omega_p V_0 \sim 0.02\omega_0$ , when  $\omega_p = 0.4\omega_0$ ,  $k \sim \omega_0/c$ , and  $V_0/c = 0.1$ ;  $V_0$  is the electron quiver velocity.

We have, in addition, performed a number of numerical experiments on Raman scattering from a magnetized plasma using an electromagnetic 11-D particle code.<sup>12</sup> In the first experiment we set  $\omega_{p} = 0.4\omega_{0}$ ,  $\Omega = 0.16\omega_{0}$ , and  $V_{e}/c = 0.1$ , thus allowing comparison with the results in Fig. 1. In the simulations, a constant density plasma of length L = 140 c/ $\omega_0$  was modeled. Constant imposed magnetic fields were applied perpendicular to the direction of the incident laser light. Figure 3 reproduces the spectrum of light, of frequency  $\omega_{em}$ , backscattered from the plasma. This spectrum shows two distinct harmonic lines with frequencies corresponding to the excitation of the n = 2,3 Bernstein waves, i.e.,  $\omega_{em} \sim 0.50 \omega_0$  and  $\omega_{em} \sim 0.58 \omega_0$ , respectively (the n = 1mode did not materialize because of its comparatively small growth rate). The theoretical results in Fig. 1 predict that the growth rates of these modes should be nearly equal. This was in fact the case early in the simulations; however, the time development of the instability revealed a lower saturation level for the n = 2 mode than for the n = 3 wave. Figure 4 shows the effect of increasing the strength of the magnetic field slightly to a level corresponding to  $\Omega = 0.164\omega_0$ . For this value, theory predicts a reduced growth rate for the wave in the cyclotron harmonic band  $3\Omega < \omega < 4\Omega$ . This again was observed in the simulations but now the saturation levels for the various excited modes are closer together than the corresponding levels in Fig. 3. A further run with  $\Omega = 0.17\omega_0$  produced a single-peaked spectrum with a frequency of approximately  $0.58\omega_0$ , as one would expect from theory (cf. Fig. 1).

To test the dependence of the growth rates of the instabilities on the plasma temperature, we set  $V_e/c = 0.06$  to allow comparison with Fig. 2. A single-peaked spectrum observed near saturation, and centered at a frequency of  $\omega_{em} \sim 0.58\omega_0$ , was taken to be evidence of the temperature effects described by Fig. 2. Early in the run, a small peak in intensity was observed at lower frequency; however, this fea-



FIG. 3. Backscattered light spectrum measured in the plasma simulation code. Here  $\omega_p = 0.4\omega_0$ ,  $\Omega/\omega_0 = 0.16$ ,  $V_0/c = V_e/c = 0.1$ . The plasma was homogeneous with fixed ions.



FIG. 4. Backscattered light spectrum measured in the plasma simulation code. Here  $\omega_{\rho} = 0.4\omega_{0}$ ,  $\Omega/\omega_{0} = 0.164$ ,  $V_{0}/c = 0.1$ ,  $V_{e}/c = 0.1$ . The plasma was homogeneous with fixed ions.

ture was no longer apparent as the  $n = 2 \mod e$ -folded during the time evolution. This indicates slower growth of the n = 3 Bernstein wave caused by the temperature reduction. Finally, in Figs. 1 and 4 we note that despite the relatively low plasma density (0.16 of the critical density), the backscattered Raman light spectrum can extend to below  $\omega_0/2$ . Fluid theory predicts that this should only occur near the quarter-critical density where temperature effects in the plasma wave dispersion relation cause a red shift<sup>8,13</sup> determined from  $\omega_{em} \sim (1/2 - 9/8V_e^2/c^2)\omega_0$ . At lower magnetic field strengths ( $\Omega \sim 0.05 \omega_0$ , i.e., 5 MG for 1  $\mu$ m light) the above effects disappear.

#### IV. SUMMARY

The Raman decay of an extraordinary light wave into a scattered extraordinary wave together with an electrostatic mode has been examined theoretically and by means of a simulation code. In relatively hot plasmas with  $V_e/c \sim 0.1$ , significant departures from the predictions of fluid theory occur when large magnetic fields are present, i.e., for  $\Omega > 0.15\omega_0$ . At field strengths corresponding to this value, the upper-hybrid wave can *separate* into discrete spectral components corresponding to the excitation of electron-Bernstein waves. For  $0.15\omega_0 < \Omega < 0.20\omega_0$ , waves with fre-

quencies within the first three cyclotron harmonic bands are the only ones that have nonzero growth rates. The n = 2harmonic band is, in this case, the one in which the upperhybrid frequency occurs and, indeed, as the electron temperature is reduced, this is the only growing mode. When  $V_e/c \sim 0.1$ , Bernstein waves in the n = 2.3 cyclotron-harmonic bands can have almost equal growth rates. At relatively low plasma densities, i.e., for  $\omega_p \leq 0.4\omega_0$  with  $\Omega$  /  $\omega_0 > 0.15$ , the backscattered Raman light wave associated with the n = 3 Bernstein mode is shifted to frequencies less than  $\omega_0/2$ . This contrasts with the results of fluid theory, which predict that the scattered light frequency should increase above  $\omega_0/2$  as density reduces; the latter behavior is observed with the n = 2 Bernstein wave. Thus, the combination of long-wavelength laser light and large self-generated magnetic fields, requiring that finite Larmor radius effects be included in a theoretical description, may give rise to scattered light at frequencies less than would be predicted by fluid theory. The present theory, therefore, is relevant for laser wavelengths of  $10\,\mu m$  but is unlikely to be important at wavelengths of 1  $\mu$ m or less, in view of the unrealistically large magnetic fields (> 10 MG) that would be required.

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