Workshop on Mathematical Challenges in Brittle Material Failure

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ABSTRACT

The Army Research Office funded an invitation-only workshop entitled “Identifying Mathematical Challenges Associated with Failure of Brittle Materials” at the Johns Hopkins University, Maryland on May 20-21, 2019. The workshop brought together mathematicians, statisticians, and mechanics of materials researchers with diverse academic and research backgrounds to discuss the state-of-the-art in brittle material failure prediction and to identify new directions for future research. Three specific goals of the workshop were: (1) to identify the state-of-the-art for modeling failure of brittle materials (e.g., ceramics, glasses); (2) to discuss the major mathematical and statistical challenges experienced by academics and scientists studying brittle failure; and (3) to propose novel and unexplored research collaborations between mechanics researchers and mathematicians to address the identified challenges. This document provides a summary of workshop presentations, discussions, and recommendations for future work (and research funding) that emerged from the workshop. The recommendations for future work are organized into four major thrusts: (i) defining robust quantities of interest; (ii) understanding and modeling variability and stochasticity; (iii) model parameter importance and calibration; and (iv) transitioning from discrete to continuum behaviors. For each thrust, specific future work discussed in the workshop is described.

INTRODUCTION

Failure of brittle materials by fracture, fragmentation, and comminution is an issue with fundamental academic, industrial, geological, and societal importance. For example, brittle failure is central to geological processes including fault rupture (Scholz 2019), to the design and modeling of protective materials including armor ceramics (Karandikar et al. 2009; Chen et al. 2007), and to the performance of structural and quasi-brittle materials including concrete (Bazant 2019). Practices for predicting the failure of brittle materials by numerical modeling vary dramatically across academia and industry. Such differences include the choice of spatial discretization scheme, numerical solver, and the level of resolved physical detail. There are also challenges with capturing the complex physics and material heterogeneity that are fundamental to brittle failure problems with current mathematical models.

To address these challenges, the Army Research Office (ARO) sponsored a workshop at Johns Hopkins University (JHU) on May 20-21, 2019, titled "Workshop on Identifying Mathematical Challenges Associated with Failure of Brittle Materials". The specific goals of the workshop were to: (1) identify the state-of-the-art for modeling failure of brittle materials (e.g., ceramics, glasses); (2) discuss the major mathematical or statistical challenges experienced by researchers studying brittle failure; and (3) propose *novel and unexplored* collaborations between mechanics researchers and mathematicians to address the identified challenges. This document describes workshop presentations, identifies the major mathematical challenges discussed by participants, and makes recommendations for future work (and research funding) that may address these challenges. Note that the Army has previously organized workshops focused on dynamic failure of brittle materials, including a 2016 Dynamic Failure Forum at Aberdeen Proving Ground, Maryland (Aydelotte et al. 2016). The distinction between prior workshops and the workshop hosted at JHU was the emphasis of the JHU workshop was on identifying paradigm-shifting mathematical approaches from the pure and applied mathematics and statistics community.

The remainder of this document is organized as follows. First, a list of workshop participants and the results of a pre-workshop survey are provided. The pre-workshop survey was intended to help workshop chairpersons design the workshop schedule. Next, a chronological summary of the workshop is provided, describing the workshop sessions and major open questions discussed by participants throughout the workshop. Next, results of a final exit-survey are discussed. Finally, a summary and future-funding recommendations are provided. The recommendations are based on workshop presentations, more than three hours of structured discussions, a one-hour focused discussion at the end of the workshop, and an exit-survey completed by participants. The recommendations for future work (and research funding) are organized into four major thrusts: (i) defining robust quantities of interest; (ii) understanding and modeling variability and stochasticity; (iii) model parameter importance and calibration; and (iv) transitioning from discrete to continuum behaviors. For each thrust, specific future work discussed at the workshop is described.

WORKSHOP PARTICIPANTS AND PRE-WORKSHOP SURVEY

The workshop was chaired by Prof. Ryan Hurley (Johns Hopkins University), Prof. James Hogan (University of Alberta), and Prof. Surya Kalidindi (Georgia Institute of Technology). The workshop was attended by 24 total participants - 16 university faculty members, 6 scientists from national laboratories, and 2 program officers from the ARO. Although the workshop chairpersons selected participants to provide presentations related to brittle failure mechanics (ME) or mathematical techniques (MA), many participants were experts in both domains. The workshop participants and their primary expertise were:

- Michael Bakas (ME), Army Research Office
- Richard Becker (ME), Army Research Laboratory
- Florin Bobaru (ME), University of Nebraska-Lincoln
- Tan Bui-Thanh (MA), University of Texas at Austin
- Maria Cameron (MA), University of Maryland, College Park
- Wai-Tong (Louis) Fan (MA), Indiana University, Bloomington
- George Gazonas (ME), Army Research Laboratory
- Roger Ghanem (MA), University of Southern California
- Lori Graham-Brady (ME), Johns Hopkins University

- Michael Homel (ME), Lawrence Livermore National Laboratory
- James Hogan (ME), University of Alberta
- Ryan Hurley (ME), Johns Hopkins University
- Surya Kalidindi (ME), Georgia Institute of Technology
- Jia-Liang Le (ME), University of Minnesota
- Yongming Liu (ME), Arizona State University
- Bruce Pitman (MA), University at Buffalo
- Michael Shields (MA), Johns Hopkins University
- David Stepp (ME), Army Research Office
- Samy Tindel (MA), Purdue University
- Andrew Tonge (ME), Army Research Laboratory
- Dongbin Xiu (MA), Ohio State University
- Min Zhou (ME), Georgia Institute of Technology

Prior to the workshop, participants were requested to complete a brief survey by email that included several prompts. These prompts were designed to motivate workshop participants to understand the goals of the workshop, to think about what they wished to get out of the workshop, and to help the chairpersons design the workshop schedule. To inform the reader of the mindset of participants prior to the work, two prompts and summarized responses from non-chairperson participants are provided below:

1. Prompt: Identify up to three challenges you face in theory, modeling, or experiments related to brittle failure that are mathematical in nature (for mechanics researchers) **or** three mathematical tools that you believe are underutilized in modeling physical systems in general or brittle fracture in particular (for mathematics researchers).
    - Summarized responses from mechanics researchers: Quantifying uncertainty and modeling across spatiotemporal scales; deterministic and probabilistic modeling; scalar or statistical quantities of interest; tools for quantifying complex spatiotemporal patterns in 3D.

- Summarized responses from mathematics researchers: Spanning spatiotemporal scales with limit theories and stochastic partial differential equations; combining uncertainties across parameters and scales; machine learning; surrogate modeling.

2. Prompt: Identify up to three topics that you would like to discuss or learn at the workshop.
   - Mechanics researchers: Uncertainty quantification (UQ) across spatiotemporal scales; machine learning; reduced order modeling; transition between states (e.g., fracture to granular flow); relating microscopic heterogeneities to macroscopic properties; developing new quantities of interest.
   - Mathematics researchers: Basics of fracture modeling across scales; Gaussian process learning; bridging scales and collapsing dimensionality; machine learning.

As noted later in this document, survey responses that became primary themes of workshops discussions included: (1) establishing scalar or statistical quantities of interest for characterizing failure; (2) uncertainty quantification in bridging spatiotemporal scales; and (3) new tools for quantifying complex spatiotemporal patterns (e.g., using machine learning, signature functions, etc.).

SUMMARY OF PRESENTATIONS AND DISCUSSIONS BY SESSION

The workshop was organized into six sessions, each of which involved presentations by two to four workshop participants. Sessions were paired, with one focused on material failure followed by one focused on mathematics and statistics. Each pair of sessions was followed by a 45-60 minute discussion by all workshop participants to summarize challenges and potential future research directions. The final workshop schedule is shown in table 1 and a chronological summary of discussions is provided next.

Day 1, Morning: Computational Methods for Modeling Fracture & Theoretical and Applied Math and Statistics

The morning of Day 1 first featured an introduction to the workshop goals and basic fracture mechanics concepts by the workshop chairs. The introduction was followed by presentations by three mechanics and two mathematics researchers in sessions titled "Computational Methods for

Modeling Fracture" and "Theoretical and Applied Mathematics and Statistics", respectively, as shown in table 1. A talk by a third mathematics researchers (Wai-Tong (Louis) Fan) was provided on the second day due to travel complications but is considered in the summary provided here, as intended in the original schedule.

The mechanics talks during the "Computational Methods for Modeling Fracture" session provided an overview of various methods for modeling blast and impact, constitutive relationships and mesoscale modeling involving the Material Point Method (MPM), and comparisons of simulations and experimental results for model calibration. Considerable attention was given to modeling fracture, fragmentation, and granular behavior across multiple length scales (nm to km) and time scales (ns to ms). Some major mathematical questions related to brittle fracture that emerged from the discussions and transcended any particular modeling approach included:

1. How to properly define or initialize material or mechanical property variability and associated uncertainty in simulations (e.g., fracture energy, strength)?
2. How to quantitatively characterize and compare complex failure patterns in simulations and experiments to determine if they are equivalent?
3. How to mathematically describe local transitions between material states (intact, damaged, granular) and interactions between neighboring material states in a partially damaged, granular, or fragmented material?

The mathematics talks during the "Theoretical and Applied Mathematics and Statistics" session provided an introduction to rough paths and stochastic differential equations (SDEs), a description of quasi-potential solvers for dynamical systems and complex networks, a discussion of rough path signature functions, and a presentation on methods for solving problems involving expanding wavefronts. The mathematics talks motivated discussions around several mathematics-focused questions related to brittle failure, including:

1. Can the governing equations of mechanics be recast as stochastic partial differential equations (SPDEs) to leverage SPDE-solving techniques for incorporating variability?
2. Can rough path signature functions be used to characterize and compare complex failure

patterns in 2D images?

3. Can high dimensional problems be collapsed into smaller parameter spaces for simulations, as is done for rough complex fields?

Additional questions and discussions related to mathematical challenges in brittle failure modeling that arose during this session are listed in Table 2.

Day 1, Afternoon: Computational Methods for Modeling Fracture & Stochastic Simulations

The afternoon of Day 1 involved presentations provided or led by three mechanics researchers and two mathematics researchers in sessions titled "Computational Methods for Modeling Fracture" and "Stochastic Simulations". The mechanics talks in the "Computational Methods for Modeling Fracture" sessions focused on cohesive zone modeling, fundamentals of peridynamic simulations for brittle failure simulations, and mesoscale simulations using realistic microstructures. The major questions emerging from the discussion included:

1. What are the major quantities of interest that can be used to characterize a material response across multiple geometries or loading conditions?
2. Do correlations exist between local material strength and local microstructural features in problems involving brittle failure?
3. How can one appropriately decompose a material response into contributions from all governing material and processing variables?

The mathematics talks in the "Stochastic Simulations" session described data-driven modeling and probabilistic frameworks for multiscale simulations involving brittle fracture. The major mathematics-related brittle failure questions emerging from the presentations and ensuing discussions included:

1. Can material and mechanical variability and uncertainty below sensor capacity during material characterization be considered white noise?
2. Can methods such as A-optimality, D-optimality, and quasi-optimality be employed to determine which experiments to perform for variable calibration when resources and the number of available experiments is severely limited?

Additional questions and discussions related to mathematical challenges in brittle failure modeling that arose during this session are listed in Table 2.

Day 2, Morning: Probabilistic and Computational Studies of Defects and Fractures & Uncertainty Quantification

The morning of Day 2 featured talks by four mechanics researchers and four mathematics researchers in sessions titled "Probabilistic and Computational Studies of Defects and Fractures" and "Uncertainty Quantification". The mechanics presentations described the role of defects across length scales, including their distributions and effects on fracture nucleation and coalescence, introduced crack band models for quasi-brittle materials, detailed dual high dimensionality requirements of physics modeling and uncertainty quantification in brittle failure modeling, and provided a demonstration of peridynamics approaches for modeling fracture.

Key questions related to brittle failure modeling that emerged from ensuing discussions included:

1. Can both 2D and 3D images be analyzed together using probabilistic relationships to learn how to characterize 3D failure from 2D images?
2. How can one rigorously transition from diffused to localized cracks in the absence of any closed-form solutions?

The mathematics presentations in the "Uncertainty Quantification" session provided an introduction to UQ methods, a description of data and uncertainty-driven reduction models for PDE-based models, and a discussion of UQ and material model uncertainty.

Key questions that emerged from the discussions included:

1. Can a simple forecasting model that avoids complex physics but includes simple scaling statistics perform well in predicting material failure?

Additional questions and discussions related to mathematical challenges in brittle failure modeling that arose during this session are listed in Table 2.

WORKSHOP EXIT SURVEY

An exit survey was completed by participants prior to the end of the workshop. Three survey prompts and summarized responses from non-chairperson participants are provided below:

1. Prompt: Identify areas where you see potential for new collaborations between mechanics and mathematics researchers.
   - Summarized participant responses: Identifying quantities of interest; solving inverse problems; UQ; machine learning; reduced-order modeling and UQ; efficient SPDE solving; classification of failure patterns using rough paths signature functions; quantifying spatiotemporal fracture metrics from experiments to validate simulations.
2. Prompt: Identify new ideas or directions you developed during the workshop.
   - Summarized participant responses: Inverse problem framework for parameter calibration in brittle failure simulations; quantifying uncertainty and developing reduced order modeling for brittle failure; signature function analysis of crack patterns; round robins of dynamic fractire.
3. Prompt: Identify any specific mathematical or other tools you would like to learn more about following the workshop, either through subsequent workshops, collaborations, or conference symposia.
   - Summarized participant responses: Gaussian Markov random fields; SPDEs and signature functions; fractional PDEs.

SUMMARY AND RECOMMENDATIONS

Figure 1 illustrates four major themes that emerged from the workshop presentations, discussions, and surveys. These four major themes capture the general thrust of recommended future work (and research funding). The following summary and recommendations focus on these themes and the future work that supports their exploration. Table ? contains additional questions that were raised by workshop participants during discussions or surveys but were not deemed to be the highest priority recommendations when considering goal number (3) of the workshop (see Abstract and Introduction). The remainder of this section summarizes the four themes conveyed in Fig. 1 and provides more detail regarding future research directions.

Theme 1: Defining Robust Quantities of Interest (QoIs)

Workshop participants agreed that clear quantities of interest (QoIs) must be defined for a given application related to brittle material failuure. A QoI can be a scalar, a set of scalars, or a field whose value correlates with a desired material performance. The sentiment that QoIs must be defined for a given application emerged in the morning of Day 1 of the workshop and was echoed continuously throughout the workshop both by mechanics and mathematics researchers. However, it became clear throughout the workshop that there are no widely-agreed-upon QoIs for many applications involving brittle material failure, either because material response is material-dependent, or because of challenges in establishing simple scalars that correlate well with desired performance. For instance, in confined compression applications, the value of a frictional strength parameter may be the appropriate QoI (Chocron et al. 2010). In applications such as sphere impact, the presence and angle of cone cracks, the number of radial cracks, or the area of comminuted material have all been studied (Leavy et al. 2010) as QoIs. However, many of these features (e.g., the presence of comminuted material) are strongly material dependent (LaSalvia et al. 2005; LaSalvia et al. 2009) or have not been clearly correlated to material performance. The workshop participants identified three *novel* ways in which mathematical techniques not previously applied to brittle failure may aid in the development of QoIs. These included:

1. Relating 2D or 3D fields (e.g., images containing fractures or damaged area) to scalar QoIs via rough path signature functions (Boedihardjo et al. 2016).
    - Signature functions are the main ingredients of rough paths theory. Signatures characterize 1D paths and 2D images and have unique mathematical and invariance properties (Boedihardjo et al. 2016). Signatures may provide a simple way of constructing a scalar representation of complex patterns such as the 2D fracture or damage patterns typically studied in the context of brittle material failure (e.g., cone cracks, radial cracks, comminuted area). New mechanics research may explore the generation of these datasets in a controlled manner, while parallel mathematics research may explore extensions of signatures to 2D and their use in translating fracture and damage images to scalar QoIs.

2. Comparing performance across loading conditions using rough path signature functions.
    - Beyond relating 2D and 3D fields to scalar QoIs using signature functions, an important goal of brittle failure research is developing QoIs applicable to multiple loading geometries and conditions. Future mechanics research may explore how brittle material performance can be deemed "equivalent" in different loading and failure scenarios. Parallel mathematics research may employ signature functions that return mathematically equivalent signatures.
3. Developing robust QoIs from multiple experimental fields (e.g., temperature, stress).
    - A distinct but related avenue for future research is the development of QoIs incorporating multiple fields. For instance, temperature, stress, and damage fields may all limit material performance and are each now measurable *in-situ* (e.g., (Keyhani et al. 2019a; Keyhani et al. 2019b)). Establishing new multi-field QoIs is a possible future mechanics research direction, while developing signature functions that can produce similar scalar metrics from consideration of multiple fields could be a supporting mathematics research direction.

THEME 2: ROLES OF VARIABILITY AND STOCHASTICITY

Workshop participants agreed that incorporating material and mechanical property variability into modeling is a paramount challenge for predicting the brittle failure of materials. This challenge is often addressed by including an initial defect or strength distribution (e.g., via Weibull modulus) in a simulation (e.g., (Tonge and Ramesh 2016)). The mathematics participants at the workshop had particular expertise in stochastic partial differential equations (SPDEs), which helped identify two *novel* ways in which mathematical techniques not previously applied to brittle failure prediction may aid in understanding the roles of material variability and stochasticity. These included:

1. Using stochastic formulations of governing equations to capture material variability and stochasticity.
    - Several mathematical methods exist for solving SPDEs with various types of variability and stochasticity. A new direction combining novel mechanics and mathematics

research may examine reformulations of governing equations (e.g., balance of linear momentum, wave equations) that contain physically-meaningful stochastic terms. Mathematics approaches to solution may yield new insight into how specific types of material variability or stochasticity give rise to specific varieties of macroscopic responses.

2. Exploring white noise and other stochastic processes to capture sub-measurement-resolution uncertainty.
   - Similar to stochastic formulations of governing equations, treating sub-measurement-resolution uncertainty via stochastic terms in governing equations was raised by workshop participants as a future research direction. Research in this direction may involve identifying resolution limits to typical measurement techniques (e.g., of defects in tomography images) and constructing appropriate white noise terms in governing equations to capture their potential effects on material performance.

THEME 3: MODEL PARAMETER IMPORTANCE, CALIBRATION

Several workshop participants gave presentations related to quantifying the relative importance of model and processing parameters. This task is very important to material performance and material processing simulations, both of which typically involve complex models containing many parameters (e.g., (Tonge and Ramesh 2016)). Workshop discussions covered a variety of methods for identifying the importance or quantitative value of parameters, such as polynomial chaos expansions (Crestaux et al. 2009), surrogate modeling (Queipo et al. 2005), quasi-optimality (Shin and Xiu 2016a; Shin and Xiu 2016b), and failure forecast modeling (Voight 1987; Voight 1988a; Voight 1988b). These discussions led to the identification of at least three *novel* ways in which mathematical techniques not previously applied to brittle failure prediction may aid in studies of model parameter importance and calibration. These included:

1. Exploring A-, D-, and quasi-optimality methods for experiment design.
   - These methods fall within the discipline of design of experiments (Pukelsheim 2006) and allow material model parameters to be estimated without statistical bias and with as few

measurements as possible. Employing these methods for designing future experiments for specific applications of brittle materials may aid researchers in extracting as much information as possible from scarce data when testing resources are limited, as is typical in experiments performed to study brittle failure.

2. Comparing surrogate modeling, deep learning, and other methods for identification of dominant material and process parameters.
   - A number of distinct approaches for identifying the relative contribution of material and process parameters on simulation results were discussed, including surrogate modeling, deep learning, and polynomial chaos expansions. A thorough comparison of these methods and their relative performance would yield insight into which of them may be optimal for identifying parameters with the strongest influence on material behavior or processing.
3. Exploring the performance of failure forecasting prediction models that do not contain detailed physics.
   - Methods have been proposed for predicting the occurrence of geologic events such as volcanic ruptures that do not contain detailed physical laws and instead employ simple empirical scaling relationships (e.g., (Voight 1987; Voight 1988a; Voight 1988b). Exploring whether similar methods can accurately predict the failure of brittle materials may provide a powerful and simple alternative to development and use of complex constitutive laws. Many workshop participants were interested in exploring this approach to brittle failure modeling.

THEME 4: BRIDGING SPATIOTEMPORAL SCALES

A common theme of presentations by both mechanics and mathematics researchers was that of bridging spatiotemporal scales. For instance, mechanics presentations discussing Weibull distributions of material strength, capturing "effective" continuum properties of discrete defects, and models such as crack band models all discussed the importance of making connections across spatial length scales. Similarly, a mathematics presentation on individual-based discrete models for solving stochastic wave equations provided an interesting method of building an understanding of connections between processes operating at different length scales. From these presentations, workshop discussions led to the identification of at least one novel way in which mathematical techniques not previously applied to brittle failure modeling may aid in the an understanding of spatiotemporal scale-bridging. This approach was:

1. Developing particle and individual-based discrete models to understand spatiotemporal scale-bridging transitions in brittle failure.
    - Particle and individual-based discrete models were discussed as tools for solving continuum equations governed by PDEs and SPDEs and understanding transitions between length scales. Specific examples were provided in the context of solving a stochastic reaction-diffusion equation that arises throughout ecology, physiology, combustion, and plasma physics, known as the FKPP (Fisher-Kolmogorov-Petrovsky-Pskunov) equation (e.g., see related work in (Houchmandzadeh and Vallade 2017)). Examples of solving other SPDEs are also found in the literature (e.g., (Durrett et al. 2016)). Many workshop participants were interested in exploring such particle or individual-based discrete models for solving stochastic versions of the PDEs governing mechanical systems (e.g., balance of linear momentum or the wave equation) in order to understand transitions from discrete (local) to continuum (effective) behavior.

In conclusion, themes and recommendations discussed in this article provide exciting and impactful directions for future inter-disciplinary and collaborative research opportunities. The advancements made through joint activities between mechanics and mathematicians will greatly improve the fundamental understanding and simulations of brittle failure. Together, this will lead to the development of improved brittle materials across many industrial sectors (e.g., security, construction, natural resources).

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TABLE 1. Workshop schedule.

Day	Presentation Title	Speaker
1	Introduction to workshop and fracture mechanics concepts	Ryan Hurley
	Session I: Computational Methods for Modeling Fracture	
1	Computational Methods for Fracture Modeling: Introduction and Overview	Richard Becker
1	Simulating comminution problems with the Material Point Method	Michael Homel
1	Attempts to use uncertainty quantification and calibration approaches in brittle systems subjected to impact loading	Andrew Tonge
	Session II: Theoretical and Applied Mathematics and Statistics	
1	Some applications of stochastic processes	Samy Tindel
1	Computing the quasipotential for nongradient SDEs	Maria Cameron
1	Introduction to rough paths techniques and applications	Samy Tindel
1	Discussions of challenges and possible future research directions	All participants
	Session III: Computational Methods for Modeling Fracture	
1	Introduction to some ARO challenges	Michael Bakas
1	Peridynamic modeling of dynamic fracture in solids	George Gazonas
1	Quantitative relations between macroscopic fracture behavior and mesoscale heterogeneous structures	Min Zhou
	Session IV: Stochastic Simulations	
1	Introduction to stochastic simulation approaches	Roger Ghanem
1	Data driven modeling	Dongbin Xiu
1	Probabilistic frameworks for multiscale simulations of brittle fracture	Roger Ghanem
1	Discussions of challenges and possible future research directions	All participants
	Session V: Probabilistic and Computational Studies of Defects and Fractures	
2	Probabilistic and Computational Studies of Defects and Fractures: Introduction	Lori-Graham Brady
2	Generalized crack band model for static and dynamic quasi-brittle fracture	Jia-Liang Le
2	Dual high-fidelity requirement of physics modeling and uncertainty quantification for brittle failure prediction	Yongming Liu
2	Stochasticity and homogenization in modeling brittle fracture when the microstructure matters	Florin Bobaru
	Session VI: Uncertainty Quantification	
2	Introduction to Uncertainty Quantification	Michael Shields
2	Data and Uncertainty-Driven Reduction Methods for PDE-Constrained Parameter Calibration Problems	Tan Bui-Thahn
2	Uncertainty Quantification and Material Models	Bruce Pitman
2	Stochastic spatial models for expanding wavefronts	Wai-Tong (Louis) Fan
2	Discussions of challenges and possible future research directions	All participants

TABLE 2. Important workshop discussions or questions distinct from main recommendations.

Question or Discussion
Can we develop a canonical model for important input and output parameters to help focus our efforts for materials development?
What new type of standard validation experiments can we perform (Kalthoff, edge-on-impact, expanding rings, thin plate perforation, modified sharpy, Brazilian disk experiments, crack speed versus G_c)?
How can neural networks be applied to analyze crack paths?
How do we model crack coalescence and interactions through direct or dynamic perturbation models?
Can we make a common data set available for validation?
How do we combined very accurate simulations with lower-order modelling to yield insightful results in a reasonable time?
How do we compare results across the different modelling approaches?
What are methods for distinguishing between material vs. experimental variability?
How to optimize the experimental sample size when attempting to determine variability parameters?
How many tests need to be performed before a reasonable match with simulations is decided?
What is the next-most-challenging problem after the ones we can currently solve (damage evolution laws through load-unload experiments with fragmentation characterization)?

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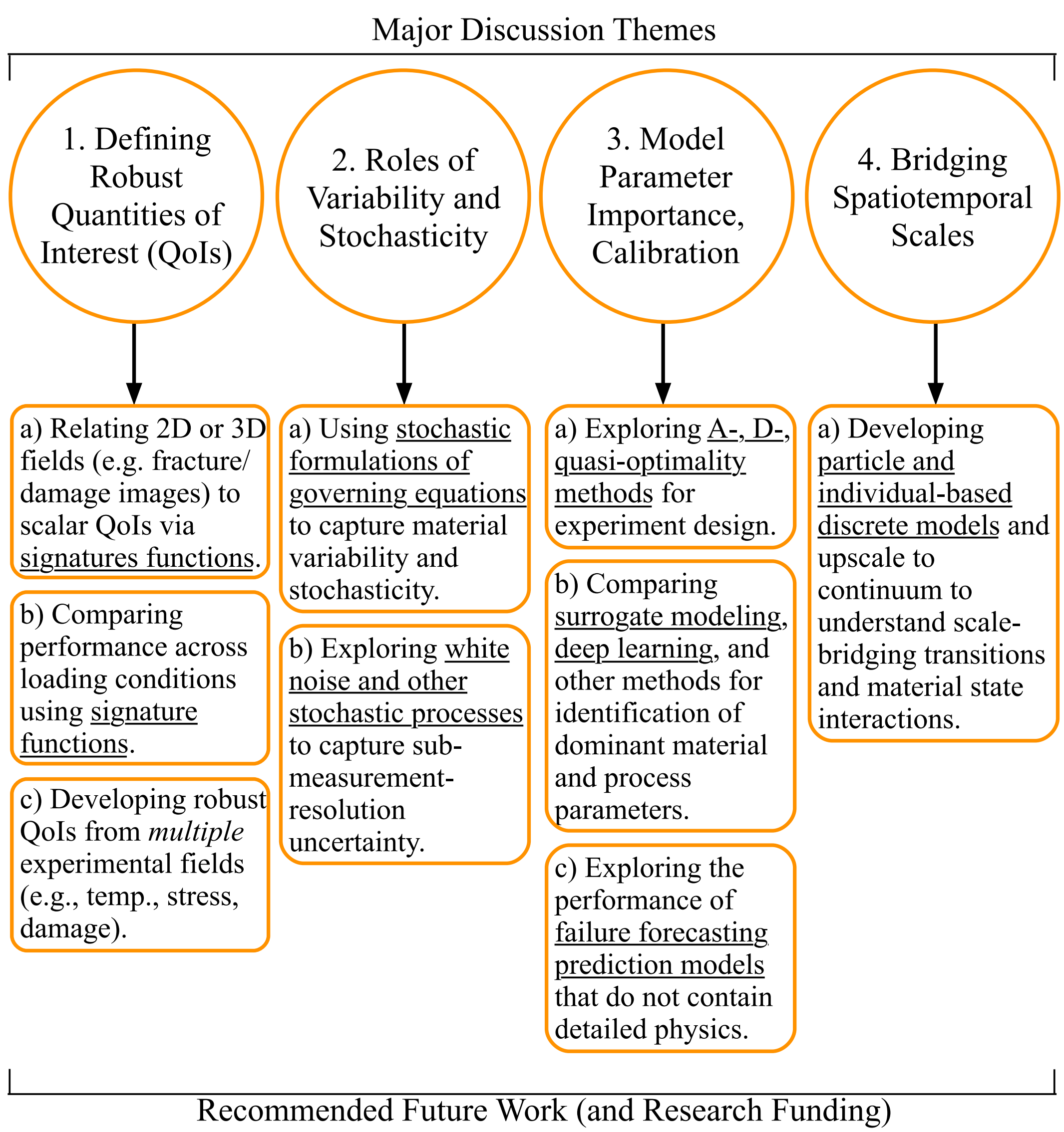

Fig. 1. Major discussion themes and recommended future work and research funding.