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MOMENT REDISTRIBUTION AND SECONDARY MOMENTS IN TWO-SPAN PRESTRESSED CONCRETE BEAMS

by

(C) Bruno SAM YUE CHI

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

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ABSTRACT

The object of this study is to investigate the flexural behavior characteristics of continuous prestressed concrete beams and to determine the effect of inelastic behavior on the secondary moment. The analysis uses conventional moment-curvature relationships and compatibility of geometry in predicting the complete moment-load curve to failure for a given beam. The analysis was compared with experimental results. It was found that a variation of secondary moment is likely once cracking occurs. Finally, from this investigation a design proposal involving the magnitude of the secondary moment at the ultimate limit state is recommended.

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1. INTRODUCTION

1.1 General Remarks

The subject of moment redistribution in continuous prestressed concrete beams, and its effect on the secondary moment produced by prestressing, has become a matter of much discussion.

A variety of approaches has been used to demonstrate that the secondary moment should be considered when calculating the ultimate load capacity of the beam, and can be neglected if and only if full redistribution of moment is achieved. Complete redistribution of moment is not likely in most practical cases. There is no experimental evidence that the full inclusion of the secondary moment in such instances yields a safe design.

1.2 Object and Scope

The purpose of this research is to study the flexural behavior characteristics of continuous prestressed concrete beams from the post-cracking stage up to ultimate in order to establish the relationship existing between the inelastic behavior and the secondary moment.

The procedure used is to set a theoretical model that is capable of tracing the post-cracking behavior of such beams, based on fundamental concepts outlined by previous investigators. The analysis described in Chapter 4 involves the use of moment-average curvature relationships and principles of geometry to determine the distribution of moment at any location in the beams at various stages of loading. A check on the accuracy of the theory is performed in Chapter 5 by comparison with available test data. Design recommendations are suggested as a result of the above analysis.

2. DEFINITIONS AND PROBLEM STATEMENT

2.1 Statically indeterminate construction

Although most prestressed concrete construction at the present time consists of statically determinate beams and girders, there are important advantages associated with indeterminate structures of prestressed concrete:

- Design moments are smaller for given spans and loads than for determinate structures;
- Stiffness is increased and deflection is reduced;
- By continuing post-tensioning tendons over several spans, fewer anchorages are required;
- Joint rigidity available in continuous frames is an important mechanism to resist horizontal loads such as are induced by wind, or seismic forces;
- Many ingenious arrangements have been developed to avoid the high friction losses of prestress.

As a result of these advantages, the applications of continuous prestressed construction are expanding, and this trend may be expected to continue (Ref. 1, 16, 17). Two-way, continuous flat plate slabs are widely used, and have proven both functional and economical. For medium and long span bridges, the economic and esthetic advantages of continuity are dominant considerations.

2.2 Secondary Moments

When an eccentric prestressing force is applied to a statically determinate beam as shown in Fig. 2.1, bending moments $P.e$ are induced; e is the eccentricity of the resultant tendon force P with respect to the centroid of the cross-section. The beam will deflect when prestressed, usually cambering upward, but no external reactions are produced by the prestressing force.

For a statically indeterminate beam as shown in Fig. 2.2, the action is more complex. The moment just described, which will be referred to as the primary moment, induces a deflection as before, but the beam is restrained by the redundant system of supports. Reactions are produced at those supports, giving rise to secondary moments in the beam. In this case, the total moments produced at any section by prestressing is the sum of the primary and the secondary moments.

The magnitude of the secondary moments in any given case depends on the particular tendon profile selected. For special cases such as the concordant tendon[1] case, the secondary moments may be zero. They are usually comparable to the primary moments and in many cases may be larger, even though they are called secondary.

----

[1] When the tendon profile selected produces no reactions due to prestressing, no secondary moments are developed. The thrust line produced by prestressing coincides with the steel centroid line, as would be the case for a single span, statically determinate beam. Such a tendon is called a concordant tendon.

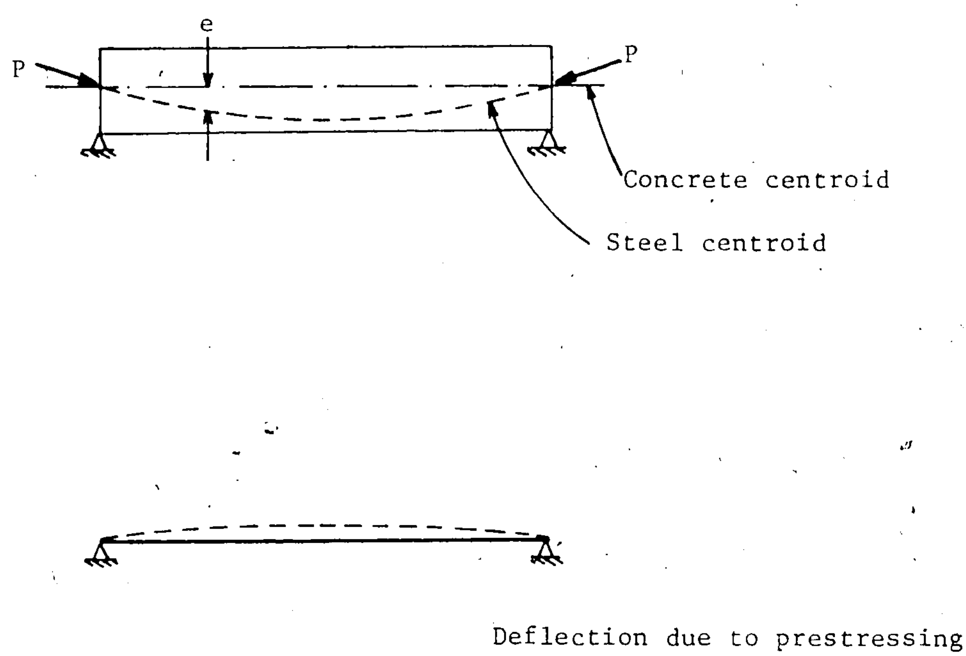


Figure 2.1 Statically determinate beam

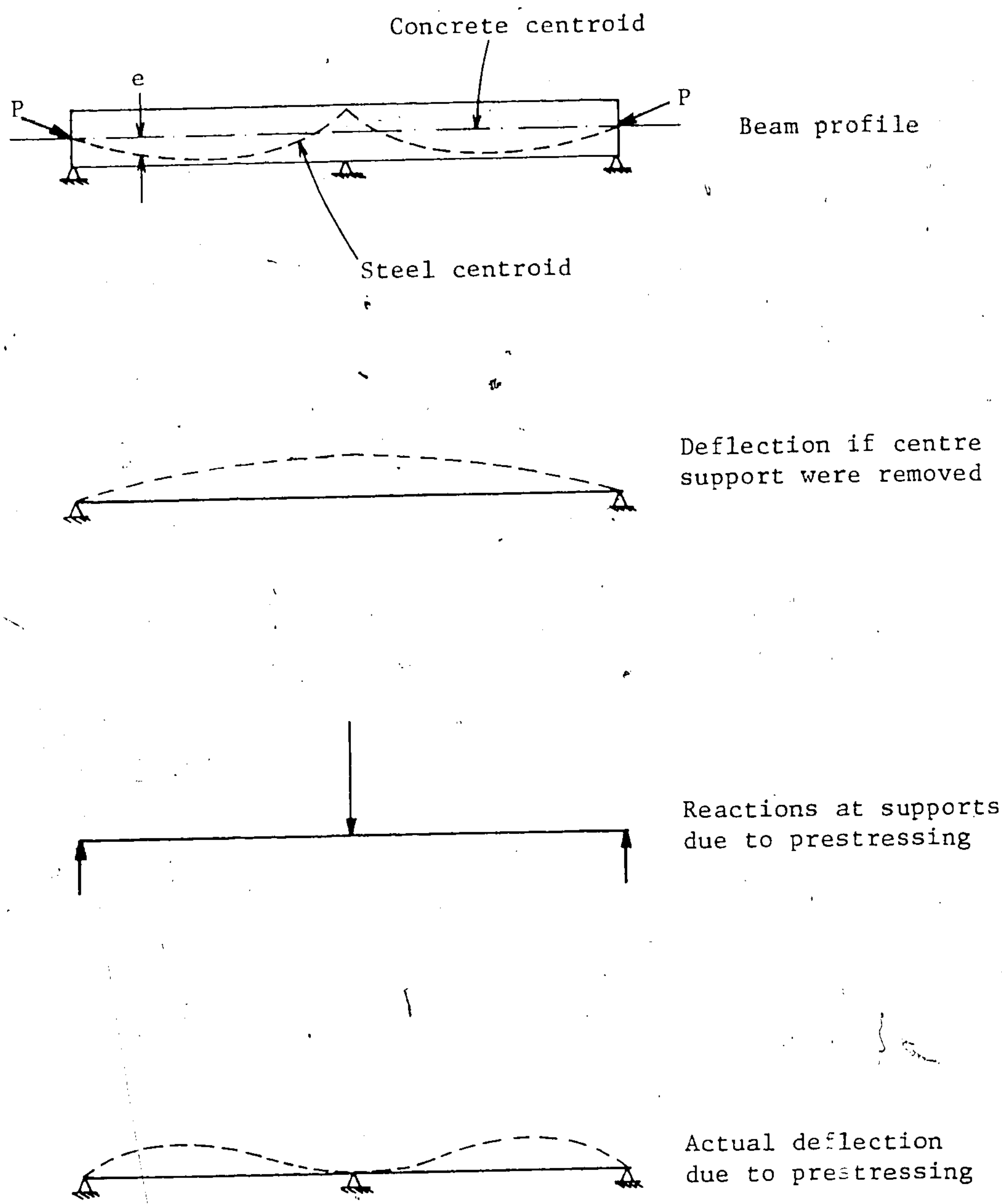


Figure 2.2 Statically indeterminate beam

2.3 Treatment of Secondary Moments due to prestressing in the ACI Building Code

The proper treatment of secondary moments in the ultimate load analysis has been the subject of much debate (Ref. 18, 21, 22).

In the 1971 edition of the ACI Building Code, it was stated that the effects of moments due to prestressing, including secondary moments, shall be neglected when calculating the moments corresponding to factored loads. It was also stated that the behavior shall be determined by an elastic analysis, with only a modest amount of redistribution of moments due to plastic behavior permitted. The accompanying ACI Code Commentary stated that the secondary moments produced by the prestressed force in a non-concordant tendon disappear at the capacity at which, because of plastic hinge formation, the structure becomes statically determinate.

The 1977 Code, however, requires the consideration of secondary moments, using a load factor of 1.0, up to and including the ultimate load.

2.4 Problem statement

The contradiction arising from the Code consideration of secondary moments in ultimate state calculations may be misleading to the designer. Secondary moments need definitely be included in the elastic analysis. However, the load carrying capacity of a continuous member is not affected by the secondary moment if complete moment redistribution can take place at ultimate. If plastic hinges do not fully develop, then ultimate load capacity will lie between the load resulting from an elastic analysis and the load at full redistribution (Lin and Thornton, ref. 18).

As required by the present Code, secondary moments are to be considered up to and including the ultimate load. Since certain circumstances exist when some redistribution of moments does occur and is, in fact, allowed in design, do the secondary moments vary under these conditions? The question arises due to the fact that, depending on the system of supports and the tendon profile, the full inclusion of secondary moments at ultimate may lead to:

- uneconomical design in sections where the secondary moment is unfavorable to the nominal capacity;
- unsafe design because the contribution of secondary moments to the beam capacity may have been improperly assumed to exist.

It is worth noting that, from a design standpoint, the ultimate stage does not necessarily correspond to the formation of a mechanism, but rather to the loading at which ultimate capacity has been reached at a critical section.

3. LITERATURE REVIEW

3.1 Nonlinear analysis of continuous beams

The 1971 ACI Building Code provisions concerning moment redistribution in continuous prestressed beams are definitely inconsistent (Lin and Thornton, 1972, ref. 18). According to the Code, full moment redistribution at ultimate is not permitted, while secondary moments must be neglected at the same time. By means of examples, it is demonstrated that neglecting secondary moments may yield a non-conservative result. A method for determining the ultimate load capacity of a continuous beam is proposed. The method takes into account the secondary moments without calculating for them and the final moment configuration is an intermediate stage between the elastic case and the full redistribution case. The method is conservative due to lack of analytical and experimental research concerning the plastic behavior of prestressed concrete beams with non-concordant cables.

It is believed that an exact solution can only be obtained when the moment-curvature relation for the entire beam is analyzed beyond the [illegible]astic range and up to failure.

A series of tests of seven [illegible]le-span beams and three beams continuous over two spa[illegible]s [illegible] ft each has been conducted (Mattock, 1971, ref. 21[illegible] [illegible]hough the primary variable of the study was the effe[illegible] [illegible] bond on the behavior of post-tensioned concrete beams, a c[illegible]ble amount of redistribution of support moment has been observed at ultimate. A large portion of redistribution (up to 85 percent) has been attributed to the action of the non-concordant tendon, because the test beams had a net reinforcement index ($\omega + \omega_p - \omega'$) that did not allow any adjustment of support design moments according to any edition of the ACI Code. Therefore the secondary moments have been assumed to have a direct effect on the amount of redistribution available. It was concluded that for a downwards transformed tendon profile, "redistribution of design support ultimate moments by an amount equal to the positive secondary prestress moment should be allowed in design, without a special limitation on the amount of reinforcement". This reduction in support moment does not require any inelastic deformation at the support section.

These findings led to subsequent changes to the 1977 ACI Code, which required the inclusion of secondary moments, using a load factor of 1.0, up to and including the ultimate state.

Many theoretical approaches were developed to enable the distribution of moments at ultimate to be related to the physical properties of the beams and the pattern of loading. These ranged from Guyon's general analysis which takes into account the actual distribution of curvature along the length of the beam (1960, ref. 14), to Baker's simplified approach in which the inelastic deformation is considered concentrated at the critical sections, i.e. the concept of a "plastic hinge" theory with the "hinges" having limited rotational capacity.

It is shown that yield of reinforcement can provide advantageous moment redistribution. However, the small amount of steel yield available in prestressed concrete beams may reduce the moment redistribution possible as compared with ordinary reinforced concrete (Baker, 1949, ref. 2).

In general, the theories require a knowledge of the moment-curvature relationships for the beam sections. The theory developed by Priestley et al. (1971, ref. 24) takes into consideration the variation of curvature between cracks caused by concrete tension. It also showed close agreement with experimental data. The relationships between moment and average curvature have been used to determine the moment-load curves for continuous beams up to the onset of concrete crushing (Priestley and Park, 1972, ref. 25).

3.2 Rotational capacity of hinging regions in reinforced concrete beams

Instances can occur in which the strain capacity of a reinforced concrete hinging section is exhausted before full redistribution of bending moments is achieved in the structure as a whole. It is therefore necessary to consider the deformation of the hinging regions in any theory of limit design for structural concrete, and more specifically to limit their rotation to known safe values (Mattock, 1964, ref. 20).

The main factors relating to rotations are: moment gradient, concrete strength, reinforcement yield stress, beam effective depth, amount of tension reinforcement and confinement of the concrete in compression (Corley, 1966, ref. 12; Roy, 1964, ref. 26).

It is demonstrated that similar approaches could be used for both reinforced concrete and partially prestressed concrete in evaluating moments and curvatures (Bishara and Brar, 1974, ref. 3).

Computer-simulated flexural tests carried out to identify all major variables that affect the behavior of partially prestressed concrete sections, confirmed earlier findings concerning the ductility of ordinary reinforced concrete sections. For prestressed sections, ductility showed considerable sensitivity to the effective prestressing. However, variables such as cross-section shape and high-grade steel stress-strain relationship have a relatively minor effect on the inelastic behavior (Cohn, 1982, ref. 11).

3.3 Experimental programmes

Only few experimental data were available in the literature. Major extensive test programmes on prestressed concrete beams were conducted by Warwaruk on simply supported beams (1962, ref. 28) and by Hawkins on two-span continuous beams (1964, ref. 15). In general, flexural cracking and bond were major factors in the behavior of test beams. It was observed that moment redistribution in continuous beams is initiated by flexural cracking and relative reduction of stiffness over the interior support. Pronounced redistribution occurs only after the moment over the interior support reaches the nearly flat portion of its moment-curvature relationship. The development of inclined tension cracks reduced both the load carrying capacity and the ductility of test beams failing in shear. The basic mechanisms of failure in shear or in flexure were similar to those in simply-supported beams.

4. DESCRIPTION OF THEORY

4.1 General remarks

In order to obtain a better understanding of the behavior characteristics of continuous prestressed concrete beams subjected to bending, a theoretical model has been developed to study the phenomena occurring in the beams when loaded up to failure.

The theory requires a knowledge of the moment-curvature relationships for the sections to determine the distribution of moments throughout the structure at a specified load. The moment-curvature relationship formulated by Priestley has been adopted. The theory takes into consideration the variation of curvature between cracks caused by concrete tension and makes possible the prediction of both the curvature at a crack and the average curvature along the length of the member.

A computer program using Fortran statements and based on the above theory has been written. Further details of the computer program are enclosed in the Appendix. Comparison with experimental data is presented in the following Chapter.

4.2 Analytical model

4.2.1 Assumptions

1) Quasi-static loading;
2) Bonded beams;
3) Negligible shear effects;
4) Plane sections remain plane (linear strain distribution);
5) Any known material stress-strain relationships;
6) Effective (after losses) prestressing;
7) Linear-elastic behavior up to decompression of concrete;
8) Partially prestressed beams in which limited cracking is permitted by the designer at service loads.

4.3 Moment-curvature relationships

4.3.1 Prestressing steel strain and stresses

In prestressed concrete beams, the prestressing steel strain is not zero even though no external load has been applied. It is therefore necessary to include the initial strain in the analysis.

Strains in the concrete and steel at loading stages of interest are shown in Fig. 4.1. Strain distribution (1) of Fig. 4.1 results from the application of prestress force P_e acting alone. At this stage the stress in the steel and the

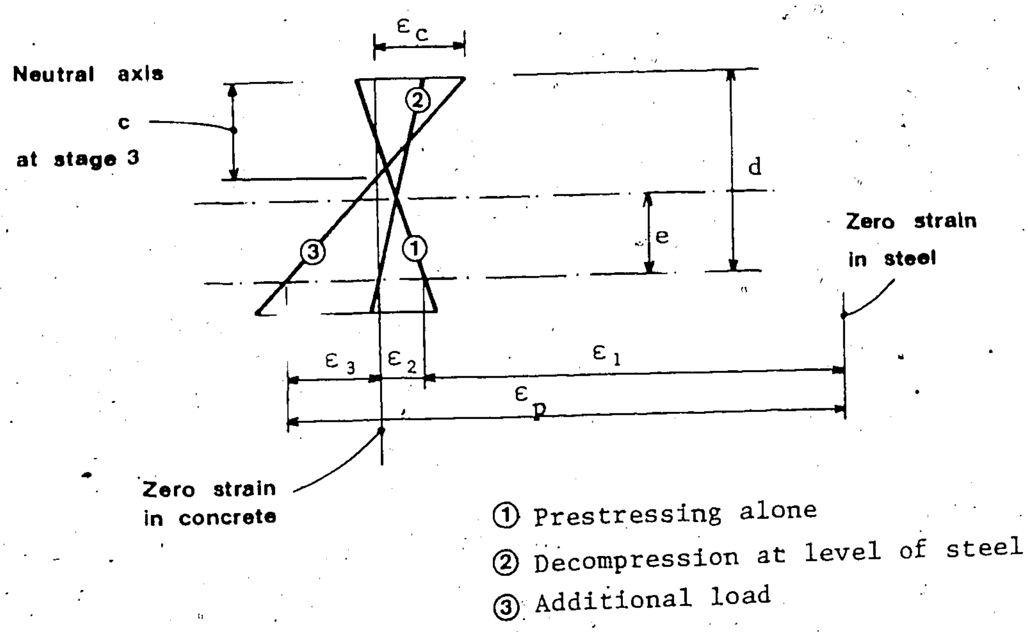


Figure 4.1 Strains in concrete and steel

associated strain are, respectively,

$$f_1 = \frac{P_e}{A_p} \tag{4.1}$$

$$\epsilon_1 = \frac{f_1}{E_p} \tag{4.2}$$

The steel strain is shown with respect to its own separate origin. Stage (2) corresponds to the decompression of concrete at the level of the steel centroid. Assuming that bond remains intact between the concrete and steel, the increase in steel strain produced as loads pass from stage (1) to stage (2) is the same as the decrease in concrete strain at that level of the beam. It is given by the expression:

$$\epsilon_2 = \frac{P_e}{A_c E_c}\left(1 + \frac{e^2}{g^2}\right) \tag{4.3}$$

in which g is the radius of gyration of the cross-section. When the load is increased further to the stage (3), the neutral axis is at a distance c below the top of the beam. The increment of strain is:

$$\epsilon_3 = \epsilon_c\left(\frac{d - c}{c}\right) \tag{4.4}$$

The total strain is the sum of the three components

$$\epsilon_p = \epsilon_1 + \epsilon_2 + \epsilon_3 \tag{4.5}$$

and the corresponding steel stress is given by the stress-strain relationship of the particular steel grade.

4.3.2 Cracking point

One loading stage that is of interest in a section analysis is the one at which cracking moment is reached. A significant change in slope can be observed at that particular loading stage in a typical moment-curvature curve as shown in Fig. 4.2. The model will therefore consider the cracking moment as a benchmark between elastic and inelastic behavior.

4.3.3 Conditions just before cracking

Assuming that the concrete has a tensile strength, the conditions just before cracking can be defined as the stage at which the extreme concrete fibre in tension reaches the flexural tensile strength f_t', so that cracking would certainly occur should the load be increased by a slight amount.

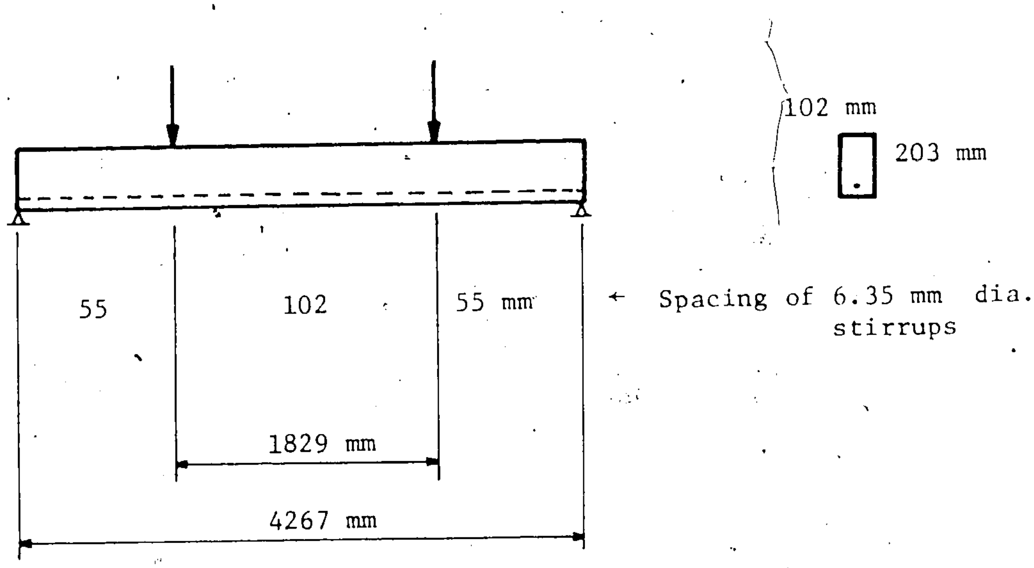
For a rectangular section as shown in Fig. 4.3,

Concrete compressive force

$$C = b \int_0^c f_c \, dy \tag{4.6}$$

Concrete tensile force

$$T_c = \frac{1}{2} f_t' (h - c) b \tag{4.7}$$



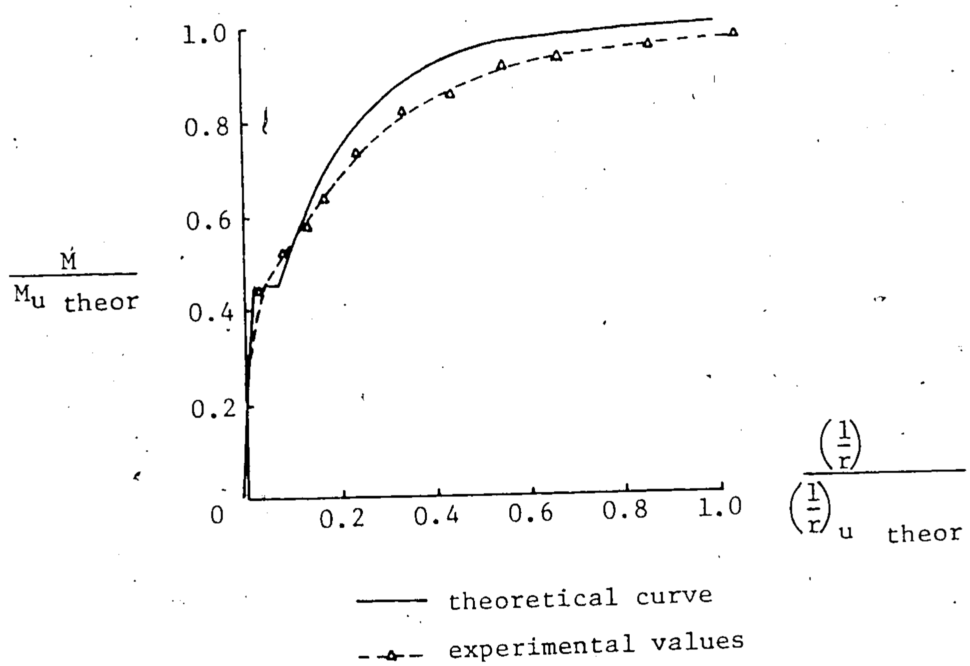


Figure 4.2 Moment-curvature relationships at a section in a constant-moment zone (Ref. 24)

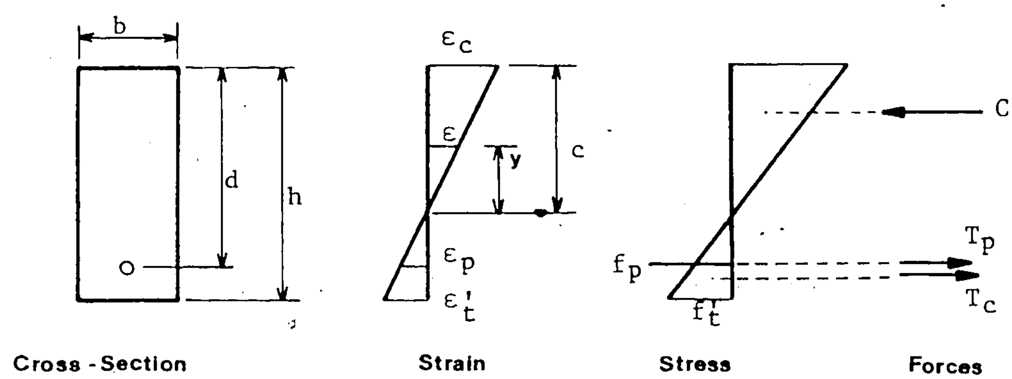


Figure 4.3 Conditions just before cracking

Steel tensile force

$$T_p = A_p\ f_p \tag{4.8}$$

The concrete stress-strain relationship is given by the expression of f_c in terms of ϵ. The equilibrium equation is:

$$C = T_c + T_p \tag{4.9}$$

After each component of the equation is substituted with an expression involving only the neutral axis c, and the steel strain ϵ_p, the equilibrium equation is solved simultaneously along with the steel strain equation 4.5.

The curvature is derived from the strain distribution

$$\left(\frac{1}{r}\right)_{cr} = \frac{\epsilon_t' + \epsilon_c}{h} \tag{4.10}$$

and the corresponding moment

$$M_{cr} = b\int_0^c f_c\ y\,dy + b\ f_t'\ \frac{1}{3}(h-c)^2 + A_p f_p\ (d-c) \tag{4.11}$$

4.3.4 Conditions after cracking

When the stress in concrete extreme fibre in tension has exceeded the tensile strength, cracking occurs in the section. As a result, the component T_c no longer exists.

The location c of the neutral axis can be determined from the general cracked section analysis developed by K. Shushkewich (Ref. 27). The equation of neutral axis is:

$$\frac{1}{6}bNc^3 + \frac{1}{2}bMc^2 + (\beta N+\alpha M)c - (\gamma N+\beta M) = 0 \qquad (4.12)$$

in which the different coefficients can be calculated from the general transformed section shown in Fig. 4.4:

$$\alpha = (b-b_w)h_f + n_p A_p \qquad (4.13)$$

$$\beta = \frac{1}{2}(b-b_w)h_f^2 + n_p A_p d \qquad (4.14)$$

$$\gamma = \frac{1}{3}(b-b_w)h_f^3 + n_p A_p d^2 \qquad (4.15)$$

where $b_w = b$ for a rectangular section.

$$N = F = A_p E_p\ (\epsilon_1 + \epsilon_2) \qquad (4.16)$$

$$M = M' - Fd \qquad (4.17)$$

The stresses are:

Concrete $$f_c = \frac{M c}{\gamma - \beta c - \frac{1}{6}bc^3} \qquad (4.18)$$

Steel $$f_p = n_p \frac{d-c}{c} f_c + \frac{N}{A_p} \qquad (4.19)$$

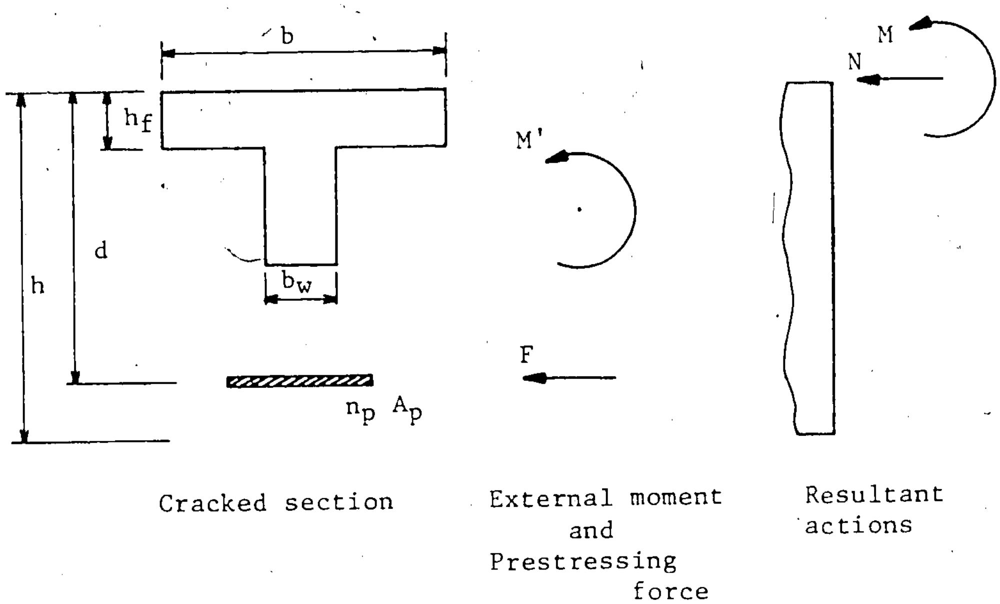


Figure 4.4 General cracked section analysis

4.3.5 Average curvature

4.3.5.1 Bond stress

It is generally agreed that the concrete-steel bond stress is a maximum very close to the crack and decreases in some fashion further away from the crack.

For this analysis, it will be assumed that, at first cracking, the bond stress decreases linearly from a maximum u_m at the crack to zero at a distance away from the crack as shown in Fig. 4.5.

4.3.5.2 Bond length

Immediately after the first crack forms, when $M = M_{cr}$, in a region of constant moment, a stress condition that varies between two limits exists.

Let A be the section at a crack, and B be a section some distance away from the crack where the stresses have not been affected by the formation of the crack. The stresses and the stress resultants at section B are as described by equations 4.6 to 4.11. At section A the stresses are found from equations 4.12 to 4.19 with $M = M_{cr}$.

At section A, all the tensile stress is carried by the steel; between sections A and B, tension is transferred from the steel to the concrete by bond. If f_{sa} and f_{sb} are respectively the steel stresses at sections A and B, then the minimum distance ℓ_b from the crack over which sufficient tension can be transferred

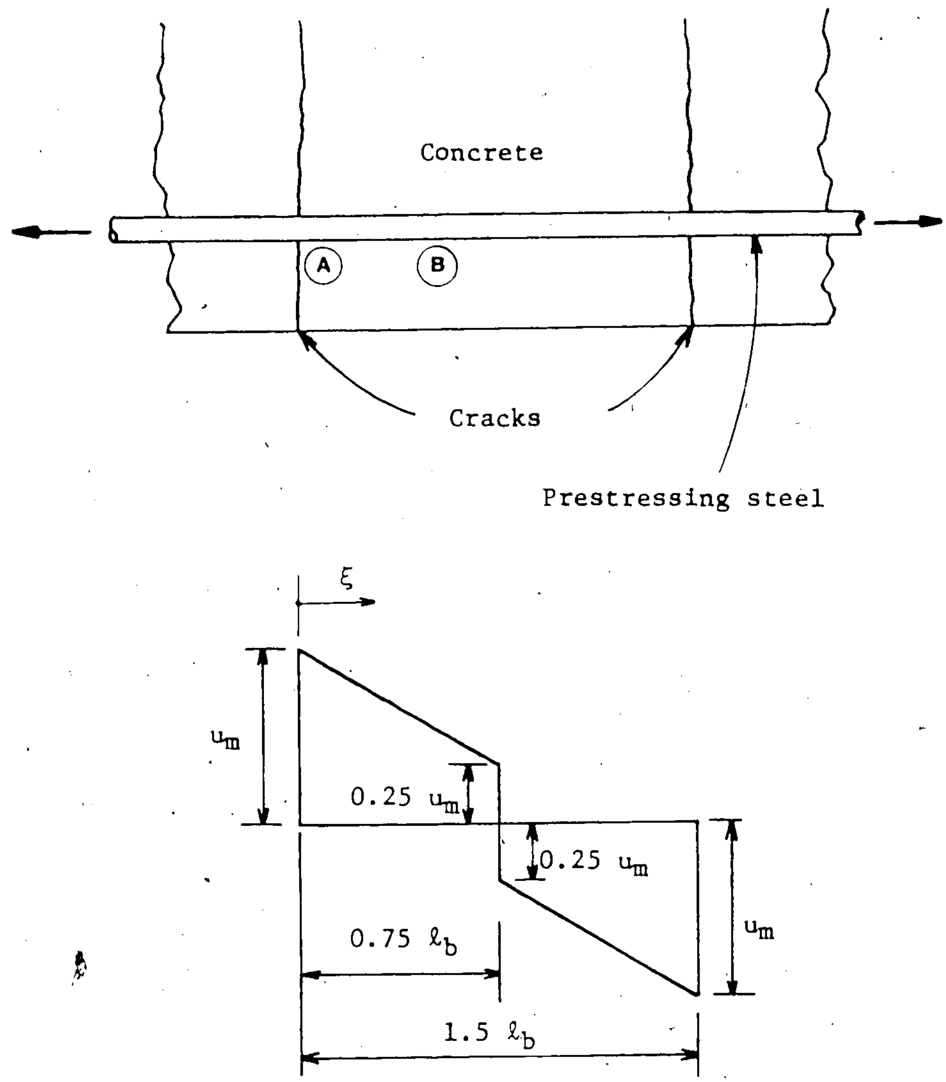


Figure 4.5 Bond stress distribution between adjacent cracks

from the steel to the concrete by bond, to cause the modulus of rupture f_t' to be just reached at B is:

$$\ell_b = \frac{A_p\left(f_{sa} - f_{sb}\right)}{u_{av}\ \Sigma O} \tag{4.20}$$

in which u_{av} is the average bond stress between the steel and the concrete, ΣO is the total surface available for bonding of the prestressing steel per unit length, A_p is the total area of prestressing steel. The maximum bond stress is:

$$u_m = 2\ u_{av} = \frac{2A_p\left(f_{sa} - f_{sb}\right)}{\ell_b\ \Sigma O} \tag{4.21}$$

4.3.5.3 Crack spacing

A new crack cannot develop between two existing cracks which formed when $M = M_{cr}$ if the spacing between these cracks are smaller than $2\ell_b$. Sufficient length is required each side of the potential crack position to build up enough concrete tension and induce a new crack. It is evident that, with initially random cracking, the individual crack spacing soon after the formation of the first crack will vary between two limits, ℓ_b and $2\ell_b$. The average crack spacing will be approximately equal to $1.5\ell_b$.

4.3.5.4 Stress distribution at a distance ℓ from a crack ($\ell \leq \ell_b$)

At a distance ℓ from the crack, the steel stress will be reduced by bond and the concrete tension will build up. The reduction of steel tension force over the length ℓ from the crack is:

$$\Delta F = \int_0^{\ell} u \, \Sigma O \, d\xi \tag{4.22}$$

Therefore, if f_{scr} is the tensile steel stress at the crack for the particular moment M acting, the tensile stress in the steel at distance ℓ from the crack is:

$$f_s = f_{scr} - \frac{\Delta F}{A_p} \tag{4.23}$$

or

$$f_s = f_{scr} - \frac{1}{A_p} \int_0^{\ell} u \Sigma O \, d\xi \tag{4.24}$$

From Fig. 4.5, the bond stress distribution can be expressed as:

$$u = u_m \left(1 - \frac{\ell}{\ell_b}\right) \tag{4.25}$$

Substituting u from eq. 4.25 into eq. 4.24 and integrating:

$$f_s = f_{scr} - \frac{1}{A_p} u_m \, \Sigma O \left(\ell - \frac{\ell^2}{2\ell_b}\right) \tag{4.26}$$

Substituting u_m from eq. 4.21 into eq. 4.26 yields:

$$f_s = f_{scr} - 2(f_{sa} - f_{sb})\left(\frac{\ell}{\ell_b} - \frac{\ell^2}{2\,\ell_b^2}\right) \qquad (4.27)$$

Note that f_s from eq. 4.27 is independent of the maximum bond stress u_m and thus the magnitude of the maximum bond stress does not affect the moment-curvature curves.

4.3.5.5 Average curvature

The assumption is made that, at a section some distance from a crack, the steel strain is still linearly related to the concrete compressive strain.

The known steel stress f_s allows the determination of the conditions in a section at a distance $\ell \leq \ell_b$ from a crack.

The action of the decompression force, along with the external moment, can be represented by a resultant force R applied with eccentricity e_{top} above the top of the section as shown in Fig. 4.6 (Nilson, ref. 23, p. 97). The portion of beam can then be analyzed as an ordinary reinforced concrete member subjected to an eccentric compression force.

Equilibrium equation:

$$R = C - T_s - T_c \qquad (4.28)$$

The concrete tensile force T_c can be expressed as:

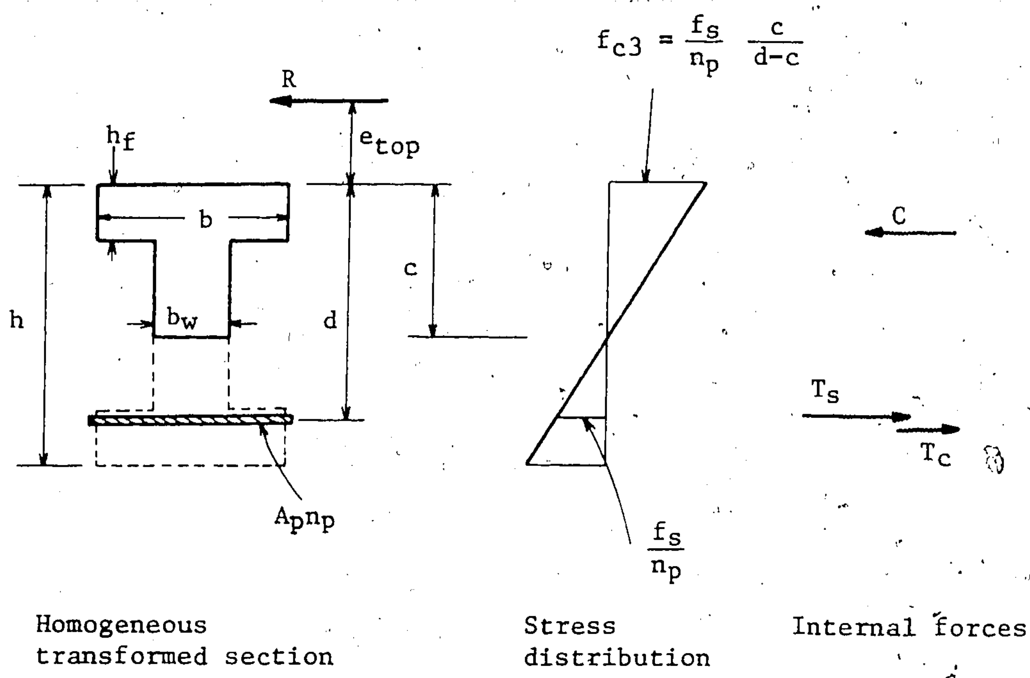


Figure 4.6 Conditions at a distance $\ell \leq \ell_b$

$$T_c = C - T_s - R \tag{4.29}$$

in which

$$C = \frac{1}{2} f_{c3} \left[b_w c + \left(\frac{2c - h_f}{c} \right) h_f (b - b_w) \right] \tag{4.30}$$

$$T_s = A_p f_s \tag{4.31}$$

$$R = A_p E_p (\epsilon_1 + \epsilon_2) \tag{4.32}$$

The neutral axis a distance *c* from the top surface, for the equivalent homogeneous transformed section, can be found from the equilibrium condition that the moment of all internal forces about the line of action of *R* must be zero:

$$\begin{aligned} (d + e_{top}) A_p f_s + T_c \left(e_{top} + c + \frac{h-c}{3} \right) \\ = f_{c3} \Bigg[\frac{1}{2} \left(\frac{c}{3} + e_{top} \right) (b_w c) \\ + h_f \left(\frac{c - h_f}{c} \right) (b - b_w) \left(\frac{h_f}{2} + e_{top} \right) \\ + \frac{1}{2} h_f \frac{h_f}{c} (b - b_w) \left(\frac{h_f}{3} + e_{top} \right) \Bigg] \end{aligned} \tag{4.33}$$

with $b_w = b$ for a rectangular section.

Solving Equation 4.33 with f_s obtained from Equation 4.27, the position of the neutral axis can be determined, and hence the concrete stress at the top surface using Equation 4.18. The curvature at this section is given by:

$$\left(\frac{1}{r}\right) = \frac{\epsilon_c + \epsilon_s}{d} \tag{4.34}$$

ϵ_c and ϵ_s are the strain corresponding respectively to the concrete stress at the top surface, and f_s.

Integration of the curvature over half the distance between the cracks gives the average curvature:

$$\left(\frac{1}{r}\right)_{av} = \int_0^{0.75\,\ell_b} \frac{\epsilon_c + \epsilon_s}{0.75\,\ell_b d}\, d\xi \tag{4.35}$$

4.4 Deformation compatibility requirements

In the analysis of continuous beams, the distribution of moments must meet the deformation compatibility requirements.

The model beam that has been adopted considers the availability of test data. This allows a check on the accuracy of the theory. A set of experiments being referred to is from Hawkins' work (ref. 15), which consists of a series of tests on two-span symmetrically loaded continuous beams. The investigation will thus be limited to a two-span symmetrically loaded continuous beam.

4.4.1 Elastic conditions

In elastic analyses of indeterminate beams, it is necessary that the slope at any interior support be continuous. Consider a continuous beam as shown in Fig. 4.7. Neglecting any differential settlement of any support, the rotation θ_{ji} occurring at j in span ij can be expressed as:

$$\theta_{ji} = \frac{1}{L_{ij}} \int_{x_i}^{x_j} \left(\frac{1}{r}\right)_x (x-x_i)\,dx \qquad (4.36)$$

where the curvatures $(1/r)_x$ are evaluated at each beam section from the corresponding values of bending moment, using the moment-curvature relationships defined in the uncracked section analysis. Similarly, the rotation θ_{jk} occurring at j in span jk is:

$$\theta_{jk} = \frac{1}{L_{jk}} \int_{x_j}^{x_k} \left(\frac{1}{r}\right)_x (x_k-x)\,dx \qquad (4.37)$$

Taking all rotations positive counterclockwise, the compatibility condition for the interior support j in Fig. 4.8 is:

$$\theta_{ji} - \theta_{jk} = 0 \qquad (4.38)$$

In the particular case of a two-span symmetrical beam loaded symmetrically, the compatibility equation reduces to:

$$\frac{1}{L_{ij}} \int_{x_i}^{x_j} \left(\frac{1}{r}\right)_x (x-x_i)\,dx = 0 \qquad (4.39)$$

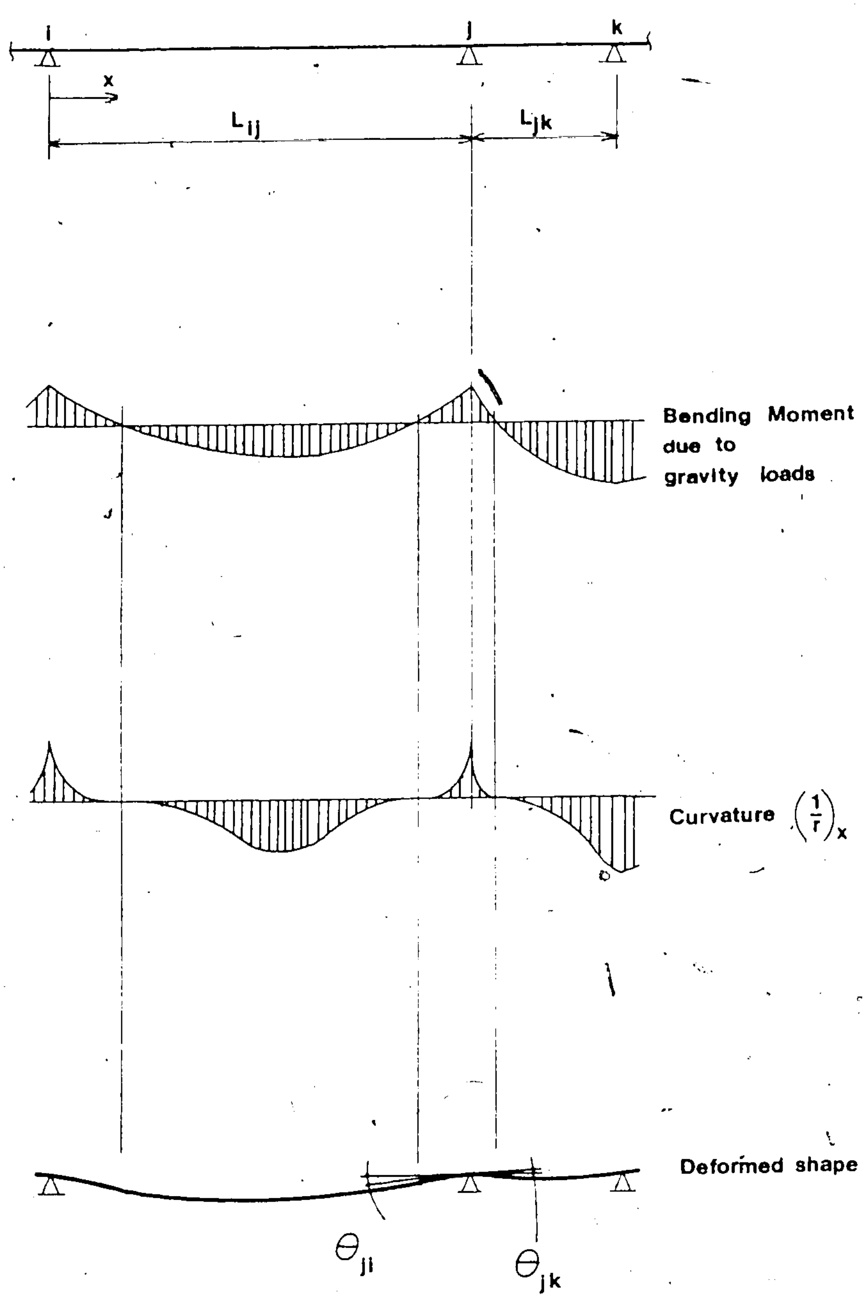


Figure 4.7 Rotation of beam within a span

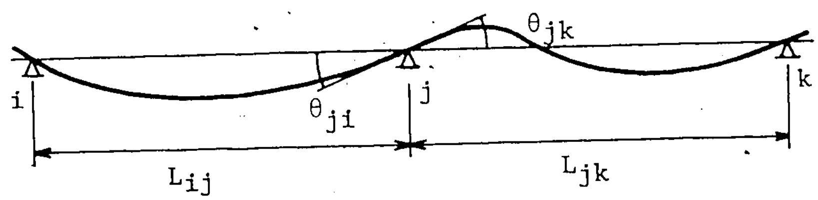


Figure 4.8 Rotation of beam at internal support

4.4.2 Non-linear conditions

Another way of formulating the compatibility equation is to ensure that the deformations occuring within a span (between two supports) are geometrically compatible. These equations are described by Guyon. Consider a symmetrical two-span continuous beam with the load applied symmetrically as shown in Fig. 4.9. The beam can be assumed to take on a polygonal shape at an advanced stage of loading. The rotation at support *B* remains zero due to symmetry, but a slope θ' occurs at some small distance *m* from *B*. The final bending moment diagram, after the iterative process described in Section 4.5, is such that the deformations resulting from it are geometrically compatible. A set of bending moments can be considered correct when it satisfies the equation

$$\phi(\lambda \ell) = \theta'(\ell - m) \qquad (4.40)$$

The quantity $(\ell - m)$ will be approximated to ℓ because of the difficulty to evaluate exactly the distance *m*, which varies with the width of the support, among other factors. Equation (4.40) is then modified to

$$\phi \, \lambda = \theta' \qquad (4.41)$$

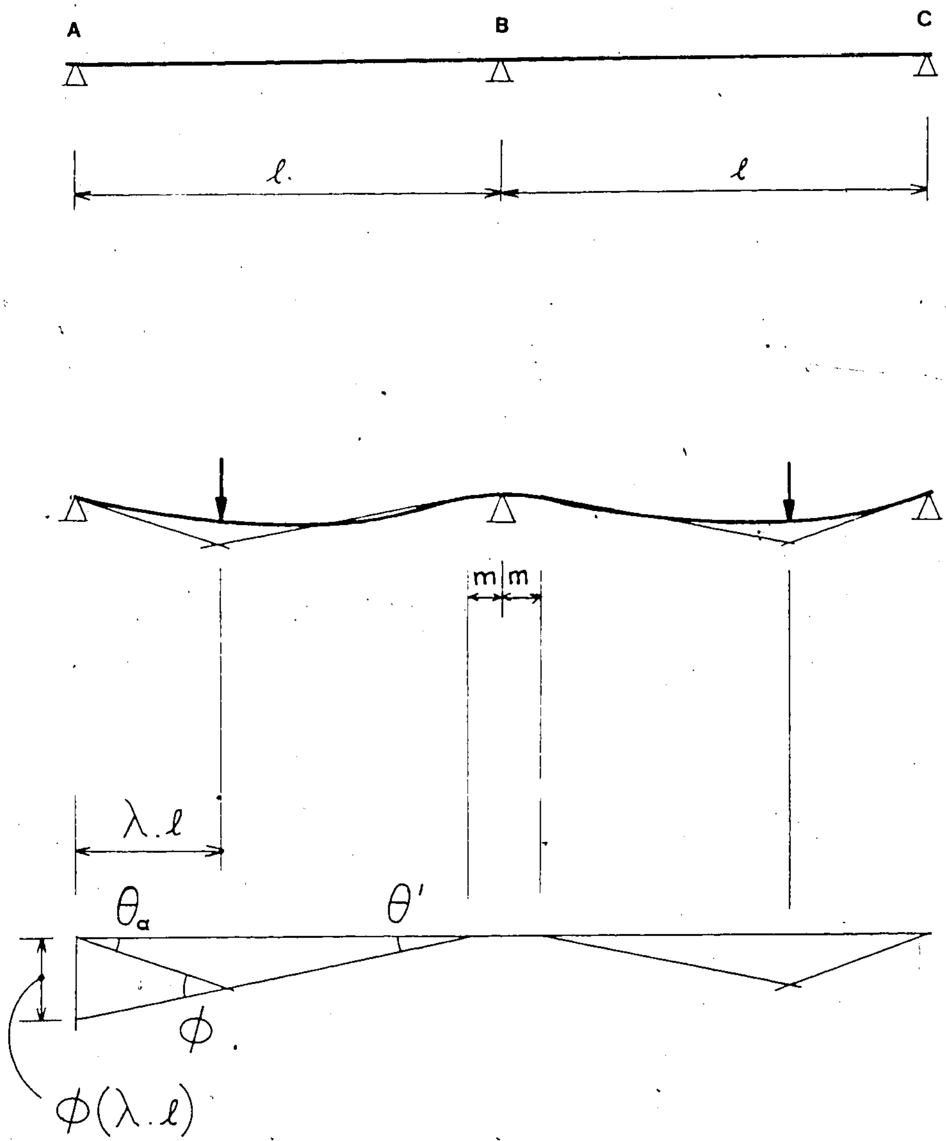


Figure 4.9 Compatibility of geometry

This compatibility equation provides a simple means of establishing an acceptable distribution of moments at a load higher than the load at first cracking. An error margin of ± 0.05 radians will be adopted as a result of the approximation described above.

4.5 Procedure of analysis

The successive steps of analysis are:

1. Compute the cracking moments and the ultimate moments at all the critical sections, based on the section properties;
2. From an elastic analysis, determine at which critical section cracking occurs first, and check the compatibility at that particular loading stage using equation 4.39, that is:
   a. From the resulting bending moment diagram, the corresponding curvature at the centre of each segment is found by referring to the moment-curvature relationships defined in Section 4.3.3;
   b. These curvature values are then used to determine the left-hand side of equation 4.39, which will equal zero if the assumed moment configuration is correct. Usually this will not be the case and adjustments are then made to the bending moment diagram by reducing either the maximum hogging moment value or the maximum sagging moment value

until the left-hand side of equation 4.39 does equal zero as required;

3. The ultimate limit state is established as the load corresponding to the final distribution of moments that satisfies equation (4.41). The first iteration is performed using the ultimate moments calculated in Step 1, one of these moments being reduced in subsequent iterations. In addition, the moment-average curvature relationships defined in Section 4.3.5 are used in cracked regions;
4. Stages between cracking and ultimate can be identified using intermediate values of moments. By modifying one of the moments and holding the other constant, compatibility can be satisfied, and the resulting distribution of bending moments can be calculated.

One can then obtain a series of points on the moment-load relationships of the critical sections.

5. RESULTS OF ANALYSIS

5.1 General remarks

The moment-load curve has been established for a series of beams, based on the theory described in the previous chapter.

In order to evaluate the accuracy of the analysis, some beam data have been taken from experimental work on continuous prestressed concrete beams, so that comparison of the results can be made.

5.2 Experimental results

Most of the experimental results available have been done on simply supported beams, due to the early research interests which focused mainly on the flexural strength and deformation characteristics, or the effects of bond.

An extensive literature search provided only one detailed testing programme (Hawkins, 1964, ref. 15) on continuous prestressed concrete beams. Tests were carried out on 22 two-span continuous beams loaded at the midspans. Because the purpose of the investigation was to study the action of both bending and shear, with the emphasis on the effects of shear, the test beams were designed with varying amounts of shear reinforcement along with flexural reinforcement. In classifying the modes of failure for the test beams, the criterion used was the crack pattern observed. The six beams which failed in flexure were selected for the present comparison. Further details on these beams are listed in Tables 5.1 through 5.3 and shown in Fig. 5.1.

Tables 5.4 and 5.5 give a summary of results from both analysis and experiment. The only known values from the experiment are the cracking moment and the ultimate moment.

5.3 Behavior of the test beams

The position of the concentrated loads implies that the positive moment was five-sixths of the negative moment as determined by an elastic analysis.

The first flexural crack was observed over the interior support where the elastic bending moment was the largest. The appearance of this crack was accompanied by an adjustment in the relative magnitudes of the exterior and interior reactions. Beyond this stage, the moments at midspan and interior support deviated from the elastic distribution. Since the interior support section cracked first, its moment was gradually redistributed to the midspan section. When the midspan also started to crack, moments were redistributed back to the interior support section. As the load was increased the moment ratio remained essentially unchanged up to the theoretical ultimate load. The moment-load curves shown in Figs. 5.2 through 5.7 include the effects of secondary moment as explained later in Section 5.4.

Beam	Designation in reference 15	Effective depth (in.)		Reinforcement ratio $\rho_p = \frac{A_p}{b\ d}$ (%)		Reinforcement index $\omega_p = \rho_p \frac{f_{ps}}{f'_c}$	
		Load Point	Centre Support	Load Point	Centre Support	Load Point	Centre Support
1	BO.08.035	8.55	10.55	0.232	0.187	0.077	0.063
2	BO.08.036	8.40	10.50	0.294	0.236	0.081	0.065
3	BO.10.043	10.35	10.10	0.190	0.195	0.096	0.096
4	BO.13.050	10.40	8.50	0.191	0.238	0.113	0.139
5	BW.10.073	10.50	10.35	0.292	0.295	0.177	0.179
6	BW.10.103	10.35	10.55	0.388	0.392	0.253	0.248

Table 5.1 Characteristics of test beams

Beam	Designation in reference 15	Concrete strength f'_c (psi)	Steel strength σ_u (ksi)	Reinforcement Area A_p (in.^{2})	Effective Prestress P_e (kips)
1	BO.08.035	6,700	250	0.118	14.05
2	BO.08.036	8,150	250	0.148	18.35
3	BO.10.043	4,450	250	0.118	12.85
4	BO.13.050	3,850	250	0.121	14.60
5	BW.10.073	3,990	270	0.183	20.30
6	BW.10.103	3,770	270	0.244	26.60

Table 5.2 Characteristics of test beams (cont'd)

Beam	Designation in reference 15	Web reinforcement	Secondary Moments (lbs-in.)	Action of Centre reaction (due to prestressing)
1	BO.08.035	No	7,843	Downward
2	BO.08.036	No	12,340	Downward
3	BO.10.043	No	17,573	Upward
4	BO.13.050	No	31,772	Upward
5	BW.10.073	Yes	27,400	Upward
6	BW.10.103	Yes	25,608	Upward

Table 5.3 Characteristics of test beams (cont'd)

Beam	Load (Kips)		Moments in kip-in.			
			Support Moment		Load Point Moment	
	Analysis	Experimental	Analysis	Experimental	Analysis	Experimental
1	20.62 (first cracking)		206.94	206	174.89	153
	27.18 (Ultim.)	30.2	284.33	317	224.70	238
2	25.24 (first cracking)		254.75	252	213.33	184
	33.05 (ultim.)	37.2	346.21	386	273.10	299
3	16.73 (first cracking)		169.71	178	140.97	178
	19.29		171.85		174.48	
	23.42		213.65		209.38	
	27.26		247.6		244.28	
	30.76 (Ultim.)	34.3	278.47	306	276.01	298
4	14.41 (first cracking)		146.28	153	121.35	168
	18.88		141.23		184.23	
	23.07		180.75		221.07	
	27.06		214.79		257.92	
	29.55 (Ultim.)	33.2	233.87	248	281.95	312

Table 5.4 Summary of results, beams 1, 2, 3 and 4

Beam	Load (Kips)		Moment (Kip-in.) at support		Moment (Kip-in.) at Load Point	
	Analysis	Experimental	Analysis	Experimental	Analysis	Experimental
5	22.29 (First cr.)		227.51	229	187.19	241
	25.93		237.51		231.29	
	31.04		283.01		277.54	
	36.15		328.52		323.80	
	41.26		374.02		370.06	
	46.38		419.52		416.32	
	48.05 (Ultim.)	51.1	434.41	459	431.46	450
6	27.27 (First cr.)		275.21	286	230.60	266
	30.34		281.92		268.70	
	36.49		340.26		322.44	
	42.51		395.30		376.18	
	48.52		450.34		429.92	
	54.54		505.39		483.66	
	57.12 (Ultim.)	63.4	528.93	575	506.65	557

Table 5.5 Summary of results, beams 5 and 6

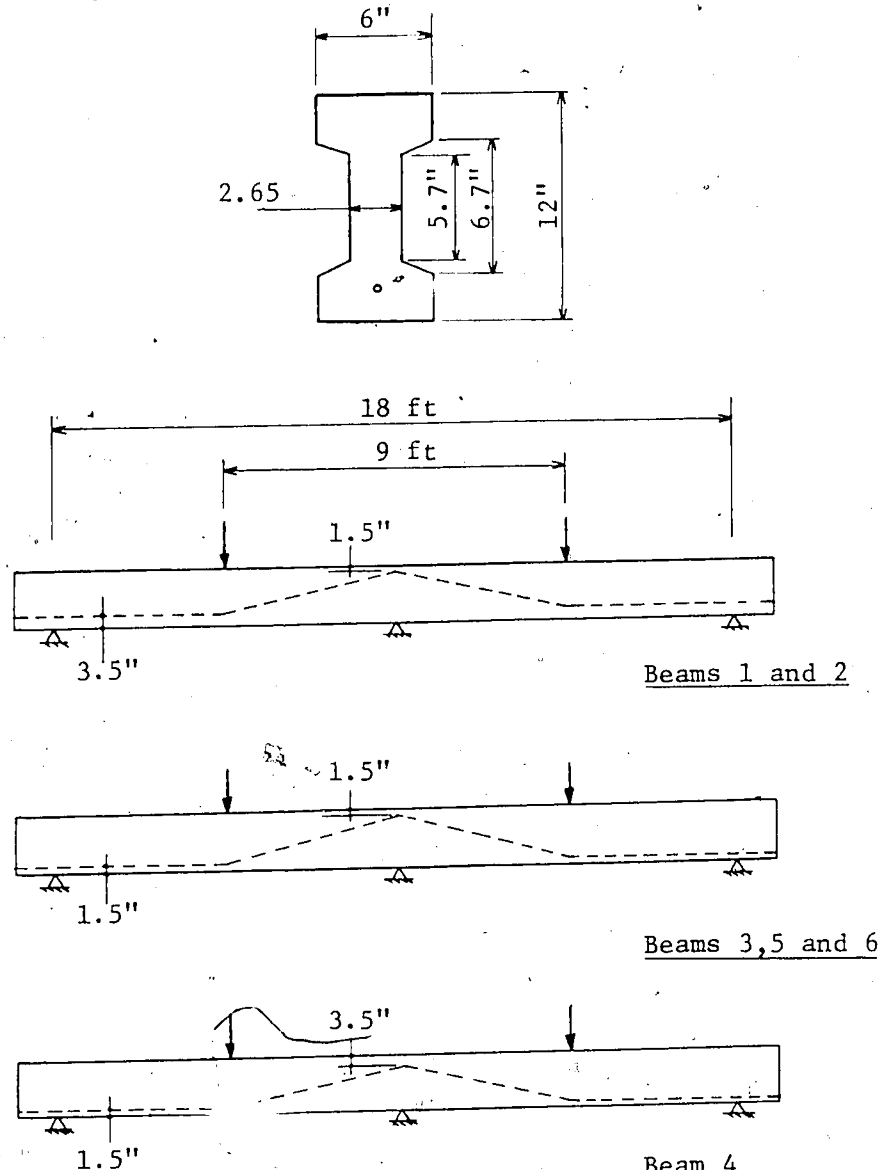


Figure 5.1 Reinforcement patterns

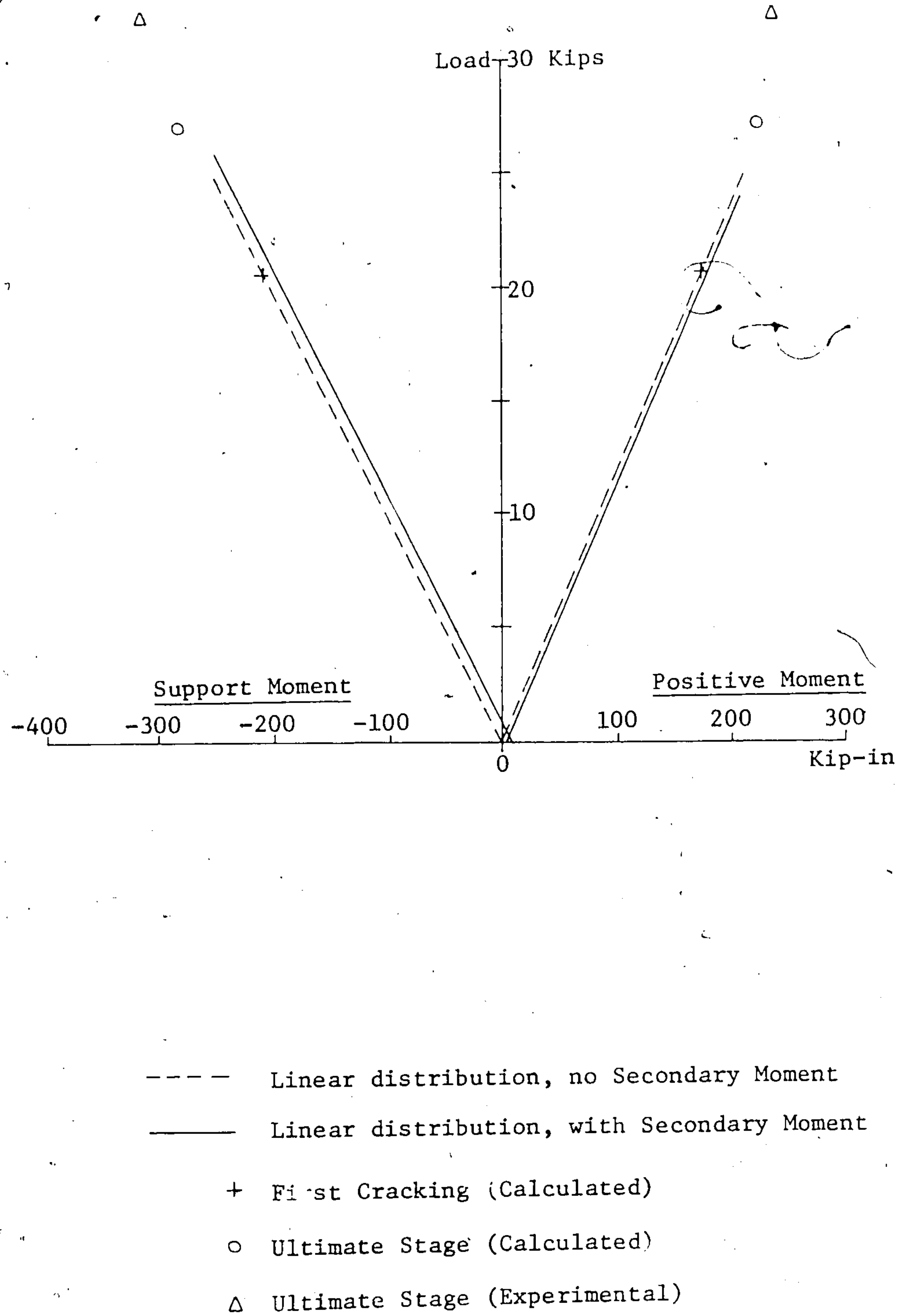
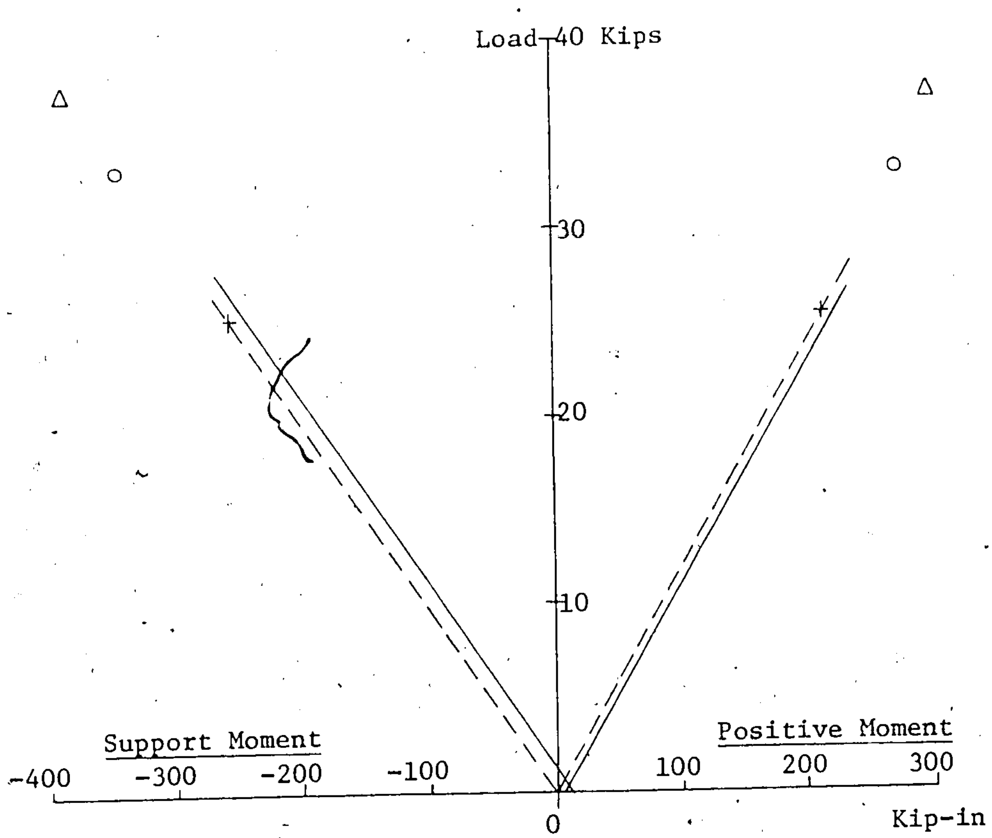


Figure 5.2 Moment-load curve for Beam 1



- - - - Linear distribution, no Secondary Moment

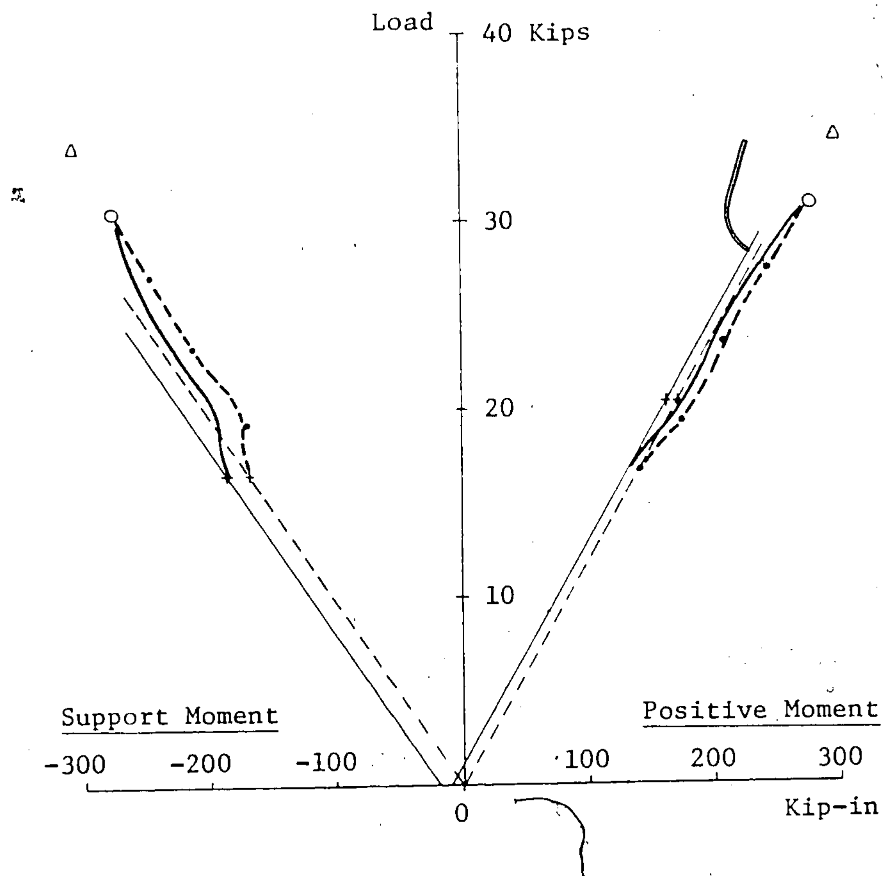
——— Linear distribution, with Secondary Moment

+ First Cracking (Calculated)

o Ultimate Stage (Calculated)

Δ Ultimate Stage (Experimental)

Figure 5.3 Moment-load curve for Beam 2



Secondary Moment neglected

- - - - Linear distribution

- - - - Inelastic behavior

△ Ultimate Stage (experimental)

○ Ultimate Stage (calculated)

• Intermediate value (calculated)

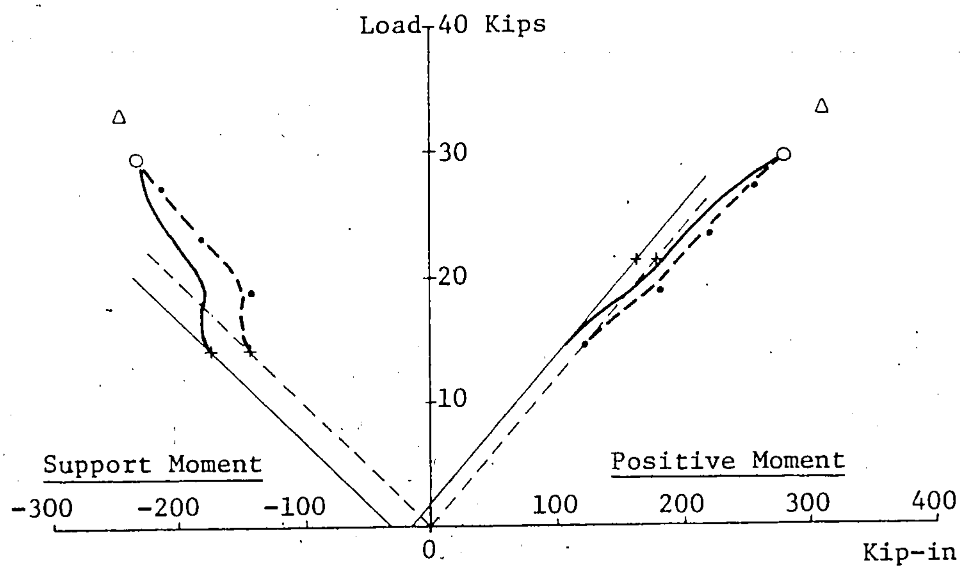
\+ First Cracking (calculated)

Secondary Moment included

——— Linear distribution

——— Inelastic behavior

Figure 5.4 Moment-load curve for Beam 3



Secondary Moment neglected

- - - - Linear distribution

—·—·— Inelastic behavior

△ Ultimate Stage (experimental)

○ Ultimate Stage (calculated)

• Intermediate value (calculated)

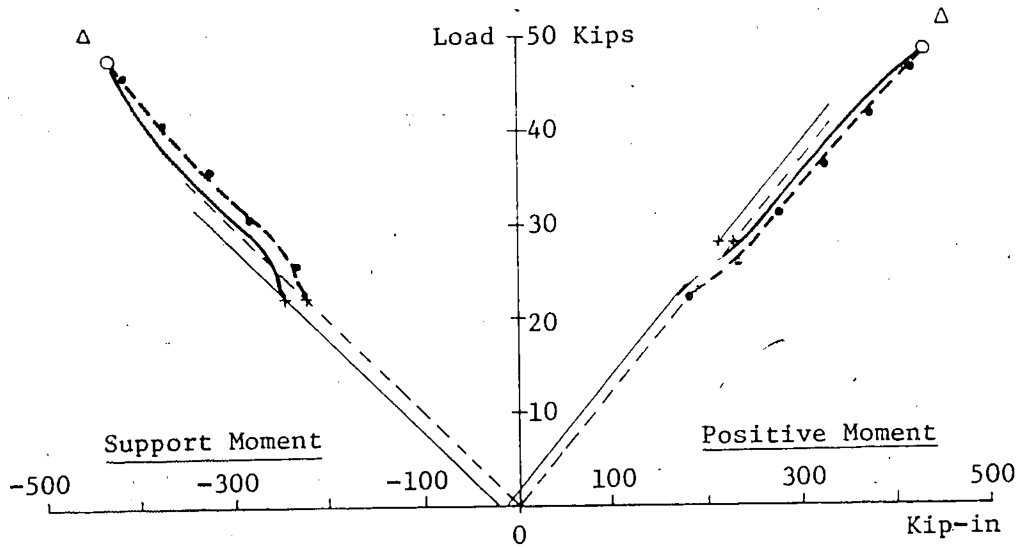
+ First Cracking (calculated)

Secondary Moment included

——— Linear distribution

——— Inelastic behavior

Figure 5.5 Moment-load curve for Beam 4



<u>Secondary Moment neglected</u>

---- Linear distribution

---- Inelastic behavior

Δ Ultimate Stage (experimental)

o Ultimate Stage (calculated)

• Intermediate value (calculated)

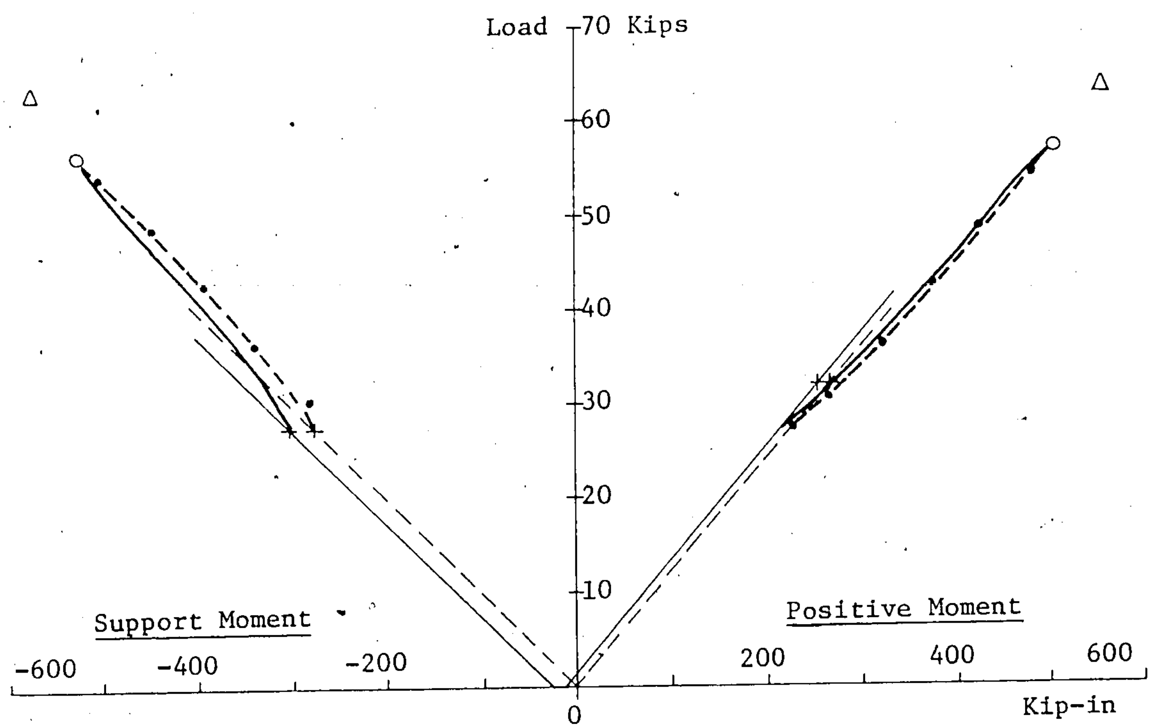
+ First Cracking (calculated)

<u>Secondary Moment included</u>

——— Linear distribution

——— Inelastic behavior

Figure 5.6 Moment-load curve for Beam 5



Secondary Moment neglected

- - - - Linear distribution

– – – – Inelastic behavior

△ Ultimate Stage (experimental)

○ Ultimate Stage (calculated)

• Intermediate value (calculated,

+ First Cracking (calculated)

Secondary Moment included

———— Linear distribution

———— Inelastic behavior

Figure 5.7 Moment-load curve for Beam 6

Beams 1 and 2: No redistribution of moment occurred because the moment capacity provided at the critical section was in accordance with the elastic distribution. However the tendon arrangement used in practice is likely to be different from the profile obtained by elastic moment analysis due to economical reasons. An example of practical design is shown in Fig. 5.8;

Beams 3, 5 and 6: The moment capacity provided in these beams was equal at both midspan and support sections. Beyond cracking at the interior support section, the positive moment to negative moment ratio rapidly reached the value of unity. The correct distribution of moment was obtained using the compatibility equation 4.40. As the load was increased up to ultimate, the variation of moment indicated an inelastic behavior which was in reasonable agreement with the experimental values at the ultimate stage. Test values of moment and load at intermediate loads, however, were not available for comparison;

Beam 4: A similar type of behavior was observed in beam 4, in which the largest amount of redistribution occurred due to the tendon profile selected which differred substantially from the elastic case: the positive moment capacity to negative moment capacity ratio was 1.3, compared to the ratio of elastic moments of 0.83.

In all six beams full redistribution of moments could be achieved as expected due to the low value of the reinforcement index ranging from 0.063 to 0.253, indicating

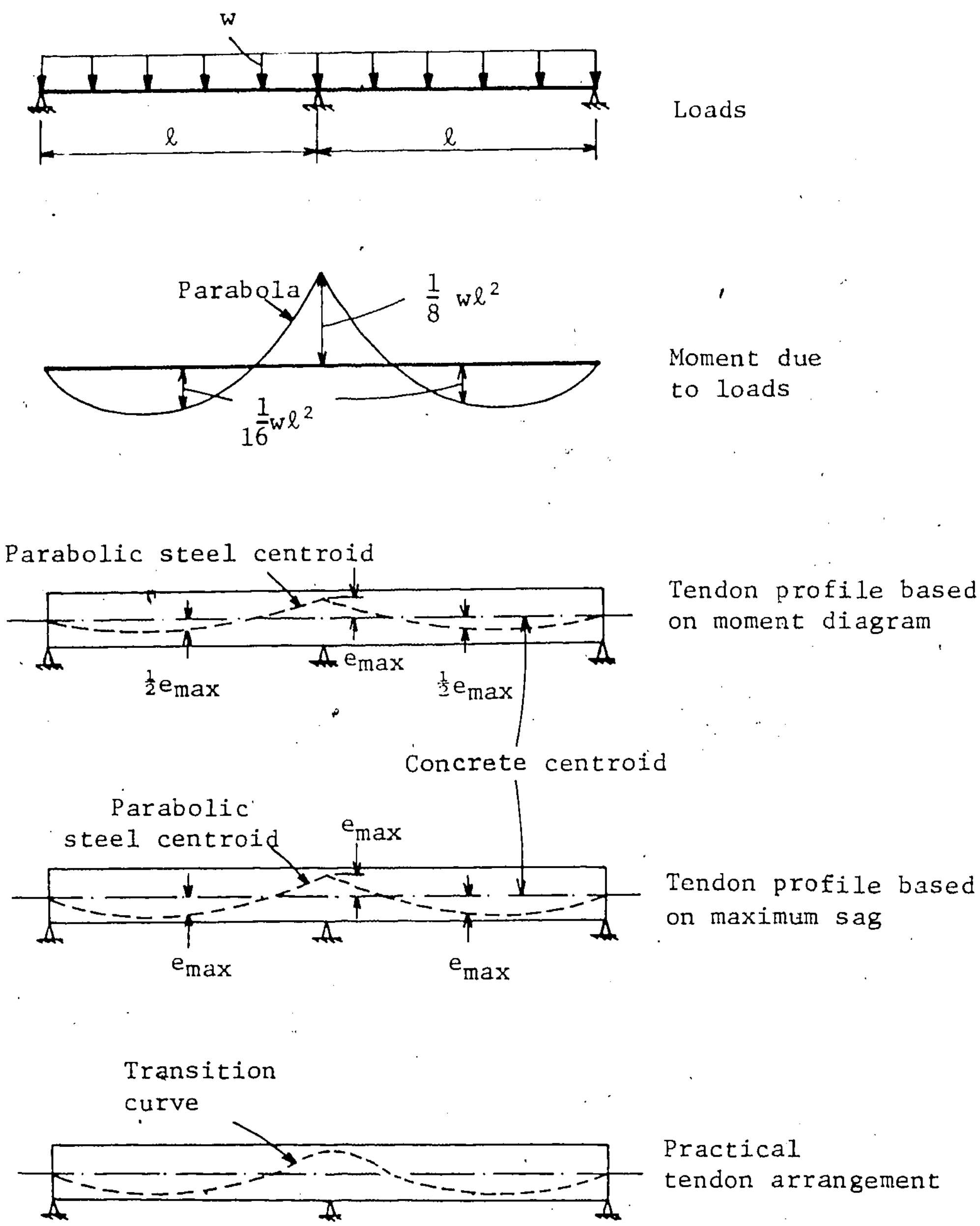


Figure 5.8 Basis for selecting tendon profile

underreinforced sections. However the effects of shear have to be considered because the development of inclined tension cracking may reduce both the load carrying capacity and the ductility of the test beams, as was observed in remaining beams of the test programme. The span-to-depth ratio of 9 indicates the dominance of shear force which may result in deep beam action. Nevertheless, flexural failure is still possible. It appears that, with sufficient web reinforcement, full redistrib[illegible] be achieved even if the critical section or sectio[illegible]ery limited rotation capacity beyond yielding, [illegible]s 5 and 6.

The higher values of moment and load observed at ultimate in the experiment may be attributed to the criterion adopted for failure: the ultimate moment calculated in the analysis used the concept of average stress of the concrete acting over the entire compressed concrete area above the neutral axis (Equation 36, ref. 28). The difference in tendon profile at the end portion of the beams between the analysis and the test might be one of the reasons for obtaining a higher load. However, this did not have much effect on the peak moment which always occurred at midspan and at the interior support due to the type of loading.

Observations on test beams described in Ref. 15 show that the significant amount of redistribution can only be achieved through extensive cracking at the critical sections. Furthermore, the use of compatibility equation 4.40 in the analysis has been made necessary in lieu of equation 4.39, due to the larger degree of deformation in the post-cracking range.

5.4 Secondary moments

As indicated in the moment-load diagrams, the amount of secondary moment existing in each of the beams was not large enough to be significant in the the over-all behavior and the load carrying capacity of the test beams.

Secondary moments are calculated in Table 5.3 on the basis of the effective prestress force P_e . While the force in the tendon does increase significantly as loads on the structure are increased as a result of bending of the member, this does not represent a change in the prestressing force that produced the secondary moments. The force resulting from prestressing is unchanged, and the secondary moments are unchanged as the load increases up to the first cracking load.

An intuitive approach can be established concerning the consideration of secondary moments: since they arise due to the restraint opposed by the redundant system of support, it is reasonable to expect a variation in the intensity of these moments as the structure exceeds the post-elastic range. There are several explanations for this:

1.) As cracking occurs at the critical sections, where peak moments are observed, the structure adjusts itself to the variation of stiffnesses.

This in turn produces a new arrangement of support reactions. The portion of the reactions due to prestressing is likely to be different from the initial one because of the decrease in stiffness induced by extensive cracking in regions surrounding the critical sections. The secondary moments should therefore vary accordingly;

2) As the loading proceeds further, one might expect the critical sections to form plastic hinges and reach the extreme stage of collapse mechanism. Secondary moments may then disappear. It is well established that they may be neglected at this particular stage (refs. 10, 18, 22);

3) In practical cases, full redistribution is not always possible because the combined action of bending and shear may cause one of the critical sections to fail by crushing of the concrete. A flexural failure, as usually defined, is not likely unless the shear-to-moment ratio is extremely small.

For these reasons, the secondary moments may be assumed to decrease from the initial value, according to the following equation:

$$\left(M_{sec}\right)_{post\text{-}crack} = \left(M_{sec}\right)_{initial}\left[1-\left(\frac{P - P_{cracking}}{P_{full\ redist.} - P_{cracking}}\right)^3\right] \qquad (5.1)$$

P is the load, usually the ultimate load, at which the secondary moment is evaluated. $P_{cracking}$ corresponds to the load at first cracking through an elastic analysis, and $P_{full\ redist.}$ is the load calculated with ultimate moments at both critical sections. Also, the cubic variation is set to reflect the gradual reduction in member stiffness. Equation 5.1 is valid under the assumptions made when establishing the analysis. Thus the limitations are:

1) (net reinforcement index) ≤ 0.30 to ensure that ductility is available;
2) bonded, partially prestressed concrete so that cracking is allowed.

Equation 5.1 is graphically represented in Fig. 5.9, and is used in Figs. 5.4 through 5.7 to indicate the possible inelastic behavior when the secondary moment is taken into consideration.

Caution has to be exercised when considering the yielding stage: although measurements of concrete strain at failure in beam tests indicate that values of ϵ_{cu} between 0.003 and 0.004 are attained, a limiting strain of 0.003 for

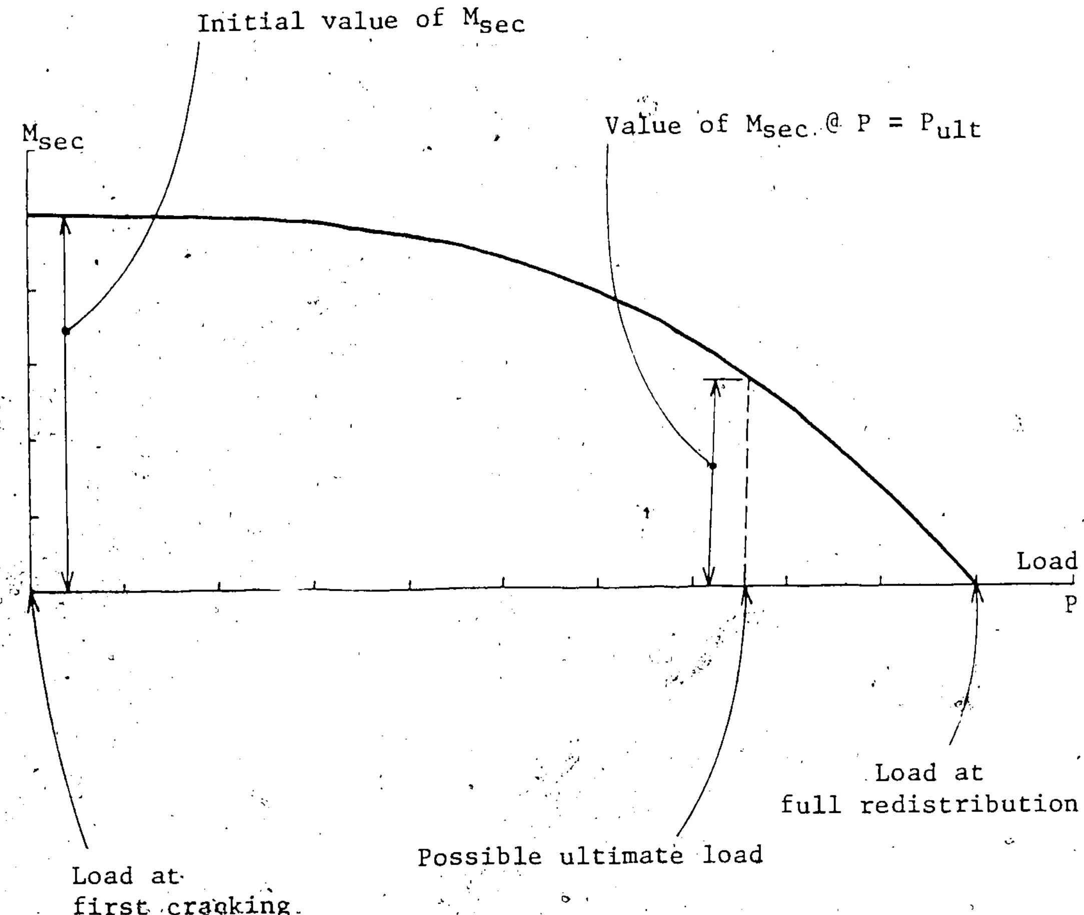


Figure 5.9 Variation of secondary moment

the concrete will be assumed. Also, prestressing steels do not show a definite yield plateau. The spread between the nominal yield strength and the ultimate tensile strength is much smaller for prestressing steels than is the spread between the corresponding values for reinforcing steel. Therefore the small amount of steel yield available in prestressed concrete beams may reduce the moment redistribution possible as compared to conventional reinforced concrete.

If the ultimate load does not correspond to full redistribution, i.e. if any of the critical sections has not attained its ultimate capacity, then a portion of the initial amount of secondary moment must be taken into account. This differs from the current Code provisions which require the inclusion of the total secondary moment.

The behavior characteristics described in the present study have been demonstrated to be in reasonable agreement with the observed ones for a particular type of beam. The cross-section shape and the high-grade steel stress-strain relationship have a relatively minor effect on the inelastic behavior (Cohn, 1982, ref. 11). However, extensive experimental work is required in the future for a better evaluation of secondary moments in beams with a non-concordant tendon.

5.5 Design method

A method for determining the ultimate load capacity of a continuous beam is proposed, which takes into account the secondary moment described by equation 5.1. Only a two-span symmetrical beam is considered, a similar approach being possibly applied to more complex structures.

This method first evaluates the value of the net reinforcement index ($\omega + \omega_p - \omega'$) at both critical sections. The limiting value of 0.20 will be used as required by section 18.10.4 of the ACI 1977 Code to allow a variation of design moments from the elastic analysis. The limitation of 0.30 is the dividing line between underreinforced and overreinforced members and this is confirmed by a parametric study (Cohn, 1982, ref. 11). Five different situations may arise as shown in Table 5.6. In cases 1, 2 and 3 the negative moments calculated by elastic analysis for any loading arrangement, may be increased or decreased by not more than

$$20 \left(1 - \frac{\omega - \omega_p - \omega'}{0.30}\right) \text{percent}$$

In cases 4 and 5, no modification to the negative moment obtained from an elastic analysis is permitted.

Cases 1 and 2: Since both critical sections are underreinforced, full redistribution is likely and hence no secondary moment remains at ultimate. The ultimate moments are those due to gravity loads only;

Case	$(\omega + \omega_p - \omega')$ at support	$(\omega + \omega_p - \omega')$ at max. positive	Moment used in design
1		$\leqslant 0.20$	
2	$\leqslant 0.20$	$0.20 < \quad \leqslant 0.30$	Modified moments according to Section 18.10.4 of ACI 1977
3		$0.30 <$	
4	$0.20 < \quad \leqslant 0.30$		Elastic moments
5	$0.30 <$		

Table 5.6 Design cases

Case 3: The section at maximum positive moment is overreinforced, which determines the ultimate load due to reduced ductility at that section. This ultimate load is calculated using the ratio of moment capacity provided, and then substituted into equation 5.1 to calculate the magnitude of secondary moment remaining at ultimate;

Case 4: Although the beam is designed according to the elastic analysis, the low value of the reinforcement index at the support section allows the ultimate moments to be reached at all critical sections as in cases 1 and 2. This is achieved by yielding of the reinforcement and extensive cracking. The secondary moment is reduced to zero at ultimate;

Case 5: The support section is overreinforced, and as a result the ultimate load corresponds to the load at which the support ultimate moment is attained through an elastic analysis. The procedure is similar to the one used in case 3.

The approach adopted in this design method satisfies at first the equilibrium condition by calculating the load from equations of statics, secondly the safety condition by ensuring that no premature failure occurs at any section of the beam, and thirdly the ductility condition to allow redistribution of forces from the elastic distribution to the assumed distribution. These three conditions of member proportioning correspond to the so-called lower bound solution, i.e. the structure is certainly capable to carry the calculated load.

As the member strength is approached the inelastic behavior at some sections results in a redistribution of moments. Recognition of this behavior can be advantageous, but a rigorous design method for moment redistribution is quite complex. The recommended design procedure is an attempt to give an account of the dependence of the secondary moment on the degree of cracking and moment redistribution. The amount of adjustment is kept within safe limits defined by current Code provisions.

6. SUMMARY, CONCLUSION AND RECOMMENDATIONS

6.1 Summary

The overall objective of this study was to investigate the flexural behavior characteristics of continuous prestressed concrete beams and determine the effects of inelastic behavior on the secondary moment.

The analysis considered the equilibrium and strain compatibility of each beam section to evaluate the conditions at a particular cross-section of the beam. In addition, a theoretical stress-strain curve was used for both concrete and prestressing steel to establish the moment-average curvature relationships. Conventional principles of geometry were applied to determine the distribution of moment at any location in the beams. The complete moment-load curves to failure for several beams were obtained from the analysis and compared with available test results.

6.2 Conclusion

The reinforcement index value of 0.30 set by the ACI Building Code to define underreinforced sections can be used as a criterion to evaluate the ductility of continuous beams. For ductile beams, full redistribution of moment is possible due to yielding of the reinforcement and extensive cracking, with the understanding that minimum bonded reinforcement is provided to prevent the early failure of the reinforcement. For non-ductile members, the ultimate limit state is defined by the load at which the moment capacity of a brittle section is attained. In such cases only partial moment redistribution is achieved.

The secondary moment varies due to the readjustment of reactions caused by cracking, and this variation is dependent upon the redistribution of moment carried out at ultimate. A cubic variation is assumed in the present study, but a different type of variation may be set in the light of further investigation.

6.3 Recommendation for future work

More experimental programmes on continuous prestressed concrete beams are needed to study the phenomenon of secondary moment. A realistic evaluation of the secondary moment is necessary because its effect on the load carrying capacity of a given beam may be favorable or unfavorable, depending on the location of a particular cross-section.

The analysis described in this investigation can be extended to more general statically indeterminate systems.

REFERENCES

1. Abeles, P.W., *Introduction to Prestressed Concrete*, Vol.2, Concrete Publications Ltd, London, 1966.

2. Baker, A.L.L., "A plastic theory of design for ordinary reinforced and prestressed concrete including moment redistribution in continuous members", *Magazine of Concrete research*, June 1949, pp. 57-66

3. Bishara, A.G. and Brar, G.S., "Rotational capacity of prestressed concrete beams", *Journal of the Structural Division*, ASCE Proceedings, ST9, Sept 1974, pp. 1883-1895

4. *Building Code Requirements for Reinforced Concrete, (ACI 318-63)*, American Concrete Institute, Detroit, June 1963, 144 pp.

5. *Building Code Requirements for Reinforced Concrete, (ACI 318-71)*, American Concrete Institute, Detroit, Feb. 1971, 78 pp.

6. *Building Code Requirements for Reinforced Concrete, (ACI 318-77)*, American Concrete Institute, Detroit, Oct. 1977, 102 pp.

7. *Commentary on Building Code Requirements for Reinforced Concrete, (ACI 318-63)*, American Concrete Institute, Detroit, 1965, 91 pp.

8. *Commentary on Building Code Requirements for Reinforced Concrete, (ACI 318-71)*, American Concrete Institute, Detroit, Feb. 1971, 96 pp.

9. Cohn, M.Z., "Rotation compatibility in the limit design of reinforced concrete continuous beams", *Proceedings*, International Symposium, Flexural mechanics of reinforced concrete, Miami, Fla., Nov 10-12, 1964, ACI SP 12, pp. 359-382

10. Cohn, M.Z. and Frostig, Y., "Nonlinear analysis of continuous prestressed concrete beams", *Proceedings*, International Symposium, Nonlinearity and Continuity in Prestressed Concrete, Waterloo, Ont., July 1983, Vol. 2, pp. 45-76

11. Cohn, M.Z. and Bartlett, M., "C puter-simulated flexural tests of partially prestressed concrete sections", *Journal of the Structural Division*, ASCE Proceedings, ST12, December 1982, pp. 2747-2765

12. Corley, W.G., "Rotational capacity of reinforced concrete beams", *Journal of the Structural Division*, ASCE Proceedings, ST5, Oct 1966, pp. 121-146

13. Gerald, Curtis F., *Applied Numerical Analysis*, Addison-Wesley Publishing Company, 2nd Ed., 1980.

14. Guyon, Y., *Prestressed Concrete, Statically Indeterminate Structures*, Contractors Record Limited, London, 1960, Chapter XXXVI, pp. 603-652

15. Hawkins, N.M., Sozen, M.A. and Siess, C.P., "Behavior of continuous prestressed concrete beams", *Proceedings*, International Symposium, Flexural mechanics of reinforced concrete, Miami, Fla., Nov 10-12, 1964, ACI SP 12, pp. 259-294

16. Leonhardt, F., "Continuous Prestressed Concrete Beams", *J. ACI*, Vol. 24, No.7, March 1953, pp. 617-634

17. Leonhardt, F., *Prestressed Concrete Design and Construction*, Wilhem Ernst and Sons, Berlin, 1964, 676 pp.

18. Lin, T.Y. and Thornton, K., "Secondary moment and moment redistribution in continuous prestressed concrete beams", *PCI Journal*, Vol. 17, No.1, January 1972, pp. 8-20

19. Mallick, S.K., "Redistribution of moments in two-span prestressed beams", *Magazine of Concrete Research*, Vol. 14, No.42, Nov. 1962, pp. 171-183

20. Mattock, A.H., "Rotational capacity of hinging regions in reinforced concrete beams", *Proceedings*, International Symposium, Flexural mechanics of reinforced concrete, Miami, Fla., Nov 10-12, 1964, ACI SP 12, pp. 143-181

21. Mattock, A.H., Yamazaki, J. and Kattula, B.T., "Comparative Study of Prestressed Concrete Beams, with and without Bond", *ACI Journal*, Proceedings, Vol. 68, Feb. 1971, pp. 116-125

22. Mattock, A.H., "Secondary Moments and Moment Redistribution in ACI 318-77 Code", *Proceedings*, International Symposium, Nonlinearity and Continuity in Prestressed Concrete, Waterloo, Ont., July 1983, Vol. 3, pp. 27-48

23. Nilson, A. H., *Design of Prestressed Concrete*, John Wiley and Sons, 1978, 521 pp.

24. Priestley, M.J.N., Park, R. and Lu, F.P.S., "Moment-curvature relationships for prestressed concrete in constant-moment zones", *Magazine of Concrete Research*, Vol. 23, No.75-76, June-Sept. 1971, pp. 69-78

25. Priestley, M.J.N. and Park, R., "Moment redistribution in continuous prestressed concrete beams", *Magazine of Concrete Research*, Vol. 24, No.80, Sept. 1972, pp. 157-166

26. Roy, H.E.H. and Sozen, M.A., "Ductility of concrete", *Proceedings*, International Symposium, Flexural mechanics of reinforced concrete, Miami, Fla., Nov 10-12, 1964, ACI SP 12, pp. 213-235

27. Shushkewich, K.W., "Simplified Cracked Section Analysis", *ACI Journal*, Oct-Nov 1983, Vol. 80, pp. 526-531

28. Warwaruk, J., Sozen, M.A. and Siess, C.P., "Strength and behavior in flexure of prestressed concrete beams", *Bulletin* No. 464, Engineering Experiment Station, University of Illinois, Urbana, August 1962, 105 pp.

APPENDIX

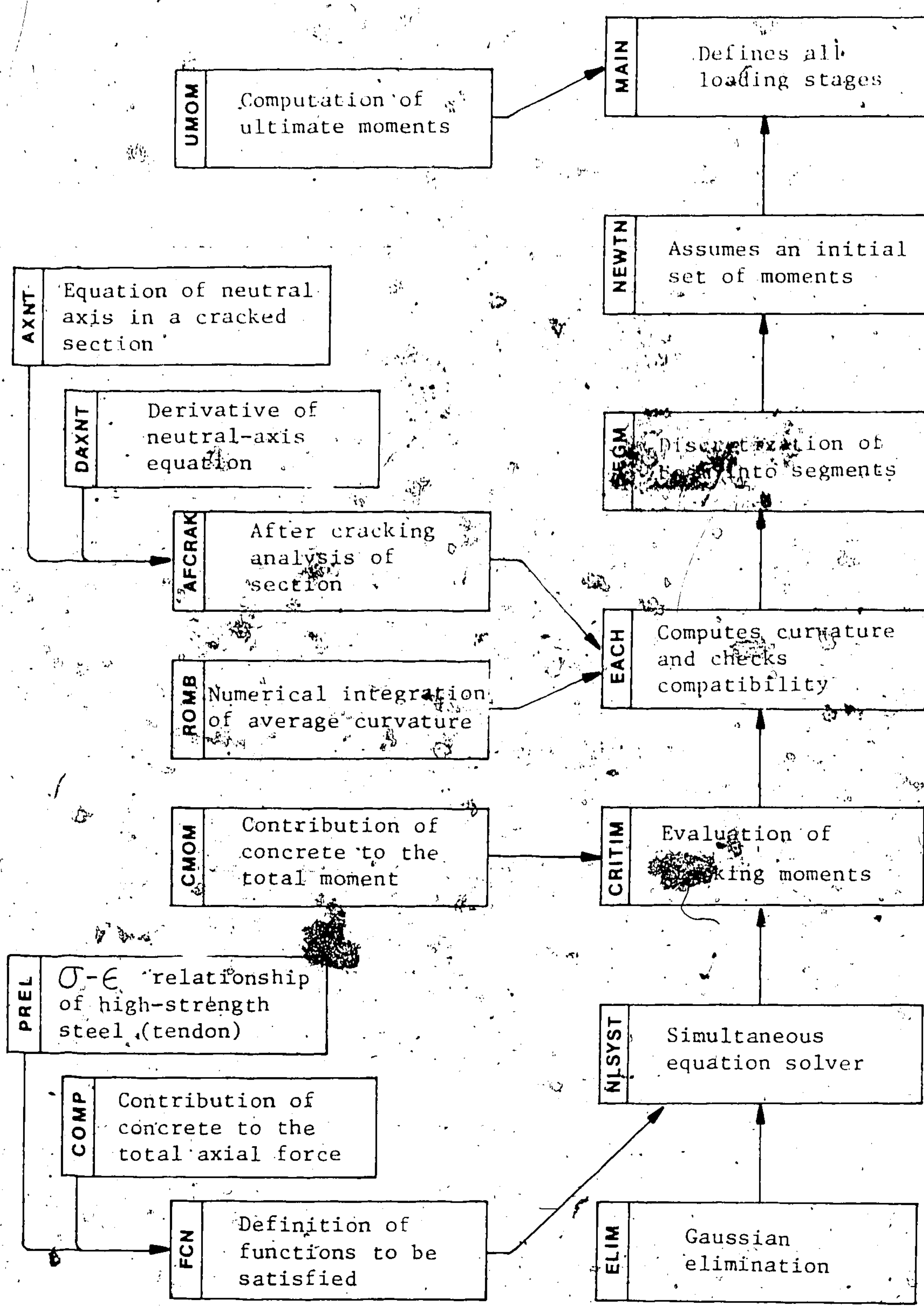
Contents

General Remarks

This Appendix gives detailed explanation on the computer program derived from the theory described in Chapter 4. The program consists of 8 subroutines and 7 functions coordinated by the Main Routine. The sequence of execution is summarized in the diagram shown in the following page. The procedure of each subprogram is explained, and the program output for 6 beams is listed at the end of the Appendix.

The limitations are:

1. Two-span symmetrical continuous prestressed beam with rotations allowed at all supports;
2. Two symmetrical concentrated loads being applied at any position;
3. Linear variation of tendon profile;
4. Rectangular or doubly-symmetric I-shaped cross-section;
5. No reinforcement other than prestressing bonded strands;
6. The stress-strain relationships are to be modelized for both concrete and steel; only Grade 250 or 270 strand are considered;
7. Secondary moments are neither evaluated nor included in the calculations;
8. Imperial units.



Sequence of execution

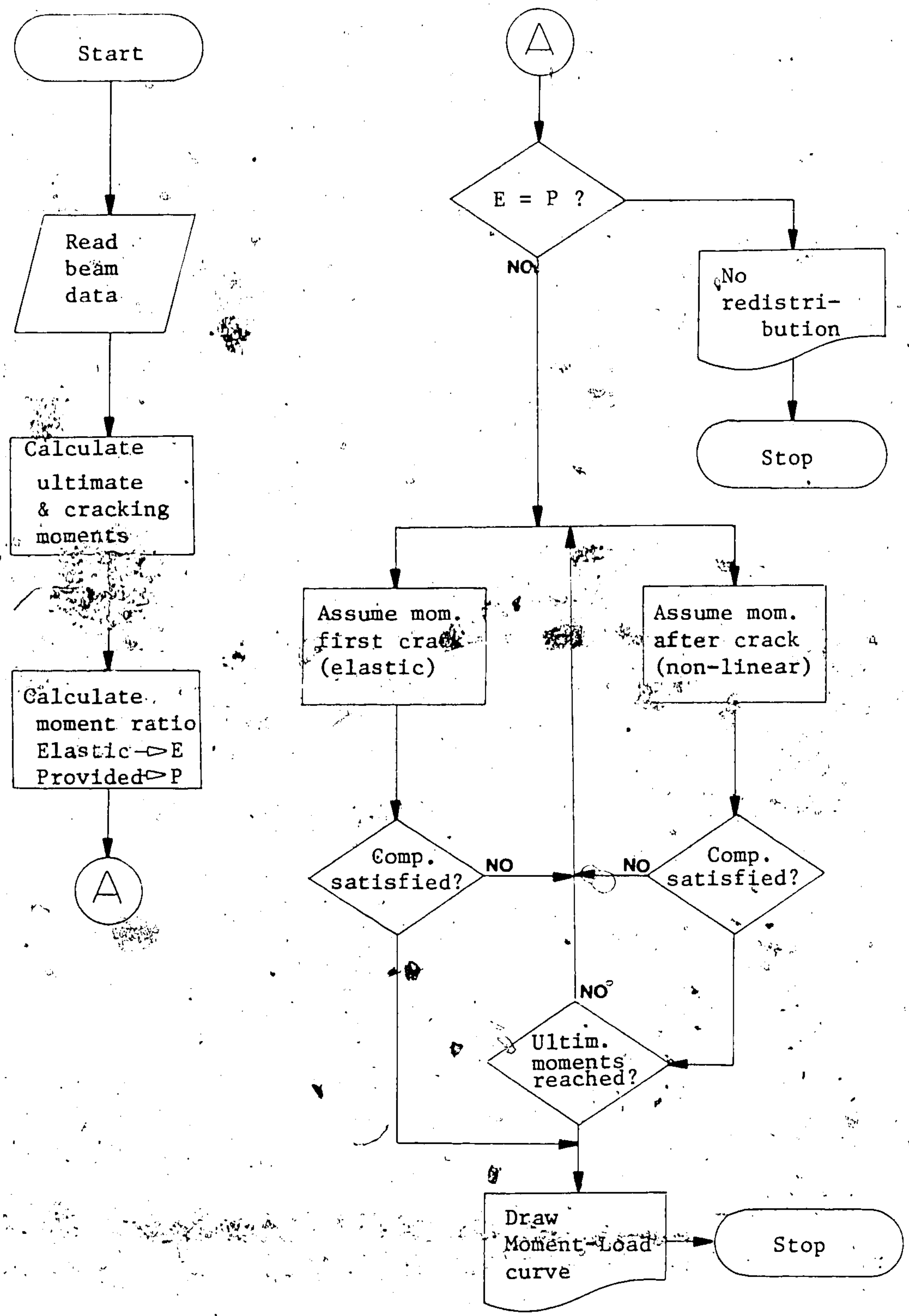
MAIN ROUTINE

The MAIN routine defines all loading stages starting from the load at which first cracking occurs, up to ultimate. The cross-section properties, the ultimate capacity and the cracking moment are evaluated at the critical sections by calling particular functions, after all the necessary informations are read.

The elastic moment distribution along the beam can be calculated knowing the position of application of the concentrated load. If the moment capacity ratio provided at the critical sections is different from the elastic ratio, then redistribution of moment is likely.

The MAIN routine assumes an initial set of moments and calls subroutine NEWTN so that this set of moments is varied until the compatibility equation is satisfied. The correct distribution of moments is then printed and another calculation is done for a higher load stage.

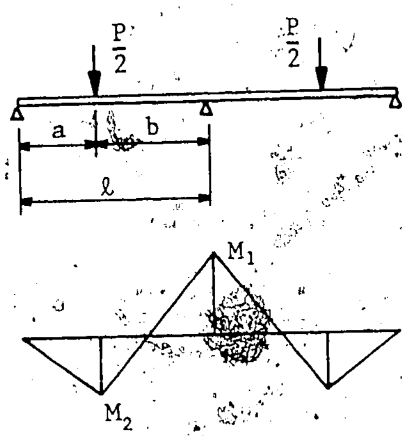
At first cracking at a critical section, the beam is still in the linear range and the compatibility equation 4.39 is used. All subsequent load stages are tested using compatibility equation 4.40.



Main routine

The load is calculated with the equation

$$P = 2\ \frac{\ell\ M_2\ -\ a\ M_1}{a\ b}$$



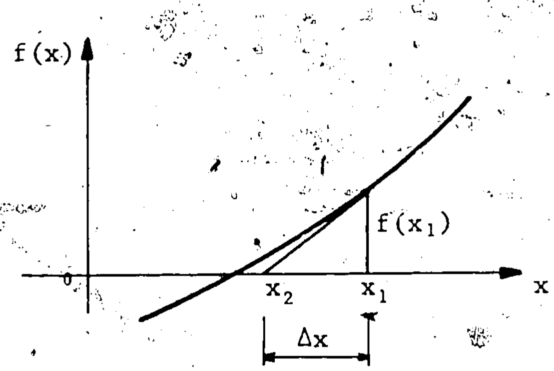
NEWTN

Newton-Raphson iterative procedure to evaluate a distribution of moments at critical sections that satisfies the compatibility conditions. The error margins adopted are:

a. Elastic case, equation 4.39: ± 0.0001 radians;

b. Non-linear case, equation 4.40: ± 0.05 radians.

If *x* represents the distribution of moments and *f(x)* the deformations resulting from that distribution of moments, expressed in the compatibility equation, then the iterations are performed as shown in the following figure. The initial guess must be close enough to the solution in order [illegible] convergence.



$$\Delta x = x_1 - x_2$$

$$\text{slope} = \frac{f(x_1)}{\Delta x}$$

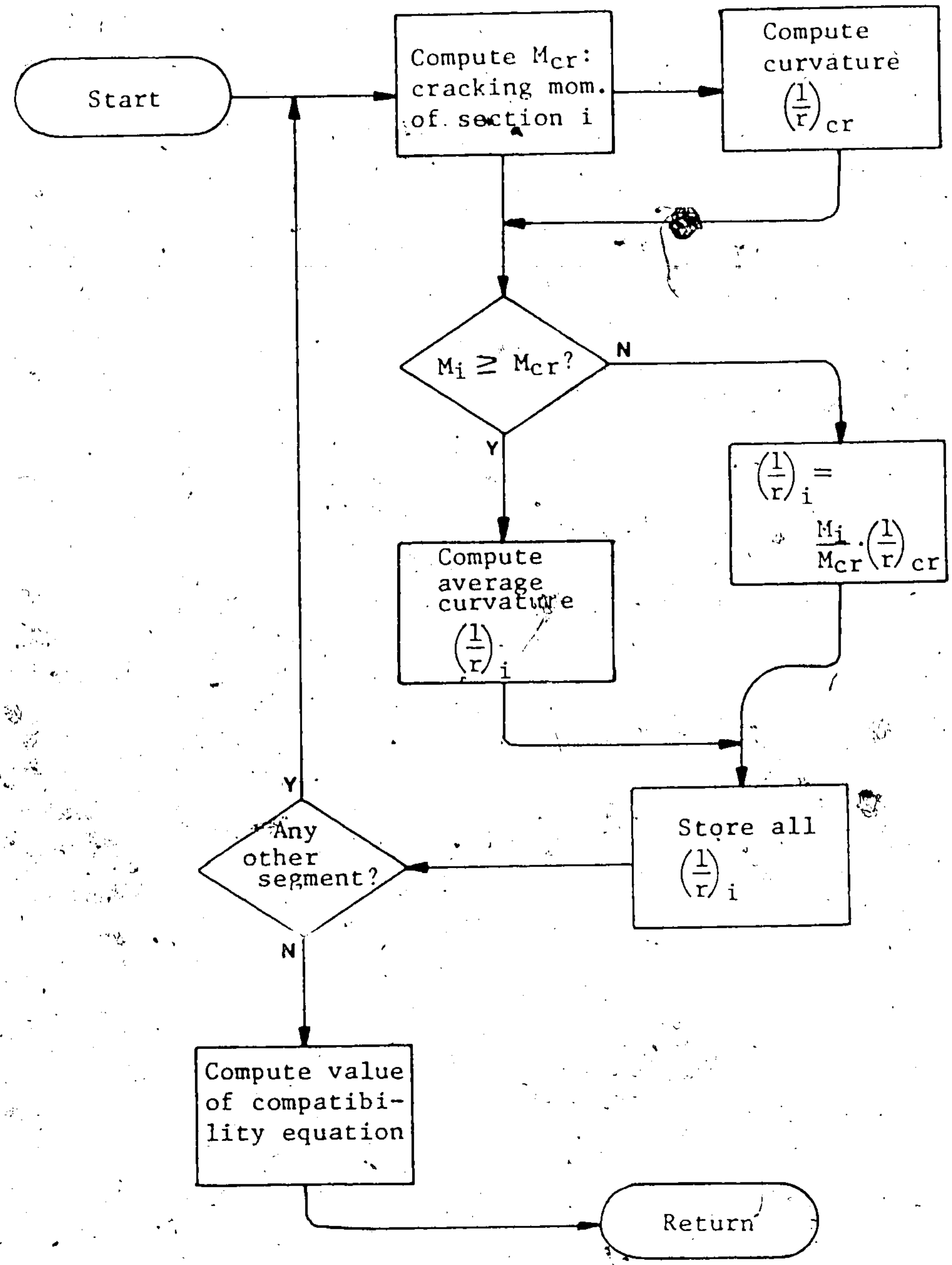
$$x_2 = x_1 - \Delta x = x_1 - \frac{f(x_1)}{\text{slope}}$$

SEGM

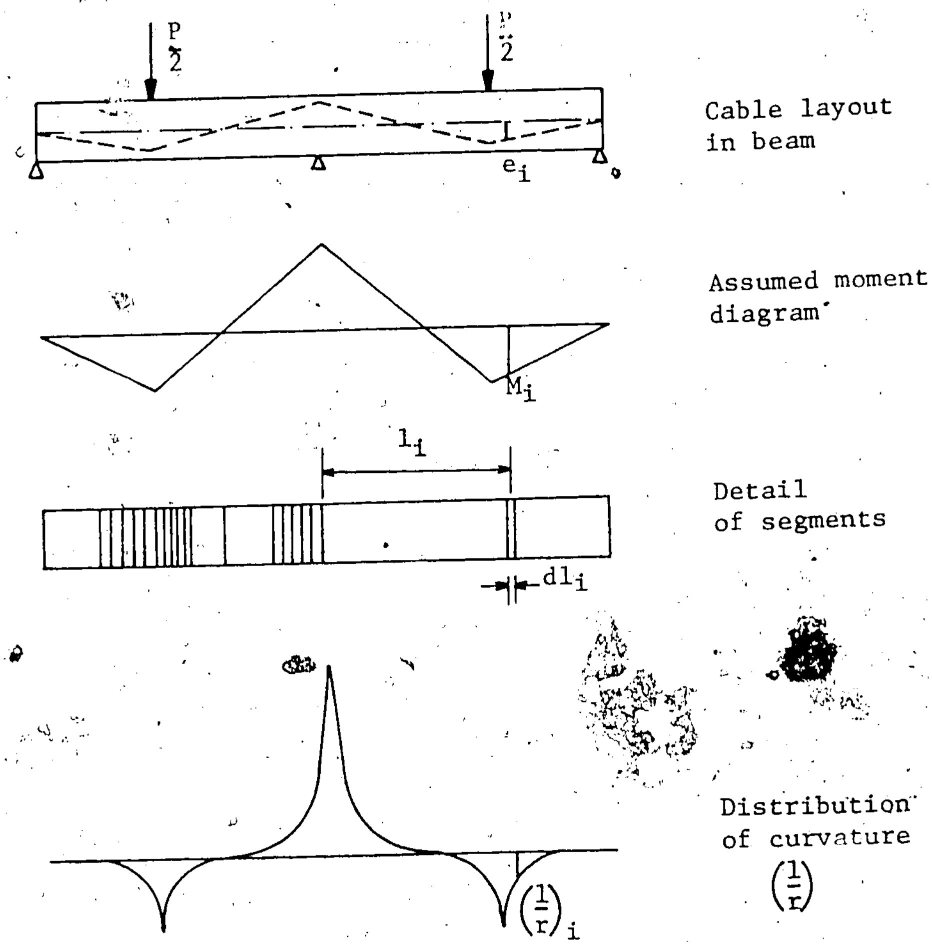
defines the portions of beam as shown in the following page. Beam segment lengths are chosen shorter in regions where the curvature reaches peak values. The distance to the end support, the eccentricity of steel and the moment at the center of each beam segment are evaluated.

EACH

evaluates the curvature and the contribution of each segment to the total deformations expressed in equations 4.39 and 4.40. If the moment at a segment is smaller than the cracking moment defined in equation 4.11, linear extrapolations are made to obtain the actual moment and curvature. Otherwise the general cracked section analysis of Section 4.3.4 is carried out to evaluate the steel stress at a crack. The average curvature in the segment in then calculated as indicated in Section 4.3.5.



Subroutine EACH



Subroutines SEGM and EACH

CRITIM

evaluates the cracking moment at a particular cross-section as described in Section 4.3.3.

UMOM

calculates the ultimate moment at a particular cross-section. If the net reinforcement index exceeds 0.30, a different equation is used as pointed out in Section 18.8.2 of the 1977 ACI Code.

NLSYST

Newton-Raphson iterative method with 2 parameters to solve 2 non-linear simultaneous equations. The derivatives of the equations considered have a contribution from each parameter. The simultaneous equations are defined in Subroutine FCN.

ELIM

Gaussian elimination to calculate the corrections to be included in each iteration in Subroutine NLSYST.

FCN

defines the equations of equilibrium and strain compatibility 4.5.

AFCRAK

General cracked section analysis defined in Section 4.3.4. The steel stress is calculated from equation 4.19.

AXNT

defines the equations of position of the neutral axis 4.12 and 4.33.

PREL

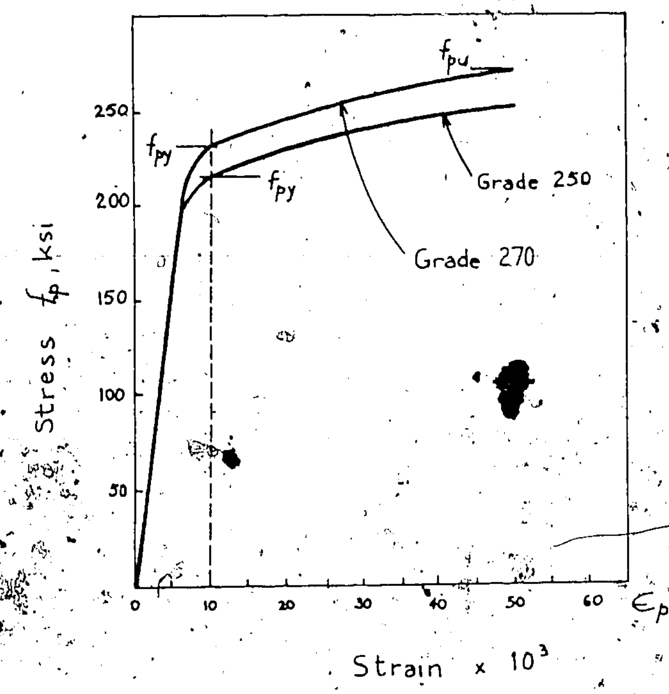
defines the stress-strain relationship for steel. The strain is expressed in terms of steel stress.

Grade 250: $\epsilon_p = \dfrac{f_p}{E_p} + 2.5\dfrac{(f_p - 195)^3}{10^7}$ $\qquad f_p > 195$ ksi

$\epsilon_p = \dfrac{f_p}{E_p}$ $\qquad f_p \leq 195$ ksi

Grade 270: $\epsilon_p = \dfrac{f_p}{E_p} + 2.0\dfrac{(f_p - 210)^3}{10^7}$ $\qquad f_p > 210$ ksi

$\epsilon_p = \dfrac{f_p}{E_p}$ $\qquad f_p \leq 210$ ksi



$E_p \approx 27 \times 10^3$ ksi

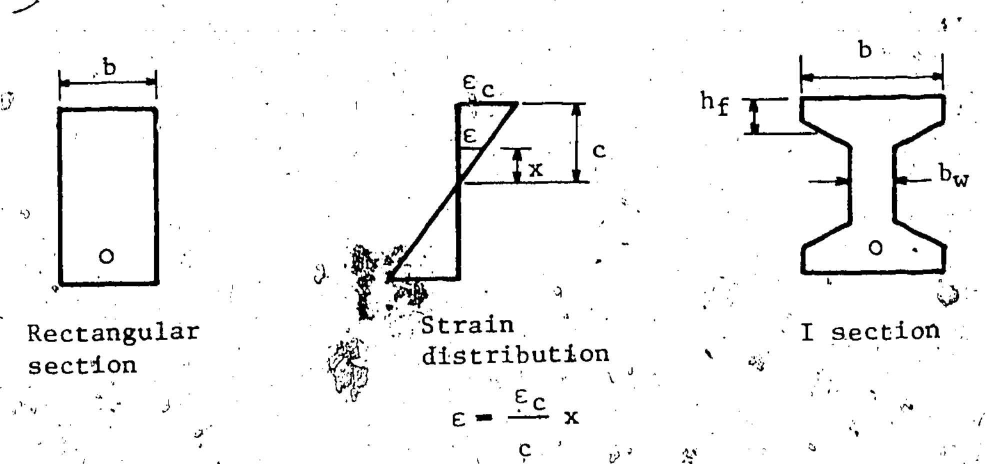
Stress-strain relationship for strands

ROMB

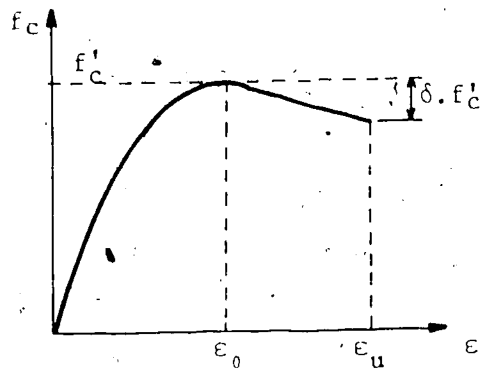
Romberg integration; performs the Simpson's rule for known values of a function. Intervals are then halved and results are extrapolated. The integration has been done on the curvatures obtained at 60 locations away from a crack to calculate the average curvature defined in equation 4.35.

CMOM

computes the contribution to the total moment, of the concrete area stressed in compression. The calculation takes into account the stress-strain relationship of concrete.



Typical cross-sections



$$f_c = f'_c \left[2 \frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2 \right] \quad , \quad \text{for} \quad 0 < \varepsilon < \varepsilon_0$$

$$f_c = f'_c \left(1 - \delta \frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0} \right) \quad , \quad \text{for} \quad \varepsilon_0 < \varepsilon < \varepsilon_u$$

Typical uniaxial compressive stress-strain curve for concrete

f'_c (psi)	ϵ_0	ϵ_u	δ
3000	0.0020	0.0040	0.37
4000	0.0019	0.0039	0.60
5000	0.0019	0.0034	0.56
6000	0.0019	0.0031	0.65
7000	0.0018	0.0025	0.27
8000	0.0017	0.0022	0.19

CMOM (cont'd)

Numerical integration (Gaussian integration)

The calculations in Functions CMOM and COMP are based on this method. The integration using 2 sample points located at $t=-\frac{1}{\sqrt{3}}$ and $t=+\frac{1}{\sqrt{3}}$, both weighted 1, can integrate exactly a cubic polynomial.

$$\int_a^b F(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f(t)\,dt$$

$$= \frac{b-a}{2}\left\{f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)\right\}$$

$$x = \frac{(b-a).t + (b+a)}{2}$$

CMOM (cont'd)

Rectangular section

for $0 < \varepsilon_c < \varepsilon_0$

$$\int_0^c f_c \, .x \, dx = \int_0^c f'_c \left[2 \frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2 \right] x \, dx$$

$$= \left(\frac{c}{2}\right)^2 f'_c \left[\frac{8}{3} \frac{\varepsilon_c}{\varepsilon_0} - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2 \right]$$

for $\varepsilon_0 < \varepsilon_c < \varepsilon_u$

$$\int_0^c f_c \, .x \, dx = \int_0^{c\frac{\varepsilon_0}{\varepsilon_c}} f'_c \left[2 \frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2 \right] x \, dx$$

$$+ \int_{c\frac{\varepsilon_0}{\varepsilon_c}}^{c} f'_c \left[1 - \delta \frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0} \right] x \, dx.$$

$$= \frac{5}{12}\left(c\frac{\varepsilon_0}{\varepsilon_c}\right)^2 f'_c + \left(\frac{c}{2}\right)^2 \left(1 - \frac{\varepsilon_0}{\varepsilon_c}\right) f'_c \cdot$$

$$\cdot \left\{ \left(1 + \frac{\varepsilon_0}{\varepsilon_c}\right) \left[2 - \frac{\delta}{\varepsilon_u - \varepsilon_0} \left(\varepsilon_c - \varepsilon_0\right) \right] - \frac{\delta}{3(\varepsilon_u - \varepsilon_0)} \varepsilon_c \left(1 - \frac{\varepsilon_0}{\varepsilon_c}\right)^2 \right\}$$

CMOM (cont'd)

I section $\quad A = c\,\frac{\varepsilon_0}{\varepsilon_c} \qquad B = c - h_f$

$\boxed{\varepsilon_c < \varepsilon_0}$

$$\int_0^c (\text{width}).f_c.x\,dx = \int_0^{c-h_f} (b-b_w)\, f'_c \left[2\frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right] x\,dx$$

$$+ \int_{c-h_f}^{c} b\, f'_c \left[2\,\frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right] x\,dx$$

$$= (b-b_w)\,\frac{f'_c}{4}\,\frac{B^3}{A}\left[\frac{8}{3} - \frac{B}{A}\right]$$

$$+ b\,h_f\,\frac{f'_c}{2A}\left[\frac{h_f^2}{3} + (c+B)^2 - \frac{1}{2A}\left((c+B)^3 + (c+B)h_f^2\right)\right]$$

CMOM (cont'd)

I section (cont'd)

$\boxed{\varepsilon_c > \varepsilon_0}$

for $\quad c\frac{\varepsilon_0}{\varepsilon_c} < c-h_f$

$$\int_0^c (\text{width}).f_c.x\,dx = \int_0^{c\frac{\varepsilon_0}{\varepsilon_c}} (b-b_w)\ f'_c\left[2\frac{\varepsilon}{\varepsilon_0}-\left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right] x\,dx$$

$$+\int_{c\frac{\varepsilon_0}{\varepsilon_c}}^{c-h_f} (b-b_w)\ f'_c\left[1-\delta\frac{\varepsilon-\varepsilon_0}{\varepsilon_u-\varepsilon_0}\right] x\,dx$$

$$+\int_{c-h_f}^{c} b\ f'_c\left[1-\delta\frac{\varepsilon-\varepsilon_0}{\varepsilon_u-\varepsilon_0}\right] x\,dx$$

$$= \frac{5}{12} f'_c\ (b-b_w)\ A^2$$

$$+\ \frac{b-b_w}{2}\ f'_c\ (B-A)\left\{B+A-\frac{\delta}{\varepsilon_u-\varepsilon_0}\left[\frac{\varepsilon_c}{2c}\left(\frac{(B-A)^2}{3}+(B+A)^2\right)-\varepsilon_0(B+A)\right]\right\}$$

$$+\ b\ f'_c\ \frac{h_f}{2}\left\{c+B-\frac{\delta}{\varepsilon_u-\varepsilon_0}\left[\frac{\varepsilon_c}{2c}\left(\frac{1}{3}\ h_f^2+(c+B)^2\right)-\varepsilon_0(c+B)\right]\right\}$$

CMOM (cont'd)

I section (cont'd)

$\boxed{\varepsilon_c > \varepsilon_0}$ (cont'd)

for $\quad c\,\frac{\varepsilon_0}{\varepsilon_c} > c-h_f$

$$\int_0^c (\text{width})\cdot f_c\cdot x\,dx = (b-b_w)\,f_c'\int_0^{c-h_f}\left[2\,\frac{\varepsilon}{\varepsilon_0}-\left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right]x\,dx$$

$$+\ b\,f_c'\int_{c-h_f}^{c\frac{\varepsilon_0}{\varepsilon_c}}\left[2\,\frac{\varepsilon}{\varepsilon_0}-\left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right]x\,dx$$

$$+\ b\,f_c'\int_{c\frac{\varepsilon_0}{\varepsilon_c}}^{c}\left[1-\delta\,\frac{\varepsilon-\varepsilon_0}{\varepsilon_u-\varepsilon_0}\right]x\,dx$$

$$= \frac{b-b_w}{4}\,f_c'\,\frac{B^3}{A}\left[\frac{8}{3}-\frac{B}{A}\right]$$

$$+\ \frac{b}{2}\,f_c'\,(A-B)\left\{\frac{1}{A}\left[(A+B)^2+\frac{(A-B)^2}{3}\right]-\frac{1}{4A^2}\left[(A+B)^3+(A+B)(A-B)^2\right]\right\}$$

$$+\ b\,f_c'\,\frac{c^2}{2}(1-P)\left[(1+P)-\frac{\delta}{\varepsilon_u-\varepsilon_0}\left\{\frac{\varepsilon_c}{2}\left[\frac{1}{3}(1-P)^2+(1+P)^2\right]-\varepsilon_0(1+P)\right\}\right]$$

$$P = \frac{\varepsilon_0}{\varepsilon_c}$$

COM

computes the contribution to the total force, of the area stressed in compression. The procedure of integration is similar to CMOM's.

Rectangular section

for $0 < \varepsilon_c < \varepsilon_0$

$$\int_0^c f_c \, dx = \int_0^c f'_c \left[2\frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2 \right] dx$$

$$= c \, f'_c \left[\frac{\varepsilon_c}{\varepsilon_0} - \frac{1}{3}\left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2 \right]$$

for $\varepsilon_0 < \varepsilon_c < \varepsilon_u$

$$\int_0^c f_c \, dx = \int_0^{c\frac{\varepsilon_0}{\varepsilon_c}} f'_c \left[2\frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2 \right] dx$$

$$+ \int_{c\frac{\varepsilon_0}{\varepsilon_c}}^{c} f'_c \left[1 - \delta \frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0} \right] dx$$

$$= \frac{2}{3} f'_c \, c \frac{\varepsilon_0}{\varepsilon_c} + \frac{1}{2} f'_c \, c\left(1 - \frac{\varepsilon_0}{\varepsilon_c}\right)\left[2 - \delta \frac{\varepsilon_c - \varepsilon_0}{\varepsilon_u - \varepsilon_0} \right]$$

COMP (cont'd)

I section $A = c\,\frac{\varepsilon_0}{\varepsilon_c}$ $B = c - h_f$

$\boxed{\varepsilon_c < \varepsilon_0}$

$$\int_0^c (\text{width}).f_c\,dx = (b-b_w)\int_0^{c-h_f} f'_c\left[2\,\frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right] dx$$

$$+ \; b\int_{c-h_f}^{c} f'_c\left[2\,\frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right] dx$$

$$= (b-b_w)\,\frac{B^2}{A}\,f'_c\left(1 - \frac{B}{3A}\right)$$

$$+ \; b\,h_f\,f'_c\left\{\frac{c+B}{A} - \left(\frac{1}{2A}\right)^2\left[(c+B)^2 + \frac{h_f^2}{3}\right]\right\}$$

$\boxed{\varepsilon_c > \varepsilon_0}$

for $c\,\frac{\varepsilon_0}{\varepsilon_c} < c - h_f$

$$\int_0^c (\text{width}).f_c\,dx = (b-b_w)\int_0^{c\frac{\varepsilon_0}{\varepsilon_c}} f'_c\left[2\,\frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right] dx$$

$$+ \; (b-b_w)\int_{c\frac{\varepsilon_0}{\varepsilon_c}}^{c-h_f} f'_c\left[1 - \delta\,\frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0}\right] dx$$

$$+ \; b\int_{c-h_f}^{c} f'_c\left[1 - \delta\,\frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0}\right] dx$$

COMP (cont'd)

I section $\boxed{\varepsilon_c > \varepsilon_0}$

for $c\,\frac{\varepsilon_0}{\varepsilon_c} < c-h_f$ (cont'd)

$$= \frac{2}{3} f_c' \,(b-b_w)\, A + (b-b_w)(B-A) f_c' \left[1 - \frac{\delta}{\varepsilon_u-\varepsilon_0}\left(\frac{\varepsilon_c}{c}(B+A)-\varepsilon_0\right)\right]$$

$$+ b\, h_f\, f_c' \left[1 - \frac{\delta}{\varepsilon_u-\varepsilon_0}\left(\frac{\varepsilon_c}{c}\,(c+B) - \varepsilon_0\right)\right]$$

for $c\,\frac{\varepsilon_0}{\varepsilon_c} > c-h_f$

$$\int_0^c (\text{width}).f_c\, dx = (b-b_w)\int_0^{c-h_f} + b\int_{c-h_f}^{c\frac{\varepsilon_0}{\varepsilon_c}} f_c' \left[2\frac{\varepsilon}{\varepsilon_0} - \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right] dx$$

$$+ b\int_{c\frac{\varepsilon_0}{\varepsilon_c}}^{c} f_c' \left[1 - \delta\,\frac{\varepsilon - \varepsilon_0}{\varepsilon_u-\varepsilon_0}\right] dx$$

$$= (b-b_w) B f_c' \left[\frac{B}{A} - \frac{1}{3}\left(\frac{B}{A}\right)^2\right] + b f_c' (A-B) \left\{ \left(1+\frac{B}{A}\right) - \frac{1}{3}\left[1 + \frac{B}{A} + \left(\frac{B}{A}\right)^2\right] \right\}$$

$$+ b\, f_c' \,\frac{c}{2}\left(1 - \frac{\varepsilon_0}{\varepsilon_c}\right)\left[2 - \frac{\delta}{\varepsilon_u-\varepsilon_0}\,\frac{1}{2}\,(\varepsilon_c - 3\,\varepsilon_0)\right]$$

```
C PROGRAM TO ANALYZE THE MOMENT REDISTRIBUTION IN
C TWO-SPAN SYMMETRICALLY LOADED CONTINUOUS
C PARTIALLY PRESTRESSED CONCRETE BEAMS.
C CONCENTRATED SYMMETRICAL VERTICAL LOADS: LINEARLY VARYING
C STEEL TENDON PROFILE.
C
C MAIN PROGRAM
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),    RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
C INPUT OF BEAM, CROSS-SECTION, AND MATERIAL DATA
C PARAMETERS ARE
C   NUMBER     REFERENCE NUMBER OF THE BEAM
C   B          OVERALL WIDTH OF CROSS-SECTION IN INCHES
C   H          OVERALL HEIGHT OF CROSS-SECTION IN INCHES
C   EXL        TENDON ECCENTRICITY AT LOAD POINT IN INCHES
C   EXS        TENDON ECCENTRICITY AT SUPPORT IN INCHES
C   PRESF      APPLIED PRESTRESSING FORCE IN LBS
C   AREAP      TOTAL AREA IN SQ. INCHES, OF PRESTRESSING TENDON
C   SIGPU      GRADE OF HIGH-STRENGTH PRESTRESSING STEEL IN KSI
C   ELMOD      MODULUS OF ELASTICITY IN PSI OF HIGH-STRENGTH STEEL
C   CSTR       CONCRETE STRENGTH IN PSI
C   EPSO       CONCRETE STRAIN AT MAXIMUM STRESS (MODEL)
C   EPSU       CONCRETE STRAIN AT CRUSHING (MODEL)
C   RDFACT     REDUCTION FACTOR OF STRENGTH OF CONCRETE (MODEL)
C   SPAN       SPAN LENGTH IN INCHES BETWEEN TWO SUPPORTS
C   FRACL      FRACTION OF SPAN FROM END SUPPORT WHERE ONE OF THE
C                 CONCENTRATED LOADS IS APPLIED
C   ISECT      = 0 FOR A RECTANGULAR CROSS SECTION
C              = 1 FOR AN I-SHAPED CROSS SECTION
C   BW         WEB THICKNESS IN INCHES (I-SECTION)
C   HF         FLANGE HEIGHT IN INCHES (I-SECTION)
C
C
       READ(5,1000) NUMBER
 1000  FORMAT (I10)
       READ(5,2000) B, H, EXL, EXS
 2000  FORMAT (4F10.5)
       READ(5,2001) PRESF, AREAP, SIGPU, ELMOD
 2001  FORMAT (3F10.5, E10.5)
       READ(5,2002) CSTR, EPSO, EPSU, RDFACT
 2002  FORMAT (4F10.5)
       READ(5,2003) SPAN, FRACL, ISECT
 2003  FORMAT(2F10.5, I10)
       IF(ISECT .EQ. 0) GO TO 10
       READ(5,2004) BW, HF
 2004  FORMAT(2F10.5)
```

```
C
C ECHO CHECK OF INPUT DATA
C
      POS = SPAN*FRACL
      WRITE(6,500) NUMBER,POS
      IF (ISECT .EQ. O) WRITE(6,501) B, H
      WRITE(6,502) B, H, BW, HF
      WRITE(6,600) EXL, EXS, SPAN
      WRITE(6,700) CSTR, EPSO, EPSU, RDFACT
      WRITE(6,800) AREAP, PRESF, SIGPU, ELMOD
  500 FORMAT(50('*'),/'MOMENT-LOAD CURVE FOR THE TWO-SPAN',
     +   ' CONTINUOUS PRESTRESSED BEAM NUMBER ',I5,' LOADED',
     +  /'SYMMETRICALLY WITH TWO CONCENTRATED VERTICAL LOADS',
     +   F7.2,' INCHES FROM THE END SUPPORTS.')
  501 FORMAT(/'RECTANGULAR CROSS-SECTION, WIDTH = ',F7.2,
     +     ' IN.,      HEIGTH = ',F7.2,' IN.')
  502 FORMAT(/'I-SHAPED SECTION, WIDTH = ',F7.2,
     +     ' IN.,   HEIGHT = ',F7.2,' IN.',
     +   /22X,'WEB ',F7.2,' IN.,       FLANGE ',F7.2,' IN.')
  600 FORMAT(//'TENDON ECCENTRICITY = ',F10.2,' IN. AT LOAD POINT',
     +   /22X,F10.2,' IN. AT SUPPORT.',
     +   /'SPAN BETWEEN SUPPORTS ',F10.2,' IN.')
  700 FORMAT(//'CONCRETE STRENGTH ',F15.2,' PSI ,',
     +        /'           STRAIN EPSO ',F15.5,
     +        /'                  EPSU ',F15.5,
     +        /'           REDUCTION FACTOR ',F10.2)
  800 FORMAT(//'TOTAL TENDON AREA ',F15.4,' SQ. IN.',
     +        /'APPLIED PRESTRESSING',F13.2,' LBS',
     +        /'GRADE               ',F13.2,' KSI',
     +        /'MOD. OF ELASTICITY  ',E13.3,' PSI',
     + /50('*'),//'THE CRACKING MOMENTS AT SUPPORT AND AT THE',
     +   ' LOAD POINT ARE, RESPECTIVELY:')
C
C  CALCULATIONS OF SECTION PROPERTIES
C
   10 FCU = CSTR/(.8 + .0001*CSTR)
      RATMOD = ELMOD/(57000.*SQRT(CSTR))
      ALPHA = (B-BW)*HF + RATMOD*AREAP
      E1 = PRESF/(AREAP*ELMOD)
C
C  FCU       AVERAGE CONCRETE STRESS (FROM WARWARUK, REF. 28)
C  RATMOD    RATIO OF STEEL MODULUS / CONCRETE MODULUS
C  ALPHA     COEFFICIENT TO BE USED IN FUNCTION "AXNT"
C  E1        INITIAL STEEL STRAIN (STAGE 1)
C
       IF (ISECT .EQ. O) GO TO 12
C
C  I         MOMENT OF INERTIA OF CROSS-SECTION
C  AREAC     AREA OF CONCRETE
C  R2        SQUARED RADIUS OF GYRATION
C  GYRL,GYRS TERMS FROM EQ. 3.3
C  E2L,E2S   STEEL STRAIN AT STAGE 2
C
```

```
      I = 2.*(B*HF**3/12.) + .5*B*HF*(H-HF)**2
     +       + BW*(H-2.*HF)**3/12.
      AREAC = 2.*B*HF + BW*(H-2.*HF)
      R2 = I/AREAC
      GYRL = EXL*EXL/R2
      GYRS = EXS*EXS/R2
      GO TO 13
   12 AREAC = B*H
      GYRL = 12.*(EXL/H)*(EXL/H)
      GYRS = 12.*(EXS/H)*(EXS/H)
   13 E2L = PRESF*(1+GYRL)/(AREAC*57000.*SQRT(CSTR))
      E2S = PRESF*(1+GYRS)/(AREAC*57000.*SQRT(CSTR))
C
C  CALCULATION OF ULTIMATE MOMENTS (ITERATIVE PROCEDURE)
C  X(1)      FIRST GUESS OF LOCATION OF NEUTRAL AXIS
C  X(2)      FIRST GUESS OF STEEL STRESS
C
      X(1) = (EXL+H/2.)/10.
      X(2) = SIGPU - 30.
      UMOML = UMOM(1, EXL)
      UCURVL = (PREL(X(2)) + EPSU)/(EXL+H/2.)
C
      X(1) = (EXS+H/2.)/10.
      X(2) = SIGPU - 30.
      UMOMS = UMOM(2,EXS)
      UCURVS = (PREL(X(2)) + EPSU)/(EXS+H/2.)
C
C  CALCULATION OF CRACKING MOMENTS
C       CTEN    MODULUS OF RUPTURE OF CONCRETE
C       ETEN    CONCRETE STRAIN AT TENSILE STRENGTH.
C
      CTEN = 7.5*SQRT(CSTR)
      ETEN = 7.5/57000.
        CSMOM = CRITIM(EXS)
        CLMOM = CRITIM(EXL)
C
C  EVALUATION OF MOMENT RATIOS
C   ELAS     ELASTIC MOMENT RATIO
C   PROVID  MOMENT CAPACITY RATIO PROVIDED
C
          ELAS = (1.-FRACL)*(2.+FRACL)/(1.+FRACL)
          PROVID = UMOML/UMOMS
          IF (ABS(ELAS-PROVID) .LE. 0.1) GO TO 80
          IF (ELAS*CSMOM - CLMOM) 50, 50, 60
C
C  EVALUATION OF THE FIRST CRACKING CONDITIONS
C
   50     WRITE (6,6000)
          CALL NEWTN (0, ELAS*CSMOM, CSMOM, 1)
          GO TO 70
   60     WRITE (6,6001)
          CALL NEWTN (0, CLMOM, CLMOM/ELAS, 1)
```

```
C
C   LOADING STAGES BETWEEN FIRST CRACKING AND ULTIMATE
C
  70    DO 51 KS = 1,5
        ADD = 0.0
        DO 51 KSS = 1,5
        SM = KS*(1.+ADD)*CSMOM
        PM = PROVID*SM
        IF ((SM.GT.UMOMS) .OR. (PM.GT.UMOML)) GO TO 30
        WRITE(6,4000) SM, PM
        ADD = ADD + 0.2
  51    CALL NEWTN (0, PM, SM, 2)
C
C   ULTIMATE STAGE
C
  30 WRITE(6,3000) UMOMS, UCURVS, UMOML, UCURVL
     CALL NEWTN (0, UMOML, UMOMS, 2)
     STOP
4000 FORMAT(/50('-'),/' THE NEXT ASSUMED SET OF MOMENTS IS:',
   + /15X,'AT SUPPORT : ',E13.6,' ,AND AT LD POINT : ',
   + E13.6,' LB-IN')
3000 FORMAT (/,50('*'),/'ULTIMATE STAGE: ASSUMED',
   +  /'  SUPPORT    MOMENT         = ',E13.6,' LB-IN.',
   +   /'               CURVATURE    = ',E13.6,
   +   /'  LOAD POINT MOMENT         = ',E13.6,' LB-IN.',
   +   /'               CURVATURE    = ',E13.6)
6000 FORMAT(/,50('-'),/'CRACKING OCCURS FIRST AT SUPPORT')
6001 FORMAT(/,50('-'),/'CRACKING OCCURS FIRST AT LOAD POINT')
     STOP
C
C   IF THE MOMENT CAPACITY RATIO PROVIDED IS EQUAL TO THE ELASTIC
C   ONE, THEN NO REDISTRIBUTION OF MOMENT OCCURS. PRINT MESSAGE.
C
  80        WC = 2.*(CLMOM+CSMOM*FRACL)/(FRACL*(1.-FRACL)*SPAN)
            WU = 2.*(UMOML+UMOMS*FRACL)/(FRACL*(1.-FRACL)*SPAN)
      WRITE (6,8000) PROVID,ELAS, WC,CSMOM,CLMOM, WU,UMOMS,UMOML
8000   FORMAT (/'CAPACITY PROVIDED, ',F7.2,' ACCORDING TO',
    +         ' ELASTIC RATIO ',F7.2,' :* NO REDISTRIBUTION. *',
    +        //'CRACKING : OCCURS AT LOAD = ',E13.6,' LBS',
    +          /'           SUPPORT MOMENT = ',E13.6,' LB-IN.',
    +          /'           LD PT MOMENT   = ',E13.6,' LB-IN.',
    +        //'ULTIMATE : OCCURS AT LOAD = ',E13.6,' LBS',
    +          /'           SUPPORT MOMENT = ',E13.6,' LB-IN.',
    +          /'           LD PT MOMENT   = ',E13.6,' LB-IN.')
      STOP
      END
C
C
```

```
C
C
C
      SUBROUTINE NEWTN (I, PLMOM, SPMOM, KANG)
C  SUBROUTINE THAT PERFORMS THE NEWTON-RAPHSON ITERATIVE
C  METHOD TO VARY EITHER THE LOAD POINT MOMENT OR THE
C  SUPPORT MOMENT UNTIL SATISFACTION OF THE COMPATIBILITY
C  EQUATION.
C  PARAMETERS ARE -
C    I        = 0 IF EACH INTERMEDIATE ITERATION IS TO BE PRINTED
C             = 1 IF NOT
C    PLMOM    INITIAL ASSUMED MOMENT AT LOAD POINT
C    SPMOM    INITIAL ASSUMED MOMENT AT SUPPORT
C    KANG     = 1 INDICATES ELASTIC COMPATIBILITY EQUATION
C             = 2 INDICATES NON-LINEAR COMPATIBILITY EQUATION
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +    RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +    ANGLEA, ANGLEB
C
C  SET TOLERANCE VALUES FOR X, F(X) TO TERMINATE ITERATIONS.
C    NLIM :    LIMIT TO NUMBER OF ITERATIONS
C
      XTOL = 9000.
      FTOL = .0001
      IF (KANG .EQ. 2) FTOL = 3.
      NLIM = 10
      DELTM = 10000.
      LOGICAL PRINT
      PRINT = .TRUE.
      IF (I .NE. 0)   PRINT = .FALSE.
      CALL SEGM (PLMOM, SPMOM, KANG)
      FX = COMPAT
        BB = 1 - FRACL
        DO 20 J = 1, NLIM
C
C  ITERATIONS ARE PERFORMED ON THE SUPPORT MOMENT FIRST
C
        CALL SEGM (PLMOM, SPMOM+DELTM, KANG)
        FXD = COMPAT
        FDER = (FXD - FX)/DELTM
        DELX = FX/FDER
        IF (ABS(DELX) .GE. 10000.) DELX     10000.
        SPMOM2 = SPMOM - DELX
      SPMOM1 = SPMOM
        CALL SEGM (PLMOM, SPMOM2, KANG)
        FX = COMPAT
      KJ = 0
      IF (.NOT. PRINT) GO TO 9
      WRITE (6,199) J, KJ, PLMOM, SPMOM2, FX
    9 IF (KJ .EQ. 0) SPMOM = SPMOM2
      IF (ABS(DELX) .LE. XTOL) GO TO 60
      IF (ABS(FX) .LE. FTOL) GO TO 70
```

```
C
C  IF TOLERANCE IS NOT MEET, ITERATIONS ARE DONE ON THE
C  LOAD POINT MOMENT.
C
      SPMOM = SPMOM1+ 1000.
      DO 20 KJ = 1, 8
        PLMOMD = PLMOM + DELTM
        CALL SEGM (PLMOMD, SPMOM, KANG)
        FXD = COMPAT
        FDER = (FXD - FX)/DELTM
        DELX = FX/FDER
        IF (ABS(DELX) .GE. 10000.) DELX = -10000.
        PLMOM = PLMOM - DELX
        CALL SEGM (PLMOM, SPMOM, KANG)
        FX = COMPAT
   80 IF (.NOT. PRINT) GO TO 5
      WRITE (6,199) J, KJ, PLMOM, SPMOM, FX
    5 IF (ABS(DELX) .LE. XTOL) GO TO 60
      IF (ABS(FX) .LE. FTOL) GO TO 70
   20 CONTINUE
C
C  WHEN LOOP IS NORMALLY COMPLETED, NLIM IS EXCEEDED..
C
      WRITE (6,200) NLIM, PLMOM, SPMOM, FX
      RETURN
C
C  THIS SECTION RETURNS AFTER MEETING TOLERANCE ON XTOL
C  (MOMENT DISTRIBUTION IS SATISFACTORY)
C
   60 WRITE (6,202) J, KJ, PLMOM, SPMOM, FX
      GO TO 300
C
C  THIS SECTION RETURNS AFTER MEETING F(X) TOLERANCE
C  (COMPATIBILITY EQUATION IS SATISFIED)
C
   70 WRITE (6,203) J, KJ, PLMOM, SPMOM, FX
C
C  CALCULATION OF LOAD CORRESPONDING TO FINAL DISTRIBUTION
C  OF MOMENTS
C
  300   W = 2.*(PLMOM+SPMOM*FRACL)/(FRACL*BB*SPAN)
        WRITE(6,205) W
  205 FORMAT(//5X,'THE TOTAL APPLIED LOAD IS= ',E13.6,' LBS')
  199 FORMAT (/5X,'AT ITER. ',I4,' ',I4,' THE MOMENTS AT ',
     + 'LD POINT = ',E13.6,' AND AT SUPPORT = ',E13.6,
     +  ' LB-IN.',/30X,'*** COMPATIBILITY EQUATION = ', E13.6)
  200 FORMAT (//5X,'TOLERANCE NOT MET. AFTER ', I4,' ITER.',
     + ' LD PT. MOM. = ', E13.6,' SUP = ', E13.6,
     + ' LB-IN.',/30X,'*** COMPATIBILITY EQ. = ',E13.6)
  202 FORMAT (//5X,'MOM. TOLERANCE MET IN ', I4,' ',I4,' ITER.',
     +  ' ,LD PT. MOM. = ', E13.6,' ,SUP = ', E13.6,
     +  ' LB-IN.',/30X'*** COMPATIBILITY EQ. = ',E13.6)
  203 FORMAT (//5X,'COMPAT. TOLERANCE MET IN ', I4,' ',I4,
     +  ' ITER., LD PT. MOM. = ', E13.6,' ,SUP = ', E13.6,
     +  ' LB-IN.',/30X,'*** COMPATIBILITY EQ. = ',E13.6)
       RETURN
       END
C
C
```

```
C
C
C
      SUBROUTINE SEGM (PLMOM, SPMOM, KANG)
C  SUBROUTINE DISCRETIZING THE BEAM INTO SEGMENTS AS SHOWN IN
C  PAGE 82. LOCATION WITH RESPECT TO THE END SUPPORT,
C  TENDON ECCENTRICITY, VALUE OF MOMENT OF EACH SEGMENT IS
C  EVALUATED.
C  THE SUBROUTINE EACH IS THEN CALLED.
C  PARAMETERS ARE IDENTICAL TO THE SUBROUTINE NEWTN'S.
C
      DIMENSION X(2), F(2),  ZL(6), ZR(6), ZRS(6)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60), RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
C  INITIALIZATION OF CUMULATIVE VARIABLES.
C       COMPAT     VALUE OF COMPATIBILITY EQUATION
C       ANGLEA     ANGLE AT END SUPPORT
C       ANGLEB     ANGLE AT ONE SIDE OF CENTRE SUPPORT
C
      COMPAT = 0.
      ANGLEA = 0.
      ANGLEB = 0.
C
C  PORTION OF BEAM BETWEEN END SUPPORT AND LOAD APPLICATION POINT.
C    ZL      DISTANCE OF THE SEGMENT CENTER TO THE END SUPPORT
C    EZ      ECCENTRICITY OF STEEL TENDON
C    DISTM   VALUE OF MOMENT
C    SLENGF  SEGMENT LENGTH
C
      ZL(1) = FRACL*SPAN/4.
      ZL(2) = 2.2*ZL(1)
      ZL(3) = 2.6*ZL(1)
      ZL(4) = 3.0*ZL(1)
      ZL(5) = 3.4*ZL(1)
      ZL(6) = 3.8*ZL(1)
        DO 200 IDIST=1,6
        EZ = EXL*ZL(IDIST)/(4.*ZL(1))
        DISTM = PLMOM*ZL(IDIST)/(4.*ZL(1))
        SLENGF = 2.*ZL(1)
        IF (IDIST .EQ. 1) GO TO 200
        SLENGF = 0.4*ZL(1)
  200   CALL EACH(ZL(IDIST), EZ, DISTM, SLENGF, KANG)
C
```

```
C
C  PORTION OF BEAM BETWEEN LOAD APPLICATION POINT AND THE POINT
C  WHERE THE TENDON ECCENTRICITY MEETS THE CROSS SECTION
C  CENTER OF GRAVITY.
C     ZR       DISTANCE OF SEGMENT TO LOAD APPLICATION POINT
C     ER       TENDON ECCENTRICITY
C     DISTM    VALUE OF MOMENT IN SEGMENT
C
      VAL = (1-FRACL)*SPAN
      ZR(1) = VAL*EXL/(20.*(EXL+EXS))
      ZR(2) = 3.*ZR(1)
      ZR(3) = 5.*ZR(1)
      ZR(4) = 7.*ZR(1)
      ZR(5) = 9.*ZR(1)
      ZR(6) = 15.*ZR(1)
        DO 300 IDIST=1,6
        ER = EXL - EXL*ZR(IDIST)/(20.*ZR(1))
        DISTM = PLMOM - ZR(IDIST)*(PLMOM+SPMOM)/VAL
        SLENGF = (2./3.)*ZR(6)
        IF (IDIST .EQ. 6) GO TO 300
        SLENGF = ZR(6)/7.5
  300   CALL EACH(4.*ZL(1)+ZR(IDIST), ER, DISTM, SLENGF, KANG)
C
C  REMAINING PORTION OF BEAM (UP TO CENTRE SUPPORT)
C    ZRS   DISTANCE OF SEGMENT TO THE LOAD APPLICATION POINT
C    ERS   TENDON ECCENTRICITY
C
      ZRS(1) = VAL*EXS/(4.*(EXL+EXS))
      ZRS(2) = 2.2*ZRS(1)
      ZRS(3) = 2.6*ZRS(1)
      ZRS(4) = 3.0*ZRS(1)
      ZRS(5) = 3.4*ZRS(1)
      ZRS(6) = 3.8*ZRS(1)
        PINF = EXL*VAL/(EXL+EXS)
        DO 400 IDIST=1,6
        ERS = -EXS*ZRS(IDIST)/(4.*ZRS(1))
        DISTM = PLMOM - (PINF + ZRS(IDIST))*(PLMOM + SPMOM)/VAL
        SLENGF = 2.*ZRS(1)
        IF (IDIST .EQ. 1) GO TO 400
        SLENGF = 0.4*ZRS(1)
  400   CALL EACH(4.*ZL(1)+PINF+ZRS(IDIST), ERS,DISTM,SLENGF,KANG)
C
C  THIS SECTION EVALUATES THE COMPATIBILITY EQUATION AFTER
C  THE CONTRIBUTION OF ALL THE SEGMENTS TO THE TOTAL ANGLES
C  HAS BEEN ACCUMULATED.
C
      IF (KANG.EQ.2) COMPAT = (180./ 3.141593) * (FRACL *
     +    (ABS(ANGLEA) + ABS(ANGLEB)) - ABS(ANGLEB))
      RETURN
      END
C
```

```
C
C
C
C
      SUBROUTINE EACH(Z, EZ, DISTM, SLENGF, KANG)
C
C  SUBROUTINE FOR CALCULATION OF CONTRIBUTION OF EACH SEGMENT
C  CURVATURE TO THE TOTAL ANGLES IN THE COMPATIBILITY EQUATIONS.
C  PARAMETERS ARE -
C    Z       DISTANCE OF CENTRE OF SEGMENT TO THE END SUPPORT
C    EZ      ECCENTRICITY OF SEGMENT
C    DISTM   VALUE OF MOMENT
C    SLENGF  LENGTH OF SEGMENT
C    KANG    =1 FOR ELASTIC COMPATIBILITY EQUATION
C            =2 FOR NON-LINEAR COMPATIBILITY EQUATION
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
C  IF THE MOMENT IN SEGMENT IS SMALLER THAN THE CRACKING
C  MOMENT CORRESPONDING TO THE ECCENTRICITY, THEN ITS
C  ACTUAL MOMENT IS EQUAL TO A PORTION OF THE CRACKING MOMENT.
C
         CRMOM = (DISTM/ABS(DISTM))*CRITIM(EZ)
         IF (ABS(DISTM) .LT. ABS(CRMOM)) GO TO 5000
C
C  GENERAL CRACKED SECTION ANALYSIS AS DESCRIBED IN
C  CHAPTER 3. EVALUATION OF THE COEFFICIENTS TO BE USED
C  IN THE EQUATION OF NEUTRAL AXIS.
C  IF THE SEGMENT SECTION IS CRACKED, THE PRESTRESSING
C  STEEL STRESS IS EVALUATED USING THE LINEAR BOND VARIATION
C  CONCEPT. THE AVERAGE CURVATURE IS CALCULATED BY INTEGRATING
C  THE CURVATURES THROUGH SUBROUTINE ROMB.
C
      BETA = (B-BW)*HF*HF/2. + RATMOD*AREAP*(ABS(EZ)+H/2.)
      GAMMA = (B-BW)*HF**3/3. + RATMOD*AREAP*(ABS(EZ)+H/2.)**2
      AXF = AREAP*ELMOD*(E1+E2X)
C
C  CONDITION AT THE CRACK, CRACKING MOMENT HAS BEEN JUST REACHED
C
      EXTMOM = ABS(CRMOM) - AXF*(ABS(EZ)+H/2.)
      Y = H/2.
      CALL AFCRAK(Y, EZ, 1, LENGTH)
        FSA = X(2)/1000.
C
C  CONDITION AT THE CRACK, CRACKING MOMENT EXCEEDED
C
      EXTMOM = ABS(DISTM) - AXF*(ABS(EZ)+H/2.)
      Y = H/2.
      CALL AFCRAK(Y, EZ, 1, LENGTH)
      FSCR = X(2)/1000.
```

```
C
C  ETOP IS THE DISTANCE FROM THE TOP OF THE SECTION TO THE
C  "FICTITIOUS" FORCE
C
         ETOP = (EXTMOM - AXF*ABS(EZ))/AXF - H/2.
C
C  THE PRESTRESSING STEEL STRESS VARIATION IS EVALUATED
C  AT 60 POINTS EVENLY SPACED WITHIN 2 HYPOTHETICAL
C  CRACKS IN THE CONCRETE.
C
         DLB = 0.
         DO 80 LENGTH=1,60
         DLB = DLB + 0.0125
         FS(LENGTH) = FSCR -2.*(FSA-FSB)*(DLB-.5*DLB*DLB)
         Y = H/2.
         CALL AFCRAK(Y, EZ, 2, LENGTH)
         STRAIN(LENGTH) = X(1) + PREL(FS(LENGTH))
   80    CONTINUE
         CALL ROMB(60, EZ, DISTM)
         GO TO 100
 5000    RESULT = (DISTM/ABS(CRMOM))*CURVCT
C
C  THE AVERAGE CURVATURE FROM THE CRACKED SEGMENTS,
C  OR THE CURVATURE FROM THE UNCRACKED SEGMENTS
C  IS ASSIGNED TO THE VARIABLE "RESULT"
C
  100 IF (KANG .NE.2) COMPAT = COMPAT + RESULT*Z/SPAN
      ANGLEA = ANGLEA + RESULT*(SPAN-Z)/SPAN
      ANGLEB = ANGLEB + RESULT*Z/SP N
      RETURN
      END
C
```

```
C
C
C
      FUNCTION CRITIM (EZ)
C
C  FUNCTION FOR THE EVALUATION OF THE CRACKING MOMENT OF
C  EACH SEGMENT. THE SINGLE PARAMETER IS THE TENDON
C  ECCENTRICITY IN THE SECTION.
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),      RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +    RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +    ANGLEA, ANGLEB
C
C  SET THE X INITIAL GUESSES
C
      X(1) = 0.0005
      X(2) = 100.
C
C  SECTION PROPERTIES
C         GYRZ : TERM OF EQUATION 3.3
C         E2X  : STEEL STRAIN AT STAGE 2
C
      IF (ISECT .EQ. O) GO TO 21
      GYRZ = ABS(EZ)**2/R2
      GO TO 23
   21 GYRZ = 12.*(ABS(EZ)/H)*(ABS(EZ)/H)
   23 E2X = EPSO*(1-SQRT(1-PRESF/(AREAC*CSTR)))*(1+GYRZ)
C
C  SIMULTANEOUS EQUATIONS OF EQUILIBRIUM AND STRAIN
C  COMPATIBILITY ARE SOLVED THROUGH SUBROUTINE "NLSYST".
C  CASE OF THE CRACKING CONDITIONS.
C
          CALL NLSYST (2, 1, 3, EZ, LENGTH)
C
C  RESULTING STEEL STRESS AND NEUTRAL AXIS
C
          FSB = X(2)
          C = H*X(1)/(ETEN+X(1))
          IF(ISECT .EQ. O) GO TO 24
          IF(H-C .LE. HF) GO TO 24
          CRITIM = ABS(CMOM(C,X(1))) + BW*CTEN*(H-C)*(H-C)/3.
     +              + (B-BW)*CTEN*(H-C-HF/2.)*HF
     +              + AREAP*X(2)*1000.*(ABS(EZ)+H/2.-C)
          GO TO 25
   24     CRITIM = ABS(CMOM(C,X(1))) + B*CTEN*(H-C)*(H-C)/3. +
     +                          A[illegible]P*X(2)*1000.*(ABS(EZ)+H/2.-C)
   25     CURVCT = (X(1)+ETEN)/H
      IF ((ABS(EZ) .EQ. EXL) .OR. [illegible](EZ) .EQ. EXS)) GO TO 500
      RETURN
  500 WRITE(6,3000) CRITIM, CURVCT
 3000 FORMAT (/'  MOMENT          =          [illegible]' LB-IN.',
     +       /'  CURVATURE       =
      RETURN
      END
C
C
```

```
C
C
C
      FUNCTION UMOM(MCASE, ECC)
C
C  FUNCTION FOR CALCULATING THE ULTIMATE MOMENT.
C  PARAMETERS ARE -
C    MCASE     = 1 FOR THE EVALUATION OF LOAD POINT MOMENT
C              = 2 FOR THE EVALUATION OF SUPPORT MOMENT
C    ECC       ECCENTRICITY OF PRESTRESSING STEEL TENDON
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
C  VALUES SET FOR USE IN SUBROUTINE "NLSYST"
C
      N = 2
      DELTA(1) = 0.001
      DELTA(2) = 2.
      EZ = 1.
      LENGTH = 1
      CALL NLSYST (N, 1, MCASE, EZ, LENGTH)
C
C  CHECK THE VALUE OF NET REINFORCEMENT INDEX
C
        OMEGA = AREAP*X(2)*1000./(B*(ECC+H/2.)*CSTR)
      IF (X(1) .LT. 0.) GO TO 70
      IF (ISECT .EQ. 0) GO TO 40
      IF (X(2)*1000.*AREAP/(FCU*B) .LE. HF) GO TO 40
C
C  DIFFERENT FORMULAS ARE TO BE USED WHEN THE REINFORCEMENT
C  INDEX EXCEEDS 0.3
C
        IF (OMEGA - 0.30) 301,301,300
  300   WRITE (6,4000) OMEGA
        UMOM = (CSTR*BW*(ECC+H/2)**2)/4.
     +        + 0.85*CSTR*(B-BW)*HF*(ECC+H/2.-HF/2.)
        RETURN
  301 ASR = AREAP - FCU*(B-BW)*HF/(X(2)*1000.)
      UMOM = (AREAP-ASR)*X(2)*1000.*(ECC+H/2.-.42*X(1))
     +          + ASR*X(2)*1000.*(ECC+H/2.-HF/2.)
      RETURN
   40 IF (OMEGA - 0.30) 201,201,200
  200   WRITE (6,4000) OMEGA
        UMOM =(CSTR*B*(ECC+H/2.)**2)/4.
        RETURN
  201 UMOM = AREAP*X(2)*1000.*(ECC+H/2.-.42*X(1))
      RETURN
   70 WRITE (6,7000) X(1)
 4000 FORMAT (/'OVERREINFORCED SECTION ***',
     +        /'****OMEGA = ',E13.5,' GT. 0.30')
 7000 FORMAT (/' NEGATIVE N.A. = ',E12.5,' (ULTIM. STAGE)')
      STOP
      END
```

```
C
C
C
      SUBROUTINE NLSYST (N, I, MCASE, EZ, LENGTH)
C
C  THIS SUBROUTINE SOLVES A SYSTEM OF N NON-LINEAR EQUATIONS BY
C  NEWTON'S METHOD. THE PARTIAL DERIVATIVES OF THE FUNCTIONS ARE
C  ESTIMATED BY DIFFERENCE QUOTIENTS WHEN A VARIABLE IS PERTUBED
C  BY AN AMOUNT EQUAL TO DELTA (DELTA IS ADDED).
C  THIS IS DONE FOR EACH VARIABLE IN EACH FUNCTION. INCREMENTS
C  TO IMPROVE THE ESTIMATES FOR THE X-VALUES ARE COMPUTED FROM
C  A SYSTEM OF EQUATIONS USING SUBROUTINE ELIM.
C  PARAMETERS ARE -
C    FCN        SUBROUTINE THAT COMPUTES VALUES OF THE FUNCTIONS.
C    N          THE NUMBER OF EQUATIONS
C    MAXIT      LIMIT TO THE NUMBER OF ITERATIONS THAT WILL BE USED
C    X          ARRAY TO HOLD THE X VALUES. INITIALLY THIS ARRAY
C               HOLDS THE INITIAL GUESSES. IT RETURNS THE FINAL
C               VALUES.
C    F          AN ARRAY THAT HOLDS VALUES OF THE FUNCTIONS
C    DELTA      A SMALL VALUE USED TO PERTURB THE X VALUES SO
C               PARTIAL DERIVATIVES CAN BE COMPUTED BY DIFFERENCE
C               QUOTIENT
C    XTOL       TOLERANCE VALUE FOR CHANGE IN X VALUES TO STOP
C               ITERATIONS. WHEN THE LATEST CHANGE IN ANY X
C               MEETS XTOL, THE SUBROUTINE TERMINATES.
C    FTOL       TOLERANCE VALUE ON F TO TERMINATE. WHEN THE
C               LATEST F VALUE IS LESS THAN FTOL, SUBROUTINE
C               TERMINATES.
      DIMENSION X(2), F(2), A(2,3), XSAVE(2),FSAVE(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),      RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
      MAXIT = 20
      XTOL = 0.0001
      FTOL = 0.0005
      LOGICAL PRINT
C
C  CHECK VALIDITY OF VALUE OF N
C
      IF (N.LT.2 .OR. N.GT.3) GO TO 999
      PRINT = .TRUE.
C
C  I INDICATES IF INTERMEDIATE RESULTS ARE TO BE PRINTED
C
      IF (I.NE.O) PRINT = .FALSE.
```

```
C
C  BEGIN ITERATIONS
C  SAVE X VALUES, THEN GET F VALUES
C
      NP= N + 1
      DO 100 IT= 1,MAXIT
        DO 10 IVBL= 1,N
          XSAVE(IVBL)= X(IVBL)
   10   CONTINUE
        CALL FCN(MCASE, EZ, LENGTH)
C
C  TEST F VALUES AND SAVE THEM
C
        ITEST= 0
        DO 20 IFCN= 1,N
          IF (ABS(F(IFCN)).GT.FTOL) ITEST= ITEST + 1
          FSAVE(IFCN)= F(IFCN)
   20   CONTINUE
C
C  PRINT CURRENT VALUES IF PRINT IS .TRUE.
        IF(.NOT.PRINT) GO TO 30
          WRITE(6,1000)IT, X
 1000     FORMAT(/' AFTER ITER. ',I3,
     +           ' X AND F VALUES ARE',/,10E13.6)
          WRITE(6,1001)F
 1001     FORMAT(/, 10E13.6)
C
C  SEE IF FTOL IS MET. IF NOT, CONTINUE. IF SO, RETURN.
C
   30   IF(ITEST.NE.0) GO TO 35
        RETURN
C
C  THIS DOUBLE LOOP COMPUTES THE PARTIAL DERIVATIVES OF EACH
C  FUNCTION FOR EACH VARIABLE AND STORES THEM IN A COEFFICIENT
C  ARRAY
C
   35   DO 50 JCOL=1,N
          X(JCOL)= XSAVE(JCOL) + DELTA(JCOL)
          CALL FCN(MCASE, EZ, LENGTH)
          DO 40 IROW=1,N
            A(IROW,JCOL)= (F(IROW) - FSAVE(IROW))/DELTA(JCOL)
   40     CONTINUE
C
C  RESET X VALUES FOR NEXT COLUMN OF PARTIALS
C
        X(JCOL)= XSAVE(JCOL)
   50   CONTINUE
C
C  PUT NEGATIVE VALUES OF F AS R.H.S. AND CALL "ELIM"
C
        DO 60 IROW= 1,N
          A(IROW,NP)= -FSAVE(IROW)
   60   CONTINUE
      CALL ELIM(A, N, NP, 2)
```

```
C
C  CHECK IF COEFFICIENT MATRIX IS NOT TOO ILL-CONDITIONED
C
        DO 70 IROW= 1,N
          IF(ABS(A(IROW,IROW)).LE.1.E-5) GO TO 998
   70   CONTINUE
C
C  APPLY THE CORRECTIONS TO THE X VALUES, ALSO SEE IF XTOL
C  IS MET.
C
        ITEST= 0
        DO 80 IVBL=1,N
          X(IVBL)= XSAVE(IVBL) + A(IVBL,NP)
          IF(ABS(A(IVBL,NP)).GT.XTOL) ITEST= ITEST +1
   80   CONTINUE
C
C  IF XTOL IS MET, PRINT LAST VALUES AND RETURN, ELSE DO
C  ANOTHER ITERATION
C
      IF(ITEST.EQ.0) GO TO 997
  100 CONTINUE
C
C  MAXIT ITERATIONS HAVE BEEN DONE.
C
      RETURN
C
C  XTOL IS MET. PRINT LAST VALUES.
C
  997 IF(.NOT.PRINT) GO TO 110
      WRITE (6,1002)IT,X
 1002 FORMAT(/,' AFTER ITER. ',I3,
     +       ' X VALUES (MEETING XTOL) ARE',/,10F13.5)
  110 RETURN
C
C  PARTIALS FORM A NEARLY SINGULAR MATRIX. PRINT MESSAGE.
C
  998 WRITE(6,1003)
 1003 FORMAT(/' CANNOT SOLVE SYSTEM. MATRIX NEARLY SINGULAR ')
      STOP
C
C  NUMBER OF EQUATIONS IS INVALID. PRINT MESSAGE.
C
  999 WRITE(6,1004)N
 1004 FORMAT(/'NUMBER OF EQNS PASSED TO NLSYST IS INVALID.',
     +' MUST BE 2<N<3. VALUE WAS ',I3)
      STOP
      END
C
C
```

```
C
C
C
      SUBROUTINE ELIM(AB, N, NP, NDIM)
C
C  THIS SUBROUTINE SOLVES A SET OF LINEAR EQUATIONS.
C  THE GAUSS ELIMINATION METHOD IS USED, WITH PARTIAL PIVOTING.
C  MULTIPLE R.H.S. ARE PERMITTED, THEY SHOULD BE SUPPLIED
C  AS COLUMNS THAT AUGMENT THE COEFFICIENT MATRIX.
C  PARAMATERS ARE -
C    AB      COEFFICIENT MATRIX AUGMENTED WITH R.H.S. VECTORS
C    N       NUMBER OF EQUATIONS
C    NP      TOTAL NUMBER OF COLUMNS IN THE AUGMENTED MATRIX
C    NDIM    FIRST DIMENSION OF MATRIX AB IN THE CALLING PROGRAM.
C  THE SOLUTION VECTOR(S) ARE RETURNED IN THE AUGMENTATION COLUMNS
C  OF AB.
C
      DIMENSION AB(NDIM,NP)
C
C  BEGIN THE REDUCTION
C
      NM1= N - 1
      DO 35 I= 1,NM1
C
C  FIND THE ROW NUMBER OF THE PIVOT ROW. INTERCHANGE ROWS TO PUT
C  THE PIVOT ELEMENT ON THE DIAGONAL
C
        IPVT= I
        IP1= I + 1
        DO 10 J= IP1, N
          IF(ABS(AB(IPVT,I)).LT.ABS(AB(J,I))) IPVT = J
   10   CONTINUE
C
C  CHECK IF THE PIVOT ELEMENT IS NOT TOO SMALL. IF SO PRINT
C  A MESSAGE AND RETURN.
C
        IF((ABS(AB(IPVT,I)).LT..00001)) GO TO 99
C
C  INTERCHANGE, EXCEPT IF THE PIVOT ELEMENT IS ALREADY ON THE
C  DIAGONAL.
C
        IF(IPVT.EQ.I) GO TO 25
        DO 20 JCOL= I,NP
          SAVE= AB(I,JCOL)
          AB(I,JCOL)= AB(IPVT,JCOL)
          AB(IPVT,JCOL)= SAVE
   20   CONTINUE
```

```
C
C   REDUCE ALL ELEMENTS BELOW THE DIAGONAL IN THE I-TH ROW.
C   CHECK FIRST TO SEE IF A ZERO ALREADY PRESENT. IF SO,
C   CAN SKIP THE REDUCTION FOR THAT ROW
C
   25   DO 32 JROW= IP1,N
          IF(AB(JROW,I).EQ.O) GO TO 32
          RATIO= AB(JROW,I)/AB(I,I)
          DO 30 KCOL= IP1,NP
            AB(JROW,KCOL)= AB(JROW,KCOL) - RATIO*AB(I,KCOL)
   30     CONTINUE
   32   CONTINUE
   35 CONTINUE
C
C   CHECK A(N,N) FOR SIZE
C
      IF(ABS(AB(N,N)).LT..00001) GO TO 99
C
C   BACK SUBSTITUTION
C
      NP1= N + 1
      DO 50 KCOL= NP1,NP
        AB(N,KCOL)= AB(N,KCOL)/AB(N,N)
        DO 45 J=2,N
          NVBL= NP1 - J
          L= NVBL + 1
          VALUE= AB(NVBL,KCOL)
          DO 40 K=L,N
            VALUE= VALUE - AB(NVBL,K)*AB(K,KCOL)
   40     CONTINUE
          AB(NVBL,KCOL)= VALUE/AB(NVBL,NVBL)
   45   CONTINUE
   50 CONTINUE
      RETURN
C
C   MESSAGE FOR A NEAR SINGULAR MATRIX
C
   99 WRITE(6,100)
  100 FORMAT(/'SOLUTION NOT FEASIBLE. A NEAR ZERO PIVOT WAS
     + ENCOUNTERED.')
      RETURN
      END
C
C
C
```

```
C
C
      SUBROUTINE FCN (MCASE, EZ, LENGTH)
C
C  THIS SUBROUTINE DEFINES ALL THE FUNCTIONS OF EQUILIBRIUM
C  AND STRAIN COMPATIBILITY TO BE SATISFIED.
C  PARAMETERS ARE -
C     MCASE     = 1  INDICATES THE ULTIMATE STAGE WHEN ANALYZING
C                    THE LOAD POINT SECTION
C               = 2  INDICATES THE ULTIMATE STAGE WHEN ANALYZING
C                    THE CENTRE SUPPORT SECTION
C               = 3  INDICATES THE UNCRACKED SECTION ANALYSIS
C     EZ             STEEL TENDON ECCENTRICITY
C     LENGTH         INDICATES THE INDEX IN THE VECTOR OF STEEL
C                    STRESS "FS" AT A DISTANCE FROM A CRACK
C
      DIMENSION  X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
C  START THE SELECTION OF CASES
C
      IF(ISECT .EQ. 0) GO TO 20
      GO TO (11,22,33),MCASE
   20 GO TO (1,2,3),MCASE
C
C  FOR MCASE   1 AND 2,
C         X(1) :  LOCATION OF NEUTRAL AXIS
C         X(2) :  STEEL STRESS
C         F(1) :  EQUATION OF EQUILIBRIUM
C         F(2) :  STRAIN COMPATIBILITY EQUATION
C  FOR MCASE = 3,
C         X(1) :  STRAIN OF EXTREME CONCRETE FIBRE IN COMPRESSION
C         X(2) :  STEEL STRESS
C         F(1) :  EQUATION OF EQUILIBRIUM
C         F(2) :  STRAIN COMPATIBILITY EQUATION
C         AN   :  LOCATION OF NEUTRAL-AXIS
C  RECTANGULAR SECTION
C
    1 F(1) = AREAP * X(2) * 1000. /(FCU*B) - X(1)
      F(2) = E1 + E2L + EPSU*(EXL+H/2.-X(1))/X(1) - PREL(X(2))
      RETURN
    2 F(1) = AREAP * X(2) * 1000. /(FCU*B) - X(1)
      F(2) = E1 + E2S + EPSU*(EXS+H/2.-X(1))/X(1) - PREL(X(2))
      RETURN
    3 AN = X(1)*H/(ETEN+X(1))
      F(1) = ABS(COMP(AN,X(1))) - 0.5*B*CTEN*(H-AN) +
     +       AREAP*X(2)*1000.
      F(2) = E1 + E2X + (ETEN+X(1))*(ABS(EZ)+H/2.)/H - PREL(X(2))
      RETURN
```

```
C
C   I-SHAPED SECTION
C        ASR : AREA OF CONCRETE WITH THE WEB WIDTH TAKEN UP TO THE
C              CROSS-SECTION TOP
C
  11    IF(X(2)*1000.*AREAP/(FCU*B) .LE. HF) GO TO 1
        ASR = AREAP - FCU*(B-BW)*HF/(X(2)*1000.)
        F(1) = 1.4*ASR*X(2)*1000./(BW*CSTR) - X(1)
        F(2) = E1 +E2L + EPSU*(EXL+H/2.-X(1))/X(1) - PREL(X(2))
        RETURN
  22    IF(X(2)*1000.*AREAP/(FCU*B) .LE. HF) GO TO 2
        ASR = AREAP - FCU*(B-BW)*HF/(X(2)*1000.)
        F(1) = 1.4*ASR*X(2)*1000./(BW*CSTR) - X(1)
        F(2) = E1 + E2S + EPSU*(EXS+H/2.-X(1))/X(1) - PREL(X(2))
        RETURN
  33    AN = X(1)*H/(ETEN+X(1))
        IF(AN .LE. HF) GO TO 3
        F(1) = ABS(COMP(AN,X(1))) - (BW*CTEN*(H-AN)/2. +
     +            (B-BW)*CTEN*HF) - AREAP*X(2)*1000.
      F(2) = E1 + E2X + (ETEN+X(1))*(ABS(EZ)+H/2.)/H - PREL(X(2))
        RETURN
      END
C
C
```

```
C
C
C
      SUBROUTINE AFCRAK(Y, EZ, KASE, LENGTH)
C
C  SUBROUTINE FOR ROOT FINDING USING NEWTON'S METHOD.
C  IT IS USED SPECIFICALLY FOR THE GENERAL CRACKED SECTION
C  ANALYSIS.
C  PARAMETERS ARE -
C      Y        LOCATION OF THE NEUTRAL AXIS
C      EZ       STEEL TENDON ECCENTRICITY
C      KASE     = 1 INDICATES THE CONDITIONS JUST AFTER CRACKING
C                   AND AT A CRACK
C               = 2 INDICATES THE CONDITIONS AT SOME DISTANCE FROM
C                   A CRACK
C      LENGHT   AS DEFINED IN SUBROUTINE "FCN"
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
C  START AN INITIAL VALUE AND CHECK IF EQUATION OF NEUTRAL AXIS
C  IS SATISFIED
C
      IF(ISECT .EQ. 0) BW=B
      FAXNT = AXNT(Y, EZ, KASE, LENGTH)
      DO 20 JC = 1,30
      DELY = FAXNT/DAXNT(Y, EZ, KASE, LENGTH)
      Y = Y - DELY
      FAXNT = AXNT(Y, EZ, KASE, LENGTH)
      IF(ABS(DELY) .LE. .0001) GO TO 70
      IF(ABS(FAXNT) .LE. .00001) GO TO 70
   20 CONTINUE
      WRITE (6,100) JC
  100 FORMAT (/' TOL. NOT MET AFTER ',I5,' ITERATIONS.')
      STOP
   70 IF(Y .LT. 0.) GO TO 30
C
C  EVALUATION OF STRESSES
C  FC    :  STRESS IN CONCRETE EXTREME FIBRE
C  X(1)  :  CORRESPONDING STRAIN IN CONCRETE
C  X(2)  :  STRESS IN PRESTRESSING STEEL
C
      FC = EXTMOM*Y/(GAMMA-BETA*Y-BW*Y**3/6.)
      X(1) = FC/(57000.*SQRT(CSTR))
      X(2) = RATMOD*(ABS(EZ)+H/2.-Y)*FC/Y + AXF/AREAP
      RETURN
C
C  NEGATIVE NEUTRAL AXIS. PRINT MESSAGE.
C
   30 WRITE (6,204) Y
  204 FORMAT (/' NEGATIVE N.A. = ',E12.5)
      STOP
      END
C
C
```

```
C
C
C
      FUNCTION AXNT(Y, EZ, KASE, LENGTH)
C
C  FUNCTION TO DEFINE THE EQUATION OF NEUTRAL AXIS
C  IN THE GENERAL CRACKED SECTION ANALYSIS TO BE USED
C  IN THE SUBROUTINE "AFCRAK"
C  PARAMETERS ARE DEFINED IN "AFCRAK"
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
      IF(ISECT .EQ. O) BW=B
      IF (KASE .EQ. 2) GO TO 2
C
C  CONDITIONS JUST AT A CRACK
C
      AXNT = BW*AXF*Y**3/6. + BW*EXTMOM*Y**2/2.
     +       + (BETA*AXF + ALPHA*EXTMOM)*Y
     +       - (GAMMA*AXF + BETA*EXTMOM)
      RETURN
C
C  CONDITIONS AT SOME DISTANCE FROM A CRACK
C  SO THAT THE STEEL STRESS AND CONCRETE STRAIN CAN BE EVALUATED
C  IN ORDER TO CALCULATE THE AVERAGE CURVATURE
C
    2 DEN = ABS(EZ)+H/2.-Y
C
C  CHECK IF THE DENOMINATOR IS NEGATIVE.
C
      IF (DEN .EQ. O.) DEN=DEN+.05
      TC1 = (BW*Y + (2*Y-HF)*HF*(B-BW)/Y)*FS(LENGTH)*Y
     +      /(2.*RATMOD*DEN)
     +       - AREAP*FS(LENGTH) - AXF/1000.
      AXNT = -AREAP*FS(LENGTH)*(ABS(EZ)+H/2.+ETOP)
     +       - TC1*(ETOP+(H+2.*Y)/3.)
     +        + (.5*(Y/3.+ETOP)*BW*Y
     +             + HF*(Y-HF)*(B-BW)*(HF/2.+ETOP)/Y
     +             + .5*HF*HF*(B-BW)*(HF/3.+ETOP)/Y)
     +          * FS(LENGTH)*Y/(RATMOD*DEN)
      RETURN
      END
C
C
```

```
C
C
C
      FUNCTION DAXNT(Y, EZ, KASE, LENGTH)
C
C  FUNCTION DEFINING THE DERIVATIVE OF THE EQUATION OF
C  NEUTRAL AXIS FOR USE IN THE SUBROUTINE "AFCRAK".
C  THE DERIVATIVE OF THE EQUATION IS REQUIRED BY NEWTON'S
C  ITERATIVE PROCEDURE (SLOPE).
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
      IF (ISECT .EQ. 0) BW=B
      IF (KASE .EQ. 2) GO TO 2
      DAXNT =BW*AXF*Y**2/2. +BW*EXTMOM*Y +(BETA*AXF +ALPHA*EXTMOM)
      RETURN
    2 DEN = ABS(EZ)+H/2.-Y
      IF (DEN .EQ. 0.) DEN=DEN+.05
      TC = (BW*Y + (2.*Y-HF)*HF*(B-BW)/Y)*FS(LENGTH)*(ABS(EZ)+H/2.)
     +      /(2.*RATMOD*DEN**2)
     +    +(BW +(B-BW)*HF*HF/Y**2)*FS(LENGTH)*Y/(2.*RATMOD*DEN)
      DAXNT = TC1*2./3. + (ETOP+(2.*Y+H)/3.)*TC
     +      - (.5*(Y/3.+ETOP)*BW*Y
     +         + HF*(Y-HF)*(B-BW)*(HF/2.+ETOP)/Y
     +         + .5*HF*HF*(B-BW)*(HF/3.+ETOP)/Y)
     +   *FS(LENGTH)*(ABS(EZ)+H/2.)/(RATMOD*DEN**2)
     +      - (.5*(BW*Y/3.+ (Y/3.+ETOP)*BW)
     +         +HF*HF*(B-BW)*(HF/2.+ETOP)/Y**2
     +         -.5*HF*HF*(B-BW)*(HF/3.+ETOP)/Y**2)
     +   *FS(LENGTH)*Y/(RATMOD*DEN)
      RETURN
      END
C
C
C
C
C
```

```
      FUNCTION PREL(SIG)
C
C  FUNCTION DEFINING THE PRESTRESSING STEEL STRESS-STRAIN
C  RELATIONSHIP FOR USE IN THE STRAIN COMPATIBILITY ANALYSIS
C  AT THE ULTIMATE AND VARIOUS LOADING STAGES.
C  THE STRESS-STRAIN CURVE HAS BEEN EXPRESSED IN AN
C  EXPONENTIAL-TYPE EQUATION.
C  PARAMETERS ARE -
C    SIG      STRESS OF STEEL
C    THE VALUE OF THE CORRESPONDING STRAIN IS RETURNED.
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
C  TWO TYPES ARE CONSIDERED:
C  250 AND 270-KSI HIGH GRADE STEEL
C
      IF(SIGPU - 270.) 10,20,20
   10 IF(SIG - 195.) 1,1,2
    2 PREL= SIG*1000./ELMOD + 2.5*((SIG-195.)**3)/10.**7
      RETURN
   20 IF(SIG - 210.) 1,1,3
    3 PREL = SIG*1000./ELMOD + 2.*((SIG-210.)**3)/10.**7
      RETURN
C
C  LINEAR RANGE
C
    1 PREL= SIG * 1000./ELMOD
      RETURN
      END
C
C
```

```
C
C
C
      FUNCTION CMOM(C,EPS)
C
C   FUNCTION CALCULATING THE CONTRIBUTION, TO THE TOTAL
C   MOMENT, OF THE COMPRESSED CONCRETE AREA IN A SECTION.
C   PARAMETERS ARE -
C       C      LOCATION OF THE NEUTRAL AXIS
C       EPS    STRAIN OF EXTREME CONCRETE FIBRE IN COMPRESSION
C   THE FUNCTION TAKES INTO CONSIDERATION THE STRESS-STRAIN
C   RELATIONSHIPS OF THE CONCRETE.
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),      RESULT,
     +  FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
      IF (C .LT. O.) GO TO 50
      IF(EPS .GT. EPSU) GO TO 3
      FF = C*EPSO/EPS
      G = C-HF
      IF((ISECT .EQ. O) .OR. (G .LE. O.)) GO TO 40
      IF(EPS .GT. EPSO) GO TO 20
C
C   I-SHAPED SECTION. EPSO IS THE STRAIN AT WHICH THERE IS
C   A CHANGE OF STRESS-STRAIN RELATIONSHIP.
C   CASE WHEN THE APPLIED STRAIN IS BELOW EPSO
C
      CMOM = (B-BW)*CSTR*G**3*(8./3. - G/FF)/(4.*FF) +
     +        B*HF*CSTR*(HF*HF/3. + (C+G)**2 -
     +        ((C+G)**3 + (C+G)*HF**2)/(2.*FF))/(2.*FF)
      RETURN
C
C   CASE WHEN THE APPLIED STRAIN IS HIGHER THAN EPSO
C
   20 IF(FF-G)21,21,23
C
C   EPSO IS IN THE WEB
C
   21 CMOM = 5.*CSTR*(B-BW)*FF*FF/12.+
     +        O.5*CSTR*(B-BW)*(G-FF)*(G+FF-RDFACT*
     +(EPS*((G-FF)**2/3.+(G+FF)**2)/(2.*C)-EPSO*(G+FF))/(EPSU-EPSO))
     +   + O.5*CSTR*B*HF*(C+G-RDFACT*
     +     (EPS*(HF**2/3.+(C+G)**2)/(2.*C)-EPSO*(C+G))/(EPSU-EPSO))
      RETURN
```

```
C
C  EPSO IS IN THE FLANGE
C
   23 P = EPSO/EPS
      CMOM = (B-BW)*CSTR*G**3*(8./3.-G/FF)/(4.*FF) +
     +    .5*B*CSTR*(FF-G)*(((FF+G)**2+(FF-G)**2/3.)/FF
     +          - ((FF+G)**3+(FF+G)*(FF-G)**2)/(4.*FF**2))
     +   .5*B*CSTR*C**2*(1.-P)*(1.+P- RDFACT*
     +   (EPS*((1.-P)**2/3.+(1.+P)**2)/2.-EPSO*(1.+P))/(EPSU-EPSO))
      RETURN
C
C  RECTANGULAR CROSS-SECTION
C
   40 IF(EPS .GT. EPSO)  GO TO 2
      FF = EPS/EPSO
      CMOM = B*((.5*C)**2)*CSTR*(FF*8./3. - FF**2)
      RETURN
    2 FF = EPSO/EPS
      G =  RDFACT/(EPSU-EPSO)
      CMOM = CSTR*5.*B*((C*FF)**2)/12. + B*((.5*C)**2)*CSTR*(1.-FF)*
     &        ((1.+FF)*(2.-G*(EPS-EPSO)) - ((1.-FF)**2)*EPS*G/3.)
      RETURN
C
C  NEGATIVE VALUE OF NEUTRAL AXIS LOCATION. PRINT MESSAGE.
C
   50 WRITE (6,300) C
  300 FORMAT (/' NEG. N.A. = ',E12.5,' IN SECT. CMOM')
      STOP
C
C  THE ULTIMATE CONCRETE STRAIN HAS BEEN EXCEEDED.
C  PRINT MESSAGE.
C
    3 WRITE(6,500) EPS
  500 FORMAT(//15X,F13.5,' ,ULT CONC STRAIN EXCEEDED.'
     &                   /26X,'END OF COMPILING IN SECT CMOM.')
      STOP
      END
C
C
C
```

```
C
C
      FUNCTION COMP(C,EPS)
C
C  FUNCTION CALCULATING THE CONTRIBUTION, TO THE TOTAL FORCE,
C  OF THE CONCRETE AREA IN COMPRESSION.
C  PARAMETERS ARE -
C      C    LOCATION OF THE NEUTRAL AXIS
C      EPS  STRAIN OF EXTREME CONCRETE FIBRE IN COMPRESSION
C  THE FUNCTION TAKES INTO CONSIDERATION THE STRESS-STRAIN
C  RELATIONSHIPS OF THE CONCRETE.
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
      IF (C .LT. O.) GO TO 50
      IF(EPS .GT.EPSU) GO TO 3
C
C  I-SHAPED CROSS-SECTION.
C
      FF = C*EPSO/EPS
      G = C-HF
      IF((ISECT .EQ. O) .OR. (G .LE. O.)) GO TO 40
      IF(EPS .GT. EPSO) GO TO 20
C
C  CASE WHEN APPLIED STRAIN IS BELOW EPSO
C
      COMP = (B-BW)*G*CSTR*(G/FF-(G/FF)**2/3.)
     +     + B*CSTR*HF*((C+G)/FF-((C+G)**2+HF**2/3.)/(4.*FF**2))
      RETURN
   20 IF(FF-G) 21, 21, 23
C
C  EPSO IN THE WEB
C
   21 COMP= 2.*(B-BW)*FF*CSTR/3. + (B-BW)*
     +    (G-FF)*CSTR*(1.-RDFACT*(EPS*(G+FF)/C-EPSO)/(EPSU-EPSO))
     + + B*HF*CSTR*(1.-RDFACT*(EPS*(C+G)/C-EPSO)/(EPSU-EPSO))
      RETURN
```

```
C
C   EPSO IN THE FLANGE
C
    23 COMP = (B-BW)*G*CSTR*(G/FF-(G/FF)**2/3.)
      + + B*CSTR*(FF-G)*(1.+G/FF-(1.+G/FF+(G/FF)**2)/3.)
      + + .5*B*CSTR*C*(1.-EPSO/EPS)*(2.-RDFACT*(EPS-3.*EPSO)/
      +    (2.*(EPSU - EPSO)))
       RETURN
C
C   RECTANGULAR CROSS-SECTION
C
    40 IF(EPS.GT.EPSO) GO TO 2
       G= EPS/EPSO
       COMP= B*C*CSTR*(G-G*G/3.)
       RETURN
     2 G= EPSO/EPS
       COMP= B*C*CSTR*(G*2./3.
      &      + .5*(1.-G)*(2.-RDFACT*(EPS-EPSO)/(EPSU-EPSO)))
       RETURN
C
C   NEGATIVE VALUE OF LOCATION OF NEUTRAL AXIS.
C   PRINT MESSAGE
C
    50 WRITE (6,300) C
   300 FORMAT (/' NEG. N.A. = ',E12.5,' IN SECT. COMP')
       STOP
C
C   ULTIMATE CONCRETE STRAIN HAS BEEN EXCEEDED.
C   PRINT MESSAGE.
C
     3 WRITE(6,501)EPS
   501 FORMAT(//15X,F13.5,' ,ULT CONC STRAIN EXCEEDED,'
      &             /26X,'END OF COMPILING IN SECT COMP.')
       STOP
       END
C
C
C
```

```
C
C
      SUBROUTINE ROMB(IBOUND, EZ, DISTM)
C
C  SUBROUTINE FOR ROMBERG INTEGRATION. PROGRAM BEGINS WITH
C  TRAPEZOIDAL INTEGRATION WITH 10 SUBINTERVALS. INTERVALS
C  ARE THEN HALVED AND RESULTS ARE EXTRAPOLATED UP TO
C  FOURTH ORDER.
C  MAXIMUM NUMBER OF SUBINTERVALS USED IN PROGRAM IS 160.
C  PARAMETERS ARE -
C      IBOUND   NUMBER OF KNOWN VALUE INTEGRATION POINTS
C      EZ       PRESTRESSING STEEL ECCENTRICITY
C      DISTM    VALUE OF MOMENT
C      RESULT   RESULT OF INTEGRATION. RETURNED.
C      TRAP     DOUBLY SUBSCRIPTED ARRAY THAT HOLDS INTERMEDIATE
C               VALUES FOR COMPARISONS AND EXTRAPOLATIONS
C       KFLG    = 0 WHEN NON-CONVERGENT
C               =1 MEANS ALL OK.
C
      DIMENSION X(2), F(2)
      COMMON X, F, B, H, EXL, EXS, PRESF, AREAP, CSTR, ELMOD,
     +FCU, E1, E2L, E2S, EPSO, SIGPU, AREAC, EPSU, COMPAT,DELTA(2),
     +  CTEN, ETEN, E2X, CRMOM, FS(60), STRAIN(60),     RESULT,
     + FRACL, SPAN, ISECT, BW, HF, R2, FSB, CURVCT, ASR, ETOP,
     +   RDFACT, RATMOD, ALPHA, BETA, GAMMA, AXF, EXTMOM, TC1,
     +   ANGLEA, ANGLEB
C
      DIMENSION TRAP(5,5)
C
C  SET FLAG AT 1 INITIALLY
C
      KFLG = 1
C
C  COMPUTE FIRST INTEGRAL WITH 10 SUBINTEGRALS AND USING TRAP
C  RULE
C
      KINT = IBOUND/10
      SUM = STRAIN(1) + STRAIN(IBOUND)
      INT = 0
      DO 10 I = 2,10
      INT = INT + KINT
   10 SUM = SUM + STRAIN(INT)*2.
      TRAP(1,1) = KINT/2*SUM
```

```
C
C  RECOMPUTE INTEGRAL WITH KINT HALVED, EXTRAPOLATE AND TEST.
C  REPEAT UP TO 4 TIMES.
C
      DO 20 I = 1,4
        KINT = KINT/2
        INT = KINT
        K = 10*2**I
        DO 30 J = 2,K,2
          SUM = SUM + STRAIN(INT)*2.
          INT = INT + KINT + KINT
   30   CONTINUE
        TRAP(1,I+1) = KINT/2*SUM
        DO 40 L = 1,I
          TRAP(L+1,I+1) = TRAP(L,I+1) + 1./(4.**L - 1.)*
     &                    (TRAP(L,I+1) - TRAP(L,I))
   40   CONTINUE
        IF(ABS(TRAP(I+1,I+1) - TRAP(I,I+1)) - .01) 50,50,20
   20 CONTINUE
C
C  IF TOLERANCE NOT MET AFTER 4 EXTRAPOLATIONS, PRINT
C  MESSAGE. SET KFLG = 0
      KFLG = 0
      WRITE (6,200)
  200 FORMAT (/'TOLERANCE NOT MET. CALCULATED VALUES WERE ')
   50 I = I + 1
      IF (KFLG .EQ. 0) STOP
C
C  CALCULATION OF FINAL RESULT
C
      RESULT = (DISTM/ABS(DISTM))*TRAP (I,I) /
     +                                    (ABS(EZ)+H/2.)
      RETURN
      END
```

```
*************************************************
MOMENT-LOAD CURVE FOR THE TWO-SPAN CONTINUOUS PRESTRESSED BEAM NUMBER     1 LOADED
SYMMETRICALLY WITH TWO CONCENTRATED VERTICAL LOADS  54.00 INCHES FROM THE END SUPPORTS.

I-SHAPED SECTION, WIDTH =     6.00 IN.,  HEIGHT =   12.00 IN.
                      WEB    2.70 IN.,    FLANGE    2.90 IN.


TENDON ECCENTRICITY =       2.55 IN. AT LOAD POINT
                            4.55 IN. AT SUPPORT.
SPAN BETWEEN SUPPORTS      108.00 IN.


CONCRETE STRENGTH         6700.00 PSI
          STRAIN EPSO         0.00190
                 EPSU         0.00260
          REDUCTION FACTOR       0.30


TOTAL TENDON AREA           0.1180 SQ. IN.
APPLIED PRESTRESSING      14050.00 LBS
GRADE                       250.00 KSI
MOD. OF ELASTICITY       0.270E+08 PSI
*************************************************

THE CRACKING MOMENTS AT SUPPORT AND AT THE LOAD POINT ARE, RESPECTIVELY:

  MOMENT          =  0.206938E+06 LB-IN.
  CURVATURE       =  0.278266E-04

  MOMENT          =  0.174896E+06 LB-IN.
  CURVATURE       =  0.276792E-04

CAPACITY PROVIDED,    0.79 ACCORDING TO ELASTIC RATIO    0.83 :* NO REDISTRIBUTION.

CRACKING : OCCURS AT LOAD =  0.206196E+05 LBS
           SUPPORT MOMENT =  0.206938E+06 LB-IN.
           LD PT MOMENT   =  0.174896E+06 LB-IN.

ULTIMATE : OCCURS AT LOAD =  0.271753E+05 LBS
           SUPPORT MOMENT =  0.284333E+06 LB-IN.
           LD PT MOMENT   =  0.224700E+06 LB-IN.
End of file
```

```
------------------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
          AT SUPPORT :  0.169712E+06 ,AND AT LD POINT :  0.174482E+06 LB-IN

   AT ITER.    1    O THE MOMENTS AT LD POINT =  0.174482E+06 AND AT SUPPORT =  0.171851E+06 LB-IN.
                         *** COMPATIBILITY EQUATION =  0.399154E-02


   MOM. TOLERANCE MET IN    1    O ITER. ,LD PT. MOM. =  0.174482E+06 ,SUP =  0.171851E+06 LB-IN.
                         *** COMPATIBILITY EQ. =  0.399154E-02


   THE TOTAL APPLIED LOAD IS=  0.192895E+05 LBS

------------------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
          AT SUPPORT :  0.203655E+06 ,AND AT LD POINT :  0.209378E+06 LB-IN

   AT ITER.    1    O THE MOMENTS AT LD POINT =  0.209378E+06 AND AT SUPPORT =  0.213655E+06 LB-IN.
                         *** COMPATIBILITY EQUATION =  0.329287E-01


   COMPAT. TOLERANCE MET IN    1    O ITER., LD PT. MOM. =  0.209378E+06 ,SUP =  0.213655E+06 LB-IN.
                         *** COMPATIBILITY EQ. =  0.329287E-01


   THE TOTAL APPLIED LOAD IS=  0.234226E+05 LBS
```

```
--------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
                AT SUPPORT :  0.237597E+06 ,AND AT LD POINT :  0.244275E+06 LB-IN

    AT ITER.    1    O THE MOMENTS AT LD POINT =  0.244275E+06 AND AT SUPPORT =  0.247597E+06 LB-IN.
                               *** COMPATIBILITY EQUATION =  0.113416E+00


    COMPAT. TOLERANCE MET IN    1    O ITER., LD PT. MOM. =  0.244275E+06 ,SUP =  0.247597E+06 LB-IN.
                               *** COMPATIBILITY EQ. =  0.113416E+00



    THE TOTAL APPLIED LOAD IS=  0.272647E+05 LBS

**************************************************
ULTIMATE STAGE: ASSUMED
  SUPPORT     MOMENT          =  0.268469E+06 LB-IN.
              CURVATURE       =  0.324270E-02
  LOAD POINT MOMENT           =  0.276014E+06 LB-IN.
            CURVATURE         =  0.323102E-02

    AT ITER.    1    O THE MOMENTS AT LD POINT =  0.276014E+06 AND AT SUPPORT =  0.278469E    LB-IN.
                               *** COMPATIBILITY EQUATION =  0.184766E+00


    COMPAT. TOLERANCE MET IN    1    O ITER., LD PT. MOM. =  0.276014E+06 ,SUP =  0.278469E+06 LB-IN.
                               *** COMPATIBILITY EQ. =  0.184766E+00


    THE TOTAL APPLIED LOAD IS=  0.307592E+05 LBS
End of file
```

```
*************************************************
MOMENT-LOAD CURVE FOR THE TWO-SPAN CONTINUOUS PRESTRESSED BEAM NUMBER      4 LOADED
SYMMETRICALLY WITH TWO CONCENTRATED VERTICAL LOADS  54.00 INCHES FROM THE END SUPPORTS

I-SHAPED SECTION, WIDTH =      6.00 IN.,  HEIGHT =    12.00 IN.
                    WEB      2.65 IN.,    FLANGE     2.90 IN.


TENDON ECCENTRICITY =         4.40 IN. AT LOAD POINT
                              2.50 IN. AT SUPPORT.
SPAN BETWEEN SUPPORTS       108.00 IN.


CONCRETE STRENGTH           3850.00 PSI
          STRAIN EPSO          0.00200
                 EPSU          0.00400
          REDUCTION FACTOR        0.58


TOTAL TENDON AREA            0.1210 SQ. IN.
APPLIED PRESTRESSING      14600.00 LBS
GRADE                       250.00 KSI
MOD. OF ELASTICITY       0.270E+08 PSI
*************************************************

THE CRACKING MOMENTS AT SUPPORT AND AT THE LOAD POINT ARE, RESPECTIVELY:

  MOMENT          =  0.145615E+06 LB-IN.
  CURVATURE       =  0.364470E-04

  MOMENT          =  0.178781E+06 LB-IN.
  CURVATURE       =  0.368537E-04

-------------------------------------------------
CRACKING OCCURS FIRST AT SUPPORT

     AT ITER.    1    0 THE MOMENTS AT LD POINT =  0.121346E+06 AND AT SUPPORT =  0.146279E+06 LB-IN.
                          *** COMPATIBILITY EQUATION = -0.863771E-04


     MOM. TOLERANCE MET IN    1    0 ITER. ,LD PT. MOM. =  0.121346E+06 ,SUP =  0.146279E+06 LB-IN
                          *** COMPATIBILITY EQ. = -0.863771E-04


     THE TOTAL APPLIED LOAD IS=  0.144063E+05 LBS
```

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--------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
              AT SUPPORT :  0.146279E+06 ,AND AT LD POINT :  0.184227E+06 LB-IN

    AT ITER.    1    0 THE MOMENTS AT LD POINT =  0.184227E+06 AND AT SUPPORT =  0.141230E+06 LB-IN.
                        *** COMPATIBILITY EQUATION = -0.432332E-02


    MOM. TOLERANCE MET IN    1    0 ITER. ,LD PT. MOM. =  0.184227E+06 ,SUP =  0.141230E+06 LB-IN.
                        *** COMPATIBILITY EQ. = -0.432332E-02


    THE TOTAL APPLIED LOAD IS=  0.188772E+05 LBS

--------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
              AT SUPPORT :  0.175534E+06 ,AND AT LD POINT :  0.221072E+06 LB-IN

    AT ITER.    1    0 THE MOMENTS AT LD POINT =  0.221072E+06 AND AT SUPPORT =  0.180746E+06 LB-IN.
                        *** COMPATIBILITY EQUATION =  0.109812E-01


    MOM. TOLERANCE MET IN    1    0 ITER. ,LD PT. MOM. =  0.221072E+06 ,SUP =  0.180746E+06 LB-IN.
                        *** COMPATIBILITY EQ. =  0.109812E-01


    THE TOTAL APPLIED LOAD IS=  0.230700E+05 LBS
```

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 -----------------------------------------------------------
  THE NEXT ASSUMED SET OF MOMENTS IS:
                AT SUPPORT :  0.204790E+06 ,AND AT LD POINT :  0.257917E+06 LB-IN

      AT ITER.    1    O THE MOMENTS AT LD POINT =  0.257917E+06 AND AT SUPPORT =  0.214790E+06 LB-IN.
                                *** COMPATIBILITY EQUATION =  0.839046E-02


      COMPAT. TOLERANCE MET IN    1    O ITER., LD PT. MOM. =  0.257917E+06 ,SUP =  0.214790E+06 LB-IN.
                                *** COMPATIBILITY EQ. =  0.839046E-02


      THE TOTAL APPLIED LOAD IS=  0.270602E+05 LBS

 ***********************************************************
 ULTIMATE STAGE: ASSUMED
   SUPPORT    MOMENT        =  0.223871E+06 LB-IN.
              CURVATURE     =  0.331049E-02
   LOAD POINT MOMENT        =  0.281948E+06 LB-IN.
            CURVATURE       =  0.318284E-02

      AT ITER.    1    O THE MOMENTS AT LD POINT =  0.281948E+06 AND AT SUPPORT =  0.233871E+06 LB-IN.
                                *** COMPATIBILITY EQUATION =  0.465541E-01


      COMPAT  TOLERANCE MET IN    1    O ITER., LD PT. MOM. =  0.281948E+06 ,SUP =  0.233871E+06 LB-IN
                                *** COMPATIBILITY EQ. =  0.465541E-01


      THE TOTAL APPLIED LOAD IS=  0.295469E+05 LBS
End of file
```

```
*************************************************
MOMENT-LOAD CURVE FOR THE TWO-SPAN CONTINUOUS PRESTRESSED BEAM NUMBER     5 LOADED
SYMMETRICALLY WITH TWO CONCENTRATED VERTICAL LOADS  54.00 INCHES FROM THE END SUPPORTS.

I-SHAPED SECTION, WIDTH =     6.00 IN.,  HEIGHT =   12.00 IN.
                      WEB     2.65 IN.,    FLANGE    2.90 IN.


TENDON ECCENTRICITY =        4.50 IN. AT LOAD POINT
                             4.35 IN. AT SUPPORT.
SPAN BETWEEN SUPPORTS      108.00 IN.


CONCRETE STRENGTH          3990.00 PSI
          STRAIN EPSO         0.00190
                 EPSU         0.00400
          REDUCTION FACTOR       0.60


TOTAL TENDON AREA           0.1830 SQ. IN.
APPLIED PRESTRESSING     20300.00 LBS
GRADE                      270.00 KSI
MOD. OF ELASTICITY       0.270E+08 PSI
*************************************************

THE CRACKING MOMENTS AT SUPPORT AND AT THE LOAD POINT ARE, RESPECTIVELY:

  MOMENT          =  0.224633E+06 LB-IN.
  CURVATURE       =  0.412315E-04

  MOMENT          =  0.228615E+06 LB-IN.
  CURVATURE       =  0.412953E-04

-------------------------------------------------
CRACKING OCCURS FIRST AT SUPPORT

     AT ITER.    1    O THE MOMENTS AT LD POINT =  0.187194E+06 AND AT SUPPORT =  0.227513E+06 LB-IN.
                             *** COMPATIBILITY EQUATION = -0.504969E-04


     MOM. TOLERANCE MET IN     1    O ITER. ,LD PT. MOM. =  0.187194E+06 ,SUP =  0.227513E+06 LB-IN.
                             *** COMPATIBILITY EQ. = -0.504969E-04


     THE TOTAL APPLIED LOAD IS=  0.222926E+05 LBS
```

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--------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
            AT SUPPORT :  0.227513E+06 ,AND AT LD POINT :  0.231287E+06 LB-IN

   AT ITER.    1    O THE MOMENTS AT LD POINT =  0.231287E+06 AND AT SUPPORT =  0.237513E+06 LB-IN.
                        *** COMPATIBILITY EQUATION =  0.429830E-04


   COMPAT. TOLERANCE MET IN    1    O ITER , LD PT. MOM. =  0.231287E+06 ,SUP =  0.237513E+06 LB-IN
                        *** COMPATIBILITY EQ. =  0.429830E-04


   THE TOTAL APPLIED LOAD IS=  0.259292E+05 LBS

--------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
            AT SUPPORT :  0.273015E+06 ,AND AT LD POINT :  0.277545E+06 LB-IN

   AT ITER.    1    O THE MOMENTS AT LD POINT =  0.277545E+06 AND AT SUPPORT =  0.283015E+06 LB-IN
                        *** COMPATIBILITY EQUATION =  0.180038E-01


   COMPAT. TOLERANCE MET IN    1    O ITER., LD PT. MOM. =  0.277545E+06 ,SUP =  0.283015E+06 LB-IN.
                        *** COMPATIBILITY EQ. =  0.180038E-01


   THE TOTAL APPLIED LOAD IS=  0.310409E+05 LBS

--------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
            AT SUPPORT :  0.318518E+06 ,AND AT LD POINT :  0.323802E+06 LB-IN

   AT ITER.    1    O THE MOMENTS AT LD POINT =  0.323802E+06 AND AT SUPPORT =  0.328518E+06 LB-IN
                        *** COMPATIBILITY EQUATION =  0.792161E-01


   COMPAT. TOLERANCE MET IN    1    O ITER., LD PT. MOM. =  0.323802E+06 ,SUP =  0.328518E+06 LB-IN
                        *** COMPATIBILITY EQ. =  0.792161E-01


   THE TOTAL APPLIED LOAD IS=  0.361527E+05 LBS
```

```
-----------------------------------------------------
THE NEXT ASSUMED SET OF MOMENTS IS:
              AT SUPPORT :  0.364020E+06 ,AND AT LD POINT :  0.370060E+06 LB-IN

    AT ITER.     1     O THE MOMENTS AT LD POINT =  0.370060E+06 AND AT SUPPORT =  0.374020E+06 LB-IN
                              *** COMPATIBILITY EQUATION =  0.172724E+00


    COMPAT. TOLERANCE MET IN     1     O ITER., LD PT. MOM. =  0.370060E+06 ,SUP =  0.374020E+06 LB-IN.
                              *** COMPATIBILITY EQ. =  0.172724E+00


    THE TOTAL APPLIED LOAD IS=  0.412644E+05 LBS

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THE NEXT ASSUMED SET OF MOMENTS IS:
              AT SUPPORT :  0.409522E+06 ,AND AT LD POINT :  0.416317E+06 LB-IN

    AT ITER.     1     O THE MOMENTS AT LD POINT =  0.416317E+06 AND AT SUPPORT =  0.419522E+06 LB-IN.
                              *** COMPATIBILITY EQUATION =  0.267402E+00


    COMPAT. TOLERANCE MET IN     1     O ITER., LD PT. MOM. =  0.416317E+06 ,SUP =  0.419522E+06 LB-IN.
                              *** COMPATIBILITY EQ. =  0.267402E+00


    THE TOTAL APPLIED LOAD IS=  0.463761E+05 LBS

*****************************************************
ULTIMATE STAGE: ASSUMED
  SUPPORT     MOMENT        =  0.424414E+06 LB-IN.
              CURVATURE     =  0.222576E-02
 LOAD POINT MOMENT          =  0.431456E+06 LB-IN.
            CURVATURE       =  0.220784E-02

    AT ITER.     1     O THE MOMENTS AT LD POINT =  0.431456E+06 AND AT SUPPORT =  0.434414E+06 LB-IN.
                              *** COMPATIBILITY EQUATION =  0.298389E+00


    COMPAT. TOLERANCE MET IN     1     O ITER., LD PT. MOM. =  0.431456E+06 ,SUP =  0.434414E+06 LB-IN.
                              *** COMPATIBILITY EQ. =  0.298389E+00


    THE TOTAL APPLIED LOAD IS=  0.480491E+05 LBS
End of file
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**************************************************
MOMENT-LOAD CURVE FOR THE TWO-SPAN CONTINUOUS PRESTRESSED BEAM NUMBER     6 LOADED
SYMMETRICALLY WITH TWO CONCENTRATED VERTICAL LOADS  54.00 INCHES FROM THE END SUPPORTS.

I-SHAPED SECTION, WIDTH =     6.00 IN.,  HEIGHT =   12.00 IN.
                    WEB     2.56 IN.,    FLANGE    2.90 IN.


TENDON ECCENTRICITY =       4.35 IN. AT LOAD POINT
                            4.55 IN. AT SUPPORT.
SPAN BETWEEN SUPPORTS     108.00 IN.


CONCRETE STRENGTH          3770.00 PSI
          STRAIN EPSO         0.00200
                 EPSU         0.00400
          REDUCTION FACTOR       0.50


TOTAL TENDON AREA           0.2440 SQ. IN.
APPLIED PRESTRESSING     26600.00 LBS
GRADE                      270.00 KSI
MOD. OF ELASTICITY      0.270E+08 PSI
**************************************************

THE CRACKING MOMENTS AT SUPPORT AND AT THE LOAD POINT ARE, RESPECTIVELY:

  MOMENT          = 0.276715E+06 LB-IN.
  CURVATURE       = 0.523262E-04

  MOMENT          = 0.269161E+06 LB-IN.
  CURVATURE       = 0.521402E-04

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CRACKING OCCURS FIRST AT SUPPORT

     AT ITER.    1    O THE MOMENTS AT LD POINT = 0.230596E+06 AND AT SUPPORT =  0.275215E+06 LB-IN.
                           *** COMPATIBILITY EQUATION = -0.461043E-04


     MOM. TOLERANCE MET IN    1    O ITER  ,LD PT. MOM. =  0.230596E+06 ,SUP =  0.275215E+06 LB IN.
                           *** COMPATIBILITY EQ.  = -0.461043E-04


     THE TOTAL APPLIED LOAD IS=  0.272743E+05 LBS
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THE NEXT ASSUMED SET OF MOMENTS IS:
              AT SUPPORT :  0.275215E+06 ,AND AT LD POINT :  0.268700E+06 LB-IN

    AT ITER.    1    0 THE MOMENTS AT LD POINT =  0.268700E+06 AND AT SUPPORT =  0.281916E+06 LB-IN.
                              *** COMPATIBILITY EQUATION =  0.676517E-02


    MOM. TOLERANCE MET IN    1    0 ITER. ,LD PT. MOM. =  0.268700E+06 ,SUP =  0.281916E+06 LB-IN.
                              *** COMPATIBILITY EQ. =  0.676517E-02


    THE TOTAL APPLIED LOAD IS=  0.303450E+05 LBS

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THE NEXT ASSUMED SET OF MOMENTS IS:
              AT SUPPORT :  0.330258E+06 ,AND AT LD POINT :  0.322440E+06 LB-IN

    AT ITER.    1    0 THE MOMENTS AT LD POINT =  0.322440E+06 AND AT SUPPORT =  0.340258E+06 LB-IN.
                              *** COMPATIBILITY EQUATION =  0.490612E-02


    COMPAT. TOLERANCE MET IN    1    0 ITER., LD PT. MOM. =  0.322440E+06 ,SUP =  0.340258E+06 LB-IN.
                              *** COMPATIBILITY EQ. =  0.490612E-02


    THE TOTAL APPLIED LOAD IS=  0.364866E+05 LBS

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THE NEXT ASSUMED SET OF MOMENTS IS:
              AT SUPPORT :  0.385301E+06 ,AND AT LD POINT :  0.376179E+06 LB-IN

    AT ITER.    1    0 THE MOMENTS AT LD POINT =  0.376179E+06 AND AT SUPPORT =  0.395301E+06 LB-IN.
                              *** COMPATIBILITY EQUATION =  0.187944E-01


    COMPAT. TOLERANCE MET IN    1    0 ITER., LD PT. MOM. =  0.376179E+06 ,SUP =  0.395301E+06 LB-IN.
                              *** COMPATIBILITY EQ. =  0.187944E-01


    THE TOTAL APPLIED LOAD IS=  0.425059E+05 LBS
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THE NEXT ASSUMED SET OF MOMENTS IS:
              SUPPORT   0.440344E+06 ,AND AT LD POINT :  0.429919E+06 LB-IN

   AT ITER    1   0 THE MOMENTS AT LD POINT =  0.429919E+06 AND AT SUPPORT =  0.450344E+06 LB-IN
                        *** COMPATIBILITY EQUATION =  0.143637E+00


   COMPAT. TOLERANCE MET IN    1   0 ITER . LD PT  MOM  =  0.429919E+06 ,SUP =  0.450344E+06 LB-IN
                        *** COMPATIBILITY EQ. =  0.143637E+00


   THE TOTAL APPLIED LOAD IS=. 0.485253E+05 LBS

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THE NEXT ASSUMED SET OF MOMENTS IS:
            AT SUPPORT   0.495387E+06 ,AND AT LD POINT :  0.483659E+06 LB-IN

   AT ITER.   1   0 THE MOMENTS AT LD POINT =  0.483659E+06 AND AT SUPPORT =  0.505387E+06 LB-IN
                        *** COMPATIBILITY EQUATION =  0.268481E+00


   COMPAT  TOLERANCE MET IN    1   0 ITER . LD PT  MOM. =  0.483659E+06 ,SUP =  0.505387E+06 LB-IN
                        *** COMPATIBILITY EQ  =  0.268481E+00


   THE TOTAL APPLIED LOAD IS=  0.545446E+05 LBS

*****************************************************
ULTIMATE STAGE  ASSUMED
  SUPPORT     MOMENT        =  0.518933E+06 LB-IN
              CURVATURE     =  0.150547E-02
  LOAD POINT MOMENT         =  0.506648E+06 LB-IN
             CURVATURE      =  0.151569E-02

   AT ITER.   1   0 THE MOMENTS AT LD POINT =  0.506648E+06 AND AT SUPPORT =  0.528933E+06 LB-IN
                        *** COMPATIBILITY EQUATION =  0.321887E+00


   COMPAT  TOLERANCE MET IN    1   0 ITER . LD PT  MOM  =  0.506648E+06 ,SUP =  0.528933E+06 LB-IN
                        *** COMPATIBILITY EQ  =  0.321887E+00


   THE TOTAL APPLIED LOAD IS=  0.571196E+05 LBS
End of file
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