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LIMITS ON FRACTIONALLY CHARGED PARTICLES

BY

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The undersigned certify that they have read, and recommended to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Limits on Fractionally Charged Particles" submitted by Sacha Davidson in partial fulfilment of the requirements for the degree of Master of Science in Theoretical Physics.

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ABSTRACT

Bounds on the mass and electric charge ϵe of a fractionally charged ($10^{-15} < \epsilon < 1$) particle (paraton) are calculated. Accelerator and Lamb shift experiments give limits for values of ϵ greater than 10^{-4}. Astrophysical arguments based on plasmon decay in red giants and white dwarfs give lower bounds on the mass, but only for $\epsilon < 10^{-5}$. Cosmology provides a lower bound on the mass from the limit on the number of neutrino flavours at nucleosynthesis, and an upper bound from requiring that the paratons not overclose the universe. The cosmology limits only apply if the particles are in equilibrium, which puts a lower bound above the upper mass bound.

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CHAPTER ONE

INTRODUCTION

This is a thesis about "funny fermions" f and $\bar{f}$ with fractional electric charge ϵ; ϵ can take on any value less than 1, although only $1 > \epsilon > 10^{-15}$ is considered here. One can include such a peculiar particle in the Standard Model by adding to the Lagrangian, by hand, kinetic and mass terms for a particle with fractional hypercharge. Alternatively, as suggested by Holdom [1], a particle can pick up an effective fractional charge ϵ via loop diagrams in a world where there are two unbroken gauged $U(1)$ symmetries.

We do not observe two $U(1)$ interactions in our low energy world, so ordinary particles do not carry the charge of the second $U(1)$. However, the idea is that if, at some large mass scale, there are particles carrying both charges (call them Ψ and Ψ'), then the gauge boson of the first $U(1)_1$ ('photon') can turn into a virtual pair of these particles and then into the gauge boson of the second $U(1)_2$ ('paraphoton'). This gives an effective interaction between the two gauge bosons, and therefore between particles carrying only the charge of the $U(1)_1$ (call these ψ_1) and particles carrying only the charge of the second $U(1)_2$ (paratons $\equiv f$). This can be described by giving the f a small effective $U(1)_1$ charge. Intuitively, virtual pairs of Ψ and Ψ' formed around a ψ_1 or f give it a small interaction with the gauge boson of the other $U(1)$. It turns out that one can always arrange to describe the $\psi_1 - f$ interaction by only giving one particle a small effective charge.

Suppose that one has a toy world populated by fermions ψ_1, f, Ψ and Ψ'

with masses m_1, μ, M and M', where

$$M > M' \gg m_1 \sim \mu \ . \tag{1.1}$$

ψ_1 is an ordinary particle and carries the charge of $U(1)_1$. f is the paraton and carries the charge of $U(1)_2$. Ψ and Ψ' interact with both gauge bosons, but Ψ' is assumed to have the opposite charge from Ψ under one of the $U(1)$s. The intrinsic charges of the four types of fermion are therefore

$$\begin{aligned} \psi_1 &\longleftrightarrow (e_1, 0) \\ \psi_2 &\longleftrightarrow (0, e_2) \\ \Psi' &\longleftrightarrow (e_1, e_2) \\ \Psi &\longleftrightarrow (e_1, -e_2) \end{aligned} \tag{1.2}$$

At some energy scale $\Lambda > M$, the gauge bosons have "diagonal" (in $U(1)$ space) kinetic terms:

$$\mathcal{L}_{kin}(\Lambda) = -\frac{1}{4}[F_{1\,\mu\nu}F_1^{\mu\nu} + F_{2\,\mu\nu}F_2^{\mu\nu}] \ . \tag{1.3}$$

But if one integrates out fluctuations down to $\Lambda' < M'$, the Lagrangian must acquire an effective interaction term to describe the Ψ and Ψ' induced photon-paraphoton mixing, because this can only happen at energy scales $E > M' > \Lambda'$. Writing $\mathcal{L}_{kin}(\Lambda')$ in terms of the fields at Λ gives

$$\mathcal{L}_{kin}(\Lambda') = -\frac{1}{4}[\chi_1 F_{1\,\mu\nu}F_1^{\mu\nu} + \chi_2 F_{2\,\mu\nu}F_2^{\mu\nu} + 2\chi F_{1\,\mu\nu}F_2^{\mu\nu}] \tag{1.4}$$

where

$$\chi_1 = 1 + \frac{e_1^2(\Lambda)}{6\pi^2}\ln\left(\frac{\Lambda^3}{\Lambda' M M'}\right) \tag{1.5}$$

$$\chi_2 = 1 + \frac{e_2^2(\Lambda)}{6\pi^2} \ln\left(\frac{\Lambda^3}{\Lambda' M M'}\right) \tag{1.6}$$

and

$$\chi = \frac{e_1(\Lambda)e_2(\Lambda)}{6\pi^2} \ln\left(\frac{M}{M'}\right) \ . \tag{1.7}$$

χ_1 and χ_2 are just the expected scaling factors for $F_{\mu\nu}F^{\mu\nu}$; the one loop contribution to Z_1 and Z_2 (vertex and fermion wavefunction renormalisation coefficients) will still satisfy the old Ward identity $Z_1 = Z_2$, even though there are two $U(1)$s. The gauge fields $A_i(\Lambda')$ therefore scale as $1/e_i(\Lambda')$. For a theory with a single $U(1)$ and one particle in the loop, this means [2]

$$A^2(\Lambda') = A^2(\Lambda)\left(1 + \frac{e^2(\Lambda)}{6\pi^2}\ln\left(\frac{\Lambda}{\Lambda'}\right)\right) \tag{1.8}$$

The obvious modifications of this for the theory considered here give (1.4), where the last term is the effective photon-paraphoton interaction. It is finite because there are two particles (Ψ and Ψ') contributing to the diagram with opposite signs.

The Lagrangian (1.4) gives a propagating photon an amplitude to turn into a paraphoton and vice-versa (from the $F_{1\mu\nu}F_2^{\mu\nu}$ term). This is not particularly attractive, so one can define new gauge bosons as linear combinations of the old ones, to get diagonal kinetic terms with the expected normalization. Since there is no reason to think that ordinary charged particles carry fractional paracharge, it would be nice to leave the 'photon' ($\equiv A_1' \propto A_1$) as the gauge boson coupling to electrically charged particles. This means that $\chi_2 F_{2\mu\nu}F_2^{\mu\nu} + 2\chi F_{1\mu\nu}F_2^{\mu\nu}$ needs to be written as $F'_{2\mu\nu}F_2'^{\mu\nu}$ = kinetic terms for some gauge boson $A'_{2\mu}$. $A'_{2\mu}$ and $A'_{1\mu}$ will then propagate independantly of each other. If $A'_{1\mu}(\Lambda') = \chi_1^{1/2}A_{1\mu}(\Lambda)$ then the photon kinetic terms can be written in the ordinary (no factor of χ_1) form. Finding $A'_{2\mu}$ is not as simple, but one can explicitely check that to lowest order in χ

$$A'_{2\mu} = \chi_2^{1/2}A_{2\mu} + \chi\chi_1^{1/2}A_{1\mu} \tag{1.9}$$

will give paraphoton kinetic terms of the desired form. (this argument is made more clearly in [1].)

One now has non-interacting gauge bosons, which has moved the effective interaction between the two $U(1)$s from the photon-paraphoton kinetic term to the gauge boson-fermion interaction term. To lowest order in χ_1, χ_2 and χ one has

$$e_2(\Lambda)A_{2\mu}(\Lambda) = e_2(\Lambda')A_{2\mu}(\Lambda') = e_2(\Lambda')(A'_{2\mu}(\Lambda') - \chi A'_{1\mu}(\Lambda')) \tag{1.10}$$

so the paraton ψ_2 interacts with the photon $A'_{1\mu}$. It has an effective fractional charge

$$\delta = \frac{e_2\chi}{e_1} = \frac{e_2^2}{6\pi^2}\ln\left(\frac{M}{M'}\right) \tag{1.11}$$

that will remain present down to arbitrarily low energies, and is independant of the scale at which the mixing takes place.

This discussion has assumed that the "real" $U(1)$ ($\equiv U(1)_1$) was electromagnetism. However, if $M, M' \geq$ a few hundred GeV (which is likely), the $U(1)$ will be hypercharge, so the fractional electric charge will be 2 to some power times (1.11).

Suppose one puts a particle with fractional hypercharge δ into the Standard Model. The origin of this charge is unimportant—it can be intrinsic, as discussed at the beginning of this introduction, or it can be an effective charge from loop diagrams. The particle is an $SU(2)$ singlet, and after symmetry breaking, will couple to the Z and the photon, since the hypercharge gauge boson can be written as a linear combination of these:

$$B_\mu = \cos\theta A_\mu - \sin\theta Z_\mu \quad . \tag{1.12}$$

(θ is the Weinberg angle.) The paraton-gauge boson interaction term in the Lagrangian is

$$\mathcal{L}_{int} = \frac{\delta}{2}g'\cos\theta\bar{f}\not{A}f - \frac{\delta}{2}g'\sin\theta\bar{f}\not{Z}f \tag{1.13}$$

and since the electromagnetic coupling constant $e = g' \cos\theta$, this implies that the paraton electric charge will be

$$\epsilon = \frac{\delta}{2} = \frac{1}{2}\frac{e_2^2}{6\pi^2}\ln\left(\frac{M}{M'}\right) = \frac{\alpha_2}{3\pi}\ln\left(\frac{M}{M'}\right) \qquad (1.14)$$

Assuming that $10^1 M' > M > 5/2M'$ (M is assumed larger than M'), this gives

$$5\alpha_2 > \epsilon > \frac{\alpha_2}{10} \qquad (1.15)$$

which will be useful later on.

Calculations in this thesis use units where $c = k = h = 1$ and $G = m_{pl}^{-2}$ ... there is a unit conversion table at the end (see table B.1). The metric (when it appears) is $(+,-,-,-)$. Four-vectors are represented by a letter (p) and three-vectors by a vector ($\vec{p}$).

CHAPTER TWO

ACCELERATORS

The most obvious place to look for particles with large fractional charge ($> 10^{-2}$) is in high energy experiments. Numerous searches have been done for the production of quark pairs [3,4,5,6,7] which rule out paratons with $\epsilon > 1/3$ and masses below current accelerator energies. There is also a possible limit from the width of the Z at LEP, but this does not appear to give interesting constraints. For $\epsilon < 1/3$, it is probably reasonable to assume that paratons would not have been detected as charged particles at ASP; their upper limit on the cross section for $e^+e^- \rightarrow$ ($\gamma +$ weakly interacting particles) [10,11] therefore excludes paratons down to $\epsilon \simeq 3 \times 10^{-2}$. And finally for masses below 1 GeV, paratons with $\epsilon > 10^{-2}$ ought to produce a "neutral current type" signal in beam dump experiments which has not been observed [12].

2.1 free quarks

a) SLAC experiment

In 1967 Bellamy et al. [3] looked for the production of fractionally charged particles produced by 12 GeV electrons incident on a copper target. The SLAC electron beam was directed onto 10 radiation lengths of copper (i.e. enough to reduce the beam energy by e^{-10} [13]) where the electrons lose their energy by bremsstrahlung. The photons can then produce pairs of charged particles in the fields of the copper nuclei and nucleons. (The photons can interact with the whole nucleus or with an individual proton.)

The beam of charged particles photo-produced in the copper was tuned to $|\vec{p}/Q = 12.5$ GeV, so that particles with momentum 12.5ϵ GeV would be transmitted through a series of five sodium iodide crystals. This avoids the large muon flux. No particles were observed in all five crystals.

Assuming that the production of paratons is not suppressed by some mechanism (as for quarks), the SLAC experiment forces them to have

$$\begin{array}{ll} \epsilon = 2/3 & \mu > 1.5 \text{ GeV} \\ \epsilon = 1/3 & \mu > 1.0 \text{ GeV} \\ \epsilon = 1/10 & \mu > 0.5 \text{ GeV} \\ \epsilon = 1/25 & \mu > 0.2 \text{ GeV} \end{array} \tag{2.1}$$

(see figure B.1).

b) e^+e^- accelerators

The Jade collaboration looked for particles with charges between 2/3 and 5/3 produced in e^+e^- annihilations at PETRA with a centre of mass energy of 27–35 GeV [4] . Exclusive production of a pair of fractionally charged particles ($e^+e^- \to f\bar{f}$) should produce two collinear tracks in the detector (drift chamber) with a smaller or larger energy loss per unit length than ordinary particles. This is because the energy loss in an ionizable material is, according to the Bethe-Bloch formula [14],

$$\frac{dE}{dx} = \epsilon^2 \left\{ \frac{(4\pi)^3 N_0 \alpha^2}{m_e v^2} \frac{Z}{A} \left[\log\left(\frac{2m_e v^2}{I(1-\beta^2)} \right) - \beta^2 \right] \right\} \tag{2.2}$$

where m_e is the electron mass, v is the paraton velocity ($= \beta$) , N_0 is Avogrado's number, I is the effective ionization potential of the material ($\approx$ 10Z eV), and Z and

A are the charge and atomic number of the material. No such tracks were observed. Jade also looked for inclusive production of fractionally charged particles($e^+e^- \rightarrow f\bar{f} + X$), and found none. Since the 90% confidence level (90% CL) experimental upper bound on the cross section for the production of particles with $\epsilon \geq 2/3$ is $\leq 10^{-2}$ of the theoretical prediction, paratons must have

$$\mu > 12 \text{ GeV} \quad \text{for } \epsilon = 2/3 \ . \tag{2.3}$$

A similiar experiment was done with the TOPAZ detector at TRISTAN [5], looking for the exclusive production of particles with Q = 2/3, 1, and 4/3 at $\sqrt{s} =$ 52 and 55 GeV. Their data force paratons to have

$$\mu > 26 \text{ GeV} \quad \text{for } \epsilon = 2/3 \tag{2.4}$$

(see figure B.1).

A group at SLACs e^+e^- storage ring PEP (positron electron project) [6], built a detector specifically to look for fractionally charged particles, so that they could extend the limit on ϵ down to 0.2 . From their results, paratons must have

$$\mu > 14 \text{ GeV} \quad \text{for } .2 \leq \epsilon \leq .8 \tag{2.5}$$

(see figure B.1).

There was also a free quark search by Mark II at SPEAR [7] but it only rules out particles with $\epsilon \geq 2/3$ and μ <1–3 GeV.

2.2 Z decay

One could hope that the width of the Z measured at LEP would give constraints on paratons in a slightly higher mass range than the free quark searches. Unfortunately, this is not the case. In the Standard Model, the Z is

$$Z^\mu = \cos\theta W^{3\mu} - \sin\theta B^\mu \tag{2.6}$$

where B is the gauge boson of hypercharge, W^3 is the third gauge boson of weak isospin and θ is the Weinberg angle. Since $\sin\theta \sim 1/4$, the Z will not couple very strongly to paratons because they only carry hypercharge. From (1.13) the paraton-Z interaction vertex factor is $i/2\,\delta g'\sin\theta\gamma^\mu = i\epsilon e\tan\theta\gamma^\mu$. The decay rate of a Z is therefore (see chapter 5 for a derivation of this)

$$\Gamma(Z \longrightarrow \bar{f}f) = \frac{\tan^2\theta\epsilon^2\alpha M_Z}{3} = \epsilon^2 \times 6.79 \times 10^{-2} \;\; \mathrm{GeV} \quad . \tag{2.7}$$

This neglects the fermion mass, but including it will only decrease Γ.

The measured decay rate of the Z from ALEPH is $\Gamma_{exp} = 2.68 \pm 0.15$ GeV [15]. If the theoretical width (with three neutrinos) is taken to be $\Gamma_{theo} = 2.487 \pm 0.027$ GeV [15], then this allows a width of roughly $\Delta\Gamma = .307$ GeV for other particles. This rules out

$$\epsilon > 2.3 \tag{2.8}$$

which is not an interesting limit.

2.3 ASP

The Anomalous Single Photon Detector (ASP) [10,11] was designed to look for events of the form

$$e^+e^- \to \gamma + \text{weakly interacting particles} \tag{2.9}$$

at PEP (SLAC's e^+e^- storage ring; $\sqrt{s} = 29$ GeV). Since ordinary charged particle detectors do not seem to be used to search for quarks with $q = 1/3$, it should be reasonable to assume that paratons with $\epsilon < .2$ would not have been detected as charged particles at ASP. This means that the ASP limit can be joined onto the lower end of the PEP limit.

ASP fit the approximately 24 events that survived all their cuts to a signal and background function, and conclude that they have 1.6 events of the form (2.9). They then do a Monte Carlo to calculate, as a function of the number of single photon events $\equiv n_{tot}$, the percentage of experiments like theirs that would observe ≤ 1.6 such interactions. Their 90% CL limit on n_{tot} is the value for which 10% of experiments would see ≤ 1.6 events; which is 4.8 events.

The number of events is $n_{tot} = \sigma_{tot} \times \mathrm{L} \times \varepsilon$, where σ_{tot} is the total cross section for (2.9), L is the luminosity and ε is the probability that a photon will be detected. ASP knows L and ε from radiative Bhabba scattering, so the limit on n_{tot} is a limit on σ_{tot}, which is

$$\sigma_{tot} \leq 0.072 \text{ pb.} \tag{2.10}$$

The "invisible" particles include 3 generations of neutrinos, which correspond to an expected 2.6 events. So subtracting n_ν from n_{tot} leaves $n_f = 2.2$ as an upper limit on the number of paraton pairs produced.

The ASP collaboration does not like this method of removing the neutrinos, because it subtracts more events ($n_\nu = 2.6$) than were actually observed ($n_{obs} = 1.6$). So they define a "likelyhood function" $\mathcal{L}$, which is proportional to the probability of observing 1.6 events when expecting 2.6 + {events from invisible particles other than neutrinos}, and is a function of the number of events due to these other particles $\equiv n_f$. They assume that the number of observed events is a poisson distribution about $2.6 + n_f$ and calculate their 90% CL limit on n_f as ℓ, where

$$\int_0^{\ell} \mathcal{L}(n_f)dn_f = .90 \quad . \tag{2.11}$$

This gives

$$n_f \leq 3.3 \tag{2.12}$$

or

$$\sigma_f \leq 0.049 \text{ pb}. \tag{2.13}$$

It appears that, roughly, the difference between the first and second methods is that in the first, the expected number of neutrino events (= 2.6) is subtracted from the upper limit on n_f (= 4.8), whereas in the second, it is the observed number (= 1.6) that is subtracted. The second method allows the the biggest cross section for the production of "other particles" so gives the weakest limit on the paraton charge. So from (2.13),

$$\sigma(e^+e^- \rightarrow f\bar{f} + \gamma) \leq 0.049 \text{ pb} = 1.26 \times 10^{-10} \text{ GeV}^2 \quad . \tag{2.14}$$

a) what is $\sigma(e^+e^- \rightarrow f\bar{f})$

It should be safe to assume that the photon was emitted by one of the incident electrons, because the paratons are ϵ^2 times less likely to do so. If the paraton has intrinsic fractional hypercharge, it couples to the Z; if it picks up an effective charge from the mixing of the paraphoton with the gauge boson of a Standard Model U(1), this is probably hypercharge, so the paraton again couples to the Z. Z exchange should therefore be included in the calculation of the amplitude for $e^+e^- \rightarrow ff + \gamma$. However, it is very convenient to ignore this contribution. Including Z exchange should increase the cross section, so the limits calculated without it should be conservative.

According to Bonneau and Martin [15], the cross section for $e^+e^- \rightarrow \gamma +$ other things, where the photon is emitted by the incident electron or positron, the mediating particle is a virtual photon, and "other things" are not an e^+e^- pair, can be factorized as

$$(A(q)) \times \sigma(e^+e^- \rightarrow \text{other things}) + O\left(\frac{m_e^2}{E^2}\right) + O\left(\frac{m_e^2}{E(E-q_0)}\right) \qquad (2.15)$$

where E is the energy of the incoming electron or positron in the centre of mass frame, q_0 is the energy of the emitted photon and $A(q)$ is a coefficient that depends on the energy and angle of the photon. At $\sqrt{s} = 29$ GeV, the second two terms will be negligeable unless the photon is very energetic. This will never be the case, because ASP did not include photons with $q_0 > 10$ GeV. (This was to avoid counting $e^+e^- \rightarrow \gamma\gamma$, where one of the photons escaped undetected, as a single photon event.) So neglecting Z exchange, photon emission by the paratons, and the correction terms in (2.15), the cross section for $e^+e^- \rightarrow \gamma f \bar{f}$ is [15]

$$d\sigma_\gamma = \frac{2e^2}{(2\pi)^3} \frac{\sin^2\theta}{(1-(\vec{p}^2/E^2)\cos^2\theta)^2} \frac{dq_0}{q_0} \, d\Omega_{\vec{q}} \, (1 - q_0/E + q_0^2/(2E^2)) \, d\sigma_{s=4E(E-q_0)} \qquad (2.16)$$

where the electron 4-momentum is $(E, -\vec{p})$, the positron 4-momentum is $(E, \vec{p})$, the photon 4-momentum is $(q_0, \vec{q})$, and $0 \le \theta \le \pi/2$ is the angle between $\vec{p}$ and $\vec{q}$.

The ASP angle cuts are $20^o \le \theta \le 160^o$, so if

$$h(\psi) = \int_0^\psi \frac{\sin^2\theta \, d(\cos\theta)}{(1-(\vec{p}^2/E^2)\cos^2\theta)^2} \qquad (2.17)$$

then doing the photon angle integral in (2.16) gives

$$d\sigma_\gamma = \frac{2e^2}{(2\pi)^3} 4\pi \{h(\pi/2) - h(\pi/9)\} \frac{dq_0}{q_0} (1 - q_0/E + q_0^2/(2E^2)) \frac{4\pi\alpha^2\epsilon^2\beta(3-\beta^2)}{24E(E-q_0)} \qquad (2.18)$$

where the cross section for paraton pair production

$$\sigma = \frac{4\pi\alpha^2\epsilon^2}{12E(E-q_0)} \frac{\beta(3-\beta^2)}{2} \qquad (2.19)$$

has been included. β is the velocity of the outgoing paraton or antiparaton. Bonneau and Martin have calculated (2.17) to be

$$h(\psi) = -1/2 + \log\frac{2E}{m_e} + \frac{m_e^2}{2E^2}\frac{\cos\psi}{1-(\vec{p}^2/E^2)\cos^2\psi} - \frac{1}{2}\log\frac{E+\mid\vec{p}\mid\cos\psi}{E-\mid\vec{p}\mid\cos\psi}. \quad (2.20)$$

$h(\psi)$ does not depend on the photon energy, so to within some constants, the q_0 integral is

$$I \equiv \int\frac{dq}{q}(1-q/E+q^2/(2E^2))\frac{4\pi\alpha^2\epsilon^2\beta(3-\beta^2)}{24E(E-q)} \quad (2.21)$$

where

$$\beta = \sqrt{1-\frac{\mu^2}{E(E-q)}} \quad (2.22)$$

$$\frac{3-\beta^2}{2} = 1+\frac{\mu^2}{2E(E-q)} \quad (2.23)$$

and the subscript has been dropped on the q_0. (2.21) can be integrated exactly ...but unfortunately ASPs experimental cuts are $q_0 < 10$ GeV and $\mid\vec{q}_T\mid > .8$ GeV. To calculate the cross section corresponding to the ASP experiment would require evaluating the energy integral in (2.16) first, with a θ dependant lower bound, and then doing the angle integral. This is not analytically feasible. A safe solution is to take the lower energy bound to be $q_{0\,min} = (\mid\vec{q}_T\mid_{min})/(\sin\theta_{min}) = 2.3$ GeV; i.e. the maximum energy possible for a detected photon with the minimum transverse momentum. This ignores photons emitted at larger angles that would have been detected, so the calculated cross section will be smaller than it truly is. (This allows ϵ to be larger, so makes the limit weaker.) The photons are emitted preferentially at low energies and small angles (see 2.16), so taking $q_{0\,min} = .8$ GeV, for instance, would overestimate the cross section.

To evaluate (2.21), define

$$y = 1-\frac{\mu^2}{E(E-q)} \quad (2.24)$$

($y = \beta^2$), so that the integral becomes

$$I = \frac{-\pi\alpha^2}{3E^2}\int \frac{\sqrt{y}}{1-y}\left(\frac{1-y}{1-y-\mu^2/E^2} - \frac{1}{2} - \frac{\mu^2}{2E^2(1-y)} + \frac{(1-y)^2}{2(1-y-\mu^2/E^2)} - \frac{1-y}{4} - \frac{\mu^2}{4E^2}\right)dy \qquad (2.25)$$

or

$$I \equiv I_1 + I_2 + I_3 + I_4 + I_5 + I_6 \qquad (2.26)$$

From Gradshteyn and Ryzhik, (G+R) [16], equation 2.213.1

$$I_1 = \frac{\pi\alpha^2}{3E^2}\left[2\sqrt{y} - \sqrt{1-\mu^2/E^2}\log\left(\frac{1-\mu^2/E^2+y-2\sqrt{(1-\mu^2/E^2)y}}{1-\mu^2/E^2-y}\right)\right] \qquad (2.27)$$

and

$$I_2 + I_6 = \left(1+\frac{\mu^2}{2E^2}\right)\frac{\pi\alpha^2}{6E^2}\left[-2\sqrt{y} - \log\left(\frac{1+y-2\sqrt{y}}{1-y}\right)\right]. \qquad (2.28)$$

I_3 can be evaluated from G+R equation 2.213.5, which gives

$$I_3 = \frac{\pi\alpha^2\mu^2}{6E^4}\left[\frac{\sqrt{y}}{1-y} + \frac{1}{2}\log\left(\frac{1+y-2\sqrt{y}}{1-y}\right)\right] \quad . \qquad (2.29)$$

I_5 is simple:

$$I_5 = \frac{\pi\alpha^2}{18E^2}y^{3/2} \qquad (2.30)$$

and I_4 can be written

$$I_4 = \frac{-\pi\alpha^2}{6E^2}\int\frac{\sqrt{y}}{1-\mu^2/E^2-y}dy + \frac{\pi\alpha^2}{6E^2}\int\frac{y\sqrt{y}}{1-\mu^2/E^2-y}dy\,. \qquad (2.31)$$

The first integral is like I_1, I_2 and I_6, and the second can be evaluated using equation 2.213.2 from G+R, so

$$I_4 = \frac{-\pi\alpha^2}{6E^2}\left[2\sqrt{y}\left(\frac{y}{3} - \frac{\mu^2}{E^2}\right) - \frac{\mu^2}{E^2}\sqrt{1-\mu^2/E^2}\,\log\left(\frac{1-\mu^2/E^2+y-2\sqrt{y(1-\mu^2/E^2)}}{1-y-\mu^2/E^2}\right)\right]. \qquad (2.32)$$

Adding all the pieces together gives

$$\sigma_\gamma = \frac{4\alpha^3\epsilon^2}{3E^2}[h(\pi/2) - h(\pi/9)] \left[\sqrt{y}\left(1 + \frac{\mu^2}{2E^2}\right) \right.$$
$$-\sqrt{1-\mu^2/E^2}\left(1 - \frac{\mu^2}{2E^2}\right)\log\left(\frac{1-\mu^2/E^2 + y - 2\sqrt{y(1-\mu^2/E^2)}}{1-\mu^2/E^2 - y}\right)$$
$$\left. -\frac{1}{2}\log\left(\frac{1+y-2\sqrt{y}}{1-y}\right) + \frac{\mu^2}{2E^2}\frac{\sqrt{y}}{1-y} - \frac{y^{3/2}}{6}\right]\Bigg|_{y_{min}}^{y_{max}} \quad (2.33)$$

which must have numbers put into it. From (2.20) and (2.14), (2.33) becomes

$$2.94 \times 10^{-2} = \epsilon^2\,[\ldots]\,|_{y_{min}}^{y_{max}} \quad (2.34)$$

where [...] is the same as the square brackets in (2.33), and

$$y_{max} = 1 - \frac{2\mu^2}{\sqrt{s}(\sqrt{s}/2 - q_{min})} \quad (2.35)$$

$$y_{min} = \begin{cases} 0 & q_{max} < 10 \text{ GeV} \\ 1 - (2\mu^2)/(s/2 - q_{max}\sqrt{s}) & q_{max} = 10 \text{ GeV} \end{cases} \quad (2.36)$$

$\sqrt{s}$ is 29 GeV, q_{min} is assumed to be 2.3 GeV and q_{max} is the maximum energy the photon can have. If the paraton is light, then q_{max} is ASPs upper energy cut of 10 GeV; if the paraton is heavy, q_{max} is the most energy the photon can have while still leaving enough energy to make the paraton pair. This could be complicated to calculate, because the rest frame of the incident e^+e^- is not the rest frame of the paratons. However, this does not matter because $y = \beta^2$, where β is the paraton velocity in their rest frame, so $y_{min} = 0$.

Finally, putting (2.35) and (2.36) into (2.34) gives

$$\begin{aligned} \mu = 1 \text{ GeV} \quad & \epsilon < .08 \\ \mu = 5 \text{ GeV} \quad & \epsilon < .08 \\ \mu = 10 \text{ GeV} \quad & \epsilon < .09 \\ \mu = 13 \text{ GeV} \quad & \epsilon < .20 \end{aligned} \tag{2.37}$$

(see figure B.1).

2.4 beam dump

Beam dump experiments have often been used to constrain the parameters of weakly interacting particles (see [12] for a list of some of these limits). In the case of paratons, this calculation was done by E. Golowich and R. W. Robinett [12] using data from the E613 experiment at Fermilab.

The idea is that paratons will be produced when the proton beam hits the target, and will reach the detector in large numbers if $\epsilon < .1$ (There are approximately 100 "$\epsilon = 1$ interaction lengths" between the target and detector—if $\epsilon = .1$ this is one.). A large paraton flux in the calorimeter would probably produce hadronic energy and no electrons or muons, so would look like neutrino scattering. The E613 Collaboration estimated that it had less than one hundred such interactions, which gives a limit on the paraton mass and charge from requiring that the predicted flux produce less than 180 (90% CL) "neutrino scattering" interactions.

Golowich and Robinett calculate paraton production in the target from vector meson decay ($pp \rightarrow V + X; V \rightarrow f\bar{f}$; where $V = \rho, \omega, \phi, \text{or} J/\psi$) and from direct Drell-Yan production ($pp \to f\bar{f}$) . They neglect pion decay to $ff + \gamma$ because the paratons would not, on average, reach the detector with enough energy to pass the experimental cut $E_{visible} > 20$ GeV. At $\sqrt{s} = 28$ GeV the average number of particles produced in a collision is 15, and many of them will be pions. Since the incident proton has $E \simeq 400$ GeV, the average pion will have $\sim 400/15 \sim 30$ GeV of energy—dividing this equally among the paratons and photon gives an average energy of 10 GeV.

The limit calculated from these production mechanisms is

$$\begin{array}{ll} \epsilon = 10^{-1} & \mu > 1.4 \text{ GeV} \\ \epsilon = 6 \times 10^{-2} & \mu > .6 \text{ GeV} \\ \epsilon = 3 \times 10^{-2} & \mu > .5 \text{ GeV} \\ \epsilon = 2.5 \times 10^{-2} & \mu > .4 \text{ GeV} \\ \epsilon = 1.8 \times 10^{-2} & \mu > .4 \text{ GeV} \end{array} \tag{2.38}$$

(see figure B.1).

CHAPTER THREE

LAMB SHIFT AND G–2

Another way of setting experimental limits on paratons is to require that their contribution to g–2 and the Lamb shift not disrupt the present agreement between theory and experiment. The Lamb shift gives an interesting constraint, but rough calculations of the g–2 limit indicate that it is within the region ruled out by accelerator experiments, so it will be ignored.

3.1 g–2

The Dirac equation implies that g = 2 for the electron; the non–relativistic limit of the Dirac equation (the Pauli equation) is [17]

$$\left[\frac{1}{2m}(\vec{P}+e\vec{A})^2+\frac{e}{2m}\vec{\sigma}\cdot\vec{B}-e\varphi\right]\psi=E\psi \tag{3.1}$$

where ψ is a two component spinor describing the non–relativistic electron, and $-e\vec{\sigma}/(2m)$ is the magnetic moment $\vec{\mu}$. Defining

$$\vec{\mu}=-g\frac{e}{2m}\vec{S} \tag{3.2}$$

(where $\vec{S}$ is the spin vector $=\frac{1}{2}\vec{\sigma}$) implies g = 2.

Calculating higher order (in α) contributions to the electron scattering amplitude gives, from the first order vertex correction, $g\rightarrow g+\alpha/(2\pi)$. Paratons do not contribute to this, since it is just an ordinary vertex diagram with a photon connecting the incoming and outgoing electrons. The paraton will contribute to only one of the seven second order diagrams—the first order correction with a paraton loop in the photon propagator. It is therefore suppressed by at least a factor of α^2. However, the anomolous magnetic moment of the electron is known to 5 decimal places past this, so it is still possible that g-2 might give an interesting limit.

The contribution of a massive paraton with $\epsilon = 1$ is [18]

$$\Delta g \simeq \frac{\alpha^2}{\pi^2}\frac{1}{45}\left(\frac{m_e}{\mu}\right)^2 \tag{3.3}$$

and if [19]

$$\begin{aligned} \left(\frac{g-2}{2}\right)_{expt} &= (1159652.4 \pm 0.4) \times 10^{-9} \\ \left(\frac{g-2}{2}\right)_{theo} &= (1159652.4 \pm 0.2) \times 10^{-9} \end{aligned} \tag{3.4}$$

then taking the paraton contribution Δg_f to be less than the maximum possible deviation between the theoretical and experimental numbers ($= 1.2 \times 10^{-9}$) gives

$$\mu > 5.0\text{MeV} \qquad (\epsilon = 1) \quad . \tag{3.5}$$

This is well below the accelerator limits. The same calculation for the muon magnetic moment gives $\mu > .1$ GeV, which is still uninteresting. If μ is very small, and $\epsilon < 1$, then the paraton contribution to g-2 is [18]

$$\epsilon^2\left(\frac{\alpha}{\pi}\right)^2\left[\frac{1}{3}\log\frac{m_e}{\mu} - \frac{25}{36} + \frac{\pi^2\mu}{4m_e} - \frac{4\mu^2}{m_e^2}\log\frac{m_e}{\mu} + 3\left(\frac{\mu}{m_e}\right)^2 + \dots\right] \tag{3.6}$$

Taking $\mu = 1$ eV gives

$$\epsilon < 1.3 \times 10^{-2} \tag{3.7}$$

which is again not worth pursuing. The g-2 limit will therefore be ignored.

3.2 Lamb shift

According to the zeroth order solution of the Schrodinger equation for an electron orbiting a proton, all the states with the same principal quantum number (n) are degenerate. One can then add pertubations that split the degeneracy [20]. The spin-orbit interaction adds a term poportional to $\vec{\sigma} \cdot \vec{L}$ that comes from the $\vec{\sigma} \cdot \vec{B}$ term in the Pauli equation and splits the $2P_{1/2}$ and $2P_{3/2}$ states (the subscript is the total angular momentum $|\vec{J}| = |\vec{L} + \vec{S}|$); this is the fine structure of the hydrogen atom. The hyperfine structure causes the separation of the 1S ground state into two levels (containing one and three states) due to the interaction of the electron and proton magnetic moments. The list of "tree level" pertubations continues (Darwin term, etc.), but none of them split the degeneracy between $2P_{1/2}$ and $2S$.

The Lamb shift does separate these two states by approximately 1063 MHz. It is caused by two effects that contribute with opposite sign. The principal one is from the vertex correction, and can be roughly understood as follows [21]. The electron can emit and reabsorb virtual photons, which means there is a "virtual" electromagnetic field surrounding the electron. This causes fluctuations in the position of the electron , so that it sees a "smeared out" (and therefore weaker) Coulomb potential. Since the potential is changing more rapidly near r = 0 where the s-state has the greatest probability of being, the " weakening" is more pronounced for $\ell = 0$ than for $\ell = 1$. Paratons do not contribute to this correction to first order.

The second contribution comes from the "vacuum polarization" correction to the photon propagator. The photon can briefly turn into a pair of charged particles on its way from the electron to the proton, creating a cloud of virtual particles around the nucleus. This shields the proton charge from the electron more effectively at larger distances, so the energy of the $2P_{1/2}$ state is increased relative to the 2S state. The paraton can contribute to this.

a) large mass limit

To order α, the photon propagator is [22]

$$\frac{-ig_{\mu\nu}}{q^2}\sum_i\left[1-\frac{\alpha Q_i^2}{3\pi}\log\frac{\Lambda^2}{m_i^2}+\frac{2\alpha Q_i^2}{\pi}\int_0^1 dz\, z(1-z)\log\left(1-\frac{q^2 z(1-z)}{m_i^2}\right)\right] \tag{3.8}$$

where the sum is over charged particles of mass m_i and charge $Q_i = e_i/e$ in the loop. The first term is just a charge renormalisation term and can be ignored. The second gives the vacuum polarization contribution to the Lamb shift.

Since the proton is far more massive than the electron, the 4-momentum transfer $= q^2$ can be taken to be $-\vec{q}^{\,2}$. The interesting term is therefore

$$\frac{ig_{\mu\nu}}{\vec{q}^{\,2}}\frac{2\alpha Q_i^2}{\pi}\int_0^1 dz\, z(1-z)\log\left(1+\frac{\vec{q}^{\,2}z(1-z)}{m_i^2}\right) \tag{3.9}$$

which gives the paraton contribution for $m_i = \mu$ and $Q_i = \epsilon$. If $\vec{q}^{\,2} \ll \mu^2$, the logarithm can be approximated as $(\vec{q}^{\,2}z(1-z))/\mu^2)$, so that the paraton correction to the photon propagator will be

$$\frac{ig_{\mu\nu}}{\vec{q}^{\,2}}\frac{2\alpha\epsilon^2}{\pi}\frac{\vec{q}^{\,2}}{30\mu^2} \quad . \tag{3.10}$$

An electron orbiting a proton sees a potential [23]

$$A_\mu(x) = \int D_F(x-y)J_\mu(y)d^4y \tag{3.11}$$

where $D_F(x-y)$ is the photon propagator from y to x and J_μ is the proton current. Since the proton is approximately stationary, this can be taken to be $J_\mu(y) = ie\,\delta_\mu^o\,\delta^3(y)$. (The γ^0 matrix is being ignored because this is a non-relativistic problem, so the anti-particle components of the wavefunction are hopefully negligible.) The paraton addition to the potential is therefore

$$\Delta A_\mu(x) = \int d^4y\int\frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\frac{ig_{\mu\nu}}{\vec{q}^{\,2}}\frac{\alpha\epsilon^2\vec{q}^{\,2}}{15\pi\mu^2}ie\,\delta_o^\nu\,\delta^3(y) \tag{3.12}$$

or

$$\Delta A_o(x) = -\frac{\epsilon^2 c\alpha}{15\pi\mu^2}\delta^3(x). \tag{3.13}$$

This is just $(\epsilon^2 m_e^2)/\mu^2 \times$ {the vacuum polarization contribution from electrons}. From first order pertubation theory, this introduces an energy shift for the state ψ of

$$\Delta E_f = <\psi \mid e\Delta A_o \mid \psi> . \tag{3.14}$$

The paraton contribution to the Lamb shift is therefore

$$\Delta E_f = \frac{\epsilon^2 m_e^2}{\mu^2} \times 27.13 \text{ MHz} \tag{3.15}$$

where 27.13 MHz is the vacuum polarization contribution from electrons calculated using (3.14).

The maximum difference between calculations and measurements of the Lamb shift is approximately 0.09 MHz [24], so (3.15) implies

$$8.7 \times 10^{-3}\epsilon < \mu_{\text{GeV}} \qquad (\mu^2 \gg \vec{q}^{\,2}) . \tag{3.16}$$

This is only applicable if $\mu^2 \gg \vec{q}^{\,2}$ because it was derived on the assumption that $\log(1+x) = x$ (where $x \propto \vec{q}^{\,2}/\mu^2$). Bethe et al. [25] have calculated that the average momentum transfer between an $2S$-state electron and the proton is 226 eV, so (3.16) probably applies for $\mu > 1$ keV, or $\epsilon > 10^{-4}$. The limit (3.16) is therefore

$$\begin{aligned} \epsilon = 1 &\quad \mu > 8.7 \text{ MeV} \\ \epsilon = 10^{-2} &\quad \mu > 87 \text{ keV} \\ \epsilon = 10^{-4} &\quad \mu > .87 \text{ keV} \end{aligned} \tag{3.17}$$

(see figure B.1).

CHAPTER FOUR

COSMOLOGY

Cosmological arguments provide two interesting constraints on the parameters of the neutrino: a limit on the number of light neutrino flavours, and a limit on the mass. Both of these can be transposed into constraints on the paraton.

Big Bang nucleosynthesis calculations predict the observed abundances of the light elements (^{4}He, ^{3}He, D, Li) in the universe today. These calculations depend, among other things, on the number of light neutrino flavours (or equivalently, on the energy density) at weak interaction freeze-out (T $\sim$ 1 MeV). One can therefore get an upper bound on N_ν by requiring that the predicted light element abundances not disagree with observation. If the paratons were relativistic at T $\sim$ 1 MeV, they would count as 'too many neutrinos', which gives the first cosmological limit on paratons.

The second constraint on neutrinos and paratons comes from requiring that the relic density in the universe today not exceed the critical density $= \rho_c$ that would make the universe flat. (Inflation predicts $\rho = \rho_c$; most observations suggest that the density is less than ρ_c.) When the temperature of the early universe drops below a given particles mass, these particles would start to annihilate. However, if the universe is expanding fast enough, they would have a small probability of finding each other, which would give a large relic density today.

Both these limits are derived on the assumption that the paratons are in thermal equilibrium with the rest of the matter in the universe. For sufficiently small ϵ, this will clearly not be the case, so for each limit there is going to be some value of ϵ below which the paratons conceivably could exist.

The first section of this chapter is a compilation of equations neccessary for the second two; for a coherent introduction to cosmology see [27,28,29]. Section 4.2 is an explanation of why primordial nucleosynthesis implies $N_\nu < 4.6$; the nucleosynthesis limits on paratons are in 4.3. 4.4 is a review of the approximation used to calculate the paraton number density today, and rough limits on the mass and charge from this are in 4.5. And finally the upper bounds on ϵ from requiring that the paratons be in thermal equilibrium are at the ends of sections 4.3 and 4.5.

4.1 introduction to cosmology

a) the Robertson-Walker metric

The cosmic microwave background radiation (MBR) is evidence that the universe was spatially isotropic (looks the same in all directions) when the photons decoupled. The MBR spectrum is consistent with that of a blackbody at T = 2.7° K, and, if one subtracts out the dipole effects due to the earths motion through the galaxy, it looks the same in all directions down to at least 1 part in 10^4. The photons decoupled from matter when the electrons (re)-combined with the protons to form hydrogen, so the isotropy of the MBR suggests that the universe was very isotropic at $T_{recomb} \simeq 4000°$ K.

On very large scales, greater than those of galaxies, clusters, and superclusters, the universe appears to be, or is assumed to be, spatially homogeneous (looks the same at all points). The galaxy–galaxy correlation function, which measures the inhomogeneity in the distribution of galaxies, gets smaller over larger distances, which suggests that the universe is homogeneous on sufficiently large scales (the correlation function would be zero in a perfectly homogeneous universe); and a universe that is isotropic about all points must be homogeneous, so unless we are located at some privileged position, the universe should be homogeneous.

The metric for a spatially homogeneous and isotropic universe is the Robertson Walker metric [27,30], which can be written

$$ds^2 = dt^2 - \mathrm{R}^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \tag{4.1}$$

where $\mathrm{R}(t)$ is the scale factor of the universe and has units of length (this means r does not). For a closed universe ($k= 1$), the equation in parentheses is just the metric for a three dimensional sphere (using $r = \sin\chi$) in which case $\mathrm{R}(t)$ is the radius of an expanding (if the universe is expanding) sphere.

b) the stress-energy tensor

The stress-energy tensor of a homogeneous and isotropic universe is that of a perfect fluid:

$$T_{\mu\nu} = (\rho + \mathrm{P})U_\mu U_\nu - g_{\mu\nu}\mathrm{P} \tag{4.2}$$

where U is the four velocity of the fluid, ρ is the energy density, and P is the pressure. The fluid can be made of anything: galaxies, dust, radiation ... and is at rest with respect to the homogeneous and isotropic space-like hypersurfaces. In the reference frame of the fluid, it is clear that $T_{\mu\nu}$ must be of the form (4.2): isotropy forces $T_{ij} = -\mathrm{P}g_{ij}$, and since T_{oi} is the flux of energy along i, it is zero because the fluid is at rest.

The conservation of energy and momentum equation is

$$T_{\mu\ ;\nu}^{\ \nu} = 0 = T_{\mu\ ,\nu}^{\ \nu} - \Gamma^\gamma_{\mu\nu} T_\gamma^{\ \nu} + \Gamma^\nu_{\nu\gamma} T_\mu^{\ \gamma} \tag{4.3}$$

where

$$\Gamma^{\nu}_{\nu\gamma} = \frac{1}{\sqrt{-g}}(\sqrt{-g})_{,\gamma} \tag{4.4}$$

and g is the determinant of the metric. One therefore has

$$T_{o\ ;\nu}^{\ \nu} = \frac{1}{\sqrt{-g}}(\sqrt{-g}T_{o}^{\ \nu})_{,\nu} - \Gamma^{\gamma}_{o\nu}T_{\gamma}^{\ \nu} \tag{4.5}$$

which becomes, substituting in the values of the connection coefficients and using the fact that $T_{\mu\nu}$ is diagonal,

$$0 = T_{o\ ;o}^{\ o} = \frac{1}{\sqrt{-g}}(\sqrt{-g}\,T_{o}^{\ o})_{,o} - \frac{\dot{R}}{R}g_{ij}\,T^{ij} \quad . \tag{4.6}$$

This gives

$$0 = (R^3\rho)_{,t} + \mathrm{P}(R^3)_{,t} \tag{4.7}$$

which looks more familiar and meaningful if written

$$0 = \frac{\partial U}{\partial t} + \mathrm{P}\frac{\partial V}{\partial t} \tag{4.8}$$

where U is the energy in some comoving volume V.

c) the Einstein equations

The Einstein equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi \mathrm{G}T_{\mu\nu} \tag{4.9}$$

where $R_{\mu\nu}$ is the Ricci tensor and R is the scalar curvature, not the scale factor R. It can be shown [31] that for a Robertson-Walker metric, they give two equations: a 'time-time' component

$$3\ddot{\mathrm{R}} = -4\pi\mathrm{G}(\rho + 3\mathrm{P})\mathrm{R} \tag{4.10}$$

and a 'space-space' component

$$R\ddot{R} + 2\dot{R}^2 + 2k = 4\pi G(\rho - P)R^2 \qquad (4.11)$$

where $\dot{R} = dR/dt$. Substituting (4.10) into (4.11) gives

$$H^2 \equiv \frac{\dot{R}^2}{R^2} = -\frac{k}{R^2} + \frac{8\pi G}{3}\rho \qquad (4.12)$$

This equation describes the evolution of a homogeneous and isotropic universe. With the conservation of energy equation, it contains all the information in the Einstein equations because (4.10) and (4.11) are related by the Bianchi identities: substituting $\dot{R}$ from the conservation of energy equation (4.7) into (4.12) gives the 'space-space' component of the Einstein equations, and the 'time-time' component can be derived from this using (4.12).

Both H (the Hubble parameter) and ρ (the energy density), are in principle measurable today, so it should be possible to determine whether the universe is open (k= −1), flat (k= 0), or closed (k = +1) from (4.12). Theoretical prejudice favours a flat (k = 0) universe because (a 'long enough' period of) inflation predicts it, and the cold dark matter theory of galaxy formation seems to require it. Measurements of ρ are therefore often written as a fraction

$$\Omega = \frac{\rho}{\rho_c} \qquad (4.13)$$

where ρ_c is the critical density that gives k = 0:

$$\rho_c = \frac{3H^2}{8\pi G} \qquad (4.14)$$

Measurements of Ω go as high as $4\rho_c$, but most are clustered around $\Omega = .1$ [32]. (These are measurements of the total mass density on the scale of galaxies and clusters; the luminous mass on the same scale is an order of magnitude smaller, and Ω could be larger on larger scales.)

a, the early universe

In a universe filled with an ideal gas of non-relativistic ($m \gg$ T) particles ('matter')

$$\mathrm{P}_{mat} = n_{mat}\mathrm{T} \quad \ll \quad \rho_{mat} = m n_{mat} \tag{4.15}$$

where n_{mat} is the number density of particles of mass m. The energy conservation equation (4.7) therefore implies

$$n_{mat} \propto \mathrm{R}^{-3} \quad . \tag{4.16}$$

However, if the particles are relativistic ($m \ll$ T), their masses are negligible and they behave like radiation, so

$$\mathrm{P}_{rad} = \frac{\rho_{rad}}{3} \qquad \rho_{rad} \propto \mathrm{T}^4 \quad . \tag{4.17}$$

Equation (4.7) then gives

$$\mathrm{R}^4 \dot{\rho}_{rad} + 4\rho\,\mathrm{R}^3\dot{\mathrm{R}} = 0 \quad \Longrightarrow \rho_{rad} \propto \mathrm{R}^{-4} \tag{4.18}$$

For R $\rightarrow 0$, $\rho_{rad} \gg \rho_{mat}$, which just says that the energy density of the hot early universe will be dominated by relativistic particles, as expected. In the radiation dominated era, the Einstein equation (4.12) can be taken to be

$$H^2 = \left(\frac{\dot{\mathrm{R}}}{\mathrm{R}}\right)^2 = \frac{8\pi\mathrm{G}}{3}\frac{c}{\mathrm{R}^4} \tag{4.19}$$

(where c is a constant relating the energy density to the scale factor) because

$$\frac{k}{\mathrm{R}^2} \ll \frac{8\pi\mathrm{G}}{3}\frac{c}{\mathrm{R}^4} \tag{4.20}$$

for small R.

For an ideal gas of fermions (+) or bosons (−) in equilibrium at temperature T, the number density per unit momentum range is

$$n_i(\vec{q}) = g_i \frac{1}{e^{(E-\nu)/\mathrm{T}} \pm 1} \tag{4.21}$$

where $E = \sqrt{\vec{q}^2 + m^2}$, ν is the chemical potential (to avoid confusion with the paraton mass μ), i labels the particle species and g_i is the number of spin states of the particle. The chemical potential is taken to be zero, on the assumption that the universe has no net number density of any conserved quantum numbers. g_i can be confusing to count; I will treat the particles and anti-particles as one species, so that $g_i = 2$ for Majorana fermions and 4 for Dirac fermions. The number density is therefore

$$n_i = \frac{4\pi g_i}{(2\pi)^3} \int_0^\infty \frac{q^2\, dq}{e^{E/\mathrm{T}} \pm 1} \tag{4.22}$$

and the energy density is

$$\rho_i = \frac{4\pi g_i}{(2\pi)^3} \int_0^\infty \frac{E\, q^2\, dq}{e^{E/\mathrm{T}} \pm 1} \tag{4.23}$$

In the relativistic limit the mass can be neglected, so

$$n_i = \frac{g_i}{2\pi^2} \mathrm{T}^3 \int_0^\infty \frac{x^2\, dx}{e^x \pm 1} \quad (x = q/\mathrm{T}) \tag{4.24}$$

$$= \frac{g_i}{\pi^2} \zeta(3) \mathrm{T}^3 \quad \text{(for bosons)} \tag{4.25}$$

$$= \frac{g_i}{\pi^2} \frac{7}{8} \zeta(3) \mathrm{T}^3 \quad \text{(for fermions)} \tag{4.26}$$

where $\zeta(3) \simeq 1.202$. In the non-relativistic limit, $e^{E/\mathrm{T}} \gg 1$ and $E \simeq m + \vec{q}^2/(2m)$, which gives

$$n_i = \frac{g_i}{2\pi^2} e^{-m/\mathrm{T}} (2m\mathrm{T})^{3/2} \int_0^\infty e^{-x^2} x^2\, dx \quad (x^2 = \vec{q}^2/(2m\mathrm{T}))$$

$$= g_i e^{-m/\mathrm{T}} \left(\frac{m_i \mathrm{T}}{2\pi} \right)^{3/2} \quad . \tag{4.27}$$

The energy density for non-relativistic particles is just

$$\rho_i = m_i n_i \tag{4.28}$$

and for relativistic particles it is

$$\rho_i = \frac{g_i}{2\pi^2} T^4 \int_0^\infty \frac{x^3\,dx}{e^x \pm 1} \quad (4.29)$$

$$= \frac{g_i T^4}{2\pi^2}\left(\frac{\pi^4}{15}\right) \quad \text{(bosons)} \quad (4.30)$$

$$= \frac{g_i T^4}{2\pi^2}\frac{7}{8}\left(\frac{\pi^4}{15}\right) \quad \text{(fermions)} \ . \quad (4.31)$$

The total energy density of all the relativistic particles is

$$\rho = \frac{g_{eff}(T)}{2}\frac{\pi^2 T^4}{15} = \frac{g_{eff}(T)}{2}\rho_\gamma \quad (4.32)$$

where $g_{eff}(T)$ is the effective number of spin degrees of freedom of the relativistic particles in equilibrium at temperature T, counting 1 for each boson spin state and 7/8 for each fermion spin state:

$$g_{eff} = \frac{7}{8} g_F \quad \text{for fermions}$$

$$g_{eff} = g_B \quad \text{for bosons} \quad (4.33)$$

See table B.2 for a list of g_{eff} as a function of temperature. Relativistic particles that are not in equilibrium will also add to g_{eff}, but their contribution will be multiplied by $(T/T_\gamma)^4$, where T is their temperature.

e) entropy

The second law of thermodynamics for particles in equilibrium in a volume V at a temperature T with zero chemical potential is

$$T dS = dE + P\,dV \ . \quad (4.34)$$

The right hand side is $d(\rho V) + P\,dV$, and the conservation of energy equation (4.7) gives

$$\frac{d}{dt}(\rho V) + P\frac{dV}{dt} = 0. \quad (4.35)$$

One would therefore expect the entropy in a comoving volume to be constant. This is in fact the case. Taking S to be a function of V and T, (4.34) becomes

$$\mathrm{T}dS(V,\mathrm{T}) = d(\rho(\mathrm{T})V) + \mathrm{P}(\mathrm{T})dV \qquad (4.36)$$

so that

$$\frac{\partial S}{\partial V} = \frac{1}{\mathrm{T}}(\rho + \mathrm{P}) \qquad \frac{\partial S}{\partial \mathrm{T}} = \frac{V}{\mathrm{T}}\frac{d\rho}{d\mathrm{T}} \qquad (4.37)$$

and requiring that $\partial^2 S/(\partial T \partial V)$ be the same, calculated from either of these equations, gives

$$\frac{\partial \mathrm{P}}{\partial \mathrm{T}} = \frac{1}{\mathrm{T}}(\rho + \mathrm{P}). \qquad (4.38)$$

The energy conservation equation (4.7) can be written $V\partial \mathrm{P}/\partial t = \partial/\partial t[V(\rho + \mathrm{P})]$, and using (4.38), this implies

$$\frac{\partial}{\partial t}\left[\frac{V}{\mathrm{T}}(\rho + \mathrm{P})\right] = 0. \qquad (4.39)$$

This is the conservation of entropy equation. Taking the differential of what is in brackets and using (4.38) gives

$$d\left[\frac{V}{\mathrm{T}}(\rho + \mathrm{P})\right] = \frac{1}{\mathrm{T}}d(\rho V) + \frac{\mathrm{P}}{\mathrm{T}}dV = dS \qquad (4.40)$$

so $V(\rho + \mathrm{P})/\mathrm{T}$ is really the entropy S. In a radiation dominated universe, $\rho + \mathrm{P} \propto \mathrm{T}^4$, so from (4.39)

$$\mathrm{T} \propto \frac{1}{\mathrm{R}} \quad \text{(radiation dominated).} \qquad (4.41)$$

This assumes that all the particles present are in thermal equilibrium, i.e. that the density and pressure in the thermodynamic equation (4.34) are the same as those in the conservation of energy equation (4.3). If there are n non-interacting gases, so that the gravitational density is

$$\rho = \sum_{i=1}^{n} \rho_i \qquad (4.42)$$

and the thermodynamic identity becomes

$$\mathrm{T}_i dS_i = d(\rho_i V) + P_i dV \quad \forall\ i \tag{4.43}$$

then the conservation of entropy equation is

$$\sum_i \mathrm{T}_i \frac{dS_i}{dt} = 0 \ . \tag{4.44}$$

Assuming that the entropy of each non-interacting gas "i" can not decrease, this still implies that the entropy of each gas is constant.

Entropy conservation means that the logarithm of the number of states accessible to the particles contained in a comoving volume is constant. As the universe expands, the different particle species will become non-relativistic and annihilate when the temperature drops below their mass. To keep the entropy constant, the temperature of the remaining relativistic particles in equilibrium must increase by some amount, which can be calculated from (4.39). If $n_{eff}(\mathrm{T}_b)$ and $n_{eff}(\mathrm{T}_a)$ are the effective number of spin degrees of freedom before and after particle species "i" annihilates, then

$$\frac{V}{\mathrm{T}_a}\frac{4}{3}\,\frac{n_{eff}(\mathrm{T}_a)}{2}\,\frac{\pi^2\mathrm{T}_a^4}{15} = \frac{V}{\mathrm{T}_b}\frac{4}{3}\,\frac{n_{eff}(\mathrm{T}_b)}{2}\,\frac{\pi^2\mathrm{T}_b^4}{15} \tag{4.45}$$

or

$$\mathrm{T}_a = \left[\frac{n_{eff}(\mathrm{T}_b)}{n_{eff}(\mathrm{T}_a)}\right]^{1/3} \mathrm{T}_b \ . \tag{4.46}$$

It is important to note that n_{eff} is somewhat different from the g_{eff} in (4.32). T_a and $n_{eff}(\mathrm{T}_a)$ are the temperature and effective number of spin degrees of freedom of the particles in equilibrium with whatever the "i"s annihilate to, and T_b and $n_{eff}(\mathrm{T}_b)$ are for the same particles plus the "i"s. g_{eff} is the total number of effective degrees of freedom of all the particles present—including those not in equilibrium with the "i"s.

For instance, below T $\sim$ 1 MeV the neutrinos are no longer in equilibrium with the photons and electrons because they interact to weakly (see next section). So when the electrons annihilate, the photon temperature will increase from T_b to T_a, where

$$T_a = \left[\frac{2+\frac{4\times7}{8}}{2}\right]^{1/3} T_b \quad . \tag{4.47}$$

The neutrinos will remain at T_b.

The temperature of a gas of massless particles always scales as $1/R$ because the energy is inversely proportional to the wavelength. So if the neutrinos are massless, there should be a 'neutrino microwave background' at $T_\nu = (4/11)^{1/3} T_\gamma$ (where $T_\gamma \simeq 2.7^\circ$ K).

f) equilibrium

A particle species is assumed to be in equilibrium if $\tau \equiv$ the time scale for the ensemble to change its energy (not including rest mass) by an amount comparable to its initial value, is less than the age of the universe.

In the early radiation-dominated universe where $\rho \propto R^{-4}$ and k is negligible:

$$H = \frac{\dot{R}}{R} = \sqrt{\frac{8\pi G\rho}{3}} = \frac{1.66}{m_{pl}}\sqrt{g_{eff}}\,T^2 \tag{4.48}$$

(using 4.32). One can also write

$$H = -\frac{\dot{\rho}}{4\rho} \tag{4.49}$$

which gives

$$t = \left[\frac{32\pi G\rho}{3}\right]^{-1/2} = \frac{1}{2H} \quad \text{(early universe)}. \tag{4.50}$$

The age of the universe is therefore roughly one over the expansion rate. Putting units into (4.50) gives

$$t_{sec} \simeq \frac{1}{T^2_{MeV}} \quad \text{(early universe).} \tag{4.51}$$

The energy exchanged in an interaction between relativistic particles is of the order the energy of the particles, so the time scale for a particle "i" to change its energy by an order of magnitude is

$$\tau \sim \frac{1}{\Gamma_i} \tag{4.52}$$

where Γ_i is the interaction rate for particle species "i" with everything else. Technically, Γ_i should be

$$\Gamma_i = \sum_j \int n_j(\vec{q})\sigma_{ij}\beta\, d^3q \tag{4.53}$$

where the sum is over the various particles that the "i"s can interact with, who are assumed to be in equilibrium and have number densities per unit momentum range, (see 4.21), equal to $n_j(\vec{q})$. β is the relative velocity of the two particles, σ_{ij} is the cross section for an "i" to interact with a "j", and the integral is over the distribution of "j"s in momentum space. I am going to approximate this as

$$\Gamma_i(T) = \sum_j n_j(T)\sigma_{ij}\beta \tag{4.54}$$

where $n_j(T)$ is now the "j" number density, and $\beta \sim 1$ in the relativistic limit. Therefore, the "i" particles are in equilibrium if

$$\frac{1}{\Gamma_i} \ll \frac{1}{2H} \quad \text{or} \quad \Gamma_i \gg H \tag{4.55}$$

As an example, consider the interaction of light neutrinos with other light leptons. For $T \ll m_W, m_Z$, weak interaction cross sections are roughly

$$\sigma \sim \frac{g^2 T^2}{m_W^4} \tag{4.56}$$

(where g is the weak isospin coupling constant) so the interaction rate for a neutrino will be (using 4.26 and 4.54)

$$\Gamma \sim \sum_i \left(\frac{g_i \mathrm{T}^3}{\pi^2}\right) \frac{g^2 \mathrm{T}^2}{m_W^4} \tag{4.57}$$

where the sum is over the other leptons.

If the neutrinos are in equilibrium with the thermal bath of leptons and other particles when $\Gamma \gg H$, and decouple when $\Gamma \approx H$, then (for $\mathrm{T} < m_W$) they will be in equilibrium at high temperatures and decouple as the temperature drops. This is because $\Gamma \sim \mathrm{T}^5$ and $H \sim \mathrm{T}^2$. To estimate the temperature at which this happens, set $\Gamma = H$, which gives, using (4.48) and (4.57) with $g_i = 4$ for the electrons,

$$\frac{1.66\sqrt{g_{eff}}}{m_{pl}} \mathrm{T}_d^2 \approx \frac{4\mathrm{T}_d^3}{\pi^2} \frac{g^2 \mathrm{T}_d^2}{m_W^4} \quad . \tag{4.58}$$

This gives a neutrino decoupling temperature of

$$\mathrm{T}_d \sim 2 \text{ MeV} \quad . \tag{4.59}$$

The actual temperature is closer to 3.5 MeV for ν_τ and ν_μ, who only interact via neutral currents, and 2 MeV for ν_e [33].

4.2 nucleosynthesis

There are two pieces of good observational evidence for the Hot Big Bang model: the isotropy of the cosmic microwave background radiation and the observed abundances of the light elements. Primordial nucleosynthesis calculations predict the observed mass fraction of ^{4}He ($Y_p \equiv (4n_{He})/n_B \sim .25$, where n_B is the baryon number density), and number densities relative to hydrogen of ^{3}He ($\sim 10^{-5}$), D ($\sim 1 - 5 \times 10^{-5}$), and Li ($\sim 10^{-10}$) [33]. This agreement between theory and observation, across eight orders of magnitude, is impressive support for the theory, particularily since there are no other scenarios (at present) that can account for these abundances.

The theoretical calculations depend, among other things, on the number of relativistic particle species present when the weak interactions freeze out. By requiring that the predictions not disagree with observation, one can get a constraint on the number of relativistic particle species at freeze out, which is usually written as an upper bound on the number of (light) Majorana neutrino flavours [33,34,35], and which I will take to be 4.6 [35]. Assuming that there are three massless ($m_\nu \ll$ 1 MeV) neutrinos, this means that the paratons can not be relativistic and in equilibrium when the weak interactions freeze out.

a) making Helium

As calculated at the end of section 1, neutrinos stay in equilibrium with the rest of matter until $T_d \sim$ a few MeV. Above this temperature the neutron to proton ratio will be kept at its equilibrium value by the reactions

$$\begin{aligned} n + e^+ &\longleftrightarrow p + \bar{\nu} \\ n + \nu &\longleftrightarrow p + e^- \\ n &\longleftrightarrow p + e^- + \bar{\nu} \end{aligned} \tag{4.60}$$

At some temperature slightly below T_d, the reactions (4.60) can no longer keep up with the expansion rate, and the neutron to proton ratio will 'freeze-out' at

$$\frac{n_n}{n_p} \propto e^{-\Delta m/T_f} \tag{4.61}$$

where $\Delta m = m_n - m_p \simeq 1.3$ MeV and $T_f \simeq .8$ MeV is the freeze-out temperature. n_n/n_p then decreases slowly as the neutrons decay to protons.

Baryons are sufficiently rare in the early universe that three or four body interactions are very unlikely. Nuclei therefore have to be made by a series of two-body processes, the first of which is the making of a deuterium nucleus. The deuterium binding energy $\equiv E_B$ is 2.2 MeV, so they can be made about the time the neutrinos decouple. However, there are far more photons in the universe than there are baryons—the baryon to photon ratio $\equiv \eta$, is

$$10^{-9} > \eta > 10^{-10}. \tag{4.62}$$

For temperatures just below 2.2 MeV, the average photon energy will be less than the deuterium binding energy, but there will be enough high energy photons to disassociate any deuterium as soon as it is formed. If Γ_1 is the rate, per neutron, at which deuterium is formed, and Γ_2 is the rate per deuterium nucleus at which it is photo-disassociated, then an appreciable amount will start to form when $\Gamma_1 > \Gamma_2$:

$$n_n > n_\gamma(E_\gamma > E_B) \simeq n_\gamma e^{-E_B/\mathrm{T}} \tag{4.63}$$

or at a temperature T_n (nucleosynthesis temperature), where

$$\log \eta > E_B/\mathrm{T}_n. \tag{4.64}$$

This happens at $\mathrm{T}_n \sim .1$ MeV. The binding energy of the other light nuclei is larger than that of deuterium, so their equilibrium abundance is much higher (it takes a more energetic photon to dissassociate them). However, since the heavier nuclei must be made by two-body processes, nucleosynthesis must wait until deuterium is formed. It therefore proceeds very rapidly once the temperature is past the "deuterium bottleneck", via reactions like

$$n + p \longrightarrow \mathrm{D} + \gamma$$

$$\mathrm{D} + \mathrm{D} \longrightarrow {}^3\mathrm{He} + n$$

$$\mathrm{D} + \mathrm{D} \longrightarrow \mathrm{T} + p$$

$$\mathrm{D} + \mathrm{D} \longrightarrow {}^4\mathrm{He} + \gamma \qquad (4.65)$$

$$\mathrm{T} + \mathrm{D} \longrightarrow {}^4\mathrm{He} + n$$

$$\mathrm{D} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + p$$

$${}^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + 2p$$

and almost all the neutrons become part of helium nuclei. Since there are no stable nuclei with atomic masses 5 and 8, it is difficult to make nuclei heavier than ^{4}He by two-body processes, and as the temperature drops, it becomes increasingly difficult for the protons to penetrate the Coulomb barriers of the nuclei. This is why primordial nucleosynthesis only makes light nuclei, principally ^{4}He.

If the neutron to proton ratio freezes out at T $\sim$ .8 MeV, $n_n/n_p \sim 1/6$ at this time. The neutron to proton ratio then decreases due to β decay, so at T $\sim$ .1 MeV (nucleosynthesis)

$$\frac{n_n}{n_p} \sim \frac{1}{6} e^{-t/\tau} \qquad (4.66)$$

where τ is the neutron half life and t $\simeq$ 100 seconds (see 4.51) is the time between freeze-out and nucleosynthesis ($\simeq$ the age of the universe at nucleosynthesis, since it is only $\simeq$ 1.5 seconds old at freeze-out). This gives $n_n/n_p \simeq 1/7$ at nucleosynthesis. If all the neutrons end up in helium, the primordial helium mass fraction will be

$$Y_p = \frac{2n_n/n_p}{1 + n_n/n_p} \simeq .25 \qquad (4.67)$$

which agrees well with the observational estimates $.22 < Y_p < .26$ [33].

b) parameters this calculation depends on

The value of Y_p predicted by numerical calculations depends on $\tau_{1/2} \equiv$ the neutron half life, N_ν = the number of light Majorana neutrino flavours, and η = the baryon to photon ratio [33].

Almost all the neutrons are incorporated into ^{4}He at nucleosynthesis, so Y_p is essentially determined by the neutron to proton ratio at that time. This depends on the temperature at which n_n/n_p freezes out and on the fraction of neutrons that decay to protons between freeze-out and nucleosynthesis. Both of these effects depend on $\tau_{1/2}$, where in the first case $\tau_{1/2}$ is a measure of the strength of the charged current weak interactions.

The neutrinos decouple when the weak interaction rate ($\equiv A\mathrm{T}^5$; see 4.57) can no longer keep up with the expansion rate ($\equiv B\mathrm{T}^2$; see 4.48), or when

$$\mathrm{T}_d^3 \approx \frac{B}{A}. \tag{4.68}$$

The more strongly interacting the neutrinos are (the larger A is), the longer they will stay in equilibrium so the lower T_d (and therefore T_f) will be. Since $n_n/n_p \sim e^{-\Delta m/\mathrm{T}_f}$, this means that the number of neutrons at freeze out will decrease, giving less ^{4}He . If $\tau_{1/2}$ decreases, then the weak interaction rate increases, so that

$$\mathrm{T}_d \propto [\tau_{1/2}]^{1/3} \tag{4.69}$$

As $\tau_{1/2}$ increases, the number of neutrons at freeze out increases.

The neutrons decay to protons between freeze out and nucleosynthesis, so as $\tau_{1/2}$ increases, a smaller fraction of them decay, which will increase the neutron to proton ratio at nucleosynthesis. This is insignificant compared to the first effect: if the neutron half life is increased by an amount $\Delta\tau_{1/2}$, then the argument of the exponential in (4.66) changes by an amount

$$t/\tau \rightarrow t/\tau(1-\Delta\tau/\tau) \tag{4.70}$$

and the argument of the exponential in (4.61) becomes (using 4.69)

$$\frac{\Delta m}{T_f} \rightarrow \frac{\Delta m}{T_f}(1-\frac{\Delta\tau}{3\tau}) \tag{4.71}$$

where $\Delta m \sim T_f$,and $t \sim 100$ seconds, $\tau \simeq 10.4$ minutes. ($\Delta\tau \sim .2$ minutes = an upper bound on the experimental uncertainty on the neutron half-life).

So as $\tau_{1/2}$ increases, the weak interaction rate decreases, the neutron to proton ratio freezes out earlier and more ^{4}He will be produced.

T_d can be changed (see 4.68) by varying A ($\propto$ the strength of the weak interactions), or B ($\propto$ the energy density at freeze out). As the energy density increases, the universe expands faster, so the weak interactions freeze out earlier. This increases the neutron to proton ratio and produces more ^{4}He. An upper bound on Y_p therefore gives an upper bound on the energy density at freeze out, if all the other parameters are fixed.

At T $\sim$ a few MeV, the universe is radiation dominated, so the density is (see 4.32) $\rho = \frac{1}{2}g_{eff}\rho_\gamma$. An increase in ρ is therefore equivalent to an increase in g_{eff}, since ρ_γ depends only on the temperature. $g_{eff}(T_f)$ is usually calculated by considering only the photons, the electrons and some number N_ν of (light Majorana) neutrinos. So an upper bound on ρ is an upper bound on N_ν.

The third parameter that the predicted helium mass fraction depends on is the baryon to photon ratio η. As it increases, the number of 'energetic' photons per nucleon decreases, so the temperature $\equiv T_n$ at which an appreciable amount of deuterium can be formed increases. This allows less time for the neutrons to decay (see 4.51) , and makes it slightly easier for the baryons to find each other at T_n because the density is higher. T_n depends logarithmically on η (see 4.64), so Y_p is not strongly dependant on the baryon to photon ratio. This is fortunate, because it is difficult to measure.

Before considering how to measure η, it is important to know whether a lower or upper bound is wanted. Increasing the number of baryons per photon increases the amount of ^{4}He produced. But Y_p can be measured—what is wanted is an upper bound on N_ν, from the measured values of Y_p, $\tau_{1/2}$, and η. So one needs an upper bound on Y_p (because Y_p increases with N_ν), and a lower bound on $\tau_{1/2}$ and η, because Y_p decreases as they do, so more neutrino flavours would be allowed for small values of the neutron half life and the baryon to photon ratio.

Since the baryon density of the universe is not known, it is very difficult to calculate η directly. However, the amount of ^{3}He and D left over after nucleosynthesis depends on the competition between the reaction rates ($\propto \eta$, since the D and ^{3}He number density increases with η) and the expansion rate. $H(T_n) \propto T_n^2$, and T_n increases as η does, but this dependance is only logarithmic, and can be ignored in comparison with that of the reaction rates. So as η decreases, the D and ^{3}He are less efficiently transformed into ^{4}He, and an upper bound on their primordial abundance gives a lower bound on η, as desired.

Unfortunately, we can only measure the abundances today, not the primordial ones ... this makes the upper bound on (D+ ^{3}He)/H difficult to calculate. It is the cause of the discrepancy between the upper limits on N_ν of [34] ($N_\nu < 5.5$–6) and of [35] ($N_\nu < 4.6$). ([34] were using an anomalous measurement of the present

(D + ^{3}He)/H ratio that was a factor of 5 larger than [35]s upper bound, and which was subsequently decreased by an order of magnitude.) The idea is that although D is easily destroyed (it is burned to ^{3}He in stars), ^{3}He is less so. By estimating the fraction of ^{3}He that will survive stellar burning, one can get a (rough) upper bound on the primordial value of (D+^{3}He)/H [33,35].

The numerical calculations of Y_p give a value that is well approximated by [35]

$$Y_p = .230 + .011 \log \eta_{10} + .013(N_\nu - 3) + .014(\tau_{1/2} - 10.6) \tag{4.72}$$

where $\eta = \eta_{10} \times 10^{-10}$ and $\tau_{1/2} \approx 10.6$ minutes is measured in minutes. Rearranging, and using $y_{23P} \equiv$ the primordial value of (D + ^{3}He)/H instead of η gives [35]

$$N_\nu = 3 + (10.6 - \tau_{1/2}) + (Y_p - .243)/.014 + 9/7 \log(10^4 y_{23P}) \tag{4.73}$$

which becomes, using $\tau_{1/2} > 10.2$ minutes, $Y_p < .26$ and $y_{23P} < 10^{-4}$ [35]

$$N_\nu < 4.6 \quad . \tag{4.74}$$

4.3 limits on paratons from nucleosynthesis

a) lower bound on the paraton mass

Assuming three light neutrinos, (4.74) allows 1.6 extra Majorana neutrinos, or their equivalent in other particles.

In the model where a paraton has intrinsic fractional hypercharge (no para-photon), a paraton and anti-paraton with $\mu < 1$ MeV would be relativistic when the neutron to proton ratio freezes out, and would count as two neutrinos. This is ruled out by (4.74), so if the paratons are in thermal equilibrium with the photons at T> T_f (so that they contribute an energy density $\propto T_\gamma^4$), one has (see B.2)

$$\mu > 1 \text{ MeV} \quad (\text{without } \gamma'). \tag{4.75}$$

For small ϵ, the paratons will not be in equilibrium and this limit will not apply. This will be discussed later.

(4.75) is a rough limit on μ; the neutron to proton ratio freezes out around .8 MeV, and it is difficult to calculate the energy density of a particle in equilibrium at $\mu \sim$ T (one can not approximate the integral in 4.23).

In the model with a paraphoton, the mass limit is considerably more stringent. The paratons annihilate principally to paraphotons, which raises the paraphoton temperature with respect to that of the photons, and means that the γ' contribute (see 4.32)

$$g_{eff} = 2\left(\frac{T_{\gamma'}}{T_\gamma}\right)^4 \tag{4.76}$$

where $T_{\gamma'}$ is determined from (4.46) to be

$$T_{\gamma'} = \left[\frac{2+\frac{7\times 4}{8}}{2}\right]^{1/3} T_\gamma. \tag{4.77}$$

1.6 neutrinos contribute

$$g_{eff}(1.6\ \nu) = \frac{7\times 1.6\times 2}{8} = 2.8 \tag{4.78}$$

which is less than $7.7 = g_{eff}(\gamma')$ from (4.76). The photon gas must therefore be heated by annihilations between T $\sim \mu$ and weak interaction freeze out.

The temperature at which the paratons must annihilate is calculated by requiring that the paraphotons contribute less than 1.6 neutrinos to g_{eff} at weak interaction freeze out:

$$g_{eff}(\gamma', T_f) = 2\left(\frac{T_{\gamma'}}{T_\gamma}\right)_f^4 \leq 2.8 \tag{4.79}$$

or

$$\left(\frac{T_{\gamma'}}{T_\gamma}\right)_f^4 \leq 1.4 \tag{4.80}$$

From (4.76),

$$\left(\frac{T_{\gamma'}}{T_\gamma}\right)_a^4 = 3.9 \tag{4.81}$$

just after the paratons annihilate, so the photon temperature must increase with respect to that of the paraphotons by $(3.9/1.4)^{1/4}$. But from (4.46) this is just

$$\left(\frac{3.9}{1.4}\right)^{1/4} = \left(\frac{n_{eff}(T_a)}{n_{eff}(T_d)}\right)^{1/3} \tag{4.82}$$

where $n_{eff}(T_a)$ { $n_{eff}(T_d)$ } is the effective number of degrees of freedom of the gas of particles in equilibrium with the photons just after the paratons annihilate { just before the neutrinos decouple }. $n_{eff}(T_d) = 2 + 7/2 + 3 \times 7/4$ (for photons, electrons, and neutrinos) so from (4.82)

$$n_{eff}(T_a) = \left(\frac{3.9}{1.4}\right)^{3/4} \frac{43}{4} = \frac{93}{4} \tag{4.83}$$

which implies $T_a > T_c$ (see table I), where T_c is the quark–hadron transition temperature, 200 MeV $< T_c <$ 400 MeV. So one has (see B.30

$$\mu > 200 \text{ MeV} \quad (\text{with } \gamma'). \tag{4.84}$$

b) upper bound on ϵ

The lower bounds on μ assume that the paratons would be in equilibrium with the electrons and photons at T $\sim$ a few MeV. For very small ϵ this will not be the case.

The paratons are assumed to be in equilibrium when their interaction rate with ordinary matter is much greater than the expansion rate of the universe. However, unlike the weak interaction cross sections below m_W, paraton cross sections decrease as the temperature rises, so the paraton interaction rate increases more slowly with temperature than the expansion rate. This suggests that paratons will behave in exactly the opposite way from neutrinos—instead of being in equilibrium at high temperatures and freezing out as the temperature drops, they will be in equilibrium at low temperatures but not at high ones.

For sufficiently small ϵ, the paratons will not be in equilibrium at T $\sim$ a few MeV, their temperature will be unknown, and the lower bound on μ from $N_\nu < 4.6$ may not apply. To calculate this value of ϵ, assume that the paratons come into thermal equilibrium when $\Gamma = H$, and that this must happen before T = 5 MeV for the mass limit to apply.

At T $\sim$ a few MeV, the only charged particle available for the paratons to interact with is the electron, so temporarily ignoring collisions with photons, the interaction rate is (see 4.54)

$$\Gamma \sim n_e \sigma(fe \to fe)\beta + \frac{n_f}{4}\sigma(f\bar{f} \to e^- e^+)\beta \tag{4.85}$$

(see Appendix A for an explanation of the factor of 4). Assuming that the electrons and paratons are relativistic,

$$n_e = n_f = \frac{\mathrm{T}^3}{2.3} \tag{4.86}$$

(see 4.26), and [36]

$$\sigma(f\bar{f} \to e^+ e^-) \simeq \frac{\alpha^2 \epsilon^2}{\mathrm{T}^2} \ . \tag{4.87}$$

The interaction rate for annihilations is therefore

$$\Gamma \sim \frac{\alpha^2 \epsilon^2}{2}\mathrm{T} \ . \tag{4.88}$$

$\sigma(fe \to fe)$ is infrared divergent, so must be delt with more carefully. The differential scattering cross-section is [37]

$$\frac{d\sigma_{scat}}{d\Omega} = \frac{\epsilon^2 \alpha^2}{2s} \frac{s^2 + t^2}{u^2} \tag{4.89}$$

which is roughly (taking $E = |\vec{p}| = \mathrm{T}$ and neglecting t^2)

$$\frac{d\sigma_{scat}}{d\Omega} \simeq \frac{c^2\alpha^2}{8\mathrm{T}^2}\frac{|\Delta\vec{p}|^4}{|\vec{p}|^4} \tag{4.90}$$

where $|\Delta\vec{p}|$ is the momentum transfer. This gives a differential interaction rate

$$\frac{d\Gamma_{scat}}{d\Omega} = n_e\frac{d\sigma_{scat}}{d\Omega}\beta \simeq \frac{\mathrm{T}^3}{2.3}\frac{\epsilon^2\alpha^2}{8\mathrm{T}^2}\frac{s^2+t^2}{u^2} \quad . \tag{4.91}$$

It should be acceptable to approximate the electron number density as that of a relativistic gas, even though $\mathrm{T}_f \sim 2m_e$.

A particle was defined to be in equilibrium if $\Gamma \gg H$ because this meant that it could change its energy as fast as the universe was cooling (see section 4.1.f). However, in this argument, it was assumed that the energy exchanged in an interaction was of order the energy of the particles, and it is precisely because this is not true that the scattering cross-section diverges. For a fixed momentum transfer $|\Delta\vec{p}|$ (or scattering angle θ), it will take $\sim |\Delta\vec{p}|^2/|\vec{p}|^2$ interactions (squared because it is a random process) to change the momentum by an order of magnitude. One therefore needs

$$\int \frac{|\Delta\vec{p}|^2}{|\vec{p}|^2}\frac{d\Gamma_{scat}}{d\Omega}d\Omega \gg H \tag{4.92}$$

to keep the particle in equilibrium. Using

$$\frac{|\vec{p}|^2}{|\Delta\vec{p}|^2} \simeq \sqrt{\frac{s^2+t^2}{u^2}} \tag{4.93}$$

in equation (4.92) gives

$$\frac{\mathrm{T}^3}{2.3}\frac{\epsilon^2\alpha^2}{8\mathrm{T}^2}\int\frac{(4+(1+cos\theta)^2)^{1/2}}{(1+cos\theta)}d\Omega \gg H \tag{4.94}$$

as the condition for paratons to be in equilibrium. θ is the centre-of- mass scattering angle, and masses have been neglected in calculating s, t and u. One can then rewrite the integral in the tractable form

$$2\pi\int\frac{\sqrt{8-4y+y^2}}{y}dy \tag{4.95}$$

where $y = 1 + \cos\theta$. This still diverges as the momentum transfer $u \sim T^2 y$ goes to zero. However, in a thermal bath, the photon acquires an effective mass $m_\gamma \sim eT$, so scattering interactions with $u < e^2T^2$ are forbidden. This gives a lower bound on the y integral of $y_{min} = 4\pi\alpha$, so (4.94) becomes

$$2.6\epsilon^2\alpha^2 T \gg \frac{1.66\sqrt{g_{eff}}}{m_{pl}} T^2 \quad . \tag{4.96}$$

The left hand side of (4.96) is the scattering interaction rate $\equiv \Gamma_{scat}$, so an upper bound on ϵ (below which paratons will not be in equilibrium at nucleosynthesis and therefore could exist) can be calculated by setting $H = \Gamma = \Gamma_{ann} + \Gamma_{scat}$ at T = 5 MeV. So

$$3\alpha^2\epsilon^2 T = \sqrt{\frac{8\pi G}{3}}\sqrt{\frac{g_{eff}(T)}{2}\frac{\pi^2}{15}}\, T^2\Bigg|_{T=5\ \mathrm{MeV}} \tag{4.97}$$

or

$$\epsilon^2 = \frac{1}{3\alpha^2 m_{pl}}\sqrt{\frac{4\pi^3 g_{eff}(T)}{45}}\, T\Bigg|_{T=5\ \mathrm{MeV}} \tag{4.98}$$

where m_{pl} is the plank mass, and $g_{eff}(T = 5\ \mathrm{MeV})$ is 43/4 (see table I), which means that the mass limit does not apply for

$$\epsilon < 4 \times 10^{-9} \tag{4.99}$$

(see figure B.2 and figure B.3).

This calculation ignored the scattering of paratons off photons. In the model without paraphotons, this is not a problem because paraton–photon interactions are $O(\epsilon^4)$. In the model with paraphotons, this is still acceptable; a paraton can interact with a photon and a paraphoton, which is $O(\epsilon^2\alpha\alpha')$, but since $\alpha' \approx \epsilon$ this is still unimportant.

4.4 $\Omega < 1$

The second cosmological limit on paratons comes from requiring that they not overclose the universe. Since $\Omega \simeq 1$ is attractive to theorists, and most observations suggest $\Omega \leq 1$, it should be reasonable to expect the paraton density today to be less than the critical density

$$\rho_c = 1.88 \times 10^{-29} h^2 \text{ g/cm}^3 \tag{4.100}$$

where $1/2 < h < 1$ comes from the uncertainty in the Hubble parameter (H = $100h$ (km/sec)/Mpc).

In the hot very early universe, the paratons are assumed to be in thermal equilibrium with ordinary particles for T $> \mu$ (this assumption will be discussed later). As the temperature drops to $\leq \mu$, they become non-relativistic and their equilibrium number density will drop very fast (it is suppressed by $e^{-\mu/\mathrm{T}}$; see 4.22 and 4.27): when the paratons meet, they annihilate, but it is difficult to find a pair of particles with enough energy to make an $f + \bar{f}$. Meanwhile, the universe is rapidly expanding, so it becomes increasingly difficult for the paratons to find each other. When T$\sim \mu/20$, this expansion has sufficiently decreased the paraton number density that they no longer can find each other and the number per comoving volume stays approximately constant until today.

The more strongly interacting paratons are, the more efficiently they will find each other to annihilate, so the lower the relic density will be today. Paraton cross sections go as $\sim \epsilon^2/E_f^2$, where E_f is the paraton energy, so they become more "weakly interacting" as ϵ gets smaller or as μ gets larger. This means that requiring $\Omega < 1$ will give a lower bound on ϵ and an upper bound on μ. The same argument for neutrinos gives a lower bound on m_ν, because (for $E_\nu < m_W$), the cross sections go as $\sim E_\nu^2/M_W^4$.

a) Lee and Weinbergs approximation to the rate equation

If there are N particles in a volume V at time t, and $n = N/V$ is the number density, then

$$\frac{dn}{dt} = \frac{1}{V}\frac{dN}{dt} - \frac{N}{V^2}\frac{dV}{dt} \quad . \tag{4.101}$$

In the early universe, $V \propto \mathrm{R}^3(t)$, and $dN/dt = N\times$ {creation rate – anihilation rate}. In the usual approximation (see 4.54), the annihilation rate is

$$\Gamma_{ann} = n\sigma_{ann}\beta. \tag{4.102}$$

The creation rate would be difficult to calculate directly; however if the particles were in equilibrium in a box, the number created per unit time would have to be equal to the number that annihilated, so

$$\frac{dN}{dt} = (Nn - N_{eq}n_{eq})\frac{\sigma\beta}{4} \tag{4.103}$$

where n is the actual number density, n_{eq} is the equilibrium number density, and σ is the annihilation cross section. This gives

$$\frac{dn}{dt} = -\frac{3\dot{\mathrm{R}}}{\mathrm{R}}n - \frac{\sigma\beta}{4}(n^2 - n_{eq}^2) \tag{4.104}$$

(which differs from the Weinberg and Lee equation [38] by the factor of 4, because their n is the particle number density, and mine is for particles and anti-particles; see Appendix A).

The interesting physics is in the decrease in n due to annihilations – not due to the expansion of the universe. So instead of calculating the change in the number density with time, consider the change in the number of particles in a comoving volume with temperature (i.e. $dN/d\mathrm{T}$). The decrease in n due to the expansion, which causes the particles to freeze out, will still be taken into account because dN/dt depends on n (see 4.102).

It should be possible to label a comoving volume by the number of photons it contains, because the photon number density is proportional to T_γ^3 (see 4.25), and the wavelength scales as $\mathrm{R}(t)$, so $n(t)\mathrm{R}^3(t)$ = a constant. There are two possible problems with this— it neglects the heating of the photons due to other particles annihilating into them, and the temperature will not scale as $1/\mathrm{R}(t)$ if the photons are in equilibrium with non-relativistic particles in a matter dominated universe, because the matter temperature will not scale as $1/\mathrm{R}(t)$. The second problem is not serious: in the standard model cosmology, the universe does become matter-dominated some time before the electrons and protons 're-combine', and therefore decouple from the photons (how soon before depends on the value of Ω). However, the temperature is so low, and the universe is expanding so slowly, that it does not matter. The first problem can be partially solved by calculating the amount of heating due to annihilations (see 4.46), and putting it into the calculation by hand at the end.

If

$$f(\mathrm{T}) = \frac{n(\mathrm{T})}{\mathrm{T}^3} \tag{4.105}$$

$$f_{eq}(\mathrm{T}) = \frac{n_{eq}(\mathrm{T})}{\mathrm{T}^3} \tag{4.106}$$

where T is the temperature of a fictitious gas of photons that is not heated by any annihilations after the paratons freeze out, then f and f_{eq} are proportional to the number of particles, and the equilibrium number of particles, in a comoving volume. Differentiating f gives

$$\frac{df}{d\mathrm{T}} = \mu \left[\frac{1}{\mathrm{T}^3}\frac{dn}{d\mathrm{T}} - \frac{3n}{\mathrm{T}^4}\right] \tag{4.107}$$

and from (4.104)

$$\frac{dn}{d\mathrm{T}} = \frac{1}{\dot{\mathrm{T}}}\left(-3\frac{\dot{\mathrm{R}}}{\mathrm{R}}n - \frac{\sigma\beta}{4}(n^2 - n_{eq}^2)\right) \tag{4.108}$$

where $\dot{\mathrm{T}} = \partial\mathrm{T}/\partial t$ and $\dot{\mathrm{R}} = \partial\mathrm{R}/\partial t$. If the temperature is measured in units of paraton mass

$$x \equiv \mathrm{T}/\mu \tag{4.109}$$

this gives

$$\frac{df}{dx} = \mu\left[\frac{\mathrm{T}}{\dot{\mathrm{T}}}\frac{1}{\mathrm{T}^4}\left(-3\frac{\dot{\mathrm{R}}}{\mathrm{R}}n - \frac{\sigma\beta}{4}(n^2 - n_{eq}^2)\right) - 3\frac{n}{\mathrm{T}^4}\right]. \tag{4.110}$$

If $\mathrm{T} \propto 1/\mathrm{R}$, then

$$\frac{\dot{\mathrm{T}}}{\mathrm{T}} = -\frac{\dot{\mathrm{R}}}{\mathrm{R}} \tag{4.111}$$

so the first and third terms in (4.110) cancel. This is true for the fictitious photon gas, but for real photons, the 'constant' that relates T to 1/R will be temperature dependant (because of the heating by annihilations) so (4.111) will not be strictly true. This will be ignored, because it is not a very important effect, and the differential equation is not analytically solvable without this correction, let alone with it. (It has been done numerically for a variety of particles with this correction [39].)

Finally, from (4.111) and (4.48)

$$\frac{\mathrm{T}}{\dot{\mathrm{T}}} = -\frac{1}{H} = -\sqrt{\frac{3}{8\pi\mathrm{G}}}\sqrt{\frac{30}{\pi^2 g_{eff}}}\frac{1}{\mathrm{T}^2} \tag{4.112}$$

so that

$$\frac{df}{dx} = \frac{\mu\sigma\beta m_{pl}}{8\sqrt{g_{eff}}}\sqrt{\frac{45}{\pi^3}}(f^2 - f_{eq}^2) \tag{4.113}$$

For relativistic particles $n_{eq} \propto \mathrm{T}^3$, (see 4.24), so that $f = f_{eq}$ = a constant is a solution of this equation. This makes sense because for $\mu \ll \mathrm{T}$, the same number of paratons will be created and destroyed, so the number of particles in a comoving volume will remain constant. As T drops below the paraton mass, the approximation $n_{eq} \propto \mathrm{T}^3$ (4.26) does not hold, and $n = n_{eq}$ is no longer a solution.

Lee and Weinberg integrated (4.113) numerically for heavy neutrinos of various masses. They say that their results can be approximated, to within an order of magnitude, by assuming that the neutrino (paraton in this case) has its equilibrium number density down to some freeze out temperature x_f, (assumed $< \mu$, so that the particles are non-relativistic) and only annihilates below that temperature. This neglects the increase in n with respect to n_{eq} above x_f due to the paratons having difficulty finding each other, and neglects any creation of particles below x_f.

For $x < x_f$,

$$\frac{df}{dx} = bf^2 \tag{4.114}$$

and at x_f

$$\left.\frac{df_{eq}}{dx}\right|_{x_f} = bf_{eq}^2\Big|_{x_f} \tag{4.115}$$

where

$$b = \frac{\mu\sigma\beta m_{pl}}{8\sqrt{g_{eff}}}\sqrt{\frac{45}{\pi^3}} \quad . \tag{4.116}$$

Equation (4.114) gives the number density as a function of temperature for $x < x_f$, and (4.115) defines x_f. df/dx is positive, because although the number density is decreasing in time, it increases with temperature. (4.114) can be solved by setting $f(x) = A(x+a)^m$:

$$mA(x+a)^{m-1} = bA^2(x+a)^{2m} \tag{4.117}$$

which implies $m = -1$, and $A = -b^{-1}$. This gives

$$f(x) = \frac{-1}{b(x+a)} \tag{4.118}$$

where a is a constant to be determined (later). From (4.27),

$$f_{eq} = \frac{g_f}{(2\pi)^{3/2}} e^{-1/x} x^{-3/2} \tag{4.119}$$

where g_f = the number of paraton spin degrees of freedom = 4. Substituting (4.119) into (4.115) gives

$$\frac{4}{(2\pi)^{3/2}}e^{-1/x_f}x_f^{-3/2}\left(\frac{-3}{2x_f}+\frac{1}{x_f^2}\right)=b\left(\frac{4}{(2\pi)^{3/2}}e^{-1/x_f}x_f^{-3/2}\right)^2. \quad (4.120)$$

x_f is assumed to be small (of order 1/10), so $3/(2x_f)$ can be neglected in comparison with $1/x_f^2$, and (4.120) becomes

$$f(x_f)=f_{eq}(x_f)=\frac{1}{bx_f^2}. \quad (4.121)$$

This determines the constant in (4.118), implying that

$$f(x)=\frac{1}{b(x_f^2+x_f-x)} \quad . \quad (4.122)$$

In the limit where $x \rightarrow 0$ (the present "zero temperature" universe), the paraton number density will be

$$n(today)=\frac{\mathrm{T}^3}{bx_f^2(1+1/x_f)} \quad (4.123)$$

where T is the temperature of the fictitious photon gas that was not heated by annihilations, and x_f has yet to be determined from (4.120). Neglecting the $3/(2x_f)$ in (4.120), and taking the logarithm of both sides, gives

$$-\frac{1}{2}\log x_f=\log\left(\frac{4b}{(2\pi)^{3/2}}\right)-\frac{1}{x_f} \quad (4.124)$$

or, neglecting the $\log(x_f)$,

$$x_f=\left[\log\left(\sqrt{\frac{45}{32}}\frac{\mu\sigma\beta m_{pl}}{\pi^3\sqrt{g_{eff}}}\right)\right]^{-1}=\left[41+\log\left(\frac{\mu\sigma\beta}{\sqrt{g_{eff}}}\right)\right]^{-1} \quad (4.125)$$

x_f is not very sensitive to the mass and annihilation cross section of the paraton, and tends to take on values between 1/10 and 1/30.

To get the paraton number density today in terms of known numbers, T (the temperature of the fictitious photon gas) must be expressed in terms of the present temperature of the microwave background radiation. But this is simple using entropy conservation: from (4.46)

$$T_\gamma = \left(\frac{n_{eff}(T_f)}{n_{eff}(today)}\right)^{1/3} T \tag{4.126}$$

where $n_{eff}(T_f)$ is the effective number of spin degrees of freedom of the relativistic particles in equilibrium with the paratons at freeze out, and $n_{eff}(today) = 2$. n_{eff} is taken to be equal to g_{eff}. The paraton energy density today is therefore, from (4.123)

$$\rho_f(today) = \frac{2}{\sqrt{g_{eff}(T_f)}} \frac{T_\gamma^3}{\sigma\beta m_{pl}} \sqrt{\frac{64\pi^3}{45}} \frac{1}{x_f^2(1+1/x_f)}. \tag{4.127}$$

If $\sigma\beta$ is in units of GeV^{-2}, then this is

$$\rho_f = \frac{3.2 \times 10^{-39}}{\sigma\beta\sqrt{g_{eff}(T_f)}} \frac{1}{x_f^2(1+1/x_f)} \text{ g/cm}^3. \tag{4.128}$$

Wolfram [41] has numerically integrated (4.113) for charged particles ($\epsilon = 1$) and says, like Lee and Weinberg, that the numerical results are within an order of magnitude of an analytic approximation that he does not describe in detail. It sounds like this one, except that the relic density that he calculates is a factor of $\sim$ 8/5 larger than (4.128). This is unimportant for a rough calculation such as this, but since the lower density gives a weaker limit on μ, I will use (4.128). (Wolfram gives the relic number density as

$$\frac{8 \times 10^{-8}}{\mu\sigma\beta\sqrt{N_{eff}}} \text{ m}^{-3} \tag{4.129}$$

where μ is in GeV, $\sigma\beta$ is in GeV^{-2} and N_{eff} is $g_{eff}/2$. Transforming (4.128) to the same units, and using $x_f = 1/20$, gives

$$\frac{3.7 \times 10^{-8}}{\mu\sigma\beta\sqrt{g_{eff}}} \text{ m}^{-3} \tag{4.130}$$

so the two equations differ by less than a factor of 2.)

4.5 limits on paratons from $\Omega < 1$

a) upper bound on μ

Paratons can annihilate to 1) a pair of paraphotons, 2) a photon and a paraphoton, 3) two photons or 4) a pair of charged particles. The first two are obviously only possible in the model with paraphotons. In this case the cross sections for 2) and 4) will respectively be suppressed by a factor $\epsilon^2\alpha/\alpha_2$ and $\epsilon^2\alpha^2/\alpha_2^2$ with respect to 1), and since $\epsilon < 5\alpha_2$ (see 1.15), these can be neglected. 3) is even less important because it is proportional to ϵ^4. In the model without a paraphoton, only 3) and 4) are possible, so the paratons will annihilate to charged particle pairs.

The non-relativistic cross section for $f\bar{f} \to \gamma'\gamma'$ is [40]

$$\sigma = \frac{\pi\alpha_2^2}{\beta\mu^2} \tag{4.131}$$

so requiring that the paraton density today $= \rho_f$ be less than ρ_c implies that (from (4.128) and (4.100))

$$\frac{3.2 \times 10^{-39}\mu^2}{x_f^2(1+1/x_f)\sqrt{g_{eff}(\mathrm{T}_f)}\pi\alpha_2^2} \ \mathrm{g/cm^3} < 1.88 \times 10^{-29}h^2 \ \mathrm{g/cm^3} \tag{4.132}$$

or

$$\frac{\mu_{\mathrm{GeV}}^2}{\alpha_2^2} < 1.88 \times 10^{10}h^2\sqrt{g_{eff}(\mathrm{T}_f)}x_f^2(1+1/x_f) \quad . \tag{4.133}$$

One can get a rough idea of what this limit implies by setting $x_f = 1/20$ (see (4.125) and following paragraph), $h = 1$, and $\sqrt{g_{eff}} = 13$ (see table 1) which gives

$$\mu < 10^5\alpha_2 \ \mathrm{GeV} \tag{4.134}$$

in agreement with [42]. From (1.15), $\alpha_2/10 < \epsilon < 5\alpha_2$, so the weakest limit on μ as a function of ϵ will be approximately

$$\mu < 10^6\epsilon \ \mathrm{GeV} \ \ (\epsilon < 10^{-2}). \tag{4.135}$$

This limit is cut off at $\epsilon = 10^{-2}$ because this corresponds to $\alpha_2 \leq 1/10$, i.e. above this value the $U(1)_2$ would become strongly coupled. I do not know how such a thing would behave, so I will assume that $\alpha_2 \leq 1/10$, which implies

$$\mu < 10^4 \text{ GeV} \quad (\epsilon > 10^{-2}). \tag{4.136}$$

This gives a rough idea of what the limit looks like, so the calculations can now be redone using the proper μ and ϵ dependant values of g_{eff} and x_f. Using $\alpha_2 = \min(1/10,\ 10\epsilon)$, $\sqrt{g_{eff}(1 \text{ TeV})} = 13$ and lower temperature values from table B.2, gives

$$\left.\begin{array}{ll} \epsilon \gg 10^{-2} & \mu < 10^4 \text{ GeV} \\ \epsilon = 10^{-3} & \mu < 10^3 \text{ GeV} \\ \epsilon = 10^{-4} & \mu < 80 \text{ GeV} \\ \epsilon = 10^{-5} & \mu < 8 \text{ GeV} \\ \epsilon = 10^{-6} & \mu < 600 \text{ MeV} \\ \epsilon = 10^{-7} & \mu < 60 \text{ MeV} \end{array}\right\} \quad (\text{with} \gamma') \tag{4.137}$$

(see figure B.3).

If there is no paraphoton, the paratons do not interact as strongly (because $\epsilon < 5\alpha_2 \Rightarrow \epsilon\alpha \ll \alpha_2$). As the temperature drops below their mass, they have more difficulty finding each other, which gives a higher relic density today. The upper bound on μ from $\rho_f < \rho_c$ should therefore be lower than (4.137).

A paraton with intrinsic fractional hypercharge should annihilate to a pair of charged particles, providing $\mu > m_e$. If the paraton was lighter than the electron, it would have to annihilate to a pair of photons, which is $O(\epsilon^4)$ and therefore very difficult. However, nucleosynthesis rules out $\mu < 1$ MeV, so it is reasonable to assume $\mu > m_e$.

The cross section for a pair of non-relativistic particles to annihilate to a pair of relativistic ones is [41]

$$\frac{\pi\epsilon^2\alpha^2}{\mu^2\beta}N(T_a) \tag{4.138}$$

where $N(T_a)$ is the effective number of relativistic charged particle species present when the paratons annihilate:

$$N(T_a) = \sum_i Q_i^2 \tag{4.139}$$

and T_a is now the temperature at which the paratons annihilate. The sum is over particles of charge Q_i and mass $< \mu$. As usual, a 'particle' includes both the particle and anti-particle, so the electron and positron together contribute 1, and each quark flavour contributes $3Q_i^2$. $N(T)$ is calculated up to $x_f \geq m_W$ in table B.2.

From (4.128) and (4.100), requiring $\rho_f < \rho_c$ gives

$$\frac{\mu^2}{\epsilon^2} < 10^6 h^2 \sqrt{g_{eff}(T_f)}\, x_f^2(1 + 1/x_f)N(T_a) \tag{4.140}$$

where μ is in GeV, as usual. Taking $h = 1$, $\sqrt{g_{eff}} = 10$, $x_f = 1/20$, and $N(T_a) = 7$ gives a rough upper bound:

$$\mu < 2 \times 10^3 \epsilon \text{ GeV} \tag{4.141}$$

or, using the correct values of x_f, $N(T_a)$ and $g_{eff}(T_a)$:

$$\begin{array}{ll} \epsilon = 1 & \mu < 2 \times 10^3 \text{ GeV} \\ \epsilon = 10^{-1} & \mu < 200 \text{ GeV} \\ \epsilon = 10^{-2} & \mu < 20 \text{ GeV} \\ \epsilon = 10^{-3} & \mu < 1 \text{ GeV} \\ \epsilon = 10^{-4} & \mu < 50 \text{ MeV} \\ \epsilon = 10^{-5} & \mu < 6 \text{ MeV} \end{array} \quad (\text{no } \gamma') \tag{4.142}$$

(see figure B.2).

b) lower bound on μ

These limits (4.137 and 4.142) assume that the paratons are in thermal equilibrium with ordinary matter at T > μ. This is only going to be true for 'sufficiently large' values of ϵ — if I give the paraton a fractional charge of 10^{-20}, it is not, for all practical purposes, going to interact with ordinary matter (other than gravitationally). It will still be present in the expanding universe at some temperature of its own = T_{ff}, and when T_{ff} drops below μ the paratons will start to annihilate. How effectively they do this will depend on the expansion rate of the universe at that time, which is controlled by the total energy density. In the early universe, all the relativistic particles at a given temperature contribute approximately equally to the total energy density, (even if the paratons give $\Omega = 1$ today) so the expansion rate will depend on the temperature of the ordinary matter (assuming it is in the majority). This means that to predict a relic number density today, one needs to know the ratio of the paraton temperature to that of ordinary matter. (The ratio of the entropies is actually more useful, because it is not changed when a particle species annihilates ... see [43] for a discusion of "shadow matter" in the universe.) Since this ratio is unknown, it is not possible to predict a relic density today unless the paratons are in equilibrium with ordinary matter before they freeze out.

The paratons are assumed to be in equilibrium when their interaction rate with ordinary matter is greater than the expansion rate, or roughly, when

$$n(\mathrm{T})(\Gamma_{ann} + \Gamma_{scat}) > \frac{1.66}{m_{pl}}\sqrt{g_{eff}}\mathrm{T}^2 \quad . \tag{4.143}$$

Using Γ_{ann} and Γ_{scat} from (4.97) and(4.96), this becomes

$$3n(\mathrm{T})\epsilon^2\alpha^2\mathrm{T} > \frac{1.66}{m_{pl}}\sqrt{g_{eff}}\mathrm{T}^2 \quad . \tag{4.144}$$

If the paraton mass is above this temperature, then conceivably the $\rho < \rho_c$ limits no longer apply. Solving (4.144) for T gives

$$\mathrm{T} < \frac{\epsilon^2 N(\mathrm{T})}{\sqrt{g_{eff}(\mathrm{T})}} \times 1.2 \times 10^{15}\ \mathrm{GeV} \tag{4.145}$$

For large ϵ this temperature is so many orders of magnitude above the energy levels at which physics is 'well understood' (< 100 GeV), that it is meaningless. However, for $\epsilon < 10^{-6}$ (4.145) gives $\mathrm{T} \leq 1$ TeV, assuming $N(\mathrm{T}) = \sqrt{g_{eff}}$ (zeroth order approximation in the now familiar iterative way of avoiding the implicit dependance of N and g_{eff} on μ and ϵ).

At temperatures below T the paratons will be in equilibrium, if $\mu \ll \mathrm{T}$. The $\rho < \rho_c$ limit will therefore apply. For $\mu \gg \mathrm{T}$, it is not clear what the paratons do, so there is a tentative upper bound to the masses ruled out by the $\rho < \rho_c$ limit; i.e. paratons may be allowed if they are sufficiently heavy. The dividing line between the masses for which the $\rho < \rho_c$ limit applies and does not apply is $\mu \sim$ T, which gives a lower bound on the paraton mass of

$$\mu > \frac{\epsilon^2 N(\mathrm{T})}{\sqrt{g_{eff}(\mathrm{T})}} 1.2 \times 10^{15}\ \mathrm{GeV} \tag{4.146}$$

(see 4.145), or using the proper values of N and g_{eff},

$$\begin{aligned} \epsilon &= 10^{-5} & \mu &> 100\ \mathrm{TeV} \\ \epsilon &= 10^{-6} & \mu &> 1\ \mathrm{TeV} \\ \epsilon &= 10^{-7} & \mu &> 6\ \mathrm{GeV} \\ \epsilon &= 10^{-8} & \mu &> 40\ \mathrm{MeV} \\ \epsilon &= 4 \times 10^{-9} & \mu &> 6\ \mathrm{MeV} \end{aligned} \tag{4.147}$$

(see figure B.2 and figure B.3).

This limit also "cuts off" the top right hand corner of the rectangle in $\mu - \epsilon$ space that was ruled out by nucleosynthesis for the model with a paraphoton.

The limits calculated in this section assume that the paraton only interacts electromagnetically with the other particles in the Universe. Below T $\sim$ 200 GeV this is a reasonable approximation, since the paraton coupling to the Z is a factor of $\tan^2\theta_w$ ($\sim$ .04) smaller than its coupling to the photon. Above T $\sim$ 200 GeV, SU(2)×U(1) will be unbroken and the paraton will interact with particles carrying hypercharge. This changes the interaction rates by less than a factor of 3/2, so it is reasonable to pretend that electromagnetism lasts up to arbitrarily high energies (or at least as long as SU(2)×U(1) does).

CHAPTER FIVE

RED GIANTS AND WHITE DWARFS

The usual way to get constraints from stars on 'unobserved particles from the theorists zoo' is to calculate the rate at which the star would lose energy to these 'other particles', and then get limits on the coupling constant and/or mass of the particle by requiring that the star not lose more energy via "others" than via ordinary radiation (see references in [44]). Red giants are a popular place to calculate these constraints because they have hot dense cores (which means that they are good at making strange particles), and they are observed to exist. White dwarfs can give a useful limit because of their high densities, and of course there is the supernova. Red giants and white dwarfs give interesting constraints on paratons, but the supernova does not appear to be very helpful.

The principal method of paraton production in red giants will be plasmon decay ($\gamma \rightarrow f\bar{f}$). Pair production in the field of a nucleus is $O(\epsilon^4)$, which makes it negligible. The cross section for trident production is rather messy, so the 'red giant limit' on paratons is calculated solely from plasmon decay. This is also the mechanism for paraton production in white dwarfs, so in the very simplistic models used here, a great deal of the white dwarf analysis can be borrowed from the red giant calculations.

I have treated the plasmon as a massive vector boson. Its decay rate can therefore be calculated and is very large; essentially a plasmon will decay if it can. This implies $\mu > m_\gamma/2$ down to very small values of ϵ, assuming that the paratons escape from the star. (If the charge is large enough that they are trapped inside the star, this limit no longer applies.) How small ϵ needs to be is calculated by requiring that the energy lost to paratons in the core be less than the energy generated by helium burning. This should provide a very conservative upper bound on ϵ.

The first thing calculated is the plasmon decay rate Γ; then the rate of energy loss of the red giant core is calculated to get an upper bound on ϵ. Finally there is an estimate of when the paratons will be in equilibrium in the star (giving a lower bound on ϵ), a similiar set of calculations for the white dwarf limit, and a short discussion of why the supernova has been ignored.

5.1 plasmon decay in red giants

a) the decay rate Γ

The plasmon is assumed to be a photon with a mass m_γ, so it has a rest frame and can decay to a pair of charged particles while conserving energy and momentum. In the plasmon rest frame

$$\begin{aligned} k &= \text{photon momentum} = (m_\gamma, \vec{0}\,) \\ p_\pm &= \text{(anti)paraton momentum} = (E, \pm\vec{p}\,) \end{aligned} \tag{5.1}$$

(where the '$-$' corresponds to the paraton), so that, using the conventions of Bjorken and Drell

$$\begin{aligned} \psi_f(x) &= \sqrt{\tfrac{\mu}{EV}}\; u(p_-, s_-)e^{-ip_-\cdot x} \\ \psi_f(x) &= \sqrt{\tfrac{\mu}{EV}}\; v(p_+, s_+)e^{ip_+\cdot x} \\ A_\mu(x) &= \tfrac{1}{\sqrt{2m_\gamma V}}\; \varepsilon_\mu e^{-ik\cdot x} \end{aligned} \tag{5.2}$$

The S-matrix element for plasmon decay is, up to a phase,

$$S_{fi} = i \int \bar{\psi}_f(x)\, \not{A}(x) \psi_f(x)\, d^4x \tag{5.3}$$

or

$$S_{fi} = i \frac{\epsilon e \mu}{EV\sqrt{2m_\gamma V}} \bar{u}\, \not{\epsilon} v\, \delta^4(k - p_- - p_+)(2\pi)^4 \tag{5.4}$$

So the transition rate per unit volume $\equiv |\, S_{fi} \,|^2 /(VT)$, is

$$\frac{\epsilon^2 e^2 \mu^2}{2m_\gamma E^2 V^3} \,|\, \bar{u}\, \not{\epsilon} v \,|^2\, \delta^4(k - p_+ - p_-)(2\pi)^4 \tag{5.5}$$

This must be divided by the photon density (1 per unit volume, with the normalization 5.2), and multiplied by the density of final states

$$\frac{V^2 d^3p_+ d^3p_-}{(2\pi)^6} \tag{5.6}$$

to get the decay rate Γ

$$\Gamma = \int \frac{\epsilon^2 e^2 \mu^2}{2m_\gamma E^2} \,|\, \bar{u}\, \not{\epsilon} v \,|^2\, \delta^4(k - p_+ - p_-) \frac{d^3p_+ d^3p_-}{(2\pi)^2} \tag{5.7}$$

or, doing some of the integrations

$$\Gamma = \int \frac{\epsilon^2 \alpha \mu^2}{2\pi m_\gamma^2} \,|\, \bar{u}\, \not{\epsilon} v \,|^2 \,|\, \vec{p} \,|\, d\Omega \ . \tag{5.8}$$

The matrix element is, assuming the plasmon has three spin states,

$$|\, \bar{u}\, \not{\epsilon} v \,|^2 = \left(\frac{1}{3} \sum_{\gamma\ spins} \varepsilon^\mu \varepsilon^{*\nu} \right) \left(\sum_{f\bar{f}\ spins} u \gamma_\mu v v \gamma_\nu u \right) \ . \tag{5.9}$$

If the plasmon is assumed to be like an intermediate vector boson, then its spin sum is $-(g_{\mu\nu} - k_\mu k_\nu / m_\gamma^2)$ [45], so that the matrix element is

$$\begin{aligned} |\, \bar{u}\, \not{\epsilon} v \,|^2 &= -\frac{1}{3} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{m_\gamma^2} \right) \mathrm{Tr}\left[u u \gamma^\mu v v \gamma^\nu \right] & (5.10) \\ &= \frac{1}{3\mu^2} (3E^2 + \vec{p} \cdot \vec{p} + 3\mu^2) & (5.11) \end{aligned}$$

Substituting (5.11) into (5.8), doing the angle integral, and noting that $E = m_\gamma/2$, gives

$$\Gamma = \frac{\epsilon^2 \alpha m_\gamma}{3} \frac{\beta(3-\beta^2)}{2} \tag{5.12}$$

where β is the velocity of the outgoing paraton.

The photon acquires its 'mass' from its constant interactions with the electron gas, so m_γ depends on the energy and number density of electrons and positrons in a complicated way [46]. However in the limit of a non-relativistic electron gas, T$\ll m_e$, [47]

$$m_\gamma^2 = \frac{4\pi\alpha n_e}{m_e} \quad . \tag{5.13}$$

The red giant is very naively assumed to be a homogeneous core of density $\rho = 10^4$ g/cm^3, temperature T $= 10^8$ °K (8.6 keV), mass .5 $M_\odot$ (.5 solar masses), radius $r = 3 \times 10^9$ cm. and energy generation rate $R_n = 10^6$ ergs/(cm^3 sec). (This is copied from [44].) This density gives

$$m_\gamma = 2 \text{ keV} \tag{5.14}$$

which in turn gives a plasmon lifetime in red giants of (from 5.12, assuming $\beta \sim 1$)

$$\tau = \Gamma^{-1} = \frac{1.4}{\epsilon^2} \times 10^{-16} \text{ sec} \quad . \tag{5.15}$$

Unless ϵ is very small, a plasmon will decay if it can. And it can unless

$$\mu > 1 \text{ keV} \tag{5.16}$$

(see figure B.4) which is the lower bound on the paraton mass from plasmon decay in red giants.

b) upper bound on ϵ

If ϵ is very small, the plasmons will decay very slowly and the star will not lose too much energy to paratons. For instance, if $\epsilon \leq 3 \times 10^{-17}$, it will take a plasmon longer than the age of the universe ($\sim 10^{17}$ sec) to decay. The upper bound on ϵ is calculated by requiring that the rate of energy loss per unit volume to paratons be less than the nuclear energy generation rate per unit volume, which is [44]

$$R_n = 100 \text{ ergs/(g sec)} = 10^6 \text{ergs/(sec cm}^3) \quad . \tag{5.17}$$

The rate of energy loss per unit volume to paratons will be $dE/(dV\,dt) =$ {energy of a plasmon}×{plasmon number density}×{decay rate}, or

$$\frac{d^2E}{dV\,dt} = \int_0^\infty \frac{\omega}{(2\pi)^3} \frac{3\,d^3k}{e^{\omega/T} - 1} \Gamma \tag{5.18}$$

where $\omega = \sqrt{m_\gamma^2 + \vec{k}^{\,2}}$ is the plasmon energy and $\vec{k}$ is its momentum. The decay rate is independant of the photon energy, so defining $x = \omega/T$ and $x_o = m_\gamma/T$, (5.18) becomes

$$\frac{d^2E}{dV\,dt} = \frac{3\Gamma}{2\pi^2} T^4 \int_{x_o}^\infty \frac{x^2\sqrt{x^2 - x_o^2}}{e^x - 1} dx \quad . \tag{5.19}$$

This is not simple to evaluate; writing

$$\frac{1}{e^x - 1} = \sum_{n=1}^\infty e^{-nx} \tag{5.20}$$

gives

$$\frac{d^2E}{dV\,dt} = \frac{3\Gamma}{2\pi^2} T^4 \sum_{n=1}^\infty \int_{x_o}^\infty x^2 \sqrt{x^2 - x_o^2} e^{-nx} dx \equiv \frac{3\Gamma}{2\pi^2} T^4 \sum_{n=1}^\infty I_n(x_o) \quad . \tag{5.21}$$

$I_n(x_o)$ can be written, after integration by parts, as

$$I_n(x_o) = -\frac{1}{3} \int_{x_o}^\infty (x^2 - x_o^2)^{3/2} e^{-nx} dx + \frac{n}{3} \int_{x_o}^\infty x (x^2 - x_o^2)^{3/2} e^{-nx} dx \tag{5.22}$$

and both these integrals can be found in tables. Using G+R [16] 3.389.4 and 3.387.6, (5.19) becomes

$$\frac{d^2E}{dV\,dt} = \frac{3\Gamma}{2\pi^2}\mathrm{T}^4 x_o^2 \sum_{n=1}^{\infty} \frac{1}{n^2}[x_o n K_3(x_o n) - K_2(x_o n)] \tag{5.23}$$

where $x_o = m_\gamma/\mathrm{T}$ and K_j is a bessel function of imaginary argument. Using $\Gamma = \epsilon^2 \alpha m_\gamma/3$, this becomes

$$\frac{d^2E}{dV\,dt} = 2.19 \times 10^{-31}\epsilon^2 \sum_{n=1}^{\infty} \frac{1}{n^2}[x_o n K_3(x_o n) - K_2(x_o n)]\ \mathrm{GeV}^5 \quad . \tag{5.24}$$

Bessel functions are tabulated for arguments .1 .2 .3 etc. [49], and there are interpolation formulas for arguments in between. However, this is only an "order of magnitude" limit on ϵ, so the Bessel functions will be taken to be linear between the tabulated values, and the series will be approximated by its first three terms (The Ks decrease very rapidly as n increases). With these assumptions the sum is ≈ 225, which implies (see 5.24)

$$\frac{d^2E}{dV\,dt} = \epsilon^2 \times 1.6 \times 10^{34}\ \mathrm{ergs/(cm^3\,sec)} \quad . \tag{5.25}$$

Recall that the nuclear energy generation rate was 10^6 ergs/(cm^3 sec). So the upper bound on ϵ is 8×10^{-15}, or

$$\epsilon < 10^{-14} \tag{5.26}$$

(see figure B.4).

c) lower bound on ϵ

The limits calculated so far (5.16 and 5.26) tacitly assume that the paratons escape the star when they are produced. This will not be the case if ϵ is large; the paraton density will build up until the creation rate is balanced by the rate at which paratons annihilate to photons or diffuse to the surface and escape. The paraton luminosity will depend on this diffusion rate, which I do not know how to calculate. It is therefore assumed that the lower bound on the paraton mass (5.16) only holds if the energy of the paratons can escape freely. If there is no paraphoton, this means 'if the paratons can escape freely'; if there are paraphotons, the paratons are more likely to annihilate to them than to photons because $\alpha_2 > \epsilon^2\alpha$. One must therefore check that the paraphotons are trapped in the star if the paratons are. (This is true: see section 5.1.d.) Otherwise the paraphotons could carry the energy out of the red giant, even if the paratons could not escape.

The principal interactions that should interfere with a paratons escape from the star are scattering off electrons and helium nuclei (assuming that all the protons are in helium). For the model without paraphotons, scattering off photons and other paratons is $O(\epsilon^4)$, and therefore negligible. For the model with a paraphoton, scattering off a pair of paraphotons can be neglected because the paraphotons will not be trapped in the star unless the paratons already are, and it is precisely when they become trapped that needs to be calculated. Scattering off another paraton via virtual paraphoton exchange can also be ignored, because although $\alpha_2^2 \gg \epsilon^2\alpha^2$, the paraton number density will be much smaller than the electron number density if they escape the star at approximately the speed of light as soon as they are produced. The paraton mean free path is therefore

$$\bar{\ell}_f = \frac{1}{n_e\sigma_e + n_{He}\sigma_{He}} \tag{5.27}$$

where σ_e is the cross section for paraton–electon scattering, and σ_{He} is the cross section for paraton–helium nucleus scattering. Since the energy density in the star is 10^4 g/cm^3, the electron and helium number densities are

$$n_e = 2.8 \times 10^{27}\ \mathrm{cm}^{-3} \tag{5.28}$$

$$n_{He} = 1.4 \times 10^{27} \text{ cm}^{-3}$$

(This assumes that all the energy is contained in the rest mass of helium nuclei.)

Both the electron and nucleus are non-relativistic, so the cross sections can be calculated using a screened Coulomb potential. This gives [50]

$$\sigma = \frac{16\pi\hat{\mu}^2 Z^2 \epsilon^2 e^4 a^4}{(4k^2a^2+1)} \tag{5.29}$$

where $\hat{\mu}$ is the reduced mass, Z is the electron or helium nucleus charge, k is the (three) momentum of the incident paraton and a is the screening length. The paratons are relativistic, so

$$k \approx \mathrm{T} \tag{5.30}$$

and the screening length is [48]

$$a = \sqrt{\frac{\mathrm{T}}{4\pi\alpha n_e}} \tag{5.31}$$

so

$$\sigma_j = \frac{4\pi Z_j^2 \epsilon^2 \mu^2}{\mathrm{T}^3/(\pi\alpha n_e)+1}\left(\frac{\mathrm{T}^2}{n_e^2}\right) \tag{5.32}$$

or

$$\sigma_e = \epsilon^2\mu^2 \times 3.4 \times 10^{-10} \text{ cm}^2 \tag{5.33}$$

$$\sigma_{He} = \epsilon^2\mu^2 \times 1.4 \times 10^{-9} \text{ cm}^2 \quad . \tag{5.34}$$

The paraton mean free path is therefore (using 5.33, 5.34, and 5.28)

$$\bar{\ell}_f \approx \frac{1}{\epsilon^2\mu^2} \times 3 \times 10^{-19} \text{ cm} \tag{5.35}$$

with μ in GeV.

If I arbitrarily assume that paratons do not escape if the core radius is greater than 10 mean free paths, then the lower bound on ϵ is (using the radius from the paragraph following 5.13)

$$\epsilon\mu_{\mathrm{GeV}} > 3 \times 10^{-14} \tag{5.36}$$

or

$$\epsilon \geq 10^{-8} \quad \text{for} \quad \mu = 1 \text{ keV}$$
$$\epsilon \geq 10^{-5} \quad \text{for} \quad \mu = 1 \text{ eV} \tag{5.37}$$

(see figure B.4).

d) trapping of paraphotons

The limit (5.37) will apply to the model with a paraphoton unless the γ' mean free path is long enough that it can escape from the star, even if the paratons can not. If the paratons are relativistic and in equilibrium, their number density is (4.26)

$$n_f \simeq \frac{T^3}{2.3} = 3.5 \times 10^{25} \text{ cm}^{-3} \tag{5.38}$$

and the cross section for paraton-paraphoton scattering in the relativistic limit is [51]

$$\sigma_f = \frac{\pi\alpha_2^2}{\mu T}\left(\log\frac{2T}{\mu} + \frac{1}{2}\right) \tag{5.39}$$

where the paraphoton energy is taken equal to the temperature. So the paraphoton mean free path is

$$\bar{\ell}_{\gamma'} = \frac{1}{n_f\sigma_f} = \frac{2 \times 10^{-4}\mu}{\alpha_2^2(\log 2T/\mu + 1/2)} \text{ cm} \tag{5.40}$$

with μ in GeV. For $\mu = 1$ keV, this is

$$\bar{\ell}_{\gamma'} = \frac{6 \times 10^{-11}}{\alpha_2^2} \text{ cm} \tag{5.41}$$

and for $\mu = 1$ eV it is

$$\bar{\ell}_{\gamma'} = \frac{2 \times 10^{-14}}{\alpha_2^2} \text{ cm} \tag{5.42}$$

Assuming that paraphotons remain in the red giant if the core radius is greater than 10 $\bar{\ell}_{\gamma'}$ and using $\alpha_2 < 10\epsilon$ (see 1.15), the lower bound on ϵ is

$$\epsilon > 4 \times 10^{-11} \quad \mu = 1 \text{ keV}$$
$$\epsilon > 8 \times 10^{-13} \quad \mu = 1 \text{ eV} \tag{5.43}$$

The paraphotons would be trapped in the red giant before the paratons are (if the paratons were trapped). So if the paratons can not escape from the star, the energy they carry can not either and (5.37) also applies when there is a paraphoton.

5.2 white dwarfs

At first sight it is strange that a dying star should give useful limits on paratons. However, the 'mass' of a plasmon in a degenerate electron gas is

$$m_\gamma = \sqrt{\frac{4\pi\alpha n_e}{\varepsilon_F}} \tag{5.44}$$

where ε_F is the Fermi energy including rest mass, and n_e is the electron number density. In a white dwarf of one solar mass with a radius of 5000 km, one has $n_e = 2 \times 10^{30}$ cm^{-3} , $\varepsilon_F = 600$ keV, and therefore $m_\gamma \simeq 40$ keV, which could increase the lower bound on the paraton mass to $\mu > 20$ keV: the present theory of white dwarf cooling seems to agree with the observed luminosity distribution of the white dwarf population [52], so white dwarves can not be losing substantial amounts of energy to paratons.

a) white dwarf cooling

A white dwarf is a star that is not large enough to build up the central temperature necessary to burn heavier elements (for instance carbon). When core helium burning ends, the star can no longer generate the radiation pressure necessary to support itself against gravitational contraction, so will shrink until it acquires a new pressure source. For stars of approximately one solar mass (2×10^{33} g), this happens when the radius has decreased to $\simeq 5000$ km: the electrons become degenerate and support the star. The white dwarf then gradually radiates away the kinetic energy stored in its ions over a period of roughly 10^9 years.

Since electrons are fermions, they must all be in distinct states. As the star contracts, the space volume occupied by the electrons decreases, forcing them into higher and higher momentum states. When there is so little volume available to the electrons that they fill all the accesible states up to an energy $\varepsilon_F \gg \mathrm{T}$, the electron number density (4.21) (with chemical potential ε_F) can be approximated as

$$n_e = \frac{8\pi}{(2\pi)^3} \int_0^{p_F} p^2 dp \tag{5.45}$$

where $\varepsilon_F = \sqrt{p_F^2 + m_e^2}$. This gives

$$\varepsilon_F = \sqrt{(3\pi^2 n_e)^{2/3} + m_e^2} = 600 \text{ keV} \tag{5.46}$$

Using this value of ε_F in (5.44) gives a plasmon mass of

$$m_\gamma = 39 \text{ keV} \tag{5.47}$$

Assuming that the plasmons can be treated as massive bosons, their number density will be

$$n_\gamma = 3e^{-m_\gamma/\mathrm{T}} \left(\frac{m_\gamma \mathrm{T}}{2\pi}\right)^{3/2} \tag{5.48}$$

Since m_γ is greater than and independant of the temperature, the plasmon number density will be exponentially suppressed as the star cools. Any limits on paratons from plasmon decay must therefore come from the early stages of white dwarf cooling, since $n \simeq 6$ cm^{-3} by the time $\mathrm{T} = 10^7$ °K (.9 keV).

In a world without paratons, white dwarfs cool by emitting photons. If the star is assumed to be a degenerate core at uniform temperature, surrounded by a thin surface layer of non-degenerate matter through which the photons must diffuse, the white dwarf luminosity can be shown to be [53]

$$L = 2 \times 10^{6} \frac{M \mathrm{T}_{^\circ K}^{7/2}}{M_\odot} \text{ ergs/sec} \tag{5.49}$$

where T is the temperature of the core in degrees Kelvin, M is the mass of the star, and $M_\odot$ is the mass of the sun. The principal source of energy that the core can lose by radiation is the heat in the ions; the star is prevented from collapsing any further by the electron pressure, so no gravitational energy, and the electrons can not cool because they are already in the lowest available energy state. If the ions are treated as an ideal monoatomic gas with a heat capacity of $c_V = \frac{3}{2}$ per ion (k=1 units), then the total thermal energy of the white dwarf is

$$U = \frac{3}{2} \mathrm{T} N \tag{5.50}$$

where N is the total number of carbon ions $= M/m_c$. Equating the luminosity (5.49) with $-dU/dt$ gives

$$-\frac{3}{2m_c}\frac{d\mathrm{T}}{dt} = \frac{2 \times 10^{6}}{M_\odot} \mathrm{T}_{^\circ K}^{7/2} \text{ ergs/sec} \tag{5.51}$$

which implies, when integrated, that the time to cool from T_i to T_f should be

$$\tau = \frac{3}{5}\frac{M}{m_c}\frac{\mathrm{T}_f}{L_f} \tag{5.52}$$

(assuming $\mathrm{T}_i^{-5/2} \ll \mathrm{T}_f^{-5/2}$). T is again the core temperature, related to the luminosity by (5.49). The observed distribution of hot $(10^{-1}L_\odot > L > 10^{-3}L_\odot)$ white dwarfs, combined with a model for the rate of star formation, agrees with this prediction[54,55]. One can therefore require that the paratons do not cool a white dwarf to, say, $10^{-2}L_\odot$ ($L_\odot = 3.9 \times 10^{33}$ ergs/sec) in less than 3.3 $\times 10^{15}$ seconds $\simeq 10^{8}$ years, which is what (5.52) predicts.

b) paraton cooling of white dwarfs

The rate at which the star loses energy to paratons should be the volume integral of {plasmon number density} × {plasmon decay rate} × {energy carried away by paratons}, which can be approximated as

$$L_{ff} \simeq n_\gamma \Gamma m_\gamma V \tag{5.53}$$

because the star is assumed to be a sphere of uniform density and temperature. Using (5.48),(5.12), and (5.44) gives

$$L_{f\bar{f}} = \epsilon^2 D e^{-m_\gamma/\mathrm{T}} \mathrm{T}^{3/2} \tag{5.54}$$

where

$$D = \sqrt{\frac{2}{\pi}} \frac{\alpha}{3} R^3 m_\gamma^{7/2} \ . \tag{5.55}$$

For a white dwarf radius R of 5000 km, D is 4.3×10^{73} GeV/sec, if T is measured in GeV. Setting this equal to $-dU/dt$ (see 5.50), gives

$$\frac{2m_c\epsilon^2 D}{3M}\tau = \frac{1}{\sqrt{m_\gamma}} \int_{x_i}^{x_f} \frac{e^x}{\sqrt{x}} dx \tag{5.56}$$

where $x = m_\gamma/\mathrm{T}$. If one now takes the $\sqrt{x}$ out of the integral because it varies much more slowly than the exponential (for m_γ not $\ll$ T) this becomes

$$\tau = \frac{1}{\epsilon^2} \frac{\sqrt{\mathrm{T_f}}}{m_\gamma} \frac{3M}{2m_c D} e^{m_\gamma/\mathrm{T}_f} \tag{5.57}$$

where τ is the time it takes to cool to T_f (assumed much smaller than T_i). Using $M = M_\odot = 1.9 \times 10^{33}$ grams, $m_c = 12$ GeV (pure carbon star), $m_\gamma = 39$ keV and $\mathrm{T}_f = 1.5$ keV (which corresponds to $L_f = 10^{-2} L_\odot$), this gives

$$\tau \simeq \frac{5 \times 10^{-5}}{\epsilon^2} \ \mathrm{sec} \tag{5.58}$$

The observations require $\tau \simeq 3.4 \times 10^{15}$ seconds, which implies that the paraton mass must be greater than $m_\gamma/2$, or that ϵ must be small:

$$\text{for } \epsilon > 1.2 \times 10^{-10} \quad \mu > 20 \text{ keV} \tag{5.59}$$

In this calculation, the cooling time increases exponentially with the plasmon mass, so m_γ has been taken as large as possible. This gives the weakest limit on ϵ, but perhaps makes the mass limit too high. It is safer to use a smaller plasmon mass for the limit on μ (this corresponds to a larger radius and smaller mass for the star), which means the white dwarf limit is

$$\text{for } \epsilon > 10^{-10} \quad \mu > 10 \text{ keV} \tag{5.60}$$

(see figure B.4).

c) paraton trapping in white dwarfs

As in the case of red giants, paratons with large ϵ will be trapped in the star. They are unlikely to interact with the electrons, because these are degenerate, but will scatter off the carbon ions. So copying the arguments made for red giants gives a paraton mean free path in white dwarves of

$$\bar{\ell}_f = \frac{1}{n_c \sigma_c} \tag{5.61}$$

where σ_c is the paraton-carbon scattering cross-section and n_c is the number density of carbon ions, which is

$$n_c = 1.8 \times 10^{29} \text{ cm}^{-3} \tag{5.62}$$

if all the mass is in the carbon. Using a screened Coulomb potential (see 5.32) for the ions, with a screening length of [56]

$$a = \sqrt{\frac{\varepsilon_F}{24\pi n_c \alpha}} \tag{5.63}$$

gives

$$\sigma_c \simeq 1.7 \times \epsilon^2 \mu^2 \times 10^{-16} \ \text{cm}^2 \tag{5.64}$$

for a one solar mass white dwarf at a temperature of 2.8 keV (this corresponds to $L = 10^{-1} L_\odot$). The mean free patn is therefore

$$\ell_f \simeq \frac{3.4 \times 10^{-14}}{\epsilon^2 \mu^2} \ \text{cm} \tag{5.65}$$

Assuming again that the paratons are trapped if their mean free path is less than 1/10 of the white dwarf radius, one has that paratons are allowed for

$$\begin{aligned} \mu &= 10 \ \text{keV} \quad \epsilon > 10^{-6} \\ \mu &= 1 \ \text{eV} \quad \epsilon > 10^{-2} \end{aligned} \tag{5.66}$$

(see figure B.4).

5.3 supernova

The 1987 supernova in the Large Magellanic Cloud (LMC), SN1987A, has been used to get limits on the axion mass and the neutrino lifetime, mass, and number of flavours (see [57] for references). One would hope that the very high temperature ($\sim$ 10 MeV) and enormous amount of energy available ($\sim 10^{53}$ ergs ... better than 100 times the energy the sun will emit in its lifetime), would give intresting constraints on paratons. This is a considerably more difficult problem than the red giant limit because of the short time scales involved (milliseconds $\rightarrow$ seconds), so the results could easily be wrong by more than an order of magnitude.

About ten million years ago, an eighteen solar mass star was born in the LMC, and for the following ten million years, burned hydrogen to helium. When it ran out of hydrogen in its central region, the helium core contracted, heated and started to burn to carbon (the star is now a red giant); when the helium at the centre was exhausted, the process repeated itself again and again, until the star became a giant onion with a central core of iron surrounded by layers of burning silicon and sulfer, oxygen and neon, helium and finally hydrogen. Since energy is required to make both lighter and heavier nuclei from iron there is no energy generation process available to the core to balance the force of gravity. The core contracts and heats—however this decreases the pressure instead of increasing it because the temperature has become high enough for the protons to capture the degenerate electrons (who were providing most of the pressure that supported the core), and energy gets used up photodisintegrating the nuclei. The core therefore collapses very rapidly for a few tenths of a second—until the density exceeds that of a nucleus, at which point the nuclear force becomes repulsive and the core rebounds, sending a shock wave out through the onion. This shock (perhaps with the help of the neutrinos) explodes the outer layers of the star, creating a supernova.

Meanwhile the core has become a very hot lump of neutrons, protons, electrons and neutrinos with a radius of $\sim$ 100 km ... to cool and contract to the radius of a neutron star ($\sim$ 10 km) it needs to lose a great deal of energy, which it does by emitting neutrinos. However the density is so high that even neutrinos can not escape freely, which is why there is still a significant fraction of electrons and protons in the core. There will be a very short (millisecond) energetic (luminosity $\sim 10^{53}$ ergs/sec) burst of neutrinos as the core starts to collapse, emitted while the density was still low enough for the neutrinos to be able to escape, followed by a longer ($\sim$ 20 second) 'tail' of neutrinos who had to diffuse out.

IMB and Kamioka observed neutrinos over a period of 11 seconds, indicating that the proto neutron star did, in fact, cool by emitting neutrinos. One can therefore get a limit on paratons by requiring that they not cool the star within the first few seconds.

To be efficiently produced and emitted by the core, paratons need ϵ sufficiently big to be made in large quantities, and a small enough charge to escape freely. This means that there will be a lower bound on ϵ, above which paratons are allowed because they could not escape freely, and an upper bound below which they are allowed because they would not be produced in sufficient numbers to cool the neutron star within a few seconds. To get a useful limit from the supernova, the lower bound obviously needs to be a number of orders of magnitude above the upper bound—and it does not appear to be.

Although the temperature is well above the electron mass, there are very few positrons because the electrons are degenerate: the chemical potential is $\nu \sim$ 150 MeV and the positron number density is suppressed by a factor $e^{-\nu/T}$. Pair production of paratons from e^+e^- annihilations is therefore not very efficient. The interaction rates for more exotic processes, such as $e + Z \rightarrow e + Z + f + f$ and $p+p \rightarrow p+p+f+\bar{f}$, are difficult to approximate, but rough estimates indicate that they are respectively well above the lower bound on ϵ (which is $\epsilon >? 10^{-7} \rightarrow 10^{-8}$?) and just below it. And finally plasmon decay probably gives an upper bound on ϵ of $10^{-10} \rightarrow 10^{-8}$, which is too close to the lower bound to be worth pursuing. So no useful limits on paratons are expected from the supernova.

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CHAPTER SIX

THE WILD GOOSE CHASE

6.1 the problems

There are numerous ways of trying to rule out fractionally charged particles in the central area of figures B.5 and B.6. However, the experimental numbers involved tend to be limits on number densities, so one needs to be able to calculate "gold-plated" lower limits on the paraton relic density.

a) halo cooling

The only way to predict the abundance of paratons is to assume that they have the relic cosmological density calculated from their mass and charge (see chapter 4, section 4), and then try to predict how many will be on the earth today. This involves understanding what they will do during galaxy formation. If cold dark matter galaxy formation (CDM) is right, then paratons need to contribute $\Omega < 1$ because they are not "dark", so they probably would not work as 'the' dark matter of CDM. However this only matters for a narrow band just below the $\rho_f < \rho_c$ limit (and possibly only for 'large' values of ϵ). Ordinary weakly interacting (dark matter) particles are assumed to clump gravitationally with ordinary matter, but to get left behind in a halo when the baryons collapse into the disk of the galaxy. This is because the weakly interacting particles can not radiate away their gravitational potential energy. One can then make standard assumptions about the density of the halo (~ 1 GeV/cm^3) and the velocity of the particles it is made of ($\sim 3 \times 10^5$ m/s), to predict a particle flux at the earth.

Paratons interact electromagnetically, and the value of ϵ ranges from 1 to 10^{-7} in the central areas. If $\epsilon \sim 1$, presumeably the paratons will cool and collapse with the baryons—if $\epsilon \sim 10^{-7}$ they are probably weakly interacting enough to remain in the halo. What happens in between is unclear ... the principal energy loss mechanism is probably paraton-electron scattering, so the cooling time scale is inversely proportional to the number density of free electrons. This in turn depends on the electron temperature which is complicated to calculate. (The gas is heated by shocks and star formation, but when ionized can probably radiate energy away faster than it gains energy from gravitational collapse, and therefore cools. [59])

b) the earth's magnetic field

The paratons have to get through the earth's dipole field in some manner. For momenta greater than some critical value given approximately by the Stormer formula [60]

$$p_c = \frac{\epsilon\, 59 \cos^4 \lambda}{r^2(1 + \sqrt{1 - \cos^3 \lambda \sin \theta \sin \phi})^2} \tag{6.1}$$

they go straight through. In this equation r is the distance from the earths centre in units of earth radii, λ is the magnetic latitude (slightly different from ordinary latitude because magnetic north is not the north pole), θ is the zenith angle and ϕ is the azimuth measured clockwise from magnetic north (this makes sense standing on the earth looking out). For $p < p_c$, incident paratons can get in directly at the poles, spiral around the field lines and again come down at the poles, or be deflected away.

c) getting through the atmosphere

Ordinary dark matter particles are sufficiently weakly interacting that they can easily penetrate several hundred feet underground to dark matter detectors. They are truly weakly interacting in the sense that they interact via the exchange of a massive intermediate vector boson; this means that WIMP-ordinary matter scattering is a point interaction, and the WIMPS can [illegible] l to scatter off nuclei, but not electrons.

Paratons are very different in that they interact via a long range force; electromagnetic cross sections diverge at low energies. Furthermore, light charged particles in a medium lose vertical momentum by scattering of nuclei, but lose energy by scattering of electrons [61]. (One might guess that a particle heavier than a nucleus would also lose energy by scattering of nuclei.) So the range of a paraton in the atmosphere should somehow be calculated starting from the Bethe-Bloch formula, not from the cross section for paraton-nucleus scattering. Holdom used the first meth[illegible] Goldberg and Hall [62] and Caldwell et al. [63] used the second.

Go[illegible] and Hall calculate the number of particles that will arrive at a germanium detector 4000 mwe ([illegible]s water equivalent) underground by taking the paraton-nucleus scattering cross section to be constant (a screened Coulomb cross section is inversely proportional to the paraton velocity squared), and calculating how many collisions, on average, it would take for a paraton to be left with too little energy to trigger the detector. This gives them an average depth at which the paratons are no longer detectable of $\approx$ 2000 mwe (no ϵ dependance indicated...$10^{-3} \rightarrow 10^{-4}$? $\mu \sim$ TeV), and they assume that the average depth of the paraton flux is a poisson distribution about this average. They claim that the counting rate at 4000 mwe constrains $\mu_{TeV}\epsilon^{-2} \times 10^{-8} \leq 1$. From modifying the

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The time-of-flight experiment gives an upper limit on the number density in sea water of one in $\sim 10^{28} \rightarrow 10^{29}$ (for particles with charge +1 and mass beween 8 and 1200 GeV). This is a very stringent limit, but unfortunately, three problems apply here.

Paratons with $\epsilon \sim .1 \rightarrow 1$ and $\mu \sim$ GeV $\rightarrow$ TeV would probably cool into the disk of the galaxy. What they do next is not clear. They could condense into stars with the baryons, in which case most of them would annihilate in the star and leave no traces. They could still be floating around in the disk, which could put too much dark matter in the disk for $\mu \sim$ TeV, but if it is a thick disk it might not be a problem. It is probably safe to assume that the paraton density is at least $\Omega_f/\Omega_b \times$ { the baryon density in the local interstellar medium}. If all the paratons incident on the earth at this density were evenly distributed on the hydrogen in the ocean, and if they stayed bound to the proton during the time-of-flight experiment, then the predicted number density is at least eight orders of magnitude greater than the experimental upper bound.

However, paratons with parameters that could be ruled out by the time-of-flight experiment would not have sufficient momentum to get through the earths magnetic field (assuming the paraton velocity is the standard WIMP velocity ~ 300 km/sec). Some fraction would be deflected away from the earth, some fraction would spiral around the field lines and come down at the poles, and of course they can always get in at the poles because the field there is vertical. It is possible to calculate a minimum number of paratons incident on the poles, but I could not begin to guess what kind of number density this would give in the oceans.

There is the additional problem that according to Holdoms range calculations, paratons with $\epsilon \sim .1 \rightarrow 1$ are going to stop a long way above the earths surface. The negative ones will presumeably bind to the molecules in the atmosphere, and I do not know at what rate the paratons would be transfered to the hydrogen or oxygen in the oceans.

I do not know how to quantify either of the above problems, so I do not think I can use this limit.

For large ϵ, the D_2O density limit also suffers from the halo cooling problem. Assuming, as above, that the density of paratons in the solar system is $\Omega_f/\Omega_b\times$ {mass density of the interstellar medium}, then the predicted number density in the oceans (neglecting the magnetic field and assuming uniform mixing in all the water in the oceans) is within 1 to 3 orders of magnitude of the experimental limit. I think this is a little too close to rule out paratons with $\epsilon \sim .1 \rightarrow 1$ and $\mu > 1$ TeV. It is also not reasonable to neglect the magnetic field.

For smaller values of ϵ, the paratons could be in the halo, which would increase the flux by $\sim 10^3$; however it is also possible that they are not ...

Stopping in the atmosphere will also be a problem for this limit; if the paratons do not reach the surface, they will presumeably bind equally well to the nitrogen ($\sim$ 80%), oxygen ($\sim$ 20 %) and water ($\sim 0 \rightarrow 4$ %) in the atmosphere. If only the paratons that bind to the water molecules end up in the ocean, this decreases the predicted number density by $\geq 10^2$.

De Rujula, Glashow and Sarid [67] have more anomolous nuclei search references, but they are all for rocks and metals, which makes them even more difficult to use (at least the ocean washes around).

b) dark matter searches

Rich, Rocchio and Spiro [65] sent a silicon semi-conductor detector up near the top of the atmosphere in a ballon to look for "strongly interacting" dark matter particles. This (probably) avoids the problem of getting through the atmosphere, and paratons with the parameters that this experiment could rule out would get through the earths magnetic field, so the only question is whether they would stay in the halo.

If our galaxy was initially a large cloud of ionized gas, the ordinary matter (baryons and electrons) could probably cool faster than it could collapse into a disk [66]. It might therefore recombine into hydrogen, leaving some fraction of free electrons. If the number density of free electrons was less than 10^{-2} cm^{-3} ($\simeq 1\%$ of the electrons unbound), then the paratons in the 'area' (in $\mu - \epsilon$ space) considered by this experiment could not have cooled in the age of the universe. However, this assumes that the hydrogen is not kept ionized by shock heating, star formation, or the existence of the paratons (if the electrons just try and cool the baryons , many of them might recombine; if they also try to cool the paratons, more of them would have to stay unbound so that they could do this.)

The timescale for paratons to collapse into the disk of the galaxy is [58]

$$\tau \sim \frac{2 \times 10^8 \mu_{GeV}}{\epsilon^2 n_e} \text{ sec} \tag{6.3}$$

where n_e is the electron number density in cm^{-3}. If all the electrons are assumed to remain unbound ($n_e = 1$ cm^{-3}), then paratons with parameters that could be ruled out by this experiment could have collapsed into the disk within the life-time of the universe.

Underground dark matter detection experiments are not interesting for the model of paratons cosidered here. Goldberg and Hall tried to rule out an 'area' (contained by the ballon experiment) using surface and underground detector results, but Holdom maintains that they greatly overestimated the distance paratons would travel in the earth due to their neglect of the Bethe-Bloch mechanism of energy loss. The results from the underground germanium detectors could only rule out an 'area' considerably to the 'left' (smaller ϵ) of the balloon experiment results because ϵ would have to be very small for the paratons to penetrate that far.

c) annihilation in 'proton decay' detectors

There is a small 'area' in the left hand corner of the triangle where the paratons are probably sufficiently weakly interacting to remain in the halo, and would have a small enough charge to get through the earths magnetic field. For $\epsilon < 10^{-4} \rightarrow 10^{-5}$ they would also not bind to nuclei. If a sufficient number of them diffused down to proton decay detectors, their annihilation there would be detectable. This is only true for the model with a paraphoton, because paratons in the 'triangle' on the graph without paraphotons do not have enough momentum to get through the magnetic field.

The branching ratio to something visible for paraton annihilation is

$$\frac{\sigma(f\bar{f} \rightarrow e^+e^-)}{\sigma(f\bar{f} \rightarrow \gamma'\gamma')} \sim \frac{\epsilon^2\alpha^2}{\alpha'^2} \tag{6.4}$$

or

$$\frac{\sigma(f\bar{f} \rightarrow \gamma\gamma')}{\sigma(f\bar{f} \rightarrow \gamma'\gamma')} \sim \frac{\epsilon^2\alpha\alpha'}{\alpha'^2} \tag{6.5}$$

If one assumes that $\alpha_2 \sim \alpha$, at least one annihilation in ϵ^{-2} should produce something visible.

If the incident paratons were evenly spread through the whole earth, and if they annihilate at the rate at which they are incident, then there should be a very observable signal in the underground detectors. However, the paratons stop near the surface and diffuse downwards, annihilating as they go. Results are then sensitive to details of the process of diffusion and annihilation.

d) annihilation to gamma rays

If the paratons remain in the halo, they do not annihilate often enough to produce a detectable gamma ray flux. If they collapse into the disk, whether or not they annihilate in sufficient numbers to give a detectable gamma ray flux will depend on whether they cool into a lumpy or a homogeneous gas.

If most (> 90 %) of the paratons do annihilate, the predicted gamma ray flux would be in contradiction with the observed one for a large range of ϵ and μ, IF the predicted flux is isotropic. On this assumption, it would be possible to ressurect the dark matter search (and heavy water) limit: either the paratons are in the halo and the dark matter limit applies, or they annihilated which is ruled out by the observed gamma ray spectrum. (Heavy water would be more difficult because there are still problems with the magnetic field and the atmosphere.)

I do not know whether it is reasonable to assume that the flux is isotropic – if the paratons annihilated in stars, there would be no flux at all. If the universe is imagined to be an expanding ballon, with spots on it where the galaxies are, then the gamma ray flux would be a ring moving out from each spot. If the rings are wide, the distribution would be isotropic; if they are narrow, it would not be. Limits of this type are clearly dependant on details of galaxy formation and evolution.

CHAPTER SEVEN

CONCLUSION

One can get useful constraints on paratons from accelerator experiments, stellar evolution and cosmological arguments. The limits for the model without paraphotons are plotted in figure B.5; those for the model with paraphotons are in figure B.6. It is unfortunate that these calculations leave a central triangular window where paratons are allowed, but there do not seem to be any simple physical arguments to rule this region out.

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APPENDIX A

FACTORS OF 2 IN INTERACTION RATES

(This is copied virtually word for word from [39].) Suppose a particle comes in g different spin states, where a particle is counted as itself and its antiparticle—so Majorana spinors have $g = 2$ and Dirac spinors have $g = 4$. Then the number density can be written

$$n = \sum_{i=1}^{g} n_i \tag{A.1}$$

where n_i is the density of particles in a specific spin and particle/antiparticle state.

The rate per unit volume at which the i-state particles annihilate will be

$$R_i = \sum_j < n_i n_j \sigma_{ij} \beta > \tag{A.2}$$

where σ_{ij} is the cross section for an "i" to annihilate with a "j" and the average is over the distribution in momentum space of the "j"s. The rate at which the whole species of particle annihilates will be

$$R = \sum_{i,j} < n_i n_j \sigma_{ij} \beta > \quad . \tag{A.3}$$

Assuming $n_i = n_j = n/g$, this is

$$R = \frac{n^2}{g^2} \sum_{i,j} < \sigma_{ij} \beta > \tag{A.4}$$

When cross sections are calculated, they are averaged over the spins of the incoming particles, so that σ_{ij} will not depend on the spins of "i" and "j", but only on whether they are particles or anti-particles. So

$$R = \frac{n^2}{g^2} \left[\frac{g}{2} < (\sigma_{??} + \sigma_{?\bar{?}} + \sigma_{\bar{?}?} + \sigma_{\bar{?}\bar{?}}) \beta > \right] \tag{A.5}$$

where ? is the particle.

Applying this to paratons, where $\sigma_{ff} = \sigma_{ff} = 0$ and $g = 4$, gives

$$R = \frac{n^2}{4} < \sigma\beta > \tag{A.6}$$

where n is the density of paratons and antiparatons. This is just

$$R = n_f n_f < \sigma\beta > \tag{A.7}$$

as expected.

APPENDIX B

TABLES AND FIGURES

Table B.1: **unit conversions**

In units where $\hbar = c = k = 1$:

$G = 1/m_{pl}^2$ and $m_{pl} = 1.22 \times 10^{19}$ GeV

1 GeV $= 1.16 \times 10^{13}$ ^{o}K

$= 1.77 \times 10^{-24}$ grams

$= 1.60 \times 10^{-3}$ ergs

1 GeV^{-1} $= 1.97 \times 10^{-14}$ cm

$= 6.58 \times 10^{-25}$ sec

Table B.2: **g_{eff} and N as a function of temperature**

mass		$4g_{eff}$	$N(T)$
$< m_e$	$<$ .5MeV	13.5 (8) *	1
$m_e \leftrightarrow T_d$	.5 $\leftrightarrow$ 1 MeV	43 (22) *	1
$T_d \leftrightarrow m_\mu$	.5 $\leftrightarrow$ 100 MeV	43	1
$m_\mu \leftrightarrow m_\pi$	100 $\leftrightarrow$ 137 MeV	57	2
$m_\pi \leftrightarrow T_c$ **	137 MeV $\leftrightarrow$?	69	3
$T_c \leftrightarrow m_s$	? $\leftrightarrow$ 480 MeV	205	11/3
$m_s \leftrightarrow m_c$	.48 $\leftrightarrow$ 1.65 GeV	247	4
$m_c \leftrightarrow m_\tau$	1.65 $\leftrightarrow$ 1.80 GeV	289	16/3
$m_\tau \leftrightarrow m_b$	1.8 $\leftrightarrow$ 5.2 GeV	303	19/3
$m_b \leftrightarrow m_t$	5.2 $\leftrightarrow$?40? GeV	345	20/3
$m_t \leftrightarrow m_W$	?40? $\leftrightarrow$ 90 GeV	387	8
$> m_W$	$>$ 90 GeV	423	9

* the quantity in parentheses is the effective number of interacting spin degrees of freedom $\equiv 4n_{eff}$ (...so does not include neutrinos below the weak interaction freeze out temperature)

** T_c is the quark–hadron transition temperature, and is somewhere between 200 and 400 MeV

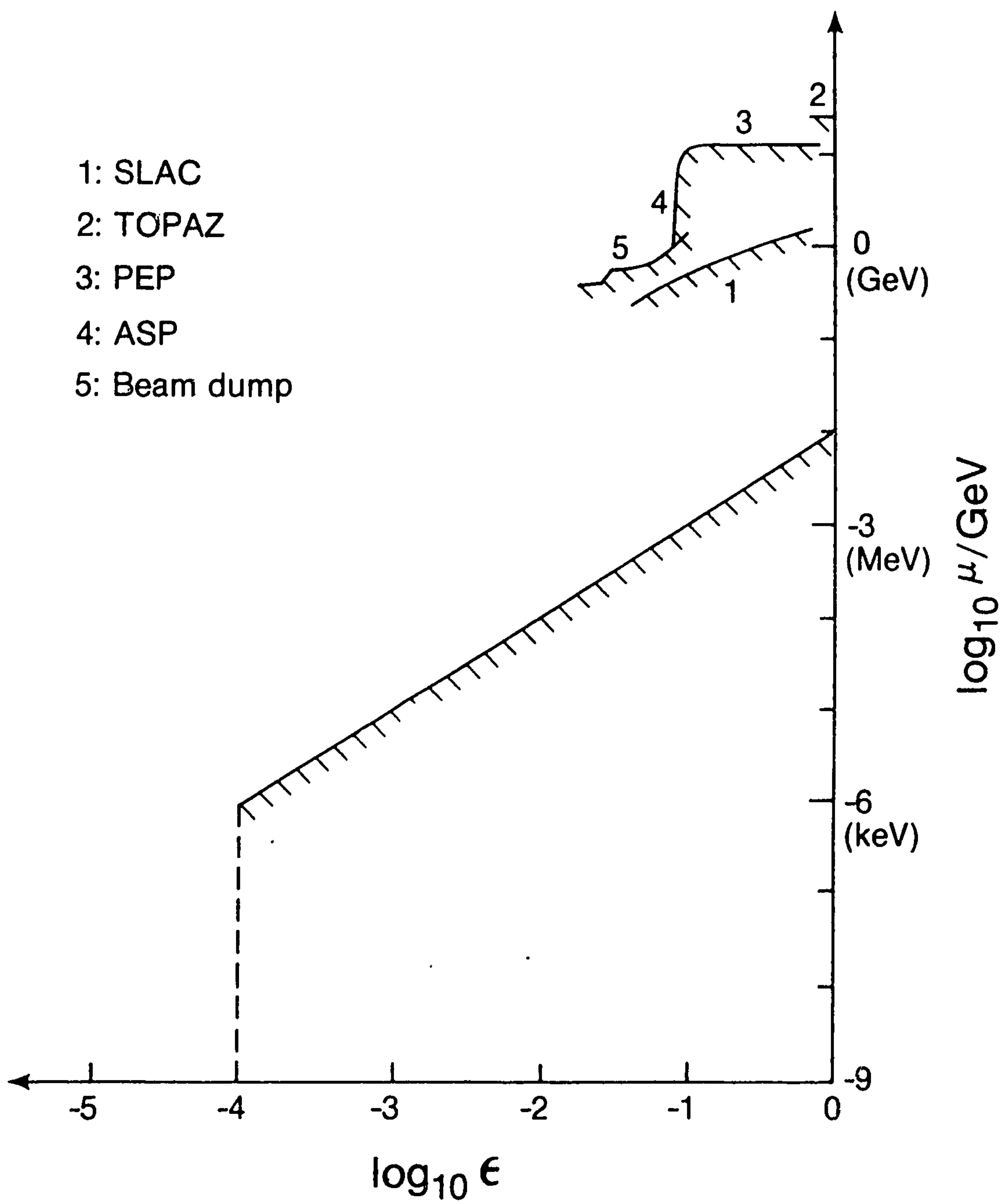

Figure B.1: region of $\mu-\epsilon$ space ruled out by accelerator and Lamb shift experiments

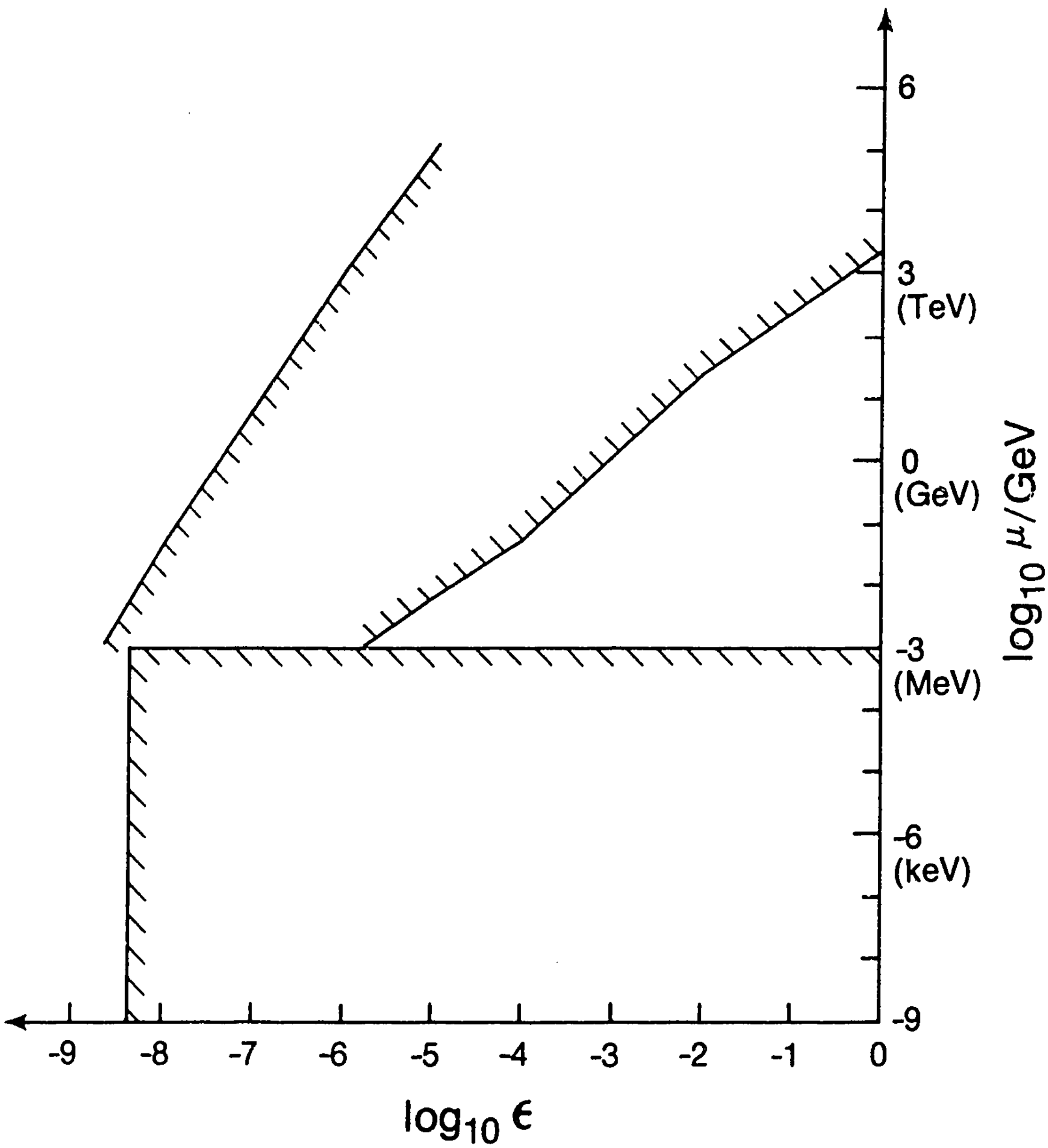

Figure B.2: region of $\mu - \epsilon$ space ruled out by cosmological arguments, if there is no paraphoton

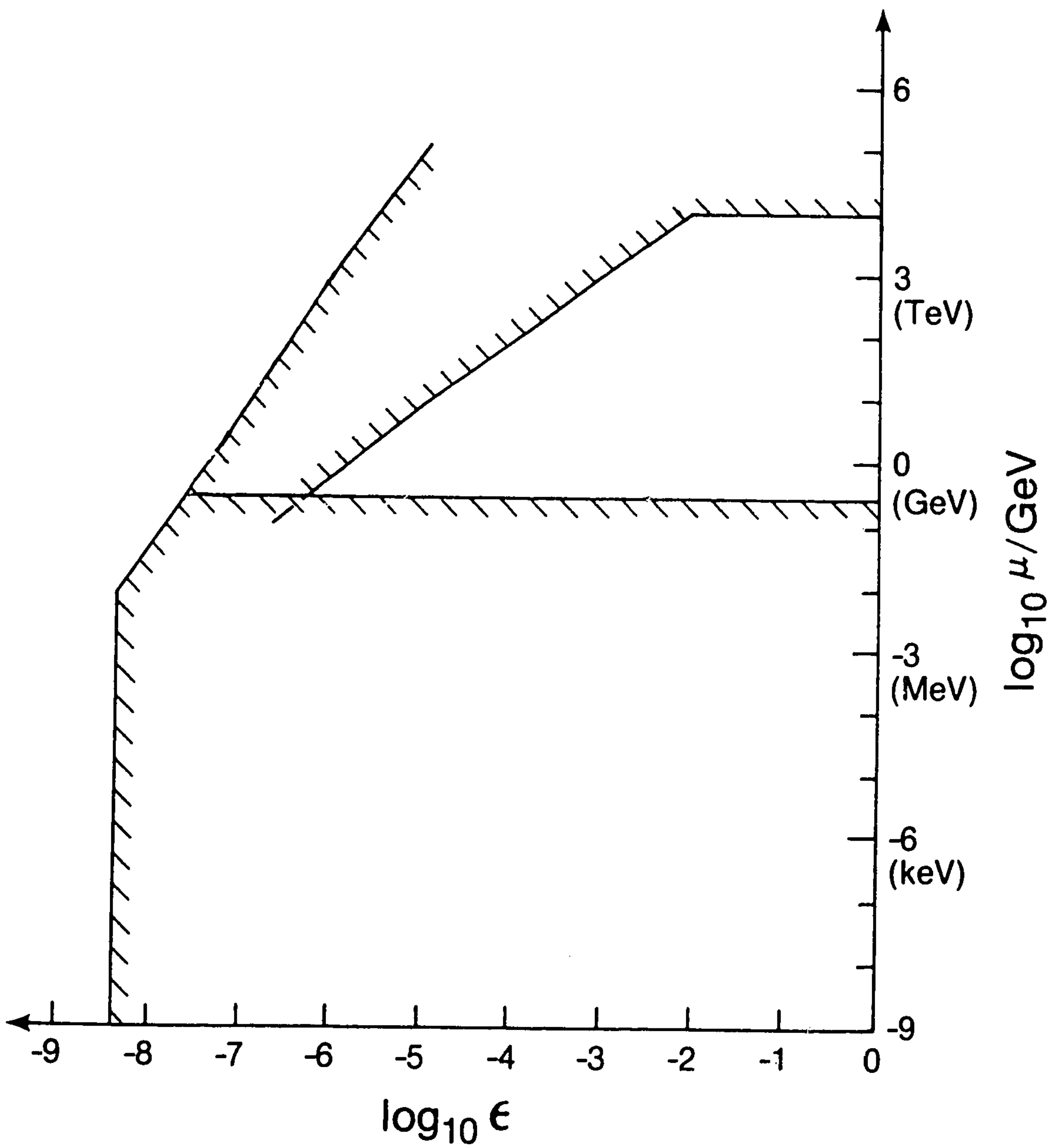

Figure B.3: region of $\mu - \epsilon$ space ruled out by cosmological arguments, if there is a paraphoton

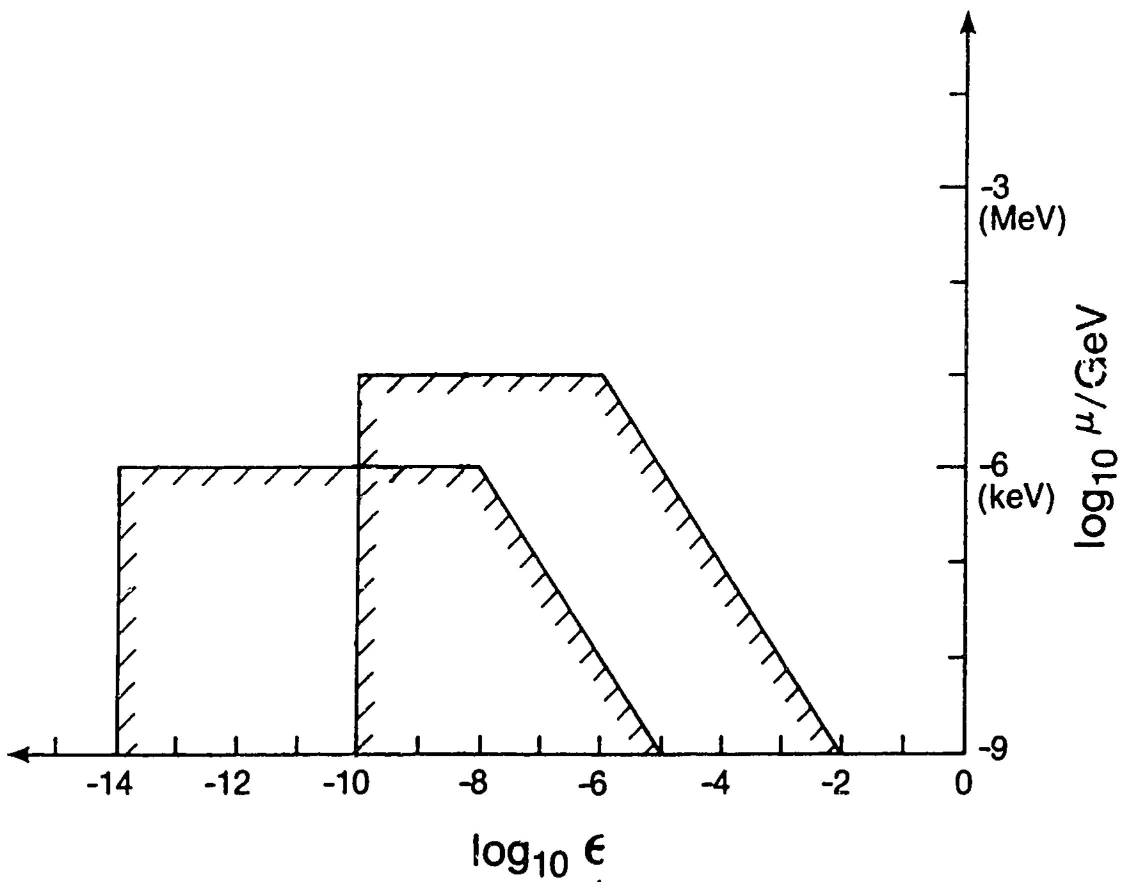

Figure B.4: region of $\mu - \epsilon$ space ruled out by red giants and white dwarfs

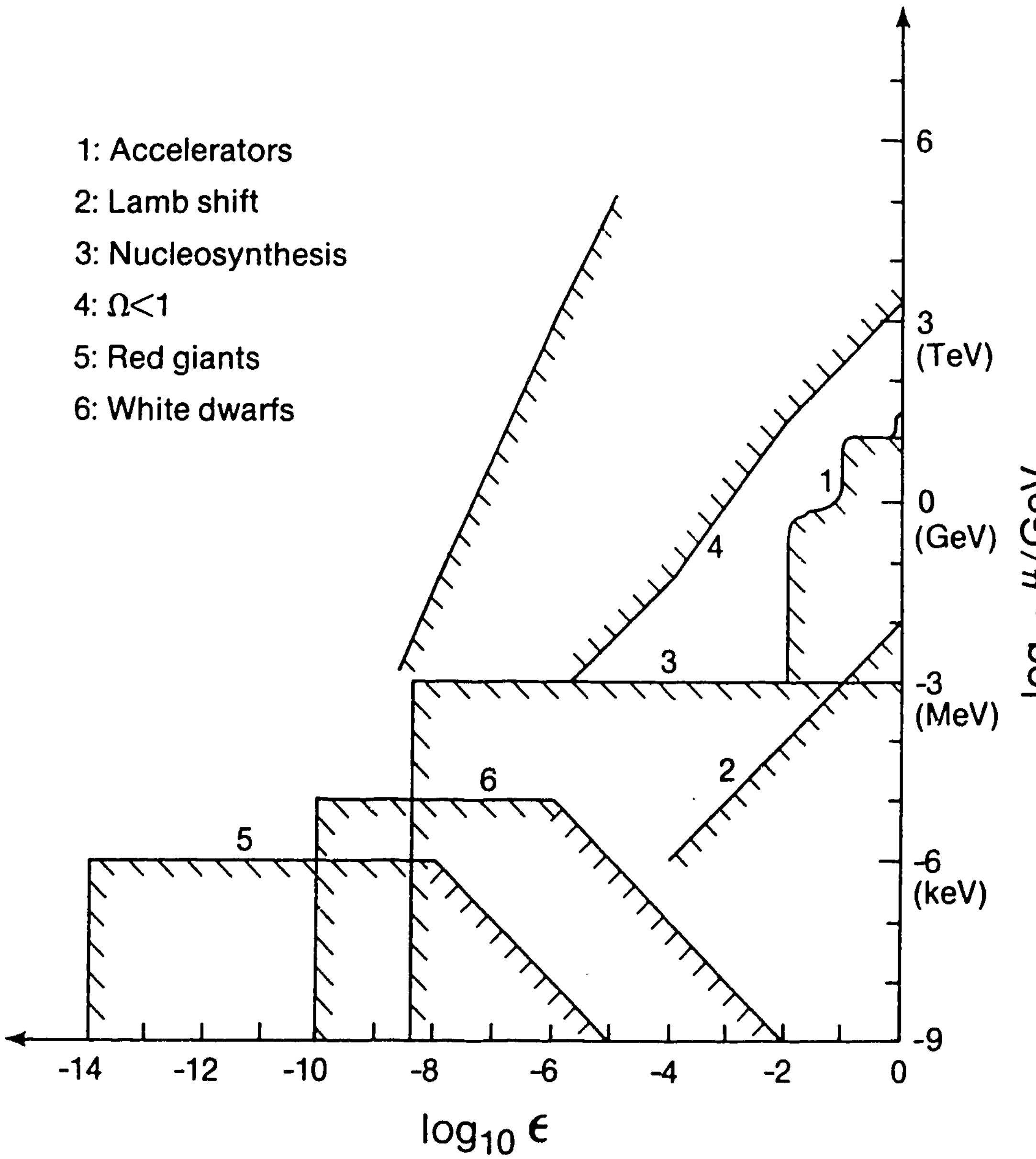

Figure B.5: region of $\mu - \epsilon$ space ruled out for the model without a paraphoton

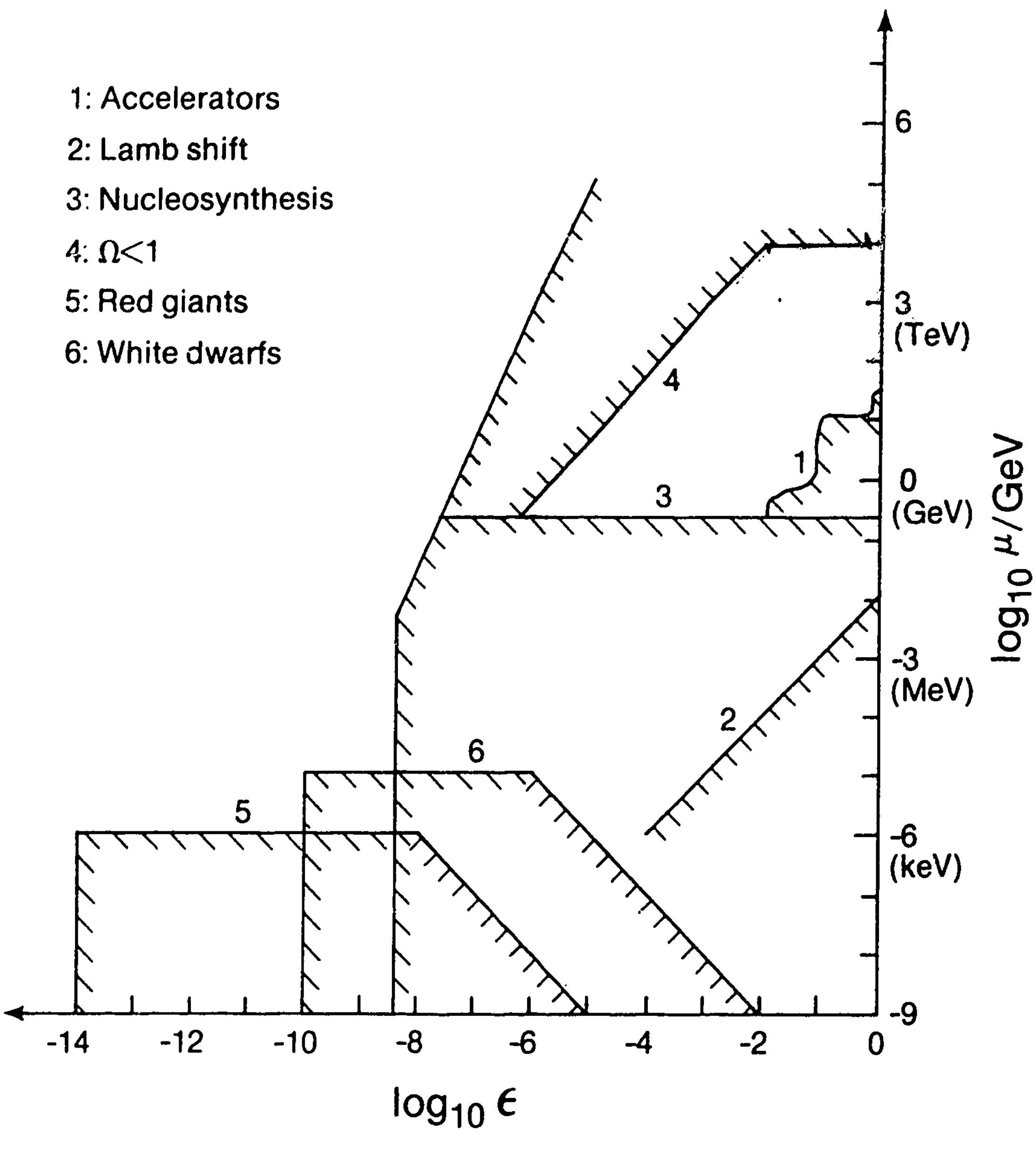

Figure B.6: region of $\mu - \epsilon$ space ruled out for the model with a paraphoton