Three Essays on The Application of The Markov Switching Multifractal Model

by

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Abstract

The overall purpose of this thesis is to extend and apply the Markov Switching Multifractal (MSM) model to various economic problems. To this extent, Chapter 1 lays the ground work for the next chapters by reviewing the MSM model, discussing its properties and outlining its estimation procedures. The chapter also reviews the distributional properties of several commodity markets that make them amenable to the MSM model.

Chapter 2 extends the MSM model by incorporating a vector error correction component, which includes in the conditional mean equation, the cointegrating relationship between spot and futures prices. The VECM-MSM model has two distinctive features that incorporate the empirical properties of asset prices. First, it includes an error correction mechanism in the mean equation that incorporates the long-run relationship between spot and futures prices. Second, the model specifies the conditional second moments as a bivariate Markov Switching Multifractal (MSM) model.

The VECM-MSM model is applied to study the problem of risk hedging in the futures market. The hedging effectiveness of the proposed VECM-MSM model is evaluated, using a value-at-risk (VaR) approach. Specifically, we compare the hedging effectiveness of the proposed model to those of alternative models by assessing their unconditional and conditional VaR coverages. Models are then ranked in terms of the adequacy and accuracy of their hedged portfolio VaR. The in-sample and out-of-

sample hedge effectiveness shows that the VECM-MSM hedged portfolio outperforms alternative hedging strategies in terms of having the lowest rate of VaR violations among the different strategies. Statistical tests of unconditional and conditional coverages also show that the VECM-MSM model better predicts an investor's downside risk in that the VaR predictions are more accurate than the predictions from the alternative models.

Chapter 3 of this thesis investigates the excess commodity comovement phenomenon, using the MSM model. One of the stylized facts of commodity prices is their tendency for comovement. The phenomenon implies that seemingly unrelated commodities tend to move together beyond what can be attributed to fundamentals, such as demand and supply conditions, exchange rates, interest rates, industrial production etc. Excess commodity comovement bears significant welfare and risk management implications. For an instance, a synchronous rise in prices of commodities exerts significant inflationary pressure on commodity import dependent countries, and limits their ability to maintain economic stability and resist inflationary pressures. Moreover, to the extent that comovement measures, such as correlation and covariance among commodities, comprise an essential ingredient in risk assessment, pricing, portfolio management and hedging, failure to account for such excess comovement can lead to sub-optimal economic decisions.

Therefore within the debate on excess commodity comovement, the objective of this chapter is twofold. First, it analyzes the degree of excess commodity comovement across a variety of commodities. Second, it analyzes the frequency-dependent nature of comovement across related (e.g. crude and heating oil) and unrelated commodities (e.g. copper and corn). First, we find that there is significant comovement between commodity prices, beyond what can simply be explained by macroeconomic fundamentals. Second, decomposing comovements into multiple frequencies, we find that all commodities exhibit long-run excess comovements which are driven by low frequency fundamentals such as weather, demographic and macroeconomic factors. But some commodities also exhibit significant short-run excess comovements that may be attributable to short-run factors such as liquidity constraints, indexation, etc. Third, the dynamic correlations show that excess comovements are higher in periods of high volatility and vice-versa.

The final chapter applies a new class of model, the Autoregressive Markov switching multifractal model, for forecasting spot electricity prices. Three variants of the model are examined. The first variant, the ARX-MSM, is a simple mean-reversion model that allows for a Markov switching multifracal (MSM) volatility process. This model incorporates several of the main characteristics of electricity prices, namely mean-reversion, conditional volatility and price spikes. Price jumps with heterogeneous durations are introduced through switches in the volatility components with heterogeneous frequencies. This second variant allows for regime switches in both drift and volatility. This is motivated by earlier empirical findings that electricity prices are characterized by regime dependent drifts and volatility. The last variant of the model introduces a risk premium into the mean equation to capture the impact of volatility on prices.

Employing hourly prices from the AESO market, the parameters of the ARX-MSM models are estimated, and one-step-ahead hourly forecasts are obtained. To put the performance of the ARX-MSM models into perspective, the results are compared to those of other notable models used in the literature, namely the AR(1), ARX, ARX-GARCH, mean-reverting jump and the 2-state independent Markov regime switching models. Goodness-of-fit tests indicate that the ARX-MSM models fit the data significantly better than the competing models. Likewise, out-of-sample results show that the ARX-MSM models provide better forecast accuracy.

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Chapter 1

A Review of The Markov-Switching Multifractal Model

1.1. Introduction

Early studies on commodity prices are based on the mainstream literature of financial markets of the time, with the fundamental assumption that financial returns are normally distributed and that financial prices follow a random-walk process Gibson and Schwartz (1990); Schwartz (1997). However empirical studies suggest that many financial returns exhibit characteristics that significantly differ from those of a normal distribution or a random-walk process. Therefore, effectively modeling the statistical characteristics of financial series has been the subject of many academic studies in the last few decades. To better understand the characteristics of financial returns and to motivate the modeling choice of subsequent chapters, we discuss the so called stylized facts of financial returns in what follows. The stylized facts are as follows:

1. Volatility clustering: Financial returns volatility exhibit strong autocorrelation

over a long horizon, a property also known as persistence. Large negative or positive returns on a given day tend to be followed by substantial return movements. To accommodate this feature, the generalized autoregressive conditional heteroscedasticity(GARCH)model was introduced by Engle (1982) and Bollerslev (1986), and has since become the standard stochastic volatility model. Persistent and variable volatility has significant implications for risk pricing and management, therefore cannot be over emphasized.

2. Heavy tails: The unconditional distributions of financial returns are characterized by heavy tails when compared to the normal distribution i.e. the probability of extreme returns is much higher than that suggested by a normal distribution. This phenomenon is also known as excess kurtosis. To accommodate this characteristic, diffusion jump models have been employed in continuous time settings (Hilliard and Reis, 1999; Merton, 1976). In discrete time, a GARCH model with a student-t distributed errors can be employed to achieve the same general effect (Bollerslev, 1987).

3. Aggregational gaussianity: This property implies the unconditional distribution of returns varies nonlinearly as the frequency of observations changes. At short horizons, the distribution of returns is far from normality. But it becomes progressively closer to following a normal distribution as the frequency of observations decreases (Campbell et al., 1996).

4. Power-law scaling: The moments of the absolute value of returns tend to vary as a power function of the frequency of observations, a phenomenon often described as power-law variation or scaling. It also implies that the growth rate of the q^{th} moment is a non-linear function of q, a feature that is consistent with the nonlinear variations of financial returns distribution with the time horizon. Evidence of power-law scaling has been documented in equity and exchange rate

markets (Calvet and Fisher, 2002; Lux, 2008), interest rate markets (Jamdee and Los, 2005) and commodity markets (Li and Lu, 2011).

In order to accommodate the aforementioned characteristics of financial returns, several models have been proposed in the extant literature. Prominent among these is the GARCH class of models introduced by Engle (1982) and Bollerslev (1986). In this class of models, volatility follows a smooth autoregressive process. In order to further capture outliers in returns, standard GARCH models have been augmented with error distributions other than the Gaussian type, such as the student-t distribution. GARCH models have been found to produce good short-run forecasts of volatility dynamics but they suffer from major drawbacks. It is well known that GARCH models often have difficulty capturing low-frequency volatility cycles. Also, GARCH models have been found to produce volatility forecasts that are too persistent (Calvet and Fisher, 2008; Lien and Tse, 2002). Given these short comings, several other models have been proposed. These include the fractionally integrated GARCH model (Baillie et al., 1996; Ding and Granger, 1996), long-memory stochastic volatility models (Breidt et al., 1998; Comte and Renault, 1998; Robinson and Zaffaroni, 1998), diffusion jump models (Merton, 1976) and Markov regime-switching models (Hamilton, 1989, 1990).

Notwithstanding the significant advancements that have been made, none of the aforementioned models is able to simultaneously accommodate the full set of stylized facts related to financial returns discussed earlier. Moreover, and perhaps most importantly, none of the above models characterize the multifrequency nature of the persistence and variability of financial returns volatility. Yet, an in-depth understanding of financial returns cannot be achieved without investigating their features at different frequencies. A casual observation of financial returns at intra-day, daily, weekly, monthly or even at longer intervals reveals the heterogeneity in durations and occurrence of shocks to returns. This observation is consistent with the intuition that economic shocks are highly heterogeneous in their degree of occurrence and persistence. High frequency shocks such as liquidity uncertainty, macroeconomic data releases, weather reports and analyst reports affect the dynamics of financial returns (Gennotte and Leland, 1990; Roll, 1984; Womack, 1996). At low frequencies, exhaustible energy resource uncertainty, political policy feedbacks, technological advancements, demographics, terrorist attacks, war, depression and macroeconomic uncertainties simultaneously drive financial returns (Bansal and Yaron, 2004; Fong, 2002; Wilson et al., 1996).

It is this multifrequency nature of financial volatility, coupled with the aforementioned properties of financial returns, that motivated the development of the Markov-Switching Multifractal (MSM) model in a series of articles by Calvet et al. (1997), Calvet and Fisher (2001, 2002, 2004) and Calvet et al. (2006). The MSM model is based on the regime-switching models of Hamilton (1989, 1990). The MSM model assumes that volatility comprises multiple components with differing degrees of persistence. Each volatility component switches randomly and independently of other components over time, generating volatility shocks of multiple frequencies. Hence, the MSM captures the thick-tails, long memory features and volatility clustering which are characteristics of many financial returns.

The objective of this chapter is to lay the ground work for the next chapters by reviewing the MSM model, discussing its properties and outlining its estimation procedures. The rest of this chapter is structured as follows. Section 2 reviews both the univariate and bivariate MSM models and provides some simulations to demonstrate the construction of the MSM model. Section 3 discusses the properties of commodity markets and how they can be characterized by the MSM model. Section 4 concludes.

1.2. The Markov-Switching Multifractal Model

1.2.1. Univariate Markov-Switching Multifractal Model

The univariate MSM model was introduced by Calvet and Fisher (2002) in the context of modeling financial returns. Let r_t denote log-return defined in discrete time on a regular grid $t = 0, 1, 2, ...\infty$. Following Calvet and Fisher (2002), r_t is defined as

$$r_t = \sigma(M_t)\varepsilon_t \tag{1.1}$$

where ε_t is *i.i.d.* standard Gaussian $\mathcal{N}(0, 1)$. Stochastic volatility is defined as

$$\sigma(M_t) = \overline{\sigma}\left(\prod_{k=1}^{\bar{k}} M_{k,t}\right)^{1/2} \tag{1.2}$$

where $\overline{\sigma}$ is a constant and coincides with the unconditional standard deviation of returns. Stochastic volatility is driven by a first-order Markov state vector M_t with k components

$$M_t = (M_{1,t}; M_{2,t}....; M_{\bar{k},t}) \tag{1.3}$$

At any period t, the dynamics of each volatility component is as follows,

$$M_{k,t} = \begin{cases} \text{is replaced} & \text{with probability } \gamma_k \\ M_{k,t-1} & \text{with probability } 1 - \gamma_k \end{cases}$$

Each component of M_t corresponds to a particular frequency $k = \{1, 2...\overline{k}\}$, with each frequency representing a shock of heterogeneous duration. $k = 1$ represents the lowest frequency (most persistent) volatility component while $k = \overline{k}$ represents the highest frequency (least persistent) volatility component. The volatility components $M_{k,t}$ have the same marginal distribution M, but switch at different frequencies with probability γ_k, where the switching events are assumed to be independent across k and t. γ_k is defined as:

$$\gamma_k = 1 - (1 - \gamma_{\bar{k}})^{b^{k-\bar{k}}},$$
$$\gamma_1 \leq \gamma_2 \leq, ...\gamma_{\overline{k}} \leq 1$$

where $b \in (0, \infty)$ controls the spacing between different volatility components.

The marginal distribution of volatility components, M, can take on any specification that ensures it has a positive support and a unit mean. A simple specification that has been used by Calvet and Fisher (2004), Calvet et al. (2006) and Lux (2008) is the binomial distribution. In a binomial MSM, each volatility component can take on two possible values $m_0 \in (1, 2)$ or $m_1 = 2 - m_0$ with probability 0.5. With this specification, each volatility component can switch independently between two possible states(high and low) while the state vector M_t can switch between $2^{\overline{k}}$ possible states. Also since all volatility components are replaced from the same marginal distribution, increasing the state space does not lead to an increase in the parameter space. This feature greatly contributes to the parsimony of the MSM model. The binomial MSM parameter vector is given by

$$\theta = (m_0, \overline{\sigma}, b, \gamma_{\overline{k}})$$

The size of the parameter m_0 determines the size of each volatility component. Therefore, the larger m_0 is, the larger the size of shock induced by switching in each volatility component. Likewise, the more volatile a returns series is, the larger $\overline{k}$ is going to be. Therefore, more volatile return series will require more volatility components to be included in the MSM specification in order to match the returns volatility.

The parameter γ_k controls the persistence of the shock induced by switching in each volatility component. The lower γ_k is, the higher the duration of the shock. Hence, γ_1 controls the persistence of the most persistent volatility component while $\gamma_{\overline{k}}$ controls the persistence of the least persistent volatility component, since $\gamma_1 \leq \gamma_2 \leq, ...\gamma_{\overline{k}}$. The parameter b controls the spacing between each transition probability, γ_k. Lastly, $\overline{\sigma}$ corresponds to the unconditional standard deviation of the returns series.

The MSM model is able to capture many of the properties exhibited by financial returns as discussed in the previous section. This is demonstrated in Figure 1.1. The figure illustrates the construction of univariate MSM when the marginal distribution M is binomial, $\overline{k}$ equals 10 and $(m_0, \overline{\sigma}, b, \gamma_{\overline{k}}) = (1.4, 0.5, 3, 0.85)$.

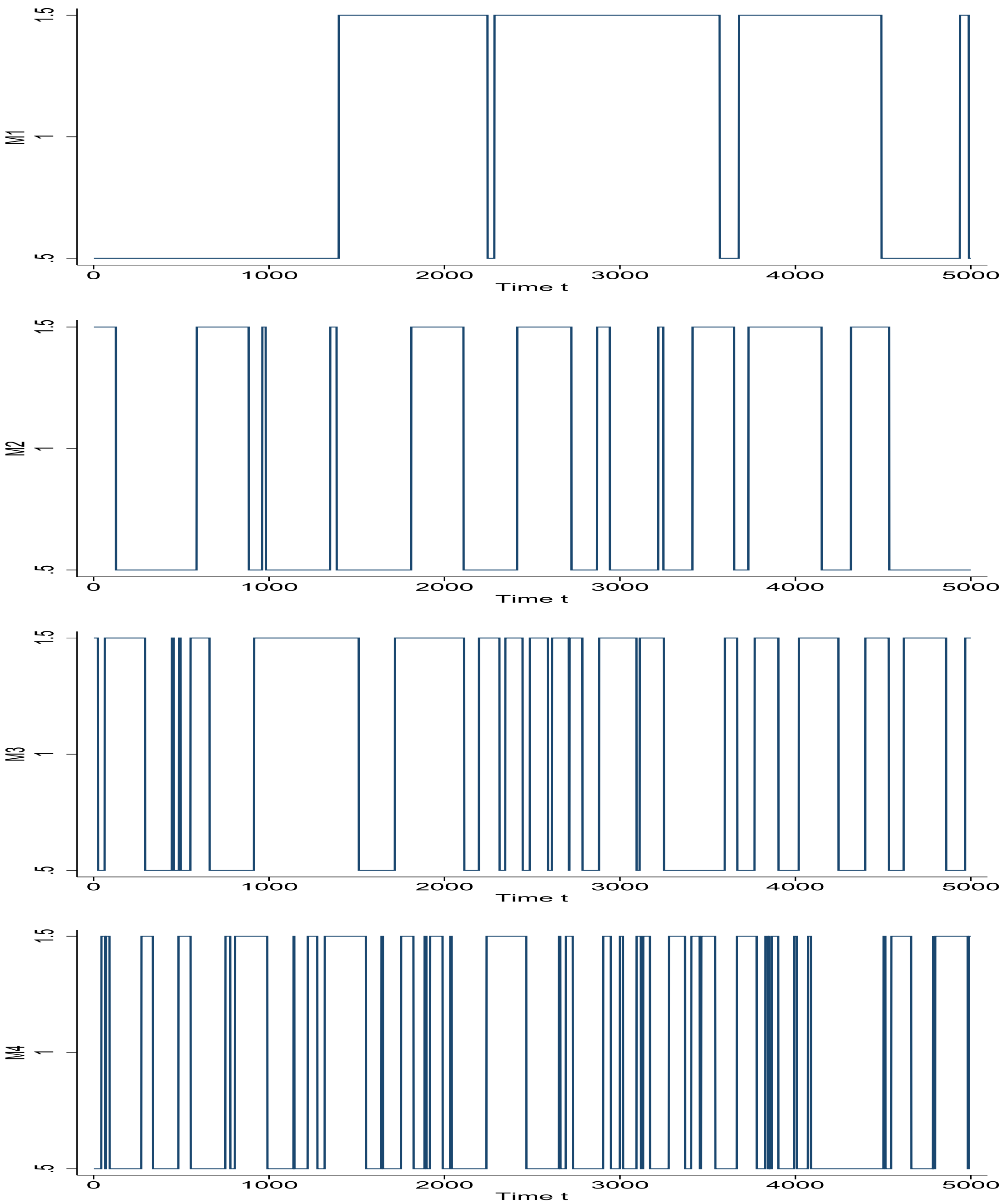
The volatility components are depicted beginning with M_1 and ending with M_{10}. The last two panels of 1.1 show the resultant daily volatility and return series. The return series shows extreme outliers and pronounced heterogeneity in volatility levels. Likewise, the volatility series reflects substantial peaks, clustering and intermittent bursts, accommodating a broad range of low, medium and high frequency dynamics. Also comparing the simulated MSM return series to a variety of energy, agricultural and metal commodity return series shown on Figure 1.2, it is easy to see substantial similarity between them as the commodity returns series are also characterized by volatility clustering, intermittent bursts and extreme outliers.

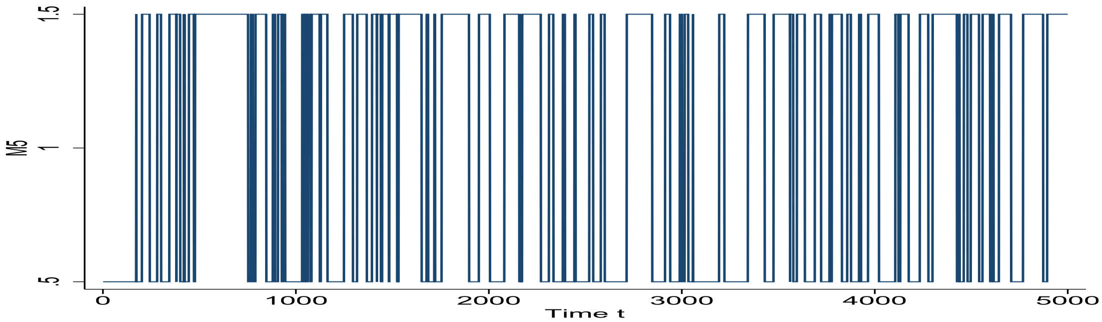
Univariate MSM Parameter Estimation

Since the econometrician only observes return r_t and not the state vector, M_t is therefore a latent state vector that must be inferred using Bayesian updating. Let

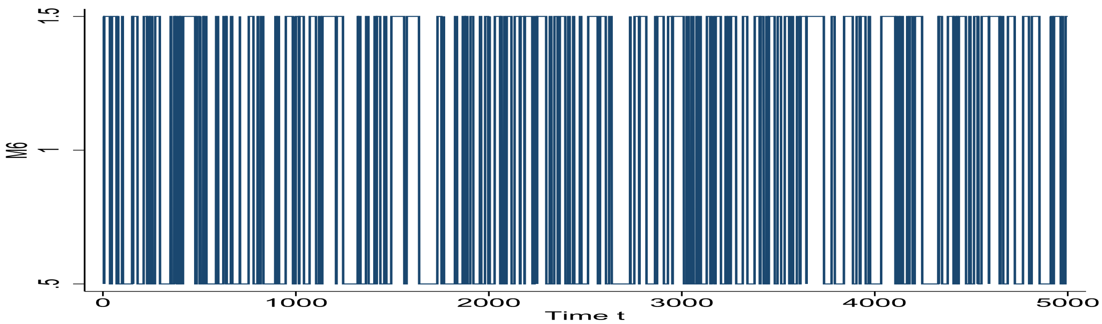
$$M_t = (M_{1,t} * M_{2,t} ..... * M_{\overline{k},t}) = (m^1, m^2, ..., m^d)$$

Figure 1.1: Simulated MSM Volatility Components

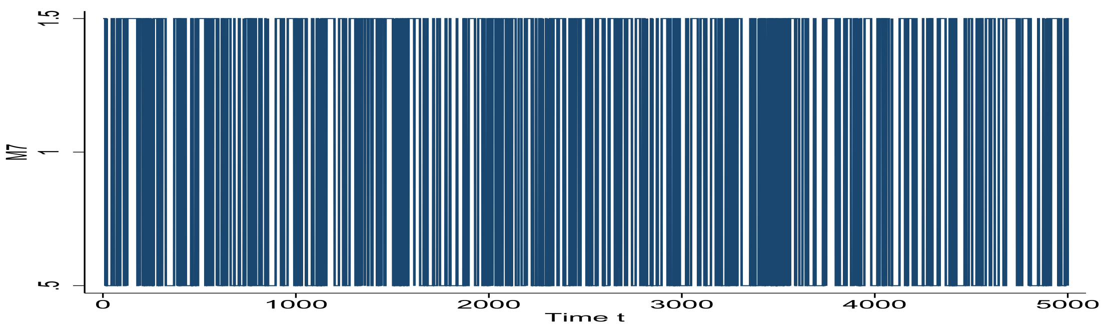




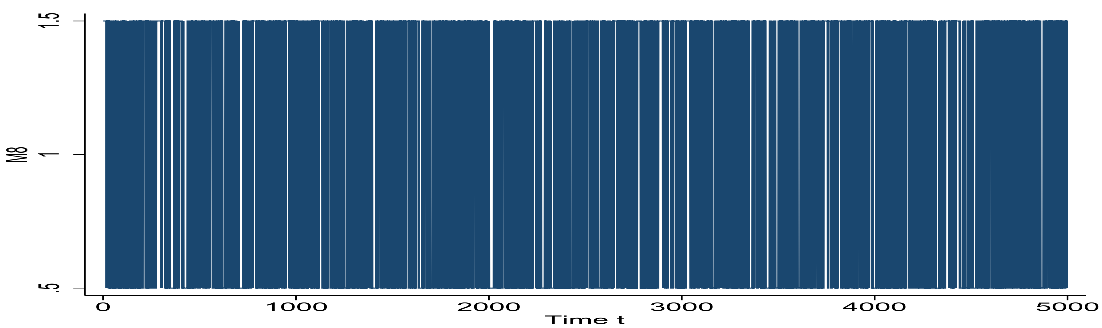
M5
1.5
1
.5
0
1000
2000
3000
4000
5000
Time t



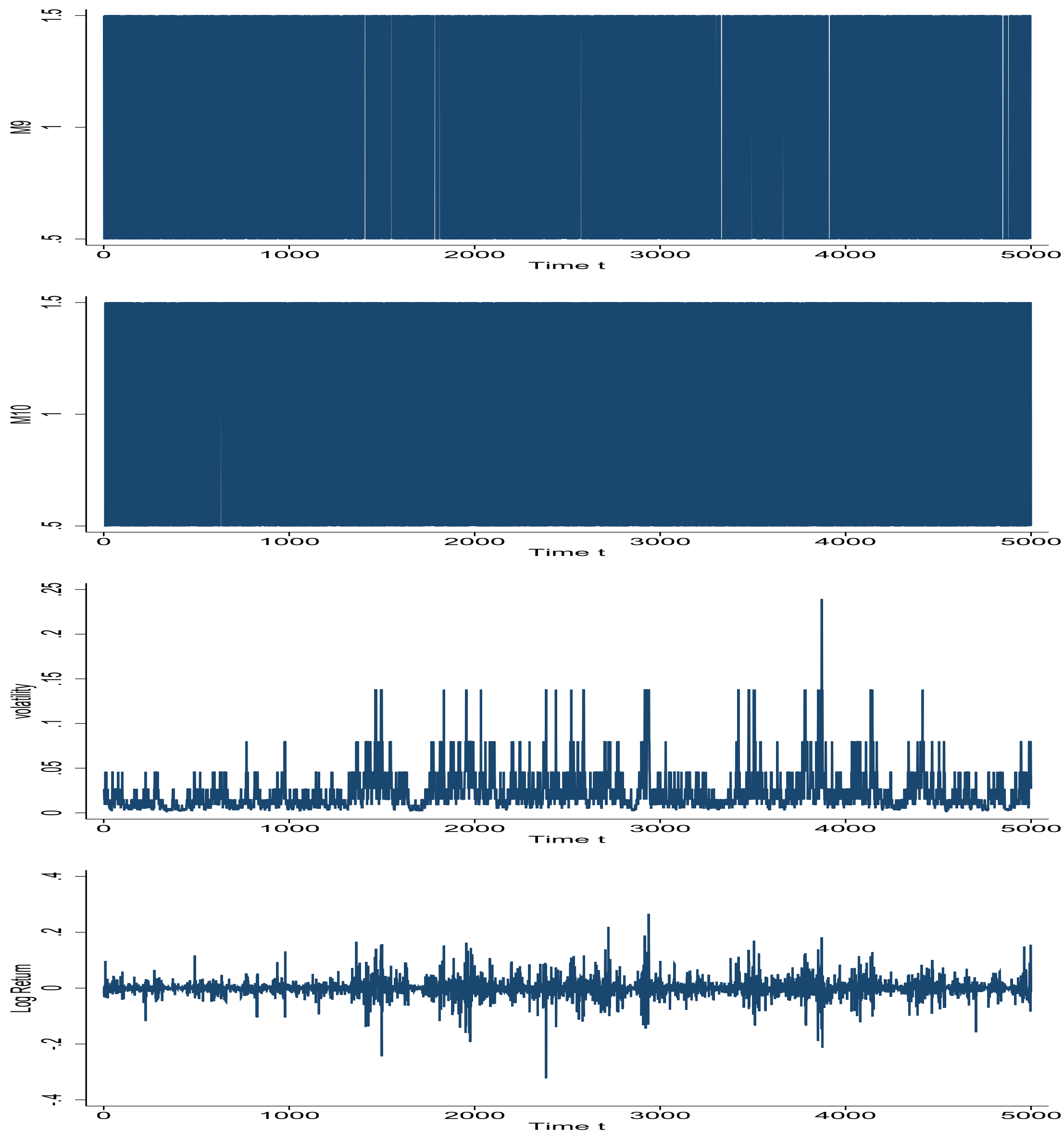
M6
1.5
1
.5
0
1000
2000
3000
4000
5000
Time t



M7
1.5
1
.5
0
1000
2000
3000
4000
5000
Time t

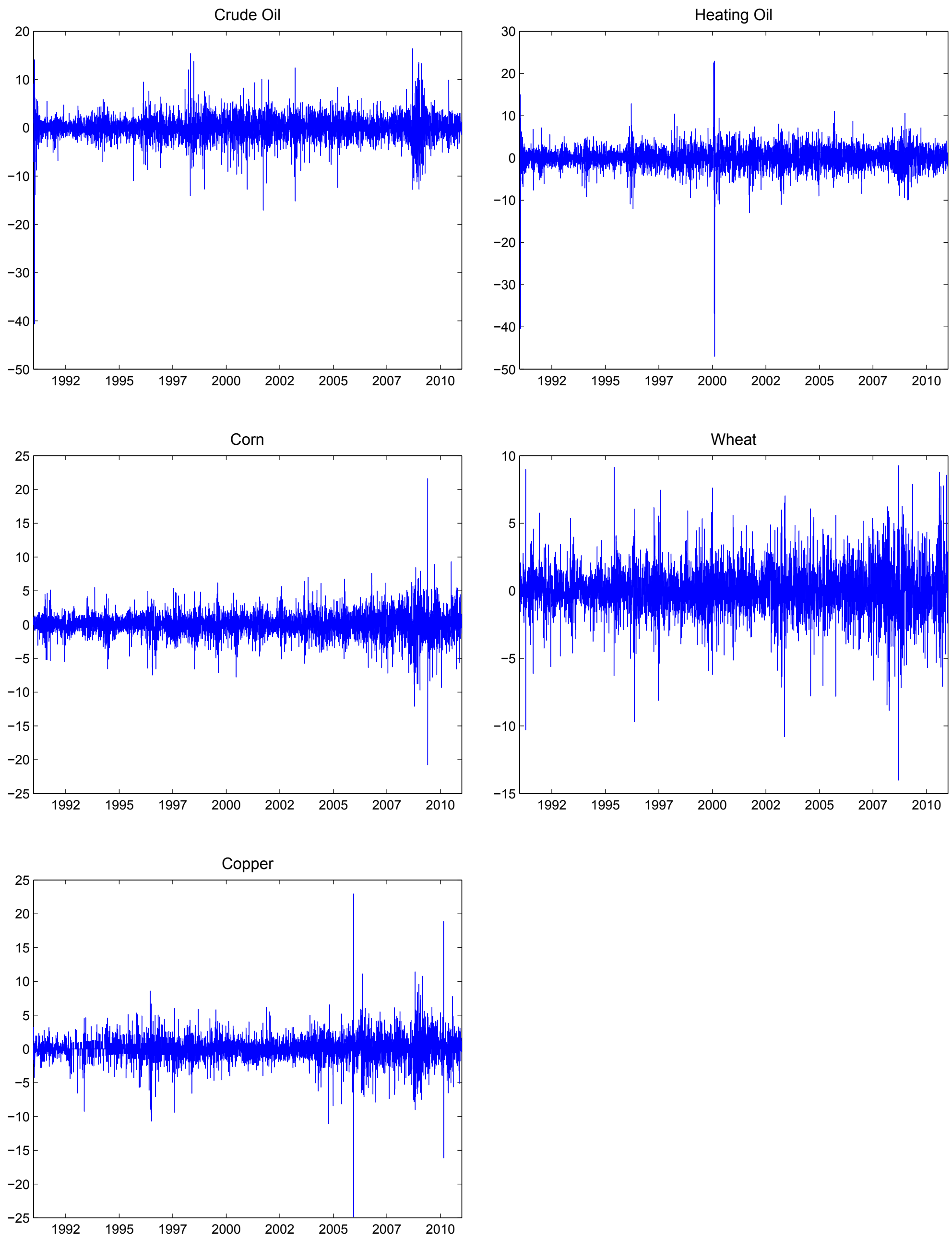


M8
1.5
1
.5
0
1000
2000
3000
4000
5000
Time t



Notes: The figure illustrates a simulated MSM model with 10 volatility components. The parameter vector used for the simulation is $(m_0, \overline{\sigma}, b, \gamma_{\overline{k}}) = (1.4, 0.5, 3, 0.85)$. The first ten panels show the plots of the volatility components, while the last two panels show the plots of the associated volatility and log returns.

Figure 1.2: Commodity Returns Series(%)



be the sample space of M_t, where $d = 2^{\overline{k}}$. For example, when $\overline{k} = 1$, there is one volatility component, which can take two possible values. Hence, $M_t = (m_0 \ \ m_1)$. Therefore, volatility can take two possible values, $\sigma(M_t)^{1/2} = (\sigma(m_0)^{1/2} \ \ \sigma(m_1)^{1/2})$. When $\overline{k} = 2$, there are two volatility components, and each can take two possible values. Hence, $M_t = (m_0 m_0 \ \ m_0 m_1 \ \ m_1 m_0, \ \ m_1 m_1)$. Therefore, volatility can take four possible values, $\sigma(M_t)^{1/2} = (\sigma(m_0 m_0)^{1/2} \ \ \sigma(m_0 m_1)^{1/2} \ \ \sigma(m_1 m_0)^{1/2} \ \ \sigma(m_1 m_1)^{1/2})$. Continuing this pattern, when there are $\overline{k}$ volatility components, and each can take two possible values, M_t has $2^{\overline{k}}$ elements. Hence, volatility can switch between $2^{\overline{k}}$ possible values.

We also define the vector of conditional probabilities $\xi_t = (\xi_t^1, \xi_t^2, ...\xi_t^d) \in \mathbb{R}^d$, where each component is defined as

$$\xi_t^j \equiv \mathbb{P}(M_t = m^j | \mathbf{r}_t)$$

Here $\mathbf{r}_t = (r_1, r_2, ...r_t)$. Lastly, we define the vector $f(r_t) = (f^1, f^2, ...f^d) \in \mathbb{R}^d$ with typical elements

$$f^j = \phi(r; \sigma^2(m^j)) = \frac{1}{\overline{\sigma}(m^j)\sqrt{2\pi}} \exp\{-\frac{r_t^2}{2(\overline{\sigma}(m^j))^2}\} \in \mathbb{R}^d$$

Using Bayes' rule, the conditional probability vector can be computed using the following recursion

$$\xi_{t+1} = \frac{f(r_{t+1}) * (\xi_t P)}{(f(r_{t+1}) * (\xi_t P))\mathbf{1'}}, \tag{1.4}$$

where $*$ denotes an Hadamard product, $\mathbf{1} = (1, 1, ...1) \in \mathbb{R}^d$ and P is the transition matrix of the Markov chain, with element

$$P_{i,j} = \mathbb{P}(M_t = m^j | M_{t-1} = m^i) = \prod_{k=1}^{\overline{k}} [(1 - \gamma_k) 1_{\{m_k^i = m_k^j\}} + \gamma_k \mathbb{P}(M = m_k^j)]$$

where $1_{\{m_k^i = m_k^j\}}$ is an indicator variable taking value of one if $m_k^i = m_k^j$. The recursion begins with the ergodic distribution given by $\xi_0^j = \prod_{k=1}^{\bar{k}} \mathbb{P}(M = m_k^j)$. Calvet and Fisher (2004) show that the closed form likelihood function is given by

$$lnL(\mathbf{r}_t;\theta) = \sum_{t=1}^{T} ln[f(r_t)\cdot(\xi_{t-1}P)] \tag{1.5}$$

The univariate MSM model has been applied to many kinds of financial returns and has often been found to outperform alternative models. Using data on exchange rates, Calvet and Fisher (2004) compared the performance of univariate MSM to univariate GARCH, Markov-switching GARCH and FIGARCH models and find that the MSM model outperforms the alternative models in both in- and out-of-sample volatility forecasts. Using data on exchange rates, stock indices and gold, Lux (2008) also find that univariate MSM model outperforms the GARCH and FIGARCH models in forecasting volatility.

Inspite of the empirical successes of univariate MSM, it is however limited in its ability to capture other critical issues in financial markets such as market integration, contagion and volatility comovement, all of which play essential roles in portfolio selection and risk management. To accommodate these other features of financial markets, the bivariate MSM model was developed by Calvet et al. (2006) and is discussed in the next section.

1.2.2. Bivariate Markov-Switching Multifractal Model

Let the vector of returns for markets α and β be denoted by

$$x_t = \begin{bmatrix} r_t^\alpha \\ r_t^\beta \end{bmatrix} = \begin{bmatrix} \sigma_\alpha(M_t^\alpha)\varepsilon_{\alpha,t} \\ \sigma_\beta(M_t^\beta)\varepsilon_{\beta,t} \end{bmatrix}$$

where the residual vector $\varepsilon_t \in \mathbb{R}^2$ is bivariate *IID* Gaussian $\mathcal{N}(0, \Sigma)$ with variance-covariance matrix given by:

$$\Sigma = \begin{bmatrix} 1 & \rho_\varepsilon \\ \rho_\varepsilon & 1 \end{bmatrix} \tag{1.6}$$

Similar to the univariate set up, stochastic volatility is driven by a bivariate first-order Markov state vector M_t with $2\texttt{x}k$ volatility components:

$$M_t = (M_{1,t}; M_{2,t} ....; M_{\bar{k},t})$$

where

$$M_{k,t} = \begin{bmatrix} M^{\alpha}_{k,t} \\ M^{\beta}_{k,t} \end{bmatrix} \in \mathbb{R}^2_+$$

The first row of M_t contains the volatility components of α returns while the second row contains the volatility components of β returns. As in the univariate model, at any period t, each column of M_t corresponds to a particular frequency $k = \{1, 2...\overline{k}\}$, with each frequency representing a shock of heterogeneous duration. $k = 1$ represents the lowest frequency (most persistent) volatility component while $k = \overline{k}$ represents the highest frequency (least persistent) volatility component. The volatility components $M_{k,t}$ have the same marginal distribution M but switch at different frequencies with probability γ_k, where the switching events are assumed to be independent across k and t. Also, corresponding to the economic intuition that volatility arrivals might be correlated across return series, the bivariate MSM allows for correlation in the switching events across volatility components and this is given by the correlation coefficient $\lambda \in [0, 1]$.

To define the evolution of $M_{k,t}$, let 1^{τ}_k denote a random variable for $\tau \in \{\alpha, \beta\}$. In each period t, 1^{τ}_k takes a value of 1 if there is a switch in $M^{\tau}_{k,t}$ and 0 otherwise. 1^{τ}_k is assumed to be *IID* and symmetric:

$$\begin{bmatrix} 1^{\alpha}_{k,t} \\ 1^{\beta}_{k,t} \end{bmatrix} \overset{d}{=} \begin{bmatrix} 1^{\beta}_{k,t} \\ 1^{\alpha}_{k,t} \end{bmatrix}$$

Given the realization of the arrival vector $1_{k,t}$, the joint dynamics of the volatility components can be summarized as:

$$\begin{aligned} \mathbb{P}\left[1^{\alpha}_{k,t} = 1 | 1^{\beta}_{k,t} = 1\right] &= \gamma_k \left[(1-\lambda)\gamma_k + \lambda\right] = \mathbb{P}\left[1^{\beta}_{k,t} = 1 | 1^{\alpha}_{k,t} = 1\right] \\ \mathbb{P}\left[1^{\alpha}_{k,t} = 1 | 1^{\beta}_{k,t} = 0\right] &= \gamma_k (1-\lambda)(1-\gamma_k) = \mathbb{P}\left[1^{\beta}_{k,t} = 1 | 1^{\alpha}_{k,t} = 0\right] \\ \mathbb{P}\left[1^{\alpha}_{k,t} = 0 | 1^{\beta}_{k,t} = 0\right] &= (1-\gamma_k)\left[1 - \gamma_k (1-\lambda)\right] = \mathbb{P}\left[1^{\beta}_{k,t} = 0 | 1^{\alpha}_{k,t} = 0\right] \end{aligned} \tag{1.7}$$

γ_k is the transition probability of the k^{th} frequency component and is defined as:

$$\gamma_k = 1 - (1 - \gamma_{\overline{k}})^{b^{k-\overline{k}}}$$

where $\gamma_k \in (0,1)$ and $b \in (0, \infty)$.

The parameter γ_k controls the persistence of the k^{th} frequency component, while the parameter b governs the growth rate of the transition probabilities of low frequency components, with $\gamma_1 \prec \gamma_2 \prec, ...\gamma_{\overline{k}} \prec 1$. As in the univariate case, the construction of bivariate MSM can accommodate any distribution of the vector M with minimal restrictions. Specifically, it requires that M has a positive support and a unit mean: $\mathbb{E}(M) = 1$ and $M \geq 0$. As in Calvet and Fisher (2004) and Calvet et al. (2006), the empirical work that follows in this thesis adopts a simple bivariate binomial distribution where each volatility component is drawn from $M = \left(M^{\alpha}, M^{\beta}\right)'$. Assuming that the volatility vector has been constructed up to time t. In time $t+1$, each element of $M_{k,t} = (M^{\alpha}_{k,t}, M^{\beta}_{k,t})$ takes values $m^{\tau}_0 \in [1,2]$ or $m^{\tau}_1 = 2 - m_0 \in [0,1]$ with probability 0.5, and stays constant otherwise. Therefore, the volatility component vector M can take four possible values, $(m^{\alpha}_0, m^{\beta}_0), (m^{\alpha}_0, m^{\beta}_1), (m^{\alpha}_1, m^{\beta}_0)$ and $(m^{\alpha}_1, m^{\beta}_1)$

with probability matrix $(p_{i,j}) = \mathbb{P}\left(M_k = (m_i^\alpha, m_j^\beta)\right)$ defined as

$$\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} = \begin{bmatrix} \dfrac{1+\rho_m}{4} & \dfrac{1-\rho_m}{4} \\ \dfrac{1-\rho_m}{4} & \dfrac{1+\rho_m}{4} \end{bmatrix}$$

where $\rho_m \in [0,1]$ is the correlation coefficient between M_k^α and M_k^β. Under the above framework, the conditional variance-covariance matrix is

$$\begin{bmatrix} \overline{\sigma}_\alpha^2 \prod_{k=1}^{\bar{k}} \mathbb{E}_t\left(M_{k,t}^\alpha\right) & \rho_\varepsilon \overline{\sigma}_\alpha \overline{\sigma}_\beta \prod_{k=1}^{\bar{k}} \mathbb{E}_t\left[\left(M_{k,t}^\alpha M_{k,t}^\beta\right)^{\frac{1}{2}}\right] \\ \rho_\varepsilon \overline{\sigma}_\alpha \overline{\sigma}_\beta \prod_{k=1}^{\bar{k}} \mathbb{E}_t\left[\left(M_{k,t}^\alpha M_{k,t}^\beta\right)^{\frac{1}{2}}\right] & \overline{\sigma}_\beta^2 \prod_{k=1}^{\bar{k}} \mathbb{E}_t\left(M_{k,t}^\beta\right) \end{bmatrix} \tag{1.8}$$

The bivariate MSM parameter vector is then characterized by

$$\Theta = \left(m_0^\alpha, m_0^\beta, \overline{\sigma}_\alpha, \overline{\sigma}_\beta, b, \gamma_{\overline{k}}, \rho_\varepsilon, \lambda, \rho_m\right)$$

The MSM model specification implies that volatility is stochastic and hit by shocks of heterogeneous frequencies indexed by $k \in \left\{1, 2, ..., \overline{k}\right\}$. Jumps in low-level volatility components cause volatility to vary discontinuously and exhibit strong persistence, while switches in high-frequency components produce substantial outliers in returns.

Estimation of Bivariate Markov-Switching Multifractal Model

Finally, to complete the specification, we discuss the maximum likelihood estimation of the bivariate MSM. The dynamics of the volatility state vector M_t are governed by a $4^{\overline{k}}\text{x}4^{\overline{k}}$ transition probability matrix P. The econometrician observes the history of past returns $R_t = \left\{r_s^\alpha,\ r_s^\beta\right\}_{s=1}^t$ but does not observe the volatility states. The vector of conditional joint density of returns is defined by

$$f\left(R_t | M_t = m^i\right) = \phi\left(R_t; \sigma g\left(m^i\right)\right)$$

where $g(m^i)$ is the $\mathbf{2x1}$ vector $M_{1,t} * M_{2,t} * .... * M_{\bar{k},t}$ and $\phi(.)$ is the bivariate standard normal density. The conditional probability vector over the unobserved states are $\xi_t = (\xi_t^1, ...\xi_t^d) \in \mathbb{R}_+^k$ where

$$\xi_t^i = \mathbb{P}\left(M_t = m^i | R_t\right)$$

Using Bayes' rule, the conditional probabilities are computed recursively as

$$\xi_t = \frac{f\left(R_t\right) \odot \xi_{t-1} P}{\left[f\left(R_t\right) \odot \xi_{t-1} P\right] \mathbf{1}'} \tag{1.9}$$

where ξ_0 is chosen to follow the ergodic distribution. The likelihood function is simply the sum of the log conditional densities:

$$lnL(R_t, ; \Theta) = \sum_{t=1}^{T} \left[ln\left(f(R_t) \cdot \xi_{t-1} P\right)\right] \tag{1.10}$$

1.3. Commodity Market

Evidence abounds in the literature that points to the similarities between the price and return behavior of commodity assets and other traditional financial assets such as equities, interest rates and exchange rates. Studying a monthly data set of 15 different commodities over the period between 1960 and 1994, Cromwell et al. (2000) find evidence of power-law scaling and long-memory in commodity returns and variability in the persistence of volatility factors. Kat and Oomen (2007a) and Kat and Oomen (2007b) study daily settlement prices on 142 different commodity futures trading on 26 different exchanges from 1965 to 2005. They find that, similar to equities, commodity returns exhibit long memory, significant kurtosis and skewness. Most importantly, they also find that volatility exhibits considerable variation over different phases of business cycles and under different monetary conditions.

Empirical evidence (Bhardwaj and Dunsby, 2011; Brooks and Prokopczuk, 2013) shows that different commodities behave differently and can not be treated as one. We examine the energy, agricultural and metal markets in turn. The commodity data consists of daily observations on spot prices for five commodities: West Texas Intermediate (WTI) crude oil, heating oil, wheat, corn and copper. The heating oil and crude oil datasets were obtained from Energy Information Administration (U.S. Department of Energy) while other data sets were obtained from the Datastream database.

1.3.1. Energy Markets

It is doubtless the case that some of the global recessions the world has experienced were preceded by oil shocks. Therefore, the importance of oil in the economy at large cannot be overemphasized. Looking at Figure 1.3, we observe that after lingering at about \$18 to \$20 per barrel during the early 1990s, the price of oil declined sharply to \$10 in 1999. It then increased to a record high of \$147 per barrel in July of 2008. This was followed by a steady decline up until December 2008 when the price was \$32 per barrel. Such exceptional oil price volatility affects many other variables and related products. Crude oil price fluctuation has significant effects on consumers and producers in terms of cost and incentives to invest in new technologies, and it makes planning more difficult. The effect of oil price fluctuation is also reflected in broader energy price changes as other forms of energy (e.g heating oil, natural gas, coal etc.) that are sometimes priced in relation to crude oil.

Energy price volatility can be (at least partially) explained by forces of demand and supply, but it is highly contentious as to which exact factors might be driving energy price volatility. Some economists argue that extreme oil price fluctuation can be attributed to growing consumption of emerging economies and the increasing cost of exploration and drilling (Krugman, 2008), low price elasticity of demand and the failure of global production to increase rapidly (Hamilton, 2008, 2009) and exchange rates fluctuation (Amano and Norden, 1998a,b; Chen et al., 2008; Cuaresma and Breitenfellner, 2008; Geman, 2005). Other economists contend that the effect of speculation on oil prices is not insignificant. Using monthly data from January 1998 to March 2008, Stevans and Sessions (2008) find that spot oil prices are driven by real supply, while longer term prices are dominated by futures prices. Employing a VAR framework, Kaufmann and Ullman (2009) find that oil price fluctuations are driven by both changes in market fundamentals and speculative pressures.

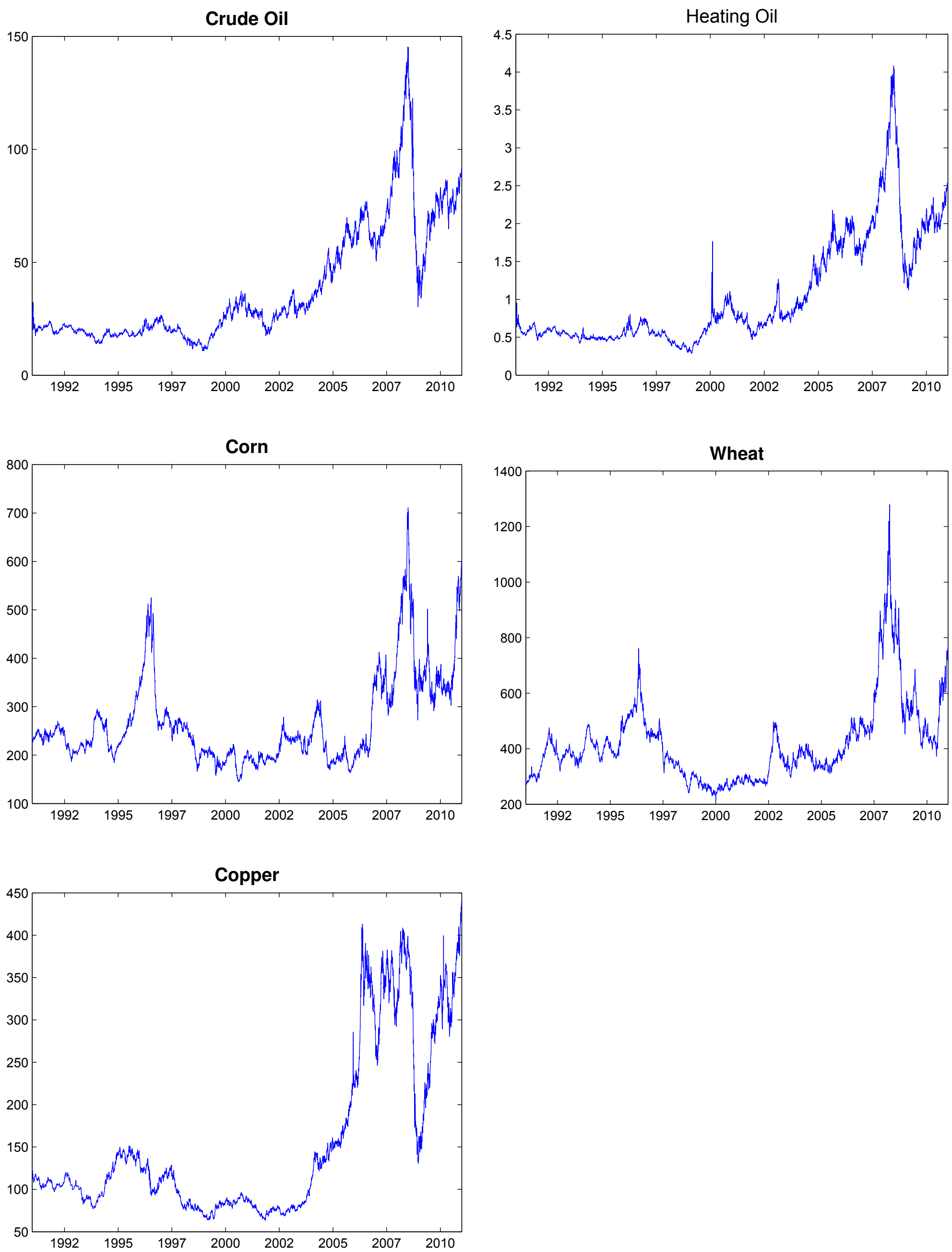
1.3.2. Agricultural Markets

Prices of corn and wheat, as with other agricultural commodities, are naturally volatile because they are constantly affected by unpredictable and uncontrollable weather conditions. But other than weather as a significant source of volatility in agricultural markets, the world's increase in demand for food and alternative energy has been driving up food prices. At the end of December 2010, corn prices were up 36% and wheat prices were up 40% from a year earlier. Higher demands for corn stems from its consumption in livestock, food processing and biofuel energy. Price of agricultural commodities also react to feedbacks from policy and political cycles. For instance, since the passage of the Energy Independence and Security Act by the United States congress in January 2007, the price of corn has risen by 55%, due to a significant proportion of corn production been directed to ethanol production.

1.3.3. Metal Markets

Demand for copper is dominated by industrial requirements such as in the production of electrical power cables, data cables, automobiles, ammunitions and jewelries. Since

Figure 1.3: Commodity Spot Price Series ($US)



the transition from producer pricing to exchange pricing in late 1970, volatility has increased significantly in the copper market. The price of copper increased almost three fold between March 2005 and May 2006, when commodity markets experienced one of its most remarkable booms. But the price of copper has also been rising steadily since the start of 2009. Much of the price increase can be attributed to significant increase in the world's demand for copper. In 2010 and 2011, world's demand for copper exceeded production by 144,000 and 99,000 tonnes respectively (International Copper Study Group, 2014). Such a demand deficit has in turn led to higher prices as commodity traders out-bid each other for the limited supply.

1.3.4. General Remarks

As is evident from the discussion above, commodity prices are simultaneously driven by multiple factors, ranging from low frequency factors such as biofuel mandate to high frequency factors such as weather conditions. Given the heterogeneity in frequencies, durations and gradations of such factors, it therefore seems reasonable to model commodity prices as being subjected to multifrequency risks as is assumed throughout this thesis. Jump models and two-state Markov-switching models can not effectively accommodate this extent of heterogeneity in volatility factors and variability in shock durations. But the MSM framework is able to do so in a very parsimonious way, irrespective of the number of volatility factors included in the model.

1.4. Conclusion

This chapter provides a review of the Markov-Switching Multifractal(MSM) model and its estimation procedures. The MSM model is a Markov-regime switching model, with multiple volatility components. Each component varies in its degree of variability and persistence. Switches in low frequency components represent rare volatility shocks such as demographics, war, depression etc that cause extreme movements in volatility that can be maintained over a long period of time. On the other hand, switches in high frequency components represent common shocks such as macroeconomic news releases, liquidity uncertainty, maturity effects etc. Such volatility shocks cause extreme tail movements in asset returns. Therefore, the MSM model is able to to capture such gradations in volatility shocks in a parsimonious way.

This chapter also lays the ground work for subsequent chapters that focus on the application of MSM in different markets. As discussed in earlier sections, commodity markets are characterized by both high frequency (maturity effects in futures markets, weather events, bad yields, economic news releases etc) and low frequency (e.g terrorist attacks on oil rigs, trade restrictions on certain international commodities, exhaustible natural resource uncertainty demand/supply shocks etc) shocks that simultaneously drive returns. While the low frequency volatility shocks matters for long term investors or market makers, high frequency shocks have implications for short term investors. Therefore, it seems consistent with this intuition to model commodity returns using MSM.

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Chapter 2

Dynamic Futures Hedging In The Presence of Multifrequency Risk

2.1. Introduction

The hedging of risk is an important factor in the day-to-day operations of derivative market participants. Hence, the search for a methodology for reducing risk has motivated the rapid development of the the theory of hedging with futures contracts[1]. One such methodology for effective hedging strategy is the futures hedge ratio which is defined as the number of futures contracts to buy or sell per unit of the underlying asset on which the hedger bears risk. In a minimum-variance setting where an agent seeks to minimize the variance of his portfolio, the optimum hedge ratio is the ratio of the unconditional covariance between cash and futures returns over the variance of futures returns. This definition implies that the hedge ratio is a constant. An estimate of the constant hedge ratio can be obtained as the slope coefficient from an Ordinary Least Square (OLS) regression of spot on futures prices. The OLS hedge ratio, however, has two major drawbacks.

[1]A future contract is a standardized, transferable, exchange-traded contract that requires delivery of a commodity, bond, currency, or stock index, at a specified price, on a specified future date.

First, the OLS hedge ratio ignores the cointegrating relationship between the spot and futures prices. Evidence abounds suggesting that omitting the long run equilibrium relationship will lead to a downward bias in the estimated hedge ratio. Moreover, changes in spot prices in many markets are partially predictable. For example, natural gas and heating oil prices are normally lower in summer due to decreased demand. Likewise, spot prices in grain markets are normally expected to fall at harvest times due to an increase in supply. Consequently, if markets are efficient, the spread between spot and futures prices (the basis) will reflect expected future price changes for the underlying asset. But the OLS hedge ratio ignores this additional information. Consequently, the OLS hedge ratio includes a partially predictable component of spot prices, whereas the optimal hedge ratio should reflect only unanticipated changes in prices. Ederington and Salas (2008) shows that when this is the case, OLS regression estimates of the minimum variance hedge ratio are inefficient.

The second drawback of the OLS hedge ratio is that it ignores the last 30 years of research in finance that has documented the time varying nature of the distributions of asset prices. Volatility of financial assets exhibits clustering and heteroscedasticity. In other words, a period of high volatility is likely to be followed by another period of high volatility, and vice-versa. If this is the case, assuming a constant variance of asset prices (hence a constant hedge ratio) does not provide adequate risk hedging.

Several methodologies have since been proposed in the literature to model the time-varying optimal hedge ratio as a function of the time-varying conditional distributions of returns. A number of studies apply multivariate generalized autoregressive conditional heteroscedasticity (GARCH) to model the joint distributions of spot and future returns (Baillie and Myers, 1991; Bracker and Smith, 1999; Brunetti and Gilbert, 2000; Engle and Kroner, 1995; Kroner and Sultan, 1993; Moschini and Myers, 2002; Smith and Bracker, 2003). These studies find that GARCH hedge ratios, on average, outperform constant hedge ratios in terms of risk reduction. However, as observed by Lien and Tse (2002), the gains in risk reduction from GARCH hedge ratios are minimal and market specific. Besides, although GARCH can be modeled in terms of any higher order of moving average and autoregressive components, in practice only GARCH(1,1) is widely used because the parameters of higher-order GARCH models are notoriously difficult to estimate. Calvet et al. (1997) also argue that the finite memory property of the discrete time GARCH process prevents it from replicating some important characteristics of financial series such as time- and frequency scaling and long memory.

To account for thick tails observed in the unconditional distributions of returns and to capture infrequent but extreme events, another approach used in the hedging literature is to introduce jumps in spot and futures prices (Chan and Maheu, 2002; Chan and Young, 2006; Chang et al., 1996; Chang and Chang, 2003). This approach, while providing some gains in hedging in terms of variance reduction, is not without its drawbacks. The jump models isolate normal events from rare but extreme events and assume that all jumps at any instant have the same expected size and frequency. However from an intuitive point of view, gradations in size and frequency exist among events such as war, depression, natural disasters etc. Therefore models that dichotomize normal from extreme events can miss such regularities.

In order to account for these shortcomings, this paper introduces a vector error correction model with a Markov Switching Multifractal error structure (VECM-MSM). The model has two distinctive features that incorporate the empirical properties of asset prices. First, it includes an error correction mechanism in the mean equation that incorporates the long-run relationship between spot and futures prices. Second, the model specifies the conditional second moments as a bivariate Markov Switching Multifractal (MSM) model (Calvet et al., 2006). In the bivariate MSM setting, the error structure is driven by a bivariate Markov state vector with multiple components, whose rescaled product defines volatility. Each of the volatility components can switch to a new level with a different probability per unit time, generating volatility shocks of multiple frequencies. Hence, the MSM captures the thick-tails, long memory features and volatility clustering which are characteristics of many financial returns. Moreover, modeling the error structure as a bivariate MSM is consistent with the intuition that asset prices are simultaneously subjected to multiple shocks of heterogeneous durations and frequencies such as demand and supply shocks, technological innovations, macroeconomic uncertainty, intermediate contributions from the political cycle as well as weather and other natural phenomena, with the relative importance of various shocks varying according to the particular asset or commodity under consideration.

The contributions of this paper are three-fold. First we extend the Markov Switching Multifractal model by introducing, for the first time, a vector error correction component, which includes in the conditional mean equation the cointegrating relationship between spot and futures prices. Evidence abounds suggesting that if the spot and futures prices are cointegrated, omitting the long run equilibrium relationship will lead to a downward bias in the estimated hedge ratio (Kroner and Sultan, 1993; Lien, 2004, 2006). Second, while the MSM model has been applied to study volatility forecasting in exchange rate market (Calvet and Fisher, 2004; Calvet et al., 2006; Lux, 2008) and equity markets (Calvet and Fisher, 2007, 2008; Chuang et al., 2013; Lux et al., 2011), no application of the model has been conducted to study the problem of risk hedging in the futures market. Hence, we also extend the MSM literature in this direction. Third, also for the first time, we evaluate the hedging effectiveness of the proposed VECM-MSM model, using a value-at-risk (VaR) approach.[2] Specifically, we compare the hedging effectiveness of the proposed model

[2]For the sake of completeness and in line with an earlier version of this paper, we also report the

to those of alternative models by assessing their unconditional and conditional VaR coverages. Models are then ranked in terms of the adequacy and accuracy of their hedged portfolio VaR. Although a few studies also evaluate hedging performance in terms of portfolio VaR (Alizadeh et al., 2008; Cotter and Hanly, 2006), these studies simply compared the size of the VaR from alternative models. The model with the smallest VaR is ranked best. But as shown later in this paper, a model's VaR can be inadequate and inaccurate if such VaR is continuously violated. Therefore, simply ranking models in terms of the size of their portfolio VaR can be grossly inaccurate.

To anticipate our results, in-sample and out-of-sample hedge effectiveness shows the VECM-MSM hedged portfolio outperforms alternative hedging strategies in terms of having the lowest rate of VaR violation among the different strategies. Statistical tests of unconditional and conditional coverages also show that the VECM-MSM model better predicts an investor's downside risk in that the VaR predictions are more accurate than the predictions from the alternative models.

The structure of this chapter is as follows. Section 2 reviews the relevant literature on the derivation and estimation of optimal futures hedge ratios. Section 3 presents the VECM-MSM model of spots and futures returns. Section 4 presents the data, the VECM-MSM estimation results, in- and out-of-sample hedging exercises, as well a comparison of the MSM hedging strategies to naive, OLS, MRS-GARCH and GARCH hedging strategies. Section 5 concludes.

unconditional hedged portfolio variances for all models considered. But for reasons argued later in this paper, we do not evaluate hedging effectiveness of alternative models in terms of their hedged portfolio variances

2.2. Literature Review

The primary reason for hedging is to reduce, or if possible eliminate, the risk exposure of agents arising from the variability of asset prices. To this extent a great amount of attention has been dedicated to the issue of hedging, as evidenced by the large number of articles written in this area. Of particular interest to this study is the concept of futures hedging. There are two major issues surrounding the concept of futures hedging. First is the determination of the optimal hedge ratio. The determination of the optimal hedge ratio depends on the objective function to be optimized. The objective function, in turn, reflects the goal of the agent as either a speculator, who cares about both risk and return, or as a hedger, who simply hedges against price fluctuation that might disrupt the delivery of the underlying asset. Whatever the objective function of an agent, the resultant optimal hedge ratio is usually a function of the distribution of the underlying asset price. This leads to the second issue. Modeling the underlying price dynamics and estimation of the optimal hedge ratio from the underlying price data.

The extant literature has dedicated a significant amount of effort to addressing these issues. This section focuses on the review of the different techniques that have been adopted for deriving and estimating the optimal futures hedge ratio.[3] It is important to note that modeling stochastic commodity prices and the valuation of their derivative contracts have long been a focus in the field of financial economics. In the commodity pricing literature, the general approach is to specify stochastic dynamics for commodity prices, and derive the valuation formulas of various derivative contracts whose payoff depends on the realization of the underlying asset value. For example, models for futures, forwards and option contracts can be derived from

[3]We do not however attempt an exhaustive review of the literature, as hedging is a broad concept that has spawned a very extensive literature.

stochastic models of commodity prices. Most notably, Schwartz (1997) propose and empirically compare three stochastic models of commodity prices. The first model is a simple one-factor mean-reverting model of the logarithm of spot commodity prices. The second model introduces stochastic convenience yield as the second factor, which also follows a mean-reverting process. The third model extends the second model by introducing a stochastic interest rate process as the third factor. Schwartz (1997) find strong evidence of mean-reversion in commodity prices. They also analyze the implications of the proposed models for pricing futures contracts and for hedging forward commitments.

Likewise, Schwartz and Smith (2000) consider a two-factor model in which the logarithm of spot commodity prices is a linear combination of a long-run equilibrium price process and a short-run deviation from the long-run equilibrium price. The long-run process follows a Geometric Brownian Motion, while the short-run process is mean-reverting. They show that this model is equivalent to the stochastic convenience yield model proposed by Gibson and Schwartz (1990). Several other models of commodity prices have since been proposed, that consider additional latent factors and more flexible stochastic processes of each factor (Casassus and Collin-Dufresne, 2005; Hilliard and Reis, 1998; Neuberger, 1999; Richter and Sørensen, 2002; Schwartz, 1998; Schwartz and Smith, 2000; Veld-Merkoulova and de Roon, 2003; Yan, 2002). These models have been applied widely to study the term structure of commodity prices, hedging commodity risk and for valuing commodity contingent claims.

However, the aforementioned models have three common features that render them inapplicable to the kind of futures hedging problem studied in this paper. First, the factors or state variables contained in these models are often not directly observable. For example, the instantaneous convenience yield is not directly observable, and must be inferred using latent variable estimation techniques. Sometimes, futures prices with different maturities are used to compute it. Also, the instantaneous interest rate is not directly observable. Therefore, the common approach for estimating these models is to employ state space estimation procedures such as a Kalman Filter. Second, like other state variables, the spot price is also assumed to be latent, and must be inferred from existing futures prices. This assumption is somewhat justified considering that spot prices for some commodities can sometimes be uncertain or illiquid (Schwartz, 1997). But the kind of risk hedging considered in this paper assumes that a hedger or speculator has a spot position, and must hedge his risk exposure from such position using futures prices. Therefore, the problem requires that prices be available for both spot and futures contract on the commodity. Lastly, the aforementioned models are specified for commodity prices, whereas the problem considered in this study requires that prices be specified for commodity returns. Therefore, this section focuses on the review of various models for commodity returns, and their application to the problem of futures hedging.

2.2.1. Derivation of the Optimal Hedge Ratio

Consider an agent with a spot position (long/short) in an underlying asset who aims to combine this with another position in the futures market of the asset. The primary objective is to eliminate or reduce fluctuations in the value of the portfolio. Therefore, the agent seeks an optimal amount of futures positions per unit of spot position that minimizes fluctuations in the portfolio value. Specifically, let the spot and futures prices of the underlying asset be denoted by S_t and F_t, respectively. Define the returns on the spot and futures positions as follows:

$$R_{s,t} = \ln S_t - \ln S_{t-1} \tag{2.1}$$

$$R_{f,t} = \ln F_t - \ln F_{t-1} \tag{2.2}$$

The return on the hedged portfolio is defined as

$$R_{h,t} = R_{s,t} - h_{t-1}R_{f,t}, \tag{2.3}$$

where h_{t-1} is the hedge ratio, which is the number of futures contracts to buy or sell for each unit of spot contract of the underlying asset on which the agent bears the risk. The main objective of the agent is to choose the optimal hedge ratio h^*, which in turn depends on the objective function of the agent. In what follows, we review several objective functions that have been proposed in the literature.

Minimum Variance (MV) Hedge Ratio

The MV hedge ratio is the most widely considered hedging strategy. Ederington (1979) and Myers and Thompson (1989) derive the MV hedge ratio by minimizing the variance of the portfolio return given in Equation (2.3). The portfolio variance is given by

$$Var_t(R_{h,t}) = Var_t(R_{s,t}) + h_{t-1}^2 Var_t(R_{f,t}) - 2h_{t-1}Cov_t(R_{s,t}, R_{f,t}) \tag{2.4}$$

Minimizing Equation (2.4) with respect to h gives the optimal hedge ratio as

$$h_{t-1}^* = \frac{Cov_t(R_{s,t}, R_{f,t})}{Var_t(R_{f,})} = \frac{\sigma_{t,sf}}{\sigma_{t,f}} \tag{2.5}$$

The MV hedge ratio is widely used by academics and practitioners due to its simplicity and ease of computation.

Mean-Extended-Gini (MEG) Coefficient Hedge Ratio

The MEG approach as an investment decision tool was developed by Yitzhaki (1982), Yitzhaki (1983) and Shalit and Yitzhaki (1984) and has since been applied to the futures hedging decision problem (Cheung et al., 1990; Kolb and Okunev, 1992; Lien and Luo, 1993; Lien and Shaffer, 1999; Shalit, 1995). The approach involves minimizing the MEG coefficient, $\Gamma_\alpha(R_h)$, given by

$$\Gamma_\alpha(R_h) = -\alpha Cov\{R_h, [1 - F_R(R_h)]^{\alpha-1}\}, \tag{2.6}$$

where $F_R(.)$ is the cumulative probability distribution function and α is a measure of risk aversion. The MEG hedge ratio has been shown to be consistent with the stochastic dominance framework (Shalit and Yitzhaki, 1984; Yitzhaki, 1982, 1983).[4] Furthermore, if spot and future prices are jointly normally distributed, it can be shown that the MEG hedge ratio is equivalent to the MV hedge ratio (Shalit, 1995). The main drawback of the MEG approach is that evaluating the derivative of $\Gamma_\alpha(R_h)$ with respect to h is difficult, as $\Gamma_\alpha(R_h)$ is a complicated function of h.

Generalized Semi-variance (GSV) Hedge Ratio

Another hedging strategy consistent with the stochastic dominance framework that has been adopted in the literature is the GSV approach (Chen et al., 2001; De Jong et al., 1997; Lien and Tse, 1998, 2000). The GSV hedge ratio is based on the $\alpha - \delta$ model of Fishburn (1977), who describes the expected disutility of an outcome under a target return, δ, weighted by a measure of risk aversion, α. The measure of risk is given by the GSV and defined as

$$G_\alpha(\delta) = \int_{-\infty}^{\delta} (\delta - R_h)^\alpha dF(R_h), \quad \alpha > 0 \tag{2.7}$$

[4]Stochastic dominance is a form of stochastic ordering. The term is used in decision theory and decision analysis and often used to determine the preference of an expected utility maximizer between some lotteries with minimal knowledge of the decision makers utility function.

where $F(R_h)$ is the probability distribution function of R_h. The GSV hedge ratio is derived from minimizing the sample analog of Equation (2.7) with respect to h as follows

$$h^* = argmin_h G_{N,\alpha}(\delta) = \frac{1}{N}\sum_{i=1}^{N}(\delta - R_{i,h})^{\alpha}\mathbf{1}_{R_{i,h}\leq\delta} \tag{2.8}$$

where N is the number of returns and $\mathbf{1}$ is an indicator function equal to 1 if $R_{i,h} \leq \delta$ and 0 otherwise. Therefore, the GSV approach only considers downside risk and has been shown to be consistent with the risk perceived by managers (Crum et al., 1981). Moreover, Lien and Tse (1998) show that the GSV hedge ratio is equivalent to the MV hedge ratio if spot and futures returns are jointly normally distributed and the futures price is a martingale.

The three hedging strategies outlined above ignore the potential trade-offs between risk and return, and are therefore not consistent with the mean-variance framework. For the risk minimizing hedge ratio to be consistent with the mean-variance framework, one needs to either assume that agents have infinite risk aversion or that the futures price follows a pure martingale process. These assumptions are hard to justify in practice. This has led to the emergence of hedging strategies that consider both risk and return . These strategies are discussed next.

Sharpe Hedge Ratio

Among the studies that consider both risk and return, Howard and D'Antonio (1984) propose optimal hedge ratios based on the Sharpe index. With this strategy, the utility function of an agent is extended from minimizing risk to optimizing with a risk-return trade-off. With an existing position in a spot market, an agent's optimization problem is to choose a hedge ratio that maximizes the ratio of a portfolio's excess return to its volatility:

$$Max_h \theta = \frac{\mathbb{E}(R_h) - i}{\sigma_h}, \tag{2.9}$$

where i is the domestic risk-free rate. The optimal hedge ratio is derived from the first-order condition for Equation (2.9) and is given by

$$\begin{aligned} h &= \frac{\varphi - \rho}{1 - \varphi\rho} \cdot \frac{\sigma_s}{\sigma_f} \frac{s_t}{f_t}, \\ \varphi &= \frac{\mathbb{E}(R_f)}{\mathbb{E}(R_s) - i} \end{aligned} \tag{2.10}$$

Note that if the futures price process is a martingale, then the Sharpe hedge ratio collapses to the MV hedge ratio. The Sharpe index hedging strategy, however, has two major drawbacks. First, the Sharpe ratio can be a proper measure of portfolio performance only when excess returns are positive. However, negative excess returns are common in practice, and are particularly unavoidable in some contexts such as foreign exchange hedging. This may lead to optimal hedge ratios that minimize rather than maximize the Sharpe ratio. Second, Chen et al. (2001) find that it is possible for the optimal hedge ratio to be undefined, in which case, the Sharpe ratio monotonically increases with the hedge ratio. Third, the Sharpe hedge ratio does not consider an investor's risk aversion.

HKL Mean-Variance Hedge Ratio

A mean-variance hedge ratio that explicitly models risk aversion was proposed by Hsln et al. (1994)(hence the name HKL). They assume that the investor has a negative exponential utility function with constant risk-aversion. The optimal hedge ratio is derived by maximizing the following expected utility function

$$Max_h \theta(\mathbb{E}(R_h), \sigma; \alpha) = \mathbb{E}(R_h) - \frac{1}{2}\alpha\sigma_h^2 \tag{2.11}$$

where α is the parameter of absolute risk aversion. The optimal hedge ratio can be derived from the first order condition for Equation (2.11) and is given by

$$h = \frac{\rho\sigma_s}{\sigma_f} - \frac{\mathbb{E}(R_f)}{\alpha\sigma_f^2} \tag{2.12}$$

It is obvious from Equation (2.12) that if the futures price process follows a pure martingale process, then the HKL hedge ratio collapses to the MV hedge ratio. Also, if the investor is infinitely risk averse (i.e if $\alpha \to \infty$), then the HKL hedge ratio is equivalent to the MV hedge ratio.

Mean-generalized semi-variance (M-GSV) Hedge ratio

Chen et al. (2001) extend the GSV hedge ratio of De Jong et al. (1997) by incorporating expected returns into the GSV objective function. The M-GSV hedge ratio is derived by maximizing the following risk return function

$$Max_h U(R_h) = \mathbb{E}(R_h) - \int_{\infty}^{\infty} (\delta - R_h)^{\alpha} dF(R_h) \tag{2.13}$$

It can be shown that the M-GSV hedge ratio will be equivalent to the MV hedge ratio if both spot and futures prices are jointly normal and the futures price follows a martingale process.

Mean Minimum Extended-Gini Hedge Ratio

Kolb and Okunev (1993) extend the MEG hedging strategy by incorporating expected return. Specifically, their optimal hedge ratio is derived by maximizing the following utility function:

$$Max_h U(R_h) = \mathbb{E}(R_h) + \alpha Cov\{R_h, [1 - F_R(R_h)]^{\alpha-1}\} \tag{2.14}$$

The M-MEG hedge ratio is similar to the MEG hedge ratio except that the M-MEG hedging strategy explicitly considers the risk-return trade-off.

So far, we have reviewed the different hedging strategies proposed in the literature in terms of possible investors objective functions. The next step in applying any hedging strategy is to estimate the hedge ratio. There are various ways of estimating the hedge ratios, as documented in the literature. But these ultimately depend on the investor's objective function. To facilitate a systematic exposition of the different approaches and to motivate the contributions of this paper, we focus on the different estimation approaches that have been proposed for the MV hedge ratio. Moreover, it can be observed from the previous section that all other hedge ratios include the MV hedge ratio as a special case.

2.2.2. Estimation of Minimum-Variance Hedge Ratio

Ordinary Least Squares(OLS) Hedge Ratio

Recall from Equation (2.5) that the minimum variance hedge ratio is the ratio that minimizes the portfolio uncertainty, and is defined as the ratio of the conditional covariance of spot and futures returns to the conditional variance of futures returns. Ederington (1979) proposes that the minimum variance hedge ratio can be obtained as the slope coefficient estimate from an OLS regression of spot returns on futures returns as follows

$$R_{s,t} = \alpha_0 + \alpha_1 R_{f,t} + \varepsilon_t \tag{2.15}$$

where the optimal hedge ratio is obtained as $\hat{\alpha}_1$. Therefore, the OLS hedge ratio is defined as the sample analogue of

$$h^* = \frac{Cov(R_{s,t}, R_{f,t})}{Var(R_{f,t})} = \frac{\sigma_{sf}}{\sigma_f} \tag{2.16}$$

The OLS hedge ratio is simple, easy to compute, and has become what is termed as the "conventional" hedge ratio in the literature. The OLS hedge ratio however has several drawbacks. First, it is biased downwards because it ignores the long-run equilibrium relationship between spot and futures prices (Bell and Krasker, 1986; Kroner and Sultan, 1993; Lien, 2004, 1996). Second, it is inefficient when prices are partially predictable, as in energy or grain markets (Ederington and Salas, 2008). Third, it ignores the presence of volatility clustering in asset returns that has been widely documented in the finance literature. In order to see the problem with the OLS hedge ratio, note that the population moments in Equations (2.5) are conditional on the information available up to time $t-1$, while the OLS model replaces them with their unconditional counterparts. Specifically, the following conditional moments are replaced as follows:

$$\begin{aligned}
\mathbb{E}_t(R_{s,t}) &= \mathbb{E}(R_{s,t}) \\
\mathbb{E}_t(R_{f,t}) &= \mathbb{E}(R_{f,t}) \\
\mathbb{E}_t[R_{s,t} - \mathbb{E}_t(R_{s,t})]^2 &= \mathbb{E}[R_{s,t} - \mathbb{E}(R_{s,t})]^2 \\
\mathbb{E}_t[R_{f,t} - \mathbb{E}_t(R_{f,t})]^2 &= \mathbb{E}[R_{f,t} - \mathbb{E}(R_{f,t})]^2 \\
\mathbb{E}_t[(R_{s,t} - \mathbb{E}_t(R_{s,t}))(R_{f,t} - \mathbb{E}_t(R_{f,t}))] &= \mathbb{E}[(R_{s,t} - \mathbb{E}(R_{s,t}))(R_{f,t} - \mathbb{E}(R_{f,t}))]
\end{aligned}$$

First, Bell and Krasker (1986) and Lien (1996) show that if $\mathbb{E}_t(R_{s,t})$ does not equal $\mathbb{E}(R_{s,t})$, then OLS regression produces a biased estimate of the minimum variance hedge ratio. Lien (2004) further corroborates this finding by proving that if an error correction term is omitted from the regression model, then the estimated hedge ratio is biased downward. Second, prices are partially predictable in some markets. For example, natural gas and heating oil prices are normally lower in summer due to decreased demand. Likewise, spot prices in grain markets are normally expected to fall at harvest times due to an increase in supply. Consequently, if markets are efficient, the spread between spot and futures prices (the basis) will reflect expected future price changes for the underlying asset. Therefore, $\mathbb{E}_t(R_{s,t})$ is likely to differ substantially and predictably from $\mathbb{E}(R_{s,t})$.

Ederington and Salas (2008) show that if markets are efficient and $\mathbb{E}_t(R_{s,t})$ does not equal $\mathbb{E}(R_{s,t})$, then the regression estimate of the minimum variance hedge ratio is inefficient. Lastly, given the widespread evidence of volatility clustering, it is likely that $\mathbb{E}_t[R_{s,t} - \mathbb{E}_t(R_{s,t})]^2 > (<)\mathbb{E}[R_{s,t} - \mathbb{E}_t(R_{s,t})]^2$ when volatility has been relatively high (low) in the recent past. The implications from the foregoing is that first, the estimate of the optimal minimum variance hedge ratio should be time varying because the distribution of the underlying asset price is time varying. Second, an efficient and unbiased estimate of the minimum variance hedge ratio should incorporate the long-run relationship between the spot and the futures prices. These have led to the emergence of hedging strategies that incorporate these features, and are discussed in what follows.

GARCH Hedge Ratio

The GARCH model was proposed by (Bollerslev, 1986, 1990; Engle, 1982) and has since been widely adopted for estimating conditional hedge ratios in commodity markets (Baillie and Myers, 1991; Bera et al., 1997; Lien and Tse, 2002; Lim, 1996; Myers, 1991), foreign exchange markets (Gagnon and Lypny, 1995; Kroner and Claessens, 1991; Kroner and Sultan, 1993) and stock markets (Brooks et al., 2002; Floros and Vougas, 2004; Park and Switzer, 1995; Tong, 1996). Following Baillie and Myers (1991), the GARCH model is specified as

$$
\begin{aligned}
R_{s,t} &= \mu_s + \varepsilon_{s,t} \\
R_{f,t} &= \mu_f + \varepsilon_{f,t}, \\
\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix} &= \varepsilon_t|\Omega_{t-1} \sim N(0, H_t), \\
H_t &= \begin{bmatrix} \sigma_{ss,t} & \sigma_{sf,t} \\ \sigma_{fs,t} & \sigma_{ff,t} \end{bmatrix}, \\
Vech(H_t) &= C + AVech(\varepsilon_{t-1}, \varepsilon'_{t-1}) + BVech(H_{t-1})
\end{aligned} \tag{2.17}
$$

The corresponding conditional optimal hedge ratio is defined as

$$
h_{t-1}|\Omega_{t-1} = \frac{\sigma_{sf,t}}{\sigma_{ff,t}} \tag{2.18}
$$

where Ω_{t-1} is the information available up to time $t-1$. The GARCH model has become the standard tool for dealing with the problem of heteroscedasticity and leptokurtosis in asset returns. In spite of the empirical successes of the GARCH hedge ratios that have been documented in the literature, there are contrary results as well. Lien and Tse (2000), in a study of ten spot and futures markets covering foreign exchange, commodities and stock markets, find that the OLS hedge ratios perform better than the GARCH hedge ratios, with the OLS providing as much as 20% less portfolio variance than the GARCH portfolio. This is further supported by Bystrom (2003), Butterworth and Holmes (2000) and Holmes (1995), who all find that the OLS hedge ratio outperforms GARCH hedge ratios. Lien and Tse (2002) attribute the failure of GARCH models to the fact that they produce variance forecasts that are too variable and persistent. Also, GARCH models cannot capture infrequent but extreme news events that drive financial returns. But such news events cause discontinuities in asset returns, evidence of which has been documented in large variety of studies (Ball and Torous, 1985; Jarrow and Rosenfeld, 1984; Jorion, 1988; Schwert, 1989).

Error Correction Hedge Ratio

In order to avoid the mis-specification problem that may arise from a possible cointegrating relationship between spot and futures prices, both series must be tested for cointegration. If both series are found to be cointegrated, then the hedge ratio must be estimated from an error correction model. This can be done by first running the following cointegrating regression and extracting the residuals:

$$S_t = \beta_0 + \beta_1 F_t + e_t \tag{2.19}$$

The extracted residuals are then included in the following error correction model (Chou et al., 1996; Ghosh, 1993; Lien and Luo, 1993) :

$$R_{s,t} = \phi_0 e_{t-1} + \theta R_{f,t} + \sum_{i=1}^{I} \phi_i R_{f,t-1} + \sum_{j=1}^{J} \delta_i R_{s,t-1} + u_t, \tag{2.20}$$

where the optimal hedge ratio is given by $\hat{\theta}$.

In order to simultaneously account for the long-run cointegrating relationship between spot and futures price series and their time-varying distributions, Kroner and Sultan (1993) combine the bivariate error correction model with a GARCH error structure in estimating the optimal hedge ratio. Their model is given by

$$\begin{aligned} R_{s,t} &= \beta_{0,s} + \beta_{1,s}(\ln S_{t-1} - \delta \ln F_{t-1}) + \varepsilon_{s,t} \\ R_{f,t} &= \beta_{0,f} + \beta_{1,f}(\ln S_{t-1} - \delta \ln F_{t-1}) + \varepsilon_{f,t}, \\ \begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix} &= \varepsilon_t | \Omega_{t-1} \sim N(0, H_t), \\ H_t &= \begin{bmatrix} \sigma_{s,t} & \sigma_{sf,t} \\ \sigma_{fs,t} & \sigma_{f,t} \end{bmatrix}, \end{aligned} \tag{2.21}$$

where $(\ln S_{t-1} - \delta \ln F_{t-1})$ is the error correction term that imposes the long-run relationship in the model. The optimal hedge ratio is given by

$$h_{t-1}|\Omega_{t-1} = \frac{\sigma_{sf,t}}{\sigma_{ff,t}}$$

Jump Process Hedge Ratios

Several studies have shown that jump processes can provide a good characterization for a wide variety of asset returns. In estimating optimal hedge ratios, (Chan, 2008, 2010; Chan and Young, 2006) assume that the spot and futures returns (or basis, as it may apply) follow a bivariate GARCH-jump process, where the common jump component follows an Autoregressive Jump Intensity process. This model has the additional advantage (over plain multivariate GARCH model) of allowing for a rich unconditional leptokurtosis in the underlying returns series. It has been applied in commodity market (Chan and Young, 2006) and foreign exchange market settings (Chan, 2008, 2010). In these applications the GARCH-jump hedge ratios outperform both the OLS and the standard GARCH hedge ratios.

Chang and Chang (2003) and Chang et al. (1996) also assume that returns follow a jump-diffusion process and estimated the associated optimal hedge ratios. Consistent with the previous studies, they find that the jump-diffusion process provide a better hedging strategy.

Markov Regime-Switching Hedge Ratios

Another approach that has been used for estimating optimal hedge ratios is to assume that the underlying returns series follow a Markov Regime-Switching (MRS) process. The motivation for adopting MRS models stems from the empirical evidence that suggests that the dynamic relationship between spot and futures returns may be characterized by regime shifts. Fong (2002) documents regime shifts in the volatility of crude oil futures returns, and associates such regime shifts to periods of severe shortages and backwardation in the crude oil market. Wilson et al. (1996) also document sudden regime shifts in the unconditional volatility of crude oil futures that are associated with events such as the Gulf war, Iran-Iraq conflict, OPEC policy changes, and extreme weather changes. This evidence suggest that the dynamic relationship between spot and futures returns may be regime dependent, and models that incorporate such regime dependent dynamics may provide superior hedge ratios compared to GARCH and OLS hedge ratios.

Following Lee and Yoder (2007a), the bivariate Markov Regime Switching GARCH(l,1) model is specified as follows:

$$\begin{aligned} R_{s,t} &= \mu_{s,i_t} + \varepsilon_{s,i_t} \\ R_{f,t} &= \mu_{f,i_t} + \varepsilon_{f,i_t} \\ \begin{bmatrix} \varepsilon_{s,i_t} \\ \varepsilon_{f,i_t} \end{bmatrix} &= \varepsilon_t | \Omega_{t-1} \sim BN(0, H_{t,i_t}), \end{aligned} \tag{2.22}$$

where $i_t = \{1, 2\}$ is the state variable indicating the market regime at time t, and follows a first-order two-state Markov process. H_{t,i_t} is a state-dependent time-varying positive definite conditional covariance matrix specified as:

$$H_{t,i_t} = \begin{bmatrix} \sigma_{s,t,i_t} & \sigma_{fs,t,i_t} \\ \sigma_{fs,t,i_t} & \sigma_{f,t,i_t} \end{bmatrix} = C_{i_t} C'_{i_t} + A_{i_t} \varepsilon_{t-1} A'_{i_t} + B_{i_t} H_{t-1} B'_{i_t} \tag{2.23}$$

where C is a lower triangular matrix, A and B are 2-dimensional parameter matrices. Lee and Yoder (2007a) apply the MRS-GARCH model to corn and nickel markets. They find that their model outperforms both OLS and GARCH in-sample, but only OLS in out-of-sample analysis. Alizadeh et al. (2008) extend the bivariate MRS-GARCH model by including a vector error correction term in the conditional mean equations, and allowing the speed of adjustment to long term mean to be dependent on the volatility state. Their representation of the model is

$$R_{s,t} = \alpha_{s,i_t} + \beta_{s,i_t}(\ln S_{t-1} - \delta \ln F_{t-1}) + \varepsilon_{s,i_t} \tag{2.24}$$

$$R_{f,t} = \alpha_{f,i_t} + \beta_{f,i_t}(\ln S_{t-1} - \delta \ln F_{t-1}) + \varepsilon_{f,i_t} \tag{2.25}$$

Note that the speed of adjustment to the long-run equilibrium level, β_{c,i_t}, $c \in \{s, f\}$, is also state dependent. This introduces an informative link between volatility and cointegration, and allows for time dependency and asymmetric behaviour across different states of the economy. Alizadeh et al. (2008) apply the VECM-MRS-GARCH model to estimate hedge ratios for crude oil, gasoline and heating oil markets. They find that their model outperforms OLS and GARCH models in four out of six cases of in- and out-of-sample analysis.

Alizadeh and Nomikos (2004) apply a univariate MRS model to estimate a MV hedge ratio using data on FTSE100 and S&P500 stock indices. They find evidence in favor of the MRS model in terms of variance reduction and increases in portfolio utility. Lee et al. (2006) develop a Random Coefficient Autoregressive MRS model for estimating optimal hedge ratios. Using data on aluminum and lead markets, they find that the model outperforms both OLS and GARCH models in terms of portfolio variance reduction. Lee and Yoder (2007b) apply a Markov regime-switching Time-varying Correlation GARCH model to the Nikkei225 and Hang-Seng indices, and report similar results.

General Comments

Although several alternative methodologies have been proposed for estimating time-varying optimal hedge ratios, the gains in terms of portfolio risk reduction compared to the simple OLS method are often minimal and sometimes economically insignificant. Sometimes, the OLS hedging strategy even performs better, as in the case of Lee and Yoder (2007b). But given the complexity and cost of hedging based on these alternative models, the superiority of these models over a simple naive or OLS hedge strategy may be hard to justify. Furthermore, the alternative models suffer from a variety of structural problems. First, it is well known that GARCH models tend to produce variance forecasts that are too variable. Therefore, GARCH models are not able to replicate some important characteristics of financial times series.

Second, jump models isolate normal events from rare but extreme events and assume that all jumps at any instant have the same expected size and frequency. However from an intuitive point of view, gradations in size and frequency exist among events such as war, depression, natural disasters etc. Therefore, models that dichotomize normal from extreme events can miss such regularities. Third, the MRS models often impose two state volatility regimes on the data: a high state, corresponding to periods of extreme market movements and a low state, corresponding to extremely calm market periods. This implies that markets are either extremely volatile or extremely calm. However, there are periods when markets can be characterized by not in either of these extremes and the standard MRS models will miss such regularities. Thus, the current state of the literature suggests that there is a need to improve our understanding of estimating futures hedge ratios.

Therefore, this paper introduces a vector error correction Markov-Switching Multifractal (VECM-MSM) model as a suitable and empirically viable alternative to model spot and futures returns. We assume that spot and futures prices are subject to multifrequency risk, and derive the resulting time-varying hedge ratios. The model incorporates the long-run relationship between spot and futures prices by including a vector error correction term in the conditional means. The conditional variance-covariance matrix is driven by a bivariate Markov state vector with multiple components, whose rescaled product defines volatility. Each of the volatility components can switch to a new level with a different probability per unit time, generating volatility shocks of multiple frequencies. Hence, the VECM-MSM approach captures the thick-tails, long memory features and volatility clustering which are characteristics of many financial prices. Modeling the variance-covariance process as a bivariate MSM is consistent with the intuition that asset prices are simultaneously subjected to multiple shocks of heterogeneous durations and frequencies such as demand and supply shocks, technological innovations, macroeconomic uncertainty, intermediate contributions from the political cycle as well as weather and other natural phenomena.

2.3. Vector Error Correction Markov-Switching Multifractal Model and Hedging

The joint distributions of spot and futures returns are assumed to follow the VECM-MSM process as follows

$$R_{s,t} = \beta_s + \alpha_s(\ln S_{t-1} - \delta \ln F_{t-1}) + \varepsilon_{s,t} \tag{2.26}$$

$$R_{f,t} = \beta_f + \alpha_f(\ln S_{t-1} - \delta \ln F_{t-1}) + \varepsilon_{f,t},$$

$$\begin{bmatrix} \varepsilon_{s,t} \\ \varepsilon_{f,t} \end{bmatrix} = \varepsilon_t | \Omega_{t-1} = \begin{bmatrix} \overline{\sigma}_s(M_t^s)\eta_{s,t} \\ \overline{\sigma}_f(M_t^f)\eta_{f,t} \end{bmatrix} \tag{2.27}$$

where the scaled residual $\eta_t \in \mathbb{R}^2$ is bivariate *IID* Gaussian $N(0, \Sigma)$ with variance-covariance matrix given by

$$\Sigma = \begin{bmatrix} 1 & \rho_\eta \\ \rho_\eta & 1 \end{bmatrix} \tag{2.28}$$

First, note that the model includes an error correction component to accommodate the long-run relationship between spot and futures prices, for the reasons discussed in section 2.2.1. Second, the conditional second moments are assumed to follow the bivariate Markov Switching Multifractal process of Calvet et al. (2006). Specifically, the volatility matrix is driven by a bivariate first-order Markov state vector, M_t, given by

$$M_t = \begin{bmatrix} M_t^s \\ M_t^f \end{bmatrix} \in \mathbb{R}_+^2$$

Note that M_s and M_f are each state vectors on their own. Each state vector contains $\overline{k}$ state variables, $M_{k,t}^c$ for $c \in \{s, f\}$, as follows

$$\begin{aligned} M_t^s &= (M_{1,t}^s; M_{2,t}^s ....; M_{\overline{k},t}^s) \\ M_t^f &= (M_{1,t}^f; M_{2,t}^f ....; M_{\overline{k},t}^f), \end{aligned} \tag{2.29}$$

therefore

$$M_t = \begin{bmatrix} M_t^s \\ M_t^f \end{bmatrix} = \begin{bmatrix} M_{1,t}^s; M_{2,t}^s ....; M_{\overline{k},t}^s \\ M_{1,t}^f; M_{2,t}^f ....; M_{\overline{k},t}^f \end{bmatrix} \tag{2.30}$$

M_t can be viewed as a state matrix, containing 2x$\overline{k}$ state variables. The first row contains the state variables for the spot returns, while the second row contains the state variables for the futures returns. Under the MSM framework, each state variable, $M_{k,t}^c$, is referred to as a volatility component, volatility multiplier or volatility frequency. The intuition is that switches in each volatility frequency represents a volatility shock and each volatility frequency has its own duration, $1/\gamma_k$, or switching probability, γ_k. The ranking is such that $M_{1,t}^c$ represents the most persistent (lowest frequency) volatility component, while $M_{\overline{k},t}^c$ represents the least persistent (highest frequency) component.

The volatility state variables are characterized by the same marginal distribution, $M = [M^s M^f]'$, but these variables are assumed to be statistically independent across t. In other words, $M^c_{k,t}$ is independent of $M^c_{k',t}$, for $k \neq k'$. Therefore, at every time t, $M_{k,t}$ may switch independently with probability γ_k for $k = 1, 2, ...\overline{k}$, where

$$\gamma_k = 1 - (1 - \gamma_{\overline{k}})^{b^{k-\overline{k}}}$$

where $\gamma_k \in (0, 1)$ and $b \in (0, \infty)$ are parameters to be estimated. For simplicity, we follow Calvet et al. (2006) and assume that

$$(\gamma^s_1, \gamma^s_2, ...\gamma^s_{\overline{k}}) = (\gamma^f_1, \gamma^f_2, ...\gamma^f_{\overline{k}})$$

The parameter γ_k controls the persistence of the k^{th} frequency component, while the parameter b governs the growth rate of the transition probabilities. Also corresponding to the economic intuition that volatility arrivals might be correlated across series,[5] the bivariate MSM allows for correlation in the switching probabilities, with the correlation coefficient given by $\lambda \in [0, 1]$.

The construction of the MSM volatility model can accommodate a wide spectrum of distributions for the vector M with minimal restrictions.[6] Specifically, it requires that M has a positive support and a unit mean: $\mathbb{E}(M) = 1$ and $M \geq 0$.

As in Calvet and Fisher (2004) and Calvet et al. (2006), this paper adopts a simple bivariate binomial distribution where each volatility component is drawn from $M = \left(M^s, M^f\right)'$. Assuming that the volatility vector has been constructed up to time t. At time $t + 1$, each element of $M_{k,t} = (M^s_{k,t}, M^f_{k,t})$ takes values $m^c_0 \in$

[5]This is especially true in the case of the spot and futures returns of an asset.

[6]MSM allows for flexible parametric and non-parametric, discrete or continuous distributions for M. See Calvet and Fisher (2004) for more details.

$[1,2]$ or $m_1^c = 2 - m_0 \in [0,1]$ with probability 0.5, and stays unchanged otherwise. Therefore, the volatility component vector M_t can take four possible values, $(m_0^s, m_0^f), (m_0^s, m_1^f), (m_1^s, m_0^f)$ and (m_1^s, m_1^f) with probability matrix $(p_{i,j})$ defined as

$$p_{i,j} = \mathbb{P}\left(M_k = (m_i^s, m_j^f)\right) = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} = \begin{bmatrix} \dfrac{1+\rho_m}{4} & \dfrac{1-\rho_m}{4} \\ \dfrac{1-\rho_m}{4} & \dfrac{1+\rho_m}{4} \end{bmatrix}$$

where $\rho_m \in [0,1]$ is the correlation coefficient between M_k^s and M_k^f. Following Calvet et al. (2006), we impose the restriction that $\rho_m = 1$. Under this framework, the volatility state vector M_t therefore takes $d = 4^{\overline{k}}$ possible values $m_1, m_2 ... m_d$.

Under the above assumptions, volatility is stochastic and is defined as the rescaled product of the volatility components:

$$\sigma_s(M_t) = \overline{\sigma}_s \left(\prod_{k=1}^{\overline{k}} M_{k,t}^s \right)^{\frac{1}{2}}$$

$$\sigma_f(M_t) = \overline{\sigma}_f \left(\prod_{k=1}^{\overline{k}} M_{k,t}^f \right)^{\frac{1}{2}}$$

and the conditional variance-covariance matrix is

$$\begin{bmatrix} \overline{\sigma}_s^2 \prod_{k=1}^{\overline{k}} \mathbb{E}_t\left(M_{k,t}^s\right) & \rho_\varepsilon \overline{\sigma}_s \overline{\sigma}_f \prod_{k=1}^{\overline{k}} \mathbb{E}_t\left[\left(M_{k,t}^s M_{k,t}^f\right)^{\frac{1}{2}}\right] \\ \rho_\varepsilon \overline{\sigma}_s \overline{\sigma}_f \prod_{k=1}^{\overline{k}} \mathbb{E}_t\left[\left(M_{k,t}^s M_{k,t}^f\right)^{\frac{1}{2}}\right] & \overline{\sigma}_f^2 \prod_{k=1}^{\overline{k}} \mathbb{E}_t\left(M_{k,t}^f\right) \end{bmatrix} \tag{2.31}$$

The VECM-MSM parameter vector is then characterized by

$$\theta = \left(\alpha_s, \alpha_f, \beta_s, \beta_f, \delta, m_0^s, m_0^f, \overline{\sigma}_s, \overline{\sigma}_f, b, \gamma_{\overline{k}}, \rho_\eta, \lambda\right)$$

The MSM specification of volatility implies that volatility is stochastic and hit by shocks of heterogeneous frequencies indexed by $k \in \left\{1, 2, ..., \overline{k}\right\}$. Jumps in low-level volatility components (e.g uncertainty regarding exhaustible energy resources supplies, weather, depression, war, etc.) cause volatility to vary discontinuously and exhibit strong persistence, while switches in high-frequency components (e.g liquidity uncertainty, maturity effects, macroeconomic and corporate news releases, political feedbacks, etc.) produce substantial outliers in returns. While the former source of risk is important for long-term hedgers in the futures market, the latter has serious implications for short-term hedgers. The parsimonious specification of bivariate shocks of heterogeneous frequencies by MSM also corresponds to the intuition that even though the spot and futures returns series may share the same fundamentals, such fundamentals may have different innovation frequencies. The bivariate MSM specification above also allows for correlation in volatility across series through the bivariate binomial distribution M as well as correlation in spot and futures returns through the bivariate *IID* Gaussian random variable $\eta_{s,t}$ and $\eta_{f,t}$.

Once the joint distributions of the return series have been adequately specified, the information can be used to construct the dynamic optimal hedge ratios. Using Equations (2.5) and (2.31) the dynamic optimal hedge ratio is defined as:

$$h_{t-1}|\Omega_{t-1} = \frac{Cov_t(R_{s,t}, R_{f,t})}{Var_t(R_{f,t})} = \frac{\rho_\eta \overline{\sigma}_s \overline{\sigma}_f \prod_{k=1}^{\overline{k}} \mathbb{E}_t \left[\left(M_{k,t}^s M_{k,t}^f\right)^{\frac{1}{2}}\right]}{\overline{\sigma}_f^2 \prod_{k=1}^{\overline{k}} \mathbb{E}_t \left(M_{k,t}^f\right)} \tag{2.32}$$

2.3.1. Estimation and Inference

Recall that $M_{k,t} = [M_{k,t}^s \;\; M_{k,t}^f]'$ is assumed to follow a bivariate binomial distribution, where $M_{k,t}^s$ and $M_{k,t}^f$ can each take two possible values and consequently, $M_{k,t}$ can take four possible values at each time t. Therefore, there exist a finite number of volatility states, and standard filtering methods apply for the estimation process.

Suppose there are $\overline{k}$ volatility components included in the model.[7] Then $M_t = (M_{1,t} * M_{2,t} ..... * M_{\overline{k},t})$ can take $4^{\overline{k}} = d$ possible values $(m_1, m_2, ...m_d) \in \mathbb{R}_+^{\overline{k}}$. The dynamics of M_t are then characterized by a dxd transition matrix A, with elements $a_{ij} = \mathbb{P}(M_{t+1} = m_j | M_t = m_i)$.

Note that the econometrician only observes the set of past returns, but not the volatility state vector. The vector M_t is therefore latent and must be inferred by Bayesian updating. Let $\Pi_t = (\Pi_t^1, \Pi_t^2, ...\Pi_t^d) \in \mathbb{R}_+^d$ be the vector of state probabilities, where

$$\Pi_t^j = \mathbb{P}(M_t = m_j | R_t) \tag{2.33}$$

where $R_t = [R_{s,t} \;\; R_{f,t}]'$ is the return matrix. The conditional probability state vector is computed recursively by Bayes updating. By Bayes rule, Π_t can be expressed as a function of the previous belief Π_{t-1} and the bivariate Gaussian density as follows

$$\Pi_t = \frac{f(R_t) \odot \Pi_{t-1} A}{[f(R_t) \odot \Pi_{t-1} A]\mathbf{1}'}, \tag{2.34}$$

where

$$\mathbf{1} = [1, 1, ...1] \in \mathbb{R}^d, \tag{2.35}$$

The Gaussian density function is given by[8]

$$f(R_t) = \frac{1}{2\pi \overline{\sigma}_s(m^s) \overline{\sigma}_f(m^f) \sqrt{1 - \rho_\eta^2}} exp\left[-\frac{z}{2(1 - \rho_\eta^2)} \right], \tag{2.36}$$

[7] The choice of $\overline{k}$ is a model selection problem to be discussed later.

[8] Note that m^s and m^f refer to the first row and second row, respectively, of $(M_1 * M_2 ..... * M_{\overline{k}}) = (m_1, m_2, ...m_d)$.

where

$$z \equiv \frac{\varepsilon_{s,t}^2}{(\overline{\sigma}_s(m^s))^2} + \frac{\varepsilon_{f,t}^2}{(\overline{\sigma}_f(m^f))^2} - \frac{2\rho_\eta \varepsilon_{s,t}\varepsilon_{f,t}}{\overline{\sigma}_s(m^s)\overline{\sigma}_f(m^f)}$$

The Bayes recursion is initiated with Π_0 , using the ergodic distribution. The log-likelihood then has a closed form expression and is given by

$$lnL(R_1, ...R_T; \theta) = \sum_{t=1}^{T} \ln[f(R_t) \cdot (\Pi_{t-1}A)] \tag{2.37}$$

The estimates of the VECM-MSM parameters are obtained using the two-step procedure of Calvet et al. (2006). They show that the two-step procedure is a special case of the Generalised Methods of Moment (GMM) approach, implying consistency and asymptotic normality of the parameter estimators. The first stage of the procedure entails maximizing the combined univariate log-likelihood functions, given by

$$lnL_1 = lnL_s(R_{s,t}; \alpha_s, \beta_s, \delta_s, m_0^s, \overline{\sigma}_s, b, \gamma_{\overline{k}}) + lnL_f(R_{f,t}; \alpha_f, \beta_f, \delta_f, m_0^f, \overline{\sigma}_f, b, \gamma_{\overline{k}})$$

This produces consistent estimates of the first stage parameters. In the second stage, the first stage parameters are used to analytically calculate the bivariate log-likelihood function in Equation (2.37). Maximization of the bivariate log-likelihood function then produces the estimates of (ρ_m, λ), which are unique to the bivariate MSM.

Estimating the optimal hedge ratio using the VECM-MSM model outlined above overcomes both the limitations of OLS model the traditional GARCH or GARCH-jump models. First, by including a vector error correction component, we allow for adjustment to the long-run equilibrium between spot and futures prices. Second, by allowing for bivariate shocks to volatility, we relax the restrictive assumption of common jump components in the traditional jump models.[9] Even though arrivals on

[9]For example, Chan (2008)

volatility occur simultaneously in both spot and futures returns (as imposed by the restriction that $\gamma^s_{\bar{k}} = \gamma^f_{\bar{k}}$), the size of jumps in both series will be different due to the difference in other volatility parameters, $m^s_0, m^f_0, \overline{\sigma}_s$ and $\overline{\sigma}_f$. Third, by introducing multiple volatility components of different frequencies, the VECM-MSM model is able to accommodate shocks of heterogeneous durations and frequencies through the different frequency components. This is a substantial improvement over other jump models that simply dichotomize between normal and rare events. Another significant improvement of the VECM-MSM hedge ratio is the ability to accommodate many states. This is in contrast to the previous Markov switching models such as the MRS-GARCH, where volatility of spot and futures returns switch between high and low states (Alizadeh et al., 2008; Lee and Yoder, 2007a,b) or the four states model of Li (2009). The VECM-MSM model can accommodate $4^{\bar{k}}$ possible states parsimoniously and the number of parameters does not increase with the number of states. Consequently, it is expected that the VECM-MSM hedge ratio will outperform other alternative hedge ratios considered in this study. Using data on a variety of assets ranging from energy, agricultural, metal and equity to foreign exchange markets ensures that our results are not market specific as can be the case in previous studies. Also, using assets from distinct markets is consistent with the empirical evidence that different asset prices behave differently and can not be treated as one (Bhardwaj and Dunsby, 2011; Brooks and Prokopczuk, 2013).

2.4. Empirical Analysis

2.4.1. Data

The data consist of weekly observations on spot and futures prices for nine assets: West Texas Intermediate (WTI) crude oil, heating oil, wheat, corn, gold, silver, S&P500 stock index, Canadian-U.S. dollars (CAD) and Pounds Sterling-U.S. dollars (GBP) exchange rates. The different price data series used start on different dates, but all data end on 12/26/2012.[10] All data sets are publicly available. The heating oil and crude oil data sets were obtained from the Energy Information Administration (U.S. Department of Energy) while other data sets were obtained from the Quandl database.[11] Data from the beginning to the end of 2010, inclusive, are used for model estimation.[12] The remaining two years of observations are reserved for out-of-sample exercises.

WTI crude oil and heating oil are traded on the New York Mercantile Exchange (NYMEX), wheat is traded on the Kansas Board of Trade (KCBT), corn is traded on the Chicago Board of Trade (CBOT) and gold and silver are traded on the Commodity Exchange of New York (COMEX). The CAD and GBP are traded on the Chicago Mercantile Exchange (CME). All prices are Wednesday settlement prices. When a holiday occurs on Wednesday, Tuesdays prices are used instead. The futures prices are front month contract, rolled on expiration.[13,14]

Table 2.1 presents the summary statistics for log prices and log returns series for the

[10]The starting dates for the price series are necessitated by the availability of data. Forcing the time series to start on the same date would result in significant loss of useful observations. The WTI data start on 01/08/1986 and heating oil on 06/04/1986. The metals data start on 01/03/1979. The agricultural data start on 01/02/1980. The S&P500 data starts on 04/28/1982. The CAD data start on 01/19/1977 and the GBP on 02/19/1975.

[11]Quandl is a collaboratively-curated portal to millions of time-series data sets, and are publicly available for downloading.

[12]The choice of 2010 as the end of the in-sample period is partly in response to a suggestion that the chaotic financial period between 2008 and 2009 might unduly favor the the MSM model in the out-of-sample analysis.

[13]The front month contract is the contract for the trading month that will expire next. Using the wheat contract for example, in February 2004 the front month would be the March 2004 contract as it is the next contract to expire. Once the March 2004 contract expires, the next front month contract is the contract that will expire next, the May 2004 contract.

[14]There are several methods of constructing continuous futures price data. A popular method is "Roll on expiration". This implies that when the current contract expires (at which point the price data terminates), the data rolls to the next contract that will expire.

entire sample, where log returns are defined as $\ln(P_{t-1}/P_t) * 100$. Both price and returns series exhibit significant excess kurtosis in all assets, with silver spot prices providing the largest for returns, with a value of 12.12. The Jarque and Bera (1987) test of normality is also reported in Table 2.1. The test statistics indicate significant departures from normality for all price and returns series. Also reported in Table 2.1, is the Ljung and Box (1978) Q statistic on the first six lags of log prices and squared returns series. As has been documented in the literature, the results show significant presence of auto correlation in both log prices and squared returns.

These characteristics of the data support the need for a model that incorporates both long memory and fat-tail features of the data, such as is the case for the proposed VECM-MSM model. Next, in order to determine whether a vector error correction component belongs in the model or not, we test for the existence of a cointegrating relationship between log spot and futures prices for all assets. In order to achieve this, we first test both log spot and log futures prices for the presence of a unit root. The results are reported in columns 3 and 4 of Table 2.2. The augmented Dickey-Fuller unit root test indicate that both series are non-stationary for all assets. the second stage of the test requires that log spot be regressed on log futures and the residuals extracted. The coefficients from the regression is the is normalized cointegrating vector, which is presented in column 2 of Table 2.2. It can be observed from the table that estimates of δ are very close to 1 for all series. Therefore, in order to reduce the number of parameters to be estimated, we set $\delta = 1$ and assume that the cointegrating vector is the basis. Hence, in the second stage of the cointegration test, we applied the augmented Dickey-Fuller test to the basis, and the results are presented on the last column of Table 2.2.

The test results indicate that the basis is stationary for all series. The null hypothesis that all assets spot prices are cointegrated with their respective futures prices cannot

Table 2.1: Summary Statistics of Log Prices and Log Returns

		Mean	Std.Dev	Min	Max	Skewness	Kurtosis	JB Test	$Q(6)$	N
Log Prices										
WTI	S	3.4312	0.6448	2.3823	4.9680	0.6388	-0.9535	148.68	1353.56	1406
	F	3.4311	0.6456	2.3768	4.9668	0.6402	-0.9562	149.41	1357.24	1406
Heating	S	-0.1347	0.6613	-1.2483	1.4024	0.6706	-0.9001	150.33	1252.69	1385
	F	-0.1349	0.6640	-1.2107	1.4041	0.6826	-0.9158	155.68	1274.88	1385
Corn	S	5.5940	0.3603	4.8081	6.7429	1.0371	1.0029	377.12	575.99	1719
	F	5.6411	0.3434	4.9715	6.7217	1.1516	1.0573	456.45	565.34	1719
Wheat	S	6.0290	0.3060	5.4670	7.2492	0.9264	0.7950	288.77	630.63	1719
	F	5.9485	0.3170	5.4250	7.1546	1.0365	0.7114	341.57	647.55	1719
Gold	S	6.1136	0.4768	5.3977	7.5058	1.4206	1.2020	698.22	551.88	1773
	F	6.1162	0.4761	5.4090	7.5113	1.4197	1.2017	697.43	552.05	1773
Silver	S	6.6456	0.5920	5.8777	8.4553	1.0998	0.3376	363.80	774.86	1773
	F	6.6463	0.5927	5.8749	8.4329	1.0881	0.2963	354.40	784.99	1773
CAD	S	-0.2202	0.1250	-0.4752	0.0869	0.0581	-0.6896	38.11	1417.85	1852
	F	-0.2210	0.1247	-0.4753	0.0802	0.0657	-0.6839	37.80	1411.82	1852
GBP	S	0.5224	0.1356	0.0695	0.8945	0.3316	0.4592	52.22	800.33	1953
	F	0.5199	0.1349	0.0672	0.8926	0.3174	0.4529	48.76	796.22	1953
S&P 500	S	6.4199	0.7549	4.6308	7.3540	-0.5329	-1.0688	151.81	1783.58	1600
	F	6.4236	0.7532	4.6313	7.3609	-0.5311	-1.0672	151.09	1782.63	1600
Log Returns										
		Mean	Std.Dev	Min	Max	Skewness	Kurtosis	JB Test	$Q^2(6)$	N
WTI	S	0.0892	5.2380	-29.2136	30.3046	-0.2248	3.0103	533.65	34.82	1406
	F	0.0893	5.0907	-37.2877	24.3899	-0.4011	4.1363	1024.32	33.61	1406
Heating	S	0.1509	5.3193	-30.0945	41.6999	0.2028	6.8029	2642.72	25.55	1385
	F	0.1547	4.8665	-26.6083	27.4067	-0.1558	3.3402	638.71	22.62	1385
Corn	S	0.0565	3.8074	-18.2322	21.6545	-0.1073	3.1627	709.93	40.29	1719
	F	0.0514	3.7014	-31.1830	20.9092	-0.3778	6.1554	2723.20	22.24	1719
Wheat	S	0.0317	3.6118	-18.2748	20.9555	0.2122	3.2481	758.26	28.11	1719
	F	0.0322	4.0225	-25.0677	22.7058	-0.0489	4.1000	1189.47	11.26	1719
Gold	S	0.1145	2.6170	-13.9475	22.4460	0.3783	6.6746	3297.08	27.51	1773
	F	0.1132	2.6768	-13.1057	20.2097	0.4111	6.0728	2743.64	36.41	1773
Silver	S	0.0908	4.6601	-43.7847	40.4142	-0.3295	12.1190	10772.33	17.18	1773
	F	0.0906	4.4671	-29.5406	21.8094	-0.3477	4.5381	1538.80	41.96	1773
CAD/USD	S	0.0009	0.9280	-5.9131	5.9619	-0.1454	6.1237	2869.25	15.16	1852
	F	0.0014	0.9402	-6.1672	5.2556	-0.1929	5.3041	2158.47	16.13	1852
GBP/USD	S	-0.0203	1.4043	-8.6689	7.3974	-0.3881	3.5702	1074.05	23.48	1953
	F	-0.0185	1.4369	-12.0095	8.0914	-0.4210	4.7021	1837.32	31.16	1953
S&P 500	S	0.1559	2.3153	-16.6634	10.1824	-0.7392	5.1869	1915.57	27.18	1600
	F	0.1551	2.3789	-17.4818	10.3134	-0.7452	5.1060	1863.07	31.47	1600

Notes: For each commodity, this table presents the summary statistics for the log prices and the percentage log returns, defined as $ln(P_{t-1}/P_t) * 100$. JB refers to the Jarque and Bera (1987) test statistics for normality, and has a χ^2 distribution with 2 degrees of freedom under the null. The test strongly rejects the null hypothesis of normality in all log prices and returns. $Q(6)$ refers to the Ljung and Box (1978) test statistics for autocorrelation of up to order 6 in log prices and $Q^2(6)$ refers to the test of autocorrelation in squared returns. Both tests have a χ^2 distribution with 6 degrees of freedom under the null. The tests strongly reject the null hypothesis of no serial correlation in all log prices and squared returns.

Table 2.2: **Test of Cointegration Between Log Spot and Futures Prices**

	Normalized CV	Augmented Dickey-Fuller Unit-Root Test		
	(1 β_0 δ)	Log Spot	Log Futures	Basis
WTI	(1 0.0051 0.9986)	-0.9510	-0.9010	-26.6640
		[0.7707]	[0.7878]	[0.0000]
Heating	(1 -0.0005 0.9947)	-1.2100	-0.9070	-13.6910
		[0.6695]	[0.7856]	[0.0000]
Corn	(1 -2.8554 1.0422)	-1.4540	-1.5010	-9.7490
		[0.5560]	[0.5332]	[0.0000]
Wheat	(1 0.5158 0.9268)	-1.9360	-2.0280	-5.8070
		[0.3152]	[0.2745]	[0.0000]
Gold	(1 -0.011 1.0014)	-0.0370	-0.0420	-29.6410
		[0.9554]	[0.9549]	[0.0000]
Silver	(1 0.0103 0.9983)	-0.9600	-0.9050	-23.1790
		[0.7677]	[0.7863]	[0.0000]
CAD/USD	(1 0.0015 1.0028)	-1.4910	-1.4670	-16.1700
		[0.5379]	[0.5498]	[0.0000]
GBP/USD	(1 -0.0001 1.0048)	-3.1180	-3.0160	-14.9480
		[0.0252]	[0.0335]	[0.0000]
S&P 500	(1 -0.0186 1.0023)	-2.1030	-2.1040	-11.1080
		[0.2432]	[0.2429]	[0.0000]

Notes: This table presents the results from the test of cointegration between log spot and log futures prices. The normalized CV column refers to the cointegrating vector, where β_0 and δ are obtained by regressing log spot on log futures prices for each series i.e $\ln S_t = \beta_0 + \delta \ln F_t + \mu_t$. The last three columns report the test statistics from the Augmented-Dickey Fuller unit root tests for the log spot, log futures and basis (difference between log spot and log futures) for each series, using one lag. The associated p-values are reported on brackets. The null hypothesis is that the associated series is non-stationary. The presence of a cointegrating relationship between spot and futures prices requires that both series be non-stationary and the basis be stationary. The tests indicate that all series are cointegrated.

be rejected at all conventional levels, and at the 1% significance level for GBP. These in turn indicate that there exists a long-run equilibrium relationship between spot and futures prices of the assets under consideration. Therefore, the inclusion of the vector error correction components into the MSM model is statistically appropriate.

2.4.2. VECM-MSM Model Estimates

Table 2.3 presents the parameter estimates from the two-step estimation procedure. The VECM-MSM models are estimated for k equals 1 to 8. Several observations from Table 2.3 merit attention. First, we find that the coefficients on the error correction terms are consistent, in most cases, with those implied by the theoretical model.[15] The speeds of adjustment of spot prices to their long-run relationship, measured by α_s, are mostly consistently negative and statistically significant, with the exception of CAD and S&P 500. On the other hand, the speeds of adjustment of futures prices have mixed signs and are mostly insignificant. This means that in the estimated error correction coefficients are in accordance with convergence towards a long-run equilibrium relationship. In other words, in response to a positive spread at period $t-1$ (i.e., $S_{t-1} > F_{t-1}$), the spot price in the next period will decrease while the futures price will either be unresponsive or less responsive than spot prices, thus restoring the long-run equilibrium.

Second, according to Fama (1984), if the current basis is an unbiased estimator of the future spot price, $\alpha_s = 1$ and $\beta_s = 0$ should hold. From a theoretical point of view, as maturity approaches, the spot and futures prices should converge if markets are efficient. We find that this hypothesis holds for corn, wheat, CAD, GBP and S&P 500, while it does not hold for the other assets.

Third, looking at the volatility multiplier parameters $\hat{m}_0^s$ and $\hat{m}_0^f$, they tend to decline as the number of frequency components increases. The intuition is that less variability is required in each individual component in order to match the volatility fluctuation of the data. Estimates of $\hat{\sigma}_s$ and $\hat{\sigma}_f$ fluctuate across k without any apparent pattern. Fourth, as k increases, the switching probability of the highest volatility component $\hat{\gamma}_{\bar{k}}$ increases,[16] whereas $\hat{b}$ does not exhibit any particular pattern. From the estimated values of $\hat{\gamma}_{\bar{k}}$, we can infer the duration of the volatility components. Taking, corn for instance, when $\overline{k} = 1$ the single volatility component has a duration

[15] Note that some values of α_s and α_f are large. This is partly due to the fact that returns are defined as $ln(P_{t-1}/P_t) * 100$. Also, we do not restrict the upper and lower bounds of the estimates of α_s and α_f during the optimization process. We observe that the model fits are better (as indicated by the log-likelihood values) when the upper and lower bounds are not restricted.

[16] This is with the exception of crude oil.

of approximately 13 weeks ($1/\hat{\gamma}_1$). When $\overline{k} = 8$, the highest frequency component $M_{8,t}$ switches every one and a half weeks, while the lowest frequency component $M_{1,t}$ has a duration of approximately 145 weeks. Therefore, the VECM-MSM model of spot and futures returns is able to capture not only the frequent but transient shocks, but also captures rare and extreme events. Thus, the lowest frequency shock in corn occurs only approximately eleven times during the data period.

Next, we observe that the correlation between the Gaussian innovations $\hat{\rho}_\varepsilon$ is always positive with almost no variation across k. Values of $\hat{\rho}_\varepsilon$ are also similar to the sample correlation observed between spot and futures returns. The correlation coefficient between volatility arrivals is also positive with no apparent pattern as k changes.

The results so far have important implications for dynamic hedging strategies. Switches in the lowest frequency components parsimoniously capture the effects of rare but extreme and unanticipated news arrivals on asset prices. This constitutes an important source of risk for long term hedgers pricing long-lived futures and must be considered in their hedging decision. Jumps in high frequency components capture normal random news events in the market, which matters most for pricing short-lived futures contracts.

Table 2.3: Vector Error Correction-MSM Parameter Estimates

	$k=1$	2	3	4	5	6	7	8
				WTI				
$\hat{\beta}_s$	0.2488	0.1887	0.2010	0.2141	0.2050	0.2171	0.2194	0.2192
	(0.1145)	(0.1029)	(0.1135)	(0.1154)	(0.2902)	(0.1108)	(0.1089)	(0.1082)
$\hat{\beta}_f$	0.2215	0.1600	0.1678	0.1787	0.1717	0.1818	0.1829	0.1829
	(0.1157)	(0.1115)	(0.1169)	(0.1204)	(0.2645)	(0.1199)	(0.1144)	(0.1144)
$\hat{\alpha}_s$	-107.8678	-112.2647	-107.4020	-107.7128	-107.5846	-108.6111	-108.4814	-108.5218
	(9.1244)	(6.8269)	(6.1526)	(6.5116)	(27.1052)	(5.6054)	(5.8291)	(5.6934)
$\hat{\alpha}_f$	-6.4974	-9.5716	-6.9609	-6.6860	-6.8148	-7.8495	-7.3448	-7.6152
	(7.0479)	(10.3978)	(9.2696)	(8.5788)	(13.3874)	(8.2693)	(8.4413)	(8.3133)
$\hat{m}_{0,s}$	1.6520	1.5112	1.4198	1.4068	1.4028	1.3073	1.2959	1.2709
	(0.0596)	(0.0488)	(0.0782)	(0.0297)	(0.0510)	(0.4753)	(0.0623)	(0.0518)
$\hat{m}_{0,f}$	1.7179	1.5150	1.4245	1.4101	1.4067	1.3134	1.3005	1.2762
	(0.0170)	(0.1225)	(0.0705)	(0.0321)	(0.0492)	(0.4096)	(0.0704)	(0.0577)

Continued on next page

Table 2.3 – Continued from previous page

	$k=1$	2	3	4	5	6	7	8
$\hat{\bar{\sigma}}_s$	6.6326	6.3729	5.7257	4.9422	6.1940	5.4069	5.3260	5.2837
	(1.1904)	(0.4409)	(0.3209)	(0.4826)	(0.5932)	(6.1862)	(0.9314)	(0.7723)
$\hat{\bar{\sigma}}_f$	7.7246	6.3181	5.8361	5.0175	6.3261	5.3703	5.3338	5.2914
	(0.4582)	(0.3515)	(0.3806)	(0.4252)	(0.6847)	(5.1500)	(0.7233)	(0.6539)
$\hat{\gamma}_{\bar{k}}$	0.0251	0.0396	0.0240	0.0222	0.0160	0.0293	0.0275	0.0286
	(0.0098)	(0.0159)	(0.0127)	(0.0150)	(1286.4)	(0.1138)	(0.0210)	(0.0182)
$\hat{b}$	-	2.9235	1.0001	1.0000	1.0000	1.0001	1.0000	1.0000
	-	(3.4035)	(0.1390)	(0.1130)	(40603)	(0.2498)	(0.0516)	(0.0512)
$\hat{\rho}_e$	0.9864	0.9895	0.9884	0.9881	0.9896	0.9885	0.9886	0.9881
	(0.0003)	(0.0002)	(0.0003)	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0004)
$\hat{\lambda}$	0.7435	0.8088	0.8825	0.9090	0.9271	0.9656	0.9739	0.9863
	(0.0752)	(0.0617)	(0.0485)	(0.0427)	(0.0459)	(0.0342)	(0.0356)	(0.0338)
lnL	-5588.75	-5437.53	-5437.91	-5437.25	-5431.36	-5451.85	-5444.84	-5451.36
				Heating Oil				
$\hat{\beta}_s$	0.1409	0.1859	0.1616	0.1603	0.1653	0.1643	0.1642	0.1641
	(0.1221)	(0.1231)	(0.1217)	(0.1246)	(0.1248)	(0.1245)	(0.1248)	(0.1245)
$\hat{\beta}_f$	0.1711	0.2259	0.2096	0.2135	0.2127	0.2142	0.2149	0.2154
	(0.1160)	(0.1180)	(0.1163)	(0.1170)	(0.1182)	(0.1182)	(0.1182)	(0.1183)
$\hat{\alpha}_s$	-23.6033	-25.3902	-24.2017	-24.6552	-24.5622	-24.5710	-24.6177	-24.6335
	(3.8724)	(4.6204)	(4.4648)	(4.5112)	(4.4771)	(4.5076)	(4.4901)	(4.4908)
$\hat{\alpha}_f$	-4.7587	-5.5268	-5.0764	-5.0504	-5.0717	-5.0003	-5.0552	-5.0312
	(2.9016)	(4.0125)	(3.7950)	(3.8109)	(3.8728)	(3.9097)	(3.8968)	(3.9321)
$\hat{m}_{0,s}$	1.7005	1.5432	1.4523	1.4070	1.3564	1.3244	1.2990	1.2805
	(0.0276)	(0.0516)	(0.0557)	(0.0639)	(0.0481)	(0.0486)	(0.0479)	(0.0497)
$\hat{m}_{0,f}$	1.6355	1.5286	1.4126	1.3669	1.3229	1.2949	1.2734	1.2564
	(0.0281)	(0.0424)	(0.0370)	(0.0511)	(0.0403)	(0.0407)	(0.0401)	(0.0411)
$\hat{\bar{\sigma}}_s$	7.3566	6.1971	6.1649	5.6835	5.5434	5.4203	5.3731	5.3559
	(0.6184)	(0.4830)	(1.0557)	(0.6666)	(0.6025)	(0.5570)	(0.6063)	(0.6490)
$\hat{\bar{\sigma}}_f$	6.4173	5.4573	5.3666	5.1357	4.9966	4.9800	4.9774	4.9597
	(0.4218)	(0.3675)	(0.5641)	(0.3669)	(0.3627)	(0.4130)	(0.4051)	(0.4208)
$\hat{\gamma}_{\bar{k}}$	0.0327	0.0455	0.0497	0.0524	0.0591	0.0618	0.0640	0.0649
	(0.0103)	(0.0138)	(0.0240)	(0.0275)	(0.0260)	(0.0299)	(0.0439)	(0.7609)
$\hat{b}$	-	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	-	(0.1697)	(0.1679)	(0.1023)	(0.0700)	(0.0712)	(0.1475)	(3.3751)
$\hat{\rho}_e$	0.9656	0.9582	0.9598	0.9621	0.9616	0.9636	0.9650	0.9661
	(0.0000)	(0.0001)	(0.0011)	(0.0009)	(0.0010)	(0.0010)	(0.0010)	(0.0011)
$\hat{\lambda}$	0.9125	0.9749	0.9771	0.9884	0.9904	0.9929	0.9945	0.9957
	(0.0280)	(0.0312)	(0.0235)	(0.0154)	(0.0137)	(0.0135)	(0.0122)	(0.0102)
lnL	-6023.69	-6002.17	-5942.61	-5906.33	-5900.94	-5881.21	-5868.13	-5857.88
				Corn				
$\hat{\beta}_s$	-0.2567	-0.1683	-0.1765	-0.1836	-0.1859	-0.1854	-0.1909	-0.1916
	(0.2479)	(0.2331)	(0.2336)	(0.2188)	(0.2187)	(0.1848)	(0.2199)	(0.2209)
$\hat{\beta}_f$	0.1824	0.1788	0.1975	0.1845	0.1713	0.1724	0.1690	0.1656
	(0.2195)	(0.2271)	(0.2382)	(0.2214)	(0.2219)	(0.1706)	(0.2324)	(0.2224)
$\hat{\alpha}_s$	-8.7720	-7.4777	-7.8384	-8.0409	-8.0245	-7.4520	-8.0794	-8.0836
	(4.1806)	(4.2187)	(4.2227)	(3.9514)	(3.9113)	(3.4590)	(3.9523)	(3.9739)
$\hat{\alpha}_f$	2.3112	2.9050	2.9280	2.8561	2.7078	2.8557	2.6607	2.6141
	(3.3150)	(3.5695)	(3.6675)	(3.4911)	(3.4357)	(2.5444)	(3.5725)	(3.4765)
$\hat{m}_{0,s}$	1.7046	1.6220	1.5224	1.4720	1.4229	1.6064	1.3635	1.3429
	(0.0171)	(0.0218)	(0.0351)	(0.0284)	(0.0270)	(0.0209)	(0.0971)	(0.0332)
$\hat{m}_{0,f}$	1.6894	1.5981	1.5026	1.4524	1.4046	1.5963	1.3508	1.3327
	(0.0239)	(0.0335)	(0.0454)	(0.0352)	(0.0378)	(0.0328)	(0.1252)	(0.0426)

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Table 2.3 – Continued from previous page

	$k=1$	2	3	4	5	6	7	8
$\hat{\bar{\sigma}}_s$	4.0454	4.3632	3.9430	3.8186	3.7759	3.4852	3.8269	3.8205
	(0.2325)	(0.2853)	(0.4334)	(0.4340)	(0.4297)	(0.2567)	(1.5000)	(0.4790)
$\hat{\bar{\sigma}}_f$	4.2402	4.3641	4.2506	3.9418	3.7941	3.4202	3.7773	3.8226
	(0.3622)	(0.4460)	(0.4508)	(0.3103)	(0.3907)	(0.4954)	(1.5721)	(0.4966)
$\hat{\gamma}_{\overline{k}}$	0.0757	0.2025	0.1735	0.3310	0.4732	0.2224	0.5744	0.6475
	(0.0260)	(0.0523)	(0.1337)	(0.2525)	(0.6391)	(0.0666)	(1.0536)	(0.5304)
$\hat{b}$	-	10.7519	3.0479	3.2181	2.7071	11.4144	2.1563	2.0468
	-	(5.4404)	(1.6031)	(1.7648)	(2.0537)	(5.1308)	(2.9130)	(0.8853)
$\hat{\rho}_e$	0.8775	0.8829	0.8755	0.8796	0.8842	0.8957	0.8860	0.8874
	(0.0067)	(0.0056)	(0.0062)	(0.0055)	(0.0058)	(0.0036)	(0.0052)	(0.0053)
$\hat{\lambda}$	0.9116	0.9369	0.9341	0.9573	0.9688	0.9437	0.9846	0.9852
	(0.0188)	(0.0073)	(0.0051)	(0.0037)	(0.0209)	(0.0067)	(0.0377)	(0.0445)
lnL	-7312.75	-7210.11	-7235.50	-7194.76	-7176.20	-7193.73	-7165.12	-7163.64
				Wheat				
$\hat{\beta}_s$	0.2276	0.1958	0.1918	0.1918	0.1917	0.1917	0.1956	0.1974
	(0.1677)	(0.1621)	(0.1718)	(0.1401)	(0.1197)	(0.1400)	(0.1295)	(0.1588)
$\hat{\beta}_f$	-0.0875	-0.0626	-0.0765	-0.0766	-0.0760	-0.0762	-0.0808	-0.0728
	(0.1928)	(0.2135)	(0.1888)	(0.1559)	(0.1873)	(0.1541)	(0.1487)	(0.1707)
$\hat{\alpha}_s$	-2.3286	-1.8683	-1.9600	-1.9605	-1.9605	-1.9605	-1.9464	-1.8904
	(1.3335)	(1.3012)	(1.4341)	(1.0749)	(0.9915)	(1.0771)	(1.1097)	(1.4455)
$\hat{\alpha}_f$	1.2404	1.0085	0.9515	0.9515	0.9446	0.9463	1.1918	1.0169
	(1.4486)	(1.7422)	(1.3261)	(1.0268)	(1.3067)	(1.0104)	(1.0959)	(1.1667)
$\hat{m}_{0,s}$	1.6824	1.6501	1.5660	1.5661	1.5661	1.5661	1.5610	1.4548
	(0.0185)	(0.0229)	(0.0311)	(0.0282)	(0.0272)	(0.0395)	(0.0249)	(0.0244)
$\hat{m}_{0,f}$	1.6069	1.4903	1.4949	1.4950	1.4956	1.4955	1.4373	1.3572
	(0.0304)	(0.0400)	(0.0402)	(0.0396)	(0.0453)	(0.0387)	(0.0853)	(0.0475)
$\hat{\bar{\sigma}}_s$	3.8618	3.2454	4.0326	3.2227	2.5752	3.9093	3.0944	2.9893
	(0.2586)	(0.2530)	(0.2945)	(0.2154)	(0.1594)	(0.3902)	(0.2370)	(0.4623)
$\hat{\bar{\sigma}}_f$	4.6930	4.9669	3.9533	3.2344	4.5432	3.7168	3.2695	3.6385
	(0.2950)	(0.3843)	(0.3000)	(0.2242)	(0.3968)	(0.2841)	(0.2341)	(0.6180)
$\hat{\gamma}_{\overline{k}}$	0.0573	0.1262	0.2004	0.2006	0.2014	0.2011	0.1826	0.3375
	(0.0179)	(0.0427)	(0.0734)	(0.0746)	(0.0728)	(0.0711)	(0.0564)	(0.1794)
$\hat{b}$	-	18.4195	18.4599	18.6374	18.9916	18.8916	9.5792	4.6261
	-	(10.6954)	(10.5063)	(10.5673)	(9.7472)	(10.2389)	(5.1785)	(2.6592)
$\hat{\rho}_e$	0.7659	0.7747	0.7794	0.7794	0.7880	0.7879	0.7968	0.7912
	(0.0131)	(0.0102)	(0.0097)	(0.0098)	(0.0090)	(0.0081)	(0.0069)	(0.0117)
$\hat{\lambda}$	0.9299	0.6475	0.8809	0.8826	0.8458	0.8726	0.9235	0.8609
	(0.0115)	(0.0612)	(0.0176)	(0.0174)	(0.0233)	(0.0188)	(0.0104)	(0.0127)
lnL	-7850.25	-7820.95	-7762.05	-7762.86	-7764.10	-7760.28	-7753.72	-7769.96
				Gold				
$\hat{\beta}_s$	-0.1421	-0.1882	-0.1715	-0.1761	-0.1816	-0.1761	-0.1770	-0.1816
	(0.0612)	(0.0508)	(0.0502)	(0.0468)	(0.0467)	(0.0498)	(0.0804)	(0.0461)
$\hat{\beta}_f$	0.1149	0.0459	0.0509	0.0426	0.0408	0.0426	0.0447	0.0407
	(0.0852)	(0.0622)	(0.0591)	(0.0614)	(0.0608)	(0.0522)	(0.0736)	(0.0588)
$\hat{\alpha}_s$	-76.6394	-75.6211	-78.5632	-78.6091	-79.1449	-78.6161	-77.9197	-79.1205
	(8.2851)	(8.8539)	(8.1984)	(8.4519)	(8.4696)	(11.3939)	(9.3163)	(9.8575)
$\hat{\alpha}_f$	19.3579	14.4339	13.3420	12.0579	12.2903	12.0555	12.8787	12.2920
	(9.4030)	(10.8290)	(9.6828)	(10.6063)	(10.3830)	(8.7456)	(9.6216)	(9.7187)
$\hat{m}_{0,s}$	1.7558	1.6348	1.6070	1.5408	1.4619	1.5407	1.5407	1.4602
	(0.0147)	(0.0272)	(0.0238)	(0.0895)	(0.0312)	(0.0315)	(0.0258)	(0.0270)
$\hat{m}_{0,f}$	1.7548	1.6448	1.6133	1.5597	1.4598	1.5595	1.5574	1.4593
	(0.0164)	(0.0215)	(0.0244)	(0.0788)	(0.0325)	(0.0245)	(0.0280)	(0.0289)

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Table 2.3 – Continued from previous page

	$k=1$	2	3	4	5	6	7	8
$\hat{\bar{\sigma}}_s$	3.1214	2.8294	2.6315	3.2735	2.5609	2.1244	3.1685	2.3745
	(0.2417)	(0.2160)	(0.1531)	(0.6283)	(0.2553)	(0.1863)	(0.2146)	(0.2715)
$\hat{\bar{\sigma}}_f$	3.1148	2.9164	2.7086	3.6196	2.7178	2.3204	3.4940	2.5281
	(0.3623)	(0.3065)	(0.1738)	(0.6164)	(0.4053)	(0.1371)	(0.2918)	(0.3347)
$\hat{\gamma}_{\overline{k}}$	0.0629	0.0699	0.8660	0.8086	0.8615	0.8065	0.7269	0.8608
	(0.0343)	(0.0458)	(0.0766)	(0.0966)	(0.0605)	(0.1064)	(0.0538)	(0.0640)
$\hat{b}$	-	11.5143	68.4520	34.9596	7.5179	34.6651	29.2239	7.4669
	-	(11.2883)	(33.6961)	(17.7257)	(1.3941)	(16.8373)	(7.9282)	(1.4756)
$\hat{\rho}_e$	0.9537	0.9483	0.9513	0.9535	0.9515	0.9535	0.9539	0.9535
	(0.0014)	(0.0018)	(0.0004)	(0.0024)	(0.0028)	(0.0024)	(0.0024)	(0.0028)
$\hat{\lambda}$	0.9887	0.9872	0.9768	0.9847	0.9983	0.9855	0.9876	0.9989
	(0.0113)	(0.0157)	(0.0199)	(0.0381)	(0.0265)	(0.0370)	(0.0334)	(0.0252)
lnL	-5433.45	-5311.93	-5248.66	-5234.84	-5224.09	-5236.16	-5232.37	-5221.70
				Silver				
$\hat{\beta}_s$	0.0270	-0.0309	-0.0189	-0.0115	-0.0116	-0.0116	-0.0116	-0.0200
	(0.0792)	(0.0769)	(0.0652)	(0.0629)	(0.0654)	(0.0670)	(0.0687)	(0.0667)
$\hat{\beta}_f$	0.0258	-0.0229	-0.0115	-0.0121	-0.0122	-0.0122	-0.0122	-0.0184
	(0.0797)	(0.0731)	(0.0827)	(0.0724)	(0.0714)	(0.0720)	(0.0724)	(0.0717)
$\hat{\alpha}_s$	-47.0888	-55.7556	-58.3168	-61.6782	-61.6852	-61.6835	-61.6834	-49.3053
	(16.3491)	(13.3300)	(11.4395)	(13.7896)	(36.3800)	(22.9987)	(32.9459)	(14.0995)
$\hat{\alpha}_f$	31.4786	29.3729	28.5457	28.2166	28.2152	28.2164	28.2157	32.6633
	(6.5430)	(7.4017)	(8.9995)	(6.4514)	(6.9506)	(6.4475)	(8.5653)	(6.0249)
$\hat{m}_{0,s}$	1.7655	1.6626	1.6077	1.5400	1.5398	1.5399	1.5399	1.5423
	(0.0177)	(0.0641)	(0.0337)	(0.0468)	(0.0388)	(0.0384)	(0.0428)	(0.0456)
$\hat{m}_{0,f}$	1.6982	1.5713	1.5376	1.5305	1.5303	1.5303	1.5303	1.5558
	(0.0159)	(0.0245)	(0.0889)	(0.0309)	(0.0348)	(0.0315)	(0.0302)	(0.0334)
$\hat{\bar{\sigma}}_s$	5.3120	6.2340	5.7379	6.6104	5.3255	4.2919	3.4587	5.1845
	(0.5352)	(1.5843)	(0.7579)	(1.2126)	(0.8571)	(0.7487)	(0.6765)	(0.9363)
$\hat{\bar{\sigma}}_f$	4.8068	5.0084	4.7040	3.7605	5.4837	4.4338	3.5845	5.9064
	(0.2388)	(0.2934)	(3.7763)	(0.4224)	(0.7738)	(0.5528)	(0.4180)	(0.8632)
$\hat{\gamma}_{\overline{k}}$	0.0456	0.0626	0.8587	0.6816	0.6782	0.6789	0.6794	0.7637
	(0.0126)	(0.0189)	(0.1228)	(0.2112)	(0.2147)	(0.2282)	(0.2417)	(0.3797)
$\hat{b}$	-	9.0231	39.1909	19.4022	19.2252	19.2664	19.2911	21.8948
	-	(8.6335)	(15.8415)	(13.1659)	(13.5689)	(13.5608)	(13.8369)	(34.5255)
$\hat{\rho}_e$	0.9543	0.9499	0.9488	0.9649	0.9659	0.9659	0.9659	0.9627
	(0.0002)	(0.0003)	(0.0003)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$\hat{\lambda}$	0.9665	0.9400	0.9384	0.9703	0.9759	0.9761	0.9763	0.9730
	(0.0023)	(0.0086)	(0.0098)	(0.0024)	(0.0051)	(0.0055)	(0.0061)	(0.0064)
lnL	-7159.65	-7116.26	-7056.27	-6877.86	-6870.66	-6871.40	-6872.13	-6903.51
				CAD				
$\hat{\beta}_s$	-0.0159	-0.0193	-0.0112	-0.0134	-0.0135	-0.0136	-0.0144	-0.0140
	(0.0201)	(0.0215)	(0.0282)	(0.0212)	(0.0228)	(0.0238)	(0.0310)	(0.0212)
$\hat{\beta}_f$	-0.0523	-0.0463	-0.0480	-0.0463	-0.0443	-0.0443	-0.0447	-0.0455
	(0.0217)	(0.0218)	(0.0255)	(0.0184)	(0.0185)	(0.0189)	(0.0332)	(0.0175)
$\hat{\alpha}_s$	-7.5529	-2.2622	-2.8562	-2.9992	-3.1100	-3.1208	-2.5084	-2.4877
	(5.1509)	(8.0664)	(10.9700)	(8.7990)	(9.0208)	(11.3588)	(2.9523)	(8.1885)
$\hat{\alpha}_f$	26.9400	29.6304	28.7778	27.4717	27.0146	26.9422	27.1701	27.2789
	(5.7440)	(8.0319)	(9.0162)	(6.2767)	(6.1702)	(5.6840)	(6.4472)	(5.0485)
$\hat{m}_{0,s}$	1.7426	1.6161	1.5494	1.4822	1.4643	1.4625	1.4160	1.3250
	(0.0222)	(0.0490)	(0.8613)	(0.0368)	(0.0287)	(0.0451)	(0.0350)	(0.0610)
$\hat{m}_{0,f}$	1.7175	1.6045	1.5224	1.4452	1.4242	1.4224	1.3631	1.2991
	(0.0215)	(0.0532)	(0.3808)	(0.0320)	(0.0383)	(0.0376)	(0.0352)	(0.1065)

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Table 2.3 – Continued from previous page

	$k=1$	2	3	4	5	6	7	8
$\hat{\bar{\sigma}}_s$	1.1829	1.1384	1.2170	1.1926	1.0047	1.3583	1.9791	0.9525
	(0.0670)	(0.1076)	(0.8960)	(0.1396)	(0.1256)	(0.2416)	(0.3124)	(0.0969)
$\hat{\bar{\sigma}}_f$	1.1809	1.1015	1.1969	1.2249	1.0588	0.8893	1.3968	0.9934
	(0.0684)	(0.1024)	(0.2068)	(0.1614)	(0.1288)	(0.1066)	(0.1685)	(0.2988)
$\hat{\gamma}_{\overline{k}}$	0.0097	0.0986	0.0711	0.0646	0.0525	0.0530	0.0851	0.0972
	(0.0079)	(0.0471)	(0.3215)	(0.0359)	(0.0256)	(0.0283)	(0.0230)	(0.0542)
$\hat{b}$	-	87.7050	15.3211	4.1653	2.6504	2.6894	2.9143	1.9032
	-	(75.9132)	(214.6779)	(1.5210)	(0.8988)	(1.0398)	(0.5271)	(0.3975)
$\hat{\rho}_e$	0.9578	0.9565	0.9599	0.9578	0.9576	0.9572	0.9538	0.9590
	(0.0011)	(0.0013)	(0.0015)	(0.0018)	(0.0018)	(0.0018)	(0.0003)	(0.0019)
$\hat{\lambda}$	0.9999	0.9994	0.9999	0.9999	0.9999	0.9920	0.8769	0.9999
	(0.1181)	(0.0239)	(0.0491)	(0.1201)	(0.1218)	(0.0348)	(0.0119)	(0.0841)
lnL	-1946.62	-1822.96	-1785.41	-1803.85	-1826.54	-1847.27	-1959.00	-1785.91
				GBP				
$\hat{\beta}_s$	0.0499	0.0488	0.0526	0.0638	0.0584	0.0641	0.0642	0.0641
	(0.0362)	(0.0450)	(0.0366)	(0.0404)	(0.0509)	(0.0440)	(0.0483)	(0.0449)
$\hat{\beta}_f$	-0.0243	-0.0166	-0.0229	-0.0108	-0.0232	-0.0110	-0.0111	-0.0110
	(0.0373)	(0.0378)	(0.0407)	(0.0456)	(0.0545)	(0.0464)	(0.0440)	(0.0470)
$\hat{\alpha}_s$	-16.0128	-15.4706	-13.8430	-14.2435	-12.6179	-14.1007	-14.0412	-14.1037
	(5.2085)	(5.7573)	(4.8302)	(6.4174)	(4.5436)	(7.5585)	(8.8240)	(7.8463)
$\hat{\alpha}_f$	17.1389	14.1569	17.6326	15.6895	17.6949	15.7006	15.7176	15.6934
	(5.5266)	(7.6057)	(8.0092)	(10.1668)	(5.2650)	(10.6764)	(9.9204)	(11.0425)
$\hat{m}_{0,s}$	1.5982	1.5031	1.5104	1.4663	1.5122	1.4658	1.4657	1.4658
	(0.0298)	(0.0623)	(0.0601)	(0.0333)	(0.0362)	(0.0390)	(0.0358)	(0.0674)
$\hat{m}_{0,f}$	1.5951	1.4932	1.4923	1.4504	1.4871	1.4499	1.4496	1.4499
	(0.0343)	(0.0632)	(0.0267)	(0.0377)	(0.0377)	(0.0405)	(0.0390)	(0.0475)
$\hat{\bar{\sigma}}_s$	1.6214	1.5773	1.6502	1.2671	1.9506	1.4342	1.1857	1.6208
	(0.1022)	(0.3122)	(0.0985)	(0.0992)	(0.1557)	(0.1318)	(0.1066)	(0.2399)
$\hat{\bar{\sigma}}_f$	1.6469	1.6255	1.6870	1.3325	1.9273	1.4913	1.2396	1.6690
	(0.1160)	(0.1769)	(0.0944)	(0.1306)	(0.1590)	(0.1397)	(0.1274)	(0.1866)
$\hat{\gamma}_{\overline{k}}$	0.0287	0.0971	0.0316	0.7188	0.0363	0.7061	0.7002	0.7070
	(0.0093)	(0.1156)	(0.0696)	(0.2809)	(0.0088)	(0.3221)	(0.3598)	(0.3275)
$\hat{b}$	-	14.9619	1.8917	13.8048	2.4076	13.3462	13.1032	13.3907
	-	(21.1034)	(4.3635)	(6.8499)	(0.6566)	(8.3437)	(8.8835)	(8.6959)
$\hat{\rho}_e$	0.9661	0.9681	0.9660	0.9673	0.9663	0.9673	0.9673	0.9673
	(0.0009)	(0.0010)	(0.0012)	(0.0014)	(0.0012)	(0.0014)	(0.0014)	(0.0014)
$\hat{\lambda}$	0.9830	0.9918	0.9821	0.9964	0.9825	0.9992	0.9984	0.9994
	(0.0416)	(0.0265)	(0.0233)	(0.0165)	(0.0326)	(0.0257)	(0.0227)	(0.0246)
lnL	-3824.98	-3800.07	-3790.41	-3771.08	-3792.22	-3769.14	-3769.90	-3768.86
				S&P 500				
$\hat{\beta}_s$	0.2341	0.2417	0.2677	0.2615	0.2690	0.2673	0.2660	0.2611
	(0.1019)	(0.1140)	(0.1110)	(0.1088)	(0.0828)	(0.0952)	(0.1075)	(0.0894)
$\hat{\beta}_f$	0.3322	0.3412	0.3655	0.3654	0.3663	0.3653	0.3698	0.3659
	(0.1006)	(0.1135)	(0.1076)	(0.1077)	(0.0832)	(0.0935)	(0.1086)	(0.0910)
$\hat{\alpha}_s$	-0.7678	-0.9254	-0.5634	-1.0630	-0.5809	-0.8727	-0.3600	-1.1601
	(12.4904)	(15.5510)	(15.5964)	(15.1564)	(10.6774)	(12.4243)	(14.1116)	(11.0473)
$\hat{\alpha}_f$	21.4393	19.8508	20.7015	20.8974	20.4119	20.3254	21.5499	20.8959
	(12.3810)	(15.6350)	(14.9304)	(14.9225)	(10.8647)	(12.0077)	(13.9030)	(11.4653)
$\hat{m}_{0,s}$	1.6494	1.5394	1.4977	1.4115	1.4982	1.4947	1.2982	1.4163
	(0.0275)	(0.0575)	(0.0412)	(0.0427)	(0.0432)	(0.0459)	(0.0411)	(0.0309)
$\hat{m}_{0,f}$	1.6518	1.5575	1.5146	1.4254	1.5149	1.5123	1.3072	1.4320
	(0.0261)	(0.0733)	(0.0338)	(0.0484)	(0.0356)	(0.0309)	(0.0430)	(0.0299)

Continued on next page

Table 2.3 – Continued from previous page

	$k=1$	2	3	4	5	6	7	8
$\hat{\bar{\sigma}}_s$	2.5560	2.7595	2.4989	2.5426	1.6782	2.3298	2.2911	2.0303
	(0.1615)	(0.3517)	(0.3174)	(0.2918)	(0.2397)	(0.4756)	(0.2570)	(0.1757)
$\hat{\bar{\sigma}}_f$	2.5747	2.9315	2.5246	2.6558	1.6691	2.3753	2.3493	2.1200
	(0.2023)	(0.4631)	(0.2663)	(0.3742)	(0.1542)	(0.2762)	(0.3050)	(0.1916)
$\hat{\gamma}_{\overline{k}}$	0.0439	0.0568	0.0616	0.0731	0.0802	0.0780	0.1596	0.0908
	(0.0164)	(0.0434)	(0.0593)	(0.0968)	(0.0634)	(0.0586)	(0.4003)	(0.0849)
$\hat{b}$	-	4.1015	2.4638	1.9014	3.9388	3.7643	1.5972	2.5927
	-	(4.7385)	(1.9924)	(1.4100)	(1.9812)	(2.0251)	(1.0841)	(1.3856)
$\hat{\rho}_e$	0.9894	0.9878	0.9888	0.9883	0.9889	0.9889	0.9884	0.9883
	(0.0004)	(0.0005)	(0.0005)	(0.0006)	(0.0005)	(0.0005)	(0.0006)	(0.0006)
$\hat{\lambda}$	0.9999	0.9999	0.9833	0.9999	0.9781	0.9800	0.9999	0.9965
	(0.0659)	(0.0550)	(0.0393)	(0.0357)	(0.0506)	(0.0496)	(0.0522)	(0.0481)
lnL	-3738.70	-3685.81	-3686.79	-3668.88	-3692.05	-3681.25	-3655.18	-3669.97

Notes: This table reports the maximum likelihood estimates of the VECM-MSM parameter estimates for each asset. Log returns are defined as $ln(P_{t-1}/P_t) * 100$. The VECM-MSM(k) model is fitted for $\overline{k}$ equals 1 to 8, where each column corresponds to the given number of frequency components $\overline{k}$ in the MSM specification. When $\overline{k} = 1$, the VECM-MSM specification corresponds to a standard Markov-switching Vector Error Correction model, with only two possible states of volatility and $\gamma_{\overline{k}} = \gamma_1$. b is therefore unidentified and omitted. Asymptotic standard errors, reported in parenthesis, are computed using the *Outer Product Gradient* estimate of the information matrix. See Hamilton (1994), page 143.

2.4.3. Model Selection

It can be observed from the results above that the log-likelihood increases non-monotonically as the number of frequency components increases. This implies that the fit of the model increases as the number of frequency components increases only up to a certain point. Therefore for hedging purposes, we need to select the value of $\overline{k}$ that fits the model best. We formalize this by following Calvet and Fisher (2004) and employ the likelihood ratio based test of Vuong (1989).

To apply the Vuong test, we assume two non-nested models VECM-MSM(k) and VECM-MSM(k'), with densities f and f' respectively. The log-likelihood difference is given by

$$LR_v = T^{-1/2}(lnL_T^f(\hat{\theta}_T) - lnL_T^{f'}(\hat{\theta}'_T)) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} ln \frac{f(r_t|R_{t-1})}{f'(r_t|R_{t-1})}$$

We consider the null hypothesis that models VECM-MSM(k) and VECM-MSM(8) have identical unconditional expected log-likelihood (VECM-MSM(k) and VECM-MSM(8) fit the data equally well), against the alternative that VECM-MSM(k) performed worst. Specifically, we test the following conditions

$$H_0 : lnL_T^k - lnL_T^8 = 0, \text{ for } k \in 1, 2, ..., 7$$
$$H_1 : lnL_T^k - lnL_T^8 < 0$$

Under the null hypothesis,

$$T^{-1/2}(lnL_T^f(\hat{\theta}_T) - lnL_T^8(\hat{\theta}'_T)) \xrightarrow{d} N(0, \sigma_*^2)$$

where

$$\sigma_*^2 = Var\left(ln\left[\frac{f^k(r_t|R_{t-1})}{f^8(r_t|R_{t-1})}\right]\right)$$

The t-ratios and corresponding one-sided p-values are reported in Table 2.4. All models are rejected in favor of VECM-MSM(8), for heating oil, corn, gold and GBP. But the null hypothesis can not be rejected for other assets. While other models may fit the data equally well, we find that VECM-MSM(8) performed best for hedging purposes, both in-sample and out-of-sample. Therefore, we employ VECM-MSM(8) for subsequent analysis.

Using the parameter estimates from the VECM-MSM model for each asset, we compute the smoothed marginal probabilities of being in high state for each volatility component, defined as $\hat{\Psi}_t^{M(k)} \equiv \mathbb{P}(M_{k,t} = m_0|R_T)$. The marginal probabilities for corn are plotted on panel a to panel h of Figure 2.1 for $M_{1,t}$ to $M_{8,t}$ respectively, while the basis (log spot price - log futures price) and the log spot price are plotted on panels i and j, for illustration. Several points from the Figure 2.1 merit discussion. First, the plot of marginal probabilities of each frequency component clearly depicts

Table 2.4: MSM Model Selection (Vuong (1989) Test)

$\bar{k}$	WTI	Heating	Corn	Wheat	Gold	Silver	CAD	GBP	S&P 500
1	-5.4401	-7.3400	-9.9209	-6.7240	-7.5240	-6.0622	-10.2830	-2.9996	-6.7897
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0014)	(0.0000)
2	0.8292	-11.6180	-12.5123	-12.2675	-7.2436	-11.1989	-4.9533	-3.8080	-4.9130
	(0.7964)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0001)	(0.0000)
3	1.9357	-14.0511	-40.7782	2.4850	-4.3106	-13.2592	0.1275	-2.1624	-7.0819
	(0.9734)	(0.0000)	(0.0000)	(0.9935)	(0.0000)	(0.0000)	(0.5507)	(0.0154)	(0.0000)
4	2.2211	-13.3214	-46.9395	2.2391	-6.2620	8.2141	-9.1641	-6.4238	2.5939
	(0.9867)	(0.0000)	(0.0000)	(0.9874)	(0.0000)	(1.0000)	(0.0000)	(0.0000)	(0.9952)
5	4.9944	-31.6390	-42.3787	4.8813	-3.6968	12.8639	-25.4579	-2.2555	-8.4249
	(1.0000)	(0.0000)	(0.0000)	(1.0000)	(0.0001)	(1.0000)	(0.0000)	(0.0121)	(0.0000)
6	-1.2492	-28.2293	-9.5561	7.5267	-6.9185	12.6348	-35.5939	-87.4079	-5.6768
	(0.1059)	(0.0000)	(0.0000)	(1.0000)	(0.0000)	(1.0000)	(0.0000)	(0.0000)	(0.0000)
7	30.4458	-45.0784	-51.7519	17.1385	-5.7935	12.3541	-74.7230	-43.7547	5.3561
	(1.0000)	(0.0000)	(0.0000)	(1.0000)	(0.0000)	(1.0000)	(0.0000)	(0.0000)	(1.0000)

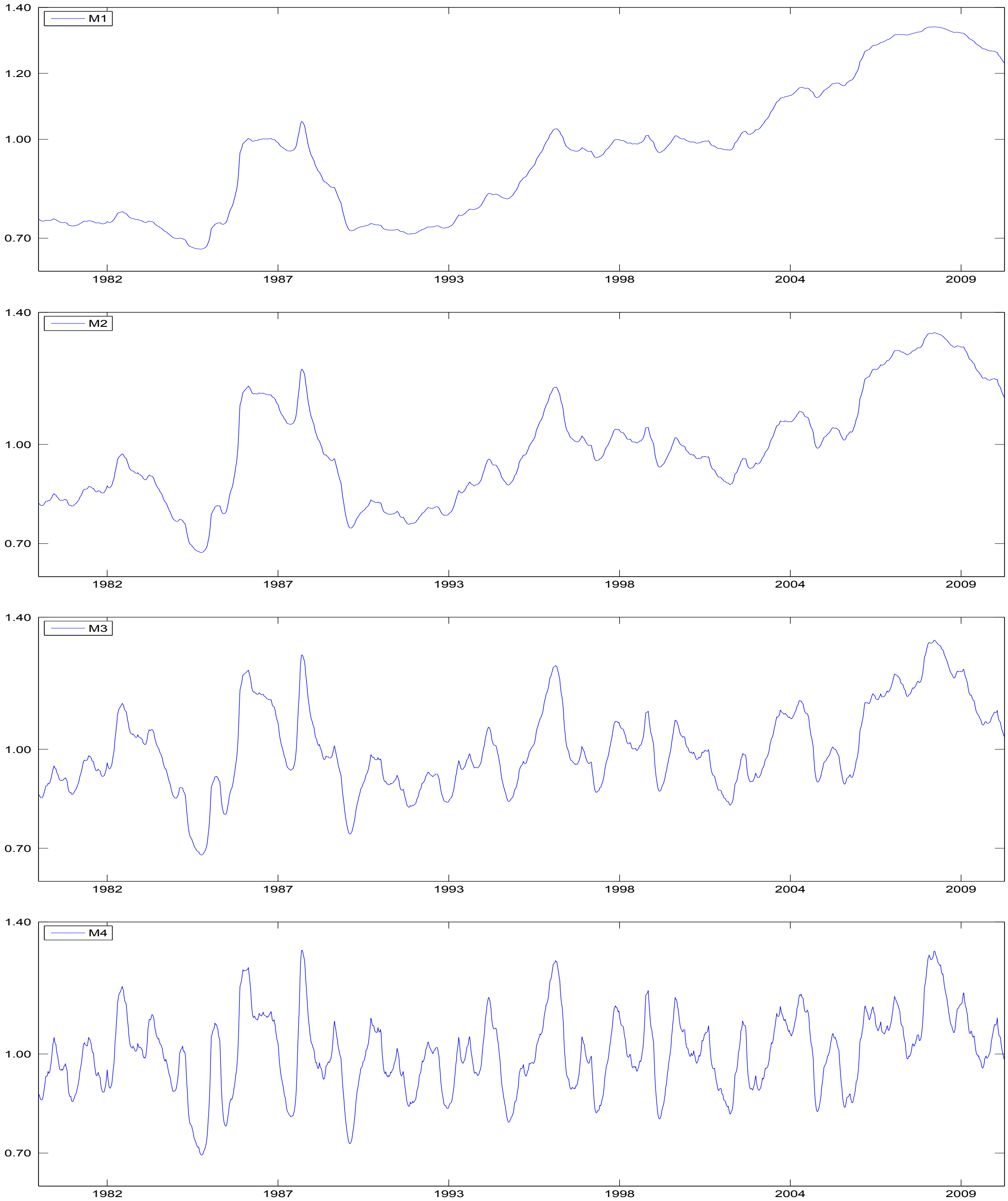
Notes: This table reports the t-ratios and the corresponding lower-tail p-values from the test of the null hypothesis that VECM-MSM(k) and VECM-MSM(8) fit the data equally well ($H_0 : lnL_T^k - lnL_T^8 = 0$), against the alternative hypothesis that VECM-MSM(k) performed worst ($H_1 : lnL_T^k - lnL_T^8 < 0$). Each row corresponds to the number of frequencies in the alternative VECM-MSM model being compared to VECM-MSM(8). For example, row $k = 1$ compares VECM-MSM(1) with VECM-MSM(8), for each asset. A low p-value indicates that the corresponding VECM-MSM model will be rejected in favour of VECM-MSM(8).

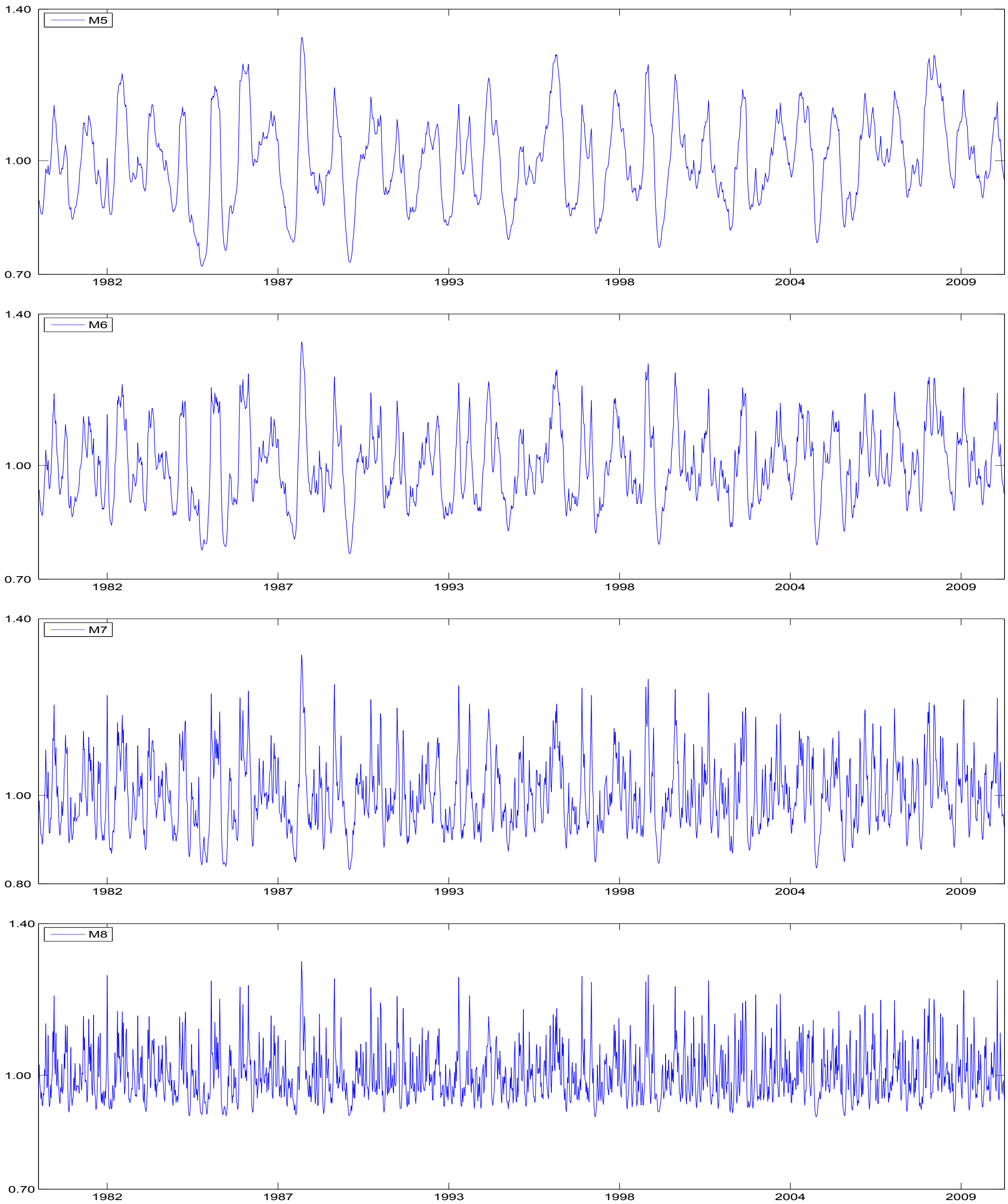
their responses to past shocks. The run-ups in agricultural commodity prices between 1994 and 1996 are associated with switches in all frequency volatility components.

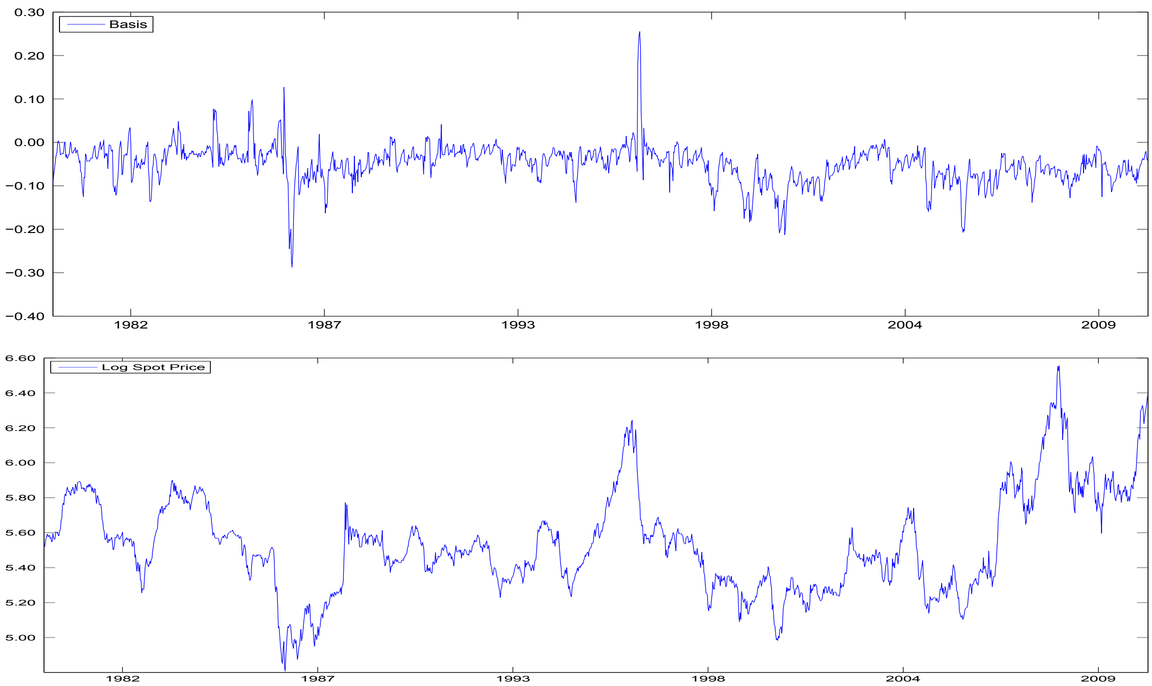
This price hike is due to a combination factors that include the burgeoning food demand in emerging economies, increase in production cost of agricultural commodities due to a rise in energy prices, increased demand for corn for bio-energy, decrease in the value of U.S. dollar and production shortfall due to extreme weather conditions. The impact of the 2008 financial crisis is also reflected across all frequency components, with larger jumps in low and medium frequency components. Although not shown here for the sake of brevity, the impact of the 2008 financial crisis is reflected across all assets.

Second, looking at the dynamics of the volatility components in relation to the basis, we observe that periods of high volatility generally coincide with periods where there is substantial deviation of the basis away from zero (i.e wider spread between the

Figure 2.1: Smoothed Probability, Basis and Spot Price For Corn




M5
M6
M7
M8
1.40
1.00
0.70
0.80
1982
1987
1993
1998
2004
2009



Notes: This figure presents the smoothed probability of being in the high state ($\Psi_t(M_k = m_0)$) for each MSM volatility component for corn returns. The figure illustrates the dynamics of the volatility components and how they respond to different price shocks.

spot and the futures prices). Although not reported in the table, we find that the correlation between the spot price volatility and the absolute values of the basis for corn is 0.4183, indicating a positive relationship between volatility and the magnitude of the basis. This is consistent with evidence from earlier studies such as Lee (1994), Choudhry (1997) and Alizadeh et al. (2008). Finally, we also observe that high volatility regimes of M_1 to M_3 (i.e medium and high frequency components) typically span periods of rising as well as falling spot prices.

2.4.4. Dynamic Hedge Ratios

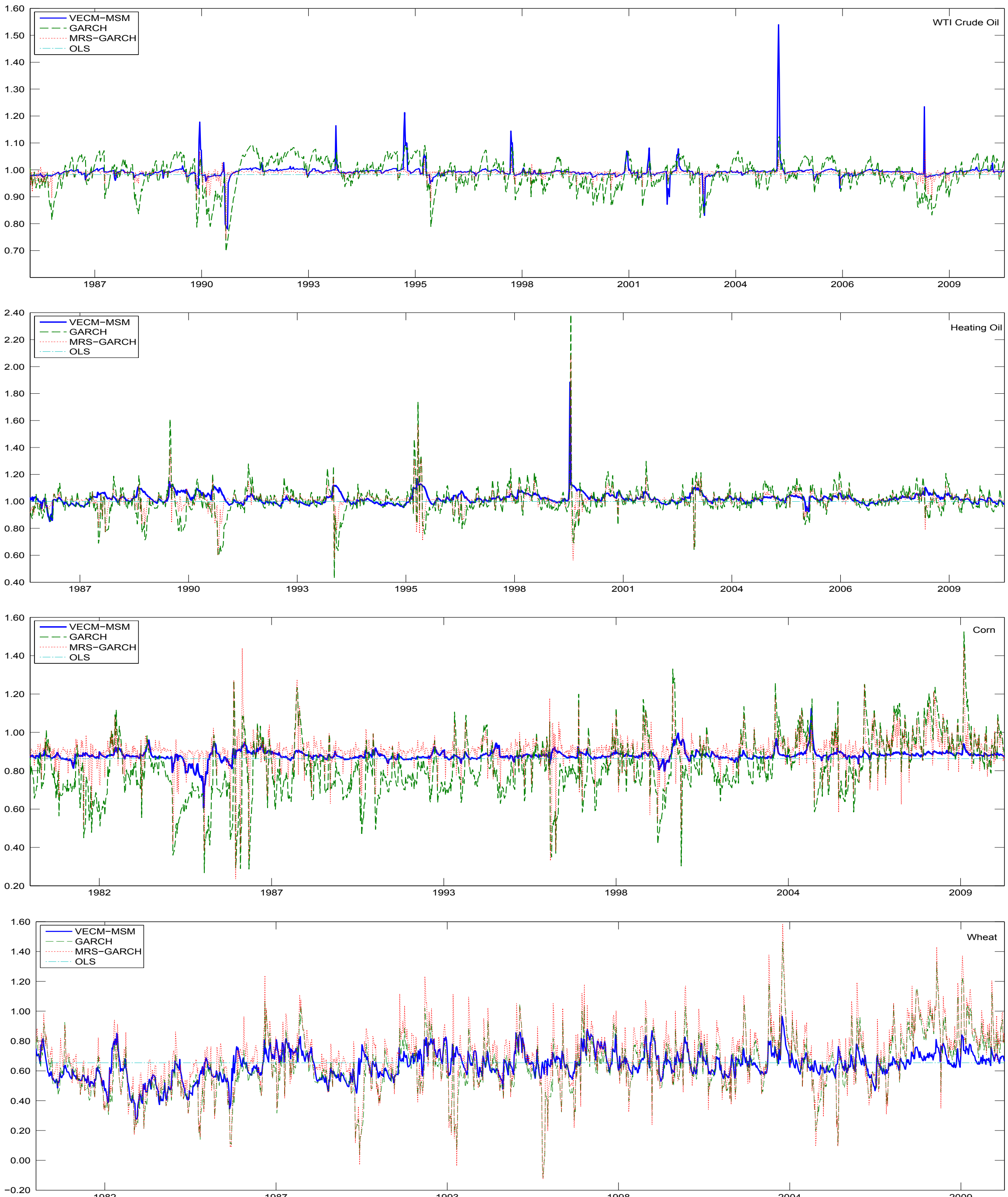
We examine hedge ratios under alternative hedging strategies. We consider (i) the conventional hedge ratio obtained as the slope coefficient from an OLS regression of spot returns on futures returns; (ii) a naive hedge ratio of one; (iii) the GARCH dynamic hedge ratios obtained from the diagonal BEKK specification[17] of Engle and Kroner (1995); (iv) dynamic hedge ratios obtained from the MRS-GARCH; and (v) dynamic hedge ratios obtained from the VECM-MSM(8) specification. Following the estimation of the VECM-MSM and MRS-GARCH parameters, we estimate the smoothed regime probabilities by applying the Kim (1994) algorithm to the filtered conditional densities. The smoothed regime probabilities are then used to calculate the state dependent variance-covariance matrix.

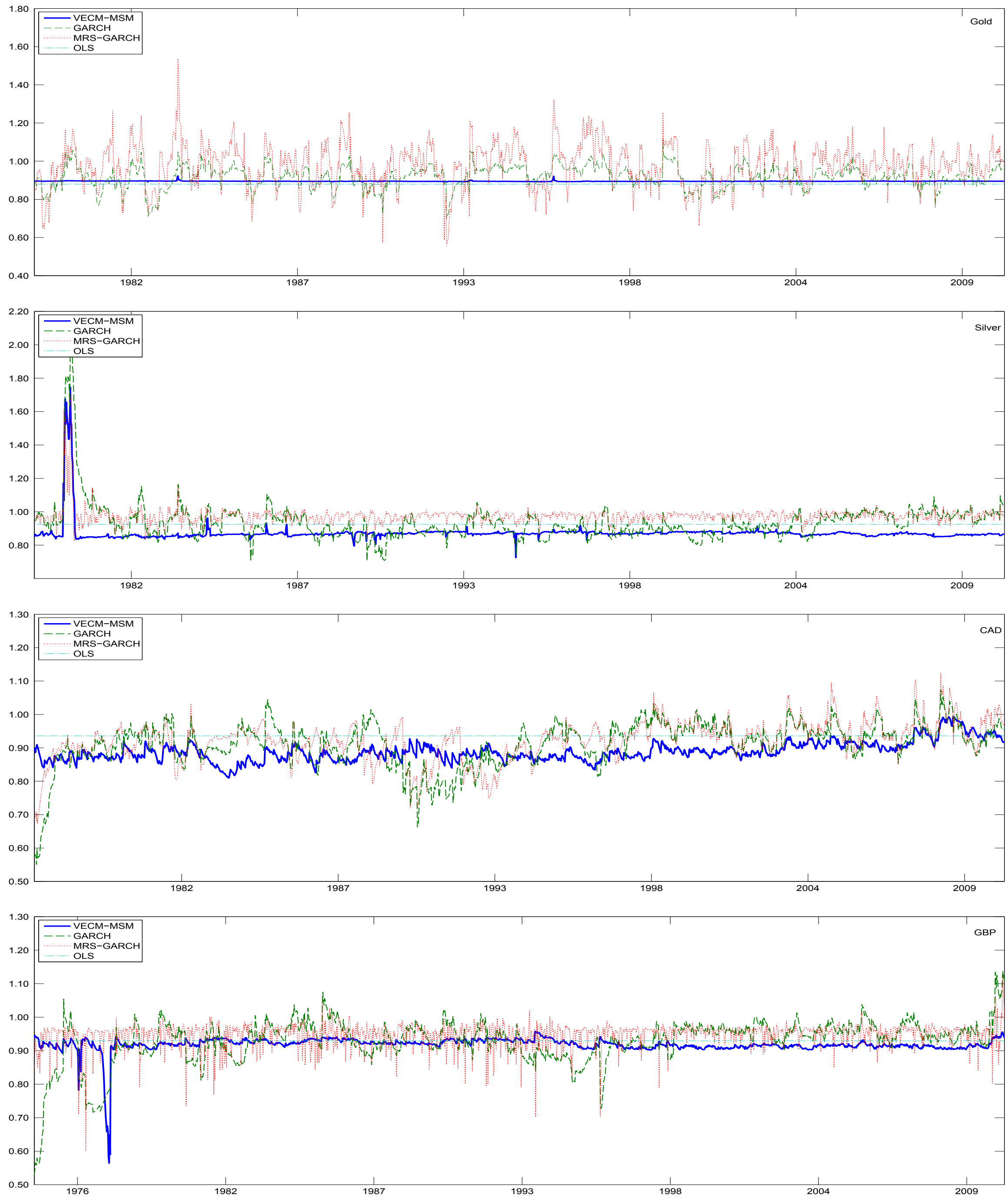
The VECM-MSM dynamic hedge ratios, along with the hedge ratios from other models are plotted for all assets and are depicted in Figure 2.2. The rich dynamics of the MSM hedge ratios clearly reject the appropriateness of the constant hedge ratios. More importantly the variation in the MSM hedge ratios also indicates the need for frequent updating of the portfolio of spot and futures positions in response to volatility movements in the commodity markets. Several other points are also worth noting from the plots. First, we observe that the hedge ratios tend to be at extreme ends the further away the basis is from zero. These periods generally coincide with periods of extreme market volatility, as noted earlier.

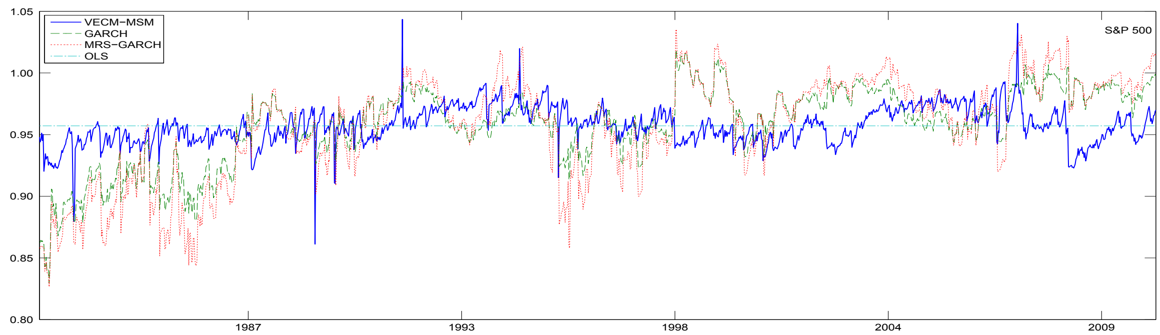
Second, the impact of past extreme shocks can also be observed on the dynamics of the hedge ratios. For example, the impact of the extreme market volatility following the 1990-1991 Gulf war can be seen in the hedge ratios for crude oil and heating oil. This period is characterized by extreme backwardation in which the spot price

[17]The BEKK GARCH is used in place of the constant correlation GARCH, in response to a suggestion from the supervisory committee.

Figure 2.2: VECM-MSM, MRS-GARCH, GARCH and OLS Hedge Ratios




VECM−MSM
GARCH
MRS−GARCH
OLS
Gold
Silver
CAD
GBP
1976
1982
1987
1993
1998
2004
2009



deviates significantly from the futures price. This leads to an increase in hedge ratios in order to minimize risk exposure from such extreme market movements. The impact of the extreme market movements of 1994-1996 and the commodity boom of early 2000s is reflected across all commodities. Likewise, the impact of the financial crises of 2008 following the collapse of Lehman Brothers is very noticeable in the stock market and the crude oil market. The hedge ratios of crude oil and S&P 500 rose substantially around this period, responding to increased volatility in the markets.

2.4.5. Hedging Effectiveness

Ederington (1979) proposed an hedging effectiveness measure that is used to measure the risk reduction effect of a minimum variance hedging strategy. This measure is defined as the percentage reduction in the variance of a hedged portfolio returns, relative to the unhedged portfolio returns. The variance reduction metric is defined as:

$$VR = 1 - \left[\frac{\sigma_{p,a}}{\sigma_{p,b}}\right] \tag{2.38}$$

$VR = 1$ implies a 100% reduction in variance while $VR = 0$ implies no reduction. Therefore, abstracting from the cost of implementing complex hedging strategies, the best hedging strategy is the strategy with the smallest portfolio variance. This unconditional variance has been adopted as a benchmark for hedging performance and as a criterion to compare and select the best among competing hedging strategies. There are, however, several problems with the use of unconditional variance as a measure of comparing hedging strategies. First, portfolio returns variance attaches equal weights to both positive and negative returns. But this assumption is not likely to hold in practice because agents care more about downside risk, which assumes that returns below a certain threshold involve risk while returns above such threshold is perceived as better investment opportunities (Grootveld and Hallerbach, 1999; Unser, 2000). Furthermore, Lien and Tse (2002) argue that a one-sided risk measure is more representative of an agent's risk perception in the context of hedging, than the traditional variance measure. Second, Lien (2006) demonstrates analytically that the variance method produces a downward biased estimator of the true hedging effectiveness. Lien (2007) further quantifies the magnitude of the bias and finds that, although the bias is small, the variance of the estimator is sometimes too large to be reliable.

Third, the unconditional variance is a criterion designed specifically to determine the hedging effectiveness of the OLS minimum variance hedge ratio by Ederington (1979), and does not apply to other hedge ratios. Therefore, it is inappropriate for comparing other hedge ratios against the OLS hedge ratios. Lien (2005) demonstrates that a strict application of the unconditional variance as a criterion for comparing hedging strategies almost always leads to an incorrect conclusion that the OLS hedging strategy outperforms other strategies. The study further demonstrates that the exception occurs only when the number of observations in the estimation sample or the out-of-sample data is small, or when there is a structural change between the in-sample period and the out-of-sample period. Lastly, using an unconditional performance measure to assess the performance of a dynamic hedging strategy that minimizes the conditional portfolio variance is inadequate and theoretically incoherent.

Therefore, several other alternative performance measures have been applied in the literature. These include unconditional semi-variance, lower partial moments (Alizadeh et al., 2008; Cotter and Hanly, 2006), certainty equivalence (Lien and Lee, 2012), value at risk (VaR) and conditional value at risk (CVaR) (Alizadeh et al., 2008; Cotter and Hanly, 2006). However, the unconditional semi-variance and lower partial moment suffer from the same criticism as the unconditional variance measure. Likewise, the unconditional VaR and the CVaR employed in the literature so far are also computed using unconditional variance (Alizadeh et al., 2008; Cotter and Hanly, 2006), thereby inheriting some of the drawbacks of the unconditional variance measure. Moreover, the earlier studies that employed the VaR criterion simply ranked models by the size of their VaR. Such an application of VaR is grossly inaccurate. A model may have a small portfolio VaR, but may also produce portfolio returns that continuously violate such VaR. Such models will be considered inadequate for risk management because they fail to accurately protect investors from downside risks. Lastly, while the certainty equivalence measure does not necessarily favor the OLS, Lien and Lee (2012) shows that, similarly to the variance method, the certainty equivalent measure is biased.

In the light of all of these, this study adopts a different approach for evaluating hedging strategies, therefore, making a significant contribution to the literature. Since hedging itself is a financial risk management tool, we employ the conditional VaR,[18] which is a risk management loss function that is widely accepted as a gold standard among financial practitioners, and constitutes an important tool within several financial regulatory compliance frameworks. No other studies in the literature, to the

[18]The use of VaR in this study is different from the way it has been applied in the hedging literature. The literature so far has simply ranked models based on the size of their unconditional VaR, thereby favoring the model with the smallest VaR. This approach is rather inadequate as shall be argued later.

best of our knowledge, have employed the conditional VaR as a measure for hedging effectiveness. A portfolio VaR quantifies, in monetary terms, the maximum expected loss (or worst case scenario) on the hedged portfolio, over a given time period, t, and given a specified degree of confidence, α. Recall from Equation (2.3) that the return on the hedged portfolio is R_h. Then the VaR measure at time t, of model g with confidence level, $\alpha\%$, is defined as the conditional quantile,$F^g_{t|t-1}(\alpha)$, where

$$F^g_{t|t-1}(\alpha)|\Omega_{t-1} = \mu^g + \Phi^{-1}(\alpha)\sigma^g_t \tag{2.39}$$

where $\Phi(.)$ is a cumulative distribution function and σ^g_t is the conditional portfolio volatility estimate for model g. With $\alpha = 0.05$, $\Phi^{-1}(\alpha) = -1.64$.[19] For example if $F_{t|t-1}(0.05) = \$10$, it means with 95% confidence level, the maximum expected loss on the portfolio value is \$10. Also, note that the conditionality of the VaR is very important here. Unlike earlier studies in the futures hedging literature, the VaR here is time varying, and is dependent on the available information in the previous period.

When the value of a hedged portfolio drops below the VaR level, a failure is said to occur. The accuracy of a hedging strategy can be easily verified by recording the failure rate of its VaR. If the number of empirical violations significantly exceeds the predicted failure rate, a model is said to be inadequate because the predicted VaR is too low and the available capital would not be sufficient to cover future losses. For example, with a 95% confidence level, if the number of times a model violates its VaR is more than 5% of the sample, then the model is considered inadequate. Conversely, if the number of empirical violations is significantly lower than the predicted failure rate, the model is equally considered inadequate because the estimated VaR is too

[19]For the VaR purpose, it is assumed that returns follow a normal distribution. The normality assumption simplifies the VaR calculations because all percentiles are assumed to be known multiples of the standard deviation. Thus, the VaR calculation requires only an estimate of the standard deviation of the portfolios change in value over the holding period.

conservative (too high), which leads to unprofitable tie down of capital. Therefore, a VaR that is either too high or too low is equally detrimental to an investor. A model is deemed adequate if its proportion of violations is close to the nominal value of $\alpha\%$.

But how close enough should the violation rate be to the nominal value? To answer this question, Kupiec (1995) introduced a simple likelihood ratio test. Unconditionally, a VaR is said to be efficient if $\mathbb{E}(I_t) = \alpha$, where $I_t = 1$ when there is a failure, and 0 otherwise. In other words, a VaR is said to be efficient if the proportion of portfolio returns less than the VaR is equal to α. Under the assumption that the failure rate is *iid* and binomially distributed, the likelihood ratio test of unconditional coverage is

$$LR_{uc} = -2\ln\left[\frac{\alpha^{n1}(1-\alpha)^{n0}}{\hat{\alpha}^{n1}(1-\hat{\alpha})^{n0}}\right] \sim \chi^2(1) \tag{2.40}$$

where α is the nominal failure rate chosen (usually 1% or 5%), $\hat{\alpha} = n1/(n0+n1)$ is the maximum likelihood estimate of α, $n1$ is the number of 1's in I_t and $n0$ is the number of 0's in I_t. The null and alternative hypotheses are

$$\begin{aligned} &H_0 : \mathbb{E}(I_t) = \alpha \\ &H_1 : \mathbb{E}(I_t) \neq \alpha \end{aligned} \tag{2.41}$$

Therefore, when the null hypothesis is rejected, a model is deemed to be inadequate. The null hypothesis will be rejected either when the empirical violation rate is too high, or too low compared to the nominal violation rate.

Christoffersen (1998) however suggests that a correctly specified VaR model should generate pre-specified violation rates conditionally at every point in time, a property referred to as conditional coverage. The author contends that given the well documented clustering of volatility in financial markets, a VaR forecast should be an interval forecast, rather than a point forecast. The interval forecast should be wide in volatile periods and narrow in tranquil periods. Also, the VaR violations should be spread independently over the entire sample rather than clustered. The author, thus, presents a framework that combines the test of the independence of violations with the test conditional coverage. Under the null hypothesis of an independent violation process with violation probability α, against the alternative of a first order Markov violation process, the likelihood ratio statistic is

$$LR_{cc} = -2 \ln \left[\frac{\alpha^{n1}(1-\alpha)^{n0}}{(1-\hat{\pi}_{01})^{n00} \;\; \hat{\pi}_{01}^{n01} \;\; (1-\hat{\pi}_{11})^{n10} \;\; \hat{\pi}_{11}^{n11}} \right] \sim \chi^2(2) \tag{2.42}$$

where n_{ij} is the number of i values followed by a j value, for $i, j = 0, 1$. $\pi_{ij} = \mathbb{P}(I_t = i | I_{t-1} = j)$, $\hat{\pi}_{01} = n_{01}/(n_{01} + n_{00})$, $\hat{\pi}_{11} = n_{11}/(n_{10} + n_{11})$, and $\hat{\pi} = (n_{01} + n_{11})/(n_{01} + n_{00} + n_{10} + n_{11})$. A model is deemed to fail the test of conditional VaR coverage if the null hypothesis is rejected. Note that the test of conditional coverage is stronger than the test of unconditional coverage, in that the former encompasses the latter. A model may pass the test of unconditional coverage, but fail the test of conditional coverage. In other words, even if a model records VaR violations that are close to the nominal violation rate α, the model may be deemed inadequate if such violations occur in clusters.

To assess the the performance of the different hedging strategies considered in this study, we employ the two VaR based tests discussed above. Specifically, we consider the 99% and the 95% VaR and perform the likelihood ratio tests outlined in Equations (2.40) and (2.42) for each asset. Recall from Equation (2.4) that the portfolio variance is given by

$$\sigma_{h,t}^2 = \sigma_{s,t}^2 + \hat{h}^2 \sigma_{f,t}^2 - 2\hat{h}\sigma_{sf,t} \tag{2.43}$$

Table 2.5: In-Sample Hedging Effectiveness of Alternative Models

Model	WTI	Heating	Corn	Wheat	Gold	Silver	CAD	GBP	S&P 500
Unconditional Coverage of 99% Value-at-Risk Assessment									
VECM - MSM									
PF	0.0414	0.0101	0.0099	0.0062	0.0246	0.0228	0.0097	0.0076	0.0194
LR_{uc}	72.9220	0.0028	0.0014	2.7369	25.4379	20.1870	0.0140	1.2071	10.4446
	(**0.0000**)	(0.9576)	(0.9700)	(0.0981)	(**0.0000**)	(**0.0000**)	(0.9058)	(0.2719)	(**0.0012**)
MRS - GARCH									
PF	0.0622	0.1460	0.0663	0.1133	0.1300	0.0959	0.0252	0.0730	0.0668
LR_{uc}	163.712	679.373	228.2174	572.840	737.957	449.502	28.573	311.199	214.857
	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)
GARCH									
PF	0.0353	0.0125	0.0266	0.0180	0.0377	0.0312	0.0206	0.0184	0.0388
RL_{uc}	50.9575	0.7436	30.9715	8.3553	76.0592	48.3291	15.1546	10.5157	72.3704
	(**0.0000**)	(0.3885)	(**0.0000**)	(**0.0038**)	(**0.0000**)	(**0.0000**)	(**0.0001**)	(**0.0012**)	(**0.0000**)
OLS									
PF	0.0215	0.0164	0.0254	0.0192	0.0168	0.0108	0.0132	0.0162	0.0307
LR_u	13.0716	4.4335	27.0837	10.8665	6.4317	0.1013	1.5953	6.0780	41.9152
	(**0.0003**)	(**0.0352**)	(**0.0000**)	(**0.0010**)	(**0.0112**)	(0.7503)	(0.2066)	(**0.0137**)	(**0.0000**)
Naive									
PF	0.0223	0.0164	0.0229	0.0142	0.0204	0.0102	0.0126	0.0157	0.0321
LR_{uc}	14.6609	4.4335	19.9189	2.5938	13.9474	0.0058	1.0859	5.1328	46.5814
	(**0.0001**)	(**0.0352**)	(**0.0000**)	(0.1073)	(**0.0002**)	(0.9394)	(0.2974)	(**0.0235**)	(**0.0000**)
Conditional Coverage of 99% Value-at-Risk Assessment									
VECM - MSM									
LR_c	72.9220	2.4367	0.0014	2.7369	25.4379	20.1870	0.0140	1.2071	10.4446
	(**0.0000**)	(0.2957)	(0.9993)	(0.2545)	(**0.0000**)	(**0.0000**)	(0.9930)	(0.5469)	(**0.0054**)
MRS - GARCH									
LR_c	163.712	695.513	228.4201	806.690	754.974	463.270	28.573	325.610	214.857
	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)
GARCH									
LR_c	50.9575	0.7436	30.9919	8.3553	76.0592	48.3291	15.1546	10.5157	72.3704
	(**0.0000**)	(0.6895)	(**0.0000**)	(**0.0153**)	(**0.0000**)	(**0.0000**)	(**0.0005**)	(**0.0052**)	(**0.0000**)
OLS									
LR_c	13.0716	24.1669	32.3867	16.1545	6.9024	6.1982	2.6347	6.0780	41.9152
	(**0.0015**)	(**0.0000**)	(**0.0000**)	(**0.0003**)	(**0.0317**)	(0.0451)	(0.2678)	(0.0479)	(**0.0000**)
Naive									
L_c	14.6609	24.1669	26.5955	6.7386	14.0726	1.9412	1.0859	5.1328	46.5814
	(**0.0007**)	(**0.0000**)	(**0.0000**)	(**0.0344**)	(**0.0009**)	(0.3788)	(0.5810)	(0.0768)	(**0.0000**)
Unconditional Coverage of 95% Value-at-Risk Assessment									
VECM - MSM									
PF	0.0767	0.0336	0.0502	0.0279	0.0803	0.0665	0.0417	0.0373	0.0515
LR_{uc}	16.9861	8.1941	0.0008	19.7040	27.4517	8.7142	2.6566	6.8647	0.0675
	(**0.0000**)	(**0.0042**)	(0.9772)	(**0.0000**)	(**0.0000**)	(**0.0032**)	(0.1031)	(**0.0088**)	(0.7950)
MRS - GARCH									
PF	0.0913	0.2217	0.1034	0.1684	0.2019	0.1642	0.0635	0.1449	0.1103
LR_{uc}	38.0560	448.420	75.1385	303.055	476.592	294.283	6.1742	237.325	86.5220
	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0130**)	(**0.0000**)	(**0.0000**)

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Table 2.5 – Continued from previous page

Model	WTI	Heating	Corn	Wheat	Gold	Silver	CAD	GBP	S&P 500
GARCH									
PF	0.0560	0.0406	0.0563	0.0440	0.0851	0.0695	0.0560	0.0459	0.0635
LR_{uc}	0.9599	2.5432	1.3179	1.2895	36.0578	11.9803	1.2916	0.6572	5.3097
	(0.3272)	(0.1108)	(0.2510)	(0.2561)	(**0.0000**)	(**0.0005**)	(0.2558)	(0.4175)	(**0.0212**)
OLS									
PF	0.0368	0.0242	0.0477	0.0402	0.0336	0.0204	0.0366	0.0335	0.0575
LR_{uc}	5.2094	21.9976	0.1861	3.4549	10.6962	39.3701	7.2698	11.9171	1.6876
	(**0.0225**)	(**0.0000**)	(0.6662)	(0.0631)	(**0.0011**)	(**0.0000**)	(**0.0070**)	(**0.0006**)	(0.1939)
Naive									
PF	0.0376	0.0242	0.0502	0.0458	0.0359	0.0234	0.0349	0.0330	0.0608
LR_{uc}	4.5923	21.9976	0.0008	0.6103	7.6568	30.8011	9.3753	12.7684	3.4649
	(**0.0321**)	(**0.0000**)	(0.9772)	(0.4347)	(**0.0057**)	(**0.0000**)	(**0.0022**)	(**0.0004**)	(0.0627)
Conditional Coverage of 95% Value-at-Risk Assessment									
VECM - MSM									
LR_c	16.9861	11.5577	0.9036	19.7626	36.3170	18.2880	4.6459	8.1985	0.0675
	(**0.0002**)	(**0.0031**)	(0.6365)	(**0.0001**)	(**0.0000**)	(**0.0001**)	(0.0980)	(**0.0166**)	(0.9668)
MRS-GARCH									
LR_c	54.8782	476.660	75.9555	305.072	522.389	345.477	9.3867	264.102	119.278
	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0000**)	(**0.0092**)	(**0.0000**)	(**0.0000**)
GARCH									
LR_c	0.9599	3.3228	1.4757	1.2949	38.9983	22.8796	7.2150	1.8844	5.3097
	(0.6188)	(0.1899)	(0.4781)	(0.5234)	(**0.0000**)	(**0.0000**)	(**0.0271**)	(0.3898)	(0.0703)
OLS									
LR_c	5.2094	38.4000	8.7138	7.0251	11.2227	41.1005	8.3259	11.9203	4.1775
	(0.0739)	(**0.0000**)	(**0.0128**)	(**0.0298**)	(**0.0037**)	(**0.0000**)	(**0.0156**)	(**0.0026**)	(0.1238)
Naive									
LR_c	4.5923	38.4000	5.1011	2.4367	7.6695	31.8300	9.3838	13.4297	6.7671
	(0.1006)	(**0.0000**)	(0.0780)	(0.2957)	(**0.0216**)	(**0.0000**)	(**0.0092**)	(**0.0012**)	(**0.0339**)
Unconditional Portfolio Variance									
VECM - MSM									
	2.4576	4.7494	4.0366	4.8498	1.3400	5.0886	0.0800	0.1904	0.1408
MRS-GARCH									
	2.6027	6.3678	4.4911	5.1834	1.4924	5.6221	0.0835	0.1940	0.1415
GARCH									
	2.7155	7.0320	4.4762	5.0352	1.3818	6.6002	0.0824	0.1918	0.1394
OLS									
	2.5989	5.0831	4.2110	5.5480	1.3395	4.8494	0.0810	0.1904	0.1423
Naive									
	2.6076	5.0831	4.4593	7.4393	1.4441	4.9607	0.0846	0.2010	0.1527

Notes: This table displays the frequency of in-sample hedged portfolio returns that exceed the 1-day VaR forecasted by the alternative hedging strategies. The VECM-MSM specification uses $\overline{k} = 8$ components. For quantile α%, PF reports the frequency of hedged portfolio returns below quantile α predicted by the model. That is PF is the proportion of $R^g_{h,t+1}, R^g_{h,t+2}, ...R^g_{h,t+n} < F^g_{t|t-1}(\alpha)|\Omega_{t-1}$, for model g. LR_{uc} and LR_c report the likelihood ratios of the test of unconditional and conditional coverage, respectively. The values in parentheses are the corresponding p-values. If the VaR forecast is correct, the observed failure rate PF should not be statistically different from the predicted failure rate α, where $\alpha = 1\%$ and 5% for 99% and 95% VaR, respectively. A boldface number indicates a rejection of the null hypothesis at the 5% significance level, that $PF = \alpha$. Note that $PF = \mathbb{E}(I_t)$.

For the conditional models (VECM-MSM, MRS-GARCH and GARCH), we compute Equation (2.43) using their in-sample conditional volatility estimates. For out-of-sample analysis, conditional volatility forecasts are used instead. For the OLS and the naive model, we use the unconditional sample variance of spot and futures returns. For out-of-sample analysis, the unconditional variances of spot and futures returns (hence portfolio variance) are updated for each week after adding the new observation. Once the portfolio variance is calculated for each model, this can then be substituted into Equation (2.39) to derive the portfolio VaR. Although, we continue to report the unconditional portfolio variance for the sake of completeness, we do not rank models based on the unconditional variance criterion. The results for the hedging effectiveness assessment are reported in Table 2.5

Two conclusions can be drawn from Table 2.5. First, the VECM-MSM hedging strategy is more conservative in terms of futures risk hedging, because it tends to fail less. This is indicated by the proportion of hedge portfolio returns that were below the 99% VaR level, under the VECM-MSM strategy. The average failure rate across the 9 portfolios is 0.0169 for the VECM-MSM model. This is smaller than the average failure rate for the other strategies: 0.0865 for MRS-GARCH, 0.0266 for GARCH, 0.0189 for OLS and 0.0185 for the naive strategy. Second, the VECM-MSM hedging strategy has a more accurate VaR prediction, compared to other strategies. For example, when the predicted failure rate is 1%, actual portfolio losses exceed the VECM-MSM VaR forecast more than 1% of the time for 4 (WTI, gold, silver and S&P 500) out of 9 portfolios. The numbers are 9, 8, 7 and 6 portfolios for the MRS-GARCH, GARCH, OLS and the naive hedging strategies, respectively.

As discussed earlier, an excessively conservative model does not necessarily lead to superior risk management. It rather leads to sub-optimal capital allocation to investments. For each portfolio, we test the null hypothesis that the empirical failure rate, PF, is equal to the expected failure rate, α. The likelihood ratio test statistics for both the unconditional coverage and conditional coverage are also reported in Table 2.5. For the VECM-MSM model, the failure rates are statistically different from the 1% prediction for 4 out of the 9 hedged portfolios, both unconditionally and conditionally. The numbers are 9, 8, 7 and 6 portfolios for the MRS-GARCH, GARCH, OLS and the naive hedging strategies, respectively, for unconditional coverage. The numbers are 9, 8, 6 and 6 portfolios for the MRS-GARCH, GARCH, OLS and the naive hedging strategies, respectively, for conditional coverage . The statistical tests therefore suggest that the VECM-MSM hedging strategy is not overly conservative. In other words, investors who care about downside risks will be better off adopting the VECM-MSM hedging strategy because it provides better hedging against downside risks.

Looking at the 95% VaR however, the results are slightly different. It can be observed that the GARCH portfolio performs better, in that it records failure rates that are closest to the predicted failure rates. The GARCH hedge portfolio failure rates are statistically different from the 5% prediction for only 3 out of the 9 hedged portfolios, both unconditionally and conditionally. The corresponding value for the VECM-MSM is 6 out of 9 hedged portfolios. The average failure rates for the OLS and the naive strategies are 0.0367 and 0.0384, respectively, for the 95% VaR. These indicate that these strategies are too conservative. This is further corroborated by the likelihood ratio tests. The OLS and naive hedge portfolios failure rates are statistically different from the 5% prediction for 6 out of the 9 hedged portfolios, both unconditionally and conditionally. The MRS-GARCH hedged strategy is the least performing model in that it records VaR failure rates that are too large to be reliable, both at the 99% level and 99% level, and for all assets.

Lastly, the unconditional portfolio variances are also reported in Table 2.5. It can be observed that the VECM-MSM records the least portfolio variance in 5 out of 9 portfolios, while the OLS strategy produces the least portfolio variance in 2 out of 9 cases. But this is limited to the in-sample analysis.

The in-sample analysis of the various hedging strategies is only indicative of historical performances. However, investors are more concerned with how well they can do in the future using different hedging strategies. Consequently, we conduct an out-of-sample hedging analysis using data from January 2011 to December 2012. For the OLS hedge ratios, we re-estimate the regression model for each out-of-sample week after adding the new observation.

In the case of the GARCH based model, the BEKK-GARCH model is re-estimated each week and the optimal hedge ratios are computed from the one-week ahead forecast of the variance-covariance matrix. For the MRS-GARCH and the VECM-MSM models, the models are estimated each week after adding the new observation. The parameters are used to back out the conditional filtered probabilities Π_t and the transition matrix P for time t. Using, the filtered probabilities and the transition matrix, we compute a one-week ahead forecast of the regime probabilities at time $t+1$ as $\Pi_{t+1} = \Pi_t P$. The forecasted regime probabilities are then used to compute the one-week ahead forecast of the variance-covariance matrix, from which the dynamic hedge ratios are computed. The process is repeated every week with new observation added to the data set. The out-of-sample results are reported in Table 2.6.

Table 2.6: Out-of-Sample Hedging Effectiveness of Alternative Models

Model	WTI	Heating	Corn	Wheat	Gold	Silver	CAD	GBP	S&P 500
Unconditional Coverage of 99% Value-at-Risk Assessment									
VECM-MSM									
PF	0.0098	0.0000	0.0000	0.0097	0.0291	0.0097	0.0098	0.0000	0.0097
LR_{uc}	0.0004	2.0704	2.0704	0.0009	2.5126	0.0009	0.0004	2.0503	0.0009
	(0.9841)	(0.1502)	(0.1502)	(0.9762)	(0.1129)	(0.9762)	(0.9841)	(0.1522)	(0.9762)

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Table 2.6 – Continued from previous page

Model	WTI	Heating	Corn	Wheat	Gold	Silver	CAD	GBP	S&P 500
MRS-GARCH									
PF	0.0000	0.0000	0.0000	0.0194	0.0291	0.0194	0.0294	0.0000	0.0194
LR_{uc}	2.0503	2.0704	2.0704	0.7236	2.5126	0.7236	2.5519	2.0503	0.7236
	(0.1522)	(0.1502)	(0.1502)	(0.3950)	(0.1129)	(0.3950)	(0.1102)	(0.1522)	(0.3950)
GARCH									
PF	0.0000	0.0000	0.0097	0.0097	0.0388	0.0388	0.0392	0.0000	0.0097
LR_{uc}	2.0503	2.0704	0.0009	0.0009	5.0012	5.0012	5.0608	2.0503	0.0009
	(0.1522)	(0.1502)	(0.9762)	(0.9762)	(**0.0253**)	(**0.0253**)	(**0.0245**)	(0.1522)	(0.9762)
OLS									
PF	0.0000	0.0000	0.0000	0.0485	0.0194	0.0000	0.0392	0.0000	0.4175
LR_{uc}	2.0503	2.0704	2.0704	8.0154	0.7236	2.0704	5.0608	2.0503	257.281
	(0.1522)	(0.1502)	(0.1502)	(**0.0046**)	(0.3950)	(0.1502)	(**0.0245**)	(0.1522)	(**0.0000**)
Naive									
PF	0.0000	0.0000	0.0000	0.0291	0.0194	0.0000	0.0196	0.0000	0.0000
LR_{uc}	2.0503	2.0704	2.0704	2.5126	0.7236	2.0704	0.7429	2.0503	2.0704
	(0.1522)	(0.1502)	(0.1502)	(0.1129)	(0.3950)	(0.1502)	(0.3887)	(0.1522)	(0.1502)
Conditional Coverage of 99% Value-at-Risk Assessment									
VECM-MSM									
LR_c	0.0041	3.0704	3.0704	0.0046	2.5126	0.0046	0.0041	3.0503	0.0046
	(0.9980)	(0.2154)	(0.2154)	(0.9977)	(0.2847)	(0.9977)	(0.9980)	(0.2176)	(0.9977)
MRS-GARCH									
LR_c	3.0503	3.0704	3.0704	0.7237	2.5126	0.7237	2.5519	3.0503	0.7237
	(0.2176)	(0.2154)	(0.2154)	(0.6964)	(0.2847)	(0.6964)	(0.2792)	(0.2176)	(0.6964)
GARCH									
LR_c	3.0503	3.0704	0.0046	0.0046	5.0012	5.0012	5.0608	3.0503	0.0046
	(0.2176)	(0.2154)	(0.9977)	(0.9977)	(0.0820)	(0.0820)	(0.0796)	(0.2176)	(0.9977)
OLS									
LR_c	3.0503	3.0704	3.0704	9.5775	0.7237	3.0704	5.0608	3.0503	261.013
	(0.2176)	(0.2154)	(0.2154)	(**0.0083**)	(0.6964)	(0.2154)	(0.0796)	(0.2176)	(**0.0000**)
Naive									
LR_c	3.0503	3.0704	3.0704	2.5126	0.7237	3.0704	0.7430	3.0503	3.0704
	(0.2176)	(0.2154)	(0.2154)	(0.2847)	(0.6964)	(0.2154)	(0.6897)	(0.2176)	(0.2154)
Unconditional Coverage of 95% Value-at-Risk Assessment									
VECM-MSM									
PF	0.0196	0.0000	0.0291	0.0291	0.0680	0.0680	0.0490	0.0196	0.0291
LR_{uc}	2.5538	10.5664	1.1046	1.1046	0.6320	0.6320	0.0021	2.5538	1.1046
	(0.1100)	(**0.0012**)	(0.2933)	(0.2933)	(0.4266)	(0.4266)	(0.9637)	(0.1100)	(0.2933)
MRS-GARCH									
PF	0.0000	0.0000	0.0291	0.0583	0.0485	0.0583	0.0686	0.0098	0.0485
LR_{uc}	10.4638	10.5664	1.1046	0.1406	0.0046	0.1406	0.6709	5.1126	0.0046
	(**0.0012**)	(**0.0012**)	(0.2933)	(0.7077)	(0.9457)	(0.7077)	(0.4127)	(**0.0238**)	(0.9457)
GARCH									
PF	0.0098	0.0194	0.0194	0.0583	0.0583	0.0777	0.0980	0.0392	0.0291
LR_{uc}	5.1126	2.6169	2.6169	0.1406	0.1406	1.4309	3.9190	0.2689	1.1046
	(**0.0238**)	(0.1057)	(0.1057)	(0.7077)	(0.7077)	(0.2316)	(0.0477)	(0.6041)	(0.2933)
OLS									
PF	0.0000	0.0000	0.0097	0.0971	0.0291	0.0000	0.0882	0.0098	0.4175
LR_{uc}	10.4638	10.5664	5.1956	3.8162	1.1046	10.5664	2.5828	5.1126	123.819
	(**0.0012**)	(**0.0012**)	(**0.0226**)	(0.0508)	(0.2933)	(**0.0012**)	(0.1080)	(**0.0238**)	(**0.0000**)

Continued on next page

Table 2.6 – Continued from previous page

Model	WTI	Heating	Corn	Wheat	Gold	Silver	CAD	GBP	S&P 500
Naive									
PF	0.0000	0.0000	0.0291	0.0485	0.0291	0.0000	0.0490	0.0000	0.0194
LR_{uc}	10.4638	10.5664	1.1046	0.0046	1.1046	10.5664	0.0021	10.4638	2.6169
	(**0.0012**)	(**0.0012**)	(0.2933)	(0.9457)	(0.2933)	(**0.0012**)	(0.9637)	(**0.0012**)	(0.1057)
Conditional Coverage of 95% Value-at-Risk Assessment									
VECM-MSM									
LR_c	2.5538	11.5664	1.1046	1.1046	0.6320	0.6320	0.0021	2.5538	1.1046
	(0.2789)	(**0.0031**)	(0.5756)	(0.5756)	(0.7291)	(0.7291)	(0.9990)	(0.2789)	(0.5756)
MRS-GARCH									
LR_c	11.4638	11.5664	1.1046	0.1406	0.0046	0.1406	0.6709	5.1163	0.0046
	(**0.0032**)	(**0.0031**)	(0.5756)	(0.9321)	(0.9977)	(0.9321)	(0.7150)	(0.0774)	(0.9977)
GARCH									
L_c	5.1163	2.6170	2.6170	0.1406	0.1406	1.4309	3.9190	0.2689	1.1046
	(0.0774)	(0.2702)	(0.2702)	(0.9321)	(0.9321)	(0.4890)	(0.1409)	(0.8742)	(0.5756)
OLS									
LR_c	11.4638	11.5664	5.1992	3.8167	1.1046	11.5664	2.5828	5.1163	127.550
	(**0.0032**)	(**0.0031**)	(0.0743)	(0.1483)	(0.5756)	(**0.0031**)	(0.2749)	(0.0774)	(**0.0000**)
Naive									
LR_c	11.4638	11.5664	1.1046	0.0046	1.1046	11.5664	0.0021	11.4638	2.6170
	(**0.0032**)	(**0.0031**)	(0.5756)	(0.9977)	(0.5756)	(**0.0031**)	(0.9990)	(**0.0032**)	(0.2702)
Unconditional Portfolio Variance									
VECM-MSM									
	0.1344	0.2174	2.2698	7.0960	0.9763	1.4524	0.0918	0.0562	0.0608
MRS-GARCH									
	0.1526	0.2638	2.3485	6.9473	0.0576	1.4760	0.0895	0.0576	0.0508
GARCH									
	0.1817	0.2254	2.3056	7.0073	0.9983	1.5805	0.0885	0.0550	0.0532
OLS									
	0.1290	0.2163	2.3024	7.9108	0.9452	1.4393	0.0887	0.0560	0.0633
Naive									
	0.1261	0.2167	2.1967	7.2513	1.0646	1.5721	0.0860	0.0518	0.0469

Notes: This table displays the frequency of out-of-sample hedged portfolio returns that exceed the 1-day VaR forecasted by the alternative hedging strategies. The VECM-MSM specification uses $\overline{k} = 8$ components. For quantile α%, PF reports the frequency of hedged portfolio returns below quantile α predicted by the model. That is PF is the proportion of $R^g_{h,t+1}, R^g_{h,t+2}, ...R^g_{h,t+n} < F^g_{t|t-1}(\alpha)|\Omega_{t-1}$, for model g. LR_{uc} and LR_c report the likelihood ratios of the test of unconditional and conditional coverage, respectively. The values in parentheses are the corresponding p-values. If the VaR forecast is correct, the observed failure rate PF should not be statistically different from the predicted failure rate α, where $\alpha = 1\%$ and 5% for 99% and 95% VaR, respectively. A boldface number indicates a rejection of the null hypothesis at the 5% significance level, that $PF = \alpha$. Note that $PF = \mathbb{E}(I_t)$.

The out-of-sample analysis further corroborates the results in Table 2.5. First, the VECM-MSM hedging strategy is more conservative in in that it tends to fail less, with an average failure rate of 0.0087 for the 99% VaR. This is smaller than the average failure rate for the other strategies: 0.0130 for MRS-GARCH, 0.0162 for GARCH, 0.0583 for OLS and 0.0076 for the naive strategy. Second, the VECM-MSM hedging strategy has a more accurate VaR prediction, compared to other strategies. For example, when the predicted failure rate is 1%, actual VECM-MSM portfolio losses exceed the 99% VaR forecast for only 1 (gold) out of the 9 portfolios. The actual portfolio losses exceed the 99% VaR for 5, 3, 4 and 3 portfolios in the case of MRS-GARCH, GARCH, OLS and the naive strategies.

For each portfolio in the out-of-sample analysis, we also test the null hypothesis that the empirical failure rate, PF, is equal to the expected failure rate, α. For the VECM-MSM model, the null hypothesis cannot be rejected for all the 9 hedged portfolios, both unconditionally and conditionally. The same holds true for the MRS-GARCH and the naive hedge strategy, indicating that both strategies perform better in the out-of-sample analysis. However, they record higher average violation rates than the VECM-MSM model. On the other hand, the null hypothesis of unconditional coverage is rejected for the GARCH and the OLS strategies in 3 out of the 9 hedged portfolios. The statistical tests therefore suggest that the VECM-MSM hedging strategy is not overly conservative in the out-of-sample VaR forecast.

The results for the 95% VaR show that the naive hedging strategy records the lowest average violation rates at 0.0140, while the OLS strategy records the highest violation rate at 0.0724. Also, the naive hedged portfolio losses do not exceed the 95% VaR for any of the 9 portfolios. Testing the null hypothesis of unconditional and conditional coverages however, we find that naive strategy is overly conservative. The null hypotheses of unconditional and conditional coverages are rejected for 5 and 4 portfolios, respectively. For the VECM-MSM model, the actual portfolio losses exceed the 95% VaR for only 2 out of 9 hedged portfolios. The numbers are 3, 4 and 3 for the MRS-GARCH, GARCH and the OLS strategies. Also, the null hypothesis of unconditional coverage is rejected for only 1 VECM-MSM hedged portfolio. The same holds true for the GARCH model. But the null hypothesis is rejected for 3 and 6 portfolios for the MRS-GARCH and the OLS strategies.

Lastly, the out-of-sample unconditional portfolio variances are also reported in Table 2.6. It can be observed that the naive strategy dominates, with the least portfolio variance in 5 out of 9 portfolios. If the portfolio variance was used as a criterion for hedging evaluation, the naive model would certainly perform better than other models. But as discussed earlier on, this approach is inaccurate and theoretically incoherent for evaluating conditional models.

2.5. Conclusion

This paper applies the VECM-MSM model to the futures hedging decision problem of a mean-variance investor. We assume that spot and futures returns follow a VECM-MSM process and derive the dynamic optimal hedge ratios under a mean-variance framework. The VECM-MSM hedge model was applied to 9 assets from five different markets: energy, agricultural, metal, foreign exchange and stock markets. The VECM-MSM model seems to capture the data reasonably well. To evaluate hedging effectiveness, we adopt the VaR approach because it captures better the downside risk that investors care about, and it is also an important evaluation metric used by financial regulators.

In-sample and out-of-sample hedge effectiveness shows the VECM-MSM hedged portfolio outperforms alternative hedging strategies in terms of providing the best coverage against downside risks. The statistical tests also show that the VECM-MSM hedging strategy is conservative in terms of protecting portfolio returns against downside risk, but not overly conservative. In other words, investors who care about downside risks will be better off adopting the VECM-MSM hedging strategy because it provides better hedging against downside risks. Statistical tests of unconditional and conditional coverages also show that the VECM-MSM model better predicts an investors downside risk in that the VaR predictions are more accurate than the predictions from the alternative models.

There is, however, room for improvements in terms of advancing the VECM-MSM model. The proposed VECM-MSM model only considers regime switching in conditional volatility, not conditional mean. The model could be improved to allow for the speed of adjustment to long-run equilibrium to be dependent on the state of the economy. This is consistent with empirical evidence pointing to the regime dependent nature of the speed of adjustment between spot and futures prices (Baillie and Bollerslev, 2000; Baillie and Kilic, 2006; Beckmann and Czudaj, 2014; Maynard and Phillips, 2001). Non-linearity in the speed of adjustment of spot and futures prices arises as a result of high transaction costs, the role of noise traders and the existence of threshold carrying cost which makes investors indifferent between buying a spot commodity or a futures contract (Chen and Wuh Lin, 2004; Huang et al., 2009; Lin and Liang, 2010; Silvapulle and Moosa, 1999). Incorporating this phenomenon into the VECM-MSM model may yield substantial improvements in hedging performance.

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Chapter 3

Excess Commodity Comovement: A Multifrequency Approach

3.1. Introduction

One of the stylized facts of commodity prices is their tendency for co-movement. This observation was first investigated by Pindyck and Rotemberg (1990) (henceforth PR) who note that the prices of seemingly unrelated commodities tend to move together beyond what can be attributed to fundamentals such as demand and supply conditions, exchange rates, interest rates, industrial production etc. This phenomenon is termed excess commodity comovement. PR posit that such excess comovement may at least partially be explained by herding behaviour among speculators in commodity markets i.e many traders tend to long or short in all commodities without plausible reasons.

Excess commodity comovement bears significant welfare and risk management implications. For instance, a synchronous rise in prices of commodities exerts significant inflationary pressure on commodity import dependent countries, and limits their ability to maintain economic stability and resist inflationary pressures. Moreover, to the extent that comovement measures, such as correlation and covariance among commodities, comprise an essential ingredient in risk assessment, pricing, portfolio management and hedging, failure to account for such excess comovement can lead to sub-optimal economic decisions. Lastly, the presence of excess comovement also casts doubts on the efficient market hypothesis and competitive-storage models of commodity pricing. This would imply that agents act irrationally in markets that are supposedly competitive.

For these reasons, there has been a plethora of empirical studies directed towards the analysis of excess comovement in commodity markets, yielding mixed findings with regards to the presence or otherwise of excess commodity comovement.

Therefore within the the debate on excess commodity comovement, the aim of this chapter is twofold. First, it analyzes the degree of excess commodity comovement across a variety of commodities. Second, it analyzes the frequency-dependent nature of comovement across related (e.g. crude and heating oil) and unrelated commodities (e.g. copper and corn). To this end, we address the following questions:

i Is there excess comovement across commodities?

ii Is excess comovement a temporary or long term phenomenon, or a combination of both?

In order to address these questions, we employ the Markov-switching Multifractal (MSM) model of Calvet and Fisher (2001), Calvet and Fisher (2004) and Calvet et al. (2006). Therefore, this study makes a significant contribution to the commodity comovement literature, by being the first to analyse the problem using the MSM model. The MSM model was first applied by Calvet et al. (2006) in the analysis of exchange rate volatility comovement, where it was found that exchange rate volatility comovement is frequency-specific. In the MSM framework, volatility and covariance are scaled multiplicative products of several random and statistically independent volatility components with heterogeneous cycles. The MSM model is particularly well-suited for addressing the questions raised in this study because it is able to decompose pairwise comovement into several frequency-specific components. Decomposing comovements into frequency-specific components (e.g low, medium and high frequency) allows us to investigate which components of a pair of series comove together, and whether excess comovement is a long or short term phenomenon.

The second contribution of this study to the literature is that it employs daily data, rather than monthly, quarterly or yearly data that has been used in the previous studies. The use of higher frequency data makes it possible to capture more rapid cycles of propagation of shocks across seemingly unrelated commodities, an important element that has been missing from earlier studies and might have contributed to some failures to detect excess commodity comovement.

To anticipate the results of this study, we find that there is significant comovement between commodity prices, beyond what can simply be explained by macroeconomic fundamentals. Second, decomposing comovements into multiple frequencies, we find that all commodities exhibit long-run excess comovements which are driven by low frequency fundamentals such as weather, demographic and macroeconomic factors. But some commodities also exhibit significant short-run excess comovements that may be attributable to short-run factors such as liquidity constraints, indexation, etc. Third, the dynamic correlations show that excess comovements are higher in periods of high volatility and vice-versa.

The rest of this chapter is structured as follows. Section 2 reviews the literature on commodity comovement. Section 3 presents the research methodology and the comovement testing framework, while section 4 presents the data and the empirical results. Section 5 provides further robustness analyses and section 6 concludes.

3.2. Literature Review

The excess comovement of commodities as a phenomenon was first analysed by Pindyck and Rotemberg (1990) in their seminal paper "The Excess Comovement of Commodity Prices". They perform a formal analysis by regressing log returns of seemingly unrelated commodities (wheat, cocoa, lumber, cotton, crude oil, gold and copper) on a set of U.S. macroeconomic variables (industrial production, consumer price index, interest rate, exchange rate, money supply and stock index) over the period April 1960 to November 1985, and compute the simple pairwise correlation between the residuals. They find the residuals to be highly correlated, which suggests that commodity prices show a persistent tendency to move together beyond what can be explained by common macroeconomic shocks. The authors argue that such excess comovement may be due to herding behaviour among commodity traders. Following the work of Pindyck and Rotemberg (1990), however, several other studies have emerged that sometimes confirmed and sometimes refute such excess comovement phenomena.

Notably, Deb et al. (1996) suggest that the Pindyck and Rotemberg (1990) approach fails to control for structural breaks and conditional heteroscedasticity in commodity returns. In order to control for the time varying conditional distribution of commodity returns, they assume that commodity returns follow a Garch process. Using data from the same period as Pindyck and Rotemberg (1990) and applying several score and likehood ratio tests, they document only weak evidence of excess comovement in commodity prices.

Palaskas and Varangis (1991) test for excess comovement among 42 primary commodities, for the period 1959 to 1980. Adopting an Error-Correction Model and cointegration techniques, they document evidence of excess comovement among 9 annual commodity prices and 14 monthly commodity prices. They suggest, however, that the observed comovement may largely be a consequence of the non-normality of many of the commodity prices. They also observe that the power of macroeconomic variables to explain variations in commodity prices falls as the frequency of observations increases.

A potential drawback of the aforementioned studies is that they fail to consider the possibility that commodity prices may be driven by other fundamental factors beyond macroeconomic indicators. It is, in fact, counterintuitive to assume that most of the demand and supply shocks that impact commodity prices can be captured by a few macroeconomic indicators, as is assumed in the above studies. Besides, macroeconomic indicators are more likely to reflect demand side than supply side conditions (Ai et al., 2006).

In order to account for this shortcoming, some studies have approached the testing of excess commodity comovement by adopting a market oriented approach. Ai et al. (2006) use quarterly time series for five agricultural commodities (wheat, corn, barley, oats and soybeans) to test for excess comovement in a partial equilibrium setting. Using data on harvests and inventory levels to control for shared fundamental shocks, they find that comovements are at best marginal, and nonexistent in some cases. They therefore rule out the influence of speculation and conclude that supply factors explain the majority of the comovement observed in commodity prices.

Lescaroux (2009) focuses on the comovement between crude oil and other commodities. The author's tests of excess comovement considered are between crude oil and metals (aluminium, copper, lead, zinc, nickel and tin) for which inventory level data are available. Moreover, the study focuses on cyclical comovements by decomposing commodity prices into trend and cyclical components using the Hodrick and Prescott (1997) filter. The effects of fundamentals are further filtered out by regressing the cyclical components of commodities on the cyclical components of their respective inventory levels. The study documents significant evidence of long term comovement between oil and metals, with only rather weak evidence of short term excess comovement.

Applying an Error Correction Model to six agricultural commodity futures prices, Malliaris and Urrutia (1996) test for long-run and short-run comovement among these commodities. The study finds significant evidence of long-term relationships between the prices of agricultural commodities. But no evidence of short-term comovement was found. Leybourne et al. (1994) combine the Error Correction approach of Malliaris and Urrutia (1996) and the regression approach of Pindyck and Rotemberg (1990) in an analysis of fifteen agricultural and metal commodity prices. They found only weak evidence of comovement among four of the fifteen commodities. Cashin et al. (1999) test for excess comovement among the same seven seemingly unrelated commodities that were examined in Pindyck and Rotemberg (1990), using a concordance approach. The concordance statistic measures the extent of synchronicity between the cycles of two random variables. The application is used to measure the extent to which the prices of two commodities are concurrently in a boom or a slump. They found no significant evidence of comovement among the commodities, with the exception of gold and crude oil.

On the other hand, using nonstationary panel methods combined with a Factor Augmented VAR approach, Byrne et al. (2012) analyzed a set twenty-four commodities from 1900 to 2008, and document a statistically significant degree of excess comovement among the commodities. Similarly, Le Pen and Sévi (2010) employ approximate factor analysis to test for excess comovement in both returns and volatility for eight seemingly unrelated commodities. They document significant evidence of comovement in returns, but also conclude that no such evidence of excess comovement is present in volatilities.

Taken together, these studies suggest that the issue of whether there is excess comovement among commodities is at best an unsolved puzzle. There are however several reasons that suggests the presence of significant comovement among commodities beyond what can be explained by fundamentals. Some of these factors are, however, not as easily quantifiable as other macroeconomic factors.

One such factor is weather and other climatological factors. Given that world grain production is relatively concentrated in certain geographical areas, adverse weather conditions that lead to disruptions in supply from these areas will induce a simultaneous increase in prices of grains. According to the International Grain Council (IGC), Australia, Argentina, Canada, the EU, Kazakhstan, Russia, Ukraine and the US accounted for 52% of total world grain production in the 2010/2011 period (International Grains Council, 2012). The impact of adverse weather can be seen in the IGC's daily grains and oilseeds index which rose by 11% in June of 2012 in response to concerns over deteriorating yields prospects caused by overly hot and dry conditions in the US. Moreover, Stevens (1991) and Anderson and Danthine (1983) document evidence of weather persistence effects on grain contract prices.

A somewhat related factor that could be important in explaining commodity comovement relates to the profit maximizing behavior of farmers. Farmers, like other economic agents, seek to maximize profit by allocating their limited resources (e.g land, fertilizers, capital, and fuel etc) to planting the most profitable crops. Therefore, they respond to relative price movements in commodity markets. For example, because maize and soybeans can be grown on the same land, a relative increase in the price of Soybeans in the spring of 2006 led to an allocation of 3.5 million acres away from planted acreage of maize in favour of soybeans (Simone and Maria, 2008). This reallocation of crop land would be expected to put upward pressure on the prices of maize and such trade-offs between two commodities could be partly responsible for comovements in their prices.

Another important factor that might be a driver of commodity comovement is the impact of oil prices. This can happen through two different channels. First, an increase in oil prices will lead to an increase in the cost of production through higher costs of inputs such as fuels (used for heating, transportation and the operation of equipments), pesticides and fertilizers. Second, higher oil prices will lead to substitution away from oil to alternative sources of energy such as ethanol and biodiesel, which are produced from commodities such as maize and oilseeds. For example, the US Department of Agriculture reports that 33% and 26% of total US corn and grain production, respectively, were used in ethanol production in 2009. This undoubtedly exerts an upward pressure on the demand for grains and oilseed, which will in turn lead to higher prices across this commodity group. Using time series price data on several vegetable oils from 1999-2006, Yu et al. (2006) find significant evidence of long term relationships between different vegetable oils and crude oil prices. Likewise, Campiche et al. (2007) document evidence of increasing comovement between petroleum and agricultural commodity prices due to increasing diversion of agricultural products towards the production of ethanol.

Lastly, commodity price comovement can also be attributed to spreading and financialization of commodity markets. Spreading is a form of arbitrage-based trading strategy whereby traders buy and sell similar commodities in various proportions in order to take advantage of perceived mispricing across the commodities. Such a trading strategy not only serves as a mechanism for restoring price relationships between a group of commodities but also leads to increased comovement among the commodities. Second, a combination of the availability of deep and liquid exchange traded commodity derivatives, and recent findings from academic papers (Egelkraut et al., 2005; Erb and Harvey, 2006; Gorton and Rouwenhorst, 2006) pointing to substantial risk premium from commodity markets, has led to a significant upsurge in commodity index funds investments. This phenomenon is termed a "financialization" of commodity markets by Domanski and Heath (2007) and has been blamed as a major driver of the 2007/2008 commodity market boom (Baffes and Haniotis, 2010). The financialization of commodity markets has also been linked to increased commodity price comovements, precipitated by the rapid growth of commodity index investment (Bahattin et al., 2001; Kyle, 2001; Tang and Xiong, 2010).

All of these factors taken together suggest that correlations may exist among commodities beyond what can be explained by basic macroeconomic variables such as exchange rates, interest rates, industrial production etc. Second, some of these factors are not easily quantifiable in a form that can be employed in direct analysis of excess commodity comovement. Therefore, a latent variable approach may be warranted. Third, and perhaps most importantly the impact of these factors on commodity comovements are likely to differ in their degree of occurrence and persistence i.e. they are likely to be frequency-specific. For example, a fall in the price of one commodity may cause significant liquidity constraints for speculators which may cause them to close their positions on other commodities. Such liquidity constraints can be regarded as a high frequency factor and its effect on commodity comovement is likely to be short-lived. On the other hand, agricultural commodity comovements precipitated by the introduction of a biofuel mandate is more likely to be persistent, evidence of which is documented by Campiche et al. (2007). This intuition is consistent with the results of Calvet et al. (2006) who find that correlation between certain commodity prices and exchange rate volatility is primarily a low frequency phenomenon and find no evidence of a long term comovement.

Lastly evidence abounds that points to the dynamic nature of comovement among financial assets (Bollerslev et al., 1992, 1988; Erb et al., 1994; Moskowitz, 2003). Assets that are seemingly uncorrelated in tranquil periods may become excessively correlated in periods of high uncertainty in the economy. Commodities, in particular, are viewed as a safe haven during crisis periods when traditional financial assets become too risky to hold. Besides, price movements of commodities such as gold and oil are viewed as proxies for global economic and political risk. Therefore, it is essential to not only consider the cross-sectional comovements among commodities but also the dynamic and regime-dependent nature of such comovements.

3.3. Research Methodology

In order to address our questions related to the presence and causes of excess co-movement across commodities, we adopt a two-step methodology that combines the approach of Pindyck and Rotemberg (1990) with the MSM model. A major drawback of the Pindyck and Rotemberg (1990) approach is the failure to account for the apparent heteroscedasticity and non-normality that characterize commodity returns (Beck, 1993; Deb et al., 1996; Palaskas and Varangis, 1991). Such heteroscedasticity emanates, for instance, from the storable nature of commodities. In other words, price variations in one period are transferred to the next period through variations in inventory levels. Our two-step approach incorporates these characteristics. Specifically, for each commodity, we estimate a linear regression model as follows

$$R_{i,t} = \beta_0 R_{i,t-1} + \sum_{j=0}^{J} \beta_{i,j} X_{t-j} + \varepsilon_{i,t}, \quad i = 1, 2....., N, t = 1, 2, ...T \tag{3.1}$$

where N is the number of commodities, $R_{i,t}$ is the log-return on commodity i at time t and X is a vector of macroeconomic variables. The structural relationship specified in Equation (3.1) was derived by Pindyck and Rotemberg (1990) as a linear approximation to a theoretical model of commodity prices. The macroeconomic variables are used as proxies to filter out the influence of shared economic fundamentals on the return of each commodity. Commodities may share common fundamentals as a result of movements in macroeconomic variables that affect demand and supply for commodities. For example, Frankel (2006) stresses the role of interest rate changes on commodity prices. A rise in interest rates may, for example, cause mineral extraction firms to intensify their activities in order to invest the proceeds in a high interest yielding investment. This leads to an increase in the supply of natural resources, and subsequently to a decrease in price. Moreover, an increase in interest rates leads to a decrease in inventory demand, and to a drop in commodity prices.

Likewise, prices of industrial commodities such as copper, crude oil, etc, are affected by the rate of industrial production. An increase in the demand for industrial commodities leads to an increase in income. Such an increase in income leads to an increase in demand for non-industrial commodities, which causes the prices of the latter to also rise. Global demand and supply shifts as drivers of commodity prices have also been emphasized by Svensson (2008) and Wolf (2008). They argue that increasing demand for natural resources from emerging economies contributed to the recently observed rises in commodity prices.

Lastly, exchange rates are also a significant driver of commodity prices. Recently, Chen et al. (2010) find that a carefully chosen set of exchange rates (essentially the market-based floating exchange rates of commodity exporting countries such as Australia, New Zealand, Canada, etc) can accurately forecast future global commodity price movements. The forecasting power of exchange rates in predicting commodity prices has its root in market participant expectations of future commodity price fluctuations. These expectations are in turn priced into the current exchange rate through their anticipated impact on future exchange rate values.

In the second step of our approach, the regression residuals, $\hat{\varepsilon}_{i,t}$, terms derived from Equation (3.1) are then investigated via an MSM specification to model commodity comovements. We discuss the details of the MSM model and tests of excess commodity comovement in what follows.

3.3.1. The Markov-Switching Multifractal Model

Under the univariate MSM framework,[1] the innovations $\hat{\varepsilon}_{i,t}$ are modeled as a function of $\overline{k}$ statistically independent random components as follows

$$\hat{\varepsilon}_{i,t} = \overline{\sigma}_i(M_t^i)\eta_{i,t} = \overline{\sigma}_i \left(\prod_{k=1}^{\overline{k}} M_{k,t}^i \right)^{1/2} \eta_{i,t} \tag{3.2}$$

where the random variables $\eta_{i,t}$ are *IID* standard Gaussian $\mathcal{N}(0,1)$. The frequency components $M_{k,t}^i$ are independent across t and k but have the same marginal distribution M that satisfies $\mathbb{E}(M) = 1$ and $M \geq 0$. Each frequency component is characterized by its own switching and decay frequency parameter, γ_k. Therefore, the innovations are defined as a scaled multiplicative product of several stochastic volatility components, with heterogeneous durations. For example, specifying $\varepsilon_{i,t}$ as an MSM(3) implies that it is composed of three volatility components which can be described as low ($M_{1,t}^i$), medium ($M_{2,t}^i$) and high ($M_{3,t}^i$) frequency components. Such a frequency decomposition can shed light on the nature of comovements across different commodities. Moreover, this approach allows us to distinguish between long-run and short-run comovements. It is this unique property of the MSM model that we exploit in answering question (*ii*) above. This approach was also adopted by Calvet et al. (2006), where they find that the correlation between crude oil and exchange rates is mainly a low frequency phenomenon.

[1]See Chapter 1 of this thesis for a review of of both univariate and bivariate MSM models.

The construction of the bivariate MSM model follows a similar setup. Specifically, for any pairwise innovation structure for commodities i and j, the bivariate MSM is specified as

$$\hat{\varepsilon}_{i,t} = \overline{\sigma}_i(M_t^i)\eta_{i,t} = \overline{\sigma}_i \left(\prod_{k=1}^{\overline{k}} M_{k,t}^i \right)^{1/2} \eta_{i,t}$$
$$\hat{\varepsilon}_{j,t} = \overline{\sigma}_j(M_t^j)\eta_{j,t} = \overline{\sigma}_j \left(\prod_{k=1}^{\overline{k}} M_{k,t}^j \right)^{1/2} \eta_{j,t} \tag{3.3}$$

where the vector $\eta_t = [\eta_{i,t} \quad \eta_{j,t}]^T$ is bivariate *IID* Gaussian $\mathcal{N}(0, \Sigma)$ with variance-covariance matrix:

$$\Sigma = \begin{bmatrix} 1 & \rho_\eta \\ \rho_\eta & 1 \end{bmatrix}$$

As in the univariate case, the frequency components for each series are assumed to be statistically independent across k and t. The frequency dynamics are however restricted as follows

$$(\gamma_1^i, \gamma_2^i, ...\gamma_{\overline{k}}^i) = (\gamma_1^j, \gamma_2^j, ...\gamma_{\overline{k}}^j) = (\gamma_1, \gamma_2, ...\gamma_{\overline{k}})$$

Under this specification, the unconditional correlation between volatility components is given by

$$Corr(M_{k,t}^i, M_{k,t}^j) = \rho_m \frac{(1-\lambda)\gamma_k + \lambda}{2 - [(1-\lambda)\gamma_k + \lambda]} \tag{3.4}$$

Comovements are quantified by the conditional covariance given by

$$Cov_t(\hat{\varepsilon}_{i,t+n}, \hat{\varepsilon}_{j,t+n}) = \rho_\eta \overline{\sigma}_i \overline{\sigma}_j \prod_{k=1}^{\overline{k}} \mathbb{E}_t[(M_{k,t+n}^i M_{k,t+n}^j)^{1/2}] \tag{3.5}$$

and the conditional correlation given by

$$Corr_t(\hat{\varepsilon}_{i,t+n}, \hat{\varepsilon}_{j,t+n}) = \rho_\eta \prod_{k=1}^{\overline{k}} \frac{\mathbb{E}_t[(M^i_{k,t+1} M^j_{k,t+1})^{1/2}]}{[\mathbb{E}_t\left(M^i_{k,t+1}\right) \mathbb{E}_t\left(M^j_{k,t+1}\right)]^{1/2}} \tag{3.6}$$

These measures of conditional comovement are time varying. Moreover, they are high in periods of high volatility (when $\{M^i_k\}_{k=1}^{\overline{k}}$ and $\{M^j_k\}_{k=1}^{\overline{k}}$ are high), and vice-versa. In other words, commodities that are otherwise less correlated in tranquil periods may become highly correlated in periods of high uncertainty. This is a crucial piece that has generally been missing from the commodity comovement literature. There is large evidence pointing to the fact that covariances and correlations across asset returns change over time, and are high in periods of recessions or financial crises (Bollerslev et al., 1988; Moskowitz, 2003).

Moreover, commodity comovements are also driven to various degrees by events such as terrorist attacks on oil rigs, weather disasters affecting a region where multiple commodities are grown (e.g. drought in the U.S. in summer of 2011), introduction of economic and political policy instruments (Energy Policy Act of 2007, Renewable Fuel Standards Program of 2010), etc. Therefore, an accurate measure of commodity comovement should incorporate the time-varying dynamics of such comovement and how it is affected by various economic events. This constitutes an innovative part of this study.

3.3.2. Tests of Excess Comovement

The primary hypotheses of interest are based on comovements between $\hat{\varepsilon}_{i,t}$ and $\hat{\varepsilon}_{j,t}$. To this end, we formulate the following two hypothesis between any pairwise series based on the innovations $\hat{\varepsilon}_{i,t}$ and scaled residuals $\hat{\eta}_{i,t}$ from the MSM process.

Hypothesis 1 (Test of zero scaled covariance):

$$H_0 : \mathbb{E}(\hat{\eta}_{i,t}, \hat{\eta}_{j,t}) = 0 \tag{3.7}$$

The null hypothesis states that the commodity pair(i, j) displays zero contemporaneous excess comovement, where $\hat{\eta}_{i,t}$ is the scaled residual from a univariate MSM process and is defined as

$$\hat{\eta}_{i,t} = \frac{\hat{\varepsilon}_{i,t}}{\overline{\sigma}_i(M_t^i)} \tag{3.8}$$

Following Deb et al. (1996), the Lagrange Multiplier test of the moment condition implied by the null hypothesis is given by

$$LM_{i,j} = T\hat{\rho}_{i,j}^2 \sim \chi^2(1)$$

where $\hat{\rho}_{i,j}$ is the correlation between the scaled residuals of commodity pair(i, j). The Lagrange Multiplier test is implemented as follows:

i Equation (3.1) is estimated for each commodity, and the parameter estimates are used to generate the residuals $\hat{\varepsilon}_{i,t}$.

ii A univariate MSM model is fitted to each residual vector generated in step **i**.

iii Using the MSM parameters estimated in step **ii**, the filtered probabilities are estimated. Using the algorithm of Kim (1994), the smoothed probabilities $\hat{\Psi}_t^l \equiv \mathbb{P}(M_t = m^l | r_1 ... r_T)$, for $l \in \{1, 2, ...2^k\}$, are estimated from the filtered probabilities.

iv The smoothed probabilities are then used to generate Equation (3.2) as follows for each commodity

$$\mathbb{E}(\varepsilon_{i,t}) = \hat{\overline{\sigma}}(\prod_{k=1}^{\overline{k}} \mathbb{E}_t[M_{k,t+1}^i]^{1/2})$$

v Lastly, the scaled residual $\hat{\eta}_{i,t}$ is generated as in Equation (3.8). Then, the scaled residuals are used to generate the LM statistic for each possible commodity pair.

Hypothesis 2 (Test of zero conditional covariance):

$$H_0 : \mathbb{E}_t(\hat{\varepsilon}_{i,t+1}, \hat{\varepsilon}_{j,t+1}|\Omega_t) = 0 \tag{3.9}$$

where Ω_t is the information available up to time t. In this test, the conditional covariance is constant and zero under the null, but time varying under the alternative. The test is implemented by fitting a bivariate MSM model to a pair of commodity residual vectors derived from Equation (3.1). Under the MSM framework, the test of zero conditional covariance implies testing the null hypothesis that $\rho_\eta = 0$ in Equation (3.5). Note that as long as the conditional variance is not constant, neither $\overline{\sigma}_i$ nor $\overline{\sigma}_j$ can be substituted for ρ_η. Since ρ_η is one of the estimated parameters of the bivariate MSM, and is reported along with its standard error, conducting the test is straightforward.

Lastly, it is important to emphasize the difference between hypotheses 1 and 2. Hypothesis 1 tests for zero scaled covariance. The test assumes that the covariances of the scaled errors are zero under the null and constant under the alternative. It makes no distinction between conditional and unconditional covariance. On the other hand, hypothesis 2 tests for zero conditional covariance. The covariance is assumed to be constant and zero under the null but is time-varying under the alternative. Therefore, the test of hypothesis 2 is a much stronger test than hypothesis 1. The comovement of a pair of commodity may pass hypothesis 1 but fail hypothesis 2. Also, the LM test of hypothesis 1 is based on the stringent assumption of normal errors, which may be violated in practice. When applied to non-normal data, the LM test based on normality may suffer from size distortion (Deb et al., 1996).

3.4. Empirical Analysis

3.4.1. Data

The data used in this study comprises daily time series for fifteen commodity spot prices, from January 4, 1994 to December 30, 2011. The sources of the data and the markets where the commodities are traded are listed in Appendix A. Commodities is a broad term, and comprises a broad set of products that are heterogenous by nature. Therefore, we take a disaggregated approach by considering three commodity categories, namely energy, metals and agricultural commodities. This ensures that our results are not market-specific but apply to a wide range of commodities. The commodity list comprises six agricultural products (cotton, sugar, corn, wheat, soybeans, oats) , seven metals products (copper, aluminium, lead, tin, zinc, nickel, gold), and two energy products (crude oil and heating oil). With the exception of metals, all commodities are traded in the U.S and quoted in U.S. dollars. Gold is also traded in the U.S and quoted in the U.S. dollars. All other metals are traded on the LME and quoted in the U.S dollars. Lastly, this study follows the extant literature and does not consider any equity or other financial assets in the analysis. Claessens et al. (2001) provide an excellent review on comovements and contagion in financial markets.

While previous studies have used low frequency data sets such as monthly and quarterly data, we use daily data for two reasons. First, monthly or quarterly data have the shortcoming of substantially reducing the number of observations. Second, and perhaps most importantly, monthly or quarterly observations may fail to reflect more rapid cycles of propagation of shocks across seemingly unrelated commodities. This becomes apparent later in the study when we decompose commodity comovements into different frequencies, ranging from decades to daily. This is an important element that has been missing from earlier studies and might have contributed to some failures to detect excess commodity comovement.

The data on macroeconomic fundamentals consist of variables used in the extant literature,[2] namely the Standard & Poor's 500 return, the nominal 3-month U.S. Treasury bill rate, the equally weighted average of the Yen, Deutsche mark[3] and British pound sterling vis-a-vis the U.S dollar and the Baltic Dry Index (BDI). We also include the Chicago Board Options Exchange (CBOE) Volatility Index (VIX), which is a measure of the market expectations of short term market volatility and investment sentiment.[4] These macroeconomic variables are used to capture the link between commodity markets and equity, interest rates, foreign exchange markets and the level of economic activity. The Baltic Dry Index is published by the Baltic Exchange in London and measures the cost of maritime transportation for major raw materials across the world. Changes in the Baltic Dry Index are largely driven by global demand for industrial commodities. Moreover, a positive correlation between maritime transportation rates and economic activities is widely documented in the economic literature (Kilian, 2009; Stopford, 2009). Therefore, the BDI is widely accepted as an indicator of the level of economic activity, similar to the index of industrial production. We use the BDI to control for the level of world economic activity because it is available at higher frequencies than the index of industrial production, which is only available on quarterly basis. Lastly, the BDI is also used

[2] These are the standard macroeconomic variables that have been used in the commodity comovement literature. See Pindyck and Rotemberg (1990) and Deb et al. (1996) for example. Money supply and inflation rates are excluded because they are not available on a daily basis.

[3] The Deutsche mark was replaced by the Euro at the beginning of 1999. Thus, the Deutsche mark data ended on December 31, 1998 and the Euro was used in its place.

[4] Thanks to the comments from a member of the supervisory committee, who pointed this out.

to control for the effect of the growth surge in the emerging economies, most notably China. This has been widely cited as a source for the the recent rise in the prices of commodities (Helbling et al., 2008; Irwin et al., 2009; Silvennoinen and Thorp, 2013; Trostle, 2010). The correlation value between the BDI and the Chinese GDP is 0.4502. Also, Figure 3.1 shows a scatter plot of BDI against the Chinese GDP. The plot and the positive correlation value clearly show a strong positive comovement between the BDI and Chinese growth.

Figure 3.1: Comovement Between BDI and Chinese GDP

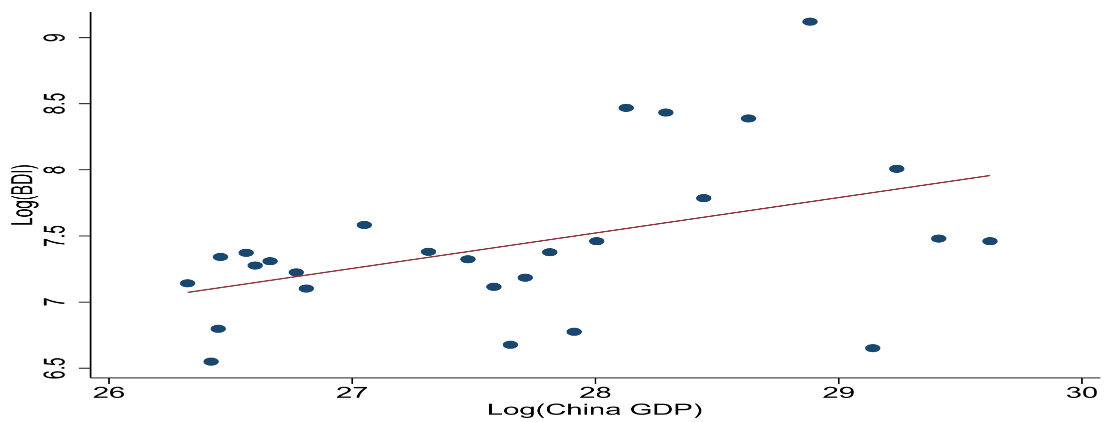

Table 3.1 presents summary statistics for the logarithms of commodity returns. The returns series exhibit significant non-normality in all cases. The Jarque-Bera (Bera et al., 1997) test statistics are all greater than the critical value of 5.9794, leading to a rejection of the null hypothesis of normality for all commodity returns. It is therefore important that we incorporate this observation into our analysis.

Table 3.2 reports the correlations of daily log returns for all commodities used in the main analyses. The pairwise correlation coefficients range between 0.0499 for oats and heating oil to 0.6237 for crude oil and heating oil. Moreover, as expected, correlations are stronger between commodities in the same group (agricultural, metals

Table 3.1: **Summary Statistics (Log Returns)**

	Mean	Standard Deviation	Minimum	Maximum	Skewness	Kurtosis	JB-Stat
Gold	0.0315	1.0531	-6.2840	7.3820	-0.1415	8.9408	6481
Copper	0.0336	1.7940	-10.4755	11.7259	-0.1666	7.4648	3673
Aluminium	0.0161	1.2474	-12.6752	11.7146	-0.2109	12.3140	15926
Tin	0.0311	1.5581	-13.8094	18.8100	0.1313	17.6951	39576
Zinc	0.0138	1.8889	-12.6185	9.6100	-0.2816	6.7223	2597
Nickel	0.0292	2.3958	-18.3586	22.0072	0.0079	7.9030	4404
Lead	0.0334	2.1086	-13.1992	13.0072	-0.1508	6.3078	2021
Cotton	0.0073	1.9399	-8.6850	12.7701	0.0378	5.1178	823
Wheat	0.0116	1.9490	-12.2987	12.5405	0.0290	6.4516	2183
Oats	0.0167	2.3393	-23.7850	23.3711	0.0324	15.9896	30913
Sugar	0.0177	2.2741	-23.5036	14.2124	-0.5084	8.7451	6236
Soybeans	0.0121	1.6551	-16.7294	8.5180	-0.7294	9.2528	7553
Corn	0.0182	1.9160	-12.1116	10.8879	-0.1549	6.2410	1942
Heating Oil	0.0420	2.7324	-47.5669	23.4507	-1.3271	36.2782	204182
Crude Oil	0.0434	2.5533	-17.0918	19.1438	-0.1331	7.9513	4504
Exchange Rate	36.8653	4.7157	25.7000	49.8500	-0.3266	2.8326	83.3178
T-Bill(3M)	3.1339	2.0362	0.0000	6.2400	-0.2875	1.5063	469
S&P500 Return	0.0225	1.2708	-9.4695	10.4236	-0.2577	10.5207	10409
VIX	2.9884	0.3641	2.2915	4.3927	0.4111	3.0942	125.427
BDI(Log)	7.6204	0.6382	6.4968	9.3753	0.7179	2.7438	390

Notes: For each commodity, this table presents the summary statistics for the percentage logarithmic returns, defined as $ln(P_{t-1}/P_t) * 100$, and the macroeconomic variables. T-Bill(3M) refers to the nominal 3-month U.S. Treasury bill rate while BDI(Log) refers to the natural logarithm of the Baltic Dry Index. The sample period is from January 04 1994 to December 30 2011, resulting in 4398 observations. JB-Stat refers to the Jarque-Bera Bera et al. (1997) test of normality statistic. Under the null hypothesis that the variable under consideration is normally distributed, the test has an asymptotic chi-squared distribution with two degrees of freedom. At 5% level of significance, the test has a critical value of 5.9794.

and energy) than between commodities in different groups.

The summary statistics indicate that many of the commodities are significantly correlated. But such correlations may be driven by shared macroeconomic fundamentals through their effects on demand and supply. The major question is whether significant comovement exists between these commodities in excess of what can be explained by fundamental factors. We address this question in the next section.

Table 3.2: **Simple Correlations of Commodity Log Returns**

	Gold	Copper	Cotton	Wheat	Oats	Sugar	Soybeans	Corn	Heating	WTI
Gold	1.0000	0.2148	0.0643	0.1054	0.0755	0.0612	0.0913	0.1051	0.0779	0.1386
Copper		1.0000	0.1710	0.1813	0.0745	0.1408	0.1982	0.1805	0.1649	0.2446
Cotton			1.0000	0.1550	0.0841	0.1179	0.2348	0.2080	0.1014	0.1300
Wheat				1.0000	0.2300	0.1031	0.3532	0.4630	0.1127	0.1458
Oats					1.0000	0.0697	0.2514	0.3153	0.0499	0.0701
Sugar						1.0000	0.1278	0.1299	0.0848	0.1405
Soybeans							1.0000	0.5748	0.1528	0.1868
Corn								1.0000	0.1424	0.1888
Heating									1.0000	0.6237
WTI										1.0000

Notes: This table presents the simple bivariate correlation coefficients between each commodity pair. The correlation coefficient r has a test statistic $t = r\sqrt{N-2}/\sqrt{1-r^2}$, which has an asymptotic $t-$distribution with $N-2$ degrees of freedom, where N is the number of observations. For $N = 4398$, the critical correlation coefficients are 0.0388, 0.0296 and 0.0248 for 1%, 5% and 10% significance levels, respectively. All correlation coefficients are significant at 1%, 5% or 10% significant levels.

3.4.2. Test Of Excess Comovement

The data are divided into two sets. The first set comprises gold, copper , cotton, wheat, oats, sugar, soybeans, corn, heating oil and crude oil. This set is used in the main analysis, as they have been widely analysed in previous studies, thereby fostering an easy comparison of our results with previous studies. The other set comprises crude oil, copper, aluminium, tin, zinc, nickel and lead. With, the exception of crude oil, all of these are traded on the LME. This set is used for robustness analysis in section 5. The inclusion of related commodities, such as corn and wheat, is to serve as useful controls for checking the power of the excess comovement tests.

In order to address the question of excess comovement among commodities, it is imperative that the effects of macroeconomic fundamentals be filtered out. We achieve this by estimating Equation (3.1) by regressing each commodity return on the current and one period lag of the macroeconomic variables and one period lag of the commodity return. The results are reported in Table 3.3. First, the R^2 is very low for all the commodities, ranging from 0.0071 to 0.0734. This is also the case with earlier studies as well.[5] Palaskas and Varangis (1991) associated this weak power of the macroeconomic variables to the non-normality of commodity returns. Equation (3.1) is also estimated using different lags of returns and the macroeconomic variables, ranging from 2 to 6. The results are not qualitatively different from those presented in Table 3.3, and the power of the regression does not improve either. However, to keep the analysis simple and for ease of presentation, we continue the analysis using only the first lag. Second, the results indicate that the S&P500 return and the Baltic Dry Index variables are significant for most commodities, while the exchange rate, interest rate and the VIX index are rarely significant. Likewise, the sum of the coefficients on each current and lagged macroeconomic variables are neither statistically nor economically significant. But the variables as a whole have significant impact on commodity returns, as indicated by the Wald-statistics and the associated p-values.

Nonetheless, summing the coefficients on current and lagged macro variables, we observe that when the interest rate is significant the net effect on commodity prices is negative, consistent with *a priori* expectations.

An increase in the interest rate leads to an increase in the required rate of return on storage. This in turn leads to a fall in inventory demand and subsequently to a fall in commodity prices. Likewise, we observe that the relationship of the equity market with commodity prices is significant in most cases. This is consistent with the financialization of commodities hypothesis, which states that the correlation between stocks and commodities is increasing due to a rapid increase in investor activities in the commodity market (Juvenal and Petrella, 2011; Masters, 2008; Tang and Xiong, 2010). Moreover, in bad times when stocks become more risky, investors diversify away to commodities in order to reduce their risk exposure to stocks.

[5]For example, see Pindyck and Rotemberg (1990) and Palaskas and Varangis (1991).

Table 3.3: Regression Results For Commodities

	Commodity Returns($R_{i,t}$)									
	Gold	Copper	Cotton	Wheat	Oats	Sugar	Soybeans	Corn	Heating Oil	WTI
E_t	-0.7581***	-0.0077	0.2260*	-0.1585	-0.2327	0.1798	-0.1709*	-0.0723	-0.1378	0.0749
	(0.0742)	(0.1107)	(0.1156)	(0.1228)	(0.1426)	(0.1425)	(0.1038)	(0.1209)	(0.1862)	(0.1739)
E_{t-1}	0.7608***	0.0163	-0.2240*	0.1503	0.2252	-0.1894	0.1681	0.0620	0.1428	-0.0689
	(0.0744)	(0.1104)	(0.1151)	(0.1225)	(0.1425)	(0.1427)	(0.1038)	(0.1209)	(0.1872)	(0.1729)
I_t	-0.4380	1.2884**	0.9476	0.7725	-0.9980	0.3858	1.3884**	0.8206	-0.4531	0.7129
	(0.5192)	(0.5828)	(0.7086)	(0.7268)	(1.0338)	(0.5877)	(0.6668)	(0.7676)	(0.7283)	(1.0622)
I_{t-1}	0.4191	-1.3195**	-0.9652	-0.7520	1.0102	-0.3863	-1.3897**	-0.8100	0.4358	-0.7326
	(0.5192)	(0.5835)	(0.7085)	(0.7269)	(1.0331)	(0.5882)	(0.6673)	(0.7676)	(0.7282)	(1.0628)
B_t	4.6849**	4.3605*	3.6300*	3.3024	3.9484	-4.8671*	2.5864	-0.0691	3.6779*	5.5756**
	(1.8434)	(2.6235)	(2.0914)	(2.3199)	(2.8860)	(2.5056)	(2.2751)	(2.5789)	(2.1974)	(2.6151)
B_{t-1}	-4.6614**	-4.3062	-3.6136*	-3.2761	-3.9101	4.9129*	-2.5540	0.1243	-3.6127	-5.5307**
	(1.8416)	(2.6204)	(2.0894)	(2.3191)	(2.8827)	(2.5092)	(2.2747)	(2.5769)	(2.1965)	(2.6147)
V_t	0.5035	-1.2231*	-0.9044	-1.2218	-0.1763	-0.4189	0.4699	-1.0604	-0.9559	0.9123
	(0.4846)	(0.6984)	(0.7359)	(0.8034)	(0.8898)	(0.8172)	(0.6169)	(0.7465)	(1.0587)	(1.0538)
V_{t-1}	-0.5678	1.0194	0.8573	1.2379	0.1623	0.4272	-0.5135	1.0398	0.7567	-1.0684
	(0.4801)	(0.6940)	(0.7375)	(0.8072)	(0.8826)	(0.8143)	(0.6132)	(0.7388)	(1.0450)	(1.0583)
SR_t	0.0159	0.2771***	0.1363***	0.1174***	0.0882*	0.0825**	0.1590***	0.1278***	0.2022***	0.3042***
	(0.0319)	(0.0413)	(0.0437)	(0.0440)	(0.0523)	(0.0391)	(0.0352)	(0.0433)	(0.0578)	(0.0614)
SR_{t-1}	0.0525**	0.1814***	0.0579**	0.0035	0.0606*	0.0435	0.0860***	0.0856***	0.1167***	0.1366***
	(0.0205)	(0.0320)	(0.0271)	(0.0314)	(0.0339)	(0.0307)	(0.0263)	(0.0323)	(0.0385)	(0.0416)
$R_{i,t-1}$	-0.0162	-0.0669***	-0.0077	-0.0301	-0.0461*	-0.1021***	-0.0300	0.0049	-0.0236	-0.0220
	(0.0246)	(0.0231)	(0.0187)	(0.0231)	(0.0249)	(0.0187)	(0.0193)	(0.0204)	(0.0512)	(0.0224)
N	4,394	4,394	4,394	4,394	4,394	4,394	4,394	4,394	4,394	4,394
R^2	0.0492	0.0734	0.0179	0.0144	0.0071	0.0170	0.0203	0.0170	0.0159	0.0252
$W-stat$	12.3700	19.9000	5.2300	4.3400	1.8900	5.0600	4.7400	4.9300	5.7600	5.5000
$P_w-value$	0.0000	0.0000	0.0000	0.0000	0.0361	0.0000	0.0000	0.0000	0.0000	0.0000

Notes: This table presents the results from estimating Equation (3.1) for the first 10 commodities. R_t is the percentage logarithmic returns, defined as $ln(P_{t-1}/P_t)*100$. E, I, SR, V and B refer to the exchange rate, interest rate, S&P500 return, the Vix index and the Baltic Dry Index respectively. W-stat reports the Wald test statistics of the overall significance of the regressors. P_w reports the corresponding p-values for the Wald tests. These indicate that the macroeconomic variables and the lagged returns are jointly significant at the 10% level for the commodities under consideration. All standard errors are heteroscedasticity consistent. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Lastly, we observe that commodity returns are positively related to increased economic activity. In periods of high economic activity, the demand for raw materials increases, leading to an increase in prices of industrial commodities. But the prices of non-industrial commodities also rise through the income effects channel.

3.4.3. MSM Model Estimates

Using the the parameter estimates from Equation (3.1), the regression residuals, $\hat{\varepsilon}_{i,t}$ are extracted and a univariate MSM is fitted to these residuals, for each commodity. The results are reported in Table 3.4. The results are reported for $\overline{k}$ varying from 1 to 10, where $\overline{k}$ is the number of volatility components included in the model.[6] When $\overline{k} = 1$, the model is identical to a standard Markov regime-switching model, with only two possible states of volatility levels. As $\overline{k}$ increases, the possible number of volatility states increases by 2^k.

Table 3.4: Univariate MSM Estimates

	$\overline{k} = 2$	3	4	5	6	7	8	9	10
Gold									
$\hat{b}$	9.7376	43.6951	34.1191	11.2143	11.8105	11.6957	7.2994	4.5044	3.5729
	(3.8574)	(19.6378)	(17.5576)	(2.0804)	(5.5545)	(6.1309)	(1.6932)	(1.0370)	(0.8680)
$\hat{m}_0$	1.6575	1.6054	1.5581	1.4814	1.4594	1.4592	1.4155	1.3634	1.3391
	(0.0263)	(0.0228)	(0.0240)	(0.0148)	(0.0175)	(0.0212)	(0.0178)	(0.0152)	(0.0140)
$\hat{\gamma}_{\overline{k}}$	0.0762	0.7786	0.8416	0.9114	0.9955	0.9950	0.9999	0.9999	0.9999
	(0.0276)	(0.1680)	(0.1034)	(0.1378)	(0.0257)	(0.0317)	(0.0007)	(0.0008)	(0.0009)
$\hat{\sigma}$	1.1664	1.1451	0.9736	1.0030	1.2201	1.0098	0.9907	0.9935	1.0130
	(0.0764)	(0.0990)	(0.0422)	(0.0642)	(0.0663)	(0.0526)	(0.0410)	(0.0501)	(0.1290)
lnL	-5533.74	-5484.26	-5457.92	-5450.24	-5443.71	-5444.32	-5443.51	-5442.80	-5442.32
Copper									
$\hat{b}$	18.1761	8.0713	16.7146	16.7005	9.4039	6.1463	4.1042	4.0322	3.3355
	(17.5358)	(5.6676)	(5.9697)	(6.4195)	(2.0105)	(1.4906)	(0.7801)	(0.7573)	(0.5691)
$\hat{m}_0$	1.5239	1.4477	1.4259	1.4259	1.3806	1.3413	1.2945	1.2938	1.2740
	(0.0203)	(0.0243)	(0.0172)	(0.0174)	(0.0145)	(0.0147)	(0.0142)	(0.0142)	(0.0124)
$\hat{\gamma}_{\overline{k}}$	0.0825	0.0682	0.7050	0.7044	0.9664	0.9855	0.9992	0.9987	0.9997
	(0.0318)	(0.0383)	(0.1721)	(0.1829)	(0.0529)	(0.0481)	(0.0046)	(0.0068)	(0.0023)

Continued on next page

[6]Though we estimate MSM with $\overline{k}$ equals 1 to 10 for all commodities, we only report results for $\overline{k}$ equals 2 to 10 due to space constraints and ease of presentation.

Table 3.4 – Continued from previous page

	$\bar{k}=2$	3	4	5	6	7	8	9	10
$\hat{\sigma}$	1.9359	2.1938	2.0900	1.7505	1.6935	1.6540	2.0095	1.7670	1.7456
	(0.1563)	(0.1277)	(0.0905)	(0.0701)	(0.0587)	(0.0770)	(0.1233)	(0.0996)	(0.0782)
lnL	-8095.35	-8069.11	-8056.25	-8056.90	-8055.56	-8056.51	-8054.34	-8054.69	-8054.77
Cotton									
$\hat{b}$	28.4112	35.5032	22.4808	13.8977	7.1911	13.9056	4.1975	3.3757	3.3863
	(11.1548)	(11.2497)	(7.2673)	(13.7661)	(0.7892)	(21.8109)	(0.5437)	(0.3002)	(0.3032)
$\hat{m}_0$	1.4767	1.4589	1.4152	1.3698	1.3292	1.3697	1.2819	1.2615	1.2611
	(0.0223)	(0.0216)	(0.0194)	(0.0269)	(0.0138)	(0.0369)	(0.0150)	(0.0126)	(0.0123)
$\hat{\gamma}_{\bar{k}}$	0.1300	0.8529	0.9089	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	(0.0596)	(0.0843)	(0.1299)	(0.0025)	(0.0001)	(0.0039)	(0.0001)	(0.0001)	(0.0001)
$\hat{\sigma}$	1.9863	1.9996	1.7638	1.8027	1.7983	1.9397	1.9390	1.9008	2.2009
	(0.0792)	(0.0772)	(0.0590)	(0.0717)	(0.0542)	(0.0738)	(0.1629)	(0.1034)	(0.1267)
lnL	-8765.62	-8743.22	-8737.14	-8733.37	-8733.01	-8734.02	-8732.82	-8732.84	-8733.34
Wheat									
$\hat{b}$	22.1850	14.7948	8.9490	5.4685	4.2461	3.8029	3.7169	3.0442	2.4033
	(9.9786)	(5.3615)	(1.7683)	(1.0524)	(0.8004)	(0.8288)	(0.6912)	(0.7274)	(0.2764)
$\hat{m}_0$	1.5437	1.4831	1.4145	1.3616	1.3289	1.3068	1.3055	1.2823	1.2538
	(0.0193)	(0.0195)	(0.0167)	(0.0149)	(0.0133)	(0.0138)	(0.0134)	(0.0137)	(0.0134)
$\hat{\gamma}_{\bar{k}}$	0.1916	0.2148	0.5421	0.6515	0.7897	0.8338	0.8192	0.8836	0.9094
	(0.0494)	(0.0790)	(0.1152)	(0.2155)	(0.1962)	(0.1958)	(0.1686)	(0.2226)	(0.1688)
$\hat{\sigma}$	2.1405	2.5298	2.2642	2.1792	2.0881	2.2495	1.9766	1.9426	2.0308
	(0.1530)	(0.1408)	(0.0979)	(0.1244)	(0.0883)	(0.1149)	(0.0918)	(0.0745)	(0.1561)
lnL	-8667.34	-8644.43	-8639.04	-8638.44	-8637.92	-8637.80	-8638.22	-8637.70	-8637.55
Oats									
$\hat{b}$	11.0794	5.5590	4.7237	5.6225	4.9869	3.8299	3.1850	5.1585	2.4917
	(4.1134)	(1.2099)	(0.9065)	(1.0994)	(0.9051)	(0.3377)	(0.1909)	(0.9714)	(0.1575)
$\hat{m}_0$	1.8899	1.8392	1.7900	1.7527	1.7020	1.6575	1.6312	1.7022	1.5782
	(0.0082)	(0.0106)	(0.0125)	(0.0111)	(0.0129)	(0.0118)	(0.0132)	(0.0117)	(0.0134)
$\hat{\gamma}_{\bar{k}}$	0.5243	0.6102	0.7814	0.9600	0.9996	0.9999	0.9999	0.9998	0.9999
	(0.0561)	(0.1158)	(0.1611)	(0.0525)	(0.0020)	(0.0002)	(0.0001)	(0.0012)	(0.0001)
$\hat{\sigma}$	2.1841	2.1438	2.1477	2.4794	2.0028	2.1704	2.1655	2.1456	2.1849
	(0.1907)	(0.2365)	(0.5418)	(0.1510)	(0.1083)	(0.2516)	(0.2479)	(0.1239)	(0.2139)
lnL	-8683.60	-8562.17	-8526.29	-8494.85	-8488.12	-8487.50	-8482.90	-8489.66	-8479.89
Sugar									
$\hat{b}$	50.0000	49.3590	13.4310	7.1741	13.5595	4.9849	3.9454	3.9968	2.4730
	(25.1258)	(380.2184)	(1.6099)	(0.7345)	(1.5821)	(0.3733)	(0.4879)	(0.4373)	(0.1098)
$\hat{m}_0$	1.5995	1.5219	1.4572	1.4057	1.4554	1.3685	1.3423	1.3431	1.2869
	(0.0331)	(0.0503)	(0.0192)	(0.0182)	(0.0175)	(0.0171)	(0.0170)	(0.0176)	(0.0146)
$\hat{\gamma}_{\bar{k}}$	0.4869	0.9986	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	(0.3078)	(0.0936)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
$\hat{\sigma}$	2.1314	2.1611	2.1372	2.1171	2.3892	1.8247	2.6394	2.3344	2.1375
	(0.1170)	(0.1766)	(0.0797)	(0.1090)	(0.0957)	(0.1057)	(0.3084)	(0.2246)	(0.1305)
lnL	-9402.69	-9365.23	-9356.70	-9357.38	-9357.71	-9358.31	-9358.13	-9358.09	-9357.22
Soybeans									
$\hat{b}$	10.8055	37.0169	22.5784	13.4434	6.9186	4.6817	3.6300	3.1028	3.5963
	(5.4207)	(12.8430)	(8.7622)	(2.6238)	(0.6835)	(0.2892)	(0.2095)	(0.1988)	(0.1868)
$\hat{m}_0$	1.5209	1.4965	1.4737	1.4161	1.3682	1.3346	1.3093	1.2907	1.3065
	(0.0269)	(0.0182)	(0.0268)	(0.0191)	(0.0150)	(0.0133)	(0.0126)	(0.0126)	(0.0127)
$\hat{\gamma}_{\bar{k}}$	0.1000	0.8710	0.8285	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	(0.0440)	(0.0774)	(0.0702)	(0.0004)	(0.0002)	(0.0001)	(0.0001)	(0.0001)	(0.0001)

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Table 3.4 – Continued from previous page

	$\overline{k}=2$	3	4	5	6	7	8	9	10
$\hat{\sigma}$	1.8941	1.9956	1.6809	1.6364	1.6441	1.6491	1.6527	1.6773	1.7531
	(0.1320)	(0.0853)	(0.0636)	(0.0591)	(0.0609)	(0.0603)	(0.0796)	(0.1170)	(0.0847)
lnL	-7943.04	-7906.71	-7900.91	-7893.95	-7892.83	-7892.80	-7893.13	-7892.93	-7893.74
Corn									
$\hat{b}$	9.6358	12.3959	8.5892	7.7915	4.6282	3.6355	3.0340	2.6665	2.4306
	(4.5355)	(3.8524)	(2.7663)	(2.1867)	(0.9046)	(0.5663)	(0.4118)	(0.3161)	(0.2690)
$\hat{m}_0$	1.5561	1.5069	1.4678	1.4145	1.3604	1.3288	1.3055	1.2868	1.2722
	(0.0185)	(0.0209)	(0.0229)	(0.0211)	(0.0170)	(0.0148)	(0.0143)	(0.0132)	(0.0136)
$\hat{\gamma}_{\overline{k}}$	0.1191	0.1649	0.1778	0.7073	0.7704	0.8457	0.8776	0.9019	0.9178
	(0.0374)	(0.0489)	(0.0524)	(0.1871)	(0.2138)	(0.1763)	(0.1583)	(0.1491)	(0.1262)
$\hat{\sigma}$	1.9177	2.4571	1.9889	1.8872	1.8770	1.8954	1.9287	1.9126	1.8853
	(0.0544)	(0.1291)	(0.0854)	(0.0712)	(0.0864)	(0.1113)	(0.1397)	(0.1478)	(0.1529)
lnL	-8589.31	-8576.49	-8572.19	-8567.28	-8566.83	-8566.34	-8566.00	-8565.72	-8565.46
Heating Oil									
$\hat{b}$	7.5072	4.8262	5.4105	9.5907	2.6148	2.7258	1.3548	1.4826	1.8831
	(5.1934)	(5.0121)	(6.4814)	(6.6103)	(1.2118)	(3.5000)	(0.3601)	(0.7348)	(0.6708)
$\hat{m}_0$	1.5124	1.4170	1.4953	1.4538	1.4306	1.4492	1.3640	1.3797	1.2775
	(0.0582)	(0.0844)	(0.0626)	(0.0402)	(0.0412)	(0.0425)	(0.0263)	(0.0265)	(0.0422)
$\hat{\gamma}_{\overline{k}}$	0.0591	0.0370	0.0370	0.2203	0.0234	0.0244	0.0114	0.0127	0.1188
	(0.0354)	(0.0406)	(0.0255)	(0.2644)	(0.0090)	(0.0334)	(0.0074)	(0.0170)	(0.1966)
$\hat{\sigma}$	3.1497	3.1497	3.1497	3.1034	3.1497	2.7650	3.1497	2.8145	3.1497
	(0.4389)	(0.8360)	(0.2809)	(0.4054)	(0.1890)	(0.1518)	(0.1883)	(0.1619)	(0.5350)
lnL	-10059.26	-10023.70	-9996.64	-9989.28	-9974.18	-9973.35	-9967.62	-9967.55	-9972.15
WTI Crude Oil									
$\hat{b}$	24.3817	11.7393	11.5559	9.9917	9.9242	6.8946	3.9963	3.4023	2.9494
	(9.7297)	(6.5441)	(2.9966)	(2.2527)	(2.6694)	(7.2996)	(0.7118)	(0.6075)	(0.5518)
$\hat{m}_0$	1.5323	1.4797	1.4280	1.3913	1.3911	1.3494	1.3008	1.2795	1.2621
	(0.0242)	(0.0226)	(0.0204)	(0.0178)	(0.0187)	(0.0342)	(0.0138)	(0.0129)	(0.0124)
$\hat{\gamma}_{\overline{k}}$	0.2080	0.2150	0.8952	0.9519	0.9501	0.9996	0.9952	0.9994	0.9999
	(0.0633)	(0.1416)	(0.1258)	(0.0517)	(0.0635)	(0.0108)	(0.0188)	(0.0038)	(0.0010)
$\hat{\sigma}$	3.0220	2.7899	2.6732	3.0905	2.6202	2.5723	2.7047	2.6889	2.6667
	(0.1399)	(0.1739)	(0.1089)	(0.1117)	(0.0889)	(0.1175)	(0.0983)	(0.0857)	(0.0873)
lnL	-9874.91	-9849.85	-9841.81	-9833.96	-9834.48	-9833.95	-9831.66	-9830.68	-9830.12

Notes: This table reports the maximum likelihood estimates of binomial univariate MSM for each commodity return residuals. Residuals are extracted after fitting Equation (3.1) to each commodity return series. The univariate MSM is fitted for $\overline{k}$ equals 1 to 10, where each column corresponds to the given number of frequency components $\overline{k}$ in the MSM specification. Results for $\overline{k}=1$ are omitted for ease of presentation. Besides, $MSM(1)$ is never optimal for any commodity. Therefore, there is no real loss of information in omitting the results. Asymptotic standard errors, reported in parentheses, are computed using the *Outer Product Gradient* estimate of the information matrix. See Hamilton (1994), page 143.

Several observations from Table 3.4 merit attention. First, looking at the volatility component parameters, we observe that as $\overline{k}$ increases, $\hat{m}_0$ tends to decline as the number of volatility states increases by $2^{\overline{k}}$. The intuition is that less variability is required in each individual component in order to match the volatility fluctuation of the data. Estimates of $\hat{\sigma}$ fluctuate across $\overline{k}$ without any apparent pattern.

Second, as $\overline{k}$ increases, the switching probability of the highest volatility component $\hat{\gamma}_{\overline{k}}$ increases, while the growth rate $\hat{b}$ fluctuates without any pattern. From the estimated values of $\hat{\gamma}_{\overline{k}}$, we can infer the duration of the volatility components, defined as $(1/\hat{\gamma}_k)$. Consider crude oil for example. When $\overline{k} = 1$, the single volatility component ($\hat{\gamma}_1 = 0.2080$) switches every 5 days. But when $\overline{k} = 10$, the highest frequency components $M_{10,t}$ switches every day while the lowest frequency components $M_{1,t}$ has a duration 1,735 days or approximately 5 years. Therefore, the MSM model of commodity returns is able to capture not only frequent but transient shocks, but also captures rare and extreme events.

It can be observed from Table 3.4 that the log-likelihood increases non-monotonically as the number of frequency components increases. This implies that the fit of the model increases as the number of frequency components increases for most commodities, except for soybeans, copper, sugar, cotton and heating oil for which the log-likelihood peaked at $\overline{k}$ equals 5, 8, 4, 8 and 9 respectively. Therefore for further testing purposes, we need to select the value of $\overline{k}$ that best fits the model for each commodity.

We formalize this by following Calvet and Fisher (2004) and employ the likelihood ratio based test of Vuong (1989). For each commodity, and for each $\overline{k} \in \{1, ...9\}$, we test the null hypothesis that MSM(k) and MSM(10) fit the data equally well. The t-ratios and the one-sided p-values for each commodity is reported in Table 3.5. The results point to MSM(10) as the best model for most commodities, except for soybeans, cotton, sugar and heating oil. For subsequent analysis, we use MSM(10) for all commodities, since this is in fact the case for more than half of the commodities. This simplifies the analysis for frequency correlations that will be examined in

Table 3.5: MSM Model Selection (Vuong (1989) Test)

	$k = 1$	2	3	4	5	6	7	8	9
Gold	35.4609	36.2907	32.6450	22.1724	24.3488	7.3691	10.1158	10.1200	24.6326
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)
Copper	35.4609	36.2907	32.6450	22.1724	24.3488	7.3691	10.1158	10.1200	24.6326
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)
Cotton	31.3312	35.1696	26.9191	11.7848	0.2078	-4.2654	4.5933	-73.6646	-122.6326
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(0.5823)	(0.0000)	(1.0000)	(0.0000)	(0.0000)
Wheat	30.1439	31.6031	15.7399	11.8135	18.8707	18.6051	8.6388	20.6040	8.5423
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)
Oats	13.6299	19.1540	24.3873	37.4850	28.3026	40.5632	76.9155	81.5789	36.5245
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)
Sugar	12.5601	12.5039	6.8164	-1.3402	1.2103	1.9476	11.8140	27.1674	16.1607
	(1.0000)	(1.0000)	(1.0000)	(0.0901)	(0.8869)	(0.9742)	(1.0000)	(1.0000)	(1.0000)
Soybeans	14.7655	23.4738	18.1866	19.8916	1.4802	-15.7462	-25.0945	-16.0129	-21.5310
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(0.9306)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Corn	32.6087	22.2052	19.9819	17.0098	21.0894	63.2952	108.3995	146.4311	230.8582
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)
Heating	5.6273	5.1286	4.0475	8.5625	13.2418	3.7506	2.6438	-9.2963	-7.9567
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(0.9999)	(0.9959)	(0.0000)	(0.0000)
WTI	27.1841	36.9773	31.0284	38.6327	24.4031	32.5094	73.0767	145.0490	258.6135
	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)	(1.0000)

Notes: This table reports the t-ratios and the corresponding lower-tail p-values from the test of the null hypothesis that MSM(k) and MSM(10) fit the data equally well ($H_0 : lnL_T^k - lnL_T^{10} = 0$), against the alternative hypothesis that MSM(k) performed worst ($H_1 : lnL_T^k - lnL_T^{10} < 0$). Each column corresponds to the number of frequencies in the alternative MSM model being compared to MSM(10). For example, column $k = 1$ compares MSM(1) with MSM(10), for each commodity. A low p-value indicates that the corresponding MSM model will be rejected in favour of MSM(10). As the results indicate, all alternative MSM(k) models are rejected in favour of MSM(10) for each commodity, except for copper, cotton and sugar and heating oil.

Section 4.4. We also note that the comovement results are not sensitive to the choice of $\overline{k}$. The results are very similar, irrespective of the number of frequency components included in the MSM model. The question then becomes why isn't MSM(2) employed instead? The advantage of using MSM(10) is that it makes it possible to analyze comovements at greater number of frequencies (at 10 different frequencies, for MSM(10)), thereby making it possible to discern even the most subtle level of comovements. Also, relying on the Vuong test as a guide to model selection eliminates the arbitrariness of employing other MSM(k) models. The disadvantage however, is that it is difficult to provide a convincing economic intuition to justify 10 different kinds of shocks simultaneously affecting commodity prices, compared to when

MSM(2) is employed for instance.

We use the parameter estimates from MSM(10) for each commodity to compute the scaled residuals, $\hat{\eta}_{i,t}$ and $\hat{\eta}_{j,t}$, as defined in Equation (3.8). The correlation coefficients, $E(\hat{\eta}_{i,t}, \hat{\eta}_{j,t})$, and the associated p-values for the test of *hypothesis 1* are reported in Table 3.6. The statistics show a strong evidence of excess comovement between all of the commodities. We find that the largest absolute correlation is between crude oil and heating oil (0.7370), while the smallest absolute correlation is between gold and sugar (0.0625). More importantly, we find that the pairwise excess correlations are barely reduced from the simple correlations reported in Table 3.2. Also, note that the sign of the correlation statistics remain unchanged. All correlation values are positive, same as the simple correlation values reported in Table 3.6. Therefore, significant excess comovement remains among the commodities after controlling for the effects of macroeconomic fundamentals.

Table 3.6: Test of Zero Scaled Covariance (Hypothesis 1)

	Gold	Copper	Cotton	Wheat	Oats	Sugar	Soybeans	Corn	Heating	WTI
Gold		0.2106	0.0665	0.0691	0.0748	0.0625	0.0878	0.0877	0.1029	0.1260
Copper	0.0000		0.1097	0.1178	0.0758	0.1181	0.1445	0.1142	0.1568	0.1913
Cotton	0.0000	0.0000		0.1299	0.0906	0.1013	0.2173	0.1762	0.1075	0.1105
Wheat	0.0000	0.0000	0.0000		0.2807	0.0821	0.3555	0.4570	0.1083	0.1170
Oats	0.0000	0.0000	0.0000	0.0000		0.0870	0.3204	0.3764	0.0756	0.0856
Sugar	0.0000	0.0000	0.0000	0.0000	0.0000		0.1014	0.0959	0.0919	0.1199
Soybeans	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		0.5768	0.1551	0.1591
Corn	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		0.1423	0.1542
Heating	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		0.7370
WTI	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	

Notes: This table reports the correlation statistics, $\rho_{i,j} = \mathbb{E}(\eta_{i,t}, \eta_{j,t})$, and the associated p-values for the test of *hypothesis 1*. The correlation statistics are reported in the upper diagonal, while the p-values are reported in the lower diagonal. For example, $corr(Gold, Copper)$ is reported in row 1 and column 2, while the corresponding p-value is reported in column 1 and row 2. The null hypothesis is $H_0 : E(\eta_{i,t}, \eta_{j,t}) = 0$. The Lagrange multiplier test of the moment condition implied by the null hypothesis is given by $LM_{i,j} = T\rho^2_{i,j}$. This test statistic has an asymptotic chi-square distribution, with 1 degree of freedom. The associated p-values are reported in the lower diagonal. The results indicate that the correlation statistics are statistically significant for all commodity pairs.

A few more points are worth noting about the results. First, consistent with a priori expectations, we find that correlations are stronger between related commodities than between unrelated commodities. For example, the largest correlation with gold is associated with copper as both are precious metals, and the largest correlation with crude oil is associated with heating oil as both are energy commodities. This observation is not surprising, as related commodities are more likely to be substitutes, complements or in the case of agricultural commodities, even grown in the same geographical area, thereby subjected to the same weather conditions.

Second, we find a significant evidence of comovement between crude oil and other biofuel commodities (corn, sugar and soybean). Such significant correlations could arise from the substitution link between crude oil and other biofuel commodities. An increase in the price of crude oil leads to an increase in the demand for substitute biofuels. This in turn increases the demand for feedstock, which consequently leads to an increase in the equilibrium price of commodities such as corn, soybeans and sugar.

Lastly, we find that our results corroborate those of Pindyck and Rotemberg (1990). We find significant evidence of comovements between non-energy related commodities. For example, we find that copper is correlated with corn, wheat, sugar and cotton. But it is difficult to think of any fundamental links between copper and the aforementioned commodities, other than the fact that they are traded in the same index, e.g. Dow-Jones-UBS commodity index. An economically plausible explanation, however, is financialization and indexation. The indexation of commodities such as the Goldman Sachs Commodities Index (GSCI) and the Dow-Jones-UBS commodity index and increasing investments in commodities as a way to diversify risk away from traditional assets leads to increased comovements among commodities, irrespective of whether they are related or not (Barberis and Shleifer, 2003; Silvennoinen and Thorp, 2013; Tang and Xiong, 2010).

Financialization can induce comovement in commodities in two ways. First, if a large set of commodities are held by several investors with similar portfolios, in a period of bad news, these investors are likely to liquidate their asset holdings at the same time, causing comovement among commodity prices, irrespective of whether or not they are related by economic fundamentals. Second, Barberis and Shleifer (2003) argue that investors often allocate funds based on asset categories, a phenomenon they term "style investing". The investment problem of allocating funds across many individual assets is often complex and difficult. To overcome this, investors make decisions by first splitting assets into categories, and then allocating funds across these asset categories. Furthermore, investors choose their investment levels across asset categories by moving funds away from categories that have recently performed poorly, and into categories that have recently done well. Such a "style" investment approach exerts pressure on the demand for the performing asset categories, thereby causing their prices to comove beyond what can be explained by fundamentals.

In order to test *hypothesis 2*, we fit a bivariate MSM to each possible commodity pair, for $\overline{k}$ equals 1 to 8.[7] Using the Vuong (1989) test, we select the best MSM(k) model, which is MSM(8).[8] The results are reported in Table 3.7.[9] The estimates of $\hat{\rho}_\eta$ and their corresponding standard errors corroborate the evidence of excess comovement reported in Table 3.6. Recall that the test of hypothesis 2 is a much stronger test than hypothesis 1. The commovement of a pair of commodity may pass hypothesis 1 but fail hypothesis 2 because the former makes no distinction between conditional and unconditional comovement. The results indicate that there is significant evidence of conditional covariance between all the unique commodity pairs. Similar to the

[7] Due to computational constraints, $\overline{k} > 8$ was not considered for any commodity pairs.

[8] As in the univariate model, the best MSM(k) model varies across commodity pairs, ranging from 5 to 8. But we use $\overline{k} = 8$ for all commodity pairs, since this is in fact the case for 53% of the possible commodity pairs.

[9] Only the estimates and the corresponding p-values for ρ_η are reported in this Table. The full parameter estimates of the bivariate MSM models are reported in Appendix B.

results of the test of zero scaled covariance, the values in Table 3.7 are barely lower than the simple correlation values reported in Table 3.2.

Table 3.7: Test of Zero Conditional Covariance (Hypothesis 2)

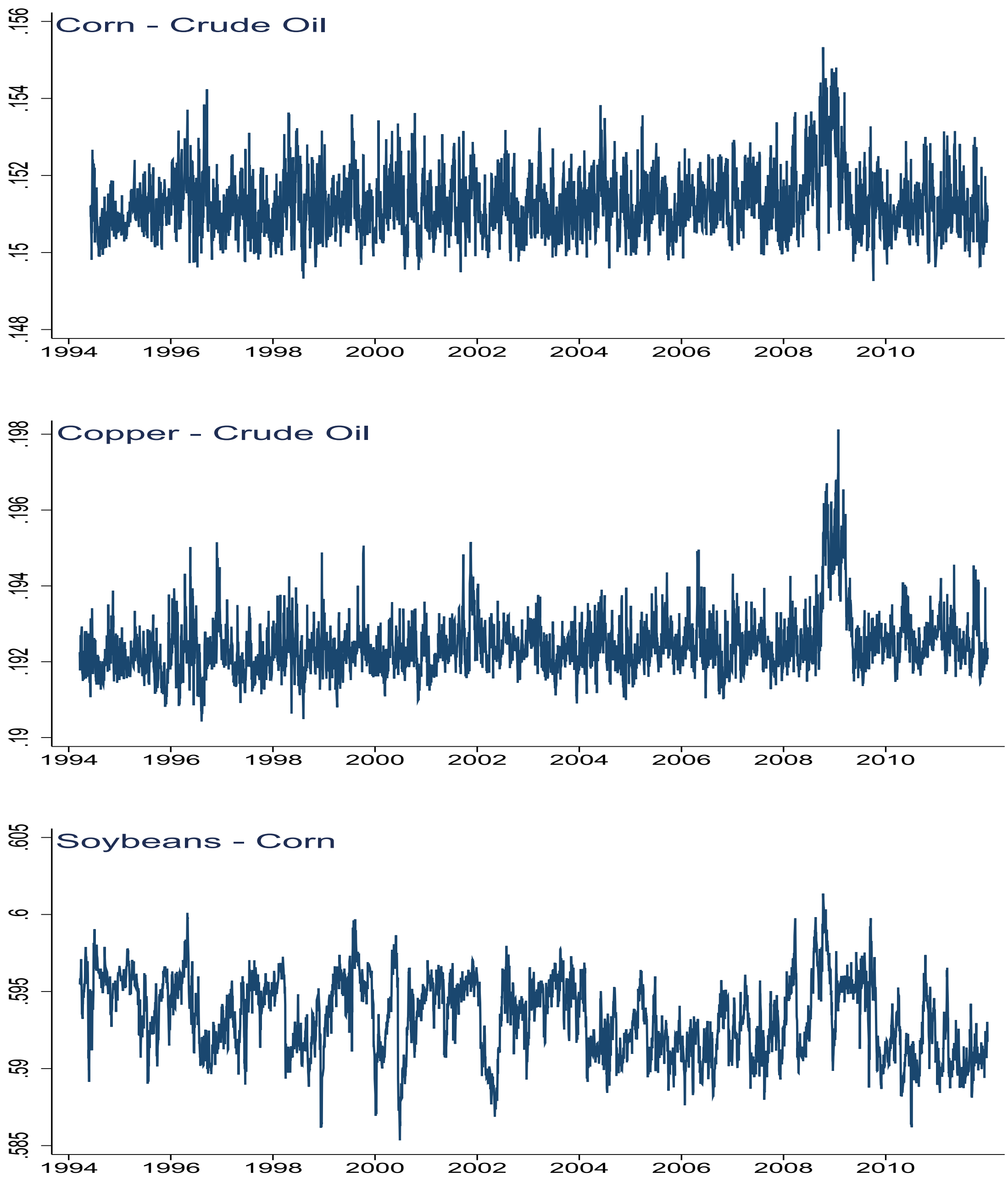
	Gold	Copper	Cotton	Wheat	Oats	Sugar	Soybeans	Corn	Heating	WTI
Gold		0.2250	0.0719	0.0730	0.0890	0.0675	0.0969	0.0945	0.1101	0.1365
Copper	0.0155		0.1149	0.1229	0.0852	0.1253	0.1536	0.1205	0.1670	0.2051
Cotton	0.0164	0.0161		0.1378	0.1052	0.1077	0.2311	0.1889	0.1128	0.1167
Wheat	0.0160	0.0154	0.0156		0.3204	0.0904	0.3763	0.4837	0.1123	0.1245
Oats	0.0172	0.0170	0.0172	0.0156		0.1000	0.3607	0.4267	0.0908	0.0977
Sugar	0.0167	0.0161	0.0166	0.0160	0.0173		0.1095	0.1035	0.0974	0.1269
Soybeans	0.0163	0.0157	0.0156	0.0149	0.0153	0.0167		0.6047	0.1647	0.1720
Corn	0.0164	0.0157	0.0158	0.0133	0.0144	0.0165	0.0114		0.1503	0.1639
Heating	0.0162	0.0164	0.0156	0.0154	0.0169	0.0162	0.0152	0.0159		0.7689
WTI	0.0167	0.0156	0.0160	0.0158	0.0170	0.0162	0.0156	0.0157	0.0058	

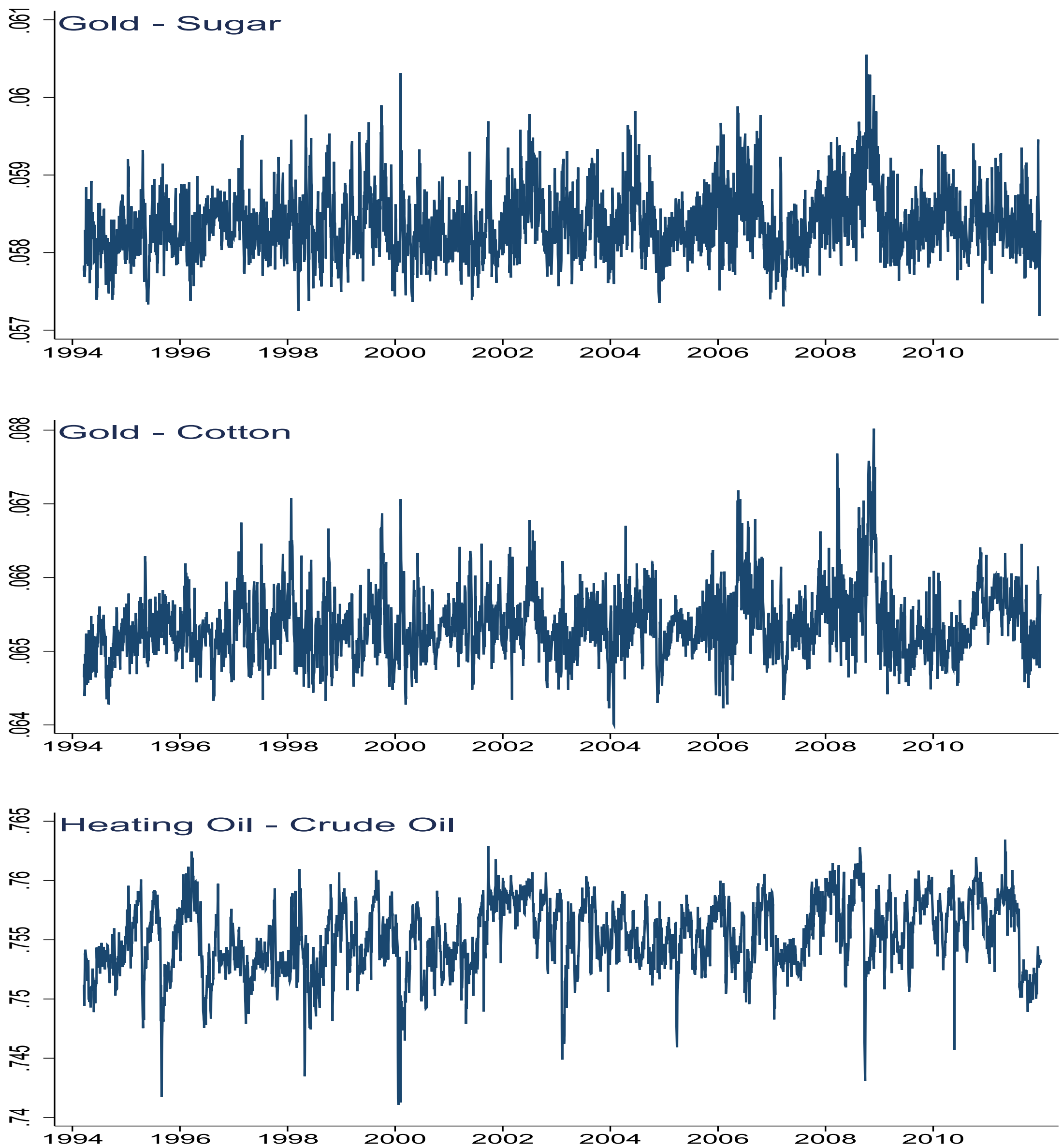
Notes: This table reports the estimates of ρ_η, and the associated p-values for the test of *hypothesis 2*. For each commodity pair, ρ_η is reported in the upper diagonal, while the associated p-value is reported in the corresponding lower diagonal. The null hypothesis is $H_0 : \mathbb{E}_t(\hat{\varepsilon}_{i,t+1}, \hat{\varepsilon}_{j,t+1}) = 0$. The null implies that the conditional covariance between commodities i and j, $Cov_t(\hat{\varepsilon}_{i,t+n}, \hat{\varepsilon}_{j,t+n})$, is constant and equal to zero. This implies that $\rho_\eta = 0$ in Equation (3.5). ρ_η is estimated by fitting a bivariate MSM(8) model to each commodity pair. The results indicate that the correlation statistics are statistically significant for all of the unique commodity pairs.

The correlation matrix from the scaled residuals only present the unconditional correlations at a snapshot in time. It does not show how the correlation dynamics evolve over time, and how they are affected by events in commodity markets. In order to see this, we use the parameter estimates from the bivariate MSM to compute the conditional correlations among the different commodity pairs. Figure 3.2 shows the conditional correlations for selected commodity pairs, namely corn - crude oil, heating oil - crude oil, soybeans - corn, copper - crude oil, gold - sugar and gold - cotton.

Looking at the corn - crude oil and copper crude - crude oil correlation dynamics, the graphs shows a major spike between 2007 and 2008. These periods coincide with periods of significant increases in oil prices, from \$51.91 in Jan 2007 to \$145.31 in July 2008, and from \$33.14 in Dec 2008 to \$112.38 in April 2011. Since crude oil is an essential input in the production of other commodities, an increase in the price of crude oil will lead to an increase in the price of other commodities. Moreover,

Figure 3.2: Dynamic Conditional Correlations Between Commodity Pairs





Notes: This figure illustrates the dynamics of the conditional correlations between selected commodity pairs. The conditional correlations, given by Equation (3.6), are calculated using the bivariate MSM(8) parameter estimates.

the spikes in correlation between corn and crude oil may also be explained, to some extent, by the substitution link between corn and crude oil. Since biofuels are made from corn (among other commodities), an increase in the price of crude oil will lead to an increase in the equilibrium quantity demanded for crude oil alternatives. An increased supply of biofuels requires an increase in the demand for inputs such as corn. Therefore, increase in crude oil prices will consequently lead to an increase in the price of corn.

Figure 3.2 also shows that the correlation between other non-energy commodity pairs increased significantly in 2008. This can be observed on the soybeans - corn, gold - cotton and gold - sugar conditional correlation charts. This spikes coincide with the 2008 financial crisis that led to a sustained period of high uncertainty in the global economy. Commodities are often viewed as a safe haven during crisis periods when traditional financial assets become too risky to hold. Moreover, the observation corroborates existing evidence that points to the dynamic nature of comovement among financial assets (Bollerslev et al., 1992, 1988; Erb et al., 1994; Moskowitz, 2003). Assets that are seemingly uncorrelated in tranquil periods may become excessively correlated in periods of high uncertainty in the economy. High comovements during periods of high volatility can be as a result of correlated information that arises in attempts by rational agents to infer information from price changes in other markets (King and Wadhwani, 1990). Also, high comovements can also be as a result of correlated liquidity shocks that arise as uninformed but rational investors try to extract information from informed investors that are liquidating their existing asset holdings (Calvo, 1999). For example, the possibility that informed investors are forced to sell securities in a particular market due to margin calls may be perceived by uninformed investors as a signal of low returns in such market. Consequently, the uninformed investors may also follow suit by liquidating their existing positions in such market.[10]

Lastly, we also observe that the conditional correlation between crude oil and heating oil is consistently high, consistent with *a priori* expectations. Heating oil is a by-product of crude oil, hence the strong comovement between the two energy commodities.

3.4.4. Frequency Decomposition

The analysis so far only addresses the question of whether or not significant excess comovement exist among commodities. Having established that excess comovement does exist among commodities, we then address the question of whether such comovement is a short term or long term phenomenon, or a combination of both. In order to address this question, the maximum likelihood estimates from the univariate MSM(10) models are used to compute, for each of the commodities, the smoothed state probabilities and the conditional expectations of each frequency component i.e. $\hat{M}^i_{k,t} = \mathbb{E}(M^i_{k,t}|r_1, r_2, ...r_T)$, for $i = 1, 2, ..., N$ and $k = 1, 2, ..., 10$. This approach is similar to the band-pass filter of Baxter and King (1999) or the H-P filter of Hodrick and Prescott (1997). The difference between these filters and the MSM frequency decomposition is that the band-pass and the H-P filters decompose a time-series into only a trend and cyclical components, while the MSM frequency decomposition is able to decompose the series into many frequencies, as may be inferred from the data.

To illustrate our findings, we present results for several pairs heating oil - crude oil (related commodities), soybeans - corn (related commodities), copper - crude oil

[10]Several other explanations have been proposed for the reasons why comovements increase in periods of high volatility (De Gregorio and Valdes, 2001; Eichengreen et al., 2001; Forbes and Rigobon, 2002; Lagunoff and Schreft, 2001; Van Rijckeghem and Weder, 2003).

(unrelated commodities), soybeans - cotton (unrelated commodities), gold - sugar (unrelated commodities) and heating oil - cotton (unrelated commodities).[11] The correlations of smoothed frequency components, $Corr(\hat{M}^i_{k,t}, \hat{M}^j_{k,t})$, for $i, j = 1, 2, ..., N$ and $k = 1, 2, ..., 10$ are presented in Table 3.8.

First, we observe that correlations are stronger at lower frequencies, and tend to decline as k increases. For example, consider soybeans and corn. The correlation between their lowest volatility frequencies ($Corr(\hat{M}^{soybeans}_{1,t}, \hat{M}^{corn}_{1,t})$) is 0.9428 (the element at the intersection of row $Soy1$ and column $Corn1$), while the correlation between their highest volatility frequencies ($Corr(\hat{M}^{soybeans}_{10,t}, \hat{M}^{corn}_{10,t})$) is 0.3890 (the element at the intersection of row $Soy10$ and column $Corn10$). Similarly, consider copper and crude oil. Although categorized as unrelated, they are both industrial commodities, whose prices tend to fluctuate with the level of economic activities. The correlation between their lowest volatility frequencies ($Corr(\hat{M}^{copper}_{1,t}, \hat{M}^{crude-oil}_{1,t})$) is 0.6745 (the element at the intersection of row *cop*1 and column $WTI1$), while the correlation between their highest volatility frequencies ($Corr(\hat{M}^{copper}_{10,t}, \hat{M}^{crude-oil}_{10,t})$) is 0.0839 (the element at the intersection of row *cop*10 and column $WTI10$).

Table 3.8: Correlation of Smoothed Frequency Components

Gold - Copper										
	Cop1	Cop2	Cop3	Cop4	Cop5	Cop6	Cop7	Cop8	Cop9	Cop10
Gold1	0.8822	0.7754	0.3269	0.1117	0.0424	0.0663	0.0329	0.0144	0.0071	0.0060
Gold2	0.8818	0.7776	0.3324	0.1142	0.0473	0.0687	0.0345	0.0151	0.0074	0.0062
Gold3	0.8553	0.7739	0.3804	0.1672	0.1067	0.0921	0.0475	0.0213	0.0101	0.0085
Gold4	0.3275	0.3565	0.3775	0.3363	0.2997	0.1652	0.0892	0.0428	0.0201	0.0166
Gold5	0.0167	0.0482	0.1539	0.2941	0.2870	0.1752	0.1034	0.0511	0.0243	0.0202
Gold6	0.0943	0.1137	0.1280	0.2240	0.2624	0.2577	0.1816	0.0941	0.0460	0.0384
Gold7	0.0767	0.0869	0.0820	0.1206	0.1527	0.1956	0.1773	0.1135	0.0626	0.0536
Gold8	0.0320	0.0360	0.0318	0.0461	0.0620	0.0878	0.1022	0.0955	0.0745	0.0692
Gold9	0.0135	0.0150	0.0124	0.0190	0.0275	0.0405	0.0538	0.0707	0.0842	0.0855
Gold10	0.0113	0.0125	0.0103	0.0160	0.0234	0.0348	0.0473	0.0662	0.0848	0.0874

Continued on next page

[11] Although we report results for only a few commodity pairs, the results are similar across the other pairs not reported.

Table 3.8 – Continued from previous page

Soybeans - Corn										
	Corn1	Corn2	Corn3	Corn4	Corn5	Corn6	Corn7	Corn8	Corn9	Corn10
Soy1	0.9428	0.8414	0.6391	0.2880	0.1593	0.0999	0.0566	0.0280	0.0142	0.0084
Soy2	0.9147	0.8159	0.6319	0.2903	0.1654	0.1059	0.0605	0.0304	0.0156	0.0092
Soy3	0.8513	0.7537	0.6406	0.3216	0.1999	0.1342	0.0792	0.0418	0.0221	0.0130
Soy4	0.2062	0.3334	0.5480	0.4745	0.3603	0.2673	0.1714	0.0984	0.0544	0.0326
Soy5	0.1512	0.2860	0.5464	0.6562	0.6084	0.5027	0.3358	0.1984	0.1118	0.0675
Soy6	0.0889	0.1865	0.4187	0.6289	0.6739	0.6483	0.4854	0.3108	0.1849	0.1147
Soy7	0.0445	0.0925	0.2198	0.3509	0.4183	0.4966	0.4907	0.4082	0.2926	0.2015
Soy8	0.0211	0.0447	0.1026	0.1622	0.2011	0.2680	0.3362	0.3920	0.3930	0.3377
Soy9	0.0110	0.0218	0.0474	0.0743	0.0935	0.1316	0.1873	0.2720	0.3636	0.3924
Soy10	0.0095	0.0188	0.0405	0.0633	0.0799	0.1134	0.1646	0.2473	0.3464	0.3890
Heating Oil - Crude Oil										
	WTI1	WTI2	WTI3	WTI4	WTI5	WTI6	WTI7	WTI8	WTI9	WTI10
Ht1	-0.2057	0.4554	0.6170	0.2853	0.0731	0.0267	0.0112	0.0062	0.0037	0.0034
Ht2	0.0628	0.6086	0.6836	0.3120	0.0948	0.0445	0.0227	0.0125	0.0076	0.0070
Ht3	0.3175	0.7220	0.7130	0.3703	0.1590	0.0905	0.0499	0.0269	0.0159	0.0146
Ht4	0.5619	0.7515	0.6635	0.4880	0.2999	0.1892	0.1067	0.0568	0.0330	0.0302
Ht5	0.5617	0.6173	0.5510	0.5902	0.4338	0.2825	0.1601	0.0855	0.0495	0.0454
Ht6	0.4593	0.5025	0.4980	0.6535	0.5391	0.3599	0.2076	0.1124	0.0655	0.0600
Ht7	0.3340	0.3920	0.4339	0.6387	0.6102	0.4428	0.2687	0.1493	0.0881	0.0809
Ht8	0.2172	0.2646	0.3209	0.5311	0.6026	0.5202	0.3497	0.2032	0.1224	0.1126
Ht9	0.1143	0.1471	0.1992	0.3751	0.5218	0.5727	0.4471	0.2775	0.1720	0.1588
Ht10	0.0623	0.0787	0.1150	0.2437	0.4015	0.5557	0.5169	0.3520	0.2275	0.2112
Copper - Crude Oil										
	WTI1	WTI2	WTI3	WTI4	WTI5	WTI6	WTI7	WTI8	WTI9	WTI10
Cop1	0.6745	0.0192	-0.3064	-0.1514	0.0052	0.0253	0.0182	0.0089	0.0051	0.0047
Cop2	0.5808	-0.0439	-0.3460	-0.1384	0.0293	0.0399	0.0251	0.0120	0.0066	0.0061
Cop3	0.3472	0.0217	-0.1791	0.0636	0.1186	0.0860	0.0473	0.0225	0.0122	0.0111
Cop4	0.2409	0.1527	0.0649	0.2628	0.2267	0.1490	0.0784	0.0377	0.0209	0.0190
Cop5	0.1493	0.1475	0.1241	0.2698	0.2954	0.1983	0.1046	0.0522	0.0299	0.0274
Cop6	0.1275	0.1154	0.0963	0.1753	0.2490	0.2108	0.1238	0.0659	0.0389	0.0357
Cop7	0.0643	0.0609	0.0547	0.0990	0.1530	0.1556	0.1055	0.0661	0.0441	0.0412
Cop8	0.0301	0.0305	0.0291	0.0520	0.0772	0.0850	0.0759	0.0764	0.0667	0.0644
Cop9	0.0144	0.0144	0.0139	0.0254	0.0378	0.0447	0.0520	0.0762	0.0835	0.0832
Cop10	0.0120	0.0119	0.0115	0.0211	0.0316	0.0379	0.0464	0.0727	0.0837	0.0839
Soybeans - Cotton										
	Cot1	Cot2	Cot3	Cot4	Cot5	Cot6	Cot7	Cot8	Cot9	Cot10
Soy1	0.9369	0.9058	0.8159	0.1674	0.0810	0.0374	0.0297	0.0138	0.0078	0.0068
Soy2	0.9423	0.9287	0.8609	0.2111	0.0688	0.0402	0.0362	0.0167	0.0089	0.0078
Soy3	0.5940	0.5834	0.5493	0.3864	0.2485	0.1230	0.0764	0.0335	0.0161	0.0138
Soy4	0.1373	0.1324	0.1607	0.3273	0.3343	0.1840	0.0874	0.0363	0.0166	0.0142
Soy5	0.0961	0.0958	0.1090	0.2081	0.3037	0.2173	0.1073	0.0468	0.0214	0.0183
Soy6	0.0394	0.0411	0.0560	0.0989	0.2044	0.1970	0.1127	0.0531	0.0257	0.0221
Soy7	0.0135	0.0149	0.0254	0.0651	0.1192	0.1228	0.0827	0.0584	0.0371	0.0332
Soy8	0.0051	0.0059	0.0113	0.0312	0.0556	0.0598	0.0569	0.0757	0.0704	0.0669
Soy9	0.0028	0.0032	0.0060	0.0152	0.0274	0.0304	0.0362	0.0689	0.0876	0.0879
Soy10	0.0025	0.0029	0.0054	0.0133	0.0239	0.0267	0.0328	0.0658	0.0879	0.0889

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Table 3.8 – Continued from previous page

Heating Oil - Cotton										
	Cot1	Cot2	Cot3	Cot4	Cot5	Cot6	Cot7	Cot8	Cot9	Cot10
Ht1	-0.7188	-0.4807	-0.2460	0.0277	0.1606	0.1622	0.1182	0.0746	0.0354	0.0195
Ht2	-0.6152	-0.3572	-0.1168	0.1389	0.2353	0.2117	0.1425	0.0884	0.0408	0.0223
Ht3	-0.2990	-0.0966	0.0678	0.2106	0.2642	0.2493	0.1802	0.1120	0.0511	0.0236
Ht4	-0.0774	-0.0144	0.0376	0.0930	0.1453	0.1630	0.1259	0.0803	0.0407	0.0190
Ht5	0.0008	0.0444	0.0672	0.0612	0.0379	0.0219	-0.0032	-0.0144	-0.0042	0.0004
Ht6	-0.0058	0.0135	0.0293	0.0416	0.0447	0.0424	0.0300	0.0284	0.0488	0.0488
Ht7	-0.0012	0.0051	0.0123	0.0222	0.0286	0.0306	0.0295	0.0366	0.0526	0.0427
Ht8	-0.0004	0.0020	0.0054	0.0111	0.0153	0.0168	0.0168	0.0199	0.0257	0.0189
Ht9	-0.0004	0.0007	0.0025	0.0058	0.0082	0.0092	0.0095	0.0112	0.0144	0.0121
Ht10	-0.0003	0.0006	0.0022	0.0051	0.0073	0.0082	0.0084	0.0100	0.0128	0.0109
Gold - Sugar										
	Sug1	Sug2	Sug3	Sug4	Sug5	Sug6	Sug7	Sug8	Sug9	Sug10
Gold1	0.6201	0.5678	0.3062	0.1385	0.0554	0.0297	0.0166	0.0092	0.0061	0.0058
Gold2	0.6163	0.5660	0.3070	0.1394	0.0557	0.0300	0.0169	0.0093	0.0062	0.0059
Gold3	0.5774	0.5524	0.3118	0.1443	0.0597	0.0334	0.0191	0.0104	0.0068	0.0065
Gold4	0.0908	0.1341	0.0933	0.0622	0.0365	0.0248	0.0159	0.0090	0.0058	0.0054
Gold5	-0.0211	0.0408	0.0813	0.0937	0.0694	0.0416	0.0253	0.0150	0.0104	0.0099
Gold6	0.0160	0.0824	0.1537	0.1662	0.1350	0.0892	0.0534	0.0303	0.0209	0.0199
Gold7	0.0132	0.0481	0.0995	0.1331	0.1277	0.1014	0.0694	0.0413	0.0286	0.0273
Gold8	0.0073	0.0220	0.0443	0.0605	0.0661	0.0674	0.0583	0.0383	0.0269	0.0255
Gold9	0.0047	0.0112	0.0202	0.0264	0.0302	0.0342	0.0326	0.0225	0.0153	0.0144
Gold10	0.0042	0.0097	0.0172	0.0224	0.0257	0.0294	0.0282	0.0194	0.0128	0.0119

Notes: This table reports the correlations from a frequency decomposition of univariate MSM with $k = 10$ components for commodity pairs heating oil - crude oil, soybeans - corn, copper - crude oil, soybeans - cotton and heating oil - cotton. First, a univariate MSM(10) is fitted to each commodity residual vector derived from Equation (3.1). Second, using the parameter estimates for each commodity, the smoothed state probabilities and the conditional expectations of each frequency component i.e. $\hat{M}^i_{k,t} = \mathbb{E}(M^i_{k,t}|r_1, r_2, ...r_T)$ are calculated, where $\hat{M}^i_{k,t}$ is a T(number of observations) by 10 matrix and $i = 1, 2, ...N$. Each element in the table corresponds to $Corr(\hat{M}^i_{k,t}, \hat{M}^j_{k,t})$ for i, j commodity pair. For example, consider heating oil and crude oil. The correlation between $\hat{M}_{1,t}$ for heating oil and $\hat{M}_{1,t}$ for crude oil is 0.1880. This is the element under $Ht1$ and $WTI1$. The correlation between $\hat{M}_{1,t}$ for heating oil and $\hat{M}_{10,t}$ for crude oil is 0.0376. This is the element under $Ht1$ and $WTI10$. Note that the correlation matrix for each pair is not symmetric. Again, consider heating oil and crude oil. $Corr(\hat{M}_{1,t}, \hat{M}_{2,t})$ (the element under $Ht1$ and $WTI2$) does not equal $Corr(\hat{M}_{2,t}, \hat{M}_{1,t})$ (the element under $Ht2$ and $WTI1$). The former is the correlation between $\hat{M}_{1,t}$ of heating oil and $\hat{M}_{2,t}$ of crude oil, while the latter is the correlation between $\hat{M}_{2,t}$ of heating oil and $\hat{M}_{1,t}$ of crude oil.

This observation also applies to commodities that exhibit weak correlations in Table 3.6, such as gold and sugar. Although the correlation between gold and sugar prices reported in Table 3.6 is statistically significant, it is economically small in absolute value compared to other commodity pairs. The results of the frequency decomposition further reveals that such correlation is driven by low frequency fundamentals, and that the correlations are significantly small at higher frequencies.

Second, we find that high frequency correlations are stronger between related commodities (soybeans - corn, heating oil - crude oil and gold - copper) than between unrelated commodities (soybeans - cotton, heating - oil cotton and gold - sugar), which is consistent with a priori expectations. Such high frequency correlation may be driven by high frequency factors such as spreading and liquidity constraints.

Lastly, we observe that all commodity pairs exhibit strong low frequency correlations, irrespective of whether they are related or not. For example, the gold - sugar correlation (0.0625) is economically weak in Table 3.6. Their low frequency correlations is high (0.6201), although it rapidly declines from $k = 4$ to $k = 10$. Therefore, we infer that all commodities exhibit strong long-run comovements which are driven by low frequency fundamentals which may involve factors such as weather, demographic and macroeconomic variables. But related commodities also exhibit strong short-run comovements that may arise from liquidity constraints, indexation, etc.

These results corroborate the findings of Lescaroux (2009) and Malliaris and Urrutia (1996), who also find significant evidence of long term comovement between the prices of commodities. But our results differ from theirs in terms of short term comovement. While we document the presence of short term comovements between related commodities, Lescaroux (2009) find only a weak evidence of short term comovement and Malliaris and Urrutia (1996) find no evidence of short term comovement.

3.5. Robustness Analysis

The tests of excess comovement conducted in section 4 are based on the standard approach for testing excess commodity comovement. Moreover, the R^2 values reported in Table 3.3 indicate that the fundamental variables do not explain much of the observed variations in commodity prices.[12] This suggests that there might be other factors driving the observed correlations, but that are not captured in the regression model. As mentioned in Section 2, these include such factors as weather, the profit maximizing behaviours of farmers, and supply side factors such as inventories, yields, etc.

In order to ensure that this is not driving our results, we adopt a second approach for filtering out the effects of shared fundamental factors. The approach is based on the theory of storage.[13] The Kaldor (1939) theory of storage states that the spread between futures and spot prices is driven by fundamental demand and supply factors. In other words, the dynamics of spot and futures prices should reflect the convenience yield, cost of storage and stock levels, where the convenience yield is a decreasing function of the stock level and the storage cost is an increasing function of the stock level. By implication, macroeconomic fundamentals should affect commodity prices through their effects on stock levels in the short run. Furthermore, the distinction between short and long run variations in commodity prices dynamics are stressed by Schwartz and Smith (2000) by modeling commodity prices as comprising an equilibrium factor and a mean-reverting stochastic factor.

This implies that the excess comovement hypothesis can be tested by filtering out the effects of shared fundamentals through the stock levels. To achieve this, we decompose the logarithm of commodity prices and stock levels into a trend and cyclical component using the Kydland and Prescott (1990) filter. Equation (3.1) is reformulated in terms of log-prices and re-estimated as follows

[12] This is also the case with earlier studies in the literature.

[13] A similar approach was used by Lescaroux (2009) and Ai et al. (2006) but arrived at different conclusions from ours.

$$P_{i,t}^c = \beta_0 P_{i,t-1}^c + \sum_{j=0}^{J} \beta_{i,j} S_{t-j}^c + \varepsilon_{i,t} \tag{3.10}$$

where $P_{i,t}^c$ is the cyclical component of the log-price of commodity i and $S_{i,t}^c$ is the cyclical component of the logarithm of the stock level for commodity i. We do this for commodities for which daily stock level data are available, namely, WTI crude oil, copper, aluminium, tin, zinc, nickel and lead. All of these commodities are traded on the London Metal Exchange and quoted in U.S. dollars. The stock level data do not correspond to global stock levels, which are unknown. Rather, the metal stocks are stocks in LME warehouses and the oil stock corresponds to the U.S. oil stock level. To serve as a benchmark for interpreting the excess comovement tests, the simple correlations between the commodities are reported in Table 3.9. As expected, there are strong correlations between the metal commodities as well as between metals and crude oil. All correlations are significant at the 1% level, ranging from 0.5940 (tin and zinc) to 0.9220 (copper and lead).

Table 3.9: Simple Correlations of Metals and Crude Oil

	WTI	Copper	Aluminium	Tin	Zinc	Nickel	Lead
WTI		0.9171	0.8428	0.8791	0.6821	0.7602	0.8718
Copper			0.9117	0.9050	0.8335	0.8460	0.9220
Aluminium				0.7640	0.8055	0.8141	0.8036
Tin					0.5940	0.6872	0.8830
Zinc						0.8967	0.7532
Nickel							0.8134
Lead							

Notes: This table presents the simple bivariate correlation coefficients between each commodity pair. The correlation coefficient r has a test statistic $t = r\sqrt{N-2}/\sqrt{1-r^2}$, which has an asymptotic $t-$distribution with $N-2$ degrees of freedom, where N is the number of observations. For $N = 4397$, the critical correlation coefficients are 0.0388, 0.0296 and 0.0248 for 1%, 5% and 10% significance levels, respectively. All correlation coefficients are significant at 1%, 5% or 10% significant levels.

In order to test for excess comovement, we estimate Equation (3.10) for the commodities and calculate the residuals. The regression result are presented in Table 3.10. As can be observed from the results, the R^2 increased significantly compared

Table 3.10: Regression Results For Commodities With Stocks

	Cyclical Components of Log-Commodity Prices ($P^c_{i,t}$)						
	WTI	Copper	Aluminium	Tin	Zinc	Nickel	Lead
$P^c_{i,t-1}$	0.9936***	0.9961***	0.9976***	0.9970***	0.9982***	0.9967***	0.9942***
	(0.0027)	(0.0017)	(0.0017)	(0.0021)	(0.0020)	(0.0018)	(0.0024)
$S^c_{i,t}$	0.0826*	-0.0704***	-0.0222	-0.0365***	-0.1008**	-0.0583***	-0.0734***
	(0.0497)	(0.0179)	(0.0302)	(0.0122)	(0.0405)	(0.0180)	(0.0193)
$S^c_{i,t-1}$	-0.0826	0.0695***	0.0235	0.0367***	0.1015**	0.0581***	0.0712***
	(0.0503)	(0.0179)	(0.0302)	(0.0123)	(0.0403)	(0.0179)	(0.0192)
N	4,397	4,397	4,397	4,397	4,397	4,397	4,397
R^2	0.9872	0.9949	0.9945	0.9932	0.9957	0.9940	0.9929
$\overline{R}^2$	0.9872	0.9949	0.9945	0.9932	0.9957	0.9940	0.9929
$F-stat$	55863	205305	135803	183562	202118	146887	145544
$P-value$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Notes: This table presents the results from regressing the cyclical components of the logarithm of commodity prices on the cyclical components of the logarithm of stock levels, where $P^c_{i,t}$ and $S^c_{i,t}$ are the cyclical components of the logarithm of price and the logarithm of stock level respectively. The cyclical components are derived by applying the Hodrick and Prescott (1997) filter to the logarithm of commodity prices and the logarithm of stock levels. The smoothing parameter λ is determined using the Ravn and Uhlig (2002) rule that sets λ to $1600p_q^4$, where p_q is the number of periods per quarter. Standard errors are heteroscedasticity and autocorrelation consistent. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

to those reported in Table 3.3. Consider copper for example. The same copper price series used in Table 3.3 are also used in Table 3.10. The R^2 increased from 0.0531 to 0.9949. This indicates that this approach is better able to filter out the effects of shared macroeconomic fundamental factors, including the effects of the supply side factors. *Ceteris paribus*, we should observe a lower level of comovement between the commodities in this group.

In the next step, we fit univariate MSM models to the residuals from Equation (3.10) and repeat the tests of zero scaled covariance for the commodities. The pairwise correlation results are reported in Table 3.11.[14] The results are in fact identical to those in Table 3.6. The null hypothesis of zero-comovement is strongly rejected in all of the possible cases. Although, the pairwise correlations decreased substantially from their original levels in Table 3.10 ranging from 0.1107 (crude oil and tin) to 0.6291 (copper

[14] A univariate MSM is fitted for all the commodities for $\overline{k} = 1$ to $\overline{k} = 10$. The Vuong (1989) tests point to optimal $\overline{k} = 10$ for all the commodities.

Table 3.11: **Robustness Analysis (Test of Zero Scaled Covariance)**

	WTI	Copper	Aluminium	Tin	Zinc	Nickel	Lead
WTI		0.1914	0.1552	0.1099	0.1599	0.1561	0.1638
Copper	0.0000		0.5144	0.2710	0.6287	0.5612	0.5327
Aluminium	0.0000	0.0000		0.1837	0.4683	0.3926	0.3852
Tin	0.0000	0.0000	0.0000		0.2616	0.2550	0.2272
Zinc	0.0000	0.0000	0.0000	0.0000		0.5213	0.5903
Nickel	0.0000	0.0000	0.0000	0.0000	0.0000		0.4522
Lead	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	

Notes: This table reports the correlation statistics, $\hat{\rho}_{i,j} = \mathbb{E}(\hat{\eta}_{i,t}, \hat{\eta}_{j,t})$, and the associated p-values for the test of *hypothesis 1*, for commodities with stock level data. The correlation statistics are reported in the upper diagonal, while the p-values are reported in the lower diagonal. The null hypothesis is $H_0 : E(\eta_{i,t}, \eta_{j,t}) = 0$. The Lagrange multiplier test of the moment condition implied by the null hypothesis is given by $LM_{i,j} = T\hat{\rho}^2_{i,j}$. This test statistic has an asymptotic chi-square distribution, with 1 degree of freedom. The associated p-values are reported in the lower diagonal. The results indicate that the correlation statistics are statistically significant for all commodity pairs.

and zinc), they remain strongly significant. Therefore, the results corroborate our initial findings from Section 4 that there is significant excess comovement between commodities. We note, however, that our results contradict those documented by Lescaroux (2009) and Ai et al. (2006) who adopted similar approaches but find weak or no evidence of comovement. While they used monthly and quarterly data, we employed daily data. Therefore, our data is able to accommodate more frequent propagation of shocks between the commodities. Hence, the difference in results.

In order to test *hypothesis 2*, we fit bivariate MSM(8) to all the residuals from Equation (3.10). The results from the test of zero conditional covariance are reported in Table 3.12. The results essentially corroborate the initial evidence of significant comovement between commodities, beyond what can be explained by shared macroeconomic fundamentals. All the estimates of ρ_η are significant at the 5% significance level, indicating that the conditional covariance between the commodity pairs are significantly different from zero, and time varying.

Table 3.12: Robustness Analysis (Test of Zero Conditional Covariance)

	WTI	Copper	Aluminium	Tin	Zinc	Nickel	Lead
WTI		0.2046	0.1635	0.1214	0.1738	0.1642	0.1770
Copper	0.0155		0.5374	0.3059	0.6609	0.5902	0.5690
Aluminium	0.0159	0.0113		0.2079	0.4925	0.4135	0.4137
Tin	0.0168	0.0156	0.0168		0.2915	0.2880	0.2591
Zinc	0.0159	0.0082	0.0120	0.0171		0.5422	0.6193
Nickel	0.0158	0.0100	0.0133	0.0159	0.0118		0.4758
Lead	0.0156	0.0111	0.0137	0.0158	0.0102	0.0121	

Notes: This table reports the estimates of ρ_η, and the associated p-values for the test of *hypothesis 2*. For each commodity pair, ρ_η is reported in the upper diagonal, while the associated p-value is reported in the corresponding lower diagonal. The null hypothesis is $H_0 : \mathbb{E}_t(\hat{\varepsilon}_{i,t+1}, \hat{\varepsilon}_{j,t+1}) = 0$. The test implies that the conditional covariance between commodities i and j, $Cov_t(\hat{\varepsilon}_{i,t+n}, \hat{\varepsilon}_{j,t+n})$, is constant and equal to zero. This implies that $\rho_\eta = 0$ in Equation (3.5). ρ_η is estimated by fitting a bivariate MSM(8) model to each commodity pair. The results indicate that the correlation statistics are statistically significant for all commodity pairs.

3.6. Conclusion

This study addresses the question of whether there is excess comovement among commodity prices, as described by Pindyck and Rotemberg (1990). We adopt a two-step approach that combines the Pindyck and Rotemberg (1990) structural model with an MSM model that accounts for heteroscedasticity, non-normality and the multi-frequency nature of volatility in commodity prices. Moreover, unlike earlier studies, we use daily commodity prices that are able to capture more rapid propagation of shocks among different commodity markets. Our findings suggest that there is significant comovement between commodity prices, beyond what can be explained by shared macroeconomic fundamentals.

Furthermore, decomposing comovements into multiple frequencies, we find that all commodities exhibit strong long-run excess comovements which are driven by low frequency fundamentals such as weather, demographic and macroeconomic factors.

But some commodities also exhibit significant short-run excess comovements that may be attributable to short-run factors such as liquidity constraints, indexation, etc.

These findings have significant welfare and risk management implications. On the welfare side, Runge and Senauer (2007) warn that increasing comovement between oil and food prices, as a result of the diversification of resources away from food to biofuel, could profoundly disrupt the relationships between food producers and consumers in the long run, with potentially devastating implications for both global poverty and food security. On the risk management side, when the comovement structure between assets are ignored in a portfolio allocation or dynamic hedging process, investors tend to allocate more resources to risky assets, thereby exposing them to more risk.

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3.7. Appendix

3.7.1. Data Description

Table 3.13: Data Description

Commodity	Exchange	Price	Category	Source
West Texas Intermediate Crude Oil	NYMEX	\$US/Barrel	Energy	Energy Information Administration
New York Harbor No. 2 Heating Oil	NYMEX	\$US/Gallon	Energy	Energy Information Administration
Raw Sugar, ISA Daily Price	NYBOT	Cents/LB	Soft	Datastream Database
Cotton	NYBOT	Cents/LB	Soft	Datastream Database
Corn, No.2 Yellow	CBOT	Cents/Bushel	Grains	Datastream Database
Soybeans, No.1 Yellow	CBOT	Cents/Bushel	Grains	Datastream Database
Oats, No.2 Milling Minneapolis	CBOT	Cents/Bushel	Grains	Datastream Database
Wheat, No.2 Hard Kansas	KCBT	Cents/Bushel	Grains	Datastream Database
Gold, Handy & Harman	COMEX	\$US/Troy Oz	Metals	Datastream Database
Copper, High Grade	COMEX	\$US/LB	Metals	Datastream Database
Copper , Grade A	LME	\$US/Metric Ton	Metals	Datastream Database
Aluminium	LME	\$US/Metric Ton	Metals	Datastream Database
Zinc	LME	\$US/Metric Ton	Metals	Datastream Database
Nickel	LME	\$US/Metric Ton	Metals	Datastream Database
Lead	LME	\$US/Metric Ton	Metals	Datastream Database
Tin	LME	\$US/Metric Ton	Metals	Datastream Database

3.7.2. Model Selection

To apply the Vuong test, we assume two non-nested models MSM(k) and MSM(k'), with densities f and f' respectively. The log-likelihood difference is given by

$$LR_v = T^{-1/2}(lnL_T^f(\hat{\theta}_T) - lnL_t^{f'}(\hat{\theta}'_T)) = \frac{1}{\sqrt{T}}\sum_{t=1}^{T} ln\frac{f(r_t|R_{t-1})}{f'(r_t|R_{t-1})}$$

We consider the null hypothesis that models MSM(k) and MSM(10) have identical unconditional expected log-likelihood (MSM(k) and MSM(10) fit the data equally well), against the alternative that MSM(k) performed worst. Specifically, we test the following conditions

$$H_0 : lnL_T^k - lnL_T^{10} = 0, \text{ for } k \in 1, 2, ..., 9$$
$$H_1 : lnL_T^k - lnL_T^{10} < 0$$

Under the null hypothesis,

$$T^{-1/2}(lnL_T^f(\hat{\theta}_T) - lnL_t^{10}(\hat{\theta}'_T)) \xrightarrow{d} N(0, \sigma_*^2)$$

where

$$\sigma_*^2 = Var\left(ln\left[\frac{f^k(r_t|R_{t-1})}{f^10(r_t|R_{t-1})}\right]\right)$$

3.7.3. Bivariate MSM Parameter Estimates

Table 3.14: Parameter Estimates from Bivariate MSM(8) for All Commodity Pairs

Commodity Pairs	$\hat{m}_{0,1}$	$\hat{m}_{0,2}$	$\hat{\bar{\sigma}}_1$	$\hat{\bar{\sigma}}_2$	$\hat{\gamma}_{\overline{k}}$	$\hat{b}$	$\hat{\rho}_\eta$	$\hat{\lambda}$	$\hat{\rho}_m$
Gold-Copper	1.3947	1.3220	0.9886	1.8490	0.9946	5.4959	0.2250	0.9504	0.3454
	(0.0128)	(0.0142)	(0.0420)	(0.0889)	(0.0125)	(0.7104)	(0.0155)	(0.3231)	(0.1954)
Gold-Cotton	1.4167	1.3305	0.9887	1.9026	0.9999	7.7193	0.0719	0.0001	0.6042
	(0.0105)	(0.0153)	(0.0365)	(0.0722)	(0.0001)	(1.2974)	(0.0164)	(0.8268)	(0.4264)
Gold-Wheat	1.3903	1.3118	0.9871	1.9954	0.9798	4.9001	0.0730	0.2394	0.1666
	(0.0128)	(0.0153)	(0.0452)	(0.1107)	(0.0344)	(0.6069)	(0.0160)	(0.5185)	(0.2000)
Gold-Oats	1.3803	1.6994	1.1335	2.7822	0.9998	5.0348	0.0890	0.0001	0.0461
	(0.0121)	(0.0100)	(0.0561)	(0.1597)	(0.0004)	(0.5079)	(0.0172)	(0.7625)	(0.1186)
Gold-sugar	1.3816	1.3689	1.1346	2.2657	0.9999	5.2349	0.0675	0.0001	0.1557
	(0.0121)	(0.0135)	(0.0551)	(0.1344)	(0.0001)	(0.7333)	(0.0167)	(0.7546)	(0.1698)
Gold-Soybeans	1.4162	1.3694	0.9852	1.7716	0.9999	7.5624	0.0969	0.3617	0.0514
	(0.0104)	(0.0135)	(0.0367)	(0.0639)	(0.0001)	(1.2490)	(0.0163)	(0.4636)	(0.1337)
Gold-Corn	1.4108	1.3680	0.9985	1.9995	0.9844	6.0174	0.0945	0.6629	0.0117
	(0.0114)	(0.0161)	(0.0416)	(0.0924)	(0.0268)	(0.9140)	(0.0164)	(0.6157)	(0.1251)
Gold-Heating	1.3888	1.3224	1.0024	3.3156	0.9338	4.6383	0.1101	0.9999	0.1629
	(0.0364)	(0.0306)	(0.0452)	(0.2357)	(0.0299)	(1.0666)	(0.0162)	(0.3900)	(0.1165)
Gold-WTI	1.3793	1.3045	1.1191	2.7081	0.9997	4.6818	0.1365	0.9999	0.1400

Continued on next page

Table 3.14 – Continued from previous page

Commodity Pairs	$\hat{m}_{0,1}$	$\hat{m}_{0,2}$	$\hat{\bar{\sigma}}_1$	$\hat{\bar{\sigma}}_2$	$\hat{\gamma}_{\overline{k}}$	$\hat{b}$	$\hat{\rho}_\eta$	$\hat{\lambda}$	$\hat{\rho}_m$
	(0.0122)	(0.0128)	(0.0576)	(0.1419)	(0.0011)	(0.6405)	(0.0167)	(0.5075)	(0.1254)
Copper-Cotton	1.3155	1.3102	1.7005	2.3920	0.9999	5.8278	0.1149	0.0647	0.6392
	(0.0134)	(0.0148)	(0.0737)	(0.1218)	(0.0002)	(0.9447)	(0.0161)	(0.3005)	(0.2510)
Copper-Wheat	1.3134	1.3096	1.7852	1.9887	0.9330	4.3495	0.1229	0.8484	0.2582
	(0.0247)	(0.0202)	(0.1717)	(0.0987)	(0.0182)	(0.4988)	(0.0154)	(0.6935)	(0.2771)
Copper-Oats	1.2931	1.6451	1.8714	2.1403	0.9999	3.7731	0.0852	0.0001	0.2824
	(0.0121)	(0.0122)	(0.0895)	(0.1601)	(0.0000)	(0.3357)	(0.0170)	(1.5242)	(0.2833)
Copper-Sugar	1.2994	1.3673	2.0696	2.2499	0.9999	4.8785	0.1253	0.0001	0.4467
	(0.0134)	(0.0139)	(0.1171)	(0.1412)	(0.0001)	(0.6958)	(0.0161)	(0.6999)	(0.2520)
Copper-Soybeans	1.2985	1.3335	2.0537	1.4341	0.9999	4.7285	0.1536	0.2276	0.6377
	(0.0133)	(0.0134)	(0.1168)	(0.0703)	(0.0001)	(0.6994)	(0.0157)	(0.2397)	(0.2251)
Copper-Corn	1.2898	1.3105	1.9760	1.9392	0.9719	3.4766	0.1205	0.4521	0.5340
	(0.0212)	(0.0170)	(0.1138)	(0.1720)	(0.0036)	(0.3239)	(0.0157)	(0.2842)	(0.2807)
Copper-Heating	1.3709	1.3670	1.7988	2.9418	0.7575	6.7712	0.1670	0.9999	0.3046
	(0.0194)	(0.0269)	(0.0708)	(0.1945)	(0.1345)	(1.4517)	(0.0164)	(0.3338)	(0.1739)
Copper-WTI	1.3119	1.3041	1.6650	2.7165	0.9946	4.3968	0.2051	0.5015	0.6659
	(0.0146)	(0.0133)	(0.0996)	(0.1516)	(0.0154)	(0.6555)	(0.0156)	(0.3252)	(0.3248)
Cotton-Wheat	1.2813	1.2896	1.9391	2.2170	0.9998	4.1392	0.1378	0.5109	0.3776
	(0.0143)	(0.0138)	(0.1031)	(0.1375)	(0.0012)	(0.5983)	(0.0156)	(0.3886)	(0.2354)
Cotton-Oats	1.2787	1.6395	1.8335	2.1434	0.9999	3.6205	0.1052	0.5523	0.2589
	(0.0133)	(0.0121)	(0.0905)	(0.1679)	(0.0000)	(0.3219)	(0.0172)	(0.7128)	(0.0977)
Cotton-Sugar	1.2848	1.3674	1.9552	2.2508	0.9999	4.8910	0.1077	0.9999	0.2646
	(0.0145)	(0.0139)	(0.0945)	(0.1420)	(0.0001)	(0.7107)	(0.0166)	(0.3924)	(0.1677)
Cotton-Soybeans	1.2784	1.3119	1.8373	1.6652	0.9999	3.9507	0.2311	0.6985	0.4981
	(0.0138)	(0.0128)	(0.0966)	(0.0926)	(0.0001)	(0.5722)	(0.0156)	(0.3672)	(0.2705)
Cotton-Corn	1.2765	1.3112	1.8946	1.9073	0.9978	3.7496	0.1889	0.4428	0.6314
	(0.0143)	(0.0139)	(0.1119)	(0.1230)	(0.0078)	(0.4907)	(0.0158)	(0.3172)	(0.2941)
Cotton-Heating	1.3177	1.3307	1.8930	3.2807	0.9537	5.2129	0.1128	0.3244	0.4030
	(0.0189)	(0.0262)	(0.0858)	(0.2948)	(0.0058)	(0.5282)	(0.0156)	(0.5369)	(0.4316)
Cotton-WTI	1.2840	1.3035	1.9561	2.6924	0.9999	4.5196	0.1167	0.2314	0.4475
	(0.0142)	(0.0126)	(0.1003)	(0.1390)	(0.0001)	(0.6710)	(0.0160)	(0.2652)	(0.2618)
Wheat-Oats	1.2868	1.6375	2.0425	2.1292	0.9999	3.5524	0.3204	0.4978	0.5457
	(0.0123)	(0.0121)	(0.0918)	(0.1688)	(0.0000)	(0.3032)	(0.0156)	(0.3303)	(0.1702)
Wheat-Sugar	1.2866	1.3262	2.0629	2.1415	0.9999	3.6863	0.0904	0.6569	0.1565
	(0.0127)	(0.0122)	(0.0979)	(0.1455)	(0.0001)	(0.4604)	(0.0160)	(0.5360)	(0.1171)
Wheat-Soybeans	1.2881	1.3126	2.1902	1.7229	0.9994	3.8062	0.3763	0.3840	0.9999
	(0.0137)	(0.0137)	(0.1385)	(0.1124)	(0.0027)	(0.5130)	(0.0149)	(0.1925)	(0.2540)
Wheat-Corn	1.2843	1.3064	2.1703	1.9355	0.8857	3.0893	0.4837	0.6370	0.9999
	(0.0189)	(0.0175)	(0.0756)	(0.1406)	(0.0651)	(0.3344)	(0.0133)	(0.2178)	(0.3551)
Wheat-Heating	1.3588	1.3541	2.0025	2.9594	0.5386	5.0878	0.1123	0.0001	0.8472
	(0.0203)	(0.0277)	(0.1340)	(0.2136)	(0.1399)	(1.3642)	(0.0154)	(0.3893)	(1.0742)
Wheat-WTI	1.2873	1.2973	2.2007	2.7074	0.9738	3.5488	0.1245	0.4589	0.1973
	(0.0197)	(0.0169)	(0.0705)	(0.1422)	(0.0077)	(0.2947)	(0.0158)	(1.0243)	(0.4328)
Oats-Sugar	1.6295	1.3211	2.1306	2.1410	0.9999	3.3269	0.1000	0.9999	0.1879
	(0.0114)	(0.0110)	(0.1731)	(0.1127)	(0.0000)	(0.2676)	(0.0173)	(0.3575)	(0.0864)
Oats-Soybeans	1.6359	1.3082	2.1426	1.6442	0.9999	3.5192	0.3607	0.5562	0.5932
	(0.0120)	(0.0120)	(0.1715)	(0.0805)	(0.0000)	(0.3006)	(0.0153)	(0.2489)	(0.1336)
Oats-Corn	1.6396	1.3072	2.1433	1.8631	0.9999	3.6227	0.4267	0.7788	0.5097
	(0.0123)	(0.0128)	(0.1681)	(0.1012)	(0.0000)	(0.3201)	(0.0144)	(0.2399)	(0.1330)
Oats-Heating	1.7030	1.3065	2.7655	3.5335	0.9893	4.2118	0.0908	0.0412	0.5163

Continued on next page

Table 3.14 – Continued from previous page

Commodity Pairs	$\hat{m}_{0,1}$	$\hat{m}_{0,2}$	$\hat{\bar{\sigma}}_1$	$\hat{\bar{\sigma}}_2$	$\hat{\gamma}_{\overline{k}}$	$\hat{b}$	$\hat{\rho}_\eta$	$\hat{\lambda}$	$\hat{\rho}_m$
	(0.0112)	(0.0094)	(0.1682)	(0.1352)	(0.0129)	(0.3815)	(0.0169)	(0.2793)	(0.1827)
Oats-WTI	1.6369	1.2898	2.1433	2.7976	0.9999	3.5452	0.0977	0.0001	0.4337
	(0.0120)	(0.0116)	(0.1707)	(0.1432)	(0.0000)	(0.3159)	(0.0170)	(0.6112)	(0.2036)
Sugar-Soybeans	1.3244	1.3087	2.1511	1.6445	0.9999	3.5865	0.1095	0.9999	0.2376
	(0.0119)	(0.0123)	(0.1373)	(0.0816)	(0.0001)	(0.4362)	(0.0167)	(0.4455)	(0.1549)
Sugar-Corn	1.3435	1.3126	1.9048	1.8833	0.9999	4.1662	0.1035	0.5017	0.5244
	(0.0134)	(0.0136)	(0.1203)	(0.1083)	(0.0001)	(0.5578)	(0.0165)	(0.2651)	(0.1590)
Sugar-Heating	1.4521	1.3769	2.6699	2.9385	0.9975	11.4826	0.0974	0.0001	0.4870
	(0.0136)	(0.0099)	(0.0978)	(0.0767)	(0.0064)	(2.0314)	(0.0162)	(0.5314)	(0.3069)
Sugar-WTI	1.3682	1.3057	2.2577	2.7015	0.9999	5.0163	0.1269	0.0001	0.4850
	(0.0140)	(0.0130)	(0.1433)	(0.1344)	(0.0001)	(0.7693)	(0.0162)	(0.5614)	(0.2648)
Soybeans-Corn	1.3098	1.3086	1.6761	1.8804	0.9983	3.5685	0.6047	0.8651	0.9999
	(0.0132)	(0.0138)	(0.1108)	(0.1250)	(0.0056)	(0.4259)	(0.0114)	(0.1098)	(0.1623)
Soybeans-Heating	1.4054	1.3709	1.5164	2.9438	0.8802	7.8755	0.1647	0.0001	0.2633
	(0.0180)	(0.0268)	(0.0674)	(0.1781)	(0.0495)	(1.0776)	(0.0152)	(0.4641)	(0.3012)
Soybeans-WTI	1.3113	1.2919	1.6600	2.9114	0.9999	3.9023	0.1720	0.0001	0.2789
	(0.0127)	(0.0123)	(0.0906)	(0.1549)	(0.0001)	(0.5758)	(0.0156)	(0.5463)	(0.2691)
Corn-Heating	1.4039	1.3635	1.7387	2.9417	0.5671	6.0213	0.1503	0.0001	0.2001
	(0.0225)	(0.0294)	(0.0794)	(0.1928)	(0.1919)	(1.6651)	(0.0159)	(0.7247)	(0.6726)
Corn-WTI	1.3102	1.2958	1.9464	2.7097	0.9563	3.4047	0.1639	0.0001	0.7456
	(0.0181)	(0.0168)	(0.1793)	(0.1640)	(0.0039)	(0.3451)	(0.0157)	(0.3113)	(0.4346)
Heating-WTI	1.3692	1.3888	2.9668	2.8588	0.7728	7.0866	0.7710	0.8866	0.9999
	(0.0272)	(0.0284)	(0.1635)	(0.1840)	(0.1287)	(1.6782)	(0.0069)	(0.0126)	(53.0079)
Heating-Cotton	1.3713	1.3562	2.9678	1.6627	0.8526	7.6960	0.1162	0.3650	0.3737
	(0.0265)	(0.0188)	(0.1566)	(0.0722)	(0.0675)	(1.2419)	(0.0156)	(0.6866)	(0.5844)
Heating-WTI	1.3024	1.2972	3.5079	2.7113	0.8754	3.4517	0.7689	0.8657	0.9999
	(0.0287)	(0.0208)	(0.2692)	(0.0912)	(0.0815)	(0.4108)	(0.0058)	(0.1374)	(0.2130)

Chapter 4

Forecasting Hourly Electricity Prices: A Multifrequency Approach

4.1. Introduction

Following the liberalization of many power markets in the late 1990s, several electricity markets have experienced fundamental changes in the behavior of wholesale spot prices. This liberalization has been characterized by a transfer of highly regulated government controlled electricity systems to competitive and deregulated wholesale markets. This in turn has led to the emergence of energy exchanges such as the European Energy Exchange in Germany and The Energy Exchange of the UK, where electricity spot and futures prices as well as forward and swap contracts are traded. One implication of this restructuring is the exposure of producers and consumers to increased risks arising from price fluctuations.

Moreover electricity, unlike other commodities, is a pure flow commodity that cannot easily be stored and generally requires instant deliveries. The demand side of the market is highly variable and depends strongly on weather, business cycles and macroeconomic uncertainty. On the supply side, low marginal cost of production, limited storability, system breakdown and outages, limited interconnection between markets, transmission (un)reliability and even speculative pressures induce substantial price volatility. The resultant spot prices are characterized by seasonality, mean-reversion, high volatility and transient and unexpected extreme price movements known as spikes in the electricity energy literature.

Electricity, like most other commodities, is traded on both regulated markets and over the counter. In the wholesale markets or power pools, buyers and sellers participate in a uniform price auction, where they submit their price and quantity bids 24 hours ahead. Since bids are accepted in ascending orders, a generator for example that is better able to anticipate the future prices can adjust its price/quantity schedule to maximize profit. But electricity is also traded either bilaterally (over the counter) or centrally on an exchange such as the Natural Gas Exchange (NGX) or the Chicago Mercantile Exchange (CME). Participants in these markets range from producers and consumers of energy who are interested in physical delivery and hedging of risks, to speculators and energy day traders who trade in the market to earn profit by speculation or by providing insurance, but want to avoid any physical delivery. For example, a day trader might take a long position in the futures market by buying a 3-month futures contract, with the hope that the value of such contact would appreciate by the end of the day, and liquidate his position for profit. For these high frequency traders, the knowledge of the hourly dynamics of spot and futures prices is of paramount importance. Hence, short-term price and load forecasting has become a valuable tool in this regard. This is the focus of the models proposed in this paper.

Several approaches have been proposed for modeling and forecasting electricity spot prices. These include simple mean-reverting processes, jump-diffusion mean-reverting processes and Markov regime switching processes. However, none of these classes of models has been able to characterize electricity prices effectively, with each class suffering from one or more deficiencies.

The objective of this paper is to apply a new class of model to the forecasting problem that can effectively characterize electricity prices, yet remain tractable for pricing and managing electricity price risk. In particular, we propose the Autoregressive Markov switching multifractal model, for forecasting spot electricity prices. Three variants of the model are examined. The first variant, the ARX-MSM, is a simple mean-reversion model that allows for a Markov switching multifractal (MSM) volatility process. This model already incorporates several of the main characteristics of electricity prices, namely mean-reversion, conditional volatility and price spikes. Price jumps with heterogeneous durations are introduced through switches in the volatility components with heterogeneous frequencies. This second variant allows for regime switches in both drift and volatility. This is motivated by earlier empirical findings that electricity prices are characterized by regime dependent drifts (Deng, 2000; Ethier and Mount, 1998). The last variant of the model introduces a risk premium into the mean equation to capture the impact of volatility on prices.

Therefore, this study makes significant contributions into the electricity pricing literature. First, we propose a new class of model for forecasting electricity prices that simultaneously incorporates several characteristics of electricity prices notably, mean-reversion, conditional volatility, jumps, multiple cycles and regime switches in both mean and volatility. The Markov-regime switching model inherits the parsimonious and long memory properties of the MSM model of Calvet and Fisher (2004), and can therefore accommodates many states in both mean and volatility. This constitutes a substantial improvement over previous regime switching models that have been proposed in the electricity literature (De Jong and Huisman, 2002; Deng, 2000;

Ethier and Mount, 1998; Huisman and Mahieu, 2003; Weron et al., 2004), that can only accommodate two or three states in either mean or volatility.

Second, while the MSM model has been applied to study the exchange rate market (Calvet and Fisher, 2004; Calvet et al., 2006; Lux, 2008) and equity markets (Calvet and Fisher, 2007, 2008; Chuang et al., 2013; Lux et al., 2011), no application of the model has been conducted in electricity markets. Hence, given the ability of the MSM model to accommodate many cycles, we extend the MSM literature by applying it to the forecast of electricity prices. Third, while several studies have analysed the various characteristics of Alberta electricity prices (Gogas and Serletis, 2009; Hinich and Serletis, 2006; Serletis and Andreadis, 2004; Serletis and Shahmoradi, 2006) and the Alberta electricity market structure (Doucet et al., 2013; Serletis and Bianchi, 2007), no attention has been paid to providing actual forecast of prices in Alberta. This study is the first, to the best of our knowledge, to provide hourly forecast of Alberta electricity prices, and compared the effectiveness of several time-series models in doing so.

Employing hourly prices from the AESO market over the period of January 1, 2011 to December 31, 2012, the parameters of the ARX-MSM models are estimated, and one-step-ahead hourly forecasts are obtained. To put the performance of the ARX-MSM models into perspective, the results are compared to those of other notable models that have been applied in the literature, namely the AR(1), ARX, ARX-GARCH, mean-reverting jump, and the 2-state independent Markov regime switching models. Goodness-of-fit tests indicate that the ARX-MSM models fit the data significantly better than the competing models. Likewise, out-of-sample results show that the ARX-MSM models provide better forecast accuracy than the existing models.

The structure of the remainder of this study is as follows. Section 2 explores the main characteristics of electricity prices in deregulated markets. Section 3 reviews the relevant literature on electricity price models. Section 4 presents the three variants of the ARX-MSM model and the estimation approach. Section 5 presents the data, the ARX-MSM estimation results, in-sample goodness-of-fit tests and out-of-sample forecast exercises, as well as a comparison of the ARX-MSM models to other competing models. Section 6 concludes.

4.2. Stylized Facts of electricity Prices

Some common characteristics of electricity prices are summarized in what follows.

Volatility - The fact that electricity is not easily storable implies that demand and supply have to be balanced instantaneously. This induces high volatility, in ranges well beyond the levels observed in other commodities and other financial assets. Comparing the annualized historical volatility of electricity markets with natural gas, oil and stock markets, the U.S. Federal Energy Regulatory Commission (2004) find volatility in electricity markets in the range of 300%, compared to 100% in commodity markets and less than 20% in equity markets. Similarly, Booth (2004) finds volatility in the Australian electricity market in the range of 900%. Booth (2004) also finds that a large proportion of the observed volatility is driven by price spikes that occur for less than 1% of the total hours in a year. Such levels of volatility in the electricity market are also driven by unforeseen factors such as power plant outages, fluctuating production capacity of renewable energy generators and other unexpected capacity constraints. Electricity prices are also characterized by volatility clustering that is largely attributed to instantaneous production processes, highly variable demand and non-storability.

Seasonality - Spot electricity prices exhibit substantial seasonality due to the cyclical nature of demand and supply (Knittel and Roberts, 2005). Cyclicality in demand arises due to variation in weather and other climatological conditions (e.g. daylight hours) and changing levels of economic activities. On the supply side, seasonal variations in output can arise when there is heavy reliance of power generators (e.g power dams) on precipitation and flooding which themselves exhibit seasonal variations. A combination of seasonal variations arising from demand and supply sides translates to seasonal fluctuations in electricity prices beyond what is observed in any other commodity.

Price spikes - Electricity prices are also characterized by both small and extreme price jumps (Geman and Roncoroni, 2006). This is due to a combination of instantaneous balancing of demand and supply and the non-storability of electricity output. Often, electricity price jumps occur due to power plant outages and other capacity bottlenecks, as well as peak loads which can sometimes result in demands in excess of power generator capacities.

Mean-reversion - In addition to jumps, electricity prices are also characterized by fast rates of mean-reversion (Bhanot, 2000; Knittel and Roberts, 2005; Lucia and Schwartz, 2002). In an equilibrium setting, when there is an increase in demand, supply is increased by turning on high marginal cost power generators, putting upward pressure on prices. This upward price pressure dissipates as demand returns to normal and high cost generators are turned off. This kind of demand-supply interaction is what induces mean-reversion in electricity markets.

4.3. Literature Review

These peculiar characteristics of electricity prices have led to a multitude of studies attempting to develop useful models of electricity price processes for the purpose of risk and portfolio management. These models can be categorized into three classes, namely: mean reverting models, jump models and Markov regime-switching models. Each of these classes of models is motivated by one or more statistical properties exhibited by short term electricity prices. We provide an overview of each of these models in what follows.

4.3.1. Mean Reverting Models

Mean-reversion models have been applied widely in the electricity price modeling literature (Bhanot, 2000; Knittel and Roberts, 2005; Lucia and Schwartz, 2002). Weather is a dominant driver of electricity demand, and to some extent, the supply. Since weather dynamics are cyclical and mean reverting (Alaton et al., 2002; Richards et al., 2004), such tendency to revert back to the mean level will also be reflected in the demand and prices of electricity. Therefore, one of the most documented characteristics of electricity prices is mean-reversion (Bhanot, 2000; Knittel and Roberts, 2005; Lucia and Schwartz, 2002). For this reason, a standard specification for modeling electricity prices is the first order autoregressive process, AR(1), which can be shown to be a discrete time version of the Ohrstein-Uhlenbeck mean-reverting process. The AR(1) process for electricity prices is specified as,

$$\begin{aligned} &p_t = \alpha + \beta p_{t-1} + \varepsilon_t \qquad (4.1)\\ &\varepsilon_t \sim N(0, \sigma^2) \end{aligned}$$

where p_t is the price of electricity at time t and ε_t is a Gaussian white noise shock. The AR(1) process can be extended to accommodate higher autoregressive orders and moving average components (hence, an autoregressive moving average process or ARMA). The mean-reversion model can also be generalised to include fundamental variables such as demand, weather, hourly, weekly and seasonal dummy variables.

In this case, we have an ARX or ARMAX process, where the X represents the fundamental regressors.

While these models are able to reproduce the mean-reversion and some of the autocorrelations inherent in electricity prices, they suffer from two major drawbacks. First, they cannot accommodate the spikes that characterize electricity prices. Electricity is not storable, except in the presence of substantial hydropower capacity. Therefore, there is little opportunity to smooth price spikes. Second, the model assumes that the error structure is homoscedastic. But electricity prices are also characterized by volatility clustering that is largely attributed to instantaneous production processes, highly variable demand and non-storability.

4.3.2. Mean Reverting Jump Diffusion Processes

An important characteristic of electricity prices is the presence of price spikes. To capture this feature, mean reverting jump models are widely used for modeling and forecasting electricity prices (Crespo Cuaresma et al., 2004; Knittel and Roberts, 2005) because they are able to address the mean-reversion and spiky nature of electricity prices. The discrete time version of the mean-reverting jump diffusion process is given by

$$p_t = \alpha + \beta p_{t-1} + \varepsilon_{t,i} \tag{4.2}$$

where i can be either 0 (if no jump occurred in time t) or 1 (if there was a jump), with $\varepsilon_{t,1} \sim N(0, \sigma^2)$ and $\varepsilon_{t,2} \sim N(\mu, \sigma^2 + \gamma^2)$. Note that the mean reverting jump model can also be extended to include fundamental regressors.

Mean reverting jump models are, however, limited in two respects. First, these jump models assume that all jumps have the same decay rate. But this is not likely to be the case in electricity markets. Economic intuition will suggest that larger shocks will revert back to normal price levels faster, due to the forces of demand and supply. On the other hand, smaller jumps are more likely to persist longer. Second, the jump process is usually assumed to be constant over time. But electricity prices typically exhibit jumps of varying magnitudes and of heterogeneous durations.

4.3.3. Markov Regime Switching Models

There are two types of Markov regime switching (MRS) models that have been applied in the electricity price modeling literature: dependent regime and independent regime MRS models. The dependent regime MRS model was first applied to electricity price modeling by Deng (2000) and Ethier and Mount (1998).

In a two-state dependent regimes MRS model, prices are assumed to follow a mean reverting AR(1) process in both states, with shared innovations but different parameters in each state. Generally, the model is specified as

$$\begin{aligned} &p_t = \alpha_i + \beta_i p_{t-1} + \sigma_i |p_{t-1}|^{\gamma_i} \varepsilon_t \\ &\varepsilon_t \sim N(0,1) \end{aligned} \tag{4.3}$$

where $i = 1$ for the "low" price regime and $i = 2$ for the "high" price regime. The two-state MRS model was extended by Huisman and Mahieu (2003), who propose a three-state MRS model, where it is assumed that any initial jump regime will be immediately followed by the reversing regime, and then by the normal regime. This is a very strong assumption that is uncharacteristic of electricity prices. This assumption was further relaxed by Andreasen and Dahlgren (2006), who assume that prices could jump up or down. But any jump is immediately followed by a reversion to the normal regime.

This class of MRS models suffers from the restrictive assumption of non-consecutive jumps in electricity prices. First, as shall be seen in the empirical section, electricity prices often exhibit consecutive price jumps and vice-versa. Second, such jumps are often characterized by heterogeneous durations. For example, heat waves or cold spells can last from several hours to several days. Also, plant outages and transmission congestions that often lead to price spikes can be very short lived. Therefore, it is not surprising that this class of MRS models have been found to perform poorly in modeling and forecasting electricity prices (Christensen et al., 2009; De Jong, 2006; Heydari and Siddiqui, 2010).

A second class of MRS models relaxes a restrictive assumptions of non-consecutive jumps by assuming independent regimes. De Jong and Huisman (2002) propose a two-state independent regime MRS model, where the base regime is modeled as an AR(1) mean-reverting process. The spike regime is modeled as a normally distributed random variable with higher mean and variance than the base regime. Generally, this class of MRS model for electricity prices is specified as

$$\begin{aligned} p_{t,1} &= \alpha + \beta p_{t-1,1} + \sigma_1 |p_{t-1,1}|^{\gamma} \varepsilon_t \\ p_{t,2} &\sim N(\mu, \sigma_2^2) \\ \varepsilon_t &\sim N(0,1) \end{aligned} \tag{4.4}$$

Such a model was applied to electricity prices from the European Energy Exchange (EEX), and was found to produce a significantly better fit and forecast accuracy than the dependent regime models, in terms of both short and medium term forecasts. Since introduced, several extensions of the model have been considered. These extensions center around the distribution of the random variable that characterizes the spike regime. Weron et al. (2004) models the spike regime as log-normally distributed. Bierbrauer et al. (2007) consider exponentially distributed spikes, while

De Jong (2006) selects autoregressive poisson distributed spikes. Using daily average spot and futures price data from the German EEX power market, for the period of 2002 to 2003, Bierbrauer et al. (2007) find that the independent regimes MRS models outperform the mean-reverting and jump models in terms of data fitting and forecast accuracy.

Despite the empirical successes of the MRS models, however, they all suffer from a common limitation: the separation of base from jump regimes. The consequence of this is often a misclassification of prices into regimes (Janczura and Weron, 2010; Weron, 2009). From an intuitive point of view, gradations in size and frequency exist in the context of electricity price spikes. Hence, simply classifying prices as being in normal or spike regimes is inconsistent with the empirical data. For example, price spikes caused by temporary transmission lines congestion, thermal limits or voltage constraints tend to be temporary, but frequent. But price spikes caused by heat waves, cold spells or other extreme weather conditions can often last from a few hours to several days. Therefore, models that simply separate normal prices from spike prices will miss such regularities. As a consequence, Weron (2009) finds the MRS models discussed above often generate negative expected price spikes, especially when log prices are fitted.

To conclude the literature review, it is important to also review other studies that have been conducted on the Alberta electricity market, on which this study focuses. Hinich and Serletis (2006) employ a Randomly Modulated Periodicity model proposed by Hinich (2000), to test for the presence of periodic signals in hourly price and loads from the Alberta's spot wholesale electricity market. Using hourly spot data over the period from 1996 to 2003, the study documents the presence of a relatively stable weekly and daily cycles in electricity loads, but finds that such cycles are less stable in prices. Serletis and Shahmoradi (2006) investigate the relationship between Alberta electricity and natural gas price changes and their volatilities, using a multivariate GARCH-M model. Employing daily data over the period from January, 1996 to November, 2004 from Alberta's (deregulated) spot power and natural gas markets, they find that there is bidirectional (linear and nonlinear) causality between natural gas and electricity prices. They further interpret the evidence of the bidirectional causality as an indication of an effective arbitrage mechanism in Albertas natural gas and power markets, raising questions about the efficient markets hypothesis.

In a bid to explain the fluctuations in electricity prices in Alberta, Serletis and Andreadis (2004) apply various tools from dynamical system theory to average daily on-peak prices, from 1996 to 2002. They find that Alberta electricity prices exhibit a multiscaling behavior of hurst exponent, consistent with a persistent fractal structure with long memory. The authors also find that Alberta electricity prices exhibit a homogeneous random multifractal behaviour. Therefore, an adequate model of Alberta electricity prices should incorporate these multifractal characteristics.

The foregoing discussion suggests that there is still a need for a model that better matches electricity price characteristics and produces accurate forecast of electricity prices. From the existing literature, it is evident that an adequate model of electricity spot prices should incorporate the following characteristics:

1. Mean-reversion
2. Conditional volatility and volatility clustering
3. Daily, weekly and seasonal effects
4. Jumps of multiple frequencies

Therefore, this study improves on the existing literature by proposing a new class of model for forecasting electricity prices that simultaneously incorporates several characteristics of electricity prices notable, mean-reversion, conditional volatility, jumps, multiple cycles and regime switches in both mean and volatility. The Markov-regime switching model inherits the parsimonious and long memory properties of the MSM model of Calvet and Fisher (2002), and can therefore accommodate many states in both mean and volatility. This constitutes a substantial improvement over previous regime switching models that have been proposed in the electricity literature (De Jong and Huisman, 2002; Deng, 2000; Ethier and Mount, 1998; Huisman and Mahieu, 2003; Weron et al., 2004), that can only accommodate two or three states in either mean or volatility.

4.4. Autoregressive Markov Switching Multifractal Model

In an attempt to simultaneously incorporate all of the characteristics discussed in section 2, this study proposes the use of an Autoregressive Markov Switching Multifractal (ARX-MSM) model. In its most basic form, the ARX-MSM assumes that the mean equation follows a mean-reverting AR(q) process, while the variance equation is specified as a Markov Switching Multifractal (MSM) process. Specifically, the ARX-MSM process is specified as

$$\begin{aligned} p_t &= \alpha_0 + \sum_{q=1}^{Q} \beta_q p_{t-q} + \sum_{j=1}^{J} \alpha_j X_{t,j} + \varepsilon_t, \\ \varepsilon_t &\sim N(0, \sigma^2(M_t)) \end{aligned} \tag{4.5}$$

where p_t is the logarithm of electricity price, X_t is a matrix or fundamental controls whose elements are to be discussed later in section 4, q is the order of the autoregressive process. When q is equal to 1 and ε_t is a standard normal Gaussian variable, the model simplifies to the discrete time version of the Ohrstein-Uhlenbeck mean-reverting process. Hence, the ARX-MSM model nests a mean-reverting process with homoscedastic variance. Consistent with the conditional volatility observed in electricity prices however, we assume that the error structure is governed by a MSM process (Calvet and Fisher, 2002). The volatility equation is specified as

$$\sigma(M_t) = \overline{\sigma} \left(\prod_{k=1}^{\overline{k}} M_{k,t} \right)^{1/2} \tag{4.6}$$

Under the MSM specification, volatility is driven by a first order Markov state vector M_t, with k volatility components:

$$M_t = (M_{1,t}; M_{2,t}; ...; M_{\overline{k},t})$$

Each volatility component, $M_{k,t}$, can take one of two possible values at each point in time t.[1] Each $M_{k,t}$ will equal $m_0 \in (1, 2)$ or $m_1 = 2 - m_0$, with equal probabilities. At time t, $M_{k,t}$ will switch and take on a new value with probability γ_k, where switching events and new draws are assumed to be independent across k and t.

Each volatility component is mutually independent but all components are drawn from the same marginal distribution M. The switching probabilities of the volatility components are related as follows

$$\begin{aligned} \gamma_1 < \gamma_2 <, ... < \gamma_{\overline{k}}, \\ \gamma_k = 1 - (1 - \gamma_{\overline{k}})^{b^{k-\overline{k}}} \end{aligned}$$

where $\gamma_{\overline{k}}$ and b are parameters to be estimated.

[1] For simplicity, we assume a binomial distribution for the volatility components. But the MSM model can accommodate any distribution that satisfies $M \geq 1$ and $\mathbb{E}(M) = 1$.

For illustration, when $\overline{k}$ is equal to 1, there is only 1 volatility component, $M_t = M_{1,t} = [m_0,\ m_1]$. Therefore, volatility can either be in a low state, $\overline{\sigma}(m_1)^{1/2}$ or high state $\overline{\sigma}(m_0)^{1/2}$. This corresponds to a standard 2-state Markov process. Likewise, when $\overline{k} = 2$, there are two volatility components and each volatility component can take one of two possible values at each time t. Consequently, volatility can take four possible values at each time t, $[\overline{\sigma}(m_0 m_0)^{1/2};\ \overline{\sigma}(m_0 m_1)^{1/2};\ \overline{\sigma}(m_1 m_0)^{1/2};\ \overline{\sigma}(m_1 m_1)^{1/2}]$. Continuing this pattern, when there are $\overline{k}$ volatility components, M_t (hence volatility) can take $2^{\overline{k}}$ possible values.

The MSM specification of volatility implies that volatility is stochastic and is hit by shocks of heterogeneous frequencies indexed by $k \in \left\{1, 2, ..., \overline{k}\right\}$. Switches in low-frequency volatility components (e.g prolonged heat waves or cold spells) cause volatility to vary discontinuously and exhibit strong persistence, while switches in high-frequency components (e.g, generator outages, transmission line congestion etc.) produce substantial outliers in prices.

The model discussed above is a simple mean reverting process, with conditional volatility. It already accommodates the salient characteristics of electricity prices: mean-reversion, conditional volatility and volatility clustering, multiple jumps of heterogenous frequencies and seasonality. Note that as discussed so far, the model allows for regime switching only in the variance equation, not in the mean equation. But Ethier and Mount (1998), among others, find strong empirical support for the existence of regime dependent drift and volatility in electricity prices. This earlier finding motivates the following extension of our electricity price model. Specifically, we allow for regime shifts in both drift and volatility as follows

$$
\begin{aligned}
p_t &= \alpha_0 + \sum_{q=1}^{Q} \beta_q(M_t) p_{t-q} + \sum_{j=1}^{J} \alpha_j X_{t,j} + \varepsilon_t, \qquad (4.7)\\
\varepsilon_t &\sim N(0, \sigma^2(M_t)),\\
\beta_q(M_t) &= \beta_q \sum_{k=1}^{\bar{k}} (M_{k,t} - 1)
\end{aligned}
$$

We refer to this model as the ARX(M)-MSM model, where the ARX(M) indicates regime shifts in mean. When $\beta_q > 0$, a high $M_{k,t}$ implies both a higher volatility and a higher rate of mean-reversion. Therefore, the further away price is from its long-run equilibrium level, the faster the speed of mean-reversion, and vice-versa. In the regime switching parlance, the model is characterized by different drift and volatility dynamics under the $2^{\bar{k}}$ different regimes. But this is achieved in a much smoother and parsimonious manner than in previous regime switching models that have been applied in the electricity literature. Yet, the ARX-MSM model is able to accommodate finitely many states as may be inferred from the data, compared to previous MRS models that are only able to accommodate two or three states.

The final extension we propose for the ARX-MSM model is the introduction of a time varying risk premium into the conditional mean equation. Risk averse economic agents (e.g, generators, hedgers and speculators) require adequate compensation for holding risky assets. Therefore, as the degree of uncertainty in asset returns varies overtime, so does the premium required for holding the assets. Although electricity is not storable, a substantial amount of evidence has been documented on the presence of risk premia in electricity markets. Longstaff and Wang (2004) analyze a high-frequency data set of hourly spot and day-ahead forward prices from the Pennsylvania New Jersey Maryland (PJM) Interconnection LLC electricity market, and find evidence of significant risk premia in the forward prices. Likewise, Torro and Lucia (2008) analyze a decade of weekly closing prices data from the Nordic Power

Exchange, and find that there are significant positive risk premia in short term electricity prices. They further document that the risk premia they identify are related to the variance and skewness of future spot prices.

Motivated by these findings, we consider the ARX-MSM(M) model, where the M stands for MSM-*in-Mean* . The model is specified as

$$p_t = \alpha_0 + \sum_{q=1}^{Q} \beta_q p_{t-q} + \sum_{j=1}^{J} \alpha_j X_{t,j} + \theta\sigma(M_t) + \varepsilon_t, \tag{4.8}$$
$$\varepsilon_t \sim N(0, \sigma^2(M_t))$$

where θ is the risk premium parameter, and measures the impact of volatility on the conditional mean of prices. Note that when $\theta = 0$, this model reduces to the ARX-MSM model presented in equation (4.5).

4.4.1. Estimation and Inference

Since there exist a finite number of volatility states, standard filtering methods apply for the estimation process (Calvet and Fisher, 2002). Suppose there are $\overline{k}$ volatility components included in the model.[2] Then $M_t = (M_{1,t} * M_{2,t} ..... * M_{\overline{k},t})$ can take $2^{\overline{k}} = d$ possible values $(m_1, m_2, ...m_d) \in \mathbb{R}_+^d$. The dynamics of M_t are then characterized by a dxd transition matrix A, with elements (Calvet and Fisher, 2002)

$$a_{ij} = \mathbb{P}(M_{t+1} = m_j | M_t = m_i) = \prod_{k=1}^{\overline{k}} \left[(1 - \gamma_k) \, 1_{\{m_k^i = m_k^j\}} + \frac{1}{2}\gamma_k \right]$$

Note that the econometrician only observes the set of past prices, but not the volatility state vector. The vector M_t is therefore latent and must be inferred by Bayesian updating. Let $\Pi_t = (\Pi_t^1, \Pi_t^2, ...\Pi_t^d) \in \mathbb{R}_+^d$ be the vector of state probabilities, where

[2] The choice of $\overline{k}$ is a model selection problem to be discussed later.

$$\Pi_t^j = \mathbb{P}(M_t = m_j | P_t) \tag{4.9}$$

where $P_t = [p_1, p_2, ..., p_{t-1}]'$ is the price vector. The conditional probability state vector is computed recursively by Bayes updating. By Bayes rule, Π_t can be expressed as a function of the previous belief Π_{t-1} and the Gaussian density as follows

$$\Pi_t = \frac{f(p_t) \odot \Pi_{t-1} A}{[f(p_t) \odot \Pi_{t-1} A]\mathbf{1'}}, \tag{4.10}$$

where

$$\mathbf{1} = [1, 1, ...1] \in \mathbb{R}^d, \tag{4.11}$$

The Gaussian density function is given by

$$f(p_t) = \frac{1}{\overline{\sigma}(m)\sqrt{2\pi}} \exp\{-\frac{\varepsilon_t^2}{2(\overline{\sigma}(m))^2}\} \tag{4.12}$$

where ε_t is defined by either equation (4.5), (4.7) or (4.8), and $\overline{\sigma}(m) = \overline{\sigma}(m_1, m_2, ...m_d)$. The Bayes recursion is initiated with the ergodic distribution Π_0, with $\Pi_0^j = 1/d, \forall j$. The log-likelihood then has a closed form expression and is given by

$$lnL(p_1, ...p_T; \Phi) = \sum_{t=1}^{T} \ln[f(p_t) \cdot (\Pi_{t-1} A)], \tag{4.13}$$

where Φ indicates the vector of parameters to be estimated.

4.5. Empirical Analysis

The empirical analysis focuses on hourly prices from the Alberta Electric System Operator (AESO) pool. The AESO pool was established in 1996 as the first competitive energy market in Canada, with the primary function of enabling the sale and purchase of electric power 24 hours a day, 7 days a week in Alberta. The non-storability nature of electricity implies that demand and supply have to be balanced instantaneously. The AESO plays a vital role in ensuring this, while at the same time is responsible for ensuring that the market operates in a fair, efficient and competitive manner (Alberta Electric System Operator, 2010; Market Surveillance Administrator, 2010).

The scale of the AESO energy markets and the system's reputation for reliability and competitiveness have helped to attract many market participants. As at the end at 2012, the market had 170 participants and handled approximately \$6.4 billion in annual energy transactions.

4.5.1. The AESO Market Structure

The AESO market system consists of three types of markets in which participants may trade electricity. The first is the spot or real time market, also referred to as the AESO pool. In the AESO pool, generating units (producers or owners of electricity generators) offer their power supply into the pool at their own chosen hourly prices, up to a ceiling of \$999.99 per megawatt hour (MWh) on a day-ahead basis (Alberta Electric System Operator, 2010). The ask prices for each hour are then sorted in ascending merit order, on a daily basis. Power from the lower-priced supply offers are dispatched before those with the higher price, moving up the merit order until total dispatched volume equals total demand for that hour. The price of the last megawatt that clears the market is the System Marginal Price (SMP). The hourly electricity spot price is then the average of the 60 SMPs in an hour.

The second market in the AESO system is the forward physical market. In the forward physical market, the delivery of electricity from sellers to buyers still passes through the pool in real time, as with the spot market. But the payment from buyers to sellers takes place outside the pool. The third market is the forward financial market, which only involves the flow of financial transactions and no physical delivery of electricity takes place.

The contracts traded on the forward financial markets are referred to as "Contract for Difference" (CFD). The CFD is similar to financial swaps in that, at settlement, the two parties involved only exchange the difference between the strike price and the hourly pool price. Therefore, the hourly pool price also serves as the index for settling financial transactions in the forward financial markets. As such, the forward financial market provides a mechanism for managing price risks and speculating in the Alberta electricity markets. The participants in the forward financial markets include not only power producers and consumers, but also include power marketers, proprietary traders, speculators, hedge funds and other financial institutions. Therefore, compared to the forward physical market, the forward financial market is more liquid and provides a consensus view on market expectations of future pool prices. Examples of the forward financial markets are the Natural Gas Exchange and the Chicago Mercantile Exchange.

4.5.2. Data

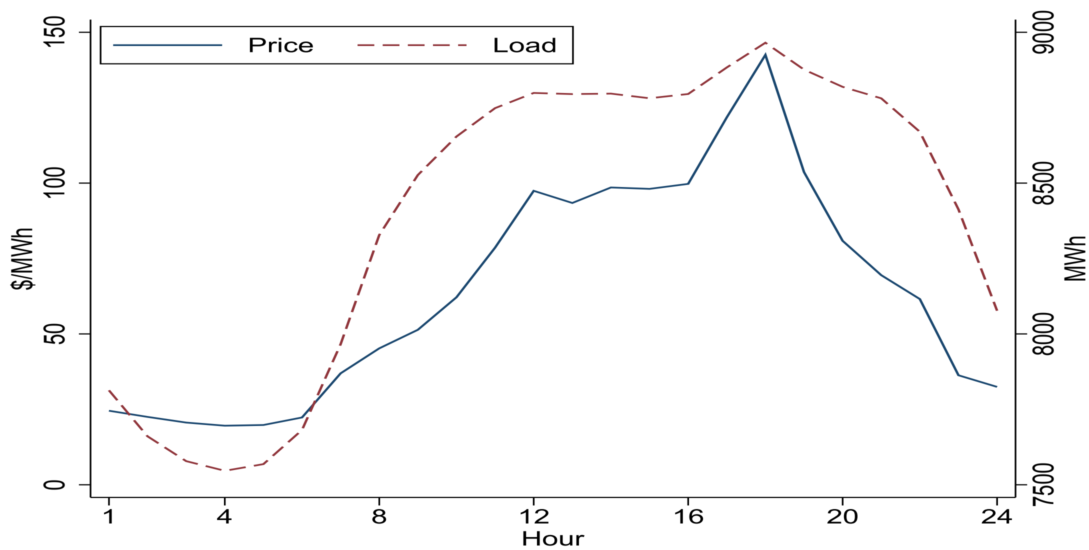
The data used in this empirical analysis consist of hourly spot electricity prices (measured in dollars per megawatt hour or \$/MWh) from the AESO pool,[3] for the period from January 1, 2011 to January 30, 2012. We also collect data on hourly electricity load forecasts, measured in megawatt hours. The use of load forecast data, rather than actual load data is motivated by the fact that actual load observation is an *ex-post* observation that does not become available until the bidding process for the corresponding hour has already taken place. Therefore, market participants do

[3] The data is publicly available at www.ets.aeso.ca.

not bid based on the actual load. Bids are submitted based on hourly load forecast, which is publicly available.

Table 4.1 presents summary statistics for the electricity spot prices. The table shows that the average spot price varies throughout the day, ranging from $18.98 for the early morning hour of 04:00 to $178.93 for the peak[4] evening hour 18:00. This is further corroborated by Figure 4.1 that shows the average hourly prices over the sample period. Prices begin to increase at about 6:00, as the workday begins. The price increase continues throughout the day as demand increases, peaking at about 18:00 and prices begin to decline thereafter. The right axis of Figure 4.1 plots the average hourly electricity load. The figure shows that variation in prices closely follows the variation in demand.

Figure 4.1: Average Hourly Prices and Demand



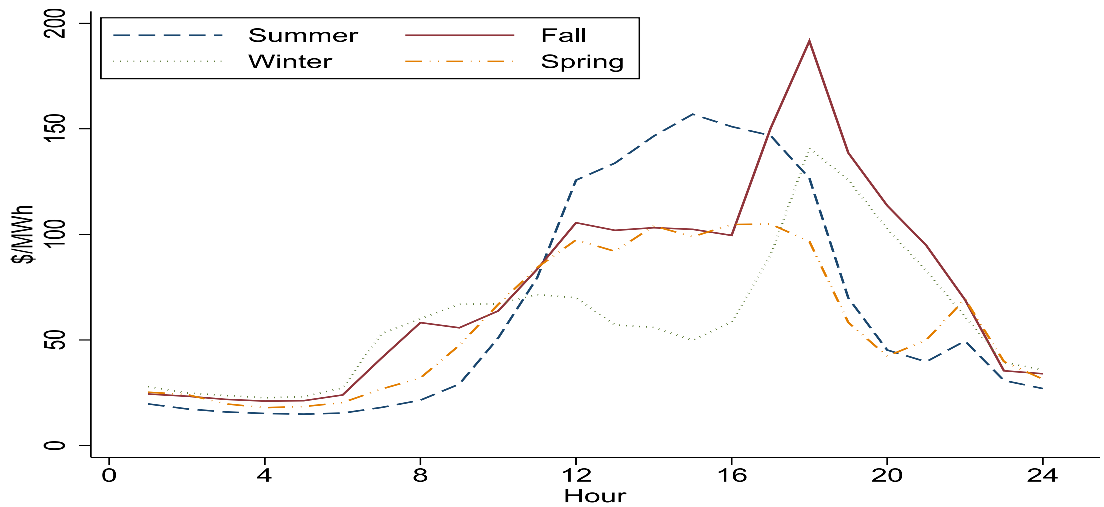
Electricity prices are also characterized by strong seasonal fluctuations. This is driven

[4]Peak hours in AESO market are defined as hours ending 08:00 to 23:00, Monday through Saturday.

Table 4.1: **Summary Statistics of Hourly Electricity Prices and Load**

Hour	Mean	Std.Dev	Min	Max	Skewness	Kurtosis
1	25.0168	26.4118	3.0800	412.6700	10.9374	145.5075
2	22.2191	19.1225	0.0000	331.5200	12.6263	198.5459
3	19.7653	7.7895	0.0000	90.2200	2.7471	21.6851
4	18.9812	6.4420	0.0000	45.0100	0.8428	4.1894
5	19.4388	7.3126	0.0000	69.6900	1.8087	11.2975
6	21.0625	8.9468	0.0000	97.1700	3.0957	22.3647
7	37.1181	86.4106	3.4100	869.5000	8.0771	71.1821
8	52.7935	113.4922	11.7500	898.2800	5.7185	37.2122
9	57.0934	110.0719	12.2800	968.7900	5.8225	40.4848
10	76.2794	153.6131	12.0800	992.0000	4.3494	22.3751
11	91.5535	173.0606	12.7600	999.9900	3.6210	15.7923
12	124.2897	224.3982	13.0700	999.9900	2.6798	8.9784
13	113.2924	207.7717	17.5500	996.4100	2.9764	10.7885
14	114.0870	206.2437	17.3000	998.5200	2.9428	10.6453
15	116.9569	218.8147	14.9600	999.9900	2.9112	10.3561
16	117.5065	215.4235	12.5500	999.9900	2.8882	10.4358
17	147.5311	241.8015	13.3300	999.9900	2.2971	7.1098
18	178.9307	285.3624	18.0900	999.9900	1.9308	5.1735
19	139.1089	244.8205	13.6800	999.9900	2.4396	7.7206
20	110.0023	199.5562	13.0400	999.9900	2.9391	11.0354
21	93.9884	163.5084	12.7000	999.2800	3.3132	14.2238
22	74.4680	131.9267	12.8400	941.0200	4.3145	23.1360
23	37.5310	38.2148	12.5100	358.5800	6.8110	53.2789
24	30.6114	27.9486	12.0100	333.9900	6.8264	59.7609
Overall	76.6575	165.3511	0.0000	999.9900	4.0489	19.1976
			Hourly AESO Load			
1	8.9601	0.0536	8.8378	9.0761	0.0826	2.3064
2	8.9399	0.0555	8.8169	9.0621	0.1020	2.3043
3	8.9290	0.0569	8.8069	9.0553	0.1232	2.2932
4	8.9245	0.0579	8.8037	9.0542	0.1303	2.2724
5	8.9277	0.0587	8.8096	9.0602	0.1213	2.2409
6	8.9419	0.0633	8.7982	9.0792	-0.0245	2.2745
7	8.9773	0.0721	8.7989	9.1201	-0.1777	2.3715
8	9.0260	0.0821	8.7972	9.1768	-0.2541	2.4338
9	9.0496	0.0738	8.8374	9.1938	-0.2634	2.5577
10	9.0649	0.0637	8.8736	9.1940	-0.2825	2.6742
11	9.0759	0.0580	8.8975	9.1989	-0.2920	2.7338
12	9.0816	0.0544	8.9130	9.2015	-0.3018	2.7692
13	9.0812	0.0519	8.9211	9.1977	-0.3071	2.7672
14	9.0810	0.0518	8.9217	9.1957	-0.3357	2.7692
15	9.0792	0.0519	8.9212	9.1930	-0.3368	2.7383
16	9.0804	0.0528	8.9240	9.1931	-0.3113	2.6861
17	9.0896	0.0576	8.9333	9.2085	-0.1422	2.5298
18	9.0985	0.0671	8.9377	9.2425	0.0952	2.2966
19	9.0881	0.0713	8.9240	9.2329	0.0480	2.1070
20	9.0814	0.0710	8.9177	9.2210	-0.0604	2.1632
21	9.0768	0.0673	8.9121	9.2063	-0.2261	2.3316
22	9.0641	0.0584	8.9220	9.1802	-0.2179	2.4373
23	9.0343	0.0501	8.9174	9.1448	0.0195	2.3233
24	8.9939	0.0506	8.8772	9.1062	0.0644	2.3278
Overall	9.0311	165.3511	8.7972	9.2425	-0.2545	2.4320

Figure 4.2: Average Hourly Prices by Season



by demand fluctuations that reflect seasonal heating or cooling needs, as well as seasonal variations in day light savings hours. This feature is reflected on Figure 4.2, which plots average hourly prices for each of the four seasons. It is worth noting that, contrary to most other electricity markets that have been analysed in the literature, AESO peak prices are higher in the Fall than in the summer. Summer peak prices occur only between the hours 12:00 and 16:00, reflecting the timing of high summer temperature levels in Alberta, therefore cooling needs. Winter prices are higher between the hours of 01:00 and 10:00 and fall prices are the highest seasonal prices, between the hours of 18:00 and 24:00.

This unusual pattern of seasonal electricity price fluctuation reflects the extremely cold weather conditions that characterize Alberta. Extremely negative weather conditions are more prevalent in Alberta than extremely positive weather conditions. Therefore, the high winter and fall prices reflect the need for heating during those periods. Table 4.1 also shows that there is substantial difference in price variation throughout each day. For example, the standard deviation of prices are higher for the peak hours, with hour 18:00 being the highest at \$285.36.

Figure 4.3: AESO Hourly Prices

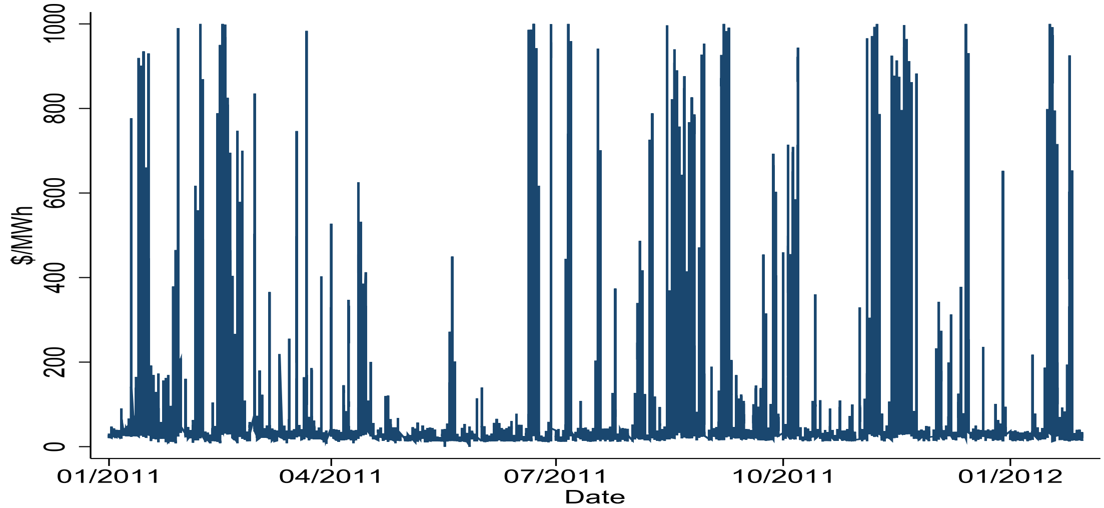
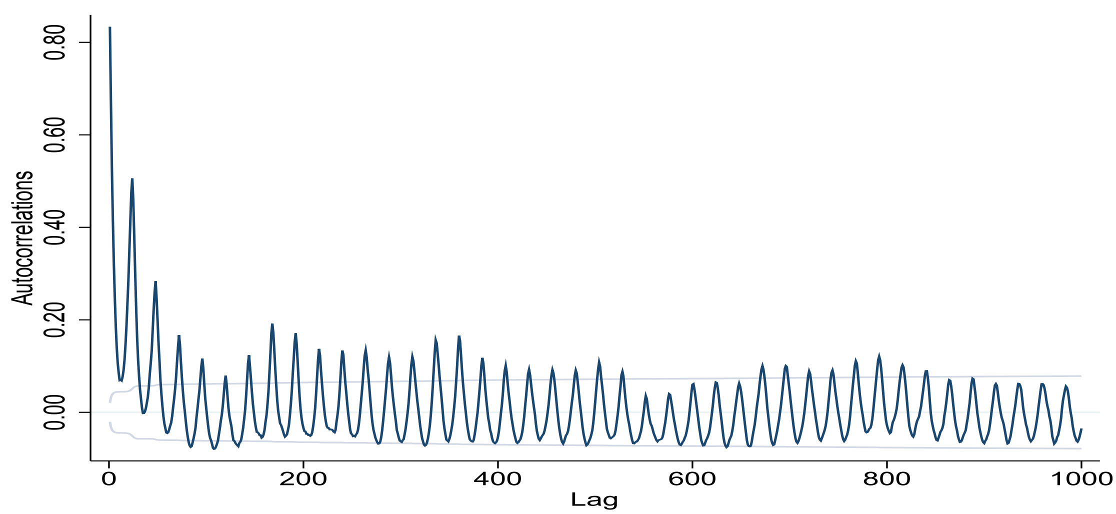


Figure 4.4: Autocorrelation Function for AESO Prices



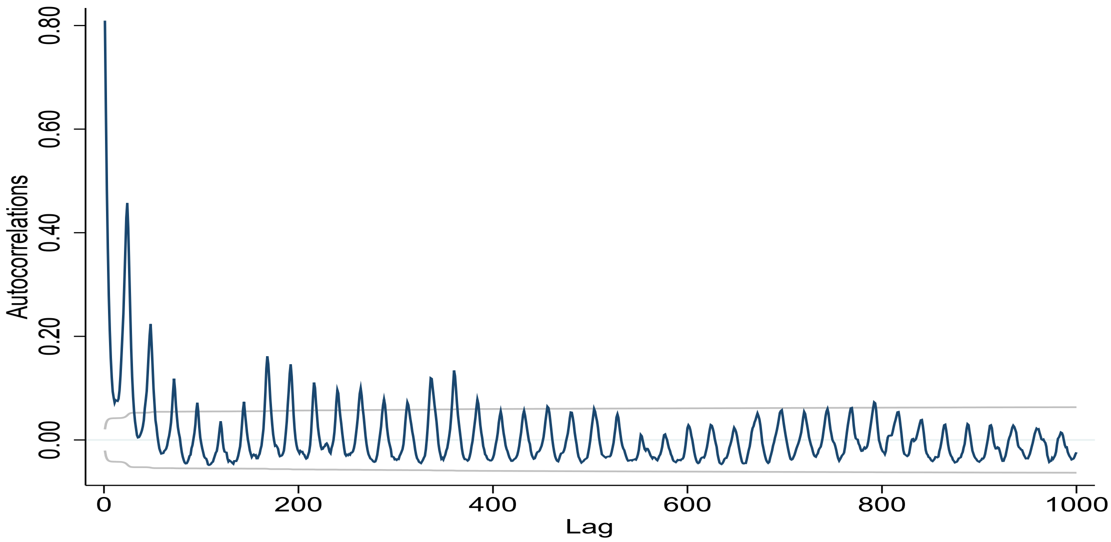
The entire set of hourly prices for the sample period is plotted in Figure 4.3. Two points are worth noting from the figure. First, electricity prices exhibit spikes and the spikes tend to occur in clusters. This is often due to demand approaching or exceeding generator supplies. Therefore, models that don't allow for consecutive price jumps will miss this feature. Second, electricity prices can be zero. This is often due to a combination of significant start up costs for generators and the inability to freely dispose off electricity (Knittel and Roberts, 2005).

Figure 4.4 plots the autocorrelation function for prices. The correlogram shows that intraday and weekly cycles are present in electricity prices. Although not reflected on this plot for ease of presentation, the autocorrelation functions remain significant beyond l000 lags. It can also be observed that prices do not appear to be exploding despite their long memory characteristics. The t-statistics from the Augmented Dickey-Fuller test of the presence of a unit root in log prices,[5] for lags 0 to 2 are -24.2274, -26.3571 and -26.6331, respectively. These are lower than the 1% critical value of -3.96, therefore, rejecting the null hypothesis that log prices are not stationary. Likewise, the t-statistics of log demand, for lags 0 to 2 are -12.3586, -36.8673 and -26.6088, respectively. These are also lower than the 1% critical value of -3.96, therefore, rejecting the null hypothesis that log demand are not stationary.

Lastly, Figure 4.5 plots the correlogram of squared returns. The figure shows that electricity price volatility is time varying and exhibits strong persistence. The figure also shows that the correlogram of squared prices exhibit cyclical patterns similar to those of prices, indicating that volatility variation is also characterized by several cycles ranging from intraday to seasonal cycles.

[5]The test includes a constant and a trend component.

Figure 4.5: **Autocorrelation Function for AESO Squared Prices**



4.5.3. ARX-MSM Parameter Estimates

The parameters for the ARX-MSM, ARX(M)-MSM and the ARX-MSM(M) models are estimated using the maximum likelihood approach. The dependent variable is the logarithm of electricity prices. The regressors include the logarithm of AESO load forecast and dummy variables for peak hours, weekday, winter, spring and summer.[6] The parameter estimates from the ARX-MSM , ARX(M)-MSM and the ARX-MSM-M are presented in Tables 4.2, 4.3 and 4.4 respectively. The three models are estimated for $\overline{k}$ equals 1 to 8. In the mean equation, only lags 1, 24 and 25 are included.[7]

[6] We define Fall season as months of September to November, winter as December to March, spring as April to May and summer as June to August.

[7] Several other specifications incorporating other lags were considered. While the estimated coefficients are statistically significant, their contributions to the forecast accuracy are either negative or positive but insignificant.

Table 4.2: Parameter Estimates for ARX-MSM Model

	ARX-MSM Model							
	$\overline{k}=1$	2	3	4	5	6	7	8
Constant	-5.4196	-2.0054	-3.2178	-2.4429	-2.6362	-2.0610	-2.4488	-2.3695
	(0.4153)	(0.4568)	(0.6351)	(0.8623)	(0.7392)	(0.6707)	(0.8473)	(0.8523)
AR(1)	0.7989	0.9259	0.8809	0.9043	0.8940	0.9206	0.9012	0.9030
	(0.0047)	(0.0087)	(0.0180)	(0.0274)	(0.0214)	(0.0094)	(0.0256)	(0.0254)
AR(24)	0.1639	0.1522	0.1611	0.1545	0.1629	0.1578	0.1595	0.1591
	(0.0044)	(0.0134)	(0.0156)	(0.0148)	(0.0158)	(0.0122)	(0.0167)	(0.0159)
AR(25)	-0.1485	-0.1348	-0.1448	-0.1392	-0.1451	-0.1409	-0.1424	-0.1419
	(0.0046)	(0.0134)	(0.0148)	(0.0129)	(0.0138)	(0.0113)	(0.0147)	(0.0141)
Log Load	0.6701	0.2438	0.3954	0.3009	0.3252	0.2518	0.3019	0.2924
	(0.0470)	(0.0527)	(0.0766)	(0.1055)	(0.0895)	(0.0775)	(0.1032)	(0.1037)
Weekday	-0.0373	-0.0168	-0.0236	-0.0189	-0.0189	-0.0148	-0.0172	-0.0167
	(0.0062)	(0.0049)	(0.0062)	(0.0061)	(0.0057)	(0.0071)	(0.0060)	(0.0057)
Peak	0.0499	0.0209	0.0295	0.0247	0.0258	0.0221	0.0247	0.0247
	(0.0062)	(0.0069)	(0.0072)	(0.0069)	(0.0069)	(0.0073)	(0.0068)	(0.0065)
Winter	-0.0371	-0.0196	-0.0293	-0.0228	-0.0242	-0.0195	-0.0224	-0.0221
	(0.0058)	(0.0075)	(0.0077)	(0.0094)	(0.0081)	(0.0072)	(0.0092)	(0.0090)
Spring	*0.0015*	*-0.0006*	*-0.0059*	*-0.0045*	*-0.0053*	*-0.0012*	*-0.0043*	*-0.0043*
	(0.0064)	(0.0125)	(0.0065)	(0.0064)	(0.0061)	(0.0055)	(0.0067)	(0.0062)
Summer	*-0.0029*	*-0.0039*	-0.0101	*-0.0086*	*-0.0101*	*-0.0016*	*-0.0088*	*-0.0088*
	(0.0056)	(0.0080)	(0.0060)	(0.0068)	(0.0064)	(0.0047)	(0.0067)	(0.0065)
$\hat{b}$		7.0795	2.8785	2.0331	1.7137	9.6216	1.3898	1.3139
		(0.9551)	(0.3038)	(0.2375)	(0.2134)	(1.3459)	(0.1221)	(0.1078)
$\hat{m}_0$	1.9332	1.9164	1.7965	1.7576	1.7135	1.8975	1.6329	1.6049
	(0.0016)	(0.0078)	(0.0106)	(0.0141)	(0.0174)	(0.0086)	(0.0131)	(0.0142)
$\hat{\gamma}_{\overline{k}}$	0.1240	0.4746	0.5125	0.5435	0.5168	0.5070	0.5255	0.5198
	(0.0072)	(0.0288)	(0.0477)	(0.0453)	(0.0479)	(0.0349)	(0.0607)	(0.0658)
$\hat{\sigma}$	0.5342	0.4867	0.4358	0.4122	0.4674	0.5342	0.4253	0.4362
	(0.0042)	(0.0171)	(0.0339)	(0.0265)	(0.0711)	(0.0228)	(0.0172)	(0.0224)
$\ln L$	-386	-19	-68	-36	-37	4	-27	-29
R^2	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79

Notes: This table reports the maximum likelihood parameter estimates for the ARX-MSM model. The models are fitted for $\overline{k}$ equals 1 to 8, where each column corresponds to the given number of frequency components $\overline{k}$ in the MSM variance equation. When $\overline{k}=1$, the model corresponds to a standard Markov-switching model, with only two possible states of volatility and $\gamma_{\overline{k}} = \gamma_1$. b is therefore unidentified and omitted. Heteroscedasticity and Autocorrelation consistent (HAC) standard errors are reported in parenthesis. Italicized figures are insignificant at 10% significance level.

Starting with the ARX-MSM model, it can be observed that all the autoregressive parameters are strongly significant. The AR(1) parameter measuring the rate of mean-reversion is significant and positive across all $\overline{k}$. This indicates that prices return rapidly from extreme positions to their equilibrium level. The half-life ($\ln 0.5/|\ln \beta_1|$) of the autoregressive process ranges from 3 hours for $\overline{k}=1$ to 9 hours for $\overline{k}=2$.

Table 4.3: Parameter Estimates for ARX(M)-MSM Model

	ARX(M)-MSM Model							
	$\overline{k}=1$	2	3	4	5	6	7	8
Constant	-12.3646	-37.8687	-43.9437	-37.7848	-33.3335	-31.1634	-30.0524	-32.7572
	(0.9083)	(3.3140)	(1.8822)	(1.9410)	(3.5303)	(1.6558)	(1.4161)	(2.4881)
AR(1)	0.9217	*0.0666*	0.1064	0.1337	0.1677	0.1389	0.1262	0.1126
	(0.0156)	(0.0520)	(0.0109)	(0.0153)	(0.0244)	(0.0086)	(0.0063)	(0.0097)
AR(24)	0.3315	*0.0140*	*0.0071*	0.0413	0.1000	0.0432	0.0359	0.0404
	(0.0128)	(0.0157)	(0.0062)	(0.0139)	(0.0232)	(0.0104)	(0.0072)	(0.0124)
AR(25)	-0.2718	*0.0170*	*-0.0089*	-0.0375	0.1000	-0.0766	-0.0649	-0.0489
	(0.0129)	(0.0287)	(0.0100)	(0.0142)	(0.0221)	(0.0123)	(0.0084)	(0.0104)
Log Load	1.4301	4.5969	5.2703	4.5858	4.0390	3.8053	3.6652	3.9812
	(0.1023)	(0.3691)	(0.2094)	(0.2174)	(0.3949)	(0.1851)	(0.1591)	(0.2765)
Weekday	*0.0094*	*-0.0515*	-0.0921	-0.0501	-0.1141	-0.0883	-0.0818	-0.1139
	(0.0197)	(0.0344)	(0.0225)	(0.0251)	(0.0463)	(0.0179)	(0.0168)	(0.0399)
Peak	-0.0273	*0.0189*	*-0.0231*	*0.0056*	*-0.0276*	*0.0292*	*0.0267*	*0.0309*
	(0.0163)	(0.0295)	(0.0203)	(0.0177)	(0.0477)	(0.0193)	(0.0196)	(0.0268)
Winter	-0.0908	-0.2421	-0.1990	-0.3593	-0.2355	-0.2903	-0.2913	-0.2798
	(0.0159)	(0.0632)	(0.0285)	(0.0474)	(0.0620)	(0.0214)	(0.0245)	(0.0375)
Spring	*0.0281*	0.1334	0.2193	0.1130	0.5106	0.1097	0.0901	0.1563
	(0.0188)	(0.0670)	(0.0353)	(0.0296)	(0.0474)	(0.0207)	(0.0201)	(0.0626)
Summer	*0.0216*	*0.0617*	*0.0929*	0.0838	*0.0882*	0.0766	0.0589	0.0559
	(0.0168)	(0.0796)	(0.0578)	(0.0361)	(0.0543)	(0.0359)	(0.0235)	(0.0338)
$\hat{b}$		3.4669	1.4630	1.4163	3.0316	2.0622	2.1763	1.6384
		(1.2147)	(0.2433)	(0.1573)	(0.1802)	(0.1116)	(0.1046)	(0.0683)
$\hat{m}_0$	*1.8708*	1.7616	1.6229	1.4941	1.2643	1.5544	1.5641	1.4750
	(0.0150)	(0.0543)	(0.0188)	(0.0320)	(0.0157)	(0.0149)	(0.0135)	(0.0156)
$\hat{\gamma}_{\overline{k}}$	*0.0001*	0.1480	0.1274	0.1280	0.1649	0.1779	0.1871	0.1891
	(0.9998)	(0.0164)	(0.0151)	(0.0152)	(0.0072)	(0.0101)	(0.0096)	(0.0142)
$\hat{\sigma}$	*0.2932*	0.5342	0.4051	0.2923	0.2230	0.1863	0.1479	0.1552
	(0.2445)	(0.0924)	(0.0188)	(0.0102)	(0.0039)	(0.0054)	(0.0040)	(0.0082)
$\ln L$	-4421	-3549	-2870	-2246	-3111	-1450	-1230	-1182
R^2	0.80	0.68	0.77	0.86	0.93	0.87	0.87	0.89

Notes: This table reports the maximum likelihood parameter estimates for the ARX(M)-MSM model. The models are fitted for $\overline{k}$ equals 1 to 8, where each column corresponds to the given number of frequency components $\overline{k}$ in the MSM variance equation. When $\overline{k}=1$, the model corresponds to a standard Markov-switching model, with only two possible states of volatility and $\gamma_{\overline{k}} = \gamma_1$. b is therefore unidentified and omitted. Heteroscedasticity and Autocorrelation consistent (HAC) standard errors are reported in parenthesis. Italicized figures are insignificant at 10% significance level.

The coefficient estimate for load is positive and significant, consistent with a priori expectations. Likewise, the weekend and peak hour effects on electricity prices are strongly significant and are rightly signed. Electricity prices are higher during peak hours than off-peak hours, and are higher during weekdays than weekends.[8] Also,

[8]The weekday dummy variable equals 0 for weekdays and 1 for weekends. Hence the negative sign.

Table 4.4: Parameter Estimates for ARX-MSM-M Model

	ARX-MSM-M Model							
	$\overline{k}=1$	2	3	4	5	6	7	8
Constant	-6.2460	-2.1984	-4.1683	-3.9911	-5.0720	-3.9481	-4.0805	-3.7117
	(0.4155)	(0.4672)	(0.6639)	(0.7505)	(1.2678)	(0.9186)	(0.7911)	(0.9608)
AR(1)	0.7528	0.9197	0.8424	0.8436	0.8083	0.8466	0.8373	0.8524
	(0.0051)	(0.0106)	(0.0220)	(0.0235)	(0.0750)	(0.0333)	(0.0222)	(0.0338)
AR(24)	0.1548	0.1500	0.1555	0.1490	0.1531	0.1542	0.1506	0.1561
	(0.0044)	(0.0165)	(0.0138)	(0.0122)	(0.0144)	(0.0126)	(0.0124)	(0.0132)
AR(25)	-0.1399	-0.1359	-0.1462	-0.1411	-0.1465	-0.1468	-0.1430	-0.1474
	(0.0045)	(0.0145)	(0.0136)	(0.0121)	(0.0144)	(0.0127)	(0.0122)	(0.0135)
Log Load	0.7707	0.2679	0.5146	0.4951	0.6269	0.4895	0.5070	0.4608
	(0.0470)	(0.0550)	(0.0801)	(0.0911)	(0.1588)	(0.1128)	(0.0944)	(0.1181)
Weekday	-0.0438	-0.0188	-0.0313	-0.0301	-0.0338	-0.0297	-0.0334	-0.0285
	(0.0061)	(0.0053)	(0.0071)	(0.0075)	(0.0144)	(0.0084)	(0.0083)	(0.0087)
Peak	0.0550	0.0223	0.0374	0.0347	0.0382	0.0348	0.0388	0.0344
	(0.0062)	(0.0067)	(0.0084)	(0.0080)	(0.0140)	(0.0107)	(0.0094)	(0.0105)
Winter	-0.0403	-0.0208	-0.0322	-0.0288	*-0.0324*	-0.0306	-0.0281	-0.0292
	(0.0057)	(0.0067)	(0.0075)	(0.0081)	(0.0221)	(0.0083)	(0.0087)	(0.0084)
Spring	*0.0036*	*-0.0003*	*-0.0026*	*-0.0034*	*0.0015*	*-0.0034*	*-0.0027*	*-0.0040*
	(0.0064)	(0.0066)	(0.0066)	(0.0074)	(0.0154)	(0.0076)	(0.0069)	(0.0068)
Summer	*-0.0012*	*-0.0047*	*-0.0088*	*-0.0098*	*-0.0071*	*-0.0099*	*-0.0084*	*-0.0101*
	(0.0056)	(0.0062)	(0.0068)	(0.0072)	(0.0140)	(0.0072)	(0.0073)	(0.0069)
$\hat{\alpha}$	0.4938	0.0897	0.2493	0.2398	0.3240	0.2396	0.2639	0.2226
	(0.0309)	(0.0310)	(0.0550)	(0.0580)	(0.2297)	(0.0771)	(0.0649)	(0.0745)
$\hat{b}$		7.2955	2.9978	2.1296	7.4331	1.6257	2.7764	1.3752
		(0.9605)	(0.3124)	(0.2072)	(4.7713)	(0.1611)	(0.3475)	(0.0933)
$\hat{m}_0$	1.9309	1.9126	1.7831	1.7390	1.8680	1.6642	1.7254	1.5867
	(0.0020)	(0.0086)	(0.0106)	(0.0144)	(0.0731)	(0.0160)	(0.0175)	(0.0158)
$\hat{\gamma}_{\overline{k}}$	0.1183	0.4820	0.4599	0.4730	0.3200	0.4675	0.5314	0.4869
	(0.0071)	(0.0296)	(0.0457)	(0.0393)	(0.1938)	(0.0548)	(0.0413)	(0.0625)
$\hat{\sigma}$	0.5342	0.4834	0.4474	0.3907	0.5342	0.4444	0.3894	0.4223
	(0.0058)	(0.0174)	(0.0328)	(0.0204)	(0.1754)	(0.0226)	(0.0120)	(0.0237)
$\ln L$	-264	-9	-21	6	-40	13	-8	5
R^2	0.81	0.79	0.80	0.80	0.81	0.80	0.80	0.80

Notes: This table reports the maximum likelihood parameter estimates for the ARX-MSM-M model. The models are fitted for $\overline{k}$ equals 1 to 8, where each column corresponds to the given number of frequency components $\overline{k}$ in the MSM variance equation. When $\overline{k}=1$, the model corresponds to a standard Markov-switching model, with only two possible states of volatility and $\gamma_{\overline{k}}=\gamma_1$. b is therefore unidentified and omitted. Heteroscedasticity and Autocorrelation consistent (HAC) standard errors are reported in parenthesis. Italicized figures are insignificant at 10% significance level.

the winter effect is negative and strongly significant. Generally, relative to the fall season, prices are lower in winter. The summer and spring effects, although rightly signed, are mostly insignificant.

The estimated MSM parameters for the variance equation are all strongly significant across all $\overline{k}$, indicating that the conditional variance equation does belong in the model of electricity prices. Lastly, looking at the volatility multiplier parameters $\hat{m}_0$, they tend to decline as the number of frequency components increases. The intuition is that less variability is required in each individual component in order to match the volatility fluctuations of the data. Estimates of the other volatility parameters fluctuate across $\overline{k}$ without any apparent pattern.

Turning to the ARX(M)-MSM results on Table 4.3 , we note that the results generally mimic those of the ARX-MSM discussed above. For the ARX(M)-MSM model, the load variable and winter effect are strongly significant while the weekday effect is mostly significant, with only a few exceptions. The spring effect, although significant, has the wrong sign for all $\overline{k}$. The peak hour and summer effects are mostly insignificant.

Also, the ARX-MSM-M results on Table 4.4 follow closely those of the ARX-MSM discussed above. Most importantly, the risk premium parameter $\hat{\alpha}$ is positive and strongly significant across $\overline{k}$. This observation indicates the presence of positive risk premium in the AESO electricity market. This is a subject that requires further investigation, but is outside the scope of this study.

It can be observed from the results in Tables 4.2 to 4.4 that the log-likelihood changes non-monotonically as the number of frequency components increases. For example, with the ARX-MSM model, the log-likelihood increases by more than 390 when $\overline{k}$ goes from 1 to 6, but begins to decline afterwards. This implies that the fit of the model increases as the number of frequency components increases only up to a certain point. Likewise, for the ARX-MSM-M model, the maximum log-likelihood occurs at $\overline{k} = 6$, whereas it occurs at $\overline{k} = 8$ for the ARX(M)-MSM model.

Therefore for forecasting purposes and for subsequent analysis, we use $\overline{k} = 6$ for the ARX-MSM and the ARX-MSM-M models. But for the ARX(M)-MSM model, even though the model fit increases as $\overline{k}$ increases, the out-of-sample forecast accuracy actually declines with $\overline{k}$. This indicates a tell-tail sign of the classical data over-fitting problem. Therefore for forecasting purposes and for subsequent analysis, we simply use $\overline{k} = 1$ for the ARX(M)-MSM model. This corresponds to a 2-state Markov regime switching model.

4.5.4. Comparisons With Alternative Models

Next, we compare our three models from section 5.3 to the various models proposed in the previous literature in terms of in-sample goodness-of-fit and out-of-sample forecast accuracy. The simplest model proposed for electricity prices is the AR(1) model, specified in equation (4.1). This model is often considered the benchmark model used to judge other potential models. The second model considered is the ARX (1, 24, 25) model which extends the AR(1) model by including the same exogenous variables included in the ARX-MSM models. This model is specified as

$$
\begin{aligned}
&p_t = \alpha_0 + \sum_{q=1}^{Q} \beta_q p_{t-q} + \sum_{j=1}^{J} \alpha_j X_{t,j} + \varepsilon_t, \qquad (4.14)\\
&\varepsilon_t \sim N(0, \sigma^2)
\end{aligned}
$$

Next, we consider the jump diffusion model, that have been widely applied in the electricity pricing literature (Crespo Cuaresma et al., 2004; Kaminski, 1997; Knittel and Roberts, 2005; Weron and Misiorek, 2008). The jump models are popular because they incorporate the basic characteristics of electricity prices (spikes and mean-reversion) and are tractable for deriving electricity derivative prices. The discretized jump diffusion model is specified as in equation (4.2). But we also augment the model by including the same set of exogenous regressors included in the ARX-MSM models

Another notable model of electricity prices we consider is the Markov regime switching (MRS) model, with two independent regimes. Several variants of the MRS model have been applied in the electricity pricing literature, with varying degrees of success (De Jong, 2006; Higgs and Worthington, 2008; Janczura and Weron, 2010; Mount et al., 2006). The 2-regime model we consider is specified as in equation (4.4). Note that unlike the first three models considered, the MRS model allows for conditional volatility.

Lastly, we consider the ARX-GARCH(1,1) model that is also popular in the extant literature. Several variants of GARCH models have been applied to electricity prices (Bowden and Payne, 2008; Garcia et al., 2005; Hickey et al., 2012; Liu et al., 2011), with indistinguishable successes among the different variants. For simplicity, this study considers only the ARX-GARCH (1,1) model. The ARX-GARCH (1,1) model is specified as follows

$$p_t = \alpha_0 + \sum_{q=1}^{Q} \beta_q p_{t-q} + \sum_{j=1}^{J} \alpha_j X_{t,j} + \varepsilon_t, \tag{4.15}$$

$$\varepsilon_t \sim N(0, \nu_t)$$

$$\nu_t = \theta + \omega \nu_{t-1} + \phi \varepsilon_{t-1}^2 \tag{4.16}$$

All models are estimated by maximum likelihood, and the parameter estimates are presented in Table 4.5.

In-Sample Comparison

The objective of this section is not to evaluate the performance of the alternative models considered, as this has been done in the earlier studies cited in section 3. Instead, we evaluate the performance of the proposed ARX-MSM models, relative

Table 4.5: Parameter Estimates for Comparison Models

	AR(1)	ARX	MRJD	MRS	ARX-GARCH(1,1)
Constant	0.4360	-13.1422	*-0.0003*	-0.0355	-8.4965
	(0.0188)	(0.8705)	(0.0003)	(0.0020)	(0.7053)
AR(1)	0.8790	0.7892	0.8239	0.8625	0.7556
	(0.0051)	(0.0070)	(0.0297)	(0.0072)	(0.0076)
AR(24)		0.2890			0.2470
		(0.0110)			(0.0096)
AR(25)		-0.2322			-0.1996
		(0.0109)			(0.0093)
log Load		1.5199	0.3931	0.3931	1.0193
		(0.0983)	(0.0032)	(0.0032)	(0.0801)
Weekday		*0.0100*	-0.3527	-0.3527	-0.0317
		(0.0162)	(0.0292)	(0.0292)	(0.0085)
Peak		-0.0301	0.8770	0.8770	0.0340
		(0.0151)	(0.0208)	(0.0208)	(0.0105)
Winter		-0.0962	-0.1044	-0.1044	-0.0902
		(0.0212)	(0.0241)	(0.0241)	(0.0110)
Spring		*0.0280*	-0.4153	-0.4153	*-0.0100*
		(0.0174)	(0.0285)	(0.0285)	(0.0106)
Summer		*0.0231*	-0.1624	-0.1624	-0.0460
		(0.0142)	(0.0255)	(0.0255)	(0.0099)
$\hat{\sigma}_1^2$	0.2047	0.1756	0.4720	0.0376	
	(0.0032)	(0.0029)	(0.0782)	(0.0007)	
$\hat{\mu}$			-0.0015	0.8044	
			(0.0004)	(0.0035)	
$\hat{\sigma}_2^2$				1.4742	
				(0.0525)	
$\hat{\gamma}$			1.0000	*-0.0361*	
			(0.1666)	(0.1900)	
$\hat{\lambda}$			*0.0118*		
			(0.0083)		
$\hat{\theta}$					0.0287
					(0.0015)
$\hat{\omega}$					0.5892
					(0.0270)
$\hat{\phi}$					0.4108
					(0.0185)
$\ln L$	-5285	-4638	-5186	-2101	-2878
R^2	0.77	0.80	0.75	0.56	0.80

Notes: The table presents the maximum likelihood parameter estimates for the comparison models. For the ARX and ARX-GARCH models, all parameters are estimated simultaneously. For the Markov regime switching (MRS) and the mean-reversion jump-diffusion (MRJD) model, the load and seasonal effects parameters are first estimated, and the residuals from the first stage are then used in the main estimation routine. All standard errors are HAC. Italicized figures are insignificant at 10% significance level.

to the alternative models. We achieve this by comparing the in-sample goodness-of-fit and out-of-sample forecast accuracy of the ARX-MSM models with those of the alternative models.

Table 4.6: In-Sample Model Comparison

	AR(1)	ARX	MRJD	MRS	ARX-GARCH(1,1)
ARX-MSM	-13.2728	-10.4185	-47.6225	-12.4887	-6.9253
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ARX(M)-MSM	-6.9957	-1.6628	-6.1811	6.9730	5.1939
	(0.0000)	(0.0482)	(0.0000)	(1.0000)	(1.0000)
ARX-MSM-M	-14.0375	-10.9967	-58.6611	-14.0981	-7.2257
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Notes: The table presents t-ratios and one-sided p-values for the test of likelihood difference between the model in each column and the corresponding model in each row. A low p-value indicates a rejection of the null hypothesis of no likelihood difference, in favor of the alternative hypothesis that the model in each column performed worse than the model in the corresponding row.

As suggested by Vuong (1989), we compare the in-sample goodness-of-fit of all models by evaluating the statistical significance of their log-likelihood differences. We consider the null hypothesis that models $f(p_t, \beta)$ and $g(p_t, \beta')$ have identical unconditional expected log-likelihood (e.g, that ARX-MSM and AR(1) fit the data equally well), against the alternative that model $g(p_t, \beta')$ performed worst.

Specifically, we test the following conditions

$$H_0 : \ln L^g - \ln L^f = 0$$
$$H_1 : \ln L^g - \ln L^f < 0$$

Under the null hypothesis,

$$t_{LR} = \frac{LR_T(\hat{\beta}', \hat{\beta})}{\sqrt{T}\hat{\sigma}_T} \xrightarrow{d} N(0,1),$$

where

$$LR_T(\hat{\beta}', \hat{\beta}) = \ln L^g - \ln L^f - \left(\frac{n_1 - n_2}{2}\right) \ln T,$$
$$\hat{\sigma}_T^2 = \frac{1}{T}\sum_{t=1}^{T}\left(\ln\left(\frac{g(p_t, \hat{\beta}')}{f(p_t, \hat{\beta})}\right)\right)^2 - \left(\frac{1}{T}\sum_{t=1}^{T}\ln\left(\frac{g(p_t, \hat{\beta}')}{f(p_t, \hat{\beta})}\right)\right)^2$$

where T is the number of observations in the sample, n_1 and n_2 are the number of parameters in model g and model f respectively. The t-ratios and corresponding one-sided p-values are reported in Table 4.6. Generally, the three ARX-MSM models dominate the alternative models. In other words, the ARX-MSM models provide better goodness-of-fit than their counterparts for the AESO prices. The exceptions are the MRS and ARX-GARCH models that provide better fit than the ARX(M)-MSM model. Lastly, comparing the different variants of the ARX-MSM models, we find that the ARX-MSM(6)-M provides the best goodness of fit overall.

Out-of-Sample Forecasts

To investigate the out-of sample forecast accuracy, we conduct one-step-ahead hourly forecasts for electricity prices from January 1 to 31, 2012 (720 point forecasts). First, the forecast accuracy of the competing models is evaluated using the diebold-mariano (DM) test (Diebold and Mariano, 2002) of equal predictive accuracy. Second, we evaluate the different models based on the standard measures used in the electricity literature. The DM test is based on the forecast errors from two different models defined as

$$\varepsilon^1_{t+h|t} = y_{t+h} - \hat{y}^1_{t+h|t}$$
$$\varepsilon^2_{t+h|t} = y_{t+h} - \hat{y}^2_{t+h|t}$$

The accuracy of each model is measured by a particular loss function $L(\varepsilon^i_{t+h|t})$. For the purpose of this study, we adopt the most popular loss function, which is the squared error loss, specified as

$$L(\varepsilon^i_{t+h|t}) = (\varepsilon^i_{t+h|t})^2, \quad i = 1, 2 \tag{4.17}$$

To determine if one model predicts better than another, we test null hypothesis of equal predictive accuracy as follows

$$H_0 : \mathbb{E}(d_t) = 0, \tag{4.18}$$

$$H_1 : \mathbb{E}(d_t) \neq 0, \tag{4.19}$$

where

$$\tag{4.20}$$

$$d_t = L(\varepsilon^1_{t+h|t}) - L(\varepsilon^2_{t+h|t}) \tag{4.21}$$

Under the null hypothesis, the DM test statistic has an asymptotic student-t distribution as follows

$$t_d = \frac{\overline{d}_t}{\sigma_d} \xrightarrow{d} N(0,1) \tag{4.22}$$

Table 4.7: Diebold-Mariano Test of Equal Predictive Accuracies (Winter)

	ARX-MSM	ARX(M)-MSM	ARX-MSM(M)	AR(1)	ARX	ARX-GARCH	MRS	MRJD
ARX-MSM		1.6391	0.7918	-2.8969	1.1177	-0.3644	-9.5848	-4.5211
ARX(M)-MSM	0.0508		-1.5117	-2.4283	-0.2981	-1.3474	-10.2342	-4.3633
ARX-MSM(M)	0.2144	0.0655		-2.8200	0.9444	-0.5414	-9.7647	-4.5520
AR(1)	0.0019	0.0077	0.0025		2.4910	0.8760	-9.2999	-3.3470
ARX	0.1320	0.3828	0.1727	0.0065		-1.6235	-10.0212	-4.6623
ARX-GARCH	0.3578	0.0891	0.2942	0.1907	0.0525		-10.0451	-3.0849
MRS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		8.9936
MRJD	0.0000	0.0000	0.0000	0.0004	0.0000	0.0011	0.0000	

Notes: The table presents results from the diebold-mariano test of equal predictive accuracy between the different models considered. The null hypothesis is that the forecast errors from model A are not statistically different from those of model B, i.e. $H_0 : \mathbb{E}(d_t) = 0$ against $H_1 : \mathbb{E}(d_t) \neq 0$. The t-statistics are reported on the upper diagonal of the table, while the associated one-sided p-values are reported in the corresponding lower diagonal. For example, the t-statistic from the comparison of the forecast errors from ARX-MSM and ARX(M)-MSM is on the cell corresponding to row 1 and column 2, while the associated p-value is on column 1 row 2. A negative t-value in cell(i,j) suggests that the model on row i provides better forecasts than the model on column j. A positive t-value suggests the opposite. The associated p-value on cell(j,i) indicates whether the t-value is statistically significant.

The results from the DM test are presented in Table 4.7. Results from all models are compared on a pairwise basis. The t-statistics are reported on the upper diagonal of the table, while the associated one-sided p-values are reported in the corresponding lower diagonal. For example, the t-statistic form the comparison of the forecast errors from ARX-MSM and ARX(M)-MSM is on the cell corresponding to row 1 and column 2, while the associated p-value is on column 1 row 2.

A few points are noteworthy from the table. First, we observe that the ARX(M)-MSM model provides the best out-of-sample forecast among all the models. At the 10% significance level, it provides a significantly better forecast than most other models, except the ARX model. Second, the predictive accuracy of the ARX-MSM and the ARX-MSM(M) models are not statistically different at any conventional significance level. Likewise, both models do not provide significantly better forecast than the ARX and the ARX-GARCH models. But they both provide significantly better forecasts than the AR(1), MRS and the MRJD models. Third, the MRS and the MRJD models provide the lowest forecast accuracy based on the DM test. Even the relatively simpler AR(1) model possesses better predictive accuracy than the MRS and the MRJD models. It is also noteworthy that the ARX model produce forecast errors that are not statistically different than those of all the MSM models, despite that it is relatively less complex.

This conclusion however changes when we consider more traditional approaches of forecast evaluation used in the electricity price forecasting literature. The mean absolute error (MAE), mean absolute percentage error (MAPE), root mean square error (RMSE) and the Theil's inequality index (TIC) are defined as follows

$$MAE = \frac{1}{N}\sum_{n=1}^{N}|\hat{p}_n - p_n|,$$

$$MAPE = \frac{1}{N}\sum_{n=1}^{N}|\frac{\hat{p}_n - p_n}{p_n}|,$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\hat{p}_n - p_n)^2},$$

$$TIC = \frac{\sqrt{\frac{1}{N}\sum_{n=1}^{N}(\hat{p}_n - p_n)^2}}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\hat{p}_n^2} + \sqrt{\frac{1}{N}\sum_{n=1}^{N}p_n^2}}$$

where N is the number of observations in the forecast sample.

The results on Table 4.8 show that the ARX-MSM model dominates other models in terms of MAE and MAPE , while the ARX(M)-MSM model dominates in terms of RMSE and TIC, although by a small margin. The MAE and MAPE for the ARX-MSM are 0.2161 and 0.0550. These are followed closely by the MAE of ARX(M)-MSM (0.2253) and MAPE of ARX-GARCH (0.0580). ARX-MSM(M) has both the least RMSE of 0.4270 and TIC of 0.0568. These are followed closely by the RMSE and TIC of the ARX model, 0.4230 and 0.0571. These results further corroborate the goodness-of-fit results from section 5.4.1, indicating that the ARX-MSM models are superior to the other models in forecasting electricity prices. The MRS model is the worst performing model, followed by the jump model.

Table 4.8: Other Evaluations of Out-of-Sample Forecasts (Winter)

Model	MAE	MAPE	RMSE	TIC
ARX-MSM	**0.2161**	**0.0550**	0.4321	0.0588
ARX(M)-MSM	0.2376	0.0619	**0.4207**	**0.0568**
ARX-MSM(M)	0.2253	0.0580	0.4301	0.0583
AR(1)	0.2353	0.0606	0.4472	0.0608
ARX	0.2373	0.0620	0.4230	0.0571
ARX-GARCH	0.2296	0.0571	0.4363	0.0596
MRS	0.4758	0.1252	0.6867	0.0925
MRJD	0.2768	0.0730	0.4750	0.0643

Notes: The table presents results from the out-of-sample forecast evaluations. Under the columns for MAE, MAPE, RMSE and TIC, the smallest value for each column appears in bold. This signify the best model based on the criteria corresponding to that column.

The out-of-sample results presented in Tables 4.7 and 4.8 are for the predictions for

Table 4.9: Diebold-Mariano Test of Equal Predictive Accuracies (Summer)

	ARX-MSM	ARX(M)-MSM	ARX-MSM(M)	AR(1)	ARX	ARX-GARCH	MRS	MRJD
ARX-MSM		0.0756	0.2758	-0.4073	1.5981	0.7255	-4.9166	-0.2696
ARX(M)-MSM	0.4699		-0.0062	-0.2869	1.0965	0.4433	-5.1273	-0.2817
ARX-MSM(M)	0.3914	0.4975		-0.7419	2.2378	0.8878	-5.0610	-0.5219
AR(1)	0.3420	0.3871	0.2292		2.5875	1.0070	-5.0652	0.0371
ARX	0.0553	0.1366	0.0128	0.0049		-1.6476	-5.3986	-3.2606
ARX-GARCH	0.2342	0.3288	0.1875	0.1571	0.0499		-5.2937	-1.0976
MRS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000		5.2459
MRJD	0.3938	0.3891	0.3009	0.4852	0.0006	0.1364	0.0000	

Notes: The table presents results from the diebold-mariano test of equal predictive accuracy between the different models considered. The null hypothesis is that the forecast errors from model A are not statistically different from those of model B, i.e. $H_0 : \mathbb{E}(d_t) = 0$ against $H_1 : \mathbb{E}(d_t) \neq 0$. The t-statistics are reported on the upper diagonal of the table, while the associated one-sided p-values are reported in the corresponding lower diagonal. For example, the t-statistic from the comparison of the forecast errors from ARX-MSM and ARX(M)-MSM is on the cell corresponding to row 1 and column 2, while the associated p-value is on column 1 row 2. A negative t-value in cell(i,j) suggests that the model on row i provides better forecasts than the model on column j. A positive t-value suggests the opposite. The associated p-value on cell(j,i) indicates whether the t-value is statistically significant.

the month of January (Winter) 2012. As a robustness check and to ensure that the results are not specific to any season, we also conduct out-of-sample predictions for the month of June (summer) 2012. The results are presented in Tables 4.9 and 4.10. The results from the DM test on Table 4.9 are less clear cut than the results in Table 4.7. The forecast accuracy of the MSM models are not statistically different from the results of most other models, except the ARX model. The ARX model actually produces significantly better forecasts than the ARX-MSM and the ARX-MSM(M) models. The MRS model is the worst performing of all the models. Considering the more traditional measures of forecast accuracy on Table 4.10, the ARX-MSM model produces the least MAE and the least MAPE, while the ARX model produces the least RMSE and the least TIC.

Lastly, Figures 4.6 to 4.13 depict the price forecasts from the different models. It can be observed that both the ARX-MSM models are able to forecast price more accurately in both low and high volatility regimes. The MRS model on the other hand generates prices that are too high in low regimes and too low in high regimes.

Table 4.10: Other Evaluations of Out-of-Sample Forecasts (Summer)

Model	MAE	MAPE	RMSE	TIC
ARX-MSM	0.4559	0.1759	1.1276	0.1735
ARX(M)-MSM	0.4999	0.1867	1.1253	0.1735
ARX-MSM(M)	0.4760	0.1820	1.1254	0.1722
AR(1)	0.4943	0.1888	1.1337	0.1742
ARX	0.4983	0.1858	1.0961	0.1691
ARX-GARCH	0.5034	0.1849	1.1134	0.1725
MRS	0.9474	0.3144	1.8102	0.2710
MRJD	0.5277	0.1972	1.1333	0.1742

Notes: The table presents results from the out-of-sample forecast evaluations. Under the columns for MAE, MAPE, RMSE and TIC, the smallest value for each column appears in bold. This signify the best model based on the criteria corresponding to that column.

Similarly, the MRJD and the ARX-GARCH models generate too many price jumps than are required to adequately forecast future prices.

Figure 4.6: ARX-MSM Price Forecast

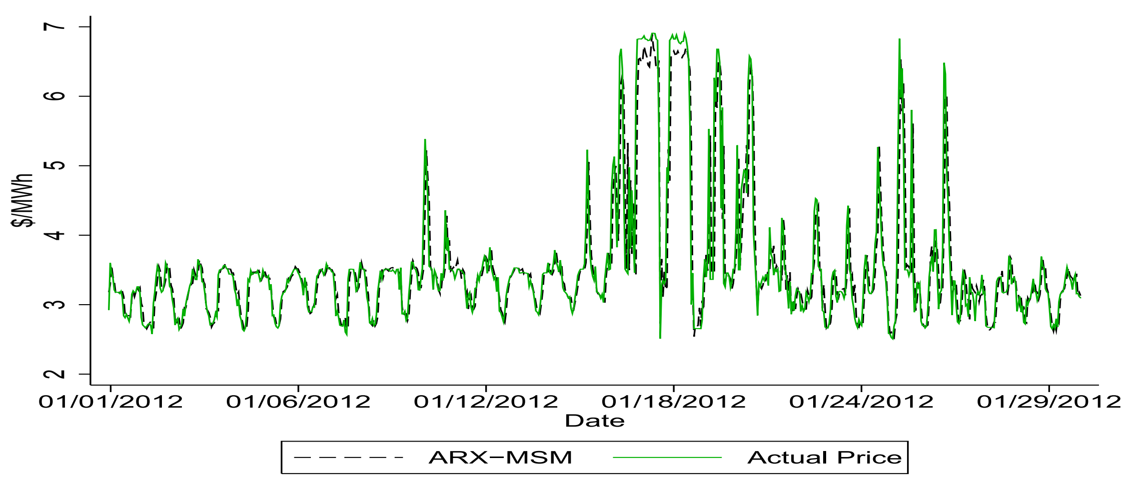


Figure 4.7: ARX(M)-MSM Price Forecast

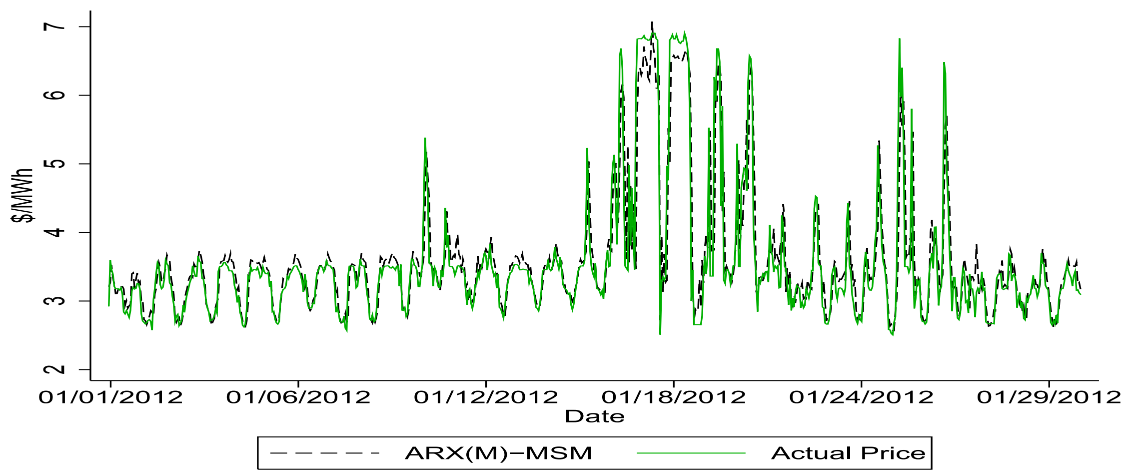


Figure 4.8: ARX-MSM-M Price Forecast

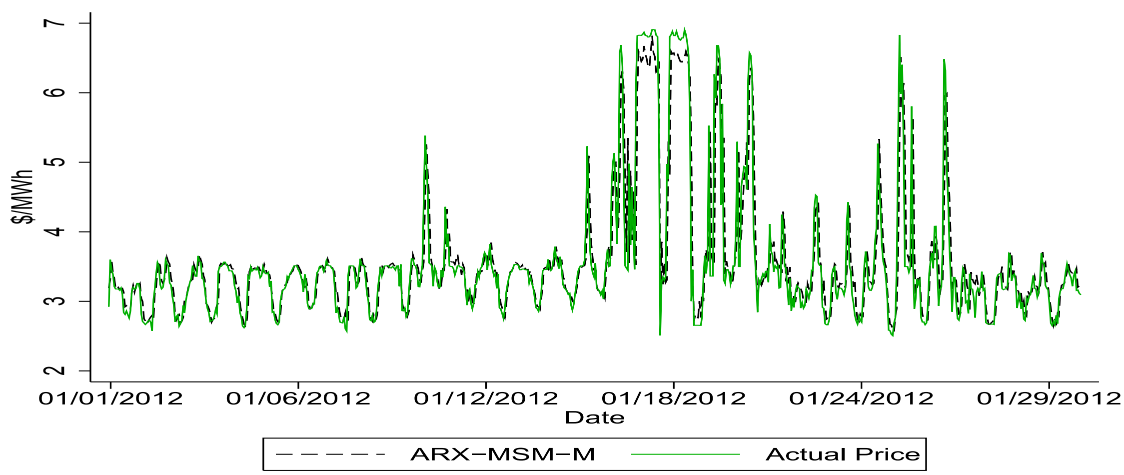


Figure 4.9: AR(1) Price Forecast

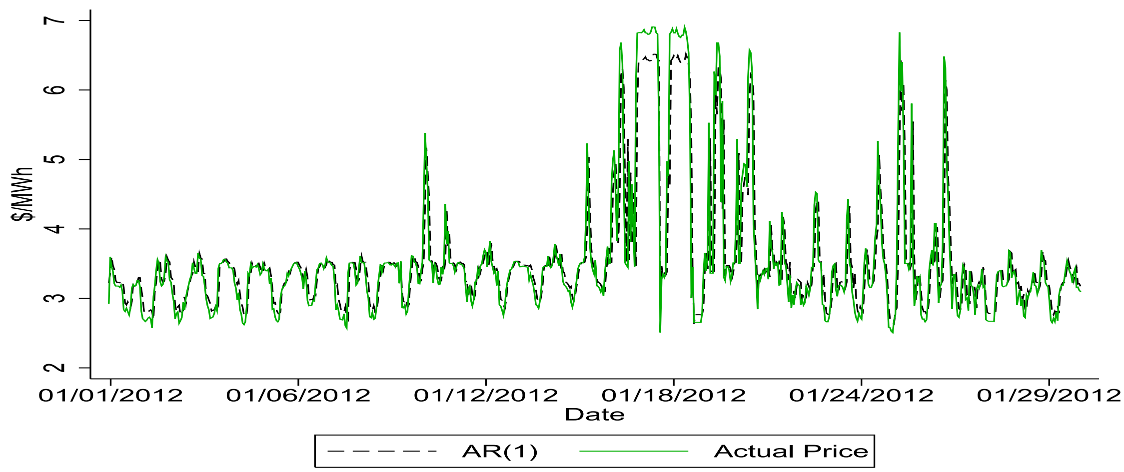


Figure 4.10: ARX Price Forecast

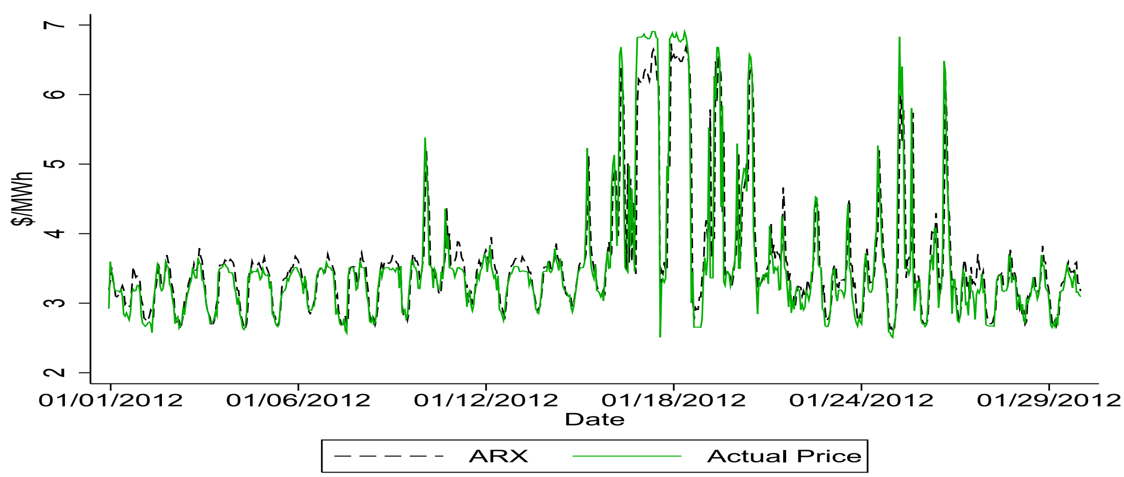


Figure 4.11: MRJD Price Forecast

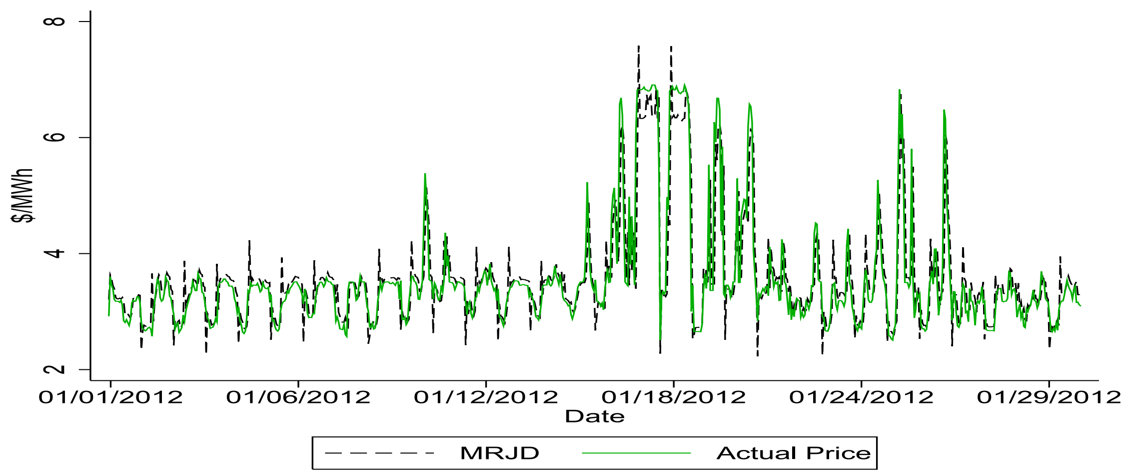
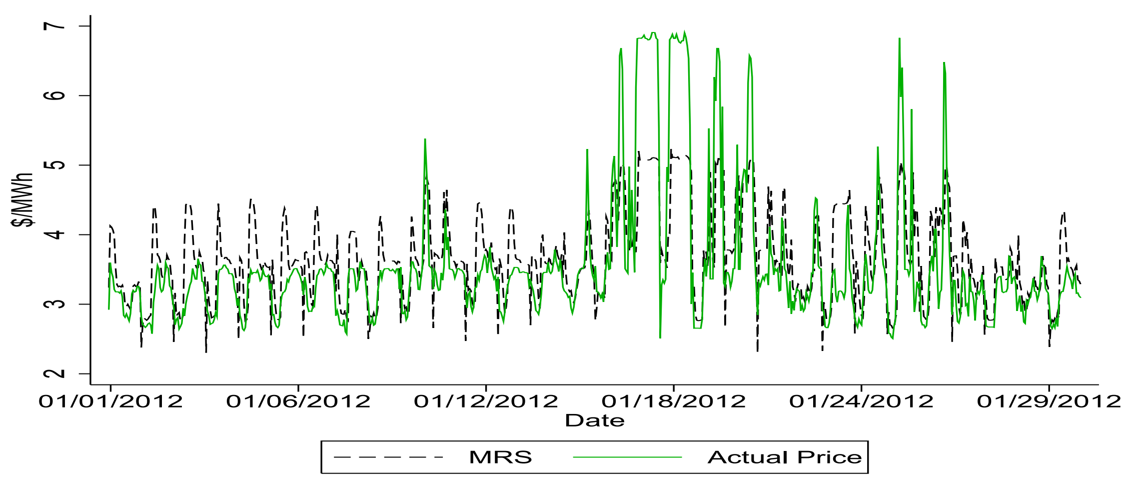


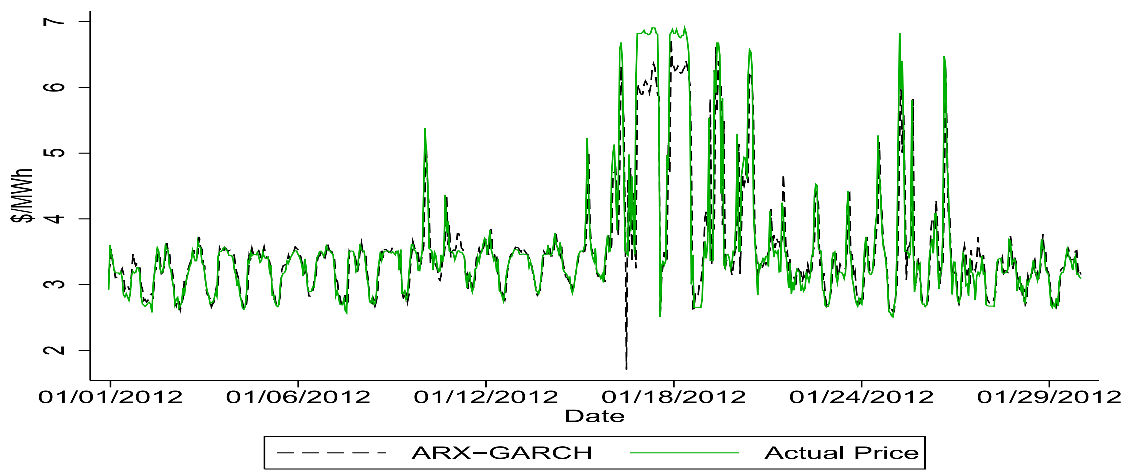
Figure 4.12: MRS Price Forecast



4.6. Conclusion

This paper introduces a new class of model, the Autoregressive Markov switching multifractal model, for forecasting spot electricity prices. Three variants of the model

Figure 4.13: ARX-GARCH Price Forecast



are examined. The first variant, the ARX-MSM, is a simple mean-reversion model that allows for a Markov switching multifractal (MSM) volatility process. This model already incorporates the main characteristics of electricity prices, namely mean-reversion, conditional volatility and price spikes. Price jumps with heterogeneous durations are introduced through switches in the volatility components with heterogeneous frequencies. The second variant allows for regime switches in both drift and volatility. This is motivated by earlier empirical findings that electricity prices are characterized by regime dependent drifts. The last variant of the model introduces risk premium into the mean equation.

Employing hourly prices from the AESO market, the parameters of the ARX-MSM models are estimated, and one-step-ahead hourly forecasts are obtained. To put the performance of the ARX-MSM models into perspective, the results are compared to those of other notable models in the literature, namely the AR(1), ARX, ARX-GARCH, mean-reverting jump and the 2-state independent Markov regime switching models. The goodness-of-fit tests indicate that the ARX-MSM models fit the data significantly better than the competing models. Likewise, out-of-sample results show that an ARX-MSM models provides always the best forecast accuracy, although by small margins.

There are however, rooms for for improvements in terms of advancing the ARX-MSM models and the empirical analysis. First, none of the variants of the ARX-MSM model incorporates the inverse leverage effect that is widely documented in the literature (Bowden and Payne, 2008; Hickey et al., 2012; Knittel and Roberts, 2005). The intuition is that positive price shocks tend to have a larger impact on volatility than negative price shocks, due to convex marginal cost curves. Second, although not present in the sample used in this study, electricity prices can be negative due to a combination of non-trivial start-up costs associated with generators and the inability to freely dispose electricity. Third, on the empirical side, the focus of this study has been on the AESO market. But the models need to be applied to other markets with different characteristics in other to generalise the validity or otherwise of the models. Fourth, this study employs hourly data for hourly price forecasting. Although this is relevant for day traders and speculators, it is of limited relevance for day ahead generators and consumers. Therefore, another potential extension of this study will be the application of the models to day ahead and multi-step forecasts.

Lastly, besides forecasting prices, the other most important application of electricity pricing models is for derivatives pricing and risk management. In the latter case, a measure conditional volatility is readily available after the estimation of the ARX-MSM models, and the filtered and transition probabilities can be used for one-step or multi-step-ahead volatility forecast for risk management purposes. For deriving derivative prices, the models are tractable and naturally generalize to both discrete and continuous time frameworks used for asset pricing.

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