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PROPOSITIONAL LOGIC, CONSERVATION AND TRANSITIVITY IN SIX TO NINE YEAR OLDS

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Propositional Logic, Conservation and Transitivity in Six to Nine Year Olds", submitted by Dexter Roland Amend in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Psychology.

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Abstract

This research was prompted by Brainerd's (1975) argument that, on logical grounds and contrary to Piaget's theory of logical growth, the acquisition of propositional logic should precede the acquisition of class and relational logic. Two samples (Canadian and American) of primary grade children (aged 6-9 yrs.) were presented with two concrete operations tasks based on class and relational logic, and a twenty-item questionnaire which assessed childrens' understanding of two inference rules and three axioms which form the basis of propositional logic. The main purpose was to find out if young children understand the basic principles of propositional logic, and to evaluate the developmental relationship between the understanding of propositional logic and the acquisition of concrete operations. Results from both samples suggested that primary grade children, regardless of age, understand the basic principles of propositional logic and that this understanding is acquired before concrete operations, or class and relational logic.

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Introduction

This research involved the assessment of primary grade childrens' (aged 6-9 years) understanding of propositional logic. Although the rationale and methods employed herein are similar to those used by others who have studied logical abilities (e.g., Burt, 1919; Hill, 1961; Ennis and Paulus, 1965), they differ markedly from Piaget's approach (Piaget, 1957; Inhelder and Piaget, 1958; Flavell, 1963) the influence of which is responsible for the prevailing view that an understanding of propositional logic is not acquired until adolescence (age 12 years and after). In order to place the present study in its proper context it is necessary to discuss Piaget's approach to propositional logic and to consider some of the work which has come in its wake.

Propositional Logic and Adolescent Thinking

In 1958, in collaboration with B. Inhelder, Piaget published "The Growth of Logical Thinking from Childhood to Adolescence". This volume, which describes and illustrates the transition from the concrete operations stage to the formal operations stage, contains virtually all of the information Piaget has provided regarding his view that propositional logic is uniquely associated with adolescent thought. This book provides the content for the following discussion of Piaget's view.

The concrete operations stage is associated with middle childhood (approximately 7-11 yrs.). This stage is based on separate cognitive structures which correspond to the logic of classes and the logic of relations. Thinking during this stage is in terms of actualities. Class and relational logic enable simple operations of correspondence, classification and ordering upon physical objects which can be observed and manipulated, but do not permit consideration of more complex object relationships which are not concretely evident in a situation. On the other hand, the formal operations stage is associated with adolescence (age 12 yrs. and after). This stage is based on an integrated cognitive structure which corresponds to a complete system of propositional logic. This system is comprised of the 16 possible relationships obtainable among the propositions p, q and their negatives (Appendix A tables these relationships). Thinking during this stage is in terms of possibilities. The 16 propositional relationships ("binary operations") express all the ways in which the objects (variables) of a complex situation may interact to produce a particular outcome. Taken together, these propositional relationships enable hypothetico-deductive reasoning which involves the systematic consideration of all variables and combinations thereof which may figure in the outcome of a complex situation.

To illustrate the differences in thinking which occur at the concrete and formal stages children and adolescents were presented with complex situations similar to demonstrations used in junior high school science classes. These demonstrations involved such things as the equality of angles of incidence and reflection the equilibrium of a balance, the role of invisible magnetism and the combination of colorless liquids to form a colored solution. In each case, subjects were asked to explain a particular outcome or solve a particular problem. The result was that children were not able to provide correct explanations or solutions, while adolescents were. The verbal responses each subject made when explaining an outcome or solving a problem were recorded and subsequently analyzed by Piaget. His analysis of the response protocols showed that childrens' failure was due to their unsystematic use of correspondence, classification and ordering procedures which failed to exhaust the possible combinations of variables involved in particular outcomes and solutions. He attributed the younger subjects' failure to the separate, unintegrated structures of class and relational logic which underlie concrete operations. On the other hand, Piaget's analysis of adolescents' responses revealed that their correct explanations and solutions stemmed from a systematic consideration of all the ways in which the variables entailed could interact. He attributed the older subjects' success to the integrated structure or complete combinatorial system of propositional logic which underlies formal operations.

According to Piaget understanding propositional logic involves the use of all of the propositional relationships in the combinatorial system. Piaget's analysis of the responses one adolescent (Gou) made when explaining the role of invisible magnetism showed that, in the course of his explanation, Gou gave responses which expressed each and all of the 16 propositional relationships.

Although the Inhelder and Piaget book presents an elaborate rationale and some illustrative support for the view that propositional logic uniquely characterizes adolescent thinking, the information it provides does not constitute sound empirical evidence. The authors make no attempt to define their sample, to adequately describe their apparatus and procedures, or to subject any of their findings to statistical treatment. Nevertheless readers of this book are left with the impression that the authors regard the acquisition of propositional logic at the formal operations stage as a distinct, invariable and observable phenomenon common to all adolescents.

Replication and Reanalysis

Lovell (1961) conducted a follow up study employing ten of the science demonstrations described by Inhelder and Piaget. Lovell defined his sample of subjects (aged 8-18 yrs.) described more explicitly the apparatus and procedures involved, and tabulated his findings. Lovell's results showed that very few, and only his ablest and oldest subjects fully achieved the stage of formal operations. Lovell suspected that Piaget's sample consisted of "able children".

Recently, Dulit (1972) performed a replication of two of the Inhelder and Piaget demonstrations, viz. the Rings experiment and the Liquids experiment. His sample consisted of average adolescents (aged 14-17 yrs.), gifted adolescents (aged 16-17 yrs.) and adults (aged 20-55 yrs.). Dulit reported that roughly 60% of his gifted adolescents functioned at the fully formal level, while fewer than 30% of his average adolescents and adults did so. Dulit's main conclusion was that formal stage thinking was far from commonplace or routine among normal adolescents and adults.

Two logicians and a psychologist (Bynum, Thomas and Weitz, 1972) reanalyzed the protocol of Dou, which Piaget presented to illustrate the use of all 16 propositional relationships. These investigators found that Piaget's interpretation and analysis of Dou's protocol was faulty. They showed that in fact only eight of Con's responses could be construed as expressions of the propositional relationships in question and that the rest were incorrectly interpreted by Piaget. They also suggested that at least six of the 16 propositional relationships are inexpressible in common language.

Weitz, Bynum and Thomas (Weitz, 1971) constructed English expressions for ten of Piaget's 16 binary combinations ("WBT Test for Propositional Logic") and administered them to three groups of subjects: 9 year olds, 12 year olds and 15 year olds. Although the grade level differences Weitz reported were few and not great, his results showed that older subjects solved more problems and used more binary operations than younger subjects. Weitz hypothesized that older subjects who scored high on the WBT test would also use more binary operations when solving the Role of Invisible Magnetism task. After administering this task, Weitz found that not one of his subjects used more than five operations, and that all age groups used the same five operations when solving the magnetism task. In addition, Weitz noted that 80% of his subjects immediately solved the magnetism problem correctly without using any binary operations. Most subjects responded simply by saying, "It must be a magnet".

The results of the preceding studies seriously undermine Piaget's approach to propositional logic.

Besides showing that Piaget's combinatorial system is an inappropriate model for adolescent thought, the results of these studies question the means, both analytic and procedural, Piaget has employed to infer that his system of propositional logic is present in adolescent thinking.

Analysis and Research in a Different Paradigm

Prior to the research reported above, Parsons (1960) analyzed the logical basis of Piaget's formal operations stage and concluded that the combinatorial system proposed by Piaget did not conform in many ways to what logicians ordinarily call propositional logic.

Ennis and Paulus (1965) also noted the uniqueness of Piaget's combinatorial system. They pointed out that there is nothing in propositional logic, as logicians view it, which requires individuals to use all of Piaget's binary combinations before it can properly be said that they understand propositional logic. These investigators developed a test to assess deductive reasoning in adolescence. Their test consisted of questions pertaining to class logic which required syllogistic reasoning, and questions pertaining to propositional logic which required conditional reasoning. Ennis and Paulus expressed the opinion that their test was a more direct and appropriate measure of the understanding of propositional logic than the methods employed by Piaget.

In a recent critique of Piaget's propositional logic Ennis (1975) argued that Piaget's system was not merely unique, but "defective". Ennis contested Piaget's claim that young children (under 12 yrs.) cannot handle propositional logic on the grounds that the claim is either untestable or false. On the one hand, Ennis pointed out that Piaget seems to be concerned with some kind of wholistic logical ability which can't be assessed, rather than with the ability to reason according to a set of principles of propositional logic. On the other hand, Ennis pointed to several studies which suggest that young children do understand some principles of propositional logic.

Hill (1961) conducted an investigation of the logical abilities of young children (aged 6-9 yrs.). Among the logical problems which she presented to her subjects were questions expressing two inference rules commonly employed by logicians when axiomatizing propositional logic, viz. Modus Ponens and Modus Tollens. Both of these basic principles of propositional logic entail the inference of conclusions from hypothetical premises. Hill's results showed that childrens' solution rates on these inference rules ranged from 74 to 92%. Subsequently, Suppes (1965) cited Hill's results to support his view that young children are able to reason in a hypothetico-deductive manner and are not limited to concrete operations as Piaget proposed.

O'Brien and Shapiro (1968) questioned Suppes' use of Hill's findings as conclusive evidence that young children are able to think in a hypothetico deductive manner. They pointed out that in Hill's study children were required only to recognize the validity or invalidity of necessary conclusions drawn from hypothetical premises. Children were not required to "test" the validity of inferences, which O'Brien and Shapiro supposed should be a better measure of hypothetico deductive reasoning. These investigators administered two tests to two matched groups of primary grade children. The first group was presented with Hill's test, which required a "yes" or "no" answer to each question. The second group was presented with an altered version of Hill's test in which one third of the items were "opened-up" so that the conclusions did not necessarily follow from the premises. Unlike Hill's original test, the questions in the altered test were open to three possible answers: "yes", "no" and "not enough clues". Performance by children who were presented with Hill's test was at the same high level initially reported by Hill. But performance by children presented with the altered test was significantly lower. O'Brien and Shapiro observed that, for one reason or other, children who received the test with "open-questions" avoided the "not enough clues" answer.

Although O'Brien and Shapiro succeeded in constructing a more difficult test of hypothetico deductive reasoning, their confirmation of Hill's initial findings, lends support to the possibility that young children are aware of the necessary relationships expressed in principles of propositional logic like Modus Ponens and Modus Tollens.

Kodroff and Roberge (1975) also presented primary grade children with instances of Modus Ponens and Modus Tollens. They varied mode of presentation in terms of concrete and verbal, and type of content in terms of related and unrelated. Their results showed that children performed well on Modus Ponens, but less well on Modus Tollens. In addition, they found that performance was better with the concrete presentation and with the related content. The Kodroff and Roberge study is the most recent in the series of studies discussed herein, each of which entails the assessment of propositional logic in a way different from that of Piaget. Further, these studies suggest that when the basic principles of propositional logic, such as the inference rules, are presented and assessed in a direct and appropriate manner, young children appear to understand them.

An Argument for Early Acquisition

Brainerd (1975) has very recently contended that propositional logic cannot be an appropriate model for adolescent thinking. He has pointed out a determinant relationship between the logic of propositions and the other two branches of mathmatical logic, viz. the logic of relations and the logic of classes. This relationship is similar in principle to the relationship between arithmetic and algebra. Just as arithmetic is more basic than algebra in that the operations and theorems of algebra are derived from more primitive arithmetic notions and not the other way around, so propositional logic is more basic than the logic of classes and relations. For, all the axioms, inference rules and theorems of propositional logic are employed in class and relational logic, but not conversely. Brainerd argues that if the logic of classes and relations are used as a model for a given stage of logical growth (concrete operations) then the succeeding stage, (formal operations) cannot, logically, be based on the logic of propositions. If anything, propositional logic should characterize cognitive acquisitions which precede the concrete operations stage.

The Present Study

The principle impetus behind the present study was Brainerd's argument that, on logical grounds, the acquisition of propositional logic should precede the acquisition of class and relational logic, or concrete operations. Empirically, the work by Hill (1961), O'Brien and Shapiro

(1968) and Fodroff and Roberge (1975) seemed to indicate that primary grade children have a grasp of Modus Ponens and Modus Tollens. Obviously, however, success with two inference rules is not enough evidence to warrant the suggestion that young children thoroughly understand propositional logic. To provide a more complete measure of young childrens' logical abilities, one which would be recognized as sufficiently thorough, it was decided to assess their understanding of all the axioms and inference rules employed in the axiomatization of a commonly acknowledged formal system of propositional logic.

The Basic Principles of Propositional Logic. Reference to most any introductory text on symbolic logic (e.g., Basson and O'Conner, 1953 or Massey, 1970) will make evident the important basic and general properties of axioms and inference rules. Axioms are fundamental composite propositions which are always true. Inference rules are the means by which it can be said that a given proposition follows from one or more other propositions. Taken together, these basic principles determine the validity of any proposition or chain of propositions in a system. Among the few commonly acknowledged systems of propositional logic are those of Whitehead and Russell (1910) and Alonzo Church (1956). The axioms and inference rules employed by Church were chosen for study in the present investigation for two reasons: a, they are fewer in number and b. they are all expressed in terms of conditionals (i.e. they are formed by use of the Horse shoe connective "⊃"). This second feature makes the axioms and inference rules more easily expressible in English.

The axioms which were employed in the present study are: Affirmation of the Consequent, Self distributive Law of the Horseshoe and Converse Law of Contraposition. These axioms are to propositional logic what the first laws of arithmetic are to arithmetic (cf. Brainerd, 1976). Like commutativity, associativity and distributivity which define how numbers are connected by addition and multiplication, these axioms define how propositions are connected by use of the "horseshoe". Affirmation of the Consequent (written $p \supset (q \supset p)$) expresses the fact that: Given a proposition (p), then p is the consequent of another proposition (q). Self distributive Law of the Horseshoe (written $(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$) defines the distributive property of the horseshoe. This law expresses the fact that: when a proposition (p) is the antecedent of two other propositions linked by a horseshoe ($p \supset r$), if p is the antecedent of the first proposition ($p \supset q$), then p is the antecedent of the second proposition ($p \supset r$). Converse Law of Contraposition (written $(\sim p \supset \sim q) \supset (q \supset p)$) expresses the fact that: If an antecedent is composed of a conditional relationship between two negative propositions ($\sim p \supset \sim q$), then the consequent is a conditional relationship formed by changing the signs of the two propositions and reversing their order ($q \supset p$). The two inference rules which were employed in this study are: Modus Ponens and Substitution. Modus Ponens (written $(p \supset q \cdot p) \supset q$) expresses the fact that: If a conditional relationship exists between two propositions ($p \supset q$) and the first proposition (p) is true, then the second proposition (q) is true. Substitution (written $((p \supset q) \cdot (r = p)) \supset (r \supset q)$) expresses the fact that: If a conditional relationship exists between two propositions ($p \supset q$), and a third proposition (r) is equivalent to the first proposition (p), then a conditional relationship exists between the third and second propositions ($r \supset q$), i.e., if $r = p$, r can be substituted for p.

Rationale Behind the Construction of the Questionnaire. Both inference rules and axioms involve a necessary relationship between two groups of propositions. For inference rules, the first group of propositions is the premises of an argument and the second group is the conclusion. An example is the following argument which is valid via Modus Ponens:

Example 1

Logical Form			English Expression
Group 1	$p \supset q$	(Premises)	If Mary wears her boots, Then Mary's feet will stay dry.
	p		Mary wears her boots
Group 2	q	(Conclusion)	Mary's feet will stay dry.

Similarly, axioms express a relationship between a group of propositions on the left hand side of a major logical connective, and a group of propositions on the right-hand side. An example is the Converse Law of Contraposition:

Example II

Logical Form		English Expression
$(\sim p \supset \sim q) \supset (q \supset p)$	Group 1	If, when Bill is not thirsty, Bill never drinks water.
Group 1 Group 2	Group 2	Then, if Bill drinks water, then Bill is thirsty.

To recognize that the conclusion follows from the premises in Example I implies an understanding of the inference rule, Modus Ponens. Similarly, to see that the second group of propositions follows from the first in Example II implies an understanding of the axiom, Converse Law of Contraposition. On the basis of this rationale, a test was constructed to assess young childrens' understanding of each of the five basic principles of propositional logic.

To assess young childrens' understanding of the five basic principles of propositional logic, each principle was translated into English and expressed in content common to the experience of primary grade children, as in the two examples previously given. Each principle was presented as a question prefaced by the phrase "Suppose you know that....". And, each question required either a "yes" or a "no" answer. Although the questions were posed in an hypothetical manner. the concern in the present study was not so much with the intracacies of hypothetico deductive reasoning, as in O'Brien and Shapiro's (1968) work, but more simply and directly with young childrens' ability to recognize the necessity and truthfulness entailed in the five specific principles which form the basis of propositional logic. In line with this concern, no "opened-up" items requiring "maybe" or "not enough clues" answers were constructed. Ennis and Paulus (1965) and others have shown that not only children but also adolescents and adults perform less well on logical problems for which there are no necessary conclusions. Further, an attempt was made to provide familiar and related content, as opposed to unfamiliar, symbolic or unrelated content which are known to increase difficulty on logical problems for adults as well as children (Wilkins, 1928; and Kodroff and Roberge, 1975). The questionnaire containing all the questions presented in this study appears in Appendix C.

Propositional Logic and Concrete Operations. The age-range of the young children employed as subjects in this study (6-9 yrs.) corresponds, within a Piagetian context, to the period of transition between the pre-operational and concrete operational stages. Thus, some of the children were expected to possess concrete operations while others were not. To obtain empirical evidence regarding Brainerd's (1975) contention that the acquisition of propositional logic should precede the acquisition of concrete operations the subjects in this study were administered two tasks which assess the presence of concrete operations. These tasks are detailed in the next section.

The Principle Questions of Interest. The major objective of this study was to find out whether young children (aged 6 9 yrs.) understand the basic principles of propositional logic, or not. Related questions of interest which this research was designed to answer were: 1., Is there an age trend in young childrens' understanding of logical principles? 2., do young children find some logical principles more difficult to understand than others, and if so, which? 3., Is there a developmental relationship between young childrens' understanding of logical principles and their performance on Piagetian concrete operational tasks?

Method

Subjects

Two samples of 72 children served as subjects in this study. Both samples consisted of 12 boys and 12 girls from each of grades 1, 2 and 3. Subjects in the first sample were drawn from the class lists of a middle class, urban elementary school in Edmonton, Canada. The age range of first graders was 6 yrs. to 7 yrs., 1 mn., and the mean age was 6 yrs., 6 mns.. The age range of second graders was 7 yrs. to 7 yrs. 9 mns., and the mean age was 7 yrs., 5 mns.. The age range of third graders was 8 yrs. to 8 yrs. 11 mns., and the mean age was 8 yrs., 5 mns.. The experimenter for this sample was a male caucasian, aged 32 yrs.. Subjects in the second sample were drawn from the class lists of a lower middle class rural elementary school in Deer Park, Washington. The age range of the first graders was 6 yrs., 6 mns. to 7 yrs., 5 mns., and the mean age was 6 yrs., 11 mns.. The age range of the second graders was 7 yrs., 5 mns. to 8 yrs., 5 mns., and the mean age was 7 yrs., 11 mns.. The age range of the third graders was 8 yrs., 4 mns. to 9 yrs., 4 mns., and the mean age was 8 yrs., 11 mns.. The experimenter for this sample was a female caucasian aged 24 yrs..

Materials

The Concrete Operations Tasks. To assess transitivity of length and conservation of length materials were prepared after Brainerd (1973). For length transitivity, three sticks were cut from ½" dowling stock: a 27.5 cm. stick (painted blue), a 27.5 cm. stick (painted yellow), and a 28.5 cm. stick (painted blue). For length conservation, two 28 cm. lengths were cut from ¼" yellow nylon cord.

The Questionnaire. To assess understanding of the principles of propositional logic, a questionnaire was constructed consisting of 20 items, four pertaining to each of the five logical principles (see Appendix C). According to the rationale mentioned previously, each item had the general form:

> "If you know the first group of statements, is the second group of statements true"?

The four items pertaining to each principle consisted of two examples which differed in content, and two expressions (one positive and one negative) for each example (e.g., items 1-4 pertaining to Substitution, in Appendix C). Each item was typed on an index card for presentation.

Treatment Conditions

Age. Three age levels, six-seven year-olds, seven-eight year olds, and nine year olds from grades 1, 2 and 3 respectively were studied.

Sex. Equal numbers of boys and girls from each grade were studied.

Order. Two of the 5! orders in which the principles can occur were randomly chosen for presentation. Order I was the sequence: Substitution (S), Modus Ponens (M), Converse Law of Contraposition (C), Self-distributive Law of the Horseshoe (D), Affirmation of the Consequent (A). Order II was the sequence: M, A, D, S, C. One half of the subjects were presented with the principles in Order I, and one half of the subjects were presented with Order II.

Conditions. One-half of the items of the questionnaire expressed the principles in a positive way and required a "yes" answer. One half of the items expressed the principles in a negative way and required a "no" answer. One half of the subjects received the positive expression of each principle first followed by the negative expression (Condition P). One half of the subjects received the negative expression first, followed by the positive expression (Condition N).

Randomizations. Equal numbers of boys and girls from the three age levels were randomly assigned to the two orders and two conditions.

Procedure

The concrete operations tasks and the questionnaire were presented to each subject (S) individually during school hours in a room separate from the class rooms. Each S was presented with the concrete operations tasks first and the questionnaire last. During the assessments the S and the experimenter (E) sat across from each other at a small table. To avoid distraction and confusion all test materials were maintained under the table, out of sight. Only those materials pertaining to the particular task at hand were visible to S at any one time. The total assessment session lasted approximately 15 minutes per S.

Administering the Concrete Operations Tasks. Each S was presented with the length transitivity task first, followed by the length conservation task.

The procedure for assessing transitivity was as follows. First, E took all three sticks from under the table. Then, holding the sticks together in one of his hands, E showed them to S and said, "There are three sticks here, two are blue and one is yellow, right?". When S gave his assent, E put all the sticks beneath the table, out of sight. Next, E took the blue and yellow sticks of equal length from under the table and placed them on the table horizontally, before S. E moved the sticks close together so that S could see they were of equal length. Then E asked "are these sticks the same length?". When S gave his assent E took the blue stick and put it beneath the table out of sight. Next E took the longer blue stick from under the table and placed it on the table beside the yellow stick. E moved the sticks close together so that S could see that the blue stick was longer than the yellow stick. Then E asked, "is the blue stick longer than the yellow stick?". When S gave his assent, E took the yellow stick and put it beneath the table, out of sight. Finally, E took both blue sticks, one in each hand, and while holding them vertically about three feet apart before S, E asked the following (randomly ordered) questions: (a), "are these two sticks the same length?" (b), "is one of these sticks longer? (if so, which one?)" (c), "is one of these sticks shorter? (if so, which one?)".

The procedure for assessing conservation was as follows. First, E took the two pieces of cord of equal length from under the table and placed them on the table, horizontally before S. E moved the pieces close together so that S could see that they were of equal length. Then E asked, "are these pieces of cord the same length?". When S gave his assent, E bent the piece of cord nearer E to form a "U" shape. Then E asked the following (randomly ordered) questions: (a), "are these two pieces of cord the same length?" (b), "is one of these pieces of cord longer (If so, which one?)" (c) "Is one of these pieces of cord shorter (If so, which one.)".

Administering the Questionnaire. Following presentation of the concrete operations tasks, E took the index cards with the items typed on them and said to S:

> "I have a stack of cards here. Each card tells something about a boy named Bill (girl named Mary, for girls) and each card asks a question. I'm going to read these cards to you one at a time. Each time I read a question, you're supposed to tell me the right answer. The first card says..."

During presentation of the questionnaire, when S hesitated after E had read a particular card, or when S asked for the card to be re-read, E re-read the card. S's answers to the concrete operations tasks and his answers to the questionnaire were recorded by E during the assessment session.

Dependent Variables

The dependent variables of interest were the correctness-incorrectness of the subject's answers on the Piagetian tasks and on the questionnaire.

Answers on the Piagetian Tasks. Subjects' correct answers to the three questions pertaining to transitivity and to the three questions pertaining to conservation were recorded on a score sheet (see Appendix D) by marking an "✓". Incorrect answers were recorded by marking an "0".

Answers on the Questionnaire. When answering the items of the questionnaire subject's responded by saying "yes" or "no". These two categories of response were indicated opposite each item on the score sheet. During the assessment for logical understanding, the experimenter simply checked (marked a "✓") in the category corresponding to the subject's response.

Results

Each correct answer on the twenty item questionnaire was assigned a score of 1. With four items pertaining to each of the five principles of propositional logic, the minimum possible score on each principle was 0, and the maximum possible score was 4. The means and standard deviations of the number of correct answers on each principle are reported by grade level for both samples in Table 1.

Analyses of Variance

Separate 3 (Grade Level) X 2 (Sex) X 2 (Order of Presentation) X 2 (Condition) X 5 (Logical Principle) mixed model analyses of variance were computed for the Canadian and American samples. Two main effects attained significance for the Canadian sample: the between-subjects effect for Grade Level ($F=11.10$, $df=2/48$, $p < .001$), and the within-subjects effect for Logical Principle ($F=86.98$, $df=4/192$, $p < .0001$). In addition, a second-order interaction, Sex X Condition X Logical Principle, attained significance ($F=4.35$, $df=4/192$, $p < .005$). Newman-Kuels post hoc comparisons of the Grade Level means revealed a significant ($p < .01$) difference between grade 3 and both grades 1 and 2, but no difference between grades 1 and 2 (see Table 2). Orthogonal

Table 1

Means and Standard Deviations of the Number of Correct Answers on Each Principle by Grade Level for the Canadian and American Samples

Grade Level		Substitution	Modus Ponens	Affirmation of the Consequent	Self-distributive Law of the Horseshoe	Converse Law of Contraposition
Canadian Sample						
1	M	3.83	3.92	2.71	2.83	2.25
	SD	0.38	0.27	0.79	0.85	0.92
2	M	4.00	3.92	2.83	2.42	2.21
	SD	0.00	0.27	0.88	0.91	0.82
3	M	4.00	4.00	3.33	3.17	2.[illegible]
	SD	0.00	0.00	0.80	0.69	0.89
American Sample						
1	M	3.83	3.88	2.71	2.42	2.36
	SD	0.56	0.44	0.94	0.95	0.60
2	M	3.71	3.96	3.00	2.67	2.00
	SD	0.46	0.21	1.04	0.80	0.76
3	M	4.00	3.96	2.88	2.42	2.38
	SD	0.00	0.21	0.83	0.95	0.70

Table 2

Grade Level and Logical Principle Means

for the Canadian and American Samples

Source	Mean
Canadian Sample	
Grade Level	
1	15.54
2	15.38
3	17.21
Logical Principle	
Modus Ponens	3.94
Substitution	3.94
Affirmation of the Consequent	2.96
Self-distributive Law of the Horseshoe	2.81
Converse Law of Contraposition	2.39
American Sample	
Grade Level	
1	15.21
2	15.34
3	15.63
Logical Principle	
Modus Ponens	3.93
Substitution	3.85
Affirmation of the Consequent	2.86
Self-distributive Law of the Horseshoe	2.50
Converse Law of Contraposition	2.25

Polynomials analysis of the Grade Level effect showed that the linear component accounted for the greatest share of variation ($F=13.18$, $df=1/69$, $P < .005$). The F-test for the quadratic component was nonsignificant. Newman Kuels post hoc comparisons of the Logical Principle means (see Table 2) disclosed the following: Converse Law of Contraposition was significantly ($P < .01$) more difficult than each of the other principles; Self-distributive Law of the Horseshoe and Affirmation of the Consequent were equally difficult, but both were significantly ($P < .01$) more difficult than Modus Ponens and Substitution; Modus Ponens and Substitution were equally difficult. Newman-Kuels post hoc comparisons of the means for the Sex X Condition X Logical Principle interaction revealed that boys in the negative condition scored significantly ($P < .01$) higher than boys in the positive condition on Affirmation of the Consequent, and girls in the positive condition scored significantly ($P < .01$) higher than girls in the negative condition on Self-distributive Law of the Horseshoe.

The results for the American sample showed no significant Grade Level effect and no significant interactions. The only effect which attained significance was the within subjects main effect for Logical Principle ($F=96$, $df=4/192$, $P < .0001$). Newman-Kuels post hoc comparisons of the Logical Principle means (see Table 2) disclosed the following: Converse Law of Contraposition and Self distributive Law of the Horseshoe were equally difficult, but both were significantly ($P < .01$) more difficult than each of the other principles; Affirmation of the Consequent was significantly ($P < .01$) more difficult than Modus Ponens and Substitution; Modus Ponens and Substitution were equally difficult. Inspection of the Logical Principle means for the Canadian and American samples reported in Table 2 indicates that the order of difficulty of the five principles of propositional logic was the same for both samples.

Ordinal Analyses

An ordinal analysis was conducted to evaluate the developmental relationship between the acquisition of propositional logic (as measured by overall performance on the questionnaire) and the acquisition of concrete operations (as measured by overall performance on the tasks for length transitivity and conservation). As with answers on the questionnaire, each correct answer on the tasks for length transitivity and length conservation was assigned a score of 1. With three questions pertaining to each task, the minimum possible score on both tasks combined was 0, and the maximum possible combined score was 6. To conduct the ordinal analysis, pass-fail criteria were established to partition all subjects.

independent of grade level into two categories: a. subjects who had acquired propositional logic but had not acquired concrete operations and b. subjects who had acquired concrete operations but had not acquired propositional logic. Two sets of criteria were employed. Criterion A required a minimum total score of 14 (70% correct) on the questionnaire for propositional logic to be considered present, otherwise it was considered absent; and, a minimum total score of 4 (67% correct) on the length transitivity and conservation tasks for concrete operations to be considered present, otherwise it was considered absent. Criterion B differed from A in that a minimum total score of 15 (75% correct) was required on the questionnaire. Both sets of criteria were applied to the data of the Canadian and American samples and the results appear in Table 3. The relationships shown in Table 3 were tested for significance by use of the binomial test (Siegel, 1956). The probability values below Table 3 indicate that, for both samples and according to both sets of criteria, propositional logic is acquired before concrete operations.

In the present study, the Piagetian tasks were employed only as a reference variable to enable the evaluation of the developmental relationship between propositional logic and concrete operations. For this reason, no analyses pertaining solely to length transitivity or

Table 3

The Developmental Relationship Between

Propositional Logic and Concrete Operations

for the Canadian and American Samples

Propositional Logic		Concrete Operations	
Canadian Sample			
Criteria		Present	Absent
A	Present	40	31***
	Absent	0***	1
B	Present	33	21*
	Absent	7*	11
American Sample			
Criteria		Present	Absent
[illegible]	Present	23	41***
	Absent	2***	6
B	Present	19	27**
	Absent	6**	20

Note. Criterion A = 70% correct on the questionnaire and 67% correct on the concrete operations tasks; Criterion B = 75% correct on the questionnaire and 67% correct on the concrete operations tasks.

*p = .014

**p < .001

***p < .0001

length conservation were planned. But inspection of the scores on the concrete operations tasks showed that approximately 97% of the subjects in both samples correctly answered all the questions pertaining to transitivity, while 44% of the subjects in the Canadian sample and 22% of the subjects in the American sample correctly answered all the questions pertaining to conservation. Owing to the ceiling effects reflected in the high solution rates on the inference rules and the perfect performance by nearly all subjects on transitivity, the inference rules and transitivity were omitted from the following analyses which evaluate the developmental relationships between conservation and each of the three axioms. Pass-fail criteria for each of these analyses required a minimum score of 3 (or 75% correct) for each axiom to be considered present, otherwise it was considered absent, and a minimum score of 2 (or 67% correct) for conservation to be considered present, otherwise it was considered absent. The results of these analyses are presented in Table 4. The binomial test was also used to test the relationships shown in Table 4. The probability values below Table 4 indicate the following: for both samples, Converse Law of Contraposition and conservation are acquired synchronously; for the Canadian sample, Self-distributive Law of the Horseshoe and conservation are acquired synchronously, but for the American sample, Self-distributive Law of the

Table 4

The Developmental Relationships Between Each of the Three Axioms and Conservation for the Canadian and American Samples

Axioms		Conservation	
Canadian Sample			
		Present	Absent
Converse Law of Contraposition	Present	15	10
	Absent	17	30
Self-distributive Law of the Horseshoe	Present	21	20
	Absent	11	20
Affirmation of the Consequent	Present	26	19*
	Absent	6*	21
American Sample			
		Present	Absent
Converse Law of Contraposition	Present	8	18
	Absent	8	38
Self-distributive Law of the Horseshoe	Present	8	22*
	Absent	8*	34
Affirmation of the Consequent	Present	9	30**
	Absent	7**	26

*$p < .01$

**$p < .001$

Horseshoe precedes conservation; for both samples.

Affirmation of the Consequent precedes conservation.

Discussion

With respect to the major question posed in this study viz. whether or not young children understand the basic principles of propositional logic, two findings seem to suggest a positive answer. First, taking over all performance on the twenty item questionnaire as a general measure of understanding, the means of all grades from both samples evidenced solution rates in excess of 75% well beyond the 50% objective chance solution rate. Second, except in one case, where the American second graders solution rate on Converse Law of Contraposition was 50%, solution rates on all the principles by all the grades from both samples were greater than 50%.

Results pertaining to the question of an age trend in childrens' understanding of propositional logic were equivocal. While an age (Grade Level) effect was evidenced in the Canadian sample, none was noted in the American sample. That an age trend was observed in the Canadian sample but not in the American sample may be due in part to socio-economic factors. The rural area from which the American sample was drawn is populated by individuals of low socio-economic status. Typically, children with relatively depressed backgrounds show slower improvement (and hence, fewer and more minimal differences in performance) in the primary grades than their counterparts in middle class urban areas such as that from which the Canadian sample was drawn. Although the Canadian sample is more representative of middle class North America, the conclusion that an age trend is generally evident in primary grade childrens' understanding of propositional logic is probably not warranted. That performance by all subjects from both samples was uniform on each principle, independent of grade level, (i.e., no Grade Level X Logical Principle interactions attained significance) lends support to the suggestion that no age trend exists in primary grade childrens' understanding of propositional logic. Further, except for Canadian third graders, mean performance on the questionnaire was almost identical for all grades from both samples. Finally, it should be noted that age trends in performance on tasks like the questionnaire have been observed in primary grade children by some investigators (e.g., Hill, 1961, and Kodroff and Roberge, 1975) but not by others (e.g., O'Brien and Shapiro, 1968).

Concerning the question of whether or not some principles are more difficult than others, it is clear that children in this study found the axioms more difficult than the inference rules. Whereas all children showed near perfect performance on Modus Ponens and Substitution, with solution rates ranging from 90 to 100%, solution rates on the axioms were much lower, ranging from 50 to 80%. The order of increasing difficulty of the five principles was the same for both samples, viz. Modus Ponens, Substitution, Affirmation of the Consequent, Self distributive Law of the Horseshoe, Converse Law of Contraposition. Although no A Priori hypotheses were formulated regarding the relative difficulty of the principles, the previous work by Hill (1961) and Kodroff and Roberge (1975) had shown that primary grade children find Modus Ponens quite easy. At least two factors are known to increase difficulty on logical problems such as those posed in the questionnaire. Neisser and Weene (1962) have shown, and it has been confirmed by others (e.g., Haygood and Bourne, 1965), that logical problems which are structurally more complex (in terms of more propositions and more connectives) are more difficult to solve. A second factor is the presence of negations in the propositions of logical problems. Hill (1961) found that the presence of negations was a better predictor of difficulty than complexity. In an attempt to account for the order of difficulty evidenced in the present study, each of the five logical principles was analyzed in terms of number of propositions, number of binary connectives and number of negations. Table 5 shows that these factors taken separately, or together, fail to provide a good prediction of the relative difficulty of the five principles. The presence of negations in the positive

Table 5

The Number of Propositions, Connectives and Negations in the Expressions of the Five Principles of Propositional Logic

Principle	Number of Propositions	Number of Connectives	Number of Negations	TOTAL
		Positive Expression		
Modus Ponens	4	3	0	7
Substitution	6	5	0	11
Affirmation of the Consequent	3	2	0	5
Self distributive Law of the Horseshoe	7	6	0	13
Converse Law of Contraposition	4	3	2	9
		Negative Expression		
Modus Ponens	4	3	3	10
Substitution	6	5	3	14
Affirmation of the Consequent	3	2	1	6
Self distributive Law of the Horseshoe	7	6	1	14
Converse Law of Contraposition	4	3	1	8
		Combined Expressions		
Modus Ponens	8	6	3	17
Substitution	12	10	3	25
Affirmation of the Consequent	6	4	1	11
Self distributive Law of the Horseshoe	14	12	1	27
Converse Law of Contraposition	8	6	3	17

expressions of the principles distinguishes Converse Law of Contraposition as most difficult, but this distinction breaks down when the negative expressions of the principles (all of which contain negations) are considered. The complexity factors give a rough indication of the order of difficulty of Modus Ponens, Substitution and Self- distributive Law of the Horseshoe, if Affirmation of the Consequent and Converse Law of Contraposition are excluded from the analysis. Further examination of the principles (See Appendix B) reveals that Affirmation of the Consequent and Converse Law of Contraposition possess unique features which are not assessed in the analysis presented in Table 5. Affirmation of the Consequent differs from all the rest of the principles in that a binary relationship ($q \supset p$) is inferred from a single proposition (p). Although Affirmation of the Consequent is least complex, the presence of entirely new information in its right-hand statement very likely increases its difficulty. Converse Law of Contraposition is unique in that the propositions which appear in the left-hand statement are negated and reversed in the right-hand statement. Although Converse Law of Contraposition contains relatively few propositions and connectives, enabling easier encoding of the information necessary to assess its validity, more processing is required to decode the information, which undoubtedly increases the difficulty of Converse Law of Contraposition.

Concerning the developmental relationship between the understanding of propositional logic and the acquisition of concrete operations, the ordinal analysis of overall scores from both samples showed that propositional logic precedes concrete operations. Even when the more stringent propositional logic criterion (requiring 75% correct answers) was employed, a significantly greater proportion of children showed an understanding of propositional logic, in the absence of concrete operations, than the reverse. Further, the separate ordinal analyses between each of the axioms and conservation showed that the axioms were acquired before conservation, or in synchrony with conservation. That Converse Law of Contraposition and Self-distributive Law of the Horseshoe (in the Canadian sample) did not emerge before conservation, does not warrant the conclusion that these axioms are not acquired before the stage of concrete operations. Brainerd (1973) demonstrated that the order of acquisition (or difficulty) of the three major indices of the concrete operations stage is: transitivity, conservation and class inclusion. Conservation was employed as a measure of concrete operations in the separate ordinal analyses because all but six of the children from both samples showed perfect performance on transitivity. But, while conservation is more difficult than transitivity, it is less difficult than class inclusion, and so represents a quite conservative measure of the concrete operations stage. In addition, it is recalled that while the criterion for the presence of conservation was a minimum of 2 correct answers (67%) the criterion for the presence of the axioms was a minimum of 3 correct answers (75%).

Before proceeding to the general discussion, brief consideration should be given to the fact that no sex or random (Order, Condition) main effects attained significance in this study. Differences in conceptual and language ability between boys and girls aged 6-9 yrs. are occasionally but not always observed. That Order or Condition of principle presentation should make no difference was certainly desired, but not necessarily expected. The single Sex X Condition X Principle interaction of the Canadian sample seems subject to no clear or meaningful explanation, except that out of nearly sixty possible interactions at least one should be significant by chance.

General Discussion and Implications

It is recalled that the presence of negations failed to distinguish the relative difficulty of the five principles of propositional logic. Yet (excluding the two inference rules on which all children showed near perfect performance) t-tests for correlated measures revealed that performance on the axioms was significantly

($P < .001$) better with positive expressions which required "yes" answers than with negative expressions which required "no" answers. For each sample, 56% of the correct responses were "yes" answers to positive expressions of the axioms, and 44% of the correct responses were "no" answers to negative expressions of the axioms. Since positive expressions presented instances in which the axioms were true, while negative expressions presented instances in which the axioms were false, the better performance on positive expressions reflects and confirms what has been reported by others (e.g., Hill, 1961 and Ennis and Paulus, 1965) that true or valid expressions of logical principles are more easily recognized than false or invalid expressions. It should also be mentioned that the greater percentage of correct "yes" answers (and hence, the better performance on positive expressions) probably reflects, to some extent, the commonly observed tendency of children to agree, or say "yes" more often than "no" when they are not sure of the correct answer to a question. Since Hill's (1961) questions were constructed in the same manner as the questions in the present research, (that is, those questions which contained negations were also those which required "no" answers) some of the poor performance which she attributed to the presence of negations was likely the result of childrens' tendency to agree.

Socio economic factors were pointed to as a probable explanation for the better performance on the questionnaire by Canadian third-graders. The relatively poor performance on the conservation task by children in the American sample (22% of the American sample gave correct answers to all of the conservation questions, while 44% of the Canadian sample did so) is also probably accountable in terms of the socio-economic differences between the American and Canadian samples. Besides the better performance by Canadian third-graders and the discrepancy in scores on the conservation task, the results obtained from both samples employed in the present study were quite consistent. Apart from the geographical and socio-economic differences, two different experimenters were employed in this research. While the writer served as experimenter for the Canadian sample, his sister-in-law, who had no previous research experience and no knowledge of the present study besides that required to administer the tasks, served as experimenter for the American sample. In this context, the consistency of the results attest to their generality, and suggest as well the efficacy and reliability of the measures employed to assess concrete operations and propositional logic.

The principle findings of the present study suggest that primary grade children understand the basic principles of propositional logic; and, that this understanding is acquired before concrete operations. These results lend empirical support to Brainerd's (1975) contention that the acquisition of propositional logic should precede the acquisition of class and relational logic.

Some investigators (including logicians) have argued that Piaget's combinatorial system of propositional logic is an inappropriate and unnecessarily complex formulation. Piaget's model of adolescent thought seems to mix and confound the understanding of propositional logic with certain other aspects of formal reasoning, such as the ability to generate all possible combinations of variables relevant to outcome, or the ability to isolate a single variable and consider its influence while holding other variables constant. In the context of research which has been reported since publication of "The Growth of Logical Thinking from Childhood to Adolescence", Linker (1973) "re-appraised" Piaget's concrete and formal stages, and concluded that many of the distinctions Piaget has drawn between childrens' and adolescents' thinking, including logical abilities, do not hold-up under close scrutiny. While certain cognitive abilities may characterize adolescent, formal operational thinking and distinguish it from the thinking of younger children, the results of the present study suggest that the understanding of propositional logic, per se, does not. Instead it appears that even primary grade children have a reasonably thorough grasp of propositional logic, in so far as they show awareness of the inherent truthfulness and validity of [illegible]xioms and inference rules on which propositional lo[illegible]sed.

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Appendix A

The sixteen binary combinations of the propositions p, q and their negatives (p q) and the corresponding propositional relationships which these combinations express.

1.	pq	Conjunction
2.	pq	Nonimplication of q by p
3.	pq	Nonimplication of p by q
4.	pq	Conjunctive Negation
5.	pq and pq	Affirmation of p
6.	pq and pq	Affirmation of q
7.	pq and pq	Equivalence
8.	pq and pq	Reciprocal Exclusion
9.	pq and pq	Negation of q.
10.	pq and pq	Negation of p
11.	pq and pq and pq	Dysjunction
12.	pq and pq and pq	Reciprocal Implication
13.	pq and pq and pq	Implication
14.	pq and pq and pq	Incompatibility
15.	pq and pq and pq and pq	Complete Affirmation
16.	0	Negation

Appendix B

The Five Basic Principles of Propositional Logic

	Positive Expression
Modus Ponens	$((p \supset q) \cdot p) \supset q$
Substitution	$((p \supset q) \cdot (r \supset p)) \supset (r \supset q)$
Affirmation of the Consequent	$p \supset (q \supset p)$
Self distributive Law of the Horseshoe	$(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$
Converse Law of Contraposition	$(\sim p \supset \sim q) \supset (q \supset p)$

	Negative Expression
Modus Ponens	$((\sim p \supset \sim q) \cdot \sim p) \supset q$
Substitution	$((\sim p \supset \sim q) \cdot (r \supset p)) \supset (r \supset q)$
Affirmation of the Consequent	$\sim p \supset (q \supset p)$
Self distributive Law of the Horseshoe	$(p \supset (q \supset \sim r)) \supset ((p \supset q) \supset (p \supset r))$
Converse Law of Contraposition	$(\sim p \supset q) \supset (q \supset p)$

Appendix C

Questionnaire

(Substitution)

1. Suppose you know that:
   Whoever does good work
   Will get a prize

   If Bill does good work

   Is it true that:
   Bill will get a prize?

2. Suppose you know that:
   Whoever does not do good work
   Will not get a prize

   If Bill does not do good work

   Is it true that:
   Bill will get a prize?

3. Suppose you know that:
   Whoever is the first person in line
   Will be the first person to go home

   If Bill is the first person in line

   Is it true that:
   Bill will be the first person to go home?

4. Suppose you know that:
   Whoever is not the first person in line
   Will not be the first person to go home

   If Bill is not the first person in line

   Is it true that:
   Bill will be the first person to go home?

(Modus Ponens)

5. Suppose you know that:
   If Bill wears his boots
   Then Bill's feet will stay dry

Appendix C (continued):

5. (continued)

And you know that:
Bill wears his boots

Is it true that:
Bill's feet will stay dry?

6. Suppose you know that:
If Bill does not wear his boots
Then Bill's feet will not stay dry

And you know that:
Bill does not wear his boots

Is it true that:
Bill's feet will stay dry?

7. Suppose you know that:
If Bill eats all his dinner
Then Bill will get dessert

And you know that:
Bill eats all his dinner

Is it true that:
Bill will get dessert?

8. Suppose you know that:
If Bill does not eat all his dinner
Then Bill will not get dessert

And you know that:
Bill does not eat all his dinner

Is it true that:
Bill will get dessert?

(Converse Law of Contraposition)

9. Suppose you know that:
When it is not cold outside
Bill never plays in the house

Is it true that:
If Bill plays in the house
Then it is cold outside?

Appendix C (continued):

10. Suppose you know that:
    When it is not cold outside
    Bill sometimes plays in the house

    Is it true that:
    If Bill plays in the house
    Then it is cold outside?

11. Suppose you know that:
    When Bill is not thirsty
    Bill never drinks water

    Is it true that:
    If Bill drinks water
    Then Bill is thirsty?

12. Suppose you know that:
    When Bill is not thirsty
    He sometimes drinks water

    Is it true that:
    If Bill drinks water
    Then Bill is thirsty?

(Self distributive Law of the Horseshoe)

13. Suppose you know that:
    When Bill is hungry
    If someone gives him some bread
    He will eat it

    Is it true:
    If when Bill is hungry
    Someone gives him some bread
    Then when Bill is hungry
    He will eat it?

14. Suppose you know that:
    When Bill is hungry
    If someone gives him some bread
    He will not eat it

    Is it true:
    If when Bill is hungry
    Someone gives him some bread
    Then when Bill is hungry
    He will eat it?

Appendix C (continued):

15. Suppose you know that:
    When Bill goes to the store
    If he takes some money
    His mother will let him buy candy

    Is it true:
    If when Bill goes to the store
    He takes some money
    Then when Bill goes to the store
    His mother will let him buy candy?

16. Suppose you know that:
    When Bill goes to the store
    If he takes some money
    His mother will not let him buy candy

    Is it true:
    If when Bill goes to the store
    He takes some money
    Then when Bill goes to the store
    His mother will let him buy candy?

(Affirmation of the Consequent)

17. Suppose you know that:
    Bill is a good cook

    Is it true that:
    If Bill makes some cookies
    Then Bill is a good cook?

18. Suppose you know that:
    Bill is not a good cook

    Is it true that:
    If Bill makes some cookies
    Then Bill is a good cook?

19. Suppose you know that:
    Bill is very tired

    Is it true that:
    If Bill takes off his coat
    Then Bill is very tired?

Appendix C (continued):

20. Suppose you know that:
    Bill is not very tired

    Is it true that:
    If Bill takes off his coat
    Then Bill is very tired?

Appendix D

Score Sheet

Name

Sex — Age

Grade — Birthdate

Order

Condition

Questions

Questions	Answers Yes	No
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		
11.		
12.		
13.		
14.		
15.		
16.		
17.		
18.		
19.		
20.		

Length Transitivity a( ) b( ) c( )

Length Conservation a( ) b( ) c( )