# Design, Analysis and Implementation of New Durable Streetlighting LED Driver Topology 

by<br>Daniel Darvishrahimabadi

A thesis submitted in partial fulfillment of the requirements for the degree of

Master of Science
in

Energy Systems

Department of Electrical and Computer Engineering

University of Alberta
(C) Daniel Darvishrahimabadi, 2021

## Abstract

Light Emitting Diode (LED) fixtures are widely used in streetlighting owing to their durability, high luminous efficacy, and low power consumption. LED drivers which are ac/dc converters are required as an interface between the LEDs and the grid voltage. There are certain requirements mandated by standards for the safe operation and requirements relevant to the performance of streetlighting LED drivers. Most important ones include a Power Factor Correction (PFC) circuit that is essential for every high power streetlighting LED driver to achieve high Power Factor (PF), and low Total Harmonic Distortion (THD), and output light flickers of a streetlighting fixture that must be maintained under a certain limit to create a healthy lighting environment.

This thesis presents a high PF, flicker-free LED driver consisting of a novel buckboost typed PFC circuit integrated with an asymmetric half-bridge LLC resonant converter. The proposed buck-boost PFC unit operates in Discontinuous Conduction Mode (DCM), and achieves almost unity PF and acceptable THD. The proposed topology limits the dc bus voltage to nearly the peak value of input sinusoidal voltage reducing the voltage stress of MOSFETs.

The control system of the LED driver is designed such that the output light flickers are mitigated without using any electrolytic capacitors in the circuit. Furthermore, the proposed LED driver achieves high efficiency throughout a wide operating range owing to the Zero Voltage Switching (ZVS) obtained in LLC switches, and optimal magnetic designs. Simulation and experimental results based on a 200 W laboratory prototype are presented to verify the effectiveness of the design and control systems. The laboratory prototype provides the maximum efficiency of $94.6 \%$, power factor greater than 0.98 at full-load, and below $15 \%$ output current ripples.

## Acknowledgments

I would like to thank my supervisor Dr. Sayed Ali Khajehoddin for the opportunity he gave me to develop my skills and pursue my aspirations. He is the primary reason I could start a wonderful new chapter of my life in a great country like Canada, and that I could leave behind the difficult life I had. To Canada and Canadians!

I would also like to thank my great friends at UAPEL for their encouragement and support during the past two years, and for all the sweet memories we made. I especially thank Mr. Mohammad Ebrahimi for his mentorship during the past two years. I also thank Mr. Morteza Esteki and Mr. Mohammad Shahabbasi for their help and support along the way of completing my thesis. Morteza helped me with theoretical design, and Mohammad helped me with formula derivations and magnetic simulations. I thank Mr. Morteza Mahdavifard as well for his useful remarks.

Last but not the least, I dedicate this thesis to my family especially my mother for her unwavering support and encouragement throughout my years of study. She raised and supported me and my twin sister as a single mother, and without her efforts, this journey could not have been possible. You are my world, maman. I also dedicate this thesis to my father who albeit I never met, I feel his presence, and to my loving grandmother, mamani.

## Contents

Abstract ..... ii
Acknowledgments ..... iii
List of Tables ..... viii
List of Figures ..... ix
Abbreviations ..... xii
List of Symbols ..... xiii
1 Introduction ..... 1
1.1 LED Driver Topologies ..... 1
1.1.1 Single-Stage LED Drivers ..... 2
1.1.2 Two-Stage LED Drivers ..... 2
1.1.3 Integrated Single-Stage LED Drivers ..... 3
1.2 LED Driver Requirements ..... 3
1.2.1 Power Decoupling in LED Drivers ..... 3
1.2.1.1 Active Energy Storage Techniques ..... 4
1.2.1.2 Control Techniques ..... 5
1.2.2 Current Balancing in LED Strings ..... 6
1.2.3 Input Power Quality ..... 7
1.2.3.1 Buck PFC ..... 8
1.2.3.2 Boost PFC ..... 8
1.2.3.3 Buck-Boost PFC ..... 9
1.3 Description of the Proposed LED Driver ..... 10
1.4 Thesis Objectives ..... 11
1.5 Thesis Outline ..... 11
2 Power Factor Correction ..... 13
2.1 The Proposed PFC Topology ..... 14
2.2 Principles of Operation ..... 15
2.2.1 Interval I, $\left[t_{0}-t_{1}\right]$ ..... 17
2.2.2 Interval II, $\left[t_{1}-t_{2}\right]$ ..... 17
2.2.3 Interval III, $\left[t_{2}-t_{3}\right]$ ..... 18
2.2.4 Interval IV, $\left[t_{3}-t_{4}\right]$ ..... 19
2.2.5 Interval V, $\left[t_{4}-t_{5}\right]$ ..... 19
2.2.6 Interval VI, $\left[t_{5}-t_{6}\right]$ ..... 20
2.2.7 Interval VII, $\left[t_{6}-t_{7}\right]$ ..... 20
2.3 Theoretical Calculation of PF and THD ..... 21
2.4 Simulation and Experimental Results ..... 23
2.5 Summary ..... 28
3 LLC Resonant Converter and Control System ..... 29
3.1 LLC Overview ..... 30
3.1.1 Voltage Gain of Resonant Tank ..... 31
3.1.2 Zero Voltage Switching ..... 32
3.2 Design of the LLC resonant circuit ..... 33
3.3 Limited DC Bus Voltage ..... 36
3.4 Control System ..... 39
3.5 Simulation and Experimental Results ..... 40
3.6 Summary ..... 46
4 Magnetic Design ..... 47
4.1 Magnetic Integration of $T_{a}$ and $L_{a}$ ..... 49
4.1.1 Core Material ..... 50
4.1.2 Core Size ..... 50
4.1.3 Structure of the Magnetic Design ..... 50
4.1.4 Heat Dissipation Capability and Total Loss Budget ..... 51
4.1.5 Optimal Core/Copper Loss Ratio ..... 52
4.1.6 Allocation of Core Loss Budget ..... 52
4.1.7 Core Loss per Volume ..... 53
4.1.8 Maximum Flux Density ..... 54
4.1.9 Number of Turns ..... 55
4.1.9.1 Number of Turns of $T_{a}$ Windings ..... 55
4.1.9.2 Number of Turns of $L_{a}$ Winding ..... 56
4.1.10 Magnetizing Inductance $L_{m a}$ ..... 58
4.1.11 Flux Calculation ..... 58
4.1.12 Core Loss ..... 60
4.1.13 Allocation of Copper Loss Budget ..... 60
4.1.14 Wire Diameters ..... 61
4.1.15 Skin and Proximity Effects ..... 63
4.1.15.1 Skin Effect ..... 63
4.1.15.2 Proximity Effect ..... 63
4.1.16 Arrangement of Wires ..... 64
4.1.17 Selected Wires ..... 64
4.1.18 Copper Loss ..... 65
4.1.18.1 DC Ohmic Loss ..... 65
4.1.18.2 High Frequency AC Loss ..... 66
4.1.18.2.1 AC Loss of $T_{a}$ ..... 66
4.1.18.2.2 AC Loss of $L_{a}$ ..... 68
4.1.19 Total Calculated Loss ..... 70
4.1.20 Experimental Prototype ..... 70
4.2 Magnetic Integration of $T_{r}$ ..... 71
4.3 Summary ..... 72
5 Design and Analysis ..... 73
5.1 Design of $L_{a}$ ..... 74
5.2 Design of $C_{1}$ ..... 76
5.3 Design of $C_{o}$ ..... 77
5.4 Design of $C_{2}$ ..... 77
5.5 Stress Analysis ..... 81
5.5.1 Voltage Stress of $S_{1}$ and $S_{2}$ ..... 81
5.5.2 Voltage Stress of Input Diode Bridge and $D_{a}$ ..... 81
5.5.3 Voltage Stress of Output Rectifying Diodes ..... 82
5.5.4 Current Stress of $S_{1}$ and $S_{2}$ ..... 83
5.5.5 Current Stress of Input Diode Bridge ..... 84
5.5.6 Current Stress of $D_{a}$ ..... 84
5.5.7 Current Stress of Output Rectifying Diodes ..... 85
5.6 Loss Analysis ..... 85
5.6.1 Loss of $S_{1}$ and $S_{2}$ ..... 87
5.6.2 Loss of Input Diode Bridge ..... 88
5.6.3 Loss of $D_{a}$ ..... 89
5.6.4 Loss of Output Rectifying Diodes ..... 89
5.6.5 The Lowest Efficiency at Full-Load ..... 89
5.7 Experimental Results ..... 90
5.8 Summary ..... 91
6 Summary and Future Work ..... 92
6.1 Summary of Contributions ..... 92
6.2 Suggested Improvements ..... 93
Bibliography ..... 94
Appendix A Current Balancing ..... 102
A. 1 The Adopted Current Balancing Scheme ..... 103
A. 2 Modes of Operation ..... 104
A.2.1 Mode I ..... 104
A.2.2 Mode II ..... 104
A. 3 Voltage of Balancing Capacitors ..... 104
A. 4 Simulation and Experimental Results ..... 105
A. 5 Summary ..... 106

## List of Tables

2.1 IEC 61000-3-2 Class C standard limits ..... 13
4.1 Experimental prototype parameters ..... 70
4.2 Leakage and magnetizing inductance of the experimental prototype ..... 72
5.1 System parameters ..... 86

## List of Figures

1.1 Different LED driver topologies ..... 2
1.2 Different types of PFC circuits operating in DCM ..... 7
1.3 The proposed LED driver ..... 10
2.1 (a) Equivalent circuit of the proposed LED driver, and (b) the simpli- fied circuit ..... 14
2.2 Buck-boost circuit ..... 15
2.3 Theoretical key waveforms of the proposed LED driver ..... 16
2.4 Interval I of the converter operation ..... 17
2.5 Interval II of the converter operation ..... 17
2.6 Interval III of the converter operation ..... 18
2.7 Interval IV of the converter operation ..... 19
2.8 Interval V of the converter operation ..... 19
2.9 Interval VI of the converter operation ..... 20
2.10 Interval VII of the converter operation ..... 20
2.11 PFC section of the proposed converter ..... 22
2.12 Input voltage and current waveforms at nominal input voltage at (a) full-load, (b) light-load ..... 23
2.13 Input voltage and current waveforms at rated power at (a) minimum input voltage, (b) maximum input voltage ..... 24
2.14 Input voltage and current waveforms at (a) full-load, (b) light-load ..... 25
2.15 Input voltage and current waveforms at rated power at (a) minimum input voltage, (b) maximum input voltage ..... 26
2.16 (a) Current of the primary winding of $T_{a}$, and (b) interaction between $D_{a}$ and $T_{a}$ during switching periods ..... 27
2.17 Measured input current harmonics of the experimental prototype at rated power at three different input (grid) voltage ..... 28
3.1 (a) Half-bridge LLC resonant converter, and (b) the simplified circuit ..... 30
3.2 The curves of LLC resonant converter depending on different quality factors; Picture courtesy of [67] ..... 32
3.3 ZVS shown in (a) $S_{1}$, and (b) LLC resonant tank ..... 33
3.4 Half-bridge LLC resonant section of the proposed LED driver ..... 34
3.5 LLC curves; Picture courtesy of [67] ..... 35
3.6 The variation of $V_{\text {bus }}$ with respect to $V_{m}$ and $P_{\text {out }}$ ..... 38
3.7 Control system of the proposed LED driver ..... 39
3.8 Simulation result showing ZVS at nominal input (grid) voltage at (a) full-load, (b) light-load ..... 40
3.9 Simulation result showing ZVS at rated power at (a) minimum input (grid) voltage, (b) maximum input (grid) voltage ..... 41
3.10 Experimental result showing ZVS at full-load for (a) $S_{1}$, and (b) $S_{2}$. ..... 42
3.11 Experimental result showing ZVS at light-load for (a) $S_{1}$, and (b) $S_{2}$ ..... 43
3.12 Current and voltage waveforms of $C_{r}$ ..... 44
3.13 DC bus voltage at full load at maximum input voltage ..... 44
3.14 Average dc bus voltage with respect to $V_{g, r m s}$ at rated power ..... 45
3.15 Output current at nominal input voltage $\left(V_{g, r m s}=220 \mathrm{~V}\right)$ at rated $\operatorname{power}\left(P_{\text {out }}=200 \mathrm{~W}\right)$ ..... 45
4.1 Flowchart of non-saturated thermally limited magnetic design ..... 48
4.2 PFC section of the proposed LED driver ..... 49
4.3 (a) The 3D model in Ansys Maxwell and (b) equivalent magnetic circuit of the magnetic integration of $L_{a}$ and $T_{a}$ ..... 49
4.4 Core loss per volume versus maximum flux density and frequency [71] ..... 54
4.5 Maximum flux density of (a) outer legs, and (b) middle leg shown in Ansys Maxwell ..... 59
4.6 Copper loss in a wire ..... 62
4.7 Arrangement of $T_{a}$ and $L_{a}$ windings ..... 64
4.8 Transformer factor $k_{t f}$, [68]; (1) and dotted line for $m_{E}=0.5,(2)$ and solid line for $m_{E}=1$, (3) and dashed for $m_{E}=2$, (4) and dash-dot for $m_{E}>2, \mathrm{LF}=$ low frequency approximation. ..... 67
4.9 Arrangement of $L_{a}$ winding ..... 69
4.10 The constructed prototype of PFC integrated magnetic $\left(T_{a}+L_{a}\right)$ ..... 71
4.11 The constructed prototype of LLC integrated magnetic $\left(T_{r}\right)$ ..... 72
5.1 (a) PFC circuit of the proposed converter, and (b) input power wave- form during a switching period ..... 74
5.2 Input and output power in one line period ..... 76
5.3 (a) The proposed converter with equivalent circuit of $T_{a}$, and (b) wave- form of $i_{C 2}$ during a switching period ..... 78
5.4 (a) Interval VII, and (b) Interval III of the converter operation ..... 82
5.5 (a) The experimental prototype, and (b) operating temperature after half an hour ..... 85
5.6 Efficiency of the prototype with respect to input voltage ..... 90
5.7 Efficiency of the prototype with respect to output power ..... 91
A. 1 The adopted current balancing circuit [63] ..... 103
A. 2 Simulation result showing the output currents ..... 105
A. 3 Experimental result showing the output currents ..... 106

## Abbreviations

ZVS Zero Voltage Switching
PFC Power Factor Correction
PF Power Factor
THD Total Harmonic Distortion
DCM Discontinuous Conduction Mode
SSL Solid State Lighting
PV Photo Voltaic
HPS High Pressure Sodium
PC Power Conversion
LED Light Emitting Diode
CRI Color Rendering Index
RCC Ripple Cancellation Converter
EMC Electro Magnetic Compatibility
EMI Electro Magnetic Interference
PFM Pulse Frequency Modulation
PWM Pulse Width Modulation
RMS Root Mean Square
KCL Kirchoff's Current Law
KVL Kirchoff's Voltage Law

## List of Symbols

| $\alpha_{i}$ | copper loss allocation coefficient | $[-]$ |
| :--- | :--- | ---: |
| $\Delta E_{C 1}$ | energy change of $C_{1}$ during $\frac{T_{l}}{4}$ | $[\mathrm{~J}]$ |
| $\Delta t_{d}$ | conduction time of $D_{a}$ | $[\mathrm{~s}]$ |
| $\Delta V_{b u s}$ | double-frequency voltage ripple of $C_{1}$ | $[\mathrm{~V}]$ |
| $\Delta v_{C 2}$ | switching frequency voltage ripple of $C_{2}$ | $[\mathrm{~V}]$ |
| $\delta$ | skin depth | $[\mathrm{m}]$ |
| $\eta$ | converter efficiency | $[-]$ |
| $\hat{B}$ | maximum flux density | $[\mathrm{T}]$ |
| $\hat{i}_{S}$ | maximum current of $S_{1-2}$ during $T_{s w}$ | $[\mathrm{~A}]$ |
| $\hat{i}_{C 2}$ | maximum of $i_{C 2}$ during $T_{s w}$ | $[\mathrm{~A}]$ |
| $\hat{i}_{i n}$ | maximum of $i_{i n}$ during $T_{s w}$ | $[\mathrm{~A}]$ |
| $\hat{p}_{i n}$ | maximum input power during $T_{s w}$ | $[\mathrm{~W}]$ |
| $\kappa$ | inductor parameter | $[-]$ |
| $\lambda$ | ferrite core exponent | $[-]$ |
| $\left\|v_{g}\right\|$ | rectified input voltage | $[\mathrm{V}]$ |
| $\mathcal{P}$ | number of strands of a Litz wire | $[-]$ |
| $\mathcal{R}$ | magnetic reluctance | $\left[\mathrm{H}^{-1}\right]$ |
| $\mathcal{V}_{e}$ | magnetic volume | $\left[\mathrm{m}^{3}\right]$ |
| $\mu_{0}$ | air permeability | $[\mathrm{H} \mathrm{m}$ |
| $\omega_{l}$ | angular line frequency | $[\mathrm{rad} \mathrm{s}$ |
| $\left.\omega_{r}\right]$ | angular resonance frequency | $[\mathrm{rad} \mathrm{s}$ |
| $\left.\omega_{s}\right]$ | angular switching frequency | $[\mathrm{rad} \mathrm{s}$ |
| $\Phi$ | magnetic flux | $[\mathrm{Wb}]$ |
| $\Psi$ | magnetic flux linkage | $[\mathrm{Wb}]$ |
| $\rho$ | copper resistivity | $[\Omega \mathrm{m}]$ |
| $\theta_{s}$ | voltage lag angle of LLC resonant tank | $[\mathrm{rad}]$ |
| $A_{e}$ | magnetic area of middle leg | $\left[\mathrm{m}^{2}\right]$ |
| $A_{f}$ | ferrite coefficient | $[-]$ |
| $a_{c h}$ | largest dimension of ferrite core | $[\mathrm{m}]$ |


| $A_{\text {eo }}$ | magnetic area of outer legs | [ $\mathrm{m}^{2}$ ] |
| :---: | :---: | :---: |
| D | duty cycle | [-] |
| $d$ | wire diameter | [m] |
| $d_{\text {wg }}$ | winding distance from ferrite core | [m] |
| $F$ | normalized switching frequency | [-] |
| $f_{e q}$ | equivalent frequency of a winding | [Hz] |
| $f_{l}$ | line frequency | [Hz] |
| $f_{p}$ | pole frequency | [Hz] |
| $f_{r p}$ | peak resonance frequency | [Hz] |
| $f_{r}$ | resonance frequency | [Hz] |
| $f_{s w}$ | switching frequency | [Hz] |
| $g$ | airgap | [m] |
| $i_{C 2,0}$ | value of $i_{C 2}$ at turn-on of $S_{1}$ | [A] |
| $i_{C 2,1}$ | value of $i_{C 2}$ at turn-on of $S_{2}$ | [A] |
| $I_{C 2, r m s}$ | rms of $i_{C 2}$ | [A] |
| $i_{C 2}$ | current of secondary winding of $T_{a}$ | [A] |
| $I_{\text {Da, max }}$ | maximum of $i_{D a}$ | [A] |
| $i_{D a}$ | current of $D_{a}$ | [A] |
| $I_{\text {Dr, max }}$ | maximum current of $D_{1-2}$ | [A] |
| $I_{g, r m s}$ | rms of input current | [A] |
| $i_{g}$ | input current | [A] |
| $i_{\text {in,ave }}$ | average of $i_{i n}$ over $T_{s w}$ | [A] |
| $I_{\text {in,max }}$ | maximum of $i_{i n}$ | [A] |
| $I_{\text {in,rms }}$ | rms of $i_{\text {in }}$ | [A] |
| $i_{\text {in }}$ | current of primary winding of $T_{a}$ | [A] |
| $I_{\text {La,max }}$ | maximum of $i_{L a}$ | [A] |
| $i_{L a, p p}$ | peak-to-peak current of $L_{a}$ | [A] |
| $I_{\text {La,rms }}$ | rms of $i_{L a}$ | [A] |
| $i_{L a}$ | current of $L_{a}$ | [A] |
| $I_{L r, \text { max }}$ | maximum of $i_{L r}$ | [A] |
| $i_{L r}$ | resonant current | [A] |
| $i_{n r 2}$ | current of secondary winding of $T_{r}$ | [A] |
| $i_{o}$ | output current | [A] |
| $I_{\text {ref }}$ | reference output current | [A] |
| $I_{r m s, S 1-2}$ | rms current of $S_{1-2}$ | [A] |
| $I_{S, \text { max }}$ | maximum current of $S_{1-2}$ | [A] |
| $i_{S 1-2}$ | current of $S_{1-2}$ | [A] |


| $J$ | current density | [ $\mathrm{mm}^{-2}$ ] |
| :---: | :---: | :---: |
| $k_{e}$ | core loss constant | [-] |
| $K_{\text {sy }}$ | symmetry factor | [-] |
| $k_{c p}$ | coupling factor | [-] |
| $k_{f}$ | inductor factor | [-] |
| $k_{t f}$ | transformer factor | [-] |
| $l_{e}$ | magnetic length | [m] |
| $l_{T}$ | mean-length-per-turn | [m] |
| $L_{l l c}$ | $L_{m r}$ to $L_{r}$ ratio | [-] |
| $m$ | number of layers of a winding | [-] |
| $M_{g}$ | voltage gain of LLC resonant tank | [-] |
| $m_{E}$ | effective number of layers of a winding | [-] |
| $M_{g, D C}$ | dc voltage gain of LLC resonant tank | [-] |
| $N_{i}$ | number of turns of a winding | [-] |
| $n_{(r)}$ | transformer turns ratio | [-] |
| $P_{h}$ | heat dissipation capability of a core | [W] |
| $P_{a c}$ | ac loss of a winding | [W] |
| $p_{\text {ave }}$ | instantaneous input power | [W] |
| $P_{\text {cond,S1-2 }}$ | conduction loss of $S_{1-2}$ | [W] |
| $P_{c u}$ | copper loss | [W] |
| $P_{\text {Da }}$ | loss of $D_{a}$ | [W] |
| $P_{\text {Db }}$ | loss of each diode used in diode bridge | [W] |
| $P_{d c}$ | dc loss of a winding | [W] |
| $P_{D r}$ | loss of $D_{1-2}$ | [W] |
| $P_{f e, \max }$ | maximum core loss | [W] |
| $P_{f e, s p}$ | core loss per volume | [ $\mathrm{W} \mathrm{m}^{-3}$ ] |
| $P_{f e}$ | core loss | [W] |
| $P_{h, c u}$ | copper loss budget | [W] |
| $P_{h, f e}$ | core loss budget | [W] |
| $P_{\text {in }}$ | average input power | [W] |
| $P_{\text {loss }}$ | total converter loss | [W] |
| $P_{\text {out }}$ | output power | [W] |
| $P_{S 1-2}$ | total loss of $S_{1-2}$ | [W] |
| $P_{T a+L a}$ | total loss of $L_{a}$ and $T_{a}$ combined | [W] |
| $P_{T r}$ | total loss of $T_{r}$ | [W] |
| $Q$ | quality factor | [-] |
| $Q_{C 2}$ | charge stored in $C_{2}$ during $T_{s w}$ | [C] |


| $R_{\text {ac }}$ | ac resistance of a winding | $[\Omega]$ |
| :---: | :---: | :---: |
| $R_{d c}$ | dc resistance of a winding | $[\Omega]$ |
| $r_{D S, \text { on }}$ | ON resistance of MOSFETs | $[\Omega]$ |
| $S_{t o t}$ | total volt-amp rating | [ V A] |
| $t_{\text {w }}$ | winding thickness | [m] |
| $T_{l}$ | line period | [s] |
| $t_{r r}$ | diode reverse recovery time | [s] |
| $T_{s w}$ | switching period | [s] |
| $V_{F}$ | diode forward voltage | [V] |
| $V_{m}$ | peak value of input sinusoidal voltage | [V] |
| $V_{\text {bus, max }}$ | maximum of $V_{\text {bus }}$ | [V] |
| $V_{\text {bus }}$ | average dc bus voltage | [V] |
| $v_{\text {bus }}$ | dc bus voltage | [V] |
| $v_{C 2}$ | voltage of $C_{2}$ | [V] |
| $v_{C r}$ | voltage of $C_{r}$ | [V] |
| $v_{D 2}$ | voltage of $D_{2}$ | [V] |
| $V_{D a, \max }$ | maximum voltage of $D_{a}$ | [V] |
| $v_{D a}$ | voltage of $D_{a}$ | [V] |
| $V_{\text {Dr,max }}$ | maximum voltage of $D_{1-2}$ | [V] |
| $V_{g, r m s}$ | rms of input voltage | [V] |
| $v_{g}$ | input (grid) voltage | [V] |
| $V_{\text {La,rms }}$ | rms of $v_{L a}$ | [V] |
| $v_{L a}$ | voltage of $L_{a}$ | [V] |
| $V_{\text {Lma,rms }}$ | rms of $v_{\text {Lma }}$ | [V] |
| $v_{L m a}$ | voltage across secondary winding of $T_{a}$ | [V] |
| $v_{L m r}$ | output voltage of resonant tank | [V] |
| $V_{M, r m s}$ | rms of $v_{M}$ | [V] |
| $v_{M}$ | voltage across primary winding of $T_{a}$ | [V] |
| $v_{n r 2}$ | voltage across secondary winding of $T_{r}$ | [V] |
| $V_{o}$ | output voltage | [V] |
| $V_{\text {rect,max }}$ | maximum voltage of input diode bridge | [V] |
| $v_{\text {rect }}$ | output voltage of diode bridge | [V] |
| $V_{S, \text { max }}$ | maximum voltage of $S_{1-2}$ | [V] |
| $v_{S 1-2}$ | drain-source voltage of $S_{1-2}$ | [V] |
| $v_{\text {tank }}$ | input voltage of resonant tank | [V] |
| $w$ | winding width | [m] |
| $z$ | wire diameter to skin depth ratio | [-] |

## Chapter 1

## Introduction

LED fixtures have become more popular in Solid State Lighting (SSL) because of their long lifespan, ultra low power consumption, good Color Rendering Index (CRI), and high luminous efficacy [1-4]. Due to their reliability, flexibility, and low maintenance costs, LEDs are widely used in both residential and commercial applications, and in a variety of lighting fixtures ranging from indoor lighting bulbs and background lighting of display panels, to decorative lighting fixtures, automotive lighting modules, and streetlighting [5-10].

The LED fixtures used in streetlighting have a variety of advantages over traditional High Pressure Sodium (HPS) lamps, such as higher luminous efficacy, longer lifespan, and higher quality output light [8]. Ongoing reduction in the price of LEDs along with environmental awareness and increasing energy costs make them a viable and promising option for streetlighting.

LEDs are inherently dc loads. Therefore, an LED driver which is an ac/dc converter is required as an interface between them and the grid voltage. Based on their application, LED drivers are implemented using different topologies, and in a vast range of power from 3 W to more than 500 W . The safe operation of LED drivers is very important that several regulatory standards have been created by industry to ensure they meet certain requirements.

### 1.1 LED Driver Topologies

Considering their topology, LED drivers are divided into three categories as follows:

1. Single-stage LED drivers
2. Two-stage LED drivers
3. Integrated single-stage LED drivers


Figure 1.1: Different LED driver topologies

### 1.1.1 Single-Stage LED Drivers

The typical configuration of a single-stage LED driver which is an ac/dc converter with PFC capability is shown in Fig. 1.1(a). Several LED drivers based on this topology have been proposed in literature [11-15]. Single-stage LED drivers are very compact and highly efficient, but they have relatively large output current ripples that cause the LEDs to flicker. The output light flickers make the lighting environment unhealthy, and cause side effects such as fatigue and vision impairment over time $[16,17]$. To limit the light flickers, the output current ripples must be reduced which is not cost effective using this configuration, especially for high power applications. Therefore, this type of LED drivers is not suitable for streetlighting. To address this problem, the PFC stage of an LED driver must be followed by a Power Conversion (PC) stage whose task is to eliminate the output current ripples.

### 1.1.2 Two-Stage LED Drivers

In a two-stage LED driver as shown in Fig. 1.1(b), the PC stage which is a dc/dc converter is cascaded with the PFC stage to remove the output ripples. Various twostage LED drivers have been proposed in literature [18-22]. Despite their advantages, they have certain drawbacks, such as high component count, extra costs, and an additional control system.

### 1.1.3 Integrated Single-Stage LED Drivers

The configuration of an integrated single-stage LED driver is shown in Fig. 1.1(c), where the PFC and PC stages are integrated to form a single unit. To overcome the shortcomings of single-stage and two-stage LED drivers, a variety of integrated single-stage LED drivers have been proposed in literature [23-34]. This type of LED drivers is designed to achieve high PF, low THD, and reduced output light flickers with relatively lower component count, simpler control system, and lower cost. As a conclusion, this topology is the most promising choice for streetlighting.

### 1.2 LED Driver Requirements

Depending on the application, different requirements must be satisfied for LED drivers. Power decoupling, current balancing, and input power quality are among the most important ones that are discussed in this thesis. There are also other requirements, such as complying with Electro Magnetic Compatibility (EMC) standards, providing galvanic isolation, dimming capability, limited output voltage range, e.g. $V_{o}<70 \mathrm{~V}$, IP rating, over current/voltage protections, etc.

### 1.2.1 Power Decoupling in LED Drivers

The instantaneous input power of an LED driver contains double-frequency oscillations, while the output power is ideally constant. The input/output power difference must be processed in the LED driver that is defined as power decoupling. As a result of poor power decoupling, double-frequency ripples appear on the output causing light flickers. As mentioned earlier, excessive light flickers in a lighting environment give rise to several possible health issues over time, and it is recommended that they be restricted as much as possible $[16,17]$.

Electrolytic capacitors are among the most straightforward options for power decoupling because they are compact and cost-effective. High-capacitance film capacitors on the other hand are bulky and relatively more expensive. The downside of electrolytic capacitors is the short lifetime in the range of $15,000-25,000$ hours, whereas film capacitors can last up to more than 50,000 hours, at least twice as that of electrolytic ones. Reliability and long lifetime play an integral role in streetlighting LED drivers as the maintenance costs associated with replacing broken fixtures at high altitudes are prohibitive. LED modules themselves benefit from a long lifetime in the range of at least 50,000 hours which this is one of the reasons they have dominated the market. Therefore, it is important that the lifetime of LED drivers matches
that of LED modules. Utilizing electrolytic capacitors in an LED driver reduces its durability and the reliability of the whole fixture. Thus, many researchers have been seeking for alternative solutions of power decoupling to remove electrolytic capacitors.

Various approaches have been developed to eliminate electrolytic capacitors, such as pulsating driving current [35], and input current shaping techniques [36,37] that focus on manipulating the output or input current respectively to decrease the input/output power difference. These techniques might yield satisfactory results, but at the expense of additional complexity in control and aggravated quality of either the output or input of the LED driver. The use of a magnetic element has been proposed in [18] in which by using a coupled inductor, the more optimized power flow path has been provided. In this technique, a portion of the input power reaches the load directly through the coupled inductor and without being processed twice. The problem of magnetic elements compared with capacitors is their large size and increased losses. Among all power decoupling approaches, active energy storage techniques [38-47], and control techniques $[48-54]$ have received more attention because they yield more satisfactory reults.

### 1.2.1.1 Active Energy Storage Techniques

An active energy storage technique is when an auxiliary circuit is added to a regular LED driver to process the input/output power difference, or when the LED driver is modified in such a way that power processing becomes more efficient. Either way, it requires additional control systems or extra components.

A non-cascaded topology has been proposed in [43] to eliminate electrolytic capacitors by designing an energy buffer unit added to a flyback converter. Whenever the instantaneous input power is lower than the output power, additional energy is supplied from the buffer capacitor, and whenever the instantaneous power is greater than the output power, the surplus energy is stored in the buffer capacitor. In the LED driver proposed in [42], a bidirectional buck-boost converter has been connected in parallel with the output of PFC stage. The bidirectional buck-boost circuit absorbs the double-frequency current ripples existing at the output of PFC stage allowing the dc power to be delivered to LEDs. In the LED driver proposed in [44], a buck converter has been utilized on the tertiary winding of a flyback circuit as the Ripple Cancellation Converter (RCC). The LED driver is controlled such that the dc power is delivered to the load via the flyback circuit through one stage of power conversion, and the ac power is processed through both the flyback and buck stage. Another usage of the flyback topology has been proposed in [45] where a bidirectional buck
ripple-cancellation unit has been added to the tertiary winding of a flyback converter. The converter proposed in [46] is a manipulation of a single-switch flyback-forward converter in which a buck circuit has been placed between the forward converter and load to make the power processing more efficient. The objective is to improve the efficiency by making an improved 1.5-stage forward-flyback converter. The control strategy is devised such that most of the power is delivered to the load directly via the flyback stage, and the rest is delivered through the forward converter and cascaded buck stage. A non-cascaded integrated LED driver has been proposed in [47] which consists of a buck-boost stage to deliver the dc power to the load, and a boost stage for low frequency ripple cancellation. Due to the opposite polarity between the output voltage of each stage, the double-frequency voltage ripples of the boost circuit is subtracted from those of the buck-boost circuit resulting in reduced output ripples.

Some of the most notable drawbacks of active energy storage converters is their poor efficiency, complicated control systems for low-frequency ripple cancellation, and the need for extra high voltage rating MOSFETs that increase the total cost of the LED driver.

### 1.2.1.2 Control Techniques

The major storage elements used in an LED driver is the dc bus capacitor at the output of PFC stage and the output capacitors across LEDs. An interesting way to remove the output double-frequency ripples is by transferring them to dc bus where by accepting high voltage ripples, a small film capacitor can be chosen as dc bus capacitor. In addition, since the low frequency ripples are removed from the output, low-capacitance film capacitors can be selected as output capacitors as well.

This method was first applied to converters of Photo Voltaic (PV) solar systems [48, 52, 53], and then to LED drivers [49, 54]. In the LED driver proposed in [49], a feed-forward controller is designed for the asymmetric half-bridge stage. The FeedForward controller is quite effective in low-frequency ripple cancellation. However, the optimal design is challenging, and it requires a complicated digital control system that increases the hardware cost.

In the proposed LED driver, similar to [54], a control system much simpler than feed-forward controller is designed. The proposed control system removes the output double-frequency ripples and place them on the dc bus capacitor where by accepting high voltage ripples, a small durable film capacitor is selected. Finally, a 200 W electrolytic-capacitor-free LED driver is achieved. The advantage of this method is that it does not require any additional circuits or complexity in control.

### 1.2.2 Current Balancing in LED Strings

For a higher reliability and performance, multiple LED strings are usually paralleled with each other at the output of lighting fixtures. However, because of the non-linear I-V characteristic of LEDs, current imbalance might happen between different LED strings in parallel resulting in inconsistent output light, and reduced lifetime of the lighting fixture.

Different active and passive techniques have been proposed to address this problem, such as active balancing techniques based on either linear current regulators [55, 56] or switching-mode current regulators [57]. Active current balancing techniques fall short in comparison with passive techniques as they suffer from increased cost and component count, as well as complex control and reduced efficiency. On the other hand, passive current balancing methods use generally simple structures based on passive elements to address the current balancing issue.

Passive techniques are divided into resistive, inductive, and capacitive approaches. Resistive schemes [58] are used to linearize LED loads with the help of series resistors. Their main shortcoming is the extra power dissipation in resistors making this method impractical for high power LED drivers. Inductive schemes use magnetic components like inductors and transformers [59]. They improve the converter in terms of thermal capability and lifetime, but they reduce the power density. In addition, they suffer from extra winding and core losses that worsen the overall efficiency. Finally, capacitive schemes seem to be the most promising solution for current balancing as not only are they almost lossless, but they also improve the overall lifetime and power density of the LED driver.

Various capacitive current balancing schemes have been proposed [15,23,60-63] which most of them rely on the charge-balance property of capacitors. Current balancing is discussed in Appendix.

### 1.2.3 Input Power Quality

Based on Energy Star program for commercial lighting, streetlighting LED drivers must have a PF greater than 0.9, [64]. In addition, IEC 61000-3-2 Class C requirements set certain limits for input current harmonics [65]. These two obligations make it imperative to utilize a PFC circuit for streetlighting LED drivers.

In integrated single-stage LED drivers, the PFC circuit is usually designed to operate in DCM, and the PC stage is responsible for regulating the output. Based on the topology of PFC circuit, integrated single-stage LED drivers are divided into three main categories as stated below:

1. Buck PFC LED drivers [23].
2. Boost PFC LED drivers [24-26, 28, 29, 50, 66].
3. Buck-boost PFC LED drivers [27,30-34].


Figure 1.2: Different types of PFC circuits operating in DCM

### 1.2.3.1 Buck PFC

Consider the buck PFC circuit shown in Fig. 1.2(a). In this circuit, the average of input rectified current denoted as $i_{S x}$ is a function of both input voltage $\left(v_{g}\right)$ and output voltage $\left(V_{o}\right)$. When $V_{o}$ is much smaller than $v_{g}$, the dependency of $i_{g}$ on $V_{o}$ becomes weaker, and it nearly becomes a linear function of only $v_{g}$ which this results in acceptable PF and THD.

Although lowering the output voltage of buck PFC cicuit helps in improving the input power quality, the efficiency is compromised due to increased conduction losses of the buck inductor and diode. In addition, as illustrated in Fig. 1.2(a), the dead angle appearing on the input current waveform is another problem associated with DCM buck PFC circuits that might cause the LED driver not to meet the required PF and THD regulatory standards.

An LED driver with a hybrid switched-inductor buck PFC circuit instead of a conventional one has been proposed in [13] to lower conduction losses. An inverted buck LED driver achieving a small dead angle has been proposed in [12], and a buck PFC circuit integrated with a resonant dc-dc unit has been proposed in [23].

### 1.2.3.2 Boost PFC

Consider the boost PFC circuit shown in Fig. 1.2(b). Because of the hatched area on $i_{L x}$ waveform during a switching period $\left(T_{s w}\right)$, the average of input rectified current denoted as $i_{L x}$ is a function of both $v_{g}$ and $V_{o}$. When $V_{o}$ is much greater than $v_{g}$, the dependency of input current $\left(i_{g}\right)$ on $V_{o}$ becomes weaker, and it nearly becomes a linear function of only $v_{g}$ resulting in acceptable PF and THD.

Several integrated single-stage LED drivers based on DCM boost PFC circuit have been proposed in literature, such as the integrated boost-flyback converter proposed in [25], the LED drivers integrating a boost-typed PFC circuit with a half-bridge converter proposed in [29,50], and the integrated bridgeless boost PFC half-bridge LED drivers proposed in $[24,26,28,66]$.

The downside of boost PFC circuits is that to get acceptable PF, the output voltage must be at least twice the input voltage. This in turn increases the voltage stress of components, and the total cost of the LED driver. This issue is more severe when the LED driver is designed for the universal input voltage range, or input voltage range of $(185-265) \mathrm{V}_{r m s}$ adopted by European and some other countries. As a result, DCM boost PFC circuits are not normally utilized in integrated single-stage LED drivers that are designed for this input voltage range.

### 1.2.3.3 Buck-Boost PFC

Consider the buck-boost PFC circuit shown in Fig. 1.2(c). Unlike the other two types of PFC circuits, in a buck-boost PFC circuit, the average of input rectified current denoted as $i_{S x}$ is a linear function of only $v_{g}$ which this results in unity PF and zero THD in theory.

Several integrated single-stage LED drivers based on DCM buck-boost PFC circuit have been proposed in literature. Buck-boost PFC circuits are commonly integrated with flyback, class E, or half-bridge resonant converters. A buck-boost PFC circuit integrated with a flyback converter has been proposed in [34]. Albeit this structure is very compact, the power switch suffers from hard-switching which has decreased the efficiency to below $90 \%$. The integration of buck-boost PFC circuit with an isolated $\mathrm{dc} / \mathrm{dc}$ converter with voltage doubler rectifier has been proposed in [30] in which below $90 \%$ efficiency has been achieved for a 120 W prototype due to hard-switching losses. An integration of a buck-boost PFC circuit with a class E resonant converter has been proposed in [32] in which an almost unity PF, $5 \%$ THD, and around $91 \%$ efficiency have been achieved for a 100 W prototype.

Owing to the soft-switching capability of half-bridge resonant converters and their simpler structure compared with class E resonant converters, the integration of a buckboost PFC circuit with a half-bridge resonant converter yields the highest efficiency, and is the best possible configuration for streetlighting LED drivers. However, it has certain problems that are addressed in the LED driver proposed in this thesis.

A single-stage LED driver by integrating a buck-boost PFC circuit with a half-bridge LLC resonant converter has been proposed in [33] where to ensure the proper operation of the buck-boost PFC circuit, extra diodes have been added that not only increase the costs, but also decrease the converter efficiency. A buck-boost typed PFC circuit integrated with a half-bridge CLCL resonant converter has been proposed in [27] where even though this LED driver is bridgeless, it still contains four extra diodes. Finally, [31] has proposed a SEPIC PFC circuit integrated with a halfbridge LLC resonant converter where again additional diodes have been utilized to ensure the buck-boost typed PFC operation.


Figure 1.3: The proposed LED driver

### 1.3 Description of the Proposed LED Driver

The proposed LED driver in thesis shown in Fig. 1.3, is composed of a buck-boost typed PFC circuit integrated with a half-bridge LLC resonant converter in which transformer $T_{a}$ acts like the buck-boost switch. Therefore, the extra diodes used in previous buck-boost half-bridge converters $[27,31,33]$ are eliminated. With a fixed duty cycle, switching frequency is used as the control variable. Therefore, Pulse Frequency Modulation (PFM) is used to regulate the converter. Since high PF and low THD are achieved with the DCM operation of input current, $S_{1}$ and $S_{2}$ are utilized to regulate the output current and make a flicker-free LED driver. Owing to the proposed control system [48,52-54], output current ripples are attenuated without using any low-lifetime electrolytic capacitors. The LLC resonant circuit is designed such that the ZVS of $S_{1}$ and $S_{2}$ is ensured for a wide operating range. On the LED side, a passive current balancing scheme is adopted which is not depicted in Fig. 1.3, but will be discussed in the "Current Balancing" chapter in Appendix.

The transformer and inductor of PFC circuit are integrated on the same core to improve the efficiency and power density. In addition, the average dc bus voltage denoted as $V_{\text {bus }}$ is limited to approximately the peak value of input sinusoidal voltage which helps in lowering the voltage stress of $S_{1}$ and $S_{2}$. The circuit is implemented and for a wide input voltage and power range, the peak efficiency of $94.6 \%$ is measured.

### 1.4 Thesis Objectives

The main objectives of this thesis are summarized as follows:

1. To design an integrated single-stage LED driver suitable for input voltage range of $(185-265) \mathrm{V}_{r m s}$ with high PF, low THD, and low output ripples.
2. To design a buck-boost typed PFC circuit that results in unity PF and zero THD in theory with minimum component count.
3. To design an LED driver with very limited dc bus voltage to lower the voltage stress of MOSFETs.
4. To design a robust controller that eliminates output ripples, and makes it possible to remove electrolytic capacitors.
5. To design an optimal magnetic integration to minimize corresponding losses and increase the total efficiency.
6. To verify the design through an experimental prototype.

### 1.5 Thesis Outline

This thesis is organized as follows:
Chapter 2 explains briefly about PF and THD requirements of streetlighting LED drivers. The proposed PFC topology is presented, and the topology and principles of operation are comprehensively investigated. In addition, theoretical PF and THD of the proposed LED driver are calculated. Finally, simulation and experimental results are presented to verify the performance of PFC circuit. Moreover, by using a Tektronix power analyzer model PA4000, the harmonic content of the input current is measured for three different input voltage.

Chapter 3 begins by explaining briefly about LLC resonant converters, and ZVS as one of the benefits. The design parameters of the LLC resonant circuit are also obtained. Subsequently, the formula of average dc bus voltage is obtained in terms of circuit parameters to verify the limited dc bus voltage range. The control system of the proposed LED driver is also presented that helps in eliminating electrolytic capacitors. At the end, simulation and experimental results are provided to validate the effectiveness of the LLC resonant circuit, limited dc bus voltage, and proposed control sytem.

Chapter 4 thoroughly investigates the optimum design of PFC integrated transformer and inductor. This chapter discusses the design steps essential for every magnetic design in details, and verifies the proposed magnetic design by calculating the maximum flux density, core loss, and copper loss including both dc ohmic and high frequency ac losses. It also gives insight into the importance of Litz wires in high frequency applications, skin effect and proximity effect concepts, and the importance of good winding arrangements to minimize corresponding losses. In addition, the magnetic integration of resonant inductor and transformer is briefly discussed. At the end, the experimental prototypes of both magnetic integrations are presented.

Chapter 5 discusses the design steps of different components. Loss and stress analyses of all component are also carried out to determine their restrictions, and expected losses. Based on stress analysis, the best possible components are selected for the experimental prototype, and based on loss analysis, the lowest efficiency of the converter at full-load is calculated. At the end, by using a Tektronix 5 series Mixed Signal Oscilloscope model MS058, the efficiency of the experimental prototype is measured and presented for several operating points.

Chapter 6 summarizes the contributions, and provides suggestions for further improvement of the design.

Ultimately in Appendix, the importance of current balancing in LED drivers is briefly discussed, and the adopted current balancing scheme and the modes of operation are analyzed. At the end, simulation and experimental results of the four outputs are presented to verify the current balancing performance.

## Chapter 2

## Power Factor Correction

Power Factor (PF) is an index determining the quality of power utilization. The lower the PF of a certain electrical device, the higher the current drawn from the grid for the same amount of power. This increased current that can also be accompanied by different orders of harmonics, creates excessive power loss in power system. Lighting applications account for about $19 \%$ of total electrical energy consumption in the world [34]. Therefore, they play an integral role in reducing the total electrical power consumption. This has given rise to increasing share and popularity of LED lighting products because of their low power consumption. Nevertheless, LEDs are inherently non-linear loads with an adverse effect on power utilization. Certain standards, such as IEC 61000-3-2 Class C [65], and U.S. Energy Star program [64] have set certain requirements for PF and input current harmonics of LED lighting products. According to Energy Star standard, the PF of high-power commercial LED lighting products must be greater than 0.9. In addition, regarding IEC standard, the input current harmonics must be within the IEC 61000-3-2 Class C standard limits shown in Table. 2.1.

Table 2.1: IEC 61000-3-2 Class C standard limits

| Harmonic order | Harmonic current/Input current (\%) |
| :---: | :---: |
| 2 | 2 |
| 3 | $\approx 30$ |
| 5 | 10 |
| 7 | 7 |
| 9 | 5 |
| 11 | 3 |
| 13 | 3 |
| 15 | 3 |

To comply with these requirements, high power LED drivers like the ones used in streetlighting must utilize a PFC circuit to ensure meeting the aforementioned standards.
In this chapter, a novel DCM buck-boost typed PFC circuit is firstly proposed. Then the converter and operating principles are analyzed, and the theoretical PF and THD are calculated as well. Finally, simulation and experimental results are provided to verify the performance of PFC operation.

### 2.1 The Proposed PFC Topology

The 200 W streetlighting LED driver proposed in this thesis is a single-stage, softswitching, high PF converter with no electrolytic capacitors that is composed of a buck-boost typed PFC rectifier integrated with an asymmetric half-bridge LLC resonant converter as shown in Fig. 2.1.


Figure 2.1: (a) Equivalent circuit of the proposed LED driver, and (b) the simplified circuit


Figure 2.2: Buck-boost circuit

In the proposed LED driver, $T_{a}$ acts like the buck-boost switch. Therefore, the extra diodes utilized in previous integrated buck-boost half-bridge LED drivers are removed. This results in lower component count, higher power density, and higher efficiency. Corresponding to the buck-boost circuit shown in Fig. 2.2, $T_{a}$ acts like $S_{x}$, $D_{a}$ acts like $D_{x}$, and $L_{a}$ acts like $L_{x}$. The proposed PFC circuit might seem like a buck converter, but as shown in (2.4), if $n=\frac{N_{P}}{N_{S}}=2$ is selected, in the first half of switching period, the voltage across $L_{a}$ denoted as $v_{L a}$ becomes a function of only $v_{g}$ and independent of $v_{b u s}$. By mimicking the PFC behavior of a DCM buck-boost converter, the best possible PF and THD are obtained without designing any control system for input current shaping.

### 2.2 Principles of Operation

Depending on the ON/OFF state of switches, seven operating intervals are conceivable for the proposed LED driver. To simplify the analysis, following assumptions are made:

1. The MOSFETs and diodes are ideal which means their ON-state voltage equals zero, and they resemble an infinity resistor in OFF-state.
2. The current balancing scheme of the LED driver is different from what is presented here, and will be analyzed in Appendix. In addition, the LED driver is composed of four outputs. Since neither of these two affects the converter analysis, for simplicity they are not considered.

The key waveforms of the converter are illustrated in Fig. 2.3.


Figure 2.3: Theoretical key waveforms of the proposed LED driver

### 2.2.1 Interval I, $\left[t_{0}-t_{1}\right]$

Before this interval, $S_{2}$ was ON having discharged $C_{S 2}$. When $S_{2}$ turns OFF and before $S_{1}$ turns ON, $C_{S 1}$ discharges completely. Input and output rectifying diodes are OFF in this interval.


Figure 2.4: Interval I of the converter operation

### 2.2.2 Interval II, $\left[t_{1}-t_{2}\right]$

At the beginning of this interval, the body diode of $S_{1}$ begins to conduct to ensure the ZVS of switch before the MOSFET of $S_{1}$ turns ON.


Figure 2.5: Interval II of the converter operation

By writing a KVL, it is obtained that:

$$
\begin{equation*}
v_{M}(t)=n\left(v_{C 2}-v_{\text {bus }}\right) \tag{2.1}
\end{equation*}
$$

For an asymmetric half bridge converter assuming $D=0.5$, it can be written that:

$$
\begin{equation*}
v_{C 2}=\frac{v_{\text {bus }}}{2} . \tag{2.2}
\end{equation*}
$$

Then (2.1) can be rewritten as:

$$
\begin{equation*}
v_{M}(t)=n\left(v_{C 2}-v_{b u s}\right)=-\frac{n}{2} v_{\text {bus }} . \tag{2.3}
\end{equation*}
$$

By writing another KVL, it is obtained that:

$$
\begin{equation*}
v_{L a}(t)=\left|v_{g}\right|-v_{M}(t)-v_{b u s}=\left|v_{g}\right|+\left(\frac{n}{2}-1\right) v_{b u s} . \tag{2.4}
\end{equation*}
$$

To get unity PF, $v_{L a}$ in (2.4) must be independent of $v_{b u s}$. Therefore, $n=2$ must be selected as the primary to secondary turns ratio of PFC transformer. Then, the last two equations are changed as follows:

$$
\begin{equation*}
v_{L a}(t)=\left|v_{g}\right|, \tag{2.5}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{M}(t)=-v_{\text {bus }} . \tag{2.6}
\end{equation*}
$$

In this interval, the current of $L_{a}$ denoted as $i_{L a}$ starts to increase linearly from zero. The current of the secondary winding of LLC transformer denoted as $i_{n r 2}$ is positive, and thus $D_{2}$ conducts. This interval continues until $i_{n r 2}$ becomes zero.

### 2.2.3 Interval III, $\left[t_{2}-t_{3}\right]$

At this interval, because $i_{n r 2}$ becomes zero, $D_{2}$ stops conducting. MOSFET of $S_{1}$ is still ON, and the rest of components operate the same as previous interval.


Figure 2.6: Interval III of the converter operation

### 2.2.4 Interval IV, $\left[t_{3}-t_{4}\right]$

Before this interval, $S_{1}$ was ON having discharged $C_{S 1}$. When $S_{1}$ turns OFF and before $S_{2}$ turns ON, $C_{S 2}$ discharges completely. Because $i_{i n}$ is still positive, $D_{\text {rect }}$ continues to conduct, and since $i_{n r 2}$ is still zero, $D_{1}$ and $D_{2}$ do not conduct.


Figure 2.7: Interval IV of the converter operation

### 2.2.5 Interval $\mathbf{V},\left[t_{4}-t_{5}\right]$

At the beginning of this interval, the body diode of $S_{2}$ begins to conduct to ensure the ZVS of switch before the MOSFET of $S_{2}$ turns ON.


Figure 2.8: Interval V of the converter operation

By writing a KVL, and since $n=2$, it is obtained that:

$$
\begin{equation*}
\frac{v_{M}(t)}{2}=v_{C 2} \tag{2.7}
\end{equation*}
$$

which can be stated as:

$$
\begin{equation*}
v_{M}(t)=2 v_{C 2}=v_{\text {bus }}>0 \tag{2.8}
\end{equation*}
$$

With the change of voltage polarity across the primary winding of $T_{a}$, input current $i_{i n}$ becomes zero causing $D_{\text {rect }}$ to stop conducting. In this interval, the voltage across $L_{a}$ is determined as:

$$
\begin{equation*}
v_{L a}(t)=-v_{b u s} . \tag{2.9}
\end{equation*}
$$

This causes $D_{a}$ to start conducting. Moreover, since $i_{n r 2}$ is negative, $D_{1}$ conducts. This interval continues until $i_{n r 2}$ becomes zero.

### 2.2.6 Interval VI, $\left[t_{5}-t_{6}\right]$

At this interval, because $i_{n r 2}$ becomes zero, $D_{1}$ stops conducting. MOSFET of $S_{2}$ is still ON, and the rest of components operate the same as previous interval.


Figure 2.9: Interval VI of the converter operation

### 2.2.7 Interval VII, $\left[t_{6}-t_{7}\right]$

At this interval, and since both $i_{L a}$ and $i_{i n}$ are zero, $D_{a}$ stops conducting.


Figure 2.10: Interval VII of the converter operation

During $t_{1}<t<t_{4}$, and according to (2.5), $v_{L a}$ is obtained as:

$$
\begin{equation*}
v_{L a}(t)=\left|v_{g}\right|, \tag{2.10}
\end{equation*}
$$

during $t_{4}<t<t_{6}$, and according to (2.9), $v_{L a}$ is obtained as:

$$
\begin{equation*}
v_{L a}(t)=-v_{b u s}, \tag{2.11}
\end{equation*}
$$

and during $t_{6}<t<t_{7}$, it is obtained as:

$$
\begin{equation*}
v_{L a}(t)=0 . \tag{2.12}
\end{equation*}
$$

According to the volt-second balance of $L_{a}$, the time duration $\Delta t_{d}=t_{6}-t_{4}$ when $D_{a}$ conducts is determined as:

$$
\begin{equation*}
\Delta t_{d}=\frac{\left|v_{g}\right|}{2 f_{s w} v_{b u s}}=\frac{V_{m}}{2 f_{s w} v_{b u s}}\left|\sin \omega_{l} t\right|, \tag{2.13}
\end{equation*}
$$

where $V_{m}$ is the peak value of input sinusoidal voltage, and $\omega_{l}$ is angular line frequency. Moreover, in this interval, because $i_{n r 2}$ is still zero, $D_{1}$ and $D_{2}$ do not conduct. This interval continues until $S_{2}$ turns OFF.

### 2.3 Theoretical Calculation of PF and THD

By considering Fig. 2.3, the maximum of $i_{i n}$ during a switching period which is denoted as $\hat{i}_{\text {in }}$ is calculated. During $t_{1}<t<t_{4}$, since $D_{a}$ does not conduct, it can be written that:

$$
\begin{equation*}
i_{i n}(t)=i_{L a}(t), \tag{2.14}
\end{equation*}
$$

and according to (2.5), $v_{L a}$ can be written as:

$$
\begin{equation*}
v_{L a}(t)=\left|v_{g}\right|=V_{m}\left|\sin \omega_{l} t\right| \tag{2.15}
\end{equation*}
$$

Then by writing the voltage-current relationship of $L_{a}$, it is obtained that:

$$
\begin{equation*}
\left|v_{g}\right|=L_{a} \frac{\hat{i}_{i n}}{\left(t_{4}-t_{1}\right)}, \tag{2.16}
\end{equation*}
$$

where

$$
\begin{equation*}
t_{4}-t_{1}=\frac{T_{s w}}{2} \tag{2.17}
\end{equation*}
$$

Then, $\hat{i}_{i n}$ is obtained as:

$$
\begin{equation*}
\hat{i}_{i n}=\frac{\left|v_{g}\right|}{2 f_{s w} L_{a}}=\frac{V_{m}\left|\sin \omega_{l} t\right|}{2 f_{s w} L_{a}} \tag{2.18}
\end{equation*}
$$

where $f_{s w}$ is switching frequency.

The PFC section of the proposed LED driver is shown in Fig. 2.11. To calculate the PF, first the average of $i_{i n}$ over a switching period which is denoted as $i_{\text {in,ave }}$ is determined based on its waveform shown in Fig. 2.3 as:

$$
\begin{equation*}
i_{i n, a v e}=\frac{\hat{i}_{i n} \frac{T_{s w}}{2}}{2 T_{s w}} \stackrel{(2.18)}{=} \frac{V_{m}\left|\sin \omega_{l} t\right|}{8 f_{s w} L_{a}} . \tag{2.19}
\end{equation*}
$$

Then, the Root Mean Square (RMS) of input current $\left(i_{g}\right)$ is calculated as:

$$
\begin{equation*}
i_{g}=\frac{V_{m} \sin \omega_{l} t}{8 f_{s w} L_{a}} \Longrightarrow I_{g, r m s}=\frac{V_{m}}{8 \sqrt{2} L_{a} f_{s w}} \tag{2.20}
\end{equation*}
$$

Finally, the theoretical PF of the proposed converter is determined according to the formula of $P_{\text {in }}$ obtained in (5.5) as:

$$
\begin{equation*}
P F=\frac{P}{S}=\frac{P_{i n}}{V_{g, r m s} \times I_{g, r m s}}=\frac{\frac{V_{m}^{2}}{16 f_{s w} L_{a}}}{\frac{V_{m}}{\sqrt{2}} \times \frac{V_{m}}{8 \sqrt{2} L_{a} f_{s w}}}=1 . \tag{2.21}
\end{equation*}
$$

Consequently, theoretical THD $=0 \%$ is inferred.


Figure 2.11: PFC section of the proposed converter

### 2.4 Simulation and Experimental Results

The simulation results of the PFC circuit are shown in Fig. 2.12 and Fig. 2.13 for different operating points in which the parasitic elements of the converter are considered.


Figure 2.12: Input voltage and current waveforms at nominal input voltage at (a) full-load, (b) light-load


Figure 2.13: Input voltage and current waveforms at rated power at (a) minimum input voltage, (b) maximum input voltage

The experimental results are shown in Fig. 2.14 and Fig. 2.15 for different operating points.

(a)

(b)

Figure 2.14: Input voltage and current waveforms at (a) full-load, (b) light-load

(a)

(b)

Figure 2.15: Input voltage and current waveforms at rated power at (a) minimum input voltage, (b) maximum input voltage

The current of the primary winding of $T_{a}$ denoted as $i_{i n}$, and the voltage across $D_{a}$ denoted as $v_{D a}$ are shown in Fig. 2.16. The current behaves like the input current of a DCM buck-boost PFC circuit, and $D_{a}$ switches as expected according to the converter operating principles. However, the leakage inductance of $T_{a}$, delays the falling time
of $i_{\text {in }}$ slightly in each switching period as shown in Fig. 2.16(a). In addition, the resonance between this inductance and the parasitic capacitance of $D_{a}$ appears on $i_{i n}$ as shown in Fig. 2.16(b). These two factors result in imperfect PF and THD different from (2.21).

(a)

(b)

Figure 2.16: (a) Current of the primary winding of $T_{a}$, and (b) interaction between $D_{a}$ and $T_{a}$ during switching periods

Using a Tektronix power analyzer model PA4000, input current harmonics of the prototype have been measured, and compared with IEC61000-3-2 class C requirements as shown in Fig. 2.17. The experimental setup can pass the required standards. Nevertheless, designing a transformer with a lower leakage inductance, and a snubber circuit for $D_{a}$ will help in further decreasing the harmonics.

### 2.5 Summary

This chapter gives an insight into the importance of PFC circuits for LED drivers to comply with required PF and THD standards. Subsequently, the novelty and advantages of the proposed PFC topology which is a DCM buck-boost typed circuit over other integrated buck-boost half-bridge LED drivers are explained. The key waveforms and operating principles of the converter are discussed, and the theoretical PF and THD are also calculated. At the end, simulation and experimental results at different operating points are presented to verify the feasibility of the proposed PFC circuit. Furthermore, by using a power analyzer, the PF and THD of the experimental prototype are measured, and the compliance with regulatory standards is verified.


Figure 2.17: Measured input current harmonics of the experimental prototype at rated power at three different input (grid) voltage

## Chapter 3

## LLC Resonant Converter and Control System

Resonant converters in power electronics have gained more popularity in recent years owing to their higher efficiency and power density. The LLC resonant converter presents many extra benefits over other types of resonant converters, such as the ability to operate in near no-load condition with a reasonable switching frequency, and the ability to regulate the output throughout a wide operating range with relatively a small change in switching frequency. In addition, if designed properly, LLC resonant converters can achieve ZVS for the entire operating range [67].

Consider the Fig. 3.1(a) showing a typical half-bridge LLC resonant converter. The followings can be observed:

1. The LLC switches, $S_{1}$ and $S_{2}$ are utilized to produce a unipolar square-wave voltage on the input terminal of resonant tank which is denoted as $v_{\text {tank }}$. A certain amount of deadtime must be considered during the commutation of switches to ensure ZVS.
2. The resonant tank consists of a series capacitor denoted as $C_{r}$, a series inductor denoted as $L_{r}$, and a transformer with magnetizing inductance of $L_{m r}$. The voltage across the primary winding of transformer denoted as $v_{L m r}$ is a bipolar square-wave voltage. The transformer provides both galvanic isolation and turns ratio which is a variable in design.
3. On the secondary side of transformer, a rectifier network is provided for delivering dc power to the load.


Figure 3.1: (a) Half-bridge LLC resonant converter, and (b) the simplified circuit
In this chapter, the LLC resonant converter and its advantages are briefly discussed, and the design parameters of the utilized half-bridge LLC resonant circuit are obtained. One of the advantages of the proposed LED driver is the limited dc bus voltage which is formulated in this chapter as a function of converter parameters. In addition, the proposed control system is discussed which effectively mitigates the output current ripples. At the end, simulation and experimental results are provided to verify the ZVS of LLC switches, limited range of dc bus voltage, and effectiveness of the proposed control system.

### 3.1 LLC Overview

The simplified circuit of a half-bridge LLC resonant converter is shown in Fig. 3.1(b) where the input voltage of LLC resonant tank denoted as $v_{\text {tank }}$ is a square-wave signal. The impedance of LLC resonant tank consisting of $C_{r}, L_{r}$, and $L_{m r}$ varies by changing the switching frequency, and this adjusts the amount of current/power delivered to the load. The fact that the amount of delivered power is controlled by changing the switching frequency makes that a viable option for control system.

The peak resonance frequency of a resonant converter denoted as $f_{r p}$ is defined as the switching frequency that minimizes the resonant tank impedance, or in other words maximizes the amount of power delivered to the load [67]. For a series-resonant converter which is an LLC circuit without the parallel inductance $L_{m r}$, peak resonance frequency is the same as resonance frequency $\left(f_{r}\right)$, and is independent of load as described below:

$$
\begin{equation*}
f_{r p}=f_{r}=\frac{1}{2 \pi \sqrt{C_{r} L_{r}}} . \tag{3.1}
\end{equation*}
$$

However, in an LLC resonant converter, and because of $L_{m r}, f_{r p}$ becomes a function of load as follows:

$$
\begin{equation*}
f_{p}=\frac{1}{2 \pi \sqrt{C_{r}\left(L_{r}+L_{m}\right)}}<f_{r p}<f_{r}=\frac{1}{2 \pi \sqrt{C_{r} L_{r}}} \tag{3.2}
\end{equation*}
$$

where $f_{p}$ is defined as pole frequency [67].
At no load condition, $f_{r p}$ is the same as $f_{p}$, and by increasing the load, it becomes larger moving towards $f_{r}$, and finally at short-circuit condition where the LLC resonant converter is like a series-resonant converter, $f_{r p}$ becomes nearly equal to $f_{r}$. These three frequencies have been shown on Fig. 3.2.

### 3.1.1 Voltage Gain of Resonant Tank

The voltage gain of resonant tank is defined as:

$$
\begin{equation*}
M_{g}=\frac{V_{L m r}}{V_{\text {tank }}}, \tag{3.3}
\end{equation*}
$$

where $V_{L m r}$ is the RMS of $v_{L m r}$, and $V_{\text {tank }}$ is the RMS of $v_{t a n k}$.
Two important variables used in the analysis of LLC resonant converters are normalized switching frequency denoted as $F$, and quality factor denoted as $Q$ that are defined as follows:

$$
\begin{equation*}
F=\frac{f_{s w}}{f_{r}} \tag{3.4}
\end{equation*}
$$

and

$$
\begin{equation*}
Q=\frac{\omega_{r} L_{r}}{R_{e q}^{2}}=\frac{2 \pi f_{r} L_{r}}{\left(\frac{V_{o}^{2}}{P_{\text {out }}}\right)\left(\frac{8 n_{r}^{2}}{\pi^{2}}\right)}=\left(\frac{\pi^{2}}{8 n_{r}^{2}}\right)\left(\frac{P_{\text {out }}}{V_{o}^{2}}\right) \sqrt{\frac{L_{r}}{C_{r}}} . \tag{3.5}
\end{equation*}
$$



Figure 3.2: The curves of LLC resonant converter depending on different quality factors; Picture courtesy of [67]

There is a family of curves as shown in Fig. 3.2 associated with an LLC resonant converter depending on different loads or equivalently different quality factors according to (3.5). In addition, the voltage gain of resonant tank defined in (3.3) varies by changing the switching frequency for every specific load.

### 3.1.2 Zero Voltage Switching

One benefit of the LLC resonant converter is the ability to achieve ZVS provided that it is designed to operate in the inductive region shown in Fig. 3.2. By achieving ZVS, the turn-on switching loss of MOSFETs becomes close to zero.

ZVS is achieved when the MOSFET turns ON after its drain-source voltage is clamped to zero. In the LLC resonant converter when ZVS is achieved, at the turn-on, before the gate-source voltage is applied, the LLC circuit forces the reversal of current to flow through the MOSFET's body diode as illustrated in Fig. 3.3(a).

When the LLC resonant converter is designed to operate in inductive region, the resonant current denoted as $i_{L r}$ lags $v_{t a n k}$. In other words, $i_{L r}$ will have a negative (positive) value on the rising (falling) edge of $v_{\text {tank }}$ that ensures turn-on ZVS for $S_{1}\left(S_{2}\right)$ as illustrated in Fig. 3.3(b). It is worth mentioning that turn-off ZVS is achieved by adding large enough parallel capacitors on switches.


Figure 3.3: ZVS shown in (a) $S_{1}$, and (b) LLC resonant tank

### 3.2 Design of the LLC resonant circuit

The half-bridge LLC resonant circuit of the proposed LED driver is shown in Fig. 3.4 which is designed for dc bus voltage range of $(265-425) \mathrm{V}$, output voltage of 50 V , and output power range of $(20-200) \mathrm{W}$. The other constraint is that ZVS must be obtained for both $S_{1}$ and $S_{2}$. Because achieving ZVS is mandatory, the LLC resonant circuit must operate in inductive region throughout the whole operating range.


Figure 3.4: Half-bridge LLC resonant section of the proposed LED driver

According to Fig. 3.2, when the LLC resonant circuit operates in inductive region, the voltage gain of resonant tank decreases by increasing the switching frequency for a given $Q$. In addition, for a given dc bus voltage, and since the output voltage is constant, $Q$ and accordingly the output power decreases by increasing the switching frequency. As a result, not only the resonance frequency but also the range of switching frequency are design variables. The other design variables are the transformer turns ratio denoted as $n_{r}$, and resonant tank variables including $C_{r}, L_{r}$, and $L_{m r}$. It is worth mentioning that $i_{C 2}$ helps in achieving ZVS, therefore, designing the LLC variables is easier compared with a regular LLC resonant circuit. The value of $i_{C 2}$ at the turn-on of $S_{1}$ and $S_{2}$ is respectively obtained in (5.23) and (5.24).

The resonance frequency is set to:

$$
\begin{equation*}
f_{r}=132 \mathrm{kHz} \tag{3.6}
\end{equation*}
$$

The dc voltage gain of resonant tank is defined as [67]:

$$
\begin{equation*}
M_{g, D C}=\frac{V_{L m r}^{+}}{\frac{V_{\text {bus }}}{2}}=\frac{n_{r} V_{o}}{\frac{V_{\text {bus }}}{2}}=\frac{2 n_{r} V_{o}}{V_{\text {bus }}}, \tag{3.7}
\end{equation*}
$$

where $V_{L m r}^{+}$is the positive value of $v_{L m r}$ in half a switching period.
The dc voltage gain of resonant tank at resonance frequency is equal to 1 . In addition, the LLC converter is designed to operate at resonance frequency when the dc bus voltage is around 380 V to make sure the highest efficiency is achieved near the nominal input (grid) voltage. The turns ratio of LLC transformer is obtained as follows:

$$
\begin{equation*}
\frac{2 n_{r} V_{o}}{380}=1 \Longrightarrow \frac{1}{n_{r}} \approx 0.26 \tag{3.8}
\end{equation*}
$$

The LLC transformer turns ratio is selected as $n_{r}=\frac{N_{r 1}}{N_{r 2}}=\frac{27}{7}$.


Figure 3.5: LLC curves; Picture courtesy of [67]

The magnetizing to series inductance ratio of resonant tank is selected as:

$$
\begin{equation*}
L_{l l c}=\frac{L_{m r}}{L_{r}}=7.5 \tag{3.9}
\end{equation*}
$$

to minimize circulating currents, and facilitate the integration of LLC transformer and inductor.

The LLC curves are shown in Fig. 3.5. $a_{1}$ and $a_{3}$ are two critical points of the design. First, the minimum switching frequency happens at minimum dc bus voltage equal to 265 V , and maximum output power which is 200 W (point $a_{1}$ ). Second, the maximum switching frequency happens at maximum dc bus voltage equal to 425 V , and minimum output power which is 20 W (point $a_{3}$ ).

The optimal point for $a_{1}$ is near and at the right side of the curve to minimize conduction losses. For this purpose, $Q_{\max }=0.44$ is selected resulting in $f_{\min }=$ 80 kHz . In addition, according to (3.5), (3.6) and (3.8), $C_{r}$ and $L_{r}$ will be determined, and consequently according to (3.9), $L_{m r}$ will be also obtained. The resonant tank variables are designed as follows:

$$
\left\{\begin{array}{l}
C_{r}=18.1 \mathrm{nF}  \tag{3.10}\\
L_{r}=80 \mu \mathrm{H} \\
\\
L_{m r}=600 \mu \mathrm{H}
\end{array}\right.
$$

Then $Q_{\text {min }}=0.044$ is obtained resulting in $f_{\max }=150 \mathrm{kHz}$. Thus, the range of switching frequency is set as:

$$
\begin{equation*}
80 \mathrm{kHz} \leq f_{s w} \leq 150 \mathrm{kHz} . \tag{3.11}
\end{equation*}
$$

### 3.3 Limited DC Bus Voltage

The output voltage is constant because the LEDs are assumed to be ideal. Furthermore, the output current is assumed to be dc based on the proper control system. As a result, the input power absorbed by the LLC and sent to the output will be dc, and consequently the double frequency oscillations can be ignored.

The voltage range of $V_{b u s}$ is important since it determines the voltage stress of $S_{1}$ and $S_{2}$. Given the peak value of input sinusoidal voltage that can be as high as 375 V , it is critical that $V_{\text {bus }}$ have a reasonable range to lower the voltage stress of components. To determine $V_{\text {bus }}$ in terms of the converter parameters, the voltage gain of half-bridge LLC resonant converter is obtained based on the Fourier series of $v_{L m r}$ and $v_{t a n k}$, and is used to relate $P_{\text {out }}$ to $V_{\text {bus }}$ as follows:

$$
\begin{equation*}
\frac{V_{o}}{V_{b u s}}=\frac{4 n_{r} F}{\pi^{2} Q} \sum_{k=\text { odd }} \frac{\sin k \theta_{s}}{k\left((k F)^{2}-1\right)}, \tag{3.12}
\end{equation*}
$$

where $\theta_{s}$ is the lag angle between $v_{\text {tank }}$ and $v_{L m r}$. In addition according to (3.4),(3.6), and (3.11), the range of $F_{2}$ is obtained as:

$$
\begin{equation*}
0.36<F^{2}<1.3 \tag{3.13}
\end{equation*}
$$

The series in (3.12) can be expanded as follows:

$$
\begin{equation*}
\sum_{k=o d d} \frac{\sin k \theta_{s}}{k\left(k^{2} F^{2}-1\right)}=\frac{\sin \theta_{s}}{F^{2}-1}+\frac{\sin 3 \theta_{s}}{3\left(9 F^{2}-1\right)}+\frac{\sin 5 \theta_{s}}{5\left(25 F^{2}-1\right)}+\ldots \tag{3.14}
\end{equation*}
$$

According to (3.13),

$$
\begin{align*}
\left|\frac{\sin 3 \theta_{s}}{3\left(9 F^{2}-1\right)}\right| & <\frac{1}{4}\left|\frac{\sin \theta_{s}}{F^{2}-1}\right|, \\
\left|\frac{\sin 5 \theta_{s}}{5\left(25 F^{2}-1\right)}\right| & <\frac{1}{18}\left|\frac{\sin \theta_{s}}{F^{2}-1}\right|, \\
& \vdots  \tag{3.15}\\
\left|\frac{\sin k \theta_{s}}{k\left((k F)^{2}-1\right)}\right| & \ll\left|\frac{\sin \theta_{s}}{F^{2}-1}\right| \quad \text { for } \quad k>5 .
\end{align*}
$$

Considering (3.15), and to simplify the formula of $V_{\text {bus }}$, only the first term of (3.14) is considered, and (3.12) can be rewritten as:

$$
\begin{equation*}
\frac{V_{o}}{V_{\text {bus }}}=\left(\frac{4 n_{r} F}{\pi^{2} Q}\right)\left(\frac{\sin \theta_{s}}{F^{2}-1}\right), \tag{3.16}
\end{equation*}
$$

where $\sin \theta_{s}$ depends on the variables of LLC resonant converter, and since $i_{L r}$ is in phase with $v_{L m r}$, and according to (5.47) is obtained by solving the following equation:

$$
\begin{gather*}
i_{L r}\left(t=\theta_{s}\right)=0 \Longrightarrow \\
\left(\frac{8 n_{r}^{2} F}{\pi^{2} Q}\right)\left(\sum_{k=o d d} \frac{\sin k \theta_{s}}{k\left(k^{2} F^{2}-1\right)}\right)\left(\sum_{k=o d d} \frac{\left(1+L_{l l c}\right) k^{2} F^{2}-1}{L_{l l c} k^{2} F^{2}\left(k^{2} F^{2}-1\right)}\right)-\sum_{k=o d d} \frac{\cos k \theta_{s}}{k^{2} F^{2}-1}=0 \tag{3.17}
\end{gather*}
$$

For simplicity, only the first terms of Fourier series of (3.17) are considered, and (3.17) can be rewritten as:

$$
\begin{equation*}
\left(\frac{8 n_{r}^{2} F}{\pi^{2} Q}\right)\left(\frac{\sin \theta_{s}}{F^{2}-1}\right)\left(\frac{\left(1+L_{l l c}\right) F^{2}-1}{L_{l l c} F^{2}\left(F^{2}-1\right)}\right)=\frac{\cos \theta_{s}}{F^{2}-1} \tag{3.18}
\end{equation*}
$$

By using (3.18), it can be written that:

$$
\begin{equation*}
\tan \theta_{s}=\frac{1}{A_{1} B_{1}} \tag{3.19}
\end{equation*}
$$

where

$$
\left\{\begin{array}{l}
A_{1}=\frac{8 n_{r}^{2} F}{\pi^{2} Q}  \tag{3.20}\\
B_{1}=\frac{\left(1+L_{l l c}\right) F^{2}-1}{L_{l l c} F^{2}\left(F^{2}-1\right)}
\end{array}\right.
$$

Finally, $\sin \theta_{s}$ is obtained in terms of $A_{1}$ and $B_{1}$ as:

$$
\begin{equation*}
\sin \theta_{s}=\frac{1}{\sqrt{1+\left(A_{1} B_{1}\right)^{2}}} \approx \frac{1}{A_{1} B_{1}} \tag{3.21}
\end{equation*}
$$

By replacing (3.21) into (3.16), and according to (3.20), $V_{\text {bus }}$ is obtained as follows:

$$
\begin{equation*}
V_{b u s}=\frac{2 n_{r} V_{o}}{L_{l l c}}\left(1+L_{l l c}-\frac{1}{F^{2}}\right), \tag{3.22}
\end{equation*}
$$

where

$$
\left\{\begin{array}{rl}
F & =\frac{f_{s w}}{f_{r}}  \tag{3.23}\\
f_{r} & =\frac{1}{2 \pi \sqrt{L_{r} C_{r}}}
\end{array} \Longrightarrow F^{2}=4 \pi^{2} L_{r} C_{r} f_{s w}^{2}\right.
$$

Assuming a lossless converter, and according to the input power formula obtained in (5.5), it can be written that:

$$
\begin{equation*}
f_{s w}=\frac{V_{m}^{2}}{16 P_{\text {out }} L_{a}} \Longrightarrow F^{2}=\left(\frac{\pi V_{m}^{2}}{8 P_{\text {out }} L_{a}}\right)^{2} L_{r} C_{r} . \tag{3.24}
\end{equation*}
$$

Then, (3.22) is related to the peak value of input sinsusoidal voltage $\left(V_{m}\right)$ and output power as follows:

$$
\begin{equation*}
V_{b u s}=\frac{2 n_{r} V_{o}}{L_{l l c}}\left(1+L_{l l c}-\frac{1}{\left(\frac{\pi V_{m}^{2}}{8 P_{\text {out }} L_{a}}\right)^{2} L_{r} C_{r}}\right) \tag{3.25}
\end{equation*}
$$

By replacing the known parameters, the formula of $V_{b u s}$ is obtained as:

$$
\begin{equation*}
V_{\text {bus }}=51.43\left(8.5-\frac{P_{\text {out }}^{2}}{V_{m}^{4} \times 2.566 \times 10^{-6}}\right) . \tag{3.26}
\end{equation*}
$$

The maximum of (3.26) happens at $V_{m}=375 \mathrm{~V}$, and $P_{\text {out }}=20 \mathrm{~W}$, and is equal to:

$$
\begin{equation*}
V_{b u s, \max }=436.74 \mathrm{~V} \tag{3.27}
\end{equation*}
$$

The variation of $V_{\text {bus }}$ with respect to $V_{m}$ and $P_{\text {out }}$ is depicted in Fig. 3.6.


Figure 3.6: The variation of $V_{\text {bus }}$ with respect to $V_{m}$ and $P_{\text {out }}$

### 3.4 Control System

The input power of an LED driver has double-frequency oscillations, whereas the output power is ideally constant. The LED driver is responsible for decoupling these two powers. As a result of imperfect power decoupling, double-frequency ripples appear on the output causing light flickers. To comply with safety regulations, the output current ripples of the LED driver must be minimized.

As discussed in $[48,52,53]$, one way to remove the output double-frequency ripples is by accepting them on the output of PFC stage and controling the LLC to produce constant dc current at the output. As a result, a low-capacitance highly durable film capacitor can be chosen as dc bus capacitor. Moreover, since the ripples are removed from the output, film capacitors can be also chosen as output capacitors.

In proposed LED driver in this thesis, similar to the proposed control system of [48] for microinverters of PV solar systems, a very fast PI controller is designed to remove the output double-frequency ripples, and place them on the dc bus capacitor. The advantage of this power decoupling method over other techniques is that it does not require any additional circuits, or complexity in control.

The control system of the proposed LED driver is depicted in Fig. 3.7.


Figure 3.7: Control system of the proposed LED driver

### 3.5 Simulation and Experimental Results

The simulation results of the half bridge LLC resonant circuit are shown in Fig. 3.8 and Fig. 3.9 displaying the current and scaled drain-source voltage waveforms of both switches, as well as the resonant current $\left(i_{L r}\right)$ and $v_{t a n k}$ waveforms.

(a)

(b)

Figure 3.8: Simulation result showing ZVS at nominal input (grid) voltage at (a) full-load, (b) light-load


Figure 3.9: Simulation result showing ZVS at rated power at (a) minimum input (grid) voltage, (b) maximum input (grid) voltage

The experimental results are also shown in Fig. 3.10 and Fig. 3.11.

(b)

Figure 3.10: Experimental result showing ZVS at full-load for (a) $S_{1}$, and (b) $S_{2}$

(a)

(b)

Figure 3.11: Experimental result showing ZVS at light-load for (a) $S_{1}$, and (b) $S_{2}$

The current and voltage waveforms of the resonant capacitor are shown in Fig. 3.12.


Figure 3.12: Current and voltage waveforms of $C_{r}$
The dc bus voltage waveform is shown in Fig. 3.13 which at rated power and maximum input voltage $\left(V_{g, r m s}=265 \mathrm{~V}\right)$, the average is only 387.1 V . In addition, the variation of $V_{\text {bus }}$ at rated power with respect to the rms of input voltage $\left(V_{g, r m s}\right)$ is depicted in Fig. 3.14.


Figure 3.13: DC bus voltage at full load at maximum input voltage


Figure 3.14: Average dc bus voltage with respect to $V_{g, r m s}$ at rated power

Finally, the proposed control system is implemented using the Texas Instrument TMS320F28335 microcontroller. One of the output currents is shown in Fig. 3.15. As explained, by accepting high voltage ripples across the dc bus capacitor, the output current ripples can be reduced which for the experimental prototype could be restricted as to below $15 \%$.


Figure 3.15: Output current at nominal input voltage $\left(V_{g, r m s}=220 \mathrm{~V}\right)$ at rated power ( $P_{\text {out }}=200 \mathrm{~W}$ )

### 3.6 Summary

This chapter starts by explaining briefly about LLC resonant converters, and their advantages over other types of resonant converters. The voltage gain function of LLC resonant tank, the LLC curves, and ZVS concept are explained, and finally the design parameters of the LLC resonant stage are obtained based on which the ZVS of both switches is guaranteed throughout the whole operating range.

In addition, the formula of average dc bus voltage is obtained in terms of the converter parameters. It is shown that $V_{\text {bus }}$ is not higher than 436.74 V during the whole operating range. This is a great advantage of the proposed LED driver since lowvoltage rating power MOSFETs can be selected.

Subsequently, the control system of the design is discussed where by designing a very fast controller, the output current ripples are minimized, and electrolytic capacitors are removed.

At the end, simulation and experimental results are provided to verify the ZVS of both switches, limited range of dc bus voltage, and effectiveness of the proposed control system which could restrict the output current ripples to below $15 \%$.

## Chapter 4

## Magnetic Design

Magnetic design plays a pivotal role in power electronics due to their effect on the overall efficiency and power density of power electronic converters. There are four magnetic components in the proposed LED driver, the inductor and transformer of the PFC circuit, as well as the inductor and transformer of the LLC resonant circuit. Each inductor and transformer pair will be integrated on the same core. Therefore, there will be only two magnetic designs.

There are three general magnetic design categories as discussed in [68] as follows:

1. Saturated thermally limited design,
2. Non-saturated thermally limited design,
3. Signal quality limited design.

The choice of magnetic design is critical, and depending on the design measurements like RMS and peak value of winding currents, insulation requirements, and maximum flux density, one of the aforementioned design categories must be selected [68]. Nonsaturated thermally limited design is chosen for the magnetic components of the proposed LED driver because of high frequency and large ac components of winding currents. The flowchart of non-saturated thermally limited design is depicted in Fig. 4.1.


Figure 4.1: Flowchart of non-saturated thermally limited magnetic design


Figure 4.2: PFC section of the proposed LED driver

### 4.1 Magnetic Integration of $T_{a}$ and $L_{a}$

The PFC section of the proposed LED driver is shown in Fig. 4.2. To achieve a higher power density and efficiency, the PFC transformer $\left(T_{a}\right)$ and inductor $\left(L_{a}\right)$ are integrated on the same core as shown in Fig. 4.3(a).


Figure 4.3: (a) The 3D model in Ansys Maxwell and (b) equivalent magnetic circuit of the magnetic integration of $L_{a}$ and $T_{a}$

### 4.1.1 Core Material

The first step of magnetic design is to choose the core material. Material 3C94 is chosen due to its high permeability, high saturation limit, and suitability for the selected operating frequency range.

### 4.1.2 Core Size

The core size is determined using the following equation comparing the component total volt-amp rating with core dimensions to assess the core capacity for withstanding expected losses [68]

$$
\begin{equation*}
S_{t o t}=\sum V_{i, r m s} I_{i, r m s}=A_{f} a_{c h}^{\lambda} \Longrightarrow a_{c h}=\left(\frac{S_{t o t}}{A_{f}}\right)^{\frac{1}{\lambda}} \tag{4.1}
\end{equation*}
$$

where $A_{f}$ is the ferrite coefficient in the range of $(5-25) \times 10^{6}$ if $a_{c h}$ is in [ m ] which for high frequency designs, $A_{f}=(20-25) \times 10^{6}$ is normally selected, $a_{c h}$ is the largest dimension of magnetic core, $S_{\text {tot }}$ is the component volt-amp rating, and $\lambda$ is an exponent characterizing the material, and shape of the core, $\lambda=3$.

Based on the rms voltage and current of the windings at rated power, the maximum dimension of core is determined as follows:

$$
\left\{\begin{array} { l } 
{ V _ { M , r m s } = 3 4 7 . 1 \mathrm { V } }  \tag{4.2}\\
{ I _ { \text { in,rms } } = 1 . 4 5 \mathrm { A } } \\
{ V _ { L m a , r m s } = 1 8 8 \mathrm { V } } \\
{ I _ { C 2 , r m s } = 2 . 5 8 \mathrm { A } } \\
{ V _ { L a , r m s } = 2 3 1 . 2 8 \mathrm { V } } \\
{ I _ { L a , r m s } = 1 . 5 9 \mathrm { A } }
\end{array} \stackrel { A _ { f } = 2 0 \times 1 0 ^ { 6 } } { \Longrightarrow } \left\{\begin{array}{l}
S_{t o t}=1356.9 \mathrm{~V} \\
a_{c h}=40.79 \mathrm{~mm}
\end{array}\right.\right.
$$

As the largest dimension of core is calculated as $a_{c h}=40.79 \mathrm{~mm}$, the ferrite core EE $42 / 20$ which is made of two E 42/21/20 cores placed on each other is selected for the PFC integrated magnetic component.

### 4.1.3 Structure of the Magnetic Design

As illustrated in Fig. 4.3(a), the transformer windings are wrapped around the middle leg, and the inductor winding is wrapped around the outer legs of the core that will be identically designed in terms of air gap, and number of winding turns. The equivalent magnetic circuit of the proposed structure is shown in Fig. 4.3(b).

Using two KVLs, and one KCL, it can be written that:

$$
\left\{\begin{array}{l}
-\frac{N_{L}}{2} i_{L a}(t)+\mathcal{R}_{\text {outer }} \Phi_{1}(t)-\mathcal{R}_{\text {center }} \Phi_{3}(t)-N_{P} i_{\text {in }}(t)+N_{S} i_{C 2}(t)=0  \tag{4.3}\\
N_{P} i_{i n}(t)-N_{S} i_{C 2}(t)+\mathcal{R}_{\text {center }} \Phi_{3}(t)-\mathcal{R}_{\text {outer }} \Phi_{2}(t)-\frac{N_{L}}{2} i_{L a}(t)=0 \\
\Phi_{3}(t)=-\Phi_{1}(t)-\Phi_{2}(t)
\end{array}\right.
$$

and consequently the magnetic fluxes shown on Fig. 4.3(b) are obtained as follows:

$$
\left\{\begin{array}{l}
\Phi_{1}(t)=-\frac{\Phi_{3}(t)}{2}+\frac{N_{L} i_{L a}(t)}{2 \mathcal{R}_{\text {outer }}}  \tag{4.4}\\
\Phi_{2}(t)=-\frac{\Phi_{3}(t)}{2}-\frac{N_{L} i_{L a}(t)}{2 \mathcal{R}_{\text {outer }}}
\end{array}\right.
$$

where $\Phi_{3}(t)$ is the flux created by $T_{a}$, and according to (4.3) and (4.4) can be calculated as:

$$
\begin{equation*}
\Phi_{3}(t)=\frac{N_{S} i_{C 2}(t)-N_{P} i_{\text {in }}(t)}{\frac{\mathcal{R}_{\text {outer }}}{2}+\mathcal{R}_{\text {center }}} . \tag{4.5}
\end{equation*}
$$

The results shown in (4.4) and (4.5) indicate that the flux created by $L_{a}\left(\frac{N_{L} i_{L a}(t)}{2 \mathcal{R}_{\text {outer }}}\right)$ does not pass through the middle leg. In addition, half of the flux created by $T_{a}$ $\left(\frac{\Phi_{3}(t)}{2}\right)$ existing in each outer leg is in the opposite direction, and the voltage created by them will be cancelled with each other not affecting $L_{a}$. In conclusion, there is no interaction between $L_{a}$ and $T_{a}$ in this structure. Therefore, they can be independently designed.

### 4.1.4 Heat Dissipation Capability and Total Loss Budget

Calculating the operating temperature of a magnetic component is essential for every power electronic device as there is a heat dissipation capability for every wire, and the ferrite core that must be considered. The heat dissipation capability of a magnetic component denoted as $P_{h}$ is defined as [68]:

$$
\begin{equation*}
P_{h}=k_{A} a b \tag{4.6}
\end{equation*}
$$

where $P_{h}$ is the allowable loss budget that guarantees the temperature rise under $50^{\circ} \mathrm{C}$ in $\mathrm{W}, k_{A}$ is a coefficient, a typical value is $2500 \mathrm{~W} \mathrm{~m}^{-2}$, and $a$ and $b$ are the two largest dimensions of magnetic core in [m].

For the chosen EE 42 core,

$$
\begin{equation*}
P_{h}=2500 \times 0.042 \times 0.042=4.41 \mathrm{~W} \tag{4.7}
\end{equation*}
$$

### 4.1.5 Optimal Core/Copper Loss Ratio

The total loss of every magnetic component consists of core loss which is related to core characteristics such as dimension, material, etc., and copper loss which is the ohmic loss dissipated in windings. For every magnetic design, there is a trade-off between these two losses.
It is proven that the maximum efficiency of any magnetic design happens when the core loss and copper loss are nearly the same [69]. Therefore, to achieve the optimal design, it is assumed that

$$
\begin{equation*}
P_{h, f e}=P_{h, c u}=\frac{P_{h}}{2} \tag{4.8}
\end{equation*}
$$

where $P_{h, f e}$ is the core loss budget, and $P_{h, c u}$ is the copper loss budget of the design.
For the selected core, it can be written that:

$$
\begin{equation*}
P_{h, f e}=P_{h, c u}=\frac{4.41}{2}=2.205 \mathrm{~W} \tag{4.9}
\end{equation*}
$$

Since this magnetic component consists of an inductor and a transformer, the core and copper loss budgets must be divided between $L_{a}$ and $T_{a}$.

### 4.1.6 Allocation of Core Loss Budget

The core loss of a certain magnetic component denoted as $P_{f e}$ is proportional to its magnetic volume denoted as $\mathcal{V}_{e}$, and is determined by Stienmetz equation as:

$$
\begin{equation*}
P_{f e}=k_{e} f_{s w}^{\gamma} \hat{B}^{\beta} \mathcal{V}_{e} \tag{4.10}
\end{equation*}
$$

where $k_{e}$ is a constant, $\gamma$ and $\beta$ are exponents, and $\hat{B}$ is maximum flux density.
According to (4.10), and assuming that the maximum flux density of $L_{a}$ and $T_{a}$ is nearly the same, the core loss budget obtained in (4.9) is divided between inductor and transformer according to their corresponding magnetic volumes.

As illustrated in Fig. 4.3(a), the flux created by $L_{a}$ only passes through the outer legs (path 1) while the flux created by $T_{a}$ passes through all three legs (both path 1 and path 2).

The total magnetic volume of a certain core is defined as:

$$
\begin{equation*}
\mathcal{V}_{e}=A_{e} \times l_{e} \tag{4.11}
\end{equation*}
$$

where $A_{e}$ is the magnetic area of middle leg, and $l_{e}$ is the total magnetic length of the core.

Both $A_{e}$ and $l_{e}$ can be found in the ferrite core datasheet, or by simple calculations based on different measures of the core given in the datasheet.

As for the EE 42 core [70], the magnetic volume of inductor magnetic path (path 1) and middle leg (path 2) shown in Fig. 4.3(a) are calculated respectively as follows:

$$
\begin{align*}
& \mathcal{V}_{e 1}=18152 \mathrm{~mm}^{3} .  \tag{4.12}\\
& \mathcal{V}_{e 2}=8784 \mathrm{~mm}^{3} \tag{4.13}
\end{align*}
$$

The total magnetic volume of the EE 42 core is obtained as:

$$
\begin{equation*}
\mathcal{V}_{e}=\mathcal{V}_{e 1}+\mathcal{V}_{e 2}=26936 \mathrm{~mm}^{3} \tag{4.14}
\end{equation*}
$$

In this design for simplicity, the core loss budget associated with middle leg (path 2) is not involved in calculations. Then, assuming that the maximum flux density of $L_{a}$ and $T_{a}$ is nearly the same, the core loss budget associated with inductor magnetic path (path 1) is divided between inductor and transformer equally, and the core loss budget of each of them is determined as follows:

$$
\begin{equation*}
\frac{\mathcal{V}_{e 1}}{\mathcal{V}_{e}}=67.3 \% \quad \stackrel{(4.10)}{\Longrightarrow} P_{h, f e, L}=P_{h, f e, T}=\frac{0.673}{2} P_{h, f e}=0.743 \mathrm{~W}, \tag{4.15}
\end{equation*}
$$

where $P_{h, f e, L}$ is the core loss budget of $L_{a}$, and $P_{h, f e, T}$ is the core loss budget of $T_{a}$.

### 4.1.7 Core Loss per Volume

The core loss per volume denoted as $P_{f e, s p}$ is defined as [68]:

$$
\begin{equation*}
P_{f e, s p}=\frac{P_{h, f e}}{1000 \times \mathcal{V}_{e}}\left[\mathrm{~kW} \mathrm{~m}^{-3}\right] \tag{4.16}
\end{equation*}
$$

According to (4.12) and (4.15), the core loss per volume of $L_{a}$ is obtained as:

$$
\begin{equation*}
P_{f e, s p, L}=\frac{0.743}{1000 \times 18152 \times 10^{-9}}=40.93 \mathrm{~kW} \mathrm{~m}^{-3} \tag{4.17}
\end{equation*}
$$

and according to (4.14) and (4.15), the core loss per volume of $T_{a}$ is obtained as:

$$
\begin{equation*}
P_{f e, s p, T}=\frac{0.743}{1000 \times 26936 \times 10^{-9}}=27.58 \mathrm{~kW} \mathrm{~m}^{-3} . \tag{4.18}
\end{equation*}
$$

### 4.1.8 Maximum Flux Density

In the datasheet of ferrite materials, the graphical dependency of $P_{f e, s p}$ is shown versus maximum flux density, and frequency as shown in Fig. 4.4 which is the datasheet of 3C94 material [71]. Based on this graph, for a given frequency, one can find the maximum flux density, and thus that of $L_{a}$ and $T_{a}$ are calculated respectively as follows:

$$
\begin{equation*}
\hat{B}_{L}=93.12 \mathrm{mT}, \tag{4.19}
\end{equation*}
$$

and

$$
\begin{equation*}
\hat{B}_{T}=80.9 \mathrm{mT} \tag{4.20}
\end{equation*}
$$

The assumption made earlier that $\hat{B}_{L}$ and $\hat{B}_{T}$ were considered almost the same is justified.


Figure 4.4: Core loss per volume versus maximum flux density and frequency [71]

### 4.1.9 Number of Turns

Calculating the number of turns of transformer and inductor windings is slightly different, yet both are calculated according to their maximum flux density.

### 4.1.9.1 Number of Turns of $T_{a}$ Windings

The number of turns of the primary winding denoted as $N_{P}$ is defined as [68]:

$$
\begin{equation*}
N_{P}=\frac{\Psi_{p p, T}}{\Phi_{p p, T}}, \tag{4.21}
\end{equation*}
$$

where $\Psi_{p p, T}$ is the magnetic flux linkage of $T_{a}$ defined as the area under the voltage waveform of the primary winding for half a switching period, and $\Phi_{p p, T}$ is the peak-to-peak magnetic flux of $T_{a}$.

The peak-to-peak magnetic flux linkage of $T_{a}$ is obtained as:

$$
\begin{equation*}
\Psi_{p p, T}=\int_{0}^{\frac{T_{s w}}{2}} v_{M}(t) d \tau=v_{b u s} \times \frac{T_{s w}}{2} . \tag{4.22}
\end{equation*}
$$

This integrated magnetic is designed at full-load, and minimum switching frequency which results in the highest magnetic field to ensure saturation is avoided for the whole operating range.

For this operating point, the average dc bus voltage is calculated according to (3.26) as:

$$
\begin{equation*}
V_{b u s}=266.05 \mathrm{~V} \tag{4.23}
\end{equation*}
$$

The approximation used in (3.21) to determine (3.26) is fairly accurate, but for the selected operating point, the actual value of $V_{\text {bus }}$ is slightly higher. Therefore, a $10 \%$ increase is regarded for (4.23). Furthermore, considering the worst case scenario, the double-frequency voltage ripples ( $40 \%$ peak-to peak is allowed across $C_{1}$ ) are considered, and the switching interval when the peak value of $v_{\text {bus }}$ happens is evaluated. Then, $\Psi_{p p, T}$ is calculated as:

$$
\begin{equation*}
\Psi_{p p, T}=1.1 \times 1.2 \times V_{b u s} \times \frac{T_{s w}}{2}=0.002195 \mathrm{~Wb} \tag{4.24}
\end{equation*}
$$

The peak-to-peak magnetic flux of $T_{a}$ is also calculated as:

$$
\begin{equation*}
\Phi_{p p, T}=2 \times \hat{B}_{T} \times A_{e}=3.78 \times 10^{-5} \mathrm{~Wb} \tag{4.25}
\end{equation*}
$$

Using (4.21), the number of turns of the primary winding is calculated as:

$$
\begin{equation*}
N_{P}=58.1 \tag{4.26}
\end{equation*}
$$

Since according to $V=4 N \Phi f$ equation for transformers, higher number of turns helps in avoiding saturation, the primary number of turns is rounded up to $N_{P}=60$, and then the number of turns of the secondary winding is determined as:

$$
\begin{equation*}
N_{S}=\frac{N_{P}}{2}=30 \tag{4.27}
\end{equation*}
$$

### 4.1.9.2 Number of Turns of $L_{a}$ Winding

The number of turns of $L_{a}$ winding denoted as $N_{L}$ is defined as [68]:

$$
\begin{equation*}
N_{L}=\frac{\Psi_{p p, L}}{\Phi_{p p, L}} \tag{4.28}
\end{equation*}
$$

where $\Psi_{p p, L}$ is the peak-to-peak magnetic flux linkage of $L_{a}$, and $\Phi_{p p, L}$ is the peak-to-peak magnetic flux of $L_{a}$.

The peak-to-peak magnetic flux linkage of $L_{a}$ is obtained as:

$$
\begin{equation*}
\Psi_{p p, L}=L_{a} \times i_{L a, p p}, \tag{4.29}
\end{equation*}
$$

where $i_{L a, p p}$ is the peak-to-peak current of $L_{a}$.
According to (5.6) and (5.7), for the chosen operating point, $L_{a}=267 \mu \mathrm{H}$, and $i_{L a, p p}$ is determined as:

$$
\begin{equation*}
i_{L a, p p}=\hat{i}_{i n} \stackrel{(2.18)}{=} \frac{V_{m}\left|\sin \omega_{l} t\right|}{2 f_{s w} L_{a}} \tag{4.30}
\end{equation*}
$$

The switching time instant when $\left|\sin \omega_{l} t\right|=1$ is considered. Then for the chosen operating point, it can be written that:

$$
\begin{equation*}
I_{L a, \max }=\frac{V_{m}}{2 f_{s w} L_{a}}=6.12 \mathrm{~A} \tag{4.31}
\end{equation*}
$$

where $I_{L a, \text { max }}$ is the maximum current of $L_{a}$ and is equal to its peak-to-peak current since it operates in DCM. Then, $\Psi_{p p, L}$ is calculated as:

$$
\begin{equation*}
\Psi_{p p, L}=L_{a} \times I_{L a, \max }=0.00163 \mathrm{~Wb} \tag{4.32}
\end{equation*}
$$

Similar to transformer, $\Phi_{p p, L}$ is calculated as:

$$
\begin{equation*}
\Phi_{p p, L}=2 \times \hat{B}_{L} \times A_{e o}=2.33 \times 10^{-5} \mathrm{~Wb} \tag{4.33}
\end{equation*}
$$

where $A_{e o} \approx \frac{A_{e}}{2}$, and is the magnetic area of outer legs around which $L_{a}$ is wrapped. Finally, according to (4.28), the number of turns of $L_{a}$ winding is calculated as:

$$
\begin{equation*}
N_{L}=69.96 \tag{4.34}
\end{equation*}
$$

For $L_{a}$, a certain amount of airgap denoted as $g$ must be considered to avoid saturation that is obtained as:

$$
\begin{equation*}
g=\frac{\mu_{0} A_{e o} N_{L}^{2}}{L_{a}}=2.88 \mathrm{~mm} . \tag{4.35}
\end{equation*}
$$

The airgap $g_{1}=g_{2}=1.4 \mathrm{~mm}$ is then chosen on each outer leg.
The inductor $\left(L_{a}\right)$ is composed of two identical sub-inductors on each outer leg that are in series with each other. According to the arrangement of these two sub-inductors shown in Fig. 4.3(a) in which their fluxes are in the same direction, the total inductance is calculated as:

$$
\begin{equation*}
L_{a}=L_{a 1}+L_{a 2}+2 M_{a} \tag{4.36}
\end{equation*}
$$

where $L_{a i}$ is the self inductance of each sub-inductor, $M_{a}=k_{c p} \sqrt{L_{a 1} L_{a 2}}$, is the mutual inductance between the sub-inductors, and $k_{c p}$ is their coupling factor.

Perfect coupling happens when all the flux passing through $L_{a 1}$ passes through $L_{a 2}$ as well resulting in $k_{c p}=1$. However, the coupling of sub-inductors in this configuration is not perfect. According to (4.36), constructing the inductor with $N_{L}=70$ obtained in (4.34) will result in an inductance higher than the inductance range of (5.7) which is not desired. Based on simulation in Ansys Maxwell, and through verifying in practice, $N_{L}=62$ is selected. The measured inductance ( $\left.L_{a}=L_{a 1}+L_{a 2}+2 M_{a}\right)$ of the laboratory prototype is $295 \mu \mathrm{H}$ which satisfies the inductance range of (5.7). While on the subject, fewer number of turns helps in decreasing the flux density according to the $N i=\mathcal{R} \Phi$ equation for inductors which in turn reduces the core loss, and there will be more window area in the core for all three windings as well.

### 4.1.10 Magnetizing Inductance $L_{m a}$

Considering Fig. 4.3(b), the magnetizing inductance of $T_{a}$ denoted as $L_{m a}$ is calculated as follows:

$$
\left\{\begin{array}{l}
L_{m a}=\frac{N_{P}^{2}}{\mathcal{R}_{\text {center }}+\frac{\mathcal{R}_{\text {outer }}}{2}}  \tag{4.37}\\
\mathcal{R}_{\text {outer }}=\frac{g_{1}}{\mu_{0} A_{e o}}=8.9 \times 10^{6} \quad \Longrightarrow L_{m a}=399 \mu \mathrm{H} \\
\mathcal{R}_{\text {center }}=\frac{g_{1}}{\mu_{0} A_{e}}=4.56 \times 10^{6} \\
N_{P}=60
\end{array}\right.
$$

### 4.1.11 Flux Calculation

The maximum flux density of $T_{a}$ and $L_{a}$ calculated in (4.20) and (4.19) respectively are based on the number of turns obtained in (4.26), (4.27), and (4.34). For transformers, according to $V=4 N \Phi f$ which is for a square-shaped excitation, $N$ and $\hat{B}$ are inversely related, while for inductors, according to $N i=\mathcal{R} \Phi, N$ and $\hat{B}$ are proportionally related. Therefore, based on the chosen number of turns for $T_{a}$ and $L_{a}$ in practice, the maximum flux density of $T_{a}$ and $L_{a}$ is changed respectively as follows:

$$
\begin{gather*}
\hat{B}_{T}=80.9 \mathrm{mT} \times \frac{58.1}{60}=78.34 \mathrm{mT}  \tag{4.38}\\
\hat{B}_{L}=93.12 \mathrm{mT} \times \frac{62}{69.96}=82.52 \mathrm{mT} \tag{4.39}
\end{gather*}
$$

One of the challenges of magnetic integration is the saturation problem that might occur in the core. According to (4.4), (4.38), and (4.39), the maximum flux passing through the core denoted as $\hat{\Phi}_{\text {tot }}$ happens in outer legs, and is the summation of the flux created by $L_{a}$ and half of the flux created by $T_{a}$ which is obtained as:

$$
\begin{equation*}
\hat{\Phi}_{t o t}=\hat{B}_{L} \times A_{e o}+\frac{\hat{B}_{T} \times A_{e}}{2}=1.987 \times 10^{-5} \mathrm{~Wb} . \tag{4.40}
\end{equation*}
$$

Then, the maximum flux density of the design denoted as $\hat{B}_{\text {tot }}$ is obtained as:

$$
\begin{equation*}
\hat{B}_{t o t}=\frac{\hat{\Phi}_{t o t}}{A_{e o}}=159 \mathrm{mT} \tag{4.41}
\end{equation*}
$$

The other way to calculate $\hat{B}_{t o t}$ is by obtaining the formula of $\hat{\Phi}_{t o t}$. According to (4.4) and (4.5), and the waveforms of $i_{C 2}, i_{i n}$, and $i_{L a}$ during a switching period shown in Fig. 2.3, it can be inferred that the maximum flux passing through the core denoted as $\hat{\Phi}_{\text {tot }}$ happens in outer legs, and according to (4.4), (4.5),(5.22),(5.53),(5.55), and by considering $\left|\sin \omega_{l} t\right|=1$, and neglecting the bus voltage ripples, can be obtained as:

$$
\begin{equation*}
\hat{\Phi}_{\text {tot }}=\left(\left|\frac{N_{S}\left(\frac{V_{\text {bus }}}{L_{m a}}+\frac{3 V_{m}}{L_{a}}\right)-N_{P} \frac{2 V_{m}}{L_{a}}}{\mathcal{R}_{\text {outer }}+2 \mathcal{R}_{\text {center }}}\right|+\frac{N_{L} V_{m}}{\mathcal{R}_{\text {outer }} L_{a}}\right) \frac{T_{s w}}{4}, \tag{4.42}
\end{equation*}
$$

which for the chosen operating point is calculated as:

$$
\begin{equation*}
\hat{\Phi}_{t o t} \approx 2.05 \times 10^{-5} \mathrm{~Wb}, \tag{4.43}
\end{equation*}
$$

and $\hat{B}_{\text {tot }}$ is obtained as:

$$
\begin{equation*}
\hat{B}_{t o t}=\frac{\hat{\Phi}_{t o t}}{A_{e o}}=164 \mathrm{mT} \tag{4.44}
\end{equation*}
$$

The value of $\hat{B}_{\text {tot }}$ obtained in (4.41) and (4.44) is nearly the same and lower than the saturation flux density of 3C94 ferrite material at 80 kHz which is 300 mT . Therefore, working in the linear region of the core is guaranteed. To verify this, the simulation of the design in Ansys Maxwell is conducted and shown in Fig. 4.5 which is compatible with calculations. The maximum flux density of the middle leg and outer legs does not happen at the same time.


Figure 4.5: Maximum flux density of (a) outer legs, and (b) middle leg shown in Ansys Maxwell

### 4.1.12 Core Loss

Using the $P_{f e, s p}-\hat{B}-f$ graph of 3C94 ferrite material shown in Fig. 4.4, the core loss per volume of the design denoted as $P_{f e, s p, \text { tot }}$ at $\hat{B}_{t o t}=159 \mathrm{mT}$, and $f_{s w}=80 \mathrm{kHz}$ is obtained as:

$$
\begin{equation*}
P_{f e, s p, t o t} \approx 120 \mathrm{~kW} \mathrm{~m}^{-3}, \tag{4.45}
\end{equation*}
$$

and the maximum core loss of the design is calculated as:

$$
\begin{equation*}
P_{f e, \max }=P_{f e, s p, t o t} \times \mathcal{V}_{e}=3.2 \mathrm{~W} \tag{4.46}
\end{equation*}
$$

This means if the design steps of the flowchart shown in Fig. 4.1 are followed, the core loss will be less than 3.2 W .

### 4.1.13 Allocation of Copper Loss Budget

The copper loss budget of the design is 2.205 W according to (4.9). Extra caution should be considered for selecting the wires as choosing inappropriately causes the copper loss to exceed the budget which in turn invalidates the whole design. The copper loss budget ( $P_{h, c u}$ ) must be distributed between the three windings;

1. The primary winding of $T_{a}$ with 60 turns.
2. The secondary winding of $T_{a}$ with 30 turns.
3. The winding of $L_{a}$ with 62 turns.

The copper loss allocation coefficient denoted as $\alpha_{i}$ which determines the allocated copper loss budget to the $i_{\text {th }}$ winding is defined as [68]:

$$
\begin{equation*}
\alpha_{i}=\frac{N_{i} I_{r m s, i}}{\sum N_{i} I_{r m s, i}}, \tag{4.47}
\end{equation*}
$$

where $N_{i}$ is the number of turns and $I_{r m s, i}$ is the rms current of the $i_{t h}$ winding.

Then, $P_{h, c u, i}$ which is the copper loss budget of the $i_{t h}$ winding is obtained as:

$$
\begin{equation*}
P_{h, c u, i}=\alpha_{i} P_{h, c u} \tag{4.48}
\end{equation*}
$$

### 4.1.14 Wire Diameters

After determining the copper loss budget of all three windings, their minimum wire diameter is obtained according to the power equation, as follows:

$$
\left\{\begin{array}{l}
P_{h, c u, i}=R_{d c, i} \times I_{r m s, i}^{2}  \tag{4.49}\\
R_{d c, i}=\frac{\rho l_{T i} N_{i}}{\frac{\pi d_{i}^{2}}{4}}
\end{array} \Longrightarrow d_{i} \geq \frac{2}{\sqrt{\pi}} I_{r m s, i} \sqrt{\frac{\rho l_{T i} N_{i}}{P_{h, c u, i}}},\right.
$$

where $R_{d c, i}$ is the dc resistance of the $i_{t h}$ winding, $l_{T i}$ is the mean-length-per-turn of the $i_{t h}$ winding, and $\rho$ is copper resistivity equal to $\rho=1.68 \times 10^{-8}$.

To determine the minimum wire diameter of each winding, first of all, according to (4.47), $\alpha_{L}, \alpha_{P}$, and $\alpha_{S}$ are calculated as the copper loss allocation coefficient of $L_{a}$ winding, primary and secondary windings of $T_{a}$ respectively, as follows:

$$
\left\{\begin{array}{l}
\alpha_{L}=\frac{N_{L} I_{L a, r m s}}{N_{P} I_{i n, r m s}+N_{L} I_{L a, r m s}+N_{S} I_{C 2, r m s}} \stackrel{(4.2)}{=} 0.375  \tag{4.50}\\
\alpha_{P}=\ldots=0.33 \\
\alpha_{S}=\ldots=0.295
\end{array}\right.
$$

Then according to (4.48), the copper loss budget of $L_{a}$ winding is obtained as:

$$
\begin{equation*}
P_{h, c u, L}=0.83 \mathrm{~W}, \tag{4.51}
\end{equation*}
$$

and the copper loss budget of $T_{a}$ primary winding is calculated as:

$$
\begin{equation*}
P_{h, c u, P}=0.73 \mathrm{~W}, \tag{4.52}
\end{equation*}
$$

and the copper loss budget of $T_{a}$ secondary winding is calculated as:

$$
\begin{equation*}
P_{h, c u, S}=0.65 \mathrm{~W} . \tag{4.53}
\end{equation*}
$$

By determining the copper loss budget of each winding, according to (4.49), the wire diameter of $L_{a}$ winding is calculated as:

$$
\begin{equation*}
d_{L} \geq \frac{2}{\sqrt{\pi}} I_{L a, r m s} \sqrt{\frac{\rho l_{T L} N_{L}}{P_{h, c u, L}}}=0.52 \mathrm{~mm} \tag{4.54}
\end{equation*}
$$

the wire diameter of $T_{a}$ primary winding is obtained as:

$$
\begin{equation*}
d_{P} \geq 0.58 \mathrm{~mm} \tag{4.55}
\end{equation*}
$$

and the wire diameter of $T_{a}$ secondary winding is obtained as:

$$
\begin{equation*}
d_{S} \geq 0.832 \mathrm{~mm} \tag{4.56}
\end{equation*}
$$

In (4.49), only the dc loss of wires is considered, whereas due to high frequency content of currents, ac loss associated with skin effect and proximity effect also exists in windings as illustrated in Fig. 4.6.

In practice, the diameter of wires is selected much larger than the obtained value from (4.49) to reduce the total loss. However, there are some restrictions associated with larger wire diameters as listed below:

1. The wires can be selected as thick as possible as long as the window area of the chosen core has enough space for them.
2. By enlarging the wire diameters, although the dc ohmic loss decreases, the ac loss associated with skin effect will increase significantly if single wires are used due to the increase of $z=\frac{d}{\delta}$ ratio of wires where $\delta$ is defined as skin depth.
3. Another challenge regarding the magnetic design is proximity effect. Normally, when a winding is wrapped in several layers, the ac loss associated with proximity effect becomes too large. A good practice is to enlarge the wire diameter as long as the winding will not become more than three layers.

Therefore, there is a trade-off between dc ohmic and ac losses. Extra caution must be considered for selecting appropriate wires.


Figure 4.6: Copper loss in a wire

### 4.1.15 Skin and Proximity Effects

### 4.1.15.1 Skin Effect

The area near the wire surface where most of the ac current flows is called skin depth $(\delta)$ which for copper at $100^{\circ} \mathrm{C}$, is calculated as:

$$
\begin{equation*}
\delta \approx \frac{7.5 \mathrm{~cm}}{\sqrt{f_{s w}}} \tag{4.57}
\end{equation*}
$$

For this magnetic design, it can be obtained that:

$$
\begin{gather*}
80 \mathrm{kHz} \leq f_{s w} \leq 150 \mathrm{kHz} \Longrightarrow \\
0.194 \mathrm{~mm} \leq \delta \leq 0.265 \mathrm{~mm} \tag{4.58}
\end{gather*}
$$

As a rule of thumb, selecting wires with diameter to skin depth ratio of greater than 1.6 results in higher ac loss [68]. Therefore, for this magnetic design, the diameter of wires denoted as $d$ should be selected as:

$$
\begin{equation*}
z=\frac{d}{\delta}<1.6 \Longrightarrow d<1.6 \delta<0.31 \mathrm{~mm} . \tag{4.59}
\end{equation*}
$$

By noticing the conflict between (4.54),(4.55),(4.56) that consider the dc ohmic loss, and (4.59) which considers the high frequency ac loss, it can be inferred that using single wires for this design is not appropriate because the combined dc ohmic and high frequency ac losses will exceed the copper loss budget. Therefore, Litz wires that their strands satisfy the $z=\frac{d_{\text {strand }}}{\delta}<1.6$ constraint should be chosen.

### 4.1.15.2 Proximity Effect

The proximity effect results in the poor utilization of the wire cross section area even if a good Litz wire is chosen. This in turn increases the ac loss significantly.

The proximity effect happens not only between adjacent wires but also between the wire and ferrite core. A similar increase in losses appears when wires are tightly wrapped around the core legs and very close to them. This issue becomes more severe near the airgap area because of the fringing effect.


Figure 4.7: Arrangement of $T_{a}$ and $L_{a}$ windings

### 4.1.16 Arrangement of Wires

In this design, and to tackle the proximity effect related to the windings of $T_{a}$, the transformer wires are selected such that the windings will not become more than three layers by utilizing the full length of the middle leg. In addition and to avoid the proximity effect between the windings and ferrite core, a certain distance is provided between the primary winding and middle leg.

The inductor winding is wrapped around the outer legs. To address the proximity effect caused by $L_{a}$, the inductor wire is chosen with the purpose that firstly the winding will not become more than two layers, and secondly there will be enough window area in the core for providing a distance between the winding and core legs which becomes larger closer to the airgap area.

The arrangement of the windings is shown in Fig. 4.7.

### 4.1.17 Selected Wires

According to the available wires, and by considering (4.59) for the strands, and (4.54), (4.55), and (4.56) for the whole wire, the Litz wire with 7 strands of AWG30 $(7 / 30)$ is chosen for the primary winding of $T_{a}$ and $L_{a}$ winding, and the Litz wire with 360 strands of AWG44 (360/44) is chosen for the secondary winding of $T_{a}$.

The rule of thumb in selecting the wires is to calculate their current density which is denoted as $J\left[\mathrm{Am}^{-2}\right]$. A good design will result in a current density of:

$$
\begin{equation*}
3 \mathrm{Am}^{-2}<J<5 \mathrm{Am}^{-2} \tag{4.60}
\end{equation*}
$$

The current density of a wire is defined as:

$$
\begin{equation*}
J=\frac{I_{r m s, i}}{\mathcal{P} \times A_{\text {single }}}, \tag{4.61}
\end{equation*}
$$

where $\mathcal{P}$ is the number of strands, and $A_{\text {single }}$ is the cross sectional area of each strand. The wire current density of $L_{a}$ winding is calculated as:

$$
\begin{equation*}
J_{L}=\frac{I_{\text {La,rms }}}{7 \times A_{\mathrm{AWG} 30}}=4.47 \mathrm{~A} \mathrm{~m}^{-2} \tag{4.62}
\end{equation*}
$$

and the wire current density of the primary winding of $T_{a}$ is obtained as:

$$
\begin{equation*}
J_{P}=\frac{I_{\text {in, rms }}}{7 \times A_{\mathrm{AWG} 30}}=4.07 \mathrm{~A} \mathrm{~m}^{-2} \tag{4.63}
\end{equation*}
$$

and the wire current density of the secondary winding of $T_{a}$ is obtained as:

$$
\begin{equation*}
J_{S}=\frac{I_{C 2, r m s}}{360 \times A_{\mathrm{AWG} 44}}=3.58 \mathrm{Am}^{-2} \tag{4.64}
\end{equation*}
$$

### 4.1.18 Copper Loss

The copper loss of the design includes dc ohmic and high frequency ac losses.

### 4.1.18.1 DC Ohmic Loss

The dc resistance of a winding with Litz wire is obtained as [68]:

$$
\begin{equation*}
R_{d c}=\frac{\rho l_{T} N}{\frac{\pi}{4} d_{\text {single }}^{2} \mathcal{P}} \tag{4.65}
\end{equation*}
$$

where $d_{\text {single }}$ is the diameter of each strand.
Using this formula, the dc resistance of the three windings are obtained as follows:

$$
\begin{align*}
& R_{L, d c}=0.162 \mathrm{~m} \Omega  \tag{4.66}\\
& R_{P, d c}=0.197 \mathrm{~m} \Omega  \tag{4.67}\\
& R_{S, d c}=0.056 \mathrm{~m} \Omega \tag{4.68}
\end{align*}
$$

Then, the dc ohmic loss corresponding to these three windings are obtained as follows:

$$
\begin{align*}
& P_{L, d c}=R_{L, d c} \times I_{L a, r m s}^{2}=0.41 \mathrm{~W} .  \tag{4.69}\\
& P_{P, d c}=R_{P, d c} \times I_{i n, r m s}^{2}=0.414 \mathrm{~W} .  \tag{4.70}\\
& P_{S, d c}=R_{S, d c} \times I_{C 2, r m s}^{2}=0.372 \mathrm{~W} . \tag{4.71}
\end{align*}
$$

### 4.1.18.2 High Frequency AC Loss

The high frequency ac loss of the design is divided between the transformer windings and inductor winding.
4.1.18.2.1 AC Loss of $T_{a}$ To determine the ac copper loss of $T_{a}$, the ac resistance of its windings will be obtained.

The equivalent frequency of transformer windings is defined as [68]:

$$
\begin{equation*}
f_{e q}=f_{s w}\left(\frac{d_{\text {single }}}{0.5 \mathrm{~mm}}\right)^{2}, \tag{4.72}
\end{equation*}
$$

which is used to determine the ac loss of windings.
Using (4.72) and by considering $f_{s w}=80 \mathrm{kHz}$, the equivalent frequency of the primary winding of $T_{a}$ for which AWG30 is selected is calculated as:

$$
\begin{equation*}
f_{e q, P}=80 \mathrm{kHz}\left(\frac{d_{\mathrm{AWG} 30}}{0.5 \mathrm{~mm}}\right)^{2}=20.74 \mathrm{kHz}, \tag{4.73}
\end{equation*}
$$

and the equivalent frequency of the secondary winding for which AWG44 is selected is obtained as:

$$
\begin{equation*}
f_{e q, S}=80 \mathrm{kHz}\left(\frac{d_{\text {AWG44 }}}{0.5 \mathrm{~mm}}\right)^{2}=0.8 \mathrm{kHz} . \tag{4.74}
\end{equation*}
$$

The ac to dc resistance ratio of transformer windings is obtained as follows [68]:

$$
\begin{equation*}
\frac{R_{a c}}{R_{d c}}=m_{E}^{2} k_{t f}=\mathcal{P} m^{2} k_{t f}, \tag{4.75}
\end{equation*}
$$

where $m_{E}$ is the winding's effective number of layers, $m$ is the winding's actual number of layers, and $k_{t f}$ is the transformer factor obtained from the graph shown in Fig. 4.8.


Figure 4.8: Transformer factor $k_{t f}$, [68]; (1) and dotted line for $m_{E}=0.5$, (2) and solid line for $m_{E}=1$, (3) and dashed for $m_{E}=2$, (4) and dash-dot for $m_{E}>2$, LF $=$ low frequency approximation.

For this design, the actual number of layers of the primary winding is $m_{P}=3$, and that of the secondary winding is $m_{S}=2$. Then according to (4.73) and (4.74), and using Fig. 4.8, $k_{t f}$ of both windings will be determined.

After obtaining all the required parameters and according to (4.75), the ac resistance of the primary winding of $T_{a}$ is calculated as:

$$
\begin{equation*}
\frac{R_{P, a c}}{R_{P, d c}} \approx 0.78 \quad \stackrel{(4.67)}{\Longrightarrow} R_{P, a c}=0.153 \mathrm{~m} \Omega, \tag{4.76}
\end{equation*}
$$

and the ac resistance of the secondary winding is calculated as:

$$
\begin{equation*}
\frac{R_{S, a c}}{R_{S, d c}} \approx 0.55 \quad \stackrel{(4.68)}{\Longrightarrow} R_{S, a c}=0.03 \mathrm{~m} \Omega . \tag{4.77}
\end{equation*}
$$

Finally, the ac loss of the primary winding is calculated as:

$$
\begin{equation*}
P_{P, a c}=R_{P, a c} \times I_{i n, r m s}^{2}=0.32 \mathrm{~W} \tag{4.78}
\end{equation*}
$$

and the ac loss of the secondary winding is determined as:

$$
\begin{equation*}
P_{S, a c}=R_{S, a c} \times I_{C 2, r m s}^{2}=0.2 \mathrm{~W} . \tag{4.79}
\end{equation*}
$$

4.1.18.2.2 AC Loss of $L_{a}$ The ac to dc resistance ratio of inductor windings is obtained as follows [68]:

$$
\begin{equation*}
\frac{R_{a c}}{R_{d c}}=\left(\frac{z^{4}}{16}\right)\left(\frac{N^{2} \mathcal{P}^{2} d_{\text {single }}^{2}}{3 w^{2}}\right)\left(\frac{\pi^{2}}{4}\right)\left(\frac{k_{f}}{\sqrt{1+\frac{(z+0.37)^{6}}{1024}}}\right) \tag{4.80}
\end{equation*}
$$

where $w$ is the winding width, and $k_{f}$ is the inductor factor defined as:

$$
\begin{equation*}
k_{f}=\frac{3.5(0.5-\kappa)^{2}+0.69}{\kappa} \tag{4.81}
\end{equation*}
$$

where

$$
\begin{equation*}
\kappa=\frac{d_{\mathrm{wg}}+\frac{t_{\mathrm{w}}}{3}}{\frac{w}{K_{\mathrm{sy}}}} \tag{4.82}
\end{equation*}
$$

in which $d_{\mathrm{wg}}$ is the distance between the winding and core leg, $t_{\mathrm{w}}$ is the winding thickness, and $K_{\text {sy }}$ is the symmetry factor. $K_{\text {sy }}=2$ for a center-gapped inductor.

The importance of the distance between the winding and ferrite core previously discussed in "Proximity Effect" section is validated because according to (4.80),(4.81) and (4.82), by increasing $d_{\text {wg }}$, the ac resistance of the winding decreases.

The approximate arrangement of $L_{a}$ winding is shown in Fig. 4.9.


Figure 4.9: Arrangement of $L_{a}$ winding

The optimum value of $\kappa$ is around 0.6 or more that could not be achieved for this integrated magnetic because of the limited window area. For the constructed inductor $\left(L_{a}\right), \kappa \approx 0.2$ is calculated by using (4.82). Then according to (4.81), the inductor factor is calculated as:

$$
\begin{equation*}
k_{f}=5.025 \tag{4.83}
\end{equation*}
$$

Finally, according to (4.80), and because AWG30 strands are selected with $z=0.96$, at $f_{s w}=80 \mathrm{kHz}$, the ac resistance of $L_{a}$ winding is calculated as:

$$
\begin{equation*}
\frac{R_{L, a c}}{R_{L, d c}}=\left(\frac{z^{4}}{16}\right)\left(\frac{N_{L}^{2} \mathcal{P}^{2} d_{\mathrm{AWG} 30}^{2}}{3 w^{2}}\right)\left(\frac{\pi^{2}}{4}\right)\left(\frac{k_{f}}{\sqrt{1+\frac{(z+0.37)^{6}}{1024}}}\right)=1.068 \quad \stackrel{(4.66)}{\Longrightarrow} R_{L, a c}=0.173 \mathrm{~m} \Omega, \tag{4.84}
\end{equation*}
$$

and the ac copper loss of $L_{a}$ is calculated as:

$$
\begin{equation*}
P_{L, a c}=R_{L, a c} \times I_{L a, r m s}^{2}=0.438 \mathrm{~W} . \tag{4.85}
\end{equation*}
$$

### 4.1.19 Total Calculated Loss

The copper loss of a winding is the summation of dc ohmic and ac losses. According to (4.70), (4.71), (4.78), and (4.79), the copper loss of the primary and secondary windings of $T_{a}$ is calculated respectively as follows:

$$
\begin{align*}
& P_{P, c u}=P_{P, d c}+P_{P, a c}=0.41+0.32=0.73 \mathrm{~W} \leq P_{h, c u, P}=0.73 \mathrm{~W}  \tag{4.86}\\
& P_{S, c u}=P_{S, d c}+P_{S, a c}=0.372+0.2=0.572 \mathrm{~W}<P_{h, c u, S}=0.65 \mathrm{~W} \tag{4.87}
\end{align*}
$$

The calculated copper loss is less than the copper loss budgets calculated in (4.52) and (4.53). Therefore, the design is acceptable.

Similarly, according to (4.85), and (4.69), the copper loss of $L_{a}$ winding is calculated as:

$$
\begin{equation*}
P_{L, c u}=P_{L, d c}+P_{L, a c}=0.41+0.438=0.848 \mathrm{~W} \approx P_{h, c u, L}=0.83 \mathrm{~W} \tag{4.88}
\end{equation*}
$$

The calculated copper loss is almost equal to the copper loss budget calculated in (4.51). Therefore, the design is acceptable.

The total copper loss of the design is then calculated as:

$$
\begin{equation*}
P_{c u}=P_{L, c u}+P_{P, c u}+P_{S, c u}=0.848+0.73+0.572=2.15 \mathrm{~W} . \tag{4.89}
\end{equation*}
$$

Finally, according to (4.46), and (4.89), the total estimated loss of the design including copper and core losses is calculated as:

$$
\begin{equation*}
P_{T a+L a}=P_{f e, \max }+P_{c u}=3.2+2.15=5.35 \mathrm{~W} . \tag{4.90}
\end{equation*}
$$

### 4.1.20 Experimental Prototype

The magnetic design is then constructed as displayed in Fig. 4.10, and the measured parameters are shown in Table. 4.1.

Table 4.1: Experimental prototype parameters

| Parameter | Value |
| :--- | :--- |
| $L_{a}$ | $295 \mu \mathrm{H}$ |
| $L_{m a}$ | $415 \mu \mathrm{H}$ |



Figure 4.10: The constructed prototype of PFC integrated magnetic $\left(T_{a}+L_{a}\right)$

### 4.2 Magnetic Integration of $T_{r}$

To achieve a higher power density, the LLC transformer and inductor are also integrated. The design of this integrated magnetic is similar to the previous one. Ferrite material 3C94, and magnetic core EE30 which is comprised of two E30/15/7 cores placed on each other are selected.

The number of turns of the primary winding of $T_{r}$ is chosen as $N_{r 1}=27$, and using the transformer turns ratio obtained in (3.8), $N_{r 2}=7$ is selected. Furthermore, based on the available Litz wires, the Litz wire with 360 strands of AWG44 (360/44) is chosen for the primary winding of $T_{r}$, and the Litz wire with 1000 strands of AWG46 (1000/46) is chosen for the secondary winding of $T_{r}$.

The estimated total loss of the design including core and copper losses is obtained as:

$$
\begin{equation*}
P_{T r}=P_{f e, \max , l l c}+P_{c u, l l c} \approx 1.125+0.85=1.975 \mathrm{~W} . \tag{4.91}
\end{equation*}
$$

The main difference of this design is that the leakage inductance of $T_{r}$ is utilized as $L_{r}$. Therefore, it must be as large as $80 \mu \mathrm{H}$ according to (3.10). To achieve this, the primary and secondary windings are separated from each other using a magnetic ferrite sheet. The primary winding is wrapped around the upper part of the middle leg, while the secondary winding is wrapped at the bottom, with a ferrite sheet placed between them. The thickness of the ferrite sheet is selected such that the desired leakage inductance is obtained.

The prototype of this magnetic design is constructed as shown in Fig. 4.11, and the measured parameters are shown in Table. 4.2.

Table 4.2: Leakage and magnetizing inductance of the experimental prototype

| Parameter | Value |
| :--- | :--- |
| $L_{r}$ | $80 \mu \mathrm{H}$ |
| $L_{m r}$ | $600 \mu \mathrm{H}$ |



Figure 4.11: The constructed prototype of LLC integrated magnetic $\left(T_{r}\right)$

### 4.3 Summary

This chapter provides optimal magnetic designs for the proposed LED driver. The magnetic integration of $T_{a}$ and $L_{a}$ based on the structure according to which they share the same core without any flux interaction, is investigated. The design flowchart of a magnetic design is followed, and accordingly appropriate core material, core size, number of turns and wires for windings are selected. Finally, the total loss of the design is calculated, and the prototype based on design calculations is constructed and verified in practice.

Following the same design steps, the magnetic integration of $T_{r}$ is also carried out. The primary and secondary windings of $T_{r}$ are separated using a magnetic ferrite sheet to utilize the leakage inductance as $L_{r}$. Finally, the total loss of the design is calculated, and the prototype based on design calculations is constructed and verified in practice.

## Chapter 5

## Design and Analysis

Design and thorough analysis of every power electronics converter is essential to achieve proper performance and avoid iterations in design process and over-design of converters leading to high cost of products.

The PFC inductance $\left(L_{a}\right)$ is designed using the power equation, and the capacitances are designed by allowing $40 \%$ peak-to-peak voltage ripples across them, and using either the amount of power they need to process or their charge balance property. During the design of $C_{2}$, the formula of $i_{C 2}$ and its important values are obtained illustrating why it helps both $S_{1}$ and $S_{2}$ in achieving turn-on ZVS.

The stress and loss analyses of components are also carried out. Based on stress analysis, the voltage and current limits of components are determined, and based on loss analysis, the lowest efficiency of the converter at full-load is calculated. Finally, the efficiency of laboratory prototype in different operating points is measured which is compatible with the calculated expected efficiency.

### 5.1 Design of $L_{a}$

The proposed PFC circuit is shown in Fig. 5.1(a). The inductance of PFC circuit denoted as $L_{a}$ is designed for nominal output power equal to 200 W , and input voltage range of $(185-265) \mathrm{V}_{r m s}$. The value of $L_{a}$ is determined assuming $\eta=1$, where $\eta$ is the converter efficiency.

The input power waveform during a switching period is depicted in Fig. 5.1(b). During the second half of switching period when $S_{2}$ is ON, the input power is zero because $i_{i n}$ is zero, and during the first half when $S_{1}$ is ON, by neglecting the fundamental voltage ripples during a switching period, the shape of input power is similar to the waveform of $i_{\text {in }}$ shown in Fig. 2.3.


Figure 5.1: (a) PFC circuit of the proposed converter, and (b) input power waveform during a switching period

To obtain the average input power denoted as $P_{\text {in }}$, the instantaneous input power denoted as $p_{\text {ave }}$ must be determined which is the average of input power over $T_{s w}$.

During $0<t<\frac{T_{s w}}{2}$ when $S_{1}$ is ON, it can be written that:

$$
\begin{equation*}
v_{\text {rect }}(t)=\left|v_{g}\right|=V_{m}\left|\sin \omega_{l} t\right| \tag{5.1}
\end{equation*}
$$

Then, the maximum of input power during a switching period denoted as $\hat{p}_{i n}$ is determined as:

$$
\begin{equation*}
\hat{p}_{i n}=v_{\text {rect }} \times \hat{i}_{i n} \tag{5.2}
\end{equation*}
$$

where $\hat{i}_{i n}$ is the maximum of input current previously calculated in (2.18).
According to (5.1), (5.2) and (2.18), $\hat{p}_{i n}$ is determined as:

$$
\begin{equation*}
\hat{p}_{i n}=\frac{V_{m}^{2} \sin ^{2} \omega_{l} t}{2 f_{s w} L_{a}} \tag{5.3}
\end{equation*}
$$

and the instantaneous input power is obtained as:

$$
\begin{equation*}
p_{a v e}=\frac{\hat{p}_{i n} \frac{T_{s w}}{2}}{2 T_{s w}} \stackrel{(5.3)}{=} \frac{V_{m}^{2} \sin \omega_{l} t^{2}}{8 f_{s w} L_{a}}=\frac{V_{m}^{2}}{16 f_{s w} L_{a}}\left(1-\cos 2 \omega_{l} t\right) \tag{5.4}
\end{equation*}
$$

The average input power is the average of (5.4) over line period $\left(T_{l}\right)$, and is obtained as:

$$
\begin{equation*}
P_{i n}=\frac{\int_{0}^{T_{l}} \frac{V_{m}^{2}}{16 f_{s w} L_{a}}\left(1-\cos 2 \omega_{l} t\right) d t}{T_{l}}=\frac{V_{m}^{2}}{16 f_{s w} L_{a}}=P_{o u t} \tag{5.5}
\end{equation*}
$$

Finally, from (5.5), $L_{a}$ is obtained in terms of the known parameters as:

$$
\begin{equation*}
L_{a}=\frac{V_{m}^{2}}{16 P_{\text {out }} f_{s w}} \tag{5.6}
\end{equation*}
$$

where since $L_{a}$ is designed for rated power, $P_{\text {out }}=200 \mathrm{~W}$ is considered, and $f_{s w}$ is ranging from 80 kHz to 150 kHz .

Based on (5.6), the PFC inductance range is obtained as:

$$
\begin{gather*}
\frac{1 \times(\sqrt{2} \cdot 185)^{2}}{16 \times 200 \times 80 \mathrm{k}}<L_{a}=\frac{\eta V_{m}^{2}}{16 P_{\text {out }} f_{s w}}<\frac{1 \times(\sqrt{2} \cdot 265)^{2}}{16 \times 200 \times 150 \mathrm{k}} \\
\Longrightarrow 267 \mu \mathrm{H}<L_{a}<293 \mu \mathrm{H} \tag{5.7}
\end{gather*}
$$

The inductance of the experimental prototype is measured as $295 \mu \mathrm{H}$.


Figure 5.2: Input and output power in one line period

### 5.2 Design of $C_{1}$

The input/output power difference during $\left[0, \frac{T_{l}}{4}\right]$ shown in Fig. 5.2 is the energy change of $C_{1}$ denoted as $\Delta E_{C 1}$ which is calculated as:

$$
\begin{align*}
\Delta E_{C 1} & =\int_{0}^{\frac{T_{l}}{4}}\left(p_{\text {ave }}(t)-P_{\text {out }}\right) d t=2 \int_{0}^{\frac{T_{l}}{8}}\left(p_{\text {ave }}(t)-P_{\text {out }}\right) d t \\
& =\frac{1}{2} C_{1}\left(V_{\text {bus }}+\frac{\Delta V_{\text {bus }}}{2}\right)^{2}-\frac{1}{2} C_{1}\left(V_{\text {bus }}-\frac{\Delta V_{\text {bus }}}{2}\right)^{2}, \tag{5.8}
\end{align*}
$$

where $\Delta V_{\text {bus }}$ is the peak-to-peak voltage ripple of $C_{1}$.

Based on (5.4), and (5.5), equation (5.8) can be rewritten as:

$$
\begin{equation*}
2 \int_{0}^{\frac{T_{l}}{8}}\left(\frac{V_{m}^{2}}{16 f_{s w} L_{a}} \cos 2 \omega_{l} t\right) d t=C_{1} V_{b u s} \Delta V_{b u s} \tag{5.9}
\end{equation*}
$$

and $\Delta V_{\text {bus }}$ can be calculated in terms of circuit parameters including $C_{1}$ as:

$$
\begin{equation*}
\Delta V_{b u s}=\frac{V_{m}^{2}}{16 L_{a} f_{s w} \omega_{l} C_{1} V_{b u s}} . \tag{5.10}
\end{equation*}
$$

By accepting peak-to-peak voltage ripples up to $40 \%$ of $V_{\text {bus }}$, it can be written that:

$$
\begin{equation*}
\Delta V_{b u s} \leq 0.4 V_{b u s} \Longrightarrow \frac{V_{m}^{2}}{16 L_{a} f_{s w} \omega_{l} C_{1} V_{b u s}} \leq 0.4 V_{b u s} \tag{5.11}
\end{equation*}
$$

and the minimum value of $C_{1}$ is obtained as:

$$
\begin{equation*}
C_{1} \geq \frac{1}{640 \pi L_{a} f_{s w}}\left(\frac{V_{m}}{V_{b u s}}\right)^{2} . \tag{5.12}
\end{equation*}
$$

According to the $V_{\text {bus }}$ formula obtained in (3.26),

$$
\begin{equation*}
\frac{V_{m}}{V_{b u s}} \leq 1 \tag{5.13}
\end{equation*}
$$

Then, (5.12) is rewritten as:

$$
\begin{equation*}
C_{1} \geq \frac{1}{640 \pi L_{a} f_{s w}} \tag{5.14}
\end{equation*}
$$

based on which the value of $C_{1}$ is decided.
For this converter, $L_{a}=295 \mu \mathrm{H}$ is selected, and $80 \mathrm{kHz} \leq f_{s w} \leq 150 \mathrm{kHz}$. Therefore, $C_{1}$ is calculated as:

$$
\begin{equation*}
C_{1} \geq 21.074 \mu \mathrm{~F} \tag{5.15}
\end{equation*}
$$

A $30 \mu \mathrm{~F}$ polyester film capacitor is selected as $C_{1}$.

### 5.3 Design of $C_{o}$

Due to the design of a fast controller, low frequency ripples will not appear on LEDs. Therefore, four $4.7 \mu \mathrm{~F}$ polyester film capacitors are chosen as output capacitors.

### 5.4 Design of $C_{2}$

The converter circuit is shown in Fig. 5.3(a) where each winding of $T_{a}$ is modelled with an inductor in series with a dependent source, $v_{C 2}$ contains relatively large switching frequency ripples due to the current shape of this capacitor denoted as $i_{C 2}$ shown in Fig. 5.3(b) during a switching period.


Figure 5.3: (a) The proposed converter with equivalent circuit of $T_{a}$, and (b) waveform of $i_{C 2}$ during a switching period

During the first half of switching period when $S_{1}$ is ON, by writing a KVL, and according to the waveform of $i_{i n}$ shown in Fig. 2.3, (2.18) and (2.2), it can be written that:

$$
\begin{align*}
v_{C 2} & +\frac{L_{m a}}{T_{s w}}\left(\hat{i}_{C 2}-i_{C 2,0}\right)-\frac{L_{m a}}{L_{a}} V_{m}\left|\sin \omega_{l} t\right|-v_{b u s}=0 \\
& \Longrightarrow \hat{i}_{C 2}-i_{C 2,0}=\frac{v_{b u s}}{2 f_{s w} L_{m a}}+\frac{V_{m}\left|\sin \omega_{l} t\right|}{f_{s w} L_{a}} \tag{5.16}
\end{align*}
$$

based on which the slope of $i_{C 2}$ during the first half of switching period is obtained as:

$$
\begin{equation*}
m_{1}=\frac{v_{b u s}}{L_{m a}}+\frac{2 V_{m}\left|\sin \omega_{l} t\right|}{L_{a}} . \tag{5.17}
\end{equation*}
$$

During the second half of switching period when $S_{2}$ is $\mathrm{ON}, i_{i n}$ is clamped to zero. By writing a KVL, and according to (2.2), it can be written that:

$$
\begin{equation*}
v_{C 2}+\frac{L_{m a}}{T_{s w}}\left(i_{C 2,0}-i_{C 2,1}\right)=0 \Longrightarrow i_{C 2,0}-i_{C 2,1}=\frac{-v_{b u s}}{2 f_{s w} L_{m a}}, \tag{5.18}
\end{equation*}
$$

based on which the slope of $i_{C 2}$ during the second half of switching period is obtained as:

$$
\begin{equation*}
m_{2}=\frac{-v_{b u s}}{L_{m a}} \tag{5.19}
\end{equation*}
$$

Considering Fig. 5.3(b), the following equations can be written:

$$
\left\{\begin{array}{l}
\frac{\frac{T_{s w}}{2}-t_{\alpha}}{t_{\alpha}}=\frac{\hat{i}_{C 2}}{-i_{C 2,0}}  \tag{5.20}\\
\frac{t_{\beta}-\frac{T_{s w}}{2}}{T_{s w}-t_{\beta}}=\frac{i_{C 2,1}}{-i_{C 2,0}}
\end{array}\right.
$$

based on which $t_{\alpha}$ and $t_{\beta}$ are determined in terms of $T_{s w}, \hat{i}_{C 2}, i_{C 2,0}$, and $i_{C 2,1}$.
Then according to charge balance property of $C_{2}$, it can be written that:

$$
\begin{equation*}
A_{r 1}+A_{r 2}+A_{r 3}+A_{r 4}=0 \tag{5.21}
\end{equation*}
$$

which by solving and using (5.20), $\hat{i}_{C 2}, i_{C 2,0}$, and $i_{C 2,1}$ are calculated respectively as follows:

$$
\begin{align*}
& \hat{i}_{C 2}=\left(\frac{v_{b u s}}{L_{m a}}+\frac{3 V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right) \frac{T_{s w}}{4} .  \tag{5.22}\\
& i_{C 2,0}=-\left(\frac{v_{b u s}}{L_{m a}}+\frac{V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right) \frac{T_{s w}}{4} .  \tag{5.23}\\
& i_{C 2,1}=\left(\frac{v_{b u s}}{L_{m a}}-\frac{V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right) \frac{T_{s w}}{4} . \tag{5.24}
\end{align*}
$$

It is worth mentioning that $i_{C 2,0}$ helps in achieving turn-on ZVS for $S 1$, and $i_{C 2,1}$ helps in achieving turn-on ZVS for $S_{2}$.

The formula of $i_{C 2}$ during the first half of switching period according to (5.22) and (5.23), is obtained as:

$$
\begin{equation*}
i_{C 2}(t)=\left(\frac{v_{b u s}}{L_{m a}}+\frac{2 V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right) t-\left(\frac{v_{b u s}}{L_{m a}}+\frac{V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right) \frac{T_{s w}}{4} \tag{5.25}
\end{equation*}
$$

and during the second half of switching period according to (5.24) and (5.23), is obtained as:

$$
\begin{equation*}
i_{C 2}(t)=\frac{-v_{b u s}}{L_{m a}}\left(t-\frac{T_{s w}}{2}\right)+\left(\frac{v_{b u s}}{L_{m a}}-\frac{V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right) \frac{T_{s w}}{4} . \tag{5.26}
\end{equation*}
$$

The area $A r_{2}+A r_{3}$ shown on Fig. 5.3 is the total charge stored in $C_{2}$ during a switching period which is calculated as:

$$
\begin{equation*}
Q_{C 2}=A r_{2}+A r_{3}=\left(\frac{T_{s w}}{8}\right)^{2} \frac{\left(\frac{v_{b u s}}{L_{m a}}+\frac{V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right)^{3}}{\left(\frac{v_{b u s}}{2 L_{m a}}+\frac{V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right)\left(\frac{v_{b u s}}{2 L_{m a}}\right)}=C_{2} \Delta v_{C 2} \tag{5.27}
\end{equation*}
$$

If similar to the design of $C_{1}, \Delta v_{C 2}<0.4 v_{C 2}$ is assumed, then according to (2.2),

$$
\begin{equation*}
\Delta v_{C 2}<0.2 v_{\text {bus }} \tag{5.28}
\end{equation*}
$$

and using (5.27), the minimum value of $C_{2}$ is obtained as:

$$
\begin{equation*}
C_{2}>\frac{T_{s w}^{2}}{6.4 k_{C 2}^{2}} \frac{L_{m a}\left(\frac{k_{C 2}}{L_{m a}}+\frac{1}{L_{a}}\right)^{3}}{\frac{1}{L_{a}}+\frac{k_{C 2}}{2 L_{m a}}} \tag{5.29}
\end{equation*}
$$

where

$$
\begin{equation*}
k_{C 2}=\frac{v_{\text {bus }}}{V_{m}\left|\sin \omega_{l} t\right|} . \tag{5.30}
\end{equation*}
$$

Since $k_{C 2}>1$ and the right term of (5.29) decreases by increasing $k_{C 2}$, the worst case scenario is considered assuming $k_{C 2}=1$, and (5.29) can be rewritten as:

$$
\begin{equation*}
C_{2}>\frac{T_{s w}^{2}}{6.4} \frac{L_{m a}\left(\frac{1}{L_{m a}}+\frac{1}{L_{a}}\right)^{3}}{\frac{1}{L_{a}}+\frac{1}{2 L_{m a}}} \tag{5.31}
\end{equation*}
$$

According to the value of $L_{m a}$ and $L_{a}$ obtained in (4.37) and (5.7) respectively, and by considering $f_{s w}=80 \mathrm{kHz}$, the minimum capacitance of $C_{2}$ is calculated as:

$$
\begin{equation*}
C_{2}>430 \mathrm{nF} \tag{5.32}
\end{equation*}
$$

To further decrease the ripples, and since the price difference of film capacitors is negligible for below $1 \mu \mathrm{~F}$ capacitors, a $1 \mu \mathrm{~F}$ polypropylene film capacitor is selected as $C_{2}$.

### 5.5 Stress Analysis

### 5.5.1 Voltage Stress of $S_{1}$ and $S_{2}$

The maximum voltage of either $S_{1}$ or $S_{2}$ can be written as:

$$
\begin{equation*}
V_{S, \max }=V_{b u s}+\frac{\Delta V_{b u s}}{2} \tag{5.33}
\end{equation*}
$$

and since $\Delta V_{\text {bus }} \leq 0.4 V_{\text {bus }}$ is assumed, it can be written that:

$$
\begin{equation*}
V_{S, \max } \leq 1.2 V_{b u s} \tag{5.34}
\end{equation*}
$$

According to (3.27), the voltage stress of switches is calculated as:

$$
\begin{equation*}
V_{S, \max } \leq 1.2 V_{b u s, \max } \leq 525 \mathrm{~V} \tag{5.35}
\end{equation*}
$$

This is the maximum voltage that $S_{1}$ and $S_{2}$ must withstand for the safe operation. For a similar boost-typed PFC converter operating in DCM with acceptable PF and THD, the voltage stress of switches exceeds 900 V . This is a significant advantage of the proposed LED driver that 600 V rating power MOSFETs can be chosen which are more cost effective than the higher voltage rating ones.

### 5.5.2 Voltage Stress of Input Diode Bridge and $D_{a}$

The maximum voltage of input diode bridge during a switching period happens at Interval VII that is depicted in Fig. 5.4(a). Since $v_{L a}(t)=0$ during this interval, and by writing two KVLs, the output voltage of input diode bridge denoted as $v_{\text {rect }}$ is determined as follows:

$$
\begin{gather*}
v_{C 2}-\frac{v_{M}(t)}{2}=0 \Longrightarrow v_{M}(t)=v_{b u s}, \quad \text { (Interval VII) }  \tag{5.36}\\
v_{\text {rect }}(t)=v_{M}(t)+v_{\text {bus }} \stackrel{(5.36)}{=} 2 v_{\text {bus }} . \quad \text { (Interval VII) } \tag{5.37}
\end{gather*}
$$

The maximum voltage of $D_{a}$ during a switching period happens at Interval III shown in Fig. 5.4(b). By writing two KVLs, the voltage of $D_{a}$ at this interval is determined as follows:

$$
\begin{gather*}
v_{C 2}-\frac{v_{M}(t)}{2}-v_{b u s}=0 \Longrightarrow v_{M}(t)=-v_{b u s}, \quad \text { (Interval III) }  \tag{5.38}\\
v_{D a}(t)=v_{\text {rect }}(t)-v_{M}(t) \stackrel{(5.38) \operatorname{and}(5.1)}{=} V_{m}\left|\sin \omega_{l} t\right|+v_{\text {bus }} . \quad \text { (Interval III) } \tag{5.39}
\end{gather*}
$$



Figure 5.4: (a) Interval VII, and (b) Interval III of the converter operation

Given the peak-to-peak dc bus voltage ripples that are below $40 \%$, and according to (5.37), the voltage stress of input diode bridge can be obtained as:

$$
\begin{equation*}
V_{\text {rect }, \max } \leq 1.2 \times 2 V_{\text {bus }}=2.4 V_{\text {bus }}, \tag{5.40}
\end{equation*}
$$

which according to (3.27), is calculated as:

$$
\begin{equation*}
V_{\text {rect }, \text { max }} \leq 1 \mathrm{kV} . \tag{5.41}
\end{equation*}
$$

Similarly, according to (5.39), and since $\left|\sin \omega_{l} t\right| \leq 1$, the voltage stress of $D_{a}$ is calculated as:

$$
\begin{equation*}
V_{D a, \max } \leq 1.2 V_{\text {bus }}+V_{m}<900 \mathrm{~V} \tag{5.42}
\end{equation*}
$$

### 5.5.3 Voltage Stress of Output Rectifying Diodes

By writing a KVL, the voltage of each output rectifying diode when it does not conduct can be obtained as:

$$
\begin{equation*}
V_{D r}=-2 V_{o}=-100 \mathrm{~V} \tag{5.43}
\end{equation*}
$$

### 5.5.4 Current Stress of $S_{1}$ and $S_{2}$

The current of switches can be written as follows:

$$
\begin{align*}
& i_{S 1}(t)= \begin{cases}i_{C 2}(t)+i_{L r}(t) & 0<t<\frac{T_{s w}}{2} \\
0 & \frac{T_{s w}}{2}<t<T_{s w}\end{cases}  \tag{5.44}\\
& i_{S 2}(t)= \begin{cases}0 & 0<t<\frac{T_{s w}}{2} \\
-i_{C 2}(t)-i_{L r}(t) & \frac{T_{s w}}{2}<t<T_{s w}\end{cases} \tag{5.45}
\end{align*}
$$

The maximum of $\left|i_{\mathrm{C} 2}(t)+i_{\mathrm{Lr}}(t)\right|$ happens at $t=\frac{T_{s w}}{2}$.
According to (5.25) and (5.22), the value of $i_{C 2}$ at $t=\frac{T_{s w}}{2}$ is determined as:

$$
\begin{equation*}
i_{C 2}\left(t=\frac{T_{s w}}{2}\right)=\hat{i}_{C 2}=\left(\frac{v_{b u s}}{L_{m a}}+\frac{3 V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right) \frac{T_{s w}}{4} \tag{5.46}
\end{equation*}
$$

In addition, by ignoring the dc bus voltage ripples, the Fourier series formula of $i_{L r}$ is obtained as:

$$
\begin{equation*}
i_{L r}(t)=\frac{4 n_{r} V_{o} \omega_{s} C_{r}}{\pi} \sum_{\substack{k=1 \\ o d d}}^{\infty} \frac{\cos \left(k \omega_{s} t-k \theta_{s}\right)}{(k F)^{2}-1}-\frac{2 V_{b u s} \omega_{s} C_{r}}{\pi} \sum_{\substack{k=1 \\ o d d}}^{\infty} \frac{\cos k \omega_{s} t}{(k F)^{2}-1} \tag{5.47}
\end{equation*}
$$

For simplicity, the two series are approximated with their first terms, and (5.47) is simplified as follows:

$$
\begin{gather*}
i_{L r}(t) \approx \frac{8 n_{r} f_{s w} C_{r} V_{o}}{F^{2}-1} \cos \left(\omega_{s} t-\theta_{s}\right)-\frac{4 V_{b u s} f_{s w} C_{r}}{F^{2}-1} \cos \omega_{\mathrm{s}} t \\
=\frac{8 n_{r} f_{s w} C_{r} V_{o} \cos \theta_{s}-4 V_{b u s} f_{s w} C_{r}}{F^{2}-1} \cos \omega_{s} t+\frac{8 n_{r} f_{s w} C_{r} V_{o} \sin \theta_{s}}{F^{2}-1} \sin \omega_{s} t \tag{5.48}
\end{gather*}
$$

and its value at $t=\frac{T_{s w}}{2}$ is calculated as:

$$
\begin{equation*}
i_{L r}\left(t=\frac{T_{s w}}{2}\right)=\frac{4 f_{s w} C_{r}\left(V_{b u s}-2 n_{r} V_{o} \cos \theta_{s}\right)}{F^{2}-1} \tag{5.49}
\end{equation*}
$$

According to (5.44), (5.45), (5.22), (5.49), and if a perfect switching with no deadtime is considered, the maximum current of either $S_{1}$ or $S_{2}$ during a switching period is obtained as:

$$
\begin{gather*}
\hat{i}_{S}=\left|\hat{i}_{S 1}\right|=\left|\hat{i}_{S 2}\right|= \\
\left|\left(\frac{v_{b u s}}{L_{m a}}+\frac{3 V_{m}\left|\sin \omega_{l} t\right|}{L_{a}}\right) \frac{T_{s w}}{4}+\frac{4 f_{\mathrm{sw}} C_{\mathrm{r}}\left(V_{\mathrm{bus}}-2 n_{r} V_{o} \cos \theta_{s}\right)}{F^{2}-1}\right| . \tag{5.50}
\end{gather*}
$$

Finally, if for simplicity, the dc bus voltage ripples are neglected, and $\left|\sin \omega_{l} t\right|=1$ is considered, the maximum current of either $S_{1}$ or $S_{2}$ will be obtained as:

$$
\begin{equation*}
I_{S, \max }=\left|\left(\frac{V_{b u s}}{L_{m a}}+\frac{3 V_{m}}{L_{a}}\right) \frac{T_{s w}}{4}+\frac{4 f_{s w} C_{r}\left(V_{b u s}-2 n_{r} V_{o} \cos \theta_{s}\right)}{F^{2}-1}\right| . \tag{5.51}
\end{equation*}
$$

For the worst case scenario in which $f_{s w}=80 \mathrm{kHz}, V_{m}=262 \mathrm{~V}$, and $P_{\text {out }}=200 \mathrm{~W}$, and by calculating $V_{\text {bus }}$ from (3.26), and $\cos \theta_{s}$ from (3.20) and (3.21), the current stress of each switch is calculated as:

$$
\begin{equation*}
I_{S, \max } \approx 9.8 \mathrm{~A} \tag{5.52}
\end{equation*}
$$

### 5.5.5 Current Stress of Input Diode Bridge

The output current of input diode bridge is denoted as $i_{i n}$ which its maximum during a switching period denoted as $\hat{i}_{i n}$ was previously calculated in (2.18). By considering $\left|\sin \omega_{l} t\right|=1$, the current stress of input diode bridge is determined as:

$$
\begin{equation*}
I_{i n, \max }=\frac{V_{m}}{2 f_{s w} L_{a}} . \tag{5.53}
\end{equation*}
$$

This current at $V_{m}=262 \mathrm{~V}, f_{s w}=80 \mathrm{kHz}$, and $P_{\text {out }}=200 \mathrm{~W}$, is calculated as:

$$
\begin{equation*}
I_{i n, \max }=5.55 \mathrm{~A} . \tag{5.54}
\end{equation*}
$$

### 5.5.6 Current Stress of $D_{a}$

The current stress of $D_{a}$ is similar to input diode bridge, and it can be written that:

$$
\begin{equation*}
I_{D a, \max }=I_{L_{a}, \max }=I_{i n, \max } \tag{5.55}
\end{equation*}
$$

For the worst case scenario, the current stress of $D_{a}$ is calculated as:

$$
\begin{equation*}
I_{D, \max }=5.55 \mathrm{~A} . \tag{5.56}
\end{equation*}
$$

### 5.5.7 Current Stress of Output Rectifying Diodes

The maximum current of each output rectifying diode can be written as:

$$
\begin{equation*}
I_{D r, \max }=\left|\frac{n_{r} I_{L r, \max }}{2}\right|, \tag{5.57}
\end{equation*}
$$

which using (5.48), is obtained as:

$$
\begin{equation*}
I_{L r, \max }=\frac{4 f_{s w} C_{r}}{F^{2}-1} \sqrt{V_{b u s}^{2}+\left(2 n_{r} V_{o}\right)^{2}} \stackrel{(5.57)}{\Longrightarrow} I_{D r, \text { max }}=\left|\frac{2 n_{r} f_{s w} C_{r}}{F^{2}-1}\right| \sqrt{V_{b u s}^{2}+\left(2 n_{r} V_{o}\right)^{2}}, \tag{5.58}
\end{equation*}
$$

and at $V_{m}=262 \mathrm{~V}, f_{s w}=80 \mathrm{kHz}$, and $P_{\text {out }}=200 \mathrm{~W}$, is calculated as:

$$
\begin{equation*}
I_{D r, \max }=7.94 \mathrm{~A} \tag{5.59}
\end{equation*}
$$

### 5.6 Loss Analysis

To verify the feasibility of the converter, the prototype of the proposed LED driver is built as shown in Fig.5.5, and the selected components are listed in Table. 5.1.


Figure 5.5: (a) The experimental prototype, and (b) operating temperature after half an hour

Table 5.1: System parameters

| Input End |  |  |
| :---: | :---: | :---: |
| Line Frequency | $f_{l}$ | 50 Hz |
| Input Voltage (rms) | $V_{g, r m s}$ | 185 V to 265 V |
| Input Filter |  |  |
| Capacitance | $C_{f}$ | 470 nF |
| Inductance | $L_{f}$ | $L_{f}=1 \mathrm{mH}$ |
| PFC Circuit |  |  |
| Inductance | $L_{a}$ | $295 \mu \mathrm{H}$ |
| Transformer | $T_{a}$ | $L_{m a}=415 \mu \mathrm{H}$ |
| Bus Capacitance | $C_{1}$ | $30 \mu \mathrm{~F}$ |
| Half-bridge LLC Resonant Circuit |  |  |
| Switching Frequency | $f_{s w}$ | 80 kHz to 150 kHz |
| Resonant Capacitance | $C_{r}$ | 18.1 nF |
| Half-bridge Capacitance | $C_{2}$ | $1 \mu \mathrm{~F}$ |
| Transformer | $T_{r}$ | $L_{m r}=600 \mu \mathrm{H}$ and $L_{r}=80 \mu \mathrm{H}$ |
| Rest of the Circuit |  |  |
| Balancing Capacitance | $C_{b 1}-C_{b 3}$ | $4.7 \mu \mathrm{~F}$ |
| Output Capacitance | $C_{o 1}-C_{o 4}$ | $4.7 \mu \mathrm{~F}$ |
| Output End |  |  |
| Nominal Power | $P_{\text {out }}$ | 200 W |
| Output Voltage | $V_{o}$ | 50 V |
| Output Current | $I_{o}$ | 0.1 A to 1 A |

The loss analysis is carried out at $V_{m}=262 \mathrm{~V}$, and $P_{\text {out }}=200 \mathrm{~W}$ which corresponds to $f_{s w}=80 \mathrm{kHz}$. This operating point is expected to yield the lowest efficiency at rated power since the conduction losses at this operating pont are relatively higher.

### 5.6.1 Loss of $S_{1}$ and $S_{2}$

Owing to the design of half-bridge LLC resonant circuit, ZVS is obtained for both $S_{1}$ and $S_{2}$. Therefore, the switching loss of both MOSFETs is negligible. The conduction loss of $S_{1}$ and $S_{2}$ is calculated as follows:

$$
\begin{align*}
& P_{S 1}=r_{D S, o n} I_{r m s, S 1}^{2},  \tag{5.60}\\
& P_{S 2}=r_{D S, o n} I_{r m s, S 2}^{2}, \tag{5.61}
\end{align*}
$$

where $r_{D S, o n}$ is the ON resistance of MOSFETs.

The rms current of $S_{1}$ is obtained as:

$$
\begin{equation*}
I_{r m s, S 1}=\frac{1}{2} \sqrt{2 \sum_{k=o d d}\left(1+\cos \xi_{k}\right) J_{k}^{2}-\sum_{k=\text { even }} H_{k}^{2}} \tag{5.62}
\end{equation*}
$$

where

$$
\left\{\begin{array}{l}
H_{k}=\frac{V_{\text {bus }}}{k \pi f_{s w} L_{m a}}+\frac{2 V_{m}}{k \pi f_{s w} L_{a}},  \tag{5.63}\\
J_{k}=\frac{V_{m}}{k \pi f_{s w} L_{a}}, \\
\cos \xi_{k}=\frac{2 n_{r} V_{o} \sin k \theta_{s}}{\sqrt{\left(2 n_{r} V_{o} \cos k \theta_{s}-V_{\text {bus }}\right)^{2}+\left(2 n_{r} V_{o} \sin k \theta_{s}\right)^{2}}}
\end{array}\right.
$$

By replacing $V_{\text {bus }}$ from (3.22), $\cos \xi_{k}$ can be rewritten as:

$$
\begin{equation*}
\cos \xi_{k}=\frac{\sin k \theta_{s}}{\sqrt{\left.\left(\cos k \theta_{s}-\frac{8.5-\frac{1}{F^{2}}}{7.5}\right)\right)^{2}+\left(\sin k \theta_{s}\right)^{2}}} \tag{5.64}
\end{equation*}
$$

By considering the selected operating point, $\theta_{s}=33^{\circ}$ from (3.18), and $F=0.6$ are obtained, and it can be approximated that:

$$
\begin{equation*}
2 \sum_{k=o d d}\left(1+\cos \xi_{k}\right) J_{k}^{2} \approx 4 \sum_{k=o d d} J_{k}^{2} \tag{5.65}
\end{equation*}
$$

Finally, by calculating the series of (5.62), the rms current of $S_{1}$ is determined as:

$$
\begin{equation*}
I_{r m s, S 1} \approx \frac{1}{4 \sqrt{6} f_{s w}} \sqrt{12\left(\frac{V_{m}}{L_{a}}\right)^{2}-\left(\frac{2 V_{m}}{L_{a}}+\frac{V_{b u s}}{L_{m a}}\right)^{2}} . \tag{5.66}
\end{equation*}
$$

Similarly, and by writing the Fourier series of $I_{r m s, S 2}$, the rms current of $S_{2}$ is obtained as:

$$
\begin{equation*}
I_{r m s, S 2} \approx \frac{1}{4 \sqrt{6} f_{s w}} \sqrt{12\left(\frac{V_{m}}{L_{a}}\right)^{2}-\left(\frac{V_{b u s}}{L_{m a}}\right)^{2}} . \tag{5.67}
\end{equation*}
$$

For the considered operating point, the rms currents are calculated as:

$$
\begin{align*}
& I_{r m s, S 1} \approx 2.3 \mathrm{~A} .  \tag{5.68}\\
& I_{r m s, S 2} \approx 3.8 \mathrm{~A} . \tag{5.69}
\end{align*}
$$

UJ3C065030B3 power MOSFETs are used for this converter with $r_{D S, o n}=35 \mathrm{~m} \Omega$. Using (5.60), and (5.61), the conduction loss of $S_{1}$ is determined as:

$$
\begin{equation*}
P_{S 1} \approx 0.185 \mathrm{~W} \tag{5.70}
\end{equation*}
$$

and the conduction loss of $S_{2}$ is calculated as:

$$
\begin{equation*}
P_{S 2} \approx 0.5 \mathrm{~W} \tag{5.71}
\end{equation*}
$$

### 5.6.2 Loss of Input Diode Bridge

Due to the existence of switching harmonics at input of the converter, four individual fast recovery diodes with $t_{r r}=75 \mathrm{~ns}$, and $V_{F}=1.2 \mathrm{~V}$ are utilized as diode bridge.

The current of each diode is the half-wave rectified of input current. By assuming that the resistance of diodes are negligible, the power loss of each of them is obtained as:

$$
\begin{equation*}
P_{D b} \approx \frac{2 P_{\text {out }} V_{F}}{\pi \eta V_{m}} \tag{5.72}
\end{equation*}
$$

which for the considered operating point, and assuming $\eta=0.95$, is calculated as:

$$
\begin{equation*}
P_{D b} \approx 0.61 \mathrm{~W} . \tag{5.73}
\end{equation*}
$$

### 5.6.3 Loss of $D_{a}$

The same diode as input bridge is selected as $D_{a}$ with $V_{F}=1.2 \mathrm{~V}$.
According to (2.13), and by neglecting the dc bus voltage ripples, the duration that $D_{a}$ conducts is determined as:

$$
\begin{equation*}
\Delta t_{d}=t_{6}-t_{4}=\frac{V_{m}}{2 f_{\text {sw }} V_{\text {bus }}}\left|\sin \omega_{l} t\right| \tag{5.74}
\end{equation*}
$$

In addition, according to (5.74), (5.55), and (2.18), the average current of $D_{a}$ is obtained. Finally, by assuming that the diode resistance is negligible, the loss of $D_{a}$ is determined as:

$$
\begin{equation*}
P_{D a} \approx \frac{V_{m}^{2} V_{F}}{16 f_{s w} L_{a} V_{b u s}}, \tag{5.75}
\end{equation*}
$$

which for the considered operating point, is calculated as:

$$
\begin{equation*}
P_{D a} \approx 0.76 \mathrm{~W} . \tag{5.76}
\end{equation*}
$$

### 5.6.4 Loss of Output Rectifying Diodes

Four fast recovery diodes with $t_{r r}=25 \mathrm{~ns}$, and $V_{F}=0.64 \mathrm{~V}$ are chosen as output rectifying diodes to avoid reverse recovery losses. As a result, the power loss of each diode is determined as:

$$
\begin{equation*}
P_{D r} \approx I_{o} V_{F}=\frac{P_{o u t} V_{F}}{4 V_{o}} \tag{5.77}
\end{equation*}
$$

which for the considered operating point, is calculated as:

$$
\begin{equation*}
P_{D r} \approx 0.64 \mathrm{~W} . \tag{5.78}
\end{equation*}
$$

### 5.6.5 The Lowest Efficiency at Full-Load

The total loss of the converter can be written as:

$$
\begin{equation*}
P_{l o s s}=P_{S 1}+P_{S 2}+4 P_{D b}+P_{D a}+4 P_{D r}+P_{T a+L a}+P_{T r}, \tag{5.79}
\end{equation*}
$$

where the loss of PFC integrated magnetic calculated in (4.90) is denoted as $P_{T a+L a}$, and $P_{T r}$ is the loss of LLC integrated magnetic calculated in (4.91).

The efficiency of the converter can be written as:

$$
\begin{equation*}
\eta=\frac{P_{\text {out }}}{P_{\text {out }}+P_{\text {loss }}} \tag{5.80}
\end{equation*}
$$

which for the considered operating, is calculated as:

$$
\begin{equation*}
\eta=93.56 \% \tag{5.81}
\end{equation*}
$$

This is the lowest efficiency at full-load. The measured efficiency of the experimental prototype at this operating point is lower than expected, but at rated power and nominal input voltage, $\eta=93.3 \%$.

### 5.7 Experimental Results

Using a Tektronix 5 series Mixed Signal Oscilloscope model MS058, the efficiency of the converter for different operating points have been measured as illustrated in Fig.5.6 and Fig.5.7 . Since the converter has four output channels, it was not possible to measure the efficiency using a two-channel power analyzer.


Figure 5.6: Efficiency of the prototype with respect to input voltage


Figure 5.7: Efficiency of the prototype with respect to output power

### 5.8 Summary

This chapter performs design calculations to obtain the PFC inductance value, and capacitance values. The formula of $i_{C 2}$ is also obtained which helps in achieving ZVS more comfortably. In addition, a comprehensive stress analysis is carried out to obtain the maximum current and voltage of every component. This helps in choosing appropriate components for the LED driver that can tolerate the expected voltage and current levels. The loss analysis is also provided that calculates the expected loss of MOSFETs and diodes. Based on the calculated losses, the lowest efficiency of the converter at full-load is then calculated. At the end, by using a Tektronix 5 series Mixed Signal Oscilloscope model MS058, the efficiency of the experimental prototype at different operating points is measured and presented.

## Chapter 6

## Summary and Future Work

### 6.1 Summary of Contributions

In this thesis, the analysis and design of a 200 W streetlighting LED driver are carried out, and different challenges associated with streetlighting LED drivers are addressed. Furthermore, the simulation and experimental results of the design are provided. The main contributions of the proposed LED driver are listed below:

1. An integrated single-stage LED driver is proposed which is suitable for input voltage range of $(185-265) \mathrm{V}_{r m s}$ with low THD, high PF, low output ripples, and limited dc bus voltage that lowers the voltage stress of MOSFETs.
2. A novel buck-boost typed PFC circuit is proposed in which the transformer of PFC circuit acts like the buck-boost switch, and the problem associated with other DCM buck-boost PFC circuits is resolved.
3. The control system of the proposed converter is designed such that low frequency ripples are removed from the output. As a result, electrolytic capacitors are eliminated which improves the lifespan of the LED driver.
4. The inductor and transformer of PFC circuit are integrated on the same core where the inductor winding is wrapped around the outer legs to achieve high efficiency.

### 6.2 Suggested Improvements

The proposed LED driver can be still improved in some areas that are presented as follows:

1. The actual PF and THD of the experimental prototype can pass the required standards, but they can be furthered improved. The leakage inductance of PFC transformer is largely responsible for not achieving the best PF and THD as not only does it affect the falling time of input current in each switching period, but it also resonates with the parasitic capacitance of $D_{a}$ that appears on input current. The problem can be addressed by interleaving the transformer windings more effectively to decrease the leakage inductance, and by designing an optimized snubber circuit for the diode.
2. Because of high frequency content of diode currents, and due to using SiC MOSFETs with low rising and falling times, Electro Magnetic Interference (EMI) noises exist in the circuit that must be filtered to comply with EMC requirements. Therefore, designing an appropriate EMI filter is recommended.
3. In the proposed LED driver, PFM is used to regulate the output. However, variable switching frequency operation makes it challenging to design the EMI filter. As an improvement, the Pulse Width Modulation (PWM) technique can be used to regulate the output by keeping the switching frequency constant.
4. It was shown in "Stress Analysis" section of Chapter 5 that the voltage stress of input diode bridge, and $D_{a}$ is nearly twice the dc bus voltage. Other alternative topologies or circuit modifications can be derived to reduce this voltage stress.
5. The obtained efficiency of the converter is relatively high. However, a bridgeless topology can improve the efficiency by removing some of the input diodes.
6. The power density of the converter is relatively high. However, further improvement is possible by increasing the switching frequency. This is less challenging for the proposed circuit because owing to the current of the secondary winding of PFC transformer, ZVS is achieved more comfortably in the proposed converter than in a regular LLC resonant converter.

## Bibliography

[1] T. Komine and M. Nakagawa, "Fundamental analysis for visible-light communication system using led lights," IEEE Transactions on Consumer Electronics, vol. 50, no. 1, pp. 100-107, 2004.
[2] J. Y. Tsao, "Solid-state lighting: lamps, chips, and materials for tomorrow," IEEE Circuits and Devices Magazine, vol. 20, no. 3, pp. 28-37, 2004.
[3] M. S. Shur and R. Zukauskas, "Solid-state lighting: Toward superior illumination," Proceedings of the IEEE, vol. 93, no. 10, pp. 1691-1703, 2005.
[4] P. H. Pathak, X. Feng, P. Hu, and P. Mohapatra, "Visible light communication, networking, and sensing: A survey, potential and challenges," IEEE Communications Surveys Tutorials, vol. 17, no. 4, pp. 2047-2077, 2015.
[5] H. Chiu and S. Cheng, "Led backlight driving system for large-scale lcd panels," IEEE Transactions on Industrial Electronics, vol. 54, no. 5, pp. 2751-2760, 2007.
[6] A. Farahat, A. Florea, J. L. M. Lastra, C. Branas, and F. J. A. Sánchez, "Energy efficiency considerations for led-based lighting of multipurpose outdoor environments," IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 3, no. 3, pp. 599-608, 2015.
[7] D. Gacio, J. Cardesin, E. L. Corominas, J. M. Alonso, M. Dalla-Costa, and A. J. Calleja, "Comparison among power leds for automotive lighting applications," in 2008 IEEE Industry Applications Society Annual Meeting, pp. 1-5, 2008.
[8] M. Cole and T. Driscoll, "The lighting revolution: If we were experts before, we're novices now," IEEE Transactions on Industry Applications, vol. 50, no. 2, pp. 1509-1520, 2014.
[9] I. Belyakova, V. Medvid, V. Piscio, O. Shkodzinsky, R. Mykhailyshyn, and M. Markovych, "Usage of light-emitting-diode lamps in decorative lighting," in

2019 IEEE 20th International Conference on Computational Problems of Electrical Engineering (CPEE), pp. 1-4, 2019.
[10] M. Zalesinska, S. Szwedek, and A. Pawlak, "Evaluation of lighting parameters at the workplace with the use replacements for incandescent lamps," in 2018 VII. Lighting Conference of the Visegrad Countries (Lumen V4), pp. 1-5, 2018.
[11] S. Dian, X. Wen, X. Deng, and S. Zhang, "Digital control of isolated cuk power factor correction converter under wide range of load variation," IET Power Electronics, vol. 8, no. 1, pp. 142-150, 2014.
[12] J. Baek and S. Chae, "Single-stage buck-derived led driver with improved efficiency and power factor using current path control switches," IEEE Transactions on Industrial Electronics, vol. 64, no. 10, pp. 7852-7861, 2017.
[13] Y. Liu, F. Syu, H. Hsieh, K. A. Kim, and H. Chiu, "Hybrid switched-inductor buck pfc converter for high-efficiency led drivers," IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 65, no. 8, pp. 1069-1073, 2018.
[14] I. Castro, K. Martin, A. Vazquez, M. Arias, D. G. Lamar, and J. Sebastian, "An ac-dc pfc single-stage dual inductor current-fed push-pull for hb-led lighting applications," IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 6, no. 1, pp. 255-266, 2018.
[15] X. Liu, Q. Yang, Q. Zhou, J. Xu, and G. Zhou, "Single-stage single-switch fouroutput resonant led driver with high power factor and passive current balancing," IEEE Transactions on Power Electronics, vol. 32, no. 6, pp. 4566-4576, 2017.
[16] B. Lehman and A. J. Wilkins, "Designing to mitigate effects of flicker in led lighting: Reducing risks to health and safety," IEEE Power Electronics Magazine, vol. 1, no. 3, pp. 18-26, 2014.
[17] A. Wilkins, J. Veitch, and B. Lehman, "Led lighting flicker and potential health concerns: Ieee standard par1789 update," in 2010 IEEE Energy Conversion Congress and Exposition, pp. 171-178, 2010.
[18] L. CAO, Y. ZHU, and H. WU, "A new electrolytic capacitor-less led driver with coupled-inductor," in 2020 IEEE Applied Power Electronics Conference and Exposition (APEC), pp. 1537-1543, 2020.
[19] M. K. Barwar, L. K. Sahu, P. Bhatnagar, K. K. Gupta, and A. H. Chander, "A decoupled low-frequency ripple cancellation method for high-power led driver
circuits," in IECON 2020 The 46th Annual Conference of the IEEE Industrial Electronics Society, pp. 3597-3602, 2020.
[20] M. Arias, D. G. Lamar, F. F. Linera, D. Balocco, A. Aguissa Diallo, and J. Sebastián, "Design of a soft-switching asymmetrical half-bridge converter as second stage of an led driver for street lighting application," IEEE Transactions on Power Electronics, vol. 27, no. 3, pp. 1608-1621, 2012.
[21] Y. Wang, S. Gao, S. Zhang, and D. Xu, "A two-stage quasi-resonant dual-buck led driver with digital control method," IEEE Transactions on Industry Applications, vol. 54, no. 1, pp. 787-795, 2018.
[22] S. Pervaiz, A. Kumar, and K. K. Afridi, "A compact electrolytic-free two-stage universal input offline led driver with volume-optimized ssc energy buffer," IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 6, no. 3, pp. 1116-1130, 2018.
[23] T. Liu, X. Liu, M. He, S. Zhou, X. Meng, and Q. Zhou, "Flicker-free resonant led driver with high power factor and passive current balancing," IEEE Access, vol. 9, pp. 6008-6017, 2021.
[24] Q. He, Q. Luo, K. Ma, P. Sun, and L. Zhou, "Analysis and design of a singlestage bridgeless high-frequency resonant ac/ac converter," IEEE Transactions on Power Electronics, vol. 34, no. 1, pp. 700-711, 2019.
[25] A. Abasian, H. Farzanehfard, and S. A. Hashemi, "A single-stage single-switch soft-switching (s6) boost-flyback pfc converter," IEEE Transactions on Power Electronics, vol. 34, no. 10, pp. 9806-9813, 2019.
[26] J. Liu, H. Tian, G. Liang, and J. Zeng, "A bridgeless electrolytic capacitor-free led driver based on series resonant converter with constant frequency control," IEEE Transactions on Power Electronics, vol. 34, no. 3, pp. 2712-2725, 2019.
[27] Y. Wang, X. Deng, Y. Wang, and D. Xu, "Single-stage bridgeless led driver based on a clcl resonant converter," IEEE Transactions on Industry Applications, vol. 54, no. 2, pp. 1832-1841, 2018.
[28] A. Malschitzky, F. Albuquerque, E. Agostini, and C. B. Nascimento, "Singlestage integrated bridgeless-boost nonresonant half-bridge converter for led driver applications," IEEE Transactions on Industrial Electronics, vol. 65, no. 5, pp. 3866-3878, 2018.
[29] S. Mangkalajan, C. Ekkaravarodome, K. Jirasereeamornkul, P. Thounthong, K. Higuchi, and M. K. Kazimierczuk, "A single-stage led driver based on zcds class-e current-driven rectifier as a pfc for street-lighting applications," IEEE Transactions on Power Electronics, vol. 33, no. 10, pp. 8710-8727, 2018.
[30] Q. Luo, J. Huang, Q. He, K. Ma, and L. Zhou, "Analysis and design of a singlestage isolated ac-dc led driver with a voltage doubler rectifier," IEEE Transactions on Industrial Electronics, vol. 64, no. 7, pp. 5807-5817, 2017.
[31] Y. Wang, N. Qi, Y. Guan, C. Cecati, and D. Xu, "A single-stage led driver based on sepic and llc circuits," IEEE Transactions on Industrial Electronics, vol. 64, no. 7, pp. 5766-5776, 2017.
[32] Y. Wang, J. Huang, W. Wang, and D. Xu, "A single-stage single-switch led driver based on class-e converter," IEEE Transactions on Industry Applications, vol. 52, no. 3, pp. 2618-2626, 2016.
[33] C. Chang, H. Chen, C. Cho, and J. Chiu, "A novel single-stage llc resonant ac-dc converter with power factor correction feature," in 2011 6th IEEE Conference on Industrial Electronics and Applications, pp. 2191-2196, 2011.
[34] Y. Li and C. Chen, "A novel single-stage high-power-factor ac-to-dc led driving circuit with leakage inductance energy recycling," IEEE Transactions on Industrial Electronics, vol. 59, no. 2, pp. 793-802, 2012.
[35] F. Zhang, J. Ni, and Y. Yu, "High power factor ac-dc led driver with film capacitors," IEEE Transactions on Power Electronics, vol. 28, no. 10, pp. 48314840, 2012.
[36] B. Wang, X. Ruan, K. Yao, and M. Xu, "A method of reducing the peak-toaverage ratio of led current for electrolytic capacitor-less ac-dc drivers," IEEE Transactions on Power Electronics, vol. 25, no. 3, pp. 592-601, 2009.
[37] G. G. Pereira, M. A. Dalla Costa, J. M. Alonso, M. F. De Melo, and C. H. Barriquello, "Led driver based on input current shaper without electrolytic capacitor," IEEE Transactions on Industrial Electronics, vol. 64, no. 6, pp. 4520-4529, 2017.
[38] Q. Hu and R. Zane, "Minimizing required energy storage in off-line led drivers based on series-input converter modules," IEEE Transactions on Power Electronics, vol. 26, no. 10, pp. 2887-2895, 2010.
[39] D. Camponogara, G. F. Ferreira, A. Campos, M. A. Dalla Costa, and J. Garcia, "Offline led driver for street lighting with an optimized cascade structure," IEEE Transactions on Industry Applications, vol. 49, no. 6, pp. 2437-2443, 2013.
[40] Y. Yang, X. Ruan, L. Zhang, J. He, and Z. Ye, "Feed-forward scheme for an electrolytic capacitor-less ac/dc led driver to reduce output current ripple," IEEE Transactions on Power electronics, vol. 29, no. 10, pp. 5508-5517, 2013.
[41] M. K. Barwar, L. K. Sahu, P. Bhatnagar, K. K. Gupta, and A. H. Chander, "A decoupled low-frequency ripple cancellation method for high-power led driver circuits," in IECON 2020 The 46th Annual Conference of the IEEE Industrial Electronics Society, pp. 3597-3602, IEEE, 2020.
[42] J. He, X. Ruan, and L. Zhang, "Adaptive voltage control for bidirectional converter in flicker-free electrolytic capacitor-less ac-dc led driver," IEEE Transactions on Industrial Electronics, vol. 64, no. 1, pp. 320-324, 2016.
[43] P. Fang, B. Sheng, S. Webb, Y. Zhang, and Y.-F. Liu, "Led driver achieves electrolytic capacitor-less and flicker-free operation with an energy buffer unit," IEEE Transactions on Power Electronics, vol. 34, no. 7, pp. 6777-6793, 2018.
[44] H. Valipour, G. Rezazadeh, and M. R. Zolghadri, "Flicker-free electrolytic capacitor-less universal input offline led driver with pfc," IEEE Transactions on Power Electronics, vol. 31, no. 9, pp. 6553-6561, 2015.
[45] P. Fang, Y.-F. Liu, and P. C. Sen, "A flicker-free single-stage offline led driver with high power factor," IEEE Journal of Emerging and Selected Topics in Power Electronics, vol. 3, no. 3, pp. 654-665, 2015.
[46] H. Dong, X. Xie, L. Jiang, Z. Jin, and X. Zhao, "An electrolytic capacitor-less high power factor led driver based on a "one-and-a-half stage" forward-flyback topology," IEEE Transactions on Power Electronics, vol. 33, no. 2, pp. 15721584, 2017.
[47] G. Z. Abdelmessih, J. M. Alonso, N. d. S. Spode, and M. A. Dalla Costa, "Electrolytic-capacitor-less off-line led driver based on integrated parallel buckboost and boost converter," in 2020 IEEE Industry Applications Society Annual Meeting, pp. 1-7, IEEE, 2020.
[48] S. A. J. Khajehoddin, Resonant micro-inverters for single-phase grid-connected photovoltaic systems. PhD thesis, Queen's University, 2015.
[49] M. Arias, M. F. Diaz, D. G. Lamar, D. Balocco, A. A. Diallo, and J. Sebastián, "High-efficiency asymmetrical half-bridge converter without electrolytic capacitor for low-output-voltage ac-dc led drivers," IEEE transactions on Power Electronics, vol. 28, no. 5, pp. 2539-2550, 2012.
[50] H. Ma, G. Chen, J. H. Yi, Q. W. Meng, L. Zhang, and J. P. Xu, "A singlestage pfm-apwm hybrid modulated soft-switched converter with low bus voltage for high-power led lighting applications," IEEE Transactions on Industrial Electronics, vol. 64, no. 7, pp. 5777-5788, 2017.
[51] H. Ma, J.-S. Lai, C. Zheng, and P. Sun, "A high-efficiency quasi-single-stage bridgeless electrolytic capacitor-free high-power ac-dc driver for supplying multiple led strings in parallel," IEEE Transactions on Power Electronics, vol. 31, no. 8, pp. 5825-5836, 2015.
[52] S. A. Khajehoddin, A. Bakhshai, and P. Jain, "A novel topology and control strategy for maximum power point trackers and multi-string grid-connected pv inverters," in 2008 Twenty-Third Annual IEEE Applied Power Electronics Conference and Exposition, pp. 173-178, IEEE, 2008.
[53] S. A. Khajehoddin, A. Bakhshai, and P. Jain, "Inverter for a distributed power generator, US Patent 8,670,249," 2014.
[54] M. Rahmani, "High power long lifetime led drivers," 2018.
[55] S. Li and S. R. Hui, "Self-configurable current-mirror circuit with short-circuit and open-circuit fault tolerance for balancing parallel light-emitting diode (led) string currents," IEEE Transactions on Power Electronics, vol. 29, no. 10, pp. 5498-5507, 2013.
[56] Y. Yu, F. Zhang, and J. Ni, "Capacitor clamped current-sharing circuit for multistring leds," IEEE Transactions on Industrial Electronics, vol. 61, no. 5, pp. 2423-2431, 2013.
[57] J. Zhang, T. Jiang, and X. Wu, "A high-efficiency quasi-two-stage led driver with multichannel outputs," IEEE Transactions on Industrial Electronics, vol. 64, no. 7, pp. 5875-5882, 2017.
[58] M. Khatib, "Ballast resistor calculation-current matching in parallel leds," Texas Instrument Application Report SLVA325, 2009.
[59] W. Thomas and J. Pforr, "A novel low-cost current-sharing method for automotive led-lighting systems," in 2009 13th European Conference on Power Electronics and Applications, pp. 1-10, IEEE, 2009.
[60] S. M. Baddela and D. S. Zinger, "Parallel connected leds operated at high to improve current sharing," in Conference Record of the 2004 IEEE Industry Applications Conference, 2004. 39th IAS Annual Meeting., vol. 3, pp. 1677-1681 vol.3, 2004.
[61] X. Liu, X. Li, Q. Zhou, and J. Xu, "Flicker-free single switch multi-string led driver with high power factor and current balancing," IEEE Transactions on Power Electronics, vol. 34, no. 7, pp. 6747-6759, 2018.
[62] X. Liu, Y. Wan, Z. Dong, M. He, Q. Zhou, and K. T. Chi, "Buck-boost-bucktype single-switch multistring resonant led driver with high power factor and passive current balancing," IEEE Transactions on Power Electronics, vol. 35, no. 5, pp. 5132-5143, 2019.
[63] X. Wu, J. Zhang, and Z. Qian, "A simple two-channel led driver with automatic precise current sharing," IEEE Transactions on Industrial Electronics, vol. 58, no. 10, pp. 4783-4788, 2011.
[64] A. Luminaires, "Energy star® program requirements for solid state lighting luminaires eligibility criteria-version 1.0," 2008.
[65] I. E. Commission et al., "Electromagnetic compatibility (emc)—part 3-2: Limits for harmonic current emissions (equipment input current $<16$ a per phase)," tech. rep., IEC 61000-3-2, 2005.
[66] C.-A. Cheng, C.-H. Chang, T.-Y. Chung, and F.-L. Yang, "Design and implementation of a single-stage driver for supplying an led street-lighting module with power factor corrections," IEEE Transactions on Power Electronics, vol. 30, no. 2, pp. 956-966, 2014.
[67] H. Huang, "Designing an llc resonant half-bridge power converter," in 2010 Texas Instruments Power Supply Design Seminar, SEM1900, Topic, vol. 3, pp. 20102011, 2010.
[68] A. V. d. Bossche and V. C. Valchev, Inductors and transformers for power electronics. Taylor and Francis Group, LLC, 2005.
[69] R. W. Erickson and D. Maksimovic, Fundamentals of power electronics. Springer Science \& Business Media, 2007.
[70] "E 42/21/20 core and accessories." https://www.tdk-electronics.tdk.com/ inf/80/db/fer/e_42_21_20.pdf. Accessed: 2021-03-19.
[71] "3c94 material specification." https://www.ferroxcube.com/upload/media/ product/file/MDS/3c94.pdf. Accessed: 2021-03-19.

## Appendix A

## Current Balancing

Since the output light of an LED is directly related to its forward current, the luminance uniformity of an LED fixture with several LEDs is guaranteed only if the LEDs have the same amount of current. To have a high standard lighting environment, equalizing or balancing the current of different LEDs is crucial for streetlighting fixtures. Current balancing is guaranteed for a string of LEDs connected in series. However, connecting LEDs in series, especially for high power streetlighting applications where hundreds of LEDs are required, increases the output voltage of driver. This will cause the LED driver not to comply with required safety standards. In addition, with this arrangement, if one of the LEDs breaks, the whole LED string turns OFF which this causes a significant reduction in reliability, and lifetime of the fixture.

Multiple LED strings connected in parallel with an acceptable output voltage is the best possible configuration for LED streetlighting fixtures. Due to the manufacturing process of LEDs, the forward voltage, and equivalent series impedance of LEDs are different that results in current imbalance even if the terminal voltage of each LED string is the same. This leads to unacceptable inconsistency in output light, as well as reduction in life expectancy of LEDs. Therefore, using a current balancing scheme is essential.

Different methods have been proposed in literature for current balancing among which devising a scheme based on the charge balance property of capacitors yields the best result because capacitive configurations are almost lossless, and not very complicated.

## A. 1 The Adopted Current Balancing Scheme

The current balancing scheme of the proposed LED driver is adopted from [63] that has two LED strings. Therefore, its current balancing scheme is extended to accommodate a four-output LED driver as depicted in Fig. A.1.


Figure A.1: The adopted current balancing circuit [63]

Four output rectifying diodes, and three balancing capacitors are used in this structure with no additional inductors. Due to the charge-balance property of balancing capacitors $\left(C_{b 1}, C_{b 2}\right.$, and $\left.C_{b 3}\right)$, the complete current balancing of LED pair $D_{o 1}$ and $D_{o 2}$, as well as LED pair $D_{o 3}$ and $D_{o 4}$ is achieved. This technique does not guarantee the complete current balancing between either of the LEDs at top ( $D_{o 1}$ or $D_{o 2}$ ) with the LEDs at bottom $\left(D_{o 3}\right.$ or $\left.D_{o 4}\right)$. However, their current imbalance is negligible.

## A. 2 Modes of Operation

(For this part, and for the waveforms depicted on Fig. A.1, one of the operating points in which the LLC resonant converter operates in above resonance $\left(f_{s w}>f_{r}\right)$ is considered)

Depending on the ON/OFF state of MOSFETs, two operating modes are conceivable.

## A.2.1 Mode I

When $S_{1}$ is ON, and according to (3.7), the secondary voltage of LLC transformer is determined as follows:

$$
\begin{equation*}
v_{L m r}(t)=V_{L m r}^{+}=n_{r} V_{o} \Longrightarrow v_{n r 2}(t)=\frac{1}{n_{r}} \times n_{r} V_{o}=V_{o} \tag{A.1}
\end{equation*}
$$

Since $v_{n r 2}(t)$ is positive, $D_{2}$ and $D_{4}$ conduct, whereas $D_{1}$ and $D_{3}$ are OFF.

## A.2.2 Mode II

When $S_{2}$ is ON, the secondary voltage of LLC transformer is determined as follows:

$$
\begin{equation*}
v_{L m r}(t)=-n_{r} V_{o} \Longrightarrow v_{n r 2}(t)=\frac{1}{n_{r}} \times\left(-n_{r} V_{o}\right)=-V_{o} \tag{A.2}
\end{equation*}
$$

Since $v_{n r 2}(t)$ is negative, $D_{1}$ and $D_{3}$ conduct, whereas $D_{2}$ and $D_{4}$ are OFF.

## A. 3 Voltage of Balancing Capacitors

For this design, equal capacitances $\left(C_{b 1}=C_{b 2}=C_{b 3}\right)$ is considered. By writing a KCL, and considering the symmetry of circuit, it can be written that:

$$
\begin{equation*}
i_{n r 2}(t)=i_{C b 2}(t)+i_{C b 3}(t) \quad C_{b 1}=C_{b 2}=C_{b 3} v_{C b 1}(t)=2 v_{C b 2}(t)=2 v_{C b 3}(t) . \tag{A.3}
\end{equation*}
$$

In addition, a KVL can be written as follows:

$$
\begin{equation*}
v_{n r 2}(t)-v_{C b 1}(t)-v_{C b 2}(t)-V_{o}-v_{D 2}(t)=0 \tag{A.4}
\end{equation*}
$$

where $v_{n r 2}(t)$ is a square wave signal with the positive value of $V_{o}$ during the first half of switching period, and the negative value of $-V_{o}$ during the second half, and $v_{D 2}(t)$ is a square wave signal with zero value at the first half of switching period, and the negative value of $-2 V_{o}$ during the second half.

By writing the Fourier series of $v_{n r 2}$, and $v_{D 2}$ which for simplicity only their first terms are considered, and according to (A.3), and (A.4), the voltage of balancing capacitors is obtained as follows:

$$
\begin{gather*}
\frac{4 V_{o}}{\pi} \sin \omega_{s} t-3 v_{C b 2}(t)-V_{o}-\left(\frac{4 V_{o}}{\pi} \sin \omega_{s} t-V_{o}\right)=0 \\
\Longrightarrow v_{C b 2}(t)=v_{C b 3}(t)=v_{C b 1}(t)=0 . \tag{A.5}
\end{gather*}
$$

In practice, the non idealities of diodes, leakage inductance of secondary winding, and resistance of PCB traces result in a non-zero, yet small ac voltage across the three capacitors.

## A. 4 Simulation and Experimental Results

Fig. A. 2 shows the simulation result of the current balancing circuit in which a moderate difference is considered in the characteristics of four LEDs. As one can see, the current balancing between the two LEDs at top, and the two LEDs at bottom is obtained. However, current balancing is not achieved between any of the LEDs at top with any of the LEDs at bottom, but the amount of their current imbalance is negligible.


Figure A.2: Simulation result showing the output currents

The output currents of the experimental prototype are shown in Fig.A. 3 where the average of each output current is measured. The current balancing between the four outputs is satisfactory.


Figure A.3: Experimental result showing the output currents

## A. 5 Summary

This chapter gives insight into the importance of current balancing in LED drivers, and why it is essential to devise a current balancing scheme for multiple LED strings connected in parallel. The adopted current balancing scheme is introduced, and the modes of operation are discussed. Finally, simulation and experimental results of the four outputs are provided showing an acceptable and below $10 \%$ difference between the four output currents.

