University of Alberta

Effects of climate change and uncertainty on timber benefits and optimal harvest decisions using risk-programming models

By

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ABSTRACT

This study incorporates the combined effects of climate change and yield uncertainty into estimates of the expected value and variance of future stand yields and net benefits for six management prescription options. These estimations provide the input data for three distinct mathematical programming based risk models that are in turn used to assess the effects of climate change and yield uncertainty on total economic returns and optimal harvest patterns for a stylized, 1000 hectare aspen forest located in central Alberta. The risk model formulations include a Markowitz minimum variance model, an expected value/variance – chance constraint hybrid model, and a discrete stochastic programming (recourse) model. All other factors equal, the impacts of climate change are positive for aspen timber management in central Alberta up to the year 2070. This result holds even when increased costs associated with climate risk are accounted for. A notable result is that climate risk accounts for only 25 % of the standard deviation in timber returns from the hypothetical forest. The remainder is due to variance in yield parameters. When compared to a baseline of normal climate and no uncertainty, objective function values are lower when both climate effects and yield parameter variances are included. However, if the decision maker is able to eliminate yield uncertainty in the first period, certainty equivalent values are higher than the baseline – meaning that the effects of climate change may be positive conditional on certain management response. The analysis also shows that if recourse is not permitted, solutions that permit harvesting to occur are not feasible under current sustained yield policy

regimes. Thus, AAC should not be viewed as a single target harvest volume that ensures sustained yield into the future. Rather AAC could be viewed as a decision tree representing a range of future possible harvests that are contingent on the realization of particular states of nature through the planning horizon. Flexibility in long term planning will be increasingly important for successful adaptation to not only climate but other factors that contribute to risk and uncertainty in timber management.

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CHAPTER ONE

INTRODUCTION

"With the <u>*possible*</u> *exception of the equator, everything begins somewhere"*
P.R. Fleming

Climate change and uncertainty

Although there is general agreement about global warming (IPCC 2001), there is uncertainty about the timing, magnitudes and pattern of climate change and related impacts. Henderson-Sellers (1993) and Jones (2000) refer to an "uncertainty explosion" embedded within the various stages (climate→biological →socioeconomic) of climate change impact assessments. Schneider (1983) and New and Hulme (2000) describe cascading levels of uncertainty with prediction uncertainty being compounded at each phase of impact assessment. Sources of uncertainty about impacts are described as follows. First, there is uncertainty about the magnitude and pattern of future global warming. This uncertainty is due to various factors including uncertainty in emission scenarios, climate sensitivity, strength of aerosol forcing, ocean heat uptake and mixing, and carbon cycle feedbacks (Wigley and Raper 2001). Second, there is uncertainty about how natural systems may respond to climate change. Third, there is uncertainty about how individuals, social groups, socioeconomic systems and institutions will respond to climate change (both directly and indirectly as a result of changes in natural systems and global markets) (Arrow et al. 1996).

Economic implications of climate change in forestry

Change in climatic conditions could influence productivity, growth, mortality, species distributions, disturbance frequency, plantation success and performance, disturbance intensity and the age class structure of Canadian forests (Saporta et al. 1998; Beaulieu and Rainville 2005). The expected values of key variables used in timber supply and economic analysis (such as stand yield coefficients) are anticipated to change. Given that there is uncertainty in future climate, there is also uncertainty about the future value of variables such as timber stand yields.

A number of US based studies have considered the impact of climate change on the forest sector at national and global levels (e.g. see Sohngen and Mendelsohn 1999; Perez Garcia et al. 2002). These models incorporate uncertainty by combining different general circulation model outputs with different ecosystem models and then comparing the different outcomes of different combinations of models. This scenario approach addresses uncertainty by showing outcome frontiers. These studies do not, however, consider how individual preferences for risk might be a determining factor relative to net benefits, landowner choices, and timber supply under a changing climate at local levels.

The previous two paragraphs suggest three things. First, since there is a relationship between forests and climate, climate change will affect forests at both the stand level (due to changes in growth and yield) and the landscape level (due to changes in disturbance patterns and species distributions). Second, because there is no way of knowing what the magnitude and pattern of future climate change will be with certainty – there is no way of predicting with certainty the impact on forest yields and structure. Thus, the future values of measures used in forest economic analysis (such as stand yield) are uncertain. Third, economic theory predicts that uncertainty affects optimal choices (e.g. optimal harvest timing choices). Therefore, to the extent that climate change is a source of uncertainty, we might expect that not only is there an economic cost associated with this effect but that the existence of uncertainty and/or changes in the level of uncertainty will influence the choices that a rational decision maker might make if he/she had; (a) knowledge of the risks, and (b) the flexibility to adapt. Thus, uncertainty, in and of itself, may have implications for timber supply at local levels.

Another consideration relative to understanding the economic implications of climate change in a Canadian forest management context is that much of Canada's forest land base is publicly owned and managed in order to achieve sustained yield objectives[1]. There are, however, opportunity costs associated with sustained yield and these opportunity costs will likely be affected by changes in climate and by uncertainty. Moreover, changes in climate and in uncertainty about forestry yields and benefits are likely to influence optimal harvest schedules under a sustained yield management regime. Thus, the institutional context for forest management may also have important implications for the benefits of forest management and optimal harvest plans.

Analytical context

This study investigates how climate change and uncertainty affects economic returns from timber management and the optimal harvest for an individual private firm managing a public forest subject to regulatory constraints (e.g. sustained yield constraints). The approach involves the creation of a stylized hypothetical central Alberta located forest of aspen (populus tremuloides Michx.). Although the forest is stylized, the yield functions are based on regression analysis using actual mensuration data for aspen sites across western Canada.

The objective of the firm is to maximize the expected value of benefits from harvesting while accounting for risk. The manager is also required to fulfill other objectives (i.e. sustained yield objectives) and these additional objectives are incorporated as constraints. This study will focus on aspen management because deciduous species are becoming increasingly valuable as a feedstock for various forest products in Alberta, including oriented strand board and chemical-thermal mechanical pulps. Studies by forest scientists also indicate that aspen productivity and health will be affected by climate change (Hogg, 1994)[2].

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[1] Even though forest management in Canada is moving toward sustainable forest management, sustained yield is still an important objective in forest management (Luckert and Williamson 2005).
[2] The focus on aspen in this study is not intended to imply that deciduous trees will be affected more or less than coniferous trees. Coniferous species are also economically important and will also be affected by climate change. The magnitude and nature of the effects on forests in general and specific tree species will likely vary from location to location.

Analytical objectives

Forest management provides a unique context for climate change risk and uncertainty analysis. First, significant and sometimes irreversible investments are made by society and landowners to establish forest capital. Second, forests have long growth cycles and the period that forest capital is vulnerable to environmental change is significant. Uncertainty about future yields may have important implications for socially optimal investment and harvest sequences.

This study will evaluate how the net benefit of forest harvesting changes when uncertainty (or risk), expected productivity effects and risk attitudes are explicitly incorporated into a timber supply model. A comparison of the present value of timber harvests with, and without, climate change effects provides a measure of the economic impact of climate change at a local forest management unit level. The first requirement is to obtain estimates of the current values for yield and net benefits, and predictions of the distributions of the future values of random variables (i.e. yield coefficients and net benefit coefficients). Thus, the study will consider how climate change, climate uncertainty, and general uncertainty affects the optimal value of choice variables (e.g. the optimal harvest area over time) and levels of economic benefits. A second issue that will be considered is to evaluate the economic implications of regulatory requirements (such as those associated with sustained yield) in a climate change context.

Methodological objectives

The first methodological objective of this study is to estimate a stand yield function that incorporates climate variables. Spittlehouse and Stewart (2003) suggest that the estimation of yield models that incorporate climate variables as predictor variables is an activity that could be immediately undertaken in order to begin adapting to climate change. The yield model will be used to predict current yield (using historical climate data) and future yield (using climate scenario data).

Uncertainty associated with climate change implies that a number of the coefficients in the timber supply / risk models will be random variables. The second methodological objective of the study is to evaluate uncertainty in harvest yields, ending inventory yields, and net benefit coefficients. The yield functions described in the previous paragraph will be incorporated into a Monte Carlo simulation model framework in order to generate sample distributions for the random variable coefficients required by the economic models.

The third methodological objective of this study is to develop, solve and compare different types of timber supply / risk models. There are three main risk model formulations that will be considered: 1. Variance minimization (i.e. a Markowitz asset allocation model), 2. Expected utility (or certainty equivalent maximization), and 3. Discrete stochastic programming (or recourse) models. These risk models use different approaches for dealing with uncertainty. The goal is to evaluate the strengths and weaknesses of each approach and the specific kinds of problems and questions that each approach is best suited to address.

In summary, there are three methodological objectives and one analytical objective. The objectives of this study are:

1. To estimate a yield model that describes functional relationships between timber yield and non-traditional yield function variables (e.g. climate variables) as well as traditional yield function variables (age, site index, stand density).
2. To employ Monte Carlo simulation to generate distributions for harvest yields, ending inventory, and net benefit values with climate change.
3. To develop, solve, compare and evaluate different types of linear and non-linear risk programming / timber supply optimization models (variance minimization, expected utility, discrete stochastic programming).
4. The general analytical objective involves conducting economic analysis:

a) To quantify and evaluate the effects of climate change and uncertainty on net benefits and optimal decisions when productivity effects, risk (or yield coefficient uncertainty) and risk preferences are incorporated into the objective function, and

b) To evaluate the degree to which climate change affects economic returns both with and without sustained yield constraints imposed.

Organization of the thesis

The theoretical basis for the analysis for this study is provided by decision theory or expected utility theory. Chapter two provides an overview of expected utility theory and related concepts and theories that pertain to the economics of risk and uncertainty.

Chapter three provides a literature review of climate change impacts on forests, and uncertainty in climate and forestry analysis. This chapter also identifies gaps in our knowledge of climate change and forestry impacts and of methodologies for impact assessment at local scales.

The data requirements for the analysis in this study include: (1) cross sectional / stand level yield and climate data across a range of sites with varying climates (for estimation of a variable density yield function), (2) climate scenario data for the study site (for prediction), and (3) other site-specific information (e.g. site index and soil features) also required for yield prediction. Chapter four describes data sources and methods for data collection and generation.

The overall methodology employed in this study involves the integration of three analytical techniques including regression analysis, Monte Carlo simulation, and mathematical programming. Chapter five provides an overview of how these various techniques are linked in order to provide a methodological approach for assessing the impacts of climate change on optimal harvest levels and forest benefits.

The first step in assessing the effects of climate change on optimal harvest (and the consequent economic impacts) for our stylized forestry case study is to estimate empirical yield equations that incorporate climate variables. A number of alternative functional forms for yield prediction models were estimated and evaluated using regression analysis. Chapter six reports the results of these various estimations and identifies a specific functional form for use as a prediction model for future yields under future climate conditions.

The next step in assessing the economic impact of climate change is to estimate statistical distributions for the future predicted values of random variables that are required for the economic models used to determine impacts. However, when random variables are functions of other random variables, the estimation of their distributions can become intractable using analytical approaches. Monte Carlo simulation methods provide a straightforward, convenient, and accepted means of estimating the distributions of continuous random variables in cases where random variables are functions of other random variables. The specific random variables required for the economic models are harvest yield, ending inventory yield, and net benefits. Chapter seven describes the methods used to estimate predictions of the future distributions of these random variables and presents the results of this analysis.

A key objective of this study is to develop and illustrate an approach for understanding the economic impact of climate change at a local forest management unit scale and to estimate how climate change affects optimal harvest. Three distinct economic risk models are developed, solved and compared. They include a portfolio variance (or risk) minimization approach (Markowitz asset allocation), a certainty equivalent maximization / chance constraint hybrid risk model, and a discrete stochastic programming (recourse) model. The structure and solutions for each of these three economic models is described in chapter 8, 9 and 10. These chapters include analysis of the impacts of climate change at a local forest management unit scale along with analysis of the implication of climate change on the social opportunity costs of constraints. Chapter 11 compares and contrasts the various risk models and provides a summary of findings. Also, areas of future research are identified.

CHAPTER TWO

AN OVERVIEW OF THE ECONOMICS OF RISK AND UNCERTAINTY

"The only certainty is uncertainty"
Pliny the Elder, AD 23-79

Introduction

The risk models developed in this study assume that agents are rational. The assumption of rational behaviour essentially means that we assume that decision makers can, and do, successfully maximize (or minimize) some objective function (e.g. utility, profits, expenditures or costs) subject to constraints (e.g. budget, expenditures, resource constraints, etc.)[3]. Economic theory and models often assume that decision makers are rational. Moreover, in many cases economic theories are premised on the assumption that agents are certain about the outcomes of their decisions. However, in some situations, a decision maker may be uncertain about the outcome of his/her decisions. In these cases theory and models that are premised on assumptions of certainty of outcomes are likely to provide disappointing results relative to predictions of behavior, choice and utility (Arrow and Lind 1970). Social science models and theories have, however, been extended to account for the effects of uncertainty on equilibrium conditions and on the behaviors of individuals and social groups (Robison and Barry 1987). In some cases these theories and models maintain the assumption of rationality (e.g. expected utility theory) while in other cases the assumption of rationality is relaxed (e.g. bounded rationality is assumed). This chapter provides an overview of some of the approaches and theories that have been developed in the social sciences for characterizing uncertainty and for understanding human behaviors and responses to uncertainty.

[3] This in turn implies that preferences are complete, reflexive, transitive, and continuous (Binger and Hoffman 1998).

Expected utility theory

von Neumann and Morgenstern (1944) and Savage (1954) introduced and developed expected utility theory in order to explain and describe behavior under uncertainty. Expected utility theory assumes that agents are rational. The principal underlying expected utility is that individuals facing uncertainty in outcomes will attempt to maximize expected utility subject to constraints where expected utility is defined as:

$$E[U] = \sum_{i=1}^{n} p_i U(x_i) \qquad (1)$$

Where;

$U(x_i)$ = utility associated with outcome i

p_i = probability of outcome i

Expected utility and the welfare effects of uncertainty are influenced by both the risk preferences of individuals and by the variance (or dispersion) of possible outcomes. Three categories of risk preferences are risk aversion, risk neutrality, and risk seeking preferences. Figure 2.1 shows a utility function characterizing risk-averse consumer preferences. The utility function for a risk-averse consumer is concave. The degree of curvature represents the consumer's relative degree of risk aversity. The curvature of the utility function is higher for more risk-averse consumers. The horizontal axis of Figure 2.1 shows three points: (E[x]-a), E[x], and (E[x]+a). If the consumer were certain that he/she was able to obtain E[x] units then his/her utility is $U[E(x)]$. With uncertainty, however, the consumer may face more than one possible outcome for x. In this case, if there is a δ_1 % chance of obtaining (E[x]-a) units and a $(1-\delta_1)$ % chance of obtaining (E[x]+a) units then the theory suggests that the consumer maximizes expected utility subject to his/her budget constraint where:

Expected Utility $= E[U(x)] = \delta_1 U(x-a) + (1-\delta_1)U(x+a)$. Figure 2.1 shows that $U[E(x)] > E[U(x)]$. Thus, the utility of the risk-averse consumer facing uncertainty is less than the utility of the certain outcome for x.

Figure 2.2 illustrates the effect of increasing the dispersion between the two possible outcomes for x (i.e. increasing uncertainty). The distance "A" represents the difference between utility from the certain outcome and utility for the uncertain outcome with low variance in x (i.e. E[x] $\pm$ a). Distance "B" represents the difference in utility from the certain outcome and utility for the uncertain outcome with higher variance in outcomes (i.e. E[x] $\pm$ b). When there is uncertainty in x, a higher variance in possible values for x results in a more significant welfare effect (i.e. a more significant reduction in utility).

A similar type of analysis is shown in Figure 2.3. This figure shows the effect of a higher degree of aversion to risk (i.e. an increase in the curvature of the expected utility function). In this case the utility of the certain outcome increases relative to the less risk-averse individual (i.e. U(E[x])'>U(E[x])). The net welfare loss associated with a given level of uncertainty is higher for the more risk-averse individual (distance B on Figure 2.3) than for the less risk averse individual (distance A on Figure2.3).

Figures 2.1 to 2.3 show the effects of uncertainty in terms of losses in utility for a risk-averse individual. An important question is: What are the relative magnitudes of these losses? Alternatively, what would the risk-averse agent be willing to pay in order to avoid uncertainty? The first step in determining this value is to ascertain the level of wealth (measured in terms of units of x) where the utility obtained with x units (obtained with certainty – $U(x_c)$) is exactly equivalent to expected utility (with uncertain outcomes). The value of x where this occurs (see Figure 2.4) is called the certainty equivalent (CE). The difference between E(x) and CE is the risk premium. Risk premium is defined as "the amount a risk-averse person is willing to pay to avoid risk" (Binger and Hoffman 1998 pg 521).

The welfare loss associated with uncertainty is a function of both the variance of outcomes and the degree of curvature of the utility function. The formula describing the curvature of the utility function is called the "absolute risk aversion function" (Pratt 1964). This function is defined as follows:

$$R(x) = \frac{-U''(x)}{U'(x)} \qquad (2)$$

The value of R(x) provides a measure of the degree of risk aversion of an individual facing uncertain outcomes. As the degree of curvature of U(x) increases the second derivative of U(x) increases relative to the first derivative resulting in a higher value of R(x) evaluated at any particular value of x. A higher value of R(x) indicates a more risk averse individual. A lower value indicates a less risk-averse individual. In the extreme case of risk neutrality, U(x) is a linear function and the value of R(x) at any value of x is zero due to the fact that the value of the second derivative of a linear function is zero.

Pratt (1964) also developed an equation for approximating the value of risk premiums. The risk premium function employs the absolute risk aversion function evaluated at the expected value of the outcome variable (i.e. R(E[x])). The approximate risk premium equation is:

$$\pi = (1/2)R(E[x])\sigma^2 \qquad (3)$$

Where:

π - risk premium

R(E[x]) – value of the risk aversion function evaluated at E[x]

σ^2 – outcome variance

The equation shows that risk premium increases as the degree of risk aversion (evaluated at E[x]) increases and as outcome variance increases (as shown in Figures 2.2 and 2.3).

As is shown in Figure 2.4, once the expected value and risk premium are known it is possible to determine the certainty equivalent according to the following relationship.

$$CE = E[x] - (1/2)R(E[x])\sigma^2 \qquad (4)$$

Where CE is the certainty equivalent and all other variables are as defined previously. The equation for certainty equivalent can be used in a risk programming formulation. For example the following basic model has been employed extensively in agricultural economics research. The optimization problem is to maximize certainty equivalent subject to constraints.

$$\begin{aligned}&Max: E[x]-0.5R(x)\sigma_x^2\\&s.t.\ \ \sum b_{ij}y_j \le c_j\\&Where \quad E[x]=\overline{a}y\end{aligned} \qquad (5)$$

This section provides a description of expected utility theory and how it can be used to understand choices of rational agents in an environment where outcomes are uncertain or risky. The previous discussion does not, however, discuss what kinds of strategies a decision maker might use as a way of managing the risk that he/she faces. Freeman (1999) identifies and discusses two separate types of risk management options: risk reduction and risk prevention. We introduce these concepts here because one of the research questions addressed in Chapter 10 pertains to measurement of the benefits of risk prevention. Risk has two main components: an adverse impact and a probability of occurrence. Accordingly, the management of risk can involve two separate types of activities. The first is to take actions that reduce the magnitude of adverse consequences. Freeman (1999) defines such activities as risk reduction activities. An example of risk reduction in a forestry context might be managing the fuels on a landscape so that when an ignition occurs fires remain controllable (assuming action is taken within a reasonable amount of time). Risk prevention, alternatively, refers to activities taken to reduce the probability of adverse consequences. An example of risk prevention in a forestry context might be restricting access to areas during highly flammable burning conditions.

Pure uncertainty

The previous section discusses the decision theory approach to risk and uncertainty. The decision theory approach requires some knowledge (objective or subjective) about the probability or likelihood of outcomes given certain choices. There are, however, cases and situations where the assignment of probabilities to outcomes is not possible. Human and biophysical systems are inherently complex. In some cases our understanding of human and biophysical systems and their interaction is so incomplete that it is not possible to reasonably assign probabilities to outcomes or future states of nature. Moreover, there may be a lack of consensus in expert opinion thereby precluding subjective estimates of probabilities. Inability to assign probabilities may be due to the number and complexity of interactions between systems or due to a lack of knowledge or due to both knowledge gaps and complexity. In cases where there is uncertainty in outcomes but it is not possible to characterize the uncertainty by assigning probabilities then uncertainty is referred to as pure uncertainty (Woodward and Bishop 1997).

Pure uncertainty is characterized by three conditions: (1) there is a lack of objective scientific data from which it might be possible to infer or assign probabilities or outcomes related to specific choices, (2) there is a lack of consensus among experts about the relative likelihood of outcomes, and (3) there is no basis for differentiating between experts in terms of the quality of their opinions (Woodward and Bishop 1997). In cases where experts disagree and where there is no basis for assigning weights to expert opinions, then it might be tempting to assign uniform probabilities to the outcome predicted by each expert. The justification for this approach is referred to as the "Principle of Insufficient Reason" (Woodward and Bishop 1997). Although this approach is commonly employed, Woodward and Bishop argue that it may result in irrational choices.

Woodward and Bishop (1997) refer to an axiomatic framework developed by Arrow and Hurwicz (1972) for explaining behavior under pure uncertainty. According to Woodward and Bishop (1997) choice criteria under pure uncertainty should satisfy the axioms described under the Arrow and Hurwicz (1972) framework (i.e. be AH consistent). These axioms include: 1. Independence of irrelevant alternatives, 2. Re-labeling, 3. Irrelevance of repetitive states, and 4. Dominance. A decision maker's choices under pure uncertainty are said to be rational "if his/her behavior is consistent with these four axioms...the only choice criteria that are AH rational will rank actions based entirely upon the maximum and/or minimum of the state space.." (Woodward and Bishop 1997, pg 496). Thus, a fundamental distinction between expected utility approaches and the pure uncertainty approach proposed by Arrow and Hurwicz (1972) is that in the expected utility approach, choice is based on expected values and variances of outcomes. Under the Arrow and Hurwicz (1972) approach, however, rational choice is based on consideration of extreme outcomes. Woodward and Bishop (1998, pg 497-498) state:

> "we can conclude, therefore, that when a decision maker is faced with pure or second-order uncertainty, then the use of a maximin-type choice criterion...would be consistent with axioms of rationality. Moreover, the use of probability based on equal weighting would not be rational under pure or second-order uncertainty."

Woodward and Bishop (1997) discuss a range of choice criterion under pure uncertainty – each based on consideration of extreme outcomes. Once such choice criteria is the maximin criteria[4]. The maximin criterion applies in cases where (a) the decision maker is pessimistic, and/or (b) decision makers exhibit uncertainty aversion.

The maximin criterion is described as follows. If there are two policy scenarios and two possible outcomes associated with each scenario, then a rational policy maker will first ascertain the minimum payoff in each scenario and choose the scenario that maximizes the minimum payoff between scenarios (Woodward and Bishop 1997).

As noted an important result of the Arrow and Hurwicz (1972) approach is that rather than focusing on measures of central tendency, a rational agent focuses on potential extreme outcomes. This means that research should attempt to develop an

[4] Woodward and Bishop (1997, pg 496) note "Maximin and the maximax are two criteria out of the set of criteria that would be AH rational. Any combination of these extremes would also be acceptable."

improved understanding of the likelihood and consequences of extreme outcomes (Woodward and Bishop 1997).

In cases where pure uncertainty exists, the maximin choice criterion generally suggests forgoing or reducing development. Such conclusions are consistent with the rationale for invoking safe minimum standards (Ciriacy-Wantrup 1968). Safe minimum standards are a form of policy mechanism for ensuring that public policies and programs take account of pure uncertainty. The underlying assumption with a safe minimum standard approach is that there exists pure uncertainty regarding our ability to substitute man-made capital for particular types of natural capital in the future (Castle et al. 1996). Based on the maximin criterion, society should adopt policies that maintain our capacity to adapt to new circumstances and situations. This capacity may require explicit policies (such as the imposition of safe minimum standards) that ensure that key attributes of natural capital are not irreversibly lost. Safe minimum standards are a form of constraint on resource development that recognizes pure uncertainty in the form of potential loss of future unknown benefits from large-scale irreversible development (Toman and Ashton 1996). Some authors (e.g. Castle et al. 1996) have suggested that a safe minimum standard (in combination with policies that encourage adaptive management) should be the basis for sustainable forest management policies. Implementation of a safe minimum standard involves: (a) the identification of resource attributes at risk to irreversible loss, (b) the identification of a critical zone for the resource or environmental feature (i.e. a level below which the feature is likely to be irreversibly lost), (c) the estimation of costs associated with implementation, and (d) the establishment of enforceable policies and regulations that ensure that the standard is maintained.

Another policy option for addressing pure uncertainty is to delay development decisions until uncertainties about future benefits of preservation are reduced through new information. Delaying decisions allows for new information or knowledge to be generated regarding unknown future values of benefits. If development proceeds and if it leads to an irreversible loss of some environmental feature, future benefits (that are presently not known) may be permanently lost. Delaying development keeps options open. Society and individuals are willing to pay to retain their options relative to future possible uses. The welfare gain associated with avoiding irreversibility and retaining the opportunity to realize future environmental benefits is called quasi-option value (Freeman 1999).

Weisbrod (1964) introduced the concept of option value. He described option value as the value people are willing to pay to keep the option of future use open. The subsequent extension of Weisbrod's concept to uncertainty analysis proceeded in two different directions. Graham (1981) interpreted option value as being the difference between option price and the expected value of the consumer surplus. This view has subsequently been shown to be arbitrary and inappropriate as a choice criterion for addressing uncertainty in cost benefit analysis. Arrow and Fisher (1974) and Henry (1974) proposed an alternative interpretation. Their interpretation of option value pertains more to pure uncertainty as opposed to Graham's application (which was incorporated into an expected utility theory construct). Arrow and Fisher (1974) and Henry (1974) considered the possibility of future benefits that are currently unknown and the role of information gathering and learning in reducing uncertainty about these benefits over time. According to Henry (1974): "The mere prospect of getting fuller information [about future values] combined with the irreversibility of the non-preservation alternative, brings forth a positive option value in favor of preservation." This type of value is referred to as "quasi-option" value (Fisher and Hanemann 1987).

What are the implications of addressing pure uncertainty by incorporating quasi-option value into cost benefit analysis? Fisher and Hanemann (1987) argue that the quasi option value of preservation is always positive[5]. Therefore, consideration of quasi-option value in economic analysis of development projects will result in lower benefit cost ratios. This occurs because quasi-option value is a type of opportunity cost of development. It is a benefit that is foregone if development proceeds and an irreversible loss occurs. Moreover, it is over and above the net present value of the stream of known environmental benefits that would be lost. Thus, the first consideration is to recognize that future information will provide clarification of

[5] It is useful to note that this finding was based on a relatively simple scenario (2 periods – passive information)

future benefits (note the basis for the recourse model developed in Chapter 10 is that for multi-period and long term problems such as timber harvest scheduling, some uncertainty is resolved before the end of the planning horizon and this may have positive economic benefits). The second consideration is recognition that there is a public welfare value associated with not foreclosing on the option of realizing these unknown future benefits by adopting decisions today that lead to irreversible loss of natural capital. The welfare gain associated with delaying development in order to take advantage of new information constitutes an opportunity cost of development. As noted by Hanemann (1989) quasi option value "is equal to the conditional value of perfect information – conditional on there being no development initially."

Adaptive management

This chapter began by providing an overview of expected value/utility-based approaches for addressing uncertainty. The previous section provides an overview of concepts related to pure uncertainty. The underlying assumptions of both sets of concepts are that: (a) knowledge can reduce uncertainty (either immediately or at some time in the future), and (b) socioeconomic systems are inherently stable. In this section we provide an overview of a third approach for characterizing, accounting for, and responding to uncertainty. This approach is based on an emerging integrative theory that is attempting to describe the performance, relative viability and processes of change in human and environmental systems. The theory is being developed by the ecologist C.S. Holling and other interdisciplinary researchers with the "Resilience Project." (Holling 2001).

The underlying premise of the theories of change being developed by the Resilience Project is that human and social systems are complex and unpredictable and in some respects unstable in that they are continually evolving and redefining themselves. Thus social and ecological systems never attain a steady state. Rather social and ecological systems are continuously changing (Holling 2001). One implication is that predicting outcomes or future states of nature is not feasible. Pure uncertainty does not just prevail in some special case. Rather, pure uncertainty is a general property of human and biological systems. Sustainability is not threatened by change and instability in social and ecological systems. Rather, sustainability requires change and instability. Important system features that facilitate evolutionary change of social and ecological systems over time are: (a) functional diversity, (b) management systems and institutional structures that recognize and account for uncertainty and unpredictability, and (c) social structures that encourage adaptive management (Holling 2001). In fact Holling (2001) states: "For linked ecological/social/economic systems, slow variables, multistable behaviors, and stochasticity cause adaptive management to outperform optimization approaches that seek stable targets" (pg 403). `

The emphasis on adaptive management promoted by this new theory is consistent with the emerging views of some natural resource economists regarding competing paradigms on sustainable development. For example, Castle et al. (1996) argue that the axiomatic foundations of both weak and strong definitions of sustainability are flawed because they assume that the outcomes are predictable. Castle et al. (1996) argue that the future cannot be predicted and that sustainable development should focus on maintaining flexibility and adaptive capacity. They recommend that greater attention be paid to encouraging adaptive management approaches as a way of achieving sustainable development.

Bounded rationality and risk perceptions

The behavioral assumptions of utility and profit maximization are fundamental to economic theory. These assumptions imply that decision makers are rational relative to their choices and therefore predictable in terms of their behavior and the outcome of their decisions. Rationality of decision makers in turn implies that the contexts for decision problems are clearly defined and that there are no limitations in terms of the capacity of individuals to understand, interpret, and evaluate the full range of options and choices available to them. Simon (1959) was among the first to question the legitimacy of rationality and expected utility theory. He noted:

> "the classical economic theory of markets with perfect competition and rational agents is deductive theory that requires almost no contact with

empirical data once its assumptions are accepted. Undoubtedly there is an area of human behavior that fits these assumptions to a reasonable approximation, where the classical theory with its assumptions of rationality is a powerful and useful tool. Without denying the existence of this area, or its importance, I may observe that it fails to include some of the central problems of conflict and dynamics with which economics has become more and more concerned.....Economics has been moving steadily into new areas where the power of the classical equilibrium model has never been demonstrated, and where it adequacy must be considered anew. Labor markets is such an area, oligopoly or imperfect competition theory another, decision making under uncertainty a third, and the theory of economic development a fourth."

The issue for Simon (1959) was the extent to which the assumption of rational agents is relevant in inherently complex multidimensional decision-making environments. His research focused on the need to develop a more complete and realistic understanding of the psychological motivations, processes and strategies used by individuals to make decisions in complex and rapidly changing environments. Simon (1959) introduced the concept of "bounded rationality" (Slovic 2002). Bounded rationality proposes that instead of being rational in terms of decisions, individuals are adaptive (i.e. they reach their objective in small steps instead of immediately achieving their objective as a result of a single decision) and instead of making choices to maximize an objective function they make choices in order to obtain some level of satisfaction that may be less than an optimal or maximized level of a particular objective (Slovic 2002).

Tversky and Kanemann (1974) had similar misgivings about assumptions of rationality and initiated a series of studies trying to better understand how people make decisions in complex environments where they are subject to risk and uncertainty. They approached the problem from a social psychology perspective. Specifically, they were interested in understanding the kinds of heuristic strategies that people use to evaluate and make choices given uncertainty and whether these heuristic strategies lead to systematic biases in assessing risk. Their findings suggest that individuals do not rely on all the information that is available to them. Nor do they attempt (or are they able) to fully understand all aspects of complex risk issues. Rather, they tend to simplify complex risk problems by applying a limited number of strategies that allow them to reduce complex problems to something that is easier to comprehend and judge. These heuristic rules are useful in that they facilitate decision-making. However, they can also result in "systematic errors" in evaluating risks. Tversky and Kahneman (1974) identify three specific categories of heuristic rules that are used to simplify decision making under uncertainty: representativeness, availability, and anchoring.

Representativeness refers to the use by individuals of similarities between events and/or processes to evaluate outcomes. For example, an individual might associate particular personality traits with a particular occupation and then rely on this association to make an evaluation about the likelihood that a particular individual has a certain occupation. However, this simple assessment ignores a significant amount of information that could influence the real probability including for example, the number of people in society with that occupation. So there is the potential for the introduction of systematic errors by relying on this type of heuristic.

The availability heuristic refers to "situations in which people assess the probability of an event by the ease with which instances or occurrences can be brought to mind." (Tversky and Kahneman 1974, pg 1127). For example, a person's perceptions of risk of a car accident could be influenced by the fact that an acquaintance had recently been involved in a car accident. This type of heuristic can lead to a number of sources of bias. Even though the probability over any given time period is constant, the same individual may make very different judgments about a particular risk based on the availability of personal knowledge or experience with that risk.

The anchoring heuristic refers to the strategies employed by individuals of evaluating risks by making some initial judgment (i.e. the anchor) and then adjusting the assessment as the individual acquires new information. In this case, the final assessments or judgments are significantly influenced by the initial anchoring assessment. Thus, if the initial assessment is in error, and if there is insufficient adjustment, the final evaluation can be significantly biased. Anchoring bias occurs when individuals have pre-conceived notions about a risk but these preconceived ideas are in error. Moreover, the adjustments that the individual makes over time are insufficient to compensate for the initial misconceptions.

Tversky and Kahneman's (1974) research suggests that economic agents are not rational. Rather, because of the complexity of risk problems and the need to simplify these problems by applying heuristic rules, systematic errors can be introduced in processing information about risks. In other words, final choices may not be rational in the sense that they optimize some objective function value. Arrow (1982) provides a number of additional examples of individual irrationality in financial and economic markets.

In a later article, Kahneman and Tversky (1979) evaluated and specifically critiqued the axioms of expected utility theory. Their critique was based on the results of a series of experiments that were conducted on the preferences of individuals relative to various combinations of risk scenarios. They discovered a number of cases where actual behavior and choices contradicted the predictions of the expected utility hypothesis. The first contradiction discovered by Kahneman and Tversky (1979) was that people attached variable weights to probabilities that are out of proportion to the actual probabilities. They tended to exaggerate some outcomes because they are relatively more certain. Expected utility predicts that people maximize expected utility based on the actual probability of a state of nature multiplied by the utility associated with that state (i.e. objective risk). Kahneman and Tversky's (1979) findings suggest that people actually assign subjective probabilities and they maximize utility using these subjective measures. So in cases where expected utility calls for indifference between two gambles, Kahneman and Tversky's experiment finds that there are clear preferences of one gamble over another.

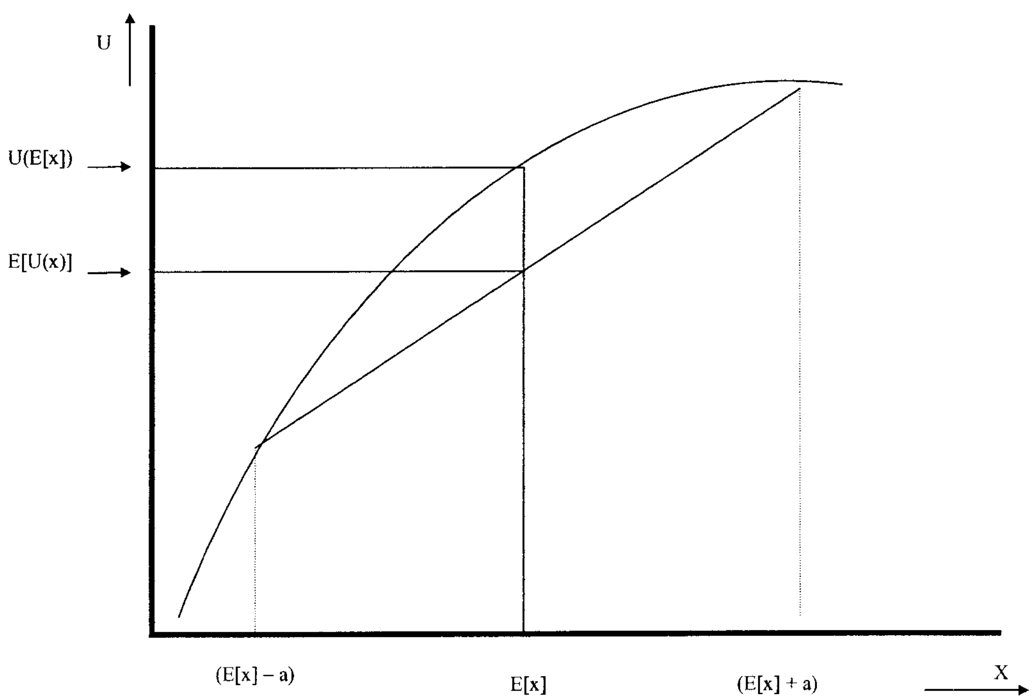
Viscusi (1985) also discusses biases in risk evaluations. His study finds there is a high correlation between wage compensation and occupational risk. Moreover, the subjective evaluation of workers regarding job risk is highly correlated to technical risk indexes. So in the context of job choice, behaviors do seem rational. Second, in cases where failures to correctly assess risk do occur it is likely a temporary condition because people are continuously learning and updating judgements about prior probabilities with new information. Thus economic agents are adaptive and even if behaviour appears to be irrational, this may be a temporary result and that agents move toward more rational choices through adaptation.

It would be remiss not to mention recent papers that challenge the expected utility theory and offer alternative explanations for risk aversion. Rabin and Thaler (2001) suggest that the expected utility model does not consistently explain risk aversion in cases where only a portion of a decision maker's wealth is a stake. These authors provide an alternative theory for explaining risk aversion that builds on Kahneman and Tversky's (1979) prospect theory. Rabin and Thaler (2001) suggest that risk aversion behaviour is better explained by considering the concepts of loss aversion and mental accounting. The arguments presented by Rabin and Thaler (2001) are compelling. Their theories, however, have not tipped the scales in terms of full rejection of the expected utility hypothesis. Moreover, they offer no alternatives in terms of normative behavioral approaches that permit the explicit consideration of risk and risk preferences in models of agents' choices and decisions. The limits of expected utility are well recognized by applied economists (Hardaker et al. 1997). Applied economists do not dispute the underlying inability of expected utility to consistently explain every individual's behaviour. However, as a general theory, efficiency analysis and expected utility theory do provide reasonable approximations of the expected behaviors of decision makers facing risk (Hardaker et al. 1997). Moreover, expected utility theory continues to be the theory of choice for explaining behaviour under uncertainty in advanced microeconomics theory text books (e.g. see Jehle and Reny 1998).

Conclusion

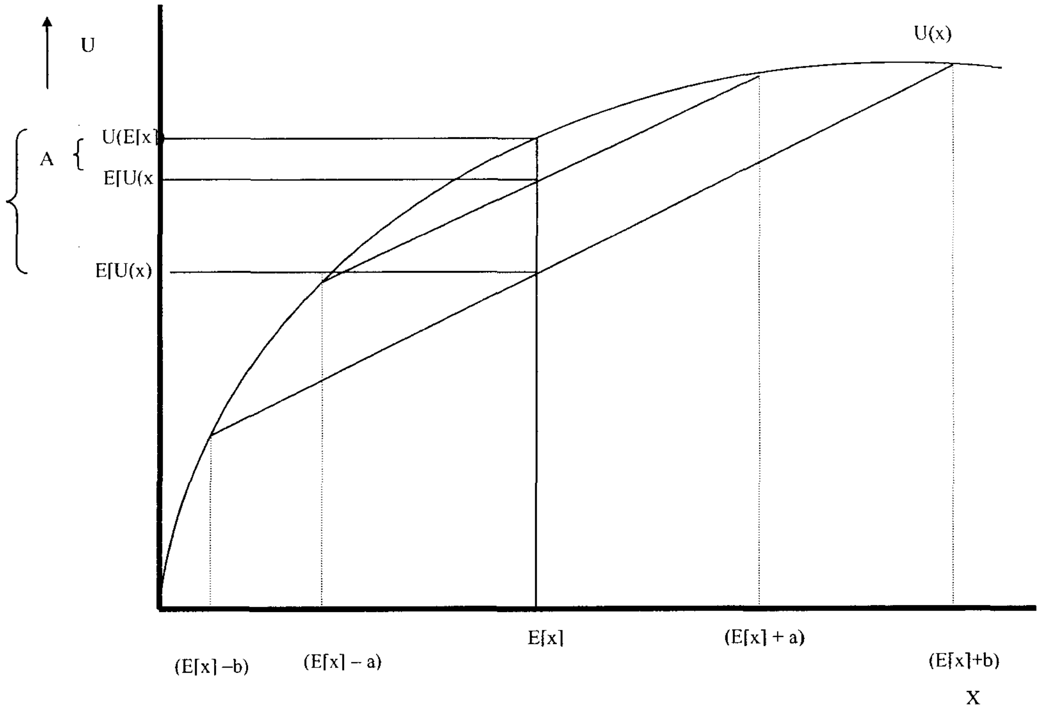
The remainder of the analysis in this study is based on expected utility theory. As noted, there are a number of different views and perspectives concerning: (a) approaches for understanding behavioral response to uncertainty, and (b) the validity of assumptions regarding rational agents and therefore the validity of decision theory and expected utility theory. The advantage of the expected utility approach is that it provides clear, explicit, concrete conceptual base that provides a foundation for model construction and testing. The expected utility approach provides a way of "organizing our thinking about economic decision making under conditions of uncertainty" (Schotter 1994, pg 458). Also, as Viscusi (1985) suggests, decision makers are, in many cases rational relative to evaluation of risks and when they are not, patterns of failure to correctly assess risks are fully consistent with a Bayesian learning process. The point of this is to note that expected utility theory is a general theory and that it does have some limitations. The theory may be suitable as a general conceptual framework but its limitations also need to be recognized and acknowledged.

Figure 2.1 Expected utility for risk averse consumers.



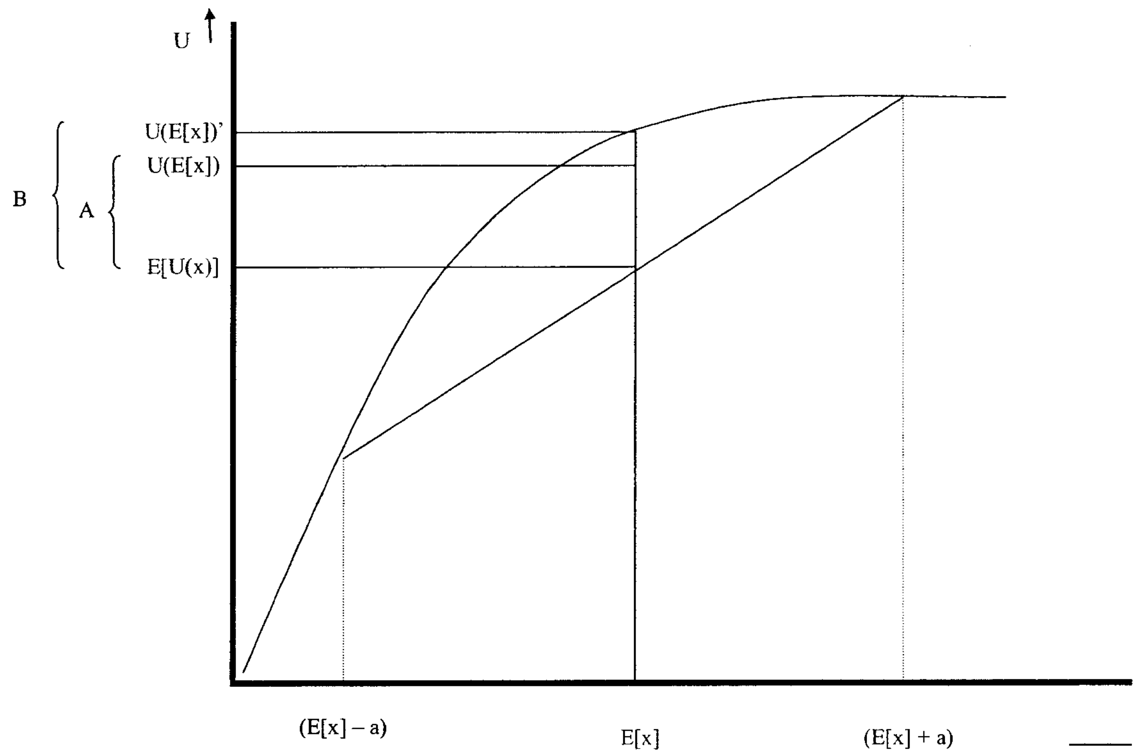
Source: Zerbe and Dively 1994

2.2. Effects of variance in outcomes on the difference between utility of the certain outcome and utility of the uncertain outcome.



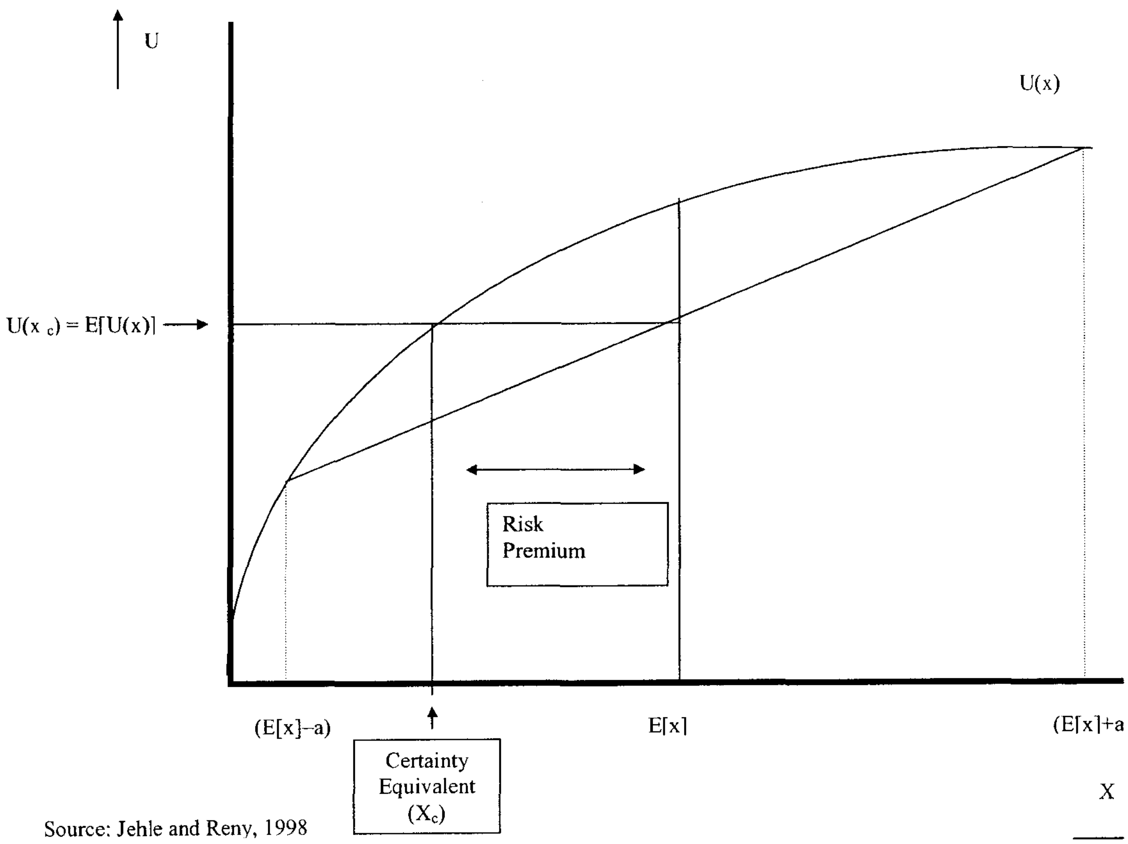
Source: Zerbe and Dively

Figure 2.3. The effect of degree of risk aversion on the difference between utility of the certain outcome and utility of the uncertain outcome.



Source: Jehle and Reny, 1998

Figure 2.4. Certainty equivalents and risk premiums with expected utility theory.



Source: Jehle and Reny, 1998

CHAPTER THREE

A REVIEW OF CLIMATE CHANGE AND UNCERTAINTY RESEARCH IN FORESTRY

This chapter provides an overview of two topic areas. First, an overview of the effects of climate change on Canadian forests is provided. Second, the applied literature on uncertainty analysis in forestry is reviewed. The last section of this chapter identifies knowledge gaps.

Climate change and forestry impacts

This section provides a brief overview of the effects of climate change on forests and the forest sector. The literature on the impacts of climate change on forest ecosystems is not extensive[6]. There is considerably less material addressing the socioeconomic impacts – particularly in a Canadian context. This section does not provide a comprehensive overview of the literature on climate change and forest impacts. References that provide useful overviews of the range of possible impacts on forests and the forest sector in Canada include Singh and Wheaton (1991), Binkley and van Kooten (1994), Saporta et al. (1998), Hauer et al. (2001), Climate Impacts and Adaptation Directorate (2002), Spittlehouse and Stewart (2003) and Hogg and Bernier (2005). Rather, a brief summary of potential biophysical effects and socioeconomic impacts is provided here.

According to the IPCC Third Assessment Report (IPCC 2001) the global average surface air temperature is projected to increase between 1.4 and 5.8 degrees centigrade by the year 2100. Temperatures in northern latitudes are predicted to increase more than in southern latitudes. Winter temperatures will increase more than summer temperatures and nighttime minimums will increase more than daytime maximums. Rates of precipitation may increase in some areas and decrease in other

[6] The amount of research looking at impacts and adaptation is starting to increase thanks to the Canadian Climate Impacts and Adaptation program and to the Canadian Climate Impacts and Adaptation Research Network (CCIARN) and the Canadian Model Forest Program.

areas, as will moisture regimes. Growing seasons will become longer. Storm frequency and intensity may increase.

Changes in climatic variables will impact forests by influencing physiological and ecological processes. First, forest productivity and health will be affected. In some areas, productivity may increase as a result of CO_2 fertilization, longer growing season, and warmer soil temperature. In other areas productivity may decrease as a result of moisture deficits (i.e. the case where evapotranspiration exceeds precipitation), increased exposure to existing pathogens, or exposure to new pathogens (Hogg et al. 2002). Second, the frequency and intensity of disturbance events (wildfire, insect and diseases, wind throw, drought) may increase (Flannigan 2004, Hogg et al. 2002). A significant increase in disturbance will result in structural changes in forest ecosystems as new forests better suited to the new growing conditions replace old forests following disturbances. Third, in some cases, climate change will manifest as a more gradual change in forest types over time. The ability of species to resist (i.e. their inherent resiliency) or adapt to climate change depends on the severity and rate of climate change and individual species tolerances. As noted in Hauer et al. (2001 pg 3) "If climate change is beyond the limit of trees physiological tolerance, forest dieback and ecosystem changes are inevitable, particularly at the margins of different forest ecosystems." Fourth, forest boundaries may shift (Hogg 1994). The southern boundary of the boreal forest is predicted to move northward and the northern boundary may also move north. However, movement of the northern boundary will be constrained by adequacy of northern soils to support forests.

Canadian society has significant economic and social ties to forests. Canadians place significant value on the knowledge that forests and wildlife are being sustained and that representative natural areas are protected. Climate change will result in shifts in ecosystem types and may threaten the existence of some species (Gray 2005). Climate change may curb our capacity to manage forests in ways that are consistent with current views of sustainable forest management. Canadians also utilize forest environments as destinations for outdoor recreation. Summer recreation activities may be positively influenced by longer season length or fewer rainy days but negatively influenced if forest aesthetics are changed or water bodies are negatively affected (e.g. a lowering of lake levels, impacts on fish population or increases in weeds). Winter recreation may suffer negative consequences due to reduced season length, milder temperatures and possibly by reductions in snow packs. Many forest based communities may be vulnerable because of strong linkages to surrounding forests, relatively low adaptive capacity (compared to larger urban centers), and socio-cultural circumstances that may contribute to a tendency to underestimate changes in climate related risks (Davidson et al. 2003).

Timber is an important natural resource that will be affected by climate change. Changes in biophysical factors affecting tree growth and timber supply include increased growing seasons, increased precipitation in some regions (decreased in other regions), changes in moisture regimes (due to changes in rates of evapotranspiration), increased site productivity (due to increased heat units and CO2 fertilization effects – assuming moisture is not limiting), increases in disturbance frequency and intensity (wildfire, insects and disease, wind damage), and increased incidence of drought. In addition to the direct impacts of climate change on productivity and growth, there will also be market effects caused by expected lower prices for forest products and timber due to a general increase in timber supply in the global market (van Kooten and Arthur 1989; Sohngen and Sedjo 2005). Some US studies forecast increased regional supply in some areas and decreased supply in other regions but overall climate change leads to an increase in US timber supply (Sohngen and Sedjo 2005; Shugart et al. 2003) and global timber supply (Perez-Garcia et al. 2002). However, the findings of these studies may not be applicable to Canada because: (a) climate change is expected to be more extreme at northern latitudes, (b) climate change may result in a shift in species distributions from higher valued, long fibered coniferous species to shorter fibered deciduous species, (c) most forest land in Canada is under public ownership which means that the kinds of autonomous adaptations that occur within competitive markets will not occur on Canadian forest land (Hauer et al. 2001).

To some extent the impacts of climate change on the Canadian forest sector can be reduced by adaptation (Hauer et al. 2000). Duinker (1990) and Spittlehouse

(2005) suggest that there is a need to review forest policies to ensure that they permit adaptation. Spittlehouse and Stewart (2003) identify a number of specific types of adaptation measures that could be considered in forest management. Because of the long-term nature of forestry and because of increasing vulnerability to timber capital with climate change, it is important to begin incorporating climate change considerations and adaptation strategies into current forest management planning. One adaptation strategy suggested by Spittlehouse and Stewart (2003) is to include climate variables in growth and yield models.

Uncertainty in forestry analysis

This section reviews some of the applied forest science and forest economics literature on uncertainty, risk modeling and related concepts such as risk preferences. Prior to the late 1980's, risk and uncertainty was not a significant topic in the forest science and forest economics literature. Fight and Bell (1977) suggest that this was primarily due to the complexities of uncertainty analysis. Lack of recognition of the relevance of uncertainty analysis may also have been a factor (Dempster 1987). However, after the late 1980's the number of published studies addressing risk and uncertainty in forestry increased (Brazee and Newman 1999). The increased attention paid to risk and uncertainty was due to increased recognition of the long periods associated with forest investment and the high levels of variance that are experienced in variables (such as patterns of disturbance, growth, product prices and stumpage prices) that are important for forest investment analysis (Brazee and Newman 1999). Kangas and Kangas (2004) provide a comprehensive review of approaches for considering risk in forestry. They identify and discuss various sources of uncertainty relevant to forestry decision making (e.g. disturbance risk, growth and yield uncertainty, price uncertainty), various ways that uncertainty is classified (e.g. due to lack of information, conflicting evidence, ambiguity, measurement error, etc), and the wide range of approaches (e.g. classical frequentist approaches, Baysien methods, and fuzzy set theory) that have been employed to study uncertainty in forestry research.

Some studies have looked at risk management in forestry contexts and mechanisms for incorporating risk into forest policy and planning. Dempster (1987) notes that a general lack of understanding of risk in Canadian forestry and of approaches and concepts for incorporating risk management into planning pose a barrier to application. The Dempster (1987) study recommends that a stronger emphasis should be placed on incorporating risk into operational decisions and long-term planning. Montgomery (1996) offers an assessment of the implications of modern forest policies for public exposure to risk and calls for explicit consideration of public preferences and perceptions of risk when looking at tradeoffs between various forest outputs.

Pukkala and Kangas (1995) describe a scenario approach for generating outcomes for alternative forest management plans. The authors then evaluate the effect of risk attitudes on preferred management plan scenarios. They find that when uncertainty exists, risk attitudes have a significant influence on preferred management strategies. Shaw (1999) also describes a scenario-based approach for incorporating risk into forest planning in the Tongass National Forest in Alaska. The process involved the establishment of 16 risk assessment panels. The panels were asked to evaluate outcomes of different management planning scenarios. The results of the panels provide a defined range of potential outcomes. As noted by Shaw (1999) in (the abstract of his report) "The panel results provided estimates of the relative risk that implementation of a range of alternative approaches to management of the Tongass National Forest would pose to the continued existence across the landscape of an array of species or resources and estimates of potential socioeconomic effects on communities."

Mendoza and Sprouse (1989) introduced a new approach to planning and decision making in forestry under uncertainty. They introduced a method called fuzzy set theory and an analytical method called fuzzy programming. Fuzzy approaches permit the incorporation of complexity and lack of clearly defined objectives into forest planning. Ells et al. (1997) apply fuzzy set theory to analyze optimal ways of allocating public forestlands given uncertainty (or lack of clear definition) in management objectives and uncertainty regarding the relationship between actions and outcomes.

Buongirono (2001) also looks at how uncertainty influences land use. His study however, incorporates stochastic growth and prices into a Faustmann land valuation model. The study uses a Markov decision process model that includes future prices and states as probability distributions.

A number of studies have considered risk and uncertainty in the context of sustainable forest management. Toman and Ashton (1996) note that consideration of quasi-option values (i.e. the value associated with avoiding irreversibility under pure uncertainty) provide added reason for preservation and/or the establishment of safe minimum standards approaches relative to defining and implementing sustainable forest management policies. Montgomery (1996) makes a case for a more direct and explicit treatment of risk in forest policy. As noted, incorporating risk into policy means that more attention needs to be paid to public risk perceptions and preferences and that policy should be reoriented to allow for flexibility and adaptability in order to manage and plan for risk. More specifically, there are public goods associated with forests that may be at risk as a result of any number of human interventions or natural processes. Market failure occurs relative to both the amount of public good to provide as well as in terms of determining the allocation of resources to reduce or manage risk to forest related public goods. Haener and Adamowicz (2000) identify the need to incorporate risk into measures or indicators of forest sustainability. They discuss a methodology for incorporating fire risk and price risk into forest resource accounts.

One of the earliest subject areas where risk concepts were applied in a forestry context was to the determination of the impacts of fire risk on stand and forest level timber supply. A number of stand level studies considered the impacts of fire risk on optimal rotation (Martell 1980; Routledge 1980, Reed 1984). Other studies adopted a broader perspective. These studies considered the impact of fire risk on timber supply at a forest level (Van Wagner 1979; Van Wagner 1983; Reed and Errico, 1986; Boychuk and Martell 1996). Blattenberger et al. (1984) incorporate risk and uncertainty into a cost plus net value change model for the purpose of identification of socially efficient levels of investment in fire management.

As noted previously, forest management and forest investment are long-term in nature. Significant effort is put into obtaining data for estimating growth and yield for stands through the establishment of temporary and permanent sample plots. The data from these plots are in turn used to estimate yield functions for the purposes of predicting future production. These predictions are used in long-term forest management planning and timber supply analysis. Generally, in operational planning, predictions are point estimates. Thus, consideration of uncertainty by looking at density functions for yield predictions in operational planning is not common. However, there have been a number of research studies looking at uncertainty relative to model predictions. Nillson (2003), for example, develops a framework for evaluating the relative benefits of different options for reducing model uncertainty.

The majority of studies that have sought to incorporate uncertainty in growth and yield prediction have applied some form of a Bayesian approach. In general, Bayesian methods entail updating prior knowledge regarding uncertainty of the parameters of a particular model with new information to obtain posterior estimates of probability distributions of parameters and model predictions. Early studies used a method called empirical Bayes (Green and Strawderman 1985; Green et al. 1992). Empirical Bayes methods soon were replaced by hierarchical Bayes approaches (Green and Strawderman 1992). Hierarchical Bayes was shown to provide more accurate estimates of the marginal posterior distributions of model parameters. Three studies applied Bayesian methods to evaluate uncertainty in growth and yield parameters between 1994 and 1997. Green et al. (1994) used a Bayesian model to evaluate the distribution of tree diameters in forest stands. Green and Strawderman (1996) developed a Bayesian version of a slash pine yield model to estimate probability distributions for various stand variables. Green and Valentine (1998) compared least squares estimates of model parameters of a linear model to estimates inferred by a Bayesian model. The estimates provided by the Bayesian model are close to maximum likelihood estimates and these models provide the added benefit of providing posterior probability distributions of model parameters. A method called Bayesian synthesis (or Bayesian melding) was applied to ascertain marginal posterior distributions of model parameters and model outputs (or predictions) (Green et al 1999; Green et al. 2000; Radtke et al 2002). The method is applied to simulation models where there is uncertainty associated with model parameters. Nystrom and Stahl (2001) apply a Bayesian approach to develop a better understanding of uncertainty in yield predictions of Scots Pine and Norway spruce. Similarly, Gertner et al. (1999) use a Bayesian methodology to evaluate the posterior distribution of parameters of a forest process model. The majority of studies considering uncertainties in yield modeling have utilized a Bayesian oriented approach. Kangas (1999) evaluated various methods for assessing uncertainty in growth and yield. None were based on a Bayesian approach but each of the methods evaluated by this author was based on a related Monte Carlo simulation type approach.

A number of papers in the forestry literature consider uncertainty and its effects on stand management, harvest scheduling, and land use. Kao (1984) uses a dynamic programming approach to evaluate the effects of uncertainty on the joint optimization of thinning regimes and rotation length. Incorporating uncertainty reduces the optimal Mean Annual Increment (MAI) by around 6 %. Gong (1998) uses an expected utility model to ascertain the optimal harvest policy of a private forestland owner under stumpage price uncertainty. The model incorporates the risk preferences of the landowner. They find that harvest age for risk averse land owners is lower than harvest age for risk neutral landowners. They also find that risk averse landowners prefer adaptive harvest strategies to harvest rates based on optimal rotation calculations. A number of other authors have considered the effets of risk preferences on agent behaviour in forest management. The results of these studies tend to be mixed. Peltola and Knapp (2001) for example, find that risk preferences have little effect on harvest sequences. Gong (1998), Pukkala and Kangas (1996), and Uusivuori (2002) find that risk preferences have a significant effect on harvesting behavior. Loonstedt and Svensson (2000) suggest that preferences are sensitive to sources of risk. They find, for example, that Swedish private landowners are more risk averse to price risk than they are to sources of variability in incomes that result from biological factors.

A subject area that has attracted a significant number of papers in the forestry literature is the consideration of stochastic coefficients in harvest scheduling models. This general approach provides a means of directly incorporating uncertainty into timber supply analysis at operational levels. One approach is to apply fuzzy set theory to harvest scheduling problems (Bare and Mendoza 1992). A much more common approach in the literature is to incorporate the density functions of coefficients directly into the optimization model. Studies that follow this latter approach include: Hoganson and Rose (1987), Marshall (1987), Gassmann (1989), Hof et al. (1988), Hof et al. (1992), Hof et al. (1995), Uusivuori (2002) and Weintraub and Abramovich (1995). The methodologies for incorporating uncertainty in coefficients into timber supply models are well established. In most cases, however, these studies have ignored risk preferences (i.e. they have assumed risk neutrality).

A relatively new methodological approach for capturing uncertainty is the real options approach. This approach originated in the finance literature and has recently been applied to forestry. If a forest manager has the option to revise or modify harvest levels in response to stochastic fluctuations in random variables (such as lumber price) then this flexibility has a certain value that should be included in decisions regarding when and how much to harvest from a land base. This approach is applied by Insley (2002) to determine "the value of the option to harvest a stand of trees and the optimal cutting time when lumber prices are assumed to follow some known stochastic process." (pg 485). Insley and Rollins (2003) apply a real options approach to evaluate the opportunity cost of limiting options by imposing sustained yield regulation constraints. Insley and Rollins (2005) also apply a real options method to determine the value of forest stands when there is complete flexibility in selection of harvest timing vs. when harvest timing is dictated by regulations. A number of studies (Conrad 1997; Forsyth 2000; Reed 1993) have applied the real options approach to evaluate option values relative to preservation of natural forests.

Uncertainty is particularly germane to investment analysis – especially in a forestry context given the long time periods associated with forestry investment. Hyldahl and Baumgartner (1991) review the forestry literature on risk and investment up to 1989. They divided studies into the following categories: capital asset pricing, portfolio theory, stochastic dominance, forestry investment analysis, decision theory, and option pricing. One common approach for accounting for risk in forestry investment is to add a risk premium to the opportunity cost of capital discount rate. Klemperer et al. (1994) looks at issues related to whether risk premiums differ between short and long term investments. He finds that for short-term investments, a risk premium of around 7 percent is appropriate but for longer-term investments a lower risk premium may be more appropriate.

Knowledge gaps

As the effects of climate change become more prominent and/or more widely recognized and understood, it is expected that forest managers and policy makers will begin to seek answers relative to long-term impacts and adaptation options. Some companies (e.g. Millar Western at Whitecourt, Alberta and Louisiana Pacific at Swan River Manitoba) have already initiated research programs to better understand climate change effects so that they can incorporate climate effects into their long-term planning.

The forest economics literature on climate change impacts on the Canadian forest sector is not well developed. Part of the issue is that there is a lack of knowledge of the implications that climate change may have for growth and yield. Estimations of growth and yield are a basic requirement for economic analysis. Basic information on growth and yield is also required for timber supply analysis. Spittlehouse and Stewart (2003) identify the development of yield relationships that consider the effects of future climate as an area that needs to be addressed.

Another significant gap relative to research on the effects of climate change on the Canadian forest sector pertains to the lack of previous analysis that recognizes the fact that in an environment of climate change, the future values of all variables of importance to benefit cost analysis, long term forest planning and harvest scheduling are random variables. The previous section describes a number of different studies in the forestry literature dealing with uncertainty. Although a few of these studies recognize risk preferences, the majority assumes risk neutrality and therefore they ignore (or simplify) the costs of risk and uncertainty and the impacts that uncertainty may have on the optimal choices of risk averse decision makers. Moreover, none of the literature on uncertainty in forestry has explicitly given consideration to uncertainty in a climate change context. This might be considered to be a general weakness in studies of the economic impacts of climate change on the forest sector to date. There are three main implications. First, methodologies that recognize, estimate and incorporate the stochastic nature of variables of importance to decision-making under climate change must be identified or developed. Second, the methodologies should be applied in some specific forest management context in order to illustrate: (a) the relative economic impacts (of both productivity effects and uncertainty effects), and (b) the implications of climate change and climate change uncertainty for optimal choices. Third, the methodologies should be tailored so that they can be applied to evaluate the implications of climate change given particular institutional contexts. In the case of Canadian forestry this means that the models should be able to analyze the economic impacts of climate change recognizing that the objective function of private sector loggers operating on public forest lands may be constrained by public forest land management objectives – namely sustained yield objectives.

If decision makers are risk averse then the uncertainties associated with climate change may have an economic cost that must be considered not only in terms of developing a better understanding the economic impacts of climate change but also because the existence of this uncertainty may influence adaptation strategies and the choices and optimal decisions made by rational forest managers. Climate change may also have implications for the opportunity costs of sustained yield and our ability to manage forests in order to achieve sustained yield objectives. Thus, there are gaps both in terms of methodological approaches for assessing the economic impacts of climate change in local forest management contexts and in terms of understanding the magnitudes and nature of economic impacts.

CHAPTER FOUR

METHODOLOGY OVERVIEW

Introduction

This chapter provides an overview of the methods used for this study. There are four stages for the analysis presented in this study. They include:

1. Estimation of a yield model that incorporates climate variables.
2. Prediction of statistical distributions for coefficients required for risk models (including present value of net benefits, harvest yields and ending inventory volumes).
3. Development of mathematical programming based risk models to provide the ability to assess the economic consequences of climate change.
4. Application of the risk models in order to assess the economic implications of climate change from a local forest management perspective.

A flow chart illustrating the linkages between these various stages is provided in Figure 4.1. The remainder of this Chapter provides an overview of these various components. More details on the methodology for each element are provided in Chapters 6 to 10.

The stylized forest and the harvest scheduling problem context

This section provides an overview of the stylized forest and the problem context. The stylized forest for this study is a 1000-hectare forest of pure aspen located near Calling Lake, Alberta. The forest is comprised of two stand types. Stand type one is a collection of 40-year old stands. There are 250 hectares of stand type one at the start of the planning horizon. Stand type 2 is a collection of 80-year old stands. There are 750 hectares of stand type two at the start of the planning horizon. The planning horizon is 60 years. There are two 30-year planning periods and two harvest decisions. The forest manager is a private sector individual operating on public land (although for comparison purposes we have also run the models assuming private land ownership). Therefore the manager's primary objective is to maximize the present value of net benefits from the forest subject to sustained yield constraints (i.e. including even flow and ending inventory constraints). There are three possible prescriptions. They include: (a) leave a hectare uncut, (b) cut in period 1, and (c) cut in period 2. The three management prescription options and the cutting schedules are provided in Table 4.1. The stand age at harvest for each prescription is shown in Table 4.2.

One issue to be aware of is that the length of the planning horizon incorporated into the model may affect the results. Typically in operational timber supply analysis, the planning horizon for timber supply analysis is two rotations. For the purposes of this analysis the length of the feasible planning horizon is limited by a lack of availability and/or low reliability of climate scenario information 160-200 years in the future, and increases in computing requirements for analysis over longer time periods. The results of this study should be interpreted with this limitation in mind.

Estimation of yield models with climate predictors

The first methodological objective is to estimate a model that can be used to predict aspen stand yield (see step 1 in Figure 4.1) under present and future climate conditions. The approach is to estimate and evaluate different functional forms and then select a functional form for prediction. The detailed methodology and results are presented in Chapter 6.

A general model for yield prediction for the climate model includes age, site class, density, climate variables and other variables as independent variables. The general model is as follows:

Y = f (age, site, density, climate)

Where:

Y – stand yield (cu. m. per ha)

Site – site index value and site variables (such as soil characteristics)

Density – some measures of density such as trees per ha. or crown closure

Climate – growing season length, monthly average temperature, climate moisture indexes, seasonal average precipitation, and possibly other variables.

A number of specific functional forms are available including simple linear, exponential (log-linear), double-log, reciprocal and Schumacher functional forms. Fekedulegn et al. (1999) summarize a number of traditional functional forms used in growth and yield modeling including negative exponential, logistic, Chapman-Richards, and Weibull functional forms. For this study three separate and somewhat distinct functional forms are evaluated. They include the reciprocal functional form, the Schumacher functional form and the Chapman-Richards functional form.

The approach for incorporating climate variables is to estimate a yield function using data that covers a range of sites. This analogue approach means that it is necessary to obtain data at a number of different geographic locations that are differentiated by unique climate conditions. This approach is similar to Ricardian models used to evaluate the impacts of climate change on agriculture (Mendelsohn et al. 1994; Reinsborough 2003; Weber and Hauer 2003). Cross-sectional aspen yield data are available through the Canadian Forest Service's Climate Impacts on the Productivity and Health of Aspen (CIPHA) project. A description of the CIPHA project is provided in Chapter 5.

One factor that is not considered is CO2 fertilization effects. Measuring the effects of CO2 fertilization would require a database covering a time span long enough that increasing CO2 in the atmosphere would be measurable. The data base used for yield estimation in this study is cross-sectional. Moreover, data on atmospheric CO2 concentrations at each site are not available, and even if the data was available, there is likely limited variation in CO2 concentration across sites.

Expected values and variances of risk model coefficients

Given that there is uncertainty in future climate, predictions of future yields and benefits that are based on climate variables are also uncertain. In other words, net benefits, harvest yields and ending inventory values are random variables. Random variables are characterized by their expected values, the form of their distributions, and measures of dispersion (e.g. their variance). Moreover, since in some cases random variables are interrelated, there may exist covariance between random variables.

Climate change will have two fundamental effects on risk model coefficients for forest management problem analysis. First, it will change expected values of stand yields (and therefore net benefits). Second, it will change the variance around expected values (and therefore the degree of risk associated with forest management). Both of these responses may affect management decisions and timber supply planning under various types of objective functions – particularly if those making choices about timber supply are risk averse.

Thus, the second methodological objective is to estimate: (a) expected values of risk model coefficients, (b) variances around risk model coefficients, and (c) covariances between risk model coefficients (steps 2 – 5 in Figure 4.1). The approach will be to simulate distributions for random variables of interest using Monte Carlo simulation. The software program @RISK (Palisade Corporation 2002) will be used to conduct these simulations.

The variables required for the risk models that will be used to address the analytical objectives include: (a) net benefits (for each stand type, prescription combination), (b) harvest yields (for each combination), and (c) ending inventory values (for each combination). The risk models require measures of expected values and variance for each of the above random variables and a matrix with the covariances between the random variables.

As shown in Figure 4.1, four separate sets of results for the random variables will be used as input data in the risk models. These are referred to as scenarios one, two, three and four. Scenario one assumes climate normal data and it does not include uncertainty. The results using scenario one data on benefits, yields, and ending inventory provide a baseline for comparison with results using predictions that include climate change and various sources of uncertainty. Scenario two predictions are based on predictions of the distribution of future climate variables. Yield model parameters are considered fixed with this scenario so the only source of uncertainty is with respect to the climate variables. Scenario three predictions are also based on predictions of the distribution of future climate variables. However, scenario three also considers yield model parameters to be random variables (i.e. for this scenario we adopt the Bayesian perspective that the parameters of the estimated yield model are random). Therefore, the Monte Carlo simulations include uncertainty in both climate variables and yield model parameters. Scenario four is similar to scenario three with respect to sources of uncertainty and climate effects (i.e. the predictions include climate change productivity effects plus this scenario considers uncertainty in both yield parameters and climate variables). The aspect where scenario four deviates from scenario three is in terms of assumptions regarding period one harvest yields. As will be described in more detail later in this chapter, scenario four assumes that the manager has obtained detailed information about the first period harvest yields (i.e. the uncertainty in the first harvest period is eliminated by a detailed stand inventory for example). A more detailed description of the methodology employed is provided in Chapter 7.

Risk programming analysis of timber supply

The third methodological objective is to solve and compare different types of risk programming timber supply optimization models. The approach for this objective is to incorporate the fixed values, expected values and covariance matrices on net benefits, harvest yield, ending inventory yield coefficients, and various deterministic scalars into risk programming models (step 6 in Figure 4.1). Three types of risk models are developed. The first is called the Markowitz asset allocation model. This model minimizes the variance of a portfolio subject to earning a minimum return. A more detailed description of this model is provided in Chapter 8.

The second model is an expected value-variance- chance constraint hybrid model. The objective function for this model is to maximize certainty equivalent (where certainty equivalent is equal to net benefits minus a penalty for risk - see Chapter two) subject to area constraints, flow constraints, and ending inventory constraints. This model incorporates risk preferences and behavioral response to risk. In this case the risk model is based on a Model I timber harvest scheduling timber supply model. A more detailed description of this formulation is provided in Chapter 9.

The third type of model is a discrete stochastic programming (DSP) model. The objective function is to maximize net present value of benefits subject to area, flow and ending inventory constraints. DSP models allow the decision maker to adapt as uncertainty becomes resolved over time. Therefore, the DSP model developed for this study is used to evaluate the influence of recourse on estimation of the net economic impacts of climate change in a forest management context. A more detailed description of the specific DSP formulation and the results provided by this model are provided in Chapter 10.

Sources of uncertainty for each risk model formulation

We have developed four different sets of model coefficients and covariance matrices. These sets provide the input data for the risk models. They are referred to as scenarios one, two, three and four. Each of the input data sets reflects a unique set of assumptions about climate and about sources of variability in predictions. The objective function coefficients for scenario one are based on climate normals for the study area. Also, the variables are viewed as fixed (i.e. there is no uncertainty relative to the predicted values). The objective function and constraint coefficients for scenarios two, three and four are considered to be random variables. Their distributions are estimated using Monte Carlo simulation (see Chapter 7). The estimates of future values for specific coefficients are the expected values obtained from the estimated sample distributions. The covariance matrices are also derived from the sample distributions generated by the Monte Carlo simulation. In the case of scenario 2, the coefficient distributions are based on predictions of the distribution of climate variables only. In the case of scenario 3, the coefficient distributions are based on predictions of the distribution of climate variables and on estimates of the distributions of yield model parameters. Thus, the scenario 3 values incorporate an additional source of uncertainty compared to scenario 2. In the case of scenario four, we have considered the possibility that variances in variables in the first planning period will be removed through some action taken by the decision maker to intensively measure the forest. Thus, yield for first period harvest is fixed (i.e. there is no variance associated with the first period yield). Scenario four does, however, still assume that there is uncertainty in second period harvest and in ending inventory yields.

The three risk models discussed above use different scenarios and/or combinations of scenarios for their data. The Markowitz model is primarily concerned with risk/return tradeoffs. We assume that the decision maker is a rational investor with an interest in incorporating uncertainty in his/her decisions and choices. For this model we have used scenarios 2 and 3 as input data (Table 4.3). The objective function for the expected value-chance constraint model is to maximize certainty equivalent values. The expected value – chance constraint model determines the optimal solution based on risk and returns as well as the explicit risk preferences of the decision maker. All four scenarios have been used to generate objective function values and solutions using this formulation (Table 4.3). In the case of the discrete stochastic programming model we have used scenarios 2 and 3 estimations for the input data. In this case the decision maker faces uncertain outcomes but he/she is risk neutral. The main contribution of this model is that it permits the decision maker to adjust his/her decisions as uncertainty becomes resolved over time.

A final point to note regarding sources of uncertainty that are incorporated into the models developed and discussed in this study is that potential sources of uncertainty are restricted to climate uncertainty and uncertainties relative to yield estimations (for example drought, insects and diseases, and physiological factors affecting growth and yield). The analysis in this study does not consider uncertainty in economic variables such as prices and discount rates. The assumption is that these values are fixed. A useful extension of the analysis undertaken in this dissertation would be to consider price trends and price and discount rate variability as additional sources of impacts and uncertainty relative to understanding climate effects at a local scale.

Economic analysis

The approach for assessment of the net economic impacts of climate change and climate uncertainty will be to run separate models that are differentiated by the extent to which they consider climate effects and uncertainty effects (i.e. the models will be run with different scenarios). The first model (the Markowitz asset allocation model) is designed to consider tradeoffs between portfolio risk and potential returns. Some specific questions that are addressed with this model are:

1. What is the shape of the return-risk frontier for forest management with climate change and how do assumptions regarding sources of uncertainty affect the this frontier?
2. What are the relative magnitudes of climate variance vs. yield parameter variance as sources of variance for this problem?
3. How might biased perceptions of real risk influence choices?

The second model (the EV-chance constrained hybrid model) is designed to identify the optimal harvest pattern given changes in productivity over time and changes in degree of uncertainty in model coefficients over time. Some questions that are considered using the results of this model include:

1. What are the economic impacts of climate change and uncertainty with and without sustained yield constraints?
2. How sensitive are the model results to different assumptions about parameter values?

The third model (Discrete Stochastic Programming with recourse) is designed to evaluate the present value of net benefits under climate change where uncertainty is sequential and where uncertainty about some variables is resolved at certain points in time within the planning horizon. In effect, this model incorporates the adaptive responses of decision makers over time in assessing the net economic impacts of climate change. Some questions that are addressed using the DSP model formulation are:

1. What is the effect of recourse (ex post adaptation) on economic returns from forest management?
2. What is the effect of risk prevention (ex ante adaptation) on economic returns from forest management?

Figure 4.1 Sequence of analysis and linkages between models.

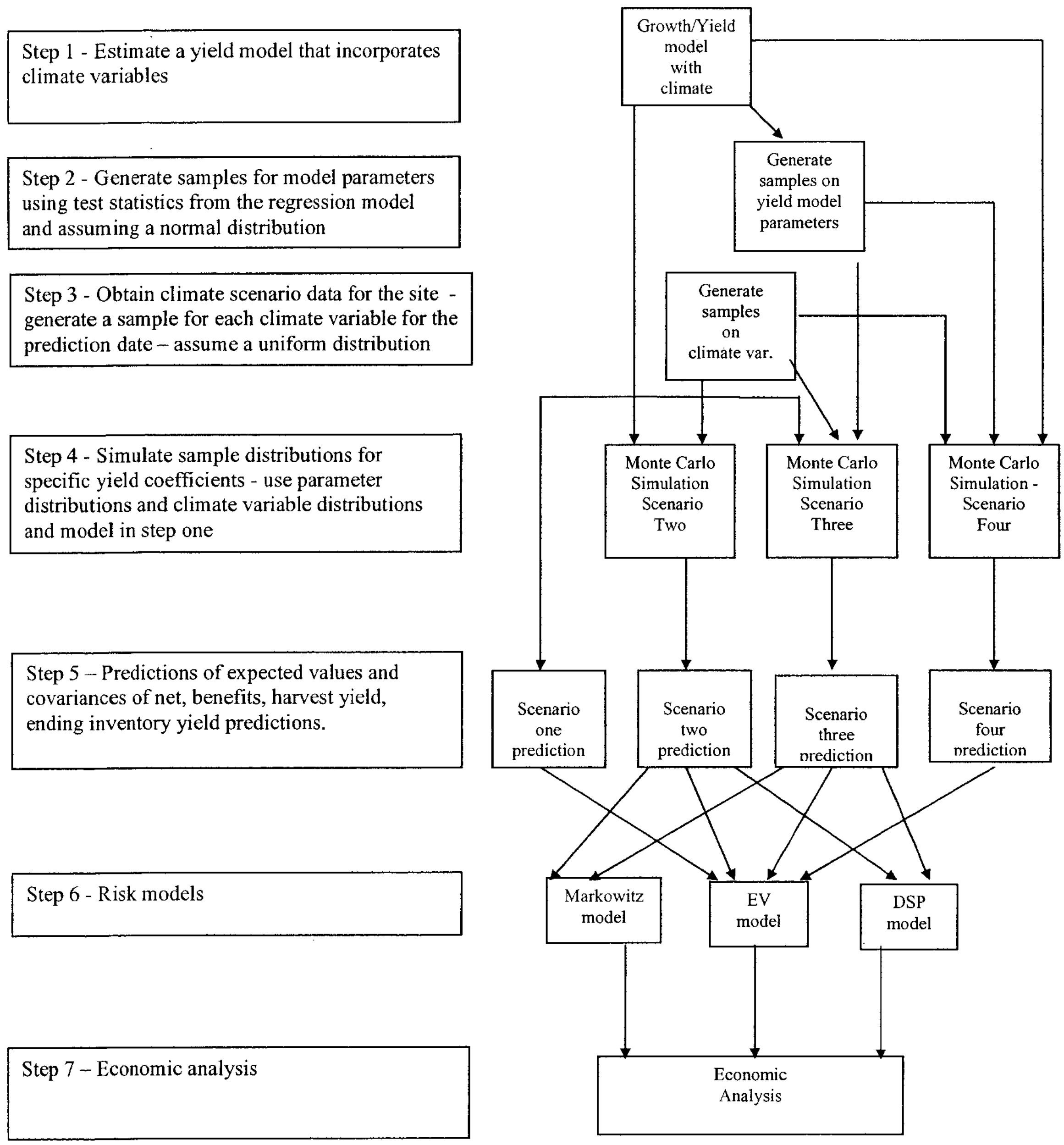


Table 4.1. Description of management prescriptions		
	Period 2010 – 2039	Period 2040 – 2070
Management Prescription	Base year 2025	Base year 2055
1	-	-
2	X	-
3	-	X

Where "X" indicates harvest and reforest and "-" indicates no action.

Table 4.2. Age and year of harvest combinations

	Starting age		
	40	80	
Management Prescription	Age at harvest		Year of harvest
1	-	-	-
2	55	95	2025
3	85	125	2055

Table 4.3 Scenarios used for each risk model

	Markowitz	EV-Chance constraint	Recourse model
Scenario 1		X	
Scenario 2	X	X	X
Scenario 3	X	X	X
Scenario 4		X	

CHAPTER FIVE

DATA

"It is a capital mistake to theorize before one has data."
Sir Arthur Conan Doyle

Three types of data are required for this study. First, cross sectional mensurational and climate history data are required for estimating a yield model that includes climate variables. Second, climate scenario data (i.e. climate predictions) are required for prediction of future yields and benefits under different climatic conditions. Third, data specific to the study site (Calling Lake, Alberta) are required for yield and benefit predictions. This chapter describes the data, identifies data sources and provides a description of methods used for data derivation.

Data used for yield model estimation [7] [8]

As was described in Chapter four, the first step in this study is to estimate a yield model. Variable density yield models relate stand yield $(m^3 \cdot ha^{-1})$ to predictor variables. Predictor variables traditionally included in empirical yield models are age, stand density (measured as stems per hectare or basal area), and site index (Clutter et al 1983). For this study we also consider the influence of other site variables such as soil characteristics and location (i.e. a boreal forest location vs. an aspen parkland location). Because the estimated yield model will be used as a predictive model for future timber supply, it is also necessary to incorporate climate variables into the yield model. The set of climate variables that are considered for each estimation model include: annual average temperature, annual average precipitation, average precipitation from May to September, and soil moisture. A description of all variables, the rationale for their consideration in the yield model, and data sources is provided in the remainder of this chapter.

[7] Mike Michaelian and Ted Hogg at the Northern Forestry Centre provided immeasurable assistance in terms of understanding factors affecting aspen yields, understanding the CIPHA data and in developing the methodologies for determining merchantable volume and site index for the CIPHA plots.
[8] The data used for estimating the yield model and for obtaining yield predictions is available upon request.

The yield model for this study is estimated using cross sectional data representing a gradient of climate conditions under which aspen currently grows. The source of the mensurational data used to estimate the yield model is CFS research plots that are part of the Climate Impacts on Productivity and Health of Aspen (CIPHA) study (Study leaders: Ted Hogg, James Brandt – Northern Forestry Centre, Edmonton). Detailed data on forest stand characteristics, individual tree measurements within plots, climate histories, defoliation histories, and drought history are contained in the CIPHA database. The study is described as follows:

> "The CIPHA study includes a network of long-term research plots in pure, undisturbed aspen stands across the western Canadian interior, extending from the Northwest Territories to southern Manitoba. The CIPHA study design for this core region consists of 25 study areas (nodes), with three stands per node and two plots per stand, with 13 nodes located in the boreal forest and 12 nodes in the more prairie-like aspen parkland zone)." (Hogg, et al. 2002) Figure 5.1 illustrates the research design for the CIPHA study.

The geographic location of CIPHA plot sites is shown in Figure 5.2. For the purposes of this study there are 140 useable observations in the CIPHA sample database. The variables obtained for each observation are defined and described in the remainder of this section.

The dependent variable for the yield model is merchantable timber stand yield **(MVOL)**[9]. Timber stand yield is defined as the merchantable volume $(m^3 \cdot ha^{-1})$ of standing timber in a homogenous stand on a particular site at a particular age (note stands are assumed to be even age stands of pure aspen). A common standard for determining merchantability in Alberta is the 15 – 10 rule. Merchantability is defined as the total stem volume, down to a top diameter inside bark of 10 cm, on all trees that are larger than 15 cm diameter at breast height (DBH). The method used to derive merchantable volume per ha for the individual observations in the database

[9] It was determined in later stages of this study that the use of merchantable volume for the dependent variable has important implications for yield model functional form. Specifically, restricting the dependent variable to larger and older trees means that young age classes are not represented. In turn this means that our data is representative of only the concave portion of the "s" shaped biological growth relationship. This is discussed in more detail in Chapter 6 and Chapter 11.

used for this study follows methods outlined in Huang (1994). First, all trees that met the 15-10 merchantability standard within each CIPHA plot were identified. Second, merchantable volume per tree was estimated using taper equations (Kozak 1988) and tree volume equations (i.e. Newton's formula (Huang 1994)) parameterized for aspen growing in various ecoregions in Alberta (see Huang 1994). This procedure essentially involves dividing the merchantable stem into a series of disks, calculating the volume of each disc and them summing the volume in each disk to obtain a total volume for the merchantable portion of the stem. Third, the volume for each merchantable tree in the plot was summed to obtain a total merchantable volume for the plot. Fourth, the merchantable volume per plot was converted to merchantable volume per ha[10].

The previous paragraph describes the method used to derive the dependent variable for the yield model for each CIPHA plot. The remaining paragraphs in this section identify and describe the independent variables required for yield model estimation. A standard and obvious variable for yield model estimation is **AGE.** The average age for trees in CIPHA plots is determined by increment boring.

It can be reasonably hypothesized that plots located in the northern boreal forest are qualitatively different than plots located on southern aspen parkland sites. In northern latitudes growing season length is shorter but the length of daily photoperiod during the growing season is longer. With respect to boreal forest sites, these sites are often wetter and/or have different types of soil structures. Therefore, there may be qualitative differences between northern boreal sites and southern aspen parkland sites that are not captured by the other independent variables. A dummy variable called **ZONE** is included to account for qualitative differences between the boreal forest and aspen parkland sites. The variable has a value of "1" for boreal sites and "0" for aspen parkland sites.

Soil characteristics can affect stand productivity by affecting drainage, moisture holding capacity and oxygen availability within root layers. Two separate variables are included for consideration in the estimations. They include **CLAY**

[10] The estimation of merchantable volume per plot and site index was conducted by Michael Michaelian at the Northern Forestry Centre.

(percent clay within the soil) and **SAND** (percent sand within the soil). Some sand within a soil contributes to drainage. However, a high percentage of sand in soil results in limited water holding capacity. Therefore, the expected sign on the sand coefficient is negative. In contrast, some clay within a soil may increase water-holding capacity. However, high percentages of clay within soils may limit the ability of soils to absorb moisture. Therefore, a moderate proportion of clay within a soil is beneficial for tree growth. However, as the percentage of clay increases, site productivity is expected to decline.

Site index is often used to evaluate differences in productivity for different stand locations. Site index measures the height of dominant and co-dominant trees at some reference age. Site index is a relative measure of the productivity of a site for a particular species where site is described as "the totality of environmental conditions (biotic, edaphic, and climate) existing at a particular location" (Clutter et al. 1983, pg 31). Site index was calculated for each CIPHA plot location by cutting down two representative trees adjacent to each plot and determining the height at age 50. Site index is affected by a number of factors – many of which are incorporated as separate independent variables (e.g. soil structure). However, it is also possible that site index will be an indicator of factors determining stand yield that are not picked up by other variables – nutrient availability for example. Therefore the variable **SITE** is also included in the estimations for consideration as an independent variable.

The density of trees on the site also affects stand yield and productivity. If stands are over stocked, trees growth is suppressed. A fundamental premise behind thinning as a silviculture treatment is that such treatments can either increase overall yield or redistribute yield volume from a large number of small trees to fewer larger trees. Theory and silvicultural practices, therefore suggest that **DENSITY** of stands may influence yields.

The influence of climate variables on stand yield is a primary interest for this study. It is recognized that the relationships between particular climate variables and stand growth and yield is complex and that a large number of interrelated factors probably should be considered. However, there is little precedence in the literature for studies relating climate variables to stand yield across large geographic areas.

Hogg (1994) notes that temperature, precipitation, and moisture are key variables. For the purposes of this study we have incorporated the following local climate variables for estimation purposes: **ANTEMP** (for average annual temperature), **ANPREC** (for average annual precipitation), **GSPRECIP** (for average precipitation during the May to September growing season) and **MOIST** (a moisture index calculated by the Jenson-Haise method) (note that the source of the climate normal data for each CIPHA plot is historical climate data for weather stations nearest each plot – in some cases multiple weather station data is used). The effects (and therefore the signs) on the climate coefficients is difficult to predict prior to estimation. There are complex interactions between the variables themselves and between the variables and aspen stand response. For example, increased annual average temperature could increase stand productivity by being associated with higher rates of metabolism or lengthened growing seasons. However, higher temperature is associated with higher levels of evapotranspiration. If evapotranspiration exceeds precipitation, moisture deficits may result.

In summary the following variables are considered in the various estimations undertaken.

MVOL:	Merchantable volume (m^3) per hectare (15-10 utilization standard)
AGE:	Stand age (years)
ZONE:	Dummy variable (aspen parkland = 0, boreal = 1)
SITE:	Site index (ht in meters at reference age 50)
LAT:	Latitude (degrees)
DENSITY:	Stems per hectare (#)
SAND:	Percent sand in soils
CLAY:	Percent clay in soils
ANTEMP:	Average annual temperature (degree centigrade)
ANPREC:	Average annual precipitation (millimeters)
GSPRECIP:	Average precipitation between May and September (mm)
MOIST:	Moisture index (Jenson-Haise).

Data for each of the above variables was obtained for the 140 useable CIPHA plots used in the estimation of the yield model. Table 5.1 provides descriptive statistics for these variables. Merchantable volume per hectare (MVOL) falls within the expected range of values. The forest industry generally expects mature stands of aspen to have 200 – 250 cubic meters per hectare on average sites in the northern boreal forest[11]. In some cases aspen yield is over 400 cubic meters per hectare of merchantable volume. In the sample the maximum value for merchantable volume is 454 cubic meters per hectare and the minimum value for stand volume is 5 cubic meters. Therefore the sample represents a good range of stand types. The mid-point value for the range of values for merchantable volume is 229.5 cubic meters per hectare. A comparison of the mid-point (229.5) to the mean (162.4) indicates that the observations are skewed toward the lower end of the range of values (probably because the merchantable volumes for aspen parkland sites tend to be lower at particular ages compared to boreal forest sites). The standard deviation for MVOL is 102.4. Thus sixty-eight percent of the observations on merchantable volume are between 60 and 264 cubic meters per hectare and therefore the majority of observations are within the normal operability range for harvesting operations.

Plot age (AGE) ranges from 28 years to 97 years. Therefore, a good range of ages is represented within the data. However, 68 % of the observations are between 51 years and 76 years (i.e. one standard deviation on each side of the mean). So the observations are somewhat grouped around the mean age of 63.6 years. The mid-point age (62 years) is close to the mean age (63.6) so the observations do not seem to be skewed.

Climate scenario data

Since yield is a function of climate and since future climate will be different than present climate, predictions of future yields require predictions of future climate. However, future climate cannot be known with any degree of certainty. Future climate variables are random variables. It is possible to obtain information regarding the potential range of values for climate variables at future points in time by considering the range of predictions provided by various combinations of general circulation models (GCMs) and future emission scenarios.

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[11] Source: Grant Williamson: Timber Operations Forester with Ainsworth in Grande Prairie.

The source of climate scenario data used for this study is the Canadian Institute for Climate Studies (CICS) web site.[12] Sixteen predictions of future values of annual temperature, annual precipitation and growing season precipitation were developed for the Calling Lake study site. Predictions are obtained for the 2020's, 2050's and 2080's. The predictions are based on different combinations of general circulation models driven by different sets of future emission scenarios. We use the outputs of three models including 1. The Canadian General Circulation Model, 2. The Australian CSIRO model, and 3. The UK Hadley Centre GCM. The models are driven by eight possible greenhouse gas emissions scenarios recently developed by the Intergovernmental Panel on Climate Change (2000). The eight scenarios are part of four families of emissions scenarios: A11 is from the A1 family; A21, A22, A23, and A2X are from the A2 family; B11 is from the B1 family; and B21, B22 are from the B2 family. These various families of scenarios are based on different storylines of future economic development based on different assumptions about population growth, energy use, technological change, and income distributions. Basically the A2 scenarios result in highest levels of GHG emissions. The B1 scenario family results in the lowest levels of emissions. IPCC (2000) provide a more detailed description of the specific storylines underlying each emissions scenario.

For the purposes of this study, the first six predictions are based on predictions by the Canadian General Circulation Model using the following IPCC Special Report on Emission Scenarios (SRES): A21, A22, A23, A2X, B21, and B22. The next four predictions are based on the Australian CSIRO GCM using the following SRES's: A11, A21, B11, B21. The next six predictions are based on the UK Hadley GCM model using the following SRESs: A21, A22, A23, A2X, B21, B22.

Obtaining data from the CICS interface site involves downloading change data for the variable of interest, for the future time period of interest, and for the particular combination of general circulation models (GCMs) and emission scenarios (SRES) combinations. The change data are then applied to climate normal data (i.e. average value of climate variables for the period 1961 to 1990) for the Calling Lake site. For the purposes of this study the high and low prediction values for the 16

[12] located at: http://www.cics.uvic.ca/

predictions define the boundaries of a uniform distribution for the climate variable of interest. The distributions of the climate variables obtained from the procedure described above are shown in Table 7.2 in Chapter Seven.

Other values and variables

The final data requirements for modeling purposes include fixed values for price, discount rate, soil features, site index, and stand density. The price of aspen is assumed to be a constant value of $ 2.50 per cubic meter over the planning horizon (Table 7.1). This price is based on prices provided on Alberta SRD's timber damage appraisal tables.[13] The assumed value of the real discount rate is 4 % (see Row et al. 1981 and Thomson 1992). Soil characteristics, site index, and climate normals are obtained directly from a specific CIPHA plot at Calling Lake. The percent sand is 34 % and the percent clay is 19 %. The site index is 19.8. Stand density by age (see Table 7.1) is based on information provided in Tables 8 and 9 in Peterson and Peterson (1992).

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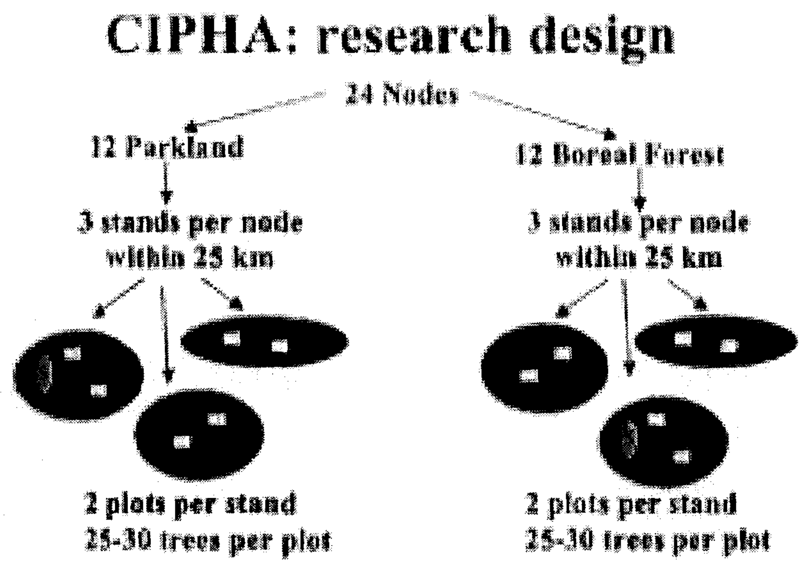
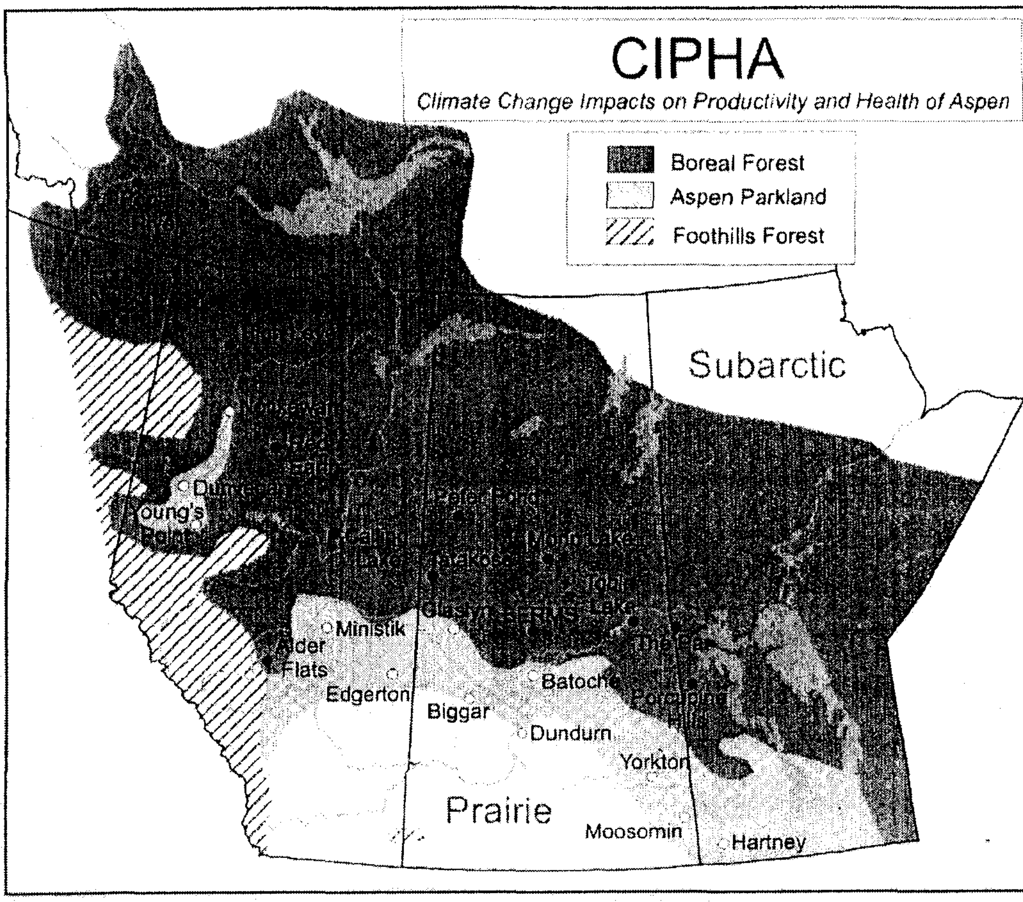
[13] web site located at: http://www3.gov.ab.ca/srd/land/m_li_timberdamage.html

Figure 5.1. Diagram showing the research design for the CIPHA study.



Source Hogg et al 2002

Figure 5.2 Map showing the location of nodes for the CIPHA study.



Source: Hogg et al 2002

Table 5.1. Summary statistics of the variables

Variable	Mean	St. Dev	Minimum	Maximum
MVOL	162.40	102.37	5.32	454.22
AGE	63.63	12.47	28	97
SITE	16.75	3.75	9.01	26.83
LAT	54.36	2.70	49.47	61.34
DENSITY	1531.40	669.73	473.68	3600
SAND	44.74	23.64	5	90.02
CLAY	21.59	14.37	5.85	67.66
ANTEMP	0.80	1.53	-3.80	3.60
ANPREC	439.61	42.82	354	518.80
GSPREC	287.77	30.76	208.30	326.30
MOIST	1.91	8.51	-21.76	19.86

n=140

MVOL – Merchantable volume per hectare (cu. m. per ha) – 15 cm DBH – 10 cm top
AGE – In years
SITE – Height at age 50 (meters)
LAT – Latitude of the plot (degrees)
DENSITY – Stems per hectare (#)
SAND – Percent sand in soil (%)
CLAY – Percent clay in soil (%)
ANTEMP – Average annual temperature (deg C)
ANPREC – Average annual precipitation. (millimeters)
GSPREC – Average precipitation from May to September (millimeters)
MOIST – Average moisture index using the Jenson-Haise calculation method.

CHAPTER SIX

YIELD FUNCTION ESTIMATION RESULTS

Introduction

This chapter presents the results of the yield equation estimations. Three different prediction models are described, estimated and compared. Two of these models are traditional functional forms for forestry yield models: the Schumacher functional form (Schumacher 1939; Sullivan and Clutter 1972) and the Chapman Richards functional form (Richards 1959; Chapman 1961; Liu and Li 2003). The shape of the yield-age relationship with these functional forms is a curve that is sigmoid shaped. The third model (the reciprocal functional form) is a more general functional form (Griffiths et al. 1993). The reciprocal functional form is concave with respect to the x-axis and it intercepts the x-axis as $x \to 0$. To our knowledge the reciprocal functional form has not been previously used in forestry applications. However, as will be described in this chapter, the reciprocal functional form is very similar to the Schumacher functional form.

This chapter is organized as follows. First, various options for yield prediction are identified and the reasons for the approach adopted for this study are discussed. Second, the origins and properties of each of the three functional forms are discussed. Third, the estimation results for the three functional forms are provided. Fourth, the estimation results are compared and assessed and a model is selected for refinement and further analysis. In the final section, further refinements are made to the selected model (the reciprocal model) in order to ensure an appropriate specification for simulations of yield response to climate variables at a boreal forest site in central Alberta.

Yield prediction methodologies

There are a number of different approaches for stand yield modeling (Clutter et al. 1983). These methods include:

1. Whole stand normal yield tables.
2. Variable density whole stand yield models (or empirical regression based

modeling approaches where the whole stand is one unit of observation).

3. Diameter-distribution based yield models (i.e. the stand is broken down into a series of diameter classes and growth and yield within each diameter class is modeled).

4. Individual tree models – distance dependent or independent (growth/yield of individual trees is modeled or simulated and then aggregated to the stand level).

5. Multiple / simultaneous equation simulation models.

The normal yield table approach is deterministic and therefore it is not suited to the kinds of probabilistic predictions of future yields that we require for this analysis. Methods 3, 4 and 5 are more rigorous approaches for simulating and predicting future stand yields. However, these methods require data for estimating aspen response to climate that is not currently available (e.g. – tree distributions by diameter class, distance between trees, mortality functions). Method two has a number of advantages. First, the data required for estimating empirical yield models are available from the CIPHA database (described in Chapter 5). Second, it is relatively straightforward to specify and estimate models that directly incorporate climate variables. Third, this method is commonly used in operational timber-supply analysis and forest management planning. Therefore, for the purposes of this study we have adopted a variable density - whole stand empirical yield modeling approach for estimating stand yield prediction functions.

This study relies on the use of stand level inventory (or yield) prediction functions to account for the effects of climate change on future expected stand yields. An alternative approach would be to estimate growth functions. Growth and yield functions are closely related. Yield functions describe the accumulated volume of inventory (either merchantable or total volume) on a site at various stand ages (a kind of stand level production function). A growth function describes the incremental volume (again merchantable or total volume) that is added as the stand grows one period. Thus, a yield function is the integral of a growth function. Similarly, the derivative of a yield function provides a growth function. Growth and yield functions can be estimated separately or jointly. For this study, a lack information on growth means that it is only possible to estimate a yield function.

Functional forms

Selection of a functional form that is supported by the data and is consistent with theory is important for modeling. Prior knowledge of the relationships between variables will have a bearing on functional form selection and model specification. Forest stands are biological systems. The expected pattern of growth in biological systems is sigmoidal. However, the estimation of sigmoidal yield functions requires that all age classes be represented in the data (Fekedulegn et al. 1999). In our case the dependent variable is not total volume. Rather, merchantable volume is used. In the case of merchantable volume, the dependent variable only reflects the volume that is present on larger and older trees (i.e. those trees that are > 15 cm dbh). Yield functions (estimated with merchantable volume as the dependent variable) must be able to capture the portion of the sigmoid yield curve that is concave with respect to increasing age and should not be forced to pass through the origin[14]. Three functional forms are suited to estimation of either sigmoid shaped yield functions or functions that are concave with respect to age. These functions include: reciprocal functional form, Schumacher (log-inverse) functional form, and Chapman-Richards functional form.

Reciprocal functional form

The reciprocal functional form is not a standard functional form for timber yield modeling. This type of functional form is, however, suited to estimation of models where the algebraic form of a relationship is a concave curve that intercepts the x axis at a point other than the origin (Griffiths, et al. 1993). The general form of a reciprocal relationship between age and yield is as follows:

$$Y = \beta_0 + \beta_1 \frac{1}{Age} \qquad [6.1]$$

[14] Another data related issue is that we have estimated the yield functions using cross sectional data. Use of cross sectional data may, however, may be problematic because of the effects of unobserved factors that vary spatially. Thus, predictions of yield at a particular location using yield functions that are estimated from cross sectional data that does not capture all effects at a specific site could potentially be biased.

The properties of the reciprocal functional form are as follows. The reciprocal function is concave if $\beta_1 < 0$. Figure 6.1 illustrates the case where $\beta_1 < 0$. If $\beta_1 < 0$, then β_0 can be interpreted as the upper asymptote of the function (i.e. $Y \to \beta_0$ as $Age \to \infty$). The reciprocal functional form intercepts the age axis at the point $(-\frac{\beta_1}{\beta_0})$. The above properties suggest that this functional form may be well suited to modeling yield / age relationships in cases where the data are restricted to trees above a certain size (or implicitly for stands that are above a certain age).

In addition to being influenced by age, timber stand yield is influenced by site (represented by site index), soil characteristics (represented by percent sand, silt and clay), stem density, geographic location (e.g. boreal forest vs aspen parkland), and local climatic factors. In theory, each of these variables has the potential to influence maximum yield potential (i.e. the asymptote of the yield function). Thus, in order to incorporate these variables into the reciprocal function, it is necessary to redefine the asymptote term (i.e. the constant).

The reciprocal variable density yield function with a redefined constant (that includes site, soils, location, climate, density and a dummy variable for boreal forest plots) within the asymptote is shown as follows:

$$Y = \beta_0 + \beta_1 ZONE + \beta_2 SITE + \beta_3 LAT + \beta_4 DENSITY + \beta_5 SAND + \beta_6 CLAY + \beta_7 ANTEMP + \beta_8 ANPREC + \beta_9 GSPREC + \beta_{10} MOIST + \beta_{11} \frac{1}{AGE}$$

[6.2]

Schumacher functional form

One of the earliest variable density yield model functional forms used in forestry was the Schumacher-type yield model. MacKinney et al. (1937) and Schumacher (1939) were the first to suggest this type of functional form. More recently this functional form has been incorporated into simultaneous equation systems that jointly estimate growth and yield functions (Buckman 1962; Sullivan and Clutter 1972).

The general form of the Schumacher-type yield model (Clutter et al. 1983) is as follows:

$$LnY = \beta_0 + \beta_1 \frac{1}{Age} + \beta_2 f(Site) + \beta_3 g(Density) \qquad [6.3]$$

The first thing to note about the above functional form is its similarity to the reciprocal function described in the previous section. The functions are similar in all respects except that the value for the dependent variable is "LnY" for the Schumacher type yield model and it is "Y" in the case of the reciprocal functional form. The second thing to note is that the third term is a function of site and the fourth term is a function of density. Thus, site and density variables may be defined in terms of their logarithms, their reciprocals, or in some cases, the equation may be estimated with these variables un-transformed.

Although the mathematical expressions of the Schumacher and reciprocal functions are similar, the shapes of these two functions are quite distinct. As noted earlier, the shape of the reciprocal yield function in a two-dimensional yield / age plane is a curve that is concave to the x-axis and that intersects the x-axis. The Schumacher-type yield model, alternatively, has a sigmoid shape and it goes through the origin.

The Schumacher-type variable density stand yield functional form has some parallels in empirical economic analysis. The log-inverse function is a functional form that is sometimes used to model company sales as a function of advertising effort (Griffiths et al. 1993). The non-linear form of the log-inverse function is:

$$Y = \exp\left\{\beta_0 + \beta_1 \frac{1}{X}\right\} \qquad [6.4]$$

And the linear form of this model is:

$$LnY = \beta_0 + \beta_1 \frac{1}{X} \qquad [6.5]$$

The algebraic relationship between yield and age with the Schumacher yield function is s-shaped (or sigmoid shaped) (i.e. if $\beta_1 < 0$ then

$\beta_1 \frac{1}{X} \to -\infty$ *as* $X \to 0$ *and* $\beta_1 \frac{1}{X} \to 0$ *as* $X \to \infty$). Thus, equation [6.5] is asymptotic to β_0. As was the case for the reciprocal functional form, climate variables are incorporated into the asymptotic (β_0) parameter of the model. A re-specified Schumacher model with climate variables is provided as follows:

$$LnY = \beta_0 + \beta_1 ZONE + \beta_2 SITE + \beta_3 LAT + \beta_4 DENSITY + \beta_5 SAND + \beta_6 CLAY$$
$$+\beta_7 ANTEMP + \beta_8 ANPREC + \beta_9 GSPREC + \beta_{10} MOIST + \beta_{11} \frac{1}{AGE} \quad [6.6]$$

Chapman Richards functional form

A more contemporary functional form for yield prediction modeling (compared to the Schumacher functional form) is the Chapman-Richards model (Richards 1959; Chapman 1961). Studies that have used a Chapman-Richards formulation include Pienarr and Turnbull (1973), Pienarr (1979), Zhang et al. (2002), Liu and Li (2003).

The Chapman-Richards model is the integral form of a basic differential growth equation for measuring rates of growth of biological organisms or systems (Clutter et al. 1983). The equation upon which the Chapman-Richards model is based is represented by the following differential:

$$\frac{dY}{dt} = \alpha Y^{\beta} - \gamma Y \quad [6.7]$$

The first right hand side term is the "anabolic growth rate" or "constructive metabolism." The second right hand side term is the "catabolic growth rate" or "destructive metabolism" (Clutter et al. 1983). Both terms are proportional to the size of the organism or biological system (such as a forest stand for example). The first term dominates when the biological system is relatively small (young) but as the system grows (ages) the second term has a more significant influence.

The integral of equation [6.7] provides the generalized form of the Chapman-Richards yield model (Liu and Li 2003). This general functional form is provided as follows:

$$Y_t = \theta\{1 - \tau \cdot \exp(-\gamma Age)\}^{(1/1-\beta)} \qquad [6.8]$$

The Chapman-Richards functional form has a sigmoid shape and therefore it is a form that is consistent with the expected pattern of growth of a forest stand as it ages over time. Before estimating a relatively complex, non-linear model such as Chapman-Richards it is important to have an understanding of the meaning of the parameters of the model. Fekedulegn et al. (1999) and Liu and Li (2003) describe the correct interpretation of the parameters in Chapman-Richards types models. These interpretations are summarized as follows:

- θ - Represents the maximum value that stand yield can attain (i.e. it represents the upper asymptote of the growth curve).
- τ - Represents the biological constant.
- γ - Represents the rate parameter. This parameter defines the rate of growth – or the rate at which yield approaches its upper asymptote.
- β - Represents the "allometric" constant. This parameter defines the shape of the function and its inflection point.

For the purposes of this study we are interested in extending the basic model to incorporate additional explanatory variables (location, site, density, soils, local climate). Liu and Li (2003) show that the upper asymptote for the Chapman Richards model is related to the three parameters:

$$\theta = \left|\frac{\alpha}{\gamma}\right|^{\frac{1}{1-\beta}} \qquad \text{(Note: } \alpha \text{ is from equation [6.7])} \qquad [6.9]$$

Our expectation is that the traditional yield variables (site index, density, zone) and the climate variables affect both the anabolic and catabolic growth of stands. Since the parameters that determine these rates of growth are embedded within the parameter describing the upper asymptote of the yield function (θ), it is logical that these additional parameters be incorporated into the parameter θ. Therefore, we redefine θ as follows:

$$\theta = f(\beta_0, ZONE, SITE, DENSITY, SAND, CLAY, ANTEMP, ANPREC, GSPREC, MOIST, LAT) \qquad [6.10]$$

The above variables were defined and described in Chapter 5. The re-specified Chapman Richards model is as follows:

$$
\begin{aligned}
&LnY = (\beta_0 + \beta_1 ZONE + \beta_2 SITE + \beta_3 DENSITY + \beta_4 SAND + \beta_5 CLAY \\
&+\beta_6 ANTEMP + \beta_7 ANPREC + \beta_8 GSPREC + \beta_9 MOIST + \beta_{12} LAT) \\
&+\beta_{10} \ln(1 - \beta_{13} e^{(-\beta_{11} AGE)}) \\
&Where: \\
&\theta = \exp[\beta_0 + \beta_1 ZONE + \beta_2 SITE + \beta_3 DENSITY + \beta_4 SAND + \beta_5 CLAY \\
&+\beta_6 ANTEMP + \beta_7 ANPREC + \beta_8 GSPREC + \beta_9 MOIST + \beta_{12} LAT]
\end{aligned}
\qquad [6.11]
$$

Estimation procedures and results

The Reciprocal model

The reciprocal model is linear in the parameters and is therefore a fairly straightforward model. The reciprocal yield model was estimated using ordinary least squares regression with SHAZAM (Version 10) (Northwest Econometrics 2004). Table 6.1 provides the estimation results for the reciprocal functional form. Two models are presented. One model estimates yield as a function of ZONE, SITE, LAT, DENSITY, SAND (percent sand), CLAY (percent clay), and AGEINV (the inverse of age). The second model adds in the following climate variables: ANTEMP, ANPREC, GSPREC, MOIST (all variables were described and defined in Chapter 5). Equation [6.2] shows the specification for the second model. The first model is nested within this specification. The reason for estimating two models is to compare and contrast the estimation results for yield models with and without climate variables.

The reciprocal yield model provides a reasonably good fit to the data. The R-squared for model 1 (no climate variables) is 0.71. Including climate variables increases the R-squared value to 0.74. Model 2 has 12 coefficients. All variables (except DENSITY, ANPREC and MOIST) are significant at the 5 % level. The p-value shows that DENSITY is significant at the 16.7 % level. All the variables with this estimation have expected signs. AGE, ZONE, SITE, ANTEMP and MOIST have positive effects on stand yield. LAT, DENSITY, SAND, CLAY, and GSPREC have negative effects on stand yield. One result that is intriguing is the negative coefficient value on GSPRECIP. Intuitively one would expect that average precipitation during the growing season would have a positive effect on yield. However, aspen productivity is, in fact, sensitive to excess precipitation. High moisture levels may contribute to increased decay and pathogen mortality in aspen stands (Source: Dr. R. Yang, Forest mensuration researcher, Canadian Forest Service – personal communication – April 2004). Moreover, the sign on annual precipitation is positive so that it may this variable that is capturing the positive effects of higher precipitation.

The Schumacher model

The Schumacher model is also linear in its parameters and can be estimated by ordinary least squares. The Schumacher model was estimated using SHAZAM V.10 (Northwest Econometrics 2004). Table 6.2 provides the estimation results for the Schumacher functional form. As was the case with the reciprocal model, two separate models are presented: with and without climate variables included.

The Schumacher model also provides a fairly good fit to the data. The R squared for the regression without climate variables is 0.69. Including climate variables increases the R-squared to 0.70. Therefore, adding the climate variables does not improve the overall fit of the model significantly. As was the case with the reciprocal model, the coefficients on DENSITY, ANPREC, and MOIST are not significant with the Schumacher estimation. In addition, the coefficients on ZONE, SAND, and CLAY are not significant. Therefore an issue with the Schumacher functional form estimation is that a number of variables are not significant.

The Chapman Richards model

The Chapman Richards model is a more contemporary yield model but it is also a more complex model than the previous two models. The Chapman Richards model is not linear in its parameters and therefore parameter estimation requires a non-linear estimation procedure. The model was estimated using SHAZAM's V.10 (Northwest Econometrics 2004) non-linear regression routine. SHAZAM uses a numerical optimization method called the quasi-Newton method to estimate the parameters of non-linear models. This is an iteration procedure that requires the specification of starting values for the parameters. These starting values can have an important influence on whether the results converge to local or global maxima and/or whether the model converges at all. Thus, depending on the starting values, the estimation method may provide different results or no results whatsoever. The estimation of the Chapman Richards model for this study proved to be highly sensitive to starting values. The estimation was especially sensitive to starting values selected for β_{10} *and* β_{11} in equation [6.11]. The procedure used for this study was to use the results of the Schumacher model as a guide for determining starting values for the coefficients in the asymptote term. For selecting starting values for β_{10} *and* β_{11} we first looked at the following three papers for general guidance: 1. Liu and Li (2003), 2. Fekedulegn et al. (1999), and 3. Pienaar and Turnbull (1973). These papers provide general indications of the correct sign and relative magnitudes for starting values for the parameters. A set of starting values were defined. However, the initial starting values failed to result in models that converged. The model was then re-estimated with different combinations of starting values. This process was repeated until the model converged and did not provide error messages. The set of starting values selected are as follows: B0:4.0, B1:0.04, B2:0.2, B3:-0.001, B4:-0.02, B5:-0.05, B6:0.1, B7:0.01, B8:-0.01, B9:-0.01, B10:7, B11: 0.001, B12:-0.1.

Table 6.3 provides the estimation results for the Chapman-Richards functional form[15]. Including climate variables into this specification increases the R-squared from 0.69 to 0.71. However, a number of the coefficients are insignificant including the constant and the coefficients associated with the variables ZONE, DENSITY, SAND, CLAY, ANPRECIP, MOIST and AGE. An insignificant coefficient on AGE is not consistent with the results of the reciprocal model or the Schumacher model. Moreover, insignificance of the coefficient on AGE raises questions about the degree to which this specification fits the data.

[15] It should be noted that numerous attempts were made to estimate the specification represented by equation [6.11]. The SHAZAM runs failed to converge. It was determined that the problem pertained to the biological constant parameter (i.e. the parameter directly in front of the exponent term in equation [6.11]). This parameter was dropped and the models were rerun. Convergence was achieved.

Model comparisons and selection

In this section the yield model estimation results are compared in four ways. First, a comparable R-squared value for each model is calculated and compared. Second, predictions of yield over age for a hypothetical aspen site are derived and graphed. Third, the general estimation results are evaluated and compared on the basis of numbers of significant coefficients and consistency of signs on coefficients with theoretical expectations. Finally, we compare our estimation results of the Chapman Richards functional form with results from a study by Fekedulegn et al. (1999). This study used a similar type of data to estimate a Chapman Richards yield function.

The dependent variable for the reciprocal model is merchantable volume. In the case of the Schumacher and the Chapman Richards functions, the dependent variable is the natural log of merchantable volume. Thus, the dependent variables for the three models are not the same. In cases where the dependent variables are different – R-squared values cannot be used to compare and contrast models. In order to obtain comparable R-squared values it is necessary to transform predicted values for all models to a common basis and then determine the degree to which the transformed predicted values explain the variance in the comparable actual values of the dependent variables. This was done by first converting the predicted values for the Schumacher and Chapman Richards models from predictions of the log of merchantable volume to predictions of merchantable volume. The conversion was conducted by generating a new variable (PMVOL) that was determined by calculating the exponent of LNMVOL. This is equivalent to an untransformed predicted value for merchantable volume. Auxiliary regressions were then estimated for MVOL as a function of PMVOL with the "noconstant" option invoked. The resulting R-squared values for these auxiliary regressions are comparable to the R-squared value for the reciprocal model estimation. The comparable R-squared values are provided in Table 6.4. Based on comparable R-squared values, the reciprocal model explains a higher percentage of the variance in merchantable volume. The reciprocal model also showed the greatest increase in R-squared value with the addition of the climate variables.

Figure 6.2 shows the yield over age relationships for each of the three models. As expected, the shape of the reciprocal-yield-age relationship is a concave curve. Also, as expected the Schumacher and Chapman Richards functions are sigmoidal with respect to yield over age. These figures are based on estimation results for Equations 6.2, 6.6 and 6.11. The only variable (other than age) that is allowed to vary is stand density (Clutter et al. 1983). For the purposes of incorporating changes in density for each age we have used the stand density results in Kirby et al.'s (1957) stand density table for average site aspen stands (Table 6.5). The values for all other variables are treated as constants. The values used were the average variable values for the sample (see Table 5.1).

The reciprocal model predicts that the age at which the mean annual increment for the sample stand culminates (i.e. the age at which the average growth rate is maximized) occurs at around 55 years. This is consistent with other studies that have found that the MAI for aspen culminates at around 60 years (Heeney et al. 1980) on medium sites. Mean annual increment appears to culminate at around 100 years for the Schumacher simulation and MAI culminates well past 100 years for the Chapman Richards simulation. The late culmination of MAI with the Schumacher estimation and the Chapman Richards simulation brings into question the degree to which these functional forms provide an acceptable fit of the data used in this study. These models predict continued growth increases at ages that normally are associated with stand breakup[16].

The third consideration for model selection is to assess the number of significant coefficients for each model and the general fit of each model to the data. The variables selected for the initial estimations were selected because they have some *apriori* justification for being included as predictors in yield equations. For the reciprocal model, 9 out of 12 coefficients were statistically significant. The

---

[16]One feature of aspen yield that is not reflected in the models estimated in this study is that after a certain period of time aspen stand yields will begin to decline as a result of stand breakup, stem decay and tree mortality. None of the models estimated here reflect declines in aspen volumes in over-mature stands. The reciprocal model is approaching its asymptote at age 100.

estimation results for the Schumacher and Chapman Richards models had fewer significant coefficients. For example, the coefficient for ZONE is insignificant in the Schumacher model. Similarly, the soil characteristic variables (SAND, CLAY) are not significant. An insignificant coefficient for the ZONE coefficient is a concern because previous research has shown that there are qualitative differences between aspen in the aspen parkland and boreal zones (Hogg 1994). Similarly, soil characteristics are an important determinant of site productivity. There are also a number of insignificant coefficients in the Chapman-Richards estimations. Similar to the Schumacher model estimation, the coefficients on ZONE, SAND, and CLAY are not significant. In addition, however, the coefficient on the variable AGE is also not significant.

The shape of the curves in Figure 6.3 indicate that the Chapman Richards and Schumacher models do not fit the data particularly well and do not provide reasonable prediction models. Moreover, there are a number of insignificant coefficients on variables that *apriori* we would have expected to be significant for these models. One reason for the poor results for these two models may be due to the fact that the data are restricted to larger and older trees and data on growth in juvenile stands is not represented in this particular sample. Therefore, these functional forms may be attempting to force sigmoidal type relationships between the dependent and independent variables based on data that is only representative of the concave portion of the yield-age relationship. Fekedulgen et al. (1999) had similar results with a data set that lacked observations in young age classes. They note that:

"Investigation of the differential forms and second derivatives of the Chapman-Richards and von Bertalanffy models indicate that the functions are suitable to model a system that encompasses the entire range of the life cycle of a biological response variable...This clearly illustrates that significance of the parameters of the Chapman-Richards and von Bertalanffy growth models depends on the range of the growth data." (pg. 333 and 334)

In order to proceed to the next phase of this study, one of the three models estimated in this chapter must be selected. The Schumacher function is a traditional functional form for yield prediction model estimation. More sophisticated non-linear models (such as the Chapman Richards functional form) are currently more commonly used. These functional forms are grounded in forest science. The reciprocal model has not been previously used in forest yield modeling. However, the results presented in this chapter show that the simpler reciprocal model appears to fit the data better than the Schumacher and the Chapman Richards functional forms. The reciprocal model is therefore selected as the functional form most suited for use in the next phases of this study.

Refinements to the reciprocal model

The previous section presents estimation results for the purpose of comparing and selecting a suitable model for yield estimation with climate variables included. The results suggest that a reciprocal model may be the most applicable for the analysis proposed in this study. Having selected the reciprocal model it was decided to try and further refine the specification and the estimation method.

In order to reduce multicollinearity, and make the prediction model more applicable to a boreal forest site some modifications of the basic model presented in Table 6.1 were considered. First, it was decided to drop the variable LAT (latitude) from the specification. The reason for dropping the variable LAT was due to likely strong correlations with climate variables of interest (namely ANTEMP). Second it was decided to drop the moisture variable due to likely collinearity with the precipitation variables and temperature variables. Finally, in order to make the model specific to a boreal forest site, interaction variables between ZONE and the other exogenous variables were incorporated into the model. This provides the ability to consider the effect of zone on both the intercept parameter as well as the response parameters. A "Z" in front of the relevant coefficient identifies the interaction terms. For example, the interaction term for density (i.e. DENSITY*ZONE) is referred to as ZDENSITY in the results tables (Table 6.6). Similarly, the coefficient for the interaction term (SITE*ZONE) is referred to as ZSITE.

The refined specification for the model is as follows:

$$
\begin{aligned}
MVOL = {} & \beta_0 + \beta_1 ZONE + \beta_2 DENSITY + \beta_3 (DENSITY * ZONE) + \beta_4 SITE \\
& + \beta_5 (SITE * ZONE) + \beta_6 SAND + \beta_7 (SAND * ZONE) + \beta_8 CLAY \\
& + \beta_9 (CLAY * ZONE) + \beta_{10} ANTEMP + \beta_{11} (ANTEMP * ZONE) \\
& + \beta_{12} ANPREC + \beta_{13} (ANPREC * ZONE) + \beta_{14} GSPREC + \beta_{15} (GSPREC * ZONE) \\
& + \beta_{16} AGEINV
\end{aligned}
\qquad [6.12]
$$

The above general model provides two separate prediction models: one for aspen parkland (ZONE=0) and one for boreal sites (ZONE=1). The boreal model will be used for prediction purposes.

$$
MVOL = \begin{cases}
(\beta_0 + \beta_1) + (\beta_2 + \beta_3) DENSITY + (\beta_4 + \beta_5) SITE \\
+ (\beta_6 + \beta_7) SAND + (\beta_8 + \beta_9) CLAY \\
+ (\beta_{10} + \beta_{11}) ANTEMP + (\beta_{12} + \beta_{13}) ANPREC & \text{When ZONE=1} \\
+ (\beta_{14} + \beta_{15}) GSPREC + \beta_{16} AGEINV \\
\\
\beta_0 + \beta_2 DENSITY + \beta_4 SITE + \beta_6 SAND \\
+ \beta_8 CLAY + \beta_{10} ANTEMP + \beta_{12} \mathrm{ANPREC} & \text{When ZONE = 0} \\
+ \beta_{14} GSPREC + \beta_{16} AGEINV
\end{cases}
$$

One feature of the data that needs to be addressed in obtaining suitable estimates of parameters and their variances is that the dependent variable in our model (merchantable volume per hectare) is always a positive number. Therefore, the dependent variable for our model has a truncated distribution. The density function for our dependent variable is given as:

$$
f(y \mid y > 0) = \frac{f(y)}{\mathrm{Prob}(y > 0)}
$$

Estimation of yield functions without accounting for the fact that the dependent variables are non-negative may result in a prediction model that overestimates volume in young stands and underestimates volume in older stands. Therefore, in addition to refining the specification of the reciprocal functional form (as described above) a procedure that accounts for the fact that the dependent variable is always a non-negative number was used to estimate the parameters. This procedure is called truncated regression (Greene 1997).

Truncated regression models are estimated using maximum likelihood estimation. The likelihood function for a truncated regression model (where the dependent variable is truncated at 0) is as follows:

$$f(y_i \mid y > 0) = \frac{\frac{1}{\sigma}\phi[(y_i - \beta' x_i)/\sigma]}{1 - \Phi[(0 - \beta' x_i)/\sigma]} \qquad [6.13]$$

Where:

$\phi[(y_i - \beta' x_i)/\sigma]$ is the standard normal density function, and

$\Phi[(0 - \beta' x_i)/\sigma]$ is the standard normal cumulative distribution function truncated at 0

Source: Greene (1997)

Estimates of "β" are obtained by maximizing the log-likelihood of equation 6.13. The truncated regression model was estimated using LIMDEP: Version 7.0 (Econometric Software Inc. 1995).

The results of the truncated regression estimation for the reciprocal yield model are provided in Table 6.6. For comparison, parameter estimates for the same model using ordinary least squares (i.e. not using truncated regression) are also provided in Table 6.6. The use of truncated regression does not have drastic effects on the values of the parameters or on their standard errors. In most cases the signs are the same with both estimations. In general, the standard errors for the truncated regression are slightly higher. The result is that one extra coefficient (ZANPREC) becomes insignificant. Also there is no significant loss in explanatory power with the truncated regression model. Auxiliary regressions were run with actual volumes as the dependent variable and predicted volume as the independent variable and with the "no constant" option invoked. Since we are interested in a model that predicts volume on a boreal site, only the data for boreal sites (n=70) were used for the auxiliary regressions. The R-squared values for the un-truncated refined reciprocal model and the truncated model were 0.76 and 0.77 respectively.

Finally, yield over age relationships using the un-truncated model and the truncated model were plotted to determine if in fact the un-truncated model provides an overestimate of volume in young stands and an underestimate of volume in old stands. The results are shown in Figure 6.3. Yield for young stands in the truncated model is clearly lower than yield predicted by the un-truncated model. Similarly, yield for old stands in the truncated model is clearly higher than yield predicted by the un-truncated model. These results show that failing to account for the fact that the dependent variable is truncated at zero may result in biased predictions of yield.

Thus based on the analysis presented in this chapter, the specific model selected for analysis in the next phase of analysis for this study is the refined reciprocal model estimated using truncated regression with the dependent variable truncated to values greater than zero.

Figure 6.1 A reciprocal stand yield growth/age relationship.

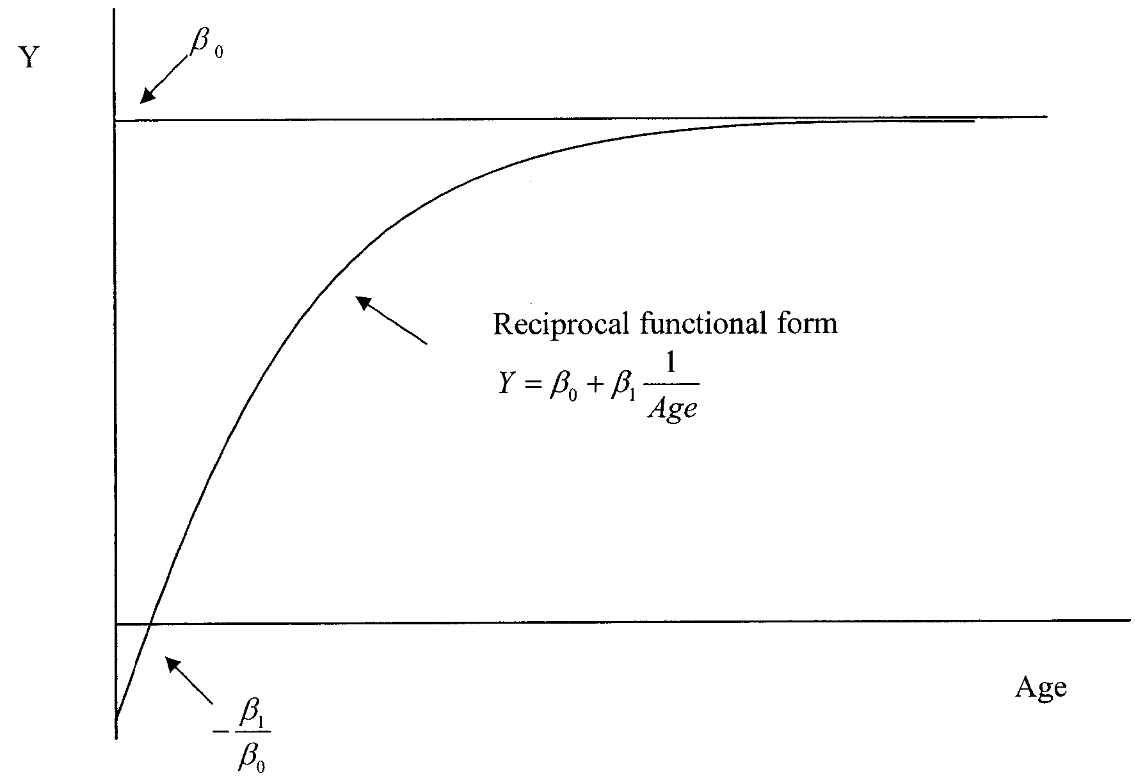


Figure 6.2. Simulation of yields using the three functional forms.

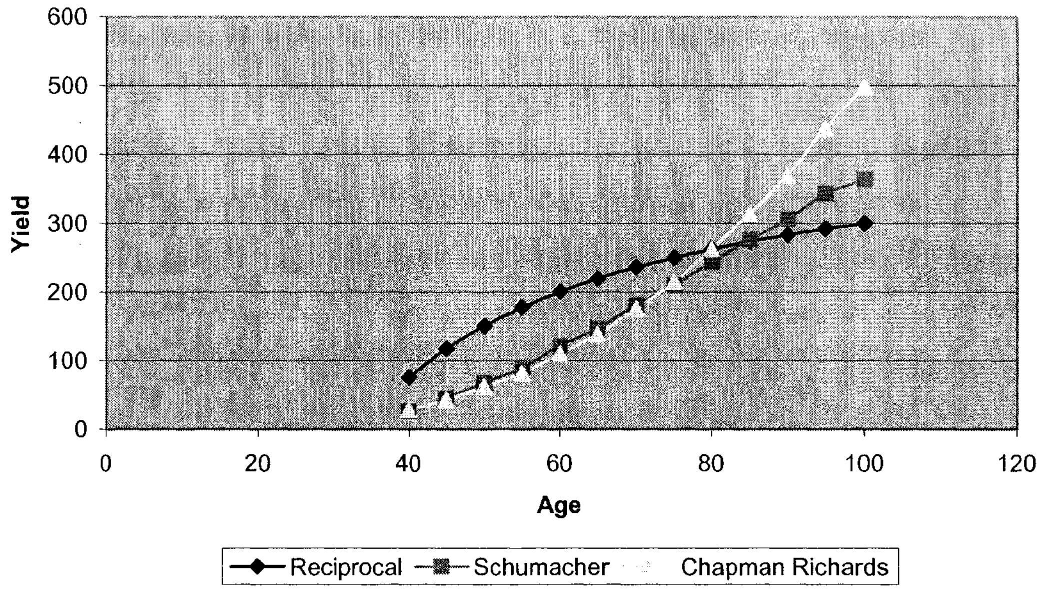


Figure 6.3 Comparison of refined reciprocal model yield curves estimated with truncated and untruncated regression using average values for boreal sites.

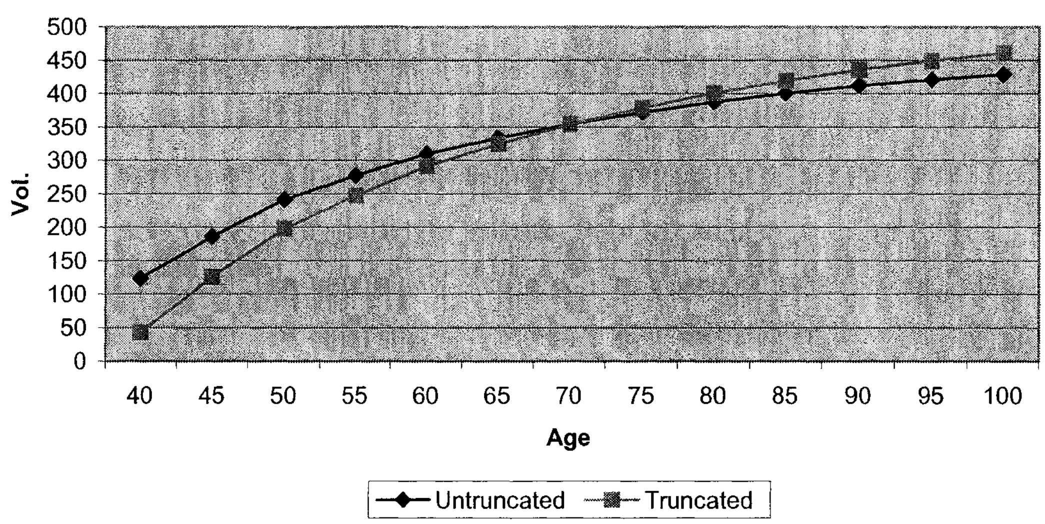


Table 6.1 Estimation results for the reciprocal timber stand yield model

	Model 1 - Without Climate			Model 2 - With Climate		
Variable Name	Estimated Coefficient	Standard Error	P-value	Estimated Coefficient	Standard Error	P-value
CONSTANT	1161	148.7	0	1334.5	232.3	0
ZONE	24.69	13.14	0.062	57.651	18.02	0.002
LAT	-18.594	2.662	0	-19.057	3.857	0
SITE	23.201	2.088	0	23.32	2.061	0
DENSITY	-1.35E-02	9.67E-03	0.167	-1.35E-02	9.75E-03	0.169
SAND	-1.6697	0.3833	0	-1.2797	0.4036	0.002
CLAY	-1.5932	0.5349	0.003	-1.2894	0.5313	0.017
ANTEMP				13.953	6.586	0.036
ANPREC				0.28508	0.2397	0.236
GSPREC				-1.2054	0.3879	0.002
MOIST				0.39244	1.114	0.725
AGEINV	-15824	1717	0	-14687	1690	0
R-squared	0.71			0.74		
R-squared -adjusted	0.69			0.71		
SE of estimate (sigma)	56.80			54.86		
F-statistic	182.97			131.91		
Significance	<1%			<1%		

Table 6.2 Estimation results for the Schumacher timber stand yield model

	Model 1 - Without Climate			Model 2 - With Climate		
Variable Name	Estimated Coefficient	Standard Error	P-value	Estimated Coefficient	Standard Error	P-value
CONSTANT	8.9711	1.316	0	8.8639	2.1	0
ZONE	-0.13012	0.1163	0.265	0.12173	0.1629	0.456
SITE	0.22214	1.85E-02	0	0.22131	1.86E-02	0
LAT	-8.83E-02	2.36E-02	0	-7.42E-02	3.49E-02	0.035
DENSITY	-1.52E-04	8.56E-05	0.078	-1.68E-04	8.82E-05	0.059
SAND	-4.64E-03	3.39E-03	0.174	-2.17E-03	3.65E-03	0.553
CLAY	-7.24E-03	4.74E-03	0.128	-5.54E-03	4.80E-03	0.251
ANTEMP				0.12037	5.96E-02	0.045
ANPREC				3.27E-03	2.17E-03	0.134
GSPREC				-8.87E-03	3.51E-03	0.013
MOIST				3.09E-04	1.01E-02	0.976
AGEINV	-147.42	15.2	0	-139.93	15.28	0
R-squared	0.69			0.70		
R-squared - adjusted	0.67			0.68		
SE of estimate (sigma)	0.50			0.49		
F - statistic	1642			1125		
Significance	<1%			<1%		

Table 6.3 Estimation results for the Chapman-Richards timber stand yield model

	Model 1 – Without Climate				Model 2 – With Climate			
Coefficent (See equation 6.11)	Estimated Value	Standard Error	t-ratio	Significance 5 %	Estimated Value	Standard error	t-ratio	Significance 5 %
B0 (CONSTANT)	11.302	10.753	1.051	NS	31.802	23.462	1.3555	NS
B1 (ZONE)	-0.14963	0.1011	-1.48	NS	0.14025	0.15808	0.88717	NS
B2 (SITE)	0.22968	1.82E-02	12.64	SIG	0.22804	1.75E-02	13.037	SIG
B8(LAT)	-8.21E-02	2.28E-02	-3.60	SIG	-5.73E-02	2.95E-02	-1.9447	SIG
B3 (DENSITY)	-1.05E-04	8.54E-05	-1.23	NS	-1.27E-04	8.65E-05	-1.4727	NS
B4 (SAND)	-4.81E-03	3.36E-03	-1.43	NS	-2.39E-03	0.33E-02-	0.71926	NS
B5 (CLAY)	-6.28E-03	4.72E-03	-1.33	NS	-4.61E-03	4.29E-03	-1.0747	NS
B6 (ANTEMP)					0.14569	5.81E-02	2.5065	SIG
B7 (ANPREC)					3.29E-03	2.03E-03	1.6231	NS
B8 (GSPREC)					-9.04E-03	3.37E-03	-2.6837	SIG
B9 (MOIST)					-6.95E-04	0.98E-02-	7.05E-02	NS
B10	3.1051	1.4711	2.11	SIG	2.70E+00	2.86E-01	9.41E+00	SIG
B11 (AGE)	3.24E-03	1.55E-02	0.21	NS	9.23E-07	7.98E-06	0.11565	NS
R-squared	0.69				0.71			

Table 6.4 Comparable R-squared values between models

	Model 1 without climate	Model 2 with climate
Reciprocal model	0.71	0.74
Schumacher model	0.62	0.62
Chapman Richards	0.62	0.63

Table 6.5 Stems per ha by age

Age	Number
40	3743
50	2296
60	1596
70	1198
80	912
90	724
100	623

Source: Kirby et al. 1957

Table 6.6 Estimation results for the refined reciprocal model estimated with truncated regression

	Ordinary least squares estimation			Truncated regression estimation		
Variable	Coefficient value	Standard error	P value	Coefficient value	Standard error	P value
CONSTANT	-56.64	110	.607	4.59	115.49	.968
ZONE	425.75	182.8	.021	406.41	179.76	.024
DENSITY	0.015	0.014	0.302	.0227	.0149	.129
ZDENSITY	-0.059	0.019	0.002	-.065	.020	.0009
SITE	17.609	2.92	0.0	25.12	3.47	0.0
ZSITE	-0.112	3.842	0.977	-.643	4.11	.876
SAND	0.774	0.538	0.153	.845	.545	.121
ZSAND	-3.37	0.880	0.0	-3.080	.864	.0004
CLAY	-0.119	0.864	0.89	-.373	.942	.692
ZCLAY	-2.060	1.144	0.074	-1.351	1.184	.254
ANTEMP	-0.683	9.808	0.945	8.303	10.038	.4081
ZANTEMP	43.811	13.29	0.001	33.73	13.43	.012
ANPREC	-0.132	0.317	0.677	.011	.319	.973
ZANPREC	1.377	0.627	0.03	.802	.616	.193
GSPREC	0.376	0.560	0.503	-.135	.572	.814
ZGSPREC	-2.548	0.9123	0.006	-1.637	.918	.075
AGEINV	-11254	1676	0.0	-19183	2624	0.0
Sigma		54.38			50.53	
R squared		0.75				

CHAPTER SEVEN

ESTIMATION OF RISK MODEL COEFFICIENT VALUES, VARIANCES AND COVARIANCES

Introduction

The general objective of this study is to assess the effects of climate change on the benefits of timber production and on decisions regarding optimal harvest choices. However, embedded within this general objective are a number of more specific questions (see Chapters 8, 9 and 10). In order to address the general objective and the sub-questions of interest we require a range of different types of models. Chapters 8, 9 and 10 look specifically at how different types of risk models and different formulations of these models can be used to address various types of climate change impact questions. However, as described in Chapter 4, an additional consideration that will affect the range of questions that can be answered and/or the range of analytical contexts that can be addressed pertains to the definitions of coefficients and input data used in the optimization models and the assumptions that these values are based on. The input data for the optimization models in Chapters 8, 9, and 10 are in the form of coefficient values (or expected values for random variables) and covariance matrices. The variables of interest include net benefits, harvest yields, and ending inventory yields. There are a number of factors that affect coefficient values and variances and covariances. Some of these factors include: (a) whether yields are based on climate history or climate futures, (b) whether climate uncertainty[17] is included in the covariance matrices, (c) whether yield uncertainty is included in the covariance matrices, and (d) alternative assumptions that can be made about whether there is uncertainty in first period harvest yields. For this study we specify four scenarios that include various combinations of assumptions regarding the factors noted above. The values of the coefficients, the variances of the coefficients, and the

[17] For the purpose of this study climate uncertainty refers to uncertainty in the expected values of climate variables in future years. This is not the same as climate variability which generally refers to the distribution for a particular climate variable in any given year.

covariances between the coefficients vary for each of these four scenarios. This Chapter describes these four scenarios; describes the methodology used to estimate expected coefficient values and covariances under each scenario; and presents results of the coefficient, variance and covariance estimations for each scenario.

The first step in assessing the effects of climate change is to obtain a baseline estimate of maximum economic returns and the optimal harvest pattern for our hypothetical forest when there are no climate effects and there is no uncertainty about yields and benefits. Scenario one provides baseline values for coefficient values. For scenario one, the estimate of coefficient values is based on the assumptions that yields are not impacted by climate (climate normals are used for prediction purposes) and that there is certainty with respect to all objective function and constraint coefficients in the risk models. Again, model results from this scenario will provide baseline results against which model runs using input data from scenarios 2, 3, and 4 can be compared for the purposes of estimating relative changes in economic returns and optimal harvest patterns with climate change.

Scenario two input data incorporates climate change impacts on stand productivity. The only source of uncertainty in coefficient estimates is with respect to the future values of climate. Climate variables within the yield function are assumed to be random variables. However, under scenario two, the parameters of the yield model are assumed to be constants. Therefore, scenario two incorporates climate uncertainty (i.e. uncertainty in climate variables) but this scenario assumes that the parameters of the yield models are known with certainty.

Scenario three input data also incorporates climate change impacts on stand productivity. However, there are two sources of uncertainty under this scenario. For scenario three both climate variables in the yield function and yield function parameters themselves are assumed to be random variables. Therefore, for this scenario, we have extended the analysis to look at climate change in the context of other potential sources of uncertainty – namely uncertainty in the parameters of the yield prediction equation.

Our final scenario (scenario 4) incorporates a new assumption about the variances. For scenarios 2 and 3, we assume that the period 1 harvest is uncertain.

This uncertainty comes from the fact that the logger is initially unsure about how much inventory is on the land and/or how much timber he/she will be able to harvest from the inventory in period one. However, in practice it may be possible for the logger to estimate (with high confidence) the volume of timber that he/she could harvest in period one, even before he/she actually harvests the stand. For example, this could be done with an operational inventory of the current standing forest. If a forestland owner were to pursue this option (which is entirely plausible if costs are not prohibitive) then uncertainty regarding period one harvest yields could be eliminated. For a risk-averse decision maker, it can be hypothesized that this added information will influence benefits and harvest choices. Thus, in summary, scenario 4 maintains the assumption that climate change occurs and this will affect productivity in future periods. We also continue to include uncertainty about climate and yield parameters as sources of uncertainty. However, the new assumption under scenario four is that uncertainty about period one harvest is eliminated through an operational cruise. Areas where there continues to be uncertainty include period two harvest yield (and associated net benefits), the ending inventory yields, and soil expectation values.

The objective function and constraint coefficients required for the risk models (in Chapter 8, 9 and 10) include net benefit ($ per ha) for each option in the choice set, harvest yield (cu. m. per ha) for each option in the choice set, and ending inventory (cu. m. per ha.) for each option in the choice set. This chapter presents predictions for the above coefficients for each scenario. As noted, for scenario one these coefficients are based on climate normals and they are deterministic values. The estimation and presentation of these deterministic values is relatively straightforward. However, the coefficients in scenarios 2, 3 and 4 are random variables. Thus, there is not a single value for these coefficients. In general, random variables are described by distribution type, expected value, and variance. For multivariate problems, knowledge of covariance between random variables may also be required. If knowledge of distribution types, expected values, variances and covariances are known ahead of time, then random variables can be fully described and included in the risk models. However, for this study, the distributions of random variables of interest for the risk models are not known *apriori*. They must be estimated. This chapter describes how sample distributions of random variables and parameters describing these sample distributions (namely expected values, variances, and covariances) are estimated. The process of estimating sample distributions for these random variables is complicated by the fact that the random variables of interest (net benefits, harvest yields, and ending inventory) are often functions of other random variables (e.g. the random objective function coefficients are functions of random yield coefficient variables and the random yield coefficient variables are functions of random model parameters and random climate variables). Therefore, the problem of estimating sample distributions for random variables of interest is hierarchical. A common technique for estimating distributions of outcome variables when there are multiple random input variables is Monte Carlo simulation (Saluga and Kicki 2002, Gill 2002, Geweke 1996).

The remainder of this chapter is organized as follows. First, a brief overview of probability and distribution theory and concepts is provided. This is followed by an overview of the theoretical basis for Monte Carlo simulation. The next section provides the theoretical context for assumptions about sources of variance and distributions of random variables is provided. This section is followed by a description of the specific Monte Carlo simulation procedure used for this study. Finally, the prediction results for net benefits, harvest yields, and ending inventory yield for each of the three scenarios are presented.

General statistical concepts

Probability concepts and distribution theory are commonly used for describing random variables and for characterizing degrees of uncertainty associated with random variables. There are two main types of random variables: discrete and continuous. The random variables employed in this study are, in general, continuous[18]. Therefore, the discussion in this section pertains to continuous random variables.

For the purposes of this chapter upper case letters are used to identify random variables and lower case letters are used to notionally identify values within the distribution of a random variable (recognizing that for any single real number "y" P(Y=y)=0 for continuous random variables). Continuous random variables are represented by their probability density and cumulative distribution functions. The probability density function (PDF) of a single random variable is denoted as $\phi(x)$. The joint multivariate probability density function is denoted as $\phi(x_1,...x_n)$. The cumulative distribution function (CDF) of a random variable (X) is denoted as $\Phi(x)$ where $0 \leq \Phi(x) \leq 1$. The relationship between a PDF and CDF is as follows: $\Phi(x) = \int_{-\infty}^{x} \phi(t)dt$.

The expected value (E[x]) of a continuous single random variable (X) is:

$$E[x] = \int_{-\infty}^{\infty} x\phi(x)dx \qquad [7.1]$$

A value that is a function of random variables is also a random variable. For example if X is a random variable and y = g (x), then Y is a random variable with a specific distribution. In some cases the distribution function for X is known (or can be assumed with some degree of justification) and the functional form for g (x) is known but the expected value, variance and distribution of Y are not known *apriori*. In cases where Y is the principal variable of interest, we require a method of estimating the expected value, variance and distribution for Y based on what we know about X and the functional form g(x).

The expected value of a measure "Y" that is a function of a random variable (X) is:

[18] There is one exception. The random variables for the DSP model in Chapter 10 are discrete. The method used for transforming the continuous random variables presented in this chapter into discrete random variables is described in Chapter 10.

$$E[y] = \int_{-\infty}^{\infty} g(x)\phi(x)dx \qquad [7.2]$$

It may be that Y is a function of a vector of random variables "**X**". The expected value of a function of "n" random variables is:

$$E[y] = \int_{x1}....\int_{x_n} g(x_1....x_n)\phi(x_1....x_n)dx_1.....dx_n \qquad [7.3]$$

Knowledge of Y's expected value is important but if we are interested in knowledge of the relative uncertainty (or variance) surrounding Y we also need to know how Y is distributed. In cases where the dimension of g(**x**) is low and where $\phi(x_1...x_n)$ is not complex, it may be feasible to analytically determine the distribution and density functions for Y ($\Phi(y)$ and $\phi(y)$). Wackerly et al. (1996) for example describe the "Methods of Distributions Functions" technique for estimating the probability distribution of a random variable that is a function of other random variables. In summary, this method involves determining $\Phi(y)$ by integration of $\phi(x_1...x_n)$ and then solving for $\phi(y)$ by differentiating $\Phi(y)$.

Monte Carlo simulation

For complex problems, the ability to rely on analytical procedures to determine the distribution of a random variable is limited by the complexity of the integrals that are involved. In many cases, an analytical solution is impractical or impossible. An alternative to an analytical procedure is Monte Carlo simulation. Monte Carlo simulation is designed to simulate the distributions of random variables that are functions of other random variables. In many of these types of situations a simulation procedure (such as Monte Carlo) may be the only option.

The principle behind Monte Carlo simulation is to draw samples of size "n" from right hand side (RHS) random variables that have a known distribution and then use individual draws from each distribution to calculate a value for the left hand side variable. If the random variables on the right hand side are not independent, then the draws are conditioned by taking account of correlations between the right hand side random variables. A sample distribution for the left hand side variable is generated. From this sample it is possible to estimate the expected value, variance and distribution of the dependent variable.

Equation 7.4 shows a case where Y is a function of the "k" vector of random variables ($X_1 \ldots X_k$).

$$Y = f(X_1 \ldots X_k) \qquad [7.4]$$

If the specific distribution for each RHS random variable is known and if the interdependence (or correlation) between the RHS random variables is known it is possible to draw a sample of size "n" from the density functions for each RHS random variable. In cases where the random variables are interdependent, draws from individual density functions for individual RHS random variables are conditional on the draws of other random variables. This can be accounted for by considering correlations between random variables during the sampling phase.

For each sample drawn from the distributions for the Xs, an individual value from the distribution of Y is calculated.

$$y_i = f(x_{1i} \ldots x_{ki}) \qquad [7.5]$$

The generated sample of values for the random variable "Y" can then be used to calculate estimators for the expected value of Y and its variance. For example, an estimate of the expected value of Y using Monte Carlo simulation is as follows:

$$\hat{E}[y] = \frac{1}{n}\sum_{i=1}^{n} f(x_{1i} \ldots x_{ki}) \qquad [7.6]$$

Each draw of x_i is from its own distribution.

The estimated variance of Y is

$$VarY = \frac{1}{(n-1)}\sum_{i=1}^{n} (f(x_{1i} \ldots . x_{ki}) - \hat{E}[y])^2 \qquad [7.7]$$

The generated sample of Y represents the distribution of Y assuming the functional form is correct, the assumptions regarding the distributions of the right hand side variables are correct, interdependencies between the random right hand side variables are accounted for, and the simulation sample size is sufficiently large that sampling bias is acceptable.

Monte Carlo simulation has been employed in many different types of applications. Monte Carlo analysis is an accepted methodology for characterizing uncertainty in environmental risk assessment (Environmental Protection Agency 1997). A number of authors have also recommended Monte-Carlo (or alternatively Bayesian) methods for characterizing uncertainties in climate impact studies (New and Hulme 2000; Katz 2002; Hobbs 1994; Shackley et al. 1998; Dowlatabadi 1998; Wigley and Raper 2001; Jones 2000). However, to our knowledge, no studies have applied Monte Carlo methods to investigate the impacts of climate and climate uncertainty in a forest management context.

An overview of the statistical context for the Monte Carlo simulations in this study

The goal of the analysis in this Chapter is to estimate expected values, variances, and covariances for net benefits, harvest yields, and ending inventory yields for the various management prescription options under the different scenario assumptions. This Chapter provides a bridge between the yield function estimation in Chapter 6 and the mathematical programming models presented in Chapters 8, 9, and 10. The input data for the models in the next three Chapters is based on the Monte Carlo simulation results presented in this Chapter. The Monte Carlo simulation results presented in this Chapter depend on the truncated regression model estimation results presented in Chapter 6.

There are two types of random variables that are discussed in this section. They are (a) random variables where the distributions are known (or can be directly inferred) and (b) random variables that we require for the risk models but for which we do not have any prior information. The first type pertains to random variables that are on the RHS of the yield prediction equations. The expected values and variances for these random variables are provided either from the regression models (in the case of the parameters) or are provided from external sources (in the case of the climate variables). The Monte Carlo approach in effect involves taking draws from the known distributions (with draws being conditioned by correlations between random variables) and generating a sample distribution for the unknown random yield variable. In this section we describe the statistical basis underlying our assumptions about the known RHS variable distributions. The second type of random variable pertains to variables where we do not know the distribution ahead of time and where it is therefore necessary to estimate expected values and variances by generating samples using Monte Carlo simulation. This section also provides more detail pertaining to what is included in the sample distributions for the unknown random variables.

In the sections that follow, the specific methods and equations used to conduct the Monte Carlo simulations of the risk model coefficients required in Chapters 8, 9, and 10 are described and the results are presented. Prior to providing the details relative to the specific equations used in the Monte Carlo simulations, it is important to clarify some of the underlying statistical assumptions and adopted for the purposes of simulating distributions of variables of interest for this study.

For the purposes of describing the underlying statistical assumptions and the general random variable estimation approach we will use a general version of the yield models estimated in Chapter 6. This general yield model is presented as follows:

$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$

Where

$Y =$ stand yield

$\beta =$ a parameter [7.8]

$X_1 =$ some climate variable

$X_2 =$ some non-climate variable affecting stand yield (e.g. age)

The equivalent statistical model is as follows:

$$Y_i = \widehat{\beta}_0 + \widehat{\beta}_1 X_{1i} + \widehat{\beta}_2 X_{2i} + e_i$$

Where

$i = 1...n$ (n= number of observations)

$Y_i =$ stand yield for observation i

$\widehat{\beta}$= a parameter estimated by truncated regression [7.9]

X_{1i} = some climate variable for observation i

X_{2i} = some non-climate variable affecting stand yield (e.g. age) for obs. i

e_i = residual error term

The first part of the Monte Carlo simulation procedure involves the estimation of a sample distribution for the random variable "Y" through simulation. Equation 7.9 provides the stylized version of the formula used within the simulations. The simulation essentially involves taking draws from the known distributions of RHS random variables where the draws are conditioned by known correlations between the RHS variables. Before providing a description of the various assumptions underlying the simulation of a sample distribution of "Y" it is useful to describe which of the terms in equation 7.9 are considered random (with known and unknown distributions), which are considered fixed, and which are considered irrelevant for this study. As noted, one of the goals[19] of the Monte Carlo procedure for this study is to simulate the distribution for future values of Y based on the following assumptions: (a) Y's future distribution is not known, (b) the betas may be random (i.e. they are random for scenarios 3 and 4) and in cases where they are random their distributions can be inferred from the truncated regression results, (c) the future value of the X_1 term (some climate variable) may be random (i.e. they are random for scenarios 2, 3, and 4) and in cases where they are random their distributions can be inferred by looking at ranges of predictions from various climate model-emission scenario combinations (note: the value of X_2 is deterministic (for all scenarios) and is known for the specific location of the forest of interest), and (d) the model error variance (e_i) is irrelevant. We now sequentially describe the basis and rational for each of these assumptions.

[19] It is important to keep in mind that the estimated distributions for Y are in turn used to estimate distributions for net benefits.

Y is a random variable whose distribution is not known

Future values of Y depend on future climate. However, there is uncertainty about future values of climate and therefore the future values of climate variables are random variables. Since climate variables are random, Y is a random variable. However, in addition to the uncertainty due to climate variables there may also be uncertainty about the parameters of the yield functions (particularly if one adopts a Bayesian perspective). Since Y is potentially a function of multiple uncertain random variables, its distribution is not known. Moreover, Y's distribution likely cannot be analytically derived because of the complexity of the integrals (as described in a previous section). One option (the classical approach) is to infer a distribution of Y from the error distribution provided by the regression. However, this approach would ignore the fact that the climate variables are also random variables and that future expected values will be different than the historical expected values. This approach would also ignore changes that might occur in variances in climate variables in different time periods. A second option is to assume that the betas are random and the data is fixed. This is the Bayesian approach. The problem here, however, is that it again ignores, and is unable to capture, uncertainty in the exogenous climate variables. Thus, a more general Monte-Carlo simulation approach is employed for this study. This more general approach is part Bayesian in that we adopt the Bayesian perspective that the betas are random variables and part classical in that we also assume that for prediction purposes, some of the independent variables are random variables.

The betas in the yield prediction equation are random variables

The truncated regression reciprocal yield model results presented in Chapter 6 provides estimates of the betas, variances for the betas, a covariance matrix, and the overall model standard error (sigma). Under the classical / frequentist approach to estimation, Y is a random variable and beta is fixed[20]. Y's probability density function is $f(Y \mid \beta)$. The relevant distributions in this case are as follows:

Y is distributed $N(\beta\bar{X}, \sigma^2)$

and

e is distributed $N(0, \sigma^2)$

However, because of uncertainty in yields we require an approach that assumes that there is uncertainty about the betas. It is the uncertainty in the parameters that partly results in uncertainty in predictions of stand yields in the future. Thus, we are interested in density functions of the form $f(\beta \mid Y)$. The Bayesian approach provides both the theoretical basis and an approach for obtaining $f(\beta \mid Y)$. The fundamental premise of the Bayesian approach (and the aspect where the Bayesian approach deviates from classical frequentist statistics) is that betas are random and the data is fixed. Moreover, the Bayesian approach provides a way of obtaining $f(\beta \mid Y)$ by combining observed data with prior knowledge of the distribution of a particular parameter. The relationship is defined by Bayes theorem.

$$f(\beta \mid Y) = \frac{f(Y \mid \beta) f(\beta)}{f(Y)}$$

Where

$f(\beta \mid Y)$ is the post sample density of β

$f(Y \mid \beta)$ is the sampling distribution from which sampling variances for β are obtained [7.10]

$f(\beta)$ is the prior. For a non-informative prior $f(\beta)$= 1

$$f(Y) = \int f(\beta) f(Y \mid \beta) d\beta$$

$f(Y)$ is a normalizing constant for a set of observed values

(see Gelman et al. 2000 pg. 8 and Griffiths et al. 1993 pg 791)

Equation 7.10 implies that it is possible to make Bayesian type inferences directly from the sample variance of the parameter estimates obtained from the truncated

[20] Beta is a fixed parameter. Estimates of beta are, however, normally distributed random variables. The variances of the betas are based on sample variance. Sample variance is the degree to which estimated betas will vary over large numbers of hypothetical future samples of similar size as the current sample.

regression estimation (given the fact that we have no prior knowledge of the distribution of the parameters (i.e. $f(\beta)=1$)). Thus, for the purposes of this study, our underlying assumption regarding the post sample density functions of RHS random variables in equation 7.9 are that the parameters are normally distributed, that the expected values are equal to the parameter estimates provided by the regression model presented in Chapter 6 (see Tables 6.6 and 7.2) and that the variances of the parameters can be also be inferred from the sample variances for the estimated betas obtained from the regression output (see Tables 6.6 and 7.2). The betas and sigma for the truncated regression model are estimated by maximum likelihood. Thus, the expected values of the beta's in equation 7.9 are approximated by selecting the betas and sigma that maximize the following likelihood function:

$$f(y_1, y_2, \ldots y_t \mid x_1, x_2 \ldots x_t, \beta_0, \beta_1, \beta_2, \sigma^2) =$$

$$(2\pi\sigma^2)^{\frac{-T}{2}} \exp\left[-\frac{\sum_{i=1}^{T} (y_t - \beta_0 - \beta_1 x_{1t} - \beta_2 x_{2t})}{2\sigma^2} \right] \qquad [7.11]$$

An approximation for the variance-covariance matrix for the betas is given by:

$$\mathrm{cov}(\beta) = \hat{\sigma}^2 (X'X)^{-1} \qquad [7.12]$$

One important aspect to note is that the standard deviations obtained from the regression output (see Table 7.2) are adjusted to account for the fact that the estimates are obtained from a truncated regression model. For the purposes of estimation, the distribution of Y is truncated at 0. Thus, the likelihood function for the truncated model is given by:

$$f_{truncated}(\beta \mid Y) = \frac{f_{non-truncated}(\beta \mid Y)}{P_N(Y > 0)}$$

Where [7.13]

$P_N(Y > 0)$ is the probability that Y is greater than zero based on a normal distribution

The future values of X_1 may be a random variable

A complicating factor for estimation of future expected values and variances of Y in equation 7.9 is that in addition to the betas being random variables, the climate variable (X_1) may also be a random variable. For example, if we are predicting what the harvest yield will be in the year 2055, we need to acknowledge that climatic factors contributing to growth between now and 2055 will be different than they were historically. We may, for example, want to incorporate our best guess about what the value of the climate variable for the year 2020 might be into the prediction equation. However, as noted in Chapter 5, we cannot predict with certainty what the value of the climate variable will be in 2020. Therefore this variable is a random variable. The source and type of information used to determine the distribution for future climate variables was described in Chapter 5. The distributions for climate variables are assumed to be uniform. The upper and lower limits of the distributions are based on the best and worst case outcomes from a range of general circulation model predictions. Thus, in addition to draws being made from the distributions for the random Beta's, draws are also made from the distributions from the random climate variables.

The model error variance is not relevant for estimation of future variances

Equation 7.9 includes an error term. When considered over the entire population the assumption regarding the distribution of the error term is that: $e \sim N(0,\sigma^2)$. If the goal of this study was to provide predictions of future yield of a specific individual stand, then model variance (i.e. the variance of "e") should be incorporated into the Monte Carlo simulation as a separate random variable. However, for this study we adopt the view that we are not making a yield prediction for any single stand. Rather, the prediction is being made for a particular class of stand types where this class is actually made up of a number of different stands. For example, the 250 hectares of age class 40 aspen in our forest may actually be made up of five separate 50 hectares stands of 40 year old aspen. For the purpose of this study we are primarily interested in the average predicted value for this class of stands. This has significant implications for the basic model. For obtaining predictions of the average yields over a group of stands we use a "conditional mean forecasting" (CMF) approach (Griffiths, et al 1993). To save space we will simply provide the relevant formulas (see Griffiths et al. 1993 pg. 244 for a discussion of CMF) for the expected values and variance of predicted value using a conditional mean forecasting approach (i.e. generating a prediction of the mean value of the dependent variable instead of the predicted value for any specific observation). First, for comparison, we provide the relevant prediction error equations and prediction error variance for prediction of yield for a single stand. The starting equations are:

$$y_0 = \beta_0 + \beta_1 x_{10} + \beta_2 x_{20} + e_0$$
$$and \qquad [7.14]$$
$$\hat{y}_0 = \hat{\beta}_0 + \hat{\beta}_1 x_{10} + \hat{\beta}_2 x_{20}$$

The prediction error is defined as: $PE = (\hat{y}_0 - y_0)$. PE is, in fact, a random variable with an expected value and a variance. The expected value is: $E[\hat{y}_0 - y_0] = 0$ and the prediction error variance is:

$$Var(\hat{y}_0 - y_0) = x'_0[\text{cov}(\hat{\beta})]x_0 + \sigma^2 \qquad [7.15]$$

Equation 7.15 shows that the model error variance (σ^2) contributes to the prediction error variance. Thus, if we were estimating the yield of a single stand, it would be appropriate to include draws from the distribution $e \sim N(0,\sigma^2)$ in simulating the distribution of Y. However, in our case, equation 7.14 is not the correct prediction equation. The correct prediction equation (based on conditional mean forecasting) is given by:

$$E[y_0] = \hat{\beta}_0 + \hat{\beta}_1 x_{10} + \hat{\beta}_2 x_{20} \qquad [7.16]$$

In this case, the variance of the prediction of E[y}is given as:

$$Var(E[y_0]) = x'_0[\text{cov}(\hat{\beta})]x_0 \qquad [7.17]$$

Note that the only difference between Equations 7.15 and 7.17 is that σ^2 is included as a separate term in 7.15 and it is not included in equation 7.17. The main result here, therefore, is that the error variance is not necessary for assessing the variance of the predicted value of the mean with a conditional mean forecasting approach. The only measures of relevance for the variance of the prediction are the sample variances of the estimators.

The above arguments are provided to provide a rationale for not including the error as an additional random variable in the Monte Carlo simulation. The question may arise: if one can calculate the variance of the prediction of y from equation 7.17 – why bother with Monte Carlo simulation? However, it is important to keep in mind that for our models, we also have to deal with uncertainty in the exogenous variables. It is because of the fact that for some scenarios both the exogenous variables and the model parameters are random variables that we rely on Monte Carlo simulation to obtain sample distributions for Y from which estimates of expected values, variances and covariances are obtained.

Setting up the prediction models

In the case of scenario one the estimates of net benefits, harvest yield and ending inventory values are deterministic. For scenarios two, three, and four these coefficients are random variables. However, whether the coefficients are deterministic or probabilistic, we require prediction models in order to estimate future values. Therefore, the initial steps are to identify the structure of the problem, provide the specification of the prediction equations, identify which variables in the problem are random and which are constant under each scenario, identify which random variables have a known distribution, and identify where Monte Carlo simulation is required in order to estimate a distribution for a particular random variable. The discussion in the previous section addressed these issues for a general model. The discussion in this section revisits these questions in the context of the specific equations required for estimating the variables required for the risk models in Chapters 8, 9 and 10. In order to address these issues it is useful to start by providing a brief overview of the optimization problem. More details about the problem context are provided in Chapter 4.

As described in Chapter 4, the hypothetical starting forest for this study is a 1000 ha stand of pure aspen forest near Calling Lake, Alberta. The forest is comprised of two age classes. Two hundred and fifty hectares is in the form of even-age, 40 year old aspen. This stand type is referred to as initial age class 1 (IAC1). Seven hundred and fifty hectares is in the form of even-age 80 year old aspen. The

80-year old forest is referred to as initial age class 2 (IAC2). The planning period and prescription options were also described in Chapter four (see Tables 4.1 and 4.2). The planning horizon is a 60-year period starting in the year 2010 and ending in 2070. The 60 year period is divided into two 30 year sub-periods with period 1 spanning the years 2010-2039 and period 2 spanning the years 2040 – 2070. The three management options available to the logger are: (a) leave the stand uncut (prescription 1), (b) cut in period one (prescription 2), (c) cut in period two (prescription 3). Thus, there are six possible values for each coefficient.

The nomenclature for identification of net benefit coefficients is provided as follows:

NB_{ij} = Net Benefit for presciption i and initial age class j
i = 1 for prescription one (no cut),
2 for prescription 2 (cut in period 1),
3 for prescription 3 (cut in period 2)
j = 1 for initial age class 1 (i.e. 40 year old stand at time=0)
2 for initial age class 2 (i.e. 80 year old stand at time=0).

A similar nomenclature is used to identify the harvest yield and ending inventory coefficients.

The objective functions in the risk models in Chapter 8, 9 and 10 require an estimate of the net benefit of forest management for each initial age class (IAC) and management prescription under each scenario. A constant price of $ 2.50 per cu. m. is assumed and the discount rate is fixed at 4 % for this study. The net benefit coefficient prediction equations for all six age-prescriptions options are defined as follows:

1. Objective function value for IAC1 and prescription 1:

$$NB_{11} = \frac{(V_{EI1.1})(2.5)}{(1.04)^{60}} + \frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}}$$

Where "t" in this and the remaining equations refers to optimal rotation [7.18]

This is the net benefit associated with leaving the 40-year old stand uncut for the entire planning period. NB_{11} incorporates the present value of the ending inventory (where $V_{EI1.1}$ is the volume of ending inventory) and the present value of the soil expectation value[21] (where V_{SEV} is the volume of stands at optimal rotation) at the end of the planning period. The two RHS yield coefficients that are incorporated in this coefficient are random variables[22] for scenarios 2, 3 and 4.

2. Objective function formula for IAC2 and prescription 1:

$$NB_{12} = \frac{(V_{EI1.2})(2.5)}{(1.04)^{60}} + \frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}} \qquad [7.19]$$

This is the net benefit associated with leaving the 80-year old stand uncut for the entire planning period. NB_{12} includes the value of the ending inventory for the 80-year old stand ($V_{EI1.2}$ in the year 2070 discounted to present value) plus the present value of soil expectation value. The two RHS yield coefficients are random variables for scenarios 2, 3 and 4.

3. Objective function formula for IAC1 and prescription 2:

$$NB_{21} = \frac{(V_{21})(2.5)}{(1.04)^{15}} + \frac{(V_{EI2.0})(2.5)}{(1.04)^{60}} + \frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}} \qquad [7.20]$$

This formula calculates the net benefit of harvesting the 40-year old stand in period one. NB_{21} includes the present value of the harvest in period one (cut in 2025) (where V_{21} is the harvest volume), the present value of the ending inventory (i.e. the aspen inventory that accumulates after harvest) (where $V_{EI2.0}$ is the volume of ending inventory), and the present value of soil expectation value. The RHS yield coefficients are random variables for scenarios 2, 3 and 4.

4. Objective function formula for IAC2 and prescription 2:

$$NB_{22} = \frac{(V_{22})(2.5)}{(1.04)^{15}} + \frac{(V_{EI2.0})(2.5)}{(1.04)^{60}} + \frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}} \qquad [7.21]$$

[21] Note that the SEV is not constrained by flow and inventory constraints for the purposes of this study.

[22] The term coefficient begins to be used here. This term is used in the context of these random variables because these terms are coefficients for the mathematical programming models presented in Chapters 8, 9 and 10. These terms are, however, in reality variables in the context of the analysis presented in this chapter. Depending on the scenario, the net benefit and yield terms may be random variables or fixed.

This formula calculates the net benefit of harvesting the 80-year old stand in period one. NB_{22} includes the present value of the harvest of the IAC2 stand in period one (cut in 2025), the present value of inventory that accumulates after harvest, and the present value of soil expectation value.

5. Objective function formula for IAC1 and prescription 3:

$$NB_{31} = \frac{(V_{31})(2.5)}{(1.04)^{45}} + \frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}} \qquad [7.22]$$

This formula calculates the net benefit of harvesting the 40-year old stand (IAC1) in period two (harvested in the year 2055). NB_{31} includes the present value of the harvest revenue in period two plus the present value of the soil expectation value. In this case there is no ending inventory value because it is assumed that the stand is cut in the year 2055. This is only 15 years before the end of the planning period. This is an insufficient time for developing merchantable timber on the site.

6. Objective function formula for IAC2 and prescription 3:

$$NB_{32} = \frac{(V_{32})(2.5)}{(1.04)^{45}} + \frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}} \qquad [7.23]$$

This formula calculates the net benefits of harvesting the 80-year old stand (IAC2) in period two. NB_{32} includes the present value of the harvest revenue in period two plus the present value of the soil expectation value. Again there is no ending inventory for this prescription.

As noted, each of the above objective function coefficients is a constant for scenario one and a random variable for scenarios two, three and four. The distributions for the coefficients are not known and therefore they are estimated by Monte Carlo simulation. However, before these simulations can take place, knowledge of the distributions of the random yield variables is required. Each of the yield variables included within the objective function equations shown above is described as follows:

1. Equation for V_{21}

$$V_{21} = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)1800 + (\beta_4 + \beta_5)19.8 + (\beta_6 + \beta_7)34$$
$$+(\beta_8 + \beta_9)19 + (\beta_{10} + \beta_{11})Antemp_{2020s} + (\beta_{12} + \beta_{13})Anprec_{2020's} \qquad [7.24]$$
$$+(\beta_{14} + \beta_{15})Gsprec_{2020s} + \beta_{16}0.018$$

V_{21} is the harvest yield for IAC 1 and prescription 2. This is a 55-year old stand harvested in 2025. For scenario 1, climate normal data are used and all coefficients and variables are constants. For scenario 2, climate variables are uncertain (i.e. the climate variables in the yield equation are random variables) but the values of yield parameters are known (yield parameter values are constants). The distribution of climate variables is uniform with a lower and upper limit based on high and low projections for the 2020s. For scenario 3, the climate variables and the beta's are random variables. As previously noted, for scenario 4, V_{21} is a fixed value. Uncertainty about first period harvest yields under scenario 4 is eliminated through measurement.

2. Equation for V_{22}

$$V_{22} = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)500 + (\beta_4 + \beta_5)19.8 + (\beta_6 + \beta_7)34$$
$$+(\beta_8 + \beta_9)19 + (\beta_{10} + \beta_{11})Antemp + (\beta_{12} + \beta_{13})Anprec \qquad [7.25]$$
$$+(\beta_{14} + \beta_{15})Gsprec + \beta_{16}0.0105$$

V_{22} is the harvest yields for IAC 2 and prescription 2. This is a 95-year old stand harvested in 2025 (estimated with the 2020 climate variables for scenarios 2,3, and 4). For scenario 2, the climate variables are random. For scenario 3, the climate variables and the betas are random. For scenario 4, V_{22} is a constant.

3. Equation for V_{31}

$$V_{31} = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)700 + (\beta_4 + \beta_5)19.8 + (\beta_6 + \beta_7)34$$
$$+(\beta_8 + \beta_9)19 + (\beta_{10} + \beta_{11})Antemp + (\beta_{12} + \beta_{13})Anprec \qquad [7.26]$$
$$+(\beta_{14} + \beta_{15})Gsprec + \beta_{16}0.012$$

V_{31} is the harvest yields for IAC 1 and prescription 3. This is an 85-year old stand harvested in 2055 (estimated with the 2020/2050 climate variables for scenarios 2,3 and 4). For scenario 2, the climate variables are random. For scenarios 3 and 4 both the climate variables and the betas are random.

4. Equation for V_{32}

$$V_{32} = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)500 + (\beta_4 + \beta_5)19.8 + (\beta_6 + \beta_7)34$$
$$+ (\beta_8 + \beta_9)19 + (\beta_{10} + \beta_{11})Antemp + (\beta_{12} + \beta_{13})Anprec \qquad [7.27]$$
$$+ (\beta_{14} + \beta_{15})Gsprec + \beta_{16}0.008$$

V_{32} is the harvest yield for IAC 2 and prescription 3. This is a 125-year old stand harvested in 2055 (estimated with the 2020/2050 climate variables). For scenario 2 the climate variables are random. For scenarios 3 and 4 both the climate variables and the betas are random.

5. Equation for $V_{EI1.1}$

$$V_{EI1.1} = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)500 + (\beta_4 + \beta_5)19.8 + (\beta_6 + \beta_7)34$$
$$+ (\beta_8 + \beta_9)19 + (\beta_{10} + \beta_{11})Antemp + (\beta_{12} + \beta_{13})Anprec \qquad [7.28]$$
$$+ (\beta_{14} + \beta_{15})Gsprec + \beta_{16}0.01$$

$V_{EI1.1}$ is the ending inventory yield for IAC 1 and prescription 1. This is a 100-year-old stand in 2070 (estimated with the 2020/2050 climate variables). For scenario 2 the climate variables are random. For scenarios 3 and 4 both the climate variables and the betas are random.

6. Equation for $V_{EI1.2}$

$$V_{EI1.2} = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)500 + (\beta_4 + \beta_5)19.8 + (\beta_6 + \beta_7)34$$
$$+ (\beta_8 + \beta_9)19 + (\beta_{10} + \beta_{11})Antemp + (\beta_{12} + \beta_{13})Anprec \qquad [7.29]$$
$$+ (\beta_{14} + \beta_{15})Gsprec + \beta_{16}0.007$$

$V_{EI1.2}$ is the ending inventory yields for IAC 2 and prescription 1. This is a 140-year-old stand in 2070 (estimated with the 2020/2050 climate variables). For scenario 2, the climate variables are random. For scenarios 3 and 4 both the climate variables and the yield model parameters are random.

7. Equation for $V_{EI2.0}$

$$V_{EI2.0} = (\beta_0 + \beta_1) + (\beta_2 + \beta_3)2250 + (\beta_4 + \beta_5)19.8 + (\beta_6 + \beta_7)34$$
$$+(\beta_8 + \beta_9)19 + (\beta_{10} + \beta_{11})Antemp + (\beta_{12} + \beta_{13})Anprec \quad [7.30]$$
$$+(\beta_{14} + \beta_{15})Gsprec + \beta_{16}0.022$$

$V_{EI2.0}$ is the ending inventory yields for IACs 1 or 2 and prescription 2. This is a 45-year-old stand in 2070 (estimated with the 2050 climate variables). For scenario 2, the climate variables are random. For scenarios 3 and 4 both climate variables and yield model parameters are random.

8. Equation for $V_{EI3.0}$

$V_{EI3.0}$: This is a 15-year-old stand in 2070. It is assumed that merchantable volume is zero.

9. Equation for V_{SEV}

V_{SEV}: Yield at optimum economic rotation (estimated with the 2080 climate variables). Optimal economic rotation is based on the Faustmann equation. Stand age varies from draw to draw within the sample.

The distributions of the yield coefficients defined above are not known for scenarios 2, 3 and 4. A sample distribution is estimated for each variable with Monte Carlo simulation.

The starting values for the constants required for the yield prediction and net benefit prediction models are shown in Table 7.1. The climate normal data for the scenario one predictions is also provided in Table 7.1. The estimations of the coefficients under scenario 2 require distribution information for the climate variables. This is shown in Table 7.2. For scenarios 3 and 4 the coefficient predictions require baseline information about the distributions of the yield model parameters and the climate variables. This information is also shown in Table 7.2.

The distributions for the yield model parameters (shown in Table 7.2) are based on the estimation results from the truncated reciprocal regression model reported in Chapter 6. Each parameter is assumed to have a normal distribution with an expected value equal to the value of the estimator and a standard deviation equal to the standard error from the regression output. The climate variables are assumed to have a uniform distribution. The reason for a uniform distribution is that according to the IPCC (2000) all GCM projections are equally likely and therefore it is not possible to assign different probabilities to different GCM forecasts. The source of the climate data and methods used to determine their distributions are described in Chapter 5. Climate forecast data for the variables ANTEP, ANPREC, and GSPREC are generated for the 2020's, 2050's and 2080's for the Calling Lake study location.

Estimation method for scenario one coefficient values

The estimation method for scenario one is straightforward. First, the estimators for the yield model parameters, the site data and the climate normal data are incorporated into equations 7.24 to 7.30 to estimate harvest yield and ending inventory. Then climate normal data is used to estimate the present value of soil expectation value. Finally, the estimated yield information and SEV results are incorporated into equations 7.18 to 7.23 to estimate deterministic net benefit values for each of the six combinations of IAC and prescription.

Monte Carlo simulation method for values in scenarios two, three, and four

Monte Carlo simulation of the distributions of the random yield variables and random net benefit coefficient values is conducted in this study using @RISK (Palisade Corporation 2002). This software conducts Monte Carlo simulation for hierarchical problems (such as this study) in an integrated way. For example, @RISK simultaneously generates distributions for yield variables and net benefit coefficients in one simulation run. The program accounts for interdependencies between random variables in the yield equations through the use of a correlation matrix. The sampling method employed is Latin Hypercube sampling[23]. The -selected sample size for the simulations is 5,000. The procedure for conducting the simulations is as follows.

[23] The approach with Latin Hypercube is to first stratify the input probability distribution into a set of intervals. A sample is then created by sampling from each intervals. A more detailed explanation of Latin Hypercube sampling is provided in Palisade Corporation (2002).

1. The first step involves setting up the simulation model. The relevant equations are defined and entered into an @RISK Monte Carlo simulation model spreadsheet (see equations defined above) and constant values and the known distributions for random variables are entered into the model.

2. Step two is to estimate a distribution for the Soil Expectation Value (SEV) random variable. This is done using @RISK. The calculation of SEV follows the usual Faustmann formulation:

$$SEV_t = \frac{(V_t)(2.5)}{e^{0.04*t} - 1} \qquad [7.31]$$

The optimal rotation occurs at the time "t" where SEV_t is maximized. Eight separate samples of SEV for ages 40, 50, 60, 70, 80, 90, 100 and 110 are created. For each observation the SEV is calculated for each age and the maximum SEV is selected and copied to a separate column. This results in a 5000 entry long vector of SEV values. This sample is used to fit a distribution using the "curve fit" function within @RISK. The resulting distribution is entered into cell L3. The distribution of SEV is Beta General (2.41, 8.19, -2.01, 798.7). Draws are made from this distribution for equations 7.18 and 7.23.

3. The third step is to set up a correlation matrix to take account of interdependencies between random variables in the yield equations. @RISK accounts for interdependent random variables with a correlation matrix that is incorporated into the Monte Carlo model. Correlations between the parameters are derived from the covariance matrix generated by LIMDEP-Version 7.0 (Econometric Software Inc. 1995). The correlation matrix was expanded to account for correlations between climate variables. The correlations between the climate variables were determined by creating sample distributions for each climate variable. Then, EXCEL was used to determine the correlation between the observations in the sample. An underlying assumption is that historical correlations between climate variables will continue into the future.

4. The fourth step is to truncate the distributions at zero (to ensure non-negative yields). The adjustment factor is a component of the truncated estimation model (Greene 1997 pg.954). The adjustment factor is as follows:

$$E[V_i \mid V_i > 0] = \beta' x_i + \sigma \frac{\phi[(0-\beta' x_i)/\sigma]}{1-\Phi[(0-\beta' x_i)/\sigma]},$$

Where:

V_i is the adjusted yield for observation i (truncated at zero),

$\beta' x_i$ is the unadjusted predicted yield for observation i,

σ is model standard error,

$\phi[(0-\beta' x_i)/\sigma]$ is a probability density function value for observation i,

$\Phi[(0-\beta' x_i)/\sigma]$ is a cumulative distribution function value for observation i, and

$\phi(0-\beta' x_i)/\sigma \sim N(0,1)$

[7.32]

This adjustment factor is incorporated into the simulation model. The resulting distributions are truncated at zero.

5. The fifth step is to conduct a Monte Carlo simulation (using @RISK) to generate samples from estimated probability distributions for each variable of interest (V21, V22, V31, V32, VEI1.1, VEI1.2, VEI2.0, VEI3.0, VSEV, NB11, NB12, NB21, NB22, NB31, NB32) under each scenario (scenarios 2, 3, and 4). As previously described Monte Carlo simulation involves drawing samples for right hand side (RHS) variables with known distributions to generate a sample for left hand side (LHS) yield variables. For this study, the @RISK software simultaneously determines the sample of the yield random variables and the net benefit random variables by: (a) drawing from the known yield parameter and climate distributions (conditioned by the correlation matrix), (b) incorporating each draw into the relevant yield equation, (c) generating a sample value for all yield coefficients, (d) adjusting the sample of yield estimates to ensure it is truncated at zero, and (e) incorporating the sample yield estimate into the relevant net benefit equation to generate a sample for net benefits. A total of 5000 samples are obtained. Thus, a sample of size n=5000 for each yield coefficient and each net benefit coefficient is obtained.

6. The sixth step is to utilize the generated sample distributions to obtain estimates of expected values, variances, and covariances for each coefficient under each scenario.

Simulation procedure for values in scenario two

As noted scenario two includes the climate future data in the coefficient estimations and it considers them to be random variables. As a result, all yield estimates and net benefits are also random variables. The estimation, therefore, is done using the Monte Carlo simulation procedure described above. The only variables that are random at the start are climate variables. Samples are drawn from the known distributions for the climate variables. A distribution for each of the dependent variables in equations 7.18 to 7.30 is estimated.

Simulation procedure for values in scenario three

Scenario three includes the climate future data in the coefficient estimations plus it considers both the climate variables and the yield model parameters to be random variables. Therefore, the Monte Carlo simulation model is run with the assumption that climate change occurs and that climate variables and yield parameters are uncertain. The distributions of both the future climate variables and the yield model parameters are known ahead of time. These assumed distributions are shown in table 7.2. The Monte Carlo simulation is set up to make draws from the distributions of both the climate variables and the yield parameter distributions in deriving a sample distribution for all yield and net benefit coefficients.

Simulation procedure for values in scenario four

Scenario four includes the climate future data in coefficient estimations plus it considers both the climate variables and the yield model parameters to be random variables. However, under scenario four, the first period harvest yield (V_{21} and V_{22}) are known with certainty. The Monte Carlo simulations are run assuming all parameters used for prediction of first period harvests are deterministic. The parameters used for prediction of the other yield and net benefit coefficients remain uncertain. The Monte Carlo simulation produces distributions for all coefficients other than the first period harvest yields.

Expected values and variances

Equations 7.18 to 7.30 are used within the Monte Carlo simulation models to generate samples for each variable for scenarios two, three and four. The samples (e.g. 5000 estimates of NB_{11}) are in turn utilized to determine expected values and variances. Also, the estimated samples are combined in order to estimate covariances between the random variables (within EXCEL). This leads to the question: What is the theoretical basis for estimation of expected values, variances and covariances? In order to illustrate the theoretical basis for estimation of expected value and variance for our random variables we provide the relevant rules for determining expected value and variance for the random variable NB_{21}.

The deterministic version of equation NB_{21} is repeated here for convenience.

$$NB_{21} = \frac{(V_{21})(2.5)}{(1.04)^{15}} + \frac{(V_{EI2.0})(2.5)}{(1.04)^{60}} + \frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}} \qquad [7.33]$$

As noted, for scenarios two, three, and four the yield coefficients are random variables and therefore NB_{21} is a random variable with some expected value and variance. Applying the rule for determining the expected value of the weighted sum of random variables (Griffiths et al. 1993) the expected value version of equation [7.33] is as follows:

$$E[NB_{21}] = E\left[\frac{(V_{21})(2.5)}{(1.04)^{15}}\right] + E\left[\frac{(V_{EI2.0})(2.5)}{(1.04)^{60}}\right] + E\left[\frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}}\right] \qquad [7.34]$$

Since the price and discount factor are constants they can be factored from the RHS expected values to provide:

$$E[NB_{21}] = \frac{(2.5)}{(1.04)^{15}} E[V_{21}] + \frac{(2.5)}{(1.04)^{60}} E[V_{EI2.0}] + \frac{(2.5)}{(e^{it}-1)(1.04)^{60}} E[V_{SEV}] \qquad [7.35]$$

Thus, with information on the expected values of the yield variables on the RHS it is possible to estimate the expected value of NB_{21}.

Another random variable parameter of interest is variance. The variance of the random variable NB_{21} can be shown by applying the rule for the variance of a weighted sum of random variables (Griffiths et al. 1993).

$$Var(NB_{21}) = Var\left(\frac{(V_{21})(2.5)}{(1.04)^{15}} + \frac{(V_{EI2.0})(2.5)}{(1.04)^{60}} + \frac{(V_{SEV})(2.5)}{(e^{it}-1)(1.04)^{60}}\right) \qquad [7.36]$$

$$= \left(\frac{(2.5)}{(1.04)^{15}}\right)^2 Var(V_{21}) + \left(\frac{(2.5)}{(1.04)^{60}}\right)^2 Var(V_{EI2.0}) + \left(\frac{(2.5)}{(e^{it}-1)(1.04)^{60}}\right)^2 Var(V_{SEV})$$

$$+2\left(\frac{(2.5)}{(1.04)^{15}}\right)\left(\frac{(2.5)}{(1.04)^{60}}\right) Cov(V_{21}, V_{EI2.0})$$

$$+2\left(\frac{(2.5)}{(1.04)^{15}}\right)\left(\frac{(2.5)}{(e^{it}-1)(1.04)^{60}}\right) Cov(V_{21}, V_{SEV})$$

$$+2\left(\frac{(2.5)}{(1.04)^{60}}\right)\left(\frac{(2.5)}{(e^{it}-1)(1.04)^{60}}\right) Cov(V_{EI2.0}, V_{SEV})$$

Thus, the variance of NB21 depends on the variances and covariances of V21, VEI2.0, and VSEV. The expected value, variance and covariance equations for the remaining random variable coefficients will not be repeated here. The main point here is to show the relationship between the Monte Carlo simulation results and general rules for determining expected values and variances of random variables that are functions of other random variables using a coefficient that is specific to this study.

Results

This section reports on the prediction results. There are three management prescriptions (no cut, cut in period 1, cut in period 2) and two starting forest types (initial age class (IAC1) 40 year old aspen stands, and initial age class (IAC2) 80 year old aspen stands). Thus, there are six choices in the choice set for each coefficient and for each scenario. The six choices are:

1. IAC1 (40 yrs) – no cut
2. IAC2 (80 yrs) – no cut

3. IAC1 (40 yrs) – period 1 cut (age at cut = 55)
4. IAC2 (80 yrs) – period 1 cut (age at cut = 95)
5. IAC1 (40 yrs) – period 2 cut (age at cut = 85)
6. IAC2 (80 yrs) – period 2 cut (age at cut = 125)

Scenario 1

Scenario 1 is the baseline scenario. Predictions of objective function and constraint coefficients are based on climate normals. The prediction results under this baseline scenario are provided in Table 7.3. The highest present value net benefit occurs when the IAC2 stand is harvested in period one (prescription two). The IAC2 stand is 95 years when harvested in period one. Similarly, economic return from the IAC1-stand is highest when the stand is harvested at age 55 (harvested in period one). Waiting until period two to harvest these stands reduces the present value of returns significantly. Harvest yields are highest when the stands are harvested in period two. In period two the harvest age for the IAC1 stand is 85 and the harvest age for the IAC2 stand is 125 years. As would be expected, ending inventory values are highest under prescription 1 (no cut).

Scenario 2

Tables 7.4 and 7.5 provide the scenario 2 expected values, variances and covariances for net benefits, harvest yields, and ending inventory for each age class-prescription combination. In general, the pattern of expected values for net benefits, harvest yield and ending inventory are similar to what was shown for scenario 1. However, it should be noted that the coefficient values for this scenario are expected values only and they may not be the values that actually occur. The actual returns are not known *apriori*. The net benefits with climate change may be lower or higher than the values reported in Table 7.3 (without climate change). The degree of uncertainty in the random variables under this scenario is shown in the estimated covariance matrix (Table 7.5). The variances of the net benefit results for prescription two tend to be the highest. Thus, although the returns under prescription two are the highest, the degree of uncertainty in these returns is also higher than prescription one or three.

Scenario 3

The expected values for net benefits, harvest yields and ending inventory for scenario 3 are shown in Table 7.6. Here again, expected values for net benefits are highest for the period one harvest. Harvest yields are highest for the period two harvest. Ending inventory values are highest for the no cut option. Coefficient variances and covariances under scenario 3 are shown in Table 7.7. Including yield parameter variance results in a very high variance for the period one harvest net benefit. The variance in net benefits associated with the period one harvest is significantly higher than the variances in net benefits for the period two harvests or the no cut option. There are two reasons for this result. First, the period one net benefit includes harvest revenues, soil expectation value and the present value of the ending inventory. Each of these values is uncertain and the variances from these three sources of variance is additive (see equation 7.36). The second reason is due to discounting. As will be discussed in later chapters, the reason why the net benefits are lower in future periods compared to the current period is that all the values in the sample distributions are discounted. This means that future distributions are discounted at 4 %. This tends to reduce both the expected values and variances. A more in depth discussion of this issue is provided in Chapter 8.

Scenario 4

As noted in the discussion in the previous section, when both climate uncertainty and yield parameter uncertainty are considered as sources of uncertainty, the variances of net benefits in period one harvest are very high compared to variances in net benefits for period two harvests or the no cut option. In some ways, the period one harvest can be interpreted as the current harvest (or at least the short term harvest). If this is the case, then the forest manager may not be satisfied with the high variance of benefits of the current harvest compared to variance in future harvest. As noted previously, this may motivate the manager to try and reduce or eliminate this variability by obtaining information about current inventory that results in less uncertainty about period one-harvest benefits. Thus, the set of assumptions for this scenario are the same as scenario three except that for this scenario we assume that the uncertainty of harvest yield for the period one cut is eliminated.

The results for the expected value of net benefits for scenario four are shown in Figure 7.8. Again, the expected value of net benefits are highest for prescription 2 (cut in period one), the expected value of harvest yields are highest for prescription 3 (cut in period two), and the expected value of ending inventory is highest for prescription 1 (no cut). The interesting result for this scenario pertains to the pattern of the variances for the various prescriptions. The variances for each coefficient for scenario four are provided in Table 7.9. The trend over time of the variance for net benefits is unique under scenario four. In this case, the variance of the net benefit coefficient is lowest for the period one harvest, then it increases in period two, then it decreases for the no cut option. However, the variance of net benefits for the no cut option is still higher than for the period one cut option.

Comparison of the results across the scenarios

Figure 7.1 compares the value of net benefits for scenario one and the expected values of net benefits for scenarios two, three and four. In terms of benefit measures only (not accounting for costs of uncertainty and/or benefits of diversification) climate change has a positive effect on the coefficient for net benefits per hectare for each prescription-IAC combination. The expected values of the net benefits for scenarios that include future climate change (scenarios two, three and four) values are higher than for scenario one (where no climate change occurs).

Figure 7.2 compares the variances for net benefits across scenarios two, three and four. The coefficients where different scenario assumptions have the most dramatic affect is for the period one harvest (prescription two) net benefits. Here, the variances for scenario four are the lowest, the variances for scenario two are in the middle, and the variances for scenario three are very high. Figure 7.2 also shows that for the period two cut and no cut prescriptions, the variances for net benefits for scenarios three and four are significantly higher than for scenario two. This begins to suggest that the contribution of variability in climate variables is low compared to variability contributed by uncertainty in yield parameters.

Figure 7.3 compares the value of harvest yield for scenario one and the expected values of harvest yield for scenarios two, three and four. Here again it can be seen that expected values under future climate (scenarios two, three and four) are higher than the harvest yield value under a normal climate. This suggests that for this study site, climate change may increase stand yield (although it is important to note that the values for future climate are "expected values" and that in reality these values are random variables with a positive probability that future yields may be less than current yields). Figure 7.3 also shows that the relative increase in harvest yield between period 1 and 2 (prescription 2 and 3) is higher for the IAC1 stand (HY21 → HY31) than the IAC2 stand (HY22 → HY32).

Figure 7.4 shows the variances of harvest yield for each scenario / prescription combination. For HY21 and HY22 we can see that the variance of harvest yield for scenario four is zero. A comparison of scenarios two and three shows that the inclusion of uncertainty in yield parameters results in a significant increase in variances of harvest yield.

Discussion

An issue to note for the analysis in this chapter pertains to scale. Uncertainty and variance are in some respects dependent on scale of analysis (Katz 2002). For example, the variance of a small number of 60 year-old stands of aspen will tend to be greater than the variance of a large sample of 60 year old aspen stands within a forest level analysis. This has potentially important implications for this study. First, although our hypothetical forest is relatively small and it is specific to a particular location – we are nonetheless looking at a forest level analysis for this study. As previously noted, our yield model is predicting a conditional mean yield value (a mean merchantable yield over a range of sites in a forest). Therefore, the variance of the error term is not required as a source of prediction variance (Griffiths 1993 pg. 250). If we were predicting the expected value and variance for a specific stand within the forest, then the error variance would be relevant as a factor that contributes to the overall prediction variance. Therefore, one implication of conducting a forest level analysis is that we are not required to incorporate the error variance of the prediction model as a source of prediction error.

A second scale related issue is that our future climate predictions are based on average predictions over a large area (Northeastern Alberta). Therefore, the variances of our climate variables are based on a much higher level of aggregation than our prediction of yield parameter variances. The main implication is that the variances of the climate variables are based on a different level of spatial aggregation. A higher level of aggregation tends to reduce sample variances. Thus, the variances we are employing in this study will likely underestimate the true variance in predicted climate futures at the spatial level that pertains to this study. Other than arbitrarily adjusting these variances, there is very little else that can be done to remedy this scale effect. It may be possible to obtain climate futures at higher levels of spatial resolution (for example using regional climate models). However, it is not possible to generate a range of estimates using regional climate models. This can only be done using the higher-level general circulation models. Therefore, the method that we use to assess variability in future climate predictions can only be applied at high levels of aggregation. Thus, for the purposes of this study we adopt the variances of future climate variables as given. These variances should, however, be considered to be lower bound estimates of the true variance in climate variables.

Another issue related to the determination of climate variances pertains to the source of information we have used for determining the distributions for our climate variables. As noted, predictions of future distributions of climate variables are based on obtaining a range of future values for climate variables from different combinations of general circulation models and SRES scenarios. At this point in time, there are no alternatives relative to obtaining measures of uncertainty in future climate. Thus, the uncertainty in future climate variables is a function of the degree to which the models embody different assumptions about atmospheric physics or employ different approaches for modeling climate response to GHG concentrations. However, it should also be recognized that it is possible that over time models tend to converge. The reason is that there is a tendency for modelers to discuss and debate areas where models deviate from each other. As a result of this scientific dialogue – models begin to conform with each other over time. The implication is that uncertainty (as measured by model disagreement) may decrease. In some respects, the process of model refinement may reduce uncertainty because this process may be a reflection of modelers having a better understanding of how climate will respond to changes in atmospheric chemistry. At the same time we need to keep in mind that it is possible that there are forces that are not included in models that may affect future climate. To the extent that these unknown factors are not considered or included, it is possible that measures of real uncertainty of future climate may not be decreasing. As a result, it is possible that our estimates of uncertainty in future climate variables are underestimated. Here again, the implications may be that our estimates of uncertainty in the future value of climate variables should be viewed as lower bound estimates.

Another issue to note relative to our predictions concerns the range of the temperature variables in the database used for the yield prediction model. As noted, the source of data for the yield function estimation results presented in this section is the CIPHA database. The CIPHA project includes plots ranging from southern Manitoba to the Yukon (see Figure 5.2). A comparison of the range of average annual temperatures for plots in the CIPHA data base with the predicted ranges of future average annual temperatures shows that the maximum average annual temperature for plots in the CIPHA database is lower than the maximum annual temperatures predicted in the years 2050 and 2080. Thus, our yield model is being used to predict future yields at temperature values that are outside the range of the data from which the yield model was estimated. This might be a significant concern if future predicted temperatures were outside the range of aspen in North America. However, aspen in North America has a wide range. It occurs from Virginia to Alaska. Thus aspen occurs on sites that are well within the range of future predicted temperatures predicted for the Calling Lake site.

As noted there are potential sources of uncertainty that are not included in the variance estimates presented in this chapter. One possible source of uncertainty is price uncertainty. Future studies should look into sources of data on price uncertainty and ways of modeling this uncertainty. Another source of uncertainty is catastrophic mortality. Major system failures could occur if there are non-linearities, non-convexities, and thresholds within growth, yield and survival functions that are breached as a result of change climate. Possible causes of major ecosystem failure (however unlikely) include major infestation by exotic pests or by pest that expanded their range, massive drought, extreme wildfire conditions, extreme weather, or dieback due to changed climatic conditions exceeding physiological tolerances. Risks due to these types of catastrophic impacts are not incorporated into the biophysical model in this study. The reason for excluding these effects was that our study site is not in an area close to a transition zone and because we are only considering impacts up to the year 2070. Widespread mortality and dieback is therefore less likely. A third possible source of uncertainty is growth uncertainty. This study estimates a stand yield function and uses this function to provide a prediction of the distribution of stand yields in future time periods. As a result, our estimates of the distributions of the parameters of the yield curve are invariant with respect to age and time. As we have noted, this results in a model where the only source of variance that changes with respect to time is the variance in the climate variables. Variances in the yield model parameters are invariant to time. If, however, we were able to estimate a growth function, then it is possible that variances in yield parameters would be a function of time and may actually increase with respect to prediction time. This may have resulted in a far higher rate of change in yield variances with respect to prediction period. This is an area that merits further investigation.

As previously noted, climate change has two potentially important implications for timber management. First, there are implications for growth rates and stand yields (productivity effects). Second, climate change introduces additional sources of uncertainty about future returns from timber production (uncertainty effects). Both of these effects have the potential to influence the economic benefits of timber production, choices regarding harvest rates and timing, timber investment, and the social opportunity costs of forest management policies (e.g. sustained yield type policies such as ending inventory constraints and even flow constraints). The next three chapters present the results of three separate models that are designed to assess the consequences of productivity change and uncertainty on total benefits and optimal decisions for our hypothetical forest management case study situation. The objective function and constraint coefficients and the covariance matrices included in the risk models in the next three chapters rely on the estimates of expected values, variances and covariances provided in this chapter.

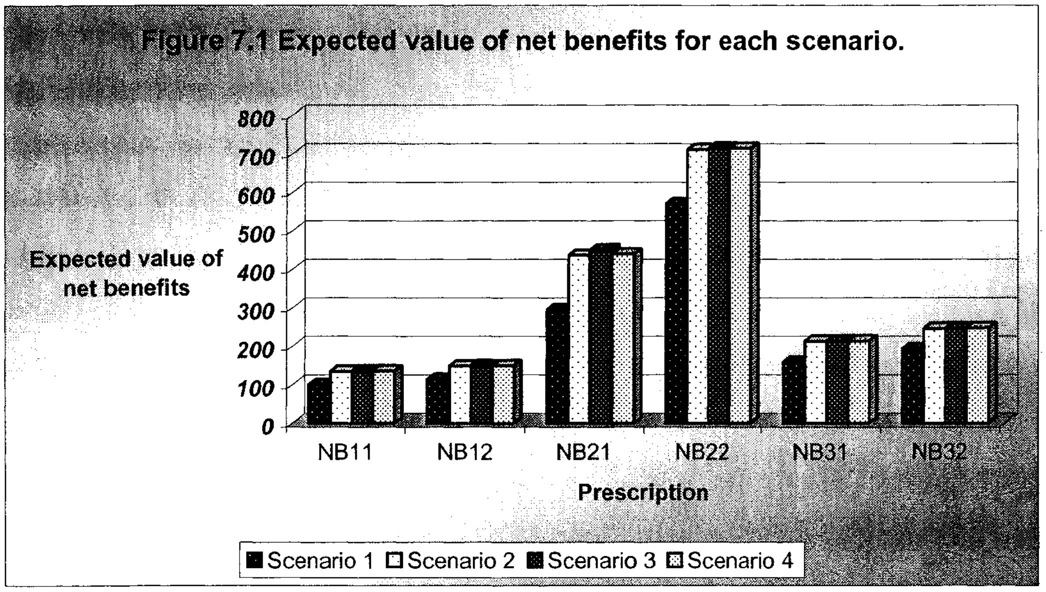


Figure 7.1 Expected value of net benefits for each scenario.

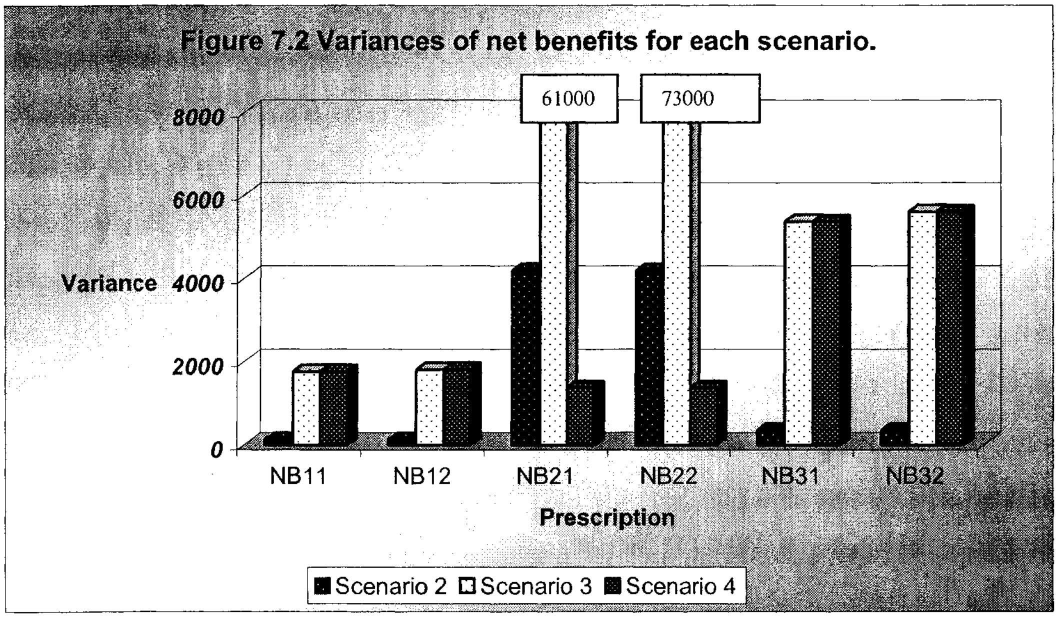


Figure 7.2 Variances of net benefits for each scenario.

Figure 7.3 Expected value of harvest yields for each scenario.

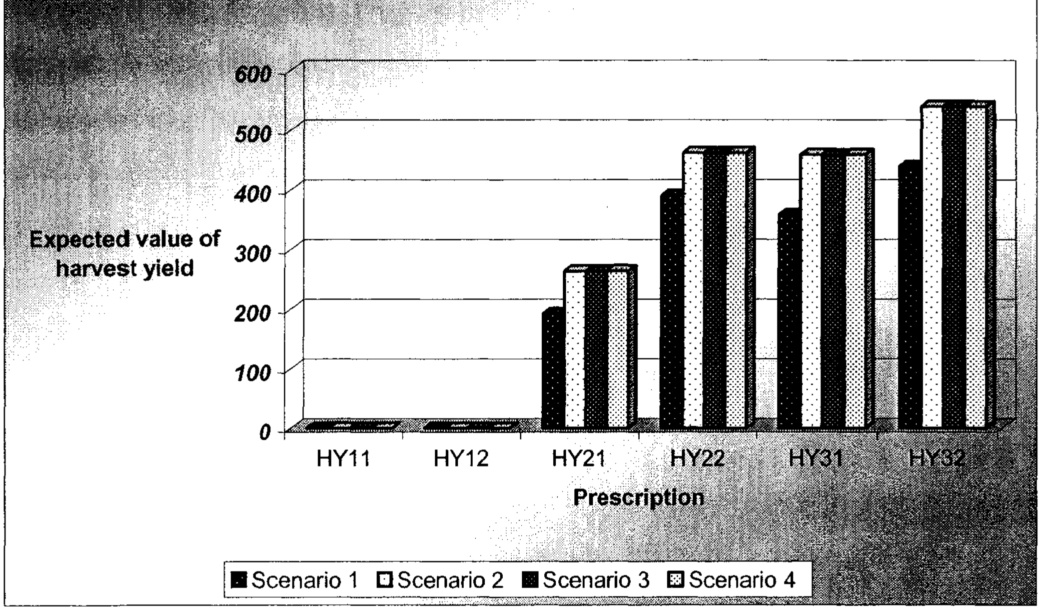


Figure 7.4 Variances of harvest yield for each scenario.

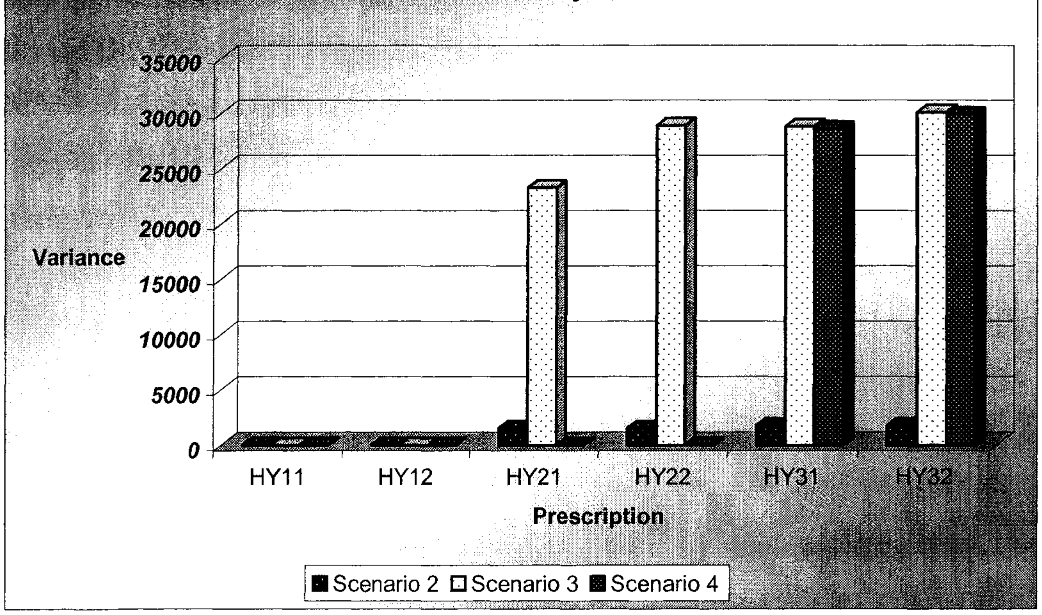


Table 7.1 Values of constants in yield predictions: Calling Lake study site

Variable	Value
Stumpage price	$ 2.50 per cu. m.
Discount rate	4.0 %
Climate normal data	
Annual temperature	0.7 deg C
Annual precipitation	463 mm
Growing season precip.	326 mm
Sand	34 %
Clay	19 %
Site index	19.8 meters at 50 yrs

Density and reciprocal of age by age class

Age	Density	Reciprocal of age
30	3000	0.033
40	2,500	0.029
45	2,250	0.025
50	2,000	0.022
55	1,800	0.020
60	1,600	0.018
65	1,425	0.017
70	1,250	0.014
80	900	0.013
85	700	0.012
90	500	0.011
100	500	0.010
110	500	0.009
120	500	0.008
125	500	0.008
130	500	0.008
140	500	0.007

Source for the stand density data: Tables 8 and 9 in Peterson and Peterson (1992)

Table 7.2 Known distributions for random variables in scenarios two, three and four (Model parameters from Table 6.6)

Coefficient	Distribution Type	Expected value	Standard deviation
Constant	Normal	4.59	115.5
Zone	Normal	406.41	179.76
Density	Normal	0.023	0.015
Zdensity	Normal	-0.065	0.02
Site	Normal	25.12	3.47
Zsite	Normal	-0.643	4.11
Sand	Normal	0.845	0.545
Zsand	Normal	-3.08	0.864
Clay	Normal	-0.373	0.942
Zclay	Normal	-1.35	1.183
Antemp	Normal	8.3	10.04
Zantemp	Normal	33.73	13.43
Anprec	Normal	0.011	0.319
Zanprec	Normal	0.802	0.616
Gsprec	Normal	-0.135	0.572
Zgsprec	Normal	-1.64	0.918
Ageinv	Normal	-19183	2624

Climate variables

2020's

Variable	Distribution	Lower	Upper
Antemp	Uniform	1.03	4.03
Anprec	Uniform	411.41	561.23
Gsprec	Uniform	285.84	394.06

2020's / 2050's

Variable	Distribution	Lower	Upper
Antemp	Uniform	1.37	4.87
Anprec	Uniform	421.65	563.5
Gsprec	Uniform	293.85	387.41

2050's

Variable	Distribution	Lower	Upper
antemp	Uniform	1.71	5.72
anprec	Uniform	431.9	565.78
gsprec	Uniform	301.87	380.76

2080's

Variable	Distribution	Lower	Upper
antemp	Uniform	2.79	8.78
anprec	Uniform	430.59	591.51
gsprec	Uniform	300.31	390.37

Table 7.3 Predictions of net benefits (NB_{ij}), harvest yields (HY_{ij}), and ending inventory yield (EIij) for scenario one.

Prescription (i=1,2,3)	Initial age class (j=1,2)	Net benefit (NB_{ij}) ($ per ha)	Harvest yield (HY_{ij}) (cu. m. per ha)	Ending inventory (EI_{ij}) (cu. m. per ha)
1 - (no cut)	1 (40 yrs)	101	0	400
1 - (no cut)	2 (80 yrs)	114	0	458
2 - (period 1 cut)	1 (40 yrs)	295	192	97
2 - (period 1 cut)	2 (80 yrs)	570	390	97
3 - (period 2 cut)	1 (40 yrs)	159	358	0
3 - (period 2 cut)	2 (80 yrs)	193	439	0

Table 7.4 Predictions of expected values of net benefits (NB_{ij}), harvest yields (HY_{ij}), and ending inventory yield (EIij) for scenario two.

Prescription (i=1,2,3)	Initial age class (j=1,2)	Net benefit $E[NB_{ij}]$ ($ per ha)	Harvest yield $E[HY_{ij}]$ (cu. m. per ha)	Ending inventory $E[EI_{ij}]$ (cu. m. per ha)
1 - (no cut)	1 (40 yrs)	135	0	501
1 - (no cut)	2 (80 yrs)	149	0	558
2 - (period 1 cut)	1 (40 yrs)	436	264	225
2 - (period 1 cut)	2 (80 yrs)	711	462	225
3 - (period 2 cut)	1 (40 yrs)	212	459	0
3 - (period 2 cut)	2 (80 yrs)	247	539	0

Table 7.5 Covariance matrices for net benefits (NB_{ij}), harvest yields (HY_{ij}) and ending inventory (EI_{ij}) for scenario two

Net Benefits

	NB11	*NB12*	*NB21*	*NB22*	*NB31*	*NB32*
NB11	119					
NB12	119	119				
NB21	651	651	4183			
NB22	651	651	4183	4184		
NB31	198	198	1156	1156	341	
NB32	198	198	1156	1156	341	341

Harvest Yield

	HY11	*HY12*	*HY21*	*HY22*	*HY31*	*HY32*
HY11	0					
HY12	0	0				
HY21	0	0	1534			
HY22	0	0	1534	1534		
HY31	0	0	1584	1584	1752	
HY32	0	0	1584	1584	1752	1752

Ending Inventory

	EI11	*EI12*	*EI21*	*EI22*	*EI31*	*EI32*
EI11	1752					
EI12	1752	1752				
EI21	1925	1925	2201			
EI22	1925	1925	2201	2201		
EI31	0	0	0	0	0	
EI32	0	0	0	0	0	0

Table 7.6 Predictions of expected values of net benefits (NB_{ij}), harvest yields (HY_{ij}), and ending inventory yield (EIij) for scenario three

Prescription (i=1,2,3)	Initial age class (j=1,2)	Net benefit $E[NB_{ij}]$ ($ per ha)	Harvest yield $E[HY_{ij}]$ (cu. m. per ha)	Ending inventory $E[EI_{ij}]$ (cu. m. per ha)
1 - (no cut)	1 (40 yrs)	136	0	500
1 - (no cut)	2 (80 yrs)	150	0	558
2 - (period 1 cut)	1 (40 yrs)	450	264	225
2 - (period 1 cut)	2 (80 yrs)	715	461	225
3 - (period 2 cut)	1 (40 yrs)	213	458	0
3 - (period 2 cut)	2 (80 yrs)	248	539	0

Table 7.7 Covariance matrices for net benefits (NB_{ij}), harvest yields (HY_{ij}) and ending inventory (EI_{ij}) for scenario three

Net Benefits

	NB11	*NB12*	*NB21*	*NB22*	*NB31*	*NB32*
NB11	1754					
NB12	1778	1806				
NB21	9942	10036	60902			
NB22	11072	11220	65629	72795		
NB31	3052	3092	17707	19642	5366	
NB32	3118	3166	17944	20048	5474	5603

Harvest Yield

	HY11	*HY12*	*HY21*	*HY22*	*HY31*	*HY32*
HY11	0					
HY12	0	0				
HY21	0	0	23262			
HY22	0	0	25443	28890		
HY31	0	0	25475	28739	28807	
HY32	0	0	25838	29385	29400	30105

Ending Inventory

	EI11	*EI12*	*EI21*	*EI22*	*EI31*	*EI32*
EI11	29486					
EI12	29915	30412				
EI21	24431	24597	22204			
EI22	24431	24597	22204	22204		
EI31	0	0	0	0	0	
EI32	0	0	0	0	0	0

Table 7.8 Predictions of expected values of net benefits (NB_{ij}), harvest yields (HY_{ij}), and ending inventory yield (EIij) for scenario four

Prescription (i=1,2,3)	Initial age class (j=1,2)	Net benefit $E[NB_{ij}]$ ($ per ha)	Harvest yield $E[HY_{ij}]$ (cu. m. per ha)	Ending inventory $E[EI_{ij}]$ (cu. m. per ha)
1 - (no cut)	1 (40 yrs)	136	0	501
1 - (no cut)	2 (80 yrs)	150	0	558
2 - (period 1 cut)	1 (40 yrs)	440	264	236
2 - (period 1 cut)	2 (80 yrs)	714	462	236
3 - (period 2 cut)	1 (40 yrs)	214	459	0
3 - (period 2 cut)	2 (80 yrs)	248	539	0

Table 7.9 Covariance matrices for net benefits (NB_{ij}), harvest yields (HY_{ij}) and ending inventory (EI_{ij}) for scenario four

Net Benefits

	NB11	*NB12*	*NB21*	*NB22*	*NB31*	*NB32*
NB11	1757					
NB12	1781	1808				
NB21	1484	1493	1363			
NB22	1484	1493	1363	1363		
NB31	3053	3092	2587	2587	5358	
NB32	3117	3164	2608	2608	5464	5590

Harvest Yield

	HY11	*HY12*	*HY21*	*HY22*	*HY31*	*HY32*
HY11	0					
HY12	0	0				
HY21	0	0	0			
HY22	0	0	0	0		
HY31	0	0	0	0	28616	
HY32	0	0	0	0	29197	29887

Ending Inventory

	EI11	*EI12*	*EI21*	*EI22*	*EI31*	*EI32*
EI11	29282					
EI12	29702	30189				
EI21	24462	24621	22336			
EI22	24462	24621	22336	22336		
EI31	0	0	0	0	0	
EI32	0	0	0	0	0	0

CHAPTER EIGHT

MARKOWITZ PORTFOLIO FRONTIER MODEL

The complexity comes from the nature of the real system, not from some weakness in model formulation. The task for the analyst is to find a simplification of the real system that is good enough for the purpose at hand. The decision is inevitably subjective, emphasizing once again the artistic nature of decision analysis.

Hardaker et al. 2004

Introduction

In this chapter, we apply efficiency analysis methods to our stylized forestry problem by incorporating the distributions developed in Chapter 7 into a Markowitz portfolio frontier model. The particular formulation used in the analysis in this chapter is one that estimates the expected value-variance (EV) frontier for our stylized forest management problem (Note: the formulation presented in Chapter 9 is also a Markowitz model, however, in that case the model is formulated to identify the specific portfolio that maximizes expected utility).

The Markowitz model has been used in a number of agriculture applications. Hardaker et al. (2004) and Brealy and Myers (2003) review the underlying theory and a number of applications in agriculture. The model has also been used in forestry applications. Mills and Hoover (1982) apply this model to look at the economic benefits of institutional investment in forestland as a portfolio diversification strategy. Thomson (1991) compares the results of single period models (such as the Markowitz model) with a multi-period "power utility function" where the investor has access to forest land purchase and non-timber financial investment alternatives for inclusion in ones portfolio. Heikkinen (1999) was the first to apply the Markowitz approach in a forest management decision-making context. He used the Markowitz model to evaluate the effect of harvesting rules on portfolios made up of forest stands and stocks. The study described in Reeves and Haight (2000) is the closest to the study presented here. They incorporate a Model I timber harvest scheduling model into a Markowitz risk model and look at the implication of price uncertainty on harvest timing and forest stand composition. Finally, Heikkinen (2003) compare the results of a single period Markowitz model to the results of a multi-period discrete stochastic programming (i.e. recourse) model applied to a land owner whose decision problem is to harvest and invest the resulting income or delay harvesting. To our knowledge, the methodology has not been used to evaluate the potential implications of climate change and climate change related uncertainty for forest management.

This chapter is organized as follows. In section two we introduce and define the terms: portfolios and prospects. Section three describes how the concept of portfolios and prospects apply to the stylized decision analysis problem context presented in this study. Section four describes the concepts and criteria underlying efficiency analysis. Section five introduces the Markowitz portfolio model and presents the particular specification used. Section six presents and provides a discussion of the estimation results. Finally, the last section provides a discussion of some of the possible implications of the results for broader forest management policy issues.

Prospects and portfolios

An investment prospect represents investment in a single asset such as a particular stock, bond, piece of real estate, or as will be described later, a particular forest management prescription. Investors are generally not restricted to a single prospect. Rather, they may prefer to invest in a number of prospects in order to diversify their investments and manage risk (Elton and Gruber 1995). The collection of prospects that an investor chooses is referred to as a portfolio. It is the expected value of returns from the portfolio that is of particular interest to the investor and it is the variance associated with the overall portfolio that provides the investor with a measure of the level of investment risk (Binkley et al. 1996). The choice facing the decision maker is: What group of prospects should be included in the portfolio in order that one's investment objectives or preferences are optimal (i.e. the optimal combination of return and risk)?

One forest sector example of portfolio construction and diversification is the increase in institutional holdings of U.S. timberlands that occurred during the 1990s

(Binkley et al. 1996; Caulfield and Newman 1999). The main reason for the inclusion of forest timberland holdings in the portfolios of large pension funds, insurance companies, banks, and endowments was the opportunity these types of assets offered in terms of both return and portfolio diversification.

The main rationale for looking at returns and variances of portfolios rather than single prospects is that the interrelationships between prospects in a portfolio influence the level of risk (or variance) of the portfolio. The variance associated with a portfolio of prospects tends to be less than the weighted average of the variances of the individual prospects (Zerbe and Dively 2004). In fact, if there are numerous prospects in a portfolio, the variance of the portfolio is largely influenced by the covariances between prospects and the variances of individual prospects have less importance. In undiversified portfolios, the variances of the individual prospects will dominate the variance of the portfolio. Thus, in general, investors manage risk by including a diverse range of prospects in their portfolios.

The choice set of available prospects and a general description of the management problem

This section describes the management problem developed for this study and identifies the set of investment options that are available to the decision maker. The analytical construct described in this section pertains to the analysis presented and discussed in this Chapter as well as the analysis presented in Chapters 9 and 10.

For this study we are applying the notion of prospects and portfolios to a particular forest management decision-making problem. The decision maker's initial assets for this study are 250 hectares of 40-year old aspen and 750 hectares of 80-year old aspen. The decision maker has already invested in this forest. Available prospects for this study are limited to individual harvest prescriptions (defined as prospects for this study). The general problem is one of selecting the optimal portfolio (and therefore an optimal set of prospects) in an environment where: (a) yield curves are shifting as a result of climate change, (b) there is uncertainty as a result of climate change, and (c) there is uncertainty in yield model predictions.

Chapter 7 provides estimates of expected values and a covariance matrix for net benefits for six specific prospects for each of four scenarios. The six prospects that are available to the decision maker in this study are defined as follows:

1. Leave initial age class 40 (IAC1) uncut for the entire period
2. Leave initial age class 80 (IAC2) uncut for the entire period
3. Cut initial age class 40 (IAC1) in period one
4. Cut initial age class 80 (IAC2) in period one
5. Cut initial age class 40 (IAC1) in period two
6. Cut initial age class 80 (IAC2) in period two.

Each prospect has a unique expected net benefit, a unique variance, and there exist a set of unique covariances between the six prospects (note these values constitute the input data for the risk models presented in this Chapter and in Chapters Nine and 10). The portfolios (i.e. the solution set provided by the risk models), therefore, are comprised of various combinations of the six prospects or management prescriptions (where the weight of each prospect in a portfolio is based on the number of hectares assigned to the prospect). There exist a large number of possible portfolios (i.e. differences in portfolios are based on variations in number of hectares assigned to each prospect) and each has a unique combination of expected return and variance.

Some forestry-based applications of the Markowitz portfolio approach provide the decision maker with the opportunity to invest in other financial market investments in addition to forest management (e.g. see Heikkenen 1999; Thomson 1991). These studies assume that the investor is starting with a pool of capital to invest and is not constrained in terms of where the capital can be invested. This study assumes that the decision-maker has already invested in forestland. The objective of the decision maker for this chapter is not to minimize risk (subject to minimum return) of his/her entire wealth holdings. Rather, the objective of the decision maker for this chapter is to minimize portfolio risk (subject to a minimum return for the forestry asset). The objective function in Chapter 9 is to maximize certainty equivalent but again the decision maker is restricted to the six prospects described here. This construct is admittedly somewhat restricted. However, optimizations within type problems, such as the one posed in this study, are not without precedence. For example, the approach adopted in this study is similar to demand system models. Demand system models use assumptions about two-stage budgeting, utility trees, and weak separability to model demand for commodity groups (Deaton and Muellbauer 1998). Demand system models choose the basket of goods that optimizes utility for some sub-group of commodities.

Thus, the available set of prospects defined for this study does not include other types of financial market investments (e.g. stocks, bonds, treasury bills, etc.). Moreover, the model in this chapter (and in Chapters 9 and 10) assumes that harvest revenues (i.e. revenues from period 1 and 2 harvest) are invested in risk free investments that earn an annual return of exactly 4 % (i.e. our assumed discount rate) and that mature at the end of the planning horizon. Thus, the real returns from reinvestment of harvest income have no effect on prospect choices. The reason for limiting the set of investment and reinvestment opportunities for this study is to keep the problem context as simple as possible so that we can focus on analyzing the effects of climate change and uncertainty in a forest management context without complicating the analysis by considering other investment types and other potential land uses. The question that we are principally interested in for this study is: How does climate change and uncertainty affect returns to forest management and manager choices relative to harvest timing? The set of six prospects defined in this chapter are also used for the risk models developed in chapters nine and ten.

It is useful at this stage to provide a bulleted summary of the stylized forest and an overview of the management problem adopted for this study. The construct described below applies to the models developed in this Chapter as well as in Chapters 9 and 10.

- The investor's starting assets for this study are 250 hectares of 40-year old aspen and 750 hectares of 80-year old aspen.
- The objective functions in this study focus on optimization of outcomes by selecting optimal portfolios where portfolios are comprised of various combinations of six possible management prescriptions. Individual management prescriptions are viewed as investment prospects. The investor

does not have access to other financial market investment options and/or the opportunity to invest in alternative land types. Moreover, reinvestment of harvest revenues is restricted to investment in risk free assets that have an annual return of exactly 4 % and that mature at the end of the planning period.

- The models in Chapters 8 and 9 are single-period models. They do not permit the consideration of dynamic risk and they do not provide opportunities to adjust management choices over time. Time is not an explanatory variable in these models. The models are static. The optimal solution is determined at the beginning of the planning horizon and this solution is fixed over the entire period.
- There are a maximum of two sources of variance that can be considered (depending on which scenario is selected): 1. Variance in climate variables, and 2. Variances in yield parameters. Variances in climate variables are permitted to change for different prediction periods in the future. Variances in yield model parameters are constant.
- The planning horizon is 60 years (2010 – 2070) and this planning horizon is divided into two 30-year planning periods.
- Stumpage prices are fixed at $ 2.50 per cu meter and the discount rate is fixed at 4%. Both of these values are constants.

Efficiency analysis and criteria

The expected utility theory described in Chapter two relies on having some understanding of the shape of the utility function for the individual in question (Hardaker et al. 2004). However, the elicitation of utility functions is inherently complex and requires significant data collection. The determination of a specific utility function is beyond the scope of this study. Alternatively, we will employ approaches that allow for the assessment and ranking of risky portfolios based only on information about expected returns and variances. The approach is referred to as "efficiency analysis" (Hardaker et al. 2004).

Efficiency analysis starts by making certain assumptions about the form of the utility function and about the extent to which the decision-maker's subjective assessment of likelihoods matches the actual probability distribution of outcomes (i.e. the individual is fully informed and rational). Given these assumptions, the set of all possible portfolios can be subdivided into the efficient set and the inefficient set (Hardaker et al. 2004). In order to characterize efficient and inefficient sets it is useful to characterize portfolios in terms of expected returns and variance (EV). The EV efficiency rule states that for two portfolios ("A" and "B") if E[A]>E[B] and Var A is less than or equal to the Var B then A is preferred to B. Similarly if E[A] is less than or equal to E[B] and Var A > Var B then B is preferred to A. Using these criteria, a frontier of efficient portfolios (n expected income – variance space) can be identified such that for any efficient portfolio with a given expected return, it is not possible to obtain a portfolio with the same return but lower variance. The inefficient set contains all portfolios that are dominated by portfolios in the efficient set.

The EV efficiency rule requires that: (a) portfolio returns are normally distributed, and (b) the utility function is concave for risk averse decision makers (Hardaker et al. 2004; Hazell and Norton 1986). However, functional forms such as quadratic utility have been shown to be somewhat contrary to theory and returns are not necessarily normally distributed (Hardaker et al. 2004; Hazell and Norton 1986). Therefore, the EV efficiency approach is generally considered to provide an approximate criterion for ranking risky choices (Heikkinen 1999; Elton and Gruber 1995). Its main advantage is that portfolios can be evaluated with only information on means and variances - without having to estimate utility functions. Thus, the approach provides a convenient method for looking at risk issues such as the one posed in this study. Hardaker et al. (2004) state: "Portfolio analysis in an EV framework is a widely used...method of decision analysis...Where direct maximization of utility is possible, it is to be preferred to the EV approximation...However, the convenience of EV analysis means that it is likely to remain in the tool-kit of agricultural economists for some time to come." (pg. 147).

The Markowitz portfolio model

An important modeling method used in EV analysis is called Markowitz portfolio optimization (Reeves and Haight 2000; Hardaker et al. 2004). One application of this type of model is to estimate the frontier of efficient portfolios by finding the set of prospects that minimizes portfolio variance subject to a minimum return. Parametric programming is used to define the frontier of efficient portfolios by varying the minimum return in the constraint and repeatedly rerunning the model. The remainder of this chapter outlines a model and presents results for this application. A second application of the Markowitz portfolio model is to find the portfolio that maximizes certainty equivalent. This application is discussed in the next chapter.

The asset allocation (or portfolio) model (Markowitz 1952) was one of the first risk models to be developed. Markowitz shared the 1990 Nobel Prize in Economics for this contribution to economic theory. The Markowitz model is described by Zenios (1996) as follows:

> "Asset allocation decisions are, usually, made based on the principle of diversification. Assuming that the risk of the asset classes is captured by the variance in their returns, the asset allocation model will diversify risk by selecting securities whose returns are not highly correlated with each other."

The Markowitz portfolio model provides for explicit consideration of the relationship between returns and risk for our stylized forest management decision context. For the purposes of this model, we assume that the decision maker is a rational economic agent whose only concern relative to his/her choice of portfolio of investments is return and risk (i.e. the EV efficiency rule).

When estimated using quadratic programming methods, the Markowitz model is a single period model (Thomson 1991). In our case, the period is 60 years long. This means that a single solution is estimated for the entire period. There is no reinvestment over the planning horizon and there is no recourse. In effect, a solution is obtained regarding how much area to harvest in periods one and two and how much area to leave uncut and this solution is fixed for the entire harvest period.

As noted, there are only six prospects available to the investor. Individual prospects are based on combinations of harvest prescriptions (prescriptions 1,2, and 3) and initial age classes (40 and 80). For convenience, the six prospects (i.e. 3 alternatives for the area in each starting age class) are repeated here. They include:

1. Leave initial age class 40 (IAC1) uncut for the entire period
2. Leave initial age class 80 (IAC2) uncut for the entire period
3. Cut initial age class 40 (IAC1) in period one
4. Cut initial age class 80 (IAC2) in period one
5. Cut initial age class 40 (IAC1) in period two
6. Cut initial age class 80 (IAC2) in period two.

In terms of portfolio construction, the forest landowner can construct a wide range of different portfolios by assigning his/her forestland to the above six prospects in varying proportions. The only constraint is that the total amount of hectares assigned must equal the starting area available (i.e. 250 hectares of IAC1 timber and 750 hectares of IAC2 timber).

The objective function for the Markowitz risk model presented in this chapter is to minimize portfolio variance of net benefits. Model constraints include: (a) a minimum portfolio return, (b) area constraints, and (c) non-negativity constraints. The perspective we are adopting for the model developed in this chapter is that of a private investor. Therefore, harvest constraints (i.e. ending inventory and flow constraints) are not imposed.

The model structure is as follows:

$$\begin{aligned}
&\underset{\{X_{ij}\}}{Min} \quad \sum_{i=1}^{3}\sum_{j=1}^{2} X'_{ij}\, Cov\{NB_{ij}\} X_{ij} \\
&st \\
&\sum_{i=1}^{3}\sum_{j=1}^{2} X'_{ij}\, E[NB_{ij}] \geq \text{E[Minimum Return]} \\
&\sum_{i=1}^{3} X_{i1} \leq 250 \qquad\qquad [8.1] \\
&\sum_{i=1}^{3} X_{i2} \leq 750 \\
&X_{ij} \geq 0
\end{aligned}$$

Where $\sum_{i=1}^{3}\sum_{j=1}^{2} X'_{ij}\, Cov\{NB_{ij}\} X_{ij}$ measures portfolio variance

And where:

X_{ij} is the area of compartment j that receives prescription i.

$E[NB_{ij}]$ is the expected net benefit from prescription i and initial age class j.

The data for this model include both the scenario two and three estimates of expected net benefits, variances, and covariances of the six prospects (see Chapter 7). One set of results is generated by running the model using the scenario two data and a second set of results is generated by running the model using the scenario three data. We selected scenarios two and three because one of the goals of this Chapter is to look at the contribution of climate risk relative to overall risk in a forest management context. The two scenarios differ only in terms of the fact that scenario three includes both yield parameter uncertainty and climate uncertainty while scenario two includes climate uncertainty only.

Parametric programming is used to estimate the frontier of portfolios for each scenario. The optimal investment portfolios (i.e. the optimal number of hectares allocated to each prospect) are estimated for each of eleven different levels of minimum returns starting at \$ 147,000 (minimum possible return for this forest) then going to \$ 200,000 then increasing by increments of \$50,000 up to a level of \$650,000 (maximum possible return for this forest). The objective function is a non-linear quadratic functional form and therefore a non-linear optimization routine is required for solving the model. The optimization problem is solved using CONOPT in GAMS (Brooke et al. 1998)[24].

Analysis

There are four questions that will be addressed in this chapter using the results of the Markowitz model. They are:

1. What is the shape of the return-risk frontier for this particular forest management problem and how does portfolio composition vary along the frontier?
2. What are the relative magnitudes of climate variance vs. yield parameter variance as sources of variance for this problem?

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[24] The GAMS program code is available upon request

3. What are the implications of discounting?

4. How might biased perceptions of risk influence choices?

The return-risk frontier

Figure 8.1 shows the EV portfolio frontiers for scenarios 2 and 3. The shapes of these curves are roughly consistent with the mean value – variance (EV) frontier predicted by expected utility theory (i.e. $\partial R/\partial SD > 0$; $\partial^2 R / \partial SD^2 < 0$ - where R is expected return and SD is standard deviation). The curves shown in Figure 8.1 are in fact, quite similar in shape to the expected value-variance frontier for a problem provided in Hardaker et al. (2004) (see Figure 8.2). The decision maker for our stylized forestry case study, has significant flexibility relative to substituting expected returns for risk. A highly risk averse decision maker will tend to prefer portfolios on the lower and steeper portions of the frontier. However, if the decision maker is prepared to accept higher levels of risk, then it is possible to increase expected returns significantly.

This study was intentionally structured to keep the problem scenario as simple as possible. This was done in order to isolate climate effects and also in order to look at climate change and risk using different risk model constructs. However, as we have noted in a previous section, one consequence is that the number of investment prospects available to the decision maker is limited to six forestry prescriptions. The limited number of prospects available for portfolio selection has implications relative to the solutions of the Markowitz model. Table 8.1 provides the set of model solutions for scenario 3. If this particular decision maker is so risk averse that he/she is willing to sacrifice any amount of return to reduce portfolio risk then he/she will decide to leave almost the entire forest uncut – with the exception of 6 hectares of initial age class 2 (Table 8.1). As shown in Table 7.5, the variances for prospects 1 and 2 (i.e. leave the stands uncut for initial age class 1 and 2) are lower than variances for the other four prospects (prescriptions). At the other extreme, if the decision maker is close to risk neutral and wants to maximize returns regardless of risk, then he/she will choose to cut the entire forest in period 1. Table 7.5 shows that again the variances for prescriptions (prospects) 2 and 3 are the highest of the six options. Thus, it would appear that at the extreme ends of the frontier, the portfolio is made up of those prospects where the variance of the individual prospects (not covariances between prospects) is the dominant consideration. This finding may be the result of the fact that there are a relatively small number of prospects for the particular forest management portfolio selection problem being considered in this study. Zerbe and Dively (1994) note that a major benefit of having a larger number of prospects in a portfolio is diversification.

Diversification of a portfolio has two effects. First, it tends to reduce portfolio variance. Second, as the number of prospects in a portfolio increases, the covariance terms become relatively more important with respect to overall portfolio variance. If the number of prospects is relatively small, the portfolio variance is dominated by the variances of the individual prospects. If the number of prospects is large, then portfolio variance is primarily affected by the covariances. Zerbe and Dively (1994) illustrate this as follows. If there are 5 prospects in a portfolio (i.e. N=5) then there are 5 variance terms and 20 covariance terms (i.e. $N^2 - N$) that are contributing to portfolio variance. If the number of prospects is equal to 10 (i.e. N=10) then there are 10 variance terms and 90 covariance terms that are contributing to portfolio variance.

Sources of variance

One way to assess the relative contribution of climate variance to total variance is to compare the position of the EV frontier when climate variance is the only source of uncertainty to the position of the EV frontier when both climate and yield variances are considered. As described in Chapter 7, scenario 2 is based on changes in climatic variables and climate variances only. Yield parameters are considered to be deterministic in scenario 2. Scenario 3 results, on the other hand, include uncertainty (variance) in both yield model parameters and in climate variables. Thus the positions of the scenario 2 and 3 EV frontier may be used to compare the relative contribution of climate variance to total variance. Figure 8.1 shows that climatic factors account for a relatively small portion of total portfolio variance in that the scenario 3-curve is much further to the right of the y-axis than the scenario 2-curve.

Table 8.2 compares the portfolio standard deviations for scenarios 2 and 3. The results in Table 8.2 confirm the findings in Figure 8.1. Climate variance accounts for about 25 % of the standard deviation in minimum expected return. This result suggests that the relative contribution of climatic factors to variance of forestry portfolios may not be large – at least within the 60 year planning horizon defined for this study and given the range of variables considered.

A potentially important implication of the relatively low contribution of climate variance to total risk is that uncertainty in yields may not increase significantly over time. *A priori* one might expect that the further into the future one is predicting – the higher the predicted variance will be. One objective of the study is to consider climate as a factor that potentially increases risk in forestry investments because predictions of future yields are more uncertain given uncertain climate variables. For the models estimated in this study, climate is the only source of variance that is allowed to change relative to predictions of future values of variables. The variance that is contributed by uncertainty in yield model parameters is constant for predictions of future values of yields. The underlying premise is that (ignoring climate effects) inventory variance of a stand of age "G" is the same in year 2070 as it is in the year 2010. This does not seem to be an unreasonable assumption. Since climate is the only factor that contributes to changes in variances for future predicted yields, and since climate contributes a relatively small amount to overall variance - the net effect may be that the increase in variance of predicted yields associated with increases in prediction period may not be particularly significant.

In order to test the degree to which variances of inventory yields for 40 and 80 year old stands are sensitive to prediction period (when climate effects are included), yield variances for 40 and 80 year old stands were estimated for three future time periods. Table 8.3 shows that the variances for stand yields do increase as the length of the prediction period increases, however, the amount of increase in variance of stand yield is not large. As noted, this may be partly because of the relatively small contribution of climate variances to overall portfolio variances.

Another possible reason that the variances are somewhat insensitive to climate change is that the variances of the climate variables themselves do not increase over time to the degree expected. Table 8.4 shows the first differences (i.e. high prediction value minus low prediction value) for the three climate variables used in the yield prediction models (i.e. temperature, annual precipitation and growing season precipitation). These high and low climate variable predictions are based on the upper and lower prediction values from various GCM model and emission scenario combinations. The predictions were obtained for the specific geographic location of interest for this study (i.e. Calling Lake). As seen in Table 8.4, uncertainty associated with temperature does increase with time. However, contrary to the temperature results and contrary to expectations, the spread between the high and low values for annual precipitation and growing season precipitation actually decreases after 2020 and increases significantly in the 2080 prediction data. There are two possible reasons. First, for the 2010 – 2070 period, the general circulation models are in general agreement about precipitation responses to increases in greenhouse gas concentrations. Second, precipitation response is not as sensitive as temperature to differences in atmospheric GHG concentrations for the study period.

The main conclusion from these results is that for the defined study period climate change does affect uncertainty and risk in forestry analysis but it is not a strong factor. It does appear from Table 8.4 that uncertainty in climate variables increases significantly in the 2070 prediction period. Thus, the effects of climate change in terms of uncertainty and risk may be more pronounced for analysis that covers longer time periods. This conclusion, however, should be qualified by the fact that the further into the future the analysis goes, the more that distributions of net benefits become discounted. Somewhat large increase in yield variance would be required in order to offset the effects of exponential discounting of future net benefit distributions. The issue of discounting is discussed further in the following section.

One final point about sources of variance in portfolios bears mentioning. The analysis presented in this study does not consider price uncertainty. Price is one of the variables in the net benefit equation. For this study we have assumed that price is constant. However, some forestry studies (e.g. Thomson 1992, Brazee and Mendelsohn 1988) consider stumpage price to be a source of variance in outcomes and incorporate price into forestry analysis as a random variable (note these studies generally ignore yield variance). Moreover, some recent studies predict that climate change will influence future stumpage prices (Sohngen and Sedjo 2005). Thus, the expected value and variance of price may be impacted by climate change and therefore price may not only be a source of variance in terms of short term investments it may become a relatively more important as a source of variance in medium and longer term investments as a result of climate change. The main issue for analysis in a Canadian context is that because stumpage prices are not determined in open competitive markets in Canada, data on the distribution of current stumpage price now and in the future (with climate change effects incorporated) are unavailable. For the purposes of this study we restricted our analysis to areas where it was possible to develop empirical relationships between climate and response variables. Our data support an analysis of how climate change might influence the distribution of stand yields at points in the future. We do not, however, have data that would support extending the analysis to consider how climate change will affect variances in Canadian stumpage prices. Thus, although we can speculate that climate change will result in lower future expected prices and increased variance over time - there is no way to incorporate price variability considerations into our quantitative risk model.

Implications of discounting

Climate change and climate change risk raise a number of dynamic issues. Some of these issues have already been the topic of significant discussion within the economics literature. Lind (1995), Toth (1995), and Tol (2003) discuss the applicability of cost-benefit analysis and discounting to climate change impact assessment. They note that the application of market discount rates can lead to drastically reduced estimates of long-term future impacts (thereby reducing the threshold for levels of costs justified for mitigation). Two key issues that arise from this literature are: (a) the extent to which use of market discount rates is equitable (i.e. does the use of market discount rates result in present value estimates of environmental damage from climate change that favor current generations and penalize future generations), and (b) the extent to which it is justified to use discount rates given the possibility of unbounded variance (i.e. the possibility of extreme calamity) surrounding predictions of future impacts.

The counter argument for the use of discounting is that it is required to ensure an efficient allocation of capital over time. For this study we assume that uncertainty around forestry benefits is not unbounded (at least for the time period of interest here) and that discounting does not result in inequitable distribution. Discount rates adopted in the forestry literature for long-term investment analysis range from 3 % (Klemperer et al. 1994; Brazee and Mendelsohn 1988) to 5 % (Thomson 1991; Berck 1979). Row et al. (1981) and Thomson (1992) recommend the use of a 4 % discount rate for forest investment analysis. We have assumed a long-term risk free discount rate of 4 % throughout this study.

Table 8.1 shows that the lowest risk portfolios are comprised of the no cut prospects. This suggests that portfolios with a weighting of long-term prospects (i.e. associated with harvesting in period two or leaving the area uncut for the entire planning period) are less risky (with respect to net benefits) than portfolios with prospects based on harvesting in period one. Thus, delaying the harvest reduces portfolio risk. As noted in a previous section, in situations where there are relatively few prospects that can be included in a portfolio, variance is dominated by the variances of the individual prospects. In this study individual prospects are the equivalent of management prescriptions. Table 7.7 provides the variance of net benefits of individual prescriptions under scenario 3. The table shows that with scenario 3, the variance in net benefit associated with cutting the IAC1 stand in 2025 is much larger than the variance of the net benefit associated with harvesting the IAC1 stand in period two. Thus, delaying the harvest reduces the variance of net benefits for that stand. There are two combined reasons for this result. The first reason is that, as noted in a previous section, increases in yield variance as prediction period increases are modest. The second reason relates to discounting. The distributions for future net benefits are discounted to present value using a 4 % discount rate. The compound nature of discounting means that future values in the sample distribution are discounted to present value at an exponential rate. Discounting the sample distribution of future values results in a lower expected value and lower variance of net benefits. Thus, for this study, discounting of net benefits significantly outweighs increases in yield uncertainty from climate change. The net result is lower variances for portfolios with a high percentage of area in the no cut or cut in period two prescriptions.

Although discounting does lead to lower variance for long-term future net benefits, it also leads to lower expected values (Table 7.6). A fundamental premise of the EV efficiency criterion is that investment portfolios are ranked on the basis of both expected values and relative risk. Therefore, low variance for portfolios dominated by long-term investment does not mean that these portfolios will be selected.

The finding that time is a risk reducing input (albeit only in terms of the present values of net benefits) was a cause of concern. Therefore, the remainder of this section considers this result in more detail. One question to consider is: should the variances of future net benefits be discounted? Discounting of future variances is supported by expected utility theory. For example, Zerbe and Dively (1994) suggest that the correct measure for an uncertain future benefit is discounted certainty equivalent where discounted certainty equivalent is estimated by the following equation:

$$PV = \frac{CE}{(1+i)^t} = \frac{\sum X_i * E[NB_i] - \beta X' VarX}{(1+i)^t} \qquad [8.2]$$

This implies that variance and expected value of net benefits are both discounted. This is basically the same as saying that to convert the future value of a random variable to present value, each and everyone one of the observations in the sample distribution of the future value should be discounted to present value. Discounting reduces both the expected value and the variance.

A second way to evaluate our result that time is risk reducing (with respect to net benefits) is to consider if this result is logical and whether the decision to delay harvest in order to reduce risk is rational. In the context of the stylized forest that we have created for this study, delaying harvest in order to reduce risk may be logical and rational for two reasons. First, low risk ventures are generally associated with low return and high return investments often come with significant risk. In our study the lowest expected value of net benefits come from holding the stands until the end of the planning period. These stands also have the lowest variances of net benefits. If our prospects had high returns and low variances for period one harvest, and lower returns and higher variances associated with delaying the harvest then clearly harvesting in period one will always be preferred over harvesting in later periods.

The second reason time is risk reducing with respect to net benefits is that individuals may prefer to delay risky decisions. A justification for discounting future variance is that the cost of risk in the future is lower than the present. Therefore, decision makers are prepared to discount future risk compared to immediate risk because they consider the cost of future risk to be lower than immediate risk.

Perceptions of risk

One of the goals of this study is to evaluate how climate effects and uncertainty effects influence the choices of a utility maximizing decision maker. As noted in a previous section, efficiency analysis and the EV efficiency criterion are based on the assumption that the decision maker's subjective assessment of outcome probabilities matches actual probabilities (i.e. the decision maker is fully informed). However, what if the decision maker is not informed? In Chapter 2, Tversky and Kahneman's (1974) tests on bounded rationality were introduced. Their theories suggest that people exposed to complex risk may use strategies that can result in biased perceptions and potentially suboptimal responses to risk. Thus, the responses of an individual with bounded rationality may be different from those of an informed or rational investor.

Thus, it is possible to conceive of many types of decision makers where each is differentiated by what they consider relevant in terms of sources of uncertainty in forestry decision-making. For example, some might be take yield uncertainty into account, some might take price uncertainty into account, some might take climate uncertainty into account, some might take risk of catastrophic losses into account, and some might take various combinations of the above sources of uncertainty into account. The fully rational and informed decision maker would take all sources of uncertainty into account.

In this section we are considering two types of decision makers. The first type of decision maker is one: (a) who is primarily concerned about climate risk, (b) who considers yield relationships to be deterministic, and (c) who ignores uncertainties associated with all other factors. This decision maker is referred to as John Doe for the purposes of this study. He would base his decisions about prospect choices (i.e. relative proportions of management options selected) based on the frontier for scenario two.

The second type of decision maker is one who incorporates both yield and climate risk in decisions. We shall refer to this decision maker as Jane Doe. Jane would tend to base her asset mix (or harvest pattern) on the scenario 3-portfolio frontier. This scenario provides a more realistic representation of risk facing the decision maker.

For the purpose of this study we consider John to have bounded rationality. Jane (a U of A alumnus) on the other hand is informed and rational. The main consequence of having bounded rationality is that choices may be suboptimal.[25] For example, assume John selects point "A" (with a standard deviation of 46,000) on Figure 8.1. Under scenario two the harvest pattern associated with point "A" involves harvesting all 1000 hectares in period 1 (i.e. this result is reported in the text only). However, under scenario three the estimated standard deviation associated with a portfolio that involves harvesting the entire area in period one is $ 186,000 (Table 8.1). If John was Jane and if he/she was aware that this was the variance associated with this particular portfolio, then depending on his/her relative risk preferences he/she might have selected something like point "B" on Figure 8.1 instead of a portfolio that resulted in harvesting the entire forest. The optimal harvest pattern in this case is to harvest 237 hectares of stand type 2 in period 1 and harvest the remaining 763 hectares of stand types 1 and 2 in period 2. Thus, basing the

[25] It is possible to obtain suboptimal choices even with rational choices if there are transaction costs and these transaction costs are ignored. See various discussions on "Coase theory" for more elaboration. Also, according to the theory of second best, it is also possible that bounded rationality is still rational Black (1997).

harvest pattern (asset mix) on scenario 2 curve results in a sub-optimal decision. The suboptimal selection of prospects (or harvest prescriptions in our case) results in an economic cost in the form of reduced utility.

The discussion above considers portfolio choice from the perspective of a decision maker who only accounts for climate uncertainty (with and without uncertainty in yield predictions). The reverse situation is also possible. That is to say the choices of an individual who only considers uncertainty in yield from traditional sources (i.e. variability in growing conditions and biological response) and ignores climate related uncertainty will also tend to be suboptimal (as we have noted earlier – climate uncertainty accounts for 25 % of the standard deviation in expected returns). The point here is that not accounting for all sources of uncertainty (climate or otherwise) can result in higher economic costs (and sub-optimal decisions) than might have been the case had investors been fully informed about their investment risk.

Summary and discussion of the results

The analysis and results in this chapter illustrate that both climate and yield uncertainty contribute to forestry investment risk (although climate risk is less significant). It is important that decision makers acknowledge and take account of all sources of risk and uncertainty in their decision-making. Ignoring some sources of risk may lead to choices that are not consistent with utility maximization.

One finding that is confirmed by the results presented in this chapter is that forest managers do have the opportunity to manage risk by varying harvest patterns and management prescriptions. It is also generally recognized that diversification can significantly reduce the risk associated with a particular portfolio. Thus, two features that may be important relative to managing future forestry risk are flexibility (in terms of the ability to choose different prospects for inclusion within a portfolio) and diversity (in terms of the number of prospects that are available for inclusion in a portfolio).

The results presented in this chapter show that climate change will increase the level of uncertainty associated with forestry operations. The section on adaptive management in Chapter 2 suggests that in complex, rapidly changing, and risky decision environments, adaptive management strategies become more important. Risk management might be viewed as an important adaptive management strategy in a forestry and climate change context. As timber supply uncertainties and risk becomes more evident, there might be higher demand for (and a higher premium on) having the flexibility to manage this risk (i.e. option price). The results provided in Figure 8.1 and Table 8.1 show that it is possible to manage risk by adjusting harvesting patterns. This result implies that increasing the diversity of management options in forestry as well as increasing the degree of flexibility that forest managers have relative to the utilization and management of the forest has the potential of reducing the economic costs of uncertainty and avoiding suboptimal choices. Moreover, given the significance of non-climate factors as sources of risk, adopting a risk management strategy based on diversification of management options and flexibility may be justified irrespective of the implications of climate change.

It might be argued that contemporary Canadian forestry is analogous to our simplified forestry problem from the point of view of having limited number of available forest management options available to managers. Just as there are relatively few available prospects for inclusion in our stylized case study, there are relatively few options available to industrial forest managers who are managing large areas of public forestland. For example, there are few options relative to rotation age. There are limited options relative to species for reforestation. And there are constraints on land conversion (e.g. from one species to another or from one use to another). This leads to the question of whether or not current forest management policy could be (or should be) modified to increase the number and diversity of available forest management options (prospects) and to increase the flexibility to allow forest managers to select alternative approaches. For example, could modification of sustained yield policy increase diversity and flexibility? Should forest managers be allowed greater flexibility with respect to selecting rotation age, harvest volume, reforestation species, and even land use? Is it possible to change tenures in a way that provides a greater role for markets to determine resource allocation under climate change and that provides more incentives for forestry companies to implement adaptation strategies to climate change?

An even more fundamental question pertains to the extent to which sustained yield should even be pursued as a part of public forest policy given climate change. Strong sustainability policies, like sustained yield, may be constraining relative to the degree of flexibility they permit. A number of studies are beginning to point to the importance of flexibility in institutions and the need for adaptive management approaches to forest management as the only way to ensure sustainable forest management (e.g. see Castle et al. 1996; and Holling 2001). Luckert and Williamson (2005) note:

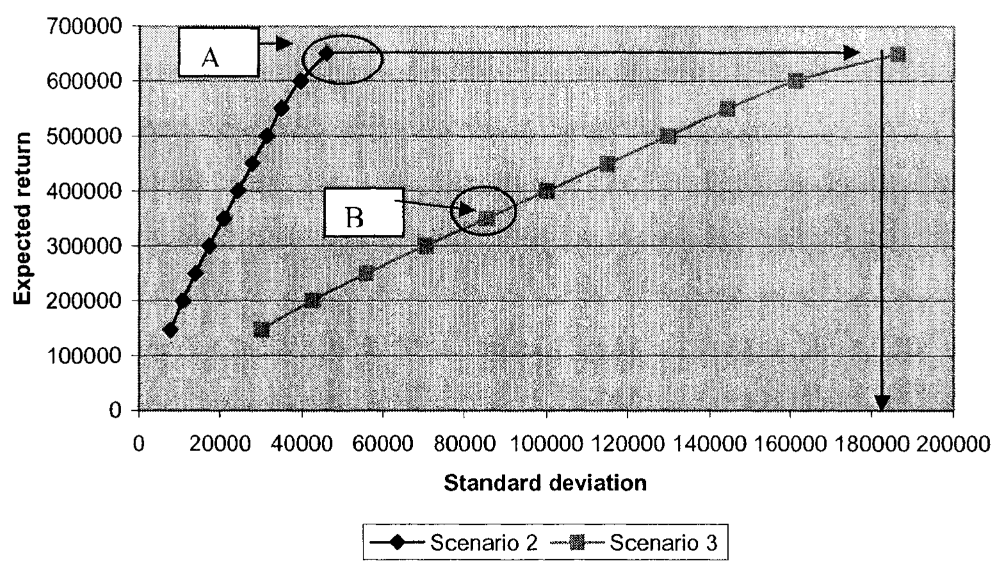
> "opportunity costs arise because SY institutionalizes inflexibility and market insensitivity. Values of forest products may change markedly over time following cycles and trends. As discussed above, a constant stream of timber production within such an environment may exacerbate market fluctuations. As this process unfolds, there may be significant losses incurred because of an inability of firms to respond to changing prices by varying quantities supplies" (pg 361.) and "the focus and orientation of sustainable development turns to defining and preserving critical zones with safe minimum standards, and the development of social institutions that promote flexibility and adaptability." (pg 358).

The analysis and results provided here suggest there may be social benefits associated with greater diversity in management options and flexibility in terms of their implementation. There may, of course, also be social costs that would also have to be considered. These types of questions and issues are left for follow up analysis.

There are a number of possible directions for extending the analysis described in this chapter. One possible direction would be to include price uncertainty as an additional source of uncertainty. A number of studies (Klemperer et al. 1994; Brazee and Mendelsohn 1988; Reed and Haight 1996 Brazee et al. 1999) find that price variance increases over time. A model formulation would be possible that would allow for systematic dynamic uncertainty in both price and growth to be incorporated. One issue concerning the Markowitz model is that it is a single period (i.e. static) model. A single period model means that decisions are fixed for the entire planning horizon. Thus, a dynamic model that permits multi-period analysis would be required. Another issue pertains to availability of stumpage price data and changes over time. Understanding the temporal properties of price variance would require modeling using time series data. Generally, long term time series of competitively determined stumpage prices in Canada is not available.

Another extension would be to broaden the diversity of investment options available to the investor. This study assumes that the decision to invest in a forestry asset has already been made. The problem is one of optimizing harvest choices in order to maximize returns from the forestry asset given climate change and yield uncertainty. However, an interesting problem might be to consider the possibility of purchasing different types of land and/or the possibility of switching land use over time in response to climate change. Or, one might be interested in looking at how the opportunity to invest in non-forest financial market assets (e.g. stocks, bonds, treasury bills, etc) affects the optimal forest management choices. Heikkenen (1999), for example, incorporates the opportunity to purchase stocks as a prospect option that could be included in the portfolio of assets. For example, one could assume that the starting asset is a quantity of cash (instead of forestry land). The prospects would include various options related to: (a) purchasing and managing forest land and possibly reinvesting harvest income in forest assets or other types of assets, or (b) various options that may not have anything to do with forestry (e.g. stocks, bonds, commodity futures, real estate, etc). Another and possibly more relevant extension from a climate change perspective would be to consider the possibility of conversion to other land-uses (e.g. grazing, crops, renewable energy plantations) following harvesting at the end of periods one and two.

Figure 8.1 Expected return vs standard deviation for scenarios 2 and 3.



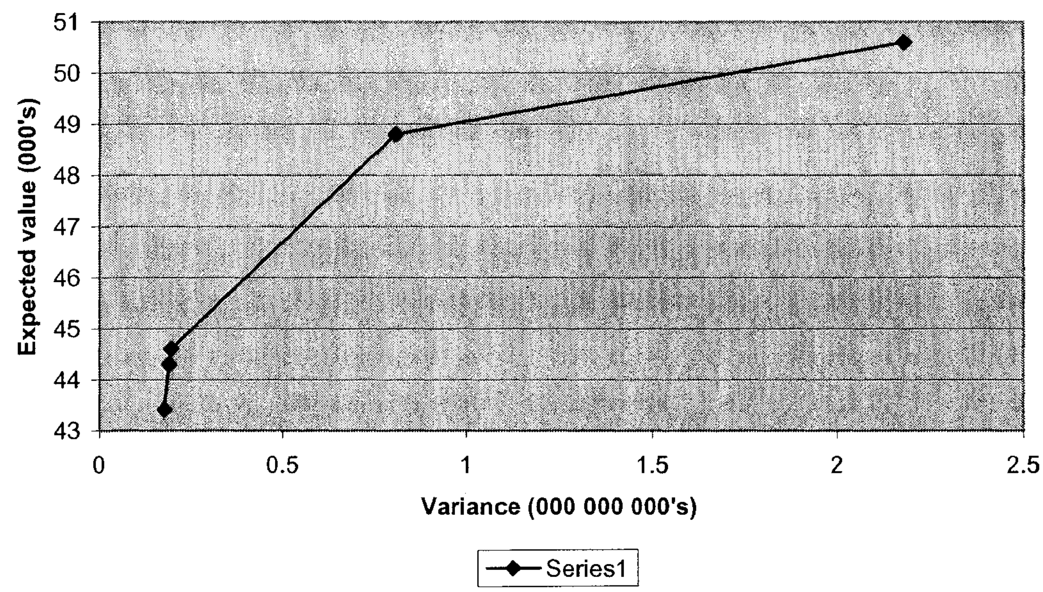
Scenario 2 is the Risk-return frontier when only uncertainty in climate variables is considered

Scenario 3 is the risk-return frontier when uncertainty in climate variables and uncertainty in yield parameters is considered.

Point A: The risk-return selection of a decision maker who only considers climate risk.

Point B: The risk-return selection of the same decision maker who considers both climate risks and yield risk.

Figure 8.2 Mean-variance frontier from Hardaker et al. 2004.



Source: Hardaker et al. 1994.

Table 8.1 Optimal investment portfolios for different levels of minimum required returns using scenario 3 data

	Minimum return required from the portfolio										
	147000	200000	250000	300000	350000	400000	450000	500000	550000	600000	650000
Investment option (prospects)	Optimal portfolio (i.e. # of ha under each initial age class / prescription combination)										
Leave 40 year old stand uncut	250	250	0	0	0	0	0	0	0	0	0
Leave 80 year old stand uncut	744	203	0	0	0	0	0	0	0	0	0
Cut 40 year old stand in period 1	0	0	0	0	0	0	0	0	0	45	250
Cut 80 year old stand in period 1	0	0	23	130	237	344	451	558	665	750	750
Cut 40 year old stand in period 2	0	0	250	250	250	250	250	250	250	205	0
Cut 80 year old stand in period 2	6	547	727	620	513	406	299	192	85	0	0
Portfolio variance	9.05E+08	1.80E+09	3.11E+09	4.96E+09	7.26E+09	9.99E+09	1.32E+10	1.68E+10	2.08E+10	2.60E+10	3.47E+10
Standard deviation	30,078	42,403	55,767	70,449	85,176	99,935	114,891	129,615	144,222	161,245	186,279

The minimum expected return of \$ 147,000 results from the ending inventory value of stands and soil expectation value

The maximum expected return of \$ 650,000 results when the entire forest is cut in period one.

Table 8.2. Relative contribution of climate variance to total portfolio variance

Minimum return	Scenario 2 standard deviation ($)	Scenario 3 standard deviation ($)	(Scen 2 / Scen 3)*100
147,000	7792	30078	25.91
200,000	10644	42403	25.10
250,000	13784	55776	24.71
300,000	17231	70449	24.46
350,000	20724	85176	24.33
400,000	24243	99935	24.26
450,000	27779	114891	24.18
500,000	31326	129615	24.17
550,000	34886	144222	24.19
600,000	39497	161245	24.49
650,000	45738	186279	24.55

Table 8.3 Scenario 3 sample variances in predictions of future values of yields and net benefits based on Monte Carlo simulation

Initial age class	Prediction year	Variance of stand yield
40	2025	23262
40	2055	28807
40	2070	29486
80	2025	28890
80	2055	30105
80	2070	30412

Table 8.4 Differences between high and low prediction values for climate variables for different future time periods

Prediction year	Temperature (deg. cent.)	Annual precipitation (mm)	Annual growing season precipitation (mm)
2020	3.0	149.82	108.22
2035	3.5	141.85	93.56
2050	4.0	133.88	78.89
2080	6.0	160.92	90.06

Table 8.5 Scenario 3 expected present value of net benefits and sample variances based on Monte Carlo simulation

Initial age class	Prediction year	Expected present value of net benefit ($ per ha)	Variance of present value of net benefit
40	2025	450	60902
40	2055	213	5366
40	2070	136	1754
80	2025	715	72795
80	2055	248	5603
80	2070	150	1806

CHAPTER NINE

EXPECTED VALUE–VARIANCE / CHANCE CONSTRAINT HYBRID MODEL

Introduction

The previous chapter described and estimated a model that identified the frontier of efficient EV portfolios for our forest management problem from the set of feasible portfolios. However, in order to assess the economic impact of climate change and in order to understand the implications of climate change for short and long term rates of harvest, we require a model that predicts the decision maker's actual portfolio choices (note: the behavioral assumption is that the decision maker selects the optimal portfolio and therefore the decision maker is rational and informed). Thus, a new type of risk model formulation is required. The first goal of this chapter is to develop a model that identifies the specific portfolios that maximize expected utility (with and without climate change). The second goal is to use the model to estimate the economic impacts of climate change and risk and to assess the implications of sustained yield on the ability of forest managers to adapt to climate change and risk.

The analysis in the previous chapter was based on efficiency theory using a Markowitz portfolio approach. The objective function was to minimize portfolio variance subject to a minimum return constraint. The model in this chapter is also based on efficiency analysis. Moreover, the Markowitz portfolio model provides the basis for the risk model – but with some significant structural modifications. The objective function for the model in this chapter is to maximize certainty equivalent. This type of objective function differs from the objective function of the previous chapter in two main ways. First, expected portfolio returns, variance, and covariances are jointly considered within the objective function. Second, the objective function requires that a measure of the individual's risk preference be included (Hazell and Norton 1986).

The inclusion of risk preferences in the risk model for this chapter requires that we adopt some initial assumptions about these risk preferences. In particular, we require some initial assumptions about how risk preferences are affected by the decision maker's wealth. There are two aspects to consider including the decision maker's initial stock of wealth, and the degree to which the decision maker's risk preferences are sensitive to incremental changes in wealth. For this study we assume that incremental changes in wealth over the range of potential portfolios does not affect preferences. Thus, risk preferences are constant for all EV combinations available to the decision maker. Risk preferences are represented by a single constant called the constant absolute risk aversion coefficient (CARA) (Hardaker et al. 2004).

The optimal portfolio for a decision maker is at the point where the slope of the indifference curve of the expected utility function is equal to the slope of the portfolio frontier. This is shown graphically in Figure 9.1. For a highly risk-averse decision maker CARA is relatively large and the indifference curve (iso-utility curve) is steeper than that of a less risk-averse decision maker. Therefore, the more risk averse an individual is – the more he/she will prefer points lower on the efficient portfolio curve.

Another fundamental way that the model in this chapter differs from the model in the previous chapter is that in this chapter we introduce forest management policy constraints into the model (described in the next section). The type of decision maker we are interested in modeling in this chapter is an individual who is a rational utility maximizer (where utility is a function of returns from the hypothetical forest), and who is subject to forest harvesting constraints because the forest in question is publicly owned. This requires further extensions to the basic EV risk model. First, we require a model that incorporates the kind of harvest constraints typically imposed for management of public forestland. Second, because harvest constraints also include yield parameters, and because there is uncertainty in yield parameters, we require a model that accounts for uncertainty in both the objective function and the constraints. The usual approach for modeling the former is timber harvest scheduling. The usual approach for modeling the latter is chance constraint risk modeling (McCarl and Spreen 1997).

The remainder of this chapter is organized as follows. First, the timber supply-modeling framework is introduced and discussed. Second, the specific formulation for the EV – Chance Constraint optimization model is provided. Third, the model is used to address questions pertaining to; sensitivity of the results to parameters, the economic impacts of climate change and risk, and the effects of climate change and risk on harvest timing.

Application in a timber management context

The Markowitz portfolio model application in the previous chapter assumes that the decision maker is a rational investor whose choices are not restricted by objectives such as having to ensure sustained yield. In the case presented in this chapter, the decision maker is a rational utility-maximizing logger who has some flexibility to determine his/her own harvest schedule. However, given that he/she is operating on crown land, his/her harvest plan is subject to harvest constraints. Harvest scheduling models are required for problems of this nature (Johnson and Scheurman 1977; Dykstra 1984). The foundation for the model discussed in this chapter is a Model I type harvest scheduling optimization model (Dykstra 1984) modified to take account of risk and risk preferences. The objective in Model I harvest scheduling models is to maximize some measure related to harvesting (economic returns in our case) subject to harvest constraints such as ending inventory constraints and flow constraints. Marshall (1988) and Reeves and Haight (2000) also employ a Model I formulation to assess the effects of uncertainty on timber management.

The formulation described and evaluated in this section extends the basic Model I harvest-scheduling model in two ways. First, it takes account of uncertainty in objective function coefficients and the risk preferences of the decision-maker. It does this by modifying the usual timber harvest scheduling objective function from maximizing benefits to maximization of certainty equivalent. An additional dimension of our forest management problem is that the constraints for the problem require yield coefficients. Since for the purposes of our analysis, yield coefficients are random variables, there exists uncertainty with respect to the constraints. Therefore, a second main feature of the model described in this chapter is that it takes account of uncertainty in the constraint coefficients using a chance constraint type of approach (Hazell and Norton 1986, McCarl and Spreen 1996). Thus, the existence of uncertainty in both the objective function and in the constraints requires the development of a hybrid risk model that incorporates aspects of both EV types of risk models (for uncertainty in objective function coefficients) and chance constrained risk models (for uncertainty in constraints).

Model structure

The theoretical context and general framework for EV risk models is discussed in Hazell and Norton (1986) and Hardaker et al. (2004). They note that if income (Y) is normally distributed and if the following functional form[26] (representing constant risk preferences) represents the decision maker's utility:

$$U(Y) = 1 - e^{-\Phi Y} \qquad [9.1]$$

then the formulation for the objective function in an EV risk model (i.e. the individuals expected utility function) is as follows[27]:

$$Max: E[U(Y)] = CE = E[Y] - 0.5\Phi Var(Y) \qquad [9.2]$$

Where:

Maximization of E[U(Y)] is equivalent to maximization of certainty equivalent, Y is portfolio income, Φ is the Pratt constant absolute risk aversion (CARA) parameter, and Var(Y) is portfolio variance.

For the purposes of obtaining a model capable of assessing the implications of climate change on our forest management case study, we are interested in a formulation that can be used to determine the maximum risk adjusted return (or certainty equivalent return) that is possible from our 1000 hectare forest given predictions of changes in stand yield and uncertainty related to climate change. For convenience, the management problem is restated. There are two initial age classes:

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[26] Hazell and Norton (1986) note that the utility function can also take other forms such as the quadratic utility function. The functional form provided in equation 9.1 implies that the risk aversion coefficient is constant over varying levels of wealth- i.e. CARA.

[27] In fact it is possible to derive the portfolio frontier (discussed in Chapter 8) from the EV model by running the model a number of times with a different value of the risk aversion parameter used in each run. The set of solutions can be used to map out the portfolio frontier.

(1) 250 hectares of 40-year old aspen (IAC1 or j=1); and (2) 150 hectares of 80-year old aspen (IAC2or j=2). There are three prescriptions: (1) no cut (i=1), (2) cut in period 1 in 2025 (i=2), and (3) cut in period 2 in 2055 (i=3). The planning horizon starts in the year 2010 and ends in 2070. The planning horizon is 60 years and there are two planning periods (each 30 years long). The specific formulation of the EV / chance constrained risk model evaluated in this study is provided as follows[28]:

$$\underset{\{X_{ij}\}}{Max}:\sum_{i=1}^{3}\sum_{j=1}^{2}E[NB_{ij}]X_{ij}-0.5\Phi\sum_{i=1}^{3}\sum_{j=1}^{2}X'_{ij}Cov\{NB_{ij}\}X_{ij} \qquad [9.3]$$

Where:

- E[NB_{ij}] is the expected present value of net benefit coefficients (defined in equations 7.18 to 7.23 in chapter 7),
- X_{ij} is the activity level for the choice variable associated with prescription "i" and initial age class "j".
- Φ is the risk aversion coefficient, and
- Cov(NB_{ij}) is the covariance matrix for the variable "net benefits".

There are fours sets of constraints in this model. The first two sets of constraints are area constraints (i.e. limit area of IAC1 harvested to 250 hectares and area of IAC2 harvested to 750 hectares) and non-negative choice variable constraints. The remaining two sets of constraints incorporate random variables. They are ending inventory constraints and flow constraints. The yield coefficients within these latter two sets of constraints are random variables. Hazell and Norton (1986) outline an approach for incorporating uncertainty in constraints in mathematical programming models called chance-constrained programming. For constraints where "a_{ijk}" is the random coefficient value, the chance constraint is formulated as follows:

$$\sum_{j}\sum_{k}E[a_{ijk}]X_{jk}+K_{\alpha}[\sum_{j}\sum_{k}X'_{jk}\operatorname{cov}(a_{ijk})X_{jk}]^{1/2}\leq b_i \qquad [9.4]$$

K_{α} is a constant obtained from a cumulative normal distribution table. The value of K_{α} is obtained as follows. In cases where Z is normally distributed with a mean of 0 and a variance of 1 (standard normal distribution), then Z_{α} is the value of z such that

[28] The model is non linear. The CONOPT solver in GAMS (Brooke et al. 1998) was used to run the model. The GAMS program code is available upon request.

Probability ($z \leq K_\alpha$) = α. Thus, if the decision maker requires that the constraint be feasible 90 % of the time, then $K_\alpha = 1.28$.

Applying the chance constrained programming formulation to the ending inventory constraint for our hypothetical forest management problem leads to the following equation:

$$\sum_{i=1}^{3}\sum_{j=1}^{2} E[VEI_{ij}]X_{ij} - K_\alpha[\sum_{i=1}^{3}\sum_{j=1}^{2} X'_{ij} Cov\{VEI_{ij}\}X_{ij}]^{1/2} \geq E^m$$

[9.5]

Applying the chance constrained programming formulation to the flow constraints for our optimization problem leads to the following equations:

$$(1-\alpha)\sum_{j=1}^{2} E[V_{2j}]X_{2j} - \sum_{j=1}^{2} E[V_{3j}]X_{3j} + K_\alpha[\{0, 0, (1-\alpha)X_{21}, (1-\alpha)X_{22}, -X_{31}, -X_{32}\} *$$

$$Cov\{V_{ij}\}\begin{Bmatrix} 0 \\ 0 \\ (1-\alpha)X_{21} \\ (1-\alpha)X_{22} \\ -X_{31} \\ -X_{32} \end{Bmatrix}]^{1/2} \leq 0$$

[9.6]

$$(1+\beta)\sum_{j=1}^{2} E[V_{2j}]X_{2j} - \sum_{j=1}^{2} E[V_{3j}]X_{3j} + K_\alpha[\{0, 0, (1+\beta)X_{21}, (1+\beta)X_{22}, -X_{31}, -X_{32}\} *$$

$$Cov\{V_{ij}\}\begin{Bmatrix} 0 \\ 0 \\ (1+\beta)X_{21} \\ (1+\beta)X_{22} \\ -X_{31} \\ -X_{32} \end{Bmatrix}]^{1/2} \geq 0$$

[9.7]

Estimation of expected values, variances and covariances for all the random variables (E[NBij], E[VEIij], E[Vij]) was discussed in chapter 7. There are five remaining parameters that require values:

1. The maximum allowable fractional reduction in between period flow (α) (i.e. the maximum % decrease allowed in harvests in period two compared to period one),
2. The maximum allowable fractional increase in between period flow (β) (i.e. the maximum % increase allowed in harvests in period two compared to period one),
3. The risk aversion coefficient value (Φ),
4. The standard normal statistic (K_α) corresponds to the percentage of times that the constraint must be satisfied given the selected values of the choice variables, and
5. The ending inventory target (E^m).

Values for these parameters are context specific. For example, if the decision maker is risk averse, a certain value for Φ is implied. The strategy adopted here for addressing variations in parameter values is to conduct sensitivity analysis by running the models for different values of the parameters and comparing the results.

Harvest flows are determined by the fractional increase (β) or decrease (α) that is permitted between periods one and two. Three sets of values are considered in this study: $\alpha=\beta=0.1$, $\alpha=\beta=0.25$ and $\alpha=\beta=0.50$. A value of 0.1 implies that harvest flow is allowed to vary between period one and two harvest by 10 %.

The conceptual and theoretical basis of the risk aversion parameter was described in chapter two (see equation 2.2). The parameter Φ is a constant ***absolute*** risk aversion (CARA) parameter (Pratt 1964; Hardaker et al. 2004). This measure is a function of (and is therefore sensitive to) absolute levels of wealth (or income), units of measurement, and to the risk preferences of the decision maker (or the slope of the individual's utility function). The measurement of a specific CARA parameter is beyond the scope of the study. Hardaker et al. (2004) describe a method for inferring the CARA from measures of ***relative*** risk aversion and measures of wealth.

In order to obtain plausible values for Φ, all that is required is information on relative risk aversion values for different types of risk preferences and a measure of wealth that is specific to the problem context of the study (in terms of wealth and measurement units). Anderson and Dillon (1992) suggest that a value of the relative risk aversion coefficient of 0.5 represents an individual who is hardly risk averse while a value of 4.0 represents an individual who is very risk averse. Arrow (1965) suggests a value of 1 as generally representing the relative risk aversion coefficient of the average individual, although Hardaker et al. (2004) suggest that values higher than one are likely more typical. They note "while it is a matter for individual judgement,...values of relative risk aversion somewhat higher than 1.0 may be more common than has been implied in the literature." (pg 109).

In terms of alternative values for wealth for this study we use the high and low objective function values for the net present value for our 1000-hectare forest. These values range from \$100,000 to \$650,000 depending on assumptions regarding climate scenarios and constraints. In terms of relative risk aversion, we consider values of 0.5 (hardly risk averse) and 4 (very risk averse) as a representative range. The formula for converting the relative risk aversion parameter to an absolute risk aversion parameter is as follows (Hardaker et al. 2004):

$$\begin{aligned} \Phi_a(w) &= U''(w)/U'(w) \\ \Phi_a(w) &= \Phi_r(w)/w \end{aligned} \qquad [9.8]$$

With two possible values for $\Phi_r(w)$ (0.5 and 4.0) and two values for wealth (\$100,000 and \$650,000) there is the possibility of four values for the constant absolute risk aversion parameter for this study. We also employ a general value of 0.00001 as a measure roughly representing the absolute risk aversion for the average forestry decision maker. The full range of values for CARA used in this study are shown in Table 9.1.

The ending inventory and flow constraints are also controlled by the selected value of the normal statistic (K_α) (i.e. the critical value for the chance constraint). The normal statistic in the objective function determines the percentage of times that the constraint must be satisfied (Hazell and Norton 1986). For example, a value of 0.53 means that values of "X" (in equation 9.6 and 9.7) are selected such that there is a 70 % likelihood that the ending inventory and flow constraints will be satisfied given the known distributions of the yield variables. A value of K_α of 1.28 means that values of "X" (equations 9.6 and 9.7) are selected such that there is a 90 % chance that the ending inventory and flow constraints will be satisfied given the distributions of these variables. For this study, we estimate the model over a range of values of the normal statistic starting at 0.0 (50 % chance constraints are satisfied), then 0.1 (54 % probability that constraints are satisfied), and increasing by increments up to a value of 2.05 (98 % probability that constraints are satisfied)[29].

The final parameter that requires a value is the ending inventory constraint target (E^m). The previous paragraphs identify a range of possibilities for values of the flow constraint parameters, the CARA parameters, and the chance constraint parameter. The analysis in the following sections will be based on a wide range of different model structures – each based on different combinations of values of the parameters. A large number of models are anticipated. Incorporating a range of possible values for the target ending inventory parameter would have increased the number of combinations of parameter values and the number of models that would be estimated. For the purposes of this study it was decided to adopt a single value of the target ending inventory parameter and use this value for all models that include harvest constraints. The rationale was to avoid further increases in the number of models developed and solved. The target ending inventory value is set at 220,000 cubic meters. This is roughly based on having about 500 hectares of 40-year old aspen (140 cu. m. per ha) and 500 hectares of 80-year old aspen on the site (300 cu. m. per ha.) at the end of the planning horizon.

Analysis

There are four questions that will be addressed using the Expected Value – Variance / Chance Constraint hybrid model. They are:

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[29] In some circumstances the chance constraint and the value of CARA may be interrelated in the sense that an individual's degree of risk aversity might also influence the appropriate value of the chance constraint parameter (K). For this study we are looking at a situation where the risk aversion parameter pertains to the logger who is making the harvesting decision and the value of K applies to the public forest land manager. So they are not interdependent. But for other types of problem constructs it is possible that the risk aversion parameter and K are jointly determined.

1. How sensitive are certainty equivalent values and portfolio composition to variations in the critical value of the normal statistics used for the chance constraints and to the allowable increase or decrease in between period harvest?
2. How sensitive are certainty equivalent values and portfolio composition to variations in the constant absolute risk aversion (CARA) coefficient?
3. What are the economic impacts of climate change and uncertainty?
4. What are the implications of climate change and uncertainty for harvest scheduling?

Sensitivity to chance and flow constraints

The purpose of the analysis undertaken with the EV/Chance constraint model in this section is to conduct a sensitivity analysis of how the risk model results vary with different combinations of even flow constraints and chance constraints. Analysis was conducted using the scenario three input data (i.e. climate change productivity effects, climate change uncertainty, and yield uncertainty all included). The value of CARA for the models in this section was set to 0.00001 (i.e. the representative average value for a moderately risk averse decision maker). The model formulation with sustained yield constraints imposed was used. The ending inventory constraint was set at 220,000 cubic meters. The range of values tested for the chance constraints and flow parameters are described below[30].

Three separate models were estimated. The first model sets the value for alpha and beta at 0.1 (i.e. 10 % allowable deviation in harvest between periods one and two). The second model set the value for alpha and beta at 0.25 (i.e. 25 % allowable deviation in harvest between periods one and two). The third model set the value for alpha and beta at 0.5 (i.e. 50 % allowable deviation in harvest between periods one and two).

Each of the three models above was initially estimated with a value for the standard normal statistic (i.e. "K" in equations 9.5, 9.6, and 9.7) set at 0 (implying

[30] The GAMS (Brooke et al. 1998) program code for the basic model used in this chapter is available on request.

that there is a 50-50 chance that the constraint will be met). The models were then re-estimated for a range of increasing values for the standard normal statistic up to 2.05 (i.e. implying that the constraints are satisfied 98 % of the time). The certainty equivalent value results are shown graphically in Figure 9.2. The solutions for the nine models that are identified in groupings A,B, and C in Figure 9.2 are provided in Table 9.2.

The model presented in this section is a traditional harvest-scheduling model with sustained yield constraints. Sustained yield constraints are often associated with public forestland management. Thus, the scenario being modeled is one of a private harvester making harvest decisions on public lands to maximize his/her certainty equivalent subject to harvest constraints that the harvester is required to meet as part of his/her harvesting obligations. It seems reasonable to assume that a private sector logger will only be interested in obtaining a lease for a parcel of public land if there is some possibility that he/she will have the opportunity to harvest timber from that land. If the risk model provides a no harvest (or a low harvest) solution (i.e. an optimal portfolio that holds most of the 1000 hectare forest until the end of the planning period), then the forest has limited value to a potential leaseholder. Thus, for the purpose of this analysis we assume that harvest restrictions that result in no harvest solutions (or limited harvest opportunities) are outside the boundary of operability.

Figure 9.2 shows that the certainty equivalent value for each model without chance constraints (i.e. the three models shown in group A) are roughly equivalent. Permitting greater flexibility in flows between periods does increase certainty equivalent slightly but the effect is not large (see Table 9.2). In fact, as shown in Figure 9.2, differences in certainty equivalent values as a result of changes in allowable deviation in flow constraints are small for all levels of the chance constraint. Furthermore, as the required likelihood of meeting the constraint increases, the differences in certainty equivalent values between the three models (i.e. 10 %, 25 % and 50 % allowable deviations in between period flows) narrows. There are two main conclusions that derive from these results. First, significant tradeoffs between chance constraints and allowable flow deviations are not evident from these results. Second, increasing the allowable deviation in between period harvest flow does not appear to be particularly effective as a strategy for adapting to uncertainty in harvest yields. This is the case, at least for the model presented here which is a static one period model without recourse. This issue is addressed again using a recourse model in Chapter 10.

One of the reasons for undertaking the sensitivity analysis in this section is to see what effect assumptions about parameter values have on the results. The analysis provided above suggests that the models are relatively insensitive to modifications in allowable flow deviations. Therefore, for the purposes of the models developed in the next two sections we will adopt the middle value for allowable flow deviations (i.e. 25 %) for the flow parameter (alpha and beta).

Figure 9.2 illustrates that increasing the required likelihood of meeting the harvest flow constraint does change certainty equivalent values. Initially certainty equivalent values decrease gradually as the required likelihood increases. However, certainty equivalent values decrease at an increasing rate as required likelihood increases. For example, for the model that allows a 10 % deviation in between period flows, increasing the required likelihood of meeting the flow constraint from 50 % to 73 % results in a 9 % decrease in certainty equivalent (Table 9.2). Increasing the required likelihood from 73 % to 90 % results in a 20 % decrease in certainty equivalent. Therefore, the marginal cost of increasing the required likelihood of meeting the constraints is increasing. The implications of these results are that reducing the required likelihood for meeting the flow constraints can have significant effects on returns. These gains would need to be viewed in the context of the social costs or relaxing the constraints.

Table 9.2 shows that the reason that certainty equivalent values increase as chance constraint requirements become less stringent is that the area harvested increases. For example, when the required likelihood of meeting the constraints is set at 98 % (i.e. see group D in Figure 9.2) there is no harvesting (note – this result is reported in the text only and is not shown in Table 9.2). In cases where the flow constraints must be satisfied 90 % of the time and the allowable deviation in flow between period 1 and 2 is 25 %, then about 32 % of the 1000 hectare forest is harvested (Table 9.2). In the case where the flow constraint must be satisfied 73 % of the time and the allowable deviation in flow between period 1 and 2 is 25 %, then 59 % of the 1000 hectare forest is harvested over the planning horizon. Thus, the incentive for a logger (whose primary interest in obtaining a lease is to maximize utility from harvest income), to obtain a lease for the forest area is much higher if he/she only needs to satisfy the flow constraints 73 % of the time as opposed to 90 % of the time. An important implication of this result is that adjusting likelihood requirements can increase flexibility relative to adapting to climate related uncertainty and general uncertainty in forest management.

As noted, one of the reasons for undertaking the sensitivity analysis in this section is to identify suitable values for parameter values that will be used in models in later sections. Hazell and Norton (1986), suggest that in agriculture economics problems, the required likelihood for chance constraints are typically 90 % or higher. However, as shown in Table 9.2 a required value of 90 % results in only 32 % of the area being harvested (with a 25 % allowable deviation in flow). A required likelihood at a level of 73 % results in 59 % of the 1000 hectares being harvested over the planning horizon. On the basis of the fact that the marginal cost of a 73 % required likelihood is significantly lower than the marginal cost of a 90 % required likelihood and a 73 % likelihood permits a higher harvest we feel that the 73 % threshold best suits the analysis to follow. Therefore, for the purpose of the models developed in later sections, we will adopt a required likelihood threshold value of 73 %. The equivalent value of "K" is 0.6.

It should be noted that there is some divergence between current forest management policies and the methods and results presented in this section. This is mostly due to the fact that uncertainties in yield coefficients and chance constraints are generally not an explicit part of operational harvest scheduling and timber supply analysis. There are, therefore, no real standards relative to what likelihood threshold is most appropriate for the purpose of chance constraint models. However, the fact that potential uncertainty in constraints is generally not explicitly considered in operational planning does not necessarily make it less relevant. Generally, forest managers probably do recognize that uncertainty in yields does exist. They may employ a number of strategies to take this uncertainty into account in their choices. For example, they may base their final decisions about allowable annual harvests on a range of different model outputs. A second strategy is that managers generally continuously update their information and regularly recalculate AACs. A third strategy is that they may incorporate conservative values into planning models resulting in conservative estimates of allowable harvests. The premise of the approach for this thesis is that the array of strategies that managers may use to deal with risk implicitly will have roughly an equivalent result as the explicit way that risk and uncertainty are addressed in the specific models presented in this dissertation. A parallel can be drawn to the agriculture economics research literature. Generally, farmers probably do not base their decisions on risk models. At the same time, risk modeling is used in agricultural economics research to evaluate the effects of risk and uncertainty on agricultural decision-making.

Sensitivity to variations in risk aversion

One purpose of this study is to understand what the implications of climate change and risk are for optimal harvest choices and economic benefits. The model described in this chapter incorporates decision maker risk preferences. We are not, however, modeling the choices of a specific decision maker with specific preferences. This means that the risk preferences of our decision maker could range anywhere from being risk neutral to being extremely risk averse. This in turn leads to the question: How important might differences in risk preferences be with respect to the results of this study and future studies looking at risk and climate change in a forestry context? The analysis in this section looks at the sensitivity of risk model solutions and objective function values to variations in the degree of risk aversity of the decision maker. The modeling approach is to re-estimate the model under various combinations of values for the CARA parameter and chance constraints (i.e. with and without chance constraints).

The values of parameters and the basic model structure used for the analysis in this section are as follows:

- Modeling is based on scenario three input data.

- The objective function is to maximize certainty equivalent subject to all harvesting constraints (i.e. ending inventory and even flow constraints), area constraints, and non-negativity constraints.

- Chance constraints are set at two levels: 0 (50 % likelihood) and 0.6 (73% likelihood).

- The even flow constraint allows for a 25% deviation in between period harvests.

- The target ending inventory parameter is set at 220,000 cubic meters.

- The model is estimated for the following values of CARA – 0.0, 0.00000077, 0.0000055, 0.00001, 0.000025, 0.00004 – with and without chance constraints imposed.

The results for the models described above are presented in Figure 9.3 and Table 9.3. The baseline value for comparison purposes is the objective function value (i.e. certainty equivalent value) when chance constraints are not imposed and when CARA is set to zero (implying risk neutral decision makers). The objective function value and solution for this scenario are shown in the first column of Table 9.3. The certainty equivalent for the stand is $439,863. Moreover, 77 % of the total 1000 hectares is harvested in either period 1 or 2. If one looks only at the effect of increasing risk aversion (i.e. ignoring chance constraints for the time being) then, as would be expected, certainty equivalent decreases with increasing risk aversion but at a decreasing rate (top line in Figure 9.3). There is a 28 % decrease in the certainty equivalent value for the average decision maker (CARA=0.00001) compared to the risk neutral decision maker (CARA-0.0). Thus, failure to account for risk in the objective function may lead to significant overestimates of the benefits of forestry production because of failure to account for a significant economic cost (i.e. the risk penalty).

Similar trends are evident in the results when the chance constraints are imposed (i.e. the normal statistic is set at 0.6 which implies that the constraint must be satisfied at least 73 % of the time). The imposition of the chance constraints shifts the CE / CARA curve downward (particularly at low values of CARA). Including chance constraints (in addition to risk preferences) magnifies the bias in estimation of benefits that occurs if uncertainty is not considered. For example, the certainty equivalent value for the average decision maker (CARA=0.00001) when both objective function and constraint risk is considered is $285,775 (Table 9.3). This value is 35 % lower than the certainty equivalent value when risk is ignored.

Figure 9.3 shows that as the value of CARA increases, the influence of chance constraints on certainty equivalent decreases. Including chance constraints has a large impact on certainty equivalent at low levels of risk aversion but as the risk aversity of the decision maker increases, risk aversion becomes more dominant. The reason for this result seems to be that as risk aversion increases, less and less area is harvested. As less and less area is harvested, the even flow constraints (and therefore the chance constraints) become less important in terms of their effect on certainty equivalent. As noted in Table 9.3, the optimal harvest solutions for the model with chance constraints are almost identical to the optimal harvest solution for the model without chance constraints when CARA is 0.00004. In both cases about 83 % of the land area remains un-harvested over the planning horizon.

Although optimal harvest solutions (both with and without chance constraints) at high levels of risk aversion are similar, the same is not true at low levels of risk aversity. Table 9.3 shows that for decision makers with relatively low levels of risk aversion, certainty equivalent values and optimal solutions are very different between the models with, and without, chance constraints imposed. For example, for decision makers with average risk aversion (CARA=0.00001) certainty equivalent is 10 % higher in the model without chance constraints. The imposition of chance constraints also affects the optimal portfolio. Seventy seven percent of the area is harvested in the model without chance constraints. When chance constraints are included, the percent of the area harvested in periods one and two drops to 59 %. So in addition to affecting objective function values, the imposition of chance constraints also has significant implications relative to optimal solutions (i.e. portfolio selection). At lower levels, risk aversion is relatively less important, but constraint risk has a significant influence on both certainty equivalent and on solutions (i.e. optimal harvest solutions or the optimal portfolio).

A number of significant findings come out of the analysis in this section. First, accurate measurement of forestry benefits requires that risk and uncertainty be taken into account. Second, consideration of risk and uncertainty is important irrespective of the specific risk preferences of decision makers. At high levels of risk aversity, risk preferences have a major influence on benefits and portfolio selection. However, risk and uncertainty considerations are also important even when the risk aversity of decision makers is somewhat low. In this case, it is the uncertainty in the constraints that has a significant effect on benefits and on portfolio choice. These results imply that risk and uncertainty have important implications for forestry analysis irrespective of whether or not risk preferences are viewed as low or high.

Economic impacts of climate change and uncertainty

A feature of the EV / Chance constraint model developed for this chapter is that it is flexible and can be modified to analyze different contexts and scenarios. Changing the values of parameters leads to new models. Also, changing the types of constraints imposed can permit application of the model to different types of problems. The previous two sections have used the model to assess the sensitivity of certainty equivalent and optimal portfolio selection to variations in chance constraints, flow constraints and risk aversity. The situation presented was one of a private harvester obtaining timber rights for public lands (i.e. a leaseholder) and maximizing certainty equivalent subject to sustained yield harvest constraints. In this section we are interested in looking at both the impacts of climate change and uncertainty as well as the interrelationships between climate change and sustained yield forest policy. The goal of the analysis in this section is to evaluate the economic impacts of uncertainty and climate both with, and without sustained yield constraints imposed. We also consider the economic implications of climate change and uncertainty in a case where the decision maker takes action to reduce or eliminate the uncertainty associated with the period one harvest yield.

Another feature of the analysis in this section is that we compare the results using the scenario one, two, three and four input data. Scenario one provides the baseline input data (no climate effects and yield predictions are deterministic). Comparisons of model runs using the scenario two input data (future random yields based on climate change) with model runs using the scenario one input data are used to estimate the pure effects of climate change (without uncertainty in yield parameters). Comparisons of model results using the scenario three input data (predictions based on future uncertain yields with both climate and yield variance as sources of uncertainty) with model runs using the scenario one input data are used to compare choices of individuals who ignore uncertainty and climate with the choices of individuals who take full account of climate change and yield uncertainty in their decision making. Comparisons of model results using scenario four input data (i.e. uncertainty in period one harvest yields is eliminated by conducting an inventory) with model results using scenario three input data, allows us to look at the implications of eliminating uncertainty in period 1 harvest yields on benefits and portfolio choices.

First we look at the case without sustained yield constraints. In this case harvesting decisions are not constrained by forest policies (i.e. this is the case of a private landowner). The objective function is similar to what was used in the previous two sections (i.e. maximize certainty equivalent). However, the constraints for this problem are limited to area constraints and non-negativity constraints. We drop the ending inventory and flow constraints for this initial model.

Since there are no harvest constraints in the first model used in this section, the model is referred to strictly as an EV model (as opposed to the EV – Chance constraint model discussed later). The objective function for the EV model is unchanged. The decision maker's goal remains to maximize certainty equivalent. As previously noted, certainty equivalent is defined as expected return minus a risk premium. The modeling approach for the analysis in this section is to solve the EV model for each of the four sets of input data (i.e. scenarios one, two, three and four) over the same range of CARA values used in the analysis in the previous section. The results of this analysis are provided in Figure 9.4 and Table 9.4.

The maximum certainty equivalent value without sustained yield constraints for the 1000 ha forest based on current yields and ignoring uncertainty (i.e. using scenario one input data) is about $ 500,000 (Figure 9.4). Since the scenario one input data are deterministic, there is no reduction for risk. The scenario two input data represents the productivity effects of climate change and the effects of climate uncertainty. The pure productivity effects of climate change can be seen by comparing CE values resulting from use of scenario two input data with CE values resulting from use of scenario one input data when the CARA parameter is set at zero (i.e. setting CARA at zero essentially makes the risk premium equal to zero). Figure 9.4 shows that the pure productivity effects of climate change from a private landowner perspective are positive. Certainty equivalent values are 28 % higher with climate change when the CARA is set to zero (Table 9.4). The effect of climate uncertainty can be seen by viewing the scenario two curve in Figure 9.4. Certainty equivalent values decrease somewhat as CARA increases, however, the effects are not large. Even for the most risk averse decision maker, the certainty equivalent value using the scenario two input data is higher than the baseline value (scenario one). Thus, the overall effects of climate change on economic benefits from a private landowner perspective (i.e. without sustained yield constraints) are positive. Uncertainty in climate variables does decrease certainty equivalent somewhat at high levels of risk aversity but not enough to offset the positive productivity effects of climate change.[31]

The results change significantly when yield uncertainty is included in the model. The effect of risk aversity on certainty equivalent when both climate effects and yield uncertainty effects are included (i.e. scenario three input data) is also shown in Figure 9.4. When yield uncertainty is considered, certainty equivalent tends to be below the baseline certainty equivalent at values of CARA above 0.000005. This

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[31] It should be noted, however, that as previously stated, the variances of the climate variables may be conservatively estimated.

result suggests that not considering yield uncertainty results in a significant overestimation of benefits from forestry operations (for private landowners). The positive climate change productivity effects actually dampen the negative effects of yield uncertainties. However, the economic costs of uncertainty in yield parameters are relatively large. Moreover, these costs increase as the degree of risk aversity increases[32].

The decline in certainty equivalent values with increasing risk aversion under scenario three is rather dramatic. A factor contributing to this decrease is the high levels of variance in net benefits associated with period one (i.e. short term) harvest. However, because the period one harvest is close to the present, managers may have the option to reduce uncertainty in period one harvest benefits by measurement (e.g. by conducting an intensive inventory or by employing some other stand yield measurement option that eliminates uncertainty regarding period one harvest yields). This is the basis for the scenario four input data. The scenario four input data is premised on the assumption that harvest yields in period one are known with certainty. Figure 9.4 shows that eliminating uncertainty in period one harvest yield results in relatively large increases in certainty equivalent values for the unconstrained model. In fact, certainty equivalent values for scenario four are higher than scenarios one, two and three for all positive values of risk aversion. Thus, there appear to be relatively large economic benefits associated with eliminating uncertainty in period-one harvest yields. In fact, Table 9.4 shows that the certainty equivalent for a logger with moderate risk aversion (i.e. CARA=0.00001) is 184 % higher under scenario four compared to scenario three. The difference in absolute dollar terms is $ 292,427. This is approximately the benefit of new information about period one harvest yield.[33] The economic gain of eliminating uncertainty in period-

[32] Note these results are contrary to the results suggested by Pannell et al. (2000) who note that by and large the costs of risk in a farm management context tend to be low. This may be due to the fact that a) our assumed risk aversion coefficients are higher, b) the variances associated with forestry production are higher, or c) a combination of the two.

[33] This amount pertains to the private landowner case. In our simple hypothetical forest case, the landowner might be prepared to pay up to this amount to reduce first period harvest yield uncertainty. In reality a large number of additional factors would likely affect the amount the owner would be willing to pay including profit requirements, tax considerations, the relative extent to which period one harvest yield is fully eliminated vs partially reduced by operational inventories, etc.

one harvest yield increases with increases in the degree of risk aversion of the logger. The economic return from eliminating period-one harvest yields is highest for an individual who is very risk averse (i.e. CARA = 0.00004). In this case, eliminating uncertainty in period-one harvest yield results in a 470 % increase in certainty equivalent value.

Another question of interest regards the impacts of climate change and uncertainty on portfolio choice (i.e. optimal solutions). The optimal portfolios for the various scenario / risk aversion value combinations are provided in Table 9.4. For the baseline case, the entire 1000 hectares is harvested in period 1. This is not surprising because: (a) the expected value of net benefit per ha is highest for period one harvest, (b) there are no harvest constraints, (c) there is no premium on risk, and (d) the planning periods are 30 years long and therefore as long as the economically optimal rotation is less than 70 yrs (i.e. 40 yrs (starting age) plus 30 yrs (planning horizon)) the unconstrained solution will be to harvest in period one. A similar result is evident using the scenario two input data.

Portfolio adjustments to risk do, however, start to become evident in the model using the scenario three input data. For example, at somewhat moderate values of CARA (0.0000055) the composition of the portfolio begins to change. Maximization of certainty equivalent requires that 71 hectares in IAC1 is cut in period 2 (Table 9.4). For very risk-averse forest landowners, all of the area in IAC2 is cut in period 2 and all of the area in IAC1 remains un-harvested. Therefore, as decision makers become more averse to risk, their general response is to want to defer harvesting to later periods. As noted and described in Chapter 8, variances of net benefits in later periods tend to be smaller than variances in net benefits in early periods. Therefore, a decision maker looking for ways to reduce risk will defer harvesting. The motivation for this type of decision may be that by delaying harvesting the decision maker is delaying risk. She/he may prefer future risk to present risk thereby justifying the discounting of risk and choices that result in putting risk off to some future date.

Under scenario four with the unconstrained model, much of the uncertainty in net benefits is eliminated. Thus, the penalties for risk also become smaller. The optimal portfolio under scenario four is to harvest the entire forest in period one.

For the next model in this section we return to the case with sustained yield constraints. Assumptions about parameter values and the basic modeling approach are described as follows:

- Modeling is based on scenario one, two, three and four input data.
- The objective function is to maximize certainty equivalent subject to all harvesting constraints (i.e. ending inventory and even flow constraints), area constraints, and non-negativity constraints.
- Chance constraints are set at 0.6 (73% likelihood).
- The even flow constraint allows for a 25 % deviation in between period harvests.
- The target ending inventory value is set at 220,000 cubic meters.
- The model is estimated for the following values of CARA – 0.0, 0.00000077, 0.0000055, 0.00001, 0.000025, 0.00004

Figure 9.5 shows the overall and relative effects of productivity changes and uncertainty on the economic benefit with sustained yield constraints. A comparison of certainty equivalent values when CARA = 0.0 shows the pure productivity effects of climate change for this scenario. The present value of net benefits using climate normal data to predict harvest yields and ending inventory yields (scenario 1) is about $300,000. The present value of net benefits using scenario 2 input data (based on pure climate change related productivity effects with the risk premium ignored - chance constraints are still in effect) is about $ 419,164 (Figure 9.5 and Table 9.5). Thus, productivity increases associated with climate change have a significant positive impact in terms of economic benefits – even when sustained yield constraints are incorporated. These results are similar to the private forestland management case.

Figure 9.5 also shows the degree of sensitivity of objective function values to changes in degree of risk aversion of the decision maker. Certainty equivalent values for model results using the scenario two input data are not particularly sensitive to risk preferences and chance constraints. In fact, even at the highest level of risk aversion, certainty equivalent values with climate change (scenario two) are much higher than certainty equivalent values without climate change (scenario one results). This confirms previous findings that climate change is not particularly prominent as a source of risk and uncertainty. Certainty equivalent values for model results using the scenario three input data are, however, much more sensitive to risk aversion. Certainty equivalent values decrease from $ 371,201 to $ 120,680 (67% decrease) with increasing risk aversity (Figure 9.5 and Table 9.5).

The results shown in Figure 9.5 for scenario four are interesting. These results show that under the assumption that a leaseholder is able to eliminate uncertainty in period-one harvest yield, then certainty equivalent values with climate change and all sources of uncertainty included (i.e. scenario four) are higher than certainty equivalent values without climate change and uncertainty (scenario one) at all levels of risk aversion for the model with sustained yield constraints. The difference in benefits between scenario three and scenario four (i.e. the economic benefit of new information about period-one harvest yield) for the moderately risk averse decision maker (i.e. CARA=0.00001) is $ 60,896. The absolute difference between the scenario three and four results for the unconstrained model was $ 292,427. Thus, the imposition of sustained yield dramatically reduces the benefits of reducing uncertainty in period one harvest yield.

A comparison of Figures 9.4 and 9.5 suggests that the overall pattern of results for objective function values for the public forestland management scenario are similar to the results observed for the private forestland scenario. In absolute terms, objective function values are lower when sustained yield constraints are imposed. A potential question of interest is: What are the relative opportunity costs of sustained yield for the various scenarios? The relative opportunity costs of sustained yield can be seen by comparing relative differences in certainty equivalent with and without sustained yield constrained models for each scenario for a decision maker with a particular level of risk aversion. In this case we compare certainty equivalent values for the moderately risk averse decision maker (i.e. CARA=0.00001). The imposition of sustained yield constraints results in the following relative decreases in certainty equivalent value. For scenario one (the baseline case), certainty equivalent decreases 44 % when sustained yield is imposed. For scenario two, certainty equivalent decreases 34 %. For scenario three, certainty equivalent decreases 18 %. For scenario four, certainty equivalent decreases 46 %. These results are mixed. Comparing the scenario one and two results would suggest that climate change tends to reduce the opportunity costs of sustained yield (i.e. the relative decrease declines when climate change effects are included). A similar result is evident with the scenario three. However, the results from comparing scenario four and scenario one, suggest that when the decision maker has the opportunity to eliminate uncertainty in period-one harvest yields, then the opportunity costs of sustained yield slightly increase under climate change. Thus, the results are dependent on the underlying assumptions made about sources of variance and about actions that a decision might take to reduce variance in areas where it is feasible to do so. If the logger does have the flexibility and capability to eliminate uncertainty in period-one harvest yields, then the benefits of climate change may be more pronounced without sustained yield than with sustained yield and therefore, the relative opportunity costs of sustained yield under climate change increases somewhat. If, however, the logger is not able to eliminate uncertainty in period-one harvest yield, then the opportunity costs of sustained yield appear to be lower under climate change.

The pattern of results pertaining to portfolio choice (i.e. harvest solutions) for the model with sustained yield constraints is different from the model without sustained yield constraints case. The solutions to the model with sustained yield constraints are shown in Table 9.5. Comparing the optimal portfolios for each scenario with the optimal portfolios provided in Table 9.4 (i.e. solutions without sustained yield constraints) shows that the imposition of harvest constraints results in a more diversified forestry portfolio compared to what a private landowner would select.

The results in Table 9.5 show that risk aversion does have some affect on portfolio composition (with the public forestland case). For scenario three, the preferred portfolio for decision makers with moderate to low risk aversion (i.e. less than 0.00001) is to harvest the majority of the 1000-hectare forest in periods 1 and 2. However, the preferred portfolio for loggers with higher levels of risk aversity for scenario three is to not harvest the majority of the area. The motivation may be similar to the motivations described for a private decision maker. If a harvester is highly risk averse, he/she may prefer to postpone harvesting (and therefore risk) until sometime in the future (as was shown in Chapter 8). The pattern of solution results for scenario four is not the same as scenario three. For scenario four, the optimal portfolio is unchanged for all levels of risk aversion. Thus, in cases where the decision maker has the opportunity to eliminate uncertainty in period one harvest yield; differences in decision maker risk preferences have no effect on portfolio selection. Questions related to the implications of climate change, uncertainty, and assumptions about period-one harvest yield on total areas harvested and harvest timing will be addressed in more detail in a later section.

The results presented up to this point leads to additional questions of interest. First, what are the direct implications of climate change productivity change in a deterministic management setting? Second, is it possible to satisfy sustained yield constraints for public forestland timber management if we combine risk and uncertainty with parameter assumptions that more closely reflect current sustained yield requirements? Third, what are the implications of climate change and risk if we consider the risk preferences of society instead of the risk preferences of an individual in the risk model? These questions are addressed in the remainder of this section using results obtained by further modifying the model structure in a way that allows us to specifically address each question.

Effects of climate change on sustained yield objectives in a deterministic setting

In many respects current forest management planning is deterministic because forest managers generally do not consider variables upon which decisions are made in a probabilistically. A question of interest might be: What are the direct implications of climate change productivity effects if we ignore all the risk aspects of our harvest-scheduling problem? In order to obtain results that pertain to this question we can compare the model results when using scenario 1 and 2 input data with chance constraints and risk aversion values set at zero. In effect the model becomes deterministic. An additional modification of the model is that we have imposed a flow constraint that allows a more typical 10 % (as opposed to 50 %) deviation in between period flows. This structure permits us to isolate the effects of climate change productivity effects relative to the extent to which sustained yield constraints become more or less constraining. The specific model outputs that will provide some indication of whether sustained yield is more or less constraining under climate change are differences in the marginal stand values of the two stand types (i.e. IAC1 and IAC2), and differences in the optimal portfolio. Marginal stand values are obtained from the GAMS output. These values represent the marginal increase in objective function values with a 1 hectare increase in each stand type. Table 9.6 provides a summary of model results based on the modeling approach described above. Table 9.6 shows that the marginal values of the two stand types for this particular construct are significantly higher with climate change than without. Also, the percentage of the area that can be harvested while still satisfying the constraints is higher with climate change than without. These results show that in a deterministic setting, climate change increases the ability to satisfy sustained yield constraints while increasing forestry benefits. Increases in net portfolio benefits occur as a result of being able to harvest a larger percentage of available area in periods one and two. As shown in Table 9.6, climate change productivity effects allow for a larger area to be harvested in periods 1 and 2. This is because productivity is increasing in each period in the planning horizon. Therefore, flow constraints and ending inventory constraints are easier to satisfy.

Effects of climate change on sustained yield in a stochastic setting

As emphasized throughout this study, forest management is not deterministic. Moreover, with climate change, forest management is becoming more (not less) uncertain (although as we have shown perhaps not to the degree we might have expected). The second sub-question of interest for this section is: Can we satisfy SY constraints if we combine risk and uncertainty along with parameter assumptions that more closely resemble current sustained yield requirements? For the models in the previous sections somewhat flexible assumptions about parameter values were adopted. For example, the analysis in previous sections allows harvest levels to deviate by 25 % between periods one and two. Also, the models have a low threshold for satisfying the chance constraint. For example, flow and ending inventory constraints only need to be satisfied 73 % of the time. A related question of interest is: Is it possible to satisfy harvesting constraints for public forestland timber management if we use a more restrictive set of assumptions about flow constraints and chance constraints in the risk model? The model for answering this question is structured as follows:

- 10 % deviation in flow constraints
- Chance constraint requires that constraints are satisfied 90 % of the time.
- Risk aversion coefficient set at 1.0E-05
- Model results are based on scenario 1, 2, 3 and 4 input data.

The solution for a model based on these assumptions is shown in Table 9.7. As was noted previously, a solution that suggests that only a small portion of the 1000-hectare forest is harvested is considered to be an inoperable solution. Table 9.7 shows that only 31 % of the forest is harvested under scenario three and 0 % is harvested under scenario four given the above assumptions. Thus, when risk and uncertainty considerations are included in a harvest-scheduling model with parameter values reflecting current sustained yield policy requirements, then harvesting becomes at best marginally operable – even with productivity enhancements from climate change and even when the logger has the opportunity to eliminate uncertainty in period one harvest yield. This raises the question of the feasibility of sustained yield in increasingly uncertain operating environments. At minimum it suggests that as the operating environment becomes more uncertain (as a result of climate change or any other number of factors) forest management agencies may find it beneficial to continually review management policies and standards and to consider ways of making forest management more flexible. A more assertive statement about the implications of the results of the analysis in this section is that sustained yield may actually prevent tenure holders from considering risk because of the possibility that satisfying relatively rigid sustained yield constraints may not be possible or feasible when considering the inherent uncertainty in yield forecasts.

Risk preferences of society

The third sub-question of interest is: What are the implications of climate change and risk if we consider the risk preferences of society instead of the risk preferences of an individual in the public forestland management risk model? Hardaker et al. (2004) note "risk aversion should seldom be assigned much importance in public decision making." (pg 115). They argue that since the wealth of society is large, then the absolute risk aversion coefficient for society may be infinitesimal. Therefore, the risk premium will be close to zero. We can show that when CARA is low, the implications of risk and uncertainty are negligible by looking at the results in Table 9.5. The model results using scenario 3 input data show that the CE values are only slightly lower with a CARA value of 7.7E-07 than they are when CARA = 0.0. Moreover, portfolio choices are identical in both cases. Therefore, if it is society in general (instead of the logger) that is being exposed to yield and climate risk (for example if the loggers lease guarantees him a certain volume of timber in each period), then risk aversion may not be a particularly important aspect to consider in decision making – even with climate change. There are some interesting implications associated with the question of who bears the risk. One way loggers might be exposed is because of the way regulations are implemented. In some cases, forest management agencies deal with new information by regularly updating annual allowable cuts (AACs). If this process of continual revision and updating results in changes in the amount of timber available to loggers, then the logger may be exposed to risk and uncertainty. If on the other hand, lease arrangements are long term and if management agencies endeavor to avoid changes in available supply, then risk may be moot. Thus, the impacts of climate change and uncertainty are fundamentally dependent on property rights and institutional structures. The design of actual vulnerability assessments and/or climate impacts assessments must take these considerations into account.

Implications of climate change and uncertainty for harvest schedules

An important question in climate analyses is: How will decision makers adapt? (Hauer et al. 2001). In a forest management context we want to know how forest managers should adapt to climate change (Spittlehouse and Stewart 2003). Particular adaptation strategies will in turn be affected by the effects of climate change on productivity, by levels of uncertainty (in climate and yields), by the degree of risk aversion of the decision maker, by patterns of variance facing the decision maker in different time periods, and by the institutional setting (i.e. is harvesting constrained by sustained yield and what values are assumed relative to sustained yield parameters within the harvest model). This section discusses the implications of climate change and uncertainty in terms of what these mean relative to adaptation. The analysis in this section considers two levels of risk aversion (CARA=0.00001 and CARA=0.00004), two institutional settings (with and without sustained yield constraints), two levels of chance constraints (K=0.6 and K=0.0), and looks at how optimal harvest schedules change across the four scenarios.

It is important to acknowledge at the outset that the crude structure of the stylized management problem developed for this study results in some significant limitations in the kinds of adaptation questions that can be addressed. For example, the individual planning periods are 30 years long. This means that for this study we cannot look at how climate change affects optimal rotation. The best we can do is to consider the implications of climate change and uncertainty relative to adaptive changes in short-term vs. long-term harvest.

The first situation considered is that of adaptation to climate change and uncertainty for a private landowner (i.e. sustained yield constraints are not imposed).

Figure 9.6 provides the harvest schedule for each of the four scenarios under various assumptions about risk preferences and chance constraints. Figure 9.6 (a) shows the harvest schedule for a moderately risk-averse decision maker. Figure 9.6 (b) shows the harvest schedule for a decision maker who is highly risk-averse. With or without climate change, in general, the manager prefers to harvest everything in the short term (i.e. period one). The one exception is for scenario three. In this case, high levels of uncertainty in period one net benefit lead to reductions in period one harvest. In the case of the moderately risk averse decision maker, a little over 20 % of the harvest is deferred to period two. In the case of the highly risk averse decision maker the entire period one harvest is either shifted to period two (about 75 %) or held uncut (about 25 %). The option to eliminate uncertainty in period one harvest yield (scenario four) results in harvest shifting back to period one. Thus, the general result shown from this case is that a private forestland decision maker exposed to climate change and uncertainty may prefer to delay some portion of the harvest to later periods in the planning horizon. However, if this decision maker is able to eliminate uncertainty in period one harvest yields, then the optimal harvest schedule is to harvest the entire forest in period one.

The implications of climate change and uncertainty on adaptation in the case where sustained yield constraints are imposed is more interesting. The implications of climate change and uncertainty on harvest timing (i.e. adaptation) for a forest subject to sustained yield constraints are shown in Figure 9.7. This figure shows the optimal harvest schedules for each of the four scenarios under various assumptions about risk aversion and chance constraints. Figure 9.7 (a) looks at total area allocated to period one harvest, period two harvest, and no harvest for a moderately risk averse decision maker (CARA-0.00001) with sustained yield constraints and with chance constraints set so that there is a 73 % likelihood of satisfying the constraint. A comparison of scenarios one and two for this case shows that climate change by itself tends to result in an increase in area allocated to period one and two. In fact the area allocated to periods one and two harvest is equal to or greater than the scenario one harvest for all three scenarios that include climate change effects. Thus, two general results appear to be that for moderately risk averse decision makers operating under sustained yield constraints, climate change permits a general increase in short and long-term harvests within the planning horizon and it permits a reduction in the amount of area that needs to be held uncut in order to satisfy ending inventory constraints. A qualification is that this result applies for only aspen management in central Alberta and for a planning period that spans the period 2010 to 2070. Analysis in different locations or for different time spans may have different results.

An interesting result from the analysis discussed in the previous paragraph is that variance patterns have very little effect on the optimal harvest schedules. For example, even though variances of net benefits decline as time progress for scenario 3 and increase between period one and two for scenario 4, the harvest schedules are virtually identical (Table 9.5). This lack of sensitivity to differences in temporal variance patterns may be the result of (a) the crude structure of the management problem (i.e. long planning periods), and (b) the fact that significantly higher levels of uncertainty may be required in order trigger different solutions when the planning periods are so large. This is not the case, however, when the decision maker is highly risk-averse. Figure 9.7 (b) shows that the amount harvested in periods one and two is very low for highly risk-averse decision makers under scenario three. The risk-averse decision maker operating under scenario three variance assumptions prefers to delay the harvest in order to minimize the degree of uncertainty he/she faces. This result is largely due to the fact that time is risk reducing relative to variance in net benefits for scenario three.

What happens to harvest choices, when the variance patterns facing the highly risk averse decision maker change? For example, as previously noted, in scenario four, the variances of net benefits are low for period one harvest (due to elimination of period one harvest yield), then increase for period two harvest benefits (due to a combination of climate and yield model uncertainty), then decrease again for the ending inventory (due to discounting of future benefit distributions). This modified variance pattern has significant implications for the preferred harvest schedule and for the type of adaptation response that a risk averse manager should pursue. Under scenario four, the highly risk averse decision maker takes advantage of new knowledge about harvest yields and as a result shifts more hectares into periods one and two (compared to S3) and reduces the number of hectares held uncut. Thus, for the highly risk averse decision maker, reducing period one benefit variances increases the period one harvest.

Figure 9.7 (c) shows the case of a moderately risk averse decision maker where chances constraints are not binding (i.e. uncertainty in the constraints is eliminated). Under this set of assumptions the recommended harvest schedule for scenarios two, three and four are very similar. The majority of the 1000 hectares forest is harvested in period one for all three scenarios. Scenario 4 calls for a small increase in period one harvest relative to scenario 3 but the increase is not significant. The main effect pertains to the difference in the optimal harvest schedule under climate change compared to scenario one. Under scenario one, almost 45 % of the area remains uncut, about 34 % is cut in period one, and about 21 % is cut in period two. For scenarios two, three and four over 45 % of the area is cut in period one, around 30 % is cut in period two, and around 25 % of the total area remains uncut. Here again, climate change has a positive effect in terms of increasing both the area that can be harvested in the short term and reducing the total amount of area that needs to remain uncut for the entire planning period.

Summary and conclusions

The first main finding of the analysis presented in this chapter is that all other factors equal, the economic impact of climate change would appear to be positive for aspen timber management for this experimental forest – even with costs associated with higher climate related risk accounted for (for our location and for a period up to 2070).

A second finding is that for managed forests subject to sustained yield constraints, increasing risk aversion reduces benefits, and the incremental decline varies depending on relative degrees of uncertainty. This confirms the expected result that accounting for risk preferences becomes increasingly important as levels of uncertainty increase. The fact that climate change increases uncertainty reinforces the importance of accounting for risk preferences and the economic costs of uncertainty when considering the impacts of climate change.

A third finding is that uncertainty associated with yield predictions and the economic costs of this uncertainty is more significant than risk caused by uncertainty in climate variables. For this study, yield uncertainty accounts for about 84 % of the risk premium and uncertainty related to future climate variables accounts for the remaining 16 % (Note this roughly compares to the finding in Chapter 8 that climate uncertainty accounted for 25 % of total portfolio standard deviation). It is important to qualify this finding somewhat because the distributions for the climate variables and the yield parameters are assessed differently. For example, uncertainty around a future climate variable is usually based on the range of predictions from various combinations of climate models and emission scenarios whereas uncertainty around yield parameters is based on cross sectional variability in yield data used to estimate the yield models. As noted, variances in climate variables are likely conservatively estimated and price variance is not included. Thus the economic costs of uncertainty from climate change are likely conservatively estimated in this study.

A fourth finding is that with the exception of scenario four, the relative gains expected from climate change are higher under a forest managed with sustained yield constraints than under a forest with no sustained yield constraints. This may be a result of the fact that higher growth rates in future periods make the satisfaction of flow constraints and ending inventory constraints relatively easier to achieve. At the same time, it is important to note that there is a significant opportunity cost associated with sustained yield – even with climate change. In the case of the logger with average risk aversion (CARA=0.00001) and subject to scenario three input data, the total certainty equivalent for the forest subject to sustained yield constraints is 18 % lower than the certainty equivalent value for the forest without sustained yield constraints (i.e. \$285,775 compared to \$346,258) (Tables 9.4 and 9.5). In the case of scenario four, certainty equivalent value with sustained yield are 46 % lower than certainty equivalent values without sustained yield constraints imposed. This exceeds the decrease in certainty equivalent values of 44 % associated with the scenario one (i.e. no climate) input data.

A fifth finding is that for moderately risk averse decision makers operating under sustained yield constraints, climate change increases the area that can be harvested in the short and long-term and decreases the area that is required to be held as uncut (this result depends on the existence of flexibility in flow and chance constraints). Moreover, differences in variance patterns (i.e. scenario three vs. scenario four) have no effect on how the harvest is scheduled. This result, however, only applies in the case where the decision maker has a moderate level of risk aversion. In the case of a decision maker who is very risk averse, then the optimal harvest pattern is to hold the majority of the stand uncut under scenario three. If, on the other hand, the decision maker has the opportunity to eliminate period one harvest yields (i.e. the scenario four case) then higher levels of harvesting in periods one and two are permitted. A related result is that under rigid flow constraints (i.e. allowable harvest can only deviate by 10 % between harvest periods and high chance constraints) then sustained yield effectively eliminates the incentive to consider risk (as indicated by the fact that there is little period one and two harvest when sustained yield constraints are inflexible) and reduces harvest feasibility.

A final finding of the results in this chapter is that levels of impact are affected by institutional settings and property rights configurations. If the institutional structure and property rights configuration are such that it is the leaseholder who is exposed to yield and climate uncertainties, then it might be expected that this exposure will have some influence on optimal harvesting decisions and on the stream of economic benefits that forest harvesting provides. In this case, accounting for risk aversion relative to climate change impact assessment may be important and needed. If on the other hand property rights are configured in such a way that it is society who is primarily exposed (i.e. society bears the risk associated with public forestland management), then accounting for risk aversion in decisions and choices about harvesting may be less important. One interesting possibility for future research might be to look at this problem as a principal agent problem. If indeed the public (the principal) has ownership of forestland but assigns property rights for timber management and if harvesting to the private sector (the agent), then the study of climate impacts and effects on decisions could be treated as a principal agent type of problem. Investigating the climate change impact analysis problem using a principal agent construct would permit consideration of other issues such as how does stumpage fee structures affect incentives for managing risk or becoming more informed (Nilsson 2003). If becoming more informed simply results in higher stumpage fees, then leaseholders may have little incentive to pursue this as an option for managing risk.

Finally, it is important to note some of the limitations in what can be done with the crude model structure developed for this study. The planning period for this study is only 60 years in length. Moreover there are only two 30-year planning periods within this planning period. Also there is only one species (aspen) and two initial age classes (40 year and 80 years). This is a highly simplified problem context compared to more typical decision analysis problems in forestry. One consequence of keeping the problem context simple, however, is that some potentially interesting questions cannot be addressed and the course model set up obscures potentially interesting results. For example, it is not possible to consider potentially important adaptation questions such as optimal rotation questions with this model.

Figure 9.1 The EV model formulation with a graphical representation of the key terms and optimal points on the EV frontier for different levels of risk aversity.

Expected utility function:

$E[U(NB)] = CE = E[NB] - 0.5\Phi Var(NB)$

Risk Premium=$0.5\Phi Var(NB)$

Terms:

CE1, E[NB1], and Variance 1 correspond to a decision maker who has a relatively low aversity to risk.

CE2, E]NB2], and Variance 2 correspond to a decision maker who has a relatively high aversity to risk.

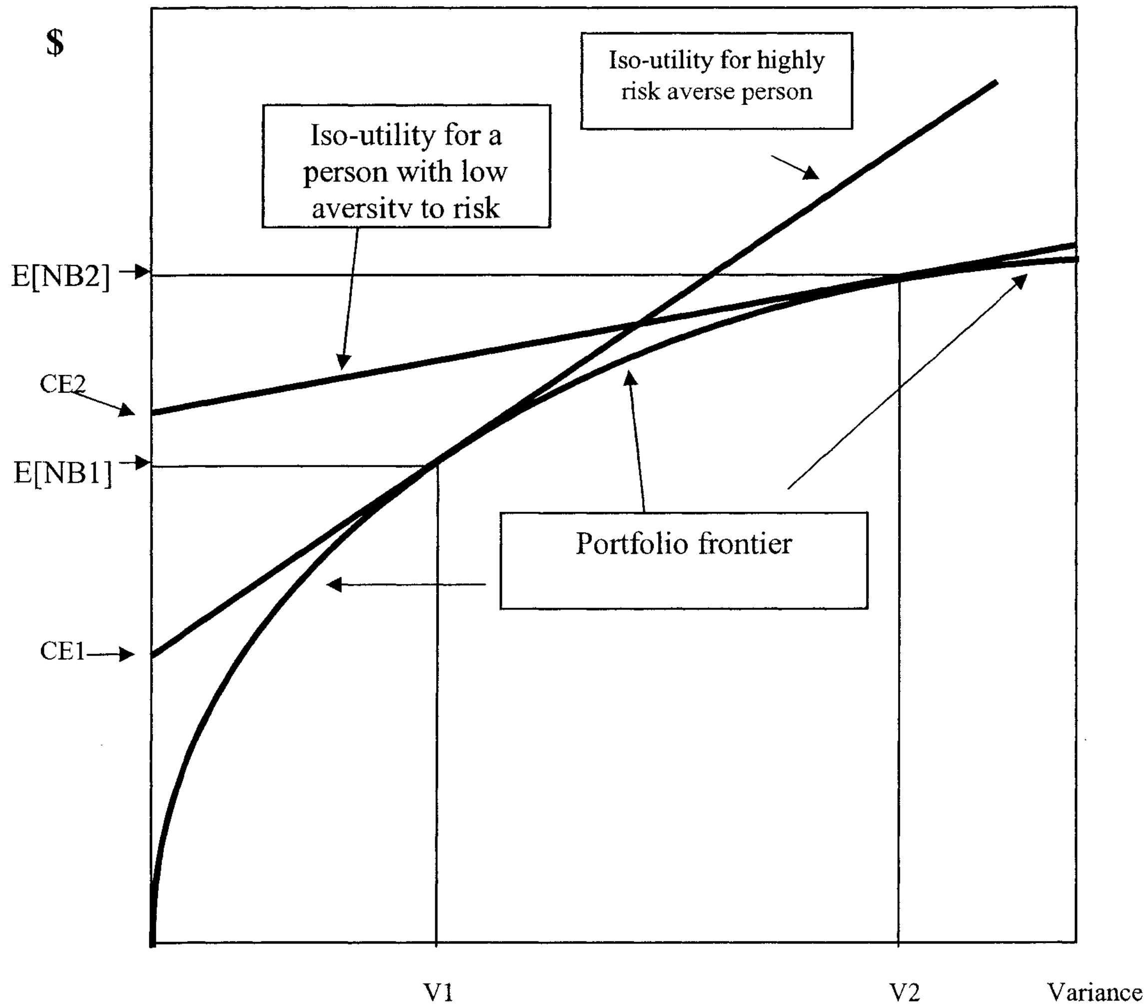
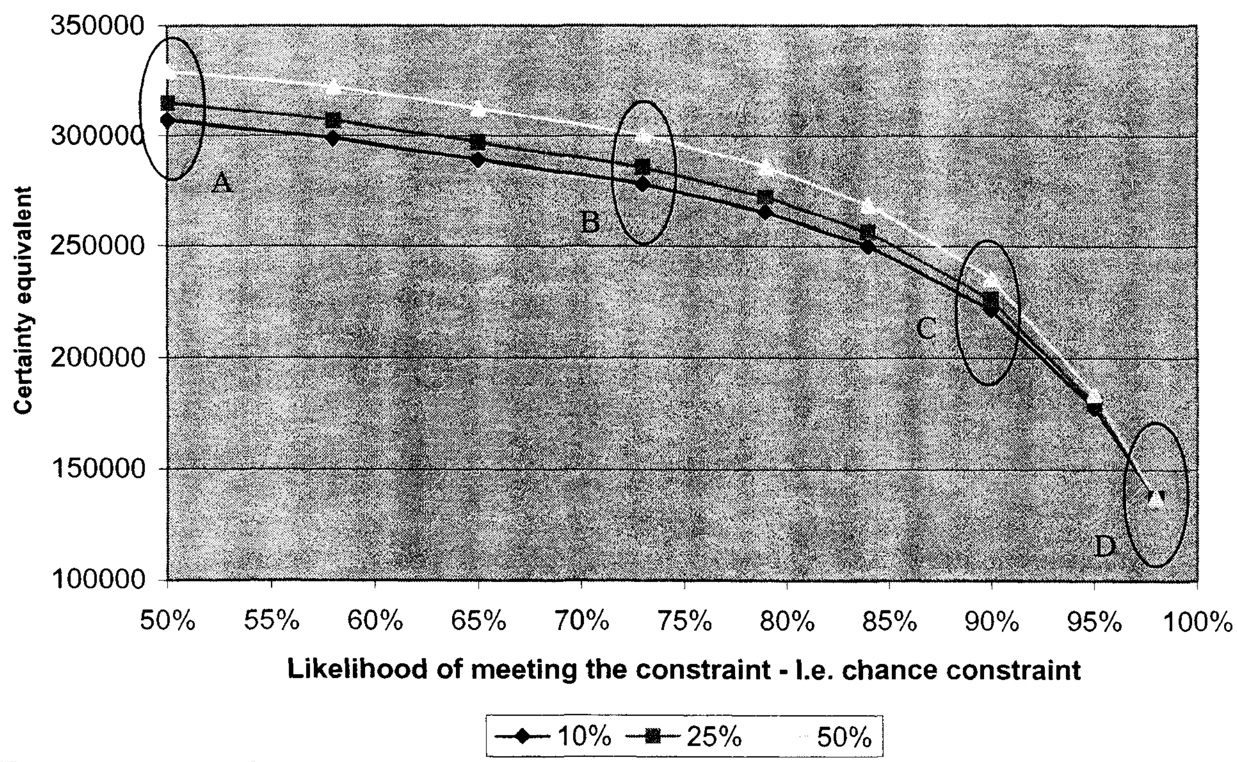
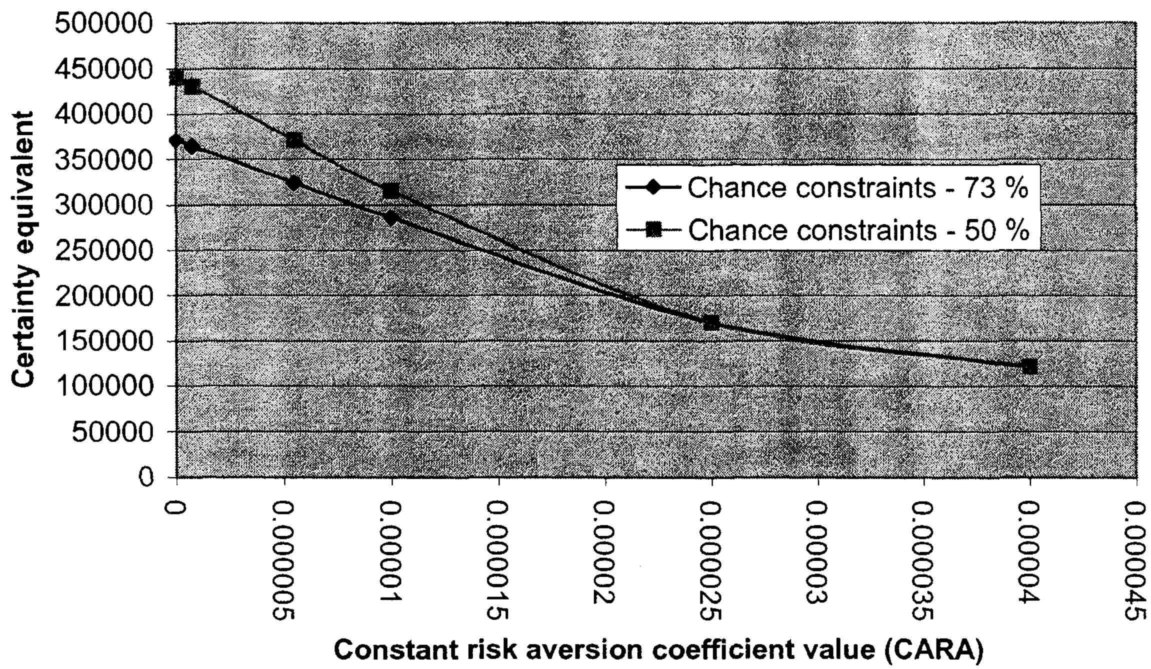


Figure 9.2 Sensitivity anaysis showing the effects of variations in likelihood of meeting flow constraints and variations in allowable flow deviations on objective function values.



Results based on scenario 3 input data - CARA=0.00001 - The 10 %, 25 %, and 50 % values represent the alpha and beta values (i.e. the allowable deviation in between period harvest)

Figure 9.3 Sensitivity analysis showing the effects of chance constraints and risk aversion on objective function values.



Results are based on scenario 3 input data. / Alpha and beta are set at 25%

Figure 9.4 Effects of climate change, uncertainty, variance assumptions, and risk preferences on objective function values - without sustained yield constraints.

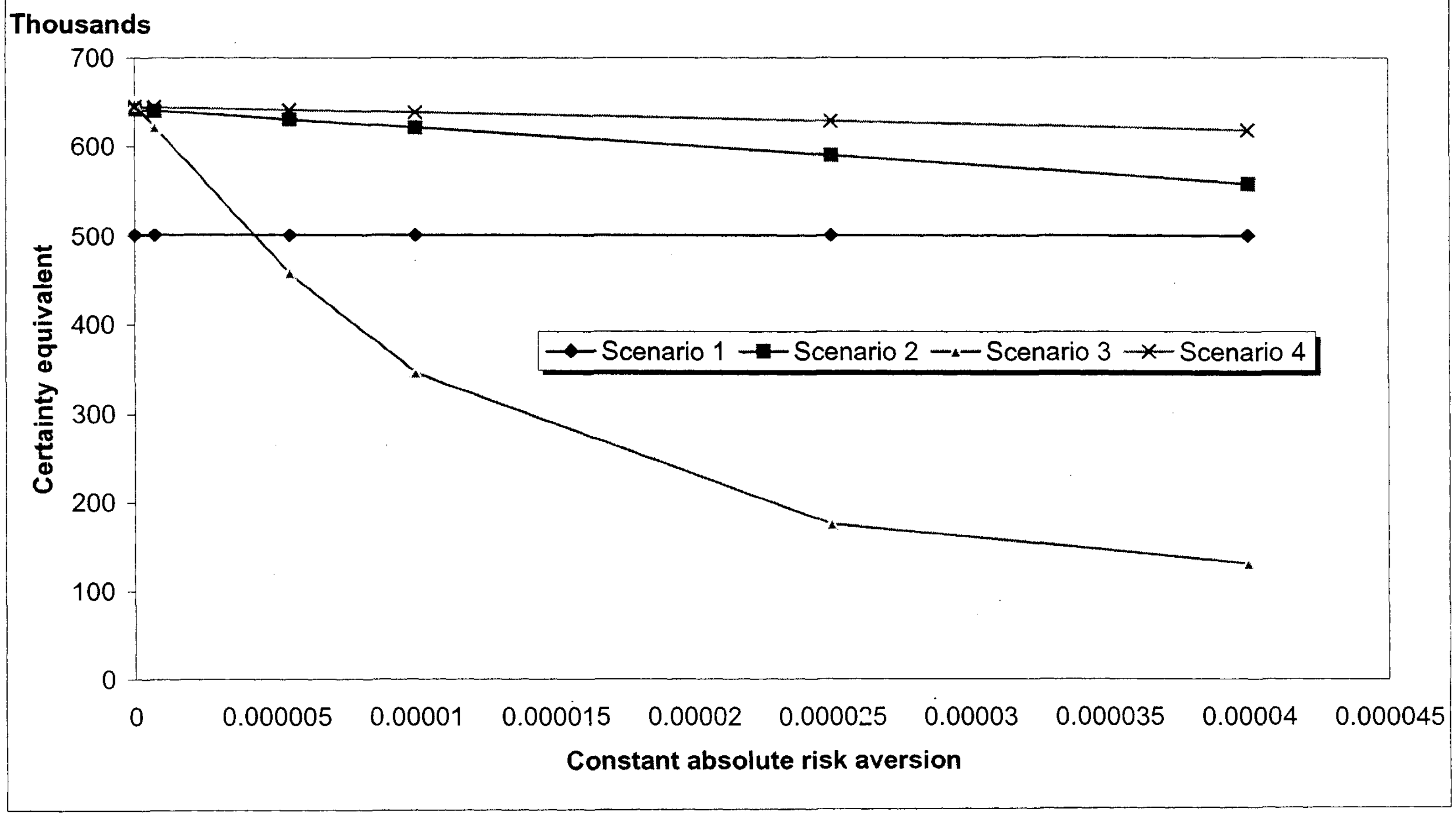
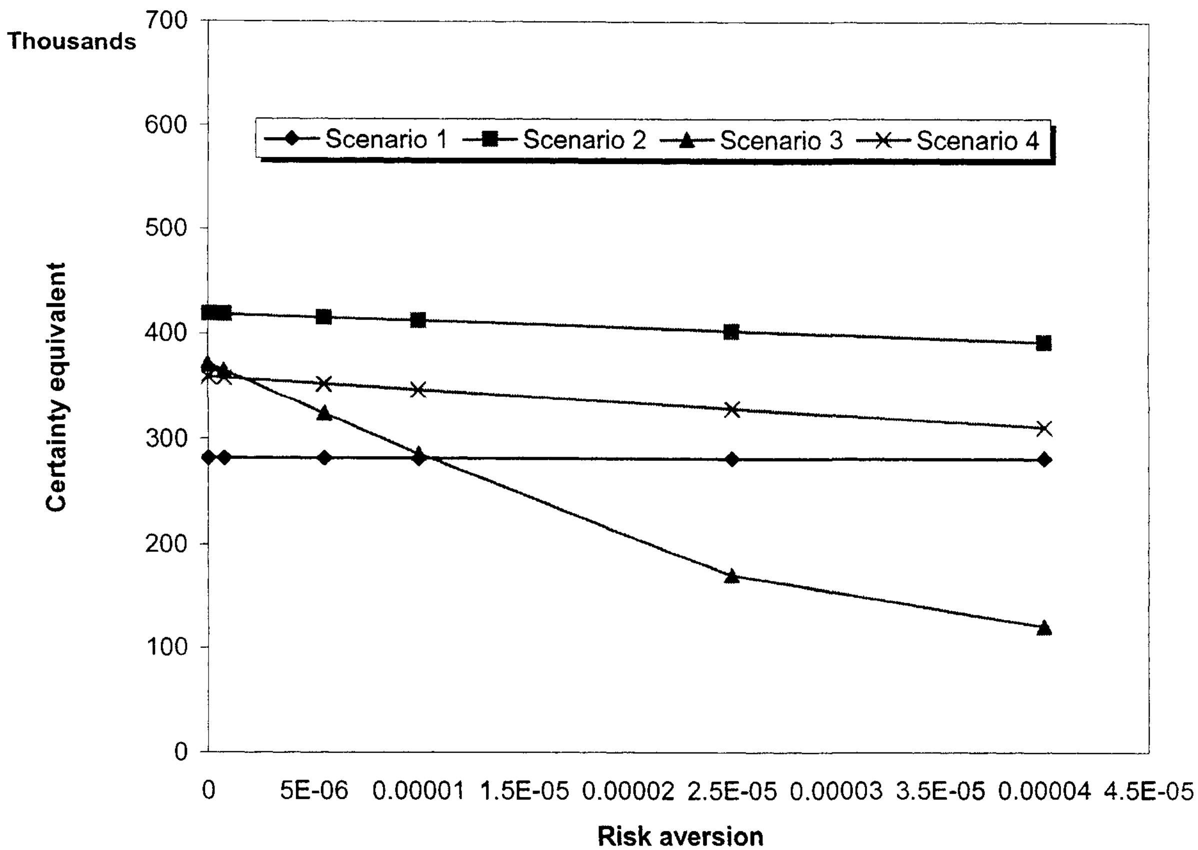
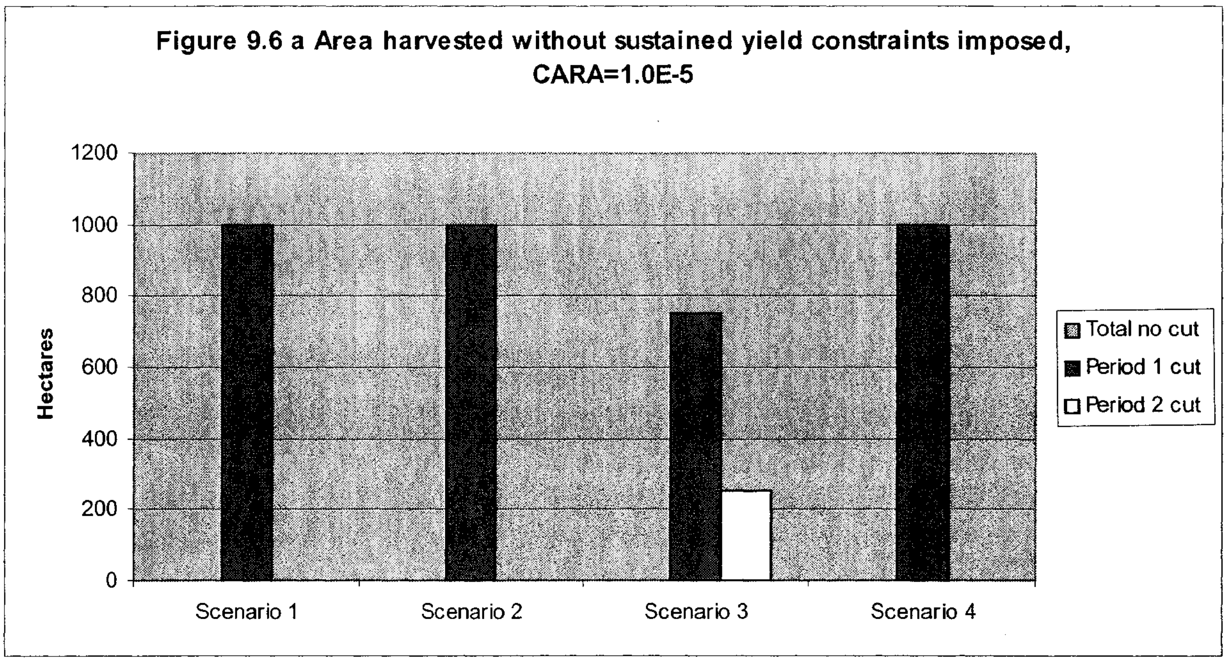


Figure 9.5 Effects of climate change, uncertainty, variance assumptions and risk preferences on objective function values with sustained yield constraints.



Sustained yield assumptions: Ending inventory = 220,000 cu. m., K=0.6, allowable deviations in flow constraints=25 %.

Figure 9.6 Effects of climate change and variance assumptions on harvest timing without sustained yield constraints imposed.



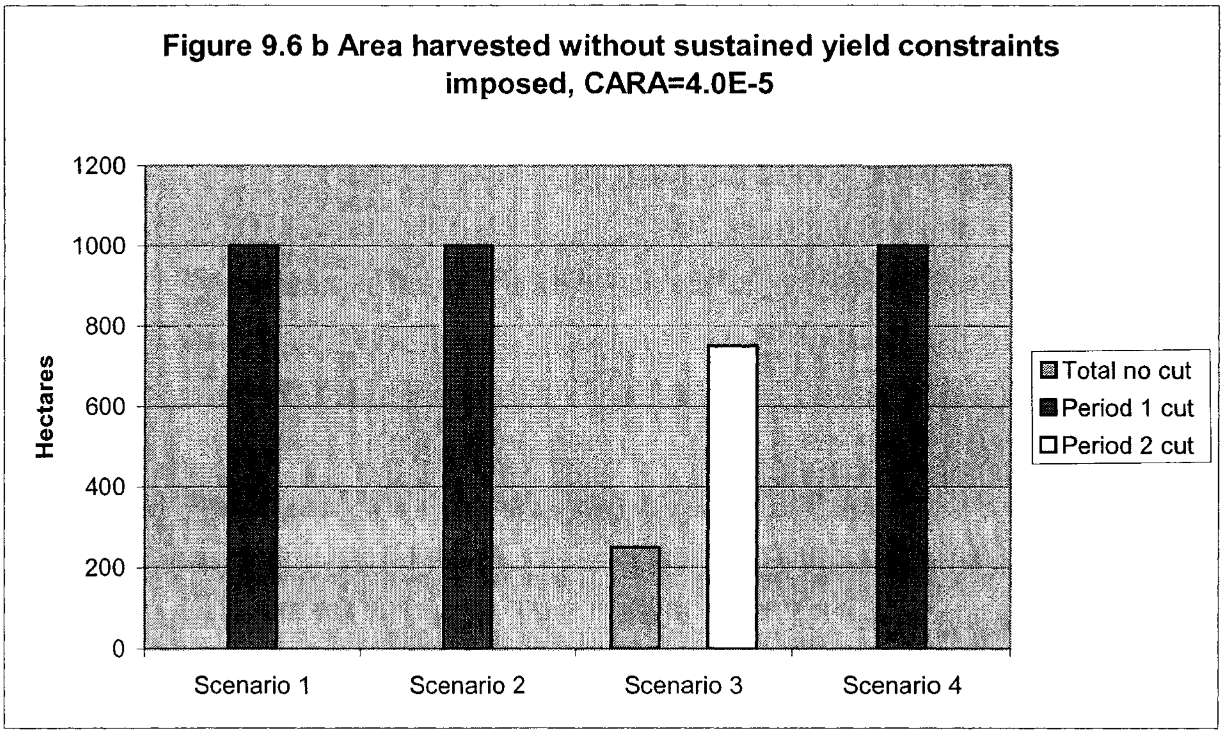


Figure 9.7 Effects of climate change and variance assumptions on harvest timing with sustained yield constraints imposed (ending inventory = 220,000 cu.m., allowable deviations in flow = 25 %, chance constraint = 0.6).

Figure 9.7 a Area harvested by period with sustained yield constraints imposed, CARA=1.0E-5 and K=73 %

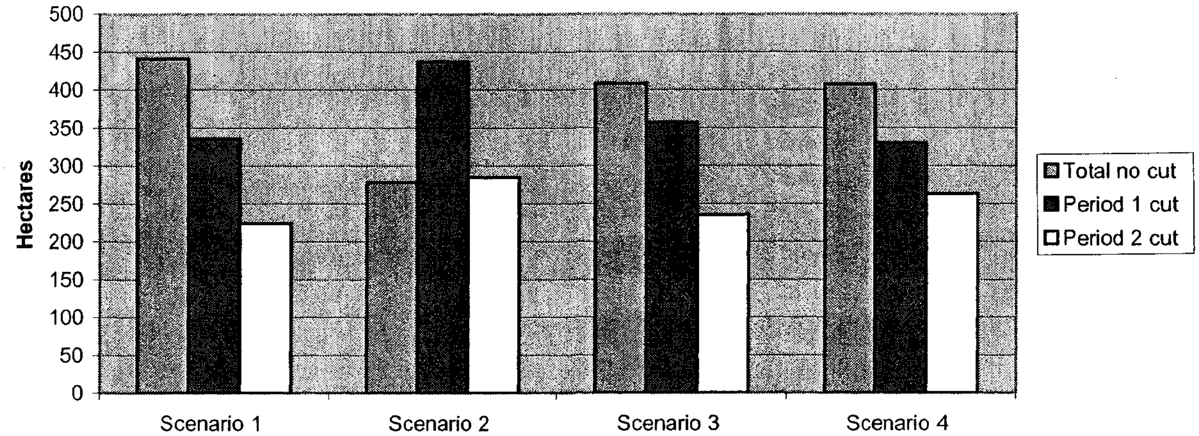


Figure 9.7 b Area harvested by period with sustained yield constraints imposed, CARA=4.0E-7, K=73%

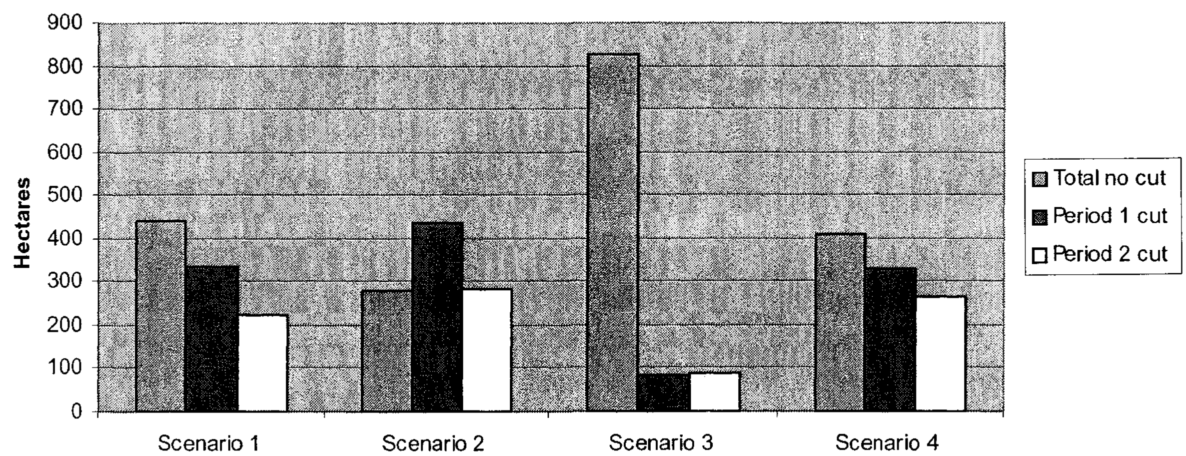


Figure 9.7 (continued)

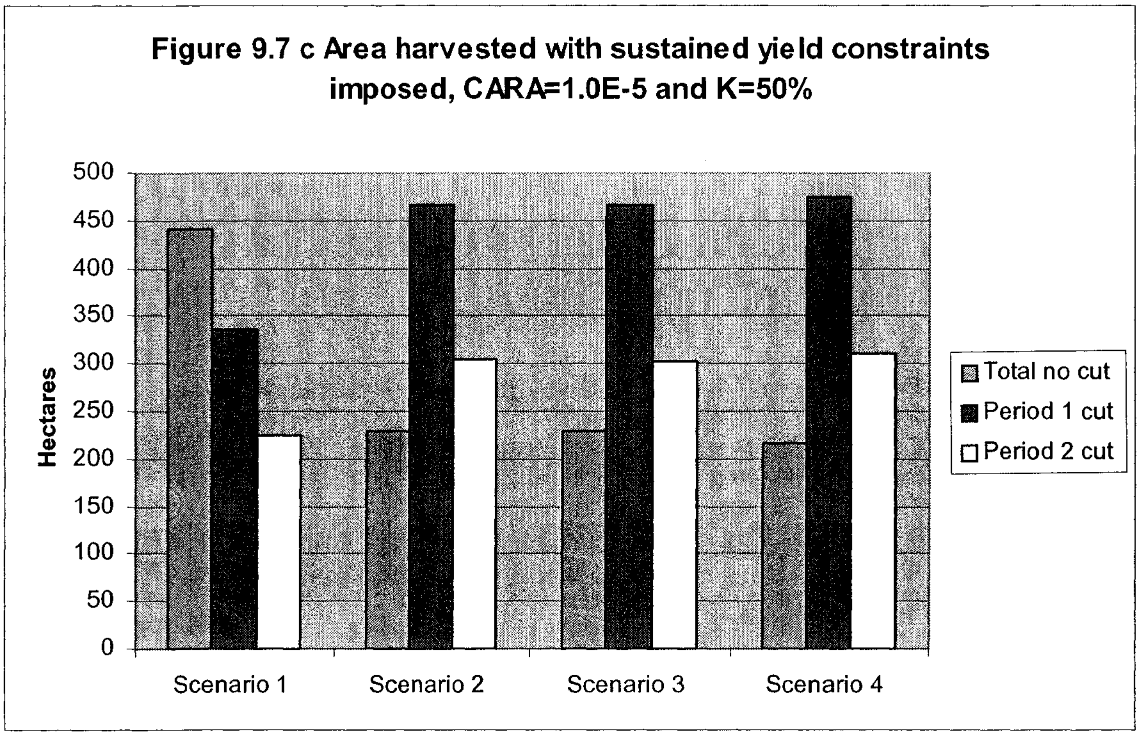


Table 9.1 Values of the CARA $(\Phi_a(w))$ parameter utilized in this study

	Wealth	
	\$650,000	\$100,000
	CARA Values	
Hardly risk averse $\Phi_r(w) = 0.5$	0.00000077	0.000005
Very risk averse $\Phi_r(w) = 4.0$	0.000006	0.00004

CARA – Constant absolute risk aversion - $\Phi_a(w)$

CRRA – Constant relative risk aversion - $\Phi_r(w)$

Note 1: For modeling purposes we have used a common CARA value of 0.0000055 that occurs for decision makers with low wealth and low-risk aversion and decision makers with high wealth and high-risk aversion.

Note 2: For some of the models in this chapter we have used a CARA value of 0.00001 as a representative value for the risk preferences of an average private logger.

Table 9.2 Objective function values and solutions (hectares) for various combinations of chance constraints and allowable deviations

	Likelihood of satisfying the constraint - 50 % (Group A)			Likelihood of satisfying the constraint - 73 % (Group B)			Likelihood of satisfying the constraint - 90 % (Group C)		
	Allowable deviation								
	50%	25%	10%	50%	25%	10%	50%	25%	10%
	Objective function values								
Certainty equivalent	329201	314909	307125	300540	285775	278332	235716	226284	221686
IAC1 – no cut	187	230	249	250	250	250	228	240	250
IAC2 – no cut	0	0	0	137	158	167	448	441	434
IAC1 - cut period 1	0	0	0	0	0	0	0	0	0
IAC2 - cut period 1	563	467	424	426	357	325	222	190	174
IAC1 - cut period 2	63	20	1	0	0	0	22	10	0
IAC2 - cut period 2	187	283	326	187	235	258	80	119	141

Analysis based on scenario 3 input data
CARA=0.00001 for the results presented in Table 9.2. – ending inventory = 220,000
Likelihood of 50 % - K = 0.0
Likelihood of 73 % - K=0.6
Likelihood of 90 % - K=1.3
Group A, B, and C refer to A, B, and C in Figure 9.2.

Table 9.3 Objective function values and solutions for constant absolute risk aversion coefficient sensitivity analysis

	Without chance constraints CARA values						With chance constraints (K=0.6) CARA values					
	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
Certainty equivalent	439863	430241	371138	314909	169992	120732	371201	364623	324216	285775	169850	120680
	Harvest solutions (hectares)											
IAC1 – no cut	230	230	230	230	250	250	250	250	250	250	250	250
IAC2 – no cut	0	0	0	0	274	574	158	158	158	158	282	577
IAC1 - cut period 1	0	0	0	0	0	0	0	0	0	0	0	0
IAC2 - cut period 1	467	467	467	467	230	85	357	357	357	357	231	85
IAC1 - cut period 2	20	20	20	20	0	0	0	0	0	0	0	0
IAC2 - cut period 2	283	283	283	283	246	91	235	235	235	235	237	88

CARA – Constant absolute risk aversion

Model assumptions:

1. Scenario 3 used as input data
2. Chance constraint set at 0.6 (i.e. 73 % likelihood of satisfying the constraint)
3. All harvest constraints (i.e. ending inventory and flow constraints) applied.
4. Allowable deviations in periodic flow = 25 %
5. Ending inventory = 220,000 cu m.

Table 9.4 Certainty equivalent values and optimal portfolios without sustained yield constraints

Scenario 1	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
	Risk aversion coefficient					
Certainty equivalent	501250	501250	501250	501250	501250	501250
	Harvest solutions (hectares)					
IAC1 – no cut	0	0	0	0	0	0
IAC2 – no cut	0	0	0	0	0	0
IAC1 - cut period 1	250	250	250	250	250	250
IAC2 - cut period 1	750	750	750	750	750	750
IAC1 - cut period 2	0	0	0	0	0	0
IAC2 - cut period 2	0	0	0	0	0	0

Scenario 2	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
	Risk aversion coefficient					
Certainty equivalent	642250	640639	630745	621332	589955	558579
	Harvest solutions (hectares)					
IAC1 – no cut	0	0	0	0	0	0
IAC2 – no cut	0	0	0	0	0	0
IAC1 - cut period 1	250	250	250	250	250	250
IAC2 - cut period 1	750	750	750	750	750	750
IAC1 - cut period 2	0	0	0	0	0	0
IAC2 - cut period 2	0	0	0	0	0	0

Scenario 3	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
	Risk aversion coefficient					
Certainty equivalent	648750	622045	458428	346258	176049	131389
	Harvest solutions (hectares)					
IAC1 – no cut	0	0	0	0	0	250
IAC2 - no cut	0	0	0	0	0	0
IAC1 - cut period 1	250	250	179	0	0	0
IAC2 - cut period 1	750	750	750	750	112	0
IAC1 - cut period 2	0	0	71	250	250	0
IAC2 - cut period 2	0	0	0	0	638	750

Scenario 4	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
	Risk aversion coefficient					
Certainty equivalent	645500	644975	641752	638685	628462	618240
	Harvest solutions (hectares)					
IAC1 - no cut	0	0	0	0	0	0
IAC2 - no cut	0	0	0	0	0	0
IAC1 – cut period 1	250	250	250	250	250	250
IAC2 - cut period 1	750	750	750	750	750	750
IAC1 - cut period 2	0	0	0	0	0	0
IAC2 - cut period 2	0	0	0	0	0	0

Table 9.5 Certainty equivalent values and optimal portfolios with sustained yield[1] constraints

	Risk aversion coefficient					
Scenario 1	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
Certainty equivalent	281400	281400	281400	281400	281400	281400
	Harvest solutions (hectares)					
IAC1 - no cut	250	250	250	250	250	250
IAC2 - no cut	191	191	191	191	191	191
IAC1 - cut period 1	0	0	0	0	0	0
IAC2 - cut period 1	336	336	336	336	336	336
IAC1 - cut period 2	0	0	0	0	0	0
IAC2 - cut period 2	224	224	224	224	224	224

	Risk aversion coefficient					
Scenario 2	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
Certainty equivalent	419164	418658	415549	412592	402734	392876
	Harvest solutions (hectares)					
IAC1 - no cut	250	250	250	250	250	250
IAC2 - no cut	28	28	28	28	28	28
IAC1 - cut period 1	0	0	0	0	0	0
IAC2 - cut period 1	437	437	437	437	437	437
IAC1 - cut period 2	0	0	0	0	0	0
IAC2 - cut period 2	285	285	285	285	285	285

	Risk aversion coefficient					
Scenario 3	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
Certainty equivalent	371201	364623	324216	285775	169850	120680
	Harvest solutions (hectares)					
IAC1 - no cut	250	250	250	250	250	250
IAC2 - no cut	158	158	158	158	282	577
IAC1 - cut period 1	0	0	0	0	0	0
IAC2 - cut period 1	357	357	357	357	231	85
IAC1 - cut period 2	0	0	0	0	0	0
IAC2 - cut period 2	235	235	235	235	237	88

	Risk aversion coefficient					
Scenario 4	0	7.70E-07	5.50E-06	1.00E-05	2.50E-05	4.00E-05
Certainty equivalent	358431	357525	351963	346671	329031	311392
	Harvest solutions (hectares)					
IAC1 - no cut	250	250	250	250	250	250
IAC2 - no cut	157	157	157	157	157	157
IAC1 - cut period 1	0	0	0	0	0	0
IAC2 - cut period 1	330	330	330	330	330	330
IAC1 - cut period 2	0	0	0	0	0	0
IAC2 - cut period 2	263	263	263	263	263	263

[1] Sustained yield parameter assumptions: Ending inventory = 220,000 cu.m., allowable flow deviation = 25 %, K=0.6,

Table 9.6. Effects of climate change productivity effects on ability to satisfy sustained yield constraints

Variable	Scenario 1	Scenario 2
Certainty equivalent	$270,313	$415,843
Marginal stand value		
IAC1	$ 387 per ha	$ 518 per ha
IAC2	$ 441 per ha	$ 606 per ha
	Harvest solutions (hectares)	
IAC1 - no cut	250	249
IAC2 - no cut	197	0
IAC1 - cut period 1	0	0
IAC2 - cut period 1	307	424
IAC1 - cut period 2	0	1
IAC2 - cut period 2	246	326

Parameter assumptions: CARA=0.0, K=0.0, allowable deviations in flow=10%, ending inventory =220,000 cu.m.
Scenario 1 – Predictions based on climate normals.
Scenario 2 – Predictions based on climate futures.

Table 9.7 Certainty equivalent value and harvest solutions using a risk model with parameters based on current forest management standards

Variable	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Certainty equivalent	$270,313	$373,561	$221,686	$137,527
Marginal stand value				
IAC1	$ 387 per ha	$ 506 per ha	$ 310 per ha	$ 118 per ha
IAC2	$ 441 per ha	$ 568 per ha	$ 365 per ha	$ 132 per ha
		Harvest solutions (hectares)		
IAC1 - no cut	250	250	250	250
IAC2 - no cut	197	96	434	750
IAC1 - cut period 1	0	0	0	0
IAC2 - cut period 1	307	365	174	0
IAC1 - cut period 2	0	0	0	0
IAC2 - cut period 2	246	289	141	0

Parameter assumptions: Allowable deviations in flow=10%, K=1.3, CARA=1.0E-5, ending inventory = 220,000 cu.m.

CHAPTER TEN

DISCRETE STOCHASTIC PROGRAMMING (RECOURSE) MODEL

Introduction

The EV / Chance Constraint risk model developed in the previous chapter shows how the combined effects of productivity changes, forest management constraints, uncertainty in objective function and constraint coefficients and risk preferences influence returns from timber harvesting and choices of forest landowners and/or managers. The model presented in the previous chapter, however, is a static, one-period model. It does not permit taking account of the sequential nature of decision-making in forest management contexts and it does not allow the decision maker to modify his/her decisions and choices over time as new knowledge is gained and uncertainties that occur early in the planning horizon become resolved.

Forest harvest scheduling problems involve making choices at the beginning of the planning horizon about harvests in various planning periods over the planning horizon. In our case, the decision maker faces uncertainty about potential benefits in periods 1, 2 and at the end of the planning horizon. However, it is possible that the uncertainty that exists relative to period 1 yields and benefits may be resolved once the decision maker reaches period two. Assuming that the decision maker knows what choice she/he would make in period 2 (given the realization of a particular state of nature in period 1), the decision maker will develop alternative harvest plans for period 2 and subsequent periods based on whatever state of nature actually occurs in period 1. Modeling this type of multi-period sequential decision-making problem requires a different approach. It requires an approach that accounts for the fact that decision makers will obtain new knowledge as time progresses and will consequently adjust their decisions as uncertainties are resolved (Hardaker et al. 2004).

The type of mathematical programming model used for optimization problems where learning occurs and uncertainty is sequentially resolved over time is discrete stochastic programming (DSP) (or stochastic programming with recourse) (Hardaker et al. 2004, McCarl and Spreen 1997, Apland and Hauer 1993). Discrete stochastic programming models permit periodic adaptation based on new information. For example, a logger might be uncertain about what the yield of a stand will be in the first cutting period. However, once she/he harvests the stand, she/he will know with certainty what the yield was. This new knowledge, in turn, may affect his/her choices relative to future harvest levels. New information results in new choices over time. Moreover, a purely rational decision-maker will recognize that levels of uncertainty for some variables that may be apparent today will be eliminated in the future and he/she will incorporate this knowledge into his/her current choices.

An aspect of the DSP model presented in this chapter is that the model assumes risk neutrality. The objective function maximizes returns without penalties for risk (i.e. there is no risk premium in the objective function). There is no adjustment for differences in relative degree of risk preferences between different decision makers. It is possible to include risk preferences into a DSP model. Apland and Hauer (1993) describe a formulation that incorporates risk premiums. The approach outlined in Apland and Hauer (1993) requires the determination of the covariance matrix between the objective function coefficient values in the DSP objective function. However, the two reasons for estimating the DSP model in this chapter are to illustrate the kind of models used for optimization problems with sequential risk, and show how sequential adaptation can influence climate impacts and harvest choices. The incorporation of risk premiums into the DSP objective function was considered to be unnecessary relative to the above objectives. In effect, the analysis presented in this chapter is representative of a public forestland management situation where the government assumes all risk associated with timber harvesting. Risk preferences, therefore, are close to neutral and the value of the risk aversion coefficient is negligible.

DSP models provide a structure for sequential multi-period decision-making under uncertainty (Apland and Hauer 1993). Dantzig (1963) is credited with the early development of DSP. Rae (1971 a and b) introduced DSP to agriculture decision-making problems. Hardaker et al. (2004) and McCarl and Spreen (1997) discuss techniques for conducting DSP. Hoganson and Rose (1987), Gassman (1989), Boychuck and Martell (1996)[34] and Hekkinen (2003) discuss various forestry applications. To our knowledge there are no applications of DSP models to climate change and forestry related uncertainty problems.

One of the reasons why a recourse model may be of interest for climate change and forest management problems is that these types of models explicitly incorporate sequential adaptation. As noted in the previous chapter, adaptation is an important part of climate impact studies. The previous chapter looked at adaptation with a static model. This chapter considers adaptation using a dynamic model. Use of a dynamic model permits us to consider the possibility that decision makers will adjust their choices as uncertainty becomes resolved. As the uncertainty arising from climate change and yield risk becomes resolved over time, decision makers will adapt by adjusting their harvest choices. Thus, for dynamic models, adaptation is explicitly incorporated into the model.

Another type of adaptation considered in this chapter is risk prevention. One potential adaptation strategy to climate change and uncertainty could be to reduce the probabilities of low yield states and increase the probability of high yield states. Freeman (1999) refers to this type of adaptation measure as risk prevention[35].

A final point to note about the analysis in this chapter is that it focuses on looking at how economic returns and harvest solutions vary with different assumptions about recourse, uncertainty and sustained yield parameters. The models presented in this chapter are based on scenarios 2 and/or 3 input data. Each of these scenarios has climate effects already incorporated in them. Thus, we are not considering the impact of climate change by comparing returns and feasible solutions with and without climate change in this chapter. Analysis of the impact of climate change was presented in Chapter 9. The issue of primary interest in this chapter is: How important is recourse in terms of returns from timber harvesting and in terms of optimal harvest choices?

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[34] This paper introduces the novel concept of incorporating a penalty into the objective function in order to avoid situations where solutions are dominated by low probability outcomes.

[35] In addition to risk prevention Freeman (1999) also discusses risk reduction. Risk reduction activities are actions taken to reduce negative consequences when undesirable states of nature occur

Model structure

The DSP model developed for this study is a three-stage model with two states of nature or outcomes in each stage (Figure 10.1). The two states of nature are: 1. High harvest yields and returns, and 2. Low harvest yields and returns. The decision tree in Figure 10.1 provides a visual representation of the DSP model developed for this study.

There are a number of terms in Figure 10.1 that require definition. The term e_{ij_tta} (in Figure 10.1) represents the value of an uncertain model parameter given the occurrence of state of nature "i" (i = H for high yield state and L for low yield state), in stage "t" (t = stage 1,2, or 3), for initial age class "a" (where a = Y is for the IAC 40 year old stands and O is for the IAC 80 year old stands). The term "j_t" identifies the decision alternative in stage "t". For example if a high yield state actually occurs in stage one, then the decision alternatives for stage two will be 1 and if a low yield state occurs in stage one then the value of the decision variable in stage two is 2.

Combinations of values for "i" over the three stages define an event history (Apland and Hauer 1993). Figure 10.1 shows that for a three-stage model with two states of nature at each stage there are 8 possible event histories for the IAC1 stand and an additional 8 possible event histories for the IAC2 stand. For example, the first event history that could occur is represented by the following sequence $\{e_{H11Y}, e_{H12Y}, e_{H13Y}\}$. Using the value of "i" for each stage – this event history is identified as HHH (high yield state in stage one, high yield state in stage two, and high yield state in stage three). Other event histories for IAC1 (i.e. a = Y) are HHL, HLH, HLL, LHH, LHL, LLH, and LLL. A similar set of event histories exist for the IAC2 stand (i.e. a = O). The event histories for stand types Y and O are linked in that solutions assume that returns are based on the same event history applying to both stand types. Each event history has a probability of occurring. A separate set of solutions for the choice variables is determined for each possible event history.

The term e_{ij_tta} in Figure 10.1 represents two types of model parameters that are uncertain for the purposes of this study. The first uncertain model parameter (C_{ij_tta}) is the uncertain objective function parameter (see equation 10.2 later in this section). This is defined as net benefit per hectare. The second parameter ($Y_{ij_t ta}$) represents the uncertain constraint parameter (see equations 10.4, 10.5, and 10.7 later in this section). This is defined as stand yield (in cubic meters) per hectare.

Figure 10.1 shows the choice variables ($X_{j_t ta}$) (note: this is the area of stand type Y and O harvested in stage "t" under decision alternative "j"). Permissible values for the choice variables depend on assumptions about the underlying information structure. This study assumes that the decision maker has perfect knowledge of the past but not the present. This implies that the decision maker's choices across states for any particular stage are fixed. However, the decision maker does have the flexibility to adjust choices in response to new knowledge gained in the previous stage. Perfect knowledge of the past is a specific type of information structure. There are, however, other types of information structures. For example, for some types of problems the decision maker might have perfect knowledge of the past and present (Apland and Hauer 1993). However, for the problem considered in this Chapter we assume that the decision maker will not know the exact yields from stand types 1 and 2 until she/he has actually harvested the stands.

As noted, there are two possible values for net benefits and yield for each stage: high and low. At the start of the planning horizon the decision maker is uncertain about yield and net benefits of harvesting in future periods because these values are functions of uncertain future climate variables and uncertain yield parameters. In stage one the decision maker faces two possible states of nature – high benefits and yields and low benefits and yields. Irrespective of this uncertainty, the decision maker makes a choice about the area of each stand type that will be harvested. The DSP model provides a solution for the amount of area of IAC1 (Young) and IAC2 (Old) to harvest in stage 1. Since she/he is uncertain what the stand yields will be in stage 1 – the optimal solution for area harvested for both states is fixed (e.g. see X_{11Y} in Figure 10.1). Following harvest in stage one; the decision maker will have determined what state of nature actually occurred. The solution for the stage two harvest will then be based on the values of the objective function and yield parameters that actually occurred in stage one. If state "H" in stage 1 occurs, then the harvest choice will be X_{12Y} (Figure 10.1). If state "L" occurs in stage 1, then the harvest choice is X_{22Y} (Figure 10.1). Here again, the decision maker's choices in stage 2 are not affected by what state of nature occurs in stage 2. The choices in stage 2 are fixed for each state of nature.

Finally, Figure 10.1 shows a set of arrows that lead from the four stage one boxes to a central box. This central box represents the total area of IAC1 and IAC2 forest harvested in stage one. Areas harvested in stage one grow for 45 years before the end of the planning horizon is reached. These areas may, therefore, contribute to ending inventory. The actual ending inventory yields that will be realized from stage one harvest will depend on future event histories. The ending inventory yields from stage one harvest are, therefore, also uncertain. However, a dilemma for this problem structure is that it is necessary that uncertainty in stage one parameters be resolved at the end of stage one (i.e. in order to determine a solution for stage one harvest area, uncertainty in yield parameters associated with stage one harvest – including ending inventory yield - must be resolved at the beginning of stage two). This means that it is necessary to use the expected value of ending inventory yield as the measure of ending inventory yield associated with hectares harvested in stage one. The expected value of ending inventory yield from areas harvested in stage one is 225 cubic meters. This is based on estimating ending inventory yields from stage one harvest under each possible event history and taking the average value as representative of an expected value.

Another random variable where it is necessary to include a single expected value is soil expectation value. Soil expectation value is the present value of bare land at the end of the planning horizon. This value is included in the net benefit parameter (i.e. the objective function coefficient). Soil expectation value is a function of future climate and yields. Because future climate and yields are uncertain, this variable is also random. It is not, however, possible to incorporate uncertainties in variables that extend past the planning horizon into the DSP problem formulation. Therefore, we were required to consider a single expected value for soil expectation value. The expected present value of soil expectation value is $ 13.6 per cubic meter. This is the expected value from the soil expectation value distributions estimated in Chapter 7.

In the introduction it was suggested that one of the reasons for wanting to use a DSP model is that it is a dynamic model. The previous two paragraphs suggest that the model developed in this study is only partly dynamic. Some values that are uncertain are incorporated as deterministic expected values and therefore the uncertainty in these variables does not influence the solutions. However, yields of ending inventory from stage one harvests are relatively low. Also, the present value of soil expectation value is low relative to overall net benefit values. Thus, we do not feel that including a deterministic value for these random variables will introduce large biases in the estimations.

The next step in this section is to provide the formulation for the DSP model. The DSP model presented in this Chapter is linear. The various versions and runs of the model presented in this chapter were solved using CPLEX within GAMS (Brooke et al. 1998)[36].

The formulation of the DSP model is as follows:

$$\underset{\{X_{ijk}\}}{Max} \quad Z = \sum_{S=1}^{16} p_s Y_S \qquad [10.1]$$

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[36] The GAMS program code for the basic model used in this chapter is available on request.

$$\begin{aligned}
&Where:\\
&C_{H11Y}X_{11Y}+C_{H12Y}X_{12Y}+C_{H13Y}X_{13Y}=Y_1\\
&C_{H11Y}X_{11Y}+C_{H12Y}X_{12Y}+C_{L13Y}X_{13Y}=Y_2\\
&C_{H11Y}X_{11Y}+C_{L12Y}X_{12Y}+C_{H23Y}X_{23Y}=Y_3\\
&C_{H11Y}X_{11Y}+C_{L12Y}X_{12Y}+C_{L23Y}X_{23Y}=Y_4\\
&C_{L11Y}X_{11Y}+C_{H22Y}X_{22Y}+C_{H33Y}X_{33Y}=Y_5\\
&C_{L11Y}X_{11Y}+C_{H22Y}X_{22Y}+C_{L33Y}X_{33Y}=Y_6\\
&C_{L11Y}X_{11Y}+C_{L22Y}X_{22Y}+C_{H43Y}X_{43Y}=Y_7\\
&C_{L11Y}X_{11Y}+C_{L22Y}X_{22Y}+C_{L43Y}X_{43Y}=Y_8\\
&C_{H110}X_{110}+C_{H120}X_{120}+C_{H130}X_{130}=Y_9\\
&C_{H110}X_{110}+C_{H120}X_{120}+C_{L130}X_{130}=Y_{10}\\
&C_{H110}X_{110}+C_{L120}X_{120}+C_{H230}X_{230}=Y_{11}\\
&C_{H110}X_{110}+C_{L120}X_{120}+C_{L230}X_{230}=Y_{12}\\
&C_{L110}X_{110}+C_{H220}X_{220}+C_{H330}X_{330}=Y_{13}\\
&C_{L110}X_{110}+C_{H220}X_{220}+C_{L330}X_{330}=Y_{14}\\
&C_{L110}X_{110}+C_{L220}X_{220}+C_{H430}X_{430}=Y_{15}\\
&C_{L110}X_{110}+C_{L220}X_{220}+C_{L430}X_{430}=Y_{16}
\end{aligned} \qquad [10.2]$$

Subject to:

Area constraints (8 equations):

$$\begin{aligned}
&X_{11Y}+X_{12Y}+X_{13Y}\le 250\\
&X_{11Y}+X_{12Y}+X_{23Y}\le 250\\
&X_{11Y}+X_{22Y}+X_{33Y}\le 250\\
&X_{11Y}+X_{22Y}+X_{43Y}\le 250\\
&X_{11O}+X_{12O}+X_{13O}\le 750\\
&X_{11O}+X_{12O}+X_{23O}\le 750\\
&X_{11O}+X_{22O}+X_{33O}\le 750\\
&X_{11O}+X_{22O}+X_{43O}\le 750
\end{aligned} \qquad [10.3]$$

Upper flow constraints (4 equations):

$$\begin{aligned}
&(1-\alpha)(Y_{H11Y}X_{11Y}+Y_{H11O}X_{11O})-(Y_{H12Y}X_{12Y}+Y_{H12O}X_{12O})\le 0\\
&(1-\alpha)(Y_{H11Y}X_{11Y}+Y_{H11O}X_{11O})-(Y_{L12Y}X_{12Y}+Y_{L12O}X_{12O})\le 0\\
&(1-\alpha)(Y_{L11Y}X_{11Y}+Y_{L11O}X_{11O})-(Y_{H22Y}X_{22Y}+Y_{H22O}X_{22O})\le 0\\
&(1-\alpha)(Y_{L11Y}X_{11Y}+Y_{L11O}X_{11O})-(Y_{L22Y}X_{22Y}+Y_{L22O}X_{22O})\le 0
\end{aligned} \qquad [10.4]$$

Lower flow constraints (4 equations):
Lower flow constraints (4 equations):

$$(1+\beta)(Y_{H11Y}X_{11Y}+Y_{H11O}X_{11O})-(Y_{H12Y}X_{12Y}+Y_{H12O}X_{12O})\geq 0$$
$$(1+\beta)(Y_{H11Y}X_{11Y}+Y_{H11O}X_{11O})-(Y_{L12Y}X_{12Y}+Y_{L12O}X_{12O})\geq 0 \qquad [10.5]$$
$$(1+\beta)(Y_{L11Y}X_{11Y}+Y_{L11O}X_{11O})-(Y_{H22Y}X_{22Y}+Y_{H22O}X_{22O})\geq 0$$
$$(1+\beta)(Y_{L11Y}X_{11Y}+Y_{L11O}X_{11O})-(Y_{L22Y}X_{22Y}+Y_{L22O}X_{22O})\geq 0$$

Area cut in period 1 that contributes to ending inventory

$$X_{11Y}+X_{11O}=Area \qquad [10.6]$$

Ending inventory constraints (8 equations):

$$\text{Tarend}-(Area*EIVOL)-(Y_{H13Y}X_{13Y})-(Y_{H13O}X_{13O})\leq 0$$
$$\text{Tarend}-(Area*EIVOL)-(Y_{L13Y}X_{13Y})-(Y_{L13O}X_{13O})\leq 0$$
$$\text{Tarend}-(Area*EIVOL)-(Y_{H23Y}X_{23Y})-(Y_{H23O}X_{23O})\leq 0$$
$$\text{Tarend}-(Area*EIVOL)-(Y_{L23Y}X_{23Y})-(Y_{L23O}X_{23O})\leq 0 \qquad [10.7]$$
$$\text{Tarend}-(Area*EIVOL)-(Y_{H33Y}X_{33Y})-(Y_{H33O}X_{33O})\leq 0$$
$$\text{Tarend}-(Area*EIVOL)-(Y_{L33Y}X_{33Y})-(Y_{L33O}X_{33O})\leq 0$$
$$\text{Tarend}-(Area*EIVOL)-(Y_{H43Y}X_{43Y})-(Y_{H43O}X_{43O})\leq 0$$
$$\text{Tarend}-(Area*EIVOL)-(Y_{L43Y}X_{43Y})-(Y_{L43O}X_{43O})\leq 0$$

Non-negativity constraints (i.e. $\forall X_{jta}, X_{jta}\geq 0$)

Where:

Z is the expected value of net income,

p_s is probability of a particular event history,

Y_s is net income associated with a particular event history (Note Y in equation 10.2 represents income – Y in equations 10.4, 10.5, and 10.7 represent yield),

C_{H11Y} is net present value of benefits for state "high yield", decision 1, stage 1, IAC 40 years,

X_{11Y} is the area of the IAC 40 area harvested in stage one,

Y_{H11Y} is stand yield – state "high yield, decision 1, stage 1, IAC 40,

α - maximum percent increase allowed in period 2 harvest,

β - maximum percent decrease allowed in period 2 harvest,

Tarend = target ending inventory,

EIVOL - expected value of volume of ending inventory from period one harvest - (225 cu. m. per ha).

The input data for the model in this chapter are based on the scenario two and/or three estimates of yield and net benefits described in chapter 7 (note: scenario two incorporates climate variables into yield and includes climate uncertainty only while scenario three includes climate change productivity effects, climate uncertainty and uncertainty in yields). A fundamental difference is that the random variables estimated in Chapter 7 are continuous random variables. The coefficients used for the model in this chapter are discrete random variables. Thus, a procedure was required to convert the estimates of continuous random variables for scenarios 2 and 3 provided in Chapter 7 into a discrete form (see equations 7.8 to 7.13 for a definition of how benefits are estimated and equations 7.14 to 7.20 for a definition of specific yield variables). The correlation between the variables reported in Chapter 7 and the model developed here is as follows. Stage one of the model developed in this chapter corresponds to the harvest in planning period one in the data presented in Chapter 7. Similarly, stage two of the model presented in this chapter corresponds to the harvest in planning period two in the data presented in Chapter 7. Finally, stage three of the model presented in this chapter corresponds to the no cut prescription for the data presented in Chapter 7. Chapter 7 provides estimates of expected values and variances for harvest yields and net benefits for each IAC for each planning period. The model for this chapter requires an estimate of high and low value for harvest yield and net benefits for each stage of the recourse model. The discrete data for this chapter were derived from the data in Chapter 7 by adding (for high yield outcomes) or subtracting (for low yield outcomes) one standard deviation to the expected value for each variable. The resulting discrete outcome data for yield and net benefits for each stand, for each outcome, for each decision alternative, for each stage, and for scenarios two and three are provided in Table 10.1, Figure 10.2, and Figure 10.3.

Analysis

There are four questions that will be addressed using the discrete stochastic programming model developed in this chapter. They are:

1. What is the effect of recourse on returns and harvest choices?

2. How sensitive are economic returns and harvest choices to differences in levels of uncertainty of outcomes when using a recourse model approach?
3. How sensitive are economic returns and harvest choice to variations in sustained yield constraints?
4. What effect does increasing the probabilities of high yield states have on returns and on harvest choices with recourse?

Effects of recourse on returns and harvest choices

This section considers the effects of recourse on returns and harvest choices - both with and without sustained yield constraints imposed. The first model in this section estimates returns and provides solutions using a model with recourse but without sustained yield constraints. The results of this model are reported in the text only. The model predicts that total returns without sustained yield constraints are approximately \$648,750 (compared to \$398,960 when sustained yield is included – see Figure 10.4) and that the entire 1000-hectare forest is cut in stage 1. However, recourse is largely irrelevant in this model because states of nature that occur in stages two and three have no influence on the decision makers choices. This is not a particularly surprising result but it does reinforce that the use of a recourse type model in a forestry and climate change assessment context would appear to be best suited to situations or cases where decisions are long term and sequential, and/or where there is an incentive to delay taking action until later stages.

The remainder of this section considers the effects of recourse in a climate change context with sustained yield constraints imposed. The assumptions for the DSP formulation with sustained yield constraints (see Equations 10.1 – 10.7) are as follows:

- Scenario 3 input data (in discrete form) is used (see Table 10.1),
- There is a 50 % likelihood of each discrete outcome (high yield state vs. low yield state) for each stage (resulting in an equal probability of 0.125 for each event history – see Table 10.2),
- The decision maker is risk neutral (i.e. there is no risk premium in the objective function),

- Flow constraints are imposed - $\alpha = \beta = 0.25$ (similar to the EV model in Chapter 9), and
- Ending inventory constraints are imposed - the target-ending inventory is 220,000 cubic meters (similar to the EV model in Chapter 9).

In order to assess what effect recourse has on economic returns and harvest schedules we first estimated the harvest patterns and returns that would occur in the absence of recourse. A restricted version of the DSP model in equations 10.1 – 10.7 was estimated[37]. The model restricts choices so that harvest levels are fixed at each stage over all states that occurred in the previous stage. Thus, the decision maker cannot adjust harvest levels as uncertainties about states are resolved. The results for the model with no recourse are provided in Figure 10.4. The expected value of income for the restricted model is approximately \$ 146,500. The solution requires that all 250 hectares of IAC1, and all 750 hectares of IAC2 stay uncut over the planning period. Thus, without recourse, solutions that permit harvesting to occur in stage one or two are not feasible[38]. In order to evaluate what factors caused the model to provide a no harvest solution the model without recourse was rerun with different values for the constraint parameters. First, we lowered the ending inventory requirement to 100,000 cubic meters. The no harvest solution continued to occur. Then we increased the allowable deviation in flow constraints from 25 % to 50 %. A solution that permitted harvesting in stage one and two resulted. Thus, the factor driving the no harvest solution in the without recourse model seems to be the flow constraint.

The results of the DSP formulation when recourse is permitted are provided in Figure 10.5. The figure shows that the expected value of returns with the recourse model is \$ 398,960. The optimal solution is for 157 hectares of IAC1 and 310 hectares of IAC2 area to be harvested in stage one. If the high yield state of nature occurs in stage 1 then the optimal solution is for 327 hectares of IAC2 to be harvested in stage two. If, however, the low yield state occurs in stage one then the decision

[37] The formulation for the restricted model is not repeated here but can be seen by looking at the GAMS program code in Appendix 2.

[38] Armstrong (2004) finds that a forest manager's ability to set a harvest that can be sustained with certainty withinin a stochastic fire regime is low to non-existent.

maker adjusts his/her choices and reduces the amount of IAC2 harvested in stage two to 221 hectares. There is no harvest of IAC1 in stage two. Ninety-three hectares of IAC1 are left uncut for all states. In the case of the IAC2 stand, 113 hectares remain uncut if the high yield state occurs in stage one and 220 hectares remain uncut if the low yield state occurs in stage one.

The remainder of this section explores these results in more detail by comparing the effects of recourse on economic returns, and considering the implications of recourse on harvest schedules. A comparison of Figure 10.4 and 10.5 shows that allowing for recourse (sequential adaptation) increases the expected value of income by about 270 % (i.e. net benefits increase from \$146,500 to \$ 398,960). This result occurs mainly because without the flexibility of recourse, it is not possible to satisfy sustained yield constraints and still harvest – given uncertainty in outcomes from climate and yield. Recourse means that the decision maker has more information upon which to base his/her choices. In single period models, and/or in models where recourse is not permitted, this type of learning does not occur. Thus, choices with a recourse model will be based on relatively more complete information[39]. As shown here, this flexibility can have significant implications for harvest solutions.

What are the implications of recourse relative to harvesting in the short run and long run? For this study, the short run pertains to the harvest solution that is provided in the first stage. The long run solution is the solution provided in stage 2. The short run solution is particularly important because these solutions are permanent, irreversible and they provide the basis for immediate actions. Long-run solutions are relatively less permanent in the sense that new solutions can be obtained over time as new information becomes available and model solutions are recalculated. Nevertheless, knowledge of the long run may be important to investors who are interested in returns over the life time of the investment and/or who may require information on future supply in order to construct the right type and scale of manufacturing facility.

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[39] This could be viewed as being analogous to quasi-option value.

One question of interest for this section is: Does recourse have any direct implications for short run solutions (i.e. the optimal harvest schedule in stage one)? As shown in Figures 10.4 and 10.5 the difference between the short run solutions with and without recourse are substantial. With recourse, 157 hectares of the IAC1 stand are harvested and 310 hectares of the IAC2 stand are harvested in stage one. Without recourse, there is no harvest in the short term. Thus, recourse is necessary for feasibility of solutions that permit harvesting under sustained yield and when there is uncertainty in yields and returns - even with climate change productivity effects included (as they are in scenario three).

Another question of interest is: What does recourse mean with respect to long run solutions? This question is also of interest for this study because under an information structure of perfect information of the past but not the present, it is only the harvest in stage two that is flexible. The harvest in stage one is fixed, irrespective of what state of nature occurs. Figure 10.5 shows that the state of nature that occurs in stage one has significant implications for how much IAC2 is harvested in stage 2 (i.e. the long run). The area of the IAC2 stand harvested in the long run is 48 % higher in stage two if the high yield outcome occurs in stage one. Also, the area of IAC2 left uncut is significantly lower if the high yield outcome occurs in stage one. Thus, the knowledge gained in stage two regarding what state of nature actually occurred in stage one has significant implications for harvest rates in stage two.

There are two main reasons why the results presented in this section are interesting. The first reason is to reinforce that understanding how decision makers will adapt is an important aspect of climate change impact analysis and climate change policy. Climate change will, to some degree, result in increased uncertainty. Moreover, there is considerable existing uncertainty in future stand yields, irrespective of climate effects. Having the flexibility to adjust choices over time as uncertainty becomes resolved has important implications for forestry decision-making in uncertain environments. In fact, as shown in the results in this section, without the flexibility of recourse, feasible solutions to the management problem given sustained yield requirements preclude harvesting.

The second reason that the results presented in this section are interesting is in terms of how they might be applied by a decision maker. All the solutions presented up to this point in this study have been in terms of area harvested under each prescription. A tenure holder is probably also interested in the question: What does climate change and uncertainty mean with respect to the potential range of volumes that can be harvested in the short term and long term? For example, as previously noted, the decision maker might be interested in purchasing a small portable sawmill and the decision about what size to purchase and the type of technology to purchase might depend on the variability of harvest volumes in the short and long term. In the case of the problem presented in this section, the tenure holder harvests 157 hectares of IAC1 and 310 hectares of IAC2 in stage one. Taking the low and high yields for each age class (from Figure 10.3) and multiplying by the hectares harvested (Figure 10.5) suggests that the decision maker can expect to harvest anywhere from 107,794 cubic meters ((157*112)+(310*291)) to 259,980 cubic meters ((157*416)+(310*631)) in stage one. In stage two the decision maker harvests 221 hectares of IAC2 in stage two if a low yield outcome occurs in stage one. If a low yield outcome also occurs in stage two the yield for the IAC2 stand is 366 cubic meters. Thus if the event history is LL then the maximum the decision maker can harvest in stage two is 80,886 cubic meters. If a high yield outcome occurs in stage one, then the decision maker harvests 327 hectares of IAC2 land in stage two. If the high yield state of nature repeats in stage two then the maximum amount the decision maker harvests in stage two is 232,824 cubic meters. Therefore, at the start of the planning period the decision maker faces the possibility that the amount available for harvesting in the long term (i.e. stage two) will range from almost 81,000 cubic meters to 233,000 cubic meters. However, the decision maker also knows that once he/she reaches the end of the first stage, his/her level of uncertainty about stage two harvest rates will decrease. He/she will know that if a low yield state occurs in stage one, then the amount available for harvesting in stage two will range from 81,000 cubic meters (366 cu. m. per ha*221 ha) to 106,522 cubic meters (482 cu. m per ha *221 ha). If on the other hand the high yield state occurs in stage one then the range of harvest volume available in stage two will be from 195,219 cubic meters (597 cu. m. per ha*327 ha) to 233,000 cubic meters (712 cu m. per ha * 327 ha).

The previous paragraph illustrates how a recourse type model can be used to identify the range of volumes a decision maker could potentially harvest in various stages and at the start of the planning horizon and how the range of predicted volumes changes as one moves from the short term to the long term in the planning horizon.

Sensitivity to differences in levels of uncertainty

The analysis in this section considers the effects of different levels of uncertainty on economic returns and harvest choice using a recourse model formulation. In the analysis in Chapter 9 the effects of different variance assumptions were evaluated using the EV – Chance Constraint model formulation. The approach was to estimate the model using scenarios one to four input data and compare the results. A similar approach is adopted for the analysis presented in this section except that only scenarios 2 and 3 input data are used. As noted in Chapter 7, the scenario 2 input data incorporates climate change as a driver of change in projected harvest and ending inventory yields and it also includes uncertainty in climate variables. Scenario 3, includes productivity effects of climate change, uncertainty effects of climate change and also uncertainty in yield parameters. Thus, the variances associated with scenario 3 input data are considerably higher than the variances associated with the scenario 2 input data.

The basic model assumptions for the models presented in this section are similar to the assumptions defining the models in the previous section (i.e. flow deviations = 25 %, ending inventory requirement – 220,000 cubic meters). The expected returns and harvest solutions using the scenario 3 input data were presented and discussed in the previous section (see Figure 10.5). The expected returns and harvest solutions using the scenario 2 input data are shown in Figure 10.6. The expected return using the scenario 2 input data is $427,437. The expected return for scenario 3 is 7 % lower than expected return for scenario 2. This is not surprising or interesting in and of itself. Nor is it inconsistent with the results presented in Chapter 9. So this result reinforces the expected result that increasing uncertainty reduces the expected benefits of timber operations even when the decision maker is risk neutral (note: despite this result – the costs of uncertainty are still often ignored in economic analysis of forestry investments).

A more interesting question is: Does the increased uncertainty in model parameters associated with the scenario 3 input data have a significant impact on harvest solutions? Figure 10.6 shows that when using the lower variance scenario 2 input data, almost all of the IAC1 land is held as ending inventory and almost all of the IAC2 land is harvested in either stage one or stage two. Figure 10.5 shows that when using the scenario 3 input data, more of IAC1 is harvested in stages one and two, and less of IAC2 is harvested in stages one and two. Generally, with a lower variance in outcomes, the decision maker relies more on IAC2 for harvesting and on IAC1 for satisfying the ending inventory constraint. With higher variance in yields, the decision maker relies more on IAC1 for harvest revenues and more on IAC2 to satisfy ending inventory requirements. Another notable result is that recourse has a larger effect in terms of harvest possibilities in stage two. Under the low variance scenario – the difference in harvest area for decision alternative 1 and 2 for IAC2 is relatively small. For the high variance scenario, the difference is quite large. Thus, the increased uncertainty does have a significant impact on optimal harvest solutions. An irrational decision maker who utilizes scenario 2 input data to determine a harvest schedule when the actual input data should be scenario 3 will either be inefficient or unable to satisfy sustained yield constraints for some event histories. Another implication is that recourse becomes more important as variance in outcomes increases. As noted previously, variances from climate change are relatively small for this study. This may be because of the relatively short length of the planning horizon. If the time horizon of this problem was longer, then it is possible to speculate that yield variances related to climate change would be more substantial. Thus, a recourse approach might be even more applicable in a climate change context for longer-term forestry problems.

Sensitivity to sustained yield constraints

In the previous section it was noted that flow constraints are the determining factor relative to a no harvest solution when the model without recourse is run. This suggests that flow constraints may play an important role relative to the results in the recourse model. In the previous chapter the effect of increasing flexibility in harvest flow constraints on objective function values in an EV-Chance Constraint model was assessed. The results suggested that increasing harvest flow constraint flexibility does not have a large effect on objective function values (see Figure 9.2). However, it is possible that increasing flexibility in flow constraints could be much more effective in cases where the logger can adjust harvest rates over time in response to new information. It is possible that having the opportunity to adjust harvest makes increased flow flexibility more beneficial. The purpose of the analysis in this section is to evaluate the effect that increasing flexibility in flow constraints has on objective function values and harvest selections in a decision setting where recourse is permitted.

The underlying assumptions for the model used in this section are as follows:

- The recourse model in equations 10.1 to 10.7 is the base model,
- Scenario 3 input data (in discrete form) were used,
- There is a 50 % likelihood of each discrete outcome for each stage. This results in an equal probability of 0.125 for each event history,
- The decision maker is risk neutral (i.e. there is no risk premium in the objective function),
- Two sets of flow constraints are tested. In the first model, the allowable deviation in flow constraints is plus or minus 10 %. In the second model the allowable deviation in flow constraints is plus or minus 50 %,
- Ending inventories are set at 220,000 cubic meters.

Results showing the effect of increasing allowable deviations in periodic flow from 10 % to 50 % are provided in Figure 10.7. Increasing the allowable deviation in periodic flows in a recourse setting has a significant effect on expected income using the recourse model. Expected income values increase from \$146,500 to \$445,789 when the allowable flow deviation is increased from 10 % to 50 %. In terms of harvest choices, when allowable flow deviations are at 10 % then harvesting in stages one and two is not feasible. Thus, even with recourse and improvement in productivity from climate change, achieving sustained yield is problematic in a stochastic setting. When allowable flow deviations are increased to 50 %, however, harvesting occurs in both stage one and stage two. It is also possible to assess the effect of increasing allowable flow deviations from 25 % to 50 % by comparing the results of Figure 10.5 with Figure 10.7. Increasing allowable flow deviations from 25 % to 50 % increases expected returns by 12 % ($398,960 to $445,789) and it increases the area harvested in both stages one and two.

Hoganson and Rose (1987) point out that flexibility is inherent in sequential decision making processes. As noted in the previous section, allowing for recourse – by itself - has positive implications for objective function values. Also, the analysis in the previous chapter found that increasing flexibility in flow constraints had some effect on objective function values but the effect was relatively small. In this section, we have looked at the effects of recourse and increased constraint flexibility in a combined way. The results obtained in this section suggest that when these two sources of flexibility are combined, the results are more significant. The message that comes from this analysis is that a desirable property of decision-making processes when uncertainty exists is flexibility and that there are a number of ways to improve flexibility. To the extent that climate change increases uncertainty about future yields and returns, planners and policy makers may find it useful to explore ways of incorporating increased levels of flexibility in timber management planning. Moreover, an approach that both allows recourse and relaxes flow constraints has significant positive economic benefits.

Effects of change in probabilities

The current climate change literature refers to various types of adaptation. One type of adaptation is autonomous adaptation (Intergovernmental Panel on Climate Change 2001). This type of adaptation occurs as an automatic response to some climate induced signal (e.g. a price effect, a yield effect, a fire risk effect, etc).

Another type of adaptation is planned adaptation. Planned adaptations are measures taken ahead of time to reduce the impacts of future changes or reduce the likelihood of impacts. For example, a particular decision maker might be able to take some action ahead of time that influences the levels of benefits derived under the best case and/or worst-case scenarios. Or alternatively, the decision maker might be in a position to take some action ahead of time that influences the probabilities of future states of nature. The next section, investigates the effect of modification of the probabilities of high yield vs. low yield outcomes. Actions taken to influence these probabilities can be considered to be a form of planned adaptation.

Freeman (1999 pg. 221) differentiates between risk reduction and risk prevention activities. Risk reduction activities are actions that are taken to reduce the magnitude or impacts of low yield outcomes. Risk prevention activities are actions taken that result in a reduction in the probabilities of low yield outcomes. The kind of adaptation we are addressing in this section is risk prevention.

The purpose of the analysis in this section is to analyze what a decision maker is willing to pay for increasing the probability of the high yield outcome (or equivalently reducing the probability of low yield outcomes), and how increasing the probability of high yield outcomes affects harvest choices. In order to assess the effects of increasing the probability of high yield outcomes we require two sets of probabilities. For the analysis in this section, one model is run with the assumption that the probability of a high yield outcome in each stage is 50 %. A second model is run where the probability of a high yield outcome at each stage is increased to 90 %. When applied over the entire planning horizon, changing the probabilities of high yield and low yield outcomes at each stage results in a change in the probability of each event history. The probabilities of each event history under each risk situation are shown in Table 9.2.

The assumptions underlying the models used for this section are as follows:

- The recourse model in equation 10.1 – 10.7 is the base model,
- Scenario 3 input data (in discrete form) is used,
- The model is run for two sets of probabilities. One set is based on an assumed 50 % likelihood of a high yield state at each stage. The second is based on an

assumed 90 % likelihood of a high yield stage at each stage. The probabilities are shown in Table 10.2,

- The decision maker is risk neutral (i.e. there is no risk premium in the objective function),
- Allowable deviations in periodic flow constraints are plus and minus 25 %, and
- Ending inventory constraints are imposed - the target-ending inventory is 220,000 cubic meters.

For the remainder of this section, we refer to actions taken to increase the probability of high yield states as risk prevention. However, a cautionary note is warranted here. The model presented in this chapter assumes risk neutrality on the part of the decision maker. The incentive for reducing uncertainty and managing risk comes from increases in expected values of returns. If the decision maker were risk averse, he/she would have an added incentive to prevent risk because risk prevention would reduce the risk premium and therefore increase the agent's utility. However, additional potential benefits in the form of reduced risk premiums are not included in this analysis. This assumes, therefore, that the decision maker is the public forestland manager (as noted – if it is the public land management agency that is exposed to risk then risk preferences are low and risk premiums have a negligible effect on choice).

Figure 10.8 shows the effects of risk prevention on objective function values for our hypothetical forest management problem. Objective function values increase as the likelihood of the high yield outcome increases (i.e. the likelihood of the low yield outcome decreases). Figure 10.8 shows that for our hypothetical case study, the net present value of benefits increases by about 132 % (i.e. $398,960 to $ 526,443) when the probability of the high yield outcome increases from 0.50 to 0.9. This result shows that risk prevention has the potential for increasing the expected value of returns in forest management under climate change. Thus, a decision maker would be willing to pay about $ 127,000 to increase the likelihood of high yield to 90 % at each stage (Note: here again that other factors such as stumpage payments changes are not accounted for here). The critical question here becomes do the returns from risk prevention justify the costs. If the present value of costs of risk prevention is lower, then risk prevention activity is a viable investment.

The recourse model was also used to show the effects of probability change on harvest choices. Probabilities for each event history were recalculated (see Table 10.2) and the DSP model was run for two probabilities (50 % and 90 % likelihood of good states). The solutions are insensitive to variations in the probabilities of the event histories (Figure 10.8). Optimal harvest solutions remained unchanged for the two probability situations. Therefore, in summary, the main effect of risk prevention in a climate change and uncertainty context is that it can significantly increase the expected value of returns. There is, however, no effect with respect to harvest solutions – at least for the problem defined for this study.

A related practical question pertaining to risk prevention is: What kinds of strategies might a forest manager utilize in order to prevent risk (i.e. reduce probabilities of low yield outcomes)? The empirical data that the yield curves in this study are based on is cross sectional. These yields will reflect historical insect defoliation events, droughts, and the effects of other pathogens. Therefore, risk prevention strategies could come in the form of forest management strategies that reduce exposure to these kinds of stochastic events. Reducing the likelihood of these disturbances is one example of a strategy that may reduce the probability of low yield outcomes in the discrete climate scenarios presented in this chapter.

Summary and conclusions

A recourse model approach applies in situations where decision-making is sequential and where some portion of the uncertainty in variables is resolved part way through a planning horizon. The ability to model adjustment part way through the planning horizon means that DSP models (i.e. recourse models) are dynamic. Timber harvest planning is by nature a multi-period sequential decision process where uncertainties may be resolved through time. In some respects, therefore, the decision problem defined for this study is suited to the application of a recourse type approach. However, as noted, the problem for this study is only partly dynamic. Moreover, the static models presented in the previous chapter also have features that make them applicable to this study (e.g. they take account of covariance between management prescriptions). In our view no single model is preferred, or more applicable, or best in all respects for problems related to looking at the impacts of climate change and uncertainty in forestry decision making contexts. Thus, the approach we have taken for this study is to estimate a suite of different models one of which is the recourse model presented in this chapter.

As noted, timber harvest-scheduling problems can be viewed as sequential decision problems where the resolution of uncertainty part way through the planning horizon results in alternative choices. A recourse model would apply to a timber harvest-scheduling problem even without a climate change dimension. However, the addition of sequential climate change productivity and uncertainty effects makes a recourse approach even more potentially valuable as a decision support tool or as an approach for looking at climate change impacts and adaptation.

The main findings of the analysis presented in this chapter are summarized as follows. The first main finding is that without some way of managing risk (through the flexibility of being able to adjust harvest) or addressing uncertainty, harvesting and sustained yield objectives may not be compatible in uncertain operating environments – even where climate change is increasing productivity over time (as was the case for our study site). This conclusion is supported by the fact that when the no recourse model is run using the scenario 3 input data, the entire forest remains uncut over the 60 year planning horizon. When the model is run where recourse is permitted (using the same input and the same set of model parameters), a solution is obtained that allows harvesting to occur in stage one and two. With recourse, forty seven percent of the forest is harvested in stage one. In stage two, an additional 33 % is harvested if the high yield state occurs in stage one and 22 % of the area is harvested if the low yield state occurs in stage one. Thus, the decision maker adjusts the area harvested in stage two after discovering what state of nature actually occurs in stage one.

A second general finding is that when uncertainty in outcomes is relatively high, forest managers are required to retain a higher proportion of their existing mature forest uncut in order to satisfy the constraints. Lower uncertainty means that forest managers can harvest a higher proportion of mature stands in the short term and rely more on immature stands to satisfy ending inventory requirements. The analysis supporting this conclusion was provided in the section "Sensitivity to differences in levels of uncertainty" where it was found that using the scenario two input data, 450 hectares of IAC2 forest is harvested in stage one while when the scenario three input data is used (i.e. higher variance) 310 hectares of IAC2 forest is harvested in stage one.

A third finding is that, the benefits of timber operations in a setting that permits recourse are considerably higher when considered in conjunction with the effects of policy adjustments aimed at making forest policies more flexible. The analysis pertaining to the impact of increasing the allowable deviation from periodic flows from 10 % to 50 % shows significant increases in expected income when using a recourse model. Similar sensitivity analysis using the EV – Chance Constraint model in Chapter 9, suggested that results were somewhat insensitive to variations in flow constraints. Thus, it would appear that flow policy adjustments, in conjunctions with recourse, offer greater potential in terms of management options in a climate change adaptation context.

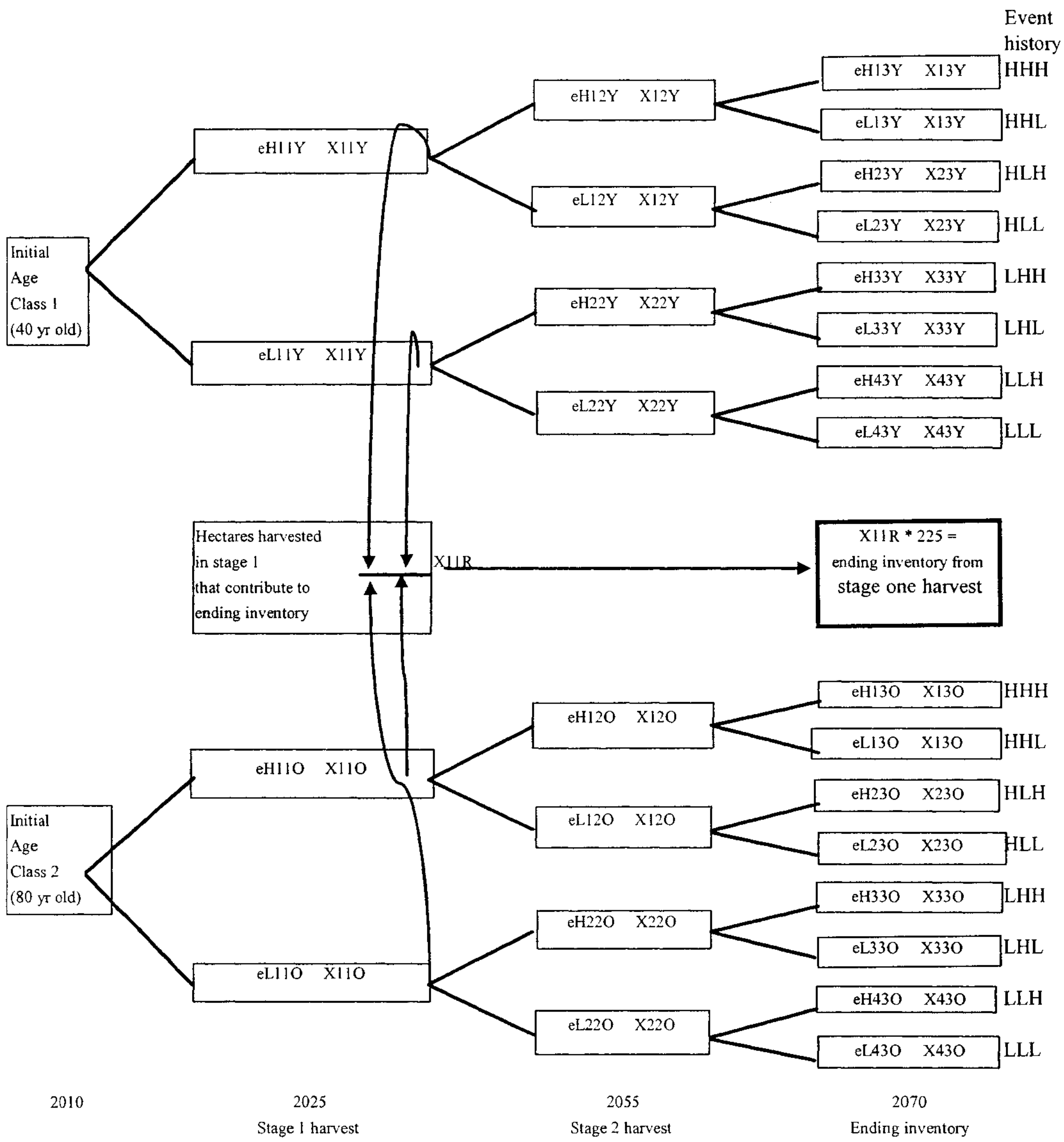
Finally, the analysis in this chapter shows that there are significant economic benefits associated with risk prevention. Risk prevention activities include activities that reduce the probability of low yield states and increase the probability of high yield states. Such activities include reducing the incidence of pathogens, reducing susceptibility to drought, and/or other forms of increasing productivity through management (e.g. thinning, spacing, fertilization) and increased research (in which case the benefits can be viewed as a return to research). Although the expected value of timber returns can increase significantly with risk prevention activities, the optimal harvest solutions are generally not affected by risk prevention – at least for the stylized forestry scenario presented in this study.

As noted in the introduction to this final section, discrete stochastic programming models have some applicability to the problem context outlined in this study. However, discrete stochastic programming models also have limitations. The main limitation is that these models suffer from what is called the "curse of dimensionality" (McCarl and Spreen 1997). This refers to the fact that network models in general, and discrete stochastic programming models in particular, quickly explode with the addition of more stages and/or the addition of more outcome alternatives. This limitation significantly restricts the applicability of the approach to climate impact studies conducted at larger scales (e.g. more stand types, more age classes, more periods, more prescriptions).

Another limitation of the DSP approach relative to the EV-Chance constraint model is that covariances between management prescriptions have no bearing on objective function values or solutions. This feature of the DSP approach makes it difficult to consider benefits that might arise from portfolio diversification. Ignoring the role of covariances in potentially reducing overall variance could mean that the impacts of uncertainty are somewhat overstated in a DSP framework.

The analysis presented in this chapter has led to a number of additional questions of interest for possible future research. Two such questions include 1. What are the effects on net benefits and harvest choices if the forest manager is able to gain better insights into yields and benefits in the current harvest period (i.e. invest in the kind of new information and knowledge that provides perfect information of the present and the past)? And 2. What are the implications of including risk preferences into the objective function of the recourse model? There was insufficient time to address these questions for the purposes of this thesis. They are left for future study.

Figure 10.1 Decision tree for the discrete stochastic programming problem.



Variable identifier = eijta	Where	i = state	H for high yield state, L for low yield state
		j = decision alternative	j = 1 in stage 1
Area harvested = Xjta			j = 1 or 2 in stage 2 j = 1,2,3 or 4 in stage 3
		t = stage	t = 1,2 or 3
		a = initial age class	Y for IAC 40 yr old stands O for IAC 80 yr old stands R for regenerated stands

Figure 10.2 Data for the discrete stochastic programming problem – Scenario two.

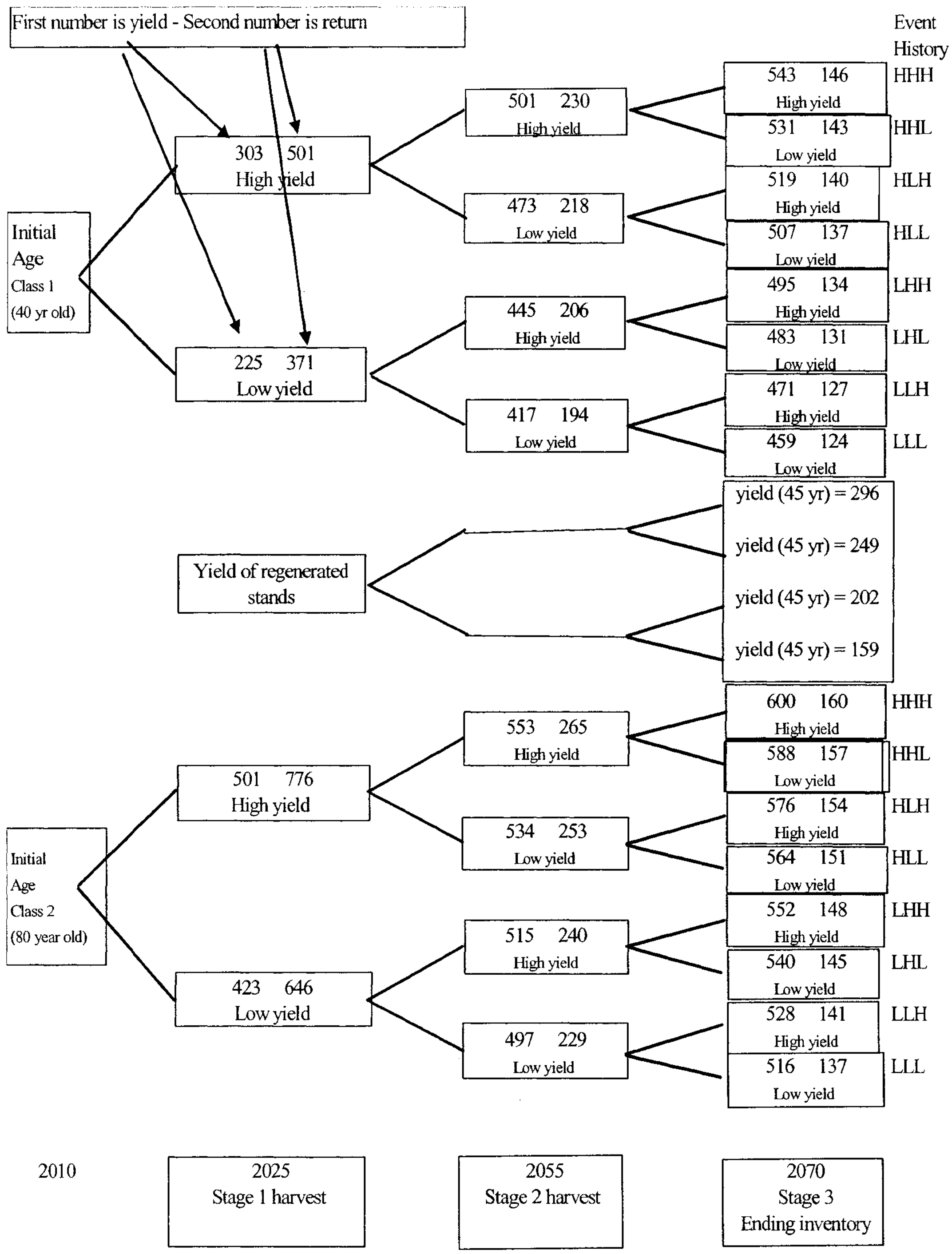


Figure 10.3 Data for the discrete stochastic programming problem – Scenario three

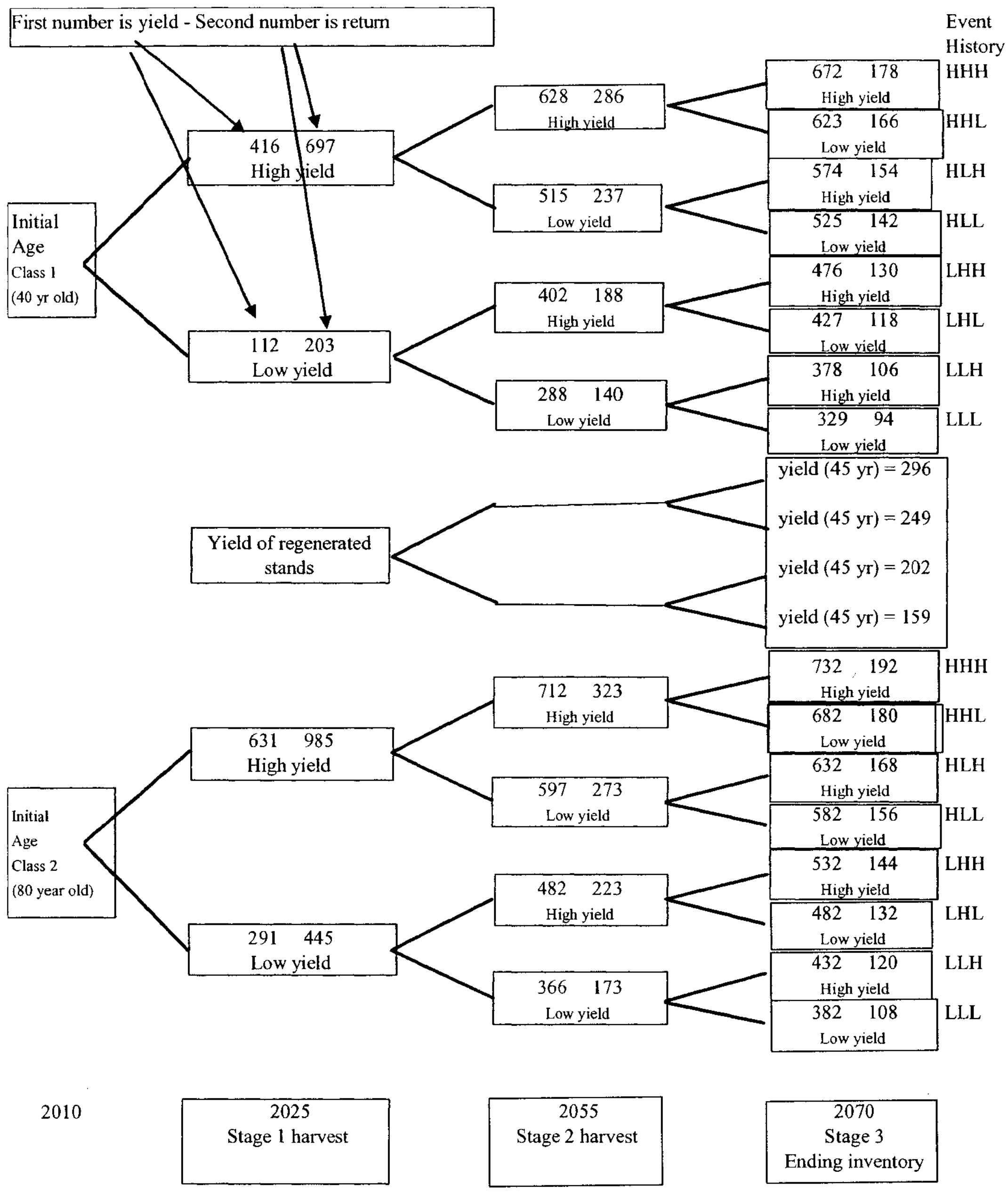


Figure 10.4 Solutions for the DSP problem without recourse using the scenario 3 input data.

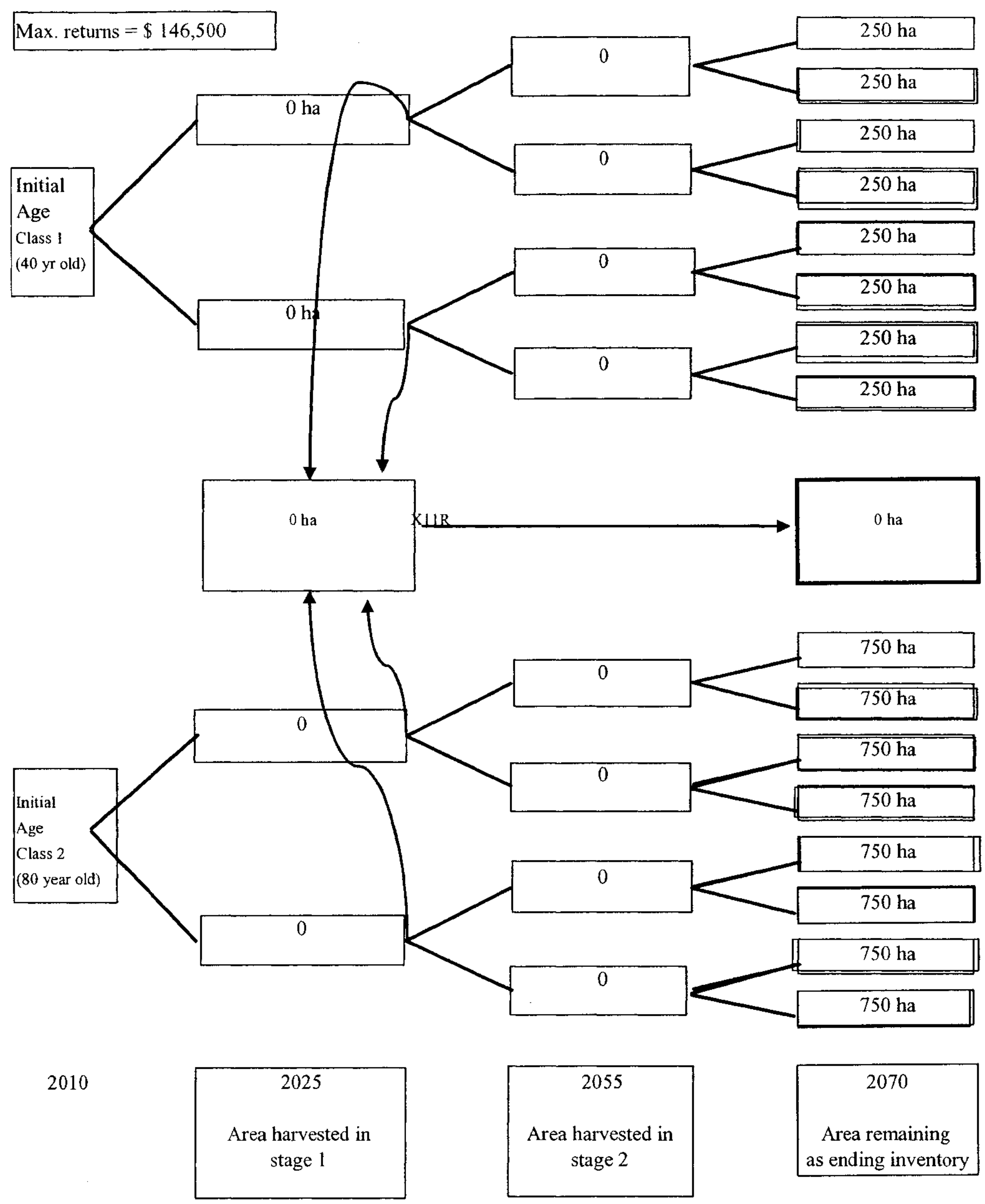


Figure 10.5 Solutions for the DSP problem with recourse using the scenario 3 input data.

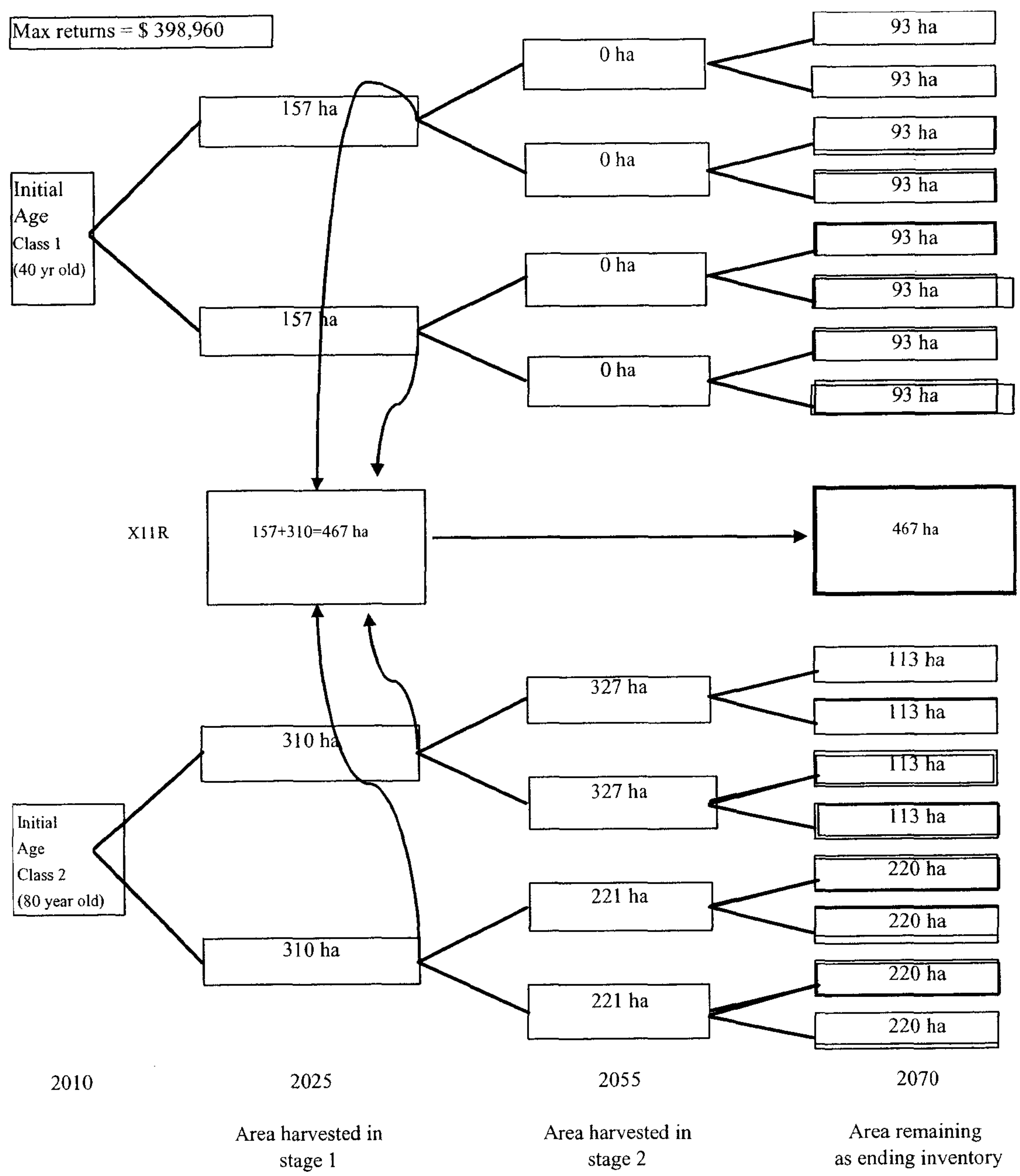


Figure 10.6 Solutions for the DSP problem with recourse using the scenario 2 input data.

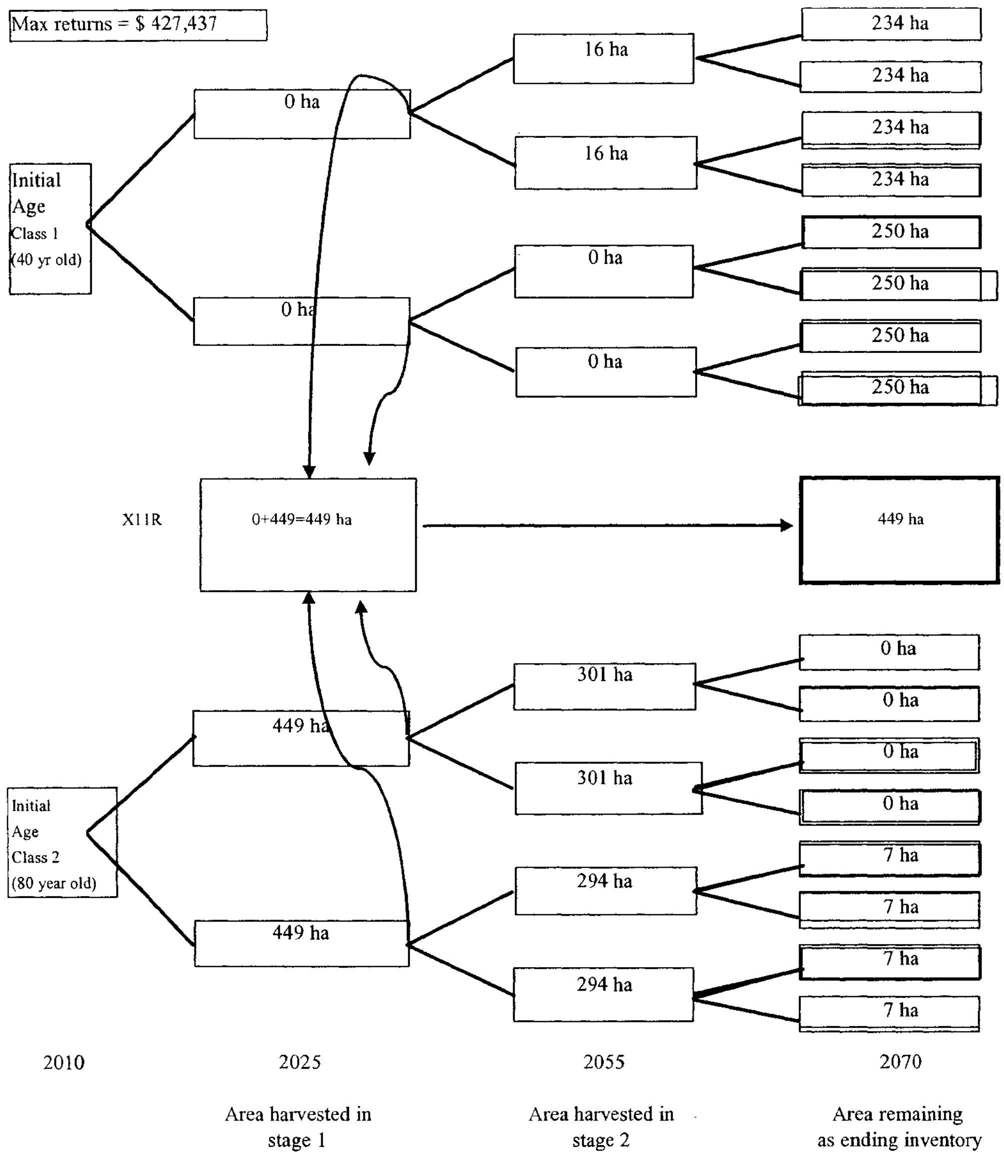


Figure 10.7 Comparison of solutions with two levels of flexibility in flow constraints (10 % and 50 %) based on scenario 3 input data.

Objective function value (10 % deviation) = $ 146,500
Objective function value (50% deviation) = $ 445,789

Top numbers shows solution when harvest is allowed to deviate by 10 % between periods
Bottom number shows solutions when harvest is allowed to deviate by 50 % between periods.

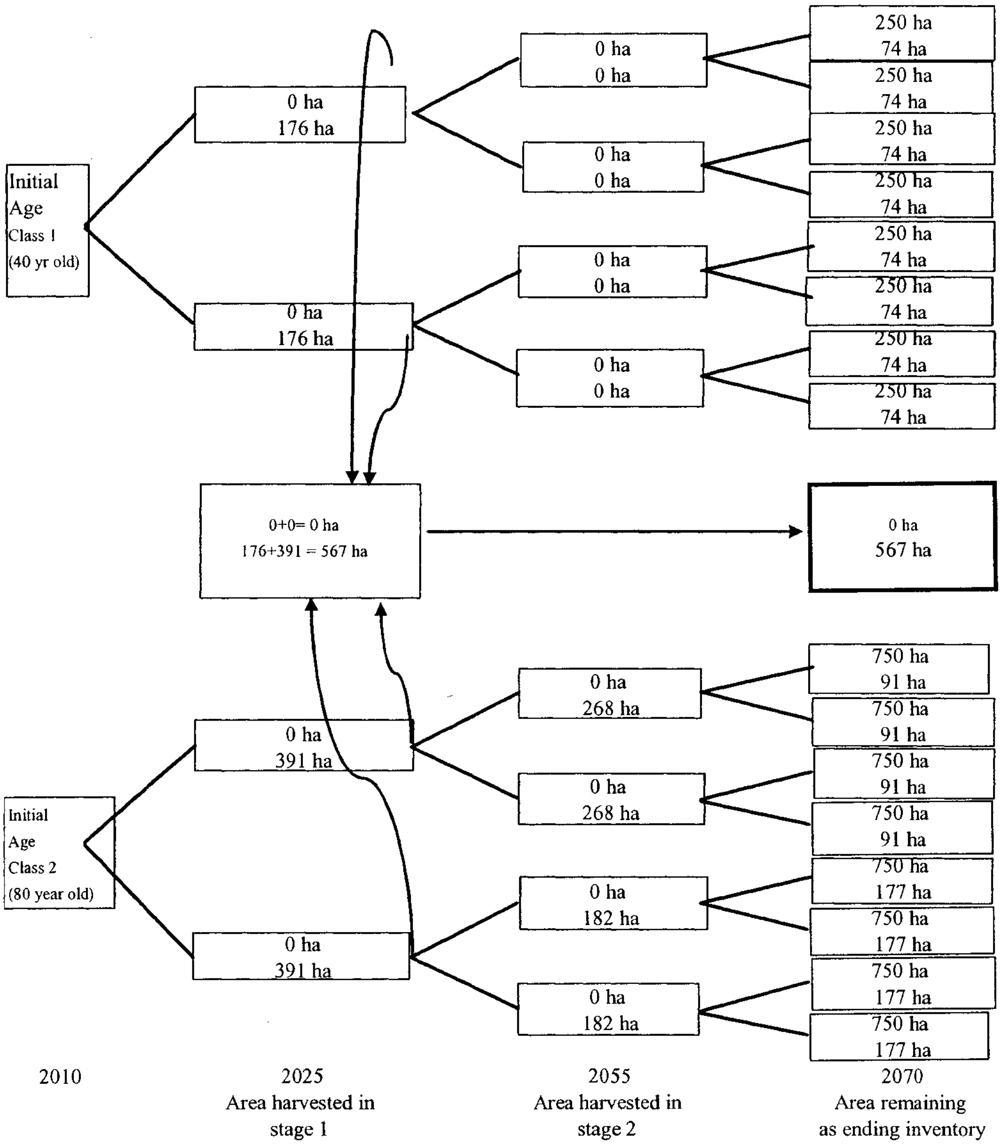


Figure 10.8 Comparison of solutions with 50 % likelihood of good state vs solutions with a 90 % likelihood of good state.

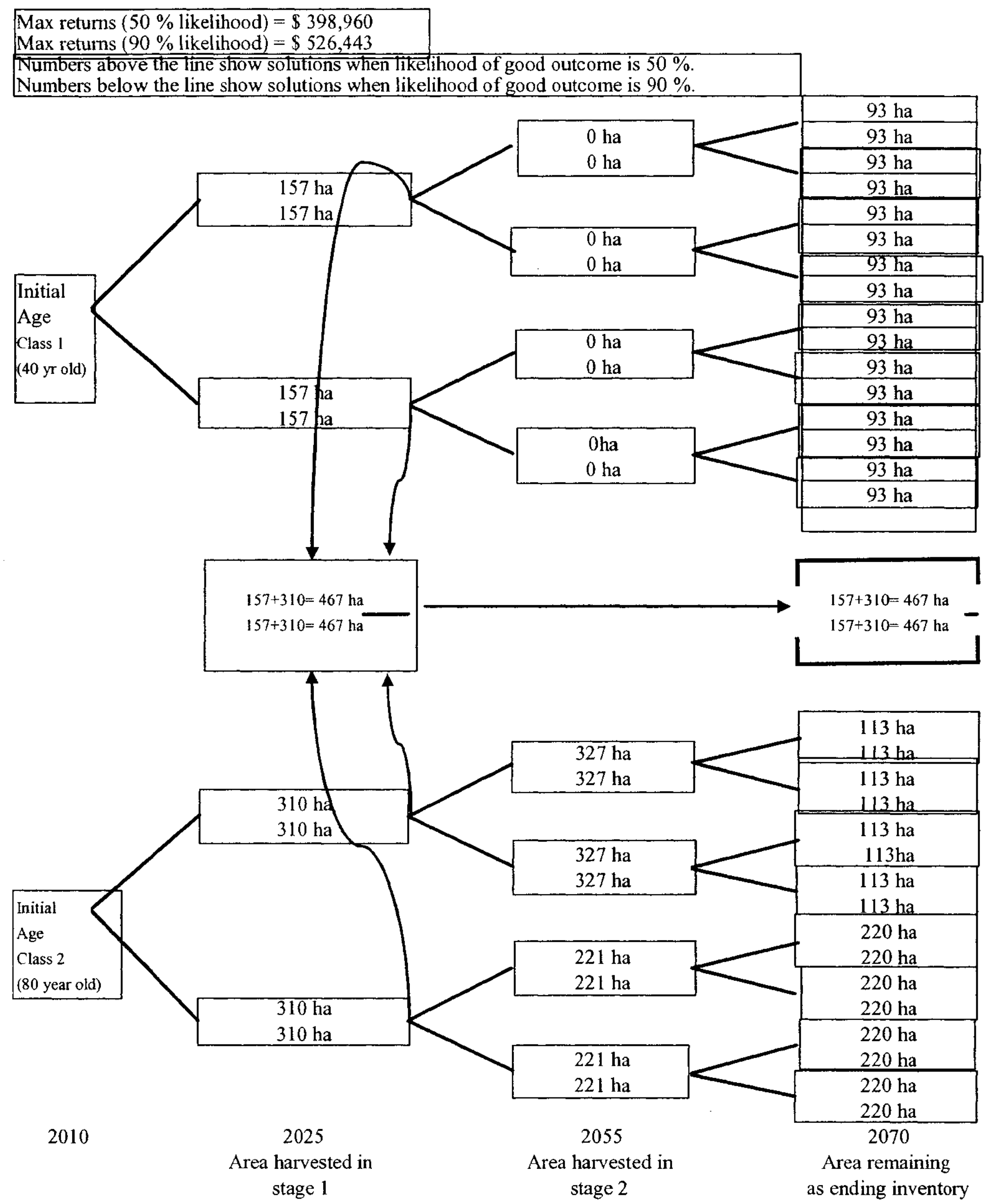


Table 10.1 Parameter values used in the basic recourse model

		Scenario two		Scenario three		
Stage	States	Yield	NPV	Yield	NPV	Choice Var.
1	e1114	303	501	416	697	X114
1	e2114	225	371	112	203	X114
1	e1118	501	776	631	985	X118
1	e2188	423	646	291	445	X118
2	e1124	501	230	628	286	X124
2	e2124	473	218	515	237	X124
2	e1224	445	206	402	188	X224
2	e2224	417	194	288	140	X224
2	e1128	553	265	712	323	X128
2	e2128	534	253	597	273	X128
2	e1228	515	240	482	223	X228
2	e2228	497	229	366	173	X228
3	e1134	543	146	672	178	X134
3	e2134	531	143	623	166	X134
3	e1234	519	140	574	154	X234
3	e2234	507	137	525	142	X234
3	e1334	495	134	476	130	X334
3	e2334	483	131	427	118	X334
3	e1434	471	127	378	106	X434
3	e2434	459	124	329	94	X434
3	e1138	600	160	732	192	X138
3	e2138	588	157	682	180	X138
3	e1238	576	154	632	168	X238
3	e2238	564	151	582	156	X238
3	e1338	552	148	532	144	X338
3	e2338	540	145	482	132	X338
3	e1438	528	141	432	120	X438
3	e2438	516	137	382	108	X438

Table 10.2 Probabilities of outcomes for different assumptions of likelihoods of good and low yield states of nature

		Likelihood		Likelihood		Likelihood		Likelihood		Likelihood	
		Good	Bad	Good	Bad	Good	Bad	Good	Bad	Good	Bad
		0.5	0.5	0.6	0.4	0.7	0.3	0.8	0.2	0.9	0.1
Event history		Probabilities of each event history									
IAC1	111	0.125		0.216		0.343		0.512		0.729	
	112	0.125		0.144		0.147		0.128		0.081	
	121	0.125		0.144		0.147		0.128		0.081	
	122	0.125		0.096		0.063		0.032		0.009	
	211	0.125		0.144		0.147		0.128		0.081	
	212	0.125		0.096		0.063		0.032		0.009	
	221	0.125		0.096		0.063		0.032		0.009	
	222	0.125		0.064		0.027		0.008		0.001	
IAC2	111	0.125		0.216		0.343		0.512		0.729	
	112	0.125		0.144		0.147		0.128		0.081	
	121	0.125		0.144		0.147		0.128		0.081	
	122	0.125		0.096		0.063		0.032		0.009	
	211	0.125		0.144		0.147		0.128		0.081	
	212	0.125		0.096		0.063		0.032		0.009	
	221	0.125		0.096		0.063		0.032		0.009	
	222	0.125		0.064		0.027		0.008		0.001	

CHAPTER ELEVEN

SUMMARY AND CONCLUSIONS

Summary of the methodology and approach

The goal of this study is to understand what the implications of climate change and uncertainty might be for economic returns from timber management and for harvest choices in the short term and long term. In order to address specific questions pertaining to this general goal, a methodology was developed and applied to a stylized forest management scenario. The methodology is unique in two ways. First, it is unique in terms of the ways that models and methods used in traditional forest economics analysis have been modified to account for climate change and uncertainty. For example, we have extended traditional yield function estimation and traditional net present value models to incorporate climate effects on stand productivity and to take account of uncertainty in climate variables and yield model parameters. The second way that the method and approach described in this study is unique is in terms of the way that various types of models and analysis are integrated. The study required an approach that predicts the effects of climate change on stand yields in the future, predicts the distributions of random variables, and takes account of the risk preferences of individuals.

The first requirement was to obtain a model that could be used to predict future yields under alternative potential climate futures. This required the estimation of a yield model that includes climate variables. The estimation of this type of model requires cross-sectional yield data that covers a range of sites with varying current climate conditions. Cross-sectional, spatial data was made available through the Canadians Forest Service's Climate Impacts on the Productivity and Health of Aspen (CIPHA) study[40]. The CIPHA data was transformed into formats suitable for yield curve estimation and a variety of yield model specifications were estimated by linear and non-linear regression methods. A reciprocal yield model functional form was selected.

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[40] T. Hogg – Principal Investigator

The second requirement was to obtain predictions of the distributions of future random variables used in the risk models. The complicating factor for this study was that the random variables required for the risk models are functions of other random variables. An approach for estimating the distributions of random variables was required. The approach adopted was Monte Carlo simulation. The approach was to incorporate the yield curves estimated in Chapter 6 into a Monte Carlo simulation model along with information on the distributions of random variables known *apriori*. The Monte Carlo simulation model provided expected values, variances, and covariances for harvest yields, net present values of returns, and ending inventory values for all stand types, all prescriptions, and for each of scenarios 2, 3 and 4 assumptions about sources of variance.

The final requirement was to develop a model that could be used to assess the implications of climate change and uncertainty on forestry returns and on harvest choices. The traditional approach is to develop a deterministic timber supply model. The timber supply model adopted for this study was a Model I type formulation. However, in order to accommodate risk and uncertainty the Model I formulation needed to be extended. The approach used for this study was to transform the Model I timber supply formulation into a probabilistic mathematical programming based risk model.

There are a variety of models for addressing risk. In this study we considered three different types of formulations. The first formulation is referred to as a Markowitz variance minimization model. This model estimates the frontier of efficient portfolios that represent different combinations of risk and return that are efficient. The second formulation estimated in this study is an expected value – chance constraint type model. The model incorporates decision maker risk preferences through the inclusion of a risk aversion parameter. The model identifies the specific portfolio that is optimal from the range of possible portfolios available to the decision maker. The third risk model estimated in this study is referred to as a discrete stochastic programming model (or recourse model). This formulation models the decision making process as a series of sequential decisions and it recognizes that with some types of information structures, some portion of the uncertainty facing decision makers is resolved over time. Thus, dynamic adjustment of the choice variables is explicitly incorporated into the modeling frame. The following section provides a more detailed comparison of these three types of modeling approaches.

Risk model comparisons

There are two reasons for estimating three separate risk models. The first reason is to develop an understanding of the types of risk questions that can be addressed with specific types of risk models in a climate change and forest management decision-making context. The second reason is to understand the strengths and weaknesses of the various approaches in different types of applications (e.g. impact assessment, policy analysis, prediction of adaptation response, etc).

The first risk model considered in this study was the Markowitz asset allocation model. This particular model is the most straightforward of the three risk models. The model looks for the particular asset mix that minimizes variance subject to a minimum expected return. One of the strengths of the Markowitz model is that the model can be employed without having to determine or derive a risk aversion coefficient. Obtaining suitable measures for the risk aversion coefficient can be challenging. An issue relative to the Markowitz approach is that it cannot be used for evaluation of the impacts of climate change. Impacts assessment requires a model that explicitly considers the agents degree of aversion to risk and uncertainty. The absence of consideration of risk preference in the Markowitz model precludes impact assessment. However, the Markowitz model could have useful applications for other types of climate change related analysis. For example, risk management is often quoted as a potentially useful adaptive response to climate change (Jones 2001). A Markowitz model approach would be appropriately applied for finding the particular mix of land uses and/or species mix configurations that minimize variance of portfolio returns under increasingly uncertain future climatic conditions.

The second risk model estimated in this study was the expected value variance - chance constraint hybrid model. This model has been widely used in studies of agricultural risk but to our knowledge there are no applications to forest management and climate change risk analysis. This risk modeling approach treats random variables as continuous random variables. Essentially the variance of a particular portfolio is derived with the aid of variance-covariance matrices between prospects within portfolios. The EV-CC model has a number of strengths. First, the formulation is explicitly and directly linked to decision theory and expected utility theory. For example, the objective function is to maximize certainty equivalent. A second advantage is that uncertainties in objective function coefficients and constraint coefficients are incorporated into the optimization framework with relative ease. Those objective function and constraint coefficients that are considered to be random variables are represented by their expected values, their variances, and covariances. A third advantage (relative to the Markowitz model) is that the model identifies a specific optimal portfolio for a decision maker with particular risk preferences. A fourth advantage (relative to the recourse model) is that random variables can be incorporated as continuous random variables[41]. One of the drawbacks of the EV / CC model is that it is a static model. This means that there is no opportunity to adjust decisions as states of nature become realized and uncertainty is resolved over time.

The third risk model approach presented in this study is the discrete stochastic programming (DSP) model (i.e. recourse model). The DSP modeling approach is designed to address problems that are sequential in nature and for which some portion of the uncertainty about future states of nature will become resolved part way through the planning period of interest. This is the main strength of this risk modeling approach. The approach treats future states of nature as discrete events. That is, there are a finite number (two in our case) of possible future states of nature at each stage of the analysis. These types of problems are quite typical in forestry – particularly when sustained yield constraints are included in the problem setting. Thus a DSP formulation is well suited to forestry management decision analysis type problems.

There are four main issues related to this type of risk model. The first is that most forestry applications of this approach have not incorporated risk preferences.

[41] The random variables of interest for this study are in fact continuous random variables. Thus, the EV-Chance constraint formulation provides a framework that is somewhat more consistent with the way the variables are defined. In the case of the DSP formulation, it was necessary incorporate additional assumptions in order to convert the continuous random variables to a discrete form.

That does not mean that it is not feasible to design a DSP that takes risk preferences into account. Apland and Hauer (1993) describe an approach for incorporating risk preferences into a DSP model. However, the approach does require the specification of a particular utility function and generally the form of such utility functions is not known. The second issue concerns what is known as the curse of dimensionality. As shown in chapter 10, DSP risk approaches are essentially decision trees. The formulation of these types of models is manageable as long as the number of states of nature and the number of stages is relatively low. However, these types of formulations explode quickly as the number of stages and or states of nature is increased. Thus, even though this type of formulation is manageable for our relatively straightforward problem (i.e. two stand types, three stages, and two states of nature at each stage) for larger and more realistic forest management problem contexts with many stand types, multiple planning periods, and multiple potential states of nature – a DSP approach may prove to be infeasible. A third issue with the DSP model approach is that it does not account for covariance between objective function coefficients or between random variables in the constraints. In Chapter 8 the potential important role of covariances between coefficients as a factor affecting choices was noted. A final issue relative to the application of the DSP approach to the specific problem in this study is that because of the structure of the approach it is necessary to ignore some sources of uncertainty. For example, the terminal values for the forest management problem (i.e. the soil expectation values) are incorporated as fixed values (i.e. these values are considered to be deterministic). Also the ending inventory values for hectares that are harvested in period one are incorporated as fixed values. The reason it is necessary to incorporate these as fixed values is that the DSP model requires that the terminal values of variables are fixed. The model assumes that the decision maker is not concerned about levels of uncertainty in variables for periods following the end of the planning period.

Summary of the main findings of this study

A finding of this study is that the contribution of climate uncertainty to total portfolio variance is relatively small. Climate variance accounts for only 25 % of the standard deviation in expected return. The remainder is due to variance in yield parameters. A cautionary note is required here. Variance in the climate variables is based on the range of predictions from different GCMs using different SRES emission scenarios. Moreover, climate variances are based on climate predictions made over large geographic areas and the resulting variance may tend to be lower than climate variances of predictions at a specific site. Yield parameter variance, on the other hand, is based on variations in the cross sectional data obtained from specific sites. Thus, part of the reason why climate variances may be relatively lower may be attributable to the methods used. Our results, therefore, likely provide a lower bound estimate of the costs of climate change due to increased uncertainty.

This study also confirms that economic returns from forestry and optimal harvesting decisions are sensitive to risk and uncertainty. For example, the analysis in Chapter 9 shows that certainty equivalent values with sustained yield constraints for highly risk averse decision makers are 57 % lower when risk preferences, climate effects, and uncertainty are taken into account. One of the implications of the results in this study is that since climate change does affect variation in expected returns, and since higher variation has an economic cost, the cost of risk and uncertainty should be included in climate impact assessments. However, such assessments also need to recognize other sources of uncertainty. Analysis of returns and harvest choices with climate change require a full accounting of all sources uncertainty for accurate assessments of impacts.

A somewhat surprising result in this study is that lower risk portfolios have higher proportions of longer-term prospects within them. The reason for this result is due to discounting. The discounting of future distributions results in lower present values of expected values and lower variances around the expected values. The net effect is that time is a risk reducing input. This implies that since decision makers are averse to risk they prefer to delay risky decisions until the future. They are prepared to discount future risk compared to immediate risk.

A main conclusion of this study is that all other factors equal, the impacts of climate change would appear to be positive for aspen timber management for areas in central Alberta up to 2070. This result holds even with the costs associated with higher climate risk accounted for. This conclusion is not inconsistent with US studies looking at the impacts of climate change on forest yields. These studies conclude that for some regions climate change results in higher productivity. The result showing positive benefits for this study must be qualified in a couple of ways. First, our analysis is regional in nature. We have assumed that the manger is a price taker and that timber demand is perfectly elastic. Thus, supply changes from climate change do not have a price effect. Prices are known with certainty and they are fixed at a single value over the entire planning period. In point of fact, climate change is likely to have important implications for timber prices and it will also likely affect price uncertainty. For example, previous studies suggest that global timber supply will increase faster than global demand resulting in a trend of decreasing prices for North American produced timber (Sohngen and Sedjo 2005).

Another important qualification is that although productivity is predicted to increase in the study area we are considering, similar effects may not be universally experienced across the boreal forest. For example, in areas in the aspen parkland or near the boreal / aspen parkland boundary – increased moisture deficits are possible under climate change. Productivity in areas that are subject to moisture deficits and increased drought frequency is more likely to decline (Hogg 1994; Hogg and Bernier 2005). For these types of areas, the economic impacts of climate change in terms of the forest sector will likely decrease.

Another consideration or qualification for this study is that the time horizon is relatively short for forestry analysis. The study covers a 60 year period. It is important to acknowledge that climate will continue to change after the end of the planning period defined for this study. It is also quite possible that changes in climate will eventually be of such magnitudes that the directions and magnitudes of impacts identified in this study will reverse.

This study has also looked at climate change from the point of view of potential implications for sustained yield forest policies. There are a number of conflicting implications of climate change for sustained yield. First, in the case where sustained yield constraints are imposed, expected net benefits are higher with climate change than they are without climate change. This is as a result of higher growth rates in future periods that make satisfying flow constraints and ending inventory constraints easier to achieve. At the same time it is important to note that there are significant opportunity costs associated with sustained yield. The total certainty equivalent of a forest with SY constraints is 18 % lower than certainty equivalent for our forest without SY constraints. Moreover, the imposition of SY tends to reduce flexibility and it tends to result in relatively undiversified sets of management options. Thus, the variances around forestry benefits are likely higher under sustained yield than would otherwise be the case and in addition risk and uncertainty becomes relatively more difficult to respond to when managers are faced with having to satisfy sustained yield constraints. One interesting result obtained from the DSP model analysis is that without recourse, harvesting is not compatible with sustained yield in uncertain decision environments. This result implies that we may need to rethink policies such as sustained yield and sustainable forest management. The models imply that without flexibility (i.e. the flexibility that is implied in a recourse type model) then our ability to satisfy sustained yield and still harvest timber is limited. There is already considerable uncertainty associated with information used for long term planning. Climate change simply augments the uncertainty. Thus, an important question to begin thinking about is: To what extent does the pursuit of sustained yield limit our ability to adapt and adjust to changing conditions?

Another finding in this thesis is that climate change impacts are a function of institutional settings and property rights configurations. Property rights configurations determine who is exposed to risk. If property rights are configured in a way such that it is society that is exposed to uncertainty, then taking account of risk in assessment models may be moot because risk aversion is of little importance for public projects. If, on the other hand, property rights are configured such that individual firms or landowners are exposed, then risk aversion is important in terms of impacts assessment and behavioral responses.

The discrete stochastic programming models provide solutions for short run and long run harvests. Short run harvests are the harvest prescriptions that are indicated for stage 1 harvest. Long run harvests are harvest solutions that are indicated for stage 2. The implication of recourse for harvesting in the short term depends on the information structure. If the individual has perfect information of the past but not the present, then recourse has little effect on harvest choices in the short run. However, recourse does influence harvest choice in the long run. Recourse under an information structure of perfect information of the past but not the present does, however, provide flexibility relative to stage 2 harvest.

A final finding of this study is that risk models can be used to look at different types of adaptation. One form of adaptation is to make adjustments in decisions as uncertainty is resolved. This is the type of ex – post adaptation that is modeled with a DSP risk model. Another example of adaptation is risk prevention. Risk prevention activities are ex-ante. Risk prevention involves actions taken by decision makers to influence the probability of a preferred state of nature occurring. In the case of this study a preferred state of nature is a high yield outcome in each stage. If the decision maker has the opportunity to influence the probability of a high yield outcome occurring, then the expected value of returns increases (see Chapter 10). If the benefits (in terms of increased expected value of returns) are higher than costs then planned adaptation investments in risk prevention are economically viable activities.

Discussion of some policy implications of climate change and risk in forest management

This section provides a general discussion of some possible policy implications of climate change and risk for forest management. Some of the discussion in this section is based on the results of this study and some of the discussion is based on application of the general risk and uncertainty literature to the issue of forest management decision making given climate change and risk.

One of the messages that comes from this study is that climate change and risk have the potential for real economic impacts and also the potential for influencing optimal harvest plans. Thus, ignoring climate change and risk may result in mis-estimation of forestry benefits and sub-optimal planning decisions. Undertaking research to better understand the impacts of climate change and risk in forestry is a start, but given the long growth cycles that are inherent in forest management it may also be opportune to begin thinking about the kinds of changes in forest policies that might be pursued in order to facilitate the identification and implementation of adaptation in the near term. For example, some possible responses to changes in risk exposure include risk prevention (discussed in Chapter 10), risk reduction (reduce the magnitudes of possible negative impacts), risk spreading (e.g. insurance schemes), portfolio diversification (discussed in Chapter 8), and adopting more flexible forest policies (e,g, build the capacity for adaptive management into forest policy).

The current risk literature shows that, in most cases, undiversified financial portfolios have higher variance than diversified portfolios. In this study we have attempted to make a link between benefits associated with diversification of a portfolio of financial assets with potential benefits of diversification of a portfolio of management options for forest management. Increasing the range of management options available to forest managers may be an important strategy for reducing uncertainty and risk resulting from climate change and other sources.

One practical example of how this might be implemented is in terms of forest practices and reforestation policies. Current policies and practices often involve clear-cutting areas and reforesting harvested stands with the same species that was harvested from the site. This strategy will likely lead to a managed forest made up of a narrow range of species (some of which may be mal-adapted to future growing conditions) growing in even age stands. But the question that needs to be addressed is what are the risks to future returns from this type of undiversified forest compared to a structurally more diversified forest and if a structurally more diversified forest is deemed desirable - what kind of policy adjustments are needed in order to provide the kinds of incentives that will result in this new type of forest. For example, an alternative strategy could be to (a) encourage the use of a broader range of forest management systems (e.g. mixed wood, agro-forestry systems, etc), and (b) encourage the reforestation of areas with a broader mix of species. Such a strategy could lead to a more diverse portfolio of forestry assets and this should reduce the risk associated with future forestry returns. The potential for reduced risk would, of course, need to be compared with whatever implications there might be relative to expected economic benefits of such a restructured forest.

Our ability to deal with the expected uncertainties inherent in climate change and forest management in general, may require some fundamental changes in our approaches to management. In chapter two, the concepts of evolutionary changes for unstable systems that are continually changing and redefining was introduced. It was noted that functional diversity, management systems and institutional structures that recognize and account for uncertainty and unpredictability, and social structures that encourage adaptive management are important system features relative to adaptability Some natural resource economists (Castle et al. 1996) argue that maintaining the quantity and quality of the stock of natural capital should not be the goal of sustainable development. Rather, the focus of sustainability should be on maintaining or increasing flexibility and adaptive capacity. These concepts did not emerge in response to uncertainties introduced by climate change. However, climate change does increase the level of uncertainty and unpredictability that we face in forest management. Therefore, the arguments for building flexibility and adaptive management into our current thinking regarding resource management and our current policies for resource management are strengthened.

Qualifications and considerations

The research in this thesis has covered a wide range of topics in a relatively new area of research. In order make the study manageable the study context was simplified, the approaches were generalized and some simplifying assumptions were adopted. Therefore the results presented in this study should be viewed with these considerations in mind. A number of specific areas that are components of this study would probably merit further analysis in order to improve and further refine the analytical foundation. In this section we identify and discuss what we view as the key qualifications and considerations relative to the analysis and results presented in the study. Some of these issues are raised again in the section following that deals with areas for future research.

One of the first issues to note is that the relationship between climate variables and yield was estimated using cross-sectional data. A similar approach (called Ricardian analysis) was used by Mendelsohn et al. (1994). The use of cross sectional data to estimate a yield function for this study assumes that future yields for our study site will be similar (all other factors equal) to current yields of stands growing on sites with climate that match future conditions. It should be recognized, however, that other factors that are not part of the estimated yield relationships may also explain yield at a particular site. These unobserved factors could result in biases with respect to yield predictions based on models estimated from cross sectional data. Three ways to reduce this bias would be to significantly expand the number of samples, expand the range of sites within the sample, and obtain data from permanent sample plots that include re-measurement data (i.e. growth data). Access to a data base that includes both stock and growth data would permit the simultaneous estimation of parameters for both yield and growth functions.

A related issue concerns the range of climate data in our cross-sectional database relative to the range of future predictions of climate variables (discussed in Chapter 6). As noted in Chapter 5, the source of data for estimating the yield function in this study is the CIPHA data base. The CIPHA project includes plots ranging from southern Manitoba to the Yukon. A comparison of the range of average annual temperatures for plots in the CIPHA data base with the predicted ranges of future average annual temperatures shows that the maximum average annual temperature for plots in the CIPHA database is lower than the maximum annual temperatures predicted in the years 2050 and 2080. Thus, our yield model is being used to predict future yields at temperature values that are outside the range of the data from which the yield model was estimated. This might be a significant concern if future predicted temperatures were outside the range of aspen in North America. However, aspen in North America has a wide range. It occurs from Virginia to Alaska. Thus aspen occurs on sites that are well within the range of future predicted temperatures predicted for the Calling Lake site.

Another issue for this study is that all of our predictions of future yields and variances are based on a stand yield model. One of the limitations that this imposes is that we are unable to consider the possibility that variances in stand yield predictions changes with time. However, if we had estimated a growth function in conjunction with a yield function, it would have been possible to model how variance changes dynamically. That is to say, we would have been able to look at the possibility that variance of stand yield are non-stationary as stands grow or as prediction period increases. One of the reasons why estimation of a growth function was not possible was due to data limitations. However, if at some point in the future data on both growth and yield becomes available for a broad cross-section of sites then a more sophisticated approach to variance modeling would be possible.

Another issue pertaining to the yield modeling component of this study is that we were not able to obtain satisfactory estimations using yield functions that are generally accepted within forest science (see Chapter 6). We were, however, able to estimate a non-traditional functional form (i.e. the reciprocal model) that provides a good fit with our data. As discussed in Chapter 6, this was mainly due to the fact that our sample was restricted to older and larger trees). Nevertheless, although the functional form estimated in this study is an acceptable start, it should not be viewed as the final answer relative to yield prediction for aspen under climate change. Further attention needs to be paid to the development of improved approaches for predicting how climate change will affect both stand yields over time and the variance around predictions of stand yields.

As noted in Chapter 7, the measures of uncertainty in climate variables for this study are obtained from ranges of predictions from different combinations of GCM models and SRES scenarios. There are two main issues that should be noted as a result of this approach. The first issue pertains to scale. GCM models provide predictions of future climate variables for large geographic areas. Thus, our approach results in a measure of uncertainty in a climate variable over a large geographic area. For this study we have applied this uncertainty to uncertainty in climate variables for a particular site. Generally, we would expect the variance in climate at a particular site to be higher than the variance in a climate variable for a large area. Thus, our approach provides a lower bound estimate of climate variance. A second issue relative to using ranges of GCM model / SRES scenarios to generate measures of uncertainty is that these models are tending to converge with each generation of new model. As noted in Chapter 7, however, it is possible that there may be factors that will affect future climate that we are currently unaware of and that are not part of the suite of current climate models. Here again, the approach used to obtain estimates of uncertainty in the future value of climate variables may be a lower bound estimate of what the true uncertainty is.

As was noted in Chapter 8, a number of previous studies have found that price uncertainty is a major source of uncertainty relative to management decisions. Moreover, some studies find that decision makers tend to be as much (if not more) concerned about price risk as uncertainty in biological factors. This study has assumed a constant price for the planning horizon. However, uncertainty in timber price risk could be an important source of uncertainty in climate impact analysis. This is an area that should be considered in future analysis of climate impacts.

An area that has not been considered within this study is the possibility that climate change in central Alberta could lead to massive mortality due to unforeseen and previously un-experienced events. Two simplifying assumptions in our biological model are that a) climate change does not change the fundamental growth relationships that are reflected in our yield function, and b) climate change does not change disturbance patterns. However, major system failures are possible if there are non-linearities, non-convexities, and thresholds within growth, yield and survival functions that are breached as a result of change climate. Possible causes of major ecosystem failure (however unlikely) include major pest and disease infestation, drought, wildfire, extreme weather, or dieback due to changed climatic conditions exceeding physiological tolerances. Risks due to these types of catastrophic impacts are not incorporated into the biophysical model in this study. The reason for excluding these effects was that our study site is not in an area close to a transition zone and because we are only considering impacts up to the year 2070. Widespread mortality and dieback is therefore less likely. However, the possibility of some unforeseen set of circumstances leading to forest ecosystem failure and mortality should not, perhaps, be entirely ruled out – especially for studies looking at longer time frames or for study sites situated in or near transition zones. One way to address this would be to include a mortality component into the biophysical models.

Future research

A number of potential extensions of the research conducted in this study have emerged. One useful extension would be to incorporate additional sources of risk and also to examine alternative ways of assessing risk. For example, as was noted in Chapter 8 and in the previous section, the sources of risk evaluated in this study are limited to uncertainty in climate variables and uncertainty in yield model parameters. Stumpage price and interest rates are incorporated deterministically. Moreover, in the case of variables that are considered to be random, only climate variable risk changes over time and the procedure for incorporating dynamic risk in these variables is limited by data availability. A useful extension of the analysis presented in this study would be to include both price risk and disturbance risk into the analysis as additional sources of risk. The inclusion of risk around these variables would necessitate the consideration of other methodologies for modeling risk and variance. For example, in the case of disturbance risk, the methods suggested by Reed (1984) where the number of major disturbance events in a planning period is considered to follow a Poisson distribution and the period of time between major disturbances follows an exponential distribution could be considered. Reed (1984) shows how the rate parameter from a Poisson distribution can be included in a Faustmann soil expectation value model. In terms of price risk, a number of studies (Reed and Haight, 1995; Conway 1999; Buongiorno 2001; Gong 1992; Haener and Adamowicz 2000) have incorporated price risk into forest economic models. These studies model price risk in various ways, however, in all cases price risk is allowed to vary over time (note studies show that price can be modeled as a random walk in which case it may be non-stationary resulting in increasing variance with respect to time). In a climate change context the consideration of price risk may be important for two reasons. First, the expected values of stumpage prices may be decreasing as a result of climate change (Sohngen and Sedjo 1995) - thereby offsetting potential benefits from positive productivity effects. Second, the variances of price may increase as a result of climate change, thereby increasing the economic costs of uncertainty for forestry returns (a lack of data on historical variation in stumpage prices or in information that might provide insights into future price risk is a significant limiting factor).

Another area that may be worth including in future studies would be to also consider the possibility that the variances of harvest yields and ending inventory yields are also time dependent. This study does not allow variances in yield parameters to change over time. The possibility that yield parameters may change over time and the resulting implications for certainty equivalent and optimal harvest choice remains an unanswered question.

Another extension of this study would be to apply similar types of models using a wider mix of species over a larger range of sites (possibly varying over some latitudinal gradient so that both negative and positive climate related site effects are captured). This could be accomplished by increasing the number of stand types in the choice set to include more species, more initial age classes, and a broader range of sites as input data for the programming models. Since the basic structure of the risk models would not change, the extension to a larger problem would be fairly straightforward. The main challenges would be to obtain high quality cross sectional mensuration data for other species and to estimate the necessary expected values, variances, and covariances for a larger set of objective function and constraint variables.

One of the issues identified in Chapter 8 was that the number of prospects included in the forest management portfolio was relatively small, and was limited to forest land-use options. If our decision maker is a private landowner, then one of the options that may be available is to not invest in forestland in the first place, and/or invest harvest revenues in alternative types of investments. A useful extension of this analysis would be to provide the decision maker with the opportunity to include non-forest investments (e.g. stocks, bonds, real estate) in their portfolio.

An aspect of the analysis conducted in this study that was only briefly touched on concerns the role of institutional contexts and property rights configurations relative to problem formulation. As was noted, assumptions regarding how property rights are configured may have significant implications for the distributions of benefits and costs and even for the need to consider risk preferences. In Canadian forestry, the context for decision-making can in some ways be viewed as a principal agent type of decision-making scenario. The forest industry (the agent) obtains the property rights for harvesting timber and in exchange agrees to manage the land base to satisfy other objectives of the crown (i.e. the principal). An interesting extension of the analysis in this thesis would be to define the problem as a principle agent type of problem with clearly defined property rights configurations and formulate the risk models accordingly.

An area that requires further study is the identification of who bears climate risks and the kinds of practical risk management tools that are available to forest managers. This study has only briefly touched on the questions of how impacts may vary depending on who bears risk. However, we have not made any attempt to assess how climate risks are actually distributed. Moreover, we have made little attempt to provide a review of the kinds of instruments that forestry managers currently use to manage risk and whether these instruments are adequate for addressing expected change in risks under a changing climate. A useful component of the institutional analysis proposed in the previous study would be to undertake an analysis of the distribution of climate and other risks to forestry stakeholders, and to provide a comparative assessment of existing approaches and instruments for managing risk across various forestry jurisdictions in Canada.

Another potential area for future research would be to consider alternative types of information structures and risk preference assumptions. The recourse model analysis provided in Chapter 10 assumes an information structure of perfect information of the past but not the present. Also, the model assumes risk neutrality. In many ways, both of these assumptions are a reasonable representation of the current reality in Canadian forestry. However, one management option that managers might want to consider is improving their knowledge of yields so that they have perfect information of the past and present. Also, in order to apply the recourse model to a private woodlot situation, it would be necessary to include an individual's risk preferences within the model. Therefore, two useful extensions of the recourse model would be to consider the case where the decision maker has perfect information of the past and the present, and to include risk preferences in the recourse model objective function.

In some respects the study presented in this dissertation might be viewed as first generation economic analysis of climate change effects on forestry. It is a first generation analysis in the sense that "prior" knowledge of parameter distributions under future climate change for variables important for forest planning and economic analysis do not exist. As a result, the methods are highly generalized, and the problem context is stylized. Also, distributions of future random variables are estimated using fairly straightforward Monte Carlo simulation methods. There may be questions about the degree to which the results of this dissertation can be directly applied for use in operational decision making. Managers, likely will require more context specific information for on the ground decision making and policy development and this will require additional research (perhaps to some degree along the lines described in the previous paragraphs in this section). Thus, we expect that as the demand for knowledge and information about climate change grows, a second generation of economic and forestry research will emerge. As we have noted throughout this study, uncertainty is an important dimension of climate change impacts analysis. Generally, including uncertainty requires some information about the distributions of parameters used in models. One option for the second generation of climate impact and adaptation models would be to continue to rely on simple Monte Carlo simulation techniques to estimate the distributions of model parameters. This approach, however, ignores the possibility that some prior knowledge about model parameters might already exist. An alternative approach would be to build on existing knowledge and refine our understanding of uncertainty in models parameters in climate impact models by using Bayesian statistical modeling methods. The Bayesian concept is based on learning. The approach starts from the premise that there is some prior knowledge of the distribution of parameters and/or model outcomes and then uses new data and knowledge to update the distribution information leading to posterior distributions of random variables and model outcomes that are based on previous knowledge but are updated with new knowledge and data collection.

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