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Forward Displacement of a 4-UPS Hybrid Cable-Driven Parallel Manipulator

Paul W. Sanford, Juan A. Carretero*
Mechanical Engineering, University of New Brunswick, Fredericton, Canada
*juan.carretero@unb.ca

***Abstract*—The forward and inverse displacement problems (FDP, IDP) are solved for a 4-DOF hybrid cable-driven parallel manipulator. This manipulator uses three cables and one extensible rod to manipulate the mobile platform, and shares some kinematic similarities to the well-studied Stewart-Gough platform. Through the use of the dual quaternion representation of a manipulator pose and the Levenberg-Marquardt algorithm, the FDP was numerically solved at a variety of poses throughout the workspace. This method was shown to be useful at determining the correct pose from an initial guess in close proximity. The inverse displacement and velocity problems are also solved for this manipulator using the standard geometric model for cable-driven manipulators and the method of reciprocal screws.

***Keywords*— cable-driven parallel manipulator; telescoping actuator; forward displacement; inverse displacement; Jacobian matrix; Levenberg-Marquardt algorithm**

I. Introduction

Advancements in both theoretical understanding and computational ability have allowed for developments in the field of robot kinematics. This has allowed new types of manipulators to be designed and implemented. This shift has driven parallel manipulators to be an increasingly viable option over their serial counterparts, especially where accuracy and speed are desired characteristics. Within the field of parallel manipulators, cable-driven parallel manipulators (CDPMs) replace conventional rigid prismatic links with cables held under tension by winches. This developing field of study has allowed for analyses to be made toward manipulators with extremely large workspaces, incredibly high accelerations, high degrees of redundancy and efficient movement due to a decreased mass of the kinematic chains attached to a tool at the end effector [1], [2]. Applications of an extremely large workspace include the Skycam [3], patented in the 1980s and used to film sporting events in stadiums ever since, as well as mechanisms to orient a large reflector for a radio telescope [4].

The kinematic analysis conducted in this work aims to solve the position problem of the manipulator two ways; a forward problem that determines the pose of the end effector as a function of the cable lengths, and an inverse problem that determines the required cable lengths at a particular pose. The inverse displacement problem (IDP) of CDPMs (and equivalently the Stewart-Gough platform [5], [6]) is simpler than the equivalent problem for standard (rigid) serial or parallel manipulators. Contrary to these other manipulators, each kinematic chain can be modelled as a straight line between two points and can be solved using something very similar to a vector loop [1]. This can be understood geometrically far more simply than even a standard 6-DOF serial manipulator such as the PUMA 560 [7]. One of the benefits of using a CDPM is that regardless of the number of cables, the IDP is equally simple. This is true for redundant CDPMs with more cables than DOF in the workspace as well as underactuated manipulators with fewer cables than DOF [1], [8]. The mechanism discussed here has four actuators in a spatial 6 DOF workspace so it will not be capable of controlling the position at the same time as arbitrary orientations.

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The forward displacement problem (FDP) of Stewart-Gough and CDPM-style mechanisms was not solved in the closed form until long after they were introduced. 30 years after Stewart's creation of the 6-DOF parallel manipulator the FDP was solved in the closed form by Husty [9] as the solution of a degree-40 polynomial, two years after Raghavan showed that there were up to 40 solutions to the problem [10]. Since then, many more examples of kinematically similar manipulators have appeared, differing by the number of cables or linear actuators as well as the configuration of the anchor points of the actuators. The commonality between these manipulators is the n-UPS configuration, where there are n kinematic chains with a universal joint (U) connection to a stationary frame, a prismatic actuator (P), and a spherical joint (S) connection to a mobile platform or end effector. For CDPMs, controlling the length of a cable under tension using a winch behaves as the prismatic actuation. Many of these configurations have been evaluated to determine the number of solutions to the FDP, or a full analytical solution has been found. With the 4-UPS manipulator of interest, the maximum possible number of solutions to the FDP has been found to be 216 [11], however the number of real solutions for a given set of cable lengths is less certain. A general mathematical model for n-UPS manipulators has been created by Wampler in 1996 [12] which uses dual quaternions and Soma coordinates to represent the pose, but this is not a full analytical solution in the closed form.

This representation of the FDP can then be solved numerically using a root finding algorithm to determine individual real solutions to the manipulator pose based on a set of cable length inputs.

This paper aims to present developments in the theory behind a 4-UPS hybrid CDPM, using 3 cables under tension and one extensible rod that can be used in tension and compression throughout its workspace. The extensible rod behaves in a kinematically similar way to the cables, but can both push and pull. One of the limitations of exclusively using cables under tension is that the maximum possible acceleration for a suspended CDPM, which has the base of each of the cables above the mobile platform (like a crane), is that of gravity. Suspended CDPMs can therefore also not provide any greater downward force than the weight of the tool and any object that it holds. By replacing one of the cables with an extensible rod it is possible to overcome that obstacle while keeping all of the attachment points above the mobile platform. This could be beneficial in an industrial setting if the manipulator were placed above a conveyor belt or work surface where cables below the surface would cause interference. Conventional CDPMs with cables above and below the mobile platform must remain under tension and act antagonistically, inducing an internal force and/or moment on the mobile platform.

This work is an extension of prior work at the Robotics and Mechanisms Laboratory at the University of New Brunswick that implemented a high packing-ratio linear actuator (HPRLA) [13] as the extensible rod. With a few basic assumptions about the geometry and kinematics of the actuators, this manipulator can be explained as an extension of standard parallel manipulator theory.

II. Cable Manipulators Theory

A. Background

In its simplest form, the kinematics of CDPMs very closely resembles the extensively studied 6-UPS Stewart-Gough platform, which uses six telescoping legs from universal joint connections on a fixed base frame to six spherical joints found on a mobile platform or end effector with each prismatic joint acting in parallel [5], [6].

A geometric model of the hybrid CDPM is shown in Fig. 1. On the stationary frame there are proximal anchor points A_i representing a universal connection where the cable winches are fixed. On the mobile platform there are distal anchor points B_i which can be modelled as spherical joints where the cables attach to the mobile platform. These cables under tension behave very similarly to rigid prismatic joints. The assumption for this analysis is based on the standard geometric model of cable manipulators [1], [14]. This set of assumptions for the displacement of the manipulator ignores effects such as cable sag, changing position of cable pulleys and cable elasticity, treating a cable under tension as a perfectly straight and rigid connection between the anchor points. In order to maintain control and for this model to be useful each of the cables must remain in tension at all times.

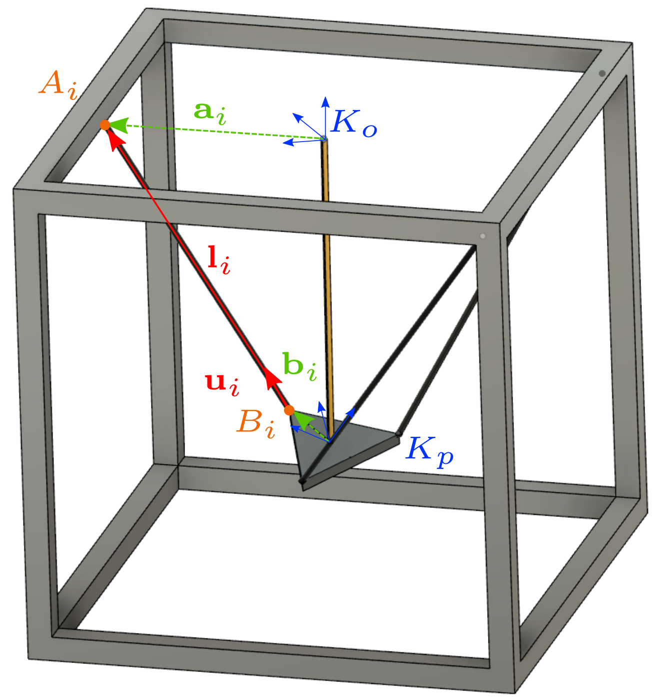

Fig. 1. Geometry of the hybrid CDPM with a central actuator and three surrounding cables

B. Inverse Displacement

The inverse displacement problem for CDPMs is fairly straightforward from vector addition and the use of a rotation matrix for a frame transformation from the fixed to the moving frame. The transformation from the base frame K_o to the moving frame K_p is defined using the pose, a combination of a position vector $\mathbf{r}$ and a rotation matrix $\mathbf{R}$). That is,

$$\mathbf{l}_i = \mathbf{a}_i - \mathbf{r} - \mathbf{R}\mathbf{b}_i \tag{1}$$

$$l_i = \|\mathbf{l}_i\| \tag{2}$$

where $\mathbf{l}_i$ is a vector from B_i to A_i, l_i is the Euclidean norm of that vector, $\mathbf{a}_i$ is the vector in K_o from the origin to A_i, and $\mathbf{b}_i$ is the vector in K_p from the mobile platform origin to point B_i.

The IDP of a CDPM finds the required cable lengths l_i that allow the mobile platform to reach the desired pose based on the geometry of the frame mostly identified by the position of the ground attachment points $\mathbf{a}_i$ as well as the geometry of the mobile platform identified by positions $\mathbf{b}_i$ expressed in terms of frame K_p. During the solution in (1), each vector quantity is separated into a three-dimensional vector so the components are known. In (2) the magnitude is found, which is the only requirement to obtain the required position of the moving platform on a CDPM and can be used to control a cable winch.

III. Jacobian and Structure Matrices

In order to fully constrain a n-UPS CDPM in an m DOF space, there is a requirement for $n \geq m+1$ cables. With fewer cables than DOF, linking the statics to the displacement problem can help obtain a solution at a given pose. The solution of the IDP provides the cable lengths and directions, which is essential to the solution of the statics as any cable can only produce force along the direction of the prismatic actuation in tension. The structure matrix is used to linearly map joint forces to external forces and torques of the mobile platform. The structure matrix is defined as the transpose of the Jacobian matrix.

The solution to an n-UPS manipulator has been found in general using the method of reciprocal screws [1], [15]. This allows for the inverse Jacobian, $\mathbf{J}_q$, to be an identity matrix $\mathbf{I}_n$ and the forward Jacobian, $\mathbf{J}_x$, to be dependent on the cable directions and the geometry of the mobile platform. That is,

$$\mathbf{J}_x^T = \begin{bmatrix} \mathbf{u}_1 & \dots & \mathbf{u}_n \\ (\mathbf{b}_1 \times \mathbf{u}_1) & \dots & (\mathbf{b}_n \times \mathbf{u}_n) \end{bmatrix} \tag{3}$$

$$\mathbf{J}_q \dot{\mathbf{q}} = \mathbf{J}_x \dot{\mathbf{x}} \quad \text{or simply} \quad \dot{\mathbf{q}} = \mathbf{J}_x \dot{\mathbf{x}} \tag{4}$$

where $\dot{\mathbf{x}}$ is the desired linear velocity of the mobile platform, $\dot{\mathbf{q}}$ groups in one vector the cable joint velocities, and $\mathbf{u}_i$ is a unit vector along cable i which is simply obtained as $\mathbf{u}_i = \mathbf{l}_i / l_i$.

Likewise, the static force problem can be solved as:

$$\mathbf{J}_x^T \mathbf{f} + \mathbf{w}_p = \mathbf{0} \tag{5}$$

where $\mathbf{w}_p$ is the external wrench applied to the mobile platform while $\mathbf{f}$ is a vector of the joint forces.

Important to note that, for typical CDPMs, all f_i, the individual components of $\mathbf{f}$, are bounded by the force limits related to the cable forces. As such, since cables are required to always be in tension, all f_i are typically strictly positive. However, for the hybrid CDPM in this work, given that the HPRLA can apply/sustain forces in tension and compression, this last constraint is relaxed for that specific actuator while, for the cables, positive tension is still required as for typical CDPMs.

IV. Forward Displacement Problem

A. Formulation

The mathematical model for the 4-UPS manipulator is very similar to that of the 6-UPS, where a pose of the mobile platform is found from a set of cable lengths. The present analysis is heavily based off of a solution of the FDP of the Stewart platform using Soma coordinates and dual quaternions to represent position and orientation [12]. With this method, one quaternion, ($\mathbf{e}$), is used to represent the orientation of the mobile platform frame ($\mathbf{R}$) while the other quaternion, ($\mathbf{g}$), is used to represent the position of the origin of the mobile platform frame ($\mathbf{r}$). Although this representation uses 8 values rather than the 6 required for a homogeneous transformation matrix (3 rotation and 3 translation), the formulation of the quadratic equations to be solved for the FDP is far simpler than other representations of the same problem, and the quaternions can be treated as standard 4×1 vectors with Wampler's formulation [12]. This solution method again uses the geometry of the proximal and distal anchor points alongside equations for the displacement from each point to search for solutions.

With only the length known for each cable, the set of kinematically-possible solutions for the location of a distal anchor point B_i from the known proximal anchor point A_i forms a sphere of radius l_i. If all of the distal anchor points coincided, the set of feasible solutions would be the intersection of each of these spheres. However, assuming the moving platform is not only a point (*i.e.*, non-zero $\mathbf{b}_i$ vectors), the solution becomes a problem of matching the geometry to the location of the spheres. Wampler [12] devised a solution to this problem using the dual quaternion representation of the pose.

Without loss of generality, the reference point on the mobile platform can be chosen to intersect the spherical joint for the central HPRLA rod, so $\mathbf{b}_1$ becomes a zero-vector. This simplifies some of the equations to reduce computational complexity. Due to the geometric basis of the problem and the information available, this becomes a solution of a system of 6 equations that are multivariable and quadratic. These equations have been converted to a form where the desired solution of each is 0 to allow for root finding computational tools to be used. For these equations, each of the quaternions ($\mathbf{e}$ and $\mathbf{g}$) are treated as 4×1 column vectors, where the first element represents the real component, and the others represent the imaginary components [12]. Therefore, the system can be written as

$$\begin{aligned} F_0 &: \mathbf{e}^T \mathbf{e} - 1 = 0 \\ F_1 &: \mathbf{g}^T \mathbf{e} = 0 \\ F_2 &: \mathbf{g}^T \mathbf{g} - l_1^2 \mathbf{e}^T \mathbf{e} = 0 \\ F_{3-5} &: \mathbf{e}^T {}^1\mathbf{M}_i \mathbf{e} + 2\mathbf{g}^T {}^2\mathbf{M}_i \mathbf{e} = 0 \end{aligned} \tag{6}$$

In the above, F_0 shows that the quaternion representing the rotation must be a unit quaternion while F_1 comes from the definition $\mathbf{r} = \mathbf{g}\mathbf{e}^T$ for the position vector $\mathbf{r}$. F_2 and F_{3-5} represent loop closure equations for the four links, knowing that $\mathbf{b}_1$ is defined as a zero vector. Finally, matrices ${}^1\mathbf{M}$ and ${}^2\mathbf{M}$ are 4×4 matrices that depend on the geometry of the system $\mathbf{a}_i$, $\mathbf{b}_i$, l_i and l_1, where ${}^2\mathbf{M}$ is a skew-symmetric matrix such that ${}^2\mathbf{M} = -{}^2\mathbf{M}^T$. More specifically,

$$\begin{aligned} {}^1\mathbf{M}_i &= (\mathbf{b}_i^T \mathbf{b}_i + \mathbf{a}_i^T \mathbf{a}_i - l_i^2 + l_1^2) I_4 \\ &- 2 \begin{bmatrix} \mathbf{a}_i^T \mathbf{b}_i & (\mathbf{b}_i \times \mathbf{a}_i)^T \\ \mathbf{b}_i \times \mathbf{a}_i & \mathbf{b}_i \mathbf{a}_i^T + \mathbf{a}_i \mathbf{b}_i^T - \mathbf{a}_i^T \mathbf{b}_i \end{bmatrix} \end{aligned} \tag{7}$$

$${}^2\mathbf{M}_i = \begin{bmatrix} 0 & (\mathbf{a}_i - \mathbf{b}_i)^T \\ \mathbf{b}_i - \mathbf{a}_i & -\text{skew}(\mathbf{a}_i + \mathbf{b}_i) \end{bmatrix} \tag{8}$$

Using computational tools in MATLAB, it is possible to solve equations $F_0 - F_5$ to find 4×1 vectors $\mathbf{e}$ and $\mathbf{g}$ that provide a feasible solution. The solution represents a root of the set of quadratic equations.

Testing and validation was completed that compares the same set of parameters with the forward and inverse displacement problems and determines whether or not this method is capable of determining complementary solutions. This was completed by beginning with a desired pose ($\mathbf{r}$, $\mathbf{R}$), using the IDP to find the corresponding cable lengths, and using this information to begin the FDP. Due to the large number of solutions, it is expected that having an initial guess being too far from the desired solution will result in the FDP finding the incorrect pose. The initial guess (in quaternion form) fed into the solver was based on a close position from prior poses ($\mathbf{e}$, $\mathbf{g}$).

B. Multiple Solutions

The complexity of the FDP for parallel manipulators comes from the large number of solutions to the problem. There are theoretically hundreds of solutions to the geometric and static problems associated with this underactuated CDPM (4 cables in a spatial application), depending on the chosen geometry as well as the current pose of the manipulator. If all of the possible solutions were found in a global sense it would be necessary to determine the correct solution for use with a physical manipulator. For this to be true, the solution must be real, in the reachable workspace based on the constraints of the physical and theoretical manipulator, and will be dependent on the motion of the system and the refresh rate of the sensors and computations. Future work will be completed toward differentiating between multiple FDP solutions and ensuring the correct pose is reached, especially when passing through singular configurations.

V. Results

The method used to solve the FDP was a built-in MATLAB function that implements the Levenberg-Marquardt algorithm for non-linear and non-square problems. In the eventual implementation of a physical manipulator, solving the pose will be completed repeatedly as the manipulator traverses its trajectory, so at this point in time the algorithm is seeded an initial guess that is very close to the desired solution. The trajectory chosen to demonstrate the solution methods is a spiral traversing the workspace in the x, y, and z directions, with a desired orientation parallel to the ground, with no changing roll, pitch, or yaw throughout the trajectory. A trajectory was made in the task space with a changing position and constant orientation, then the IDP was solved to find the set of cable lengths, $\mathbf{l}_i$, that makes each pose, ($\mathbf{r}$, $\mathbf{R}$), possible, where $\mathbf{r}$ is the translation to the end effector position and $\mathbf{R}$ is the rotation matrix representing the orientation.

The planned manipulator is approximately a cube of side length 2 m. The placement of the proximal anchor points on the top of the manipulator are one at the centre (HPRLA) and three cables attached at a distance of 1 m, equally distributed radially around the centre and shifted down 0.1 m. On the mobile platform the distal anchor points follow a similar configuration, all on the same plane and with a radius of 0.1 m from the centre attachment. Testing was completed at 2000 positions along the path.

The error in the position, $e_{\mathbf{r}}$, was measured by finding the 2-norm of the vector between the desired position and the position found using the forward displacement problem. The error in the rotation, $e_{\mathbf{R}}$ was found as a measure of radians between the two poses. More specifically,

$$e_{\mathbf{r}} = \|\mathbf{r} - \mathbf{r}_{\text{FDP}}\| \tag{9}$$

$$(\mathbf{v}, e_{\mathbf{R}}) = \mathbf{R}\mathbf{R}_{\text{FDP}}^T \tag{10}$$

where ($\mathbf{r}$, $\mathbf{R}$) is the given pose while ($\mathbf{r}_{\text{FDP}}$, $\mathbf{R}_{\text{FDP}}$) is the pose solved by the forward displacement problem.

When the two orientations are the same (*i.e.*, zero error in the orientation), the resultant rotation matrix on the right side should be an identity matrix. In (10), the left side is represented as a vector $\mathbf{v}$ and an angle $e_{\mathbf{R}}$, where the angle in radians is the absolute error between the matrices rotated about the vector axis. This measure is converted to degrees as the final results are displayed.

The first set of tests was conducted while only giving the manipulator the initial pose at the start of the path. This was fed into the FDP as the initial guess to the Levenberg-Marquardt algorithm. Each sequential pose that the algorithm tried to find was based on a linear extrapolation of the FDP solution of the two prior poses. This should in theory allow the initial guess to be close to the desired solution, but the spiral trajectory that accelerated in three directions caused that guess to be inaccurate.

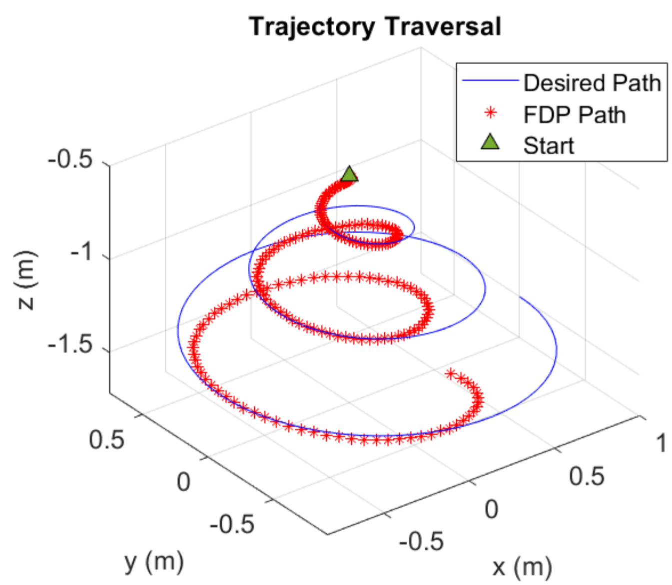

Fig. 2. Path of the manipulator solved using the FDP (no end effector pose feedback provided)

Fig. 2 shows the inaccuracies in the FDP in following a trajectory without any information about the true resultant pose of the manipulator. While using the cable lengths that should have made the manipulator follow the desired path, the deviation of the FDP path can clearly be seen. The manipulator follows a different but nearby trajectory, likely due to the solution branching out after passing through one of the many singular configurations near the beginning of the path. This

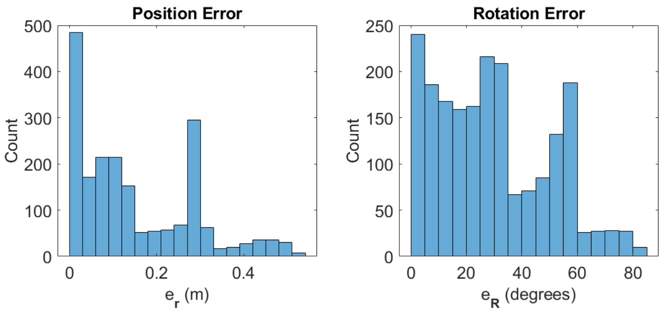

Fig. 3. Position and angle error in FDP (no end effector pose feedback provided)

error is quantified in Fig. 3 with a position error in the decimetre range and a rotation error approaching 90°.

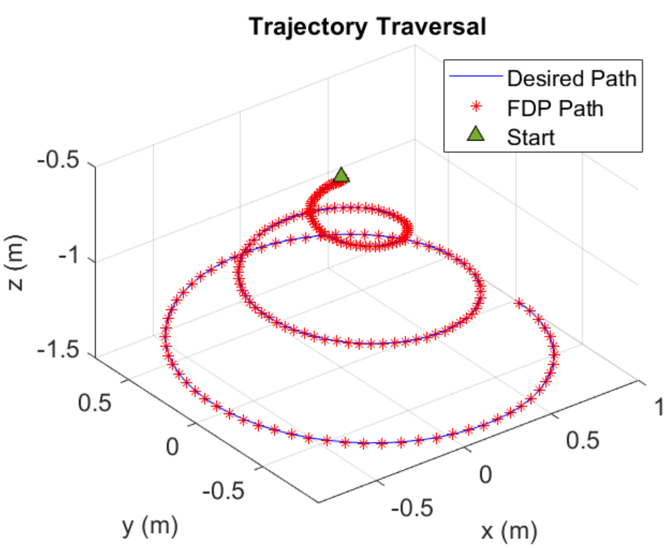

Fig. 4. Path of the manipulator solved using the FDP (end effector pose feedback provided)

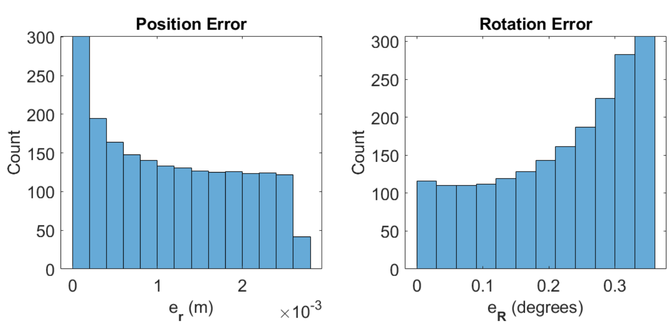

Fig. 5. Position and angle error in FDP (pose feedback provided to point algorithm toward the correct pose)

If the manipulator had access to some information about the true pose of the end effector, the propagation of error after crossing singularities could be quickly corrected. This is approximated here by offering an initial guess to the FDP solver that is skewed toward the desired path. This was completed by choosing an initial guess that was found as a linear interpolation between the prior manipulator FDP pose and the next desired pose along the trajectory. Fig. 4 shows the FDP path being almost perfectly coincident with the desired trajectory. The position and rotation error shown in Fig. 5 remain in the millimetre range and sub-degree range, respectively. This confirms that the correct pose is found each time to the accuracy level of the solver.

VI. Conclusions and Future Work

As was shown in Section V, the performance of the FDP was strongly dependent on the location of the initial guess. When no information was given about the true pose of the manipulator, the trajectory that was followed deviated from the desired course, most likely at a singular configuration of the manipulator. Information about the true pose was then given to the solver which allowed for the correct pose to be found, with each resultant pose arriving within 3 mm and 0.4° of the desired solution. This reinforces the need for a global method at determining solutions to the FDP to ensure that the correct path is followed, and supports the necessity of recording the end effector position to ensure the manipulator does not change configurations along a trajectory.

Further work will be dedicated into linking the statics and dynamics of the manipulator with the kinematics for trajectory planning, optimization, and determining the static equilibrium workspace. This manipulator will be developed into a physical apparatus therefore a thorough analysis of computational time will be conducted to develop real-time capable solutions for controlling this manipulator. A singularity analysis will be conducted in order to aid in the path planning process. The viability of trajectories for this manipulator will be studied to determine what continuous moves are possible in terms of position and orientation. This manipulator will be used to help quantify the limits of the HPRLA and how it can be used to improve an existing cable-driven parallel manipulator.

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