# Dynamic Phasor Modeling of Modular Multilevel Converters for Hybrid AC/DC Power Systems

by

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# Abstract

This thesis develops a time averaged model (TAM) and a dynamic phasor model (DPM) of a generalised modular multilevel converter (MMC) structure that can be used to represent four different MMC topologies. These four topologies include two different classes of next generation MMCs that are garnering attention for use in hybrid AC/DC power systems: (i) DC/DC/AC MMCs that combine DC/DC and AC/DC conversion stages into a single converter structure, and (ii) DC/DC MMCs. The former enables controlled power exchange between two DC systems and an AC grid while the latter is a key building block of future DC grids. A TAM is developed in the $\alpha\beta 0$ frame that includes all dynamics in a converter, barring switching harmonics. This model is useful when detailed studies of the converter dynamics are required, but the long simulation time of the switched model is an impediment. A DPM is then derived from the TAM that takes into account multiple harmonic components to ensure an accurate representation of the practical converter behaviour across a wide range of operating points. This model permits the elaboration of a steady-state solution procedure to solve for the full state solution at

an arbitrary operating point, enabling systematic study of the DC/DC/AC and DC/DC MMCs using industry standard numerical software tools. Extensive simulations and steady-state analyses are carried out to investigate the converters' behaviour, including how the current stresses and capacitor voltage ripples change due to varying parameters such as DC and AC systems power flows.

# Dedication

*To my wife, without whom all of this would not have been possible. I admire your bravery for agreeing to leave behind your life, and become my own.*

# Acknowledgments

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# Nomenclature

| Acronym | Long Form |
|---|---|
| AC | Alternating Current |
| AT | Auto-Transformer |
| BESS | Battery Energy Storage System |
| BP-DCDC | Bipolar DC/DC Converter |
| BP-DCDCAC | Bipolar DC/DC/AC Converter |
| DAB | Dual Active Bridge |
| DC | Direct Current |
| DPM | Dynamic Phasor Model |
| EMI | Electromagnetic Interference |
| F2F | Front-to-Front |
| FFT | Fast-Fourier Transform |
| FBSM | Full-Bridge Submodules |
| HVAC | High-Voltage Alternating Current |
| HVDC | High-Voltage Direct Current |
| IGBT | Insulated-Gate Bipolar Transistor |
| KVL | Kirchhoffs Voltage Law |
| LCC | Line Commutated Converters |
| MMC | Modular Multilevel Converter |
| MP-DCDC | Multiport DC/DC Converter |
| MP-DCDCAC | Multiport DC/DC/AC Converter |
| MVDC | Medium-Voltage Direct Current |
| PV | Photovoltaic |
| SPWM | Sinusoidal Pulse Width Modulation |
| TAM | Time Averaged Model |
| VSC | Voltage Sourced Converter |

# Chapter 1

# Introduction

## 1.1 Background

In the last couple decades of energy research, DC transmission has gathered a great amount of interest from academics and industry all around the world. This can be seen by the increase of this topic in recent literature; the journal papers containing DC transmission as a subject have almost tripled the last twenty years compared to the whole of the last century, going by the search results in the *IEEEXplore digital library*. The reason is not because DC transmission can supplant AC transmission, the latter being well suited for many of its current applications, but because recent research seems to indicate that the future requires a combination of both of these technologies (AC and DC). High-Voltage Direct Current (HVDC) can be used for very long transmission distances, to interconnect unsynchronised AC networks, to transmit more power than AC in a comparable corridor size, and for power quality functions in existing and future grids [1]. Another very prominent application for HVDC revolves around the ever-increasing renewable energy systems; the underwater transmission of energy harvested from off-shore

wind farms [2]–[4], the transmission of PV energy without the need of AC conversion [5], [6], and connections to BESSs [7], [8]. More recently, the development of multiterminal DC systems and DC grids using HVDC is being pursued [9]–[11]

Despite its many benefits, challenges arise for the use of HVDC when several connections are to be made. Most of the previously mentioned applications require only a sending end and a receiving end, called point-to-point, and their operation and design are well understood [12]. However, as more sources and uses for energy are added to various locations, an evolution beyond basic point-to-point connections is inevitable. This becomes even more so the case when the existing AC grid is taken into consideration. Interconnecting multiple DC sources and loads, as well as including the existing AC grid, is a very active area in recent research. Such blending of DC and AC transmission technologies has been envisioned for the future of energy distribution in Europe, dubbed a *supergrid* [13]. This type of network is projected to be able to extend through a very wide area, incorporating HVDC transmission to support the existing HVAC grid, and be able to meet the ever increasing energy needs [14].

The interconnection of AC and DC links, to form what might well be the hybrid AC/DC power system of the future, comes with several challenges. One of the most prominent of these is the power conversion needed between AC and DC systems, and between different DC voltage levels. To achieve these conversions, power electronic converters are required. There is a myriad of converters capable of achieving AC/DC conversion, but for the purpose of HVDC only recent advancements in semiconductor technology allow some to be of consideration. Line commutating converters (LCCs) have been around for decades and are a mature technology, with high efficiency, and still recently being used in transmission projects around the world [15]; however, they suffer from the fact that rely on the AC system for commutation and absorb reactive power, as well as that power reversal requires the inverting of the voltage polarity. Voltage source converters

(VSCs) don't have any of the aforementioned disadvantages; moreover, they excel at controllability of power direction are therefore well suited for DC grid applications. Two and three level VSCs have the shortcoming of only being able to endure medium voltage rating before the large amount of series-connected semiconductors, usually IGBTs, start posing challenges such as high switching losses, reduced efficiency, and EMI problems [13].

Another type of VSC that has been gaining a lot of popularity since its inception is the modular multilevel converter (MMC) [16], a converter that distributes capacitors in the converter structure, instead of having them in parallel with the DC link, as illustrated by Figure 1.1a. For each phase in the converter there are two valves, or arms, each of them made up by N series cascaded submodules, each submodule a low-voltage DC/DC chopper cell with a capacitor. The output voltage of this converter has N+1 levels, giving it the ability of providing waveforms that are very close to a sinusoidal. Thus, the required filters at the output are considerably smaller than other VSCs. It also has low $dv/dt$, given the large number of steps in the AC waveforms, and can achieve high efficiency due to low switching losses.

Each of the N stacked submodules shown in Figure 1.1a, rated for a relatively low voltage, on the order of a couple of kilovolts, provides one either positive or negative step in the AC voltage, which results in a multi-level AC waveform that is very similar to a sinusoidal one. The submodules can be half-bridge or full-bridge, Figure 1.1b, the latter having the ability to operate in all four quadrants, in which both positive and negative voltages can be obtained in the submodule terminals, regardless of current direction [17].

It is widely considered that, for the upcoming need of interconnecting DC and AC grids, as well as the connection of different DC power systems, the MMCs is one of the most important converter technologies, and it's the reason they have been eagerly researched [13], [17]–[19]. The principle of this converter can also be applied to make

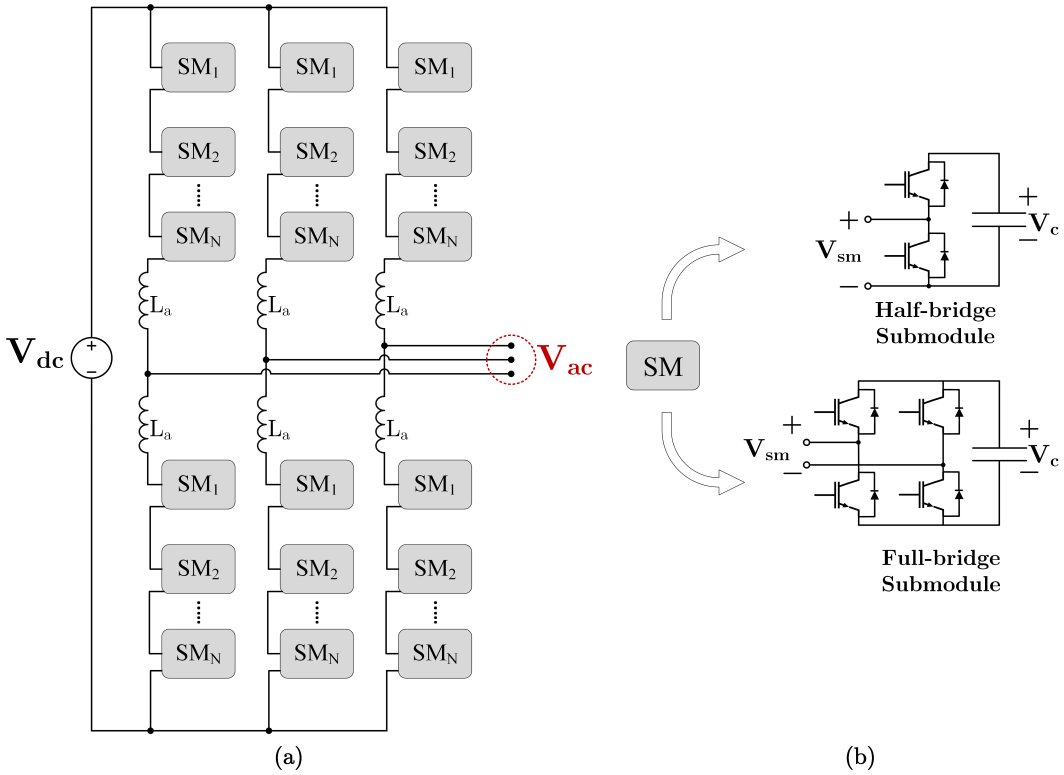



Figure 1.1: Conventional AC/DC MMC (a) three-phase topology, (b) standard submodule configurations

DC/DC converters, as will be discussed in the next section, but it is one more of the advantages making the MMC an attractive tool for the future of HVDC transmission.

## 1.2 MMC Topologies for Future DC and Hybrid AC/DC Power Systems

As previously stated, the MMC is the VSC of choice for HVDC grids. The AC/DC MMC has been well studied and its modelling and behaviour can be found in many papers, e.g., [20]–[24]. The application of the MMC concept to DC/DC conversion is,

on the other hand, much more recent and therefore not as well developed as the former. Several structures using MMCs have been investigated in published works with the objective of DC/DC conversion for use in HVDC. MMC type topologies that offer simultaneous DC/DC and AC/DC conversion have also recently emerged. These "multiport" converters are ideally suited for hybrid AC/DC power systems applications.

### 1.2.1 DC/DC Converters

One of the first MMC based DC/DC converter topologies proposed for an HVDC network is based on the well-known dual-active-bridge or front-to-front (DAB or F2F) topology [25], which was firstly investigated in the early 2010s [26]. The general topology of this converter is shown in Figure 1.2. As its name indicates, this converter is based on the principle of the dual-active-bridge, where two AC/DC MMCs are coupled through their AC sides by an AC transformer. The name front-to-front is used as a reference to the back-to-back converter configuration [27], in which two AC/DC converters are connected through their DC sides, to make an AC/AC converter. The DAB-MMC converter has been intensively studied, and many works on it can be found where its operation is described and analysed, e.g., [28]–[30]. The basic principle is that the DC power in one of the ports is converted in its totality to AC power, which then goes through voltage conversion in the transformer, then rectified back to DC by the other MMC. It can be seen that this conversion is a two-stage conversion, meaning that this converter will have twice the conduction and switching losses of a one-stage AC/DC conversion process. An advantage, however, is that it will have galvanic separation between the two DC ports. By selecting an appropriate turns ratio in the transformer, both buck and boost operation can be achieved, and since the AC section of the converter is completely internal it's not bounded by any frequency requirements of the AC grids, nor by its waveform. It also

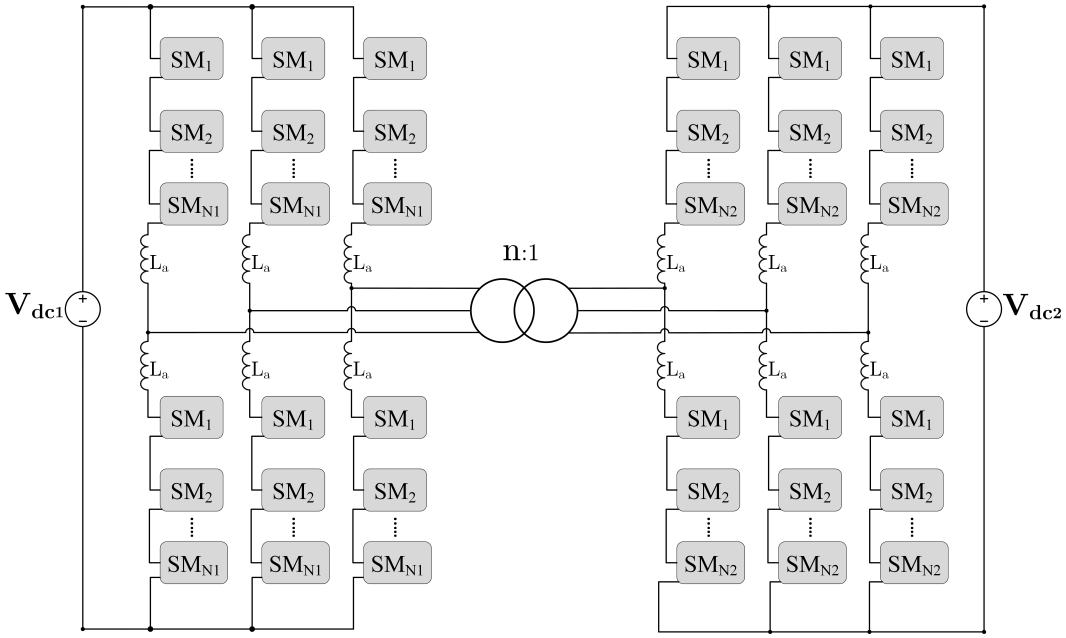



Figure 1.2: DAB (or F2F) MMC

offers bidirectional power flow.

Another topology for DC/DC conversion that uses the AC/DC MMC as a building block is the HVDC autotransformer (HVDC-AT), first introduced in [31] and whose general structure can be seen in Figure 1.3. As shown, the converter consists of two MMCs connected in series on their DC side, and their AC links are coupled together via a transformer. The first DC voltage, $V_{dc1}$ or high-level DC voltage, is formed by the sum of both MMCs' DC voltages, while the second DC voltage, $V_{dc2}$ or low-level DC voltage, is made by the lower MMC's DC voltage. The internal AC link is there to exchange average AC power between the two MMCs for capacitor voltage balancing; this power exchange, however, needs to be only a fraction of the dc power transfer, and therefore the power conversion is less than the full power conversion to be expected from the F2F converter.

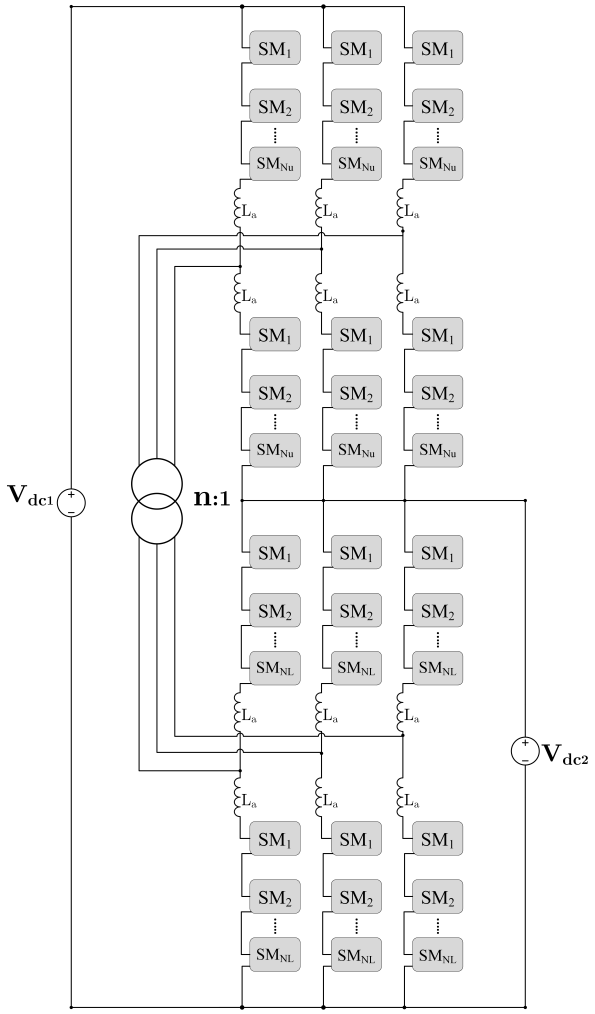



Figure 1.3: HVDC autotransformer [31]

The HVDC autotransformer is therefore a non-isolated partial power processing topology, where

$$P_{ac} = \left(1 - \frac{V_{dc2}}{V_{dc1}}\right) \cdot P_{dc} \tag{1.1}$$

is the power processed by the AC transformer. This converter is suitable for DC/DC applications in HVDC grids, while offering significant potential savings in capital cost

and losses relative to the F2F MMC; however, it does not provide galvanic separation, and DC fault blocking requires full-bridge submodules in the upper MMC [32].

In [33], a non-isolated DC/DC structure called tuned filter modular multilevel dc converter (TF-MMDCC) is proposed in which the power balance between the phase arms is maintained by a secondary power loop. The topology, shown in Figure 1.4, was also proposed by other research groups around the same time [34], [35] and is now commonly referred to as the Modular Multilevel DC Converter (M2DC). This converter bears structural similarity with the conventional MMC AC/DC converter, but requires an added filter in its output that has the purpose of blocking AC currents from going through to the second DC side. Internally circulating AC currents are needed to balance the capacitor voltages as in the case of the HVDC-AT, but without the use of an AC transformer. The filter can be implemented in different ways, including magnetics, passives, and active filtering. The magnetics solution is shown in Figure 1.4, where a zig-zag transformer is used. In [36] it is specified that the M2DC's intention is to work as a step-down, or a *buck*, converter. The M2DC can be used in a symmetrical monopole structure and employ the use of full-bridge submodules (FBSM) to achieve both step-up and step-down operation, known as buck-boost operation, and also to provide DC fault blocking [37]. Similar to the HVDC-AT, the M2DC is a non-isolated partial power processing topology where the amount of internal AC power processing is shown in Equation (1.1). A comparison between the mentioned converters (F2F, HVDC-AT, and M2DC) can be seen in [18], and a comprehensive overview of other non-isolated MMC-based DC/DC converters that share commonalities with the HVDC-AT and M2DC can be found in [19].

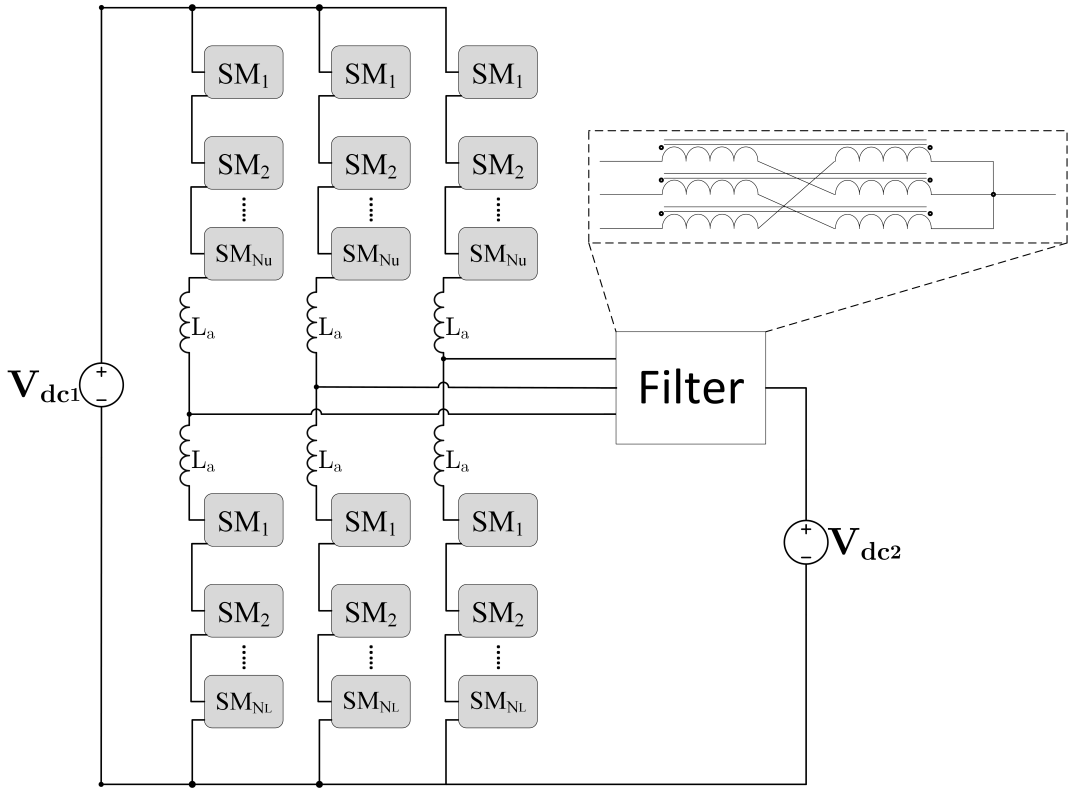



Figure 1.4: M2DC [33]–[35] with magnetics solution for the filter

### 1.2.2 DC/DC/AC Converters

All of the DC/DC converters described in Section 1.2.1 use internal AC currents to shuttle average AC power between arms for the purpose of capacitor power balancing. This suggests the potential to "tap" into this AC circuit for enabling power exchange with a local AC grid. i.e., to add an AC port to the existing DC/DC MMCs. The F2F and the HVDC-AT already have a transformer that, by adding windings, could be used to interconnect an AC grid [32], shown in Figure 1.5a and 1.5b, respectively. Of course, the price paid is increased transformer size and design complexity. The M2DC can also be endowed with an AC port, as shown in Figure 1.5c, where a transformer is added in

parallel with the filter block [36].

Patent [38] describes an M2DC where the AC port is instead taken at the converter midpoint. In [39], a delta/zig-zag transformer is used to combine the AC transformer and filter for the M2DC in Figure 1.5c. The resulting compact DC/DC/AC M2DC structure is shown in Figure 1.5d, where the total MVA rating of the delta/zig-zag transformer is less than the total MVA rating of all magnetics in Figure 1.5c. In [40], a generalised DC/DC/AC structure using a centre-tapped transformer is presented, that gives rise to two new topologies that could be used in future hybrid AC/DC power systems. The generalised topology is shown in Figure 1.5e, where three DC ports, d1-d3, can be used to connect to either a bipolar DC grid or two different DC grids. The control of these topologies allows for independent exchange of power between its DC and AC ports, and the state-space model is provided in [40]. In [41], four different multiport DC/DC/AC MMCs are presented; these include the three topologies in Figure 1.5a - 1.5c, and a fourth new topology, shown here in Figure 1.5f, which is equivalent to the buck-boost MMC in [42] with a transformer added for AC grid interconnection.

### 1.2.3 Availability of Analytical Models

For the study of new converter topologies, the mathematical modelling is a very useful tool; it allows for the creation of well-tuned controls to be made and permits the evaluation of the topology without having to resort to building a physical model. The conventional DC/AC MMC has been well studied, and has models and controls made for them that can be found in the literature, e.g. [20]–[24]. Since the HVDC DC/DC converters in Section 1.2.1 and the multiport DC/DC/AC MMCs in Section 1.2.2 are relatively new, little literature regarding their modelling and control is available.

When a converter is modelled, there is usually the question of how detailed the mod-

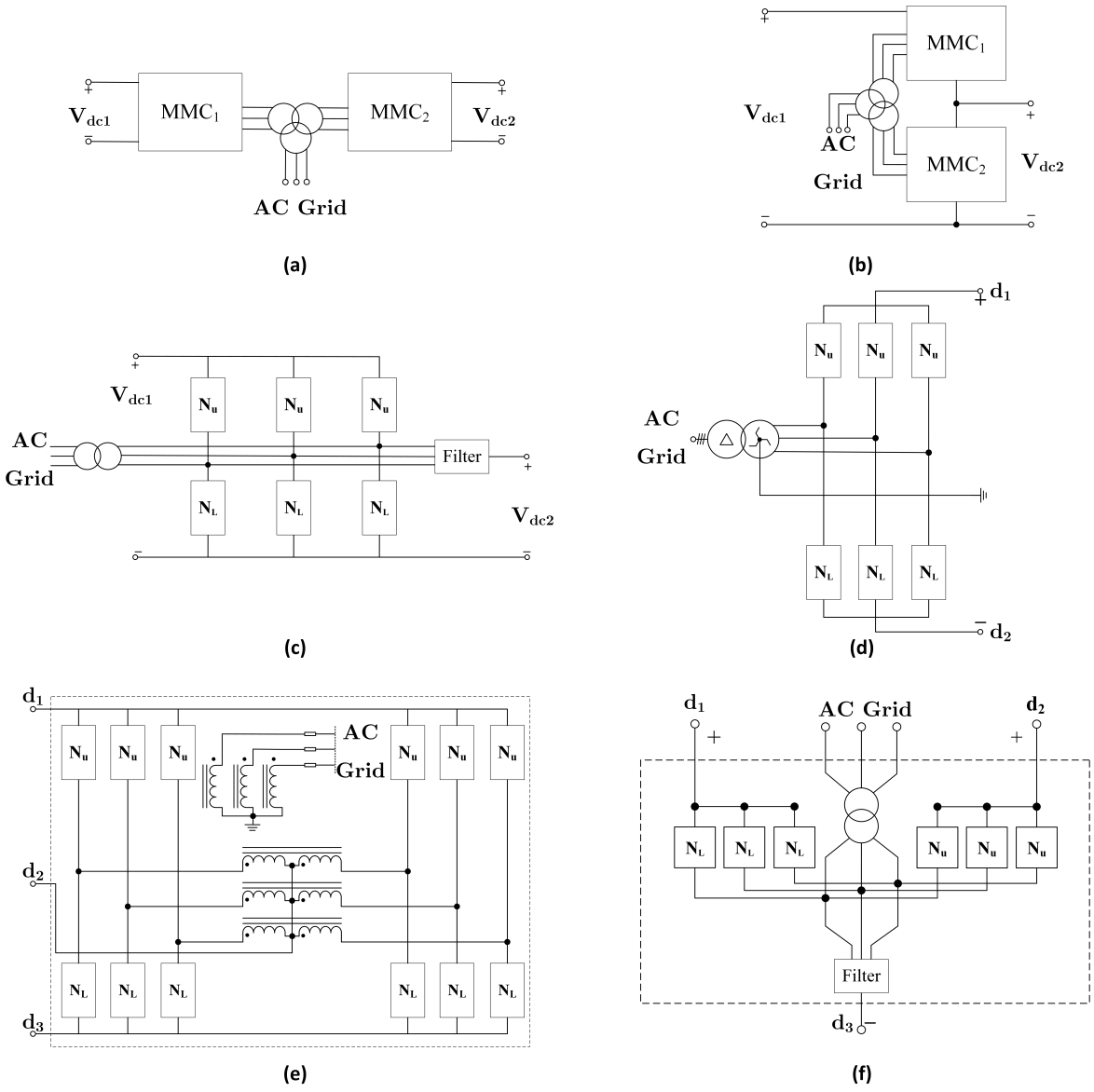



Figure 1.5: Examples of three-phase DC/DC/AC MMCs; (a) based on F2F MMC, (b) based on HVDC-AT [32], (c) based on M2DC [36], (d) Bipolar MMC with delta/zig-zag transformer [39], (e) Generalised DC/DC/AC chain-link structure [40], (f) multiport buck-boost converter [41]

elling will be, and this usually brings into question how much of the electronic components, namely switches, are to be included. Figure 1.6 [43] shows a depiction of different types of modelling accuracy that can be employed during the modelling of a converter, depending on the type of study. Making a fully detailed model, i.e. modelling every single semiconductor in the converter, would ideally be the best for all types of studies, since it would provide a completely faithful representation of the converter; nevertheless, this model would be very complex and computationally challenging even for a two or three level VSC. For an MMC, which has a much higher number of components, and therefore nodes in the admittance matrix, it would be very difficult if not impossible. Detailed equivalent models and switching function models still take into account the switching of the components, but do not model each submodule individually; these models are useful for electromagnetic transient studies and fault analyses [44]–[48]. An averaged model is made by replacing the semiconductors in a model with an equivalent controlled voltage or current source. The averaged model will naturally lose the information on individual switching cells, as well as the influence of diodes, and neglect the high order switching harmonics. However, it is a very good for accelerated simulations, controller design, and as a basis for creating a state space dynamic model [49]–[51]. In [52], a detailed explanation can be found regarding the modelling of DC/DC MMCs. This includes the large-signal dynamic model, and it is a good example of a full detailed model. Another example of modelling of DC/DC MMCs can be found in [53], where the averaged and the switched converter models of a single string of a DC-MMC are compared.

A very useful technique when modelling converters is the dynamic phasor model, first introduced for power electronic applications in [54] and more formally referred to as generalised averaging method. For linear control design and eigenvalue analysis, a linear time-invariant model is required; however, often, the averaged model of an MMC converter will have time-varying state variables at steady-state. This is because of the

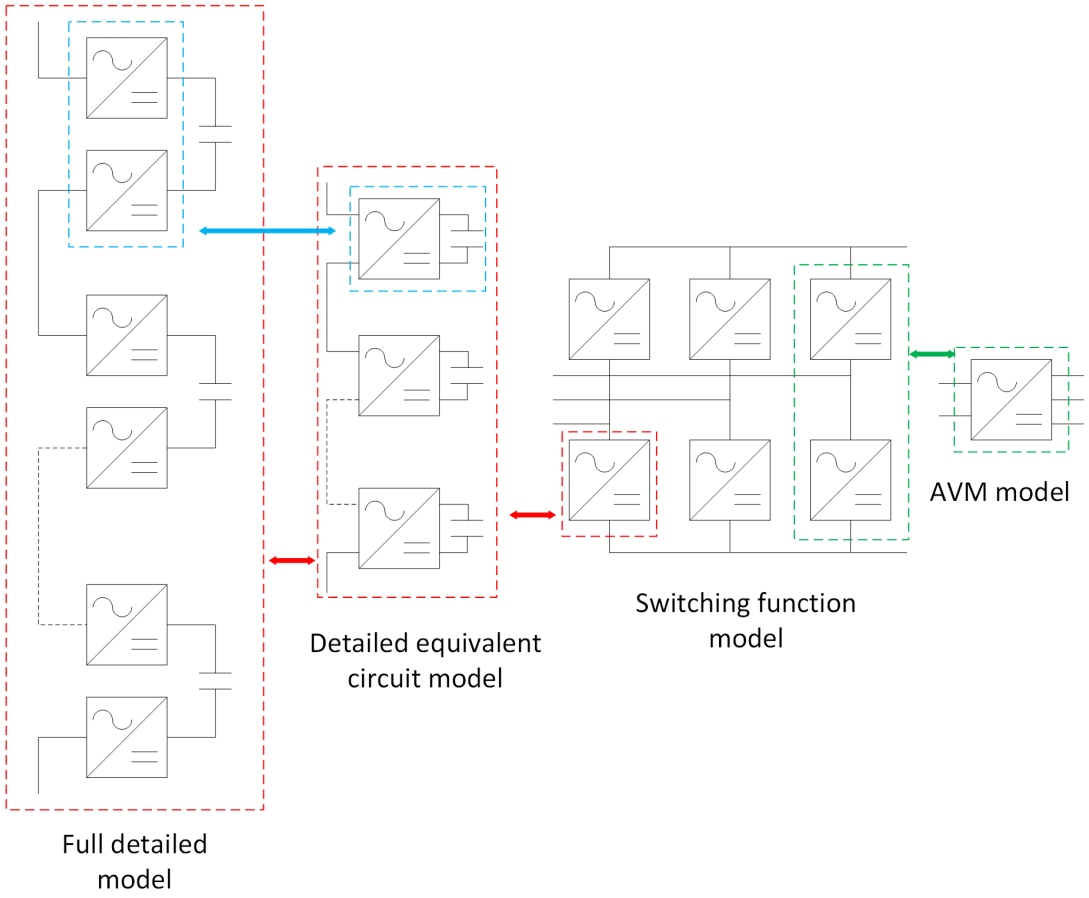
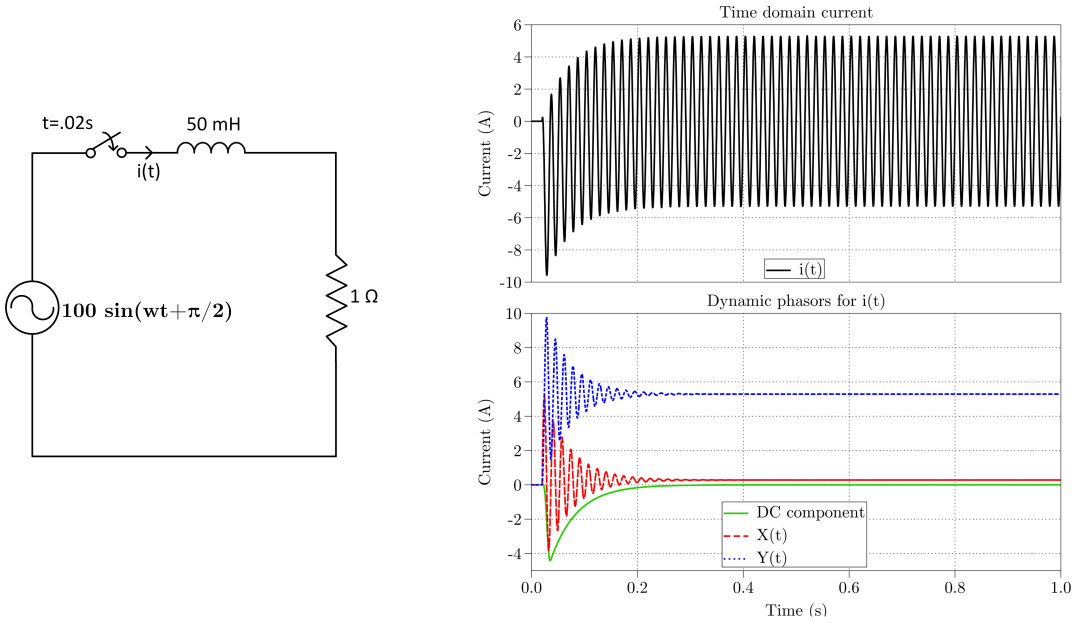



Figure 1.6: Different detail in models for converters [43]

existence of multiple frequency components in the converter. To tackle this, the concept of dynamic phasors can be used, in which the time-varying waveform is to be approximated with the coefficients from a Fourier series expansion. In other words, the sinusoidal quantities can be represented with phasors that are allowed to change over time. This becomes useful since in steady-state the equations that represent the converter dynamics will become time-invariant [55]. To illustrate, Figure 1.7a shows a simple RL circuit connected to an AC voltage source, with a switch that closes at t=0.02 seconds. Figure 1.7b shows two plots: the upper plot shows the time domain current $i(t)$; the lower plot shows

three current components that correspond to the dynamic phasor model of $i(t)$. The DC component, solid plot, can be seen overshooting after the switch closes, then converging to zero. The dashed and dotted plots show the AC fundamental components of $i(t)$ broken into cosine and sine components with amplitudes $X(t)$ and $Y(t)$, respectively, [56]. Observe the three states in the dynamic phasor model of $i(t)$ become DC valued (i.e., time-invariant) at steady-state. If the $X(t)$ and $Y(t)$ plots were to be multiplied by $cos(wt)$ and $sin(wt)$, respectively, and summed with the DC component, the resulting waveform would be $i(t)$. This method is widely used in converter analysis, for it provides very useful information of the converter operation and permits a more detailed control design than the conventional time averaged model. This modelling technique has been recently adopted in MMC studies [57]–[61].




(a) (b)

Figure 1.7: Dynamic phasors visualisation using series RL circuit

## 1.3 Thesis Scope

The most recent research in power systems suggests that interconnecting different DC and AC systems is to be a very important part of the future. However, at present, little work has been invested towards the development of analytical models for newest types of DC/DC/AC converters using MMC technology. This thesis aims to fill this gap.

This thesis first develops a time averaged dynamic model of a single-phase generalised DC/DC/AC MMC structure. A three-phase model is then synthesised from the single-phase model. A dynamic phasor model that includes multiple harmonic components is then developed from the state-space representation of the three-phase model, in which the state variables are time-invariant at steady-state. From this one model, four different

converter models are derived, two of which are DC/DC/AC, and two are DC/DC. Solution procedures, to solve for the input variables required for different operating points, are then developed for each converter. Using these solution procedures, steady-state analysis for each converter, in different operating points, are carried out.

# Chapter 2

# Power Transfer Mechanisms of DC/DC and Multiport DC/DC/AC MMCs

In this chapter a summary of the operation principles for the conventional DC/AC MMC, the DC/DC MMC, and the multiport DC/DC/AC MMC will be presented. The power transfer mechanisms for each of these topologies are also discussed.

For the purposes of this chapter only the steady-state operation of the converters is considered. Other assumptions employed in this chapter are: (I) The converter components are assumed to be ideal and lossless, by which the input power equals the output power, (II) The number of submodules employed in every converter is large enough to guarantee ideal sinusoidal waveforms, (III) The voltage sort and selection algorithm [22], [62] is used to balance the voltage in the capacitors within each arm, (IV) Only DC and fundamental frequency AC components are considered.

## 2.1 Conventional AC/DC MMC

As stated in Chapter 1, many studies on the conventional AC/DC MMC have been carried out and its operation is well understood [20]–[24], [43], [63], [64]. Here a brief overview of the principle of operation of the AC/DC MMC is presented.

Figure 2.1a shows the conventional single-phase AC/DC MMC structure, where the upper and lower arms can be seen comprising the leg of the converter. Correspondingly, each arm has a designated current, $i_u$ and $i_l$, while the current that goes into the leg from the DC source is termed $I_{dc}$ and the current that goes to the AC system is named $i_s$. To describe how the submodules are able to produce the different voltage levels that compose the AC voltage, the half-bridge submodule operation is described. The half-bridge submodule consists of two IGBTs, each of them with an antiparallel diode, and one capacitor. The two semiconductors are operated in a complementary manner, which is to say, when one has a gating signal on 1(ON), the other one has a 0(OFF). This entails that, for a half-bridge submodule, there are four useful switching states Figure 2.1b shows these four states of the submodule, while Table 2.1 provides an explanation of what happens in each of them. To form the AC voltage waveform, the capacitor on each submodule can either be inserted ($S_1$, $S_3$) or bypassed ($S_2$, $S_4$). The resulting waveform for each submodule can be seen as having two levels, and by stacking hundreds of them the many levels of the MMC can be achieved. It is also clear that the voltage provided by the submodule can only be positive, the direction of the current $i_{SM}$ notwithstanding.

While the number of capacitors to be inserted in the upper and lower arms is specified by the modulation technique, such as space vector modulation, multicarrier sinusoidal pulse-width modulation, and nearest level modulation, determining which capacitors from each arm to be inserted is also something that needs to be addressed. Given

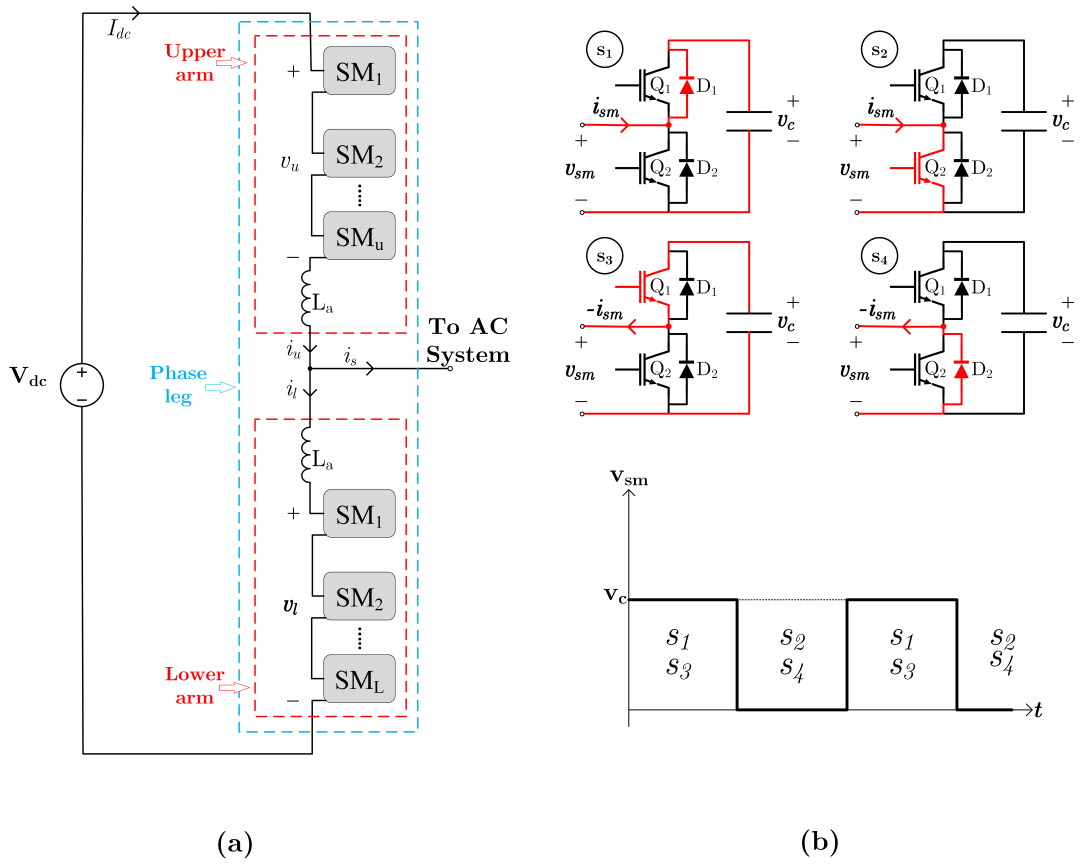



Figure 2.1: (a) single-phase DC/AC MMC, (b) useful switching states of half-bridge submodule and waveform produced by each state

Table 2.1: Practical switching states of the half-bridge submodule

| States in Figure 2.1b | Capacitor State | $D_1$ | $D_2$ | $i_{SM}$ flows through | $v_{SM}$ |
|---|---|---|---|---|---|
| $S_1(+i_{SM})$ | Inserted | 1 | 0 | $D_1$ | $+v_c$ |
| $S_2(+i_{SM})$ | Bypassed | 0 | 1 | $Q_2$ | 0 |
| $S_3(-i_{SM})$ | Inserted | 1 | 0 | $Q_1$ | $+v_c$ |
| $S_4(-i_{SM})$ | Bypassed | 0 | 1 | $D_2$ | 0 |

that there could potentially be hundreds of capacitors in each arm, these capacitors will change their voltage depending on which are inserted and the direction of current into

the capacitor. Since the goal is to maintain the capacitor's voltage as close to their rated value $\frac{V_{dc}}{N}$, there is the need for active balancing via control. Many techniques have been proposed for the balancing of the capacitor's voltage, although the most common is called "sort and selection algorithm" [22], [23], [65]. In this method the voltage of each capacitor is measured and then a sorting takes place which makes an order of the capacitors that will be inserted depending on the direction of the arm current.

The arm voltages in the MMC of Figure 2.1a can be described as:

$$v_u(t) = \frac{V_{dc}}{2} + \hat{V}cos(\omega t), \tag{2.1}$$

$$v_l(t) = \frac{V_{dc}}{2} - \hat{V}cos(\omega t), \tag{2.2}$$

where $\hat{V}$ is the peak magnitude fundamental frequency voltage, which assuming the use of half-bridge submodules can take values between 0 and $\frac{V_{dc}}{2}$. Accordingly, the arms' currents are:

$$i_u(t) = I_{dc} + \frac{\hat{I}}{2}cos(\omega t + \theta_i) \tag{2.3}$$

$$i_l(t) = I_{dc} - \frac{\hat{I}}{2}cos(\omega t + \theta_i) \tag{2.4}$$

$$i_s(t) = Icos(\omega t + \theta_i) \tag{2.5}$$

where $\hat{I}$ is the peak magnitude fundamental frequency current.

In the analysis of MMCs, a common practice is to map the voltages and currents of the MMC into a coordinate system which decouples the terms that relate to the external AC system and the internal MMC quantities, and by doing so, separates the DC

and fundamental frequency components. This system is called sum-difference or "ΣΔ" coordinate system [62], [66]. After being transformed to this new domain, the voltages and currents can be referred to as the ones that are common to both arms (Σ) and the ones that are differential to each arm (Δ). The current and voltage transformation of the quantities Equations (2.1) to (2.4) for the single-phase MMC in Figure 2.1a are defined as:

$$\begin{bmatrix} i_\Sigma \\ i_\Delta \\ v_\Sigma \\ v_\Delta \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} i_u \\ i_l \\ v_u \\ v_l \end{bmatrix}. \tag{2.6}$$

The ΣΔ domain parameters in Equation (2.6) are illustrated in Figure 2.2. For the single-phase MMC, the abstract ΣΔ quantities ideally consist of DC and fundamental frequency components as follows:

$$i_\Sigma = I_{dc} \tag{2.7}$$

$$v_\Sigma = \frac{V_{dc}}{2} \tag{2.8}$$

$$i_\Delta = \hat{I}_\Delta cos(\omega t + \theta_{i\Delta}) \tag{2.9}$$

$$v_\Delta = \hat{V}_\Delta cos(\omega t). \tag{2.10}$$

As noted by Equations (2.7) to (2.10) and Figure 2.2, the Σ terms are both common to both arms and DC quantities, whereas the Δ terms are differential to both arms and AC fundamental frequency quantities.

The ΣΔ reference frame is also useful for examining the DC/AC conversion process.

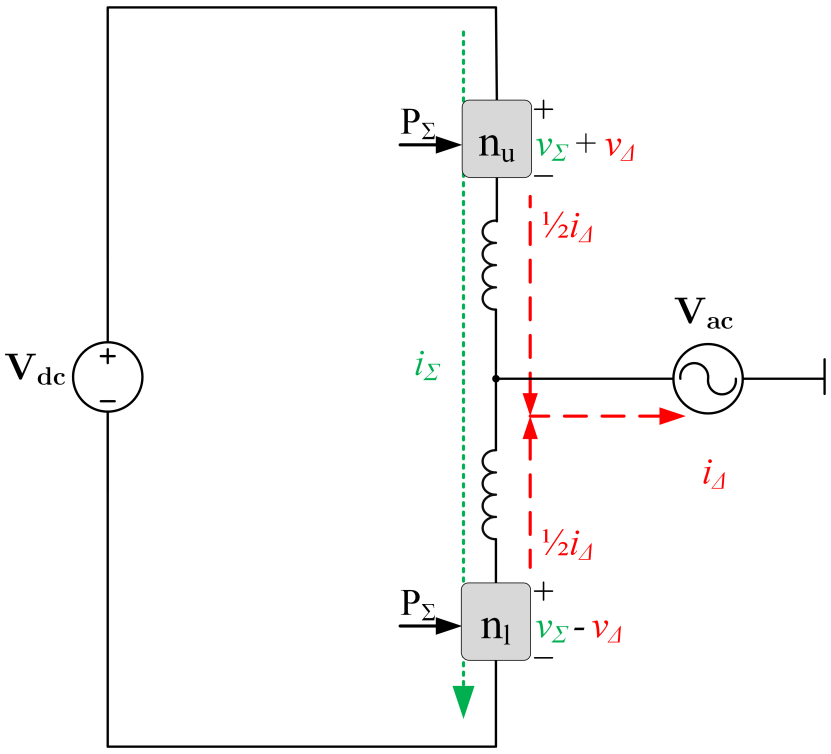



Figure 2.2: ΣΔ transformation of the single-phase MMC, where the quantities from Equations (2.1) to (2.4) are transformed through Equation (2.6). The dotted, green terms represent DC quantities and dashed, red terms represent fundamental frequency components

For the conventional AC/DC MMC, the average power absorbed by each arm can be calculated as follows:

$$P_u = \frac{1}{T}\int_0^T (v_\Sigma + v_\Delta)(i_\Sigma + \frac{\hat{I}_\Delta}{2})dt = \frac{V_{dc}I_{dc}}{2} + \frac{\hat{V}_\Delta \hat{I}_\Delta}{4}cos(\theta_{i\Delta}) \tag{2.11}$$

$$P_l = \frac{1}{T}\int_0^T (v_\Sigma - v_\Delta)(i_\Sigma - \frac{\hat{I}_\Delta}{2})dt = \frac{V_{dc}I_{dc}}{2} + \frac{\hat{V}_\Delta \hat{I}_\Delta}{4}cos(\theta_{i\Delta}). \tag{2.12}$$

Since the arms have only capacitive energy storage, at steady-state $P_u$=$P_l$=0, therefore:

$$P_u = P_l \Rightarrow \frac{V_{dc}I_{dc}}{2} = -\frac{\hat{V}_\Delta \hat{I}_\Delta}{4} cos(\theta_{i\Delta}). \tag{2.13}$$

Thus

$$P_{dc} = -P_{ac} \tag{2.14}$$

where

$$P_{dc} \triangleq V_{dc}I_{dc} \tag{2.15}$$

$$P_{ac} = P_{grid} \triangleq \frac{\hat{V}_\Delta \hat{I}_\Delta}{4} cos(\theta_{i\Delta}). \tag{2.16}$$

Equation (2.14) represents the arms power balance requirement for a single-phase AC/DC MMC. This implies that, assuming the grid is absorbing power, the arms inject average power of $\frac{P_{dc}}{2}$ per arm to the AC systems via the modulated AC quantities. It is also worth noting that both arms process the same amount of power, splitting the full input DC power between them. Thus, in the $\Sigma\Delta$ domain, the AC/DC power transfer mechanism is purely a common mode ($\Sigma$) component, as shown in Figure 2.2.

$$P_\Sigma = \frac{-P_{ac}}{2} \tag{2.17}$$

$$P_\Delta = 0. \tag{2.18}$$

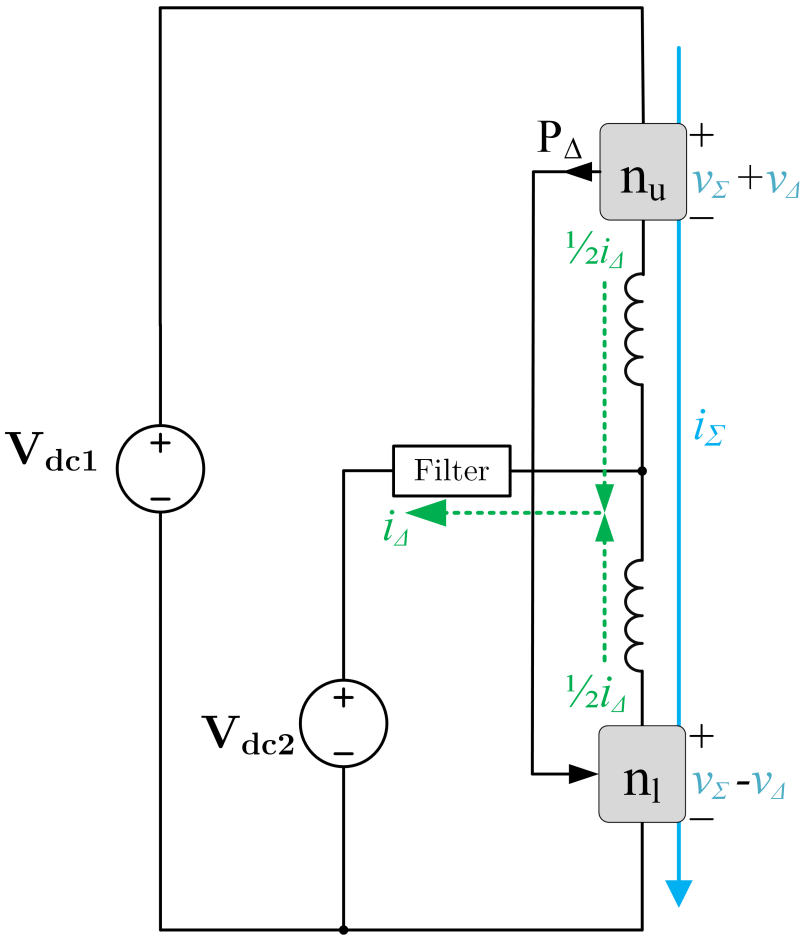



Figure 2.3: M2DC with ΣΔ currents and voltages, where dotted, green terms represent DC quantities and solid, blue terms represent a combination of DC and AC quantities

## 2.2 DC/DC MMC

The M2DC, described comprehensively in [33], [67]–[69], is a DC/DC MMC that can transfer power between two DC systems. Figure 2.3 illustrates the currents, voltages, and average power exchange between arms of the M2DC. The AC quantities synthesised in the arms do not exit through the midpoint of the converter, where a filter prevents anything but DC quantities from reaching the second voltage source, but are used to exchange

average AC power between the upper and lower arms to maintain power balance of the submodule capacitors. The DC step ratio $G_v$ is used to indicate the voltage ratio between the two DC sources as

$$G_v = \frac{V_{dc2}}{V_{dc1}}, \tag{2.19}$$

and can take values from 0 to 1 for step-down operation. The $\Sigma\Delta$ quantities in Figure 2.3 ideally consist of DC and fundamental frequency components as follows:

$$i_\Sigma = \left(1 - \frac{1}{2G_v}\right) I_{dc} + \hat{I}_\Sigma cos(\omega t + \theta_{i\Sigma}) \tag{2.20}$$

$$v_\Sigma = \frac{V_{dc1}}{2} + \hat{V}_\Sigma cos(\omega t + \theta_{v\Sigma}) \tag{2.21}$$

$$i_\Delta = \frac{I_{dc}}{G_v} \tag{2.22}$$

$$V_\Delta = \left(\frac{1}{2} - G_v\right) V_{dc1} + \hat{V}_\Delta cos(\omega t) \tag{2.23}$$

From Equation (2.20) it can be seen that, for the M2DC, $i_\Sigma$ now has both AC and DC components, while in the conventional MMC (see Equation (2.7)) it was only DC. This is because the AC quantities no longer exit through the middle point. However, it can also be seen that $i_\Sigma$ is purely AC at the step ratio of $G_v$=0.5. Equations (2.21) and (2.23) show that the arms' voltages have both $\Sigma$ and $\Delta$ terms are comprised of DC and AC components. The ratio $G_v$, as in the case of the currents, sets how the DC voltage is split between the terms.

To maintain steady-state power balance for the submodules' capacitors, the M2DC's arms have to exchange average power between them.

Assuming $\hat{V}_\Delta \gg \hat{V}_\Sigma$[1], the average power absorbed by the upper and lower arms can be calculated as follows:

$$P_u = \frac{1}{T}\int_0^T (v_\Sigma + v_\Delta)(i_\Sigma + i_\Delta)dt = (1 - G_v)V_{dc1}I_{dc} + \frac{\hat{V}_\Delta \hat{I}_\Sigma}{2} cos(\theta_{i\Sigma}) \tag{2.24}$$

$$P_l = \frac{1}{T}\int_0^T (v_\Sigma - v_\Delta)(i_\Sigma - i_\Delta)dt = -(1 - G_v)V_{dc1}I_{dc} - \frac{\hat{V}_\Delta \hat{I}_\Sigma}{2} cos(\theta_{i\Sigma}). \tag{2.25}$$

From Equations (2.24) and (2.25), it can be observed that $P_u = -P_l$. This indicates that average AC power in one arm has to be shuttled to the other arm in order to maintain submodule capacitor power balance. Since the arms have only capacitive energy storage, at steady-state $P_u = P_l = 0$, therefore:

$$(1 - G_v)V_{dc1}I_{dc} = -\frac{\hat{V}_\Delta \hat{I}_\Sigma}{2} cos(\theta_{i\Sigma}). \tag{2.26}$$

Thus, by defining:

$$P_{dc} \triangleq V_{dc1}I_{dc} \tag{2.27}$$

$$P_{ac} \triangleq -\frac{\hat{V}_\Delta \hat{I}_\Sigma}{2} cos(\theta_{i\Sigma}), \tag{2.28}$$

we have:

$$(1 - G_v)P_{dc} = P_{ac}. \tag{2.29}$$

[1]This assumption is valid for small arm choke impedances

Equation (2.29) represents the arms power balance requirement for the M2DC. Since this power is now differential mode to the arms in a single leg, $P_\Delta$ now exists in Figure 2.3, when it didn't in the conventional AC/DC MMC. No common mode power exists for this converter, therefore:

$$P_\Sigma = 0 \tag{2.30}$$

$$P_\Delta = (1 - G_v)P_{dc} \tag{2.31}$$

These equations are in contrast to Equations (2.17) and (2.18) for the AC/DC MMC.

## 2.3 DC/DC/AC MMC

As mentioned in Chapter 1, the M2DC can be adapted to include an AC port [36], [38], and several such topological variations can be found in the literature [39]–[41]. This converter fulfils the energy conversion objectives of the two converters presented in Sections 2.1 and 2.2 simultaneously. Because of this, its behaviour is also described as a simultaneous behaviour of those two. Figure 2.4 illustrates how these powers, currents, and voltages would look for a single-phase M2DC with an AC port. As it can be expected, the voltages and currents present in this converter are a combination of the quantities present in the AC/DC and DC/DC converters in Figures 2.2 and 2.3, and can be defined as:

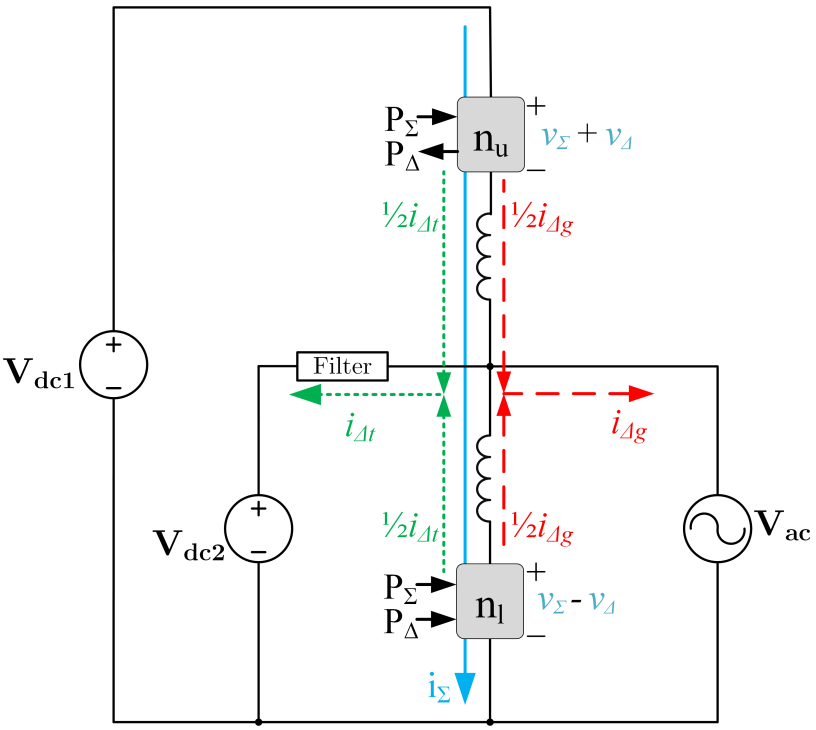



Figure 2.4: M2DC with AC port, showing ΣΔ currents and voltages, where dotted, green terms represent DC quantities; dashed, red terms represent fundamental frequency components; and solid, blue terms represent a combination of both

$$i_\Sigma = I_{dc} + \hat{I}_\Sigma cos(\omega t + \theta_{i\Sigma}) \tag{2.32}$$

$$v_\Sigma = \frac{V_{dc1}}{2} + \hat{V}_\Sigma cos(\omega t + \theta_{v\Sigma}) \tag{2.33}$$

$$i_\Delta = I_{\Delta t} + \hat{I}_{\Delta g} cos(\omega t + \theta_{i\Delta g}) \tag{2.34}$$

$$V_\Delta = \left(\frac{1}{2} - G_v\right) V_{dc1} + \hat{V} cos(\omega t). \tag{2.35}$$

In this case, to maintain the balance in the submodule capacitors, the arms have to send average power to the AC system and they need average power shuttled between

them. For these reasons, the M2DC with an AC port requires simultaneous existence of $P_\Sigma$ and $P_\Delta$ terms:

$$P_\Sigma = -\frac{P_{ac}}{2} \tag{2.36}$$

$$P_\Delta = (1 - G_v)Pdc. \tag{2.37}$$

Table 2.2: Currents' harmonic components and power transfer mechanisms for each of the conversion cases

| Current | AC/DC | DC/DC | DC/DC/AC |
|---|---|---|---|
| $i_\Sigma$ | DC | AC + DC* | AC + DC |
| $i_\Delta$ | AC | DC | DC + AC |
| **Average Power** | **AC/DC** | **DC/DC** | **DC/DC/AC** |
| $P_\Sigma$ | $\hat{V}_\Delta, \hat{I}_\Delta$ | - | $\hat{V}_\Delta, \hat{I}_\Delta$ |
| $P_\Delta$ | - | $\hat{V}_\Delta, \hat{I}_\Sigma$ | $\hat{V}_\Delta, \hat{I}_\Sigma$ |

*Exists when $G_v \neq 0.5$

Table 2.2 summarises the harmonic components of $i_\Sigma$ and $i_\Delta$ present in AC/DC, DC/DC, and combined DC/DC/AC conversion. It also shows the arms' average powers, $P_\Sigma$ and $P_\Delta$, present in each conversion, and their constituent harmonic components. Each of the currents fulfils a role in each of the conversion cases. Since $i_\Delta$ represents the current that exits through the midpoint in all cases, this current regulates external power injection in all cases, whether it be real and reactive power injection to the grid when AC conversion is present, or DC power when DC/DC conversion is being performed.

## 2.4 Regulation of Capacitor Voltage and External Network Power Injections

Regulating the capacitor voltages of the upper and lower arms is an objective that must be accomplished by the converter, as briefly mentioned in Section 2.1. This can be accomplished by appropriate control of $i_{\Sigma}$. For this purpose, the targets of this regulation are defined as:

$$\Sigma V_{cap,\Sigma} = \frac{1}{2}\Sigma V_{cap,upper} + \frac{1}{2}\Sigma V_{cap,lower} \tag{2.38}$$

$$\Sigma V_{cap,\Delta} = \frac{1}{2}\Sigma V_{cap,upper} - \frac{1}{2}\Sigma V_{cap,lower}, \tag{2.39}$$

where $\Sigma V_{cap,upper}$ and $\Sigma V_{cap,lower}$ represent the sum of the submodule capacitor voltages for the upper and lower arms, respectively.

In the case of AC/DC conversion, the objective is to regulate $\Sigma V_{cap,\Sigma}$, by means of the DC component of $i_{\Sigma}$. When DC/DC conversion is present, $\Sigma V_{cap,\Delta}$ is to be regulated by the fundamental frequency component of $i_{\Sigma}$. It can be noted that the role of either the DC or the fundamental frequency component of the currents in the AC/DC conversion is fulfilled by the other component in the DC/DC conversion. E.g. the regulation of $\Sigma V_{cap,\Sigma}$ is fulfilled by the DC component of $i_{\Sigma}$ in the AC/DC conversion, and the equivalent task in DC/DC conversion, the regulation of $\Sigma V_{cap,\Delta}$, is realised by the fundamental frequency component of $i_{\Sigma}$. The same symmetry occurs with the power injections to the external DC and AC grids. Table 2.3 summarises these points, where the superscript indicates with "dc" or a "1" if the component is the DC or the fundamental frequency component, respectively.

Table 2.3: Currents' harmonic components and powers for each of the conversion cases

| Regulation Objective | AC/DC | DC/DC | DC/DC/AC |
|---|---|---|---|
| $P_{grid}, Q_{grid}$ | $i^1_\Delta$ | - | $i^1_\Delta$ |
| $P_{dc}$ | - | $i^{dc}_\Delta$ | $i^{dc}_\Delta$ |
| $\Sigma V_{cap,\Sigma}$ | $i^{dc}_\Sigma$ | - | $i^{dc}_\Sigma$ |
| $\Sigma V_{cap,\Delta}$ | - | $i^1_\Sigma$ | $i^1_\Sigma$ |

## 2.5 Multiport MMC Structure for Simultaneous DC/ DC/ AC Conversion

This thesis is focused on simultaneous DC/DC and AC/DC conversion using MMCs, a topic of emerging interest for the future of power systems. For this purpose, the modelling of a converter that can interconnect two DC ports and an AC port, akin to the structure presented in Figure 2.4 is the first objective in this work. On the early stages of this enterprise it became clear that a single unified model could be applied to four different converter structures. Figure 2.5 illustrates these four topologies; two of them are for DC/DC/AC conversion, and two are for DC/DC conversion. Figure 2.5a and 2.5c are three-phase topologies with two DC ports and one AC port, suitable for DC/DC/AC conversion and termed multiport DC/DC/AC (MP-DCDCAC) and bipolar DC/DC/AC (BP-DCDCAC) converters, respectively. Figure 2.5b and 2.5d are two DC/DC converter topologies that result from removing the AC port from the aforementioned structures, and are called multiport DC/DC (MP-DCDC) and bipolar DC/DC (BP-DCDC) converters, respectively.

As it can be seen in Figure 2.5, the four topologies all use the exact same base converter structure, marked by a dotted square, which allows the use of the same unified

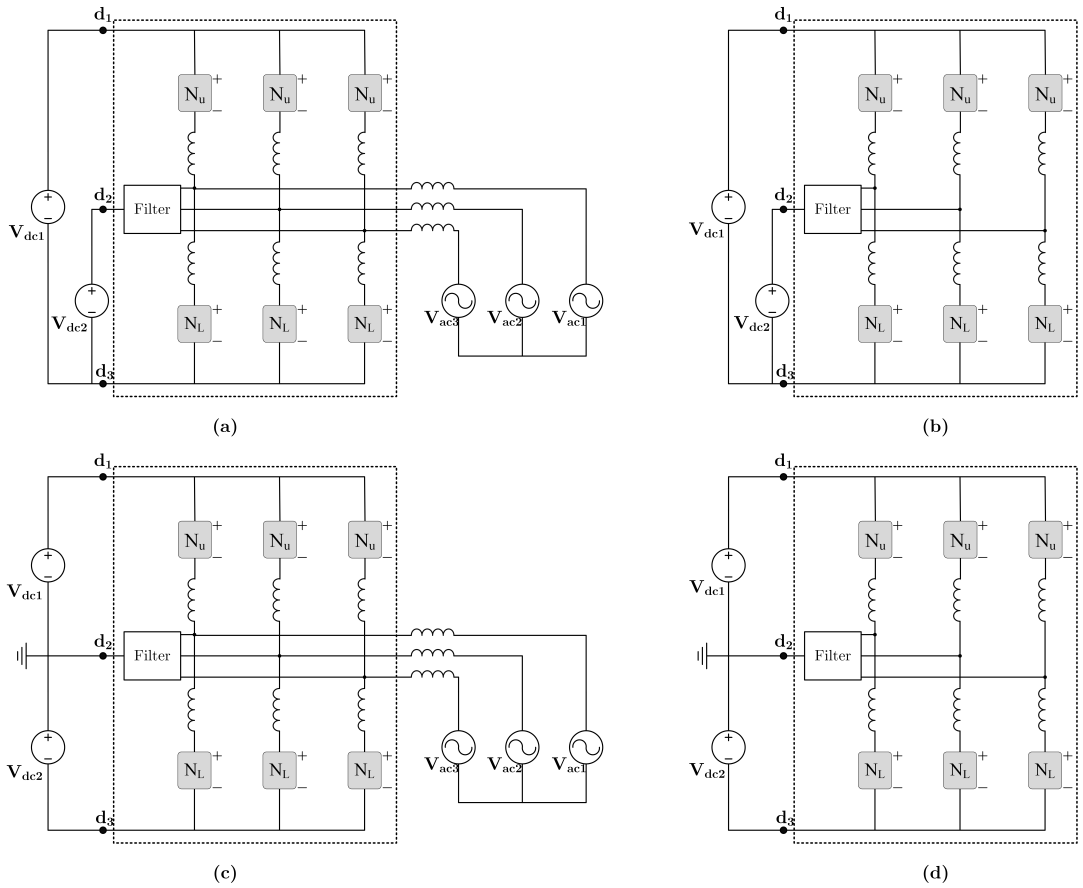



Figure 2.5: MMC topologies under study in this thesis: (a) MP-DCDCAC, (b) MP-DCDC, (c) BP-DCDCAC, (d) BP-DCDC

base model for all of them by only redefining the voltages in the DC ports (d1-d3) and redefining the external AC grid connections. This can easily be done in the model with minimal changes. The base structure identified in Figure 2.5 is the primary point of interest of the next chapter, in which a unified mathematical model is obtained for it.

# Chapter 3

# Converter Dynamic Modelling

In this chapter, a generalised time-averaged three-phase dynamic model for the four converter structures firstly identified in Figure 2.5 and now shown here in Figure 3.1a, is developed. This process is carried out starting with a single-phase leg model and progresses to a complete three phase model for a representative converter structure. From this benchmark model, a multi-harmonic dynamic phasor model (DPM) is the derived. The model derivation is carried out using the MP-DCDCAC structure, Figure 3.1b, as the base topology to obtain the unified model that is later used to simulate all four converter structures shown in Figure 3.1. The validation of the developed models is carried out using a combination of PLECS, a time-domain software for power electronics circuits simulation, and SIMULINK, a MATLAB-based graphical programming environment.

## 3.1 Time-Averaged Model

This section presents the process of developing a three-phase unified model in the $\alpha\beta 0$ reference frame that represents the general structure of Figure 3.1a. This process is

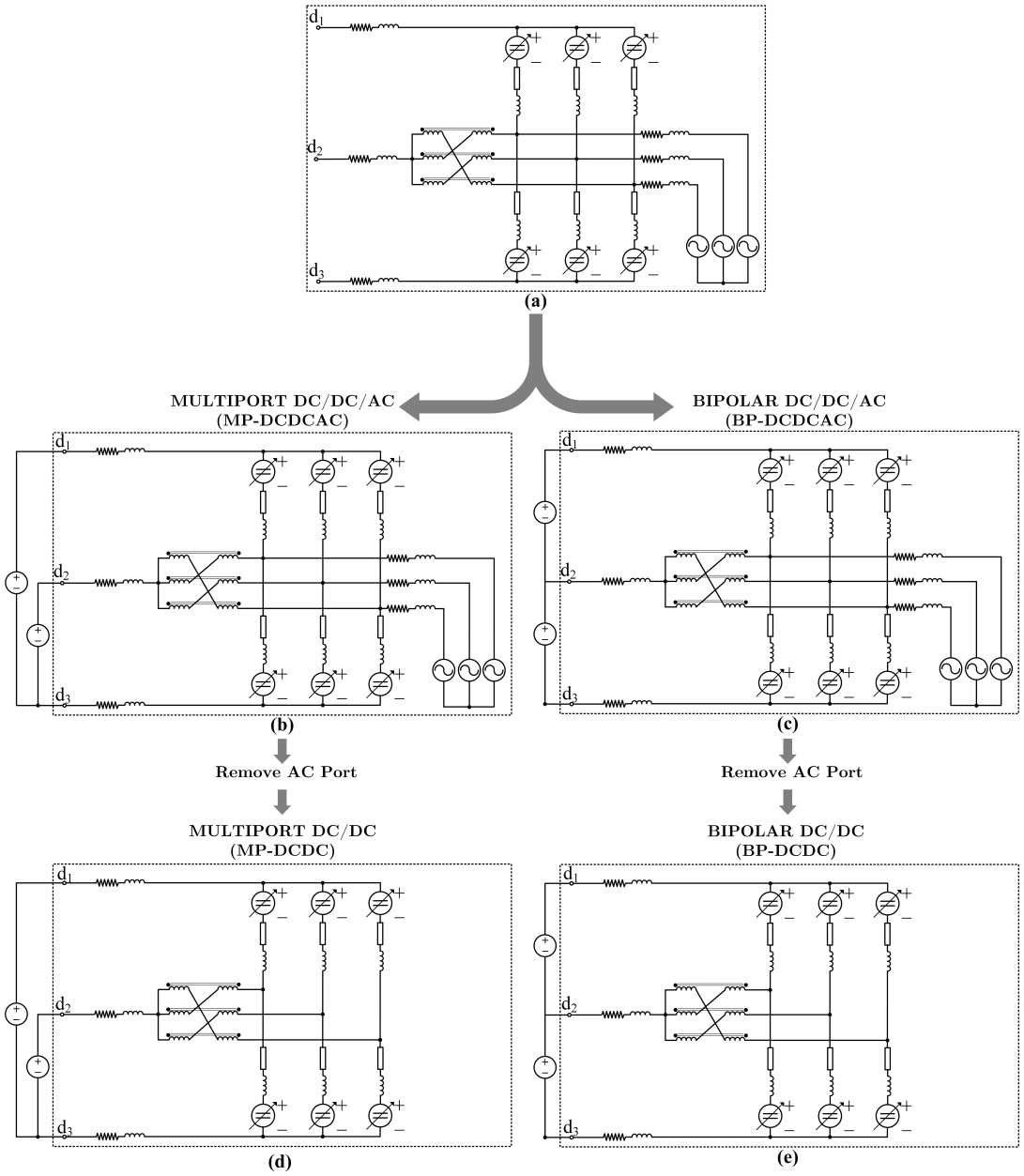



Figure 3.1: Target converter structures for modelling: (a) Unified general structure, (b) MP-DCDCAC topology, (c) BP-DCDC topology, (d) MP-DCDCAC topology, (e) BP-DCDC topology

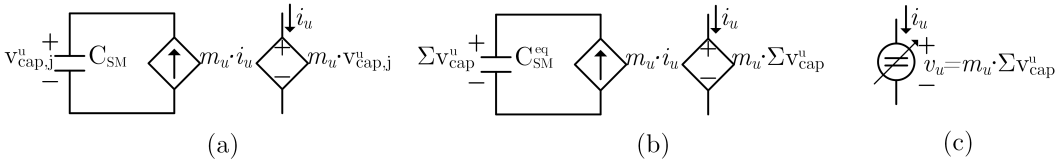



Figure 3.2: Average models for upper arm: (a) individual $j^{th}$ submodule, (b) composite arm with N series-cascaded submodules, (c) compact representation of upper arm

done by using the MP-DCDCAC topology, shown in Figure 3.1b, and the model is later adapted to the other three topologies in Figure 3.1. The developed time averaged model (TAM) is validated with a switched model.

### 3.1.1 Single Phase Leg for DC/DC/AC Conversion

A time-averaged model for the submodules in the MMC arm is shown in Figure 3.2, taking for this example the upper arm [70]. A single submodule can be averaged over one switching period as shown in Figure 3.2a, where represents the voltage of the capacitor, CSM, of the jth individual submodule in the upper arm, where j can take values from 0 to N; m represents the modulating signal for the arm, that can take values between 0 and 1, assuming the use of half-bridge submodules. Figure 3.2 illustrates how a single submodule can be represented as a voltage source controlled by the modulating signal that submodule would receive and the voltage a capacitor would have subjected to a current source controlled by the current in the arm and the modulating signal. Figure 3.2b illustrates the same methodology applied to the entire arm of N cascaded submodules, summing all the capacitance of one arm, $C^{eq}_{SM}$, and its voltage, $\Sigma V^{u}_{cap}$. Figure 3.2c shows a compact representation of the upper arm time averaged module.

The single-phase leg for DC/DC/AC conversion to be modelled is shown in Figure 3.3, where the currents and voltages are presented in the $\Sigma\Delta$ domain, as presented in Sec-

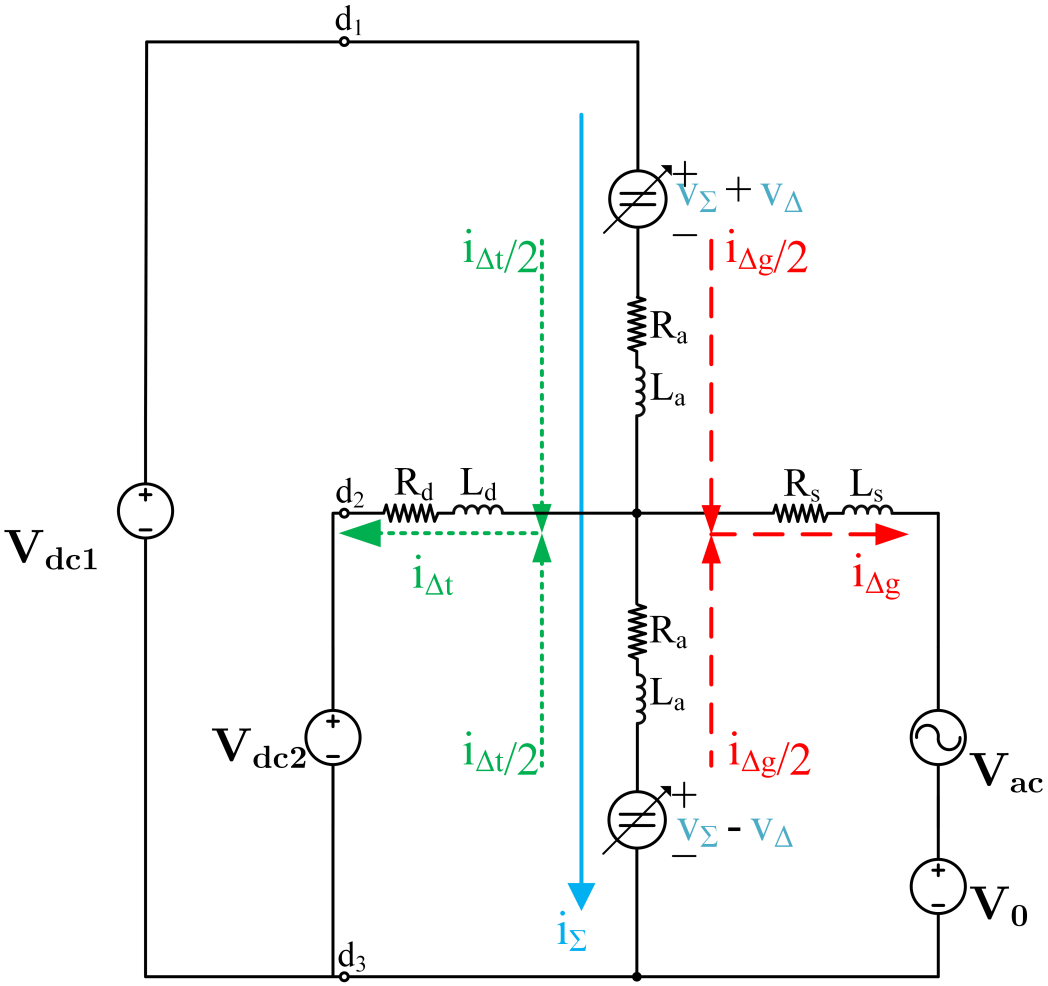



Figure 3.3: Time-averaged model of a single-phase leg for DC/DC/AC conversion

tion 2.1. Differential mode arm current $i_\Delta$, first introduced in Figure 2.2 in the previous chapter, corresponds here to $i_\Delta = i_{\Delta t} + i_{\Delta c}$ which is the sum of the external AC and DC networks' currents; $i_\Sigma$, $v_\Sigma$, and $v_\Delta$ are the same as previously introduced in Equation (2.6). Three differential equations, from three different KVL loops, are written to represent the current dynamics and their relation with the arms' voltages as:

$$L_a\frac{d}{dt}\left(i_\Sigma+\frac{i_{\Delta g}}{2}+\frac{i_{\Delta t}}{2}\right)+R_a\left(i_\Sigma+\frac{i_{\Delta g}}{2}+\frac{i_{\Delta t}}{2}\right)+L_d\frac{d}{dt}(i_{\Delta t}) \\ +R_d(i_{\Delta t})+V_{dc2}-V_{dc1}+V_\Sigma+V_\Delta=0 \tag{3.1}$$

$$L_a\frac{d}{dt}\left(i_\Sigma-\frac{i_{\Delta g}}{2}-\frac{i_{\Delta t}}{2}\right)+R_a\left(i_\Sigma-\frac{i_{\Delta g}}{2}-\frac{i_{\Delta t}}{2}\right)-L_d\frac{d}{dt}(i_{\Delta t}) \\ -R_d(i_{\Delta t})-V_{dc2}+V_\Sigma-V_\Delta=0 \tag{3.2}$$

$$L_s\frac{d}{dt}(i_{\Delta g})+R_s(i_{\Delta g})-L_d\frac{d}{dt}(i_{\Delta t})-R_d(i_{\Delta t})-V_{dc2}+V_{ac}+V_0=0. \tag{3.3}$$

In an effort to simplify the system in Equations (3.1) to (3.3) and decouple as many states as possible, three new equations are obtained by: adding Equation (3.1) and Equation (3.2); subtracting Equation (3.2) from Equation (3.1); solving for $\frac{d}{dt}(i_{\Delta t})$ in the previously obtained equation and replacing its value in Equation (3.3). The resulting equations are written in state-space form as:

$$l\frac{d}{dt}(\mathbf{x_i})=a_{11}\cdot\mathbf{x_i}+b\cdot\mathbf{u}+n\cdot\mathbf{w}, \tag{3.4}$$

where the state, input, and disturbance vectors are respectively defined as:

$$\mathbf{x_i}=\begin{bmatrix}i_\Sigma & i_{\Delta g} & i_{\Delta t}\end{bmatrix}^{\mathbf{T}},\quad \mathbf{u}=\begin{bmatrix}v_\Sigma & v_\Delta\end{bmatrix}^{\mathbf{T}},\quad \mathbf{w}=\begin{bmatrix}V_{dc1} & V_{dc2} & V_{ac} & V_0\end{bmatrix}^{\mathbf{T}}. \tag{3.5}$$

The state-space matrices in Equation (3.4), $l$, $a_{11}$, $b$, $n$, are given by:

$$
\begin{aligned}
l &= \begin{bmatrix} 2L_a & 0 & 0 \\ 0 & L_a & La + 2L_d \\ 0 & \frac{L_a L_d}{L_a + 2L_d} + L_s & 0 \end{bmatrix} \\
a_{11} &= \begin{bmatrix} -2R_a & 0 & 0 \\ 0 & -R_a & -R(R_a + 2R_d) \\ 0 & -\left(\frac{R_a L_d}{L_a + 2L_d} + R_s\right) & -\left(\frac{L_d(R_a + 2R_d)}{L - a + 2L_d} - R_d\right) \end{bmatrix} \\
b &= \begin{bmatrix} -2 & 0 \\ 0 & -2 \\ 0 & -\frac{2L_d}{L_a + 2L_d} \end{bmatrix} \\
n &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ \frac{L_d}{L_a + 2L_d} & \left(1 - \frac{2L_d}{L_a + 2L_d}\right) & -1 & -1 \end{bmatrix}.
\end{aligned}
\tag{3.6}
$$

At this moment, the system in Equation (3.4) does not include the capacitor voltage dynamics, nor the modulating signals that are the inputs of interest. The equations that govern the capacitor voltages and currents, shown in Figure 3.2b for the upper arm, can be written for either arm as:

$$
v_{u/l} = m_{u/l} \cdot \Sigma V_{cap}^{u/l} \tag{3.7}
$$

$$
m_{u/l} \cdot i_{u/l} = \frac{C_{u/l}}{N_{u/l}} \cdot \left(\Sigma V_{cap}^{u/l}\right), \tag{3.8}
$$

where $C_{u/l}$ and $N_{u/l}$ represent the upper or lower arm submodule's capacitance and number of submodules, respectively. Each of the three terms in Equation (3.7) can also

be represented in the $\Sigma\Delta$ domain with the following transformation:

$$\begin{bmatrix} x_\Sigma \\ x_\Delta \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_u \\ x_L \end{bmatrix}, \tag{3.9}$$

where x can be v, m, or $\Sigma V_{cap}$. By doing so, $v_{u/l}$ in Equation (3.7) can mapped into $\Sigma\Delta$ domain parameters $v_\Sigma$ and $_\Delta$ for both arms as:

$$\begin{bmatrix} V_\Sigma \\ V_\Delta \end{bmatrix} = \begin{bmatrix} m_\Sigma & m_\Delta \\ m_\Delta & m_\Sigma \end{bmatrix} \begin{bmatrix} \Sigma V_{cap,\Sigma} \\ \Sigma V_{cap,\Delta} \end{bmatrix}. \tag{3.10}$$

$[V_\Sigma, V_\Delta]^T$ in Equation (3.10) can replace $\mathbf{u}$ in Equation (3.4) to introduce the modulating signals into the model. The multiplication of the modulating signals matrix in Equation (3.10) with $b$ in Equation (3.6) results in:

$$a_{12} = \begin{bmatrix} -2m_\Sigma & -2m_\Delta \\ -2m_\Delta & -2m_\Sigma \\ \left(\frac{-2L_d}{L_a+2_d}\right) m_\Delta & \left(\frac{-2L_d}{L_a+2_d}\right) m_\Sigma \end{bmatrix}. \tag{3.11}$$

At this point, only the capacitor voltage dynamics are missing. By substituting the $\Sigma\Delta$ domain values into Equation (3.8) for each arm, the resulting equations become:

$$(m_\Sigma + m_\Delta)\left(i_\Sigma + \frac{i_{\Delta g}}{2} + \frac{i_{\Delta t}}{2}\right) = \left(\frac{C_u}{N_u}\right)\frac{d}{dt}\left(\Sigma V_{cap,\Sigma} + \Sigma V_{cap,\Delta}\right) \tag{3.12}$$

$$(m_\Sigma - m_\Delta)\left(i_\Sigma - \frac{i_{\Delta g}}{2} - \frac{i_{\Delta t}}{2}\right) = \left(\frac{C_u}{N_u}\right)\frac{d}{dt}\left(\Sigma V_{cap,\Sigma} - \Sigma V_{cap,\Delta}\right). \tag{3.13}$$

Equations (3.12) and (3.13) can be written in matrix form as:

$$c\frac{d}{dt}\mathbf{x}_v = a_{21} \cdot \mathbf{x}_i, \tag{3.14}$$

where $\mathbf{x}_v = [\Sigma V_{cap,\Sigma} \;\; \Sigma V_{cap,\Delta}]^T$, and:

$$\begin{aligned} c &= \begin{bmatrix} \frac{C_u}{N_u} & \frac{C_u}{N_u} \\ \frac{C_L}{N_L} & -\frac{C_L}{N_L} \end{bmatrix} \\ a_{21} &= \begin{bmatrix} m_\Sigma + m_\Delta & \frac{m_\Sigma + m_\Delta}{2} & \frac{m_\Sigma + m_\Delta}{2} \\ m_\Sigma - m_\Delta & \frac{m_\Sigma - m_\Delta}{2} & \frac{m_\Sigma - m_\Delta}{2} \end{bmatrix}. \end{aligned} \tag{3.15}$$

By including Equation (3.11) and Equation (3.14) into Equation (3.4), the set of differential equations that represent the time averaged arm current and capacitor voltage dynamics of the single-phase leg DC/DC/AC converter in Figure 3.3 can be written in state-space form as:

$$\frac{d}{dt}\begin{bmatrix} \mathbf{x_i} \\ \mathbf{x_v} \end{bmatrix} = \begin{bmatrix} l^{-1}a_{11} & l^{-1}a_{12} \\ c^{-1}a_{21} & 0_{2x2} \end{bmatrix}\begin{bmatrix} \mathbf{x_i} \\ \mathbf{x_v} \end{bmatrix} + \begin{bmatrix} l^{-1}n \\ 0_{2x4} \end{bmatrix}\begin{bmatrix} \mathbf{w} \end{bmatrix}, \tag{3.16}$$

where:

$$\mathbf{x_i} = \begin{bmatrix} i_\Sigma & i_{\Delta g} & i_{\Delta t} \end{bmatrix}^{\mathbf{T}}, \tag{3.17}$$

$$\mathbf{x_v} = \begin{bmatrix} \Sigma V_{cap\Sigma} & \Sigma V_{cap\Delta} \end{bmatrix}^{\mathbf{T}}, \tag{3.18}$$

$$\mathbf{w} = \begin{bmatrix} V_{dc1} & V_{dc2} & V_{ac} & V_0 \end{bmatrix}^{\mathbf{T}}. \tag{3.19}$$

### 3.1.2 Three-Phase Converter for DC/DC/AC Conversion

The set of differential equations in Equation (3.16), that represents the single-phase converter in Figure 3.3, can be used to develop a three-phase model as shown in Figure 3.4. The state variables $x_i$ and $x_v$ in Equation (3.16) represent a single phase; by triplicating

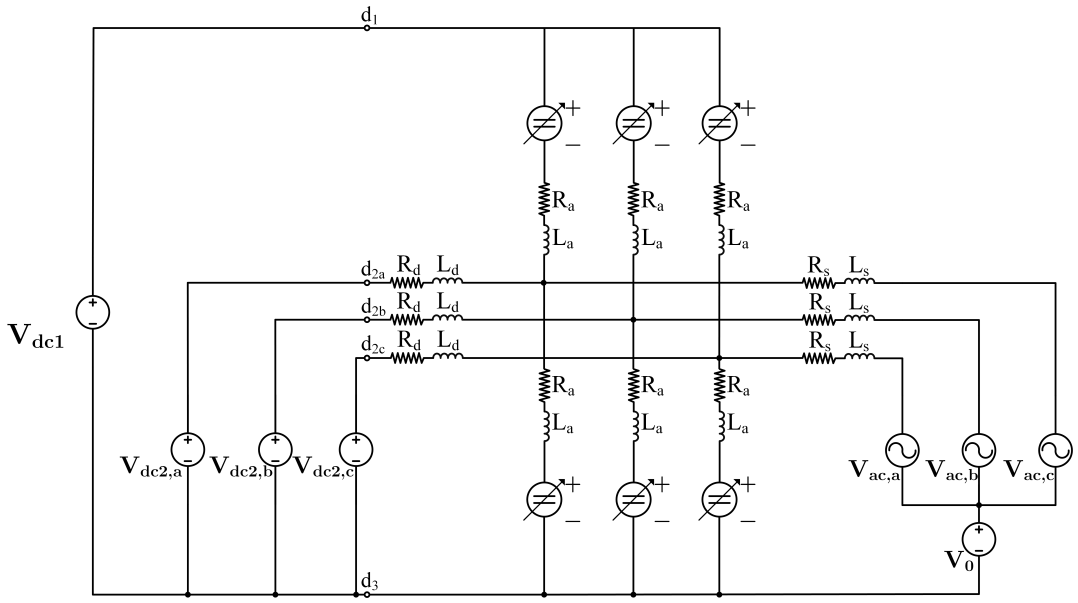



Figure 3.4: Three-phase structure for DC/DC/AC conversion

these state equations, the three-phase converter model can be expressed as:

$$\frac{d}{dt}\begin{bmatrix}\mathbf{X_i}\\ \mathbf{X_v}\end{bmatrix} = \begin{bmatrix}L^{-1}A_{11} & L^{-1}A_{12}\\ C^{-1}A_{21} & 0_{6x6}\end{bmatrix}\begin{bmatrix}\mathbf{X_i}\\ \mathbf{X_v}\end{bmatrix} + \begin{bmatrix}L^{-1}N\\ 0_{6x8}\end{bmatrix}\begin{bmatrix}\mathbf{W}\end{bmatrix}, \tag{3.20}$$

where the uppercase matrices in Equation (3.20) contain the information of their lowercase counterparts in Equation (3.16), augmented to represent the three-phase converter, in this manner:

$$U = \begin{bmatrix}u & 0 & 0\\ 0 & u & 0\\ 0 & 0 & u\end{bmatrix}. \tag{3.21}$$

The state and disturbance vectors in Equation (3.20) are now defined as:

$$\begin{aligned}
\mathbf{X_i} &= \begin{bmatrix} i_{\Sigma a} & i_{\Sigma b} & i_{\Sigma c} & i_{\Delta ga} & i_{\Delta gb} & i_{\Delta gc} & i_{\Delta ta} & i_{\Delta tb} & i_{\Delta tc} \end{bmatrix}^T \\
\mathbf{X_v} &= \begin{bmatrix} \Sigma V_{\Sigma a} & \Sigma V_{\Sigma b} & \Sigma V_{\Sigma c} & \Sigma V_{\Delta a} & \Sigma V_{\Delta b} & \Sigma V_{\Delta c} \end{bmatrix}^T \\
\mathbf{W} &= \begin{bmatrix} V_{dc1} & V_{dc2-a} & V_{dc2-b} & V_{dc2-c} & V_{ac-a} & V_{ac-b} & V_{ac-c} \end{bmatrix}^T .
\end{aligned} \tag{3.22}$$

The structure in Figure 3.4 has now three phases, which was the objective; however, the structure has at the moment three sources of DC voltage as the second DC port, which is unwanted. Now that the model has three phases, the magnetics filter (zig-zag transformer) presented in Section 2.1 is the ideal interface between the converter and the second DC port. The voltage source $V_0$, which represent a voltage common to the a,b,c phases, is also undesirable in this model. Before making any such changes, the system can more easily be modified by transforming it into the well-known $\alpha\beta 0$-frame [71].

### 3.1.3 $\alpha\beta 0$ - Frame Mapping

The Clarke transform is the method through which a system in *abc*-frame can be mapped into the $\alpha\beta 0$-frame, and is defined as:

$$T_c = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} & \frac{-1}{3} \\ 0 & \frac{1}{\sqrt{3}} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \tag{3.23}$$

Each of the quantities in Equation (3.20) that are expressed in terms of the *abc*-frame, the fifteen state-variables, the six modulating signals, and the three-phase grid voltages, are mapped into the $\alpha\beta 0$-frame as:

$$\boldsymbol{X}_{\alpha\beta 0} = T_c \cdot \boldsymbol{X}_{abc}, \tag{3.24}$$

where $\boldsymbol{X}_{\alpha\beta 0}$ and $\boldsymbol{X}_{abc}$ represent vectors of three quantities in the $\alpha\beta 0$-frame and $abc$-frame, respectively. Therefore, in Equation (3.20) every instante of $abc$ quantities can be replaced with $\alpha\beta 0$ quantities by:

$$\boldsymbol{X}_{abc} = T_c^{-1} \cdot \boldsymbol{X}_{\alpha\beta 0}. \tag{3.25}$$

After the substitution, every set of three equations is pre-multiplied by $T_C$. This will yield a model whose state variables and modulating signals are all in the $\alpha\beta 0$-frame, in the form of:

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{X}_{i\alpha\beta 0}\\ \boldsymbol{X}_{v\alpha\beta 0}\end{bmatrix} = \begin{bmatrix} L^{-1}A_{11} & L^{-1}T_3 A_{12} T_3^{-1} \\ C^{-1}T_2 A_{21} T_2^{-1} & 0_{6x6}\end{bmatrix}\begin{bmatrix}\boldsymbol{X}_{i\alpha\beta 0}\\ \boldsymbol{X}_{v\alpha\beta 0}\end{bmatrix} + L^{-1}\begin{bmatrix} I_v & N_{\alpha\beta 0}\\ 0_{6x1} & 0_{6x7}\end{bmatrix}\begin{bmatrix} V_{dc1}\\ \boldsymbol{W}_{\alpha\beta 0}\end{bmatrix}, \tag{3.26}$$

where

$$\begin{aligned}
\boldsymbol{X}_{i\alpha\beta 0} &= \begin{bmatrix} i_{\Sigma a} & i_{\Sigma b} & i_{\Sigma c} & i_{\Delta ga} & i_{\Delta gb} & i_{\Delta gc} & i_{\Delta ta} & i_{\Delta tb} & i_{\Delta tc}\end{bmatrix}^{\mathbf{T}}\\
\boldsymbol{X}_{v\alpha\beta 0} &= \begin{bmatrix} \Sigma V_{\Sigma a} & \Sigma V_{\Sigma b} & \Sigma V_{\Sigma c} & \Sigma V_{\Delta a} & \Sigma V_{\Delta b} & \Sigma V_{\Delta c}\end{bmatrix}^{\mathbf{T}}\\
\boldsymbol{W}_{\alpha\beta 0} &= \begin{bmatrix} V_{dc1} & V_{dc2-a} & V_{dc2-b} & V_{dc2-c} & V_{ac-a} & V_{ac-b} & V_{ac-c}\end{bmatrix}^{\mathbf{T}}.
\end{aligned} \tag{3.27}$$

$T_3$ and $T_2$ are 9x9 and 6x6 matrices, respectively, with diagonal $T_C$ matrices. $N_{\alpha\beta 0}$ is the same as its $abc$ counterpart in Equation (3.20), but removing the first column. $I_v$ is a column vector defined as:

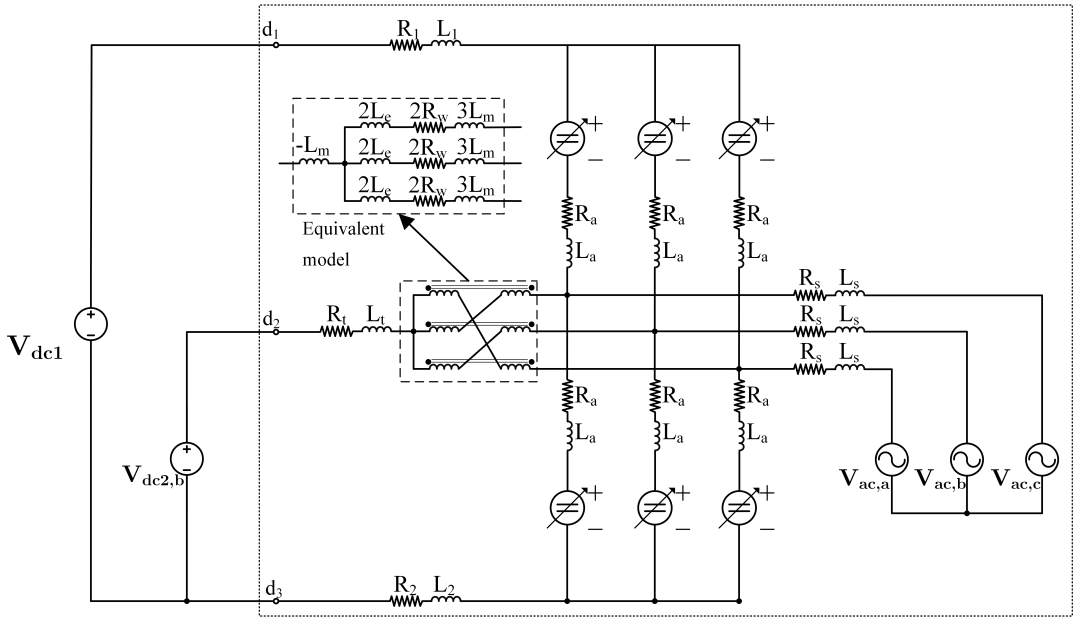



Figure 3.5: Three-phase structure for DC/DC/AC conversion

$$I_v = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}^{\mathbf{T}}. \tag{3.28}$$

In the new system of Equation (3.26), the $\alpha$ and $\beta$ components of the currents sum to zero when not in the phase legs, meaning that only *0* sequence quantities enter the DC rails. Taking advantage of this phenomenon, three important modifications are made to the topology in Figure 3.4, since now only the equations that represent the zero components of the currents will be affected. The modifications are: (i) eliminate $V_0$ and leave the AC network neutral point floating, (ii) include a zig-zag transformer as the interface between the converter structure and only one $d_2$ port, and (iii) include line impedances for each of the three DC ports.

Modification (i) is applied by deleting from the model the sixth column from $N_{\alpha\beta 0}$, as well as the input $V_0$ from $\boldsymbol{W}_{\alpha\beta 0}$. In the structure, this amounts to removing the

zero-sequence path from the AC network to the bottom DC rail of the converter. By doing this, the state variable $i_{\Delta g0}$ ceases to exist, for there is no path for it to circulate. Therefore, the sixth column of the $A_1 1$ and $A_2 1$ are removed, as well as the sixth row of the whole model. The resulting system is reduced from fifteen to fourteen state variables.

Modification (ii) requires that the existing impedances that connect the three $V_{dc2}$ sources and the converter be replaced with the equivalent model of a three-phase zig-zag transformer, shown in Figure 3.5. A zig-zag transformer is characterised by winding resistance, $R_w$, leakage inductance, $L_e$, and magnetising inductance, $L_m$. These three parameters can replace existing inductances $L_d$ and $R_d$ (in Figure 3.4) as:

$$L_d = 2L_e + 3L_m \tag{3.29}$$

$$R_d = 2R_w. \tag{3.30}$$

The -$L_m$ term in Figure 3.5 is required to properly model the zig-zag transformer, but this term is only seen by the current $i_{\Delta t0}$, as it is positioned after the three legs of the transformer have converged into one. The impedance to be added in port $d_2$ will similarly only be seen by the same current, and in the same manner, the impedances to be added in ports $d_1$ and $d_3$, modification (iii), will only be seen by current $i_{\Sigma 0}$. This makes adding them to the model relatively simple. Since the only affected parts of the model will be the third and ninth columns and rows, it's simple to rewrite the equations of the KVL loops including these terms, much like in the first section of this chapter. The final structure is shown in Figure 3.5, while the final $\alpha\beta 0$-frame model including all the aforementioned changes, is given by:

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{X}_i^* \\ \boldsymbol{X}_v^*\end{bmatrix} = \begin{bmatrix}\dot{A}_{11} & \dot{A}_{12} \\ \dot{A}_{21} & 0_{6x6}\end{bmatrix}\begin{bmatrix}\boldsymbol{X}_i^* \\ \boldsymbol{X}_v^*\end{bmatrix} + \begin{bmatrix}\dot{N} \\ 0_{6x5}\end{bmatrix}\begin{bmatrix}\boldsymbol{W}^*\end{bmatrix}, \tag{3.31}$$

where

$$\dot{A}_{11} = L^{-1}A_{11}^*;\;\; \dot{A}_{12} = L^{-1}T_3A_{12}^*T_3^{-1};\;\; \dot{A}_{21} = C^{-1}T_2A_{21}^*T_2^{-1};\;\; \dot{N} = L^{-1}N^*. \tag{3.32}$$

In Equation (3.32), the matrices with an asterisk represent their counterparts in Equation (3.26), after going through the three aforementioned modifications.

The state and disturbance vectors in Equation (3.31) are defined by:

$$\begin{aligned}\boldsymbol{X}_i^* &= \begin{bmatrix} i_{\Sigma\alpha} & i_{\Sigma\beta} & i_{\Sigma 0} & i_{\Delta g\alpha} & i_{\Delta g\beta} & i_{\Delta t\alpha} & i_{\Delta t\beta} & i_{\Delta t0}\end{bmatrix}^{\mathbf{T}} \\ \boldsymbol{X}_v^* &= \begin{bmatrix} \Sigma V_{\Sigma\alpha} & \Sigma V_{\Sigma\beta} & \Sigma V_{\Sigma 0} & \Sigma V_{\Delta\alpha} & \Sigma V_{\Delta\beta} & \Sigma V_{\Delta 0}\end{bmatrix}^{\mathbf{T}} \\ \boldsymbol{W}^* &= \begin{bmatrix} V_{dc1} & V_{dc2} & V_{ac-\alpha} & V_{ac-\beta} & V_{ac-0}\end{bmatrix}^{\mathbf{T}}.\end{aligned} \tag{3.33}$$

The $\dot{A}$ and $\dot{N}$ matrices defined in Equation (3.32) are provided in full in Appendix A.

Note that Figure 3.5 now exactly matches the model of Figure 1.1(b). The transition to the $\alpha\beta 0$-frame has permitted several changes to be made to the converter in a simple manner, as well as it permits a more streamlined analysis than in the *abc*-frame; however, the $\alpha\beta 0$-frame is a stationary reference frame, which means that the states in the model are time-varying at steady-state. Solving for an equilibrium point, $\frac{d}{dt}\boldsymbol{X}_i^* = \frac{d}{dt}\boldsymbol{X}_v^* = 0$, would not yield any useful information, as the only solution it could provide would be a trivial one, with zero power transfers.

A further transformation could be applied to the model in Equation (3.31) that would

make the states DC-valued at steady-state, the well-known *dq0* transformation. This transformation, however, would not solve the problem on its own, because although said transformation involves a rotating reference frame, it could only make one harmonic frequency DC-valued. If the *dq0*-frame were to be used, multiple synchronous *dq* reference frames would be required. Instead, the $\alpha\beta 0$-frame offers a more elegant solution while also capturing unbalanced converter operation (can capture both positive and negative sequences). The concept of "dynamic phasor modelling", as introduced in Section 1.2.3, is applied to achieve the conditions required for solving the system for equilibrium points instead. This process is presented in Section 3.2.

### 3.1.4 Three-Phase Converter for DC/DC Conversion

The model in Equation (3.31) can be used to accurately model either version of the DC/DC/AC converters with minor changes. To properly model, however, either version of the DC/DC converters, a bigger change has to be made to the model in Figure 3.5, and naturally in Equation (3.31) as well. The rows and columns that represent the AC network, i.e. all the terms related to $i_{\Delta g}$, have to be removed from the model. This results in a reduced model that is represented as:

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{X}^*_{ir}\\ \boldsymbol{X}^*_{v}\end{bmatrix} = \begin{bmatrix}\dot{A}_{11r} & \dot{A}_{12r}\\ \dot{A}_{21r} & 0_{6x6}\end{bmatrix}\begin{bmatrix}\boldsymbol{X}^*_{ir}\\ \boldsymbol{X}^*_{v}\end{bmatrix} + \begin{bmatrix}\dot{N}_r\\ 0_{6x5}\end{bmatrix}\begin{bmatrix}\boldsymbol{W}^*_{r}\end{bmatrix}, \tag{3.34}$$

where the subscript "$r$" means a reduced version of Equation (3.31), where the three-phase AC grid in Figure 3.5 has been eliminated. In Equation (3.34), matrices $\dot{A}_{12r}$ and $\dot{N}_r$ are obtained by removing the fourth and fifth row, matrix $\dot{A}_{21r}$ by removing the fourth and fifth column, and matrix $\dot{A}_{11r}$ by removing the fourth and fifth columns and rows, from their respective counterparts in Equation (3.31). The reduced state and input

vectors are defined as:

$$\begin{aligned} \boldsymbol{X}_{ir}^{*} &= \begin{bmatrix} i_{\Sigma\alpha} & i_{\Sigma\beta} & i_{\Sigma 0} & i_{\Delta t\alpha} & i_{\Delta t\beta} & i_{\Delta t0} \end{bmatrix}^{\mathbf{T}} \\ \boldsymbol{W}_{r}^{*} &= \begin{bmatrix} V_{dc1} & V_{dc2} \end{bmatrix}^{\mathbf{T}}. \end{aligned} \tag{3.35}$$

### 3.1.5 Model Validation

In this section, the validation of the developed models is carried out by comparing switched PLECS models of the MP-DCDCAC and BP-DCDC topologies in Figures 3.1(b) and 3.1(e), respectively, with their respective time averaged models in Equation (3.31) and Equation (3.34). All the simulations are run using open loop control. The open loop modulating signals are obtained by a solution procedure that will be explained in Chapter 4. Given this, the modulating signals are not specified here, and only the operating points for which the converters are set to validate them and their parameters are given. The simulations are carried out using the PLECS blockset in Simulink. The switched model used for the MMC arms was the one provided in the PLECS' demo "*HVDC Transmission System with MMCs*".

Table 3.1 shows the parameters used for the two simulations. The MP-DCDCAC converter is rated for 480 MW, and the operating point for this simulation is one in which half of its rated power is being transferred from the first DC port to the second DC port, and the other half is transferred to the AC port with a 0.9 power factor. The BP-DCDC converter is rated for 384 MW, and the operating point for this simulation is set to a transfer of 18% of rated power from the first DC port to the second DC port. In both cases, a disturbance at $t$=2s in the form of a reduction of power transferred of 50% for the AC and 10% for the second DC port is triggered to demonstrate matching

of both models during steady-state and dynamic response.

Figure 3.6 and Figure 3.7 show how the waveforms for the time averaged and switched models of the MP-DCDCAC and BP-DCDC, respectively, have an excellent match in both simulations. To provide a more compact notation in the waveforms' results, the voltage state variables, i.e. $\Sigma v_{\Sigma}$ and $\Sigma v_{\Delta}$, are referred to only as $v_{\Sigma}$ and $v_{\Delta}$. It's worth noting that the current that goes through the $d_2$ port is equal to three times the state variable $i_{\Delta t0}$. Figure 3.8 shows the upper and lower arm voltage of phase $a$ for both simulations, where the stepped voltage change from the switched model is contrasted by the smooth curve of the averaged model. Since a high degree of matching between the averaged and switched models is expected, as it has been shown in many works, i.e. [46], [51], [70], [72]–[74], the model is determined validated by confirming the excellent visual matching between the two models.

Table 3.1: Simulation parameters for the switched/averaged simulations

| Parameter | MP-DCDCAC | BP-DCDC |
|---|---|---|
| $w, f_s w^*$ | $2\pi$60 rad/s, 10kHz | $2\pi$60 rad/s, 10kHz |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH,0,40 mH, 50 mH, 20 mH | 80 mH, 80 mH, 0, 50 mH, 0 |
| $L_e, L_m$ | 9.8 mH, 28 H | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 0 Ω, 1 Ω, 0.4 Ω, 0.6 Ω | 2.1 Ω, 2.1 Ω, 0 Ω, 0.4 Ω, 0 Ω |
| $R_w$ | 0 Ω | 0.1 Ω |
| $C_u, C_L$ | 20 mF | 10 mF |
| $N_u, N_L$ | 38 | 38 |
| $V_{dc1}, V_{dc2}$ | 400 kV, 200 kV | 76 kV, 76 kV |
| *Peak L-N* $V_{ac}$ | 180 kV | - |

* Carrier frequency for the SPWM for the switched model

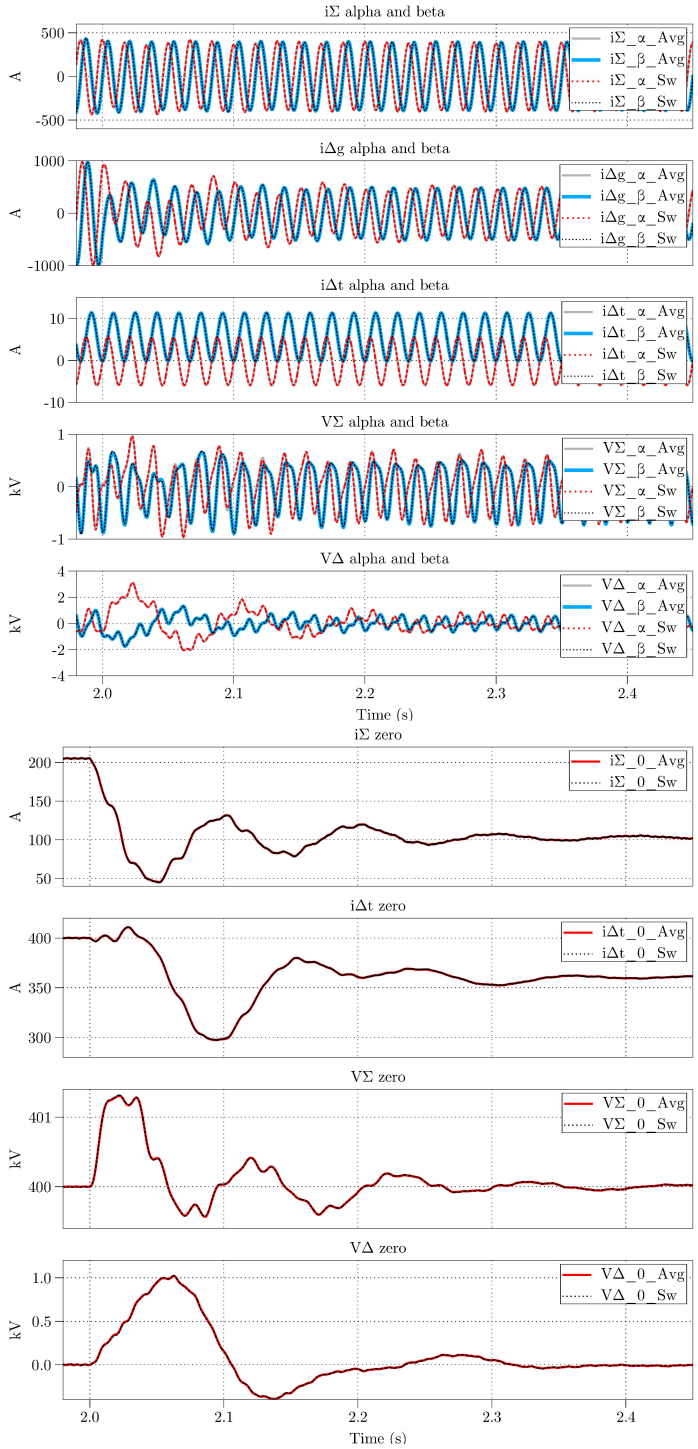



Figure 3.6: MP-DCDCAC simulation results comparing the 14 state variables in the $\alpha\beta 0$ frame for the PLECS' switched model vs the time averaged model in Equation (3.31)

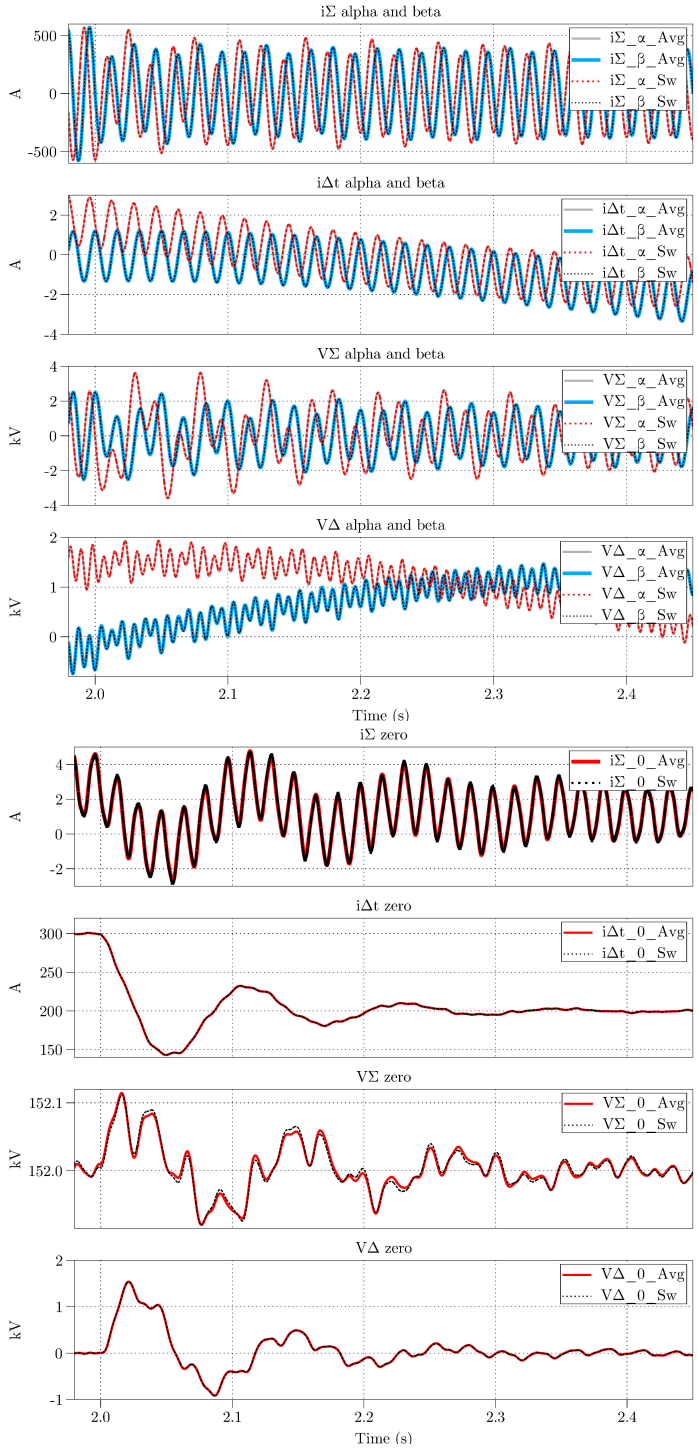



Figure 3.7: BP-DCDC simulation results comparing the 12 state variables in the $\alpha\beta$ 0 frame for the PLECS' switched model vs the time averaged model in Equation (3.34)

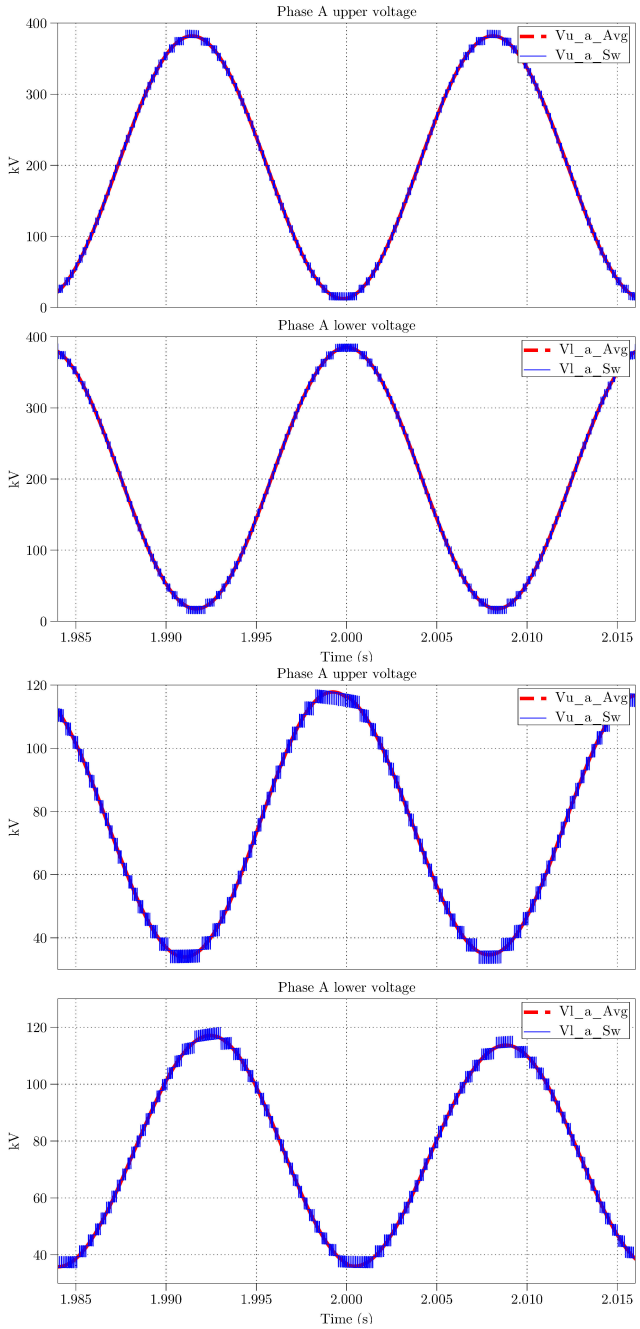



Figure 3.8: Upper and lower phase a arms' voltages for the: (a) MP-DCDCAC and (b) BP-DCDC simulations, comparing the PLECS' switched model vs time averaged model in Equation (3.31) and Equation (3.34)

## 3.2 Dynamic Phasor Model

In this section, the steps taken to develop a DPM of the MP-DCDCAC system given by Equation (3.31) are shown. This DPM can then be used, with minor modifications, to derive DPMs of the three remaining converter structures in Figure 3.1c - Figure 3.1e. All four DPMs are validated against their timeaveraged counterpart models developed in Section 3.1.

### 3.2.1 DPM Derivation for DC/DC/AC Conversion

The principle of dynamic phasors, introduced as generalised averaging method in [54] and applied to MMCs in [57]–[61], can be described as an averaging tool used to approximate time-varying waveforms with a Fourier series expansion that includes the time-varying nature of the waveforms in its complex coefficients. The approximation of a time-varying signal, $x(t)$, can be represented, for an arbitrary accuracy and on the interval ($t$-T), as [54]:

$$x(t) \approx \sum_{k=-n}^{+n} \langle x \rangle_k(t) e^{jkw_s(t-T+s)}, \tag{3.36}$$

where $k$ represents integers of the harmonics, $n$ the range of harmonics to be considered for the Fourier expansion, $\omega_s = \frac{2\pi}{T}$, $s \in (t-T, t)$, and $\langle x \rangle_k(t)$ represents the complex Fourier coefficients, or the dynamic phasor of $x(t)$. Instead of using the complex exponential representation, the use of sines and cosines is adopted in this work, to allow the direct use of real quantities.

According to Equation (3.36), it is necessary to choose which harmonics are to be modelled. To pick which harmonics are to be selected for this converter, the system in Equation (3.31) is simulated in open-loop, with the control inputs chosen to approximate

a realistic operating point. The resulting waveforms indicate that the dominant harmonics for the states in Equation (3.31) can be categorised based on their $\alpha\beta 0$ components. Table 3.2 summarises the findings for these simulations.

Table 3.2: Dominant harmonic components for each $\alpha\beta 0$ frame component in system Equation (3.31)

| $\alpha\beta 0$ **component** | **DC** | $\mathbf{1}^{st}$ | $\mathbf{2}^{nd}$ | $\mathbf{3}^{rd}$ |
|---|---|---|---|---|
| $\alpha$ | | X | X | |
| $\beta$ | | X | X | |
| 0 | X | | | X |

Therefore, the states in Equation (3.31) are to be approximated by a combination of DC and AC harmonic components as follows:

$$i_{\Sigma\alpha}(t) \approx I^{1\parallel}_{\Sigma\alpha} cos(wt) + I^{1\perp}_{\Sigma\alpha} sin(wt) + I^{2\parallel}_{\Sigma\alpha} cos(2wt) + I^{2\perp}_{\Sigma\alpha} sin(2wt) \tag{3.37}$$

$$i_{\Sigma\beta}(t) \approx I^{1\parallel}_{\Sigma\beta} cos(wt) + I^{1\perp}_{\Sigma\beta} sin(wt) + I^{2\parallel}_{\Sigma\beta} cos(2wt) + I^{2\perp}_{\Sigma\beta} sin(2wt) \tag{3.38}$$

$$i_{\Sigma 0}(t) \approx I^{dc}_{\Sigma 0} + I^{3\parallel}_{\Sigma 0} cos(3wt) + I^{3\perp}_{\Sigma 0} sin(2wt) \tag{3.39}$$

$$i_{\Delta g\alpha}(t) \approx I^{1\parallel}_{\Delta g\alpha} cos(wt) + I^{1\perp}_{\Delta g\alpha} sin(wt) + I^{2\parallel}_{\Delta g\alpha} cos(2wt) + I^{2\perp}_{\Delta g\alpha} sin(2wt) \tag{3.40}$$

$$i_{\Delta g\beta}(t) \approx I^{1\parallel}_{\Delta g\beta} cos(wt) + I^{1\perp}_{\Delta g\beta} sin(wt) + I^{2\parallel}_{\Delta g\beta} cos(2wt) + I^{2\perp}_{\Delta g\beta} sin(2wt) \tag{3.41}$$

$$i_{\Delta t\alpha}(t) \approx I^{1\parallel}_{\Delta t\alpha} cos(wt) + I^{1\perp}_{\Delta t\alpha} sin(wt) + I^{2\parallel}_{\Delta t\alpha} cos(2wt) + I^{2\perp}_{\Delta t\alpha} sin(2wt) \tag{3.42}$$

$$i_{\Delta t0}(t) \approx I^{dc}_{\Sigma t0} + I^{3\parallel}_{\Sigma t0} cos(3wt) + I^{3\perp}_{\Sigma t0} sin(2wt) \tag{3.43}$$

$$i_{\Delta t\beta}(t) \approx I^{1\parallel}_{\Delta t\beta} cos(wt) + I^{1\perp}_{\Delta t\beta} sin(wt) + I^{2\parallel}_{\Delta t\beta} cos(2wt) + I^{2\perp}_{\Delta t\beta} sin(2wt) \tag{3.44}$$

$$\Sigma v_{\Sigma\alpha}(t) \approx \Sigma V_{\Sigma\alpha}^{1\|} cos(wt) + \Sigma V_{\Sigma\alpha}^{1\perp} sin(wt) + \Sigma V_{\Sigma\alpha}^{2\|} cos(2wt) + \Sigma V_{\Sigma\alpha}^{2\perp} sin(2wt) \tag{3.45}$$

$$\Sigma v_{\Sigma\beta}(t) \approx \Sigma V_{\Sigma\beta}^{1\|} cos(wt) + \Sigma V_{\Sigma\beta}^{1\perp} sin(wt) + \Sigma V_{\Sigma\beta}^{2\|} cos(2wt) + \Sigma V_{\Sigma\beta}^{2\perp} sin(2wt) \tag{3.46}$$

$$\Sigma v_{\Sigma 0}(t) \approx \Sigma V_{\Sigma 0}^{dc} + \Sigma V_{\Sigma 0}^{3\|} cos(3wt) + \Sigma V_{\Sigma 0}^{3\perp} sin(2wt) \tag{3.47}$$

$$\Sigma v_{\Delta\alpha}(t) \approx \Sigma V_{\Delta\alpha}^{1\|} cos(wt) + \Sigma V_{\Delta\alpha}^{1\perp} sin(wt) + \Sigma V_{\Delta\alpha}^{2\|} cos(2wt) + \Sigma V_{\Delta\alpha}^{2\perp} sin(2wt) \tag{3.48}$$

$$\Sigma v_{\Delta\beta}(t) \approx \Sigma V_{\Delta\beta}^{1\|} cos(wt) + \Sigma V_{\Delta\beta}^{1\perp} sin(wt) + \Sigma V_{\Delta\beta}^{2\|} cos(2wt) + \Sigma V_{\Delta\beta}^{2\perp} sin(2wt) \tag{3.49}$$

$$\Sigma v_{\Delta 0}(t) \approx \Sigma V_{\Delta 0}^{dc} + \Sigma V_{\Delta 0}^{3\|} cos(3wt) + \Sigma V_{\Delta 0}^{3\perp} sin(2wt). \tag{3.50}$$

The control inputs follow the same pattern, with their components assigned as follows:

$$m_{\Sigma\alpha}(t) = M_{\Sigma\alpha}^{1\|} cos(wt) + M_{\Sigma\alpha}^{1\perp} sin(wt) \tag{3.51}$$

$$m_{\Sigma\beta}(t) = M_{\Sigma\beta}^{1\|} cos(wt) + M_{\Sigma\beta}^{1\perp} sin(wt) \tag{3.52}$$

$$m_{\Sigma 0}(t) = M_{\Sigma 0}^{dc} \tag{3.53}$$

$$m_{\Delta\alpha}(t) = M_{\Delta\alpha}^{1\|} cos(wt) + M_{\Delta\alpha}^{1\perp} sin(wt) \tag{3.54}$$

$$m_{\Delta\beta}(t) = M_{\Delta\beta}^{1\|} cos(wt) + M_{\Delta\beta}^{1\perp} sin(wt) \tag{3.55}$$

$$m_{\Delta 0}(t) = M_{\Delta 0}^{dc}. \tag{3.56}$$

The only disturbance terms in the model that require conversion into the DPM are the AC sources, which are assigned fundamental frequency components, as they are assumed to be purely sinusoidal. Note that each harmonic is represented by a set of parallel (cosine) and a perpendicular (sine) component. The uppercase average values, e.g. $I_{\Delta g\beta}^{1\|}(t)$ and $M_{\Sigma\beta}^{1\|}(t)$ are DC valued in steady-state. Also note that time dependence of the dynamic phasors is not explicitly shown for compact notation, e.g. $M_{\Sigma 0}^{dc}(t)$ and

$M^{1\perp}_{\Sigma\beta}(t)$ are equivalent to $M^{dc}_{\Sigma 0}$ and $M^{1\perp}_{\Sigma\beta}$, respectively.

To obtain the equations for the dynamic phasors, Equations (3.35)-(3.54) are substituted into Equation (3.31), then the corresponding multiplications are performed. After, the equations are separated into those that contain sines and cosines with the same frequency, and the system is solved for each of the dynamic phasors, e.g. $\frac{d}{dt}I^{1\|}_{\Sigma\alpha}(t)$ process is extremely tedious, albeit quite straightforward. A detailed tutorial of this process can be found in [70], [73].

The resulting DPM in the $\alpha\beta 0$-frame respresenting the structure in Figure 3.5 is presented as:

$$\frac{d}{dt}\begin{bmatrix}\widetilde{\boldsymbol{X}}^*_i\\ \widetilde{\boldsymbol{X}}^*_v\end{bmatrix} = \begin{bmatrix}\widetilde{A}_{11} & \widetilde{A}_{12}\\ \widetilde{A}_{21} & 0_{22x22}\end{bmatrix}\begin{bmatrix}\widetilde{\boldsymbol{X}}^*_i\\ \widetilde{\boldsymbol{X}}^*_v\end{bmatrix} + \begin{bmatrix}\widetilde{N}\\ 0_{22x5}\end{bmatrix}\left[\widetilde{\boldsymbol{W}}^*\right], \tag{3.57}$$

where:

$$\begin{aligned}\widetilde{\boldsymbol{X}}^*_i =[\ & I^{1\|}_{\Sigma\alpha}\ I^{1\perp}_{\Sigma\alpha}\ I^{2\|}_{\Sigma\alpha}\ I^{2\perp}_{\Sigma\alpha}\ I^{1\|}_{\Sigma\beta}\ I^{1\perp}_{\Sigma\beta}\ I^{2\|}_{\Sigma\beta}\ I^{2\perp}_{\Sigma\beta}\ I^{dc}_{\Sigma 0}\ I^{3\|}_{\Sigma 0}\ I^{3\perp}_{\Sigma 0}\ I^{1\|}_{\Delta g\alpha}\ldots \\ \ldots\ & I^{1\perp}_{\Delta g\alpha}\ I^{2\|}_{\Delta g\alpha}\ I^{2\perp}_{\Delta g\alpha}\ I^{1\|}_{\Delta g\beta}\ I^{1\perp}_{\Delta g\beta}\ I^{2\|}_{\Delta g\beta}\ I^{2\perp}_{\Delta g\beta}\ I^{1\|}_{\Delta t\alpha}\ I^{1\perp}_{\Delta t\alpha}\ I^{2\|}_{\Delta t\alpha}\ldots \\ \ldots\ & I^{2\perp}_{\Delta t\alpha}\ I^{1\|}_{\Delta t\beta}\ I^{1\perp}_{\Delta t\beta}\ I^{2\|}_{\Delta t\beta}\ I^{2\perp}_{\Delta t\beta}\ I^{dc}_{\Delta t0}\ I^{3\|}_{\Delta t0}\ I^{3\perp}_{\Delta t0}\ ]\end{aligned} \tag{3.58}$$

$$\begin{aligned}\widetilde{\boldsymbol{X}}^*_v =[\ & \Sigma V^{1\|}_{\Sigma\alpha}\ \Sigma V^{1\perp}_{\Sigma\alpha}\ \Sigma V^{2\|}_{\Sigma\alpha}\ \Sigma V^{2\perp}_{\Sigma\alpha}\ \Sigma V^{1\|}_{\Sigma\beta}\ \Sigma V^{1\perp}_{\Sigma\beta}\ \Sigma V^{2\|}_{\Sigma\beta}\ \Sigma V^{2\perp}_{\Sigma\beta}\ldots \\ \ldots\ & \Sigma V^{dc}_{\Sigma 0}\ \Sigma V^{3\|}_{\Sigma 0}\ \Sigma V^{3\perp}_{\Sigma 0}\ \Sigma V^{1\|}_{\Delta t\alpha}\ \Sigma V^{1\perp}_{\Delta t\alpha}\ \Sigma V^{2\|}_{\Delta t\alpha}\ \Sigma V^{2\perp}_{\Delta t\alpha}\ldots \\ \ldots\ & \Sigma V^{1\|}_{\Delta t\beta}\ \Sigma V^{1\perp}_{\Delta t\beta}\ \Sigma V^{2\|}_{\Delta t\beta}\ \Sigma V^{2\perp}_{\Delta t\beta}\ \Sigma V^{dc}_{\Delta t0}\ \Sigma V^{3\|}_{\Delta t0}\ \Sigma V^{3\perp}_{\Delta t0}\ ]\end{aligned} \tag{3.59}$$

$$\widetilde{\boldsymbol{W}}^* =[\ V_{dc1}\ V_{dc2}\ V^{1\|}_{ac,\alpha}\ V^{1\perp}_{ac,\alpha}\ V^{1\|}_{ac,\beta}\ V^{1\perp}_{ac,\beta}\ ]. \tag{3.60}$$

The system in Equation (3.57) contains 52 states, shown in Equation (3.58) and Equation (3.59), 6 disturbance terms, shown in Equation (3.60), and 10 modulating

signals, which are embedded in the state matrix. The states and disturbance matrices in Equation (3.57) are given in Appendix C.

### 3.2.2 DPM Derivation for DC/DC Conversion

The DPM for DC/DC conversion is obtained in the same way the DC/DC three-phase model in Section 3.1.4 is derived, which is by reducing the DC/DC/AC DPM. The rows and columns that represent the AC network, i.e. all the terms related to grid current $I_{\Delta g}$, are removed from Equation (3.57). The resulting DPM is represented as:

$$\frac{d}{dt}\begin{bmatrix}\widetilde{\boldsymbol{X}}^*_{ir}\\ \widetilde{\boldsymbol{X}}^*_{vr}\end{bmatrix} = \begin{bmatrix}\widetilde{A}_{11r} & \widetilde{A}_{12r}\\ \widetilde{A}_{21r} & 0_{22x22}\end{bmatrix}\begin{bmatrix}\widetilde{\boldsymbol{X}}^*_{ir}\\ \widetilde{\boldsymbol{X}}^*_{vr}\end{bmatrix} + \begin{bmatrix}\widetilde{N}_r\\ 0_{22x5}\end{bmatrix}\left[\widetilde{\boldsymbol{W}}^*_r\right], \tag{3.61}$$

where the subscript "$r$" means a reduced version of Equation (3.57), where the three-phase AC grid parameters have been eliminated. In Equation (3.61), matrices $\widetilde{A}_{12r}$ and $\widetilde{N}_r$ are obtained by removing rows $12^{th}$ to $19^{th}$, matrix $\widetilde{A}_{21r}$ by removing columns $12^{th}$ to $19^{th}$, and matrix $\widetilde{A}_{11r}$ by removing both columns and rows $12^{th}$ to $19^{th}$, from their respective counterparts in Equation (3.57). The reduced state and input vectors are defined as:

$$\widetilde{\boldsymbol{X}}^*_{ir} = [\ I^{1\parallel}_{\Sigma\alpha}\ I^{1\perp}_{\Sigma\alpha}\ I^{2\parallel}_{\Sigma\alpha}\ I^{2\perp}_{\Sigma\alpha}\ I^{1\parallel}_{\Sigma\beta}\ I^{1\perp}_{\Sigma\beta}\ I^{2\parallel}_{\Sigma\beta}\ I^{2\perp}_{\Sigma\beta}\ I^{dc}_{\Sigma0}\ I^{3\parallel}_{\Sigma0}\ I^{3\perp}_{\Sigma0} \ldots \tag{3.62}$$
$$\ldots\ I^{1\parallel}_{\Delta t\alpha}\ I^{1\perp}_{\Delta t\alpha}\ I^{2\parallel}_{\Delta t\alpha}\ I^{2\perp}_{\Delta t\alpha}\ I^{1\parallel}_{\Delta t\beta}\ I^{1\perp}_{\Delta t\beta}\ I^{2\parallel}_{\Delta t\beta}\ I^{2\perp}_{\Delta t\beta}\ I^{dc}_{\Delta t0} \ldots$$
$$\ldots\ I^{3\parallel}_{\Delta t0}\ I^{3\perp}_{\Delta t0}\ ]$$
$$\widetilde{\boldsymbol{X}}^*_{vr} = [\ \Sigma V^{1\parallel}_{\Sigma\alpha}\ \Sigma V^{1\perp}_{\Sigma\alpha}\ \Sigma V^{2\parallel}_{\Sigma\alpha}\ \Sigma V^{2\perp}_{\Sigma\alpha}\ \Sigma V^{1\parallel}_{\Sigma\beta}\ \Sigma V^{1\perp}_{\Sigma\beta}\ \Sigma V^{2\parallel}_{\Sigma\beta}\ \Sigma V^{2\perp}_{\Sigma\beta} \ldots \tag{3.63}$$
$$\ldots\ \Sigma V^{dc}_{\Sigma0}\ \Sigma V^{3\parallel}_{\Sigma0}\ \Sigma V^{3\perp}_{\Sigma0}\ \Sigma V^{1\parallel}_{\Delta t\alpha}\ \Sigma V^{1\perp}_{\Delta t\alpha}\ \Sigma V^{2\parallel}_{\Delta t\alpha}\ \Sigma V^{2\perp}_{\Delta t\alpha} \ldots$$
$$\ldots\ \Sigma V^{1\parallel}_{\Delta t\beta}\ \Sigma V^{1\perp}_{\Delta t\beta}\ \Sigma V^{2\parallel}_{\Delta t\beta}\ \Sigma V^{2\perp}_{\Delta t\beta}\ \Sigma V^{dc}_{\Delta t0}\ \Sigma V^{3\parallel}_{\Delta t0}\ \Sigma V^{3\perp}_{\Delta t0}\ ]$$
$$\widetilde{\boldsymbol{W}}^*_{r} = [\ V_{dc1}\ V_{dc2}\ ] \tag{3.64}$$

The system in Equation (3.61) contains 44 states, shown in Equation (3.62) and Equation (3.63), 2 disturbance terms, shown in Equation (3.63), and 10 modulating signals, which are embedded in the state matrix. The state and disturbance matrices in Equation (3.61) are given in Appendix D.

### 3.2.3 DPM Validation

In this section, the validation of the developed DPMs is carried out in the same way as the validation in Section 3.1.5. In this instance, however, the switched model is not used to compare the DPM, having demonstrated that the TAM is an excellent match for the switched model, this validation is performed by comparing the TAM and the DPM.

For this validation, one set of results is shown for each of the four topologies in Figure 3.1. Table 3.3 shows the parameters used in each simulation. The parameters have been chosen to demonstrate the versatility of the DPM when it comes to the number of submodules in the upper and lower arms, as well as the capacitance of each submodule. For all four cases, the capacitances are selected to maintain the energy storage of the

converter at an approximate of $50\frac{kJ}{MW}$. For the case of DC/DC/AC conversion in both topologies, 1 pu power is being transferred from the first DC port to the second DC port and AC port, split equally. For the DC/DC conversion in both topologies, 1 pu power is being transferred from the first DC port to the second DC port. The disturbance is also presented as a power reduction of power transferred of 50% for the AC and 10% for the second DC port, triggered at $t$=2s.

Figures 3.9 to 3.12 show the results for the four simulation cases, comparing the averaged model with the DPM. Tables 3.4 to 3.7 make a comparison of the most important harmonic components for the state variables in each case. The superscript in the state variable indicates which harmonic component is being compared.

Table 3.3: Simulation parameters for the averaged/DPM simulations

| Parameter | MP-DCDCAC | MP-DCDC |
|---|---|---|
| $P_{rated}$ | 480 MW | 480 MW |
| $w$ | $2\pi 60$ rad/s | $2\pi 60$ rad/s |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH,0,40 mH, 50 mH, 20 mH | 80 mH, 80 mH, 0, 50 mH, 0 |
| $L_e, L_m$ | 9.8 mH, 28 H | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 0 Ω, 1 Ω, 0.4 Ω, 0.6 Ω | 2.1 Ω, 0 Ω, 1 Ω, 0.4 Ω, 0 Ω |
| $R_w$ | 0 Ω | 0 Ω |
| $C_u, C_L$ | 10 mF, 12 mF | 10 mF, 12 mF |
| $N_u, N_L$ | 200, 180 | 200, 180 |
| $V_{dc1}, V_{dc2}$ | 400 kV, 200 kV | 400 kV, 200 kV |
| *Peak L-N* $V_{ac}$ | 180 kV | N/A |
| Parameter | BP-DCDCAC | BP-DCDC |
| $P_{rated}$ | 768 MW | 384 MW |
| $w$ | $2\pi 60$ rad/s | $2\pi 60$ rad/s |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH,82 mH, 0 mH, 50 mH, 20 mH | 80 mH, 80 mH, 0, 50 mH, 0 |
| $L_e, L_m$ | 9.8 mH, 28 H | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 2.1 Ω, 0 Ω, 0.4 Ω, 0.6 Ω | 2.1 Ω, 2.1 Ω, 0 Ω, 0.4 Ω, 0 Ω |
| $R_w$ | 0.1 Ω | 0.1 Ω |
| $C_u, C_L$ | 18 mF, 18 mF | 9 mF, 9 mF |
| $N_u, N_L$ | 200, 200 | 200, 200 |
| $V_{dc1}, V_{dc2}$ | 320 kV, 320 kV | 160 kV, 160 kV |
| *Peak L-N* $V_{ac}$ | 180 kV | N/A |

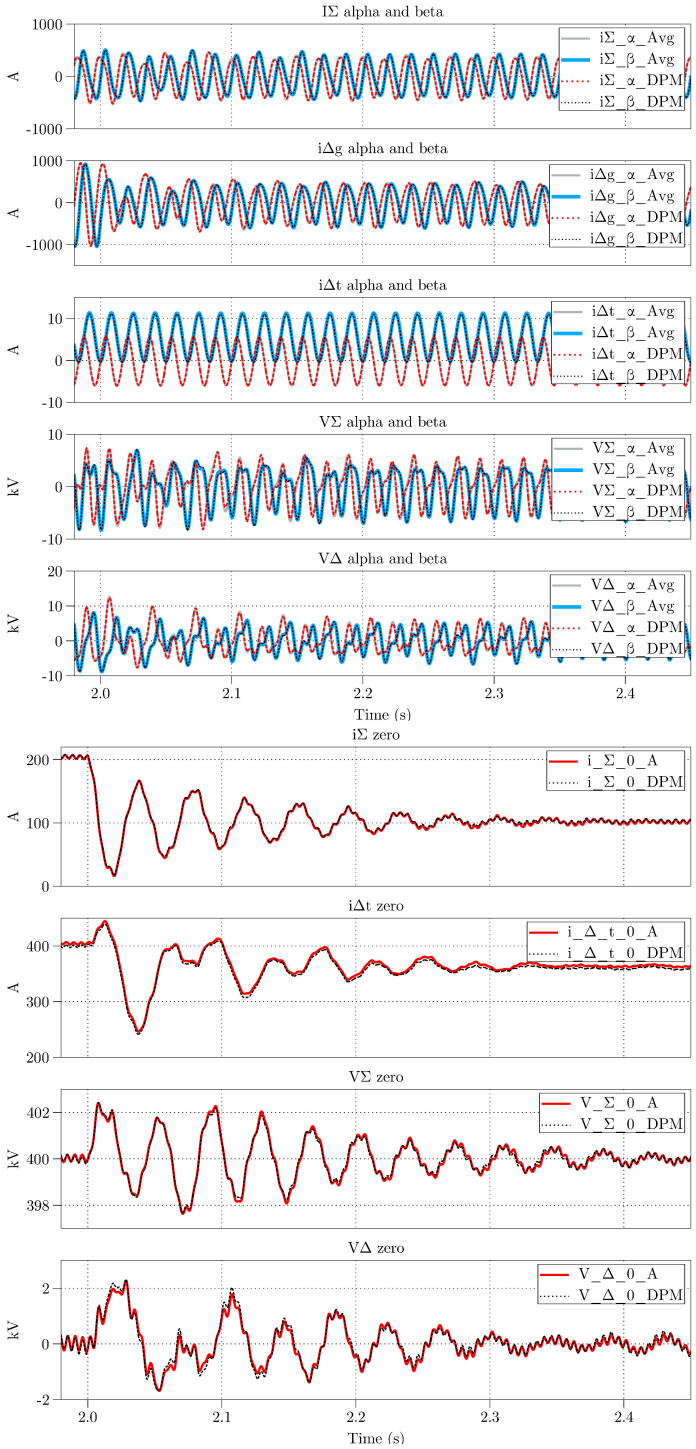



Figure 3.9: MP-DCDCAC; DPM and averaged model simulation results

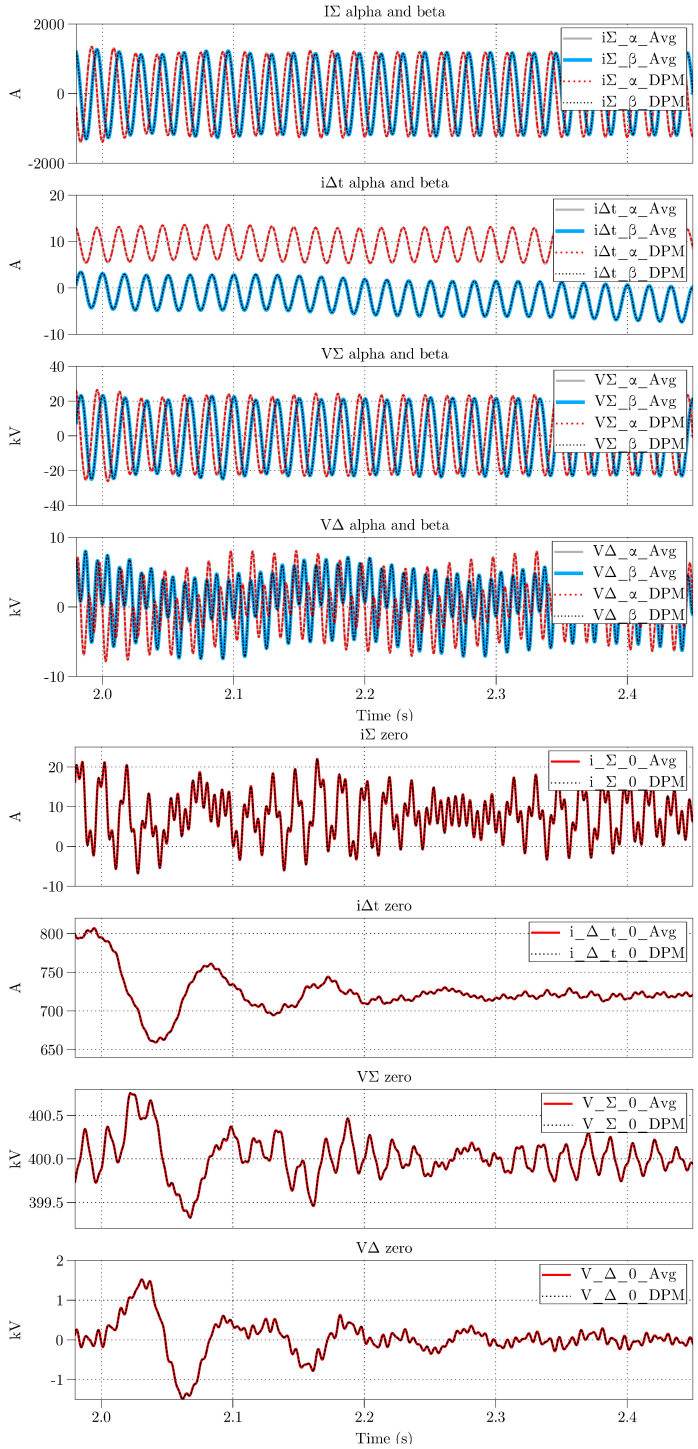



Figure 3.10: MP-DCDC; DPM and averaged model simulation results

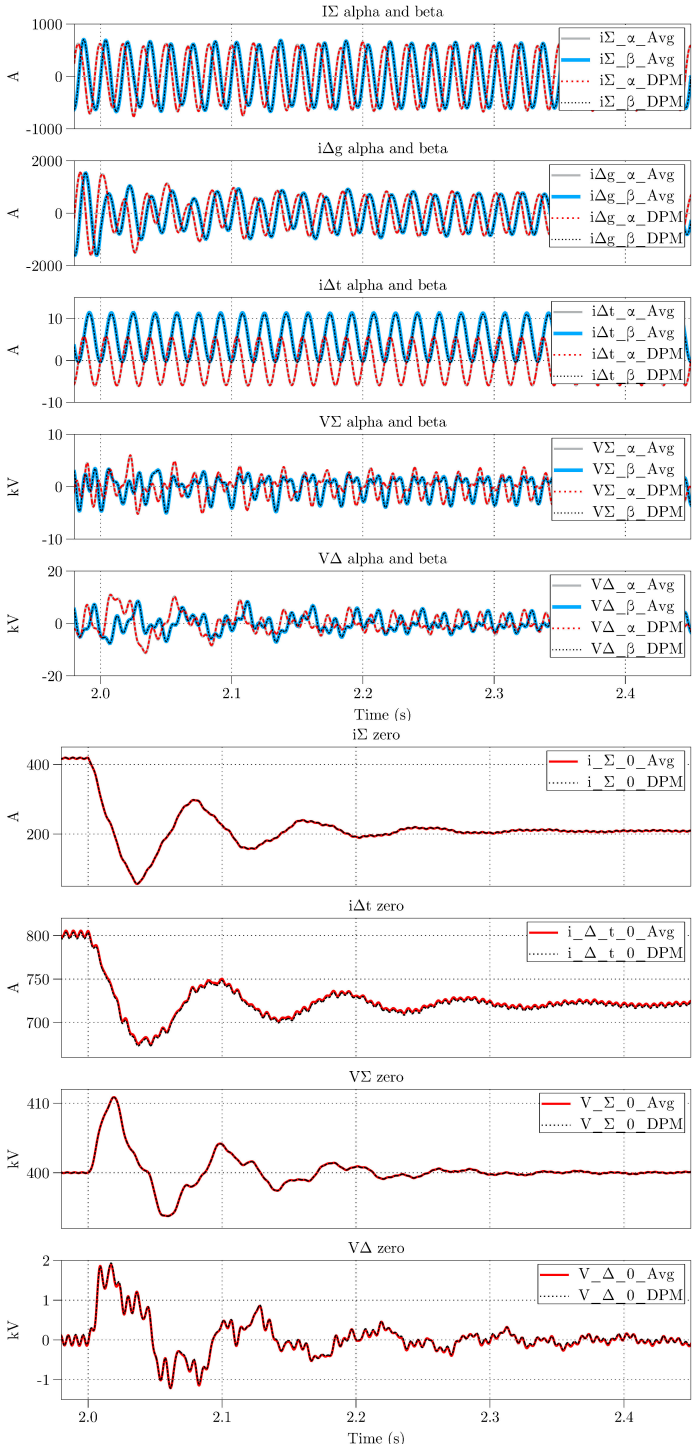



Figure 3.11: BP-DCDCAC; DPM and averaged model simulation results

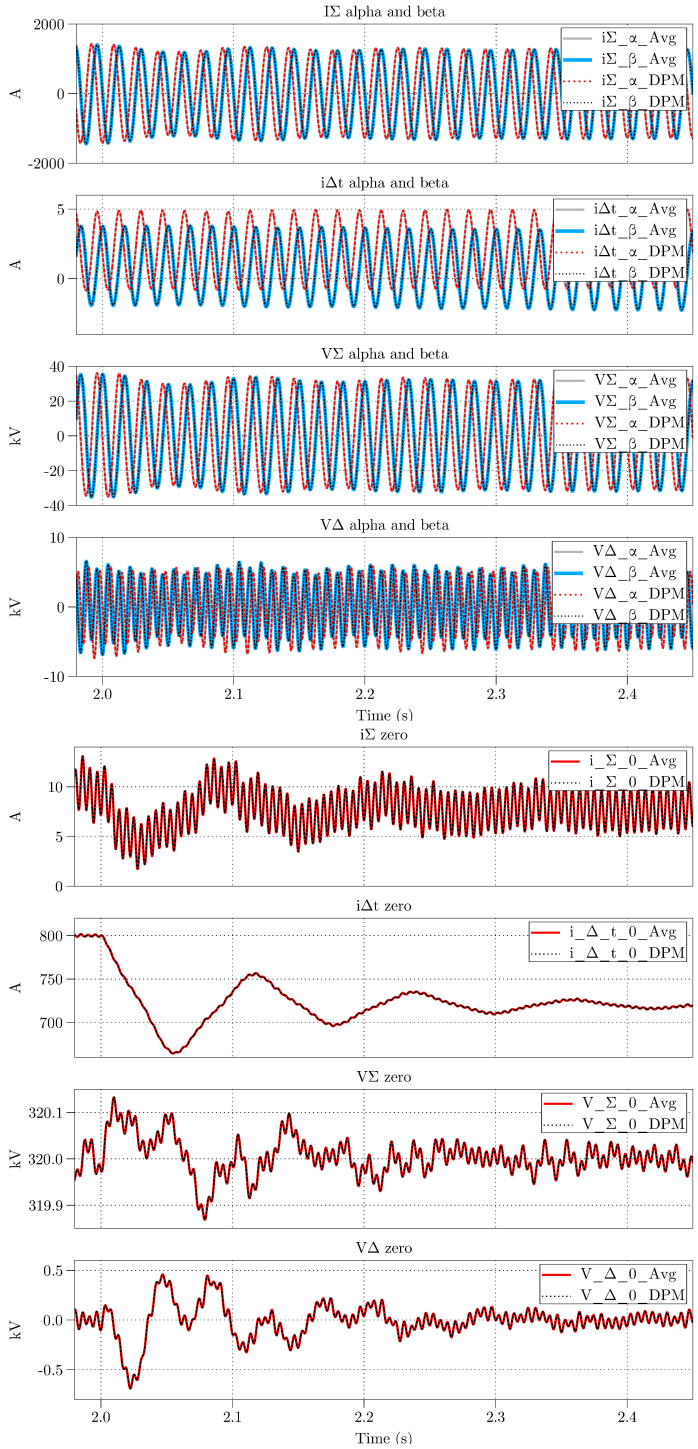



Figure 3.12: BP-DCDC; DPM and averaged model simulation results

Table 3.4: MP-DCDCAC State Variables Steady-State Comparison

| State Variable | TAM | DPM | Percent Error |
|---|---|---|---|
| $i^1_{\Sigma\alpha}[A_pk]$ | 431.434 | 423.31 | 1.92% |
| $i^1_{\Delta g\alpha}[A_pk]$ | 976.73 | 983.829 | 0.72% |
| $V^1_{\Sigma\alpha}[kV_{pk}]$ | 5.18522 | 5.08545 | 1.96% |
| $V^2_{\Sigma\alpha}[kV_{pk}]$ | 3.02994 | 3.1145 | 2.72% |
| $V^1_{\Delta\alpha}[kV_{pk}]$ | 7.13196 | 7.28104 | 2.05% |
| $V^2_{\Delta\alpha}[kV_{pk}]$ | 2.48198 | 2.33423 | 6.33% |
| $i^{dc}_{\Sigma 0}[A]$ | 204.045 | 205.262 | 0.59% |
| $i^{dc}_{\Delta t\alpha}[A]$ | 403.964 | 399.974 | 1.00% |
| $V^{dc}_{\Sigma 0}[kV]$ | 399.998 | 400.002 | 0.00% |

Table 3.5: MP-DCDC State Variables Steady-State Comparison

| State Variable | TAM | DPM | Percent Error |
|---|---|---|---|
| $i^1_{\Sigma\alpha}[A_pk]$ | 1353.38 | 1353.01 | 0.03% |
| $V^1_{\Sigma\alpha}[kV_{pk}]$ | 25.4928 | 25.4843 | 0.03% |
| $V^2_{\Sigma\alpha}[kV_{pk}]$ | 1.12191 | 1.12269 | 0.07% |
| $V^1_{\Delta\alpha}[kV_{pk}]$ | 3.04729 | 3.04717 | 0.00% |
| $V^2_{\Delta\alpha}[kV_{pk}]$ | 4.65869 | 4.65561 | 0.07% |
| $i^{dc}_{\Delta t0}[A]$ | 800.296 | 800.291 | 0.00% |
| $V^{dc}_{\Sigma 0}[kV]$ | 400 | 400 | 0.00% |

It can be noted that the DC/DC cases have excellent matching between models, and the DC/DC/AC is also very good; however, Appendix E shows the same operating points in which the capacitance is decreased. In the case of the DC/DC/AC conversion, the matching worsens with the decreased capacitance, and hence with increased capacitor

voltage ripple. The same does not occur with the DC/DC conversion cases, which suggests the accuracy of the model is not nearly as sensitive to capacitance selection. A reason for this could be the presence of decoupled harmonics in the AC/DC and DC/DC conversions that cross couple when DC/DC/AC conversion occurs. This suggests that, for an even more precise model for the DC/DC/AC conversion, a more comprehensive spectrum of harmonics has to be selected when developing the DPM beyond the frequencies considered in Equation (3.37) to Equation (3.50).

Table 3.6: BP-DCDCAC State Variables Steady-State Comparison

| State Variable | Avg | DPM | Percent Error |
|---|---|---|---|
| $i^1_{\Sigma\alpha}[A_pk]$ | 663.555 | 661.217 | 0.35% |
| $i^1_{\Delta g\alpha}[A_pk]$ | 1569.41 | 1571.61 | 0.14% |
| $V^1_{\Sigma\alpha}[kV_{pk}]$ | 1.96597 | 1.8981 | 3.58% |
| $V^2_{\Sigma\alpha}[kV_{pk}]$ | 2.74241 | 2.85204 | 3.84% |
| $V^1_{\Delta\alpha}[kV_{pk}]$ | 5.155 | 5.18667 | 0.61% |
| $V^2_{\Delta\alpha}[kV_{pk}]$ | 2.2955 | 2.18447 | 5.08% |
| $i^{dc}_{\Sigma 0}[A]$ | 417.659 | 418.142 | 0.12% |
| $i^{dc}_{\Delta t\alpha}[A]$ | 801.887 | 800.001 | 0.24% |
| $V^{dc}_{\Sigma 0}[kV]$ | 400.009 | 400 | 0.00% |

## 3.3 Chapter summary

In this chapter, two time-averaged state-space models were developed that can be used to represent the four converter structures in Figure 3.1a - Figure 3.1e. Two of the topologies are for DC/DC/AC conversion and the other two are for DC/DC conversion. The developed TAMs are time-variant and therefore solving for a steady-state operating

Table 3.7: BP-DCDC State Variables Steady-State Comparison

| State Variable | Avg | DPM | Percent Error |
|---|---|---|---|
| $i^1_{\Sigma\alpha}[A_pk]$ | 1430.35 | 1430.11 | 0.02% |
| $V^1_{\Sigma\alpha}[kV_{pk}]$ | 35.423 | 35.4162 | 0.02% |
| $V^2_{\Sigma\alpha}[kV_{pk}]$ | 0.879617 | 0.880158 | 0.06% |
| $V^1_{\Delta\alpha}[kV_{pk}]$ | 1.11742 | 1.1147 | 0.24% |
| $V^2_{\Delta\alpha}[kV_{pk}]$ | 5.89186 | 5.88948 | 0.04% |
| $i^{dc}_{\Delta t0}[A]$ | 800.016 | 800.004 | 0.00% |
| $V^{dc}_{\Sigma 0}[kV]$ | 320 | 320 | 0.00% |

point other than the trivial case of zero power transfer is not possible; moreover, classical linear state space analysis tools cannot be used given the states are time-varying. To overcome this challenge, two different multi-frequency DPMs in the alpha-beta-zero reference frame are developed from the TAMs that yield time-invariant states (i.e. DC-valued states) at steady state. The stationary reference frame is exploited as it helps decouple harmonic content within the converter.

The TAMs are validated against detailed switched converter models in the software PLECS, with excellent matching shown for all four converter structures, i.e., for both DC/DC/AC and DC/DC conversion. The DPMs are then validated against the TAMs with very good results demonstrated during both steady-state and dynamics. It was observed that the accuracy of the DPMs for DC/DC conversion is, for all practical purposes, insensitive to capacitive energy storage, while, interestingly, the accuracy of the DPMs for DC/DC/AC conversion is a function of converter internal stored energy. Therefore, capacitance selection for the DC/DC/AC models should ensure that a minimum amount of stored energy is maintained, otherwise, excessive capacitor voltage ripple will result, which can deteriorate the model accuracy.

Tables 3.8 and 3.9 provide a summary of the developed TAMs and DPMs, respectively.

Table 3.8: Summary of developed time-averaged models

| Converter | State-Space Model | Model Matrices |
|---|---|---|
| MP-DCDCAC | Equation (3.31)<br>• Set $L_2 = 0$, $R_2 = 0$ | Appendix A |
| MP-DCDC | Equation (3.34)<br>• Set $L_2 = 0$, $R_2 = 0$ | Appendix B |
| BP-DCDCAC | Equation (3.31)<br>• Set $V_{dc1}$ as the sum of the two DC networks, and $V_{dc2}$ as the second DC network<br>• Set $L_t = 0$, $R_t = 0$ | Appendix A |
| BP-DCDC | Equation (3.34)<br>• Set $V_{dc1}$ as the sum of the two DC networks, and $V_{dc2}$ as the second DC network<br>• Set $L_t = 0$, $R_t = 0$ | Appendix B |

Table 3.9: Summary of developed DPMs

| Converter | State-Space Model | Model Matrices |
|---|---|---|
| MP-DCDCAC | Equation (3.57)<br>• Set $L_2 = 0$, $R_2 = 0$ | Appendix C |
| MP-DCDC | Equation (3.61)<br>• Set $L_2 = 0$, $R_2 = 0$ | Appendix D |
| BP-DCDCAC | Equation (3.57)<br>• Set $V_{dc1}$ as the sum of the two DC networks, and $V_{dc2}$ as the second DC network<br>• Set $L_t = 0$, $R_t = 0$ | Appendix C |
| BP-DCDC | Equation (3.61)<br>• Set $V_{dc1}$ as the sum of the two DC networks, and $V_{dc2}$ as the second DC network<br>• Set $L_t = 0$, $R_t = 0$ | Appendix D |

# Chapter 4

# DC/DC/AC and DC/DC Converters Case Studies

In this chapter, a systematic solution procedure to obtain the modulating signals for a specified operating point for the two DC/DC/AC and the two DC/DC converters is firstly introduced. By using this algorithm, steady-state waveforms for each of the four converters are shown for different operating points, and an analysis of the converters' operation is given. Studies are carried out to examine the response of the converters to a step-change in one of the parameters.

## 4.1 DC/DC/AC Conversion

This section covers the systematic solution procedure and the studies carried out that relate to DC/DC/AC conversion.

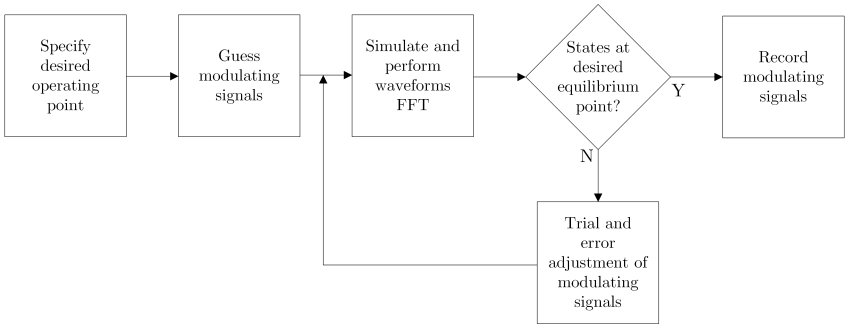



Figure 4.1: Process to obtain modulating signals using TAMs

### 4.1.1 Challenges in steady-state analysis using TAM

The TAMs developed in section 3.1 provide near perfect matching of the switched model of the generalised converter structure, and are useful for detailed converter dynamics analysis. To obtain the specific steady-state operating points in the converter, however, the TAMs are not directly useful, since their states are time-varying at steady-state and therefore analytically solving for $\frac{dx}{dt} = 0$ is not possible. Solving the TAM system of equations by setting the derivatives of the states to zero would only give the trivial answer of zero power flow between the ports. Moreover, the process of finding modulating signals for the TAMs would look something like the process shown in Figure 4.1, a process that doesn't give accurate results, and is extremely time consuming.

The states of the DPMs, on the other hand, are DC valued at steady-state, and are therefore ideal to use a solver to solve $\frac{dx}{dt} = 0$ for any desired operating point, including solving for the associated equilibria modulating signals.

### 4.1.2 Steady-State Solution Procedure

The simplest way to operate the converters, as shown in Section 3.1.5 and Section 3.2.3, is open loop modulation where fixed control inputs are used. This necessitates

the open loop modulating signals that yield the desired operating point to be known before running the simulation. To do this, a mathematical software such as Matlab, can be used to find the equilibrium point at which the system meets the specified criteria. This method is successfully applied in [70], [73].

The DPM allows solving for the desired open-loop steady-state control inputs, since the state variables are all constants at steady-state. This means that the system of differential equations that can be used to represent both DC/DC/AC converters, Equation (3.57), can be solved to obtain the control inputs by specifying that the derivatives of the states are zero, which can be represented as:

$$\begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & 0_{22x22} \end{bmatrix} \begin{bmatrix} \widetilde{\boldsymbol{X}}_i^* \\ \widetilde{\boldsymbol{X}}_v^* \end{bmatrix} + \begin{bmatrix} \widetilde{N} \\ 0_{22x5} \end{bmatrix} \left[ \widetilde{\boldsymbol{W}}^* \right] = 0. \tag{4.1}$$

To solve a system of nonlinear equations, as the one shown in Equation (4.1), the number of unknown variables has to be equal to the number of equations. The system in Equation (4.1) contains 52 equations and 68 unknown variables (52 states, 6 disturbance terms, and 10 modulating signals), assuming that all converter design parameters have been specified.

The 6 disturbance terms, i.e. the external DC and AC port voltages, are specified by the desired operating point. Since the objective is to have all 10 modulating signals as outputs of the solution procedure, 10 of the other variables need to be specified. The 10 modulating signals, shown in Section 3.2.1 in Equation (3.51) - Equation (3.56), are given here in column vector format:

$$\mathbf{M} = \begin{bmatrix} M_{\Sigma\alpha}^{1\parallel} & M_{\Sigma\alpha}^{1\perp} & M_{\Sigma\beta}^{1\parallel} & M_{\Sigma\beta}^{1\perp} & M_{\Sigma 0}^{dc} & M_{\Delta\alpha}^{1\parallel} & M_{\Delta\alpha}^{1\perp} & M_{\Delta\beta}^{1\parallel} & M_{\Delta\beta}^{1\perp} & M_{\Delta 0}^{dc} \end{bmatrix}^{\mathbf{T}}. \tag{4.2}$$

The solution procedure should yield modulating signals that produce balanced three-phase converter voltages and currents. For this to be accomplished, the eight modulating signals that are either $\alpha$ or $\beta$ quantities, must satisfy the following constraints:

$$M_{\Sigma\alpha}^{1\parallel} = M_{\Sigma\beta}^{1\perp} \tag{4.3}$$

$$M_{\Delta\alpha}^{1\parallel} = M_{\Delta\beta}^{1\perp} \tag{4.4}$$

$$M_{\Sigma\alpha}^{1\perp} = -\, M_{\Sigma\beta}^{1\parallel} \tag{4.5}$$

$$M_{\Delta\alpha}^{1\perp} = -\, M_{\Delta\beta}^{1\parallel}. \tag{4.6}$$

Only one of the four relationships in Equations (4.3) to (4.6) needs to be explicitly specified, and the remaining three relationships will be an outcome of the solution procedure. As an example, both terms in Equation (4.3) can be set as being the same variable in the model. This forces the solver to provide balanced quantities for the other three sets of modulating signals, while simultaneously reducing the number of variables that need to be specified from 10 to 9.

The remaining 9 "free" variables can be specified in whatever way is convenient for the desired converter operation. In this work, for the purpose of DC/DC/AC conversion in both MP and BP structures, the 9 variables are specified as follows:

- $I_{\Sigma\alpha}^{1\perp}$ and $I_{\Sigma\beta}^{1\parallel}$: These two states partially comprise the first harmonic component of the circulating current $i_{\Sigma}$, responsible for capacitor charge balancing between arms. The desired operation of the converter includes no reactive power circulation

between the arms to maximise conversion efficiently, and therefore:

$$I_{\Sigma\alpha}^{1\perp} = I_{\Sigma\beta}^{1\parallel} = 0. \tag{4.7}$$

- $I_{\Sigma g\alpha}^{1\parallel}$ and $I_{\Sigma g\beta}^{1\perp}$: These two states are related to the real power flow into the AC port. These are set to achieve a desired $P$ injection into the AC grid, $P_{ac,3\phi}$ as:[1]

$$I_{\Sigma g\alpha}^{1\parallel} = I_{\Sigma g\beta}^{1\perp} = \sqrt{2}\frac{P_{ac,3\phi}}{\sqrt{3}\cdot V_{ac,LL,RMS}}. \tag{4.8}$$

- $I_{\Sigma g\alpha}^{1\perp}$ and $I_{\Sigma g\beta}^{1\parallel}$: These two states are related to the reactive power flow into the AC port. These are set to achieve a desired $Q$ injection into the AC grid, $Q_{ac,3\phi}$, as[1]:

$$I_{\Sigma g\alpha}^{1\perp} = -I_{\Sigma g\beta}^{1\parallel} = \sqrt{2}\frac{Q_{ac,3\phi}}{\sqrt{3}\cdot V_{ac,LL,RMS}}. \tag{4.9}$$

- $I_{\Delta t0}^{dc}$: Represents one third of the average current that will flow into the $d_2$ DC port, controlling the desired DC power flow into that port, $P_{dc}$, according to:

$$I_{\Delta t0}^{dc} = \frac{P_{dc,2}}{3\cdot V_{dc,2}}. \tag{4.10}$$

- $\Sigma V_{\Sigma 0}^{dc}$ and $\Sigma V_{\Delta 0}^{dc}$: These states are specified to ensure balanced submodule capacitor voltages in the upper and lower arms, according to:

$$\Sigma V_{\Sigma 0}^{dc} = \frac{V_{cap}^{rated}}{2}(N_u + N_L) \tag{4.11}$$

$$\Sigma V_{\Delta 0}^{dc} = \frac{V_{cap}^{rated}}{2}(N_u - N_L). \tag{4.12}$$

[1]Voltage $V_{ac,LL,RMS}$ is directly related to the peak AC grid voltage $\alpha\beta$ components contained in the $W$ vector in Equation (4.1).

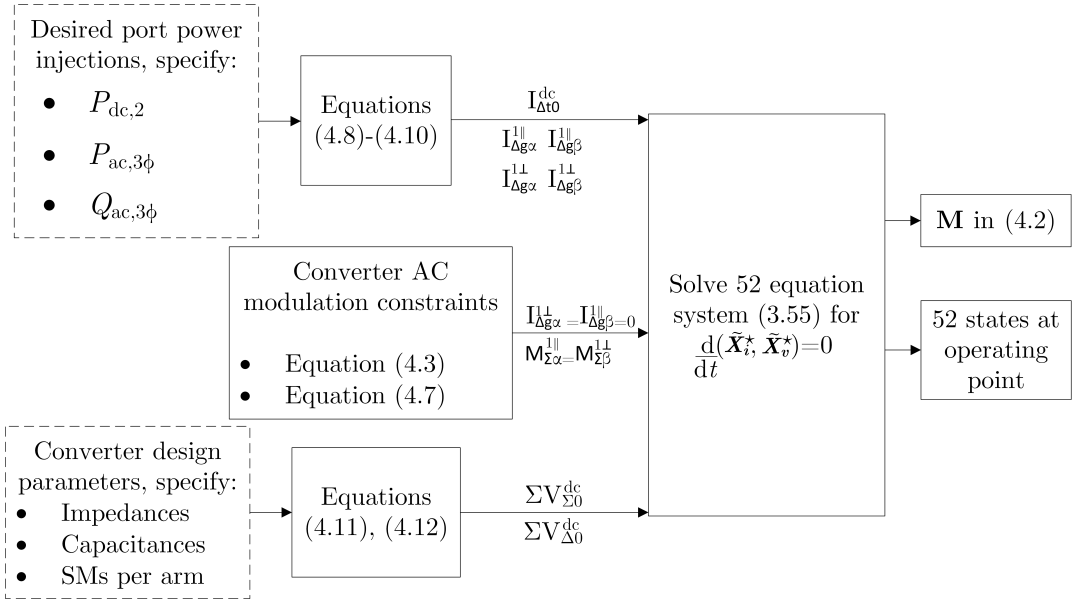



Figure 4.2: Solution procedure visualisation

Figure 4.2 provides a visualisation of the solution procedure. After specifying the nine values as shown, a mathematical solver such as Matlab can be used to solve the values for all the modulating signals that would provide the specified operating point, **M**, which can be used to run the simulation of the converter.

Another advantage of the solution procedure is that it also outputs the values for all the other unspecified states at that equilibrium point. Those values can be used to systematically study the converters' steady-state operating point, as will be elaborated below.

### 4.1.3 MP-DCDCAC Steady-State Analysis

In this section, the solution procedure presented in Section 4.1.2 is used to find the modulating signals required to perform the simulations for six arbitrary steady-state operating points for the MP-DCDCAC converter. Figure 4.3 shows the MP-DCDCAC

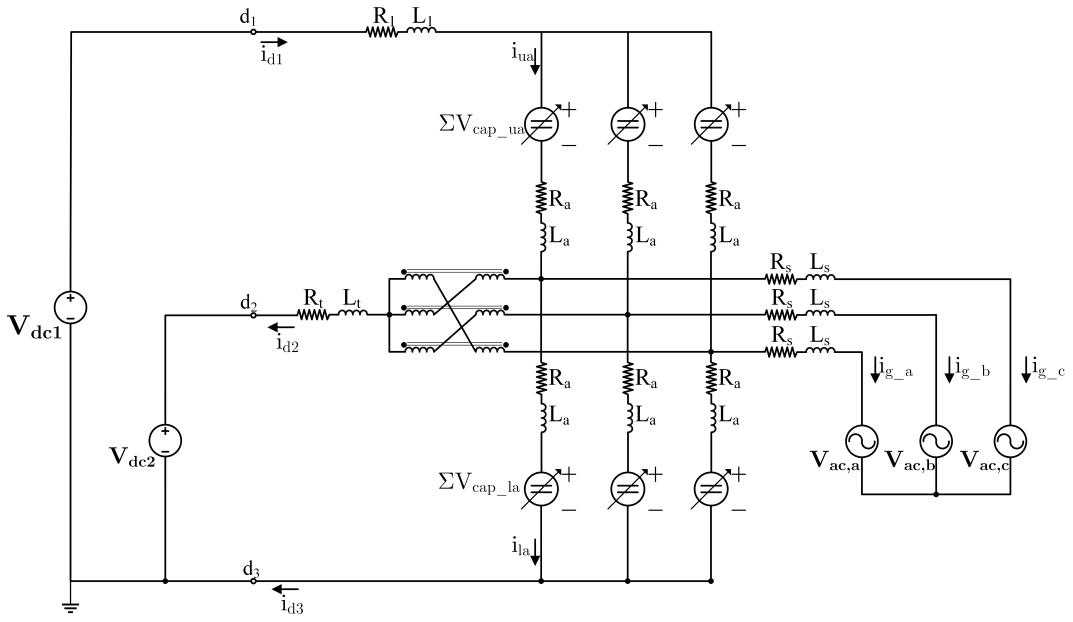



Figure 4.3: MP-DCDCAC converter with current direction convention

converter that is simulated using the DPM, including the conventions for the current directions. The first DC network, connected between ports $d_1$ and $d_3$, will henceforth be named $\mathrm{NW}_1$; in the same manner the network connected between ports $d_2$ and $d_3$ will be called $\mathrm{NW}_2$, and the AC network will be called $\mathrm{NW}_{ac}$. The convention for the powers flows to each of the networks, i.e. $P_{dc,1}$, $P_{dc,2}$, and $P_{ac}$, is that a positive sign means that network is providing power to the converter. Because of these current flows, $P_{dc,1}$ and $P_{dc,2}$ can be defined as:

$$P_{dc,1} = V_{dc,1} \cdot i_{d1}^{dc} \tag{4.13}$$

$$P_{dc,2} = V_{dc,2} \cdot i_{d2}^{dc} \tag{4.14}$$

The MP-DCDCAC is rated for 480 MW, and for all six operating points the $\mathrm{NW}_1$

voltage is set to $V_{dc1} = 400$kV. The $NW_2$ voltage, $V_{dc1}$, is set depending on the ratio $G_v$, introduced in Section 2.2 in Equation (2.19).

#### 4.1.3.1 400/200kV ($G_v$=0.5) Converter Design

Figures 4.4 to 4.6 show the simulation waveforms for three different port power injections for the MP-DCDCAC at $G_v = 0.5$. The waveforms shown are the upper and lower phase $a$ arm currents, the upper and lower phase a sum of capacitor voltages, the currents entering $NW_ac$, and the three DC port currents. The peak value of the $NW_{ac,L-N}$ voltages are set to 180 kV. Table 4.1 lists the parameters used for these simulations.

Figures 4.4 and 4.5 display simulation results where active power transfer between the external DC and AC systems is performed. Figure 4.4 shows a case where 1 pu power is being transferred from $NW_1$ to $NW_2$ and $NW_{ac}$, being split equally. Figure 4.5 shows a case in which $NW_2$ receives 1 pu power, while $NW_1$ and $NW_{ac}$ provide 0.8 and 0.2 pu power, respectively. The DC power transfers are clearly visible through the DC ports currents. In both cases, the AC grid currents are comprised of first and second harmonic currents, being the latter much more noticeable in the second case, as the first harmonic content is much lower.

The first case presents an equal combination of $P_\Sigma$ and $P_\Delta$ converter power transfer mechanisms, as expected from the explanation in Section 2.3, whereas the second case, which is mostly DC/DC conversion, shows $P_\Sigma$ as the more prominent power transfer mechanism. In both cases, the capacitor voltages are balanced at 400 kV as desired. The third case, Figure 4.6, demonstrates zero power flow between the three networks. There is practically no current flowing through the DC and AC ports; however, a small amount of current is still circulating in the arms, to maintain a balance of the capacitor's voltages. The DC component of the arm capacitor voltages is 400 kV as expected.

Table 4.1: Simulation parameters for MP-DCDCAC using $G_v$=0.5

| Parameter | Value |
|---|---|
| $w$ | $2\pi 60$ rad/s |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH, 40 mH, 50 mH, 20 mH |
| $L_e, L_m$ | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 1 Ω, 0.4 Ω, 0.6 Ω |
| $R_w$ | 0.1 Ω |
| $C_u, C_L$ | 10 mF |
| $N_u, N_L$ | 200 |
| *Peak L-N* $V_{ac}$ | 180 kV |

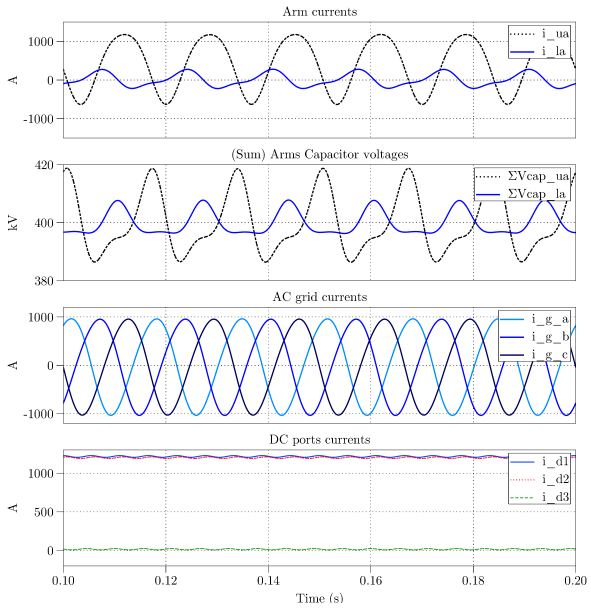



Figure 4.4: Steady-state MP-DCDCAC DPM waveforms for $G_v$=0.5 (400/200 kV); $P_{dc,1}$=1 pu, $P_{dc,2}$ =-0.5 pu, $P_{ac}$ =-0.5 pu

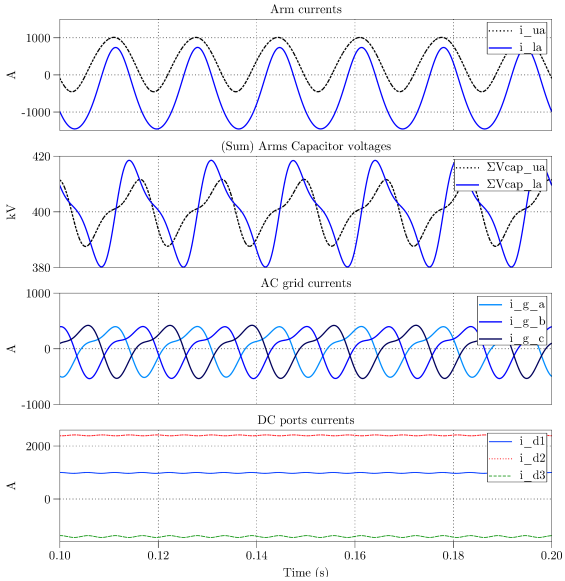



Figure 4.5: Steady-state MP-DCDCAC DPM waveforms for $G_v$=0.5 (400/200 kV); $P_{dc,1}$=0.8 pu, $P_{dc,2}$=-1 pu, $P_{ac}$ =0.2 pu

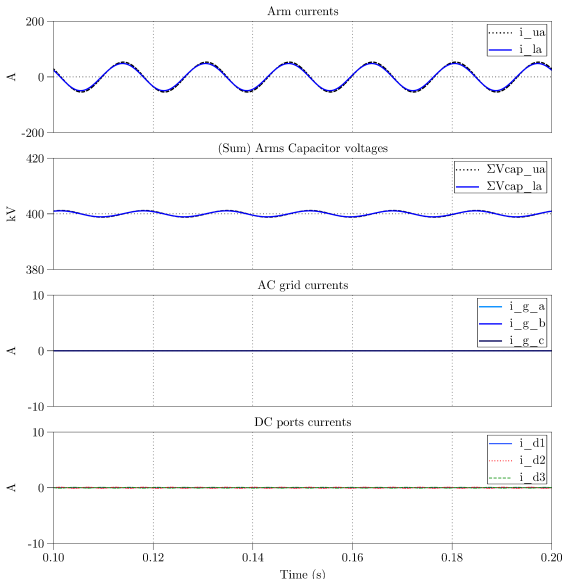



Figure 4.6: Steady-state MP-DCDCAC DPM waveforms for $G_v$=0.5 (400/200 kV); $P_{dc,1}$=0 pu, $P_{dc,2}$ =0 pu, $P_{ac}$ =0 pu

#### 4.1.3.2 400/320kV ($G_v$=0.8) Converter Design

Figures 4.7 to 4.9 show similar port power injections for the MP-DCDCAC converter as Figures 4.4 to 4.6, but now utilising a DC voltage ratio $G_{\mathrm{v}}$=0.8. These simulations demonstrate the capability of the DPM to utilize a different number of submodules for the upper and lower arms, contrasting to the previous three cases in the previous section, where they were equal. This means that the DC component of $\Sigma V_{cap}$ at which the capacitor's voltage is balanced will be different for each arm in this section. The converter parameters used in this section are listed in Table 4.2. The number of submodules used in each arm are selected to support the maximum DC+AC voltages each arm will be subjected to, at 2 kV rated voltage for each submodule. This means that the DC component of $\Sigma V_{cap}$ for the upper and lower arms is set to 160 and 400 kV, respectively, as dictated by Equation (4.11). The peak value of the $\mathrm{NW}_{ac,L-N}$ voltages are set to 80 kV, since assuming the use of half-bridge submodules this is the maximum peak that can be synthesised by the arms.

In the case of equal splitting of the power between $\mathrm{NW}_2$ and $\mathrm{NW}_{ac}$, Figure 4.7 shows a common nmode DC component in the arms' currents not present in the 400/200 kV case, given that $G_{\mathrm{v}}$ is no longer 0.5, as expected by the information in Table 2.2, in Section 2.3. It can also be noted that, in this case, there is DC current exiting through the $d_3$ DC port, given the new voltage balancing requirements for this operation. The grid currents retain the expected first harmonic component behaviour that were present in the $G_{\mathrm{v}}$=0.5 cases, although the second harmonic component is relatively much smaller. Once again, Figure 4.9 demonstrates the ability of the DPM to achieve a zero-power transfer operation, in which a small amount of AC average power is transferred between arms to maintain capacitor voltage balance, and a very small amount of third harmonic current can be seen in the DC ports.

Table 4.2: Simulation parameters for MP-DCDCAC using $G_v$=0.8

| Parameter | Value |
|---|---|
| $w$ | $2\pi 60$ rad/s |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH, 40 mH, 50 mH, 20 mH |
| $L_e, L_m$ | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 1 Ω, 0.4 Ω, 0.6 Ω |
| $R_w$ | 0.1 Ω |
| $C_u, C_L$ | 8 mF, 18 mF |
| $N_u, N_L$ | 80, 200 |
| *Peak L-N* $V_{ac}$ | 80 kV |

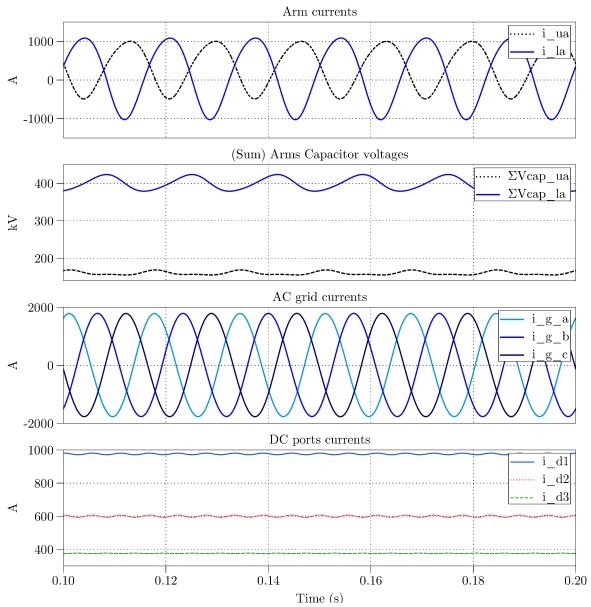



Figure 4.7: Steady-state MP-DCDCAC DPM waveforms for $G_v$=0.8 (400/320 kV); $P_{dc,1}$=0.8 pu, $P_{dc,2}$=-0.4 pu, $P_{ac}$=-0.4 pu

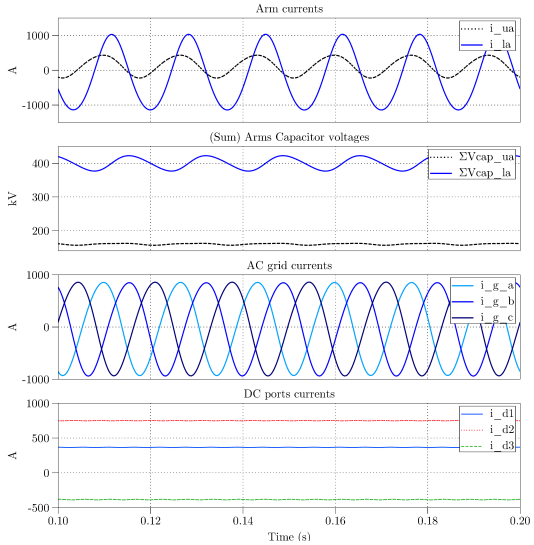



Figure 4.8: Steady-state MP-DCDCAC DPM waveforms for $G_v$=0.8 (400/320 kV); $P_{dc,1}$=0.3 pu, $P_{dc,2}$=-0.5 pu, $P_{ac}$=0.2 pu

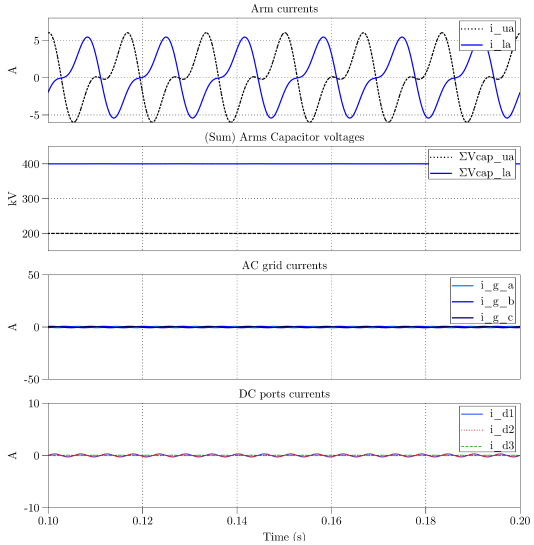



Figure 4.9: Steady-state MP-DCDCAC DPM waveforms for $G_v$=0.8 (400/320 kV); $P_{dc,1}$=0 pu, $P_{dc,2}$ =0 pu, $P_{ac}$ =0 pu

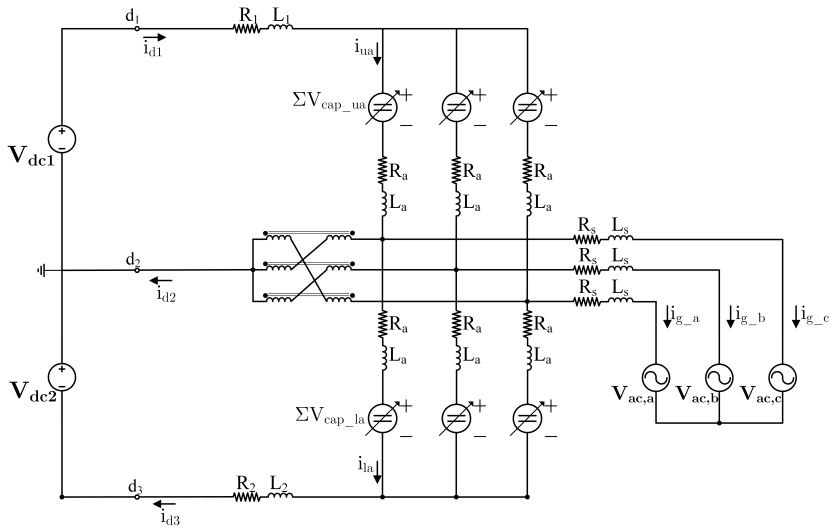



Figure 4.10: BP-DCDCAC converter with current direction convention

### 4.1.4 BP-DCDCAC Steady-State Analysis

In this section, the solution procedure presented in Section 4.1.2 is used to solve for the modulating signals required to enable open-loop simulations for three different steady-state operating points for the BP-DCDCAC converter. Figure 4.10 shows the BP-DCDCAC converter that is simulated using the DPM, including the conventions for the current directions. This topology is called bipolar because the DC port $d_1$ is at a positive potential and the DC port $d_3$ is at a negative potential, whereas in the MP topology $d_1$ and $d_2$ were at a positive potential, and $d_3$ was grounded.

The first DC network, connected between ports $d_1$ and $d_2$, will henceforth be named $\mathrm{NW}_1$; in the same manner the network connected between ports $d_2$ and $d_3$ will be called $\mathrm{NW}_2$, and the AC network will be called $\mathrm{NW}_{ac}$. The convention for the powers flows to each of the networks, i.e. $P_{dc,1}$, $P_{dc,2}$, and $P_{ac}$, is that a positive sign means that network is providing power to the converter.

The BP-DCDCAC is rated for 768 MW, and this converter is ideal for interconnecting

two DC networks that operate at the same voltage level, i.e. bipolar grids. Because of this, for all three operating points, the voltages for $\mathrm{NW}_1$ and $\mathrm{NW}_2$ are set to 320 kV, while the peak value of the $\mathrm{NW}_{ac,L-N}$ voltages are set to 320 kV.

It must be noted that a minor difference exists between how one of the specified variables in the solution procedure is set differently for the BP-DCDCAC compared to the MP-DCDCAC. The variable $I^{dc}_{\Delta t0}$ in Figure 4.2 does not represent the same port power transfer mechanism for the BP topology in Figure 4.10 as it did for the MP topology in Figure 4.3. For the latter, it was the average current that was exclusively injected into the second DC network. For the BP topology, $I^{dc}_{\Delta t0}$ represnts the average current that flows into the common point between the two DC networks. Therefore, pecifying this variable does not directly specify the power flowing in or out of the second DC network.

To address this situation, once again the $\Sigma\Delta$ domain is used to simplify the explanation and analysis of this conversion. Instead of having $P_{dc,1}$ and $P_{dc,2}$ to express the powers in and out of the first and second DC networks, $P_{dc,\Sigma}$ and $P_{dc,\Delta}$ are used to represent the common mode and differential mode of the powers exchanged between the DC networks themselves, respectively. The mapping between these quantities is defined as:

$$\begin{bmatrix} P_{dc,\Sigma} \\ P_{dc,\Delta} \end{bmatrix} = \begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix} \begin{bmatrix} P_{dc,1} \\ P_{dc,2} \end{bmatrix} \tag{4.15}$$

Given the DC current common to all three converter's legs, i.e. $3 \cdot I^{dc}_{\Sigma 0}$, and assuming both DC networks have the same voltage level, i.e. $V_{dc1} = V_{dc2} = V_{dc}$, $P_{dc,\Sigma}$ and $P_{dc,\Delta}$ can also be defined as:

$$P_{dc,\Sigma} = 2(3 \cdot I^{dc}_{\Sigma 0})V_{dc} \tag{4.16}$$

$$P_{dc,\Delta} = I^{dc}_{\Delta t0} \cdot V_{dc} \tag{4.17}$$

Since $I^{dc}_{\Delta t0}$ is proportional to $P_{dc,\Delta}$ and $I^{dc}_{\Sigma 0}$ is proportional to $P_{dc,\Sigma}$, it is now easy to set in the solution procedure $P_{dc,\Delta}$ through $I^{dc}_{\Delta t0}$, and have $I^{dc}_{\Sigma 0}$ be an outcome that will set $P_{dc,\Sigma}$. The parameters of the converter used for these simulations are shown in Table 4.3.

Table 4.3: Simulation parameters for BP-DCDCAC

| Parameter | Value |
|---|---|
| $w$ | $2\pi 60$ rad/s |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH, 82 mH, 50 mH, 20 mH |
| $L_e, L_m$ | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 2.1 Ω, 0.4 Ω, 0.6 Ω, 0.1 Ω |
| $R_w$ | 0.1 Ω |
| $C_u, C_L$ | 10 mF |
| $N_u, N_L$ | 320 |

#### 4.1.4.1 320/320kV ($G_v$=1) Converter Design

Figures 4.11 to 4.14 show the waveforms for four different port power injections for the BP-DCDCAC. The waveforms shown are the upper and lower phase $a$ arm currents, the upper and lower phase a sum of capacitor voltages, the currents entering $\text{NW}_{ac}$, and the three DC port currents. The last plot also includes the current $3(I^{dc}_{\Sigma 0})$, as it can be of use to understand the powers being transferred between ports for this converter.

Figure 4.11 presents a case in which $\text{NW}_1$ and $\text{NW}_2$ each supply 0.5 pu power (col-

lectively 1 pu) to the $NW_{ac}$. This case is analogous to a conventional AC/DC converter, and the waveforms are what would be expected of such a conversion. For example, the arms contain large negative sequence second harmonic component that sums to zero at the DC rails. It is interesting to note that, out of all operating points presented in this chapter, this is the only one in which the currents in $NW_{ac}$ do not present a second harmonic component, possibly due to the fact that no DC/DC conversion is occurring.

Figures 4.12 and 4.13 show the simulation waveforms for the operating points in which the power transfers are $P_{dc,1}$=0.25 pu, $P_{dc,2}$=-0.5 pu, $P_{ac}$=0.25 pu and $P_{dc,1}$=-0.6 pu, $P_{dc,2}$=0.5 pu, $P_{ac}$=0.1 pu, respectively. Both present the arms' currents and voltages harmonic spectrum that are expected from both theory and the results of the MP-DCDCAC converter for simultaneous DC/DC/AC conversion. The grid currents are a combination of first and second harmonic, and the converter presents both $P_{\Sigma}$ and $P_{\Delta}$ power transfer mechanisms, the latter being much more noticeable in the case shown in Figure 4.13, as DC/DC conversion is the dominant conversion in this case. It is also interesting noting the inversion of DC currents sign between the two cases, given both DC networks switching their operation, be it providing or receiving power.

Figure 4.14, as with the MP-DCDCAC, demonstrates zero power flow between the three networks. The results are similar to the zero-power flow case of the previous section, noting the small amount of first and second harmonic current in the arms to maintain the capacitor's voltage balance, and the small third harmonic current in all DC port currents. In all cases, the capacitor voltages are balanced at 640 kV, as desired.

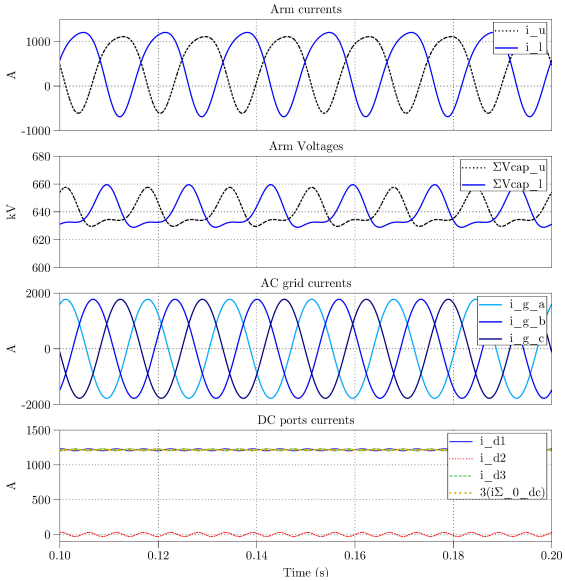



Figure 4.11: Steady-stateBP-DCDCAC DPM waveforms; $P_{dc,1}$=0.5 pu, $P_{dc,2}$=0.5 pu, $P_{ac}$=-1 pu

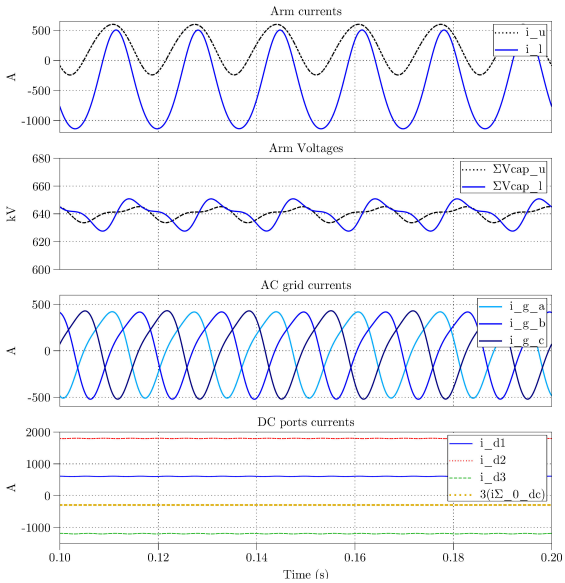



Figure 4.12: Steady-stateBP-DCDCAC DPM waveforms; $P_{dc,1}$=0.25 pu, $P_{dc,2}$=-0.5 pu, $P_{ac}$=0.25 pu

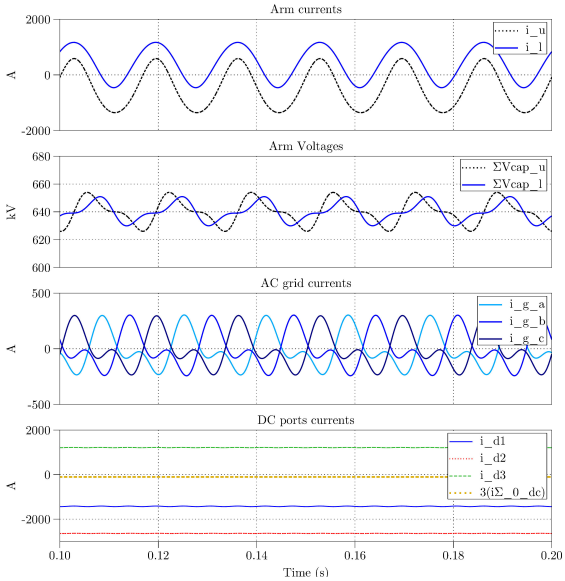



Figure 4.13: Steady-stateBP-DCDCAC DPM waveforms; $P_{dc,1}$=-0.6 pu, $P_{dc,2}$=0.5 pu, $P_{ac}$=0.1 pu

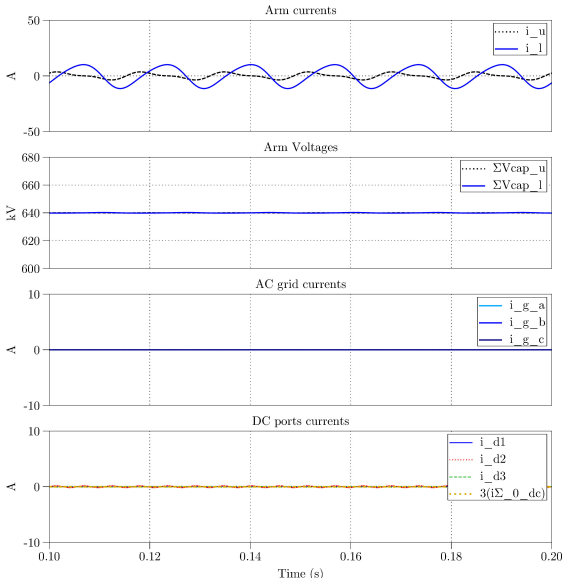



Figure 4.14: Steady-stateBP-DCDCAC DPM waveforms; $P_{dc,1}$=0 pu, $P_{dc,2}$ =0 pu, $P_{ac}$ =0 pu

### 4.1.5 Varying Parameters Studies

Another useful application for the DPM in Equation (4.1) and solution procedure developed in section Section 4.1.2 is that it can be used to explore the converters' behaviour in response to changing parameters. This can be accomplished by running the solution procedure many times while step-varying one or several of the specified quantities, and collecting the data of interest in each iteration. This section presents a few examples of this use for the MP-DCDCAC converter of section Section 4.1.3, for $G_{\mathrm{v}}$=0.5 and $G_{\mathrm{v}}$=0.8, utilising the parameters of Table 4.1.

Figure 4.15(a) and (b) show how the peak current stress and the percent capacitors voltage peak-to-peak ripple in the upper and lower phase $a$ arms, respectively, change while varying the power flow into $\mathrm{NW}_2$ while having 0.5 pu fixed power flow into $\mathrm{NW}_{ac}$, meaning that power out of $\mathrm{NW}_1$ changes accordingly, for $G_{\mathrm{v}}$=0.5. Figure 4.15(c) and (d) carry out the same analysis but having 1 pu fixed power provided by $\mathrm{NW}_1$, and varying it from 1 pu into $\mathrm{NW}_2$, to 1 pu into $\mathrm{NW}_{ac}$, meaning that in all instances $P_{dc,2} + P_{ac}$=1. Figure 4.15(a) and (c) present the peak arms' currents in two ways: (i) only the DC components and (ii) DC plus AC components. The DC only curves are useful to visualise average power transfers. Also, the difference between these two curves is the AC stress. Figure 4.15(a) shows that in both extremes of the graph the arms' peak current stress is caused only by AC components, at -0.5 pu power flow to the $\mathrm{NW}_2$ for the upper arm and 0.5 pu for the lower arm. Figure 4.15(c) demonstrates how the arms are subjected to a different stress pattern that in Figure 4.15(a), given that in this case the upper arm is always processing 1 pu power from $\mathrm{NW}_1$. The voltage ripple in the arms' capacitors in Figure 4.15(b) and (d) resemble the shape of their respective arms' currents. The converters are usually designed to keep this peak-to-peak ripple below 10%, meaning that the full loading operation of this converter might require the choosing of the capacitors

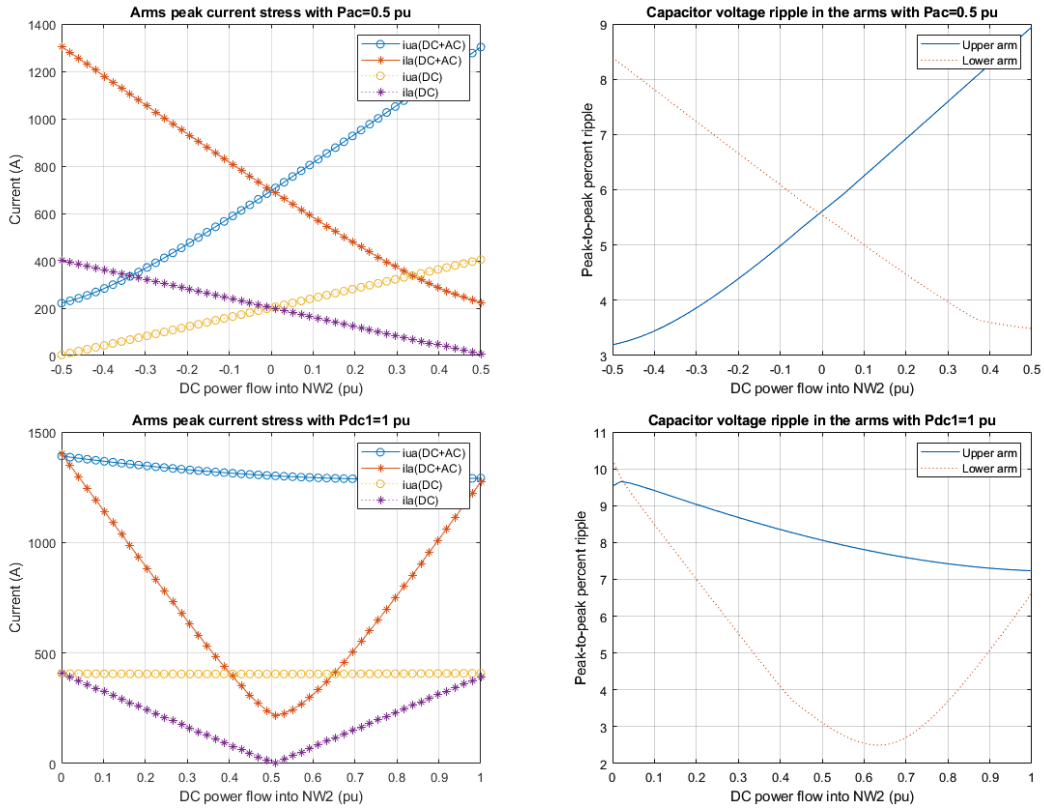



Figure 4.15: Loci of steady-state studies for $G_v$=0.5 for the phase $a$ upper and lower arms: (a) arms' peak current stress with $P_{ac}$=0.5 pu; (b) percent capacitor voltage ripple with $P_{ac}$=0.5; (c) arms' peak current stress with $P_{dc1}$=1 pu; (d) percent capacitor voltage ripple with $P_{dc1}$=1 pu

to be above the $50\frac{kJ}{MW}$ energy storage selected for these studies.

Figure 4.16 shows the studies presented in Figure 4.15 but with $G_{\mathrm{v}}$=0.8, as discussed in section 4.1.2.2. The patterns of the loci are similar in both sets, as they tell of the behaviour of the converter in those situations. The biggest differences can be found in that the lines for the upper and lower arms no longer intersect in the middle of the graphs for the $P_{ac}$=0.5 cases, and a shift towards the right side of the graph in the $P_{dc1}$=1 pu cases for the quantities representing the lower arm. All four figures demonstrate how

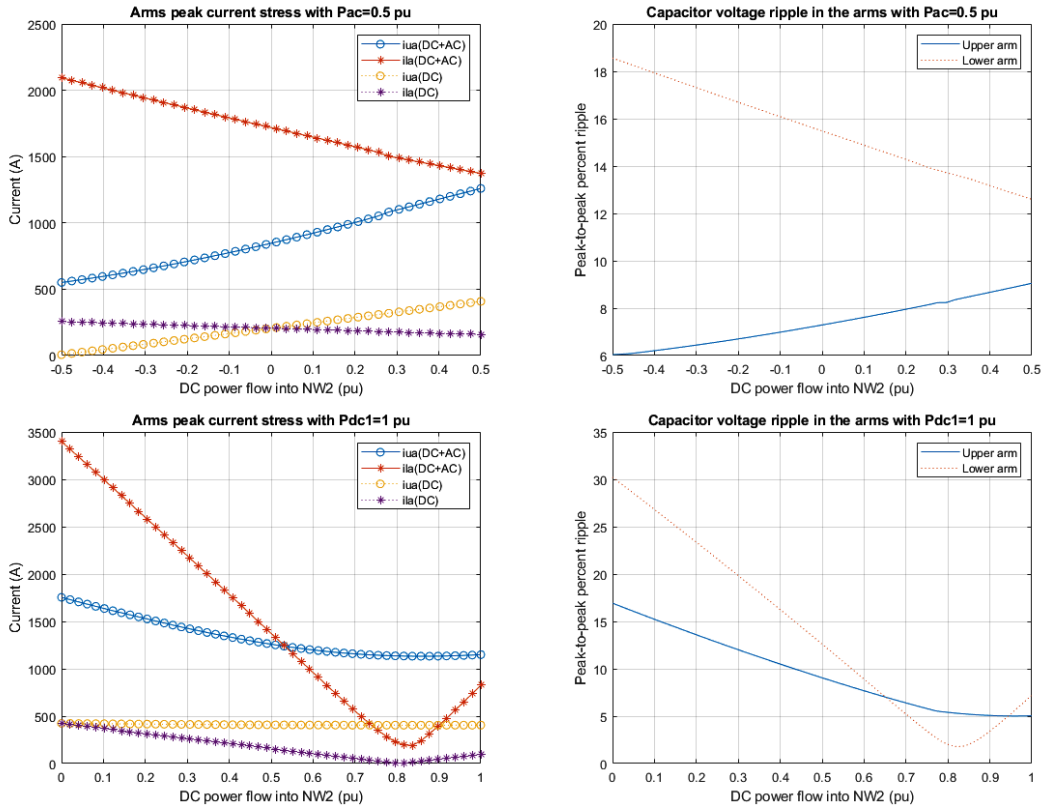



Figure 4.16: Loci of steady-state studies for $G_v$=0.8 for the phase $a$ upper and lower arms: (a) arms' peak current stress with $P_{ac}$=0.5 pu; (b) percent capacitor voltage ripple with $P_{ac}$=0.5; (c) arms' peak current stress with $P_{dc1}$=1 pu; (d) percent capacitor voltage ripple with $P_{dc1}$=1 pu

this converter design is impacted by the limited AC voltage available with half-bridge submodules in this case, since a very high current is required to satisfy the power flows. Figure 4.16(a) shows the stress in the lower arm is very high when the power flows only from $\mathrm{NW}_2$ to $\mathrm{NW}_{ac}$, while a similar thing is shown in Figure 4.16(c) when the power flows from $\mathrm{NW}_1$ to $\mathrm{NW}_{ac}$. Figure 4.16(b) and (d) demonstrate the high peak-to-peak voltage ripple in the arms in the same cases. This information can be used to take decisions during the converter design as to whether prevent these scenarios of high current stress,

or perhaps employ full-bridge submodules in the upper arm.

## 4.2 DC/DC Conversion

This section covers the solution procedure, steady-state operating point analyses, and varying parameters studies for the DC/DC converters.

### 4.2.1 Steady-State Solution Procedure

The system of differential equations that can be used to represent both DC/DC converters, originally presented in Section 3.2.2 as Equation (3.61), is represented here as:

$$\begin{bmatrix} \widetilde{A}_{11r} & \widetilde{A}_{12r} \\ \widetilde{A}_{21r} & 0_{22x22} \end{bmatrix} \begin{bmatrix} \widetilde{\boldsymbol{X}}^*_{ir} \\ \widetilde{\boldsymbol{X}}^*_{vr} \end{bmatrix} + \begin{bmatrix} \widetilde{N}_r \\ 0_{22x5} \end{bmatrix} \begin{bmatrix} \widetilde{\boldsymbol{W}}^*_r \end{bmatrix} = 0 \tag{4.18}$$

The derivatives of the states are set to zero in Equation (4.18), as this is the objective of the solver to obtain the modulating signals which serve as control inputs for the simulations. The solution procedure for the DC/DC converters is the same as the one presented in section 4.1.2 for the DC/DC/AC converters, the only difference being that the system in Equation (4.18) now contains 44 equations and 56 unknown variables (44 states, 2 disturbance terms, and 10 modulating signals), assuming that all converter design parameters have been specified.

The disturbance terms, i.e. external DC port voltages, are specified by the desired operating point. Once again, the objective is to have all 10 modulating signals as outputs of the solution procedure, which again requires 10 specified terms. The difference in this case is that the four AC grid parameters that were being specified for the DC/DC/AC,

Equations (4.8) and (4.9), can no longer be specified given the lack of an AC port. One way to reduce the number of variables that need to be specified is to use the remaining three relationships presented in section 4.1.2, equations Equations (4.4) to (4.6), since only Equation (4.3) was used in the DC/DC/AC solution procedure, reducing the number of unknowns to 11, only requiring one more to be specified.

To solve this, a constraint in the form of one more differential equation is added to the system in Equation (4.18), given by:

$$(V_{\Sigma 0}^{dc} + V_{\Delta t0}^{dc})(M_{\Sigma\alpha}^{1\|} + M_{\Delta\alpha}^{1\|}) - \hat{V}_{\Delta ac} = 0 \tag{4.19}$$

Equation (4.19) provides a constraint in the possible values of $M_{\Sigma\alpha}^{1\|}$ and $M_{\Delta\alpha}^{1\|}$ ,by stressing that the parallel component of the upper arm's AC voltage has to be collinear with the reference vector (0°). The rest of the specified values are the same as the ones given in Figure 4.2.

### 4.2.2 MP-DCDC Steady-State Analysis

In this section, the solution procedure explained in section 4.2.1 is used to find the modulating signals required to perform the simulations for six different steady-state operating points for the MP-DCDC converter. Figure 4.17 shows the MP-DCDC converter that is simulated using the DPM, including the conventions for the current directions. The first DC network, connected between ports $d_1$ and $d_3$, will henceforth be named $\mathrm{NW}_1$; in the same manner the network connected between ports $d_2$ and $d_3$ will be called $\mathrm{NW}_2$. Since in these converters only two networks are present, the definition of the operating points only provides the power being supplied by $\mathrm{NW}_1$, $P_{dc,1}$, and a positive sign means $\mathrm{NW}_1$ is supplying power.

The MP-DCDC is rated for 480 MW, and the voltages for corresponding to the DC

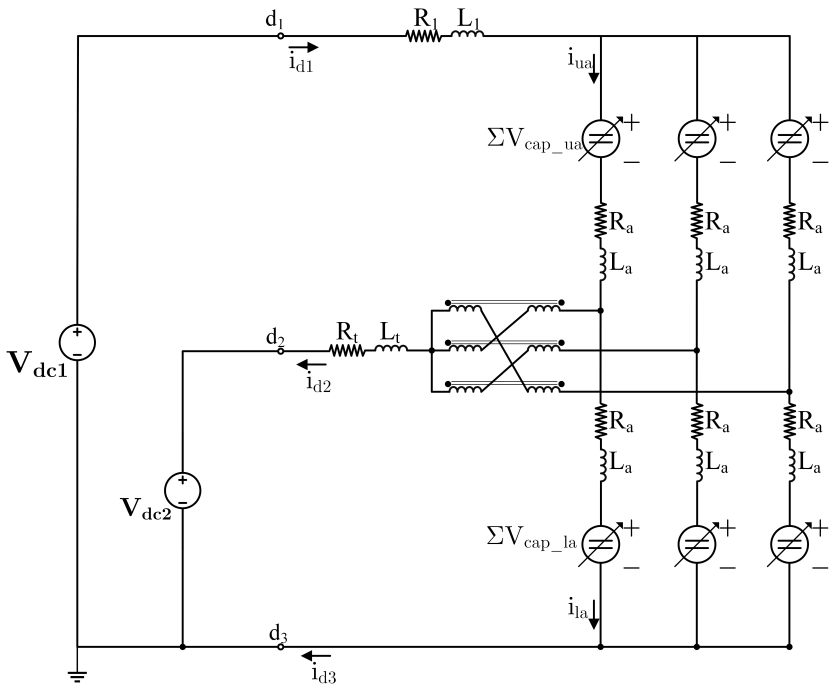



Figure 4.17: MP-DCDC converter with current direction convention

ports $NW_1$ to $NW_2$ vary in the different $G_v$ scenarios.

#### 4.2.2.1 500/250kV ($G_v$=0.5) Converter Design

Figures 4.18 to 4.20 show the simulation waveforms for three different port power injections, $P_{dc,1}$=1 pu, $P_{dc,1}$=-1 pu, and $P_{dc,1}$=0 pu, for the MP-DCDC at $G_v$=0.5. The waveforms shown are the upper and lower phase a arm currents, the upper and lower phase a sum of capacitor voltages, and the three DC port currents. Table 4.4 provides the parameters used for these simulations.

Figures 4.18 and 4.19 demonstrate active power transfer between the DC ports, as it can be appreciated by the DC port currents. The common mode arm's current is exclusively AC current, while the differential mode current is exclusively DC, as expected.

Since this is DC/DC conversion under $G_{\mathrm{v}}$=0.5, the DC component of both arms' currents is the same value, around 320 A, but opposite signs for upper and lower arms. Figure 4.20 demonstrates a situation of no power being exchanged between DC ports, and the only visible waveform is a small AC component in the arms' currents to maintain capacitor voltage balance. In all cases, the capacitor voltages are balanced at 500 kV as desired.

Table 4.4: Simulation parameters for MP-DCDC using $G_v$=0.5

| Parameter | Value |
|---|---|
| $w$ | $2\pi 60$ rad/s |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH, 40 mH, 50 mH, 20 mH |
| $L_e, L_m$ | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 1 Ω, 0.4 Ω, 0.6 Ω |
| $R_w$ | 0.1 Ω |
| $C_u, C_L$ | 8 mF |
| $N_u, N_L$ | 250 |

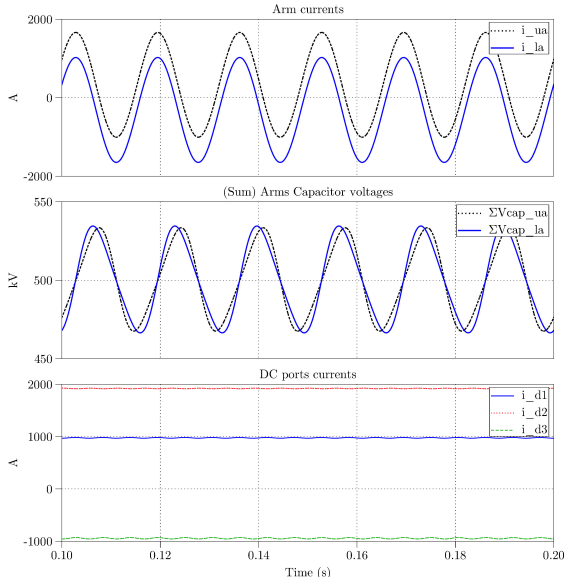



Figure 4.18: Steady-state MP-DCDC DPM waveforms for $G_v$=0.5 (500/250 kV); $P_{dc,1}$= 1 pu

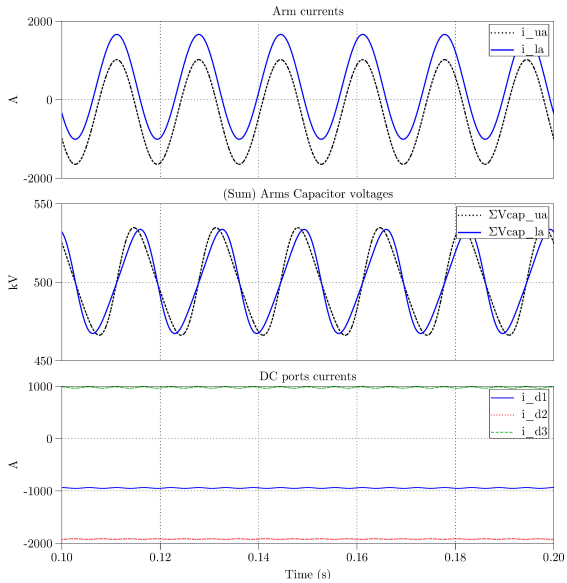



Figure 4.19: Steady-state MP-DCDC DPM waveforms for $G_v$=0.5 (500/250 kV); $P_{dc,1}$= -1 pu

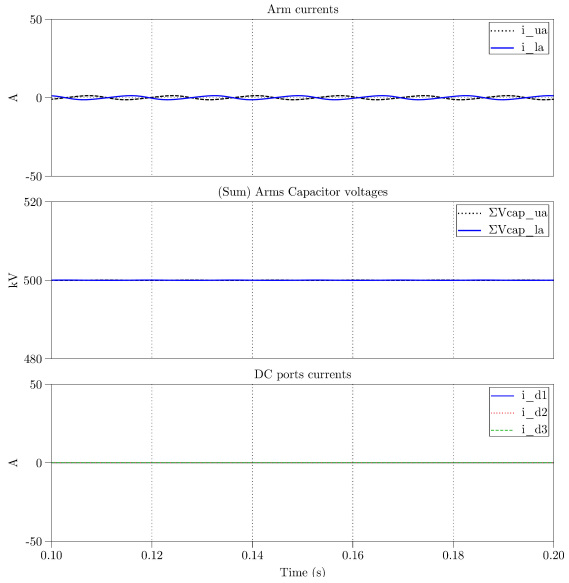



Figure 4.20: Steady-state MP-DCDC DPM waveforms for $G_v$=0.5 (500/250 kV); $P_{dc,1}$= 0 pu

#### 4.2.2.2 400/320kV ($G_v$=0.8) Converter Design

Figures 4.21 to 4.23 show the same port power injections for the MP-DCDC converter as Figures 4.18 to 4.20, but now utilising $G_{\mathrm{v}}$=0.8. The waveforms shown are the upper and lower phase $a$ arm currents, the upper and lower phase $a$ sum of capacitor voltages, and the three DC port currents. Table 4.5 provides the parameters used for these simulations.

Figures 4.21 and 4.22 show cases with active power transfer between DC ports. As in the waveforms in the previous section, the arms' currents differential mode current is exclusively DC, however in these cases there is also a DC component in the common mode current. This corresponds with the information in Table 2.2, in Section 2.3, where it was stated that a DC common mode current would exist if $G_{\mathrm{v}} \neq$0.5. A very small component of third harmonic current is present in the arms' currents and voltages, a

component often neglected in the phasor modelling of DC/DC converters, as its effect is almost negligible. Figure 4.23 demonstrates a situation of no power being exchanged between DC ports, and only AC current in the arms to maintain capacitor voltage balance is noticeable. In contrast to simulations in Section 4.1, the DC port currents show no sign of third harmonic current in the zero-power transfer scenario. In all cases, the capacitor voltages are balanced at 160 and 400 kV for the upper and lower arms, respectively, as desired.

Table 4.5: Simulation parameters for MP-DCDC using $G_v$=0.8

| Parameter | Value |
|---|---|
| $w$ | $2\pi 60$ rad/s |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH, 40 mH, 50 mH, 20 mH |
| $L_e, L_m$ | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 1 Ω, 0.4 Ω, 0.6 Ω |
| $R_w$ | 0.1 Ω |
| $C_u, C_L$ | 12 mF, 14 mF |
| $N_u, N_L$ | 80, 200 |

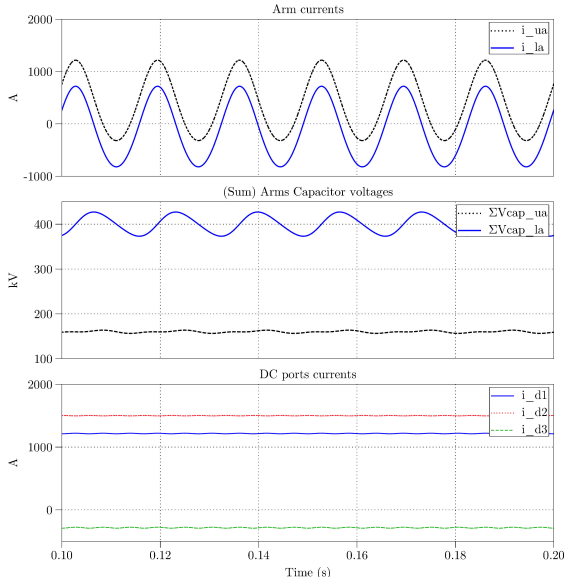



Figure 4.21: Steady-state MP-DCDC DPM waveforms for $G_v$=0.8 (400/320 kV); $P_{dc,1}$= 1 pu

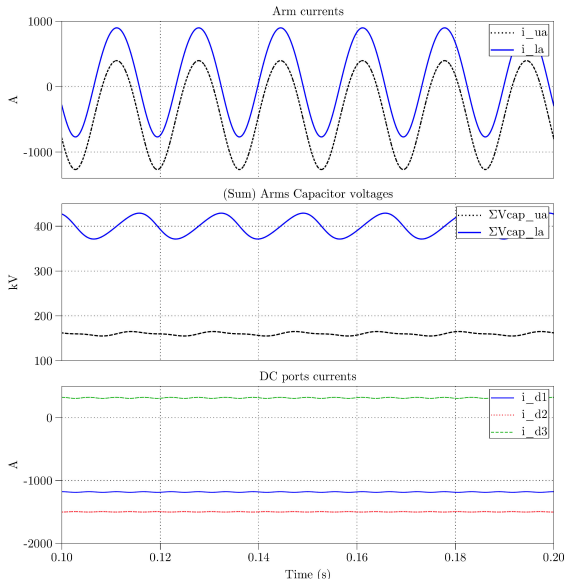



Figure 4.22: Steady-state MP-DCDC DPM waveforms for $G_v$=0.8 (400/320 kV); $P_{dc,1}$= -1 pu

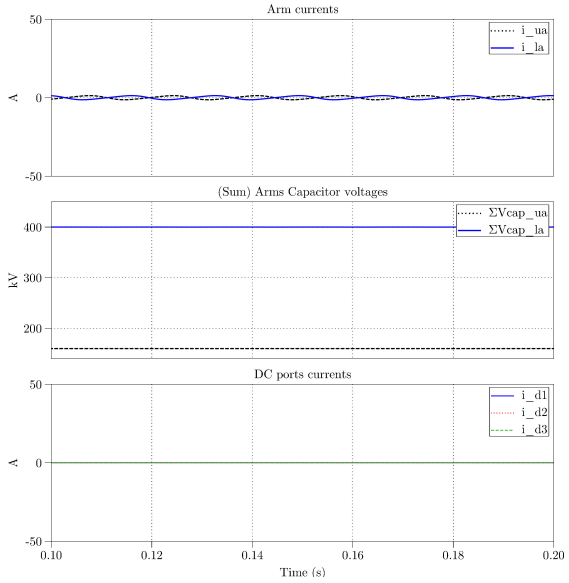



Figure 4.23: Steady-state MP-DCDC DPM waveforms for $G_v$=0.8 (400/320 kV); $P_{dc,1}$= 0 pu

### 4.2.3 BP-DCDC Steady-State Analysis

In this section, the DC/DC solution procedure presented in Section 4.2.1 is used to solve for the modulating signals required to enable open-loop simulations for three different steady-state operating points for the BP-DCDCAC converter. Figure 4.24 shows the BP-DCDC converter that is simulated using the DPM, including the conventions for the current directions. This converter topology can be considered as an inverting topology, contrasting with the more common DC/DC conversion in which the DC networks are linked by non-inverting topologies [19], [33], [34]. The inverting DC-DC topology in Figure 4.3 (without the AC port) is used as the core building block of the multiport DC/DC converter in [9].

The BP-DCDC is rated for 384 MW. For all three operating points the voltages for $NW_1$ and $NW_2$ are set to 320 kV, consistent with a bipolar DC grid application.

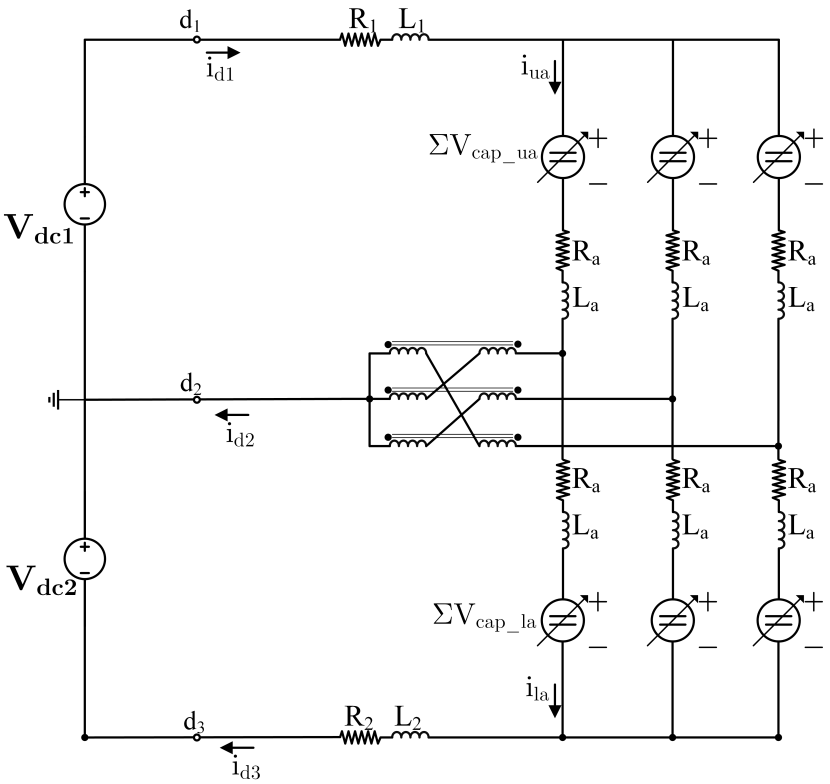



Figure 4.24: BP-DCDC converter with current direction convention

Table 4.6: Simulation parameters for BP-DCDC

| Parameter | Value |
|---|---|
| $w$ | $2\pi 60$ rad/s |
| $L_1, L_2, L_t, L_a, L_s$ | 82 mH, 82 mH, 50 mH, 20 mH |
| $L_e, L_m$ | 9.8 mH, 28 H |
| $R_1, R_2, R_t, R_a, R_s$ | 2.1 Ω, 2.1 Ω, 0.4 Ω, 0.6 Ω, 0.1 Ω |
| $R_w$ | 0.1 Ω |
| $C_u, C_L$ | 8 mF |
| $N_u, N_L$ | 320 |

The differences between the solution procedure of the MP-DCDCAC and BP-DCDCAC explained in Section 4.1.4 apply here as well. Table 4.6 shows the parameters used in these simulations.

#### 4.2.3.1 320/320kV ($G_v$=1) Converter Design

Figures 4.25 to 4.27 show the same port power injections for the BP-DCDC as the cases for the MP-DCDC. The waveforms shown are the upper and lower phase $a$ arm currents, the upper and lower phase $a$ sum of capacitor voltages, and the three DC port currents. Table 4.6 provides the parameters used for these simulations.

Figures 4.25 and 4.26 show the scenarios where full pu power is being transferred from $NW_1$ to $NW_2$ and the reversed operation, respectively. As in the previous cases in this DC/DC conversion section, the waveforms in these two cases are mirrored, demonstrating bidirectional power flow. It's interesting to note that in both cases the term $3(I_{\Sigma 0}^{dc})$ is zero. As mentioned in Section 2.3, DC/DC conversion requires only $P_{\Delta}$ as power transfer mechanism, and contrasting to the simulations for the BP-DCDCAC converter, $I_{\Sigma 0}^{dc}$ has no effect in this conversion. This means that $P_{\Delta}$ alone, and by extension $id_2$, can explain which network is supplying and which one is receiving power.

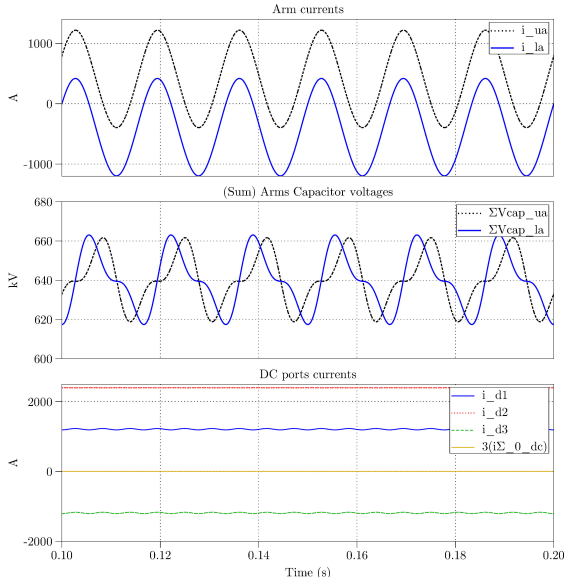



Figure 4.25: Steady-state BP-DCDC DPM waveforms for $G_v$=1 (320/320 kV); $P_{dc,1}$ = 1 pu

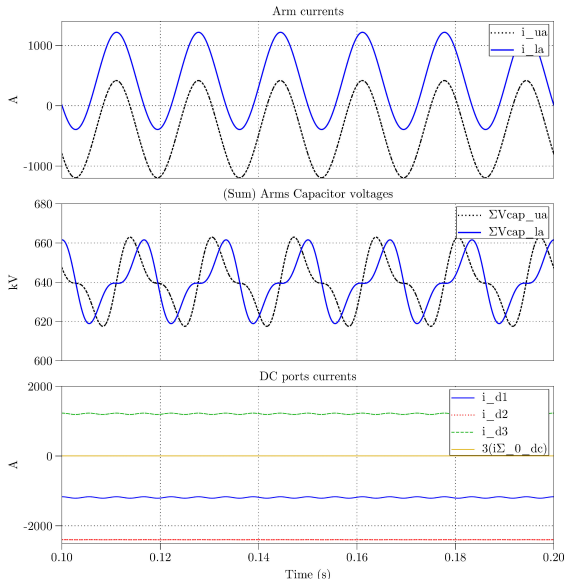



Figure 4.26: Steady-state BP-DCDC DPM waveforms for $G_v$=1 (320/320 kV); $P_{dc,1}$ = -1 pu

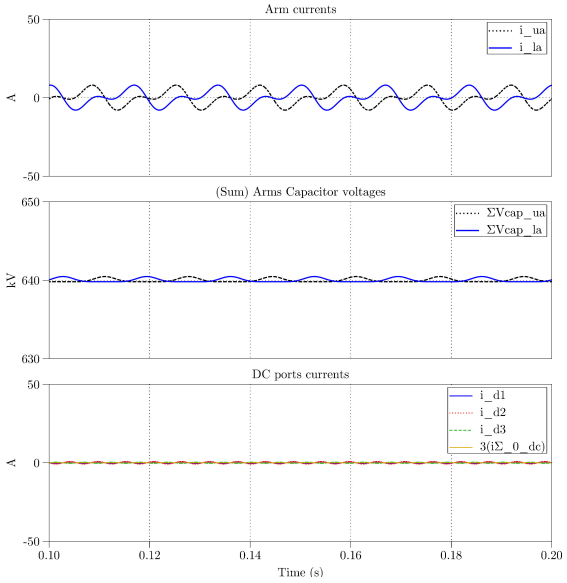



Figure 4.27: Steady-state BP-DCDC DPM waveforms for $G_v$=1 (320/320 kV); $P_{dc,1} = 0$ pu

### 4.2.4 Varying Parameters Studies

As in Section 4.1.4, the DPM in equation Equation (4.18) and the revised solution procedure is here used to demonstrate the converter's behaviour in response to changing parameters. This section presents a few examples of this use for the MP-DCDC converter of Section 4.2.2, for $G_{\mathrm{v}}$=0.5 and $G_{\mathrm{v}}$=0.8, utilising the parameters of Table 4.4 for the $G_{\mathrm{v}}$=0.5 and Table 4.5 for the $G_{\mathrm{v}}$=0.8 cases.

Figure 4.28 shows the four studies performed in this section, in all of them the power flow is varied from 1 pu power received by $\mathrm{NW}_1$ to 1 pu power received by $\mathrm{NW}_2$. Figure 4.28(a) and (b) demonstrate how the peak current stress and the percent capacitors voltage peak-to-peak ripple in the upper and lower phase a arms, respectively, change while varying the power flow, at a ratio of $G_{\mathrm{v}}$=0.5. These plots show symmetry about the $P_{dc,1}$=0 operating point, which is expected according to the literature in DC/DC

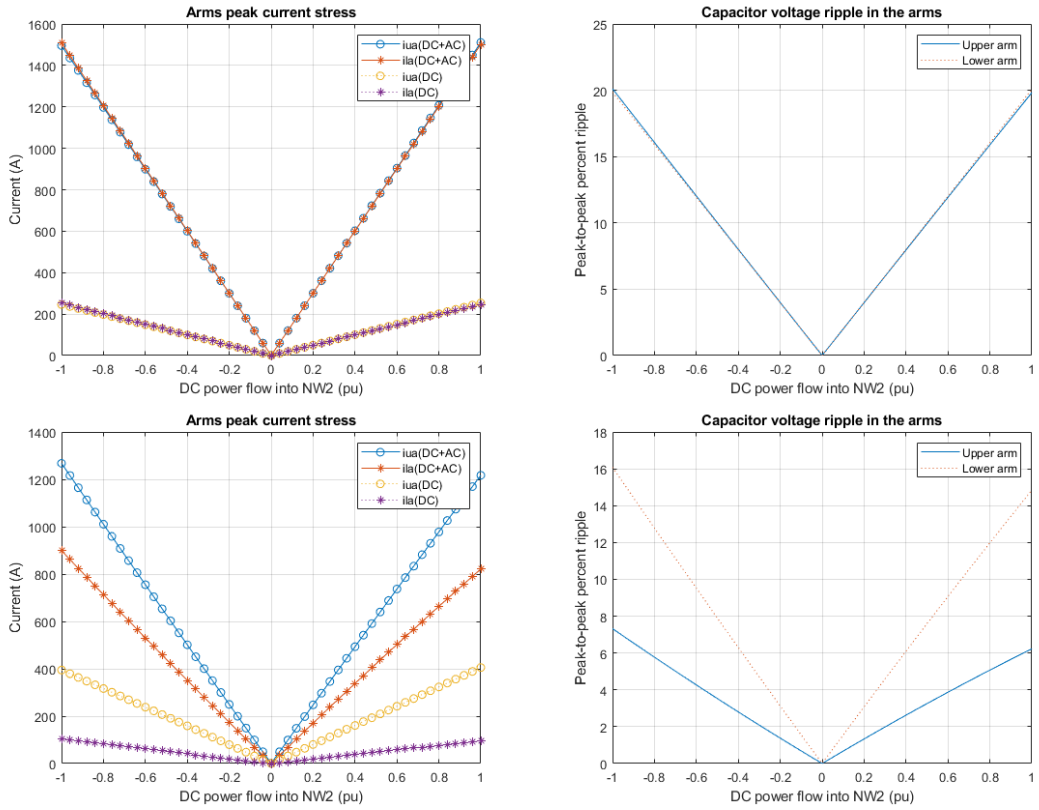



Figure 4.28: Loci of steady-state studies for the phase a upper and lower arms: (a) arms' peak current stress for $G_v$=0.5; (b) percent capacitor voltage ripple for $G_v$=0.5; (c) arms' peak current stress for $G_v$=0.8; (d) percent capacitor voltage ripple for $G_v$=0.8

conversion with this $G_{\mathrm{v}}$ ratio, as presented in [70]. Figure 4.28(c) and (d), meanwhile, show the same parameters but with a ratio of $G_{\mathrm{v}}$=0.8, and, as expected, the upper and lower arms are no longer subjected to the same stresses. These results can be used to properly design a practical converter, i.e., if the currents in the arms exceed the limits of the devices to be used, or if the voltage ripple is to be kept within a certain limit, the proper measures can be taken by analysing the operating ranges to which the converter will be subjected, and designing accordingly.

## 4.3 Chapter Summary

In this chapter, the derived DPMs for the four topologies of interest in this thesis have been subjected to steady-state and varying parameters studies, for different DC step ratios $G_{\mathrm{v}}$. For the steady-state studies, different operating points have been simulated for each converter, where the power injections to the external networks have been specified, and waveforms for several steady-state quantities in the converters are examined. For the varying parameters studies, a variable in the operating point is step-varied through a predetermined range with the goal of observing how these changes affect converter performance, i.e. capacitor voltage ripple and current stress in the arms. In all cases, the converters' behaviour validates the theoretical information presented in Chapter 2, including the harmonic components that are required in specific converter's tasks, and the power transfer mechanisms present in DC/DC and DC/DC/AC conversion processes.

The shown studies are only a small sample of all the studies that can be performed using the DPMs, which provide insight into the operation of these novel converters, and demonstrate the versatility of the developed models.

# Chapter 5

# Conclusion

As DC transmission gathers increasing interest as another viable way for transmission and distribution of power, hybrid AC/DC grids appear to be the power system of the future. The technology to link AC and DC systems, as well as different levels of DC systems, will become ever more crucial as the currently AC dominated transmission and distribution systems require interconnection with HVDC and MVDC systems. MMC converters with the ability to simultaneously perform DC/DC and AC/DC conversions are attractive devices that can help realise future highly meshed hybrid AC/DC grids. This thesis has undertaken the task to develop and validate benchmark models that the research community can utilise to study the emerging classes of DC/DC/AC and DC/DC MMCs.

## 5.1 Contributions

This thesis provides the following core contributions:

1. An $\alpha\beta 0$-frame TAM of a generalised MMC structure is derived that includes multiple harmonic components to capture the different power transfer mechanisms involved in AC/DC and DC/DC conversion processes. This unified model is used to represent two different DC/DC/AC MMC topologies; minor modifications are then made to adapt this model to represent two different DC/DC MMC topologies. All four TAMs permit the user to specify all the impedances in the converters structure and at the ports, as well as balanced or unbalanced networks' voltages, and values for arms' number of cells and capacitances, which can be different between the upper and lower arms. This provides a wide range of versatility for converter studies, and improves on the conventional modelling of converters, where upper and lower arms' capacitances and cell numbers are traditionally set to be equal. The derived TAMs show excellent agreement with PLECS switched model simulations.
2. The aforementioned benchmark $\alpha\beta 0$-frame TAM can be used by researches to study the DC/DC/AC and DC/DC classes of MMCs. This thesis uses the TAM to derive a unified $\alpha\beta 0$ DPM that accounts for DC through third harmonic components, providing a time-invariant (at steady-state) non-linear model that can be used for a wide range of studies not easily performed with the TAM. These models offer the same versatility provided by the TAMs. Similar to the TAM, the unified DPM is then manipulated to create four different models for DC/DC/AC and DC/DC MMCs, which show very good agreement with the derived TAMs. This thesis focuses on using the DPMs for steady-state operating point analyses of the four different MMC topologies. Interestingly, this work reveals the DC/DC/AC DPMs accuracy is somewhat sensitive to the chosen capacitance in the cells, suggesting an even broader spectrum of harmonic content has to be selected for the DPM when a small amount of converter internal energy storage is desired. The DC/DC DPMs did not demonstrate any such sensitivity to internal energy storage.

3. A comprehensive solution procedure is formulated that allows a user to solve for any desired operating point for the DPMs, yielding the full set of open-loop modulating signals and full state solution, while only requiring the user to input basic data such as port power injections and terminal voltages. This is not possible to achieve using only the TAMs. The solution procedure is essential to exploiting the DPMs for study of the converters' steady-state operation in a systematic manner, as demonstrated with various case studies in this work.

## 5.2 Future Work

This work presents groundwork for more research into DC/DC/AC MMCs to be made. The developed models provide a sound foundation for future work in HVDC, MVDC, and hybrid grids. A few tentative future works include:

1. Experimental validation of the DC/DC/AC and DC/DC DPMs, and investigation into the potential to adapt the solution procedure equations to real-time predictions.
2. Investigate the addition of harmonic components to the modulating signals embedded in the DPMs to study capacitor voltage reduction techniques and suppression of unwanted harmonic currents.
3. Carry out small-signal stability studies and explore the use of conventional linear control design tools such as root locus and bode plots, via linearization of the DPMs.
4. Study the capabilities of the presented models utilising full-bridge submodules.
5. Investigate the use of the DPMs for accelerating simulation of their switched-model counterparts.

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# Appendix A

# DC/DC/AC TAM Matrices

The following matrices are for the DC/DC/AC TAM given by Equation (3.31). To reduce the size of the given sub-matrices, several constants are defined as $C_{xxy}$ , where "xx" describes the type of constant, i.e., "$RL$" for a constant with units of resistance times inductance, and "y" takes values of 1,2,3... depending on how many variables share the same unit type. Since the A sub-matrices are of considerable length column-wise, they are divided in several matrices that are numbered with a superscript with only one digit, counting from left to right. The model in Equation (3.31) is repeated here for reference as:

$$\frac{d}{dt}\begin{bmatrix}\mathbf{X}_i^* \\ \mathbf{X}_v^*\end{bmatrix} = \begin{bmatrix}\dot{A}_{11} & \dot{A}_{12} \\ \dot{A}_{21} & 0_{6x6}\end{bmatrix}\begin{bmatrix}\mathbf{X}_i^* \\ \mathbf{X}_v^*\end{bmatrix} + \begin{bmatrix}\dot{N} \\ 0_{6x5}\end{bmatrix}\begin{bmatrix}\mathbf{W}^*\end{bmatrix}$$

$$\dot{A}_{11}=\begin{bmatrix} -\frac{R_a}{L_a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{R_a}{L_a} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{C_{RL1}}{2C_{LL1}} & 0 & 0 & 0 & 0 & \frac{C_{RL}}{4C_{LL1}} \\ 0 & 0 & 0 & -\frac{R_s+R_aC_{PU1}}{L_s+L_aC_{PU1}} & 0 & \frac{2L_aR_w-(2L_e+3L_m)R_a}{C_{LL2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{R_s+R_aC_{PU1}}{L_s+L_aC_{PU1}} & 0 & \frac{2L_aR_w-(2L_e+3L_m)R_a}{C_{LL2}} & 0 \\ 0 & 0 & 0 & -\frac{L_sR_a-L_aR_s}{C_{LL2}} & 0 & -\frac{L_sR_a+4L_sR_w+2L_aR_w}{C_{LL2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{L_sR_a-L_aR_s}{C_{LL2}} & 0 & -\frac{L_sR_a+4L_sR_w+2L_aR_w}{C_{LL2}} & 0 \\ 0 & 0 & -\frac{C_{RL4}}{C_{LL1}} & 0 & 0 & 0 & 0 & -\frac{C_{RL3}}{2C_{LL1}} \end{bmatrix}$$

$C_{RL1}=9L_1R_2+9L_2R_1+3L_1R_a+3L_aR_1+3L_2R_a+3L_aR_2+12L_eR_1+12L_eR_2+18L_tR_1+18L_tR_2+2L_aR_a+8L_eR_a+12L_tR_t$

$C_{LL1}=L_a^2+9L_1L_2+9L_1L_t+9L_2L_t+6L_1L_e+6L_2L_e+6L_aL_t+4L_aL_e+3L_1L_a+9L_2L_a$

$C_{RL4}=3(3L_1R_2-3L_2R_1-L_aR_1-L_2R_a+L_aR_2)$

$C_{PU1}=\frac{2L_e+3L_m}{L_a+4L_e+6L_m}$

$C_{LL2}=2L_aL_e+3L_aL_m+L_aL_s+4L_eL_s+6L_mL_s$

$C_{RL2}=9L_1R_2-9L_2R_1+3L_1R_a-3L_aR_1-3L_2R_a+3L_aR_2-12L_eR_1+12L_eR_2+18L_1R_t-18L_tR_1-18L_2R_t+18L_tR_2+12L_1R_w-12L_2R_w$

$C_{RL3}=9L_1R_2+9L_2R_1+3L_1R_a+3L_aR_1+3L_2R_a+3L_aR_2+18L_1R_t+18L_2R_t+12L_1R_w+12L_2R_w+2L_aR_a+12L_aR_t+8L_aR_w$

$$\dot{A}_{12}=[\dot{A}_{12}^1 \quad \dot{A}_{12}^2]$$

$$\dot{A}_{12}^1=\begin{bmatrix} -\frac{2M_{\Sigma0}+M_{\Sigma\alpha}}{2L_a} & -\frac{M_{\Sigma\beta}}{2L_a} & -\frac{M_{\Sigma\alpha}}{L_a} \\ -\frac{M_{\Sigma\beta}}{2L_a} & -\frac{2M_{\Sigma0}+M_{\Sigma\alpha}}{2L_a} & -\frac{M_{\Sigma\beta}}{L_a} \\ \frac{3M_{\Delta\alpha}(L_1-L_2)-2M_{\Sigma\alpha}C_L}{2L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Delta\beta}(L_1-L_2)-2M_{\Sigma\beta}C_L}{2L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Delta0}(L_1-L_2)-2M_{\Sigma0}C_L}{2L_a(L_a+4L_e)C_{PU2}} \\ -\frac{(2M_{\Delta0}+M_{\Delta\alpha})(2L_e+3L_m)}{C_{LL2}} & \frac{M_{\Delta\beta}(2L_e+3L_m)}{C_{LL2}} & -\frac{2M_{\Delta\alpha}(2L_e+3L_m)}{C_{LL2}} \\ \frac{M_{\Delta\beta}(2L_e+3L_m)}{C_{LL2}} & -\frac{(2M_{\Delta0}+M_{\Delta\alpha})(2L_e+3L_m)}{C_{LL2}} & -\frac{2M_{\Delta\beta}(2L_e+3L_m)}{C_{LL2}} \\ -\frac{L_s(2M_{\Delta0}+M_{\Delta\alpha})}{C_{LL2}} & \frac{M_{\Delta\beta}L_s}{C_{LL2}} & -\frac{2L_sM_{\Delta\alpha}}{C_{LL2}} \\ \frac{M_{\Delta\beta}L_s}{C_{LL2}} & -\frac{L_s(2M_{\Delta0}+M_{\Delta\alpha})}{C_{LL2}} & -\frac{2L_sM_{\Delta\beta}}{C_{LL2}} \\ \frac{3M_{\Sigma\alpha}(L_1-L_2)-3L_aM_{\Delta\alpha}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Sigma\beta}(L_1-L_2)-3L_aM_{\Delta\beta}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Sigma0}(L_1-L_2)-3L_aM_{\Delta0}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} \end{bmatrix}$$

$$C_{PU2}=\left(\frac{3L_1+3L_2}{2L_a+1}\right)\left(\frac{3L_1+3L_2+12L_t}{2L_a+8L_e}\right)-\left(\frac{(3L_1-3L_2)^2}{4L_a(L_a+4L_e)}\right)$$

$$C_L=\left(\frac{3L_1+3L_2+12L_t}{2L_a+8L_e}+1\right)(L_a+4L_e)$$

$$\dot{A}_{12}^{2} = \begin{bmatrix} -\dfrac{2M_{\Delta 0}+M_{\Delta\alpha}}{2L_a} & -\dfrac{M_{\Delta\beta}}{2L_a} & -\dfrac{M_{\Delta\alpha}}{L_a} \\ -\dfrac{M_{\Delta\beta}}{2L_a} & -\dfrac{2M_{\Delta 0}+M_{\Delta\alpha}}{2L_a} & -\dfrac{M_{\Delta\beta}}{L_a} \\ \dfrac{3M_{\Sigma\alpha}(L_1-L_2)-2M_{\Delta\alpha}C_L}{2L_a(2L_a+8L_e)C_{PU2}} & \dfrac{3M_{\Sigma\beta}(L_1-L_2)-2M_{\Delta\beta}C_L}{2L_a(2L_a+8L_e)C_{PU2}} & \dfrac{3M_{\Sigma 0}(L_1-L_2)-2M_{\Delta 0}C_L}{2L_a(L_a+4L_e)C_{PU}} \\ -\dfrac{(2M_{\Sigma 0}+M_{\Sigma\alpha})(2L_e+3L_m)}{C_{LL}} & \dfrac{M_{\Sigma\beta}(2L_e+3L_m)}{C_{LL2}} & -\dfrac{2M_{\Sigma\alpha}(2L_e+3L_m)}{C_{LL2}} \\ \dfrac{M_{\Sigma\beta}(2L_e+3L_m)}{C_{LL}} & -\dfrac{(2M_{\Sigma 0}+M_{\Sigma\alpha})(2L_e+3L_m)}{C_{LL}} & -\dfrac{2M_{\Sigma\beta}(2L_e+3L_m)}{C_{LL}} \\ -\dfrac{L_s(2M_{\Sigma 0}+M_{\Sigma\alpha})}{C_{LL}} & \dfrac{M_{\Sigma\beta}L_s}{C_{LL2}} & -\dfrac{2L_sM_{\Sigma\alpha}}{C_{LL}} \\ \dfrac{M_{\Sigma\beta}L_s}{C_{LL2}} & -\dfrac{L_s(2M_{\Sigma 0}+M_{\Sigma\alpha})}{C_{LL2}} & -\dfrac{2L_sM_{\Sigma\beta}}{C_{LL}} \\ \dfrac{3M_{\Delta\alpha}(L_1-L_2)-3L_aM_{\Sigma\alpha}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU}} & \dfrac{3M_{\Delta\beta}(L_1-L_2)-3L_aM_{\Sigma\beta}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} & \dfrac{3M_{\Delta 0}(L_1-L_2)-3L_aM_{\Sigma 0}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU}} \end{bmatrix}$$

$$\dot{A}_{21} = [\dot{A}_{21}^{1} \quad \dot{A}_{21}^{2} \quad \dot{A}_{21}^{3} \quad \dot{A}_{21}^{4} \quad \dot{A}_{21}^{5}]$$

$$\dot{A}_{21}^{1} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha})-\left(\frac{Nl}{4Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha}) & \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta})-\left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta}) \\ \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta})-\left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta}) & \left(\frac{Nu}{4Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha})-\left(\frac{Nl}{4Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha}+M_{\Sigma\alpha})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha}-M_{\Sigma\alpha}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \\ \left(\frac{Nl}{4Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha})+\left(\frac{Nu}{4Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha}) & -\left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \\ -\left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) & \left(\frac{Nl}{4Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha})+\left(\frac{Nu}{4Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha}+M_{\Sigma\alpha})+\left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha}-M_{\Sigma\alpha}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})+\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \end{bmatrix}$$

$$\dot{A}_{21}^{2} = \begin{bmatrix} \left(\frac{Nu}{2Cu}\right)(M_{\Delta\alpha}+M_{\Sigma\alpha})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\alpha}-M_{\Sigma\alpha}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta 0}+M_{\Sigma 0})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta 0}-M_{\Sigma 0}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\alpha}+M_{\Sigma\alpha})+\left(\frac{Nl}{2Cl}\right)(M_{\Delta\alpha}-M_{\Sigma\alpha}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})+\left(\frac{Nl}{2Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta 0}+M_{\Sigma 0})+\left(\frac{Nl}{2Cl}\right)(M_{\Delta 0}-M_{\Sigma 0}) \end{bmatrix}$$

$$\dot{A}_{21}^{3} = \begin{bmatrix} \left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha})+\left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha}) & -\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \\ -\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) & \left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha})+\left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha}) \\ \left(\frac{Nu}{8Cu}\right)(M_{\Delta\alpha}+M_{\Sigma\alpha})+\left(\frac{Nl}{8Cl}\right)(M_{\Delta\alpha}-M_{\Sigma\alpha}) & \left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})+\left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \\ \left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha})-\left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha}) & \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta})-\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta}) \\ \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta})-\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta}) & \left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha})-\left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha}) \\ \left(\frac{Nu}{8Cu}\right)(M_{\Delta\alpha}+M_{\Sigma\alpha})-\left(\frac{Nl}{8Cl}\right)(M_{\Delta\alpha}-M_{\Sigma\alpha}) & \left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \end{bmatrix}$$

$$\dot{A}_{21}^{4} = \begin{bmatrix} \left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha})+\left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha}) & -\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \\ -\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) & \left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha})+\left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha}) \\ \left(\frac{Nu}{8Cu}\right)(M_{\Delta\alpha}+M_{\Sigma\alpha})+\left(\frac{Nl}{8Cl}\right)(M_{\Delta\alpha}-M_{\Sigma\alpha}) & \left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})+\left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \\ \left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha})-\left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha}) & \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta})-\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta}) \\ \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta})-\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta}) & \left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0}+M_{\Delta\alpha}+2M_{\Sigma 0}+M_{\Sigma\alpha})-\left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0}+M_{\Delta\alpha}-2M_{\Sigma 0}-M_{\Sigma\alpha}) \\ \left(\frac{Nu}{8Cu}\right)(M_{\Delta\alpha}+M_{\Sigma\alpha})-\left(\frac{Nl}{8Cl}\right)(M_{\Delta\alpha}-M_{\Sigma\alpha}) & \left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta}+M_{\Sigma\beta})-\left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta}-M_{\Sigma\beta}) \end{bmatrix}$$

$$\dot{A}_{21}^{5} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) + \left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) + \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta 0} + M_{\Sigma 0}) + \left(\frac{Nl}{4Cl}\right)(M_{\Delta 0} - M_{\Sigma 0}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta 0} + M_{\Sigma 0}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta 0} - M_{\Sigma 0}) \end{bmatrix}$$

$$\dot{N} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{3L_2 + L_a + 4L_e + 4L_t}{2C_{LL1}} & \frac{3(L_1 - L_2)}{3L_a(L_a + 4L_e)C_{PU2}} & 0 & 0 & -\frac{3(L_1 - L_2)}{2(L_a + 2L_s)(L_a + 4L_e)C_{PU2}} \\ 0 & 0 & \frac{-1}{L_s + L_a C_{PU1}} & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{L_s + L_a C_{PU}} & 0 \\ 0 & 0 & \frac{L_a}{C_{LL}} & 0 & 0 \\ 0 & 0 & 0 & \frac{L_a}{C_{LL2}} & 0 \\ \frac{3L_2 + L_a}{C_{LL}} & \frac{-\frac{3(L_1 - L_2)}{L_a + 2}}{(L_a + 4L_e)C_{PU}} & 0 & 0 & \frac{3L_a\left(\frac{L_1 - L_2}{L_a + 2}\right)}{(L_a + 2L_s)(L_a + 4L_e)C_{PU}} \end{bmatrix}$$

# Appendix B

# DC/DC TAM Matrices

The following matrices are for the DC/DC TAM given by Equation (3.34). To reduce the size of the given sub-matrices, several constants are defined as $C_{xxy}$, where "xx" describes the type of constant, i.e., "$RL$" for a constant with units of resistance times inductance, and "y" takes values of 1,2,3... depending on how many variables share the same unit. Since the A sub-matrices are of considerable length column-wise, they are divided in several matrices that are numbered with a superscript with only one digit, counting from left to right. The model in equation Equation (3.34) is repeated here for reference as:

$$\frac{d}{dt}\begin{bmatrix}\boldsymbol{X}^*_{ir}\\ \boldsymbol{X}^*_{v}\end{bmatrix} = \begin{bmatrix}\dot{A}_{11r} & \dot{A}_{12r}\\ \dot{A}_{21r} & 0_{6x6}\end{bmatrix}\begin{bmatrix}\boldsymbol{X}^*_{ir}\\ \boldsymbol{X}^*_{v}\end{bmatrix} + \begin{bmatrix}\dot{N}_r\\ 0_{6x5}\end{bmatrix}\begin{bmatrix}\boldsymbol{W}^*_{r}\end{bmatrix}, \tag{B.1}$$

$$\dot{A}_{11r}=\begin{bmatrix} -\frac{R_a}{L_a} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{R_a}{L_a} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{C_{RL1}}{2C_{LL1}} & 0 & 0 & \frac{C_{RL2}}{4C_{LL1}} \\ 0 & 0 & 0 & \frac{-R_a-4R_w}{L_a+4L_e+6L_m} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-R_a-4R_w}{L_a+4L_e+6L_m} & 0 \\ 0 & 0 & -\frac{C_{RL4}}{C_{LL1}} & 0 & 0 & -\frac{C_{RL3}}{2C_{LL1}} \end{bmatrix}$$

$C_{RL} = 9L_1R_2 + 9L_2R_1 + 3L_1R_a + 3L_aR_1 + 3L_2R_a + 3L_aR_2 + 12L_eR_1 + 12L_eR_2 + 18L_tR_1 + 18L_tR_2 + 2L_aR_a + 8L_eR_a + 12L_tR_t$

$C_{LL1} = L_a^2 + 9L_1L_2 + 9L_1L_t + 9L_2L_t + 6L_1L_e + 6L_2L_e + 6L_aL_t + 4L_aL_e + 3L_1L_a + 9L_2L_a$

$C_{RL4} = 3(3L_1R_2 - 3L_2R_1 - L_aR_1 - L_2R_a + L_aR_2)$

$C_{RL2} = 9L_1R_2 - 9L_2R_1 + 3L_1R_a - 3L_aR_1 - 3L_2R_a + 3L_aR_2 - 12L_eR_1 + 12L_eR_2 + 18L_1R_t - 18L_tR_1 - 18L_2R_t + 18L_tR_2 + 12L_1R_w - 12L_2R_w$

$C_{RL3} = 9L_1R_2 + 9L_2R_1 + 3L_1R_a + 3L_aR_1 + 3L_2R_a + 3L_aR_2 + 18L_1R_t + 18L_2R_t + 12L_1R_w + 12L_2R_w + 2L_aR_a + 12L_aR_t + 8L_aR_w$

$$\dot{A}_{12r} = [\dot{A}^1_{12r} \quad \dot{A}^2_{12r}]$$

$$\dot{A}^1_{12r}=\begin{bmatrix} -\frac{2M_{\Sigma 0}+M_{\Sigma\alpha}}{2L_a} & -\frac{M_{\Sigma\beta}}{2L_a} & -\frac{M_{\Sigma\alpha}}{L_a} \\ -\frac{M_{\Sigma\beta}}{2L_a} & -\frac{2M_{\Sigma 0}+M_{\Sigma\alpha}}{2L_a} & -\frac{M_{\Sigma\beta}}{L_a} \\ \frac{3M_{\Delta\alpha}(L_1-L_2)-2M_{\Sigma\alpha}C_L}{2L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Delta\beta}(L_1-L_2)-2M_{\Sigma\beta}C_L}{2L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Delta 0}(L_1-L_2)-2M_{\Sigma 0}C_L}{2L_a(L_a+4L_e)C_{PU2}} \\ -\frac{L_s(2M_{\Delta 0}+M_{\Delta\alpha})}{C_{LL2}} & \frac{M_{\Delta\beta}L_s}{C_{LL2}} & -\frac{2L_sM_{\Delta\alpha}}{C_{LL2}} \\ \frac{M_{\Delta\beta}L_s}{C_{LL2}} & -\frac{L_s(2M_{\Delta 0}+M_{\Delta\alpha})}{C_{LL2}} & -\frac{2L_sM_{\Delta\beta}}{C_{LL2}} \\ \frac{3M_{\Sigma\alpha}(L_1-L_2)-3L_aM_{\Delta\alpha}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Sigma\beta}(L_1-L_2)-3L_aM_{\Delta\beta}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Sigma 0}(L_1-L_2)-3L_aM_{\Delta 0}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} \end{bmatrix}$$

$$C_{PU2} = \left(\frac{3L_1+3L_2}{2L_a+1}\right)\left(\frac{3L_1+3L_2+12L_t}{2L_a+8L_e}\right) - \left(\frac{(3L_1-3L_2)^2}{4L_a(L_a+4L_e)}\right)$$

$$C_L = \left(\frac{3L_1+3L_2+12L_t}{2L_a+8L_e}+1\right)(L_a+4L_e)$$

$$\dot{A}^{12}_{12r}=\begin{bmatrix} -\frac{2M_{\Delta 0}+M_{\Delta\alpha}}{2L_a} & -\frac{M_{\Delta\beta}}{2L_a} & -\frac{M_{\Delta\alpha}}{L_a} \\ -\frac{M_{\Delta\beta}}{2L_a} & -\frac{2M_{\Delta 0}+M_{\Delta\alpha}}{2L_a} & -\frac{M_{\Delta\beta}}{L_a} \\ \frac{3M_{\Sigma\alpha}(L_1-L_2)-2M_{\Delta\alpha}C_L}{2L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Sigma\beta}(L_1-L_2)-2M_{\Delta\beta}C_L}{2L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Sigma 0}(L_1-L_2)-2M_{\Delta 0}C_L}{2L_a(L_a+4L_e)C_{PU2}} \\ -\frac{L_s(2M_{\Sigma 0}+M_{\Sigma\alpha})}{C_{LL2}} & \frac{M_{\Sigma\beta}L_s}{C_{LL2}} & -\frac{2L_sM_{\Sigma\alpha}}{C_{LL2}} \\ \frac{M_{\Sigma\beta}L_s}{C_{LL2}} & -\frac{L_s(2M_{\Sigma 0}+M_{\Sigma\alpha})}{C_{LL2}} & -\frac{2L_sM_{\Sigma\beta}}{C_{LL2}} \\ \frac{3M_{\Delta\alpha}(L_1-L_2)-3L_aM_{\Sigma\alpha}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Delta\beta}(L_1-L_2)-3L_aM_{\Sigma\beta}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} & \frac{3M_{\Delta 0}(L_1-L_2)-3L_aM_{\Sigma 0}\left(\frac{L_1+L_2}{L_a+2}\right)}{L_a(2L_a+8L_e)C_{PU2}} \end{bmatrix}$$

$$\dot{A}_{21r} = [\dot{A}^{11}_{21r} \quad \dot{A}^{12}_{21r} \quad \dot{A}^{13}_{21r} \quad \dot{A}^{14}_{21r}]$$

$$\dot{A}_{21r}^{11} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)(2M_{\Delta 0} + M_{\Delta\alpha} + 2M_{\Sigma 0} + M_{\Sigma\alpha}) - \left(\frac{Nl}{4Cl}\right)(2M_{\Delta 0} + M_{\Delta\alpha} - 2M_{\Sigma 0} - M_{\Sigma\alpha}) & \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) - \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) \\ \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) - \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) & \left(\frac{Nu}{4Cu}\right)(2M_{\Delta 0} + M_{\Delta\alpha} + 2M_{\Sigma 0} + M_{\Sigma\alpha}) - \left(\frac{Nl}{4Cl}\right)(2M_{\Delta 0} + M_{\Delta\alpha} - 2M_{\Sigma 0} - M_{\Sigma\alpha}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ \left(\frac{Nl}{4Cl}\right)(2M_{\Delta 0} + M_{\Delta\alpha} - 2M_{\Sigma 0} - M_{\Sigma\alpha}) + \left(\frac{Nu}{4Cu}\right)(2M_{\Delta 0} + M_{\Delta\alpha} + 2M_{\Sigma 0} + M_{\Sigma\alpha}) & -\left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ -\left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) & \left(\frac{Nl}{4Cl}\right)(2M_{\Delta 0} + M_{\Delta\alpha} - 2M_{\Sigma 0} - M_{\Sigma\alpha}) + \left(\frac{Nu}{4Cu}\right)(2M_{\Delta 0} + M_{\Delta\alpha} + 2M_{\Sigma 0} + M_{\Sigma\alpha}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) + \left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) + \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \end{bmatrix}$$

$$\dot{A}_{21r}^{12} = \begin{bmatrix} \left(\frac{Nu}{2Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) - \left(\frac{Nl}{2Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{2Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta 0} + M_{\Sigma 0}) - \left(\frac{Nl}{2Cl}\right)(M_{\Delta 0} - M_{\Sigma 0}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) + \left(\frac{Nl}{2Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) + \left(\frac{Nl}{2Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta 0} + M_{\Sigma 0}) + \left(\frac{Nl}{2Cl}\right)(M_{\Delta 0} - M_{\Sigma 0}) \end{bmatrix} \quad \dot{A}_{21}^{14} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) + \left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) + \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta 0} + M_{\Sigma 0}) + \left(\frac{Nl}{4Cl}\right)(M_{\Delta 0} - M_{\Sigma 0}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ \left(\frac{Nu}{4Cu}\right)(M_{\Delta 0} + M_{\Sigma 0}) - \left(\frac{Nl}{4Cl}\right)(M_{\Delta 0} - M_{\Sigma 0}) \end{bmatrix}$$

$$\dot{A}_{21r}^{13} = \begin{bmatrix} \left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0} + M_{\Delta\alpha} - 2M_{\Sigma 0} - M_{\Sigma\alpha}) + \left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0} + M_{\Delta\alpha} + 2M_{\Sigma 0} + M_{\Sigma\alpha}) & -\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ -\left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) & \left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0} + M_{\Delta\alpha} - 2M_{\Sigma 0} - M_{\Sigma\alpha}) + \left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0} + M_{\Delta\alpha} + 2M_{\Sigma 0} + M_{\Sigma\alpha}) \\ \left(\frac{Nu}{8Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) + \left(\frac{Nl}{8Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) & \left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) + \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \\ \left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0} + M_{\Delta\alpha} + 2M_{\Sigma 0} + M_{\Sigma\alpha}) - \left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0} + M_{\Delta\alpha} - 2M_{\Sigma 0} - M_{\Sigma\alpha}) & \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) - \left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) \\ \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) - \left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) & \left(\frac{Nu}{8Cu}\right)(2M_{\Delta 0} + M_{\Delta\alpha} + 2M_{\Sigma 0} + M_{\Sigma\alpha}) - \left(\frac{Nl}{8Cl}\right)(2M_{\Delta 0} + M_{\Delta\alpha} - 2M_{\Sigma 0} - M_{\Sigma\alpha}) \\ \left(\frac{Nu}{8Cu}\right)(M_{\Delta\alpha} + M_{\Sigma\alpha}) - \left(\frac{Nl}{8Cl}\right)(M_{\Delta\alpha} - M_{\Sigma\alpha}) & \left(\frac{Nu}{8Cu}\right)(M_{\Delta\beta} + M_{\Sigma\beta}) - \left(\frac{Nl}{8Cl}\right)(M_{\Delta\beta} - M_{\Sigma\beta}) \end{bmatrix}$$

$$\dot{N}_r = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{3L_2 + L_a + 4L_e + 4L_t}{2C_{LL}} & \frac{3(L_1 - L_2)}{3L_a(L_a + 4L_e)C_{PU}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{3L_2 + L_a}{C_{LL}} & -\frac{3(L_1 - L_2)}{L_a + 2} \\ & \frac{}{(L_a + 4L_e)C_{PU2}} \end{bmatrix}$$

# Appendix C

# DC/DC/AC DPM Matrices

The following matrices are for the DC/DC/AC DPM given by Equation (3.57) . To reduce the size of the given sub-matrices, several constants are defined as $C_{xxy}$ , where "xx" describes the type of constant, i.e., "$RL$" for a constant with units of resistance times inductance, and "y" takes values of 1,2,3... depending on how many variables share the same unit. This model includes second and third harmonic components for the modulating signals, but due to time constraints they were not investigated in the thesis' body, and are always set to 0 for simulation purposes. The model in equation Equation (3.57) is repeated here for reference as:

$$\frac{d}{dt}\begin{bmatrix}\widetilde{\boldsymbol{X}}_i^* \\ \widetilde{\boldsymbol{X}}_v^*\end{bmatrix} = \begin{bmatrix}\widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & 0_{22x22}\end{bmatrix}\begin{bmatrix}\widetilde{\boldsymbol{X}}_i^* \\ \widetilde{\boldsymbol{X}}_v^*\end{bmatrix} + \begin{bmatrix}\widetilde{N} \\ 0_{22x5}\end{bmatrix}\left[\widetilde{\boldsymbol{W}}^*\right], \tag{C.1}$$

$$\tilde{A}_{11} = \begin{bmatrix}\tilde{A}_{11}^{11} & \tilde{A}_{11}^{12} & \tilde{A}_{11}^{13} & \tilde{A}_{11}^{14} & \tilde{A}_{11}^{15} & \tilde{A}_{11}^{16} & \tilde{A}_{11}^{17} \\ [0]_{22x8} & \tilde{A}_{11}^{22} & \tilde{A}_{11}^{23} & \tilde{A}_{11}^{24} & \tilde{A}_{11}^{25} & \tilde{A}_{11}^{26} & \tilde{A}_{11}^{27}\end{bmatrix}$$

$$\tilde{A}_{11}^{11}=\begin{bmatrix} -\frac{R_a}{L_a} & -\omega & 0 & 0 & 0 & 0 & 0 & 0 \\ \omega & -\frac{R_a}{L_a} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{R_a}{L_a} & -2\omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\omega & -\frac{R_a}{L_a} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{R_a}{L_a} & -\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega & -\frac{R_a}{L_a} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{R_a}{L_a} & -2\omega \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\omega & -\frac{R_a}{L_a} \end{bmatrix}$$

$$\tilde{A}_{11}^{12}=\begin{bmatrix} [0]_{8x3} \\ \frac{C_{RL1}}{2C_{LL1}} & 0 & 0 \\ 0 & -\frac{C_{RL}}{2C_{LL}} & -3\omega \\ 0 & 3\omega & -\frac{C_{RL1}}{2C_{LL}} \end{bmatrix} \tilde{A}_{11}^{22}=\begin{bmatrix} [0]_{16x3} \\ \frac{C_{RL4}}{C_{LL1}} & 0 & 0 \\ 0 & \frac{C_{RL4}}{C_{LL1}} & 0 \\ 0 & 0 & \frac{C_{RL4}}{C_{LL1}} \end{bmatrix}$$

$C_{RL1}=9L_1R_2+9L_2R_1+3L_1R_a+3L_aR_1+3L_2R_a+3L_aR_2+12L_eR_1+12L_eR_2+18L_tR_1+18L_tR_2+2L_aR_a+8L_eR_a+12L_tR_t$

$C_{LL1}=L_a^2+9L_1L_2+9L_1L_t+9L_2L_t+6L_1L_e+6L_2L_e+6L_aL_t+4L_aL_e+3L_1L_a+9L_2L_a$

$C_{RL4}=3(3L_1R_2-3L_2R_1+L_1R_a-L_aR_1-L_2R_a+L_aR_2)$

$$\tilde{A}_{11}^{13}=\begin{bmatrix} & & [0]_{11x4} & \\ -\frac{R_s+R_aC_{PU1}}{L_s+L_aC_{PU1}} & -\omega & 0 & 0 \\ \omega & -\frac{R_s+R_aC_{PU1}}{L_s+L_aC_{PU1}} & 0 & 0 \\ 0 & 0 & -\frac{R_s+R_aC_{PU1}}{L_s+L_aC_{PU1}} & -2\omega \\ 0 & 0 & 2\omega & -\frac{R_s+R_aC_{PU1}}{L_s+L_aC_{PU1}} \end{bmatrix}$$

$$\tilde{A}_{11}^{23}=\begin{bmatrix} & & [0]_{4x4} & \\ -\frac{L_sR_a-L_aR_s}{C_{LL2}} & 0 & 0 & 0 \\ 0 & -\frac{L_sR_a-L_aR_s}{C_{LL2}} & 0 & 0 \\ 0 & 0 & -\frac{L_sR_a-L_aR_s}{C_{LL2}} & 0 \\ 0 & 0 & 0 & -\frac{L_sR_a-L_aR_s}{C_{LL}} \\ & & [0]_{7x4} & \end{bmatrix}$$

$$\tilde{A}_{11}^{14} = \begin{bmatrix} & & [0]_{15x4} & \\ -\frac{R_s + R_a C_{PU}}{L_s + L_a C_{PU1}} & -\omega & 0 & 0 \\ \omega & -\frac{R_s + R_a C_{PU1}}{L_s + L_a C_{PU1}} & 0 & 0 \\ 0 & 0 & -\frac{R_s + R_a C_{PU1}}{L_s + L_a C_{PU1}} & -2\omega \\ 0 & 0 & 2\omega & -\frac{R_s + R_a C_{PU1}}{L_s + L_a C_{PU1}} \end{bmatrix}$$

$$\tilde{A}_{11}^{24} = \begin{bmatrix} & & [0]_{4x4} & \\ -\frac{L_s R_a - L_a R_s}{C_{LL2}} & 0 & 0 & 0 \\ 0 & -\frac{L_s R_a - L_a R_s}{C_{LL2}} & 0 & 0 \\ 0 & 0 & -\frac{L_s R_a - L_a R_s}{C_{LL2}} & 0 \\ 0 & 0 & 0 & -\frac{L_s R_a - L_a R_s}{C_{LL2}} \\ & & [0]_{3x4} & \end{bmatrix}$$

$$\tilde{A}_{11}^{15} = \begin{bmatrix} & & [0]_{11x4} & \\ \frac{2L_a R_w - R_a(2L_e + 3L_m)}{C_{LL2}} & 0 & 0 & 0 \\ 0 & \frac{2L_a R_w - R_a(2L_e + 3L_m)}{C_{LL2}} & 0 & 0 \\ 0 & 0 & \frac{2L_a R_w - R_a(2L_e + 3L_m)}{C_{LL2}} & 0 \\ 0 & 0 & 0 & \frac{2L_a R_w - R_a(2L_e + 3L_m)}{C_{LL2}} \end{bmatrix}$$

$$\tilde{A}_{11}^{25} = \begin{bmatrix} & & [0]_{4x4} & \\ -\frac{L_s R_a + 4L_s R_w + 2L_a R_w}{C_{LL2}} & -\omega & 0 & 0 \\ \omega & -\frac{L_s R_a + 4L_s R_w + 2L_a R_w}{C_{LL2}} & 0 & 0 \\ 0 & 0 & -\frac{L_s R_a + 4L_s R_w + 2L_a R_w}{C_{LL2}} & -2\omega \\ 0 & 0 & 2\omega & -\frac{L_s R_a + 4L_s R_w + 2L_a R_w}{C_{LL2}} \\ & & [0]_{7x4} & \end{bmatrix}$$

$$C_{PU1} = \frac{2L_e + 3L_m}{L_a + 4L_e + 6L_m}$$

$$C_{LL} = 2L_a L_e + 3L_a L_m + L_a L_s + 4L_e L_s + 6L_m L_s$$

$$\tilde{A}_{11}^{16} = \begin{bmatrix} & & [0]_{15x4} & \\ \frac{2L_a R_w - R_a(2L_e + 3L_m)}{C_{LL2}} & 0 & 0 & 0 \\ 0 & \frac{2L_a R_w - R_a(2L_e + 3L_m)}{C_{LL2}} & 0 & 0 \\ 0 & 0 & \frac{2L_a R_w - R_a(2L_e + 3L_m)}{C_{LL2}} & 0 \\ 0 & 0 & 0 & \frac{2L_a R_w - R_a(2L_e + 3L_m)}{C_{LL2}} \end{bmatrix}$$

$$\tilde{A}_{11}^{26} = \begin{bmatrix} & [0]_{4x4} & & \\ -\frac{L_sR_a + 4L_sR_w + 2L_aR_w}{C_{LL2}} & -\omega & 0 & 0 \\ \omega & -\frac{L_sR_a + 4L_sR_w + 2L_aR_w}{C_{LL2}} & 0 & 0 \\ 0 & 0 & -\frac{L_sR_a + 4L_sR_w + 2L_aR_w}{C_{LL2}} & -2\omega \\ 0 & 0 & 2\omega & -\frac{L_sR_a + 4L_sR_w + 2L_aR_w}{C_{LL2}} \\ & [0]_{3x4} & & \end{bmatrix}$$

$$\tilde{A}_{11}^{17} = \begin{bmatrix} & [0]_{8x3} & \\ \frac{C_{RL2}}{4C_{LL1}} & 0 & 0 \\ 0 & \frac{C_{RL2}}{4C_{LL1}} & 0 \\ 0 & 0 & \frac{C_{RL2}}{4C_{LL1}} \end{bmatrix} \tilde{A}_{11}^{22} = \begin{bmatrix} & [0]_{16x3} & \\ -\frac{C_{RL3}}{2C_{LL1}} & 0 & 0 \\ 0 & -\frac{C_{RL3}}{2C_{LL1}} & -3\omega \\ 0 & 3\omega & -\frac{C_{RL3}}{2C_{LL1}} \end{bmatrix}$$

$$C_{RL2} = 9L_1R_2 - 9L_2R_1 + 3L_1R_a - 3L_aR_1 - 3L_2R_a + 3L_aR_2 - 12L_eR_1 + 12L_eR_2 + 18L_1R_t - 18L_tR_1 - 18L_2R_t + 18L_tR_2 + 12L_1R_w - 12L_2R_w$$

$$C_{RL3} = 9L_1R_2 + 9L_2R_1 + 3L_1R_a + 3L_aR_1 + 3L_2R_a + 3L_aR_2 + 18L_1R_t + 18L_2R_t + 12L_1R_w + 12L_2R_w + 2L_aR_a + 12L_aR_t + 8L_aR_w$$

$$\tilde{A}_{12} = \begin{bmatrix} \tilde{A}_{12}^{11} & \tilde{A}_{12}^{12} & \tilde{A}_{12}^{13} & \tilde{A}_{12}^{14} & \tilde{A}_{12}^{15} & \tilde{A}_{12}^{16} & \tilde{A}_{12}^{17} & \tilde{A}_{12}^{18} & \tilde{A}_{12}^{19} & \tilde{A}_{12}^{110} \\ \tilde{A}_{12}^{21} & \tilde{A}_{12}^{22} & \tilde{A}_{12}^{23} & \tilde{A}_{12}^{24} & \tilde{A}_{12}^{25} & \tilde{A}_{12}^{26} & \tilde{A}_{12}^{27} & \tilde{A}_{12}^{28} & \tilde{A}_{12}^{29} & \tilde{A}_{12}^{210} \\ \tilde{A}_{12}^{31} & \tilde{A}_{12}^{32} & \tilde{A}_{12}^{33} & \tilde{A}_{12}^{34} & \tilde{A}_{12}^{35} & \tilde{A}_{12}^{36} & \tilde{A}_{12}^{37} & \tilde{A}_{12}^{38} & \tilde{A}_{12}^{39} & \tilde{A}_{12}^{310} \end{bmatrix}$$

$$\tilde{A}_{12}^{11} = \begin{bmatrix} -\frac{M_{\Sigma 0}^{dc}}{L_a} - \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} & -\frac{M_{\Sigma\alpha}^{2\perp}}{4L_a} \\ -\frac{M_{\Sigma\alpha}^{2\perp}}{4L_a} & -\frac{M_{\Sigma 0}^{dc}}{L_a} + \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Sigma 0}^{3\parallel}}{2L_a} - \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} & \frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma 0}^{3\perp}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma 0}^{3\perp}}{2L_a} & \frac{M_{\Sigma 0}^{3\parallel}}{2L_a} - \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} \\ \frac{M_{\Sigma\beta}^{2\parallel}}{4L_a} & \frac{M_{\Sigma\beta}^{2\perp}}{4L_a} \\ \frac{M_{\Sigma\beta}^{2\perp}}{4L_a} & -\frac{M_{\Sigma\beta}^{2\parallel}}{4L_a} \\ \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} & -\frac{M_{\Sigma\beta}^{1\perp}}{4L_a} \\ \frac{M_{\Sigma\beta}^{1\perp}}{4L_a} & \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} \\ \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 - L_2))}{8C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\parallel} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\perp} C_L}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 - L_2))}{8C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\parallel} C_L}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\alpha}^{2\perp} C_L}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 - L_2))}{8C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 - L_2))}{8C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\parallel} C_L}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$C_{PU2} = \left(\frac{3L_1 + 3L_2}{2L_a + 1}\right)\left(\frac{3L_1 + 3L_2 + 12L_t}{2L_a + 8L_e}\right) - \left(\frac{(3L_1 - 3L_2)^2}{4L_a(L_a + 4L_e)}\right)$$

$$C_L = \left(\frac{3L_1 + 3L_2 + 12L_t}{2L_a + 8L_e} + 1\right)(L_a + 4L_e)$$

$$\tilde{A}_{12}^{12} = \begin{bmatrix} -\frac{M_{\Sigma 0}^{3\parallel}}{2L_a} - \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} & -\frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma 0}^{3\perp}}{2L_a} \\ \frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma 0}^{3\perp}}{2L_a} & \frac{M_{\Sigma 0}^{3\parallel}}{2L_a} - \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Sigma 0}^{dc}}{L_a} & 0 \\ 0 & -\frac{M_{\Sigma 0}^{dc}}{L_a} \\ \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} & \frac{M_{\Sigma\beta}^{1\perp}}{4L_a} \\ -\frac{M_{\Sigma\beta}^{1\perp}}{4L_a} & \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} \\ 0 & 0 \\ 0 & 0 \\ \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\parallel} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\parallel} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\alpha}^{1\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\parallel} C_L}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{13} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{2\parallel}}{4L_a} & \frac{M_{\Sigma\beta}^{2\perp}}{4L_a} \\ \frac{M_{\Sigma\beta}^{2\perp}}{4L_a} & -\frac{M_{\Sigma\beta}^{2\parallel}}{4L_a} \\ \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} & -\frac{M_{\Sigma\beta}^{1\perp}}{4L_a} \\ \frac{M_{\Sigma\beta}^{1\perp}}{4L_a} & \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} \\ -\frac{M_{\Sigma0}^{dc}}{L_a} + \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} & \frac{M_{\Sigma\alpha}^{2\perp}}{4L_a} \\ \frac{M_{\Sigma\alpha}^{2\perp}}{4L_a} & -\frac{M_{\Sigma0}^{dc}}{L_a} - \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Sigma0}^{3\parallel}}{2L_a} + \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} & -\frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma0}^{3\perp}}{2L_a} \\ \frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma0}^{3\perp}}{2L_a} & \frac{M_{\Sigma0}^{3\parallel}}{2L_a} + \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Delta\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Delta\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{14} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} & \frac{M_{\Sigma\beta}^{1\perp}}{4L_a} \\ -\frac{M_{\Sigma\beta}^{1\perp}}{4L_a} & \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Sigma0}^{3\parallel}}{2L_a} + \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} & \frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma0}^{3\perp}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma0}^{3\perp}}{2L_a} & \frac{M_{\Sigma0}^{3\parallel}}{2L_a} + \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Sigma0}^{dc}}{L_a} & 0 \\ 0 & -\frac{M_{\Sigma0}^{dc}}{L_a} \\ \frac{M_{\Delta\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Delta\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} \\ \frac{M_{\Delta\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp}C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{15}=\begin{bmatrix} -\frac{M_{\Sigma\alpha}^{1\parallel}}{L_a} & -\frac{M_{\Sigma\alpha}^{2\parallel}}{2L_a} & -\frac{M_{\Sigma\alpha}^{2\perp}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{1\perp}}{L_a} & \frac{M_{\Sigma\alpha}^{2\perp}}{2L_a} & -\frac{M_{\Sigma\alpha}^{2\parallel}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{2\parallel}}{L_a} & -\frac{M_{\Sigma\alpha}^{1\parallel}}{2L_a} & -\frac{M_{\Sigma\alpha}^{1\perp}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{2\perp}}{L_a} & \frac{M_{\Sigma\alpha}^{1\perp}}{2L_a} & -\frac{M_{\Sigma\alpha}^{1\parallel}}{2L_a} \\ -\frac{M_{\Sigma\beta}^{1\parallel}}{L_a} & -\frac{M_{\Sigma\beta}^{2\parallel}}{2L_a} & -\frac{M_{\Sigma\beta}^{2\perp}}{2L_a} \\ -\frac{M_{\Sigma\beta}^{1\perp}}{L_a} & \frac{M_{\Sigma\beta}^{2\perp}}{2L_a} & -\frac{M_{\Sigma\beta}^{2\parallel}}{2L_a} \\ -\frac{M_{\Sigma\beta}^{2\parallel}}{L_a} & -\frac{M_{\Sigma\beta}^{1\parallel}}{2L_a} & -\frac{M_{\Sigma\beta}^{1\perp}}{2L_a} \\ -\frac{M_{\Sigma\beta}^{2\perp}}{L_a} & \frac{M_{\Sigma\beta}^{1\perp}}{2L_a} & -\frac{M_{\Sigma\beta}^{1\parallel}}{2L_a} \\ \frac{3M_{\Delta0}^{dc}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma0}^{dc}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta0}^{3\parallel}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma0}^{3\parallel}C_L}{2C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta0}^{3\perp}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma0}^{3\perp}C_L}{2C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{3M_{\Delta0}^{3\parallel}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma0}^{3\parallel}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta0}^{dc}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma0}^{dc}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & 0 \\ \frac{3M_{\Delta0}^{3\perp}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma\beta}^{3\perp}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & 0 & \frac{M_{\Delta0}^{dc}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma0}^{dc}C_L}{C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{16}=\begin{bmatrix} -\frac{M_{\Delta0}^{dc}}{L_a}-\frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} & -\frac{M_{\Delta\alpha}^{2\perp}}{4L_a} \\ -\frac{M_{\Delta\alpha}^{2\perp}}{4L_a} & -\frac{M_{\Delta0}^{dc}}{L_a}+\frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Delta0}^{3\parallel}}{2L_a}-\frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} & \frac{M_{\Delta\alpha}^{1\perp}}{4L_a}-\frac{M_{\Delta0}^{3\perp}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{1\perp}}{4L_a}-\frac{M_{\Delta0}^{3\perp}}{2L_a} & \frac{M_{\Delta0}^{3\parallel}}{2L_a}-\frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} \\ \frac{M_{\Delta\beta}^{2\parallel}}{4L_a} & \frac{M_{\Delta\beta}^{2\perp}}{4L_a} \\ \frac{M_{\Delta\beta}^{2\perp}}{4L_a} & -\frac{M_{\Delta\beta}^{2\parallel}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} & -\frac{M_{\Delta\beta}^{1\perp}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\perp}}{4L_a} & \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} \\ \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\alpha}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\alpha}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\alpha}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\alpha}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma\alpha}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\alpha}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\alpha}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{17} = \begin{bmatrix} -\frac{M_{\Delta 0}^{3\parallel}}{2L_a} - \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} & -\frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} \\ \frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} & \frac{M_{\Delta 0}^{3\parallel}}{2L_a} - \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Delta 0}^{dc}}{L_a} & 0 \\ 0 & -\frac{M_{\Delta 0}^{dc}}{L_a} \\ \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} & \frac{M_{\Delta\beta}^{1\perp}}{4L_a} \\ -\frac{M_{\Delta\beta}^{1\perp}}{4L_a} & \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} \\ 0 & 0 \\ 0 & 0 \\ \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\alpha}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\perp}C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{18} = \begin{bmatrix} \frac{M_{\Delta\beta}^{2\parallel}}{4L_a} & \frac{M_{\Delta\beta}^{2\perp}}{4L_a} \\ \frac{M_{\Delta\beta}^{2\perp}}{4L_a} & -\frac{M_{\Delta\beta}^{2\parallel}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} & -\frac{M_{\Delta\beta}^{1\perp}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\perp}}{4L_a} & \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} \\ -\frac{M_{\Delta 0}^{dc}}{L_a} + \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} & \frac{M_{\Delta\alpha}^{2\perp}}{4L_a} \\ \frac{M_{\Delta\alpha}^{2\perp}}{4L_a} & -\frac{M_{\Delta 0}^{dc}}{L_a} - \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Delta 0}^{3\parallel}}{2L_a} + \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} & -\frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} \\ \frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} & \frac{M_{\Delta 0}^{3\parallel}}{2L_a} + \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} \\ \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{19} = \begin{bmatrix} \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} & \frac{M_{\Delta\beta}^{1\perp}}{4L_a} \\ -\frac{M_{\Delta\beta}^{1\perp}}{4L_a} & \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Delta 0}^{3\parallel}}{2L_a} + \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} & \frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} & \frac{M_{\Delta 0}^{3\parallel}}{2L_a} + \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Delta 0}^{dc}}{L_a} & 0 \\ 0 & -\frac{M_{\Delta 0}^{dc}}{L_a} \\ \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{110} = \begin{bmatrix} -\frac{M_{\Delta\alpha}^{1\parallel}}{L_a} & -\frac{M_{\Delta\alpha}^{2\parallel}}{2L_a} & -\frac{M_{\Delta\alpha}^{2\perp}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{1\perp}}{L_a} & \frac{M_{\Delta\alpha}^{2\perp}}{2L_a} & -\frac{M_{\Delta\alpha}^{2\parallel}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{2\parallel}}{L_a} & -\frac{M_{\Delta\alpha}^{1\parallel}}{2L_a} & -\frac{M_{\Delta\alpha}^{1\perp}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{2\perp}}{L_a} & \frac{M_{\Delta\alpha}^{1\perp}}{2L_a} & -\frac{M_{\Delta\alpha}^{1\parallel}}{2L_a} \\ -\frac{M_{\Delta\beta}^{1\parallel}}{L_a} & -\frac{M_{\Delta\beta}^{2\parallel}}{2L_a} & -\frac{M_{\Delta\beta}^{2\perp}}{2L_a} \\ -\frac{M_{\Delta\beta}^{1\perp}}{L_a} & \frac{M_{\Delta\beta}^{2\perp}}{2L_a} & -\frac{M_{\Delta\beta}^{2\parallel}}{2L_a} \\ -\frac{M_{\Delta\beta}^{2\parallel}}{L_a} & -\frac{M_{\Delta\beta}^{1\parallel}}{2L_a} & -\frac{M_{\Delta\beta}^{1\perp}}{2L_a} \\ -\frac{M_{\Delta\beta}^{2\perp}}{L_a} & \frac{M_{\Delta\beta}^{1\perp}}{2L_a} & -\frac{M_{\Delta\beta}^{1\parallel}}{2L_a} \\ \frac{3M_{\Sigma 0}^{dc}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta 0}^{dc}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma 0}^{3\parallel}(3(L_1-L_2))}{2C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta 0}^{3\parallel}C_L}{2C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma 0}^{3\perp}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta 0}^{3\perp}C_L}{2C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{3M_{\Sigma 0}^{3\parallel}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta 0}^{3\parallel}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma 0}^{dc}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta 0}^{dc}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & 0 \\ \frac{3M_{\Sigma 0}^{3\perp}(L_1-L_2)}{2C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{3\perp}C_L}{C_{PU}\ (L_a^2+4L_aL_e)} & 0 & \frac{M_{\Sigma 0}^{dc}(3(L_1-L_2))}{2C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta 0}^{dc}C_L}{C_{PU}\ (L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{21} = \begin{bmatrix} -\frac{M_{\Delta 0}^{dc}(4L_e+6L_m)}{C_{LL2}} - \frac{M_{\Delta\alpha}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Delta\alpha}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Delta 0}^{dc}(4L_e+6L_m)}{C_{LL2}} + \frac{M_{\Delta\alpha}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Delta 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} \\ -\frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} & \frac{M_{\Delta 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\beta}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \end{bmatrix}$$

$$\tilde{A}_{12}^{22} = \begin{bmatrix} -\frac{M_{\Delta 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} \\ \frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} & \frac{M_{\Delta 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Delta 0}^{dc}(4L_e+6L_m)}{C_{LL2}} & 0 \\ 0 & -\frac{M_{\Delta 0}^{dc}(4L_e+6L_m)}{C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{A}_{12}^{23} = \begin{bmatrix} \frac{M_{\Delta\beta}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\beta}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel}(2L_e+3L_m)}{2C_{LL}} \\ \frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL}} & \frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL}} \\ -\frac{M_{\Delta0}^{dc}(4L_e+6L_m)}{C_{LL2}}+\frac{M_{\Delta\alpha}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Delta\alpha}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Delta0}^{dc}(4L_e+6L_m)}{C_{LL}}-\frac{M_{\Delta\alpha}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Delta0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}}+\frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL}}-\frac{M_{\Delta0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} \\ \frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL}}-\frac{M_{\Delta0}^{3\perp}(2L_e+3L_m)}{C_{LL}} & \frac{M_{\Delta0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}}+\frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \end{bmatrix}$$

$$\tilde{A}_{12}^{24} = \begin{bmatrix} \frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Delta0}^{3\parallel}(2L_e+3L_m)}{C_{LL}}+\frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}}-\frac{M_{\Delta0}^{3\perp}(2L_e+3L_m)}{C_{LL}} \\ -\frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL}}-\frac{M_{\Delta0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} & \frac{M_{\Delta0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}}+\frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Delta0}^{dc}(4L_e+6L_m)}{C_{LL}} & 0 \\ 0 & -\frac{M_{\Delta0}^{dc}(4L_e+6L_m)}{C_{LL}} \end{bmatrix}$$

$$\tilde{A}_{12}^{25}=\begin{bmatrix}
-\frac{M_{\Delta\alpha}^{1\parallel}(4L_e+6L_m)}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\parallel}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\perp}(2L_e+3L_m)}{C_{LL2}}\\
-\frac{M_{\Delta\alpha}^{1\perp}(4L_e+6L_m)}{C_{LL2}} & \frac{M_{\Delta\alpha}^{2\perp}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\parallel}(2L_e+3L_m)}{C_{LL2}}\\
-\frac{M_{\Delta\alpha}^{2\parallel}(4L_e+6L_m)}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{C_{LL2}}\\
-\frac{M_{\Delta\alpha}^{2\perp}(4L_e+6L_m)}{C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\parallel}(2L_e+3L_m)}{C_{LL2}}\\
-\frac{M_{\Delta\beta}^{1\parallel}(4L_e+6L_m)}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\perp}(2L_e+3L_m)}{C_{LL2}}\\
-\frac{M_{\Delta\beta}^{1\perp}(4L_e+6L_m)}{C_{LL2}} & \frac{M_{\Delta\beta}^{2\perp}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel}(2L_e+3L_m)}{C_{LL2}}\\
-\frac{M_{\Delta\beta}^{2\parallel}(4L_e+6L_m)}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{C_{LL2}}\\
-\frac{M_{\Delta\beta}^{2\perp}(4L_e+6L_m)}{C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\parallel}(2L_e+3L_m)}{C_{LL2}}
\end{bmatrix}$$

$$\tilde{A}_{12}^{26}=\begin{bmatrix}
-\frac{M_{\Sigma0}^{dc}(4L_e+6L_m)}{C_{LL2}}-\frac{M_{\Sigma\alpha}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\perp}(2L_e+3L_m)}{2C_{LL2}}\\
-\frac{M_{\Sigma\alpha}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Sigma0}^{dc}(4L_e+6L_m)}{C_{LL2}}+\frac{M_{\Sigma\alpha}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}}\\
-\frac{M_{\Sigma0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}}-\frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}}-\frac{M_{\Sigma0}^{3\perp}(2L_e+3L_m)}{C_{LL2}}\\
-\frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}}-\frac{M_{\Sigma0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} & \frac{M_{\Sigma0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}}-\frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}}\\
\frac{M_{\Sigma\beta}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{2\perp}(2L_e+3L_m)}{2C_{LL2}}\\
\frac{M_{\Sigma\beta}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}}\\
\frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}}\\
\frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}}
\end{bmatrix}$$

$$\tilde{A}_{12}^{27} = \begin{bmatrix} -\frac{M_{\Sigma 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} \\ \frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} & \frac{M_{\Sigma 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Sigma 0}^{dc}(4L_e+6L_m)}{C_{LL2}} & 0 \\ 0 & -\frac{M_{\Sigma 0}^{dc}(4L_e+6L_m)}{C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{A}_{12}^{28} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL}} \\ \frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL}} & \frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL}} \\ -\frac{M_{\Sigma 0}^{dc}(4L_e+6L_m)}{C_{LL2}} + \frac{M_{\Sigma\alpha}^{2\parallel}(2L_e+3L_m)}{2C_{LL}} & \frac{M_{\Sigma\alpha}^{2\perp}(2L_e+3L_m)}{2C_{LL}} \\ \frac{M_{\Sigma\alpha}^{2\perp}(2L_e+3L_m)}{2C_{LL2}} & -\frac{M_{\Sigma 0}^{dc}(4L_e+6L_m)}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{2\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Sigma 0}^{3\parallel}(2L_e+3L_m)}{C_{LL}} + \frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL}} - \frac{M_{\Sigma 0}^{3\perp}(2L_e+3L_m)}{C_{LL}} \\ \frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp}(2L_e+3L_m)}{C_{LL}} & \frac{M_{\Sigma 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}} + \frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \end{bmatrix}$$

$$\tilde{A}_{12}^{29} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Sigma 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}}+\frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} & \frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}}-\frac{M_{\Sigma 0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} \\ -\frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{2C_{LL2}}-\frac{M_{\Sigma 0}^{3\perp}(2L_e+3L_m)}{C_{LL2}} & \frac{M_{\Sigma 0}^{3\parallel}(2L_e+3L_m)}{C_{LL2}}+\frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{2C_{LL2}} \\ -\frac{M_{\Sigma 0}^{dc}(4L_e+6L_m)}{C_{LL2}} & 0 \\ 0 & -\frac{M_{\Sigma 0}^{dc}(4L_e+6L_m)}{C_{LL2}} \end{bmatrix}$$

$$\tilde{A}_{12}^{210} = \begin{bmatrix} -\frac{M_{\Sigma\alpha}^{1\parallel}(4L_e+6L_m)}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\parallel}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\perp}(2L_e+3L_m)}{C_{LL2}} \\ -\frac{M_{\Sigma\alpha}^{1\perp}(4L_e+6L_m)}{C_{LL2}} & \frac{M_{\Sigma\alpha}^{2\perp}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\parallel}(2L_e+3L_m)}{C_{LL2}} \\ -\frac{M_{\Sigma\alpha}^{2\parallel}(4L_e+6L_m)}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{C_{LL}} \\ -\frac{M_{\Sigma\alpha}^{2\perp}(4L_e+6L_m)}{C_{LL2}} & \frac{M_{\Sigma\alpha}^{1\perp}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\parallel}(2L_e+3L_m)}{C_{LL2}} \\ -\frac{M_{\Sigma\beta}^{1\parallel}(4L_e+6L_m)}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\perp}(2L_e+3L_m)}{C_{LL2}} \\ -\frac{M_{\Sigma\beta}^{1\perp}(4L_e+6L_m)}{C_{LL2}} & \frac{M_{\Sigma\beta}^{2\perp}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel}(2L_e+3L_m)}{C_{LL2}} \\ -\frac{M_{\Sigma\beta}^{2\parallel}(4L_e+6L_m)}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{C_{LL}} \\ -\frac{M_{\Sigma\beta}^{2\perp}(4L_e+6L_m)}{C_{LL2}} & \frac{M_{\Sigma\beta}^{1\perp}(2L_e+3L_m)}{C_{LL}} & -\frac{M_{\Sigma\beta}^{1\parallel}(2L_e+3L_m)}{C_{LL}} \end{bmatrix}$$

$$\tilde{A}_{12}^{31} = \begin{bmatrix} -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL}} - \frac{M_{\Delta\alpha}^{2\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta\alpha}^{2\perp} L_s}{2C_{LL}} & -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL}} + \frac{M_{\Delta\alpha}^{2\parallel} L_s}{2C_{LL}} \\ -\frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} \\ -\frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} & \frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{2\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{2\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{2\perp} L_s}{2C_{LL}} & -\frac{M_{\Delta\beta}^{2\parallel} L_s}{2C_{LL}} \\ \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL}} & -\frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL}} \\ \frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{3M_{\Sigma\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{3M_{\Sigma\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Sigma\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{3M_{\Sigma\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Sigma\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} & \frac{3M_{\Sigma\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{32} = \begin{bmatrix} -\frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL}} \\ \frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} & \frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL}} & 0 \\ 0 & -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \\ \frac{3M_{\Sigma\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{3M_{\Sigma\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Sigma\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} & \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{3M_{\Sigma\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Sigma\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{3M_{\Sigma\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{33} = \begin{bmatrix} \frac{M_{\Delta\beta}^{2\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{2\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{2\perp} L_s}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} \\ -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{2\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{2\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\alpha}^{2\perp} L_s}{2C_{LL2}} & -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} - \frac{M_{\Delta\alpha}^{2\parallel} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} \\ \frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL}} & \frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{3M_{\Sigma\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\beta}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{3M_{\Sigma\beta}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\beta}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{34} = \begin{bmatrix} \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} \\ -\frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} & \frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} & 0 \\ 0 & -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} \\ \frac{3M_{\Sigma\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\beta}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{3M_{\Sigma\beta}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\beta}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{35} = \begin{bmatrix} -\frac{2M_{\Delta\alpha}^{1\parallel}L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\parallel}L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\perp}L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\alpha}^{1\perp}L_s}{C_{LL2}} & \frac{M_{\Delta\alpha}^{2\perp}L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\parallel}L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\alpha}^{2\parallel}L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\parallel}L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\perp}L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\alpha}^{2\perp}L_s}{C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp}L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\parallel}L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\beta}^{1\parallel}L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel}L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\perp}L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\beta}^{1\perp}L_s}{C_{LL2}} & \frac{M_{\Delta\beta}^{2\perp}L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel}L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\beta}^{2\parallel}L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\parallel}L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\perp}L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\beta}^{2\perp}L_s}{C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp}L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\parallel}L_s}{C_{LL}} \\ \frac{3M_{\Sigma0}^{dc}(L_1-L_2)}{C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta0}^{dc}(3L_1+3L_2+2L_a)}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{3M_{\Sigma0}^{3\parallel}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta0}^{3\parallel}(3L_1+3L_2+2L_a)}{2C_{PU}\ (L_a^2+4L_aL_e)} & \frac{3M_{\Sigma0}^{3\perp}(L_1-L_2)}{2C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta0}^{3\perp}(3L_1+3L_2+2L_a)}{2C_{PU}\ (L_a^2+4L_aL_e)} \\ \frac{3M_{\Sigma0}^{3\parallel}(L_1-L_2)}{C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta0}^{3\parallel}(3L_1+3L_2+2L_a)}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{3M_{\Sigma0}^{dc}(L_1-L_2)}{C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta0}^{dc}(3L_1+3L_2+2L_a)}{C_{PU}\ (L_a^2+4L_aL_e)} & 0 \\ \frac{3M_{\Sigma0}^{3\perp}(L_1-L_2)}{C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{3\perp}(3L_1+3L_2+2L_a)}{C_{PU}\ (L_a^2+4L_aL_e)} & 0 & \frac{3M_{\Sigma0}^{dc}(L_1-L_2)}{C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta0}^{dc}(3L_1+3L_2+2L_a)}{C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{36} = \begin{bmatrix} -\frac{2M_{\Sigma0}^{dc}L_s}{C_{LL}} - \frac{M_{\Sigma\alpha}^{2\parallel}L_s}{2C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\perp}L_s}{2C_{LL2}} \\ -\frac{M_{\Sigma\alpha}^{2\perp}L_s}{2C_{LL}} & -\frac{2M_{\Sigma0}^{dc}L_s}{C_{LL}} + \frac{M_{\Sigma\alpha}^{2\parallel}L_s}{2C_{LL2}} \\ -\frac{M_{\Sigma0}^{3\parallel}L_s}{C_{LL}} - \frac{M_{\Sigma\alpha}^{1\parallel}L_s}{2C_{LL}} & \frac{M_{\Sigma\alpha}^{1\perp}L_s}{2C_{LL}} - \frac{M_{\Sigma0}^{3\perp}L_s}{C_{LL}} \\ -\frac{M_{\Sigma\alpha}^{1\perp}L_s}{2C_{LL2}} - \frac{M_{\Sigma0}^{3\perp}L_s}{C_{LL}} & \frac{M_{\Sigma0}^{3\parallel}L_s}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{1\parallel}L_s}{2C_{LL}} \\ \frac{M_{\Sigma\beta}^{2\parallel}L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{2\perp}L_s}{2C_{LL}} \\ \frac{M_{\Sigma\beta}^{2\perp}L_s}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel}L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\parallel}L_s}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\perp}L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\perp}L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{1\parallel}L_s}{2C_{LL}} \\ \frac{3M_{\Delta\alpha}^{1\parallel}(L_1-L_2)}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1+L_2)+2L_a)}{4C_{PU}\ (L_a^2+4L_aL_e)} & \frac{3M_{\Delta\alpha}^{1\perp}(L_1-L_2)}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1+L_2)+2L_a)}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{3M_{\Delta\alpha}^{2\parallel}(L_1-L_2)}{4C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1+L_2)+2L_a)}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{3M_{\Delta\alpha}^{2\perp}(L_1-L_2)}{4C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1+L_2)+2L_a)}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{3M_{\Delta\alpha}^{2\perp}(L_1-L_2)}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1+L_2)+2L_a)}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{3M_{\Delta\alpha}^{2\parallel}(L_1-L_2)}{4C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1+L_2)+2L_a)}{4C_{PU}\ (L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{37} = \begin{bmatrix} -\frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL2}} \\ \frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL}} & \frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL}} - \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL}} \\ -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL2}} & 0 \\ 0 & -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL}} \\ -\frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \\ \frac{3M_{\Delta\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{3M_{\Delta\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{38} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{2\parallel} L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{2\perp} L_s}{2C_{LL}} \\ \frac{M_{\Sigma\beta}^{2\perp} L_s}{2C_{LL}} & -\frac{M_{\Sigma\beta}^{2\parallel} L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL}} \\ \frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} \\ -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL2}} + \frac{M_{\Sigma\alpha}^{2\parallel} L_s}{2C_{LL}} & \frac{M_{\Sigma\alpha}^{2\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\alpha}^{2\perp} L_s}{2C_{LL}} & -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL}} - \frac{M_{\Sigma\alpha}^{2\parallel} L_s}{2C_{LL2}} \\ -\frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL}} \\ \frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL}} & \frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{3M_{\Delta\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\beta}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{3M_{\Delta\beta}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\beta}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{39} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL}} \\ -\frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL}} + \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL2}} \\ -\frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL}} & \frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL}} + \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL}} \\ -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL}} & 0 \\ 0 & -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL}} \\ \frac{3M_{\Delta\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\beta}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{3M_{\Delta\beta}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\beta}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{310} = \begin{bmatrix} -\frac{2M_{\Sigma\alpha}^{1\parallel} L_s}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\parallel} L_s}{C_{LL}} & -\frac{M_{\Sigma\alpha}^{2\perp} L_s}{C_{LL}} \\ -\frac{2M_{\Sigma\alpha}^{1\perp} L_s}{C_{LL2}} & \frac{M_{\Sigma\alpha}^{2\perp} L_s}{C_{LL}} & -\frac{M_{\Sigma\alpha}^{2\parallel} L_s}{C_{LL}} \\ -\frac{2M_{\Sigma\alpha}^{2\parallel} L_s}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\parallel} L_s}{C_{LL}} & -\frac{M_{\Sigma\alpha}^{1\perp} L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\alpha}^{2\perp} L_s}{C_{LL}} & \frac{M_{\Sigma\alpha}^{1\perp} L_s}{C_{LL}} & -\frac{M_{\Sigma\alpha}^{1\parallel} L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\beta}^{1\parallel} L_s}{C_{LL}} & -\frac{M_{\Sigma\beta}^{2\parallel} L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\perp} L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\beta}^{1\perp} L_s}{C_{LL}} & \frac{M_{\Sigma\beta}^{2\perp} L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel} L_s}{C_{LL}} \\ -\frac{2M_{\Sigma\beta}^{2\parallel} L_s}{C_{LL}} & -\frac{M_{\Sigma\beta}^{1\parallel} L_s}{C_{LL}} & -\frac{M_{\Sigma\beta}^{1\perp} L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\beta}^{2\perp} L_s}{C_{LL2}} & \frac{M_{\Sigma\beta}^{1\perp} L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\parallel} L_s}{C_{LL}} \\ \frac{3M_{\Delta 0}^{dc}(L_1 - L_2)}{C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma 0}^{dc}(3L_1 + 3L_2 + 2L_a)}{C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta 0}^{3\parallel}(L_1 - L_2)}{2C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma 0}^{3\parallel}(3L_1 + 3L_2 + 2L_a)}{2C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta 0}^{3\perp}(L_1 - L_2)}{2C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma 0}^{3\perp}(3L_1 + 3L_2 + 2L_a)}{2C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta 0}^{3\parallel}(L_1 - L_2)}{C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma 0}^{3\parallel}(3L_1 + 3L_2 + 2L_a)}{C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta 0}^{dc}(L_1 - L_2)}{C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma 0}^{dc}(3L_1 + 3L_2 + 2L_a)}{C_{PU2}(L_a^2 + 4L_aL_e)} & 0 \\ \frac{3M_{\Delta 0}^{3\perp}(L_1 - L_2)}{C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{3\perp}(3L_1 + 3L_2 + 2L_a)}{C_{PU}\ (L_a^2 + 4L_aL_e)} & 0 & \frac{3M_{\Delta 0}^{dc}(L_1 - L_2)}{C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma 0}^{dc}(3L_1 + 3L_2 + 2L_a)}{C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{21} = \begin{bmatrix} \tilde{A}_{21}^{11} & \tilde{A}_{21}^{12} & \tilde{A}_{21}^{13} & \tilde{A}_{21}^{14} & \tilde{A}_{21}^{15} & \tilde{A}_{21}^{16} & \tilde{A}_{21}^{17} & \tilde{A}_{21}^{18} & \tilde{A}_{21}^{19} & \tilde{A}_{21}^{110} & \tilde{A}_{21}^{111} & \tilde{A}_{21}^{112} & \tilde{A}_{21}^{113} & \tilde{A}_{21}^{114} \\ \tilde{A}_{21}^{21} & \tilde{A}_{21}^{22} & \tilde{A}_{21}^{23} & \tilde{A}_{21}^{24} & \tilde{A}_{21}^{25} & \tilde{A}_{21}^{26} & \tilde{A}_{21}^{27} & \tilde{A}_{21}^{28} & \tilde{A}_{21}^{29} & \tilde{A}_{21}^{210} & \tilde{A}_{21}^{211} & \tilde{A}_{21}^{212} & \tilde{A}_{21}^{213} & \tilde{A}_{21}^{214} \end{bmatrix}$$

$$\tilde{A}_{21}^{11} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{11} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{13} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{14} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{15} = \begin{bmatrix} \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & 0 \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) & 0 & \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{16} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta 0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta 0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{17} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{18} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta 0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta 0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{19} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{110} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{111} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{112} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{113} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{114} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Delta0}^{3\parallel}}{2}+M_{\Sigma0}^{3\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Delta0}^{3\parallel}}{2}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Delta0}^{3\perp}}{2}+M_{\Sigma0}^{3\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Delta0}^{3\perp}}{2}-M_{\Sigma0}^{3\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) & 0 & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{21} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{22} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} + 2M_{\Delta 0}^{dc}\right) - \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} - 2M_{\Delta 0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} + 2M_{\Delta 0}^{dc}\right) - \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} - 2M_{\Delta 0}^{dc}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{23} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\parallel} - \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\parallel} - \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma 0}^{3\parallel} - \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma 0}^{3\parallel} - \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) - \left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{24} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{25} = \begin{bmatrix} \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & 0 \\ \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) & 0 & \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{26} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$
\tilde{A}_{21}^{27} = \begin{bmatrix}
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) & 0 \\
0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\
0 & 0 \\
0 & 0 \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)
\end{bmatrix}
$$

$$
\tilde{A}_{21}^{28} = \begin{bmatrix}
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta 0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)
\end{bmatrix}
$$

$$
\tilde{A}_{21}^{29} = \begin{bmatrix}
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\
0 & 0 \\
0 & 0 \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) & 0 \\
0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)
\end{bmatrix}
$$

$$\tilde{A}_{21}^{210} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{211} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) - \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel} + \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc} + 2M_{\Delta 0}^{dc}\right) - \left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc} - 2M_{\Delta 0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc} + 2M_{\Delta 0}^{dc}\right) + \left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc} - 2M_{\Delta 0}^{dc}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{212} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc} - \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2} + \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} + 2M_{\Delta 0}^{dc} + \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma 0}^{dc} + \frac{M_{\Sigma\alpha}^{2\parallel}}{2} - 2M_{\Delta 0}^{dc} - \frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\parallel} - \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\parallel} - \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp} + \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} + M_{\Delta 0}^{3\perp} - \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma 0}^{3\perp} - \frac{M_{\Sigma\alpha}^{1\perp}}{2} - M_{\Delta 0}^{3\perp} + \frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma 0}^{3\parallel} - \frac{M_{\Sigma\alpha}^{1\parallel}}{2} - M_{\Delta 0}^{3\parallel} - \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma 0}^{3\parallel} - \frac{M_{\Sigma\alpha}^{1\parallel}}{2} + M_{\Delta 0}^{3\parallel} + \frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2} + \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2} - \frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} + \frac{M_{\Delta\beta}^{1\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2} - \frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\alpha}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2} + \frac{M_{\Delta\beta}^{2\perp}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2} - \frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} + \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) + \left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2} - \frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{213} = \begin{bmatrix}
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\
0 & 0 \\
0 & 0 \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{2\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\
0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)
\end{bmatrix}$$

$$\tilde{A}_{21}^{214} = \begin{bmatrix}
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & 0 \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) & 0 & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right)
\end{bmatrix}$$

$$\tilde{A}_{22} = \begin{bmatrix}
0 & -\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\omega & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3\omega \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\omega & 0
\end{bmatrix}$$

$$\widetilde{N} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3L_2 + L_a + 4L_e + 6L_t}{2C_{LL}} & \frac{3L_1 - 3L_2}{2L_a(L_a + 4L_e)C_{PU2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{L_s + (L_a C_{PU1})} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{L_s + (L_a C_{PU1})} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{L_s + (L_a C_{PU1})} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_s + (L_a C_{PU1})} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{L_a}{C_{LL2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{L_a}{C_{LL}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{L_a}{C_{LL}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{L_a}{C_{LL2}} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{3L_2 + L_a}{C_{LL}} & \frac{\frac{-3L_1 - 3L_2}{L_a + 2}}{(L_a + 4L_e)C_{PU2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

# Appendix D

# DC/DC DPM Matrices

The following matrices are for the DC/DC DPM given by Equation (3.61). To reduce the size of the given sub-matrices, several constants are defined as $C_{xxy}$ , where "xx" describes the type of constant, i.e., "$RL$" for a constant with units of resistance times inductance, and "y" takes values of 1,2,3... depending on how many variables share the same unit. This model includes second and third harmonic components for the modulating signals, but due to time constraints they were not investigated in the thesis' body, and are always set to 0 for simulation purposes. The model in equation Equation (3.61) is repeated here for reference as:

$$\frac{d}{dt}\begin{bmatrix}\widetilde{\boldsymbol{X}}^{*}_{ir}\\ \widetilde{\boldsymbol{X}}^{*}_{vr}\end{bmatrix} = \begin{bmatrix}\widetilde{A}_{11r} & \widetilde{A}_{12r}\\ \widetilde{A}_{21r} & 0_{22x22}\end{bmatrix}\begin{bmatrix}\widetilde{\boldsymbol{X}}^{*}_{ir}\\ \widetilde{\boldsymbol{X}}^{*}_{vr}\end{bmatrix} + \begin{bmatrix}\widetilde{N}_{r}\\ 0_{22x5}\end{bmatrix}\left[\widetilde{\boldsymbol{W}}^{*}_{r}\right], \tag{D.1}$$

$$\tilde{A}_{11r} = \begin{bmatrix}\tilde{A}^{11}_{11r} & \tilde{A}^{12}_{11r} & [0]_{11x4} & [0]_{15x4} & \tilde{A}^{15}_{11r}\\ [0]_{22x8} & \tilde{A}^{22}_{11r} & \tilde{A}^{23}_{11} & \tilde{A}^{24}_{11r} & \tilde{A}^{25}_{11r}\end{bmatrix}$$

$$\tilde{A}_{11r}^{11} = \begin{bmatrix} -\frac{R_a}{L_a} & -\omega & 0 & 0 & 0 & 0 & 0 & 0 \\ \omega & -\frac{R_a}{L_a} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{R_a}{L_a} & -2\omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\omega & -\frac{R_a}{L_a} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{R_a}{L_a} & -\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega & -\frac{R_a}{L_a} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{R_a}{L_a} & -2\omega \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\omega & -\frac{R_a}{L_a} \end{bmatrix}$$

$$\tilde{A}_{11}^{12} = \begin{bmatrix} & [0]_{8x3} & \\ \frac{-C_{RL}}{2C_{LL1}} & 0 & 0 \\ 0 & -\frac{C_{RL1}}{2C_{LL1}} & -3\omega \\ 0 & 3\omega & -\frac{C_{RL1}}{2C_{LL1}} \end{bmatrix} \tilde{A}_{11}^{22} = \begin{bmatrix} & [0]_{8x3} & \\ \frac{C_{RL4}}{C_{LL1}} & 0 & 0 \\ 0 & \frac{C_{RL4}}{C_{LL1}} & 0 \\ 0 & 0 & \frac{C_{RL4}}{C_{LL1}} \end{bmatrix}$$

$$\tilde{A}_{11}^{12} = \begin{bmatrix} & [0]_{8x3} & \\ \frac{-C_{RL}}{2C_{LL1}} & 0 & 0 \\ 0 & -\frac{C_{RL1}}{2C_{LL1}} & -3\omega \\ 0 & 3\omega & -\frac{C_{RL1}}{2C_{LL1}} \end{bmatrix} \tilde{A}_{11}^{22} = \begin{bmatrix} & [0]_{8x3} & \\ \frac{C_{RL4}}{C_{LL1}} & 0 & 0 \\ 0 & \frac{C_{RL4}}{C_{LL1}} & 0 \\ 0 & 0 & \frac{C_{RL4}}{C_{LL1}} \end{bmatrix}$$

$C_{RL1} = 9L_1R_2 + 9L_2R_1 + 3L_1R_a + 3L_aR_1 + 3L_2R_a + 3L_aR_2 + 12L_eR_1 + 12L_eR_2 + 18L_tR_1 + 18L_tR_2 + 2L_aR_a + 8L_eR_a + 12L_tR_t$

$C_{LL1} = L_a^2 + 9L_1L_2 + 9L_1L_t + 9L_2L_t + 6L_1L_e + 6L_2L_e + 6L_aL_t + 4L_aL_e + 3L_1L_a + 9L_2L_a$

$C_{RL4} = 3(3L_1R_2 - 3L_2R_1 + L_1R_a - L_aR_1 - L_2R_a + L_aR_2)$

$$\tilde{A}_{11}^{23} = \begin{bmatrix} \frac{-R_a - 4R_w}{L_a + 4L_e + 6L_m} & -\omega & 0 & 0 \\ \omega & \frac{-R_a - 4R_w}{L_a + 4L_e + 6L_m} & 0 & 0 \\ 0 & 0 & \frac{-R_a - 4R_w}{L_a + 4L_e + 6L_m} & -2\omega \\ 0 & 0 & 2\omega & \frac{-R_a - 4R_w}{L_a + 4L_e + 6L_m} \\ & & [0]_{7x4} & \end{bmatrix}$$

$$\tilde{A}_{11}^{24}=\begin{bmatrix}\frac{-R_a-4R_w}{L_a+4L_e+6L_m} & -\omega & 0 & 0\\ \omega & \frac{-R_a-4R_w}{L_a+4L_e+6L_m} & 0 & 0\\ 0 & 0 & \frac{-R_a-4R_w}{L_a+4L_e+6L_m} & -2\omega\\ 0 & 0 & 2\omega & \frac{-R_a-4R_w}{L_a+4L_e+6L_m}\end{bmatrix}$$

$$[0]_{3x4}$$

$$\tilde{A}_{11}^{17}=\begin{bmatrix}[0]_{8x3}\\ \frac{C_{RL2}}{4C_{LL1}} & 0 & 0\\ 0 & \frac{C_{RL2}}{4C_{LL1}} & 0\\ 0 & 0 & \frac{C_{RL2}}{4C_{LL}}\end{bmatrix}\tilde{A}_{11}^{22}=\begin{bmatrix}[0]_{8x3}\\ -\frac{C_{RL3}}{2C_{LL1}} & 0 & 0\\ 0 & -\frac{C_{RL3}}{2C_{LL1}} & -3\omega\\ 0 & 3\omega & -\frac{C_{RL3}}{2C_{LL1}}\end{bmatrix}$$

$C_{RL2}=9L_1R_2-9L_2R_1+3L_1R_a-3L_aR_1-3L_2R_a+3L_aR_2-12L_eR_1+12L_eR_2+18L_1R_t-18L_tR_1-18L_2R_t+18L_tR_2+12L_1R_w-12L_2R_w$

$C_{RL}=9L_1R_2+9L_2R_1+3L_1R_a+3L_aR_1+3L_2R_a+3L_aR_2+18L_1R_t+18L_2R_t+12L_1R_w+12L_2R_w+2L_aR_a+12L_aR_t+8L_aR_w$

$$\tilde{A}_{12r}=\begin{bmatrix}\tilde{A}_{12r}^{11} & \tilde{A}_{12r}^{12} & \tilde{A}_{12r}^{13} & \tilde{A}_{12r}^{14} & \tilde{A}_{12r}^{15} & \tilde{A}_{12r}^{16} & \tilde{A}_{12r}^{17} & \tilde{A}_{12r}^{18} & \tilde{A}_{12r}^{19} & \tilde{A}_{12r}^{110}\\ \tilde{A}_{12r}^{21} & \tilde{A}_{12r}^{22} & \tilde{A}_{12r}^{23} & \tilde{A}_{12r}^{24} & \tilde{A}_{12r}^{25} & \tilde{A}_{12r}^{26} & \tilde{A}_{12r}^{27} & \tilde{A}_{12r}^{28} & \tilde{A}_{12r}^{29} & \tilde{A}_{12r}^{210}\end{bmatrix}$$

$$\tilde{A}_{12r}^{11}=\begin{bmatrix}-\frac{M_{\Sigma 0}^{dc}}{L_a}-\frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} & -\frac{M_{\Sigma\alpha}^{2\perp}}{4L_a}\\ -\frac{M_{\Sigma\alpha}^{2\perp}}{4L_a} & -\frac{M_{\Sigma 0}^{dc}}{L_a}+\frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a}\\ -\frac{M_{\Sigma 0}^{3\parallel}}{2L_a}-\frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} & \frac{M_{\Sigma\alpha}^{1\perp}}{4L_a}-\frac{M_{\Sigma 0}^{3\perp}}{2L_a}\\ -\frac{M_{\Sigma\alpha}^{1\perp}}{4L_a}-\frac{M_{\Sigma 0}^{3\perp}}{2L_a} & \frac{M_{\Sigma 0}^{3\parallel}}{2L_a}-\frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a}\\ \frac{M_{\Sigma\beta}^{2\parallel}}{4L_a} & \frac{M_{\Sigma\beta}^{2\perp}}{4L_a}\\ \frac{M_{\Sigma\beta}^{2\perp}}{4L_a} & -\frac{M_{\Sigma\beta}^{2\parallel}}{4L_a}\\ \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} & -\frac{M_{\Sigma\beta}^{1\perp}}{4L_a}\\ \frac{M_{\Sigma\beta}^{1\perp}}{4L_a} & \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a}\\ \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma\alpha}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\alpha}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma\alpha}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)}\\ \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma\alpha}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\alpha}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}\\ \frac{M_{\Delta\alpha}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma\alpha}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma\alpha}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)}\end{bmatrix}$$

$$C_{PU2} = \left(\frac{3L_1 + 3L_2}{2L_a + 1}\right)\left(\frac{3L_1 + 3L_2 + 12L_t}{2L_a + 8L_e}\right) - \left(\frac{(3L_1 - 3L_2)^2}{4L_a(L_a + 4L_e)}\right)$$

$$C_L = \left(\frac{3L_1 + 3L_2 + 12L_t}{2L_a + 8L_e} + 1\right)(L_a + 4L_e)$$

$$\tilde{A}_{12r}^{12} = \begin{bmatrix} -\frac{M_{\Sigma 0}^{3\parallel}}{2L_a} - \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} & -\frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma 0}^{3\perp}}{2L_a} \\ \frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma 0}^{3\perp}}{2L_a} & \frac{M_{\Sigma 0}^{3\parallel}}{2L_a} - \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Sigma 0}^{dc}}{L_a} & 0 \\ 0 & -\frac{M_{\Sigma 0}^{dc}}{L_a} \\ \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} & \frac{M_{\Sigma\beta}^{1\perp}}{4L_a} \\ -\frac{M_{\Sigma\beta}^{1\perp}}{4L_a} & \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} \\ 0 & 0 \\ 0 & 0 \\ \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\parallel} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\parallel} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\alpha}^{1\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\perp} C_L}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 - L_2))}{8C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\parallel} C_L}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{13} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{2\parallel}}{4L_a} & \frac{M_{\Sigma\beta}^{2\perp}}{4L_a} \\ \frac{M_{\Sigma\beta}^{2\perp}}{4L_a} & -\frac{M_{\Sigma\beta}^{2\parallel}}{4L_a} \\ \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} & -\frac{M_{\Sigma\beta}^{1\perp}}{4L_a} \\ \frac{M_{\Sigma\beta}^{1\perp}}{4L_a} & \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} \\ -\frac{M_{\Sigma 0}^{dc}}{L_a} + \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} & \frac{M_{\Sigma\alpha}^{2\perp}}{4L_a} \\ \frac{M_{\Sigma\alpha}^{2\perp}}{4L_a} & -\frac{M_{\Sigma 0}^{dc}}{L_a} - \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Sigma 0}^{3\parallel}}{2L_a} + \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} & -\frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma 0}^{3\perp}}{2L_a} \\ \frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma 0}^{3\perp}}{2L_a} & \frac{M_{\Sigma 0}^{3\parallel}}{2L_a} + \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\beta}^{1\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{M_{\Delta\beta}^{2\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\beta}^{2\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{M_{\Delta\beta}^{2\perp}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\beta}^{2\parallel}(3(L_1 - L_2))}{8C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel} C_L}{4C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{14} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} & \frac{M_{\Sigma\beta}^{1\perp}}{4L_a} \\ -\frac{M_{\Sigma\beta}^{1\perp}}{4L_a} & \frac{M_{\Sigma\beta}^{1\parallel}}{4L_a} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Sigma0}^{3\parallel}}{2L_a} + \frac{M_{\Sigma\alpha}^{1\parallel}}{4L_a} & \frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma0}^{3\perp}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{1\perp}}{4L_a} - \frac{M_{\Sigma0}^{3\perp}}{2L_a} & \frac{M_{\Sigma0}^{3\parallel}}{2L_a} + \frac{M_{\Sigma\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Sigma0}^{dc}}{L_a} & 0 \\ 0 & -\frac{M_{\Sigma0}^{dc}}{L_a} \\ \frac{M_{\Delta\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Delta\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Delta\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{15} = \begin{bmatrix} -\frac{M_{\Sigma\alpha}^{1\parallel}}{L_a} & -\frac{M_{\Sigma\alpha}^{2\parallel}}{2L_a} & -\frac{M_{\Sigma\alpha}^{2\perp}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{1\perp}}{L_a} & \frac{M_{\Sigma\alpha}^{2\perp}}{2L_a} & -\frac{M_{\Sigma\alpha}^{2\parallel}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{2\parallel}}{L_a} & -\frac{M_{\Sigma\alpha}^{1\parallel}}{2L_a} & -\frac{M_{\Sigma\alpha}^{1\perp}}{2L_a} \\ -\frac{M_{\Sigma\alpha}^{2\perp}}{L_a} & \frac{M_{\Sigma\alpha}^{1\perp}}{2L_a} & -\frac{M_{\Sigma\alpha}^{1\parallel}}{2L_a} \\ -\frac{M_{\Sigma\beta}^{1\parallel}}{L_a} & -\frac{M_{\Sigma\beta}^{2\parallel}}{2L_a} & -\frac{M_{\Sigma\beta}^{2\perp}}{2L_a} \\ -\frac{M_{\Sigma\beta}^{1\perp}}{L_a} & \frac{M_{\Sigma\beta}^{2\perp}}{2L_a} & -\frac{M_{\Sigma\beta}^{2\parallel}}{2L_a} \\ -\frac{M_{\Sigma\beta}^{2\parallel}}{L_a} & -\frac{M_{\Sigma\beta}^{1\parallel}}{2L_a} & -\frac{M_{\Sigma\beta}^{1\perp}}{2L_a} \\ -\frac{M_{\Sigma\beta}^{2\perp}}{L_a} & \frac{M_{\Sigma\beta}^{1\perp}}{2L_a} & -\frac{M_{\Sigma\beta}^{1\parallel}}{2L_a} \\ \frac{3M_{\Delta0}^{dc}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{dc}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta0}^{3\parallel}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{3\parallel}C_L}{2C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta0}^{3\perp}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{3\perp}C_L}{2C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{3M_{\Delta0}^{3\parallel}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{3\parallel}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta0}^{dc}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{dc}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & 0 \\ \frac{3M_{\Delta0}^{3\perp}(L_1-L_2)}{2C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{3\perp}C_L}{C_{PU}\ (L_a^2+4L_aL_e)} & 0 & \frac{M_{\Delta0}^{dc}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{dc}C_L}{C_{PU}\ (L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{16} = \begin{bmatrix} -\frac{M_{\Delta 0}^{dc}}{L_a} - \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} & -\frac{M_{\Delta\alpha}^{2\perp}}{4L_a} \\ -\frac{M_{\Delta\alpha}^{2\perp}}{4L_a} & -\frac{M_{\Delta 0}^{dc}}{L_a} + \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Delta 0}^{3\parallel}}{2L_a} - \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} & \frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} & \frac{M_{\Delta 0}^{3\parallel}}{2L_a} - \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} \\ \frac{M_{\Delta\beta}^{2\parallel}}{4L_a} & \frac{M_{\Delta\beta}^{2\perp}}{4L_a} \\ \frac{M_{\Delta\beta}^{2\perp}}{4L_a} & -\frac{M_{\Delta\beta}^{2\parallel}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} & -\frac{M_{\Delta\beta}^{1\perp}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\perp}}{4L_a} & \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} \\ \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\alpha}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{17} = \begin{bmatrix} -\frac{M_{\Delta 0}^{3\parallel}}{2L_a} - \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} & -\frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} \\ \frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} & \frac{M_{\Delta 0}^{3\parallel}}{2L_a} - \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Delta 0}^{dc}}{L_a} & 0 \\ 0 & -\frac{M_{\Delta 0}^{dc}}{L_a} \\ \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} & \frac{M_{\Delta\beta}^{1\perp}}{4L_a} \\ -\frac{M_{\Delta\beta}^{1\perp}}{4L_a} & \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} \\ 0 & 0 \\ 0 & 0 \\ \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\alpha}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{18} = \begin{bmatrix} \frac{M_{\Delta\beta}^{2\parallel}}{4L_a} & \frac{M_{\Delta\beta}^{2\perp}}{4L_a} \\ \frac{M_{\Delta\beta}^{2\perp}}{4L_a} & -\frac{M_{\Delta\beta}^{2\parallel}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} & -\frac{M_{\Delta\beta}^{1\perp}}{4L_a} \\ \frac{M_{\Delta\beta}^{1\perp}}{4L_a} & \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} \\ -\frac{M_{\Delta 0}^{dc}}{L_a} + \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} & \frac{M_{\Delta\alpha}^{2\perp}}{4L_a} \\ \frac{M_{\Delta\alpha}^{2\perp}}{4L_a} & -\frac{M_{\Delta 0}^{dc}}{L_a} - \frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Delta 0}^{3\parallel}}{2L_a} + \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} & -\frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} \\ \frac{M_{\Delta\alpha}^{1\perp}}{4L_a} - \frac{M_{\Delta 0}^{3\perp}}{2L_a} & \frac{M_{\Delta 0}^{3\parallel}}{2L_a} + \frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} \\ \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}\,C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}\,C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}\,C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{2\perp}\,C_L}{4C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}\,C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU}\ (L_a^2+4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}\,C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{19} = \begin{bmatrix} \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} & \frac{M_{\Delta\beta}^{1\perp}}{4L_a} \\ -\frac{M_{\Delta\beta}^{1\perp}}{4L_a} & \frac{M_{\Delta\beta}^{1\parallel}}{4L_a} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Delta0}^{3\parallel}}{2L_a}+\frac{M_{\Delta\alpha}^{1\parallel}}{4L_a} & \frac{M_{\Delta\alpha}^{1\perp}}{4L_a}-\frac{M_{\Delta0}^{3\perp}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{1\perp}}{4L_a}-\frac{M_{\Delta0}^{3\perp}}{2L_a} & \frac{M_{\Delta0}^{3\parallel}}{2L_a}+\frac{M_{\Delta\alpha}^{2\parallel}}{4L_a} \\ -\frac{M_{\Delta0}^{dc}}{L_a} & 0 \\ 0 & -\frac{M_{\Delta0}^{dc}}{L_a} \\ \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\beta}^{2\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{2\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\beta}^{2\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\beta}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} & \frac{M_{\Delta\beta}^{1\perp}C_L}{4C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Sigma\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{M_{\Sigma\beta}^{1\perp}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\beta}^{1\perp}C_L}{4C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1-L_2))}{8C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta\beta}^{1\parallel}C_L}{4C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{1110} = \begin{bmatrix} -\frac{M_{\Delta\alpha}^{1\parallel}}{L_a} & -\frac{M_{\Delta\alpha}^{2\parallel}}{2L_a} & -\frac{M_{\Delta\alpha}^{2\perp}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{1\perp}}{L_a} & \frac{M_{\Delta\alpha}^{2\perp}}{2L_a} & -\frac{M_{\Delta\alpha}^{2\parallel}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{2\parallel}}{L_a} & -\frac{M_{\Delta\alpha}^{1\parallel}}{2L_a} & -\frac{M_{\Delta\alpha}^{1\perp}}{2L_a} \\ -\frac{M_{\Delta\alpha}^{2\perp}}{L_a} & \frac{M_{\Delta\alpha}^{1\perp}}{2L_a} & -\frac{M_{\Delta\alpha}^{1\parallel}}{2L_a} \\ -\frac{M_{\Delta\beta}^{1\parallel}}{L_a} & -\frac{M_{\Delta\beta}^{2\parallel}}{2L_a} & -\frac{M_{\Delta\beta}^{2\perp}}{2L_a} \\ -\frac{M_{\Delta\beta}^{1\perp}}{L_a} & \frac{M_{\Delta\beta}^{2\perp}}{2L_a} & -\frac{M_{\Delta\beta}^{2\parallel}}{2L_a} \\ -\frac{M_{\Delta\beta}^{2\parallel}}{L_a} & -\frac{M_{\Delta\beta}^{1\parallel}}{2L_a} & -\frac{M_{\Delta\beta}^{1\perp}}{2L_a} \\ -\frac{M_{\Delta\beta}^{2\perp}}{L_a} & \frac{M_{\Delta\beta}^{1\perp}}{2L_a} & -\frac{M_{\Delta\beta}^{1\parallel}}{2L_a} \\ \frac{3M_{\Sigma0}^{dc}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta0}^{dc}C_L}{C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Sigma0}^{3\parallel}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta0}^{3\parallel}C_L}{2C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Sigma0}^{3\perp}(3(L_1-L_2))}{2C_{PU}\ (L_a^2+4L_aL_e)}-\frac{M_{\Delta0}^{3\perp}C_L}{2C_{PU}\ (L_a^2+4L_aL_e)} \\ \frac{3M_{\Sigma0}^{3\parallel}(L_1-L_2)}{2C_{PU}\ (L_a^2+4L_aL_e)}-\frac{M_{\Delta0}^{3\parallel}C_L}{C_{PU}\ (L_a^2+4L_aL_e)} & \frac{M_{\Sigma0}^{dc}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta0}^{dc}C_L}{C_{PU2}(L_a^2+4L_aL_e)} & 0 \\ \frac{3M_{\Sigma0}^{3\perp}(L_1-L_2)}{2C_{PU}\ (L_a^2+4L_aL_e)}-\frac{M_{\Delta\beta}^{3\perp}C_L}{C_{PU}\ (L_a^2+4L_aL_e)} & 0 & \frac{M_{\Sigma0}^{dc}(3(L_1-L_2))}{2C_{PU2}(L_a^2+4L_aL_e)}-\frac{M_{\Delta0}^{dc}C_L}{C_{PU}\ (L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{21} = \begin{bmatrix} -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL}} - \frac{M_{\Delta\alpha}^{2\parallel} L_s}{2C_{LL}} & -\frac{M_{\Delta\alpha}^{2\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta\alpha}^{2\perp} L_s}{2C_{LL}} & -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL}} + \frac{M_{\Delta\alpha}^{2\parallel} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL}} - \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} \\ -\frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL}} & \frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL}} - \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL}} \\ \frac{M_{\Delta\beta}^{2\parallel} L_s}{2C_{LL}} & \frac{M_{\Delta\beta}^{2\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{2\perp} L_s}{2C_{LL}} & -\frac{M_{\Delta\beta}^{2\parallel} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL}} & \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{3M_{\Sigma\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{3M_{\Sigma\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Sigma\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} & \frac{3M_{\Sigma\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Sigma\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} & \frac{3M_{\Sigma\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} \end{bmatrix}$$

To maintain the same model nomenclature than in the DC/DC/AC DPM, $C_{LL2}$ remains as it was in the original model, but in this case it is defined as follows and its units are only one "$L$".

$C_{LL2} = L_a + 4L_e + 6L_m$

$$\tilde{A}_{12r}^{22} = \begin{bmatrix} -\frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL}} - \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} \\ \frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} & \frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL}} \\ -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} & 0 \\ 0 & -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL}} \\ \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL}} \\ 0 & 0 \\ 0 & 0 \\ \frac{3M_{\Sigma\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{3M_{\Sigma\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12}^{23} = \begin{bmatrix} \frac{M_{\Delta\beta}^{2\parallel} L_s}{2C_{LL}} & \frac{M_{\Delta\beta}^{2\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{2\perp} L_s}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} \\ -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{2\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{2\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Delta\alpha}^{2\perp} L_s}{2C_{LL2}} & -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} - \frac{M_{\Delta\alpha}^{2\parallel} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} \\ \frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} & \frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{3M_{\Sigma\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\beta}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{3M_{\Sigma\beta}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\beta}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{24} = \begin{bmatrix} \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Delta\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Delta\beta}^{1\parallel} L_s}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} \\ -\frac{M_{\Delta\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Delta 0}^{3\perp} L_s}{C_{LL2}} & \frac{M_{\Delta 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Delta\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} & 0 \\ 0 & -\frac{2M_{\Delta 0}^{dc} L_s}{C_{LL2}} \\ \frac{3M_{\Sigma\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\beta}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Delta\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{3M_{\Sigma\beta}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma\beta}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{25} = \begin{bmatrix} -\frac{2M_{\Delta\alpha}^{1\parallel} L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\parallel} L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\perp} L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\alpha}^{1\perp} L_s}{C_{LL2}} & \frac{M_{\Delta\alpha}^{2\perp} L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{2\parallel} L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\alpha}^{2\parallel} L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\parallel} L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\perp} L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\alpha}^{2\perp} L_s}{C_{LL2}} & \frac{M_{\Delta\alpha}^{1\perp} L_s}{C_{LL2}} & -\frac{M_{\Delta\alpha}^{1\parallel} L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\beta}^{1\parallel} L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel} L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\perp} L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\beta}^{1\perp} L_s}{C_{LL2}} & \frac{M_{\Delta\beta}^{2\perp} L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{2\parallel} L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\beta}^{2\parallel} L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\parallel} L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\perp} L_s}{C_{LL2}} \\ -\frac{2M_{\Delta\beta}^{2\perp} L_s}{C_{LL2}} & \frac{M_{\Delta\beta}^{1\perp} L_s}{C_{LL2}} & -\frac{M_{\Delta\beta}^{1\parallel} L_s}{C_{LL2}} \\ \frac{3M_{\Sigma 0}^{dc}(L_1 - L_2)}{C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta 0}^{dc}(3L_1 + 3L_2 + 2L_a)}{C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma 0}^{3\parallel}(L_1 - L_2)}{2C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta 0}^{3\parallel}(3L_1 + 3L_2 + 2L_a)}{2C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma 0}^{3\perp}(L_1 - L_2)}{2C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta 0}^{3\perp}(3L_1 + 3L_2 + 2L_a)}{2C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Sigma 0}^{3\parallel}(L_1 - L_2)}{C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta 0}^{3\parallel}(3L_1 + 3L_2 + 2L_a)}{C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Sigma 0}^{dc}(L_1 - L_2)}{C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta 0}^{dc}(3L_1 + 3L_2 + 2L_a)}{C_{PU2}(L_a^2 + 4L_aL_e)} & 0 \\ \frac{3M_{\Sigma 0}^{3\perp}(L_1 - L_2)}{C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta\beta}^{3\perp}(3L_1 + 3L_2 + 2L_a)}{C_{PU2}(L_a^2 + 4L_aL_e)} & 0 & \frac{3M_{\Sigma 0}^{dc}(L_1 - L_2)}{C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Delta 0}^{dc}(3L_1 + 3L_2 + 2L_a)}{C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{26} = \begin{bmatrix} -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{2\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Sigma\alpha}^{2\perp} L_s}{2C_{LL2}} & -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL2}} + \frac{M_{\Sigma\alpha}^{2\parallel} L_s}{2C_{LL2}} \\ -\frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL2}} \\ -\frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL2}} & \frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{2\parallel} L_s}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{2\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{2\perp} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel} L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{3M_{\Delta\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{3M_{\Delta\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Delta\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} & \frac{3M_{\Delta\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Delta\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{3M_{\Delta\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{27} = \begin{bmatrix} -\frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL}} - \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL2}} \\ \frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL2}} & \frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL2}} - \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL2}} & 0 \\ 0 & -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} \\ 0 & 0 \\ 0 & 0 \\ \frac{3M_{\Delta\alpha}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{3M_{\Delta\alpha}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Delta\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} & \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{3M_{\Delta\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} \\ \frac{3M_{\Delta\alpha}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} & \frac{3M_{\Delta\alpha}^{1\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_a L_e)} - \frac{M_{\Sigma\alpha}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_a L_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{28} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{2\parallel} L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{2\perp} L_s}{2C_{LL}} \\ \frac{M_{\Sigma\beta}^{2\perp} L_s}{2C_{LL}} & -\frac{M_{\Sigma\beta}^{2\parallel} L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL2}} \\ \frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} \\ -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL}} + \frac{M_{\Sigma\alpha}^{2\parallel} L_s}{2C_{LL2}} & \frac{M_{\Sigma\alpha}^{2\perp} L_s}{2C_{LL}} \\ \frac{M_{\Sigma\alpha}^{2\perp} L_s}{2C_{LL}} & -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL}} - \frac{M_{\Sigma\alpha}^{2\parallel} L_s}{2C_{LL}} \\ -\frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL}} + \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL}} \\ \frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL}} & \frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL}} + \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} \\ \frac{3M_{\Delta\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\beta}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{3M_{\Delta\beta}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\beta}^{2\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{29} = \begin{bmatrix} \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL2}} \\ -\frac{M_{\Sigma\beta}^{1\perp} L_s}{2C_{LL}} & \frac{M_{\Sigma\beta}^{1\parallel} L_s}{2C_{LL}} \\ 0 & 0 \\ 0 & 0 \\ -\frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL}} + \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL2}} & \frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL}} \\ -\frac{M_{\Sigma\alpha}^{1\perp} L_s}{2C_{LL2}} - \frac{M_{\Sigma 0}^{3\perp} L_s}{C_{LL}} & \frac{M_{\Sigma 0}^{3\parallel} L_s}{C_{LL2}} + \frac{M_{\Sigma\alpha}^{1\parallel} L_s}{2C_{LL}} \\ -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL}} & 0 \\ 0 & -\frac{2M_{\Sigma 0}^{dc} L_s}{C_{LL}} \\ \frac{3M_{\Delta\beta}^{2\parallel}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\beta}^{2\perp}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{2\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{M_{\Sigma\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{3M_{\Delta\beta}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} \\ \frac{3M_{\Delta\beta}^{1\perp}(L_1 - L_2)}{4C_{PU}\ (L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\perp}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} & \frac{3M_{\Delta\beta}^{1\parallel}(L_1 - L_2)}{4C_{PU2}(L_a^2 + 4L_aL_e)} - \frac{M_{\Sigma\beta}^{1\parallel}(3(L_1 + L_2) + 2L_a)}{4C_{PU2}(L_a^2 + 4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{12r}^{310} = \begin{bmatrix} -\frac{2M_{\Sigma\alpha}^{1\parallel}L_s}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\parallel}L_s}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\perp}L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\alpha}^{1\perp}L_s}{C_{LL2}} & \frac{M_{\Sigma\alpha}^{2\perp}L_s}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{2\parallel}L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\alpha}^{2\parallel}L_s}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\parallel}L_s}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\perp}L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\alpha}^{2\perp}L_s}{C_{LL2}} & \frac{M_{\Sigma\alpha}^{1\perp}L_s}{C_{LL2}} & -\frac{M_{\Sigma\alpha}^{1\parallel}L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\beta}^{1\parallel}L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel}L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\perp}L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\beta}^{1\perp}L_s}{C_{LL2}} & \frac{M_{\Sigma\beta}^{2\perp}L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{2\parallel}L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\beta}^{2\parallel}L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\parallel}L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\perp}L_s}{C_{LL2}} \\ -\frac{2M_{\Sigma\beta}^{2\perp}L_s}{C_{LL2}} & \frac{M_{\Sigma\beta}^{1\perp}L_s}{C_{LL2}} & -\frac{M_{\Sigma\beta}^{1\parallel}L_s}{C_{LL2}} \\ \frac{3M_{\Delta0}^{dc}(L_1-L_2)}{C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{dc}(3L_1+3L_2+2L_a)}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{3M_{\Delta0}^{3\parallel}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{3\parallel}(3L_1+3L_2+2L_a)}{2C_{PU2}(L_a^2+4L_aL_e)} & \frac{3M_{\Delta0}^{3\perp}(L_1-L_2)}{2C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{3\perp}(3L_1+3L_2+2L_a)}{2C_{PU2}(L_a^2+4L_aL_e)} \\ \frac{3M_{\Delta0}^{3\parallel}(L_1-L_2)}{C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{3\parallel}(3L_1+3L_2+2L_a)}{C_{PU2}(L_a^2+4L_aL_e)} & \frac{3M_{\Delta0}^{dc}(L_1-L_2)}{C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{dc}(3L_1+3L_2+2L_a)}{C_{PU2}(L_a^2+4L_aL_e)} & 0 \\ \frac{3M_{\Delta0}^{3\perp}(L_1-L_2)}{C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma\beta}^{3\perp}(3L_1+3L_2+2L_a)}{C_{PU}\ (L_a^2+4L_aL_e)} & 0 & \frac{3M_{\Delta0}^{dc}(L_1-L_2)}{C_{PU2}(L_a^2+4L_aL_e)} - \frac{M_{\Sigma0}^{dc}(3L_1+3L_2+2L_a)}{C_{PU2}(L_a^2+4L_aL_e)} \end{bmatrix}$$

$$\tilde{A}_{21} = \begin{bmatrix} \tilde{A}_{21r}^{11} & \tilde{A}_{21r}^{12} & \tilde{A}_{21r}^{13} & \tilde{A}_{21r}^{14} & \tilde{A}_{21r}^{15} & \tilde{A}_{21r}^{16} & \tilde{A}_{21}^{17} & \tilde{A}_{21r}^{18} & \tilde{A}_{21r}^{19} & \tilde{A}_{21r}^{110} \\ \tilde{A}_{21r}^{21} & \tilde{A}_{21r}^{22} & \tilde{A}_{21r}^{23} & \tilde{A}_{21r}^{24} & \tilde{A}_{21r}^{25} & \tilde{A}_{21r}^{26} & \tilde{A}_{21r}^{27} & \tilde{A}_{21r}^{28} & \tilde{A}_{21r}^{29} & \tilde{A}_{21r}^{210} \end{bmatrix}$$

$$\tilde{A}_{21r}^{12} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21}^{13} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{14} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc})+\left(\frac{Nl}{4Cl}\right)(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}) & 0 \\ 0 & \left(\frac{Nu}{4Cu}\right)(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc})+\left(\frac{Nl}{4Cl}\right)(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{15} = \begin{bmatrix} \left(\frac{Nu}{2Cu}\right)(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}) & \left(\frac{Nu}{4Cu}\right)(-M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel})-\left(\frac{Nl}{4Cl}\right)(-M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}) & \left(\frac{Nu}{4Cu}\right)(-M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel})-\left(\frac{Nl}{4Cl}\right)(-M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}) & \left(\frac{Nu}{4Cu}\right)(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel})-\left(\frac{Nl}{4Cl}\right)(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}) & \left(\frac{Nu}{4Cu}\right)(-M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel})-\left(\frac{Nl}{4Cl}\right)(-M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}) & \left(\frac{Nu}{4Cu}\right)(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp})-\left(\frac{Nl}{4Cl}\right)(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}) \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}) & \left(\frac{Nu}{2Cu}\right)(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}) & 0 \\ \left(\frac{Nu}{2Cu}\right)(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}) & 0 & \left(\frac{Nu}{2Cu}\right)(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc})-\left(\frac{Nl}{2Cl}\right)(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}) \end{bmatrix}$$

$$\tilde{A}^{16}_{21r} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(2M^{dc}_{\Sigma 0}+\frac{M^{2\parallel}_{\Sigma\alpha}}{2}+2M^{dc}_{\Delta 0}+\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M^{dc}_{\Sigma 0}+\frac{M^{2\parallel}_{\Sigma\alpha}}{2}-2M^{dc}_{\Delta 0}-\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\perp}_{\Sigma\alpha}}{2}+\frac{M^{2\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\perp}_{\Sigma\alpha}}{2}-\frac{M^{2\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\perp}_{\Sigma\alpha}}{2}+\frac{M^{2\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\perp}_{\Sigma\alpha}}{2}-\frac{M^{2\perp}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M^{dc}_{\Sigma 0}-\frac{M^{2\parallel}_{\Sigma\alpha}}{2}+2M^{dc}_{\Delta 0}-\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M^{dc}_{\Sigma 0}-\frac{M^{2\parallel}_{\Sigma\alpha}}{2}-2M^{dc}_{\Delta 0}+\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M^{3\parallel}_{\Sigma 0}+\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+M^{3\parallel}_{\Delta 0}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M^{3\parallel}_{\Sigma 0}+\frac{M^{2\parallel}_{\Sigma\alpha}}{2}-M^{3\parallel}_{\Delta 0}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M^{3\perp}_{\Sigma 0}-\frac{M^{1\perp}_{\Sigma\alpha}}{2}+M^{3\perp}_{\Delta 0}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M^{3\perp}_{\Sigma 0}-\frac{M^{1\perp}_{\Sigma\alpha}}{2}-M^{3\perp}_{\Delta 0}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M^{3\perp}_{\Sigma 0}+\frac{M^{1\perp}_{\Sigma\alpha}}{2}+M^{3\perp}_{\Delta 0}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M^{3\perp}_{\Sigma 0}+\frac{M^{2\perp}_{\Sigma\alpha}}{2}-M^{3\perp}_{\Delta 0}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M^{3\parallel}_{\Sigma 0}+\frac{M^{1\parallel}_{\Sigma\alpha}}{2}-M^{3\parallel}_{\Delta 0}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M^{3\parallel}_{\Sigma 0}+\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+M^{3\parallel}_{\Delta 0}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\parallel}_{\Sigma\beta}}{2}-\frac{M^{2\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\parallel}_{\Sigma\beta}}{2}+\frac{M^{2\parallel}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}-\frac{M^{2\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}+\frac{M^{2\perp}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}-\frac{M^{2\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}+\frac{M^{2\perp}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\parallel}_{\Sigma\beta}}{2}+\frac{M^{2\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\parallel}_{\Sigma\beta}}{2}-\frac{M^{2\parallel}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}-\frac{M^{1\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}+\frac{M^{1\parallel}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\perp}_{\Sigma\beta}}{2}+\frac{M^{1\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\perp}_{\Sigma\beta}}{2}-\frac{M^{1\perp}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\perp}_{\Sigma\beta}}{2}-\frac{M^{1\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\perp}_{\Sigma\beta}}{2}+\frac{M^{1\perp}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}-\frac{M^{1\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}+\frac{M^{1\parallel}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\parallel}_{\Sigma\alpha}}{2}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\perp}_{\Sigma\alpha}}{2}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\perp}_{\Sigma\alpha}}{2}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\parallel}_{\Sigma\alpha}}{2}+\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\parallel}_{\Sigma\alpha}}{2}-\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\perp}_{\Sigma\alpha}}{2}-\frac{M^{2\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\perp}_{\Sigma\alpha}}{2}+\frac{M^{2\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\perp}_{\Sigma\alpha}}{2}+\frac{M^{2\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\perp}_{\Sigma\alpha}}{2}-\frac{M^{2\perp}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\parallel}_{\Sigma\alpha}}{2}+\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\parallel}_{\Sigma\alpha}}{2}-\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right) \end{bmatrix}$$

$$\tilde{A}^{17}_{21r} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(M^{3\parallel}_{\Sigma 0}+\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+M^{3\parallel}_{\Delta 0}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M^{3\parallel}_{\Sigma 0}+\frac{M^{1\parallel}_{\Sigma\alpha}}{2}-M^{3\parallel}_{\Delta 0}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M^{3\perp}_{\Sigma 0}+\frac{M^{1\perp}_{\Sigma\alpha}}{2}+M^{3\perp}_{\Delta 0}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M^{3\perp}_{\Sigma 0}+\frac{M^{1\perp}_{\Sigma\alpha}}{2}-M^{3\perp}_{\Delta 0}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M^{3\perp}_{\Sigma 0}-\frac{M^{1\perp}_{\Sigma\alpha}}{2}+M^{3\perp}_{\Delta 0}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M^{3\perp}_{\Sigma 0}-\frac{M^{1\perp}_{\Sigma\alpha}}{2}-M^{3\perp}_{\Delta 0}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M^{3\parallel}_{\Sigma 0}+\frac{M^{1\parallel}_{\Sigma\alpha}}{2}-M^{3\parallel}_{\Delta 0}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M^{3\parallel}_{\Sigma 0}+\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+M^{3\parallel}_{\Delta 0}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M^{dc}_{\Sigma 0}+2M^{dc}_{\Delta 0}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M^{dc}_{\Sigma 0}-2M^{dc}_{\Delta 0}\right) & 0 \\ 0 & \left(\frac{Nu}{8Cu}\right)\left(2M^{dc}_{\Sigma 0}+2M^{dc}_{\Delta 0}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M^{dc}_{\Sigma 0}-2M^{dc}_{\Delta 0}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}-\frac{M^{1\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}+\frac{M^{1\parallel}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\perp}_{\Sigma\beta}}{2}-\frac{M^{1\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\perp}_{\Sigma\beta}}{2}+\frac{M^{1\perp}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\perp}_{\Sigma\beta}}{2}+\frac{M^{1\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\perp}_{\Sigma\beta}}{2}-\frac{M^{1\perp}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}-\frac{M^{1\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}+\frac{M^{1\parallel}_{\Delta\beta}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\parallel}_{\Sigma\alpha}}{2}+\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\parallel}_{\Sigma\alpha}}{2}-\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\perp}_{\Sigma\alpha}}{2}+\frac{M^{2\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\perp}_{\Sigma\alpha}}{2}-\frac{M^{2\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\parallel}_{\Sigma\alpha}}{2}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\perp}_{\Sigma\alpha}}{2}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\perp}_{\Sigma\alpha}}{2}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\perp}_{\Sigma\alpha}}{2}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\perp}_{\Sigma\alpha}}{2}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\parallel}_{\Sigma\alpha}}{2}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) \end{bmatrix}$$

$$\tilde{A}^{18}_{21r} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\parallel}_{\Sigma\beta}}{2}-\frac{M^{2\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\parallel}_{\Sigma\beta}}{2}+\frac{M^{2\parallel}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}-\frac{M^{2\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}+\frac{M^{2\perp}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}-\frac{M^{2\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}+\frac{M^{2\perp}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\parallel}_{\Sigma\beta}}{2}+\frac{M^{2\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\parallel}_{\Sigma\beta}}{2}-\frac{M^{2\parallel}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}-\frac{M^{1\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}+\frac{M^{1\parallel}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\perp}_{\Sigma\beta}}{2}+\frac{M^{1\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\perp}_{\Sigma\beta}}{2}-\frac{M^{1\perp}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\perp}_{\Sigma\beta}}{2}-\frac{M^{1\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\perp}_{\Sigma\beta}}{2}+\frac{M^{1\perp}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}-\frac{M^{1\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{1\parallel}_{\Sigma\beta}}{2}+\frac{M^{1\parallel}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M^{dc}_{\Sigma 0}-\frac{M^{2\parallel}_{\Sigma\alpha}}{2}+2M^{dc}_{\Delta 0}-\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M^{dc}_{\Sigma 0}-\frac{M^{2\parallel}_{\Sigma\alpha}}{2}-2M^{dc}_{\Delta 0}+\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\perp}_{\Sigma\alpha}}{2}-\frac{M^{2\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\perp}_{\Sigma\alpha}}{2}+\frac{M^{2\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\perp}_{\Sigma\alpha}}{2}-\frac{M^{2\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\perp}_{\Sigma\alpha}}{2}+\frac{M^{2\perp}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M^{dc}_{\Sigma 0}+\frac{M^{2\parallel}_{\Sigma\alpha}}{2}+2M^{dc}_{\Delta 0}+\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M^{dc}_{\Sigma 0}+\frac{M^{2\parallel}_{\Sigma\alpha}}{2}-2M^{dc}_{\Delta 0}-\frac{M^{2\parallel}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M^{3\parallel}_{\Sigma 0}-\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+M^{3\parallel}_{\Delta 0}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M^{3\parallel}_{\Sigma 0}-\frac{M^{1\parallel}_{\Sigma\alpha}}{2}-M^{3\parallel}_{\Delta 0}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M^{3\perp}_{\Sigma 0}+\frac{M^{1\perp}_{\Sigma\alpha}}{2}+M^{3\perp}_{\Delta 0}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M^{3\perp}_{\Sigma 0}+\frac{M^{1\perp}_{\Sigma\alpha}}{2}-M^{3\perp}_{\Delta 0}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M^{3\perp}_{\Sigma 0}-\frac{M^{1\perp}_{\Sigma\alpha}}{2}+M^{3\perp}_{\Delta 0}-\frac{M^{1\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M^{3\perp}_{\Sigma 0}-\frac{M^{1\perp}_{\Sigma\alpha}}{2}-M^{3\perp}_{\Delta 0}+\frac{M^{1\perp}_{\Delta\alpha}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M^{3\parallel}_{\Sigma 0}-\frac{M^{1\parallel}_{\Sigma\alpha}}{2}-M^{3\parallel}_{\Delta 0}-\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M^{3\parallel}_{\Sigma 0}-\frac{M^{1\parallel}_{\Sigma\alpha}}{2}+M^{3\parallel}_{\Delta 0}+\frac{M^{1\parallel}_{\Delta\alpha}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\parallel}_{\Sigma\beta}}{2}+\frac{M^{1\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\parallel}_{\Sigma\beta}}{2}-\frac{M^{1\parallel}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{1\perp}_{\Sigma\beta}}{2}+\frac{M^{1\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{1\perp}_{\Sigma\beta}}{2}-\frac{M^{1\perp}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\parallel}_{\Sigma\beta}}{2}+\frac{M^{2\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\parallel}_{\Sigma\beta}}{2}-\frac{M^{2\parallel}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}-\frac{M^{2\perp}_{\Delta\alpha}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M^{2\perp}_{\Sigma\beta}}{2}+\frac{M^{2\perp}_{\Delta\beta}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\perp}_{\Sigma\beta}}{2}+\frac{M^{2\perp}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\perp}_{\Sigma\beta}}{2}-\frac{M^{2\perp}_{\Delta\beta}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M^{2\parallel}_{\Sigma\beta}}{2}+\frac{M^{2\parallel}_{\Delta\beta}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M^{2\parallel}_{\Sigma\beta}}{2}-\frac{M^{2\parallel}_{\Delta\beta}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{19} = \begin{bmatrix} \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{110} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{3\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{3\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Delta0}^{3\parallel}}{2}+M_{\Sigma0}^{3\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Delta0}^{3\parallel}}{2}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Delta0}^{3\perp}}{2}+M_{\Sigma0}^{3\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Delta0}^{3\perp}}{2}-M_{\Sigma0}^{3\perp}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) & 0 & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{21} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{22} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{23} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta 0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta 0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta 0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{24} = \begin{bmatrix} \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\ 0 & 0 \\ 0 & 0 \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta 0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Sigma 0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta 0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta 0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Sigma 0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta 0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) & 0 \\ 0 & \left(\frac{Nu}{4Cu}\right)\left(2M_{\Sigma 0}^{dc}+2M_{\Delta 0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(2M_{\Sigma 0}^{dc}-2M_{\Delta 0}^{dc}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\ \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{4Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{4Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \end{bmatrix}$$

$$\tilde{A}_{21r}^{25} = \begin{bmatrix}
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{4Cu}\right)\left(-M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)+\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & 0 \\
\left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) & 0 & \left(\frac{Nu}{2Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)+\left(\frac{Nl}{2Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right)
\end{bmatrix}$$

$$\tilde{A}_{21r}^{26} = \begin{bmatrix}
\left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)
\end{bmatrix}$$

$$\tilde{A}_{21r}^{27} = \begin{bmatrix}
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}+\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)-\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\
0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\
0 & 0 \\
0 & 0 \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\perp}}{2}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)
\end{bmatrix}$$

$$\tilde{A}_{21r}^{28} = \begin{bmatrix}
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}-\frac{M_{\Delta\alpha}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\alpha}^{2\perp}}{2}+\frac{M_{\Delta\alpha}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}+2M_{\Delta0}^{dc}+\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}+\frac{M_{\Sigma\alpha}^{2\parallel}}{2}-2M_{\Delta0}^{dc}-\frac{M_{\Delta\alpha}^{2\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)
\end{bmatrix}$$

$$\tilde{A}_{21r}^{29} = \begin{bmatrix}
\left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)-\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) \\
0 & 0 \\
0 & 0 \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}-\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}+M_{\Delta0}^{3\perp}+\frac{M_{\Delta\alpha}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(M_{\Sigma0}^{3\perp}+\frac{M_{\Sigma\alpha}^{1\perp}}{2}-M_{\Delta0}^{3\perp}-\frac{M_{\Delta\alpha}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}-M_{\Delta0}^{3\parallel}-\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-M_{\Sigma0}^{3\parallel}-\frac{M_{\Sigma\alpha}^{1\parallel}}{2}+M_{\Delta0}^{3\parallel}+\frac{M_{\Delta\alpha}^{1\parallel}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) & 0 \\
0 & \left(\frac{Nu}{8Cu}\right)\left(2M_{\Sigma0}^{dc}+2M_{\Delta0}^{dc}\right)+\left(\frac{Nl}{8Cl}\right)\left(2M_{\Sigma0}^{dc}-2M_{\Delta0}^{dc}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}+\frac{M_{\Delta\beta}^{2\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\parallel}}{2}-\frac{M_{\Delta\beta}^{2\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}+\frac{M_{\Delta\beta}^{2\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{2\perp}}{2}-\frac{M_{\Delta\beta}^{2\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(-\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right) \\
\left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}+\frac{M_{\Delta\beta}^{1\perp}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\perp}}{2}-\frac{M_{\Delta\beta}^{1\perp}}{2}\right) & \left(\frac{Nu}{8Cu}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}+\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)+\left(\frac{Nl}{8Cl}\right)\left(\frac{M_{\Sigma\beta}^{1\parallel}}{2}-\frac{M_{\Delta\beta}^{1\parallel}}{2}\right)
\end{bmatrix}$$

$$\tilde{A}_{21r}^{210} = \begin{bmatrix}
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\parallel}+M_{\Sigma\alpha}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\parallel}-M_{\Sigma\alpha}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\perp}+M_{\Sigma\alpha}^{1\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\perp}-M_{\Sigma\alpha}^{1\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\alpha}^{2\perp}+M_{\Sigma\alpha}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\alpha}^{2\perp}-M_{\Sigma\alpha}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\alpha}^{1\parallel}+M_{\Sigma\alpha}^{1\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\alpha}^{1\parallel}-M_{\Sigma\alpha}^{1\parallel}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\parallel}+M_{\Sigma\beta}^{2\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\parallel}-M_{\Sigma\beta}^{2\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\perp}+M_{\Sigma\beta}^{1\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\perp}-M_{\Sigma\beta}^{1\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta\beta}^{2\perp}+M_{\Sigma\beta}^{2\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta\beta}^{2\perp}-M_{\Sigma\beta}^{2\perp}\right) & \left(\frac{Nu}{8Cu}\right)\left(-M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(-M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta\beta}^{1\parallel}+M_{\Sigma\beta}^{1\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta\beta}^{1\parallel}-M_{\Sigma\beta}^{1\parallel}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{8Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)-\left(\frac{Nl}{8Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\parallel}+M_{\Sigma0}^{3\parallel}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\parallel}-M_{\Sigma0}^{3\parallel}\right) & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right) & 0 \\
\left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{3\perp}+M_{\Sigma0}^{3\perp}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{3\perp}-M_{\Sigma0}^{3\perp}\right) & 0 & \left(\frac{Nu}{4Cu}\right)\left(M_{\Delta0}^{dc}+M_{\Sigma0}^{dc}\right)-\left(\frac{Nl}{4Cl}\right)\left(M_{\Delta0}^{dc}-M_{\Sigma0}^{dc}\right)
\end{bmatrix}$$

$$\tilde{A}_{22r} = \begin{bmatrix}
0 & -\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\omega & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\omega & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3\omega & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\omega & 0 & 0
\end{bmatrix}$$

$$\widetilde{N_r} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{3L_2 + L_a + 4L_e + 6L_t}{2C_{LL1}} & \dfrac{3L_1 - 3L_2}{2L_a(L_a + 4L_e)C_{PU2}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \dfrac{3L_2 + L_a}{C_{LL1}} & \dfrac{\dfrac{-3L_1 - 3L_2}{L_a + 2}}{(L_a + 4L_e)C_{PU2}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# Appendix E

# Accuracy of DC/DC/AC DPM

The accuracy of the developed DC/DC/AC DPMs in Chapter 3 was found to be susceptible to the amount of internal energy storage in the converter, i.e., it is influenced by the capacitance selection. The lower the capacitance selected, the worse the accuracy became. Here are presented the results of the validation simulations in Section 3.2.3, using the parameters in Table 3.3, but reducing the capacitance in the arms from 50 $\frac{kJ}{MW}$ to 30 $\frac{kJ}{MW}$. These results show how the DC/DC/AC converter models are sensitive to internal energy storage, while the DC/DC converter models are not.

Table E.1: MP-DCDCAC State Variables Steady-State Comparison Using 30 $\frac{kJ}{MW}$

| State Variable | TAM | DPM | Percent Error |
|---|---|---|---|
| $i^1_{\Sigma\alpha}[A_pk]$ | 445.747 | 412.093 | 8.17% |
| $i^1_{\Delta g\alpha}[A_pk]$ | 970.983 | 992.357 | 2.15% |
| $V^1_{\Sigma\alpha}[kV_{pk}]$ | 9.1587 | 8.18674 | 11.87% |
| $V^2_{\Sigma\alpha}[kV_{pk}]$ | 5.87866 | 6.74617 | 12.86% |
| $V^1_{\Delta\alpha}[kV_{pk}]$ | 12.2899 | 12.7744 | 3.79% |
| $V^2_{\Delta\alpha}[kV_{pk}]$ | 4.64023 | 3.75481 | 23.58% |
| $i^{dc}_{\Sigma 0}[A]$ | 201.436 | 205.38 | 1.92% |
| $i^{dc}_{\Delta t\alpha}[A]$ | 418.186 | 400 | 4.55% |
| $V^{dc}_{\Sigma 0}[kV]$ | 400.013 | 400 | 0.00% |

Table E.2: MP-DCDC State Variables Steady-State Comparison Using 30 $\frac{kJ}{MW}$

| State Variable | TAM | DPM | Percent Error |
|---|---|---|---|
| $i^1_{\Sigma\alpha}[A_pk]$ | 1610.07 | 1610.22 | 0.01% |
| $V^1_{\Sigma\alpha}[kV_{pk}]$ | 39.6904 | 39.694 | 0.01% |
| $V^2_{\Sigma\alpha}[kV_{pk}]$ | 1.73437 | 1.73496 | 0.03% |
| $V^1_{\Delta\alpha}[kV_{pk}]$ | 5.66615 | 5.66808 | 0.03% |
| $V^2_{\Delta\alpha}[kV_{pk}]$ | 5.74896 | 5.74865 | 0.01% |
| $i^{dc}_{\Delta t\alpha}[A]$ | 799.989 | 700.981 | 0.00% |
| $V^{dc}_{\Sigma 0}[kV]$ | 399.99 | 399.999 | 0.00% |

Table E.3: BP-DCDCAC State Variables Steady-State Comparison Using 30 $\frac{kJ}{MW}$

| State Variable | Avg | DPM | Percent Error |
|---|---|---|---|
| $i^1_{\Sigma\alpha}[A]$ | 660.18 | 652.9 | 1.12% |
| $i^1_{\Delta g\alpha}[A]$ | 1773.44 | 1780.03 | 0.37% |
| $V^1_{\Sigma\alpha}[kV]$ | 3.19855 | 3.00139 | 6.57% |
| $V^2_{\Sigma\alpha}[kV]$ | 5.60515 | 5.91054 | 5.17% |
| $V^1_{\Delta\alpha}[kV]$ | 10.2753 | 10.3741 | 0.95% |
| $V^2_{\Delta\alpha}[kV]$ | 3.92826 | 3.61007 | 8.81% |
| $i^{dc}_{\Sigma 0}[A]$ | 469.463 | 471.003 | 0.33% |
| $i^{dc}_{\Delta t\alpha}[A]$ | 805.826 | 800.022 | 0.73% |
| $V^{dc}_{\Sigma 0}[kV]$ | 400.27 | 400 | 0.07% |

Table E.4: BP-DCDC State Variables Steady-State Comparison Using 30 $\frac{kJ}{MW}$

| State Variable | Avg | DPM | Percent Error |
|---|---|---|---|
| $i^1_{\Sigma\alpha}[A]$ | 1288.6 | 1287.68 | 0.07% |
| $V^1_{\Sigma\alpha}[kV]$ | 55.3164 | 55.2419 | 0.13% |
| $V^2_{\Sigma\alpha}[kV]$ | 1.44153 | 1.43979 | 0.12% |
| $V^1_{\Delta\alpha}[kV]$ | 1.72325 | 1.67467 | 2.90% |
| $V^2_{\Delta\alpha}[kV]$ | 10.7167 | 10.736 | 0.18% |
| $i^{dc}_{\Delta t\alpha}[A]$ | 799.894 | 799.958 | 0.01% |
| $V^{dc}_{\Sigma 0}[kV]$ | 319.995 | 319.997 | 0.00% |

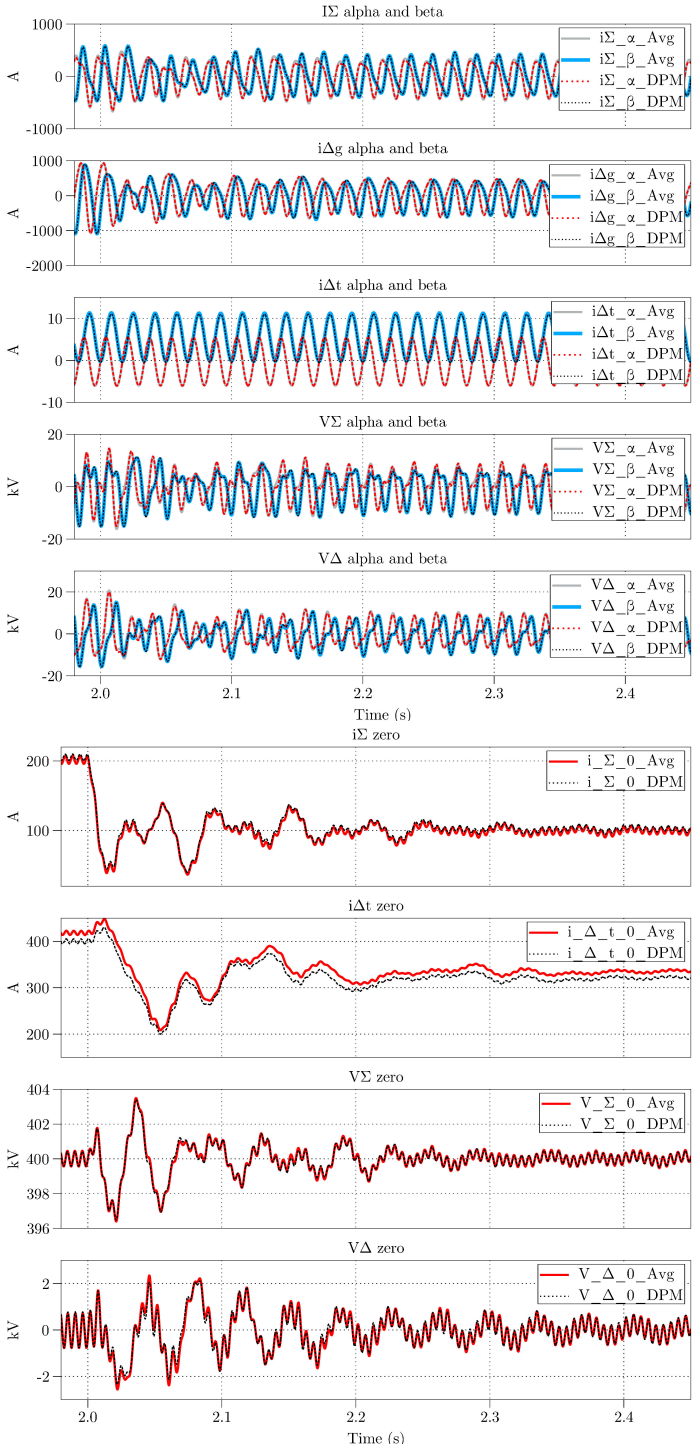


Figure E.1: MP-DCDCAC; DPM and averaged model simulation results using $30\frac{kJ}{MW}$

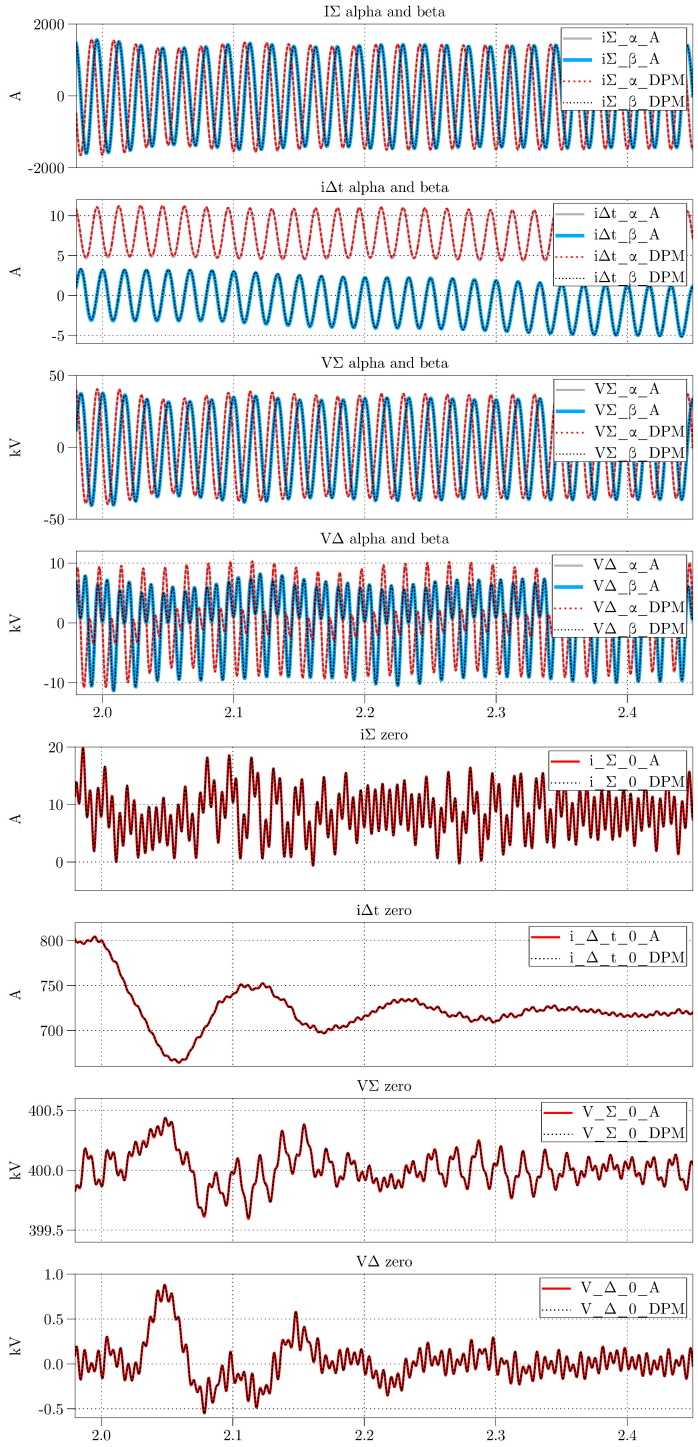


Figure E.2: MP-DCDC; DPM and averaged model simulation results using $30\frac{kJ}{MW}$

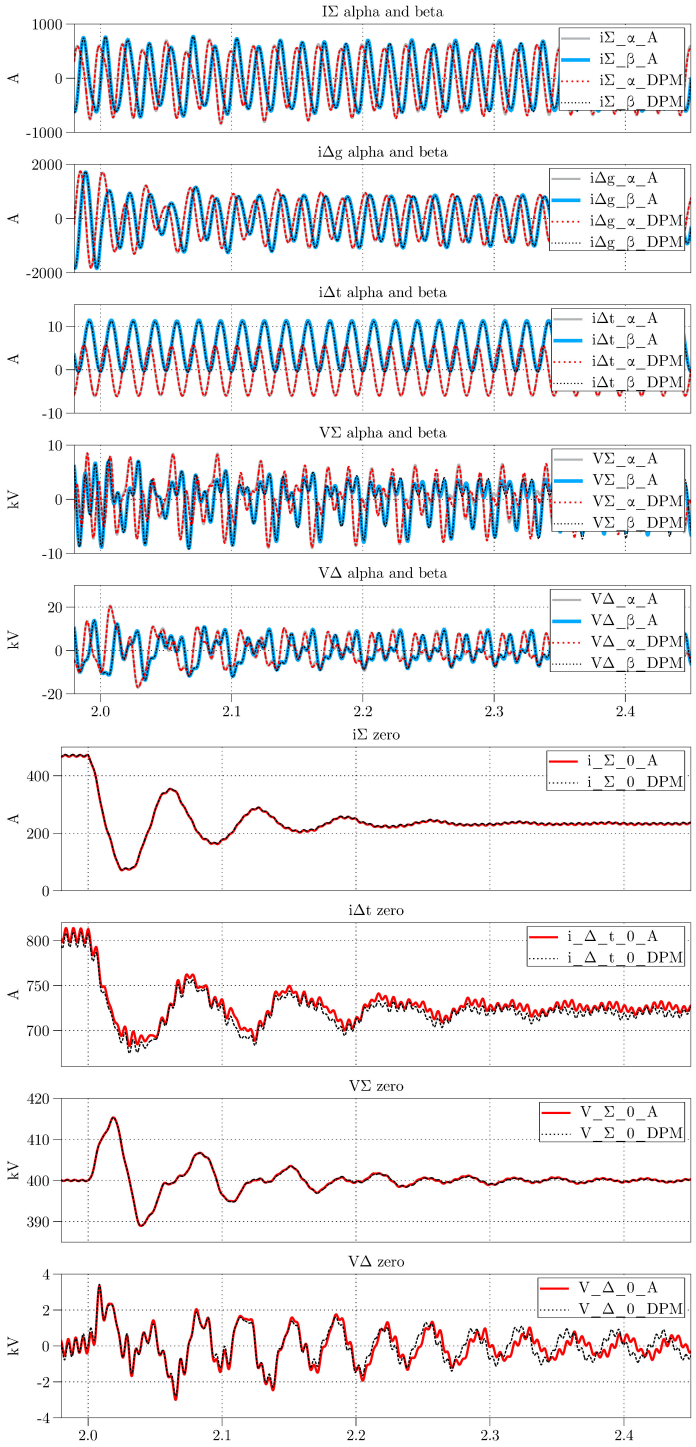


Figure E.3: BP-DCDCAC; DPM and averaged model simulation results using $30\frac{kJ}{MW}$

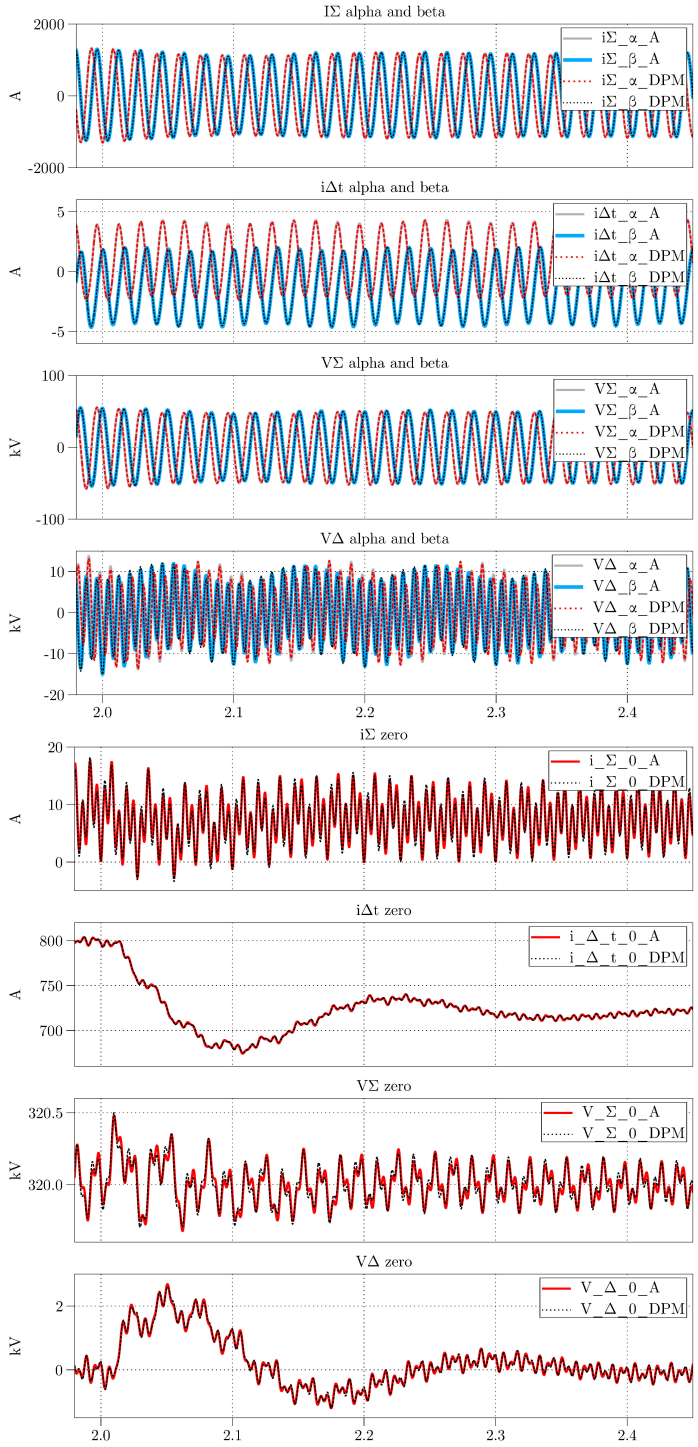


Figure E.4: BP-DCDC; DPM and averaged model simulation results using $30\frac{kJ}{MW}$