

# RURAL ECONOMY

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## **Risk Premia in Tractor and Combine Investments**

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## **Abstract**

A farmer planning to use Net Present Value (NPV) analysis on machinery requires estimates of operating benefits over time, an estimate of terminal or salvage values and a risk-adjusted discount rate. Using financial market information and related Root Mean Square Errors on machinery value forecasts, risk premia for combine and tractor investments are estimated for non-diversified investors. These risk premia can be added to the risk free rate in comparable maturity long term bonds to derive an appropriate discount rate for NPV analysis. Where machines are held as single-asset portfolios, risk premia identified for discounting terminal value vary between 5.5% and 8.3% for combines and between 2.4% and 3.6% for tractors, depending on age during the holding period. Where machines are held as parts of multi-asset portfolios, risk premia are usually lower, depending on machinery's weight in the multi-asset portfolio and its covariance with the rest of the portfolio.



## **Risk Premia in Tractor and Combine Investments**

Farm machinery is a major investment for most farmers and it may be the dominant asset held by some. Studies on optimal replacement of farm machinery assets such as Reid and Bradford (1987, 1983) that are theoretically consistent with Net Present Value investment criteria have not specified an appropriate discount rate for the NPV calculation. Reid and Bradford (1987) acknowledged that a tangible asset can be replaced with a financial alternative such as a bond or a stock portfolio. Conceptually, the appropriate discount rate is that obtainable from a financial market opportunity with expected cash flow and expected risk which is identical to a proposed farm machinery investment.

A discount rate is composed of a riskless base rate and a risk premium. A riskless rate is customarily identified as the rate of return on a default-free government security. This rate rises and falls with changes in capital supply and demand and with inflationary expectations. The risk premium is the market reward for risk bearing, the difference between rate of return on a risky investment and the riskless rate. This paper will identify an appropriate risk premium for farm machinery, which can be added to the prevailing riskless rate to establish an NPV discount rate. Appropriate discount rates have not previously been available for selected farm machinery. The results presented here will significantly improve applied farm machinery investment decisions. The analysis will be consistent with standard mean-variance capital market analysis.

The risk premium estimation method will first find a financial market investment which would, if substituted for the farm machinery investment, contribute the same amount of risk to the machinery owner's portfolio. The rate of return on this financial market investment is the relevant opportunity cost and hence the appropriate discount rate.

A machine generates a series of operating cash flows and a terminal value. Each period's expected cash flow,  $C_T$ , occurring  $T$  periods in the future with risk  $u$  are discounted to present values as  $C_T(1+r_u)^{-T}$  where  $r_u$  is the risk adjusted discount rate. Because data are available for terminal values, this paper will concentrate on finding the appropriate discount rate for that value. However, the conceptual development for  $r$  extends to all machinery cash flows.

Let  $S_{a,T}$  be the normalized expected salvage value where  $(a)$  is the age of the machine at the time a prediction is made and  $T$  is the number of periods over which salvage is forecast.  $S_{a,T}$  is the ratio of salvage value at time  $T$  to the value of the machine at time of forecast. For example, if a two year old ( $a=2$ ) tractor is worth \$50,000 and its predicted value at age six ( $T=4$ ) is 35,000 then  $S_{a,T}=0.7$ . The salvage risk ( $u$ ) dimension is presumed to be the variance around the estimated salvage value. This follows the general practice in the financial literature of characterizing investment risk by variance. The paper first considers methods for estimating the expected salvage value and the forecast variance around this value. Next, a financial asset which replicates the expectation and variance of cash flow  $S_{a,T}$  is developed using three expository steps:

- 1) Reproduce the expected salvage value and cash flow timing using either of two financial assets; riskless bonds (bonds) or a diversified common stock portfolio (stocks).

- 2) Reproduce the salvage variance by selecting a combination of stocks, and bonds from (1) above. This portfolio has the same expected salvage value, variance and timing as the machine asset and would provide an equivalent financial market opportunity for an investor who contemplates investing all wealth in a farm machine.

Reproduce the machine salvage's contribution to variance in the machinery-owner's portfolio, by selecting a combination of stocks and bonds from (1) above. This combination provides an equivalent financial market investment for the investor planning to hold a machine within a portfolio of other assets.

## Estimating Farm Machinery Terminal Value and its Variance

Several studies have estimated farm machinery depreciation rates. The more recent of these include Perry, Bayaner and Nixon (1990), Hansen and Lee (1991), Cross and Perry (1995) and Unterschultz and Mumey (1996). In the latter paper a Root Mean Squared Error (RMSE) statistic is developed to measure the dispersion around terminal value predictions. The normalized RMSE is defined as

$$(1) \quad RMSE_{aT} = \sqrt{\sum_i \left( \frac{\text{ActualTerminalValue}_i - \text{ForecastTerminalValue}_i}{\text{OriginalValue}_i} \right)^2 \div n}$$

where (a) equals the machine age at the beginning of the forecast period, T equals the length of the forecast,  $i=1 \dots n$  and n is the number of observations on age (a) assets.

The Unterschultz and Mumey estimation of RMSE can be illustrated with an example. Consider a four year (T=4) investment horizon on all one year old (a=1) combines. Assume a one year old Massey Ferguson “750” combine is worth \$40,000. The Unterschultz and Mumey terminal value estimate for this machine when it is five years old is 73.3% of the year one value or \$29,320. If the actual value in year five is \$22,000 then one term in the RMSE calculation is

$$(2) \quad RMSE_{a=1, T=4} = \sqrt{\left( \dots + \left( \frac{22,000 - 29,320}{40,000} \right)^2 + \dots \right) / n}$$

where n is the total number of four year forecasts generated on all one year old combines in the data set.

Several depreciation rates have been applied to a common tractor and combine data set from the Official Guide: Tractors and Farm Equipment, spanning the period from Spring, 1972 through Spring 1992. These depreciation rates are from:

1. Cross and Perry, based on a separate data set. (Combines and Tractors)
2. Hansen and Lee, based on a separate data set. (Tractors only)
3. Unterschultz and Mumey, based on Official Guide data above with age depreciation only. (Combines and Tractors)

4. Unterschultz and Mumeey, based on Official Guide data above with age depreciation and adjustment for time effects reflecting differing supply and demand conditions. (Combines and Tractors)

Each of these depreciation methods is used to generate a series of forecasts which, when compared with Official Guide values, enable calculation of RMSE for forecast periods extending from T= 1 to 10 years into the future on tractors and T= 1 to 7 years into the future on combines. There is a different RMSE for each starting asset age and length of forecast. This provides four alternative sources of RMSE values to serve as estimates of the terminal value variance. Summaries of tractor and combine RMSE based on the four different depreciation forecasts methods where a=1 and where T varies are shown in Figures 1 and 2 along with other information.

**Replicating the Expected Salvage Value With Financial Assets**

Two financial market investments are used to reproduce the expected money recovery from salvage. A risk free investment in strip bonds, B (an asset with terminal value only and no periodic interest payments) that matures in T periods can reproduce the expected future machine salvage S<sub>T</sub>. A simple present value calculation to give the dollar value of the initial bond investment required is  $S_0B = S_{aT} (1 + r_B^*)^{-T}$  where  $r_B^*$  is the geometric rate on riskless long bonds.

The second financial investment is in a broadly diversified stock portfolio such as a cross-section of the New York Stock Exchange with all income reinvested. Stocks are risky and normally earn a risk premium above bond income. When  $r_P^*$  is the geometric return on stock, the necessary present investment required to reproduce the expected terminal machine value at time t is  $S_0P = S_{aT} (1 + r_P^*)^{-T}$ .

Seigel (1992) estimated the annual inflation adjusted arithmetic return on long-term Government bonds as 0.5% ( $r_B$ ) and on common stocks as 7.4% ( $r_P$ ), using 1946-1990 data. He similarly estimated geometric returns of -0.1% ( $r_B^*$ ) and 6.2% ( $r_P^*$ ) for Government long bonds and common stocks respectively. The geometric rates are applicable to the calculations replicating the expected terminal machine value above. Using Siegel's long-term arithmetic bond rate as an approximation for the rate on

default-free strip bonds, the difference between Siegel's two arithmetic rates comprises an estimate of  $r_m=6.9\%$ , the annual risk premium above default-free bonds expected from stock market investment.

Any expected future cash amount can be replicated with a combination of stocks and bonds,  $X_p P_0 + (1-X_p)B_0$ , where  $X_p$  is the proportion of stocks. Since  $r > r_B$ , a smaller financial investment is required for replication as  $X_p$  increases.

### Reproducing the Salvage Value Variance For A Non-Diversified Machine Portfolio

If  $S_{aT}$  were certain, its expected value could be reproduced with bond investment,  $B_0$ . If the normalized variance of  $S_T$  were identical to the normalized variance anticipated from the market portfolio investment  $P$ , then the stock investment would also duplicate the expectation and variance. For more general results an equivalent terminal risk portfolio (ETRP) may be constructed using a linear combination of the bond and stock portfolio.

By design any linear combination of portfolio weights equal to 1 invested in the bond portfolio and the stock portfolio have a future expected value equal to the terminal machine value (i.e.  $E_0((1 - X_p)B_T + X_p P_T) = S_{aT}$ ). The ETRP is constructed by finding a weight,  $X_p$ , that sets the portfolio standard deviation equal to the RMSE of  $S$ . All the variance is contributed by the stock component. A distribution assumption on stock investment performance is required to forecast this standard deviation.

Log-normality of stock prices is a standard assumption in finance (Hull 1989) and this distribution assumption is used here. The ETRP forecast standard deviations are calculated by finding the portfolio weight  $X_p$  such that

$$(3) \quad RMSE_{aT} = \{(X_p P_0)^2 e^{2r_p T} [e^{\frac{2}{p} T} - 1]\}^{1/2}$$

where the right hand side is the standard deviation from a log-normal distribution,  $r_p$  is the arithmetic inflation adjusted return on the stock portfolio and  $\sigma_p$  is the stock portfolio returns one period standard deviation. Arithmetic rates are commonly used in the log normal distribution. The solution for  $X_p$  is the ratio,

$$(4) \quad X_p^* = RMSE_{aT} / \{P_0^2 e^{2r_p T} [e^{\frac{2}{p} T} - 1]\}^{1/2}$$

This  $X_p^*$  is the percentage that should be invested in the stock portfolio  $B_0$  to create a blended bond and stock  $ETRP = B_0 X_p^* + B_0(1-X_p^*)$  that duplicates  $S_T$  and the forecast standard deviation surrounding  $S_T$ . The risk premium earned on ETRP is  $X_p^* r_m$ .

Siegel (1992) has estimated historic inflation adjusted arithmetic stock return as  $r_p=7.4\%$  and standard deviation as  $\sigma_p = 15.6\%$  for the period 1946 to 1990. Unterschultz and Mumey results indicate an approximate geometric 5% depreciation rate for tractors and 10% depreciation rate for combines in their data set. Assuming an initial machine value normalized to 1 which corresponds to the RMSE calculations described above, these depreciation estimates provide forecasts of the terminal machine value  $S_0$  to use in the ETRP calculations described above.

The standard deviation of cash flows expected at various future times from an investment in the stock portfolio only (i.e.  $X=1$ ) are shown in Figures 1 and 2 and are comparable to the machinery salvage RMSE data included in these figures for tractors and combines respectively. The RMSE are based on age one ( $a=1$ ) machine assets. Similar results are obtained using assets of different ages and are not reported here. Tractor investment risk first increases then decreases but at all times remains below the stock market risk. Combine risk increases and even exceeds stock market risk over different investment horizons. Tractor investments exhibit lower total risk than the combine investment.

Cross and Perry's depreciation estimates have lower RMSE in short-term tractor forecasts; i.e., their depreciation model fits the data more closely. The Unterschultz and Mumey time-adjusted forecast has lower RMSE on long-term forecasts. Hansen and Lee's forecast tends to have higher RMSE, not surprisingly since their depreciation rates were estimated from small tractors and the RMSE is obtained in testing these rates against data from large tractors. For subsequent analysis, Cross and Perry's RMSE values will be used, since their depreciation forecasts are derived from out-of-sample data. However, final estimates of machinery risk premia are quite insensitive to the differences in the three forecasts.

Using the procedure outlined above to derive the portfolio weight,  $X$ , in the ETRP, profiles of machinery risk premiums over different investment horizons are

calculated and presented in Table 1 based on stock market returns, risk premia and standard deviations from Siegel. For example, the risk premia on a one year old tractor for an investment horizon of four years is estimated at 2.5%. This 2.5% would be added to the risk free long bond rate to generate the net present value discount rate. Tractor risk premia range from 2.3% to 3.6% for the Cross and Perry estimates. The mean risk premium is 2.7%. The Cross and Perry combine risk premia range from 5.5% to 8.3% with a mean of 7.4%. The level of risk premium depends on the source of the forecast.

These machinery risk premium estimates are relatively insensitive to changes in the stock portfolio expected return but are quite sensitive to changes in the stock portfolio standard deviation of returns. For example if the standard deviation of stock returns is 21.12% and the risk premium is 7.3% (Patterson p.113) then the tractor and combine risk premium ranges are 1.6%-2.7% and 4.2%-6.1% respectively. Machinery risk premiums for tractors and combines are higher than those in Table 1 if stock market risk premiums derived from short term risk free bonds are used.

The machinery risk premiums presented above are valid if machinery is the sole asset in the farm portfolio. Where other assets such as land constitute a significant proportion of the portfolio, the machine risk premiums are more difficult to evaluate. The next section examines risk premia for farm machinery when machinery constitutes less than half of the investor's portfolio.

### **Reproducing the Salvage Value Variance For A Diversified Machinery Investor**

Consider a machinery investor who also has investments in other assets ( which we shall simply denote as land). The relevant risk premia for machinery is now influenced by the covariance of machinery with land. The following describes conceptually how this risk premia is derived and places bounds on the range of likely risk premia.

The overall estimation procedure is similar to the non-diversified case. The objective is to replicate the risk contribution of the machine portfolio to the whole portfolio by identifying an ETRP of stocks and bonds. An ETRP weight,  $w_p^*$ , will be found which, cognizant of the covariance relationships between assets, maintains both the

expectation and variance of the total terminal cashflow unchanged in period T. To ease the discussion some terminology is first defined.

- $Y_{M,T}$ : Proportion of investors cashflow expected from terminal machinery value at time T. Then  $(1-Y_{M,T})$  is the proportion of cash flow attributed to land.  $Y_{M,T}$  is a known quantity.
- $X_p$ : The ETRP weight in the stocks such that  $Y_{M,T}-X_p$  is the portfolio weight in bonds.
- $\sigma_{L,T}^2$  is the forecast variance of the rest of the land cashflow at some future time period T.
- $\sigma_{M,T}^2$  is the forecast variance of the machine portion of the portfolio at some future time period T. This is proxied by the RMSE measures described above and shown in Figures 1 and 2.
- $\sigma_{P,T}^2$  is the forecast variance of the stocks at some future time period T using the log normality assumption.
- $\rho_{M,L}$  is the correlation between the value of the machine terminal value and the land cashflow at time T.
- $\rho_{P,L}$  is the correlation between the terminal value of stocks in the ETRP and the land cashflow at time T.

With this terminology the risk surrounding the investors land and machine portfolio forecast value at time T can be characterized as

$$(5) \quad \sigma_{M,L,T}^2 = Y_{M,T}^2 \sigma_{M,T}^2 + (1-Y_{M,T})^2 \sigma_{L,T}^2 + 2Y_{M,T}(1-Y_{M,T}) \rho_{M,L} \sigma_{M,T} \sigma_{L,T}$$

The total portfolio risk is composed of the risk from the machinery portion and from the land portion of the portfolio. The relationship or correlation between the two portfolios can change the overall risk of the portfolio. The objective is to replicate the machinery portion of risk in the equation above using the ETRP.

Replacing the machinery with the stock and bond portfolio gives a different total risk measure. Holding the portion invested in land fixed at  $(1-Y_{M,T})$  while allowing the portion  $X_p$  invested in stocks to vary gives a total risk measure of

$$(6) \quad \sigma_{P,L,T}^2 = X_P^2 \sigma_{P,T}^2 + (1 - Y_{M,T})^2 \sigma_{L,T}^2 + 2X_P(1 - Y_{M,T}) \sigma_{P,T} \sigma_{L,T} \rho_{P,L}$$

The ETRP finds the  $X_p^{**}$  that equates  $\sigma_{P,L,T}^2 = \sigma_{M,L,T}^2$  to give a quadratic form in  $X_p$  as shown below.

$$(7) \quad X_P^2 \sigma_{P,T}^2 + 2X_P(1 - Y_{M,T}) \sigma_{P,T} \sigma_{L,T} \rho_{P,L} - [Y_{M,T}^2 \sigma_{M,T}^2 + 2Y_{M,T}(1 - Y_{M,T}) \sigma_{M,T} \sigma_{L,T} \rho_{M,L}] = 0$$

This can be solved by the quadratic formula.

(8)

$$X_P^{**} = \frac{-2(1 - Y_{M,T}) \sigma_{P,T} \sigma_{L,T} \rho_{P,L} \pm [(2(1 - Y_{M,T}) \sigma_{P,T} \sigma_{L,T} \rho_{P,L})^2 + 4 \sigma_P^2 (Y_{M,T}^2 \sigma_{M,T}^2 + 2Y_{M,T}(1 - Y_{M,T}) \sigma_{M,T} \sigma_{L,T} \rho_{M,L})]^{0.5}}{2 \sigma_P^2}$$

This  $X_p^{**}$  is the amount invested in the stock portion of the ETRP. The only positive solution is where the second term on the RHS is positive.  $X_p^{**}$  is the weight invested in the stock portfolio of the ETRP that replaces the risk supplied by the machine portfolio where the proportion invested in land is fixed at  $1 - Y_{M,T}$ . The optimal weight of stocks is a function of the portfolio invested in land, and incorporates the relationship between stocks and land.

The appropriate risk premium depends on the correlation parameters involved. However, the upper bound on the risk premium is the corresponding risk premium for a machinery-only portfolio, and the lower bound on the risk premium is zero, assuming non-negative correlation parameters. A sample of calculated values is shown in Table 2 which illustrates the decrease in risk premia.

## Conclusions

The non-diversified machinery investment risk premium varies with the intended holding period or investment horizon. Risk premia in terminal combine values are consistent with a risk premium ranging from 5.5% to 8.3%. For tractors the risk premium range is 2.4% to 3.6% with the greatest risk over the shorter investment horizons. These risk premia can be added to the risk free rate in comparable maturity long term bonds to derive an appropriate discount rate for NPV analysis for non-diversified farm machinery portfolios. Where machinery constitutes only 30% of the future investment portfolio, risk premia vary. One estimate of this risk premia is 2.2% for tractors and 4.3% for combines.

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**Table 1**  
**Tractor and Combine Risk Premium Estimates on an Equivalent-Terminal-Risk-Portfolio For Non Diversified Investor**

Investment Horizon (years)	Tractor <sup>1</sup>				Combine			
	H&L	U&M	U&M+Time	C&P	U&M	U&M+Time	C&P	
1	5.9%	5.6%	5.5%	3.6%	5.4%	5.3%	5.5%	
2	5.0%	4.7%	4.5%	3.3%	6.9%	6.7%	7.2%	
3	4.9%	4.5%	4.1%	3.2%	7.8%	7.4%	8.2%	
4	4.1%	3.7%	3.1%	2.8%	8.0%	7.4%	8.3%	
5	3.5%	3.2%	2.5%	2.5%	7.7%	7.0%	8.2%	
6	3.0%	3.0%	2.1%	2.3%	7.1%	6.3%	7.6%	
7	2.6%	2.7%	1.8%	2.3%	<u>6.6%</u>	<u>5.6%</u>	<u>7.0%</u>	
8	2.4%	2.6%	1.6%	2.4%				
9	2.3%	2.4%	1.3%	2.4%				
10	<u>2.0%</u>	<u>2.1%</u>	<u>1.0%</u>	<u>2.4%</u>				
mean	3.6%	3.5%	2.7%	2.7%	7.1%	6.5%	7.4%	

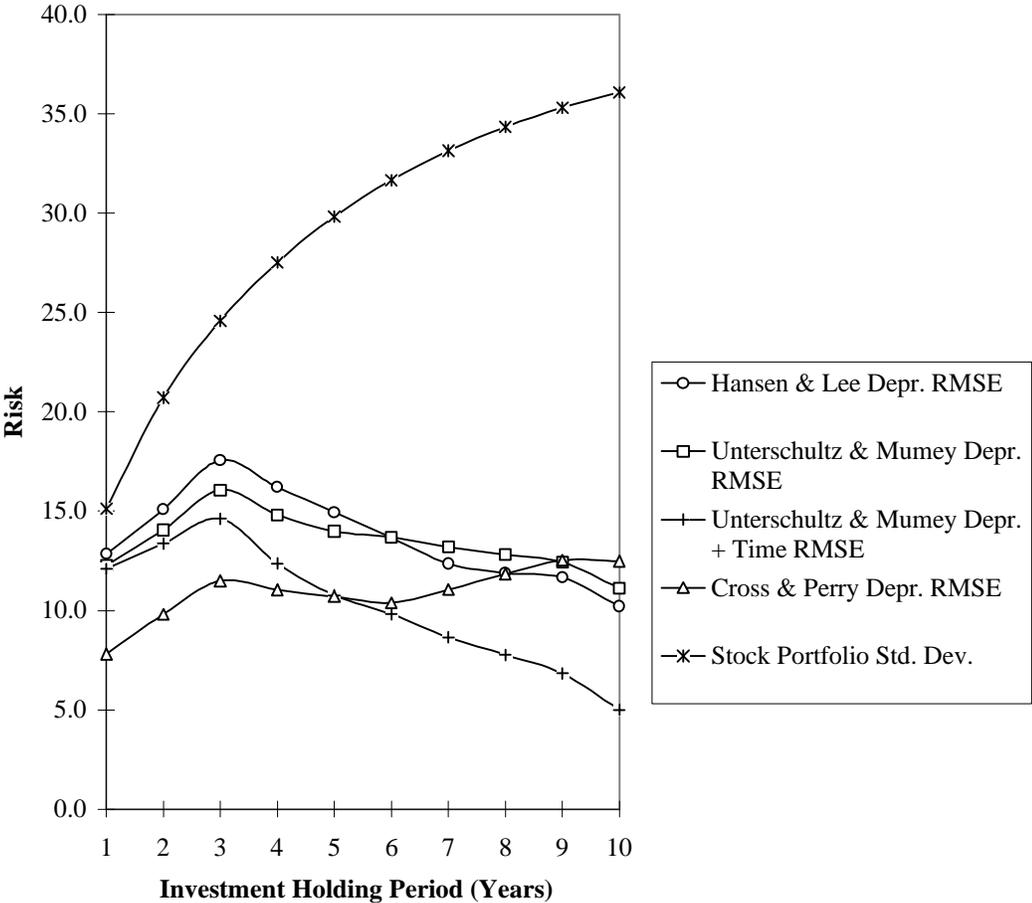
- H&L=Hansen and Lee depreciation estimates, U&M=Unterschultz and Mumey depreciation estimates, U&M+Time=Unterschultz and Mumey depreciation estimates with a time adjustment and C&P=Cross and Perry depreciation estimates. The results are based on the planned purchase of a one year old machine asset. The risk premia is  $r_m=0.069$ .

**Table 2**  
**Tractor and Combine Risk Premium Estimates on an Equivalent-Terminal-Risk-Portfolio For Partially Diversified Investors With a Four Year Investment Horizon**

<i>M.L</i>	<i>P.L</i>	Tractor Risk Premia	Combine Risk Premia
Machinery Only			
---	---	2.8%	8.3%
30% Machinery	70% Land		
0.5	0.5	0.8%	2.5%
0.5	0.0	2.2%	4.3%
0.0	0.5	0.1%	1.1%
0.0	0.0	0.8%	2.5%

- The stock market risk premium is 0.069. The land variance is assumed equal to the stock variance. Increasing the variance of the land increases the risk premia. The investment horizon is for a 4 year investment holding period and uses the Cross and Perry results to estimate the machine MSE. The weight of the machinery is  $\frac{1}{3}$  in the portfolio.

**Figure 1: Comparison of Tractor Terminal Asset Value Risk to Stocks Risk**



**Figure 2: Comparison of Combine Terminal Asset Value Risk to Stocks Risk**

