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FRAME-DIAPHRAGM INTERACTION FOR HORIZONTAL LOADS

by

(C) JAMES KEITH FALCONER

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE.

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TO MY WIFE MARIA CECILIA

ABSTRACT

Current procedures to assess the strength and stiffness of a steel deck diaphragm under in-plane loads are reviewed. A new method to calculate the stiffness of a steel deck diaphragm is developed.

The influence of different parameters that affect the behaviour of a steel deck is analyzed.

The behaviour of one storey rectangular buildings under horizontal loads is studied. A particular solution based on the beam model is applied for buildings having a limited number of bays. A general solution based on a beam on an elastic foundation is used for any number of bays. This procedure is applied for symmetrical as well as nonsymmetrical buildings.

Practical examples of analysis for lateral load resistance of buildings using the method developed are presented.

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LIST OF SYMBOLS

The notation generally corresponds with the notation commonly encountered in texts. However, in certain cases it has not been possible to maintain uniform symbology throughout.

Special Symbols

Σ	denotes a summation
d	denotes ordinary differentiation
$\int$	denotes integration

Roman Symbols

A, A_1, A_2	Constants in Appendices N and Q
A_o	Cross Sectional Area of a purlin
$\bar{A}_s$	Shear area of the horizontal diaphragm
a	Length of a shear panel
a_p	Average spacing of profile channel closures, in feet
a_s	Center to center spacing of seam welds in feet
a_w	Spacing of marginal welds in feet
B, B_1, B_2	Constants in Appendices N and Q
b	Width of a shear panel
C, C_1, C_2	Constants in Appendices N and O
$C'_1, C'_2, C'_3, C'_4, C'_5$	Constants in Appendix F
c	Constant in Equation 3.2
$C_{11}, C_{12}, C_{13}, C_{21}, C_{22}, C_{23}$	Stiffness components in Bryan's method

D, D_1, D_2	Constants in Appendices N and Q
Dx, Dy	Bending stiffness in the x and y direction
d	Constant in Appendix F
do	Pitch of corrugations
E, E_s	Young's Modulus, Young's Modulus of the horizontal diaphragm
F_i	Flexibility factors
F_{eph}, F_{epvi}	Horizontal and vertical force acting on each fastener
F_{R_i}	Resultant force defined in Appendix D
$F_{s_{ci}}$, F_{s_i}	Force taken by a shear connector, seam fastener respectively.
Fu	Ultimate strength of the fastener connection
F_{su}, F_{pu}, F_{sc}, F_{eu}	Strength of seam fasteners, purlin fasteners, end fasteners respectively
f	Constant defined in Appendix N
$f'c$	Compressive strength of concrete
f_1, f_2, f_3	Correction factors in Appendix B
G_s	Shear modulus of the horizontal diaphragm
g_F	Unitary rigidity
g_{1e}, g_{2e}, g_{1p}, g_{2p}	Quantities defined in Appendix E
h_1	Length of bar
h_o	Height of a corrugation

h	Height of the building
I_e, I_g	Quantities defined in Appendix D
I_D, I_X	Gross moment of inertia of deck unit as defined in Appendix F
I_S	Moment of Inertia of horizontal diaphragm
K_o	Sheeting constant in Appendix B
K_1	Quantity defined by Equation 3.8
K_2	Quantity defined by Appendix E
K	Stiffness of diaphragm
K_B, K_F, K_{sh}, K_w, K_s	Bending stiffness of the horizontal diaphragm, stiffness of the frame, shearing stiffness of the horizontal diaphragm, stiffness of the wall, stiffness of the slab respectively
K_2	Quantity defined in Appendix F
L, L_1, L_2	Length of the building between shear walls
L_R, L_V	Quantities defined in Appendix F
lw, l'w	Quantities defined in Appendix F.
m	Quantity defined in Appendix F
N_{cr}	Buckling load of a shear panel
n_e, n_p, n_s, n_{sh}, n_{sc}	Number of end fasteners, purlin fasteners, seam fasteners, sub-panels per panel, sheet connector fastener respectively

n_o	Quantity defined in Appendix F
n	Parameter defined by Equation 4.4
p	Number of interior purlins
Q	Applied force on Diaphragm in Appendix D
Q_{ult}	Failure load defined in Appendix E
Qw, Qw_1, Qw_2	Force taken by middle wall, right wall, left wall respectively
q_{ave}, q_{ud}, q_D, q_1, q_6, q_6', q_6''	Average, limiting, components of shear respectively
q	pitch of sheet purlin fasteners
R	Quantity defined in Appendix F
S	Section modulus of puddle weld group at supports
S_s, S_p, S_{sc}, S_e	Flexibility of seam fasteners, purlin fasteners, shear connectors, end fasteners respectively
t, t_1, t_2', t_f	Thickness of horizontal diaphragm flat sheet element, flutted element and fill over top of deck respectively
V_R, V_T	Force acting on a frame, total force acting on a building respectively
w	Uniformly distributed horizontal load acting on the building

w_f, w_s	Load taken by the frames and the slab, respectively
x	Axis in global coordinate system, also K_F/K_B
x_1	Distance defined in Appendix F1
x_i, x_p	Distance from every end fastener, purlin fastener respectively to the center of rotation of the sub-panel
x_o	Quantity defined in Appendix E
y_1, y_2	Displacement of the building
y	Axis in global coordinate system, also K_F/K_{sh}
z	Axis in global coordinate system, also V_R/V_T

Greek Symbols

α_o	Angle defined in figure 3-6
α	Constant defined in Appendix Q
β	Constant defined in Appendix Q
β_o	Coefficient of inelastic shear deformation
β_1	Constant defined in Appendix D
γ	Unit weight of fill concrete
Δ	Deflection of the horizontal diaphragm
Δ_1, Δ_e	Slip at the fasteners
δ	Coefficient defined in Appendix G
η	Ratio of Young's Modulus

θ_1	Constant defined by Equation A.2
κ	Shear deformation coefficient
λ	Multiplier defined in Appendix A
λ_1	Quantity defined by Equation D.5
ν	Poissons's ratio
ε	Parameter defined in Equation A.6
ϕ	Quantity defined by Equation A.3

CHAPTER I

INTRODUCTION

1. GENERAL REMARKS

When a structure is subjected to lateral loads, a complex distribution of forces results within the structure. The first question that arises is how the lateral forces are transferred to the vertical elements. This distribution of forces can become complicated depending on the geometry of the structure and the stiffness of the horizontal diaphragm and the vertical resisting elements.

1.1 Diaphragm Action

To show the influence of diaphragm action let us consider the one-story building shown in Figure 1.1. As shown, the wind excerts a positive pressure on the windward side of the building. The wall cladding ADKI is assumed to have enough strength to work as a loaded girder spanning vertically between the wall foundation and the perimeter roof beams AB, BC and CD. The roof surface ADEH is loaded in its own plane along its edge AD. The roof system transmits this horizontal edge load from its line of application to the frames and shear walls and hence to the building foundations.

In the case of an earthquake loading inertia forces are important. These forces depend on the mass of the element involved. The equivalent static force which must be applied to a structure in design is generally determined by building codes as

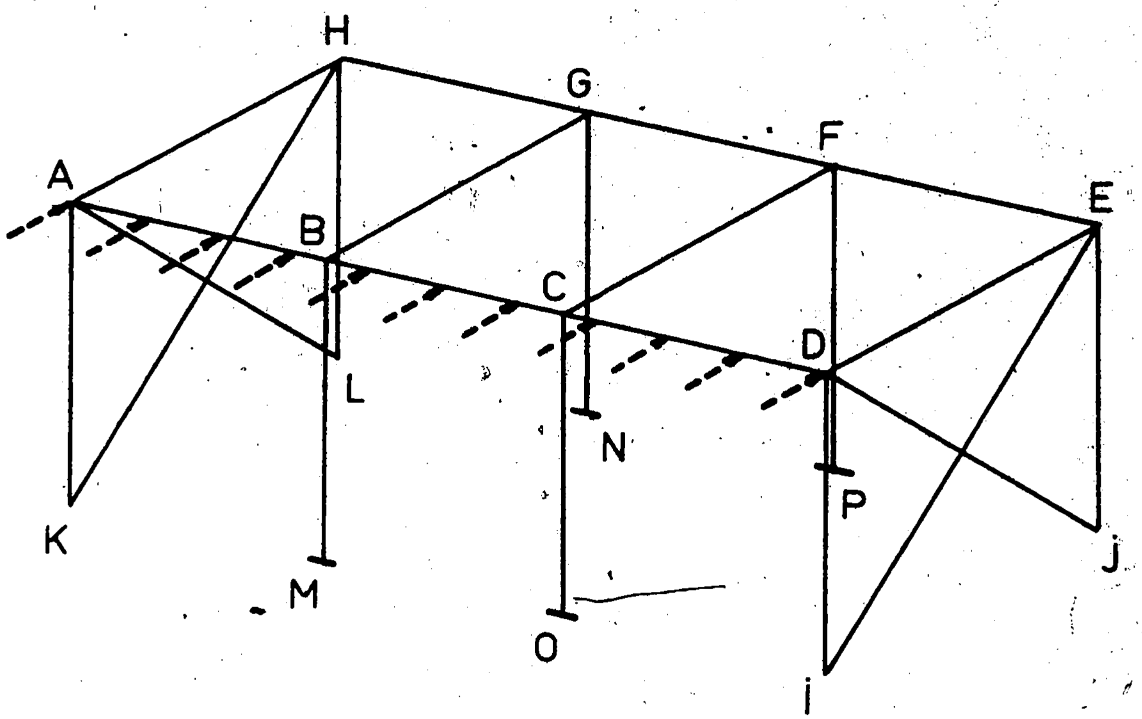


Figure I-I Horizontal Forces on a Building a percentage of the dead weight of the structure. This force is assumed to act at the roof line of a one-storey structure or at each floor line in the case of multiple stories. The roof or floor is designed as a diaphragm to transmit the force to the resisting cross walls.

In conventional earthquake and wind calculations, reinforced concrete horizontal diaphragms are assumed to be infinitely rigid relative to their vertical supports. The lateral forces applied to the diaphragm are assumed to be distributed to the vertical supports in proportion to the relative stiffness of such supports. However, in a structure with widely spaced shear walls or in a structure where steel decks or concrete filled steel decks are used as horizontal diaphragms, the deformation of the diaphragm can be significant. In these cases, the distribution of seismic forces in the vertical-resistant elements is markedly different from the distribution that would result if the horizontal diaphragms were assumed to be rigid. If the diaphragms are very flexible the vertical supports are assumed to be rigid relative to the diaphragm and lateral forces are assumed to be distributed to the vertical supports in proportion to the contributing diaphragms areas. For diaphragms intermediate between rigid and flexible, one or the other of the above methods is usually applied, depending on which idealized situation is more closely approximated. Therefore there is a need to know the distribution of forces to the vertical resisting elements when the diaphragms are neither infinitely rigid nor infinitely flexible.

1.2 OBJECTIVE AND SCOPE

The purpose of this study is to analyze the influence of the diaphragm stiffness on the distribution of lateral forces to the structural resisting elements in buildings. The study is limited to one storey symmetrical and nonsymmetrical rectangular buildings.

Chapter II reviews and analyzes different methods to calculate strength and stiffness for steel decks. Then a method is developed by the writer to assess the stiffness of steel deck diaphragms. Finally the parameters that influence the stiffness and strength of steel deck diaphragms are studied.

Chapter III is devoted to a study of the influence of diaphragm stiffness on the distribution of lateral forces to the vertical resisting elements for symmetrical buildings.

Chapter IV presents the analysis of the influence of diaphragm stiffness on lateral force distribution for nonsymmetrical buildings.

Chapter V is devoted to the application of the theory developed in Chapter III to practical examples.

CHAPTER II

CRITICAL REVIEW OF CURRENT METHODS TO EVALUATE STRENGTH AND STIFFNESS OF STEEL DECK DIAPHRAGMS

2.1 INTRODUCTION

The intent of this chapter is to analyze and compare some procedures, developed in Great Britain and North America, which are currently used in engineering practice, to calculate the strength and stiffness of steel deck diaphragms. A new approach to assess the stiffness of a steel deck diaphragm is developed and on the basis of tests performed in England is compared to the above mentioned procedures. At the end of the chapter, the parameters that influence the stiffness and strength of the steel diaphragms are studied, so that, when designing a diaphragm a swift decision may be taken.

2.2 EVALUATION OF STRENGTH AND STIFFNESS OF STEEL DECK DIAPHRAGMS

The strength and stiffness of decks can be evaluated using several methods as detailed below.

Testing. This is the most accurate way of assessing the strength and stiffness of diaphragms, but at the same time, the most expensive and time consuming. Thus, it has not been used extensively in current engineering practice.

Finite Element Method. The diaphragm is visualized as an assemblage of structural elements interconnected at a discrete number of nodal points. The corrugated steel sheeting is represented by an orthotropic plate element, purlins and edge beams are defined as beam elements and seam fasteners, end fasteners and shear connectors are represented by spring elements. This method yields results that are accurate and additionally provides information regarding internal force distributions. Furthermore, it may be applied to diaphragms with openings, which is not possible currently for other methods with the exception of testing. The finite element method has the following shortcomings:

(i) a computer is needed which is not always available to designers;

(ii) The preparation of the input data is time consuming; and,

(iii) numerical errors can occur because of the large amount of input data required.

<u>Analytical Method</u>. This approach uses either the Energy or Equilibrium methods or both. Although these methods are not exact they provide readily applicable solutions to practically any type of steel diaphragm.

<u>Empirical Method</u>. The Empirical method is based on the result of testing, so that, it may be applied to decks having similar characteristics. The results are given in tables usually supplied by the firms that manufacture deck material. Empirical equations have been provided by the U.S. Department of Army, Navy and Air Force (31) and by Luttrell (21) whose research in West

Virginia University has made available information regarding the behaviour of steel deck diaphragms. Some of the companies that make use of these equations are Westeel Rosco, Bowman-Peterson, Westland Metals, etc.

Generally what is required for design purposes is a readily available design procedure which provides a reliable assessment of the strength and stiffness of diaphragms. There are only two, of the above mentioned procedures which satisfy that requirement, i.e. the Analytical and the Empirical methods. For steel deck diaphragms both methods are available, for concrete deck diaphragms only Empirical Equations have been developed. The Analytical methods are discussed in detail in the following sections.

2.3 BEHAVIOUR OF A STEEL DECK DIAPHRAGM UNDER IN-PLANE LOADS

A typical isolated diaphragm is shown in Figure 2.1. It is made up of a number of component elements, that is, corrugated steel sheeting, purlins, rafters, shear connectors, end fasteners and seam fasteners. Different alternate arrangements are possible. The sheeting may be fixed to the supporting structure on all four sides or only on two sides. The corrugations on the individual panels may be aligned parallel or perpendicular to the applied force.

When a force is applied to a diaphragm a distribution of forces among the different elements that make up the diaphragm takes place. By means of fasteners part of the force will be

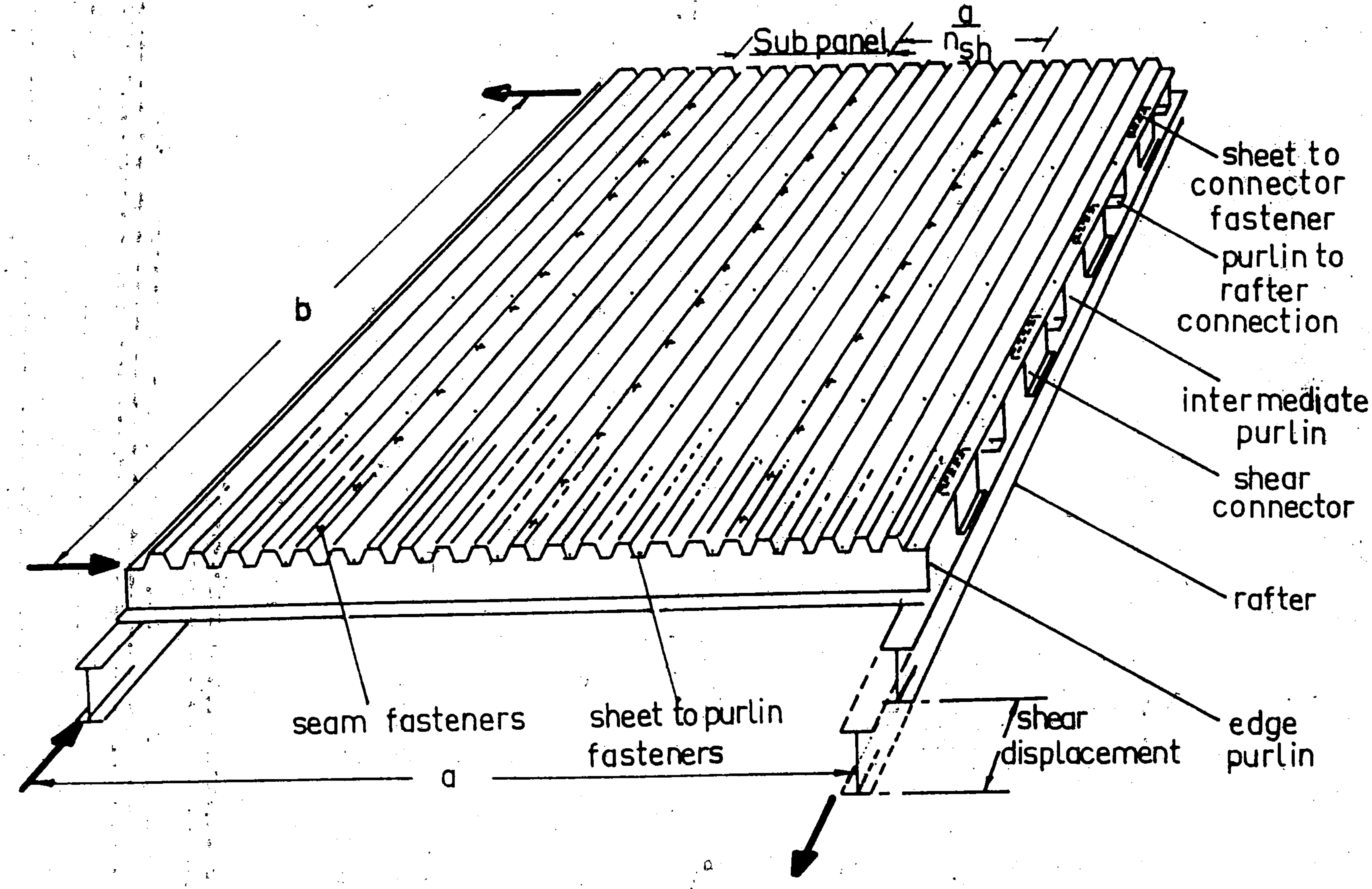


Figure 2-1 Arrangement of typical shear panel

taken by end and intermediate purlins and the remainder is carried by the subpanels.

Under these assumptions, two different modes of failure of the diaphragm may occur. First, if there are many fasteners and with enough strength not to fail before buckling of the sheeting occurs, this failure that is called "Buckling as an orthotropic plate". This kind of failure is uncommon and represents the upper bound load that a steel diaphragm could take. Figure 2-2 illustrates such a buckling failure. Second, if the number or the strength of the fasteners is low, the steel deck fails at loads below buckling by failure at the fasteners. This failure may occur by tearing of the sheeting at the fastener locations or by simple shear failure of the fasteners. This kind of failure is called "Failure at the fasteners".

2.4 ANALYTICAL METHODS

2.4.1 Failure as an orthotropic plate

There are several published approaches that assess the buckling failure of a diaphragm; mainly Bergmann and Reissner's (4) (Appendix A, Eq. A.1), Hlavacek's (21) (Appendix A, Eq. A.4), Easley-McFarland's (14) (Appendix A, Eq. A.6), Easley's (15) (Appendix A, Eq. A.6). All of them give similar solutions for the critical buckling load. Easley's (15) equations (Appendix A, Eq. A.6) are the most rigorous, but yield solutions only when the parameter ε is known; ε is only available for simple and clamped supports. Hlavacek's equations (20) (Appendix A, eq. A.4) are

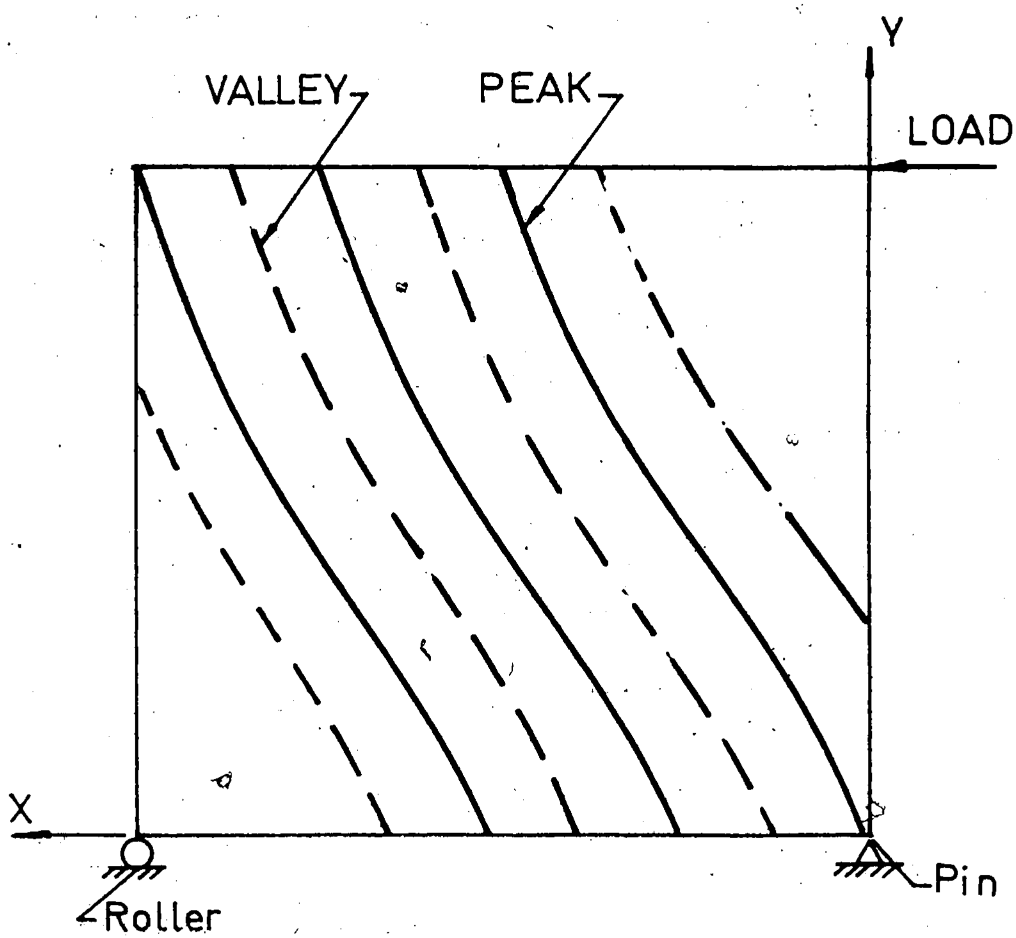


Figure 2-2 Buckled Pattern

applicable for a completely different kind of deck, that is a flat sheet which is stiffened by perpendicular plates distributed symmetrically to both faces of the sheet.

Bergmann and Reissner (4) were the first investigating buckling load and according to Easley (15) the derivation of their formula is the most rigorous of all.

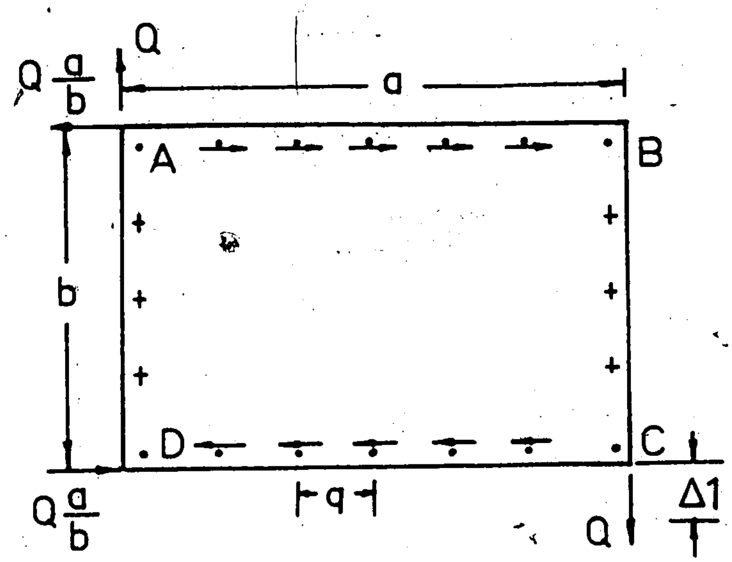
2.4.2 Failure at the fasteners

A number of researchers have provided solutions for the failure that the fasteners; among them can be mentioned Bryan (7) and Davies (11) in England being the pioneers, Easley (13) in the United States and Fazio (17) et al. in Canada. The most relevant contributions have been made by Bryan (7), Davies (11) and Easley (13), while Fazio (17) et al. have refined Easley's (13) method. In the following sections a description of the solutions proposed by the forementioned authors is presented together with the advantages and disadvantages of their methods. Finally a refined approach is presented.

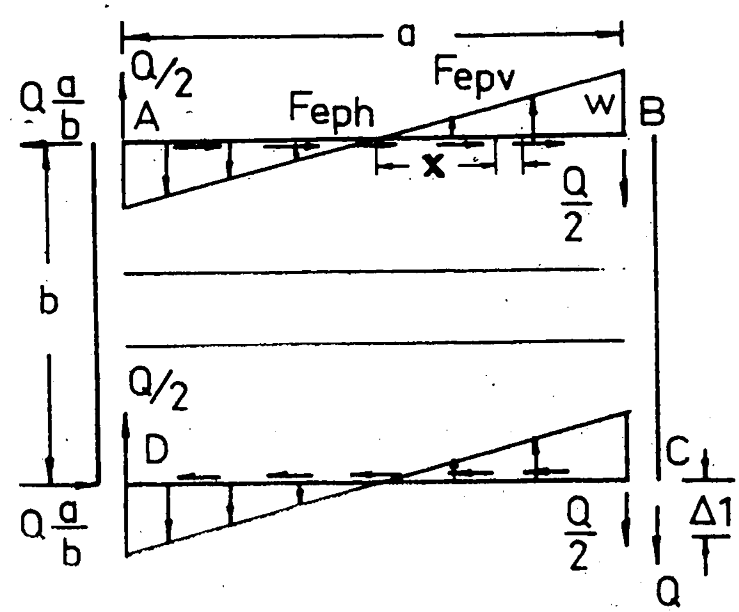
2.4.2.1 Bryan's Method

Bryan's approach (7) has been accepted as a feasible solution for steel decks in Great Britain. His method is based on the Energy Method and Equilibrium. Details of his method are presented in Appendix B.

Bryan considers the flexibility of a diaphragm instead of the stiffness. He recognizes two types of steel deck and assumes a distribution of internal forces as shown in Figure 2



(a) Sheet Fasteners on all four edges



(b) Sheet Fasteners on two edges only

Figure 2-3 Sheet Fasteners

3. If shear connectors are present (Figure 2-3(a)) there will be slip only at the end fasteners; a mode of deformation that he called "Direct Shear Transfer". Otherwise, Figure 2-3(b) applies, in which there is a linear distribution of forces at the end fastener locations. Bryan referred to this as the "Indirect Shear Transfer".

Bryan states that the overall flexibility of a steel deck is the result of the summation of the flexibilities of the components that make-up a steel deck. Consequently he analyzes the sheet deformation, shear strain of the sheet, axial strain in the purlins and sheet connection deformations. By studying the effects separately, the influence of each factor in the overall flexibility can be assessed and the less important factors may be discarded.

Bryan's solution cannot be considered exact because it requires the determination of six different parameters by testing. Rigorously his solution should only be applicable for the kind of deck and connections produced by the different fabricators in Great Britain; when applying his method to decks manufactured in North America testing should be required to obtain new parameters.

Bryan's assumed distribution of internal forces is not real according to observations made on tested panels (tilting of the sub-panels). This affects the flexibility contribution of the connections to the overall flexibility of the panel but does not affect the results for the axial forces in the purlins, the sheet deformation and the shear strain of the sheet.

Bryan did not take into account the vertical distribution of forces that exists at the purlin connections; this is an item that may be incorporated in his approach to improve his method.

The advantage of his method is that the expressions for stiffness are not difficult to calculate.

Bryan considers different possible modes of failure assuming an internal force distribution which is not realistic. The ultimate strength of the deck is obtained by considering failure at the seam connection, or, failure at the shear connectors, or, failure at the sheet-purlin connection, or, failure at the end purlins by buckling whichever is least.

2.4.2.2 Davies' Method

Davies' (11) method is based on Bryan's (7) work. He assumes on the basis of Finite element analysis, an improved distribution of internal forces. Details of his method are presented in Appendix C.

Davies' assumptions can be discussed best with reference to Figures 2.4 and 2.5. The first assumption is that the seam slip at each internal seam is equal and denoted by 2Δ. The second assumes that the total horizontal force Qa/b in the two edge purlins is equally shared between the sub-panels. Using these two assumptions vertical and horizontal equilibrium are satisfied for an interior sub-panel. When the same principle is applied to the end panel, vertical equilibrium is not satisfied, so that he makes a further assumption: the slip at the end

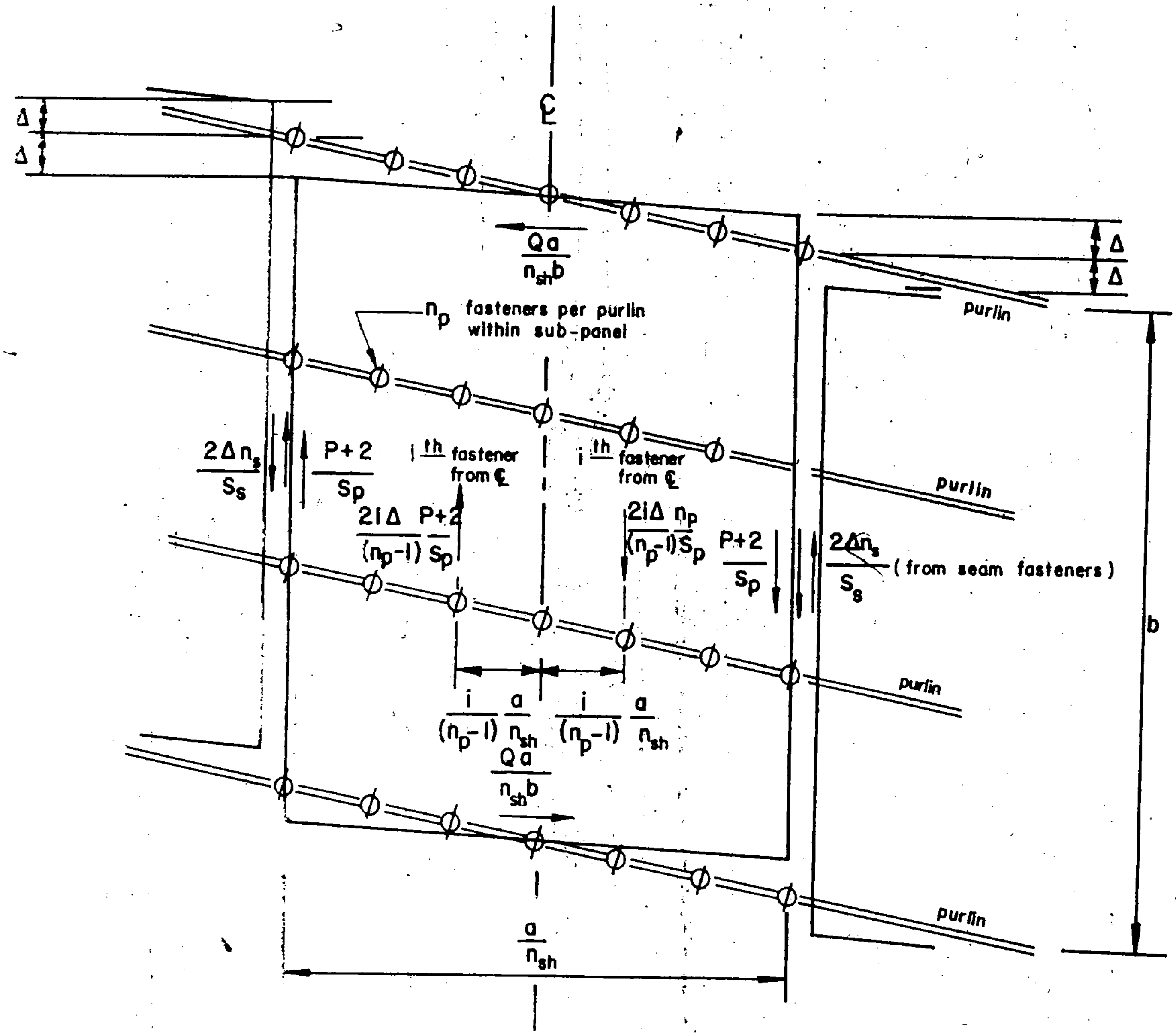


Figure 2-4 Forces and Deformations Assumed for Typical Internal Sub-Panel

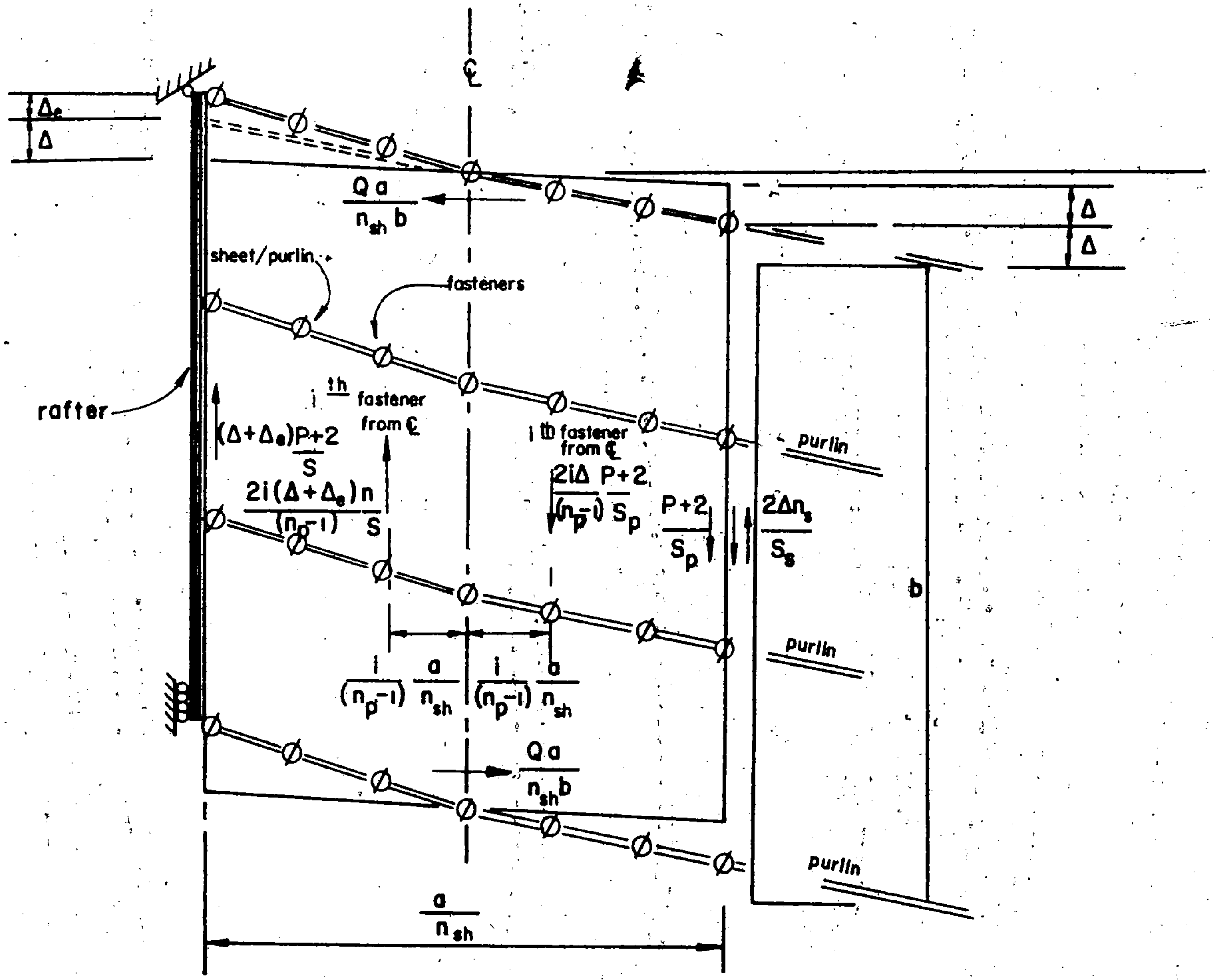


Figure 2-5 Forces and Deformations Assumed for End Sub-Panel

connectors is $\Delta e + \Delta$. This Δe is obtained from equilibrium considerations violating moment equilibrium for the end sub-panel.

2.4.2.3 Easley's Method

Easley's method (13) is basically an equilibrium approach that requires some parameters that are obtained by testing. His main assumption is that the sub-panel in a diaphragm tends to tilt in the frame, as shown in Figure 2-6(a); this was also confirmed by test observations. Davies (11) applied the same concept to his theoretical approach, and MacGregor (23) testing independently at the University of Alberta made the same observation. Easley's (13) assumptions are discussed in detail in the following paragraphs.

The assumption that sub-panels deform in the same way is a simplifying one that leads to simple equilibrium equations for the interior sub-panels, however the connections used are the same for all the sub-panels; this is an idealization not applicable to real steel deck diaphragms.

The assumption that sub-panel end fastener forces have components parallel and perpendicular to the panel ends may be easily understood with the aid of Figure 2.7, which shows that panel rotation occurs when a lateral force is applied. The magnitude of the perpendicular component of force increase proportionally to the distance from the center of rotation. This is similar to an assemblage of bolts acted upon by a moment where the force taken by any bolt is proportional to the bolt distance

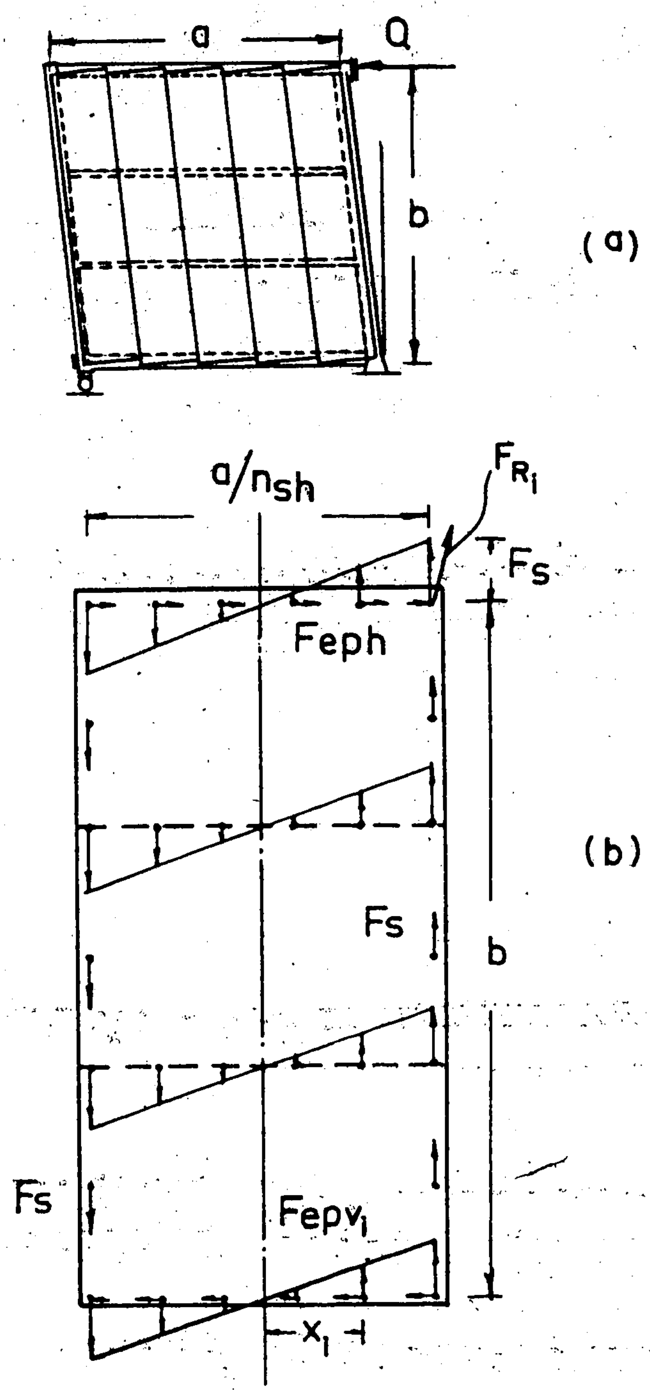


Figure 2-6 Fastener Forces

from the center of rotation.

Easley assumes that the distribution of forces for the sub-panel side fasteners are uniform in magnitude and parallel to the sides of the sub-panel. Theoretically this assumption is not correct. When the diaphargm starts to tilt (Figure 2.7) some forces perpendicular to the side of the sub-panels should be developed. Also, as it has been shown by Finite Element Analysis by Nilson (28) these side fastener forces are distributed parabolically.

Easley (13) also observed in his testing that failure of the diaphragm could occur because of local bearing failure or tear out of the sub-panels at the fasteners either those which hold sub-panels together or those connecting sub-panels to the frames, or failure in the fasteners themselves. He recognized two different types of connection failure, either failure at the end fastener located in line with the seam fastener or failure at the seam fasteners. The first assumption of failure is theoretically correct because it corresponds to the fastener taking the maximum vertical load (see Figure 2.6) according to the linear force distribution. Testing has confirmed this assumption. The second mode of failure theoretically corresponds to all the seam fasteners in a sub-panel failing simultaneously. Testing has confirmed this assumption too. Although the actual distribution of forces is parabolic, a redistribution of forces takes place before failure, confirming that the behaviour of a diaphragm is influenced by plastic deformations. The main shortcoming of this approach is that it

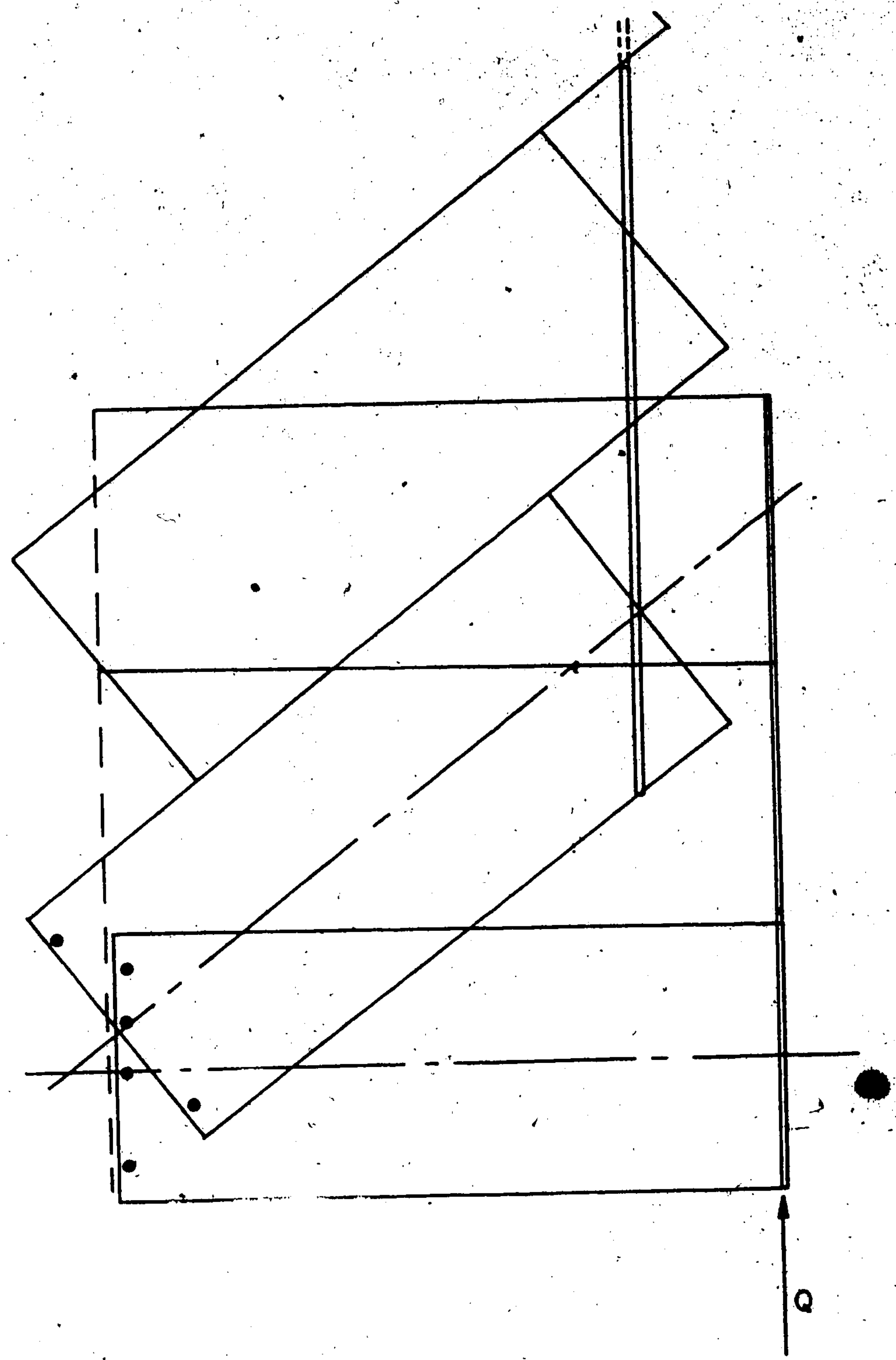


Figure 2-7 Shear Panel Rotation

is not suitable to the type of steel deck usually made in North America in which fasteners of different shear stiffnesses are used; however, his equations can be modified to include fasteners that have different load-deformation characteristics.

2.4.2.4 Fazio, Ha, El-Hakim's Method

This approach (17) is based on the methods of Bryan (7), Davies (11) and Easley (13) and it has the same shortcomings already pointed out. Nevertheless, unlike Easley's (13) method, this method considers equilibrium of an external sub-panel using different type of fasteners for the end, purlin and side attachments. Therefore the purpose of Fazio, Ha, El-Hakim's approach is to analyze steel decks that are commonly used in practice. The main assumption made is that the stress distribution at the end fasteners for the end sub-panel is not symmetrical; it is symmetrical for the internal sub-panels (see Figure 2.8). This leads to simple equations, thus making the method attractive and easy to apply. However, Fazio, Ha, El-Hakim's assumption satisfies the equilibrium of the external sub-panel but not the equilibrium of the internal sub-panels.

2.4.2.5 Writer's Approach

In this section a solution to the stiffness of a steel deck diaphragm has been developed by the writer and is discussed in detail in the following paragraphs.

The assumptions on which this method is based are:

1. The seam slip for a line of seam-fasteners is

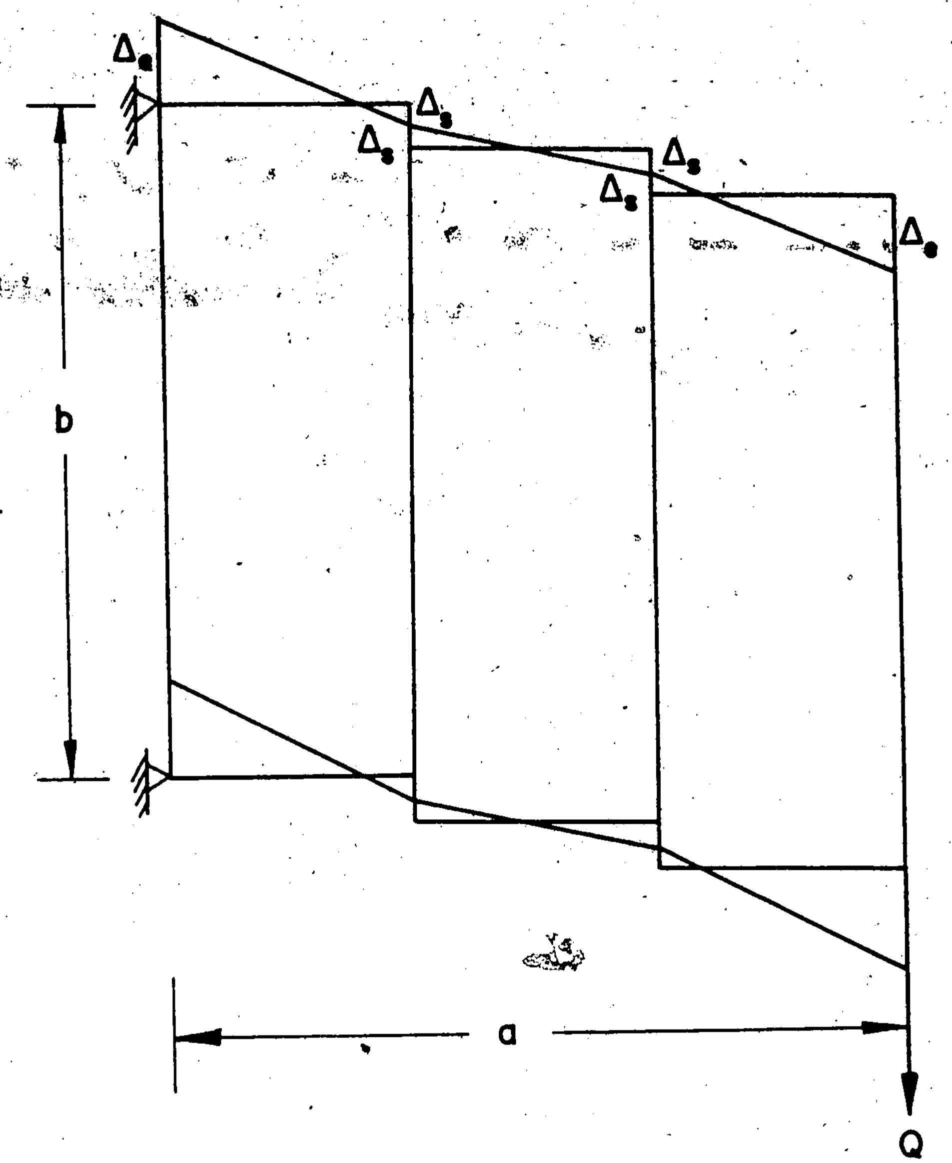


Figure 2-8 Deformation of the Panel

constant, but it varies from one line of seam-fasteners to the next according to the mode of deformation assumed in Figure 2.9.

2. The vertical component of the shear connector force is constant.

3. The horizontal component of the shear connector and seam fastener force is negligible.

4. The horizontal force taken by the end purlins is shared evenly be each sub-panel

5. The vertical components of the end fastener and the purlin fastener forces vary linearly at every sub-panel.

For every sub-panel there are two unknown displacements. Two equations are available for each sub-panel, from vertical force equilibrium and moment equilibirum, which permit the determination of displacements in each sub-panel. These equations are solved simultaneously for the complete panel to obtain the total displacement.

The necessary equations using equilibrium and compatibility are developed below with the aid of Figure 2.9.

The vertical equilibrium of the first sub-panel yields

$$n_{scl} F_{scl} - n_s F_{sl} + \frac{2\Delta_1}{Se} \sum_{i=1}^{ne} \left(1 - \frac{x_i}{x_{ol}}\right) + \frac{P\Delta_1}{Sp} \sum_{i=1}^{n_p} \left(1 - \frac{x_i}{x_{ol}}\right) = 0 \qquad (1)$$

in which

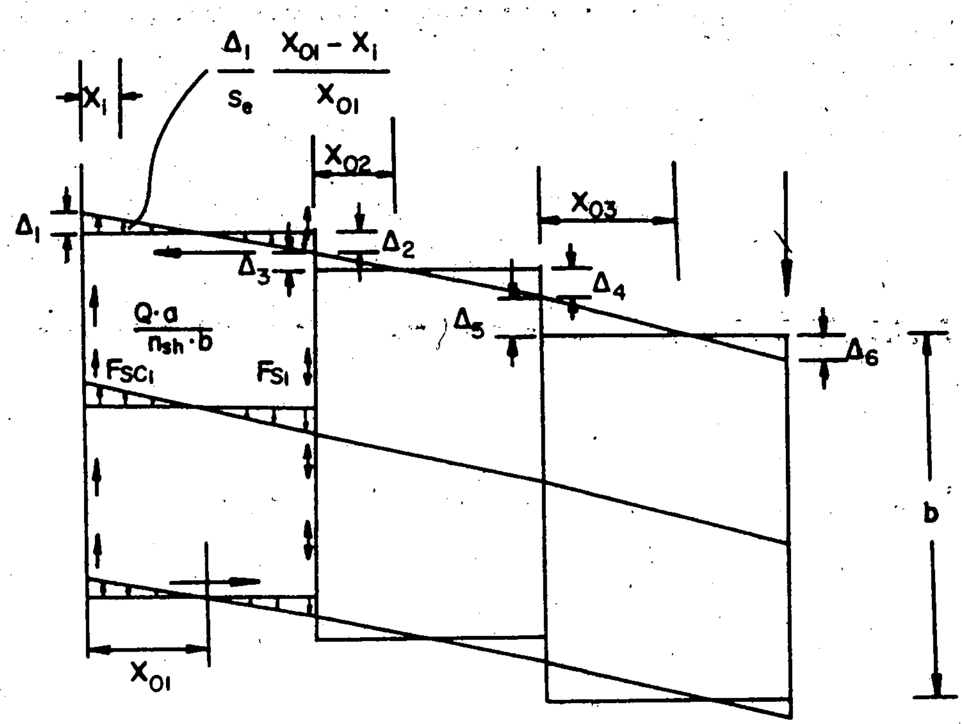


Figure 2-9 Writer's Approach

$$F_{scl} = \Delta_1/S_{sc} \tag{2}$$

$$F_{sl} = (\Delta_2 + \Delta_3)/S_s \tag{3}$$

Replacing 2 and 3 in 1 results

$$n_{sc}\frac{\Delta_1}{S_{sc}} - n_s\frac{(\Delta_2+\Delta_3)}{S_s} + \frac{2\Delta_1}{S_e}\sum_{i=1}^{n_e}\left(1-\frac{x_i}{x_{o1}}\right) + \frac{P\Delta_1}{S_p}\sum_{i=1}^{n_p}\left(1-\frac{x_i}{x_{o1}}\right) = 0 \tag{4}$$

Also, because of linear distribution of stresses within the sub-panel

$$\frac{\Delta_1}{\Delta_2} = \frac{x_{o1}}{\left(\frac{a}{n_{sh}} - x_{01}\right)} \tag{5}$$

or

$$x_{o1} = \frac{\Delta_1}{\Delta_1+\Delta_2}\,\frac{a}{n_{sh}} \tag{6}$$

Replacing 6 in 4 results

$$n_{sc}\frac{\Delta_1}{S_{sc}} - n_s\frac{(\Delta_2+\Delta_3)}{S_s} + \frac{2\Delta_1}{S_e}\sum_{i=1}^{n_e} 1 - \frac{2\Delta_1}{S_e}\sum^{n_e}\frac{x_i}{\frac{\Delta_1}{\Delta_1+\Delta_2}\,\frac{a}{n_{sh}}}$$

$$+ \frac{P\Delta_1}{S_p}\sum_{i=1}^{n_p} 1 - \frac{P\Delta_1}{S_p}\sum_{i=1}^{n_p}\frac{x_i}{\frac{\Delta 1}{\Delta_1+\Delta_2}\,\frac{a}{n_{sh}}} = 0 \tag{7}$$

Rearranging terms

$$\Delta_1\left(\frac{n_{sc}}{S_{sc}} + \frac{2n_e}{S_e} + \frac{P n_p}{S_p} - \frac{2n_{sh}}{S_e a}\sum^{ne} x_i - \frac{P n_{sh}}{S_p a}\sum^{n_p} x_i\right)$$

$$- \Delta_2\left(\frac{n_s}{S_s} + \frac{2nsh}{S_e a}\sum^{ne} x_i + \frac{P n_{sh}}{S_p a}\sum^{n_p} x_i\right) - \Delta_3 \frac{n_s}{S_s} = 0 \tag{8}$$

Moment equilibrium of the first sub-panel gives

$$n_s F_{s1} \frac{a}{n_{sh}} - \frac{2\Delta_1}{S_e}\sum_{i=1}^{n_e}\left(1 - \frac{x_i}{x_{o1}}\right) x_i - P\frac{\Delta_1}{S_p}\sum_{i=1}^{n_p}\left(1 - \frac{x_i}{x_{o1}}\right) x_i = \frac{Qa}{n_{sh}} \tag{9}$$

Replacing 6 in 9 results

$$\Delta_1\left(\frac{2n_{sh}}{S_e a}\sum_{i=1}^{n_e} x_i^2 + \frac{P n_{sh}}{S_p a}\sum_{i=1}^{n_e} x_i^2 - \frac{2}{S_e}\sum^{n_e} x_i - \frac{P}{S_p}\sum^{n_p} x_i\right) +$$

$$\Delta_2\left(\frac{2n_{sh}}{S_e a}\sum_{i=1}^{n_e} x_i^2 + \frac{P n_{sh}}{S_p a}\sum_{i=1}^{n_p} x_i^2 + \frac{n_s a}{S_s n_{sh}}\right) + \Delta_3 \frac{n_s a}{S_s n_{sh}} = \frac{Qa}{n_{sh}} \tag{10}$$

Vertical equilibrium of the second sub-panel yields

$$n_s F_{s1} - n_s F_{s2} + \frac{2\Delta_3}{S_e} \sum_{i=1}^{n_e} (1 - \frac{x_i}{x_{o2}}) + \frac{P\Delta_3}{S_p} \sum_{i=1}^{n_p} (1 - \frac{x_i}{x_{o2}}) = 0 \quad (11)$$

$$Fs_2 = \frac{\Delta_4 + \Delta_5}{S_s} \quad (12)$$

Because of linear distribution of stresses within the sub-panel

$$\frac{\Delta_3}{\Delta_4} = \frac{x_{o2}}{\frac{a}{n_{sh}} - x_{o2}} \quad (13)$$

or

$$x_{o2} = \frac{\frac{a}{n_{sh}} \quad \Delta_3}{\Delta_3 + \Delta_4} \quad (14)$$

Replacing 3, 12 and 14 in 11 yields

$$\Delta_2 \frac{n_s}{S_s} + \Delta_3 (\frac{2ne}{S_e} + \frac{n_s}{S_s} + \frac{P \quad sp}{S_p} - \frac{Pn_{sh}}{Sp_a} \sum^{n_p} x_i - \frac{2 \quad nsh}{aS_e} \sum x_i$$

$$- \Delta_4(\frac{n_s}{S_s} + \frac{2n_{sh}}{aSe} \sum_{i=1}^{n_e} x_i + \frac{Pn_{sh}}{Sp_a} \sum_{i=1}^{n_p} x_i) - \Delta_5 \frac{n_s}{S_s} = 0 \quad (15)$$

Moment equilibrium of the second sub-panel yields

$$n_s F_{s2} \frac{a}{nsh} - \frac{2\Delta_3}{S_e} \sum_{i=1}^{n_e} (1 - \frac{x_i}{x_{o2}}) x_i - \frac{P\Delta_3}{S_p} \sum^{n_p} (1 - \frac{x_i}{x_{o2}}) x_i = \frac{Qa}{n_{sh}} \quad (16)$$

Replacing 12 and 14 in 16 results

$$\Delta_3 (\frac{2n_{sh}}{S_e a} \sum^{n_e} x_i^{\,2} + \frac{Pn_{sh}}{S_p a} \sum^{n_p} x_i^{\,2} - \frac{2}{S_e} \sum^{n_e} x_i - \frac{P}{S_p} \quad x_i +$$

$$+ \Delta_4 (\frac{2nsh}{S_e a} \sum^{n_e} x_i^{\,2} + \frac{P_{n_{sh}}}{S_p a} \sum^{n_p} x_i^{\,2} + \frac{n_s a}{S_s n_{sh}}) + \frac{\Delta_5 n_s a}{S_s n} = \frac{Qa}{n_{sh}} \quad (17)$$

Vertical equilibrium of the third sub-panel yields

$$n_s F_{s2} - n_{sc} F_{sc2} + \frac{2\Delta_5}{S_e} \sum (1 - \frac{x_i}{x_{o3}}) + \frac{P\Delta_5}{S_p} \sum (1 - \frac{x_i}{x_{o3}}) = 0 \quad (18)$$

$$F_{sc2} = \frac{\Delta_6}{S_{sc}} \quad (19)$$

and

$$\frac{\Delta_5'}{\Delta_6} = \frac{x_{o3}}{\frac{a}{n_{sh}} - x_{o3}} \quad (20)$$

or

$$x_{o3} = \frac{\Delta_5}{\Delta_5+\Delta_6} \; \frac{a}{n_{sh}} \tag{21}$$

Replacing 12, 19 and 20 in 18 results

$$\Delta_4 \frac{n_s}{S_s} + \Delta_5 \left(\frac{n_s}{S_s} + \frac{2n_e}{S_e} + \frac{Pn_p}{S_p} - \frac{2n_{sh}}{S_e a} \sum x_i - \frac{Pn_{sh}}{S_p a} \sum x_i\right)$$

$$- \Delta_6 \left(\frac{n_{sc}}{S_{sc}} + \frac{2n_{sh}}{S_e a} \sum^{n_e} x_i + \frac{Pn_{sh}}{S_p a} \sum^{n_p} x_i\right) = 0 \tag{22}$$

Moment equilibrium yields

$$n_{sc}\Delta_6 \frac{a}{S_{sc}\, n_{sh}} - 2\frac{\Delta_5}{S_e} \sum \left(1 - \frac{x_i}{x_{o3}}\right) x_i - \frac{P\Delta_5}{S_p} \sum \left(1 - \frac{x_i}{x_{o3}}\right) x_i = \frac{Qa}{n_{sh}} \tag{23}$$

Replacing 21 in 23.

$$\Delta_5 \left(\frac{2n_{sh}}{S_e a} \sum x_i^2 + \frac{Pn_{sh}}{S_p a} \sum x_i^2 \quad 2 \sum \frac{x_i}{S_e} \quad \frac{P}{S_p} \sum x_i\right) \tag{24}$$

$$+ \Delta_6 \left(\frac{n_{sc}}{S_{sc}} \frac{a}{n_{sh}} + \frac{2n_{sh}}{S_e a} \sum x_i^2 + \frac{Pn_{sh}}{S_p a} \sum x_i^2\right) - \frac{Qa}{n_{sh}}$$

where:

Fs_i = Force taken by a seam fastener at line i

Fs_{ci} = Force taken by a shear connector at edge i

n_e = number of end fasteners

n_p = number of purlin fasteners

n_s = number of seam fasteners

n_{sc} = number of shear connectors

p = number of intermediate purlins

s_e = flexibility of an end fastener

s_p = flexibility of a purlin fastener

s_s = flexibility of a seam fastener

s_{sc} = flexibility of a shear connector.

x_i = distance from the left edge of sub-panel i to the end and purlin fastener as shown in Figure 2-9

x_{oi} = distance from the left edge of sub-panel i to the neutral axis of end and purlin fastener as shown in Figure 2-9

Δ_i = displacement of left end fastener of sub-panel i

The same procedure may be applied for more than 3 sub-panels.

Once the individual displacements are obtained, this component of the shear stiffness can be added to the sheet shear deformational stiffness and the stiffness resulting from axial strain in purlins, as derived by Bryan (7).

The advantages of this method over the previously mentioned methods may be summarized as follows: While Easley (13) considers only one type of connections, this method makes use of different types, which is more applicable to real steel decks. Fazio's (17) approach assures equilibrium of the external sub-panel but does not satisfy equilibrium for the interior ones. Davies' approach violates moment equilibrium for the external sub-panel. The writer's approach meets moment and force requirements for every sub-panel. The only shortcoming of the writer's approach is that it requires a computer program to solve the set of simultaneous equations.

2.5 THE EMPIRICAL METHOD OF THE U.S. DEPARTMENT OF ARMY, NAVY AND AIR FORCE

The equations used by the U.S. Department of Army, Navy and Air Force (33) (Appendix F) have an empirical basis. They are the result of information gathered by the Triservice Seismic Design Committee and received from the companies supplying the material. According the the U.S. Department of Army, Navy and Air Force (33) the equations for strength and stiffness are subjected to some revision as new data is obtained, which makes the method not applicable for a particular kind of deck, but only, for decks having similar characteristics to the ones tested.

This method is the only one that provides equations to calculate the stiffness and strength for steel concrete diaphragms. It is illustrated in [illegible] example in Chapter V

2.6 COMPARISON OF ANALYTICAL METHODS

Only a limited amount of published information regarding tests performed on steel decks is available in the literature. A series of tests by Davies (11) were selected to assist in a comparison and assessment of each of the applicable analytical methods described above. Calculations for deck flexibilities were made and are presented in Table 2.1, together with the experimental results reported by Davies (11). Parameters common to all the methods are shown in Table 2.2.

Figures 2-10 and 2-11 show the dimensions and general arrangement of steel decks tested by Davies (11).

Basically panels A and B are similar using an open corrugated profile (Amascodeck R15, Ref. 7).

Panels A1 and A2 are of direct shear transfer, that is, with shear connectors. While A1 has purlin fasteners and end fasteners in every corrugation; A2 has fasteners only in alternate corrugations.

Panels B1, B2 and B3 are of indirect shear transfer that is, without shear connectors and with purlin and end fasteners at every corrugation, at alternate corrugations and at every third corrugation respectively.

Panel C1 and C2 are similar, without shear connectors and with purlin and end fasteners at every corrugation and at alternate corrugations, respectively.

From the results summarized in Table 2.2, it is observed that the overall flexibility depends heavily on two parameters, sheet distorsion and the fastener flexibility. These two parameters account for at least 70% of the total flexibility of the steel deck.

Table 2.1 Summary of Panel Flexibilities in inch/kip

Test	Experiment	Writer's Approach	%	Fazio's	%	Davies'	%	Bryan	%
A1	0.0473	0.0338	71.5	0.0422	89.2	0.045	95.1	0.0467	98.7
A2	0.1590	0.1580	99.4	0.175	110.1	0.169	106.3	0.1720	108.1
B1	0.121	0.048	39.7	0.052	43	0.074	61.2	0.0548	45.3
B2	0.201	0.197	98.0	0.191	95.0	0.206	102.5	0.1957	97.4
B3	0.385	0.533	138.4	Not available		0.507	131.7	0.5114	132.8
C1	0.0149	0.0213	143	0.0166	111.4	0.0191	128.2	0.114	765.6
C2	0.0613	0.0686	111.9	0.0609	99.3	0.0613	100	0.198	323.

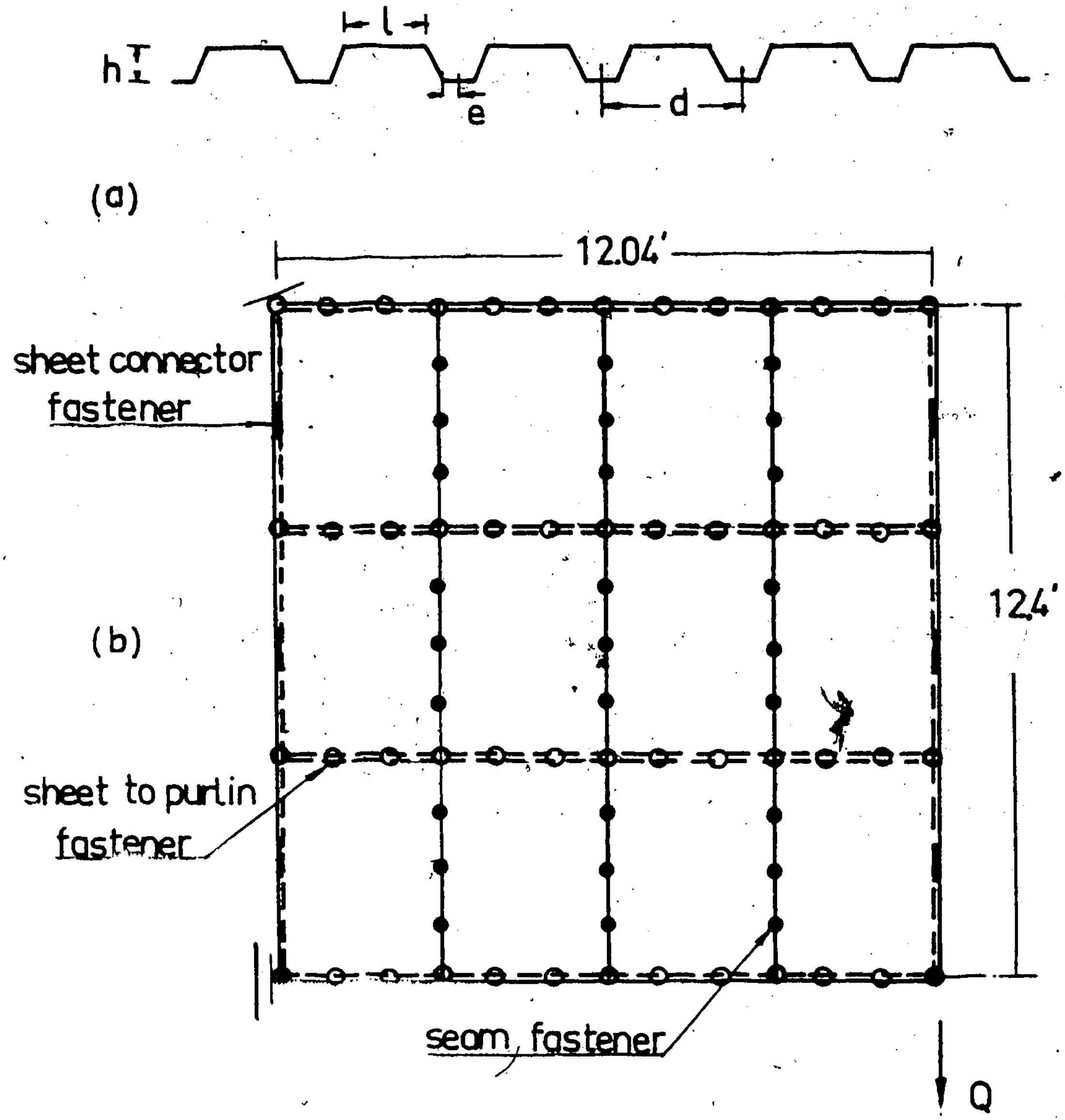


Figure 2-10 Typical Shear Panel tested (Types A&B)

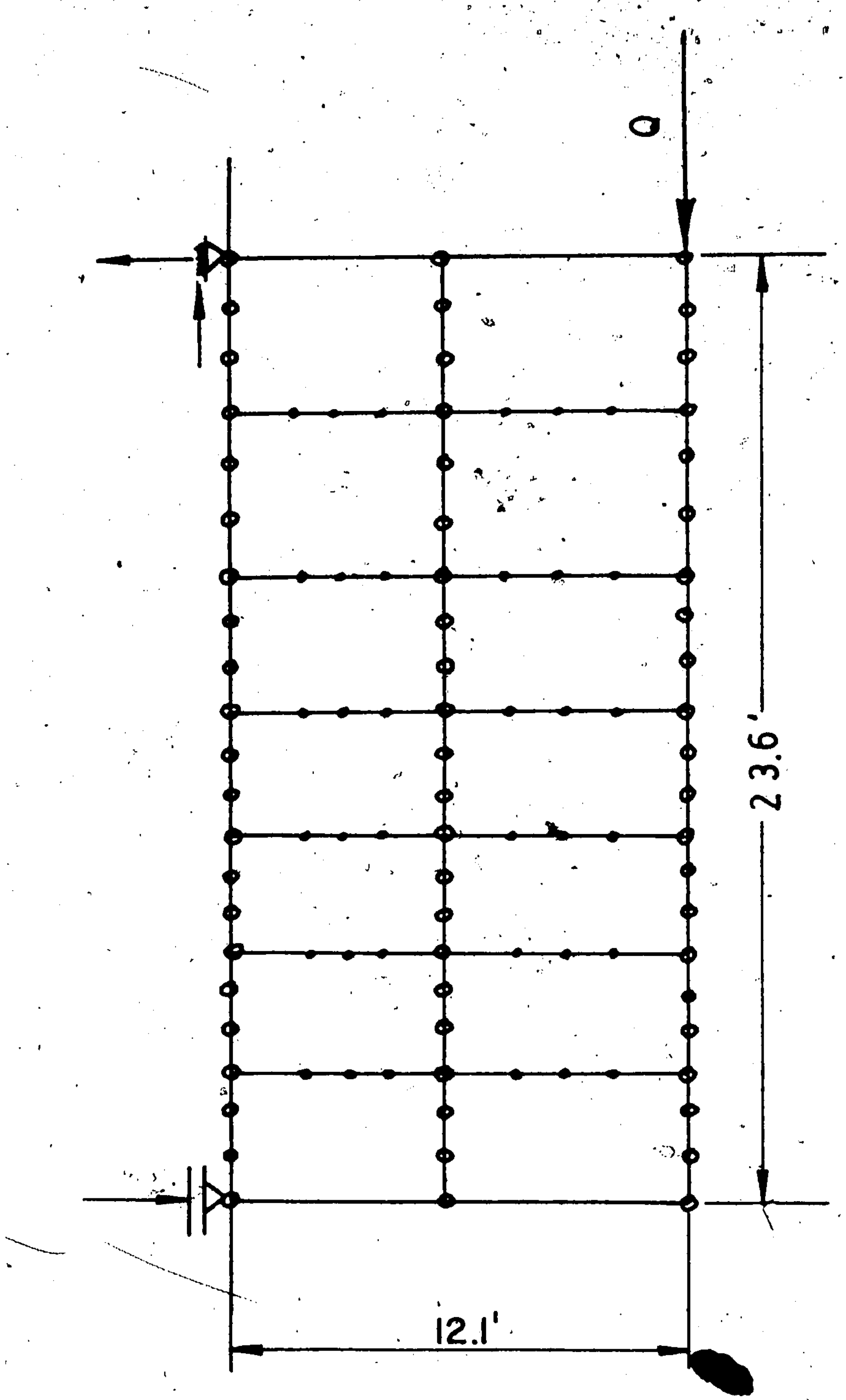


Figure 2-11 Typical Shear Panel Tested (Type C)

Table 2.2 Flexibility Factors

Test	Sheet Distortions	%	Shear Strain	%	Axial Strain	%	Connections	%	Total in/kip
A1	0.0031	9.2	0.0034	10.1	0.00125	3.7	0.02605	77.1	0.0338
A2	0.12405	78.7	0.0034	2.2	0.00125	0.8	.0290	18.4	0.1577
B1	0.0031	6.4	0.0034	7.1	0.00125	2.6	0.0403	83.8	0.0481
B2	0.12405	63.1	0.0034	1.7	0.00125	0.6	0.0680	34.6	0.1967
B3	0.4199	78.6	0.0034	0.6	0.00125	0.2	0.1098	20.5	0.5344
C1	0.0010	4.7	0.0020	9.4	0.00407	19.1	0.0142	66.7	0.0213
C2	0.0418	60.9	0.0020	2.9	0.00407	5.9	0.0207	30.2	0.0686

The sheet distortion depends on an experimental constant K_o Eq. B.1, Appendix B which should be carefully assessed. For panel B1 the test value of flexibility is not predicted by any of the methods; suggesting that the test value is not accurate.

The results given by the writer's method predict the flexibility of the diaphragm with almost the same degree of accuracy as that of the other methods. Furthermore not having the theoretical shortcomings of Fazio, Ha, El Hakim's (17) and Davies' (11) method regarding internal equilibrium and moment respectively, makes the method more acceptable.

The distribution of forces for the end fasteners is more realistic than in Bryan's (7) method.

Bryan's (7) approach for the prediction of the flexibility of panels C1 and C2 results in calculated flexibilities that are greatly different from test results, which points out that his theory for indirect shear transfer is not completely correct.

2.7 PARAMETERS THAT INFLUENCE STRENGTH AND STIFFNESS OF A STEEL DECK DIAPHRAGM

2.7.1 Introduction

When designing a building in Engineering practice it is necessary to know the strength and the stiffness of each of the elements that shape it. For a steel deck diaphragm under in-plane loads, there are several variable such as, panel length, panel width, purlin spacing and number and arrangement of connections, that affect the values of the strength and stiffness. Thus, to obtain an optimum design, the influence of each variable on the overall strength and stiffness should be known.

The Fazio, Ha, El Hakims's (17) approach, being one of the most accurate procedures according to the comparisons made in the preceding section has been selected to show how panel length, panel width, purlin spacing and number and arrangement of connections affect the behaviour of a steel panel.

Two computer programs based on Fazio, Ha, El Hakim's (17) equations were developed; the first one for strength calculations and the second one for stiffness calculations. As will be shown in Chapter III, the ratio of frame to slab stiffness needs to be known in order to calculate the amount of force taken by intermediate frames.

2.7.2 Panel Length and Width

Deck T15-24 was selected from reference 35. Figures 2-12(a) and 2-12(b) show the arrangement of connections, purlins and the profile selected. For the calculations performed the connection properties were kept constant as follows:

S_s = 0.0613 in/kip (flexibility of seam fasteners)

S_p = 0.0613 in/kip (flexibility of purlin fasteners)

S_{sc} = 0.02049 in/kip (flexibility of shear connectors)

S_e = 0.0613 in/kip (flexibility of end fasteners)

Fsu = 0.382 kips (strength of seam fasteners)

Fpu = 0.917 kips (strength of purlin fasteners)

Fsc = 2.751 kips (strength of shear connectors)

Feu = 0.917 kips (strength of end fasteners)

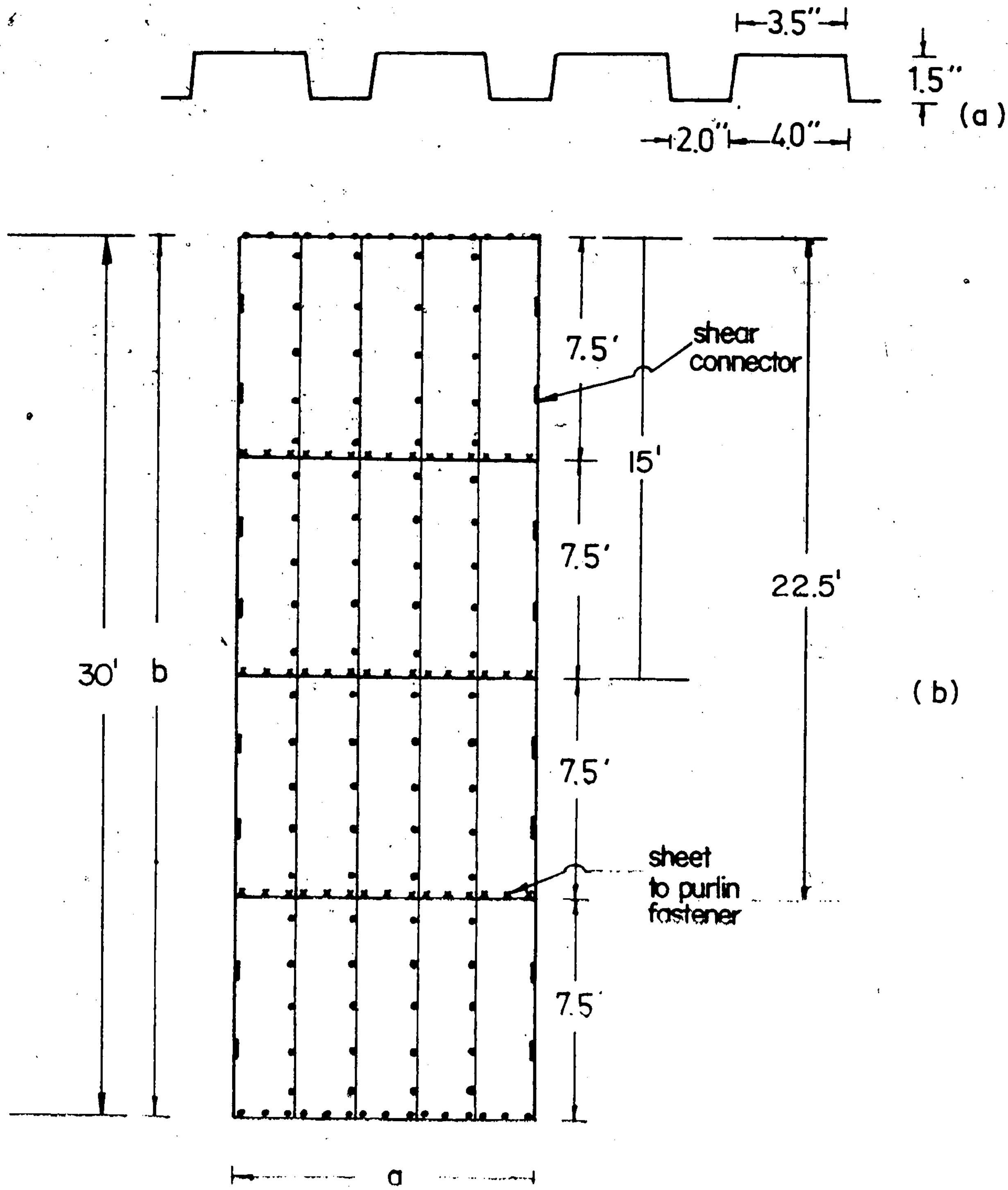


Figure 2-12 Typical Arrangement

Stiffness - The results of calculations are tabulated in Tables 2.3, 2.4 and 2.5 for 30', 22.5' and 15' width panels, respectively and are plotted in Figure 2.13.

The first observation that may be drawn from Figure 2.13 is that there is a well defined relation between stiffness and length of a shear panel; a power curve fits the data points well. The coefficient of determination ranges between 0.976 and 0.998. The power curves that best represent the data points are:

For 30' width

$$K = 421.4 \times 2^{-1.006} (k/in)$$

For 22.5' width

$$K = 259.7 \times 2^{-1.027} (k/in)$$

For 15" width

$$K = 111.1 \times 2^{-1.038} (k/in)$$

A second conclusion that can be drawn is related to Tables 2.3, 2.4 and 2.5. It is seen that the two most important parameters that effect the flexibility, or it's inverse, stiffness, of a steel deck diaphragm are sheet distortion ($c_{1.1}$, Appendix B, eq. B.1) and the flexibility of the connections. When designing, control of these two parameters is important. Therefore, if an accurate value of the shear stiffness is required, care should be taken to ensure that experimental factors, such as shear stiffness of fasteners and K_o factor for sheet distortion, involved in the determination of shear stiffness are properly obtained.

Strength - To analyze the effect of different widths on

Table 2.3. Width 30' Length's Influence in Stiffness for Width = 30'

Length	10'	%	20'	%	30'	%	40'	%	50'	%	Av.
Factor											
C1.1	0.006737	27.85	0.0134705	28.41	0.020176	28.2	0.0269044	27.54	0.033624	27.40	27.90
C1.2	0.0009112	3.77	0.0018224	3.84	0.002787	3.91	0.0037185	3.74	0.0046498	3.79	3.81
C1.3	0.000224	0.93	0.0008944	1.89	0.0021392	3.00	0.0071576	7.20	0.0084	6.84	3.97
C2.1	0.001088	4.50	0.002176	4.59	0.003328	4.67	0.00444	4.47	0.00552	4.52	4.55
C2.2 +											
C2.3	0.10523	61.76	0.02905	61.27	0.429	60.14	0.05669	57.05	0.070506	57.45	59.8
C											
(in/kip)	0.02419		0.047413		0.071331		0.0909105		0.1227317		
K											
(kipin)	41.34		21.09		14.02		10.11		8.15		

Table 2.4. Width 22.5' Length's Influence in Stiffness for Width = 22'

Length	10'	%	20"	%	30'	%	40'	%	50'	%	Av.
Factor											
C1.1	0.017403	41.88	0.034796	42.37	0.052218	42.57	0.06964	40.19	0.087043	40.47	41.50
C1.2	0.0013875	3.34	0.002775	3.38	0.004163	3.34	0.00555	3.20	0.0069345	3.22	3.30
C1.3	0.000447	1.08	0.0017789	2.18	0.004275	3.43	0.01431	8.26	0.01683	7.83	4.56
C2.1	0.002178	5.24	0.004356	5.30	0.006534	5.24	0.0087165	5.03	0.010899	5.07	5.18
C2.2 +											
C2.3	0.02014	48.47	0.0384	46.76	0.0567	45.44	0.07505	43.31	0.09336	43.41	45.48
C											
(in/kip)	0.041556		0.082116		0.2148898		0.1732667		0.2150667		
K											
(kip.in)	24.06		12.18		8.07		5.77		4.65		

Table 2.5. Width 15' Length's Influence in Stiffness for Width = 15'

Length	10'	%	20"	%	30'	%	40'	%	50'	%	Av.
Factor											
C1.1	0.060558	60.77	0.12092	60.82	0.18169	60.33	0.241837	57.36	0.302788	57.74	59.40
C1.2	0.00277	2.78	0.00555	2.79	0.00833	2.76	0.0111	2.63	0.01388	2.65	2.72
C1.3	0.001117	[illegible]	0.0047	2.25	0.01069	3.55	0.03579	8.49	0.0420	8.00	4.68
C2.1	0.0054	5.42	0.01089	5.48	0.01635	5.42	0.02178	5.17	0.0272	5.19	5.34
C2.2 +											
C2.3	0.0298	29.91	0.05693	28.66	0.0842	27.93	0.111	26.35	0.13	26.41	27.85
(A/kip)	0.099645		0.198819		0.30144		0.421607		0.52437		
(kip.in)	10.04		5.03		3.32		2.37		1.91		

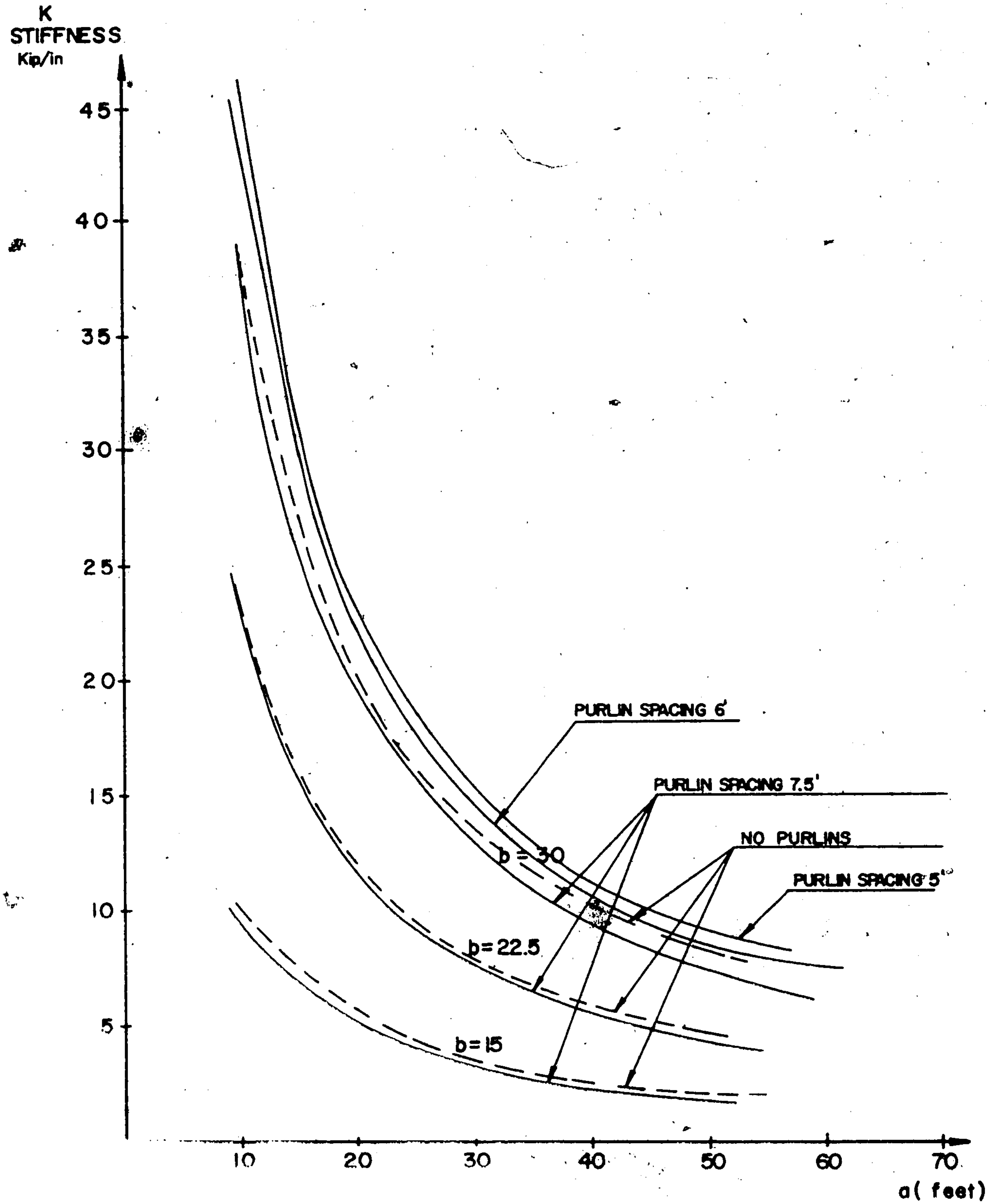


Figure 2-13 Stiffness of Diaphragms

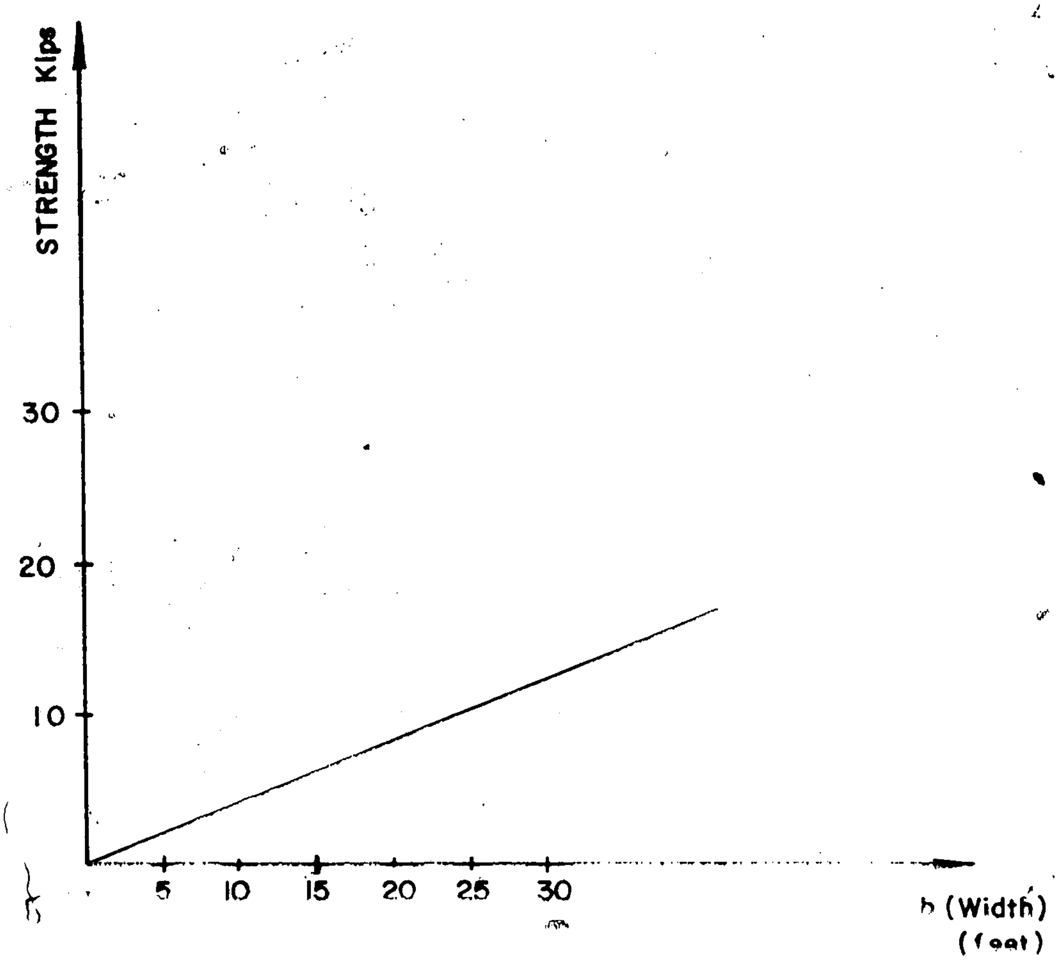


Figure 2.14 Strength of Diaphragms

Table 2.6 Width's Influence in Strength

Panel Width (Feet)	30	22.5	15
Failure at Seam (kips)	12.05	9.26	6.45
Failure at Side (kips)	26.99	20.49	13.96
Failure at End (kips)	22.55	17.06	11.55
Failure Load (kips)	12.05	9.26	6.45

strength the same panels discussed previously were used. The results are shown in Table 2.6 where the lowest value for each panel is the failure load. If these values are plotted Figure 2.14 is obtained, which shows a linear increase in strength with width.

2.7.3 Purlin Spacing

The same panel used in the previous section was selected for the purpose of analyzing the effect of purlin spacing on strength and stiffness. The dimensions of the panel, profile, purlin section, number of seam and end fasteners, and, shear connectors were kept constant throughout the analysis. In Figure 2-15 the different arrangements selected are shown. The only difference between any two of them is the number of purlins and purlin fasteners.

Stiffness - The results are shown in Figure 2-16. It is observed that stiffness and number of purlins are linearly related. Another observation is that the bigger the ratio a/b (Length/width) the less important it is to add more purlins to the panel. From Figure 2-16, the slope of the straight lines decreases when a/b increases.

Strength - As for stiffness, the strength increases as the number of purlins increased as shown in Figure 2-17.

2.7.4 Number of Seam Connectors

The effect of increasing the number of seam connectors is analyzed for shear panels of different sizes. The result of the calculations are shown in Figure 2-18 and 2-19.

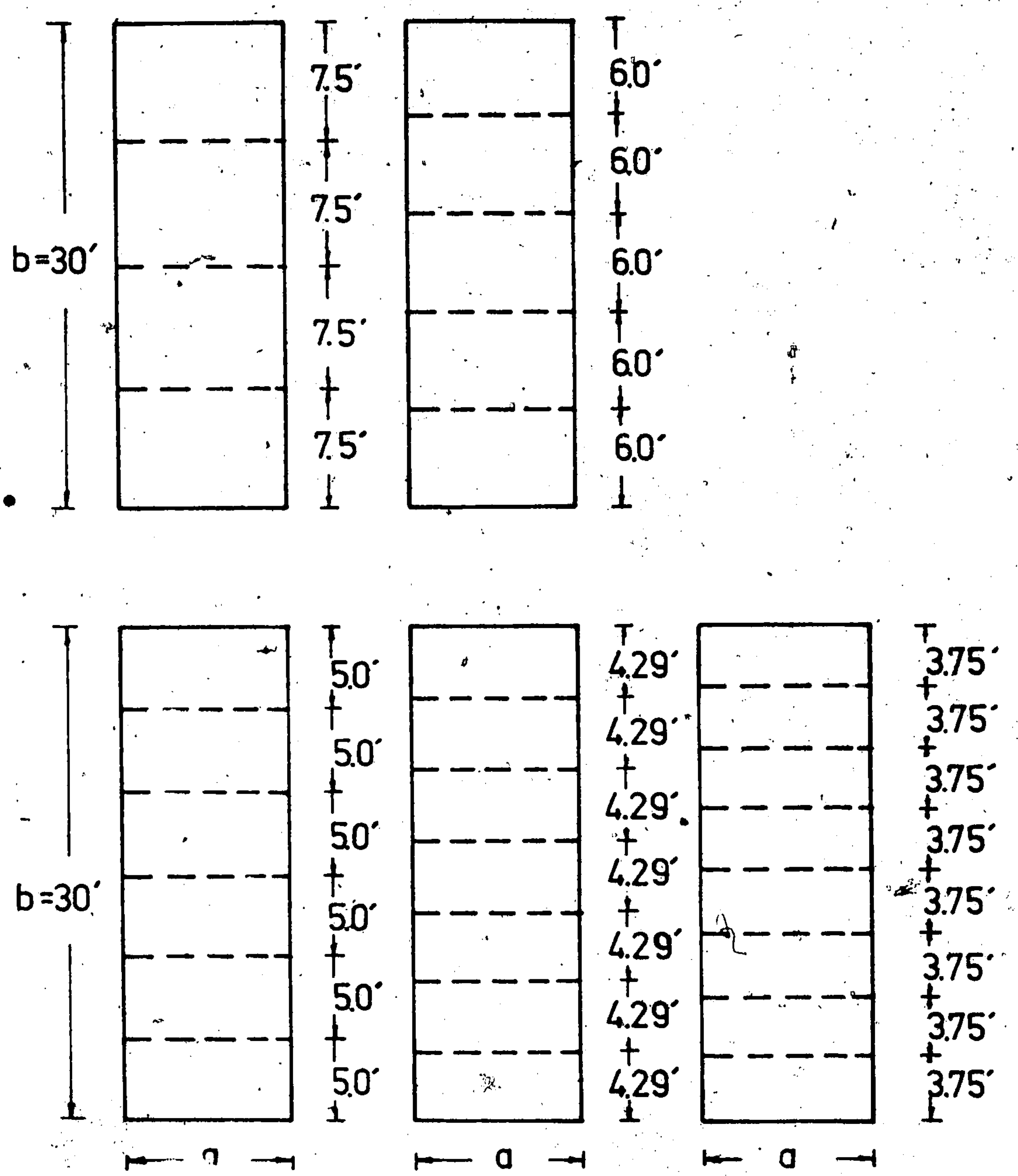


Figure 2-15 Influence of purlin spacing

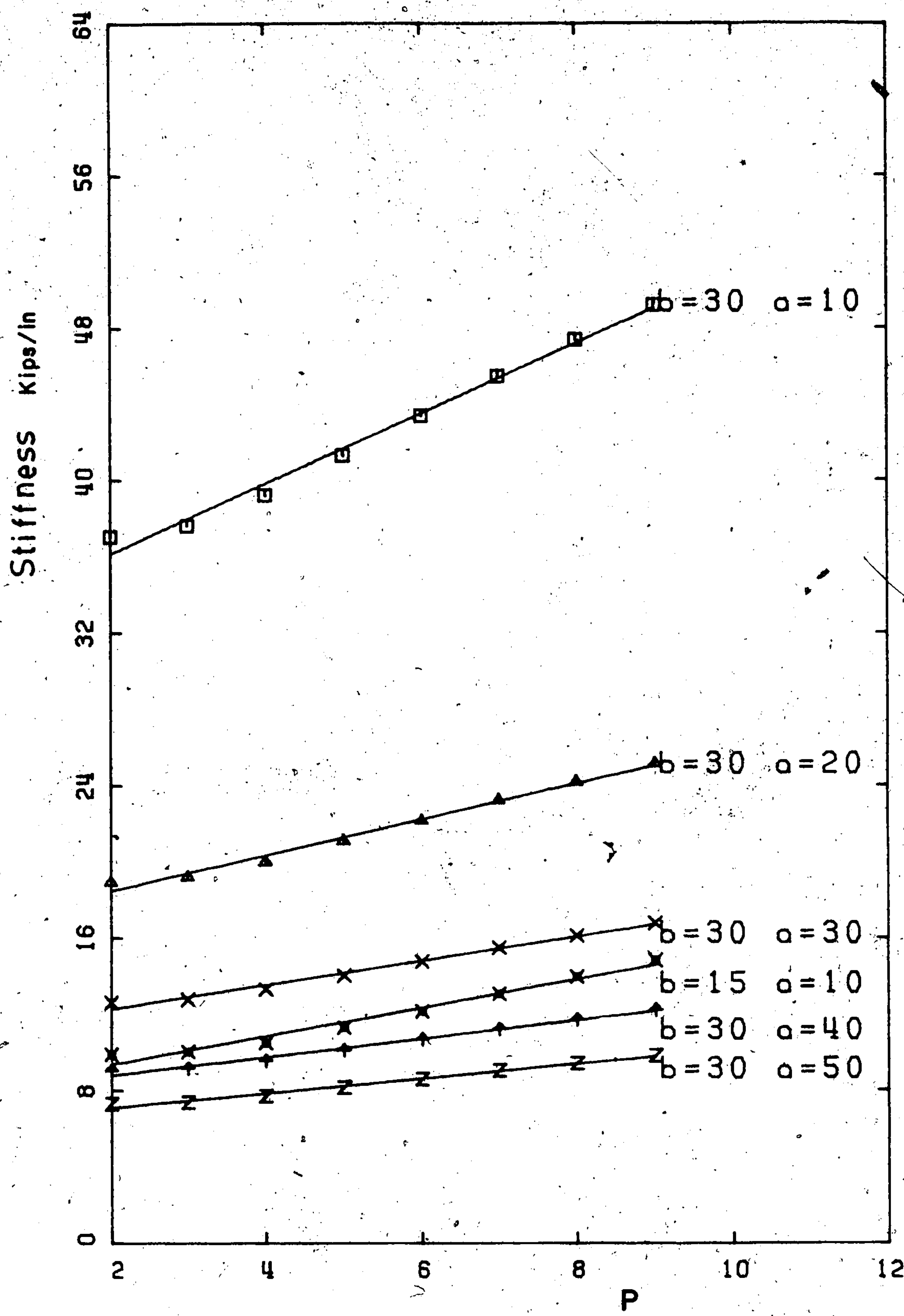


Figure 2-16 Influence of Purlin spacing on stiffness

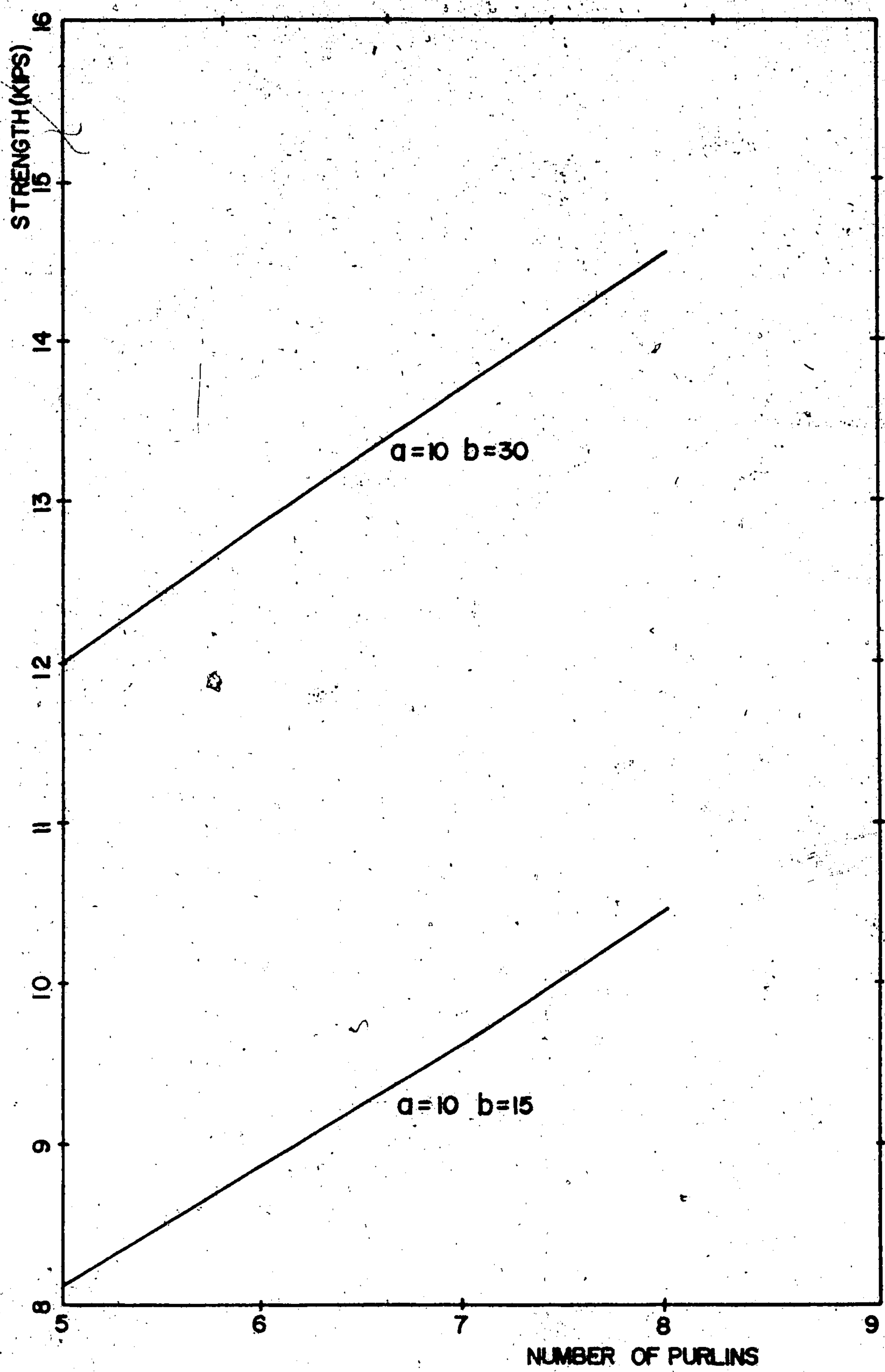


Figure 2-17 Influence of Purlin spacing on strength

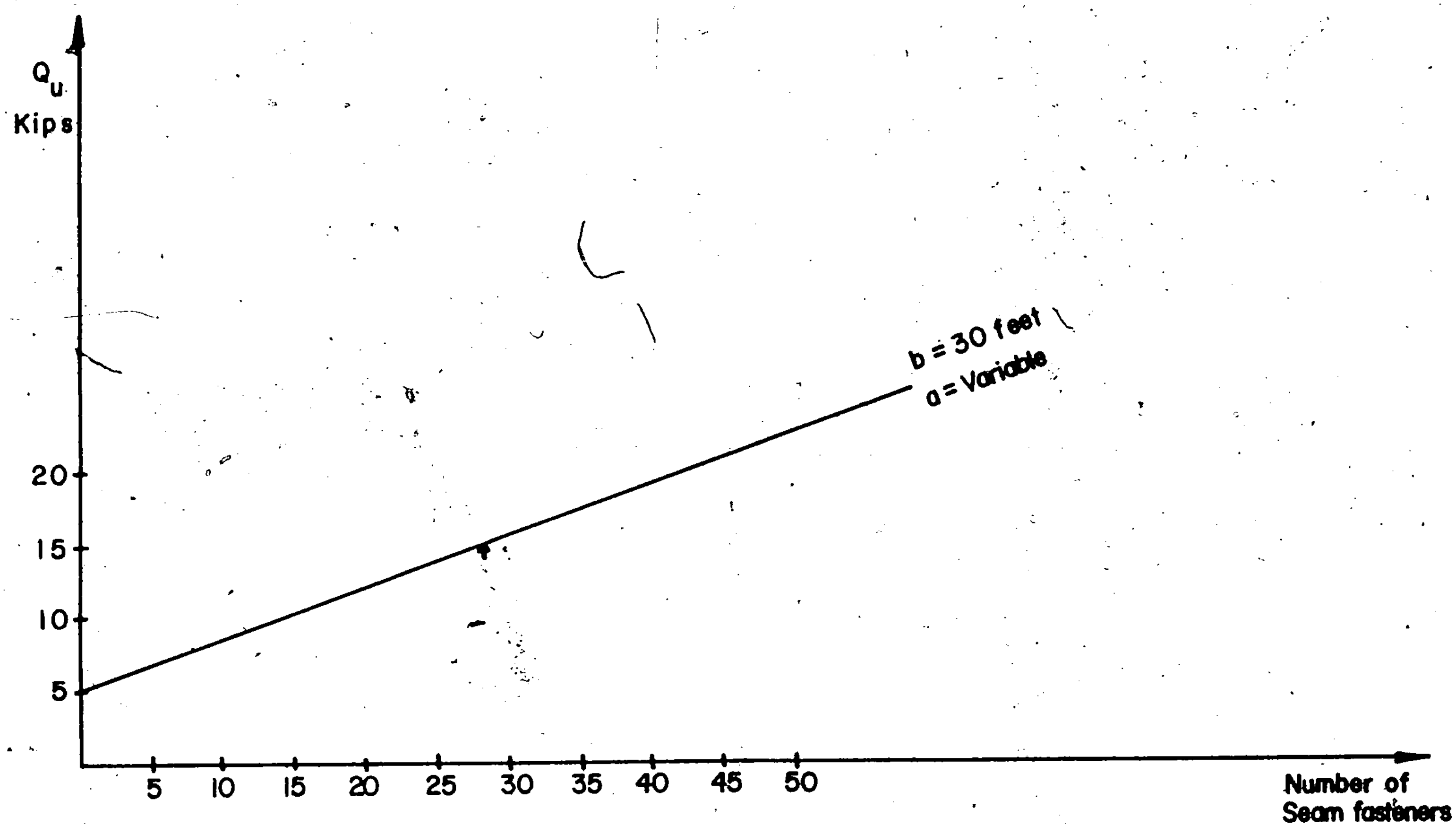


Figure 2-18 Influence of the number of Seam fasteners on strength

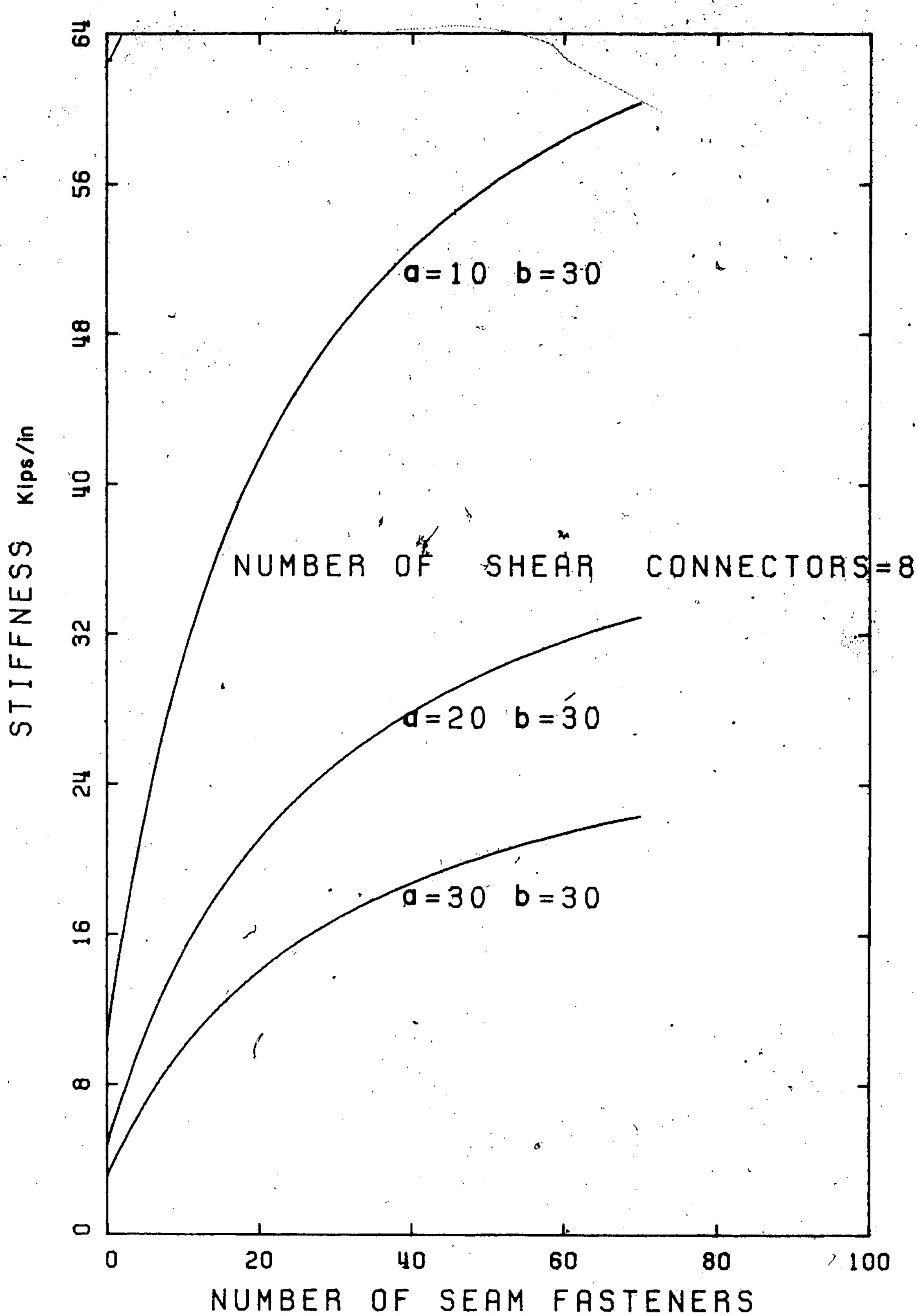


Figure 2-19 Seam Connection's Influence.

Figure 2-18 shows the influence of the number of seam connectors on strength. It is observed that a straight line fits the data points very well, and that the effect of increasing the number of seam connectors is the same for different shear panel lengths.

Figure 2-19 shows the influence of the number of seam fasteners on stiffness. A small increase in the number of seam fasteners implies a big increase in stiffness and this influence is less when the length of the shear panel increases.

2.7.5 Number of Shear Connectors

The influence of increasing the number of shear connectors is studied for shear panels of different dimensions. The results are shown in Figure 2-20, 2-21 and 2-22.

Figure 2-20 shows the influence of the number of shear connectors on strength. It is seen that only a few connectors are needed in order to reach a strength value above which any increase in the number of connectors is useless from the strength point of view.

Figures 2-21 and 2-22 show the influence of the number of shear connectors on stiffness. The same observation as that obtained for seam connectors may be pointed out, that is, a small increase in the number of shear connectors implies a big increase in stiffness and this influence is less when the length of the shear panel increases.

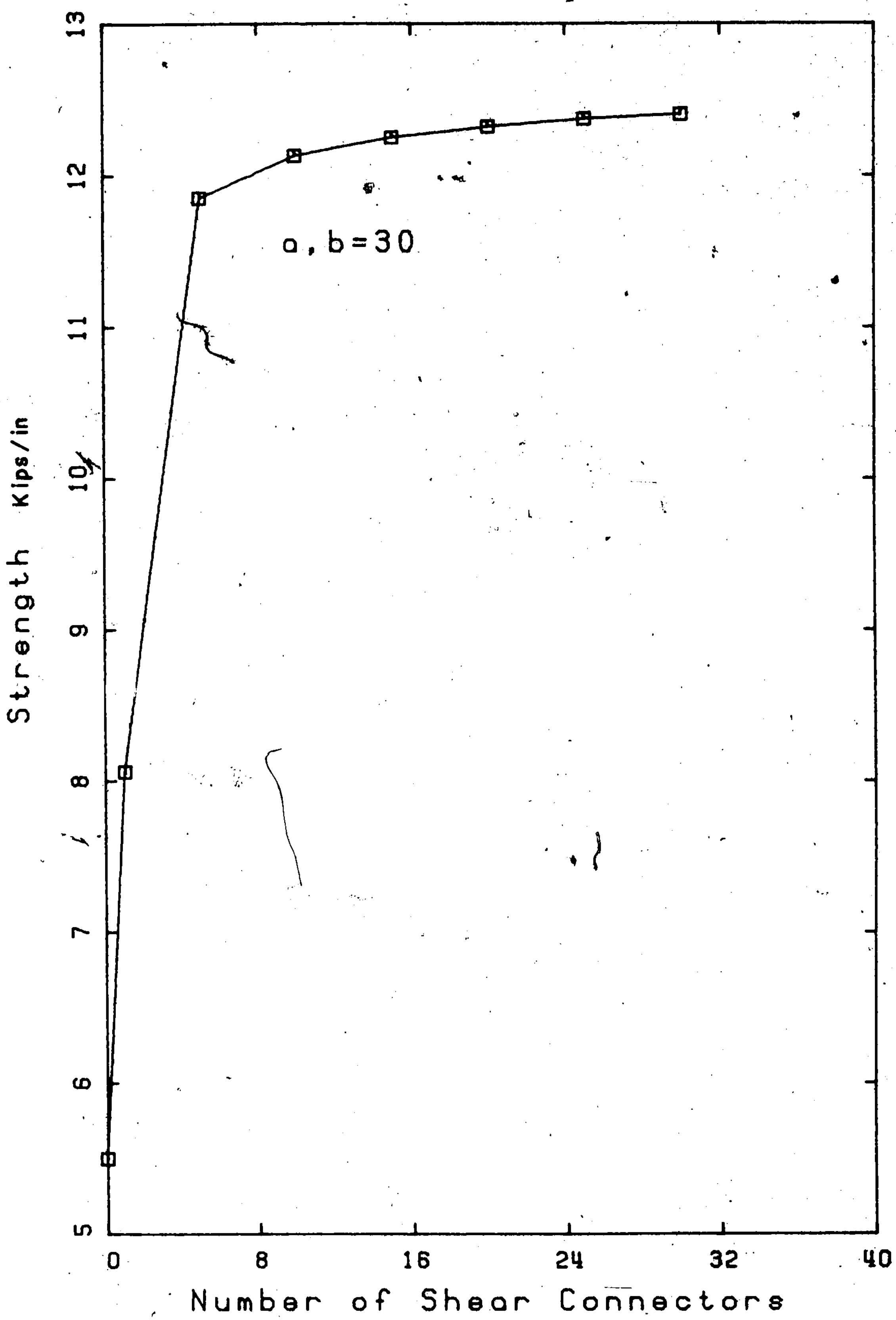


Figure 2-20 Shear Connector's Influence on Strength

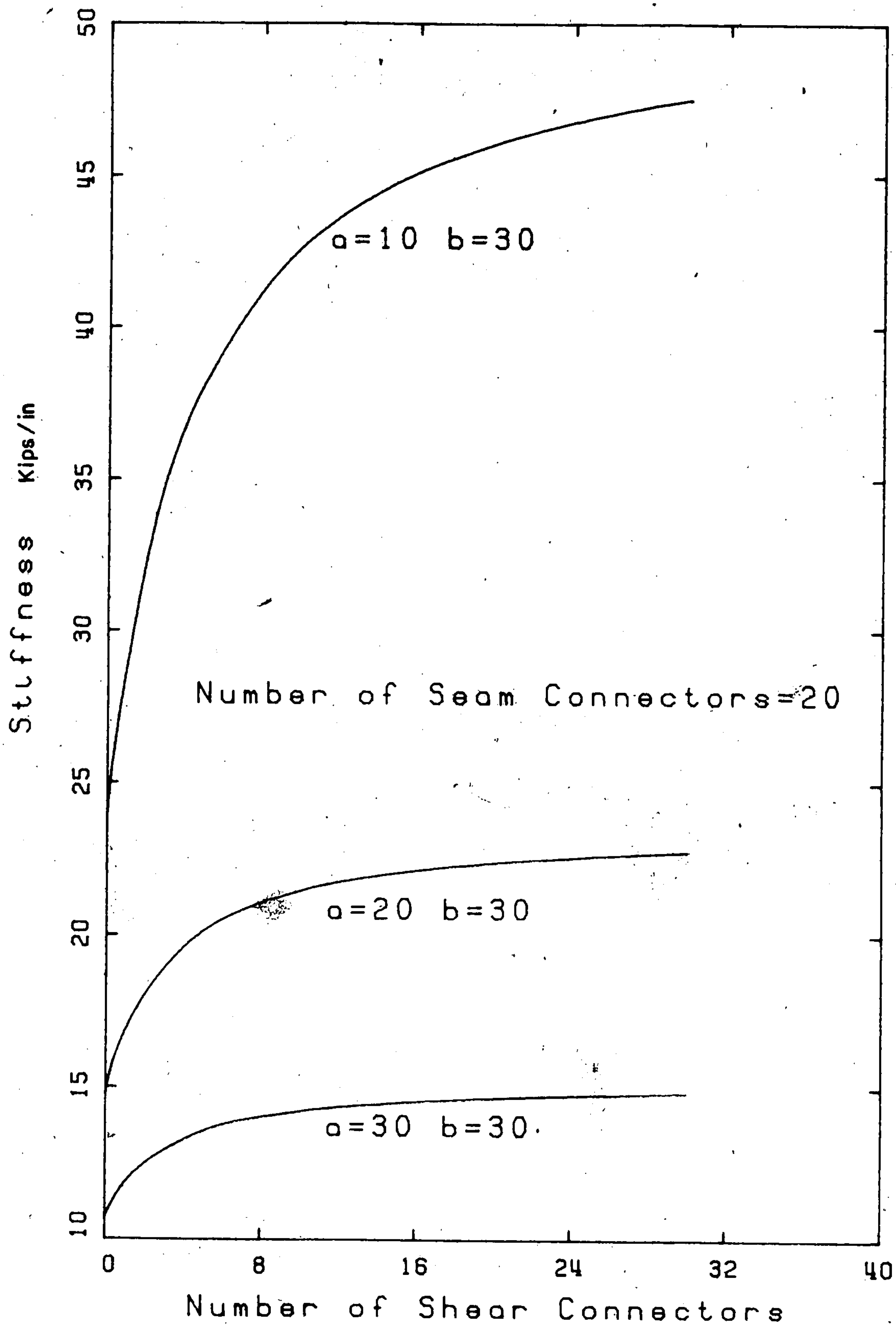


Figure 2-21 Shear Connector's Influence on Stiffness

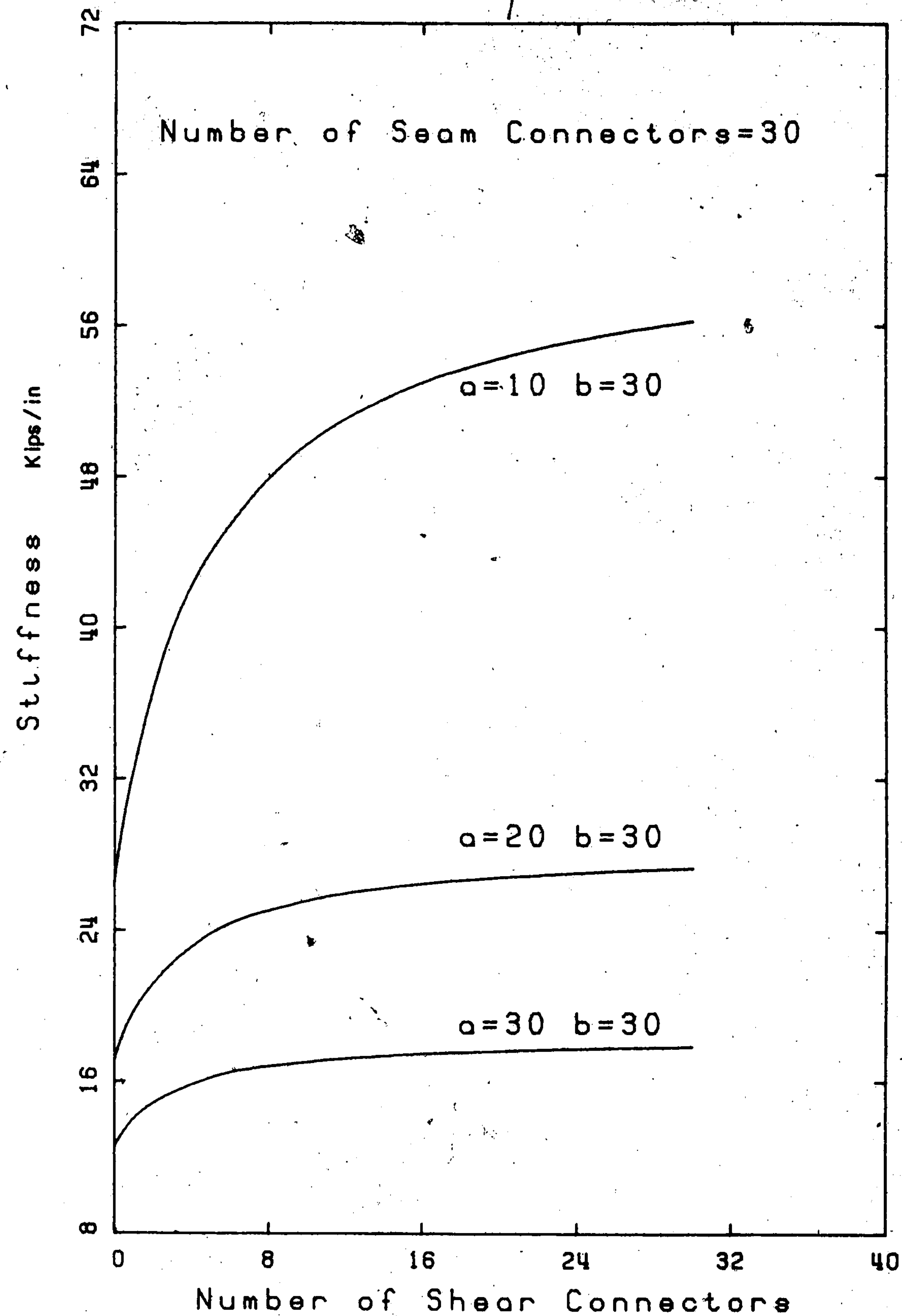


Figure 2-22 Shear Connector's Influence on Stiffness

CHAPTER III.

INTERACTION FRAME-DIAPHRAGM FOR SYMMETRICAL ONE-STOREY BUILDINGS.

3.1 INTRODUCTION

This chapter is devoted to the analysis of rectangular, symmetrical, one storey buildings subjected to uniformly distributed horizontal loads such as wind or earthquake.

Typical buildings analyzed are shown in Figure 3-1 (Types 1, 2 and 3). They all have end shear walls and parallel intermediate frames. The only difference between two of them is the number of intermediate frames. Theoretically the number of intermediate frames may vary from zero to infinity.

A typical building is shown in an isometrical view in Figure 3-2 and is comprised of several elements: a front wall ADKI, a horizontal diaphragm AHDE, two interior frames BGMN and CFOP and two end walls AHKL and DEIJ.

When horizontal forces act on a building such as the one shown in Fig. 3-2 a complex distribution of forces among the structural elements that make-up the building takes place. The lateral horizontal load is taken by the front wall ADKI (Figure 3-2); if this wall is structurally strong it transfers part of the force to the horizontal diaphragm AHDE, which in turn in combination with shear walls AHKL and DEIJ and interior frames BGMN and CFOP finally transfers the lateral loads to the foundation of the building.

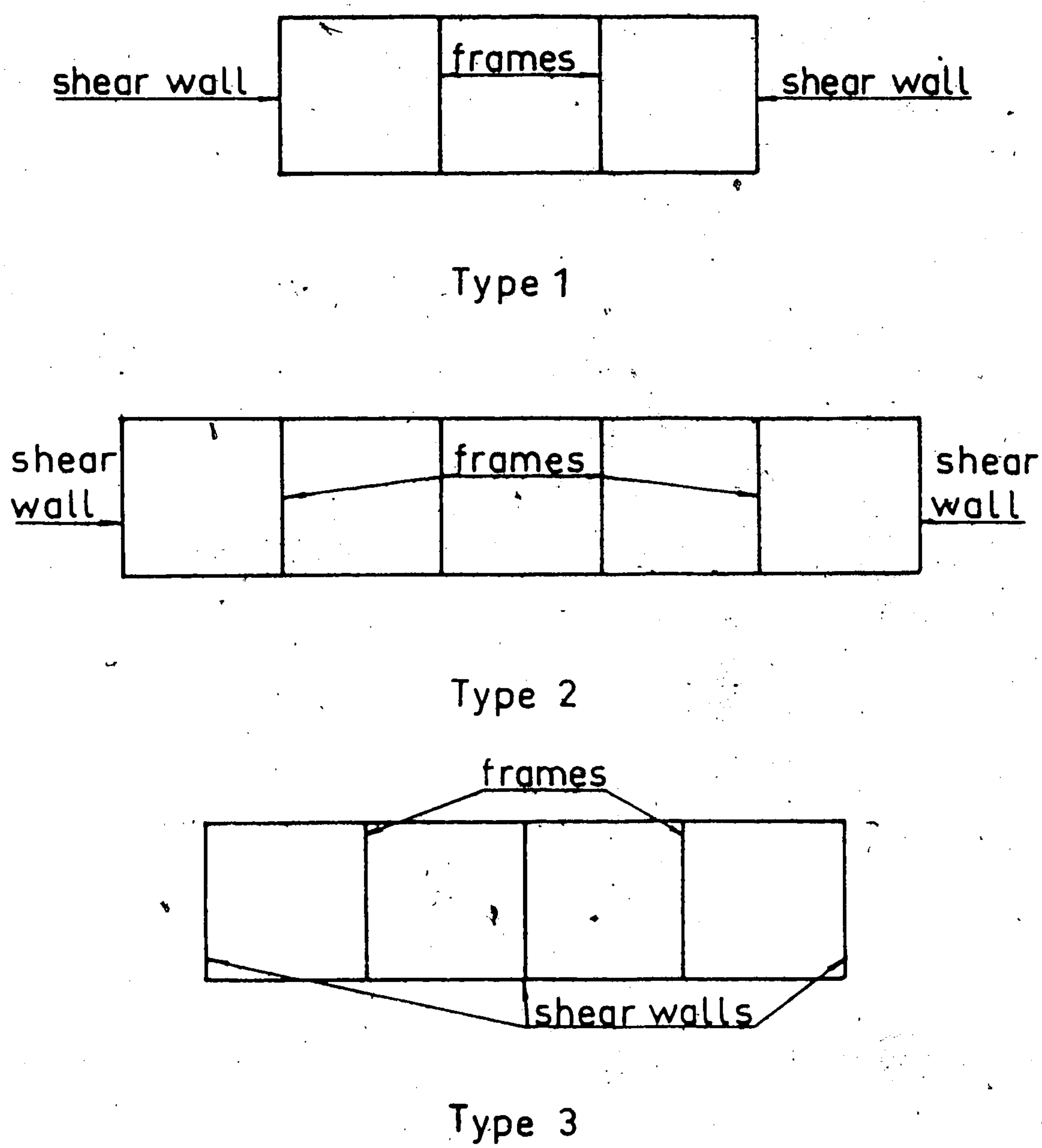


Figure 3-1 Type of Buildings

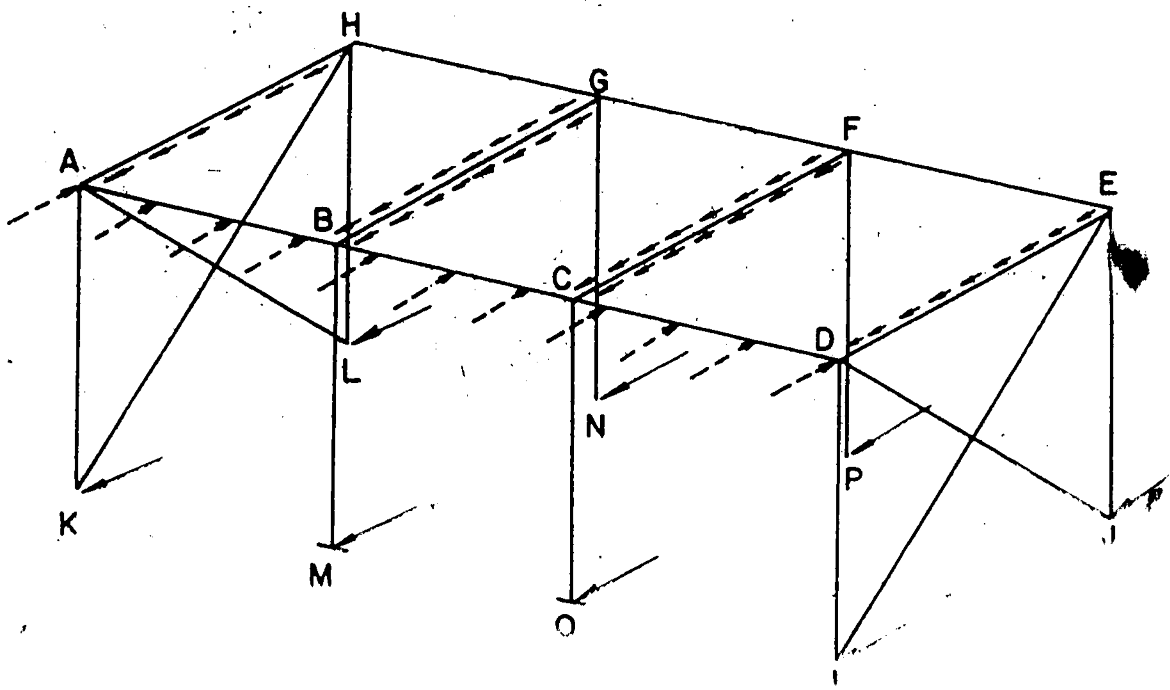


Figure 3-2 Isometric View of a Building

3.2 Method of Analysis

The structural behaviour of a building may be studied by means of models. To represent the horizontal diaphragm, two models have been selected: the beam model and the truss model. To model frames and walls, springs of different constants are used.

For buildings up to five bays, the beam model yields solutions which are not difficult to obtain. For more than five bays a solution based on the theory of beams on elastic foundations which was first used by Muto (25), is developed.

3.2.1 Beam Model

Generally, the horizontal diaphragm will behave as a deep girder, so that both bending and shear properties must be considered. However, when the building has a steel deck diaphragm the beam model is adopted using the shear properties only, because the contribution of the edge girders and the steel deck to the bending properties is negligible. When the horizontal diaphragm is made of concrete, bending as well as shear properties must be considered.

The beam model procedure was applied to determine the force distribution for frames tested by Bryan (7). Table 3-1 presents the experimental and analytical results reported by Bryan together with those obtained from the application of the beam model. In this case only shear properties are used because the horizontal diaphragm is a steel deck. As shown in Table 3.1, the beam model predicts the force taken by the frames well.

Table 3.1

STIFFNESSES

		Tests kips/in	Beam Model kips/in	% of Tests	Bryan kips/in	% of Tests
TEST I No Gable Bracing at ends	F1	10.7	9.1	85	8.7	81.3
	F2	11.8	10.5	89	10.2	86.4
TEST I Gable Bracing at Ends	F1	6.91	5.81	84.1	5.79	83.8
	F2	8.08	7.58	93.8	7.57	93.7
TEST II Gable Bracing At Ends	F1	2.76	2.74	99.3	2.76	100
	F2	3.55	3.62	102	3.60	101.4

3.2.1.1 Solution for a Limited Number of Bays

The solution for the force taken by an interior frame, on a three bay building, (Type 1, as shown in Figure 3-1) is used to explain some of the concepts involved in the beam model. Derivations are detailed in Appendix J.

The force taken by a frame is:

$$\frac{V_R}{V_T} = \frac{\frac{K_F}{K_B} + \frac{K_F}{K_{sh}} + \frac{1}{2\ K_W/K_F}}{2.725\ \frac{K_F}{K_B} + 3\ \frac{K_F}{K_{sh}} + \frac{1}{K_W/K_F} + 1} \qquad (3.1)$$

where:

V_R = Force taken by an interior frame

V_T = Total horizontal force acting on the building

K_F = Stiffness of the frame

K_B = Bending stiffness of the horizontal diaphragm

K_{sh} = Shearing stiffness of the horizontal diaphragm

K_W = Stiffness of the wall.

Simplifying further, the following quantities are replaced in Equation 3.1.

K_F/K_B = x

K_F/K_{sh} = y

K_W/K_F = c (in general c = $\Sigma K_W/\Sigma K_F$)

Thus, the solution becomes:

$$\frac{V_R}{V_T} = \frac{x + y + \frac{1}{2}c}{2.725x + 3y + \frac{1}{c} + 1} \qquad (3.2)$$

Using Fig. 3.3, Equation (3.2) represents for every c value a space surface whose intersection with the planes y = 0 and x = 0 yield the curves for the pure bending solution and the pure shear solution, respectively.

Since factor c may vary from ∞ to 0 representing the lower and upper bounds respectively, the limits for the force taken by a frame would be:

Lower bound limit

$$\lim_{c \to \infty} \frac{V_R}{V_T} = \frac{x + y}{2.725x + 3y + 1} \qquad (3.3)$$

which is a space surface shown in Figure 3.3. The intersection of this surface with the bending and shear planes yields the curves:

$$\frac{V_R}{V_T} = \frac{x}{2.725x + 1} \qquad \text{and}$$

$$\frac{V_R}{V_T} = \frac{y}{3y+1}$$

The Upper bound limit is:

$$\lim_{c \to 0} \frac{V_R}{V_T} = 0.5 \qquad (3.4)$$

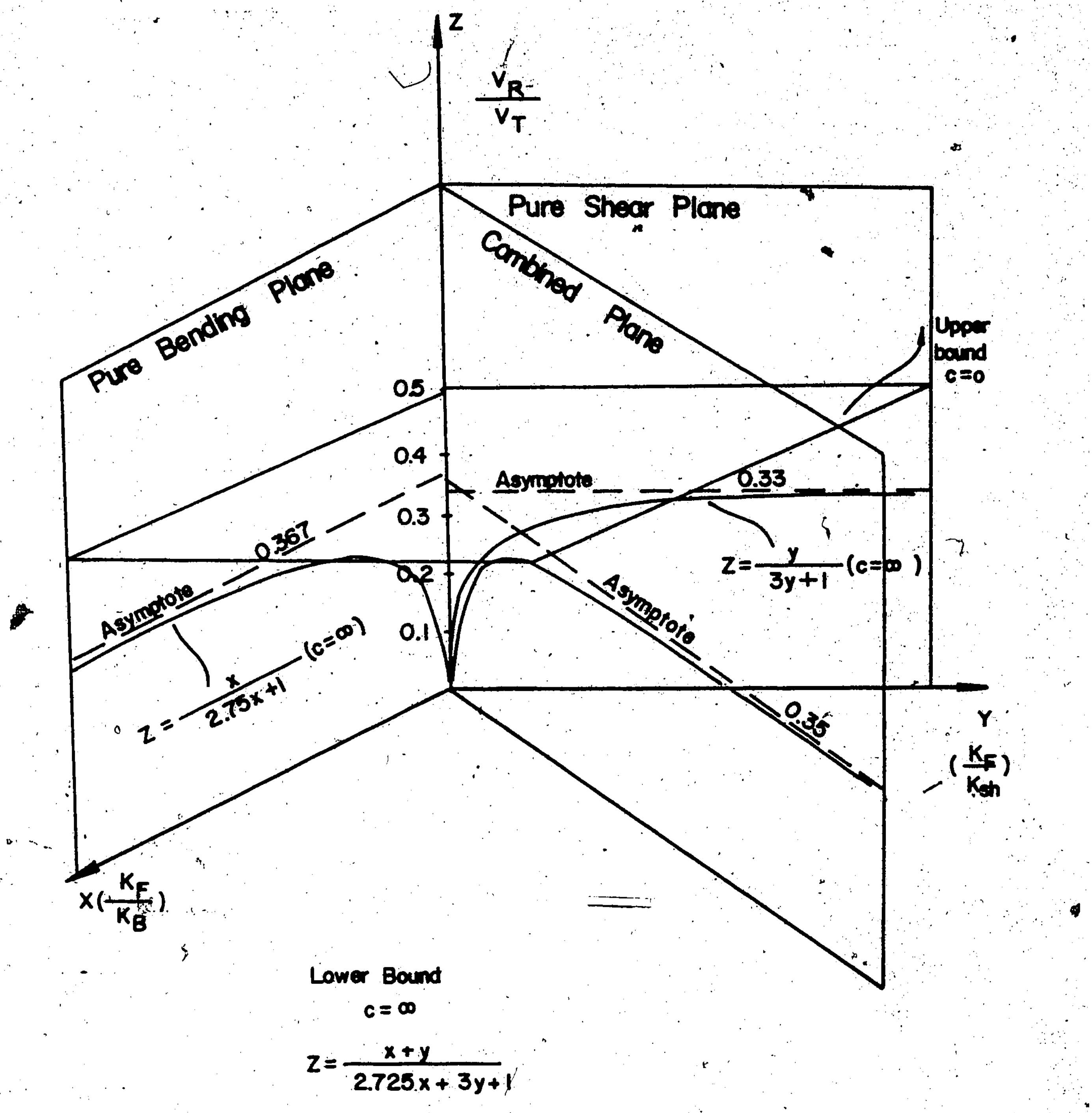


Figure 3-3 Schematic Interaction

which is a plane in the space as shown in Figure 3.3. Equations (3.3) and (3.4) are the limits for the complete set of solutions.

The plane x = 0 is the pure shear plane in that only shear properties are present; similarly the plane y = o is the pure bending plane since only bending properties are present. Any plane passing through the origin and perpendicular to the plane z = 0 yields a combined solution in which bending as well as shear properties are present in a constant ratio.

The intersection of the space surface (3.2) with the plane y = x is an example of a combined solution as shown in Figure 3.3 and given by Eqn. 3.5 below.

$$\frac{V_R}{V_T} = \frac{2x + \frac{1}{2}c}{5.725x + \frac{1}{c} + 1} \qquad (3.5)$$

The lower bound limit in this plane is

$$\lim_{c \to \infty} \frac{V_R}{V_T} = \frac{2x}{5.725x + 1} \qquad (3.6)$$

which coincides with equation (3.3) when y is replaced by x.

Similarly the upper bound limit in this plane is:

$$\lim_{c \to 0} \frac{V_R}{V_T} = 0.5 \qquad (3.7)$$

Because the pure shear plane and the pure bending plane are the limits where either only shear properties or bending properties are represented, it is important to have the solution for the force taken by an interior frame for these planes.

Figures 3.4 and 3.5 show solutions for the force taken by interior frames for a three and a four bay building respectively. In these figures the pure shear solution and the pure bending solution are presented together. To obtain a wide range of stiffness the x axis is necessarily logarithmic. The y axis represents the percentage of the total horizontal force taken by an interior frame. The parameter c has already been defined and the parameter n = 1 identifies symmetrical buildings as will be seen in the following chapter.

As it is shown in figures 3.4 and 3.5, the curves representing the fraction of the total horizontal force taken by an interior frame, range between two extremes which correspond to a relatively infinitely stiff diaphragm and an infinitely flexible diaphragm.

When the horizontal diaphragm is stiff relative to the frame, the ratio K_F/K_S (stiffness of the frame/stiffness of the slab) tends to zero and all the curves flatten out; this means that for any c the displacement of the horizontal diaphragm is constant along the building and the force taken by an interior frame is proportional to its stiffness. As the ratio K_F/K_S increases, the fraction of the total horizontal force taken by an interior frame increases, so that if it is assumed that the horizontal diaphragm is infinitely stiff the real force taken by a frame could be underevaluated. Otherwise, if it is assumed that the horizontal diaphragm is infinitely flexible, this leads

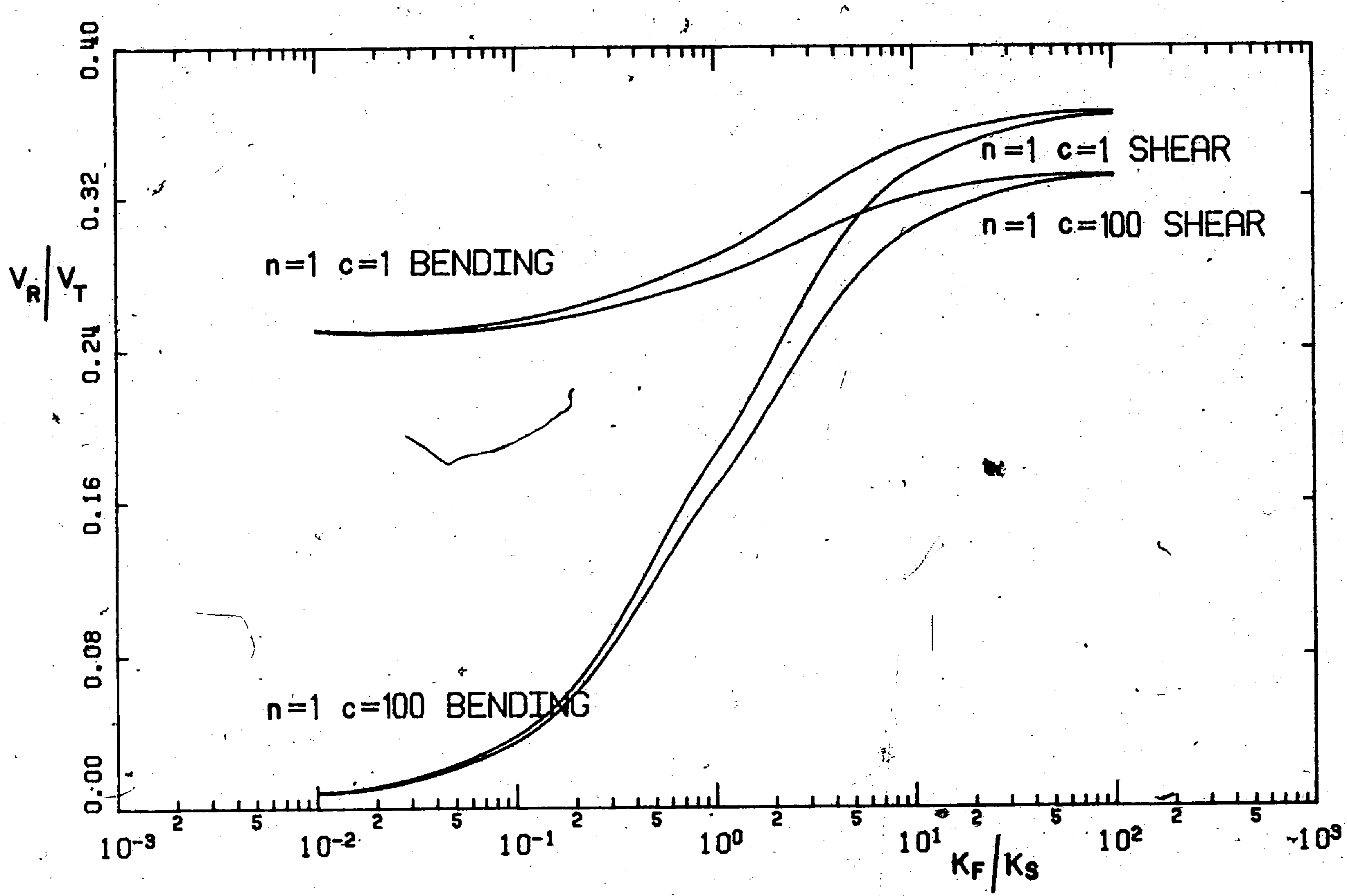


Figure 3-4 Three Bays Symmetrical

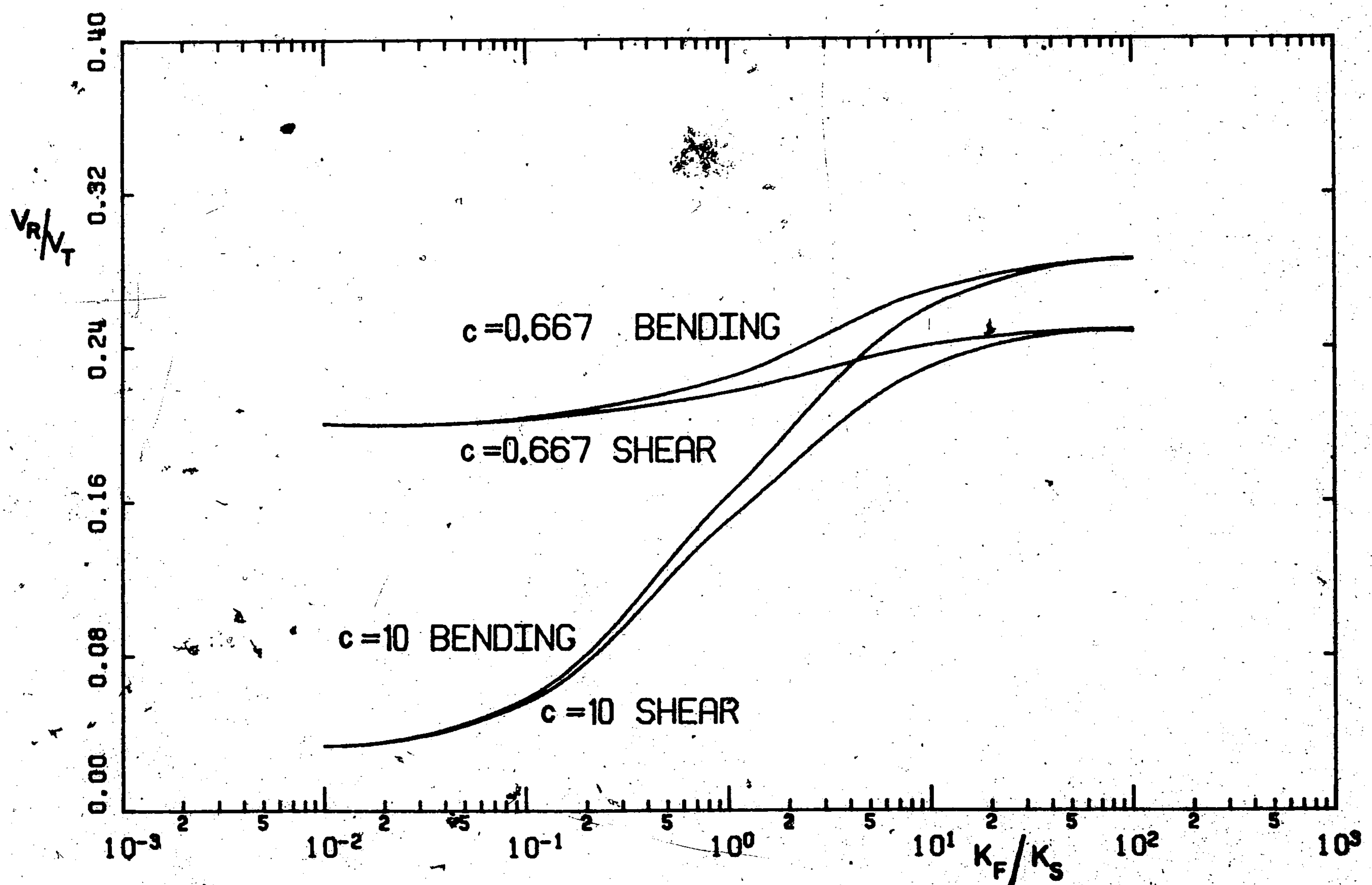


Figure 3-5 Four Bays Symmetrical Loads

to an overevaluation of the force taken by the frame.

3.2.1.2 Truss Model

This model is only applicable to buildings having steel deck diaphragms and is equivalent to the pure shear solution given by the beam model as it is explained in Appendix H, for the case of a two bay building.

As shown in Figure 3.6, for a type 1 building, the horizontal diaphragm is modelled by a diagonal bar AB in which the shear deflection is the same as in the diaphragm.

The vertical and horizontal members that shape the truss are taken as infinitely rigid, walls and intermediate frames are replaced by springs.

STIFFNESS OF THE HORIZONTAL DIAPHRAGM -- Figure 3.6 shows 3 types of buildings. The steel deck diaphragm of a typical bay of any of those buildings is replaced by a diagonal bar AB that gives an equivalent shear deflection.

Figure 3.7 shows the same typical bay under the action of a lateral force K_1

$$K_1 = P_1 \cos \theta$$

but $\cos \theta = b/h_1$

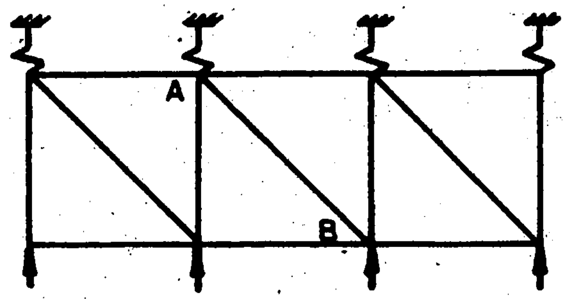
so that $P_1 = K_1 \, h_1/b$

Triangles ACB and BDE are similar thus $\frac{h_1}{b} = \frac{1}{\delta}$

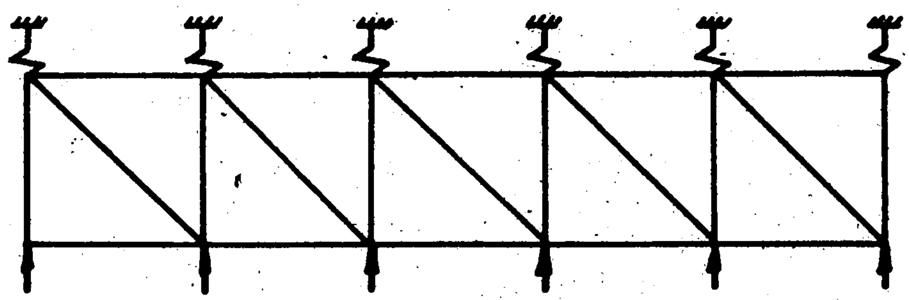
so that

$$\delta = \frac{b}{h_1}$$

Type 1



Type 2



Type 3

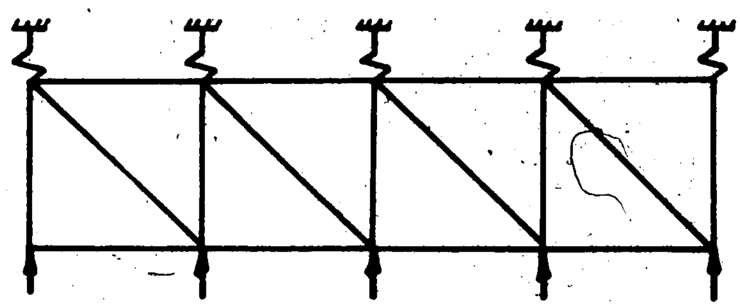


Figure 3-6 Truss Model

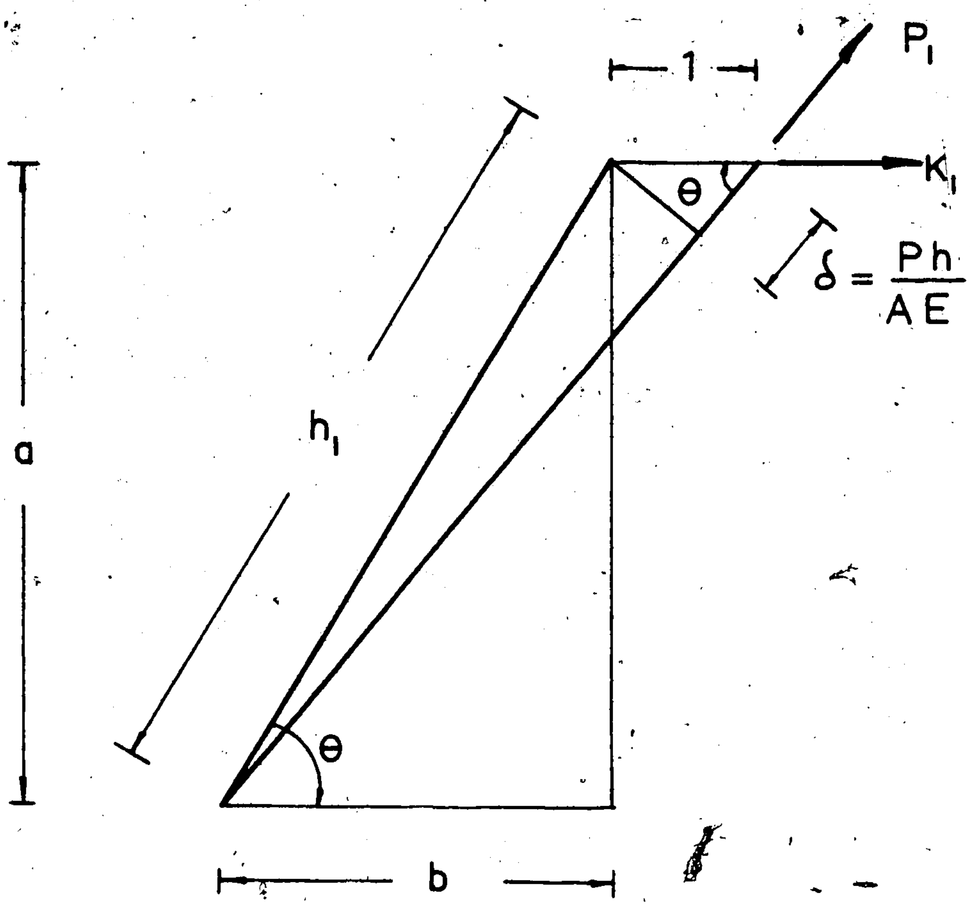


Figure 3-7 Stiffness of the Diaphragm

also

$$\delta = \frac{P_1 h_1}{AE}$$

Finally:

$$\frac{b}{h_1} = \frac{K_1 h_1^2}{AEb}$$

thus the force that causes an unitary displacement is

$$K_1 = \frac{b^2 AE}{h_1^3} \qquad (3.8)$$

and the area of the equivalent bar is

$$A = \frac{K_1 h_1^3}{b^2 E} \qquad (3.9)$$

Three bays buildings

The layout of the building is shown in Figure 3.7 (Type 1). It consists of three bays, two end shear walls and two interior frames. This distribution may be interchanged for two end frames and two interior shear walls.

The variables involved in the solution of the problem were already defined for the beam model and they are:

$$\frac{K_F}{K_s} = y$$

which is the independent variable.

The dependent variable is

$$\frac{V_R}{V_T} = Z$$

and the parameter c is defined as:

$$c = 2\ K_W/\Sigma K_F$$

These above mentioned variables are related by the set of curves shown in Figure 3.8.

It is observed from Figure 3.8 that there is a well defined relation between the relative stiffness K_F/K_S and the percentage of horizontal force taken by the frame. Depending on the value of the c constant, it can be observed that there is an upper boundary when the external walls are frames represented by c = 1. When the parameter c increases the curves get closer and most of them gather in a narrow band space, so that, from a practical viewpoint there is no difference for the force taken by the frame when the outerwalls are either concrete or masonry.

An important observation that may be obtained from Figure 3.8 is that the assumption of an infinitely rigid horizontal diaphragm may lead to erroneous results for the force carried by the frame. A value of the relative stiffness can be chosen such that the assumption of a rigid horizontal diaphragm

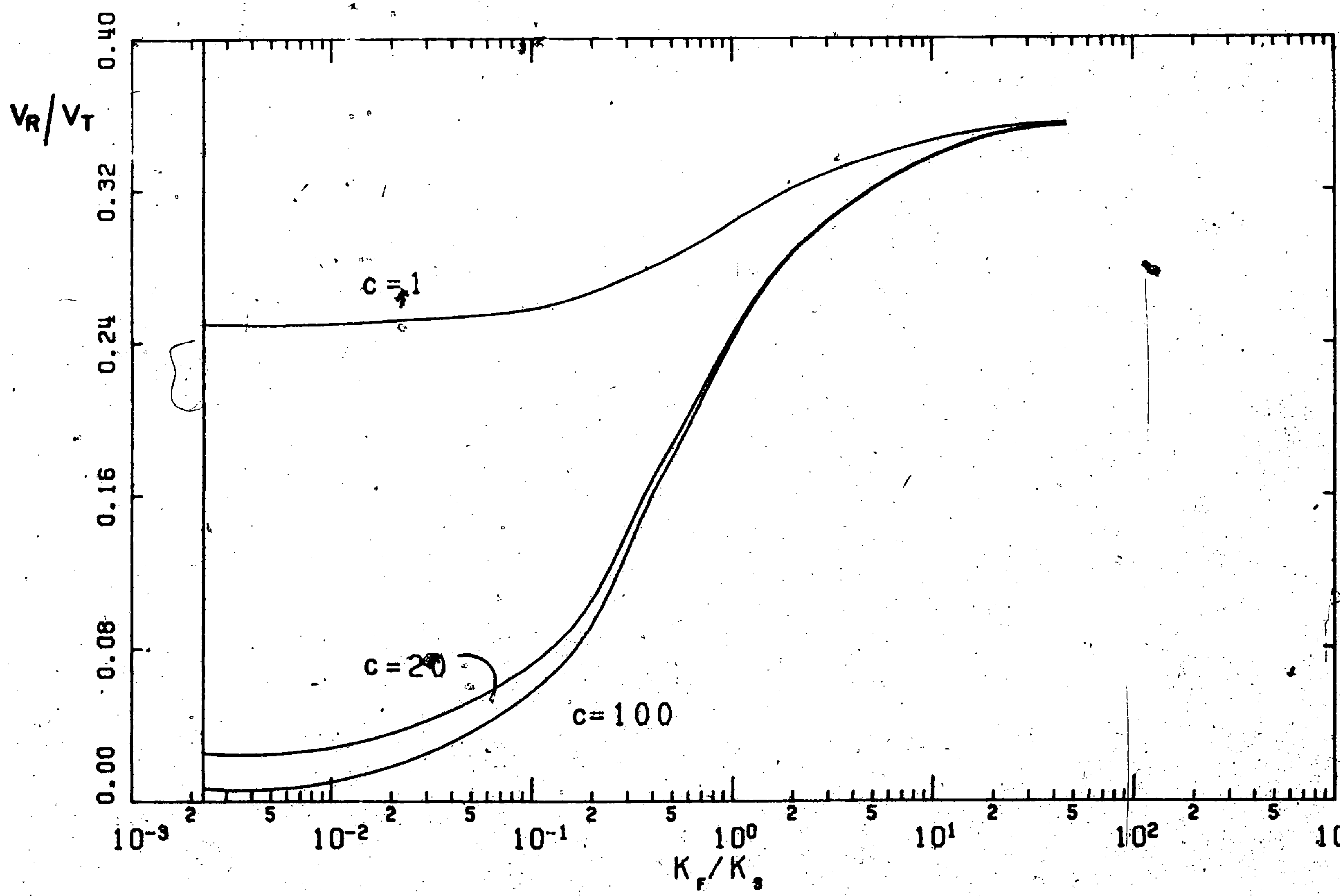


Figure 3-8 Three Bays Truss Model

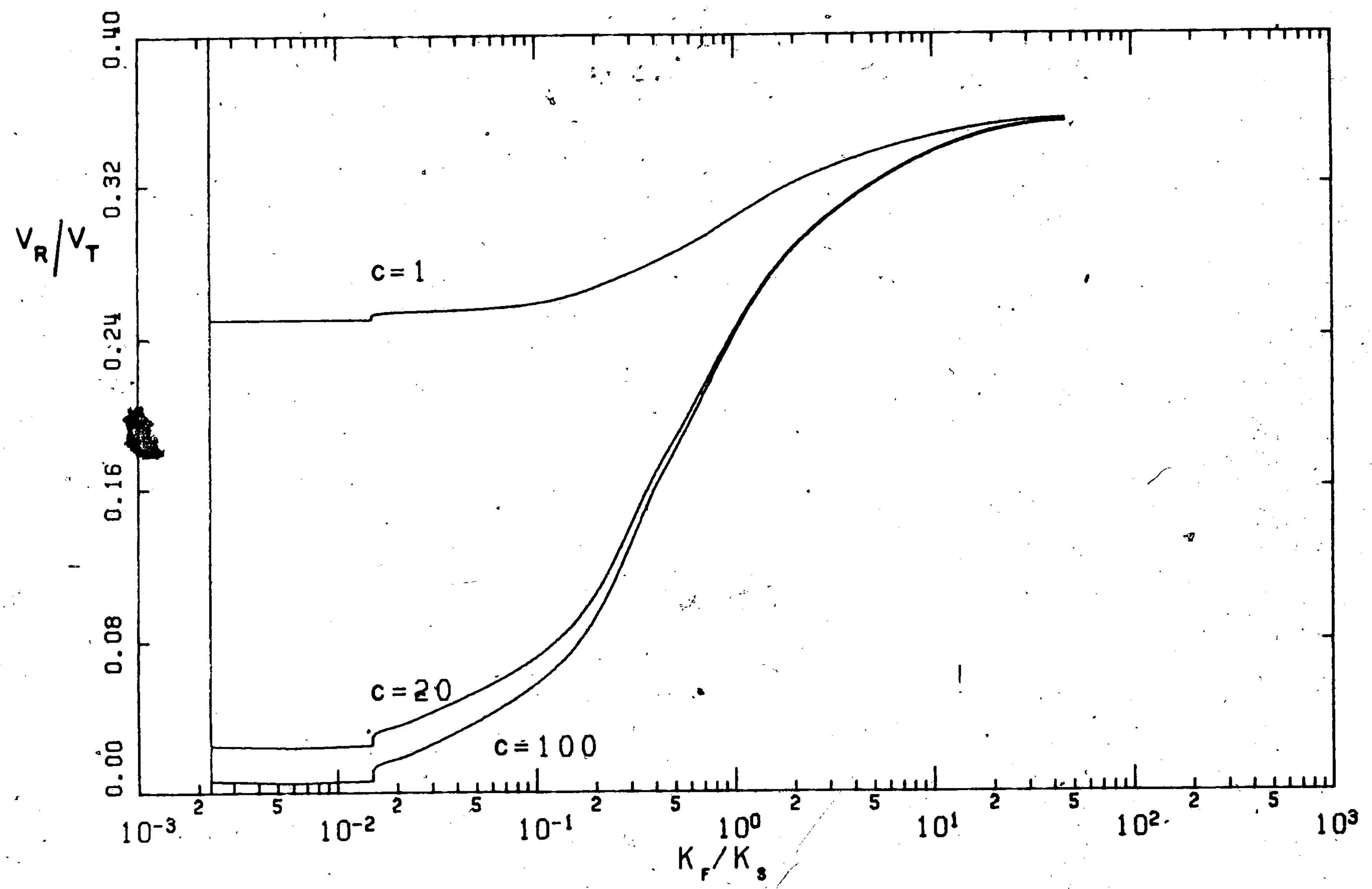


Figure 3-9 Three Bays Truss Model Simplified

is acceptable, for example

$$\frac{\frac{V_R}{V_T} - \frac{V_R}{V_{T\infty}}}{\frac{V_R}{V_{T\infty}}} < 0.20$$

This condition is satisfied for a value of K_F/K_S approximately equal to 10^{-2}, thus Figure 3.8 becomes Figure 3.9.

It is also observed that when the stiffness of the frame relative to the stiffness of the horizontal diaphragm is large, all the curves gather in a narrow space and finally for $K_F/K_S > 40$ the curves reach a constant value equal to 0.352 for infinitely flexible diaphragms which is higher than the area distribution method which yields a value 0.333.

Four and Five Bays One Storey Buildings.

The features of these buildings are shown in Figure 3.6 (Type 3 and Type 2 for four and five bays, respectively).

The same model is used as for three bays, but in these two cases for each c value two curves are obtained which represent the fraction of the total horizontal force taken by each of the interior frames. (See Figures 3.10, 3.11 and 3.12) (only one curve is shown for the case of four bays)

From these figures is observed again that there is a well defined relation between the relative stiffness and the percentage of the total horizontal force taken by an interior frame.

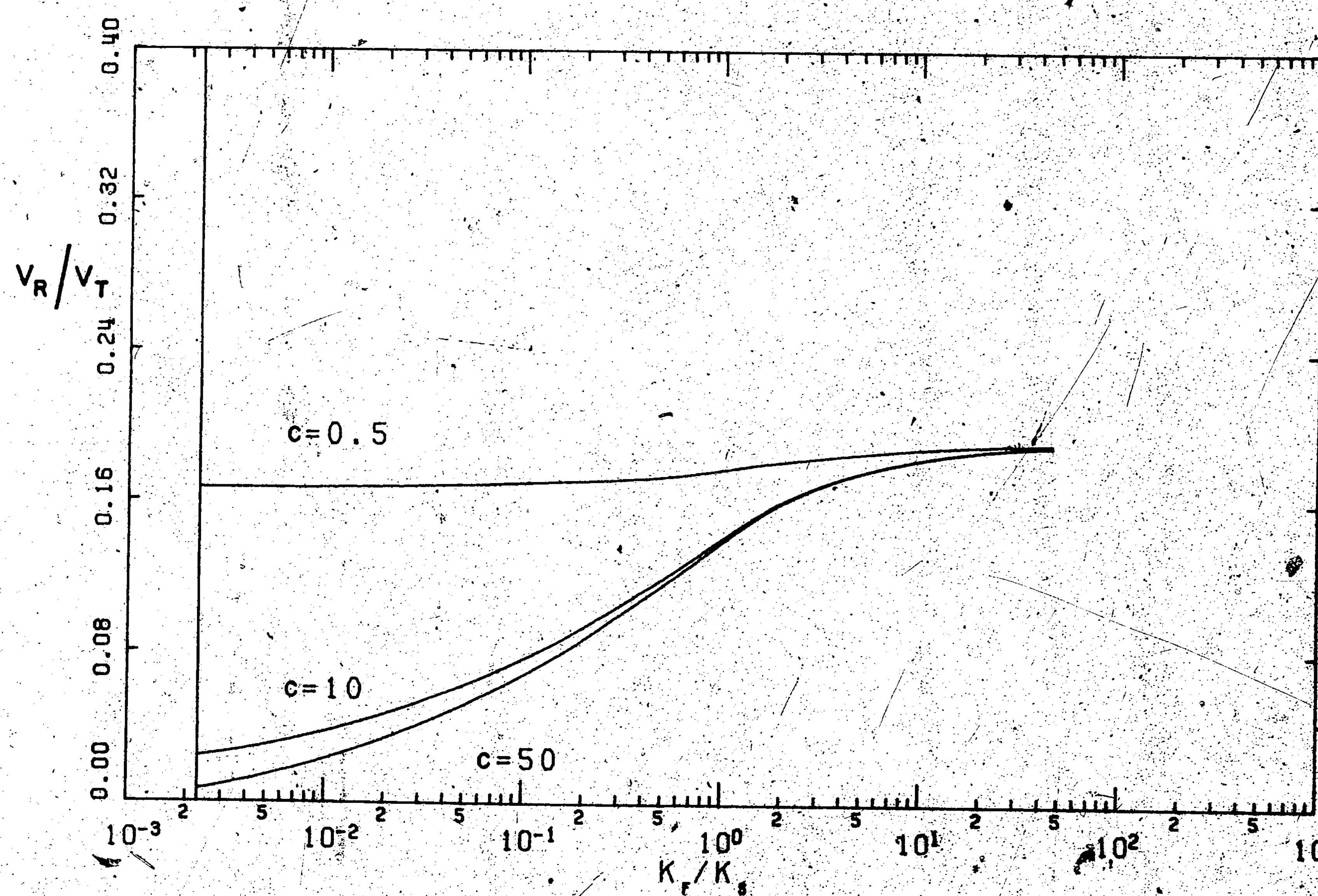


Figure 3-10 Five Bays Shear Force taken by first frame

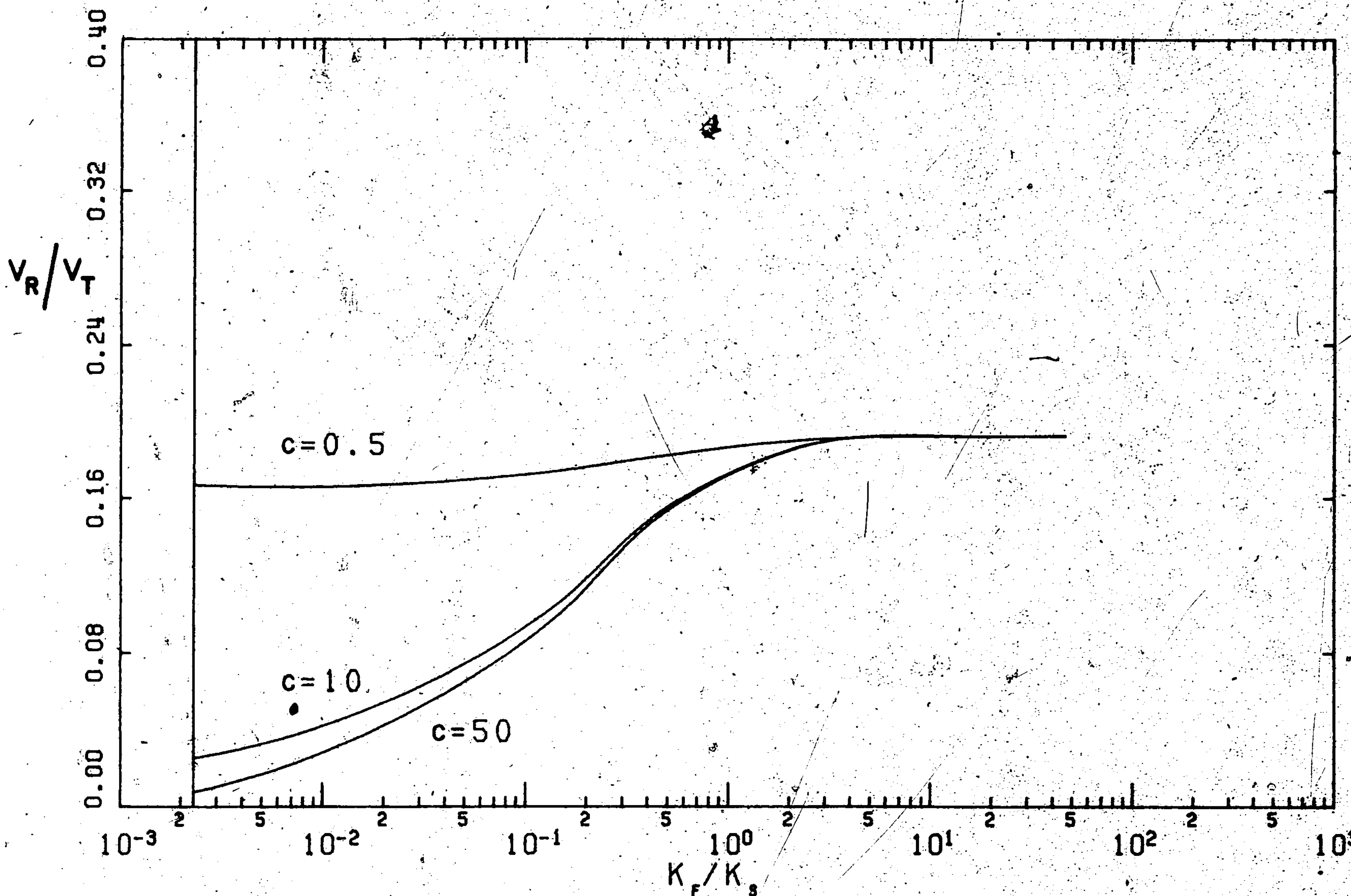


Figure 3-11 Five Bays Shear Force taken by second frame

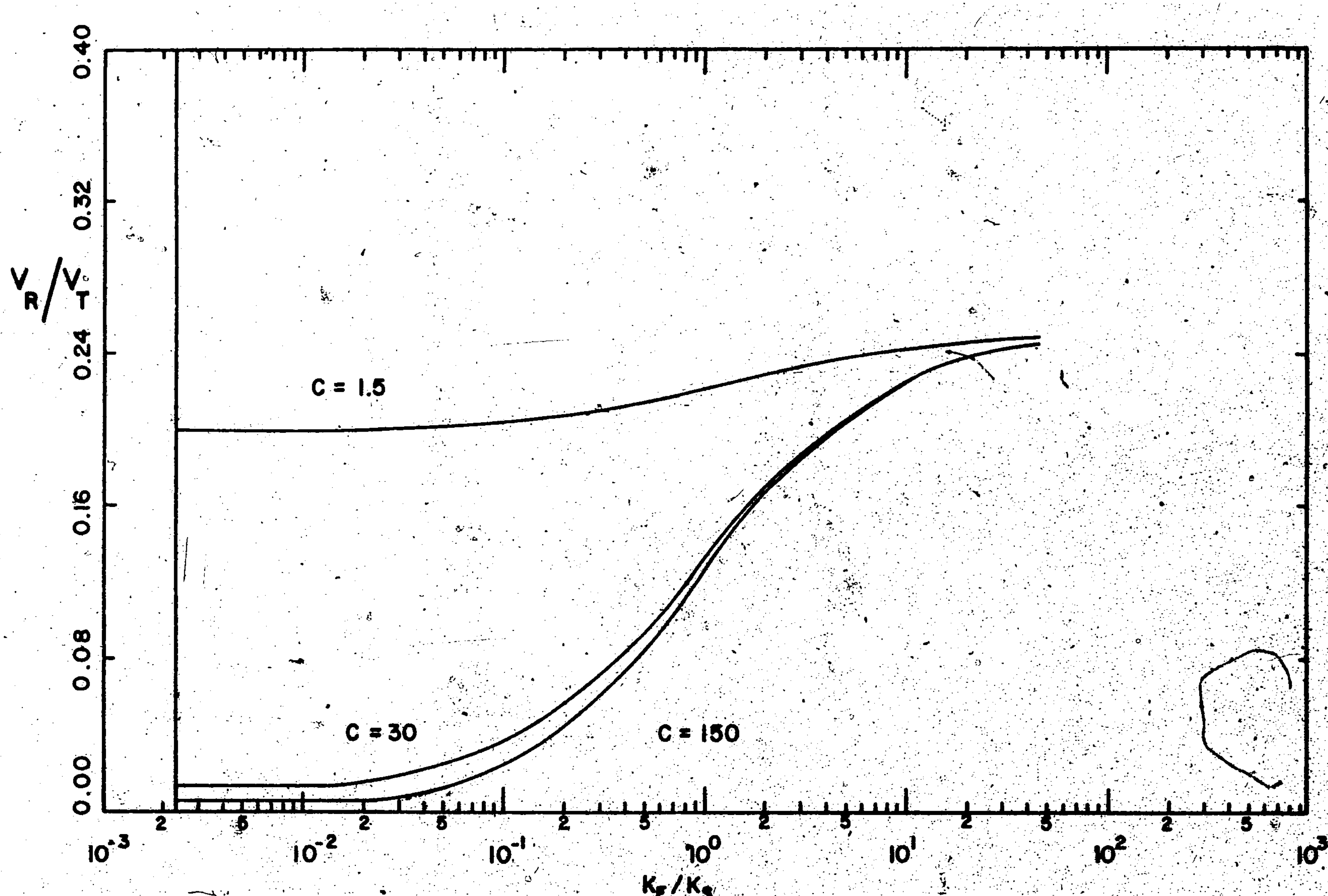


Figure 3-12 Four Bays Truss, Model Percentage of force taken by frame

3.2.2 General Solution

The general solution is based on the theory of beams on elastic foundation as first used by Muto (25).

As shown in Figure 3.13, the stiffness of the frames is converted into resisting elements distributed per unit length of the building. Development of the equations is presented in Appendix N.

The differential equation that represents this model is:

$$\frac{d^4y}{dx^4} - 2\beta^2 \frac{d^2y}{dx^2} + \alpha^4 y = \frac{W}{E_s I_s} \qquad (3.10)$$

$$\beta = \left(\frac{\beta_0 \kappa}{2G_s \bar{A}_s}\right)^{1/2}$$

$$\alpha = \left(\frac{g_F}{E_s I_s}\right)^{1/4}$$

where:

$\bar{A}_s$ = Shear area of the horizontal diaphragm

E_s = Young's Modulus of the horizontal diaphragm

g_F = Uniformly distributed resisting force

G_s = Shearing Modulus of the horizontal diaphragm

I_s = Moment of Inertia of the horizontal diaphragm

W = Uniformly distributed horizontal load

y = Displacement of the horizontal diaphragm

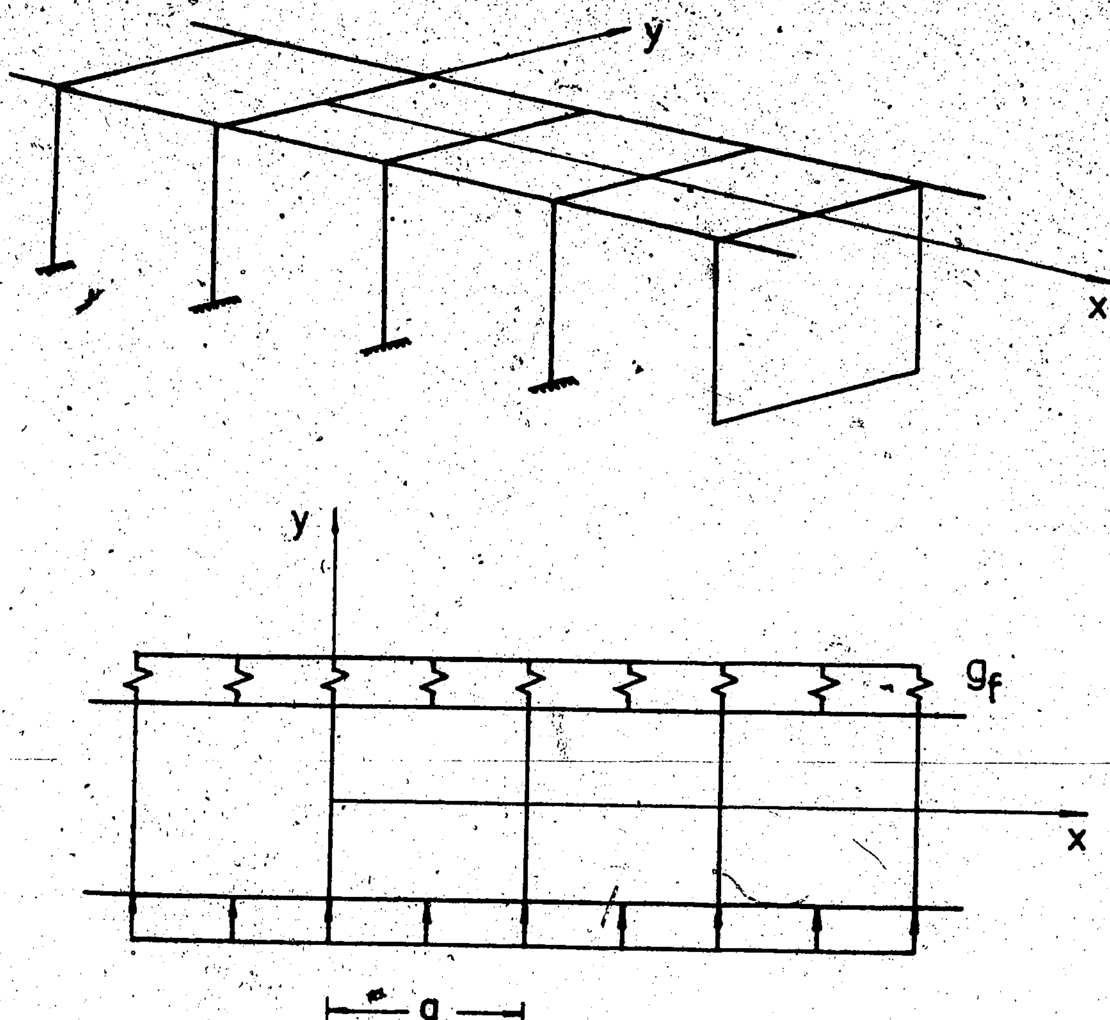


Figure 3-13 Beam on an Elastic Foundation

β_0 = Coefficient of inelastic shear deformation

κ = Shear deformation coefficient.

Solutions for the differential equation are dependent upon the factors α and β.

if $\alpha > \beta$

$$y = e^{dx}(A\cos fx + B\sin fx) + e^{-dx}(C\cos fx + D\sin fx) + W/g_F \quad (3.11)$$

where:

$$d = \left(\frac{\alpha^2+\beta^2}{2}\right)^{1/2}$$

$$f = \left(\frac{\alpha^2-\beta^2}{2}\right)^{1/2}$$

If $\alpha = \beta$

$$y = A\,e^{dx} + Bxe^{dx} + C\,e^{-dx} + Dxe^{-dx} + W/g_F \quad (3.12)$$

If $\alpha < \beta$

$$y = Ae^{(d+f)x} + Be^{-(d+f)x} + Ce^{(d-f)x} + De^{-(d-f)x} + W/g_F \quad (3.13)$$

The Boundary conditions are

$x = 1/2$, $y = Q_W/S_W$, $M_S = 0$

where

M_s = Bending moment of the horizontal diaphragm

Q_W = Force taken by an end wall

S_W = Stiffness of an end wall

The solution is developed in Appendix N. A computer solution is also given in Appendix T.

This procedure may be applied for different boundary conditions, that is, end shear walls, shear wall at the middle of the building or eccentrically arranged shear walls.

3.2.3 Comparison of the General Solution with a Standard Program

Several computer runs were made in order to evaluate the results given by the general solution and a standard program (SAPIV) (31) for a 3 bay building and a 5 bay building. When using the standard program, the horizontal diaphragm is represented by a beam element, while the vertical resisting elements are replaced by spring elements. For the general solution the intermediate frames are replaced by a continuous resisting element.

The results of these computer solutions are shown in Tables 3.2 an 3.3 The percentage of difference, between both solutions for the percentage of the total horizontal force taken by the interior frames ranges between 0% and 18% for three bays. For five bays differences range between 0.2% and 16%.

The main reason for these differences is related to the different models used to represent the frames. For longer buildings with a larger number of bays the two models should give similar results.

Table 3.2

THREE BAY BUILDING

Case	K_W/K_F	K_F/K_S	V_F/V_T	% Difference
MUTO 1 SAP 1	108.1	0.00106	0.00530 0.00560	5.36
MUTO 2 SAP 2	105.6	0.00109	0.00546 0.00575	5.04
MUTO 3 SAP 3	1.0	0.00109	0.25018 0.25017	0.00
MUTO 4 SAP 4	108.1	4.2500	0.36553 0.33961	7.09
MUTO 5 SAP 5	105.6	0.0920	0.24840 0.25571	2.86
MUTO 6 SPA 6	108.1	0.1290	0.08014 0.09795	18.20
MUTO 7 SAP 7	108.1	0.0270	0.00612 0.00664	7.80
MUTO 8 SAP 8	108.1	0.1700	0.09886 0.11826	16.40

TABLE 3.3

FIVE BAY BUILDING

Case	$\frac{2K_W}{3K_F}$	$\frac{K_F}{K_{s1}}$	$\frac{K_F}{K_{s2}}$	$\frac{V_{F1}}{V_T}$	% Difference	$\frac{V_{F2}}{V_T}$	% Difference
MUTO 1	0.5	0.00339	0.00195	0.1659	0.80	0.1728	1.62
SAP 1				0.1673		0.1700	
MUTO 2	72.08	0.00339	0.00195	0.00635	16.00	0.00897	4.60
SAP 2				0.00756		0.00942	
MUTO 3	0.95	0.00[illegible]	0.00195	0.12843	0.26	0.12977	0.20
				0.12877		0.12951	
MUTO 4	9.15	0.00339	0.00195	0.02530	3.98	0.02778	1.03
				0.02635		0.02807	

CHAPTER IV

INTERACTION FRAME-DIAPHRAGM FOR NONSYMMETRICAL ONE-STOREY BUILDINGS

4.1 INTRODUCTION

This chapter is devoted to the analysis of rectangular, nonsymmetrical one storey buildings subjected to uniformly distributed horizontal loads such as wind or earthquake.

Typical buildings studies are shown in Figure 4.1. Figure 4-1a and Figure 4-1b show buildings where the end walls have different stiffnesses. These different stiffnesses influence the displacement of the building in such a way that it is no longer only translational but is also rotational.

When the stiffness of the horizontal diaphragm is infinite as shown in Figure 4-2a, the total acting horizontal load can be concentrated at the geometric center of the building, which is defined as the center of gravity of the plan area of the building. In this particular case the building will rotate around the center of rotation which is defined as the center of gravity of the stiffnesses of the vertical elements of the building. Once the center of rotation is known the force resisted by any frame can be easily determined.

When the stiffness of the horizontal diaphragm is not infinite the total load can not be concentrated at the Geometric center and the load acting in any frame depends on the stiffness of the frame relative to the stiffness of the horizontal diaphragm and also on the location of the frame. Instead a distributed load is considered to act on the horizontal diaphragm which is modelled as a beam with internal shearing force and

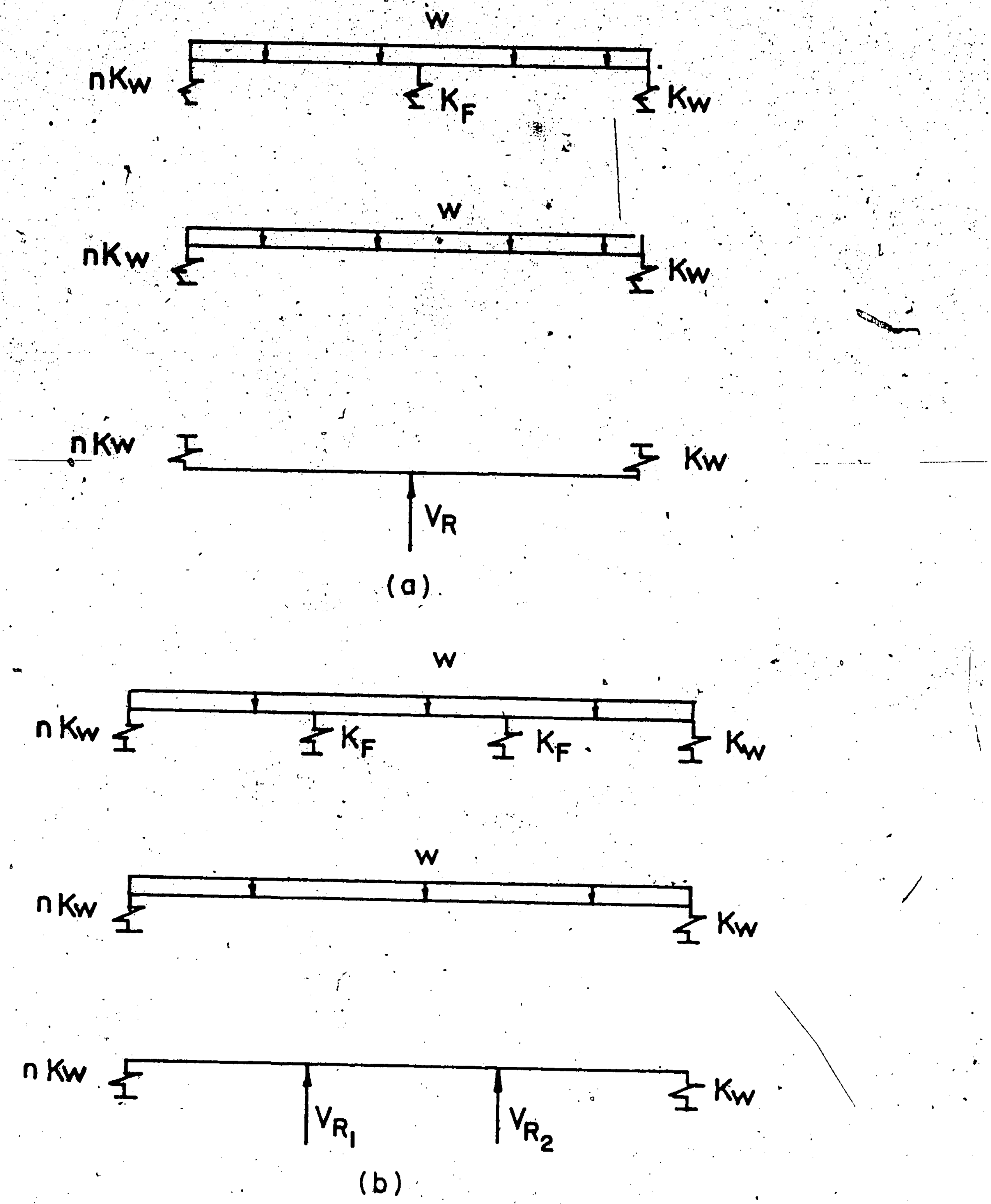


Figure 4-1 Typical Arrangements

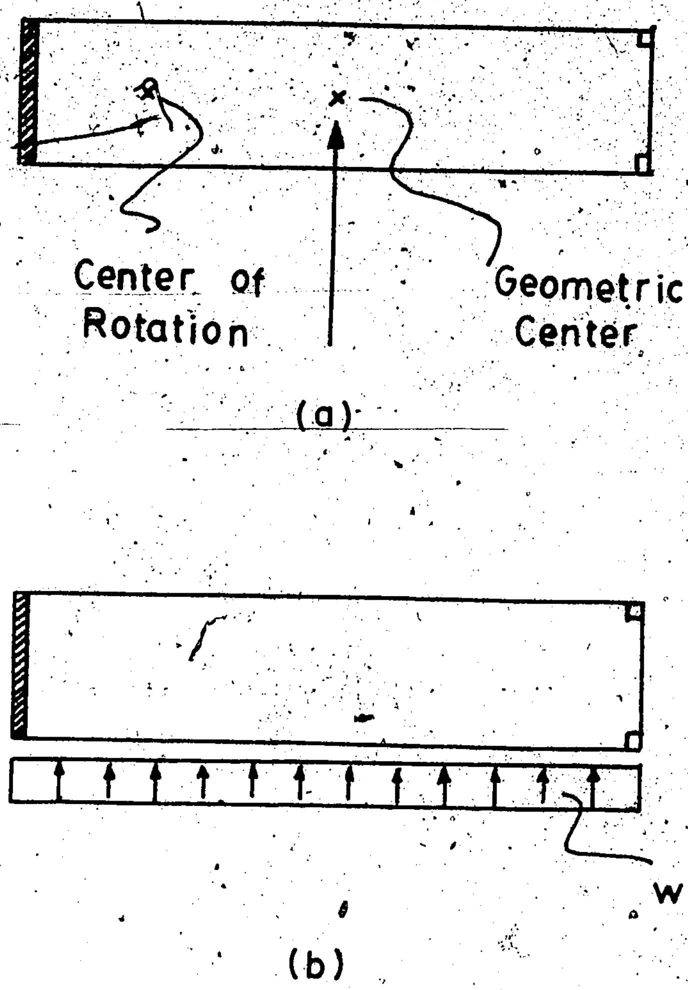


Figure 4 - 2 Torsion on Buildings

bending moment (see figure 4-2b).

When the number of bays is less than four the beam model yields results which are readily calculated. For more than three bays a solution based on the theory of beams on elastic foundations is developed.

4.2 METHOD OF ANALYSIS

As in Chapter III the beam model has been selected to represent the horizontal diaphragm. To model frames and walls, springs of different constants are used.

For buildings having up to three bays the beam model yields solutions easily obtainable. For more than three bays a solution based on the theory of beams on elastic foundations is developed.

4.2.1 BEAM MODEL

The horizontal diaphragm will behave as a deep plate girder, so that, bending and shear must be considered. When the building has a steel deck diaphragm only shear properties are considered since the bending contribution is slight. When the horizontal diaphragm is made of concrete, bending as well as shear properties must be considered.

4.2.1.1 SOLUTION FOR A LIMITED NUMBER OF BAYS

The solution for the force taken by an interior frame of a three bay building, shown in Figure 4-1b, will be used to explain how the nonsymmetrical arrangement of vertical elements affects the fraction of the total horizontal load taken by an interior frame. Derivations are detailed in Appendix M.

The force taken by frame 1 shown in Figure 4-1b is

$$\frac{V_R}{V_T} = \frac{A}{B} \qquad (4.1)$$

in which:

A and B are shown in Appendix M.

V_R = Force taken by the first interior frame

V_T = Total horizontal force acting on the building

As in the symmetrical case equation (4.1) represents for every c value a space surface whose intersection with the planes y - 0 and x = 0 yield the curves for the pure bending solution and the pure shear solution, respectively.

The limits for the force taken by a frame would be

$$\lim_{c \to \infty} \frac{V_R}{V_T} = \frac{0.1817x^2 + x + 1.1829xy + y + y^2}{0.495x^2 + 2.907x + 3.27xy + 4y + 3y^2 + 1}$$

$$= \frac{x + y}{2.725x + 3y + 1} \qquad (4.2)$$

which is the lower bound limit.

The intersection of this surface with the bending and shear planes yields the curves

$$\frac{V_R}{V_T} = \frac{x}{(2.725x + 1)}$$

and

$$\frac{V_R}{V_T} = \frac{y}{(3y + 1)}$$

The upper bound limit is

$$\lim_{c \to 0} \frac{V_R}{V_T} = \frac{\frac{0.05555(n+1)^2}{4n}}{\frac{0.1111(n+1)^2}{4n}} = 0.5 \qquad (4.3)$$

which is a plane in the space as shown in Figure 3-3.

Equations (4.2) and (4.3) are the limits for any possible set of solutions.

As in the symmetrical case the pure shear plane and the pure bending planes are the limits where either only shear properties or only bending properties are represented; it is necessary to have the solution for the force taken by an interior frame for the pure shear and the pure bending planes respectively as shown in Figure 4-3. To obtain a wide range of stiffness the x axis is necessarily logarithmic. The y axis represents the percentage of the total horizontal force taken by an interior frame. Parameter c has already been defined; n for a 3 bay building is related to c according to

$$c = (n+1)\ K_w/2K_F \qquad (4.4)$$

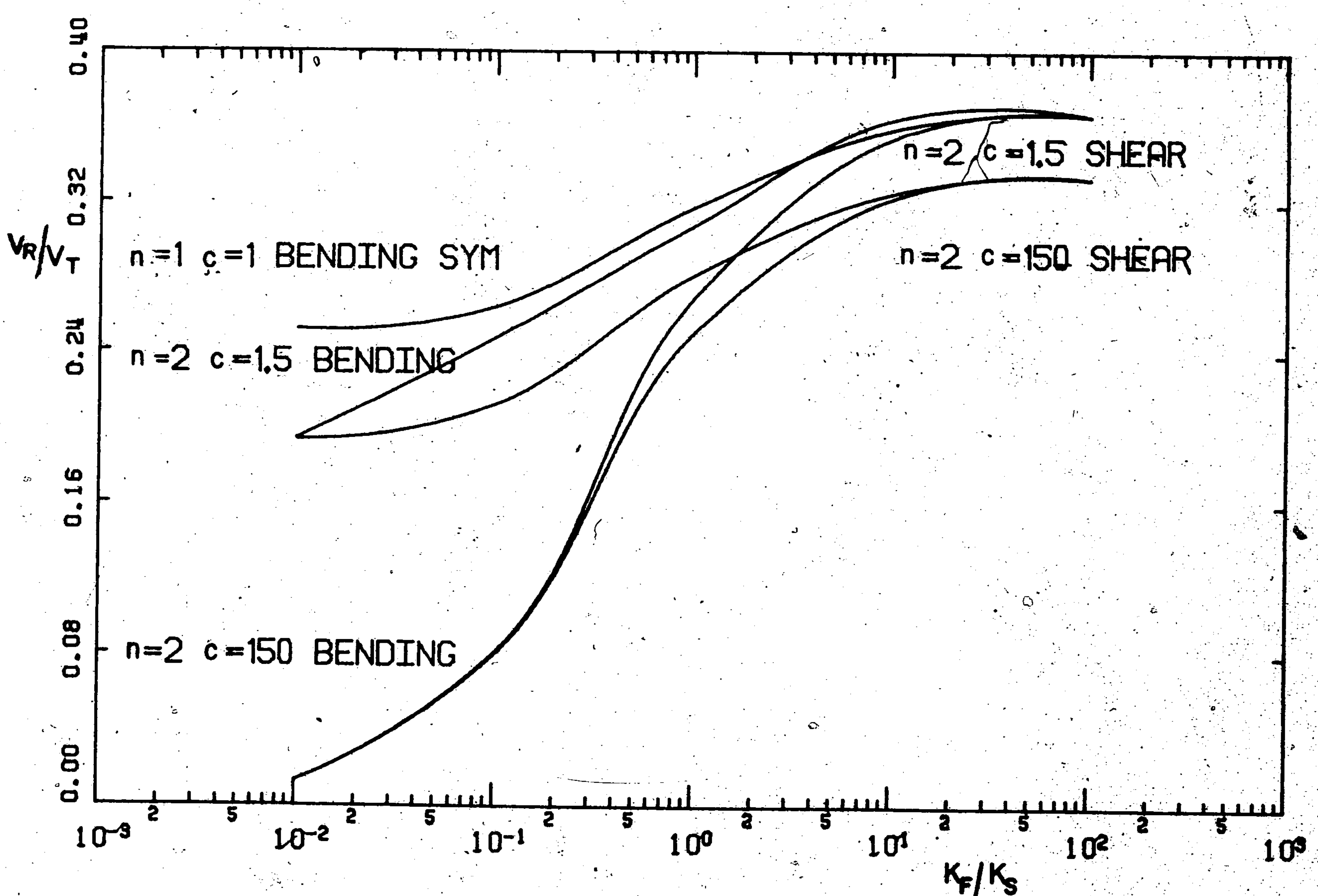


Figure 4-3 Three Bays Torsion

As in the symmetrical case the curves that represent the fraction of the total horizontal force taken by an interior frame, range between two extremes. These are an infinitely stiff diaphragm and an infinitely flexible diaphragm, relative to frame stiffness..

When the horizontal diaphragm is stiff relative to the frame, the ratio K_F/K_S tends to zero and for any c the displacement of the horizontal diaphragm is comprised of a lateral displacement and a rotation, subsequently the force taken by an interior frame is proportional to its stiffness and to the relative location with respect to the center of rotation. When the ratio K_F/K_S tends to infinity the horizontal diaphragm is not able to rotate and the distribution of loads to each intermediate frame is a function of the tributary area which is identical to the symmetrical case. When the ratio K_F/K_S is intermediate the diaphragm will be able rotate and this will increase or decrease the amount of horizontal force taken by an interior frame depending on its location.

4.2.1.2 RELATIONSHIP WITH THE SYMMETRICAL CASE

The relationship between the symmetrical and the nonsymmetrical case is best shown when comparing the results for the force taken by an interior frame for a three bay building.

For a three bay building the upper bound limit for the force taken by an interior frame is the plane Z = 0.5 which is the same limit as for symmetrical buildings. The lower bound limit is the surface

$$Z = \frac{x + y}{2.725x + 3y + 1}$$

which is the same limit for symmetrical buildings.

When applying the general solution, the solution for the differential equation is the same for the symmetrical and the nonsymmetrical cases. The only differences are the boundary conditions imposed on the solution.

4.2.2 GENERAL SOLUTION

The general solution for a nonsymmetrical arrangement of walls is based on the theory of a beam on an elastic foundation.

As shown in figure 4-4 the stiffness of the frames is converted into resisting elements distributed per unit length.

Development of the equations is explained in Appendix Q.

Solutions for the differential equation are dependent upon the factors β and α.

If $\beta < \alpha$

$$y_1 = e^{dx_1}(A_1\cos fx_1 + B_1\sin fx_1) + e^{-dx_1}(C_1\cos fx_1 + D_1\sin fx_1) + W/gF)$$

$$y_2 = e^{dx_2}(A_2\cos fx_2 + B_2\sin fx_2) + e^{-dx_2}(C_2\cos fx_2 + D_2\sin fx_2) + W/gF$$

If $\beta > \alpha$

$$y_1 = A_1 e^{(d+f)x_1} + B_1 e^{-(d+f)x_1} + C_1 e^{(d-f)x_1} + D_1 e^{-(d-f)x_1} + \frac{W}{gF}$$

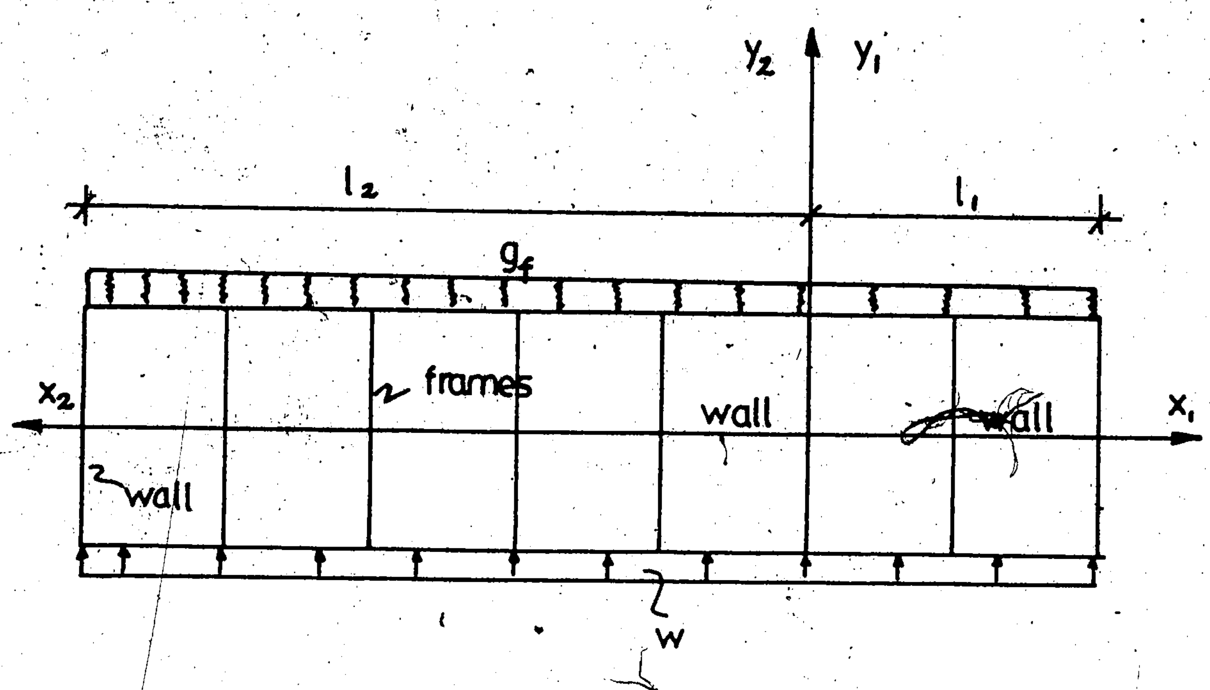


Figure 4-4 Nonsymmetrical Arrangement of Walls

$$y_2 = A_2e^{(d+f)x_2}+B_2e^{-(d+f)x_2}+C_2e^{(d-f)x_2}+D_2e^{-(d-f)x_2}+\frac{W}{gF}$$

If $\beta = \alpha$, the solutions are

$$y_1 = A_1e^{dx_1}+B_1x_1e^{dx_1}+C_1 e^{-dx_1}+D_1 x_1e^{-dx_1}+\frac{W}{gF}$$

$$y_2 = A_2e^{dx_2}+B_2x_2e^{dx_2}+C_2e^{-dx_2}+D_2x_2e^{-dx_2}+\frac{W}{gF}$$

in which

y_1 = displacement of the right part of the building

y_2 = displacement of the left part of the building

The boundary conditions are

$x_1 = l_1$ $\qquad$ $y_1 = Qw_1/K_W$

$x_1 = l_1$ $\qquad$ $Ms_1 = 0$

$x_1 = 0$ $\qquad$ $y_1 = Qw/K_W$

$x_2 = 0$ $\qquad$ $y_2 = Qw_2/K_W$

$x_1 = x_2 = 0$

$$\left(\frac{dy}{dx} - \frac{\beta_{o\kappa}}{G_s\bar{A}_s} Q_s\right)_1 = -\left(\frac{dy}{dx} - \frac{\beta_{o\kappa}}{G_s\bar{A}_s} Q_s\right)_2$$

$x_1 = x_2 = 0$

$M_{s_1} = M_{s_2}$

$x_2 = l_2$ $\qquad$ $y_2 = Qw_2/K_W$

$x_2 = l_2$ $\qquad$ $M_{s_2} = 0$

where

y_1 = Displacement of the left part of the building

y_2 = Displacement of the right part of the building

Q_{w_1} = Force taken by the right wall

K_W = Stiffness of the right wall

Q_w = Force taken by the middle wall

K_W = Stiffness of the middle wall

Q_{w_2} = Force taken by the left wall

K_W = Stiffness of the left wall

CHAPTER V

APPLICATION OF THE METHOD

5.1 Introduction

This chapter is devoted to the application of the procedure discussed in the first four chapters to two practical examples. Details of calculations are shown in Appendix R.

Both buildings analyzed have three bays, two end shear walls and two steel intermediate frames as shown in Figure 5.1. The horizontal deck for both cases is a steel deck; in the first case it is filled with concrete whereas in the second case there is no concrete fill. To obtain the stiffness of the horizontal diaphragm the procedure developed by the U.S. Department of the Army, Navy and Air Force (33) is applied. The horizontal steel deck characteristics are those given by the manufacturer (35).

5.2 THREE BAY BUILDING WITH CONCRETE DECK

The building shown in Figure 5.1 is first designed for vertical loads, then making use of the beam model to represent the horizontal diaphragm, the building is analyzed for horizontal loads. To calculate the stiffness of the horizontal diaphragm three constants are assessed:

The first one considers the contribution of the edge girders to the overall moment of inertia.

The second one accounts for the contribution of the concrete and the steel deck to the overall moment of inertia.

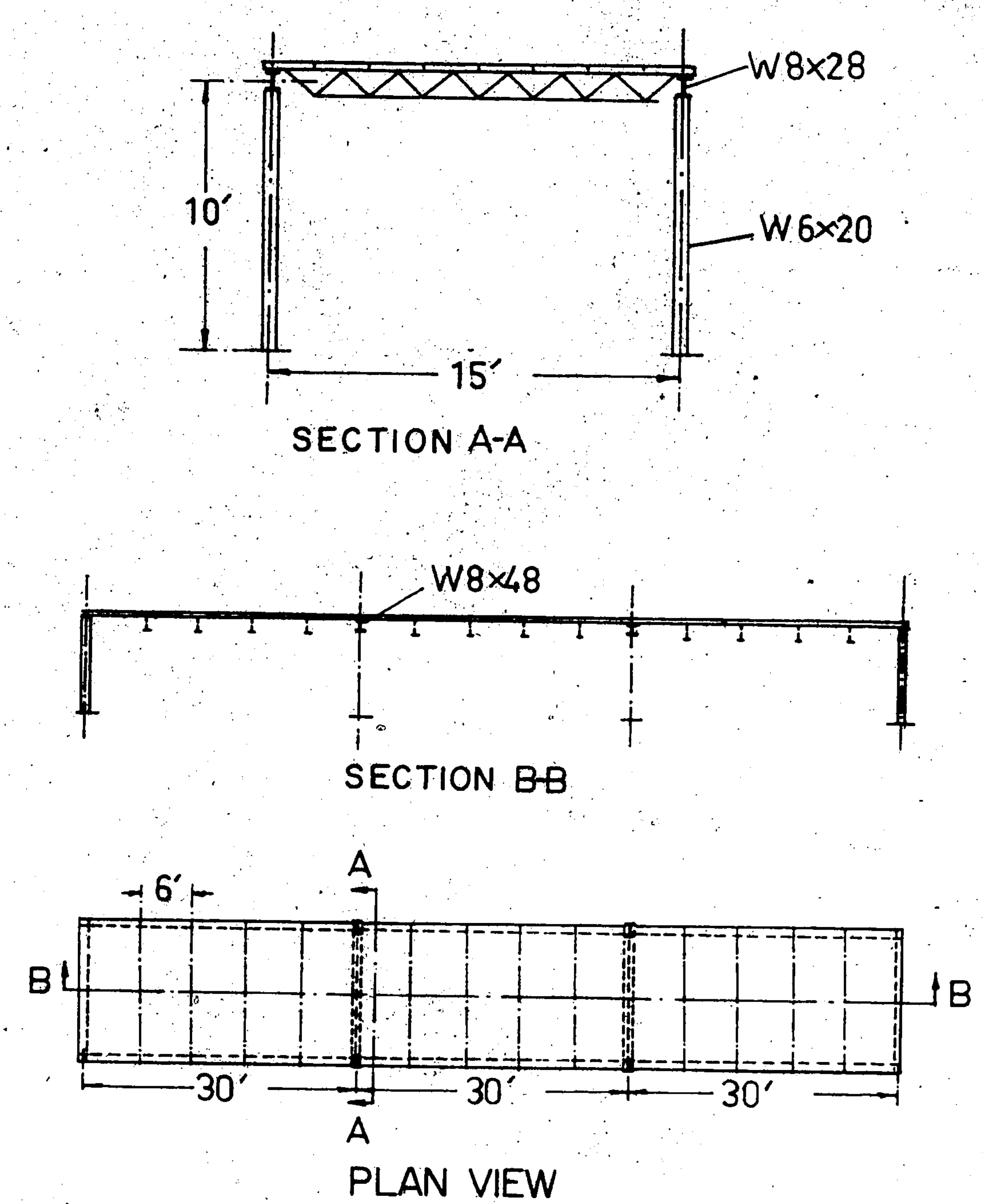


Figure 5-1 One Storey Symmetrical Buildings.

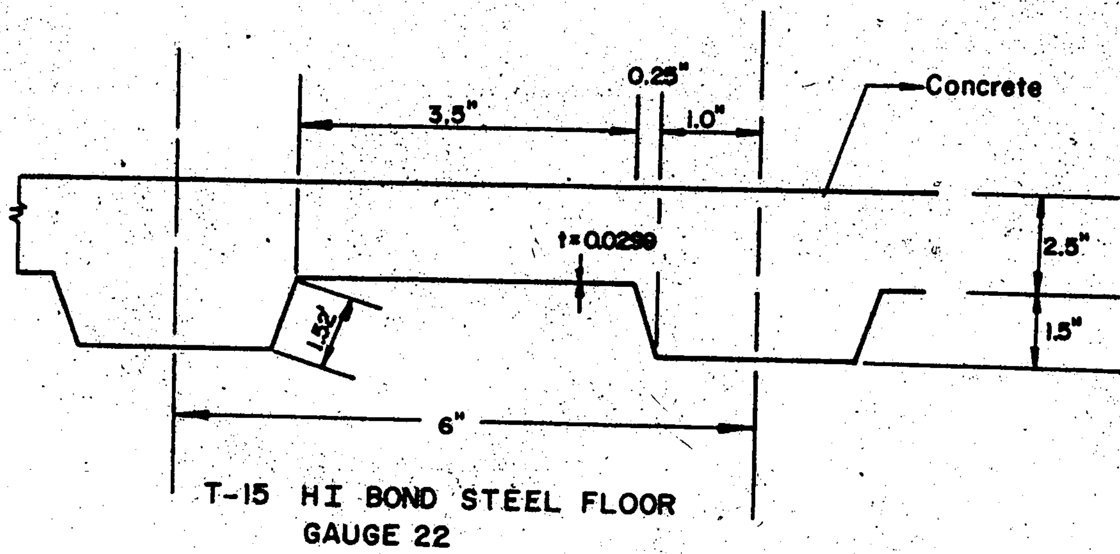


Figure 5-2 Steel Deck with Concrete Fill

The third coefficient accounts for the shear properties of the horizontal diaphragm. All calculations are presented in Appendix R.

5.3 THREE BAY BUILDING WITH STEEL DECK

The same procedure as explained in section 5.2 is applied, but in this case to calculate the stiffness of the horizontal diaphragm only two constants are assessed:

The first one considers the contribution of the edge girders to the overall moment of inertia.

The second one accounts for the shear properties of the horizontal diaphragm. The contribution of the steel deck to the overall moment of inertia is negligible. Details are given in Apendix R.

5.4 DISCUSSION

The importance of considering the stiffness of the horizontal disphragm is appreciated when displacements and horizontal forces on the vertical elements are calculated.

Several variations of end walls are considered in Appendix R and the results are summarized in Tables 5.1 and 5.2 respectively for concrete horizontal diaphragm and steel horizontal diaphragm. Three different end walls are considered and are shown as cases 1, 2 and 3 respectively.

Table 5.1 presents results for a building with the horizontal diaphragm very stiff since it is filled with concrete. The first column of Table 5.1 shows the stiffness of the frame relative to the stiffness of the slab. This value is extremely small and according to the study performed in Chapter III the horizontal diaphragm can be assumed infinitely stiff. The second column of Table 5.1 shows the stiffness of the end wall relative to the stiffness of the interior frame. As the stiffness of the end wall decreases the force taken by an interior frame increases.

In the second case, summarized in Table 5.2, the horizontal diaphragm is a steel deck with no concrete; this makes the horizontal diaphragm very flexible. The assumption that such a horizontal diaphragm is infinitely stiff is no longer valid as it is seen when comparing results of the force taken by an interior frame. When the stiffness of the horizontal diaphragm is calculated the force taken by an interior frame may be greatly increased. This increase is best shown when comparing the first two lines of Table 5.2. There is an increase of the force taken by an interior frame of more than two hundred per cent. When the stiffness of the wall relative to the stiffness of the frame is 1.0 the assumption that the horizontal diaphragm is infinitely stiff is not bad, since according to study performed in Chapter III, this case approximates the upper bound solution for the whole set of curves.

Table 5.1

Forces in frame for concrete Diaphragm

Case	$\frac{K_F}{K_S}$	$\frac{K_W}{K_F}$	$\frac{V_F}{V_T}$	Frame Displacement Inches	Remarks
1	0.008	2335.0	0.0080	0.0120	Concrete end walls
	0.000	2335.0	0.0002	0.0003	Horizontal Diaphragm infinitely stiff
2	0.008	70.0	0.0140	0.0212	Braced end walls
	0.000	70.0	0.0071	0.0107	Horizontal Diaphragm infintely stiff
3	0.008	1.0	0.254	0.1080	End wall is a frame
	0.000	1.0	0.250	0.1060	Horizontal Diaphragm inifintely stiff

Table 5.2

Forces in frame for steel deck Diaphragm

Case	$\frac{K_F}{K_s}$	$\frac{K_W}{K_F}$	$\frac{V_F}{V_T}$	Frame Displacement Inches	Remarks
1	0.745	655.0	0.2310	0.2060	Concrete end walls
	0.00	655.0	0.008	0.007	Horizontal Diaphargm assumed infinitely stiff
2	0.745	19.6	0.2360	0.2100	Braced end walls
	0.000	19.6	0.0240	0.0220	Horizontal Diaphragm assumed infinitely stiff.
3	0.745	1.0	0.2950	0.2630	End wall is a frame.
	0.000	1.0	0.2510	0.2230	Horizontal Diaphrgm assumed infinitely stiff.

CHAPTER VI

SUMMARY CONCLUSIONS AND RECOMMENDATIONS

6.1 SUMMARY

An investigation of the interaction between frame and diaphragm for horizontal loads acting on one storey rectangular buildings has been presented.

Chapter II includes a critical review of different approaches to calculate the strength and stiffness of steel deck diaphragms. A new approach to calculate the stiffness of steel diaphragms has been developed and presented. A parametric study of the different factors that govern the strength and stiffness of a steel deck diaphragm has been presented.

In Chapter III a particular solution for the force taken by an interior frame in a symmetrical one storey rectangular building has been developed. This solution may be applied to buildings that have up to five bays. A general solution based on the theory of a beam on an elastic foundation has been presented as well. This solution applies for any number of bays.

In Chapter IV a particular solution for the force taken by an interior frame for rectangular nonsymmetrical one storey buildings under horizontal loads has been presented. This solution may be applied to buildings having up to three bays. A general solution based on the theory of a beam on an elastic foundation has been developed which provides the solution for the force taken by an interior frame for buildings having any number of bays.

Chapter V presents two practical examples of the theory developed in Chapter III with variations in stiffness of end walls and interior frames. Effects of variation in diaphragm stiffness relative to end wall and frame stiffness are discussed.

6.2 CONCLUSIONS

1. The most accurate analytical method to assess the stiffness of a steel deck diaphragm under in-plane loads is the method developed by Davies (11) in Britain.
2. The most important factors that affect the strength and stiffness of a steel deck diaphragm are: panel length, purlin spacing, number of seam connections and number of shear connectors.
3. The parameter K_F/K_S (stiffness of the frame/stiffness of the horizontal diaphragm) is the first most important parameter that affects the amount of horizontal load taken by an internal frame
4. The parameter $c = \Sigma K_W/\Sigma K_F$ (summation of the stiffness of the outerwalls/summation of the stiffness of the internal frames) is the second most important parameter that affects the amount of horizontal load taken by an internal frame.
5. The curve that represents the percentage of horizontal force taken by an internal frame is dependent upon the bending and shearing properties of the horizontal diaphragm. The

solution lies in a plane which is bounded by the pure shear plane and the pure bending plane.

6. The solution in the pure shear plane is a valid solution for buildings having a steel deck horizontal diaphragm when using the beam model.
7. The solution for the force taken by an internal frame is always bounded by a lower bound space surface when $c \to \infty$ and an upper bound space surface when $c \to 0$. This two limits are dependent upon the number of internal frames.

6.3 LIMITATIONS OF THE METHOD AND RECOMMENDATIONS

The solution for the stiffness of a steel deck diaphragm under in-plane loads in Chapter II is an elastic one; non linear behaviour of connections and redistribution of stresses takes place before the failure of the diaphragm occurs. It is recommended that nonlinear behaviour be incorporated in the analysis of the diaphragm.

The solution discussed in Chapters III and IV is based in the Beam model so that the ratio between length and width of the building must be larger than 2.0. It is also a linear solution since both shear and bending deflections are superimposed. Therefore, there is a need to study this matter more thoroughly. A finite element analysis that incorporates the Geometrical and Physical properties of the horizontal diaphragm may be applied. The study may be widened for buildings of more than one storey.

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APPENDIX A

PLATE BUCKLING FAILURE

Bergmann, V.S. and Reissner's Method

Bergmann and Reissner (4) calculated buckling loads of rectangular corrugated orthotropic plates with the aid of Eq. A.1.

$$N_{cr} = \frac{4\lambda D_x^{0.25} D_y^{0.75}}{b^2} \tag{A.1}$$

where

N_{cr} = Buckling load

b = panel width

D_x = Bending stiffness per unit length of diaphragm in the x direction

D_y = Bending stiffness per unit length of diaphragm in the y direction

λ = Multiplier dependent upon θ and ϕ

$$\theta = 2\ (D_x\ D_y)^{0.5}/D_{xy} \tag{A.2}$$

$$\phi = \frac{b}{2}\left(\frac{D_x}{D_y}\right)^{0.5} \tag{A.3}$$

where

a = Panel length

The dependence of λ on θ and ϕ is given in the form of curves in Reference 30.

Hlavacek's Method

Hlavacek (21) calculated buckling loads with the aid of Equation A.4.

$$N_{cr} = \frac{41\ D_x^{0.27}\ D_y^{0.75}}{b^2} \tag{A.4}$$

Easley - McFarland's Method

Easley and McFarland (14) calculated buckling loads using Equation A.5.

$$N_{cr} = \frac{36\ D_x^{0.25}\ D_y^{0.75}}{b^2} \tag{A.5}$$

Easley's Method

Easley (15) calculated buckling loads with the aid of Eq. A.6.

$$N_{cr} = \frac{36\ \xi\ D_x^{0.25}\ D_y^{0.75}}{b^2} \tag{A.6}$$

Where ξ is a parameter that represents boundary conditions. The lower and upper boundaries for ξ are 1.0 and 1.9, but the true variation of ξ with diaphragm end restraint is unknown.

APPENDIX B

BRYAN'S METHOD

Stiffness

Bryan (7) states that the overall shear flexibility of a shear panel can be evaluated by considering the effect of separate components. These components can be summarized as follows:

Sheet Distortion

$$c_{1.1} = \frac{0.144\ a\ d_0^4\ f_1\ K_0}{Et^3 b^3} \qquad (B.1)$$

where

a = Panel Length

d_0 = Pitch of corrugations

f_1 = correction factor (Reference 7, p. 110)

K_0 = sheeting constant (Reference 7, p. 109)

E = Modulus of Elasticity

t = sheet thickness

b = Panel width

Shear strain in the sheet

$$c_{1.2} = \frac{2\ a\ f_2\ (1+\nu)(1+2h/d)}{btE} \qquad (B.2)$$

where

a = Panel Length

f_2 = correction factor (Reference 7, p. 110)

ν = Poisson's ratio

h = height of a corrugation

d = Pitch of corrugations

b = Panel width

t = sheet thickness

E = Modulus of Elasticity;

Axial strain in the Purlins

$$c_{1.3} = \frac{2\,a^3 f_3}{3b^2 A_0 E} \qquad \text{(B.3)}$$

where

f_3 = correction factor (Reference 7, p. 110)

A_0 = cross area

Sheet - Purlin Fasteners (Sheeting fixed on all four edges)

$$c_{2.1} = \frac{2\,a\,q\,S_p}{b^2}\,f_3 \qquad \text{(B.4)}$$

where

q = pitch of sheet-purlin fasteners

S_p = flexibility per sheet-purlin fastener

f_3 = correction factor

Sheet-Purlin Fasteners (sheeting fixed to the purlins)

$$c_{2.1} = \frac{2S_p q}{2} (6/p+2 + a^2 f_3/b^2) \quad (B.5)$$

Seam Fasteners (sheeting fixed on all four edges)

$$c_{2.2} = (n_{sh}-1)\ S_s/n_s \quad (B.6)$$

where

n_{sh} = number of sub-panels

S_s = flexibility of a seam fastener

n_s = number of seam fasteners

Seam Fasteners (sheeting fixed on two edges)

$$c_{2.2} = n_{sh}\ S_s/n_s \quad (B.7)$$

Sheet-Connector Fasteners

$$c_{2.3} = 2\ S_{sc}/n_{sc} \quad (B.8)$$

where

S_{sc} = flexibility of sheet-connector fasteners

n_{sc} = number of sheet-connector fasteners

APPENDIX C

DAVIES' METHOD

Stiffness

In addition to the components already obtained by Bryan, Davies (10) modifies the following coefficients

Sheet-purlin fastener

$$c_{2.1} = 2\, a\, s_p \cdot q\, f_3 / b^2 \qquad \text{(C.1)}$$

where

a = panel length

s_p = flexibility per sheet-purlin fastener

q = pitch of sheet-purlin fastener

f_3 = correction factor

b = panel width

Seam fastener

$$c_{2.2} = \frac{2\, s_s\, s_p\, (n_{sh}-1)}{2\, n_s\, s_p + g_1 (p+2)\, s_s} \qquad \text{(C.2)}$$

where

s_s = flexibility of a seam fastener

n_{sh} = number of sub-panels

n_s = number of seam fasteners

p = number of intermediate purlins

$$g_1 = \sum_{i=1}^{n_p/2} \left(\frac{2_i - 1}{n_p - 1}\right)^2 \qquad (C.3)$$

where

n_p = total number of sheet-purlin fasteners per sub-panel, n_p even

$$g_1 = \sum^{\frac{n_p - 1}{2}} \left(\frac{2_i}{n_p - 1}\right)^2 \qquad (C.4)$$

n_p odd

Shear connector (direct shear transfer)

$$c_{2.3} = \frac{2\ S_{sc}}{n_{sc}} \qquad (C.5)$$

APPENDIX D

EASLEY'S METHOD

Strength

End Fastener Failure

Easley (13) assumes that the horizontal force taken by each fastener is constant.

$$F_{eph} = \frac{Q}{n_{sh}\, n_e} \qquad (D.1)$$

The vertical component is

$$F_{epvi} = \frac{2x_i\, n_{sh}\, Qb}{a^2 \beta_1} \qquad (D.2)$$

where

$$\beta_1 = n_s + \frac{4I_e + 2\, P\, I_g}{\left(\frac{a}{n_{sh}}\right)^2}$$

where

$$I_e = \sum_{i=1}^{n_e} x_i^{\,2}$$

$$Ig = \sum_{p=1}^{n_p} x_p^{\,2}$$

where

a = Length of the panel

b = width of the panel (parallel to the direction of the corrugations)

F_{ep_h} = Horizontal force component in the fasteners

F_{epvi} = Vertical Force component

F_{R_i} = Resultant Force

n_e = number of end fasteners per sub-panel

n_p = number of purlin fasteners

n_s = number of seam fasteners

n_{sh} = number of sub-panels per panel

P = number of intermediate purlins per panel

Q = Applied force as shown in Figure 2.6a

x_j = Distance from every end fastener to the center of rotation of the sub-panel

x_p = Distance from every purlin fastener to the center of rotation of the sub-panel.

$$F_{R_i} = (Fe_{ph}^{\;2} + Fe_{pvi})^{1/2} \quad (D.3)$$

Seam Fastener Failure

From Figure 2-5

$$F_s = \frac{Qb}{a\beta_1} \quad (D.4)$$

where

F_s = side fastener force

The lowest value from equations D.3 and D.4 gives the strength of the steel deck diaphragm under in plane loads.

STIFFNESS

Easley's equation for the stiffness of a diaphragm is given following the AISI (1) recommendations for shear diaphragms.

$$K^1 = \frac{1}{\dfrac{d}{Gt\ell} + \dfrac{\lambda_1 b \; n_{sh}}{0392 \; F_u a\beta_1}} \quad (D.5)$$

In which

d = length of one corrugation measured along the corrugation

F_u = ultimate strength of the fastener connection

G = shear modulus of elasticity of the panel material

ℓ = projection of the length, d, on the plane of the panel

n_{sh} = number of sub-panels

t = material thickness

λ_1 = constant

According to Easley the parameters F_u and λ_1 are obtained by testing.

APPENDIX E

Fazio's Et Al. Approach

Strength

According to this method (17) the steel deck may have three modes of failures.

Failure at Seam

$$Q_{ult} = \frac{K_1(n_s F_{su} + P F_{pu} + 2 F_{eu})}{\left(\frac{a}{n_{sh} X_o} - 1\right)\left(\frac{2 n_s}{S_s} + \frac{P}{S_p} + \frac{2}{S_e}\right)}$$

where

$$K_1 = \frac{2 n_s}{S_s}\left(\frac{a}{n_{sh} X_o} - 1\right) + \frac{2}{S_e}\left(g_{2e} - g_{1e}\right) + \frac{P}{S_p}\left(g_{2p} - g_{1p}\right)$$

and

$$X_o = \frac{\frac{2 n_s}{S_s} + \frac{2 g_{1e}}{S_e} + \frac{P}{S_p} g_{1p}}{\frac{n_{sc}}{S_{sc}} + \frac{2 n_s}{S_s} + \frac{2 n_e}{S_e} + \frac{P n_p}{S_p}}$$

and

$$g_{1e} = \frac{n_{sh}}{a} \sum_{i=1}^{n_e} x_i$$

$$g_{2e} = \frac{n_s}{a X_o} \sum_{i=1}^{n_e} x_i^2$$

$$g_{1p} = \frac{n_{sh}}{a} \sum_{p=1}^{n_p} x_p$$

$$g_{2p} = \frac{n_{sh}}{a X_o} \sum_{p=1}^{n_p} x_p^2$$

Failure At Side

$$Q_{ult} = \frac{K_2 (n_{sc} F_{sc} + P F_{pu} + 2 F_{eu})}{\frac{n_{sc}}{S_{sc}} + \frac{n_p}{S_p} + \frac{2}{S_e}} \quad (E.2)$$

Failure At End

$$Q_{ult} = \frac{F_{eu}}{(F_{eph}^2 + F_{epv}^2)^{1/2}} \quad (E.3)$$

where

a = length of diaphragm in direction perpendicular to corrugations

F_{eph} = horizontal constant force as shown in Figure E.1

F_{epv} = maximum vertical force at the end fastener

F_{eu} = strength of end fastener

F_{pu} = strength of sheet to purlin fastener

F_{sc} = strength of side fastener

F_{su} = strength of seam fastener

n_e = number of end fasteners per panel end

n_p = number of purlin fasteners per panel

P = number of purlins

n_s = number of seam fasteners per seam line

n_{sc} = number of side fasteners per side

n_{sh} = number of sub-panel per panel

S_e = flexibility of end fastener

S_p = flexibility of purlin fastener

S_s = flexibility of seam fasteners

S_{sc} = flexibility of side fasteners

APPENDIX F

EMPIRICAL METHOD FOR STRENGTH AND STIFFNESS

U.S. DEPARTMENTS OF THE ARMY, NAVY AND AIR FORCE

CONCRETE DIAPHRAGMS

This type of diaphragm is made of a steel deck with a superimposed fill of concrete having a minimum fic of 2,500 p.s.i. at 28 days and a minimum weight of 90 p.s.f.

Galvanized steel deck is used. The method to calculate shear stiffness and the working shear is given empirically by the Department of the Army, Air Force and Navy (31).

Two types of steel deck diagphrams with concrete are categorized:

Type 1. If the diaphragm is loaded without shear stresses passing through the deck or its attachments as shown in Figure F.1.a.

Type 2. If the shear passes through the deck or its attachments as shown in Figure F.1.b.

Stiffness

The factor F is introduced to calculate the stiffness of a horizontal diaphragm and is defined as the average deflection in microinches of the diaphragm web, per foot of span, stressed with a shear of one pound per foot.

$$F = \frac{\Delta_w \cdot 10^6}{q_{ave} \cdot l_1}$$

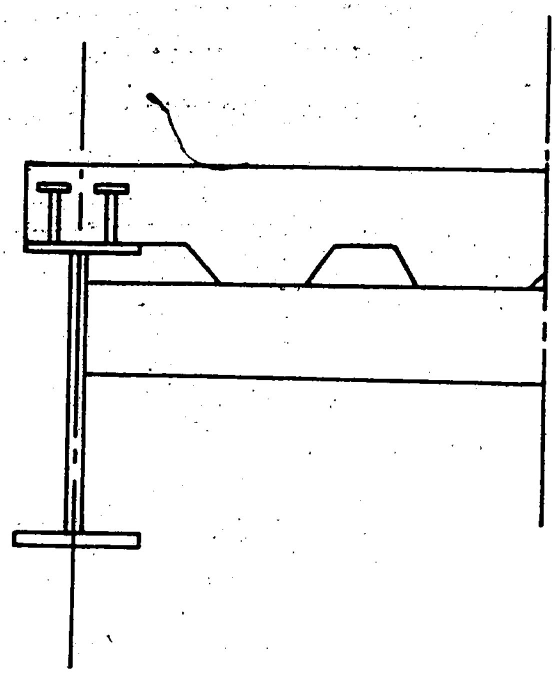


Fig. F1-A

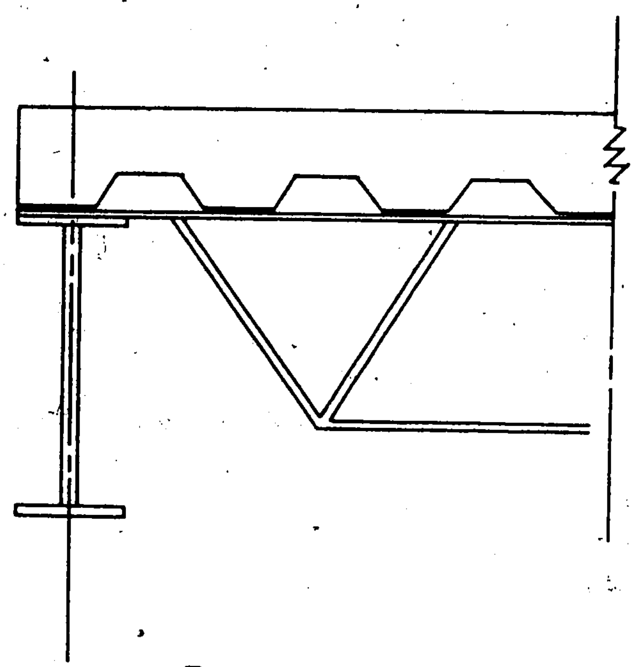


Fig. F1-B

Figure F-1 Steel Deck with Concrete Fill

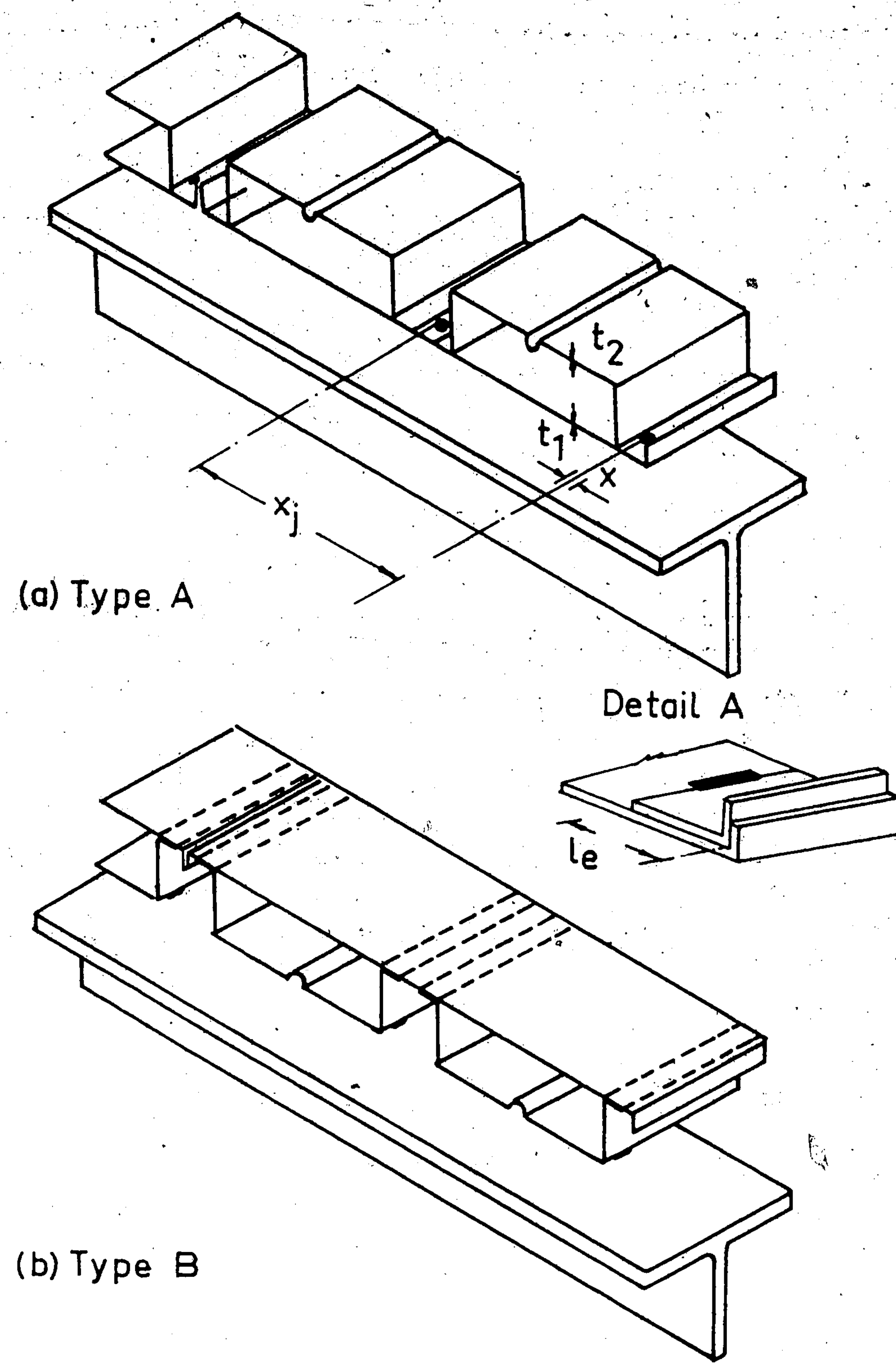


Figure F-2 Steel Deck Diaphragms

where

l_1 =Distance in ft. between vertical resisting elements and the point to which the deflection is to be determined.

q_{ave} =Average shear in diaphragm in pounds per ft. over length L_1.

Δ_w =Web deflection in inches.

TYPE 1

Strength

The criteria to calculate the strength of a concrete diaphragm is given by ACI 318-71 (2) but limited to the value of the equation

$$q_{ud} = \frac{6.0\sqrt{f'c} \; . \; t}{1 + \frac{3}{(P+1)^2} \frac{b^2}{t^2}} \quad (F.1)$$

where

q_{ud} =shear capacity in pounds per foot

f'c =compressive strength of concrete

b =width of the building

P = Number of interior purlins

t =Thickness of slab in inches

The flexibility factor F is calculated using the formula

$$F = \frac{10^6}{8.5t\gamma^{1.5} \sqrt{f'c}} \quad (F.2)$$

where

t =Thickness of the slab in inches

γ =Weight of the concrete in pounds per cubic foot.

f'c =The compressive strength of the concrete at 28 days in pounds per square inch.

TYPE 2

Strength

$$q_D + q_1 = q_6 \tag{F.3}$$

where

$$q_1 = \frac{92\ S\ (t_1+t_2')\ K_2(P+1)}{ab}\ n_{sh} \tag{F.4}$$

in which $K_2 = 1{,}000$

$$q_6 = q_6' + q_6'' \tag{F.5}$$

where

$$q_6' = \frac{t_f\ \gamma^{1.5}\ \sqrt{f'c}}{200} \tag{F.6}$$

and

$$q_6'' = z\sqrt{\frac{ka}{X_1(t_1+t_2')\ n_{sh}}} \tag{F.7}$$

where

q_D = Working shear in pounds per foot

a = Length of the diaphragm

X_1 = Distance in feet between outermost puddle welds attaching a deck unit to the supporting framing member.

S = Section modulus in feet of puddle weld group at supports

t_1 = Thickness of flat sheet elements in inches

t_2' = Effective thickness of fluted elements in inches

t_f = Thickness of fill over top of deck in inches.

The Flexibility factor F, is defined using the Formula

$$F = \frac{20\ q_6^{11}}{a^2 q_D}\ n_{sh}^{2} \tag{F.8}$$

STEEL DECK DIAPHRAGMS

Two types of steel deck diaphragms are categorized: Type A and Type B as shown in Figure F-2. Definition of the symbols used are as follows:

a = Length of the diaphragm

a_p = Averge spacing of profile channel closures, in feet.

a_s = Center to center spacing of seam welds in feet.

a_w = Spacing of margina lwelds in feet.

C_1 = 1 for galvanized decks; 0.65 for painted decks.

C_2 = 1 for button-punched seams; $40t_s^{1/2} l'_w$ for welded seams

C_3 = 1 for button-punch seams; $150\ t_s l'_w$ for welded seams.

C_4 = 1 for button-punched seams; $\frac{6(P+1)}{b}$ for welded seams.

C_5 = 1.2 for continuous angle closure; 1 for continuous zee closure; $\frac{1.44}{a_p}$ for profile channel closure.

d = Distance in feet between outermost puddle welds attaching a deck unit to the supporting framing member.

$F_1, F_2 \ldots$ = Components contributing to the flexibility factor F ($F = \Sigma F_n$).

f'_c = Compressive strength of fill concrete at 28 days in pounds per square inch.

h = Height of fluted elements in inches

I_D = Gross moment of inertia of deck unit about vertical centerline axid through unit in inches to the fourth power.

I_X = Gross moment of inertia of deck unit above the horizontal neutral axis of the deck cross-section per foot of width in inches to the fourth power.

l_1 = Distance in feet between vertical resisting element (such as shear wall) and the point to which the deflection is to be determined.

L_2 = Average length of each deck unit in feet.

l_e = Length of edge lip deck panel in inches

L_R = Distance in feet between shear transfer elements

L_v = Vertical load span of deck units in feet.

l_w = Minimum length in inches of seam weld.

l'_w = Effective length in inches of seam weld.

n = Average number of vertical deck elements per foot which are laterally restrained at the bottom by puddle welds.

n_s = Number of seam attachments in span b along a seam

q_D = Working shear in pounds per foot. The one-thir increase usually permitted on working stresses is not applilcable to this vaue.

$q_1, q_2 \ldots$ = Components or limiting vaues of working shear in pounds per foot.

q_{ave} = Average shear in diaphragm over length L_1 in pounds per foot.

R = $\frac{b}{(P+1)L_2}$

S = Section modulus in feet of puddle weld group at supports. (Each weld assumed as unit area.)

t_1 = Thickness of flat sheet elements in inches.

t_2 = Thickness of fluted element in inches.

t'_2 = Effective thickness of fluted elements in inches.

t_c = Thickness of closure element in inches.

t_f = Thickness of fill over top of deck in inches.

t_s = Thickness in inches of deck steet at seams.

γ = Unit weight of fill concrete in pounds per cubic foot.

TYPE A

Strength

$$q_D = (q_1 + q_2)\, q_3/q_2, \text{ where } q_3/q_2 < C_1 \qquad (F.9)$$

and

$$q_D < \frac{I_x \times 10^6}{2\,\dfrac{b^2}{(p+1)^2}}$$

or

$$q_D < \frac{10^4}{1.5\sqrt{\dfrac{b}{P+1}}\left(F_1 + F_2 + \dfrac{F_3 + L_2}{12}\right)}, \text{ if } l_e < 0.5"$$

$$q_1 = \frac{92\, S\, (t_1 + t_2')\, K_2 (P+1)\, n_{sh}}{ab}$$

where

$$K_2 = \frac{1{,}000}{1 + S\left[\dfrac{1}{\dfrac{(t_1 + t_2)\, t_1}{t_2{}^2} + 100\, m^{0.5}\, t_2^2\sqrt{\dfrac{43}{h}}\left(\dfrac{t_2}{t_1 + t_2}\right)^3}\right]^{2\ 0.5}}$$

$$q_2 = \frac{n_s\, a\, t_s{}^{0.5} C_2}{n_{sh}{}^2}\left[q_1\left(\frac{500}{I_D} + \frac{0.5\,(P+1)}{b\, x_1 S\, (t_1 + t_2^1)^2}\right)\right]^{0.5}$$

$$q_3 = \frac{3600 \ t_s n_s C_3 (p+1)}{b}$$

The flexibility F is defined as

$$F = F_1 + F_2 + F_3$$

where

$$F_1 = \frac{1}{12 \ (t_1 + t_2)}$$

$$F_2 = \frac{a \ b^2 \ C_4}{(P+1)^2 n_{sh} \ 160} \left(\frac{500}{I_D} + \frac{0.5 \ (P+1)}{b \ X_1 \ S \ (t_1 + t_2)^2}\right) \frac{q_1}{q_1 + q_2}$$

$$F_3 = \frac{R \ (P+1)}{\left(t_1 + \frac{12.5 \ m^2 C_1^2 T_2^3}{h}\right)}$$

TYPE B

Strength

$q_D = q_3$, q_4, or q_5 whichever is the lesser but limited to 1050 lb/ft

$$q_3 = \frac{0.6 \ t_s^2 \ n_s \ 1_w^1 \ (P+1)}{b}$$

$$q_4 = \frac{t_s}{10} \left(\frac{1}{a_s}\right)^2 \cdot 10^6$$

$$q_5 = \frac{C_5 t_c^2 \cdot 10^6}{2h^{0.5}}$$

<u>Flexibility</u>

$$F = F_1 + F_4 + F_5$$

where

$$F_1 = \frac{1}{12\ (t_1 + t_2)}$$

$$F_4 = \frac{3500}{q_3}$$

$$F_5 = \frac{20000}{L_R\ q_5}$$

APPENDIX G

STIFFNESS OF THE SLAB

The horizontal diaphragm is shown in Figure G-1. It is considered as a simple supported beam between shear walls.

Considering bending plus shear deflections

$$\Delta = \int_0^{\delta_1 l} \frac{(l-\delta_1)\left(\frac{Wlx}{2} - \frac{Wx^2}{2}\right)}{E_S I_S} dx +$$

$$\int_{\delta_1 l}^{l} \frac{\delta_1 (l-x)\left(\frac{Wlx}{2} - \frac{Wx^2}{2}\right)}{E_S I_S} dx +$$

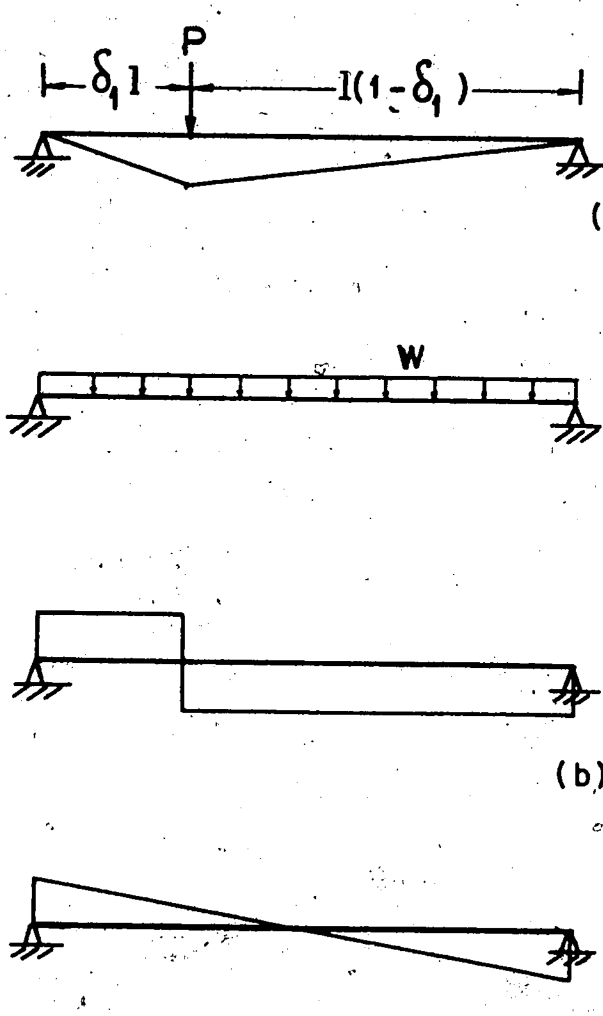
$$\int_0^{\delta_1 l} \frac{(l-\delta_1)\left(\frac{Wl}{2} - W\right)}{G_S \bar{A}_S} dx + \int_{\delta_1 l}^{l} \frac{-\delta_1\left(\frac{Wl}{2} - Wx\right)}{G_S \bar{A}_S} dx$$

The integration of these quantities yield

$$\Delta = \frac{Wl^4}{24E_S I_S}(\delta_1^4 - 2\delta_1^3 + \delta_1) + \frac{Wl^2}{2G_S \bar{A}_S}(\delta_1 - \delta_1^2)$$

Therefore the stiffness of the slab is

$$K_S = \frac{Wl}{\Delta} = \frac{1}{\frac{l^3(\delta_1^4 - 2\delta_1^3 + \delta_1)}{24\ E_S I_S} + \frac{l(\delta_1 - \delta_1^2)}{2\ G_S \bar{A}_S}}$$



(a) Bending

(b) Shear

Figure G-1 Stiffness of the slab

where

l = Length of the slab between end shear walls

$\overline{A}_S$ = Shear area of the horizontal diaphragm

E_S = Young's Modulus of the horizontal diaphragm

G_S = Shearing Modulus of the horizontal diaphragm

I_S = Moment of Inertia of the horizontal diaphragm

W = Uniformly distirbuted horizontal load

δ_1 = Location coefficient

Δ = Deflection of the horizontal diaphragm

APPENDIX H

SYMMETRICAL TWO BAY BUILDING

TRUSS SOLUTION

The structure is shown in Figure H-1

The matrix solution yields:

$$\frac{P_1}{2} - K_w U_1 = K_s U_1 - K_s U_2 + 0.U_1$$

$$P_1 - K_F U_2 = -K_s U_1 + K_s U_2 - K_s U_1$$

$$\frac{P_1}{2} - K_w U_1 = 0.U_1 - K_s U_2 + K_s U_1$$

From which

$$U_2 = \frac{2P\,(K_s + \tfrac{1}{2}\,K_w)}{2\,K_s K_w + K_F K_s + K_F K_w}$$

$$V_F = K_F U_2$$

$$\frac{V_F}{2P} = \frac{\dfrac{1}{2\,K_w/K_F} + \dfrac{K_F}{4k_s}}{1 + \dfrac{1}{c} + \dfrac{K_F}{2K_s}}$$

Replacing $\frac{K_F}{K_s}$ for x

and $\frac{V_F}{2P}$ for z

$$z = \frac{\dfrac{1}{c} + \dfrac{x}{4}}{1 + \dfrac{1}{c} + \dfrac{x}{2}}$$

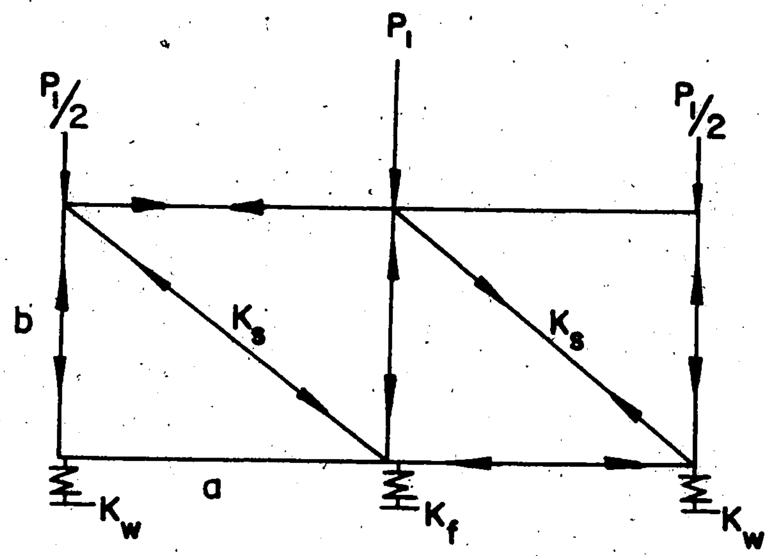


Figure H-1 Two Bay Building

which coincides with the pure shear solution given by the beam model when replacing

$$K_{sh} = \frac{8\ G_S \bar{A}_S}{l}$$

for

$$K_{sh_1} = \frac{2\ G_S \bar{A}_S}{l}$$

Two Bay Building

Referring to Figure I-1, the deflection at the center of the span for a uniformly distributed load is:

$$\Delta_1 = \frac{5}{384}\frac{Wl^4}{E_S I_S} + \frac{Wl^2}{8G_S\bar{A}_S} + \frac{Wl}{4K_w} + \frac{Wl}{4\ nK_w}$$

The Deflection at the center of the building for a point load is:

$$\Delta_2 = \frac{V_R l^3}{48E_S I_S} + \frac{V_R l}{4G_S\bar{A}_S} + \frac{V_R}{4K_w} + \frac{V_R}{4n\ K_w}$$

Equating the deflections

$$\Delta_1 - \Delta_2 = \frac{V_R}{K_F}$$

Thus

$$\frac{5}{384}\frac{Wl^4}{E_S I_S} + \frac{Wl^2}{8G_S\bar{A}_S} + \frac{Wl}{4K_w} + \frac{Wl}{4n\ K_w} - \frac{V_R l^3}{48E_S I_S} - \frac{V_R l}{4G_S\bar{A}_S}$$

$$- \frac{V_R}{4K_w} - \frac{V_R}{4n\ K_w} = \frac{V_R}{K_F}$$

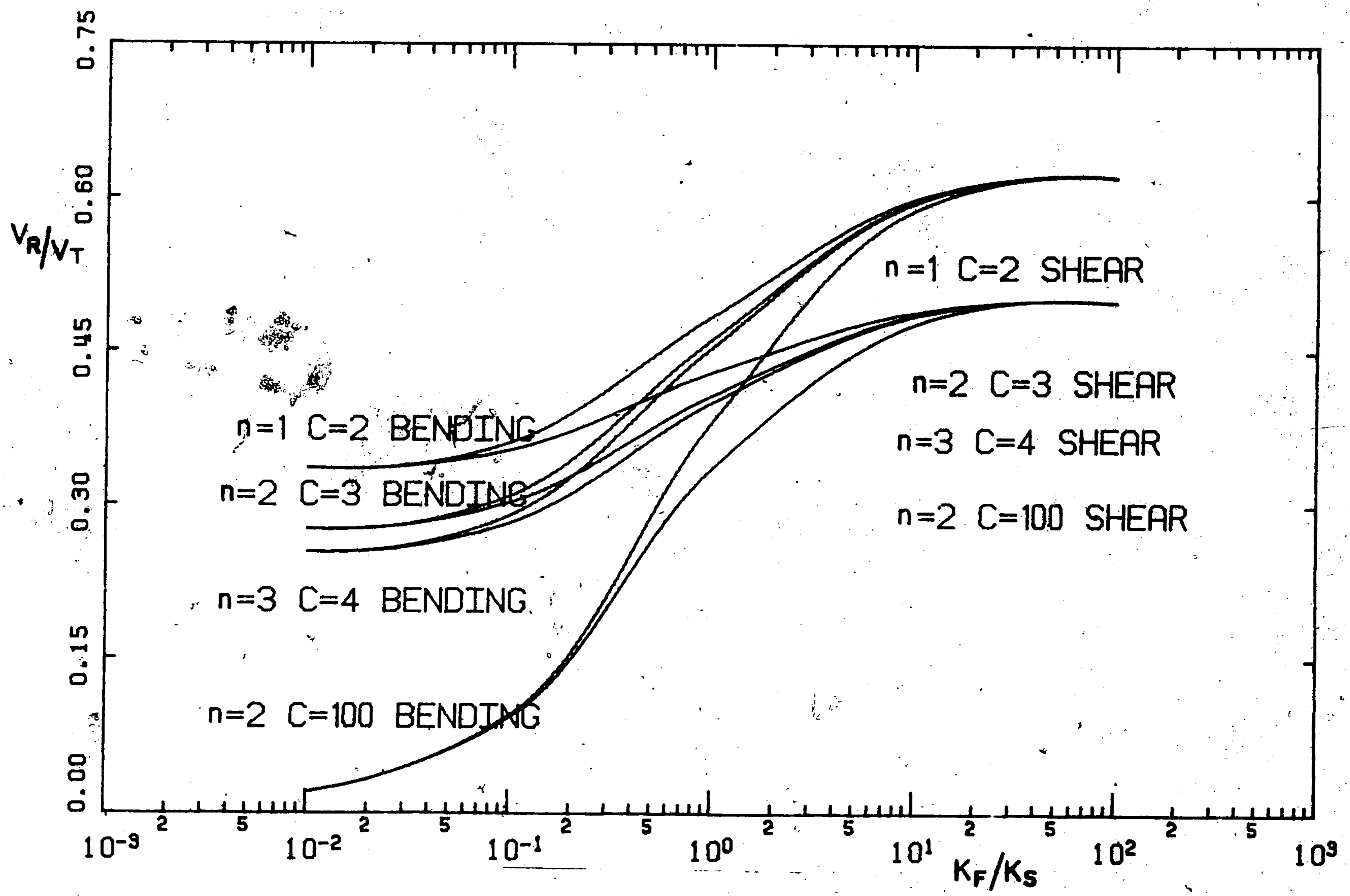


Figure I-I Two Bays Torsion

Therefore

$$V_R = \frac{\frac{5}{384}\frac{Wl^4}{E_S I_S} + \frac{Wl^2}{8G_S\bar{A}_S} + \frac{Wl}{4K_w} + \frac{Wl}{4nK_w}}{\frac{l^3}{48E_S I_S} + \frac{1}{4G_S\bar{A}_S} + \frac{1}{4K_w} + \frac{1}{4nK_w} + \frac{1}{K_F}}$$

If the Bending stiffness of the horizontal diaphragm is defined as

$$K_B = \frac{384}{5}\frac{E_S I_S}{l^3}$$

and the shear stiffness of the horizontal diaphragm defined as

$$K_{sh} = \frac{8G_S\bar{A}_S}{l}$$

The solution is

$$\frac{V_R}{V_T} = \frac{\frac{1}{K_B} + \frac{1}{K_{sh}} + \frac{1}{4K_w} + \frac{1}{4nK_w}}{\frac{1.6}{K_B} + \frac{2}{K_{sh}} + \frac{1}{4K_w} + \frac{1}{4nK_w} + \frac{1}{K_F}}$$

Replacing

$$\frac{K_F}{K_B} = x \qquad \frac{K_F}{K_{sh}} = y$$

$$c = \frac{K_w}{K_F}(n+1)$$

Thus

$$\frac{V_R}{V_T} = \frac{x + y + \frac{(n+1)^2}{4nc}}{1.6x + 2y + \frac{(n+1)^2}{4nc} + 1}$$

If n = 1 the symmetrical case is obtained

$$\frac{V_R}{V_T} = \frac{x + y + \frac{1}{c}}{1.6x + 2y + \frac{1}{c} + 1}$$

$$\lim_{c \to \infty} \frac{V_R}{V_T} = \frac{x + y}{1.6x + 2y + 1}$$

$$\lim_{c \to o} \frac{V_R}{V_T} = 1$$

These are the limiting surfaces for the complete set of surfaces.

The intersection of the above written expression with the plane x = o is represented by

$$Z = \frac{1 + \frac{1}{c}}{2y + \frac{1}{c} + 1} \qquad \text{where } 2 = \frac{V_R}{V_T}$$

which is a pure shear solution

$$\lim_{y \to o} Z = \frac{\frac{1}{c}}{\frac{1}{c} + 1}$$

$$\lim_{y \to \infty} Z = 0.5$$

The intersection with y = o is

$$Z = \frac{x + \frac{1}{c}}{1.6x + \frac{1}{c} + 1}$$

which is a pure bending solution

$$\lim_{x \to o} Z = \frac{\frac{1}{c}}{\frac{1}{c} + 1}$$

This value coincides with the value obtained for pure shear

$$\lim_{x \to \infty} Z = 0.625$$

The combined solution is represented by a set of surfaces which may be intersected by planes representing selected combinations of shear and bending stiffnesses. If the plane x = y is selected

$$Z = \frac{2x + \frac{1}{c}}{3.6x + 1 + \frac{1}{c}}$$

$$\lim_{x \to 0} Z = \frac{\frac{1}{c}}{1 + \frac{1}{c}}$$

$$\lim_{x \to \infty} Z = 0.5555$$

If the plane $y = 2x$ is selected

$$Z = \frac{3x + \frac{1}{c}}{5.6x + \frac{1}{c} + 1}$$

$$\lim_{x \to 0} Z = \frac{\frac{1}{c}}{\frac{1}{c} + 1}$$

$$\lim_{x \to \infty} Z = 0.5357$$

APPENDIX J

THREE BAYS SYMMETRICAL BUILDING

Based in Figure J.1 the deflections Δ_1 and Δ_2 are respectively

$$\Delta_1 = \frac{V_R}{486} \frac{l^3}{E_S I_S}$$

$$\Delta_2 = \frac{7}{486} \frac{V_R l^3}{E_S I_S}$$

Thus, the total bending deflection is

$$\Delta_B = \frac{15}{486} \frac{V_R l^3}{E_S I_S}$$

The shear deflection is:

$$\Delta_{sh} = \frac{V_R \; l}{3 G_S \bar{A}_S}$$

Then replacing the end supports by yielding supports, the total deflection is

$$\Delta_1 = \frac{15}{486} \frac{V_R l^3}{E_S I_S} + \frac{V_R l^3}{3 G_S \bar{A}_S} + \frac{V_R}{K_W}$$

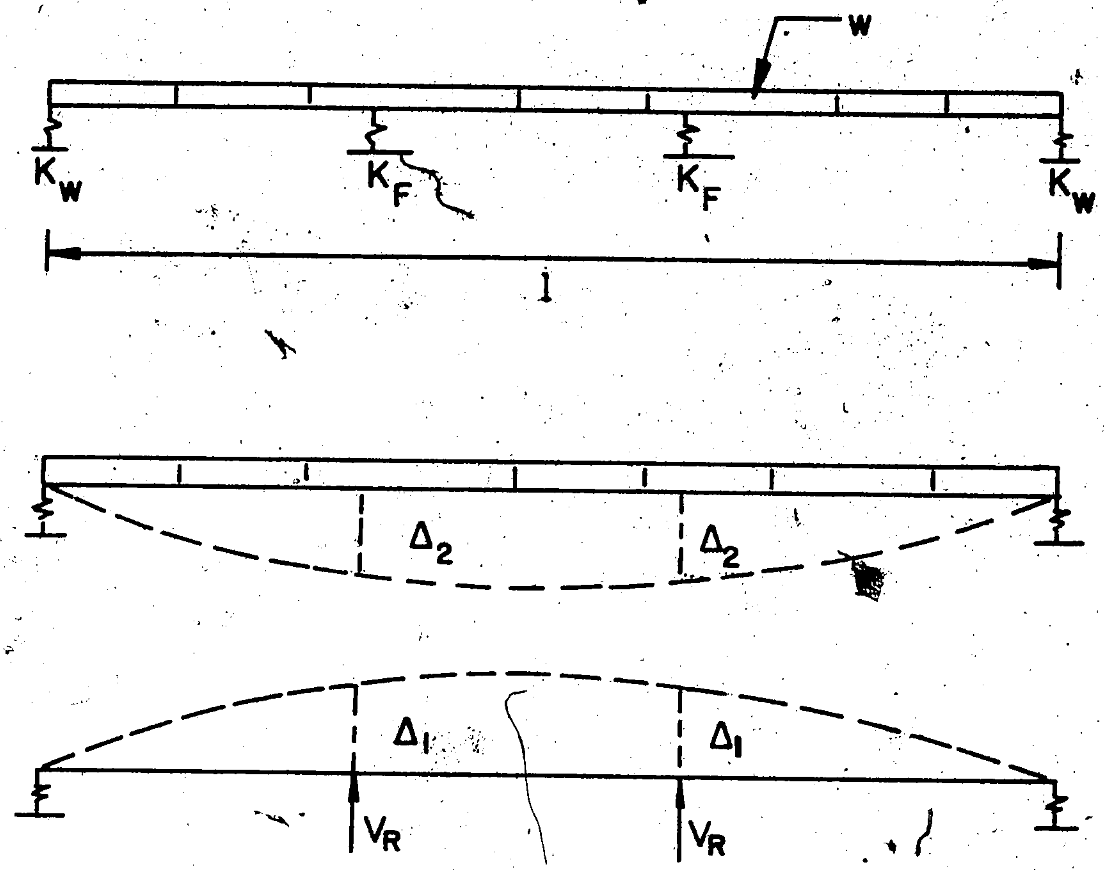


Figure J-1 Three Bays Symmetrical Building

The Deflection in the opposite direction is

$$\Delta_2 = \frac{4.35}{384} \frac{Wl^4}{E_S I_S} + \frac{Wl^2}{9G_S \bar{A}_S} + \frac{Wl}{2K_w}$$

Equating the deflections, finally

$$V_R = \frac{\frac{4.35}{384E_S I_S} Wl^4 + \frac{Wl^2}{9G_S \bar{A}_S} + \frac{Wl}{2K_w}}{\frac{15}{486} \frac{l^3}{E_S I_S} + \frac{l}{3G_S \bar{A}_S} + \frac{1}{K_w} + \frac{1}{K_F}} \qquad \text{(J-1)}$$

Defining the bending stiffness as

$$K_B = \frac{384}{4.35} \frac{E_S I_S}{l^3}$$

and the shearing stiffness as

$$K_{sh} = \frac{9G_S \bar{A}_S}{l}$$

Replacing in J.1

$$\frac{V_R}{V_T} = \frac{\frac{1}{K_B} + \frac{1}{K_{sh}} + \frac{1}{2K_w}}{\frac{2.725}{K_B} + \frac{3}{K_{sh}} + \frac{1}{K_w} + \frac{1}{K_F}}$$

Defining

$$c = (n+1)\frac{K_w}{\Sigma K_F} \; ; \text{ if } n = 1 \quad c = \frac{K_w}{K_F}$$

$$\frac{K_F}{K_B} = x$$

$$\frac{K_F}{K_{sh}} = y$$

$$\frac{V_R}{V_T} = \frac{x + y + \frac{1}{2c}}{2.725x + 3y + \frac{1}{c} + 1}$$

If a solution based in bending is sought

$$\frac{V_R}{V_T} = \frac{x + \frac{1}{2c}}{2.725x + \frac{1}{c} + 1}$$

This curve lies in the Bending Plane.

The solution based in shear yields

$$\frac{V_R}{V_T} = \frac{y + \frac{1}{2c}}{3y + \frac{1}{c} + 1}$$

Limits

Bending solution

$$\lim_{x \to o} \frac{V_R}{V_T} = \frac{\frac{1}{2c}}{1 + \frac{1}{c}}$$

$$\lim_{x \to \infty} \frac{V_R}{V_T} = 0.367$$

Shearing solution

$$\lim_{y \to o} \frac{V_R}{V_T} = \frac{\frac{1}{2c}}{1 + \frac{1}{c}}$$

$$\lim_{y \to \infty} \frac{V_R}{V_T} = 0.333$$

and

$$\lim_{c \to o} \frac{x + \frac{1}{2c}}{2.725x + \frac{1}{c} + 1} = 0.5$$

APPENDIX K

FOUR BAYS SYMMETRICAL BUILDING

Solution for pure shear and pure bending are obtained if the following procedure is used (See Figure K-1).

Equilibrium yields:

$$2\ V_w + 2\ VA + V_B = 2\ F_w + 2\ FA + F_B$$

Diaphragm shear first span

$$V_{D_1} = V_w - F_w$$

$$V_w = F_w + FA + \frac{F_B}{2} - VA\ \frac{V_B}{2}$$

Diaphragm distortion

$$\Delta_{D_1} = \Delta_A - \Delta_w$$

$$\frac{V_{D_1}}{K_{S_1}} = \frac{VA}{K_F} - \frac{V_w}{K_w}$$

Thus

$$\frac{FA + \frac{F_B}{2} - VA - \frac{V_B}{2}}{K_{S_1}} = \frac{V_A}{K_F} - \frac{V_w}{K_w} \qquad (K.1)$$

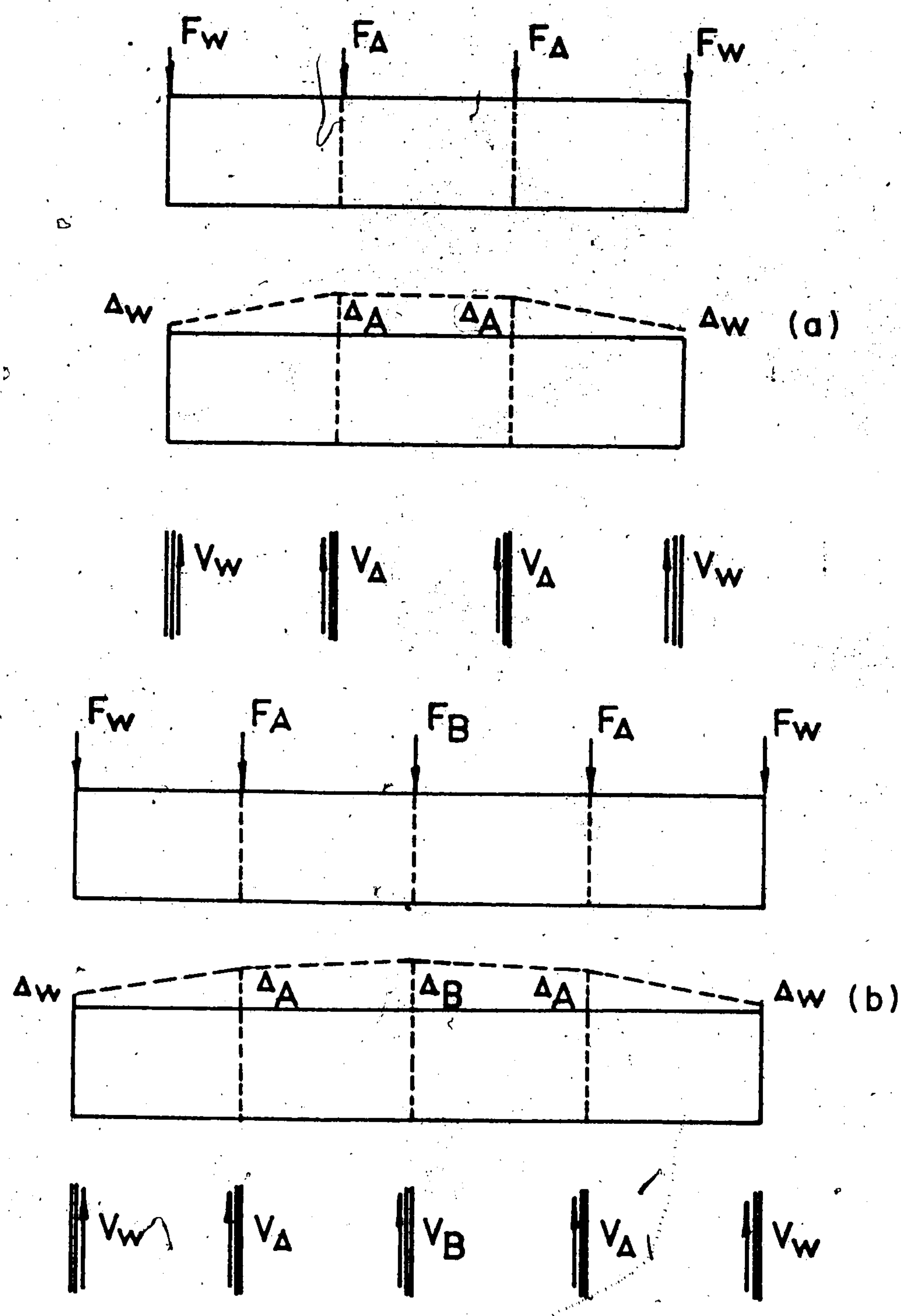


Figure K-1 Three and Four Bay Buildings.

Taking moments with respect to the wall

$$VA + \frac{V_B}{2} = \frac{Wl}{2} - V_w$$

Diaphragm shear second span

$$V_{D_2} = V_{D_1} - F_A + V_A$$

$$V_{D_2} = \frac{F_B}{2} - \frac{V_B}{2}$$

Diaphragm distortion

$$\Delta_{D_2} = \Delta_B - \Delta_A$$

$$\frac{V_{D_2}}{K_{S_1}} = \frac{V_B}{K_F} - \frac{V_A}{K_F} = \frac{1}{2K_{S_1}} (F_B - V_B)$$

From (K-1)

$$\frac{V_A}{K_F} - \frac{V_W}{K_w} = \frac{F_A}{K_{S_1}} + \frac{F_B}{2K_{S_1}} - \frac{V_A}{K_{S_1}} - \frac{V_B}{2K_{S_1}}$$

A system of three simultaneous equations is obtained

$$V_A \left[\frac{1}{K_F} + \frac{1}{K_{S_1}}\right] + V_B \frac{1}{2K_{S_1}} - \frac{1}{K_w} V_w = \frac{F_A}{K_{S_1}} + \frac{F_B}{2F_{S_1}}$$

$$-V_A \frac{1}{K_F} + V_B \left[\frac{1}{K_F} + \frac{1}{2K_{S_1}}\right] = \frac{F_B}{2K_{S_1}}$$

$$V_A + V_B\ ½ + V_w = \frac{Wl}{2}$$

Finally a solution is obtained

$$\frac{V_A}{Wl} = \frac{\frac{1}{3c}\left[1+\frac{K_F}{2K_{S_1}}\right] + FA\frac{K_F}{K_{S_1}}\left[1+\frac{K_F}{2K_{S_1}}\right] + F_B\frac{K_F}{2K_{S_1}}\left[1-\frac{1}{3c}\right]}{\frac{1}{c} + \frac{K_F}{K_{S_1}}\frac{1}{3c} + 1 + \frac{2K_F}{K_{S_1}} + \frac{K_F{}^2}{2K_{S_1}{}^2}} \qquad (K.2)$$

If a Bending solution is sought

$$FA = 1.143\ \frac{Wl}{4}$$

$$F_B = 0.929\ \frac{Wl}{4}$$

Therefore

$$\frac{V_A}{Wl} = \frac{\frac{1}{3c} + 0.128\frac{K_F}{K_{S_1}c} + 0.4019\frac{K_F}{K_{S_1}} + 0.143\frac{K_F{}^2}{K_{S_1}{}^2}}{\frac{1}{c} + \frac{K_F}{K_{S_1}3c} + 1 + \frac{2K_F}{K_{S_1}} + 0.5\frac{K_F{}^2}{K_{S_1}{}^2}}$$

in which

$c = 2K_w/3K_F$

$K_S = 107.78\ E_SI_S/l^3$

$K_{S_1} = 85.33\ E_SI_S/l^3$

$\frac{K_{S_1}}{K_S} = 0.792$

Replacing in K.2

$$\frac{V_A}{Wl} = \frac{\frac{1}{3c} + 0.612\,\frac{K_F}{K_S c} + 0.507\,\frac{K_F}{K_S} + 0.228\,\frac{K_F^2}{K_S^2}}{\frac{1}{c} = 1.263\,\frac{K_F}{K_S c} + 1 + 2.525\,\frac{K_F}{K_S} + 0.797\,\frac{K_F^2}{K_S^2}}$$

$$\lim_{\frac{K_F}{K_S} \to 0} \frac{V_A}{Wl} = \frac{\frac{1}{3c}}{1+\frac{1}{c}}$$

$$\lim_{\frac{K_F}{K_S} \to \infty} \frac{V_A}{Wl} = \frac{\frac{1}{3c}}{1 + \frac{1}{c}}$$

If a shearing solution is sought

$$F_A = 0.25\ Wl$$

$$F_B = 0.25\ Wl$$

Thus

$$\frac{V_A}{Wl} = \frac{\frac{1}{3c} + \frac{K_F}{6K_{S_1} c} + \frac{1}{4}\,\frac{K_F}{K_{S_1}} + \frac{1}{8}\,\frac{K_F^2}{K_{S_1}^2} + \frac{1}{12}\,\frac{K_F}{K_{S_1}}}{\frac{1}{c} + \frac{K_F}{K_{S_1}}\,\frac{1}{3c} + 1 + \frac{2K_F}{K_{S_1}} + \frac{K_F^2}{2K_{S_1}^2}}$$

$$K_S = 10.67\,\frac{G_S \bar{A}_S}{l}$$

$$K_{S_1} = 5.33\,\frac{G_S \bar{A}_S}{l}$$

Thus

$$\frac{V_A}{W1} = \frac{\frac{1}{3c} + \frac{K_F}{3K_S c} + \frac{2K_F}{3K_S} + \frac{K_F 2}{2K_S 2} +}{\frac{1}{c} + \frac{K_F}{1.5cK_S} + 1 + \frac{4K_F}{K_S} + \frac{2K_F 2}{K_S 2}}$$

$$\underset{\frac{K_F}{K_S} \to 0}{L_{im}} \frac{V_A}{W1} = \frac{\frac{1}{3c}}{1 + \frac{1}{c}}$$

$$\underset{\frac{K_F}{K_S} \to \infty}{L_{im}} \frac{V_A}{W1} = 0.25$$

APPENDIX L

FIVE BAYS SYMMETRICAL BUILDING

Making use of the same procedure Δ_s for four bays, solution for the force taken by the frame is obtained (see Figure L-1). Equilibrium yields

$$2V_w + 2V_A + 2V_B = 2F_w + 2F_A + 2F_B$$

$$V_A + V_B + V_w = F_A + F_B + F_w$$

Diaphragm Shear First Span

$$V_{D_1} = V_w - F_w$$

$$V_w = -F_w + F_A + FB - V_A - V_B \qquad (L.1)$$

Diaphragm distortion

$$\Delta_{D_1} = \Delta_A - \Delta_w$$

$$\frac{V_{D_1}}{K_{S_1}} = \frac{V_A}{K_F} - \frac{V_w}{K_w}$$

Moments with respect to wall

$$V_A \frac{L}{5} + V_B \frac{2}{5} L + V_B \frac{3}{5} L + V_A \frac{4}{5} L + V_w L = \frac{Wl^2}{2}$$

$$V_A + V_B + V_w = \frac{Wl}{2}$$

Diaphragm shear second span

$$V_{D_2} = V_{D_1} - F_A + V_A = F_B - V_B$$

Diaphragm distortion

$$\Delta_{D_2} = \Delta_B - \Delta_A$$

$$\frac{V_{D_2}}{K_{S_1}} = \frac{V_B}{K_F} - \frac{V_A}{K_F} = \frac{1}{K_{S_1}} (F_B - V_B)$$

From L.1

$$\frac{V_A}{K_F} - \frac{V_w}{K_w} = \frac{F_A}{K_{S_1}} + \frac{F_B}{K_{S_1}} - \frac{V_A}{K_{S_1}} - \frac{V_B}{K_{S_1}}$$

The set of simultaneous equations is obtained

$$V_A \left[\frac{1}{K_F} + \frac{1}{K_{S_1}}\right] + V_B \frac{1}{K_{S_1}} - \frac{1}{K_w} V_w = \frac{F_A}{K_{S_1}} + \frac{F_B}{K_{S_1}}$$

$$V_A + V_B + V_w = \frac{Wl}{2}$$

$$\frac{-V_A}{K_F} + V_B\left[\frac{1}{K_F} + \frac{1}{K_{S_1}}\right] = \frac{F_B}{K_{S_1}}$$

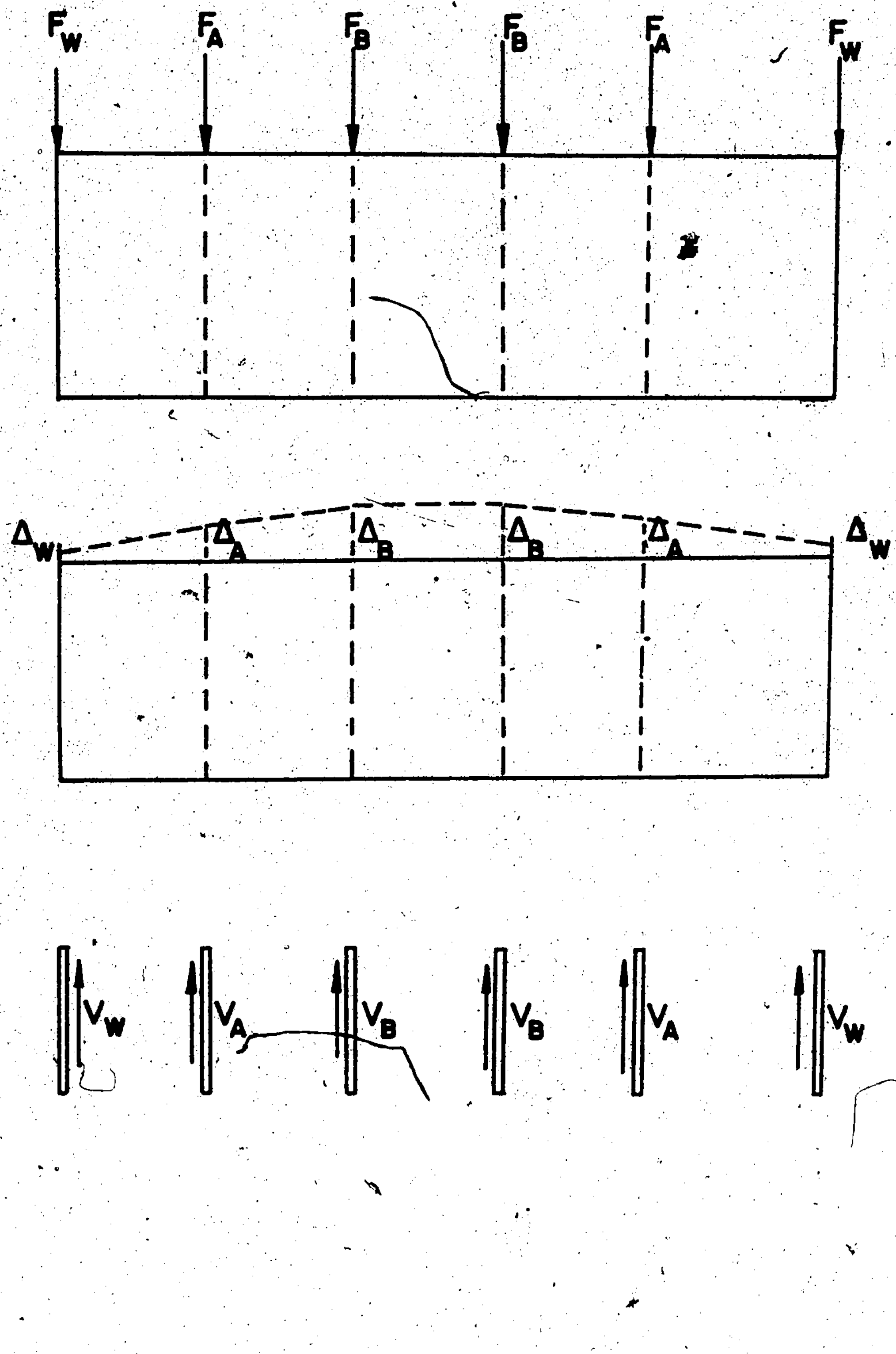


Figure L-1 Five Bays Symmetrical Building

$$V_A = \frac{FA\frac{K_F}{K_{S_1}} + FA\frac{K_F{}^2}{K_{S_1}{}^2} + F_B\frac{K_F}{K_{S_1}} + F_B\frac{K_F{}^2}{K_{S_1}{}^2} - F_B\frac{K_F}{K_{S_1}}\frac{1}{2c}}{1 + \frac{3K_F}{K_{S_1}} + \frac{K_F{}^2}{K_{S_1}{}^2} + \frac{1}{c} + \frac{K_F}{K_{S_1}}\frac{1}{2c}}$$

$$\frac{- F_B\frac{K_F{}^2}{K_{S_1}{}^2} + \frac{Wl}{4c} + \frac{Wl}{4c}\frac{K_F}{K_{S_1}}}{1 + \frac{3K_F}{K_{S_1}} + \frac{K_F{}^2}{K_{S_1}{}^2} + \frac{1}{c} + \frac{K_F}{K_{S_1}}\frac{1}{2c}}$$

APPENDIX M

NONSYMMETRICAL THREE BAY BUILDING

Referring to Figure 4.1 the third span deflections are:

$$\Delta_1 = \frac{4.35 W l^4}{384 E_S I_S} + \frac{W l^2}{9 G_S \bar{A}_S} + \frac{W l}{3 n K_w} + \frac{W l}{6 K_w}$$

$$\Delta_2 = \frac{8}{486} \frac{V_{R_1} l^3}{E_S I_S} + \frac{1}{486} \frac{V_{R_2} l^3}{E_S I_S} + \frac{6}{27} \frac{V_{R_1} l}{G_S \bar{A}_S} + \frac{3}{27} \frac{V_{R_2} l}{G_S \bar{A}_S}$$

$$+ \frac{\frac{4}{9} V_{R_1} + \frac{2}{9} V_{R_2}}{n K_w} + \frac{\frac{V_{R_1}}{9} + \frac{2}{9} V_{R_2}}{K_w}$$

$$\Delta_3 = \frac{4.35 W l^4}{384 E_S I_S} + \frac{W l^2}{9 G_S \bar{A}_S} + \frac{W l}{6 n K_w} + \frac{W l}{3 K_w}$$

$$\Delta_4 = \frac{7}{486} \frac{V_{R_1} l^3}{E_S I_S} + \frac{3}{27} \frac{V_{R_1} l}{G_S \bar{A}_S} + \frac{8}{486} \frac{V_{R_2} l^3}{E_S I_S} = \frac{6}{27} \frac{V_{R_2} l}{G_S \bar{A}_S}$$

$$+ \frac{\frac{2}{9} V_{R_1} + \frac{V_{R_2}}{9}}{n K_w} + \frac{\frac{2 V_{R_1}}{9} + \frac{4}{9} V_{R_2}}{K_w}$$

$$K_B = 88.28 \; E_S I_S / l^3$$

$$K_{sh} = 9 \; G_S \bar{A}_S / l$$

If the displacements are equalized a set of two simultaneous equations is obtained, furthermore if

$$x = \frac{K_F}{K_B}$$

$$y = \frac{K_F}{K_{sh}}$$

$$c = \frac{Kw\ (n+1)}{2K_F}$$

The force taken by the first frame is:

$$\frac{V_{R_1}}{V_T} = \frac{0.181\ 1x^2 + 1.183xy+y^2 + \frac{0.04039x(n+1)}{2c} + \frac{0.1612}{2cn}\ x\ (n+1) + x + y + \frac{0.3839y(n+1)}{2cn} + \frac{0.2222y(n+1)}{2c} + \frac{0.167(n+1)}{2c} + \frac{0.0555(n+1)^2}{n4c^2} + \frac{0.333(n+1)}{2cn}}{0.495x^2 + 3.268xy + 3y^2 + 2.907x + 4y + \frac{0.242x(n+1)}{2cn} + \frac{0.242x(n+1)}{2c} + \frac{0.666y(n+1)}{2cn} + \frac{0.666y(n+1)}{2c} + \frac{0.1111(n+1)^2}{n4c^2} + \frac{0.5555(n+1)}{2cn} + \frac{0.5555(n+1)}{2c} + 1} = \frac{A}{B}$$

When y = o, pure bending solution

$$\frac{V_{R_1}}{V_T} = \frac{181x^2 + \frac{0.04039x(n+1)}{2c} + \frac{0.1612x(n+1)}{2cn} + \frac{0.167(n+1)}{2c} + \frac{0.0555(n+1)^2}{n4c^2} + \frac{0.333(n+1)}{2cn} + x}{0.495x^2 + 2.907x + \frac{0.242x(n+1)}{2cn} + \frac{0.242x(n+1)}{2c} + \frac{0.1111(n+1)^2}{n4c^2} + \frac{0.5555(n+1)}{2cn} + \frac{0.5555(n+1)}{2c} + 1}$$

$$\lim_{c \to 0} \frac{V_{R_1}}{V_T} = 0.5$$

$$\lim_{c \to \infty} \frac{V_{R_1}}{V_T} = \frac{0.181x^2 + x}{0.495x^2 + 2.907x + 1} = \frac{x}{2.725x + 1}$$

$$\lim_{x \to \infty} \frac{V_{R_1}}{V_T} = 0.367$$

When x = o, pure shear solution

$$\frac{V_{R_1}}{V_T} = \frac{y^2 + y + \frac{0.3889y(n+1)}{2cn} + \frac{0.2222y(n+1)}{2c} + \frac{0.167(n+1)}{2c} + \frac{0.0555(n+1)^2}{n4c^2} + \frac{0.333(n+1)}{2cn}}{3y^2 + 4y + \frac{0.666y(n+1)}{2cn} + \frac{0.666y(n+1)}{2c} + \frac{0.1111(n+1)^2}{n4c^2} + \frac{0.5555(n+1)}{2cn} + \frac{0.5555(n+1)}{2c} + 1}$$

$$\lim_{c \to 0} \frac{V_{R_1}}{V_T} = 0.5$$

$$\lim_{c \to \infty} \frac{V_{R_1}}{V_T} = \frac{y^2 + 1}{3y^2 + 4y + 1} = \frac{y}{3y + 1}$$

$$\lim_{y \to \infty} \frac{V_{R_1}}{V_T} = 0.333$$

The force taken by the second frame is:

$$\frac{V_{R_2}}{V_T} = \frac{\begin{aligned} & 0.181^2 + 1.181xy + y^2 + \frac{0.1605x(n+1)}{2c} + \frac{0.3885y(n+1)}{2c} \\ & + \frac{0.2222y(n+1)}{2cn} + \frac{0.0409x(n+1)}{2cn} + x + y \\ & + \frac{0.05555(n+1)^2}{4nc^2} + \frac{0.16666(n+1)}{2nc} + \frac{0.333(n+1)}{2c} \end{aligned}}{\begin{aligned} & 0.493x^2 + 3.268xy + 3y^2 + 2.905x + 4y \\ & + \frac{0.2419x(n+1)}{2c} + \frac{0.2419x(n+1)}{2cn} + \frac{0.6666}{2cn} y(n+1) \\ & + \frac{0.1111(n+1)^2}{4nc^2} + \frac{0.6666y(n+1)}{2c} + \frac{0.5555(n+1)}{2cn} + \frac{0.5555(n+1)}{2c} + \ldots \end{aligned}}$$

When y = o, pure bending solution

$$\frac{V_{R_2}}{V_T} = \frac{0.181x^2 + \frac{0.1605x(n+1)}{2c} + \frac{0.0409x(n+1)}{2cn} + x + \frac{0.0555(n+1)^2}{4nc^2} + \frac{0.1666(n+1)}{2nc} + \frac{0.333(n+1)}{2c}}{0.495x^2 + 2.907x + \frac{0.2419x(n+1)}{2c} + \frac{0.2419x(n+1)}{2cn} + \frac{0.1111(n+1)^2}{4nc^2} + \frac{0.5555(n+1)}{2nc} + \frac{0.5555(n+1)}{2c} + 1}$$

$$\lim_{c \to 0} \frac{V_{R_2}}{V_T} = 0.5$$

$$\lim_{c \to \infty} \frac{V_{R_2}}{V_T} = \frac{0.181x^2 + x}{0.495x^2 + 2.907x + 1} = \frac{x}{2.725 + 1}$$

$$\lim_{x \to \infty} \frac{V_{R_2}}{V_T} = 0.367$$

when x = o, pure shear solution

$$\frac{V_{R_2}}{V_T} = \frac{y^2 + y + \frac{0.3885y(n+1)}{2c} + \frac{0.2222y(n+1)}{2cn} + \frac{0.05555(n+1)^2}{4nc^2} + \frac{0.1666(n+1)}{2nc} + \frac{0.333(n+1)}{2c}}{3y^2 + 4y + \frac{0.6666y(n+1)}{2cn} + \frac{0.6666y(n+1)}{2c} + \frac{0.1111(n+1)^2}{4nc^2} + \frac{0.5555(n+1)}{2cn} + \frac{0.5555(n+1)}{2c} + 1}$$

$$\lim_{c \to o} \frac{V_{R_2}}{V_T} = 0.5$$

$$\lim_{c \to \infty} \frac{V_{R_2}}{V_T} = \frac{y^2 + y}{3y^2 + 4y + 1} = \frac{y}{3y + 1}$$

$$\lim_{y \to \infty} \frac{V_{R_2}}{V_T} = 0.333$$

APPENDIX N

BEAM ON ELASTIC FOUNDATION MODEL

The rigidity of the frames is converted into resisting elements distributed in the spacing, a, therefore a rigidity g_F per unit length is used (See Figure 3.13).

The uniformly distributed load, will be shared by the slab and the frames.

Thus

$$W = W_F + W_S \tag{N.1}$$

in which

W_F = load taken by the frames

W_S = load taken by the slab

For the slab

$$\left(\frac{dy}{dx}\right) = \left(\frac{dy}{dx}\right)_V + \left(\frac{dy}{dx}\right)_B$$

where

$\left(\frac{dy}{dx}\right)_V$ = shear angle

$\left(\frac{dy}{dx}\right)_B$ = bending angle

Differentiating once

$$\frac{d^2y}{dx^2} = \left(\frac{d^2y}{dx^2}\right)_V + \left(\frac{d^2y}{dx^2}\right)_B$$

$$\left(\frac{d^2y}{dx^2}\right)_B = -\frac{M}{E_S I_S} \quad \text{and} \quad \left(\frac{d^2y}{dx^2}\right)_V = \frac{\beta_o \kappa}{G_S \bar{A}_S} W_s$$

Substituting

$$\frac{d^2y}{dx^2} = \frac{\beta_o \kappa}{G_S \bar{A}_S} W_s - \frac{M}{E_S I_S} \qquad \text{or}$$

$$\frac{d^2y}{dx^2} + \frac{M}{E_S I_S} = \frac{\beta_o \kappa}{G_S \bar{A}_S} W_s$$

Differentiating twice we get consecutively

$$\frac{d^3y}{dx^3} + \frac{dM}{dx E_S I_S} = \frac{\beta_o \kappa}{G_S \bar{A}_S} \frac{dW_s}{dx}$$

$$\frac{d^4y}{dx^4} + \frac{dV}{dx E_S I_S} = \frac{\beta_o \kappa}{G_S \bar{A}_S} \frac{d^2W_s}{dx^2}$$

$$W_s = \frac{dV}{dx}$$

$$\therefore \; W_s = E_S I_S \frac{d^4y}{dx^4} + \frac{\beta_o \kappa}{G_S \bar{A}_S} \frac{d^2W_s}{dx^2} E_S I_S \qquad \text{(N.2)}$$

From N.1

$$W = W_s + W_F = \text{constant}$$

$$\frac{dW}{dx} = 0 = \frac{dW_s}{dx} + \frac{dW_F}{dx}$$

$$\frac{dW_s}{dx} = \frac{dW_F}{dx}$$

Therefore

$$\frac{d^2W_s}{dx^2} = \frac{d^2W_F}{dx^2} \qquad \text{(N.3)}$$

The force taken by the unitary spring is

$$W_F = g_F \cdot y$$

Thus $\dfrac{dW_F}{dx} = g_F \dfrac{dy}{dx}$

Differentiating again $\dfrac{d^2W_F}{dx^2} = g_F \dfrac{d^2y}{dx^2}$

Making use of N.3 $\dfrac{d^2W_s}{dx^2} = -g_F \dfrac{d^2y}{dx^2}$

Substituting in (N.2)

$$W_s = E_S I_S \frac{d^4y}{dx^4} - \frac{\beta_o \kappa E I_S}{G_S \bar{A}_S} g_F \frac{d^2y}{dx^2}$$

and finally:

$$\frac{d^4y}{dx^2} - \frac{\beta_o \kappa}{G_S \bar{A}_S} g_F \frac{d^2y}{dx^2} + \frac{g_F y}{E_S I_S} = W \qquad \text{(N.4)}$$

where

β_o = coefficient of inelastic shear deformation

κ = shear deformation coefficient

g_F = unitary rigidity

G_S = shearing Modulus of slab

E_S = Young's Modulus of slab

I_S = Moment of Inertia of slab

$\bar{A}_S$ = shear area of slab

W = Uniformly distributed horizontal load

If

$$2\beta^2 = \frac{\beta_o \kappa}{G_S \bar{A}_S} \qquad \text{and}$$

$$\alpha^4 = \frac{g_F}{E_S I_S}$$

Replacing in N.4

$$\frac{d^4y}{dx^4} = 2\beta^2 \frac{d^2y}{dx^2} + \alpha^4 y = \frac{W}{E_S I_S} \qquad \text{(N.5)}$$

Solutions for this equation are dependent upon the value of α relative to β.

1. When $\alpha > \beta$

$$y = e^{dx}(A\cos fx + B\sin fx) + e^{-dx}(C\cos fx + D\sin fx) + \frac{W}{g_F}$$

where

$$d = \left(\frac{\alpha^2 + \beta^2}{2}\right)^{1/2} \quad \text{and}$$

$$f = \left(\frac{\alpha^2 - \beta^2}{2}\right)^{1/2}$$

2. when $\beta > \alpha$

$$y = Ae^{(d+f)x} + Be^{-(d+f)x} + Ce^{(d-f)x} + De^{-(d-f)x} + \frac{W}{g_F}$$

where

$$f = \sqrt{\frac{\beta^2 - \alpha^2}{2}}$$

3. when $\beta = \alpha$

$$y = Ae^{dx} + Bxe^{dx} + Ce^{-dx} + Dxe^{-dx} + \frac{W}{g_F}$$

APPENDIX F

ONE STOREY BUILDING WITH END SHEAR WALLS

1. $\alpha > \beta$

The Boundary conditions are

$x = 1/2$

$y = \frac{Q_w}{K_w} = -\frac{Q_s}{K_w}$

$M_s = 0$ (Neglecting torsional resistance)

Symmetrical conditions

$y(x) = y(-x)$

$y = 2A \cos fx \cosh dx + 2B \sin fx \sinh dx + W/g_F$

$\frac{dy}{dx} = 2 \sin fx \cosh dx\ (Bd-Af) + 2 \cos fx \sinh dx\ (Ad+Bf)$

$\frac{d^2y}{dx^2} = 2 \cos fx \cosh dx\ (Ad^2+2Bdf-Af^2) + 2 \sin fx \sinh dx\ (Bd^2-2Afd-Bf^2)$

$\frac{d^3y}{dx^3} = 2 \sin fx \cosh dx\ (Bd^3-3Afd^2 - 3Bf^2+Af^3) + 2 \cos fx \sinh dx\ (Ad^3+3Bfd^2-3Af^2d-Bf^3)$

Bending moment of slab

$M_s(x) = -E_S I_S \left(\frac{d^2y}{dx^2} - 2\beta^2 y + 2\beta^2 \frac{W}{g_F}\right)$

$x = 1/2 \quad Q_s(1/2) = -yS_w = -Q_w$

Shear of slab

$$Q_s = -E_S I_S \left(\frac{d^3 y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$\therefore\ y_{(1/2)} = \frac{EI_S}{S_w}\left(\frac{d^3 y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$\therefore\ 2A\cos\frac{fl}{2}\cos\frac{dl}{2} + 2B\sin\frac{fl}{2}\sinh\frac{dl}{2} + \frac{W}{g_F} =$$

$$\frac{E_S I_S}{S_w}\Big(2\sin\frac{fl}{2}\cosh\frac{dl}{2}\,(Bd^3 - 3Afd^2 - 3Bf^2 d + Af^3)$$

$$+ 2\cos\frac{fl}{2}\sinh\frac{dl}{2}\,(Ad^3 + 3Bfd^2 - 3Af^2 d - Bf^3)$$

$$- 2\beta^2\left(\sin\frac{fl}{2}\cosh\frac{dl}{2}\,(Bd - Af) + 2\cos\frac{fl}{2}\sinh\frac{dl}{2}\,(Ad + Bf)\right)\Big)$$

Substituting

$$P = \sin\frac{fl}{2}\cosh\frac{dl}{2}$$

$$Q = d^3 - 3f^2 d - 2\beta^2 d$$

$$M = \cos\frac{fl}{2}\sinh\frac{dl}{2}$$

$$N = f^3 - 3fd^2 + 2\beta^2 f$$

$$R_1 = d^2 - f^2 - 2\beta^2$$

$$S = tg\frac{fl}{2}\, tgh\frac{dl}{2}$$

$$V = \sin\frac{fl}{2}\sinh\frac{dl}{2}$$

$$T = \cos\frac{fl}{2}\cosh\frac{dl}{2}$$

Thus

$$\frac{K_w}{EI_S}\left(2A\cos\frac{fl}{2}\cosh\frac{dl}{2} + 2B\sin\frac{fl}{2}\sinh\frac{dl}{2} + \frac{W}{g_F}\right) =$$

$$2A\,(NP + MQ) + 2B\,(PQ - MN)$$

$$A\left(NP + MQ - \frac{K_w}{E_S I_S}\cos\frac{fl}{2}\cosh\frac{dl}{2}\right) + B\left(PQ - MN - \frac{K_w}{E_S I_S}\right.$$

$$\left.\sin\frac{fl}{2}\sinh\frac{dl}{2}\right) = \frac{K_w \cdot W}{2g_F E_S I_S}$$

Applying $M_S(1/2) = 0$

$$0 = 2\cos\frac{fl}{2}\cosh\frac{dl}{2}\,(Ad^2 + 2Bfd - Af^2) + 2\sin\frac{fl}{2}\sinh\frac{dl}{2}$$

$$(Bd^2 - 2Afd - Bf^2) - 2\beta^2\left(2A\cos\frac{fl}{2}\cosh\frac{dl}{2} + 2B\sin\frac{fl}{2}\right.$$

$$\left.\sinh\frac{dl}{2} + \frac{W}{g_F}\right) + 2\beta^2\,\frac{W}{g_F}$$

$$A(d^2 - f^2 - 2\beta^2) + 2Bfd - 2Afd\,tg\frac{fl}{2}\,tgh\frac{dl}{2} + B\,tg\frac{fl}{2}$$

$$tgh\frac{dl}{2}\,(d^2 - f^2 - 2\beta^2) = 0$$

$$\therefore\ A = B\,(2fd + SR_1)/(2fds - R_1)$$

$$B = \frac{K_W \cdot W \quad (2fds - R_1)}{2g_F EI_S \quad H_1}$$

$$A = \frac{K_w . W (2fd + SR_1)}{2g_F E_S I_S H_1}$$

where

$$H_1 = \left(2fd(PN+MQ+SPQ-SMN\frac{K_w . T}{E_S I_S} - \frac{S V K_w}{E_S I_S}\right)$$

$$+ R_1 (SPN + SMQ - PQ + MN)\Big)$$

The solution is

$$y = \frac{W}{g_F} \left(1 + \frac{K_w}{H_1 E_S I_S} (2fd+SR_1) \cos fx \cosh dx + \frac{K_w}{H_1 E_S I_S} (2fdS - R_1)\right.$$

$$\left. \sin fx \sinh dx \right) \quad (P.1)$$

2. When $\beta > \alpha$

$$d = \sqrt{\frac{\alpha^2 + \beta^2}{2}}$$

$$f = \sqrt{\frac{\beta^2 - \alpha^2}{2}}$$

$$y = Ae^{(d+f)x} + Be^{-(d+f)x} + Ce^{(d-f)x} + De^{-(d-f)x} + \frac{W}{g_F}$$

$$\frac{dy}{dx} = (d+f)Ae^{(d+f)x} - (d+f)Be^{-(d+f)x} + (d-f)Ce^{(d-f)x}$$

$$- (d-f)De^{-(d-f)x}$$

&

$$\frac{d^2y}{dx^2} = (d+f)^2Ae^{(d+f)x} + (d+f)^2Be^{-(d+f)x} + (d-f)^2Ce^{(d-f)x}$$

$$+ (d-f)^2De^{-(d+f)x}$$

$$\frac{d^3y}{dx^3} = (d+f)^3Ae^{(d+f)x} - (d+f)^3Be^{-(d+f)x} + (d-f)^3$$

$$Ce^{(d-f)x} - (d-f)^3De^{-(d-f)x}$$

Symmetrical conditions

$y(x) = y(-x)$

Thus

$y(x) = 2A \cosh(d+f)x + 2B \cosh(d-f)x + W/g_F$

Boundary Conditions

1. $x = l/2$

$M_s(l/2) = 0$

$$M_s = -E_S I_S \left(\frac{d^2y}{dx^2} - 2\beta^2 y + 2\beta^2 W/g_F\right)$$

$$A(d-f)^2 \cosh (d+f)\, l/2 + B(d+f)^2 \cosh(d-f)\, l/2 = 0$$

(P.2)

2. $x = l/2$

$$y(l/2) = \frac{E_S I_S}{K_w}\left(\frac{d^3y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$A\big((d+f)^3 \text{sin[illegible]} l/2 - 2\beta^2 (d+f) \cdot \sinh(d+f) l/2$$

$$- \cosh(d+f) l/2 \cdot \frac{K_w}{E_S I_S}\big) + B\big((d-f)^3 \sinh(d-f) l/2$$

$$- 2\beta^2 (d-f) \sinh (d-f) l/2 - \cosh(d-f) \frac{l}{2} \frac{K_w}{E_S I_S}\big)$$

$$= \frac{WK_w}{2g_F E_S I_S}$$

(P.3)

Replacing

$\sinh (d+f)\ l/2 = P2$

$\cosh (d+f)\ l/2 = RM2$

$\sinh (d-f)\ l/2 = V1$

$\cosh (d-f)\ l/2 = T2$

$d + f = R2$

$d - f = RN1$

$$A = \frac{W\ K_w}{2g_F E_S I_S}$$

$$\frac{R_2^2\ T_2}{R_2^2 T_2 (R_2^3 P_2 - 2\beta^2 R_2 P_2 - RM_2 S_w/E_S I_S) - RN_1^2 RM_2 (RN_1^3 V_1 - 2\beta^2 RN_1 V_1 - T_2\ K_w/E_S I_S}$$

$$B = -A\ \frac{RN_1^2\ RM_2}{R_2^2\ T_2}$$

3. When $\beta = \alpha$

$$y = Ae^{dx} + Bxe^{dx} + Ce^{-dx} + Dxe^{-dx}$$

Symmetrical conditions

$y(x) = y\ (-x)$

Therefore

$$y = 2A\ \cosh dx + 2B\ \sinh dx \cdot x + W/g_F$$

Boundary Conditions

1. $x = l/2$

$M_s(l/2) = 0$

$$A(d^2\cosh\frac{dl}{2} - 2b^2\cosh\frac{dl}{2}) + B(d^2\frac{l}{2}\sinh\frac{dl}{2} +$$

$$2d\cosh\frac{dl}{2} - 2b^2\frac{l}{2}\sinh\frac{dl}{2}) = 0$$

2. $x = l/2$

$$y\ (l/2) = \frac{E_S I_S}{S_w}(\frac{d^3y}{dx^3} - 2\beta^2\frac{dy}{dx})$$

$$A(d^3\sinh\frac{dl}{2} - 2\beta^2 d\sinh\frac{dl}{2} - \cosh\frac{dl}{2}\frac{K_w}{E_S I_S})$$

$$+ B(d^3\frac{l}{2}\cosh\frac{dl}{2} + 3d^2\sinh\frac{dl}{2} - 2\beta^2\frac{dl}{2}\cosh\frac{dl}{2} -$$

$$- 2b^2\sinh\frac{dl}{2} - \sinh\frac{dl}{2}\frac{K_w}{E_S I_S}) = \frac{WK_w}{2g_F E_S I_S}$$

Replacing

$$\cosh\frac{dl}{2} = RM_3$$

$$\sinh\frac{dl}{2} = P_3$$

$$V_2 = 2dRM_3 - \frac{d^2l}{2} P_3$$

$$T_3 = d^2RM_3$$

$$R_3 = P_3d^3 + RM_3 \frac{K_w}{E_SI_S}$$

$$RN_2 = -\frac{d^2l}{2} RM_3 + d^2P_3 - P_3 \frac{K_w}{E_SI_S}$$

Therefore

$$A = \frac{wK_wV_2}{2g_F E_S I_S (T_3RN_2 - R_3V_2)}$$

$$B = \frac{AT_3}{V_2}$$

APPENDIX Q

ECCENTRICALLY ARRANGED SHEAR WALLS

The solution is divided in two branches as it is seen in Figure 4.4.

1. $\alpha > \beta$

$$y_1 = e^{dx_1}(A_1 \cos fx_1 + B_1 \sin fx_1) + e^{-dx_1}(C_1 \cos fx_1$$

$$+ D_1 \sin fx_1) + \frac{W}{g_F}$$

$$y_2 = e^{dx_2}(A_2 \cos fx_2 + B_2 \sin fx_2) + e^{-dx_2}(C_2 \cos fx_2$$

$$+ D_2 \sin fx_2) + \frac{W}{g_F}$$

The Boundary conditions are:

1. $x_1 = l_1 \quad y_1 = \frac{Q_w}{K_w} = -\frac{Q_{s_1}}{K_w}$

2. $x_1 = l_1 \quad M_{s_1} = 0$

3. $x_1 = 0 \quad y_1 = \frac{Q_w}{K_w} = \frac{Q_{s_1} + Q_{s_2}}{K_w}$

4. $x_2 = 0 \quad y_2 = \frac{Q_w}{K_w} = \frac{Q_{s_1} + Q_{s_2}}{K_w}$

5. $x_1 = x_2 = 0$

$$\left(\frac{dy}{dx} - \frac{\beta_o}{G_S \bar{A}_S} Q_s\right)_1 = -\left(\frac{dy}{dx} - \frac{\beta_o}{G_S \bar{A}_S} Q_s\right)_2$$

that is the rotation angle is the same

6. $x_1 = x_2 = 0$

$$M_{s_1} = M_{s_2}$$

that is the Moment is continuous

7. $x_2 = l_2 \quad y_2 = \frac{Q_w}{K_w} = -\frac{Q_{s_2}}{K_w}$

8. $x_2 = l_2 \quad M_{s_2} = 0$

Thus, the constants to be determined are:

$A_1,\ B_1,\ C_1,\ D_1; \qquad A_2,\ B_2,\ C_2,\ D_2$

The bending moment equation is

$$M_s = E_S I_S\left(\frac{d^2y}{dx^2} - 2\beta^2 y + 2\beta^2 \frac{W}{g_F}\right)$$

The shear equation is

$$Q_s = -E_S I_S\left(\frac{d^3y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$\frac{x_1 = l_1}{M_{s1} = 0}$$

$$A_1\left[\cos fl_1 e^{dl_1}(d^2-f^2-2\beta^2) - 2\,fde^{dl_1}\sin fl_1\right]$$

$$+ B_1\left[\sin fl_1 e^{dl_1}(d^2-f^2-2\beta^2) + 2fde^{dl_1}\cos fl_1\right]$$

$$+ C_1\left[\cos fl_1 e^{-dl_1}(d^2-f^2-2\beta^2) + 2fde^{-dl_1}\sin fl_1\right]$$

$$+ D_1\left[\sin fl_1 e^{-dl_1}(d^2-f^2-2\beta^2) - 2fde^{-dl_1}\cos fl_1\right] = 0$$

Making the following substitutions

$$S_1 = \sin fl_1 e^{dl_1}$$

$$T_1 = \cos fl_1 e^{dl_1}$$

$$U_1 = \sin fl_1 e^{-dl_1}$$

$$V_1 = \cos fl_1 e^{-dl_1}$$

$$R = d^2 - f^2 - 2\beta^2$$

The above written expression becomes

$$A_1[T_1R-2fdS_1] + B_1(S_1R+2fdT_1] + C_1[V_1R+2fdU_1]$$

$$+ D_1[U_1R-2fdV_1] = 0$$

Another set of substitutions is

$$T_1R - 2fdS_1 = P_1$$

$$S_1R + 2fdT_1 = Q_1$$

$$V_1R + 2fdU_1 = R_1$$

$$U_1R - 2fdV_1 = W_1$$

Thus

$$A_1P_1 + B_1Q_1 + C_1R_1 + D1W1 = 0 \qquad (2)$$

Condition 8 states

$$x_2 = \ell_2 \qquad M_{s_2} = 0$$

$$A_2[\cos fl_2 e^{dl_2}(d^2-f^2-2\beta^2)-2fde^{dl_2}\sin fl_2]$$

$$+ B_2[\sin fl_2 e^{dl_2}(d^2-f^2-2\beta^2) + 2fde^{dl_2}\cos fl_2]$$

$$+ C_2[\cos fl_2 e^{-dl_2}(d^2-f^2-2\beta^2)+2fde^{-dl_2}\sin fl_2]$$

$$+D_2[\sin fl_2 e^{-dl_2}(d^2-f^2-2\beta^2)-2fde^{-dl_2}\cos fl_2] = 0$$

Making substitutions

$$S_2 = \sin fl_2 e^{-dl_2}$$

$$T_2 = \cos fl_2 e^{dl_2}$$

$$U_2 = \sin fl_2 e^{-dl_2}$$

$$V_2 = \cos fl_2 e^{-dl_2}$$

Thus

$$A_2[T_2R-2fdS_2]+B_2[S_2R+2fdT_2]+C_2[V_2R+2fdU_2]$$

$$+ D_2[U_2R-2fdV_2] = 0$$

$$P_2 = T_2R - 2fdS_2$$

$$Q_2 = S_2R + 2fdT_2$$

$$R_2 = V_2R + 2fdU_2$$

$$W_2 = U_2R - 2fdV_2$$

Thus

$$A_2P_2 + B_2Q_2 + C_2R_2 + D_2W_2 = 0 \qquad (8)$$

Condition 6 states

$x_1 = x_2 = 0$

$M_{s_1} = M_{s_2}$

$$M_{s_1} = E_SI_S[\frac{d^2y}{dx^2} - 2\beta^2y + 2\beta^2 \frac{W}{g_F}]$$

$$M_{s_2} = E_SI_S[\frac{d^2y}{dx^2} - 2\beta^2y+2\beta^2 \frac{W}{g_F}]$$

$$A_1(d^2-f^2-2\beta^2) + B_1 2fd + C_1(d^2-f^2-2\beta^2)$$

$$+ D_1(-2fd) =$$

$$A_2(d^2-f^2-2\beta^2) + B_2 2fd + C_2(d^2-f^2-2\beta^2)$$

$$+ D_2(-2fd)$$

$$A_1R - A_2R + B_1 2fd - B_2 2fd + C_1R - C_2R - 2D_1 fd$$

$$+ 2D_2 fd = 0$$

$$A_1R - A_2R + B_1 2fd - B_2 2fd + C_1R - C_2R - D_1 2fd$$

$$+ D_2\ 2fd = 0 \qquad (6)$$

Condition 5

$$x_1 = x_2 = 0$$

$$\left(\frac{dy}{dx} - \frac{\beta_o}{G_S\bar{A}_S} Q_{s_1}\right)_1 = -\left(\frac{dy}{dx} - \frac{\beta_o}{G_S\bar{A}_S} Q_{s_2}\right)_2$$

$$A_1\left(x_o + \frac{dG_S\bar{A}_S}{E_S I_S \beta_o \kappa}\right) - B_1\left(y_o - \frac{fG_S\bar{A}_S}{E_S I_S \beta_o \kappa}\right) - C_1\left(x + \frac{dG_S\bar{A}_S}{\beta_o E_S I_S \kappa}\right)$$

$$- D_1\left(y_o - \frac{fG_S\bar{A}_S}{E_S I_S \beta_o \kappa}\right) = -A_2\left(x_o + \frac{fG_S\bar{A}_S}{E_S I_S \beta_o \kappa}\right) + B_2\left(Y_o - \frac{fG_S\bar{A}_S}{E_S I_S \beta_o \kappa}\right)$$

$$+ C_2\left(x_o + \frac{dG_S\bar{A}_S}{\beta_o E_S I_S \kappa}\right) + D_2\left(y_o - \frac{fG_S\bar{A}_S}{E_S I_S \beta_o \kappa}\right)$$

In which

$$x_o + \frac{dG_S\bar{A}_S}{E_S I_S \beta_o \kappa} = E_1 \qquad x_o = d^3 - 3df^2 - 2\beta^2 d$$

$$y_o - \frac{fG_S\bar{A}_S}{E_S I_S \beta_o \kappa} = F_1 \qquad y_o = f^3 - 3d^2 f + 2\beta^2 f$$

Thus

$$A_1 E_1 + A_2 E_1 - B_1 F_1 - B_2 F_1 - C_1 E_1 - C_2 E_1 - D_1 F_1 - D_2 F_1 = 0 \qquad (5)$$

Condition 3

$$x_1 = 0 \qquad y_1 = \frac{Q_{s_1} + Q_{s_2}}{K_w}$$

$$-A_1 G_1 - A_2 X_o - B_1 y_o + B_2 y_o - C_1 H_1 + C_2 X_o + D_1 y_o + D_2 y_o = \frac{K_w \cdot W}{g_f E_s I_s} \qquad (3)$$

in which

$$G_1 = x_o + \frac{K_W}{E_S I_S}$$

$$H_1 = \frac{K_w}{E_S I_S} - x_o$$

Condition 4

$$x = 0 \qquad y_2 = \frac{Q_{s_1} + Q_{s_2}}{K_w}$$

$$-A_1x - A_2G_1 + B_1y + B_2y + C_1x - C_2H_1 + D_1y + D_2y = \frac{K_wW}{g_F E_S I_S} \quad (4)$$

in which

$$G_1 = x + \frac{K_w}{E_S I_S}$$

$$H_1 = \frac{K_w}{E_S I_S} - x$$

Condition 1

$$x_1 = l_1 \qquad y_1 = -\frac{Q_{s_1}}{K_w}$$

$$T_3 = d^3T_1 + f^3S_1 - 3d^2fS_1 - 3df^2T_1$$

$$S_3 = d^3S_1 - f^3T_1 + 3d^2fT_1 - 3df^2S_1$$

$$V_3 = d^3V_1 - f^3U_1 + 3d^2fU_1 - 3df^2V_1$$

$$U_3 = d^3U_1 + f^3V_1 - f^3V_1 - 3df^2U_1 - 3d^2fV_1$$

$$T_4 = dT_1 - fS_1$$

$$S_4 = dS_1 + fT_1$$

$$V_4 = dV_1 + fU_1$$

$$U_4 = fV_1 - dU_1$$

Also

$$T_5 = T_3 - 2\beta^2T_4 - \frac{T_1K_w}{E_SI_S}$$

$$S_5 = S_3 - 2\beta^2S_4 - \frac{S_1K_w}{E_SI_S}$$

$$V_5 = V_3 - 2\beta^2V_4 + \frac{V_1K_w}{E_SI_S}$$

$$U_5 = U_3 + 2\beta^2U_4 + \frac{U_1K_w}{E_SI_S}$$

$$A_1T_5 + B_1S_5 - C_1V_5 - D_1U_5 = \frac{WK_w}{g_FE_SI_S}$$

Condition 7

$x_2 = l_2 \qquad y_2 = \frac{Q_{s_2}}{K_w} = \frac{Q_w}{K_w}$

$$A_2 \cos fl2 e^{dl_2} + B_2 \sin fl_2 e^{dl_2} + C_2 \cos fl_2 e^{-dl_2}$$

$$+ D_2 \sin fl_2 e^{-dl_2} + \frac{W}{g_F} =$$

$$= \frac{E_S I_S}{K_w} [A_2(d^3T_2 + f^3S_2 - 3d^2fS_2 - 3df^2T_2)$$

$$+ B_2(d^3S_2 - f^3T_2 + 3d^2fT_2 - 3df^2S_2$$

$$- C_2(d^3V_2 - f^3U_3 + 3d^2fU_2 - 3df^2V_2)$$

$$-D_2(d^3U_2 + f^3V_2 - 3df^2U_2 - 3d^2fV_2)$$

$$-2\beta^2A_2(dT_2 - fS_2) - 2\beta^2B_2(dS_2 + fT_2)$$

$$+2\beta^2C_2(dV_2 + fU_2) - 2\beta^2D_2(fV^2 - dU_2)$$

In which

$$T_6 = d^3T_2 + f^3S_2 - 3d^2fS_2 - 3df^2T_2$$

$$S_6 = d^3S_2 - f^3T_2 + 3d^2fT_2 - 3df^2S_2$$

$$V_6 = d^3V_2 - f^3U_2 + 3d^2fU_2 - 3df^2V_2$$

$$U_6 = d^3U_2 + f^3V_2 - 3df^2U_2 - 3d^2fV_2$$

$$T_8 = dT_2 - fS_2$$

$$S_8 = dS_2 + fT_2$$

$$V_8 = dV_2 + fU_2$$

$$U_8 = fV_2 \not+ dU_2$$

Thus

$$A_2(T_6 - 2\beta^2T_8 - T_2\,\frac{K_w}{E_SI_S}) + B_2(S_6 - 2b^2S_8 - S_2\,\frac{K_w}{E_SI_S})$$

$$- C_2(V_6 - 2\beta^2V_8 + \frac{V_2K_w}{E_SI_S}) - D_2(U_6 + 2\beta^2V_8 + U_2\,\frac{K_w}{E_SI_S})$$

$$= \frac{W\ K_w}{g_FE_SI_S}$$

in which

$$T_6 - 2\beta^2T_8 - \frac{T_2K_w}{E_SI_S} = T_9$$

$$S_6 - 2b^2S_8 - S_2\,\frac{Kw}{E_SI_S} = S_9$$

$$V_6 - 2\beta^2 V_8 + V_2 \frac{K_w}{E_S I_S} = V_9$$

$$U_6 + 2\beta^2 V_8 + U_2 \frac{K_w}{E_S I_S} = U_9$$

$$A_2 T_9 + B_2 S_9 - C_2 V_9 - D_2 U_9 = \frac{W\ K_w}{g_F E_S I_S} \quad (M.7)$$

Thus a system of 8 simultaneous equations is obtained

(1) $A_1 T_5 + B_1 S_5 - C_1 V_5 - D_1 V_5 = \frac{W\ K_w}{g_F E_S I_S}$

(2) $A_1 P_1 + B_1 Q_1 + C_1 R_1 + D_1 W_1 = 0$

(3) $-A_1 G_1 - A_2 x + B_1 y + B_2 y - C_1 H_1 + C_2 x + D_1 y + D_2 y = \frac{W\ K_w}{g_F E_S I_S}$

(4) $-A_1 x - A_2 G_1 + B_1 y + C_1 x - C_2 H_1 + D_1 y + D_2 y = \frac{W\ K_w}{g_F E_S I_S}$

(5) $A_1 E_1 + A_2 E_1 - B_1 F_1 - B_2 F_1 - C_1 E_1 - C_2 E_1 - D_1 F_1 - D_2 F_1 = 0$

(6) $A_1 R_1 - A_2 R + B_1 2fd - B_2 2fd + C_1 R - C_2 R - D_1 2fd + D_2 2fd = 0$

(7) $A_2 T_9 + B_2 S_9 + -C_2 V_9 - D_2 U_9 = \frac{W\ K_w}{g_F E I_S}$

(8) $A_2 P_2 + B_2 Q_2 + C_2 R_2 + D_2 W_2 = 0$

Using any program that solves a set of simultaneous equations, the constants A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2 would be known and the values incorporated in the solution for y_1 and y_2.

2. $\beta > \alpha$

The solution is

$$y_1 = A_1 e^{(d+f)x} + B_1 e^{-(d+f)x} + C_1 e^{(d-f)x} + D_1 e^{-(d-f)x}$$

$$y_2 = A_2 e^{(d+f)x} + B_2 e^{-(d+f)x} + C_2 e^{(d-f)x} + D_2 e^{-(d-f)x}$$

The Boundary conditions are

1. $x_1 = l_1 \qquad y_1 = Q_w/K_w = -Q_{s_1}/K_w$

2. $x_1 = l_1 \qquad M_{s_1} = 0$

3. $x_1 = 0 \qquad y_1 = \frac{Q_w}{K_w} = (Q_{s_1} + Q_{s_2})/K_w$

4. $x_2 = 0 \qquad y_2 = Q_w/K_w = (Q_{s_1} + Q_{s_2})/K_w$

5. $x_1 = x_2 = 0 \qquad M_{s_1} = M_{s_2}$

6. $x_1 = x_2 = 0 \quad \left(\frac{dy}{dx} - \frac{\beta_o \kappa}{G_S \bar{A}_S} Q_s\right)_1 = -\left(\frac{dy}{dx} - \frac{\beta \kappa}{G_S \bar{A}_S} Q_s\right)_2$

7. $x_2 = l_2 \qquad y_2 = Q_w/K_w = -\frac{Q_{s_2}}{K_w}$

8. $x_s = l_2 \qquad M_{s_2} = 0$

Bending Moment

$$M_s = -E_S I_S \left(\frac{d^2y}{dx^2} - 2\beta^2 y + 2\beta^2 W/g_F\right)$$

Shear

$$Q_s = -E_S I_S \left(\frac{d^3y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$x_1 = l_1$$

$$M_{s_1} = 0$$

$$(d+f)^2 A_1 e^{(d+f)l_1} + (d+f)^2 B_1 e^{-(d+f)l_1} + (d-f)^2 C_1 e^{(d-f)l_1}$$

$$+ (d-f)^2 D_1 e^{-(d-f)l_1} - 2\beta^2 (A_1 e^{(d+f)l_1} + B_1 e^{-(d+f)l_1} +$$

$$C_1 e^{(d-f)l_1} + D_1 e^{-(d-f)l_1} + W/g_F) + 2b^2 W/g_F = 0$$

$$A_1 e^{(d+f)l_1}((d+f)^2 - 2\beta^2) +$$

$$+ B_1 e^{-(d+f)l_1}((d+f)^2 - 2\beta^2) +$$

$$+ C_1 e^{(d-f)l_1}((d-f)^2 - 2\beta^2)$$

$$+ D_1 e^{-(d-f)l_1}((d-f)^2 - 2\beta^2) = 0$$

8 $x_2 = l_2 \qquad M_{s_2} = 0$

$$A_2 e^{(d+f)l_2}((d+f)^2 - 2\beta^2) +$$

$$B_2\, e^{-(d+f)\ell_2}((d+f)^2 - 2\beta^2) =$$

$$C_2\, e^{-(d-f)l_2}\,((d-f)^2 - 2\beta^2) +$$

$$D_2\, e^{-(d-f)l_2}((d-f)^2 - 2\beta^2) = 0$$

5 $x_1 = x_2 = 0 \qquad M_{s_1} = M_{s_2}$

$$A_1((d+f)^2 - 2\beta^2) +$$

$$B_1((d+f)^2 - 2\beta^2) +$$

$$D_1\,((d-f)^2 - 2\beta^2) =$$

$$= A_2\,((d+f)^2 - 2\beta^2) +$$

$$B_2\,((d+f)^2 - 2\beta^2) +$$

$$C_2\,((d-f)^2 - 2\beta^2) +$$

$$D_2\,((d-f)^2 - 2\beta^2)$$

3 $x_1 = 0 \qquad y_1 = Q_w/K_w = (Q_{s_1} + Q_{s_2})/K_w$

$$Q_{s_1} = -E_S I_S \left(\frac{d^3y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$Q_{s_2} = -E_S I_S \left(\frac{d^3y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$Q_{s_1} = -E_S I_S \Big((d+f)^3 A_1 - (d+f)^3 B_1 + (d-f)^3 C_1 - (d-f)^3 D_1$$

$$-2\beta^2 \big((d+f)A_1 - (d+f)B_1 + (d-f)C_1 - (d-f)D_1\big)$$

$$Q_{s_2} = -E_S I_S \Big((d+f)^3 A_2 - (d+f)^3 B_2 + (d-f)^3 C_2 - (d-f)^3 D_2$$

$$-2\beta^2 \big((d+f)A_2 - (d+f)B_2 + (d-f)C_2 - (d-f)D_2\big)$$

$$y_1 = A_1 + B_1 + C_1 + D_1 + W/g_F = (Q_{s_1} + Q_{s_2})/S_w$$

$$A_1 \left[\frac{K_w}{E_S I_S} + (d+f)^3 - 2\beta^2 (d+f)\right]$$

$$+ B_1 \left[\frac{K_w}{E_S I_S} - (d+f)^3 + 2\beta^2 (d+f)\right]$$

$$+ C_1 \left[\frac{K_w}{E_S I_S} + (d-f)^3 - 2\beta^2 (d-f)\right]$$

$$+ D_1 \left[\frac{K_w}{E_S I_S} - (d-f)^3 + 2\beta^2 (d-f)\right]$$

$$+ A_2 \left[(d+f)^3 - 2\beta^2 (d+f)\right] + b_2 \left[-(d+f)^3 + 2\beta^2 (d+f)\right]$$

$$+ C_2[(d-f)^3-2\beta^2(d-f)] + D_2 [-(d-f)^3+2\beta^2(d-f)] =$$

$$= - \frac{K_w w}{g_F E_S I_S}$$

$x_2 = 0 \qquad y_2 = Q_w/K_w = (Q_{s_1} + Q_{s_2})/K_w$

$$A_2 + B_2 + C_2 + D_2 + W/g_F = \%$$

$$A_2 [\frac{K_w}{E_S I_S} + (d+f)^3 - 2\beta^2(d+f)] +$$

$$B_2 [\frac{K_w}{E_S I_S} - (d+f)^3 + 2\beta^2(d+f)] +$$

$$+ C_2 [\frac{K_w}{E_S I_S} + (d-f)^3 - 2\beta^2(d-f)] +$$

$$+ D_2 [\frac{K_w}{E_S I_S} - (d-f)^3 + 2\beta^2(d-f)] +$$

$$+ A_1 [(d+f)^3 - 2\beta^2(d+f)] + B_1[-(d+f)^3+2\beta^2(d+f)] +$$

$$+ C_1[(d-f)^3 - 2\beta^2(d-f0] + D_1[-(d-f)^3 + 2\beta^2(d-f)] =$$

$$= - \frac{K_w \hat{W}}{g_F E_S I_S}$$

1 $x_1 = l_1 \quad y_1 = -\frac{Q_{s_1}}{K_w}$

$$A_1 e^{(d+f)l_1} + B_1 e^{-(d+f)l_1} + C_1 e^{(d-f)l_1} + D_1 e^{-(d-f)l_1} + \frac{W}{g_F}$$

$$= \frac{Q_{s_1}}{K_w} = \frac{EI_S}{K_w}\left(\frac{d^3y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$= \frac{EI_S}{K_w}\Big((d+f)^2 A_1 e^{(d+f)l_1} - (d+f)^3 B_1 e^{-(d+f)l_1} + (d-f)^3 C_1 e^{(d-f)l_1}$$

$$-(d-f)^3 D_1 e^{-(d-f)l_1}$$

$$-2\beta^2 \big((d+f)A_1 e^{(d+f)l_1} - (d-f)B_1 e^{-(d+f)l_1}$$

$$+(d-f)C_1 e^{(d-f)l_1} - (d-f)D_1 e^{-(d-f)l_1}\big)$$

$$A_1\Big((d+f)^3 e^{(d+f)l1} - \frac{K_w}{E_S I_S} e^{(d+f)l1} - 2\beta^2 (d+f)\, e^{(d+f)l1}\Big)$$

$$+ B_1 \Big(-(d+f)^3\, e^{-(d+f)l1} - \frac{K_w}{E_S I_S} e^{-(d+f)l1} + 2\beta^2 (d+f)\, e^{-(d+f)l1}\Big)$$

$$+ C_1 \Big((d-f)^3 e^{(d-f)l1} - \frac{K_w}{E_S I_S} e^{(d-f)l1} - 2\beta^2 (d-f)\, e^{(d-f)l1}\Big)$$

$$+ D_1 \left(-(d-f)^3 e^{-(d-f)l1} - \frac{K_w}{E_S I_S} e^{-(d-f)l1} + 2\beta^2 (d-f)\; e^{-(d-f)l1}\right)$$

$$= \frac{W\; K_w}{gF\; E_S I_S}$$

7 $\quad x_2 = l_2 \qquad y_2 = -\frac{Q_{s_2}}{K_w}$

$$A_2\, e^{(d+f)l_2} + B_2\, e^{-(d+f)l_2} + C_2\, e^{(d-f)l_2} +$$

$$+ D_2\, e^{-(d-f)l_2} + \frac{W}{gF} = -\frac{Q_{s_2}}{K_w}$$

$$A_2 \left((d+f)^3 e^{(d+f)l_2} - \frac{K_w}{E_S I_S} e^{(d+f)l_2} - 2\beta^2 (d+f)\; e^{(d+f)l_2}\right)$$

$$+ B_2\left(-(d+f)^3 e^{-(d+f)l_2} - \frac{K_w}{E_S I_S} e^{-(d+f)l_2} + 2\beta^2 (d+f)\; e^{-(d+f)l_2}\right)$$

$$+ C_2 \left((d-f)^3 e^{(d-f)l_2} - \frac{K_w}{E_S I_S} e^{(d-f)l_2} - 2\beta^2 (d-f)\; e^{(d-f)l_2}\right)$$

$$+ D_2\left(-(d-f)^3 e^{-(d-f)l_2} - \frac{K_w}{E_S I_S} e^{-(d-f)l_2} + 2\beta^2 (d-f)\; e^{-(d-f)l_2}\right)$$

$$= \frac{W\ K_W}{gF\ E_S I_S}$$

6 $\underline{x_1 = x_2 = 0}$

$$\left(\frac{dy}{dx} - \frac{\beta_o \kappa}{G_S \bar{A}_S} Q_s\right)_1 = -\left(\frac{dy}{dx} - \frac{\beta_o \kappa}{G_S \bar{A}_S} Q_s\right)_2$$

$$\frac{dy_1}{dx} = (d+f)\ A_1 - (d+f)\ B_1 + (d-f)\ C_1 - (d-f)\ D_1$$

$$Q_{s_0} = -E_S I_S \left(\frac{d^3 y}{dx^3} - 2\beta^2 \frac{dy}{dx}\right)$$

$$- Q_{s_0} = +E_S I_S \Big((d+f)^3 A_1 - (d+f)^3 B_1 + (d-f)^3 C_1 - (d-f)^3 D_1$$

$$- 2\beta^2 \left[(d+f)\ A_1 - (d+f)\ B_1 + (d-f)\ C_1 - (d-f)\ D_1\right]\Big)$$

$$A_1 \left((d+f) + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d+f)^3 - \frac{2b^2 E_s I_s}{G_S \bar{A}_S} \beta_o \kappa\ (d+f)\right) +$$

$$+ B_1 \left(-(d+f) - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d+f)^3 + \frac{2\beta^2 E_s I_s}{G_S \bar{A}_S} \beta_o \kappa\ (d+f)\right)$$

$$+ C_1 \left((d-f) + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d-f)^3 - \frac{2b^2 E_s I_s}{G_S A_S} \beta_o \kappa (d-f)\right)$$

$$+ D_1 \left(-(d-f) - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d-f)^3 + \frac{2\beta^2 E_S I_S}{G_S A_S} \beta_o \kappa (d-f)\right)$$

$$= -\left(\frac{dy}{dx} - \frac{\beta_o \kappa}{G_S \bar{A}_S} Q_s\right)_2$$

$$A_1 \left((d+f) + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d+f)^3 - \frac{2\beta^2 E_S I_S}{G_S \bar{A}_S} \beta_o \kappa (d+f)\right)$$

$$+ A_2 \left((d+f) + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d+f)^3 - \frac{2\beta^2 E_S I_S}{G_S \bar{A}_S} \beta_o \kappa (d+f)\right.$$

$$+ B_1 \left(-(d+f) - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d+f)^3 + \frac{2\beta^2 E_S I_S}{G_S \bar{A}_S} \beta_o \kappa (d+f)\right.$$

$$+ B_2 \left(-(d+f) - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d+f)^3 + \frac{2\beta^2 E_S I_S}{G_S \bar{A}_S} \beta_o \kappa (d+f)\right)$$

$$+ C_1 \left((d-f) + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d-f)^3 - \frac{2\beta^2 E_S I_S}{G_S \bar{A}_S} \beta_o \kappa (d-f)\right.$$

$$+ C_2 \left((d-f) + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d-f)^3 - \frac{2\beta^2 E_S I_S}{G_S \bar{A}_S} \beta_o \kappa (d-f)\right)$$

$$+ D_1 \left(-(d-f) - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d-f)^3 + \frac{2\beta^2 E_S I_S}{G_S \bar{A}_S} \beta_o \kappa (d-f)\right)$$

$$+ D_2 \left(-(d-f) - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S (d-f)^3 + \frac{2\beta^2 E_S I_S}{G_S \bar{A}_S} \beta_o \kappa (d-f)\right) = 0$$

3 $\beta = \alpha$

The same boundary conditions apply

$x_1 = l_1$ $\quad M_{s_1} = 0$

$$A_1 d^2 e^{dl_1} + B_1 d\, e^{dl_1}(dl_1-2) + C_1 d^2 e^{-dl_1} +$$

$$+ D_1 d\, e^{-dl_1} (dl_1+2) = 0$$

$x_2 = l_2$ $\quad M_{s_2} = 0$

$$A_2 d^2 e^{dl_2} + B_2 d\, e^{dl_2} (dl_2-2) + C_2 d^2 e^{-dl_2}$$

$$+ D_2 d\, e^{-dl_2} (dl_2+2) = 0$$

$x_1 = x_2 = 0$ $\quad M_{s_1} = M_{s_2}$

$$A_1 d^2 + B_1\, d\,(dl_1 - 2) + C_1 d^2 + D_1 d(dl_1 + 2)$$

$$= A_2 d^2 + B_2 d(dl_2 - 2) + C_2 d^2 + D_2 d(dl_2 + 2)$$

$\underline{x_1 = l_1}$ $\qquad y_1 = \dfrac{Q_w}{K_w} = -\dfrac{Q_{s_1}}{K_w}$

$$A_1\left(-d^3 e^{dl_1} - e^{dl_1}\left(\frac{K_w}{E_S I_S}\right)\right) + B_1\left(d^2 e^{dl_1} - d^3 l_1 e^{dl_1} - \frac{K_w}{E_S I_S}\, e^{dl_1}\, l_1\right)$$

$$C_1\left(d^3 e^{-dl_1} - e^{-dl_1}\, \frac{K_w}{E_S I_S}\right) + D_1\left(d^2 e^{-dl_1} e^{-dl_1} \cdot \frac{K_w}{E_S I_S}\, e^{-dl_1}\, l_1\right)$$

$$= \frac{WK_w}{g_F E_S I_S}$$

$\underline{x_2 = l_2}$ $\qquad y_2 = -\dfrac{Q_{s_2}}{K_w}$

$$A_2\left(-d^3 e^{dl_2} - e^{dl_2}\, \frac{K_w}{E_S I_S}\right) + B_2\left(d^2 e^{dl_2} - d^3 l_2 e^{dl_2} - \frac{K_w}{E_S I_S}\, e^{dl_2}\, l_2\right)$$

$$+ C_2\left(d^3 e^{-dl_2}\; e^{-dl_2}\, \frac{K_w}{E_S I_S}\right) + D_2\left(d^2 e^{-dl_2} + d^3 l_2\, e^{-dl_2} - \frac{K_w}{E_S I_S}\, e^{-dl_2}\; l_2\right)$$

$$= \frac{WK_w}{g_F E_S I_S}$$

$\underline{x_1 = 0}$ $\quad y_1 = \frac{Q_{s_1} + Q_{s_2}}{K_w}$

$$A_1(d^3 - \frac{K_w}{E_S I_S}) + B_1 d^2 - C_1(d^3 + \frac{K_w}{E_S I_S}) + D_1 d^2 +$$

$$+ A_2 d^3 - B_2 d^2 - C_2 d^3 - D_2 d^2 = \frac{WK_w}{g_F E_S I_S}$$

$\underline{x_2 = 0}$ $\quad y_2 = \frac{Q_{s_1} + Q_{s_2}}{K_w}$

$$A_1 d^3 - B_1 d^2 - C_1 d^3 - D_1 d^2 + A_2(d^3 - \frac{K_w}{E_S I_S}) - B_2 d^2$$

$$- C_2 (d^3 + \frac{K_w}{E_S I_S}) - D_2 d^2 = \frac{WK_w}{g_F E_S I_S}$$

$\underline{x_1 = x_2 = 0}$

$$(\frac{dy}{dx} - \frac{\beta_o \kappa}{G_S \bar{A}_S} Q_s)_1 = - (\frac{dy}{dx} - \frac{\beta_o \kappa}{G_S \bar{A}_S} Q_s)_2$$

$$A_1\left(d - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S d^3\right) + B_1\left(1 + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S d^2\right) - C_1\left(d - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S d^3\right)$$

$$+ D_1\left(1 + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S d^2\right) = -A_2\left(d - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S d^3\right)$$

$$- B_2\left(1 + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S d^2\right) + C_2\left(d - \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S d^3\right) - D_2\left(1 + \frac{\beta_o \kappa}{G_S \bar{A}_S} E_S I_S d^2\right)$$

APPENDIX R

1.1 DESIGN OF A THREE BAY ONE STOREY BUILDING WITH END SHEAR WALLS AND CONCRETE HORIZONTAL DIAPHRAGM.

The building is shown in Figure 5.1 and is comprised of three bays, two end shear walls and two interior frames. The type of deck used is selected from Reference 33 which is T-15 Hi Bond 24" coverage, deck thickness 0.030" and a concrete cover of 2.5" (see Figure 5.2).

The design loads are shown below:

Vertical loads:

Dead loads:

Deck: 35 psf

Beams: 15 psf

Live loads:

Snow: 50 psf

Horizontal Loads

The procedure recommended by the National Building Code of Canada (25) states:

$$V = A\ S\ K_2\ I\ F\ W \tag{R.1}$$

where:

V = Minimum lateral seismic force

S = Seismic response factor

A = Acceleration ratio

K_2 = Constant that represents damping and ductility of the building

F = Foundation factor

W = Dead load including 25% of the design snow load

$W = (50 + 0.25 \times 50) \times 90 \times 15 = 84375$ lb.

Therefore, replacing in (R.1)

$V = 0.08 \times 1.0 \times 0.8 \times 1.0 \times 1.0 \times 84375 = 5400$ lb.

Thus, the uniformly distributed horizontal load is

$V/90 = 60$ lb.l.f.

Stiffness of the Horizontal Diaphragm

To calculate the stiffness of the horizontal diaphragm three constants must be assessed.

1. Moment of Inertia of the perimeter beams with respect to the centerline of the building.
2. Moment of Inertia of the transformed area of steel deck with concrete fill.
3. Shear area of the horizontal deck with concrete fill.

1. Moment of Inertia of the perimeter beams

Beams: W8 x 28 Area = 8.23 in^2

$$I_S = 2A \left(\frac{b}{2}\right)^2$$

where

I_S = Moment of Inertia

A = Cross sectional area of the perimeter beams

b = width of the building

Thus

$$I_S = 2 \times 8.23 \left(\frac{15 \times 12}{2}\right)^2 = 133,326 \text{ in}^4$$

2. Moment of Inertia of the transformed area of concrete

$$E_c = 33 \, \gamma^{1.5} \sqrt{f'c}$$

where

γ = 145 pcf

$f'c$ = 3000 psi

Thus

E_c = 3,155,924

and

$$n = E_s/E_c = 29,000,000/3,155,924 = 9.2$$

For a 1 ft. width the steel area is = 0.499 in^2 and the concrete area = 36.75 in^2.

The equivalent steel depth of the slab is

$$t = \left(\frac{36.75}{9.2} + 0.499\right)/12 = 0.375"$$

Therefore the moment of Inertia of the horizontal diaphragm

$$I_S = \frac{1}{12} t\ b^3 = \frac{1}{12} 0.375 \times 180^3 = 182,736 \text{ in}^4$$

3. Shear area of the steel deck with concrete fill

To calculate the shear deflection of the deck the flexibility factor is used (33) such that

$$\Delta_s = \frac{V\ L\ F}{2 \times 10^6} \qquad (R.2)$$

where

V = Average Shear along length L/2

L = span

F = Flexibility factor

but

$$V = \frac{wl}{4b} \qquad (R.3)$$

Thus

$$\Delta_s = \frac{w\ l^2\ F}{8b \times 10^6} \qquad (R.4)$$

The shear deflection for an equivalent beam is:

$$\Delta_s = \frac{wl^2}{8G_S \bar{A}_S} \qquad (R.5)$$

where

G_S = shear modulus of the horizontal diaphragm

$\bar{A}_S$ = shear area of the horizontal diaphragm

Web area of plate girder

$\bar{A}_s = b.t$

where

b = width of the horizontal diaphragm

t = thickness of the horizontal diaphragm

Comparing R.4 and R.5 the thickness of the equivalent slab is obtained as

$t = 10^6/FG_S$

From Reference 33 the flexibility factor F is 0.90 for the 6' spacing

Thus

$$t = \frac{1,000,000}{11,200,000 \times 0.90} = 0.099"$$

The shear area is

$\bar{A}_S = 0.099 \times 15 \times 12 = 17.9 \text{ in}^2$

Frame Design

Designing the interior frame for the vertical loads, a minimum size for beam and columns from strength and deflection point of view is obtained. Thereafter any size of beam and columns may be used with the results shown in Table R-1.

Table R-1

Stiffness of the Frames

Beam	I_B	Column	I_C	$(\frac{I_{xx}}{L})_B$	$(\frac{I_{xx}}{h})_C$	K Kips/in
W8 x 48	184	W6 x 20	41.5	1.02	0.346	3.574
W10 x 49	273	W8 x 20	69.4	1.52	0.578	5.867
W12 x 58	476	W10 x 29	158	2.64	1.317	12.740

Stiffness of the End Walls

The exterior walls may be made of any material; for this particular case concrete is used.

The stiffness of the exterior wall is obtained from equation R-6.

$$K = \frac{e}{\frac{4}{3}\left(\frac{H}{b}\right)^3 + \frac{1.2}{G}\left(\frac{H}{b}\right)} \qquad \text{(R-6)}$$

where

$e \doteq 8"$ (thickness of the wall)

$b = 180"$ (width of the building)

$H = 120"$ (Height of the building)

$E = 3{,}156$ k/in (Young's Modulus)

$G_S = 1{,}372$ k/in (Shear Modulus)

$\nu = 0.15$ (Poisson's ratio)

Solving K = 8346 k/in

The force taken by an interior frame is:

$$\frac{V_R}{V_T} = \frac{\frac{1}{K_B} + \frac{1}{K_{sh}} + \frac{1}{2K_W}}{\frac{2.725}{K_B} + \frac{3}{K_{sh}} + \frac{1}{K_W} + \frac{1}{K_F}}$$

Thus

$$\frac{V_R}{V_T} = \frac{\frac{1}{643} + \frac{1}{1661} + \frac{1}{28346}}{\frac{2.725}{643} + \frac{3}{1661} + \frac{1}{8346} + \frac{1}{3574}}$$

$$\frac{V_R}{V_T} = 5.4 \text{ kips}$$

$V_R = 0.042$ kips, $V_w = 2.66$ kips

$c = K_w/K_F = 8346/3574 = 2335$

The displacements of the frame and wall are respectively

$$\Delta_1 = \frac{0.040}{3,574} = 0.0112"$$

$$\Delta_2 = \frac{2.66}{8346} = 0.0003"$$

If the horizontal diaphragm is considered infinitely stiff

$F = k \ \Delta$

where

F = horizontal force applied to the building

Δ = Uniform displacement of the building

$\Delta = 5.4/16699 = 0.000323"$

The force taken by a frame and a wall are respectively

$V_R = 3.574 \times 0.00032 = 0.00116$ kps

$V_W = 8346 \times 0.00032 = 2.699$ kps

and $c_1 = \infty$

1.2 THREE BAY BUILDING WITH END BRACED WALLS AND CONCRETE DIAPHRAGM.

Stiffness of the End Walls

$$K_1 = AE \frac{b^2}{h_1^3}$$

where

A = Area of the diagonal element

b = width of the building

E = Modulus of Elasticity

h_1 = Length of Diagonal

$$K_1 = 2.70 \times 29000 \times \frac{(15 \times 12)^2}{(18 \times 12)^3} = 250 \text{ k/in}$$

The coefficient C is

$$c_1 = 250/3.574 = 69.9$$

The force taken by an interior frame is:

$$\frac{V_R}{V_T} = \frac{\frac{1}{643} + \frac{1}{1999} + \frac{1}{2 \cdot 250}}{\frac{2.725}{643} + \frac{3}{1999} + \frac{1}{250} + \frac{1}{3.574}}$$

Thus

$$\frac{V_R}{V_T} = 0.014$$

$V_R = 0.0757$ Kips $\quad V_w = 2.624$ kips

The displacements of the frame and wall are respectively

$$\Delta_1 = \frac{0.0757}{3.574} = 0.0212"$$

$$\Delta_2 = \frac{2.623}{250} = 0.0105"$$

If the concrete diaphragm is considered infinitely stiff, the displacement of the building is uniform:

$$\Delta = \frac{5.4}{\Sigma K} = \frac{5.4}{507.2} = 0.01065"$$

and the force taken by an interior fram and the end wall are respectively

$V_R = 0.0381$ kips

$V_w = 2.662$ kips

Reducing the stiffness of the braced walls. Let use a frame having a stiffness $K_F = 12.75$ K/in from Table R-1. Also increasing the stiffness of the interior frames.

$K_F = 12.75$ K/in

Thus, the force taken by an interior frame is:

$$\frac{V_R}{V_T} = \frac{\frac{1}{643} + \frac{1}{1999} + \frac{1}{2x12.74}}{\frac{2.725}{643} + \frac{3}{1999} + \frac{1}{12.74} + \frac{1}{12.74}} = 0.254$$

Thus

$V_R = 0.254 \times 5.4 = 1.37$ kips

$V_W = 1.33$ kips

$c = 1.$

The displacement of the frame and wall are respectively

$$\Delta_1 = \frac{1.37}{12.74} = 0.108"$$

$$\Delta_2 = 0.104"$$

For a infinitely stiff diaphragm, the displacement of the building is uniform:

$$\Delta = \frac{5.4}{50.96} = 0.106"$$

The force taken by an interior frame and an end wall are:

$V_R = 12.74 \times 0.106 = 1.35$ kips

$V_w = 1.35$ kips

2. DESIGN OF A THREE DAY ONE STOREY BUILDING WITH END SHEAR WALLS AND STEEL DECK DIAPHRAGM

The same building is designed as in the first example.

The type of deck used is selected from Reference 33 which is T-15 24" coverage, 3 transverse welds, span between purlins is 6' and the deck thickness is 0.030".

Horizontal Loads

The procedure recommended by the National Building Code of Canada (25) for wind states

$p = q\, C_e\, C_g\, C_p$

$q = 10.5$ lbsf

$C_e = 1.0$

$C_g = 2$

$C_p = 1.2$

Thus

$p = 10.5 \times 1.0 \times 2.0 \times 1.2 = 25.2$ lbsf

$V = 25.2 \times 5 = 126$ lb.l.f.

$V_T = 126 \times 90 = 11340$ lb

Stiffness of the Horizontal Diaphragm

Two constants must be assessed:

1. Moment of Inertia of the perimeter beams with respect to the centerline of the building.
2. Shear area of the horizontal disphragm

1. Moment of Inertia of the Inertia of the perimeter beams is calculated below

   Using W8 x 28 beams from Reference 8.

   The area is 8.23 in^2 and the moment of inertia is:

   $$I = 2 \times 8.23 \quad \left(\frac{15\times12}{2}\right)^2 \quad = 133{,}326 \ in^4$$

2. Shear area of the horizontal disphragm.

   The flexibility factor is used (33) such that

   $F = 23 + 185R.$

   where

   $R =$ Ration between purlin spacing and length of deck supplied

   In this case

   $F = 23 + 185 \times 6/18 = 84.7$

   and the thickness of the equivalent slab is

   $t = 10^6/FG = 10^6/84.7 \times 11{,}200{,}000 = 0.0013"$

The shear area is

$$\bar{A}_S = 0.0013 \times 15 \times 12 = 0.195 \text{ in}^2$$

Stiffness of an Interior Frame

Table R-1 provides values for the stiffness of the frame. The value selected from this example is 12.74 k/in.

Stiffness of the End Walls

The exterior walls in this case are braced walls.

The stiffness is

$$K_1 = AE \frac{b^2}{h_1^3}$$

where

A = Area of the diagonal element

b = width of the building

E = Modulus of Elasticity

h_1 = Length of Diagonal

A = 2.70 in^2

$$K_1 = 2.70 \times 29000 \times \frac{(15\text{x}12)^2}{(18\text{x}12)^3} = 250 \text{ K/in}$$

The force taken by an interior frame is

$$\frac{V_R}{V_T} = \frac{\frac{1}{K_B} + \frac{1}{K_{sh}} + \frac{1}{2K_w}}{\frac{2.725}{K_B} + \frac{3}{K_{sh}} + \frac{1}{K_w} + \frac{1}{K_F}}$$

$$K_B = 88.36 \ \frac{EI}{L^3}$$

$$I_S = 133,326 \text{ in}^4$$

$$K_B = 88.36 \times 29,000 \times 133,326/(90\text{x}12)^3 = 271 \text{ K/in}$$

$$K_{sh} = 9 \ \frac{G\overline{A}}{L}$$

$$\overline{A}_S = 0.195 \text{ in}^2$$

$$K_{sh} = 9 \times 11,200 \times 0.165/90\text{x}12 = 18.2 \text{ K/in}$$

$$c = K_w/K_F = 250/12.74 = 19.6$$

$$\frac{V_R}{V_T} = \frac{\frac{1}{271} + \frac{1}{18.2} + \frac{1}{2\text{x}250}}{\frac{2.725}{271} + \frac{3}{18.2} + \frac{1}{250} + \frac{1}{12.74}} = 0.2356$$

$$\therefore V_R = 11340 \times 0.2356 = 2672 \text{ lb}$$

$$V_w = 2.998 \text{ Kips}$$

The displacement of frames and end braced walls are respectively

$$\Delta_1 = \frac{2.672}{12.74} = 0.210"$$

$$\Delta_2 = \frac{2.998}{250} = 0.012"$$

If the horizontal diaphragm is considered infinitely stiff

$$F = K\Delta$$

$$\Delta = \frac{11.34}{525.5} = 0.022"$$

The forces taken by frames and end braced walls are respectively:

$$V_R = 12.74 \times 0.022 = 0.275 \text{ kips}$$

$$V_w = 5.500 \text{ kips}$$

Concrete end walls

$K_w = 8346$

$K_F = 12.74$

$$\frac{V_R}{V_T} = \frac{\frac{1}{271} + \frac{1}{18.2} + \frac{1}{2x8346}}{\frac{2.725}{271} + \frac{3}{18.2} + \frac{1}{8346} + \frac{1}{12.74}} =$$

$c = 655$

$$\frac{V_R}{V_T} = 0.231$$

Frame displacement

$$\Delta_1 = \frac{0.231 \times 1134}{12.74} = 0.206"$$

Considering the horizontal diaphragm infinitely stiff

$$F = K\Delta$$

$$K = 16717$$

$$\Delta = \frac{11.34}{16717} = 0.0007$$

Force taken by the frame

$$F = 0.007 \times 12.74 = 0.009 \text{ K}$$

$$\frac{V_R}{V_T} = 0.0008$$

<u>Frame End Wall</u>

K_w = 12.74 K/in

K_F = 12.74 K/in

$$\frac{V_R}{V_T} = \frac{\frac{1}{271} + \frac{1}{18.2} + \frac{1}{2 \times 12.74}}{\frac{2.725}{271} + \frac{3}{18.2} + \frac{1}{12.74} + \frac{1}{12.74}} =$$

$c_1 = 1.0$

$$\frac{V_R}{V_T} = 0.295$$

Frame displacement

$$\Delta_1 = \frac{0.295 \times 11.34}{12.74} = 0.263"$$

Considering the horizontal diaphragm infinitely stiff

$F = K\Delta$

$K = 50.96$

$$\Delta = \frac{11.34}{50.96} = 0.223"$$

Force taken by the frame

$F = 12.74 \times 0.223 = 2.84$ kips

$$\frac{V_R}{V_T} = 0.251$$

APPENDIX S

COMPUTER PROGRAMS

STRENGTH
STIFFNESS

```
      IMPLICIT REAL*8(A-Z)
      REAL*8 NP,NSH,NS,NSC,SS,SE,SSC,B,A,FSU,FPU,FEU,FSC,X(100),X1(100)
      INTEGER*4 NE,NF,KOUNT,IV
      CALL FREAD(5,'2I*4,4R*8:',NE,NF,NP,NSH,NS,NSC)
      CALL FREAD(5,'4R*8:',SS,SE,SSC,SP)
      CALL FREAD(5,'6R*8:',A,B,FSU,FPU,FEU,FSC)
      NP=NP-2.0D0
      KOUNT=NE
      IF(NF.GT.KOUNT) KOUNT=NF
      CALL FREAD(5,'R*8 V:',X,NE)
      DO 10 IV=1,NE
      G1E=G1E+X(IV)
      G2E=G2E+X(IV)**2
   10 CONTINUE
      CALL FREAD(5,'R*8 V:',X1,NF)
      DO 102 IV=1,NF
      G1P=G1P+X1(IV)
      G2P=G2P+X1(IV)**2
  102 CONTINUE
      G1P=NSH/A*G1P
      G1E=NSH/A*G1E
      XO=(2.0D0*NS/SS+2.0D0*G1E/SE+NP*G1P/SP)/(NSC/SSC+2.0D0*NS/SS+2.0D0
     &*NE/SE+NP*NF/SP)*A/NSH
      G2P=NSH/A/XO*G2P
      G2E=NSH/A/XO*G2E
      K1=2.0D0*NS*(A/NSH/XO-1.0D0)/SS+2.0D0*(G2E-G1E)/SE+NP*(G2P-G1P)/SP
C     FAILURE AT SEAM
      OULT1=K1*(NS*FSU+NP*FPU+2.0D0*FEU)/(A/NSH/XO-1.0D0)/(2.0D0*NS/SS+
     &NP/SP+2.0D0/SE)
C     FAILURE AT SIDE
      OULT2=K1*(NSC*FSC+NP*FPU+2.0D0*FEU)/(NSC/SSC+NP/SP+2.0D0/SE)
C     FAILURE AT END
      IF(XO.GT.0.5D0) KOUNT=1
      FEPV1=(1.0D0-X(KOUNT)/XO)/SE/K1
      FEPH=A/NSH/B/NE
      FR=(FEPH**2+FEPV1**2)**0.5*0
      OULT3=FEU/(FEPH**2+FEPV1**2)**0.5
      OULT=OULT1
      IF(OULT2.LT.OULT) OULT=OULT2
      IF(OULT3.LT.OULT) OULT=OULT3
      WRITE(6,100) NE,NF,NP,NSH,NS,NSC
      WRITE(6,200) SS,SE,SSC,SP
      WRITE(6,300) A,FSU,FPU,FEU,FSC
  100 FORMAT(
     &         '0',T5,'NE       ',I8,' '/
     &         '0',T5,'NF       ',I8,' '/
     &         '0',T5,'NP       ',G20.8/
     &         '0',T5,'NSH      ',G20.8/
     &         '0',T5,'NS       ',G20.8/
     &         '0',T5,'NSC      ',G20.8)
  200 FORMAT(
     &         '0',T5,'SS       ',G20.8/
     &         '0',T5,'SE       ',G20.8/
     &         '0',T5,'SSC      ',G20.8/
     &         '0',T5,'SP       ',G20.8)
  300 FORMAT(
     &         '0',T5,'A        ',G20.8/
     &         '0',T5,'FSU      ',G20.8/
     &         '0',T5,'FPU      ',G20.8/
     &         '0',T5,'FEU      ',G20.8/
     &         '0',T5,'FSC      ',G20.8)
      WRITE(6,400) G1E,G2E,G1P,G2P,XO,K1
  400 FORMAT(
     &         '0',T5,'G1E      ',G20.8/
     &         '0',T5,'G2E      ',G20.8/
     &         '0',T5,'G1P      ',G20.8/
     &         '0',T5,'G2P      ',G20.8/
     &         '0',T5,'XO       ',G20.8/
     &         '0',T5,'K1       ',G20.8)
      WRITE(6,500) OULT1,OULT2,OULT3,OULT
  500 FORMAT(
     &         '0',T5,'FAILURE AT THE SEAM',G20.8/
     &         '0',T5,'FAILURE AT THE SIDE',G20.8/
     &         '0',T5,'FAILURE AT THE END ',G20.8/
     &         '0',T5,'FAILURE LOAD       ',G20.8)
      STOP
      END
End of file
```

POOR PRINT
Epreuve illisible


```
      IMPLICIT REAL*8 (A-Z)
      REAL*8 X(1000),X1(1000)
      INTEGER*4 NE,NF,KOUNT,IV
      CALL FREAD(5,'8R*8:',A,A1,D,F1,K,E,T,B)
      CALL FREAD(5,'5R*8:',H,F2,NU,F3,P)
      CALL FREAD(5,'4R*8:',SS,SE,SP,SSC)
      CALL FREAD(5,'2I*4,4R*8:',NE,NF,NP,NSH,NSC,NS)
      NP=NP-2.0D0
      KOUNT=NE
      IF(NF.GT.KOUNT) KOUNT=NF
      CALL FREAD(5,'R*8 V:',X,NE)
      DO 10 IV=1,NE
      S1E=S1E+X(IV)
      S2E=S2E+X(IV)**2
   10 CONTINUE
      CALL FREAD(5,'R*8 V:',X1,NF)
      DO 102 IV=1,NF
      S1P=S1P+X1(IV)
      S2P=S2P+X1(IV)**2
  102 CONTINUE
      G1P=NSH/A*S1P
      G1E=NSH/A*S1E
      XO=(2.0D0*NS/SS+2.0D0*S1E/SE+NP*G1P/SP)/(NSC/SSC+2.0D0*NS/SS+2.0D0
     A*NE/SE+NP*NF/SP)*A/NSH
      IF(XO.GT.0.5D0) KOUNT=1,
      G2P=NSH/A/XO*S2P
      G2E=NSH/A/XO*S2E
      K1=2.0D0*NS*(A/NSH/XO-1.0D0)/SS+2.0D0*(G2E-G1E)/SE+NP*(G2P-G1P)/SP
      C11 =0.144D0*A*D**4*F1*K/E/T**3/B**3
      C12 =2.0D0*A*F2*(1.0D0+NU)*(1.0D0+2.D0*H/D)/B/T/E
      C13 =2.0D0*A**3*F3/3.0D0/B**2/A1/E
      C21A=2.D0*A*P*SP*F3/B**2
      C21B=2.0D0*DP*P*(5.0D0/NP+A**2*F3/B**2)/A
      C22 =2.0D0*(1.0D0+(NSH-1.0D0)*(A/NSH/XO-1.0D0))/K1
      WRITE(6,100) A,D,F1,K,E,T,B
      WRITE(6,200) H,F2,NU,F3,P
      WRITE(6,300) NP,NSH,XO,K1
      WRITE(6,400) C11,C12,C13,C21A,C21B,C22
      CCA=C11+C12+C13+C21A+C22
      CCB=C11+C12+C13+C21B+C22
      CCA=1.0D0/CCA
      CCB=1.0D0/CCB
      WRITE(6,500) CCA,CCB
  100 FORMAT(
     1'0',T5,'A',T10,G20.7/
     1'0',T5,'D',T10,G20.7/
     1'0',T5,'F1',T10,G20.7/
     1'0',T5,'K',T10,G20.7/
     1'0',T5,'E',T10,G20.7/
     1'0',T5,'T',T10,G20.7/
     1'0',T5,'B',T10,G20.7)
  200 FORMAT(
     1'0',T5,'H',T10,G20.7/
     1'0',T5,'F2',T10,G20.7/
     1'0',T5,'NU',T10,G20.7/
     1'0',T5,'F3',T10,G20.7/
     1'0',T5,'P',T10,G20.7/)
  300 FORMAT(
     1'0',T5,'NP',T10,G20.7/
     1'0',T5,'NSH',T10,G20.7/
     1'0',T5,'XO',T10,G20.7/
     1'0',T5,'K1',T10,G20.7)
  400 FORMAT(
     1'0',T5,'C11',T10,G20.7/
     1'0',T5,'C12',T10,G20.7/
     1'0',T5,'C13',T10,G20.7/
     1'0',T5,'C21A',T10,G20.7/
     1'0',T5,'C21B',T10,G20.7/
     1'0',T5,'C22',T10,G20.7)
  500 FORMAT(
     1'0',T5,'KA',T10,G20.7/
     1'0',T5,'KB',T10,G20.7)
      STOP
      END
End of file
```

APPENDIX T

COMPUTER PROGRAMS

CALC1	SYMMETRICAL BUILDINGS
CALC2	NONSYMMETRICAL BUILDINGS
CALC3	NONSYMMETRICAL BUILDINGS

```
      IMPLICIT REAL*8(A - H,O - Z)
      REAL*8 L, K
      REAL*4 XA, XB, YA, YB, XV(500,5), YV(500,5), RMOM(500,5),
     1     SHEAR(500,5)
      INTEGER*4 ND(5)
      CALL FREAD(-2, 'ENDFILE', 1)
      CALL FREAD(-2, 'ENDLINE', 0)
      CALL FTNCMD('ASSIGN SCARDS=TITLES1 2=TITLES1(+100);')
      CALL FREAD(2, '2R:', XA1, XB1)
      CALL FREAD(2, '2R:', YA1, YB1)
      CALL FREAD(2, '2R:', XA2, XB2)
      CALL FREAD(2, '2R:', YA2, YB2)
      CALL FREAD(2, '2R:', XA3, XB3)
      CALL FREAD(2, '2R:', YA3, YB3)
      NP = 0
   10 CALL FREAD(5, '3R*8:', L, SL, H, &90)
      CALL FREAD(5, '5R*8:', EF, EC, GC, EW, GW)
C     NOTE - W => HL (HORIZONTAL LOAD)
      CALL FREAD(5, 'I,4R*8:', N, E, BI, CI, HL)
      CALL FREAD(5, '4R*8:', D, T, B, K)
      CALL FREAD(5, '3R*8:', XI, XF, XINCR)
      CALL FREAD(5, 'R*8:', SI)
      NP = NP + 1
      AS = D * L
      SW = T / (4.0D0*(H/L)**3/EW + H/GW/L)
      SB = BI / L
      SC = CI / H
      S = 12.0D0 * EF * SB / H ** 2 / (1.0D0+2.0D0*SB/SC)
      SN = DFLOAT(N) * S
      GF = SN / SL
      SBL = DSQRT(B*K*GF/2.0D0/GC/AS)
      C = (GF/EC/SI) ** 0.25D0
      SD=DSQRT((C**2+SBL**2)/2.0D0)
      DL=SD*SL
      IF(SBL-C) 12,13,11
11    SF=DSQRT((SBL**2-C**2)/2.0D0)
      FL= SF* SL
      P2 = DSINH((DL+FL)/2.0D0)
      T2= DCOSH((DL-FL)/2.0D0)
      RM2=DCOSH((DL+FL)/2.0D0)
      V1=  DSINH((DL-FL)/2.0D0)
      R2= SD+SF
      RN1= SD-SF
      A2= SW*HL*R2**2*T2/((R2**2*T2*(R2**3*P2-2.0D0*SBL**2*R2*P2
     &-RM2*SW/EC/SI))
     &+RN1**2*RM2*(RN1**3*V1-2.0D0*SBL**2*RN1*V1-T2*SW/EC/SI))/
     & 2.0D0/GF/EC/SI
      B2=-A2*RN1**2*RM2/R2**2*T2
      GO TO 400
12    SF = DSQRT((C**2 - SBL**2)/2.0D0)
      FL = SF * SL
      S1 = DTAN(FL/2.0D0) * DTANH(DL/2.0D0)
      V = DSIN(FL/2.0D0) * DSINH(DL/2.0D0)
      T1 = DCOS(FL/2.0D0) * DCOSH(DL/2.0D0)
      P1 = DSIN(FL/2.0D0) * DCOSH(DL/2.0D0)
      RM = DCOS(FL/2.0D0) * DSINH(DL/2.0D0)
      R1 = SD ** 2 - SF ** 2 - 2.0D0 * SBL ** 2
      O1 = SD ** 3 - 3.0D0 * SF ** 2 * SD - 2.0D0 * SBL ** 2 * SD
      RN = SF ** 3 - 3.0D0 * SD ** 2 * SF + 2.0D0 * SBL ** 2 * SF
      H1 = 2.0D0 * SF * SD * (P1*RN + RM*O1 + S1*P1*O1 - S1*RM*RN - SW*
     1T1/EC/SI - S1*V*SW/EC/SI) + R1 * (S1*P1*RN + S1*RM*O1 - P1*O1 +
     2RM*RN)
      A1 = SW * HL * (2.0D0*SF*SD + S1*R1) / 2.0D0 / GF / EC / SI / H1
      B1 = SW * HL * (2.0D0*SF*SD*S1 - R1) / 2.0D0 / GF / EC / SI / H1
      GO TO 400
13    SF = DSQRT((C**2 - SBL**2)/2.0D0)
      P3 = DSINH(DL/2.0D0)
      RM3=DCOSH(DL/2.0D0)
      T3= SD**2*RM3
      V2=2.0D0*SD*RM3-SD**2*SL/2.0D0*P3
      R3=P3*SD**3+RM3*SW/EC/SI
      RN2=-SD**2*SL*RM3/2.0D0+SD**2*P3-P3*SW/EC/SI
      A3=HL*SW*V2/2.0D0/GF/EC/SI/(T3*RN2-R3*V2)
      B3=A3*T3/V2
400   CONTINUE
      WRITE (6,20) L, SL, H, EF, EC, GC, EW, GW, BI, CI, SI, SB, SC, SW,
     1S, SN
   20 FORMAT ('1', T5, 'Width of Building (in)'
     1          G20.7, /T5, 'Length of Building (in)
     2          G20.7/T5, 'Height of Building (in)
     3          G20.7/T5, 'Young''s Modulus for Frame (Kips/in**2)
     4          G20.7/T5, 'Young''s Modulus for Concrete (Kips/in**2)',
     5          G20.7/T5, 'Shear Modulus for Concrete (Kips/in**2)
     6          G20.7/T5, 'Young''s Modulus for the Wall (Kips/in**2)',
     7          G20.7/T5, 'Shear Modulus for the Wall (Kips/in**2)
     8          G20.7/T5, 'I of Beam (in**4)
     9          G20.7/T5, 'I of Column (in**4)
     *          G20.7/T5, 'I of Slab (in**4)
     1          G20.7/T5, 'Stiffness of Beam (in**3)
     2          G20.7/T5, 'Stiffness of Column (in**3)
     3          G20.7, /T5, 'Stiffness of Wall (in**3)
     4          G20.7/T5, 'Stiffness of Frame (Kip/in)
     5          G20.7/T5, 'Total Stiffness of Frame (Kip/in)
     6          G20.7)
      WRITE (6,30) N, E, T, AS, D, B, K, HL
   30 FORMAT (/T5, 'Number of Frames
     1          G20.7/T5, 'Spacing Between Frames (in)
     2          G20.7/T5, 'Thickness of Wall (in)
     3          G20.7/T5, 'Area of Slab (in)
     4          G20.7/T5, 'Thickness of Slab
     5          G20.7/T5, 'Beta
     6          G20.7/T5, 'Kappa
     7          G20.7/T5, 'Horizontal Load (kips/in)
     8          G20.7)
      WRITE (6,50) GF, SBL, C, SD, SF
   50 FORMAT ('0', T5, 'UNIT STIFFNESS (KIPS/IN)
     1          G20.7/T5, 'b      (dimensions)
     2          G20.7/T5, 'c (wird)
     3          G20.7/T5, 'Small d
     4          G20.7/T5, 'Small f
     5          G20.7)
      ND(NP) = 0
      X = XI
```

```
499   IF(SGL-C) 500,502,501
500   Y = HL / SF + (1.0D0+SW/H1/EC/SI*((2.0D0*SF*SD + S1*R1)
     1*DCOS(SF*X)
     2*DCOSH(SD*X) + (2.0D0*SF*SD*S1 - R1)*DSIN(SF*X)*DSINH(SD*X)))

   80 FORMAT (T5, 5G20.7)
      ND(NP) = ND(NP) + 1
      XV(ND(NP),NP) = Y
      YV(ND(NP),NP) = X
      Y1 = 2.0D0 * DSIN(SF*X) * DCOSH(SD*X) * (B1*SD - A1*SF) + 2.0D0 *
     1DCOS(SF*X) * DSINH(SD*X) * (A1*SD + B1*SF)
      Y2 = 2.0D0 * DCOS(SF*X) * DCOSH(SD*X) * (A1*SD**2 + 2.0D0*B1*SD*
     1SF - A1*SF**2) + 2.0D0 * DSIN(SF*X) * DSINH(SD*X) * (B1*SD**2 - 2.
     20D0*A1*SF*SD - B1*SF**2)
      Y3 = 2.0D0 * DSIN(SF*X) * DCOSH(SD*X) * (B1*SD**3 - 3.0D0*A1*SF*SD
     1**2 - 3.0D0*B1*SD*SF**2 + A1*SF**3) + 2.0D0 * DCOS(SF*X) * DSINH
     2(SD*X) * (A1*SD**3 + 3.0D0*B1*SF*SD**2 - 3.0D0*A1*SD*SF**2 - B1*
     3SF**3)
      GO TO 504
501   Y= 2.0D0*A2*DCOSH(R2*X)+2.0D0*B2*DCOSH(RN1*X)+HL/SF
      WRITE (6,80) X, Y
      GO TO 505
502   Y= 2.0D0*A3*DCOSH(SD*X)+2.0D0*B3*DSINH(SD*X)*X+HL/SF
      WRITE (6,80) X, Y
      GO TO 505
504   WRITE (6,80) Y1, Y2, Y3
      RMOM(ND(NP),NP) = -EC * SI * (Y2 - 2.0D0*SGL**2*Y + 2.0D0*SGL**2*
     1HL/SF)
      SHEAR(ND(NP),NP) = -EC * SI * (Y3 - 2.0D0*SGL**2*Y1)
      WRITE (6,80) X, Y, SHEAR(ND(NP),NP), RMOM(ND(NP),NP)
505   X = X + XINCR
      IF (X .LE. XF) GO TO 499
      GO TO 10
   90 CONTINUE
      STOP
99999 DO 100 IV = 1, NP
      CALL PLOTIT(XV(1,IV), YV(1,IV), ND(IV), IV, 1, 1, 5, XA1, XB1,
     1 5.0, YA1, YB1, 5.0, 1)
  100 CONTINUE
      DO 110 IV = 1, NP
      CALL PLOTIT(SHEAR(1,IV), YV(1,IV), ND(IV), IV, 1, 1, 5, XA2,
     1 XB2, 5.0, YA2, YB2, 5.0, 1)
  110 CONTINUE
      DO 120 IV = 1, NP
      CALL PLOTIT(RMOM(1,IV), YV(1,IV), ND(IV), IV, 1, 1, 5, XA3,
     1 XB3, 5.0, YA3, YB3, 5.0, 1)
  120 CONTINUE
      CALL PLOTIT(XV, YV, ND(1), 0, 1, 1, 5, XA1, XB1, 5.0, YA1, YB1
     1 , 5.0, 1)
      STOP
      END
End of file
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 L,K
      CALL FREAD(5,'4R*8:',L,SL1,SL2,H)
      CALL FREAD(5,'5R*8:',EF,EC,GC,EW,GW)
C     NOTE - W => HL (HORIZONTAL LOAD)
      CALL FREAD(5,'I,4R*8:',N,E,BI,CI,HL)
      CALL FREAD(5,'4R*8:',D,T,B,K)
      CALL FREAD(5,'R*8:',SI)
      AS = D * L
      SW = T / (4.0D0*(H/L)**3/EW + H/GW/L)
      SB = BI / L
      SC = CI / H
      S = 12.0D0 * EF * SB / H ** 2 / (1.0D0+2.0D0*SB/SC)
      SN = DFLOAT(N) * S
      SF = SN /(SL1+SL2)
      SBL = DSQRT(B*K*SF/2.0D0/GC/AS)
      C = (SF/EC/SI) ** 0.25D0
      SD = DSQRT((C**2 + SBL**2)/2.0D0)
      Z [illegible]*SW/(SF*EC*SI)
      DL1=[illegible]*SL1
      DL2=SD*SL2
      WRITE(6,200) L,SL1,SL2,H,EF,EC,GC,EW,GW,BI,CI,SI,SB,SC,SW,S,SN
200   FORMAT('1',T5,'Width of Building (in)                ',G20.7,
     &/ T5,'Length of First span                       ',G20.7 / T5,'Lengh
     &t of second span                       ',G20.7 / T5,'Height of buil
     &ding                          ',G20.7 / T5,'Young''s Modulus for Fra
     &me                  ',G20.7 / T5,'E for Concrete
     &       ',G20.7 / T5,'Shear E for concrete                      ',G20
     &.7 / T5,'E for wall                                 ',G20.7 / T5,'
     &Shear e for wall                       ',G20.7 / T5,'I for beam
     &                           ',G20.7 / T5,'I of column
     &                      ',G20.7 / T5,'[illegible]ab
     &        ',G20.7 / T5,'Stiffness of beam                          ',G20
     &.7,
     &/ T5,'Stiffness of column                        ',G20.7 / T5,'Stiff
     &ness of wall                           ',G20.7 / T5,'stiffness of
     &of Frame (in**3)                ',G20.7/
     &' total stiffness of frame ',G20.7)
      WRITE(6,400)N,E,T,AS,D,B,K,HL
400   FORMAT(/ T5,'Number of Frames
     &                  ',G20.7 / T5,'Spacing Between Frames (in)
     &    ',G20.7 / T
     &5,'Thickness of Wall (in)                        ',G20.7 / T5,'Area of
     &Slab (in)                                ',G20.7 / T5,'Thickness of Slab
     &                            ',G20.7 / T5,'Beta
     &                  ',G
     &20.7 / T5,'Kappa                                         ',G20.7 / T5,
     &'Horizontal Load (kips/in)                      ',G20.7)
      IF(SBL-C)12,13,11
12    SF=DSQRT((C**2-SBL**2)/2.0D0)
      FD2=2*SF*SD
      FL2=SF*SL2
      R=SD**2-SF**2-2.0D0*SBL**2
      FL1=SF*SL1
      S1=DEXP(DL1)*DSIN(FL1)
      T1=DEXP(DL1)*DCOS(FL1)
      U1=DEXP(-DL1)*DSIN(FL1)
      V1=DEXP(-DL1)*DCOS(FL1)
      S2=DEXP(DL2)*DSIN(FL2)
      T2=DEXP(DL2)*DCOS(FL2)
      U2=DEXP(-DL2)*DSIN(FL2)
      V2=DEXP(-DL2)*DCOS(FL2)
      P1=T1*R-2.0D0*SF*SD*S1
      O1=S1*R+2.0D0*SF*SD*T1
      R1=V1*R+2.0D0*SF*SD*U1
      W1=U1*R-2.0D0*SF*SD*V1
      P2=T2*R-2.0D0*SF*SD*S2
      O2=S2*R+2.0D0*SF*SD*T2
      R2=V2*R+2.0D0*SF*SD*U2
      W2=U2*R-2.0D0*SF*SD*V2
      X1=SD**3-3.0D0*SD*SF**2-2.0D0*SBL**2*SD
      Y1=SF**3-3.0D0*SD**2*SF+2.0D0*SBL**2*SF
      E1=X1+SD*GC*AS/(EC*SI*B*K)
      F1=Y1-SF*GC*AS/(EC*SI*B*K)
      G1=X1+SW/(EC*SI)
      H1=SW/(EC*SI)-X1
      T3=SD**3*T1+SF**3*S1-3.0D0*SD**2*SF*S1-3.0D0*SD*SF**2*T1
      S3=SD**3*S1-SF**3*T1+3.0D0*SD**2*SF*T1-3.0D0*SD*SF**2*S1
      V3=SD**3*V1-SF**3*U1+3.0D0*SD**2*SF*U1-3.0D0*SD*SF**2*V1
      U3=SD**3*U1+SF**3*V1-3.0D0*SD*SF**2*U1-3.0D0*SD**2*SF*V1
      T4=SD*T1-SF*S1
      S4=SD*S1+SF*T1
      V4=SD*V1+SF*U1
      U4=SF*V1-SD*U1
      T5=T3-2.0D0*SBL**2*T4-T1*SW/(EC*SI)
      S5=S3-2.0D0*SBL**2*S4-S1*SW/(EC*SI)
      V5=V3-2.0D0*SBL**2*V4+V1*SW/(EC*SI)
      U5=U3+2.0D0*SBL**2*U4+U1*SW/(EC*SI)
      T6=SD**3*T2+SF**3*S2-3.0D0*SD**2*SF*S2-3.0D0*SD*SF**2*T2
      S6=SD**3*S2-SF**3*T2+3.0D0*SD**2*SF*T2-3.0D0*SD*SF**2*S2
      V6=SD**3*V2-SF**3*U2+3.0D0*SD**2*SF*U2-3.0D0*SD*SF**2*V2
      U6=SD**3*U2+SF**3*V2-3.0D0*SD*SF**2*U2-3.0D0*SD**2*SF*V2
      T8=SD*T2-SF*S2
      S8=SD*S2+SF*T2
      V8=SD*V2+SF*U2
      U8=SF*V2-SD*U2
      T9=T6-2.0D0*SBL**2*T8-T2*SW/(EC*SI)
      S9=S6-2.0D0*SBL**2*S8-S2*SW/(EC*SI)
      V9=V6-2.0D0*SBL**2*V8+V2*SW/(EC*SI)
      U9=U6+2.0D0*SBL**2*U8+U2*SW/(EC*SI)
      WRITE(6,2231) T5,S5,V5,U5
2231  FORMAT(' T5=',G20.7/' S5=',G20.7/' V5=',G20.7/' U5=',G20.7/)
      WRITE(6,2232) P1,O1,R1,W1
2232  FORMAT(' P1=',G20.7/' O1=',G20.7/' R1=',G20.7/' W1=',G20.7/)
      WRITE(6,2233) G1,X1,Y1,H1
2233  FORMAT(' G1=',G20.7/' X1=',G20.7/' Y1=',G20.7/' H1=',G20.7/)
      WRITE(6,2234) E1,F1
2234  FORMAT(' E1=',G20.7/' F1=',G20.7/)
      WRITE(6,2235) R,FD2
2235  FORMAT(' R=',G20.7/' FD2=',G20.7/)
      WRITE(6,2237) T9,S9,V9,U9
2237  FORMAT(' T9=',G20.7/' S9=',G20.7/' V9=',G20.7/' U9=',G20.7/)
      WRITE(6,2238) P2,O2,R2,W2
```

```
      WRITE(6,300)
300   FORMAT('-',T15,'X',10X,'Y'///)
400   FORMAT('0',T5,'UNIT STIFFNESS (KIPS/IN)                    ',G20.7,
     &/ T5,'b      (dimensions)                        ',G20.7 / T5,'c
     &                                       ',G20.7 / T5,'Small e
     &                                 ',G20.7 / T5,'Small f
     &                        ',G20.7 / T5,'
     &          ',G20.7 / T5,'n2                                     ',G20
     &.7 / T5,'K                                        ',G20.7)
500   STOP
      END
End of file
```

```
      REAL INC
      CALL FREAD(5,'5R*4:',RL1,RL2,INC,ALPHA,BETA)
      CALL FREAD(5,'6R*4:',SD,SD1,SF,SF1,HL,EF)
      CALL FREAD(5,'4R*4:',A1,B1,C1,D1)
      CALL FREAD(5,'4R*4:',A2,B2,C2,D2)
      WRITE(6,200)
 200  FORMAT(T5,'        X1                  Y1')
      N=RL1/INC+2
      X=-INC
      DO 10 I=1,N
      X=X+INC
      IF(ALPHA-BETA)30,35,40
 30   Y=A1*EXP((SD1+SF1)*X)+B1*EXP(-(SD1+SF1)*X)+C1*EXP((SD1-SF1)*X
     1)+D1*EXP(-(SD1-SF1)*X)+HL/EF
      GO TO 50
 35   Y=A1*EXP(SD*X)+B1*X*EXP(SD*X)+C1*EXP(-SD*X)+D1*EXP(-SD*X)+HL/
     1EF
      GO TO 50
 40   Y=EXP(SD*X)*(A1*COS(SF*X)+B1*SIN(SF*X))+EXP(-SD*X)*(C1*COS(SF*
     1X)+D1*SIN(SF*X))+HL/EF
 50   WRITE(6,100) X,Y
 100  FORMAT(T5,2E15.7)
 10   CONTINUE
      WRITE(6,300)
 300  FORMAT(T5,'        X2                  Y2')
      N=RL2/INC+2
      X=-INC
      DO 20 I=1,N
      X=X+INC
      IF(ALPHA-BETA)60,65,70
 60   Y=A2*EXP((SD1+SF1)*X)+B2*EXP(-(SD1+SF1)*X)+C2*EXP((SD1-SF1)*X
     1)+D2*EXP(-(SD1-SF1)*X)+HL/EF
      GO TO 80
 65   Y=A2*EXP(SD*X)+B2*X*EXP(SD*X)+C2*EXP(-SD*X)+D2*X*EXP(-SD*X)+HL
     1/EF
      GO TO 80
 70   Y=EXP(SD*X)*(A2*COS(SF*X)+B2*SIN(SF*X))+EXP(-SD*X)*(C2*COS(SF*
     1X)+D2*SIN(SF*X))+HL/EF
 80   WRITE(6,100) X,Y
 20   CONTINUE
      STOP
      END
End of file
```