WORKING PAPER 2002-3

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Shadow Prices as Estimates of the Cost of Forest Fires

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May 17, 2002

Abstract

Linear programming based timber harvest scheduling models produce shadow prices of constraints as a standard output. We present a method whereby the shadow prices of starting inventory constraints can be used to approximate the costs of potential fires, expressed in terms of the units of measure for the objective function in the timber supply model. The approximation worked remarkably well over a large range of fire sizes, and alternative model formulations, for a study area in northeastern Alberta.

Keywords: timber harvest scheduling, forest fire, optimization, linear programming.

Introduction

Wildfire management agencies in North America are under increasing pressure to justify their expenditures. The 1998 fire season in Alberta, Canada was extreme: almost 1 700 fires burned more than 726 000 ha of forest. The provincial government spent $242 million on forest protection (mostly by fighting forest fires) that season. In response the severity of the fire season, and the amount of money expended, the Alberta Forest Protection Advisory Committee commissioned the consulting firm KPMG to review the circumstances of that fire season and to make recommendations to improve the efficiency of forest protection in Alberta. The KPMG report (Nash *et al.* 1999)[1] made several recommendations, among which was the "consideration of the level of protection appropriate for Alberta must be founded on an assessment of values-at-risk and the priorities placed on those values". The values-at-risk identified by KPMG included

1. Human lives and the health and safety of people potentially affected by wildfire.
2. Communities and homes of people living in or near the forest.
3. Private property such as buildings and cottages and others.
4. Public property and infrastructure such as power lines, communications sites, roads and others.
5. Industrial facilities such as gas plants, mine sites and forest sector infrastructure.
6. Timber — both standing timber and growing stock contributing to annual allowable cuts.
7. Non-timber resources such as recreation opportunities, wildlife, aesthetics, trapping areas, biodiversity/ecosystem integrity and others.

Timber is recognized as an important asset at risk from fire in Alberta and in many other jurisdictions. The main purpose of this paper is to propose and illustrate a method of evaluating the costs of potential fires using information generated by the solution of a standard timber supply model. This method could be used to improve the efficiency of resource allocation for fire-fighting, and to provide information which could help guide landscape design for the reduction of wildfire risk.

The basic premise of this study is that the value of a unit area of forest land is the value of its contribution to the objective for which the forest is being managed. This is particularly appropriate in situations where periodic forest-level harvest volumes are constrained by intertemporal volume flow constraints such as even-flow or non-declining yield policies. Many constrained forest management problems are modeled and solved using linear programming (LP). If a standard LP-based timber supply model (TSM) is used to optimize the level of some output from the forest, the marginal contribution of each unit area is the shadow price associated with the appropriate starting inventory constraint.

In Alberta, the potential effect of a fire on timber supply has at times been evaluated by a team consisting of a fire behaviour officer and a timber supply analyst. The fire behaviour officer identifies the potential boundary of the fire, and the timber supply analyst uses a geographic information system (GIS) to remove the area affected by the fire from the productive land base,

[1] Available online at the URL http://envweb.env.gov.ab.ca/env/forests/fpd/pdf/kpmg.pdf.

and re-solves the TSM [2]. The difference in the objective function values before and after deletion of the fire area reflects the cost of the potential fire. This is valuable information that could be used to help evaluate alternative fire fighting strategies. Fires with a potentially high cost may be more important to target with suppression resources than fires with low costs.

Shadow prices are a standard output of most LP solution packages. Because they represent the marginal value of the area of timber classes, it is conceivable that they could be used to approximate the "delete and recalculate" procedure described above. The sum of the shadow prices for the area enclosed by the burn boundary would provide this approximation. Unfortunately, shadow prices are strictly valid for small changes in one constraint. A severe fire year would typically result in large changes in many constraints.

However, if these shadow prices can provide a reasonable approximation of the true cost of a fire (as determined by the delete and recalculate procedure) a great deal of time and effort could be saved in estimating the timber supply costs of a potential fire, as these shadow prices could be stored as a static GIS data layer. The sum of the area-weighted shadow prices within a polygon representing a fire boundary would be an approximation of the cost of the potential fire. This information could also be combined with data layer representing fire risk. The combination of value and risk information could prove to be very useful to help direct fire-fighting resources, or to help develop landscape designs to minimize the timber values at risk from fire. The purpose of this study is to examine the applicability of a sum of shadow prices as an approximation to the true costs of fire.

This paper is organized as follows. First, we describe the input data used for this study. In the next section, a simple linear programming based timber supply model is developed and discussed. The concept of shadow prices is explained more thoroughly. Following this, a spatially explicit fire simulation model is described, as is its use in a Monte Carlo simulation to develop a map of annual probability of fire. In the discussion section, the shadow price method is evaluated with respect to its ability to approximate the true costs of fire.

Study Area and Input Data

The forest inventory data for this study represents Forest Management Unit L2 in northeastern Alberta. The inventory was interpreted from 1:15 000 leaf-off aerial photography flown c. 1993. This area is located in the boreal mixedwood section of the boreal forest (Rowe 1972). It is approximately bounded by 55^oN and 55^o37'N latitude, and 112^o40'W and 113^o50'W longitude. The study area's boundary encompasses 316 053 ha, of which 253 821 ha are considered to be productive timber land for this study.

The attributes of forested polygons relevant to this study were species group (aspen, mixed, white spruce, pine, and black spruce), timber productivity rating (TPR) (good, medium, fair, and unproductive), and decadal age class. Inventory cells with a TPR of unproductive are not considered in the analysis. The starting inventory area for fair sites is shown in Table 1, medium sites in Table 2, and good sites in Table 3.

Our fire models use a six-fold land-cover classification based on inventory attributes (Cumming 2001a). The four forested classes (deciduous, white spruce, pine and black spruce) are determined by the canopy species composition. The two non-forested classes are water (lakes,

[2] pers. comm. C. Tymstra. Alberta Sustainable Resource Development

ponds and large rivers) and other (predominantly wetlands, but including minor amounts of roads, clearings, and areas recently burned or harvested). For model parameterisation we required only the approximate locations and final sizes of recorded lightning fires in Alberta, over the interval 1961–1998 (Alberta Environmental Protection 1998).

The yield tables used for this study are the Alberta government's Phase 3 inventory yield tables (Alberta Forest Service 1985). The softwood yields are presented in Table 4 and the hardwood yields in Table 5.

Timber Supply Model

The base timber supply model used here was designed to represent the usual formulation of timber supply models used in Alberta (Alberta Environmental Protection 1996, 1998a,b). The objective function maximizes the timber volume harvested from a forest in the first period of the planning horizon. Many forest areas in Alberta supply several mills that predominantly use either softwood or hardwood logs as mill furnish, and many timber stands produce both softwood and hardwood logs. Even flow of harvest volume for both the softwood and hardwood components is required for the planning horizon (*i.e.* no inter-period variation is allowed in projected harvest volumes for either the softwood or hardwood components). Variants of the base model were examined where the even flow constraints were relaxed to represent non-declining yield, others where the flow constraints were removed entirely, and runs where the objective function was changed to maximization of net present value. The models were constructed using the Woodstock forest modeling system (Remsoft Inc. 2000).

The forest management problem is framed as a straightforward implementation of the Model II timber harvest scheduling formulation (Johnson and Scheurman 1977). The notation used here closely follows that used by Dykstra (1984). We will use the term development type to refer to areas of forest that follow a particular yield curve. A timber type refers to a particular age class within a development type.

The objective function used here is

$$\max Z = \sum_{i=1}^{D} \sum_{k=1}^{H} \sum_{j=-M+1}^{k-N} c_{ijk} x_{ijk} \tag{1}$$

where $Z =$ the value of the objective function,
$D =$ the number of timber development types,
$H =$ the number of periods in the planning horizon,
$N =$ the minimum number of periods between harvests,
$x_{ijk} =$ area (ha) of forest in development type i, born in period j, and harvested in period k,
$M =$ age of oldest existing timber type, in periods, and
$c_{ijk} =$ objective function coefficient associated with harvesting in period k, forest in development type i that was born in period j. For most of the runs used for this study, the objective function maximizes the total volume harvested in the first period: c_{ij1} represents yield table volumes for timber types eligible for harvest in the first period. All other c_{ijk} are set to zero. Some runs were conducted where the objective function was to maximize the total volume harvested over the entire planning horizon, and others maximized net present value. The c_{ijk} were changed appropriately.

Area constraints are incorporated to ensure that all of the area of the forest is explicitly assigned to a harvest or no-harvest activity.

$$\sum_{k=1}^{H} x_{ijk} + u_{ij} = A_{ij} \qquad i = 1, 2, \ldots, D; j = -M+1, -M = 2, \ldots, 0 \tag{2}$$

$$\sum_{l=k+N}^{H} x_{ikl} + u_{ik} = \sum_{j=-M+1}^{k} x_{ijk} \qquad i = 1, 2, \ldots, D; k = 1, 2, \ldots, H \tag{3}$$

where $A_{ij} =$ initial area (ha) of development type i born in period j, and
$u_{ij} =$ area (ha) of forest in development type i born in period j that is never harvested in the planning horizon.

Volume flow constraints are used to control the variation in the harvest of timber volume from one period to the next. Volume flow constraints were constructed for total volume, and for the softwood and hardwood components of total volume.

$$(1-\gamma)S_k - S_{k+1} \leq 0 \qquad k = 1, 2, \ldots, H-1 \tag{4}$$

$$(1+\delta)S_k - S_{k+1} \geq 0 \qquad k = 1, 2, \ldots, H-1 \tag{5}$$

$$S_k = \sum_{i=1}^{D} \sum_{j=-M+1}^{k-N} s_{ijk} x_{ijk} \qquad k = 1, 2, \ldots, H-1 \tag{6}$$

$$(1-\epsilon)D_k - D_{k+1} \leq 0 \qquad k = 1, 2, \ldots, H-1 \tag{7}$$

$$(1+\zeta)D_k - D_{k+1} \geq 0 \qquad k = 1, 2, \ldots, H-1 \tag{8}$$

$$D_k = \sum_{i=1}^{D} \sum_{j=-M+1}^{k-N} h_{ijk} x_{ijk} \qquad k = 1, 2, \ldots, H-1 \tag{9}$$

$$(1-\alpha)T_k - T_{k+1} \leq 0 \qquad k = 1, 2, \ldots, H-1 \tag{10}$$

$$(1+\beta)T_k - T_{k+1} \geq 0 \qquad k = 1, 2, \ldots, H-1 \tag{11}$$

$$T_k = \sum_{i=1}^{D} \sum_{j=-M+1}^{k-N} t_{ijk} x_{ijk} \qquad k = 1, 2, \ldots, H-1 \tag{12}$$

where $S_k =$ softwood volume (m^3) harvested in period k,
$\gamma =$ maximum proportional decrease in softwood harvest volume from one period to the next,
$\delta =$ maximum proportional increase in softwood harvest volume from one period to the next,
$s_{ijk} =$ softwood harvest volume (m^3 ha^{-1}) associated with development type i, birth period j, and harvest period k,
$D_k =$ hardwood volume (m^3) harvested in period k,
$\epsilon =$ maximum proportional decrease in hardwood harvest volume from one period to the next,
$\zeta =$ maximum proportional increase in hardwood harvest volume from one period to the next,
$h_{ijk} =$ hardwood harvest volume (m^3 ha^{-1}) associated with development type i, birth period j, and harvest period k
$T_k =$ total volume (m^3) harvested in period k,
$\alpha =$ maximum proportional decrease in total harvest volume from one period to the next,
$\beta =$ maximum proportional increase in total harvest volume from one period to the next,
$t_{ijk} =$ total harvest volume (m^3 ha^{-1}) associated with development type i, birth period j, and harvest period k.

For the even-flow models constructed, α, β, γ, δ, ϵ, and ζ were set to zero. For the non-declining yield models, Eqs. 5, 8, and 11 were dropped from the model and α, γ, and ϵ were set to zero.

Non-negativity constraints apply to each activity in the linear programming formulation.

$$x_{ijk} \geq 0; u_{ij} \geq 0; S_k \geq 0; D_k \geq 0; T_k \geq 0 \qquad \forall i, j, k \tag{13}$$

The solution to the dual of this linear programming model provides useful information on the sensitivity of the solution to changes in the right-hand sides of the constraints. There is a dual activity associated with each of the constraints, and the optimum level of the dual activity represents the shadow price of this constraint. The relaxation of a binding constraint in a linear programming problem will result in a improvement of the optimal objective function value. Conversely, tightening a binding constraint will result in a degradation of the optimal objective function value (Dykstra 1984). In the particular problem presented here, tightening any binding constraint will result in a reduction of the optimal period 1 harvest volume.

The constraints of most interest here are the starting inventory constraints (Eq. 2). The shadow prices represent the marginal cost to the solution of a reduction in the area of each of the timber types represented in the starting inventory. Because the objective function is expressed in terms of period 1 harvest volume (m^3) and the starting inventory constraints are expressed in terms of area (ha), the shadow prices on the starting inventory constraints are express in terms of volume of period 1 harvest per unit area of starting inventory (m^3 ha^{-1}).

The shadow price of a constraint gives the change in the optimal objective function value for small change in the right-hand-side of constraint, holding all other aspects of the problem constant. The range of right-hand-side values for which the shadow price of a constraint remains constant can be determined through a ranging analysis. If the RHS of more than one binding constraint changes, or if any change beyond the range, the objective function value of a new solution to the problem would be different than that determined from the sum of the shadow prices.

The Fire Model

FEENIX is a grid-based spatial dynamic model developed for boreal forests (Cumming *et al.* 1998). Model landscapes are initialised from digital from a tiled, edge-matched coverage of digital forest inventory data, gridded to a resolution of 3 ha. FEENIX is a collection of mechanistic or statistical submodels of forest management and ecological processes, of which only the fire submodel was used in the present study. FEENIX treats fire as a three stage stochastic process of arrival, escape, and growth. These processes were parameterized to simulate the frequency and size of lightning fires now prevalent in the Alberta-Pacific Forest Industries Inc. Forest Management Agreement Area (FMA) which includes our study area (Cumming and Armstrong 2001). Human caused fires have been of relatively minor importance within the FMA (Cumming 2001a).

A fire arrival is a detected wildfire (Cunningham and Martell 1973). Although many fire ignitions (*i.e.* the smoldering combustion of duff initiated by lightning) may extinguish without ever being detected, these can be ignored here, as their total size is negligible given current detection effort. In FEENIX, arrivals are modeled as a Bernoulli(p_a) process where the parameter p_a is the per-cell annual arrival probability. In the FMA, the spatial distribution of fire arrivals can be modelled as an overdispersed mixture of Poisson(λ_i) processes with class-specific intensities, λ_i (Cumming 2000). The intensities are the expected number of arrivals per unit area and time. As the probability of 2 or more arrivals per 3 $\mathrm{ha} \cdot \mathrm{yr}^{-1}$ is negligible for all classes, the λ_i are class-specific approximations of p_a (Table).

The early growth of fires is modelled as a Bernoulli(p_e) process, where the parameter p_e is the probability that an arrival will escape its cell of origin and exceed 3 ha in size. This discrete growth stage models an important aspect of forest fire management, namely attempted fire suppression by airborne initial attack crews (Hirsch *et al.* 1998; Martell 2001). Fire-size data from

1968–1998 suggest that a 1983 change in initial attack strategy (Gray and Janz 1985) has reduced p_e from 0.20 to 0.11 (Cumming, in review)[3]. We adopt the latter value to represent the present fire management regime.

The final size x of escaped fires was sampled from a truncated exponential distribution (Cumming 2001b) with cumulative density function

$$F_Z(z;\sigma,b) = \frac{1-\exp(-z/\sigma)}{1-\exp(-b/\sigma)}, 0 \leq z < b$$

where $z = \log(x/3)$ and σ and b are the shape and truncation parameters, respectively. For the FMA over the interval 1961–1998, we estimated the parameters $\hat{\sigma} = 2.41$ and $\hat{\mathrm{b}} = 10.82$, which imply a maximum fire size of 150 000 ha. Fires were grown as roughly rectangular shapes centered around the arrival cell, but avoiding barriers such as lakes. To minimize edge effects, the study landscape was padded to its bounding rectangle with cells of class Other and mapped onto a torus.

Using the fire model, we ran 10 000 independent 1 year simulations on the study area, counting the number of runs in which each cell was burned by an escaped fire. On dividing by 10 000, these counts estimate the current annual point probability of burning (p_b), at a 3 ha resolution. These estimated probabilities ignored fire arrivals that did not escape. However, the mean size of such fires in the FMA is presently less than 0.5 ha and so (from Table) their contribution to p_b is $< 2.2 \times 10\ {-5}$, which is negligible.

Results and Discussion

The base timber supply model was run using the starting inventory data and yield curves discussed earlier. The model was run for twenty periods. Each period represents 10 years. The objective was to maximize the total volume harvested in the first period. Softwood, hardwood, and total harvest volumes were constrained to even flow. The optimal first period harvest volume for the base run was 7 174 530 m^3. The shadow prices of the starting inventory constraints range from 0 to 56.4 m^3/decade/ha. The shadow prices for each combination of cover group, TPR, and age class are reported in Tables 1, 2, and 3. The units of measure for shadow price reflect the units of measure for the objective function and starting inventory constraints. Figure 1 is a map of FMU L2 indicating spatial variation in shadow prices. The dark colours on the map indicate areas with a large marginal contribution to the TSM objective function.

In the standard version of FEENIX, the sizes of escaped fire are outcomes of a percolation process (Albinet and Searby 1986), whereby fires spread with probability p_s from a burning cell to its unburnt, flammable neighbors. Spread probabilities may be a constant or vary with land cover class, as is evidently appropriate in our study region (Cumming 2001a). However, as class-specific estimates for p_s are not yet available, most previous applications of FEENIX have used a constant value, tuned so that the expected size of simulated fires matches the mean of an empirical distribution. However, this solution was inappropriate here, because large fires ($> 25,000$ ha) would be generated too infrequently. That is why we sampled fire sizes from a parametric statistical model. However, in consequence, we consider that our simulations underestimated the

[3]S. G. Cumming (in review) Effective fire suppression in boreal forests. Submitted to Canadian Journal of Forest Research.

magnitude of spatial variation in burn probability and expected loss. Before any practical application of our methods, the fire growth model should be improved. Ideally, the observed global behaviours (*e.g.* the fire-size distribution and land-class specific burn rates) would emerge from simple local rules governing the spread of individual fires. However, to determine these local rules poses a challenging problem in statistical modelling.

Of the 10 000 simulated fire years, fires occurred in 4 260, and productive timberland was burned in 4 195. In the biggest fire year, 107 688 ha of productive timberland burned. Figure 2 summarizes the results of the 10 000 fire simulations as an annual fire probability map. The darker shades on the map indicate higher probabilities of a cell burning. The highest annual probability of a cell burning is 0.0040. The average annual burn probability for forested cells is 0.0017.

The shading of each cell in Figure 3 represents the product of annual burn probability and the shadow price for each cell. This represents probability-weighted shadow prices, or the expected value of loss in harvest (m^3/decade/ha) due to fire. Maps such as this could be used to help fire managers identify high priority areas for pre-suppression activities or location of fire fighting resources. The map shown here represents annual probability, so would be most useful at a strategic level (*e.g.* for tanker base location, or fire break construction) . A similar daily map based on components of a fire weather index (Van Wagner 1987) could also be used to help allocate resources on a daily basis. The underlying thought here is that resources could be directed to areas of the forest with high values and a high risk of fire. An important component not capture with these maps is the probability of success of fire management activities. It makes little sense to allocate resources when their effectiveness is projected to be near zero.

The sum of the shadow prices (SSP) for each of the 4 195 years in which productive timber land was burned was calculated as

$$SSP = \sum_{i=1}^{T} \sum_{j=1}^{M} d_{ij} b_{ij} \tag{14}$$

where SSP represents the sum of the shadow prices (m^3), T represents the number of development types in the starting inventory, M represents the number of age classes in the starting inventory, d_{ij} is the shadow price (m^3 ha^{-1}) of the starting inventory constraint for development type i and age class j, and b_{ij} is the burned area (ha) of the development type i and age class j. The SSP for the most severe fire year simulated was 2 807 716 m^3/decade.

We selected a subset of the 10 000 simulated fire years to compare the SSP approximation to the true costs of the fire year. The empirical cumulative probability density function (EDF) for the sum of shadow prices on the 4 195 years in which timber with value burned is presented in 4. The subset was chosen based on quantiles from the EDF displayed in 4. The simulations corresponding to each of the deciles of the distribution were used for comparisons, as were each of the percentiles between 90 and 100. In this way, a number of fire years with different severities can be used to evaluate the SSP approximation.

The area of burn in each timber type from each examined quantile was removed from the starting inventory in the timber supply model. The TSM was re-solved, and the true cost of the fire was calculated as the difference between the objective function values for the base run and the post-burn run. This was compared to the estimated cost as determined by SSP. For each of the quantile runs examined, the ratio of SSP to true cost was between 0.96 and 1.01. The SSP is a remarkably accurate estimate of the true cost of the fire years examined, despite the fact that the

RHS of many starting inventory constraints are being changed simultaneously, and that many of these changes are outside of the range in which shadow prices would remain constant. For example, in the largest fire year, the RHSs of 154 of the 224 starting inventory constraints are reduced beyond their lower bounds, as determined through ranging analysis.

We examined several different formulations of the timber supply model to see how well the results hold. Runs A through D summarized in Table 11 represent comparisons of the no fire situation to the maximum fire year under slightly different formulations of the TSM. For Run A, the even flow constraints on hardwood and softwood volumes were removed, and even flow was required on total volume. Run B was a modification of Run A where the objective function was changed to maximize total volume harvested over the 20 period planning horizon and the flow constraints were relaxed to non-declining yield. Run C was the same as Run B except that the volume flow constraints were removed entirely. In Run D, non-declining yield constraints were reintroduced, and the objective function was changed to maximize net present value assuming a 5% discount rate, harvest and regeneration costs of \$5000/ha, and harvest revenues of \$60/m^3 for softwood and \$50/m^3 for hardwood. Despite these changes in TSM formulation, the ratio of SSP to true cost ranged from 0.98 to 1.00. SSP remains a good approximation of the true cost, even with these changes in problem formulation.

Run E was modified from the base run in order to examine a major change in the behaviour of fire on the landscape. Rather than behave according to the FEENIX fire model, this fire burns timber types with the highest shadow price. In this run, the total area of productive forest burned was the same as the biggest year from the simulations. The area of many of the most valuable timber types was reduced to zero. Even with this extreme change to the model, the ratio of SSP to true cost was 0.92.

The previous runs examined cases where fire is assumed to permanently remove burnt area from the land base in a timber supply model. A series of runs were conducted where fire was assumed to reset the age of burnt area to the youngest age class. In other words, the stand is assumed to regenerate immediately after a fire. The standing timber on the burnt area is lost to the solution, but the productive capacity of the land is not. We accomplish this in the true cost runs by reducing the area of the timber types to reflect fire, and increasing the area of the youngest age class in each development type to reflect the area burned in that development type. The SSP estimate of costs for fire are calculated as

$$SSP_r = \sum_{i=1}^{T}\sum_{j=1}^{M} d_{ij}b_{ij} - \sum_{i=1}^{T}\sum_{j=1}^{M} d_{ij}b_{1j} \tag{15}$$

where SSP_r represents the sum of the shadow prices (m^3) with regeneration, and the remainder of the variables are as defined for Equation 14.

Table 12 compares the true costs of fire with regeneration to the SSP$_r$ estimate for the 90th through 100th percentile fire years. The ratio of SSP$_r$ to the true cost is between 0.93 and 1.00 for the 90th through 98th percentiles. The ratio drops to 0.75 for the 99th percentile and 0.61 for the 100th percentile. For most of the range of observed fires, the SSP approach to estimating the cost of fires is remarkably good, although it weakens as fires get larger. This is probably due to the very large changes made to the right hand side constraints of the timber types in the youngest age class.

Concluding Comments

We have demonstrated that on one specific forest, the shadow prices of the starting inventory constraints in an LP-based timber supply model provides an excellent approximation of the true costs of forest fire over a large range of fire sizes and alternative model formulations. If this method is applicable to other forests, it has a number of potential uses:

1. The SSP method allows for a nearly instantaneous evaluation of the costs of individual fires in terms of the objective function used in the development of the forest management plan. This information could be useful to help decide whether or not a particular fire should be fought. In the case where multiple fires are burning simultaneously, this information could help determine the priority of individual fires.

2. The SSP method would also allow for quick evaluation of land base removals for parks, well sites, roads, seismic exploration lines, *etc.*

3. When an SSP map is coupled with a fire probability map, a probability-weighted values at risk map can be used to identify areas of high value that have a relatively high probability of burning. This information could be used to help locate and allocate fire-fighting resources, or to help design a landscape which would reduce the values at risk from fire.

We cannot draw any conclusions about the general applicability of the SSP method based on results for this one study area, but the method is promising enough to explore further. Exploration of forests with different species and age class structures is necessary to test the method's general applicability. Examination of forests that face different policy constraints is also necessary.

Acknowledgments

This research was supported by the Sustainable Forest Management Network. The forest inventory data for this study were generously provided by Alberta Pacific Forest Industries Ltd. Rick Pelletier created the grid-based representation of the forest inventory. Subhash Lele helped with the fire escape probability analysis. The authors would also like to thank their colleagues in the Boreal Ecology and Economics Synthesis Team (BEEST), especially Fiona Schmiegelow.

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Table 1: Starting inventory area (ha) and shadow price (m^3/decade/ha) by cover group and age class for fair TPR sites.

	Aspen		Mixed		White Spruce		Pine		Black Spruce	
Age	Area	SP	Area	SP	Area	SP	Area	SP	Area	SP
1	12	7.7	0	8.8	0	10.3	15	10.3	150	2.5
2	3	9.0					3	11.1		
3	3	10.5					1 104	11.8	6 423	3.3
4	651	11.4	6	10.9			207	12.5	1 239	3.6
5	309	12.1	54	11.7			27	13.0	1 938	3.7
6			447	12.6	3	13.3	12	13.9	4 170	3.7
7			282	13.3	3	14.0	33	14.7	2 238	3.7
8			159	14.1	15	15.2			1 293	4.0
9			21	14.7			18	16.6	1 365	4.0
10			132	15.3			99	17.4	1 008	4.1
11			24	15.8	9	18.3	264	18.2	1 062	4.2
12	9	0.0	3	16.2	9	19.3	12	19.0	501	4.3
13			6	16.5			57	19.6	123	4.3
14	66	0.0	90	17.0	24	20.2	84	19.9	396	4.3
15			3	17.6					150	4.3
16	18	0.0	3	17.7	3	20.6	42	20.5	57	5.0
17							9	20.5	33	5.0
18	12	0.0	15	17.7						
19									72	5.0
20									3	5.0
21										
22										
23									57	5.0

Table 2: Starting inventory area (ha) and shadow price (m^3/decade/ha) by cover group and age class for medium TPR sites.

	Aspen		Mixed		White Spruce		Pine		Black Spruce	
Age	Area	SP	Area	SP	Area	SP	Area	SP	Area	SP
1	4 353	18.4	120	19.7	78	18.6	42	20.7	426	9.4
2	1 440	20.4	588	21.6	1 305	20.3	318	22.2	336	10.4
3	1 242	21.9	945	24.3	27	21.7	270	23.5	6 171	11.4
4	10 017	23.5	915	26.6	105	23.1	804	25.1	711	12.5
5	1 542	25.2	654	28.6	6	24.7	681	26.6	6 720	12.5
6			3 264	29.3	48	26.2	1 197	28.2	11 343	12.5
7			1 485	29.6	33	27.6	1 650	29.6	12 705	12.5
8			1 647	30.5	78	29.1	384	31.0	6 909	13.7
9			870	31.6	129	30.3	105	32.2	2 655	14.9
10			1 494	31.4	258	31.5	369	33.3	5 055	15.9
11			1 434	31.1	126	32.6	282	34.3	3 429	16.8
12	849	26.8	1 434	30.7	327	33.6	351	35.2	3 252	17.7
13	69	23.5	705	30.1	219	34.5	132	36.0	1 203	18.4
14	1 506	0.0	3 288	29.4	1 020	35.2	534	36.8	1 296	18.6
15	3	0.0	246	29.5	195	36.0	138	37.5	189	19.0
16	3	0.0	1 188	29.5	573	36.0	231	37.5	165	19.0
17			111	29.5	30	36.0	9	37.5		
18			81	29.5	93	36.0	9	37.5	6	19.0
19					6	36.0	3	37.5	24	19.0
20										
21					27	36.0				

Table 3: Starting inventory area (ha) and shadow price (m^3/decade/ha) by cover group and age class for good TPR sites.

	Aspen		Mixed		White Spruce		Pine		Black Spruce	
Age	Area	SP	Area	SP	Area	SP	Area	SP	Area	SP
1	4 170	30.3	198	34.3	237	29.5	3	35.5	168	20.1
2	2 727	33.2	381	37.5	603	32.2			18	21.9
3	375	35.5	18	40.2			81	41.0	1 674	23.6
4	4 641	37.5	333	42.9	27	37.1	147	43.1	2 142	25.1
5	1 227	41.6	357	45.4	18	39.2	219	45.2	1 332	26.7
6	33	44.8	10 242	46.6	33	41.0	984	47.0	3 747	28.5
7			10 671	47.7	261	42.6	543	48.6	3 462	30.1
8			13 095	50.1	438	44.0	342	50.0	3 207	31.6
9			6 453	51.4	459	45.1	186	51.2	729	33.1
10			6 129	50.7	705	46.2	294	52.3	2 376	34.5
11			6 318	49.9	591	47.1	330	53.2	1 668	35.8
12	984	42.6	4 242	48.8	1 152	48.0	195	54.1	879	37.1
13	168	37.8	1 203	47.7	333	48.7	96	54.8	282	38.2
14	570	0.0	3 933	46.5	1 266	49.5	222	55.5	249	39.0
15	3	0.0	33	45.3	6	50.1	9	56.2	24	39.8
16			15	44.9			9	56.4	33	39.8
17					21	50.1			12	39.8
18									6	39.8
19									6	39.8

Table 4: Softwood yield (m^3/ha) by cover group, TPR, and age class.

	Mixed			White Spruce			Pine			Black Spruce		
Age	Fair	Medium	Good	Fair	Medium	Good	Fair	Medium	Good	Fair	Medium	Good
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	22	0	0	56	0	0	0
5	0	0	29	0	2	91	0	33	131	0	0	6
6	0	0	89	0	50	156	0	83	199	0	0	56
7	0	34	140	0	96	212	17	129	257	0	0	105
8	0	74	184	13	140	262	46	170	308	0	0	150
9	12	111	221	43	179	305	73	207	350	0	25	192
10	41	143	253	71	215	341	98	239	385	0	51	231
11	68	172	279	98	247	371	121	268	415	0	77	267
12	93	197	302	124	276	397	142	293	441	0	102	299
13	116	219	321	149	301	420	162	316	464	0	127	328
14	137	239	338	172	324	438	180	336	484	10	150	354
15	156	256	353	194	343	455	195	354	501	25	173	379
16	174	272	366	213	362	470	211	370	516	40	195	401
17	191	286	377	233	378	483	223	385	530	55	216	422
18	206	299	386	250	393	494	236	398	542	69	236	440

Table 5: Hardwood yield (m^3/ha) by cover group, TPR, and age class.

	Aspen			Mixed		
Age	Fair	Medium	Good	Fair	Medium	Good
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	10	0	0	0
4	0	8	98	0	0	20
5	0	68	164	0	9	60
6	10	115	215	0	37	95
7	44	154	255	13	63	121
8	72	184	284	32	87	141
9	96	210	310	51	108	154
10	116	231	331	68	124	163
11	93	186	267	59	106	140
12	71	142	204	49	89	116
13	49	98	140	39	71	93
14	27	53	76	29	53	70
15	4	9	13	20	35	47
16	0	0	0	10	18	23
17	0	0	0	0	0	0
18	0	0	0	0	0	0

Table 6: Annual fire arrival probabilities p_a for 3 ha model cells, by land-cover class.

Class	p_a
Deciduous	0
White spruce	1.46×10^{-4}
Black spruce	6.54×10^{-5}
Pine	3.63×10^{-5}
Other	3.63×10^{-5}
Water	0

Table 7: Area burned (ha) in most severe fire year on fair TPR timber classes.

Age	Aspen	Mixed	White Spruce	Pine	Black Spruce
1	3			3	105
3	3			300	5 589
4	357	6		204	993
5	72	30		12	1 371
6		99		3	2 886
7		36		15	1 107
8		21			672
9					801
10		75			594
11		12	9	39	351
12		3			444
13				6	48
14	63	60	24	54	333
15					9
16	15	3	3	21	
17				9	33
18	12				
19					3
Total	525	345	36	666	15 339

Table 8: Area burned (ha) in most severe fire year on medium TPR timber classes.

Age	Aspen	Mixed	White Spruce	Pine	Black Spruce
1	2 052	63	57	15	159
2	537	408	1 119	18	120
3	765	189		165	3 111
4	6 066	510	93	762	492
5	339	282	6	189	2 742
6		1 290	36	231	4 611
7		609	12	165	3 996
8		447	21	141	3 348
9		258	105	51	732
10		621	168	210	2 055
11		369	36	129	1 308
12	498	792	192	249	990
13	18	192	99	33	231
14	1 128	2 394	792	378	564
15		186	126		63
16		1 125	525	174	66
17		66	21	3	
18		81	93	9	
19			6	3	3
21			27		
Total	11 403	9 882	3 534	2 925	24 591

Table 9: Area burned (ha) in most severe fire year on good TPR timber classes.

Age	Aspen	Mixed	White Spruce	Pine	Black Spruce
1	696	93	114		78
2	1 269	375	546		
3	111	9		72	381
4	1 662	207	21	108	837
5	144	48	15	162	696
6	24	4 155	3	567	1 992
7		1 368	42	147	1 560
8		4 050	147	57	1 101
9		1 932	192	6	231
10		2 451	447	87	1 227
11		3 216	321	87	522
12	342	1 110	456	96	438
13	3	207	54		33
14	477	2 265	705	162	147
15					12
16		3			24
17			21		
19					6
Total	4 728	21 489	3 084	1 551	9 285

Table 10: Comparison of true costs and the SSP approximation for the remove and recalculate series. The number in the run name indicates the percentile of the run.

Run	Period 1 Harvest (m^3)	Proportion of Base Run	Difference from base (m^3)	SSP (m^3)	Ratio
base	7 174 530				
q000	7 174 520	1.00	10	10	1.00
q010	7 174 430	1.00	100	101	1.01
q020	7 174 350	1.00	180	177	0.99
q030	7 174 250	1.00	280	277	0.99
q040	7 174 120	1.00	410	410	1.00
q050	7 173 880	1.00	650	647	1.00
q060	7 173 460	1.00	1 070	1 068	1.00
q070	7 172 400	1.00	2 130	2 123	1.00
q080	7 168 550	1.00	5 980	5 974	1.00
q090	7 143 460	1.00	31 070	31 069	1.00
q091	7 136 520	0.99	38 010	37 877	1.00
q092	7 128 230	0.99	46 300	46 285	1.00
q093	7 115 430	0.99	59 100	58 514	0.99
q094	7 095 350	0.99	79 180	79 101	1.00
q095	7 075 870	0.99	98 660	98 622	1.00
q096	7 037 560	0.98	136 970	135 454	0.99
q097	6 956 460	0.97	218 070	217 178	1.00
q098	6 820 880	0.95	353 650	349 798	0.99
q099	6 455 260	0.90	719 270	692 493	0.96
q100	4 323 760	0.60	2 850 770	2 807 716	0.98

Table 11: Comparison of true costs and the SSP approximation for alternative objective functions and constraints.

Run	Description	Objective Function Value				
		Pre-fire	Post-fire	True Cost	SSP	Ratio
A	Maximize period 1 harvest volume (m^3), even flow of total volume					
		7.240×10^6	4.366×10^6	2.874×10^6	2.832×10^6	0.99
B	Maximize planning horizon harvest volume (m^3), non-declining yield of total volume					
		1.540×10^8	9.256×10^7	6.141×10^7	6.046×10^7	0.98
C	Maximize planning horizon harvest volume (m^3), no flow constraints					
		1.602×10^8	9.650×10^7	6.372×10^7	6.270×10^7	0.98
D	Maximize NPV ($), non-declining yield of total volume.					
		6.409×10^8	3.883×10^8	2.526×10^8	2.525×10^8	1.00
E	Maximize period 1 harvest volume(m^3), even flow of softwood and hardwood volumes, most valuable stands burn					
		7.175×10^6	1.981×10^6	5.194×10^6	4.786×10^6	0.92

Table 12: Comparison of true costs and the SSP approximation for the regenerate and recalculate series. The number in the run name indicates the percentile of the run.

Run	Period 1 Harvest (m^3)	Proportion of Base Run	Difference from base (m^3)	SSP (m^3)	Ratio
base	7,174,530				
r090	7,164,740	1.00	9,790	9,734	0.99
r091	7,163,220	1.00	11,310	11,269	1.00
r092	7,159,140	1.00	15,390	15,277	0.99
r093	7,158,810	1.00	15,720	15,620	0.99
r094	7,150,490	1.00	24,040	23,781	0.99
r095	7,145,940	1.00	28,590	28,050	0.98
r096	7,134,400	0.99	40,130	39,083	0.97
r097	7,110,120	0.99	64,410	60,210	0.93
r098	7,094,260	0.99	80,270	76,019	0.95
r099	6,908,840	0.96	265,690	199,198	0.75
r100	5,895,600	0.82	1,278,930	781,303	0.61

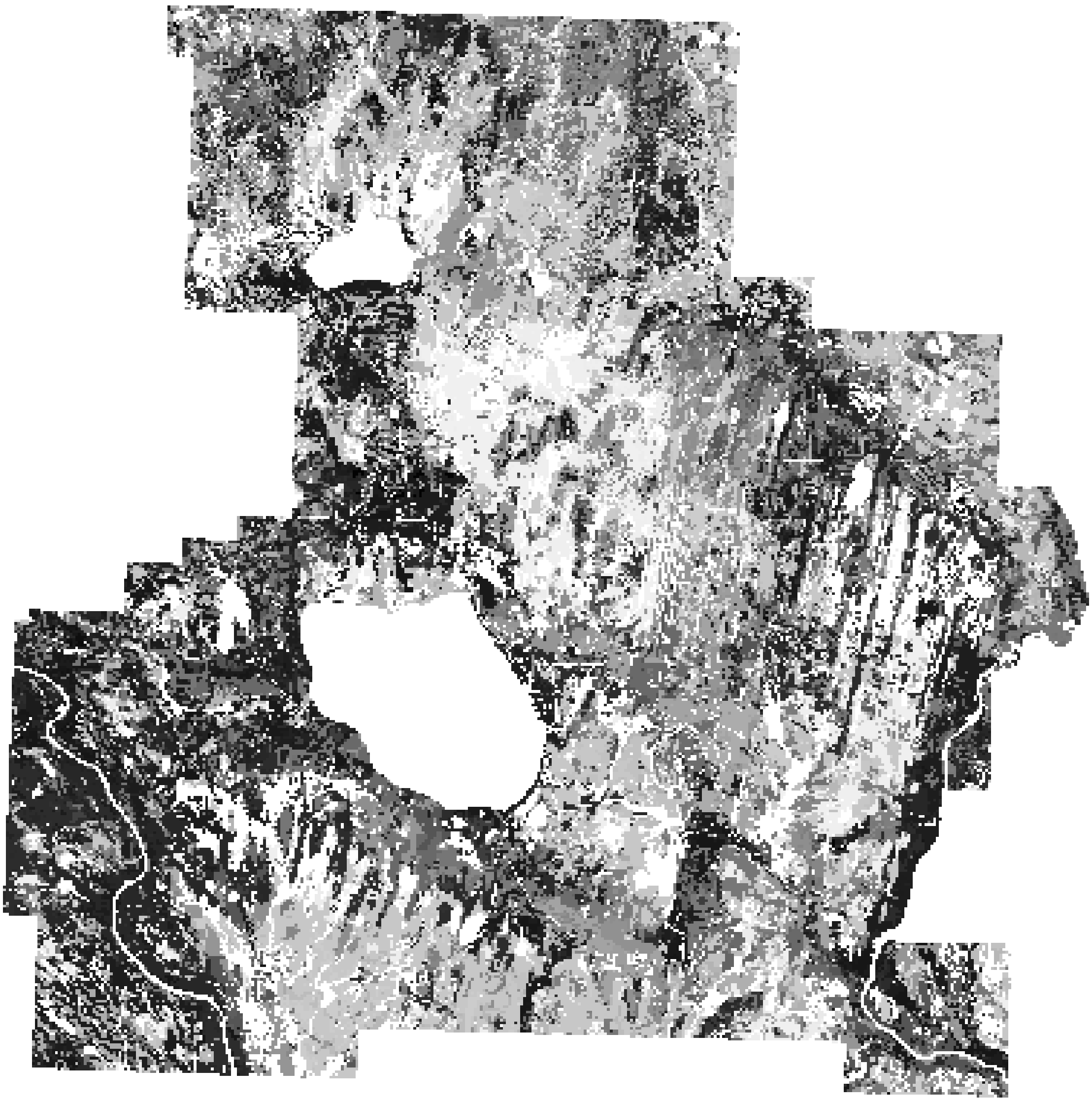

Figure 1: Map of FMU L2 showing shadow prices: white is 0 m^3/decade/ha, black is 56.4 m^3/decade/ha.

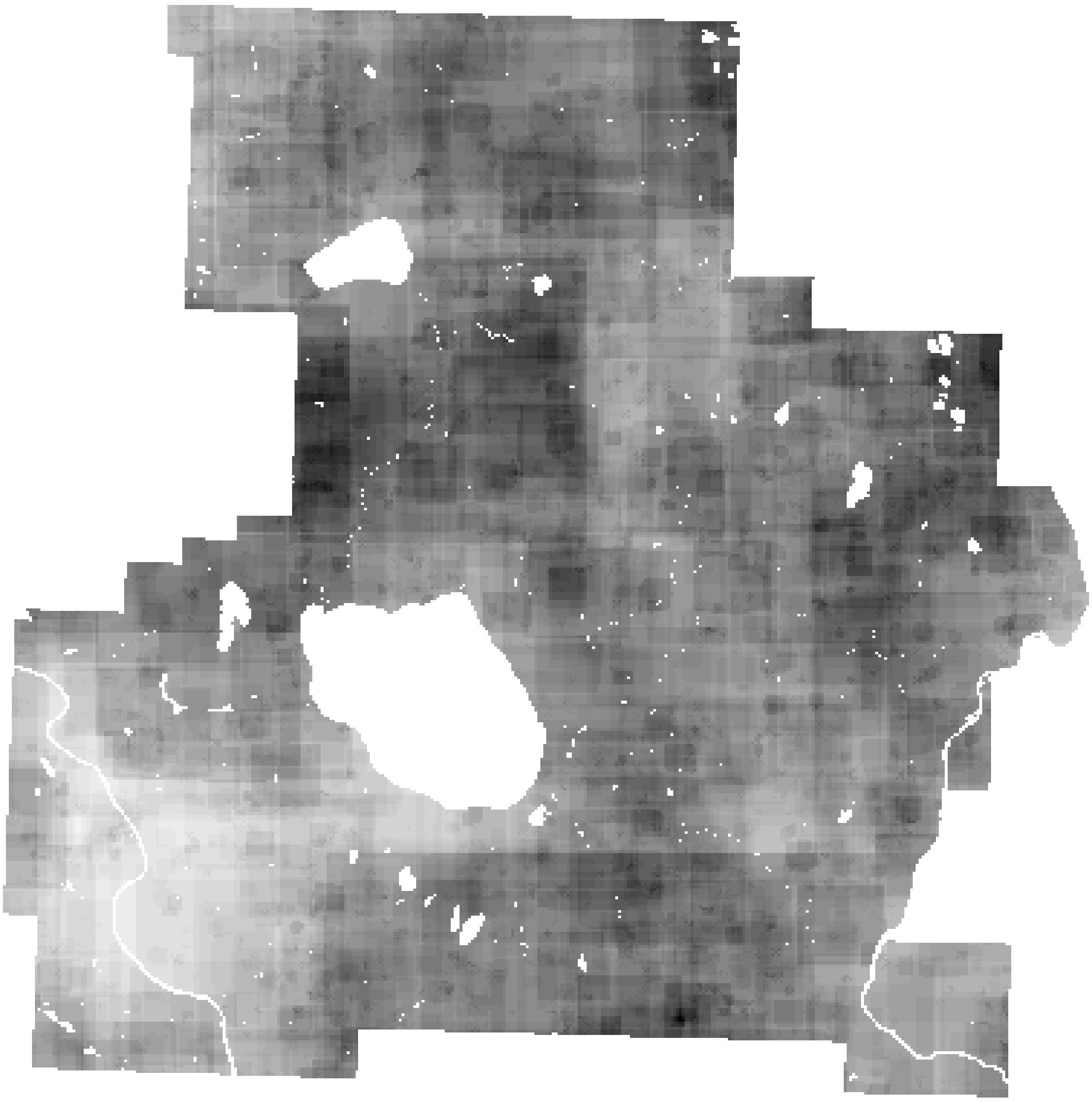

Figure 2: Map of FMU L2 showing probability of fire: white is 0, black is 0.004.

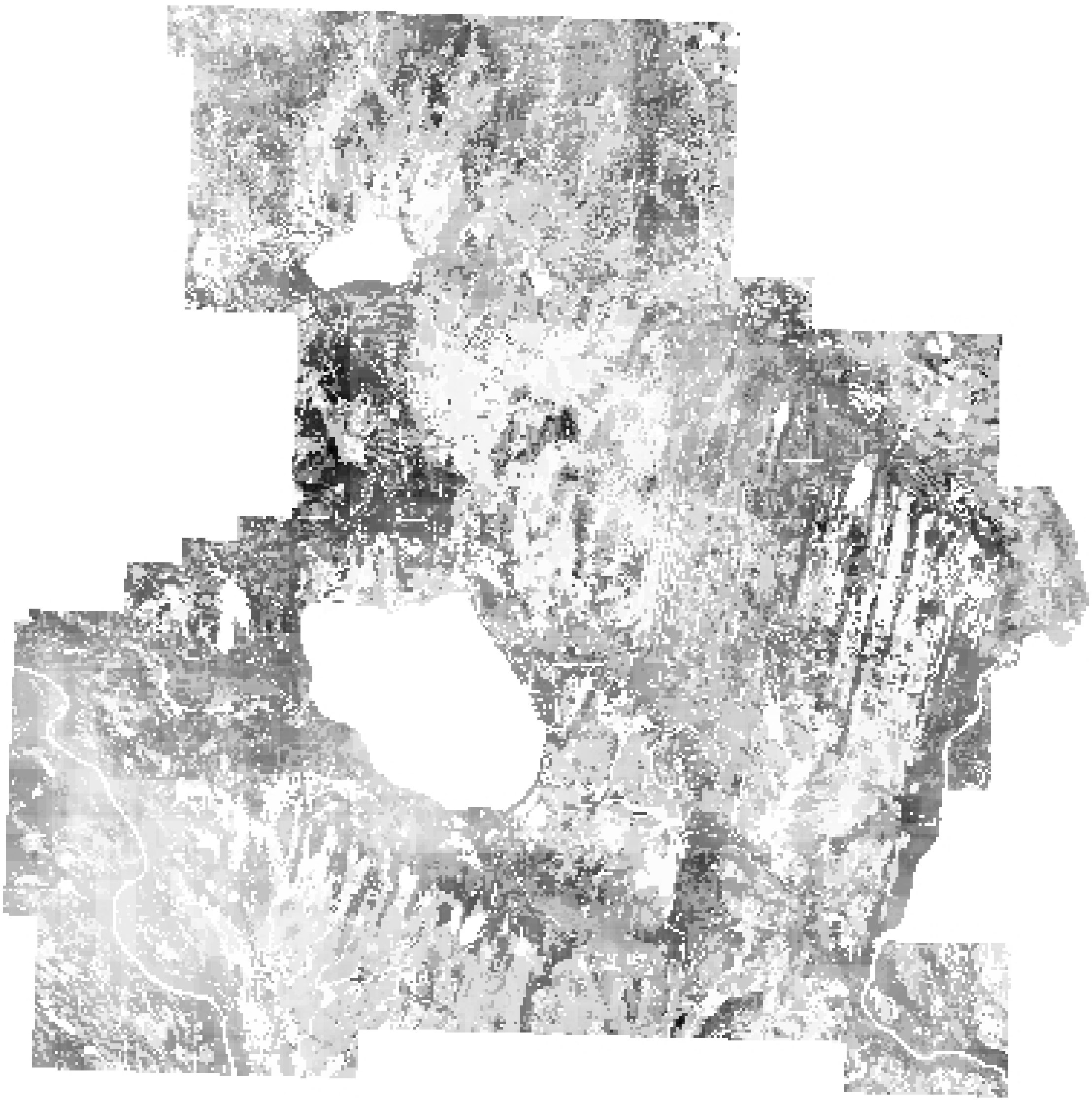

Figure 3: Map of FMU L2 showing probability weighted shadow prices: white is 0 m^3/decade/ha, black is 0.183 m^3/decade/ha.

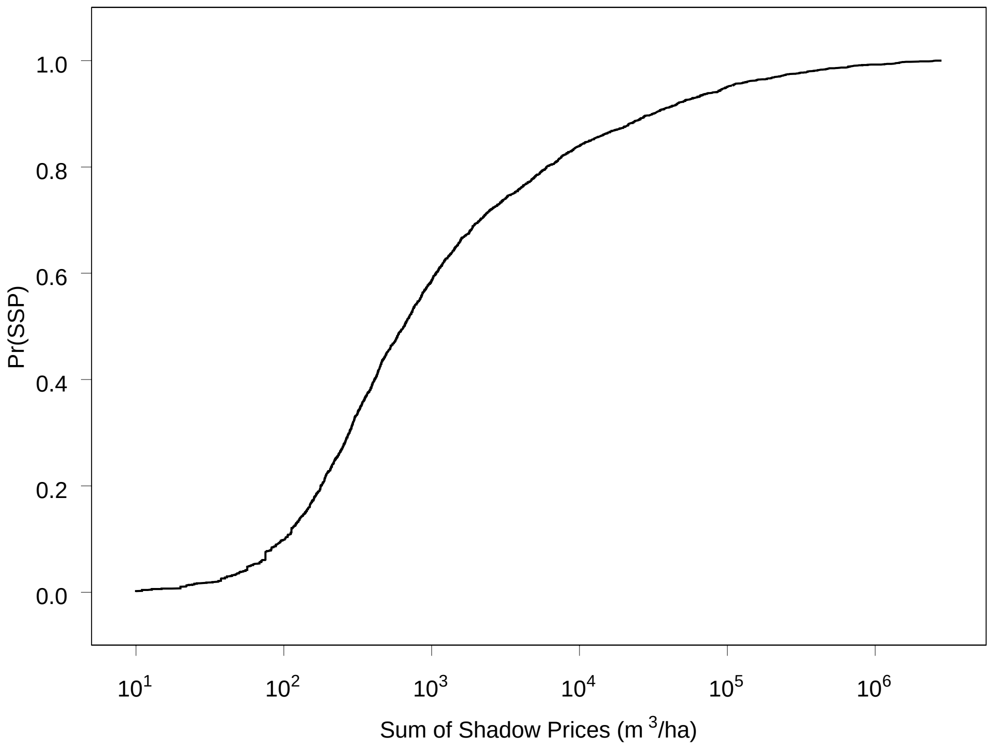

Figure 4: Empirical probability density function for sum of shadow prices for simulated years with a non-zero SSP.