The Role of Spatial Visualization in Primary Mathematics and Science: A Content Analysis of Alberta's Curriculum

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Alberta's mathematics and science curricula are in transition. In the 2023/24 school year, K–6 teachers were expected to implement the new mathematics curriculum, and K–3 teachers also undertook the implementation of the new science curriculum.

This article reports on a content analysis of the outgoing and incoming curriculum documents for both mathematics and science. Our aim is to identify and analyze changes in the curricula that may have an impact on how teachers interpret and implement them in the classroom.

To narrow our discussion, we focus on spatial visualization, which is fundamental to teaching and learning in both mathematics and science.

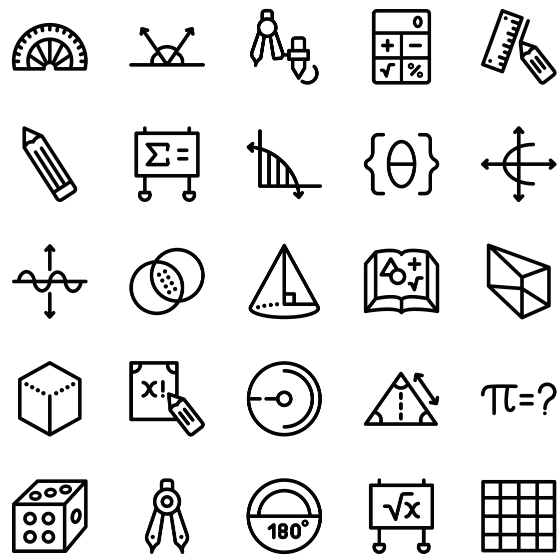
In what follows, we provide

- an interdisciplinary comparison of how visualization is taken up differently in the mathematics and science curricula,
- an intradisciplinary analysis of how the role of visualization has changed in the incoming curriculum for each subject and
- windows of opportunity for primary (Grades 1–3) teachers to attend to learning outcomes through visualization in their mathematics and science classrooms.

Context

This study was occasioned by our recent work with elementary teachers to support them in using visualization in their mathematics and science lessons.

Throughout our professional learning sessions with the teachers, we noticed that all four curricula for Grades 1–3 mathematics and science—the two outgoing curricula and the two incoming curricula—present both opportunities and challenges for incorporating visualization, but each discipline has unique opportunities and challenges.



In mathematics, the outgoing curriculum (Alberta Education 2007) includes visualization (which involves using mental imagery and external representations, such as graphs) as one of seven mathematical processes. Moreover, this mathematical process is explicitly linked to learning outcomes. However, specific guidance for teachers is not provided. Rather, linking the process of visualization to learning outcomes seems to suggest only the opportunity to employ it.

In contrast, the incoming mathematics curriculum (Alberta Education 2022) does not include mathematical processes but explicitly incorporates visualization into some learning outcomes.

The incoming (Alberta Education 2023) and outgoing (Alberta Education 1996) science curricula are more consistent, but both focus exclusively on working with external visualizations (such as drawings, graphs and models).

There are opportunities to spatialize these curricula, as called for by Davis, Okamoto and Whiteley (2015).

First, though, we felt that we needed a clearer picture of how visualization manifests in all four curriculum documents.

Visualization in Mathematics and Science

Before we proceed with our content analysis, it is worth discussing the importance of visualization itself and providing a rationale for using visualization to guide our analysis.

In mathematics education, visualization is fundamental to spatial reasoning, which occurs when students engage in "locating, orienting, decomposing/recomposing, shifting dimensions, balancing, diagramming, symmetrizing, navigating, transforming, comparing, scaling, feeling, and visualizing" mathematical objects in the classroom (Davis, Okamoto and Whiteley 2015, 140). Spatial reasoning also involves the capability to "recall, generate, manipulate, and reason about spatial relations" (Gilligan-Lee, Hawes and Mix 2022, 1).

Research in cognitive psychology and mathematics education points to the deep connection between spatial reasoning and mathematical understanding.

Spatial reasoning is associated with students' current levels of mathematical achievement (Atit et al 2022), their future mathematical achievement (Verdine et al 2017) and their achievement in other domains, such as reading comprehension (Hanline, Milton and Phelps 2010).

Moreover, spatial reasoning is susceptible to intervention, and spatial training in the classroom is associated with gains in overall achievement in mathematics (Uttal et al 2013).

In an extensive meta-analysis of the literature, Gilligan-Lee, Hawes and Mix (2022) further established evidence that this association may be causal: effective spatial training has been observed to translate into improved outcomes in mathematics.

Visualization, which we focus on here, is fundamental to spatial reasoning and, more generally, mathematical thinking.

Visualization is often defined as the capacity for mentally transforming shapes and objects, but that does not limit its usefulness to spatial contexts. Consider the role of the number line, an inherently spatial object, in teaching and learning number sense.

Moreover, visualization is often evoked through the metaphor of the mind's eye, but we have reason to appreciate the role of the entire body in spatial reasoning, generally, and in visualization, specifically. Markle (2021), for example, describes visualization as a sensorimotor phenomenon that involves bodily movement, through both physical actions (such as gesture) and imagined movement. In short, visualization draws on all our senses, not just the visual, and this presents a host of instructional and assessment opportunities for the classroom teacher.

In science education, visualization has been emphasized as an important skill for reasoning and communication. Just as visualization plays a critical role in meaning-making, explanation and communication in scientific communities, visualization in the science classroom is critical to enhancing students' reasoning and learning (Gilbert 2005). Teachers and students use visualization in diverse ways, such as shifting from physical material to abstract models and from pictorial to symbolic representations (Olson 2013).

Although visualization is critical to scientific meaning-making and communication, it has not gained much attention in the science classroom with respect to the use of mental imagery.

Traditionally, visualization in science education has been limited to interpreting or creating tables, graphs or diagrams or drawing real objects or phenomena. In this approach, students' visualization or visual representations are examined as the products or outcomes of their knowledge.

Visualization has also been recognized as a process of reasoning that explores the role of visual and spatial modalities (Gilbert 2005). Research has shown that when students engage in constructing visual representations together, they question, speculate, refine and develop scientific knowledge collectively (Tytler et al 2020; Yoon, Kim and Lee 2021). For example, while drawing the movement of air molecules in a heated container, students make meaning of the unseen and abstract phenomenon. Knowledge is discussed and negotiated through drawing collectively.

Despite the current momentum behind the use of visualization in science education, visualization is often perceived as a subordinate tool in the process of developing scientific language and knowledge.

Defining *Visualization* in an Interdisciplinary Context

Visualization is fundamental to both science and mathematics, but as we have noted, it is taken up differently in each discipline.

In mathematics education, visualization is an integral aspect of spatial reasoning and is typically conceived of as the capacity to mentally transform shapes and objects (Davis, Okamoto and Whiteley 2015).

In science education, visualization is typically associated with external representations (such as graphs, diagrams, drawings and other models) that help students explain complex or abstract scientific concepts and process their thinking as functional elements of the collective reasoning system (Tytler et al 2020).

We think of visualization as encompassing not only mental imagery and working with external representations but also the interactions between our minds, bodies and the material environment. To conduct a content analysis of Alberta's mathematics and science curriculum documents, we needed to operationalize a definition of *visualization* that reflected these commitments. Our definition is as follows:

> Visualization entails (1) the process of developing, interpreting and using mental images and (2) the process and products of developing, constructing and using spatial inscriptions.

In general, our definition of *visualization* aligns with Arcavi's (2003) influential definition. The first part of the definition preserves the importance of mental imagery. The second part attends to what might be best captured by the term *external visualizations,* such as graphs and diagrams. We use the term *spatial inscriptions* instead of *images* or *representations* because, following others (Presmeg 1986; Roth and McGinn 1998), we view spatial inscriptions (such as diagrams) as emerging out of interactions between individuals, collectives and the material world.

Analytical Process

Using our definition of *visualization,* we conducted a qualitative content analysis of Grades 1–3 learning outcomes in the outgoing mathematics (Alberta Education 2007) and science (Alberta Education 1996) curriculum documents and the incoming mathematics (Alberta Education 2022) and science (Alberta Education 2023) curriculum documents.

Content analysis involves interpreting textual data "through the systematic classification process of coding and identifying themes or patterns" (Hsieh and Shannon 2005, 1278).

We separated our definition of *visualization* into two parts: mental imagery and spatial inscriptions. We then categorized each learning outcome as one, both or neither. Only learning outcomes that explicitly reflected an element of our definition received a code. Many learning outcomes would naturally lend themselves to visualization in the classroom, and we will share some of those later, but we focused exclusively on what was present in the learning outcome. To ensure consistency in our coding, we engaged in several rounds of coding subsets of learning outcomes and continued to do so until the coders reached unanimous consensus.

As noted, our study was in part occasioned by a major shift in the curricula that Alberta's teachers and students engage with in the classroom. We highlight here some important differences between the outgoing curricula and the incoming curricula that had an impact on our analysis.

Of particular relevance, the outgoing curricula are organized at the most granular level by specific outcomes (SOs) in mathematics and specific learner expectations (SLEs) in science. This level of granularity does not exist in the incoming curricula. What are called learning outcomes (LOs) in the incoming curricula are often equivalent to what the outgoing curricula call general outcomes (GOs) in mathematics and general learner expectations (GLEs) in science. The most granular level of organization in the incoming curricula consists of the knowledge, understandings, and skills and procedures (KUSPs) for each learning outcome.

We sought to maintain consistency with regard to the curricular components, despite these differing structures. To this end, we applied codes to the SOs/SLEs in the outgoing curricula and the KUSPs in the incoming curricula. When we discuss both together in this article, we use the lowercase *outcomes.*

Our choice to code these outcomes requires three important caveats.

First, we are aware that the SOs/SLEs and the KUSPs do not align perfectly in terms of granularity.

Nevertheless, we argue that they provide comparable reflections of the extent to which visualization is present in the curricula as a tool, process or object of learning.

Second, while we acknowledge the importance of the mathematical processes and their critical role in the outgoing mathematics curriculum, no comparable structure exists in the incoming curriculum. Therefore, we focused only on whether our definition of *visualization* was explicit in the outcome, regardless of whether the process of visualization was explicitly linked to that outcome.[1]

Third, and perhaps most important, we acknowledge that a curriculum is more than a collection of outcomes. For example, most curriculum documents include front matter that provides insight into the intentions that inform the curriculum. This front matter is critical to how teachers attend to curricular objectives. One reason we restricted ourselves to analyzing outcomes is that substantive front matter is absent in the incoming curricula. We also wanted to maintain our focus on the role of visualization in how teachers teach and assess specific topics and outcomes.

Results and Discussion

Table 1 provides an overview of the results of our content analysis, in terms of visualization, of the four curriculum documents.

Table 1. Number of Outcomes Explicitly Reflecting Visualization in Alberta's Mathematics and Science Curricula

	Mathematics		Science	
	Outgoing (n = 71)	Incoming (n = 448)	Outgoing (n = 180)	Incoming (n = 419)
Mental imagery	0	2	0	0
Spatial inscriptions	14	45	8	14
Both	0	1	0	0
Neither	57	400	172	405

Interdisciplinary Comparison

Our content analysis found that visualization is prominent in neither the mathematics curricula nor the science curricula.

Mental imagery is virtually absent from all the curriculum documents. Only the incoming mathematics curriculum contains explicit references to mental imagery and even there in only 3 of the 448 KUSPs.

It is important to note that the outgoing mathematics curriculum attends to mental imagery as a process, but even had we included those curricular links, the results of our analysis would be unchanged due to our requirement that the use of mental imagery be explicit in order to be categorized as visualization.

In science, as noted, visualization is associated with external representations, not mental imagery. That said, we were surprised by the low number of outcomes in the science curricula categorized as spatial inscriptions.

Intradisciplinary Comparison: Mathematics

As noted, the structure and content of the outgoing and incoming mathematics curricula are significantly different, but in terms of explicit reference to visualization at the outcome level, the curricula are similar.

A potentially positive aspect of the incoming curriculum is that it explicitly incorporates language addressing visualization into the KUSPs for geometry, measurement, number and statistics—four of the incoming curriculum's organizing ideas (which are comparable to strands in the outgoing curriculum). Unfortunately, some of that language is ambiguous, as we discuss below.

Statistics

The incoming mathematics curriculum uses the verb *visualize* in at least three ways.

One way is in the context of statistics—"the science of collecting, analyzing, visualizing, and interpreting data" (Alberta Education 2022, 12). Data visualization does indeed draw on a variety of the skills and capabilities that fall under the umbrella of spatial reasoning, in particular spatial visualization (for example, imagining various ways to represent data). However, in the context of statistics, data visualization is about organizing and interpreting numerical information in pictorial form (for example, displaying data using a graph).

Number

Another way the incoming mathematics curriculum invokes visualization is through number, which deals with quantities "measured with numbers that enable counting, labelling, comparing, and operating" (Alberta Education 2022, 1).

As noted, spatial visualization is an important aspect of spatial thinking, in particular, and mathematical understanding, more generally. However, the verb *visualize* is again used in this context in an ambiguous and potentially limiting way.

Consider the four instances of the verb *visualize* under the organizing idea of number (Table 2). In each case, *visualize* seems to be used synonymously with *pictorial depiction*, which is connected to spatial visualization but is not constitutive of it. In fact, all four outcomes in Table 2 are explicitly linked to spatial inscriptions and were coded as such.

Table 2. Visualization in the Context of Number in Alberta's Incoming Mathematics Curriculum

Grade	Skills, procedures and understandings
1	Visualize quantities between 10 and 20 as compositions of 10 and another quantity.
2	Visualize 100 as a composition of multiples of 10 in various ways.
3	Visualize and model products and quotients as arrays.
3	Visualize fractions as compositions of a unit fraction.

Although there is great potential for incorporating spatial visualization exercises in support of these outcomes (and others under the organizing idea of number), the results of our coding suggest that using spatial language more precisely would help make specific spatial actions explicit in the curriculum and aid teachers in implementing spatial practices in the classroom.

Geometry and Measurement

The incoming mathematics curriculum also references visualization in geometry and measurement. Since visualization is fundamentally involved in thinking about shapes and objects in space (for example, through mental rotation of shapes and objects, perspective-taking, navigating and so on), this is to be expected.

Table 3 shows the relevant outcomes.

Table 3. Visualization in the Context of Spatial Reasoning in Alberta's Incoming Mathematics Curriculum

Grade	Organizing idea	Skills, procedures and understandings
2	Geometry	A shape can be visualized as a composition of other shapes.
2	Geometry	Create a picture or design with shapes from verbal instructions, visualization, or memory.
2	Measurement	Estimate length by visualizing the iteration of a referent for a centimetre.
3	Measurement	Estimate length by visualizing the iteration of a referent for a centimetre or metre.

It is worth taking up these outcomes in detail. Although they all refer to visualization, we coded them differently: the first outcome received no code, and the other three were identified as attending explicitly to mental imagery.

Table 3 also makes it evident that visualization as it pertains to spatial reasoning in the curriculum is restricted to estimating length. These outcomes provide the clearest and most coherent link to spatial visualization as a mathematical process: students must imagine a referent for a centimetre (say, a finger's width), and then imagine the number of referents associated with a given length. Visualizing length and area in this way is a valuable practical skill and a powerful spatial reasoning exercise that can enhance students' capabilities in other mathematical domains.

The other outcomes for geometry in Grade 2 are not as clear in the context of spatial visualization as a mathematical process.

In the first outcome in Table 3, for example, *visualized* could be interpreted as being synonymous with *pictorial representation* or *concrete representation.* For example, a student could use pattern blocks to show a hexagon as the composition of two trapezoids or six triangles. Of course, this activity involves the mathematical process of spatial visualization, but it does so only implicitly. To capitalize on the importance of spatial visualization in mathematical understanding, a task must be designed to explicitly elicit spatial visualization (for example, "I'd like you to imagine what shapes you might put together to make a hexagon") and open up opportunities for assessment (for example, "Will you tell me what you imagined?").

The second outcome in Table 3 introduces yet another ambiguity through its distinction between *visualization* and *memory.* On the one hand, it is likely that *visualization* is intended to be synonymous with *pictorial representation* or *concrete representation.* On the other, it raises questions about the role of memory in spatial visualization as a mathematical process.

Intradisciplinary Comparison: Science

Given how often teachers and students use spatial inscriptions in the science classroom, visualization is surprisingly not explicit in the outgoing and incoming science curricula. Only 8 outcomes (out of 180) in the outgoing curriculum and 14 outcomes (out of 419) in the incoming curriculum explicitly reflect spatial inscriptions. Considering the total number of outcomes in each curriculum, visualization is even less present in the incoming curriculum.

In both curricula, visualization is explicit in outcomes related to modelling, scientific process and scientific communication.

Modelling

Visualization for modelling includes illustrating, diagramming or modelling Earth systems or living systems (such as life cycles, water cycles or structures). The science concepts include the large scale of time and space and the complexity of interactions and changes. These concepts are challenging for young students to understand. Visualizing the interconnected time- and space-related concepts is effective for conceptual understanding.

Table 4 shows the relevant outcomes.

It is clear that there is more emphasis on modelling through visualization in the incoming science curriculum than in the outgoing curriculum.

TABLE 4. Examples of Modelling Systems in Alberta's Science Curricula

Outgoing curriculum	Incoming curriculum
Using a variety of materials and techniques, design, construct and test structures that are intended to: . . . • serve as models of particular living things, objects or buildings. (Grade 3)	Life cycles can be represented in many ways, such as • illustrations • diagrams • models • stories. (Grade 2) Describe and diagram the changes of state of water using the water cycle. (Grade 3)

Scientific Process

Scientific process skills are critical for scientific investigations and problem solving.

The subject introduction to the incoming science curriculum states that students are expected to "deepen knowledge and understanding through collaborative conversation, recording and analyzing data, and interpreting scientific texts, including diagrams, models, or digital simulations" (Alberta Education 2023). Therefore, visualizing data collection and analysis is explicitly emphasized as part of the scientific process.

Table 5 shows examples of scientific process in the science curricula.

TABLE 5. Examples of Scientific Process in Alberta's Science Curricula

Outgoing curriculum	Incoming curriculum
Record observations and measurements, using captioned pictures and charts, with guidance in the construction of charts. Computer resources may be used for record keeping and for display and interpretation of data. (Grade 3)	Observations can be recorded as data in many ways, such as • words • drawings • photographs • numbers and counts • sound and video recordings. (Grade 1)

Scientific process through visualization is explicit in both science curricula, but there is more emphasis on it in the outgoing curriculum. In the outgoing curriculum, four out of eight outcomes focus on recording observation through visual inscriptions, whereas one out of eight outcomes focuses on that skill in the incoming curriculum.

Scientific Communication

Scientific communication is multimodal, with diverse forms of communication (such as language, visual inscriptions and gestures). The outgoing and incoming science curricula both refer to visualization for communication, though the emphasis is minimal.

Table 6 shows examples of scientific communication in Alberta's science curricula.

TABLE 6. Examples of Scientific Communication in Alberta's Science Curricula

Outgoing curriculum	Incoming curriculum
Communicate results of construction activities, using oral language, captioned pictures and simple graphs (pictographs and bar graphs). (Grade 2) Communicate results of construction activities, using written and oral language and pictures. (Grade 3)	Work individually or in groups to create instructions using precise words, pictures, or diagrams. (Grade 2)

The emphasis on visualization as communication is more explicit in the outgoing science curriculum than in the incoming curriculum. The only outcome with a communication emphasis in the incoming curriculum is in Grade 2, under the organizing idea of computer science: "Students apply creativity when designing instructions to achieve a desired outcome" (Alberta Education 2023, 12).

Windows of Opportunity: Implications and Strategies for the Classroom

As discussed, our content analysis revealed gaps and ambiguities with respect to visualization in Alberta's primary mathematics and science curriculum documents.

One significant gap involves mental imagery. While visualization overall tends to be underrepresented in Alberta's curricula, the explicit use of mental imagery in outcomes is very limited compared with references to spatial inscriptions (such as diagrams and graphs).

There are also ambiguities that could lead teachers to overemphasize or neglect important aspects of visualization (for example, the use of the word *visualize* to refer to creating pictorial representations).

Despite these gaps and ambiguities, we believe there is great potential for spatializing both curriculum and pedagogy. The following are practices to consider adopting in the classroom.

Use Spatial Language Frequently, Purposefully and Precisely

Regardless of age or level of formal mathematical training, we all share the experience of movement—we are all spatial beings. But we often take this shared experience for granted. One way we can acknowledge it is to make it explicit through the effective use of spatial language.

In their wonderful resource for thinking spatially in early years mathematics, Moss et al (2016) note that spatial language includes precise positional language (*above, below*); dimensional language (*wide, long*); nominal language (*square, triangle*); and transformative language (*slide, rotate, reflect*). This is essential for learning early ideas in geometry, but we argue that visualization can be used to explore concepts across mathematics and science.

For example, we used the following visualization exercise with teachers in our study:

> Try this: Imagine you are in an empty room. You notice a ladder leaning against one of the walls. Imagine climbing the ladder to the middle rung. While you stand on the ladder, it begins to slide down the wall toward the floor. Now, can you imagine the path your body takes through the air as the ladder slides to the floor?

The absence of mathematical or scientific terminology in this visualization prompt is by design. For everyone in the classroom to be able to imagine the scenario, it must use concrete, active and plain language grounded in students' experiences. When students begin to share their visualizations (through class discussion or sketching, for example), teachers can begin to use the more precise spatial language discussed above.

Make Short Visualization Exercises a Routine in the Classroom

Many teachers already use mental mathematics routines (such as number talks). Short visualization routines can be included just as easily.

For example, while the sliding ladder exercise could be the focus of a secondary mathematics or physics lesson, teachers could also use it in the early years classroom as an opening routine in a lesson about transformations. In fact, the exercise need not be linked to mathematical or scientific content at all.

Exercises like this capitalize on the rich experiences and intuitions students bring with them into the classroom. This means that the process of visualizing can be just as important as the content to be visualized.

One way teachers can simply and effectively incorporate visualization exercises as regular routines in the classroom is to make small tweaks to the exercises. Consider the following prompt:

> Imagine a cube. Slice the cube in half vertically at a right angle. Pull one half of the cube away and look at its cross-section. What shape is the cross-section?

Most of us possess the requisite knowledge such that we do not need to imagine looking at half of a cube. We know that when we slice a cube in half as described, the cross-section will be a square. We might think we are so familiar with cubes that no visualization could yield any surprises. But consider this variation, which we presented to teachers in our study:

> Try this: Imagine a cube. Now imagine setting the cube on the desk in front of you such that it sits on a vertex and the line formed between that vertex and its diagonal opposite is at a right angle to your desk. Using a plane parallel to the desk and starting at the very top of the cube, start making thin slices. Each time you slice a piece of the cube off, look at the shape of the resulting cross-section. What do you see?

As we found, a small change in the orientation of the cube yields some surprising results.

Create a Safe Shared Space for Students to Discuss What They Imagine

We have led visualization exercises with young children, high-achieving secondary students, undergraduate students and experienced teachers. We have found that everyone visualizes differently, which is an important implication for classroom practice. Sometimes those differences can cause unnecessary anxiety and stress.

A common misconception we encounter is that visualization is about seeing crisp, static images in the mind's eye. Although some people can, in fact, see eidetically, they are the exception, not the norm. Far more often, our mental imagery is imperfect and ephemeral, but it is still useful in developing mathematical and scientific understanding—and it can be improved with use.

To foster good visualization practices, we must give students time and space to discuss what they see and allow them to sketch, make gestures and engage with the material world.

For example, when we used the cube-slicing exercise with teachers, we had them mould cubes out of clay before engaging in discussion about their visualizations. The physical cubes served as a shared point of departure for teachers to talk about what they saw, to consider each other's ideas and to subsequently refine their own.

Closing Remarks

Research suggests that strong spatial skills benefit students' overall mathematics achievement. Visualization is one way to support the development of strong spatial skills in the classroom.

Although we have highlighted several gaps and ambiguities in Alberta's outgoing and incoming curricula for mathematics and science, we see great potential for teachers at all levels to incorporate visualization practices in their classrooms.

Note

1. It is worth emphasizing how prominent the process of visualization is in the outgoing mathematics curriculum: 60 of 71 (85 per cent) outcomes for Grades 1–3 are linked explicitly to this process.

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