Interfacing Techniques for Electromagnetic Field and Circuit Simulation Programs

IEEE Task Force on Interfacing Techniques for Simulation Tools

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***Abstract*—Interfacing of disparate simulation programs is increasingly undertaken to gain a deeper understanding of their functionality and exploit their merits for modeling complex systems. In this paper, techniques for interfacing field and circuit equations of low-frequency electromagnetic apparatus are reviewed, analyzed, and classified into two main categories of direct and indirect methods according to the coupling method. Each category includes a vast array of techniques employing different methods for circuit modeling. The field analysis is restricted to 2-D finite-element method, which is a widely used numerical technique for modeling magnetic behavior of power apparatus. The main features and problems associated with each technique are summarized. Methods for coupling of mechanical equations are addressed in a separate section. A comprehensive list of references is also included at the end of this paper to provide further information to the readers.**

***Index Terms*—Circuit simulation, electromagnetic (EM) fields, finite-element methods (FEMs), interfacing.**

I. Introduction

OVER the years, circuit simulation of electrical systems has evolved to a mature and well-established field of study in electrical engineering. Several techniques, such as the loop current method, nodal analysis, modified nodal analysis (MNA), and state-variable approach [1] are widely used in different circuit simulators, including electromagnetic transients (EMT)-type programs (Alternative Transients Program (ATP), PSCAD/EMTDC, EMTP-RV, etc.), the SPICE family programs, and the MATLAB/SIMULINK. All of these simulators are able to solve circuit equations for lumped and distributed elements in steady-state and transient regimes, handle the nonlinear behavior of circuit elements, and employ different techniques to discretize the temporal derivatives during the transient simulation.

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The field analysis of EM apparatus, with the aid of numerical techniques, has been used as the most accurate and detailed method of predicting the magnetic behavior of such systems under various operating conditions. Several effects, such as space harmonics, slotting, and saturation, which are usually present in different types of electric machines, transformers, and other magnetic apparatus could be readily included in the field equations. The finite-element method (FEM) is usually the method of choice for modeling low-frequency phenomena of power apparatus because of its flexibility and accuracy.

While one of the main source terms in the FEM equations is the winding currents, EM apparatus are usually connected to linear and nonlinear voltage sources [2]. Thus, to have a precise simulation of the system, one needs to solve the FEM equations in conjunction with the circuit equations relating the voltages and currents. The coupling between field and circuit equations becomes more prominent when neither currents nor voltages at the output terminals of the apparatus are known *a priori*. This usually occurs when the apparatus is connected to a power-electronic converter and affects the behavior of the circuit (switching states) to a great extent [3]. Although the 3-D FEM is more accurate in modeling the end region of EM devices, 2-D FEM is still the method of choice when the coupling between the field and circuit equations is desired. This is mainly due to the complexity of 3-D FEM.

This paper provides an overview of different techniques for coupling 2-D FEM equations with external circuit equations of a power apparatus at low frequencies (up to a few kilohertz). The main applications of these techniques include simulation of electrical drives, transformers, actuators, and individual conductors connected to circuit elements, and the interaction phenomena between rotating machines and the network (e.g., SSR and fault studies). A significant amount of work has been accomplished in this area, which usually groups into one of the two broad categories of direct or indirect methods. In direct methods, FEM equations and circuit equations of a system are combined and solved simultaneously. In indirect methods, the FEM-based part is handled as a separate subsystem which communicates with the circuit model through coupling coefficients. In the literature [4]–[6], the terms "eddy current" method and "coupled circuit" method are used interchangeably for "direct" and "indirect" methods respectively, especially for the simulation of electrical drives.

This paper is organized as follows: Section II provides a brief overview of the EM equations and the FEM for low-frequency EM transients. Sections III and IV discuss different direct and indirect methods with their subcategories available for coupled analysis. A comparison between direct and indirect methods is provided in Section V, and the technical challenges encountered in coupled field-circuit simulations are discussed in Section VI. Section VII presents the coupling approach for including mechanical equations, and Section VIII briefly addresses the time discretization techniques for coupled problems. Conclusions appear in Section IX.

II. Finite-Element Model of EM Apparatus

A 2-D quasi-static magnetic model is described by the Maxwell's equation as

$$-\frac{\partial}{\partial x}\left(\nu\frac{\partial A}{\partial x}\right)-\frac{\partial}{\partial y}\left(\nu\frac{\partial A}{\partial y}\right)=J+\nu\frac{\partial B_{my}}{\partial x}-\nu\frac{\partial B_{mx}}{\partial y} \quad (1)$$

where A is the magnetic vector potential, ν is the reluctivity, J is the current density, and B_{mx} and B_{my} are the induction terms corresponding to the permanent magnets, respectively. It is assumed that A and J have only components in the z direction. Two types of conductors are usually distinguished in the literature:

1) Stranded (thin, fine wire) conductors which are made of thin separate filaments. In this type of conductor, the skin effect and induced eddy currents are assumed negligible, resulting in a constant current density as

$$J=\frac{Ni_f}{S_f} \quad (2)$$

where i_f is the current of a single filament, N is the number of filaments, and S_f denotes the region occupied by the filaments. The relationship between the potential difference across a stranded conductor and the conductor current is

$$v_f=R_f i_f+\frac{Nl}{S_f}\int_{S_f}\frac{\partial A}{\partial t}ds \quad (3)$$

where l is the axial length of each filament and R_f is the dc resistance

$$R_f=\frac{N^2 l}{\delta S_f}. \quad (4)$$

2) Solid (thick, massive) conductors which are conductors with a relatively large surface area. This type of conductor can exhibit significant skin effect and the relationship between J and total current i_b cannot be calculated in advance. Instead, according to the Ohm's law, the following relation is valid for the surface of the solid conductors:

$$J=-\delta\frac{\partial A}{\partial t}+\delta\frac{v_b}{l} \quad (5)$$

where δ is the electrical conductivity, v_b is the voltage across the solid bar, and l is the length of the solid conductor. The relationship between the voltage across a solid conductor and the current through it can be written as

$$v_b=R_b i_b+R_b\int_{S_b}\delta\frac{\partial A}{\partial t}ds \quad (6)$$

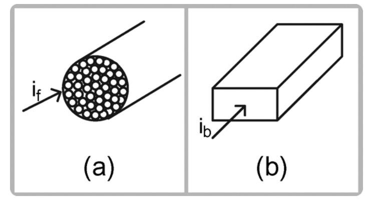

Fig. 1. (a) Stranded conductor. (b) Solid conductor.

where S_b denotes the region occupied by the solid conductor and R_b is the dc resistance

$$R_b=\frac{l}{\delta S_b}. \quad (7)$$

Fig. 1 shows cross sections of a stranded and a solid conductor, respectively. In the regions, such as air gap and iron core, when eddy current is neglected, the right-hand side of (1) is zero.

Applying the Galerkin method [7] to (1), (3) and (6), the following set of matrix equations is obtained:

$$S\mathbf{A}+N\frac{d}{dt}\mathbf{A}-P\mathbf{I_f}-P'\mathbf{V_b}=\mathbf{D} \quad (8)$$

$$Q'\frac{d}{dt}\mathbf{A}+R'\mathbf{I_b}=\mathbf{V_b} \quad (9)$$

$$Q\frac{d}{dt}\mathbf{A}+R\mathbf{I_f}+L\frac{d}{dt}\mathbf{I_f}=\mathbf{V_f} \quad (10)$$

where $\mathbf{A}$, $\mathbf{I_f}$, $\mathbf{V_f}$, $\mathbf{I_b}$, and $\mathbf{V_b}$ are vectors of the magnetic vector potential at the nodes, the currents, and voltages of stranded conductors, and the currents and voltages of the solid conductors, respectively. R is the matrix of dc resistance of the thin conductors and L is the matrix of the end-windings inductances. R' is a diagonal matrix containing the dc resistances of thick conductors. External series resistance and inductance can also be included in R, L, and R'. Matrices S, N, P, P', D, Q, and Q' are obtained by assembling elemental matrices according to the FEM. Each elemental matrix includes the geometrical information and material properties of the corresponding element inside the mesh region. Further details for an application of the FEM and the detailed expressions for the elements of the matrices can be found in [7].

It should be noted that (8) represents the magnetic field, and (9) and (10) act as the coupling equations. The next two sections focus on different methods for describing the circuit equations and solving the coupled system of equations using direct and indirect methods, respectively.

III. Direct Methods

A. Coupling Based on Simple Circuit Equations

Based on (8)–(10), if two out of four vectors $\mathbf{I_f}$, $\mathbf{V_f}$, $\mathbf{I_b}$, and $\mathbf{V_b}$ are known, the equations comprise a case of coupled magnetic fields, and circuit equations and can be solved without any need for including extra equations. Indeed, the field (8) can be solved independently if both currents of stranded conductors and voltages of the solid conductors are known. However, in some applications, it is necessary to solve the coupled system of equations together. This coupling approach is especially useful when EM apparatus are connected to independent voltage/current sources, directly or through series external RLC elements.

If there is no solid conductor in the finite-element region or when the solid conductors are assumed to be shorted ($\mathbf{V_b} = \mathbf{0}$), the system of coupled equations will reduce to (8) and (10). This set of reduced equations is used in [8] for the simulation of a capacitor motor when the main and auxiliary windings are connected to voltage sources in [9] for the simulation of a synchronous generator under load and in [10] for analyzing a fast acting pulse width a modulated (PWM) solenoid actuator. Reference [2] simplifies the coupled system further by substituting the vector of the winding currents $\mathbf{I_f}$, from (10) into (8). The field equation is then solved for the analysis of an induction machine (IM), with a smooth solid-iron rotor. Each steady-state operating point is calculated as the asymptotic solution of the EM-field equation for a fixed speed. The analysis of a saturated coil fed by a voltage source through a diode is studied in [3]. The coil is modeled as a stranded conductor and the saturated core is represented as a solid conductor with short-circuit paths ($\mathbf{V_b} = \mathbf{0}$). An extra unknown voltage due to the nonlinear voltage–current characteristic of the diode is also introduced in the equations.

The coupling based on simple circuit equations is also used in some applications where the finite-element region consists of only solid conductors. In this case, only (8) and (9) have to be considered to solve the coupled problem. This formulation is used in [11] for the analysis of an IM, where the stator windings and rotor bars are treated as solid conductors. Reference [12] investigates a special case of solid conductors supplied by current sources. The coupled system is formulated in terms of the unknown vectors $\mathbf{A}$ and the electric scalar potential gradient which is defined as

$$\mathbf{G} = \nabla \boldsymbol{\phi} = -\frac{\mathbf{V_b}}{l}. \tag{11}$$

A general coupled problem based on simple circuit equations is categorized into four different classes according to the presence or absence of eddy currents and the type of supply (voltage or current) in [13] and [14]. The coupling equations are back-substituted in the field equations so that the unknown vector in the final system of equations for all classes is only $\mathbf{A}$.

When an EM apparatus is connected to a power-electronic converter, it is still possible to use the coupling based on simple circuit equations if the switching sequence of the converter can be determined in advance. This approach is used in [15] for the simulation of permanent-magnet synchronous machines fed by current inverters. The stator voltage (10) is modified in each time step according to the state of the inverter. This is accomplished by changing the equivalent resistance of the switches (sequential method). Constant speed operation is assumed so that the switching sequence of the inverter is determined before the beginning of the simulation. A similar approach is used in [16] for the analysis of a permanent-magnet brushless dc motor drive, in [17] for a switched reluctance motor drive, and in [18] for a linear induction motor drive.

This method can only be applied to relatively simple circuits since all the circuit components have to be connected to a particular FEM-based model. To derive a set of coupled equations for the arbitrary connection of EM apparatus and circuit elements, a more general method for circuit analysis should be adopted. The following sections discuss three popular methods of circuit analysis for coupled field-circuit systems.

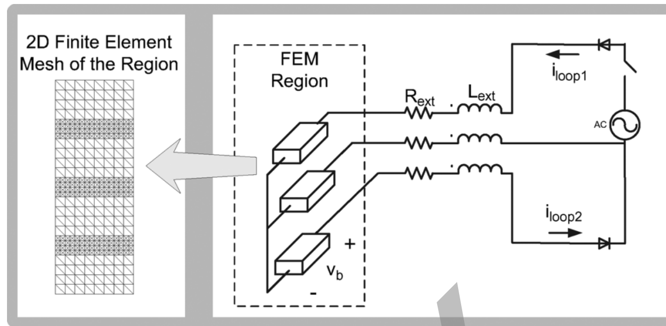

Fig. 2. System of three busbars in a rectangular domain studied in [20] and [21] based on the loop current method for coupling.

B. Coupling Based on the Loop (Mesh) Current Method

In this type of analysis, a system of n loops and fictitious loop currents are selected for the circuit. Currents in all branches can be described as the summation of a few loop currents with appropriate signs. The relationship between the currents in the EM apparatus and loop currents can be written as

$$\mathbf{I_{fem}} = P\mathbf{I_{loop}} \tag{12}$$

where P is a rectangular matrix with a few nonzero elements of $+1$ or -1 in each row. The equation is used to replace current vectors $\mathbf{I_f}$ and $\mathbf{I_b}$ in (8)–(10) with the loop current vector $\mathbf{I_{loop}}$. The Kirchhoff's voltage law is written for each loop as

$$\mathbf{U} = \mathbf{V_R} + \mathbf{V_L} + \mathbf{V_C} + \mathbf{V_{fem}} \tag{13}$$

where $\mathbf{U}$, $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ are vectors of voltage sources, resistive, inductive, and capacitive voltage drops in each loop, respectively. $\mathbf{V_{fem}}$ is the vector of voltages across the EM apparatus in each loop. This will provide an adequate number of equations to solve the coupled field-circuit system in terms of unknowns $\mathbf{A}$, $\mathbf{I_{loop}}$, $\mathbf{V_b}$, and $\mathbf{V_f}$.

This method is used in [19] for the analysis of an IM connected to a voltage source, where the field equations are solved in the rotating reference frame of the rotor. The loop current method is also used in [20] for the transient study of solid conductors connected to external circuits. Stranded conductors can also be included without external circuit connections. All of the given voltage sources and voltages across capacitors in the loops are placed in the right-hand side of loop equations. Other methods (e.g., eliminating the loop currents) for solving the equations are also discussed in [20]. The same method is also addressed in [21] in addition to a comprehensive literature review of previous work in this area. Fig. 2 shows a system of three busbars in a rectangular domain which is studied as a numerical example in [21].

In [22], the direct coupling of field-circuit equations for the steady-state sinusoidal solution, based on the loop analysis, is described for solid and stranded conductors. The commercial software FLUX2D is used to implement the formulation for an

IM in a locked rotor condition. A similar method for the axisymmetric and transient conditions is discussed in [23].

Reference [24] presents a method for the direct coupling of field-circuit equations for motor drives and semiconductor converters. Loop currents and the modified magnetic vector potential ($\boldsymbol{\varphi} = \mathbf{A}l$) are used as the unknowns in the equations. In this paper, the volt-ampere characteristics of semiconductors are approximated by piecewise linear functions to reduce the computational time. Numerical results include the transients in a permanent-magnet synchronous motor drive.

In [25], direct coupling of field and circuit equations (2-D and 3-D) based on the tree/cotree algorithm and loop analysis is presented. The tree/cotree algorithm can be used to derive the circuit equations in a systematic fashion. One advantage of the method is that the circuit equations are first solved separately to predict the state of nonlinear elements (switches) for the next time step. Thus, the coupled system can be simulated with a relatively large time step with adequate accuracy and reduced computation time. Numerical examples include a brushless dc motor, a PM synchronous motor fed by an ac–dc–ac PWM inverter, and a three-phase power transformer with a rectifier circuit.

1) Application of the Loop Current Method for Modeling the Rotor Cage Bars: In a squirrel cage IM, the rotor cage bars are solid conductors which are connected together by means of end rings. Every portion of the end ring between two bars could be considered to be an external circuit consisting of a series resistance and inductance. Damper cage bars in synchronous machines have the same configuration. Thus, to have a consistent set of equations, circuit loop equations of the rotor network are usually added to the coupled set of (8)–(10). $\mathbf{I_b}$ in (9) is also substituted by its equivalent in terms of the loop currents and then all of the equations are solved together. Fig. 3 depicts a portion of the rotor cage bars network for a squirrel cage induction motor in which loop currents and voltages across the bars are considered as the independent variables of the rotor network.

Loop equations of the rotor bars are used in [26] and [27] for coupled analysis of a shaded pole motor and an IM, respectively. This method has been developed further in [28] and [29] for coupled analysis of skewed-rotor IM using the multislice finite-element model. A multislice model is also used in [30] for the simulation of a brushless dc motor including a control loop. In this paper, the permanent magnet (PM) is represented as a squirrel cage to predict the effect of eddy current loss inside the PM. The impressed terminal voltages of the motor are determined by the controller. In the control loop simulation, the inputs are the speed command or the current reference, and the outputs are the stator phase voltages.

In the technical literature, loop equations are not directly added to the set of coupled equations. Instead, they are used to relate the current and voltage of each bar in the network. These relations are then back-substituted in (9) to eliminate one of the unknown vectors $\mathbf{I_b}$ or $\mathbf{V_b}$. This method is used in [31] for a coupled model of a squirrel cage IM under PWM supply conditions. Equation (9) is modified to eliminate $\mathbf{I_b}$ and then all three sets of equations are solved simultaneously. It is assumed that the applied PWM voltages to the stator windings

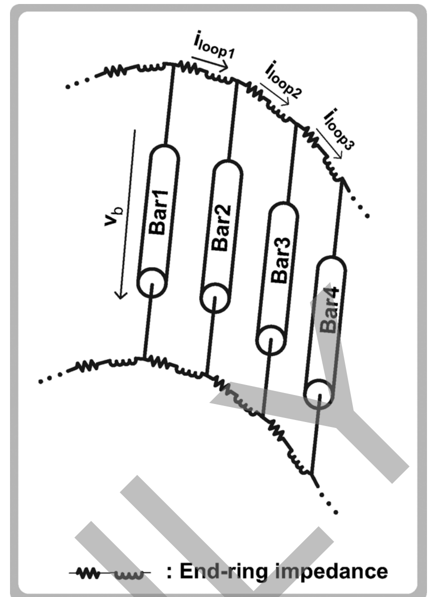

Fig. 3. Portion of the rotor cage bars network for a squirrel cage induction motor.

are known. A similar approach has been used in [32] and [33] for steady-state and transient analysis of an IM.

The direct method for coupling field-circuit equations of a skewed rotor IM is described in [34], where loop equations of the rotor bars are used to eliminate $\mathbf{V_b}$ from (9). The same approach is used in [35] for PWM inverter-fed ac motor drives. It is also assumed that the output voltages of the inverter are known.

Modeling the rotor cage bars network as a macroelement inside FLUX2D software is described in [36] for coupled analysis of an IM. In [37], direct coupling of the field, circuit, and motion equations of a cage IM are discussed. The rotor circuit equations are simplified by assuming the effect of end-ring resistance and inductance in series with each bar, and then shorted at both ends.

C. Coupling Based on the Nodal or Modified Nodal Approach (MNA)

Consider a circuit with n nodes. In nodal analysis, $n-1$ nodes are selected as independent nodes and their voltages ($\mathbf{V_{node}}$), with respect to the reference node, are deemed as unknowns to be determined.

To translate a circuit topology into equations, Kirchhoff's current law is applied to all $n-1$ nodes. This results in a set of matrix equations

$$Y\mathbf{V_{node}} = \mathbf{I}. \tag{14}$$

In the steady state, the admittance matrix Y and current source vector $\mathbf{I}$ can be constructed by a simple electric circuit inspection. The same algorithm is used for the transients study by expressing all inductances and capacitances of the circuit as equivalent conductances in the discrete-time domain. When voltage sources are present in the circuit, their currents are also assumed as extra unknowns and the equations expressing the voltage drops are added to the nodal equations. This method, which avoids the necessity of any source transformation, is

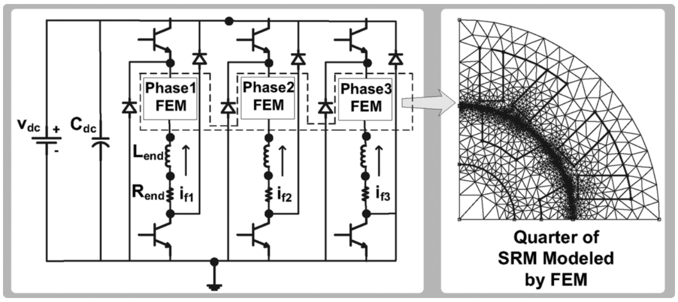

Fig. 4. Electric circuit and finite-element mesh of a switched reluctance motor analyzed in [39], based on the nodal approach for coupling.

called modified nodal analysis (MNA) and is the basis of many circuit simulators, such as SPICE.

To couple nodal equations with field equations, the voltages across the EM apparatus are related to node voltages by the aid of the transformation matrix T

$$\mathbf{V}_{\mathbf{fem}} = T\mathbf{V}_{\mathbf{node}} \tag{15}$$

where T is a rectangular matrix with a few nonzero elements of $+1$ or -1 in each row. Using (15), $\mathbf{V_b}$ and $\mathbf{V_f}$ in (8)–(10) are replaced by $\mathbf{V_{node}}$. Also, the unknown vectors $\mathbf{I_b}$ and $\mathbf{I_f}$ are placed in the right-hand side of nodal equations by the aid of the transformation matrix

$$Y\mathbf{V}_{\mathbf{node}} = \mathbf{I} - T^T\mathbf{I}_{\mathbf{fem}}. \tag{16}$$

Then, (8)–(10) and (16) can be solved simultaneously to obtain the unknowns $\mathbf{A}$, $\mathbf{V_{node}}$, and $\mathbf{I_{fem}}$ in the coupled system of equations.

Reference [38] presents a direct coupling method where circuit equations are included in terms of the admittance matrix for a squirrel cage alternator. This approach simplifies the equations so that the final equations to be solved only include $\mathbf{A}$ as the unknown.

Transient analysis of coupled systems based on the nodal approach is described in [39] where a symmetrical matrix equation is obtained and solved by a step-by-step integration technique. Reported test cases include a switched reluctance motor (SRM) and a skewed brushless dc motor with a squirrel cage in the rotor. Fig. 4 shows the inverter circuit and the finite-element mesh of the analyzed SRM where nodal voltages, the nodal values of magnetic vector potential, and currents inside the finite-element region are adopted as the unknowns.

In [40], the direct coupling approach based on MNA for electric circuit simulation is discussed. The method is applicable to 2-D and 3-D FEM and two new voltage-source-type circuit elements have been developed for magnetic-circuit coupling. The only disadvantage of the approach is the loss of matrix symmetry due to the nature of MNA. The application of the MNA for direct coupling of linear systems and solid conductors is also presented in [41] and [42]. The same method as that of [41] is used in [43] for nonlinear magnetodynamic phenomena, considering the existence of stranded conductors.

A general system of coupled equations for the nodal and the loop current methods is developed in [44]. When the nodal approach is used, an extra set of unknowns for stranded conductors is introduced to make the final coefficient matrix symmetric. For the loop analysis, an extra set of unknowns for solid conductors is added to the equations to preserve the symmetry.

D. Coupling Based on the State-Space (SS) Approach

The state-space approach is another method to construct dynamic equations of an electric circuit. Independent capacitor voltages and inductor currents are commonly assumed as the state variables of the system. The SS equations are written in a matrix form as

$$\frac{d\mathbf{x}}{dt} = G_1\mathbf{x} + G_2\mathbf{E} \tag{17}$$

where $\mathbf{x}$ is the state variable vector and $\mathbf{E}$ is the source vector, including independent voltage and current sources. The SS equations can be systematically obtained using the graph theory and Welsh's algorithm [45]. Also, any output vector $\mathbf{y}$ of variables of the electric circuit can be expressed in terms of the state variables and the vector of sources.

Coupling of the SS and the field equations is usually achieved by considering the current vector in the EM apparatus as a source term in the SS equations, as

$$\frac{d\mathbf{x}}{dt} = G_1\mathbf{x} + G_2\mathbf{E} + G_3\mathbf{I}_{\mathbf{fem}}. \tag{18}$$

The voltages at the terminals of EM apparatus can be written as

$$\mathbf{V}_{\mathbf{fem}} = G_4\mathbf{x} + G_5\mathbf{E} + G_6\mathbf{I}_{\mathbf{fem}} \tag{19}$$

where matrices G_1, G_2, G_3, G_4, G_5, and G_6 are dependent on the circuit topology and can be obtained by network analysis methods [7].

Since the vector of voltages across stranded conductors $(\mathbf{V_f})$ is not an explicit source vector in the field equations, it can be omitted from the coupled equations by substituting $\mathbf{V_f}$ in (10) according to (19), and deducing the coupled equations based on (8)–(10), (18) and (19).

The aforementioned approach is used in [46] for the direct coupling of field and circuit equations of electrical machines, taking the external circuit (voltage or current converter) into account and the eddy currents in solid conductive parts. Only stranded conductors are assumed to be connected to the external circuit. Reference [47] simplifies the coupled equations based on the SS approach by assuming that the rings are all ideal short circuits for the case of a single phase IM. Thus, variables related to the rotor bars (solid conductors) are all eliminated from the coupled equations.

Reference [48] describes a method for direct coupling of field and circuit simulations in 2-D and 3-D structures. Both of the loop currents and the SS approaches are discussed for the circuit analysis.

A more systematic approach to develop the coupling equations for the case of a converter connected to an EM apparatus is presented in [49]. The SS equations are built by using the tree/cotree approach and the Welsh algorithm and, as an example, a flyback converter equipped with a snubber and a pot core transformer is simulated. This method is further developed to include the control loop in [50] and [51]. The same coupling

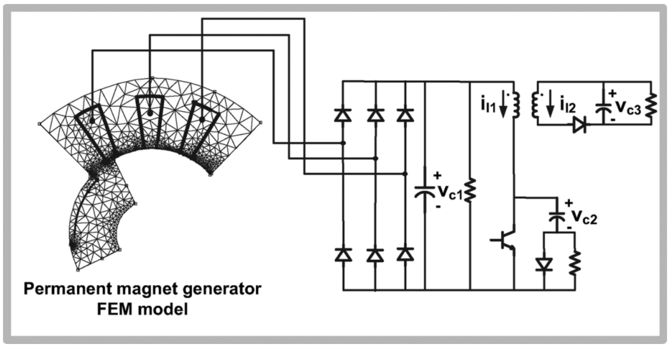

Fig. 5. PM generator feeding a flyback converter analyzed in [56], based on the SS approach for coupling.

method is also used in [52] and [53] where the inclusion of the EM device movement in the resolution of field- and circuit-coupled problem is also addressed.

Reference [54] describes an approach to develop the SS equations of a coupled problem using the signal flow graph (SFG). The SFG is a weighted and directed graph representing a system of equations. Only the steady-state condition is discussed in this paper. The method retains matrix symmetry by multiplying the equations by specific coefficients. Reference [55] discusses different methods to solve the system of coupled equations obtained in [54]. Automatic calculation of the coupled equation with the aid of the graph theory and the SS approach is also addressed in [56]. The numerical example includes a PM generator feeding a flyback converter, as shown in Fig. 5 where the indicated capacitor voltages and inductor currents are taken as the state variables for the circuit equations.

An alternative formulation of the coupled problem in terms of the SS equations is presented in [57]. This method considers EM apparatus as voltage sources in the SS equations of the circuit. Thus, the $\mathbf{V_{fem}}$ vector is added to the right-hand side of (18) instead of $\mathbf{I_{fem}}$. Also, the positions of $\mathbf{V_{fem}}$ and $\mathbf{I_{fem}}$ are interchanged in (19). The merit of this approach is that the coupled system can be simplified to include only $\mathbf{A}$ and $\mathbf{x}$ as the independent unknowns. The currents and voltage drops of the EM apparatus are calculated subsequently by means of the two output equations.

The SS approach is also used in [58] with special attention paid to polyphase structures connected to the static converters. It is shown that stranded conductors should always be considered as links to ensure the necessary number of linearly independent equations in the SS model. In situations where this is not possible, placing a large resistor in parallel with the EM apparatus to solve the problem has been proposed.

E. Comparison Between Different Direct Methods

All three circuit-analysis methods mentioned before have been used successfully in coupled field-circuit problems. However, implementing the loop current method in the existing finite-element codes is rather difficult compared to the MNA [40]. Furthermore, in a circuit with a large number of branches and complicated topology, the MNA provides a more systematic process to build the circuit equations. In [44], using the loop current method is proposed if only stranded conductors are modeled by the finite-element approach, while the nodal method is proposed if there are only solid conductors in the finite-element region.

Two difficulties associated with the inclusion of voltage sources and stranded conductors in MNA and solid conductors in the loop current analysis are addressed in [54]. It is concluded that the best approach of circuit coupling is a description of circuit equations in terms of the unknown currents and unknown voltages (SS approach). Both the nodal and loop current equations can be obtained from the SS equations by eliminating a specific set of unknowns [55].

The SS approach usually provides a more convenient way to handle all types of connections between EM apparatus and circuit elements. However, choosing the type of circuit analysis in a direct coupling method is still a matter of preference.

IV. Indirect Methods

In indirect methods, circuit and field equations of the coupled system are maintained as different subsystems and solved separately. To do this, the coupling coefficients should be exchanged back and forth between the two subsystems. The indirect methods can be divided into two subcategories according to the type of the coupling coefficients: 1) the current output approach and 2) the circuit parameter approach.

A. Coupling Based on the Current Output Approach

In this approach, (8)–(10) are considered as a single subsystem, representing the FEM-based subsystem. Assuming only stranded conductors are connected to the external circuit (which is true for most of the practical cases in electrical machines and other EM apparatus), the inputs to the FEM-based subsystem are terminal voltages $(\mathbf{V_f})$ and the outputs are currents of the stranded conductors $(\mathbf{I_f})$. Thus, for the external circuit, the FEM-based subsystem acts as a voltage-controlled current source. Additional outputs, such as flux linkages, solid conductors currents, and torque (for electrical machines) can also be calculated inside the FEM-based subsystem. It can be shown that if voltages at the terminals of EM apparatus are known, the current output approach is mathematically equivalent to direct coupling based on simple circuit equations. Fig. 6 shows a block diagram of an indirectly coupled system based on the current output approach.

The current output approach is proposed in [59], where the FEM-based subsystem for an IM is embedded inside a MATLAB/SIMULINK S-function. The inputs to the FEM-based block are stator winding voltages and the outputs are phase currents, flux linkages, and EM torque. The FEM-based subsystem is executed with a time step that is larger than the SIMULINK time step. The connection between the rotor cage bars is modeled with an impedance network inside the FEM-based subsystem. The same method is also discussed in [60]. A data-exchange scheme between a FEM-based program (FCSMECK) and a system simulator (SIMULINK) for the current output approach is presented in [61].

B. Coupling Based on the Circuit Parameter Approach

Another approach for indirect coupling of field and circuit equations is to define the coupling coefficients in terms of the

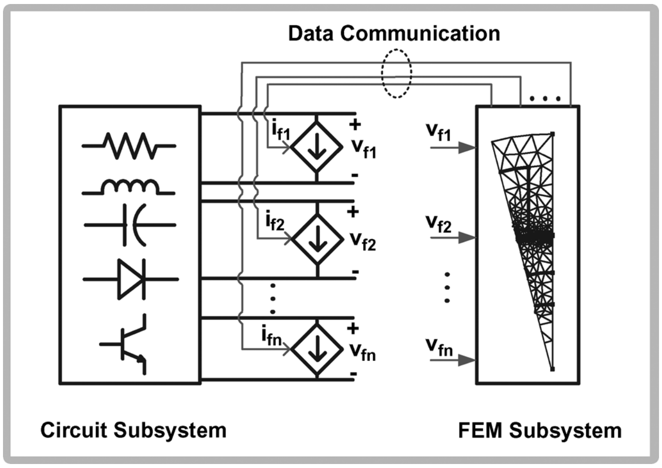

Fig. 6. Schematic of an indirectly coupled field-circuit system based on the current output approach.

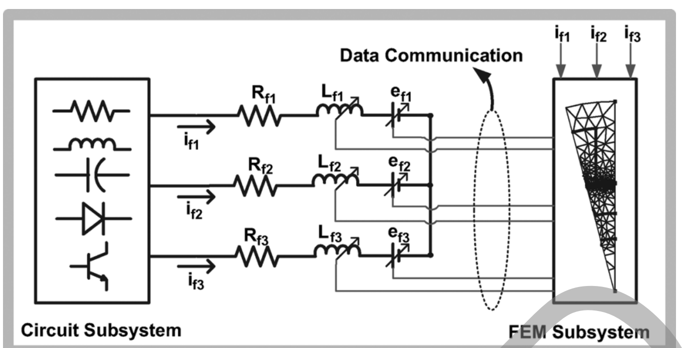

Fig. 7. Schematic of an indirectly coupled field circuit system based on the circuit parameter approach.

circuit parameters [e.g., inductances and back electromotive forces (emfs)]. Therefore, the FEM-based subsystem only includes a modified version of the field (8) which accepts current densities in all conductors as the inputs and provides inductances and/or back emfs as the outputs. The coupling (9) and (10) are also modified and the induced voltages in the conductors ($Q'(d/dt)A$ and $Q(d/dt)A$) are described in terms of inductive voltage drops and/or back emfs. Fig. 7 shows the block diagram of an indirectly coupled system based on the circuit parameter approach.

The coupling based on the circuit parameter approach usually includes several iterations between the circuit and the field equations to obtain accurate results. This iterative procedure is described in [62] for the simulation of a squirrel cage IM in the steady state. Circuit equations of the machine are developed in the impedance form $(\mathbf{V} = Z\mathbf{I})$ for the stator windings and the rotor bar network. Starting with a set of initial currents, the nonlinear field equations are solved to obtain the reluctivity of each element at the operating point. The coupling impedances are then determined by solving a series of linear field equations, one for each current, assuming that the reluctivity is constant. The solution of the circuit equations then gives an improved estimate of the winding currents which are used to solve the new nonlinear field equations. This procedure continues until convergence is obtained. To define the current flow paths in the rotor, the rotor bar currents are described by a series of harmonic currents in the rotor dq frame. The same method is used in [4] and [63] for the transient study of an IM. The circuit equations of the stator windings are described as

$$\frac{d\boldsymbol{\psi_f}}{dt} = \mathbf{V_f} - R\mathbf{I_f}, \tag{20}$$

$$\boldsymbol{\psi_f} = M\mathbf{I_f} \tag{21}$$

where $\boldsymbol{\psi_f}$ is the vector of flux linkages of the stator windings, and M is the inductance matrix of the stator winding and the rotor harmonic current distributions. A similar method for the coupled simulation of synchronous machines is used in [64] and [65] where the circuit equations are described in terms of the stator currents rather than the flux linkages. Reference [66] provides an alternative way to write the circuit equations of a coupled system based on the circuit parameter approach. The circuit analysis method is based on Kirchhoff's current law at each node and the equations relating voltage and current of each branch.

The calculation of inductance values from the solution of nonlinear field equations is a time-consuming procedure which places an extra burden on the field solver. In [67], the circuit parameter approach is used for the analysis of switched reluctance drives. The method is based on iterations between the circuit and the field equations in each step of the Newton–Raphson method. In each step, a modified estimate of the currents is extracted from the circuit equations and used in the next iteration. The procedure is repeated until changes in node potentials are below a specified limit. The estimation of the currents is performed based on (10) where the rate of change of $\mathbf{A}$ is obtained directly from the field equation; therefore, the inductance calculation is bypassed. This method is also used in [68] for the simulation of a generator suddenly connected to a resistive load via a three-phase bridge rectifier, where the iterations between the circuit and the field equations are performed in each time step until convergence is reached. Coupled analysis of a switched reluctance drive with the aid of the circuit parameter approach and an iterative loop between the field and the circuit equations without inductance calculation is also discussed in [69].

References [70] and [71] present an application of the circuit parameter approach to obtain the steady-state nonsinusoidal waveforms of electrical machines. The method is based on the state-space equations of the machine and the estimation of inductance profile over an entire 360° of rotation. Each apparent self and mutual inductance has a nonsinusoidal periodic profile, which depends on the instantaneous values of the rotor and the stator winding currents, at a given rotor position. The combined magnetic effect of the individual bar currents is represented as two magnetomotive forces in quadrature angle with respect to each other. In this method, the SS equation of the machine (in terms of its voltages and flux linkages) is integrated until periodic answers are obtained. Then, the currents are calculated in terms of the flux linkage and the inductance matrix. These current profiles are then used to update the inductance profile from a new finite-element solution. The loop equations of the rotor bars are also incorporated in the SS model. This method is also used in [72] and [73] with an improved rotor bar modeling;

in [74], for an inverter-fed IM through binary resistance representation of the switches; in [75] for the analysis of a switched reluctance motor drive system during fault conditions; and in [76], for a salient-pole synchronous generator under unbalanced conditions.

The indirect coupling method in [77] is based on handling the finite-element model as a multiport circuit element. In the multiport element, voltages and currents of solid and stranded conductors are treated as the voltages and currents of the other elements in the circuit. Nodal values of the magnetic vector potential can be seen as internal variables of the multiport element. The circuit equations are based on the modified loop formulation. It is shown that this method is mathematically equivalent to the direct coupling method.

Reference [78] uses the circuit parameter approach for coupled analysis of a brushless motor. Inputs to the magnetic-field block are the line currents and rotor position, and the outputs are the back emfs and the inductances for the circuit subsystem and torque for kinematic equations. Field equations are solved once for several steps of the circuit simulation. A similar approach is used in [79] for the simulation of a doubly fed induction generator fed by a frequency converter. In this paper, the effect of interaction between the stator and the rotor is modeled inside the emf instead of the inductance matrix for improved accuracy.

Indirect coupling based on the circuit parameter approach provides a convenient way to implement the coupled system inside a circuit simulator, or to interface the circuit simulator with a field simulator program. In [80], an indirect coupled Simulink-finite-elements model is used for the simulation of a loaded IM. Magnetizing current, stator leakage inductance, and rotor impedance are calculated for several values of the slip to take frequency dependence of the rotor parameters into account. These parameters are then used in the Simulink model by means of a lookup table. A coupled model of a switched reluctance motor inside the PSCAD/EMTDC program is described in [81] by modeling precalculated dynamic inductances from the FEM as functions of the rotor position and excitation. All of the parameters of the motor and drive (such as torque, currents, etc.) are then calculated from the circuit simulation. Reference [82] presents a method for indirect (weak) coupling of the field solvers and the Alternative Transients Program (ATP) software. The method relies on the computation of differential inductance coefficients for the magnetic structure, and the communication of those inductance values to the transient program at each time step, where the required discretization is used to update the electrical network parameters. The implementation of the inductance network in ATP is achieved through transient analysis of control systems (TACS) functions. A numerical example includes the coupled simulation of a transformer. A more general approach for the simulation of transformer energization transients through an interface between the EMTP-RV software and the FLUX3D software is presented in [83]. The dynamic link library (DLL)-based interface permits an exchange of any kind of variables, such as currents, voltages, switching instants, fluxes, and mechanical forces between the two programs.

In [84], an application of the circuit parameter approach in the real-time simulation of electrical drives is presented. After the motor design in the JMAG Studio Sofware, a behavioral model is generated with the aid of an integrated module (JMAG-RT), including all nonlinear characteristics of the motor. The new model of the machine receives the currents and speed from the circuit simulator, and provides the corresponding precomputed inductance and flux data from JMAG software.

The coupling approach in [85] is based on extracting the inductances and back emfs from the field simulation and Norton equivalents from the circuit simulation. The Norton equivalent circuit parameters include the conductance matrix and source currents. The FEM-based simulator has an embedded subroutine to support the Norton equivalent representation of the circuit subsystem.

C. Extraction of Circuit Parameters From FEM

A major step in the coupling method based on the circuit parameter approach is to obtain the coupling coefficients (inductances and/or back emfs) from the finite-element solution. This is usually achieved by calculating the flux linkage of windings from the nodal values of the magnetic vector potential $(\mathbf{A})$. The flux linkage can be split into two components.

1) A component which is produced by the coupled inductance branches and the currents through them. This only models the effects of coupling between conductors which are directly connected to the external circuit.
2) An internal flux which is produced by other sources, such as permanent magnets and induced eddy currents.

There are several methods to split up the flux linkage between these two components according to the coupling method and the formulation of the problem. Two of the methods are described in [79] and [85]. It is to be noted that if the circuit dynamic equations are formulated in terms of the flux linkage derivatives rather than the currents derivatives, then the flux linkage can be entirely described by the inductance matrix and the current vector. This method is used in [4] and [70] for indirect coupling of the IM.

Two definitions of inductance are usually used in the technical literature to formulate the circuit equations of EM apparatus (i.e., apparent and differential inductances). The apparent inductance is defined as the slope of a straight line that passes through the operating point and origin of the magnetic flux-current nonlinear characteristic. The differential inductance is defined by the slope of the nonlinear curve at the operating point. Both of these inductances could be obtained from the finite-element solution by using flux-linkage or energy perturbation methods. Further details, advantages, and drawbacks of each definition and method are provided in [86]–[88].

D. Comparison Between the Different Indirect Methods

The coupling based on the current output approach is straightforward because the FEM-based subsystem is represented by voltage-controlled current sources to the external circuit. Furthermore, since voltages and currents are passed directly between the two subsystems, the time-consuming procedure of extracting circuit parameters (e.g., inductances and back emfs calculations) is bypassed during the simulation. Thus, the computation time is shorter than that of the circuit parameter approach.

However, as reported in [79], the current output approach might cause problems when inductive or capacitive components are present in the circuit. The reason is that the FEM-based subsystem is solved with a larger time step (major) than that of the circuit model (minor). Hence, the current and voltage waveforms at the terminals of the EM apparatus are updated only at major time steps. The circuit parameter approach overcomes this problem since the outputs of the FEM-based subsystem in this method are the inductances and the back emfs, and both voltages and currents can vary during the circuit simulation.

V. Comparison of Direct and Indirect Methods

Both of the direct and the indirect methods for coupling field and circuit equations have advantages and drawbacks. Hence, the selection of an appropriate coupling method depends on the problem.

Direct methods are usually more reliable and effective in handling nonlinearities of the field and the circuit equations by applying the multidimensional Newton–Raphson iteration [77]. However, the substitution of the dense and usually negative-definite circuit equations in the finite-element equations results in the loss of sparsity, symmetry, and positive definiteness of the final system of coupled equations. These properties are important to solve the large number of finite-element equations by efficiently using sparse solvers. Thus, an extra step in formulating the direct coupled equations is to modify the equations by multiplying the circuit equations with a coefficient [54] or introducing additional unknowns into the equations [44] to retrieve some of these properties, or to use conventional dense solvers for relatively sparse coupled equations [49] and [50], which reduce the efficiency.

Also, increasing complexity of the circuit model feeding the EM apparatus requires the development of a new set of equations for each different case, especially in closed-loop control systems [79]. However, the indirect methods provide the flexibility of implementing the coupled problem inside circuit simulator programs [80]–[82].

One drawback of the indirect methods is the need for several iterations between the field and circuit subsystems due to the nonlinear nature of the equations [4]. These extra iterations could make the indirect simulation noticeably slower than the direct simulation methods [5], [6], and [89]. Several methods have been proposed to avoid excessive iterations. In [82], the coupling coefficients are communicated between the two subsystems with one time-step delay. The delay decouples the field and the circuit equations at the cost of using a smaller time step to preserve the accuracy of solution. The possibility of using a predictor to approximate the current (input of the FEM-based subsystem) over the next time step is also mentioned in the same paper. Another method to avoid the closed-loop iteration is to add a high-pass filter in the feedback path for drift compensation in the steady state [79].

An advantage of using the indirect coupling method, as described in [66], is that the number of iterations needed for the nonlinear field solution are relatively small compared to that of nonlinear circuit equations. Since the number of field equations are usually much larger than that of circuit equations, this feature results in considerable time savings. Finally, indirect coupling allows multirate simulation of a coupled system in which the circuit simulation can be performed with a much smaller time step than that of the FEM-based simulation, as described in [79].

VI. Technical Challenges of Coupled Field-Circuit Simulation

The growing complexity of EM apparatus and the circuit elements connected to them, as well as the application of advanced control algorithms, prompts the use of multidomain simulation to avoid the costly failure of the whole system after implementation. The main difficulty associated with the use of a coupled field-circuit simulation is the large computational time of the simulation due to the complexity of the field solution. Typical execution time for a few seconds of simulation of a coupled system is in the order of several hours, as reported in [90]. This is especially troublesome during the design stage when repetitive solutions of the coupled system are necessary. Research is currently ongoing on permeance network models as an alternative to the FEM-based models for coupled problems due to their relatively fast solution albeit at reduced accuracy [91].

Although direct coupling of field and circuit equations results in a robust set of equations to be solved, in practical applications, it is usually desirable to exploit the features of the available software in each domain. Newer commercially available software is usually equipped with various interfacing utilities to communicate with other programs. Compatibility and synchronization of different field and circuit simulators are important issues which have to be fully examined before successful interfacing of the programs.

VII. Coupling With Mechanical Equations

In the analysis of EM apparatus with moving or rotating parts, the mechanical equations are also coupled to the EM equations and can be expressed as

$$J\frac{d}{dt}\omega_m = T_e - T_L, \tag{22}$$

$$\frac{d}{dt}\theta = \omega_m \tag{23}$$

where J is the moment of inertia, T_e is the electrical torque, T_L is the load torque, and ω_m and θ are the rotational speed and angular position of the rotor.

The link between the mechanical and EM equations is provided through the electrical torque (T_e) which is calculated from the field analysis. Several methods, such as Maxwell stress tensor [47] and the virtual work principle [59] can be used to deduce the electrical torque. The interpolation function between the stator mesh and the rotor mesh, in the field equations, is dependent on the position of the rotor which is, in turn, a function of the EM torque. Thus, there would be an interdependence between mechanical equations and the other system equations [35].

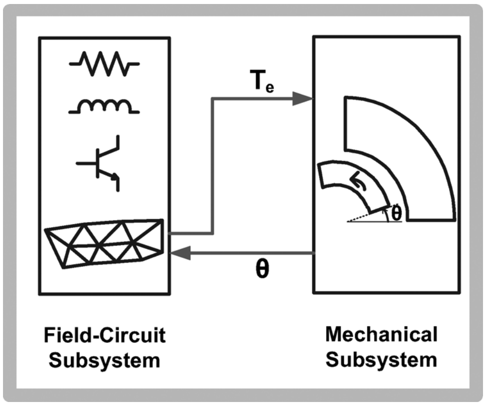

Fig. 8. Schematic of the iterative procedure between the field-circuit and mechanical subsystems.

Another approach is to add the mechanical equations to the coupled field-circuit equations by introducing rotor position [11] and [37], or rotor speed [34] or both [30] as the extra unknowns and then simultaneously solving all of the equations.

If the mechanical equations are separated from the rest of system equations, an iterative loop between the solution of the field-circuit equations and the mechanical equations is necessary. Fig. 8 shows a schematic of this approach where the electrical torque and the rotor displacement are passed back and forth between the two subsystems in each time step until the convergence is obtained. However, since the time constant of the mechanical system is usually much larger than that of the EM system, simplified approaches have been proposed to avoid or reduce the number of iterations between the large-size nonlinear field-circuit equations and the mechanical equations. In [33], [35], and [66], predictor-corrector methods are used to estimate the displacement of the rotor at the next time step from the mechanical equations. This value is then used to formulate and solve the field-circuit equations and obtain the electrical torque. To decrease numerical errors, the mechanical equations are solved again by the use of updated torque from the field solution. If the difference between the estimated and the calculated rotor displacement is large, the iteration continues. References [24] and [29] use the same method by choosing an explicit integration method to solve the mechanical equations of the system at the first step. Thus, the solution of the mechanical equations does not directly affect the finite-element equations.

VIII. Time Discretization

One major step in the transient study of a coupled field-circuit problem is proper discretization of temporal derivatives in the equations, before proceeding with the numerical simulation. These derivatives in the field and the coupling (8)–(10) are related to the induced eddy currents and voltages of the conductors in the finite-element region. Temporal derivatives also appear in the circuit equations due to the presence of energy-storing elements, such as inductors and capacitors.

Almost all of the discretization methods used in the coupled problems can be described by the general one-step θ-algorithm [7]. This method describes variable x at the time $(t+\Delta t)$ as

$$x(t+\Delta t) = x(t) + \left[(1-\theta)\frac{d}{dt}x(t) + \theta\frac{d}{dt}x(t+\Delta t)\right]\Delta t \tag{24}$$

where $x(t)$ is the value of x at time t, Δt is the time step, and $0 < \theta \leq 1$.

Different values of θ between 0 and 1 are used in the coupled problems. Most authors use $\theta = 1$, which is the well-known backward-Euler method. This method is simple and stable in nature, and by choosing a relatively small time step, the solution accuracy is also preserved [7].

Crank–Nicholson ($\theta = 0.5$) is another method [2], [12], [24], [27], [35], and [37] for coupled problems. This method is more accurate than the backward-Euler but needs more computational effort. A detailed stability analysis of the time-dependent eddy current problem, coupled with the loop equation, is presented in [92]. It is shown that the backward-Euler method is always stable while the Crank–Nicholson approach generates undamped or even divergent oscillations in some cases. A single application of the θ algorithm with $\theta = 0.75$ for transients of an IM connected to an adjustable speed drive is presented in [11].

IX. Conclusion

Coupled field-circuit analysis is a major step in the accurate design and simulation of EM devices connected to the complex external circuits. Different direct and indirect methods of coupling for low-frequency power apparatus are discussed and compared in detail in this paper. It is concluded that direct methods are more efficient with respect to the accuracy and speed of the simulation because they preserve the strong coupling between the equations and avoid the iterative loop between the field and circuit subsystems in each time step. On the other hand, indirect methods are more suitable for multirate simulation and can be easily implemented inside the circuit simulator programs by defining the FEM-based subsystem as a user-defined multiport element or by interfacing two different field and circuit analysis programs. This paper also discusses the technical challenges in field-coupled simulation, the coupling with mechanical equations, and time discretization techniques.

References

[1] L. Chua and P. Lin, *Computer-Aided Analysis of Electronic Circuits*. Englewood Cliffs, NJ: Prentice-Hall, 1975.

[2] B. Brunelli, D. Casadei, U. Reggiani, and G. Serra, "Transient and steady-state behaviour of solid rotor induction machines," *IEEE Trans. Magn.*, vol. MAG-19, no. 6, pp. 2650–2654, Nov. 1983.

[3] F. Piriou and A. Razek, "Coupling of saturated electromagnetic systems to non-linear power electronic devices," *IEEE Trans. Magn.*, vol. 24, no. 1, pp. 274–277, Jan. 1988.

[4] S. Williamson, L. H. Lim, and A. C. Smith, "Transient analysis of cage-induction motors using finite-elements," *IEEE Trans. Magn.*, vol. 26, no. 2, pp. 941–944, Mar. 1990.

[5] C. I. McClay and G. T. van der Toorn, "A comparison of time-stepped finite-element techniques for the calculation of losses in cage induction motors," in *Proc. 9th Int. Conf. Electrical Machines and Drives*, 1999, pp. 35–39.

[6] A. M. Knight and J. C. Salmon, "A comparison between finite element techniques when modelling single phase line-start permanent magnet motors," in *Proc. 9th Int. Conf. Electrical Machines and Drives*, 1999, pp. 351–355.

[7] J. P. A. Bastos and N. Sadowski, *Electromagnetic Modeling by Finite Element Methods*. New York: Marcel Dekker, 2003.

[8] T. Nakata and N. Takahashi, "Direct finite element analysis of flux and current distributions under specified conditions," *IEEE Trans. Magn.*, vol. MAG-18, no. 2, pp. 325–330, Mar. 1982.

[9] C. N. Ashtiani and D. A. Lowther, "The use of finite element in the simulation of the steady state operation of a synchronous generator with a known terminal loading condition," *IEEE Trans. Magn.*, vol. MAG-19, no. 6, pp. 2381–2384, Nov. 1983.

[10] A. M. Pawlak and T. W. Nehl, "Transient finite element modeling of solenoid actuators: The coupled power electronics, mechanical, and magnetic field problem," *IEEE Trans. Magn.*, vol. 24, no. 1, pp. 270–273, Jan. 1988.

[11] S. J. Salon, R. Palma, and C. C. Hwang, "Dynamic modeling of an induction motor connected to an adjustable speed drive," *IEEE Trans. Magn.*, vol. 25, no. 4, pp. 3061–3063, Jul. 1989.

[12] V. K. Garg and J. Weiss, "Finite element solution of transient eddy-current problems in multiply-excited magnetic systems," *IEEE Trans. Magn.*, vol. MAG-22, no. 5, pp. 1257–1259, Sep. 1986.

[13] P. Belforte, M. Chiampi, and M. Tartaglia, "A finite element computation procedure for electromagnetic fields under different supply conditions," *IEEE Trans. Magn.*, vol. MAG-21, no. 6, pp. 2284–2287, Nov. 1985.

[14] G. Meunier, D. Shen, and J.-L. Coulomb, "Modelisation of 2D and axisymmetric magnetodynamic domain by the finite elements method," *IEEE Trans. Magn.*, vol. 24, no. 1, pp. 166–169, Jan. 1988.

[15] N. Sadowski, B. Carly, Y. Lefevre, M. Lajoie-Mazenc, and S. Astier, "Finite element simulation of electrical motors fed by current inverters," *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1683–1688, Mar. 1993.

[16] R. Liu, Y. Zhang, M. Hu, and D. Yan, "Field circuit coupled time stepping finite element analysis on permanent magnet brushless DC motors," in *Proc. 8th Int. Conf. Electrical Machines and Systems*, Sep. 2005, vol. 3, pp. 2105–2108.

[17] N. Sadowski, Y. Lefevre, C. G. C. Neves, and R. Carlson, "Finite elements coupled to electrical circuit equations in the simulation of switched reluctance drives: Attention to mechanical behaviour," *IEEE Trans. Magn.*, vol. 32, no. 3, pp. 1086–1089, May 1996.

[18] S. C. Ahn, Jr., J. H. Lee, and D. S. Hyun, "Dynamic characteristic analysis of LIM using coupled FEM and control algorithm," *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1876–1880, Jul. 2000.

[19] P. G. Potter and G. K. Cambrell, "A combined finite element and loop analysis for nonlinearly interacting magnetic fields and circuits," *IEEE Trans. Magn.*, vol. MAG-19, no. 6, pp. 2353–2355, Nov. 1983.

[20] I. A. Tsukerman, A. Konrad, and J. D. Lavers, "A method for circuit connections in time-dependent eddy current problems," *IEEE Trans. Magn.*, vol. 28, no. 2, pp. 1299–1302, Mar. 1992.

[21] I. A. Tsukerman, A. Konrad, G. Meunier, and J. C. Sabonnadiere, "Coupled field-circuit problems: Trends and accomplishments," *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1701–1704, Mar. 1993.

[22] P. Lombard and G. Meunier, "A general method for electric and magnetic coupled problems in 2D and magnetodynamic domain," *IEEE Trans. Magn.*, vol. 28, no. 2, pp. 1291–1294, Mar. 1992.

[23] P. Lombard and G. Meunier, "A general purpose method for electric and magnetic combined problems for 2D, axisymmetric and transient systems," *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1737–1740, Mar. 1993.

[24] A. Demenko, "Time-stepping FE analysis of electric motor drives with semiconductor converters," *IEEE Trans. Magn.*, vol. 30, no. 5, pp. 3264–3267, Sep. 1994.

[25] P. Zhou, W. N. Fu, D. Lin, S. Stanton, and Z. J. Cendes, "Numerical modeling of magnetic devices," *IEEE Trans. Magn.*, vol. 40, no. 4, pp. 1803–1809, Jul. 2004.

[26] E. G. Strangas and K. R. Theis, "Shaded pole motor design and evaluation using coupled field and circuit equations," *IEEE Trans. Magn.*, vol. MAG-21, no. 5, pp. 1880–1882, Sep. 1985.

[27] E. G. Strangas, "Coupling the circuit equations to the non-linear time dependent field solution in inverter driven induction motors," *IEEE Trans. Magn.*, vol. MAG-21, no. 6, pp. 2408–2411, Nov. 1985.

[28] S. L. Ho, H. L. Li, and W. N. Fu, "Inclusion of interbar currents in a network-field coupled time-stepping finite-element model of skewed-rotor induction motors," *IEEE Trans. Magn.*, vol. 35, no. 5, pp. 4218–4225, Sep. 1999.

[29] S. L. Ho, H. L. Li, W. N. Fu, and H. C. Wong, "A novel approach to circuit-field-torque coupled time stepping finite element modeling of electric machines," *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1886–1889, Jul. 2000.

[30] S. L. Ho, W. N. Fu, H. L. Li, H. C. Wong, and H. Tan, "Performance analysis of brushless DC motors including features of the control loop in the finite element modeling," *IEEE Trans. Magn.*, vol. 37, no. 5, pp. 3370–3374, Sep. 2001.

[31] A. Arkkio, "Finite element analysis of cage induction motors fed by static frequency converters," *IEEE Trans. Magn.*, vol. 26, no. 2, pp. 551–554, Mar. 1990.

[32] E. Vassent, G. Meunier, and A. Foggia, "Simulation of induction machine using complex magnetodynamic finite element method coupled with the circuits equations," *IEEE Trans. Magn.*, vol. 27, no. 5, pp. 4246–4249, Sep. 1991.

[33] E. Vassent, G. Meunier, A. Foggia, and G. Reyne, "Simulation of induction machine operation using a step by step finite element method coupled with circuits and mechanical equations," *IEEE Trans. Magn.*, vol. 27, no. 6, pp. 5232–5234, Nov. 1991.

[34] S. L. Ho and W. N. Fu, "A comprehensive approach to the solution of direct-coupled multislice model of skewed rotor induction motors using time-stepping eddy-current finite element method," *IEEE Trans. Magn.*, vol. 33, no. 3, pp. 2265–2272, May 1997.

[35] S. L. Ho, S. Yang, M. Rahman, and H. C. Wong, "Transient analysis of PWM inverter-fed AC motor drives using finite element method coupling with external circuit model," in *Proc. IEEE Int. Conf. Power Electronics and Drive Systems*, 1999, pp. 591–596.

[36] Y. H. Pham, P. F. Wendling, S. J. Salon, and H. Acikgoz, "Transient finite element analysis of an induction motor with external circuit connections and electromechanical coupling," *IEEE Trans. Energy Convers.*, vol. 14, no. 4, pp. 1407–1412, Dec. 1999.

[37] C. Grabner, "Coupled electromagnetic-mechanical finite element approach for the transient analysis of squirrel cage induction motors," in *Proc. Can. Conf. Electrical and Computer Engineering*, May 2005, pp. 462–469.

[38] F. Hecht and A. Marrocco, "A finite element simulation of an alternator connected to a nonLinear external circuit," *IEEE Trans. Magn.*, vol. 26, no. 2, pp. 964–967, Mar. 1990.

[39] N. M. Abe and J. R. Cardoso, "Coupling electric circuit and 2D-FEM model with Dommel's approach for transient analysis," *IEEE Trans. Magn.*, vol. 34, no. 5, pp. 3487–3490, Sep. 1998.

[40] J. S. Wang, "A nodal analysis approach for 2D and 3D magnetic-circuit coupled problems," *IEEE Trans. Magn.*, vol. 32, no. 3, pp. 1074–1077, May 1996.

[41] M. C. Costa, S. I. Nabeta, N. M. Abe, and J. R. Cardoso, "A nodal analysis approach applied to electric circuits coupling in magnetodynamic 2D FEM," in *Proc. 11th Conf. Computation of Electromagnetic Fields*, 1997, vol. 2, pp. 753–754.

[42] M. C. Costa, R. H. Mendes, S. I. Nabeta, and J. R. Cardoso, "Simulation of induction machine by finite element method coupled with electric circuits using the modified nodal analysis," presented at the 8th Bienn. IEEE Conf. Electromagnetic Fields Computation, 1998.

[43] M. C. Costa, S. I. Nabeta, and J. R. Cardoso, "Modified nodal analysis applied to electric circuits coupled with FEM in the simulation of a universal motor," *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1431–1434, Jul. 2000.

[44] W. N. Fu, D. Lin, S. Stanton, and Z. J. Cendes, "Modeling of solid conductors in two-dimensional transient finite-element analysis and its application to electric machines," *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 426–434, Mar. 2004.

[45] D. J. A. Welsh and M. B. Powell, "An upper bound for the chromatic number of a graph and its applications to the timetabling problems," *Comput. J.*, vol. 10, pp. 85–86, 1967.

[46] B. Davat, M. Lajoie-Mazenc, and J. Hector, "Magnetic structure and feeding circuit modelling," *IEEE Trans. Magn.*, vol. MAG-19, no. 6, pp. 2471–2473, Nov. 1983.

[47] N. Sadowski, R. Carlson, S. R. Arruda, C. A. da Silva, and M. Lajoie-Mazcnc, "Simulation of single-phase induction motor by a general method coupling field and circuit equations," *IEEE Trans. Magn.*, vol. 31, no. 3, pp. 1908–1911, May 1995.

[48] F. Piriou and A. Razek, "Finite element analysis in electromagnetic systems accounting for electric circuits," *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1669–1675, Mar. 1993.

[49] P. Kuo-Peng, N. Sadowski, J. P. A. Bastos, R. Carlson, and N. J. Batistela, "A general method for coupling static converters with electromagnetic structures," *IEEE Trans. Magn.*, vol. 33, no. 2, pp. 2004–2009, Mar. 1997.

[50] P. Kuo-Peng, J. P. A. Bastos, N. Sadowski, and R. Carlson, “Analysis of a combined converter-electromagnetic device by taking into account its control loop,” *IEEE Trans. Energy Convers.*, vol. 14, no. 4, pp. 1430–1434, Dec. 1999.

[51] J. L. R. Ortiz, N. Sadowski, P. Kuo-Peng, N. J. Batistela, and J. P. A. Bastos, “Coupling static converter with control loop and non-linear electromagnetic devices,” *IEEE Trans. Magn.*, vol. 37, no. 5, pp. 3514–3517, Sep. 2001.

[52] P. Kuo-Peng, N. Sadowski, N. J. Batistela, and J. P. A. Bastos, “Coupled field and circuit analysis considering the electromagnetic device motion,” *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1458–1461, Jul. 2000.

[53] D. Deas, P. Kuo-Peng, N. Sadowski, A. M. Oliveira, J. L. Roel, and J. P. A. Bastos, “2-D FEM modeling of the tabular linear induction motor taking into account the movement,” *IEEE Trans. Magn.*, vol. 38, no. 2, pp. 1165–1168, Mar. 2002.

[54] H. De Gersem, R. Mertens, U. Pahner, R. Belmans, and K. Hameyer, “A topological method for field-circuit coupling,” *IEEE Trans. Magn.*, vol. 34, no. 5, pp. 3190–3193, Sep. 1998.

[55] H. De Gersem, R. Mertens, D. Lahaye, S. Vandewalle, and K. Hameyer, “Solution strategies for transient, field-circuit coupled systems,” *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1531–1534, Jul. 2000.

[56] A. M. Oliveira, P. Kuo-Peng, N. Sadowski, M. S. de Andrade, and J. P. A. Bastos, “A non-a priori approach to analyze electrical machines modeled by FEM connected to static converters,” *IEEE Trans. Magn.*, vol. 38, no. 2, pp. 933–936, Mar. 2002.

[57] J. F. Charpentier, Y. Lefevre, and H. Piquet, “An original and natural method for coupling electromagnetic field equations with circuit equations put in a state form,” *IEEE Trans. Magn.*, vol. 34, no. 5, pp. 2489–2492, Sep. 1998.

[58] A. M. Oliveira, P. Kuo-Peng, M. V. F. da Luz, N. Sadowski, and J. P. A. Bastos, “Generalization of coupled circuit-field calculation for polyphase structures,” *IEEE Trans. Magn.*, vol. 37, no. 5, pp. 3444–3447, Sep. 2001.

[59] S. Kanerva, S. Seman, and A. Arkkio, “Simulation of electric drive with coupled finite element analysis and system simulator,” in *Proc. 10th Eur. Conf. Power Electronics and Applications*, Toulouse, France, Sep. 2003.

[60] S. Kanerva, J. Kaukonen, A. Szucs, and T. Hautamaki, “Coupled FEM-control simulation in the analysis of electrical machines and converters,” in *Proc. 12th Int. Power Electronics and Motion Control Conf.*, Aug. 2006, pp. 1925–1930.

[61] S. Kanerva, “Data transfer methodology between a FEM program and a system simulator,” in *Proc. 5th Int. Conf. Electrical Machines and Systems*, Aug. 2001, vol. 2, pp. 1121–1124.

[62] S. Williamson and M. C. Begg, “Analysis of cage induction motors—A combined fields and circuits approach,” *IEEE Trans. Magn.*, vol. MAG-21, no. 6, pp. 2396–2399, Nov. 1985.

[63] S. Williamson, L. H. Lim, and M. J. Robinson, “Finite-element models for cage induction motor analysis,” *IEEE Trans. Ind. Appl.*, vol. 26, no. 6, pp. 1007–1017, Dec. 1990.

[64] S. Williamson and A. F. Volschenk, “Time-stepping finite element analysis for a synchronous generator feeding a rectifier load,” *Proc. Inst. Elect. Eng., Elect. Power Appl.*, vol. 142, no. 1, pp. 50–56, Jan. 1995.

[65] F. Piriou and A. Razek, “Calculation of saturated inductances for numerical simulation of synchronous machines,” *IEEE Trans. Magn.*, vol. MAG-19, no. 6, pp. 2628–2631, Nov. 1983.

[66] G. Bedrosian, “A new method for coupling finite element field solutions with external circuits and kinematics,” *IEEE Trans. Magn.*, vol. 29, no. 2, pp. 1664–1668, Mar. 1993.

[67] T. W. Preston, A. B. Reece, and P. S. Sangha, “Analysis of switched reluctance drives by the finite element time-stepping method,” in *Proc. 5th Int. Conf. Electrical Machines and Drives*, Sep. 1991, pp. 81–85.

[68] T. W. Preston, J. P. Sturgess, and M. Tarkanyi, “Finite-element simulation of a generator feeding into a rectifier load,” in *Proc. 5th Int. Conf. Electrical Machines and Drives*, Sep. 1991, pp. 311–315.

[69] L. Xu and E. Ruckstadter, “Direct modeling of switched reluctance machine by coupled field-circuit method,” *IEEE Trans. Energy Convers.*, vol. 10, no. 3, pp. 446–454, Sep. 1995.

[70] N. A. Demerdash and P. Baldassari, “A combined finite element-state space modeling environment for induction motors in the ABC frame of reference: The no-load condition,” *IEEE Trans. Energy Convers.*, vol. 7, no. 4, pp. 698–707, Dec. 1992.

[71] P. Baldassari and N. A. Demerdash, “A combined finite element-state space modeling environment for induction motors in the ABC frame of reference: The blocked-rotor and sinusoidally energized load conditions,” *IEEE Trans. Energy Convers.*, vol. 7, no. 4, pp. 710–719, Dec. 1992.

[72] N. A. Demerdash, J. F. Bangura, and A. A. Arkadan, “A time-stepping coupled finite element-state space model for induction motor drives Part I: Model formulation and machine parameter computation,” *IEEE Trans. Energy Convers.*, vol. 14, no. 4, pp. 1465–1471, Dec. 1999.

[73] J. F. Bangura, F. N. Isaac, N. A. Demerdash, and A. A. Arkadan, “A time-stepping coupled finite element-state space model for induction motor drives Part II: Machine performance computation and verification,” *IEEE Trans. Energy Convers.*, vol. 14, no. 4, pp. 1472–1478, Dec. 1999.

[74] J. F. Bangura and N. A. Demerdash, “Simulation of inverter-fed induction motor drives with pulse-width modulation by a time-stepping coupled finite element-flux linkage-based state space model,” *IEEE Trans. Energy Convers.*, vol. 14, no. 3, pp. 518–525, Sep. 1999.

[75] A. A. Arkadan and B. W. Kielgas, “The coupled problem in switched reluctance motor drive systems during fault conditions,” *IEEE Trans. Magn.*, vol. 30, no. 5, pp. 3256–3259, Sep. 1994.

[76] F. Deng and N. A. Demerdash, “CFE-SS approach for salient-pole synchronous generators under unbalances,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 33, no. 1, pp. 142–162, Jan. 1997.

[77] J. Vaananen, “Circuit theoretical approach to couple two-dimensional finite element models with external circuit equations,” *IEEE Trans. Magn.*, vol. 32, no. 2, pp. 400–410, Mar. 1996.

[78] M. Ito, K. Kawabata, F. Tajima, and N. Motoi, “Coupled magnetic field analysis with circuit and kinematics modelings of brushless motors,” *IEEE Trans. Magn.*, vol. 33, no. 2, pp. 1702–1705, Mar. 1997.

[79] S. Kanerva, S. Seman, and A. Arkkio, “Inductance model for coupling finite element analysis with circuit simulation,” *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1620–1623, May 2005.

[80] A. Dumitrescu, F. David, V. Burtea, T. Jokinen, and M. Rosu, “Combined simulink-finite element approach to inverter driven induction motor,” in *Proc. 9th Int. Conf. Electric Machines and Drives*, 1999, pp. 401–403.

[81] K. N. Srinivas and R. Arumugam, “Dynamic characterization of switched reluctance motor by computer-aided design and electromagnetic transient simulation,” *IEEE Trans. Magn.*, vol. 39, no. 3, pp. 1806–1812, May 2003.

[82] E. Melgoza, J. L. Guardado, and V. Venegas, “A method for coupling electromagnetic transients programs with finite element magnetic field solvers,” in *Proc. Int. Conf. Power System Transients*, Montreal, QC, Canada, Jun. 19–23, 2005.

[83] S. Dennetiere, Y. Guillot, J. Mahseredjian, and M. Rioual, “A link between EMTP-RV and FLUX3D for transformer energization studies,” presented at the Int. Conf. Power System Transients, Lyon, France, Jun. 4–7, 2007.

[84] S. Abourida, C. Dufour, and J. Belanger, “Hardware-in-the-loop simulation of finite-element based motor drives with RT-LAB and JMAG,” in *Proc. IEEE Int. Symp. Industrial Electronics*, Jul. 2006, vol. 3, pp. 2462–2466.

[85] P. Zhou, D. Lin, W. N. Fu, B. Ionescu, and Z. J. Cendes, “A general cosimulation approach for coupled field-circuit problems,” *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 1051–1054, Apr. 2006.

[86] N. A. Demerdash and T. W. Nehl, “Electric machinery parameters and torques by current and energy perturbations from field computations—Part I: Theory and formulation,” *IEEE Trans. Energy Convers.*, vol. 14, no. 4, pp. 1507–1513, Dec. 1999.

[87] N. A. Demerdash and T. W. Nehl, “Electric machinery parameters and torques by current and energy perturbations from field computations—PartII: Applications and results,” *IEEE Trans. Energy Conversion*, vol. 14, no. 4, pp. 1514–1522, Dec. 1999.

[88] R. Escarela-Perez, E. Campero-Littlewood, M. A. Arjona-Lopez, and A. Laureano-Cruces, “Comparison of two techniques for two-dimensional finite-element inductance computation of electrical machines,” *Proc. Inst. Elect. Eng., Elect. Power Applications*, vol. 152, no. 4, pp. 855–861, Jul. 2005.

[89] A. M. Knight, “A comparison of time stepped finite element modelling methods for induction motor analysis,” in *Proc. Int. Conf. Power Electronics, Machines and Drives*, Jun. 2002, pp. 375–380.

[90] S. Kanerva, S. Seman, and A. Arkkio, “Simulation of electric drive systems with coupled finite element analysis and system simulator,” presented at the 10th Eur. Conf. Power Electronics and Applications EPE 2003, Toulouse, France, Sep. 2003, unpublished.

[91] D. Petrichenko, M. Hecquet, P. Brochet, V. Kuznetsov, and D. Laloy, “Design and simulation of turbo-alternator using a coupled permeance network model,” *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 1259–1262, Apr. 2006.

[92] I. Tsukerman, “A stability paradox for time-stepping schemes in coupled field-circuit problems,” *IEEE Trans. Magn.*, vol. 31, no. 3, pp. 1857–1860, May 1995.