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Spontaneous Breakdown of Isospin Symmetry in Nuclei and Isobaric Analog States

By

Ivan Christov Danchev ©

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Spontaneous Breakdown of Isospin Symmetry and Isobaric Analog States" submitted by Ivan Christov Danchev in partial fulfilment of the requirements for the degree of Master of Science in Theoretical Physics

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Abstract

Assuming that the SU(2)-isospin symmetry in nuclei is spontaneously broken to U(1) we derive the observed splitting in the dispersion relations of protons and neutrons in nuclei and show that the isobaric analog state is the Nambu-Goldstone boson of the broken symmetry. The explicit isospin symmetry breaking caused by the Coulomb interaction explains the observed displacement of the isobaric analog state with respect to the parent ground state.

Acknowledgements

I am very grateful to my supervisor, Prof. Khanna, for taking me as his student, teaching me the nuclear many-body problem and offering me to work on such an interesting project as the spontaneous breakdown of isospin symmetry in nuclei. It also turned out to be a subject that was little studied and prejudices prevailed in the literature. Without Prof. Khanna's knowledge and advice I would have easily be mislead. Without his encouragement and financial support this work would have never been accomplished.

I am also grateful to Prof. H. Umezawa for teaching me the essentials of his remarkable theory of symmetry rearrangement in Nature and for helping me apply it in nuclei. I appreciate his wisdom, kindness and patience in dealing with me and consider myself lucky that I had a chance to meet him. I was truly amazed how much this man knew and yet (or maybe because of that) was not prejudiced by his knowledge. Explained by him concepts that I had found difficult to understand seemed easy, vivid and .. exciting. Exactly, exciting because after seeing the picture drawn by others, after marveling at its beauty one notices that there are details one can add or correct and ... isn't that exciting ? I had these rare moments of gazing into the emptiness, smiling, hearing Umezawa's words, smiling ...and being entirely happy.

For this rare happiness I would like to thank Prof. Khanna and Prof. Umezawa. Thank you.

Living for two years away from my fatherland among the multicultural community of the Physics Department I felt never alone. I got to know many people of different background and to me they all became embodiment of their beautiful countries: Sharon and Jannifer for beautiful, beautiful Canada, Bahman for wise and modest Iran, Alick for wild and adventurous New Zealand and Catherine for the finnesse of France, Pat for kind and clever Ireland, Beata, Jana and Tomas for sunny and warm Slovakia and Chekhia, Biao and Hanyou for the ancient talent of China, Alexei, Alex and Andrei for the vast undiscovered potential of Russia, Alfio for the amazing talant of Italy and Pierre for the cultural richness of french speaking Canada. And again: Warren for ever helping Canada, Kevin for compassionate Canada, Gloria for proud Canada, Andrzeij and Weronika for the noble and beautiful Poland and ... the list could be continued however not exhausted. I am happy that I met these people.

I would also like to thank the Department of Physics for the kind hospitality extended to me during these years. I appreciate the efforts of Prof. Sherif, Mrs. Lynn Chandler and Mr. Mark Henderson to ease the financial pressure on my family. I am also grateful to Mrs. Marion Brady and Mrs. Moira Hamilton for their excellent job as librarians and to Mrs. Phyllis Tripp for copying more articles than I could read and understand.

During my stay I missed my parents very much and I failed to be around them in difficult times but I tried to be to my family what my Mother and Father have been to me.

I am delighted with my wife and son. They have never failed to understand and forgive. To them is this thesis dedicated.

To Maria and Christo,
Who spent many evenings alone,
While I studied the Symmetry,
And loved and cared for me
Until the last day.

Edmonton, July 22 1993

The world breaks every one and afterward many are strong at the broken places. But those that will not break it kills. It kills the very good and the very gentle and the very brave impartially. If you are none of these you can be sure it will kill you too but there will be no special hurry.

E. Hemingway, "A Farewell to the Arms", 1929.

Contents

Introduction

While the existing approaches to the nuclear many-body problem exploit the assumption of SU(2)-isospin invariance of the effective nucleon-nucleon interaction, they do not explain why (in apparent defiance of the SU(2)-isospin invariance) protons and neutrons feel different effective potentials in the nucleus (See. fig.0.1).

The Bethe-Weizsäcker semiempirical binding energy formula carries this inequivalence between protons and neutrons in the (so called) "symmetry term". And derivations of it from microscopic considerations exist: in a Fermi gas [1] and liquid model [2] and the shell model [3]. Although these approaches predict the correct dependence of the symmetry energy on the neutron and proton number, their assumption 'ab initio' is different Fermi momenta for protons and neutrons. Thus the question where from protons and neutron derive their different Fermi momenta in nuclei and how this fact doesn't invalidate consequences of the nuclear SU(2) isospin invariance remains unanswered.

Consider the state obtained from a nucleus in ground state when a neutron is removed and substituted with a proton endowed with the spin and coordinates of the removed neutron.If the SU(2) isospin symmetry were realized in a Wigner-Weyl mode (i.e. as a symmetry of the Lagrangian and of the ground state) the produced nucleus(called daughter) would be degenerate in energy with the initial nucleus (called parent) (fig.0.2a). However the symmetry energy term in the Bethe-Weizsäcker formula predicts that the ground state of the daughter nucleus lies below the ground state of

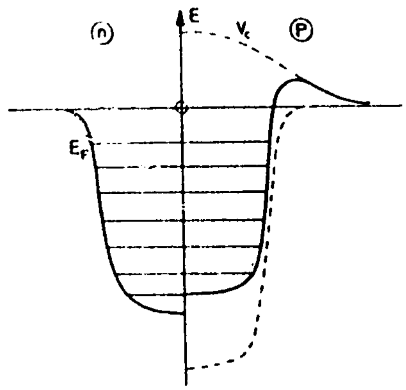

Fig.0.1.Protons and neutrons feel different nuclear effective potential (Copied from [17], *p.47)*

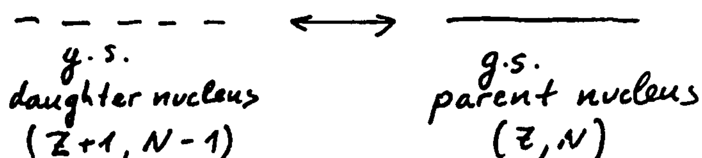

Fig.0.2. a) Imaginary situation: the dotted line shows the expected position of the g.s. of the daughter nucleus if SU(2)-isospin had been a symmetry of the ground state

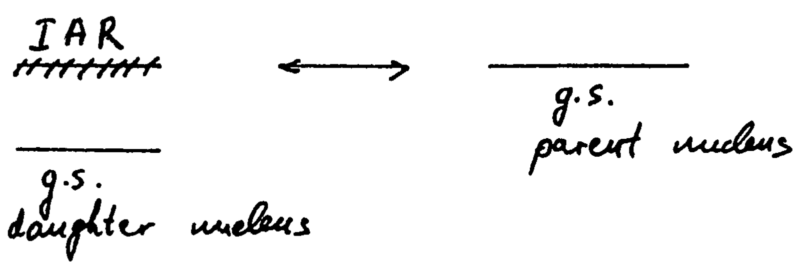

Fig.0.2. b) Actual situation: the continuous line in the daughter nucleus shows the actual position of the g.s. of the daughter nucleus while the isobaric analog state (in the daughter nucleus) lies at the level of the g.s. of the parent nucleus

the parent nucleus (fig.0.2b). Apparently even in the absence of the Coulomb interactions protons and neutrons in nuclei are not equivalent. But, although the ground state of the daughter nucleus is not degenerate with the ground state of the parent, there still exists a collective state in the energy spectrum of the daughter nucleus (fig.2,b), the isobaric analog state (IAS) of ground state of the parent nucleus, which, in the absence of the Coulomb interactions, is degenerate in energy with the ground state of the parent nucleus (and therefore carries the SU(2) symmetry).

Is it a coincidence that the symmetry lost in the quasiparticle spectrum (protons and neutrons feel different effective potential in the nucleus) is recovered though the collective excitation - the isobaric analog resonance?

A similar situation exists in ferromagnetism: as a consequence of the dynamical symmetry rearrangement of SU(2) spin symmetry the spin-up quasielectrons differ in their dispersion relations from the spin-down quasielectrons and a collective state (magnon) appears as a Nambu-Goldstone boson of the broken symmetry.

Therefore the challenge is to try to explain the "symmetry energy" term in Bethe-Weizsäcker formula (or the asymmetric nuclear potential for protons and neutrons) and the appearance of the isobaric analog states as deriving from spontaneous breakdown of nuclear isospin symmetry: protons and neutrons being the quasiparticles, which split in their dispersion relations, while the isobaric analog resonance is the Nambu-Goldstone boson, which recovers the broken symmetry.

This work is organized as follows:

Chapter 1 introduces the concept of charge symmetry and charge independence and discusses the symmetry of the ground state of nuclei.

Chapter 2 discusses the essentials of Umezawa's [6] self-consistent treatment of spontaneously broken (or rearranged) symmetries on the example of SU(2) isospin symmetry.

Chapter 3 describes the features of the isobaric analog resonances relevant to and derivable from the assumption of broken isospin invariance. Reviewed are chapters of Henley & Frauenfelder's [5] book and Lemmer's Cargese Lectures (1968) [7].

Chapter 4 develops a schematic contact model of the effective nuclear interactions with dynamically rearranged symmetry of the ground state. Obtained are the dispersion relations of the quasinucleons (protons and neutrons in nuclear matter) and it is argued that the splitting in their energy gives rise to the "symmetry energy" term in the Bethe-Weizsäcker semiempirical mass formula. Mean field approximation is employed to solve the gap equation for the order parameter in the broken symmetry phase and the coefficient of the "symmetry energy term" in the Bethe-Weizsäcker formula is calculated. The isobaric analog state is shown to be the Nambu-Goldstone gapless mode of the broken symmetry. Coulomb interaction, which breaks explicitly the isospin symmetry of the nuclear Hamiltonian provides the observed energy shift of the isobaric analog state with respect to the ground state of the parent nucleus.

In conclusion it is suggested that aspects of the dynamics of the isobaric analog resonances be addressed from the viewpoint of the low energy theorems for Nambu-Goldstone bosons: the narrowness of their decay width attributed to Adler's consistency theorem , while the emerging experimental evidence of relations between the cross sections for excitation of single and double isobaric analog states in charge exchange reactions interpreted as a manifestation of the multiple production theorem for Nambu-Goldstone bosons.

Chapter 1

Isospin Symmetry of Nuclear Interactions and The Ground State of Nuclei

When the experiments of Chadwick revealed the existence of neutron, with its spin equal and mass approximately equal to proton's, the immediate question was where that symmetry came from.

Heisenberg [8] conjectured that the interaction of protons and neutrons in nuclei (invoked to counteract the Coulomb repulsion of protons and keep nuclei from disintegration) should obey a charge symmetry, i.e. a discrete symmetry with respect to exchange of protons with neutrons and vice versa. To exemplify the concept of charge symmetry assumption, consider proton-proton scattering (fig.1.1). Apply charge symmetry to obtain neutron-neutron scattering:

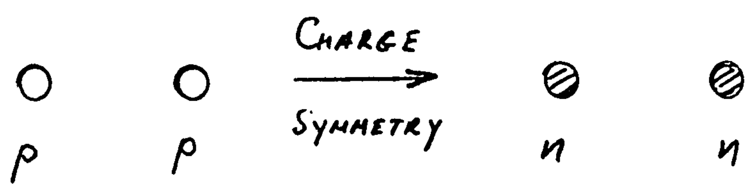

Fig.1.1. Charge symmetry relates the nn-scattering to the pp-scattering

Therefore charge symmetry implies that the strength of the nn- and pp- interaction and consequently the scattering lengths of n-n scattering and p-p scattering are equal (or close, if charge asymetric effects can be treated perturbatively).

Compare this prediction with the experimental evidence for the s-wave scattering lengths

$a_{pp} = -16.6$ to $- 16.9$ fm (corrected for the Coulomb repulsion)

$a_{nn} = -16.4 \div 1.9$ fm

to see the remarkable agreement.

Substantial evidence in support of charge symmetry of nuclear interactions comes from the the scattering of positive and negative pions from deuteron targets: they exhibit approximately equal cross sections after correction for the Coulomb scattering. As the experiment shows the $p\pi^-$ system behaves approximately as its charge symmetric counterpart $n\pi^+$ (and the same applies to $p\pi^+$ and $n\pi^-$).

Important evidence is provided by the structure (see fig.1.2) of excited states in mirror nuclei (i.e. odd A and obtainable from one another by swapping a neutron into proton or vice versa). The correction for the Coulomb repulsion puts the energy levels with respectively assigned quantum numbers in remarkable coincidence.

However the occurrence of π^0 in the π-meson triplet (i.e. showing similar properties: mass, coupling to nucleons etc.) can't be predicted from charge symmetry. To assume that π^0 is a charge singlet leaves open the question why its mass comes close to that the charged pions π^- , π^+. Charge symmetry may be a part of a larger symmetry, but what symmetry ?

As the study of the excited levels of isobar nuclei (i.e. containing the same mass number A) shows the similarity in the respective level repetition and spacing persists in nuclei with the same A, but different ratios of protons and neutrons.(See the triplet on fig.1.3 and the quadruplet on fig.1.5.)

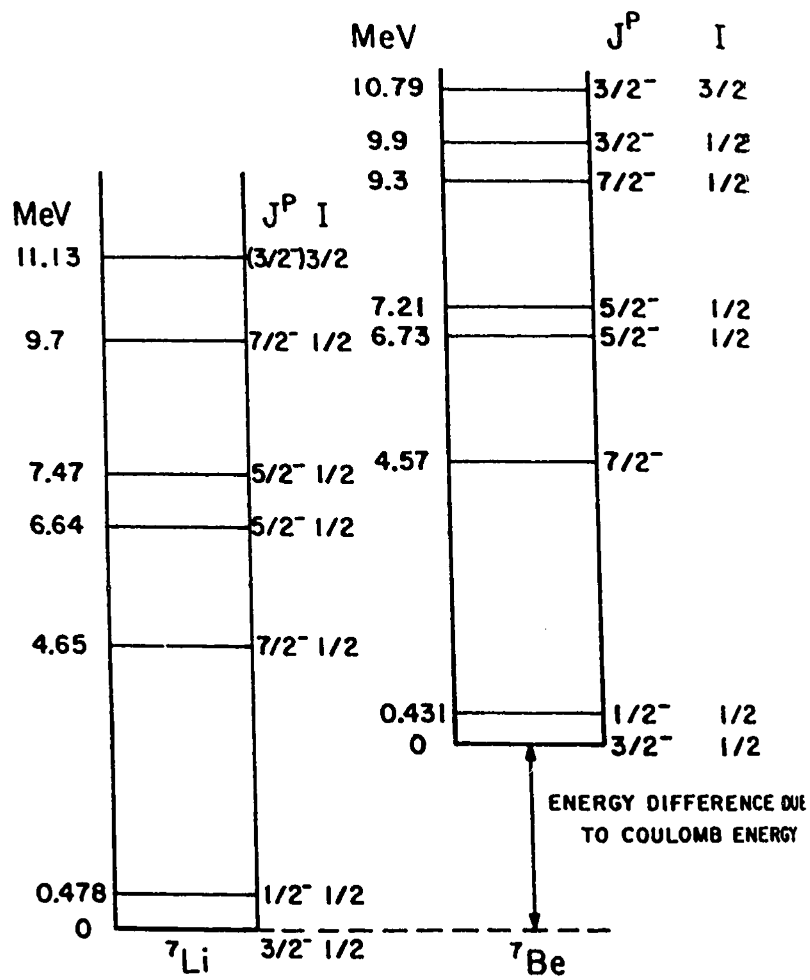

Fig.1.2. Level structure in mirror nuclei (isobars). Spins, parities and isospins of levels are shown.

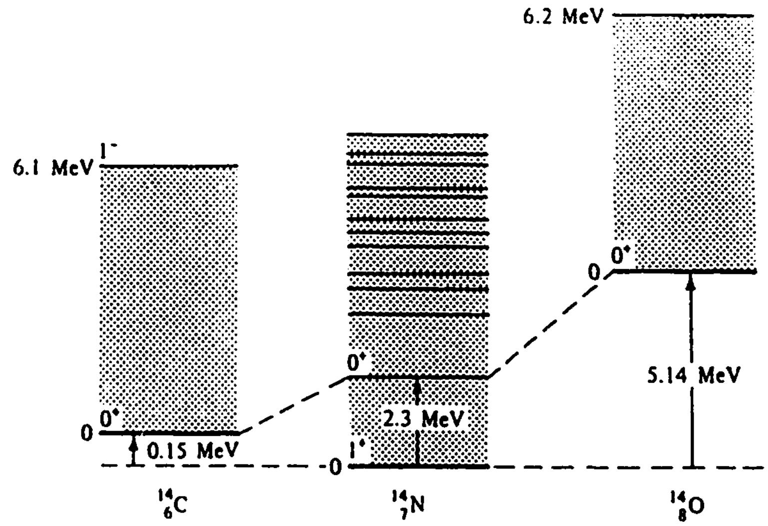

Fig.1.3.Level structure of A=14 isobars. The labels denote spin and parity, for instance 0^+. *The ground state of* ^{14}N *is an isosinglet; the first excited stsate is a member of an isospin triplet.*

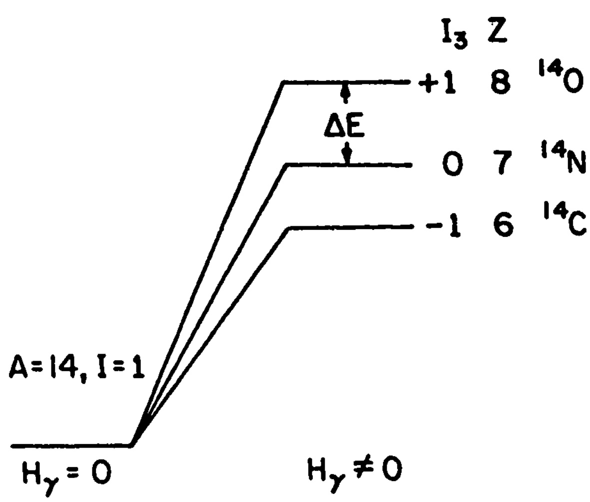

Fig.1.4. Splitting of an isospin triplet by the electromagnetic interaction.

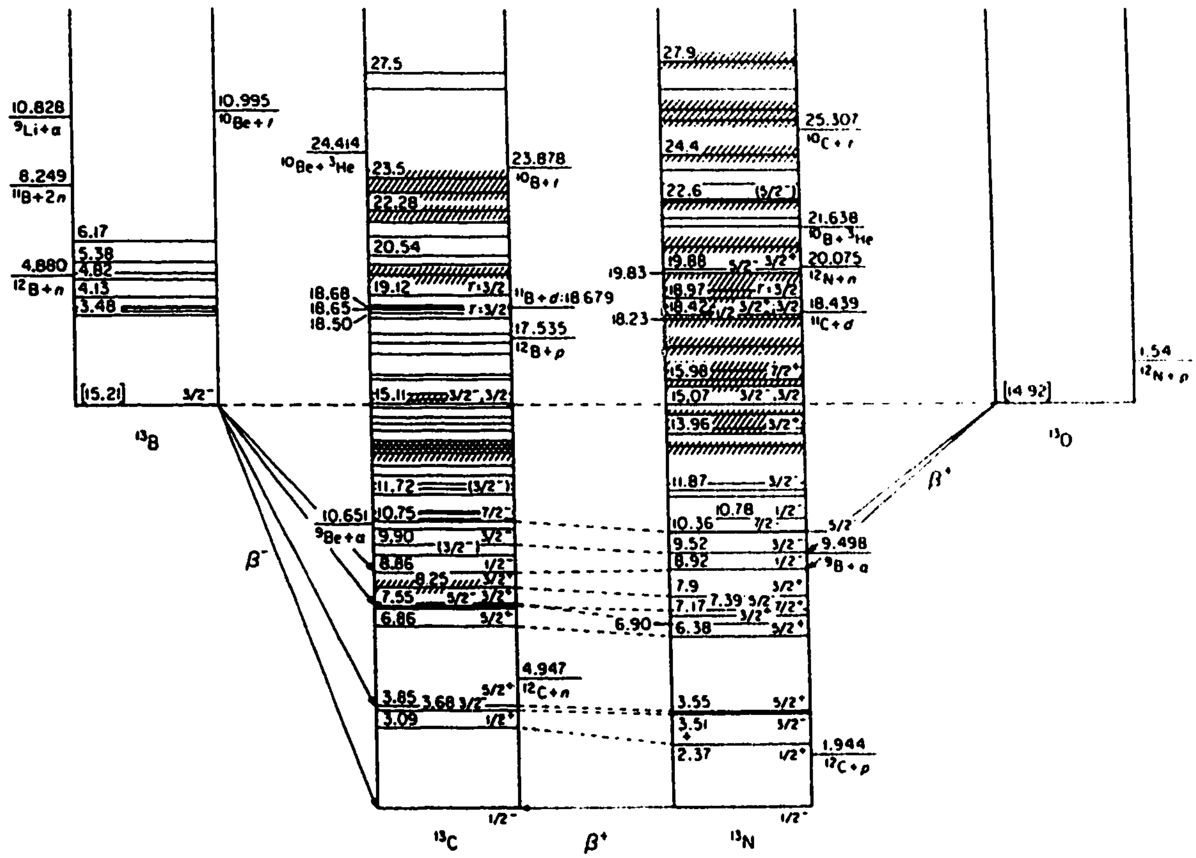

Figure 10.2 Isobar diagram, $A = 13$. The diagrams for individual isobars have been shifted vertically to eliminate the neutron–proton mass difference and the Coulomb energy, taken as $E_c = 0.60Z(Z-1)/A^{1/3}$. Energies in square brackets represent the (approximate) nuclear energy. $E_n = M(Z,A) - ZM(H) - NM(n) - E_c$, minus the corresponding quantity for ^{13}C: here M represents the atomic mass excess in MeV. Levels which are presumed to be isospin multiplets are connected by dashed lines (from *Nuclear Physics*, A152, 32, 1970)

Fig.1.5. Isobar diagram, A=13. Quadruplet states.

Clearly this suggests that not only the pp- and nn- interactions are the same, but also the nuclear pn-interaction comes close. Remark that charge symmetry cannot make any predictions about the strength of np-interaction as compared to that of nn- or pp-interactions, because under charge transformation the p-n pair transforms into itself (fig.1.6)

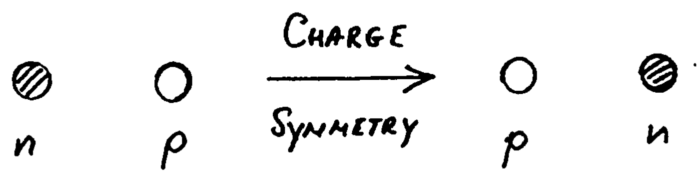

Fig.1.6. Charge symmetry transforms a np-interacting pair into itself and therefore cannot relate the strength of the np-interaction to the pp- or the nn-interactions

However the experimental value for the s-wave scattering length in the singlet state of pn-interaction apparently differs from that of pp- and nn- interactions [10]

$a_{pn} = -23.679 \pm 0.028$ fm

Examining the experimental evidence Breit, Condon and Present [9] proposed that the nn-, np- and pp- interactions are equal provided that the Coulomb interactions are substracted and states with equal orbital and spin numbers are compared.

Indeed, correction for magnetic effects removes about 1 fm of the 7 fm difference between a_{pp} and a_{pn}. The remaining discrepancy implies about 4% difference between the strengths of pp- and np- nuclear interactions [10].

There is as yet no firm evidence of any discrepancy between the pp- and nn- nuclear interactions.

The one-pion exchange part of the nuclear force provides an explanation for the greater accuracy of charge symmetry (see Fig.1.7). Whereas the p-p and n-n pairs both interact by exchanging a π^0, the p-n pair interacts exchanging a π^+ and

π^-. The pion mass difference $(m_+ - m_0)/m_0 \approx 0.03$ will thus have no effect on charge symmetry, but it destroys the charge independence. According to Henley [11], about half of the difference between a_{pp} and a_{np} can be accounted for in this way.

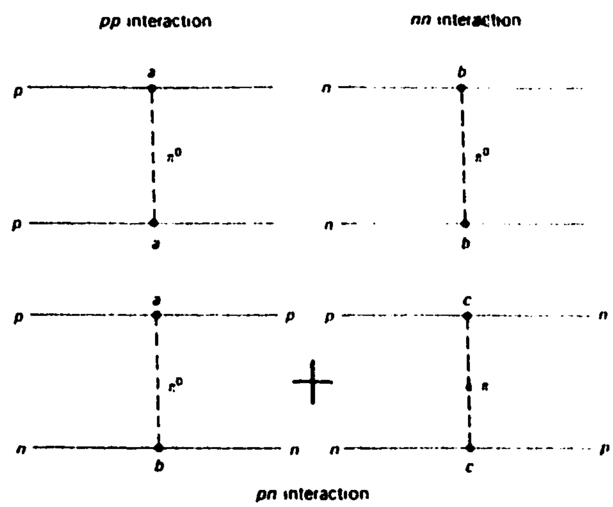

Fig.1.7. Diagrammatic representation of the contribution of the single pion exchange to the pp-, nn- and np- interactions

Formally the concept of charge independence was developed in analogy with the spinor representation of the rotational group. Crucial evidence that SU(2) provides correct description of the symmetry of nuclear forces comes from experiment - the excited states in nuclei are grouped in multiplets according the irreducible representations of SU(2)-isospin group.

Protons and neutrons are considered the doublet (basic) representation of SU(2)

$$| p\rangle = \begin{pmatrix} \uparrow \\ 0 \end{pmatrix} \qquad | n\rangle = \begin{pmatrix} 0 \\ \downarrow \end{pmatrix},$$

Rotations in isospin space are carried by the unitary operator

$$U(\boldsymbol{\alpha}) = \exp\left[i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}\right], \tag{1.1}$$

where τ's are the isospin matrices related to the Pauli matrices by definition by $\tau = \frac{\sigma}{2}$.

Isospins of individual nucleons can be added as vectors in isospin space.

$$\mathbf{T} = \sum_{i=1}^{A} \tau \qquad T_3 = \sum_{i=1}^{A} \tau_{3,i} \tag{1.2}$$

And the isospin charge T_3 is related to the electric charge through

$$Q = \sum_{i=1}^{A} q_i = e\,(T_3 + \frac{A}{2})$$

as the experimental evidence that protons couple to electromagnetic field requires.

Isospin independence of nuclear reactions is expressed through the requirement that the isospin generators commute with the nuclear Hamiltonian.

$$[H_h, \mathbf{T}] = 0 \tag{1.3}$$

The electromagnetic interactions destroy the isotropy of charge space, coupling to protons only, i.e.

$$[H_h + H_\gamma, \mathbf{T}] \neq 0\,, \tag{1.4}$$

where H_γ is the electromagnetic interaction term.

Treating the electromagnetic field as a perturbation to the isospin invariant nuclear Hamiltonian H_h one obtains the splitting of the (otherwise 2T+1 degenerate) nuclear levels in the presence of electromagnetic field (Fig. 1.4)

Weak interactions on nucleons also break the isospin invariance of the Hamiltonian. If one treats them as a perturbation to the invariant part of the Hamiltonian, one can estimate the effect of the breaking.

Obviously, the quest is to attempt to explain the observed deviations from isospin symmetry as due to the explicit breaking of the symmetry by the weak and the electromagnetic forces. The different masses of proton and neutron and between the charged and the neutral mesons can also be treated as explicit sources of symmetry breakdown, although it's believed that this difference can ultimately be attributed the electromagnetic forces.

Leaving aside the very interesting question of the experimental evidence of explicit breakdown of isospin symmetry (refer to[12] for a review), we proceed to ask:

Is the SU(2)-symmetry of the nuclear forces a symmetry of the ground state of nuclei?

Examine the Bethe-Weizsäcker semiempirical formula, proposed to characterize the energy of the gound state of nuclei:

$$B(N,Z) = a_v A + a_s A^{\frac{2}{3}} + a_c \frac{Z^2}{A^{\frac{1}{3}}} + a_I \frac{(N-Z)^2}{A} - \delta(A), \tag{1.5}$$

where $B(N,Z)$ is the binding energy defined as

$$B(N,Z) = M(A) - (NM_n + ZM_p) \tag{1.6}$$

and the values of the coefficients are [1]

a_v=-15.68 MeV (coefficient of volume energy)

a_s= 18.56 MeV (coefficient of surface energy)

a_c= 0.717 MeV (coefficient of coulomb energy)

a_I= 28.1 MeV (coefficient of the symmetry energy)

$$\delta(A) = \left(\begin{array}{c} 34A^{-\frac{3}{4}} \text{ MeV for odd-odd nuclei} \\ 0 \text{ MeV for odd-even nuclei} \\ -34A^{-\frac{3}{4}} \text{ MeV for even-even nuclei} \end{array} \right) \text{(coefficient of pairing energy)}$$

Charge symmetry may be a symmetry of the ground state, because for a fixed A and allowing N to exchange value with Z, the energy of the ground state doesn't change, provided that correction for the changed Coulomb interaction is made. Symmetry of the ground state implies degenerate energy levels, however the reverse is not always true.

If SU(2)-isospin symmetry were a symmetry of the nuclear ground state, then the energy of the ground state would not change under rotations in the isospin space.

Consider the effect of rotation in the isospin space on the ground state energy as described by the Bethe-Weizsäcker formula.

As the Bethe-Weizsäcker formula doesn't accommodate continuous changes of the direction of **T**, ($|T| = A$), where A is the mass number, consider only rotations in discrete steps (see Fig.1.8). Then the change in the projection of **T** describes going from a nucleus (Z, N) to a neighbouring nuclei $(Z+1, N-1)$, $(Z-1, N+1)$, $(Z+2, N-2)$ etc.

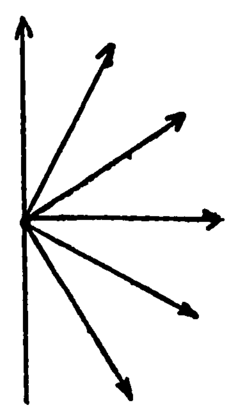

Fig.1.8. Discrete rotations in isospin space correspond to going from one nucleus to another in an isobar multiplet

Obviously the energy of the ground state changes, the variance coming from the

$$a_c \frac{Z^2}{A^{\frac{1}{3}}} \quad \text{and} \quad a_I \frac{(N-Z)^2}{A} \tag{1.7}$$

terms.

Leaving aside the first term as coming from the explicit breaking of the isospin symmetry by the Coulomb repulsion of protons in nuclei, we discuss the term

$$a_I \frac{(N-Z)^2}{A}, \tag{1.8}$$

i.e. the so called "symmetry energy" term in Bethe-Weizsäcker semiempirical mass formula.

Its non-invariance may be attributed to the inherent non-invariance of the ground state of the nucleus under SU(2) isospin transformation, i.e. the SU(2) may be broken in the ground state (spontaneous breakdown of symmetry).

It might be objected that to argue about the exact symmetry of the true ground state of a system on the basis of an approximate formula is questionable. But how can an asymmetric approximation be a *good* approximation to a symmetric state? Anyway, we were unable to find a better approximation for the energy of the ground state that is a symmetric approximation.

Therefore we conjecture that the ground state of heavy nuclei violates the SU(2)-isospin symmetry and explain in Chapter 2 how this can happen. We stress upon *two* ingredients of the mechanism of symmetry rearrangement and state that *both have been observed* in nuclei: the *splitting* in the dispersion relations of the quasiparticles and the *appearance* of a gapless collective mode.

To prepare for the ensuing discussion we explain the meaning of the words "splitting in the dispersion relations of the quasiparticles" and "gapless collective mode" on an example from ferromagnetism.

As argued by Umezawa et al. in [13], the ferromagnetic phase transition, in which the spin-up electrons are separated from the spin-down electrons by an energy gap (see fig. 1.9) and the appearance of the spin wave, is described as a dynamical symmetry rearrangement [14],[13]. The order parameter, which in a translationally invariant theory is a spatial constant, is related to the energy gap, while the collective mode (spin wave) is the zero mode (gapless) Nambu-Goldstone boson.

We proceed with discussion of the nuclear isospin phase transition stressing upon the similarity of fig.1.9 (the splitting in the dispersion relations of the quasi-electrons in a ferromagnetic) and fig.1.10, which is related to the observed and used initial approximation for the proton and neutron wave functions as deriving from an asymmetric one-body nuclear potential.

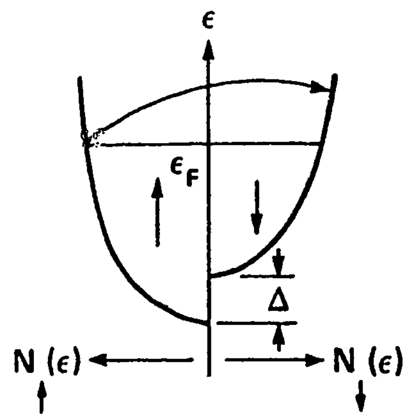

Fig.1.9. Splitting in the dispersion relations of spin-up and spin-down quasielectrons in ferromagnets.(Copied from [16]*,p.21)*

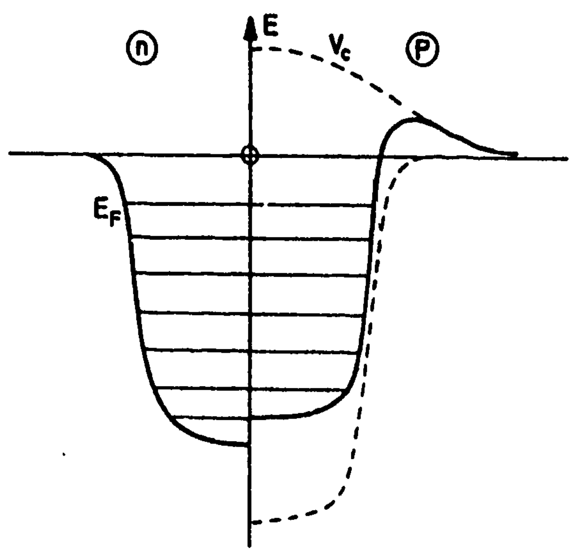

Fig.1.10. Splitting in the dispersion relations of protons and neutrons in nuclei.(Copied from [17]*, p.47)*

It is well known[15] that this asymmetry (the gap) in the effective nuclear potential for protons and neutrons is included in the Bethe-Weizsäcker semiempirical formula in the so called, "symmetry energy" term. Indeed, defining μ_n and μ_p as the neutron and proton chemical potentials

$$\mu_n \equiv \frac{\partial E}{\partial N} = -\frac{\partial B(N,Z)}{\partial N} \qquad \mu_p \equiv \frac{\partial E}{\partial Z} = -\frac{\partial B(N,Z)}{\partial Z} \tag{1.9}$$

one can prove that

$$-(\mu_n - \mu_p) = \frac{\partial B(N,Z)}{\partial N}_{A=const} = 4a_I\frac{(N-Z)}{A}, \tag{1.10}$$

which explains why (in the absence of Coulomb interactions) proton in heavy nuclei (N > Z) seems to sit in a potential well, which is deeper than that of neutron.(See fig.1.10)

Assuming that, indeed, the splitting of the dispersion relations of proton and neutron in nuclei is a sign of dynamical symmetry rearrangement, we seek the other ingredient of the phenomenon - the Nambu-Goldstone boson.

And we find it under the name of Isobaric Analog Resonance (IAR). Before we proceed to a discussion of the properties of these remarkable resonances in nuclei we devote a chapter on the general features of the dynamical symmetry rearrangement.

Chapter 2

Dynamical Rearrangement of Isospin Symmetry

Umezawa[19] and collaborators developed a consistent approach to quantum field theories with symmetry rearrangement of the ground state and applied it to pion dynamics [20], superconductivity [21], superfluidity, ferro- and para-magnetism [13] at zero and finite temperature [22].

To distinguish between the concepts of observed symmetry and symmetry of the Lagrangian Umezawa introduced the concept of dynamical map [6] between the Heisenberg (unobserved) fields and the (observed) asymptotic fields which are solutions of the linearized Heisenberg equations with boundary conditions consistent with the observational symmetry.

Dynamical map is the weak relation (i.e. satisfied for the matrix elements of the operators) between the Heisenberg (interpolating) fields and the physical (asymptotic or in-) fields, called quasiparticles

$$\phi_H(x) \stackrel{w}{=} \Phi\left[\phi^{in}(x)\right], \tag{2.1}$$

where the functional Φ is defined in section 1.2. We now discuss the implications of the weak relation, which are independent of the choice of Φ.

Suppose that the Lagrangian of the system remains unchanged (i.e. exhibits a symmetry) when the Heisenberg fields $\phi_H(x)$ are transformed to $\phi'_H(x)$ under a group transformation F, i.e.

$$\mathcal{L}\left[\phi'_H(x)\right] = \mathcal{L}\left[\phi_H(x)\right] \qquad , \quad \text{when} \quad \phi'_H(x) = F\left[\phi_H(x)\right] \tag{2.2}$$

Do the physical fields $\phi^{in}(x)$ which through the dynamical map (1.1) induce the F- transformation of the Heisenberg fields necessarily transform as

$$\phi^{in'}(x) = F\left[\phi^{in}(x)\right] \tag{2.3}$$

or the $\phi^{in}(x)$ may transform under a different transformation G

$$\phi^{in'}(x) = G\left[\phi^{in}(x)\right] \tag{2.4}$$

and *yet induce* through the dynamical map (1.1) an F-transformation of the Heisenberg fields (1.2)? Or in other words we seek to determine G from

$$F\left[\phi_H(x)\right] \stackrel{w}{=} \Phi\left[G\left[\phi^{in}(x)\right]\right], \tag{2.5}$$

where Φ is given and F is a specified transformation of the Heisenberg fields which leaves the Lagrangian 2.2 unchanged.

Because the dynamical map is a weak relation, it does not fix the transformation of the quasiparticles, when the transformation of the Heisenberg fields is chosen.

If the observed fields are F-transformed when the Lagrangian exhibits a symmetry under the F-transformation of the Heisenberg fields, it is said that the symmetry is realized in a Wigner-Weyl mode or as an unbroken symmetry.

If the observed fields transform under a different group G, when the Lagrangian is invariant under a F-transformation of the Heisenberg fields, then it is said that the F-symmetry of the Lagrangian is rearranged into a G-symmetry of the observed fields. The symmetry rearrangement is accompanied by the appearance of a nonzero vacuum expectation value of a certain local operator, called the order parameter.

Because the transformation of the quasiparticle fields induces through the dynamical map the transformation of the Heisenberg fields, G an F are parametrised by equal number of parameters, i.e. degrees of symmetry are not lost during the symmetry rearrangement. But consider what one would think if G and F contain a common subgroup S and one is prejudiced to believe that the symmetry of the Lagrangian should manifest itself as the same symmetry at the observational level. Expecting to find F, one observes only the subgroup of it S realized in a Wigner-Weyl mode, which may mislead one into believing that the F symmetry is spontaneously (for no reason) broken to S at the observational level. A careful analysis, however reveals the missing degrees of symmetry disguised as gapless fields - the Nambu-Goldstone bosons.

To exemplify the dynamical symmetry rearrangement consider a nonrelativistic model of interacting fermion fields with a Lagrangian invariant under a global SU(2) isospin transformation

$$\mathcal{L}\left[\psi'(x)\right] = \mathcal{L}\left[\psi(x)\right], \quad \text{when} \quad \psi'(x) = \exp\left[i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}\right]\psi(x) \tag{2.6}$$

Construct the generating functional of the Green functions

$$W_\varepsilon\left[\eta^\dagger,\eta\right] = \frac{1}{N}\int \mathcal{D}\psi\mathcal{D}\psi^\dagger \exp\{i\int d^4x\left[\mathcal{L}\left[\psi\right] + i\varepsilon\psi^\dagger(x)\tau_3\psi(x) + \eta^\dagger(x)\psi(x) + \psi^\dagger(x)\eta(x)\right]\}, \tag{2.7}$$

where $N = W[0,0]$ and the ε-term is added to explicitly break the SU(2)symmetry of the Lagrangian to U(1) and prescribe the boundary conditions on the Green functions, as explained in [23].

When the limit $\varepsilon \to 0$ is taken, the symmetry of the Lagrangian is recovered, however what remains from this procedure, are certain relations between the Green functions (Ward-Takahashi identities) which allow for solutions from which one can tell what pattern of symmetry breaking was used by the ε-term and how the symmetry of the Lagrangian is rearranged into the observed symmetry of the quasiparticles.

To derive the Ward-Takahashi identities we change variables in the generating functional $\psi(x) \to \exp\left[i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}\right]\psi(x)$ and observe that, because the functional integral does not depend on the integration variables, the α-dependence is fictitious

$$\frac{\partial W}{\partial \alpha} = 0, \tag{2.8}$$

from where the (basic) Ward-Takahashi identities follow

$$\int d^4x \langle \eta^\dagger(x)\tau_3\psi(x) - \psi^\dagger(x)\tau_3\eta(x)\rangle_{\eta^\dagger,\eta,\varepsilon} = 0 \tag{2.9}$$

$$\int d^4x \langle \eta^\dagger(x)\tau_+\psi(x) - \psi^\dagger(x)\tau_+\eta(x)\rangle_{\eta^\dagger,\eta,\varepsilon} = -i\varepsilon \int d^4x \langle \tau_+(x)\rangle_{\eta^\dagger,\eta,\varepsilon} \tag{2.10}$$

$$\int d^4x \langle \eta^\dagger(x)\tau_-\psi(x) - \psi^\dagger(x)\tau_-\eta(x)\rangle_{\eta^\dagger,\eta,\varepsilon} = +i\varepsilon \int d^4x \langle \tau_-(x)\rangle_{\eta^\dagger,\eta,\varepsilon} \tag{2.11}$$

The Ward-Takahashi identities allow to define the order parameter, a quantity of vital importance for theories displaying symmetry rearrangement of the ground state. Motivated by the consequences for the observables, which we discuss later, we choose

$$\langle \psi^\dagger(x)\tau_3\psi(x)\rangle_\varepsilon \tag{2.12}$$

as an order parameter.

By taking $\frac{\delta}{i\delta\eta^\dagger(y)}$ from 2.10, multiplying by τ_-, taking $\frac{\delta}{i\delta\eta(y)}$ with contraction of the isospin indices and using the isospin algebra one can derive

$$\langle \psi^\dagger(x)\tau_3\psi(x)\rangle_\varepsilon = -\varepsilon \int d^4y \langle \psi^\dagger(y)\tau_+\psi(y), \psi^\dagger(x)\tau_-\psi(x)\rangle_\varepsilon \tag{2.13}$$

If translational invariance of fermion Green functions is assumed, the order parameter is a spatial constant, depending on ε

$$\langle \psi^\dagger(x)\tau_3\psi(x)\rangle_\varepsilon = v_\varepsilon \tag{2.14}$$

Consider the limit $\varepsilon \to 0$

$$v \equiv \lim_{\varepsilon\to 0} v_\varepsilon = -\lim_{\varepsilon\to 0} \varepsilon \int d^4y \langle \tau_+(y)\tau_-(x)\rangle, \tag{2.15}$$

where $\tau_\pm(x) \equiv \langle \psi^\dagger(x)\tau_\pm\psi(x)\rangle$, etc.

Whether or not the symmetry is rearranged is decided by the vacuum expectation value of the order parameter.

If $v = 0$, then any finite integral on the right hand side of 2.14 will suffice to make it an equality in the limit $\varepsilon \to 0$ As discussed in Section 2 the symmetry is of the Heisenberg fields appears as a symmetry in quasiparticle fields (Wigner-Weyl mode of symmetry realization).

However if $v \neq 0$, then a gapless boson $(w(\mathbf{q})_{|\mathbf{q}|\to 0} \to 0)$ is required to make the equation 2.15 consistent (Goldstone Theorem), i.e. a collective excitation whose spectral function

$$\Delta(q, \mathbf{q}) = \frac{\chi(q^2)}{q - \omega(\mathbf{q}) + i\varepsilon} + continuum \qquad (2.16)$$

must contain a pole at zero (See Appendix A for derivation).

As explained earlier, the gapless mode (called Nambu-Goldstone boson) carries the degrees of symmetry which the Lagrangian loses when the ε-term is specified and which do not show up as the same symmetries in the quasiparticle sector after the symmetry of the Lagrangian is recovered through $\varepsilon \to 0$. The symmetry of the Heisenberg fields is rearranged into a different symmetry of the observed quasiparticles.

When no translational invariance of the fermion Green functions is assumed the order parameter may be observed as a collective excitation (as is the case with the σ-boson in the chiral dynamics of pions).

$$\phi(x) = \langle \psi^\dagger(x) \tau_3 \psi(x) \rangle \qquad (2.17)$$

To account for the appearance of collective excitations at the observational level (change of the observed degrees of freedom) one redefines the generating functional

$$W_\varepsilon \left[\eta^\dagger, \eta, j_+, j_-, s\right] = \frac{1}{N} \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\tau_- \mathcal{D}\tau_+ \mathcal{D}\phi \exp\{i \int d^4x \left[\mathcal{L}\left[\psi\right] + i\varepsilon\phi(x) + \eta^\dagger(x)\psi(x) + \psi^\dagger(x)\eta(x) + j_+(x)\tau_-(x) + \tau_+(x)j_-(x) + s(x)\phi(x)\right]\}, \qquad (2.18)$$

The transformations of the boson fields under an infinitesimal isospin transformation $\delta\psi(x) = [i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}]\,\psi(x)$ of the fermion fields are

$$\delta\tau_-(x) = -i\alpha^+\phi(x) + i\alpha^3\tau_-(x) \tag{2.19}$$

$$\delta\tau_+(x) = +i\alpha^-\phi(x) - i\alpha^3\tau_+(x) \tag{2.20}$$

$$\delta\phi(x) = +i\alpha^+\tau_+(x) - i\alpha^-\tau_-(x) \tag{2.21}$$

Changing variables in the generating functional and taking partial derivatives with respect to the α's we can rederive the WT-identities showing explicitly the bosons degrees of freedom

$$\int d^4x\langle\eta^\dagger(x)i\tau_3\psi(x) - \psi^\dagger(x)i\tau_3\eta(x) + ij_+(x)\tau_-(x) - i\tau_+(x)j_-(x)\rangle_{J,\varepsilon} = 0 \tag{2.22}$$

$$\int d^4x\langle\eta^\dagger(x)i\tau_+\psi(x) - \psi^\dagger(x)i\tau_+\eta(x) - ij_+(x)\phi(x) + is(x)\tau_+(x)\rangle_{J,\varepsilon} = +\varepsilon\int d^4x\langle\tau_+(x)\rangle_{J,\varepsilon} \tag{2.23}$$

$$\int d^4x\langle\eta^\dagger(x)i\tau_-\psi(x) - \psi^\dagger(x)i\tau_-\eta(x) + i\phi(x)j_-(x) - is(x)\tau_-(x)\rangle_{J,\varepsilon} = -\varepsilon\int d^4x\langle\tau_-(x)\rangle_{J,\varepsilon} \tag{2.24}$$

where J stands for the sources $\eta^\dagger, \eta, j^+, j^-, s$.

The Ward-Takahashi identities will further be used to show the splitting in the dispersion relations of the quasiparticles - yet another artifact of the dynamical rearrangement of isospin symmetry besides the appearance of the Nambu-Goldstone boson.

By taking appropriate functional derivatives (without contraction of the isospin indices) from 2.22 one obtains

$$\tau_3\langle\psi(x)\psi^\dagger(y)\rangle - \langle\psi(x)\psi^\dagger(y)\rangle\tau_3 = 0 \tag{2.25}$$

The Fourier transform of the matrix Green's function of the fermions

$$\langle\psi(x)\psi^\dagger(y)\rangle_{ab} = i\int\frac{d^4p}{(2\pi)^4}e^{i(x-y)}S_{ab}(p) \tag{2.26}$$

can be expanded in the (complete) set of matrices

$$S_{ab}(p) = A(p)\delta_{ab} + B^k(p)\tau^k_{ab}, \tag{2.27}$$

where summation over k is implied.

Using 2.25 one obtains that $B^{+,-}(p) = 0$, i.e.

$$S(p) = \begin{pmatrix} A(p) + B(p) & 0 \\ 0 & A(p) - B(p) \end{pmatrix} \tag{2.28}$$

A convenient parametrization is,

$$A(p) + B(p) = \frac{1}{e(p) - \mu(p)} \tag{2.29}$$

$$A(p) - B(p) = \frac{1}{e(p) + \mu(p)}, \tag{2.30}$$

which allows to relate $e(p)$ and $\mu(p)$ to the dispersion relations of the quasiparticles.

Whether or not $\mu(p)$ can be nonzero is decided by the WT-identities between the quasiparticle propagators and the vertex functions.

We derive this identity.

By taking $\frac{\delta}{i\delta\eta_\alpha(y)}$ from 2.24, with boson sources put to zero one obtains

$$\int d^4x \langle \eta^\dagger(x) i\tau_- \psi(x), \psi^\dagger_\alpha(y)\rangle_\eta - \langle \left[\psi^\dagger(y)\tau_-\right]_\alpha \rangle_\eta - \int d^4x \langle \psi^\dagger(x) i\tau_- \eta(x), \psi^\dagger_\alpha(y)\rangle_\eta = -\varepsilon \int d^4x \langle \psi^\dagger_\alpha(y)\tau_-(x)\rangle_\eta \tag{2.31}$$

Next, by taking $\frac{\delta}{i\delta\eta^\dagger_\beta(z)}$ and putting the fermion sources to zero one obtains

$$\langle [\tau_-\psi(z)]_\beta \psi^\dagger_\alpha(y)\rangle - \langle \psi_\beta(z) \left[\psi^\dagger(y)\tau_-\right]_\alpha \rangle = -\varepsilon \int d^4x \langle \psi_\beta(z)\psi^\dagger_\alpha(y)\tau_-(x)\rangle_\eta, \tag{2.32}$$

which for $\alpha = 1$ and $\beta = 2$ reads

$$\langle \psi_\uparrow(z)\psi^\dagger_\uparrow(y)\rangle - \langle \psi_\downarrow(z)\psi^\dagger_\downarrow(y)\rangle = -\varepsilon \int d^4x \langle \psi_\downarrow(z)\psi^\dagger_\uparrow(y)\tau_-(x)\rangle \tag{2.33}$$

Switching to the Fourier transforms of the quasiparticle propagators and the vertex functions, defined as

$$\langle\psi_\uparrow(x)\psi_\uparrow^\dagger(y)\rangle = i\int\frac{d^4p}{(2\pi)^4}e^{-i(x-y)}S_+(p) \tag{2.34}$$

$$\langle\psi_\downarrow(x)\psi_\downarrow^\dagger(y)\rangle = i\int\frac{d^4p}{(2\pi)^4}e^{-i(x-y)}S_-(p) \tag{2.35}$$

$$\langle\tau_+(x)\psi_-\rangle = i\int\frac{d^4p}{(2\pi)^4}e^{-i(x-y)}\Delta(p) \tag{2.36}$$

$$\langle\psi_\downarrow(x)\psi_\uparrow^\dagger(y)\tau_-(z)\rangle = i^2\int\frac{d^4p_1d^4p_2}{(2\pi)^8}\times \tag{2.37}$$
$$S_-(p_1)\Gamma_-(p_1;p_2,p_1-p_2)S_+(p_2)\Delta(p_1-p_2)e^{-ip_1x}e^{+ip_2y}e^{i(p_2-p_1)z}$$

and using $v = -\lim_{\epsilon\to0} i\epsilon\Delta(0)$ one can write the relation 2.33 in Fourier space as

$$S_-^{-1}(p) - S_+^{-1}(p) = v\Gamma_-(p,p,0) \tag{2.38}$$

Now we can use the result 2.28 derived from the rotational symmetry around the third axis to substitute $S_+(p)$ and $S_-(p)$ with

$$S_+(p) = \frac{1}{e(p)-\mu(p)}\,, \qquad S_-(p) = \frac{1}{e(p)+\mu(p)}\,, \tag{2.39}$$

obtaining

$$\mu(p) = \frac{v\Gamma_-(p,p,0)}{2}. \tag{2.40}$$

Clearly, $\mu(p)\neq0$ implies $v\neq0$ and $\Gamma_-(p)\neq0$, which is exactly the assumption of the spontaneously broken symmetry. Thus the splitting in the dispersion relations of the quasiparticles arises from the dynamical rearrangement of SU(2) symmetry. As argued in Chapter 4, in the case of spontaneously broken SU(2)-isospin symmetry of strong interactions in nuclear matter this effect is observed as a difference in proton and neutron effective potentials in nuclear matter.

We have mentioned that in the spontaneous breakdown of symmetry the original symmetry of the Lagrangian is manifested as a different symmetry at the observational level. Now we show that SU(2)-isospin symmetry of our model is transformed into a different symmetry of the observed fields.

As we have shown the WT-identities obeyed by the Green functions of our model require the existence of (at least) three asymptotic fields:

i) two non-relativistic fermion fields, satisfying the equations

$$D\psi^{in}(x) = 0,$$

where

$$D \equiv \begin{pmatrix} i\frac{\partial}{\partial t} - \frac{\nabla^2}{2m} - \mu_+ & 0 \\ 0 & i\frac{\partial}{\partial t} - \frac{\nabla^2}{2m} - \mu_- \end{pmatrix}.$$

ii) one non-relativistic (complex) boson field, satisfying the equation

$$K\tau_-^{in}(x) = 0,$$

where

$$K(\partial) \equiv i\frac{\partial}{\partial t} - \omega(-\nabla^2).$$

Whether or not the order parameter can propagate as a field can not be decided from considerations based on the WT-identities. However it can be shown that if such field exists it is unstable and decays into Nambu-Goldstone bosons.

The dynamical map between the Heisenberg operators and the observed in-fields is given by the in-field expansions

$$S =: \langle \exp[-i\mathcal{A}] \rangle : \tag{2.41}$$

$$S\psi_H(x) =: \langle \psi(x) \exp[-i\mathcal{A}] \rangle :, \tag{2.42}$$

where the : : stand for normal products and

$$\mathcal{A} = \int d^4z \left[\psi^\dagger(z) \overleftarrow{D}(z)\psi^{in}(z) + \psi^{\dagger in}(z) \overrightarrow{D}(z)\psi(z) + \tau_+(z) \overleftarrow{K}(z)\tau_-^{in}(z) + \tau_+^{in}(z) \overrightarrow{K}(z)\tau_-(z) \right]. \tag{2.43}$$

Equations 2.41,2.42 are equivalent to the LSZ-reduction scheme.

Now we formulate the question of the dynamical symmetry rearrangement : What transformations of the in-fields induce a SU(2)-isospin transformation of the Heisenberg fields $\psi_H(x)$? Or more precisely, we seek a transformation of

$$\psi^{in}(x) \rightarrow \psi'^{in}(x;\alpha) \qquad \tau^{in}(x) \rightarrow \tau'^{in}(x;\alpha), \tag{2.44}$$

which through the dynamical map equation

$$S\psi_H(x) =: \langle \psi(x) \exp[-i\mathcal{A}] \rangle :, \tag{2.45}$$

where

$$S =: \langle \exp[-i\mathcal{A}] \rangle :, \tag{2.46}$$

induces

$$S \rightarrow S; \qquad \psi_H(x) \rightarrow \psi'_H(x) = \exp[i\boldsymbol{\tau} \cdot \mathrm{a}]\, \psi(x). \tag{2.47}$$

In addition we require that the transformed in-fields $\psi'^{in}(x)$, $\tau'^{in}(x)$ satisfy the equations of motion.

The answer to this question will be found as we examine the conditions imposed on the dynamical map 2.42 by the WT-relations 2.22, 2.23, 2.24, which though the particular choice of the ε-term and the order parameter carry the pattern of the symmetry breaking and rearrangement.

Let $\boldsymbol{\alpha} = \boldsymbol{n}\alpha$ with $\boldsymbol{n}$ being a unit vector in the direction of the transformation. Define $\mathcal{A}(\alpha)$ as the expression obtained after the in-fields $\psi^{in}(x)$, $\tau^{in}(x)$ in $\mathcal{A}$ are substituted with the transformed in-fields $\psi'^{in}(x;\alpha)$ and $\tau'^{in}(x;\alpha)$. Then the dynamical map 2.42 and condition 2.47 give

$$S\psi'_H(x) = \exp[i\alpha\boldsymbol{n} \cdot \boldsymbol{\tau}]\, S\psi_H(x) =: \langle \psi(x) \exp[-i\mathcal{A}(\alpha)] \rangle : \tag{2.48}$$

or in differential form

$$\boldsymbol{n}\cdot\boldsymbol{\tau}\ :\ \langle\psi(x)\exp\left[-i\mathcal{A}(\alpha)\right]\rangle := -:\langle\psi(x)\frac{\partial\mathcal{A}(\alpha)}{\partial\alpha}\exp\left[-i\mathcal{A}(\alpha)\right]\rangle :,$$

which for the α^+-component reads as

$$:\langle\tau_+\psi(x)\rangle :=$$
$$\int d^4z:\langle\left[\psi^\dagger(z)\frac{\eta(z,\alpha)}{\partial\alpha^+}+\frac{\eta^\dagger(z,\alpha)}{\partial\alpha^+}\psi(z)+\frac{j_+(z,\alpha)}{\partial\alpha^+}\tau_-(z)+\tau_+(z)\frac{j_-(z,\alpha)}{\partial\alpha^+}\right]\psi(x)\rangle :, \tag{2.49}$$

and has to be compared with the operator identity

$$:\langle\tau_+\psi(x)\rangle :_{\eta j}+\int d^4z:\langle\left[\eta^\dagger(z)i\tau_+\psi(z)-\psi^\dagger(z)i\tau_+\eta(z)-ij_+(z)\phi(z)\right],\psi(x)\rangle :_{\eta j}=$$
$$+v_\varepsilon\int d^4z:\langle\tau_+(z),\psi(x)\rangle :_{\eta j}\overleftarrow{K}(x) \tag{2.50}$$

obtained from 2.23 with the special choice of sources as the following operators in the Fock space of observables:

$$\eta^\dagger(x,\alpha)=-\overleftarrow{D}(x)\psi'^{in}(x,\alpha)\qquad \eta(x,\alpha)=-\psi^{\dagger' in}(x,\alpha)\overrightarrow{D}(x) \tag{2.51}$$

$$j_+(x,\alpha)=-\tau_-'^{in}(x,\alpha)\overrightarrow{K}(x)\qquad j_-(x,\alpha)=-\overrightarrow{K}(x)\tau_-'^{in}(x,\alpha) \tag{2.52}$$

Using integration by parts to shift the action of the differential operators on the free fields under the integral on the left-hand side of equation 2.50, one can prove that the integral over z is zero.

Then the comparison of the equations 2.49, 2.50 yields the conditions

$$\frac{\eta^\dagger(z,\alpha)}{\partial\alpha^+}=0\qquad \frac{\eta(z,\alpha)}{\partial\alpha^+}=0 \tag{2.53}$$

$$\frac{j_+(z,\alpha)}{\partial\alpha^+}=0\qquad \frac{j_-(z,\alpha)}{\partial\alpha^+}=v_\varepsilon\overleftarrow{K}(z), \tag{2.54}$$

which after substitution with the special choice of sources 2.51 give differential equations for the in-fields

$$\frac{\psi^{\dagger,in}(z,\alpha)}{\partial\alpha^+}=0\qquad \frac{\psi^{in}(z,\alpha)}{\partial\alpha^+}=0 \tag{2.55}$$

$$\frac{\tau_+^{in}(z,\alpha)}{\partial\alpha^+} = 0 \qquad \frac{\tau_-^{in}(z,\alpha)}{\partial\alpha^+} = -v_\varepsilon, \tag{2.56}$$

Following the same procedure but applied to the variation of α^- one also obtains

$$\frac{\psi^{\dagger,in}(z,\alpha)}{\partial\alpha^-} = 0 \qquad \frac{\psi^{in}(z,\alpha)}{\partial\alpha^-} = 0 \tag{2.57}$$

$$\frac{\tau_-^{in}(z,\alpha)}{\partial\alpha^-} = 0 \qquad \frac{\tau_+^{in}(z,\alpha)}{\partial\alpha^-} = v_\varepsilon, \tag{2.58}$$

To summarize, we showed that the the rotations around τ_+ and τ_- leave the fermion in-fields unchanged

$$\psi^{in}(x) \overset{\alpha^+,\alpha^-}{\longrightarrow} \psi^{in}(x) \tag{2.59}$$

while the boson in-fields are translated

$$\tau_-^{in}(x) \overset{\alpha^+,\alpha^-}{\longrightarrow} \tau_-^{in}(x) - \alpha^+ v \tag{2.60}$$

$$\tau_+^{in}(x) \overset{\alpha^+,\alpha^-}{\longrightarrow} \tau_+^{in}(x) + \alpha^- v \tag{2.61}$$

One can also show that under these transformations of the in-fields the S-matrix remains unchanged, i.e.

$$S\left[\psi'^{in}(x), \tau'^{in}(x)\right] = S\left[\psi^{in}(x), \tau^{in}(x)\right] \tag{2.62}$$

thus the required invariance condition 2.47 is satisfied.

Now we will show that the rotation under the third axis which preserved the symmetry of the $\mathcal{L}_\varepsilon$-Lagrangian, remains as the same symmetry in the sector of the observed fields, i.e. we will show that the transformation of the Heisenberg fields

$$\psi_H(x) \longrightarrow \exp\left[i\tau_3\alpha_3\right]\psi_H(x) \tag{2.63}$$

is induced though the dynamical map by the same transformation of the in-fields

$$\psi^{in}(x) \longrightarrow \exp\left[i\tau_3\alpha_3\right]\psi^{in}(x) \tag{2.64}$$

$$\tau^{in}(x) \longrightarrow \exp\left[i\tau_3\alpha_3\right]\tau^{in}(x), \tag{2.65}$$

where as before $\psi(x)$ is

$$\psi(x) = \begin{pmatrix} \psi_\uparrow(x) \\ \psi_\downarrow(x) \end{pmatrix}$$

but $\tau(x)$ *now* stands for

$$\tau(x) = \begin{pmatrix} \tau_+(x) \\ \tau_-(x) \end{pmatrix}$$

The proof proceeds exactly as before, when we showed how the in-fields should transform under τ_+ and τ_- rotation in order to induce the required transformation of the Heisenberg fields. Now we use 2.22 to show that

$$: \langle \tau_3\psi(x)\rangle :_{\eta,j} +$$
$$\int d^4z : \langle \left[\eta^\dagger(z)i\tau_3\psi(z) - \psi^\dagger(z)i\tau_3\eta(z) + ij_+(z)\tau_-(z) - i\tau_+(z)j_-(z)\right], \psi(x)\rangle :_{\eta,j} = 0,$$

which when compared with the condition imposed by the dynamical map

$$: \langle \tau_3\psi(x)\rangle :=$$
$$\int d^4z : \langle \left[\psi^\dagger(z)\frac{\eta(z,\alpha)}{\partial\alpha^3} + \frac{\eta^\dagger(z,\alpha)}{\partial\alpha^3}\psi(z) + \frac{j_+(z,\alpha)}{\partial\alpha^3}\tau_-(z) + \tau_+(z)\frac{j_-(z,\alpha)}{\partial\alpha^3}\right]\psi(x)\rangle :,$$

yields

$$\frac{\psi^{\dagger,in}(z,\alpha)}{\partial\alpha^3} = -i\psi^{\dagger,in}(z,\alpha) \qquad \frac{\psi^{in}(z,\alpha)}{\partial\alpha^3} = +i\psi^{in}(z,\alpha) \tag{2.66}$$

$$\frac{\tau_-^{in}(z,\alpha)}{\partial\alpha^3} = +i\tau_-^{in}(z,\alpha) \qquad \frac{\tau_+^{in}(z,\alpha)}{\partial\alpha^3} = -i\tau_+^{in}(z,\alpha)v_\varepsilon, \tag{2.67}$$

Thus we see that the fermion and boson doublets are rotated around the τ_3 - axis:

$$\psi^{in}(x) \longrightarrow \exp\left[i\tau_3\alpha_3\right]\psi^{in}(x) \tag{2.68}$$

$$\tau^{in}(x) \longrightarrow \exp\left[i\tau_3\alpha_3\right]\tau^{in}(x), \tag{2.69}$$

i.e. the rotation of the Heisenberg fields around the τ_3-axis is induced by the rotation of the fermion and boson in-fields.

Chapter 3

Isobaric Analog States

The isobaric analog states were discovered as prominent resonances in (p,n) scattering experiments on ^{88}Sr and ^{89}Y targets [24]. As the experiment showed for certain energies of the incident protons the number of neutrons emitted increased substantially (see Fig.3.1).

Assuming a compound nucleus mechanism, in which a proton is initially absorbed by the target nucleus (Z,N)

$$
\begin{aligned}
p +^{88} Sr &\longrightarrow^{89} Y^* \longrightarrow^{88} Y + n \\
p +^{89} Sr &\longrightarrow^{90} Zr^* \longrightarrow^{89} Zr + n
\end{aligned}
\qquad (3.1)
$$

leading to the nucleus (Z+1,N) in an excited state, whose energy is later released through neutron emission, the resonances were proved to lie at the ground state level of the analog nuclei, respectively:

$$
\begin{aligned}
&^{89}Y^* \quad \text{corresponds to} \quad ^{89}Sr \quad \text{(g.s.)} \\
&^{90}Zr^* \quad \text{corresponds to} \quad ^{90}Y \quad \text{(g.s.)}
\end{aligned}
\qquad (3.2)
$$

The discovery that the SU(2)- isospin symmetry, according to which isobar multiplets are constructed, can be used as a predictive tool in heavy nuclei, came as a surprise. Clearly, in that region, the Coulomb forces are comparable to the nuclear forces and it was expected that they cannot be treated as a perturbation to the latter.

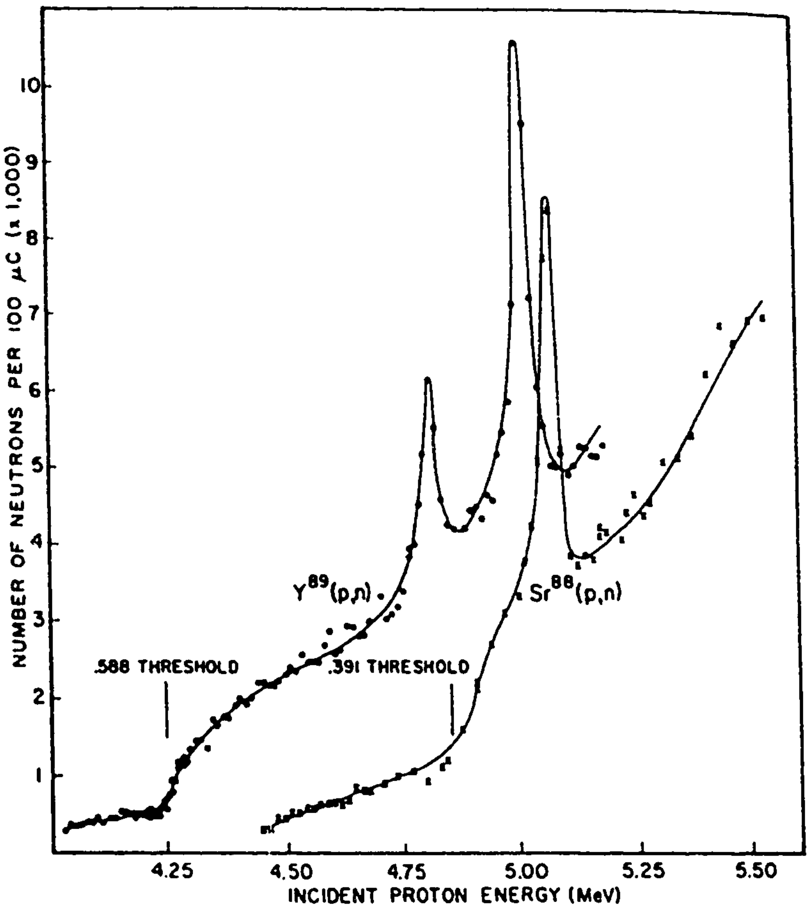

Fig.3.1. Neutron yields for $^{88}Sr(p,n)$ and $^{89}Y(p,n)$ near threshold.(from [23])

Yet, although considerable, the effect of the the Coulomb forces could be estimated by perturbation theory and substracted from experimental data to show that the predictions of the the exact SU(2)-isospin symmetry still hold. How could this happen? We return to this question in Chapter 4, where we argue that after the vacuum realignment the Coulomb force can still be treated perturbatively.

We now turn to the definition and description of the basic properties of the isobaric analog resonances [25], [5].

Assume that the Coulomb interactions are switched off, (we estimate the effect of the Coulomb interaction later) take a nucleus (Z,N), called parent, at its ground state and decide to replace a single neutron with a proton endowing it with the spin and orbital quantum numbers of the replaced neutron (even neglecting the difference in mass). Using the isospin formalism this is expressed as changing the projection of the isospin vector **T**, ($| T |= A$) of the nucleus from

$$T_> \equiv T_3 = \frac{1}{2}(N - Z) \qquad \text{to} \qquad T_< \equiv T_3 = \frac{1}{2}(N - Z) - 1 \tag{3.3}$$

As predicted from the "symmetry energy" term in Bethe-Weizsäcker formula, the ground state of the obtained nucleus (called daughter) lies below the ground state of the parent nucleus. Another argument often used, which is equivalent to the above, is that in its ground state the nucleus always picks up the smallest possible projection of its isospin vector, in this case

$$T_< = \frac{1}{2}(N - Z) - 1 \tag{3.4}$$

As the ground state of the daughter nucleus drops below the ground state of the parent (See Fig.3.2) it allows for an *excited state* in the daughter nucleus to come at the level of the ground state of the parent, thus simulating the SU(2) symmetry, although the symmetry in the ground state is broken, as discussed earlier in Chapter 1. When *this state* was found, it was called the isobaric analog state of the parent ground state (IAS), because it was observed in the neighbouring isobar nucleus and carried the quantum numbers of the ground state of the initial (parent) nucleus (apart from a different isospin projection).

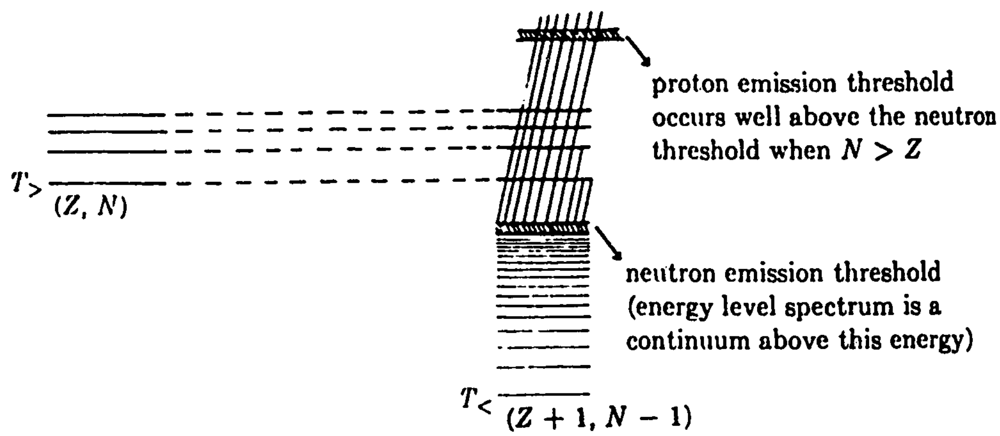

Fig.3.2.Schematic energy level diagram in the absence of the Coulomb interaction. Copied from [25])

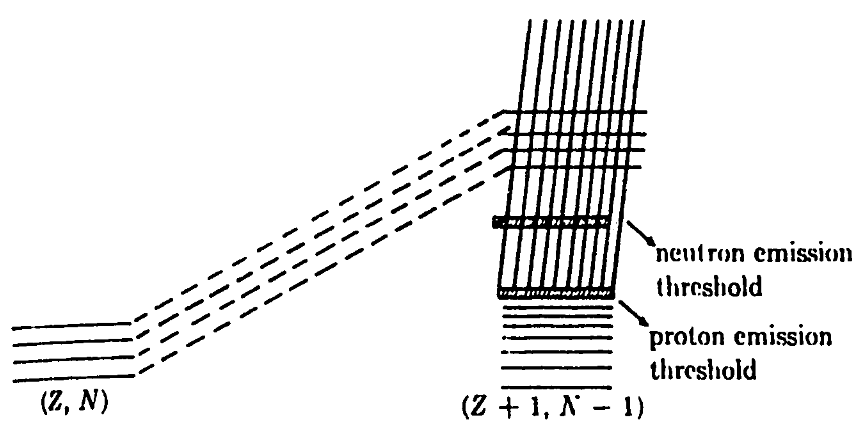

Fig.3.3. Schematic energy level diagram in the presence of the Coulomb interaction.Copied from [25])

When the influence of the Coulomb interaction is considered (See Fig.3.3), its effect is to a) shift upward the spectra as a whole of the parent and daughter nucleus and b) displace the positions of proton and neutron thresholds in both the parent and daughter nuclei. (This is also seen from a previous picture, namely Fig.1.9. The reversion of the sign of the energy gap between proton and neutron in the presence of the Coulomb field comes from the fact that protons couple to electric field and in sufficiently high field this energy can become bigger than the symmetry energy.)

The Coulomb field also causes the isobaric analog state to decay, however we will leave this very important question out of the scope of our present discussion.

Clearly, the shift in nuclear spectra due to the electromagnetic interactions of protons (called Coulomb displacement energy) is important to estimate and predict, because it gives valuable information about nuclear structure.This has been done in the conventional formalism [26]. We show in Chapter 4 how this can be accomplished in Umezawa's unified treatment of spontaneously broken symmetries.

We now proceed with discussion how the energy of inelastically scattered protons on targets containing heavy nuclei is related to the energy of the isobaric analog state, the Coulomb displacement energy in mean field approximation and the energy the ground state of the parent nucleus [7].

Consider inelastic proton scattering (p,p') from a target nucleus (Z,N) for example, $^{142}Ce^{58}_{82}$.

Assume that a resonance is observed at energy say E_k of the scattered protons (See Fig.3.4.).

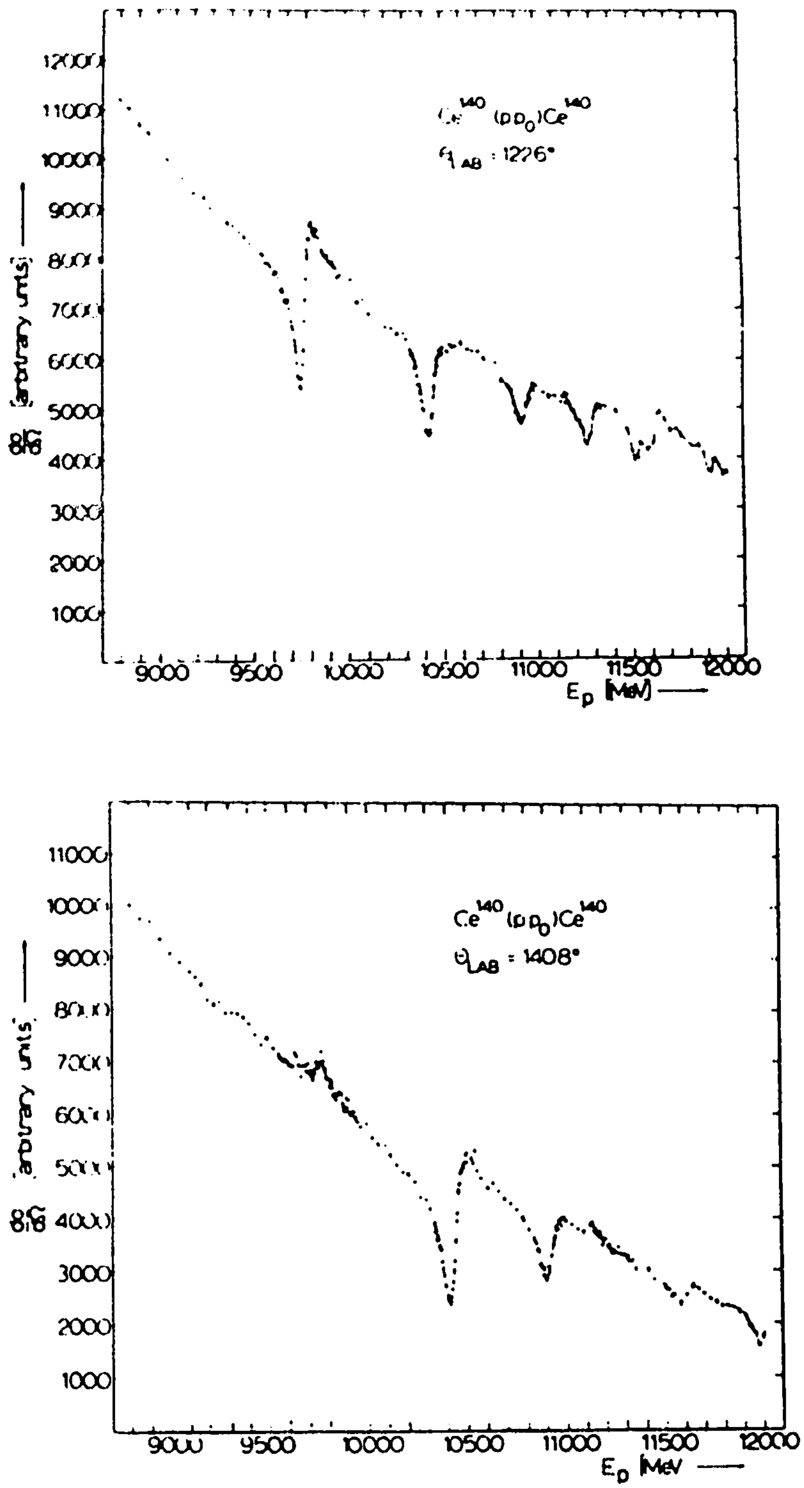

Fig.3.4. Excitation function of the differential elastic cross section for protons on ^{140}Ce at 122.6° and 140.8° (from [7])

Estimate the Coulomb displacement energy Δ_c of the ground state of the nucleus (Z,N+1), i.e. the parent nucleus of the compound nucleus, which is produced after the incident protons are absorbed by the target nucleus. The formula often used is the semiempirical formula

$$\Delta_c = -1.036 + 1.448\frac{Z}{A^{\frac{1}{3}}} \qquad \text{MeV}, \tag{3.5}$$

which treats the Coulomb interactions of protons in nuclei as a mean field produced by the a uniformly charged sphere and takes into account proton statistics by antisymmetrizing protons' wave function in nuclei[27]. The Coulomb displacement energy Δ_c gives the difference between the actual ground state of the parent nucleus (Z,N+1) and the estimate where it would lie if a proton were replaced by a neutron but is still considered as a proton in the "symmetry energy" term in the Bethe-Weizsäcker formula.

A comparison of the independently obtained quantities E_k, (experimental) and Δ_c (semiempirical estimate) suggests the following relationship. When E_k is added to the neutron emission threshold S_n in the parent nucleus , one obtains the Coulomb displacement energy of the isobaric analog state with respect to the parent ground state (an estimate using 3.5)

$$\Delta_c = E_k + S_n, \tag{3.6}$$

This is the experimental evidence.

As we have mentioned the explanation to the above-stated fact is provided by the suggestion that, if corrected for the Coulomb displacement energy (and the neutron-proton mass difference) , the isobaric analog state (observed in the daughter nucleus) appears at the level of the ground state of the parent nucleus. Following Lemmer's 1968 Cargese Lectures [7] we now show how this suggestion, based on the charge invariance of the nuclear forces, works.

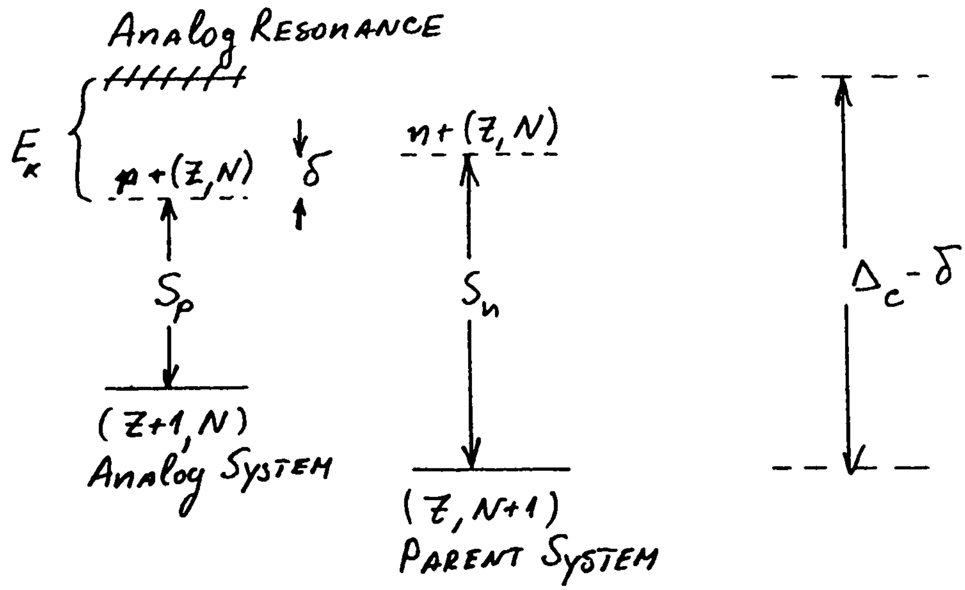

Fig.3.5. Energetic relations for an analog resonance.(from [7])

The charge independence of nuclear interactions implies that by taking away a neutron and substituting it with a proton with the same spin-space quantum numbers one obtains a state with the same energy as the energy of the state with which one started, although this may not be the ground state of the new system. Thus the energy of the isobaric analog state should be degenerate with the parent ground state. However, this is when the Coulomb interaction is absent. In reality there is an energy gain Δ_c due to Coulomb repulsion a single proton feels in the Coulomb field of the target nucleus (Z,N) and an energy loss $\delta = (m_n - m_p)$ due to the fact that a proton is lighter than a neutron. Therefore the analog state of a given parent state will be at an energy

$$E_A = \Delta_c - \delta \tag{3.7}$$

above the latter. If Δ_c is large enough then the analog state is pushed above the threshold for proton emission from the proton plus target system (called the analog system) and is seen as a resonance in proton scattering.

By examining Figure 3.5, which reflects the assumptions about the position of the isobaric analog state and the effect of the Coulomb interaction, one can write

$$E_k = E_A - S_n + \delta \tag{3.8}$$

which together with 3.7 gives

$$E_k = \Delta_c - S_n, \tag{3.9}$$

i.e. it provides an explanation of the experimentally determined relation 3.5 between the Coulomb displacement energy of the parent nucleus, the neutron threshold in the parent nucleus and the kinetic energy of the emitted protons at the resonance.

This relation is in remarkable agreement with experimental data over a wide range of nuclei [28],[27], suggesting that the basic features of the underlying physics are correctly included in this simple model. For instance, Δ_c is estimated to be 15.3 MeV in ^{140}Ce. The neutron separation energy of ^{141}Ce is S_n =5.44 MeV. Therefore the estimated value of the kinetic energy of the scattered proton at the (isobaric analog) resonance is E_k = 9.9 MeV. The experimental value is E_k^{exp} = 9.773 MeV.

However the equation 3.7 *does not provide* an explanation why the isobaric analog resonance lies exactly at the (corrected for the Coulomb repulsion of protons) ground state of the parent nucleus. Rather, the fact that it is obeyed in experiment reflects the correctness of an assumption that for a dynamical reason, which must be provided by the study of nuclear interactions in the daughter nucleus, the isobaric analog states are allowed as a collective excitation (resonance) in it. Conceived and developed by Fallieros [29],[30], this remarkable argument is inspired by the success of the particle-hole description of the giant dipole resonance by Brown and Bolsterli [31] and suggests calculation of nuclear excitation spectra in a model of nuclear structure. Because of the tremendous success of the nuclear shell model in predicting the properties of nuclei at that time, logically its power was tried on the isobaric analog states. To repeat, the objective is to build the isobaric analog resonances as a collective particle-hole excitation on the nuclear ground state. After obtaining the desired result within a shell model approach we will notice how insensitive it is to the *details* of nuclear structure. This observation will be used as a motivation for the dynamical symmetry rearrangement approach of Umezawa which we apply in Chapter 4 to the study of the IAR.

Before reviewing the approach of Falliero following the Cargese (1968) lectures of Lemmer [7], we'd like to relate the isospin formalism introduced earlier to the characteristic pictorial description of nuclear structure in the shell model.

In the shell model derivation [3] of the "symmetry term" in Bethe-Weizsäcker semiempirical mass formula, the fact that protons and neutrons have different Fermi momenta in nuclei, is attributed to the neutron excess in nuclei and Pauli principle. Thus in the language of the shell model Fig.1.10 is viewed as an adjusted to give equal Fermi momenta picture Fig.3.6, where the asymmetry does not come from the asymmetry of the effective potential, but from the excess of neutrons and the impossibility to accomodate more than one nucleon of a given species (proton or neutron) at a given quantum state.

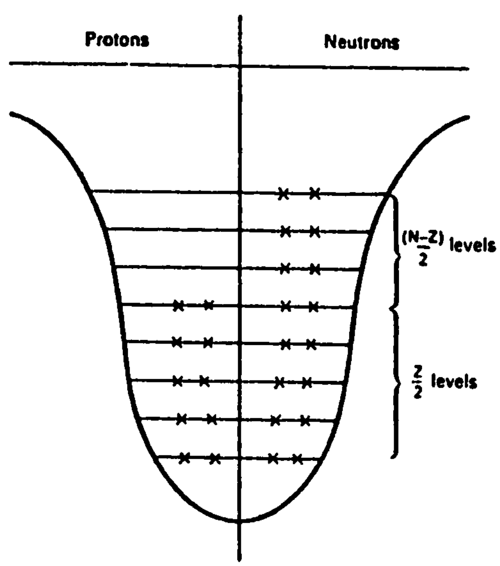

Fig.3.6. Splitting in the dispersion relations of protons and neutrons in nuclei in the shell model approach. (Copied from [4],p.221)

We will discuss the decay channels of the isobaric analog state of the ground state of ^{209}Pb whose shell picture in obvious notations is

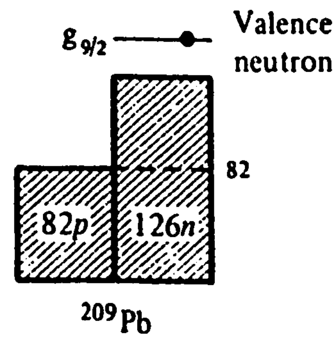

Fig.3.7. Schematic representation of the ^{209}Pb nucleus. The core is indicated by blocks (Copied from [5])

Consider the energy diagrams Fig.3.8 for the isobars ^{209}Pb and ^{209}Bi. (Compare with Fig.3.3.)

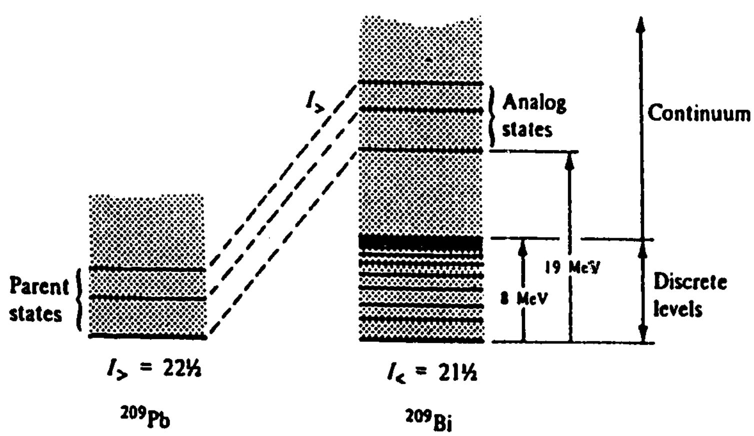

Fig.3.8. Energy level diagram for the isobars (Z,N) and (Z+1,N-1), for A=209, Z=82. Coulomb interaction included. (Copied from [5])

The concept of isospin invariance, which implies that when the nuclear isospin raising operator T_+ is applied to the ground state of the parent nucleus $| T_> , -T_> \rangle$ it gives a degenerate state in the daughter nucleus and the definition of the isobaric analog state as

$$| IAS \rangle \equiv | T_> , -T_> + 1 \rangle \tag{3.10}$$

are combined to obtain

$$| IAS \rangle = \text{const.}\, T_+ \; | T_> , -T_> \rangle \tag{3.11}$$

where the constant is readily found from the the algebra of the raising and lowering operators of isospin

$$T_+ \; | T , -T_3 \rangle = [(T - T_3)(T + T_3 + 1)]^{\frac{1}{2}} \; | T , -T_3 \rangle, \tag{3.12}$$

which for $T_3 = -T_>$ and $T = T_>$ gives

$$| \text{analog} \rangle = \frac{1}{\sqrt{2T_>}} \; | \text{parent g.s.} \rangle \tag{3.13}$$

One can factorize the action of the isospin raising operator T_+on the ground state of ^{209}Pb into a part acting on the core and another one acting on the single neutron

$$|^{209} Pb \rangle = | core \rangle \; | single \quad neutron \rangle \equiv |^{208} Pb \rangle \; | n \rangle \tag{3.14}$$

$$T_+ = T_+^c + T_+^{sp} \tag{3.15}$$

Thus for the isobaric analog state of the ground state of ^{209}Pb one obtains

$$\begin{aligned} | IAS \rangle = \frac{1}{\sqrt{2T_>}} (T_+^c + T_+^{sp}) \; |^{209} Pb \rangle = \\ \frac{1}{\sqrt{2T_>}} (T_+^c + T_+^{sp}) \; |^{208} Pb \rangle \; | n \rangle = \\ \frac{1}{\sqrt{2T_>}} \left[| n \rangle \, T_+^c \; |^{208} Pb \rangle + |^{208} Pb \rangle \, T_+^{sp} \, | n \rangle \right] = \\ \frac{1}{\sqrt{2T_>}} \left[\sqrt{2T_> - 1} \; |^{208} Bi^* + n \rangle + |^{208} Pb + p \rangle \right]. \end{aligned} \tag{3.16}$$

If represented in a pictorial form the equation reads

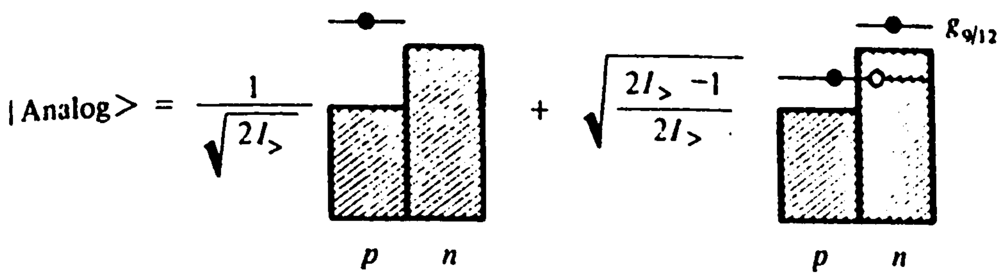

Fig.3.9. Representation of the analog state of a single-particle shell model state.(from [5]*)*

Therefore the decomposition of the isobaric analog state into the $|^{208}Bi^{*}+n\rangle$ and $|^{208}Pb+p\rangle$ points out the chanels of decay

$$\text{analog}\begin{cases}\longrightarrow {}^{208}\text{Pb}+p\\ \longrightarrow {}^{208}\text{Pb}^{*}+p\\ \longrightarrow {}^{208}\text{Bi}^{*}+n\end{cases}$$

Fig.3.10. Decay channels of the isobaric analog state (in ^{209}Bi) of the ground state of ^{209}Pb.(from [5]*)*

We now discuss the essentials of Falliero's dynamical approach to the isobaric analog resonances choosing as a particular example ^{208}Pb in the ground state and its analog in ^{208}Bi.

As we saw from the previous (preparatory) example , the T_+ operator relates the g.s. of ^{208}Pb to a $p\bar{n}$- excitation (i.e. a proton particle - neutron hole excitation) in ^{208}Bi. Calculate the energy of this excitation with respect to the ground energy of ^{208}Pb (Fig.3.12).

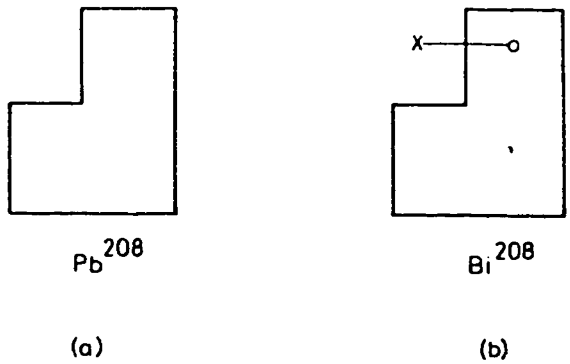

Fig.3.11. a) ^{208}Pb core b) $p\bar{n}$-excitation relative to the ground state of ^{208}Pb.(from [7])

To do that one can imagine that a neutron from the core is removed to infinity (supply energy S_n), then turned into a proton there (gain energy from the proton-neutron mass difference) and returned to the spin-orbital state previously occupied by the removed neutron (energy S_p is released in the binding). Clearly, in the rough energy balance described above we have neglected the interaction of the approaching proton-particle with the neutron-hole in the neutron shell. Note that also, because of the Pauli principle, only the excess of neutrons, occupying energy levels higher than these of protons' in the shell can be subjected to that operation. Calculating the net energy in the above Gedanken experiment one obtains the so called zero excitation energy of the $p\bar{n}$- excitation (zero because the interaction energy of the $p\bar{n}$-pair has been neglected)

$$E_{p\bar{n}} = S_{\bar{n}} - \delta - S_p \tag{3.17}$$

When one tries to predict the position of the isobaric analog state of the ^{208}Pb ground state in ^{208}Bi, one obtains $E_{p\bar{n}}$=5.6 MeV , while the needed , i.e. experimental displacement of the isobaric analog state is 17.6 MeV. (See Fig.3.13)

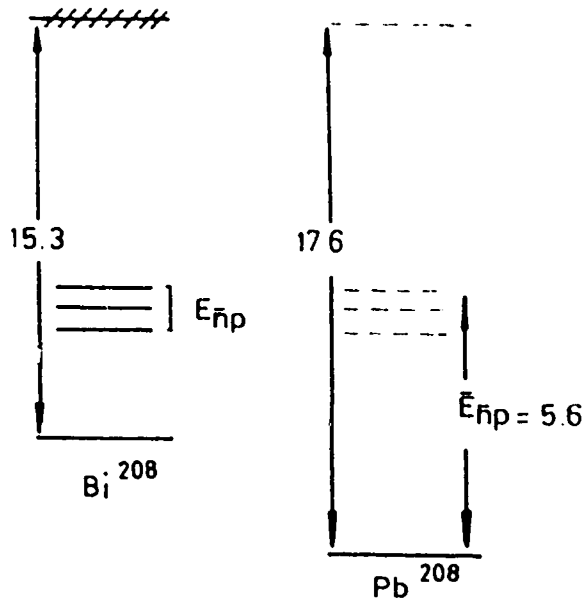

Fig.3.12. Proton-neutron hole levels in ^{208}Bi, measured relative to the ground state of ^{208}Pb. All energies are in MeV. The analog state of ^{208}Pb (g.s.) (in ^{208}Bi) lies at 17.6 MeV on this scale. (from [7])

We decide to include the $p\bar{n}$-interactions in an attempt to achieve agreement between our estimate and the experimental value.

To simplify the calculation we choose an attractive δ-interaction (contact interaction) between protons and neutrons , i.e.

$$V_{np} = -V_o\delta(\mathbf{r}_n - \mathbf{r}_p), \tag{3.18}$$

which means repulsive $p\bar{n}$-interaction with the corresponding strength.

Because the angular momentum of the filled core supplying the $p\bar{n}$-excitation is zero, these pairs must necessarily couple to zero. Thus the question is how to diagonalize the Hamiltonian of the coupled to zero angular momentum of N-Z interacting $p\bar{n}$-pairs. We sketch the method of Brown and Bolsterli, which was originally provided a particle-hole description of the giant dipole resonance. The only formal difference is that instead of $p\bar{p}$ excitations coupled to angular momentum one, we use

$p\bar{n}$ excitations coupled to angular momentum zero.

While the zero excitation contribution to the diagonal elements of the Hamiltonian is $E_{p\bar{n}}$, the calculation of the matrix elements of the coupled to zero angular momentum interacting $p\bar{n}$-pairs gives (see Appendix B)

$$\langle j'\bar{j}'(0) \mid V \mid j\bar{j}(0)\rangle = \frac{F_o}{4\pi}\frac{1}{2}(2j+1)^{\frac{1}{2}}(2j'+1)^{\frac{1}{2}}, \tag{3.19}$$

where F_o stands for

$$F_o = V_o \int drr^2 R_j^2(r) R_{j'}^2(r). \tag{3.20}$$

If one assumes that the zero excitation energies are degenerate (i.e. $E_{p\bar{n}} = \bar{E}$ for all $p\bar{n}$-pairs supplied by the excess neutrons of the shell) , then the matrix elements of the total Hamiltonian takes the form

$$H_{jj'} = \bar{E}\delta_{jj'} + \frac{F_o}{4\pi}\frac{1}{2}(2j+1)^{\frac{1}{2}}(2j'+1)^{\frac{1}{2}}. \tag{3.21}$$

If the eigenvalues of $H_{jj'}$ are calculated, in this case one finds that, while N-Z-1 of them stay at the zero excitation energy, one of them is pushed up to

$$E_A = \bar{E} + \frac{F_o}{4\pi}\frac{1}{2}\sum_j (2j+1) \tag{3.22}$$

Assuming that this is the level of the isobaric analog state measured from the ground state of the parent nucleus, one can reverse the argument and see whether nuclear interactions can provide that shift. For ^{208}Pb the sum over J is equal to the 44 (the neutron excess N-Z in ^{208}Pb) . Therefore the required shift of $17.65 - 5.63 \approx 12$ MeV may be provided by

$$\frac{F_o}{4\pi}(\frac{44}{2}) = 12 \quad \text{MeV , or } \frac{F_o}{4\pi} = 0.55 \quad \text{MeV}, \tag{3.23}$$

a value which is reasonable in the shell model.

Providing a particle-hole description of a monopole (J=0) resonance and identifying it with the isobaric analog state (IAS) by the above mentioned argument might seem questionable because, after all, there are other monopole excitations of the nucleus. What distinguishes the isobaric analog state from the other monopole resonances ?

As it will be seen below the clue comes from the specific neutron hole - proton particle nature of this monopole excitation.

Define the $p\bar{n}$-excitation

$$| p\bar{n}\rangle_j = \sum_m C(j-mjm \mid 00)(-1)^{j+m} a_{jm}(n) a^\dagger_{jm}(p) \,|^{208} Pb\rangle \tag{3.24}$$

i.e. it is a superposition of all $p\bar{n}$-pairs in which individual members of the pair carry equal in absolute value and opposite in direction angular momentum j.

Since

$$C(j-mjm \mid 00) = \frac{(-1)^{j+m}}{\sqrt{2j+1}} \tag{3.25}$$

$| p\bar{n}\rangle_j$ is equal to

$$| p\bar{n}\rangle_j = \frac{1}{\sqrt{2j+1}} a_{jm}(n) a^\dagger_{jm}(p) \,|^{208} Pb\rangle. \tag{3.26}$$

Define the isobaric analog state as the superposition of all $| p\bar{n}\rangle_j$, i.e.

$$| IAS\rangle \equiv \sum_j C_j \;| p\bar{n}\rangle_j \tag{3.27}$$

To determine C_j one uses that $| \; IAS\rangle$ is an eigenstate of the total Hamiltonian $H = T + V$, i.e.

$$H \;| IAS\rangle \;=\; E_A \;| IAS\rangle \tag{3.28}$$

or in the angular momentum basis E_A satisfies the secular equation

$$Det(E_A \delta_{jj'} - H_{jj'}) = 0 \tag{3.29}$$

which is equivalent to

$$\sum_{j'} (E_A \delta_{jj'} - H_{jj'}) C_{j'} = 0, \tag{3.30}$$

where

$$H_{jj'} = \langle j'\bar{j}'(0) \mid H \mid j\bar{j}(0)\rangle = \bar{E}\delta_{jj'} + \frac{F_o}{4\pi}\frac{1}{2}(2j+1)^{\frac{1}{2}}(2j'+1)^{\frac{1}{2}} \tag{3.31}$$

Therefore

$$(E_A - \bar{E})C_j = \frac{F_o}{4\pi}\frac{1}{2}\sum_{j'}(2j+1)^{\frac{1}{2}}(2j'+1)^{\frac{1}{2}} \tag{3.32}$$

or

$$C_j = \frac{F_o}{4\pi}\frac{1}{2}\frac{1}{(E_A - \bar{E})}\sum_{j'}(2j'+1)^{\frac{1}{2}}C_{j'}\ (2j+1)^{\frac{1}{2}} \tag{3.33}$$

i.e. $C_j \sim \sqrt{2j+1}$. But this is just the weight with which $\mid p\bar{n}\rangle_j$ comes into the building of the analog state previously defined as

$$T_+\ |^{208}Pb\rangle = \sum_j\sum_m a_{jm}(n)a^\dagger_{jm}(p)\ |^{208}Pb\rangle = \sum_j\sqrt{2j+1}\ |^{208}Pb\rangle, \tag{3.34}$$

where use of 3.26 was made.

Thus we see that if the isobaric analog state is defined as the solution of the secular equation 3.29

$$\mid IAS\rangle = \sum_j C_j \mid p\bar{n}\rangle_j, \tag{3.35}$$

where $C_j \sim \sqrt{2j+1}$, corresponding to energy E_A

$$E_A = \bar{E} + \frac{F_o}{4\pi}\frac{1}{2}\frac{1}{C_j}(2j+1)^{\frac{1}{2}}\sum_{j'}C_{j'}(2j'+1)^{\frac{1}{2}}, \tag{3.36}$$

then it turns out to be obtainable from the g.s. of ^{208}Pb by acting on it with the isospin raising operator T_+ :

$$\mid IAS\rangle = T_+\ |^{208}Pb\rangle. \tag{3.37}$$

This completes the identification of the collective excitation with the isobaric analog state (IAS).

We now turn to the very important and interesting connection between the energies of the zero excitation modes, the IAS and the "symmetry energy" term in the Bethe-Weizsäcker semiempirical mass formula. It was originally discovered by Falliero et al.[30]. We will follow the presentation of Lemmer in his Cargese lectures (1968).

Consider the effective nuclear potential V_p for a single neutron as compared to that of a single proton V_p. As discussed earlier

$$V_p = V_n - U + V_c, \tag{3.38}$$

where U is related to the symmetry energy in the Bethe-Weizsäcker semiempirical mass formula and V_c is the Coulomb potential. If evaluated for a state $\mid a\rangle$ which is presumed to be initially occupied by a neutron and then by a proton, it gives the difference between the binding energies of the proton and neutron in that state

$$S_n - S_p = -\langle a \mid U \mid a\rangle + \langle a \mid V_c \mid a\rangle \tag{3.39}$$

If the state $\mid a\rangle$ is representative of the all the states that can be occupied in turn by a neutron and then a proton (i.e. the states occupied by the excess neutrons in the ground state), then this relation may be thought as averaged, which gives

$$\Delta_c = \langle a \mid V_c \mid a\rangle \qquad \bar{U} = \langle a \mid U \mid a\rangle \tag{3.40}$$

i.e. the above matrix elements are related to the observed Coulomb displacement energy, and the (observed) difference between the proton and neutron chemical potentials derivable, as we showed, from the Bethe-Weizsäcker semiempirical mass formula.

But, as we discussed earlier, that difference is also related to the zero-excitation energy

$$S_n - S_p = \bar{E} + \delta \tag{3.41}$$

Comparing the equations 3.39 and 3.41 one obtains

$$-\bar{U} + \Delta_c = \bar{E} + \delta, \tag{3.42}$$

from where, remembering that,

$$\Delta_c - \delta = E_A \qquad \text{and} \qquad E_A = \bar{E} + \frac{F_o}{4\pi}\frac{1}{2}\sum_j (2j+1) \tag{3.43}$$

one gets

$$\bar{U} = \frac{F_o}{4\pi}\frac{1}{2}\sum_j (2j+1) = \frac{F_o}{4\pi}\frac{1}{2}(N-Z) \tag{3.44}$$

This is a remarkable result. It relates the observed difference in the proton and neutron chemical potentials to the strength of the nuclear interaction; $F_o \sim V_o$ in the schematic model discussed earlier. We will return to this point later when we derive it from a dynamical symmetry rearrangement point of view.

Now we show that the isobaric analog state is displaced at $\Delta_c - \delta$ with respect to the ground state of the parent nucleus and therefore in the absence of the Coulomb interaction should be degenerate in energy with it thus confirming the prediction of the exact isospin symmetry of the nuclear 2-body force.

Obviously, if one wishes to explain the difference in the separation energies of neutron and proton , which remains as a difference even after the Coulomb repulsion of protons is accounted for, one needs to explain why an SU(2) - invariant nuclear force gives rise to different effective potentials for protons and neutrons. For a contact (i.e. δ-) interaction the answer is provided by a remarkable property of the SU(2)-isospin invariant nuclear force: only particles of the different species can interact at one point, i.e. $V_{nn} = V_{pp} = 0$, while V_{np} is generically nonzero. This fact is directly related to the Pauli exclusion principle. (Here, as well as in the previous discussion, we have neglected the spin of protons and neutrons. However as Brown et al. [32] have shown that its inclusion does not change substantially this result.)

If this property of the SU(2)-isospin invariant δ-force is taken into account, then the difference between the separation energies of proton and neutron, or rather the contribution to that difference from the nuclear force Δ_{np}

$$\bar{E}_{p\bar{n}} = S_n - S_p = \Delta_{np} + \Delta_c - \delta \tag{3.45}$$

is easily understood: simply when a neutron is separated it interacts with Z protons, while when a proton is put in its place it interacts with N neutrons.

Therefore, applying as before the δ-interaction model 3.18 to estimate the contribution Δ_{np}, one obtains

$$\Delta_{np} = -\frac{F_o}{4\pi}\frac{1}{2}(N - Z), \tag{3.46}$$

where the negative sign comes from the fact that *now* one considers the interaction of a single nucleon with the rest of the nucleons of the opposite species in the nucleus coupled to j=0 , i.e. now the interaction is between neutron(particle) and proton(particle) and it is attractive according to the initial assumption 3.18.

We now return to the relationship between E_A and $\bar{E}_{p\bar{n}}$:

$$E_A = \bar{E} + \frac{F_o}{4\pi}\frac{1}{2}\sum_j(2j+1), \tag{3.47}$$

where the summation is over the excess neutron states. Arguing as before that this sum equals (N-Z) and inserting the value of $\bar{E}$ from 3.45

$$\bar{E}_{p\bar{n}} = -\frac{F_o}{4\pi}\frac{1}{2}(N - Z) + \Delta_c - \delta \tag{3.48}$$

we obtain

$$E_A = \Delta_c - \delta, \tag{3.49}$$

which is exactly the relation observed in experiment and which seemed to be predicted by the assumption that SU(2)-isospin symmetry remains intact in heavy nuclei (refer to 3.7). Then we asked ourselves if a dynamical explanation could be provided and here we end up saying that a schematic (i.e. δ-interaction) model of nuclear interactions provides such an explanation. At first glance one may think that the "miracle" cancelation of different effects which lead to the result 3.49 occurred entirely because of the special property of the contact force. However a closer examination reveals an interesting very general fact: If one chooses appropriately the difference between the effective one-body potentials of the protons and neutrons in nuclei, then one can put in place the isobaric analog state, i.e. make it correspond to the Coulomb displaced ground state of the parent nucleus.

We devote the next chapter to the derivation of this statement from the viewpoint of the dynamical rearrangement of isospin symmetry in nuclei.

Chapter 4

Dynamical Rearrangement of Isospin Symmetry in Nuclei in a Contact Model of Nuclear Interactions

Recently F. Khanna and H.Umezawa initiated an extensive search for nuclear features which are related and could be described within the formalism of dynamical symmetry rearrangement. They argued [1] [33] that the s- and d- bosons in the IBM model may appear from a dynamical rearrangement of Elliot symmetry. Discussing [34] the spin-isospin excitations in nuclei H.He, F.Khanna and H.Umezawa conjectured that they may appear from spontaneous breakdown of SU(4) spin-isospin symmetry and that the isobaric analog states are the Nambu-Goldstone bosons of the spontaneously broken SU(2) isospin symmetry:

"One may use SU(2) isospin symmetry for large nuclei.This would lead to excitation modes $\tau_\pm \mid 0\rangle$ that may be associated with the isobaric analog states that have been prominent in (p,n) reactions. It was a surprise to find them in heavy nuclei. SU(2) isospin symmetry is explicitly broken by the Coulomb interaction. It was anticipated that for nuclei with large value of Z , isospin would not be a good symmetry. However, rather sharp states that were related to the parent ground state by the operator $\tau_\pm \mid 0\rangle$ were found in heavy nuclei. The state is displaced by an energy that is very close to the expectation value of the Coulomb interaction. If the conjecture of spontaneous symmetry breakdown is correct, in the absence of Coulomb

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interaction, the isobaric analog state would have a structure that may be described in terms of pair of nucleons. The pair of particles in the Coulomb interaction would acquire a finite mass.It would be interesting to find if the level spacing can be obtained using such an argument."

To show how the approach of dynamical symmetry rearrangement, indeed, confirms this prediction and to compare the results with the conventional approach, discussed in Chapter 3, we specify the SU(2) isospin invariant nuclear interaction as

$$V = \int d^4x \frac{1}{2} \lambda \psi_\uparrow^\dagger(x) \psi_\downarrow^\dagger(x) \psi_\downarrow(x) \psi_\uparrow(x), \tag{4.1}$$

i.e. we choose an isospin invariant contact interaction (up and down arrows stand for neutron and proton respectively). Choosing the order parameter as before

$$I_\varepsilon = \langle \psi^\dagger(x) \tau_3 \psi(x) \rangle_\varepsilon \tag{4.2}$$

we can derive the WT-identities between the Green functions following the ε-procedure described in Chapter 2.

$$\langle \psi^\dagger(x) \tau_3 \psi(x) \rangle_\varepsilon = -\varepsilon \int d^4y \langle \tau_+(y) \tau_-(x) \rangle, \tag{4.3}$$

$$\langle \psi_\uparrow(x) \psi_\uparrow^\dagger(y) \rangle - \langle \psi_\downarrow(x) \psi_\downarrow^\dagger(y) \rangle = -\varepsilon \int d^4x \langle \psi_\downarrow(x) \psi_\uparrow^\dagger(y) \tau_-(z) \rangle. \tag{4.4}$$

The method of the derivation of the Ward-Takahashi can be used further to derive identities linking 3-point Green functions to 4-point Green functions , etc. , thus obtaining a chain of equations reflecting the basic nonlinearity of the field equations. However when one iterates the solution, for example expressing the 3-point Green functions through the 4-point Green functions and substituting the result into the relation between the 2- and 3- point Green functions , one needs to truncate the chain, i.e.to approximate the solution. Then the question arises whether one can pick up the approximation to satisfy the WT-identities for the exact Green functions. As argued by J.Goldstone , A. Salam and S. Weinberg [35], the expansion in the number of loops gives the invariant approximation, i.e. complying with the symmetry requirements of the theory reflected in the WT-identities. Umezawa et al.[13] extended the argument to include the case when bound states appear in the theory as a result of spontaneous symmetry breakdown.

We decide to see whether a Hartree-Fock approximation (HF) to the quasiparticles (i.e. truncation of the chain at the 3-point Green's function) and the corresponding random phase approximation (RPA) to the collective state (IAS) are consistent with the Ward-Takahashi identities.

Deciding to use proper vertices (instead of the improper that we used for pedagogical purposes in Chapter 2), we define

$$\langle\psi_\downarrow(x)\psi_\uparrow^\dagger(y)\tau_-(z)\rangle = i^2\int\frac{d^4p_1d^4p_2}{(2\pi)^8}S_-(p_1)S_+(p_2)e^{-ip_1(x-z)}e^{-ip_2(z-y)}$$
$$+i^2\int\frac{d^4p_1d^4p_2}{(2\pi)^8}S_-(p_1)\Gamma_-(p_1;p_2,p_1-p_2)S_+(p_2)\Delta(p_1-p_2)e^{-ip_1x}e^{+ip_2y}e^{i(p_2-p_1)z} \tag{4.5}$$

Substituting with the Fourier transforms of the Green's functions in the 4.4 and using $I = -\lim_{\epsilon\to 0} i\varepsilon\Delta(0)$ we obtain

$$S_-^{-1}(p) - S_+^{-1}(p) = I\Gamma_-(p,-p,0), \tag{4.6}$$

where $\Gamma_-(p,-p,0)$ is the proper vertex.

According to 4.2

$$I = -\lim_{\epsilon\to 0}\langle\psi_\uparrow(t-\epsilon,\mathbf{x})\psi_\uparrow^\dagger(t+\epsilon,\mathbf{x}) - \psi_\downarrow(t-\epsilon,\mathbf{x})\psi_\downarrow^\dagger(t+\epsilon,\mathbf{x})\rangle, \tag{4.7}$$

which after substitution with the Green's functions of the quasiparticles gives

$$I = -i\int\frac{d^4p}{(2\pi)^4}e^{2i\epsilon p_0}\left[S_+(p) - S_-(p)\right] = -i\int_+\frac{d^4p}{(2\pi)^4}\left[S_+(p) - S_-(p)\right], \tag{4.8}$$

where the integration path for dp_0 is along the half circle of the upper plane for complex p_0, indicated by the symbol $+$.

Combining 4.6 and 4.8 one can derive the (so called) gap equation

$$1 = -i \int \frac{d^4p}{(2\pi)^4} S_+(p) S_-(p) \Gamma_-(p; p, 0), \tag{4.9}$$

which can be used (either it or 4.8) to determine the order parameter in consistency with the approximation used for the quasiparticles.

Choosing a HF-approximation (setting $\Gamma_-(p; p, 0) = \lambda$ to the exact propagators of the quasinucleons (i.e. protons and neutrons in nuclei), we *require* that 4.6, which is valid for the exact propagators *be valid* for the HF-approximation to them:

$$\frac{1}{S^0_-(p)} - \frac{1}{S^0_+(p)} = I\lambda, \tag{4.10}$$

where λ is the unrenormalized coupling constant.

If one uses the arguments which in Chapter 2 lead to the parametrization of $S_+(p)$ and $S_-(p)$ in terms of the functions $e(p)$ and $\mu(p)$, one can see that

$$S^0_\pm(p) = \frac{\Theta(\epsilon_\pm(\mathbf{p}))}{p_0 - \epsilon_\pm(\mathbf{p})) + i\varepsilon} + \frac{\Theta(-\epsilon_\pm(\mathbf{p}))}{p_0 - \epsilon_\pm(\mathbf{p})) - i\varepsilon}, \tag{4.11}$$

where

$$\epsilon_\pm(\mathbf{p}) = \frac{1}{2m}(p^2 - k_F^2) \mp \frac{1}{2}\lambda I. \tag{4.12}$$

Thus we obtain a splitting in the dispersion relations of proton and neutron in nuclei, which is proportional to the order paremeter and the coupling constant. However the order parameter I in 4.12 must be picked up to be consistent with the approximation for the quasiparticles. To do that we can use either 4.8, which derives from the definition of the order parameter or the gap equation 4.9.

The order parameter that is consistent with the HF-approximation for the quasiparticles may be determined from the equation

$$I_{HF} = -i \int_+ \frac{d^4p}{(2\pi)^4} \left[S^0_+(p) - S^0_-(p)\right], \tag{4.13}$$

where $S_+^0(p)$ and $S_-^0(p)$ are the quasiparticle propagators in Hartree-Fock approximation, i.e.

$$\begin{aligned} I_{HF} &= -i\int_+ \frac{d^4p}{(2\pi)^4}\left[\frac{\Theta(-\epsilon_+(p))}{p_0-\epsilon_+(p)-i\varepsilon} - \frac{\Theta(-\epsilon_-(p))}{p_0-\epsilon_-(p)-i\varepsilon}\right] \\ &= \int \frac{d^3p}{(2\pi)^3}\left[\Theta(-\epsilon_+(\mathbf{p})) - \Theta(-\epsilon_-(\mathbf{p}))\right] \\ &= n(I_{HF}) - z(I_{HF}), \end{aligned} \tag{4.14}$$

where $n(I_{HF})$ and $z(I_{HF})$ stand for the neutron and proton densities.

The calculation is presented in Appendix C. Here we state the result. The nuclear density n is just above the critical nuclear density n_c, which justifies the approximation of the coupling constant with

$$\lambda = 2\frac{4}{3}\frac{\varepsilon_F}{n}. \tag{4.15}$$

Thus we see that the dispersion relations for the quasinucleons in the HF-approximation for the propagators are

$$\epsilon_\pm(p) = \frac{1}{2m}(p^2 - k_F^2) \mp 2a_I\frac{(N-Z)}{A}, \tag{4.16}$$

where a_I is the value of the symmetry energy coefficient

$$a_I = 2\frac{\varepsilon_F}{3}. \tag{4.17}$$

We saw in Chapter 1 that the splitting in the dispersion relations of proton and neutron in nuclei (proportional to N-Z) is observed in experiment and is also predicted by the "symmetry energy term" in the Bethe-Weizsäcker semiempirical mass formula. We mentioned the attempts to explain the presence of this term in different models. Here we have presented a derivation that relates this effect to the dynamical rearrangement of isospin symmetry in nuclei and we have estimated the value of the symmetry energy coefficient in Bethe-Weizsäcker semiempirical mass formula.

For $\varepsilon_F \approx 37 - 45$MeV the computed value of a_I falls within the experimental values claimed to lie between 22 MeV (earlier)[15],[16] and 28 MeV (later) estimates [1],[18].

When the quasiparticle propagators are calculated in HF-approximation, the corresponding collective state is calculated in the random phase (RPA) approximation[36].

We proceed to show that the (RPA) for the collective state satisfies the requirement (for the exact propagator) that it be a gapless mode.

Because the exact propagator of the collective state is related to the 3-point Green function as

$$\langle \tau_+(x)\tau_-(y)\rangle = -\lim_{\epsilon\to 0}\langle \psi_1(t-\epsilon,\mathrm{x})\psi_1^\dagger(t+\epsilon,\mathrm{x})\tau_-(y)\rangle, \qquad (4.18)$$

then the equation 4.5 can be used to obtain

$$\Delta(q) = -i\int \frac{d^4p}{(2\pi)^4} S_-(p+\frac{1}{2}q)S_-(p-\frac{1}{2}q) \\ -i\int \frac{d^4p}{(2\pi)^4} S_-(p+\frac{1}{2}q)S_-(p-\frac{1}{2}q)\Gamma_-(p+\frac{1}{2}q;p-\frac{1}{2}q,q)\Delta(q), \qquad (4.19)$$

which when solved for $\Delta(q)$ gives

$$\Delta(q) = -i\frac{\int \frac{d^4p}{(2\pi)^4} S_-(p+\frac{1}{2}q)S_-(p-\frac{1}{2}q)}{1+i\int \frac{d^4p}{(2\pi)^4} S_-(p+\frac{1}{2}q)S_-(p-\frac{1}{2}q)\Gamma_-(p+\frac{1}{2}q;p-\frac{1}{2}q,q)} \qquad (4.20)$$

The denominator for $q = 0$ is zero because of the gap equation for the exact propagators of the quasiparticles i.e. this confirms that the Goldstone theorem holds for the exact propagator of the collective state.

To determine whether the approximation to the exact collective state propagator satisfies the Goldstone theorem, i.e. whether it has a pole at $q = 0$, one has to insert the HF-approximation for the quasiparticle propagators.

$$\Delta(q)_{RPA} = -i\frac{\int \frac{d^4p}{(2\pi)^4} S_-^0(p+\frac{1}{2}q)S_-^0(p-\frac{1}{2}q)}{1+i\lambda\int \frac{d^4p}{(2\pi)^4} S_-^0(p+\frac{1}{2}q)S_-^0(p-\frac{1}{2}q)} \qquad (4.21)$$

But when $q = 0$ the denominator is exactly the gap equation, which may be used to determine the order parameter consistent with the HF-approximation for the quasiparticles. Instead we used the equivalent of it 4.8. Therefore the random phase approximation to the propagator of the collective state satisfies the Goldstone theorem.

To summarize, we have shown that the when the HF-approximation for the quasiparticles is picked up to satisfy the WT-identities for the propagators and the vertex function, then the RPA approximation for the collective state obeys the Goldstone theorem.

Consider the effect of the explicit breaking of isospin symmetry by the electromagnetic interactions. Treating the latter in the mean field approximation, i.e. assuming that protons move in the mean Coulomb field of the nucleus one and picking for symplicity the latter as constant one can write

$$V_c = E_c \psi^\dagger(x) \frac{1}{2}(1+\tau_3)\psi(x). \tag{4.22}$$

Factorizing the effect of the Coulomb coupling into a symmetry-obeying term $\psi^\dagger(x)\psi(x)$ and a symmetry-violating term $\psi^\dagger(x)\tau_3\psi(x)$, one can include the former into the SU(2) isospin invariant Lagrangian, and note the latter has the structure of the added ε-term in the Lagrangian, although not infinitesimal. Therefore, if one decides to derive the Goldstone theorem following the ε-procedure in Chapter 2, one would obtain

$$\lim_{\varepsilon\to 0} \langle \psi^\dagger(x)\tau_3\psi(x)\rangle_\varepsilon = +i \lim_{\varepsilon\to 0} (E_c + i\varepsilon) \int d^4y \langle \tau_+(y)\tau_-(x)\rangle, \tag{4.23}$$

which implies

$$I = \lim_{\varepsilon\to 0} (E_c + i\varepsilon) \frac{\chi(0)}{w(\mathbf{k})|_{\mathbf{k}=0} + i\varepsilon}, \tag{4.24}$$

from where it follows that

$$w(\mathbf{k})|_{\mathbf{k}=0} = E_c \tag{4.25}$$

Thus we see that the effect of the Coulomb field in the nucleus on the isobaric analog state is to shift up its position with respect to ground state of the parent nucleus. This effect is very well known and in fact (in a reversed argument) the observed shift is used to determine the mean Coulomb field of the nucleus [37].

Chapter 5

Conclusion

Assuming that the isospin symmetry of nuclear interactions is spontaneously broken in nuclei we have derived the different nuclear effective potentials of protons and neutrons in nuclei and showed that the appearance of the isobaric analog states (IAS) is required to satisfy consistency conditions imposed by the Ward-Takahashi identities.

Employing the Hartree-Fock approximation for the quasiparticles and the corresponding RPA for the collective state (IAS) we showed that the order parameter in the mean field approximation is equal to the neutron-proton number difference. The splitting in the dispersion relations of protons and neutrons in nuclei, which gives rise to the "symmetry energy" term in the Bethe-Weizsäcker semiempirical mass formula, is proportional to the coupling constant of the theory and the neutron-proton number difference.The "symmetry energy" coefficient a_I in the Bethe-Weizsäcker semiempirical mass formula is evaluated in the mean field approximation and the result agrees with phenomenological fits.

We treated the Coulomb interactions in mean field approximation (picking up an uniform mean field for simplicity) and showed that the isobaric analog state acquires a finite gap proportional to the Coulomb potential.

Other issues which may be addressed within the approach of dynamical symmetry rearrangement (the sector of low energy theorems for Nambu-Goldstone bosons, in particular [6]) are the decay of isobaric analog states and their multiple production in charge exchange experiments [38]. Maybe the narrowness of their decay width can be attributed to Adler's consistency theorem, while the emerging experimental evidence of relations between the cross sections for excitation of single and double isobaric analog states in charge exchange reactions may be a manifestation of the multiple production theorem for Nambu-Goldstone bosons. Research is currently under way along these lines.

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Appendix A

The Green's Function of the Nambu-Goldstone Boson

The causal Green's function of a nonrelativistic interacting complex scalar boson field is defined as

$$G(\mathbf{r},\mathbf{r}';t-t') \equiv \{\theta(t-t')\langle 0 \mid \tau_H(\mathbf{r},t)\tau_H^{+}(\mathbf{r}',t') \mid 0\rangle + \theta(t'-t)\langle 0 \mid \tau_H^{+}(\mathbf{r}',t')\tau_H(\mathbf{r},t) \mid 0\rangle\},$$

where τ_H and τ_H^{+} are field operators in Heisenberg representation.

If the matrix elements are expanded in the complete set of physical states $\mid \nu\rangle$, then

$$\langle 0 \mid \tau_H(\mathbf{r},t)\tau_H^{+}(\mathbf{r}',t') \mid 0\rangle = \sum_{\nu} \langle 0 \mid \tau_H(\mathbf{r},t) \mid \nu\rangle\langle \nu \mid \tau_H^{+}(\mathbf{r}',t') \mid 0\rangle \tag{A.1}$$

$$\langle 0 \mid \tau_H^{+}(\mathbf{r}',t')\tau_H(\mathbf{r},t) \mid 0\rangle = \sum_{\nu} \langle 0 \mid \tau_H^{+}(\mathbf{r}',t') \mid \nu\rangle\langle \nu \mid \tau_H(\mathbf{r},t) \mid 0\rangle \tag{A.2}$$

Assuming time translational symetry of the theory (which implies energy conservation)

$$T_t \mid 0\rangle = \mid 0\rangle \qquad T_t \mid \nu\rangle = \exp\{-i\mathcal{E}_\nu t\} \mid \nu\rangle, \tag{A.3}$$

one obtains from

$$\langle 0 \mid \tau_H(\mathbf{r},t) \mid \nu\rangle = \langle 0 \mid T_t^{-1}T_t\tau_H(\mathbf{r},t)T_t^{-1}T_t \mid \nu\rangle \tag{A.4}$$

$$\langle 0 \mid \tau_H^{+}(\mathbf{r}',t') \mid \nu\rangle = \langle 0 \mid T_t^{-1}T_t\tau_H^{+}(\mathbf{r}',t')T_t^{-1}T_t \mid \nu\rangle, \tag{A.5}$$

the relations

$$\langle 0 \mid \tau_H(\mathbf{r},t) \mid \nu\rangle = \exp(-i\mathcal{E}_\nu t)\langle 0 \mid \tau_H(\mathbf{r},0) \mid \nu\rangle \tag{A.6}$$

$$\langle 0 \mid \tau_H^{+}(\mathbf{r}',t') \mid \nu\rangle = \exp(-i\mathcal{E}_\nu t')\langle 0 \mid \tau_H^{+}(\mathbf{r}',0) \mid \nu\rangle \tag{A.7}$$

$$\langle \nu \mid \tau_H(\mathbf{r},t) \mid 0\rangle = \exp(i\mathcal{E}_\nu t)\langle \nu \mid \tau_H(\mathbf{r},0) \mid 0\rangle \tag{A.8}$$

$$\langle \nu \mid \tau_H^{+}(\mathbf{r}',t') \mid 0\rangle = \exp(i\mathcal{E}_\nu t')\langle \nu \mid \tau_H^{+}(\mathbf{r}',0) \mid 0\rangle, \tag{A.9}$$

which, when inserted into the definition for the causal Green's function give

$$G(\mathbf{r},\mathbf{r}';t-t') = \sum_\nu \theta(t-t')\exp\left[-i\mathcal{E}_\nu(t-t')\right] \times$$
$$\{\langle 0 \mid \tau_H(\mathbf{r},0) \mid \nu\rangle\langle \nu \mid \tau_H^{+}(\mathbf{r}',0) \mid 0\rangle + \langle 0 \mid \tau_H^{+}(\mathbf{r}',0) \mid \nu\rangle\langle \nu \mid \tau_H(\mathbf{r},0) \mid 0\rangle\}$$

If the representation

$$\mp 2\pi i \exp(-i\varepsilon t)\theta(\pm t) = \int dq_o \frac{\exp(-iq_o t)}{q_o - \varepsilon \pm i0} \tag{A.10}$$

is used, the Green's function may be written as

$$G(\mathbf{r},\mathbf{r}';t-t') = i\int \frac{dq_o}{2\pi}\exp\left[-iq_o(t-t')\right] \times$$
$$\sum_\nu \left\{ \frac{\langle 0 \mid \tau_H^{+}(\mathbf{r}',0) \mid \nu\rangle\langle \nu \mid \tau_H(\mathbf{r},0) \mid 0\rangle}{q_o + \mathcal{E}_\nu - i0} - \frac{\langle 0 \mid \tau_H(\mathbf{r},0) \mid \nu\rangle\langle \nu \mid \tau_H^{+}(\mathbf{r}',0) \mid 0\rangle}{q_o - \mathcal{E}_\nu + i0} \right\}.$$

If in addition the theory is translationally invariant (which implies conservation of total momentum P_ν), i.e.

$$R_\mathbf{r}\tau_H(\mathbf{r},0)R_\mathbf{r}^{-1} = \tau_H(0,0) \qquad R_{\mathbf{r}'}\tau_H^{+}(\mathbf{r}',0)R_{\mathbf{r}'}^{-1} = \tau_H^{+}(0,0) \tag{A.11}$$

$$R_\mathbf{r} \mid 0\rangle = \mid 0\rangle \qquad R_\mathbf{r} \mid \nu\rangle = \exp\{-i\mathcal{P}_\nu \cdot \mathbf{r}\} \mid \nu\rangle, \tag{A.12}$$

then using

$$\langle 0 \mid \tau_H(\mathbf{r},0) \mid \nu\rangle = \langle 0 \mid R_\mathbf{r}^{-1}R_\mathbf{r}\tau_H(\mathbf{r},0)R_\mathbf{r}^{-1}R_\mathbf{r} \mid \nu\rangle \tag{A.13}$$

$$\langle \nu \mid \tau_H^{+}(\mathbf{r}',0) \mid 0\rangle = \langle \nu \mid R_{\mathbf{r}'}^{-1}R_{\mathbf{r}'}\tau_H^{+}(\mathbf{r}',0)R_{\mathbf{r}'}^{-1}R_{\mathbf{r}'} \mid 0\rangle, \tag{A.14}$$

one obtains

$$G(\mathbf{r}-\mathbf{r}^{'};t-t^{'}) = i\int \frac{dq_o}{2\pi}\exp\left[-iq_o(t-t^{'})\right]\times$$
$$\sum_{\nu}\left\{\exp\left[+i\mathcal{P}_\nu\cdot(\mathbf{r}-\mathbf{r}^{'})\right]\frac{|\langle\nu\mid\tau_H^{+}(0,0)\mid 0\rangle|^2}{\mathcal{E}_\nu(\mathcal{P}_\nu)+q_o-i0}+\exp\left[-i\mathcal{P}_\nu\cdot(\mathbf{r}-\mathbf{r}^{'})\right]\frac{|\langle\nu\mid\tau_H(0,0)\mid 0\rangle|^2}{\mathcal{E}_\nu(\mathcal{P}_\nu)+q_o-i0}\right\}$$

If one makes the substitution $\mathcal{P}_\nu \rightarrow -\mathcal{P}_\nu$ in the second term, the exponential terms are factorized

$$G(\mathbf{r},t) = i\int \frac{dq_o}{2\pi}\exp\left[-iq_o(t)\right]\sum_{\nu}\exp\left[+i\mathcal{P}_\nu\cdot\mathbf{q}\right]\times$$
$$\left\{\frac{|\langle\nu\mid\tau_H^{+}(0,0)\mid 0\rangle|^2}{\mathcal{E}_\nu(\mathcal{P}_\nu)-q_o-i0}+\frac{|\langle\nu\mid\tau_H(0,0)\mid 0\rangle|^2}{\mathcal{E}_\nu(-\mathcal{P}_\nu)+q_o-i0}\right\}$$

From this expression after comparison with the definition of the Fourier transform of the Green function

$$G(\mathbf{r},t) = i\int \frac{d^4q}{(2\pi)^4}\exp\left[i(\mathbf{q}\cdot\mathbf{r}-q_o t)\right]\Delta(\mathbf{q},q_o), \tag{A.15}$$

one obtains

$$\Delta(\mathbf{q},q_o) = \sum_{\nu}(2\pi)^3\delta(\mathbf{P}_\nu-\mathbf{q})\times$$
$$\left\{\frac{|\langle\nu\mid\tau_H(0,0)\mid 0\rangle|^2}{+q_o+\mathcal{E}_\nu(-\mathbf{P}_\nu)-i0}-\frac{|\langle\nu\mid\tau_H^{+}(0,0)\mid 0\rangle|^2}{q_o-\mathcal{E}_\nu(\mathbf{P}_\nu)+i0}\right\} \tag{A.16}$$

Arguing that the poles of the propagator Eq.A.17 give the energies of the single particle states (i.e. those states, whose energy is entirely determined by the total momentum $\mathcal{E}_\nu(\mathcal{P}_\nu)=\mathcal{E}(\mathcal{P}_\nu)$) one obtains that close to the poles

$$\Delta(\mathbf{q},q_o) \approx \frac{|\langle 0\mid\tau_H^{+}(0,0)\mid\tau_-(\mathbf{q})\rangle|^2}{q_o+\mathcal{E}_-(\mathbf{q})-i0}-\frac{|\langle 0\mid\tau_H(0,0)\mid\tau_+(\mathbf{q})\rangle|^2}{q_o-\mathcal{E}_+(\mathbf{q})+i0} \tag{A.17}$$

where $\mid\tau_\pm(\mathbf{q})\rangle$ stand for the single particle states and $\mathcal{E}_\pm(\mathbf{q})$ for their dispersion relations.

One can argue[39] that the $i0$ term in the propagator of the quasiparticles (synonymous with single particle states) comes from a damping term in the respective Lagrangian, added to ensure the convergence of the path integral from where the Green's functions are derived by functional differentiations. However the added damping term may not observe the symmetry of the Lagrangian, thus raising the question of its rearrangement[23] in the limit $\varepsilon \to 0$.

Assuming that the damping is caused by the added $i\varepsilon\psi^\dagger(x)\tau_3\psi(x)$ term in the fermion Lagrangian one can write in the Fourier space the Ward-Takahashi identity linking the order parameter to the boson propagator

$$v = \lim_{\varepsilon\to 0} -i\varepsilon \left\{ \frac{|\langle 0 \mid \tau_H^+(0,0) \mid \tau_-(0)\rangle|^2}{\mathcal{E}_-(0) - i0} - \frac{|\langle 0 \mid \tau_H(0,0) \mid \tau_+(0)\rangle|^2}{\mathcal{E}_+(0) + i0} \right\}, \quad (A.18)$$

and see that the pole contribution to the boson propagator may ensure the non-zero value of the order parameter only if

$$\mathcal{E}_\pm(\mathbf{p}) \to 0, \quad \text{when} \quad |\mathbf{p}| \to 0. \quad (A.19)$$

Then

$$v = |\langle 0 \mid \tau_H^+(0,0) \mid \tau_-(0)\rangle|^2 - |\langle 0 \mid \tau_H(0,0) \mid \tau_+(0)\rangle|^2 . \quad (A.20)$$

If one considers a small isospin violating boson density coupling to an external field h (in energy units), i.e. $h\psi^\dagger(x)\tau_3\psi(x)$, then its effect will be to shift the poles of A.17 at zero momenta up and below the ground state energy. Arguing that the mode with energy less than the ground state energy would be unobservable (in a properly normalized vacuum) one obtains that one of the normalization coefficients in A.17 must be identically zero. Assuming without loss of generality that it is the first coefficient that is non zero one obtains that the boson propagator is

$$\Delta(\mathbf{q}, q_o) = \frac{|\langle 0 \mid \tau_H^+(0,0) \mid \tau_-(\mathbf{q})\rangle|^2}{q_o - \mathcal{E}_-(\mathbf{q}) - i0} + continuum \quad (A.21)$$

with the dispersion relation of a gapless mode Eq.A.19.

Thus we have proved that to satisfy the Ward-Takahashi identities in the case of spontaneously broken (global) symmetry one needs to assume the existence of a gapless boson (Goldstone theorem).

Appendix B

Particle-hole Calculation of the Isobaric Analog Resonance

Construct the proton (particle) - neutron (hole) wave function coupled to zero momentum

$$\psi_i(\mathbf{r}_p - \mathbf{r}_{\bar{n}}) = (-1)^{j_i} \left[Y_{j_i}(\theta_{\bar{n}}, \varphi_{\bar{n}}) Y_{j_i}(\theta_p, \varphi_p)\right]_0^0 R_{j_i}(r_p) R_{j_i}(r_{\bar{n}}), \tag{B.1}$$

where

$$[Y_{j_i}(\theta_{\bar{n}}, \varphi_{\bar{n}}) Y_{j_i}(\theta_p, \varphi_p)]_0^0 \equiv \sum_m C(j_i, j_i, 0; m, -m, 0) Y_{j_i}^m(\theta_p, \varphi_p) Y_{j_i}^{-m}(\theta_{\bar{n}}, \varphi_{\bar{n}}) \tag{B.2}$$

For a contact interaction

$$Y_{j_i}^m(\theta, \varphi) Y_{j_i}^{-m}(\theta, \varphi) =$$
$$\sum_L \left[\frac{(2j_i+1)(2j_i+1)}{4\pi(2L+1)}\right]^{\frac{1}{2}} C(j_i, j_i, L; m, -m, 0) C(j_i, j_i, L; 0, 0, 0) Y_L^0(\theta, \varphi)$$

Substituting in $B.2$ and summing over m picks up the L=0 contribution

$$[Y_{j_i}(\theta_{\bar{n}}, \varphi_{\bar{n}}) Y_{j_i}(\theta_p, \varphi_p)]_0^0 = \frac{(2j_i+1)}{\sqrt{4\pi}} C(j_i, j_i, 0; 0, 0, 0) Y_0^0, \tag{B.3}$$

where

$$C(j_i, j_i, 0; 0, 0, 0) = \frac{(-1)^{j_i}}{\sqrt{2j_i+1}} \tag{B.4}$$

Therefore

$$\psi_i(\mathbf{r}_p - \mathbf{r}_{\bar{n}}) = \sqrt{\frac{2j_i+1}{4\pi}} R_{j_i}(r_p) R_{j_i}(r_{\bar{n}}) Y_0^0 \tag{B.5}$$

and the matrix element is

$$\langle j'\bar{j}'(0) \mid V \mid j\bar{j}(0)\rangle = \frac{F_o}{4\pi}\frac{1}{2}(2j+1)^{\frac{1}{2}}(2j'+1)^{\frac{1}{2}}, \quad \text{(B.6)}$$

where F_o stands for

$$F_o = V_o \int_0^\infty dr r^2 R_j^2(r) R_{j'}^2(r). \quad \text{(B.7)}$$

Appendix C

Calculation of the Symmetry Energy Constant a_I

Carrying out the integration in the domain where the argument of the θ-functions is positive yields (the index HF in I_{HF} is omitted)

$$I = 2\int \frac{d^3p}{(2\pi)^3}\left[\Theta(-\epsilon_+(\mathrm{p})) - \Theta(-\epsilon_-(\mathrm{p}))\right] = \frac{1}{(2\pi)^3}\frac{4\pi}{3}\left[p_+^3 - p_-^3\right], \tag{C.1}$$

where

$$p_\pm = \left[2m(\varepsilon_F \pm \frac{\lambda I}{2})\right]^{\frac{1}{2}} \tag{C.2}$$

Therefore the order parameter I in the HF-approximation is self-consistently determined as a function of the coupling constant λ and the nuclear density n from the equation

$$\frac{I}{2} = \frac{n}{4\varepsilon_F^{\frac{3}{2}}}\left[(\varepsilon_F + \frac{\lambda I}{2})^{\frac{3}{2}} - (\varepsilon_F - \frac{\lambda I}{2})^{\frac{3}{2}}\right], \tag{C.3}$$

which was obtained from $C.1$ using $C.2$ and

$$n = \frac{2}{3\pi^2}(2m\varepsilon_F)^{\frac{3}{2}} \tag{C.4}$$

Expressing the nuclear density n through the densities of protons and neutrons $n = n_+ + n_-$ and using that each density is determined from

$$n_\pm = 2\int \frac{d^3p}{(2\pi)^3}\Theta(-\epsilon_\pm(\mathrm{p})) \tag{C.5}$$

one obtains

$$\frac{n}{2} = \frac{n}{4\varepsilon_F^{\frac{3}{2}}}\left[(\varepsilon_F + \frac{\lambda I}{2})^{\frac{3}{2}} + (\varepsilon_F - \frac{\lambda I}{2})^{\frac{3}{2}}\right]. \tag{C.6}$$

Further, it is convenient to redefine the order parameter I as a dimensionless parameter Δ denoting the relative isospin polarisation

$$\Delta \equiv \frac{n_+ - n_-}{n_+ + n_+} = \frac{n_+ - n_-}{n}. \tag{C.7}$$

Then by combining $C.3$ and $C.6$ one obtains an equation which relates the relative isospin polarisation to the nuclear density and coupling constant.

$$\frac{\lambda n}{2\varepsilon_F} = \frac{1}{\Delta}\left[(1+\Delta)^{\frac{2}{3}} - (1-\Delta)^{\frac{2}{3}}\right]. \tag{C.8}$$

This equation is an analog of the well known Stoner equation in the theory of ferromagnetism. The only difference is the coefficient 2 which arises from the additional isospin degree of freedom of nuclear matter.

For fixed coupling constant λ the nonzero isospin polarization occurs at nuclear densities

$$n \geq 2\frac{4\varepsilon_F}{3\lambda} \tag{C.9}$$

Considering the actual value of nuclear relative isospin polarisation $\Delta \approx 0.2$, one obtains that the actual nuclear density is just a few percent above the critical value

$$n_c = 2\frac{4\varepsilon_F}{3\lambda}. \tag{C.10}$$

Therefore one can take the critical nuclear density n_c as representitative of the actual nuclear density n. This allows us to express the coupling constant λ through the critical nuclear density

$$\lambda = 2\frac{4\varepsilon_F}{3n_c} \tag{C.11}$$

and by substitution in

$$4a_I\frac{N-Z}{A} = -(\mu_p - \mu_n) = \lambda(n_+ - n_-). \tag{C.12}$$

to obtain the value of the symmetry energy coefficient a_I

$$a_I = \frac{2}{3}\varepsilon_F. \tag{C.13}$$