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THE UNIVERSITY OF ALBERTA

A NEW TECHNIQUE FOR LEAST ABSOLUTE VALUE POWER SYSTEM STATE ESTIMATION

by

AMIR HOUSSEIN ROUHI

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

AND RESEARCH IN PARTIAL FULFILMENT OF THE

REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF ELECTRICAL ENGINEERING

EDMONTON, ALBERTA

FALL 1988

ISBN 0-315-45651-5

THE UNIVERSITY OF ALBERTA

RELEASE FORM

NAME OF AUTHOR: AMIR HOUSSEIN ROUHI

TITLE OF THESIS: A NEW TECHNIQUE FOR LEAST ABSOLUTE VALUE POWER SYSTEM STATE ESTIMATION

DEGREE: MASTER OF SCIENCE

YEAR THIS DEGREE GRANTED: 1988

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The Undersigned Certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled A NEW TECHNIQUE FOR LEAST ABSOLUTE VALUE POWER SYSTEM STATE ESTIMATION submitted by AMIR HOUSSEIN ROUHI, in partial fulfilment of the requirements for the degree of Master of Science in Electrical Engineering.

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Supervisor

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Date: Aug. 31/88

To My Parents

and

To My Grandmother Mrs. Iran Mavaddat

ABSTRACT

A state estimator plays an important role in the operation of a power system. In this thesis the development and testing of a new power system static state estimator is described. The new state estimator, which is based on a recently developed least absolute value estimation technique, produces accurate estimates. The test results, which are presented indicate that the new estimator is a viable alternative to least squares power system state estimators, which are currently being used.

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to my supervisor Dr. G.S. Christensen. Without his support, guidance and encouragement this thesis would not have been possible. I also wish to thank Dr. S.A. Soliman for his enthusiasm and frequent discussions. I consider it a great honour to have studied under the guidance of these two gentlemen.

I am grateful to Dr. A.M. Atallah for his helpful advice and counsel. Thanks is also due to Dr. S.Y. Mansour, for providing valuable information and insight. I also wish to thank Drs. D.H. Kelly and M.N. Oguztoreli for their helpful discussions.

I gratefully acknowledge the financial support that has been provided by the Department of Electrical Engineering at the University of Alberta, and the Natural Sciences and Engineering Research Council of Canada.

Finally, thanks is due to my family for their support and encouragement.

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CHAPTER I

INTRODUCTION

A power system state estimator produces an estimate of the voltages and phase angles at all of the buses in a power system. Once a power system state estimate is obtained, it can be used for various control and monitoring functions.

A number of power system state estimators have been developed. Most of these estimators are based on the concept of least squares estimation. Unfortunately least squares estimators produce inaccurate estimates when the data that they process contains one or more erroneous measurements.

A few authors have developed least absolute value based estimators that use linear programming in the estimation process. These estimators have not been widely accepted because they use excessive memory space and computing time.

In this thesis a new least absolute value state estimator is presented.

1.1 Outline of the Thesis

In chapter II the parameter estimation problem is introduced. Different types of estimates are then discussed and compared. In chapter III, linear programming based least absolute value state estimation is explained and its disadvantages are presented. A new approach to least absolute value state estimation is then introduced and discussed. In chapter IV the power system state estimation problem is defined and a brief overview of previous research is given. In the fifth chapter the new power system state estimator is described.

In chapter VI test results are given. These results are used to evaluate the new power system state estimator and support the conclusions that are made in the final chapter of this thesis.

CHAPTER II

INTRODUCTION TO LINEAR PARAMETER ESTIMATION

In this chapter the parameter estimation problem is stated. The different criteria for judging the quality of an estimate are then presented and compared. This is followed by a review of least squares estimation and the properties of least squares estimates. Finally, least squares and least absolute value estimation are compared using a couple of examples. This chapter deals exclusively with unconstrained estimation; constrained estimation is dealt with in chapter III.

2.1 Statement of the Linear Parameter Estimation Problem

The parameter estimation problem involves estimating n system parameters from m (m ≥ n) measurements, and other system information, i.e. system structure and constraints. Equation (2.1) describes the relationship between the measurements and the system parameters.

$$\underline{z} = H\underline{\theta} + \underline{v} \tag{2.1}$$

where:
- $\underline{z}$ is the mx1 (m ≥ n) vector of measurements.
- $\underline{\theta}$ is the nx1 vector of the parameters that are to be estimated.
- H is the mxn matrix which describes the mathematical relationship between the measurement vector $\underline{z}$ and the parameter vector $\underline{\theta}$, in the absence of measurement errors.
- $\underline{v}$ is the mx1 vector of unknown measurement

errors. Each element of $\underline{v}$ represents the error in a measurement.

The parameter estimation problem is to estimate the elements of the parameter vector $\underline{\theta}$, given the H matrix and the measurement vector $\underline{z}$, with the elements of $\underline{v}$ unknown.

If the number of measurements (m) equals the number of parameters (n), then an estimate of $\underline{\theta}$ can be obtained by using $\underline{\theta} = H^{-1}\underline{z}$. For this type of calculation the parameter estimate exactly fits the measurement vector, i.e. $\underline{z} - H\underline{\theta} = \underline{v} = \underline{0}$. Estimates obtained in this manner are of poor quality and not useful, since this estimation process assumes that the error vector $\underline{v}$ only contains 0's. Thus, this type of estimate does not account for, or filter out measurement errors.

In most cases the number of measurements exceeds the number of system parameters, m > n. With more measurements than unknowns, an exact fit of the parameter vector $\underline{\theta}$ to the measurement vector $\underline{z}$ is not possible if some of the measurements are inaccurate and contain errors. However, since the number of measurements exceeds the number of unknown parameters, measurement errors can be filtered out in the estimation process and a good quality estimate can be produced.

Given m > n, there are many different estimates of $\underline{\theta}$ that can be calculated. Define an mx1 vector $\underline{r}$ of residuals as:

$$\underline{r} = \underline{z} - H\underline{\theta} \tag{2.2}$$

There are three common procedures for determining an estimate of $\underline{\theta}$.

Define three cost functions as follows [1].

$$J_1(\underline{\theta}) = \sum_{i=1}^{m} |z_i - H_i\underline{\theta}| = \sum_{i=1}^{m} |r_i| \tag{2.3}$$

$$J_2(\underline{\theta}) = \sum_{i=1}^{m} (z_i - H_i\underline{\theta})^2 = \sum_{i=1}^{m} r_i^2 \tag{2.4}$$

$$J_\infty(\underline{\theta}) = \lim_{p\to\infty} \left(\sum_{i=1}^{m} |z_i - H_i\underline{\theta}|^p \right)^{1/p}$$

$$= \lim_{p\to\infty} \left(\sum_{i=1}^{m} |r_i|^p \right)^{1/p} = \max |r_i| \tag{2.5}$$

where: - r_i, z_i and H_i are the rows of $\underline{r}$, $\underline{z}$, and H, that correspond to the ith measurement.

- m is the number of measurements.

In equation (2.3) the cost function is the sum of the absolute values of the residuals. When an estimate of $\underline{\theta}$ is calculated so as to minimize $J_1(\underline{\theta})$, the estimate is called an L_1 or least absolute value (LAV) estimate. The cost function of equation (2.4) represents the sum of the squares of the residuals. When the estimate of $\underline{\theta}$ minimizes $J_2(\underline{\theta})$ the estimate is called an L_2 or least squares (LS) estimate. The cost function of equation (2.5) is equal to the absolute value of the largest residual. Minimization of $J_\infty(\underline{\theta})$ yields an estimate of $\underline{\theta}$ that is called the L_∞ or Chebyshev estimate.

Given a set of measurements, an L_1 estimate represents the median of the data, the mean or common average is represented by an L_2 estimate, and the midrange, which is calculated by taking the average of the largest and the smallest data point, is represented by an L_∞ estimate [1].

The following two examples demonstrate some of the properties of the three types of estimates.

Example 2.1 [1]

Given the following five measurements of y calculate the L_1, L_2 and L_∞ estimates of y.

$y_1 = 24.50,\ y_2 = 26.25,\ y_3 = 24.25,\ y_4 = 24.75,\ y_5 = 24.00$.

For this example the structure of the system can be represented by equation (2.6).

$$\underline{z} = Hy + \underline{v} \tag{2.6}$$

where: - *y is the parameter that is to be estimated.*
- *$\underline{z}$ is the vector of measurements.*

$$\underline{z} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 24.50 \\ 26.25 \\ 24.25 \\ 24.75 \\ 24.00 \end{bmatrix}$$

- *H relates $\underline{z}$ to y.*

$$H^T = [\ 1\ ,\ 1\ ,\ 1\ ,\ 1\ ,\ 1\ ]$$

- *$\underline{v}$ represents the measurement errors in $\underline{z}$.*

$$\underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

From the set of measurements and the system structure the following three estimates of y are calculated.

The L_1 estimate is y = 24.500.

The L_2 estimate is y = 24.750.

The L_∞ estimate is y = 25.125.

Consider that 4 of the 5 measurements lie in the range $24.00 \leq y \leq 24.75$, and the other measurement ($y_2 = 26.25$) falls well outside this range (In keeping with accepted terminology, this type of measurement will be called an outlier or a bad data point). For this example the L_1 estimate of y lies in the given range while the L_2 estimate lies on the boundary of the range and the L_∞ estimate lies outside of the range.

Example 2.2

If the second measurement in example 2:1 is changed from $y_2 = 26.25$ to $y_2 = 100.00$, then the following estimates are obtained.

The L_1 estimate is y = 24.500.

The L_2 estimate is y = 39.500.

The L_∞ estimate is y = 62.000.

Note that the L_1 estimate remains unchanged from example 2.1 and is unaffected by the outlier, whereas the other two estimates are adversely affected by the bad data point.

Of the three estimates the L_∞ is the most affected by bad data, and will not be considered further in this thesis. Least squares estimation, in spite of the fact that least squares estimates are biased by bad data, is the most commonly used type of estimation and will be investigated in the next section.

2.2 Least Squares Estimation

The first application of least squares estimation appears to have taken place in 1795, when Karl Gauss used L_2 estimation to predict the motion of several comets and planets [2].

Least squares estimation has been applied to many estimation problems since then, and continues to be widely used. Several authors [3,4] have speculated that the popularity of least squares estimation is not due to any superiority of L_2 estimates over L_1 or other types of estimates, but instead stems from the ease of computation and well known mathematical properties of least squares estimation. Numerous authors have compared L_1 and L_2 estimation; see for example [5,6,7]. However, a consensus on the superiority of either estimation technique does not appear to have been reached. The question of the supremacy of either estimation technique lies in the domain of the mathematicians and will not be speculated upon here. Nevertheless for many classes of problems, given the nature of the measurements common to the problem, a convincing case can often be made in favor of one type of estimation over the other. The next three sections discuss L_2 estimation.

2.2.1 Derivation of the Least Squares Estimation Equation

The following derivation is made under the assumption that the matrix H is of full rank.

Recall the least squares cost function given in section 2.1.

$$J_2(\underline{\theta}) = \sum_{i=1}^{m} (z_i - H_i\underline{\theta})^2 \qquad (2.4)$$

Equation (2.4) can be rewritten in the following form:

$$J_2(\underline{\theta}) = (\underline{z} - H\underline{\theta})^T(\underline{z} - H\underline{\theta}) \qquad (2.7)$$

Expanding (2.7) yields:

$$J_2(\underline{\theta}) = \underline{z}^T\underline{z} - \underline{\theta}^T H^T \underline{z} - \underline{z}^T H\underline{\theta} + \underline{\theta}^T H^T H\underline{\theta} \qquad (2.8)$$

Since $\underline{\theta}^T H^T \underline{z} = \underline{z}^T H\underline{\theta}$, (2.8) can be written as:

$$J_2(\underline{\theta}) = \underline{z}^T\underline{z} - 2\underline{z}^T H\underline{\theta} + \underline{\theta}^T H^T H\underline{\theta} \qquad (2.9)$$

The least squares estimate of $\underline{\theta}$ minimizes the cost function $J_2(\underline{\theta})$. Setting the first derivative of $J_2(\underline{\theta})$ equal to 0 yields values of $\underline{\theta}$ that either maximize or minimize the cost function.

$$\frac{dJ_2(\underline{\theta})}{d\underline{\theta}} = -2H^T\underline{z} + 2H^TH\underline{\theta} = 0 \qquad (2.10)$$

or

$$\underline{\theta} = \underline{\hat{\theta}} = (H^TH)^{-1}H^T\underline{z} \qquad (2.11)$$

$\underline{\hat{\theta}}$ represents the least squares estimate of $\underline{\theta}$. For (2.11) to be valid (H^TH) must be invertible. This is always true when H is of full rank as assumed earlier. In order to determine whether $\underline{\hat{\theta}}$ is a minimum or a maximum, the second derivative of the cost function must be calculated.

$$\frac{d^2J_2(\underline{\theta})}{d\underline{\theta}^2} = 2H^TH > 0 \qquad (2.12)$$

Note that $2H^TH$ is always positive definite because H is of full rank. Consequently $\underline{\hat{\theta}}$ minimizes $J_2(\underline{\theta})$ and represents the L_2 parameter estimate which can be calculated by using the least squares estimation equation (2.11) [2,8].

2.2.2 Weighted Least Squares Estimation

It is possible to assign weights to each measurement so that measurements which are assigned larger weights influence the least squares estimate more than measurements assigned smaller weights. The cost function for weighted least squares estimation is:

$$J_2(\underline{\theta}) = \sum_{i=1}^{m} w_i(z_i - H_i\underline{\theta})^2 \qquad (2.13)$$

where w_i is the weight assigned to the ith measurement. Equation (2.13) can be rewritten as:

$$J_2(\underline{\theta}) = (\underline{z} - H\underline{\theta})^T W(\underline{z} - H\underline{\theta}) \qquad (2.14)$$

where W is a diagonal mxm matrix that contains the weights, i.e. W = diag. { w_i , $i=\overline{1,m}$ }. It can be shown that the weighted least squares estimation equation is given by:

$$\hat{\underline{\theta}} = (H^TWH)^{-1}H^TW\underline{z} \qquad (2.15)$$

Note that equation (2.15) represents the weighted least squares estimate $\hat{\underline{\theta}}$ only if W is positive definite and H is of full rank.

The weighting matrix, for least squares estimation, is often set equal to the inverse of the mxm covariance matrix R.

$$R = \text{Expected Value of } [\ \underline{v}\ \underline{v}^T\ ] \qquad (2.16)$$

The weighting matrix is diagonal unless the measurements are not independent, i.e. the covariance matrix contains non-zero, off diagonal terms.

2.2.3. Properties of Least Squares Estimates

Least squares estimates possess a number of interesting properties. It has been shown [2] that L_2 estimates are the best estimates (maximum likelihood) when the measurement errors obey a Gaussian or normal distribution and the weighting matrix is equal to the inverse of the covariance matrix. It has also been stated [9] that in cases where the measurement error distribution is not Gaussian, but the number of measurements greatly exceeds the number of unknown parameters, the method of least squares yields very good estimates.

Another valuable feature of L_2 estimation is the ease with which least squares estimates can be calculated; equations (2.11) and (2.15) can easily be implemented on a computer. In contrast, no algorithm has yet been devised that makes the calculation of L_1 estimates as easy as the calculation of L_2 estimates.

There are many estimation problems for which the error distribution is not Gaussian and the number of measurements does not greatly exceed the number of unknown parameters. In these cases, least squares estimates are adversely affected by bad data. This problem has been recognized and addressed by several authors [3] who have proposed different ways of refining the least squares method so that L_2 estimates are less affected by bad data.

Robustness term which describes the effect of bad data on an estimation technique. The more robust an estimation algorithm, the less sensitive it is to outliers. Although the authors mentioned in [3] have had some success developing techniques that increase the robustness of L_2 estimation, L_1 estimation, without any adjustments, remains one of the most robust estimation procedures.

Despite some of the difficulties encountered by L_2 estimation when bad data is present, least squares estimation cannot be dismissed. The overwhelming majority of research in power system state estimation has used L_2 estimation techniques. Only a handful of papers have investigated L_1 state estimation. Also, L_2 estimates provide information that can be used in L_1 estimation.

2.3 Comparison of Least Squares and Least Absolute Value Estimation

It was mentioned earlier that the choice of L_1 or L_2 estimation depends on the nature of the measurements encountered in the power system state estimation problem. The two examples that are presented in this section compare L_1 and L_2 estimates, that are calculated for data sets which are similar to those encountered in the power system state estimation problem, i.e. the data set contains many accurate measurements and a few bad data points.

The actual procedure for determining L_1 estimates will be given in chapter III.

Example 2.3 [4]

Fit the following set of measurements with a straight line of the form $y = ax + b$.

x	0	1	2	3	4	5
y	2	3	7	5	6	7

The data points represent the equation $y = x + 2$ exactly, except for the third data point which contains a large error. The L_2 estimate, which does not fit any of the data points, is $y = 0.914x + 2.714$, $a = 0.914$ and $b = 2.714$. The L_1 estimate, which fits all but the third data point, is $y = x + 2$, $a = 1.0$ and $b = 2.0$. Figure 2.1 shows the least absolute value and the least squares estimates.

Example 2.4

Fit the following set of measurements with a line of the form $y = ax + b$.

x	0	1	2	3	4	5
y	-2	1	4	13	10	7

The data points represent the equation $y = 3x - 2$, exactly except for data points 4 and 6 which should be (3,7) and (5,13), but have their y values exchanged. The L_2 estimate, which does not fit any of the data points is $y = 2.314x - 0.286$, $a = 2.314$ and $b = -0.286$. The L_1 estimate, which exactly fits all but the 4th and 6th data points, is $y = 3.0x - 2.0$, $a = 3.0$ and $b = -2.0$. Figure 2.2 shows the least absolute value and least squares estimates.

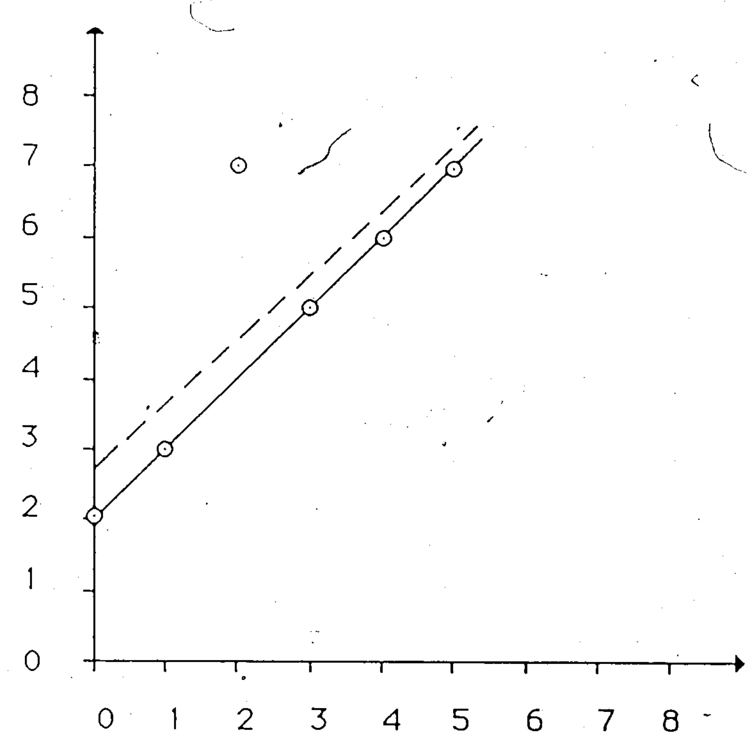

Figure 2.1 Least Squares and Least Absolute Value Estimates for Example 2.3

LEGEND	
Least Squares Estimate $y = 0.914x + 2.714$	- - - - -
Least Absolute Value Estimate $y = x + 2$	———
Data Points	⊙

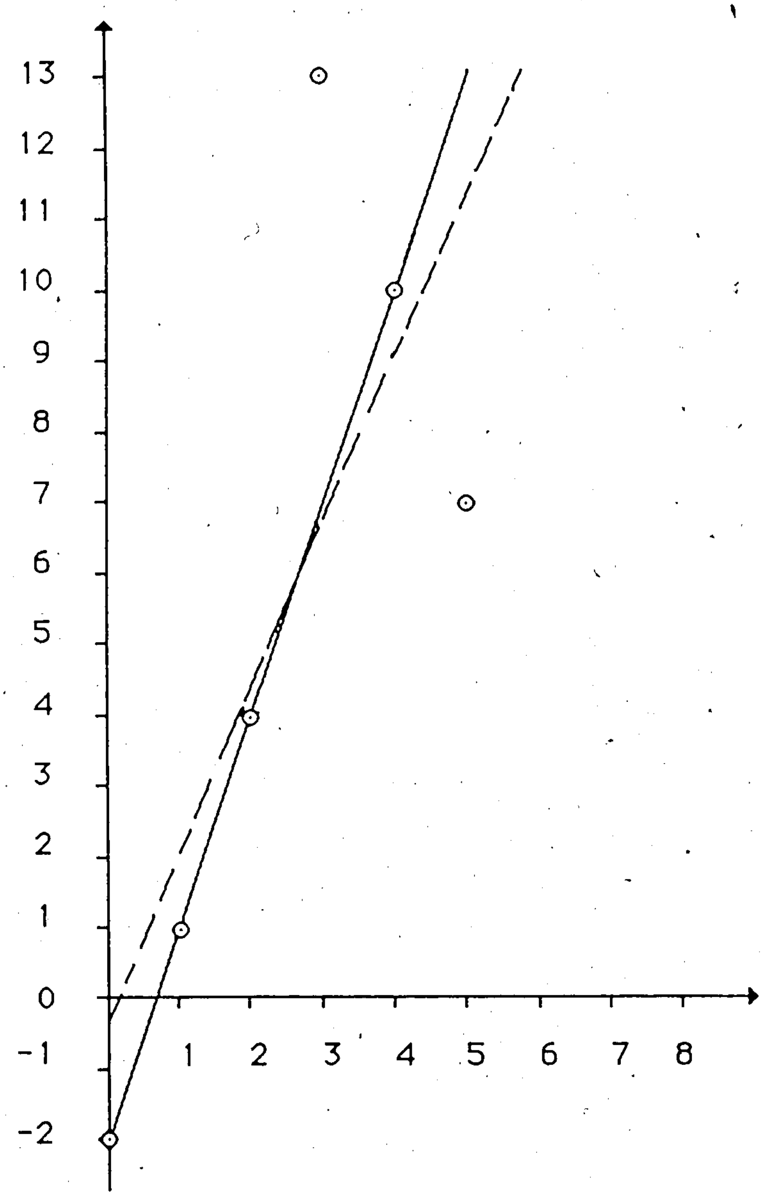

Figure 2.2 Least Squares and Least Absolute Value Estimates for Example 2.4

LEGEND

Least Squares Estimate $y = 2.314x - 0.286$ ------

Least Absolute Value Estimate $y = 3x - 2$ ——

Data Points ⊙

These two simple examples illustrate how least squares estimates can be biased by bad data. Note that the least absolute value estimates are unaffected by the bad data points. Both examples use data sets that are similar in nature to the measurement sets encountered in the power system state estimation problem. The behavior of L_1 and L_2 estimates when given these types of data sets, indicates that L_1 estimation may be better suited to the power system state estimation problem. In the following chapter least absolute value estimation will be examined in greater detail, and it will be shown how L_2 estimates can be used to provide valuable information for L_1 estimators.

CHAPTER III

LEAST ABSOLUTE VALUE ESTIMATION

This chapter begins with a discussion of linear programming based least absolute value (LAV) estimation. A new technique for L_1 estimation is then introduced and explained in great detail. Several examples are used to illustrate the simplicity and efficiency of the new technique. This chapter concludes by showing how the new estimation technique can be applied to constrained estimation problems.

Before discussing the linear programming approach to LAV estimation, a distinction should be made between the type of estimate and the estimation procedure or algorithm. In chapter II, L_1 and L_2 estimation were discussed, while in this chapter the discussion centers around two different procedures of obtaining least absolute value estimates. The two estimation procedures, although they usually produce the same estimate, differ considerably.

3.1 Linear Programming Based Least Absolute Value Estimation

The most common approach used to calculate LAV estimates, is to first formulate the estimation problem as a linear programming problem and then solve for the L_1 estimate using: the simplex method, the revised simplex method or a related technique.

A linear programming problem contains a cost function, which must be minimized or maximized, and a set of constraints. The simplex method minimizes or maximizes the cost function within the bounds imposed by the constraints. When minimizing the cost function, the simplex method begins with an initial feasible solution that does not violate any constraints and then decreases the cost function until a minimum, which represents the solution of the linear programming problem, is reached. This approach is iterative in nature and is known as a successive improvements technique. In the following section, one of several methods of formulating the L_1 estimation problem as a linear programming problem is described.

3.1.1 Formulating L_1 Estimation as a Linear Programming Problem

Given an mx1 vector of measurements $\underline{z}$, and an nx1 vector of parameters $\underline{\theta}$, the LAV estimation problem is to estimate $\underline{\theta}$ so as to minimize the cost function given by equation (3.1).

$$J_1(\underline{\theta}) = \sum_{i=1}^{m} \left| z_i - \sum_{j=1}^{n} H_{ij}\theta_j \right| \qquad (3.1)$$

where:
- z_i is the ith element of $\underline{z}$.
- θ_j is the jth element of $\underline{\theta}$.
- H_{ij} is the element in the ith row and the jth column of the mxn matrix H. The H matrix defines the relationship between $\underline{z}$ and $\underline{\theta}$.

Let $$r_i = z_i - \sum_{j=1}^{n} H_{ij}\theta_j \qquad i=1,2,\ldots,m \qquad (3.2)$$

The linear programming formulation of the LAV estimation problem is as follows [1].

$$\text{minimize} \quad \sum_{i=1}^{m} r_i \tag{3.3}$$

subject to:

$$r_i + \sum_{j=1}^{n} H_{ij}\theta_j \geq z_i \qquad i=1,2,\ldots,m \tag{3.4}$$

$$r_i - \sum_{j=1}^{n} H_{ij}\theta_j \geq -z_i \qquad i=1,2,\ldots,m \tag{3.5}$$

All of the r_i's in the formulation will be non-negative because the minimum value that any r_i can reach, without violating a constraint, is the larger of expressions (3.6) and (3.7).

$$r_i \geq z_i - \sum_{j=1}^{n} H_{ij}\theta_j \tag{3.6}$$

$$r_i \geq \sum_{j=1}^{n} H_{ij}\theta_j - z_i \tag{3.7}$$

If one of the two expressions is negative, the other is positive, and r_i must be positive in order to obey both constraint equations. If one of the two expressions equals 0 the other expression equals 0 and therefore r_i equals 0. Consequently r_i when subject to the constraint equations, represents the absolute value of the ith residual. Minimizing the sum of the r_i's in the linear programming problem produces the L_1 estimate. The following example demonstrates the formulation of a simple L_1 estimation problem as a linear programming problem.

Example 3.1 [1]

Formulate the given LAV estimation problem as a linear programming problem.

Fit the given set of measurements with a plane of the form $y = a_1 x_1 + a_2 x_2$.

y	x_1	x_2
83	1	1
125	3	-1
310	2	7
-7	1	-2
215	3	2

$$\text{let} \quad \underline{z} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 83 \\ 125 \\ 310 \\ -7 \\ 215 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 1 \\ 3 & -1 \\ 2 & 7 \\ 1 & -2 \\ 3 & 2 \end{bmatrix}, \quad \underline{\theta} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$J = \sum_{i=1}^{5} \left| z_i - \sum_{j=1}^{2} H_{ij} a_j \right| \qquad (3.8)$$

The LAV estimation problem is to calculate $\underline{\theta}$ such that J in (3.8) is a minimum. The linear programming formulation of the problem is as follows.

define $\quad r_i = z_i - \sum_{j=1}^{2} H_{ij} a_j \qquad i=1,2,\ldots,5$

minimize: $\sum_{i=1}^{5} r_i$

subject to: $\quad r_1 + a_1 + a_2 \geq 83$

$$r_2 + 3a_1 - a_2 \geq 125$$

$$r_3 + 2a_1 + 7a_2 \geq 310$$

$$r_4 + a_1 - 2a_2 \geq -7$$

$$r_5 + 3a_1 + 2a_2 \geq 215$$

$$r_1 - a_1 - a_2 \geq -83$$

$$r_2 - 3a_1 + a_2 \geq -125$$

$$r_3 - 2a_1 - 7a_2 \geq -310$$

$$r_4 - a_1 + 2a_2 \geq 7$$

$$r_5 - 3a_1 - 2a_2 \geq -215$$

In solving the problem for the L_1 estimate, feasible values of: a_1, a_2 and r_1-r_5 will be found first. Then these values will be changed, in order to iteratively decrease the value of the cost function. When the cost function reaches its minimum value, subject to the constraints, the estimation procedure is completed and the values of a_1 and a_2 represent the LAV estimate. The values of r_1-r_5 represent the absolute values of the residuals.

Most linear programming algorithms would not formulate the LAV estimation problem in the manner given in the example. Instead, an equivalent or dual problem with less constraints, but more variables per constraint equation would be solved. Nevertheless, example 3.1 is useful in showing how L_1 estimation problems can be converted into linear programming problems.

3.1.2 Disadvantages of Linear Programming Based L_1 Estimation

Linear programming is the most popular method of calculating L_1 estimates. Several different linear programm g formulations of the LAV estimation problem have been presented in the literature [10,11,12,13]. Despite its popularity linear programming based L_1 estimation has certain disadvantages:

a) It requires excessive memory storage; a typical formulatic requires the manipulation of a matrix of size (2mnxm) [14], where m is the number of measurements and n is the number of unknown parameters.

b) It is an iterative technique and thus may use considerable CPU time and be computationally inefficient [6].

c) The solutions obtained may not be unique [15].

Some of the more recent algorithms have attempted to overcome these difficulties and research continues in this area.

In the next section a new non-iterative technique for LAV estimation is presented.

7

3.2 A New Technique for Least Absolute Value Estimation

3.2.1 History of Combined L_1 And L_2 Estimation

It was mentioned earlier that a great deal of research in least absolute value estimation has involved attempts to prove the superiority of L_1 estimates over L_2 estimates and the development of more efficient linear programming based estimation algorithms. Very little research effort has been expended in finding and developing relationships between the two types of estimates.

In 1973 Schlossmacher [14] presented an iterative technique which uses successive weighted least squares estimates to find an LAV estimate. His algorithm has the following steps:

1) Obtain a weighted least squares estimate with all of the weighting factors set equal to one i.e. { $w_i = 1$, $i = 1,2,\ldots,m$; m = the number of measurements }.
2) Use the generated weighted least squares solution to calculate the residuals { r_i , $i = 1,2,\ldots,m$ }.
3) Set $w_i = 1/|r_i|$ $i = 1,2,\ldots,m$. If any $r_i \cong 0$, set $w_i = 0$.
4) Repeat steps (2) and (3) until the changes in the r_i's, between successive iterations approaches 0.

Although Schlossmacher's technique gives approximate estimates it is an iterative technique and has been criticized as being computationally inefficient [16].

In 1976-77 Sposito, Hand and McCormick [17,18] suggested that L_2 estimates be used as starting points for linear programming based L_1 estimators. Their research indicates that starting a linear programming based L_1 estimator at the L_2 estimate saves many iterations. They found that in general the total computing time (time to calculate the L_2 estimate + time to calculate the L_1 estimate) of their technique, is less than the time needed to calculate an L_1 estimate from a flat start. The main drawback of their technique is that it still requires a linear programming algorithm to calculate the L_1 estimate.

In 1987 Christensen and Soliman [5,19,20] developed a new L_1 estimation procedure. Their procedure, which does not use linear programming, manipulates a simple relationship between L_1 and L_2 estimates. Their new technique is non-iterative and uses information provided by a least squares estimate to calculate an LAV estimate.

3.2.2 The New Least Absolute Value Estimation Technique

In this section the new estimation technique that was developed by Christensen and Soliman is given. The new estimation technique utilizes the interpolation property that is stated in theorem 3.1.

<u>*Theorem 3.1*</u> [21]

If the column rank of the mxn matrix H is k $(k \leq n)$, then the L_1 estimate interpolates at least k of the m measurements.

Since LAV estimates interpolate measurements, the estimation problem reduces to selecting the k points that the estimate should interpolate. The new technique assumes that H has full column rank and therefore k = n.

Given the measurement equation (3.9), the first step of the method is to calculate the least squares estimate of $\underline{\theta}$ which is given by $\underline{\theta}^*$ in (3.10).

$$\underline{z} = H\underline{\theta} + \underline{v} \tag{3.9}$$

$$\underline{\theta}^* = (H^TH)^{-1}H^T\underline{z} \tag{3.10}$$

The residuals of the L_2 estimate are then calculated using (3.11).

$$r_i = z_i - H_i\underline{\theta}^* \qquad i = 1,2,..,m \tag{3.11}$$

where:
- r_i is the ith residual.
- z_i is the ith measurement.
- H_i is the ith row of H.

The residuals are then ranked by their absolute values and stored in the mx1 vector $\underline{r}$, with the smallest residual as r_1 and the largest as r_m.

$$\underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_n \\ \vdots \\ r_m \end{bmatrix} = \begin{bmatrix} \hat{\underline{r}} \\ \hline \underline{r}^* \end{bmatrix} \begin{array}{l} \updownarrow n \times 1 \\ \updownarrow (m-n) \times 1 \end{array} \Bigg\} m \times 1 \qquad (3.12)$$

The rows of the $\underline{z}$ vector and the H matrix are also rearranged so that all the z_i's and all the rows of H, correspond to the ranking of the absolute values of the residuals.

$$H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \\ \vdots \\ H_m \end{bmatrix} = \begin{bmatrix} \hat{H} \\ \hline H^* \end{bmatrix} \begin{array}{l} \updownarrow n \times n \\ \updownarrow (m-n) \times n \end{array} \Bigg\} m \times n \qquad (3.13)$$

$$\underline{z} = \begin{bmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ z_n \\ \cdot \\ \cdot \\ \cdot \\ z_m \end{bmatrix} = \begin{bmatrix} \hat{\underline{z}} \\ \hline \underline{z}^* \end{bmatrix} \begin{matrix} \updownarrow n \times 1 \\ \updownarrow (m-n) \times 1 \end{matrix} \; \updownarrow m \times 1 \tag{3.14}$$

The sub-vectors $\hat{\underline{r}}$ and $\hat{\underline{z}}$ and the sub-matrix $\hat{H}$ correspond to the n smallest residuals.

The final step of the algorithm is to calculate the LAV estimate from equations (3.15) and (3.16).

$$\hat{\underline{z}} = \hat{H}\hat{\underline{\theta}} \tag{3.15}$$

$$\hat{\underline{\theta}} = \hat{H}^{-1}\hat{\underline{z}} \tag{3.16}$$

Once the final estimate is calculated the LAV measurement residuals can be calculated using (3.17).

$$\underline{r} = \underline{z} - H\hat{\underline{\theta}} \tag{3.17}$$

From equations (3.12) to (3.16) it can easily be seen that the first n residuals of (3.17) will be equal to zero. The other n-m residuals will be either zero or non-zero.

The four steps in the new algorithm can be summarized as follows:

1) Calculate the least squares estimate of $\underline{\theta}$ using

$$\underline{\theta}^* = (H^T H)^{-1} H^T \underline{z} .$$

2) Calculate the residuals of the L_2 estimate using (3.11).

$$r_i = z_i - H_i \underline{\theta}^* \qquad i = 1,2,\ldots,m \qquad (3.11)$$

3) Select the n measurements that correspond to the residuals with the smallest least absolute values and form $\hat{\underline{z}}$ and $\hat{H}$.

4) Solve for the LAV solution $\hat{\underline{\theta}}$ using equation (3.16).

A useful property of L_1 estimates is given by theorem 3.2.

<u>*Theorem 3.2*</u> [17]

For a least absolute value estimate of n parameters, if n_1 is the number of positive residuals and n_2 is the number of negative residuals, then an optimal least absolute value estimate $\hat{\underline{\theta}}$ obeys the following equation.

$$|n_1 - n_2| \le n$$

Theorem 3.2 gives necessary, but not sufficient conditions for optimality. Thus L_1 estimates that are produced by the new technique, or by any other technique, can be checked if they obey theorem 3.2. If $|n_1 - n_2| \le n$, then the estimate may be optimal. If however, $|n_1 - n_2| > n$, then the estimate is definitely not optimal.

The following two examples illustrate the application of the new technique.

<u>*Example 3.2*</u>

Fit the data { (1,2),(2,4),(3,6),(4,0),(5,15),(6,12) } with a straight line of the form $y = a_1x + a_2$. (THe data represents the line $y = 2x + 0$, except for data points 4 and 5 which do not fall on the line.)

$$\underline{z} = \begin{bmatrix} 2 \\ 4 \\ 6 \\ 0 \\ 15 \\ 12 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix} \qquad \underline{\theta} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

<u>*Step 1*</u> *Calculate the least squares estimate $\underline{\theta}^*$.*

$$\underline{\theta}^* = (H^T H)^{-1} H^T \underline{z} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.20 \\ -1.20 \end{bmatrix}$$

Step 2 Calculate the residuals of the least squares estimate.

$$\underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix} = \underline{z} - H\underline{\theta}^* = \begin{bmatrix} 1.00 \\ 0.80 \\ 0.60 \\ -7.60 \\ 5.20 \\ 0.00 \end{bmatrix}$$

Step 3 Select the two measurements that correspond to the smallest residuals and form $\hat{\underline{z}}$ and $\hat{H}$.

The smallest residuals are r_6 and r_3.

$$\hat{\underline{z}} = \begin{bmatrix} 12 \\ 6 \end{bmatrix} \qquad \hat{H} = \begin{bmatrix} 6 & 1 \\ 3 & 1 \end{bmatrix}$$

Step 4 Solve for the LAV estimate.

$$\hat{\underline{\theta}} = \hat{H}^{-1}\hat{\underline{z}}$$

$$\hat{\underline{\theta}} = \begin{bmatrix} 6 & 1 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 6 \end{bmatrix}$$

$$\hat{\underline{\theta}} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 0.0 \end{bmatrix}$$

The same solution has been obtained by linear programming.

The LAV residuals, of the measurements that were not selected for interpolation are: $r_1 = 0.0$, $r_2 = 0.0$,

$r_4 = -8.0$ and $r_5 = 5.0$. Note that the least absolute value estimate fits all but the 4th and the 5th data points, i.e. the 3rd and the 6th data point are selected for interpolation and the first two data points have residuals equal to 0. Thus the two bad data points, which do not fall on the line $y = 2x + 0$, are rejected by the new estimation technique. In contrast, the least squares estimate does not interpolate any of the data points and is affected by bad data. Figure 3.1 shows the data points and the L_1 and L_2 estimates.

In example 3.3 the LAV estimation problem which was formulated as a linear programming problem in example 3.1, is solved using the new technique.

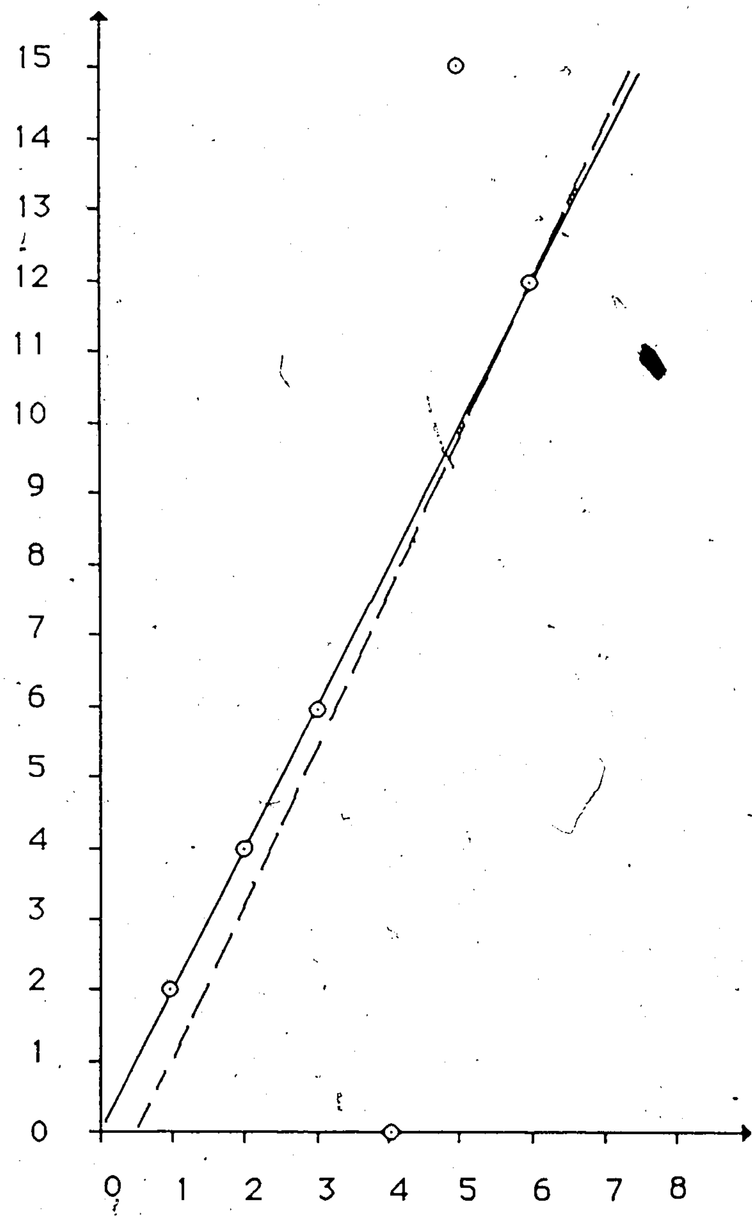

Figure 3.1 Least Squares and Least Absolute Value Estimates for Example 3.2

LEGEND

Least Squares Estimate $y = 2.20x - 1.20$ - - - - -

Least Absolute Value Estimate $y = 2x + 0$ ———

Data Points ⊙

Example 3.3 [1]

Fit the given set of measurements with a plane of the form $y = a_1x_1 + a_2x_2$.

y	x_1	x_2
83	1	1
125	3	-1
310	2	7
-7	1	-2
215	3	2

$$\text{let} \quad \underline{z} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 83 \\ 125 \\ 310 \\ -7 \\ 215 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 1 \\ 3 & -1 \\ 2 & 7 \\ 1 & -2 \\ 3 & 2 \end{bmatrix}, \quad \underline{\theta} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Step 1 Calculate the least squares estimate $\underline{\theta}^*$.

$$\underline{\theta}^* = (H^TH)^{-1}H^T\underline{z} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 51.803 \\ 29.545 \end{bmatrix}$$

Step 2 Calculate the residuals of the L_2 estimate.

$$\underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} = \underline{z} - H\underline{\theta}^* = \begin{bmatrix} 1.652 \\ -0.865 \\ -0.420 \\ 0.286 \\ 0.500 \end{bmatrix}$$

<u>Step 3</u> Select the two measurements that correspond to the smallest residuals and form $\hat{\underline{z}}$ and $\hat{H}$.

The smallest residuals are r_4 and r_3.

$$\hat{\underline{z}} = \begin{bmatrix} -7 \\ 310 \end{bmatrix} \qquad \hat{H} = \begin{bmatrix} 1 & -2 \\ 2 & 7 \end{bmatrix}$$

<u>Step 4</u> Solve for the LAV estimate.

$$\hat{\underline{\theta}} = \hat{H}^{-1}\hat{\underline{z}}$$

$$\hat{\underline{\theta}} = \begin{bmatrix} 1 & -2 \\ 2 & 7 \end{bmatrix}^{-1} \begin{bmatrix} -7 \\ 310 \end{bmatrix}$$

$$\hat{\underline{\theta}} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 51.909 \\ 29.455 \end{bmatrix}$$

The optimality of the LAV solution can be checked by calculating the residuals of three data points that are not interpolated.

$$r_1 = z_1 - H_1\hat{\underline{\theta}} = 1.636$$

$$r_2 = z_2 - H_2\hat{\underline{\theta}} = -1.273$$

$$r_5 = z_5 - H_5\hat{\underline{\theta}} = 0.364$$

Two residuals are positive and one is negative. Therefore the difference between the number of positive and the number of negative residuals is one, which is less than the number of unknown parameters. Consequently, the estimate obeys theorem 3.2 and may be optimal.

The same L_1 solution can be obtained by solving the linear programming problem that is formulated in example 3.1. However solution by the new technique appears to be superior because the new technique is non-iterative and thus should require less computing time than linear programming. In fact the LAV solution for example 3.3 could easily have been obtained by hand calculations.

3.2.3 Bias of the New Technique

For some problems the new technique will not produce the optimal LAV estimate. In these cases the estimate produced by the new method will be nearly optimal and the cost function evaluated at the estimate will have a value that is only slightly greater than the cost function evaluated at the optimal LAV estimate.

The following two examples illustrate cases in which the new technique does not produce an optimal L_1 estimate.

Example 3.4 [1]

This is the same as example 2.2 that was given in chapter II.

Given the following five measurements of y, calculate the L_1 estimate of y.

$y_1 = 24.50,\ y_2 = 100.00,\ y_3 = 24.25,\ y_4 = 24.75,\ y_5 = 24.00$

The structure of the system can be represented by (3.18).

$$\underline{z} = Hy \qquad (3.18)$$

where:
$$\underline{z} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 24.50 \\ 100.00 \\ 24.25 \\ 24.75 \\ 24.00 \end{bmatrix} \qquad H = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

and y is to be estimated.

Step 1 Calculate the least squares estimate y^*.

$$y^* = (H^T H)^{-1} H^T \underline{z} = 39.50$$

Step 2 Calculate the residuals of the least squares estimate.

$$\underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} = \underline{z} - Hy^* = \begin{bmatrix} -15.00 \\ 60.50 \\ -15.25 \\ -14.75 \\ -15.50 \end{bmatrix}$$

Step 3 Select the measurements corresponding to the smallest residual and form $\hat{\underline{z}}$ and $\hat{H}$. The smallest residual is r_4.

$$\therefore \hat{\underline{z}} = 24.75 \;, \; \hat{H} = 1$$

Step 4 Solve for the LAV estimate.

$$\hat{y} = \hat{H}^{-1} \hat{\underline{z}} = 24.75$$

The LAV estimate produced by the new method is $\hat{y}$ = 24.75. However in chapter II the optimal LAV estimate is given as $\hat{y}$ = 24.50.

The LAV cost function is given by equation (3.19).

$$J = \sum_{i=1}^{5} | z_i - H_i\hat{y} | \qquad (3.19)$$

For the new technique J = 76.75, while for the LAV estimate J = 76.50.

For this example, the new technique did not produce the optimal LAV estimate, however since the difference between the two values of the cost function is so small the LAV estimate produced by the new technique is near optimal and likely acceptable for many applications.

<u>Example 3.5</u> [17]

Calculate the L_1 estimate of a_1 and a_2, if the data represents a straight line of the form $y = a_1 x + a_2$.

x	y
-8.5299	-6.7934
-3.8751	-0.7804
-3.4707	-3.1098
-3.2683	-2.2848
-2.4099	-4.7076
0.3745	1.4024
1.1724	2.9106
1.8267	4.3988
5.8068	4.9851
7.7160	6.5859

The L_2 estimate is:

$$\underline{\theta}^* = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.86468 \\ 0.66341 \end{bmatrix}$$

The smallest residuals belong to the 1st and 4th data points, and therefore they are interpolated. The LAV estimate obtained by applying the new technique is:

$$\hat{\underline{\theta}} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.85689 \\ 0.51577 \end{bmatrix}$$

The optimal LAV estimate, calculated by linear programming is:

$$\hat{\underline{\theta}} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.80758 \\ 0.35461 \end{bmatrix}$$

The corresponding cost functions are $J = 11.15603$ for the new technique and $J = 11.01392$ for the optimal LAV estimate. The small difference in the values of the cost function indicate that either estimate is acceptable for most applications.

3.2.4 Unique Solutions of the New Method

A difficulty that is encountered by linear programming based LAV estimation is that the solution obtained may not always be unique. The following example demonstrates a case in which the linear programming based estimate is not unique.

Example 3.6

Fit the following set of data with a line of the form $y = ax + b$.

x	1	2	3	4
y	1	2	2.9	0

$$H = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}, \quad \underline{z} = \begin{bmatrix} 1.0 \\ 2.0 \\ 2.9 \\ 0.0 \end{bmatrix}, \quad \underline{\theta} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Six possible L_1 estimates of a and b can be obtained by interpolating the six different combinations of two data points. The residuals and the sum of the residuals for the six combinations are summarized in table 3.1.

The only estimate obtained by applying the new technique is estimate #1. Linear programming based estimation could produce any one of four estimates (#2 - #5). Thus the linear programming estimate is not unique and estimates of a range from -1 to +.95 while estimates of b range from +0.05 to +4.0 . From Table 3.1 , it can be seen that , the cost function value for the unique LAV estimate, that is produced by the new technique, is 4.1, whereas the four linear programming based LAV estimates all have cost function values of 3.9. Even though the linear programming estimates have smaller cost function values, it appears wiser to use the unique LAV estimate that is obtained by the new technique, rather than applying linear programming and obtaining one of four possible estimates.

The new method will almost always produce a unique estimate. In cases in which the absolute values of two residuals are equal, but there is room for only one of the corresponding measurements in the interpolated measurement set, a tie-breaking procedure is implemented. Two LAV estimates, which correspond to two different interpolated sets of measurements, are calculated. Each interpolated set contains nearly the same set of measurements (corresponding to the n smallest L_1 residuals), with the only difference

ESTIMATE NUMBER	POINTS FITTED	ESTIMATE a	b	R1	R2	R3	R4	SUM R's
1	1 an[illegible]	1.00	0.00	0.00	0.00	-0.10	-4.00	4.10
2	1 and 3	0.95	0.05	0.00	0.05	0.00	3.85	3.90
3	1 and 4	-0.33	1.33	0.00	1.33	2.57	0.00	3.90
4	2 and 3	0.90	0.20	-0.10	0.00	0.00	-3.80	3.90
5	2 and 4	-1.00	4.00	-2.00	0.00	1.90	0.00	3.90
6	3 and 4	-2.90	11.60	-7.70	5.80	0.00	0.00	13.50

Table 3.1 Summary of Data for Example 3.6 .

being that each set contains a different one of the two measurements that was involved in the tie. The value of the cost functions of both estimates are then calculated and compared. The estimate with the smaller valued cost function is the unique LAV estimate, that is produced by the new method and the tie-breaking procedure.

The following example shows a case in which the tie-breaking procedure must be implemented.

Example 3.7

Fit the data { (1,4.5),(2,7.0),(3,5.0),(4,6.0) } with a line of the form y = ax + b.

$$H = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}, \quad \underline{z} = \begin{bmatrix} 4.5 \\ 7.0 \\ 5.0 \\ 6.0 \end{bmatrix}, \quad \underline{\theta} = \begin{bmatrix} a \\ b \end{bmatrix}$$

Step 1 *Calculate the least squares solut... $\underline{\theta}^*$.*

$$\underline{\theta}^* = (H^T H)^{-1} H^T \underline{z} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.25 \\ 5.00 \end{bmatrix}$$

Step 2 *Calculate the residuals of the least squares estimate.*

$$\underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \underline{z} - H\underline{\theta}^* = \begin{bmatrix} -0.75 \\ 1.50 \\ -0.75 \\ 0.00 \end{bmatrix}$$

Step 3 Select the two measurements that correspond to the two smallest residuals and form $\hat{\underline{z}}$ and $\hat{H}$.

The smallest residual is r_4. The absolute values of the 1st and 3rd residuals are equal. Thus the tie-breaking procedure must be implemented and two estimates must be calculated. The first estimate corresponds to interpolation of the 4th and 1st measurements.

$$\hat{\underline{z}}_1 = \begin{bmatrix} 6.0 \\ 4.5 \end{bmatrix} \qquad \hat{H}_1 = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}$$

The other estimate corresponds to the interpolation of the 4th and 3rd data point.

$$\hat{\underline{z}}_2 = \begin{bmatrix} 6.0 \\ 5.0 \end{bmatrix} \qquad \hat{H}_2 = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$$

Step 4 Solve for both LAV estimates.

$$\hat{\underline{\theta}}_1 = \hat{H}_1^{-1}\hat{\underline{z}}_1$$

$$\hat{\underline{\theta}}_1 = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 6.0 \\ 4.5 \end{bmatrix}$$

$$\hat{\underline{\theta}}_1 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.5 \\ 4.0 \end{bmatrix}$$

$$J_1 = 2.5$$

$$\underline{\hat{\theta}}_2 = \hat{H}_2^{-1}\underline{\hat{z}}_2$$

$$\underline{\hat{\theta}}_2 = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 6.0 \\ 5.0 \end{bmatrix}$$

$$\underline{\hat{\theta}}_2 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix}$$

$$J_2 = 4.5$$

The cost function corresponding to $\underline{\hat{\theta}}_1$ *is less than the cost function corresponding to* $\underline{\hat{\theta}}_2$. *Therefore* $\underline{\hat{\theta}}_1$ *is the unique LAV estimate produced by the new technique.*

If a case occurs in which the two cost functions are equal then both estimates are equally valid. Under such circumstances the estimate produced by the new technique will not be unique. Extensive testing of the new technique has demonstrated that this situation rarely occurs and that the estimates are almost always unique.

If more than two residuals are equal, a tie-breaking procedure that is similar to the one demonstrated in example 3.7 is used.

3.2.5 Features of the New Technique

The new technique possesses a number of features that make it an attractive alternative to linear programming.

a) It is non-iterative in nature and thus does not have to go through many iterations to reach an estimate.

b) It does not require an initial feasible solution, whereas the linear programming approach must first calculate an initial feasible solution and then solve for the L_1 estimate.

c) In most cases it produces a unique solution.

d) The estimates are easily calculated and can often be calculated by hand for simple examples; whereas linear programming estimation almost always requires a computer.

3.3 Constrained Parameter Estimation with the New Technique

The new estimation technique can easily be modified to handle equality and inequality constraints. Furthermore, it will be shown in chapter V that the new technique is well suited to the power system state estimation problem, in which zero injection buses can be treated as equality constraints.

Recall from section 3.2 that the LAV solution of an n parameter estimation problem interpolates n of the m measurements. Given ℓ equality constraints (ℓ < n; if ℓ

equals n then the LAV estimate is equal to the solution of the system of equality constraints), the LAV estimate must interpolate the ℓ equality constraints and $(n-\ell)$ of the measurements. The total number of interpolated points is thus $(n-\ell) + \ell = n$. The new method selects the ℓ constraints and the $(n-\ell)$ measurements which correspond to the smallest L_2 residuals, for interpolation.

In calculating the L_2 estimate the equality constraints are treated as measurements. In step 2 of constrained estimation the least squares residuals of the m measurements are calculated, and in step 3 $\underline{\hat{z}}$ and $\hat{H}$ are formed from the interpolated measurements and the constraints. In step 4 the L_1 estimate is computed. All inequality constraints are then checked. If any of the inequality constraints are violated, the L_1 estimate is recomputed with all violated inequality constraints treated as equality constraints.

The following example demonstrates the method.

Example 3.8 [22]

Fit the data { (1,2),(2,2),(3,3),(4,4),(5,3) } with a straight line of the form y = ax + b, subject to the constraints y(6) = 5 and y(0) ≤ 1.2 . The estimator equation can be written as:

$$\underline{z} = H\underline{\theta}$$

where:

$$\underline{z} = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 3 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \end{bmatrix} \qquad \underline{\theta} = \begin{bmatrix} a \\ b \end{bmatrix}$$

The equality constraint can be written as:

$$C\underline{\theta} = \underline{d}$$

where: $C = [\ 6\ ,\ 1\ ]\ ,\ \underline{d} = [\ 5\ ]$

The inequality constraint can be written as:

$$E\underline{\theta} \le \underline{f} \qquad (3.20)$$

where: $E = [\ 0\ ,\ 1\ ]\ ,\ \underline{f} = [\ 1.2\ ]$.

Since the equality constraint is treated as a measurement the estimator equation and the equality constraint can be combined into the form given in equation (3.21).

$$A\underline{\theta} = \underline{B} \tag{3.21}$$

$$A = \begin{bmatrix} H \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \\ 5 & 1 \\ 6 & 1 \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} \underline{z} \\ \underline{d} \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \\ 3 \\ 5 \end{bmatrix}$$

Now A and $\underline{B}$ can be used to calculate the L_1 estimate.

<u>Step 1</u> Calculate the least squares solution $\underline{\theta}^*$.

$$\underline{\theta}^* = (A^T A)^{-1} A^T \underline{B} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.543 \\ 1.267 \end{bmatrix}$$

<u>Step 2</u> Calculate the residuals of the least squares estimate.

$$\underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{bmatrix} = \underline{z} - H\underline{\theta}^* = \begin{bmatrix} 0.1905 \\ -0.3524 \\ 0.1048 \\ 0.5619 \\ -0.9810 \end{bmatrix}$$

Note that r_6 does not need to be calculated since the equality constraint is automatically part of the interpolated set.

Step 3 Select the measurement corresponding to the smallest residual and the equality constraint for interpolation. Form $\hat{\underline{z}}$ and $\hat{H}$.

The smallest residual is r_3.

$$\hat{\underline{z}} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \qquad \hat{H} = \begin{bmatrix} 3 & 1 \\ 6 & 1 \end{bmatrix}$$

Step 4 Solve for the LAV estimate.

$$\hat{\underline{\theta}} = \hat{H}^{-1}\hat{\underline{z}}$$

$$\hat{\underline{\theta}} = \begin{bmatrix} 3 & 1 \\ 6 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\hat{\underline{\theta}} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.667 \\ 1.000 \end{bmatrix}$$

Now check the inequality constraint that is given by (3.20).

$$E\hat{\underline{\theta}} = [\ 0,\ 1\ ] \begin{bmatrix} 0.667 \\ 1.000 \end{bmatrix} = 1.0 < 1.2$$

The inequality constraint is not violated and therefore $\hat{\underline{\theta}}$ is the L_1 estimate.

If, however, the inequality constraint were $\hat{\underline{\theta}} < 0.8$ then the problem would be solved again with the inequality constraint treated as an equality constraint $E\underline{\theta} = 0.8$.

In chapter V the new estimation technique will be applied to the power system state estimation problem.

CHAPTER IV

POWER SYSTEM STATIC STATE ESTIMATION

This chapter begins with an introduction to the power system state estimation problem. The role of a state estimator in the monitoring and control of a power system is then discussed. The chapter concludes with a brief description of previous research in the application of least squares and least absolute value techniques to power system state estimation.

4.1 The Power System Static State Estimation Problem

The state of an electric power system can be completely described by the set of voltage magnitudes and phase angles at all of the buses in the system [23]. For this reason voltage magnitudes and phase angles are often called the state variables of a power system. Given the state of a power system, all other quantities of interest such as line currents and power flows, can be calculated.

The power system state estimation problem is to develop a static state estimator. A static state estimator can be described as a data processing algorithm that transforms metered measurements and information about system structure into an accurate estimate of the system state. Figure 4.1 illustrates the inputs and outputs of a state estimator.

In the next section, the nature of the measurement sets, that are processed by state estimators, is investigated.

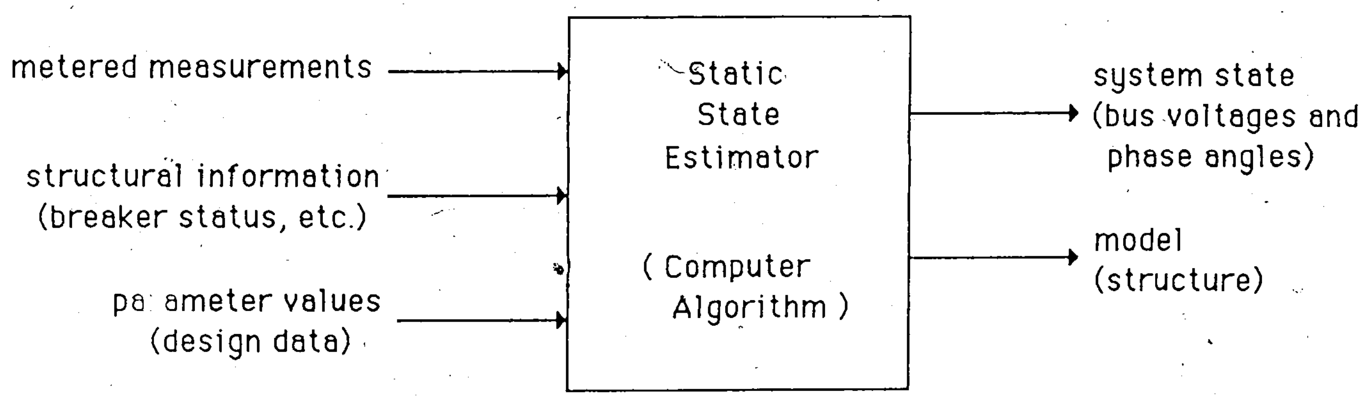

Figure 4.1 [24] Inputs and Outputs for a Static State Estimator

4.1.1 Measurement Set

The measurement sets processed by power system state estimators usually contain measurements of real and reactive power flow along system transmission lines and measurements of real and reactive power injections at the system buses.

There are many sources of measurement errors in any power system monitoring scheme. Some of these sources are: noise in instrumentation and telemetry, errors in system modelling, and malfunctioning of monitoring and communications equipment. Measurements which contain large errors are defined as bad data.

It is possible to calculate a state estimate if the number of measurements taken is equal to the number of state variables, however, such an estimate would be inaccurate because all of the measurements contain noise and the measurement set likely contains bad data. In order to reduce the negative impact of bad data and noise on the accuracy of the state estimate, the number of measurements taken (m) always exceeds the number of state variables (n). Another reason why m should be greater than n is so that the power system will be observable and will remain observable even when some measurements are unavailable.

Observability can be defined as the ability of the measurements to represent the state of the entire system. If enough measurements are not taken, it may not be possible to estimate all of the state variables, and thus the power system will be unobservable to the estimator. Also if some measurements become unavailable, a previously observable power system may become unobservable.

The ratio of m divided by n is defined as the redundancy ratio. For power system state estimation the commonly accepted range of redundancy ratios is 1.4 to 2.8 . Larger redundancy ratios are too costly and smaller ratios often render the system unobservable to the estimator.

4.1.2 The Role of a State Estimator

State estimators provide a data base that can be used for monitoring and control functions. The data from state estimates is made available to power system operators in the form of visual displays. The data can also be for automatic control, and to monitor system security without operator intervention. Such a scheme would signal the operator whenever a condition that requires his tention occurs. On line load forecasting algorithms and optimal power flow programs can also use the data base that is provided by state estimates [25]. The data could also be useful in checking system topology, anomaly detection and fault diagnosis [26]. The data is also useful for off line tasks such as: medium and long range load prediction, unit commitment, contingency analysis, data logging and maintenance scheduling [27].

When state estimators identify bad data they are detecting faulty instrumentation and thus facilitating its repair [28].

As power systems become larger and more complex, even more uses of the data base provided by state estimators will be found and the importance of state estimators will undoubtedly increase.

4.2 Least Squares Estimation Applied to Power Systems

The power system state estimation problem was formulated in 1970 by Schweppe et al [23], Larson et al [26] and Stagg et al [29]. All three of the original approaches to the power system state estimation problem used some form of weighted least squares estimation.

The following is a simple implementation of weighted least squares (WLS) estimation theory.

The cost function that must be minimized for WLS estimation is given by (4.1).

$$J = (\underline{z} - H(\underline{\theta}))^T W(\underline{z} - H(\underline{\theta})) \qquad (4.1)$$

where:
- $\underline{z}$ is an mx1 (m > n) vector of measurements.
- $H(\underline{\theta})$ is an mx1 vector which represents the non-linear relationship between $\underline{z}$ and $\underline{\theta}$.
- $\underline{\theta}$ is an nx1 vector of system parameters.
- $\underline{v}$ is an mx1 vector that contains the measurement noise.
- W is an mxm weighting matrix which is usually equal to the inverse of the covariance matrix, i.e. $W = R^{-1}$ where R = Expected Value $(\underline{v}\ \underline{v}^T)$.

Equation 4.1 can be linearized about a specific state $\underline{\hat{\theta}}$ as in (4.2).

$$J = (\Delta \underline{z} - \frac{\partial H}{\partial \underline{\theta}}|_{\hat{\underline{\theta}}} \Delta\underline{\theta})^T \; W \; (\Delta z - \frac{\partial H}{\partial \underline{\theta}}|_{\hat{\underline{\theta}}} \Delta\underline{\theta}) \tag{4.2}$$

where:

- $\Delta\underline{z} = \underline{z} - H(\hat{\underline{\theta}})$.
- $\Delta\underline{\theta}$ is the change in the state estimate.
- $\frac{\partial H}{\partial \underline{\theta}}|_{\hat{\underline{\theta}}}$ is mxn, and represents the partial derivative of H with respect to the state of the system evaluated at $\hat{\underline{\theta}}$.

The least squares estimate is obtained in the following manner:

1) Assume an initial starting point $\hat{\underline{\theta}}$. Usually a flat start (i.e. all voltages = 1 per unit and all phase angles = 0) is assumed.
2) Calculate the value of $\Delta\underline{\theta}$ which minimizes (4.2). The value of $\Delta\underline{\theta}$ can be calculated by (4.3).

$$\Delta\underline{\theta} = [\,(\frac{\partial H}{\partial \underline{\theta}}|_{\hat{\underline{\theta}}})^T W (\frac{\partial H}{\partial \underline{\theta}}|_{\hat{\underline{\theta}}})\;]^{-1} [\;\frac{\partial H}{\partial \underline{\theta}}|_{\hat{\underline{\theta}}}\;]^T W \Delta\underline{z} \tag{4.3}$$

3) Update the state estimate using (4.4).

$$\hat{\underline{\theta}} = \hat{\underline{\theta}} + \Delta\underline{\theta} \tag{4.4}$$

4) If $\Delta\underline{\theta}$ meets the convergence criteria $\hat{\underline{\theta}}$ is the weighted least squares estimate. If $\Delta\underline{\theta}$ does not meet the criteria, return to step 2.

All of the weighted least squares techniques that have been developed for power system state estimation, use some variation of the technique that has just been given.

Least squares estimators provide an optimal estimate if the components of the error vector follow a Gaussian distribution [4]. However it is well known that least squares estimators can produce biased and inaccurate estimates when the error vector does not obey a true Gaussian distribution or if bad data is present in the measurement set. Consequently considerable research has been conducted in developing methods of detecting the presence of bad data in the measurement set. Once the presence of bad data has been detected and the bad data point has been identified and deleted from the measurement set, the least squares estimate can be computed [30].

Some of the techniques that have been developed for bad data detection and identification involve prefiltering of the measurement set; while others involve inspection and testing of the least squares estimate and the error vector $\underline{v}$ [31]. Some research has also explored the possibility of adjusting the least squares cost function so that bad data can be readily identified. Other approaches to the bad data problem involve geometric transformations. Although many methods of dealing with bad data have been developed, none of the methods have been widely accepted. Research is continuing in this area.

4.3 Least Absolute Value State Estimation Applied to Power Systems

Rather than develop methods of dealing with the bad data problem encountered by least squares estimation, a few reseachers have developed LAV state estimators. The cost function that must be minimized by a least absolute value state estimator is given by equation (4.5).

$$J = \sum_{i=1}^{m} |z_i - H_i(\underline{\theta})| \qquad (4.5)$$

As mentioned in chapter II LAV estimates are relatively immune to bad data. In the context of power system state estimation this means that an LAV estimator does not require any additional bad data detection software. Also, it will be shown in chapter V that the residuals of an LAV estimate can be used to identify bad data.

The first suggestion that LAV estimation could be used as an alternative to least squares power system state estimation was made in 1978 [32]. In a well written paper Irving et al compared least squares and LAV estimates (which were obtained by linear programming). Their research showed that LAV estimates are superior to least squares estimates when the measurement set contains bad data and the least squares estimator does not use bad data detection software.

In 1982 Kotiuga and Vidyasagar [30] presented the results of their research work in LAV power system state estimation. Their results confirmed those that were presented by Irving et al. The main difference between Kotiuga and Vidyasagar's work and the work presented four years earlier, is that Kotiuga and Vidyasagar claimed that their linear programming based formulation of the estimation problem is more efficient.

In 1985 Kotiuga [33] and Falco [34] developed LAV based, power system tracking state estimators and in 1986 Lo and Mahmoud [31] presented a decoupled LAV power system state estimator.

All of the LAV state estimators that have been developed have used linear programming to calculate their estimates. Despite the advantages of LAV based estimation it has not been widely accepted as a better approach or even a reasonable alternative to least squares power system state estimation.

In the next chapter it will be shown how the new LAV estimation technique, that was presented in Chapter III, can be applied to the power system state estimation problem.

CHAPTER V

APPLICATION OF THE NEW LAV METHOD TO POWER SYSTEMS

In this chapter the new least absolute value estimation technique is applied to the power system state estimation problem. The chapter begins with a discussion of the relationship between power system measurements and state variables. The important concept of observability is then explained. This is followed by a description of the new algorithm. It will then be shown that the new algorithm can be used to identify bad data. The possibility of using the new algorithm to correct errors in system topology is also dealt with. The chapter concludes with a discussion of the constrained power system state estimation problem.

5.1 Power System Modelling

Before applying the new LAV estimation technique, a power system model must be developed. Specifically the relationship between: the measurements, the state variables and the system parameters must be described by equations. Obviously when bus voltages and phase angles are measured, the measurement equals the true value of the state variable plus noise. However, when power flows and injections are measured the relationship is not as simple. The relationships between measurements of power flows and injections, and the state variables are given by equations (5.1) - (5.4).

$$P_k = \sum_{n=1}^{N} V_k V_n Y_{kn} \cos(\theta_{kn}+\delta_n-\delta_k) \qquad (5.1)$$

$$Q_k = -\sum_{n=1}^{N} V_k V_n Y_{kn} \sin(\theta_{kn} + \delta_n - \delta_k) \qquad (5.2)$$

$$P_{i-k} = V_i^2 Ys_{ik} \cos(\theta s_{ik}) - V_i^2 Y_{ik} \cos(\theta_{ik}) + V_i V_k Y_{ik} \cos(\theta_{ik} + \delta_k - \delta_i) \qquad (5.3)$$

$$Q_{i-k} = -V_i^2 Ys_{ik} \sin(\theta s_{ik}) + V_i^2 Y_{ik} \sin(\theta_{ik}) - V_i V_k Y_{ik} \sin(\theta_{ik} + \delta_k - \delta_i) \qquad (5.4)$$

where:

- N = the number of buses in the power system.
- P_k = real power injected into bus k.
- Q_k = reactive power injected into bus k.
- P_{i-k} = real power flow from bus i to bus k.
- Q_{i-k} = reactive power flow from bus i to bus k.
- V_k = the magnitude of the voltage at bus k.
- δ_k = the phase angle of the voltage at bus k.
- Y_{kn} = the magnitude of element (k,n) of the admittance matrix.
- θ_{kn} = the phase angle of element (k,n) of the admittance matrix.
- Ys_{ik} = one-half of the magnitude of the shunt admittance of line i-k.
- θs_{ik} = the phase angle of the shunt admittance of line i-k.

Thus a power system can be modelled by state equation (5.5).

$$\underline{z} = H(\underline{\theta}) + \underline{v} \qquad (5.5)$$

where:

$\underline{z}$ is an mx1 vector that contains all of the m (m>n) measurements.

$H(\underline{\theta})$ is an mx1 vector which represents relationships (5.1) to (5.4).

$\underline{\theta}$ is an nx1 vector (n = 2(N-1) = the number of state variables).

$\underline{v}$ is an mx1 vector of measurement errors.

Note that bus 1 is usually the reference bus. The voltage at bus 1 is assumed to be known and the phase angle at bus 1 is the reference phase angle for all of the other buses, i.e. $\delta_1 = 0$. Therefore the number of state variables, and the number of elements in $\underline{\theta}$ is 2(N-1), where N is the number of buses in the system.

A simple 3 bus system is modelled in example 5.1.

Example 5.1

Given: the 3 bus system as shown in figure 5.1, the measurements indicated on the diagram and the admittance and shunt admittance matricies given by (5.6) and (5.7), model the system with an equation of the form $\underline{z} = H(\underline{\theta}) + \underline{v}$. *Note that all voltages are in per unit and all phase angles are in radians.*

$$Y_{ADMITTANCE} = \begin{bmatrix} Y_{11}|\underline{\theta_{11}} & Y_{12}|\underline{\theta_{12}} & Y_{13}|\underline{\theta_{13}} \\ Y_{21}|\underline{\theta_{21}} & Y_{22}|\underline{\theta_{22}} & Y_{23}|\underline{\theta_{23}} \\ Y_{31}|\underline{\theta_{31}} & Y_{32}|\underline{\theta_{32}} & Y_{33}|\underline{\theta_{33}} \end{bmatrix} \quad (5.6)$$

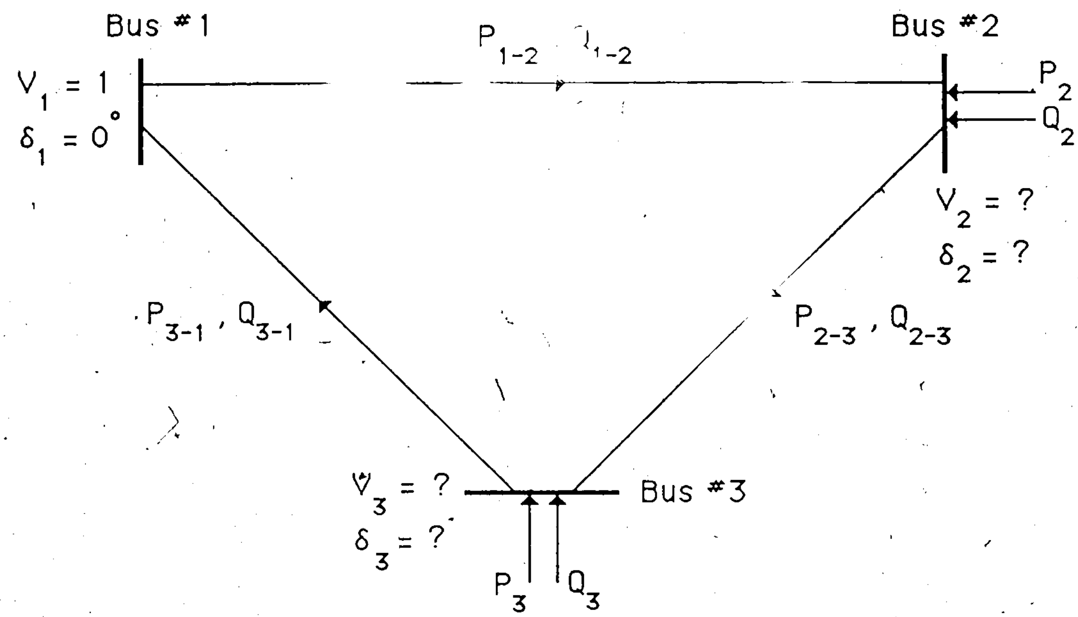

Figure 5.1 Power System for Example 5.1

$$Y_{SHUNT} = \begin{bmatrix} Y_{S11}\angle\theta_{S11} & Y_{S12}\angle\theta_{S12} & Y_{S13}\angle\theta_{S13} \\ Y_{S21}\angle\theta_{S21} & Y_{S22}\angle\theta_{S22} & Y_{S23}\angle\theta_{S23} \\ Y_{S31}\angle\theta_{S31} & Y_{S32}\angle\theta_{S32} & Y_{S33}\angle\theta_{S33} \end{bmatrix} \tag{5.7}$$

In order to represent the system in the form $\underline{z} = H(\underline{\theta}) + \underline{v}$*, the vectors:* $\underline{z}$*,* $\underline{\theta}$ *and* $\underline{v}$ *must be defined by (5.8) - (5.10).*

$$\underline{z} = \begin{bmatrix} P_2 \\ Q_2 \\ P_3 \\ Q_3 \\ P_{1-2} \\ Q_{1-2} \\ P_{2-3} \\ Q_{2-3} \\ P_{3-1} \\ Q_{3-1} \end{bmatrix} \tag{5.8}$$

$$\underline{\theta} = \begin{bmatrix} V_2 \\ V_3 \\ \delta_2 \\ \delta_3 \end{bmatrix} \tag{5.9}$$

$$\underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ v_{10} \end{bmatrix} \quad (5.10)$$

The 3 bus power system is now modelled by $\underline{z} = H(\underline{\theta}) + \underline{v}$, where the vectors in the equations are defined by (5.8) - (5.10). Note that $H(\underline{\theta})$ represents the relationships between the state variables and the state equations; $H(\underline{\theta})$ contains equations like (5.1) to (5.4).

The cost function to be minimized in order to obtain the weighted LAV power system state estimate $\hat{\underline{\theta}}$, is given by (5.11).

$$J = \sum_{i=1}^{m} w_i \, |z_i - H_i(\underline{\theta})| \qquad (5.11)$$

where: m is the number of measurements.

z_i is the ith element of $\underline{z}$.

H_i is the ith element of H.

w_i is the weight attached to the ith element.

Equation (5.11) and the state equation are non-linear, therefore (5.11) must be minimized by an iterative procedure. When (5.11) is minimized at least n of the m terms in the equation will be equal to 0 because an LAV estimate interpolates a number of measurements equal to the number of state variables. In section 5.3 the procedure of minimizing (5.11) to obtain the least absolute value estimate, will be given.

5.2 Observability

Before the new algorithm is applied to power system state estimation, the important concept of observability will be discussed. If a set of power system measurements can be used to obtain an estimate of the power system state, then the measurement set is said to be observable [35].

For least squares estimation the entire measurement set is used to determine the L_2 estimate and therefore the entire measurement set is tested for observability. In least absolute value estimation only n of m measurements are used to calculate the final L_1 estimate, and consequently only the subset of the n measurements must be tested for observability. Recall from section 4.2 that the non-linear state equation $\underline{z} = H(\underline{\theta}) + \underline{v}$ can be linearized, as in equation (5.12).

$$\Delta \underline{z} = \frac{\partial H}{\partial \underline{\theta}}\Big|_{\hat{\underline{\theta}}} \Delta \underline{\theta} + \underline{q} \qquad (5.12)$$

where: $\Delta \underline{z}$ is $\underline{z} - H(\hat{\underline{\theta}})$.

$\Delta \underline{\theta}$ is the change in the state estimate.

$\frac{\partial H}{\partial \underline{\theta}}\Big|_{\hat{\underline{\theta}}}$ is mxn and represents the partial derivative of H with respect to the state evaluated at $\hat{\underline{\theta}}$.

$\underline{q}$ is mx1 and represents noise and errors due to linearization and bad data.

An L_2 estimate of $\underline{\theta}$ can only be obtained if $\frac{\partial H}{\partial \underline{\theta}}\Big|_{\hat{\underline{\theta}}}$ has a column rank of n, at each value of $\hat{\underline{\theta}}$ encountered during the iterative process of obtaining the L_2 estimate. Now consider the nxn matrix $\frac{\partial \hat{H}}{\partial \theta}|_{\hat{\theta}}$ which contains the n of the m rows of $\frac{\partial H}{\partial \theta}|_{\hat{\theta}}$ which correspond to the n interpolated measurements. If the column rank of $\frac{\partial \hat{H}}{\partial \theta}|_{\hat{\theta}}$ is equal to n at each $\hat{\theta}$ encountered in the iterative LAV estimation procedure, then the interpolated subset of n measurements is observable. If the column rank of $\frac{\partial \hat{H}}{\partial \theta}|_{\hat{\theta}}$ is not equal to n at any of the values of $\hat{\theta}$ encountered during the estimation procedure then the subset of n measurements is not observable, however, a different subset of n measurements may be observable.

Thus, the question of observability is related to the column rank of the matrix $\frac{\partial H}{\partial \theta}|_{\hat{\theta}}$ for L_2 estimation and the column rank of $\frac{\partial \hat{H}}{\partial \theta}|_{\hat{\theta}}$ for L_1 estimation. Matricies of this form are called Jacobians.

In 1980 Krumpholz, Clements and Davis discovered and proved an extremely useful relationship that exists between the measurement set and the Jacobian matrix [35]. The relationship that they discovered made it unnecessary to calculate the column rank of the Jacobian. Instead, they showed that a power system measurement set or subset can be tested for observability by examining the measurement set and the structure of the power system.

5.2.1 Observability Terminology

Before examining the observability conditions that were established by Krumpholz et al, several key words must be defined and explained.

A power system contains a set of buses (nodes) and a set of lines (branches). A bus at which the real and reactive power injections are measured is called a measured bus, and a bus at which these quantities are not measured is an unmeasured bus. A measured line is a line whose real and reactive power flows are measured.

A tree consists of any connected loop-free collection of measured lines and all the buses that the lines are connected to. A critical tree is a tree which contains all of the unmeasured buses in a power system. If a power system measurement set contains more than one tree, the collection of trees is called a forest. A boundary injection measurement is a real or reactive power injection measurement at a bus that is part of a tree and is also connected to one or more unmeasured lines. Figure 5.2a shows a measurement that is not a boundary injection measurement, and figure 5.2b shows a measurement that is a boundary injection measurement.

A line flow measurement is redundant if its addition to the measurement set does not increase the number of unknowns that can be solved for. Consider the four buses and four line flow measurements shown in figure 5.3 . (Note that all voltages in this thesis are in per unit and all phase angles are in radians .)

If P_{2-1}, P_{3-2} and P_{4-3} are measured, δ_2, δ_3 and δ_4 can be calculated. The addition of P_{1-4} to the measurement set does not add any information in the sense that no new

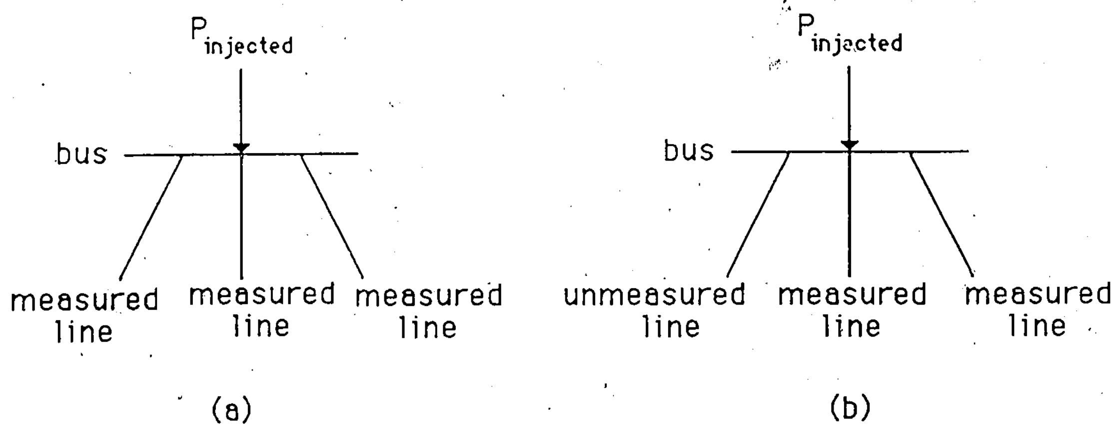

Figure 5.2 $P_{injected}$ in (a) is not a boundary injection measurement because the bus is not connected to an unmeasured line. $P_{injected}$ in (b) is a boundary injection measurement because the bus is connected, by the measured lines, to a tree and the bus is also connected to an unmeasured line.

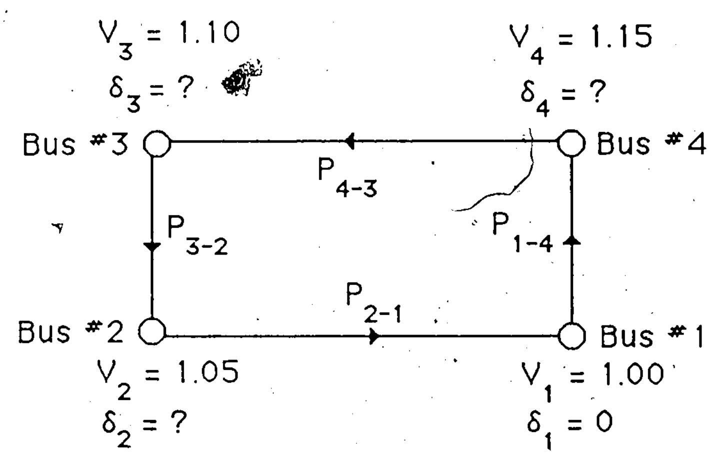

Figure 5.3 A Four Bus Power System

unknowns can be calculated. Therefore measurement P_{1-4} is redundant with respect the measurement set formed by the other three measurements. Note that the three measurements form a tree which contains four buses and three lines. The addition of the fourth measurement creates a loop. In general a measurement that forms a loop from a tree, or part of a tree, is redundant.

5.2.2 Necessary Conditions For Observability

Krumpholz et al proved that a measurement set is observable if all of the unmeasured buses are connected by a tree, i.e. a critical tree exists. They also demonstrated that an unobservable measurement set can be made observable by the addition of boundary injections to the measurement set. The following conditions must be met before a boundary injection measurement can be added to the measurement set:

a) The bus at which the measurement is made must be connected to at least one unmeasured line.

b) A path of unmeasured line(s) that lead from the injected bus to an unmeasured bus, must be available. The unmeasured bus may be part of another tree or may not belong to any tree.

c) The path of unmeasured lines must not pass through any other unmeasured buses.

If all three conditions are met the boundary injection is added to the measurement set and the lines, in the path between the injected bus and the unmeasured bus, are added to the connection diagram. If enough boundary injections are added to the measurement set the forest will eventually become a critical tree, and the measurement set will thus be observable.

The following four examples illustrate the application of the observability conditions. The power system used for all the examples is shown in figure 5.4.

Example 5.2

Given the power system shown in figure 5.4 and the measurement set (P_{1-2}, P_{2-5}, P_3, P_4, P_6), determine if the measurement set is observable.

The three buses (3,4,6) that have injection measurements are measured buses. The three unmeasured buses (1,2,5) are connected by the line flows P_{1-2} and P_{2-5}. Therefore a critical tree that connects the three unmeasured buses exits, and consequently the measurement set is observable.

Example 5.3

Given the power system shown in figure 5.4 and the measurement set (P_{1-2}, P_{1-4}, P_{2-5}, P_{3-6}, P_4) determine if the measurement set is observable.

The only measured bus is bus #4. In order for the measurement set to be observable the five unmeasured buses must be connected. From the connection diagram shown in figure 5.5 it can be seen that buses 1, 2, 4 and 5 are connected and buses 3 and 6 are connected.

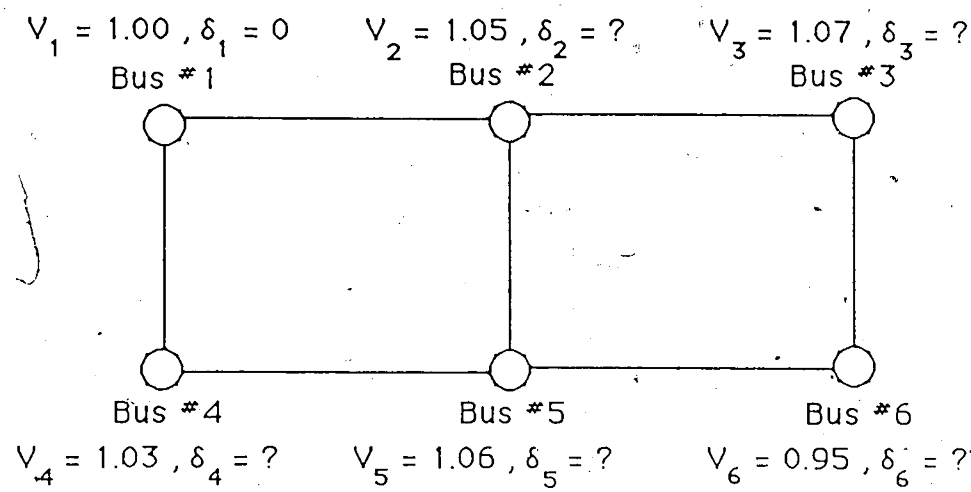

Figure 5.4 Power System for Examples 5.2 to 5.5

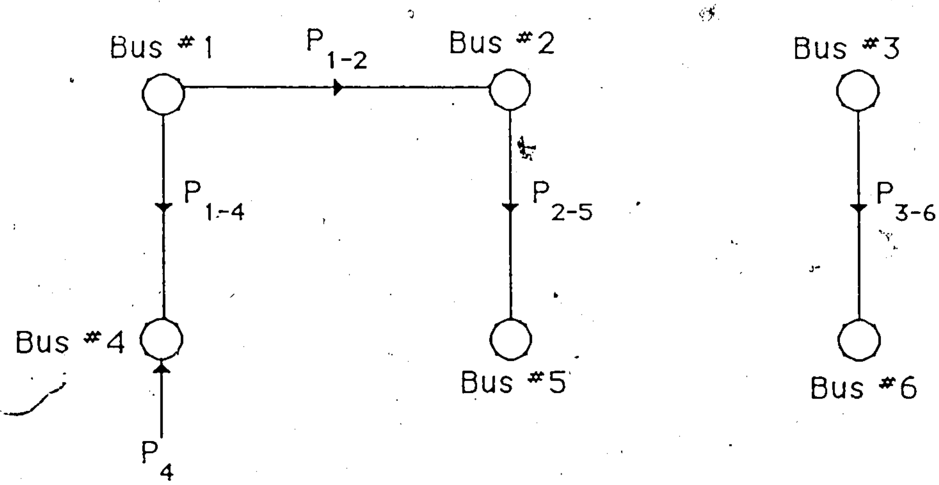

Figure 5.5 Connection Diagram for Example 5.3

However, the two trees that connect the two sets of buses, are not connected to each other. Thus, a forest composed of two trees exists. Since a critical tree does not exist the measurement set is unobservable.

Note that bus 4 is a measured bus and also belongs to a tree. Since a measured bus does not need to be part of a tree to satisfy the observability conditions, the line flow measurement that connects buses 1 and 4 is redundant and should be deleted from the measurement set. Alternatively, the power injection measurement at bus #4 can be deleted from the measurement set, thus making bus #4 an unmeasured bus. The non-redundant measurement set will not be observable, but will contain one less measurement than the redundant measurement set.

Example 5.4

Add a boundary injection measurement to the measurement set that is given in example 5.3, so that it becomes observable.

It was mentioned in the previous example that one of two measurements, P_4 and P_{1-4} should be deleted from the interpolated measurement set. Arbitrarily delete P_{1-4} from the measurement set. The connection diagram for the reduced measurement set is shown in figure 5.6. A boundary injection measurement which makes the measurement set observable must now be added to the measurement set. First consider adding a power injection measurement at bus #1. Bus #1 is part of a tree and is connected to an unmeasured line (line 1-4), therefore the measurement of power injected at bus #1 is a

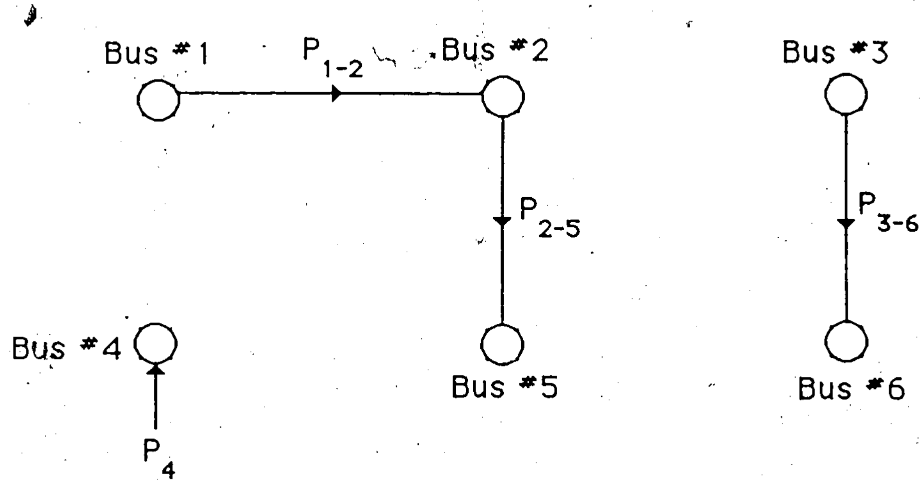

Figure 5.6 Connection Diagram for Example 5.4, with the Reduced Measurement Set

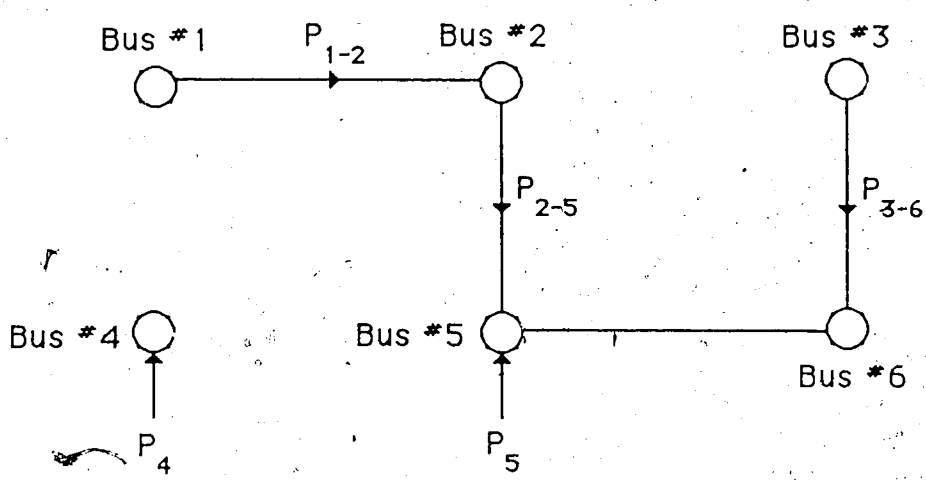

Figure 5.7 Connection Diagram for Example 5.4, with the New Measurement Set

boundary injection measurement. However a path of unmeasured lines that runs from bus #1 to an unmeasured bus that belongs to the other tree in the forest (containing buses #3 and #6), does not exist. Therefore adding the power injection measurement at bus #1 will not make the measurement set observable. Now consider adding a power injection measurement at bus #5. Bus #5 is part of a tree and is connected to two unmeasured lines, therefore a power injection measurement at bus #5 is a boundary injection measurement. One of the two unmeasured lines connects bus #5 to bus #6, which belongs to another component of the forest. Therefore adding the injection measurement at bus #5 will add line 5-6 to the connection diagram. The connection diagram of the new measurement set is shown in figure 5.7. All of the unmeasured buses are now connected by a critical tree consequently the measurement set(P_{1-2}, P_{2-5}, P_{3-6}, P_4, P_5) is observable. Note that adding P_5 to the measurement set does not add line 4-5 to the connection diagram because line 4-5 does not connect two trees.

Example 5.5

List all of the measurements which can be added to the measurement set (P_{1-2}, P_{2-3}, P_{3-6}, P_4) to make it observable. Figure 5.8 contains the connection diagram for the four measurements.

If a power injection measurement at bus #5 is added to the measurement set, then bus #5 will become a measured bus. All of the measured buses will then be connected and

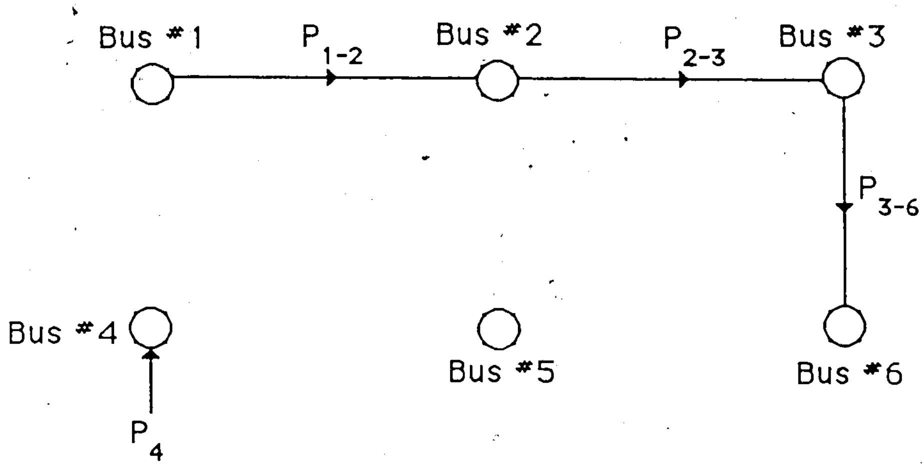

Figure 5.8 Connection Diagram for Example 5.5.

consequently the measurement set will be observable. If a power flow measurement along either line 2-5 or line 5-6 is added to the measurement set, all of the unmeasured buses will be connected and the measurement set will be observable. If a power flow measurement along line 4-5 is added to the measurement set, line 4-5 will be added to the connection diagram. However, the measurement set will remain unobservable since the unmeasured buses (1, 2, 3, 5, 6) will remain unconnected. If a boundary injection measurement at bus #2 is added to the measurement set, line 2-5 will be added to the connection diagram. The unmeasured buses will then be connected by a critical tree and the measurement set will be observable. Similarly a boundary injection measurement at bus #6 will add line 5-6 to the connection diagram and a critical tree will be formed. Thus adding a bus injection measurement at bus #6 will make the measurement set observable. So the individual measurements that can be added to the measurement set to make it observable are { P_5, P_{5-6} *or* P_{6-5} , P_{2-5} *or* P_{5-2} , P_2 *and* P_6 }.

5.2.3 A New Observability Algorithm

Krumpholz et al developed an algorithm that can be used to test a measurement set for observability [35]. If the measurement set is observable a state estimate can be calculated. If the measurement set is not observable their algorithm returns the observable sub-networks of the measurement set.

Although their algorithm is well suited to least squares estimation, it is not feasible for LAV estimation. In L_2 estimation, every measurement is used to calculate the estimate. Thus when a measurement set is tested for observability all m of the measurements are available to fulfill the observability conditions. For LAV estimation only n of the m measurements are used to calculate the estimate. Consequently for L_1 estimation, the n measurement subset must satisfy the same observability conditions that the entire set of measurements had to fulfill for L_2 estimation. Also, for L_2 estimation, if the set of m measurements is not observable a least squares estimate cannot be obtained until additional measurements are added to the measurement set. Whereas for L_1 estimation, if a particular subset of n measurements is not observable, another subset may be observable and the addition of more measurements to the measurement set may not be required.

There are $\binom{m}{n}$ (i.e. $\frac{m!}{n!(m-n)!}$) possible subsets of n measurements, many of which are not observable. If the algorithm that was developed by Krumpholz et al is applied to observability testing for LAV estimation, many subsets of measurements may have to be tested before an observable subset is found. If the first subset tested is not observable another subset would have to be formed and then tested for observability. If the second subset is not observable the process would continue until an observable subset is found or all possible subsets have been tested.

Rather than use the observability algorithm that was developed by Krumpholz et al an observability algorithm better suited to L_1 estimation was devised. The new observability algorithm constructs an observable subset of n measurements.

Recall from chapter IV that for linear LAV estimation the interpolated set of n measurements corresponds to the n measurements which have the smallest L_2 residuals. In power system state estimation the set of n measurements that corresponds to the n smallest residuals is rarely observable, so the procedure of choosing the interpolated measurements must be modified.

Due to the weak coupling between real power measurements and bus voltages and between reactive power measurements and the bus phase angles, the observability conditions that were presented in section 5.2.2 are applied to two observability sub-problems, namely P-δ and Q-V observability. This means that both the subset of real power measurements and the subset of reactive power measurements must be observable.

The observability algorithm processes one measurement at a time. If a measurement is redundant it is rejected, but if a measurement contains non-redundant information it is added to the interpolated measurement set. Once the interpolated measurement set contains n measurements the L_1 estimate can be calculated.

The order of processing the measurements is as follows. The measurements are ranked according to their L_2 residuals. The measurements are then processed according to this ranking starting from the measurement that corresponds to the smallest L_2 residual. Once n measurements have been accepted into the interpolated set the processing stops and an L_1 estimate is calculated.

The flow charts that are given in figures 5.9 and 5.10 demonstrate how real power measurements are processed. Note that for the new observability algorithm, a line is considered to be unmeasured unless a measurement of power flow along the line belongs to the interpolated measurement set. Similarly a bus is an unmeasured bus unless a power injection measurement, at the bus, belongs to the interpolated measurement set. Consequently, before any measurements are processed all lines and all buses are considered to be unmeasured.

If the entire set of real power measurements is processed, as in figure 5.9, and an observable set of measurements is not found, the real power boundary injection measurements are processed in the manner described in figure 5.10. The order of processing of the real power boundary injection measurements is determined by the L_2 residuals of each measurement, i.e. the boundary injection measurement with the smallest residual is processed first, followed by the boundary injection measurement with the next smallest residual if necessary, etc.

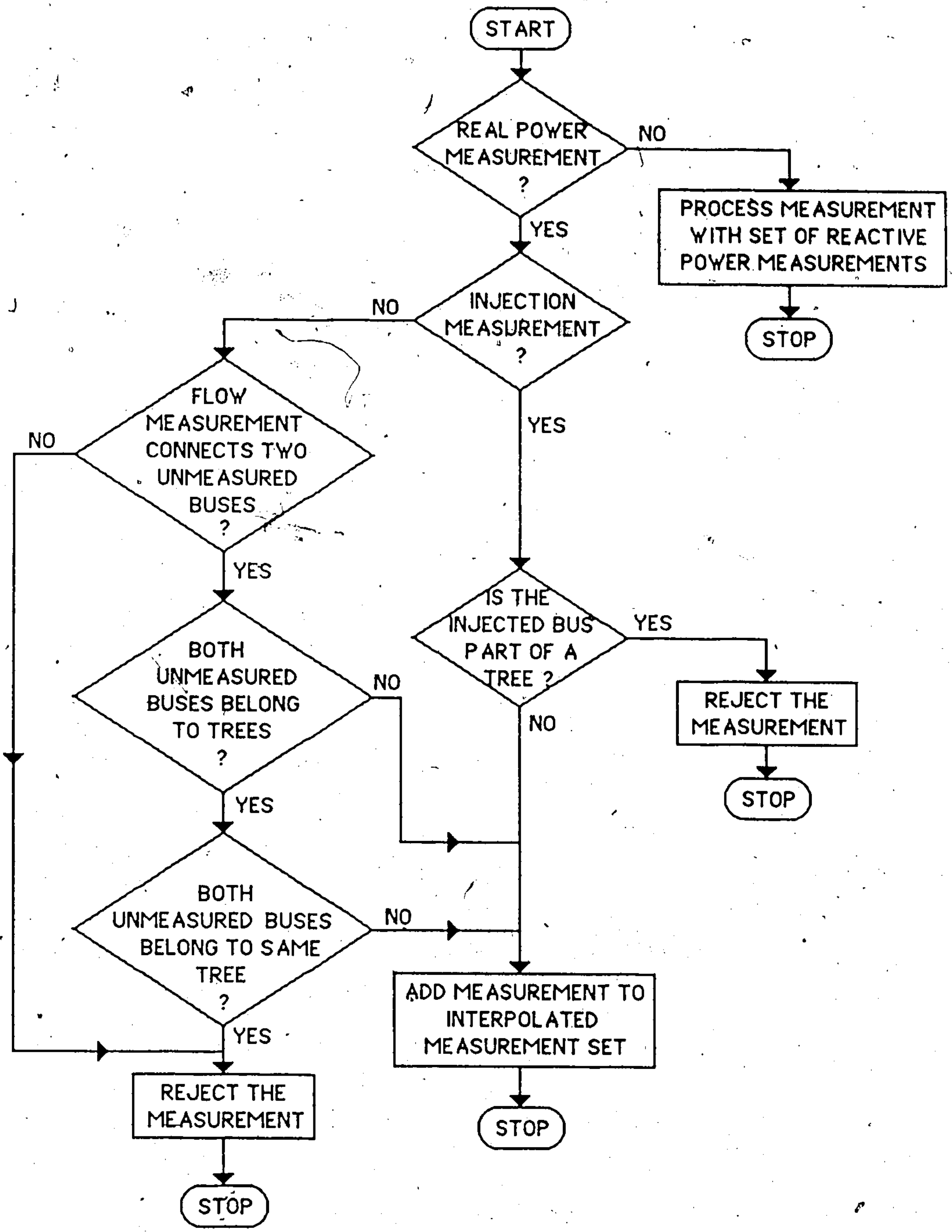

Figure 5.9 Measurement Processing Flow Chart for Real Power Measurements

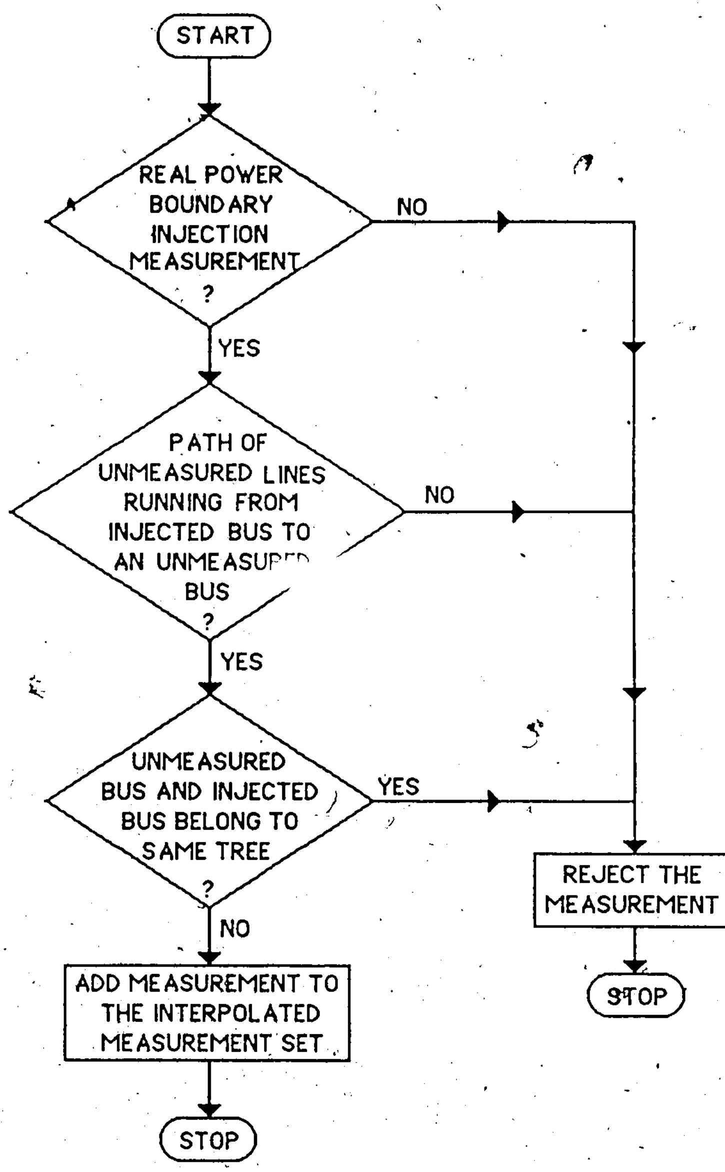

Figure 5.10 Measurement Processing Flow Chart for Real Power Boundary Injection Measurements

Consider the flow chart in figure 5.9. An injection measurement is accepted only if it is taken at a bus that is not part of a tree. Thus an unmeasured bus that belongs to a tree will remain an unmeasured bus and cannot become a measured bus. Line flow measurements are accepted only when they connect two unmeasured buses that do not belong to the same tree. A line flow measurement that connects two unmeasured buses which are already connected by a tree, is redundant and forms a loop.

Reactive power measurements are handled in the same manner as real power measurements.

The following two examples illustrate the implementation of the new observability algorithm. The power system in both examples is similar to the power system shown in figure 5.4, with the only difference being that the power system shown in figure 5.11 has unknown bus voltages.

Example 5.6

Given the power system shown in figure 5.11, apply the new observability algorithm to the measurement set (P_{1-2}, P_2, P_6, P_5, P_{4-1}, P_{2-5}, P_{3-6}, P_3, P_1, P_{5-6}, Q_{1-2}, Q_{1-4} Q_{4-5}, Q_1, Q_{2-5}, Q_5, Q_6, Q_{2-3}, Q_2, Q_3, Q_{5-6}) and determine an observable subset of meaurements if one exists. Process the measurements in the order given.

First consider the real power measurement subset (P_{1-2}, P_2, P_6, P_5, P_{4-1}, P_{2-5}, P_{3-6}, P_3, P_1, P_{5-6}). The first measurement, P_{1-2}, is accepted into the interpolated set. The next measurement , P_2, is not accepted because bus

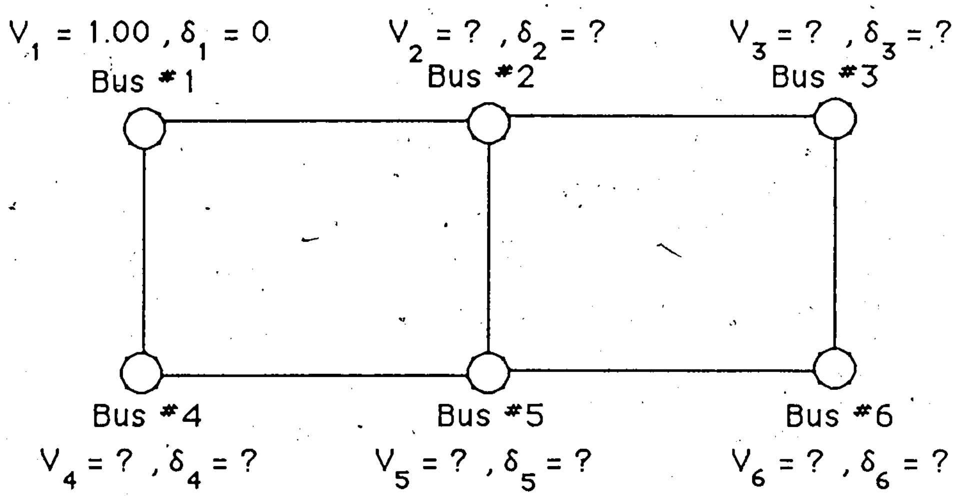

Figure 5.11 Power System for Examples 5.6 and 5.7

#2 is already part of a tree. The next two measurements P_6 and P_5, are accepted because neither bus #5 nor bus #6 belongs to a tree. Thus, buses 5 and 6 become measured buses and do not have to belong to a critical tree. The next measurement P_{4-1} is accepted because bus #4 is an unmeasured bus that does not belong to a tree. The set of unmeasured buses now contains buses 1 to 4. Three of the unmeasured buses (1,2, and 4) are connected by a tree. The only unmeasured bus that is not part of a tree is bus 3. Consequently, bus 3 must either join the set of measured buses or be connected to the tree by a line flow measurement. The next measurement, P_{2-5} does not involve bus 3 and is rejected. The next measurement to be processed is P_{3-6}; since it connnects bus 3 to a measured bus instead of an unmeasured bus it is also rejected. The following measurement, P_3, is accepted into the measurement set because it transforms bus 3 into a measured bus. The acceptance of P_3 into the measurement set leaves the remaining three unmeasured buses connected. The accepted subset of real power measurements (P_{1-2}, P_6, P_5, P_{4-1}, P_3) is thus observable and the processing of real power measurements need not continue.

Now consider the reactive power measurements (Q_{1-2}, Q_{1-4}, Q_{4-5}, Q_1, Q_{2-5}, Q_5, Q_6, Q_{2-3}, Q_2, Q_3, Q_{5-6}). The first three measurements connect four unmeasured buses and are accepted into the interpolated measurement set. The next measurement, Q_1, is an injection measurement at an unmeasured bus that already belongs to a tree, and is rejected. The following measurement, Q_{2-5}, connects buses 2 and 5 which are already connected by the tree that is formed from the first three measurements. So the measurement is rejected. The measurement of Q_5 is rejected for the same reason that Q_1 was rejected for. The measurement of Q_6 is accepted since Q_6 is an unmeasured bus and is not part of a tree. The next measurement Q_{2-3} is also accepted because it connects bus #3 to all of the unmeasured buses. A critical tree that connects all of the unmeasured buses now exists and therefore the interpolated set of reactive power measurements, (Q_{1-2}, Q_{1-4}, Q_{4-5}, Q_6, Q_{2-3}), is observable. The connection diagrams for the real and reactive measurements that were accepted are given in figure 5.12.

Example 5.7

Given the power system shown in figure 5.11, apply the new observability algorithm to the measurement set (P_{1-2}, P_2, P_6 ,P_5, P_{4-1}, P_{2-5}, P_{3-6}, Q_{1-2}, Q_{4-5}, Q_4, Q_{2-5}, Q_{1-4}, Q_6, Q_1) and determine an observable subset of measurements if one exists. Process the measurements in the order given.

First consider the real power measurement subset (P_{1-2}, P_2, P_6 , P_5, P_{4-1}, P_{2-5}, P_{3-6}). This measurement subset is the same as the first seven measurements of the real power measurement set in example 5.6. The measurements are thus processed in the same manner as the first seven real power measurements were processed in example 5.6. So the measurements (P_{1-2}, P_6, P_5, P_{4-1}) constitute the

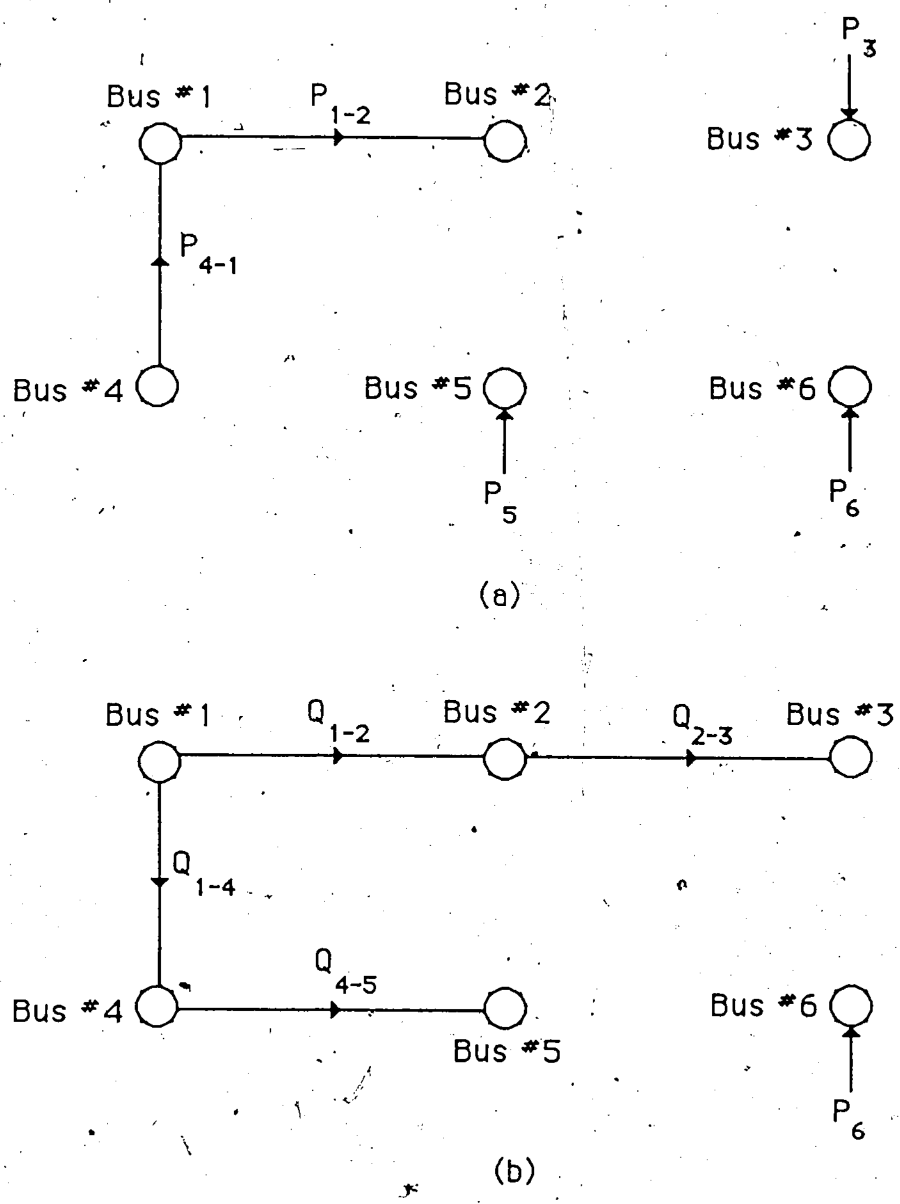

Figure 5.12 (a) Connection Diagram for Interpolated Real Power Measurements.
(b) Connected Diagram for Interpolated Reactive Power Measurements.

interpolated measurement set after all seven real power measurements have been processed. The interpolated measurement set is not observable because bus #3 is an unmeasured bus that does not belong to a tree. The only boundary injection measurement, P_2, is then processed. in the manner described by the flowchart given in figure 5.10. Since there is an unmeasured line from bus #2 to bus #3, P_2 is accepted into the measurement set and line 2-3 is added to the tree. All unmeasured buses are now connected by a tree and therefore the interpolated measurement set (P_{1-2}, P_6, P_5, P_{4-1}, P_2) is observable.

Now consider the reactive power measurements (Q_{1-2}, Q_{4-5}, Q_4 Q_{2-5}, Q_{1-4}, Q_6, Q_1). After all the measurements have been processed once, the interpolated measurement set of reactive power measurements is (Q_{1-2}, Q_{4-5}, Q_{2-5}, Q_6). The connection diagram is given in figure 5.13.

The boundary injection measurements (Q_4 and Q_1) are then processed in the manner illustrated in figure 5.10. Since a path of unmeasured lines between bus 3, and bus 1 or bus 4, does not exist, neither boundary injection will make the set of reactive power measurements observable. Since the set of reactive power measurements is unobservable, the entire measurement set is unobservable.

5.3 The New LAV Power System State Estimation Algorithm

Before the new algorithm is presented in detail several items should be discussed. First of all, the power system state estimation problem is non-linear, i.e. the

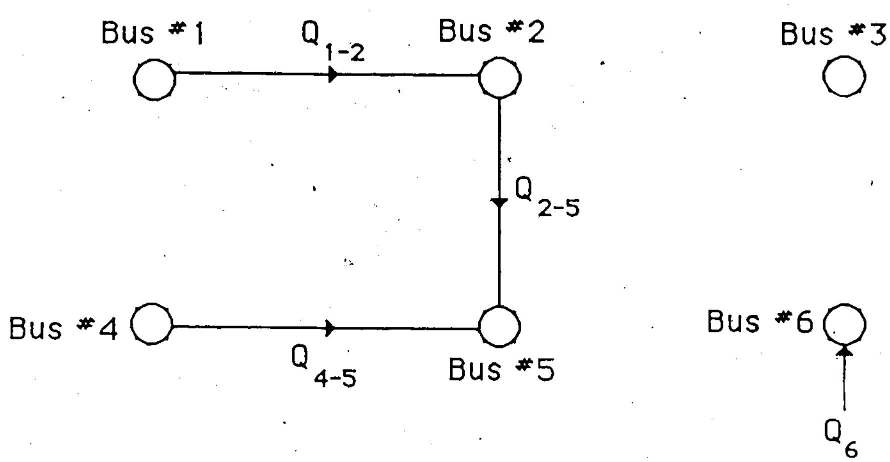

Figure 5.13 Connection Diagram for the Interpolated Reactive Power Measurements of Example 5.7

relationships between the measurements and the state variables are non-linear. The state equation must be linearized, so that least absolute value and least squares estimation can be applied to the problem. Once the equations are linearized the state estimate can be calculated iteratively.

The new algorithm assumes that the only measurements available to the estimator are measurements of real and reactive power flows and injections. If measurements of some bus voltages and phase angles are available they can be used to modify the starting point of the estimation procedure. Usually a flat start is assumed (all bus voltages = 1 per unit and all phase angles = 0), however any measured bus voltages and phase angles can replace the corresponding state variables in the flat start [31].

The new algorithm does not weigh the measurements. All measurements have a weight of 1. This is in contrast to weighted least squares estimation which usually uses a weighting matrix that is equal to the inverse of the covariance matrix. Since the entire measurement set is used to calculate a least squares estimate, a weighting matrix which emphasizes the more accurate measurements is necessary. However, in LAV estimation only n of the m measurements are used to calculate the estimate. Since all n of the measurements are interpolated, a weighting matrix which pulls the estimate closer to the more accurate measurements is unnecessary.

The steps in the new LAV power system state estimation algorithm are now given.

Step 1 Collect all system data and all measurements.

Step 2 Assume an initial estimate, $\underline{\theta}_0$, of the state vector. Usually a flat start is assumed, however any measured bus voltages and phase angles may be used as part of the initial estimate.

Step 3 Linearize the state equation about the state estimate, $\underline{\theta}$.

Step 4 Calculate the least squares solution ($\Delta\underline{\theta}$) of the linearized equation.

Step 5 Update $\underline{\theta} = \underline{\theta} + \Delta\underline{\theta}$. If $\Delta\underline{\theta}$ is smaller than the convergence criterion go to step 6, if it is not go to step 3.

Step 6 Calculate the vector of residuals using:

$$\underline{r} = \left| \Delta\underline{z} - \frac{\partial H}{\partial \underline{\theta}}\Big|_{\hat{\underline{\theta}}} \Delta\underline{\theta} \right|$$

Then rank the residuals from smallest to largest.

Step 7 Process the measurements to determine which measurements should be interpolated. Begin with the measurement that corresponds to the smallest residual and continue in order. When processing a measurement the criterion used to determine whether a measurement should be accepted or rejected for interpolation is the observabilty conditions that have been presented. Once n measurements have been selected for interpolation stop processing.

Step 8 Linearize the state estimate about $\underline{\theta}$.

Step 9 Using only the n interpolated measurements, calculate the LAV solution ($\Delta\underline{\theta}$) of the linearized equation.

Step 10 Update $\underline{\theta} = \underline{\theta} + \Delta\underline{\theta}$. If $\Delta\underline{\theta}$ is less than the convergence criterion go to step 11, if not return to step 6.

Step 11 The LAV state estimate is $\underline{\theta}$.

A flow chart of the procedure is given in figure 5.14.

5.3.1 Using the New Algorithm to Identify Bad Data and Check System Topology

As mentioned earlier, an LAV estimate interpolates n of the m measurements. The remaining (m-n) measurements are not interpolated. The residual vector of the final state estimate can be calculated using equation (5.14).

$$r = | \underline{z} - H(\hat{\underline{\theta}}) | \tag{5.14}$$

where $\hat{\underline{\theta}}$ is the final LAV estimate.

Since n measurements are interpolated n of the m components of the residual vector will be equal to zero. The other (m-n) components will be non-zero. The uninterpolated measurements that correspond to the relatively small residuals are only contaminated by noise. However, the uninterpolated measurements that lie far from the state estimate will have large residuals and can be identified as bad data.

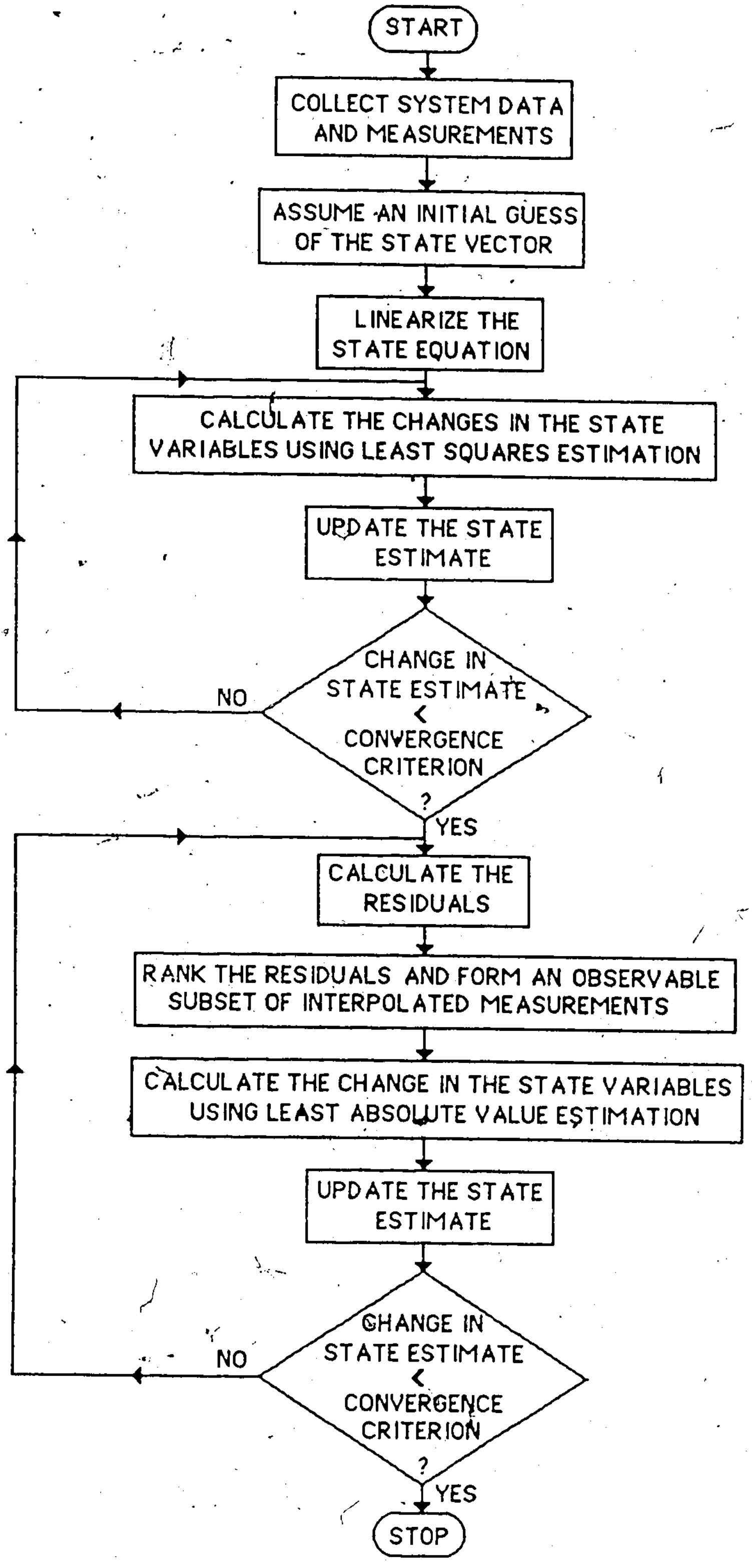

Figure 5.14 Flow Chart of the Power System State Estimation Algorithm

So all that needs to be done to identify bad data is to calculate the residuals. Once a bad data measurement is identified, steps can be taken to correct the cause of the faulty measurement. Note that the residuals need not be calculated if only an LAV estimate without bad data detection is required.

It should be mentioned that if a bad data measurement is selected as a member of the interpolated measurement set, the LAV state estimate will interpolate the bad data measurement. In this case the state estimate may be of poor quality in the region of the power system that contains the bad data measurement, and accurate measurements may be erroneously identified as bad data. Testing of the LAV estimation algorithm indicates that under most circumstances bad data measurements will not be interpolated.

The new LAV estimation algorithm can also be used to determine if a line is in service or out of service. Generally, when a state estimate is calculated it is assumed that the topology of the system is correct. However, if it is uncertain whether a line is in service, the state estimation procedure can be modified so that the correct system topology may be determined [38].

The measurements that are directly affected by the line in question are the flow measurements along the line and the injection measurements at the two buses that the line connects. Since it is uncertain whether the line is in or out of service, the flow measurements along the line should be disregarded.

The measurements of the real and reactive power injection at the two buses, that are connected by the line in question, are pivotal in determining the actual status of the line. Consider one of the two buses and assume that it is connected to three other lines and the line in question. If the line is in service the power injected at the bus will be equal to the power flows along the four lines. However, if the line is out of service the power injected at the bus will be equal to the flow along only three lines. Both possibilities can be simultaneously placed in the measurement set, so that the same measurement of power injected will be repeated. Although the two measurements will be equal, the models that they represent will be different, i.e. one injection measurement will be modelled as the sum of three flows, and the other will be modelled as the sum of four flows.

All four of the injection measurements at the two buses are repeated and modelled in the same manner. A state estimate is then calculated. If the injection measurements that correspond to the line being in service are interpolated, then the line is in service. If the injection measurements that correspond to the line being out of service are interpolated, then the line is out of service. If measurements corresponding to both possibilites are interpolated then the results are inconclusive. If none of the injection measurements are interpolated, but the LAV residuals that correspond to one of the two possibilites, are much smaller than those that correspond to the other possibility, then the line status is represented by the injection measurements that correspond to the smaller residuals.

If all four injection measurements are not available, the same procedure may be used with the available injection measurements. However the results may not be as reliable.

5.4 Constrained Power System State Estimation

Most power systems contain some bus[illegible]ich there is no generation or load. Since real and re[illegible]ve power is not injected at these buses, these buses [illegible] called zero injection buses.

The zero values of $P_{injected}$ and $Q_{injected}$, though not measured, can be treated as measurements. This increases the redundancy of the measurement set (number of measurements divided by the number of state variables) , without adding any additional metering costs.

Since the zero values of $P_{injected}$ and $Q_{injected}$ (zero injection measurements) are known to be correct, they are automatically made members of the interpolated measurement set. Thus the LAV estimator will be constrained to interpolate all of the zero injection measurements.

The handling of zero injection buses can be summarized as follows.

a) The values of real and reactive power injected at the zero injection buses ($P_{IN} = 0$, $Q_{IN} = 0$) are treated as measurements and added to the measurement set.

b) After the least squares estimate has been calculated n measurements must be selected for interpolation. All of the zero injection measurements are placed in the interpolated measurement set first. The other measurements are then processed in the same manner as they were processed for unconstrained state estimation.

c) Assuming that there are ℓ zero injection measurements, the interpolated measurement set will contain all ℓ of the zero injections and $(n-\ell)$ actual measurements.

d) The LAV estimate can then be calculated.

CHAPTER VI

RESULTS AND DISCUSSION

In this chapter, test results for the new power system state estimation algorithm are presented and discussed. The new algorithm is evaluated on the basis of the following four criteria:

1) The ability of the algorithm to reject bad data and produce quality estimates.
2) The algorithm's effectiveness in determining correct system topology.
3) The quality of the state estimates when the algorithm is used for constrained estimation.
4) The algorithms ability to filter measurement noise.

These criteria are dealt with individually in sections 6.2 to 6.5. In section 6.6 a final assessment of the algorithm is presented.

6.1 Measurement Sets and Other Test Data

The algorithm was applied to three standard power systems namely: the 5 bus system used by Stagg and El-Abiad [36], and the IEEE 14 and 30 bus systems [37]. Line parameters and generation and load data for all three systems, is given in the appendix.

For each test, both the least squares estimate and the least absolute value estimate, that is produced by the new algorithm, are given. The convergence criteria used is 0.001 per unit for the voltage state variables, and 0.001 radians for the phase angle state variables. The redundancy ratio, for each measurement set is defined as the number of actual measurements divided by the number of state variables. Zero injection measurements, though used, are not measured and thus do not affect the redundancy ratio. The following seven measurement sets were used.

Measurement Set 5a

Redundancy Ratio = 2.75

Real Power Injections: P_2, P_3, P_4, P_5

Reactive Power Injections: Q_2, Q_3, Q_4, Q_5

Real Power Flows: P_{1-2}, P_{1-3}, P_{2-3}, P_{2-4}, P_{2-5}, P_{3-4}, P_{4-5}

Reactive Power Flows: Q_{1-2}, Q_{1-3}, Q_{2-3}, Q_{2-4}, Q_{2-5}, Q_{3-4}, Q_{4-5}

Measurement Set 5b

Redundancy Ratio = 2.25

Real Power Injections: P_2, P_3, P_4, P_5

Reactive Power Injections: Q_2, Q_3, Q_4, Q_5

Real Power Flows: P_{1-2}, P_{2-3}, P_{2-5}, P_{3-4}, P_{4-5}

Reactive Power Flows: Q_{1-2}, Q_{2-3}, Q_{2-5}, Q_{3-4}, Q_{4-5}

Measurement Set 14a

Redundancy Ratio = 2.04

Real Power Injections: P_3, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}

Reactive Power Injections: Q_3, Q_8, Q_{10}, Q_{11}, Q_{12}, Q_{13}, Q_{14}

Real Power Flows: P_{1-2}, P_{1-5}, P_{2-3}, P_{2-4} P_{2-5}, P_{3-4}, P_{4-5}, P_{4-7} P_{4-9}, P_{5-6}, P_{6-11}, P_{6-12}, P_{6-13}, P_{7-8}, P_{7-9}, P_{9-10}, P_{9-14}, P_{10-11}, P_{12-13}, P_{13-14}

Reactive Power Flows: Q_{1-2}, Q_{1-5}, Q_{2-3}, Q_{2-4}, Q_{2-5}, Q_{3-4}, Q_{4-5}, Q_{4-7}, Q_{4-9}, Q_{5-6}, Q_{6-11}, Q_{6-12}, Q_{6-13}, Q_{7-8}, Q_{7-9}, Q_{9-10}, Q_{9-14}, Q_{10-11}, Q_{12-13}, Q_{13-14}

Zero Injections: P_7, Q_7, P_8

Measurement Set 14b (Constrained)

Redundancy Ratio = 1.81

Real Power Injections: P_2, P_3, P_4, P_5, P_6, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}

Reactive Power Injections: Q_2, Q_3, Q_4, Q_5, Q_6, Q_8, Q_9, Q_{10}, Q_{11}, Q_{12}, Q_{13}, Q_{14}

Real Power Flows: P_{2-1}, P_{3-2}, P_{3-4}, P_{4-5}, P_{5-1}, P_{6-11}, P_{6-12}, P_{10-9}, P_{10-11}, P_{12-13}, P_{13-14}, P_{14-9}

Reactive Power Flows: Q_{2-1}, Q_{3-2}, Q_{3-4}, Q_{4-5}, Q_{5-1}, Q_{6-11}, Q_{6-12}, Q_{10-9}, Q_{10-11}, Q_{12-13}, Q_{13-14}, Q_{14-9}

Zero Injections: P_7, Q_7, P_8 (Constraints)

Measurement Set 14c (Constrained)

Redundancy Ratio = 2.04

Real Power Injections: P_3, P_5, P_6, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14}

Reactive Power Injections: Q_3, Q_5, Q_6, Q_8, Q_9, Q_{10}, Q_{11}, Q_{12}, Q_{13}, Q_{14}

Real Power Flows: P_{1-2}, P_{1-5}, P_{2-3}, P_{2-4}, P_{2-5}, P_{3-4}, P_{4-5}, P_{4-9}, P_{5-6}, P_{6-11}, P_{6-12}, P_{6-13} P_{9-10}, P_{9-14} P_{10-11}, P_{12-13} P_{13-14}

Reactive Power Flows: Q_{1-2}, Q_{1-5}, Q_{2-3}, Q_{2-4}, Q_{2-5}, Q_{3-4}, Q_{4-5}, Q_{4-9}, Q_{5-6}, Q_{6-11}, Q_{6-12}, Q_{6-13} Q_{9-10}, Q_{9-14} Q_{10-11}, Q_{12-13} Q_{13-14}

Zero Injections: P_7, Q_7, P_8 (Constraints)

Measurement Set 30a

Redundancy Ratio = 1.98

Real Power Injections: P_2, P_3, P_4, P_5, P_7, P_8, P_{10}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{23}, P_{24}, P_{26}, P_{29}, P_{30}

Reactive Power Injections: Q_2, Q_3, Q_4, Q_5, Q_7, Q_8, Q_{10}, Q_{11}, Q_{12}, Q_{13}, Q_{14}, Q_{15}, Q_{16}, Q_{17}, Q_{18}, Q_{19}, Q_{20}, Q_{21}, Q_{23}, Q_{24}, Q_{26}, Q_{29}, Q_{30}

Real Power Flows: P_{1-2}, P_{1-3}, P_{2-4}, P_{2-5}, P_{3-4}, P_{4-6}, P_{5-7}, P_{6-7}, P_{6-8}, P_{6-9}, P_{6-28}, P_{8-28}, P_{9-10}, P_{9-11}, P_{10-17}, P_{10-20}, P_{10-21}, P_{10-22}, P_{12-13}, P_{12-14}, P_{12-16}, P_{14-15}, P_{15-18}, P_{15-23}, P_{16-17}, P_{18-19}, P_{19-20}, P_{21-22}, P_{22-24}, P_{23-24}, P_{24-25}, P_{25-26}, P_{27-28}, P_{27-29}, P_{29-30}

Reactive Power Flows: Q_{1-2}, Q_{1-3}, Q_{2-4}, Q_{2-5}, Q_{3-4}, Q_{4-6}, Q_{5-7}, Q_{6-7}, Q_{6-8}, Q_{6-9}, Q_{6-28}, Q_{8-28}, Q_{9-10}, Q_{9-11} Q_{10-17}, Q_{10-20}, Q_{10-21}, Q_{10-22}, Q_{12-13}, Q_{12-14}, Q_{12-16}, Q_{14-15}, Q_{15-18}, Q_{15-23}, Q_{16-17}, Q_{18-19}, Q_{19-20}, Q_{21-22}, Q_{22-24}, Q_{23-24}, Q_{24-25}, Q_{25-26}, Q_{27-28}, Q_{27-29}, Q_{29-30}

Zero Injections: P_6, P_9, P_{11}, P_{22}, P_{25}, P_{27}, P_{28}, Q_6, Q_9, Q_{22}, Q_{25}, Q_{27}, Q_{28}

Measurement Set 30b (Constrained)

Redundancy Ratio = 1.53

Real Power Injections: P_2, P_3, P_4, P_5, P_7, P_8, P_{10}, P_{12}, P_{13}, P_{14}, P_{15}, P_{16}, P_{17}, P_{18}, P_{19}, P_{20}, P_{21}, P_{23}, P_{24}, P_{26}, P_{29}, P_{30}

Reactive Power Injections: Q_2, Q_3, Q_4, Q_5, Q_7 Q_8, Q_{10}, Q_{11},

Q_{12}, Q_{13}, Q_{14}, Q_{15}, Q_{16}, Q_{17}, Q_{18}, Q_{19}, Q_{20}, Q_{21}, Q_{23}, Q_{24}, Q_{26}, Q_{29}, Q_{30}

Real Power Flows: P_{2-1}, P_{2-4}, P_{2-5}, P_{3-1}, P_{3-4}, P_{4-12}, P_{5-7}, P_{10-17} P_{10-20}, P_{10-21}, P_{12-13}, P_{12-14}, P_{12-15}, P_{12-16}, P_{14-15}, P_{15-18}, P_{15-23}, P_{16-17}, P_{18-19}, P_{19-20}, P_{23-24}, P_{29-30},

Reactive Power Flows: Q_{2-1}, Q_{2-4}, Q_{2-5}, Q_{3-1}, Q_{3-4}, Q_{4-12}, Q_{5-7}, Q_{10-17} Q_{10-20}, Q_{10-21}, Q_{12-13}, Q_{12-14}, Q_{12-15}, Q_{12-16}, P_{14-15}, Q_{15-18}, Q_{15-23}, Q_{16-17}, Q_{18-19}, Q_{19-20}, Q_{23-24}, Q_{29-30},

Zero Injections: P_6, P_9, P_{11}, P_{22}, P_{25}, P_{27}, P_{28}, Q_6, Q_9, Q_{22}, Q_{25}, Q_{27}, Q_{28} (Constraints)

6.2 The Effect of Bad Data

When the measurement set does not contain any bad data or noise both the least squares and the least absolute value estimates are extremely accurate. In this section the effect of bad data on the two types of estimates is investigated. All of the measurements, except the bad data, are exact, i.e. noise is not added to the measurement set. The effect of noise is investigated in section 6.5.

A representative sample of test results are shown in tables 6.1 - 6.3. Voltages are in per unit and phase angles are in radians.

TEST NUMBER MEAS. SET BAD DATA			1 5a P(2,4) reversed in sign			
	EXACT VALUES		LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000	1.0600	0.0000
2	1.0474	-0.0490	1.0482	-0.0469	1.0474	-0.0490
3	1.0242	-0.0872	1.0275	-0.0777	1.0242	-0.0872
4	1.0236	-0.0930	1.0270	-0.0830	1.0236	-0.0930
5	1.0179	-0.1073	1.1091	-0.1037	1.0179	-0.1073

TEST NUMBER MEAS. SET BAD DATA	2 5a P(3) set equal to 0			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0489	-0.0450	1.0474	-0.0490
3	1.0298	-0.0703	1.0242	-0.0872
4	1.0278	-0.0799	1.0236	-0.0930
5	1.0191	-0.1032	1.0179	-0.1073

TEST NUMBER MEAS. SET BAD DATA	3 5b Q(5) reversed in sign			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0491	-0.0497	1.0474	-0.0490
3	1.0245	-0.0876	1.0242	-0.0872
4	1.0242	-0.0935	1.0236	-0.0930
5	1.0278	-0.1107	1.0179	-0.1073

Table 6.1 State Estimates for the 5 Bus System

TEST NUMBER	4			
MEAS. SET	5a			
BAD DATA	Q(3,4) set equal to 0			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0472	-0.0489	1.0469	-0.0488
3	1.0241	-0.0872	1.0242	-0.0872
4	1.0231	-0.0928	1.0220	-0.0925
5	1.0174	-0.1072	1.0164	-0.1070

TEST NUMBER	5			
MEAS. SET	5b			
BAD DATA	P(2,3), P(3) reversed in sign P(4,5) =0.085, Q(4,5) =0.024			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0519	-0.0364	1.0474	-0.0490
3	1.0417	-0.0372	1.0403	-0.0381
4	1.0383	-0.0504	1.0360	-0.0548
5	1.0220	-0.0920	1.0179	-0.1073

TEST NUMBER	6			
MEAS. SET	5a			
BAD DATA	P(2,3), P(3) reversed in sign P(4,5) =0.085, Q(4,5) =0.024			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0507	-0.0394	1.0474	-0.0490
3	1.0386	-0.0452	1.0242	-0.0872
4	1.0351	-0.0587	1.0236	-0.0930
5	1.0202	-0.0967	1.0179	-0.1073

Table 6.1 Continued

TEST NUMBER MEAS. SET BAD DATA			7 14a P(13) reversed in sign			
BUS	EXACT VALUES V	EXACT VALUES ANGLE	LEAST SQUARES V	LEAST SQUARES ANGLE	LAV V	LAV ANGLE
1	1.0600	0.0000	1.0600	0.0000	1.0600	0.0000
2	1.0450	-0.0869	1.0450	-0.0869	1.0450	-0.0869
3	1.0100	-0.2220	1.0100	-0.2220	1.0100	-0.2220
4	1.0186	-0.1802	1.0186	-0.1802	1.0186	-0.1802
5	1.0203	-0.1533	1.0203	-0.1533	1.0203	-0.1533
6	1.0700	-0.2483	1.0706	-0.2469	1.0700	-0.2482
7	1.0620	-0.2333	1.0616	-0.2341	1.0619	-0.2333
8	1.0900	-0.2333	1.0896	-0.2341	1.0900	-0.2333
9	1.0563	-0.2609	1.0558	-0.2619	1.0563	-0.2609
10	1.0513	-0.2636	1.0509	-0.2644	1.0513	-0.2636
11	1.0571	-0.2582	1.0572	-0.2580	1.0571	-0.2582
12	1.0552	-0.2632	1.0553	-0.2583	1.0552	-0.2632
13	1.0504	-0.2646	1.0580	-0.2515	1.0504	-0.2646
14	1.0358	-0.2800	1.0366	-0.2786	1.0358	-0.2800

TEST NUMBER MEAS. SET BAD DATA	8 14a P(12) reversed in sign P(3,4), Q(13,14) = 0			
BUS	LEAST SQUARES V	LEAST SQUARES ANGLE	LAV V	LAV ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0452	-0.0866	1.0450	-0.0869
3	1.0126	-0.2176	1.0100	-0.2220
4	1.0166	-0.1858	1.0186	-0 1802
5	1.0183	-0.1583	1.0202	-0.1533
6	1.0676	-0.2534	1.0700	-0.2482
7	1.0600	-0.2393	1.0619	-0.2333
8	1.0882	-0.2393	1.0900	-0.2333
9	1.0545	-0.2670	1.0563	-0.2609
10	1.0494	-0.2696	1.0513	-0.2636
11	1.0549	-0.2639	1.0571	-0.2582
12	1.0591	-0.2592	1.0553	-0.2632
13	1.0485	-0.2679	1.0504	-0.2646
14	1.0343	-0.2857	1.0358	-0.2799

Table 6.2 State Estimates for the 14 Bus System

TEST NUMBER	9			
MEAS. SET	14a			
BAD DATA	P(3), Q(10), P(9,10), Q(6,12) all reversed in sign			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0452	-0.0871	1.0450	-0.0869
3	1.0448	-0.1110	1.0100	-0.2220
4	1.0156	-0.1781	1.0186	-0.1802
5	1.0176	-0.1512	1.0203	-0.1533
6	1.0671	-0.2431	1.0700	-0.2483
7	1.0589	-0.2336	1.0619	-0.2333
8	1.0870	-0.2337	1.0900	-0.2333
9	1.0533	-0.2621	1.0563	-0.2609
10	1.0537	-0.2621	1.0513	-0.2636
11	1.0560	-0.2531	1.0571	-0.2582
12	1.0550	-0.2595	1.0552	-0.2632
13	1.0484	-0.2606	1.0504	-0.2646
14	1.0331	-0.2791	1.0358	-0.2800

TEST NUMBER	10			
MEAS. SET	14a			
BAD DATA	P(4,5) set equal to 0			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0454	-0.0862	1.0450	-0.0869
3	1.0124	-0.2143	1.0130	-0.2084
4	1.0230	-0.1670	1.0248	-0.1552
5	1.0168	-0.1635	1.0181	-0.1531
6	1.0691	-0.2507	1.0676	-0.2485
7	1.0645	-0.2243	1.0682	-0.2077
8	1.0924	-0.2246	1.0961	-0.2077
9	1.0584	-0.2336	1.0627	-0.2349
10	1.0530	-0.2576	1.0564	-0.2423
11	1.0576	-0.2562	1.0588	-0.2475
12	1.0543	-0.2652	1.0532	-0.2615
13	1.0499	-0.2660	1.0496	-0.2612
14	1.0367	-0.2763	1.0391	-0.2636

Table 6.2 Continued

TEST NUMBER MEAS. SET BAD DATA			11 30a P(12) set to 0			
	EXACT VALUES		LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000	1.0600	0.0000
2	1.0450	-0.0692	1.0451	-0.0690	1.0450	-0.0692
3	1.0283	-0.0916	1.0285	-0.0910	1.0283	-0.0916
4	1.0206	-0.1099	1.0210	-0.1091	1.0207	-0.1099
5	1.0100	-0.2072	1.0100	-0.2070	1.0100	-0.2072
6	1.0147	-0.1386	1.0149	-0.1380	1.0147	-0.1386
7	1.0050	-0.1759	1.0051	-0.1755	1.0050	-0.1759
8	1.0100	-0.1499	1.0101	-0.1494	1.0100	-0.1499
9	1.0505	-0.1660	1.0504	-0.1650	1.0505	-0.1660
10	1.0411	-0.1804	1.0408	-0.1792	1.0411	-0.1804
11	1.0820	-0.1660	1.0818	-0.1650	1.0820	-0.1660
12	1.0645	-0.1100	1.0658	-0.1031	1.0645	-0.1100
13	1.0710	-0.0234	1.0723	-0.0177	1.0710	-0.0234
14	1.0491	-0.1327	1.0492	-0.1287	1.0492	-0.1328
15	1.0410	-0.1415	1.0409	-0.1380	1.0411	-0.1415
16	1.0459	-0.1440	1.0461	-0.1403	1.0460	-0.1440
17	1.0368	-0.1730	1.0365	-0.1712	1.0368	-0.1730
18	1.0285	-0.1672	1.0279	-0.1655	1.0285	-0.1672
19	1.0244	-0.1793	1.0238	-0.1779	1.0244	-0.1793
20	1.0276	-0.1806	1.0271	-0.1793	1.0277	-0.1806
21	1.0290	-0.1870	1.0286	-0.1858	1.0290	-0.1870
22	1.0296	-0.1863	1.0293	-0.1851	1.0296	-0.1863
23	1.0281	-0.1636	1.0276	-0.1617	1.0281	-0.1636
24	1.0200	-0.1876	1.0196	-0.1864	1.0200	-0.1876
25	1.0144	-0.1971	1.0141	-0.1965	1.0144	-0.1971
26	0.9967	-0.2045	0.9963	-0.2039	0.9967	-0.2045
27	1.0199	-0.1985	1.0198	-0.1981	1.0199	-0.1985
28	1.0101	-0.1469	1.0102	-0.1465	1.0101	-0.1469
29	1.0000	-0.2201	0.9999	-0.2199	1.0000	-0.2201
30	0.9885	-0.2356	0.9884	-0.2355	0.9885	-0.2356

Table 6.3 State Estimates for the 30 Bus System

TEST NUMBER	12			
MEAS. SET	30a			
BAD DATA	Q(24) reversed in sign			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0450	-0.0692	1.0450	-0.0692
3	1.0284	-0.0916	1.0283	-0.0916
4	1.0207	-0.1099	1.0207	-0.1099
5	1.0100	-0.2072	1.0100	-0.2072
6	1.0148	-0.1386	1.0147	-0.1386
7	1.0050	-0.1759	1.0050	-0.1759
8	1.0100	-0.1500	1.0100	-0.1499
9	1.0513	-0.1663	1.0505	-0.1660
10	1.0424	-0.1808	1.0411	-0.1804
11	1.0827	-0.1664	1.0820	-0.1660
12	1.0649	-0.1102	1.0645	-0.1100
13	1.0713	-0.0236	1.0710	-0.0234
14	1.0496	-0.1329	1.0491	-0.1327
15	1.0418	-0.1419	1.0410	-0.1415
16	1.0466	-0.1442	1.0459	-0.1440
17	1.0378	-0.1734	1.0368	-0.1730
18	1.0291	-0.1675	1.0285	-0.1672
19	1.0250	-0.1795	1.0244	-0.1793
20	1.0284	-0.1809	1.0276	-0.1806
21	1.0309	-0.1877	1.0290	-0.1870
22	1.0319	-0.1872	1.0296	-0.1863
23	1.0305	-0.1651	1.0281	-0.1636
24	1.0289	-0.1924	1.0200	-0.1876
25	1.0149	-0.1972	1.0144	-0.1972
26	0.9951	-0.2031	0.9967	-0.2045
27	1.0185	-0.1978	1.0199	-0.1986
28	1.0100	-0.1469	1.0101	-0.1470
29	0.9971	-0.2185	1.0000	-0.2202
30	0.9853	-0.2340	0.9885	-0.2357

Table 6.3 Continued

TEST NUMBER	13			
MEAS. SET	30a			
BAD DATA	P(19), Q(10), P(6,28) = 0			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0451	-0.0690	1.0450	-0.0692
3	1.0286	-0.0909	1.0283	-0.0916
4	1.0210	-0.1090	1.0207	-0.1099
5	1.0100	-0.2070	1.0100	-0.2072
6	1.0153	-0.1371	1.0147	-0.1386
7	1.0052	-0.1752	1.0050	-0.1759
8	1.0106	-0.1481	1.0100	-0.1499
9	1.0515	-0.1657	1.0505	-0.1660
10	1.0424	-0.1799	1.0411	-0.1804
11	1.0828	-0.1663	1.0820	-0.1660
12	1.0650	-0.1098	1.0645	-0.1100
13	1.0714	-0.0235	1.0710	-0.0234
14	1.0494	-0.1326	1.0491	-0.1327
15	1.0417	-0.1409	1.0410	-0.1415
16	1.0466	-0.1440	1.0459	-0.1440
17	1.0378	-0.1727	1.0368	-0.1730
18	1.0299	-0.1650	1.0285	-0.1672
19	1.0273	-0.1742	1.0244	-0.1793
20	1.0297	-0.1773	1.0276	-0.1806
21	1.0300	-0.1863	1.0290	-0.1870
22	1.0306	-0.1856	1.0296	-0.1863
23	1.0288	-0.1626	1.0281	-0.1636
24	1.0209	-0.1853	1.0200	-0.1876
25	1.0158	-0.1888	1.0144	-0.1971
26	0.9983	-0.1960	0.9967	-0.2045
27	1.0212	-0.1869	1.0199	-0.1985
28	1.0116	-0.1422	1.0101	-0.1469
29	1.0021	-0.2041	1.0000	-0.2201
30	0.9908	-0.2187	0.9885	-0.2356

Table 6.3 Continued

TEST NUMBER	14			
MEAS. SET	30a			
BAD DATA	P(5), Q(5) reversed in sign P(26) doubled and reversed in sign P(30), Q(30) doubled			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0471	-0.0595	1.0450	-0.0692
3	1.0248	-0.0927	1.0284	-0.0916
4	1.0169	-0.1107	1.0207	-0.1099
5	1.0224	-0.1065	1.0100	-0.2072
6	1.0105	-0.1373	1.0148	-0.1386
7	1.0027	-0.1491	1.0051	-0.1759
8	1.0054	-0.1506	1.0100	-0.1499
9	1.0480	-0.1799	1.0506	-0.1660
10	1.0390	-0.1973	1.0411	-0.1804
11	1.0801	-0.1829	1.0820	-0.1660
12	1.0628	-0.1268	1.0645	-0.1100
13	1.0696	-0 [illegible]412	1.0710	-0.0234
14	1.0465	-0.1524	1.0492	-0.1328
15	1.0384	-0.1610	1.0411	-0.1415
16	1.0437	-0.1623	1.0460	-0.1440
17	1.0346	-0.1909	1.0368	-0.1730
18	1.0255	-0.1878	1.0285	-0.1672
19	1.0214	-0.1997	1.0244	-0.1793
20	1.0248	-0.2006	1.0277	-0.1806
21	1.0267	-0.2048	1.0290	-0.1870
22	1.0273	-0.2042	1.0296	-0.1863
23	1.0254	-0.1834	1.0282	-0.1636
24	1.0176	-0.2055	1.0200	-0.1876
25	1.0131	-0.2100	1.0184	-0.1903
26	1.0086	-0.1994	1.0267	-0.1593
27	1.0133	-0.2163	1.0192	-0.2104
28	1.0050	-0.1491	1.0102	-0.1469
29	0.9860	-0.2480	0.9839	-0.2487
30	0.9627	-0.2793	0.9569	-0.2838

Table 6.3 Continued

6.2.1 Discussion of Bad Data Tests

In the last section results from 14 tests are presented. In ten of these tests (1,2,3,6,7,8,9,11,12,13) bad data is rejected and not interpolated. For all of these cases the LAV state estimate is completely accurate while the least squares estimate is affected by bad data. These results indicate that when an LAV estimate does not interpolate bad data it is superior to the least squares estimate.

In 4 of the tests (4,5,10,14) the LAV estimate interpolates bad data. When this occurs, the LAV state estimate is inaccurate in the region of the power system that contains the interpolated bad data.

In two of these cases (4,10) the measurement set contains only one bad data. In both of these cases, the least squares estimate is slightly more accurate than the LAV estimate. This is so simply because a least squares estimator uses the entire measurement set to calculate its estimate while a least absolute value estimator uses only n of the m measurements (m = the number of measurements and n = the number of state variables). Even though both estimators use the same bad data measurement the percentage of bad data in the interpolated measurement set ($\frac{1}{n}$x100%), is greater than the percentage of bad data in the entire measurement set ($\frac{1}{m}$x100%), consequently the bad data measurement has a greater impact on the least absolute value estimate.

In the two other cases in which bad data is interpolated (5,14), the measurement set contains several bad data points. In test #5, the measurement set contains four bad data points. Of these four points, only one (P_3 reversed) is interpolated; the other three bad data points are rejected. An examination of the results of test #5 indicates that the least absolute value estimate is more accurate than the least squares estimate. Even though the LAV estimator interpolates a bad data measurement, the least squares estimator is less accurate because it uses all 4 bad data points to calculate its estimate.

Test #6 contains exactly the same bad data as test #5, but it has a larger measurement set. In this case all 4 bad data are rejected by the LAV estimator and the LAV estimate is exact, whereas the least squares estimator continues to produce an inaccurate estimate. It thus seems possible to improve the bad data rejection properties of the new algorithm by increasing the redundancy ratio.

In case #14 there are 5 bad data measurements. The LAV estimator interpolates three of the bad data (P_{26}, P_{30}, Q_{30}). In the region of the power system that contains the interpolated bad data, the least squares estimate is more accurate than the LAV estimate. At bus #26 the least squares estimate of ($V_{26} = 1.0086$, $\delta_{26} = -0.1994$) is closer than the least absolute value estimate of ($V_{26} = 1.0267$, $\delta_{26} = -0.1593$) , to the actual values of ($V_{26} = 0.9967$, $\delta_{26} = -0.2045$). In regions of the power system that are not near the interpolated bad data measurements, the LAV estimate is more accurate. At bus #3 the least squares estimate of (V_3 = 1.0248, δ_3 -0.0927) is not as accurate as the LAV estimate of (V_3 = 1.0284, δ_3 = -0.0916). The actual values at bus #3 are (V_3 = 1.0283, δ_3 = -0.0916).

The test results that have just been presented support the following four statements:

a) When the new LAV estimator does not interpolate bad data, it gives better results than a least squares estimator.

b) When the new LAV estimator interpolates all of the bad data points it produces estimates that are less accurate than least squares estimates.

c) When some, but not all, bad data points are interpolated by the new LAV estimator, the LAV state estimate tends to be less accurate than the least squares estimate, in the region of the interpolated bad data. In other regions the LAV estimate tends to be more accurate.

d) The ability of an LAV state estimator to reject bad data increases as the number of measurements increases.

6.3 Determining System Topology

Two sets of tests were conducted. In the first set of tests the line is actually in service, and in the second set of tests the line is not in service. The measurements, that are given to the estimator, for both sets of tests are obtained from a load flow program.

A total of seven lines were tested. The results are presented in table 6.4.

SYSTEM	MEASUREMENT SET	LINE	a	b
5 bus	5a	2-5	LINE IN	INCONCLUSIVE
5 bus	5a	4-5	LINE IN	LINE OUT
14 bus	14a	3-1	LINE IN	INCONCLUSIVE
14 bus	14a	4-5	LINE IN	INCONCLUSIVE
14 bus	14a	2-3	LINE IN	INCONCLUSIVE
30 bus	30a	6-7	LINE IN	INCONCLUSIVE
30 bus	30a	8-28	LINE IN	LINE OUT

Table 6.4 . Results for line status tests.
Column a represents test that were conducted when the line was actually in service.
Column b represents test that were conducted when the line was actually out of service.

6.3.1 Discussion of Topology Test Results

The results that are presented in table 6.4 indicate that the new LAV state estimator always detects that a line is in service if it actually is in service. However, when a line is actually out of service the results were usually inconclusive, although the estimator did twice correctly predict that a line was out of service.

The technique of determining whether a line is actually in service or out of service, was first presented by Kotiuga [4,33,38]. Although he presented the technique, he never presented any test data to support his technique in either his Ph.D. thesis [38] or in the two technical papers [4,33], in which he presented the technique.

Although the LAV state estimator can reliably determine that a line is in service, it does not appear able to determine if a line is out of service. Further research in this area is required.

6.4 Results of Constrained State Estimation

In this section, test results for constrained LAV state estimation are presented. Since a constrained estimate interpolates all of the zero power injection measurements, all of the buses at which the power injection is zero are classified as measured buses. Consequently the observability criteria established in chapter V dictates that power flows along lines that are connected to measured buses, cannot be part of the interpolated measurement set. As a result, the three measurements sets used by the state estimator for constrained state estimation, do not contain any measurements of power flow along lines that are connected to zero injection buses. Test results are presented in table 6.5.

MEAS SET	BAD DATA	NUMBER OF BAD DATA INTERPOLATED
14b	P(13) REVERSED IN SIGN	0
14b	Q(9) SET EQUAL TO 0	1
14b	P(4,5) SET EQUAL TO 0	1
14b	Q(3,2) REVERSED IN SIGN	0
14b	P(5) SET EQUAL TO 0	0
14b	Q(6) SET EQUAL TO 0	0
14b	P(13,14) REVERSED IN SIGN	0
14b	Q(10,9) SET EQUAL TO 0	0
14b	P(3,4), Q(13,14) SET EQUAL TO 0 P(12) REVERSED IN SIGN	1
14b	Q(3,2) REVERSED IN SIGN, Q(10)=0 P(14,9) DOUBLED	1
14b	P(3), Q(10), P(10,9) AND Q(6,12) ALL REVERSED IN SIGN	1
14b	P(10), Q(3), Q(10,9) AND Q(6,12) ALL DOUBLED	1
14c	P(13) REVERSED IN SIGN	0
14c	Q(9) SET EQUAL TO 0	0
14c	P(4,5) SET EQUAL TO 0	0
14c	Q(2,3) REVERSED IN SIGN	0
14c	P(5) SET EQUAL TO 0	0
14c	Q(6) SET EQUAL TO 0	0
14c	P(13,14) REVERSED IN SIGN	1
14c	Q(10,9) SET EQUAL TO 0	0
14c	P(3,4), Q(13,14) SET EQUAL TO 0 P(12) REVERSED IN SIGN	0
14c	Q(3,2) REVERSED IN SIGN, Q(10)=0 P(14,9) DOUBLED	1
14c	P(3), Q(10), P(10,9) AND Q(6,12) ALL REVERSED IN SIGN	1
14c	P(10), Q(3), Q(10,9) AND P(6,12) ALL DOUBLED	1
30b	P(16,17) REVERSED IN SIGN	1
30b	Q(2,1) SET EQUAL TO 0	1
30b	P(16) SET EQUAL TO 0	0
30b	Q(24) REVERSED IN SIGN	1
30b	P(5) AND Q(5) REVERSED IN SIGN P(26) REVERSED AND DOUBLED P(30), Q(30) DOUBLED	2
30b	P(12), Q(26) SET EQUAL TO 0	1
30b	Q(10) REVERSED IN SIGN P(3,1), Q(14,12) SET EQUAL TO 0	1

Table 6.5 Results for Constrained State Estimation

6.4.1 Discussion of Constrained State Estimation Test Results

For the 14 bus system, a bad data measurement was interpolated in only three of the sixteen cases, in which a single bad data contaminated the measurement set. However for the 30 bus system in three of the four cases in which there was only one bad data measurement, the measurement was interpolated.

The reason why the two 14 bus measurement sets produce better LAV estimates can be found by examining the redundancy ratio of the three measurement sets. The two 14 bus constrained measurement sets have redundancy ratios of 1.81 and 2.04, while the 30 bus constrained measurement set has a redundancy ratio of only 1.53. The larger redundancy ratio makes the LAV estimator more effective in rejecting bad data.

Since the 30 bus system has thirteen zero injection measurements six of its buses are classified as measured buses even before the estimation process begins (the seventh bus, bus #11 is a zero injection bus for real power flow measurements only) . Consequently the constrained 30 bus measurement set does not contain any measurements of power flows along lines that are connected to zero injection buses. Of the 41 lines in the 30 bus system, only 22 of the lines are not connected to zero injection buses. Even if all of the power flows along these lines are measured the redundancy ratio is only 1.53. Thus, the constrained 30 bus measurement set does not produce estimates that reject bad data very well because of its low redundancy ratio.

The test results for the cases in which a single bad data contaminated the measurement set, indicates that constrained estimation is effective when the number of constraints does not reduce the redundancy ratio to a low value.

In the case of multiple bad data, the estimates that are produced by the 14 bus and the 30 bus measurement sets generally reject the majority of the bad data measurements.

6.5 The Effect of Noise

The feature that makes LAV state estimation attractive, is the ability of LAV estimates to reject bad data. This ability is a direct result of the interpolation property of LAV estimates. Obviously when the measurement set is contaminated by noise LAV state estimates will interpolate noisy measurements. This will decrease the accuracy of the state estimate. Least squares estimators tend to filter noise better since they do not interpolate any measurements.

In order to evaluate the performance of least squares and LAV estimators in the presence of noise, a random number generator was used to add Gaussian noise, with a 2% variance, to the measurement set. Results for the 5 bus system are presented in table 6.6.

BUS	EXACT VALUES V	ANGLE
1	1.0600	0.0000
2	1.0474	-0.0490
3	1.0242	-0.0872
4	1.0236	-0.0930
5	1.0179	-0.1073

TEST NO.	0			
MEAS SET	5a			
BAD DATA	P(3) set equal to 0			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0472	-0.0496	1.0470	-0.0502
3	1.0239	-0.0881	1.0239	-0.0885
4	1.0233	-0.0939	1.0232	-0.0942
5	1.0174	-0.1090	1.0175	-0.1086

TEST NO.	0			
MEAS SET	5a			
BAD DATA	P(3) set equal to 0			
	ERROR IN LEAST SQUARES		ERROR IN LAV	
BUS	V	ANGLE	V	ANGLE
1	-	-	-	-
2	0.0002	0.0006	0.0004	0.0012
3	0.0003	0.0009	0.0003	0.0013
4	0.0003	0.0009	0.0004	0.0012
5	0.0005	0.0013	0.0004	0.0013
SUM OF ERRORS	0.0013	0.0037	0.0015	0.0050

Table 6.6 State Estimates and Absolute Values of Errors for Gaussian Noise Added to the Measurement Set.

TEST NO.	1			
MEAS SET	5a			
BAD DATA	P(2,4) R[illegible]ERSED IN SIGN			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0479	-0.0476	1.0485	-0.0452
3	1.0272	-0.0787	1.0254	-0.0833
4	1.0267	-0.0840	1.0248	-0.0890
5	1.0185	-0.1054	1.0182	-0.1054

TEST NO.	1			
MEAS SET	5a			
BAD DATA	P(2,4) REVERSED IN SIGN			
	ERROR IN LEAST SQUARES		ERROR IN LAV	
BUS	V	ANGLE	V	ANGLE
1	-	-	-	-
2	0.0005	[illegible]	0.0011	0.0038
3	0.0030	0.0085	0.0012	0.0039
4	0.0031	0.0090	0.0012	0.0040
5	0.0006	0.0019	0.0003	0.0019
SUM OF ERRORS	0.0072	0.0208	0.0038	0.0136

Table 6.6 Continued

TEST NO.	2			
MEAS SET	5a			
BAD DATA	P(3) SET EQUAL TO 0			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0486	-0.0455	1.0485	-0.0452
3	1.0296	-0.0709	1.0254	-0.0833
4	1.0276	-0.0807	1.0248	-0.0891
5	1.0185	-0.1048	1.0197	-0.1018

TEST NO.	2			
MEAS SET	5a			
BAD DATA	P(3) SET EQUAL TO 0			
	ERROR IN LEAST SQUARES		ERROR IN LAV	
BUS	V	ANGLE	V	ANGLE
1	-	-	-	-
2	0.0012	0.0035	0.0011	0.0038
3	0.0054	0.0163	0.0012	0.0039
4	0.0030	0.0127	0.0012	0.0039
5	0.0006	0.0025	0.0018	0.0055
SUM OF ERRORS	0.0102	0.0350	0.0053	0.0171

Table 6.6 Continued

TEST NO.	3			
MEAS SET	5b			
BAD DATA	Q(5) REVERSED IN SIGN			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0487	-0.0507	1.0470	-0.0502
3	1.0240	-0.0891	1.0235	-0.0894
4	1.0237	-0.0952	1.0228	-0.0954
5	1.0270	-0.1132	1.0167	-0.1111

TEST NO.	3			
MEAS SET	5b			
BAD DATA	Q(5) REVERSED IN SIGN			
	ERROR IN LEAST SQUARES		ERROR IN LAV	
BUS	V	ANGLE	V	ANGLE
1	-	-	-	-
2	0.0013	0.0017	0.0004	0.0012
3	0.0002	0.0019	0.0007	0.0022
4	0.0001	0.0022	0.0008	0.0024
5	0.0091	0.0059	0.0012	0.0038
SUM OF ERRORS	0.0107	0.0117	0.0031	0.0096

Table 6.6 Continued

TEST NO.	4			
MEAS SET	5a			
BAD DATA	Q(3,4) SET EQUAL TO 0			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0470	-0.0495	1.0470	-0.0502
3	1.0239	-0.0880	1.0239	-0.0885
4	1.0228	-0.0938	1.0232	-0.0942
5	1.0168	-0.1088	1.0177	-0.1086

TEST NO.	4			
MEAS SET	5a			
BAD DATA	Q(3,4) SET EQUAL TO 0			
	ERROR IN LEAST SQUARES		ERROR IN LAV	
BUS	V	ANGLE	V	ANGLE
1	-	-	-	-
2	0.0004	0.0005	0.0004	0.0012
3	0.0003	0.0008	0.0003	0.0013
4	0.0008	0.0008	0.0004	0.0012
5	0.0011	0.0015	0.0002	0.0013
SUM OF ERRORS	0.0026	0.0036	0.0013	0.0050

Table 6.6 Continued

TEST NO.	5			
MEAS SET	5b			
BAD DATA	P(2,3), P(3) REVERSED SIGN P(4,5)=0.085, Q(4,5)=0.024			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0516	-0.0373	1.0470	-0.0502
3	1.0413	-0.0386	1.0400	-0.0390
4	1.0378	-0.0520	1.0357	-0.0560
5	1.0212	-0.0945	1.0177	-0.1086

TEST NO.	5			
MEAS SET	5b			
BAD DATA	P(2,3), P(3) REVERSED SIGN P(4,5)=0.085, Q(4,5)=0.024			
	ERROR IN LEAST SQUARES		ERROR IN LAV	
BUS	V	ANGLE	V	ANGLE
1	-	-	-	-
2	0.0042	0.0117	0.0004	0.0012
3	0.0171	0.0486	0.0158	0.0482
4	0.0142	0.0410	0.0121	0.0370
5	0.0033	0.0128	0.0002	0.0013
SUM OF ERRORS	0.0388	0.1141	0.0285	0.0877

Table 6.6 Continued

TEST NO.	6			
MEAS SET	5a			
BAD DATA	P(2,3), P(3) REVERSED SIGN P(4,5)=0.085, Q(4,5)=0.024			
	LEAST SQUARES		LAV	
BUS	V	ANGLE	V	ANGLE
1	1.0600	0.0000	1.0600	0.0000
2	1.0505	-0.0399	1.0470	-0.0502
3	1.0385	-0.0456	1.0255	-0.0833
4	1.0349	-0.0593	1.0245	-0.0903
5	1.0196	-0.0983	1.0180	-0.1069

TEST NO.	6			
MEAS SET	5a			
BAD DATA	P(2,3), P(3) REVERSED SIGN P(4,5)=0.085, Q(4,5)=0.024			
	ERROR IN LEAST SQUARES		ERROR IN LAV	
BUS	V	ANGLE	V	ANGLE
1	-	-	-	-
2	0.0031	0.0091	0.0004	0.0012
3	0.0143	0.0416	0.0013	0.0039
4	0.0113	0.0337	0.0009	0.0027
5	0.0017	0.0090	0.0001	0.0004
SUM OF ERRORS	0.0304	0.0934	0.0027	0.0082

Table 6.6 Continued

6.5.1 Discussion of Test Results for Added Noise

For all the tests presented in table 6.6, noise is added to the measurement set. In test #0 the measurement set is only contaminated by noise, bad data measurements are not present. For tests #1 - #6, the measurement set is contaminated by both bad data and noise. Tests #1 - #6 are the same tests that are given in table 6.1, except that for those tests, noise was not added to the measurement set.

For test #0 the sum of the absolute values of the errors for the least squares estimate is less than the corresponding sum for the least absolute value estimate, and therefore the least squares estimate is slightly more accurate. For tests #1 - #6, the sums of the errors for the LAV estimates, with one exception, are less than the corresponding sums for the least squares estimates.

The results indicate that if only noise contaminates the measurement set the least squares estimate tends to be more accurate. However, when both bad data and noise contaminate the measurement set the LAV estimate is more accurate.

In general the measurement set contains at least one bad data measurement and noise. Under these circumstances the new LAV estimator will produce superior estimates.

6.6 Algorithm Assessment

On the basis of the test results presented in this chapter, several conclusions about the new LAV power system state estimation algorithm can be made.

1) The new algorithm, in most cases, rejects bad data measurements. This is in contrast to least squares state estimators which are always affected by bad data.
2) The algorithm can reliably determine that a line is in service. However if a line is out of service the algorithm produces inconclusive results.
3) The algorithm is suitable for constrained state estimation, except when the number of zero injection buses reduces the redundancy ratio to a low level.
4) In the general case in which the measurement set is contaminated by both bad data and noise, the new LAV algorithm produces estimates that are superior to least squares estimates.
5) The new LAV power system state estimator is an attractive and viable alternative to least squares and linear programming based state estimators.

CHAPTER VII

CONCLUDING REMARKS

7.1 Conclusion

In this thesis a new LAV power system state estimator has been described and evaluated. In chapter II background material describing parameter estimation is presented. The new estimator is based on a simple relationship between least squares and least absolute value estimates. This relationship is described in chapter III. In chapter IV the power system state estimation problem is defined and a brief overview of previous research is given. In chapter V the new power system state estimator is presented and in chapter VI its test results are given. The test results indicate that the new state estimator has very good bad data rejection properties and produces accurate estimates. On the basis of the test results it is concluded that the new estimator is a viable alternative to other power system state estimators that have been developed.

7.2 Suggestions for Further Research

It may be worthwhile to investigate the following areas.

1) Establishing an easier way to choose the interpolated measurements. Before the interpolated measurements can be selected, a least squares estimate must be calculated. It may be possible to select the interpolated measurements without having to calculate the least squares estimate.
2) Developing a tracking state estimator. It may be possible to adapt the algorithm so that it can track the state of a power system. The static state estimator that has been developed must recalculate a state estimate when system changes occur. A tracking state estimator would be able to follow these changes.
3) Improving the technique of determining the correct status of a line, so that the state estimator can reliably detect that a line is out of service.
4) Developing a better observability algorithm. It may be possible to develop an observability algorithm that does not need to process the measurements individually.

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APPENDIX

THE 5 BUS TEST SYSTEM [36]

LINE	RESISTANCE (p.u.)	REACTANCE (p.u.)	LINE CHARGING (p.u.)
1-2	0.020	0.060	0.030
1-3	0.080	0.240	0.025
2-3	0.060	0.180	0.020
2-4	0.060	0.180	0.020
2-5	0.040	0.120	0.015
3-4	0.010	0.030	0.010
4-5	0.080	0.240	0.025

Table A.1 Impedance and Line Charging Data for the 5 Bus Test System (Base = 100 MVA)

BUS NO.	NET GENERATION MW	MVAR
1*	-	-
2	20.0	20.0
3	-45.0	-15.0
4	-40.0	-5.0
5	-60.0	-10.0

* Slack Bus

Table A.2 Operating Conditions for the 5 Bus Test System

THE 14 BUS TEST SYSTEM [37]

LINE	RESISTANCE (p.u.)	REACTANCE (p.u.)	LINE CHARGING (p.u.)
1-2	0.01938	0.05917	0.02640
2-3	0.04699	0.19797	0.02190
2-4	0.05811	0.17632	0.01870
1-5	0.05403	0.22304	0.02460
2-5	0.05695	0.17388	0.01700
3-4	0.06701	0.17103	0.01730
4-5	0.01335	0.04211	0.00640
5-6	0.00000	0.25202	0.00000
4-7	0.00000	0.20912	0.00000
7-8	0.00000	0.17615	0.00000
4-9	0.00000	0.55618	0.00000
7-9	0.00000	0.11001	0.00000
9-10	0.03181	0.08450	0.00000
6-11	0.09498	0.19890	0.00000
6-12	0.12291	0.25581	0.00000
6-13	0.06615	0.13027	0.00000
9-14	0.12711	0.27038	0.00000
10-11	0.08205	0.19207	0.00000
12-13	0.22092	0.19988	0.00000
13-14	0.17093	0.34802	0.00000

Table A.3 Impedance and Line Charging Data for the 14 Bus Test System (Base = 100 MVA)

BUS NO.	NET GENERATION MW	MVAR
1*	-	-
2**	18.3	-
3**	-94.2	-
4	-47.8	3.9
5	-7.6	-1.6
6**	-11.2	-
7	0.0	0.0
8**	0.0	-
9	-29.5	-16.6
10	-9.0	-5.8
11	-3.5	-1.8
12	-6.1	-1.6
13	-13.5	-5.8
14	-14.9	-5.0

* Slack Bus
** Generator Bus

Table A.4 Operating Conditions for the 14 Bus Test System

TRANSFORMER DESIGNATION	TAP SETTING
5-6	0.932
4-7	0.978
4-9	0.969

Table A.5 Transformer Data for the 14 Bus Test System

BUS NUMBER	SUSCEPTANCE (p.u.)
9	0.19

Table A.6 Static Capacitor Data for the 14 Bus Test System

THE 30 BUS TEST SYSTEM [37]

LINE	RESISTANCE (p.u.)	REACTANCE (p.u.)	LINE CHARGING (p.u.)
1-2	0.01920	0.05750	0.02640
1-3	0.04520	0.18520	0.02040
2-4	0.05750	0.17370	0.01840
3-4	0.01320	0.03790	0.00420
2-5	0.04720	0.19830	0.02090
2-6	0.05810	0.17630	0.01870
4-6	0.01190	0.04140	0.00450
5-7	0.04600	0.11600	0.01020
6-7	0.02670	0.08200	0.00850
6-8	0.01200	0.04200	0.00450
6-9	0.00000	0.20800	0.00000
6-10	0.00000	0.55600	0.00000
9-11	0.00000	0.20800	0.00000
9-10	0.00000	0.11000	0.00000
4-12	0.00000	0.25600	0.00000
12-13	0.00000	0.14000	0.00000
12-14	0.12310	0.25590	0.00000
12-15	0.06620	0.13040	0.00000
12-16	0.09450	0.19870	0.00000
14-15	0.22100	0.19970	0.00000
16-17	0.08240	0.19230	0.00000
15-18	0.10700	0.21850	0.00000
18-19	0.06390	0.12920	0.00000
19-20	0.03400	0.06800	0.00000
10-20	0.09360	0.02090	0.00000
10-17	0.03240	0.08450	0.00000
10-21	0.03480	0.07490	0.00000
10-22	0.07270	0.14990	0.00000
21-22	0.01160	0.02360	0.00000
15-23	0.10000	0.20200	0.00000
22-24	0.11500	0.17900	0.00000
23-24	0.13200	0.27000	0.00000
24-25	0.18850	0.32920	0.00000
25-26	0.25440	0.38000	0.00000
25-27	0.10930	0.20870	0.00000
28-27	0.00000	0.39600	0.00000
27-29	0.21980	0.41530	0.00000
27-30	0.32020	0.60270	0.00000
29-30	0.23990	0.45330	0.00000
8-28	0.06360	0.20000	0.02140
6-28	0.01690	0.05990	0.00650

Table A.7 Impedance and Line Charging Data for the 30 Bus Test System (Base = 100 MVA)

BUS NO.	NET GENERATION MW	MVAR
1*	-	-
2**	18.3	-
3	-2.4	-1.2
4	-7.6	-1.6
5**	-94.2	-
6	0.0	0.0
7	-22.8	-10.9
8**	-30.0	-
9	0.0	0.0
10	-5.8	-2.0
11**	0.0	-
12	-11.2	-7.5
13	70.45	-6.2
14	-6.2	-1.6
15	-8.2	-2.5
16	-3.5	-1.8
17	-9.0	-5.8
18	-3.2	-0.9
19	-9.5	-3.4
20	-2.2	-0.7
21	-17.5	-11.2
22	0.0	0.0
23	-3.2	-1.6
24	-8.7	-6.7
25	0.0	0.0
26	-3.5	-2.3
27	0.0	0.0
28	0.0	0.0
29	-2.4	-0.9
30	-10.6	-1.9

* Slack Bus
** Generator Bus

Table A.8 Operating Conditions for the 30 Bus Test System

TRANSFORMER DESIGNATION	TAP SETTING
6-9	0.978
6-10	0.969
4-12	0.932
28-27	0.968

Table A.9 Transformer Data for the 30 Bus Test System

BUS NUMBER	SUSCEPTANCE (p.u.)
10	0.190
24	0.043

Table A.10 Static Capacitor Data for the 30 Bus Test System