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BAYESIAN ANALYSIS OF IN-SITU TEST DATA FOR ESTIMATING THE COMPRESSIVE STRENGTH OF CONCRETE IN EXISTING STRUCTURES

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ABSTRACT

A simple analytical procedure, based on Bayesian statistical theory, for use with in-situ test data for predicting the compressive strength of concrete in existing structures is presented.

The underlying theory of Bayesian statistics is introduced and the fundamental theorem is derived. The application of Bayes' theorem to the problem of updating or improving prior statistical information is described and highlighted in examples. Closed form relationships are derived that permit the combination of any amount of core strength data and nondestructive test (NDT) data for predicting the mean in-situ strength of concrete in a homogeneous element. A parametric study of the relationships using data from an extensive investigation of a concrete highway bridge reveals the relative significance of the major variables in the analysis.

Two theoretical approaches for optimizing in-situ test programs are also presented.

ACKNOWLEDGEMENTS

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A_C = start up cost for coring

A_N = start up cost for NDTs

A_{NR} = cost of not renovating if the in-situ concrete strength equals f_{spec}

A_R = fixed cost of renovating

B_C = coefficient relating sample size to coring costs

B_N = coefficient relating sample size to NDT costs

C_C = total cost of core specimen data

C_N = total cost of NDT data

C_{NR} = cost of not renovating

C_R = cost of renovating

C_T = total cost of all test data

$E(x|v)$ = expected value of concrete strength given a pulse velocity measurement

EL_{NR} = expected opportunity loss if the decision is to not renovate

EL_R = expected opportunity loss if the decision is to renovate

$f(x)$ = probability density function of variable x

$f(x|y)$ = probability density function of variable x given that y has occurred

f_b = breakeven strength

f_{reqd} = minimum required mean concrete strength for a new loading condition on an existing structure

f_{spec} = specified concrete strength at time of construction

$L_N(D)$ = unit normal loss integral

L_{NR} = opportunity loss when decision is to not renovate

L_R = opportunity loss when decision is to renovate

m = NDT data sample size

n = core data sample size

P(A) = probability of event A occurring

P(A|B) = probability of event A occurring given that event B has occurred

P(A∩B) = probabilty of both event A and event B occurring simultaneously

$(s_v)^2$ = variance of pulse velocity data used to develop regression relationship

v_i = pulse velocity measurement for ith core

$\bar{v}$ = mean value of pulse velocity measurements used in forming regression relationship

x_i = compressive strength of ith core

$\bar{x}$ = mean strength of core data

α,β = constants defining shape of regression relationship between concrete strength and pulse velocity

$(\sigma_{Ei})^2$ = variance of the expected value of the mean concrete strength calculated using the pulse velocity v_i

$(\sigma_o)^2$ = variance of the concrete strength for the sampled population

$(\sigma_{po})^2$ = variance of the posterior distribution of the mean strength

$(\sigma_{pr})^2$ = variance of the prior distribution of the mean strength

μ_{po} = mean or expected value of the posterior distribution of the mean strength

μ_{pr} = mean or expected value of the prior distribution of the mean strength

CHAPTER 1

1.1 General

Estimation of the in-situ compressive strength of concrete in an existing structure can normally occur only after a field investigation is performed. The field investigation provides an opportunity to measure directly or indirectly the strength of the concrete at discrete locations. In order to ensure that a sufficiently accurate assessment of the strength can be made from the in-situ measurements, careful consideration must be given to the amount and type of data obtained and the locations from which it is obtained. Another major factor that influences the accuracy of the strength assessment is the method of data analysis.

Direct measurements of compressive strength can be obtained from tests on cores taken from the structure. Estimates of strength can also be obtained from nondestructive tests such as pulse velocity and rebound number (Malhotra, 1984). Even though NDTs, like pulse velocity and rebound hammer readings, are normally less expensive than concrete core strength measurements (Samarin and Dhir 1984) it is always necessary to obtain some concrete core data so that a regression relationship between the NDT measurements and concrete strength can be developed. In addition to economic factors, core removal is usually limited for other reasons including the desire to minimize damage to the structure for both structural and architectural reasons and the difficulty in obtaining specimens from certain locations.

When the strength of more than just one element of a structure is being investigated it is important to analyse the data from each element separately. Even if the concrete specifications were uniform throughout the structure, curing rates and other factors unique to different elements (eg. columns and slabs can have tremendously different volume-to-surface-area ratios) could cause significant variations in strength between different elements. In some cases it is not clear whether or not data from different parts of a structure should be combined. Fortunately there are methods available to address this problem (see for example Di Leo, Pascale, and Viola 1984).

As with many engineering materials, concrete strength is variable, even within one localized part of a structure. Therefore it is necessary to use statistical procedures to arrive at a rational estimate of the in-situ strength. Even though the so-called "classical" relationships for analyzing data are used most often, such relationships give reliable results only when large quantities of data are available (Ang and Tang 1975). When a strength prediction must be made from a small amount of data, which is often the case, it is better to use relationships based on Bayesian statistics (Tang 1971, Rao and Corotis 1982, Viola 1983, Bazant and Chern 1984).

1.2 Objectives and Scope

The objectives of this study are

1. To present the underlying theory and the fundamental theorem of the Bayesian statistical approach and to show how the theorem can be applied to update existing statistical distributions.

2. To develop an analytical procedure for combining various types and quantities of in-situ data for predicting the compressive strength of concrete in existing structures.

3. To study the characteristics of the analytical procedure by evaluating in-situ concrete data obtained during an extensive structural investigation.

4. To present strategies and procedures for optimizing testing programs used to obtain data for concrete strength prediction.

CHAPTER 2

An Introduction to Bayesian Statistical Theory

2.1 Introduction

In the problem solving process engineers often make use of statistical procedures to evaluate measured data. When only a limited quantity of measured data is available the so-called "classical" statistical relationships, which are most commonly used, may not provide good solutions because they should be used only when relatively large quantities of data are available (Ang and Tang 1975). When the quantity of data is limited Bayesian statistics provides a good alternative to the classical approach. Bayesian relationships can be developed that systematically combine large or small amounts of new data with previous information; previous information can be either subjective or objective in nature. Because the procedure facilitates the combination of new data with previous information it is also an ideal method to use when data become available intermittently and frequent updating of statistical characteristics of a random system is necessary.

This chapter gives the development of Bayes' theorem which is the basis for all Bayesian relationships. A general presentation is given on how to use the theorem to revise and update statistical distributions. The presentation given in this chapter is based upon information from several references (Schmitt 1969, Winkler 1972, Ang and Tang 1975, and Guttman, Wilks and Hunter 1982).

2.2 Bayes Theorem

2.2.1 For Discrete Systems

A discrete system is one from which only a finite number of different events or outcomes can possibly occur. Each of the events in the system has associated with it a probability of occurrence and the entire set of possible events has an associated discrete probability distribution known as a probability mass function (PMF). The probabilities of all discrete events in any one system must sum to equal one. An example of a discrete system is the presence of reinforcing steel in cores taken from a reinforced concrete wall. This is a discrete system because only two different outcomes are possible when taking cores; either steel is or is not present in a given core. A hypothetical PMF for this system is shown in Figure 2.1.

Bayes' theorem is basically a conditional probability relationship. Conditional probability is defined as the probability that one event will occur given that a particular second event has occurred. The conditional probability of an event A occurring given that an event B has also occurred is denoted as

$$P(A|B)$$

A relationship for calculating conditional probability for discrete systems can be developed easily with the aid of a Venn diagram such as the one shown in Figure 2.2. The large

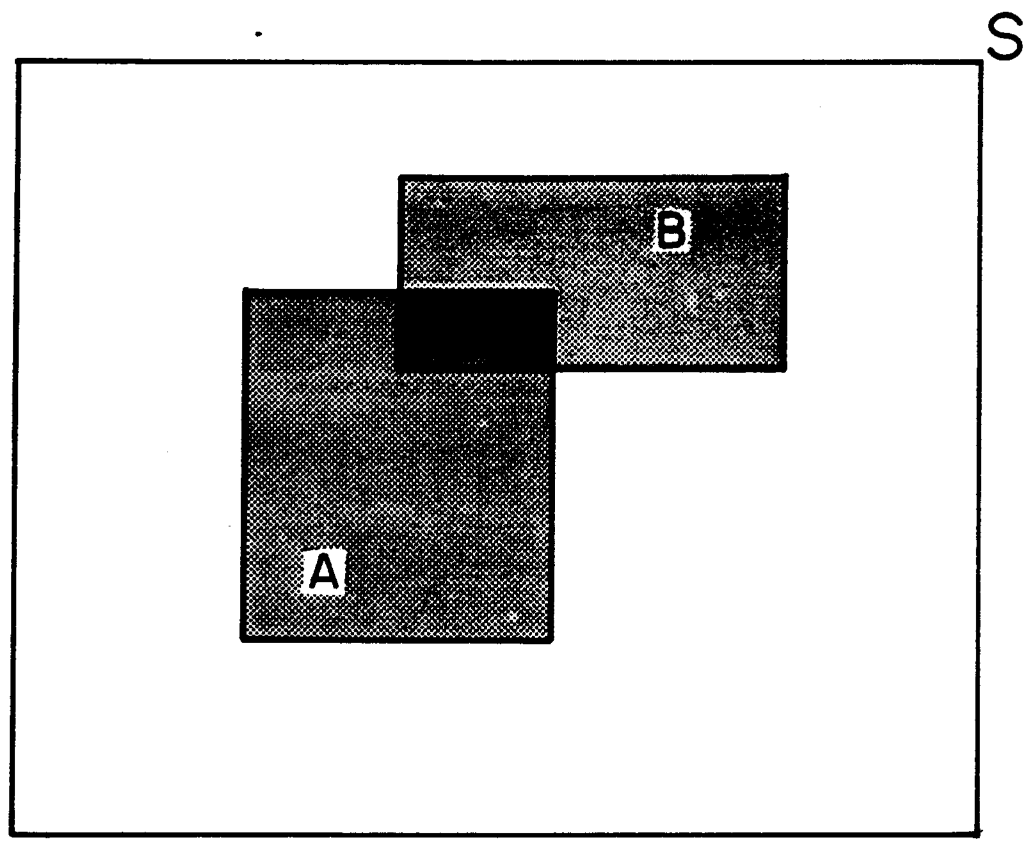

Fig. 2.2 Venn Diagram Representing the Probability of Two Events

rectangular box shown in this figure, labelled S, represents the entire discrete system or sample space from which possible events can be chosen. The squares, A and B, each represent a possible event in the total sample space. The probabilities of either of these events occurring independently is represented by the normalized area of the square representing the event being considered.

As shown in the Venn diagram the events A and B overlap. This indicates that both events can occur simultaneously. The normalized area of the overlap represents the probability of such a joint event. This condition is referred to as intersection and the intersection of events A and B is denoted as

$$A \cap B$$

The probability of intersecting events A and B is denoted as

$$P(A \cap B)$$

For the concrete core example intersecting events could occur if event A represented horizontal steel present in a core and event B represented vertical steel present in a core. Of course it is possible that both horizontal and vertical steel could be present in one core, and this is implied in the Venn diagram.

From the Venn diagram it can be seen that the conditional probability of event A happening given that event B has also happened is equal to the probability of the intersection of events A and B divided by the probability of event B occurring. The resulting relationship is

$$[2.1] \qquad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Similarly,

$$[2.2] \qquad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

In many discrete conditional probability systems there will be more than one possible type A and type B events, as depicted in the Venn diagram of Figure 2.3. The conditional probability of the k^{th} A event given that the j^{th} B event has also happened follows directly from [2.1],

$$[2.3] \qquad P(A_k|B_j) = \frac{P(A_k \cap B_j)}{P(B_j)}$$

Similarly,

$$[2.4] \qquad P(B_j|A_k) = \frac{P(A_k \cap B_j)}{P(A_k)}$$

In order to transform the conditional probability relationships developed thus far into the Bayesian form, the concept of total probability must be introduced. Referring again to the Venn diagram of Figure 2.3 it can be seen that the probability of event B_1 happening can be calculated by summing the probabilities of the intersection of event B_1 with several A

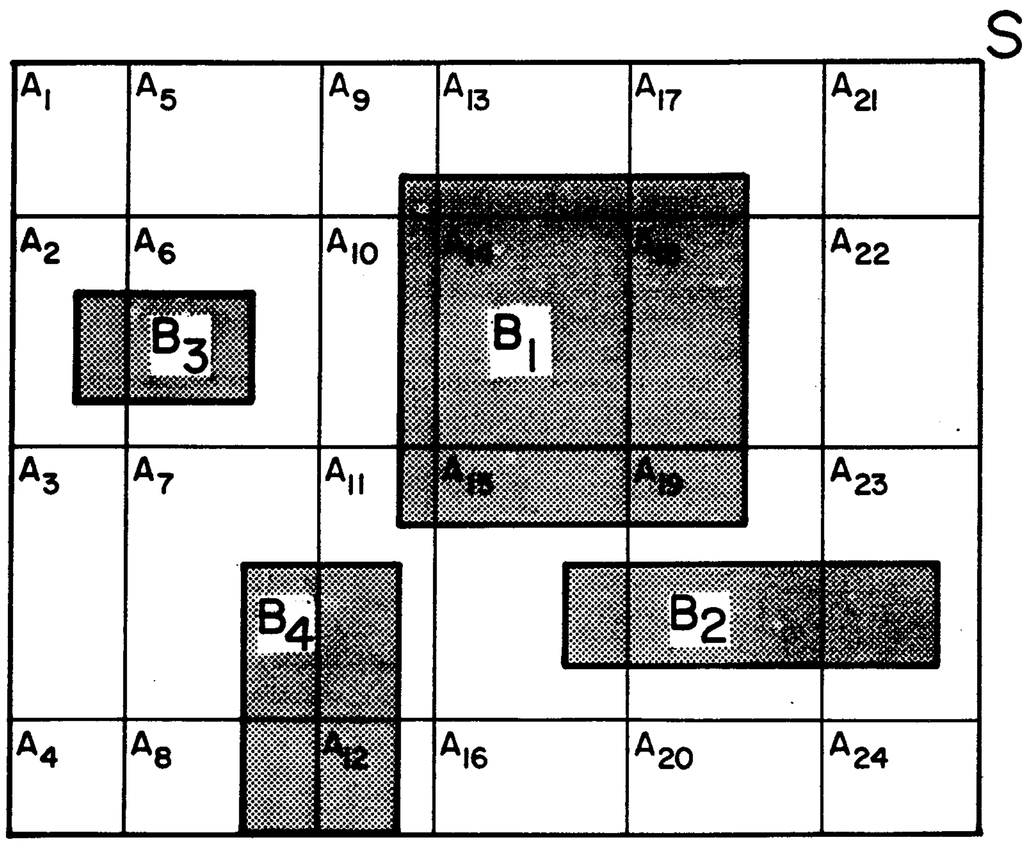

Fig. 2.3 Venn Diagram Representing the Probability of Multiple Events

events,

$$P(B_1) = P(A_9 \cap B_1) + P(A_{10} \cap B_1) + P(A_{11} \cap B_1) + P(A_{13} \cap B_1) + P(A_{14} \cap B_1) + P(A_{15} \cap B_1) + P(A_{17} \cap B_1) + P(A_{18} \cap B_1) + P(A_{19} \cap B_1)$$

Since no other A events intersect with event B_1 it is not necessary to consider them when calculating $P(B_1)$. However, a generalized expresssion for $P(B_1)$ results if all A events are considered,

$$[2.5] \qquad P(B_j) = \sum_{i=1}^{n} P(A_i \cap B_j)$$

where n = the total number of A events. Rearranging [2.4] and substituting it into [2.5] results in the total probability theorem,

$$[2.6] \qquad P(B_j) = \sum_{i=1}^{n} P(A_i) \cdot P(B_j | A_i)$$

Substituting [2.6] into [2.3] gives

$$[2.7] \qquad P(A_k | B_j) = \frac{P(A_k \cap B_j)}{\sum_{i=1}^{n} P(A_i) \cdot P(B_j | A_i)}$$

Finally, rearranging and then substituting [2.4] into [2.7] gives the conditional probability relationship for discrete systems which is commonly known as Bayes' theorem,

$$[2.8] \qquad P(A_k|B_j) = \frac{P(A_k) \cdot P(B_j|A_k)}{\sum_{i=1}^{n} P(A_i) \cdot P(B_j|A_i)}$$

where $1 \leqslant k \leqslant n$

$1 \leqslant j \leqslant m$

m = the total number of B events.

2.2.2 For Continuous Systems

A continuous system is one from which essentially an infinite number of different events or outcomes can be chosen. As a consequence of this any single event has associated with it a zero probability of occurrence; a probability of occurrence is associated with any finite sequence or range of the possible continuous events, however. Probabilty distributions used to describe the probability of occurrence of events from continuous systems are known as probability density functions (pdf). The major requirement of such functions is that the area they enclose sums to equal one. An example of a continuous system is the compressive strength of concrete in a structure. This is a continuous system because a very large number of different strengths are possible when measuring the strength throughout the structure. A hypothetical pdf for this system is shown in Figure 2.4.

Bayes' theorem for continuous systems can be developed from

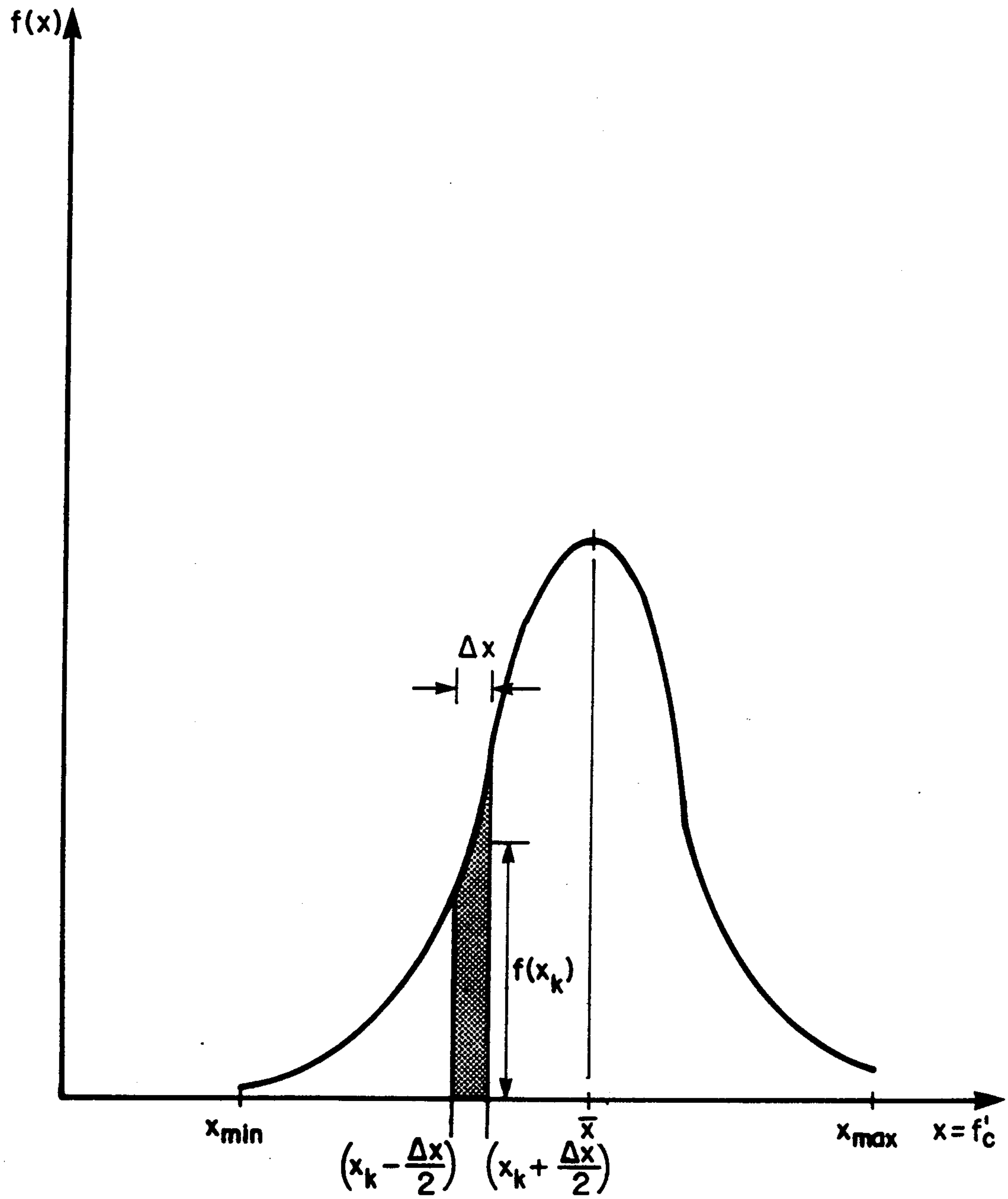

Fig. 2.4 Hypothetical Probability Density Function Showing the Distribution of Concrete Compressive Strength in a Structure

[2.8] (i.e. from Bayes' theorem for discrete systems). First, however, it is necessary to identify the expressions required for computing probabilities in continuous systems. As already indicated the probability of a single event happening in a continuous system is equal to zero. For a very small sequence of events centered on a point the probability is given as

$$[2.9] \qquad P(x_k) = P\left(x_k - \frac{\Delta x}{2} < x \leq x_k + \frac{\Delta x}{2}\right)$$

$$\approx f(x_k) \cdot \Delta x$$

where

$f(x_k)$ = the value of the pdf at x_k;

Δx = a very small range of possible events.

From Figure 2.4 it is seen that $P(x_k)$ as given by [2.9] is approximately equal to the area under the curve describing the pdf in the immediate vicinity of $x = x_k$.

Conditional probability in a continuous system is computed in a similar manner,

$$[2.10] \qquad P(x_k | y_j) = P\left(x_k - \frac{\Delta x}{2} < x \leq x_k + \frac{\Delta x}{2} \;\middle|\; y = y_j\right)$$

$$\approx f(x_k | y_j) \cdot \Delta x$$

An example of conditional probability in a continuous system is predicting in-situ compressive strength of concrete in a certain location of a structure given a pulse velocity measurement from that location. Figure 2.5 shows a series of hypothetical pdfs from which probabilities could be determined for this example.

Substituting [2.9] and [2.10] into [2.8] gives

$$[2.11] \qquad f(x_k|y_j)\Delta x = \frac{f(x_k)\Delta x \cdot f(y_j|x_k)\Delta x}{\sum_{i=1}^{n} f(x_i)\Delta x \cdot f(y_j|x_k)\Delta x}$$

In the limit as Δx approaches zero [2.11] becomes Bayes' theorem for continuous systems,

$$[2.12] \qquad f(x_k|y_j) = \frac{f(x_k) \cdot f(y_j|x_k)}{\int_{-\infty}^{\infty} f(x) \cdot f(y_j|x)\, dx}$$

It is possible to adjust [2.12] for cases where more than one experimental or field observation is available. From elementary probability theory

$$[2.13] \qquad f(a|b_1, b_2 \ldots b_m) = f(a|b_1) \cdot f(a|b_2) \;, \;\ldots f(a|b_m)$$

$$= \prod_{i=1}^{m} f(a|b_m)$$

Applying [2.13] to [2.12] results in a more general form of Bayes' theorem for continuous systems,

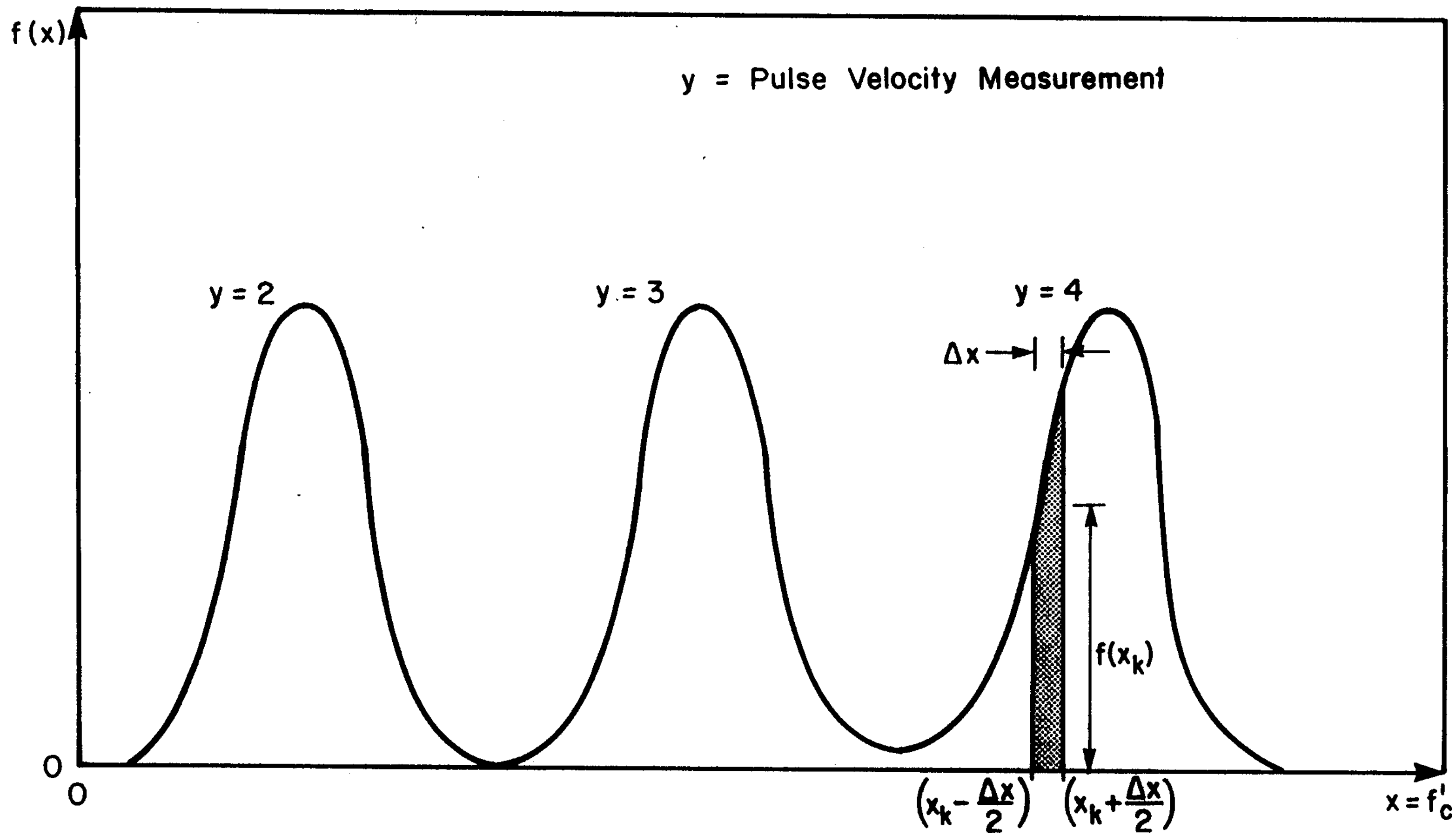

Fig. 2.5 Hypothetical Series of pdfs for Determining Conditional Probability of Concrete Compressive Strength Given a Pulse Velocity Measurement

$$[2.14] \qquad f(x_k | y_1, \ldots y_m) = \frac{f(x_k) \cdot \prod_{i=1}^{m} f(y_i | x_k)}{\int_{-\infty}^{\infty} f(x) \cdot \prod_{i=1}^{m} f(y_i | x)\, dx}$$

The major difference between Bayes' theorem for discrete systems and Bayes' theorem for continuous systems is that the former predicts new probabilities directly whereas the latter predicts ordinates to a new probability density function. To predict probabilities for the continuous case subsequent integration must be done of the area under the new pdf over the range of interest.

2.3 Bayesian Inference - Revising Statistical Distributions

2.3.1 For Discrete Systems

As seen in Section 2.2 Bayes' theorem is a special form of conditional probability. However, in this form the concept of conditional probability can be easily used to update the probability distributions of random variables. Bayes' theorem provides for the systematic combination of an existing probability distribution with new data resulting in an updated PMF based on all available information.

In [2.8] the set of A events represents all of the different events that are possible outcomes in the discrete, random system of interest. The probabilities associated with each A event constitute the PMF for this system and are together commonly referred to as the prior distribution. The probability of the

k^{th} A event before any new data are available is given in [2.8] as $P(A_k)$. Similarly all n A events have an associated probability of occurrence and all of these probabilities are required when using [2.8], as seen in the denominator.

Sometimes the prior PMF is based on good statistical or other information; this, however, is not a prerequisite to using Bayes' theorem for updating purposes. In cases where no prior information is available for defining the PMF, it is then acceptable to assign each possible event an equal probability of occurrence. As for all PMFs these probabilities must sum to equal one, and therefore the probability of each event would be equal to the inverse of the total number of events. A prior distribution of this type is known as a diffuse prior.

In order to update a prior PMF it is of course necessary to first obtain experimental, field, or some other type of data to combine with the prior statistical distribution. In [2.8] event B_j represents the set of new data and $P(B_j|A_k)$ represents the probability, observed from the test data, of event A_k happening. Similarly all n A events have an associated conditional probability based only on the new data. These conditional probabilities are known as likelihoods. Although the likelihoods seem very much like the conditional probability of an A event occurring given that event B_j has occurred, which is denoted as $P(A_k|B_j)$ for the k^{th} A event in [2.8], it is important to note the difference. $P(A_k|B_j)$ is a conditional probability given both prior information and new data. $P(B_j|A_k)$ is a conditional probability given only the new data. When using

Bayes' theorem for the current application it is essential not to confuse these two different types of probabilities. As indicated by Belz (1973) and by Guttman, Wilks and Hunter (1982), the term $P(B_j|A_k)$ of Bayes' theorem for the current application is sometimes denoted $L(A_k|B_j)$ in order to clearly symbolize the meaning to be the likelihood of event A_k given the data from event B_j.

Combining the previous or prior distribution of a random discrete variable with observed data through Bayes' theorem as given by [2.8] results in an updated distribution based upon both the existing information and the new data. The resulting PMF is called the posterior distribution. In [2.8] the posterior probability associated with the k^{th} event in the discrete system is given as $P(A_k|B_j)$. An example of updating a discrete statistical system using Bayes' theorem is given in Appendix A.

2.3.2 For Continuous Systems

Bayes' theorem for continuous random variables can be used to update pdfs just as the discrete form of the theorem can be used to update PMFs. The probability distribution of the random variable, which is known as the prior, is represented in general by f(x) in [2.12]. One ordinate of the prior pdf is given by $f(x_k)$. As in the discrete case if no prior distribution is available it is acceptable to use a diffuse prior pdf. This diffuse prior must assign a constant probability density to the entire range of possible events.

From experimental, field or other observations a new pdf for the range of events being considered can be developed. This new pdf represents the likelihood density of attaining the range of events based only on the new data. In [2.12] the likelihood function is given by $f(y_j|x_k)$ and in [2.14] by $\prod_{i=1}^{m} f(y_i|x_k)$. As described for the discrete case it is important not to confuse the likelihood function with the posterior function.

Combining the prior pdf with observed data through [2.12] or [2.14] results in a posterior pdf represented by $f(x_k|y_j)$ or $f(x_k|y_1, \ldots y_m)$ respectively. Based on all available information the posterior pdf gives the best statistical description of the continuous system. An example of updating a continuous system using Bayes' theorem is given in Appendix B.

CHAPTER 3

The Development of Bayesian Relationships for Estimating Mean Concrete Compressive Strength

3.1 Introduction

In order to predict the in-situ compressive strength of concrete in an existing structure it is normally necessary to first remove several core specimens and to take some non-destructive test (NDT) measurements such as pulse velocities and rebound hammer readings. Because the data are random, a statistical method must be employed to estimate the strength of the concrete based on the data. The analysis of the data must conside the various errors associated with transforming NDT measurements into equivalent compressive strength values. In addition, proper weighting must be given to core data and NDT data when different amounts of each of these types of in-situ measurements are used. Furthermore, when predicting concrete strength based on small amounts of data it is sometimes beneficial to include subjective information in the analysis and this must be done rationally. Data analysis of problems which include any or all of these characteristics can be done using Bayesian statistics.

In this chapter simple Bayesian relationships are developed that can be used to consistently and systematically combine direct core and indirect NDT data with other pertinent subjective or statistical information to estimate in-situ concrete strength.

3.2 Estimation of Mean Strength when Only Concrete Core Compressive Strength Data are Available

For any homogeneous section of a reinforced concrete structure it is common and reasonable (Mirza, Hatzinikolas, and MacGregor, 1979) to assume a normal or Gaussian distribution for the compressive strength of the concrete. Therefore the probability density function for the in-situ strength of the concrete in a homogeneous section can be expressed as

$$[3.1] \qquad f(x) = \left(2\pi\sigma^2\right)^{-1/2} \cdot \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

where x = random variable, in this example concrete strength;
σ^2 = variance of the random variable;
μ = mean or expected value of the random variable.

Following the presentations of Breipohl (1970) and Guttman, Wilks, and Hunter (1982), in order to use Bayes' theorem for predicting the mean value, μ, based on some core strength data and any other pertinent information it is first necessary to assume a value for the variance of the strength, say σ_o^2. As reported by Mirza, Hatzinikolas and MacGregor (1979) the coefficient of variation for the compressive strength of concrete in homogeneous elements is usually observed to be 10%, 15%, and 20% for concrete placed with excellent, good, and poor quality control respectively. (For concrete strengths greater than about 30 MPa the coefficient of variation tends to be less than these values.) Tso and Zelman (1970) describe one method that can be used to estimate the quality control attained during the construction of a concrete structure. By referring to construction documents, using the method of Tso and Zelman, or by applying engineering judgement, it is possible to determine a reasonable value for σ_o^2.

Secondly, it is necessary to consider the mean or expected value of the concrete compressive strength to itself be random, and distributed normally with a pdf of the form given by [3.1]. Bayes' theorem is used then to improve or update the pdf of the mean strength, not of the strength. Although Bayesian statistics could be used to predict both μ and σ_o (see Bazant and Chern 1984 or Guttman, Wilks and Hunter 1982) the procedure is considerably more difficult and is not presented here.

To proceed, a prior distribution for the mean strength is required. Based on engineering judgement the mean and variance of the mean strength must be assumed. These assumptions might be based on construction documents or any other information that is both pertinent and reasonable (see Jones 1977). If the statistical characteristics of the mean strength are very difficult to predict it is prudent to assign a relatively large value to the variance and a relatively small value to the mean. In extreme cases a diffuse prior may be used; however, when a diffuse prior is used with continuous Gaussian pdfs the resulting posterior pdf will be identical to the classical prediction (Ang and Tang, 1975) and no advantage will result from using the Bayesian approach. In all cases the prior pdf should give the best prediction of the mean concrete strength before any new tests have been taken. In general the prior pdf will be given as,

$$[3.2] \qquad f(\mu) = \left(2\pi\sigma_{pr}^{2}\right)^{-1/2} \cdot \exp\left\{-\frac{1}{2}\left(\frac{\mu - \mu_{pr}}{\sigma_{pr}}\right)^{2}\right\}$$

where μ_{pr} = assumed value for the prior mean of the mean compressive strength of the concrete;

σ_{pr}^2 = assumed value for the prior variance of the mean compressive strength of the concrete;

μ = random variable, i.e. the mean compressive strength of the concrete.

In order to combine core compressive strength data with the prior estimation of the mean strength of the concrete it is first necessary to define a likelihood function using the core data. This likelihood function must give the conditional probability density of obtaining the test data assuming that the mean value of the mean concrete strength distribution is μ. Because all concrete cores come from the same population the variance σ_o^2 applies for all data and the likelihood or probability density of the strength of the ith core is given as,

$$[3.3] \qquad f(x_i|\mu) = \left(2\pi\sigma_o^{2}\right)^{-1/2} \cdot \exp\left\{-\frac{1}{2}\left(\frac{x_i - \mu}{\sigma_o}\right)^{2}\right\}$$

where x_i = compressive strength of the ith core.

For the general case where strength data from n different cores are available the likelihood pdf is given as

$$[3.4] \qquad \prod_{i=1}^{n} f(x_i|\mu) = \{(2\pi\sigma_o^2)^{-1/2}\}^n \cdot \exp\{-[\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma_o^2}]\}$$

Combining [3.4] and [3.2] through Bayes' theorem ([2.14]) gives the posterior pdf, that is the pdf which predicts the most likely statistical distribution of the mean concrete strength, as,

$$[3.5] \qquad f(\mu|x) = \frac{\{(2\pi\sigma_o^2)^{-1/2}\}^n \cdot \exp\{-[\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma_o^2}]\} \cdot (2\pi\sigma_{pr}^2)^{-1/2} \cdot \exp\{-\frac{1}{2}(\frac{\mu-\mu_{pr}}{\sigma_{pr}})^2}{\int_{-\infty}^{\infty} \{(2\pi\sigma_o^2)^{-1/2}\}^n \cdot \exp\{-[\sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma_o^2}]\} \cdot (2\pi\sigma_{pr}^2)^{-1/2} \cdot \exp\{-\frac{1}{2}(\frac{\mu-\mu_{pr}}{\sigma_{pr}})^2\} \, d\mu}$$

To use [3.5] to determine the updated pdf is obviously very complicated, and fortunately unnecessary. By combining constants and rearranging variables it is possible to simplify [3.5] so that simple closed form relationships completely describing the posterior pdf can be obtained.

First expanding the numerator and denominator of [3.5] gives,

$$[3.6] \qquad f(\mu|x) = \frac{(2\pi\sigma_o^2)^{-n/2} \cdot (2\pi\sigma_{pr}^2)^{-1/2} \cdot \exp(Z_1)}{(2\pi\sigma_o^2)^{-n/2} \cdot (2\pi\sigma_{pr}^2)^{-1/2} \cdot \int_{-\infty}^{\infty} \exp(Z_1) \, d\mu}$$

where $Z_1 = \{-\frac{1}{2\sigma_o^2}[\sum_{i=1}^{n} x_i^2 - 2\mu\sum_{i=1}^{n} x_i + n\mu^2] - \frac{1}{2\sigma_{pr}^2}[\mu^2 - 2\mu\mu_{pr} + \mu_{pr}^2]\}$

Altering Z_1 gives,

$$Z_1 = \{-\frac{1}{2\sigma_o^2}[\sum_{i=1}^{n} x_i^2 - 2n\mu\bar{x} + n\mu^2] - \frac{1}{2\sigma_{pr}^2}[\mu^2 - 2\mu\mu_{pr} + \mu_{pr}^2]\}$$

[3.7] $$Z_1 = \{[-\frac{1}{2\sigma_o^2}\sum_{i=1}^{n} x_i^2 - \frac{\mu_{pr}^2}{2\sigma_{pr}^2}] + [\frac{2n\mu\bar{x}}{2\sigma_o^2} - \frac{n\mu^2}{2\sigma_o^2} - \frac{\mu^2}{2\sigma_{pr}^2} + \frac{2\mu\mu_{pr}}{2\sigma_{pr}^2}]\}$$

where $\bar{x}$ = sample mean of the core strength data.

Substituting [3.7] into [3.6] and simplifying gives,

[3.8] $$f(\mu|x) = \frac{\exp(Z_2)}{\int_{-\infty}^{\infty} \exp(Z_2)\, d\mu}$$

where $$Z_2 = \{\frac{2n\mu\bar{x} - n\mu^2}{2\sigma_o^2} + \frac{2\mu\mu_{pr} - \mu^2}{2\sigma_{pr}^2}\}$$

Altering Z_2 gives,

$$Z_2 = -\frac{1}{2}[\mu^2 X - 2\mu Y]$$

$$= -\frac{1}{2} X [\frac{\mu^2 X}{X} - \frac{2\mu Y}{X}]$$

$$= -\frac{1}{2} X [\frac{\mu^2 X}{X} - \frac{2\mu Y}{X} + \frac{Y^2}{X^2} - \frac{Y^2}{X^2}]$$

[3.9] $$Z_2 = -\frac{1}{2} X [(\mu - \frac{Y}{X})^2 - \frac{Y^2}{X^2}]$$

where $X = \left(\frac{n}{\sigma_o^2} + \frac{1}{\sigma_{pr}^2}\right)$

$Y = \left(\frac{n\bar{x}}{\sigma_o^2} + \frac{\mu_{pr}}{\sigma_{pr}^2}\right)$

Substituting [3.9] into [3.8] and simplifying gives,

$$[3.10] \qquad f(\mu|x) = \frac{\exp(Z_3)}{\int_{-\infty}^{\infty} \exp(Z_3)\, d\mu}$$

where $Z_3 = -\frac{1}{2} X \left[\left(\mu - \frac{Y}{X}\right)^2\right]$

In general (Breipohl, 1970)

$$[3.11] \qquad \int_{-\infty}^{\infty} \exp\left\{-\frac{1}{2v^2}(\lambda - b)^2\right\} d\lambda = \sqrt{2\pi} \cdot v$$

Using [3.11] to evaluate the denominator of [3.10] gives

$$[3.12] \qquad f(\mu|x) = \left(\frac{X}{2\pi}\right)^{1/2} \cdot \exp\left\{-\frac{1}{2} X \left[\left(\mu - \frac{Y}{X}\right)^2\right]\right\}$$

Comparing [3.12] with [3.1] it is clear that [3.12], the posterior distribution, is Gaussian and has an expected mean value given by

$$\mu_{po} = \frac{Y}{X}$$

$$= \frac{\left(\frac{n\bar{x}}{\sigma_o^2} + \frac{\mu_{pr}}{\sigma_{pr}^2}\right)}{\left(\frac{n}{\sigma_o^2} + \frac{1}{\sigma_{pr}^2}\right)}$$

[3.13] $$\mu_{po} = \frac{\frac{\sigma_o^2}{n} \mu_{pr} + \sigma_{pr}^2 \bar{x}}{\frac{\sigma_o^2}{n} + \sigma_{pr}^2}$$

and a variance given by

[3.14] $$\sigma_{po}^2 = \frac{1}{\left(\frac{n}{\sigma_o^2} + \frac{1}{\sigma_{pr}^2}\right)}$$

$$\sigma_{po}^2 = \frac{\sigma_o^2 \sigma_{pr}^2}{n\sigma_{pr}^2 + \sigma_o^2}$$

[3.13] and [3.14] are the updated mean and variance of the mean compressive strength of the concrete. Substitution of these values into [3.1] gives the posterior pdf.

3.3 Estimation of Mean Strength when Ultrasonic Pulse Velocity Measurements and Concrete Core Strength Data are Available

The procedure for combining indirect test data, like pulse velocity measurements, with direct test data, like core compressive strengths, in order to update a prior pdf is described by Tang (1971). Tang suggests that first the indirect test data should be combined, using Bayes' theorem, with the prior pdf. The resulting pdf is then used as a prior distribution for combination with a likelihood function derived from the direct data. The second step of this two step procedure is very similar to the method described in Section 3.2. The first step, although in general similar to the method given in Section 3.2 contains some new considerations and will be described in detail in this section.

As in the case where only core strength data are available it is first necessary to assume a value for the variance of the concrete strength, σ_o^2. As discussed in Section 3.2 this value is normally based on the quality of the concrete and can be reasonably selected using engineering judgement.

Before proceeding it is also necessary to combine all previous information and/or beliefs about the strength of the concrete being studied. This prior information must be presented in the form of a pdf like [3.2].

In order to update any statistical distribution through Bayes' theorem it is necessary to have the new data in a form identical to the prior distribution. Therefore raw pulse velocity data cannot be used directly to update a prior compressive strength distribution. The pulse velocity data must first be converted into equivalent compressive strength data. This is normally done by developing a regression relationship (see Ang and Tang 1975, for example) between the pulse velocity through the concrete and the concrete compressive strength, calibrated to the homogeneous section being studied. In general, where the regression relationship is a power function, an estimate of the mean strength of the concrete given a pulse velocity measurement can be determined using

[3.15] $E(x|v) = \alpha\ v^{\beta}$

where $E(x|v)$ = expected value of x given v;

v = pulse velocity;

α, β = constants describing shape of regression relationship.

Because regression relationships are based on random data they normally provide only a "best fit" relationship between two variables. The error in prediction is usually described by the calibration error of the regression curve, which is computed to be the conditional standard deviation or the conditional variance of the dependent variable given the independent variable. Although it is sometimes important to consider the variation in the calibration error with variation in the independent variable, for the purposes of this study a constant calibration error will be assumed using the following equation (Ang and Tang 1975):

[3.16] $$(\sigma_{x|v})^2 = \frac{1}{m-2} \sum_{i=1}^{m} (x_i - E(x_i|v_i))^2$$

where $(\sigma_{x|v})^2$ = calibration error or conditional variance of the regression relationship;

m = total number of test data used in developing the regression relationship;

x_i = compressive strength of the ith core used in calibrating the regression relationship;

$E(x_i|v_i)$ = compressive strength of the ith core calculated by substituting the pulse velocity through the ith core into the regresssion relationship.

Figure 3.1 shows a regression relationship and the associated calibration error.

In addition to the conditional variance of the regression relationship resulting from the randomness of the calibration data, the pulse velocities of the calibration specimens have associated statistical properties. Assuming these pulse velocities are normally distributed, as are the compressive strengths, it is possible to describe the set of data by its sample mean, $\bar{v}$, and its sample variance $(s_v)^2$. These statistical characteristics of the pulse velocity portion of the calibration data are shown in Figure 3.1.

One further statistical property required before developing the likelihood function is the variance of the expected value of concrete strength as given by [3.15]. It is assumed that the mean value of the concrete strength is itself random and normally distributed about the estimate $E(x|v)$. The variance of this parameter is given by Tang (1971) as,

$$[3.17] \qquad (\sigma_{Ei})^2 = \frac{(\sigma_{x|v})^2}{m} \{1 + (\frac{v_i - \bar{v}}{s_v})^2\}$$

where $(\sigma_{Ei})^2$ = variance of the expected value of the mean compressive strength at the pulse velocity v_i;

v_i = pulse velocity measurement for the ith core.

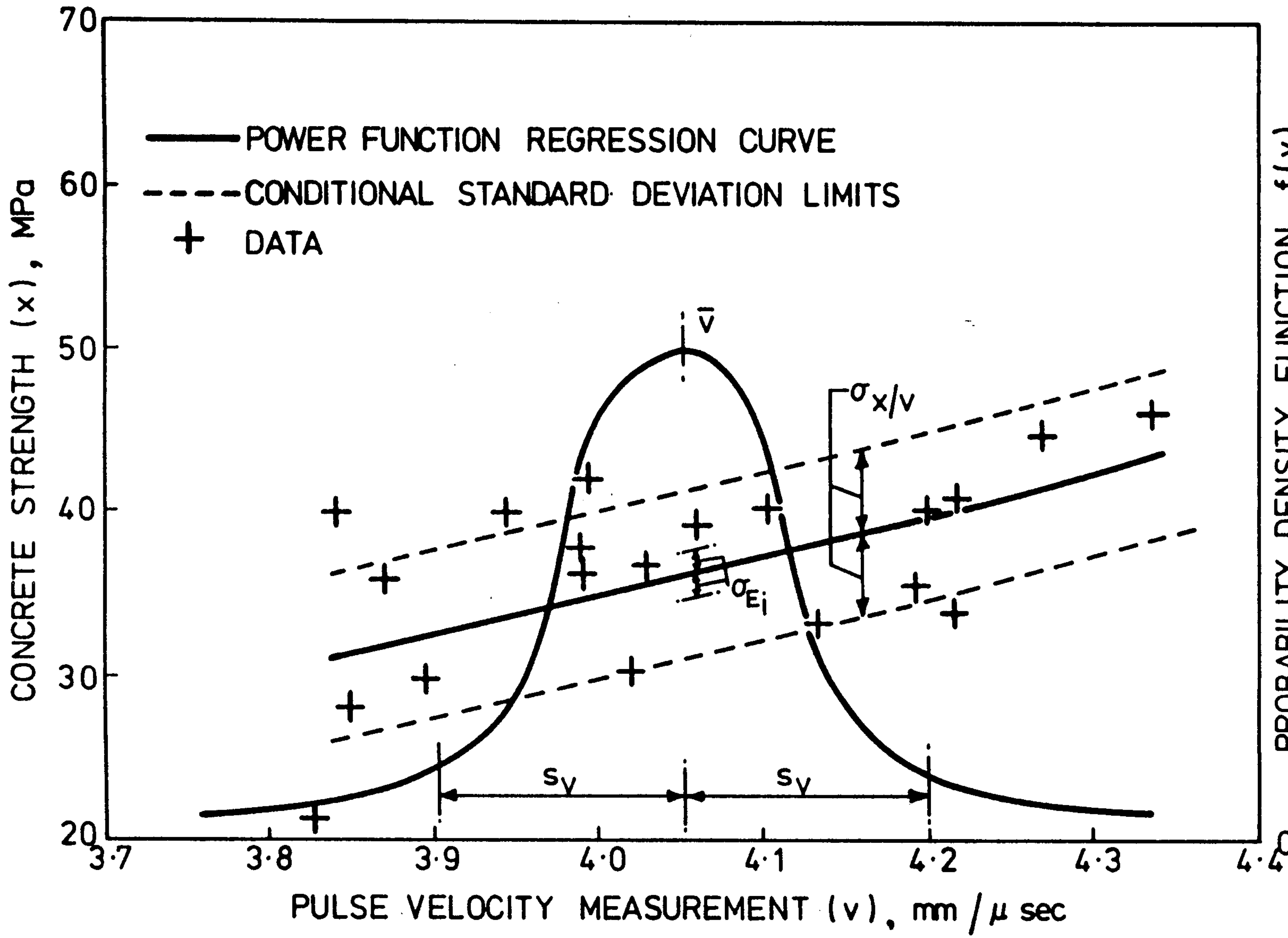

Fig. 3.1 Regression Analysis and Associated Statistical Parameters

By applying the theorem of total probability (see [2.6] for the discrete case) the pdf of the true concrete strength at the location of the ith pulse velocity measurement, $f(x_i)$, is calculated as,

[3.18] $$f(x_i) = \int_{-\infty}^{\infty} f(x_i | E_i) \cdot f(E_i)\, dE_i$$

where $f(x_i | E_i)$ = pdf of the true concrete strength in the location of the ith pulse velocity measurement given a mean of E_i;

$f(E_i)$ = pdf of the mean of the true concrete strength at the location of the ith pulse velocity measurement with a mean given by [3.15].

Substituting the two applicable Gaussian distributions into the right hand side of [3.18] gives,

$$f(x_i) = \int_{-\infty}^{\infty} \{[2\pi(\sigma_{x|v})^2]^{1/2} \cdot \exp[-\tfrac{1}{2}(\frac{x_i - E_i}{\sigma_{x|v}})^2] \cdot$$

$$[2\pi(\sigma_{Ei})^2]^{-1/2} \cdot \exp[-\tfrac{1}{2}(\frac{Ei - E(x_i|v_i)}{\sigma_{Ei}})^2]\}\, dEi$$

[3.19] $$f(x_i) = [4\pi^2(\sigma_{x|v})^2 (\sigma_{Ei})^2]^{1/2} \cdot \exp[\frac{-x_i^2}{2(\sigma_{x|v})}] - \frac{E(x_i|v_i)^2}{2(\sigma_{Ei})^2}] \int_{-\infty}^{\infty} IdE_i$$

where $$I = \exp\left[\frac{2x_iE_i - E_i^2}{2(\sigma_{x|v})^2} + \frac{2E_i\ E(x_i|v_i) - E_i^2}{2(\sigma_{Ei})^2}\right]$$

Tang (1971) indicates that the integral term in [3.19], when evaluated, is

[3.20] $$\int_{-\infty}^{\infty} IdEi = \left\{\frac{2\pi}{\frac{1}{(\sigma_{x|v})^2} + \frac{1}{(\sigma_{Ei})^2}}\right\}^{1/2} \cdot \exp\left\{-\frac{1}{2}\frac{\left(\frac{x_i}{(\sigma_{x|v})^2} + \frac{E(x_i|v_i)}{(\sigma_{Ei})^2}\right)^2}{\frac{1}{(\sigma_{x|v})^2} + \frac{1}{(\sigma_{Ei})^2}}\right\}$$

Substituting [3.20] into [3.19] gives,

$$f(x_i) = \frac{(2\pi)^{1/2} \cdot (2\pi \cdot 2\pi(\sigma_{x|v})^2\ (\sigma_{Ei})^2)^{1/2}}{\left\{\frac{(\sigma_{Ei})^2 + (\sigma_{x|v})^2}{(\sigma_{Ei})^2\ (\sigma_{x|v})^2}\right\}}$$

$$\cdot \exp\left\{-\frac{1}{2}\left[\frac{x_i^2}{(\sigma_{x|v})^2} + \frac{E(x_i|v_i)^2}{(\sigma_{Ei})^2} - \frac{[x_i(\sigma_{Ei})^2 + E(x_i|v_i)\ (\sigma_{x|v})]^2}{(\sigma_{x|v})^2(\sigma_{Ei})^2\ [(\sigma_{Ei})^2 + (\sigma_{x|v})^2]}\right]\right\}$$

[3.21] $$f(x_i) = \frac{(2\pi)^{-1/2}}{((\sigma_{Ei})^2 + (\sigma_{x|v})^2)^{1/2}} \cdot \exp\left(-\frac{1}{2} z_4\right)$$

where

$$z_4 = \left\{\frac{x_i^2}{(\sigma_{x|v})^2} + \frac{E(x_i|v_i)^2}{(\sigma_{Ei})^2} - \frac{x_i^2(\sigma_{Ei})^4 + 2x_iE(x_i|v_i)(\sigma_{Ei})^2(\sigma_{x|v})^2 + E(x_i|v_i)^2(\sigma_{x|v})^4}{(\sigma_{x|v})^2\ (\sigma_{Ei})^2\ [(\sigma_{Ei})^2 + (\sigma_{x|v})^2]}\right\}$$

$$z_4 = \{\frac{x_i - E(x_i|v_i)^2}{(\sigma_{Ei})^2 + (\sigma_{x|v})^2}\}$$

[3.21] is a Gaussian relationship with a mean equal to the expected value (as given by [3.15]) and with a variance equal to the sum of the conditional variance of the regression relationship and the variance of the expected value of the mean (that is [3.16] plus [3.17]).

Finally it is possible to develop the likelihood function. Because direct compressive strength data is not available [3.3] cannot be used. Instead a likelihood function is required that gives the conditional probability density of obtaining the expected compressive strength (as given by [3.15]) assuming that the mean value of the mean concrete strength distribution is as given by the prior. To properly consider the several statistical errors related to the use of expected compressive strength data based on indirect pulse velocity data, it is necessary to use the theorem of total probability in developing the likelihood function. For the concrete at the location of the ith pulse velocity measurement the likelihood function is given as,

[3.22] $$f(E(x_i|v_i)|\mu) = \int_{-\infty}^{\infty} f((E(x_i|v_i)|\mu)|x_i)\ f(x_i)\ dx_i$$

where $f(E(x_i|v_i)|\mu)$ = likelihood pdf of μ given the expected value of concrete strength at the location of the ith pulse velocity measurement;

$f((E(x_i|v_i)|\mu)|x_i)$ = likelihood pdf of μ given the true value of the concrete strength at the location of the ith pulse velocity measurement.

Substituting the two applicable Gaussian distributions into the right hand side of [3.22] gives,

$$f(E(x_i|v_i)|\mu) = \int_{-\infty}^{\infty} \{\frac{(2\pi)^{-1/2}}{(\sigma_o)^{-1/2}} \cdot \exp\left[-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma_o}\right)^2\right] \cdot \frac{(2\pi)^{-1/2}}{((\sigma_{Ei})^2 + (\sigma_{x|v})^2)^{1/2}} \cdot \exp\left[-\frac{1}{2}\frac{(x_i - E(x_i|v_i))^2}{(\sigma_{Ei})^2 + (\sigma_{x|v})^2}\right]\} \, dx_i$$

[3.23] $$f(E(x_i|v_i)|\mu) = \{4\pi^2\sigma_o^2 [(\sigma_{Ei})^2 + (\sigma_{x|v})^2]\}^{-1/2} \cdot \exp\left\{\frac{-\mu^2}{2\sigma_o^2} - \frac{E(x_i|v_i)^2}{2(\sigma_{Ei})^2 + 2(\sigma_{x|v})^2}\right\} \int_{-\infty}^{\infty} I dz_i$$

where $$I = \exp\left\{\frac{2x_i\mu - x_i^2}{2\sigma_o^2} + \frac{2E(x_i|v_i)x_i - x_i^2}{2(\sigma_{Ei})^2 + 2(\sigma_{x|v})^2}\right\}$$

Evaluating the integral term in [3.23] (see Tang 1971) gives,

[3.24]

$$\int_{-\infty}^{\infty} I dx_i = \frac{(2\pi)^{1/2}}{\left(\frac{1}{\sigma_o^2} + \frac{1}{(\sigma_{Ei})^2 + (\sigma_{x|v})^2}\right)^{1/2}} \cdot \exp\left\{\frac{1}{2}\left[\frac{\left(\frac{\mu}{\sigma_o^2} + \frac{E(x_i|v_i)^2}{(\sigma_{Ei})^2 + (\sigma_{x|v})^2}\right)^2}{\frac{1}{\sigma_o^2} + \frac{1}{(\sigma_{Ei})^2 + (\sigma_{x|v})^2}}\right]\right\}$$

Substituting [3.24] into [3.23] and simplifying gives,

[3.25]

$$f(E(x_i|v_i)|\mu) = \frac{(2\pi)^{-1/2}}{(\sigma_o^2 + (\sigma_{Ei})^2 + (\sigma_{x|v})^2)^{1/2}} \cdot \exp\left(-\frac{1}{2} Z_5\right)$$

where $$Z_5 = \left\{\frac{(E(x_i|v_i) - \mu)^2}{\sigma_o^2 + (\sigma_{Ei})^2 + (\sigma_{x|v})^2}\right\}$$

[3.25] is a Gaussian relationship with a mean equal to the expected value as given by [3.15] and with a variance equal to the sum of the conditional variance of the regression relationship, the variance of the expected value of the mean concrete strength, and the assumed variance of the true concrete strength.

For the general case where r different pulse velocity measurements are available the likelihood pdf is given as,

[3.26]

$$\prod_{i=1}^{r} f(E(x_i|v_i)|\mu) = \prod_{i=1}^{r} \frac{(2\pi)^{-1/2}}{(\sigma_o^2 + (\sigma_{Ei})^2 + (\sigma_{x|v})^2)^{1/2}} \cdot \exp\left\{-\frac{1}{2}\left[\frac{(E(x_i|v_i) - \mu)^2}{\sigma_o^2 + (\sigma_{Ei})^2 + (\sigma_{x|v})^2}\right]\right\}$$

Combining [3.26] and [3.2] through Bayes' theorem ([2.14]) gives,

$$[3.27] \qquad f(\mu|E(x_i|v_i)) = \frac{Z_6 \frac{(2\pi)^{-1/2}}{\sigma_{pr}} \cdot \exp\{-\frac{1}{2}(\frac{\mu - \mu_{pr}}{\sigma_{pr}})^2\}}{\int_{-\infty}^{\infty} Z_6 \frac{(2\pi)^{-1/2}}{\sigma_{pr}} \cdot \exp\{-\frac{1}{2}(\frac{\mu - \mu_{pr}}{\sigma_{pr}})^2\}\, d\mu}$$

where
$$Z_6 = \prod_{i=1}^{r} \frac{(2\pi)^{-1/2}}{((\sigma_{Ei})^2 + (\sigma_{x|v})^2 + (\sigma_o)^2)^{1/2}}$$

$$\cdot \exp\{-\frac{1}{2}[\sum_{i=1}^{r} \frac{(E(x_i|v_i) - \mu)^2}{(\sigma_{Ei})^2 + (\sigma_{x|v})^2 + \sigma_o^2}]\}$$

Simplifying and expanding [3.27] gives,

$$[3.28] \qquad f(\mu|E(x_i|v_i)) = \frac{\exp(Z_7)}{\int_{-\infty}^{\infty} \exp(Z_7)\, d\mu}$$

where
$$Z_7 = \{-\frac{1}{2}[(\frac{\mu - \mu_{pr}}{\sigma_{pr}})^2 + \sum_{i=1}^{r} \frac{(E(x_i|v_i) - \mu)^2}{(\sigma_{Ei})^2 + (\sigma_{x|v})^2 + \sigma_o^2}]\}$$

altering Z_7 gives,

$$Z_7 = \{-\frac{1}{2}[\frac{\mu^2 - 2\mu_{pr}\mu + \mu_{pr}^2}{\sigma_{pr}^2} + \sum_{i=1}^{r} \frac{E(x_i|v_i)^2 - 2E(x_i|v_i)\mu + \mu^2}{(\sigma_{Ei})^2 + (\sigma_{x|v})^2 + \sigma_o^2}]\}$$

[3.29] $$Z_7 = \{-\frac{1}{2}[\frac{\mu^2 - 2\mu_{pr}\mu}{\sigma_{pr}^2} + \sum_{i=1}^{r}\frac{\mu^2}{(\sigma_{Ei})^2} - \sum_{i=1}^{r}\frac{2\mu\ E(x_i|v_i)}{(\sigma_{si})^2}]$$

$$-\frac{1}{2}[\frac{\mu_{pr}^2}{\sigma_{pr}^2} + \sum_{i=1}^{r}\frac{E(x_i|v_i)}{(\sigma_{Si})^2}]\}$$

where $(\sigma_{Si})^2 = \{(\sigma_{Ei})^2 + (\sigma_{x|v})^2 + (\sigma_o)^2\}$

Substituting [3.29] into [3.28] and simplifying gives,

[3.29] $$f(\mu|E(x_i|v_i)) = \frac{\exp(Z_8)}{\int_{-\infty}^{\infty}\exp(Z_8)\ d\mu}$$

where $$Z_8 = \{-\frac{1}{2}[\frac{\mu^2}{\sigma_{pr}^2} - \frac{2\mu_{pr}\mu}{\sigma_{pr}^2} + \sum_{i=1}^{r}\frac{\mu^2}{(\sigma_{Si})^2} - \sum_{i=1}^{r}\frac{2\mu\ E(x_i|v_i)}{(\sigma_{Si})^2}]\}$$

Altering Z_8 gives

[3.30] $$Z_8 = -\frac{1}{2}U[(\mu - \frac{V}{U}) - \frac{V^2}{U^2}]$$

where $$U = (\frac{1}{\sigma_{pr}^2} + \sum_{i=1}^{r}\frac{1}{(\sigma_{Si})^2})$$

$$V = \left(\frac{\mu_{pr}}{\sigma_{pr}^2} + \sum_{i=1}^{r} \frac{E(x_i|v_i)}{(\sigma_{Si})^2}\right)$$

Substituting [3.30] into [3.29] and simplifying gives,

[3.31] $$f(\mu|E(x_i|v_i)) = \frac{\exp(Z_9)}{\int_{-\infty}^{\infty} \exp(Z_9)\, d\mu}$$

where $$Z_9 = -\frac{1}{2} U \left[\left(\mu - \frac{V}{U}\right)^2\right]$$

Using [3.11] to evaluate the denominator of [3.31] gives,

[3.32] $$f(\mu|E(x_i|v_i) = \left(\frac{U}{2\pi}\right)^{1/2} \cdot \exp\left\{-\frac{1}{2} U \left(\mu - \frac{V}{U}\right)^2\right\}$$

Comparing [3.32] with [3.1] it is clear that [3.32] is Gaussian with a mean value given by,

$$\mu_{po} = \frac{V}{U}$$

[3.33]

$$\mu_{po} = \left\{\frac{\mu_{pr} / \sum_{i=1}^{r} \frac{1}{(\sigma_{Si})^2} + \left[\sigma_{pr}^2 \sum_{i=1}^{r} \frac{E(x_i|v_i)}{(\sigma_{Si})^2} \Big/ \sum_{i=1}^{r} \frac{1}{(\sigma_{Si})^2}\right]}{1 \Big/ \sum_{i=1}^{r} \frac{1}{(\sigma_{Si})^2} + \sigma_{pr}^2}\right\}$$

and a variance given by

$$(\sigma_{po})^2 = \frac{1}{U}$$

$$[3.34] \qquad (\sigma_{po})^2 = \frac{\sigma_{pr}^{\ 2} \Big/ \sum_{i=1}^{r} \frac{1}{(\sigma_{Si})^2}}{\sigma_{pr}^{\ 2} + \left(\sum_{i=1}^{r} \frac{1}{(\sigma_{Si})^2} \right)^{-1}}$$

[3.33] and [3.34] are the posterior or updated mean and variance of the mean compressive strength of the concrete based on new data being indirect pulse velocity measurements. Substitution of these values into [3.1] gives the posterior pdf for the mean compressive strength of the concrete based on prior information and pulse velocity data. This posterior pdf in turn serves as a prior pdf for combination with direct core compressive strength data. In order to combine direct core strength data with this new pdf simply use [3.33] as μ_{pr} and [3.34] as $(\sigma_{pr})^2$ and proceed as outlined in Section 3.2.

3.4 Estimation of Mean Strength when Rebound Numbers, Ultrasonic Pulse Velocity Measurements, and Concrete Core Strength Data are Available

The procedure for combining two types of indirect data, like rebound hammer numbers and pulse velocity measurements, with direct test data, like core compressive strengths, in order to update a prior pdf, is described by Tang (1971). Tang suggests that in the first step of the three step procedure, one of the two sets of indirect data should be combined, using Bayes' theorem, with the original prior pdf. The simple closed form relationships required to do this are developed in Section 3.3.

The resulting posterior pdf must then be considered as a prior pdf for the second step of the procedure. The unused set of indirect data is combined with the new prior pdf developed in step one. Once again the procedure and Bayesian relationships developed in Section 3.3 must be used. The resulting posterior pdf from this second step is then used as a prior pdf for combination with the direct data. This final step of the procedure is executed using the relationships developed in Section 3.2.

CHAPTER 4

Parametric Study of Bayesian Approach for Predicting Concrete Strength

4.1 Introduction

The closed form relationships developed in the previous chapter provide a simple, convenient method for combining various types of data to predict the in-situ compressive strength of concrete. This analytical procedure is especially attractive because it can accommodate both core strength data and NDT data, in any quantity, along with other pertinent information. Even though the procedure is very flexible it is necessary to select data with some care in order to ensure reasonable strength predictions result. With a proper understanding of the relative significance of the various parameters upon which the Bayesian relationships are based it is possible to determine realistic data requirements at the outset of an investigation.

This chapter presents the results of a parametric study of the Bayesian relationships. The study was performed using data obtained in a real investigation.

4.2 Description of Study

This study investigates the parametric characteristics of the Bayesian procedure for evaluating the in-situ compressive strength of concrete in existing structures. The effects of varying the prior information, the variance of the sampled population, and the amount and type of new data are examined.

Also, a comparison is made between a random and a quasi-random data selection procedure.

Data used in the study was obtained during a recent investigation of a reinforced concrete bridge (Mikhailovsky and Scanlon 1985). The systematic data selection procedure adopted was intended to simulate as closely as possible the procedure commonly used in actual in-situ concrete strength investigations.

4.3 Description of Test Data

All test data used in this study are from an investigation of a 35 year old reinforced concrete highway bridge investigated by Mikhailovsky and Scanlon (1985). The minimum specified 28 day compressive strength for the concrete was 20.7 MPa (3000 psi). The three span bridge is composed of five parallel, continuous, cast in place T-girders. Test data were obtained only from girder stems of the middle span of the bridge. A grid, dividing each of the girders into 92 regions of approximately equal size, was marked onto the girder stems for use in identifying the test locations. Schematic details of the bridge and the grid system are shown in Figure 4.1.

Three types of data reported by Mikhailovsky and Scanlon (1985) were used in this study. These are pulse velocity measurements, minimum rebound hammer numbers, and concrete core compressive strengths. The minimum rebound hammer number refers to the lowest reading obtained in a series of 9 readings in the vicinity of a test location. Mikhailovsky and Scanlon (1985) found that this value produced better correlation with core

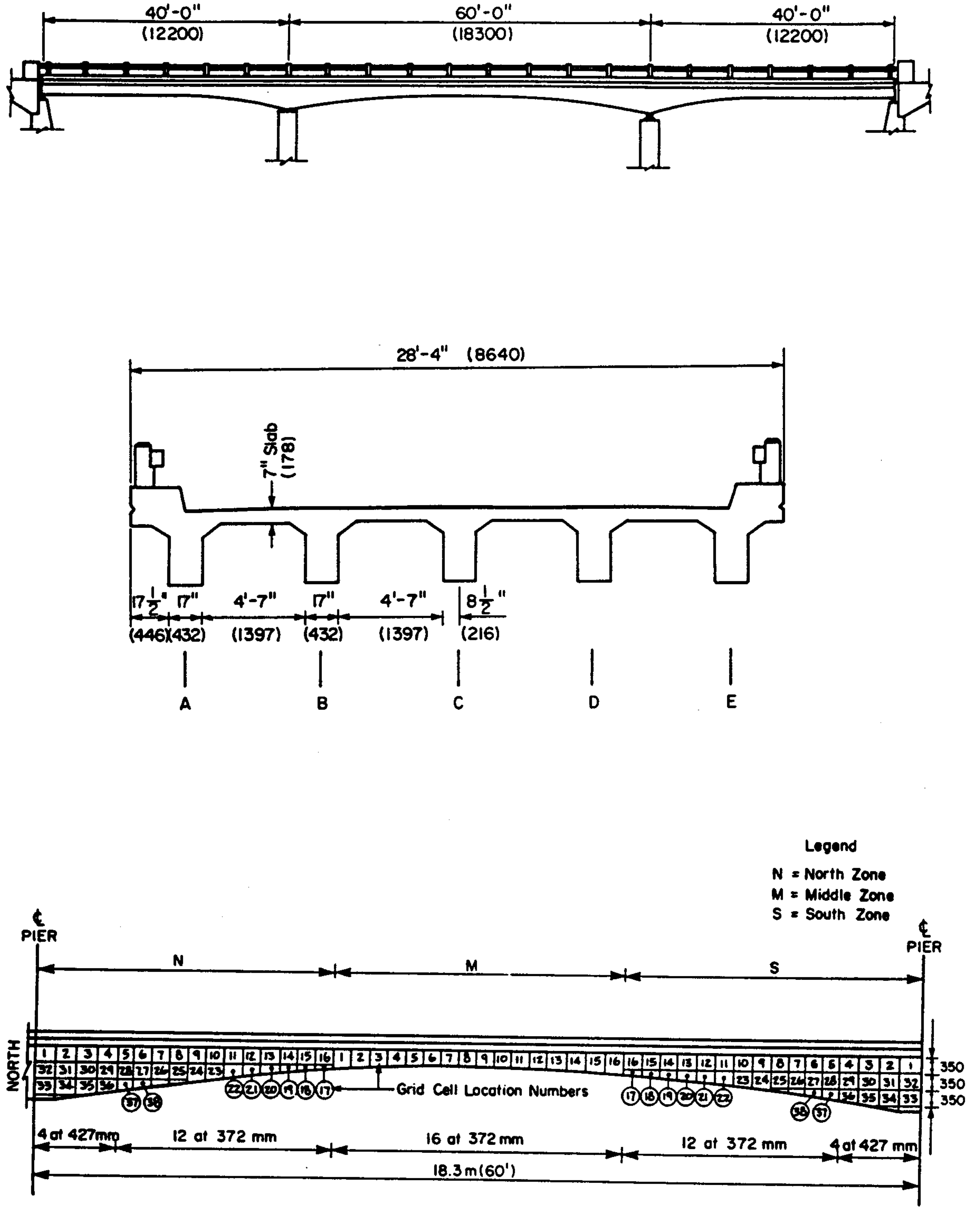

Fig. 4.1 Reinforced Concrete Bridge Details (Mikhailovsky and Scanlon 1985)

compressive strengths than did either mean or maximum rebound hammer readings.

Of the 460 total possible grid locations only 451 locations afforded field measurements that could be used to determine pulse velocities. Six of the nine locations not usable for calculating pulse velocities were also not usable for determining the minimum rebound hammer number for the concrete in those cells. To maintain consistency the rebound numbers corresponding to the three other cells without pulse velocity measurements were not used. Table C1 in Appendix C summarizes the pulse velocity and minimum rebound hammer data.

Thirty-two concrete cores were removed from the central girders at 21 randomly chosen locations. In eleven of the locations two cores were removed, one from each half of the girder stem. In this study, at these locations, the average compressive strength of the two cores was used as the actual strength for the concrete at that location. Since the amount of core strength data considered in the various analyses was always equal to a multiple of four, one of the 21 available core strengths was not used.

Although NDTs were performed on the actual core specimens this data was not used in this study. Whenever NDT data corresponding to core specimens were needed, the NDT measurements performed on the undisturbed concrete (prior to coring) were used. Table C2 in Appendix C lists the core compressive strength data and the NDT measurements observed at each core location.

For a detailed description of the sampling and test methods that were used to obtain the data, see the report by Mikhailovsky and Scanlon (1985).

4.4 Data Selection Procedure

Two data selection procedures were used in this study. Both procedures were intended to follow as closely as possible the selection process usually used in real concrete strength investigations. The only difference betwen the two procedures was the manner in which concrete core specimen data were selected. NDT data were selected in a similar manner in both procedures.

In a real investigation, after the section of concrete being studied is sufficiently delineated, the field data usually obtained first would be a series of nondestructive test (NDT) measurements taken at random locations. Depending on economics and other considerations one, two, or, on rare occasions, three types of NDT measurements would be taken at each test location. Three types of NDTs commonly used are pulse velocity measurements, rebound hammer readings, and surface penetration measurements (Malhotra 1984). In this study only the first of these types alone, and the first two of these types together are considered.

Next, at some of the NDT locations, concrete cores are removed so that the compressive strength of the concrete in-situ can be measured. These compressive strength data is then combined with the NDT data corresponding to the core locations in order to develop a regression relationship between an NDT measurement and concrete compressive strength. This regression relationship is used to predict the concrete compressive strength at test locations, where only NDT data are available. In this study, when two types of NDT measurements were considered at each test location then two regression relationships were developed, one for use with each type of NDT data. Furthermore, it was assumed that the pulse velocity data and the rebound hammer data, even at identical locations were statistically independent. Thus when both types of NDT data were considered in analysis two predictions of concrete strength were made and each prediction was treated as separate and unique.

The first data selection procedure used simulated the process just described and ensured that data were selected completely randomly. This was accomplished by first randomly arranging the NDT data and the concrete core data listed in Tables Cl and C2. This was done using a random selection process. The resulting data bases are given in Tables C3 and C4 of Appendix C.

To simulate an investigation in which NDT measurements were to be taken at n locations and core specimens were to be removed at m of the n test locations, the first (n-m) data listed in the random NDT data base (i.e. Table C3) and the first m data listed in the random concrete core data base (i.e. Table C4) were selected. In subsequent simulations where larger quantities were required of either or both direct and indirect data these data were taken from the unused portions of the data bases in the order in which it is listed.

It is normally most economical, and certainly less destructive, to obtain considerably more NDT data than core compressive strength data. Therefore it is important that the regression relationship, which is used to convert the NDT data into concrete strength estimates, be based on data which cover the complete range of NDT measurements. This is necessary because regression relationships should be applied only over the range of indirect data which were used in developing the relationship (Ang and Tang 1975).

Unfortunately, in some investigations, as few as three or four cores may be removed. With such a small amount of data, if it is selected randomly, it is not likely that the range of the resulting regression relationship will be adequate to process all of the NDT data. In a real investigation this potential problem can be avoided if core locations are selected in a systematic way rather than a random way (Mikhailovsky and Scanlon 1985). A systematic method that would achieve the desired results would begin after all NDT measurements were obtained. Before removing any core specimens the NDT data would be arranged sequentially, from the smallest measurements to the largest, and then broken down into a series of sub-groups equal in number to the number of cores that are to be removed. The coring locations would then be selected by choosing randomly one location from each sub-group. The regression relationship developed using these quasi-random data will very likely be applicable over the entire range of NDT data.

The second data selection process used in this study was identical to the first except that the selection of the core data was intended to simulate the quasi-random process just described. In order to simulate quasi-random data selection another core data base was developed. In developing this new data base it was first necessary to arrange the 20 core data sets (as listed in Table C2) in sequential order. Since all analyses performed in the study used core data quantities in multiples of four this data was then broken down into four sub-groups. Four sets of data were then selected, one from each sub-group, and placed in a new data base in this new order. This procedure was repeated four more times; all of the data were then in a quasi-random order as listed in Table C5 of Appendix C. Thus the second data selection procedure was undertaken exactly as was the first, except that Tables C3 and C5 were used instead of Tables C3 and C4.

One statistical error that occurred when the data bases were developed was noted at the conclusion of this study. The NDT measurements at locations corresponding to core specimens were used twice; these data were used in developing the regression relationship and it was used again as NDT data to be processed by the regression relationship. Since the majority of this NDT data did not come into use until relatively large amounts of NDT data were being analyzed, this error is considered to be of academic interest only and likely did not affect the results of any analyses from a practical standpoint.

4.5 Effect of Prior Distribution on Posterior Distribution of Mean Concrete Strength

From prior information an engineer can usually postulate a reasonable Gaussian distribution for describing the mean of the compressive strength of a concrete being studied. As discussed previously the degree of certainty of such an estimate is usually dependent upon the availability of preliminary test data and/or construction documents pertaining to concrete placement records and the original concrete specifications. Generally the less preliminary information available, the larger will be the variance and the smaller will be the mean for the prior distribution. However, regardless of the prior information available the selection of a prior distribution is always a subjective exercise.

To evaluate the effect that various prior distributions had on the prediction of the mean strength of the concrete being studied a series of 5 groups of analyses were performed. Each of these 5 groups had a different prior distribution and all of the analyses combined only direct core strength data with the prior information. Within each of the 5 groups 10 analyses were performed; each of these 10 analyses considered a different amount of core strength data. Random concrete core strength data (as given in Table C4) were used in each analysis and the quantity of data used varied in increments of 2 from 2 to 20 measurements.

The 5 prior distributions considered in this study were as follows:

1. A diffuse prior, which consists of a mean strength equal to any reasonable value and a variance of the mean strength equal to infinity. As discussed in Section 3.2 this form of a prior results in the Bayesian posterior distribution being identical to the classical prediction. This prior would be used when statistical predictions are to be based on new data only.

2. Prior mean strength equal to one half of the specified compressive strength and the prior variance of the mean strength equal to the variance of the sampled population. This prior might be used when the in-situ concrete compressive strength is suspected to be much less than originally specified.

3. Prior mean strength equal to the specified strength and the prior variance of the mean strength equal to the variance of the sampled population. This prior might be used when the in-situ concrete strength is considered to be as per specified and the investigation is taking place soon after construction.

4. Prior mean strength equal to 1.17 times the specified strength and the prior variance of the mean strength equal to the variance of the sampled population. This prior might be used when the concrete being investigated is 10 or more years old and exhibits a quality which indicates that specifications were met at construction time. The 1.17 factor is based on

strength gain predictions over time as given by ACI Committee 209 (1982).

5. Prior mean strength equal to 2 times the specified strength and the prior variance of the mean strength equal to the variance of the sampled population. This prior might be used when the in-situ concrete strength is suspected to be considerably greater than originally specified.

In all of the cases given above, except number 1 in which a diffuse prior was used, the values of the prior variance of the mean and the variance of the sampled population were not given. This information was not necessary because when these two variances are equal the prediction of the mean strength is not affected by the magnitudes of the variances. This is easily verified by referring to [3.13], the relationship which is used to estimate the mean strength. Although it is not possible for the variance of the mean strength to be as large as the variance of the sampled population, this assumption was used for two reasons. First it represents a limiting condition for the prior variance. Second it allows the study of the effect of varying just μ_{pr}. The effect of varying σ_{pr} relative to σ_o was not considered.

Figure 4.2 shows the variation in the estimate of the mean compressive strength (μ_{po}) with sample size for each of the 5 priors. The estimate of the mean strength when only 2 test measurements are considered varies from 27.7 MPa for μ_{pr} =

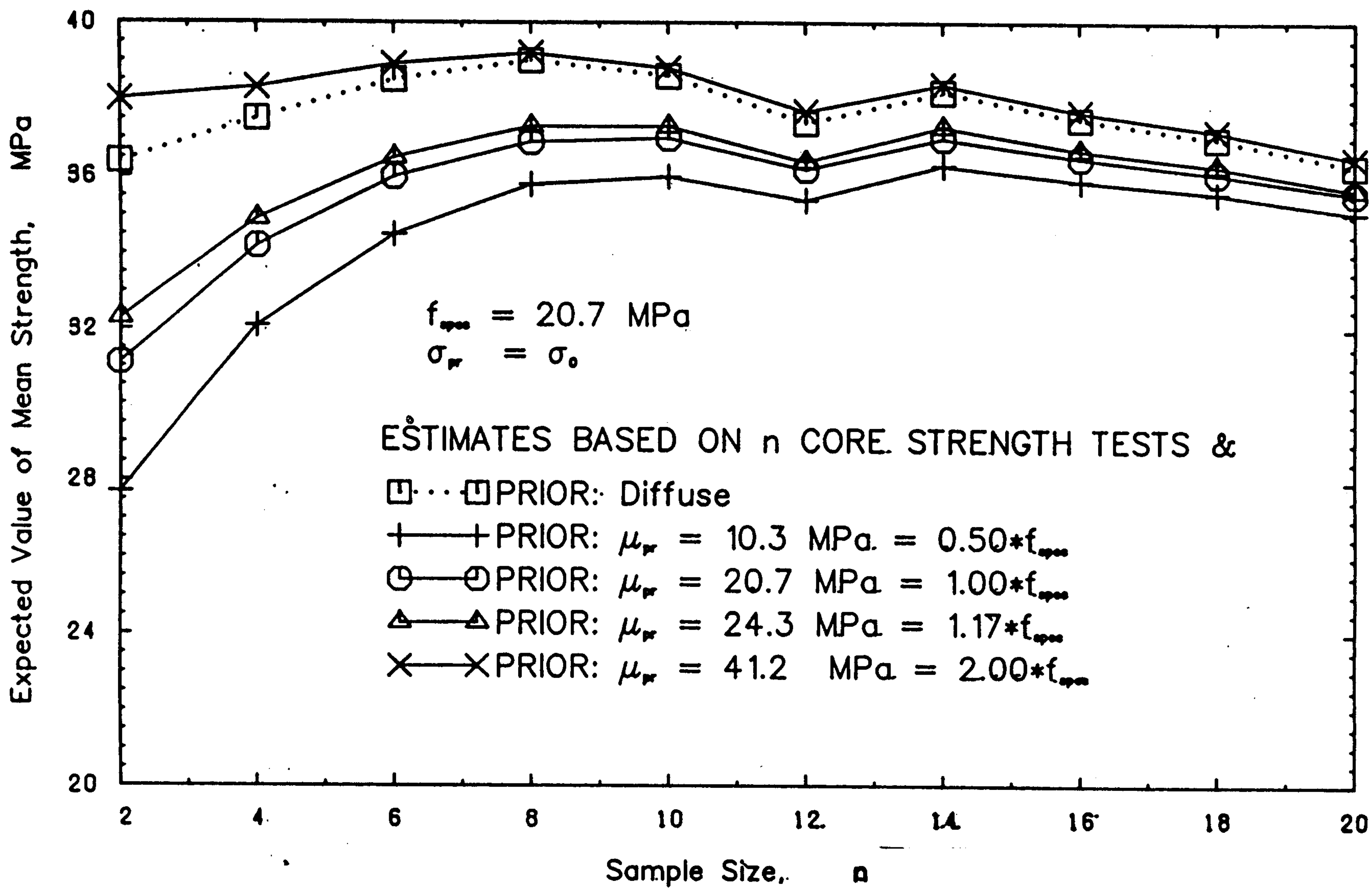

Fig. 4.2 Variation in the Estimate of the Mean Strength

$0.5f_{spec}$ to 38.0 MPa for $\mu_{pr} = 2.0f_{spec}$. The estimate of μ_{po} when 20 test measurements are considered varies from 35.1 MPa to 36.5 MPa for these two extreme cases.

These results indicate that when relatively small amounts of test data are available the prior distribution greatly influences the mean strength prediction. Conversely, when relatively large amounts of test data are available the influence of the prior distribution on the prediction of μ_{po} is small. The interpretation of relatively large and relatively small amounts of test data vary between different problems. Viola (1983) notes that the prior distribution can affect μ_{po} significantly whenever the prior variance is approximately equal to or less than the sampled population variance divided by the total number of test measurements. This observation is easily verified by referring to [3.13]. In this study, as seen in Figure 4.2, a relatively small number of test data appears to be any number less than about ten.

For the purposes of this study the best prior distribution for predicting μ_{po} appears to be either the diffuse prior or the prior with $\mu_{pr} = 2.0f_{spec}$. This conclusion is based on the fact that the value of μ_{po} varied the least with data sample size for these 2 cases. The remaining priors would be considered conservative.

To evaluate the effect that different prior distributions had on the prediction of the variance of the mean compressive strength (σ_{po}) a series of 3 sets of analyses were performed. The first set of analyses used a diffuse prior. The second set used $\sigma_{pr} = \sigma_o = 0.20f_{spec} = 4.1$ MPa. The third set used $\sigma_{pr} = 0.15f_{spec} = 3.1$ MPa. In all three of these analyses only core strength data were considered available. As seen from [3.14] (the relationship required to calculate σ_{po}) the magnitudes of the test data were not required; however, the quantity of data was required and this was varied in each set of analyses from 2 to 20 in increments of 2.

Figure 4.3 shows the variation in the estimate of the standard deviation of the mean strength with sample size for each of the three priors. The estimate of σ_{po} when only 2 test measurements are available is 2.1 MPa for σ_{pr} = 3.1 MPa and 2.9 MPa when a diffuse prior is used. When 20 test measurements are available σ_{po} converges on 0.90 MPa for all three priors.

The form of the prior distribution does not appear to affect the value of σ_{po} when a relatively large amount of data are available. This observation is consistent with the effect that the prior has on μ_{po}. However, a relatively large amount of data for the prediction of σ_{po} appears to be have a smaller effect than it does for the prediction of μ_{po} (see Figure 4.3). The relative insignificance of σ_{pr}, as observed in this study, practically will always be the case because $\sigma_{pr} \leq \sigma_o$ is always true. This observation is easily verified by referring to [3.14].

Further analyses (not summarized here) were performed in which a relatively large quantity of nondestructive test data and a relatively small quantity of core strength data were used. Two types of priors were considered in these analyses. The first

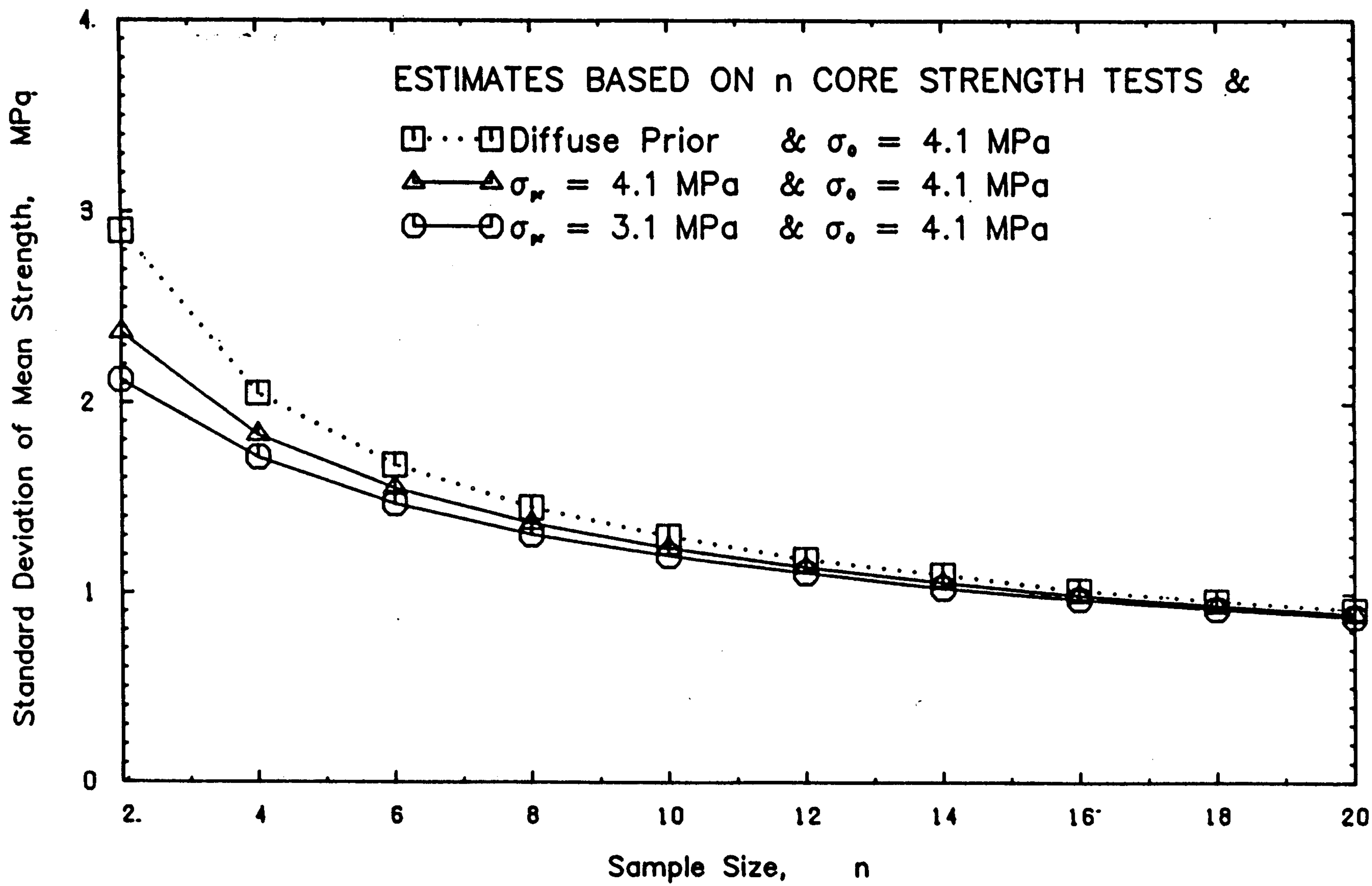

Fig. 4.3 Variation in the Estimate of the Standard Deviation of the Mean Strength

consisted of a diffuse prior and the second was characterized by $\mu_{pr} = f_{spec}$ and $\sigma_{pr} = \sigma_o = 0.2f_{spec}$. Essentially identical posterior distributions resulted in both analyses.

From the observations recorded in this study it is clear that relatively large quantities of test data should generally be obtained in order to predict in-situ concrete strength with a high degree of confidence. However, if very reliable prior information is available, small quantities of data can be used also.

4.6 Effect of Sampled Population Variance on Posterior Distribution of Mean Concrete Strength

As discussed in Section 3.2 the coefficient of variation of concrete compressive strength within a homogeneous section usually varies from approximately 10% to 20%. These values are commonly associated with concrete placed under excellent and poor levels of quality control respectively. Therefore, by reviewing available construction documents and/or by making a preliminary investigation of the concrete being studied it is normally possible to make a good estimate of σ_o.

To evaluate the effect that the population variance has on the prediction of the mean strength of the concrete, 9 groups of analyses were performed. The first six of these groups considered core strength data alone. Three of these six groups had diffuse priors and three had priors characterized by $\mu_{pr} = f_{spec} = 20.7$ MPa and $\sigma_{pr} = 0.15\ f_{spec} = 3.1$ MPa. Three different standard deviations for the sampled population were considered with each of these two groups of three. These were 3.1 MPa, 4.1 MPa and 5.2 MPa. Within each of all 6 groups 10 analyses were performed; each of these 10 analyses considered a different amount of concrete core strength data. The strength measurements were taken from the random data base (Table C4) and varied from 2 items to 20 items in increments of 2.

The remaining three groups of analyses considered both core strength data and pulse velocity measurements. All three of these analyses used a diffuse prior and the first 400 pulse velocity measurements taken from the data base given in Table C3. Three different values of σ_o were considered, one for each group. These were 3.1 MPa, 4.1 MPa and 5.2 MPa. Within each of the 3 groups 5 analyses were performed, each analysis considering a different amount of random core strength data taken from Table C4. The quantity of core strength data varied from 4 to 20 items in increments of 4.

Figure 4.4 shows the variation in the estimate of the mean compressive strength with sample size for the 6 groups of analyses which considered only core strength data. When a diffuse prior is used the mean strength (μ_{po}) is independent of σ_o; this is shown clearly in Figure 4.4 and can be discerned from [3.13]. However, for the three other analyses shown in the figure it is clear that the value of σ_o does affect μ_{po}. For these cases, when only 2 test data are used, the value of μ_{po} varies from 27.4 MPa when σ_o = 5.2 MPa to 31.1 Mpa when σ_o = 3.1 Mpa. When 20 test measurements are used μ_{po} varies from 34.4 Mpa to 35.6 MPa for the two extremes.

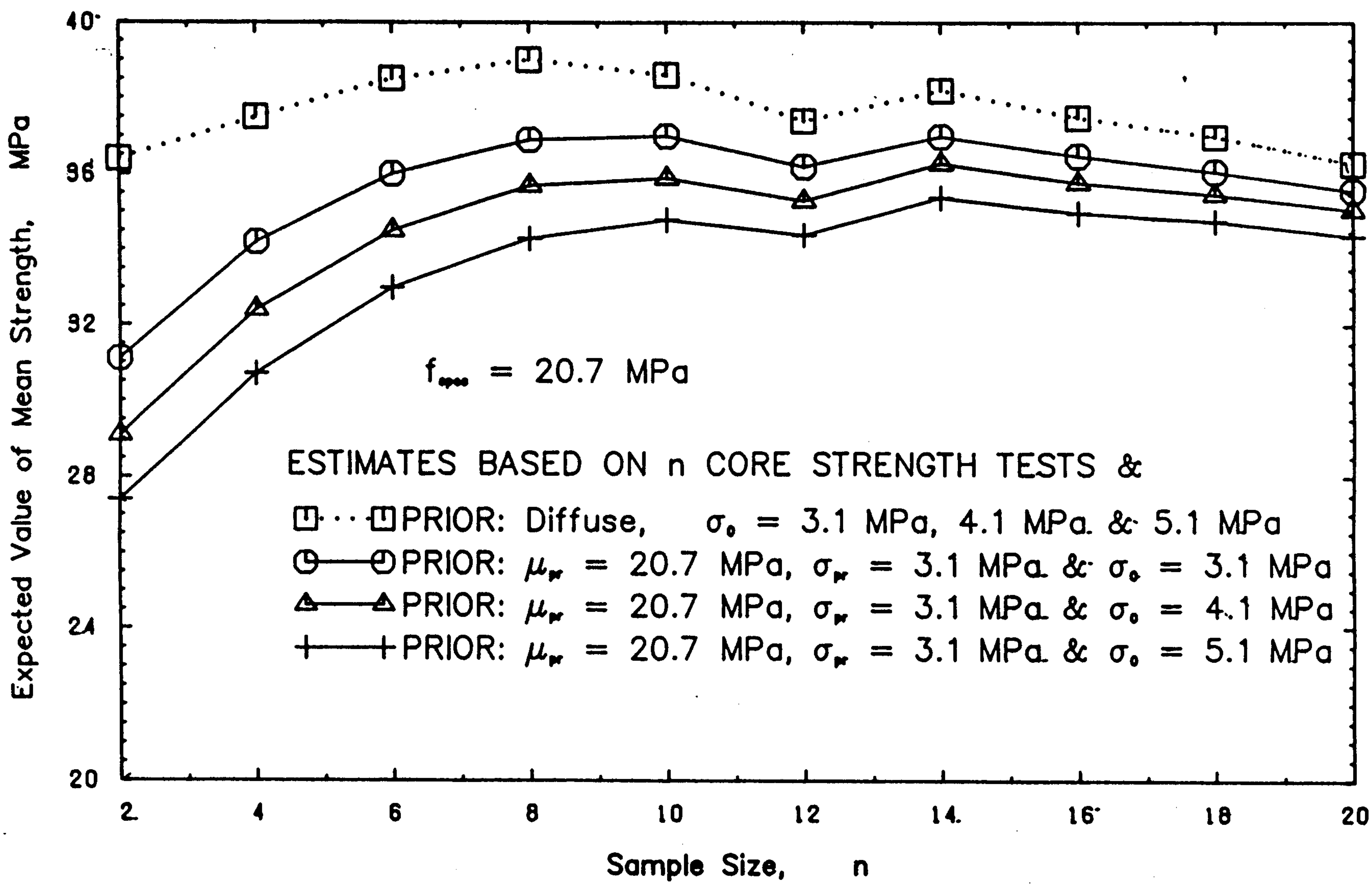

Fig. 4.4 Variation in the Estimate of the Mean Strength

Figure 4.5 shows the variation in the estimate of the mean compressive strength with core strength sample size for the 3 groups of analyses which considered 400 pulse velocity measurements. When 4 core specimens were used μ_{po} varied from 36.4 MPa to 36.3 MPa for σ_o = 3.1 MPa and σ_o = 5.2 MPa respectively. When 20 core specimens were used μ_{po} varied from 34.6 MPa to 34.3 Mpa for the two extremes.

These results indicate that the influence of σ_o on μ_{po} is significant when only relatively small amounts of test data are available. This fact holds true for direct data alone as shown in Figure 4.4 and direct data combined with indirect data, as shown in Figure 4.5. The varying effect that σ_o has on μ_{po} is similar to that noted in Section 4.5 for σ_{pr} and μ_{pr}. However, σ_o appears to influence μ_{po} over a larger range of sample size.

To evaluate the effect that σ_o had on σ_{po} several of analyses were performed using identical sets of data as described previously in this section. Figure 4.6 shows the variation in the estimate of the standard deviation of the mean strength with core specimen sample size when only direct data were considered. When a diffuse prior is used with 2 strength measurements the estimate of σ_{po} ranges from 2.2 MPa for σ_o = 3.1 MPa to 3.7 MPa for σ_o = 5.2 MPa. When 20 test measurements are considered σ_{po} ranges from 0.7 MPa to 1.2 MPa for the two extremes. When σ_{pr} = 3.1 Mpa and 2 test data are used σ_{po} ranges from 1.8 MPa for σ_o = 3.1 MPa to 2.4 MPa for σ_o = 5.2 MPa. When 20 test specimens are considered, in this later case, the estimates for σ_{po} range from about 0.70 MPa to 1.10 MPa for the

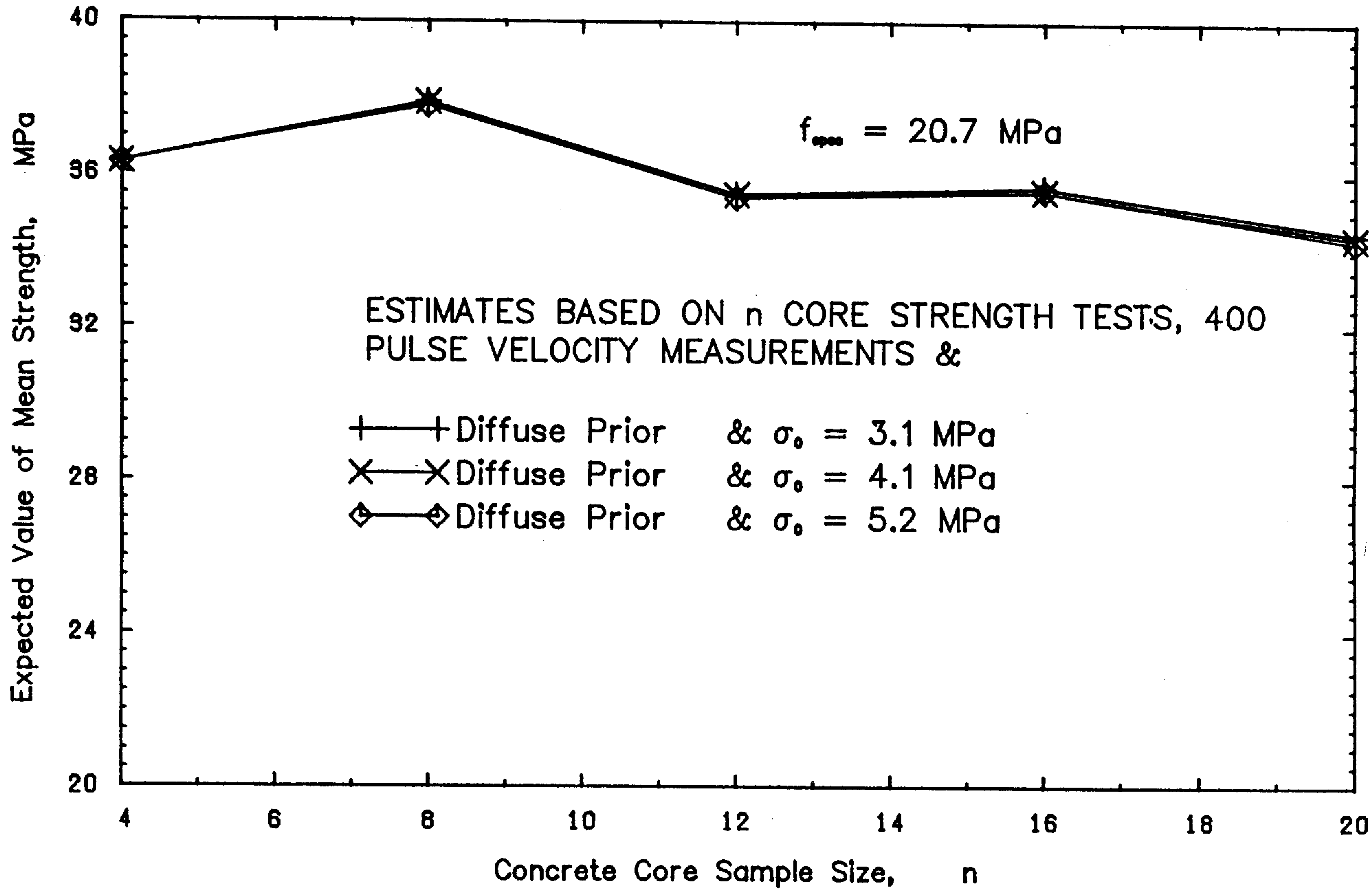

Fig. 4.5 Variation in the Estimate of the Mean Strength

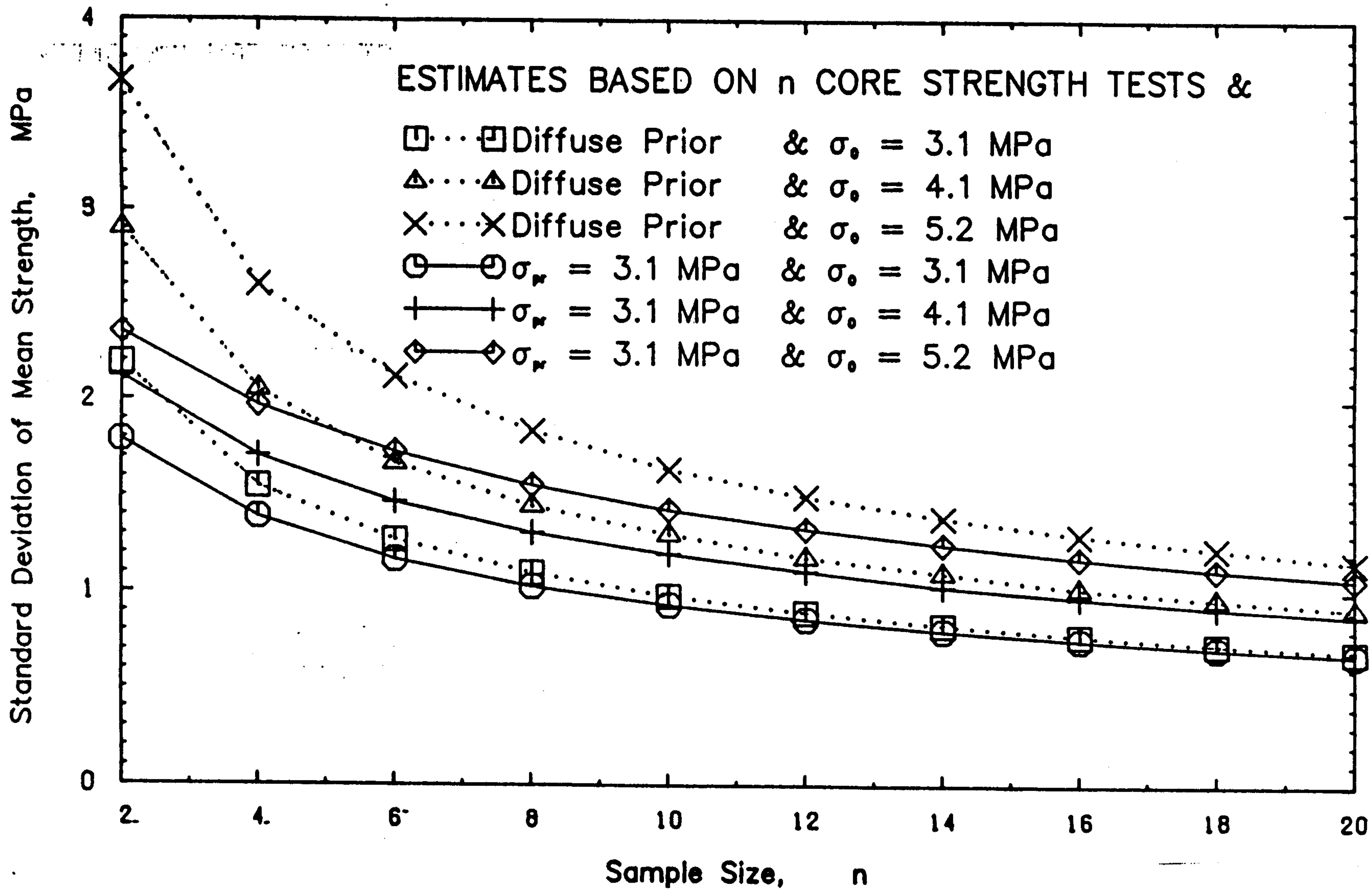

Fig. 4.6 Variation in the Estimate of the Standard Deviation of the Mean Strength

two extremes.

Figure 4.7 shows the variation in the estimate of σ_{po} with core specimen sample size when both direct and 400 indirect data were used with a diffuse prior. When 4 strength measurements were considered σ_{po} estimates ranged from 0.16 MPa for σ_o = 3.1 MPa to 0.26 MPa for σ_o = 5.2 Mpa. When 20 strength measurements were considered σ_{po} estimates ranged from 0.29 MPa to 0.36 MPa for the two extremes.

These results indicate that the effect of σ_o on σ_{po} is significant only when relatively small quantities of data are available. This is true regardless of the distribution of direct and indirect data.

4.7 Effect of Type of Data on Posterior Distribution of Mean Concrete Strength

At the outset of a concrete strength investigation a decision must normally be made regarding the amount and type of tests that should be undertaken. Intuitively it seems that in-situ strength predictions will improve most quickly as the quantity of core strength data increases. This is of course a fact and as discussed in Section 3.3 is because nondestructive or indirect measurements have more error associated with them. To nullify the greater error more indirect data is required than the direct data it replaces to ensure equivalent levels of confidence in strength predictions. Unfortunately economic considerations coupled with the need to limit destructive testing of a structure usually results in the need for extensive nondestructive testing.

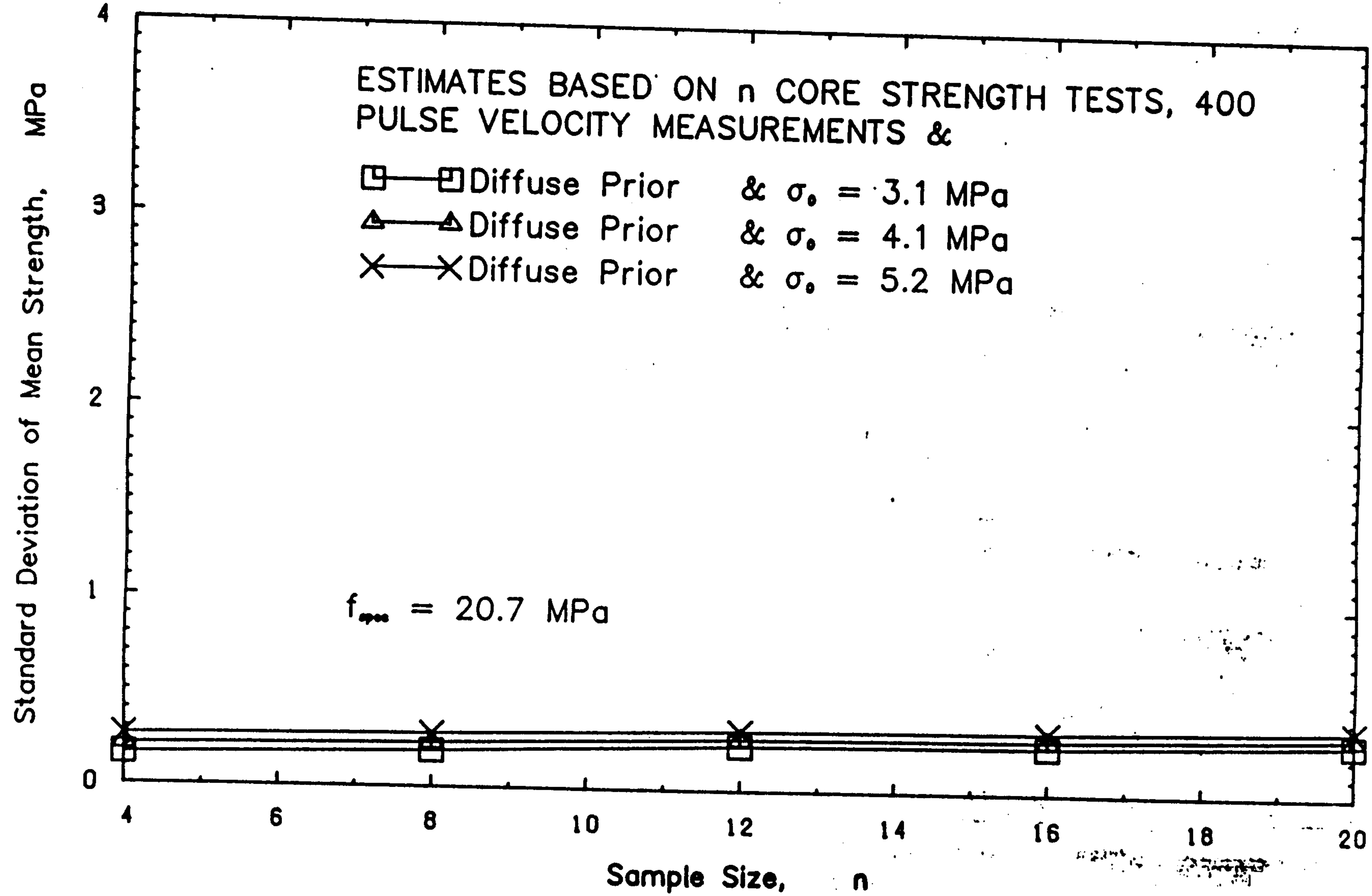

Fig. 4.7 Variation in the Estimate of the Standard Deviation of the Mean Strength

To evaluate the effect that various types of data have on the prediction of the mean strength 4 groups of analyses were performed. In all of the analyses it was assumed that μ_{pr} = 20.7 MPa, σ_{pr} = 5.2 MPa and σ_o = 5.2 MPa. Two of the four groups of analyses combined core strength data with just pulse velocity measurements. The other two groups combined core strength data with pulse velocity measurements and minimum rebound numbers. Within each of the two groups of two analyses one set of analyses considered 4 direct core measurements and the other set of analyses considered 20 direct core measurements. The core data were taken from the quasi-random data base given in Table C5. In all four groups of analyses each type of nondestructive test data used varied in quantity from 10 to 400 elements in 8 increments. These data were taken from Table C3.

Figure 4.8 shows the variation in the estimate of the mean compressive strength with nondestructive test sample size for the 4 groups analyzed. When only 10 pulse velocity measurements were combined with core strengths the value of μ_{po} varied from 33.2 MPa for 4 core strengths to 35.4 Mpa for 20 core strengths. When 400 pulse velocity measurements were considered the value of μ_{po} varied from 33.9 MPa to 34.4 Mpa for the two extremes. When 10 pulse velocity measurements and 10 minimum rebound numbers were combined with core strengths the value of μ_{po} varied from 34.1 MPa for 4 core strengths to 35.6 MPa for 20 core strengths. When 400 pairs of NDT data were used μ_{po} varied from 34.7 MPa to 35.1 Mpa for the two extremes.

These results indicate that within all four hypothetical

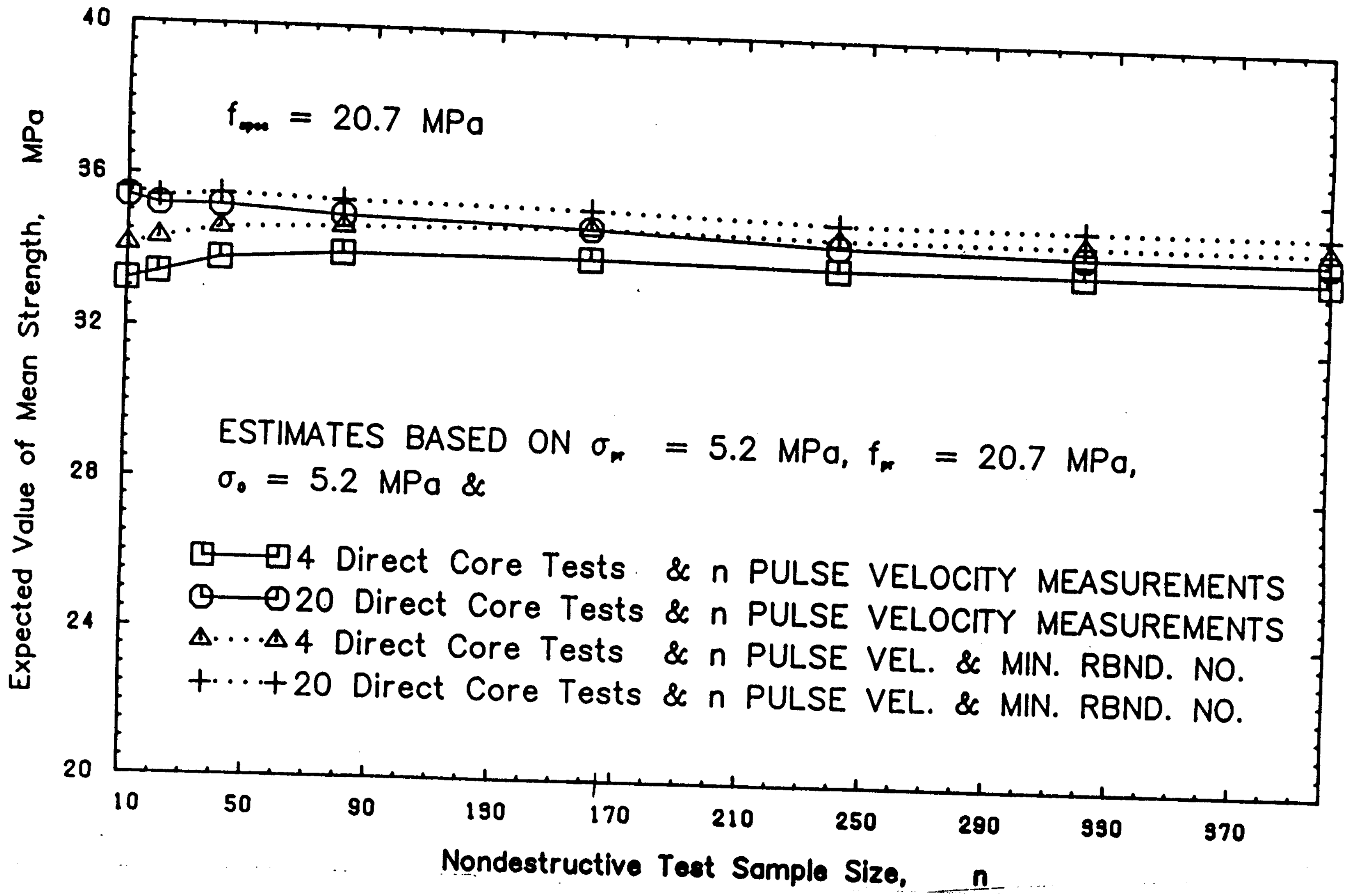

Fig. 4.8 Variation in the Estimate of the Mean Strength

testing programs considered the prediction of μ_{po} does not vary significantly, especially after about 50 NDT measurements were considered. If constancy is the basis for determining the best method then the superior set of analyses was that which considered two types of NDT data combined with 20 core strengths. The value of μ_{po} is almost constant for this case for all quantitites of NDT data considered as shown in Figure 4.8. Even without the benefit of the results shown in Figure 4.8 this method would have intuitively been selected as being best because it is based upon both types of available NDT data and a relatively large quantity of direct strength data (i.e. 20 cores instead of 4).

Although the value of μ_{po} is almost constant within each of the four groups of analyses it varies between each of the groups over the entire range of NDT data. This is caused by the permanent bias that is introduced into the regression relationships used in each group of analyses due to the varying amount and types of data used in each regression analysis. For this reason it is difficult to know if μ_{po} is predicted more accurately when 20 core measurements are used just with pulse velocity data or when 4 core measurements are used with both pulse velocity measurements and minimum rebound numbers. However, intuitively one would expect the prediction based on 20 core strengths to be most accurate.

To evaluate the effect that various types of data have on the prediction of σ_{po} 4 groups of analyses were performed applying the data used for investigating the effect on μ_{po}.

Figure 4.9 shows the variation in the estimate of σ_{po} with nondestructive test sample size for each of the four groups of analyses. When only 10 pulse velocity measurements were combined with core strengths the value of σ_{po} varied from 1.7 MPa for 4 core strengths to 1.0 MPa for 20 core strengths. When 400 pulse velocity measurements were considered the value of σ_{po} varied from 0.41 MPa to 0.36 MPa. When 10 pulse velocity measurements and 10 minimum rebound numbers were combined with core strengths the value of σ_{po} varied from 1.5 MPa for 4 core strengths to 0.96 MPa for 20 core strengths. When 400 pairs of NDT data were used μ_{po} varied from .30 MPa to 0.27 MPa for the two extremes.

As shown in Figure 4.9 above a certain threshold quantity of NDT data (in this study this quantity is about 200 measurements) there is no significant change in the value of σ_{po}. This implies that the confidence level for predicting μ_{po} does not improve significantly after 200 NDT measurements are taken. As intuitively expected the value of σ_{po} is smaller when both types of NDT data were used from each test location compared to when just pulse velocities were used, all other things being equal. When NDT data were taken from fewer than about 60 locations the value of σ_{po} was smaller for the case where pulse velocities were combined with 20 core strengths than when both types of NDT data were combined with 4 core strengths.

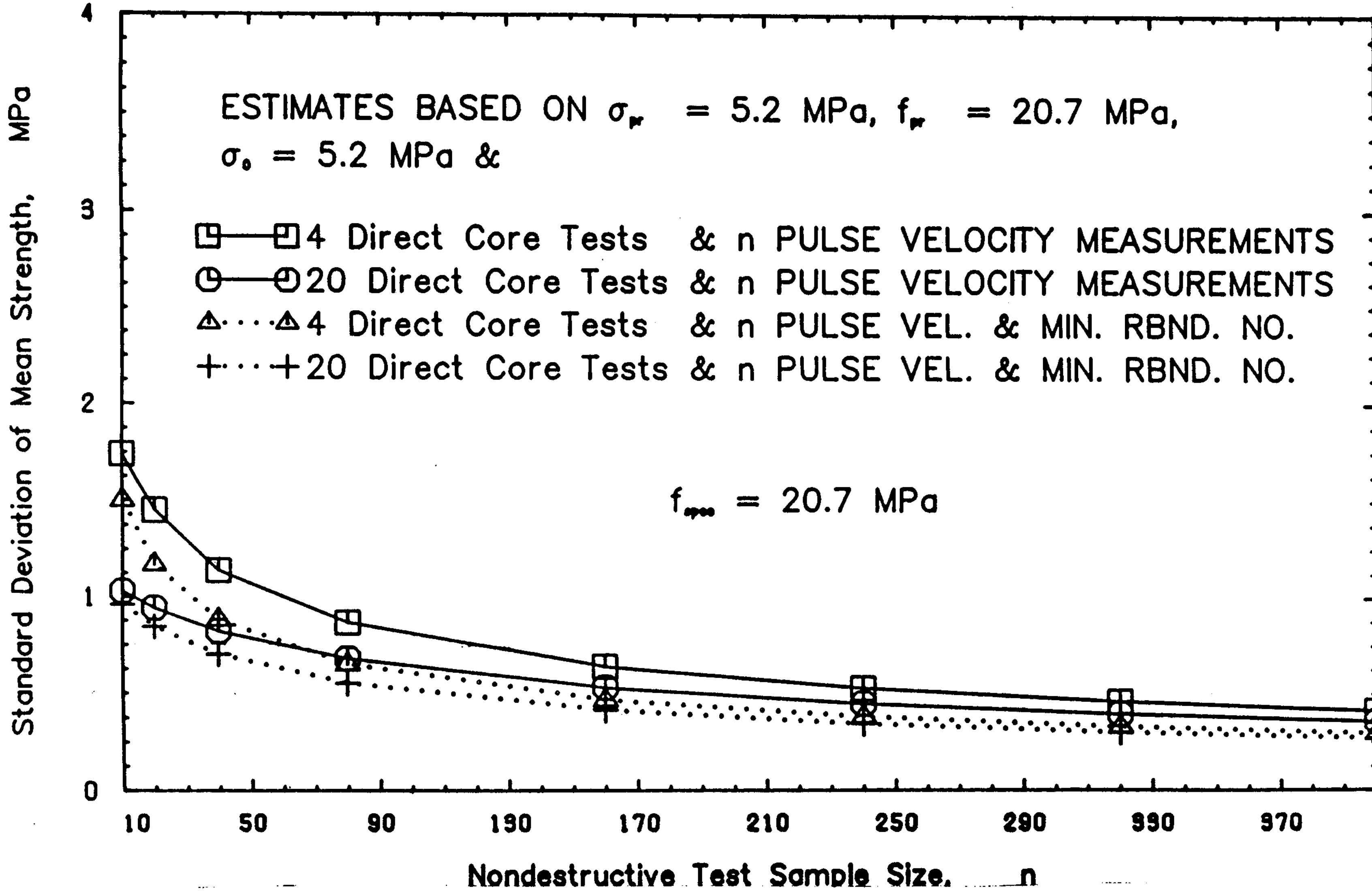

Fig. 4.9 Variation in the Estimate of the Mean Strength

4.8 Effect of Data Selection Procedure on Posterior Distribution of Mean Concrete Strength

As discussed in Section 4.4 when small quantities of randomly selected data are used to develop a regression relationship there is a high probability that the relationship will not be strictly valid over the range of NDT measurements for which it is required. Consequently small groups of randomly selected cores should not be relied upon. A quasi-random data selection procedure was proposed that appears to compensate for this potential problem. Although pure statistical data selection procedures are violated when the quasi-random method is used, its application may be justified when economic or other factors allow the removal of only a relatively small number of cores.

To evaluate the effect of the two data selection procedures on the prediction of μ_{po} 4 groups of analyses were performed. In all cases the sampled population standard deviation was considered to be 5.2 MPa and the prior distribution was described by μ_{pr} = 20.7 MPa and σ_{pr} = 5.2 MPa. Two of the four groups of analyses combined core strength data selected randomly (i.e. from Table C4) with pulse velocity measurements. The remaining two groups combined core strength data selected from the quasi-random data base (i.e. Table C5) with pulse velocity measurements. Four and twenty core strength measurements were considered for each of the two data selection procedures. In all four groups of the analyses the quantity of NDT data considered varied from 10 to 400 measurements in 8 increments. The NDT data were selected from Table C3.

Figure 4.10 shows the variation in the estimate of μ_{po} with pulse velocity sample size for the four groups of analyses. When only 10 pulse velocity measurements were combined with 4 core strengths the value of μ_{po} varied from 35.9 MPa for the random selection procedure to 33.2 MPa for the quasi-random selection procedure. When 400 pulse velocity measurements were considered the value of μ_{po} varied from 36.3 MPa to 33.9 MPa for the two extremes. When 10 pulse velocity measurements were combined with 20 core strengths the value of μ_{po} was approximately 35.5 MPa for both data selection procedures. When 400 pulse velocities were considered the value of μ_{po} was approximately 34.4 MPa for both procedures.

As seen in Figure 4.10 the value of μ_{po} is almost identical for both core data selection procedures when 20 core strengths were used. This is because only 1 of the 20 items of core data varies between the two procedures. However, as seen in the same figure, when only 4 core strengths were considered the prediction of μ_{po} is noticeably different for the two selection procedures. In this latter case only 1 of the regression analysis data is the same for both procedures.

The reason that the random procedure predicts μ_{po} to be larger than does the quasi-random procedure, when n = 4, is easily determined from Figure 4.11. This figure shows that the random regression relationship predicts concrete strength to be greater than does the quasi-random regression relationship. In addition the standard error of the random regression relationship is much smaller than it is for the quasi-random case. Both of

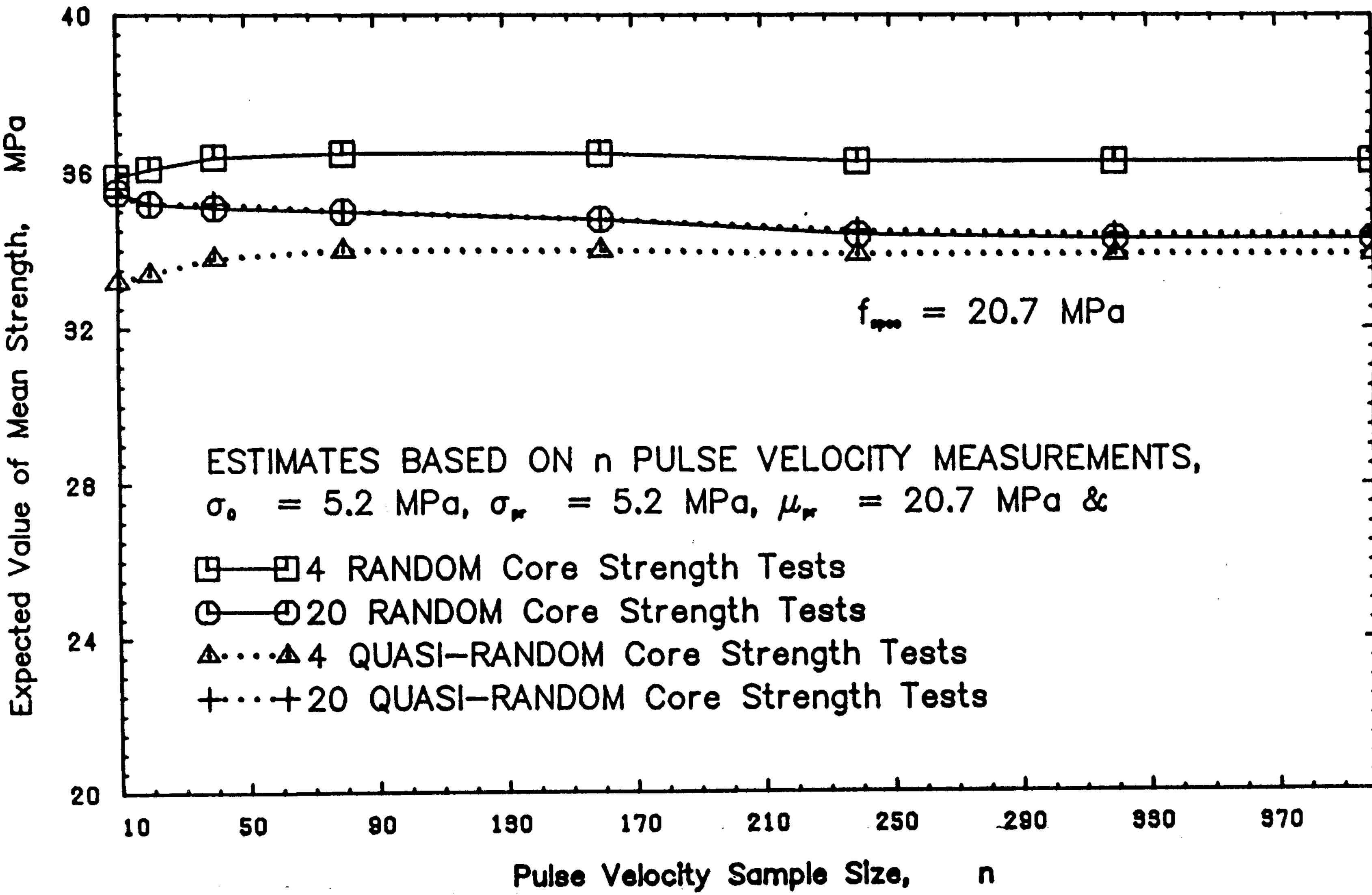

Fig. 4.10 Variation in the Estimate of the Mean Strength

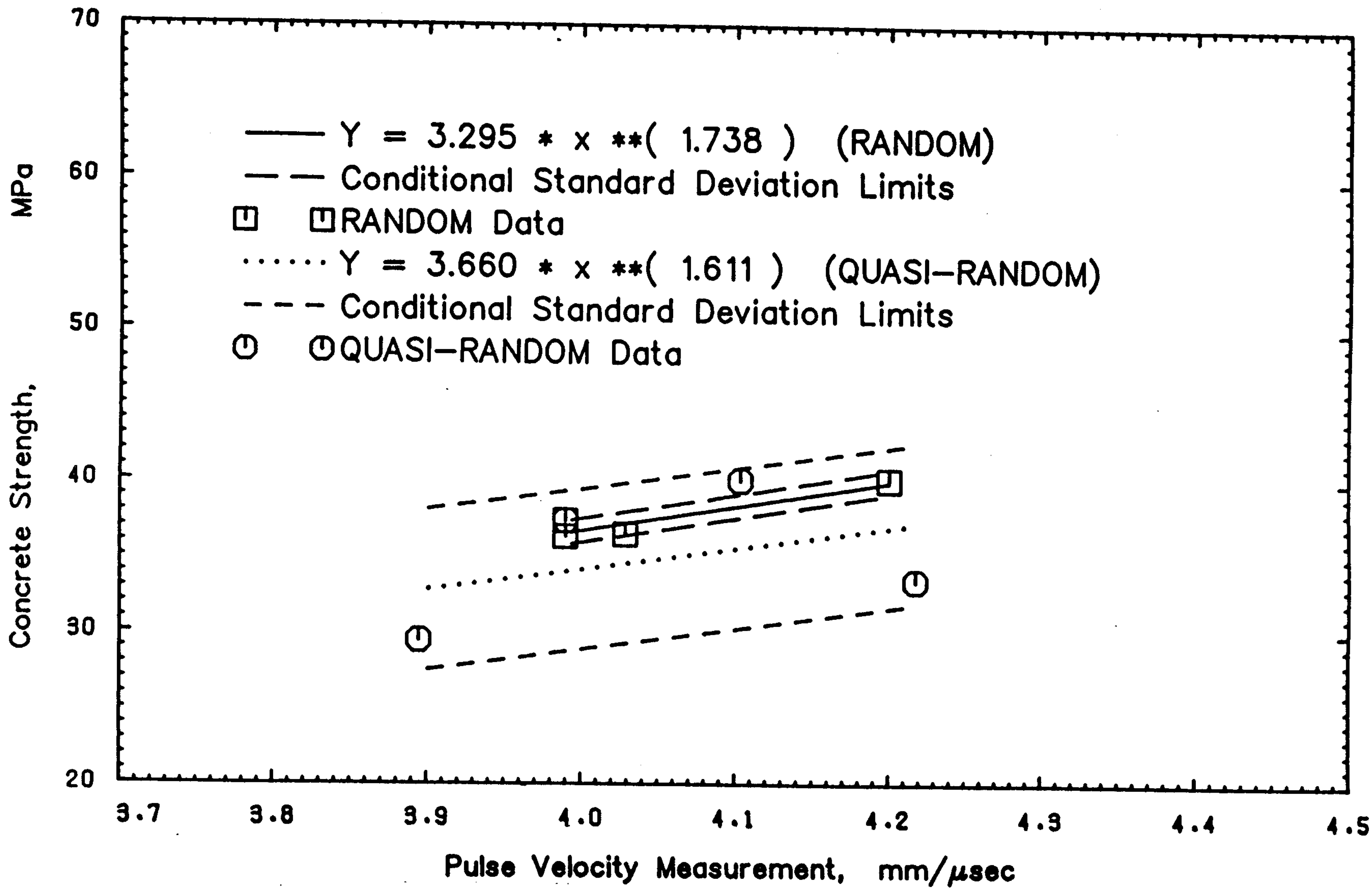

Fig. 4.11 Regression Analysis for n = 4

these factors contribute to the disparity that exists in the predicted value of μ_{po}. Regression relationships developed from 8, 12, 16, and 20 cores using the two data selection procedures are shown in Figures 4.12, 4.13, 4.14, and 4.15 respectively.

As seen in Figures 4.11 through 4.15 the quasi-random core selection procedure generally resulted in a regression relationship which was more widely applicable than in the random case. Comparison of the prediction of μ_{po} based on the two different 4 core regression relationships indicates that the quasi-random case results in more conservative estimates over the full range of NDT sample size. However one would not expect the same result for all random core selections. When more than 50 NDT measurements were considered the quasi-random approach predicted μ_{po} more closely to the 20 core case than did the random approach.

The same data used to investigate μ_{po} were used to evaluate the effect of data selection procedure on the value of σ_{po}. Figure 4.16 shows the variation in the estimate of σ_{po} with pulse velocity sample size for each of the 4 groups of analyses. When only 10 pulse velocity measurements were combined with 4 core strengths the value of σ_{po} varied from 1.36 MPa for randomly selected cores to 1.74 MPa for the quasi-random selection procedure. When 400 pulse velocities were considered the value of σ_{po} varied from 0.26 MPa to 0.41 MPa for the two extremes. When 10 pulse velocities were combined with 20 core strengths the value of σ_{po} was approximately equal to 1.00 MPa for both data selection procedures. When 400 pulse velocities were combined

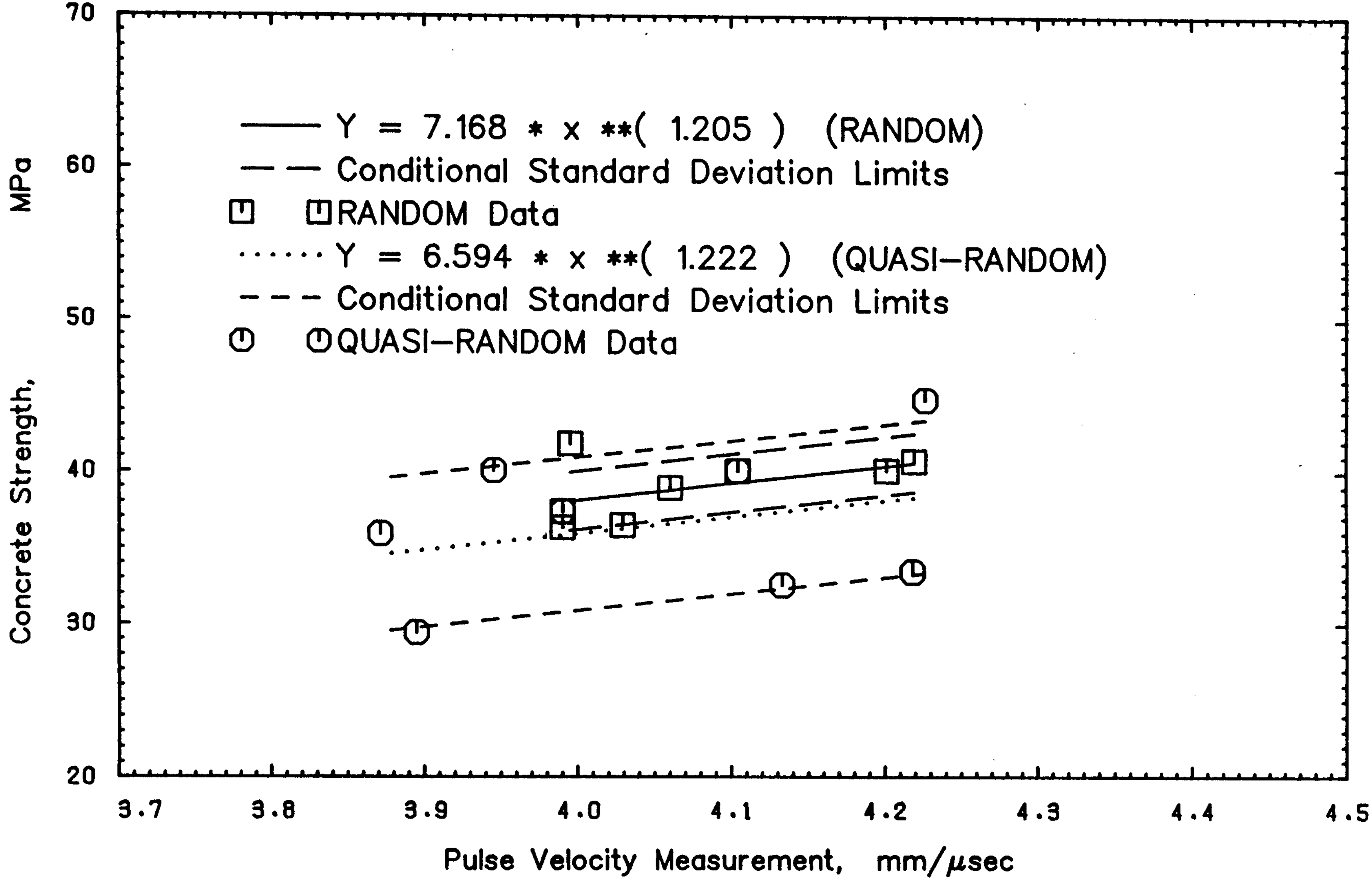

Fig. 4.12 Regression Analysis for n = 8

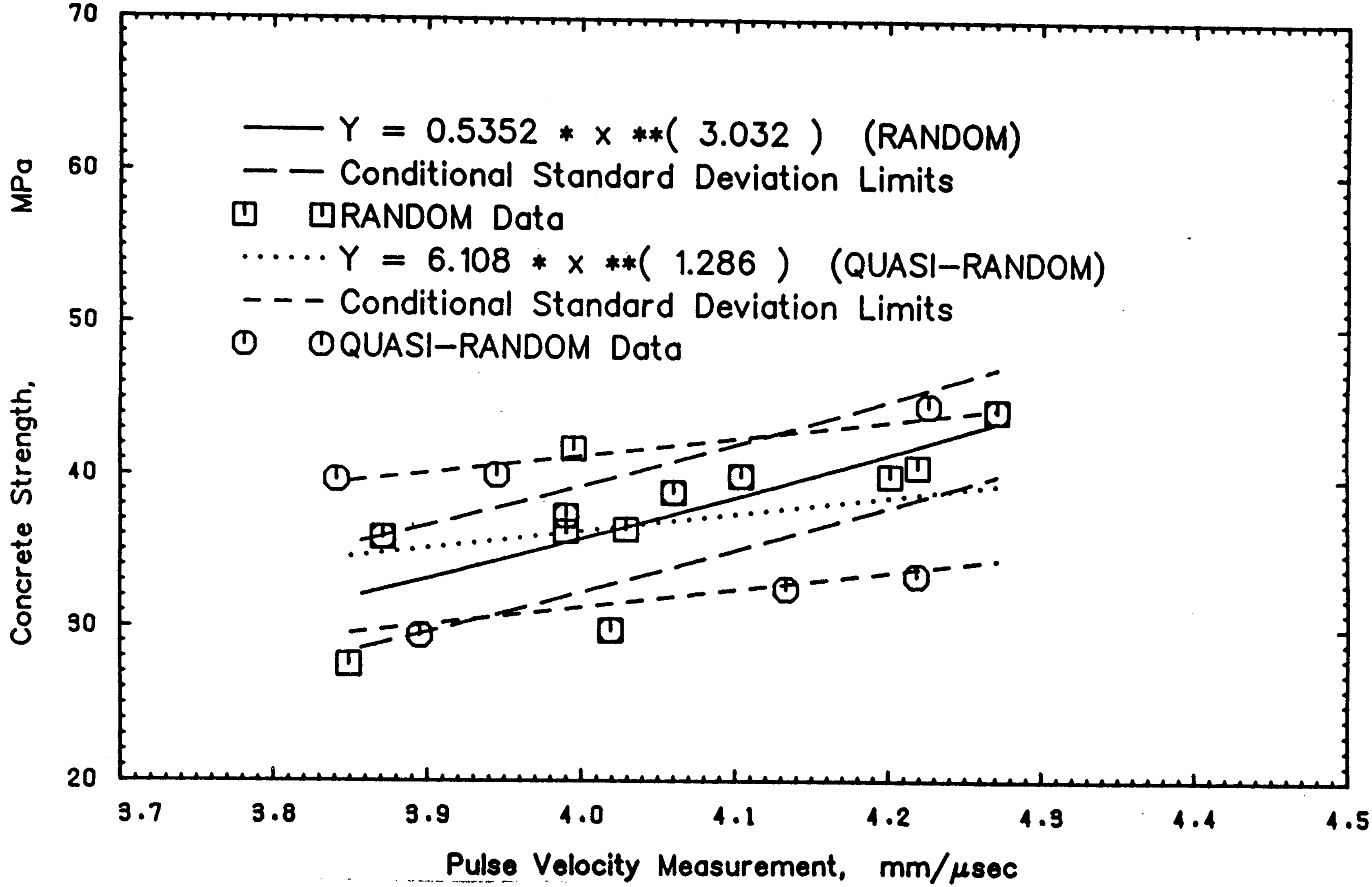

Fig. 4.13 Regression Analysis for n = 12

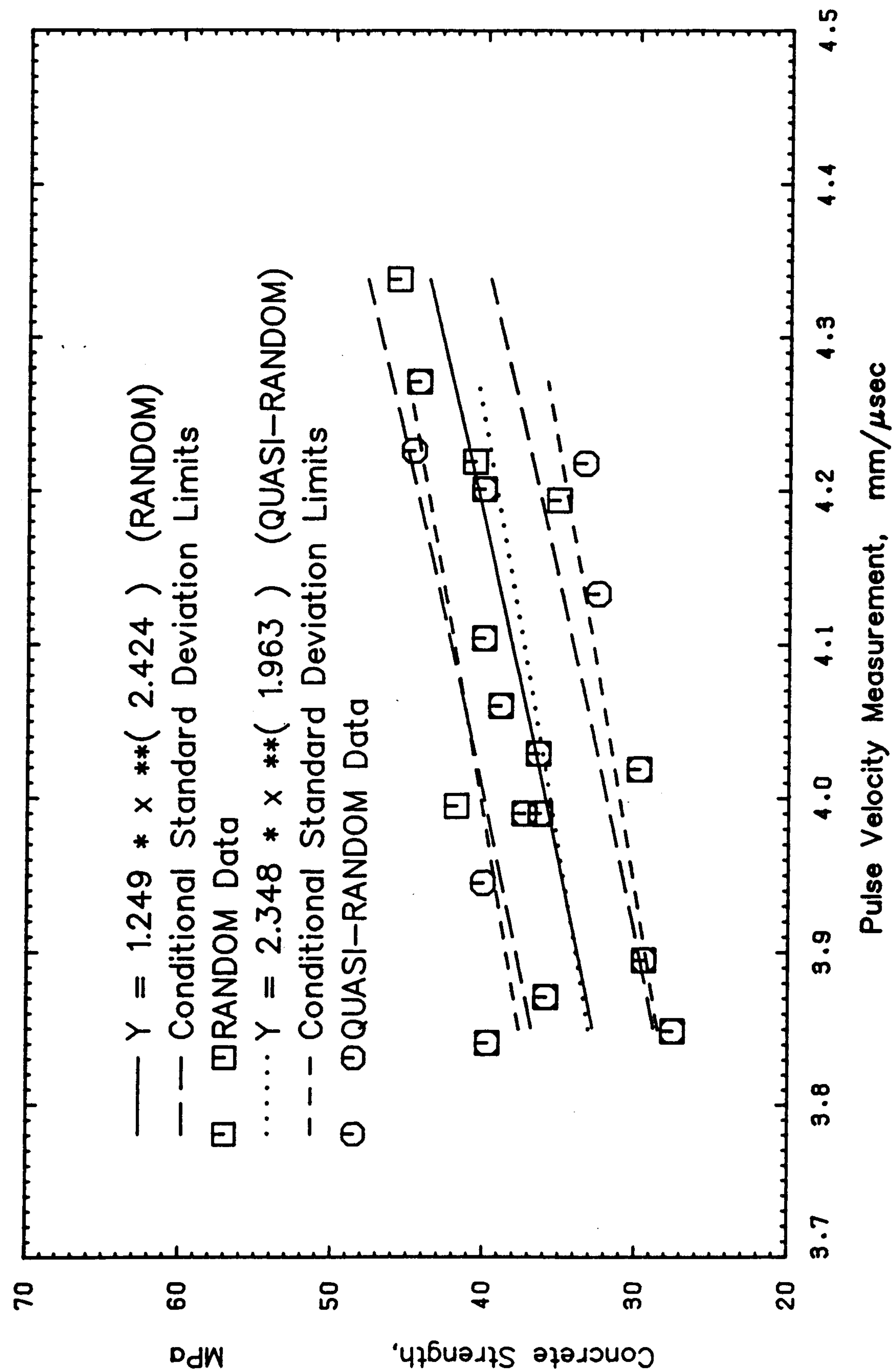

Fig. 4.14 Regression Analysis for n = 16

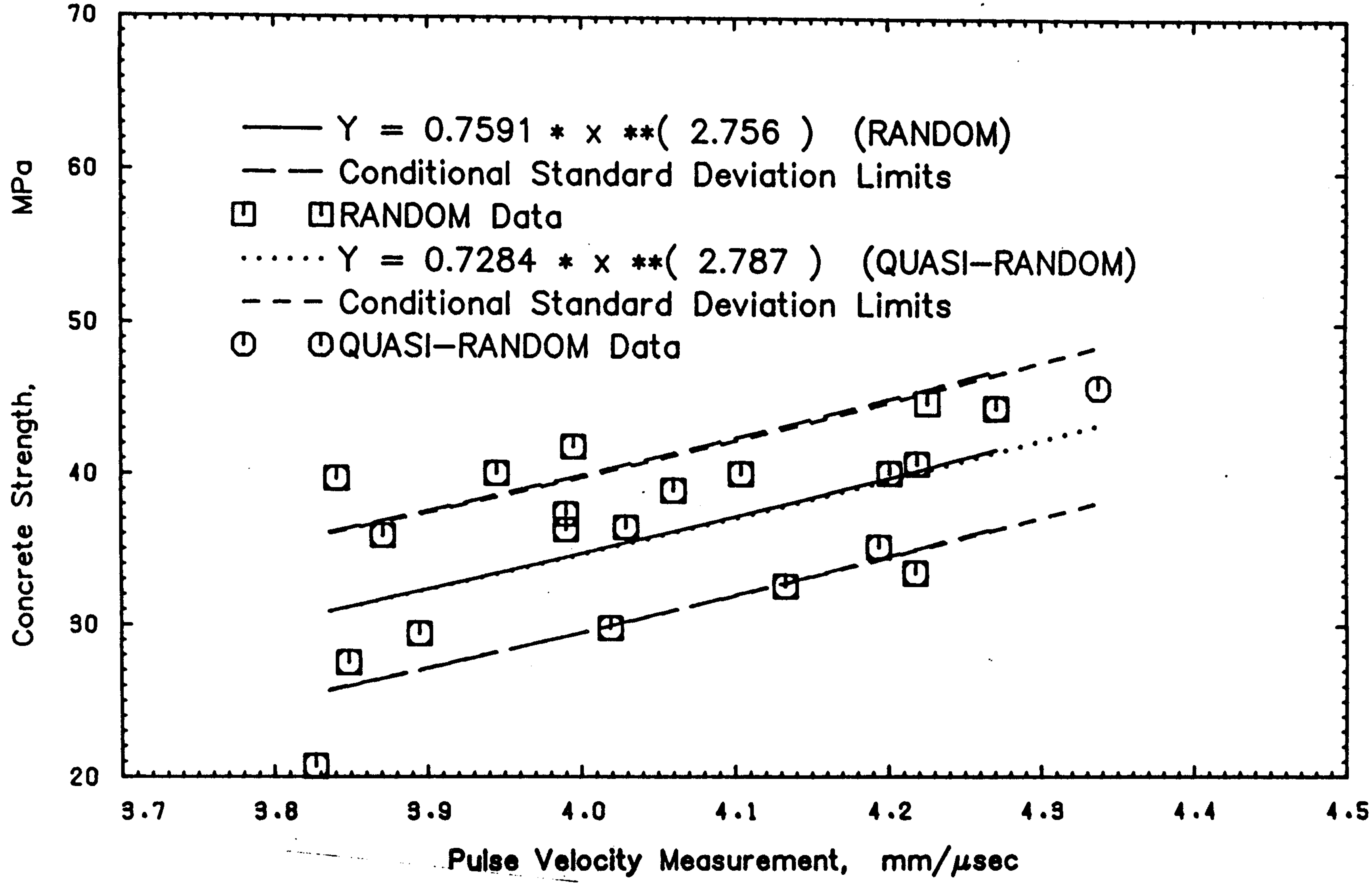

Fig. 4.15 Regression Analysis for n = 20

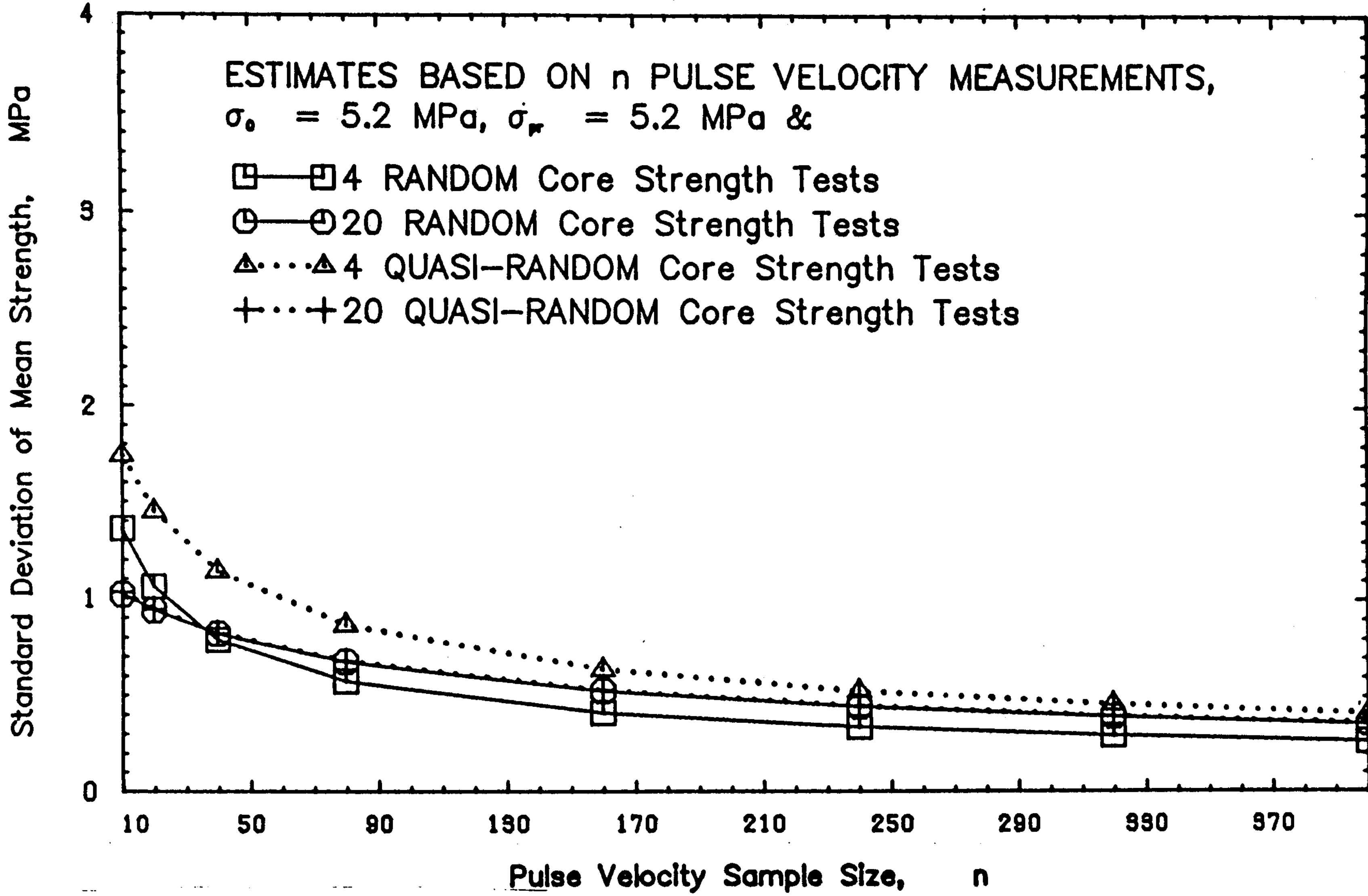

Fig. 4.16 Variation in the Estimate of the Standard Deviation of the Mean Strength

with 20 core strengths σ_{po} was approximately equal to 0.36 MPa for both data selection procedures.

The major reason why the values of σ_{po} as predicted using the random procedure were less than the quasi-random procedure was because the standard error of the regression relationship was significantly smaller for the former case. This is easily seen in Figure 4.11.

The results of the analysis shown in Figure 4.16 imply that the greatest confidence in the prediction of σ_{po} was associated with the case where pulse velocities were combined with 4 randomly selected core strengths. Intuitively the predictions associated with 20 core strengths would normally be considered better, however. This paradox did not occur when data were selected using the quasi-random procedure. For this reason the quasi-random data selection procedure appears to be superior when only small quantities of regression analysis data are used.

CHAPTER 5

Optimizing the Testing Program

5.1 Introduction

There are several approaches that can be adopted for optimizing a testing program. The best strategy to follow usually depends on the nature of the project and the purpose of the testing program. This chapter outlines optimization techniques for two different cases. The first method presented can be used when a predetermined fixed sum of money is provided for the testing program of a project. The second method given can be used when it is not clear at the outset of a project what amount of money should be invested in testing.

5.2 Approach I - Minimization of Posterior Variance

In some concrete strength investigations only a relatively small and limited amount of money is available for the testing programs from which the in-situ compressive strength prediction is made. In such situations Tang (1971) suggests that the best strength prediction will result if the testing program is planned to ensure that the posterior variance of the mean strength is minimized. Minimization of this variance is achieved by optimizing the number of core specimens and NDT measurements obtained during testing.

Due to the errors associated with transforming NDT measurements into equivalent concrete strengths a greater quantity of NDT data is required than core strength measurements in order to achieve equivalent posterior variances. Normally, however, the unit cost of NDT measurements is lower than the unit cost of core strength data. This latter factor normally offsets the effect of the former concern and makes the use of at least some NDT data a viable option in most cases. The concept of cost differences for the two types of test data is shown in Figure 5.1 where the following cost functions are plotted:

[5.1] $$C_C = A_C + \exp(n\ B_C)$$

[5.2] $$C_N = A_N + \exp(m\ B_N)$$

where C_C = total cost for core specimen data;

A_C = start up cost for coring;

B_C = coefficient relating sample size to coring costs;

n = core specimen sample size;

C_N = total cost for NDT data;

A_N = start up cost for NDTs;

B_N = coefficient relating sample size to NDT costs;

m = NDT sample size.

Since NDT data cannot be used effectively without a proper calibration relationship it is therefore always necessary to remove some cores during an investigation. Using the method outlined by Tang (1971) it is possible to determine if core strength data alone or in combination with NDT data will result in the smallest posterior variance for the mean strength. The

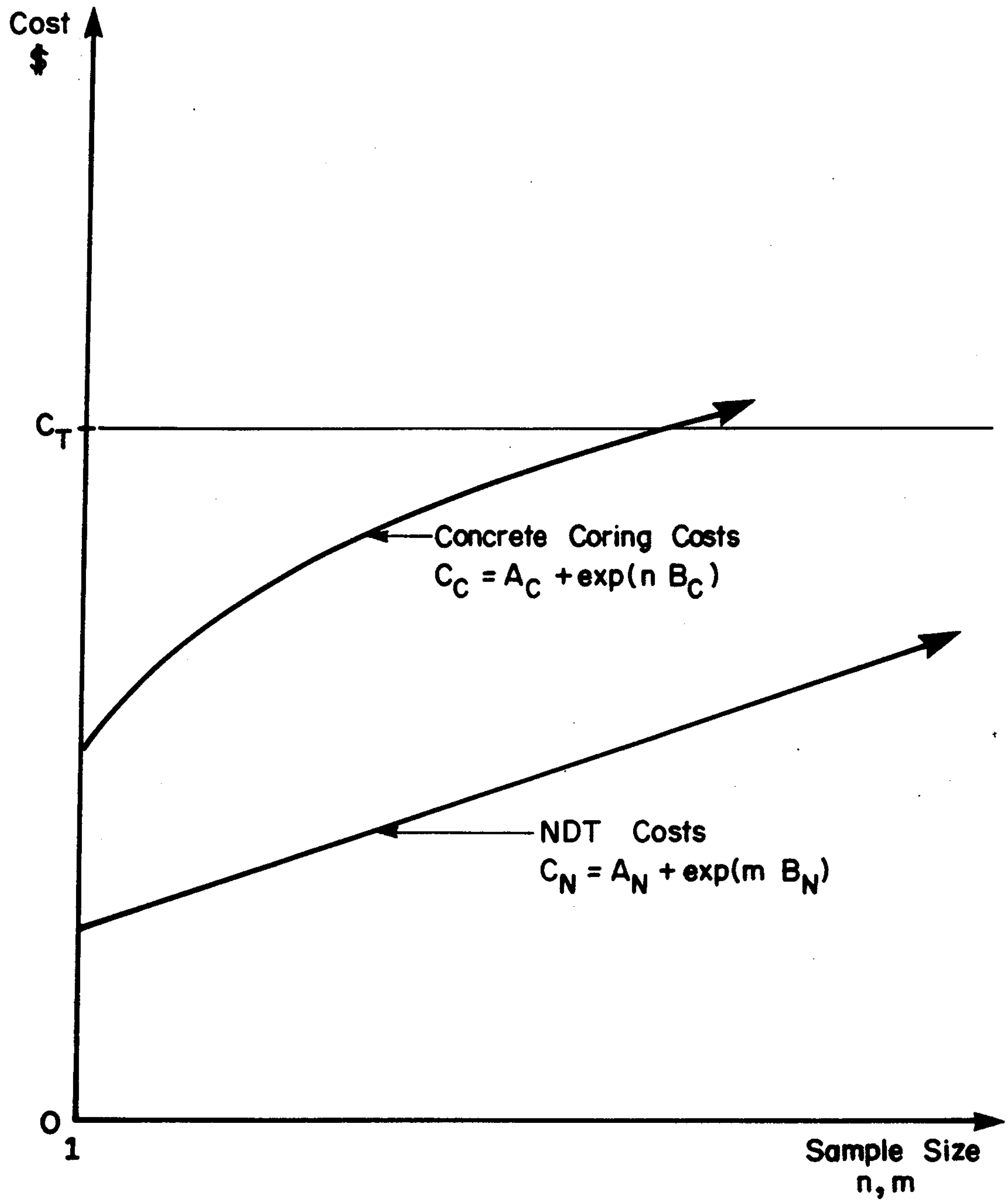

Fig. 5.1 Generalized Cost Functions for Two Concrete Test Methods

optimum number of each type of test data is determined by considering both the costs associated with each type of test and the effect that each type of test has on the posterior variance.

When only core strength data are used to predict the in-situ concrete strength then the relationship between posterior variance and the cost of testing (assuming a diffuse prior) is obtained by rearranging [5.1] and combining it with [3.14]. This relationship is given as

$$[5.3] \qquad (\sigma_{po})^2 = \frac{B_C \, (\sigma_o)^2}{\ln \, (C_T - A_C)}$$

where C_T = the total cost of all testing.

When one type of NDT data is used along with core strength data the posterior variance can be calculated approximately using

$$[5.4] \qquad (\sigma_{po})^2 = \frac{\frac{(\sigma_o)^2}{n} \, \{\frac{(\sigma_o)^2 + (\sigma_{x|v})^2}{m - n}\}}{\frac{(\sigma_o)^2}{n} + \{\frac{(\sigma_o)^2 + (\sigma_{x|v})^2}{m - n}\}}$$

Whenever NDT data are used in conjunction with core strength data the total number of NDT data, m, must be larger than the total number of core strength measurements, n. This is because n of the m NDT data are used in developing a calibration curve which is necessary for processing NDT measurements at locations where no core strength data are available. It is clear from [5.4] that if m is not greater than n, $(\sigma_{po})^2$ assumes an unacceptable value.

Equation [5.4] is derived from [3.34] and is based on the assumptions that the prior distribution is diffuse and that $(\sigma_{E_i})^2$ is negligible compared to $(\sigma_o)^2$ and $(\sigma_{x|v})^2$. The first assumption ensures that only the effect of test data on the posterior variance is considered. The second assumption simplifies [5.3] and is generally valid unless a relatively small quantity of data is used to establish the calibration relationship (see [3.17]).

The total number of NDT measurements that can be taken if a series of n core specimens are to be removed as well is determined using the following relationship which is developed by combining [5.1] and [5.2]:

$$[5.5] \qquad m = \frac{1}{B_N} \ln \{C_T - A_C - A_N - \exp(n\, B_C)\}$$

Substituting [5.5] into [5.4] and simplifying gives the relationship between the posterior variance, test costs, and core sample size as,

$$[5.6] \qquad (\sigma_{po})^2 = \frac{(\sigma_o)^2 \{(\sigma_o)^2 + (\sigma_{x|v})^2\}\, B_N}{(\sigma_o)^2 \ln\{C_T - A_C - A_N - \exp(nB_C)\} + nB_N(\sigma_{x|v})^2}$$

Whenever [5.6] provides a value for $(\sigma_{po})^2$ which is less than that given by [5.3], that is,

$$[5.7] \qquad \frac{(\sigma_o)^2 \{(\sigma_o)^2 + (\sigma_{x|v})^2\}\, B_N}{(\sigma_o)^2 \ln\{C_T - A_C - A_N - \exp(nB_C)\} + nB_N(\sigma_{x|v})^2} < \frac{B_C\, (\sigma_o)^2}{\ln(C_T - A_C)}$$

then both core specimens and NDTs should be taken. There is one exception to this rule. If $m \leqslant n$ as given by [5.5], then there is no advantage to taking any NDT measurements. The reason for this was discussed previously, and whenever this situation occurs all testing moneys should be used for coring. Simplifying [5.7] gives

$$[5.8] \qquad \frac{B_N \{(\sigma_o)^2 + (\sigma_{x|v})^2\} \ln (C_T - A_C)}{B_C(\sigma_o)^2 \ln \{C_T - A_C - A_N - \exp (nB_C)\} + nB_C B_N (\sigma_{x|v})^2} < 1$$

Equation [5.8] is a simple relationship that can be used to plan the testing program for an in-situ concrete strength investigation. However, in order to use it the following information must be available at the outset of the investigation:

an estimate of the

(i) cost function for concrete coring,

(ii) cost function for NDTs,

(iii) sampled population variance, $(\sigma_o)^2$, and

(iv) calibration relationship variance, $(\sigma_{x|v})^2$.

The relative success of using [5.8] to plan the testing program hinges directly on the accuracy of these four items. Therefore, engineers with previous experience in concrete strength investigations would benefit most in using [5.8].

Although [5.8] does indicate for a given value of n whether or not nondestructive testing should proceed, it does not indicate what value of n is optimum. As indicated in Section 4.8 the number of pairs of test data used in developing a calibration curve can affect the outcome of results measurably. Therefore n should be selected carefully following the guidelines set out in Chapter 4.

5.3 Approach II - Minimization of Potential Losses

During the lifetime of a reinforced concrete structure it is possible that the design loading will change. If the loading is increased then the structure will normally need to be upgraded unless it can be shown that the capacity of the existing structure is sufficient to resist the new loading. Often structural capacity is greater than originally designed due to the in-situ concrete strength being significantly greater than originally specified. As a consequence of this it is generally advantageous to perform in-situ strength tests prior to any renovations that are being considered. If the strength is determined to be greater than originally specified then the need for structural improvements may be minimized or negated all together.

Investigating the in-situ strength of concrete has with it associated costs. In order to determine whether or not the strength prediction will be of any value, these costs must be estimated and compared in some way with anticipated renovation costs. Since strength prediction costs increase as the accuracy of the prediction increases it is necessary to determine an acceptable level of accuracy as well. By applying a systematic procedure sometimes known as preposterior decision making (as outlined by Winkler 1972 and by Jones 1977) it is possible to predetermine the optimum testing program to use before renovations proceed. Based on the results of the testing program it is possible to determine if renovations are required or not. This procedure is described subsequently.

At the outset of a potential renovation project a primary decision that must be made is whether or not to proceed with renovations. (In most projects of this type there are often more than two possible choices from which to make the decision, however. This more general case, which is discussed by Winkler (1972), is not considered here in order to simplify the presentation.) There is a cost associated with each of these two possible decisions. If the decision to renovate was made, based on the assumption that the in-situ concrete strength was as originally specified, then the cost would be fixed regardless of what the actual strength was. If the decision was to not renovate there would be a varying cost depending upon what the actual in-situ strength was. This cost would be zero if the actual mean strength was equal to or greater than the new required mean strength and the cost would increase in some manner as the value of the in-situ strength decreased. Presumably this cost function would be based on several factors related to the cost to society of having an understrength structure. For the purposes of this study the cost function for not renovating will be considered linear. (Winkler (1972) discusses the case where cost functions are non-linear.) The cost functions for the two primary decisions are shown in Figure 5.2 and are given as follows:

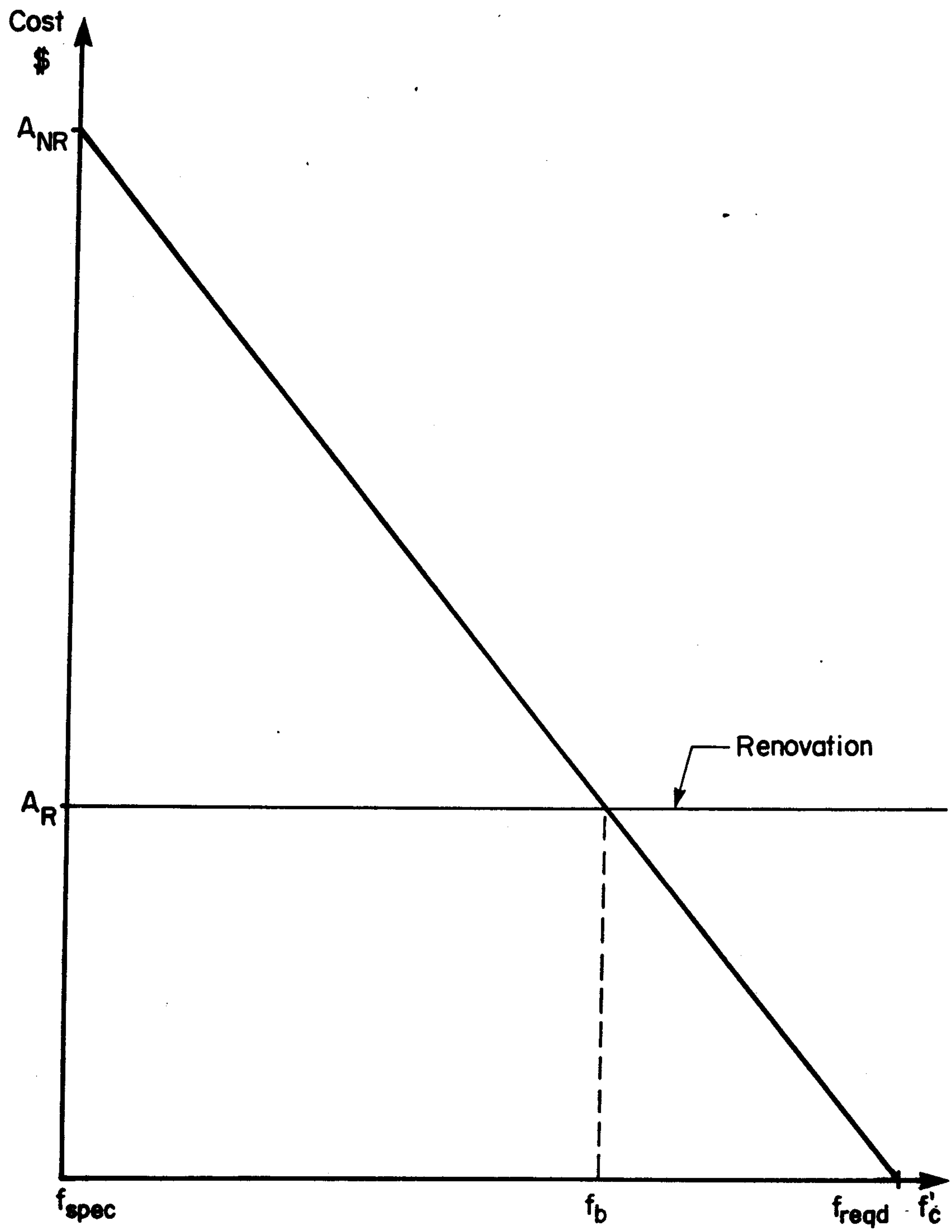

Fig. 5.2 Hypothetical Cost Functions for Structural Renovation Decision Problem

[5.9] $C_R = A_R$

and

[5.10] $C_{NR} = A_{NR} \left\{\frac{f_{reqd} - f'_c}{f_{reqd} - f_{spec}}\right\}$

where C_R = cost to renovate;

A_R = fixed cost of renovating;

C_{NR} = cost to not renovate;

A_{NR} = cost of not renovating if in-situ concrete strength equals f_{spec};

f_{reqd} = minimum required mean concrete strength for new loading conditions.

As indicated in Figure 5.2 there is a value of mean in-situ strength for which the cost of either decision is identical. This value of f'_c is known as the break-even strength, f_b, and is determined by equating [5.9] to [5.10]. The break even strength is given as

[5.11] $f'_c = f_b = f_{reqd} - \frac{A_R}{A_{NR}} (f_{reqd} - f_{spec})$

Except when $f'_c = f_b$ one of the two decisions is better to make than the other. As seen in Figure 5.2 when $f'_c < f_b$ the least expensive decision is to renovate. When $f'_c > f_b$ the least expensive decision is to not renovate. The difference in cost betwen the two decisions is known as the opportunity loss. The opportunity loss represents the financial penalty associated with making the wrong decision. Opportunity loss functions can be developed for the two possible decisions. The loss functions for this problem are shown in Figure 5.3 and are given as follows:

[5.12]

$$L_R = 0 \qquad \text{if } f_c' < f_b$$

$$L_R = A_R - A_{NR} \left\{\frac{f_{reqd} - f_c'}{f_{reqd} - f_{spec}}\right\} \qquad \text{if } f_c' > f_b$$

where L_R = opporunity loss when the decision is to renovate; and

[5.13]

$$L_{NR} = A_{NR} \left\{\frac{f_{reqd} - f_c'}{f_{reqd} - f_{spec}}\right\} - A_R \qquad \text{if } f_c' < f_b$$

$$L_{NR} = 0 \qquad \text{if } f_c' > f_b$$

where L_{NR} = opportunity loss when the decision is to not renovate.

If for a potential renovation project it was known for certain that the mean in-situ concrete strength was f_p (see Figure 5.3) then the opportunity loss or the cost of making the wrong decision would be C_p; in this example the wrong decision would be to not renovate. Unfortunately perfect information that leads to exact in-situ strength predictions cannot be realistically obtained. However, if the exact value of f_c' could be obtained for a price it would only be beneficial to purchase the information if the cost did not exceed C_p. In other words it

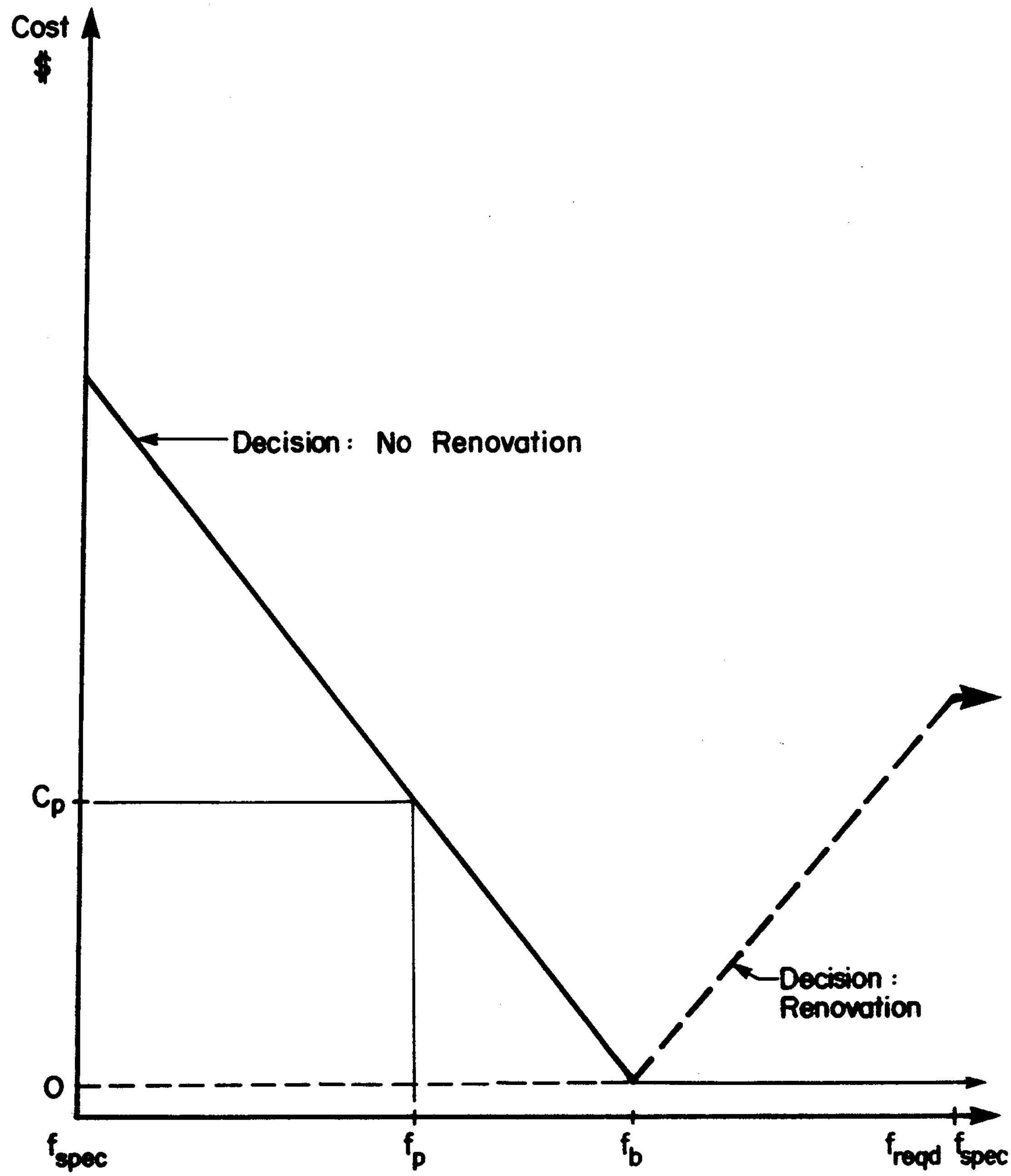

Fig. 5.3 Opportunity Loss Functions for Structural Renovation Problem

would only be worth purchasing the information describing the exact value of f'_c if the cost of this information plus the cost of renovating together did not exceed the cost of not renovating. This maximum acceptable cost, C_p, is known as the value of perfect information (VPI).

In order to determine the maximum amount of money that should be spent on investigating f'_c it is necessary to first have an estimate of the in-situ strength. Without this information it is impossible to proceed. Using the method described in Section 3.2 it is possible to establish a prior distribution for the mean in-situ strength. This estimate, which is defined by both a mean, μ_{pr}, and a standard deviation, σ_{pr}, is then compared with the two loss functions. Based on this comparison there is usually a good indication as to which of the two possible decisions is best. For the example shown in Figure 5.4 the decision to renovate has the smaller probability of opportunity loss and is therefore likely the best decision. However, as seen in the figure, there remains a probability that losses will occur even if the decision to renovate is taken. The probable losses associated with the decision to renovate give an upper limit to the amount of money that should be spent on information to improve the expected value of f'_c.

The expected opportunity loss associated with the decision to renovate is calculated by multiplying the opportunity loss function for that decision by the prior probability distribution for f'_c. Since both the loss function and the prior distribution are continuous, integration must be used in this calculation as

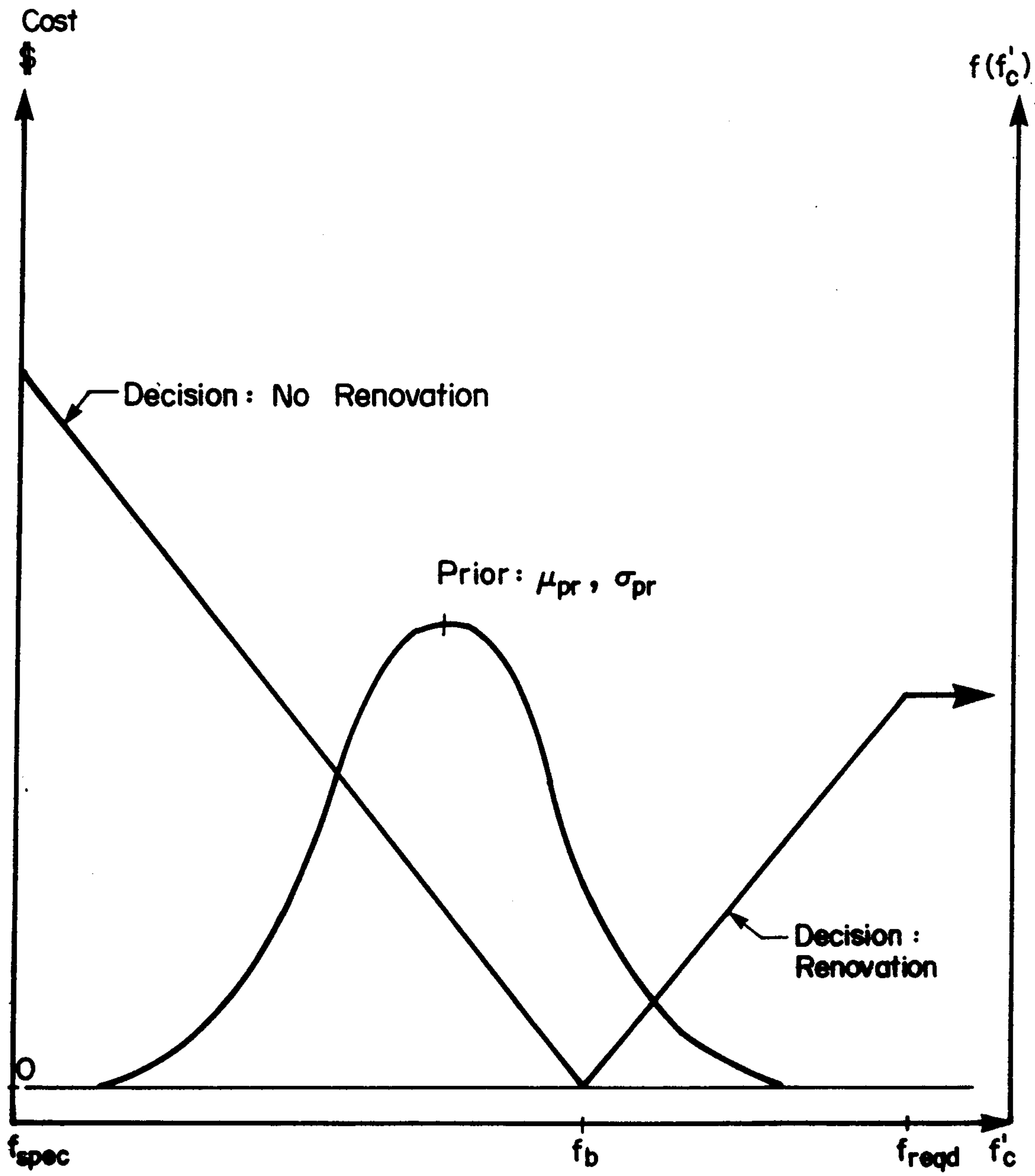

Fig. 5.4 Normal Prior Distribution Superimposed on Opportunity Loss Functions

follows:

$$[5.14] \qquad EL_R = \int_{-\infty}^{\infty} L_R \cdot f(f'_c)\, df'_c$$

where EL_R = expected opportunity loss if the decision is made to renovate;

$f(f'_c)$ = normal probability density function for the prior distribution of the mean in-situ concrete strength.

Substituting [5.12] into [5.14] gives

$$[5.15] \qquad EL_R = \int_{-\infty}^{f_b} 0\, f(f'_c)\, df'_c + \int_{f_b}^{\infty} \{A_R - A_{NR}(\frac{f_{reqd} - f'_c}{f_{reqd} - f_{spec}})\}\, f(f'_c)\, df'_c$$

From [5.11]

$$[5.16] \qquad A_{NR} = A_R\, (\frac{f_{reqd} - f_{spec}}{f_{reqd} - f_b})$$

Substituting [5.16] into [5.15] gives

$$[5.17] \qquad EL_R = \frac{A_R}{(f_{reqd} - f_b)} \int_{f_b}^{\infty} (f'_c - f_b)\, f(f'_c)\, df'_c$$

The expected opportunity loss given by [5.17] is also equal to the maximum amount of money that should be spent on improving the prediction of the mean in-situ concrete strength. This value is similar to the VPI and is known as the expected value of perfect information (EVPI).

A similar relationship to [5.17] can be developed for predicting expected opportunity loss when the best decision is to not renovate. This relationship is given as

$$[5.18] \qquad EL_{NR} = \frac{A_R}{(f_{reqd} - f_b)} \int_{-\infty}^{f_b} (f_b - f_c') \, f(f_c') \, df_c'$$

where EL_{NR} = expected opportunity loss if the decision is made to not renovate.

Equation [5.18] also gives the EVPI when the decision not to renovate appears best.

Evaluation of [5.17] or [5.18] is simplified when these equations are put into a form which contains the unit normal loss integral ($L_N(D)$). Either equation can be expressed by the following relationship:

$$[5.19] \qquad EL_R = EL_{NR} = EVPI = \left| \frac{A_R}{(f_{reqd} - f_b)} \right| \sigma_{pr} \, L_N(D)$$

where $L_N(D) = \int_D^{\infty} (f_c' - D) \, f_N(f_c') \, df_c'$

$$D = \left| \frac{f_b - \mu_{pr}}{\sigma_{pr}} \right|$$

$$f_N(f_c') = (2\pi)^{-1/2} \exp\{- \tfrac{1}{2} (f_c')^2\}$$

Reference to a table of values for $L_N(D)$ (see for instance Jones 1977 or Winkler 1972) makes it possible to evaluate [5.19] with ease.

Winkler (1972) has highlighted the following important characteristics about [5.19]:

1) The first term in the relationship, $\left| A_R / (f_{reqd} - f_b) \right|$, represents the slope of the non-zero portion of both loss functions. As the magnitude of this value increases so too does the expected loss for an incorrect decision.

2) Increases in σ_{pr} cause increases in the expected loss in two ways. First there is a direct effect as seen in [5.19]. Second, as σ_{pr} increases D decreases and $L_N(D)$ increases. The effect of increasing σ_{pr} as noted above is intuitively obvious, because σ_{pr} is a measure of error in the estimate of f'_c and as this error increases the probability of an incorrect decision and associated increased potential losses must increase as well.

3) As the difference between the breakeven strength, f_b, and the prior estimate of the concrete strength, μ_{pr}, increases then the value of D increases and $L_N(D)$ decreases resulting in a decrease in the expected losses. This observation indicates that the probability of loss decreases as the mean in-situ strength is located further into the non-zero range of one or the other loss functions.

Once the upper limit to the value of information is determined using [5.19] it is possible to develop a strategy for determining the optimal in-situ testing program. Normally a testing program would collect both NDT and concrete core data. This data would then be combined with the prior distribution following the procedure outlined in Chapter 3. To simplify the presentation that follows only core strength data will be considered.

With minor changes to [5.19] it is possible to predict the expected value of sample information (EVSI) before tests are taken. If a sample size of n cores is to be taken and it is assumed that the prior mean strength remains unchanged then the EVSI is given as

$$[5.20] \qquad EVSI = \left| \frac{A_R}{f_{reqd} - f_b} \right| \sigma_* \, L_N(D_*)$$

where $D_* = \left| \dfrac{f_b - \mu_{pr}}{\sigma_*} \right|$

$$\sigma_*^2 = \sigma_{pr}^2 - \sigma_{po}^2$$

Substitution of [3.14] into the expression for σ_*^2 gives

$$\sigma_*^2 = \frac{n\,\sigma_{pr}^4}{\sigma_o^2 + n\sigma_{pr}^2}$$

The value used for the variance in [5.20], σ_*^2, represents the reduction in the variance of the assumed mean in-situ strength due to sampling. Although the mean strength will likely change due to sampling it is not possible to predict this change until sampling has actually occurred. Thus μ_{pr} is used in

[5.20]. After the testing is performed the new estimate of f_c' must be calculated and compared with μ_{pr}. Further consideration of this detail will be discussed subsequently.

As the size of the sample increases the value of the information gets larger because the error in the estimation of the mean in-situ strength becomes smaller. With a sufficiently large number of samples the EVSI is approximately equal to the EVPI. However, due to the cost of acquiring the sample information there is a desirable upper limit to the amount of money to be spent on testing. As discussed previously testing costs should not exceed the EVPI. The maximum sample size can be determined by equating [5.19] and the cost function for testing ([5.1]) and solving for n_{max}.

The optimum sample size is determined by maximizing the difference between the EVSI and the cost of testing. By calculating the EVSI and the cost of testing for a range of sample sizes between 1 and n_{max} it is possible to determine the sample size n_{opt} which results in the largest net value. Figure 5.5 shows the relationship between sample size and the value of test informaiton for a hypothetical situation.

After taking a sample of size n_{opt} it is possible to calculate μ_{po} using [3.13]. If the new value of f_c' as given by μ_{po} is separated from μ_{pr} by f_b then the original best decision has changed. When this occurs the testing program should be continued so that in-situ strength confirmation can be made. The posterior distribution resulting from the first test sequence (i.e. μ_{po} and σ_{po}) are used as the prior distribution for the

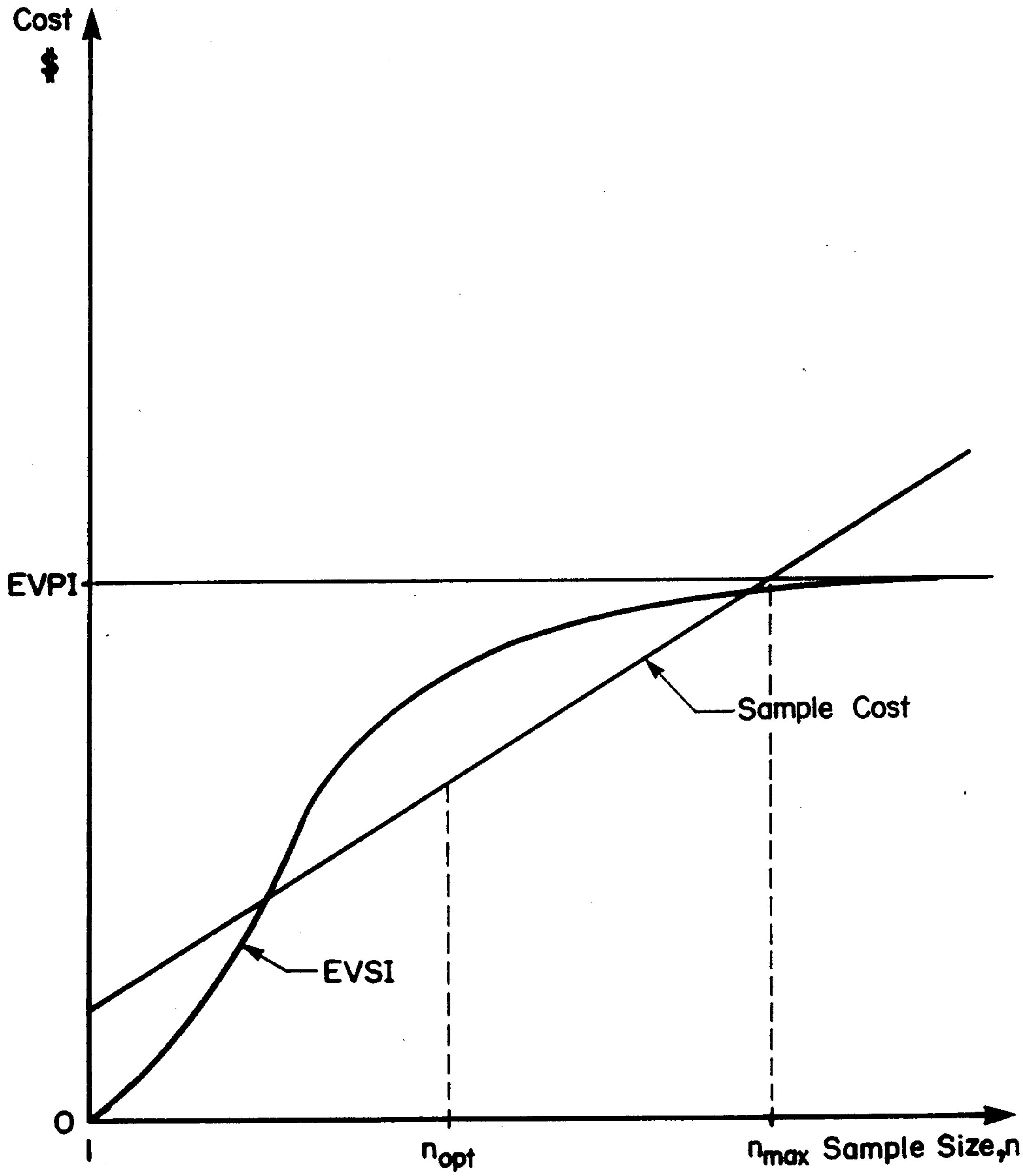

Fig. 5.5 Hypothetical Comparison of the Expected Value of Sample Information and the Cost of Testing

next iteration. Even when the best decision does not change due to testing it is sometimes desirable to continue testing, especially if the EVPI remains significant after initial tests. Ideally, preposterior analysis should be carried out until the optimal sample size n_{opt} is zero and further testing is of no value.

CHAPTER 6

Summary and Conclusions

6.1 Summary

The major goal of this research project was to present and evaluate the Bayesian statistical method for analyzing test data to predict the strength of concrete in existing structures.

An introduction to Bayesian theory was first presented. Bayes' theorem was derived for both discrete and continuous random systems. Application of the theorem to problems of updating statistical distributions was described in general terms for both the discrete and continuous cases. An example problem for each of the two cases was also given complete with solution.

Simple closed form equations were derived to evaluate in-situ test data and other pertinent information for predicting the concrete strength. Direct (core strength) data or both direct data and indirect (non-destructive test (NDT)) data can be evaluated. The errors associated with transforming NDT data into equivalent compressive strengths were accounted for in the equations. These equations were developed using Bayes' theorem for continuous random variables. Concrete strength was considered to be normally distributed, as is usually the case, and therefore the equations could be used for other Gaussian type problems. Another basic assumption used when developing the equations was that the variance of the concrete population being sampled was known. Consequently, the equations can only be used to predict the mean strength of the concrete being studied. It was shown that the variance can be reasonably estimated by other means.

A parametric study was performed to evaluate how four major variables influence the in-situ strength prediction as given by the equations. The four variables were the type of prior information, the variance of the sampled population, the amount and type of new data, and the data selection procedure. Field data from an in depth investigation of a concrete bridge was used in the study. The data included pulse velocity measurements, rebound hammer readings, and concrete core strengths.

Two procedures for optimizing field testing programs were also presented in this report. The first method was based entirely on the Bayesian equations and can be used for projects in which the testing costs are fixed. The second method was based in part on Bayesian statistics and in part on decision theory. This method can be used to determine what the costs of a testing program should be.

6.2 Conclusions

The Bayesian relationships derived herein form the basis for a simple approach for evaluating in-situ test data for the purpose of predicting the mean concrete strength. Although the procedure allows for the systematic combination of prior information with any amount of new direct data or direct and indirect data it appears that some amount of engineering judgement should be applied when planning testing programs and when interpreting test results. The following observations, based upon the parameter study reported in Chapter 4, are helpful in this regard:

1. When only a relatively small amount of test data is available non-diffuse prior information significantly influences the mean strength prediction (μ_{po}) and the standard error (σ_{po}) in this prediction. For large samples of data the effect of the prior on μ_{po} and σ_{po} disappears. These characteristics are true regardless of the type(s) of test data being used. Therefore, relatively small testing programs should be relied upon only when there is no doubt that the prior information accurately describes the strength characteristics of the concrete being studied.

2. The influence of the sampled population variance ($\sigma_o{}^2$) on the prediction of μ_{po} is significant only when a relatively small quantity of new test data is available. This observation is true for σ_{po} as well. This characteristic is similar to that observed for the prior except that the influence of σ_o extends over a larger range of sample size. When no prior information is available (i.e. a diffuse prior is specified) then σ_o only affects the error in the prediction of the mean (i.e. σ_{po}), not the magnitude of the mean (μ_{po}). Although the relative magnitude of σ_o is an insignificant factor when large quantitites of data are available, it should be selected carefully when only relatively small quantities of new data are available. As the value of σ_o increases the estimation of

μ_{po} decreases and σ_{po} increases. These observations are true for direct data alone and for direct data combined with indirect data.

3. When a set of core strength data (4 or more measurements) is combined with NDT measurements the predicted value of the mean strength does not vary significantly as the number of NDT measurements varies from 10 to 400. However, there are significant reductions in σ_{po} as the NDT sample size varies from 10 to 100. There is no advantage to taking more than about 100 NDT measurements for one homogeneous element. Also, there appears to be no real advantage to taking more than one type of NDT measurement.

4. When relatively small amounts of core strength data are being used to form a regression relationship for processing NDT data it is best to select the core locations systematically, not randomly. Systematic core location selection can ensure that the resulting regression relationship is applicable over the full range of NDT measurements. Based on this research, systematic selection also seems to prevent over-conservative predictions of μ_{po}.

6.3 Areas of Further Study

1. Bayesian statistics can be used to develop procedures for predicting two parameters instead of just one (as was done herein). Application of Bayesian theory in this way would allow

both μ_{po} and σ_o to be predicted. The implications of assuming σ_o, as was done in this study, could also be determined.

2. Introductory theory and simple procedures for optimizing the testing program of a concrete strength investigation have been presented; however, no evaluation has been performed. Verification and if necessary improvement of the procedures given would be beneficial.

3. In-situ testing of laboratory prepared structural elements in conjunction with destructive tests of the elements would provide excellent data with which the Bayesian relationships presented herein could be further evaluated.

4. Based on the research reported here and on further related studies it may be possible to develop general guidelines for in-situ concrete strength investigations.

REFERENCES

ACI Committee 209. 1982. Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures - Report No. ACI 209R-82. Designing for Creep and Shrinkage in Concrete Structures - A tribute to Adrian Pauw. American Concrete Institute, Detroit, Michigan, pp. 193-300.

Ang, A. H-S., and Tang, W.H. 1975. Probability Concepts in Engineering Planning and Design - Volume 1, Basic Principles. John Wiley and Sons, Inc., New York, New York, 409 p.

Bazant, Z.P. and Chern, J.C. 1984. Bayesian Statistical Prediction of Concrete Creep and Shrinkage. Journal of the American Concrete Institute, 81(4), pp. 319-330.

Belz, M.H. 1973. Statistical Methods for the Process Industries. John Wiley and Sons, Inc., New York, New York, 706 p.

Breipohl, A.M. 1970. Probabilistic Systems Analysis - An Introduction to Probabilistic Models, Decisions, and Applications of Random Processes. John Wiley and Sons, Inc., New York, New York, 352 p.

Di Leo, A., Pascale, G. and Viola, E. 1984. Core Sampling Size in Nondestructive Testing of Concrete Structures. In Situ/Nondestructive Testing of Concrete - ACI Publication SP-82. American Concrete Institute, Detroit, Michigan, pp. 459-477.

Guttman, I., Wilks, S.S. and Hunter, J.S. 1982. Introductory Engineering Statistics, 3rd Edition. John Wiley and Sons, Inc., New York, New York, 580 p.

Jones, J.M. 1977. Introduction to Decision Theory. Richard D. Irwin, Inc., Homewood, Illinois, 369 p.

Malhotra, V.M. (Editor) 1984. In Situ/Nondestructive Testing of Concrete - ACI Publication SP-82. American Concrete Institute, Detroit, Michigan, 831 p.

Mather, K. 1985. Preservation Technology: Evaluating Concrete in Structures. Concrete International: Design and Construction, Vol. 7, No. 10, pp. 33-41.

Mikhailovsky, L. and Scanlon, A. 1985. Evaluation of Existing Bridge Structure by Nondestructive Test Methods. Structural Engineering Report No. 128, Department of Civil Engineering, The University of Albera, 194 p.

Mirza, S.A., Hatzinikolas, M., and MacGregor, J.G. 1979.

Statistical Descriptions of Strength of Concrete. ASCE Journal of the Structural Division, 105 (ST6), pp. 1021-1037.

Rao, H.G. and Corotis, R.B. 1982. Bayesian Analysis of Regional Wind Energy Potential. ASCE Journal of the Engineering Mechanics Division, 108 (EM6), pp. 1198-1214.

Samarin, A. and Dhir, R.K. 1984. Determination of In Situ Concrete Strength: Rapidly and Confidently by Nondestructive Testing. In Situ/Nondestructive Testing of Concrete - ACI Publication SP-82. American Concrete Institute, Detroit, Michigan, pp. 77-94.

Scanlon, A. 1984. Reliability of Existing Concrete Structures. Presented at 1984 Spring Convention, American Concrete Institute, Phoenix, Arizona, 19 p.

Schmitt, S.A. 1969. Measuring Uncertainty - An Elementary Introduction to Bayesian Statistics. Addison-Wesley Publishing Company, Reading, Massachusetts, 400 p.

Tang, W.H. 1971. A Bayesian Evaluation of Information for Foundation Engineering Design. Proceedings, First International Conference on Applications of Statistics and Probability to Soil and Structural Engineering, Hong Kong University Press, pp. 174-185.

Tso, W.K. and Zelman, I.M. 1970. Concrete Strength Variation in Actual Structures. Journal of the American Concrete Institute, 67 (12), pp. 981-988.

Viola, E. 1983. On the Bayesian Approach to Estimate Compressive Strength of Concrete In Situ. Proceedings, Fourth International Conference on Applications of Statistics and Probability in Soil and Structural Engineering, Vol. 2, Department of Civil Engineering, University of Florence, Italy, pp. 1155-1166.

Winkler, R.L. 1972. An Introduction to Bayesian Inference and Decision. Holt, Rinehart, and Winston, Inc., New York, New York, 563 p.

APPENDIX

APPENDIX A

Example of Bayesian Inference for a Discrete Random System

Problem

An engineer at an ironworks factory is in charge of ordering reinforcing steel from a supplier. The factory has a stockpile of reinforcement on hand and the management tries to maintain the distribution of stock proportional to its use. The ironworks factory has been operating at capacity for the last year and will likely continue at capacity for the foreseeable future. Because of the steady-state conditions the engineer has been ordering about the same tonnage of reinforcement every month for the last year; all that changes from order to order is the distribution of the bar sizes. The engineer in completing his next order refers to the statistical distributions of bar sizes used during the last year and during the current month (see Figure A1). What distribution of bar sizes should the engineer order?

Solution

The PMF for the distribution of bar sizes required on average over the last year is considered to be the prior distribution (see Figure A1). The distribution of bar sizes required in just the last month constitutes the likelihoods for each of the possible events. A reasonable prediction of what the distribution of bar sizes should be in the upcoming order can be attained by using Bayes' theorem in the form of [2.8]. The decimal of one unit that should be ordered as 10M bars is

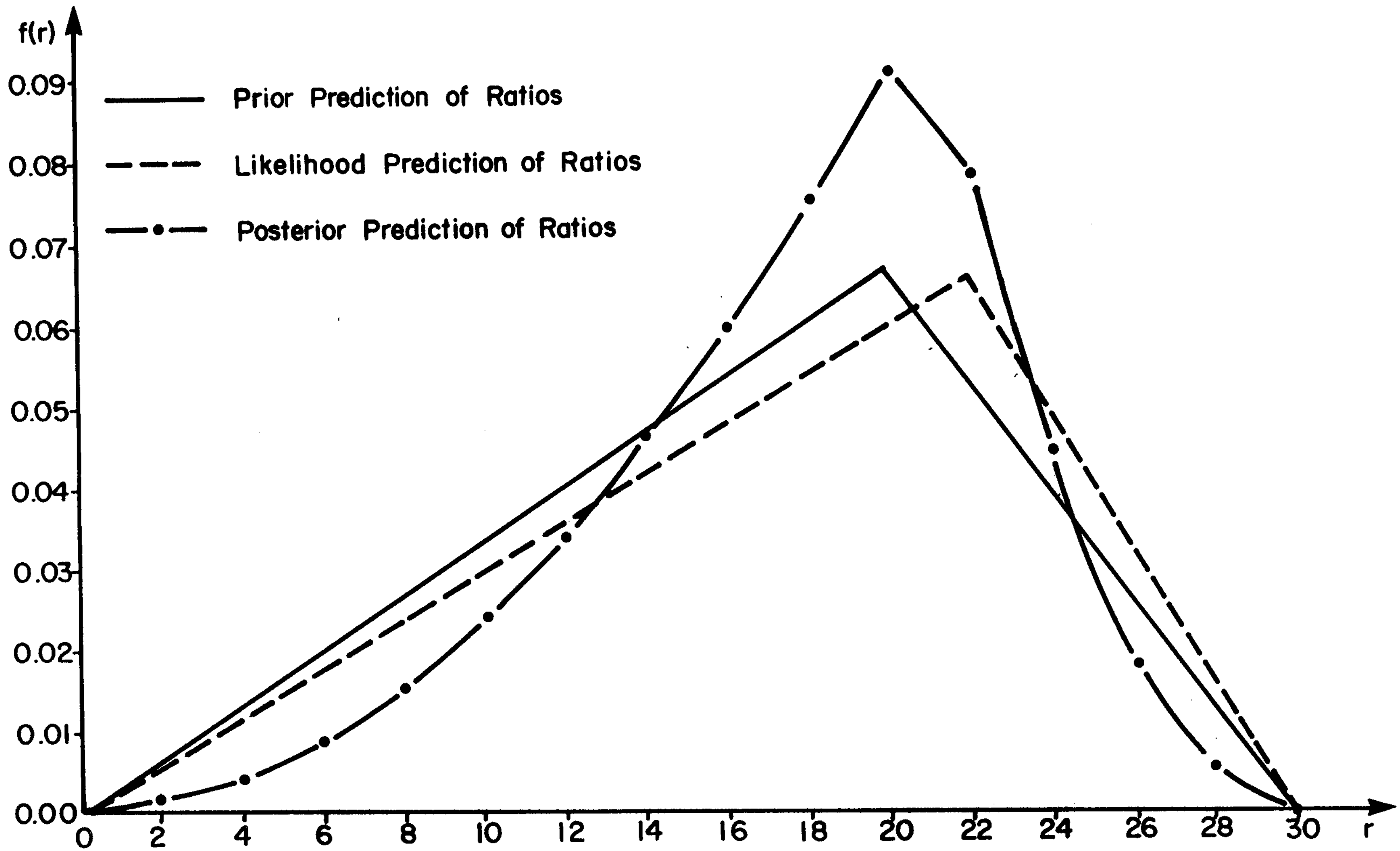

Fig. A1 Prior, Likelihood, and Posterior PMFs for Unit Reinforcing Bar Requirements calculated as,

$$P(B=10M|LM) = \frac{P(B=10M) \cdot P(LM|B=10M)}{X}$$

where

$$\begin{aligned} X = {} & P(B=10M) \cdot P(LM|B=10M) + \\ & P(B=15M) \cdot P(LM|B=15M) + \\ & P(B=20M) \cdot P(LM|B=20M) + \\ & P(B=25M) \cdot P(LM|B=25M) + \\ & P(B=30M) \cdot P(LM|B=30M) + \\ & P(B=35M) \cdot P(LM|B=35M) + \\ & P(B=45M) \cdot P(LM|B=45M) \end{aligned}$$

where LM refers to the data from last month. Substituting into this equation gives,

$$P(B=10M|B'=10M) = \frac{.20 \times .20}{\begin{array}{l} .20\text{x}.20 + .25\text{x}.30 + .25 \text{ x } 0.10 + \\ .15\text{x}.20 + .10\text{x}.15 + .05\text{x}.05 + 0 \end{array}}$$

$$= 0.213$$

In a similar way the following probabilities were calculated,

$$P(B=15M|LM) = 0.400$$

$$P(B=20M|LM) = 0.133$$

$$P(B=25M|LM) = 0.160$$

$P(B=30M|LM) = 0.080$

$P(B=35M|LM) = 0.013$

$P(B=45M|LM) = 0.000$

The posterior PMF resulting from this exercise is shown in Figure A1. This posterior PMF should be used by the engineer to determine what distribution of bar sizes to order next month. Since the summation of the probabilities equals one, this posterior PMF is theoretically acceptable.

APPENDIX B

Example of Bayesian Inference for a Continuous Random System

Problem

Over the course of several years a structural design engineer has kept track of the span-to-depth ratio of simply supported primary steel girders that he has designed. The average span-to-depth ratio is 20 and for simplicity the engineer represented the pdf describing this random system using the following bi-linear relationship:

$$f(r) = 0.00333r, \qquad 0 < r \leq 20$$

$$f(r) = -0.0067r + 0.20, \qquad 20 < r \leq 30$$

where r = the ratio of the girder span to the girder depth. These equations are shown in Figure B1 and labelled "Prior Prediction of Ratios".

After completion of a project the structural engineer wanted to combine the span-to-depth ratios of the girders used in order to update his current statistical distribution. The ratios used were 10, 15, 22, 22, 22, 24, 26, 26, 26, and 27. What does the new pdf look like?

Solution

The new beam span-to-depth ratios can be used to develop a new simple bi-linear relationship representing the likelihood function. The average of the ratios is 22. The density function

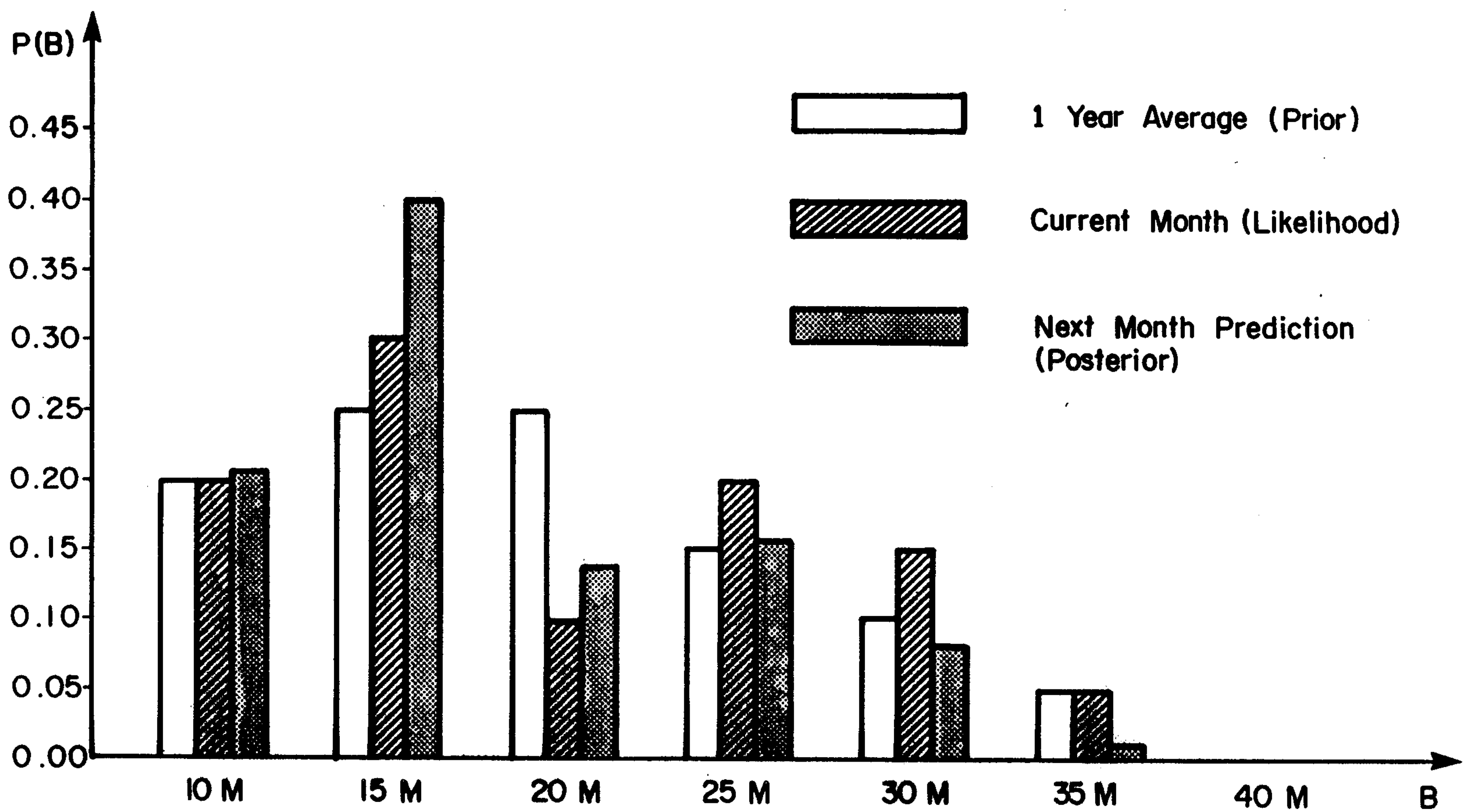

Fig. B1 Prior, Likelihood, and Posterior pdfs for Distribution of Span-to-Depth Ratios which describes the likelihood function is therefore

$$f(r'|r) = 0.00303r, \qquad 0 < r \leq 22$$

$$f(r'|r) = -0.0083r + 0.250, \qquad 22 < r \leq 30$$

where r' refers to the pdf being based on new data.

In order to predict the posterior distribution of the pdf [2.12] is used. For the event of r = 2 the ordinate of the posterior density function is calculated as,

$$f(r_2|r') = \frac{f(r_2) \cdot f(r'|r_2)}{\int_{-\infty}^{\infty} f(r) \cdot f(r'|r)\, dr}$$

In this problem it is convenient to use numerical integration as follows,

$$f(r_2|r') = \frac{f(r_2) \cdot f(r'|r_2)}{\sum_{i=0}^{15} f(r_{2i}) \cdot f(r'|r_{2i}) \cdot \Delta_r}$$

$$= \frac{0.00333r_2 \cdot 0.00303r_2}{\sum_{i=0}^{10} 0.00333r_{2i} \cdot 0.00303r_{2i} \cdot 2 + \sum_{i=11}^{15} (-0.0067r_{2i} + 0.200) \cdot (-0.0083r_{2i} + 0.250) \cdot 2}$$

where $\Delta r = 2$.

$$f(r_2|r') = \frac{0.0067 \cdot 0.0061}{2.213 \times 10^{-2} \times 2} = 0.00092$$

The ordinates of the updated pdf of r are calculated at intervals of 2 from r equals 0 to 30; these calculations are summarized in Table B1. The updated pdf is shown in Figure B1. A check that the enclosed area of the posterior pdf equals one indicates that it is theoretically an acceptable statistical description of the system.

Table B1 - Calculations for Updating f(r)

r	f(r)	f(r'|r)	f(r) • f(r'|r)	f(r|r')
0	0.0	0.0	0.0	0.0
2	0.0067	0.0061	4.087×10^{-5}	0.00092
4	0.0133	0.0121	1.609×10^{-4}	0.0036
6	0.0200	0.0182	3.640×10^{-4}	0.0082
8	0.0266	0.0242	6.437×10^{-4}	0.0145
10	0.0333	0.0303	1.009×10^{-3}	0.0228
12	0.0400	0.0364	1.456×10^{-3}	0.0329
14	0.0466	0.0424	1.976×10^{-3}	0.0446
16	0.0533	0.0485	2.585×10^{-3}	0.0584
18	0.0599	0.0545	3.265×10^{-3}	0.0738
20	0.0666	0.0606	4.036×10^{-3}	0.0912
22	0.0526	0.0666	3.503×10^{-3}	0.0791
24	0.0392	0.0508	1.991×10^{-3}	0.0450
26	0.0258	0.0342	8.824×10^{-4}	0.0199
28	0.0124	0.0176	2.182×10^{-4}	0.0049
30	0.0	0.0	0.0	0.0
Σ	0.496	0.503	2.213×10^{-2}	0.499

APPENDIX C

In-Situ Test Data and Parametric Study Data Bases

Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
1	AN#1	3.805	38.0
2	AN#2	3.925	40.0
3	AN#3	4.064	36.0
4	AN#4	4.108	42.0
5	AN#5	4.150	44.0
6	AN#6	4.076	47.0
7	AN#7	4.152	41.0
8	AN#8	4.194	46.0
9	AN#9	4.068	40.0
10	AN#10	4.125	41.0
11	AN#11	3.828	44.0
12	AN#12	3.646	41.0
13	AN#13	3.863	40.0
14	AN#14	3.974	44.0
15	AN#15	3.911	43.0
16	AN#16	3.669	42.0
17	AN#17	3.564	41.0
18	AN#18	3.615	44.0
19	AN#19	3.814	46.0
20	AN#20	3.814	42.0
21	AN#21	3.875	41.0
22	AN#22	3.862	39.0
23	AN#23	3.871	39.0
24	AN#24	3.952	44.0
25	AN#25	4.127	43.0
26	AN#26	4.089	41.0
27	AN#27	4.093	40.0
28	AN#28	4.106	43.0
29	AN#29	4.271	41.0
30	AN#30	4.023	43.0
31	AN#31	4.148	44.0
32	AN#32	4.201	42.0
33	AN#33	4.000	42.0
34	AN#34	4.100	41.0
35	AN#35	3.980	40.0
36	AN#36	4.030	40.0
37	AN#37	3.886	40.0
38	AN#38	3.416	40.0
39	AM#1	3.853	45.0
40	AM#2	3.929	41.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
41	AM#3	3.976	36.0
42	AM#4	3.966	40.0
43	AM#5	3.961	40.0
44	AM#6	3.922	36.0
45	AM#7	4.154	43.0
46	AM#8	3.859	41.0
47	AM#9	3.841	40.0
48	AM#10	3.966	42.0
49	AM#11	3.877	42.0
50	AM#12	4.033	43.0
51	AM#13	3.954	44.0
52	AM#14	4.163	41.0
53	AM#15	3.978	43.0
54	AM#16	4.003	43.0
55	AS#1	3.995	48.0
56	AS#2	4.000	48.0
57	AS#3	4.042	43.0
58	AS#4	4.108	49.0
59	AS#5	3.995	40.0
60	AS#6	4.214	42.0
61	AS#7	3.832	40.0
62	AS#8	3.990	43.0
63	AS#9	4.080	42.0
64	AS#10	4.116	38.0
65	AS#11	4.066	43.0
66	AS#12	4.150	43.0
67	AS#13	3.948	38.0
68	AS#14	3.988	38.0
69	AS#15	4.242	49.0
70	AS#16	4.268	47.0
71	AS#17	3.551	40.0
72	AS#18	3.694	40.0
73	AS#19	3.578	40.0
74	AS#20	3.914	41.0
75	AS#21	4.048	40.0
76	AS#22	4.027	44.0
77	AS#23	3.895	46.0
78	AS#24	4.103	50.0
79	AS#25	4.100	43.0
80	AS#26	3.900	41.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
81	AS#27	4.015	38.0
82	AS#28	4.081	40.0
83	AS#29	3.897	38.0
84	AS#30	4.066	36.0
85	AS#31	3.863	44.0
86	AS#32	4.043	43.0
87	AS#33	3.874	42.0
88	AS#34	3.785	38.0
89	BN#1	4.242	47.0
90	BN#2	4.059	44.0
91	BN#3	3.910	41.0
92	BN#4	4.104	38.0
93	BN#5	3.871	40.0
94	BN#6	3.926	44.0
95	BN#7	3.899	41.0
96	BN#8	4.029	41.0
97	BN#9	3.842	43.0
98	BN#10	4.023	46.0
99	BN#11	4.020	45.0
100	BN#12	4.003	43.0
101	BN#13	3.770	42.0
102	BN#14	4.171	50.0
103	BN#15	4.020	48.0
104	BN#16	3.978	47.0
105	BN#17	3.786	47.0
106	BN#18	3.986	41.0
107	BN#19	4.164	38.0
108	BN#20	4.118	38.0
109	BN#21	3.896	42.0
110	BN#22	3.944	30.0
111	BN#23	3.930	30.0
112	BN#24	4.037	36.0
113	BN#25	4.013	36.0
114	BN#26	3.967	43.0
115	BN#27	4.040	44.0
116	BN#28	3.999	47.0
117	BN#29	4.072	47.0
118	BN#30	4.009	40.0
119	BN#31	4.219	40.0
120	BN#32	4.029	38.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
121	BN#33	4.097	49.0
122	BN#34	3.944	46.0
123	BN#35	4.104	42.0
124	BN#36	4.092	44.0
125	BN#37	4.097	47.0
126	BN#38	4.028	43.0
127	BM#1	3.921	45.0
128	BM#2	3.895	40.0
129	BM#3	4.081	44.0
130	BM#4	3.695	40.0
131	BM#5	3.929	40.0
132	BM#6	3.843	41.0
133	BM#7	3.666	43.0
134	BM#8	3.867	42.0
135	BM#9	4.032	44.0
136	BM#10	3.618	44.0
137	BM#11	3.854	41.0
138	BM#12	3.996	44.0
139	BM#13	4.246	47.0
140	BM#14	4.056	46.0
141	BM#15	3.968	48.0
142	BM#16	4.030	43.0
143	BS#1	3.886	40.0
144	BS#2	4.093	40.0
145	BS#3	3.895	38.0
146	BS#4	3.896	38.0
147	BS#5	4.024	33.0
148	BS#6	3.944	37.0
149	BS#7	4.133	38.0
150	BS#8	3.947	41.0
151	BS#9	3.830	42.0
152	BS#10	4.240	41.0
153	BS#11	4.061	41.0
154	BS#12	3.941	38.0
155	BS#13	4.198	38.0
156	BS#14	4.018	40.0
157	BS#15	4.193	40.0
158	BS#16	4.064	42.0
159	BS#17	3.437	42.0
160	BS#18	3.800	46.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
161	BS#19	3.894	44.0
162	BS#20	3.910	48.0
163	BS#21	3.903	44.0
164	BS#22	3.862	38.0
165	BS#23	3.830	37.0
166	BS#24	3.761	36.0
167	BS#25	4.027	42.0
168	BS#26	3.945	40.0
169	BS#27	3.778	36.0
170	BS#28	3.895	36.0
171	BS#29	3.871	34.0
172	BS#30	3.807	37.0
173	BS#31	3.871	42.0
174	BS#32	4.081	40.0
175	BS#33	3.906	40.0
176	BS#34	3.901	40.0
177	BS#35	3.800	38.0
178	BS#36	3.799	30.0
179	CN#1	3.827	37.0
180	CN#2	3.838	38.0
181	CN#3	4.082	34.0
182	CN#4	3.930	39.0
183	CN#5	4.218	37.0
184	CN#6	3.749	37.0
185	CN#7	4.208	32.0
186	CN#8	3.784	34.0
187	CN#9	4.171	32.0
188	CN#10	4.137	37.0
189	CN#11	4.124	41.0
190	CN#12	4.101	37.0
191	CN#13	4.138	39.0
192	CN#14	3.866	37.0
193	CN#15	3.974	35.0
194	CN#16	4.113	36.0
195	CN#17	3.689	38.0
196	CN#18	3.947	46.0
197	CN#19	3.819	46.0
198	CN#20	3.751	41.0
199	CN#21	4.097	41.0
200	CN#22	4.116	45.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
201	CN#23	4.199	43.0
202	CN#24	4.089	39.0
203	CN#25	4.160	39.0
204	CN#26	4.185	30.0
205	CN#27	4.208	36.0
206	CN#28	4.047	41.0
207	CN#29	4.052	40.0
208	CN#30	4.148	32.0
209	CN#31	4.071	33.0
210	CN#32	3.733	38.0
211	CN#33	3.976	40.0
212	CN#34	4.145	38.0
213	CN#35	4.073	41.0
214	CN#36	4.008	38.0
215	CN#37	3.918	41.0
216	CN#38	3.658	39.0
217	CM#1	4.024	43.0
218	CM#2	3.907	46.0
219	CM#3	4.284	41.0
220	CM#4	3.805	45.0
221	CM#5	4.029	42.0
222	CM#6	3.544	41.0
223	CM#7	3.809	39.0
224	CM#8	3.810	44.0
225	CM#9	4.102	40.0
226	CM#10	3.816	41.0
227	CM#11	4.061	44.0
228	CM#12	3.884	44.0
229	CM#13	4.133	42.0
230	CM#14	3.998	41.0
231	CM#15	4.031	44.0
232	CM#16	4.253	40.0
233	CS#1	4.338	48.0
234	CS#2	4.166	49.0
235	CS#3	3.926	45.0
236	CS#4	4.354	39.0
237	CS#5	4.060	34.0
238	CS#6	4.345	40.0
239	CS#7	4.360	34.0
240	CS#8	4.117	40.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
241	CS#9	3.902	38.0
242	CS#10	4.081	32.0
243	CS#11	3.881	39.0
244	CS#12	4.143	42.0
245	CS#13	4.184	40.0
246	CS#14	4.299	37.0
247	CS#15	4.207	38.0
248	CS#16	4.123	36.0
249	CS#17	3.713	36.0
250	CS#18	4.001	36.0
251	CS#19	3.901	39.0
252	CS#20	3.907	40.0
253	CS#21	4.059	40.0
254	CS#22	4.085	38.0
255	CS#23	3.961	39.0
256	CS#24	4.249	45.0
257	CS#25	3.948	38.0
258	CS#26	4.190	42.0
259	CS#27	4.027	41.0
260	CS#28	3.981	36.0
261	CS#29	4.225	39.0
262	CS#30	4.202	42.0
263	CS#31	4.132	47.0
264	CS#32	4.201	48.0
265	CS#33	3.932	40.0
266	CS#34	3.914	47.0
267	CS#35	3.970	46.0
268	CS#36	3.943	34.0
269	CS#37	3.682	34.0
270	CS#38	3.791	31.0
271	DN#1	3.894	38.0
272	DN#2	4.000	36.0
273	DN#3	3.990	34.0
274	DN#4	3.628	38.0
275	DN#5	3.935	40.0
276	DN#6	3.899	40.0
277	DN#7	3.799	39.0
278	DN#8	3.849	45.0
279	DN#9	4.005	43.0
280	DN#10	3.942	43.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
281	DN#11	3.879	43.0
282	DN#12	3.907	38.0
283	DN#13	3.734	41.0
284	DN#14	3.742	36.0
285	DN#15	3.980	36.0
286	DN#16	3.980	38.0
287	DN#17	3.672	43.0
288	DN#18	3.735	49.0
289	DN#19	3.715	47.0
290	DN#20	3.830	46.0
291	DN#21	3.697	41.0
292	DN#22	3.775	41.0
293	DN#23	3.840	42.0
294	DN#24	4.009	46.0
295	DN#25	4.139	43.0
296	DN#26	3.965	41.0
297	DN#27	4.091	40.0
298	DN#28	4.014	40.0
299	DN#29	4.007	40.0
300	DN#30	4.125	41.0
301	DN#31	3.918	42.0
302	DN#32	4.162	36.0
303	DN#33	4.096	30.0
304	DN#34	4.020	38.0
305	DN#35	4.129	41.0
306	DN#36	4.009	40.0
307	DN#37	3.881	35.0
308	DN#38	3.944	36.0
309	DM#1	4.070	49.0
310	DM#2	3.795	45.0
311	DM#3	4.035	43.0
312	DM#4	3.942	47.0
313	DM#5	4.038	50.0
314	DM#6	3.675	45.0
315	DM#7	4.015	42.0
316	DM#8	3.850	47.0
317	DM#9	4.125	44.0
318	DM#10	3.302	40.0
319	DM#11	4.068	47.0
320	DM#12	3.783	48.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/µsec)	Minimum Rebound No.
321	DM#13	4.081	41.0
322	DM#14	3.864	40.0
323	DM#15	3.887	44.0
324	DM#16	3.702	40.0
325	DS#1	4.193	34.0
326	DS#2	4.172	38.0
327	DS#3	4.076	39.0
328	DS#4	4.226	38.0
329	DS#5	4.101	43.0
330	DS#6	3.996	36.0
331	DS#7	4.057	36.0
332	DS#8	3.806	36.0
333	DS#9	4.084	37.0
334	DS#10	3.911	38.0
335	DS#11	4.057	39.0
336	DS#12	3.966	40.0
337	DS#13	4.073	39.0
338	DS#14	3.872	38.0
339	DS#15	4.006	36.0
340	DS#16	4.007	37.0
341	DS#17	3.827	38.0
342	DS#18	3.902	40.0
343	DS#19	3.917	42.0
344	DS#20	3.852	47.0
345	DS#21	4.044	47.0
346	DS#22	3.955	32.0
347	DS#23	3.971	38.0
348	DS#24	3.970	42.0
349	DS#25	3.974	43.0
350	DS#26	4.069	40.0
351	DS#27	4.054	38.0
352	DS#28	3.988	36.0
353	DS#29	4.055	36.0
354	DS#30	4.291	48.0
355	DS#31	4.194	40.0
356	DS#32	4.109	39.0
357	DS#33	4.269	48.0
358	DS#34	4.291	40.0
359	DS#35	3.940	45.0
360	DS#36	3.951	45.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/µsec)	Minimum Rebound No.
361	DS#37	3.740	36.0
362	DS#38	3.813	38.0
363	EN#1	3.970	37.0
364	EN#2	3.824	38.0
365	EN#3	3.977	40.0
366	EN#4	3.940	41.0
367	EN#5	4.026	38.0
368	EN#6	4.011	42.0
369	EN#7	4.042	44.0
370	EN#8	3.945	39.0
371	EN#9	3.916	41.0
372	EN#10	4.006	39.0
373	EN#11	3.998	40.0
374	EN#12	3.907	38.0
375	EN#13	4.078	38.0
376	EN#14	3.967	40.0
377	EN#15	3.754	38.0
378	EN#16	3.659	42.0
379	EN#17	3.469	36.0
380	EN#18	3.444	38.0
381	EN#19	3.765	41.0
382	EN#20	3.698	44.0
383	EN#21	3.873	38.0
384	EN#22	3.889	43.0
385	EN#23	4.032	38.0
386	EN#24	4.031	36.0
387	EN#25	3.959	38.0
388	EN#26	3.971	40.0
389	EN#27	4.058	40.0
390	EN#28	3.930	41.0
391	EN#29	4.062	40.0
392	EN#30	4.100	40.0
393	EN#31	3.967	39.0
394	EN#32	3.890	41.0
395	EN#33	4.012	50.0
396	EN#34	3.921	43.0
397	EN#35	3.890	42.0
398	EN#36	3.883	38.0
399	EN#37	3.993	39.0
400	EN#38	4.078	43.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
401	EM#1	3.768	46.0
402	EM#2	3.940	38.0
403	EM#3	3.871	46.0
404	EM#4	3.768	40.0
405	EM#5	3.757	41.0
406	EM#6	3.837	42.0
407	EM#7	4.072	43.0
408	EM#8	4.026	43.0
409	EM#9	3.793	41.0
410	EM#10	3.913	39.0
411	EM#11	3.811	40.0
412	EM#12	3.855	42.0
413	EM#13	3.868	44.0
414	EM#14	3.682	44.0
415	EM#15	4.011	41.0
416	EM#16	4.043	40.0
417	ES#1	3.905	39.0
418	ES#2	3.913	36.0
419	ES#3	3.795	37.0
420	ES#4	3.865	36.0
421	ES#5	3.930	39.0
422	ES#6	3.942	48.0
423	ES#7	4.016	41.0
424	ES#8	4.057	41.0
425	ES#9	3.916	47.0
426	ES#10	4.017	48.0
427	ES#11	4.043	48.0
428	ES#12	4.091	44.0
429	ES#13	4.060	43.0
430	ES#14	4.077	40.0
431	ES#17	3.976	40.0
432	ES#19	4.144	40.0
433	ES#20	4.035	39.0
434	ES#21	3.928	44.0
435	ES#22	3.925	38.0
436	ES#23	4.019	44.0
437	ES#24	4.059	46.0
438	ES#25	4.045	42.0
439	ES#26	4.055	36.0
440	ES#27	3.957	36.0

cont. Table C1 Nondestructive Test Data

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
441	ES#28	3.874	37.0
442	ES#29	3.849	39.0
443	ES#30	4.049	44.0
444	ES#31	3.953	38.0
445	ES#32	4.136	42.0
446	ES#33	4.021	34.0
447	ES#34	4.167	38.0
448	ES#35	4.088	37.0
449	ES#36	3.896	36.0
450	ES#37	3.972	36.0
451	ES#38	3.920	37.0

Table C2 Concrete Core Data

No.	Location	Core Strength (MPa)	Pulse Velocity (mm/μsec)	Minimum Rebound No.
1	AN#23	35.88	3.871	39.0
2	AN#29	44.42	4.271	41.0
3	AN#32	40.10	4.201	42.0
4	AM#9	39.70	3.841	40.0
5	AS#5	41.82	3.995	40.0
6	AS#8	37.34	3.990	43.0
7	BN#4	40.05	4.104	38.0
8	BN#8	36.45	4.029	41.0
9	BN#31	40.72	4.219	40.0
10	BS#3	29.42	3.895	38.0
11	BS#7	32.56	4.133	38.0
12	CN#1	20.70	3.827	37.0
13	CN#5	33.45	4.218	37.0
14	CS#1	45.76	4.338	48.0
15	CS#5	38.93	4.060	34.0
16	DN#3	36.30	3.990	34.0
17	DS#31	35.16	4.194	40.0
18	EN#8	40.05	3.945	39.0
19	ES#23	29.79	4.019	44.0
20	ES#29	27.51	3.849	39.0

Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/µsec)	Minimum Rebound No.
1	BS#5	4.024	33.0
2	BN#2	4.059	44.0
3	AM#7	4.154	43.0
4	AS#21	4.048	40.0
5	EN#7	4.042	44.0
6	CN#19	3.819	46.0
7	AN#18	3.615	44.0
8	EN#6	4.011	42.0
9	DN#32	4.162	36.0
10	AS#24	4.103	50.0
11	BM#2	3.895	40.0
12	BS#10	4.240	41.0
13	AM#6	3.922	36.0
14	BS#3	3.895	38.0
15	DS#11	4.057	39.0
16	ES#35	4.088	37.0
17	AS#2	4.000	48.0
18	AM#1	3.853	45.0
19	AM#11	3.877	42.0
20	CS#19	3.901	39.0
21	EM#16	4.043	40.0
22	DN#14	3.742	36.0
23	CN#21	4.097	41.0
24	BN#25	4.013	36.0
25	DN#33	4.096	30.0
26	BN#10	4.023	46.0
27	BS#25	4.027	42.0
28	BN#38	4.028	43.0
29	AN#4	4.108	42.0
30	DN#36	4.009	40.0
31	CS#35	3.970	46.0
32	DS#31	4.194	40.0
33	BS#1	3.886	40.0
34	AN#36	4.030	40.0
35	DN#8	3.849	45.0
36	BS#24	3.761	36.0
37	AM#9	3.841	40.0
38	DS#33	4.269	48.0
39	DS#29	4.055	36.0
40	BN#27	4.040	44.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
41	EM#1	3.768	46.0
42	AS#6	4.214	42.0
43	DN#5	3.935	40.0
44	BN#13	3.770	42.0
45	ES#5	3.930	39.0
46	CN#20	3.751	41.0
47	DM#13	4.081	41.0
48	CN#1	3.827	37.0
49	BN#33	4.097	49.0
50	CN#25	4.160	39.0
51	CS#13	4.184	40.0
52	DS#17	3.827	38.0
53	AS#16	4.268	47.0
54	BM#8	3.867	42.0
55	EN#24	4.031	36.0
56	BM#10	3.618	44.0
57	DS#2	4.172	38.0
58	AS#8	3.990	43.0
59	CS#1	4.338	48.0
60	CN#31	4.071	33.0
61	CS#7	4.360	34.0
62	ES#17	3.976	40.0
63	DS#35	3.940	45.0
64	AS#25	4.100	43.0
65	AN#2	3.925	40.0
66	CM#4	3.805	45.0
67	CN#7	4.208	32.0
68	AM#13	3.954	44.0
69	EN#9	3.916	41.0
70	ES#32	4.136	42.0
71	EN#12	3.907	38.0
72	ES#26	4.055	36.0
73	EN#4	3.940	41.0
74	DN#2	4.000	36.0
75	DN#4	3.628	38.0
76	CS#34	3.914	47.0
77	DS#28	3.988	36.0
78	CN#13	4.138	39.0
79	AN#35	3.980	40.0
80	BN#21	3.896	42.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
81	CN#27	4.208	36.0
82	AM#8	3.859	41.0
83	EM#14	3.682	44.0
84	EN#23	4.032	38.0
85	CN#24	4.089	39.0
86	CM#12	3.884	44.0
87	DN#35	4.129	41.0
88	BN#8	4.029	41.0
89	BS#31	3.871	42.0
90	BM#1	3.921	45.0
91	BS#36	3.799	30.0
92	EM#10	3.913	39.0
93	DN#23	3.840	42.0
94	AM#10	3.966	42.0
95	DN#7	3.799	39.0
96	EN#15	3.754	38.0
97	ES#8	4.057	41.0
98	AN#20	3.814	42.0
99	EN#11	3.998	40.0
100	CN#14	3.866	37.0
101	AS#23	3.895	46.0
102	BM#4	3.695	40.0
103	BS#7	4.133	38.0
104	AM#3	3.976	36.0
105	ES#10	4.017	48.0
106	DS#13	4.073	39.0
107	ES#36	3.896	36.0
108	AN#32	4.201	42.0
109	AN#28	4.106	43.0
110	AS#27	4.015	38.0
111	AS#19	3.578	40.0
112	CN#11	4.124	41.0
113	AM#14	4.163	41.0
114	AN#38	3.416	40.0
115	EN#3	3.977	40.0
116	CS#26	4.190	42.0
117	DS#37	3.740	36.0
118	DN#30	4.125	41.0
119	CN#30	4.148	32.0
120	AN#33	4.000	42.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
121	DM#9	4.125	44.0
122	BM#6	3.843	41.0
123	AS#28	4.081	40.0
124	EN#2	3.824	38.0
125	BN#11	4.020	45.0
126	DN#28	4.014	40.0
127	AN#11	3.828	44.0
128	CN#29	4.052	40.0
129	DS#26	4.069	40.0
130	ES#20	4.035	39.0
131	ES#25	4.045	42.0
132	BN#6	3.926	44.0
133	CS#4	4.354	39.0
134	DS#30	4.291	48.0
135	CS#30	4.202	42.0
136	BN#19	4.164	38.0
137	ES#33	4.021	34.0
138	CS#31	4.132	47.0
139	BN#16	3.978	47.0
140	ES#29	3.849	39.0
141	CN#3	4.082	34.0
142	AN#7	4.152	41.0
143	AN#21	3.875	41.0
144	BS#18	3.800	46.0
145	DN#25	4.139	43.0
146	CS#8	4.117	40.0
147	EM#2	3.940	38.0
148	CS#37	3.682	34.0
149	DN#22	3.775	41.0
150	BM#11	3.854	41.0
151	DM#14	3.864	40.0
152	ES#12	4.091	44.0
153	BS#16	4.064	42.0
154	AS#7	3.832	40.0
155	AS#11	4.066	43.0
156	BS#2	4.093	40.0
157	CS#6	4.345	40.0
158	CM#2	3.907	46.0
159	DN#37	3.881	35.0
160	BS#26	3.945	40.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
161	AS#31	3.863	44.0
162	DN#24	4.009	46.0
163	CN#35	4.073	41.0
164	DS#9	4.084	37.0
165	EM#4	3.768	40.0
166	EN#25	3.959	38.0
167	DN#26	3.965	41.0
168	CM#3	4.284	41.0
169	DS#8	3.806	36.0
170	CN#36	4.008	38.0
171	EN#31	3.967	39.0
172	AS#15	4.242	49.0
173	AN#9	4.068	40.0
174	CN#22	4.116	45.0
175	DS#36	3.951	45.0
176	ES#11	4.043	48.0
177	BS#20	3.910	48.0
178	AN#6	4.076	47.0
179	CN#17	3.689	38.0
180	CN#37	3.918	41.0
181	EN#14	3.967	40.0
182	AM#2	3.929	41.0
183	EM#5	3.757	41.0
184	CN#12	4.101	37.0
185	EN#28	3.930	41.0
186	AS#29	3.897	38.0
187	EN#8	3.945	39.0
188	AS#13	3.948	38.0
189	EN#21	3.873	38.0
190	CS#17	3.713	36.0
191	CS#36	3.943	34.0
192	AS#9	4.080	42.0
193	BS#21	3.903	44.0
194	BS#14	4.018	40.0
195	DS#14	3.872	38.0
196	CN#6	3.749	37.0
197	BM#16	4.030	43.0
198	EN#27	4.058	40.0
199	DM#4	3.942	47.0
200	DS#6	3.996	36.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
201	CS#5	4.060	34.0
202	EN#22	3.889	43.0
203	CM#1	4.024	43.0
204	CS#9	3.902	38.0
205	BN#22	3.944	30.0
206	CS#38	3.791	31.0
207	EM#8	4.026	43.0
208	CM#8	3.810	44.0
209	DM#16	3.702	40.0
210	DN#29	4.007	40.0
211	CM#14	3.998	41.0
212	AN#1	3.805	38.0
213	CN#15	3.974	35.0
214	AS#22	4.027	44.0
215	ES#27	3.957	36.0
216	AS#34	3.785	38.0
217	DS#24	3.970	42.0
218	CN#10	4.137	37.0
219	AM#5	3.961	40.0
220	CN#18	3.947	46.0
221	EN#17	3.469	36.0
222	DN#15	3.980	36.0
223	AN#30	4.023	43.0
224	AS#18	3.694	40.0
225	BN#23	3.930	30.0
226	AN#14	3.974	44.0
227	EN#13	4.078	38.0
228	ES#22	3.925	38.0
229	AM#12	4.033	43.0
230	CN#4	3.930	39.0
231	CS#14	4.299	37.0
232	AN#19	3.814	46.0
233	BS#9	3.830	42.0
234	EN#36	3.883	38.0
235	BN#20	4.118	38.0
236	AS#3	4.042	43.0
237	CN#38	3.658	39.0
238	AN#10	4.125	41.0
239	BN#5	3.871	40.0
240	EM#9	3.793	41.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
241	EN#34	3.921	43.0
242	DS#20	3.852	47.0
243	EN#29	4.062	40.0
244	CS#33	3.932	40.0
245	CS#24	4.249	45.0
246	AN#23	3.871	39.0
247	BS#12	3.941	38.0
248	BN#35	4.104	42.0
249	BN#24	4.037	36.0
250	BS#4	3.896	38.0
251	CS#3	3.926	45.0
252	CS#20	3.907	40.0
253	AM#16	4.003	43.0
254	EN#5	4.026	38.0
255	DS#7	4.057	36.0
256	EN#1	3.970	37.0
257	DS#25	3.974	43.0
258	DN#6	3.899	40.0
259	AN#37	3.886	40.0
260	BN#4	4.104	38.0
261	CN#2	3.838	38.0
262	DN#16	3.980	38.0
263	DS#16	4.007	37.0
264	DS#38	3.813	38.0
265	BS#28	3.895	36.0
266	CN#23	4.199	43.0
267	DS#21	4.044	47.0
268	BM#7	3.666	43.0
269	DS#15	4.006	36.0
270	EM#6	3.837	42.0
271	CM#6	3.544	41.0
272	AS#1	3.995	48.0
273	AN#26	4.089	41.0
274	EN#30	4.100	40.0
275	ES#38	3.920	37.0
276	BS#27	3.778	36.0
277	BN#1	4.242	47.0
278	DS#3	4.076	39.0
279	BN#37	4.097	47.0
280	ES#24	4.059	46.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
281	CM#15	4.031	44.0
282	DM#3	4.035	43.0
283	DS#1	4.193	34.0
284	DS#10	3.911	38.0
285	EM#3	3.871	46.0
286	BM#3	4.081	44.0
287	AM#4	3.966	40.0
288	DS#12	3.966	40.0
289	DM#5	4.038	50.0
290	CM#16	4.253	40.0
291	ES#30	4.049	44.0
292	AN#25	4.127	43.0
293	EN#38	4.078	43.0
294	DS#34	4.291	40.0
295	BM#5	3.929	40.0
296	CS#11	3.881	39.0
297	DM#6	3.675	45.0
298	DN#13	3.734	41.0
299	BS#17	3.437	42.0
300	CN#26	4.185	30.0
301	DN#12	3.907	38.0
302	EN#19	3.765	41.0
303	BS#34	3.901	40.0
304	EM#12	3.855	42.0
305	ES#4	3.865	36.0
306	BS#22	3.862	38.0
307	AS#14	3.988	38.0
308	AN#15	3.911	43.0
309	DM#1	4.070	49.0
310	DS#27	4.054	38.0
311	BN#30	4.009	40.0
312	BS#29	3.871	34.0
313	BM#15	3.968	48.0
314	BN#26	3.967	43.0
315	EN#32	3.890	41.0
316	AS#5	3.995	40.0
317	CM#11	4.061	44.0
318	ES#31	3.953	38.0
319	ES#13	4.060	43.0
320	EM#11	3.811	40.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/µsec)	Minimum Rebound No.
321	DS#23	3.971	38.0
322	CN#33	3.976	40.0
323	CS#12	4.143	42.0
324	AN#8	4.194	46.0
325	DN#21	3.697	41.0
326	EM#7	4.072	43.0
327	ES#1	3.905	39.0
328	CN#16	4.113	36.0
329	DM#15	3.887	44.0
330	DN#38	3.944	36.0
331	DS#4	4.226	38.0
332	CS#18	4.001	36.0
333	EN#10	4.006	39.0
334	CS#32	4.201	48.0
335	ES#37	3.972	36.0
336	AS#30	4.066	36.0
337	AS#26	3.900	41.0
338	BN#17	3.786	47.0
339	BN#34	3.944	46.0
340	ES#21	3.928	44.0
341	BN#3	3.910	41.0
342	DN#20	3.830	46.0
343	CS#28	3.981	36.0
344	AN#17	3.564	41.0
345	BN#9	3.842	43.0
346	AN#13	3.863	40.0
347	AN#27	4.093	40.0
348	AM#15	3.978	43.0
349	CS#21	4.059	40.0
350	DN#10	3.942	43.0
351	DN#1	3.894	38.0
352	CM#9	4.102	40.0
353	AN#3	4.064	36.0
354	CN#8	3.784	34.0
355	EN#20	3.698	44.0
356	ES#28	3.874	37.0
357	BM#14	4.056	46.0
358	DS#18	3.902	40.0
359	CN#5	4.218	37.0
360	BN#15	4.020	48.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
361	CN#9	4.171	32.0
362	BN#36	4.092	44.0
363	DN#34	4.020	38.0
364	CS#27	4.027	41.0
365	AS#12	4.150	43.0
366	BS#6	3.944	37.0
367	BS#15	4.193	40.0
368	DN#17	3.672	43.0
369	BS#23	3.830	37.0
370	BM#13	4.246	47.0
371	EN#18	3.444	38.0
372	DN#27	4.091	40.0
373	DS#32	4.109	39.0
374	ES#3	3.795	37.0
375	BS#11	4.061	41.0
376	AN#24	3.952	44.0
377	EN#33	4.012	50.0
378	BS#19	3.894	44.0
379	CN#34	4.145	38.0
380	CN#28	4.047	41.0
381	EN#16	3.659	42.0
382	BS#8	3.947	41.0
383	DM#2	3.795	45.0
384	BS#35	3.800	38.0
385	ES#34	4.167	38.0
386	AN#5	4.150	44.0
387	BS#13	4.198	38.0
388	AN#29	4.271	41.0
389	ES#14	4.077	40.0
390	AN#16	3.669	42.0
391	CS#23	3.961	39.0
392	ES#2	3.913	36.0
393	DM#12	3.783	48.0
394	BN#29	4.072	47.0
395	DN#3	3.990	34.0
396	DN#19	3.715	47.0
397	AN#34	4.100	41.0
398	CM#13	4.133	42.0
399	EM#15	4.011	41.0
400	DM#7	4.015	42.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
401	ES#23	4.019	44.0
402	BN#32	4.029	38.0
403	BN#14	4.171	50.0
404	CS#29	4.225	39.0
405	BN#31	4.219	40.0
406	AN#31	4.148	44.0
407	ES#7	4.016	41.0
408	DS#19	3.917	42.0
409	BM#12	3.996	44.0
410	CN#32	3.733	38.0
411	DS#22	3.955	32.0
412	AN#22	3.862	39.0
413	BS#30	3.807	37.0
414	EN#37	3.993	39.0
415	BN#12	4.003	43.0
416	CS#10	4.081	32.0
417	DN#11	3.879	43.0
418	AN#12	3.646	41.0
419	BM#9	4.032	44.0
420	CS#25	3.948	38.0
421	ES#9	3.916	47.0
422	AS#33	3.874	42.0
423	AS#4	4.108	49.0
424	CM#10	3.816	41.0
425	BN#28	3.999	47.0
426	EN#26	3.971	40.0
427	AS#10	4.116	38.0
428	AS#20	3.914	41.0
429	DM#10	3.302	40.0
430	BN#18	3.986	41.0
431	DN#9	4.005	43.0
432	DN#31	3.918	42.0
433	CM#5	4.029	42.0
434	CS#16	4.123	36.0
435	ES#6	3.942	48.0
436	EM#13	3.868	44.0
437	ES#19	4.144	40.0
438	CS#22	4.085	38.0
439	DS#5	4.101	43.0
440	CS#15	4.207	38.0

cont. Table C3 Random Nondestructive Test Data Base

No.	Location	Pulse Velocity (mm/μsec)	Minimum Rebound No.
441	EN#35	3.890	42.0
442	CS#2	4.166	49.0
443	AS#17	3.551	40.0
444	DN#18	3.735	49.0
445	AS#32	4.043	43.0
446	DM#8	3.850	47.0
447	BS#32	4.081	40.0
448	BN#7	3.899	41.0
449	DM#11	4.068	47.0
450	BS#33	3.906	40.0
451	CM#7	3.809	39.0

Table C5 Quasi-Random Concrete Core Data Base

No.	Location	Core Strength (MPa)	Pulse Velocity (mm/μsec)	Minimum Rebound No.
1	BS#3	29.42	3.895	38.0
2	AS#8	37.34	3.990	43.0
3	BN#4	40.05	4.104	38.0
4	CN#5	33.45	4.218	37.0
5	AN#23	35.88	3.871	39.0
6	EN#8	40.05	3.945	39.0
7	BS#7	32.56	4.133	38.0
8	DS#4	44.74	4.226	38.0
9	AM#9	39.70	3.841	40.0
10	ES#23	29.79	4.019	44.0
11	CS#5	38.93	4.060	34.0
12	AN#29	44.42	4.271	41.0
13	ES#29	27.51	3.849	39.0
14	DN#3	36.30	3.990	34.0
15	BN#8	36.45	4.029	41.0
16	AN#32	40.10	4.201	42.0
17	CN#1	20.70	3.827	37.0
18	AS#5	41.82	3.995	40.0
19	DS#31	35.16	4.194	40.0
20	BN#31	40.72	4.219	40.0