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A QUASI-OPTIMAL EXCITATION CONTROL FOR POWER SYSTEM STABILITY

by

© A.H.M. ABDUR RAHIM

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled A QUASI-OPTIMAL EXCITATION CONTROL FOR POWER SYSTEM STABILITY submitted by A.H.M. Abdur Rahim in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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ABSTRACT

This thesis develops and evaluates practical methods of 'online' excitation control for power system stabilization.

The equations of a power system are expressed in a state space form. A method of finding the closed loop quasi-optimal excitation control is presented. The linearized model of a single machine infinite bus problem is considered and the method is then extended to a nonlinear model. The problem of implementing the quasi-optimal control is studied and a practical compromise is suggested. The results obtained with the quasi-optimal scheme are compared with those obtained by a steepest descent method. It is found that the quasi-optimal scheme gives optimal solution for linear systems. The effect of time delay on the stability of the system is also considered. Finally, the closed loop (quasi-optimal) control has been applied to a two machine system and the effect of local control investigated.

TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF TABLES

NOTATION

D	Damping coefficient
E_{fd}	Generator open circuit field voltage
E_o	Operating value of $E_{fd}(E_{fdo})$
e_d	Direct axis component of armature voltage
e_q	Quadrature axis component of armature voltage
e_t	Generator terminal voltage
e_{tr}	Reference terminal voltage
i_d	Direct axis armature current
i_{fd}	Field current
i_{kd}	Direct axis amortisseur winding currents
i_{kq}	Quadrature axis amortisseur winding currents
i_q	Quadrature axis armature current
K_r	Regulator gain
M	Generator inertia constant
n	Speed deviation
$\dot{n}$	Generator acceleration
P_{in}	Net input power
P_o	Power output (P_{out})
p	Differentiation operator
Q_o	Reactive power output
R	Armature resistance
R_e	Equivalent resistance from generator terminal to the busbar
r_{fd}	Field winding resistance

T_{in}	Net input torque
T_m	Inertia constant (2H) seconds
$u(t)$	Generator input voltage (E_{fd})
$u_s(t)$	Stabilizing signal
v	Busbar voltage
v_{fd}	Applied field voltage
x_{afd}	Mutual reactance between field and armature
x_e	Equivalent reactance from machine terminal to the busbar
x_{ffd}	Self reactance of generator field winding
x_d	Direct axis armature reactance
x_q	Quadrature axis armature reactance
δ	Rotor angular position, radian
ψ_d	Direct axis armature flux linkage
ψ_{fd}	Field flux linkage
ψ_{kd}	Direct axis amortisseur winding flux linkage
ψ_{kq}	Quadrature axis amortisseur winding flux linkage
ψ_q	Quadrature axis armature flux linkage
ω	Angular frequency, rad/sec
τ_r	Regulator time constant, seconds

All quantities normalized except as indicated

CHAPTER 1

INTRODUCTION

The growth of loads as well as increased demand for reliability have led to the development of complex, highly interconnected power systems. Disturbances on such a system may propagate throughout the rest of the system, resulting in major failures of power supply in absence of adequate safeguards. For reliable production and transmission of electrical energy, it is essential that the power system be stable in the sense described in the following section. System stability can be enhanced by suitable control techniques, some of which are described in Section (1.2). One of the more important of these — control of machine field voltage or excitation control is investigated in this thesis. A corresponding strategy for state feedback quasi-optimal excitation control is developed and evaluated in the chapters to follow.

1.1 Power System Stability

The definition of power system stability as given by Kimbark[1] is "Power system stability is a term applied to alternating current electric power systems, denoting a condition in which the various synchronous machines of the system remain in synchronism or 'in step' with each other. Conversely, instability denotes a condition involving loss of synchronism or falling 'out of step'". Depending on the magnitude and type of disturbances, power system stability may be classified into three types. These are described below.

Steady state stability is the ability of the power system to maintain power transfer over the system without loss of stability or synchronism when the magnitude of power transfer is increased gradually. It is assumed that the increase in power level upto this limit occurs slowly enough to allow regulating devices to respond with their steady state characteristics, and that inertia effects are negligible.

Transient stability is the ability of the power system to maintain stability in the presence of a sudden large change in load occasioned by system switching (as may be caused by load switchings or circuit breaker operations), or by a fault. It is usually assumed that the regulating devices do not have time to function during the transient period and that non-linear modes of system operation are encountered during the period. Inertia is important.

Dynamic stability is the ability of the power system to maintain stability for small disturbances and to prevent growth of oscillations. Dynamic instability generally occurs due to lack of damping torque. Due to the small disturbance assumption, this type of stability is usually investigated through the use of linearized models[2,3].

1.2 Basic "Swing equation" and Methods to Improve Stability

The "Swing equation" for a synchronous machine can be written as

$$M \frac{d^2\delta}{dt^2} + D \frac{d\delta}{dt} = P_{in} - P_o \tag{1.1}$$

where the output power P_o is a function of the rotor angular position, machine internal voltages and transfer (or mutual) admittances of the system. The right hand side of equation (1.1) represents the accelerating power P_a. At a steady state operating point of the machine, both the velocity of the machine $\dot{\delta}$ and the accelerating power P_a are zero while the rotor angle and other machine currents are constant. This is also called an equilibrium point. The following approaches have been used[4] to help stabilize a system after a perturbation from the equilibrium point takes place due to a disturbance.

1. Changing transfer admittance: Any switching of the system configuration to improve power transfer between sources is beneficial. The faster this is done the better. (a) The most commonly used method involves detection and clearing of faults by protective relays and circuit breakers. The time required has been decreased through the years due to the use of improved equipment, to the order of 3 cycles at the present time. Any further improvement in this figure is not likely. (b) The transfer admittance of the power system can be changed by switching in additional capacitors into the transmission network. Control of power system transients by capacitor switching has been reported in literature[5,6] in recent years. Rama Rao[7] and Miniesy[8] have considered the problem of finding the optimal value of capacitance for particular disturbances. The difficulty encountered by these investigators has been in pre-determining the optimal value of capacitance and the duration of switching independent of the type of disturbance. (c) Braking resistors are also used to change the effective conductance of the system and for dissipating the extra energy available during a fault. Load shedding is useful in decreasing load on heavily loaded generators.

2. Change in mechanical input: P_{in} varies in accordance with the speed governor action following deviations from scheduled frequency or as a result of supervisory control action. Since mechanical systems have large time constants, stabilization by controlling P_{in} is not very effective.

However, "fast valving" may be helpful in this regard[9].

3. Change in field excitation: This directly controls the internal voltage of the machine thus controlling its power output. This approach will be considered in detail in the next section since this type of control for stabilization is used in this thesis.

1.3 Historical Development of Excitation Control

Until recently most synchronous machines were equipped with rotating type of exciters having relatively long time constants and low "ceiling" voltages. As a result they were of limited effectiveness in improving the stability of the power system.

The introduction of static excitation systems has revolutionized the technology of excitation control. In recent years, considerable emphasis has been placed on the use of static excitation schemes with relatively fast response and high ceiling voltages to improve power system stability. Two major effects of high speed voltage regulators are well known[3,10]. The first is the increase in the restoring synchronizing forces made possible through the forcing of both excitation and internal-machine fluxes. This improves transient or "first swing" stability. The second is the deterioration of machine damping resulting in dynamic instability. The use of additional feedback signals, particularly velocity, through the excitation system has been suggested for satisfactory operation[11,12]. Most studies[10,12] have considered linear models even when dealing with first swing stability. For large disturbances a non-linear model is needed. It has been found possible to select control signals such as the magnitude of acceleration that effectively control large disturbances but degrade dynamic stability[13]. Dineley[12] found that by a 'continuous judicious combination' of velocity and acceleration signals a maximum effect on rotor oscillations can be produced. Jones[14] and Smith[15] considered the problem of transient removal with bang-bang excitation control.

1.4 Research Objective

The equations governing the power system dynamics are of high order and nonlinear. To obtain the optimal control for stabilization (the performance index for such problems is either time, or the norm of the deviation of some or all of the states and controls from the nominal value) for such a problem by standard optimization procedures, one has to resort to iterative techniques even for disturbances known a-priori[7,8,16,17]. However, disturbances which appear on a power system are in general not known in advance. Some system parameters such as the transfer admittances are dependent on the location and type of disturbance, and have to be measured immediately after a disturbance appears on the system. The control must again act within a very small fraction of a second to stop 'first swing' instability for large disturbances. By the time the optimal control is obtained for the particular system configuration by means of 'off-line' computations, it may be too late and instability may result. The time limitation is not very severe for small disturbances, however, since the oscillations grow slowly.

Thus, in general, a truly optimal excitation control scheme may not be implementable due to the complexity of the controller and time limitations. For this reason it is considered worthwhile to explore quasi-optimal excitation schemes which are implementable on real systems.

The main objective of this study is to explore means for stability enhancement by eliminating system transients in the least possible time. For such control schemes to be realizable 'on-line' it is necessary to develop techniques for efficient determination of control in terms of system state and parameters. Secondary objectives are investigation of the implementability of such control and consideration of practical compromises needed to apply the control to real systems.

1.5 Scope of Thesis

The quasi-optimal excitation control for the linearized single machine infinite bus problem is found directly as a function of the system states. Pontryagin's minimum principle is used to obtain the control from a transformed simplified system. A proportional control is suggested as a practical compromise which for small disturbances is equivalent to a velocity feedback signal. The scheme is then extended to a nonlinear machine model and a two machine system. Results indicate that the quasi-optimal excitation control is very effective in removing transients, though a bang-bang excitation scheme may not be realizable. The proportional control which is a modification of the bang bang control is realizable and effective. A summary of results and conclusions as well as suggestions for future research will be presented in Chapter 6 .

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CHAPTER 2

MATHEMATICAL MODEL OF A POWER SYSTEM

The synchronous generator, exciter and load models are presented in the first three sections of this chapter. The mathematical models for a power system are derived in the state space form in Sections 4 and 5. Two cases are considered: (a) A single machine connected to an infinite bus (hereafter referred to as single machine case), (b) two machines feeding a load from a busbar.

2.1 Synchronous Generator Equations

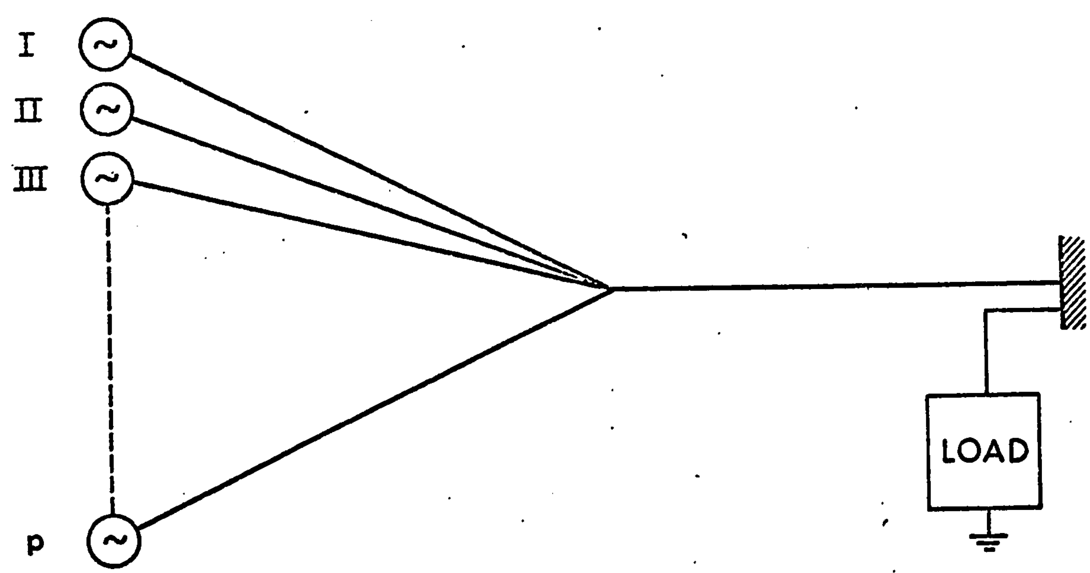

Fig. 2.1 Multi-machine System Configuration

Assume that p machines are interconnected by a transmission network so that they feed a single equivalent load. A multi-load system can be reduced to this form. The equations for the mth machine are[1,2]: (The list of symbols is given in page xiii)

a) Voltage and flux linkage equations for the armature

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix}_{m^*} = \frac{\omega}{\omega_o}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix}_m - R_m\begin{bmatrix} i_d \\ i_q \end{bmatrix}_m + \frac{1}{\omega_o}\begin{bmatrix} p\psi_d \\ p\psi_q \end{bmatrix}_m \qquad (2.1)$$

$$\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix}_m = \begin{bmatrix} x_{afd} & x_{akd1} \cdots x_{akdj} & 0 \cdots\cdots 0 \\ 0 & \cdots\cdots 0 & x_{akq1} \cdots\cdots x_{akqj} \end{bmatrix}_m \begin{bmatrix} i_{fd} \\ i_{kd1} \\ \vdots \\ i_{kdj} \\ i_{kq1} \\ \vdots \\ i_{kqj} \end{bmatrix}_m - \begin{bmatrix} x_d & 0 \\ 0 & x_q \end{bmatrix}_m \begin{bmatrix} i_d \\ i_q \end{bmatrix}_m \qquad (2.2)$$

---

* Subscript m refers to the quantities of the mth machine.

b) Voltage and flux linkage relations for the rotor

$$\begin{bmatrix} E_{fd} \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}_m = (x_{afdo})_m \begin{bmatrix} \frac{1}{\omega_o r_{fd}} & & & & & & \\ & \frac{1}{\omega_o r_{kd1}} & & & [0] & & \\ & & \ddots & & & & \\ & & & \frac{1}{\omega_o r_{kdj}} & & & \\ & & & & \frac{1}{\omega_o r_{kq1}} & & \\ & [0] & & & & \ddots & \\ & & & & & & \frac{1}{\omega_o r_{kqj}} \end{bmatrix}_m \begin{bmatrix} p\psi_{fd} \\ p\psi_{kd1} \\ \vdots \\ p\psi_{kdj} \\ p\psi_{kq1} \\ \vdots \\ p\psi_{kqj} \end{bmatrix}_m + (x_{afdo})_m \begin{bmatrix} i_{fd} \\ i_{kd1} \\ \vdots \\ i_{kdj} \\ i_{kq1} \\ \vdots \\ i_{kqj} \end{bmatrix}_m$$

(2.3)

$$
\begin{bmatrix} \psi_{fd} \\ \psi_{kd1} \\ \vdots \\ \psi_{kdj} \\ \psi_{kq1} \\ \vdots \\ \psi_{kqj} \end{bmatrix}_m
=
\left[\begin{array}{cccc|cccc}
x_{ffd} & x_{fkd1} & \cdots & x_{fkdj} & & & & \\
x_{fkd1} & x_{kkd1} & \cdots & x_{kdij} & & & & \\
\vdots & x_{kd21} & \cdots & x_{kd2j} & & [0] & & \\
 & \vdots & & \vdots & & & & \\
x_{fkdj} & x_{kdj1} & \cdots & x_{kkdj} & & & & \\
 & & & & x_{kkq1} & \cdots & & x_{kq1j} \\
 & & & & x_{kq21} & \cdots & & x_{kq2j} \\
 & [0] & & & \vdots & & & \\
 & & & & x_{kqj1} & \cdots & & x_{kkqj}
\end{array}\right]_m
\begin{bmatrix} i_{fd} \\ i_{kd1} \\ \vdots \\ i_{kdj} \\ i_{kq1} \\ \vdots \\ i_{kqj} \end{bmatrix}_m
-
\begin{bmatrix}
0 & x_{afd} \\
\vdots & x_{akd1} \\
\vdots & \vdots \\
0 & x_{akdj} \\
x_{akq1} & 0 \\
\vdots & \vdots \\
x_{akqj} & 0
\end{bmatrix}_m
\begin{bmatrix} i_q \\ i_d \end{bmatrix}_m
\qquad (2.4)
$$

Here by definition[3]

$$E_{fd} = \frac{x_{afdo}}{r_{fd}} \; v_{fd} \tag{2.5}$$

$$m = 1, 2, \ldots\ldots\ldots\ldots p$$

It is assumed that there are j rotor windings on each axis and a field winding on the direct axis.

c) The equation for the terminal voltage of the mth machine is

$$e_{tm} = \sqrt{e_{dm}^2 + e_{qm}^2} \tag{2.6}$$

d) The system connection relationships for the mth machine with the bus bar are

$$\begin{bmatrix} e_d \\ e_q \end{bmatrix}_m = \begin{bmatrix} R_e & -\frac{\omega}{\omega_o} x_e \\ \frac{\omega}{\omega_o} x_e & R_e \end{bmatrix}_m \begin{bmatrix} i_d \\ i_q \end{bmatrix}_m + \left(\frac{x_e}{\omega_o}\right)_m \begin{bmatrix} pi_d \\ pi_q \end{bmatrix}_m + \begin{bmatrix} v_d \\ v_q \end{bmatrix} \tag{2.7}$$

where

$$\begin{aligned} v_d &= v \sin(\delta_m + \theta_v) \\ v_q &= v \cos(\delta_m + \theta_v) \end{aligned} \tag{2.8}$$

θ_v is the angle between bus bar voltage and a common reference frame.

e) The torque equations for the mth machine are

$$p\delta_m = \omega_o \, n_m \tag{2.9}$$

$$(T_m pn)_m = (T_{in})_m - \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix}_m^T \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}_m \tag{2.10}$$

In order to distinguish between reference frames, subscripts d and q are used to denote components along the two axes of machine reference frames while subscripts D and Q refer to components of the common reference frame. The currents and voltages of the machines are related to the network (having common reference frame) by the transformations

$$\begin{bmatrix} V_D \\ V_Q \end{bmatrix}_m = \begin{bmatrix} \cos\delta_m & -\sin\delta_m \\ \sin\delta_m & \cos\delta_m \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix}_m \tag{2.11}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix}_m = \begin{bmatrix} \cos\delta_m & \sin\delta_m \\ -\sin\delta_m & \cos\delta_m \end{bmatrix} \begin{bmatrix} I_D \\ I_Q \end{bmatrix}_m \tag{2.12}$$

2.2 The Excitation System

There are various types of voltage regulators in use but only the direct acting continuous type of regulator and exciter is considered here. The power amplification for modern continuous regulators is provided by such devices as: magnetic amplifiers, amplidynes and controlled rectifiers. Since the voltage regulator controls the reactive power of a generator, it usually includes certain limiting and compensating circuits. These additions are not considered here. When a linearized model of the system is used, the exciter and regulator may be represented mathematically by a series of gains and time constants[4]. In this study, however, the voltage regulator and exciter is represented by a single gain and time constant even for nonlinear modes of system operation.

Static exciters or electronic exciters are now almost universally used for large generators. The maximum output voltage (or ceiling voltage) for these exciters is as large as 7 per unit[5] compared to about 2 per unit for conventional type of exciters. The corresponding time constants are also of the order of 10-20 milliseconds compared to about .5 seconds for that of an amplidyne type of exciter. This type of exciter may be very reasonably represented by a single gain and time constant. The voltage regulator equation is written as

$$\Delta E_{fd} = \frac{K_r}{1 + \tau_r p} \left(e_t - e_{tr} - u_s(t)\right) \tag{2.13}$$

where K_r is less than zero.

2.3 Load representation

Various load representations are used for stability studies. Constant impedance, constant current or constant power and constant power factor load models may be used. Until recently loads in stability studies have been customarily represented as impedances which are constant under changing voltage and frequency conditions. This practice has been accepted for one or more of the following reasons[6]

(a) For studies of the first transient swing of a small system characterized by a generating plant feeding radially into a load area, the load representation is not very critical.

(b) The constant impedance elements lend themselves to simple models. In this case loads may be included in the admittance matrix of the system. Until the advent of large scale digital computers, it has been impractical to make any other assumption for load response.

(c) Data on the response of system load to disturbances has been generally unavailable in the literature and it is almost impossible to obtain such data on the system.

(d) It is possible to achieve a completely closed form[7] simulation algorithm by making the assumption that all loads have a constant impedance current-voltage-frequency characteristic which saves computational time and effort.

From the above considerations, the simplest type of load representation, constant impedance, is used throughout the study. The load is described by the complex matrix equation

$$[V] = [Z][I] \tag{2.14}$$

Or by its inverse

$$[I] = [Y][V] \tag{2.15}$$

Each of these equations can be expanded to a set of the form

$$\begin{bmatrix} V_{D1}\\ V_{Q1}\\ \vdots \\ V_{Dr}\\ V_{Qr}\end{bmatrix} = \begin{bmatrix} R_{11} & -X_{11} & \cdots & R_{11} & -X_{1r}\\ X_{11} & R_{11} & \cdots & X_{1r} & R_{1r}\\ \vdots & & \ddots & & \vdots\\ R_{r1} & -X_{r1} & \cdots & R_{rr} & -X_{rr}\\ X_{r1} & R_{r1} & \cdots & X_{rr} & R_{rr}\end{bmatrix}\begin{bmatrix} I_{D1}\\ I_{Q1}\\ \vdots\\ I_{Dr}\\ I_{Qr}\end{bmatrix} \qquad (2.16)$$

2.4 Single machine infinite bus system

In stability studies, the interval of time of interest is relatively small compared to the time constants of the governor and the prime mover. While governor and prime mover responses are important, they are ineffective in the early part of the machine swing and will, if considered, contribute to stability. For simplicity, the torque variations due to governor control are neglected. The amortisseur windings are known to aid stability, so these are also not considered. The system studied is shown in figure 2.2.

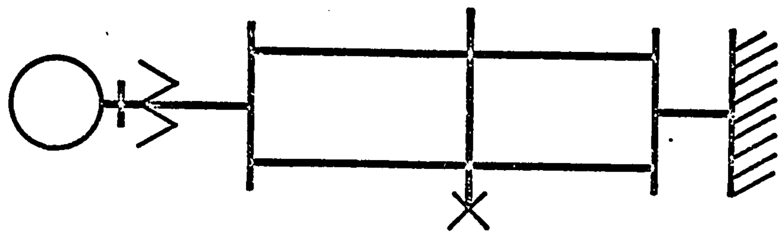

Fig. 2.2 System configuration for single machine case.

For this case θ_v in equation (2.8) is zero. Combining the terminal constraint with the machine flux linkage equations and replacing E_{fd} by u(t), the following set of ordinary nonlinear differential equations is obtained. (Details are given in Section A2-1 of appendix A2).

$$\dot{\underline{X}} = \underline{f}(\underline{X}) + \underline{d}\, u(t) \tag{2.17}$$

where

$$\underline{X} = [i_{fd}, i_d, i_q, n, \delta]^T \tag{2.18}$$

$\underline{d}$ is a vector of constant elements.

The voltage regulator equation is

$$pu(t) = \frac{K_r}{\tau_r} e_t - \frac{1}{\tau_r} u(t) - \frac{K_r}{\tau_r} u_s(t) + (E_o - K_r e_{tr})/\tau_r \tag{2.19}$$

The linearized state equations for the machine given in section A2-2 of appendix A2 are

The torque equations, in particular, are

$$(T_m \dot{n})_m = (T_{in})_m - (x_{afd}\ i_{fd}\ i_q)_m + (x_d - x_q)_m\ (i_d\ i_q)_m \tag{2.23}$$

$$\dot{\delta}_m = \omega_o n_m \tag{2.24}$$

where $m = 1, 2$

Equation (2.22) is similar to equation (A2-1.13) except that a_{15} is a function of the variable bus voltage v and arguments of sine and cosine terms change from δ to $\delta+\theta_v$. The bus bar voltage is obtained by the relation

$$v = Z\ i_\ell \tag{2.25}$$

where i_ℓ is the load current.

$$\Delta\underline{\dot{X}} = \underline{C}\,\Delta\underline{X} + \underline{d}\,\Delta u(t) \tag{2.20}$$

The corresponding linearized regulator equation is

$$\Delta\dot{u}(t) = \sum_{j=1}^{5} c_{6j}\,x_j + c_{66}\,\Delta u(t) - \frac{K_r}{\tau_r}\,u_s(t) \tag{2.21}$$

2.5 Two machine system

The following system configuration is considered

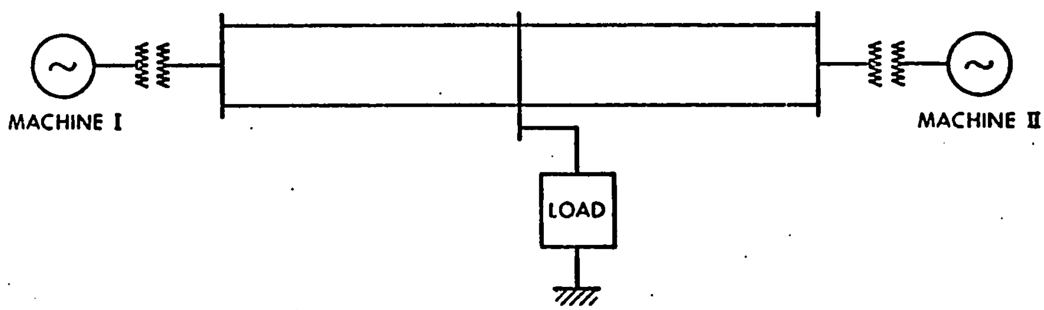

Fig. 2.3 System configuration for two machine case.

As in the case of the single machine, the governor action and amortisseur windings are neglected. The following set of differential equations is obtained

$$\underline{\dot{X}}_m = \underline{f}(\underline{X}_m, v) + \underline{d}_m\,u_m(t) \tag{2.22}$$

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APPENDIX A2-1

The differential equations for the single machine infinite bus case are developed in this section. For this case θ_v in equation (2.8) is zero. It is assumed that there is only one winding on the rotor - the field winding.

Substituting the terminal constraints equations (2.7) and (2.8) in equation (2.1) and rewriting equation (2.3) gives (neglect saturation)

$$p\psi_{fd} = E_{fd}\frac{\omega_o r_{fd}}{x_{afd}} - \omega_o r_{fd} i_{fd} \tag{A2-1.1}$$

$$p\psi_d = \omega_o[\frac{x_e}{\omega_o} pi_d + R_e i_d - \frac{\omega}{\omega_o} x_e i_q + v\sin\delta]$$

$$+ \omega_o R i_d + \omega\psi_d \tag{A2-1.2}$$

$$p\psi_q = \omega_o[\frac{x_e}{\omega_o} pi_q + R_e i_q + \frac{\omega}{\omega_o} x_e i_d + v\cos\delta]$$

$$+ \omega_o R i_q - \omega\psi_d \tag{A2-1.3}$$

Substituting the flux linkage relations (2.2) and (2.4) in the above three equations, the following simultaneous equations are obtained

$$\begin{bmatrix} x_{ffd} & -x_{afd} & 0 \\ x_{afd} & -(x_d+x_e) & 0 \\ 0 & 0 & -(x_q+x_e) \end{bmatrix} \begin{bmatrix} pi_{fd} \\ pi_d \\ pi_q \end{bmatrix} = \begin{bmatrix} -\omega_o r_{fd} i_{fd} + E_{fd}\frac{\omega_o r_{fd}}{x_{afd}} \\ \omega_o(R_e+R) i_d + \omega_o(R_e+R) i_q \\ -\omega x_{afd} i_{fd} + \omega(x_d+x_e) i_d \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ -\omega x_q \, i_q + \omega_o v \sin\delta \\ \omega_o(R_e+R) i_q + \omega_o v \cos\delta \end{bmatrix} \qquad \text{(A2-1.4)}$$

which in turn gives the following nonlinear differential equations

$$pi_{fd} = \frac{r_{fd}(x_d+x_e)i_{fd}}{\Delta_d} + \frac{x_{afd}(R_e+R)}{\Delta_d} i_d - \frac{x_{afd}(x_q+x_e)}{\Delta_d}(1+n)i_q$$

$$+ \frac{x_{afd}\, v}{\Delta_d} \sin\delta - \frac{r_{fd}(x_d+x_e)}{\Delta_d} E_{fd} \qquad \text{(A2-1.5)}$$

$$pi_d = \frac{r_{fd}\, x_{afd}}{\Delta_d} i_{fd} + \frac{x_{ffd}(R_e+R)}{\Delta_d} i_d - \frac{x_{ffd}(x_q+x_e)}{\Delta_d}(1+n)i_q$$

$$+ \frac{x_{ffd}\, v}{\Delta_d} \sin\delta - \frac{r_{fd}}{\Delta_d} E_{fd} \qquad \text{(A2-1.6)}$$

$$pi_q = \frac{x_{afd}}{\Delta_q}(1+n)i_{fd} - \frac{(x_d+x_e)}{\Delta_q}(1+n)i_d - \frac{(R_e+R)}{\Delta_q} i_q - \frac{v}{\Delta_q}\cos\delta$$

(A2-1.7)

where
$$\Delta_d = \frac{x_{afd}^2 - x_{ffd}(x_d+x_e)}{\omega_o}$$

$$\Delta_q = \frac{x_q+x_e}{\omega_o}$$

Substituting pi_d and pi_q in equation (2.7) gives

$$e_d = \frac{x_e\, r_{fd}\, x_{afd}}{\omega_o\, \Delta_d}\, i_{fd} + \left[\frac{x_e\, x_{ffd}(R_e+R)}{\omega_o\, \Delta_d} + R_e\right] i_d$$

$$-\left[\frac{x_e\, x_{ffd}(x_q+x_e)}{\omega_o\, \Delta_d} + x_e\right] (1+n) i_q - \frac{x_e\, r_{fd}}{\omega_o\, \Delta_d}\, E_{fd}$$

$$+ \left[v + \frac{x_{ffd}\, x_e\, v}{\omega_o\, \Delta_d}\right] \sin\delta \qquad \text{(A2-1.8)}$$

$$e_q = \frac{x_e\, x_{afd}}{\omega_o\, \Delta_q} (1+n) i_{fd} + \left[x_e - \frac{x_e(x_d+x_e)}{\omega_o\, \Delta_q}\right](1+n) i_d$$

$$+\left[R_e - \frac{x_e(R_e+R)}{\omega_o\, \Delta_q}\right] i_q + \left[v - \frac{x_e v}{\omega_o\, \Delta_q}\right] \cos\delta \qquad \text{(A2-1.9)}$$

The torque and voltage regulator equations can be written as

$$pn = \frac{T_{in}}{T_m} - \frac{x_{afd}}{T_m}\, i_{fd}\, i_q + \frac{(x_d-x_q)}{T_m}\, i_d i_q \qquad \text{(A2-1.10)}$$

$$p\delta = \omega_o n \qquad \text{(A2-1.11)}$$

$$pE_{fd} = \frac{K_r}{\tau_r}\, e_t - \frac{1}{\tau_r} E_{fd} + (E_o - K_r\, e_{tr})/\tau_r - \frac{K_r}{\tau_r}\, u_s(t) \qquad \text{(A2-1.12)}$$

where $e_t^2 = e_d^2 + e_q^2$ (A2-1.12a)

Equations (A2-1.5) to (A2-1.7) and (A2-1.10), (A2-1.11) can be grouped as

$$\begin{bmatrix} pi_{fd} \\ pi_d \\ pi_q \\ pn \\ p\delta \end{bmatrix} = \begin{bmatrix} a_{11}\, i_{fd} + a_{12}\, i_d + a_{13}(1+n)i_q + a_{15}\sin\delta \\ a_{21}\, i_{fd} + a_{22}\, i_d + a_{23}(1+n)i_q + a_{25}\sin\delta \\ a_{31}(1+n)i_{fd} + a_{32}(1+n)i_d + a_{33}i_q + a_{35}\cos\delta \\ a_{41}\, i_{fd}\, i_q + a_{42}\, i_d\, i_q + K \\ \omega_o n \end{bmatrix}$$

$$+ \begin{bmatrix} d_1 \\ d_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} E_{fd} \qquad \text{(A2-1.13)}$$

APPENDIX A2-2

For small perturbations, the system equations can be linearized about an operating point. In such cases, the change in speed deviation is negligibly small, so that $\omega/\omega_o = 1$. To include the voltage regulator action in the general linearized model, it is necessary to obtain an expression for perturbation of terminal voltage in terms of the states chosen.

From equation (A2-1.12a)

$$\Delta e_t = \frac{e_{qo}}{e_{to}} \Delta e_d + \frac{e_{qo}}{e_{to}} \Delta e_q \qquad \text{(A2-2.1)}$$

The voltage regulator equation in the linearized form is

$$p\Delta E_{fd} = \frac{K_r}{\tau_r} \Delta e_t - \frac{1}{\tau_r} \Delta E_{fd} - \frac{K_r}{\tau_r} u_s(t) \qquad \text{(A2-2.2)}$$

Substituting the linearized forms of Δe_d and Δe_q in (A2-2.1) one gets

$$p\Delta E_{fd} = c_{61}\,\Delta i_{fd} + c_{62}\,\Delta i_d + c_{63}\,\Delta i_q + c_{65}\,\Delta\delta + c_{66}\,\Delta E_{fd} - \frac{K_r}{\tau_r} u_s(t) \qquad \text{(A2-2.3)}$$

where

$$c_{61} = \frac{K_r}{\tau_r}\frac{e_{do}}{e_{to}}\,\frac{r_{fd}\,x_{afd}\,x_e}{\omega_o\Delta_d} + \frac{K_r}{\tau_r}\frac{e_{qo}}{e_{to}}\,\frac{x_e\,x_{afd}}{\omega_o\Delta_q}$$

$$c_{62} = \frac{K_r}{\tau_r}\frac{e_{do}}{e_{to}}\left[R_e + \frac{x_e\, x_{ffd}(R_e+R)}{\omega_o\Delta_d}\right] + \frac{K_r}{\tau_r}\frac{e_{qo}}{e_{to}}\left[x_e - \frac{x_e(x_d+x_e)}{\omega_o\Delta_q}\right]$$

$$c_{63} = -\frac{K_r}{\tau_r}\frac{e_{do}}{e_{to}}\left[x_e + \frac{x_e\, x_{ffd}(x_q+x_e)}{\omega_o\Delta_d}\right] + \frac{K_r}{\tau_r}\frac{e_{qo}}{e_{to}}\left[R_e - \frac{x_e(R_e+R)}{\omega_o\Delta_q}\right]$$

$$c_{65} = -\frac{K_r}{\tau_r}\frac{e_{qo}}{e_{to}}\sin\delta_o\left[v - \frac{x_e\, v}{\omega_o\Delta_q}\right] + \frac{K_r}{\tau_r}\frac{e_{do}}{e_{to}}\cos\delta_o\left[v + \frac{x_e\, x_{ffd}\, v}{\omega_o\Delta_d}\right]$$

$$c_{66} = -\frac{K_r}{\tau_r}\frac{e_{do}}{e_{to}}\frac{r_{fd}\, x_e}{\omega_o\Delta_d} - \frac{1}{\tau_r}$$

(A2-2.4)

The linearized voltage current torque relations are written as

$$\begin{bmatrix} p\Delta i_{fd} \\ p\Delta i_d \\ p\Delta i_q \\ pn \\ p\Delta\delta \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & c_{15} \\ c_{21} & c_{22} & c_{23} & 0 & c_{25} \\ c_{31} & c_{32} & c_{33} & 0 & c_{35} \\ c_{41} & c_{42} & c_{43} & 0 & 0 \\ 0 & 0 & 0 & \omega_o & 0 \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_d \\ \Delta i_q \\ n \\ \Delta\delta \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} E_{fd}$$

(A2-2.5)

where $c_{ij} = a_{ij}$; $i = 1, 2, 3$

$j = 1, 2, 3$

$$c_{15} = a_{15} \cos\delta_o$$

$$c_{25} = a_{25} \cos\delta_o$$

$$c_{35} = -a_{35} \sin\delta_o \qquad \text{(A2-2.6)}$$

$$c_{41} = a_{41}\, iq_o$$

$$c_{42} = a_{42}\, iq_o$$

$$c_{43} = a_{41}\, i_{fdo} + a_{42}\, i_{do}$$

CHAPTER 3

CLOSED LOOP QUASI-OPTIMAL EXCITATION CONTROL FOR A SINGLE MACHINE

3.1 Introduction

The disturbances appearing on a power system are, in general, not known in advance. To the best knowledge of the author, the available methods of optimization involve iteration even for a known disturbance. This is costly in terms of computing time. The time involved in finding the optimal control by digital computers, after a fault appears on a system, may be so long that the system becomes unstable before a control strategy is determined. A suitable control must overcome or avoid this. Secondly, a predetermined strategy (found by iterative schemes) will not be optimal for other than the design disturbance. A closed loop control in which the excitation depends on actual conditions and requires a minimum of computation in its implementation is desirable. This chapter discusses such a control.

The stability problem for a single machine infinite bus system is formulated in Section 2. A quasi-optimal state feedback excitation control for the linearized system is suggested in Section 3. Section 4 investigates the closeness of the quasi-optimal solution to the optimal one. In Section 5, the method is extended to the nonlinear machine model. For implementing the quasi-optimal closed loop control on real systems, a practical compromise is suggested in Section 6. The properties of a functional which depends on the states of the system is discussed in Section 7. The results obtained for the single machine infinite bus system are discussed in Section 8.

3.2 Statement of the problem

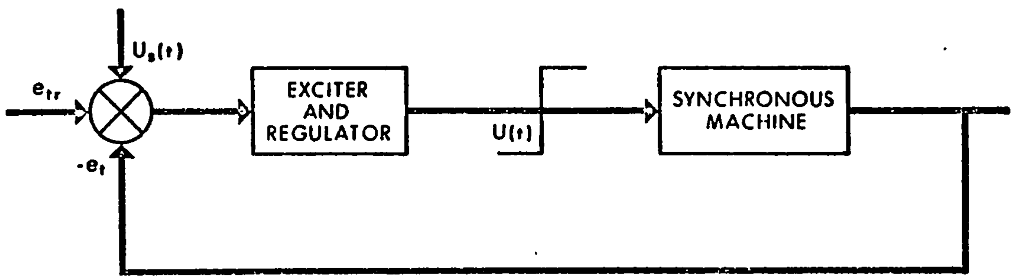

Fig. 3.1 System block diagram

Assuming that the synchronous machine is equipped with a static excitation system, the regulator time constant is very small with respect to the machine time constants and initially τ_r (equation 2.19) is taken as zero. Then the voltage regulator equation becomes a static equation. The exciter in such a case can be driven to the ceilings almost instantaneously. The dynamic equations for a single machine case, as given in the previous chapter, are

$$\dot{\underline{X}}(t) = f(\underline{X}) + \underline{d}\, u(t) \tag{3.1}$$

The magnitude of the field voltage is bounded by the exciter ceiling so that the normalized scalar control u(t) is constrained in magnitude by the relation $|u(t)| \leq 1$. It is assumed that u(t) is a piecewise continuous function of time.

The system under consideration returns to the stable equilibrium point (see the definition given in Chapter 1) if both velocity and acceleration (n, $\dot{n}$) of the machine decreases to zero following a disturbance while the final rotor angle remains constrained in the range from 0 - $\pi/2$ radians.

The optimization problem can be stated as: Find the admissible control u(t) which transfers system (3.1) from the set of given initial states $\underline{X}(0)$ to the desired final states

$$n_{(t_f)}[x_{4(t_f)}] = 0$$

$$\dot{n}_{(t_f)}[\dot{x}_{4(t_f)}] = 0 \tag{3.2}$$

$$0 \leq \delta_{(t_f)}[\omega_o x_{5(t_f)}] \leq \pi/2$$

So that the cost functional

$$J = \int_{t_o}^{t_f} dt \tag{3.3}$$

is minimized, since it is desired that the transients are eliminated in the least possible time.

3.3 The linearized system

The linear system is valid for only small disturbances. For such cases, the rotor angle does not traverse far beyond the operating point. It is likely that the rotor angle will always remain between 0 and $\pi/2$ radians. For stable operation following a disturbance, it is desired that the machine velocity and acceleration should drop to zero in the smallest possible time. Rewriting the linearized machine equations from equation (2.20) (disregarding the regulator time constant) leads to

$$\Delta\dot{\underline{X}} = \underline{C}\,\underline{\Delta X} + \underline{d}\,u(t) \tag{3.4}$$

and the swing equations become

$$\Delta\dot{\delta} = \omega_o n \tag{3.5}$$

$$\dot{n} = c_{41}\,\Delta i_{fd} + c_{42}\,\Delta i_d + c_{43}\,\Delta i_q \tag{3.6}$$

Step 1

Differentiate equation (3.6) with respect to time to get

$$\ddot{n} = c_{41}\,p\Delta i_{fd} + c_{42}\,p\Delta i_d + c_{43}\,p\Delta i_q \tag{3.7}$$

Substitute the equations for $p\Delta i_{fd}$, $p\Delta i_d$ and $p\Delta i_q$ from equation (3.4) into (3.7) to obtain

$$\begin{aligned}\ddot{n} = {} & (c_{41}\,c_{11} + c_{42}\,c_{21} + c_{43}\,c_{31})\Delta i_{fd} + (c_{41}\,c_{12} + \\ & c_{42}\,c_{22} + c_{43}\,c_{32})\Delta i_d + (c_{41}\,c_{13} + c_{42}\,c_{23} + c_{43}\,c_{33})\Delta i_q \\ & + (c_{41}\,c_{15} + c_{42}\,c_{25} + c_{43}\,c_{35})\Delta\delta + (c_{41}d_1 + c_{42}d_2)u(t)\end{aligned} \tag{3.8}$$

or

$$\ddot{n} = L(\Delta i_{fd}, \Delta i_d, \Delta i_q, \Delta\delta) + b\,u(t) \tag{3.9}$$

Since Δi_{fd}, Δi_d, Δi_q and $\Delta\delta$ are functions of time, the above equation can be written as

$$\ddot{n} = L(t) + b\,u(t) \tag{3.10}$$

where
$$L(t) = (c_{41}\, c_{11} + c_{42}\, c_{21} + c_{43}\, c_{31})\Delta i_{fd} + (c_{41}\, c_{12} + c_{42}\, c_{22}$$
$$c_{43}\, c_{32})\Delta i_d + (c_{41}\, c_{13} + c_{42}\, c_{23} + c_{43}\, c_{33})\Delta i_q$$
$$+ (c_{41}\, c_{15} + c_{42}\, c_{25} + c_{43}\, c_{35})\Delta\delta$$

$$b = c_{41}\, d_1 + c_{42}\, d_2 \tag{3.11}$$

Equation (3.10) can be represented by the block diagram in figure 3.2.

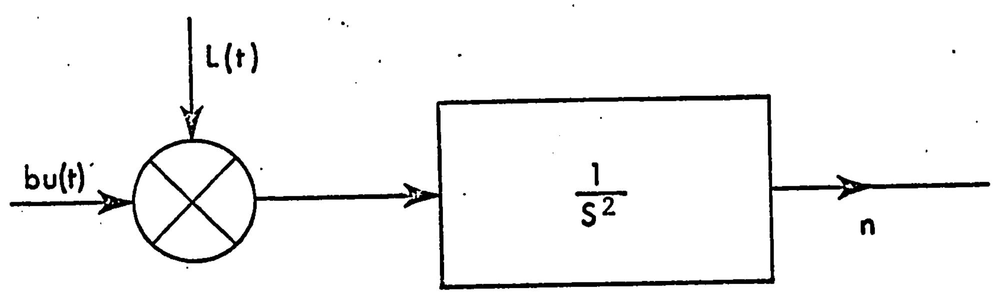

Fig. 3.2 Double integral plant with disturbance L(t).

This is a so called double integral plant where the input to the plant is modified by an additive term L(t), the value of which is generally unknown. (Properties of L(t) are given in Section (3.7)). The system

(3.10) is called controllable[1] only if

$$|L(t)| \leq |b\ u(t)| \tag{3.12}$$

or

$$|L(t)/b| \leq 1 \tag{3.13}$$

For small disturbances, the quantity L(t) is small enough. It is assumed that L(t) is at least piecewise continuous.

Step 2

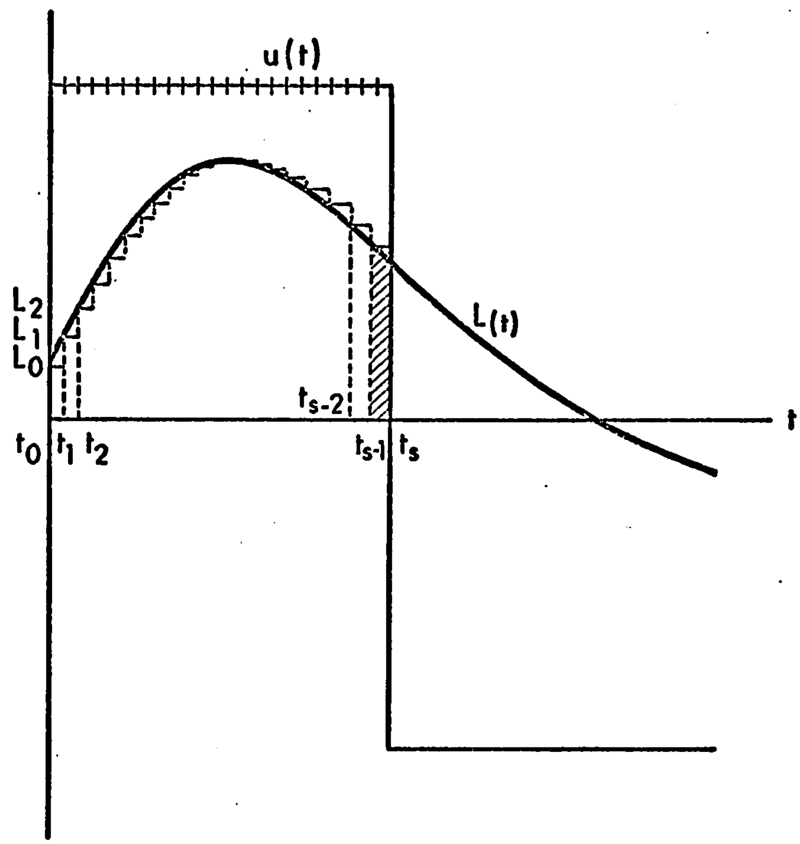

Fig. 3.3 A portion of L(t) and control u(t)

L(t) depends on the states of the system (3.4) and its initial value is known. Suppose at $t = t_o$, L(t) is L_o. The optimal control problem can be reformulated as follows:

Given the system

$$\ddot{n} = u_o(t), \quad u_{omin} \leq u_o(t) \leq u_{omax} \tag{3.14}$$

where

$$u_o(t) = L_o + b\, u(t) \tag{3.15}$$

Find the admissible control that forces the system (3.14) from any initial state $\{n_{(o)}, \dot{n}_{(o)}\}$ to the origin in shortest possible time.

The analysis, using Pontryagin's minimum principle (P.M.P) for time optimal control of the system (3.14) is given in several references[2] and is briefly outlined here for completeness only.

For convenience, assume $x_1 = n$; then system (3.10) can be rewritten as

$$\dot{x}_1 = x_2 \tag{3.16}$$

$$\dot{x}_2 = u_o(t)$$

[These x's may not have any relation with $\underline{X}$ in equation (3.4)].

The Hamiltonian for (3.16) is given by

$$H = 1 + x_2(t)\, p_1(t) + u_o(t)\, p_2(t) \tag{3.17}$$

If $x^*(t)$, $p^*(t)$ and $u_o^*(t)$ correspond to the optimal state, costate and control respectively, then by the minimum principle

$$1 + x_2^*(t)\, p_1^*(t) + u_o^*(t)\, p_2^*(t) \leq 1 + x_2^*(t)\, p_1^*(t) + u_o(t)\, p_2^*(t) \tag{3.18}$$

holds for all admissible $u_o(t)$ and for $t \in [t_o, t_f]$. Equation (3.18) gives

$$u_o^*(t) = \begin{cases} u_{omax}, \text{ if } p_2(t) < 0 \\ u_{omin}, \text{ if } p_2(t) > 0 \end{cases} \tag{3.19}$$

The costate variables satisfy the relation

$$\dot{p}_1(t) = -\frac{\partial H}{\partial x_1(t)} = 0 \tag{3.20}$$

$$\dot{p}_2(t) = -\frac{\partial H}{\partial x_2(t)} = -p_1(t) \tag{3.21}$$

Let π_1 and π_2 be the initial values of the costates, then

$$p_1(t) = \pi_1 = \text{constant} \tag{3.22}$$

$$p_2(t) = \pi_2 - \pi_1 t \tag{3.23}$$

Note that $p_2(t)$ is a straight line in the p_2 - t plane and can intersect the time axis once only. So from relation (3.19), the optimal control is piecewise constant and can switch at most once. Assume $u_o(t) = \alpha$ (which is either u_{omax} or u_{omin}) and let $x_1(o) = \xi_1$ and $x_2(o) = \xi_2$. Equation (3.16) can be solved with these initial conditions for constant control to obtain the relations

$$x_2(t) = \xi_2 + \alpha t \tag{3.24}$$

$$x_1(t) = \xi_1 + \xi_2 t + \frac{1}{2}\alpha t^2 \tag{3.25}$$

Next, eliminate t to find

$$x_1 = \xi_1 + \frac{1}{2\alpha} x_2^2 - \frac{1}{2\alpha}\xi_2^2 \tag{3.26}$$

where $$t = (x_2 - \xi_2)/\alpha \tag{3.26a}$$

Equation (3.26) is the equation of the trajectory in the $x_1 - x_2$ plane originating at (ξ_1,ξ_2) as affected by the action of the control α. The switch curve is the locus of all points (x_1,x_2) which can be forced to zero by the control $u_o = \alpha$ and is given by

$$\gamma^\circ = \left\{(x_1,x_2) : x_1 - \frac{x_2^2}{2[L_o - |b|\mathrm{Sgn}\{x_2\}]} = 0\right\} \tag{3.27}$$

If $$\Sigma^\circ = x_1 - \frac{x_2^2}{[L_o - |b|\mathrm{Sgn}\{x_2\}]} \tag{3.28}$$

then the control law can be given as

$$u_o^*(t) = \begin{cases} u_{omin} & \text{if } \Sigma^\circ > 0 \\ u_{omax} & \text{if } \Sigma^\circ < 0 \end{cases} \tag{3.29}$$

If $\Sigma^\circ = 0$, then $u_o^*(t)$ is u_{omin} for $x_2 > 0$, otherwise $u_o^*(t)$ is u_{omax}. But for a physical system, the trajectory does not continue along the curve

$\sum^{\circ} = 0$ for a finite interval of time[2].

Step 3

Given the system of equations (3.16), for any set of initial states (ξ_1, ξ_2) the optimal control $u_0^*(t)$ is obtained by control law (3.29). The value of the optimal control $u^*(t)$ is decided by the relation (3.15). If b is positive, u_{omax} implies $u = 1$ and if b is negative u_{omax} implies $u = -1$ and vice versa.

The term L(t) depends on Δi_{fd}, Δi_d, Δi_q and $\Delta\delta$ (and derivative of the disturbance function, if there is a sustained disturbance). The initial value of L(t) is calculated depending on the initial values of currents and rotor angle. The optimal control is then obtained for the constant value L_0. This control u(t) (* for the optimal values are dropped for the rest of the analysis) is then used to solve the system of equations (3.4) for a small time step.

Step 4

At the end of the first interval, at $t = t_1$ (fig. 3.3), suppose that the states in the $x_1 - x_2$ plane are $\underline{X}_1$ and the corresponding values of other states are $\{\Delta i_{fd(1)}, \Delta i_{d(1)}, \Delta i_{q(1)}$ and $\Delta\delta_{(1)}\}$. Using these new values of currents and angle recalculate L(t). Suppose $L(t) = L_1$ at $t = t_1$. If $L_1 = L_0$, the control law (3.29) still holds good and the process is continued. However, if $L_1 \neq L_0$, then the second problem is:

Given the system of equations

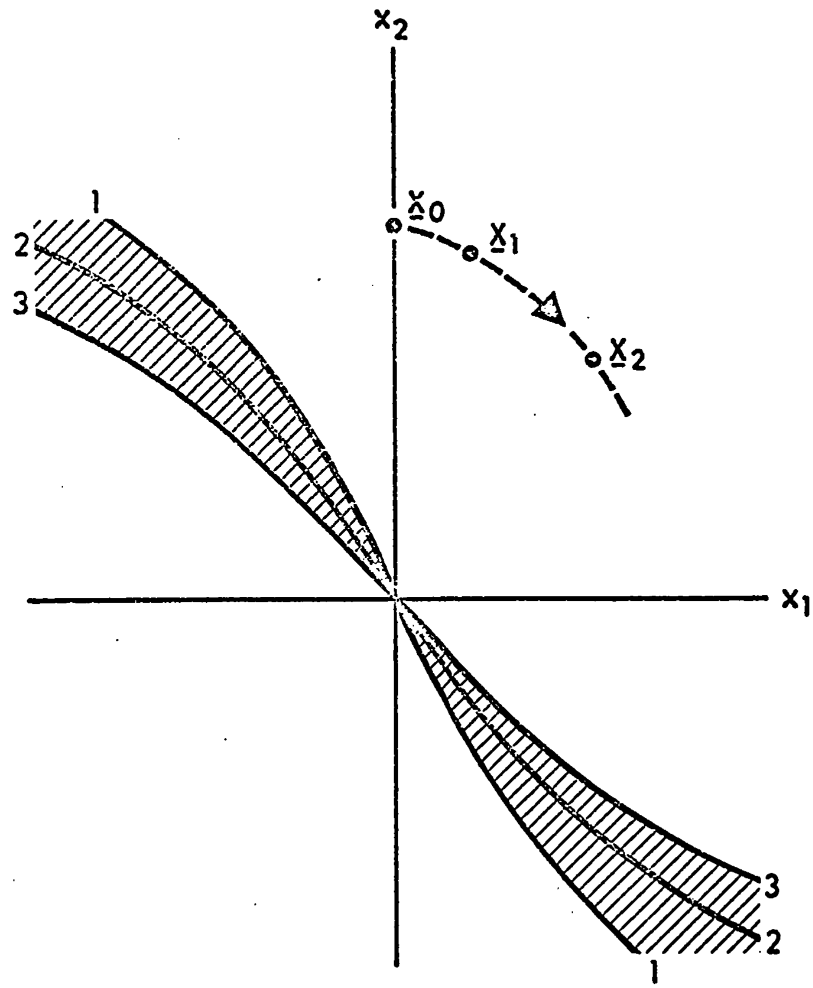

Fig. 3.4 Moving switch curves in phase plane.

$$\ddot{n} = L_1 + bu(t)$$

$$= u_1(t), \qquad u_{1min} \leq u_1(t) \leq u_{1max} \tag{3.30}$$

Find the admissible control that forces the system (3.30) from the given initial states $\underline{X}_1$ to the origin in minimum possible time. Proceeding as before, the switch curve is given by

$$\gamma^1 = \{(x_1,x_2) : x_1 - \frac{x_2^2}{2[L_1 - |b|Sgn\{x_2\}]} = 0\} \tag{3.31}$$

and the switching function and control scheme is similar to (3.28) and (3.29) respectively with L_0 replaced by L_1. Similarly at $t=t_p$, the switch curve for $L(t) = L_p$ is given by

$$\gamma^p = \{(x_1,x_2) : x_1 - \frac{x_2^2}{2[L_p - |b|Sgn\{x_2\}]} = 0\} \tag{3.32}$$

Figure 3.4 shows the set of moving switch curves in the phase plane.

3.4 Optimality of the control

It can be seen that for finding the control u(t), the quantity L(t) has been approximated by a staircase function. Since L(t) is being approximated, it would appear the control scheme (3.29) would give rise to a sub-optimal solution. In fact, if equation (3.10) was solved by applying this control scheme, only a sub-optimal solution would be found, the nearness to optimality being decided by the size of the step.

However, a close look would reveal that equation (3.10) is just a dummy equation which is needed to find the optimal control. The information obtained from (3.10) is deciding the control at the sample points $\{t_1, t_2, ...t_s, ...\}$ which in turn is being fed to the system (3.4). It will be shown that irrespective of the variation of L(t) in the range $t \in [t_0, t_{s-1}]$, the control u is +1 (figure 3.3).

Observe that at $t = t_0$, the control is +1 in figure 3.3 and it is optimal since by proposition L(t) does not change instantaneously. Again at $t = t_1$, the control is +1;by the control law (3.29) at that instant also the control is optimal. Since right hand side of equation (3.1) is linear in the control u, the time optimal control for system (3.4) is bang bang i.e., either it is +1 or -1. The only other possibility in this particular case is that the control switches from +1 to -1 and then switches back to +1 in the interval $\{t_0,t_1\}$. This means that the control switches twice in the first time step. But this interval can be chosen small enough so that the control really cannot have two intermediate switches. Moreover, the number of switchings for system (3.4) is finite. It is improbable that the control will switch twice in each subinterval. It can be seen in the figure that in the intervals $\{t_0,t_1\}$, $\{t_1,t_2\}$, ...$\{t_{s-2},t_{s-1}\}$, irrespective of the variation of L(t) the control is +1 i.e., in this interval, the staircase approximation of L(t) does not affect the optimal scheme.

However, the control switches at $t = t_s$ and this switching is determined by a constant value L_s. But since L(t) does not remain constant in $\{t_{s-1},t_s\}$, the correct switching can be anywhere in this interval. The error in switching will be negligible since the subintervals can be made very small as allowed by numerical integration procedures with sufficient accuracy.

The control obtained by the above scheme is optimal over each subinterval, except possibly in the neighbourhood of the switch points. As the solution is time optimal in each subinterval, it is optimal over the whole interval[3]. (A similar work done by Oldenburger[4] is given in Section A3-1 of appendix A3).

3.5 The nonlinear model

For large disturbances, the linearized machine model gives a poor approximation. In such cases, the rotor angle might swing past 90 degrees and there is a possibility that the machine will settle momentarily at the unstable equilibrium point. For stable operation of the machine, the velocity and acceleration of the machine should drop to zero following a disturbance while the final rotor angle should be constrained between 0 and $\pi/2$ radians.

In finding the control function, it is recognized that n is very small. So the terms where n (speed deviation) is coupled to other system states are neglected. (This is not necessary but simplifies the algorithm). Differentiating the nonlinear swing equation (A2-1.13) yields

$$p^3(\delta/\omega_o) = (a_{41}\, i_{fd} + a_{42}\, i_d) p i_q + a_{41}\, i_q\, p i_{fd} + a_{42}\, i_q\, p i_d \tag{3.33}$$

Substituting the nonlinear voltage current relations from equation (A2-1.13) in equation (3.33) gives

$$p^3(\delta/\omega_o) = a_{41}\, a_{31}\, i_{fd}^{\;2} + a_{42}\, a_{32}\, i_d^{\;2} + (a_{41}\, a_{13} + a_{42}\, a_{23}) i_q^{\;2}$$

$$+ (a_{41}\, a_{32} + a_{42}\, a_{31}) i_{fd}\, i_d + (a_{41}\, a_{11} + a_{42}\, a_{21} + a_{41}\, a_{33})$$

$$i_{fd}\, i_q + (a_{41}\, a_{12} + a_{42}\, a_{22} + a_{42}\, a_{33})\, i_d\, i_q$$

$$+ (a_{41}\, i_{fd} + a_{42}\, i_d)\, a_{35} \cos\delta + (a_{41}\, a_{15} + a_{42}\, a_{25}) i_q \sin\delta$$

$$+ (a_{41}\, d_1 + a_{42}\, d_2) i_q\, E_{fd} \qquad (3.34)$$

Equation (3.34) can be written as

$$p^3(\delta/\omega_o) = L(t) + b(t)\, u(t) \qquad (3.35)$$

where $L(t) = a_{41}\, a_{31}\, i_{fd}^{\;2} + a_{42}\, a_{32}\, i_d^{\;2} + (a_{41}\, a_{13} + a_{42}\, a_{23}) i_q^{\;2}$

$$+ (a_{41}\, a_{32} + a_{42}\, a_{31}) i_{fd}\, i_d + (a_{41}\, a_{11} + a_{42}\, a_{21} + a_{41}\, a_{33})$$

$$i_{fd}\, i_q + (a_{41}\, a_{12} + a_{42}\, a_{22} + a_{42}\, a_{33}) i_d\, i_q$$

$$+ (a_{41}\, i_{fd} + a_{42}\, i_d)\, a_{35} \cos\delta + (a_{41}\, a_{15} + a_{42}\, a_{25})\, i_q \sin\delta$$

(3.36)

$$b(t) = (a_{41}\, d_1 + a_{42}\, d_2) i_q$$

Assume that L(t) and b(t) are constant at their initial values L_0 and b_o respectively and let $x_1 = \delta/\omega_o$, then (3.35) can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= u_o(t) \end{aligned} \tag{3.37}$$

where $u_o(t) = L_o + b_o\, u(t)$

and $u_{omin} \leq u_o(t) \leq u_{omax}$ (3.38)

The optimization problem is to find the admissible control that forces the system (3.37) from any set of initial states $\{\xi_1(x_1(o)), \xi_2(x_2(o)), \xi_3(x_3(o))\}$ to

$$\begin{aligned} 0 \leq x_{1(t_f)} &\leq \pi/2\omega_o \\ x_{2(t_f)} &= 0 \\ x_{3(t_f)} &= 0 \end{aligned} \tag{3.39}$$

in the shortest possible time.

As in the case for the second order system (3.16), the Hamiltonian can be constructed and it can be shown that the H-minimal control is piecewise constant and can switch, at most, twice. The system of equations (3.37) can be solved for constant $u_o(t) = \alpha$ to give

$$x_3 = \alpha t + \xi_3 \tag{3.40}$$

$$x_2 = \frac{\alpha t^2}{2} + \xi_3 t + \xi_2 \tag{3.41}$$

$$x_1 = \frac{\alpha t^3}{6} + \frac{\xi_3 t^2}{2} + \xi_2 t + \xi_1 \tag{3.42}$$

from (3.40) $$t = (x_3 - \xi_3)/\alpha \tag{3.43}$$

Substituting for t in (3.41) one gets

$$x_2 = \xi_2 + \frac{x_3^{\,2}}{2\alpha} - \frac{\xi_3^{\,2}}{2\alpha} \tag{3.44}$$

$$x_1 = \frac{\alpha}{6}\left(\frac{x_3 - \xi_3}{\alpha}\right)^3 + \frac{\xi_3}{2}\left(\frac{x_3 - \xi_3}{\alpha}\right)^2 + \xi_2\left(\frac{x_3 - \xi_3}{\alpha}\right) + \xi_1 \tag{3.45}$$

This leads to

$$x_1 = \frac{1}{3\alpha^2}\xi_3^{\,3} - \frac{\xi_2\xi_3}{\alpha} + \xi_1 + \frac{1}{6\alpha^2}x_3^{\,3} + \frac{x_3}{\alpha}\left(\xi_2 - \frac{1}{2\alpha}x_3^{\,2}\right) \tag{3.46}$$

Substituting (3.44) in this expression yields

$$x_1 = \frac{1}{3\alpha^2}\xi_3^{\,3} - \frac{\xi_2\xi_3}{\alpha} + \xi_1 - \frac{1}{3\alpha^2}x_3^{\,3} + \frac{x_2 x_3}{\alpha} \tag{3.47}$$

Or

$$x_1 + \frac{1}{3\alpha^2}x_3^{\,3} - \frac{x_2 x_3}{\alpha} = \xi_1 + \frac{1}{3\alpha^2}\xi_3^{\,3} - \frac{\xi_2\xi_3}{\alpha} \tag{3.48}$$

Expression (3.48) is the equation of a surface in the x_1-x_2-x_3 plane for any initial state (ξ_1, ξ_2, ξ_3). The equation of the trajectory passing through $x_1 = \pi/2\omega_0$, $x_2 = 0$ and $x_3 = 0$ is given by

$$x_1 - \frac{\pi}{2\omega_o} = \frac{x_2 x_3}{\alpha} - \frac{1}{3\alpha^2} x_3^3 \quad (3.49)$$

which is one of the switching surfaces in the x_1-x_2-x_3 plane.

The other surface which passes through the origin is given by

$$x_1 = \frac{x_2 x_3}{\alpha} - \frac{1}{3\alpha^2} x_3^3 \quad (3.50)$$

where $$\alpha = L_o + b_o \, \mathrm{Sgn}\left\{ x_2 - \frac{x_3^2}{2[L_o + b_o \mathrm{Sgn}\{x_3\}]} \right\}; \; b_o < 0 \quad (3.51)$$

In the region between these two surfaces, the control is decided by the states x_2-x_3 and the switching curve is given by

$$x_2 - \frac{x_3^2}{2[L_o + b_o \mathrm{Sgn}\{x_3\}]} = 0 \quad ; \quad b_o < 0 \quad (3.52)$$

The following steps are involved in finding the control

1. Determine x_2 and x_3.
2. Determine $\sum = x_2 - \dfrac{x_3^2}{2[L_o + b_o \mathrm{Sgn}\{x_3\}]}$; $b_o < 0$ (3.53)
3. If $\sum > 0$, $\alpha = u_{omin}$ otherwise $\alpha = u_{omax}$.
4. If $\sum = 0$ and $x_3 > 0$, $\alpha = u_{omin}$ otherwise $\alpha = u_{omax}$.
5. Determine $\sum_1 = x_1 - \dfrac{\pi}{2\omega_o} - \dfrac{x_2 x_3}{\alpha} + \dfrac{1}{3\alpha^2} x_3^3$ (3.54)

   $\sum_2 = x_1 - \dfrac{x_2 x_3}{\alpha} + \dfrac{1}{3\alpha^2} x_3^3$ (3.55)
6. If $\sum_1 \leq 0$ and $\sum_2 \geq 0$

$$u(t) = \text{Sgn}\left[x_2 - \frac{x_3^{\,2}}{2[L_0 + b_0 \text{Sgn}\{x_3\}]}\right] ; \; b_0 < 0 \tag{3.56}$$

7. If $\sum_1 > 0$, $u(t) = 1$; $b_0 < 0$

If $\sum_2 < 0$, $u(t) = -1$; $b_0 < 0$ (3.57)

The system does not continue along the switching surfaces for a finite interval of time due to system imperfections[4]. The process outlined above is continued until the desired final states are reached.

3.6 Proportional Control

So far it has been considered that the regulator time constant τ_r is zero, so that the exciter can be driven to the ceiling voltages instantaneously. This means that for small deviations of velocity and acceleration, the exciter will still be repeatedly driven to the limits. This causes unnecessary wear and tear to the equipment and is equivalent to 'firing a cannon to kill a bug'. It deteriorates the machine internal voltage and worsens the terminal voltage of the machine. Moreover, the reproduction of complicated control functionsby physical equipment in industry is generally costly and practical compromises must be made[4]. To allow normal voltage regulator action, a small deadzone has to be provided so that when the oscillations are small enough, the voltage regulator takes over. It is usually not convenient in practice to use

$$u(t) = \text{Sgn} \sum \tag{3.58}$$

as the control equation but to employ the proportion relationship

$$u = k\Sigma \tag{3.59}$$

where k is as large as possible. For $k = \infty$, equation (3.59) reduces to (3.58). An optimum value of k has to be found by measurement or computer simulation. Too high a gain throws the system into a steady hunting. Equation (3.59) gives approximate bang bang control for large disturbances. When the oscillations gradually die out, normal voltage regulator action takes over.

For small disturbances the acceleration term in Σ is negligible compared to the velocity term, so that proportional control gives

$$\begin{aligned} u(t) &= k\Sigma \\ &\simeq kn \end{aligned} \tag{3.60}$$

which is the velocity signal as used at present. For large disturbances a combination of velocity and acceleration terms is determined by L(t), i.e., by the various currents and the rotor angle. References [5,6,7,8] bear this out. The proportional control is not quasi-optimal and, in general, quasi-optimal or optimal control may not be implementable.

3.7 Some comments on L(t)

Rewriting equation (3.9)

$$\ddot{n} = L(\Delta i_{fd}, \Delta i_d, \Delta i_q, \Delta\delta) + b\, u(t) \tag{3.61}$$

The arguments of L are functions of time, so without loss of generality equation (3.61) may be expressed as

$$\ddot{n} = L(t) + b\,u(t), \quad |u(t)| \le 1 \tag{3.62}$$

The block diagram for system (3.62) is given in figure 3.2.

The concept of controllability, as given in reference [2] is that a system is controllable if it is possible to drive any state of the system to the origin in finite time. Here controllability means complete controllability (global controllability). For example, the system

$$\ddot{n} = u(t), \quad |u(t)| \le 1 \tag{3.63}$$

is completely controllable. But if the upper and lower bounds of the control u(t) is such that $0 \le u(t) \le 1$, then system (3.63) is not completely controllable since there are some states in the state space which can never be brought to the origin with the application of this control. It is obvious that the system (3.62) is controllable only if[4]

$$|L(t)/b| \le 1 \tag{3.64}$$

The term L(t) could be considered an additive disturbance term to the input of system (3.63) which is governed by a set of differential equations. It can be seen that L(t) does not contain any output or derivative of the output feedback term and also L(t) can have any value satisfying constraint (3.64).

For the nonlinear system (3.35) L(t) contains feedback terms such as

δ and n. Moreover, for large disturbances, it may not satisfy the constraint (3.64) for all $t \in [t_0, t_f]$. In such cases, the solution obtained with the control scheme is not optimal.

To summarize, L(t) should have the following properties, so that the solution will be closest to optimal.

1. It should be continuous in the time interval, at least it should not have any jump discontinuities.

2. It should satisfy the relation

$$|L(t)/b| < |u(t)| \quad \text{for all } t \in [t_0, t_f] \qquad (3.65)$$

3. Output feedback terms should not dominate L(t) and L(t) should not have any other constraint except (3.65).

A sub-optimal solution is obtained if

1. L(t) contains output or derivatives of output feedback which are dominant.
2. The uncontrollable region is followed by a long controllable section[4].

An example, showing that a sub-optimal solution is obtained when L(t) contains output terms, is given in Section A3-2 of appendix A3.

3.8 Discussion of results

A power system comprised of synchronous generators and transmission network, whose parameters are given in section A3-3 of appendix A3, was simulated on a digital computer. Damper windings and saturation of the machine and governor action are not considered. The system configuration is given in figure 2.2.

i) The linearized model - 10% torque step.

For small disturbances, the nonlinear differential equations of the system can be linearized about an operating point. A 10% input torque step is considered for such a system.

Figure 3.5 shows the effect of exciter ceiling on the response of the system. With the bang bang control the time for stabilization and the angular deviation increases, as the exciter ceiling is decreased; that is, the effectiveness of the control decreases.

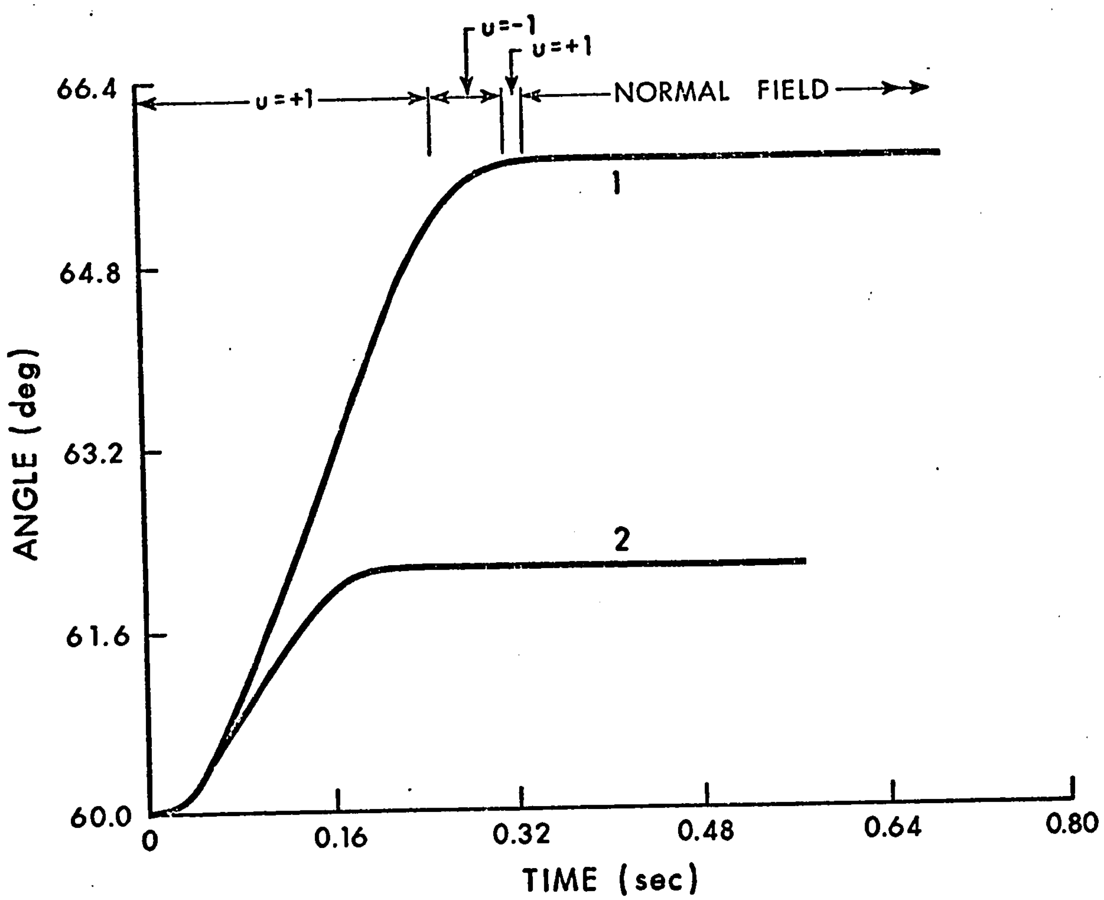

Fig. 3.5 Rotor angle time characteristics for 10% torque step (Bang bang control).

1. Ceiling voltage ±3 p.u.
2. Ceiling voltage ±5 p.u.

The optimal bang bang control results in a deadbeat response. While the use of proportional control provides a well damped stabilization, it is not optimal and takes more time to stabilize. Voltage regulator action alone leads to dynamic instability. These are shown in figures 3.6, 3.7 and 3.8.

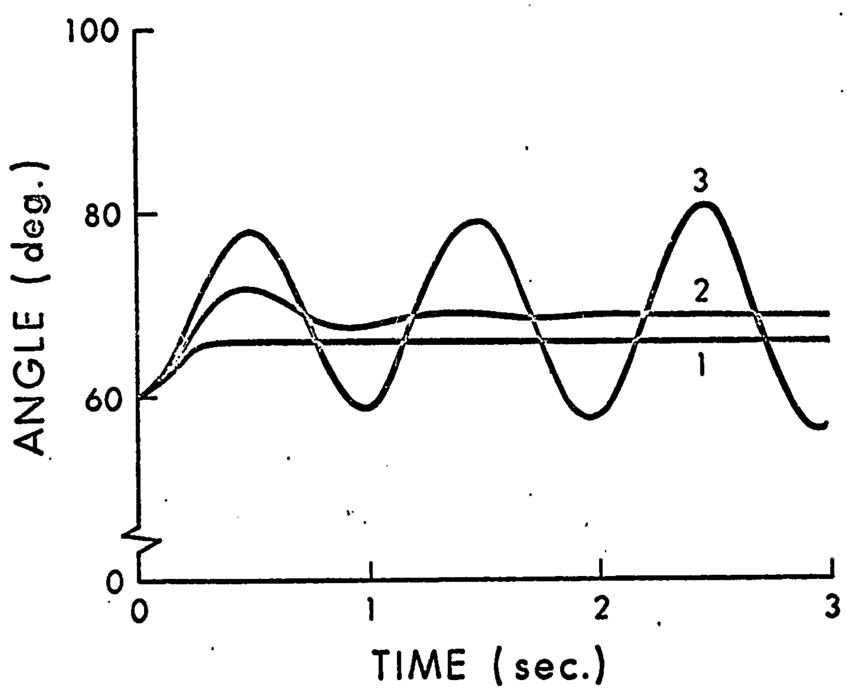

Fig. 3.6 Angle time characteristics for 10% torque step.

1. Optimal control.
2. Proportional control.
3. Voltage regulator alone.

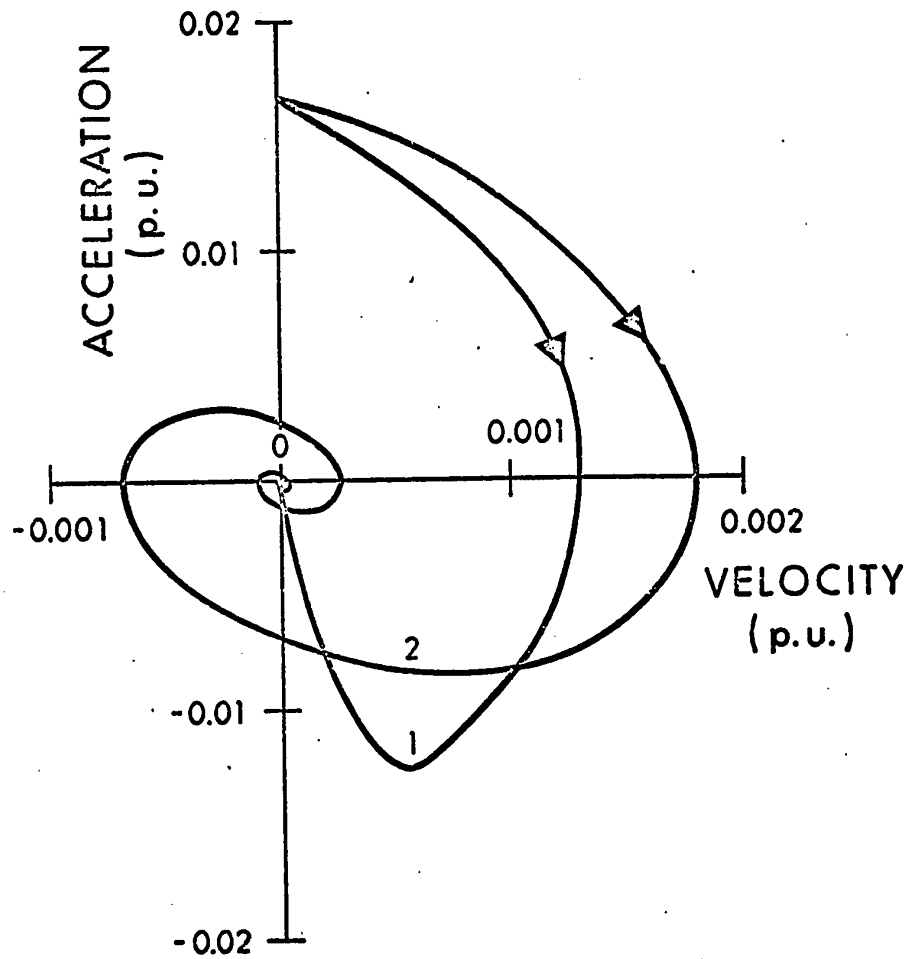

Fig. 3.7 Velocity vs acceleration plot corresponding to Fig. 3.6.

1. Optimal control.

2. Proportional control.

driven too hard but effective stabilization is obtained. Increasing the regulator gain implies a shift of the roots of the linearized system to the right hand side of the complex frequency plane. The bang bang control assumes that the regulator gain is infinite.

Case	Reg.Gain	Control Signal	Ceiling Voltage (p.u.)	Stability	Time to stabilize (sec)	Final rotor angle
1	-4.5	Proportional	±5	Stable	-	68°
2	-4.5	Voltage Regulator only	±5	Unstable	-	-
3	-10	Prop	±5	Stable	-	68°
4	-10	V.R.	±5	Unstable	-	-
5	-100	Prop	±5	Stable	-	68°
6	-100	V.R.	±5	Unstable	-	-
7	-1000	Prop	±5	Stable	-	68°
8	-1000	V.R.	±5	Unstable	-	-
9	—	Bang bang	±5	Stable	.18	62.1°
10	—	Bang bang	±3	Stable	.35	65.7°

Table 3.1 10% Torque Step, linearized system.

ii) Nonlinear model - 30% torque step.

This is a relatively large disturbance and the system has to be

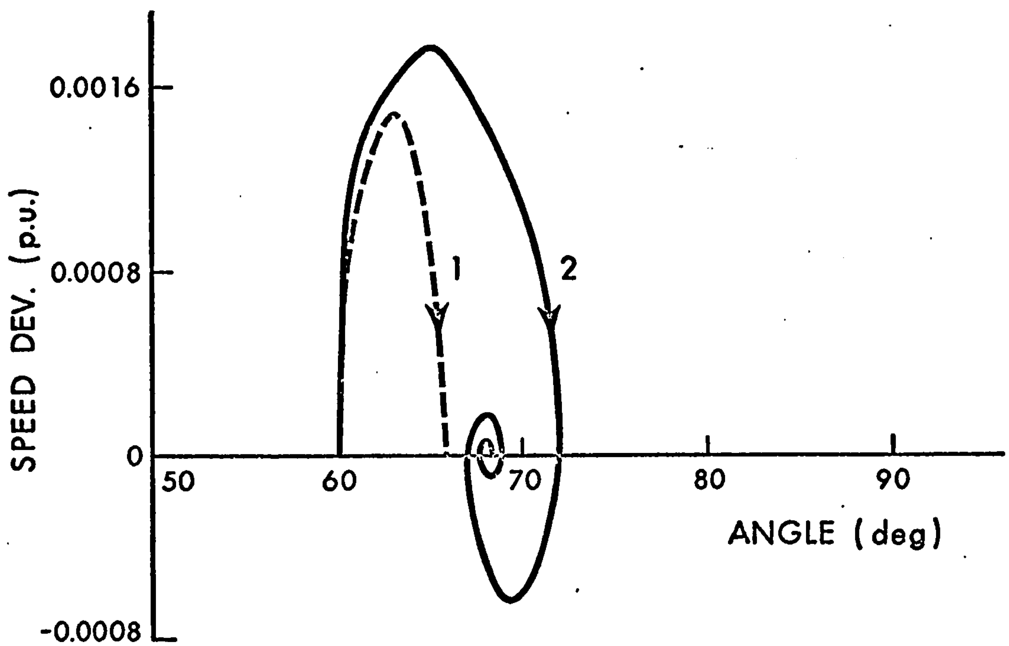

Fig. 3.8 Angle vs velocity plot corresponding to Fig. 3.6.

1. Optimal control.
2. Proportional control.

One factor of interest in stability studies is the gain of the voltage regulator. The regulator gain was varied from -4.5 to -1000 with the results shown in Table 3.1. It is found that a gain of -1000 drives the exciter almost in bang bang fashion even for a small disturbance. With the proportional control, a higher gain regulator takes less time to stabilize. A gain of -100 appears to be optimal in that the exciter is not represented by non-linear equations. The term $L(t)/b(t)$ exceeds, in magnitude, the control $u(t)$ for a very small period of time. According to the definition of controllability introduced in section 3.7, the system is uncontrollable for that period of time.

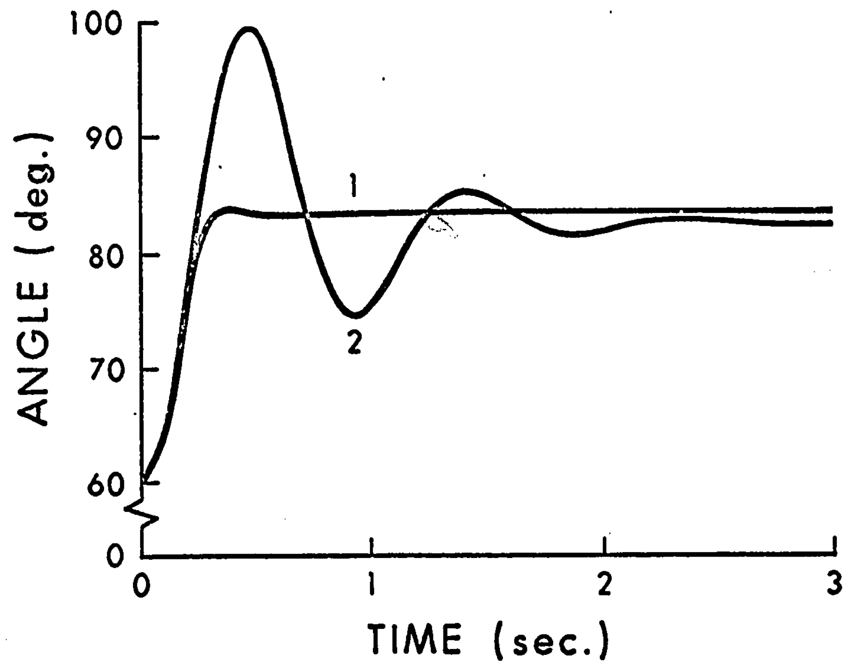

Fig. 3.9 Angle time characteristics for 30% torque step.

1. Bang bang control.
2. Proportional control.

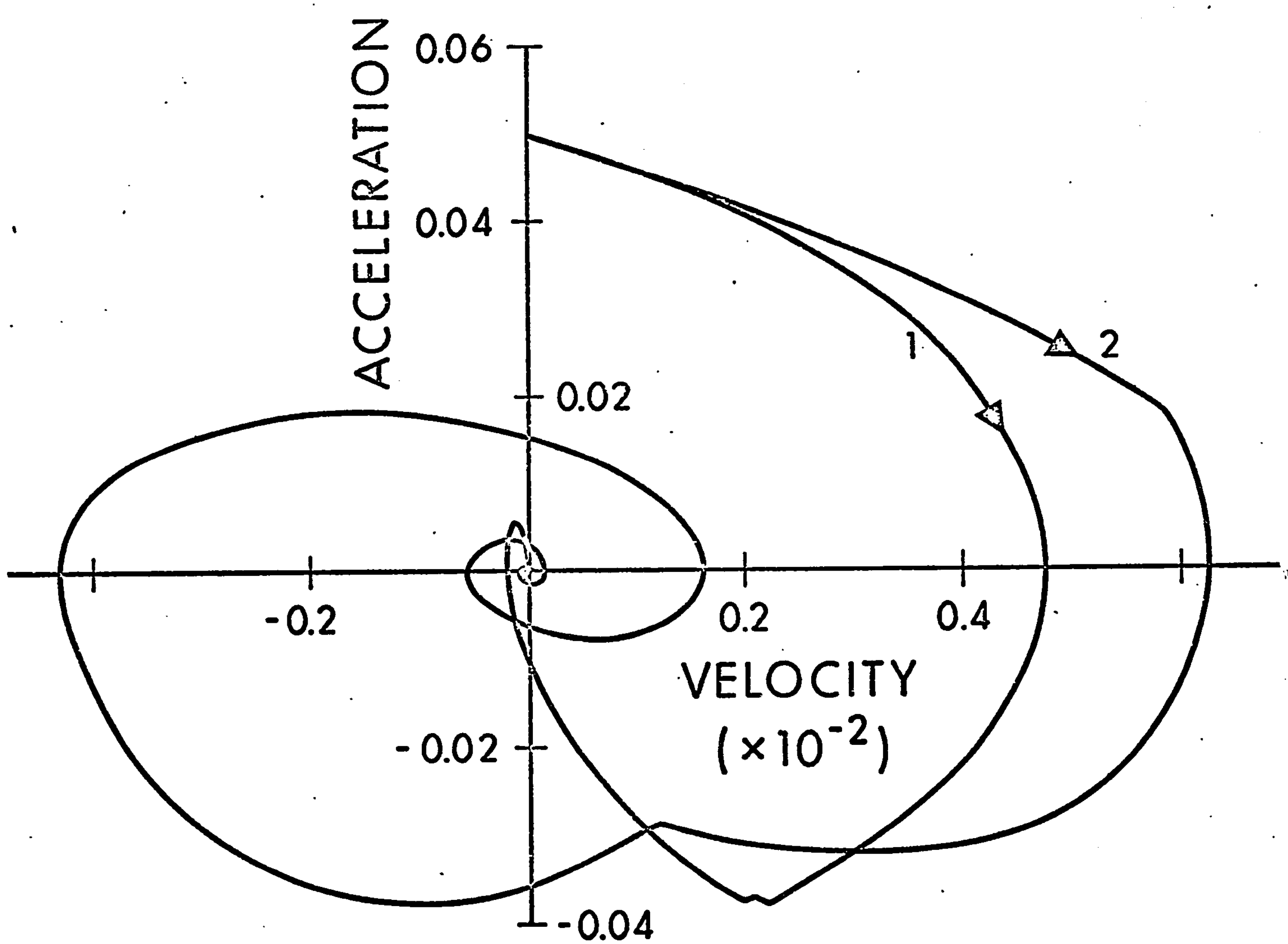

Fig. 3.10 Velocity vs acceleration plot corresponding to Fig. 3.9

1. Bang bang control.

2. Proportional control.

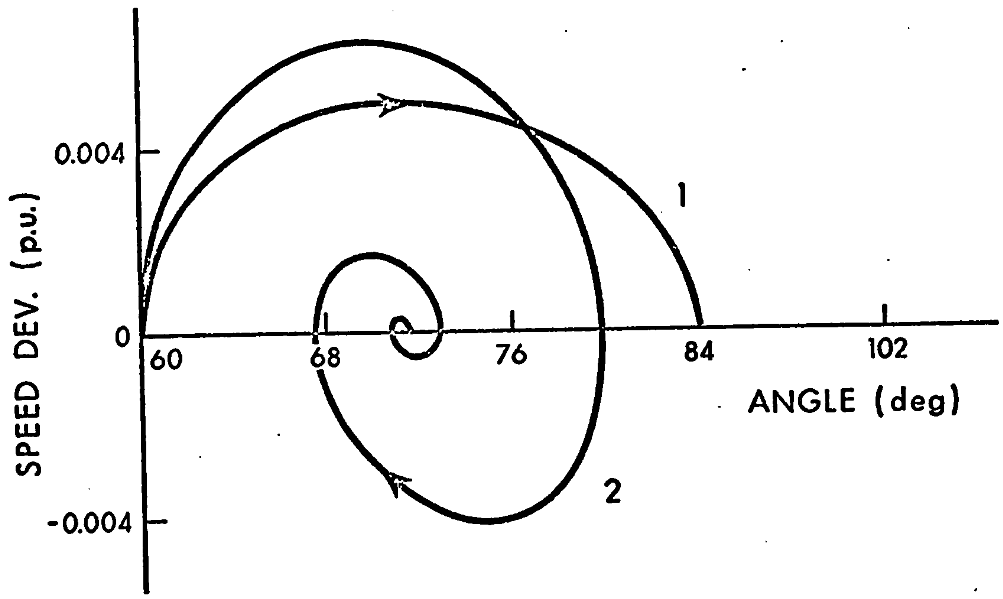

Fig. 3.11 Angle vs velocity plot corresponding to Fig. 3.9.
1. Bang bang control.
2. Proportional control.

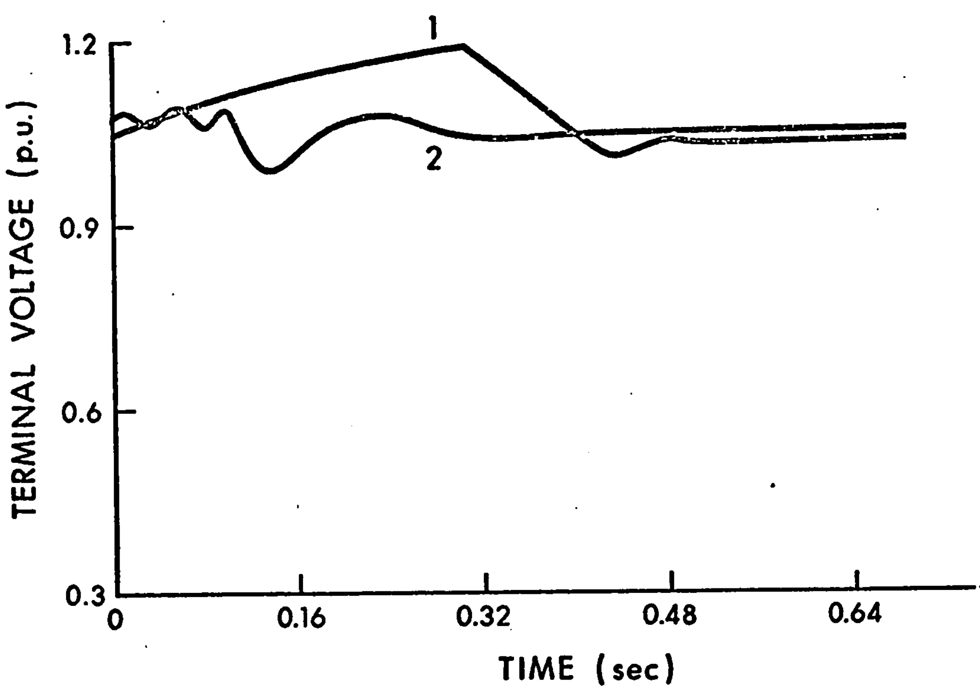

Fig. 3.12 Terminal voltage characteristics for 30% torque step.
1. Bang bang control.
2. Proportional control.

The bang bang control obtained by the closed loop scheme is sub-optimal. This results in an overshoot in the angle time characteristics (fig. 3.9). The time required for stabilization is fairly small. The proportional control leads to a heavily damped oscillatory response. The corresponding phase plane plots are shown in figures 3.10 and 3.11. Figure 3.12 shows the variation of terminal voltage with time for the bang bang as well as proportional control. Proportional control is better in this regard.

iii) Nonlinear model - 100% torque pulse for 3 cycles.

This is a severe disturbance. The term L(t)/b(t) remains very large with respect to u(t) for a considerable period of time. The system initially is uncontrollable. However, this uncontrollable section of L(t) is followed by a long controllable section, giving a sub-optimal solution. The angle time characteristics (fig. 3.13) show a large overshoot followed by an undershoot. Proportional control produces a highly damped response with a longer settling time. The two curves are nearer to each other than in the 30% torque step case. In figure 3.13 it is observed that the proportional control gives a lower first peak than the bang bang control. The bang bang control causes a large initial slope in the states and when the disturbance ceases after 3 cycles, leads to a large peak in angle due to inertia. On the other hand, the proportional control is slower and by the time it becomes effective, the disturbance is not in effect, requiring less effort and consequently the first peak is lower. The phase plane plots are shown in figures 3.14 and 3.15. If in this case, the torque pulse were 150% for 3 cycles, the system would be uncontrollable for the particular ceiling voltage.

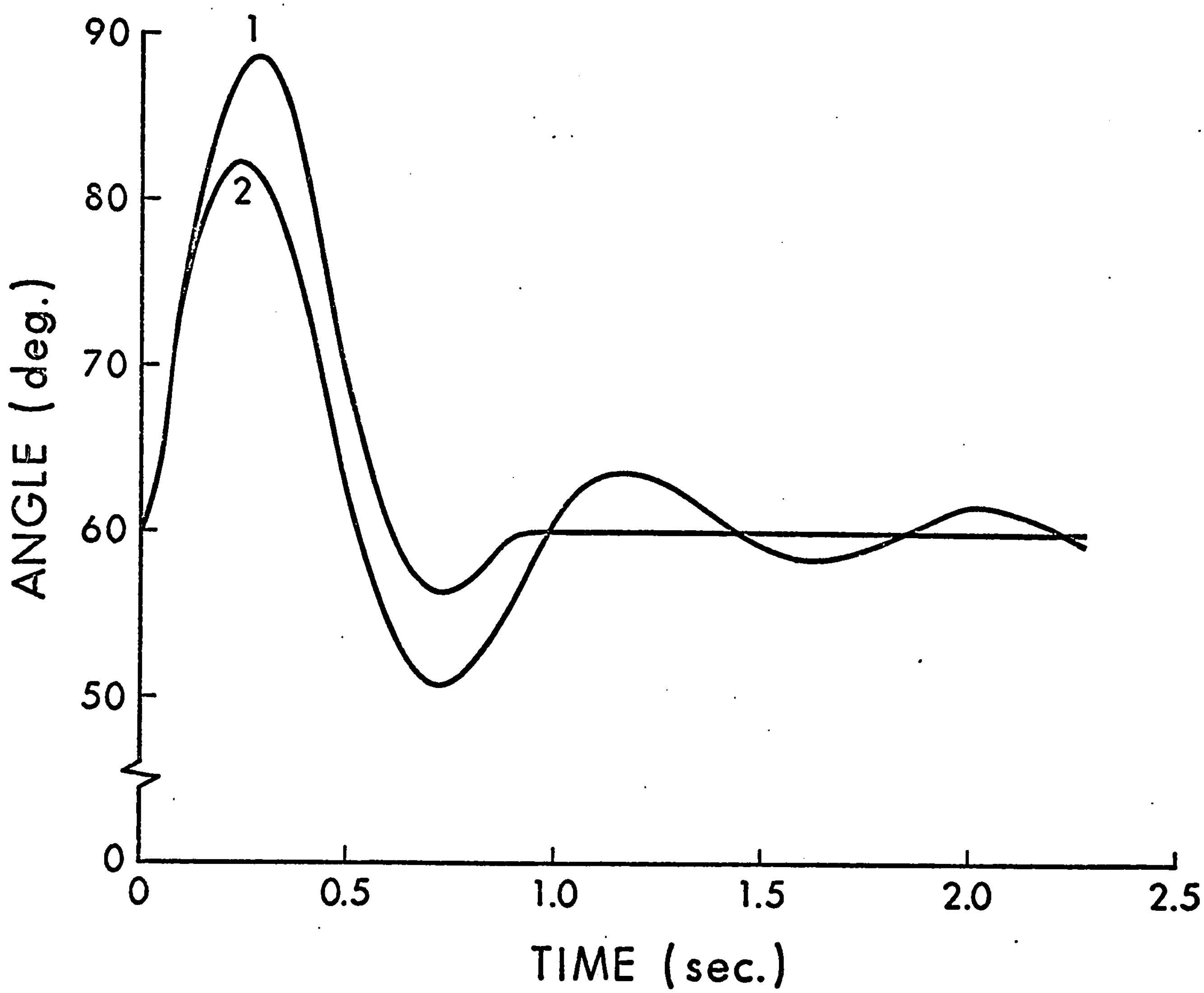

Fig. 3.13 Angle time characteristics for 100% torque pulse.

1. Bang bang control.

2. Proportional control.

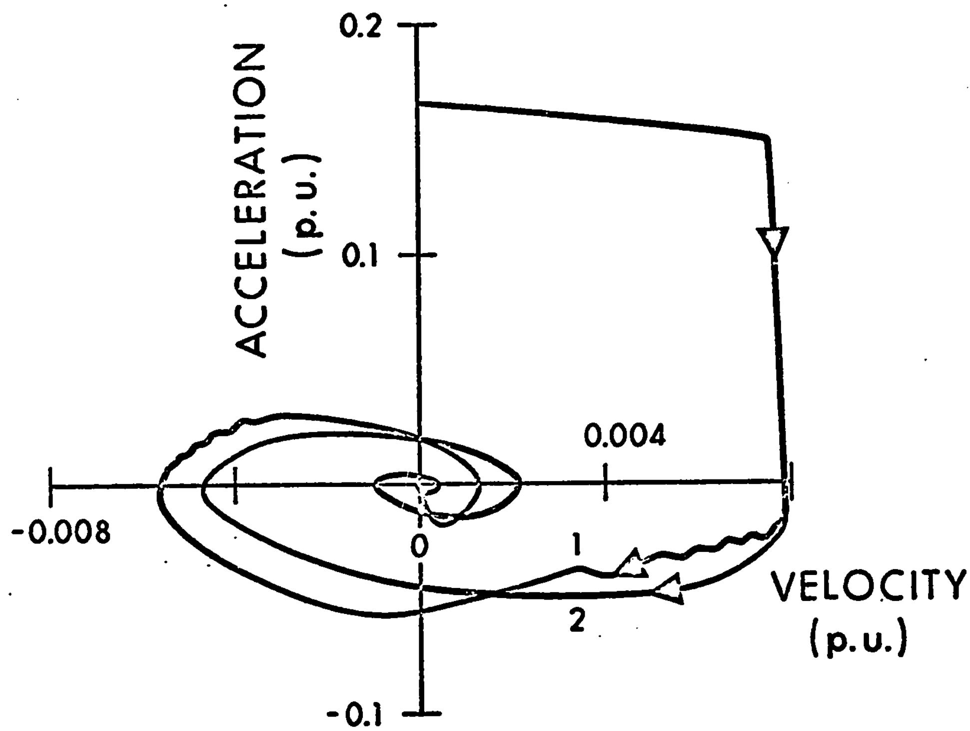

Fig. 3.14 Velocity vs acceleration plot corresponding to Fig. 3.13.

1. Bang bang control.
2. Proportional control.

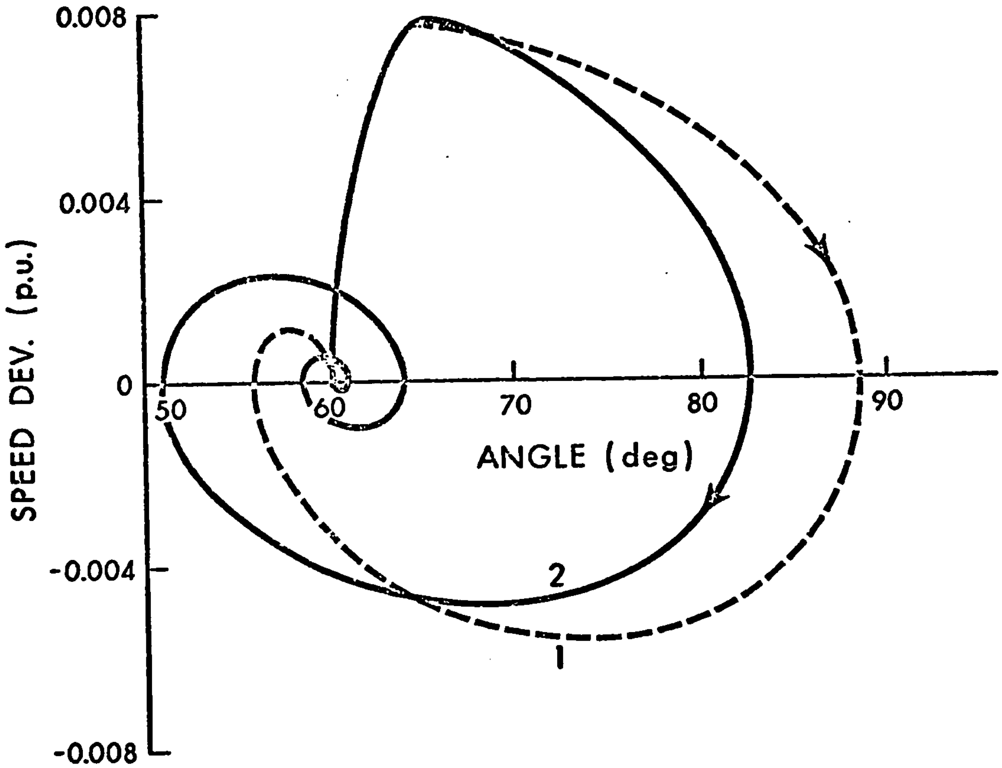

Fig. 3.15 Angle vs speed deviation plot corresponding to Fig. 3.13.

1. Bang bang control
2. Proportional control.

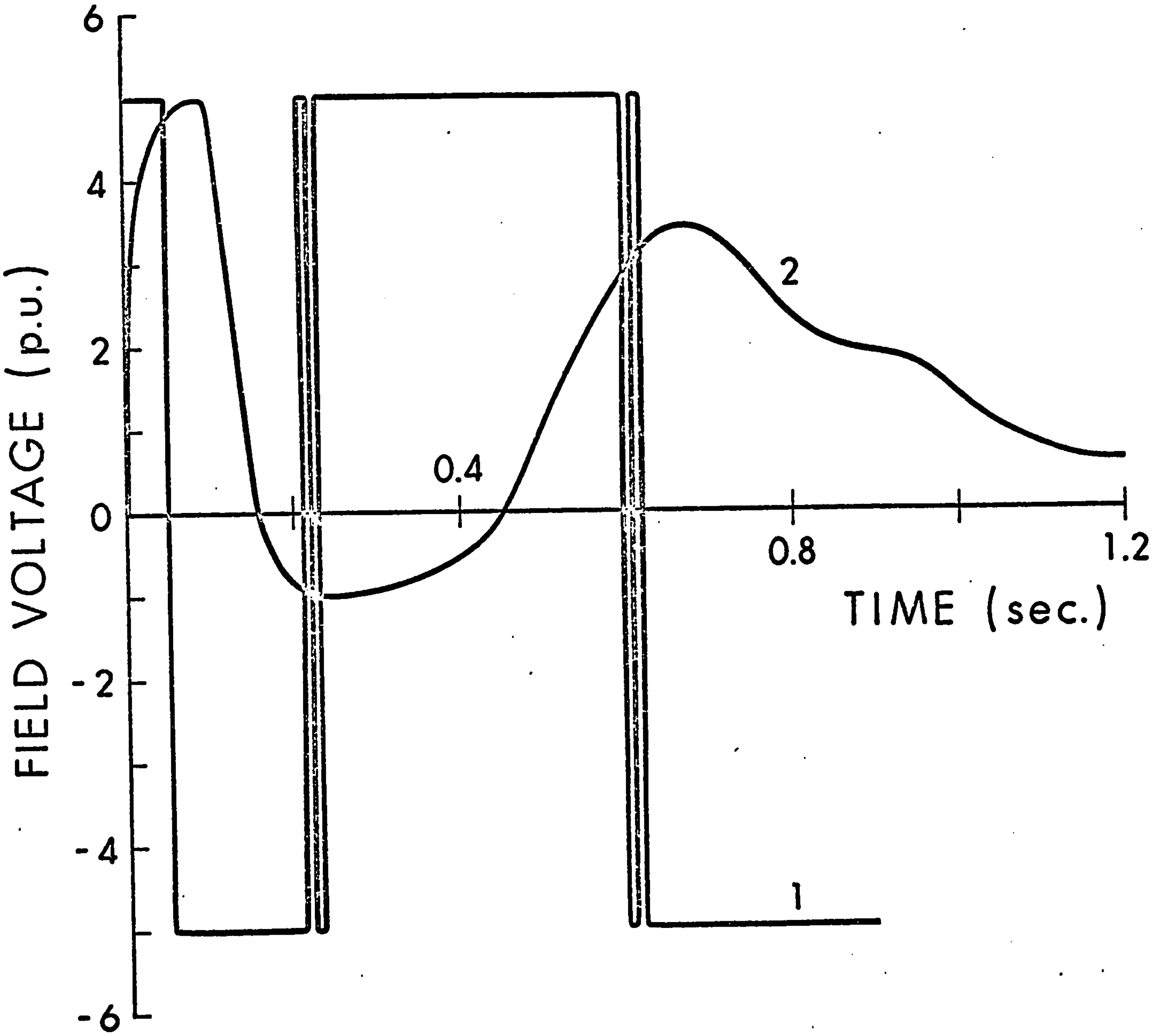

Fig. 3.17 Field voltage variation for 100% torque pulse.

1. Bang bang control.
2. Proportional control.

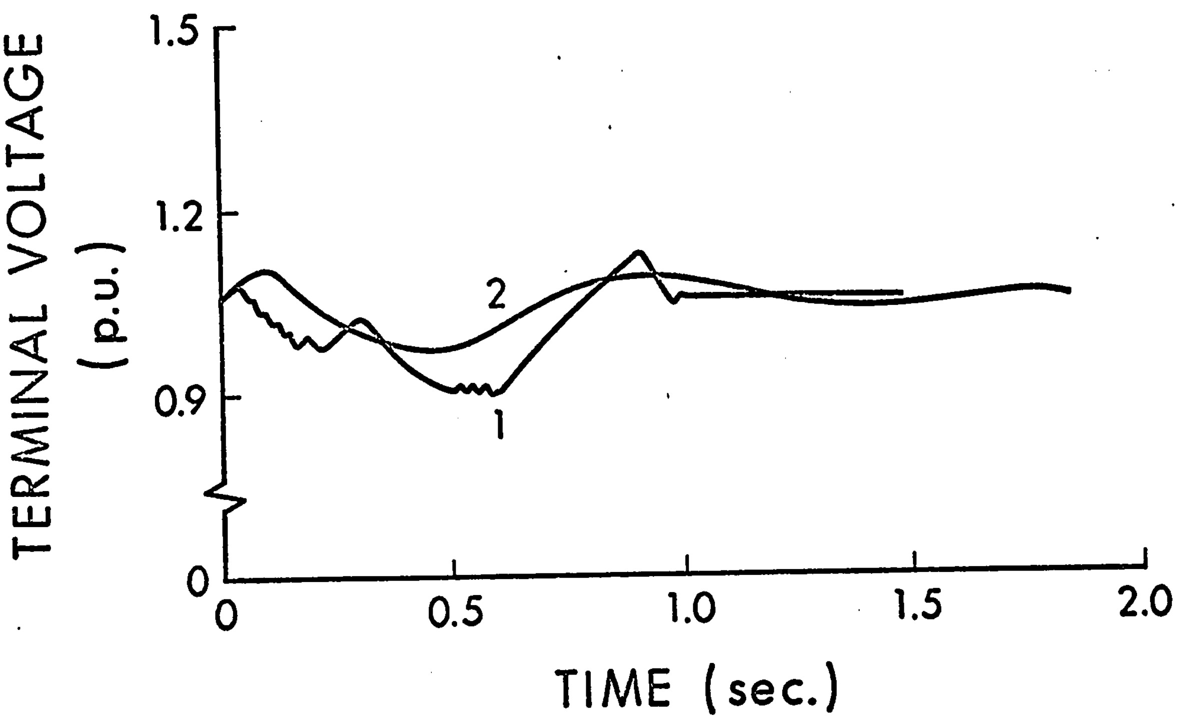

Fig. 3.16 Terminal voltage characteristics for 100% torque pulse.

1. Bang bang control.
2. Proportional control.

A factor of interest is the terminal voltage of the machine (fig. 3.16). As would be expected, the bang bang control deteriorates the voltage regulation, since the voltage regulator action is completely overridden by the bang bang control. This is offset to some extent by faster elimination of the transients. Figure 3.17 shows multiple switching in the field voltage. This is partly due to sudden variations in L(t) and b(t) and partly because of quasilinearization of these qualities. This is dealt with in more detail in the next chapter.

Apart from 30% torque step and 100% torque pulse, several other cases have been considered. These are summarized in Table 3.2. With the voltage regulator alone, all cases are unstable.

Case	Mode of disturbance	Control Signal	Stability	Time to stabilize (sec)	Final rotor angle
1	20% torque step	Bang bang	Stable	.355 sec	70°
2	20% torque step	Proportional	Stable	-	74°
3	30% torque step	Bang bang	Stable	.48	84°
4	30% torque step	Proportional	Stable	-	82°
5	50% pulse for 3 cycles	Bang bang	Stable	.668	59.5°
6	50% pulse for 3 cycles	Proportional	Stable	-	60°
7	100% pulse for 3 cycles	Bang bang	Stable	.988	59.5°
8	100% pulse for 3 cycles	Prcportional	Stable	-	60°

Table 3.2 Nonlinear system, torque step and pulses (K_r = - 100, celing ± 5 p.u.)

iv) Nonlinear model - Three phase fault at intermediate bus; cleared in 3 cycles by opening a line.

The bang bang control gives one overshoot and a small undershoot (fig. 3.18). While the fault is on and a small period afterwards, the system is uncontrollable. If the final rotor angle is considered free, the system attempts to stabilize at the unstable equilibrium point and then slowly creeps further out of phase. The constraint on the final rotor angle prevents this. With the proportional control, the system has a heavily damped stable response and on the voltage regulator control alone, is unstable (first swing). The phase plane plot (Fig. 3.19) shows that the uncontrollable period continues until the maximum velocity is reached and the loops produced are apparently due to harmonic torques. Figures 3.20 through 3.25 show the plots of velocity vs angle phase plane, field and terminal voltage characteristics, field and different armature currents. Figure 3.26 shows the angle time characteristics for a ceiling voltage of ± 4.3 p.u. A ceiling voltage lower than this causes instability for the particular disturbance.

The three phase short circuit is also considered for a relatively lightly loaded condition. This and the summary of the case considered is given in Table 3.3.

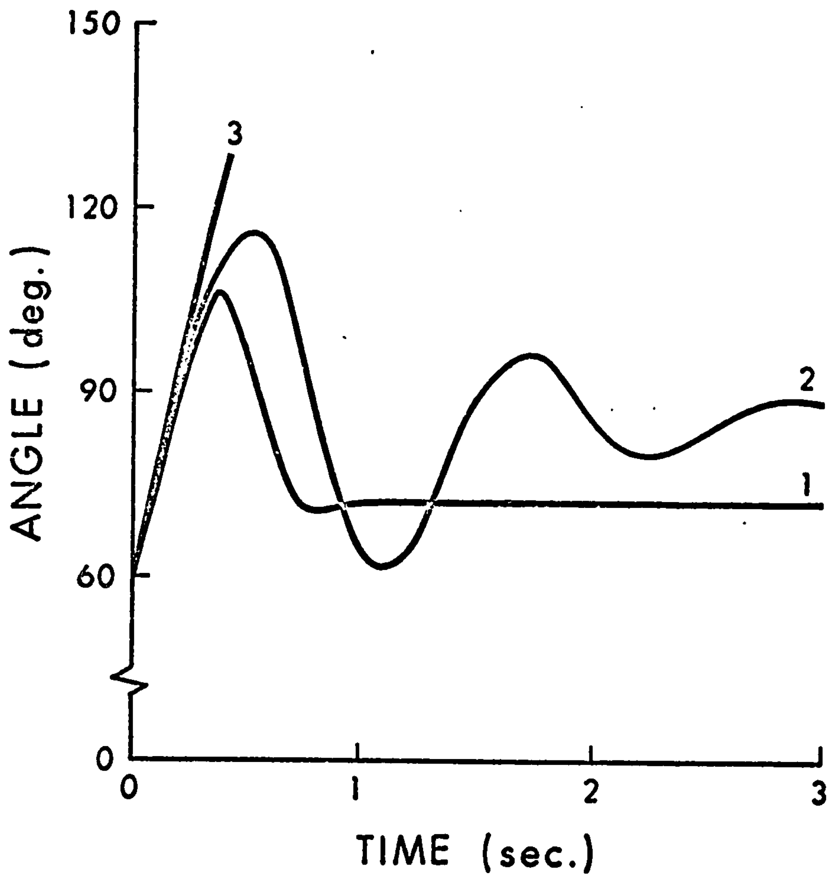

Fig. 3.18 Angle time characteristics for 3 phase fault.

1. Bang bang control.
2. Proportional control.
3. Voltage regulator only.

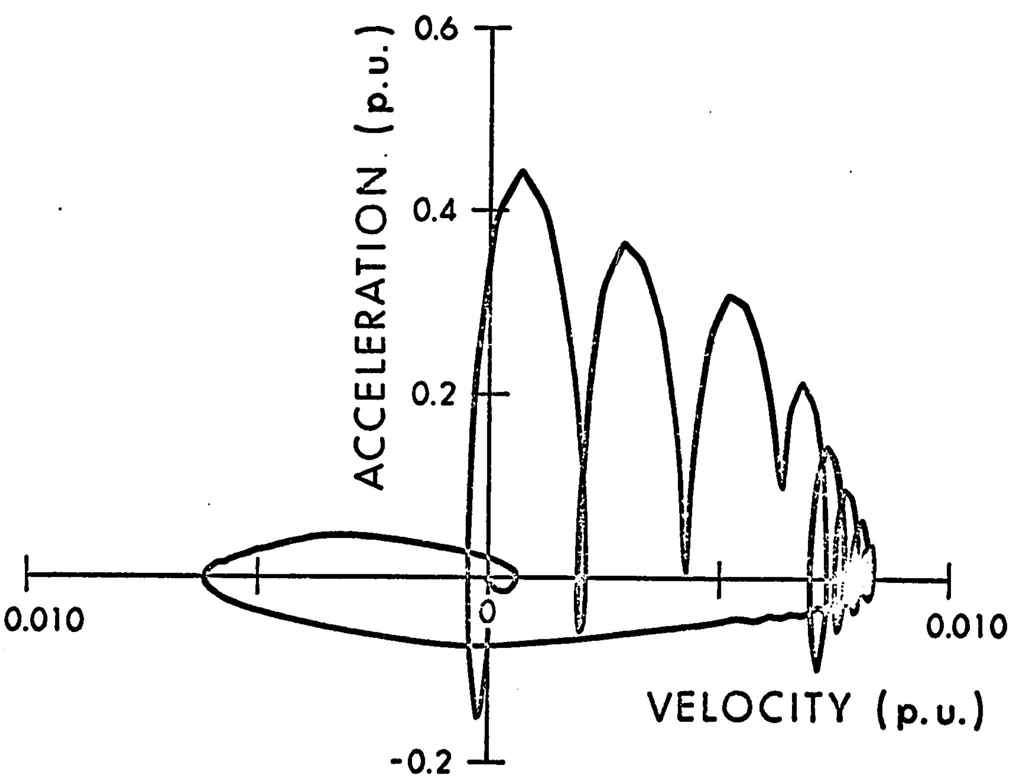

Fig. 3.19 Velocity vs acceleration plot corresponding to Fig. 3.18 for bang bang control only.

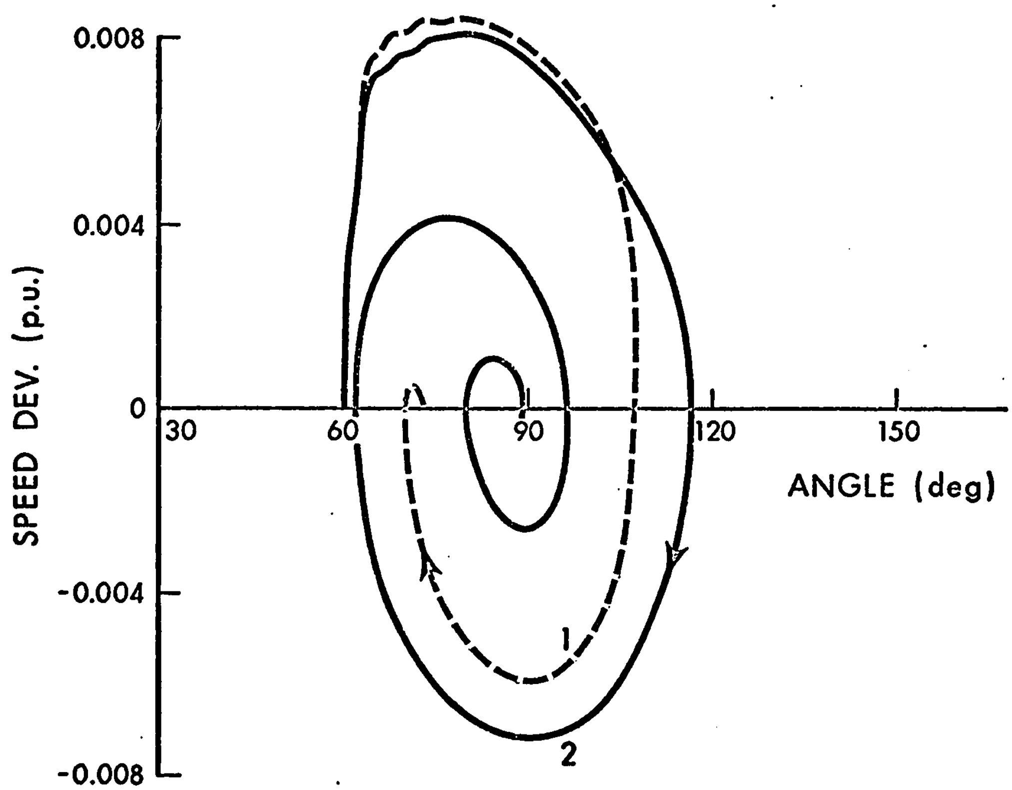

Fig. 3.20 Angle vs velocity plot corresponding to Fig. 3.18.

1. Bang bang control.

2. Proportional control.

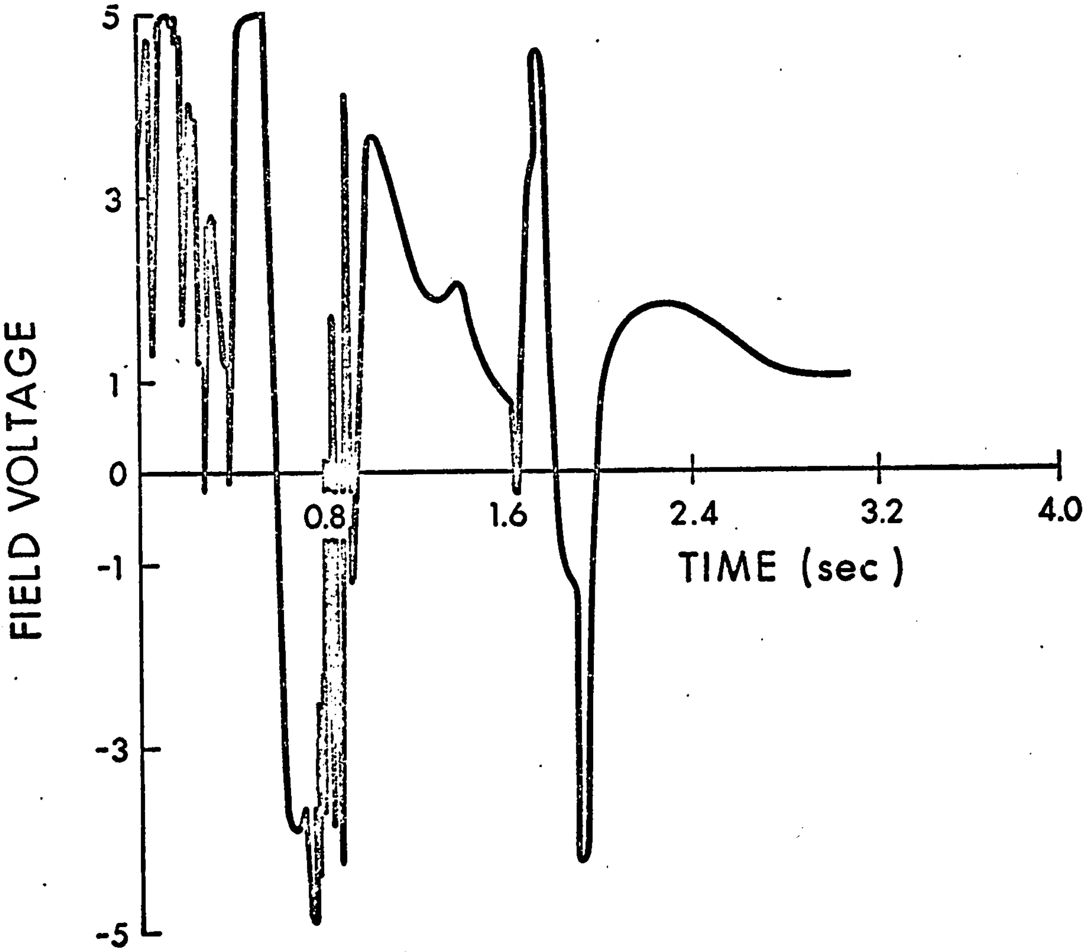

Fig. 3.21 Field voltage characteristics, proportional control.

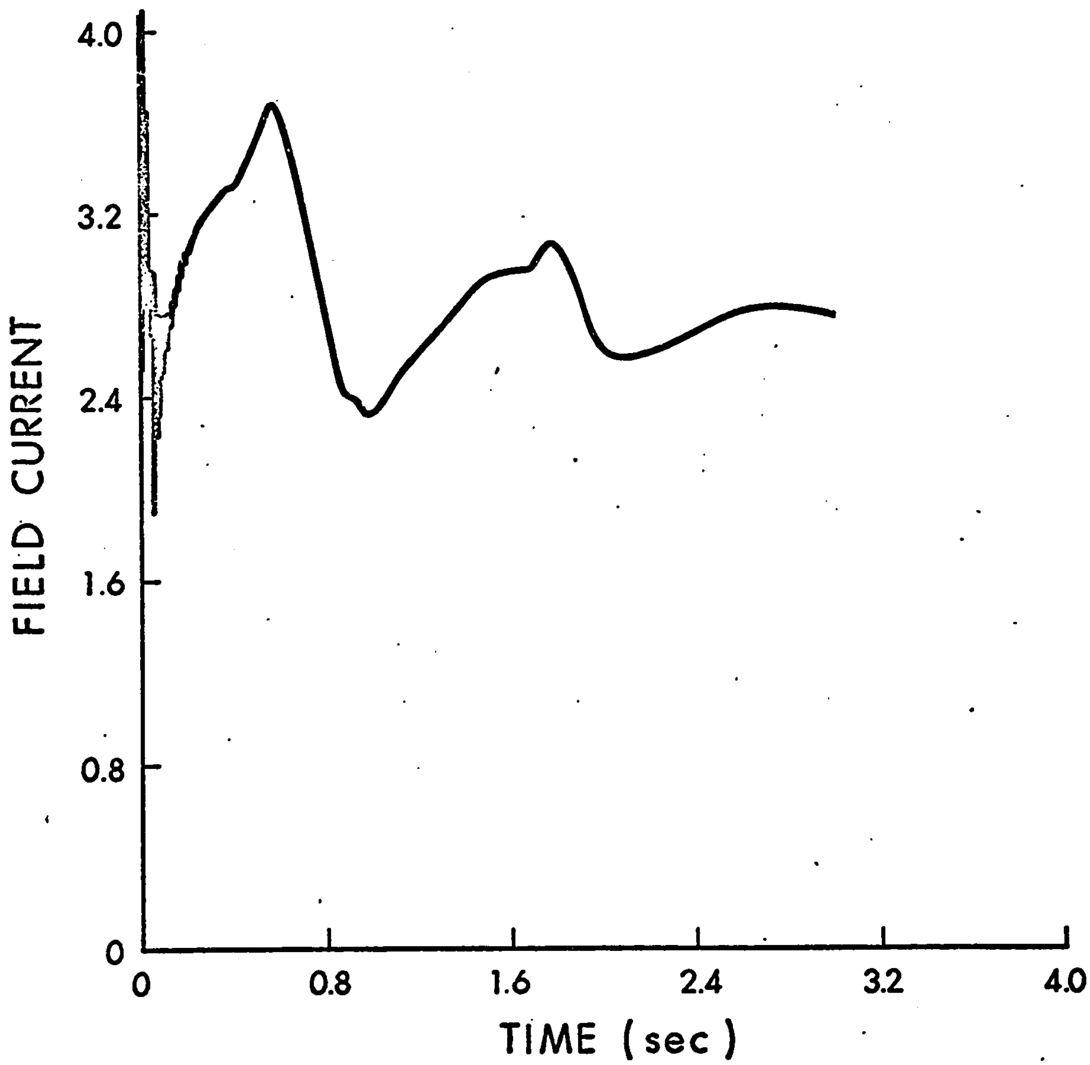

Fig. 3.22 Field current time characteristics, proportional control.

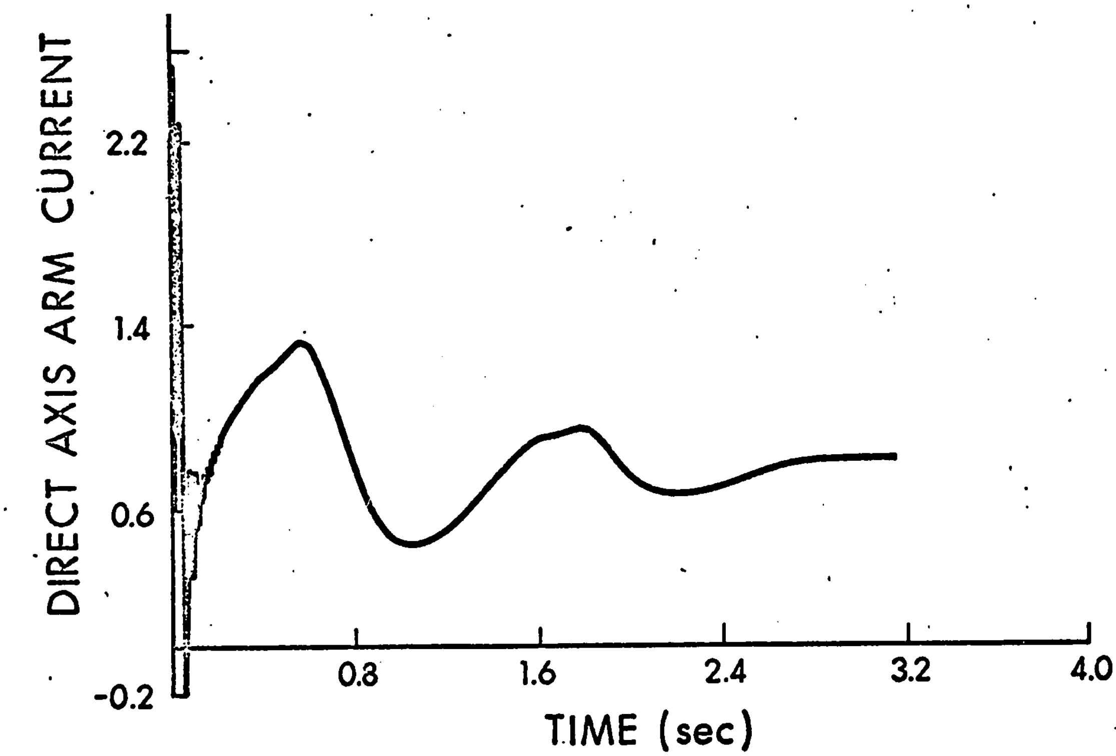

Fig. 3.23 Direct axis armature current - time characteristics, proportional control.

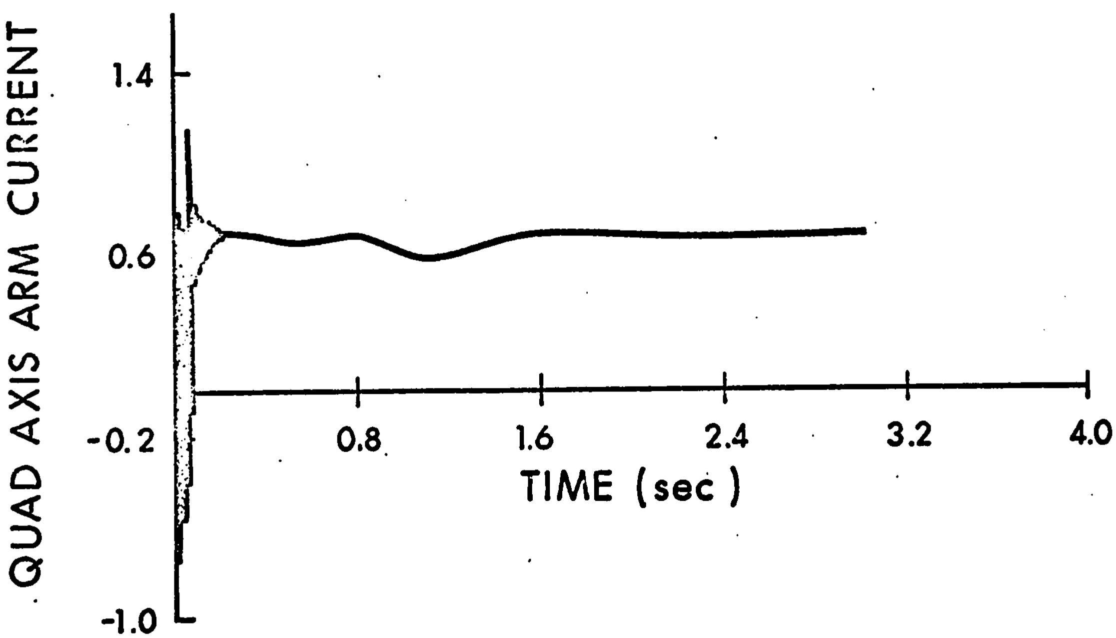

Fig. 3.24 Quadrature axis armature current - time characteristics proportional control.

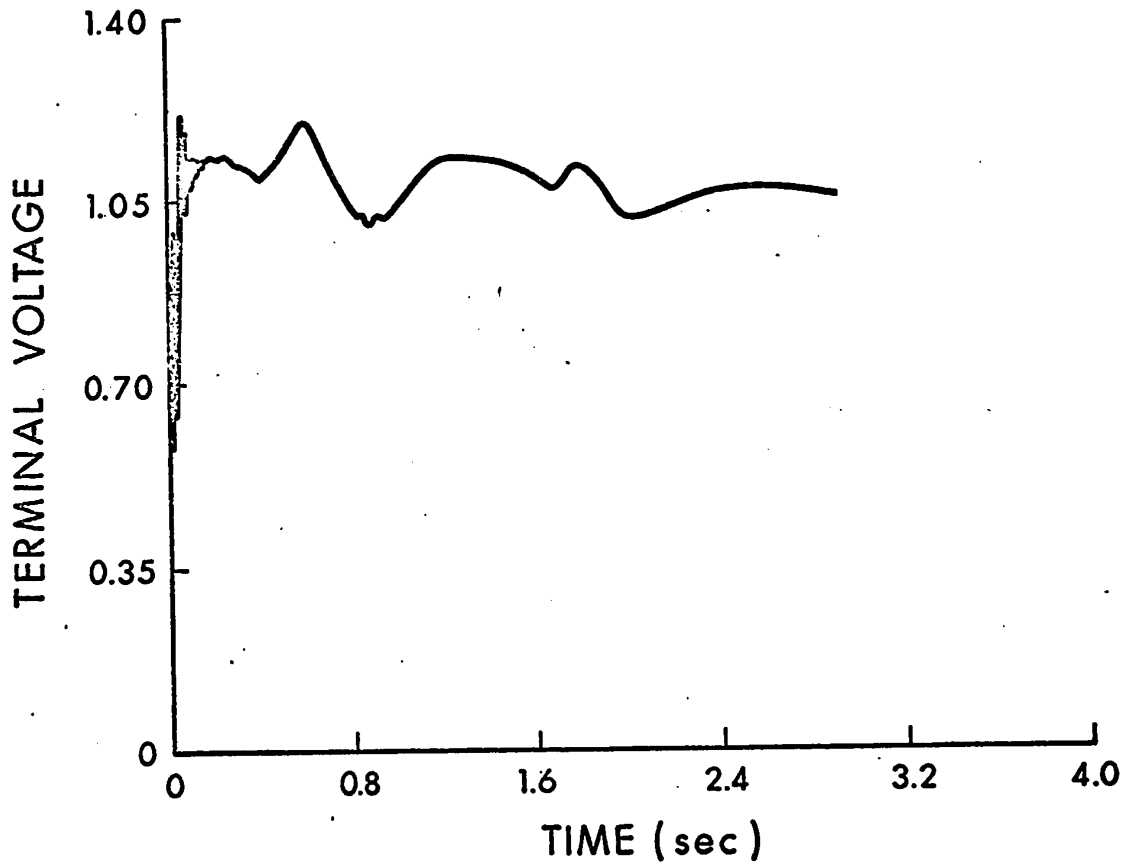

Fig. 3.25 Terminal voltage variation with time, proportional control.

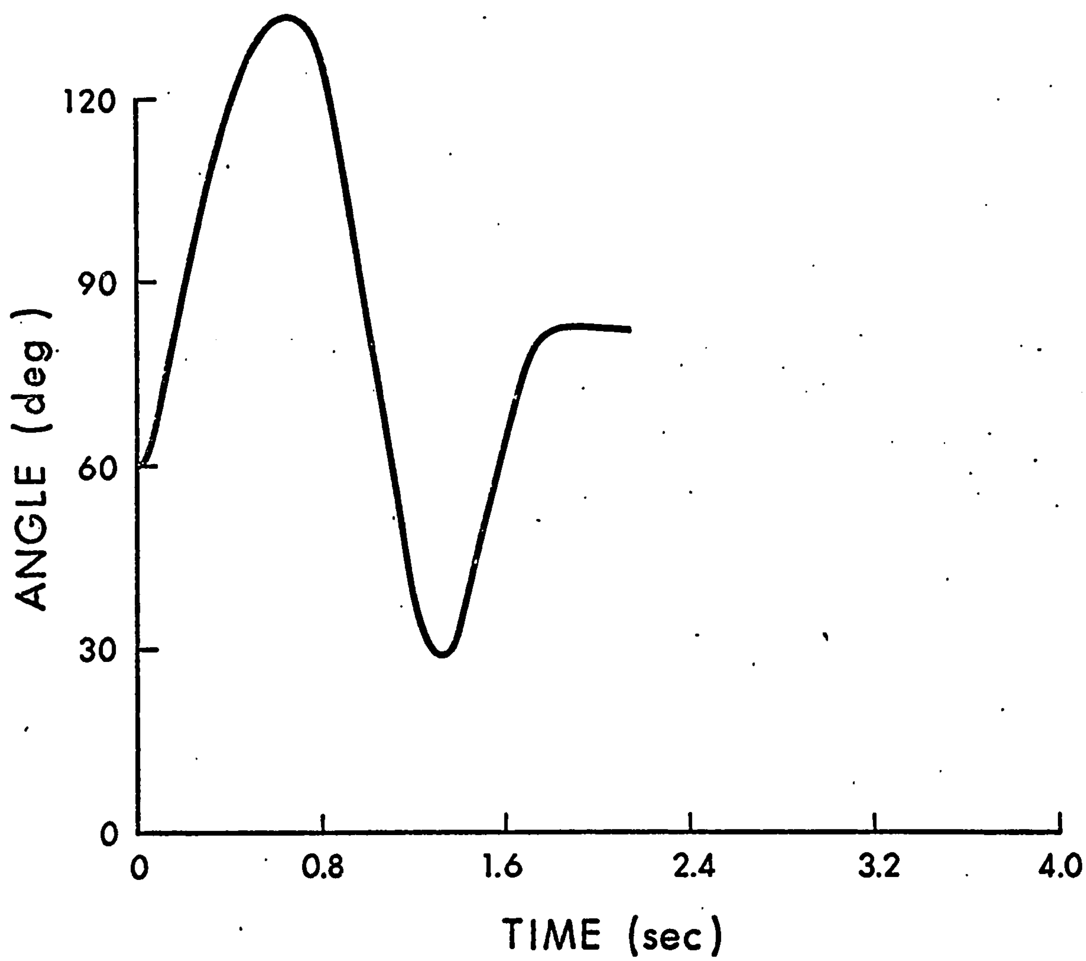

Fig. 3.26 Angle time characteristic for the minimum allowable ceiling voltage.

Operating point	Case	Control signal	Ceiling	Stability	Time to stabilize (sec)	Final rotor angle
δ_o = 60°	1	Bang bang	±7	Stable	1.011	72°
P_{out} = 92%	2	Bang bang	±4.3	Stable	2.04	82°
	3	Proportion -al	±5	Stable	-	85°
	4	Voltage regul- ator only	±5	Unstable	-	-
δ_o = 50°	5	Bang bang	±5	Stable	.694	83°
P_{out} = 73%	6	Proportion -al	±5	Stable	-	68°

Table 3.3 Three phase short circuit cleared in 3 cycles by opening a line

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4. R. Oldenburger, "Optimal Control", Holt, Rinehart and Winston, 1966.

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APPENDIX A3-1

Oldenburger[4] considered a system

$$\ddot{x} = u(t) - L(t) \tag{A3-1.1}$$

where L(t) is an arbitrary disturbance term and the control is constrained in magnitude by $|u| \leq 1$.

By a geometrical construction and also applying direct method of optimization, he found a switch curve

$$\gamma = \left\{ (x_1, x_2) \;:\; x_1 + \frac{x_2|x_2|}{2[1 + L\mathrm{Sgn}\{x_2\}]} = 0 \right\} \tag{A3-1.2}$$

Let equation (A3-1.2) be investigated now. For $L(t) = L_o$, at $t = t_o$

$$\begin{aligned} u_{omax} &= 1 - L_o \\ u_{omin} &= -(1 + L_o) \end{aligned} \tag{A3-1.3}$$

which means that for $x_2 > 0$

$$\sum^{\circ} = x_1 - \frac{x_2^{\,2}}{2u_{omin}} \tag{A3-1.4}$$

and for $x_2 < 0$, $\quad \sum^{\circ} = x_1 - \frac{x_2^{\,2}}{2u_{omax}}$ (A3-1.5)

Equation (A3-1.4) and (A3-1.5) are identical to equation (3.45). He considered the solution to system (A3-1.1) using optimal scheme found from equation (A3-1.2) to be suboptimal.

APPENDIX A3-2

This section considers an example where the term L(t), considered in Section 3.3, contains output feedback. The problem is solved by the technique suggested in Section 3.3 and the switch curves obtained are compared with those obtained by a standard optimization method.

The equation of the harmonic oscillator is

$$\ddot{x} + \omega^2 x = u(t) \tag{A3-2.1}$$

The control u(t) is constrained in magnitude such that $|u(t)| \leq 1$. The problem is to find the admissible control which transfers system (A3-2.1) from the given initial states to the origin in smallest possible time.

Considering $x = x_1$, $\dot{x} = x_2$ the first set of switch curves for the system (A3-2.1) obtained by applying Pontryagin's Minimum Principle (Reference [2]) in the range $|x_1| \leq 2$ is

$$(\omega x_1 - 1)^2 + (\omega x_2)^2 = 1 \; ; \quad \omega x_2 \leq 0 \tag{A3-2.2}$$

$$(\omega x_1 + 1)^2 + (\omega x_2)^2 = 1 \; ; \quad \omega x_2 \geq 0 \tag{A3-2.3}$$

which are two semicircles shown in figure A3-2.1 (curve 1).

Next, consider $-\omega^2 x = L(t)$, then equation (A3-2.1) can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= L(t) + u(t) \end{aligned} \tag{A3-2.4}$$

Constraining L(t) in magnitude, so that

$$|L(t)| \leq 1 \tag{A3-2.5}$$

or

$$|\omega^2 x| \leq 1 \tag{A3-2.6}$$

and applying the technique presented in Section 3.3, the switch curve obtained is

$$x_1 + \frac{x_2|x_2|}{2[1 + \omega^2 x_1 \mathrm{Sgn}\{x_2\}]} = 0 \tag{A3-2.7}$$

For $x_2 > 0$ and $\omega = 1$, the equation of the switch curve is

$$x_1 + \frac{x_2^2}{2[1 + x_1]} = 0 \tag{A3-2.8}$$

or

$$2x_1^2 + 2x_1 + x_2^2 = 0 \tag{A3-2.9}$$

which is curve 2 in figure A3-2.1 in the range $|x_1| < 1$. By the constraint (A3-2.6) no information is available for $|x_1| > 1$.

Here the quantity L(t), i.e. -x, does not satisfy the restrictions imposed on it in Section 3.8. (a) Here, L(t) is a pure output feedback term, but as indicated, L(t) should not be dominated by output feedback terms. (b) The final value of L(t) is zero in this case, which puts an extra restriction on it in addition to (A3-2.5).

Still it can be seen from figure A3-2.1 that for states sufficiently near the origin, the scheme (A3-2.7) gives a near optimal solution implying that if L(t) is small compared to u(t), the solution will be close to optimal. It can be seen that for $\omega^2 < .5$, the switch curves (1) and (2) in figure A3-2.1 are very close to each other for $|x_1| \leq 1$. Switch curve 3

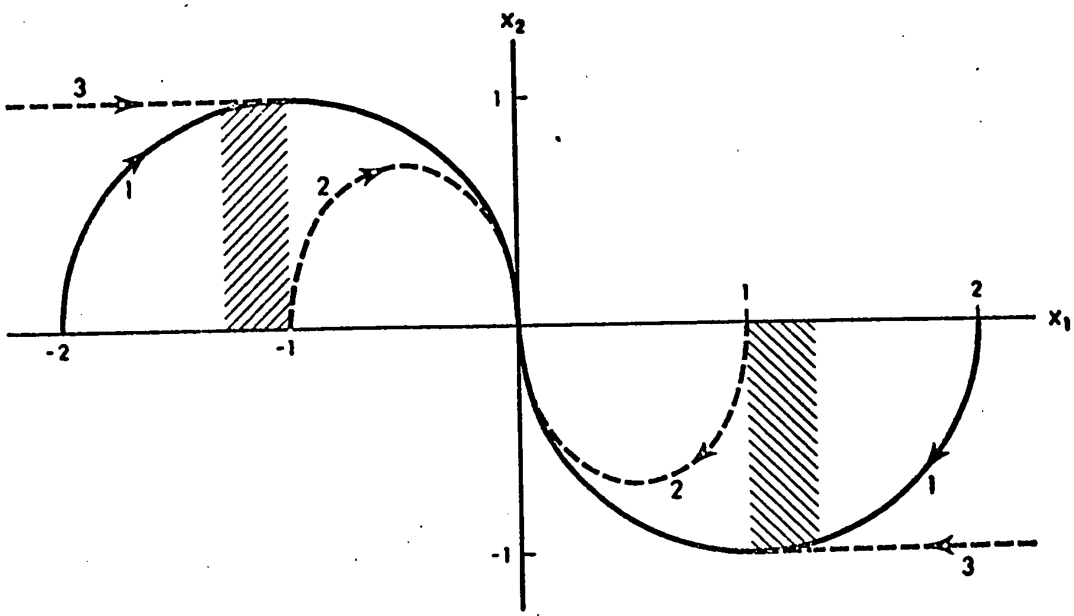

Fig. A3-2.1 The harmonic oscillator and sub-optimal control.

gives another sub-optimal solution[2] for system A3-2.1.

From the above analysis, it can be seen that as long as the term L(t) is constrained in magnitude, the control scheme gives a reasonably good sub-optimal solution even if it contains output feedback. Of course, sub-optimality of the scheme does not effect the solution of the original power system equations as explained in Section 3.4.

APPENDIX A3-3

Hamming's modified predictor corrector method was used to solve the differential equations for the single machine case. The starting values were found by Runge-Kutta Gill numerical integration routine. The data for the machine was taken from Roy[7]. The system configuration is given in figure 2.2.

Machine rating 46 MVA, 13.8 kv

x_{afd}	.562	x_{tie}	= .7/section
x_d	.6803	r_{tie}	= .07/section
x_q	.4363	x_{transf}	= .1
r_{fd}	.000508	x_e	= .8
x_{ffd}	.692	R_e	= .07
H	3	τ_r	= .0125 sec
R	.00435	K_r	= -4.5 to -1000

The operating point for the linear model (underexcited) are

$i_{fdo} = 1.77935$	$\delta_o = 60°$
$i_{do} = .30167$	$v = 1$
$i_{qo} = .718637$	$P_o = .662$

e_{to} = .85077 $\qquad$ Q_o = .0147

E_{fd} = 1

The operating points for the nonlinear model (normally excited) are

i_{fdo} = 2.473309 $\qquad$ δ_o = 60°

i_{do} = .564341 $\qquad$ v = 1

i_{qo} = .734433 $\qquad$ P_o = .916

e_{to} = 1.05208 $\qquad$ Q_o = .3324

E_{fd} = 1.39

CHAPTER 4

EVALUATION OF A QUASI-OPTIMAL STATE FEEDBACK EXCITATION SCHEME

4.1 Introduction

As established in the previous chapter, the quasi-optimal control is optimal for the linearized system model. Use of this model is clearly justified when L(t) is small compared to the control u(t). For large disturbances L(t) is quite appreciable, however, and the quasi-optimal scheme may give a solution quite different from the optimal ones. In order to evaluate the effectiveness of the quasi-optimal scheme, the results obtained by this method are compared with those obtained by a standard optimization technique.

The optimal control problem stated in Section (3.2) of Chapter 3 is solved in Section 2 using the steepest descent method. The results obtained with the quasi-optimal scheme (closed loop scheme) are compared with those obtained by the steepest descent method in Section 3. Section 4 investigates the effect of time delays on the optimal and quasi-optimal schemes.

4.2 Numerical example using steepest descent

The steepest descent algorithm constructed by Bryson and Denham[1,2] and later modified by Vachino[3] to take into consideration control problems with inequality constraints has been used here*. For starting the algorithm a first guess for the optimal control (which is parametized by the switching

* Details of the steepest descent technique is given in Section A4.1 of Appendix A4.

times for this particular problem) and final time is required. The control provided by the quasi-optimal scheme is very effective in this regard; in general, it produces much more rapid convergence to the optimal solution than controls only slightly different from it.

i) Linearized model

The linearized system equations rewritten from chapter 2 (dropping Δ for convenience)are

$$\dot{\underline{X}} = \underline{C}\ \underline{X} + \underline{d}\ u(t) \tag{4.1}$$

The operating values of u(t) considered for linearization are 1 per unit. For small disturbances the exciter may not be driven too hard and it is assumed that it will be driven to ± 3 per unit only, i.e.,

$$-4 \leq u(t) \leq 2 \tag{4.2}$$

The terminal constraints are that velocity and acceleration of the machine at $t = t_f$ are zero, that is,

$$\psi_1^{+} = - x_4 = 0 \tag{4.3}$$

$$\psi_2 = - \dot{x}_4 = - c_{41}\ x_1 - c_{42}\ x_2 - c_{43}\ x_3 = 0 \tag{4.4}$$

Usually, one of the terminal conditions is used as a stopping condition of the form $\Omega[x(t_f), t_f] = 0$. This is not done here since both velocity and acceleration of the machine are alternating. Satisfying

+ Different symbols used in this section are defined in appendix A4.

only one of the terminal conditions does not necessarily mean that the process should be discontinued there. For such problems, the terminal time t_f is generally chosen as a stopping condition[4]. The time required for stabilization by the quasi-optimal scheme is used as the stopping time. Assuming that the quasi-optimal scheme only gives a sub-optimal solution, the optimum time will be less than that obtained by it. The stopping condition for a 10% torque step is chosen as

$$\Omega = .35 - t_f \tag{4.5}$$

The performance index is

$$\phi = - t_f \tag{4.6}$$

Evaluating the values of the parameters c_{ij} at this operating point, the terminal conditions are

$$\psi_1 = x_4(t_f) = 0 \tag{4.7}$$

$$\psi_2 = .06708\, x_1(t_f) - .02913\, x_2(t_f) + .15437\, x_3(t_f) = 0 \tag{4.8}$$

Assuming two switchings of the control, the nominal control is

$$u_o(t) = 2 \qquad t_o \leq t \leq t_1$$

$$u_o(t) = -4 \qquad t_1 \leq t \leq t_2 \tag{4.9}$$

$$u_o(t) = 2 \qquad t_2 \leq t \leq t_f$$

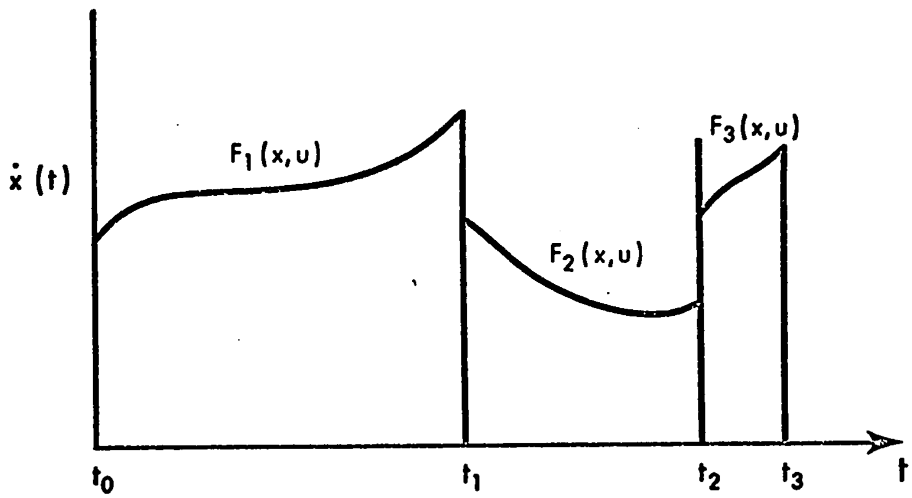

Fig. 4.1 System equations

The rewritten system equation is

$$\dot{\underline{X}} = f_1[1-h(t-t_1)] + f_2[h(t-t_1) - h(t-t_2)] + f_3[h(t-t_2) - h(t-t_f)] \tag{4.10}$$

which yields the equations of variation

$$\delta\dot{\underline{X}} = \delta f_1[1 - h(t-t_1)] + \delta f_2[h(t-t_1) - h(t-t_2)]$$

$$+ \delta f_3[h(t-t_2) - h(t-t_f)] + (f_1-f_2)\Delta(t-t_1)\delta t_1$$

$$+ (f_2-f_3)\Delta(t-t_2)\delta t_2 + f_3\Delta(t-t_f)\delta t_f \tag{4.11}$$

This vector equation can be written in the component form

$$\delta\dot{\underline{X}} = F_X\,\delta X + L\delta\omega \tag{4.12}$$

and $$\dot{\lambda} = -F_X^T\,\lambda \tag{4.13}$$

where $$F_X = \underline{C} \tag{4.14}$$

$$L = \begin{bmatrix} 6d_1\Delta(t-t_1) & -6d_1\Delta(t-t_2) & 2d_1\Delta(t-t_f) \\ 6d_2\Delta(t-t_f) & -6d_2\Delta(t-t_2) & 2d_2\Delta(t-t_f) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4.15}$$

$$\delta\omega = [\delta t_1,\ \delta t_2,\ \delta t_f]^T \tag{4.16}$$

From equation (4.5), (4.6) and (4.7)

$$\lambda\phi(t_f) = \frac{\partial\phi}{\partial X}\bigg|_{t_f} = [0] \quad (4.17)$$

$$\lambda\Omega(t_f) = \frac{\partial\Omega}{\partial X}\bigg|_{t_f} = [0] \quad (4.18)$$

$$\lambda\psi_1(t_f) = \frac{\partial\psi_1}{\partial X}\bigg|_{t_f} = [0 \quad 0 \quad 0 \quad -1 \quad 0]^T \quad (4.19)$$

$$\lambda\psi_2(t_f) = \frac{\partial\psi_2}{\partial X}\bigg|_{t_f} = [.06708 \quad -.02913 \quad .15437 \quad 0 \quad 0]^T \quad (4.20)$$

$$\lambda^T_{\phi\Omega}(t) = \lambda^T_{\phi}(t) - \frac{\dot{\phi}(t_f)}{\dot{\Omega}(t_f)} \lambda^T_{\Omega}(t) \quad (4.21)$$

or $$\lambda^T_{\phi\Omega}(t) = [0] \quad (4.22)$$

$$\lambda^T_{\psi\Omega}(t) = \lambda^T_{\psi}(t) - \frac{\dot{\psi}(t_f)}{\dot{\Omega}(t_f)} \lambda^T_{\Omega}(t) \quad (4.23)$$

or $$\lambda^T_{\psi\Omega}(t) = \lambda^T_{\psi}(t) + \dot{\psi}(t_f)\, \lambda^T_{\Omega}(t) \qquad (\text{since } \dot{\Omega}(t_f) = -1) \quad (4.24)$$

or $$\lambda_{\psi\Omega}(t) = \lambda^T_{\psi}(t) \quad (4.25)$$

Substituting equation (4.17) through (4.25) into equation (A4-1.15), (A4-1.16) and (A4-1.17) gives

$$I_{\phi\phi} = [0] \quad (4.26)$$

$$I_{\psi\phi} = [0] \quad (4.27)$$

$$I\psi\psi = \sum_{s=1}^{3} \int_{t_{s-1}}^{t_s} \lambda\psi\Omega^T(t)\ L_s\ W^{-1}\ L_s^T\ \lambda\psi\Omega d\tau \qquad (4.28)$$

Equations (4.26), (4.27) and (4.28) substituted in equation (A4-1.11) gives the final variational equation for the control as

$$\delta\omega(t) = \sum_{s=1}^{3} W^{-1}\ L_s^T(t)\ \lambda\psi\Omega(t)\ I\psi\psi^{-1}\ d\psi \qquad (4.29)$$

where $d\psi = [d\psi_1 \mid d\psi_2]$ (4.30)

Equation (4.29) is considerably simpler than the original expression given in equation (A4-1.11). Expressions similar to equation (4.29) were obtained by other workers also[4].

ii) The nonlinear model

The procedure for the nonlinear model is similar to that for the linear one. Here F_x^T is given as

$$F_x^T = \begin{bmatrix} a_{11} & a_{21} & a_{31}(1+x_4) & a_{41}x_3 & 0 \\ a_{12} & a_{22} & a_{32}(1+x_4) & a_{42}x_3 & 0 \\ a_{12}(1+x_4) & a_{23}(1+x_4) & a_{33} & (a_{41}x_1+a_{42}x_2) & 0 \\ a_{13}x_3 & a_{23}x_3 & (a_{31}x_1+a_{32}x_2 & 0 & \omega_o \\ a_{15}\cos x_5 & a_{25}\cos x_5 & -a_{35}\sin x_5 & 0 & 0 \end{bmatrix}$$

(4.31)

where the parameters a_{ij} are as given in section A2-1 of appendix A2.

$$L = \begin{bmatrix} 10d_1\Delta(t-t_1) & -10d_1\Delta(t-t_2) & 10d_1\Delta(t-t_3) & -5d_1\Delta(t-t_f) \\ 10d_2\Delta(t-t_1) & -10d_2\Delta(t-t_2) & 10d_2\Delta(t-t_3) & -5d_2\Delta(t-t_f) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.32}$$

where the nominal control sequence is taken as

$$\begin{aligned} u_o(t) &= 5 \quad , \quad t_0 \le t \le t_1 \\ u_o(t) &= -5 \quad , \quad t_1 \le t \le t_2 \\ u_o(t) &= 5 \quad , \quad t_2 \le t \le t_3 \\ u_o(t) &= -5 \quad , \quad t_3 \le t \le t_f \end{aligned} \tag{4.33}$$

The terminal constraints are

$$\psi_1 = -x_4 = 0 \tag{4.34}$$

$$\psi_2 = .0936667\, x_1\, x_3 - .0406667\, x_2\, x_3 = 0 \tag{4.35}$$

so,

$$\lambda\psi_1 = [0 \quad 0 \quad 0 \quad -1 \quad 0]^T \tag{4.36}$$

$$\lambda\psi_2 = [.0936667\ x_3(t_f),\ \ -.0406667\ x_3(t_f),\ \ (.0936667\ x_1(t_f) - .0406667\ x_2(t_f)),\ \ 0,\ \ 0]^T \tag{4.37}$$

The rest of the procedures follow that for the linear system

4.3 Results comparing steepest descent with quasi-optimal schemes

For a 10% torque step (figures 4.2 and 4.3) it can be seen that there is virtually no difference in the response between the closed loop scheme and the optimal scheme (by S.D. method) except near the point where the control switches.

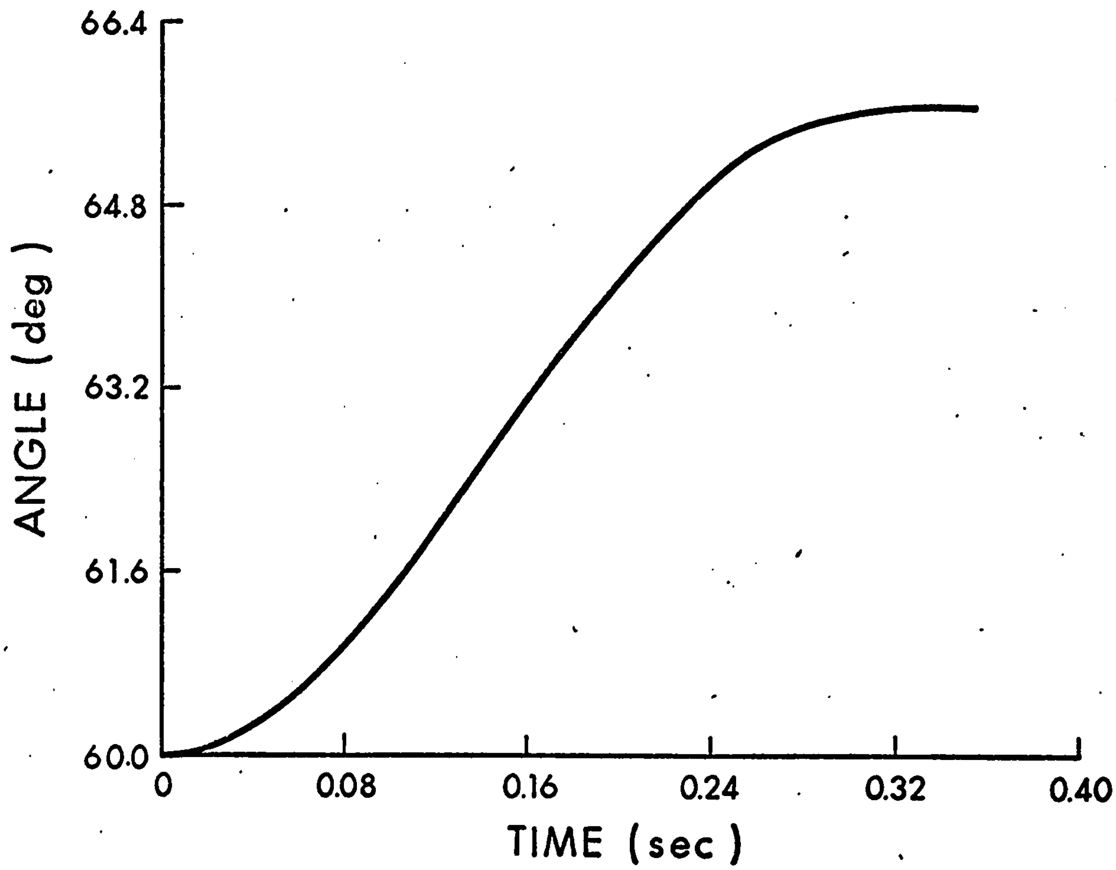

Fig. 4.2 Angle time characteristics for a 10% torque step (linearized system).

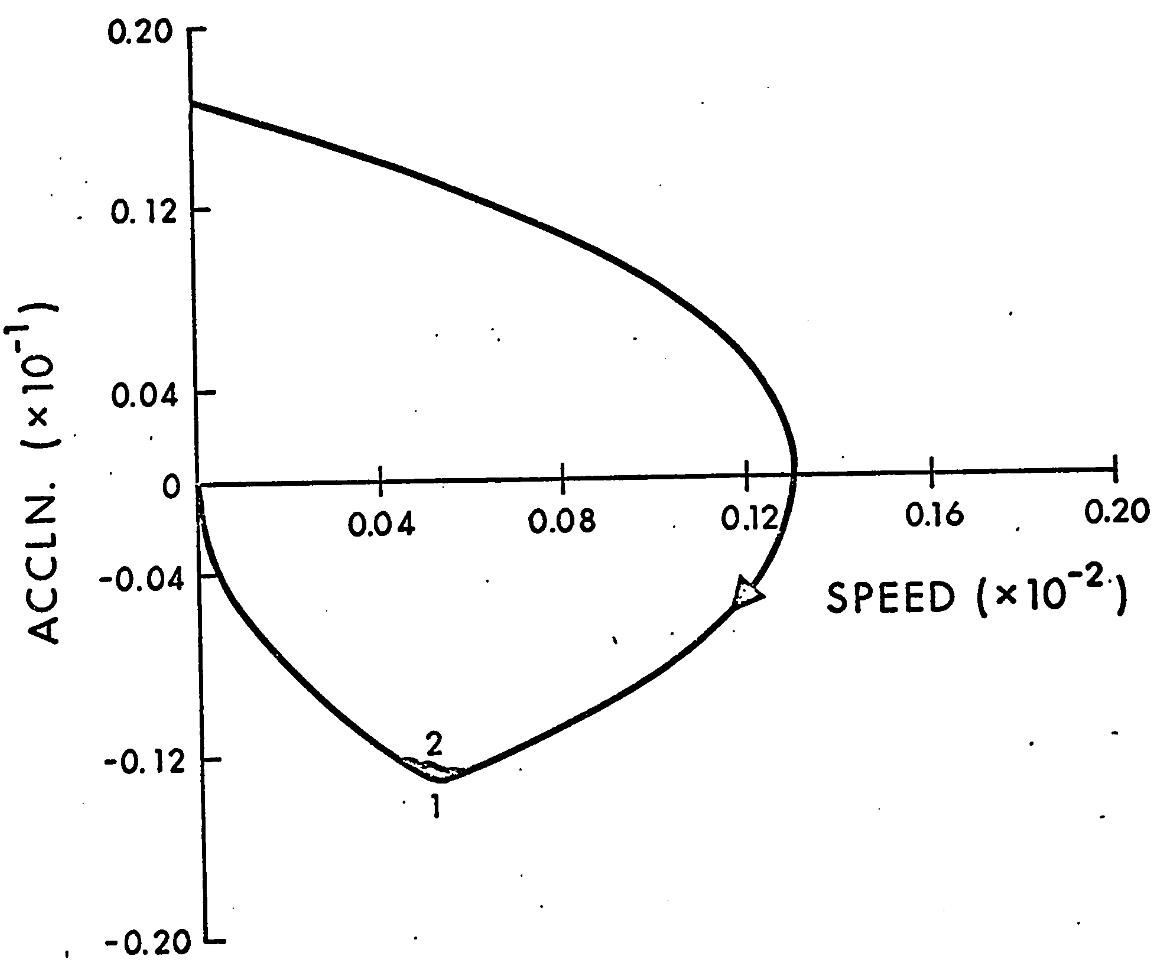

Fig. 4.3 Phase plane corresponding to Fig. 4.2.

1. Optimal control.
2. Closed loop control.

There is no doubt that the closed loop (C.L.) scheme gives optimal control when L(t) is zero. For small disturbances, the magnitude of the term L(t) is sufficiently small compared to the input term to yield an almost optimal control for a piecewise constant approximation.

Table 4.1 shows that as the integration step size is reduced, the time required for stabilization is less. It is obvious that the closed loop scheme depends on the integration step size and does not foresee the switch curve so that it is quite likely that the control switches several times near the correct switch point (figure 4.3). This can be avoided by taking larger step sizes, in which case the 'switch miss' may be quite large resulting in at least another extra switch. The results of Table 4.1 and 4.2 show this. The best way to avoid the intermediate switches is to provide a 'deadzone' so that the control does not switch if the magnitude of the switch function $\sum$ becomes smaller than a certain pre-determined quantity ε.

Scheme	Step size (sec)	No. of switches	Total cost (sec)	% Error	Remarks
Steepest descent (S.D.)	.0005	1	.3415	-	-
Closed-loop (C.L.)	.0001	1	.3427	.352	Intermediate switches (I.S)
Closed-loop	.0002	1	.3428	.382	Fewer I.S.
Closed-loop	.0005	1	.343	.44	Still fewer I.S.
Closed-loop	.001	2	.366	6.7	Almost no I.S.

Table 4.1 10% torque step, linearized system (ceiling voltage ±3 p.u.)

For large disturbances, the results do not agree very closely to the optimal ones as shown in figures 4.4, 4.5 and 4.6, which are for a 30% input torque step. This is approximately the maximum load the machine can deliver.

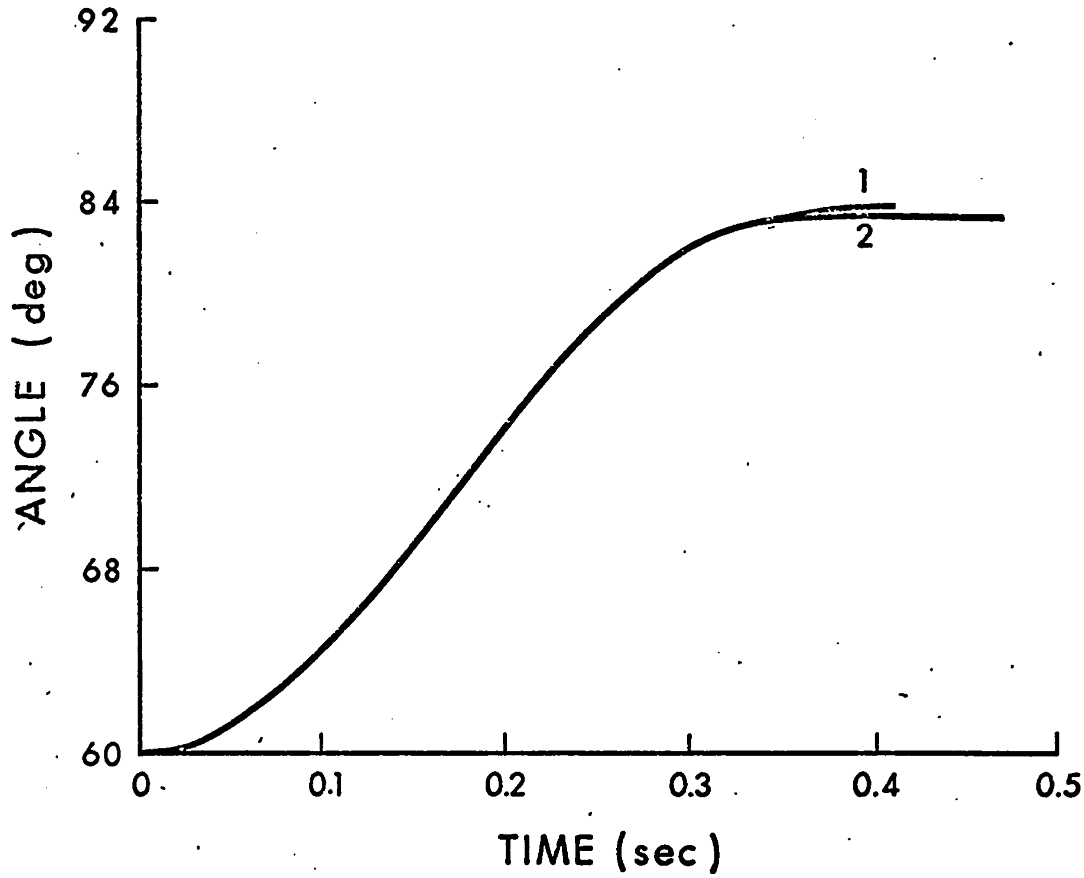

Fig. 4.4. Angle time characteristics, 30% torque step (nonlinear model).

1. Optimal control.

2. Closed loop control.

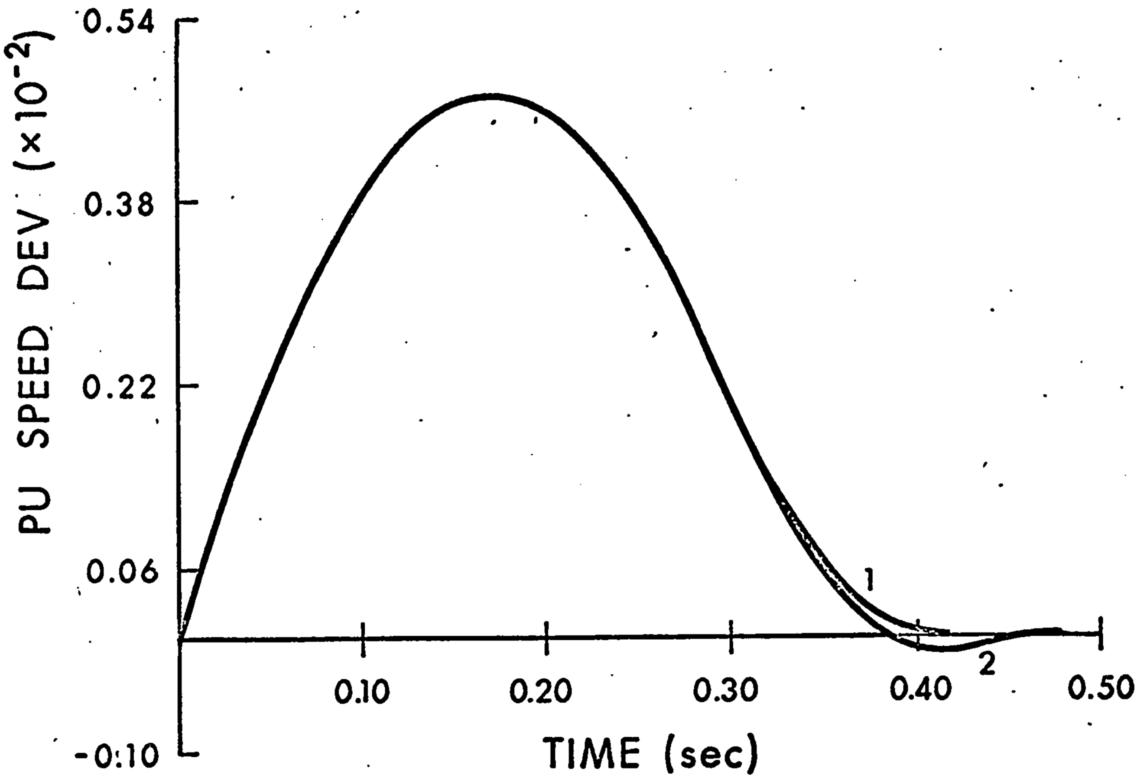

Fig. 4.5 Speed deviation vs time corresponding to Fig. 4.4.

1. Optimal control.
2. Closed loop control.

During and immediately after a large disturbance on the system, the constraint, that $|L(t)/b(t)|\leq 1$, may not be met. It was observed in reference [5] that if for a small fraction of the total time under consideration, L(t) does not satisfy the constraint, the scheme will yield a sub-optimal solution. However, if the constraint is not met for

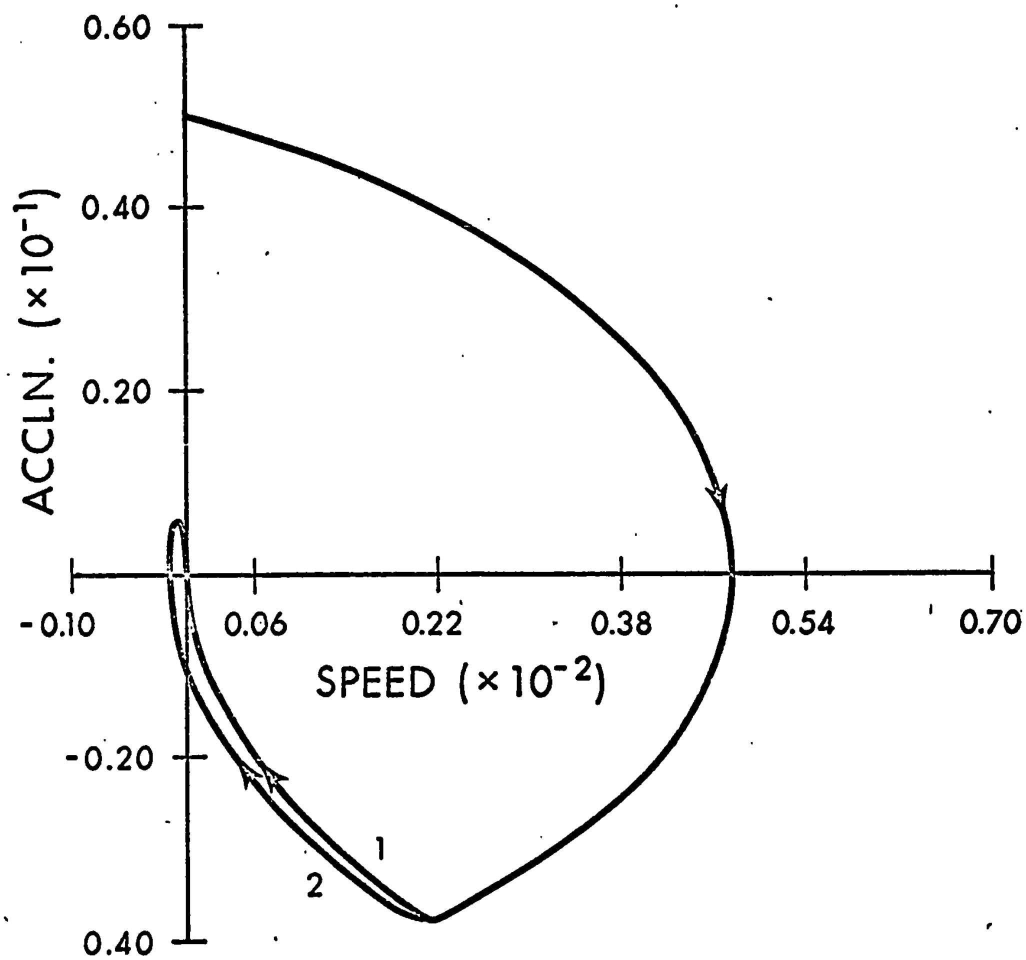

Fig. 4.6 Phase plane plot corresponding to Fig. 4.4.

1. Optimal control.
2. Closed loop control.

all $t \in [t_0, t_f]$, the scheme fails to give any information about the optimal control. Another factor to be considered here is that L(t) cannot be considered as a disturbance term as in reference [5] since it contains output feedback δ . In such a case the solution will be close to optimal only if the dependence of L on δ is small. However, the nonlinear system can always be linearized and the optimal control can be applied on the basis of a linearized system.

Disturbance	Scheme	Step size	No. of switches	Total cost (sec)	Error	Remarks
20% torque step	S.D.	.001	1	.33144	-	-
20% torque step	C.L.	.001	2	.386	16.3%	I.S.*
30% torque step	S.D.	.001	1	.409912	-	-
30% torque step	C.L.	.001	2	.4788	16.8%	I.S.

* Refer to Table 4.1.

Table 4.2 Nonlinear system (ceiling voltage ±5 p.u.).

4.4 Effect of time delay

So far it has been considered that the control can be instantly applied to the system immediately after it is found either by optimal or quasi-optimal schemes. In general, the switching times of the control are changed from the nominal ones, by the delay introduced by computation (includes instrumentation delay), by the actuator (includes exciter) and by the variation of the parameters of the system. The first two delay the switching of the control and the last one might cause the switching to occur ahead of the nominal value or after it. However, it is considered that the effect of variation of parameters on the control is small compared to the others. In this section the effect of time delays on the stability limits of a synchronous generator is investigated.

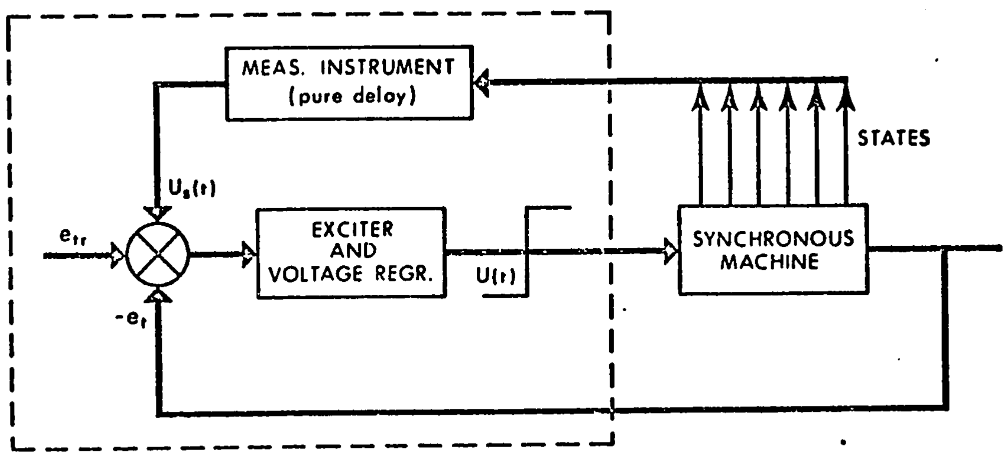

Fig. 4.7 The system block diagram.

Two different cases are considered here - the effect of time delays with fixed excitation strategy (or the so called open loop optimal control); and the effect on closed loop excitation control.

i) Fixed strategy

The switch curve for a particular disturbance is found by the closed loop scheme. The switch curve crosses the time axis at several points and these are stored (this is equivalent to storing the control). It is also necessary to store the switch points in the 'deadzone' (where the magnitude of the switching function is less than 10^{-6}) since observation for a longer period of time is needed when the control is delayed.

The second step is to delay the control by τ seconds from the predetermined control strategy making sure that the field voltage remains normal for $t \leq \tau$sec. Different values of τ have been considered for the nonlinear machine model with 20% torque step. It is observed that $\tau = .215$ seconds is in the neighbourhood of the critical delay when the system appears to be in a limit cycle. For τ greater than .25 seconds, the system is unstable. The phase plane plots are shown in figures 4.8, 4.9, 4.10 and 4.11 for different values of τ.

The system without any time lag returns to the equilibrium point in about .4 seconds. The field voltage after that simply bangs around in the 'deadzone' with no net effect. The disturbance considered is small and in the absence of any stabilizing signal, the machine may remain stable up to .92 seconds. Due to these reasons a relatively large time does not seriously deteriorate the stability of the system for the fixed excitation strategy which has little importance for on-line applications.

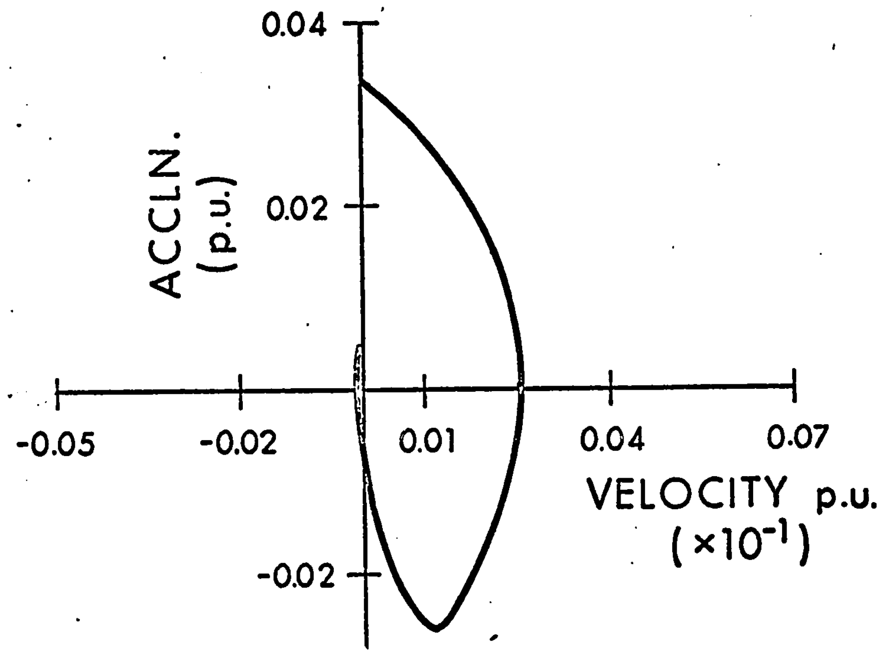

Fig. 4.8 Velocity vs acceleration plot for no time lag (τ = 0).

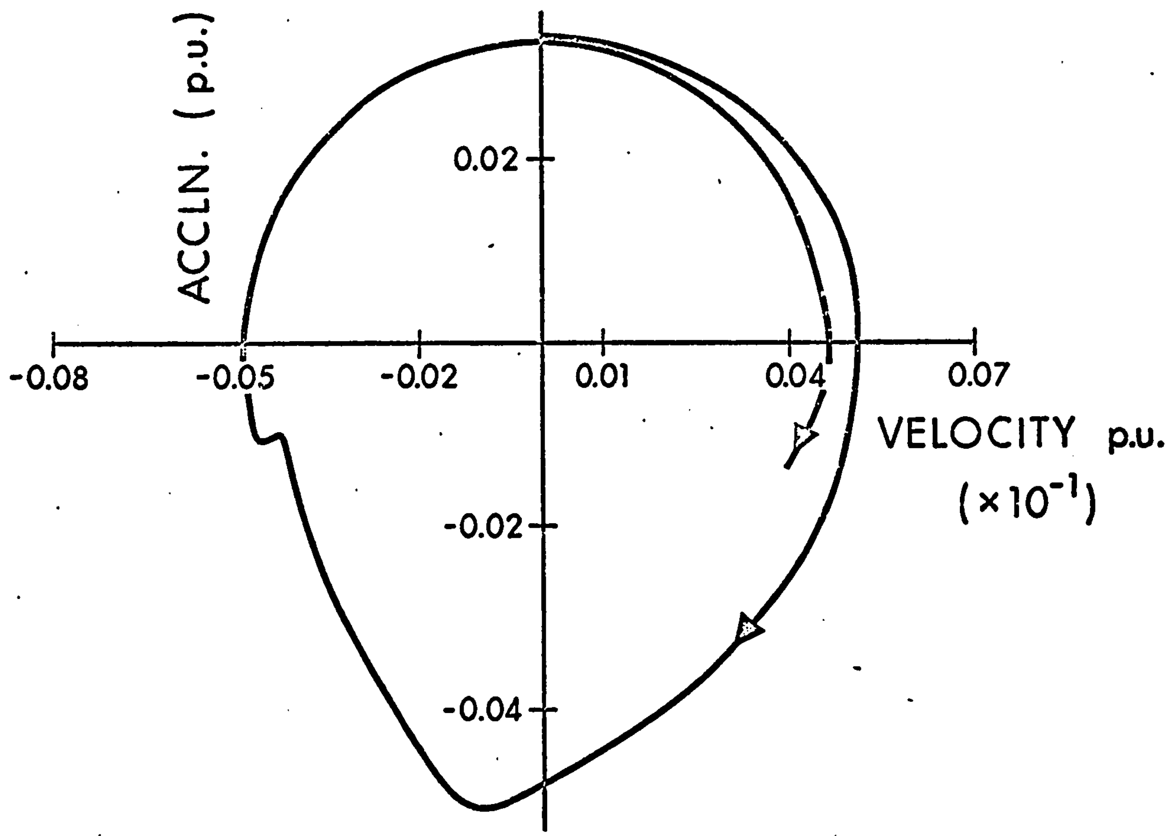

Fig. 4.9 Phase plane plot for τ = .185 Sec.

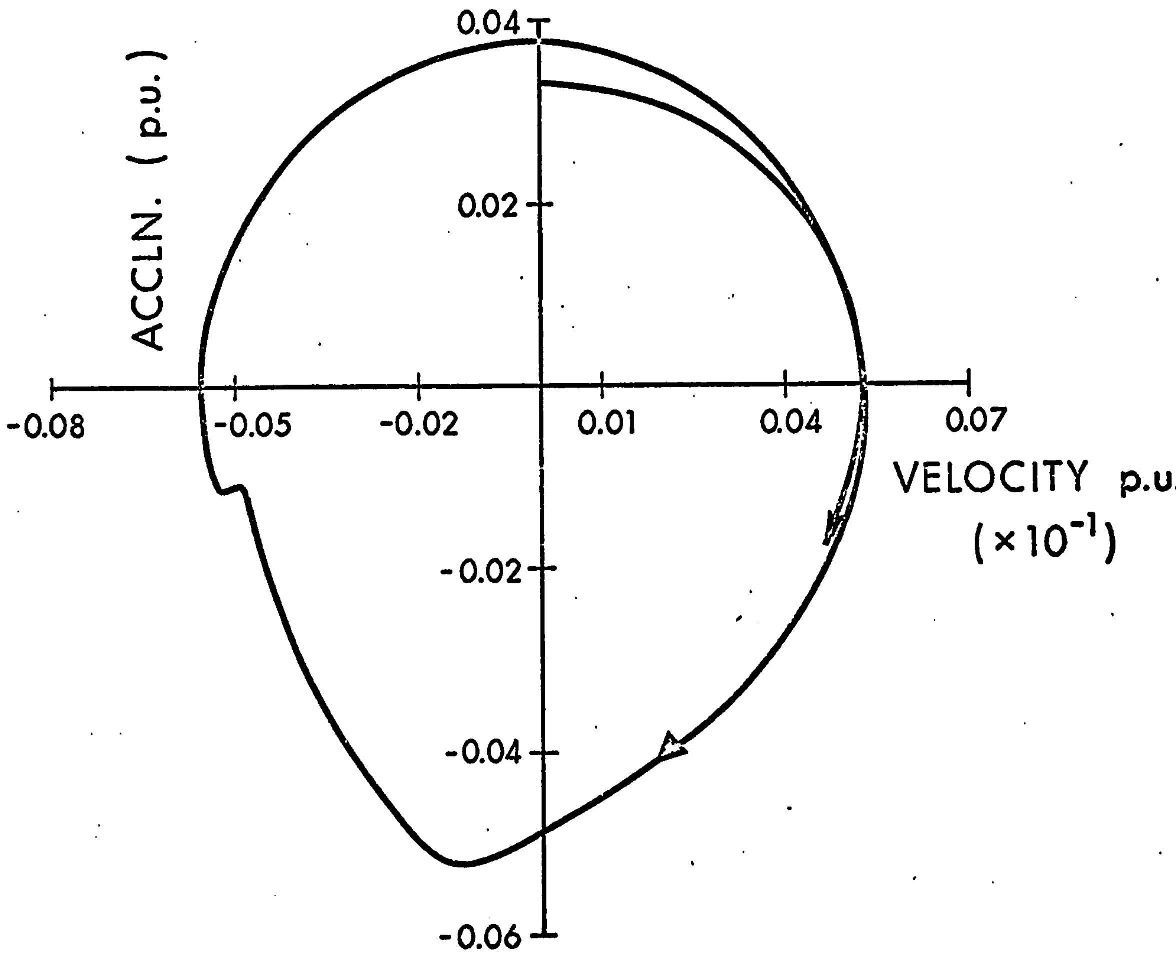

Fig. 4.10 Phase plane plot for τ = .215 Sec.

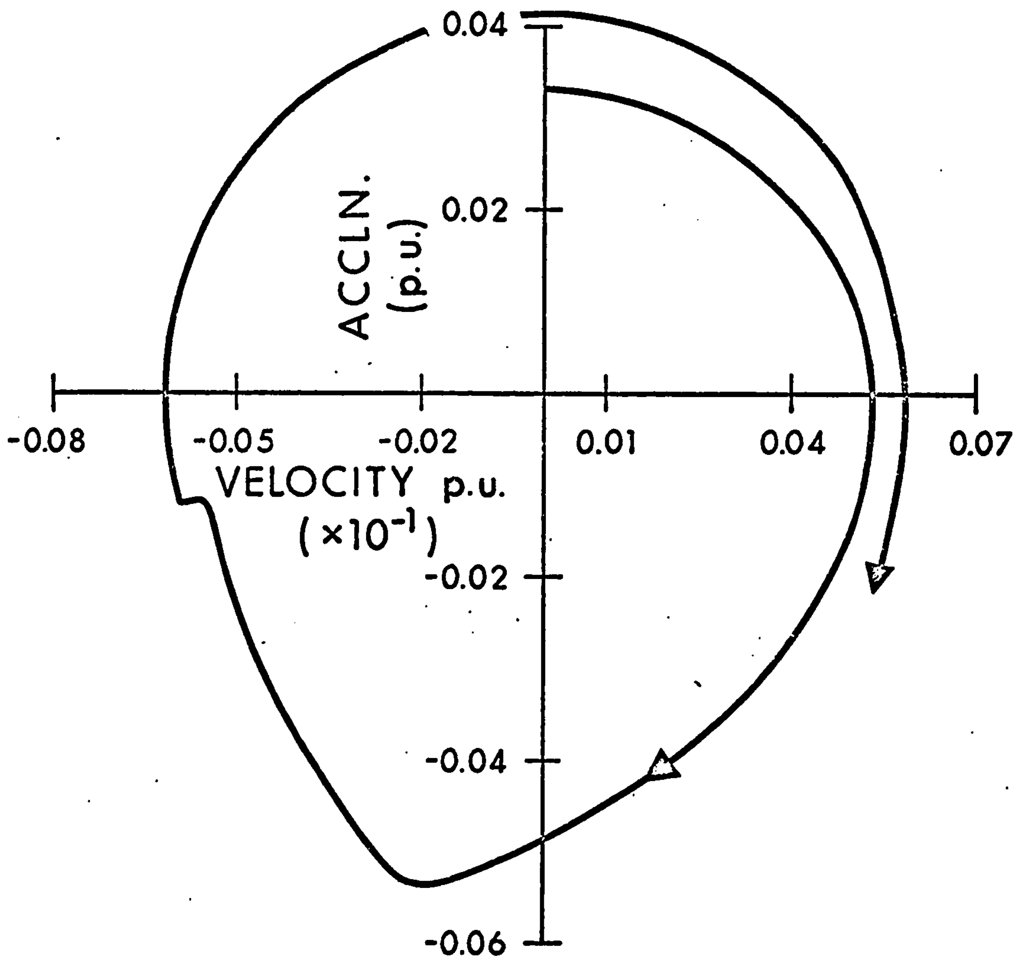

Fig. 4.11 Phase plane plot for τ = .25 Sec.

ii) Closed loop strategy

The quasi-optimal control based on the system states is computed and stored at each point of integration. The control is applied to the system after τ seconds. The delayed control results in new system states and the control in the subsequent steps is found on that basis. Since the control depends on the measurement of actual states, it is expected that the time lag allowable is less in this case. For the same disturbance as in the previous case (open loop case), a delay of about 50 ms is critical. The system becomes unstable for a time lag of 60 ms. For modern excitation systems and measurement equipment, this is an appreciable time from a stability viewpoint. The phase plane plots are given in figures 4.12 and 4.13 for different values of time lags.

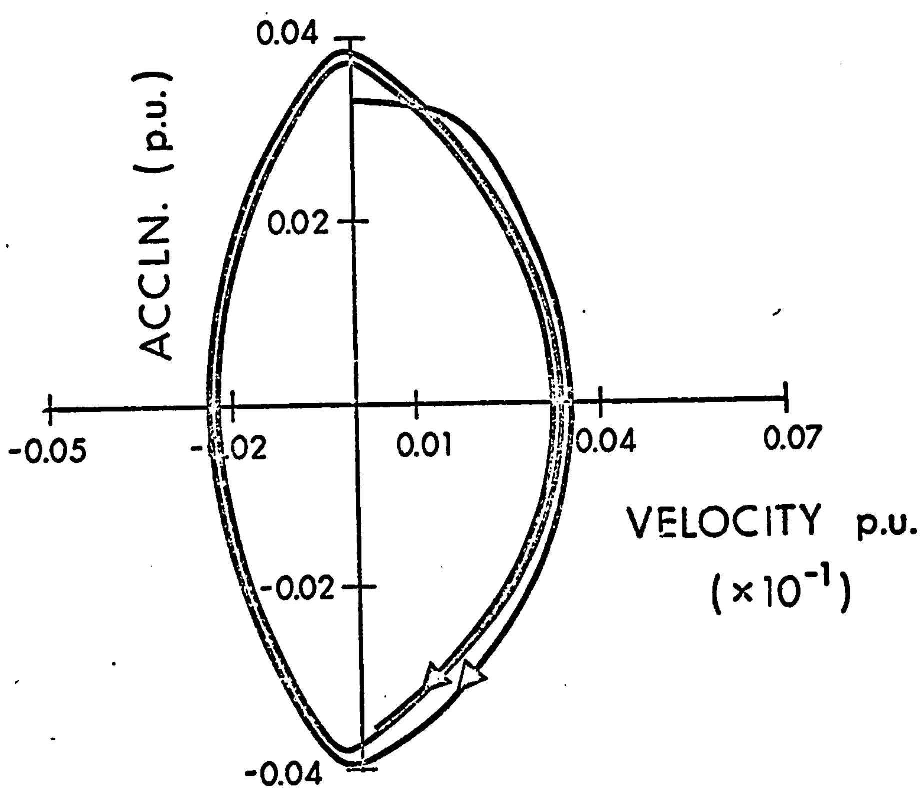

Fig. 4.12 Velocity vs acceleration plot for τ = .05 Sec.

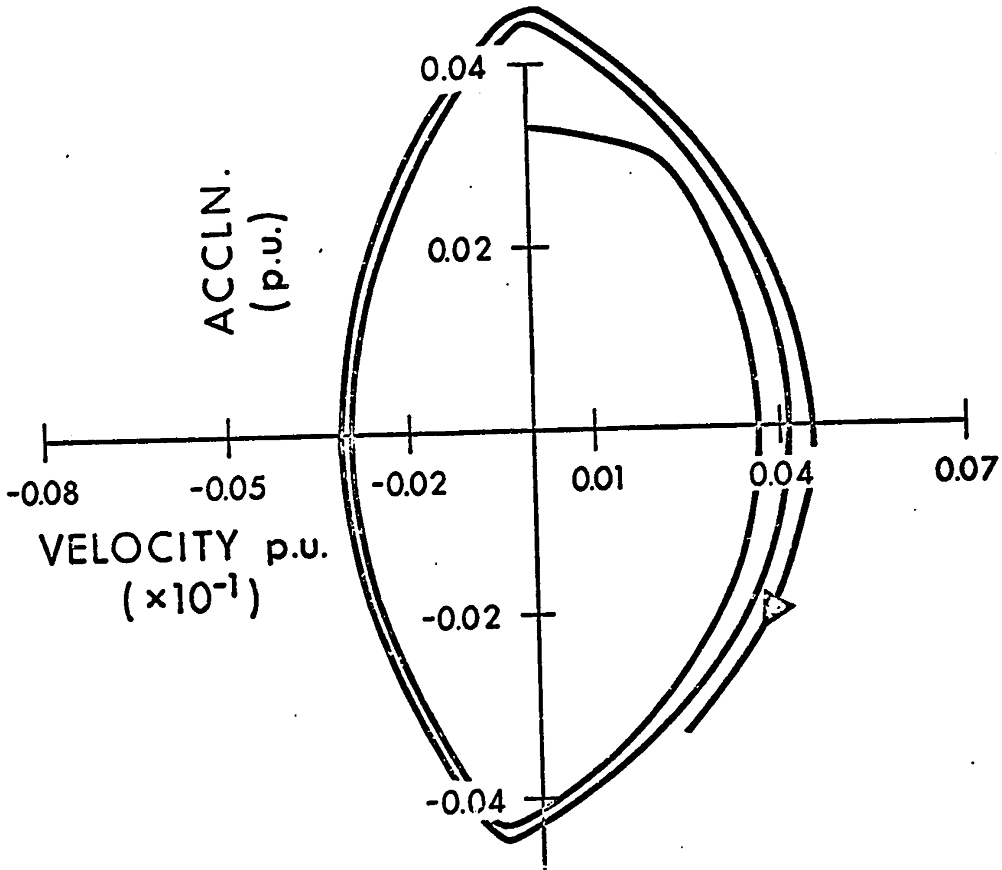

Fig. 4.13 Phase plane plot for τ = .06 Sec.

The last case considered is that no control was applied up to about .92 seconds followed by a no-lag bang bang control. The system required about 2.5 seconds to stabilize. This shows that a longer initial delay is allowable even with the closed loop scheme. The critical initial delay is seen to be about .95 seconds.

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APPENDIX A4-1

METHOD OF STEEPEST DESCENT

Many computer procedures for solving the problem stated in Section (3.2) are available in the literature[1,2,3,6]. Van Slyke[7] in his Ph.D. thesis made a detailed study of mathematical programming techniques for optimal control problems. These are not presented here.

Both Pontryagin's minimum principle (P.M.P) and the method of calculus of variations can be applied to solve the problem stated in the previous section. These are indirect methods in that the control is a function of some multipliers,values of which are not known in general. P.M.P uses a set of adjoint variables initial values of which are generally not known. Newton Raphson method may be used to search these initial values. Convergence is fast but the guess for the initial costate variables should be quite close to the optimal ones. This method is seriously handicapped in that it is usually difficult to make a good guess for the initial adjoint variables.

Iterative methods such as steepest descent and conjugate gradient methods based on the calculus of variations deal directly with the control and yield the optimal solution in the limit. One disadvantage with these methods is that a local maximum (or minimum) solution may be reached instead of the global one. The conjugate gradient method is faster in convergence, but needs an additional set of influence functions if the final values of the states are not free.

Due to the simplicity of the steepest descent method, the problem stated is solved with this method. It can be seen that the right hand side of equation (3.1) is linear in the control variable u(t). The optimal control for such a problem is bang bang by P.M.P. The only unknown for these problems is the optimum value of the switching times of the control u(t). The steepest descent algorithms constructed by Bryson and Denham[1,2] and later modified by Vachino[3] to take into consideration control problems with inequality constraints, have been used here. The problem is formulated as follows:

Choose the control function u(t) from a class of piecewise continuous functions of time such that $|u(t)| \leq 1$ which takes the system described by the vector valued differential equation

$$\dot{\underline{X}} - \underline{F}(\underline{X}, u) = 0 \qquad \text{(A4-1.1)}$$

from its initial state $\underline{X}(o)$ at time t_0, to its intended final state, such that it satisfies the vector valued terminal condition

$$\psi(\underline{X}(t_f), t_f) = 0 \qquad \text{(A4-1.2)}$$

and minimizes the cost index (Mayer formulation)

$$\phi(\underline{X}(t_f), t_f) \qquad \text{(A4-1.3)}$$

The time t_f is chosen as the first time that one of the terminal conditions, referred to as a stopping condition,

$$\Omega(\underline{X}(t_f), t_f) = 0 \qquad \text{(A4-1.4)}$$

is satisfied.

Using the Heaviside step function[3], the equation of variation for equation (A4-1.1) can be written as

$$\delta\dot{x}(t) = F_x\ \delta x + L\delta\omega \qquad (A4\text{-}1.5)$$

where $$L = -[(F_{s(i+1)} - F_{si})\Delta(t-t_s)] \qquad (A4\text{-}1.6)$$

$s = 1,2, \ldots\ldots.N$

$i = 1,2, \ldots\ldots.n$

N is the total no of distinct discontinuities in u(t) for the system of order n

$\delta\omega = [\delta t_1, \delta t_2, \ldots\ldots.\delta t_N]^T$

$\Delta(t-t_s)$ is a delta function occuring at the switching instants $t = t_s$

The influence functions are given by

$$\dot{\lambda} = -F_x^T\ \lambda \qquad (A4\text{-}1.7)$$

Choosing the terminal conditions on the adjoint vector

$$\lambda_\phi(t_f) = \frac{\partial\phi}{\partial x}\bigg|_{t_f} \qquad (A4\text{-}1.8)$$

$$\lambda_\psi(t_f) = \frac{\partial\psi}{\partial x}\bigg|_{t_f} \qquad (A4\text{-}1.9)$$

$$\lambda_\Omega(t_f) = \frac{\partial\Omega}{\partial x}\bigg|_{t_f} \qquad (A4\text{-}1.10)$$

the variation of control ω (the switch times) in the final form is obtained as

$$\delta\omega(t) = \pm\sum_{s=1}^{N+1} W^{-1} L_s^T(t)[\lambda_{\phi\Omega}(t) - \lambda_{\psi\Omega}(t)\, I_{\psi\psi}^{-1}\, I_{\psi\phi}]\sqrt{\frac{(dP)^2 - d\psi^T\, I_{\psi\psi}^{-1}\, d\psi}{I_{\phi\phi} - I_{\psi\phi}^T\, I_{\psi\psi}^{-1}\, I_{\psi\phi}}}$$

$$+\sum_{s=1}^{N+1} W^{-1} L_s^T(t)\, \lambda_{\psi\Omega}(t)\, I_{\psi\psi}^{-1}\, d\psi \qquad \text{(A4-1.11)}$$

where

$$\lambda_{\phi\Omega}^T(t) = \lambda_\phi^T(t) - \frac{\dot\phi(t_f)}{\dot\Omega(t_f)}\lambda_\Omega^T(t) \qquad \text{(A4-1.12)}$$

$$\lambda_{\psi\Omega}^T(t) = \lambda_\psi^T(t) - \frac{\dot\psi(t_f)}{\dot\Omega(t_f)}\lambda_\Omega^T(t) \qquad \text{(A4-1.13)}$$

$$(dP)^2 = \int_{t_o}^{t_f} \delta\omega(\tau)\, W\, \delta\omega(\tau)\, d\tau \qquad \text{(A4-1.14)}$$

$$I_{\phi\phi} = \sum_{s=1}^{N+1}\int_{t_{s-1}}^{t_s} \lambda_{\phi\Omega}^T\, L_s\, W^{-1}\, L_s^T\, \lambda_{\phi\Omega}\, d\tau \qquad \text{(A4-1.15)}$$

$$I_{\psi\phi} = \sum_{s=1}^{N+1}\int_{t_{s-1}}^{t_s} \lambda_{\psi\Omega}^T\, L_s\, W^{-1}\, L_s^T\, \lambda_{\phi\Omega}\, d\tau \qquad \text{(A4-1.16)}$$

$$I_{\psi\psi} = \sum_{s=1}^{N+1}\int_{t_{s-1}}^{t_s} \lambda_{\psi\Omega}^T\, L_s\, W^{-1}\, L_s^T\, \lambda_{\psi\Omega}\, d\tau \qquad \text{(A4-1.17)}$$

Computing Procedure

a) Compute the nominal path by integrating the differential equations with a nominal control variable program and fixed initial conditions. Store the solution.

b) Compute the λ_ψ functions all at the same time by integrating the adjoint differential equations backwards, evaluating the partial derivatives on the nominal path by reference to (a).

c) Simultaneously with (b) calculate the quantity $\lambda_{\psi\Omega}$ and perform integrations (backwards) leading to the number $I_{\psi\psi}$.

d) Select desired terminal conditions $d\psi$ to bring the next solution closer to $\psi = 0$ than were achieved by nominal path.

e) Using the values of $d\psi$ chosen, so that the final time does not exceed the predetermined value, calculate $\delta\omega(t)$. If t_f exceeds the predetermined value go over again to (d).

f) Obtain a new nominal path by using $\omega_{new} = \omega_{old} + \delta\omega$ and repeat process (a) through (f) until terminal conditions $\psi = 0$ are satisfied.

CHAPTER 5

AN EXTENDED CASE

This chapter considers an extension of the method developed in chapter 3 to a two machine system. The closed loop control scheme for the two machine system is developed in section 1. The results obtained by such control, for a number of cases, is discussed in section 2.

5.1 Closed loop control for two machine system

For stable operation of synchronous generators in a power system, the relative velocity and acceleration between any two machines in the system should drop to zero following a disturbance while the relative rotor angular positions should remain between 0 and $\pi/2$ radians. So for two machine system, the optimal control problem for stability can be formulated as follows:

Given the initial states $\underline{X}_m(o)$; m = 1,2 and the following information on the target states

$$
\begin{aligned}
n_1(t_f) - n_2(t_f) &= 0 \\
\dot{n}_1(t_f) - \dot{n}_2(t_f) &= 0 \\
0 \leq \delta_1(t_f) - \delta_2(t_f) &\leq \pi/2
\end{aligned}
\tag{5.1}
$$

where $n_m = x_{4m}$ and $\delta_m = \omega_o\, x_{5m}$

Find the admissible control $u_m(t)$ which forces system (2.22) from the given initial states to the desired final states in the minimum possible time.

The procedure for the two machine system is similar to that for the single machine case and is briefly outlined as follows:

For each machine, differentiate the swing equations (2.23) with respect to time and substitute the voltage current relations. Then the following set of differential equations is obtained

$$p^3(\delta_1/\omega_o) = L_1(t) + b_1(t)\, u_1(t) \tag{5.2}$$

$$p^3(\delta_2/\omega_o) = L_2(t) + b_2(t)\, u_2(t) \tag{5.3}$$

Subtracting equation (5.3) from (5.2) yields

$$p^3(\delta_1/\omega_o) - p^3(\delta_2/\omega_o) = L_1(t) - L_2(t) + b_1(t)\, u_1(t) - b_2(t)\, u_2(t) \tag{5.4}$$

or $$p^3(\delta/\omega_o) = L(t) + u(t) \tag{5.5}$$

where
$$\begin{aligned} \delta &= \delta_1 - \delta_2 \\ L(t) &= L_1(t) - L_2(t) \\ u(t) &= b_1(t)\, u_1(t) - b_2(t)\, u_2(t) \end{aligned} \tag{5.6}$$

As before, assuming $x_1 = \delta/\omega_o$, equation (5.5) gives the set of differential equations

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3 \qquad (5.7)$$
$$\dot{x}_3 = u_o(t)$$

where $u_o(t) = L_{1(o)} - L_{2(o)} + b_{1(o)}\, u_1(t) - b_{2(o)}\, u_2(t)$ (5.8)

and $u_{omin} \leq u_o(t) \leq u_{omax}$

Defining the following quantities as

$$\Sigma = x_2 - \frac{x_3^{\,2}}{2u_{omin}} \quad , \text{ if } x_3 > 0 \qquad (5.9)$$

$$\Sigma = x_2 - \frac{x_3^{\,2}}{2u_{omax}} \quad , \text{ if } x_3 < 0 \qquad (5.10)$$

$$\Sigma_1 = x_1 - \frac{2\pi}{w_o} - \frac{x_2 x_3}{\alpha} + \frac{1}{3\alpha^2} x_3^{\,3} \qquad (5.11)$$

$$\Sigma_2 = x_1 - \frac{x_2 x_3}{\alpha} + \frac{1}{3\alpha^2} x_3^{\,3} \qquad (5.12)$$

where $\alpha = L(o) + u_{omin}$ if $\Sigma > 0$ (5.13)

$\alpha = L(o) + u_{omax}$ if $\Sigma < 0$ (5.14)

Then the control scheme is

$u_o(t) = u_{omax}$ if $\Sigma_1 < 0$ (5.15)

$u_o(t) = u_{omin}$ if $\Sigma_2 > 0$ (5.16)

Otherwise, $u_o(t) = u_{omax}$ if $\Sigma < 0$ (5.17)

$$u_o(t) = u_{omin} \quad \text{if } \sum > 0 \tag{5.18}$$

As in the case of a single machine system, the constraint on L(t) that

$$|L(t)| < |u(t)| \tag{5.19}$$

should be satisfied. This implies that $u_{omin} < 0$.
Substituting equation (5.15) through (5.18) the initial value of the quasi-optimal control $u_1(t)$ and $u_2(t)$ is obtained from equation (5.8). The process is continued until the desired final states are reached. The proportional control for the two machine case is obtained in a similar way as for the single machine case.

5.2 Discussion of results

The system configuration as given in figure 2.3 is considered. The machines are interconnected by a double circuit transmission line and feed an impedance load from the intermediate bus. The two machines considered are identical but carry different loads. Machine II has less load than machine I. Faults and disturbances are considered on machine I only. The following cases are considered. (Data for the two machine system are given in section A5-1 of appendix A5).

i) 100% torque pulse for 3 cycles.

The angle time characteristics in figure 5.1 shows that the application of bang bang control results in a near deadbeat response. This changes the original load sharing between the two machines. The proportional control based on the relative velocity and acceleration of the system gives a heavily damped response, bringing the system to the original operating point. The intermediate bus voltage returns to normal. Though the relative velocity of the machines becomes zero, the system speed up by about .4% (figure 5.4). On a real system, this will not happen since the governing system will act to bring down the speeds of the individual machine. Figures 5.2 and 5.3 give the phase plane plot and figure 5.5 shows the variation of the terminal voltage of the individual machines.

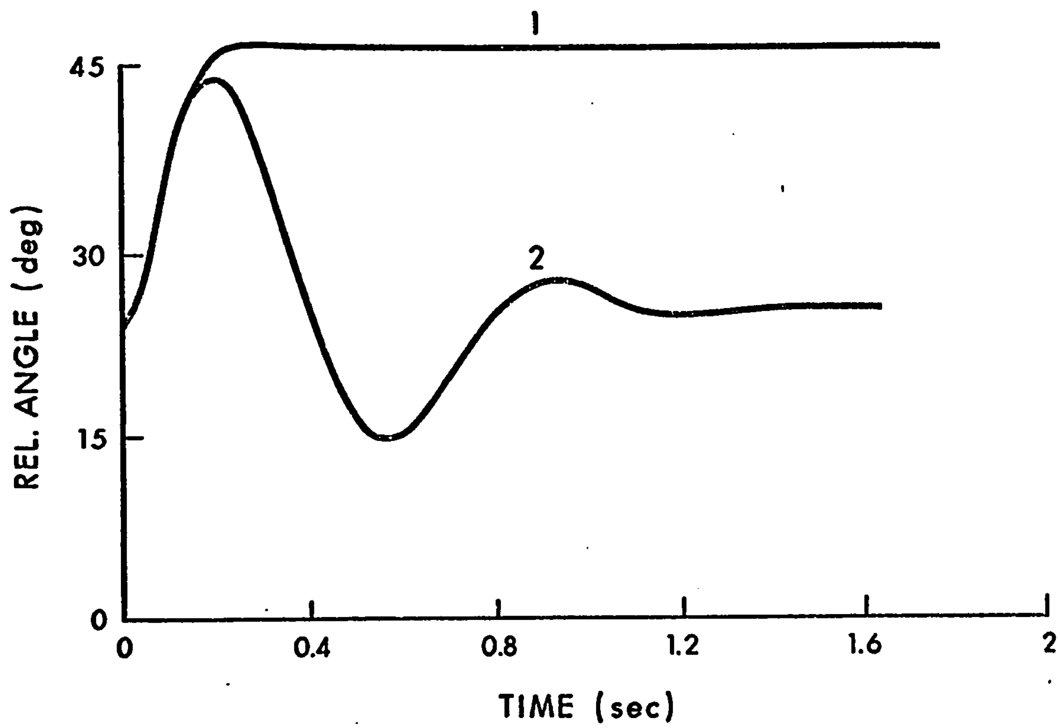

Fig. 5.1 Relative angle time characteristics for 100% torque pulse

1. Bang bang control.
2. Proportional control.

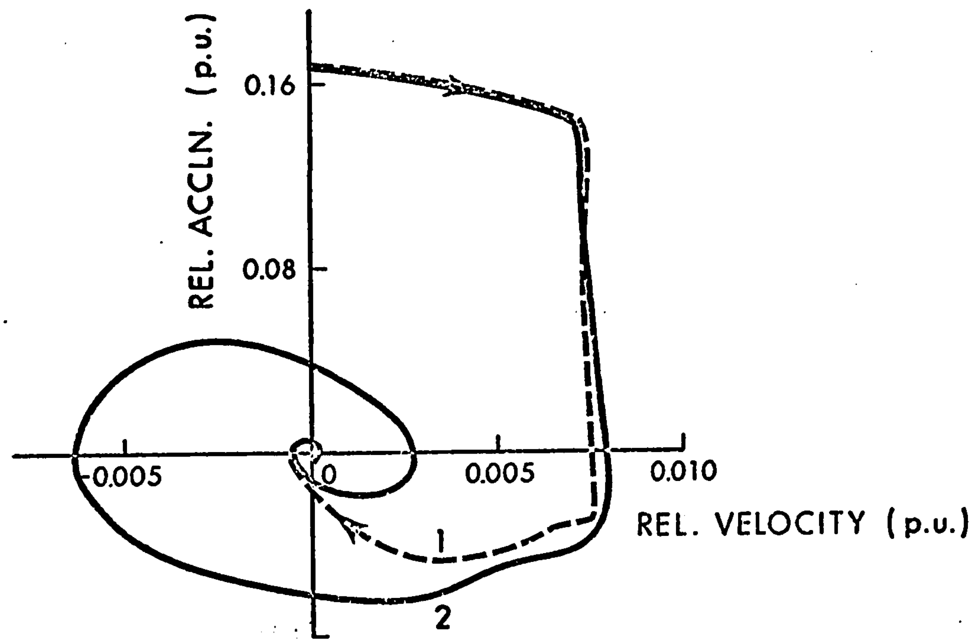

Fig. 5.2 Relative velocity vs. relative acceleration plot corresponding to Fig. 5.1.

1. Bang bang control.

2. Proportional control.

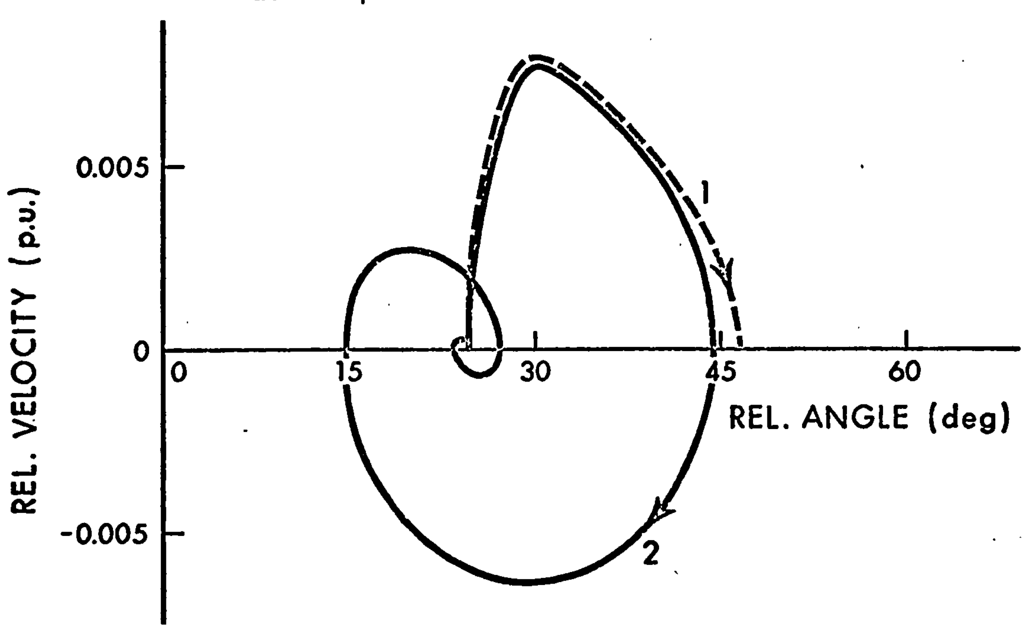

Fig. 5.3 Relative angle vs relative velocity plot corresponding to Fig. 5.1.

1. Bang bang control.

2. Proportional control.

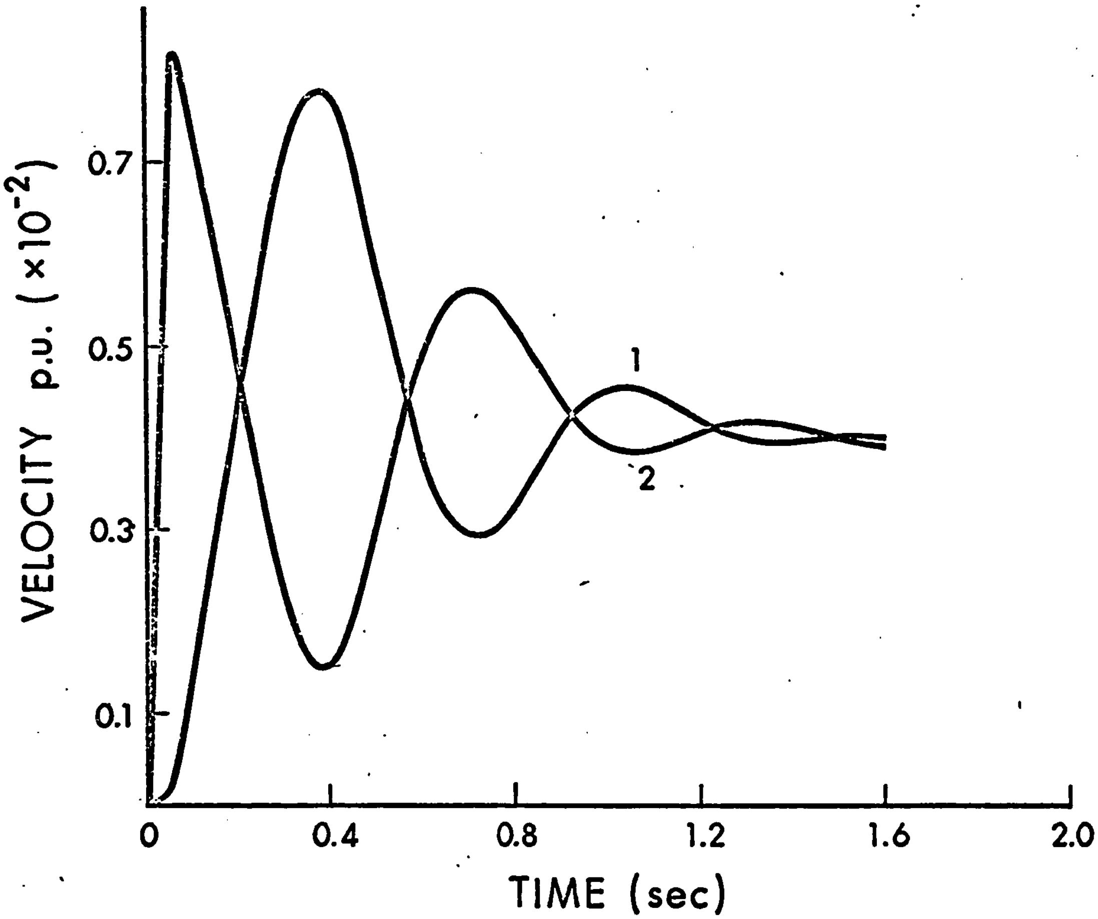

Fig. 5.4 Velocity vs time with proportional control, 100% torque pulse.

1. Machine I

2. Machine II

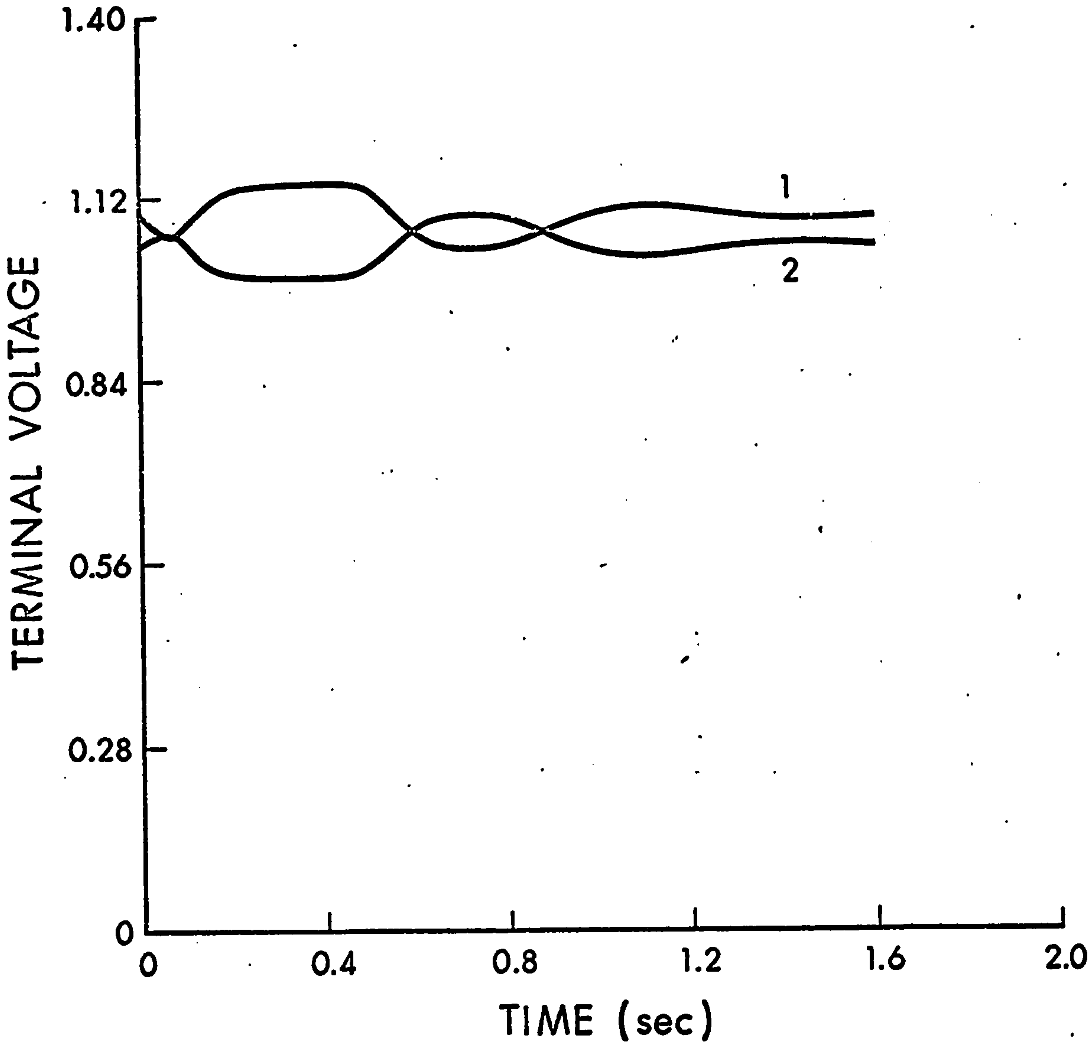

Fig. 5.5 Terminal voltage variation with proportional control, 100% torque pulse.

1. Machine I

2. Machine II

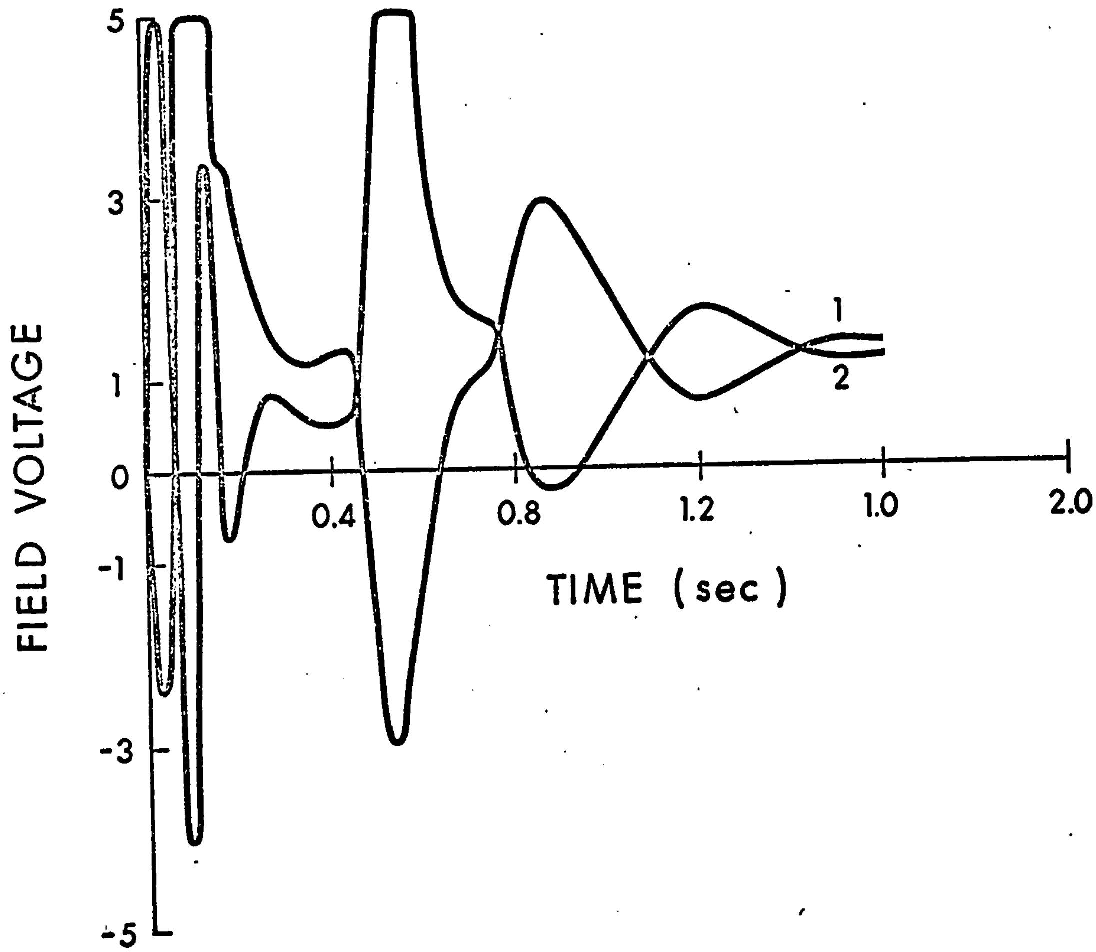

Fig. 5.6 Field voltage characteristics with proportional control, 100% torque pulse.

1. Machine I

2. Machine II

ii) 150% torque pulse for 3 cycles

This is a very severe disturbance. The bang bang control results in an overshoot and an undershoot. The intermediate bus (load bus) voltage is very much reduced resulting in a totally different load sharing between the machines. The proportional control results in a heavily oscillatory response (fig. 5.7) and a return to nearly normal load sharing. Figure 5.8 gives the phase plane plot.

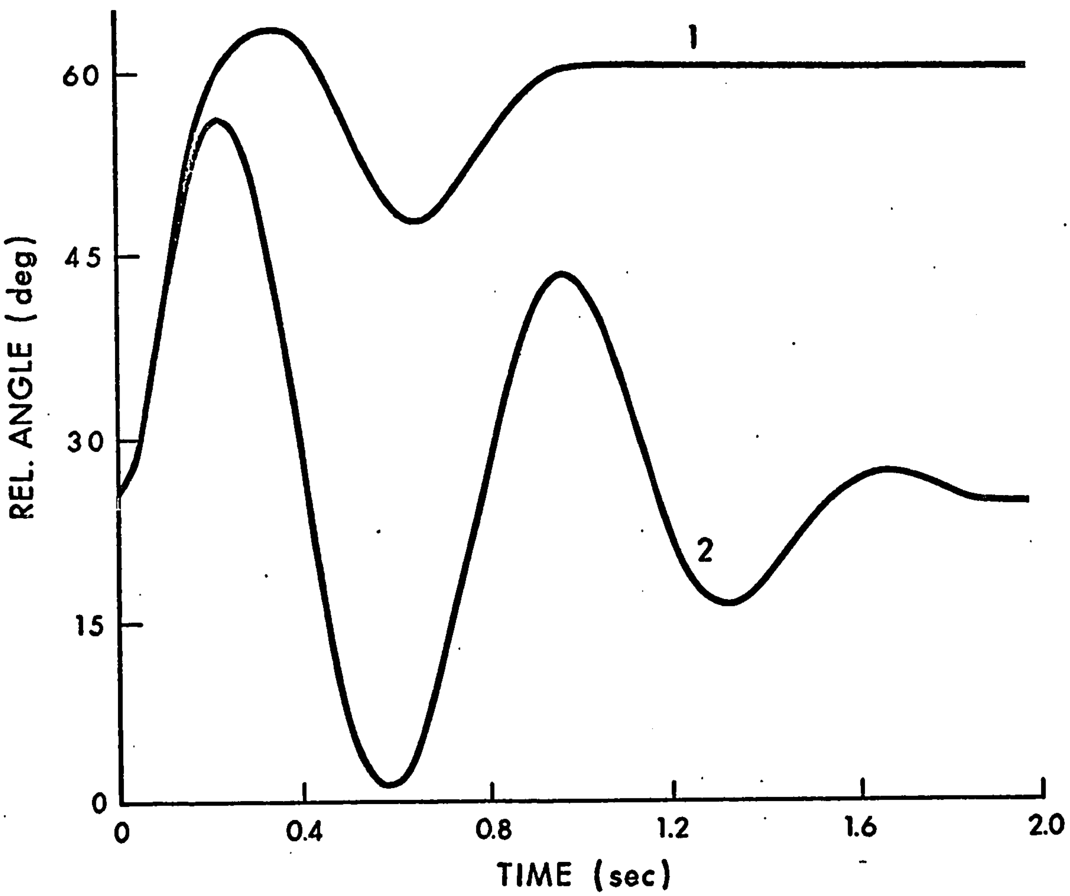

Fig. 5.7 Relative angle time characteristics for 150% torque pulse.

1. Bang bang control.
2. Proportional control.

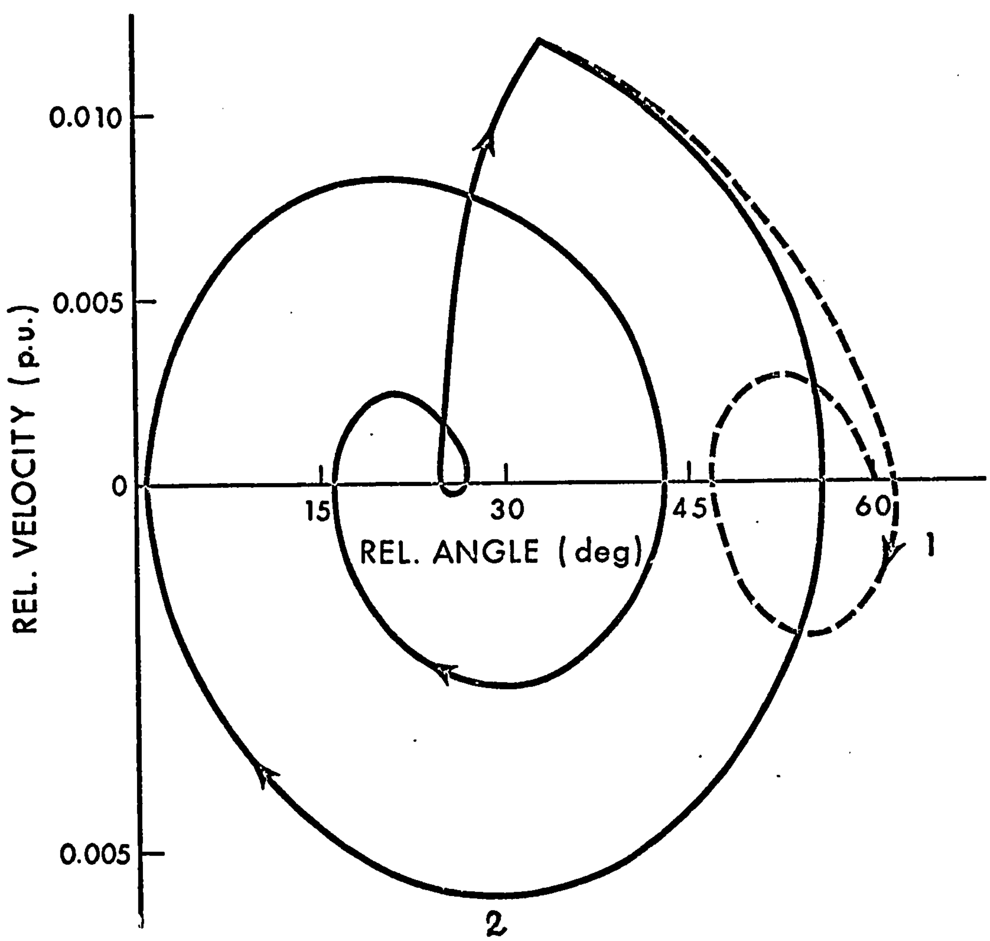

Fig. 5.8 Phase plane plot corresponding to Fig. 5.7

1. Bang bang control.

2. Proportional control.

iii) Three phase short circuit at the transformer end of the first machine cleared in 3 cycles (underexcited case)

The second machine is normally excited. Both the machines are relatively lightly loaded. (Machine I is loaded at .73 per unit). Figure 5.9 shows that both bang bang and proportional control results in similar type of oscillations. The proportional control restores the load bus voltage and machine internal voltage, while the bang bang control reduces them. Figure 5.10 shows the phase plane plot and figure 5.11 shows the variation of the individual rotor angles with proportional control only. It can be seen that the rotor angles continue to increase with time; this is due to the off-set in velocity of the individual machines. Again, this will not occur if governor action is taken into consideration.

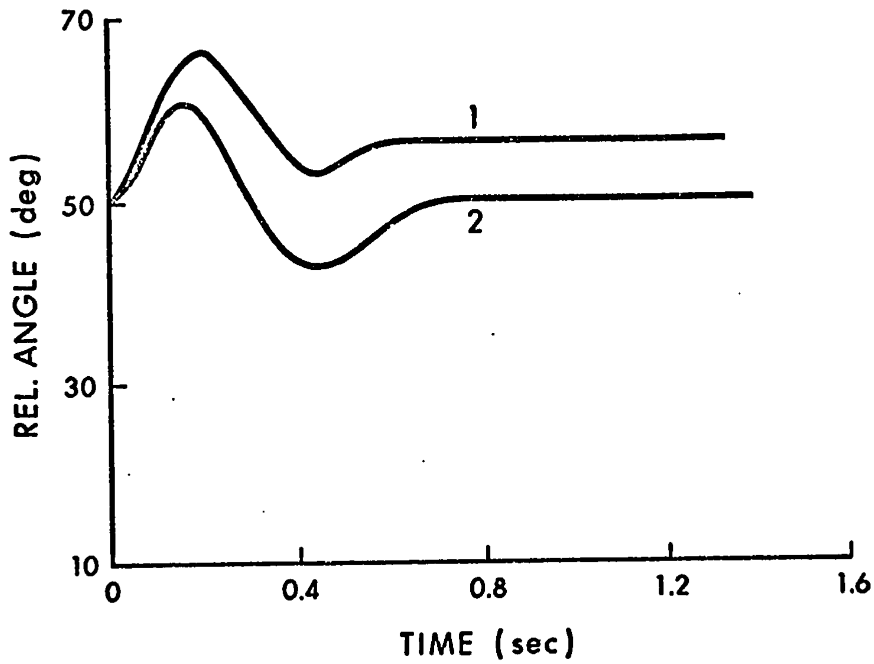

Fig. 5.9 Relative angle-time characteristics for 3 - ϕ fault on under excited machine.

1. Bang bang control.
2. Proportional control.

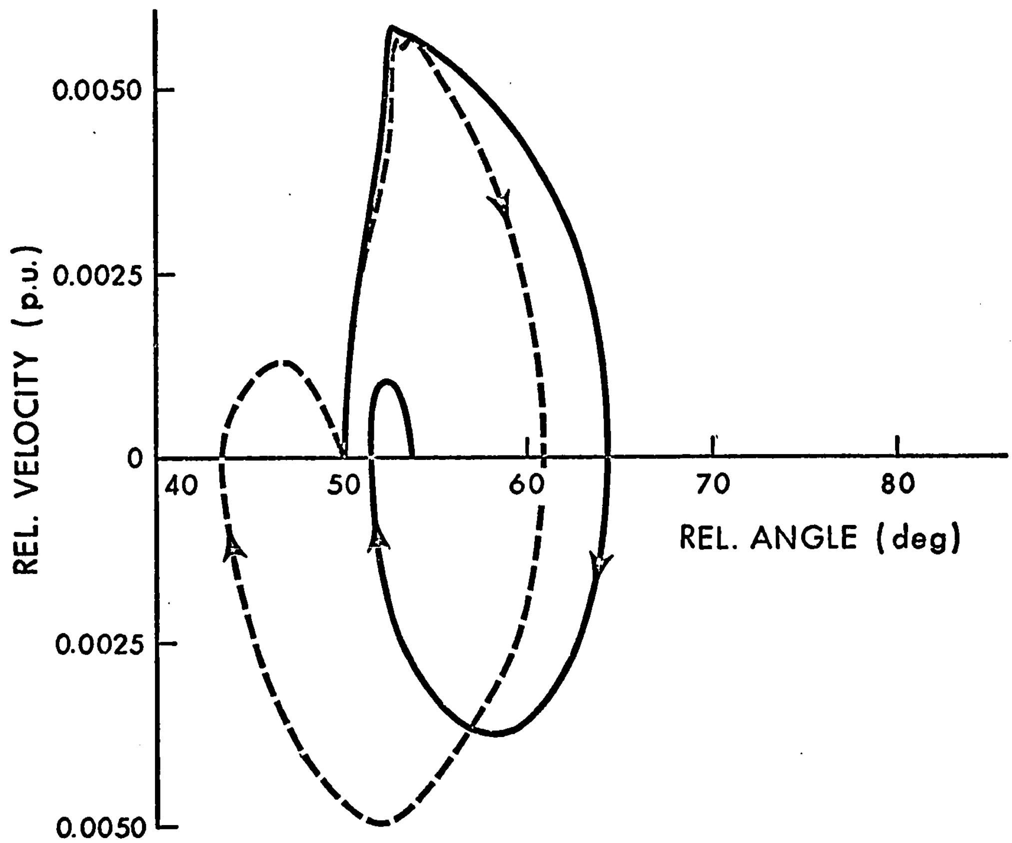

Fig. 5.10 Relative angle vs velocity plot corresponding to Fig. 5.9.

1. Bang bang control.

2. Proportional control.

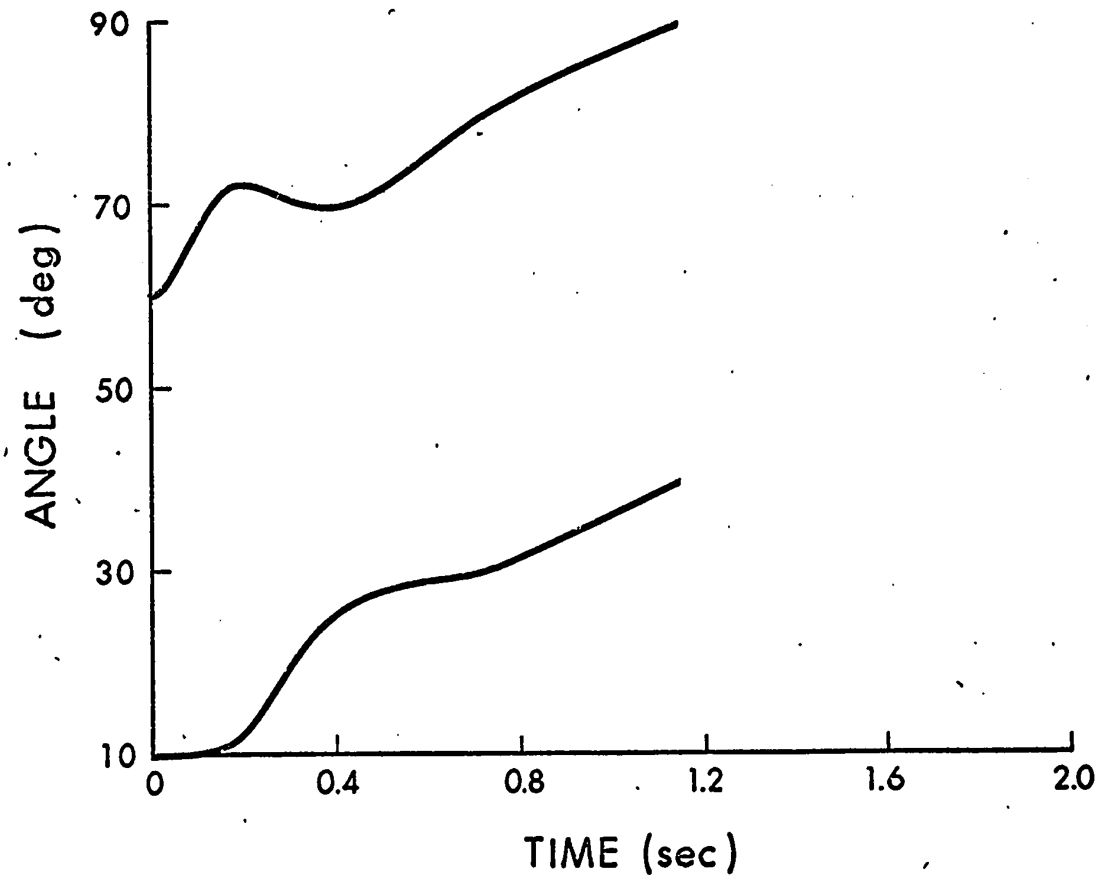

Fig. 5.11 Angle time characteristics with proportional control.

1. Machine I
2. Machine II

iv) Three phase short circuit on first machine (normal excitation).

In this case machine I is loaded about .85 per unit and machine II about .4 per unit. Figure 5.12 shows that bang bang control deteriorates the machine and bus voltages so much that when the system tends to stabilize after about .8 seconds, the maximum power delivered by the two machines(at these voltages)is much less than that demanded by the load, resulting in instability. Proportional control, however, retains the machine as well as bus voltage, so that the system stabilizes in about 1.6 seconds.

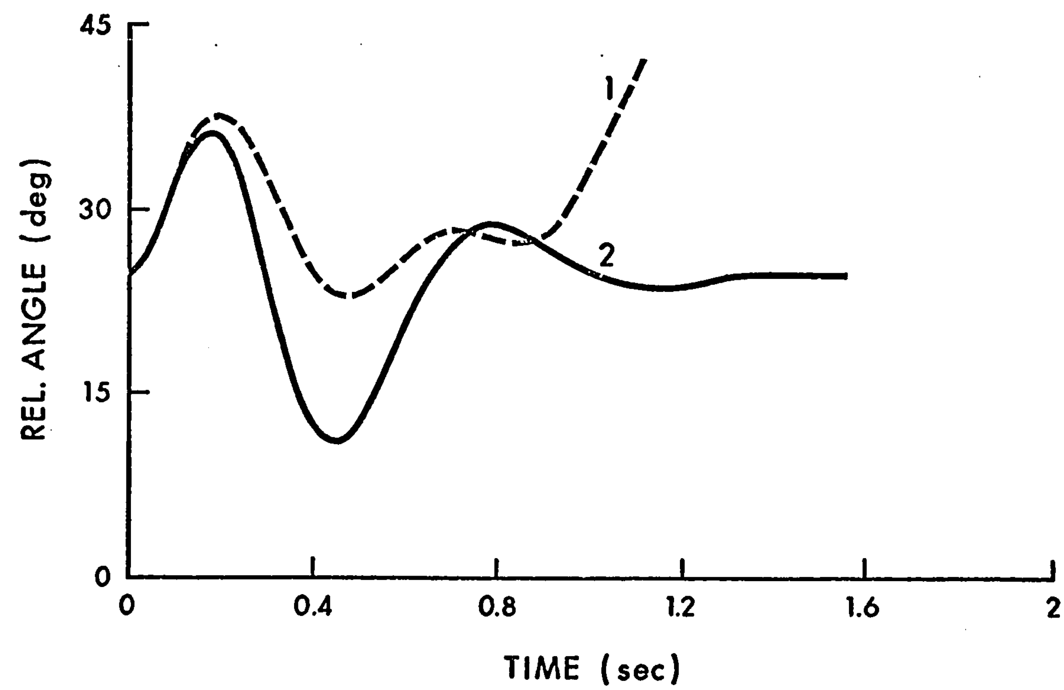

Fig. 5.12 Relative angle time characteristics for a 3 - ϕ fault on the normally excited machine.

1. Bang bang control.
2. Proportional control.

v) The effect of local control.

A 100% torque pulse for 3 cycles is again considered on the first machine. Figure 5.13 shows the relative angle vs time characteristics.

Curve 1 is with bang bang control on both the machines, whereas curve 2 is with control on the first machine only, second machine with normal field voltage. Curve 3 shows the response when control is applied on machine II only. It can be seen that applying control on the first machine only takes less time as compared to that when control is applied on both of them. This demonstrates that local disturbances are better taken care of by local control. In this case the local control also depends on the global variables. Figure 5.14 shows the phase plane plot corresponding to curves 2 and 3 in figure 5.13.

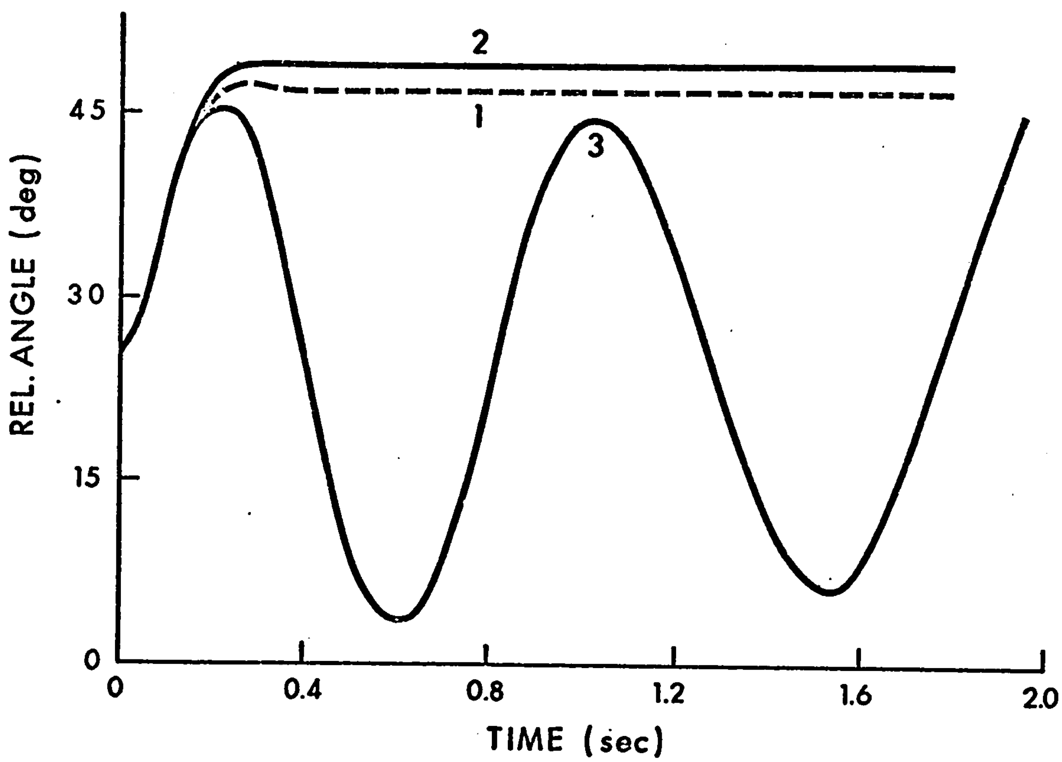

Fig. 5.13 Relative angle-time characteristics for 100% torque pulse.

1. Control on both the machines.
2. Control on Machine I only.
3. Control on Machine II only.

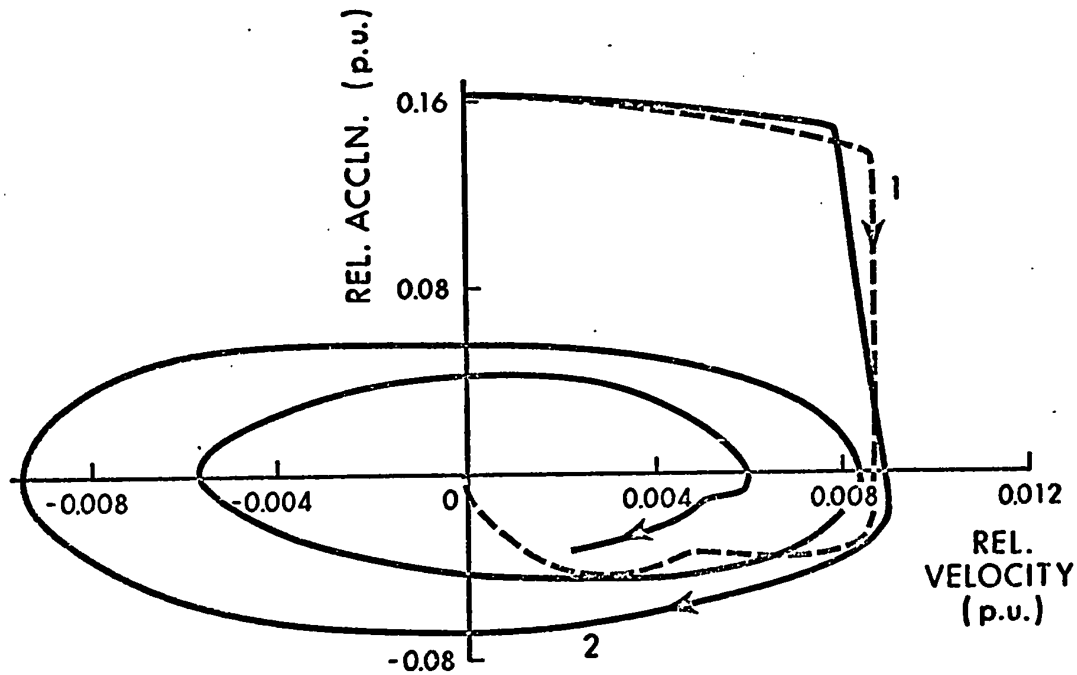

Fig. 5.14 Phase plane plot corresponding to Fig. 5.13.

1. Control on Machine I only.

2. Control on Machine II only.

APPENDIX A5-1

TWO MACHINE SYSTEM DATA

The two machines considered are identical and their parameters are given in section A3-3 of appendix A3. Two system conditions were considered - (1) both machines normally excited, and (2) machine I under excited.

The operating quantities for the under excited case is given as

M/C I		M/C II	
i_{fd}	1.42348	i_{fd}	1.77935
i_d	.231041	i_d	.0066098
i_q	.987381	i_q	.19621
δ	60°	δ	10°
e_t	.7696	e_t	.9983
P_o	.72976	P_o	.1957
Q_o	-.27684	Q_o	-.01021

$$v = 1$$

$$Z_L = .852 \angle -.712 \text{ rad.}$$

Operating quantities for the normally excited case are

M/C I		M/C II	
i_{fd}	2.3131	i_{fd}	2.224
i_d	.4464	i_d	.2407
i_q	.745	i_q	.3027
δ	40°	δ	15°
e_t	1.044	e_t	1.0927
P_o	.8841	P_o	.3599
Q_o	.2025	Q_o	.2215

$$v = 1$$

$$Z_L = .8246 \angle .0142 \text{ rad.}$$

CHAPTER 6

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

6.1 Summary and Conclusions

In this thesis, a quasi-optimal excitation control for power system stabilization is found directly as a function of state and parameters of the system. It is suited to online controls requiring relatively little hardware, either digital or analog, as it is not necessary to solve the system differential equations. Based on the closed loop scheme, two types of excitation control have been investigated:

i) Bang bang control scheme - This provides dead beat response for small disturbances (figures 3.5 and 3.6). However, for large disturbances there may be an uncontrollable period resulting in a sub-optimal solution. The exciter ceiling voltage, which limits the control signal, has a direct bearing on both the uncontrollable period and the speed of response during the controllable period. Increasing the exciter ceiling provides better damping of the transient (figure 3.5). If a bang bang control is used, some deadband must be provided to allow normal voltage regulator action to take place. This also can take care of any switching chatter.

ii) Proportional control - In the case of a large disturbance, the exciter is rapidly driven to its ceiling and the control action is almost bang bang. As the system settles down, the exciter voltage variations decrease and the effects of voltage regulator action become dominant. Dynamic stability is retained. Although the transient is longer with greater excursions in rotor angle, the transition to normal voltage regulator action is smooth and does not require additional control equipment. The demand on the excitation system is lower. The regulator time constants may be taken into account. The results obtained with these two types of control are presented in Chapters 3 and 5.

The results obtained with the closed loop quasi-optimal scheme are compared with those obtained by a steepest descent method. For small disturbances, the closed loop scheme is found to yield an optimal solution. It may be noted that although the steepest descent technique was used to evaluate the closed loop scheme, the amount of computation required for steepest descent methods precludes online control. This may give worse results than the closed loop scheme for other than the design disturbances. For the application of steepest descent methods, the closed loop scheme offers computational advantages in that the results provide a good set of initial switching times for the steepest descent algorithms, so that convergence is rapid. The comparison of closed loop scheme with the steepest descent method is given in Section 4.3 of Chapter 4.

The effect of instrumentation and related time delays is not as severe for predetermined control strategies as for the closed loop scheme. This is shown in Section 4.4 of Chapter 4. Since the sensing and measuring instruments as well as the static excitation systems have very small time constants, their effect is less appreciable. Even a relatively long initial time delay followed by a bang bang excitation strategy retains system stability.

For multimachine systems, the control is obtained based on the information from all machines in the system. For local disturbances, local control may be more effective, in which case the amount of computation is also minimized.

6.2 Suggestions for further research

Further areas of investigation include the extension of these methods to include governor and prime mover dynamics and consideration of damper windings. Tests on real machines should be performed to compare the results with computer simulation. Methods for determining L(t) and b(t) or reasonable approximation of these quantities with a minimum of computation and equipment should be investigated. The sensitivity of these quantities to parameter variations are also of concern.

Modern power systems are very complex and the possibility of monitoring the optimal control from a central computer should be explored. Study of local control involving the local variables only, needs concern.

Further investigation of local versus central control should be made as, if local control is satisfactory from a stability viewpoint, it would be superior in terms of cost, equipment and complexity and speed of control.

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