

**Comparison of the Prediction Accuracy of Isothermal Compressibility and Isobaric Thermal Expansivity by Different Volume-Translated Equations of State**

by

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## ABSTRACT

Isothermal compressibility,  $\kappa_T$ , and isobaric thermal expansivity,  $\alpha_P$ , are two important parameters that are used in the simulation and modeling of many petroleum and chemical processes.  $\kappa_T$  and  $\alpha_P$  can be calculated by empirical correlations or thermodynamic models, but these predicting methods are not sufficiently accurate. Cubic equations of state (CEOSs) are widely used in the petroleum and chemical industry to describe the phase behavior of fluids. It is recognized that one of the deficiencies of CEOSs is their inaccuracy in predicting liquid-phase volumes. The volume translation (VT) strategy was then proposed by researchers to overcome this deficiency, which could significantly improve the performance of CEOSs in predicting volumetric properties. With recent developments in the volume-translated equations of state (VT-EOSs),  $\kappa_T$  and  $\alpha_P$  should also be predicted more accurately by VT-EOSs. In this work,  $\kappa_T$  and  $\alpha_P$  of two example fluids (i.e., methane and carbon dioxide) are predicted by seven different VT models: one constant VT model, two linear temperature-dependent VT models, two exponential temperature-dependent VT models, and two temperature-pressure-dependent VT models (i.e., models based on a distance function). The accuracy of each model is evaluated by comparing the predictions with the pseudo-experimental data for the liquid phase, the vapor phase, and the supercritical phase. The predicted results show that the distance-function-based temperature-pressure-dependent VT models exhibit relatively better performance in predicting  $\kappa_T$  and  $\alpha_P$  than the temperature-dependent and the constant VT models. Overall, the VT-PR EOS model proposed by Abudour *et al.* (2012) provides the most

accurate predictions of  $\kappa_T$ , while the VT-SRK EOS model proposed by Chen and Li (2020) provides the most accurate predictions of  $\alpha_P$ .

## **DEDICATION**

This dissertation is dedicated to my dearest parents, Mr. Bing Guan and Mrs. Rong Li, and my girlfriend Minger Guo.

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# CHAPTER 1 INTRODUCTION

## 1.1. Research Background

Phase behavior of fluids encountered in the petroleum and chemical industry plays an important role in the efficiency of various industrial processes. The volume changes of a fluid at different temperature and pressure conditions are critical to the design and simulation of these industrial processes.<sup>1,2</sup> Isothermal compressibility ( $\kappa_T$ ) and isobaric thermal expansivity ( $\alpha_P$ ) are two important properties that describe the volumetric behavior of a given fluid in response to the changes in pressure and temperature, respectively. For instance, the knowledge of  $\kappa_T$  is of importance in reservoir engineering since it is a useful parameter that can be used to identify the reservoir fluid type.<sup>3</sup> Reservoir engineers also need an accurate estimate on  $\kappa_T$  of reservoir fluids as  $\kappa_T$  is a required input parameter in the governing equations of fluid flow within reservoirs.<sup>4-6</sup>  $\alpha_P$  is a very useful parameter that finds applications in many chemical and industrial processes involving heat transfer, thermal processing of materials, and the design of high-pressure injection equipment.<sup>2,7</sup>  $\kappa_T$  and  $\alpha_P$  also play critical roles in the steady-state and transient simulations of fluid flow in pipes since they are used in empirical correlations that relate *in-situ* flow rates to those at standard conditions.<sup>8</sup>

$\kappa_T$  can be defined as:<sup>9</sup>

$$\kappa_T = -\frac{1}{v} \left( \frac{\partial v}{\partial P} \right)_T \quad (1)$$

$\alpha_P$ , also called the coefficient of thermal expansion, can be defined as:<sup>9</sup>

$$\alpha_P = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_P \quad (2)$$

where  $v$ ,  $P$ , and  $T$  are molar volume, pressure, and temperature, respectively.  $\kappa_T$  describes the change in the molar volume of a fluid due to a change in pressure at a constant temperature, while  $\alpha_P$  represents the change in the molar volume of a fluid caused by a temperature change at a constant pressure.

## 1.2. Literature Review of the Determination of $\kappa_T$ and $\alpha_P$ and the Volume-Translated EOS

### 1.2.1. Determination of $\kappa_T$ and $\alpha_P$

There are mainly two methods to determine  $\kappa_T$  and  $\alpha_P$ , namely, the experimental methods and the theoretical methods. Experimentally, robust laboratory tests are relied on to acquire pressure-volume-temperature (PVT) data, and then the acquired PVT data are used to obtain reliable and accurate results of  $\kappa_T$  and  $\alpha_P$ .<sup>6</sup> Some correlations have been proposed to estimate  $\kappa_T$  and  $\alpha_P$ . Trusler and Lemmon<sup>10</sup> proposed empirical correlations for  $\kappa_T$  and  $\alpha_P$  based on the measurements of speed of sound.<sup>11</sup> Baonza *et al.*<sup>12</sup> proposed a method to extrapolate  $\kappa_T$  and  $\alpha_P$  for mesitylene based on the measured molar densities. However, the correlations based on speed of sound and the extrapolation method are not accurate enough.<sup>12,13</sup> Cerdeiriña *et al.*<sup>14</sup> adopted a fitting equation to fit the measured density data with temperature and then analytically differentiated the equation with respect to temperature to calculate  $\alpha_P$ . However, fitting equations obtained by different fitting strategies could result in different equations, thus leading to different temperature dependences of  $\alpha_P$  for the same density data.<sup>14</sup>

### 1.2.2. Volume-Translated EOS

Accurate  $\kappa_T$  and  $\alpha_P$  can be calculated based on accurate PVT relations.<sup>15</sup> Cubic equations of state (CEOSs) have been widely used to describe PVT relations. Peng-Robinson (PR) EOS<sup>16</sup> and Soave-Redlich-Kwong (SRK) EOS<sup>17</sup> are the most used CEOSs, due to their simplicity and reliability.<sup>18,19</sup> Although PR EOS and SRK EOS are capable of calculating the molar volumes of various substances over different temperature and pressure ranges, it is recognized that one of their deficiencies is the inaccurate liquid-phase volume prediction.<sup>20</sup> To overcome this shortcoming, Martin<sup>21</sup> first proposed the concept of volume translation (VT) for CEOSs in 1979. In this method, a volume translation is applied in the volume-pressure diagram to make the isotherms shift along the volume axis without leading to any change in the vapor-liquid equilibrium calculations.<sup>22</sup> Many VT models have been proposed for PR EOS and SRK EOS, leading to the so-called VT-PR EOSs and VT-SRK EOSs, respectively.

VT-EOS models can be generally divided into three categories: constant volume translations, temperature-dependent volume translations, and volume translations that are temperature-pressure-dependent.<sup>23</sup> In the constant volume translation models, the volume translation term is not a function of temperature, but a constant corresponding to the difference between the volume predicted by an EOS and the measured saturated liquid volume at a given reduced temperature ( $T_r$ ). In 1982, P eneloux *et al.*<sup>22</sup> applied the constant volume translations acquired at  $T_r = 0.7$  to SRK EOS, which significantly improved the predictions of liquid density at low reduced temperatures. Based on the CEOS models updated by Le Guennec *et al.*<sup>24</sup>, Pina-Martinez *et al.*<sup>25</sup> updated the volume translation constants for more than 1000 pure substances based on the pseudo-experimental saturated

liquid molar volumes at  $T_r = 0.8$  as reported by the DIPPR database. However, a larger volume translation is required as the temperature gets closer to the critical point, and the largest volume translation is required at the critical point.<sup>26</sup>

Temperature-dependent VT models can be in a linear form or an exponential form.<sup>27-35</sup>

Ungerer and Batut<sup>34</sup> proposed a volume translation for PR EOS that has a linear relationship with temperature and molecular weight. Baled *et al.*<sup>35</sup> developed a linear temperature-dependent volume translation model for both PR EOS and SRK EOS to correct density predictions at high-temperature and high-pressure conditions, which provides better density predictions in the single-phase region than in the saturation region.<sup>23</sup> An exponential temperature-dependent VT model developed by Magoulas and Tassios<sup>29</sup> was applied to PR EOS for alkanes. However, temperature-dependent volume translations might cause the crossover of PV isotherms, implying that a lower molar volume is predicted by the VT model at a higher temperature under an isobaric condition.<sup>36</sup> The Ungerer and Batut model<sup>34</sup> and the Magoulas and Tassios model<sup>29</sup> were reported to induce the crossover of PV isotherms at relatively low pressures.<sup>35-38</sup> This crossover phenomenon is inconsistent with the basic assumptions of thermodynamics and therefore limits the application of these temperature-dependent VT models.<sup>36</sup> To address such an issue, Shi and Li<sup>36</sup> developed a criterion to determine when the crossover phenomenon of PV isotherms appears and subsequently proposed a VT-PR EOS to avoid the crossover phenomenon within a large pressure range.<sup>32</sup>

Since the required volume translation increases as the critical point is approached, constant volume translations and temperature-dependent volume translations have relatively poor performance.<sup>23</sup> Based on the fact that the required corrections for the single-phase liquid

densities are not only temperature-dependent but also volume-dependent,<sup>26</sup> Chou and Prausnitz<sup>39</sup> and Mathias *et al.*<sup>40</sup> introduced a volume-dependent distance function ( $d$ ) and proposed temperature-pressure-dependent VT models. Based on this method, improved VT-PR EOS and VT-SRK EOS were proposed by Abudour *et al.*<sup>23</sup> and Frey *et al.*,<sup>41,42</sup> respectively. The Frey *et al.* model<sup>41,42</sup> only showed modest improvement on density predictions, while the Abudour *et al.* model<sup>23</sup> was reported to be the best performed VT model for saturated liquid and compressed liquid density predictions.<sup>43</sup> In 2020, Chen and Li<sup>26</sup> developed a distance-function-based VT-SRK EOS, obtaining slightly better performance than the Abudour *et al.* model<sup>23</sup> in predicting the molar volumes of the saturated and single-phase liquids for 56 substances.

Because of the significant improvement made towards the volume translation models in CEOSs, some authors have attempted to use CEOSs to predict thermodynamic properties that require volumetric data.<sup>1,6,44-46</sup> Avasthi and Kennedy<sup>1</sup> developed a method to predict  $\kappa_T$  and  $\alpha_P$  for hydrocarbons by differentiating CEOSs. Trujillo *et al.*<sup>45</sup> applied PR EOS to computational fluid dynamics simulations to predict thermodynamic relationships including PVT behavior and other thermodynamic derivative properties. Adepoju *et al.*<sup>44</sup> proposed a mathematical method to predict  $\kappa_T$  for oil samples based on PR EOS. Regueira *et al.*<sup>46</sup> reported that the volume translation versions of PR EOS and SRK EOS yielded better performance than the original EOSs in predicting  $\kappa_T$  for high-pressure and high-temperature reservoir fluids. Burgess *et al.*<sup>6</sup> extended the VT-SRK EOS of Baled *et al.*<sup>35</sup> to conditions up to 533 K and 276 MPa. But they found that the predictions of  $\kappa_T$  could deviate from the experimental data by over 50%, and these predictions were inaccurate over the entire temperature and pressure ranges.<sup>6</sup>



Being designed to correct molar volume calculations, a VT-EOS can lead to a saturated molar volume different from the one by the untranslated EOS at a given temperature. As a result, some fluid properties can be affected by volume translation.<sup>37</sup> Jaubert *et al.* focused on the Péneloux-type VT models<sup>22</sup> and analytically studied the influence of the temperature-dependent and the temperature-independent VT models on several thermodynamic properties, including  $\kappa_T$  and  $\alpha_p$ .<sup>37</sup> They found that, for a pure substance, applying a temperature-independent VT model to a given EOS affects neither the product of molar volume and isothermal compressibility ( $v\kappa_T$ ) nor the product of molar volume and isobaric thermal expansivity ( $v\alpha_p$ ), while applying a temperature-dependent VT model affects the value of  $v\alpha_p$ , but leaves the value of  $v\kappa_T$  unchanged.<sup>37</sup>

### 1.3. Problem Statement

The experimental determination of  $\kappa_T$  and  $\alpha_p$  can be reliable but expensive and time-consuming, and they are hardly conducted at *in-situ* conditions.<sup>6</sup> Since the existing theoretical methods for calculating  $\kappa_T$  and  $\alpha_p$  are not sufficiently accurate,<sup>12,13</sup> there is still a need to further improve the prediction accuracy of  $\kappa_T$  and  $\alpha_p$ . As the predictions of molar volume can be improved by different kinds of VT-EOS models, predicting  $\kappa_T$  and  $\alpha_p$  based on the molar volumes obtained from a VT-EOS should also be improved and become more accurate over wide ranges of temperature and pressure. Due to the wide use of PR EOS and SRK EOS in the simulation of phase behavior in the petroleum and chemical industry, applying VT-PR EOS or VT-SRK EOS to predict the basic PVT properties as well as  $\kappa_T$  and  $\alpha_p$  becomes a natural and consistent choice. However, the prediction accuracy of different VT-EOSs for  $\kappa_T$  and  $\alpha_p$  has not been comprehensively investigated, and it is not known which VT model is the most accurate in predicting  $\kappa_T$

and  $\alpha_p$ . Therefore, the performance of different types of VT-EOSs in predicting  $\kappa_T$  and  $\alpha_p$  needs to be evaluated.

#### 1.4. Research Objectives

The main objective of this research is to determine the most accurate VT model for predicting  $\kappa_T$  and  $\alpha_p$ . The detailed objectives include the following:

- To develop a numerical procedure for calculating  $\kappa_T$  and  $\alpha_p$  using the temperature-pressure-dependent VT-EOS models that adopt the temperature-pressure-dependent distance function;
- To more accurately predict  $\kappa_T$  and  $\alpha_p$  for pure fluids by using different VT-EOS models, including one constant VT model, two linear temperature-dependent VT models, two exponential temperature-dependent VT models, and two temperature-pressure-dependent VT models; and
- To evaluate the prediction accuracy of  $\kappa_T$  and  $\alpha_p$  by different VT-EOS models and select the most accurate model for predicting  $\kappa_T$  and  $\alpha_p$  thereof.

#### 1.5. Thesis Structure

The thesis contains four chapters:

- **Chapter 1** introduces the research background, literature review, problem statement, research objectives, and thesis structure.
- **Chapter 2** presents the methodology employed in this thesis, including the untranslated EOSs and the VT models used for predicting  $\kappa_T$  and  $\alpha_p$ , and the

numerical procedure for calculating  $\kappa_T$  and  $\alpha_P$  by using the distance-function-based VT models.

- **Chapter 3** shows the sensitivity analysis on the numerical calculations of  $\kappa_T$  and  $\alpha_P$ , and evaluates the performance of seven representative VT-EOSs in predicting  $\kappa_T$  and  $\alpha_P$  for two example pure fluids, carbon dioxide (CO<sub>2</sub>) and methane (CH<sub>4</sub>), in different phase regions by comparing the predicted results with the pseudo-experimental data.
- **Chapter 4** summarizes the conclusions of this study and gives recommendations for future work.

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## CHAPTER 2 METHODOLOGY

The prediction accuracy of  $\kappa_T$  and  $\alpha_p$  by seven representative VT-EOS models is examined. These VT models include one constant VT for the SRK EOS updated by Pina-Martinez *et al.*,<sup>1</sup> two linear temperature-dependent VT-PR EOSs including the Ungerer and Batut model<sup>2</sup> and the Baled *et al.* model,<sup>3</sup> two exponential temperature-dependent VT-PR EOSs including the Magoulas and Tassios model<sup>4</sup> and the Shi *et al.* model,<sup>5</sup> and two temperature-pressure-dependent models including the VT-PR EOS proposed by Abudour *et al.*<sup>6</sup> and the VT-SRK EOS proposed by Chen and Li.<sup>7</sup>

### 2.1. PR EOS

PR EOS<sup>8</sup> is given as:

$$P^{PR} = \frac{RT}{v^{PR} - b^{PR}} - \frac{a^{PR}(T)}{v^{PR}(v^{PR} + b^{PR}) + b^{PR}(v^{PR} - b^{PR})} \quad (3)$$

$$a^{PR}(T) = \frac{0.457535R^2T_c^2}{P_c} \alpha^{PR}(T) \quad (4)$$

$$b^{PR} = \frac{0.077796RT_c}{P_c} \quad (5)$$

where  $P^{PR}$  and  $v^{PR}$  are pressure and molar volume in PR EOS, respectively,  $T$  is temperature,  $R$  is the gas constant,  $a^{PR}$  and  $b^{PR}$  are EOS parameters in PR EOS, which can be expressed in terms of critical temperature ( $T_c$ ) and critical pressure ( $P_c$ ). The  $\alpha^{PR}(T)$  term in Equation (4) is the dimensionless  $\alpha$ -function for PR EOS, which has been developed into various modifications to improve the property predictions for pure substances. The  $\alpha$ -function used in the original PR EOS can be written as:<sup>9</sup>

$$\alpha^{PR}(T) = [1 + m(1 - \sqrt{T_r})]^2 \quad (6)$$

$$m = 0.37464 + 1.54226\omega - 0.26992\omega^2 \quad (7)$$

where  $\omega$  is acentric factor and  $T_r$  is the reduced temperature ( $T_r = \frac{T}{T_c}$ ). Different versions of  $\alpha$ -function in the PR EOS are available in the literature.

## 2.2. SRK EOS

SRK EOS<sup>9</sup> is given as:

$$P^{SRK} = \frac{RT}{v^{SRK} - b^{SRK}} - \frac{a^{SRK}}{v^{SRK}(v^{SRK} + b^{SRK})} \quad (8)$$

$$a^{SRK}(T) = \frac{1}{9(\sqrt[3]{2}-1)} \frac{R^2 T_c^2}{P_c} \alpha^{SRK}(T) \quad (9)$$

$$b^{SRK} = \frac{\sqrt[3]{2}-1}{3} \frac{RT_c}{P_c} \quad (10)$$

where  $P^{SRK}$  and  $v^{SRK}$  are pressure and molar volume in SRK EOS, respectively,  $a^{SRK}$  and  $b^{SRK}$  are EOS parameters in SRK EOS, and  $\alpha^{SRK}(T)$  is the  $\alpha$ -function for SRK EOS. The  $\alpha^{SRK}(T)$  term used in the SRK EOS and the VT-SRK EOS models studied in this work is the  $\alpha$ -function proposed by Twu *et al.*<sup>10</sup>, which can be expressed as:

$$\alpha^{SRK}(T) = T_r^{N^{SRK}(M^{SRK}-1)} \exp[L^{SRK}(1 - T_r^{M^{SRK}N^{SRK}})] \quad (11)$$

where  $L^{SRK}$ ,  $M^{SRK}$ , and  $N^{SRK}$  are substance-dependent parameters. The values of  $L^{SRK}$ ,  $M^{SRK}$ , and  $N^{SRK}$  updated by Pina-Martinez<sup>11</sup> are used in this work.

### 2.3. Volume Translation Methods

A general form of the volume translation term in an EOS can be expressed as:<sup>12</sup>

$$v^{VT-EOS} = v^{EOS} - c \quad (12)$$

where  $v^{VT-EOS}$  and  $v^{EOS}$  are the corrected molar volume after volume translation and the untranslated molar volume calculated by an EOS, respectively, and  $c$  is the volume translation term.

#### 2.3.1. Constant Volume Translation Updated by Pina-Martinez *et al.*<sup>11</sup>

Pina-Martinez *et al.*<sup>11</sup> updated the value of the volume translation term based on the pseudo-experimental saturated liquid volume at  $T_r = 0.8$  and optimized the Twu  $\alpha$ -function for SRK EOS with three parameters,  $L^{SRK}$ ,  $M^{SRK}$ , and  $N^{SRK}$ , as shown in Equation (11). The volume translation in this model is a constant for a given substance, which is given by:

$$c = v_L^{SRK} - v_L^{exp} \quad (13)$$

where  $v_L^{SRK}$  is the molar volume calculated from the untranslated SRK EOS and  $v_L^{exp}$  is the pseudo-experimental saturated liquid volume obtained from the DIPPR database.<sup>11</sup>

#### 2.3.2. VT-PR EOS Proposed by Ungerer and Batut<sup>2</sup>

Ungerer and Batut<sup>2</sup> proposed a VT model for PR EOS for paraffin, naphthenic, and aromatic hydrocarbons at temperatures under 200°C and pressures below 120 MPa. The volume translation term is linearly dependent on temperature, as given by:<sup>2</sup>

$$c = -34.5 + 0.46666MW + (0.023 - 0.00056MW)T \quad (14)$$

where  $MW$  is molecular weight. The  $\alpha$ -function used in this model is calculated by Equations (6) and (7).

### 2.3.3. VT-PR EOS Proposed by Baled *et al.*<sup>3</sup>

Baled *et al.*<sup>3</sup> proposed a VT model for PR EOS to predict the properties of reservoir fluids under high-temperature and high-pressure conditions. The volume translation term in this model is linearly temperature-dependent and is given by:<sup>3</sup>

$$c = -(A_b + B_b \frac{T}{T_c}) \quad (15)$$

$$A_b, B_b = f(MW, \omega) = k_{b0} + k_{b1} \exp\left(\frac{-1}{k_{b2}MW\omega}\right) + k_{b3} \exp\left(\frac{-1}{k_{b4}MW\omega}\right) + k_{b5} \exp\left(\frac{-1}{k_{b6}MW\omega}\right) \quad (16)$$

where  $A_b$  and  $B_b$  are model parameters that depend on molecular weight and acentric factor;  $k_{b0}$  through  $k_{b6}$  are model parameters used to determine  $A_b$  and  $B_b$ . For determining  $A_b$ ,  $k_{b0}$  to  $k_{b6}$  should have the values of  $-4.1034$ ,  $31.723$ ,  $0.0531$ ,  $188.68$ ,  $0.0057$ ,  $20.196$ , and  $0.0003$ , respectively; for determining  $B_b$ ,  $k_{b0}$  to  $k_{b6}$  should have the values of  $-0.3489$ ,  $-28.547$ ,  $0.0687$ ,  $-817.73$ ,  $0.0007$ ,  $-65.067$ , and  $0.0076$ , respectively.<sup>3</sup> The  $\alpha$ -function calculated by Equations (6) and (7) is also used in this model.

### 2.3.4. VT-PR EOS Proposed by Magoulas and Tassios<sup>4</sup>

Magoulas and Tassios<sup>4</sup> developed an exponential temperature-dependent volume translation for PR EOS for  $n$ -alkanes with carbon numbers between 1 and 20:

$$c = - \left[ c_{m0} + (\delta_c - c_{m0}) \exp \left( \beta \left| 1 - \frac{T}{T_c} \right| \right) \right] \quad (17)$$

$$c_{m0} = \frac{RT_c}{P_c} (k_{m0} + k_{m1}\omega + k_{m2}\omega^2 + k_{m3}\omega^3 + k_{m4}\omega^4) \quad (18)$$

$$\beta = l_{m0} + l_{m1}\omega^2 \quad (19)$$

$$\delta_c^{PR} = \frac{RT_c}{P_c} (Z_c^{PR} - Z_c^{exp}) \quad (20)$$

$$Z_c^{exp} = 0.289 - 0.0701\omega - 0.0207\omega^2 \quad (21)$$

where  $k_{m0}$  to  $k_{m4}$ ,  $l_{m0}$  and  $l_{m1}$  are model parameters. The values of  $k_{m0}$  to  $k_{m4}$  are  $-0.014471$ ,  $0.067498$ ,  $-0.084852$ ,  $0.067298$ , and  $-0.017366$ , respectively. The values of  $l_{m0}$  and  $l_{m1}$  are  $-10.2447$  and  $-28.6312$ , respectively.  $\delta_c^{PR}$  is the volume correction at the critical temperature in PR EOS,  $Z_c^{PR}$  is the critical compressibility factor in PR EOS with a universal value of  $0.3074$ , and  $Z_c^{exp}$  is the experimental critical compressibility factor that can be estimated by Equation (21). Magoulas and Tassios<sup>4</sup> suggested using Equation (6) to calculate the  $\alpha$ -function with the  $m$  term calculated by Equation (22) instead of Equation (7):

$$m = d_{m0} + d_{m1}\omega + d_{m2}\omega^2 + d_{m3}\omega^3 + d_{m4}\omega^4 \quad (22)$$

where  $d_{m0}$  to  $d_{m4}$  are model parameters with the values of  $0.384401$ ,  $1.52276$ ,  $-0.213808$ ,  $0.034616$ , and  $-0.001976$ , respectively.<sup>4</sup>

### 2.3.5. VT-PR EOS Proposed by Shi *et al.*<sup>5</sup>

Shi *et al.*<sup>5</sup> developed an exponential temperature-dependent VT model for PR EOS with the constraint proposed by Shi and Li.<sup>13</sup> The constraint was applied to avoid the PV

isotherm crossover phenomenon at temperatures up to 1000 K and pressures up to 100 MPa. The expression is given as:<sup>5</sup>

$$c = \frac{z_c^{exp} RT_c}{P_c} \left\{ A_s \exp \left[ -\frac{\left(\frac{T}{T_c} - 1\right)^2}{2B_s^2} \right] + C_s \right\} \quad (23)$$

where  $A_s$ ,  $B_s$ , and  $C_s$  are substance-dependent model parameters. The  $\alpha$ -function revised by Le Guennec *et al.*<sup>14</sup> is applied together with this VT model:

$$\alpha^{PR}(T) = \left(\frac{T}{T_c}\right)^{2(M_s-1)} \exp \left\{ L_s \left[ 1 - \left(\frac{T}{T_c}\right)^{2M_s} \right] \right\} \quad (24)$$

$$M_s = 0.1760\omega^2 - 0.2600\omega + 0.8884 \quad (25)$$

$$L_s = 0.1290\omega^2 + 0.6039\omega + 0.0877 \quad (26)$$

### 2.3.6. VT-PR EOS Proposed by Abudour *et al.*<sup>6</sup>

The VT model proposed by Abudour *et al.*<sup>6</sup> is presented as below:

$$c = - \left[ c_{a0} - \delta_c^{PR} \left( \frac{0.35}{0.35 + d^{PR}} \right) \right] \quad (27)$$

$$c_{a0} = \frac{RT_c}{P_c} [c_{a1} - (0.004 + c_{a1}) \exp(-2d^{PR})] \quad (28)$$

$$d^{PR} = \frac{1}{RT_c} \left( \frac{\partial P^{PR}}{\partial \rho} \right)_T = - \frac{v^{PR^2}}{RT_c} \left( \frac{\partial P^{PR}}{\partial v^{PR}} \right)_T = \frac{v^{PR^2}}{RT_c} \left[ \frac{RT}{(v^{PR} - b^{PR})^2} - \frac{2a^{PR}(v^{PR} + b^{PR})}{(v^{PR^2} + 2v^{PR}b^{PR} - b^{PR^2})^2} \right] \quad (29)$$

where  $c_{a1}$  is a substance-dependent parameter,  $d^{PR}$  is the dimensionless distance function in PR EOS, 0.35 is a universal constant for all substances, and  $\rho$  is molar density. To avoid iterative solutions,  $d^{PR}$  is calculated from the untranslated PR EOS. The

distance function represents the distance between the critical point and the point of interest on the PV isotherm.<sup>6</sup> In this model, the  $\alpha$ -function developed by Gasem *et al.*<sup>15</sup> is used:

$$\alpha^{PR}(T) = \exp \left\{ \left( A_a + B_a \frac{T}{T_c} \right) \left[ 1 - \left( \frac{T}{T_c} \right)^{c_a + D_a \omega + E_a \omega^2} \right] \right\} \quad (30)$$

where  $A_a$  to  $E_a$  are model parameters with the values of 2.0, 0.836, 0.134, 0.508, and  $-0.0467$ , respectively.<sup>6</sup>

### 2.3.7. VT-SRK EOS Proposed by Chen and Li<sup>7</sup>

Chen and Li<sup>7</sup> developed a VT model with three substance-dependent parameters for SRK EOS by modifying the distance function:

$$c = c_{c1} \left( \frac{RT_c}{P_c} \right) + \delta_c^{SRK} \left( \frac{1}{c_{c2} + c_{c3} d^{SRK}} \right) \quad (31)$$

$$\delta_c^{SRK} = \frac{RT_c}{P_c} (Z_c^{SRK} - Z_c^{exp}) \quad (32)$$

where  $c_{c1}$ ,  $c_{c2}$ , and  $c_{c3}$  are substance-dependent parameters,  $\delta_c^{SRK}$  is the volume correction at the critical temperature in SRK EOS,  $Z_c^{SRK}$  is the critical compressibility factor in SRK EOS with a universal value of  $\frac{1}{3}$ , and  $d^{SRK}$  is the distance function calculated by the untranslated SRK EOS:

$$d^{SRK} = \frac{1}{RT_c} \left( \frac{\partial P^{SRK}}{\partial \rho} \right)_T = - \frac{v^{SRK^2}}{RT_c} \left( \frac{\partial P^{SRK}}{\partial v^{SRK}} \right)_T = \frac{v^{SRK^2} T}{T_c (v^{SRK} - b^{SRK})^2} - \frac{a^{SRK} (2v^{SRK} + b^{SRK})}{RT_c (v^{SRK} + b^{SRK})^2} \quad (33)$$

## 2.4. Numerical Procedure for Calculating $\kappa_T$ and $\alpha_P$ Using the Distance-Function-Based VT Models

Because the molar volume calculated with the distance-function-based VT models is a function of both temperature and pressure, analytical expressions of  $\kappa_T$  and  $\alpha_P$  cannot be obtained using the distance-function-based VT-EOS models proposed by Abudour *et al.*<sup>6</sup> and Chen and Li.<sup>7</sup> Therefore, a numerical method should be used for calculating  $\kappa_T$  and  $\alpha_P$  based on the distance-function-based VT models. The central finite difference method is the simplest way to numerically calculate the derivatives by approximating the partial derivative as the slope of the secant line of two adjacent points at the point of interest (i.e.,  $(T_i, P_i)$  in this case).<sup>16</sup> With temperature and pressure condition,  $(T_i, P_i)$ , at the point of interest as input information, the secant line slope used to calculate  $\kappa_T$  at this input point,  $\frac{\Delta v}{2\Delta P}$ , can be obtained from the molar volumes calculated at two adjacent points (i.e.,  $(T_i, P_i + \Delta P)$  and  $(T_i, P_i - \Delta P)$ ):

$$\frac{\Delta v}{2\Delta P} = \frac{v(T_i, P_i + \Delta P) - v(T_i, P_i - \Delta P)}{2\Delta P} \quad (34)$$

where  $\Delta v$  is the molar volume change and  $\Delta P$  is the pressure difference. Similarly, the secant line slope for calculating  $\alpha_P$  at  $(T_i, P_i)$ ,  $\frac{\Delta v}{2\Delta T}$ , can be obtained from the molar volumes at the adjacent points (i.e.,  $(T_i + \Delta T, P_i)$  and  $(T_i - \Delta T, P_i)$ ):

$$\frac{\Delta v}{2\Delta T} = \frac{v(T_i + \Delta T, P_i) - v(T_i - \Delta T, P_i)}{2\Delta T} \quad (35)$$



where  $\Delta T$  is the temperature difference. The slope of the secant line converges to the slope of the tangent line at the point of interest (i.e.,  $(T_i, P_i)$ ) only when  $\Delta T$  or  $\Delta P$  approaches zero. Therefore, the following relations can be obtained:

$$\left(\frac{\partial v}{\partial P_i}\right)_{T_i} = \lim_{\Delta P \rightarrow 0} \frac{v(T_i, P_i + \Delta P) - v(T_i, P_i - \Delta P)}{2\Delta P} \quad (36)$$

$$\left(\frac{\partial v}{\partial T_i}\right)_{P_i} = \lim_{\Delta T \rightarrow 0} \frac{v(T_i + \Delta T, P_i) - v(T_i - \Delta T, P_i)}{2\Delta T} \quad (37)$$

The sensitivity of  $\Delta v$  to the values of  $\Delta T$  and  $\Delta P$  should be evaluated to determine the appropriate values of  $\Delta T$  and  $\Delta P$  to ensure that the calculations by Equations (36) and (37) are sufficiently accurate. To calculate  $\left(\frac{\partial v}{\partial P_i}\right)_{T_i}$  at a given temperature and pressure point,  $(T_i, P_i)$ , the molar volumes at  $(T_i, P_i + \Delta P)$ ,  $(T_i, P_i)$ , and  $(T_i, P_i - \Delta P)$  are obtained from a VT-EOS model with an initial value of  $\Delta P$ . The secant line slope at  $(T_i, P_i)$  is calculated by Equation (34). By gradually reducing  $\Delta P$ , the value of the secant line slope at each  $\Delta P$  can be evaluated. When  $\Delta P$  is reduced to a sufficiently small value and the slope of the secant line does not change significantly even if  $\Delta P$  is further reduced, the convergence of the secant line slope is considered to be reached. Consequently, this converged value of the secant line slope can be considered as the value of  $\left(\frac{\partial v}{\partial P_i}\right)_{T_i}$  at  $(T_i, P_i)$ . As such, the value of  $\kappa_T$  at  $(T_i, P_i)$  can be evaluated by using Equation (1). A similar approach should be implemented to determine  $\left(\frac{\partial v}{\partial T_i}\right)_{P_i}$  and  $\alpha_p$ . A sensitivity analysis is needed to determine appropriate  $\Delta T$  and  $\Delta P$  that can be used as the maximum allowable temperature and pressure differences to approximate the partial derivatives of molar volume.

CO<sub>2</sub> and CH<sub>4</sub> are not only greenhouse gases but also two fluids frequently encountered in petroleum engineering. CO<sub>2</sub> injection has been widely used in hydrocarbon reservoirs for enhancing oil recovery.<sup>17</sup> CH<sub>4</sub> is the main component of natural gas, and it can be used as an injection gas in enhanced oil recovery methods as well.<sup>18,19</sup> CO<sub>2</sub> can also be used to replace CH<sub>4</sub> in natural gas hydrates without causing damage to the gas hydrate reservoirs.<sup>20,21</sup> Therefore, CO<sub>2</sub> and CH<sub>4</sub> are selected as two pure example fluids to evaluate the prediction accuracy of  $\kappa_T$  and  $\alpha_P$  by different VT models.

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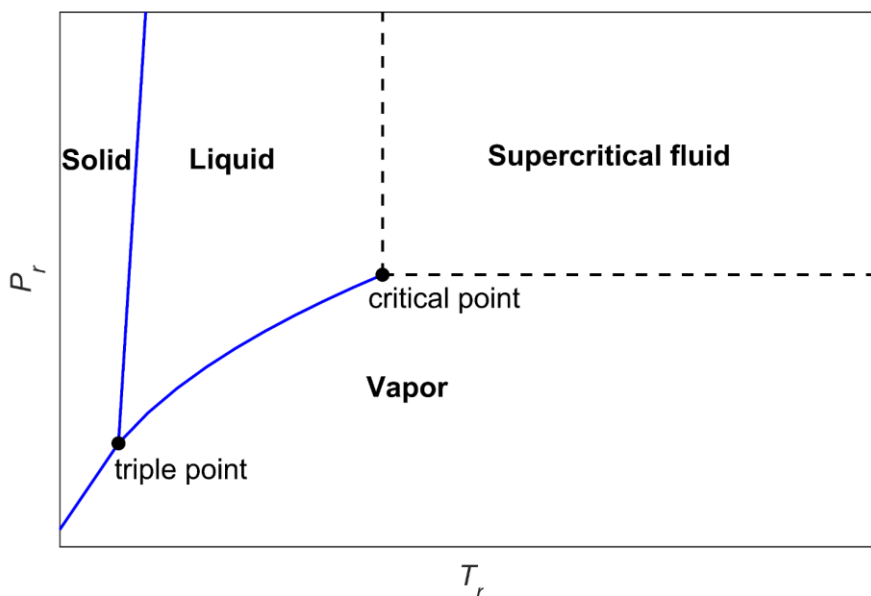
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## CHAPTER 3 RESULTS AND DISCUSSION

The predictions of  $\kappa_T$  and  $\alpha_P$  are conducted by the VT models studied in this work for pure CO<sub>2</sub> and CH<sub>4</sub>. Figure 1 shows the temperature and pressure ranges examined in this study. Since  $\kappa_T$  and  $\alpha_P$  can vary in different phase states,<sup>1,2</sup> three phase regions are considered, which are the liquid-phase region, the vapor-phase region, and the supercritical-phase region. The predictions are made between the triple point temperature ( $T_{triple}$ ) and  $T_r = 3$  for each substance. The pressure range starts from the reduced pressure ( $P_r = \frac{P}{P_c}$ ) of 0.1 to  $P_r = 3$ . Table 1 lists the properties of CO<sub>2</sub> and CH<sub>4</sub> (including critical temperature, critical pressure, critical compressibility factor, acentric factor, and molecular weight), the temperature and pressure ranges examined in this study as well as the number of the predicted data. Table 2 lists the substance-dependent parameters in the studied VT models that are retrieved from the literature.<sup>3-6</sup>



**Figure 1** Different regions in the phase diagram of a pure substance.

**Table 1** Physical properties of CO<sub>2</sub> and CH<sub>4</sub>.<sup>7</sup>

Substance	$T_c$ (K)	$P_c$ (MPa)	$\omega$	$Z_c$	$MW$ (g/mol)	Temperature range (K)	Pressure range (MPa)	Number of data points
CO <sub>2</sub>	304.128	7.3770	0.22394	0.27493	44.0095	217-912	0.7377-22.1310	20810
CH <sub>4</sub>	190.564	4.5992	0.01140	0.28640	16.0425	91-571	0.45992-13.7976	14371

**Table 2** Model parameters in the various VT-EOS models for CO<sub>2</sub> and CH<sub>4</sub>.

Substance	Parameters of the Shi <i>et al.</i> VT-PR EOS <sup>6</sup>			Parameters of the Abudour <i>et al.</i> VT-PR EOS <sup>3</sup>	Parameters of the Chen and Li VT-SRK EOS <sup>5</sup>			Constant VT updated by Pina-Martinez <i>et al.</i> <sup>4</sup>	Parameters of the $\alpha$ -function in the SRK EOS <sup>4</sup>		
	$A_s$	$B_s$	$C_s$	$c_{a1}$	$c_{c1}$	$c_{c2}$	$c_{c3}$	$c$ (cm <sup>3</sup> /mol)	$L^{SRK}$	$M^{SRK}$	$N^{SRK}$
CO <sub>2</sub>	0.0320	0.1053	-0.0173	0.00652	0.00608	0.92912	2.65917	4.1585	0.2806	0.8684	2.2782
CH <sub>4</sub>	0.0228	0.1288	-0.0429	0.01313	-0.00195	0.79540	2.13497	2.0509	0.2170	0.9082	1.8172

To evaluate the performance of different VT models, the predicted  $\kappa_T$  and  $\alpha_p$  are compared with the pseudo-experimental data provided by the National Institute of Standards and Technology (NIST) Web Thermo Tables (WTT) Standard Reference Database Version 2-2012-1-Pro.<sup>7</sup> The absolute average percentage deviation (%AAD) is used as a performance indicator:

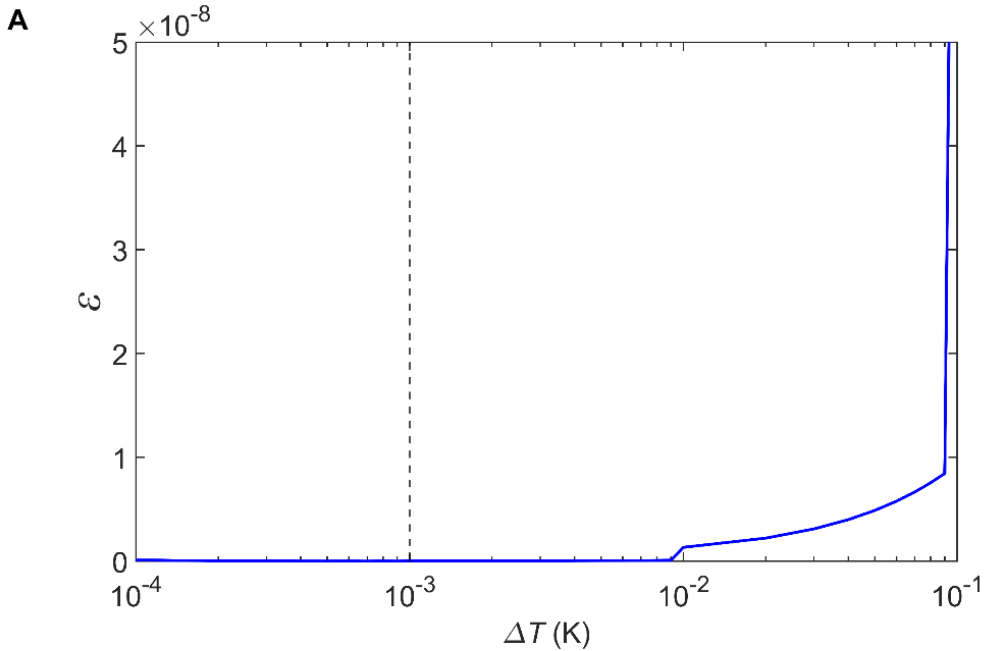
$$\%AAD = \frac{100}{N} \sum_{i=1}^N \left| \frac{Y^{EOS} - Y^{exp}}{Y^{exp}} \right|_i \quad (38)$$

where  $N$  is the number of data points,  $Y^{EOS}$  is the property calculated by the EOS model, and  $Y^{exp}$  is the pseudo-experimental value obtained from NIST.<sup>7</sup> In addition, the relative deviation (%RD) is used to demonstrate the model performance for the predicted  $\kappa_T$  along an isotherm and the predicted  $\alpha_p$  along an isobar:

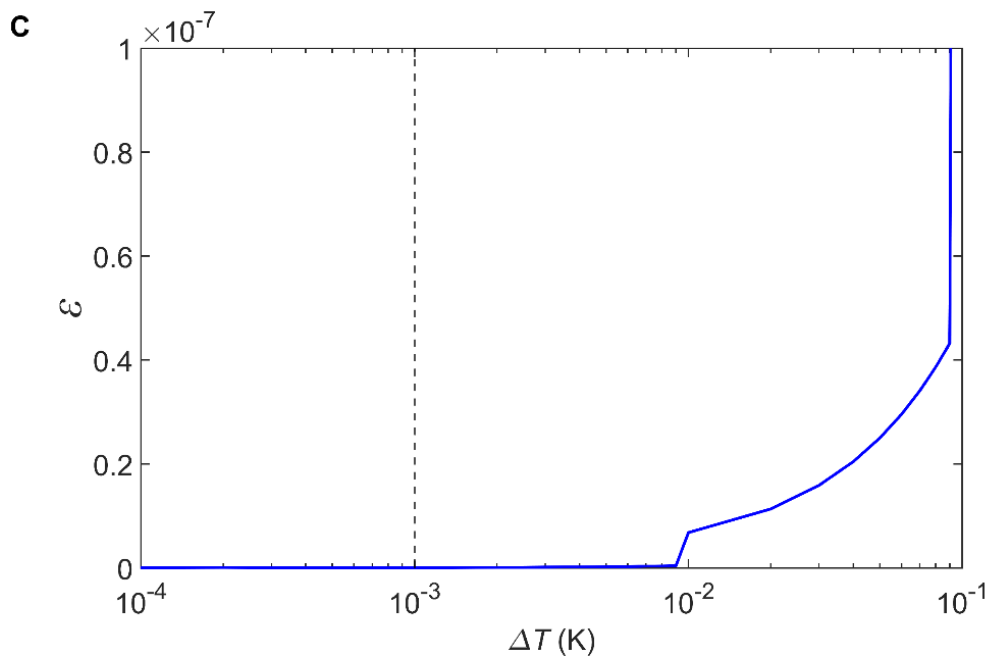
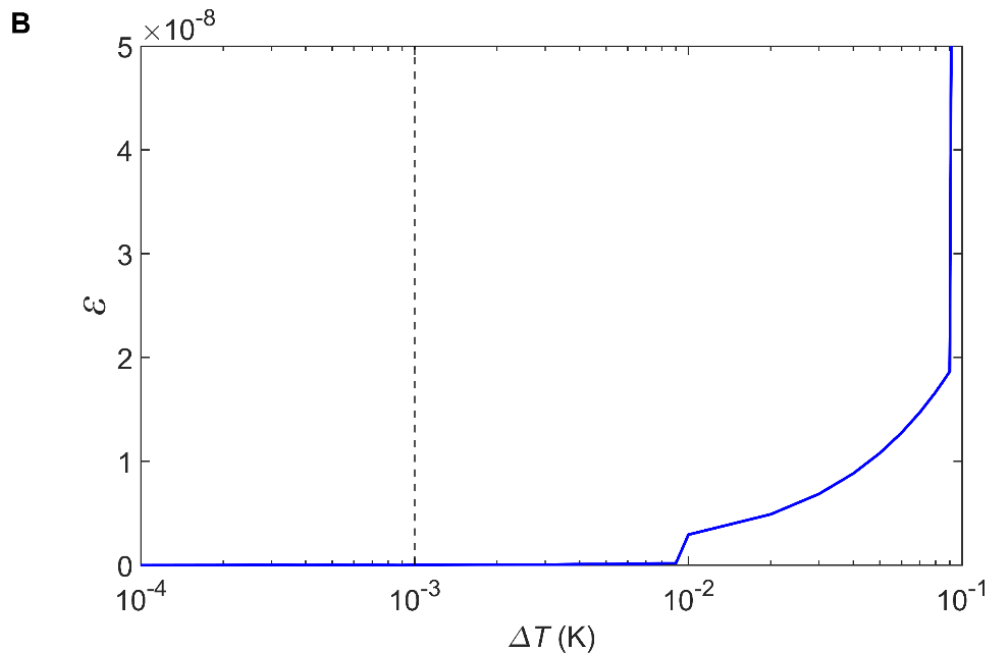
$$\%RD = 100 \times \left( \frac{Y^{EOS} - Y^{exp}}{Y^{exp}} \right) \quad (39)$$

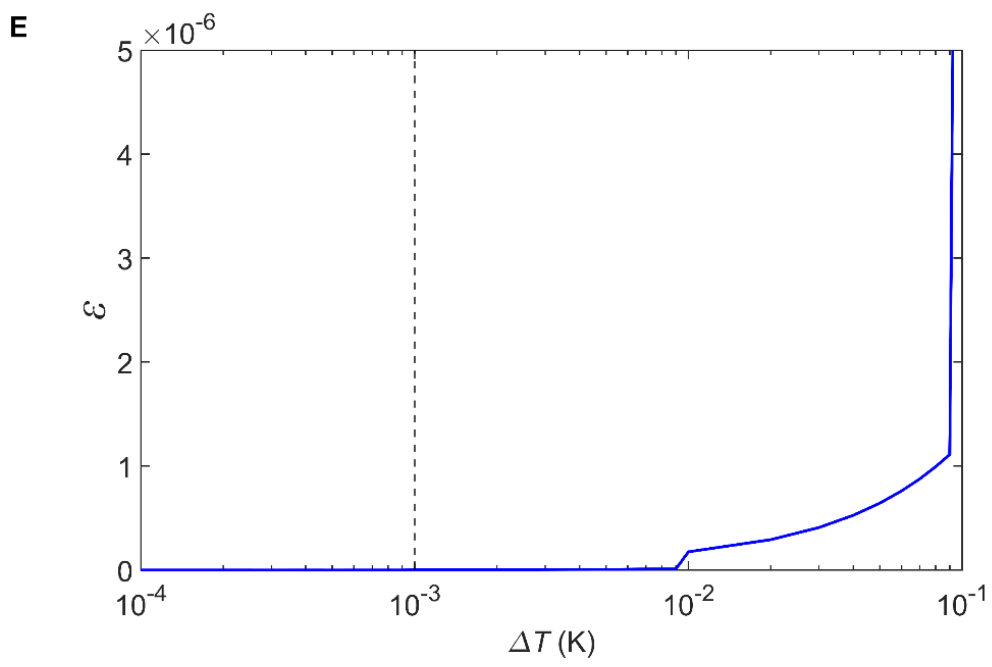
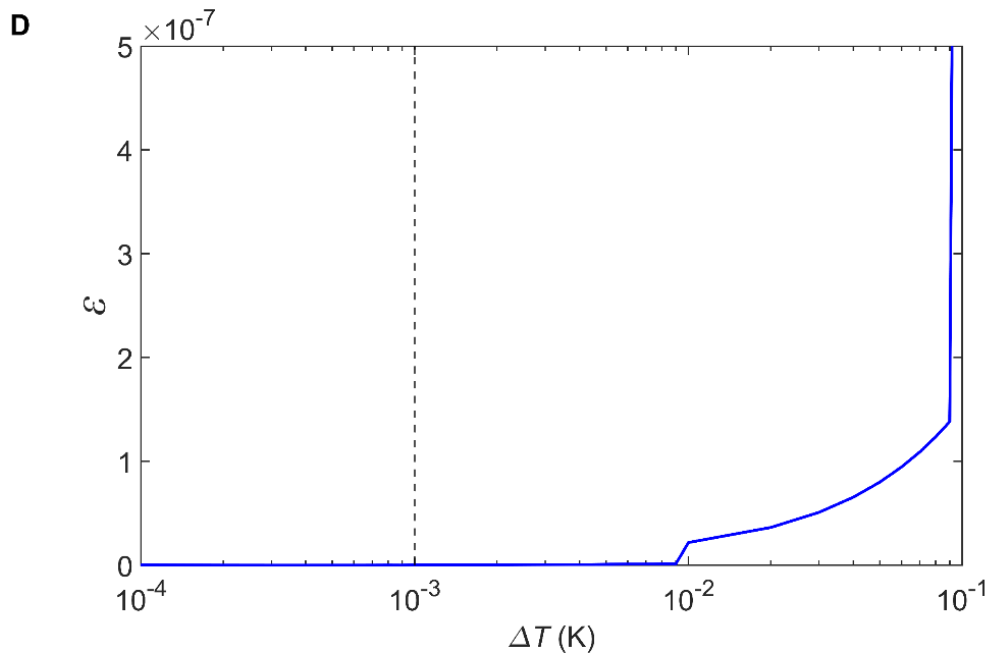
### 3.1. Sensitivity Analysis on the Numerical Calculations of $\kappa_T$ and $\alpha_P$

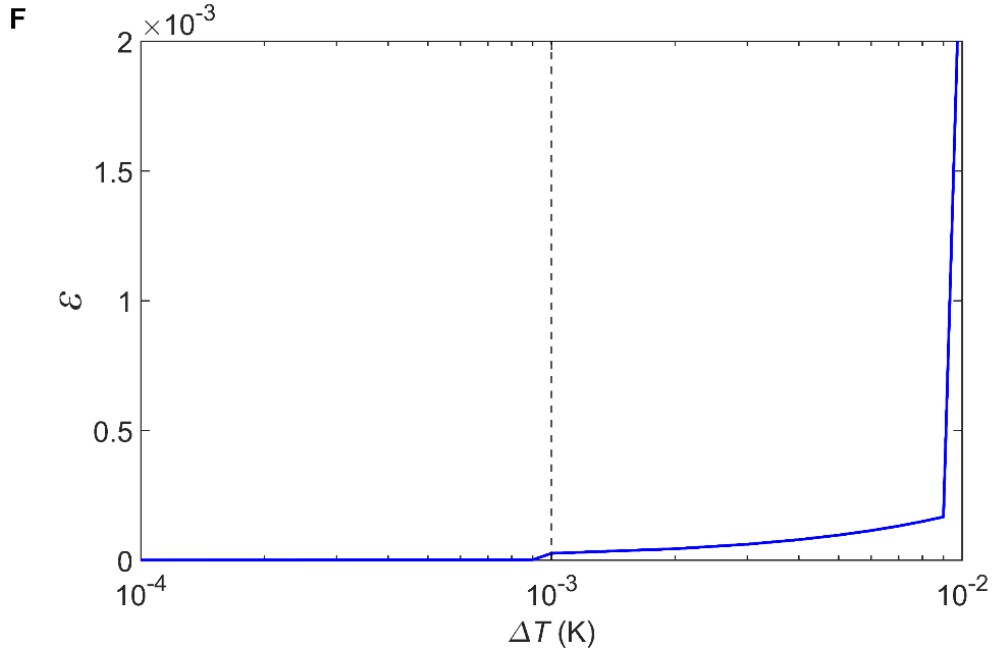
To determine appropriate  $\Delta T$  and  $\Delta P$  that can be used as the maximum allowable temperature and pressure differences to approximate the partial derivatives of molar volume, the sensitivity analysis on the numerical calculations of  $\kappa_T$  and  $\alpha_P$  is conducted based on the calculations for liquid  $\text{CH}_4$  by the untranslated SRK EOS<sup>8</sup>. Figure 2 shows the difference between  $\frac{\Delta v_i}{2\Delta T_i}$  and  $\frac{\Delta v_{i-1}}{2\Delta T_{i-1}}$  (i.e.,  $\varepsilon = \left| \frac{\Delta v_i}{2\Delta T_i} - \frac{\Delta v_{i-1}}{2\Delta T_{i-1}} \right| \text{ cm}^3 \text{ mol}^{-1} \text{ K}^{-1}$ ) calculated at  $P_r = 1$  and different temperatures with a decreasing  $\Delta T$ . It is obvious that the value of  $\frac{\Delta v}{2\Delta T}$  can converge to a constant when  $\Delta T$  is reduced to  $1 \times 10^{-3}$  K and below.





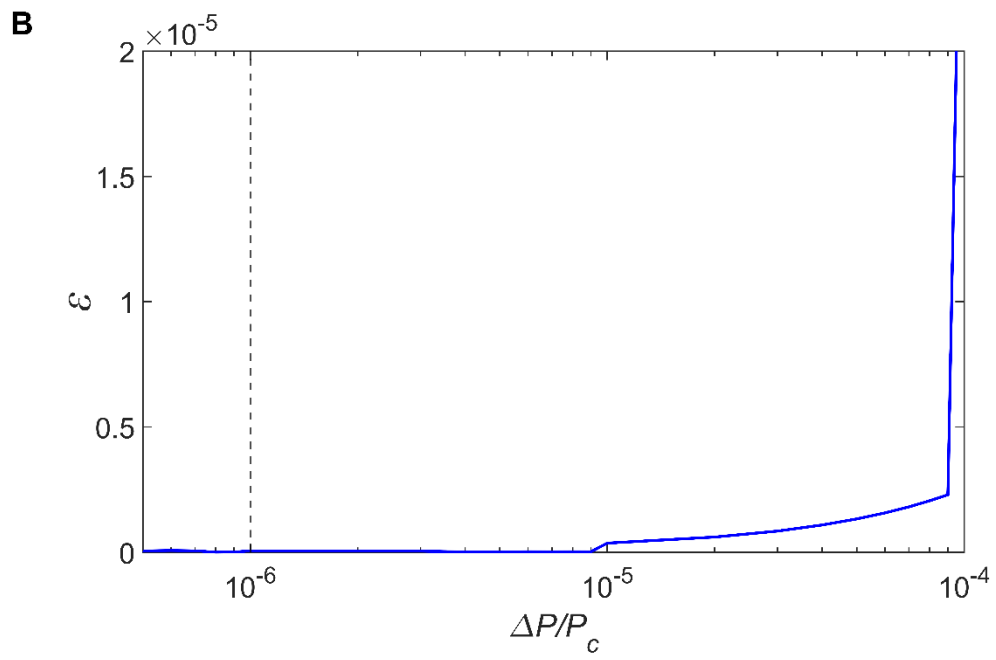
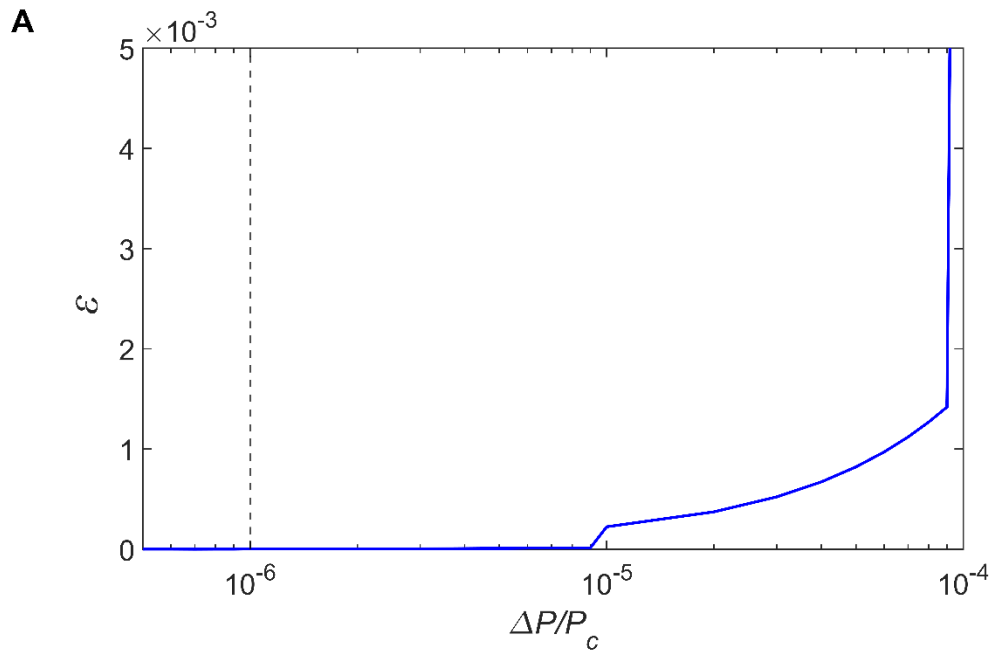


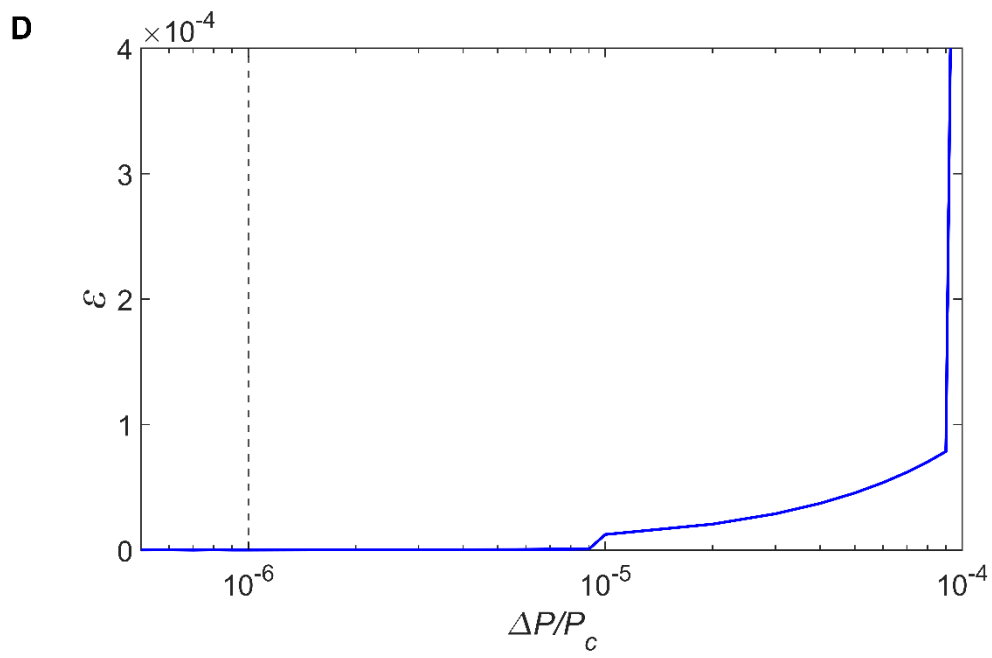
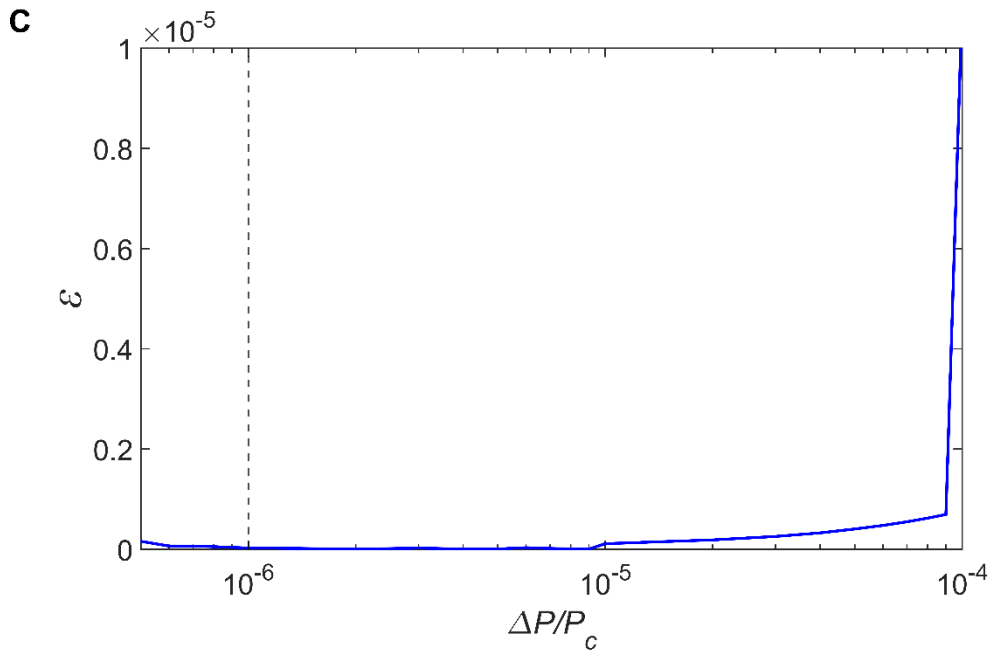


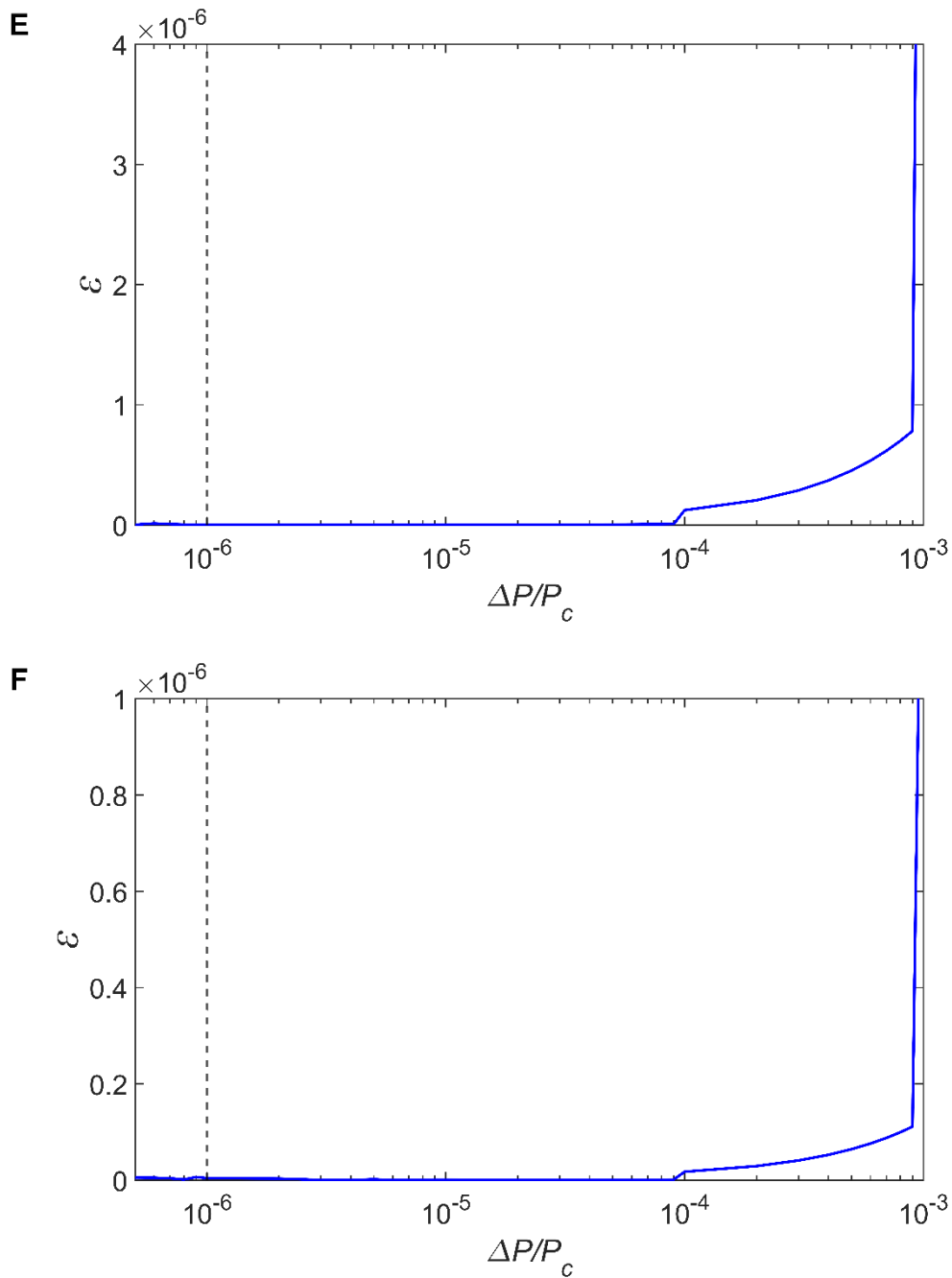


**Figure 2** The difference between  $\frac{\Delta v_i}{2\Delta T_i}$  and  $\frac{\Delta v_{i-1}}{2\Delta T_{i-1}}$  ( $\varepsilon = \left| \frac{\Delta v_i}{2\Delta T_i} - \frac{\Delta v_{i-1}}{2\Delta T_{i-1}} \right| \text{ cm}^3 \text{ mol}^{-1} \text{ K}^{-1}$ ) calculated by using the untranslated SRK EOS<sup>8</sup> for the liquid-phase CH<sub>4</sub> when the temperature difference ( $\Delta T$ ) is reduced. The calculations are done at  $P_r = 1$  and different temperatures: the triple point temperature (A),  $T_r = 0.6$  (B),  $T_r = 0.7$  (C),  $T_r = 0.8$  (D),  $T_r = 0.9$  (E), and  $T_r = 1$  (F).

Figure 3 shows the difference between  $\frac{\Delta v_i}{2\Delta P_i}$  and  $\frac{\Delta v_{i-1}}{2\Delta P_{i-1}}$  (i.e.,  $\varepsilon = \left| \frac{\Delta v_i}{2\Delta P_i} - \frac{\Delta v_{i-1}}{2\Delta P_{i-1}} \right| \text{ cm}^3 \text{ mol}^{-1} \text{ MPa}^{-1}$ ) at  $T_r = 1$  and different pressures when  $\Delta P$  is reduced. It can be seen from Figure 3 that the convergence of  $\frac{\Delta v}{2\Delta P}$  can be reached as  $\Delta P$  is reduced to  $1 \times 10^{-6} \times P_c \text{ MPa}$  and below. Therefore,  $\Delta P = 1 \times 10^{-6} \times P_c \text{ MPa}$  and  $\Delta T = 1 \times 10^{-3} \text{ K}$  are adopted to approximate the partial derivatives appearing in Equations (36) and (37), respectively.







**Figure 3** The difference between  $\frac{\Delta v_i}{2\Delta P_i}$  and  $\frac{\Delta v_{i-1}}{2\Delta P_{i-1}}$  ( $\varepsilon = \left| \frac{\Delta v_i}{2\Delta P_i} - \frac{\Delta v_{i-1}}{2\Delta P_{i-1}} \right| \text{ cm}^3 \text{ mol}^{-1} \text{ MPa}^{-1}$ ) calculated by using the untranslated SRK EOS<sup>8</sup> for the liquid-phase CH<sub>4</sub> when the pressure difference ( $\Delta P$ ) is reduced. The calculations are done at  $T_r = 1$  and different reduced pressures:  $P_r = 0.1$  (A),  $P_r = 0.5$  (B),  $P_r = 0.7$ , (C),  $P_r = 1$  (D),  $P_r = 1.5$  (E), and  $P_r = 2$  (F).

### 3.2. Predictions of $\kappa_T$ and $\alpha_P$

Table 3 lists the %AADs of the  $\kappa_T$  predictions yielded by each EOS model for the liquid-phase regions, the vapor-phase regions, and the supercritical-phase regions of CO<sub>2</sub> and CH<sub>4</sub>. The calculation results of  $\alpha_P$  are presented in Table 4. Tables 3 and 4 show that the VT-PR EOS proposed by Abudour *et al.*<sup>3</sup> yields the best overall results for  $\kappa_T$  predictions with a %AAD of 5.11%, while the VT-SRK EOS proposed by Chen and Li<sup>5</sup> yields the best overall results for  $\alpha_P$  predictions with a %AAD of 2.77%.

**Table 3** Comparison of the prediction accuracy of  $\kappa_T$  yielded by different VT-EOS models.

Substance	Phase region	%AADs yielded by PR EOS models						%AADs yielded by SRK EOS models		
		PR EOS <sup>9</sup>	Ungerer and Batut model <sup>10</sup>	Baled <i>et al.</i> <sup>11</sup> model	Magoulas and Tassios model <sup>12</sup>	Shi <i>et al.</i> <sup>6</sup> model	Abudour <i>et al.</i> model <sup>3</sup>	SRK EOS <sup>8</sup>	Chen and Li model <sup>5</sup>	Constant VT updated by Pina-Martinez <i>et al.</i> <sup>4</sup>
			Linear temperature-dependent	Exponential temperature-dependent	Temperature-pressure-dependent	Temperature-pressure-dependent	Constant VT			
CO <sub>2</sub>	Liquid	52.27	<b>14.20</b>	74.00	50.57	50.86	15.93	52.16	18.22	39.73
	Vapor	0.49	1.01	1.29	0.59	<b>0.35</b>	0.62	1.12	0.70	1.51
	Supercritical	2.21	5.64	4.90	2.04	2.66	<b>1.11</b>	5.31	2.67	6.21
CH <sub>4</sub>	Liquid	26.86	25.38	27.11	34.37	21.76	11.05	36.59	<b>6.81</b>	30.49
	Vapor	0.72	1.47	0.77	1.02	<b>0.32</b>	0.55	0.63	0.56	0.82
	Supercritical	2.84	7.30	2.97	3.52	3.09	<b>1.41</b>	4.42	2.26	4.85
Overall		14.23	9.17	18.51	15.35	13.17	<b>5.11</b>	16.71	5.20	13.94

**Table 4** Comparison of the prediction accuracy of  $\alpha_p$  yielded by different VT-EOS models.

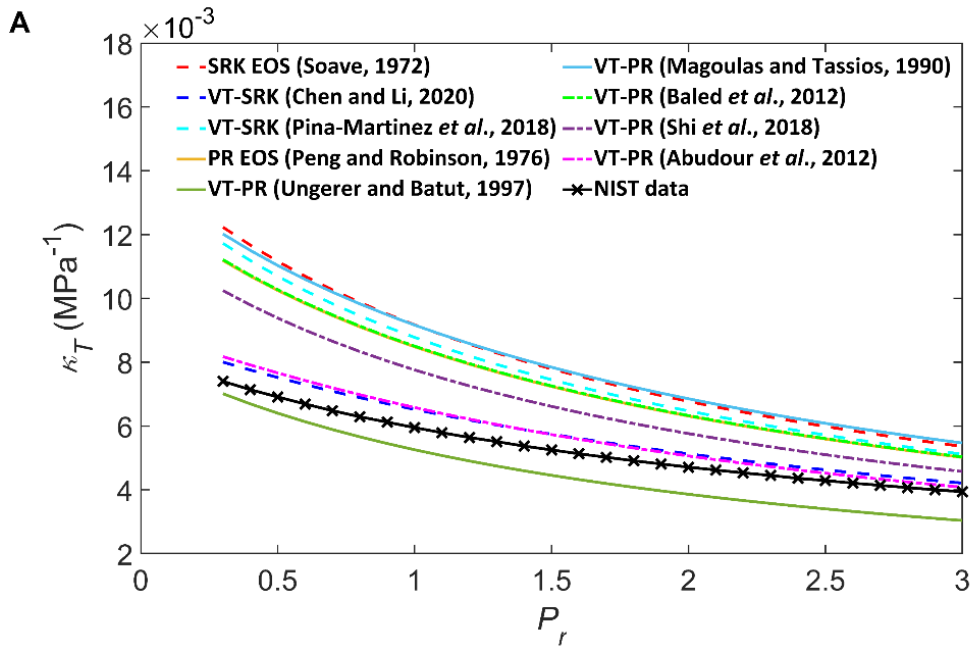
Substance	Phase region	%AADs yielded by PR EOS models						%AADs yielded by SRK EOS models		
		PR EOS <sup>9</sup>	Ungerer and Batut model <sup>10</sup>	Baled <i>et al.</i> model <sup>11</sup>	Magoulas and Tassios model <sup>12</sup>	Shi <i>et al.</i> model <sup>6</sup>	Abudour <i>et al.</i> model <sup>3</sup>	SRK EOS <sup>8</sup>	Chen and Li model <sup>5</sup>	Constant VT updated by Pina-Martinez <i>et al.</i> <sup>4</sup>
			Linear temperature-dependent	Exponential temperature-dependent	Temperature-pressure-dependent	Temperature-pressure-dependent	Constant VT			
CO <sub>2</sub>	Liquid	18.48	12.60	26.75	67.16	13.21	<b>3.34</b>	16.72	4.21	9.27
	Vapor	1.39	<b>0.86</b>	1.64	1.75	1.48	1.67	1.28	1.59	1.37
	Supercritical	<b>2.29</b>	4.70	3.36	4.59	2.35	2.42	5.09	2.88	6.12
CH <sub>4</sub>	Liquid	18.09	34.23	17.51	64.44	9.88	7.49	16.60	<b>5.10</b>	12.64
	Vapor	1.44	1.35	1.43	1.84	1.33	1.55	0.73	1.19	<b>0.70</b>
	Supercritical	3.06	8.10	3.04	4.97	2.58	2.03	2.99	<b>1.65</b>	3.44
Overall		7.46	10.30	8.96	24.12	5.14	3.08	7.23	<b>2.77</b>	5.59

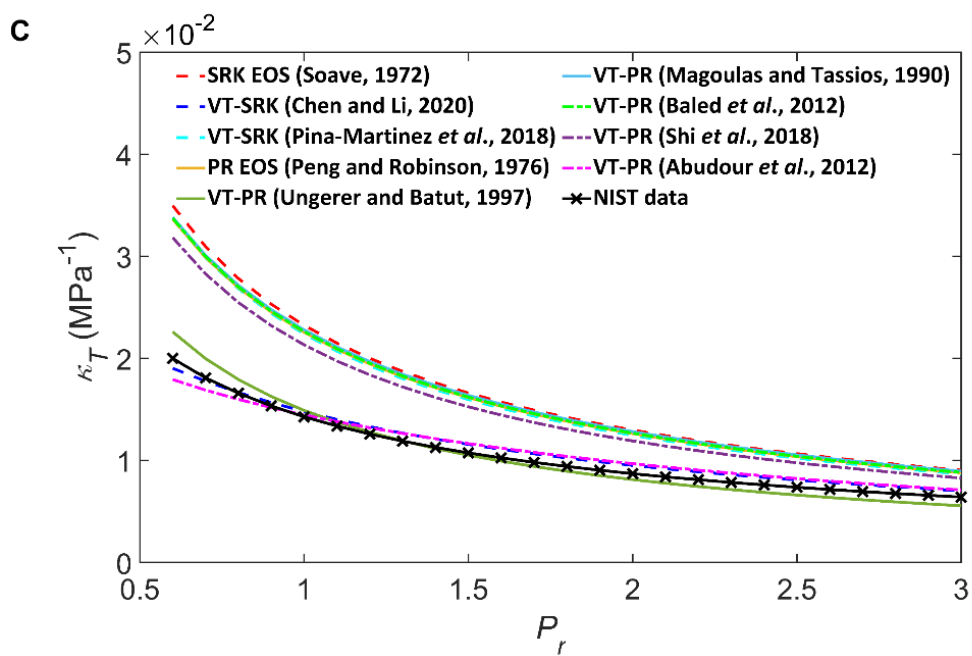
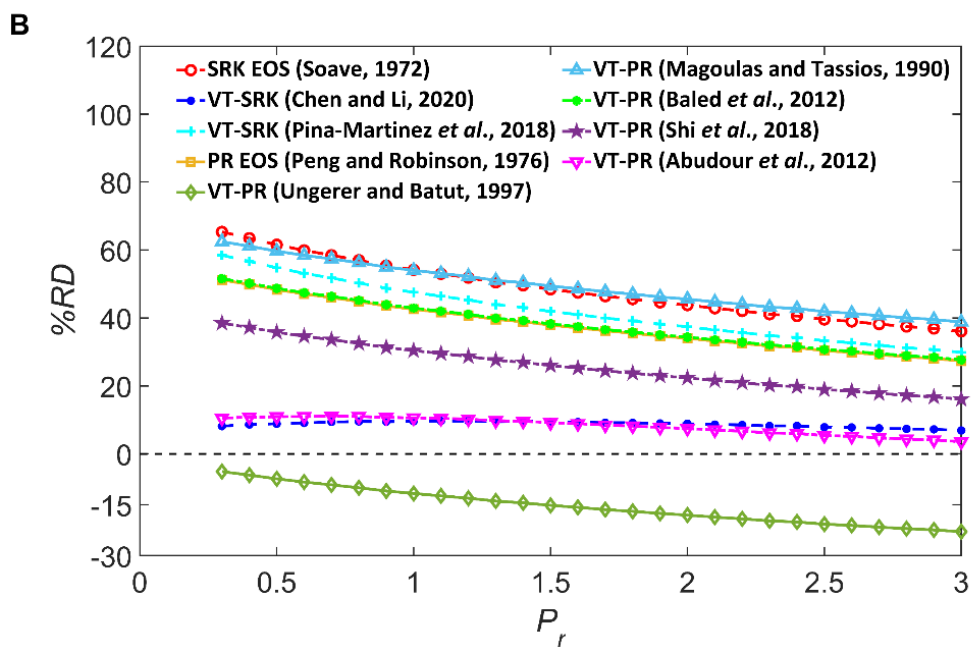
### 3.2.1. Predicted Results for the Liquid-phase $\kappa_T$ and $\alpha_p$

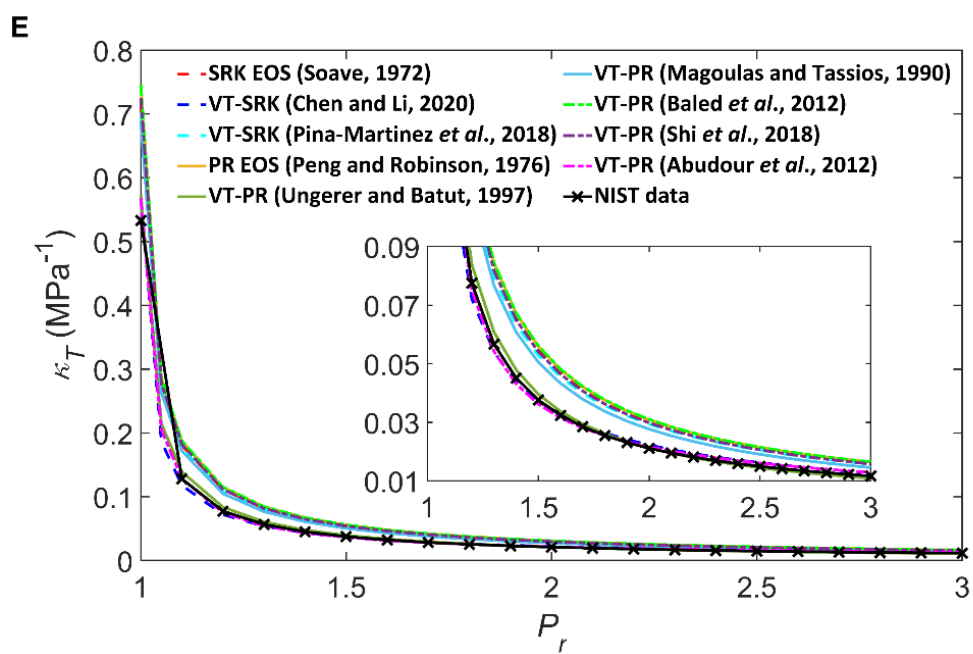
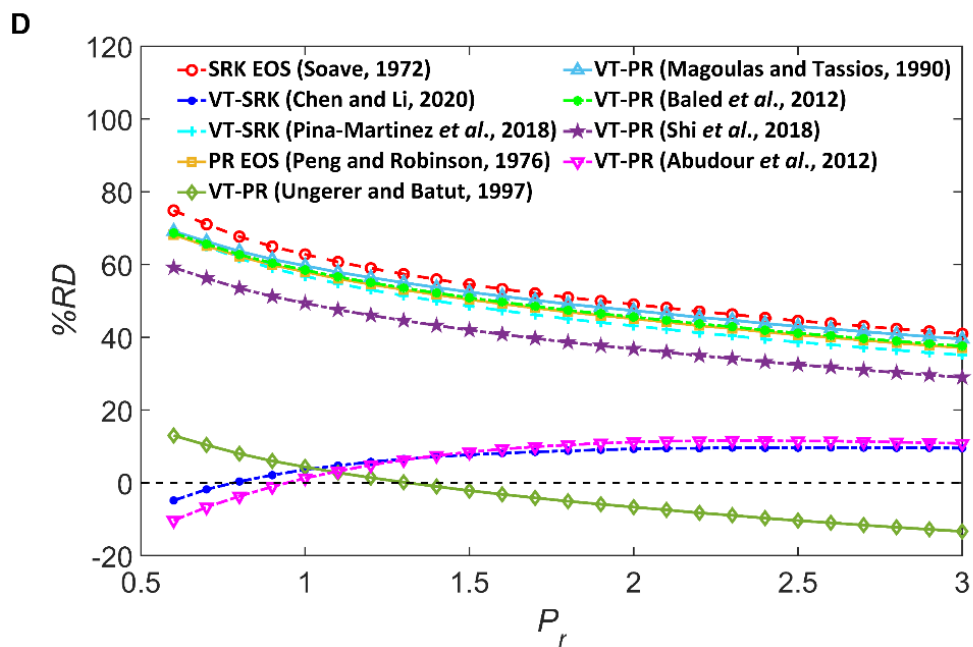
For the predictions of the liquid-phase  $\kappa_T$ , the Ungerer and Batut VT-PR EOS<sup>10</sup> gives the lowest %AAD of 14.20% for CO<sub>2</sub>. The Abudour *et al.* VT-PR EOS<sup>3</sup> and the Chen and Li VT-SRK EOS<sup>5</sup> also present good results for CO<sub>2</sub>, yielding %AADs of 15.93% and 18.22%, respectively. As for the liquid-phase  $\kappa_T$  of CH<sub>4</sub>, the Ungerer and Batut model<sup>10</sup> performs poorly with a %AAD of 25.38%, while the Abudour *et al.* model<sup>3</sup> and the Chen and Li model<sup>5</sup> maintain their decent accuracy. The VT-PR EOS of Baled *et al.*<sup>11</sup> yields higher %AADs of the liquid-phase  $\kappa_T$  predictions than the PR EOS for both CO<sub>2</sub> and CH<sub>4</sub>. The Shi *et al.* VT-PR EOS<sup>6</sup> and the constant VT-SRK EOS updated by Pina-Martinez *et al.*<sup>4</sup> only have slight improvement to their corresponding untranslated EOS models. Figure

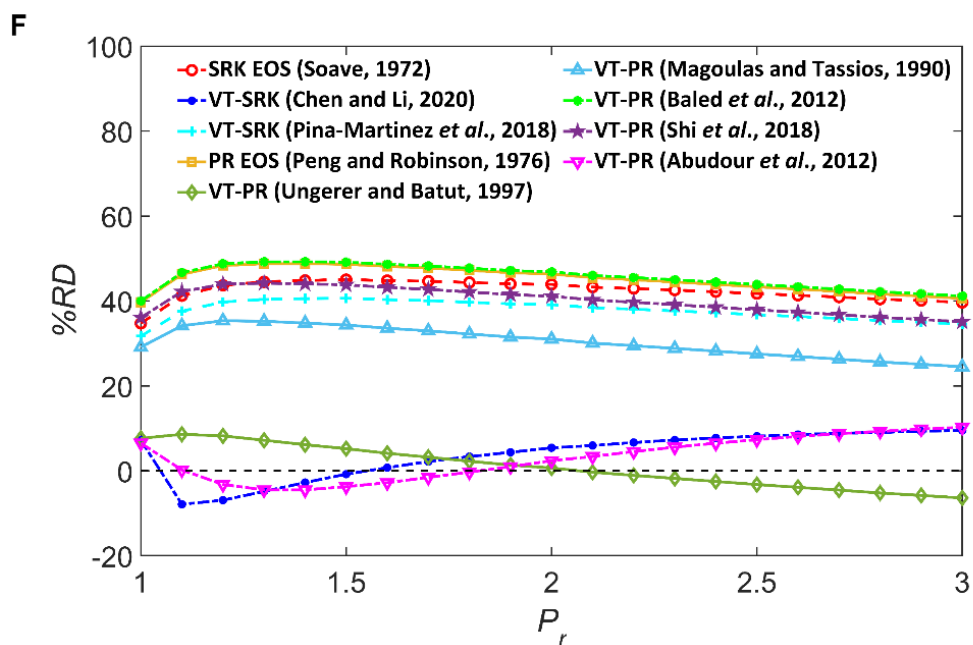


4 compares the  $\kappa_T$  predicted by each model against the NIST data<sup>7</sup> at different isotherms for the liquid-phase CH<sub>4</sub>. Figure 5 shows the same comparison for liquid CO<sub>2</sub>. The distribution of %RDs as a function of pressure is also illustrated. It is clear that all models give a similar trend of  $\kappa_T$  versus pressure at different temperatures that  $\kappa_T$  decreases with pressure. As temperature approaches the critical temperature, the change of  $\kappa_T$  with pressure becomes more significant. The prediction errors of the liquid-phase  $\kappa_T$  by most VT models tend to be larger at lower pressures for a given temperature.

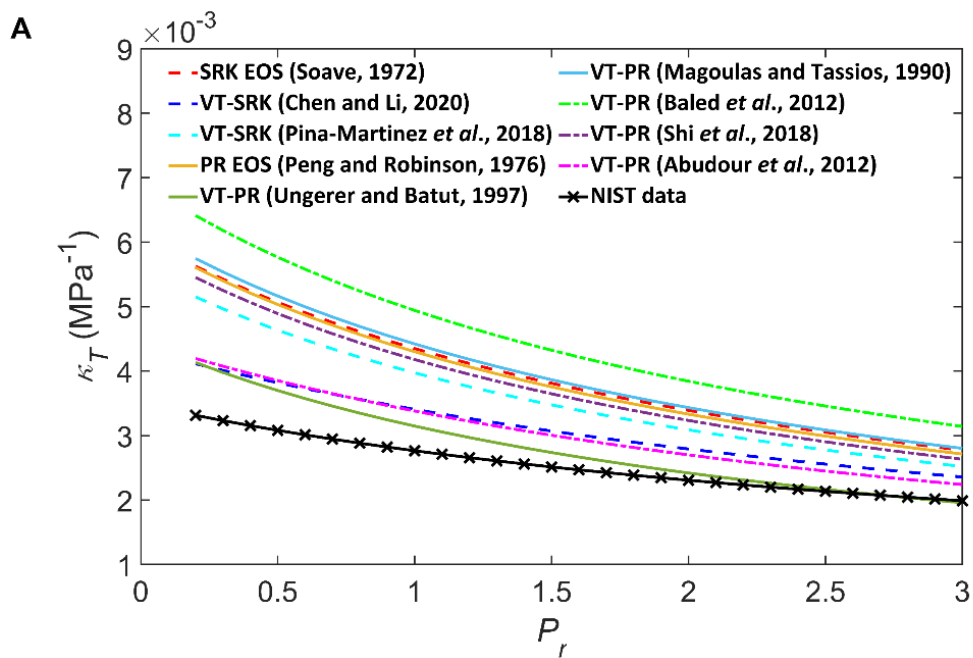


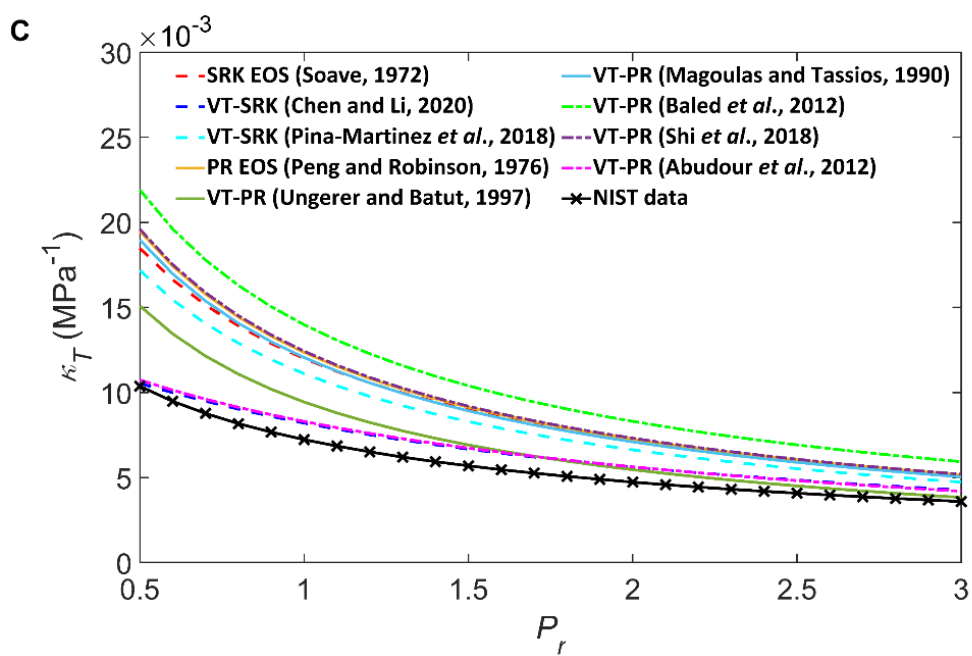
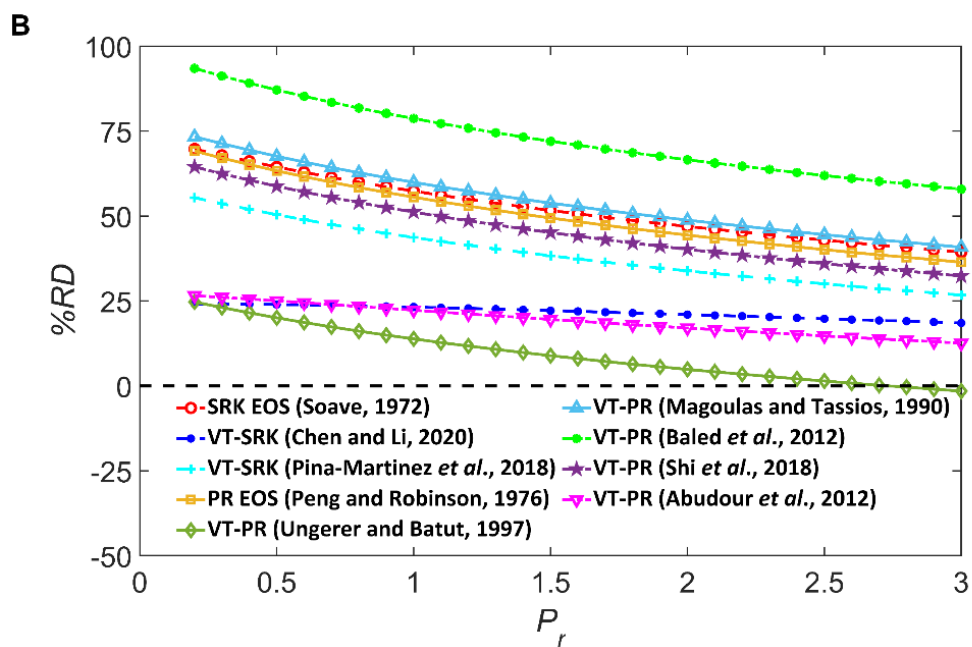


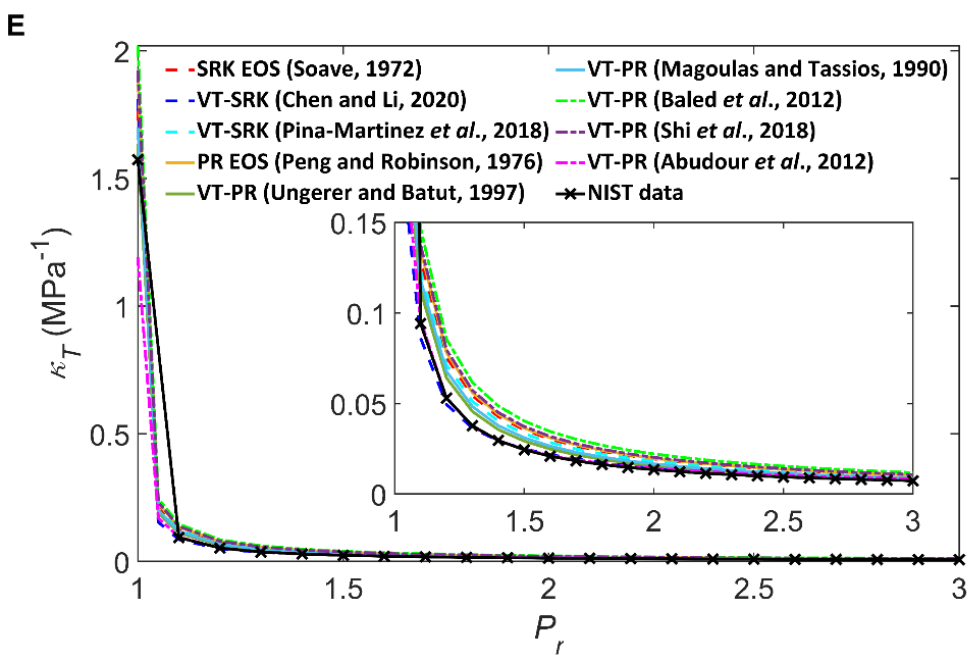
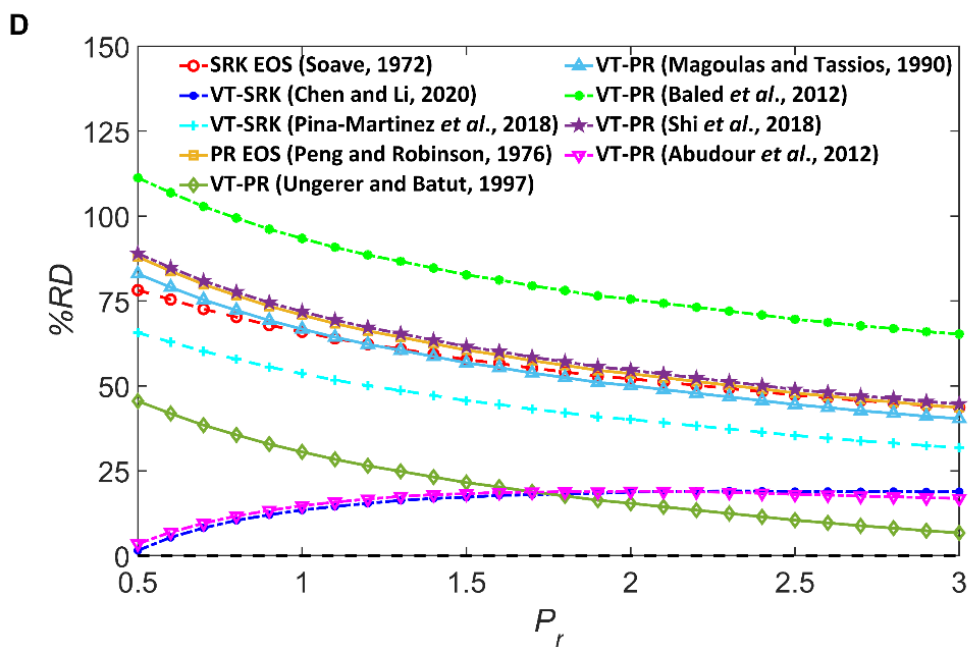


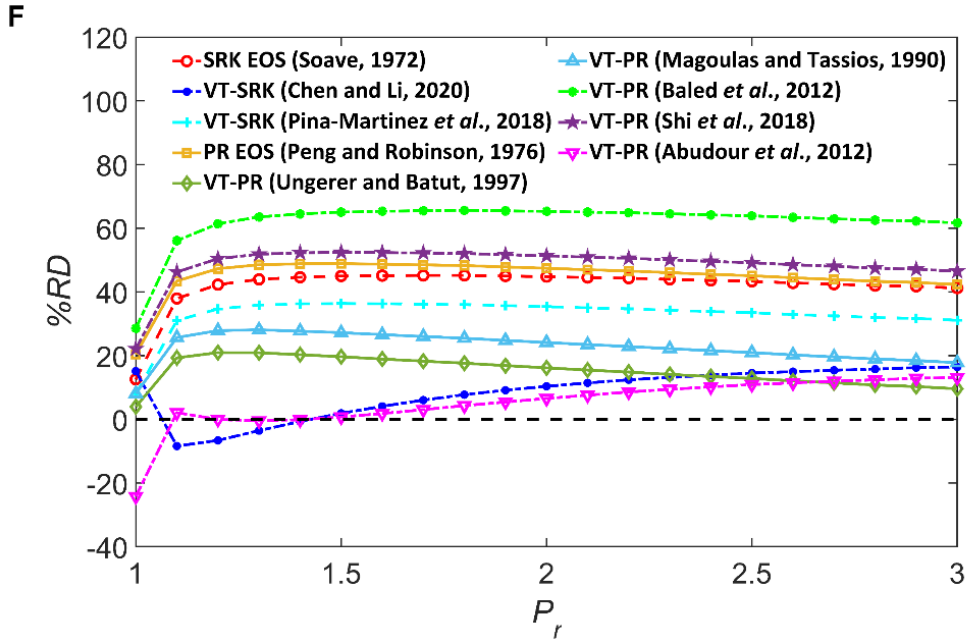


**Figure 4** Comparison of the calculated  $\kappa_T$  for the liquid-phase  $\text{CH}_4$  against the NIST data (A, C, and E) and  $\%RDs$  yielded by the models studied in this work (B, D, and F) at pressures from the saturated vapor pressures to  $P_r = 3$  and different temperatures:  $T_r = 0.8$  (A and B),  $T_r = 0.9$  (C and D), and  $T_r = 1$  (E and F).





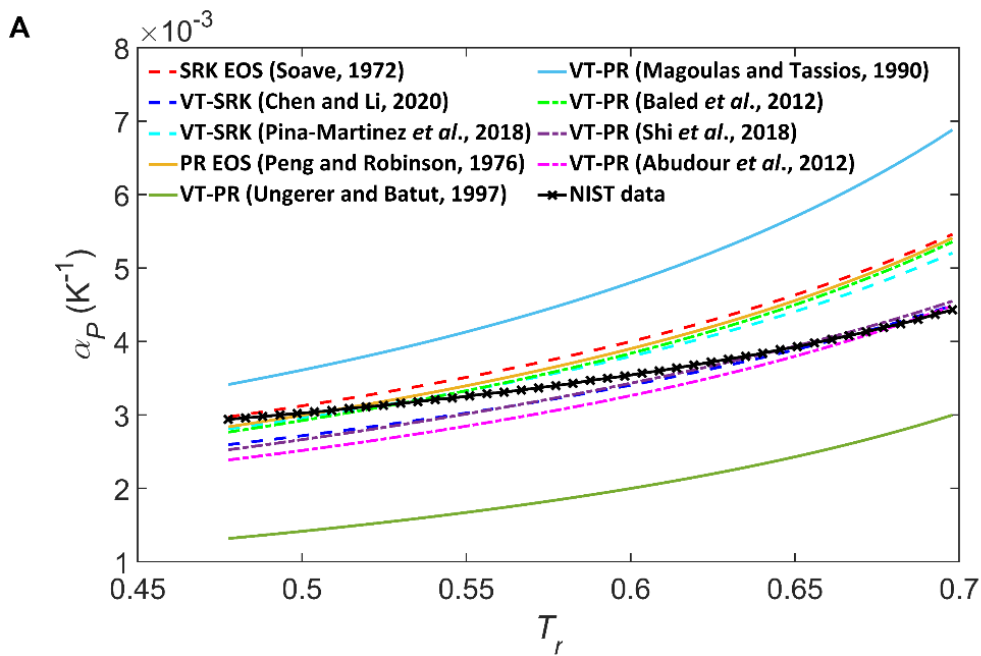




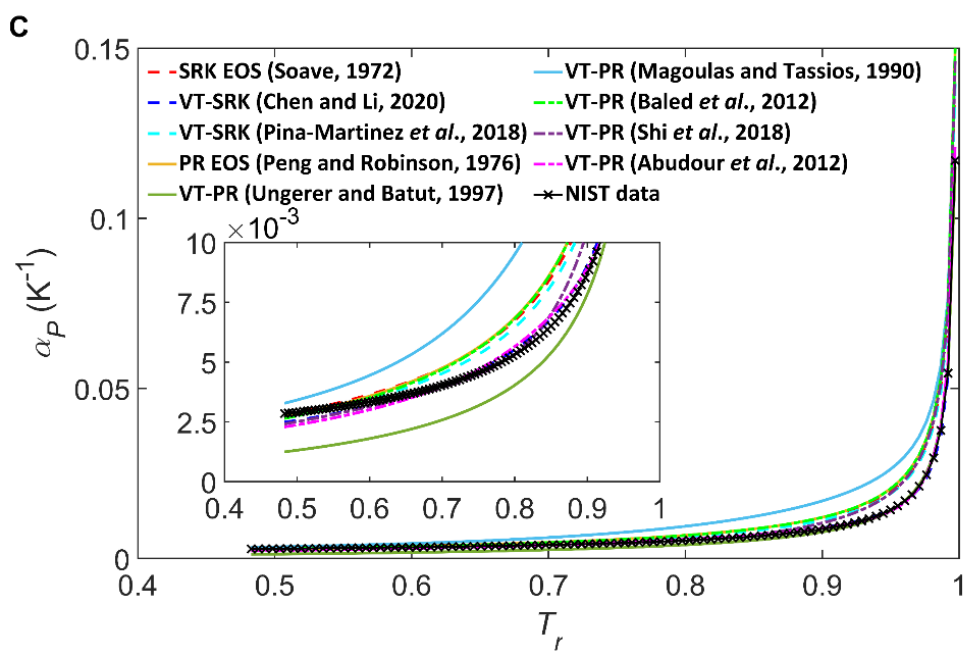
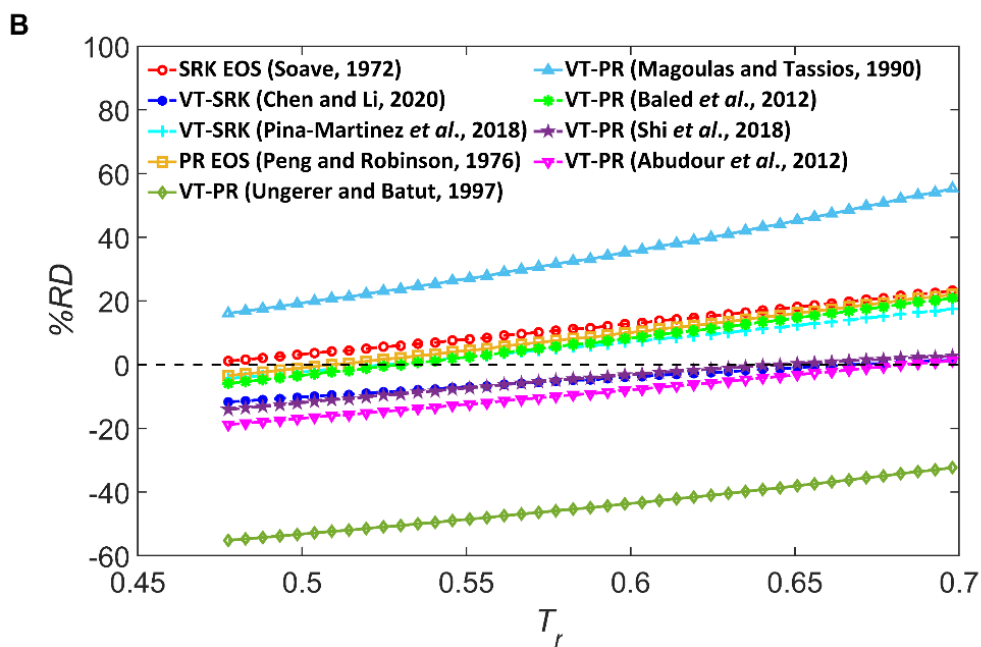
**Figure 5** Comparison of the calculated  $\kappa_T$  for the liquid-phase CO<sub>2</sub> against the NIST data (A, C, and E) and %RDs yielded by the models studied in this work (B, D, and F) at pressures from the saturated vapor pressures to  $P_r = 3$  and different temperatures:  $T_r = 0.8$  (A and B),  $T_r = 0.9$  (C and D), and  $T_r = 1$  (E and F).

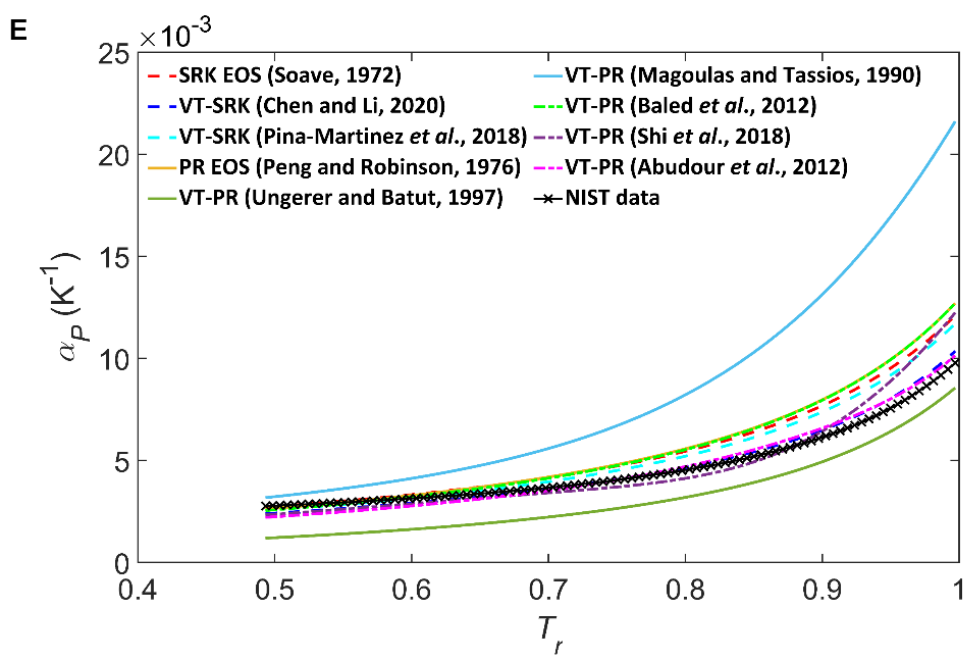
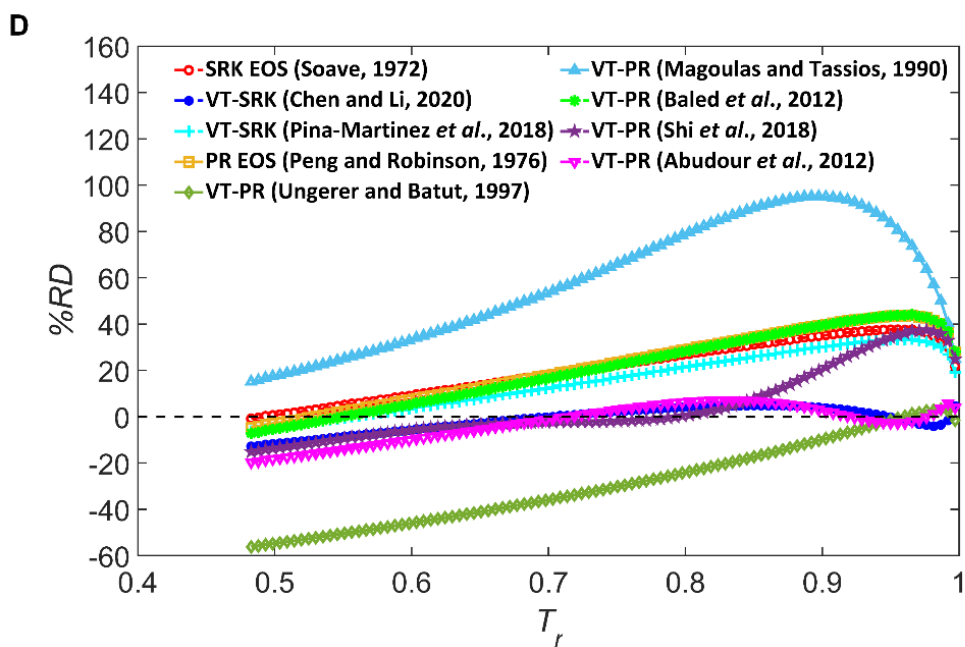
As for the liquid-phase CH<sub>4</sub> and CO<sub>2</sub>, the prediction accuracy of  $\alpha_p$  by different VT models is relatively better than those of  $\kappa_T$ . The Abudour *et al.* model<sup>3</sup> yields the lowest %AAD of 3.34% for CO<sub>2</sub>, while the Chen and Li model<sup>5</sup> yields the lowest %AAD of 5.10% for CH<sub>4</sub>. Similar to the liquid-phase  $\kappa_T$  results, the Ungerer and Batut model<sup>10</sup> performs better for CO<sub>2</sub> than CH<sub>4</sub> in predicting the liquid-phase  $\alpha_p$ . The VT-PR EOS of Shi *et al.*<sup>6</sup> and the constant VT-SRK EOS updated by Pina-Martinez *et al.*<sup>4</sup> also show relatively accurate results for the liquid-phase  $\alpha_p$ . The VT-PR EOSs of Baled *et al.*<sup>11</sup> and Magoulas and Tassios<sup>12</sup> show poorer performance than the untranslated PR EOS. Figure 6 compares the predicted  $\alpha_p$  against the NIST data<sup>7</sup> and shows the distribution of %RDs for the liquid-phase CH<sub>4</sub>. Figure 7 shows the same results for the liquid-phase CO<sub>2</sub>. The constant VT-SRK EOS updated by Pina-Martinez *et al.*<sup>4</sup> and the models of Abudour *et al.*<sup>3</sup> and Chen and Li<sup>5</sup> exhibit decent accuracy over the entire temperature and pressure ranges

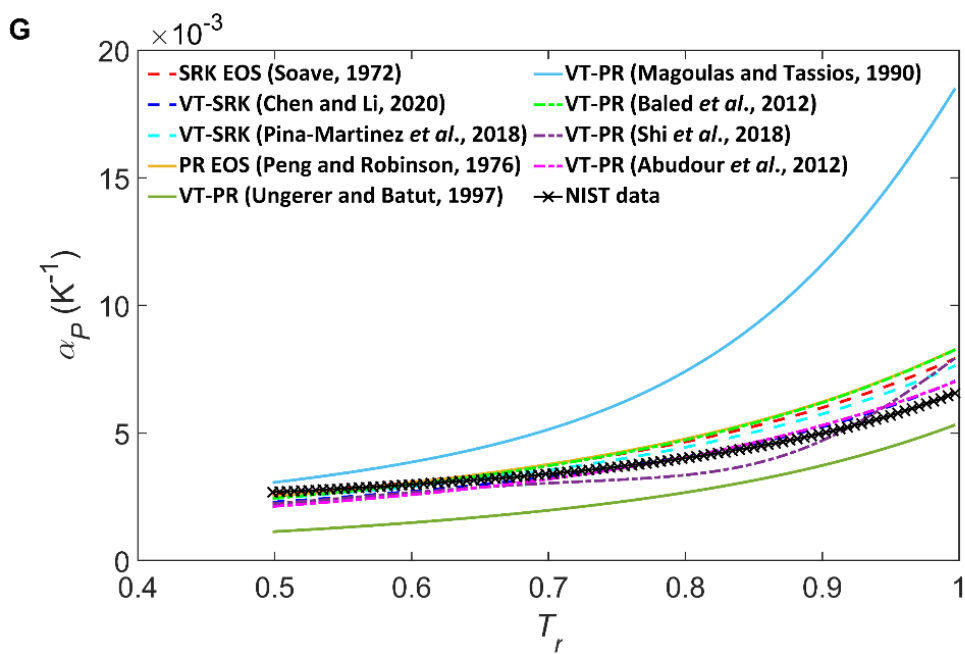
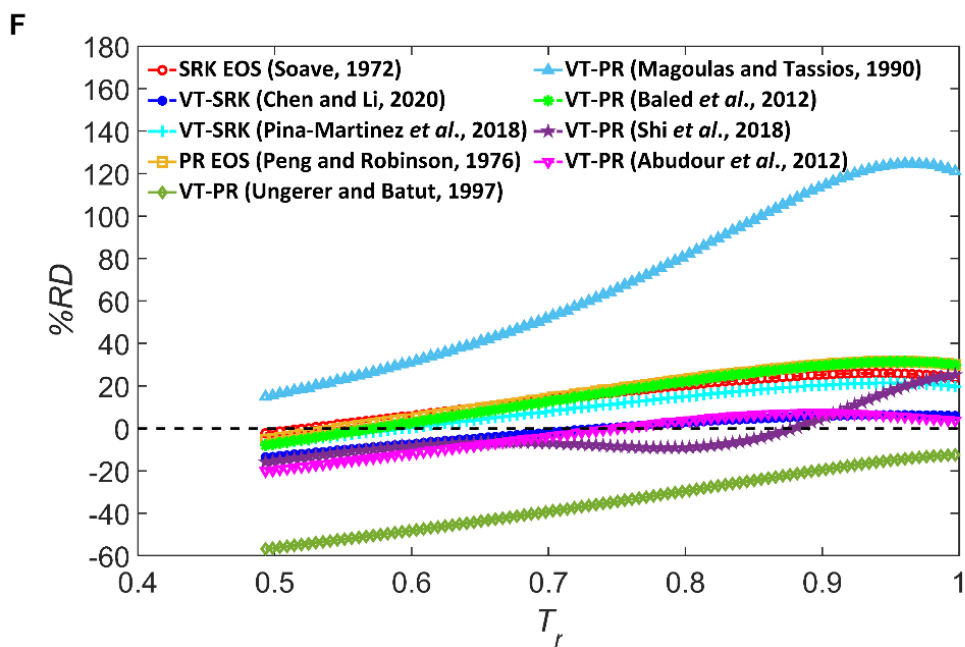
in the liquid-phase region. The other models can predict a similar trend of  $\alpha_p$  changing with temperature, but their %RDs tend to be larger as pressure increases. The Magoulas and Tassios VT-PR EOS<sup>12</sup> yields poor prediction accuracy of  $\alpha_p$  for the liquid phases. The %RDs yielded by this model can be greater than 100% at pressures larger than the critical pressure.

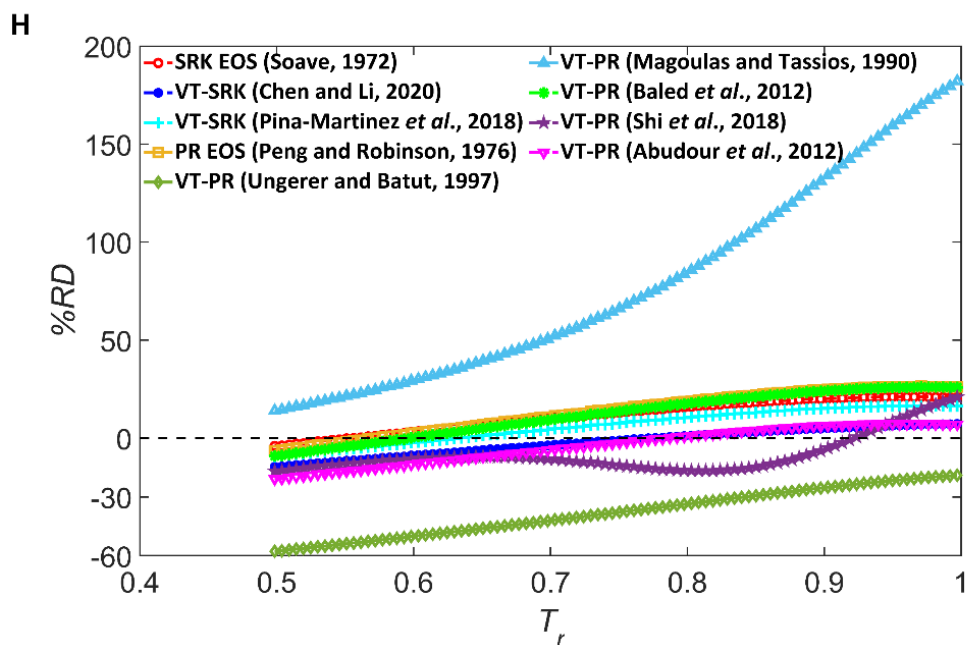




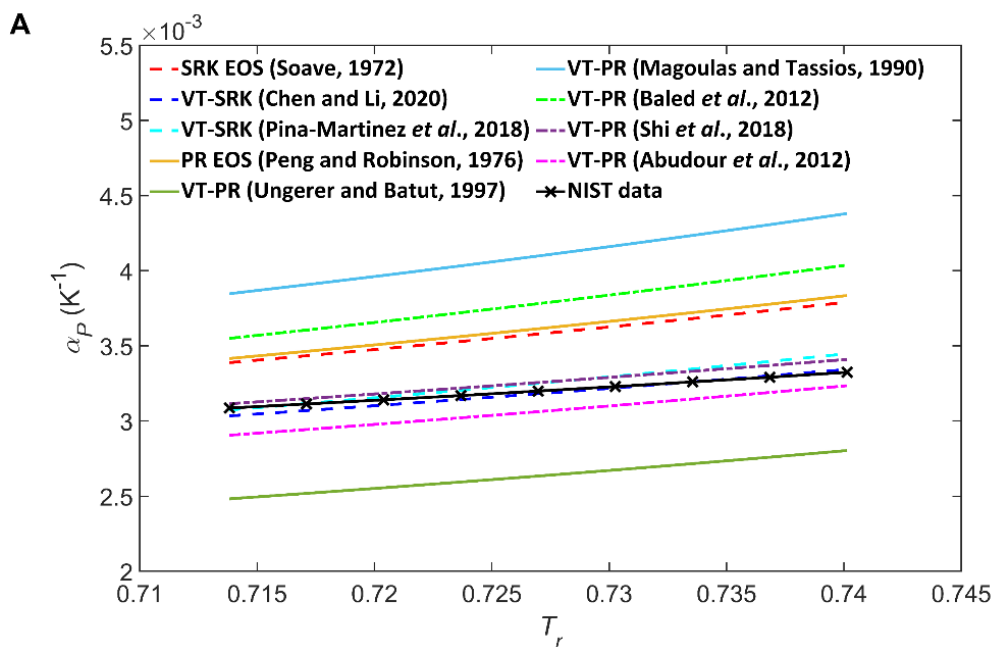


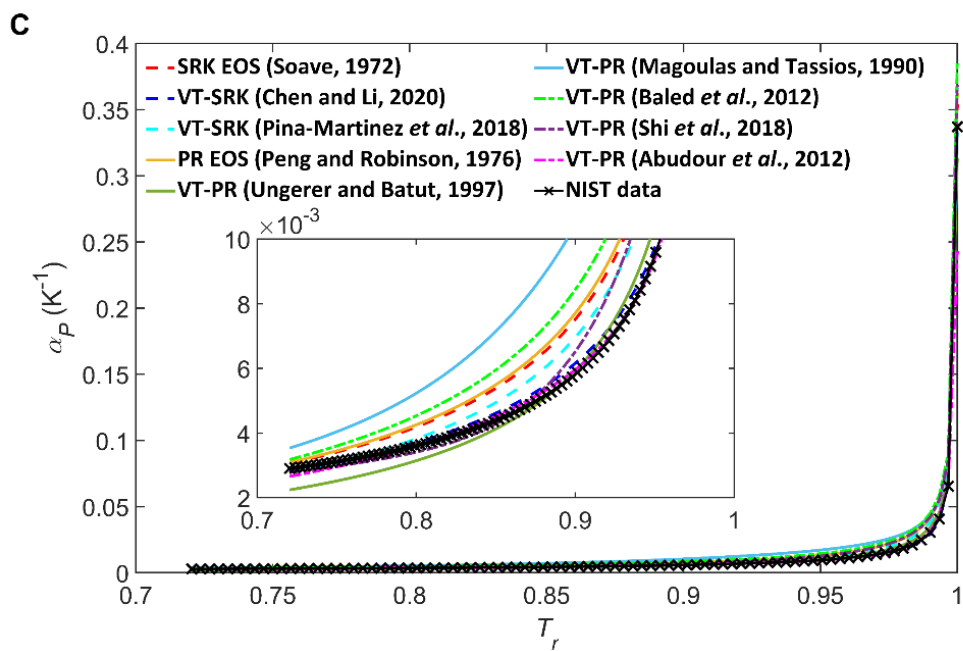
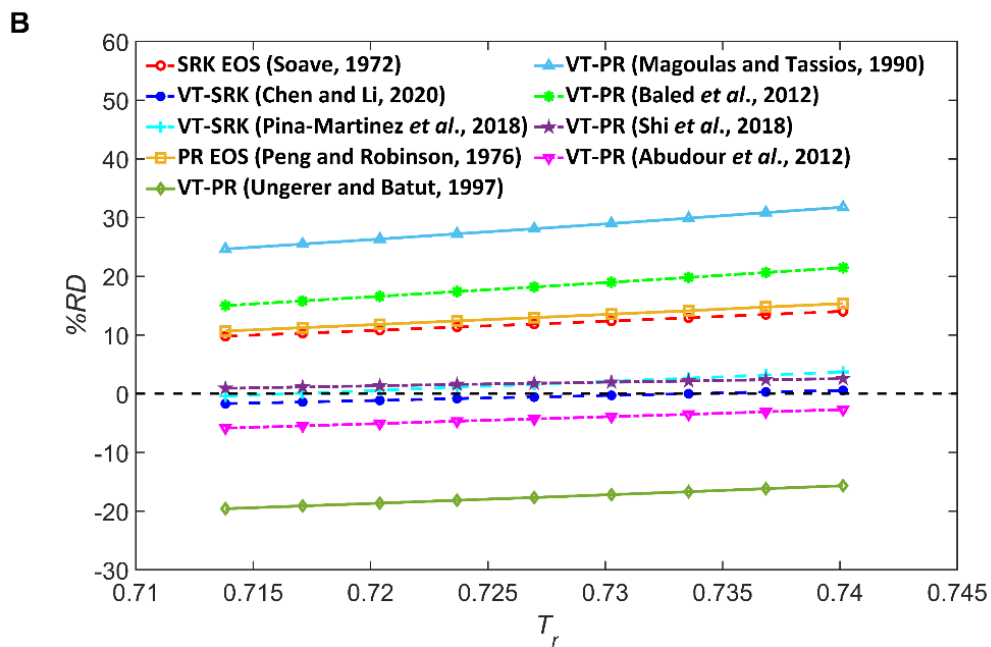


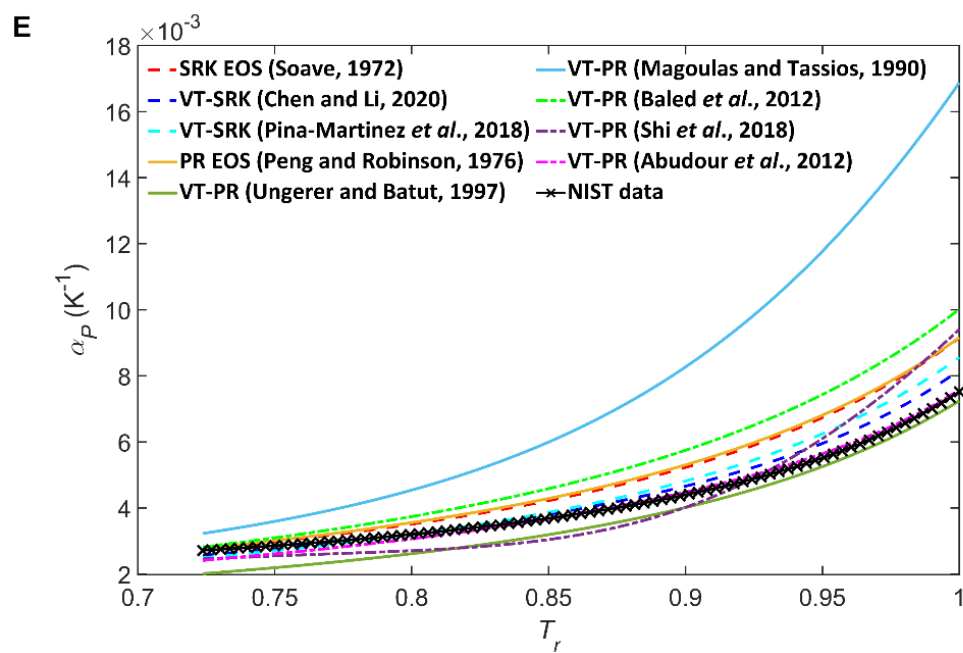
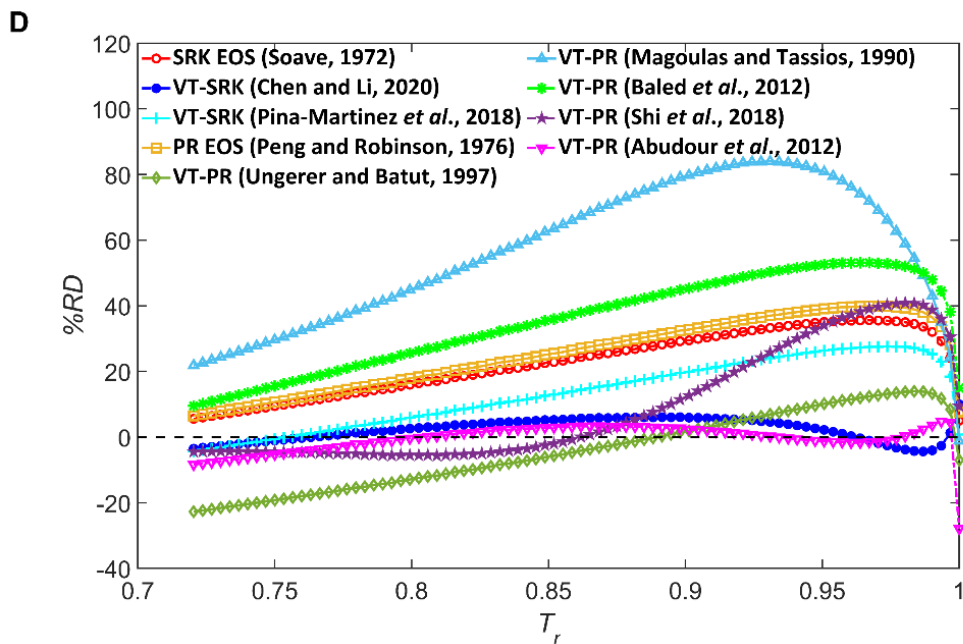




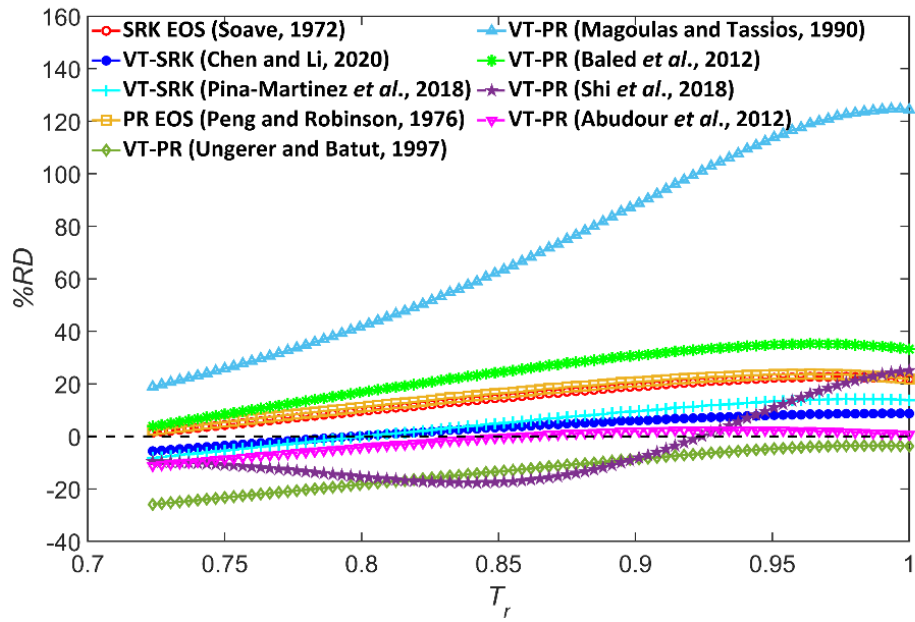
**Figure 6** Comparison of the calculated  $\alpha_p$  for the liquid-phase  $\text{CH}_4$  against the NIST data (A, C, E, and G) and  $\%RDs$  yielded by the models studied in this work (B, D, F, and H) at temperatures from the triple point temperature to  $T_r = 1$  and different pressures:  $P_r = 0.1$  (A and B),  $P_r = 1$  (C and D),  $P_r = 2$  (E and F), and  $P_r = 3$  (G and H).



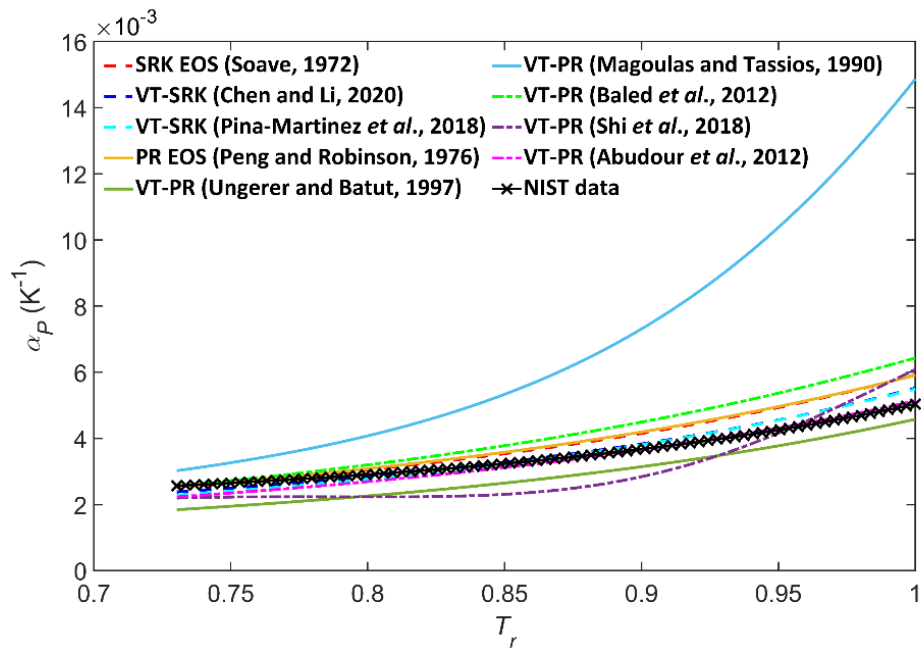


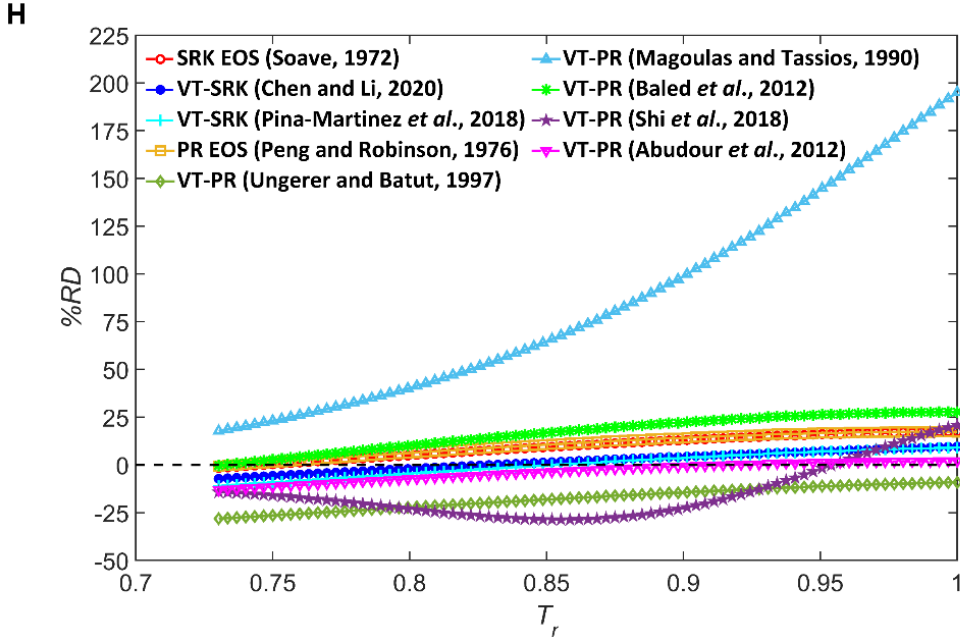


**F**



**G**





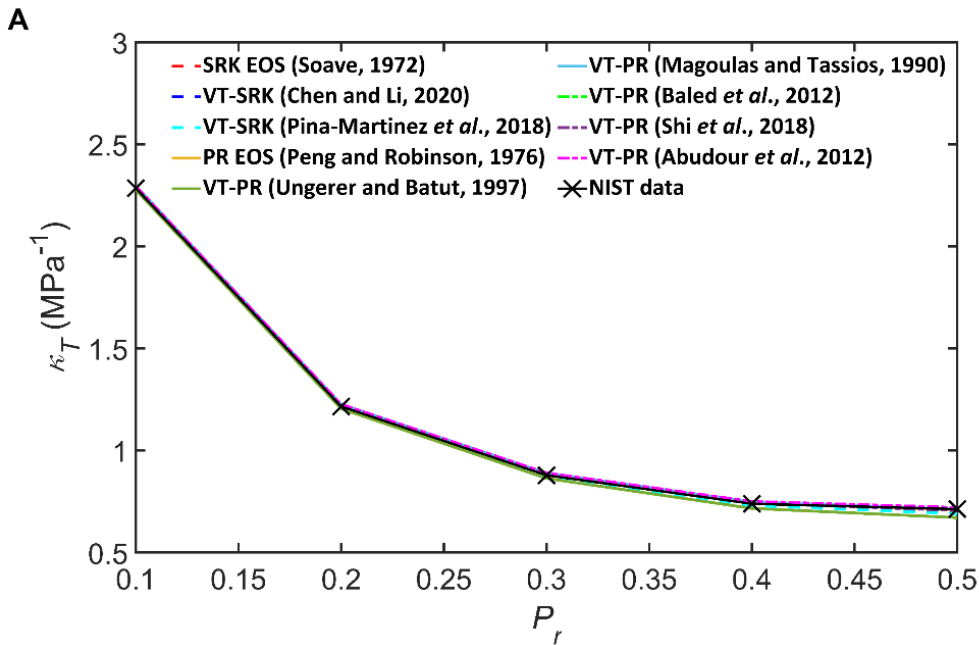
**Figure 7** Comparison of the calculated  $\alpha_p$  for the liquid-phase  $\text{CO}_2$  against the NIST data (A, C, E, and G) and  $\%RDs$  yielded by the models studied in this work (B, D, F, and H) at temperatures from the triple point temperature to  $T_r = 1$  and different pressures:  $P_r = 0.1$  (A and B),  $P_r = 1$  (C and D),  $P_r = 2$  (E and F), and  $P_r = 3$  (G and H).

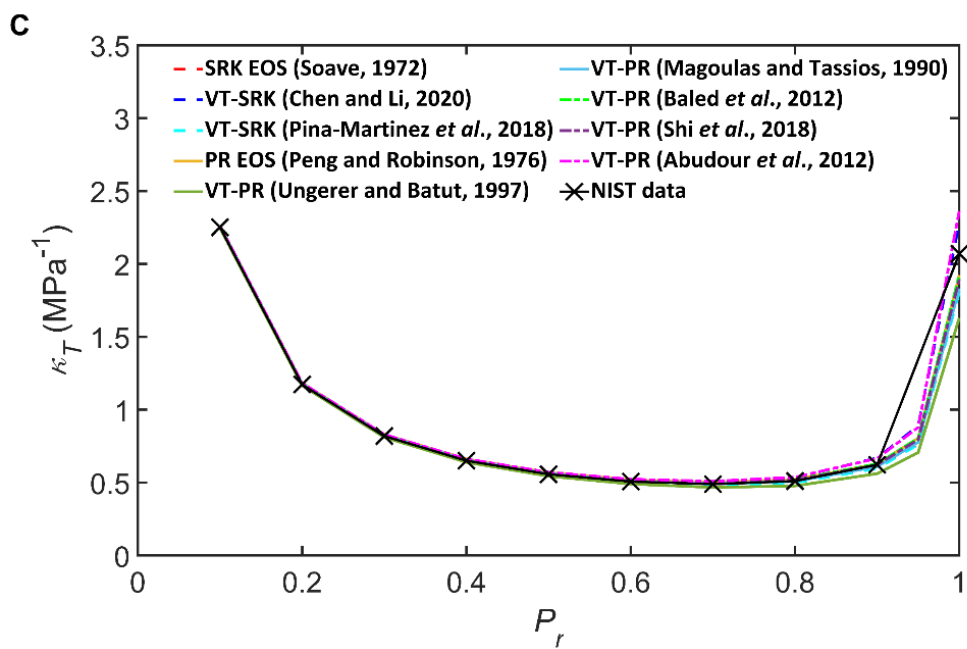
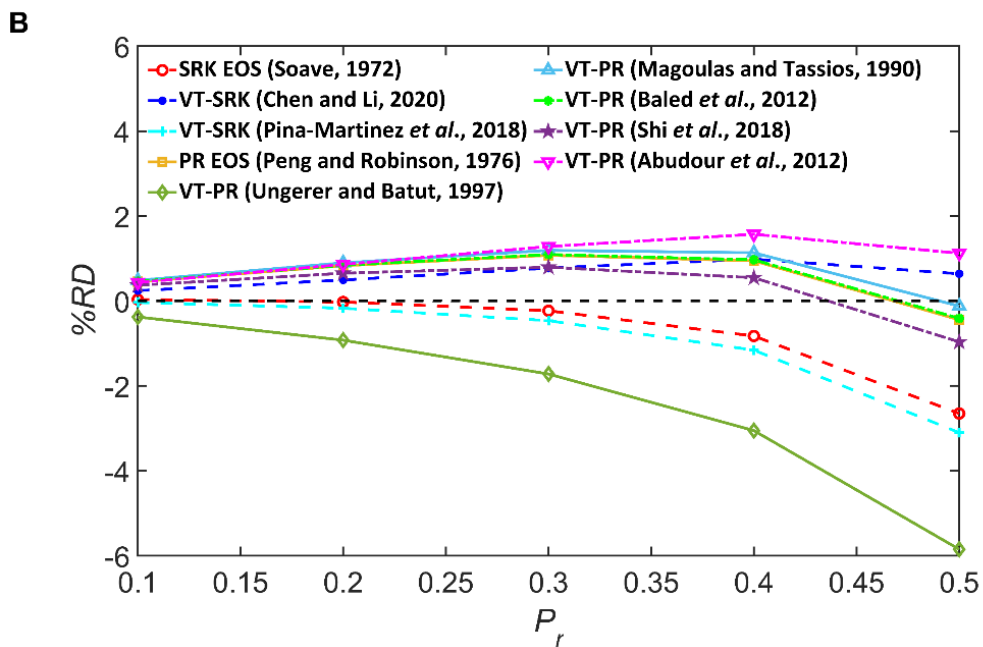
### 3.2.2. Predicted Results for the Vapor-phase $\kappa_T$ and $\alpha_p$

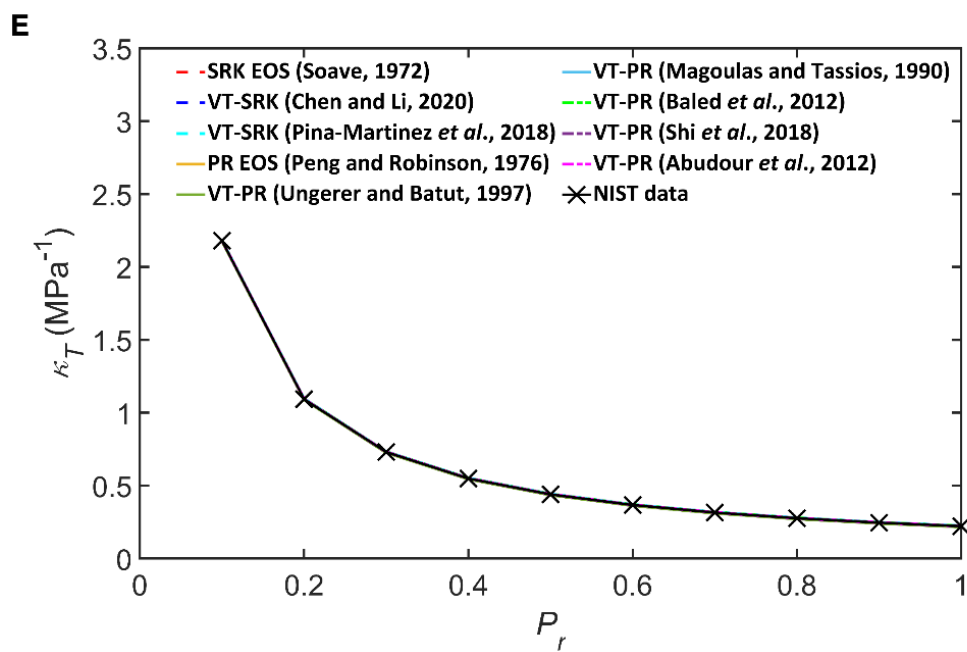
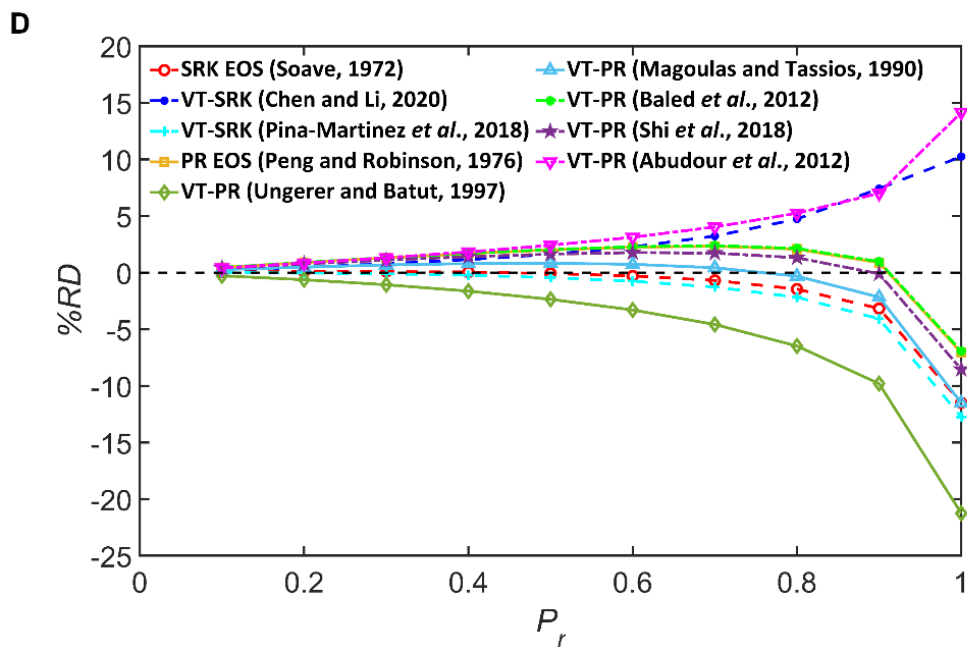
It is noted that the original untranslated PR EOS and SRK EOS already give pretty accurate predictions for the vapor-phase  $\kappa_T$  and  $\alpha_p$ . Most of the VT models lead to larger deviations of the vapor-phase  $\kappa_T$  predictions. Only the VT models of Shi *et al.*,<sup>6</sup> Abudour *et al.*,<sup>3</sup> and Chen and Li<sup>5</sup> result in improvement over their corresponding untranslated EOSs. The model of Shi *et al.*<sup>6</sup> yields the most accurate  $\kappa_T$  predictions for the vapor phases of both  $\text{CH}_4$  and  $\text{CO}_2$ , with  $\%AADs$  of 0.35% and 0.32%, respectively. As for the vapor-phase  $\alpha_p$  predictions, the Ungerer and Batut model<sup>10</sup> yields the lowest  $\%AAD$  of 0.86% for  $\text{CO}_2$ , while the constant VT-SRK EOS updated by Pina-Martinez *et al.*<sup>4</sup> performs relatively better for  $\text{CH}_4$  with a  $\%AAD$  of 0.70%.

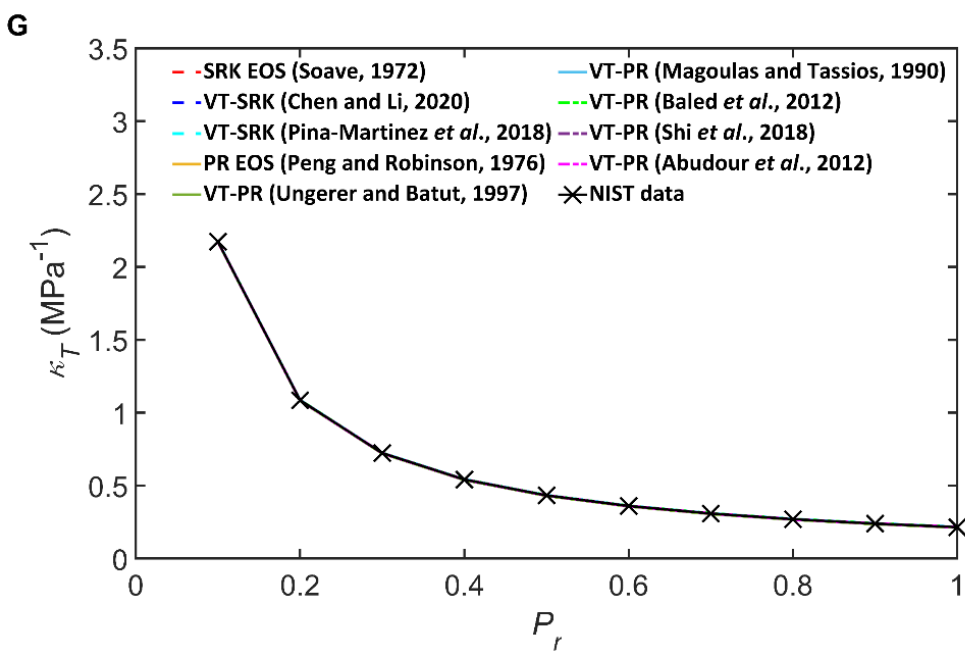
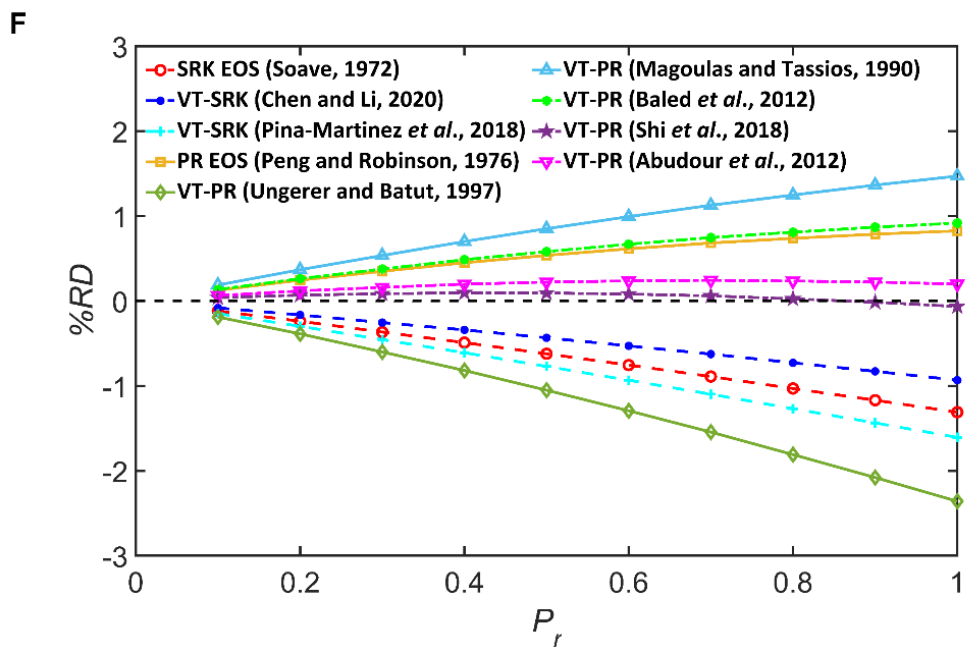


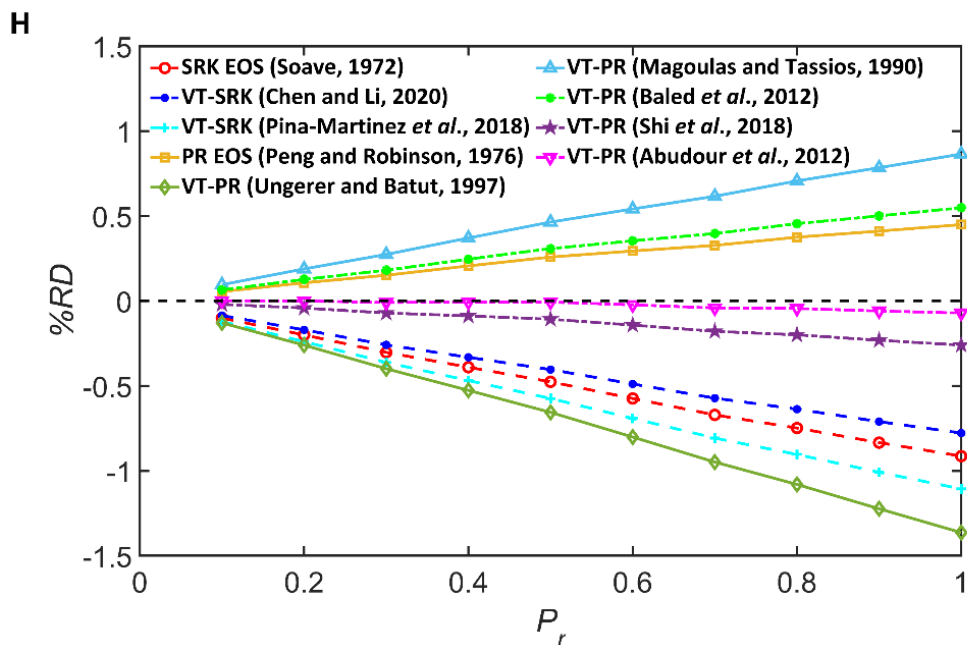
Most VT models fail to improve the vapor-phase  $\kappa_T$  and  $\alpha_p$  predictions. A vapor phase has a relatively much larger volume than a liquid phase, which renders the volume corrections provided by VT-EOS models insignificant.<sup>11</sup> Figure 8 compares the calculated  $\kappa_T$  for the vapor-phase CH<sub>4</sub> against the NIST data. It also shows the distribution of %RDs over pressure. The same results for CO<sub>2</sub> are shown in Figure 9. It is clear that the vapor-phase  $\kappa_T$  predictions for both CH<sub>4</sub> and CO<sub>2</sub> made by each model are in close agreement with the NIST data. The vapor-phase  $\kappa_T$  decreases with an increasing pressure, except at conditions around the critical point. At  $T_r = 1$ , the vapor-phase  $\kappa_T$  becomes larger as  $P_r$  approaches 1. The %RDs yielded by different VT models are relatively low, but they tend to grow as pressure increases.



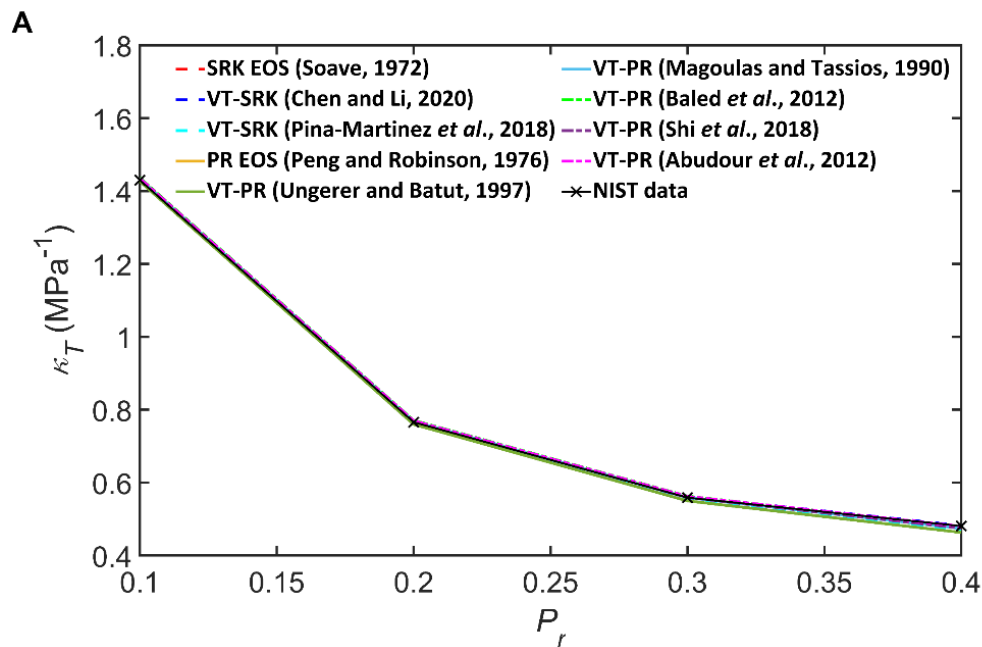


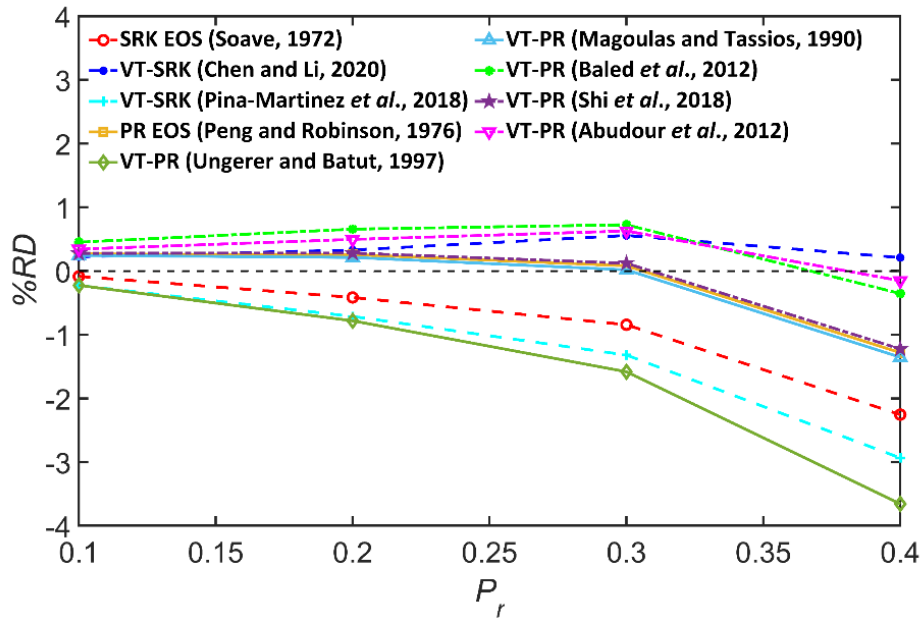
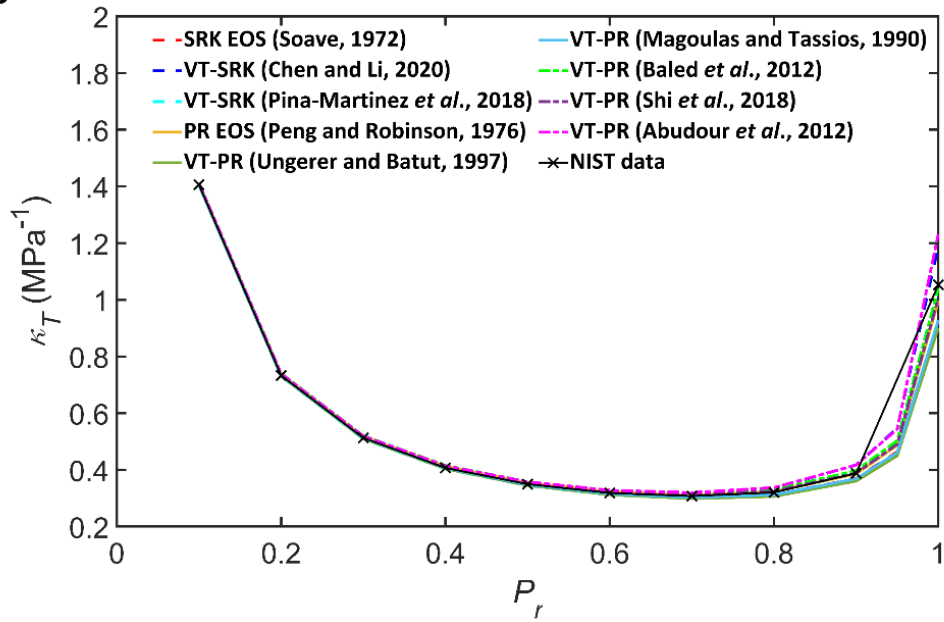


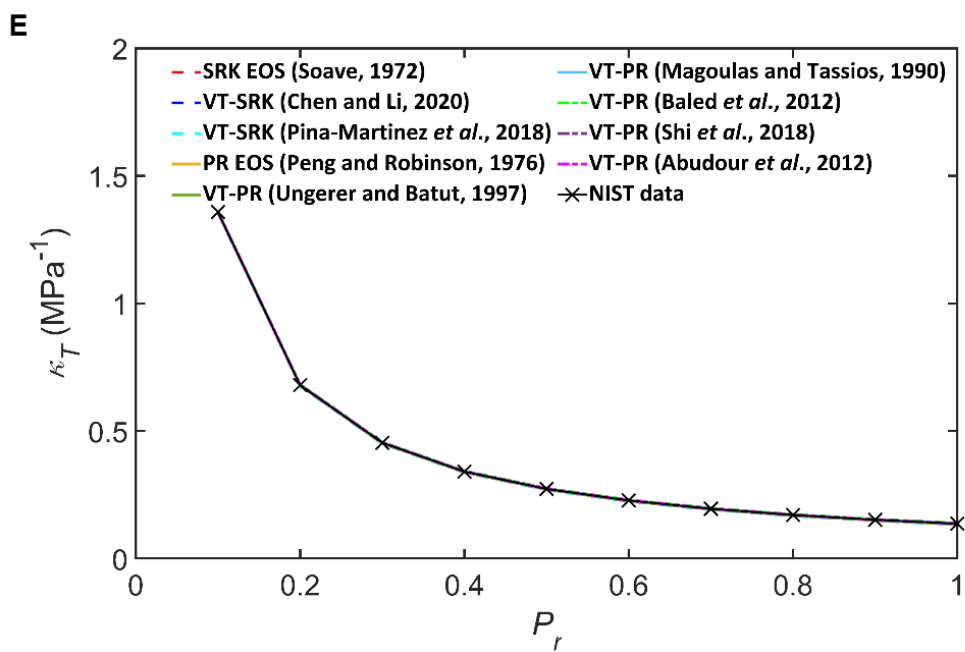
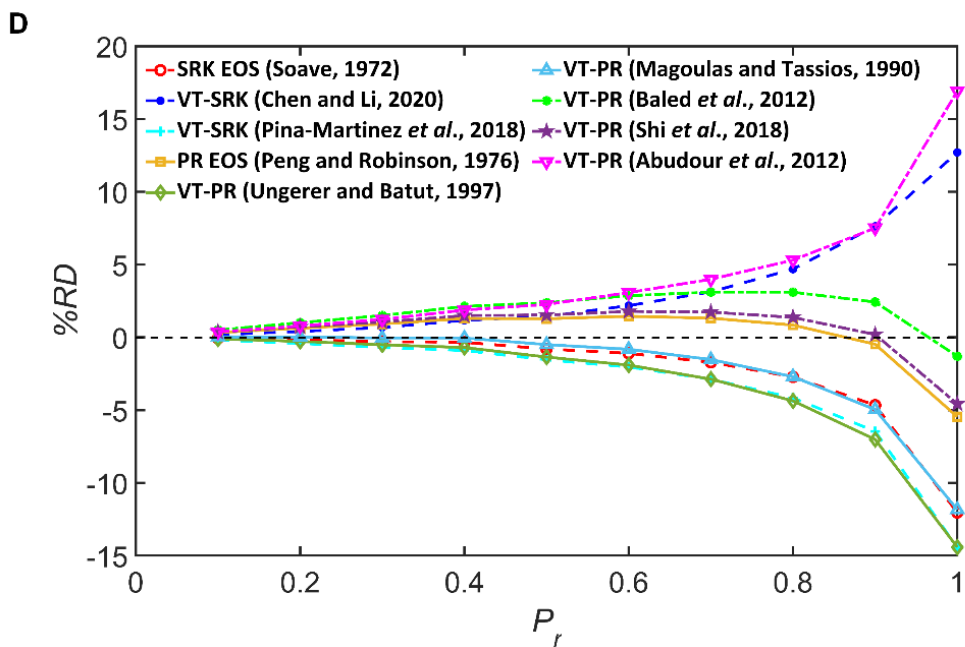


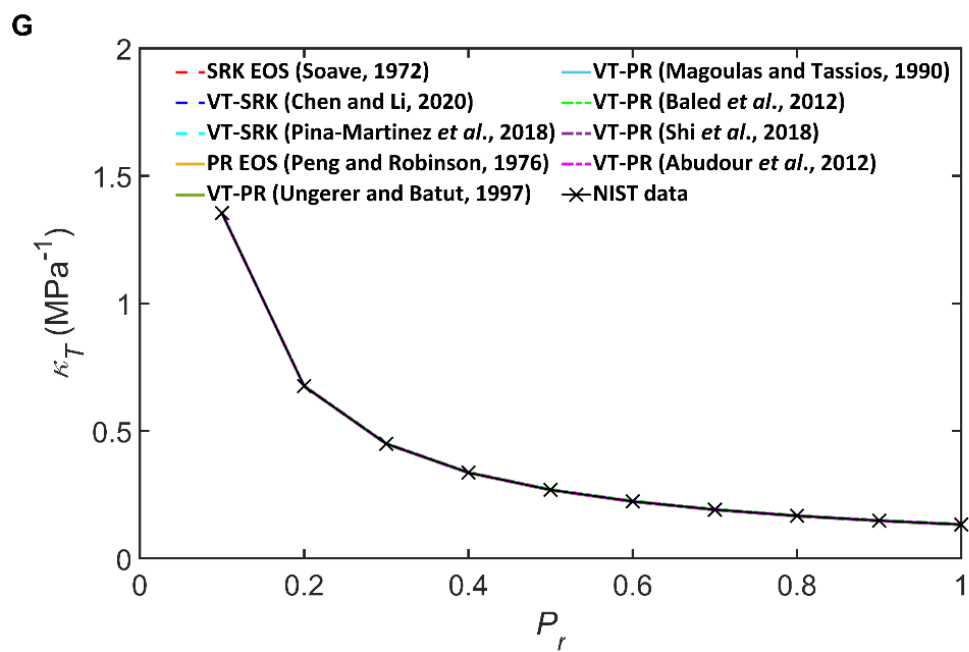
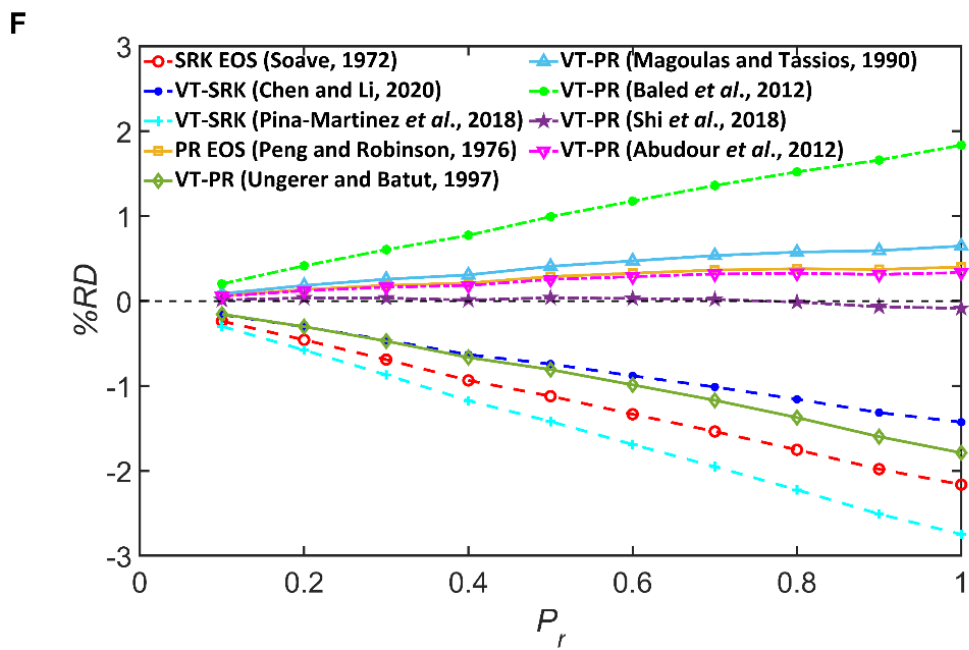


**Figure 8** Comparison of the calculated  $\kappa_T$  for the vapor-phase  $\text{CH}_4$  against the NIST data (A, C, E, and G) and %RDs yielded by the models studied in this work (B, D, F, and H) at pressures from  $P_r = 0.1$  to  $P_r = 1$  and different temperatures:  $T_r = 0.9$  (A and B),  $T_r = 1$  (C and D),  $T_r = 2$  (E and F),  $T_r = 3$  (G and H).

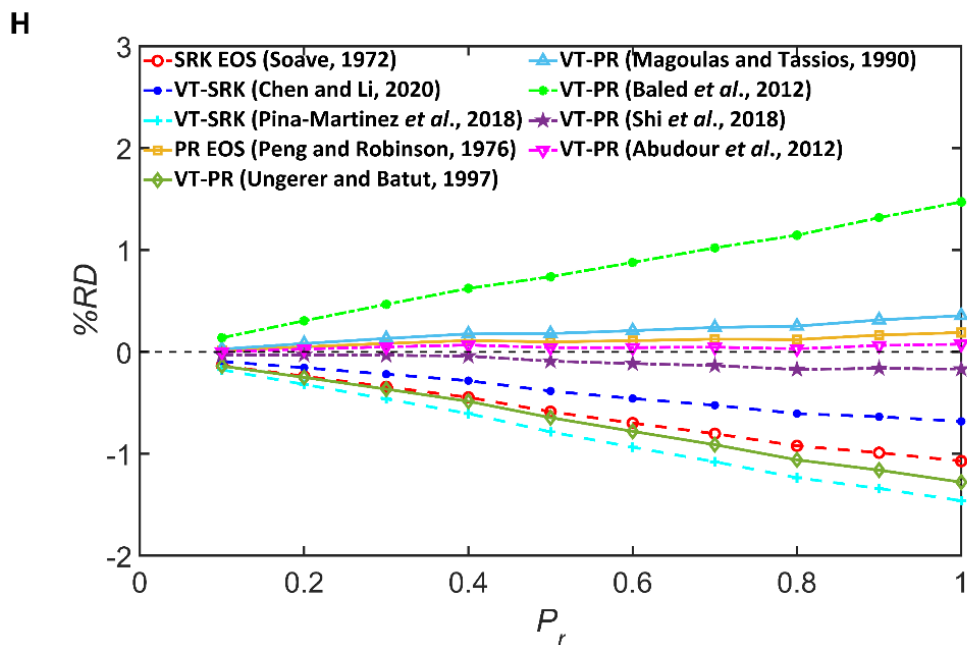


**B****C**



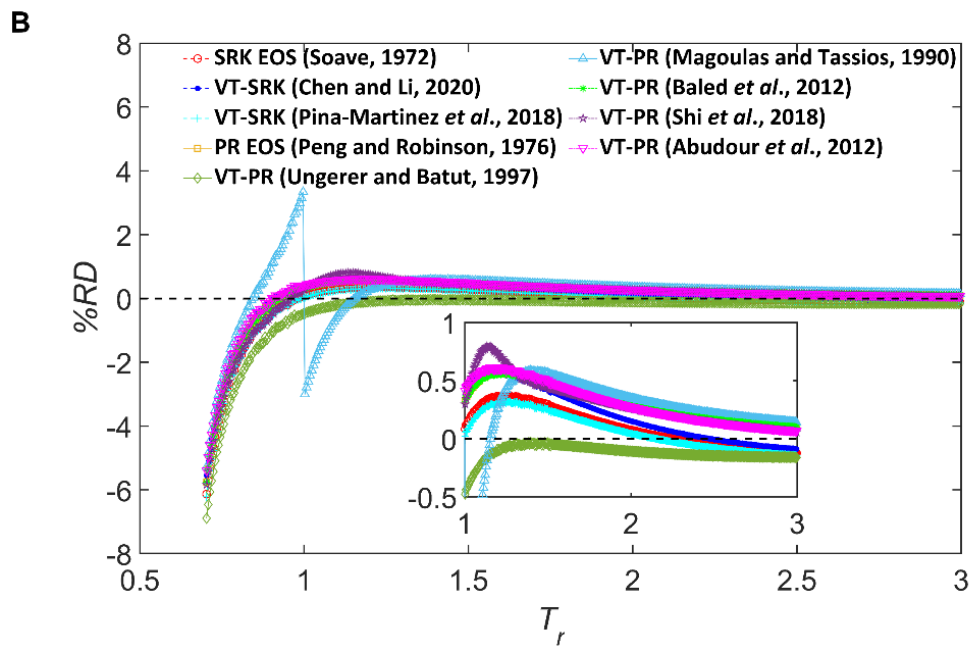
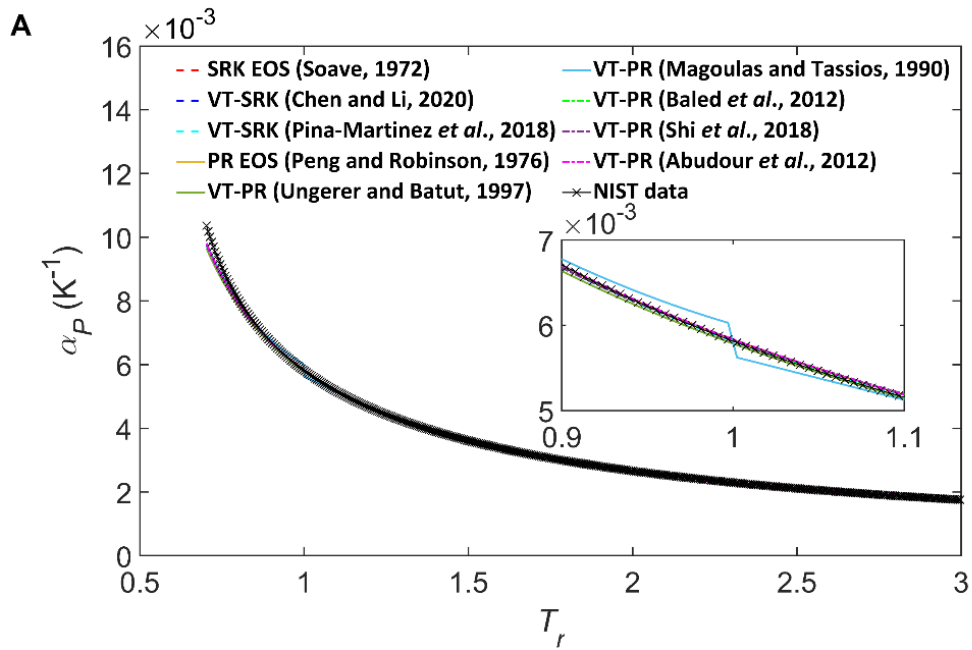


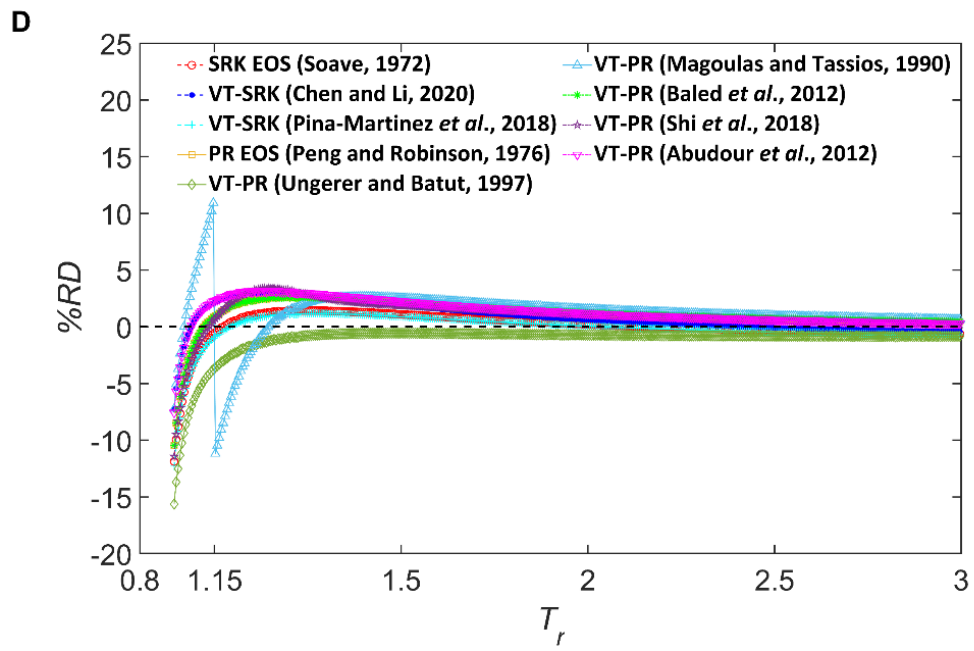
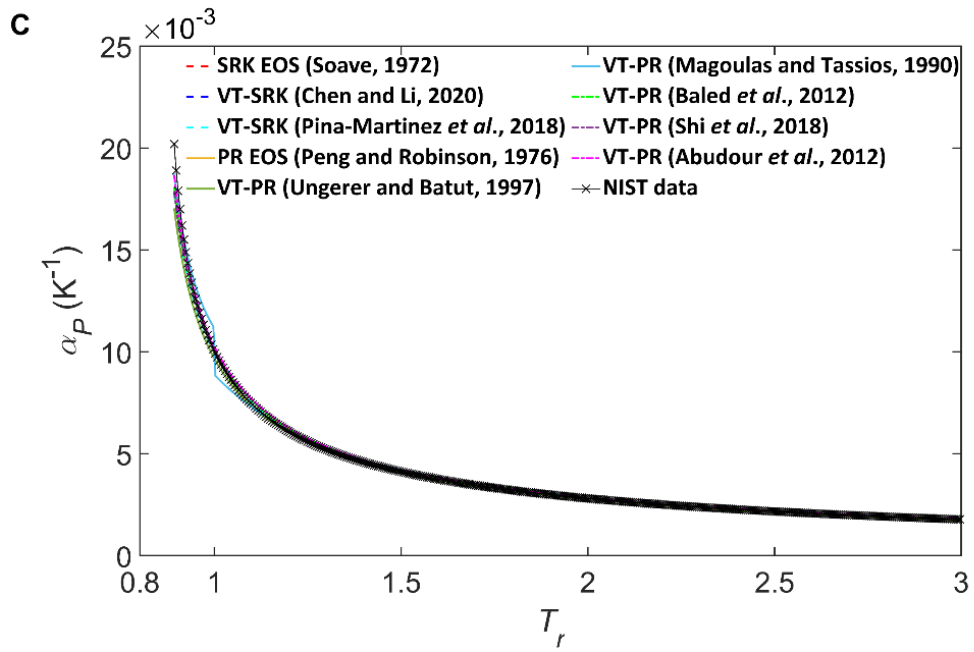


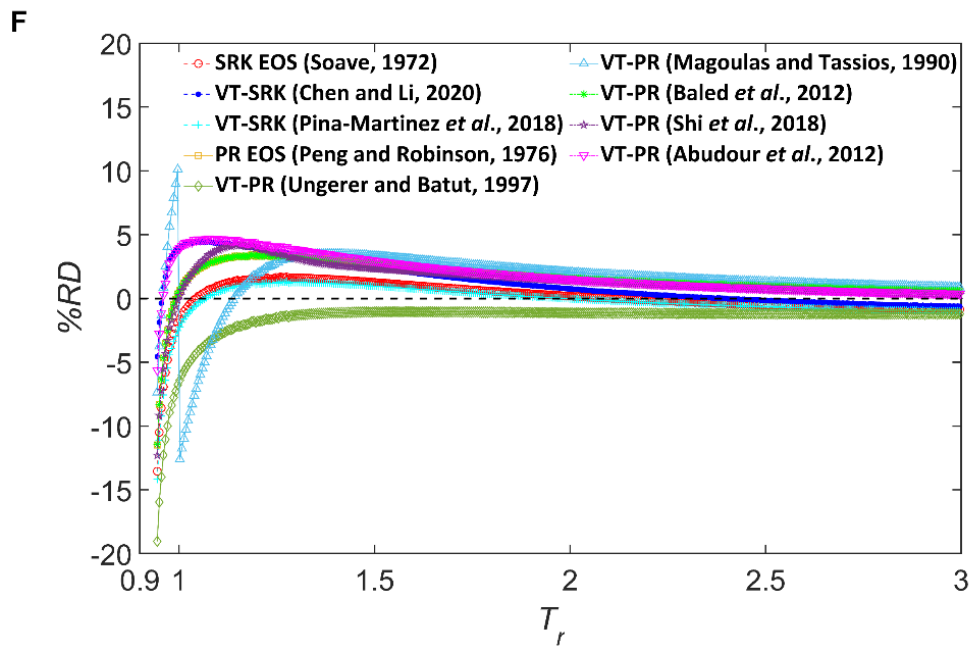
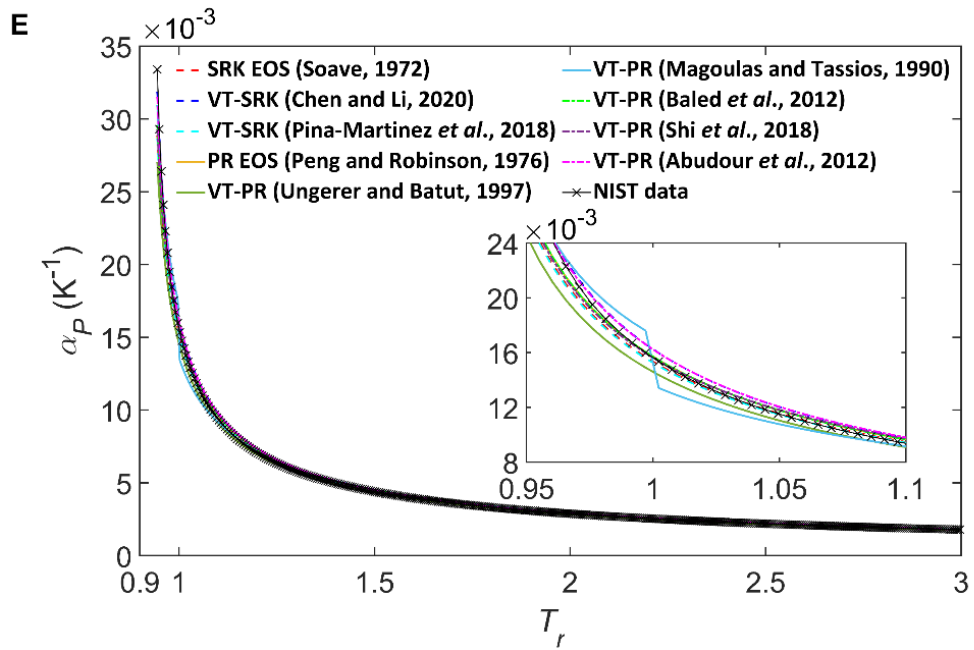


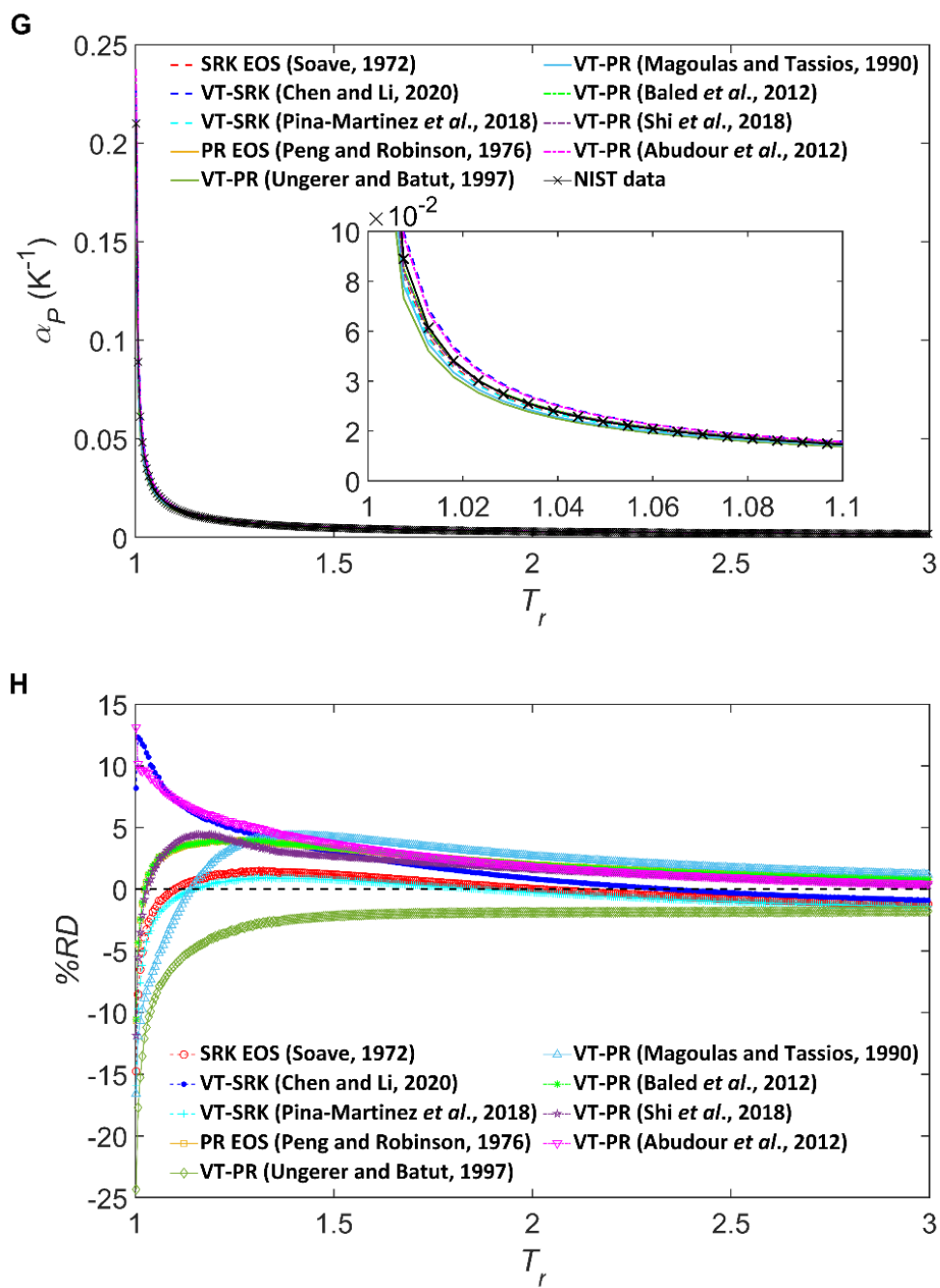
**Figure 9** Comparison of the calculated  $\kappa_T$  for the vapor-phase  $\text{CO}_2$  against the NIST data (A, C, E, and G) and  $\%RDs$  yielded by the models studied in this work (B, D, F, and H) at pressures from  $P_r = 0.1$  to  $P_r = 1$  and different temperatures:  $T_r = 0.9$  (A and B),  $T_r = 1$  (C and D),  $T_r = 2$  (E and F),  $T_r = 3$  (G and H).

Figures 10 and 11 illustrate the performance of each model in predicting the vapor-phase  $\alpha_p$  at different pressures for  $\text{CH}_4$  and  $\text{CO}_2$ , respectively. It can be seen that the predictions are also consistent with the NIST data.<sup>7</sup> However, there is a dramatic drop in the vapor-phase  $\alpha_p$  calculated by the Magoulas and Tassios model<sup>12</sup> for both  $\text{CH}_4$  and  $\text{CO}_2$  around the critical temperature. Based on the  $\%RDs$  distribution yielded by each model for the vapor-phase  $\text{CH}_4$  and  $\text{CO}_2$ , the predictions of vapor-phase  $\alpha_p$  become more accurate as temperature increases along an isobar and are relatively more accurate at temperatures above the critical temperature.

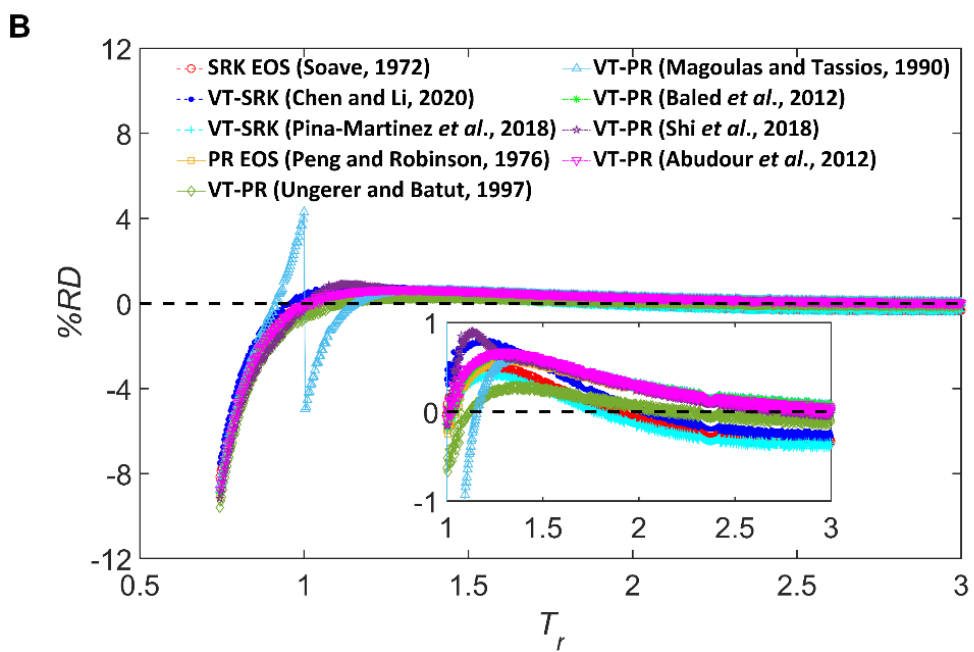
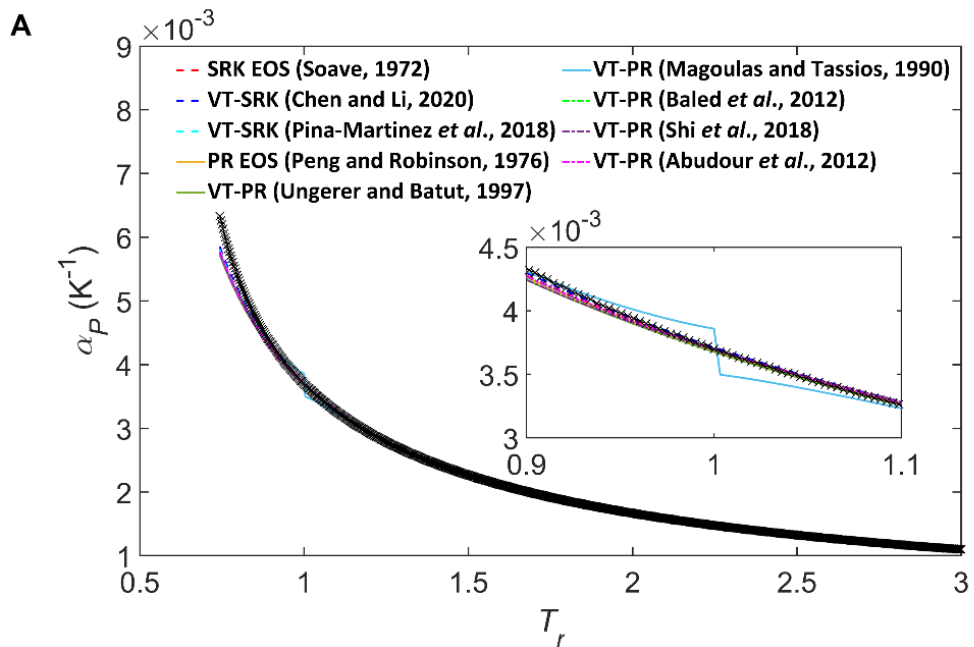


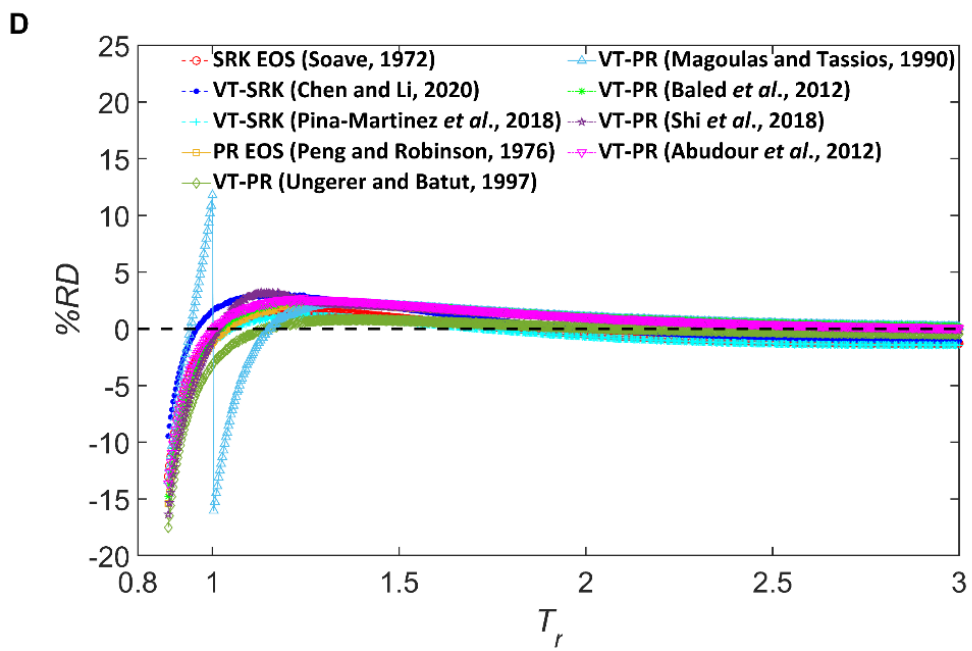
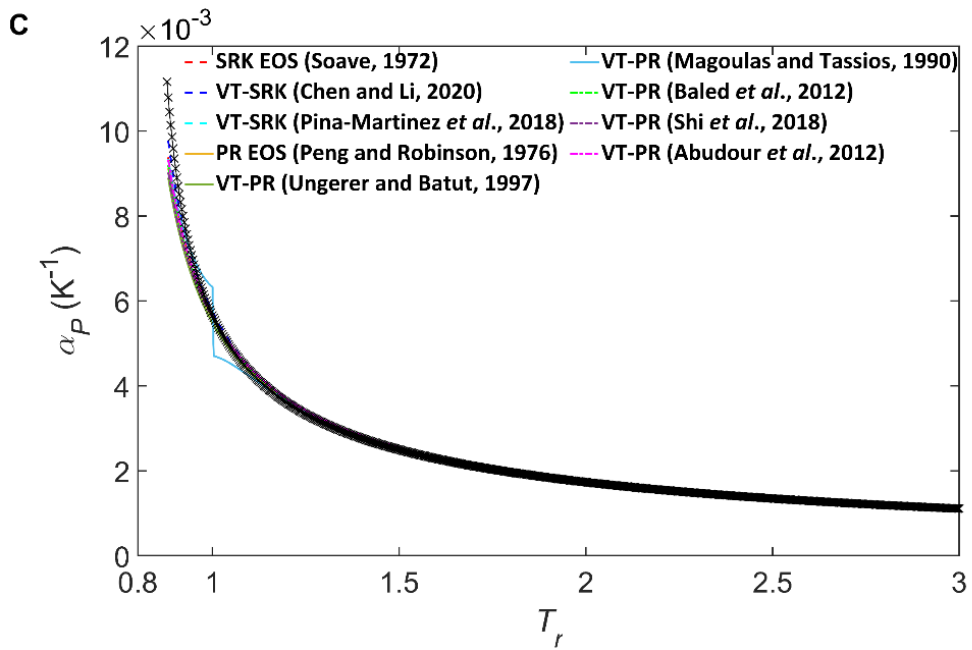


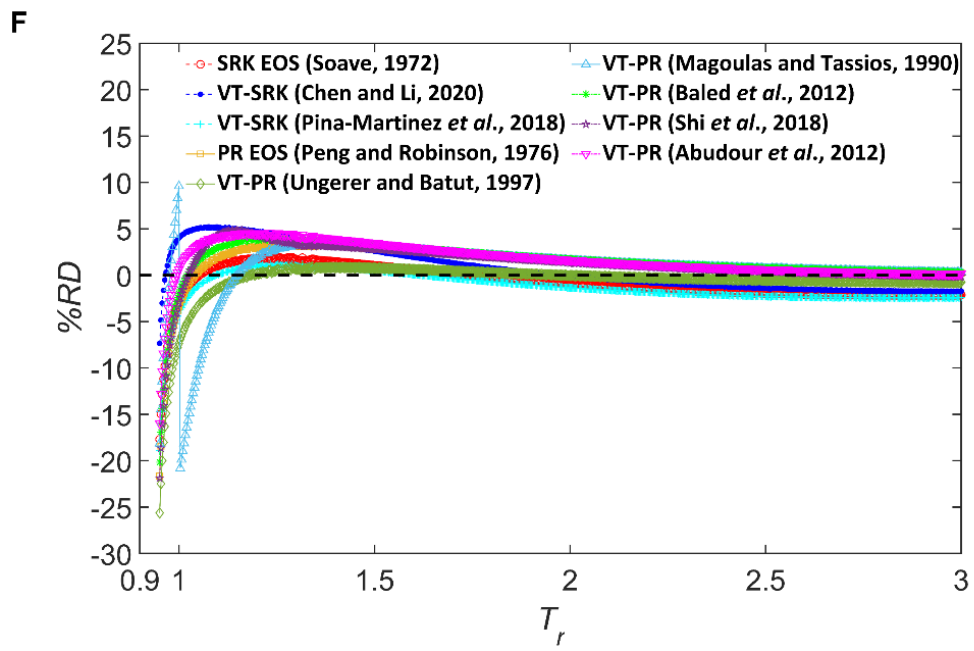
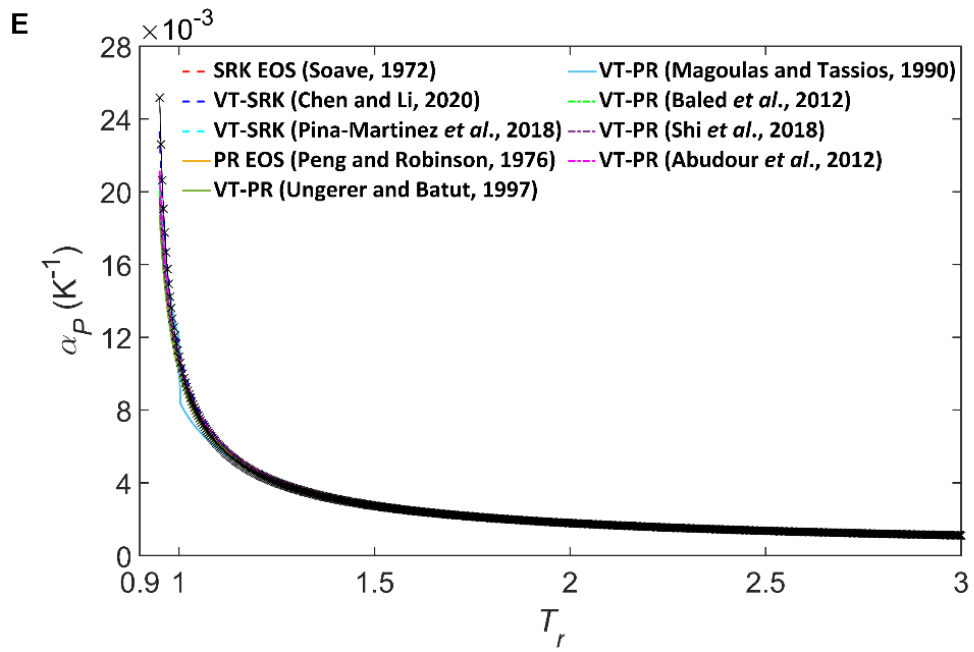




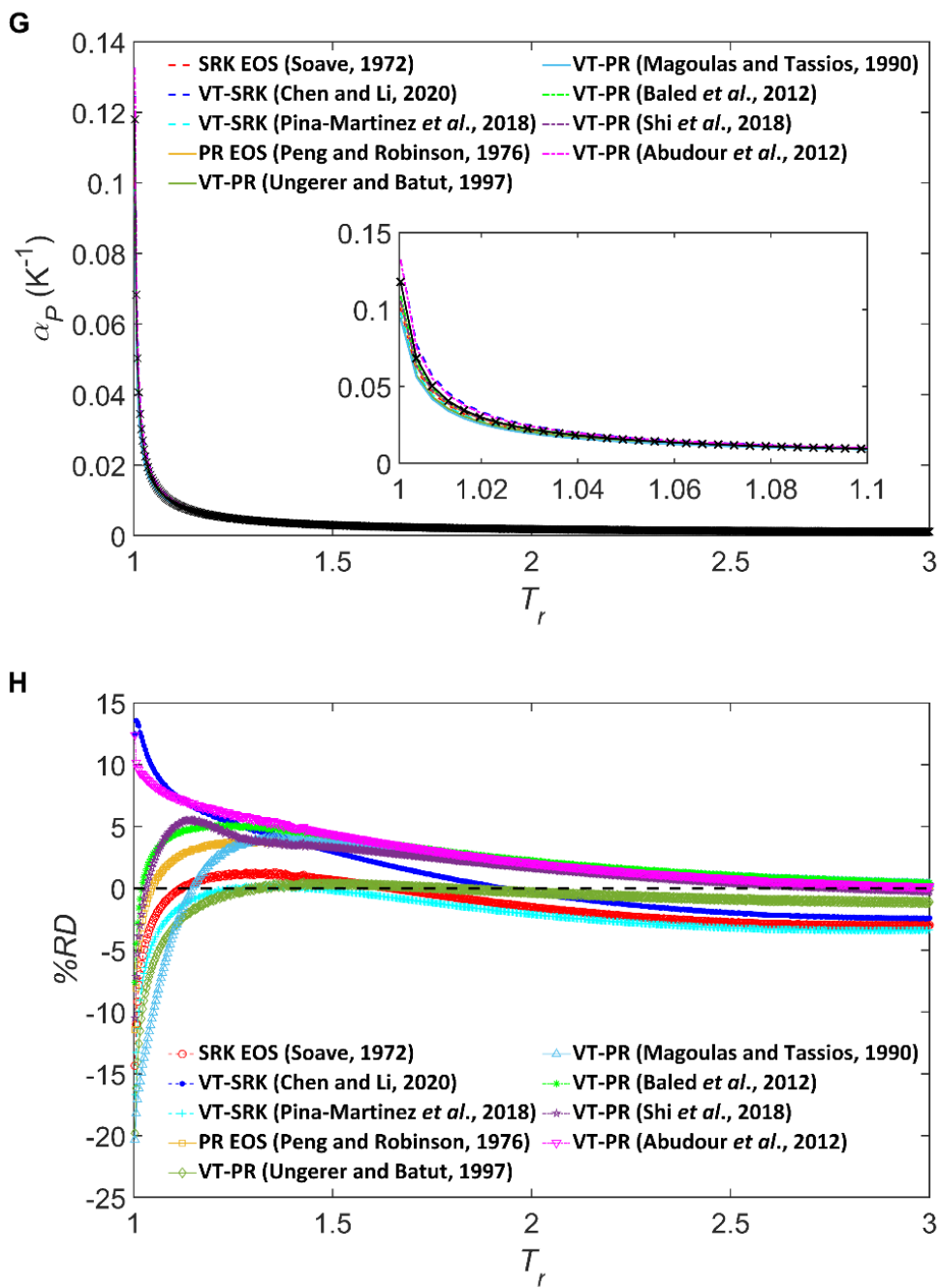
**Figure 10** Comparison of the calculated  $\alpha_p$  for the vapor-phase CH<sub>4</sub> against the NIST data (A, C, E, and G) and %RDs yielded by the models studied in this work (B, D, F, and H) at temperatures from the saturated vapor temperature to  $T_r = 3$  and different pressures:  $P_r = 0.1$  (A and B),  $P_r = 0.4$  (C and D),  $P_r = 0.7$  (E and F), and  $P_r = 1$  (G and H).







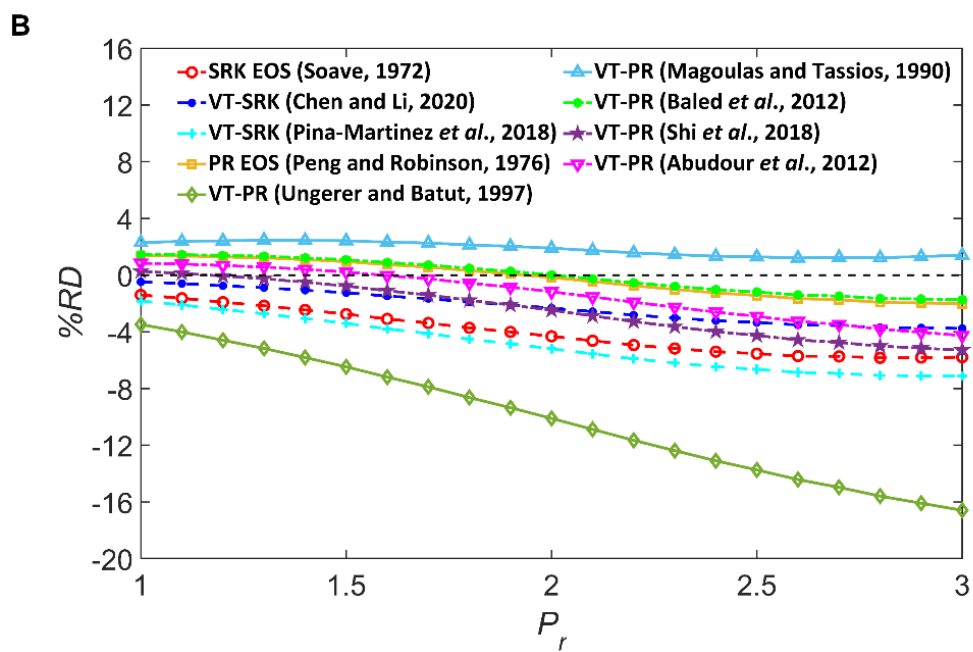
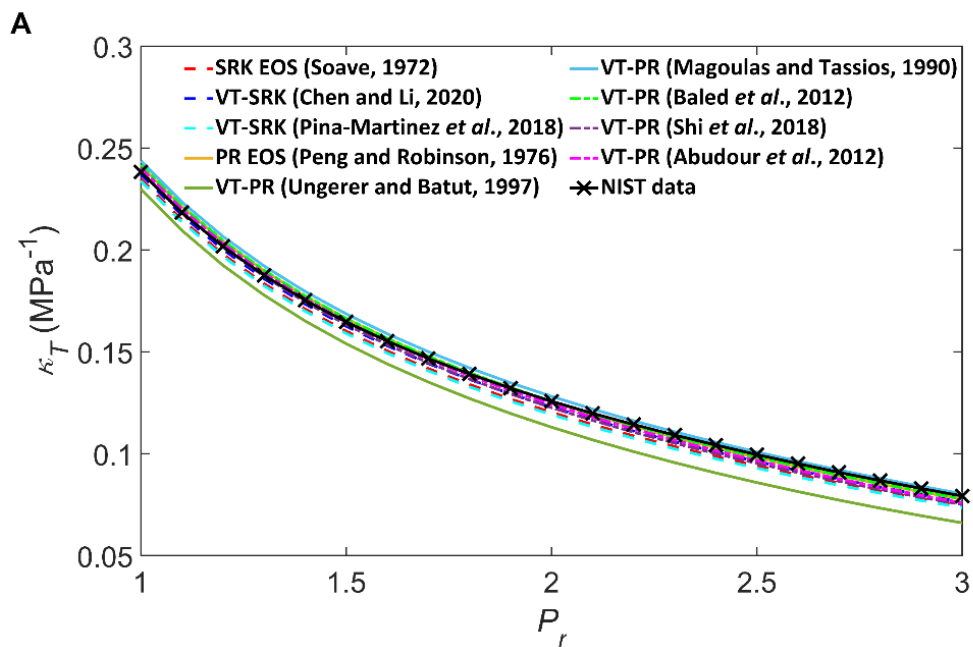


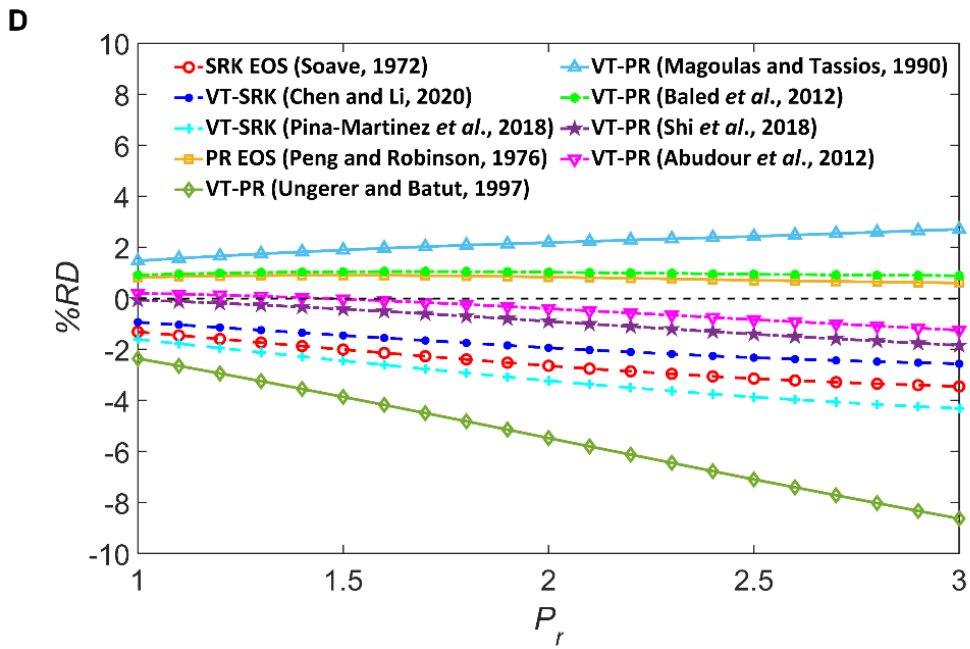
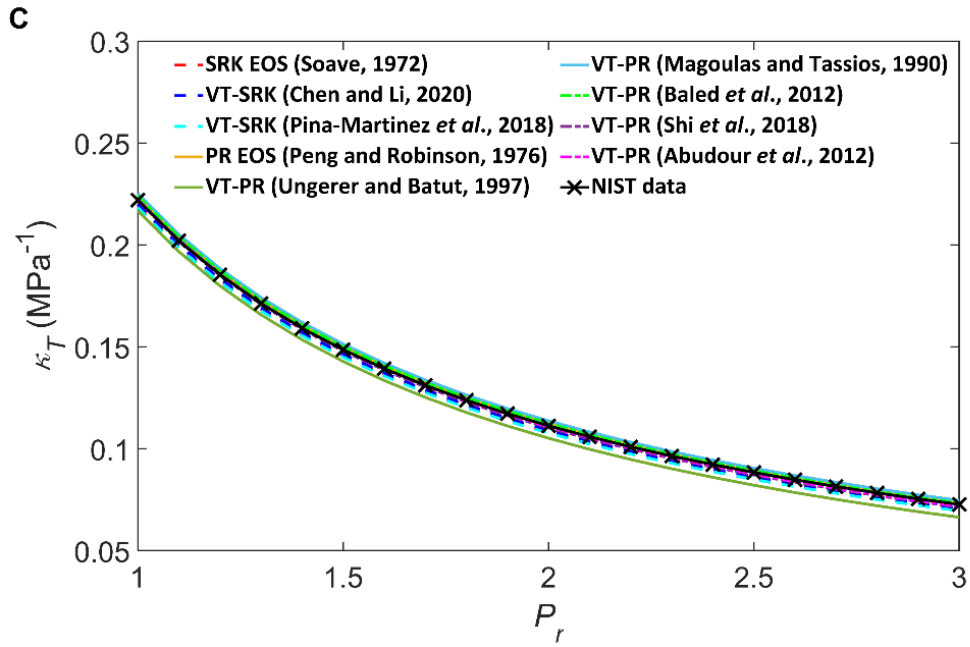


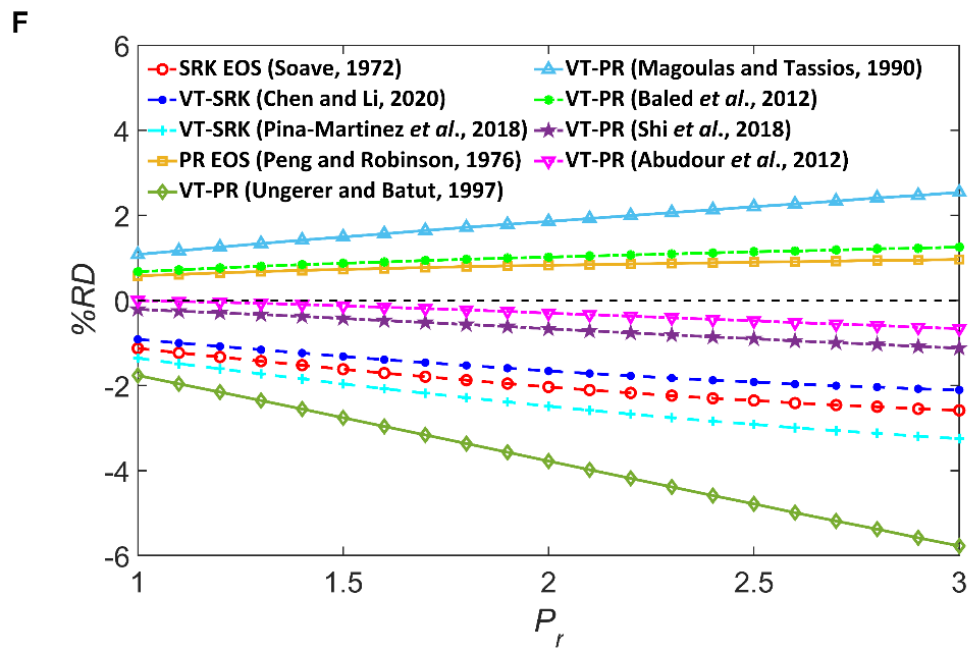
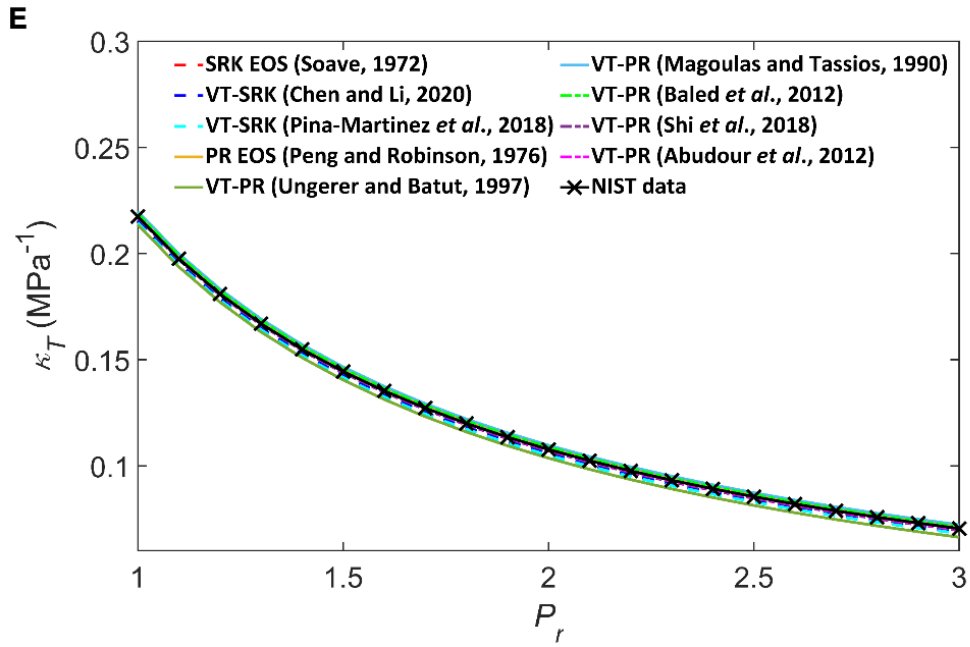
**Figure 11** Comparison of the calculated  $\alpha_p$  for the vapor-phase CO<sub>2</sub> against the NIST data (A, C, E, and G) and %RDs yielded by the models studied in this work (B, D, F, and H) at temperatures from the saturated vapor temperature to  $T_r = 3$  and different pressures:  $P_r = 0.1$  (A and B),  $P_r = 0.4$  (C and D),  $P_r = 0.7$  (E and F), and  $P_r = 1$  (G and H).

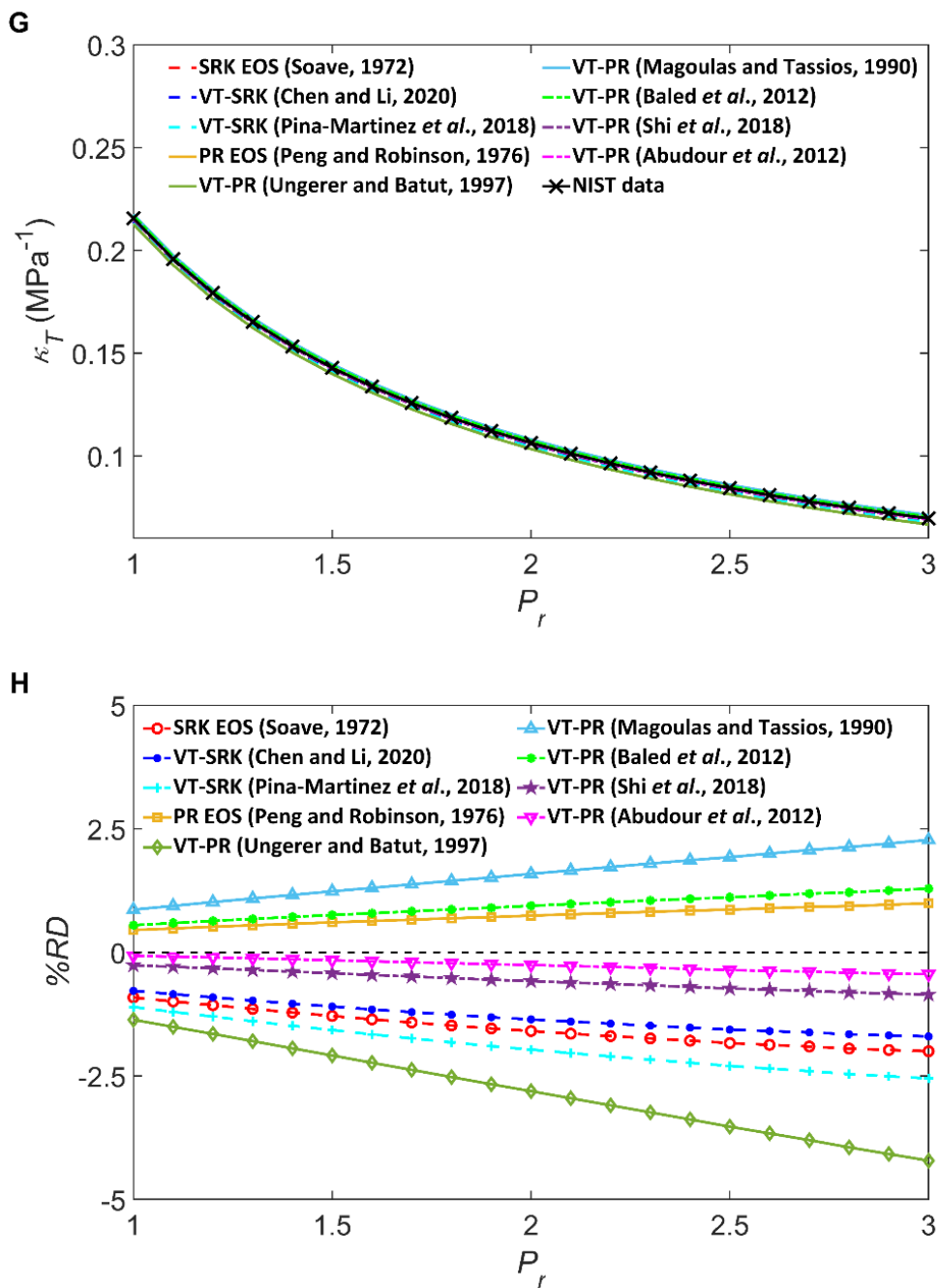
### 3.2.3. Predicted Results for the Supercritical-phase $\kappa_T$ and $\alpha_P$

The predictions of the supercritical-phase  $\kappa_T$  and  $\alpha_P$  by different VT models are still relatively accurate. This can be partially attributed to the relatively good performance of the untranslated PR EOS and SRK EOS in the supercritical-phase region. According to the %AADs of  $\kappa_T$  predictions for the supercritical CO<sub>2</sub> and CH<sub>4</sub>, the VT models of Abudour *et al.*<sup>3</sup> and Chen and Li<sup>5</sup> can double the accuracy yielded by their corresponding untranslated EOSs. The VT-PR EOS by Abudour *et al.*<sup>3</sup> yields the lowest %AADs of 1.11% and 1.41% of  $\kappa_T$  predictions for the supercritical phases of CO<sub>2</sub> and CH<sub>4</sub>, respectively. The Chen and Li VT-SRK EOS<sup>5</sup> yields the second-lowest %AAD of 2.26% for the supercritical-phase  $\kappa_T$  of CH<sub>4</sub>. The Magoulas and Tassios VT-PR EOS<sup>12</sup> provides relatively accurate results for the supercritical phase of CO<sub>2</sub> with a %AAD of 2.04% but a larger %AAD of 3.52% for CH<sub>4</sub>. The other VT models yield higher %AADs than their corresponding untranslated EOSs for the supercritical phases of CO<sub>2</sub> and CH<sub>4</sub>. Figure 12 compares the predicted  $\kappa_T$  for the supercritical CH<sub>4</sub> against the NIST data,<sup>7</sup> as well as the corresponding %RDs yielded by each EOS model. Figure 13 shows the same information for the supercritical CO<sub>2</sub>. Compared to the NIST data,<sup>7</sup> all models can correctly capture the variation trend of  $\kappa_T$  versus pressure change.  $\kappa_T$  in the supercritical-phase region tends to decrease with an increasing pressure at a given temperature. The calculated %RDs are relatively low but tend to become enlarged as pressure increases at a fixed temperature.

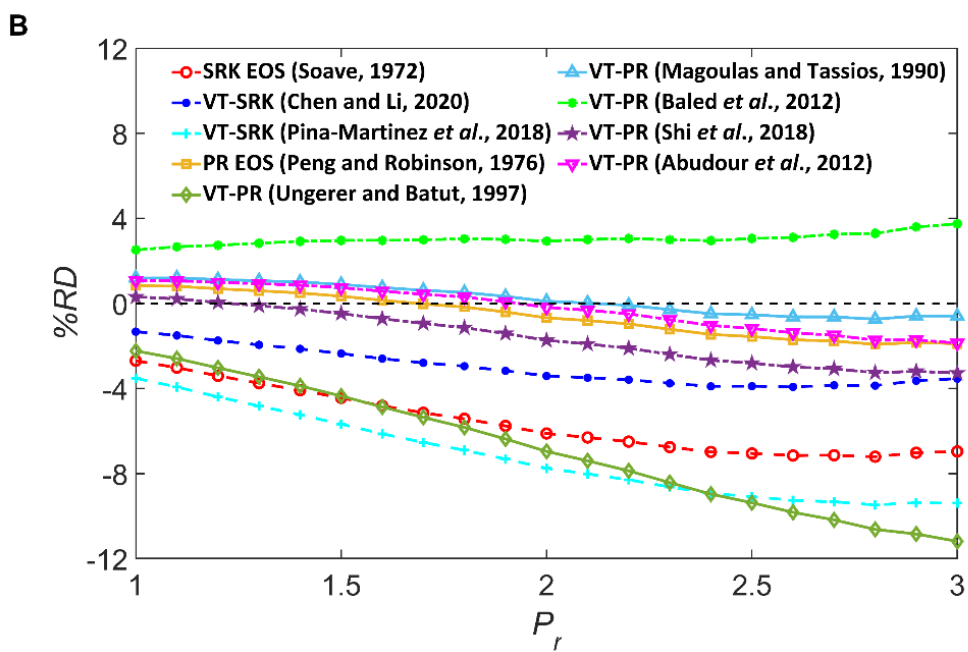
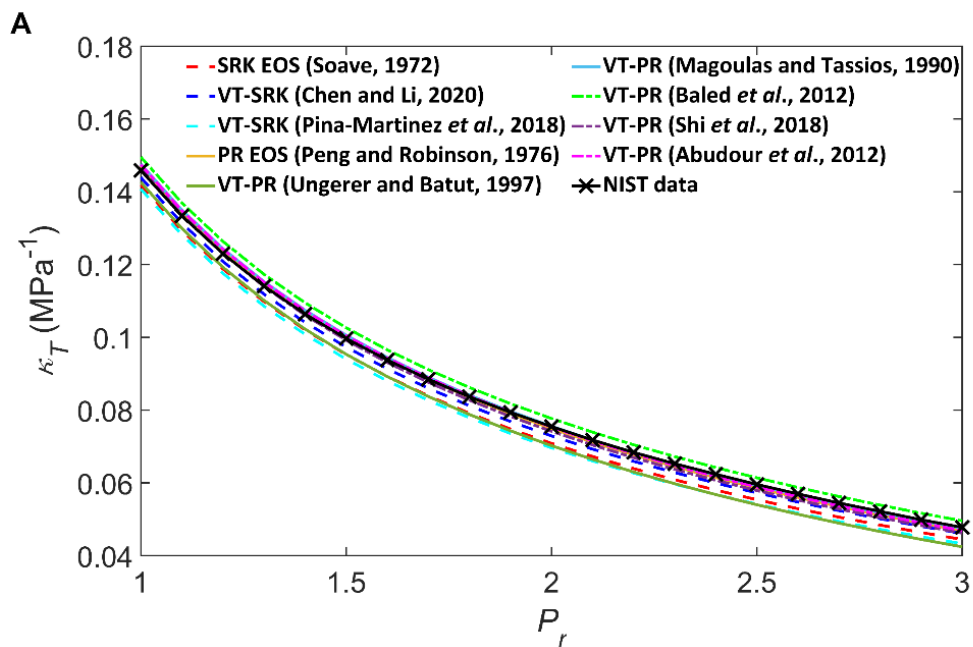


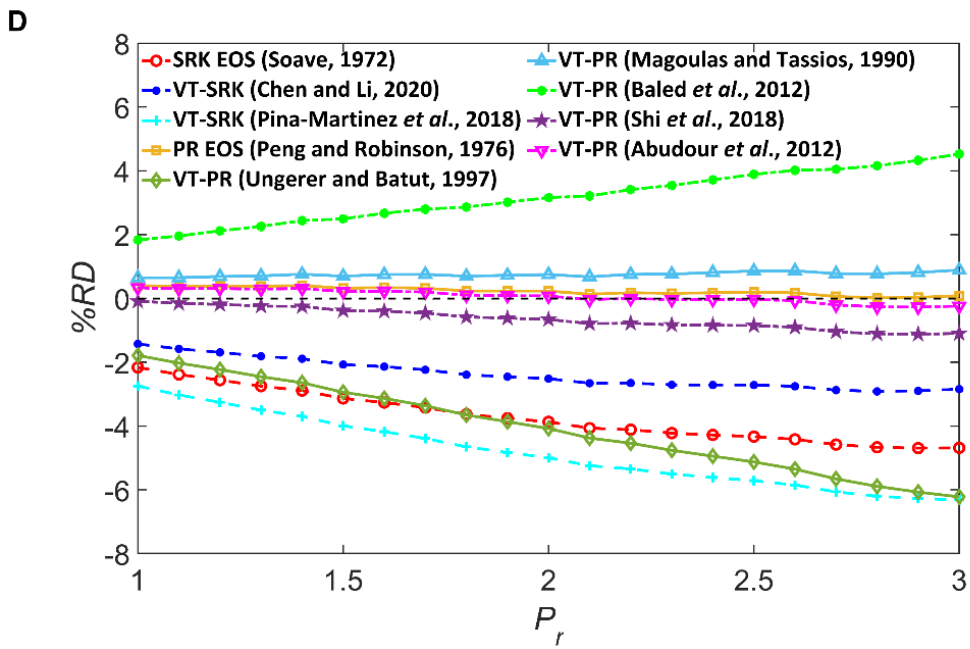
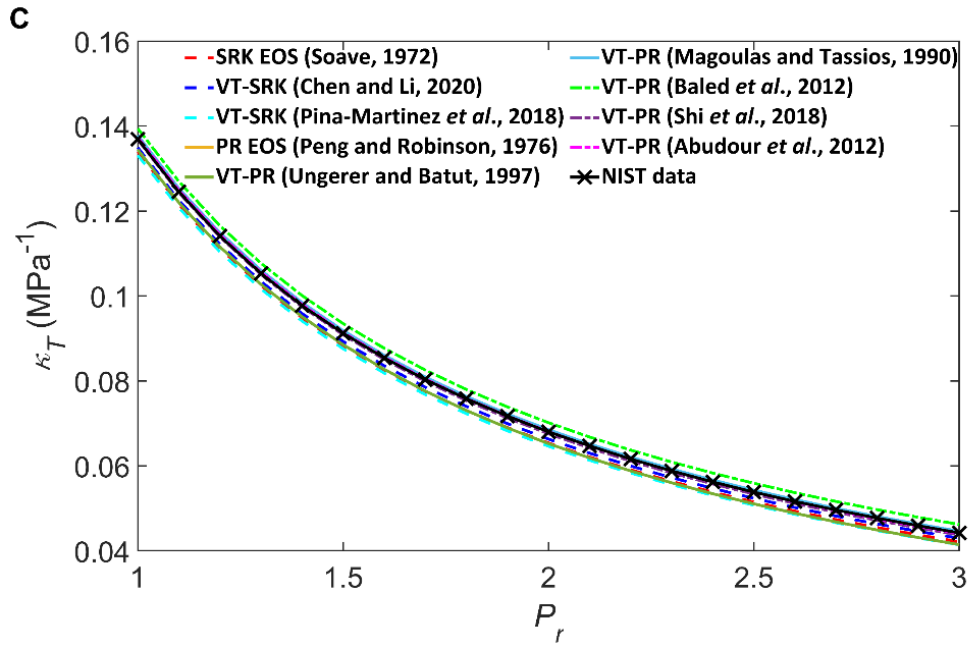




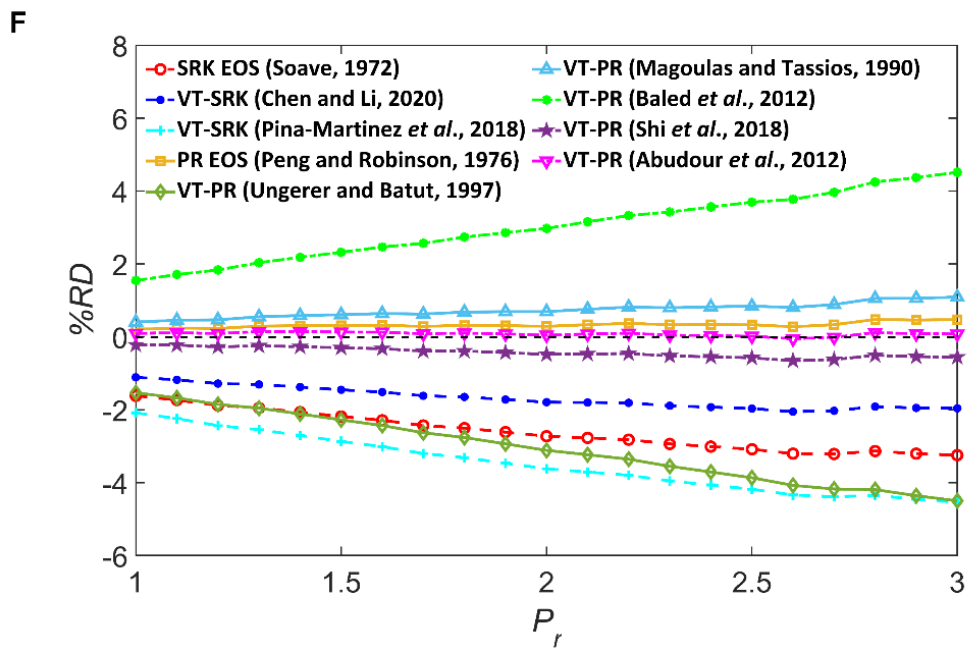
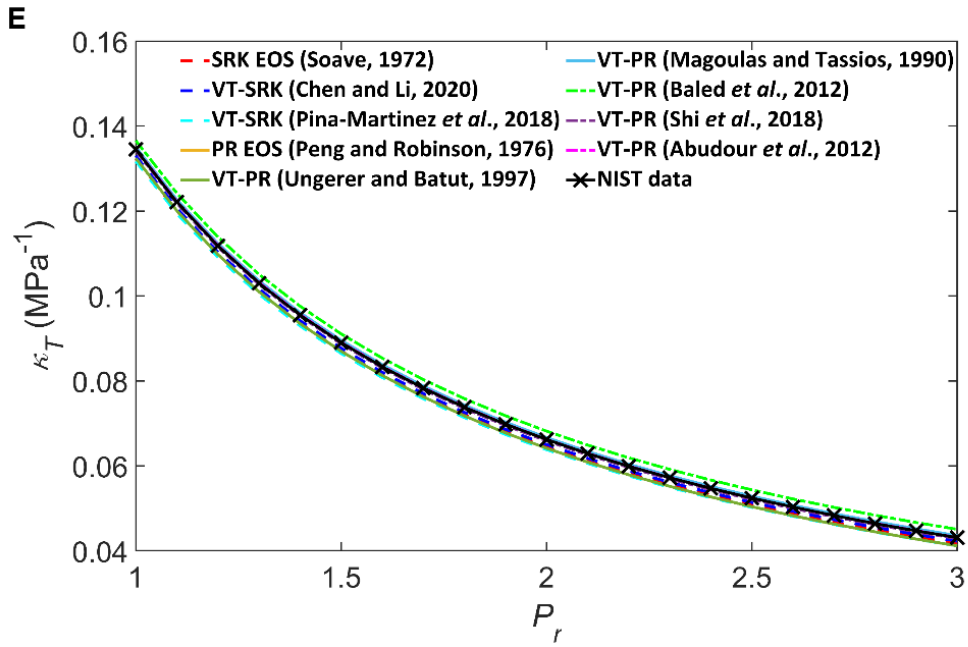


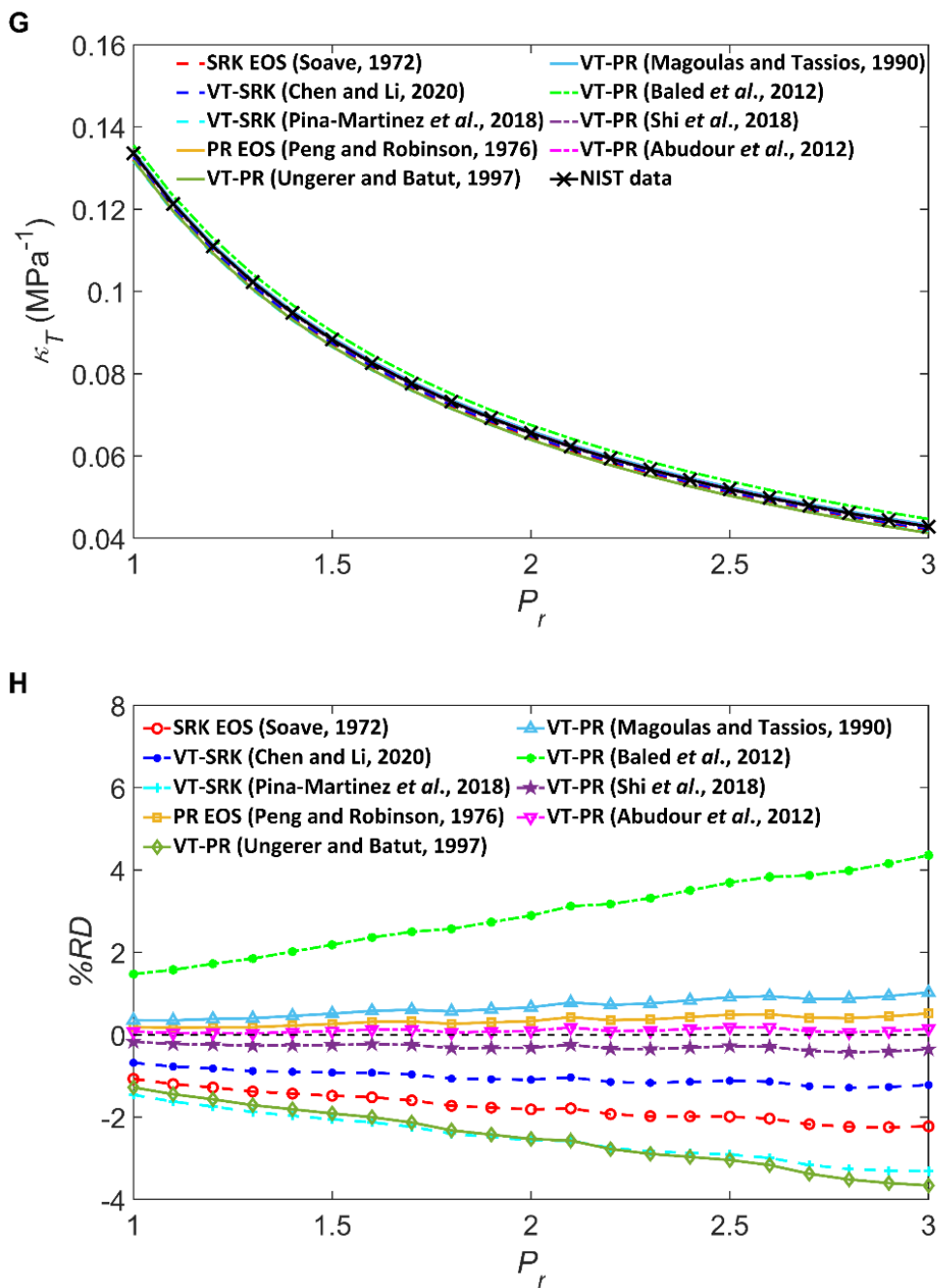
**Figure 12** Comparison of the calculated  $\kappa_T$  for the supercritical-phase CH<sub>4</sub> against the NIST data (A, C, E, and G) and %RDs yielded by the models studied in this work (B, D, F, and H) at pressures from  $P_r = 1$  to  $P_r = 3$  and different temperatures:  $T_r = 1.5$  (A and B),  $T_r = 2$  (C and D),  $T_r = 2.5$  (E and F), and  $T_r = 3$  (G and H).







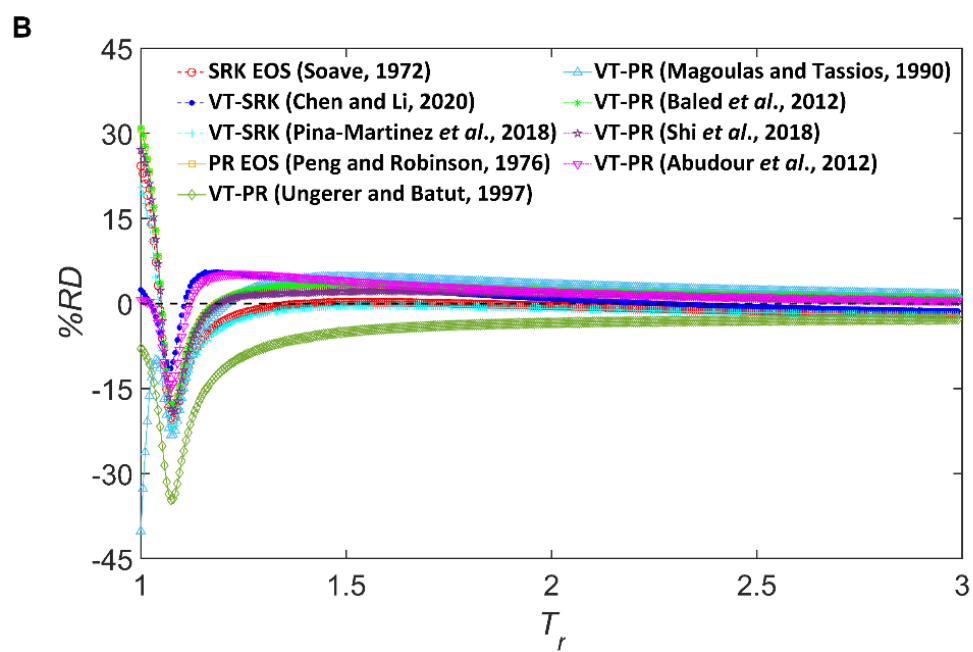
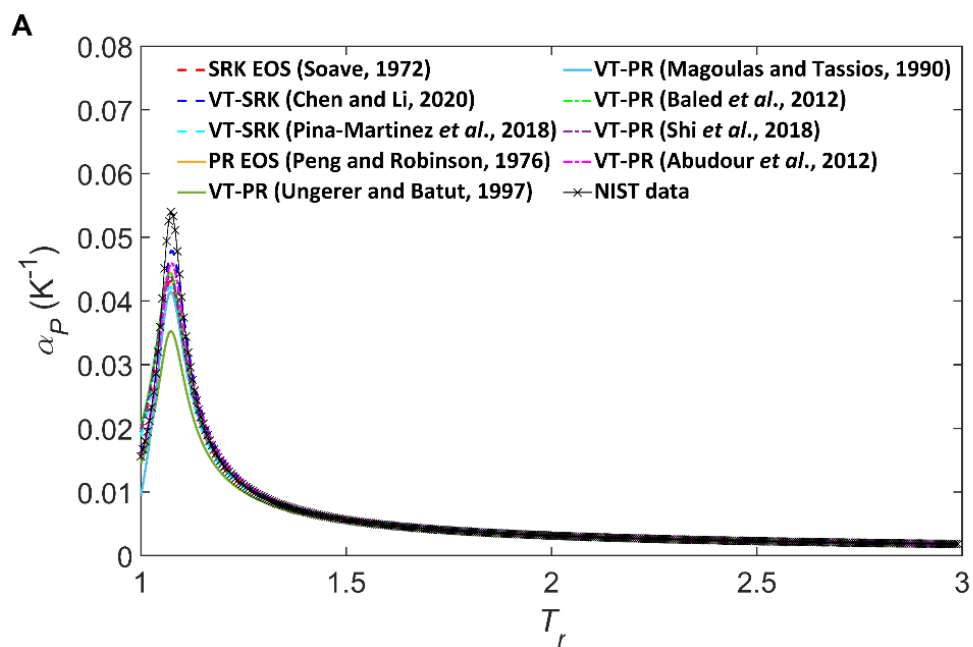


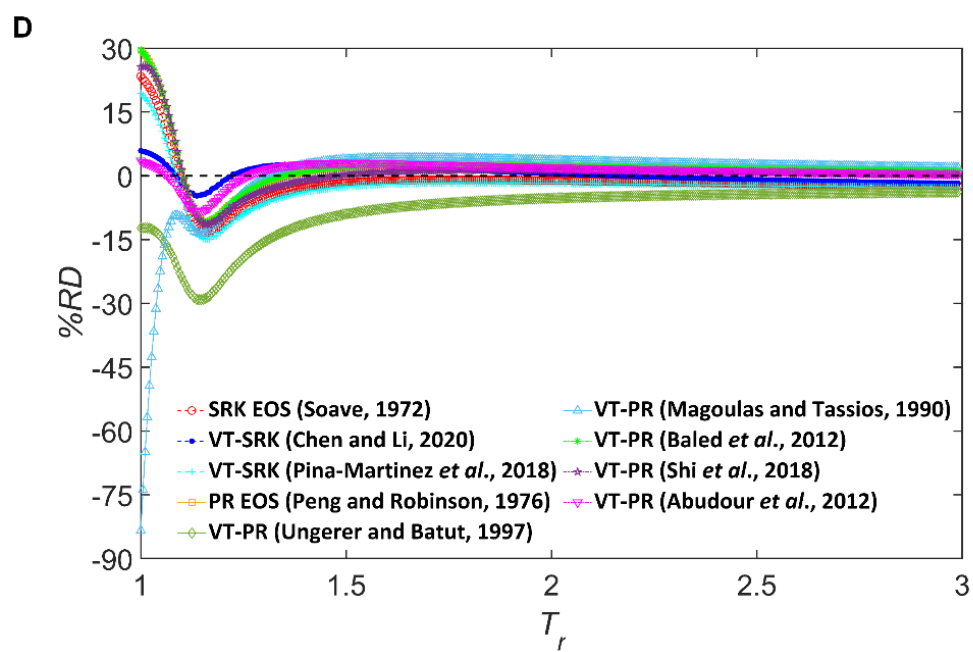
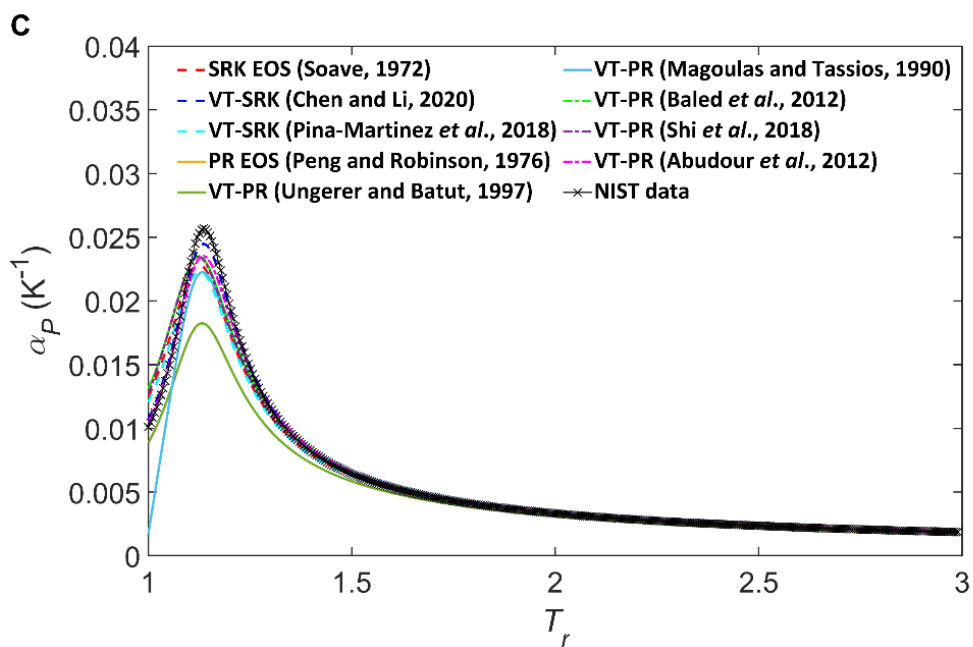


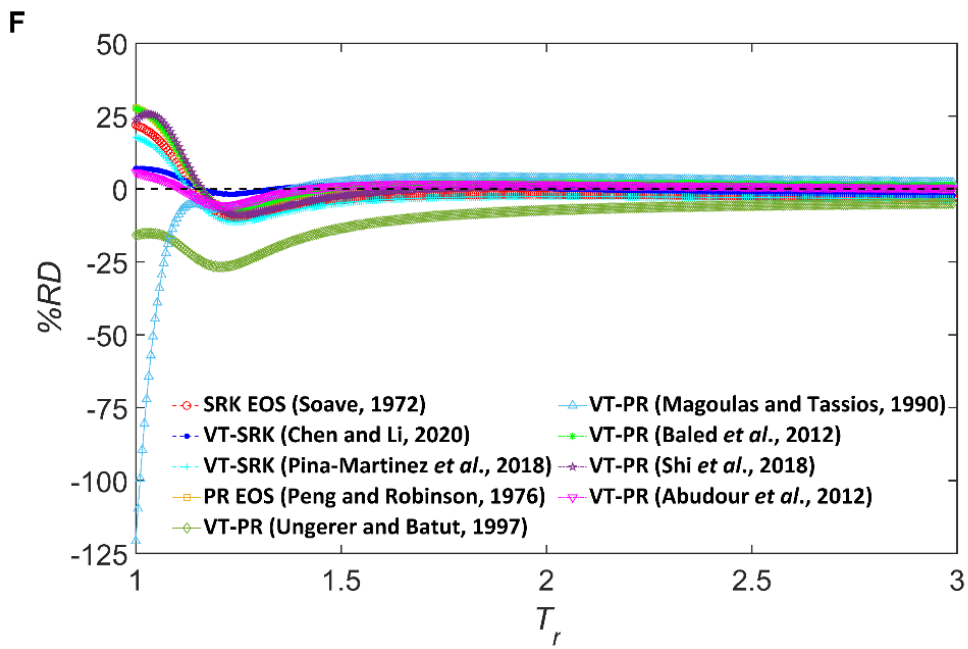
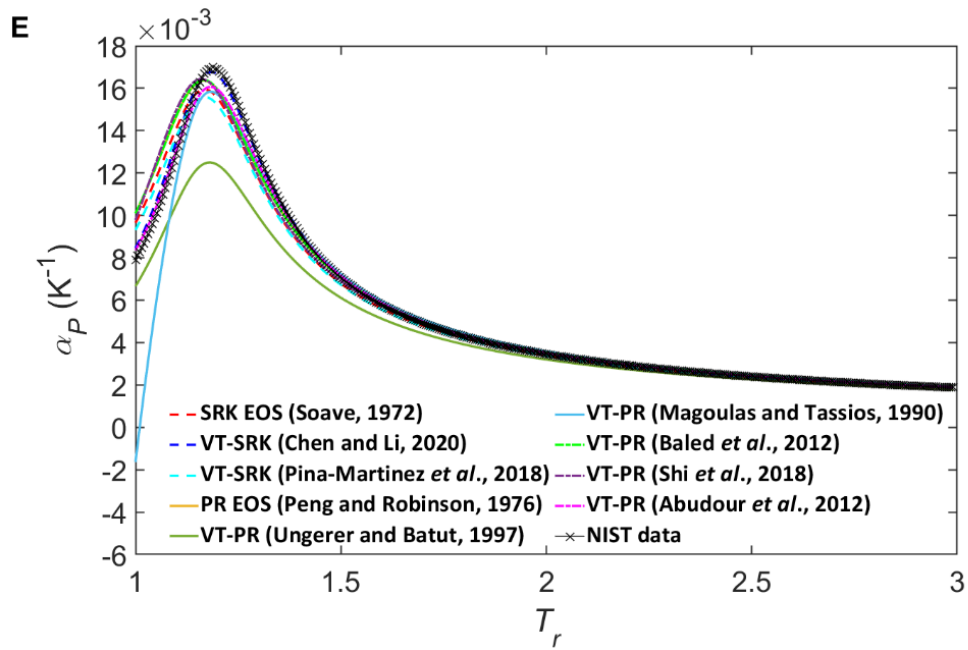
**Figure 13** Comparison of the calculated  $\kappa_T$  for the supercritical-phase CO<sub>2</sub> against the NIST data (A, C, E, and G) and %RDs yielded by the models studied in this work (B, D, F, and H) at pressures from  $P_r = 1$  to  $P_r = 3$  and different temperatures:  $T_r = 1.5$  (A and B),  $T_r = 2$  (C and D),  $T_r = 2.5$  (E and F),  $T_r = 3$  (G and H).

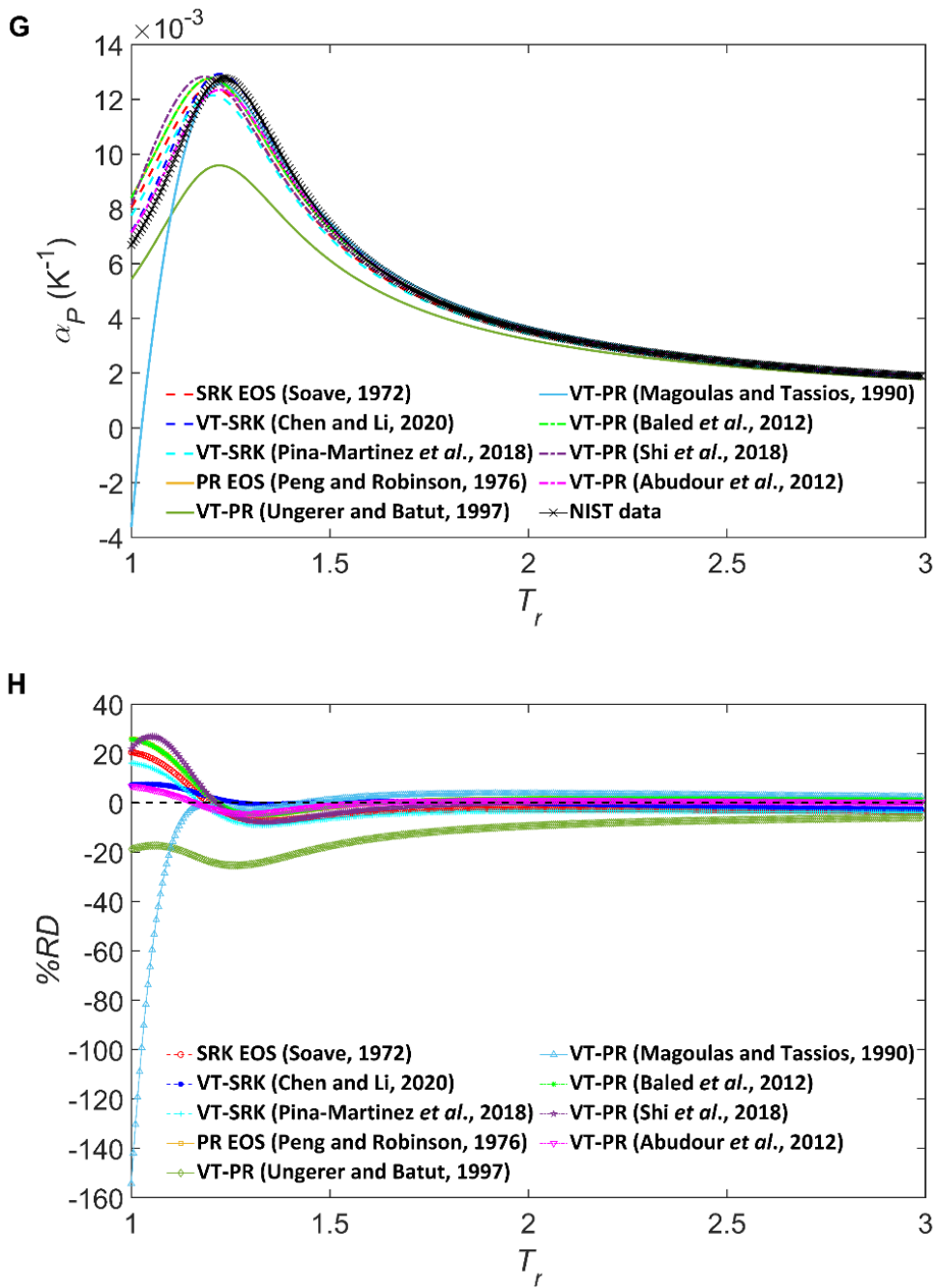
The predictions of  $\alpha_p$  for the supercritical phase by each EOS model are compared to the NIST data,<sup>7</sup> and the corresponding %RDs are also calculated. The results for CH<sub>4</sub> and CO<sub>2</sub> are shown in Figures 14 and 15, respectively. As shown in Figures 14 and 15, the predicted

values of  $\alpha_p$  tend to peak at temperatures above the critical temperature. Such variation trend of  $\alpha_p$  could be attributed to the existence of the Widom line.<sup>13</sup> The Widom line can be considered as an extension of the vapor pressure line.<sup>13</sup> The upper left side of the Widom line can be considered as a liquid-like supercritical phase, while the lower right side of the Widom line can be considered as a vapor-like supercritical phase.<sup>13</sup> Along an isobaric line in the liquid-like region,  $\alpha_p$  increases with an increasing temperature. Along an isobaric line in the vapor-like region,  $\alpha_p$  decreases with an increasing temperature. Therefore,  $\alpha_p$  can reach a maximum at the pseudo-saturation line in the supercritical phase. All models can predict the occurrence of the peaks but tend to yield relatively larger deviations of  $\alpha_p$  predictions around the peaks. It is worthwhile noting that the  $\alpha_p$  predicted by the Magoulas and Tassios model<sup>12</sup> for the supercritical phases of both CH<sub>4</sub> and CO<sub>2</sub> can be less than zero. These negative  $\alpha_p$  values appear around the critical temperature at  $P_r$  above 2.5. This can be attributed to the occurrence of the isotherm crossover phenomenon. Shi *et al.*<sup>14</sup> also reported that the Magoulas and Tassios model<sup>12</sup> has an isotherm crossover issue.

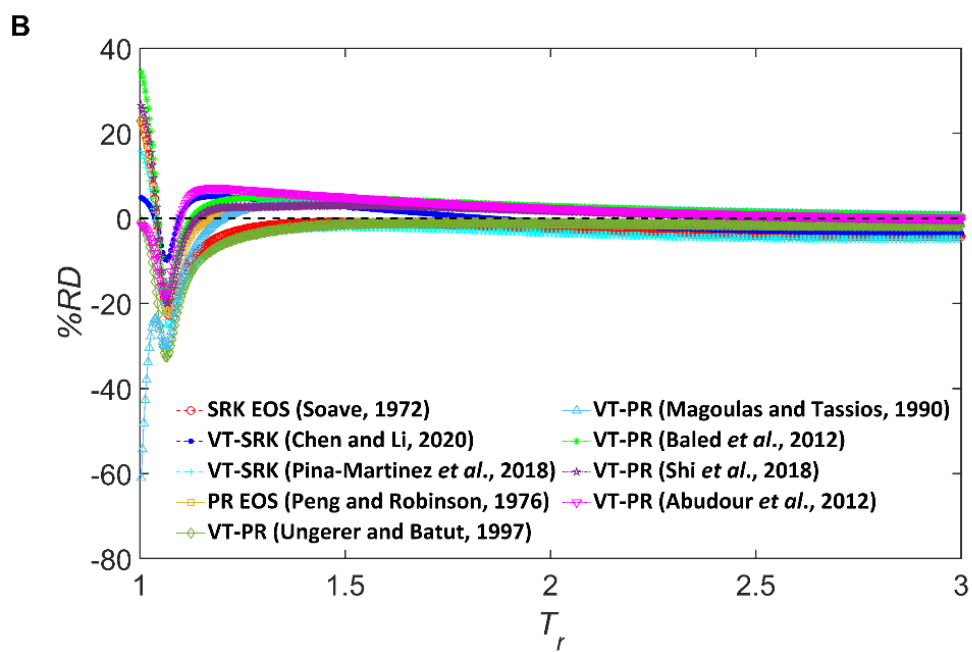
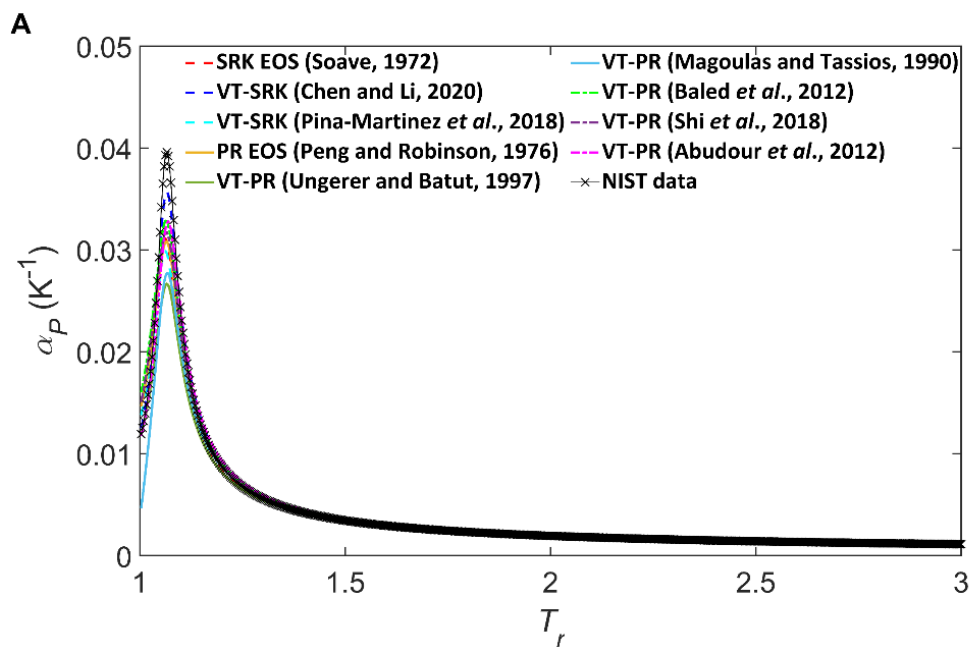




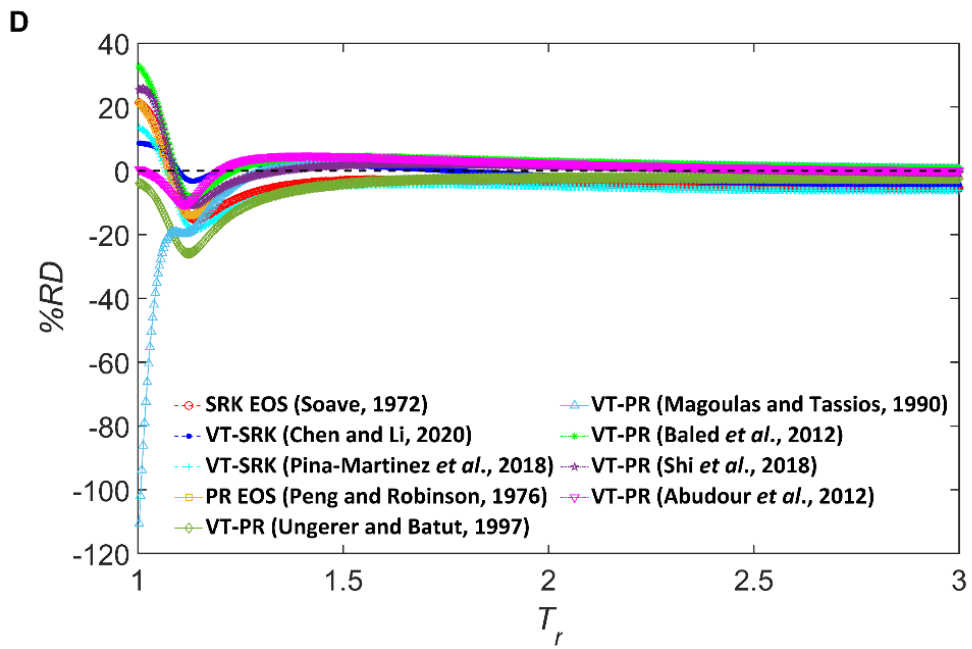
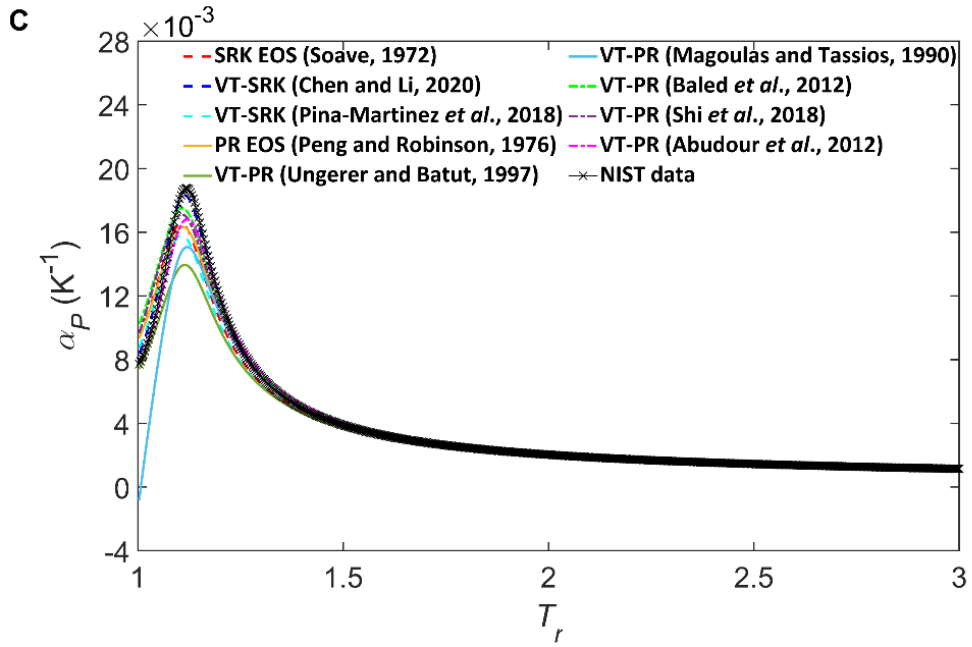


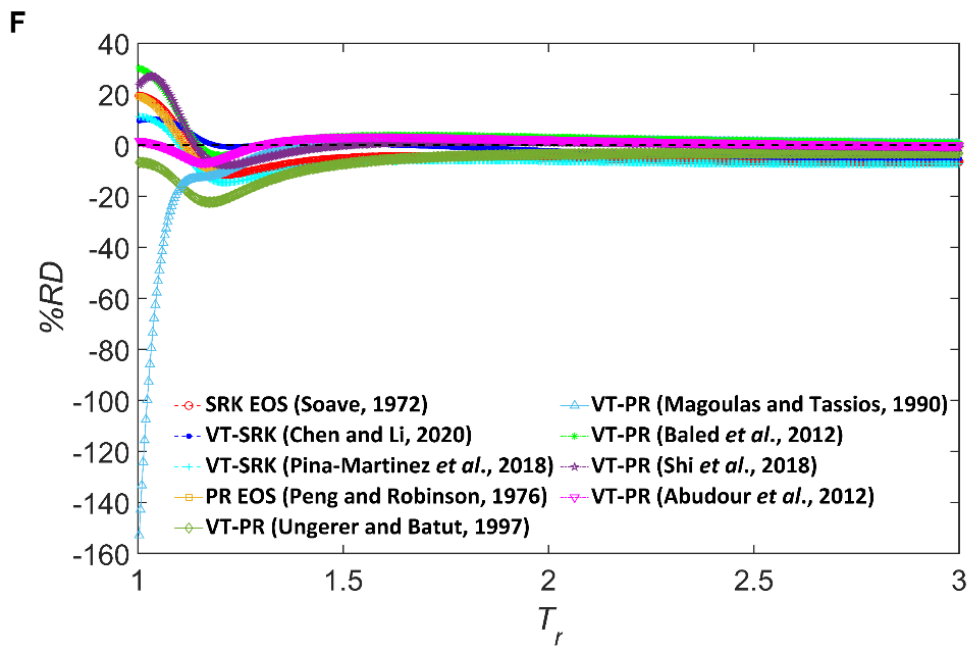
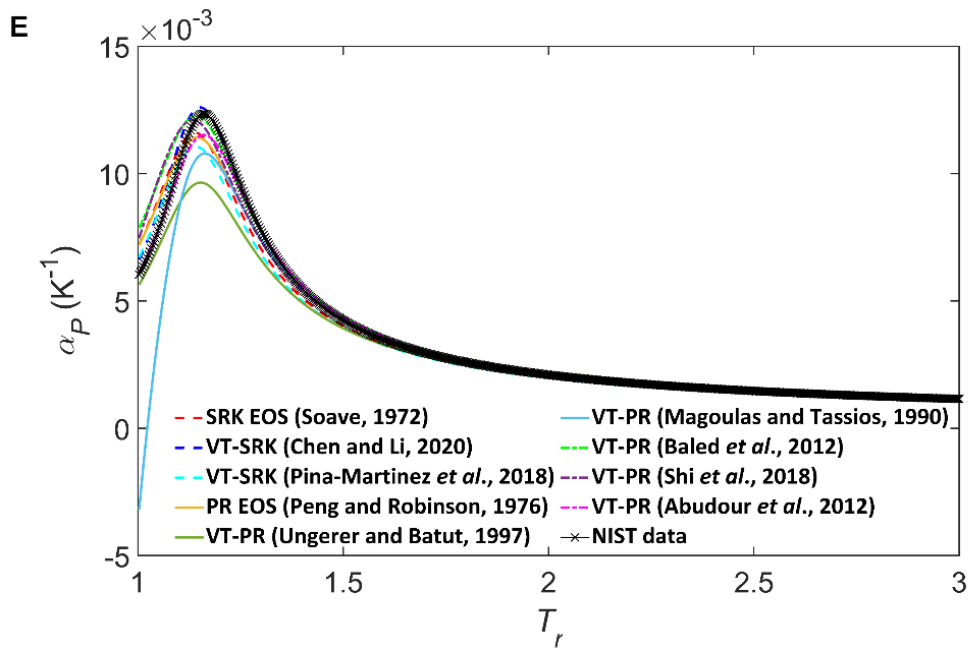


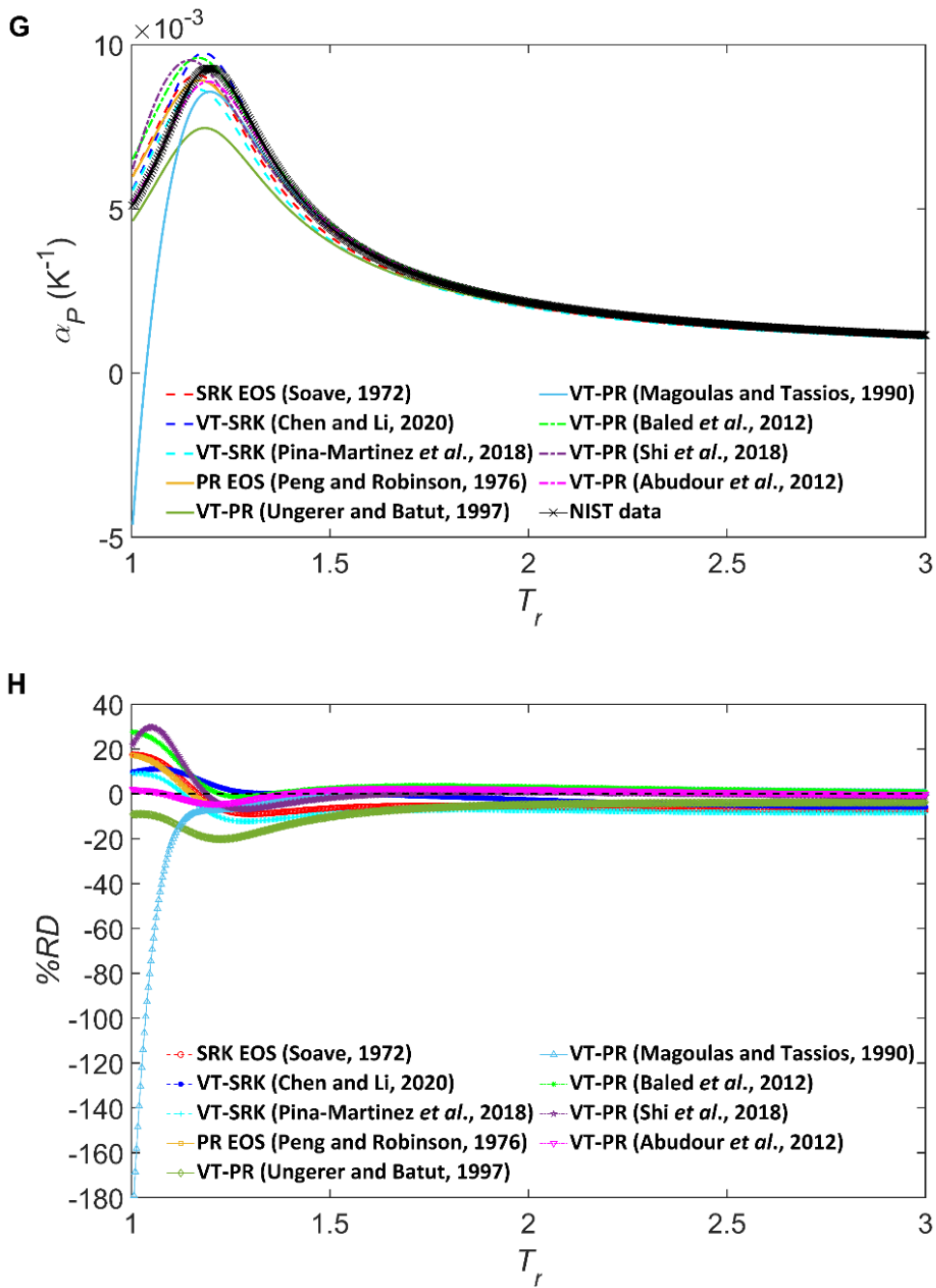
**Figure 14** Comparison of the calculated  $\alpha_p$  for the supercritical-phase CH<sub>4</sub> against the NIST data (A, C, E, and G) and %RDs yielded by the models studied in this work (B, D, F, and H) at temperatures from the  $T_r = 1$  to  $T_r = 3$  and different pressures:  $P_r = 1.5$  (A and B),  $P_r = 2$  (C and D),  $P_r = 2.5$  (E and F), and  $P_r = 3$  (G and H).









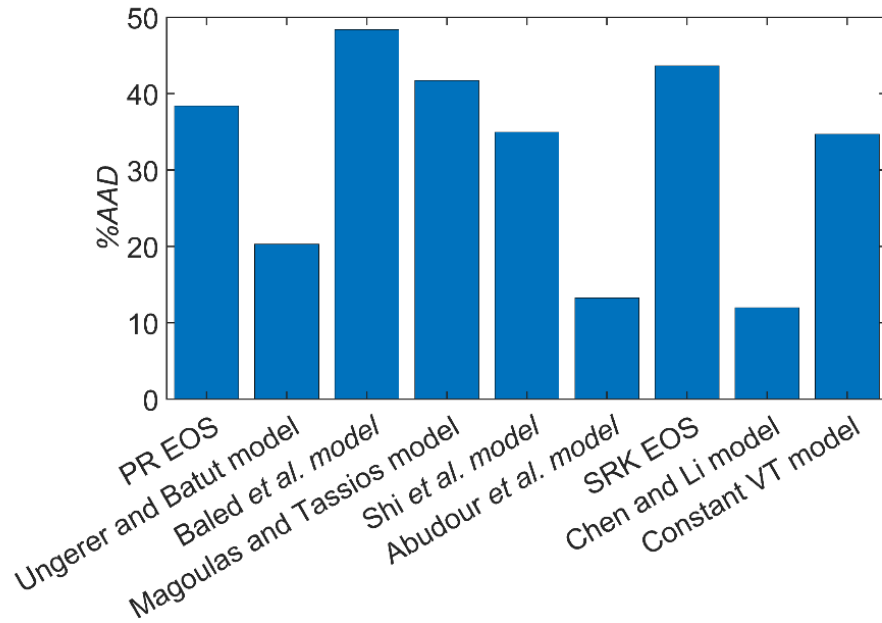


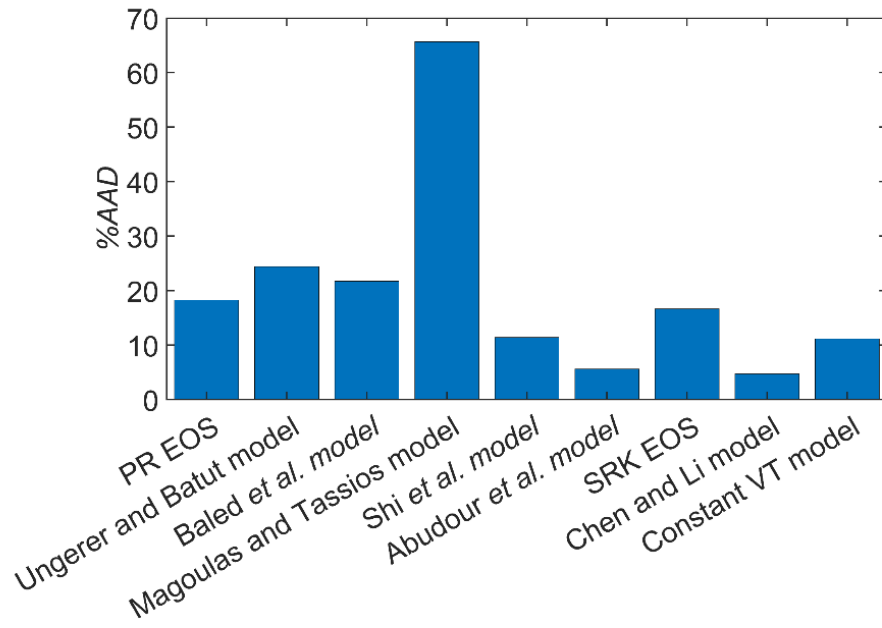
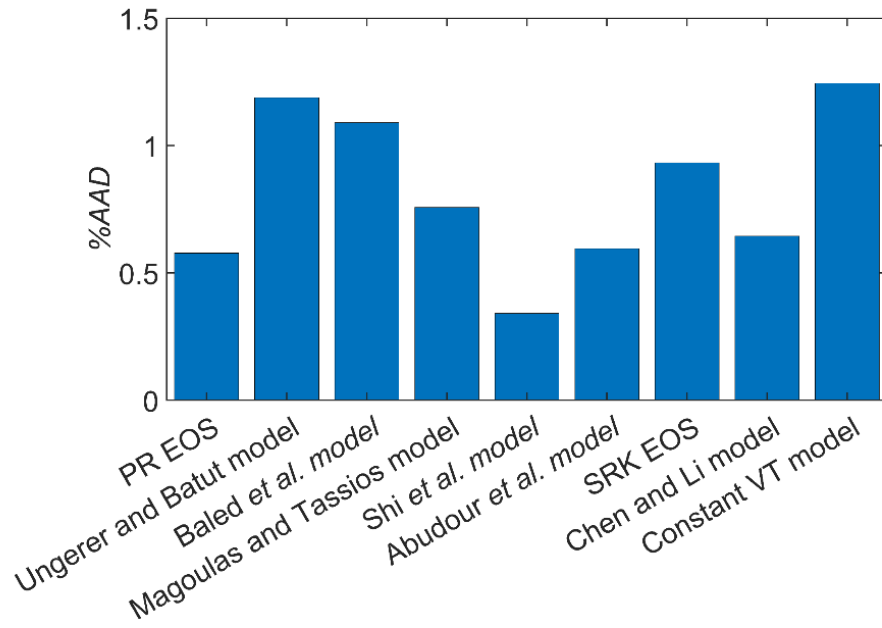
**Figure 15** Comparison of the calculated  $\alpha_p$  for the supercritical-phase CO<sub>2</sub> against the NIST data (A, C, E, and G) and %RDs yielded by the models studied in this work (B, D, F, and H) at temperatures from the  $T_r = 1$  to  $T_r = 3$  and different pressures:  $P_r = 1.5$  (A and B),  $P_r = 2$  (C and D),  $P_r = 2.5$  (E and F), and  $P_r = 3$  (G and H).

The overall %AADs yielded by different EOS models for the liquid, vapor, and supercritical phases are summarized in Figure 16. The %AADs yielded by all EOS models in predicting  $\kappa_T$  and  $\alpha_p$  for the vapor phase are much lower than those for the liquid

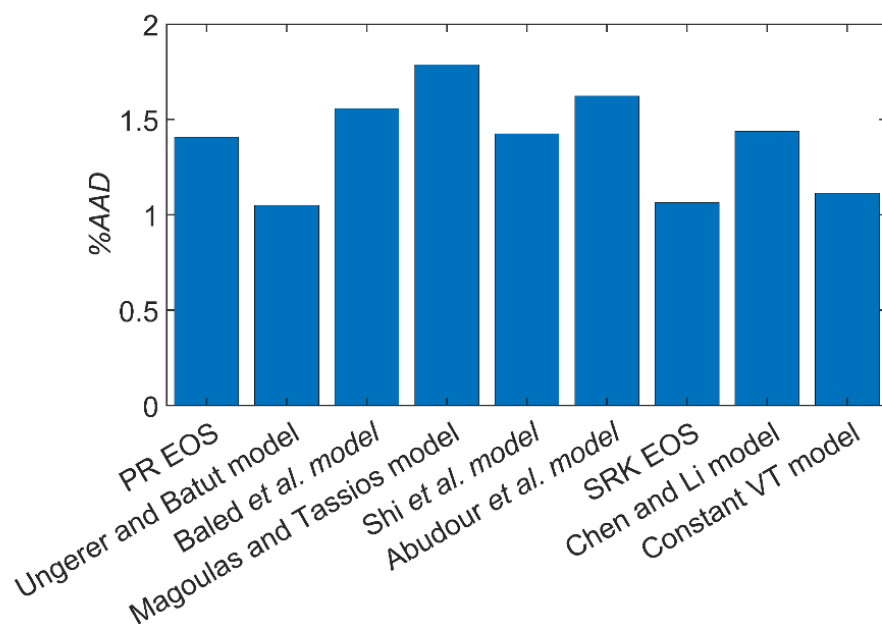
phase. However, when being compared to the untranslated EOSs, most VT models have inferior performance in predicting the vapor-phase  $\kappa_T$  and  $\alpha_P$ . Regarding the  $\kappa_T$  and  $\alpha_P$  predictions for the liquid and supercritical phases, it is clear that the Abudour *et al.* model<sup>3</sup> is the most accurate VT-PR EOS model and the Chen and Li model<sup>5</sup> is the most accurate VT-SRK EOS model. This is because both models can provide more accurate molar-volume predictions by using the distance-function-based volume translations. On the contrary, in the temperature-dependent and the constant VT models, only the same volume translation value is used for the same PV isotherm without considering the pressure change, so these two types of VT models provide relatively poorer predictions of molar volume.

A

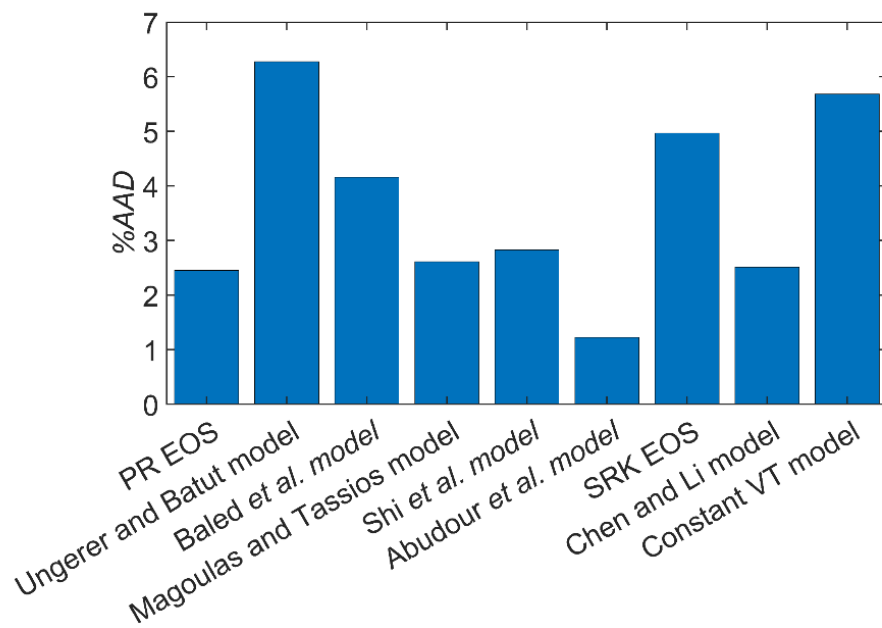


**B****C**

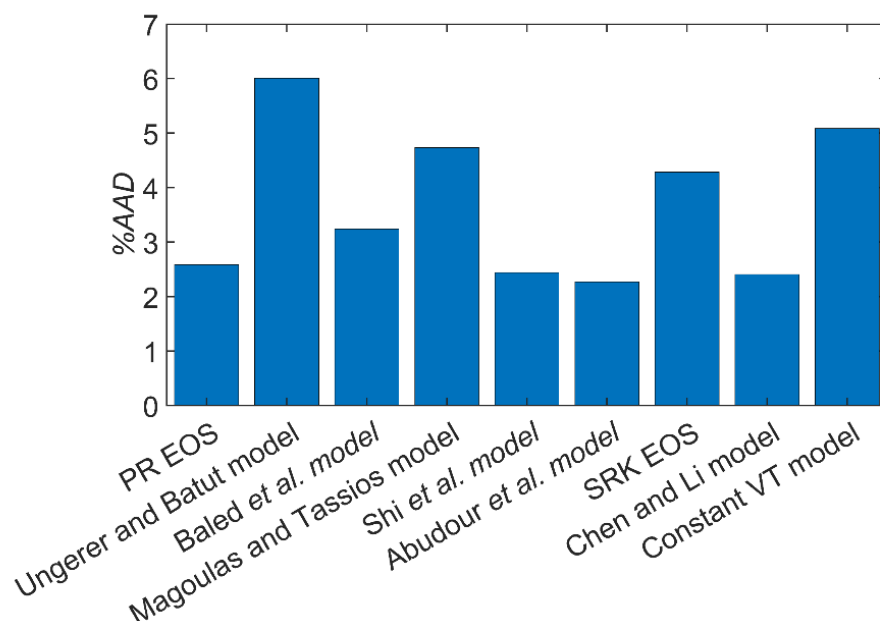
**D**



**E**



F



**Figure 16** The overall %AADs of the predicted  $\kappa_T$  (A, C, and E) and  $\alpha_p$  (B, D, and F) yielded by the models studied in this work for the liquid-phase region (A and B), the vapor-phase region (C and D), and the supercritical-phase region (E and F).

Considering the prediction accuracy of  $\kappa_T$  and  $\alpha_p$  within the entire examined temperature and pressure ranges for the liquid, vapor, and supercritical phases, overall, the VT-PR EOS proposed by Abudour *et al.*<sup>3</sup> provides the most accurate  $\kappa_T$  predictions, while the VT-SRK EOS proposed by Chen and Li<sup>5</sup> provides the most accurate  $\alpha_p$  predictions. In summary, it can be concluded that a VT-EOS that can provide more accurate PVT predictions can also yield more accurate predictions of  $\kappa_T$  and  $\alpha_p$ .

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## CHAPTER 4 CONCLUSIONS AND RECOMMENDATIONS

### 4.1. Conclusions

Seven VT-EOS models (including the constant VT-SRK EOS updated by Pina-Martinez *et al.*,<sup>1</sup> two linear temperature-dependent VT-PR EOS models including the Ungerer and Batut model<sup>2</sup> and the Baled *et al.* model,<sup>3</sup> two exponential temperature-dependent VT-PR EOS models including the Magoulas and Tassios model<sup>4</sup> and the Shi *et al.* model,<sup>5</sup> and two temperature-pressure-dependent models including the VT-PR EOS model proposed by Abudour *et al.*<sup>6</sup> and the VT-SRK EOS proposed by Chen and Li<sup>7</sup>), together with the untranslated PR EOS<sup>8</sup> and SRK EOS<sup>9</sup>, are implemented in this study in an attempt to obtain more accurate predictions of  $\kappa_T$  and  $\alpha_P$ . Since the molar volume is treated as a function of both temperature and pressure in the distance-function-based VT models, analytical expressions of  $\kappa_T$  and  $\alpha_P$  cannot be obtained by using the VT-EOS models that adopt the distance function. To predict  $\kappa_T$  and  $\alpha_P$  by these distance-function-based VT-EOS models developed by Abudour *et al.*<sup>6</sup> and Chen and Li<sup>7</sup>, a numerical method is proposed to approximate the partial derivatives of the molar volume with respect to temperature and pressure, respectively. The predictions of  $\kappa_T$  and  $\alpha_P$  are made in the liquid-phase region, the vapor-phase region, and the supercritical-phase region for pure CH<sub>4</sub> and CO<sub>2</sub> within the temperature range from their corresponding  $T_{triple}$  to  $T_r = 3$  and the pressures from  $P_r = 0.1$  to  $P_r = 3$ . The predictions of  $\kappa_T$  and  $\alpha_P$  by each model are compared with the pseudo-experimental data of NIST<sup>10</sup>. The prediction accuracy of each model is evaluated by comparing the %AADs and the %RDs. The predicted results show that the distance-function-based VT models exhibit relatively better performance in

predicting  $\kappa_T$  and  $\alpha_P$  than the temperature-dependent and the constant VT models. Overall, the distance-function-based VT-PR EOS model proposed by Abudour *et al.*<sup>6</sup> is the most accurate VT model for predicting  $\kappa_T$  with an overall %AAD of 5.11%, while the distance-function-based VT-SRK EOS model proposed by Chen and Li<sup>7</sup> is the most accurate VT model for predicting  $\alpha_P$  with an overall %AAD of 2.77%. In addition, it can be concluded that the more accurate the PVT relation is, the more accurate the predictions of  $\kappa_T$  and  $\alpha_P$  can be.

## 4.2. Recommendations

The best overall prediction accuracy of  $\kappa_T$  and  $\alpha_P$  are provided by the temperature-pressure-dependent VT models proposed by Abudour *et al.*<sup>6</sup> and Chen and Li,<sup>7</sup> respectively. In future studies, more experimental data of  $\kappa_T$  and  $\alpha_P$  at high-temperature and high-pressure conditions should be made available so that they can be used to check if the superior performance of these two models in predicting  $\kappa_T$  and  $\alpha_P$  can be preserved at high-temperature and high-pressure conditions.

In this thesis, the prediction accuracy of VT-EOS models is only evaluated for two example pure substances. However, the processes encountered in the petroleum and chemical industry often involve more complex fluids. Therefore, future studies should examine the predictions of  $\kappa_T$  and  $\alpha_P$  for more pure substances and mixtures by the VT models of Abudour *et al.*<sup>6</sup> and Chen and Li<sup>7</sup>. In addition, appropriate mixing rules need to be implemented to apply VT-EOSs to predict  $\kappa_T$  and  $\alpha_P$  for mixtures.

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