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Power Allocation in Training for Amplify-and-Forward Relay Network

by

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A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Science

in

Computer Engineering

Department of Electrical and Computer Engineering

Spring 2014
Edmonton, Alberta

Abstract

This thesis focuses on the power allocation during the channel training process for three cooperative relay networks: a one-way single-relay network, a one-way multi-relay network and a two-way single-relay network. For these three networks, under the amplify-and-forward (AF) scheme, we investigate the power allocation problem during the channel training process for the destination or both terminals in the case of the two-way single-relay network to estimate the global channel state information (CSI) of the whole network. Linear minimum-mean-square-error (LMMSE) estimation is adopted. The mean-square-error (MSE) of the channel estimation and the outage probability (OP) of the network with channel estimation error are derived. Closed-form solutions for the power allocation problems based on MSE minimization and OP minimization are obtained. Our simulation results demonstrate that the proposed MSE-based and OP-based power allocation schemes are superior to even power allocation.

Acknowledgements

I would like to express my eternal gratitude towards my supervisor, Dr. Yindi Jing, for providing me this valuable opportunity to work under her supervision. Her encouragement and guidance have been a solid foundation in making this work a success.

I shall extend my sincere thanks to my colleague, Saman Atapattu, for giving me advice in this research field.

I owe my deepest gratitude to my paraents and my husband. Their unconditional supports and love give me power to overcome any challenge in my life.

Last but not least, I am grateful for my friends who give me endless support and help in my life.

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Chapter 1

Introduction

Wireless communication is the exchange of information between two or more nodes that are not connected by electrical conductors. It has undergone a remarkable development in the communication industry, since the first analog cellular telephone system deployed in Chicago in 1983. Wireless communication has drawn great attention of the public and the media. Currently, from the report of the International Telecommunication Union, there are 6.8 billion cell phone users with the world population of 7.1 billion. And this number is growing. The international telecommunication union predicts that there will be more than 7 billion users in early 2014. The report also pointed out that more than a third of the world's population use the Internet. Many people indicated that they could not imagine the life without their cellphones, which have become part of everyday life. Indeed, the wireless communication revolution has become a great help for the development of the world. There are plenty of applications of wireless communication technology for education, health care, government, banking, environment and business. Many new applications and products have changed the way people work and play, including automated factories, video conference system, online games, smart phones, and laptops. With the huge market demand of wireless systems, we can tell that there is a bright future for wireless applications in household, personal products and the larger worldwide networking infrastructure. However, there are still many problems that remain to be solved to make current research ideas of future wireless communication applications realities.

In this chapter, we will discuss the details of wireless channel and channel models. Presented next is the introduction of cooperative network, including relaying strategies, one-way relay network, two-way relay network, and relay selection. Further, we will explain channel estimation and introduce two channel estimators: the minimum-mean-square-error (MMSE) estimator and the linear-minimum-mean-square-error (LMMSE) estimator. Then there is the discussion of existing literature on channel estimation in cooperative relay network. In addition, we will also elaborate the motivations and contributions of this thesis. Lastly, we explain the outline of this thesis.

1.1 Wireless Channel

Wireless channel refers to the medium between the source antenna and the destination antenna. We can obtain the information of the received signal from that of the transmitted signal if we have a model of the channel.

The power of the transmitted signal experiences a reduction as it propagates through space. We call the reduction path-loss. Path-loss can be represented by a path-loss exponent, which is normally in the range of 2 to 4. Theoretically, the path-loss exponent is 2 for propagation in free-space. In practice, the power falls off more quickly in relatively lossy environments.

In reality, there are objects such as buildings and trees between the source and the destination or around the paths between them. In this situation, the transmitted signal experiences a loss through absorption, reflection, scattering, and diffraction. We call this effect shadowing.

The transmitted signal may also travel along many different paths to a destination simultaneously. In this situation, these signals from different paths combine at the destination, which may result in constructive or destructive interference, increasing or decreasing the signal power at the destination. We call this effect multi-path.

All these effects cause channel fading. A deep fade is caused by strong destructive interference. Fading is often modeled as a random process. Mathematically, fading is usually modeled as a time-varying random change in the amplitude and phase of the transmitted signal.

Here we introduce several types of wireless fading channels in Table 1.1. The velocity of a moving reflectors relative to the source can cause a shift in the frequency of the signal transmitted along each path. We call this Doppler shift. Thus, signals from different paths can have different Doppler shifts. The Doppler spread D_s is the difference in Doppler shifts between different signal components contributing to a single fading channel tap. The coherence time T_c of the channel is related to the Doppler spread with the following relationship:

$$T_c = \frac{1}{4D_s}. \tag{1.1}$$

Channels with a large Doppler spread have a short coherence time.

Slow Fading

Slow fading occurs when the coherence time T_c of the channel is much larger than the delay constraint of the channel. Slow fading can be caused by shadowing. For slow fading, the amplitude and phase change of the signal imposed by the channel can be considered roughly as a constant over the period of use.

Fast Fading

Fast fading occurs when the coherence time T_c of the channel is much smaller than the delay constraint of the channel. For fast fading, the amplitude and phase change of the signal imposed by the channel varies considerably over the period of use.

On the other hand, the transmitted signal traveling in different paths reach the destination using different time. The delay spread T_d is the difference between the longest and shortest propagation time in different paths. The coherence bandwidth W_c of the channel is related to the delay spread with the following relationship:

$$W_c = \frac{1}{2T_d}. \tag{1.2}$$

Flat Fading

Flat fading occurs when the coherence bandwidth W_c of the channel is much larger than the bandwidth W of the signal. For flat fading, all frequency components of

Types of channel	Characteristic
Slow fading	$T_c \gg$ delay constraint
Fast fading	$T_c \ll$ delay constraint
Flat fading	$W_c \gg W$
Frequency-selective fading	$W_c \ll W$

Table 1.1: The types and defining characteristics of wireless channels.

the signal will experience the same magnitude of fading.

Frequency-Selective Fading

Frequency-selective fading occurs when the coherence bandwidth W_c of the channel is much smaller than the bandwidth W of the signal. For frequency-selective fading, different frequency components of the signal will experience uncorrelated fading.

In this thesis, we consider flat fading and slow fading channel following Rayleigh distribution. Rayleigh distribution is a very reasonable model when there is no dominant propagation and there are many small reflectors that scatter the signal during its travel. It is employed primarily for its simplicity in typical cellular situations with a relatively small number of reflectors. With Rayleigh fading, the envelope of the channel response will follow the Rayleigh probability density function:

$$P_X(x) = \frac{2x}{\Omega} e^{-x^2/\Omega}, x \geq 0, \tag{1.3}$$

where $\Omega = \mathbb{E}(X^2)$. The phase of the channel response is uniformly distributed. Another representation is that the real and imaginary parts of the channel response are independent and identically distributed zero-mean Gaussian processes. In other words, the channel response follows a circularly symmetric complex Gaussian distribution. We denote the circularly symmetric complex Gaussian distribution whose mean is m and whose variance is σ^2 as $\mathcal{CN}(m, \sigma^2)$. Let $\mathcal{N}(0, \sigma^2)$ denotes the real Gaussian distribution with zero-mean and variance σ^2. Thus the channel response can be represented as

$$h = x + jy, \tag{1.4}$$

where $j = \sqrt{-1}$. x and y are independent and follow $\mathcal{N}(0, \sigma^2/2)$, equivalently, $h \sim \mathcal{CN}(0, \sigma^2)$

1.2 Cooperative Relay Network

Cooperative communication promises significant performance improvements in the coverage, capacity, and transmission reliability [1] of wireless communication systems. A conventional single hop system only considers the direct transmission, which means that the destination node decodes the information only based on the signal from the source while regarding the signals from other nodes as interference. In cooperative communication, nodes in a network other than the transmitter can help relaying the signals. The destination combines the direct signal and the relayed signal to achieve improved performance.

The simplest cooperative relay network consists of three nodes: one source node, one destination node, and one relay node supporting the communication between the source node and the destination node. A general relay network can have multiple relays.

1.2.1 One-Way and Two-Way Relay Network

In a one-way relay network, the communication is unidirectional, from the source to the destination. The data communication usually takes two steps. In the first step, the source node sends out signal to the relay nodes. In the second step, the relay nodes broadcast a version of the received signal to the destination node. Two one-way relay network models are considered in this thesis, the one-way single-relay network and the one-way multi-relay network. In Fig. 1.1, a one-way network with two relays is shown.

In a two-way relay network, the communication is bidirectional, where two terminals exchange information via the help of intermediate relays [2]. The data transmission takes two steps. In the first step, two terminals send out signals to the relays. In the second step, the relays broadcast a version of the received signals to both the terminals. In this thesis, we consider a two-way single-relay network as shown in Fig. 1.2. By allowing both terminals concurrently sending information to each other, a two-way relay network can recover the spectral efficiency loss caused by the half-duplex mode of relays. The two-way relay network has attracted considerable

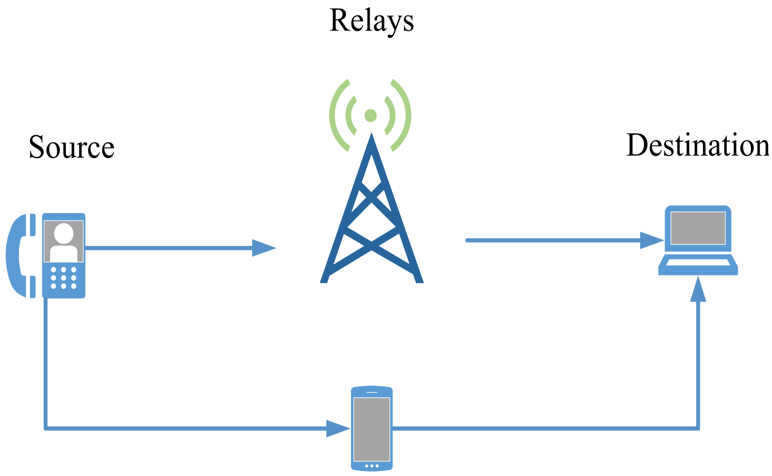

Fig. 1.1: One-way two-relay network.

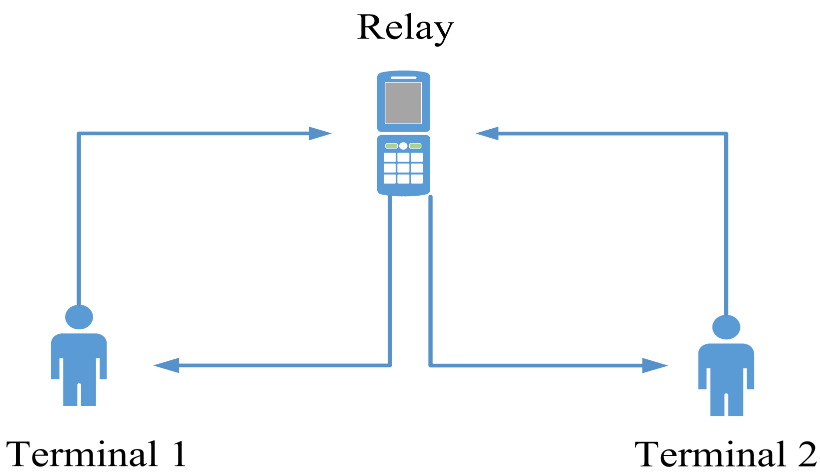

Fig. 1.2: Two-way single-relay network.

attention [2] in the last decade due to its efficient and practical importance.

1.2.2 Relaying Strategies

Major relaying strategies include amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF). In AF relaying, the relay nodes amplify the received signal from the source node and forward the amplified signal to the destination node. There are fixed gain and variable gain amplification factors at relays [3, 4]. The amplification factor is a constant for fixed gain relays. It requires the knowledge of the average power received by the relays. For variable gain relays, the amplification factor requires the instantaneous channel knowledge [5]. Fixed gain relays are simpler and easier to implement. In DF relaying, the relay nodes decode the information from the source node and in case of correct decoding, forward re-encoded signal to the destination node. The CF strategy allows the relay nodes to quantize the received signal from the source node and forward encoded versions of the quantized signal to the destination node without decoding the signal.

1.2.3 Relay Selection

In dense wireless networks, there are typically many relays between the source and the destination, where relay selection has attracted considerable attention [6–8]. Relay selection is a low complexity and low overhead strategy since it avoids the need of synchronization among relays.

Next we will introduce some single relay selection schemes [9, 10]. The widely used ones are: best relay selection, nearest neighbor selection, best worse channel selection, and best harmonic mean selection. [11, 12] proposed the nearest neighbor selection, in which the relay lies nearest to the base station is selected. For dual-hop protocols, each relay has two channels, denote the channel from the source to the relay as h_1 and the channel from the relay to the destination as h_2. As introduced in [12, 13], the best worse channel selection is to select the relay whose worse channel, $\min(|h_1|, |h_2|)$, is the best. In [13, 14], the best harmonic mean selection was proposed, in which the relay with the largest harmonic mean $(|h_1|^{-2} + |h_2|^{-2})^{-1}$ cooperates. In [6–8, 15], the best relay selection was proposed, where the relay that provides the best received signal-to-noise ratio (SNR) will be selected to forward information. Best relay selection was shown to achieve full diversity order [15] and has advantage in spectral efficiency compared with schemes where relays transmit in orthogonal channels [16]. Best relay selection is used in Chapter 3 of this thesis. In [15], the authors generalized the idea of multi-relay selection allowing more than one relay to cooperate. All the mentioned works are for one-way relay network. Relay selection for two-way relay network were investigated in [17–19].

1.3 Channel Estimation

In wireless communication, channel state information (CSI) refers to properties of a communication link from the source to the destination. It describes the combined propagation effect of all elements, for example, diffraction, refraction, path-loss tropospheric and ionospheric scintillation, and rain attenuation. When CSI is available, wireless communication can be designed to be adaptive to current channel conditions for the best performance. In reality, CSI is obtained via a training process and estimation at the destination. And estimated CSI is always subject to error.

There are a number of methods to estimate the CSI parameters. For Bayesian estimation methods, CSI parameters are viewed as random with a specified probability distribution. A loss function is employed to express the cost of estimating x as $\hat{x}$. The estimation is obtained via the minimization of the loss function. When the mean-square-error (MSE) is used as the loss function, we obtain the class of MMSE estimators. With the Gaussian observation model, MMSE estimators are linear and thus easy to implement. But for non-Gaussian case, the structure of the MMSE estimator can be complicated. Sometimes, the MMSE estimation can be intractable. In this case, we can retain the MMSE criterion but constrain the estimator to be linear. Thus, the obtained estimator is the LMMSE estimator.

In what follows, we introduce results on MMSE and LMMSE estimations, which will be used in later chapters.

1.3.1 MMSE Estimation

First, we will introduce some notation to help the formulation of MMSE estimation. Let $\mathbf{Y} \in \mathbb{R}^n$ be the observations in function of the value $\mathbf{x} \in \mathbb{R}^m$ from the parameter vector $\mathbf{X}$ to be estimated. Then the MMSE estimator is

$$\hat{\mathbf{x}}_{MMSE} = \mathbb{E}\{\mathbf{X}|\mathbf{Y} = \mathbf{y}\}, \tag{1.5}$$

which is the conditional mean of $\mathbf{X}$ given $\mathbf{Y} = \mathbf{y}$. The MSE of the MMSE estimator is

$$\text{MSE}(\hat{\mathbf{x}}_{MMSE}) = trace\{K_E\}, \tag{1.6}$$

where the error covariance matrix K_E is given by

$$K_E = \mathbb{E}\{[\mathbf{X} - \mathbb{E}(\mathbf{X}|\mathbf{Y})][\mathbf{X} - \mathbb{E}(\mathbf{X}|\mathbf{Y})]^T\}. \tag{1.7}$$

The trace of an $n \times n$ square matrix A is defined to be the sum of the elements on the main diagonal of A.

1.3.2 LMMSE Estimation

The LMMSE estimator is the estimator achieving minimum MSE among all estimators of a linear form. Using the same notation as that in Subsection 1.3.1, the LMMSE estimate [20] is

$$\hat{\mathbf{x}}_{LMMSE} = \mathbb{E}\{\mathbf{X}\} + C_{\mathbf{XY}} C_{\mathbf{YY}}^{-1}(\mathbf{y} - \mathbb{E}\{\mathbf{Y}\}), \tag{1.8}$$

where $C_{\mathbf{XY}}$ is the cross-correlation of $\mathbf{X}$ and $\mathbf{Y}$, and $C_{\mathbf{YY}}$ is the auto-correlation of $\mathbf{Y}$. The MSE on $\hat{\mathbf{x}}_{LMMSE}$ is

$$\text{MSE}(\hat{\mathbf{x}}_{LMMSE}) = C_{\mathbf{XX}} - C_{\mathbf{XY}} C_{\mathbf{YY}}^{-1} C_{\mathbf{YX}}. \tag{1.9}$$

where $C_{\mathbf{XX}}$ is the auto-correlation of $\mathbf{X}$.

1.4 Literature on Channel Estimation in Cooperative Relay Network

Cooperative communication promises significant performance improvements in the coverage, capacity, and transmission reliability [1] of wireless communication systems. There are a lot of work proposing cooperative schemes and analyzing their performance, e.g., [15, 21–39]. Most of these aforementioned work needed CSI for either transmission or reception, or both. Thus training design and channel estimation are important research problem for cooperative network.

In recent years, there are many work on the designs of the training and channel estimation schemes, as well as the analysis of the impact of CSI error for cooperative relay networks, e.g., [28, 30, 40–54, 54–60]. In [40–43], relay networks with DF were considered; while [28, 30, 44–54, 54–60] were on relay networks with AF.

Among these papers, [28, 30, 45, 49, 50, 52, 60] studied one-way single-relay network with single or multiple antennas. In [45], the authors proposed algorithms to compute the optimal source pilot matrix and the optimal relay pilot matrix. [28] formulated a capacity lower bound for the dual-hop wireless relay channel with channel estimation. In [52], the channel estimations in a relay network with multiple transmit and receive antennas, including the estimation of the end-to-end channel matrix and the individual estimation of the transmitter-relay channels and the relay-receiver channels were investigated. In [30], the authors investigated the error rate performance with imperfect channel estimation. In [49], both the end-to-end channel estimation and the individual source-to-relay and relay-to-destination channel estimation methods were investigated. [50] studied the optimal pilot symbol spacing and derived the bit error rate for the relay network with fast-fading channels.

[44, 46–48, 55] were on one-way multi-relay network. [44] developed cost-effective algorithms for adaptive joint power allocation, and estimation of the parameters of the receiver and the channels. In [46], schemes for the estimation of the end-to-end channel coefficients were proposed. The optimal training sequences and precoding matrices were also derived. In [48], for the general multiple-input and multiple-output (MIMO) relay networks, schemes for the receiver to estimate the individual source-to-relay and relay-to-destination channels were proposed. The requirement on the training time for full diversity in data transmission with mismatched maximum likelihood (ML) decoding was also derived. In [47], both mismatched and matched decodings for multi-relay MIMO networks with CSI estimation error were investigated. [55] was on the channel estimation over doubly-selective channels.

[51, 53, 54, 57–59] were on two-way relay network. In [54], for a multiple-relay network, the authors investigated the impact of CSI estimation error on the performance, in the sense of OP and bit error rate. [51, 53, 57–59] studied the training and channel estimation schemes for two-way single-relay network. [53] proposed a channel training algorithm based on the least square principle, and a method to design the optimal training sequences based on minimization of the MSE. In [57], channel estimation schemes and training designs based on Bayesian Cramér-Rao bound were investigated. [51] proposed a new channel estimator based on the minimization of average SNR, and designed the optimal training sequence by resorting to Cramér-Rao lower bound.

1.4.1 Power Allocation in Training for Cooperative Relay Network

All these aforementioned work focused on the training design and the effect of channel estimation error on network performance, while assuming fixed training powers at the relays or the source. In this subsection, we review the literature on training power allocation for cooperative relay network.

For one-way single-relay network, the power allocation was considered in [56, 61–63]. In [61] and [62], the authors focused on the end-to-end channel estimation. Based on the maximization of the average received SNR at the destination node, [61] investigated the joint power allocation between the training and the data transmission periods, and between the source and the relay for both the training and the data transmission periods, under a total power constraint. [62] investigated a similar power allocation problem but in terms of maximizing a mutual information lower bound. [56, 64] studied the estimation of the individual source-to-relay channel and relay-to-destination channel. In [64], the authors investigated the power allocation between the estimation of the source-to-relay channel and the relay-to-destination channel at the relay during training, while the source power was fixed. Both the minimization of the MSE of the source-to-relay channel estimation and the maximization of the average effective SNR were considered. [63] was on the end-to-end channel estimation for fixed gain relays and individual channel estimation for variable gain relays. The power allocation was between the training and the data transmission, and between the source and the relay.

[58, 59] investigated the power allocation for two-way single-relay network. In [58, 59], for the end-to-end channel estimation, the authors investigated the joint power allocation between data-transmission and training and between the two terminals and the relay. [58] aimed at minimizing an OP upper bound, while [59] aimed at maximizing an average sum-rate lower bound, minimizing the MSE, and

minimizing the Bayesian Cramér-Rao lower bound.

1.5 Thesis Motivations and Contributions

This thesis is on the power allocation problem during channel training and estimation for AF relay network. The motivations are as follows.

In early studies of cooperative relay communication, perfect CSI is usually assumed to be available. In reality, as explained in Section 1.3, CSI is obtained via training and estimation. Estimated CSI is always subject to error, which degrades communication performance. Thus it becomes important to have quality training designs and channel estimation schemes to optimize the whole network performance.

We focus on the power allocation problem in the training process of cooperative relay network. Intelligent power allocation schemes can enhance the performance substantially. A dynamic power allocation scheme can take full advantage of the channel diversity among users and thus can make the best use of limited resources. The training power allocation problem starts to attract considerable attention in recent years. Currently, there are limited results of power allocation as explained in Subsection 1.4.1. And all the aforementioned work assumed fixed training powers at the relays or the source. Many problems are open. In this thesis, we consider the power allocation between the source and the relay, and between different channel estimation stages.

For the channel estimation, different to [61] and [62], where the end-to-end channel estimation was investigated, we focus on the estimation of the individual source-to-relay channel and relay-to-destination channel. The reasons of estimating individual channels instead of end-to-end ones are two-fold. First, the equivalent end-to-end channel values are concatenations of the individual channels. We can always calculate estimates of the end-to-end channels with estimates of individual channels. Second, the structure and values of the end-to-end channels are different for different protocols used in data transmission (e.g., relay/antenna selection, beamforming, distributed space-time coding). It also changes with other designs such as relay power amplification factor and precoding. Thus, if there is a change in the protocol, with end-to-end channel estimates, training needs to be conducted again. With individual channel estimates, however, the end-to-end channel estimates can be calculated and there is no need to repeat the training.

This thesis studies power allocation during channel training for three AF network scenarios. The main contributions are detailed as follow. For a one-way single-relay network, we study the power allocation between the training of different channels and between the source and the relay during the training of the source-to-relay channel. For a one-way multi-relay network, we study the power allocation among all the relay paths, between the training of different channels for every relay path, and between the source and every relay during the training of the source-to-relay channel. For a two-way single-relay network, we study the power allocation between the relay-training stage and the terminal-training stage, and between the terminals and the relay during the terminal-training stage. For all three network scenarios, we use the total MSE of channel estimations and the outage probability (OP) of the network as the design objectives. For the high power regime, closed-form analytical solutions for the training power allocation are derived. Simulation results on the MSE and the OP are demonstrated to show the superiority of the proposed schemes to an even power allocation.

1.6 Thesis Outline

This thesis is organized as follows. Chapter 1 provides the background of wireless channel, cooperative relay network, channel estimations, related literature, as well as the motivations, contributions, and outline of the thesis. Chapter 2 is on the training power allocation for a one-way single-relay network. Chapter 3 is on the training power allocation for a one-way multi-relay network. Chapter 4 is on the training power allocation for a two-way single-relay network. Chapter 5 gives the conclusions of the thesis and proposes several possible future work.

Chapter 2

Power Allocation in Training for One-Way Single-Relay Network

2.1 Introduction

In this chapter, we consider a one-way single-relay network under AF protocol and study the power allocation problem during the training of the individual source-to-relay and relay-to-destination channels. This includes two parts: 1) the power allocation between the training of the source-to-relay channel and the relay-to-destination channel, and 2) in the training of the source-to-relay channel, the power allocation between the source transmission and the relay transmission. Two objective functions, the total MSE of estimations of all individual channels and the OP of the whole network, are considered. Simulation results show performance improvement of the proposed schemes over even power allocation.

In what follows, we clarify the difference of our work to existing ones [56, 62, 65]. While in [62, 65], estimations of the end-to-end channels were investigated, we work on the estimation of the individual source-to-relay and relay-to-destination channels. Compared with [56], our work is different in the following aspects. First, the power allocation problem is different. While [56] worked on the allocation between the training and data transmission periods for a fixed relay power, we work on the allocation between the training of the two channels and between the source and the relay during the estimation of source-to-relay channel. Second, we consider both OP and total MSE, while in [56] only the OP was studied. Further, we provide

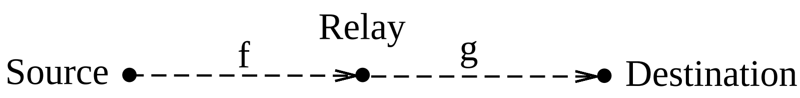

Fig. 2.1: One-way single-relay network model.

analytical solutions to the power allocation problems for the high power regime.

The rest of this chapter is organized as follows. The system model and channel estimation scheme are explained in Section 2.2. In Section 2.3, a closed-form power allocation is derived based on the minimization of the total MSE of the estimations. In Section 2.4, we derived another closed-form power allocation based on the minimization of the OP of the network under channel estimation error. Simulation results are shown in Section 2.5. Finally, concluding remarks are given in Section 2.6.

2.2 System Model and Training Scheme

The one-way single-relay AF network model is shown in Fig. 2.1. Each node is equipped with one antenna. Denote the channel from the source to the relay as f and that from the relay to the destination as g. There is no direct link between the source and the destination. f and g have the distribution $\mathcal{CN}(0, \sigma_f^2)$ and $\mathcal{CN}(0, \sigma_g^2)$, respectively.

2.2.1 Training and LMMSE Estimators

The channel training and estimation problem is to estimate the individual channel coefficients f and g at the destination. The training period is divided into two stages: first the training of g and then the training of f.

To estimate g, a pilot symbol is sent from the relay. Without loss of generality, the unit symbol is used. The received symbol at the destination is

$$x = \sqrt{P_g} g + n_g, \tag{2.1}$$

where P_g is the relay power used for the training of g. n_g is the complex Gaussian noise at the destination whose distribution follows $\mathcal{CN}(0, 1)$. Using the results in Section 1.3, the LMMSE estimate of g is

$$\hat{g} = \frac{\sqrt{P_g}\sigma_g^2}{1 + P_g\sigma_g^2}x. \tag{2.2}$$

This is also the MMSE estimation. Denote the estimation error as

$$\triangle g \triangleq g - \hat{g}. \tag{2.3}$$

$\triangle g$ follows the distribution $\mathcal{CN}\left(0, \frac{\sigma_g^2}{1+P_g\sigma_g^2}\right)$. Thus, from Section 1.3, we get the MSE on g:

$$\text{MSE}(g) = \frac{\sigma_g^2}{1 + P_g\sigma_g^2} \approx \frac{1}{P_g}, \tag{2.4}$$

where the approximation is valid for $P_g \gg 1$.

Then to estimate f at the destination, a pilot symbol is sent from the source and forwarded by the relay. This training stage takes two steps. Again, without loss of generality, the unit symbol is used as the pilot. The received signal at the relay is

$$y_1 = \sqrt{P_{f,S}}f + n_1, \tag{2.5}$$

where $P_{f,S}$ is the transmitted power of the source and n_1 is the complex Gaussian noise at the relay, respectively. Here we consider fixed gain relay. The relay amplification factor is [3]

$$A = \sqrt{\frac{P_{f,R}}{1 + P_{f,S}\sigma_f^2}}. \tag{2.6}$$

Thus, the received signal at the destination is

$$y = y_1 A g + n_2 = \sqrt{\alpha}gf + \sqrt{\beta}gn_1 + n_2, \tag{2.7}$$

where $\alpha \triangleq \frac{P_{f,S}P_{f,R}}{1+P_{f,S}\sigma_f^2}$ and $\beta \triangleq \frac{P_{f,R}}{1+P_{f,S}\sigma_f^2}$ with $P_{f,R}$ the power the relay uses during this training stage. n_2 is the complex Gaussian noise at the relay and the destination. n_1 and n_2 are assumed to be independent and their distribution follow $\mathcal{CN}(0, 1)$. Since this training stage is after the training of g, thus the estimation $\hat{g}$ is known at the destination. From (2.3), we can rewrite (3.3) as

$$y = \sqrt{\alpha}\hat{g}f + (\sqrt{\alpha}f + \sqrt{\beta}n_1)\triangle g + \sqrt{\beta}\hat{g}n_1 + n_2. \tag{2.8}$$

Let $n \triangleq (\sqrt{\alpha}f + \sqrt{\beta}n_1)\triangle g + \sqrt{\beta}\hat{g}n_1 + n_2$, which is the equivalent noise in the observation model (2.8). Different to the traditional linear Gaussian observation model, in (2.8), the equivalent noise term n depends on f. Also given $\hat{g}$ and f, it can be shown straightforwardly that n is non-Gaussian. Thus the model in (2.8) is non-linear and non-Gaussian. The MMSE estimate of f is difficult to derive. By using the results in Section 1.3, after straightforward calculations, the LMMSE estimate of f can be calculated to be:

$$\hat{f} = \frac{\sigma_f^2\sqrt{\alpha}\hat{g}^* y}{1 + (\sigma_f^2\alpha + \beta)[|\hat{g}|^2 + \mathrm{MSE}(g)]}. \quad (2.9)$$

Denote the estimation error of f as

$$\triangle f \triangleq f - \hat{f}. \quad (2.10)$$

By using (1.9), the MSE of the LMMSE estimation can be calculated to be

$$\mathrm{MSE}(f) = \sigma_f^2 - \frac{\sigma_f^4\alpha|\hat{g}|^2}{1 + (\sigma_f^2\alpha + \beta)\left[|\hat{g}|^2 + \mathrm{MSE}(g)\right]}. \quad (2.11)$$

2.2.2 Power Allocation Problem Statement

In this chapter, we investigate the power allocation problem for the channel training process explained in Section 2.2.1. To help the problem formulation, we introduce a few new notation on power. Let P_f be the total source and relay power used in the training of f and P_t be the total power used for the overall training. Thus

$$P_f = P_{f,S} + P_{f,R} \quad \text{and} \quad P_t = P_f + P_g.$$

We use η to represent the percentage of power allocated to the source during the training of f, i.e., $P_{f,S} = \eta P_f$. Thus the power allocated to the relay during the training of f satisfies $P_{f,R} = (1-\eta)P_f$. We use λ to indicate the power allocated to the training of f, i.e., $P_f = \lambda P_t$. Thus the power used for the training of g satisfies $P_g = (1-\lambda)P_t$. Our power allocation problem is to find the optimal η and λ for a given total training power P_t. Two objective functions are considered in this chapter, the total MSE of the channel estimations and the OP of the relay network under channel estimation error.

2.3 MSE-Based Power Allocation

In this section, we aim to derive the optimal power allocation based on the minimization of the total MSE of the estimations of f and g. The problem can be represented as

$$\begin{aligned}&\arg\min_{\lambda,\eta}[\mathrm{MSE}(g)+\mathrm{MSE}(f)]\\=&\arg\min_{\lambda}[\mathrm{MSE}(g)+\min_{\eta}\mathrm{MSE}(f)],\end{aligned} \tag{2.12}$$

where the equality is because MSE(g) is independent of η.

We first try to understand the behavior of MSE(g) and MSE(f) from (2.4) and (2.11), respectively. MSE(g) is a decreasing function of P_g and $\lim_{P_g\to\infty}\mathrm{MSE}(g)=0$. On the other hand, MSE(f) depends not only on $P_{f,S}, P_{f,R}$ (powers of the source and the relay during the training of f) via α and β, but also on P_g (the power of the relay during the training of g) via MSE(g). For a fixed P_g, even for infinite $P_{f,S}$ and $P_{f,R}$, the training of f cannot be perfect. It can be shown that

$$\lim_{P_{f,S},P_{f,R}\to\infty}\mathrm{MSE}(f)=\frac{\sigma_f^2\mathrm{MSE}(g)}{|\hat{g}|^2+\mathrm{MSE}(g)}.$$

Hence, to improve the training quality of f, not only the power of the second training stage but also the power of the first training stage must be increased. Also notice that MSE(f) depends on $\hat{g}$, the estimation of the relay-to-destination channel. These make the power allocation problem challenging.

We first work on the inner minimization problem of (2.11), $\min_\eta \mathrm{MSE}(f)$. Define $a\triangleq|\hat{g}|^2+\mathrm{MSE}(g)$. From (2.11) and the definition of η, we have

$$\mathrm{MSE}(f)=\sigma_f^2-\frac{\eta(1-\eta)P_f^2|\hat{g}|^2{\sigma_f}^4}{(1+\sigma_f^2\eta P_f)[1+a(1-\eta)P_f]}. \tag{2.13}$$

By calculating the derivative of MSE(f) with respect to η and making it zero, we have

$$\eta^\star=\frac{\sqrt{1+aP_f}}{\sqrt{1+aP_f}+\sqrt{1+\sigma_f^2P_f}}. \tag{2.14}$$

Notice that (2.14) is a function of $\hat{g}$ and MSE(g) via a. Thus, the optimal power allocation for the two steps of the training of f depends on both the value and the quality of the estimation of g. With this $\eta^\star$, we have

$$\mathrm{MSE}(f)|_{\eta=\eta^\star} = \sigma_f^2 \frac{2(1+\rho_1\rho_2) + P_f(\sigma_f^2 + a) + \sigma_f^2 \mathrm{MSE}(g) P_f^2}{(1+\rho_1\rho_2)^2}, \tag{2.15}$$

where $\rho_1 \triangleq \sqrt{1+aP_f}$, $\rho_2 \triangleq \sqrt{1+\sigma_f^2 P_f}$.

Next we consider the optimization of λ, i.e., the power allocation between the training of f and training of g. Note that in the optimization of λ, $\hat{g}$ is unknown. Thus we should use the average value of (2.15) over $\hat{g}$. However, because of complexity of the function (2.15), the average cannot be found in a tractable form. In the following, for tractable analysis and closed-form solution, we approximate $|\hat{g}|^2$ with its average value, i.e., $\frac{P_g\sigma_g^4}{1+P_g\sigma_g^2}$ (this is equivalent to using $a \approx \sigma_g^2$). In addition, we consider the high power region, i.e., $P_t \gg 1$. By only omitting the lower order terms of P_t, we have

$$\mathrm{MSE}(f)|_{\eta=\eta^\star} \approx \frac{(P_t - P_f)(\sigma_g + \sigma_f)^2 + P_f\sigma_f^2}{\sigma_g^2 P_f (P_t - P_f)}. \tag{2.16}$$

By using (2.16) and (2.4) in (2.12), the MSE-based power allocation problem reduces to

$$\arg\min_{\lambda} \left[\frac{(\sigma_g+\sigma_f)^2}{\sigma_g^2}\frac{1}{P_f} + \frac{\sigma_f^2+\sigma_g^2}{\sigma_g^2}\frac{1}{P_t - P_f} \right].$$

By calculating the derivative of the objective function and making it zero, we find the optimal λ to be

$$\lambda^\star = \left(1 + \frac{\sqrt{\sigma_f^2+\sigma_g^2}}{\sigma_f+\sigma_g}\right)^{-1}. \tag{2.17}$$

We can derive from (2.17) that $\lambda^\star \le \sqrt{2}(\sqrt{2}-1) \approx 59\%$. This means that the power used in the training of f is no larger than 59% of the total power.

From (2.17) we can also see that $\lambda^\star$ depends on the channel variances only. But from (2.14), we see that $\eta^\star$ depends on not only the channel variances but also $\hat{g}$ via a (the estimation of the relay-to-destination channel). Thus, with the proposed power allocation scheme, the powers the source and the relay used for the training of f are actually adaptive to the channel quality. Also $\eta^\star$ shows that with a better g-channel quality, more power should be allocated to the source. For the implementation of the proposed scheme, a central controller is needed.

2.4 OP-Based Power Allocation

Total MSE-based power allocation aims at the minimum variance on the channel error. However, it is not a direct measure of the network performance. Further, it weights the errors on the two channels MSE(f) and MSE(g) equally, which may not be optimal. In this section, we aim to minimize the OP of the network under channel estimation error, denoted as P_{out}. We use AF with fixed gain relay power coefficient for data transmission. Let $P_{d,S}$, $P_{d,R}$ be the powers used by the source and the relay during the data transmission. The following lemma on the OP for the high power regime is obtained.

Lemma 1: *Assume that the training power and data power have the same order, i.e., $P_{f,S}, P_{f,R}, P_g, P_{d,S}, P_{d,R} \sim P$ and $P \gg 1$. Given the SNR-threshold γ_{th} and MSEs of channel estimates, we have,*

$$P_{\text{out}} \approx \gamma_{th} \left[\frac{\text{MSE}(g)}{\sigma_g^2} + \frac{\text{MSE}(f)}{\sigma_f^2} \right] + p + \mathcal{O}\left(\frac{\ln P}{P^2} \right), \tag{2.18}$$

where $p = \mathcal{O}(\frac{\ln P}{P})$ is independent of $P_{f,S}$, $P_{f,R}$, and P_g.

Proof. First we explain some mathematical operators. $x \sim y$ means that x and y are of the same order of magnitude. $f(x) = \mathcal{O}(g(x))$ means that $\lim_{x\to\infty} \frac{f(x)}{g(x)} = c$ with c a non-zero constant. In other words, $f(x) = \mathcal{O}(g(x))$ means that $f(x)$ and $g(x)$ functions have the same scaling for large x.

Next we will provide the proof. The data transmissions take two steps. In the first step, the source sends a data symbol s which has unit power. The received signal at the relay is

$$r = \sqrt{P_{d,S}} f s + n_{d,R}. \tag{2.19}$$

In the second step, the relay amplifies and forwards the received signal to the destination. Fixed relay gain amplification factor is used at the relay. Similar to the derivation of (2.6). Thus the transceiver equation of the data transmission period can be written as

$$y = \sqrt{\frac{P_{d,R}}{1 + P_{d,S}}} r g + n_{d,D} = \sqrt{\alpha_d} g f s + n_d, \tag{2.20}$$

where $\alpha_d \triangleq \frac{P_{d,S}P_{d,R}}{1+P_{d,S}\sigma_f^2}$, $\beta_d \triangleq \frac{P_{d,R}}{1+P_{d,S}\sigma_f^2}$, and

$$n_d \triangleq \sqrt{\beta_d}(\hat{g} + \triangle g)n_{d,R} + n_{d,D},$$

with $n_{d,R}$ and $n_{d,D}$ the complex Gaussian noise at the relay and the destination during data transmission, following $\mathcal{CN}(0,1)$. Considering that only channel estimates are available at the destination, using (2.3) and (2.10), we can rewrite (2.20) as

$$y = \sqrt{\alpha_d}\hat{g}\hat{f}s + \omega, \tag{2.21}$$

where

$$\omega \triangleq \sqrt{\alpha_d}[\triangle f(\hat{g} + \triangle g) + \hat{f}\triangle g]s + \sqrt{\beta_d}(\hat{g} + \triangle g)n_{d,R} + n_{d,D}.$$

$\omega$is the noise-plus-channel-error term. Recall that $\triangle g$ follows the distribution $\mathcal{CN}\left(0, \frac{\sigma_g^2}{1+P_g\sigma_g^2}\right)$. Compared to $\hat{g}$, $\triangle g$ is a lower order term of P_t.

When the training power is high, we can neglect the lower order terms in ω, which are $\sqrt{\alpha_d}\triangle f\triangle gs$ and $\sqrt{\beta_d}\triangle gn_{d,R}$. The received SNR can be approximated as

$$\begin{aligned}\frac{\mathbb{E}(\alpha_d|\hat{g}|^2|\hat{f}|^2)}{\mathbb{E}(|\omega|^2)} &\approx \frac{\alpha_d|\hat{g}|^2|\hat{f}|^2}{\mathbb{E}[\sqrt{\alpha_d}(\triangle f\hat{g} + \hat{f}\triangle g)s + \sqrt{\beta_d}\hat{g}n_{d,R} + n_{d,D}]} \\ &= \frac{\alpha_d|\hat{g}|^2|\hat{f}|^2}{\alpha_d\text{MSE}(f)|\hat{g}|^2 + \alpha_d\text{MSE}(g)|\hat{f}|^2 + \beta_d|\hat{g}|^2 + 1} \triangleq \gamma. \end{aligned} \tag{2.22}$$

Denote $Q_1 \triangleq \gamma_{th}\text{MSE}(g)$ and $Q_2 \triangleq \gamma_{th}\left[\text{MSE}(f) + \frac{1}{P_{d,S}}\right]$. The OP can be calculated as

$$P_{\text{out}} = \mathbb{P}(\gamma \leq \gamma_{th}) \approx P_1 + P_2 + P_3, \tag{2.23}$$

where

$$\begin{aligned} &P_1 \triangleq \mathbb{P}(|\hat{g}|^2 \geq Q_1 \ \& \ |\hat{f}|^2 \leq Q_2), \\ &P_2 \triangleq \mathbb{P}(|\hat{g}|^2 \leq Q_1 \ \& \ |\hat{f}|^2 \geq Q_2), \\ &P_3 \triangleq \mathbb{P}\left(|\hat{g}|^2 \geq Q_1 \ \& \ Q_2 \leq |\hat{f}|^2 \leq \frac{\gamma_{th}}{\alpha_d(|\hat{g}|^2 - Q_1)} + Q_2\right). \end{aligned}$$

Recall that $\lim_{P_g\to\infty}\text{MSE}(g) = 0$. Thus when $P_g \gg 1$, we can approximate $\hat{f}$ as Gaussian following the distribution $\mathcal{CN}(0, \text{MSE}(f))$. Then the probability density function of $|\hat{g}|^2$ and $|\hat{f}|^2$ are

$$f_{|\hat{g}|^2}(x) = \frac{1}{\text{MSE}(g)} e^{\frac{-x}{\text{MSE}(g)}}, \tag{2.24}$$

$$f_{|\hat{f}|^2}(y) = \frac{1}{\text{MSE}(f)} e^{\frac{-y}{\text{MSE}(f)}}, \tag{2.25}$$

respectively. By using (2.24) and (2.25), P_1 can be calculated as

$$\begin{aligned}\mathbb{P}(|\hat{g}|^2 \geq Q_1 \ \& \ |\hat{f}|^2 \leq Q_2) &\approx \int_{Q_1}^{\infty}\int_{0}^{Q_2} f_{|\hat{g}|^2}(x) f_{|\hat{f}|^2}(y) dx dy \\ &\approx e^{-\frac{Q_1}{\sigma_g^2}}(1 - e^{-\frac{Q_2}{\sigma_f^2}}) + \mathcal{O}\left(\frac{1}{P^2}\right).\end{aligned}$$

Using the same mathematical manipulation, we can show that

$$P_2 \approx e^{-\frac{Q_2}{\sigma_f^2}}(1 - e^{-\frac{Q_1}{\sigma_g^2}}) + \mathcal{O}\left(\frac{1}{P^2}\right),$$

$$P_3 \approx e^{-\frac{Q_1}{\sigma_g^2}-\frac{Q_2}{\sigma_f^2}}(1 - c) + \mathcal{O}\left(\frac{1}{P^2}\right),$$

where $c \triangleq \sqrt{\frac{4\gamma_{th}}{\alpha_d \sigma_g^2 \sigma_f^2}} K_1\left(\sqrt{\frac{4\gamma_{th}}{\alpha_d \sigma_g^2 \sigma_f^2}}\right)$ and $K_1(\cdot)$ is the modified Bessel function of the second kind. Thus,

$$P_{\text{out}} \approx e^{-\frac{Q_1}{\sigma_g^2}} + e^{-\frac{Q_2}{\sigma_f^2}} - (1+c)e^{-\frac{Q_1}{\sigma_g^2}-\frac{Q_2}{\sigma_f^2}} + \mathcal{O}\left(\frac{1}{P^2}\right). \tag{2.26}$$

When $P_{d,R} \gg 1$, we have $c = 1 + q + \mathcal{O}\left(\frac{\ln P_{d,R}}{P_{d,R}^2}\right)$, where $q = \mathcal{O}(\frac{\ln P_{d,R}}{P_{d,R}})$. By using this in (2.26), (3.30) can be obtained.

With Lemma 2, the OP minimization problem becomes

$$\arg\min_{\lambda}\left[\sigma_f^2 \text{MSE}(g) + \sigma_g^2 \arg\min_{\eta} \text{MSE}(f)\right]. \tag{2.27}$$

The inner minimization problem is the same as that in Section 2.3, hence we have the same optimal solution $\eta^\star$ in (2.14). For the optimization of λ, by using (2.16) and (2.4) in (2.27), the problem reduces to

$$\arg\min_{\lambda}\left[\frac{(\sigma_g + \sigma_f)^2}{P_f} + \frac{2\sigma_f^2}{P_t - P_f}\right].$$

By calculating the derivative of the objective function and making it zero, we find the optimal λ to be

$$\lambda^+ = \left(1 + \frac{\sqrt{2}\sigma_f}{\sigma_f + \sigma_g}\right)^{-1}. \tag{2.28}$$

We can see from (2.28) that $\lambda^+ > \sqrt{2} - 1 \approx 41\%$, which says that the power allocated to the training of f is always no less than 41%. Also. when σ_f^2 increases, less power should be allocated to the training of f, while when σ_g^2 increases, more power should be allocated to the training of f.

2.5 Simulation Results and Discussions

In this section, we compare the proposed power allocation schemes with an even power allocation where $P_{f,S} = P_{f,R} = P_f/2$ and $P_g = P_f/2 = P_t/3$, i.e., $\eta = 1/2$ and $\lambda = 2/3$. In this even power allocation, the powers allocated to the source and the relay during the training of f and that allocated to the training of g are the same. To capture the effect of pass-loss, we model $\sigma_f^2 = (\frac{\phi}{d_{SR}})^\tau$ and $\sigma_g^2 = (\frac{\phi}{d_{RD}})^\tau$ where τ is the path-loss exponent and ϕ is a constant. d_{SR}, d_{RD}, and d_{SD} are the distances between the source and the relay, the relay and the destination, and the source and the destination, respectively. Here we introduce a parameter μ to relate d_{SR} and d_{SD} as $d_{SR} = \mu d_{SD}$. Thus $\sigma_f^2 = (\frac{\phi}{\mu d_{SD}})^\tau$ and $\sigma_g^2 = [\frac{\phi}{(1-\mu)d_{SD}}]^\tau$. In our simulations, we assume $\tau = 3$ and $\phi/d_{SD} = 1$.

Fig. 2.2 shows the total MSE in the logarithmic scale. The training power is set to be $P_t = 20$dB. When the relay is closer to the source, we can see that the MSE-based scheme is slightly better than the OP-based one and both proposed schemes are superior to the even power allocation. When the relay is closer to the destination, the gap between MSE-based scheme and the even power allocation scheme becomes larger. For $\mu = 0.1$, the total MSE reduces by about 24.2% with the proposed MSE-based scheme. For $\mu = 0.9$, the total MSE improvement of the proposed MSE-based scheme is about of 26.1%. The total MSE performance improves when μ increases. This indicates that we have better channel estimation quality as the relay is closer to the destination. The proposed OP-based power allocation is superior to even power allocation for small μ but inferior for large μ.

Fig. 2.3 and Fig. 2.4 show the network OP for different μ and P_t. We set $\gamma_{th} = 0.1$ and $P_{d,S} = P_{d,R} = 20$dB. In Fig. 2.3, we have training power $P_t = 20$dB. From Fig. 2.3, we see that performance improvement of 24.5% and 33.3%

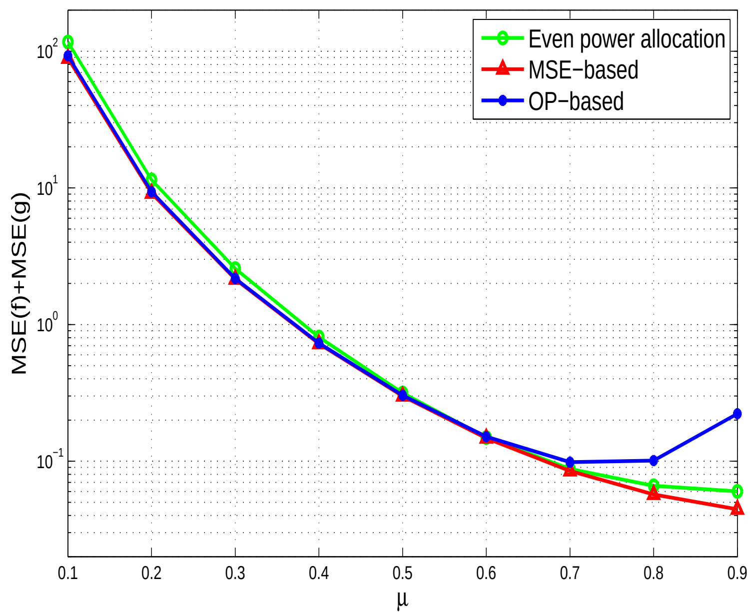

Fig. 2.2: Total MSE of channel estimations for one-way single-relay network where $P_t = 20$dB.

over the even power allocation are obtained with the proposed OP-based power allocation for $\mu = 0.1$ and $\mu = 0.9$, respectively. MSE-based and OP-based schemes have comparable performance for small μ. But as μ is closer to 1, the OP-based scheme is significantly better. The figure also shows that the OP improves when μ increases. This is due to the better channel estimation quality as the relay is closer to the destination. Fig. 2.4 shows that the OP improves as the total training power P_t increases. This is because with higher training power, the effect of channel estimation error is less significant. When the relay is closer to the destination node (μ is larger), the advantage of the OP-based scheme is more prominent.

2.6 Conclusions

In this chapter, we investigated the power allocation during training for a one-way single-relay network, which included the power allocation between the training of different channels and between the source and the relay during the training of the source-to-relay channel. For the high power regime, analytical solutions for the power allocation based on the minimization of the total MSE and the OP were found.

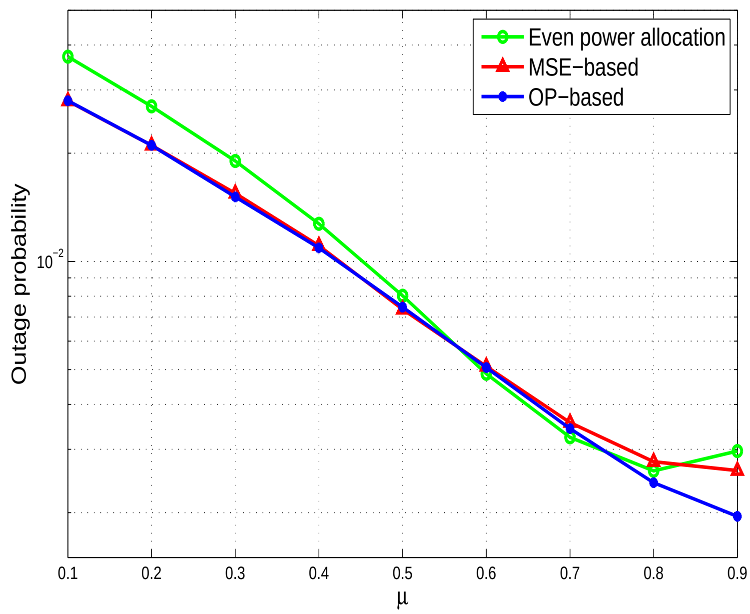

Fig. 2.3: Outage probability for different relay locations for one-way single-relay network where $P_t = 20$dB.

Simulations on the MSE and the OP were demonstrated to show the superiority of the proposed schemes to even power allocation.

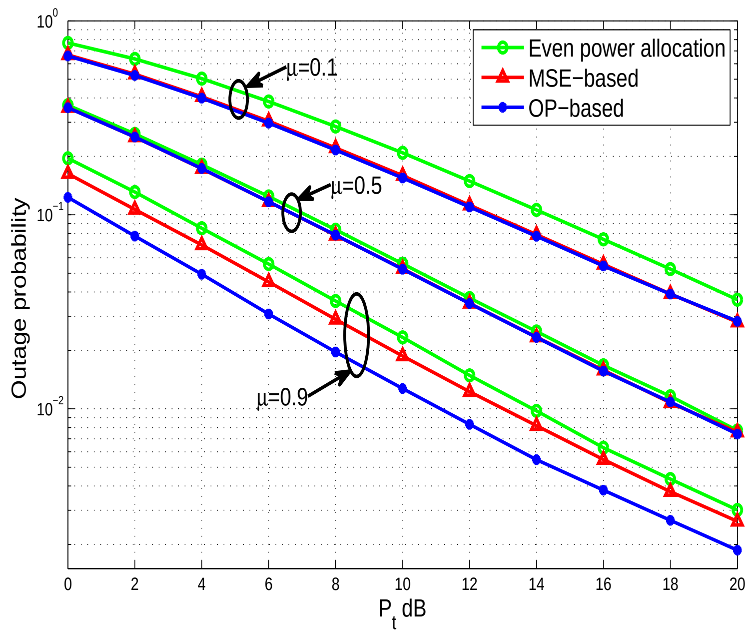

Fig. 2.4: Outage probability for different training power values for one-way single-relay network.

Chapter 3

Power Allocation in Training for One-Way Multi-Relay Network

3.1 Introduction

In this chapter, we consider a one-way multi-relay AF network and study the power allocation problem during the training of the individual source-to-relay and relay-to-destination channels. This includes three parts: 1) the power allocation among the training of channels on different relay paths, 2) for each relay path, the power allocation between the training of the source-to-relay channel and the relay-to-destination channel, and 3) in the training of the source-to-relay channel, the power allocation between the source transmission and the relay transmission. Two objective functions, the total MSE of estimations of all individual channels and the OP of the whole network, are considered. Simulation results of the proposed schemes are shown and compared to even power allocation.

In what follows, we clarify the difference of our work to existing ones [56, 61–64]. First of all, we consider networks with multiple relays, while [56, 61–64] were on single relay networks. Different to [61] and [62], where the end-to-end channel estimation was investigated, we focus on the estimation of the individual source-to-relay channel and relay-to-destination channel. In addition, our objective functions in the power allocation are different to those in [61, 62]. Compared with [56, 63, 64], we consider different system model and power allocation problem. [56] worked on the power allocation between the training and the data transmission at the source for a fixed relay power, [64] studied the power allocation between the estimation of different channels at the relay for a fixed source power, [63] worked on the power allocation between the training and the data transmission, and between the source and the relay. Our work is on the joint power allocation among the relay paths, between the two channels of each relay path, and between the source transmission and the relay transmission. Besides, [63] considered variable relay gain while our work considers fixed relay gain. Moreover, we provide closed-form analytical solutions to the power allocation problems.

The rest of the chapter is organized as follows. The system model, training scheme and the power allocation problem are described in Section 3.2. The MSE-based power allocation and the OP-based power allocation are investigated in Section 3.3 and Section 3.4, respectively. Simulation results are shown and discussed in Section 3.5. Concluding remarks are given in Section 3.6.

3.2 System Model and Training Scheme

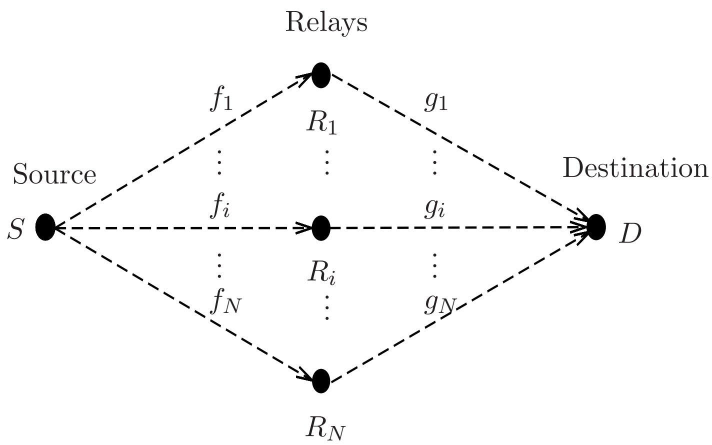

Fig. 3.1: One-way multi-relay network model.

We consider a one-way multi-relay network where there are one source node S, one destination node D, and N ($N \geq 2$) relay nodes R_1, R_2, $\cdots$, R_N. The network model is depicted in Fig. 3.1. Each node is equipped with single antenna which can be used for both transmission and reception. Denote the channel between S

and the ith relay as f_i and that between the ith relay and D as g_i. Both g_i and f_i are independent zero-mean complex Gaussian random variables with variance $\sigma_{g,i}^2$ and $\sigma_{f,i}^2$, respectively. Thus, the channels are assumed to be independent Rayleigh flat-fading. There is no direct link between the source and the destination due to large path-loss.

3.2.1 Training Scheme

In this subsection, we demonstrate the training scheme for the estimation of the individual channels at the receiver. Since all channels are independent, we consider the training and estimation of channels on different relay paths separately and sequentially. In other words, the training of g_1, f_1 is conducted first, followed by the training of g_2, f_2, so on so forth, and finally the training of g_N, f_N. Without loss of generality, in what follows, we elaborate the training of g_i, f_i. The procedure follows our work in Chapter 2. It contains two stages.

The first stage is for the training of g_i, which takes 1 time slots. Relay i sends a training symbol to the destination. Without loss of generality, we use the unit symbol as the training symbol. The received symbol at the destination is

$$x_i = \sqrt{P_{g,i}}g_i + n_g, \tag{3.1}$$

where $P_{g,i}$ is the power the ith relay uses for the training of g_i and n_g is the noise at the destination whose distribution follows $\mathcal{CN}(0, 1)$.

The second stage is for the training of f_i, which takes 2 time slots. A two-step cooperative strategy is used, where first the source sends the unit symbol to Relay i, then Relay i amplifies and forwards its received signal with fixed-gain relay power coefficient explained in Section 2.2.1. Denote the power used at the source and Relay i for the training of f_i as $P_{S,i}$ and $P_{R,i}$, respectively. Define

$$\alpha_i \triangleq \frac{P_{S,i}P_{R,i}}{1 + P_{S,i}\sigma_{f,i}^2} \quad \text{and} \quad \beta_i \triangleq \frac{P_{R,i}}{1 + P_{S,i}\sigma_{f,i}^2}. \tag{3.2}$$

The same as the description in Chapter 2, the received symbol at the destination is

$$y_i = \sqrt{\alpha_i}g_i f_i + \sqrt{\beta_i}g_i n_i + n_f, \tag{3.3}$$

where n_i and n_f are the noises at the ith relay and the destination respectively, following the distribution $\mathcal{CN}(0,1)$.

Denote the total transmission power used for the training of f_i as $P_{f,i}$. We have

$$P_{f,i} = P_{S,i} + P_{R,i}.$$

Denote the total transmission power used for the training of channels related to Relay i (f_i and g_i) as P_i. We have

$$P_i = P_{f,i} + P_{g,i}.$$

If the total power used in the overall training process is P_t. We have

$$P_t = \sum_{i=1}^{N} P_i.$$

We illustrate the training scheme and the power used for each training step in Fig. 3.2.

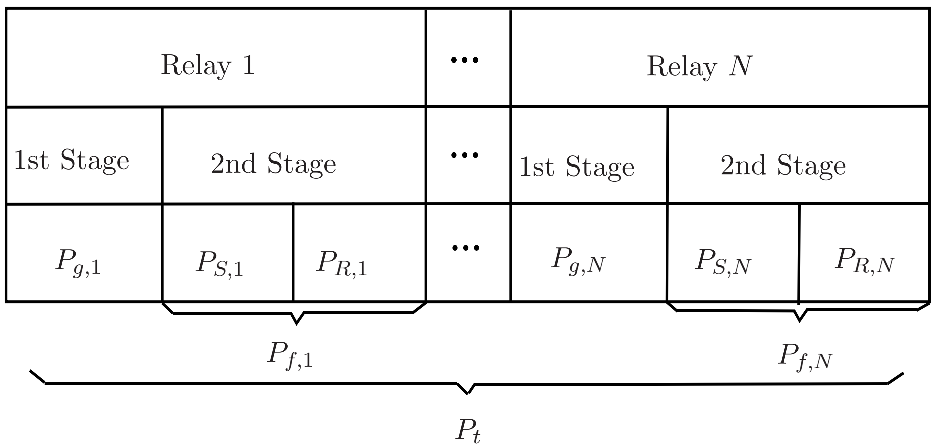

Fig. 3.2: Training scheme and power for one-way multi-relay network.

3.2.2 LMMSE Estimators

In this subsection, we explain the LMMSE estimates of the individual channel coefficients f_i and g_i ($i = 1, \cdots, N$) at the destination. Again, without loss of generality, we elaborate the estimations of f_i and g_i.

First for g_i, from (3.1), similarly to Chapter 2, we get the LMMSE estimate as

$$\hat{g}_i = \frac{\sqrt{P_{g,i}}\sigma_{g,i}^2}{1 + P_{g,i}\sigma_{g,i}^2} x_i. \tag{3.4}$$

Since the observation model in (3.1) is linear and Gaussian, (3.4) is also the MMSE estimation of g_i. Denote the estimation error as $\triangle g_i \triangleq g_i - \hat{g}_i$. $\triangle g_i$ follows the distribution $\mathcal{CN}\left(0, \frac{\sigma_{g,i}^2}{1+P_{g,i}\sigma_{g,i}^2}\right)$. The MSE on g_i is:

$$\text{MSE}(g_i) = \frac{\sigma_{g,i}^2}{1 + P_{g,i}\sigma_{g,i}^2} \approx \frac{1}{P_{g,i}}, \tag{3.5}$$

where the approximation is under the condition $P_{g,i} \gg 1$.

To estimate f_i, we use the observation y_i in (3.3). Notice that g_i is unknown and only the estimation $\hat{g}_i$ is known at the destination. We rewrite (3.3) as

$$y_i = \sqrt{\alpha_i}\hat{g}_i f_i + (\sqrt{\alpha_i} f_i + \sqrt{\beta_i} n_i)\triangle g_i + \sqrt{\beta_i}\hat{g}_i n_i + n_f. \tag{3.6}$$

Let

$$w_i \triangleq (\sqrt{\alpha_i} f_i + \sqrt{\beta_i} n_i)\triangle g_i + \sqrt{\beta_i}\hat{g}_i n_i + n_f,$$

which is the equivalent noise in the observation model in (3.6). The MMSE estimate of f_i is intractable due to the unknown distribution of w_i. Using the results in Subsection1.3.2, after straightforward calculations, the LMMSE estimate of f_i is:

$$\hat{f}_i = \frac{\sigma_{f,i}^2\sqrt{\alpha_i}\hat{g}_i^* y_i}{1 + (\sigma_{f,i}^2\alpha_i + \beta_i)[|\hat{g}_i|^2 + \text{MSE}(g_i)]}. \tag{3.7}$$

Denote the estimation error as $\triangle f_i \triangleq f_i - \hat{f}_i$. By using (1.9), the MSE on f_i is:

$$\text{MSE}(f_i) = \sigma_{f,i}^2 - \frac{\sigma_{f,i}^4\alpha_i|\hat{g}_i|^2}{1 + (\sigma_{f,i}^2\alpha_i + \beta_i)[|\hat{g}_i|^2 + \text{MSE}(g_i)]}. \tag{3.8}$$

3.2.3 Power Allocation Problem Statement

We investigate the power allocation problem during the channel training process for a given total training power. This includes the power allocation among different relay paths, and for each relay path, the power allocation between the training of the relay-to-destination channel and the training of the source-to-relay channel, and for the source-to-relay channel training, the power allocation between the source transmission and the relay transmission.

To help the problem formulation, we introduce a few new notation on power. Recall that P_t is the total training power and P_i is the training power for the ith relay path. Define θ_i such that

$$P_i = \theta_i P_t \quad \text{and} \quad \sum_{i=1}^{N} \theta_i = 1. \tag{3.9}$$

Thus θ_i is the percentage of power allocated to the ith relay path. Recall that for the ith relay path, $P_{g,i}$ is the power for the training of g_i and $P_{f,i}$ is the power for the training of f_i. Define λ_i such that

$$P_{f,i} = \lambda_i P_i \quad \text{and} \quad P_{g,i} = (1 - \lambda_i) P_i. \tag{3.10}$$

Thus λ_i is the percentage of power allocated for the training of f_i. Recall that $P_{S,i}$ and $P_{R,i}$ are the source power and relay power used for the training of f_i. Define η_i such that

$$P_{S,i} = \eta_i P_{f,i} \quad \text{and} \quad P_{R,i} = (1 - \eta_i) P_{f,i}. \tag{3.11}$$

Thus η_i is the percentage of power allocated to the source in the two-step training of f_i. The notation and relationship for the training power of each training stages and steps are also represented in Fig. 3.2.

Under an overall training power constraint P_t, our power allocation problem is to derive the optimal θ_i, λ_i and η_i. This joint power allocation problem is challenging due to the complex nature of multi-relay networks. There are conflicts among the relays and among different channels for training power. Further, due to the two-step transmission, the performance of the source-to-relay channel estimate depends on the estimate of the relay-to-destination channel. From (3.8) and (3.2), we can see that $\text{MSE}(f_i)$ is a function of not only $P_{S,i}, P_{R,i}$ (the power used in the training of f_i), but also $\text{MSE}(g_i)$ and $|\hat{g}_i|$. In addition, $\text{MSE}(f_i)$ depends on $\hat{g}_i$, the estimation value of g_i.

In the following two sections, we propose closed-form solutions for the power allocation based on two objective functions: the total MSE of the LMMSE estimates of all channels and the OP of the network with channel estimation error. For tractable analysis, we consider the high power region only, i.e., $P_t \gg 1$.

3.3 MSE-Based Power Allocation

In this section, we use the total MSE of all channel estimates as the criterion for the power allocation.

3.3.1 MSE-Based Power Allocation Solution

The total MSE on all channels is

$$\text{MSE}_{total} = \sum_{i=1}^{N}[\text{MSE}(g_i) + \text{MSE}(f_i)]. \tag{3.12}$$

Our power allocation problem can be represented as

$$\arg\min_{\theta_i,\lambda_i,\eta_i} \sum_{i=1}^{N}[\text{MSE}(g_i) + \text{MSE}(f_i)] = \arg\min_{\theta_i} \sum_{i=1}^{N} \min_{\lambda_i}[\text{MSE}(g_i) + \min_{\eta_i} \text{MSE}(f_i)], \tag{3.13}$$

where the equality is because $\text{MSE}(g_i)$ is independent of η_i.

There are three layers of optimization. First, we solve the optimization of η_i. The problem is $\min_{\eta_i} \text{MSE}(f_i)$. By using (3.11), (3.5), and (3.2) in (3.8), we have

$$\text{MSE}(f_i) = \sigma_{f,i}^2 - \frac{\eta_i(1-\eta_i)P_{f,i}^2|\hat{g}_i|^2\sigma_{f,i}^4}{(1+\sigma_{f,i}^2\eta_i P_{f,i})[1+\left(|\hat{g}_i|^2 + P_{g,i}^{-1}\right)(1-\eta_i)P_{f,i}]}. \tag{3.14}$$

By calculating the derivative of $\text{MSE}(f_i)$ with respect to η_i and making it zero, the optimal η_i can be found to be:

$$\eta_i^* = \frac{\sqrt{1+\left(|\hat{g}_i|^2 + P_{g,i}^{-1}\right)P_{f,i}}}{\sqrt{1+\left(|\hat{g}_i|^2 + P_{g,i}^{-1}\right)P_{f,i}} + \sqrt{1+\sigma_{f,i}^2 P_{f,i}}}. \tag{3.15}$$

Now, we consider the optimization of λ_i. The problem is

$$\begin{aligned} &\arg\min_{\lambda_i}[\text{MSE}(g_i) + \min_{\eta_i} \text{MSE}(f_i)] \\ &= \arg\min_{\lambda_i}\left[\text{MSE}(g_i) + \text{MSE}(f_i)|_{\eta=\eta^*}\right]. \end{aligned} \tag{3.16}$$

Define

$$\rho_{1,i} \triangleq \sqrt{1+\left(|\hat{g}_i|^2 + P_{g,i}^{-1}\right)P_{f,i}} = \sqrt{1+\lambda_i[(1-\lambda_i)^{-1} + |\hat{g}_i|^2 P_i]},$$
$$\rho_{2,i} \triangleq \sqrt{1+\sigma_{f,i}^2 P_{f,i}} = \sqrt{1+\sigma_{f,i}^2\lambda_i P_i}.$$

By using the optimal η_i in (3.15) and the definition of λ_i in (3.10), we can show with straightforward calculations from (3.14) that

$$\mathrm{MSE}(f_i)|_{\eta_i=\eta_i^*}=\sigma_{f,i}^2\left[\frac{2}{1+\rho_{1,i}\rho_{2,i}}+\frac{\lambda_i P_i[\sigma_{f,i}^2+|\hat{g}_i|^2+(1-\lambda_i)^{-1}P_i^{-1}]+\sigma_{f,i}^2\lambda_i^2(1-\lambda_i)^{-1}P_i}{(1+\rho_{1,i}\rho_{2,i})^2}\right] \tag{3.17}$$

Recall that λ_i represents the power allocation between the training of f_i and g_i for the ith relay path. Its optimization is before the training of g_i, so the value of $\hat{g}_i$ is not available in the optimization of λ_i. Thus, we should use the average value of (3.17) over $\hat{g}_i$ in the objective function in (3.16). However, because of the complexity of the function, the average cannot be found in a tractable form. In the following, for tractable analysis and closed-form solution, we approximate $|\hat{g}_i|^2$ with its average value, i.e., $|\hat{g}_i|^2 \approx \mathbb{E}\{|\hat{g}_i|^2\} = \frac{P_{g,i}\sigma_{g,i}^4}{1+P_{g,i}\sigma_{g,i}^2}$. Using this approximation and also considering the high power region ($P_i \gg 1$), we have

$$\rho_{1,i} \approx \sqrt{1+\sigma_{g,i}^2\lambda_i P_i},$$

$$|\hat{g}_i|^2+(1-\lambda_i)^{-1}P_i^{-1} \approx \sigma_{g,i}^2.$$

Then, (3.17) is approximated as

$$\mathrm{MSE}(f_i)|_{\eta_i=\eta_i^*} \tag{3.18}$$

$$\approx \sigma_{f,i}^2\left[\frac{2}{1+\sqrt{1+\sigma_{g,i}^2\lambda_i P_i}\sqrt{1+\sigma_{f,i}^2\lambda_i P_i}}+\frac{\lambda_i P_i(\sigma_{g,i}^2+\sigma_{f,i}^2)+\sigma_{f,i}^2\lambda_i^2(1-\lambda_i)^{-1}P_i}{\left(1+\sqrt{1+\sigma_{g,i}^2\lambda_i P_i}\sqrt{1+\sigma_{f,i}^2\lambda_i P_i}\right)^2}\right]$$

$$=\frac{1}{\sigma_{g,i}^2 P_i}\left[\frac{(\sigma_{f,i}+\sigma_{g,i})^2}{\lambda_i}+\frac{\sigma_{f,i}^2}{1-\lambda_i}\right]+\mathcal{O}\left(\frac{1}{P_i^2}\right). \tag{3.19}$$

Thus using (3.5) and (3.19), we have

$$\mathrm{MSE}(f_i)|_{\eta_i=\eta_i^*}+\mathrm{MSE}(g_i) \approx \frac{1}{\sigma_{g,i}^2 P_i}\left[\frac{(\sigma_{f,i}+\sigma_{g,i})^2}{\lambda_i}+\frac{\sigma_{f,i}^2+\sigma_{g,i}^2}{1-\lambda_i}\right]+\mathcal{O}\left(\frac{1}{P_i^2}\right) \tag{3.20}$$

Define

$$h_i(\lambda_i) \triangleq \left[\frac{(\sigma_{f,i}+\sigma_{g,i})^2}{\lambda_i}+\frac{\sigma_{f,i}^2+\sigma_{g,i}^2}{1-\lambda_i}\right]. \tag{3.21}$$

The λ_i optimization problem in (3.16) can thus be approximated as $\arg\min_{\lambda_i} h_i(\lambda_i)$. By calculating the derivative of the objective function and making it zero, the optimal λ_i is

$$\lambda_i^* = \left(1+\frac{\sqrt{\sigma_{f,i}^2+\sigma_{g,i}^2}}{\sigma_{g,i}+\sigma_{f,i}}\right)^{-1}. \tag{3.22}$$

By using the optimal solution λ_i^* in (3.21), we have

$$h_i(\lambda_i^*) = \left(1 + \sigma_{f,i}\sigma_{g,i}^{-1} + \sqrt{1 + \sigma_{f,i}^2\sigma_{g,i}^{-2}}\right)^2 P_i^{-1}. \tag{3.23}$$

Finally, as the last step, we solve the optimization of θ_i. Notice that with the approximation mentioned above, from (3.13), the optimization problem becomes

$$\arg\min_{\theta_i} \sum_{i=1}^{N} h_i(\lambda_i^*), \text{subject to} \sum_{i=1}^{N} \theta_i = 1. \tag{3.24}$$

Since the objective function is a monotonic function of each θ_i and the constraint is linear in θ_i, (3.24) is a convex optimization problem. Thus we can use the method of Lagrange multipliers to find the global optimal. We introduce a Lagrange multiplier z, and the Lagrange function is defined as

$$\Lambda_1(\theta_1, \cdots, \theta_N, z) \triangleq \sum_{i=1}^{N} \frac{\left[(\sigma_{g,i} + \sigma_{f,i}) + \sqrt{\sigma_{f,i}^2 + \sigma_{g,i}^2}\right]^2}{\sigma_{g,i}^2\theta_i} + z\left(\sum_{i=1}^{N}\theta_i - 1\right). \tag{3.25}$$

By calculating the gradient of (3.25) and making them zero, the optimal θ_i can be derived to be

$$\theta_i^* = \frac{1 + \sigma_{f,i}\sigma_{g,i}^{-1} + \sqrt{1 + \sigma_{f,i}^2\sigma_{g,i}^{-2}}}{\sum_{i=1}^{N}\left(1 + \sigma_{f,i}\sigma_{g,i}^{-1} + \sqrt{1 + \sigma_{f,i}^2\sigma_{g,i}^{-2}}\right)}. \tag{3.26}$$

To sum-up, the proposed power allocation solution is specified by (3.15), (3.22), and (3.26).

3.3.2 Discussions

One possible implementation of this power allocation is as follows. For a given fixed training power P_t. The nodes (source, relays, and destination) in the network calculate the power allocated to the training of the channels on each relay path $P_1, \cdots, P_N$ from (3.9) using the optimal θ_i^* provided in (3.26). Notice that θ_i^* depends on the channel variances only. In wireless communication, the variance of a channel does not change or change significantly slower than the instantaneous channel value. Thus, it is easy for the nodes in a network to know the values of the channel variances. Then the source and the ith Relay calculate the powers allocated to the training of g_i and the training of f_i, as from (3.10) using the optimal λ_i^* in

(3.22). Notice that similar to θ_i^*, λ_i^* also depends on the channel variances only. Then Relay i uses power $P_{g,i}$ to conduct the training of g_i and the receiver obtains the channel estimate $\hat{g}_i$. The receiver then calculate the optimal η_i^* from (3.15) and broadcasts it back to the source and Relay i. The source and Relay i can then calculate the power $P_{g,i}$ and $P_{f,i}$ to use in the two-step training of f_i from (3.11).

To implement the power allocation between the source and the relay in the training f_i, we need 1) to conduct the training of g_i first; and 2) to use the receiver as the master controller and feedback the power coefficients. This is because η^* depends on $\hat{g}_i$, the instantaneous relay-to-destination channel estimate.

The proposed power allocation solution is adaptive to the channel variances. In the training of the source-to-relay channels, the powers are also adaptive to the estimates of the relay-to-destination channels. Thus, the proposed design can achieve better performance as will be seen in the simulation section. However, to implement the design, feedback from the receiver is needed, which causes extra overhead.

3.4 OP-Based Power Allocation

The MSE-based power allocation scheme proposed in this section aims at the minimum total variance of the channel estimation error. However, total MSE does not directly depict the performance of the network, e.g., the throughput or the accuracy of the data transmission. In addition, in the total MSE formula, the MSE of all channels, $\text{MSE}(f_i), \text{MSE}(g_i), i = 1, \cdots, N$ are weighted equally, which may not be optimal for relay network performance. These motivate the OP-based power allocation.

In this section, we consider the OP of the network, denoted as P_{out}, and use it as the objective function in the training power allocation problem. The OP depends on not only the CSI precision, but also the data-transmission protocol. In this work, we choose the best relay selection (Section 1.2.3) for the data-transmission, where the relay that provides the best received SNR will be selected to forward information. In [66], the effect of feedback delay and channel estimation errors were investigated for relay selection with DF. For two-way relay networks with relay selection, the impact of channel estimation error on the performance was studied in [67]. In this work, we consider the OP under channel estimation error with relay selection.

The best relay selection scheme under channel estimation error is as follows. First, the source sends out a data symbol s which has unit power. Then given the channel estimates, the relay with the highest received SNR is selected to forward using fixed gain power coefficient. Denote the power the source and the ith relay use for data-transmission as $Q_{S,i}$ and $Q_{R,i}$, respectively. Let

$$\alpha_{di} \triangleq \frac{Q_{S,i}Q_{R,i}}{1+Q_{S,i}\sigma_{f,i}^2}, \quad \beta_{di} \triangleq \frac{Q_{R,i}}{1+Q_{S,i}\sigma_{f,i}^2}.$$

If the ith relay is selected, using the results in Chapter 2, the transceiver equation is

$$y_{d,i} = \sqrt{\alpha_{di}}\hat{g}_i\hat{f}_i s + \nu_i, \tag{3.27}$$

where

$$\nu_i \triangleq \sqrt{\alpha_{di}}[\triangle f_i(\hat{g}_i + \triangle g_i) + \hat{f}_i \triangle g_i]s + \sqrt{\beta_{di}}(\hat{g}_i + \triangle g_i)n_{di} + n_d.$$

ν_i is the noise-plus-channel-error term. n_{di} and n_d are the noises at the ith relay and the destination, following $\mathcal{CN}(0,1)$. Recall that $\triangle g_i$ follows the distribution $\mathcal{CN}\left(0, \frac{\sigma_{g,i}^2}{1+P_{g,i}\sigma_{g,i}^2}\right)$. Compared to $\hat{g}_i$, $\triangle g_i$ is a lower order term of P_t. When the training power is high, we can neglect the lower order terms in v_i which are $\sqrt{\alpha_{di}}\triangle f_i \triangle g_i s$ and $\sqrt{\beta_{di}}\triangle g_i n_{di}$. The received SNR via the ith relay path can be approximated as

$$\begin{aligned}\frac{\mathbb{E}(\alpha_{di}|\hat{g}_i|^2|\hat{f}_i|^2)}{\mathbb{E}(|\nu_i|^2)} &\approx \frac{\alpha_{di}|\hat{g}_i|^2|\hat{f}_i|^2}{\mathbb{E}[\sqrt{\alpha_{di}}(\triangle f_i\hat{g}_i + \hat{f}_i\triangle g_i)s + \sqrt{\beta_{di}}\hat{g}_i n_{di} + n_d]} \\ &= \frac{\alpha_{di}|\hat{g}_i|^2|\hat{f}_i|^2}{[\alpha_{di}\text{MSE}(f_i)+\beta_{di}]|\hat{g}_i|^2 + \alpha_{di}\text{MSE}(g_i)|\hat{f}_i|^2 + 1} \triangleq \gamma_i. \end{aligned} \tag{3.28}$$

The relay with the highest γ_i is chosen for information forwarding. The received SNR after relay selection is:

$$\gamma = \max_i\{\gamma_i\}. \tag{3.29}$$

We have proved that the following lemma on the OP of best relay selection under channel estimation error.

Lemma 2: *Assume that all powers have the same order, i.e.,* $P_t, Q_{S,i}, Q_{R,i} \sim P$ *and* $P \gg 1$*. Assume that the MSEs of the channel estimations for* f_i *and* g_i *are* $\mathrm{MSE}(f_i)$ *and* $\mathrm{MSE}(g_i)$*, which are fixed values. With the SNR-threshold* γ_{th}*, the outage probability of the network is*

$$P_{\text{out}} = \gamma_{th}^N \prod_{i=1}^{N} \left(\frac{\mathrm{MSE}(g_i)}{\sigma_{g,i}^2} + \frac{\mathrm{MSE}(f_i)}{\sigma_{f,i}^2} + T_i \right) + \mathcal{O}\left(\frac{\ln P}{P^N}\right), \tag{3.30}$$

where

$$T_i \triangleq \frac{1}{Q_{S,i}\sigma_{f,i}^2} + \frac{1}{Q_{R,i}\sigma_{g,i}^2}\left(1 - 2\gamma - \ln \frac{\gamma_{th}}{Q_{R,i}\sigma_{g,i}^2}\right).$$

with γ *the Euler-Mascheroni Constant.*

Proof. From (3.28) we can see that γ_i is a function of $|\hat{g}_i|$ and $|\hat{f}_i|$, which has the same structure as the SNR in [68]. Also, when the training power is high, $\hat{f}_i$ can be approximated to be Gaussian. By using Theorem 1 in [68], the CDF of γ_i is:

$$F_{\gamma_i}(\gamma_{th}) = 1 - e^{-\gamma_{th}\left(\frac{\mathrm{MSE}(f_i)}{\sigma_{f,i}^2} + \frac{\mathrm{MSE}(g_i)}{\sigma_{g,i}^2} + \frac{\beta_{di}}{\alpha_{di}\sigma_{f,i}^2}\right)} \sqrt{\frac{4\gamma_{th}}{\alpha_{di}\sigma_{g,i}^2\sigma_{f,i}^2}} K_1\left(\sqrt{\frac{4\gamma_{th}}{\alpha_{di}\sigma_{g,i}^2\sigma_{f,i}^2}}\right) \tag{3.31}$$

where $K_1(\cdot)$ is the modified Bessel function of the 2nd kind. When $Q_{R,i} \gg 1$, we have

$$\sqrt{\frac{4\gamma_{th}}{\alpha_{di}\sigma_{g,i}^2\sigma_{f,i}^2}} K_1\left(\sqrt{\frac{4\gamma_{th}}{\alpha_{di}\sigma_{g,i}^2\sigma_{f,i}^2}}\right) = 1 - \frac{\gamma_{th}}{Q_{R,i}\sigma_{g,i}^2}\left(1 - 2\gamma - \ln\frac{\gamma_{th}}{Q_{R,i}\sigma_{g,i}^2}\right) + \mathcal{O}\left(\frac{\ln Q_{R,i}}{Q_{R,i}^2}\right),$$

from which we can represent (3.31) as

$$F_{\gamma_i}(\gamma_{th}) = \gamma_{th}\left(\frac{\mathrm{MSE}(f_i)}{\sigma_{f,i}^2} + \frac{\mathrm{MSE}(g_i)}{\sigma_{g,i}^2} + T_i\right) + \mathcal{O}\left(\frac{\ln P}{P^2}\right). \tag{3.32}$$

From the received SNR formula in (3.29), the OP can be calculated as:

$$P_{\text{out}} = \mathbb{P}[\gamma \le \gamma_{th}] = \prod_{i=1}^{N} F_{\gamma_i}(\gamma_{th}). \tag{3.33}$$

After straightforward mathematical manipulation, (3.30) can be acquired. □

The OP-based power allocation problem is to find θ_i, λ_i and η_i that minimize the OP in (3.30). It is noteworthy that in the total MSE optimization, as shown in (3.12), the sum-MSE of every relay path and the MSEs of the two channels on every path have the same weighting. With the OP optimization, from (3.30), we can see that the MSEs of different relay paths and channels have different weights based on the channel variances, $\sigma_{g,i}^2$ and $\sigma_{g,i}^2$.

For high power range, we can ignore the lower order term of P to obtain the OP-based power allocation problem as follows:

$$\begin{aligned} &\arg\min_{\theta_i,\lambda_i,\eta_i} \prod_{i=1}^{N} \left(\frac{\text{MSE}(g_i)}{\sigma_{g,i}^2} + \frac{\text{MSE}(f_i)}{\sigma_{f,i}^2} + T_i \right) \\ &= \arg\min_{\theta_i} \prod_{i=1}^{N} \left[\arg\min_{\lambda_i} \left(\frac{\text{MSE}(g_i)}{\sigma_{g,i}^2} + \arg\min_{\eta_i} \frac{\text{MSE}(f_i)}{\sigma_{f,i}^2} \right) + T_i \right], \end{aligned} \tag{3.34}$$

where the equality is because $\text{MSE}(g_i)$ is independent of η_i and T_i is independent of the training powers P_i, $P_{g,i}$, $P_{S,i}$ and $P_{R,i}$.

Similar to the derivations in Section 3.3, there are three layers of optimization. First, we solve the optimization of η_i: $\arg\min_{\eta_i} \text{MSE}(f_i)$, which is the same as that in Section 3.3. Thus the optimal solution η_i^* is given in (3.15).

Next, we solve the second layer optimization of λ_i in (3.34), the problem is

$$\arg\min_{\lambda_i} \left(\frac{\text{MSE}(g_i)}{\sigma_{g,i}^2} + \frac{\text{MSE}(f_i)|_{\eta_i=\eta_i^*}}{\sigma_{f,i}^2} \right). \tag{3.35}$$

We consider the same approximation as that in Section 3.3. Similarly from (3.19) and (3.5), we have

$$\frac{\text{MSE}(g_i)}{\sigma_{g,i}^2} + \frac{\text{MSE}(f_i)|_{\eta_i=\eta_i^*}}{\sigma_{f,i}^2} \approx \frac{(\sigma_{g,i}+\sigma_{f,i})^2}{\lambda_i P_i} + \frac{2\sigma_{f,i}^2}{(1-\lambda_i)P_i} \triangleq \tilde{h}_i(\lambda).$$

The problem in (3.35) reduces to

$$\arg\min_{\lambda_i} \tilde{h}_i(\lambda).$$

By calculating the derivative of the objective function and making it zero, the optimal λ_i is

$$\lambda_i^+ = \left(1 + \frac{\sqrt{2}\sigma_{f,i}}{\sigma_{f,i}+\sigma_{g,i}} \right)^{-1}. \tag{3.36}$$

Finally, we solve the optimization of θ_i. For the very high power regime, $\ln P \gg 1$, with the optimal λ_i^+ in (3.36), we have

$$\tilde{h}_i(\lambda^+) = \frac{(\sigma_{g,i}+\sigma_{f,i}+\sqrt{2}\sigma_{f,i})^2}{\sigma_{g,i}^2\sigma_{f,i}^2\theta_i P_t} + T_i$$

and

$$\prod_{i=1}^{N}\tilde{h}_i(\lambda^+) = \prod_{i=1}^{N}\left(\frac{(\sigma_{g,i}+\sigma_{f,i}+\sqrt{2}\sigma_{f,i})^2}{\sigma_{g,i}^2\sigma_{f,i}^2\theta_i P_t}+T_i\right)$$
$$= \sum_{i=1}^{N}\frac{(\sigma_{g,i}+\sigma_{f,i}+\sqrt{2}\sigma_{f,i})^2\prod_{j=1,j\neq i}^{N}T_j}{\sigma_{g,i}^2\sigma_{f,i}^2\theta_i P_t}+\mathcal{O}\left(\frac{\ln^{N-2}P}{P^N}\right). \quad (3.37)$$

The first term in (3.37) scales as $\mathcal{O}\left(\frac{\ln^{N-1}P}{P^N}\right)$. Thus, after omitting the lower order term of P, the θ_i optimization reduces to

$$\arg\min_{\theta_i}\sum_{i=1}^{N}\frac{(\sigma_{g,i}+\sigma_{f,i}+\sqrt{2}\sigma_{f,i})^2\prod_{j=1,j\neq i}^{N}T_j}{\sigma_{g,i}^2\sigma_{f,i}^2\theta_i P_t}, \text{subject to} \sum_{i=1}^{N}\theta_i = 1. \quad (3.38)$$

It is obvious that the objective function is a monotonic function, and the constraint is linear in θ_i. Thus we use the same method in Section 3.3 to obtain the optimal θ_i as

$$\theta_i^+ = \frac{[\sigma_{f,i}^{-1}+(1+\sqrt{2})\sigma_{g,i}^{-1}]\prod_{j=1,j\neq i}^{N}(T_j)^{1/2}}{\sum_{i=1}^{N}\{[\sigma_{f,i}^{-1}+(1+\sqrt{2})\sigma_{g,i}^{-1}]\prod_{j=1,j\neq i}^{N}(T_j)^{1/2}\}}. \quad (3.39)$$

To sum-up, the solution of the OP-based training power allocation is specified by (3.15), (3.36), and (3.39).

For the OP-based power allocation scheme, we get the same optimal solution η_i as that for the MSE-based scheme, but for different optimal λ_i and θ_i solutions. From (3.36), we can see that λ_i^+ depends on the channel variances only. When $\sigma_{f,i}^2$ increases or $\sigma_{g,i}^2$ decreases, less power should be allocated to the training of f_i, and vice versa. From (3.39), we can see that θ_i^+ depends on not only the channel variances but also the data transmission power and SNR-threshold via T_j. To implement the power allocation, the same procedure as explained in Section 3.3.2 can be used.

3.5 Simulation Results and Discussions

In this section, we show the simulation results. The total MSE of all channel estimations and the OP of the network under channel estimation error are simulated. We compare the two proposed schemes, MSE-based power allocation and OP-based power allocation, with an even power allocation, where $P_{S,i} = P_{R,i} = P_{g,i} = P_i/3$

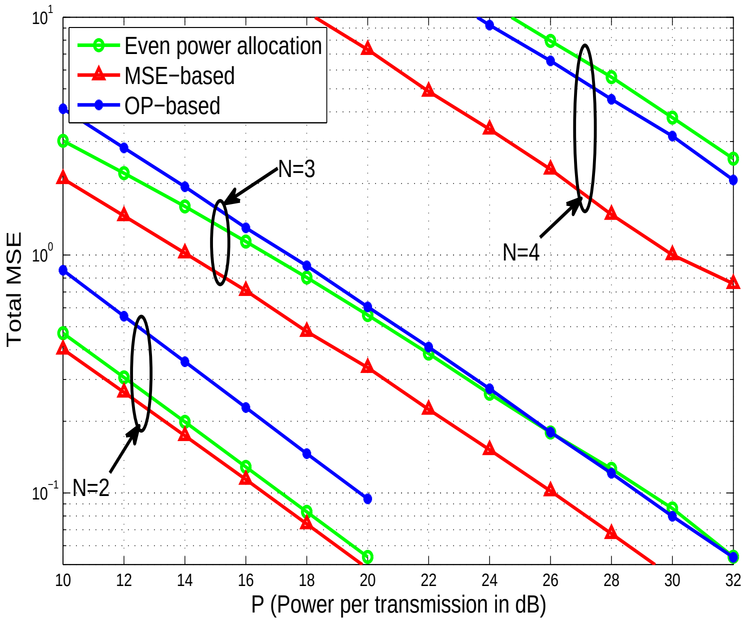

Fig. 3.3: Total MSE for different training power values for one-way networks with 2, 3, and 4 relays.

and $P_i = P_t/N$, i.e., $\theta_i = 1/N$, $\lambda_i = 2/3$, and $\eta_i = 1/2$. With this power allocation, the transmission powers for every training step and every node are the same. Similar as that in Chapter 2, we use μ_i to represent the location of the ith relay for the ith path. We assume the path-loss exponent $\tau = 2$ and $\phi/d_{SD} = 1$. Thus $\sigma_{f,i}^2 = (\frac{1}{\mu_i})^2$ and $\sigma_{g,i}^2 = (\frac{1}{1-\mu_i})^2$

Fig. 3.3 shows the total MSE in the logarithmic scale versus the training power for networks with $N = 2, 3$ and 4. Power P in Fig. 3.3 means the average training power for every transmission. In other words, $P_t = 3NP$. We use P instead of P_t for the horizontal axis because when N is large, with the same P_t, the power allocated to each step decreases, which may result in misleading MSE comparison. We draw the total MSEs for the power range 10-32dB. In this simulation, the locations of the relays are random, where μ_i are generated as a uniform random variable on $(0, 1)$. From Fig. 3.3, we can see that both proposed schemes achieve lower MSEs than the even power allocation, while the MSE-based scheme has lower total MSE than the OP-based scheme for $N = 3, 4$. For $N = 2$, the OP-based scheme has the largest MSE, while the MSE-based scheme has the lowest MSE. The figure also shows that the total MSE reduces when training power increases. We also observe that the gap between the MSE-based scheme and the even scheme becomes larger when training power increases. For $N = 4$, the improvements of the MSE-based scheme over even power allocation are about 68.4% and 73.5% at $P = 20$dB and $P = 30$dB, respectively. In addition, the MSE increase when N increases, which is because there are more channels and the total MSE is the summation of errors of all channels.

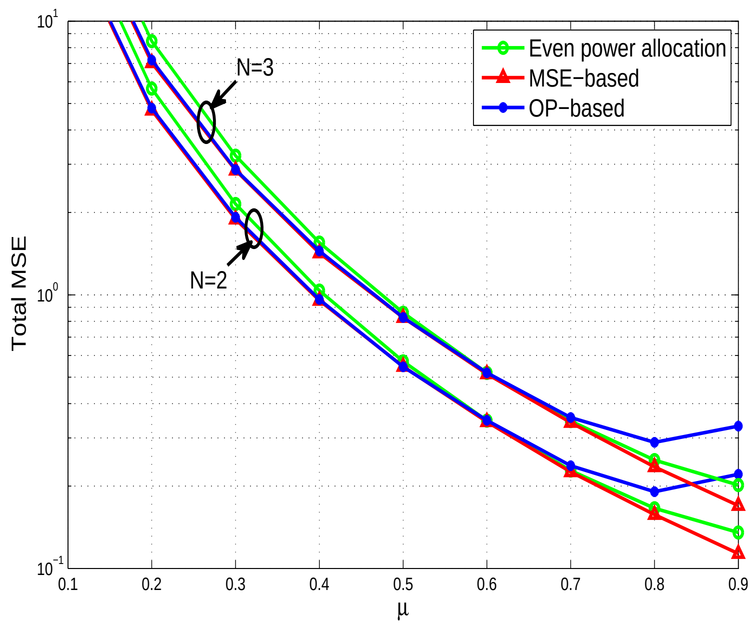

Fig. 3.4: Total MSE for different relay locations for one-way networks with 2 and 3 relays where $P = 15$dB.

Fig. 3.4 shows the total MSE in the logarithmic scale for different relay locations, determined by the parameter μ. We fix $P = 15$dB and study networks with $N = 2$ and $N = 3$. We consider fixed network topology and assume that all relays are close to each other with the same μ value. In Fig. 3.4, we can see that when the relays are closer to the source (small μ), the MSE-based and the OP-based schemes have comparable performance and both outperform the even power allocation. When the relays are closer to the destination (large μ), the MSE-based scheme is superior to both the even and OP-based schemes. For $N = 3$, the total MSE reduction of the proposed MSE-based scheme over even power allocation is about 16.5% and 16.2%

at $\mu = 0.2$ and $\mu = 0.9$, respectively. The total MSE performance improves when μ increases which indicates that channel estimation quality gets better as the relays are closer to the destination.

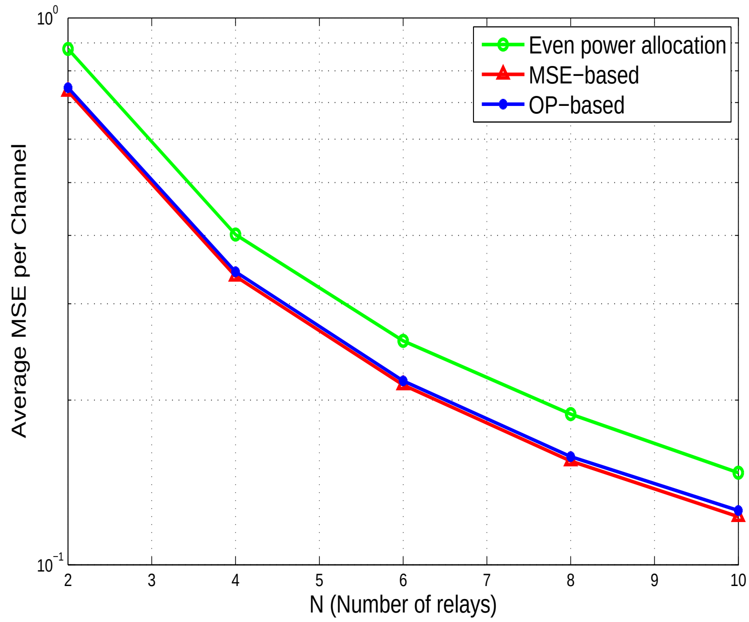

Fig. 3.5: Average MSE per channel for different relay numbers where $P = 15$dB and $\mu = 0.2$.

Fig. 3.5 shows the average MSE per channel versus the number of relays for a fixed average training power per transmission where $P = 15$dB. In this simulation, we consider fixed relay location and let $\mu = 0.2$. The average MSE per channel is defined as the total MSE divided by number of channels. We can see that both proposed schemes outperform even power allocation, while the MSE-based scheme is slightly better than the OP-based scheme. We can also see that the average MSE decreases when there are more relays.

In Figs. 3.6-3.8, we show the OP of different relay networks with relay selection and channel estimation error. The SNR threshold is set to be $\gamma_{th} = 0.5$ and the data powers are set as $Q_{S,i} = Q_{R,i} = 15$dB.

Fig. 3.6 shows the network OP for different average training power per transmission for networks with $N = 2, 3$ and 4. From Fig. 3.6, we can see that the proposed OP-based scheme performs much better than the MSE-based scheme and

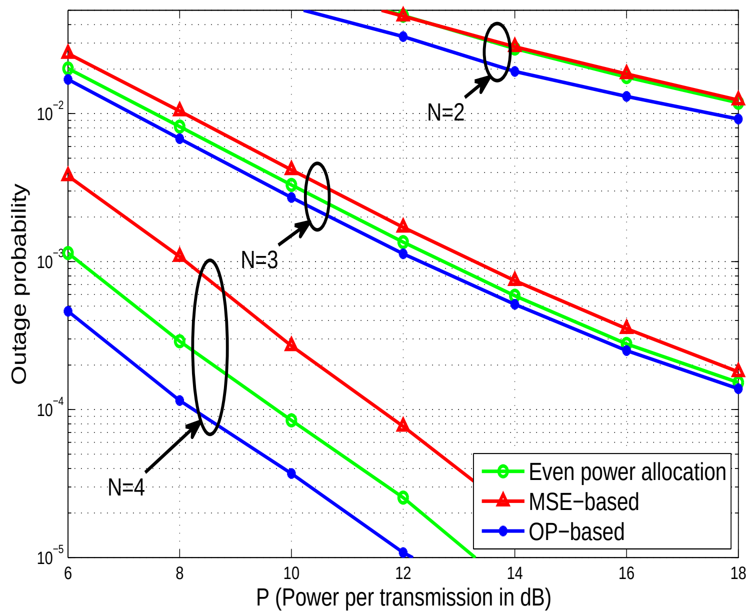

Fig. 3.6: Outage probability for different training power values for one-way networks with 2, 3, and 4 relays.

the even power scheme. The advantage of the OP-based scheme over the other two gets bigger when the training power increases. For $N = 4$, the improvement of the proposed OP-based scheme over the even power allocation is about 57.5% and 60.3% at $P = 6$ dB and $P = 12$dB, respectively. The figure also shows that the OP decreases when the training power increases because with higher power, we have better training quality.

Fig. 3.7 shows the OP versus relay location for networks with $N = 2, 3$. We consider fixed network topology where the relays are located close to each other and have same μ value. Also, we set $P = 15$dB. In Fig. 3.7, both proposed schemes are superior to the even power allocation scheme. When the relays are closer to the source or the destination, the gap between the OP-based scheme and even power allocation scheme becomes larger. For $N = 3$, we see that the OP-based scheme is about 41% and 34.6% better obtained at $\mu = 0.1$ and $\mu = 0.9$, respectively. For small μ, the OP-based scheme is slightly superior to the MSE-based scheme. The figure also reveals that OP performance improves when μ increases. This is due to better relay-to-destination channel estimation quality when the relays are closer to

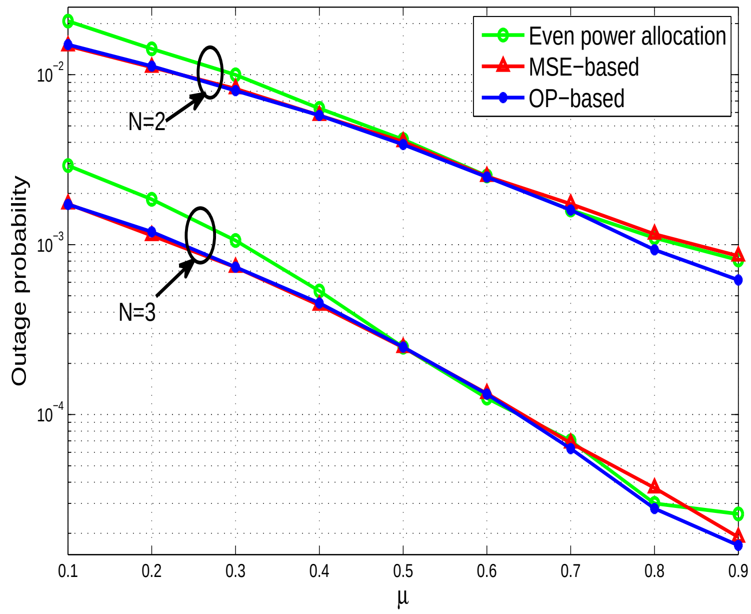

Fig. 3.7: Outage probability for different relay locations for one-way networks with 2 and 3 relays where $P = 15$dB.

the destination, which also infers better source-to-relay channel estimation quality.

Fig. 3.8 shows the OP for different numbers of relays for a fixed average power $P = 10$dB per transmission and a fixed $\mu = 0.9$. We can see from Fig. 3.8 that the OP performance improves when the number of relay increases. The OP-based power allocation is largely better than the other two schemes, while the even power allocation performs the worst. With more relays, the gap between the OP-based scheme and the even power allocation becomes larger.

3.6 Conclusions

In this chapter, we investigated the power allocation during the training process for a one-way multi-relay AF network. The power allocation was among all the relay paths, between the training of different channels for every relay path, and between the source and every relay during the training of the source-to-relay channel. Using the total MSE and the OP as the design objectives, we derived closed-form analytical solutions for the training power allocation in the high power regime. Simulation results on the MSE and the OP were demonstrated to show the superiority of the

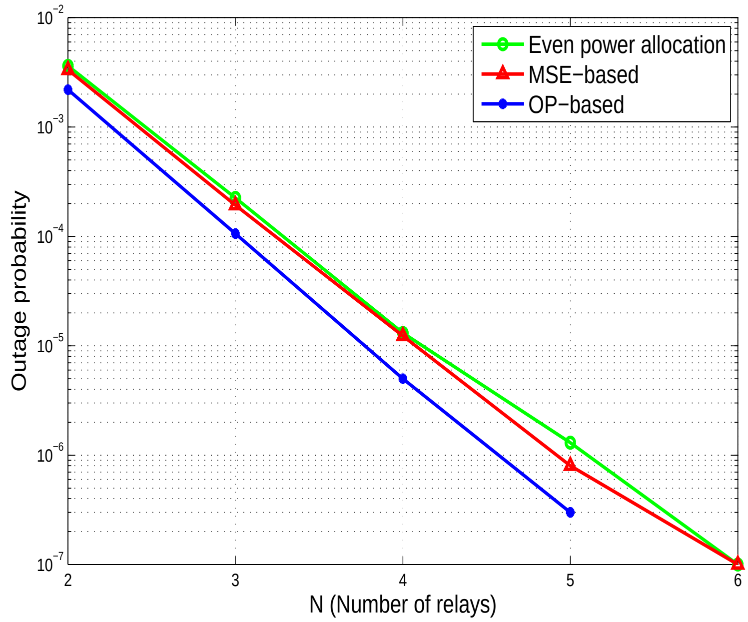

Fig. 3.8: Outage probability for different relay numbers where $P = 10$dB and $\mu = 0.9$.

proposed schemes to an even power allocation.

Chapter 4

Power Allocation in Training for Two-Way Single-Relay Network

4.1 Introduction

Chapter 2 and Chapter 3 are both on one-way relay network. Since two-way relay network can recover the spectral efficiency loss caused by the half-duplex mode of relays, in this chapter, we consider a two-way single-relay network. We investigate the power allocation problem during the channel training process for both terminals to estimate the global CSI of the whole network. This includes the power allocations between different training steps and among different nodes. The LMMSE channel estimates are adopted. Two power allocation schemes are proposed: a MSE based scheme and an OP based scheme. Our simulation results demonstrate that the proposed power allocations are superior to even power allocation.

Now we clarify the difference of our work to existing literature. Different to [58, 59], we study the estimation of the individual channels between either terminal and the relay. Our work is different to [60] not only in the power allocation problem but also in the objective functions. We consider both the MSE and the OP. Further, we consider channels with different variances.

The rest of the chapter is organized as follows. Section 4.2 describes the system model, training scheme, channel estimation rules, and the power allocation problem. In Section 4.3, we investigate the MSE of the channel estimations and propose an MSE-based power allocation. In Section 4.4, we investigate the OP of the network with CSI error, and propose an OP-based power allocation. Section 4.5 shows simulation results. Section 4.6 concludes the paper.

4.2 System Model and Training Scheme

We consider a two-way single-relay network, where two terminals T_1 and T_2 exchange information through a relay R. The network model can be seen from Fig. 4.1. Each node has one antenna which can be used for both transmission and reception. Denote the channel between T_1 and the relay as g and that between T_2 and the relay as f. Both g and f are independent zero-mean complex Gaussian random variables with variance σ_g^2 and σ_f^2, respectively. There is no direct link between the two terminals. We assume that the channel reciprocity holds, which means the channel estimate of the uplink direction can directly be utilized for the downlink. For $i = 1, 2$, the estimations of f and g at T_i are denoted as f_i and g_i, respectively. Their MSEs are denoted as $\text{MSE}(f_i)$ and $\text{MSE}(g_i)$.

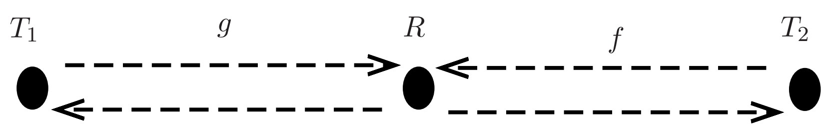

Fig. 4.1: Two-way single-relay network model.

4.2.1 Training and LMMSE Estimators

The training goal is for both terminals T_1 and T_2 to obtain estimates of both channel coefficients f and g. Without loss of generality, we only look into T_1 while results of T_2 can be obtained similarly. A two-stage training scheme is used, including the relay-training stage and the terminal-training stage.

In the relay-training stage, the relay sends a training symbol to T_1 and T_2. Without loss of generality, we use the unit symbol. The received signal at T_1 is

$$x_1 = \sqrt{P_1} g + n_1,$$

where P_1 is the power the relay uses in this step and n_1 is the noise at T_1, following $\mathcal{CN}(0,1)$. Using the results from Section 1.3, the LMMSE estimate of g at T_1 is thus

$$g_1 = \frac{\sqrt{P_1}\sigma_g^2}{1+P_1\sigma_g^2} x_1, \tag{4.1}$$

which is also the MMSE estimate. Denote the estimation error as

$$\triangle g_1 \triangleq g - g_1, \tag{4.2}$$

which follows $\mathcal{CN}(0, \mathrm{MSE}(g_1))$, where

$$\mathrm{MSE}(g_1) = \frac{\sigma_g^2}{1+P_1\sigma_g^2} \approx \frac{1}{P_1}, \tag{4.3}$$

The approximation is valid when $P_1 \gg 1$.

The terminal-training stage takes 2 steps. First, T_1 and T_2 send the unit symbol to the relay at the same time. Let the power used by each terminal for terminal-training stage be $P_{2,T}$, the received signal at the relay is

$$r = \sqrt{P_{2,T}} g + \sqrt{P_{2,T}} f + n_R. \tag{4.4}$$

Second, the relay amplifies and forwards its received signal to T_1 and T_2 with a fixed-gain relay amplification factor as explained in Section 2.2.1. Let the power used by the relay for this step be $P_{2,R}$. The received symbol at T_1 is

$$\begin{aligned} y_1 &= r\sqrt{\frac{P_{2,R}}{1+P_{2,T}(\sigma_g^2+\sigma_f^2)}} g + n_2 \\ &= \sqrt{\alpha} g^2 + \sqrt{\alpha} g f + \sqrt{\beta} g n_R + n_2, \end{aligned} \tag{4.5}$$

where

$$\alpha \triangleq \frac{P_{2,T}P_{2,R}}{1+P_{2,T}(\sigma_g^2+\sigma_f^2)}, \quad \beta \triangleq \frac{P_{2,R}}{1+P_{2,T}(\sigma_g^2+\sigma_f^2)}.$$

n_R and n_2 are noises at the relay and T_1. They are independent and follow $\mathcal{CN}(0,1)$. In reality, since T_1 only knows g_1, its estimation of g, it uses g_1 in the self-interference cancellation. By using (4.2), we can obtain

$$\tilde{y}_1 = y_1 - \sqrt{\alpha} g_1^2 = \sqrt{\alpha} g_1 f + n, \tag{4.6}$$

where

$$n \triangleq \sqrt{\alpha} \triangle g_1 f + \sqrt{\beta}(\triangle g_1 + g_1) n_R + \sqrt{\alpha}(\triangle g_1^2 + 2 g_1 \triangle g_1) + n_2.$$

n is the equivalent noise in the observation model (4.6), containing both the noise and the channel estimation error terms.

An estimate of f needs to be obtained at T_1 based on $\tilde{y}_1$. The MMSE estimate of f is intractable due to the unknown distribution of n, thus we resort to the LMMSE estimate. Using the results in Section 1.3, after straightforward calculations, the LMMSE estimate is obtained as

$$f_1 = \frac{\sigma_f^2 \sqrt{\alpha} g_1^* \tilde{y}_1}{1 + (\sigma_f^2 \alpha + \beta) a_g + 2\alpha \mathrm{MSE}(g_1)(a_g + |g_1|^2)}, \tag{4.7}$$

where $a_g \triangleq |g_1|^2 + \mathrm{MSE}(g_1)$. The MSE of this channel estimation can be calculated to be

$$\mathrm{MSE}(f_1) = \sigma_f^2 - \frac{\sigma_f^4 \alpha |g_1|^2}{1 + (\sigma_f^2 \alpha + \beta) a_g + 2\alpha \mathrm{MSE}(g_1)(a_g + |g_1|^2)}. \tag{4.8}$$

Similarly, we can obtain estimates of the channels at T_2 (f_2 and g_2). Their MSEs are as follows:

$$\mathrm{MSE}(f_2) = \frac{\sigma_f^2}{1 + P_1 \sigma_f^2} \approx \frac{1}{P_1}, \tag{4.9}$$

$$\mathrm{MSE}(g_2) = \sigma_g^2 - \frac{\sigma_g^4 \alpha |f_2|^2}{1 + (\sigma_g^2 \alpha + \beta) a_f + 2\alpha \mathrm{MSE}(f_2)(a_f + |f_2|^2)}, \tag{4.10}$$

where $a_f \triangleq |f_2|^2 + \mathrm{MSE}(f_2)$.

4.2.2 Power Allocation Problem Statement

In this chapter, we investigate the power allocation problem during the training for the two-way single-relay network. Let P_t be the total power used for training and P_2 be the total power the terminals and the relay use in the terminal-training stage.

Thus $P_t = P_1 + P_2$ and $P_2 = 2P_{2,T} + P_{2,R}$. Let λ be the percentage of the total training power used in the terminal-training stage. We have

$$P_2 = \lambda P_t, \quad P_1 = (1-\lambda)P_t. \tag{4.11}$$

For the terminal-training stage, let η be the fraction of the power allocated to each terminal. We have

$$P_{2,T} = \eta P_2, \quad P_{2,R} = (1-2\eta)P_2. \tag{4.12}$$

The power allocation problem is thus to find the optimal η and λ for an arbitrary training power P_t. We consider two objective functions: the total MSE of the channel estimations and the OP of the relay network.

4.3 MSE-Based Power Allocation

In this section, we investigate the total MSE of all channel estimations and propose an MSE-based power allocation.

The total MSE of all channel estimations is as follows:

$$\mathrm{MSE}_{total} = \mathrm{MSE}(g_1) + \mathrm{MSE}(f_1) + \mathrm{MSE}(g_2) + \mathrm{MSE}(f_2).$$

The goal is to design λ and η, defined in (4.11) and (4.12), to minimize the MSE_{total}. For the tractability of analysis, we only consider the high power region, i.e., $P_t \gg 1$. Notice that $\mathrm{MSE}(g_1)$ and $\mathrm{MSE}(f_2)$ are independent of η. For an arbitrarily λ, the optimization of η that minimizes MSE_{total} is as follows:

$$\min_{\eta \in [0,1]} (\mathrm{MSE}(f_1) + \mathrm{MSE}(g_2)).$$

Recall that $\triangle g_1$ follows the distribution $\mathcal{CN}(0, \mathrm{MSE}(g_1))$. Compared to g_1, $\triangle g_1$ is a lower order term of $P_t \gg 1$. When the training power is high, we can neglect the lower order terms in $a_g \mathrm{MSE}(g_1)$ which is $\mathrm{MSE}(g_1)^2$. By using (4.12) in (4.8) and (4.10) and calculating the derivative of $(\mathrm{MSE}(f_1) + \mathrm{MSE}(g_2))$ to η and making it zero, the optimal η can be obtained as (lower order terms of P_t are omitted):

$$\eta^* = \left(2 + \sqrt{\frac{(\sigma_f^2 + \sigma_g^2)(|g_1|^2 + |f_2|^2)}{|g_1|^2 |f_2|^2}}\right)^{-1}. \tag{4.13}$$

Now we consider the optimization of λ. By using (4.3), (4.8)-(4.10) in MSE_{total} and also the optimal η in (4.13), via straightforward calculations, we have

$$\begin{aligned} \text{MSE}_{total} &= \text{MSE}(g_1) + \text{MSE}(f_2) + [\text{MSE}(f_1) + \text{MSE}(g_2)]_{\eta=\eta^*} \\ &\approx \frac{10}{P_1} + \frac{\left(|g_1|^2 + |f_2|^2\right)\left[2\sqrt{|g_1|^2|f_2|^2} + \sqrt{(|g_1|^2 + |f_2|^2)\sigma^2}\right]}{P_2|g_1|^2|f_2|^2}, \end{aligned} \tag{4.14}$$

where $\sigma^2 \triangleq \sigma_f^2 + \sigma_g^2$. Similar to the previous chapters, for tractable closed-form solution, we replace $|g_1|^2$ and $|f_2|^2$ with their average values in (4.14), then the following MSE-based objective function is obtained:

$$h_1(P_1, P_2) \triangleq \frac{10}{P_1} + \frac{1}{P_2}\frac{\sigma^2(\sigma_g + \sigma_f)^2}{\sigma_f^2\sigma_g^2}. \tag{4.15}$$

By using (4.11) in (4.15) and solving $\frac{\partial h_1}{\partial \lambda} = 0$, we obtain the following solution of λ:

$$\lambda^* = \left(1 + \sqrt{\frac{10\sigma_g^2\sigma_f^2}{(\sigma_g^2 + \sigma_f^2)(\sigma_g + \sigma_f)^2}}\right)^{-1}. \tag{4.16}$$

From (4.16) we can see that λ^* depends on the channel variances only. But from (4.13), we see that η^* depends on not only the channel variances but also g_1 and f_2. Thus, similar to the results in Chapters 2 and 3, the training powers of the nodes in the terminal-training stage actually adapt to the channel quality.

4.4 OP-Based Power Allocation

The power allocation derived in the previous section weights MSEs of all channels equally is neither a direct network performance measure nor be optimal. In this section, we work on the OP, denoted as P_{out}. We use AF with fixed gain relay power coefficient for the data transmission. Let $P_{d,T}$, $P_{d,R}$ be the powers used by either terminal and the relay. Again, we only consider the high power region, i.e., $P_t \gg 1$. The following lemma on the OP is acquired.

Lemma 3: *Assume that all powers have the same order, i.e., $P_t, P_{d,T}, P_{d,R} \sim P$ and $P \gg 1$. Given the SNR-threshold γ_{th} and MSEs of channel estimates, the OP* *of the two-way single-relay network is:*

$$P_{\text{out}} = p + \gamma_{th}\left[\text{MSE}(g_1)\left(\frac{1}{\sigma_g^2}+\frac{4}{\sigma_f^2}\right)+\text{MSE}(f_2)\left(\frac{1}{\sigma_f^2}+\frac{4}{\sigma_g^2}\right)\right]$$
$$+\gamma_{th}\left(\frac{\text{MSE}(f_1)}{\sigma_f^2}+\frac{\text{MSE}(g_2)}{\sigma_g^2}\right)+\mathcal{O}\left(\frac{\ln P}{P^2}\right), \tag{4.17}$$

where $p = \mathcal{O}(\frac{\ln P}{P})$ *is independent of* P_1, $P_{2,T}$ *and* $P_{2,R}$.

Proof. For the data-transmission, let

$$\alpha_d \triangleq \frac{P_{d,T}P_{d,R}}{1+P_{d,T}\sigma^2},\ \beta_d \triangleq \frac{P_{d,R}}{1+P_{d,T}\sigma^2}.$$

By similar derivations to those in Section 4.2.1, after self-interference cancellation, T_1 gets

$$\tilde{y}_1 = \sqrt{\alpha_d}g_1 f_1 s + \omega_1,$$

where s is the data symbol with unit power. Then the noise-plus-channel-error term is

$$\omega_1 \triangleq \sqrt{\alpha_d}[\triangle f_1(g_1+\triangle g_1) + f_1\triangle g_1 + 2g_1\triangle g_1 + \triangle g_1{}^2]s + \sqrt{\beta_d}(g_1+\triangle g_1)n_{d,R} + n_3.$$

where $n_{d,R}$ and n_3 are independent noises at the relay and T_1, respectively, following $\mathcal{CN}(0,1)$. After omitting lower order terms of P ($\sqrt{\alpha_d}\triangle f_1\triangle g_1$, $\sqrt{\alpha_d}\triangle g_1{}^2$ and $\sqrt{\beta_d}\triangle g_1 n_{d,R}$), the SNR at T_1 is

$$\frac{\mathbb{E}(\alpha_d|g_1|^2|f_1|^2)}{\mathbb{E}(|\omega_1|^2)}$$
$$\approx \frac{\alpha_d|g_1|^2|f_1|^2}{\mathbb{E}[\sqrt{\alpha_d}[\triangle f_1(g_1+\triangle g_1) + f_1\triangle g_1 + 2g_1\triangle g_1 + \triangle g_1{}^2]s + \sqrt{\beta_d}(g_1+\triangle g_1)n_{d,R} + n_3]}$$
$$\approx \frac{\alpha_d|g_1|^2|f_1|^2}{c_1|g_1|^2 + \alpha_d\text{MSE}(g_1)|f_1|^2+1} \triangleq \gamma_1 \tag{4.18}$$

where $c_1 \triangleq \alpha_d(\text{MSE}(f_1)+4\text{MSE}(g_1))+\beta_d$. f_1 is approximately Gaussian for $P_1 \gg 1$. By using the result from [69] and [70], we get the CDF of γ_1:

$$F_{\gamma_1}(x) = 1 - e^{-\frac{xc_1}{\alpha_d}\left(\frac{1}{\sigma_f^2}+\frac{\alpha_d\text{MSE}(g_1)}{c_1\sigma_g^2}\right)}c, \tag{4.19}$$

where $c \triangleq \sqrt{\frac{4x}{\alpha_d\sigma_g^2\sigma_f^2}}K_1\left(\sqrt{\frac{4x}{\alpha_d\sigma_g^2\sigma_f^2}}\right)$. Similarly let $c_2 \triangleq \alpha_d(\text{MSE}(g_2)+4\text{MSE}(f_2)) + \beta_d$. The CDF of the SNR at T_2, γ_2, is

$$F_{\gamma_2}(x) = 1 - e^{-\frac{xc_2}{\alpha_d}\left(\frac{1}{\sigma_g^2}+\frac{\alpha_d\text{MSE}(f_2)}{c_2\sigma_f^2}\right)}c, \tag{4.20}$$

The OP can be calculated as

$$\begin{aligned}
P_{\text{out}} &= \mathrm{P}[\min(\gamma_1, \gamma_2) \leq \gamma_{\text{th}}] \\
&= 1 - [1 - F_{\gamma_1}(\gamma_{th})][1 - F_{\gamma_2}(\gamma_{th})] \\
&= 1 - e^{-\gamma_{th}\left(\frac{c_1}{\alpha_d \sigma_f^2} + \frac{\mathrm{MSE}(g_1)}{\sigma_g^2} + \frac{c_2}{\alpha_d \sigma_g^2} + \frac{\mathrm{MSE}(f_2)}{\sigma_f^2}\right)c^2}. \qquad (4.21)
\end{aligned}$$

When $P_{d,R} \gg 1$, $c = 1 + q + \mathcal{O}\left(\frac{\ln P_{d,R}}{P_{d,R}^2}\right)$, where $q = \mathcal{O}\left(\frac{\ln P_{d,R}}{P_{d,R}}\right)$. By using this in (4.21), (4.17) can be acquired.

The goal of the OP-based power allocation is to design λ and η to minimize the OP in (4.17). Similar to the derivations in Section 4.3, notice that $\mathrm{MSE}(g_1)$ and $\mathrm{MSE}(f_2)$ are independent of η. For an arbitrarily given λ, the optimization of η that minimizes the OP is as follows:

$$\arg\min_{\eta}(\sigma_g^2 \mathrm{MSE}(f_1) + \sigma_f^2 \mathrm{MSE}(g_2)). \qquad (4.22)$$

We use (4.8), (4.10), and (4.12) in (4.22) and neglect lower order terms of P_t. By calculating the derivative of the objective function and making it zero, the optimal η is obtained as:

$$\eta^+ = \frac{1}{2}\left(1 + \sqrt{\frac{\sigma_f^2|g_1|^2 + \sigma_g^2|f_2|^2}{2|g_1|^2|f_2|^2}}\right)^{-1}. \qquad (4.23)$$

For the optimization of λ, by using (4.23), (4.3), (4.8)-(4.10) in (4.17), and replacing $|f_2|^2$ and $|g_1|^2$ with their mean values, we have the following objective function from the OP formula:

$$h_2(P_1, P_2) \triangleq \frac{\sigma_f^2\sigma_g^2(2\sigma^2 P_1 + 4\sigma_g^2 P_t)}{P_2 P_1 \sigma_f^2 \sigma_g^2 + 2\sigma^2 P_1 + 4\sigma_g^2 P_t} + \frac{\sigma_f^2\sigma_g^2(2\sigma^2 P_1 + 4\sigma_f^2 P_t)}{P_2 P_1 \sigma_f^2 \sigma_g^2 + 2\sigma^2 P_1 + 4\sigma_f^2 P_t} + \frac{5\sigma^2}{P_1}. \qquad (4.24)$$

By using (4.11) in (4.24) and solving $\frac{dh_2}{d\lambda} = 0$, we obtain the following solution of λ:

$$\lambda^+ = \left(1 + \sqrt{\frac{9P_t^2\sigma_g^4\sigma_f^4 + 20P_t\sigma^2\sigma_g^2\sigma_f^2 + 10\sigma^4}{2P_t\sigma_g^2\sigma_f^2}}\right)^{-1}. \qquad (4.25)$$

4.5 Simulation Results and Discussions

In this section, we compare the two proposed power allocation schemes with an even power allocation where $P_{2,T} = P_{2,R} = P_2/3$ and $P_1 = P_2 = P_t/2$, i.e., $\eta = 1/3$ and

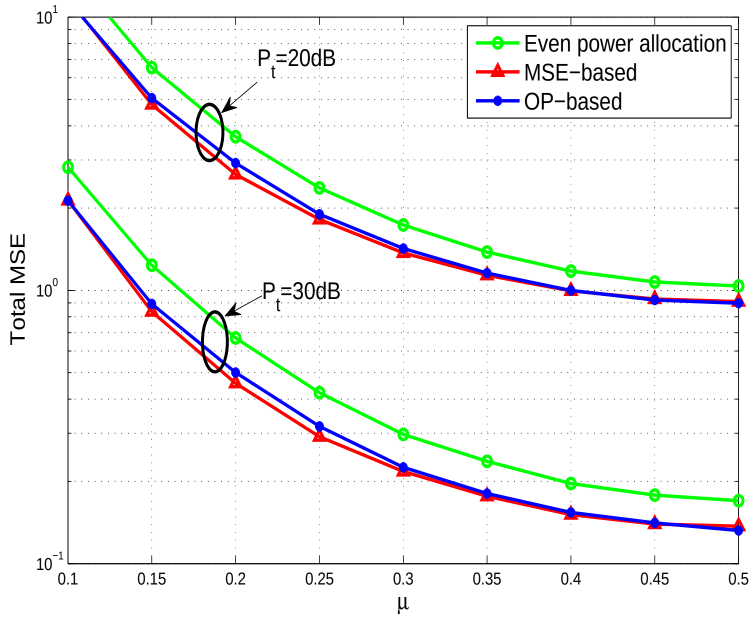

Fig. 4.2: Total MSE for different training power values for two-way single-relay network.

$\lambda = 1/2$. It means that the relay-training stage and the terminal-training stage use the same power, and each terminal and the relay use the same power during the terminal-training stage. Denote $d_{T_1T_2}$ as the distance between T_1 and T_2. Similar as that in Chapter 2, we use the parameter μ to represent the location of the relay. In th simulations, we assume the path-loss exponent is 2 and $\phi/d_{T_1T_2=1}$. Thus $\sigma_g^2 = (\frac{1}{\mu})^2$ and $\sigma_f^2 = (\frac{1}{(1-\mu)})^2$.

Fig. 4.2 shows the total MSE versus μ in the logarithmic scale for different training powers. Due to the symmetry of the two terminals, we only show the range $\mu \in [0, 0.5]$. We can see that both proposed schemes outperform even power allocation; while the MSE-based scheme is slightly superior to the OP-based scheme. For $P_t = 20$dB, the total MSE improvements of the proposed MSE-based power allocation over even power allocation are about 27.2% and 15.3% for $\mu = 0.2$ and $\mu = 0.4$, respectively. The total MSE reduces when μ increases up to 0.5, indicating that better channel estimates can be obtained as the relay is closer to the midpoint.

Fig. 4.3 shows the network OP with $\gamma_{th} = 0.1$ for different training powers. The powers used in data-transmission are fixed as $P_{d,T} = P_{d,R} = 10$dB. From Fig. 4.3, we

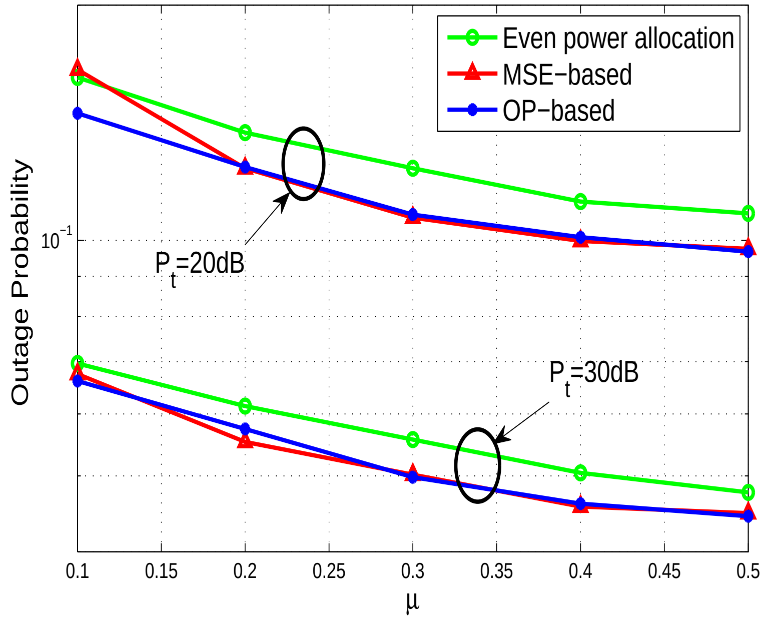

Fig. 4.3: Outage probability for different training power values for two-way single-relay network.

can see that the OP-based scheme performs much better than even power allocation for all values of μ. For $\mu = 0.4$, the outage probability reduces by about 13% and 10.5% with the proposed OP-based scheme for $P_t = 20$dB and $P_t = 30$dB, respectively. The OP-based and MSE-based schemes have comparable performance for a wide range of μ. But when $P_t = 20$dB and $\mu < 0.15$, the former performs noticeably better. Fig. 4.3 also shows that the OP decreases when μ gets closer to 0.5.

4.6 Conclusions

In this chapter, a two-step training scheme for a two-way single-relay AF network is proposed. We investigate the power allocations between the relay-training step and the terminal-training step, and between the terminals and the relay. For the high power regime, we study the total MSE of channel estimations and network OP, from which power allocation solutions are derived in closed-form. Simulation results illustrate that the proposed power allocations perform better than even power allocation in both the MSE and the OP.

Chapter 5

Conclusions and Future Work

In this chapter, we give the conclusions of the thesis and propose possible future work.

5.1 Conclusions

This thesis has investigated the power allocation problem during the channel training process for AF relay networks. Three network scenarios are considered: the one-way single-relay network, the one-way multi-relay network, and the two-way single-relay network.

For a one-way single-relay network, the power allocation is between the training of source-to-relay channel and relay-to-destination channel, and between the source and the relay during the training of the source-to-relay channel.

For a one-way multi-relay network, the power allocation is among all the relay paths, between the training of source-to-relay channel and relay-to-destination channel for every relay path, and between the source and every relay during the training of the source-to-relay channel.

For a two-way single-relay network, the power allocation is between the relay-training step and the terminal-training step, and between the terminals and the relay.

For all three network scenarios, with the aid of high SNR approximations, closed-form analytical solutions are found for the power allocation based on the minimization of the total MSE of channel estimations and the network OP. Simulation results on the MSE and the OP are demonstrated to show the superiority of the proposed schemes to an even power allocation.

The proposed power allocation schemes with closed-form analytical solutions can help increase the performance of cooperative relay network.

5.2 Future Work

A few future research directions have been recognized to extend the studies in this thesis.

In Chapter 2, during the data transmission, it is assumed that the relay forwards the received signal using AF relaying with a fixed gain amplification factor. It is interesting and also challenging to study the case with another relaying method: AF relaying with a variable gain amplification factor. Then the power allocation problem with the OP of the network as the objective function will be more complicated. But the solution may achieve better network performance since the variable gain relays can achieve better performance with a design adjustable to the instantaneous channel knowledge.

In Chapter 3, for the OP-based power allocation, we choose the best relay selection for the data-transmission, where the relay that provides the best received SNR is selected to forward information. In [15] the idea of relay selection was generalized to allow more than one relay to cooperate. And it has been shown that the SNR-optimal multiple relay selection perform much better than the corresponding single relay selection methods. Thus it is interesting and also challenging to employ the multiple relay selection scheme in our study and to investigate the training power allocation strategies.

In Chapter 4, only a single-relay network has been considered. We can extend the single-relay network to the more general multi-relay network. In addition, the training power allocation and relay selection can be jointly carried out.

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