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# THE UNIVERSITY OF ALBERTA

# EXPERIMENTAL STUDY OF TRANSVERSELY LOADED CONTINUOUS STEEL PLATES

by

KUIT PAUL RATZLAFF



# A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

IN STRUCTURAL ENGINEERING

DEPARTMENT OF CIVIL ENGINEERING

EDMONTON, ALBERTA
SPRING 1995



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This thesis is dedicated to my mother and father,

Mary and David Ratzlaff

#### ABSTRACT

Structures such as bottom-founded petroleum exploration and production platforms and ice-breaking vessels are required to resist extreme loads. The loads, primarily due to ice, must be resisted by the hull or exterior wal, that is generally made up of a rectangular grid of frames and stiffeners covered by steel plate. The results of the experimental work reported here, supported by the predictions of various analyses, show that thin steel plates with span to thickness ratios of between 65 and 155 are indeed suitable to resist extreme transverse loads without the need for stiffening elements when the resistance provided by membrane behaviour is utilized.

Four tests were carried out on plates with nominal thicknesses between 6 mm and 16 mm and having an aspect ratio, with/length, of 0.34. The plates were loaded transversely by fluid pressure until failure occurred. Deflections and strains at numerous locations were recorded throughout the loading history. The plates all failed at the edge near mid-length where the greatest bending occurred. Although they were considered to have failed prematurely within the heat affected zone of the weld at the plate edges, the plates were loaded for the most part into the inelastic flexural membrane range of behaviour. The failure loads were many times that corresponding to yield-

line collapse mechanisms. Three failure mechanisms have been identified.

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# List of Symbols

A initial cross-sectional area

At = final cross-sectional area

d = distance

e = eccentricity

D = flexural rigidity =  $Eh^3/12(1-v^2)$ 

E = modulus of elasticity

n = plate thickness

L = length

 $L_{\rm O}$  = initial or original length

 $L_{f} = final length$ 

M = bending moment per unit length

 $M_{\rm O}$  = bending moment per unit length at plate edge

 $M_D$  = fully plastic moment per unit length

 $M_U$  = fully plastic ultimate moment per unit length

 $M_V$  = yield moment per unit length

N = axial force per unit length

 $P = P_0 = horizontal component of force S$ 

 $P_{U} = \sigma_{U}h = ultimate force per unit length$ 

q = uniform load per unit area

 $q_C$  = critical value of uniform load per unit

area =  $16 \text{ Mp/L}^2$ 

Q = total load per unit area due to q

S = axial membrane force per unit length

U = nondimensional membrane stress

parameter =  $[SL^2/4D]^{1/2}$ 

V = vertical (shear) component of S

w = transverse deflection

 $[W_m]_y$  = maximum transverse deflection at load

corresponding to first yielding

 $W_m$  = maximum transverse delflection

x,y,z = cartesian coordinates

 $\varepsilon$  = engineering strain

 $\varepsilon_{\rm f}$  = engineering strain at fracture

 $\epsilon_{D}$  = engineering strain corresponding to  $P_{O}$ 

 $\varepsilon$  = axial strain corresponding to S

 $\varepsilon_{st}$  = engineering strain at onset of strain hardening

 $\varepsilon_{t}$  = crue strain

 $\varepsilon_u$  = engineering strain corresponding to  $\sigma_u$ 

 $\varepsilon_{V}$  = uniaxial yield strain

 $\theta$  = angle of inclination, degrees

v = Poisson's ratio

 $v_p$  = 0.5 = inelastic strain ratio with zero

volume change

 $v_s$  = inelastic strain ratio = absolute value of ratio

of lateral strain to longitudinal strain

 $\sigma$  = engineering stress

 $\sigma_t$  = true stress

 $\sigma_{\rm tr}$  engineering ultimate stress in uniaxial tension

 $\sigma_{\text{y}}$  engineering yield stress in uniaxial tension

ρ = radius of curvature

 $ho_{\rm C}$  = critical radius of curvature corresponding to  $\epsilon_{\rm u}$ 

 $\rho_{\text{min}}$  = minimum radius of curvature

 $\varphi$  = curvature  $\approx \frac{2}{3}$  w/dx<sup>2</sup>

 $\phi_y$  = curvature corresponding to My

#### 1. INTRODUCTION

# 1.1 General

In the design of transversely loaded flat plates which are continuous over a rectangular grid of beams and stiffeners the structural usefulness of the system is often considered to be the flexural strength of the plate panels alone. This flexural strength may be even limited to that based on first yield, neglecting the strength developed when a mechanism forms, where for the latter the strength is taken as that given by classical yield-line theory. Such limits are irrelevant for any system in which the deflections are not critic—and in which the increase in strength due to membrane accion can be considered.

Some of the structures that have been designed or proposed for oil exploration and production in the Arctic Ocean consist of steel caissons clad with continuous thin steel plating. Interior plate panels are continuous over the support grillage on all four edges and can therefore by modelled as being rotationally and translationally fixed along the edges. Because the skin plates may be subjected to enormous local lateral forces from the floe ice, economic design requires that the full strength of the plating be utilized.

To benefit from the increased strength provided by the development of membrane behaviour, a complete understanding of the load-deflection behaviour to failure of the system is

required. Of particular significance are: (i) the material characteristics of the steel plate, (ii) the geometry and strength characteristics of the supporting grillage, (iii) the method construction of the system, (iv) the type and severity of loading, and (v) the determination of possible failure modes and loads. With this knowledge advantage can be taken of the increased load-carrying capacity provided by membrane action while still providing any desired level of safety. At present, no clear design procedures are available. Such procedures should be confirmed experimentally.

# 1.2. Objectives

The overall objective of this work was to provide experimental evidence on the behaviour of transversely loaded continuous steel plates to failure and to develop analytical models to predict this behaviour. Failure modes were to be investigated.

## 1.3 Scope

The literature has been reviewed and the assumptions and limitations of the analyses and the results of experimental work discussed. In particular the previous work of the aut' with Kernedy (Ratzlaff and Kennedy, 1985; 1986) is thoroughly reviewed. It describes the behaviour throughout leading and predicts some failure modes and loads.

Four large scale tests that included an extensive array of measurements were performed. The tests were done by

subjecting commonly available mild structural steel plates with an aspect ratio (defined as the width divided by the length) of about one-third to fluid pressure. A plate having an aspect ratio of one-third and thickness, h, is shown in Fig. 1.1. Two finite element models that modelled the behaviour of two of the experimental plates are presented and the results compared with those of the tests.

In this work only long and initially flat plates with aspect ratios that can be considered to approach zero are considered. The effects of restraint at the edges and material strain hardening have been included and are shown to be of significance.

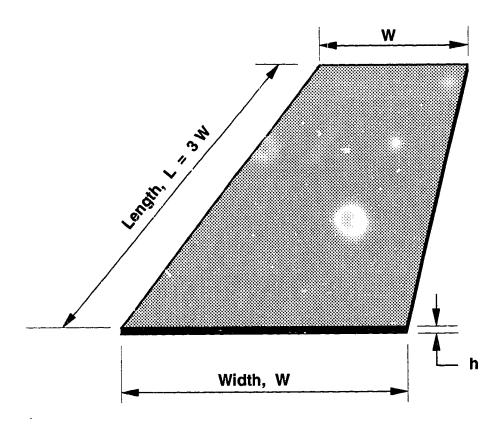


Figure 1.1 Plate having aspect ratio of one-third and thickness, h

#### 2. LITERATURE REVIEW

### 2.1 General

Analyses to predict the load-deflection behaviour of transversely loaded continuous plates have considered steel plates of varying aspect ratios. These analyses have commonly assumed the stress-strain characteristics of the plate material to be elasto-plastic, ignored strain hardening, and have neglected the contribution to load-carrying of the membrane forces that develop when the edges of the place are restrained translationally.

Boobnoff (1902) determined the exact closed form solution to the differential equations describing the large elastic deflection behaviour of uniformly loaded clamped rectangular flat infinitely long plates. The aspect ratio, the ratio of the width to the length, is therefore zero. The differential equations for plates of any aspect ratio were first determined by von Karman in 1910.

It would be expected that the maximum deflection that occurs at mid-width and mid-length would not be significantly different for a plate with an aspect ratio of less than about 1/2 than that for a plate with an aspect ratio of zero (Clarkson, 1956; Hooke, 1970).

For an infinitely long flat plate the elastic loaddeflection equation of von Karman, when shear deformations are neglected, becomes

$$[2.1] \qquad \frac{q}{D} = \frac{d^4w}{dx^4} - \frac{N_x}{D} \frac{d^2w}{dx^4}$$

If the term containing the membrane force  $N_{\mathrm{X}}$  in [2.1] is neglected, the equation reduces to the elastic small deflection equation of Lagrange (1811). Some approximate solutions to the large deflection equations of von Karman for rectangular plates with different boundary or edge conditions were first presented by Way (1938). Levy (1942) used a Fourier series method to solve the same equations for a plate with an aspect ratio of 1. To the author's knowledge, no closed-form solution to the complete von Karman equations exists for a plate with an aspect ratio other than zero. A multitude of approximate solutions exist for the load-deflection behaviour of transversely loaded clamped plates of any aspect ratio when the plate material is assumed to exhibit elasto-plastic behaviour. These solutions are generally considered too rigourous mathematically for use in design. One of these analyses, for example, is the well known upper-bound yield line method of Ingerslev (1923) and Johansen (1943, 1946).

For the elastic and elasto-plastic analyses that follow, it is assumed that the tensile and compressive characteristics of the plate are bilinear as shown in Fig. 2.1. This results in a moment-curvature relationship for a rectangular cross-section of unit width, although curved as shown dotted in Fig. 2.2, that may be idealized as being bilinear. Because a plate with an aspect ratio approaching

zero is restrained under load from deforming in the direction of its length the state of stress is one of plane strain or constrained plane stress (Lay, 1982). With restraint the yield stress and corresponding field strain are increased above the uniaxial values and depend on Poisson's ratio. If it is assumed that the maximum principal strain theory is valid and Poisson's ratio is taken as 0.30, the yield stress under plane strain conditions is  $\sigma_{\rm Y}/(1-{\rm v}^2)$  = 1.099  $\sigma_{\rm Y}$ . The corresponding yield strain remains unchanged from that found in a uniaxial state of stress and is equal to  $\epsilon_{V}$ . The tensile and compressive stress-strain curves based on the transformed values of yield strain and yield stress are shown in Fig. 2.1 and exhibit larger stresses for a given strain than for the condition of uniaxial tension or compression. Although other yield theories are not considered in this study, the maximum principal strain theory gives a yield stress under plane strain conditions, and taking Poisson's ratio to be 0.30, that is about 2% above that given by the maximum energy of distortion theory formulated by von Mises.

Yielding occurs because of shear distortions. If it is assumed, upon yielding, that the inelastic strain ratio is 0.50, consistent with zero volume change, the stress-strain curve would rise to 1.333  $\sigma_{\rm Y}$  as shown in Fig. 2.1. Ratzlaff et al. (1988) show, however, that after yielding the inelastic strain ratio may reach values of 0.40 to 0.45, somewhat less than the zero volume change value of 0.50, and

with subsequent straining there is a gradual reduction to a value of about 0.35.

# 2.2 Elastic and Elasto-plastic Behaviour

# 2.2.1 Theoretical Behaviour

Many analyses are based on the simplifying assumptions that the stress-strain relationship is bilinear, neglecting strain hardening, and also that the moment-curvature relationship is bilinear. As well, the effect of the variation in Poisson's ratio in the inelastic range is neglected.

Figure 2.3 developed by Ratzlaff and Kennedy (1985) presents nondimensionalized load-deflection responses based on different assumptions in behaviour of transversely loaded flat plates. To nondimensionalize the curves, the load intensity per unit width, q, has been divided by that corresponding to a plastic collapse mechanism, qc, and the mid-width deflections,  $W_{m}$ , divided by the initial plate thickness, h. The curves are drawn for a steel plate of zero aspect ratio subject to a load of uniform intensity, q, with a span to depth ratio, L/h, equal to 100 (h = 10 mm). The steel is characterized by a yield strain of 1500  $\mu\epsilon$  and a modulus of elasticity of 200000 MPa . With the exception discussed subsequently both the stress-strain and momentcurvature relationships are assumed bilinear. Poisson's ratio is assumed to be 0.30 and on yielding the inelastic strain ratio is taken as 0.50 as discussed previously.

Curve F in Fig. 2.3 gives the first two of the three segments of the load-deflection response when the plate is assumed to carry loads in flexure only. The formation of the last hinge at midspan leading to mechanism formation occurs at a deflection greater than the maximum shown. The load to cause the collapse mechanism to form,  $q_{\rm C}$ , (used to nondimensionalize the load intensity) equals  $16{\rm Mp/L^2}$ .

When the edges of a plate are restrained against translation as well as rotation the flexural behaviour as given by curve F must represent a lower-bound.

Another lower-bound solution, established by considering elastic membrane action only and neglecting bending, is given by curve  $M_{\rm e}$ . The deflections result solely from the elastic straining of the plate. The curve terminates at point B when the maximum membrane stress at the edges reaches yield.

Curve M<sub>i</sub>, extending beyond point B, gives the inelastic membrane response as yielding gradually progresses from the plate edges towards the mid-width when Poisson's ratio is assumed to be 0.30. The increased load-carrying capacity results from the increasing inclination of the membrane forces at the edges. Not entirely consistent with the assumption of a bilinear stress-strain curve, this analysis requires the membrane stress at the edge, for the plate characteristics given, to rise to about 1.05  $\sigma_{\rm Y}$  at a value of q/q<sub>C</sub> equal to 12.4. Curve M<sub>i</sub> projects backward to the origin.

Curve II in Fig. 2.3 represents the inelastic membrane response based on the same assumptions as those for curve M; except that the inelastic membrane response is based on an inelastic strain ratio of 0.50. Beyond point B there is no flexural action and the response would be expected to gradually move from curve M; towards curve N as yielding progresses from the edges towards the centre and the inelastic strain ratio applies to more and more of the membrane.

The closed-form solution by Timoshenko (1940) for elastic flexural membrane behaviour is shown as curve E in Fig. 2.3 and in reality should not be drawn beyond point A' representing the onset of yielding of the extreme fibres at the edges of the plate in combined tension and flexure. Point A represents the onset of yielding if the moment-rotation curve in the presence of axial load is assumed to be elasto-plastic. The extension of curve E beyond point A assumes that yielding has not occurred. Curve E, taking both membrane and flexural contributions into account is an upper-bound.

If it is assumed that a collapse mechanism forms with zero deflection and then that deflections occur due to inelastic membrane action, as suggested by Kennedy and Hafez (1984), curve Y joining Mi asymptotically, results.

Within the limits of the assumptions made the shaded areas in Fig. 2.3 represent a permissible domain of behaviour as the curves provide, in general, upper and

lower-bound solutions. Curve R is a possible response drawn from the point representing full yielding at the edge, partially in tension and partially in compression, to point B representing full yielding in tension at the edges due to membrane action. Beyond point B there is no flexural action and the response would be expected to move gradually from curve Mi towards curve N as yielding progresses from the edges towards the centre and the inelastic strain ratio applies to more and more of the membrane. When straining beyond the strain hardening strain occurs, the response could rise above curve N.

#### 2.2.2 Experimental Behaviour

Plotted in Fig. 2.4 are test results of Young (1959) for a plate with an aspect ratio of 1/3 and a width to thickness ratio of 57.1. These data are in good agreement with curves E and R for a plate having an aspect ratio of zero. Although falling slightly below point B, to the right of point B the experimental results gradually move from curve Mi to N and even exceed it. The data that fall below and to the right of point B can be attributed to the assumption that the moment-rotation curve is elasto-plastic and to the fact that to get full yielding of the plate in membrane action (curves Mi and N) a small portion of the plate near the edge must be strained into the strain would therefore increase the deflections beyond those assumed.

# 2.3 Behaviour Considering Strain Hardening

In Fig. 2.5 inelastic membrane behaviour beyond point B, as described by Ratzlaff and Kennedy (1986), is depicted for a variety of assumptions. Curves Mi and N have been described in the previous section and are both drawn for a uniformly loaded membrane. Curve V is similar to curve N but is drawn for a plate subjected to fluid pressure.

Curve U in Fig. 2.5 predicts the load-deflection behaviour under uniform loading when the stress-strain curve beyond the onset of strain hardening is approximated by five linear segments as given by Ratzlaff and Kennedy (1986). At point B the plate is fully yielded in tension. With increased loading and as yielding progresses from the edges towards the mid-width, the inelastic strain ratio is computed based on the dilitation that occurs as suggested by Stang et al. (1946). The curve bends sharply upward at a value of  $W_m/h$  of about 8.2 when the membrane strains enter the strain hardening region and the corresponding stresses increase. Curve U continues until the ultimate tensile strain is achieved at some location in the plate.

The results of 2 tests reported by Ratzlaff and Kennedy (1986) are shown in Fig. 2.6 together with curves V and U as well 's curve U' taking into account the contribution of observed shear deformations to the total deflection, and the results of a finite element analysis. Ratzlaff and Kennedy (1986) suggested two possible failure modes as indicated by the maximum strain limit and shear limit on curve U. The

maximum tensile strain occurs at the edges of the plate on the loaded side but could not be reached if the shear limit governs. The proposed shear limit is reached when the average shear stress along the edges equals the ultimate shear stress, taken as  $0.577\sigma_{\rm u}$ . Specimen 2 failed prematurely due to loss of restraint while an examination of the failure surface along one edge of specimen 1 revealed that it failed in shear (Vitovec, 1984, 1989). For test 1 the test-to-predicted load ratio was 0.97 and the test-to-predicted deflection ratio based on curve U' was 1.08.

The finite element analysis, which did not model the shear limit, follows curve U closely but does not exceed it at high loads. This difference is attributed to the use of a limited number of elements through the thickness. The edge, increasing the restrain there.

#### 2.4 Other Work

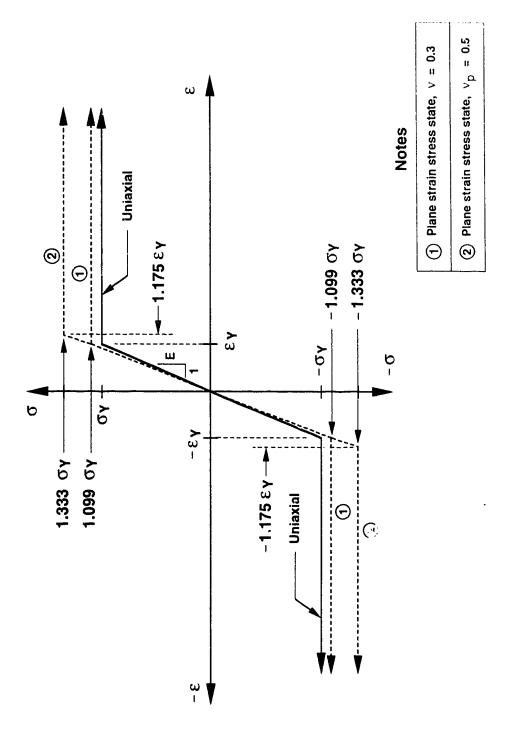
In a series of tests, Taylor (1986) tested six plates under lateral fluid pressure. The load-deflection behaviour into the inelastic membrane range followed closely the predictions of Ratzlaff and Kennedy (1985) but unfortunately the plates were not loaded to failure.

Kennedy and Cheng (1992a,b) tested a series of steel-concrete sandwich panels, essentially completely restrained at the edges, with 5 transverse loads. The behaviour was dominated by membrane action with failure initiating in one or both of the steel plates at the support, in what was described as a shear-tension failure. Based on an extension

of the von Mises-Huber-Hencky yield condition to ultimate and assuming an inelastic strain ratio in this condition of 0.50 the mean test-to-predicted ratio for 6 tests was 1.13 with a coefficient of variation of 0.037. As the authors state, however, the test points are clustered because only one load configuration was used. Moreover the failure criterion was based on only axial force and shear acting at the edge of the membrane, i.e., no bending moments were considered to exist there.

#### 2.5 Summary

The literature review reveals that those tests that have been conducted were generally of relatively small scale when compared to actual structures in which membrane behaviour could be utilized effecti/ely. Moreover, with the exception of tests by the author (Ratzlaff and Kennedy, 1986) laterally loaded steel plate tests have not been taken to failure, not have failure mechanisms been proposed. Therefore the need exists for tests on plates of substantial scale and with a range of width/thickness ratios. These would serve to confirm the failure mechanisms that have been proposed or suggest others and to establish preferred proportioning of membrane structures, i.e. the ratio of the distance between supporting frames and the plate thickness. Implied in this is whether intermediate stiffeners and orthogonal frames should be used at all.



BAGINES 2.2 BILLINGAY STRESS-STREST RELACIONALING

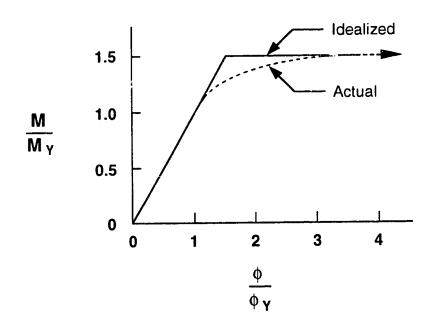
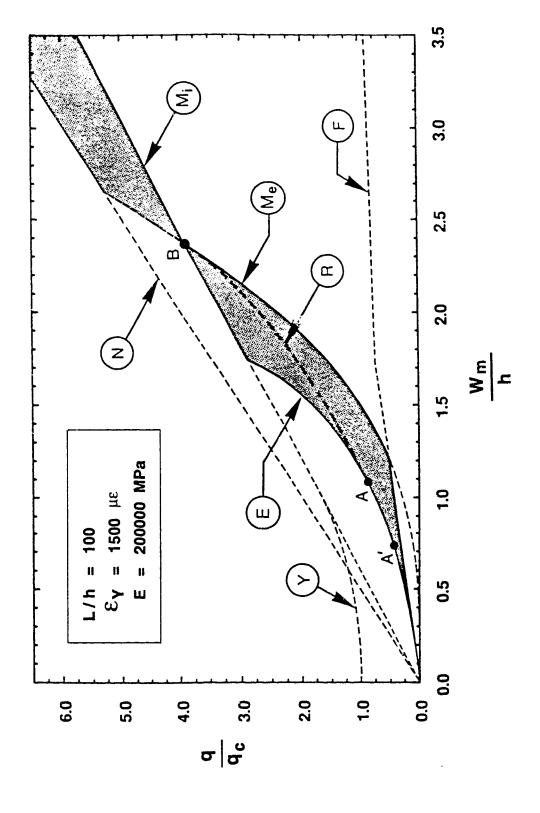
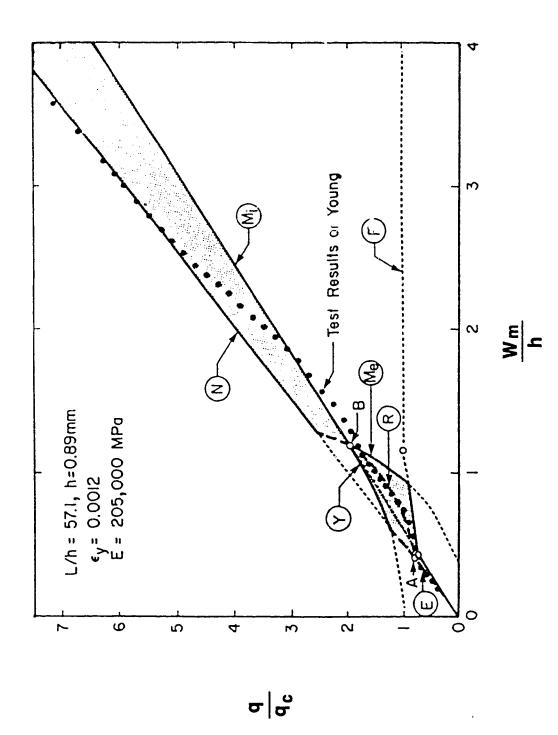


Figure 2.2 Elasto-plastic moment-curvature relationship

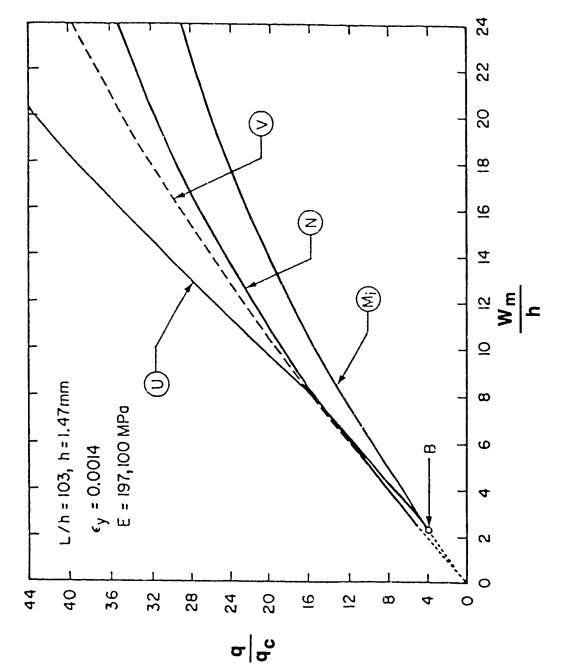
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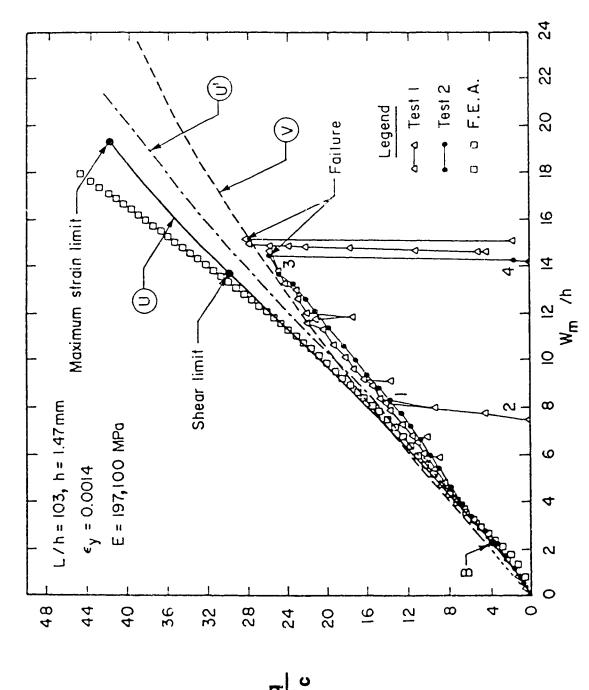
Behavioural domain without stain hardening Figure 2.3



Test results of Young (1959) and behavioural domain Figure 2.4



Load-deflection behaviour of inelastic membrane (Ratzlaff and Kennedy, 1986) Figure 2.5



Test and predicted load-deflection curves (Ratzlaff and Kennedy, 1986) Figure 2.6

### 3. EXPERIMENTAL PROGRAM

#### 3.1 General

The primary objectives of the experimental program were:

- to verify the predicted load-deflection behaviour of full-scale transversely loaded continuous flat steel plates;
- 2. to determine the influence of the corners on the overall behaviour;
- to investigate failure modes.

Four tests were performed on steel plates having nominal thicknesses of 6, 12 and 16 mm. The overall width and length, out-to-out of welds, of all specimens were 1064 mm and 3064 mm, respectively, giving an aspect ratio of the loaded surface after deducting the size of the edge welds of about 1/3. With this aspect ratio it was anticipated that the stress condition, across the width at the middle of the long sides, would approach that of plane strain. geometry chosen provided nominal width-to-thickness ratios of about 174, 86, and 64 for the three plate thicknesses. width/thickness ratio of ship hull plating between intermediate stiffeners varies between 30 and 80 (Hughes, 1981). This range is also common in caisson-type steel icerelisting walls in offshore structures used for oil exploration in the Arctic ocean (Comyn, 1985). The greater width/thickness ratios of plates used in the tests were selected in anticipation that, with increased spacing of

intermediate stiffeners, the local behaviour would be satisfactory for steel caisson-type structures such as ships and other offshore structures.

Material properties of the test plates were determined by means of a series of uniaxial tension tests. The effect of welding on the geometry and microstructure of the restraint zone near the edges of the plates was investigated in small-scale welding tests and small-scale bending tests.

The chronological sequence of tests that developed follows. The ancillary tests to determine the stress-strain properties of the steels, Poisson's ratio and the inelastic strain ratio were first carried out. Plate test P6-1, tested first, failed at a lower load than expected and examination of the fracture surface showed copper contamination from the copper backing bar. This led to a series of weld tests using ceramic backing bars to develop a procedure to give contaminant-free welds with superior profiles and characteristics.

After completion of the weld tests, plate specimens P6-2, P12 and P16 were tested in that sequence. Because even these tests failed at lower than expected loads, a series of small scale bend tests were conducted to examine the ductility of the heat-affected zones and to determine the effect of the heat-affected zone on the failure loads and modes.

### 3.2 Plate Tests

#### 3.2.1 General

In all tests the four edges of the test plates were fixed against rotation and translation by groove welding the edges of the specimens to a 152 mm thick reaction plate as shown in Fig. 3.1. The depth of the groove weld, made from one side, was 1.5 times the nominal test plate thickness. The multipass weld was designed to contain the oil under pressure and to transmit bending moments, shear forces and in-plane axial forces from the plate specimens to the relatively rigid reaction plate. The thickness of the reaction plate was determined such that its behaviour would be elastic when the test plates acting as membranes with a 30 degree inclination at the edges failed in shear. The reaction plate was re-used from test to test.

The measured thicknesses of the test plates is given in Table 3.1.

#### 3.2.2 Test Specimens

The 4 test specimens, sheared from larger plates, were supplied by C.W. Carry Ltd., Edmonton, Alberta. Specimen P6-2 was hot rolled by Altos Hornos De Mexico S.A. whereas the 3 others were hot rolled by Stelco Inc., Hamilton, Ontario. All plates were specified to conform with CSA Standard CAN3-G40.21-M81 "Structural Quality Steels", (CSA, 1981), grade 300W. Chemical analyses, mill test reports, and the requirements of CSA Standard CAN3-G40.21-M81 are given

in Table 3.2. Additional material from each heat was supplied to obtain material properties.

After identifying and marking each test plate the ancillary test material was sheared from one end and the test plate geometry determined. Electrical resistance strain gauges were applied to the lower surface and, as this surface was to be the loaded surface, the strain gauges were protected with flowable silicon (see Fig. 3.1). The lead wires were then passed through holes in the reaction plate into which copper tubes had been threaded to protect the wires during welding of the test plate to the reaction plate. After transporting the test plate-reaction plate assembly to the structural engineering laboratory at the University of Alberta, the strain gauges were placed on the top (free) surface of the test plate and a few on the lower surface of the reaction plate.

### 3.2.3 Reaction Plate

The mechanical properties of the reaction plate, hot-rolled by Stelco Inc., exceeded the requirements of ASTM A36-84A plate with a yield strength of 336 MPa and an ultimate tensile strength of 510 MPa as given on the mill test certificates.

Twelve 12.7 mm diameter holes were drilled through the thickness of the reaction plate to accommodate pressure piping fittings and plugs so that strain gauge wires could be brought from the underside of the test specimens, as discussed previously, and to provide ports for injecting or

releasing oil and bleeding off air. Four lifting lugs were attached to the side of the reaction plate.

To seal the holes in the reaction plate through which the lead wires from the strain gauges passed, the lead wires were threaded through machined piping plugs as shown on Fig. 3.2. The wires were twisted counter clockwise and the conical nole filled with "5-minute" epoxy which was allowed to harden. The piping plugs were then threaded into the reaction plate holes.

The edges of the reaction plate were prepared for welding by flame cutting as shown in Fig. 3.3. Figures 3.4 and 3.5 show the backing bars used in the four tests. To maintain the gap between the test plate and the reaction plate additional backing bars were placed between the two plates. The copper backing bar used in test P6-1 was fastened to the reaction plate by counter sunk machine screws. Because in the first plate test P6-1 copper from the backing bar had infiltrated into the weld metal and was evident on the failure surface, after extensive testing and experimentation, ceramic backing bars were used for the remainder of the tests. To hold the ceramic backing in place, continuous thin steel plates were tack welded to the reaction plate as shown in Fig. 3.4(b) and Fig. 3.5.

## 3.2.4 Welding Materials And Procedures

All shielded metal arc welding used for the root passes was performed using type E48018 covered electrodes meeting the requirements of AWS A5.1-81, specification for Carbon Steel Covered Arc Welding Electrodes, (American Welding Society, 1981). Type E 48018 electrodes provide a minimum specified yield strength of 480 MPa and an ultimate tensile strength of between 550 MPa and 650 MPa. The flux cored welding was done with type LA T-9 flux cored electrode supplied by Liquid Air Canada Ltd., Montreal, Quebec, to meet the requirements of AWS A5.20-79 class E70T-1, specification for Carbon Steel Electrodes for Flux Cored Arc Welding (American Welding Society, 1979). The LA T-9 electrode is specified to have a minimum yield strength of 450 MPa and a minimum ultimate tensile strength of 550 MPa. Both filler metals were chosen for their strength and similarity in chemical composition to the base metal of the reaction plate and test plates. The weld deposit sequences are shown on Figs. 3.4 and 3.5.

Welding for test P6-1 was done in the shop of C.W. Carry Ltd., Edmonton. A number of steel frames were welded to the reaction plate to hold the test plate in position. These frames are shown in Fig. 3.6. Before welding began, the reaction plate was preheated using, acetylene gas flame, to a global temperature of 180°C with local temperatures occasionally exceeding 250°C. The first and second root passes were made in the horizontal position and the steel

frames removed. The root passes were started at opposite corners of the assembly and were deposited along the long sides first. The surfaces of the first two root passes were ground before making additional passes to minimize slag entrapment. With the test assembly in the vertical position, while maintaining the preheat temperature, the remaining four passes of the groove weld were placed in the downhand position.

The procedures used for welding specimens P6-2, P12 and P16 were the same, with the root pass made at the University of Alberta and subsequent flux cored welding at C.W. Carry Ltd. Mechanical clamps were used to hold the test plate against the backing bars for the root passes. The reaction plate was preheated to 150°C to 200°C. The root pass was placed with the clamped assembly in a vertical plane but with one end 22° above the other as shown in Fig. 3.7. Welding proceeded from the lower to the upper corner. The long sides were again welded first. Based on the results of small scale tests done at the Welding Research Lab, Dept. of Mining and Metallurgical Engineering, University of Alberta, a welding speed of about 60 mm/min. and welding machine settings of 170 Amps and 22 Volts were used for the root pass.

#### 3.2.5 Test Set-Up

The test set-up consisted of the self equilibrating test
-reaction plate assembly, support columns, a pneumatically
activated hydraulic oil pump with reservoir and control
valves, an enclosure structure, mechanical and electronic

instrumentation, and data acquisition systems. A number of the components can be seen in Fig. 3.8. An enclosure, visible in Fig. 3.8, was provided to limit the area of the laboratory sprayed by oil when rupture occurred. The enclosure was placed along the long sides of the test assembly where rupture was anticipated. Figure 3.9 shows schematically a partial cross-section of the test-reaction plate assembly.

## 3.2.6 Instrumentation And Measurement

### 3.2.6.1 Strain Measurements

Assuming symmetrical behaviour, the bulk of the strain gauges were placed in one quadrant only. Strain gauges were generally applied at the same locations on each surface of late. The strain gauge alphanumeric reference code, concribed in detail in Table 3.3, specifies the location, the orientation, the type of gauge and the surface upon which the gauge was mounted.

The location of grid lines for test plates P6-1, P6-2, P12 and P16 are given in Figs. 3.10, 3.11, 3.12, and 3.13, respectively, and the strain gauge codes, in Tables 3.4, 3.5, 3.6, and 3.7, respectively.

The strain gauges were applied using Armstrong type A-12 epoxy adherive and subsequent to lead wire attachment protected with Dow Corning flowable silicon type RTV-340. In Fig. 3.14 are shown a pair of strain gauges on the loaded surface of specimen P6-1 and blanketed with silicon. The

free ends of lead wires were connected to the lata acquisition system to form a 1/4 Wheatstone bridge configuration. The remaining 3/4 of the bridge configuration was made up of electrical resistances contained within the data acquisition equipment.

Prior to tests on plates P12 and P16, three strain gauges were mounted vertically on the outside of weld metal located at the centre of the long side to measure the strains in the weld.

#### 3.2.6.2 Load Measurements

Fluid pressure, measured by an electronic pressure transducer on-line between the test set-up and the pump, was monitored by the data acquisition equipment.

#### 3.2.6.3 LVDT Measurements

Nine linear variable displacement transformers (LVDT's) were mounted directly across the width at the centre of the long sides, as shown in Figs. 3.15 and 3.16, to measure the vertical displacement of the centreline of the plate. The LVDT output signals were recorded electronically. The LVDT located at the mid-width together with the pressure transducer were connected to an x-y plotter to give a visual display of the load-deflection behaviour during testing.

## 3.2.6.4 Surface Level Indicator (SLI) Measurements

The surface level indicator (SLI), (see Figs. 3.17 and 3.18), bolted to the lifting lugs of the reaction plate, was fabricated using steel channels, hard rubber wheels with steel axles, plastic chains, gears and bearings, two electrical motors, a spring-loaded cable LVDT, two electrical potentiometers, a power supply, control switches, and numerous other pieces. At any load step the SLI traversed the width of the test specimen at 14 locations along the half length as shown in Fig. 3.19 to provide continuous displacement measurements of the plate surface from one long edge to the long edge opposite. The LVDT and potentiometer gave horizontal and vertical displacement readings that were recorded graphically with an x-y plotter.

### 3.2.7 Test Procedure

The testing procedure generally consisted of the following steps:

- Oil was pumped into the pressure cavity between the reaction plate and test plate while allowing entrapped air to escape;
- 2. Initial surface levels were recorded with the SLI at zero oil pressure. The spindles of the LVDT's were checked to ensure they were in contact with and in the correct location on the test plate surface;
- 3. Initial strain gauge reading, pressure readings and LVDT readings were taken and stored in the data acquisition system;

- 4. Up to the theoretical collapse pressure,  $q_{\rm C}$ , the oil pressure was increased in increments of about 0.05  $q_{\rm C}$ , taking pressure, strain and LVDT readings at each step. SLI profiles were recorded at a pressure of  $q_{\rm C}$ ;
- 5. Beyond the pressure  $q_C$ , increments of about 0.1  $q_C$  were used and the corresponding electronic readings were recorded. Profiles of the plate surface and photographs were taken at random pressure levels;
- 6. After fracture, photographs were taken and the surface profiles were recorded.

Each test took about 8 hours to complete. Due to the volume of oil required to fill the pressure cavity as the test plate deformed, it was necessary to add oil periodically to the reservoir. Except for one occurrence while testing plate P12, the pressure was increased continuously or main ained and not reduced during the test. Visual inspection of the test set-up was made periodically to ensure that all test equipment and instrumentation were performing as planned.

#### 3.3 Calibrations

### 3.3.1 LVDT'S And Potentiometers

The LVDT's were calibrated at a near constant supply voltage against a linear micrometer having a resolution of 0.001 mm. These calibrations also defined the linear range of the LVDT's. The relevant data are given in Table 3.8.

After installing the two potentiometers in the SLI (see Fig. 3.18) they were calibrated using a hand-held digital caliper. The 10-turn potentiometer used to determine transverse displacements in the x-z plane was found to have a constant of proportionality of 128.55 mm/V at a supply excitation voltage of 9.555 Volts. The spring-loaded cable potentiometer used to determine vertical displacements was found to have a calibration constant of 56.30 mm/V at a supply excitation of 9.555 Volts. These calibrations were then used to calibrate the x-y plotter that recorded graphically the pressure and displacement readings during testing.

#### 3.3.2 Pressure Transducer

The electronic pressure transducer, used to monitor fluid pressure during testing and having a capacity of 13.8 MPa, was calibrated by 2 different methods.

First the transducer was attached on-line between a hand activated pump and a loading jack that was placed between the heads of a 1000 KN capacity MTS loading machine. A preset AC excitation voltage was used on the transducer. Using the effective area of the jack piston to translate the MTS load to a pressure, corresponding values of a pressure and the output voltage were determined. A linear least squares analysis of these data gave a calibration factor of 1415 kPa/V.

Second, a calibration was done using dead weights to give corresponding pressures and output voltages directly. The least squares fit of these data gave a calibration factor of 1418 kPa/V. The average of the two values was used during testing.

Table 3.1 Test Plate Designation

Plate Designation	Nominal Thickness (mm)		Measured	Measured Thickness	
		Number of Measurements	Mean (mm)	Standard Deviation(mm)	Coefficient of Variation, V
P6-1	9	27	6.8	0.07	0.011
P6-2	9	18	7.1	90.0	0.008
P12	12	30	13.4	0.10	0.008
P16	16	30	16.8	0.04	0.003

Chemical Composition And Mechanical Properties From Mill Reports Table 3.2

	H 0		hemical	Chemical Composition (%)	ion (%)			Mechanical Properties	ties
Plate	No.	U	Mn	Δ,	w	si	Yield Strength (MPa)	Tensile Strength % elongation in (MPa)	% elongation in 200mm gauge
P6-1	<b>P6-1</b> 316533	0.17	1.29	0.011	0.010	0.04	313.2	450.3	26.0
P6-2	78328	0.19	1.10	0.012	0.025	90.0	316.3	464.5	26.0
P12	621764	0.16	1.40	0.012	0.012	0.24	324.9	509.7	23.0
P16	346039	0.19	1.30	0.005	0.022	0.18	352.5	539.0	22.0
CSA G40	CSA G40.21-M81	>0.05	<i.50< th=""><th>&lt;0.01</th><th>&lt;0.05</th><th>&lt;0.40</th><th>&gt;300</th><th>&gt;450 and &lt;620</th><th>&gt;20.0</th></i.50<>	<0.01	<0.05	<0.40	>300	>450 and <620	>20.0

Table 3.3 Strain Gauge Reference Code

	CODE :	a b -	c d e
Code letter designator	Options		Significance
a	A through M		Located on grid lines ${f A}$ through ${f M}$
b	1 through 7		Located on grid lines 1 through 7
c	T or L	<b>T</b> =	Oriented transverse to long side
		L =	Oriented parallel to long side
đ	H or P	Н =	High elongation foil strain gauge: Micro - Measurements type EP-08-250BG-120 [£ limits = ± 20 % or less]
·		P =	foil strain gauge: Showa type N11-FA-5-120-11 $[\epsilon \text{ limits} = \pm 1.5 \%]$
e	I or O	<b>I</b> =	Mounted on loaded surface
		0 =	Mounted on free surface
Example:	D3-THI	lines trans elonga	ed at intersection of grid  D and 3. Oriented in  verse direction. High  ation type gauge.  ed on loaded surface.

Table 3.4 Strain Gauges On Test Plate P6-1

Longitudinal grid lines	linal ies <b>A</b>	۵۷	υ	Д	p	Ĕ	<b>o</b>	Ħ	H	ט	×	ឯ	<b>×</b>
trans. grid lines													
м	A1-THI A1-THO A1-LHI AX-THO	B1-THI B1-THO	C1-THI C1-THO C1-LPI C1-LPO	D1-THI D1-THO	E1-THI E1-THO E1-LPI E1:-LPO	F1 - TH1 F1 - TH0	G1-TH1 G1-TH0 G1-LPI G1-LPO	H1 - TH1 H1 - TH0	11-THI 11-THO 11-LPI 11-LPO	JI-THI JI-THO	K1-THI K1-THO K1-LPI K1-LPO	L1-THI L1-THO	M1-THI M1-THO M1-LHI MX-THO
М	A2-THI A2-THO	B2-THI B2-THO		D2-THI D2-THO D2-LPI D2-LPO			G2-THI G2-THO G2-LPI G2-LPC		12-THI 12-THO		P2 -THI R2-THO		
m	A3-THI A3-THO A3-LHI			D3-THI D3-THO			G3-THI G3-THO G3-LPI G3-LPC						
4							C4-THI G4-THO G4-LHI G4-LHO						
S	A5-THI A5-THO A5-LHI			DS-THI DS-THO DS-LFI DS-LPO			G5-THI G5-THO G5-LHI G5-LHO						٠
y	А6-7НІ А6-ТНО			26-781 26-783 26-181 D6-180			G6-TH1 G6-TH0 G8-LH1 G6-LH0						
7	A7-SHI A7-SHO						G7-LHI G7-LHC						

Table 3.5 Strain Gauges On Test Plate P6-2

Longitudinal grid lines	dinal nes A	Ø	υ	Д	ធ	Du	<b>U</b>	Ħ	н	þ	×	ы	×
Trans. grid lines													!
ч	A1-THI A1-THO	B1-THI B1-THO	C1-THI C1-THO	D1-THI D1-THO	E1-THI E1-THO	F1-1HI F1-THO	G1-THI G1-THO G1-LPI	H1 - THI H1 - THO	11-THI 11-THO 11-1.PI	J1-TH1 J1-TH0	K1 -THI K1 -THO K1 -LPI	Li - THI Li - THO	M1-THI M1-THO
	AX-THO		C1-LPO		E1-LPO		G1-LPO		11-LPO		K1-LPO		MX-THO
Q	A2-THI A2-THO	B2-THI B2-THO		D2-THI D2-THO D2-LPI D2-LPO			G2-THI G2-THO G2-LPI G2-LLO		12-THI 12-THO		K2-TH1 K2-T40		
m	A3-THI A3-THO			D3-THI D3-THO			G3-THI G3-THO G3-LPI G3-LPO						
4	·						G4-THI G4-THO G4-LHI G4-LHO						
ம	AS-THI AS-THO			DS-THI DS-THO DS-LPI DS-LPO			G5-THI G5-THO G5-LHI G5-LHO					·	
v	A6-THI A6-THO			D6-TH1 D6-TH0 D6-LH0 D6-LH0			047-95 1H1-95 04L-95						
7	A7-SHI A7-SHO			D7-LHI D7-LHO			G7-ГНІ G7-ГНО				,		

Table 3.6 Strain Gauges On Test Plate P12

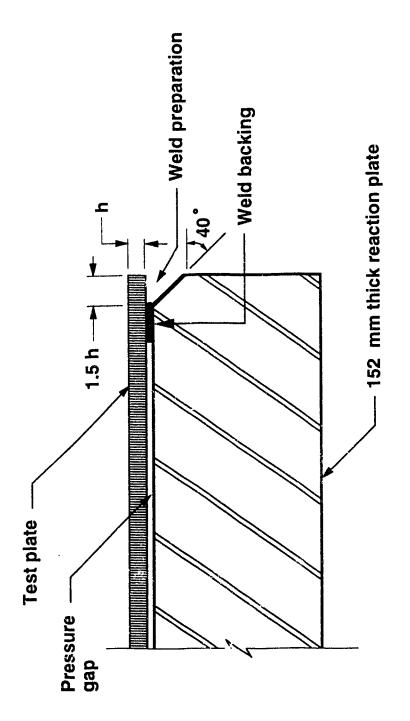
								-					
Longitudinal grid lines	dinal nes <b>A</b>	Д	υ	Ð	ធ	Dia .	Ġ	Ħ	н	b	×	IJ	Ä
Trans. grid													
<u> </u>	A1-THI A1-THO	B1-THI B1-THO	C1-THI C1-THO	D1-THI D1-THO	E1-THI E1-THO E1-T PT	F1-THI F1-THO	G1-THI G1-THO	H1 -THI H1 -THO	11-THI 11-THO 11-1.PT	JI-THI	K1-THI K1-THO K1-LPI	L1-THI L1-THO	M1-TH1 M1-THO
_	AX-THO		C1-LP0		E1-LPO		G1-LPO		11-LPO		K1-LP0		MX-THO
8	A2-THI A2-THO	B2-THI B2-THO		D2-THI D2-THO D2-LPI D2-LPO			G2-THI G2-THO G2-LPI G2-LPO		12-THI 13-THO		K2-THI K2-THO		
m	A3-THI A3-THO			D3-THI D3-THO			G3-THI G3-THO G3-LPI G3-LPO						
4							G4-THI G4-THO G4-LHI G4-LHO						
ហ	AS-THI AS-THO			D5-THI D5-THO D5-LPI D5-LPO			GS-THI GS-THO GS-LHI GS-LHO						
v	A6-THI A6-THO			D6-THI D6-THO D6-LHI D6-LHO			G6-THI G6-THO G6-LHI G6-LHO						
7	A7-SHI A7-SHO			D7-LHI D7-LHO			G7-LHI G7-LHO						

Table 3.7 Strain Gauges On Test Plate P16

Longitudinal grid lines	linal ies <b>A</b>	ф	7)	Q	ធ	D4	<b>O</b>	Ħ	н	ب	×	ı	×
Trans. grid lines	A1-THI A1-THO	B1-fi.	7 - THI 11-THO C1-LPI	D1-THI D1-TPO	E1-THI E1-THO E1-LPI	F1-TH1 F1-TPO	G1-THI G1-THO G1-LPI	HI THI HI-TPO	71-THI 11-THO 11-LPI	J1-7HI J1-7PO	KI - THI KI - THO KI - LPI	L1 -TH1 L1 -TP0	M1-THT M1-THO
a	AZ THI AZ THO	B2-THI B2-THO	C1-LPO	D2-THI D2-THO D2-LPI D2-LPI	E1-LPO		G1-LPO G2-TH1 G2-TH0 G2-LP1 G2-LP0		II-LPO 12-58I 12-THO		K1-LPO E2-THI K2-THO		NX-THO
m	АЗ-ТНІ АЗ-ТНО			D3-THI D3-THO			G3-THI G3-TPO G3-LPI G3-LPO						
4							G4-TH1 G4-TPO G4-LH1 G4-LPO						
w	AS-THI AS-THO			DS-THI OS. JO DS-LPI DS-LPO			GS-THI GS-TPO GS-LHI GS-LPO						
vo	A6-THI A6-THO			D6-THI D6-THO D6-LHI D6-LHO			G6-THI G6-THO G6-LHI G6-LPO						
7	A7-SHI A7-SHO			D7-LH1 D7-LH0			G7-LHI G7-LHO						

Table 3.8 LVDT Characterist.cs

Laboratory .eference code	Location number in frame	Nominal operating range (mm)	Excitation Voltage (V)	;ivity .n/V)	Calibration constant (mm/V)
7T0502	-	± 12.7	5.998	± 0.0003	3.875
6T1004	2	± 25.4	5.999	+ 0.0005	4.977
6T3006	m	± 76.2	5.999	± 0.002	17.525
6T3005	4	± 76.2	5.999	± 0.002	17.305
6T3001	S	± 76.2	5.999	± 0.002	16.800
6T3016	9	± 76.2	5.999	± 0.002	17.732
6T3045	7	± 76.2	5.998	± 0.002	16.354
6T1006	ω	± 25.4	5.999	± 0.0005	5.950
7T0501	<u>ი</u>	+ 12.7	5.998	± 0.0003	3.897



Partial cross-section through test and reaction plates at edge Figure 3.1

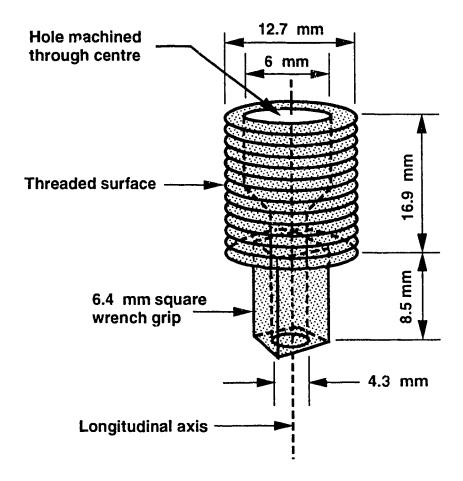
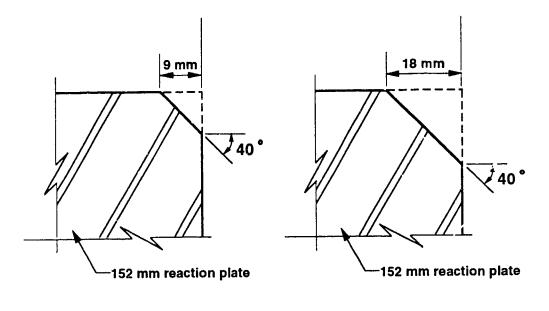
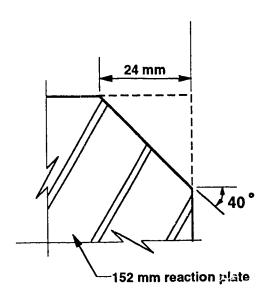


Figure 3.2 Machined steel piping plug



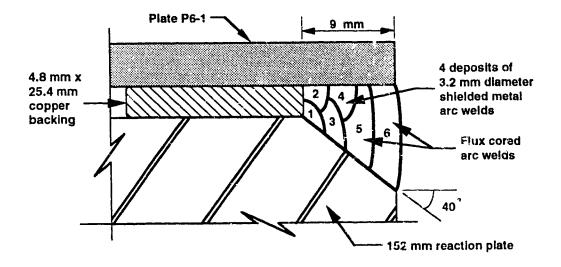
(a) Tests P6-1 and P6-2

(b) Test P12

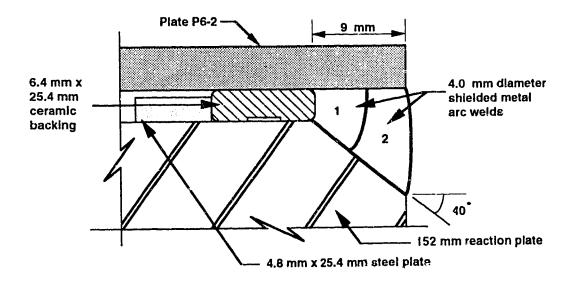


(c) Test P16

Figure 3.3 Reaction plate edge preparations

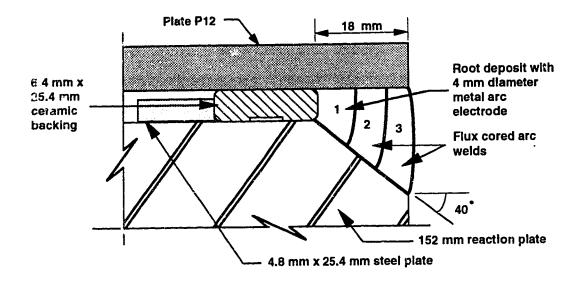


## (a) Specimen P6-1

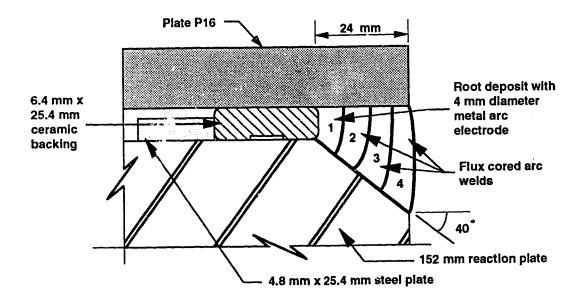


## (b) Specimen P6-2

Figure 3.4 Edge preparations, backing bars and welding sequences

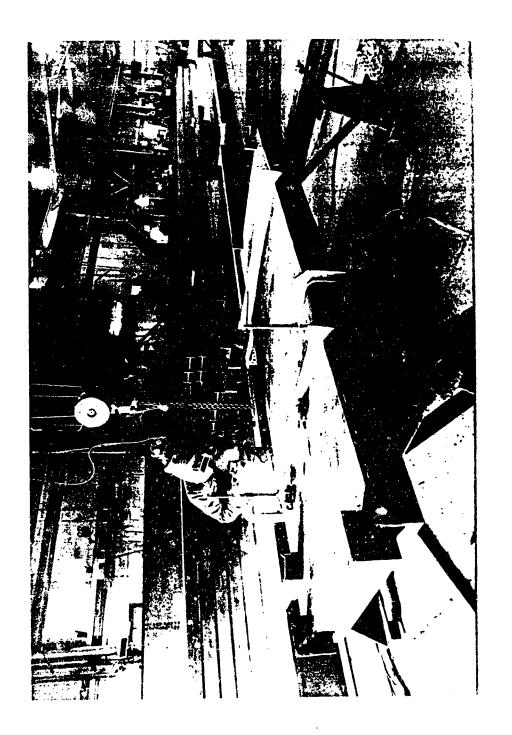


## (a) Specimen P 12



## (b) Specimen P 16

Figure 3.5 Edge preparations, backing bars and welding sequences



Clamping frames on specimen P6-1 and reaction plate assembly Figure 3.6



Clamped assembly elevated at one end, before welding Figure 3.7

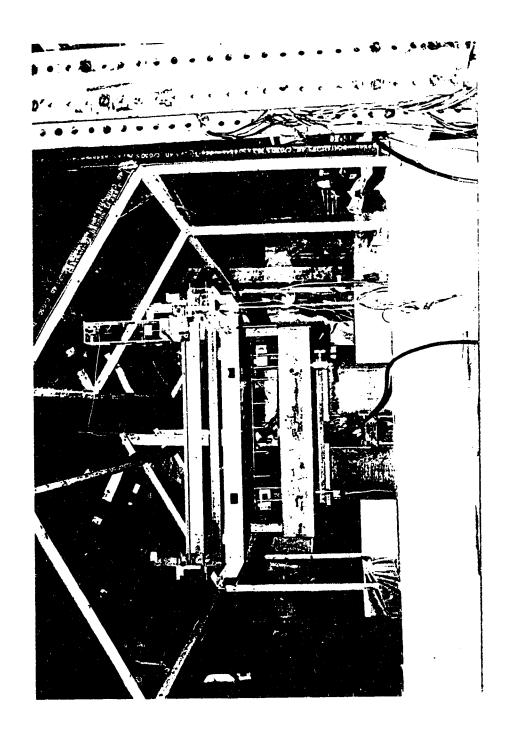
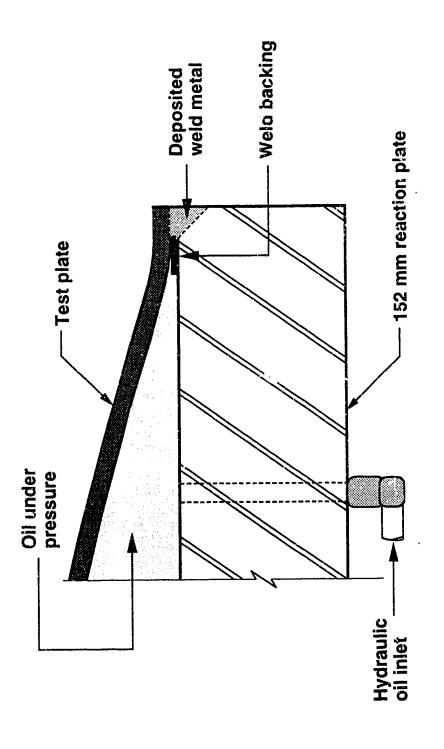
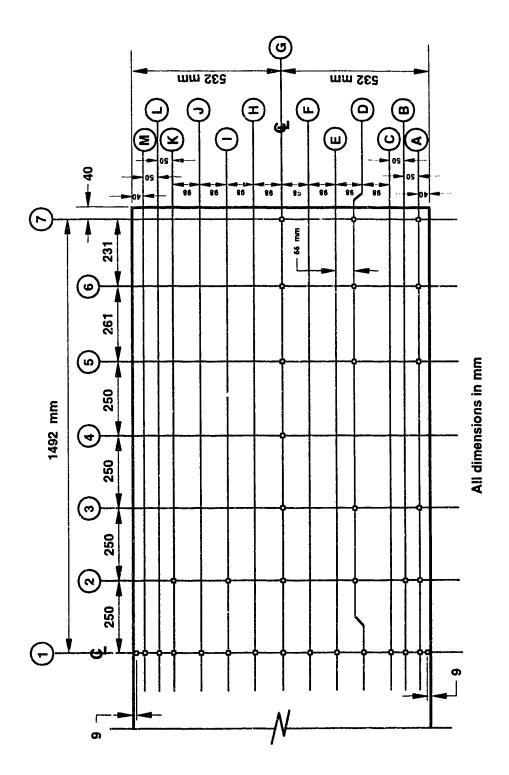


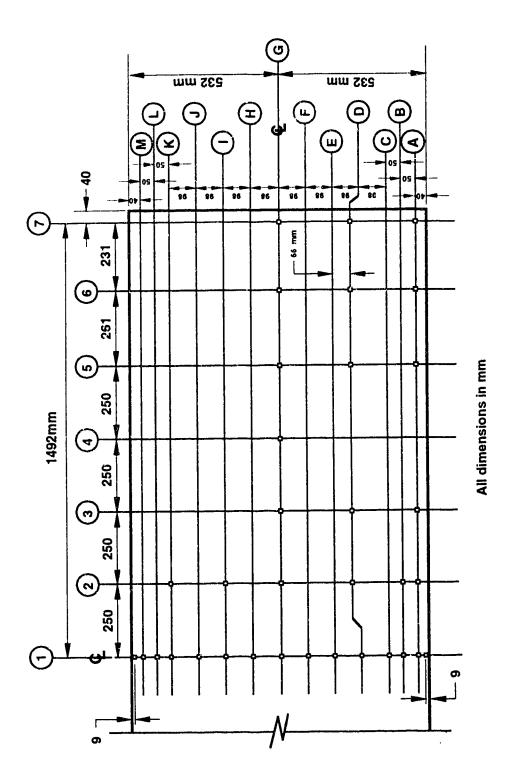
Figure 3.8 End view of test set-up



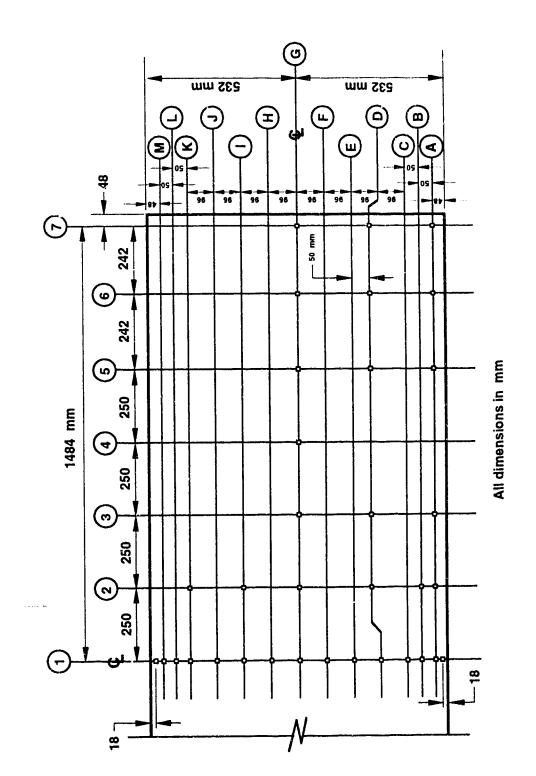
Partial cross-section at edge of test apparatus Figure 3.9



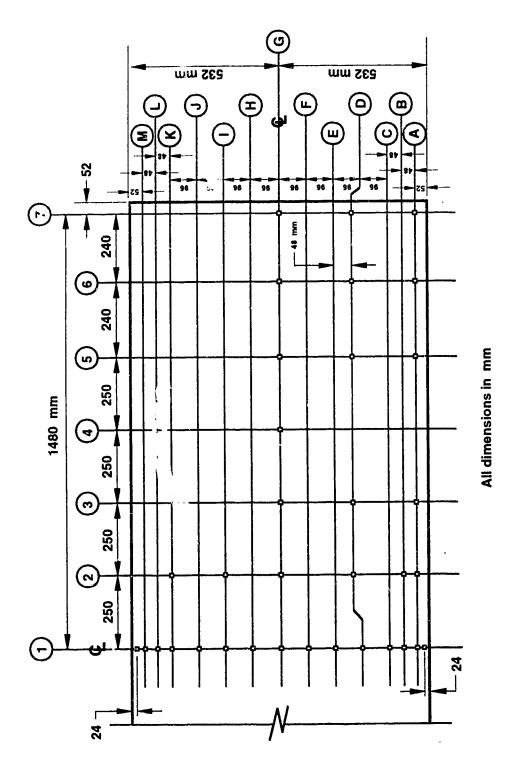
Grid lines for strain gauge locations on test specimen P6-1



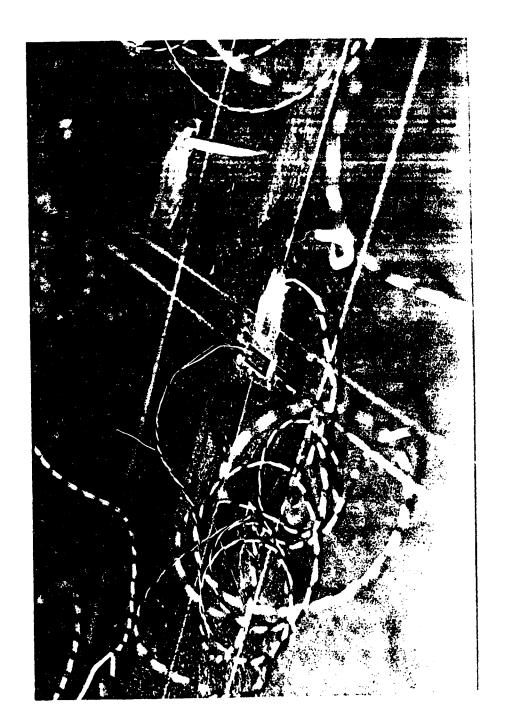
Grid lines for strain gauge locations on test specimen P6-2Figure 3.11



Grid lines for strain gauge locations on test specimen P12 Figure 3.12



Grid lines for strain gauge locations on test specimen P16 Figure 3.13



Strain gauges blanketed with silicon on specimen P6-1 Figure 3.14

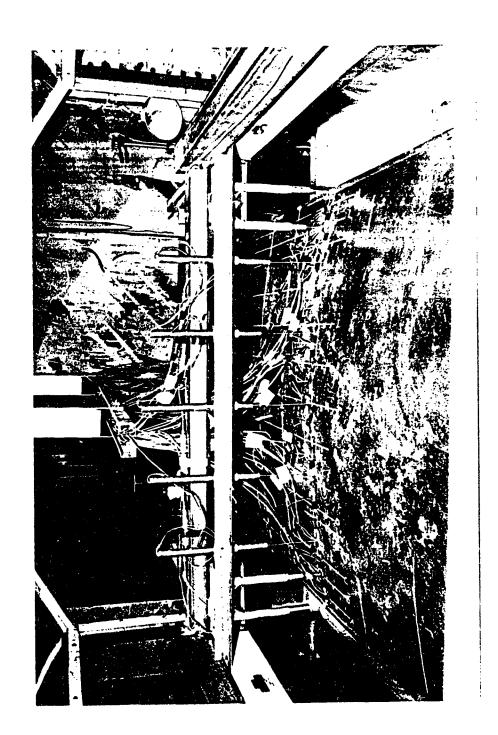
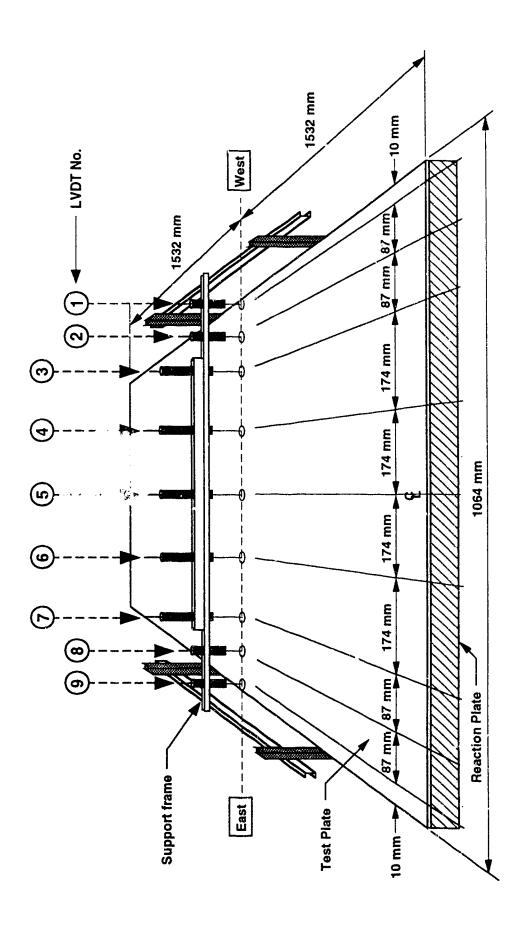


Figure 3.15 LVTD's in support frame above test plate



LVDT locations in support frame above test plate Figure 3.16

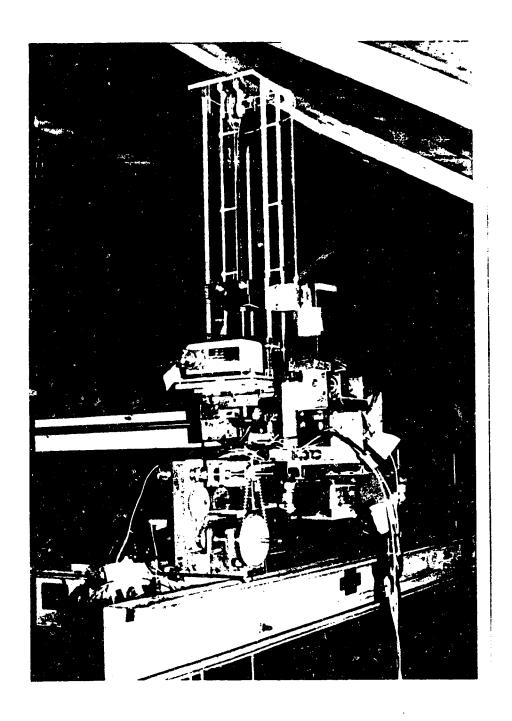
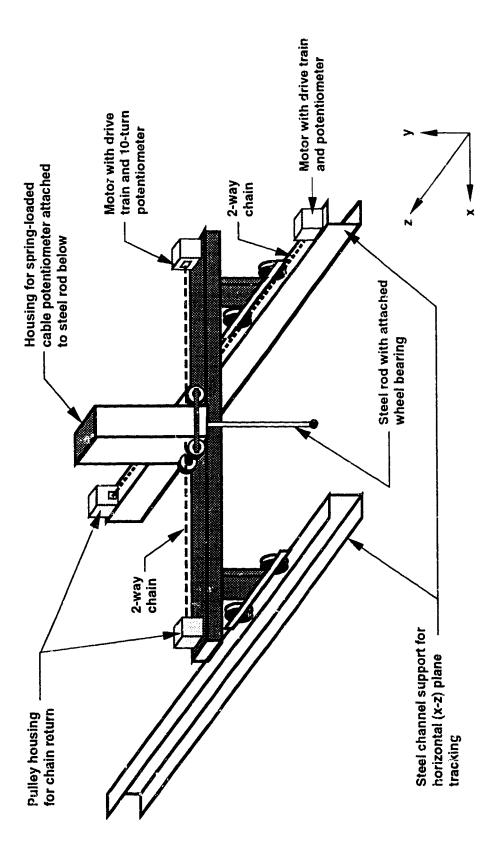


Figure 3.17 Surface level indicator (SLI) above test plate



Surface Level Indicator (SLI) Schematic diugram of Figure 3.18

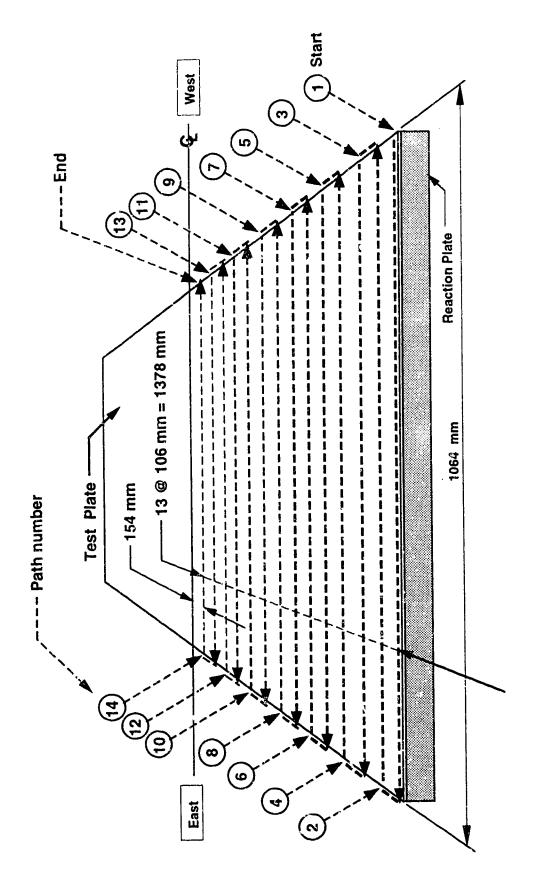


Figure 3.19 Path of Surface Level Indicator (SLI)

#### 4. MATERIAL PROPERTIES AND BEHAVIOUR

#### 4.1 Tension Tests

A total of 37 uniaxial tension coupon tests were performed on specimens from the four parent plates to establish the stress-strain characteristics, Poisson's ratio, and the inelastic strain ratio. With these data, the behaviour in other stress states could be determined. As shown in Fig. 4.1 five coupons were oriented parallel to and five transversely to the direction of rolling of the parent plate. Table 4.1 gives the alphanumeric identification of the test coupons. After cutting from the larger plate the full thickness coupons were milled to provide a reduced section.

Table 4.1 indicates which coupons were mounted with strain gauges to measure longer than trains on both faces and which had longitudinal and transverse gauges on both faces in order to determine Poisson's ratio. The table also notes which specimens had high elongation strain gauges so that inelastic strain ratios could be determined. As well, calipers were used to measure large strains. Figure 4.2 shows a tension coupon with longitudinal and transverse gauges in the Baldwin testing machine.

Figures 4.3 and 4.4 give typical stress-strain curves for the four test plates. The coupon test results are summarized in Tables 4.2 through 4.5 for plates P6-1, P6-2, P12 and P16, respectively. No significant difference exists

in the material properties parallel and transverse to the direction of rolling.

In Figs. 4.5, 4.6 and 4.7, the strain ratio is plotted versus axial strain for the 6 mm, 12 mm and 16 mm plates, respectively. The strain ratio is defined as the absolute value of the mean transverse strain divided by the mean longitudinal strain. The strain ratio, so defined, is sometimes called the total strain ratio because it is based on engineering or total strains. In the elastic range, this ratio is, of course, Poisson's ratio. Because these strains could amount to as much as 20% the strain data were corrected for both the nonlinearity in electrical output of the one-quarter Wheatstone Bridge circuit and for the effect of transverse strains on the electrical resistance of the strain gauge. (Ratzlaff et al, 1988).

The value of Poisson's ratio obtained for the 6 mm and 12 mm plates was 0.28 and for the 16 mm plate, 0.29. In the yield plateau the strain ratio would appear to fluctuate widely. However, this is attributed to the fact that yielding is a discontinuous phenomenon and the gauges, of finite length, give average strains and not those at a point.

For the 6 mm and 12 mm plates the inelastic strain ratio beyond the yield plateau has a maximum value approaching 0.4 at a longitudinal strain of about 2% and then decreases almost linearly to a value of about 0.33 at an axial strain of about 20%. Beyond the yield plateau the inelastic strain ratio for the 16 mm plate begins at values as high as 0.47,

decreases at first rapidly to about 0.44 and then linearly to about 0.37 at an axial strain of about 15%.

Instantaneous values of the inelastic strain ratio can be obtained by using true strains rather than engineering strains to determine this ratio. From Figs. 4.5, 4.6, and 4.7, instantaneous strain ratios so obtained were found to be from 0.5% to 5.7% greater than the total inelastic strain ratio, increasing as the longitudinal strain increases.

### 4.2 Weld Tests

After plate specimen P6-1 was tested it was determined that copper from the copper backing strips had infiltrated the weld metal during welding. As the copper had appeared on the fracture surface is as considered responsible for crack initiation and sublequent fracture of the plate. Therefore, a non-metallic chemically neutral non-hygroscopic ceramic backing bar was selected for subsequent use. Small scale shielded metal arc welding tests were done to stablish a welding procedure that, when used for the plate tests, would provide a favourable geometry and microstructure at the heavily stressed weld root. As well, the procedure should minimize the possibility of entrapment of slag and allow the root weld to be completed in one pass. Figure 4.8 illustrates the test set-up, fabrication and evaluation procedures. various ceramic weld backing configurations assessed and other test parameters are given in Table 4.6 and 4.7. The 6 mm plate came from the ancillary material of specimens P6-1 and P6-2.

All welding was performed using a constant excitation of 21 Volts and 4 mm diameter type E48018 shielded metal arc electrodes meeting the requirements of AWS A5.1-81 (AWS, 1981). Stop and start locations were marked on the edges of the plates. As shown in Fig. 4.8, each test specimen was cut into 4 or more smaller pieces, the sides of which were ground smooth and polished for visual inspection.

Ceramic weld backing bar 3, even at start and stop locations, gave a smooth weld root face and almost always complete root penetration. By inclining the test set-up at an angle  $\theta$  of 20° to the vertical as shown in Fig. 4.8 (b) the root pass was deposited most easily.

Some of the pieces were pried apart as shown in Fig. 4.8 (c) to evaluate the performance of the welds. The welds were very ductile. Failure occurred through the heat-affected zone initiating at the root face of the weld as shown in Fig. 4.8(c) but not before the plate had been bent through an argle of more than 15°.

# 4.3 Bending Tests

Plates of tests PG-2, P12 and P16 all failed when cracks developed through the thickness of the plate at the toe of the weld in the mid-length of a long side, even though ceramic backing bars with a rounded nose had been used to eliminate the possibility of copper contamination and to provide a favourable weld profile. It appeared that the tensile and flexural straining at this location combined with the strain concentration due to the change in geometry

exceeded the considerable ductility of the heat-affected zone next to the toe of the weld as established in the weld tests. Of the three small scale restrained bending tests that were performed on specimens fabricated using the 12 mm plate steel, two simulated the conditions of edge fixity in the plate tests. Figure 4.9 shows these latter test specimens in the restraining frame positioned for welding. After welding, the run-off tabs were removed. The dimensions and instrumentation of the three test specimens are given in Fig. 4.10. Figure 4.11 shows a longitudinal section. The long direction of the specimens were both parallel to and perpendicular to the rolling direction of the parent plate with the latter modelling the condition on the long side of the plate tests.

Figures 4.11(a) and 4.11(b) show how specimens 1 and 2, and specimen 3, respectively, were loaded. For specimen 3 it is noted that no weld exists on the upper side of the specimen at the point where the bending and tensile strains are a maximum as is the case for specimens 1 and 2. Figure 4.12 is an isometric view of the restraining frame for testing specimen 3. The restraining frames were quite rigid relative to the test specimens and therefore provided rotational and translational restraint at the ends. All welding was done with type E48018 shielded metal arc electrodes as was the case for the root pass in plate tests P6-2, P12 and P16.

Load-displacement curves for the three tests are given in Fig. 4.13. The maximum load of 63.3 kN for specimen 1 with the rolling direction parallel to the span was 11% greater than that for specimen 2 with the rolling direction transverse to the span. The curves are otherwise similar. Longitudinal sections of specimens 1 and 2 at failure, given in Fig.'s 4.14 and 4.15, show that failure initiated with cracks starting at the weld toe and progressed through the thickness. For specimen 1 the crack deflected longitudinally in a manner similar to lamellar tearing.

Specimen 3 attained a maximum load of 121 kN, about twice that of specimens 1 and 2 and ruptured on the tension side underneath the load point as shown in Fig. 4.16. mean strains of gauges mounted on the convex side of beam 3, immediately adjacent to the support where the maximum tensile strains occurred, at loads corresponding to the failure of specimens 1 and 2, was about 5.4% as compared to a fracture strain of the plates of about 31% on a 150 mm gauge length. Fracture of specimens 1 and 2 is therefore considered to have occurred as the result of pre-necking cracking in the heat-affected zone due to a loss in relative ductility and exacerbated by the strain concentration due to a change in cross section. Lay (1982) relates the drop in ductility there to the reduction in the amount of ductile ferrite present because the relatively rapid cooling rate after welding increases the perlite percentage. Matic and Jolles

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(1987) reported a critical fracture strain of 7% at one location in the heat-affected zone of an HY-100 steel.

Table 4.1 Tensile Coupon Identification

	Coupon d	irection
Parent plate	Plate rollin Transverse	g direction Parallel
P6-1	6T1-1 <sup>2</sup> 6T2-1 6T3-1 6T4-1 <sup>2</sup>	6P6-1 1 6P7-1 2 6P8-1 6P9-1 1 6P10-1 1
P6-2	6T1-2 1 6T2-2 6T3-2 2 6T4-2 1	6P6-2 6P8-2 <sup>2</sup> 6P10-2
P12	12T1 <sup>2</sup> 12T2 12T3 12T4 <sup>2</sup> 12T5	12P6 <sup>1</sup> 12P7 <sup>2</sup> 12P8 12P9 <sup>1</sup> 12P10 <sup>1</sup>
P16	16T1 <sup>2</sup> 16T2 16T3 16T4 <sup>2</sup> 16T5	16P6 <sup>1</sup> 16P7 <sup>2</sup> 16P8 16P9 <sup>1</sup> 16P10 <sup>1</sup>

Modes: 1 Poisson's ratio determined using Showa type N11-FA-5-120-11 strain gauges.

Poisson's ratio determined using Micro-Measurements type EP-08-250BG-120 strain gauges.

Tensile Coupon Test Results For Specimen P6-1 Steel Plate Table 4.2

parameter	σ <sub>y</sub> (MPa)	E (MPa)	ε <sub>γ</sub> (με)	ε <sub>st</sub> (με)	ςn (πε)	σ <sub>u</sub> (MPa)	ξξ (με)	>
6T1-1	-	1	;	12900	193000	485.2	306000	0.294
6T2-1	295.0	202700	1460	17600	194000	478.5	29100	ı
<b>6T3-1</b> 2	296.2	201300	1440	13400	195000	481.2	305000	ţ
6T4-1	296.2	208800	1420	16400	192000	467.6	292000	0.282
6T5-1 2	293.0	205700	1420	13200	182000	473.6	279000	1
Mean 2	294.4	204600	1440	14700	191000	477.2	295000	0.288
Coefficient of variation 0.	0.0093	0.0141	0.0116	0.1462	0.0246	0.0128	0.0340	0.0208
6P6-1	293.1	211500	1390	15800	196000	477.8	296000	0.302
6F7-1 2	288.9	210000	1380	16400	193000	467.8	293000	0.290
658-1	0.062	200300	1450	14200	189000	477.7	296000	!
6P9-1	6.682	200000	1450	11800	191600	473.3	291000	0.288
6P10-1 2	267.9	207200	1290	14760	192000	175.2	258006	0.281
Mean	286.0	205800	1390	33600	192000	474.4	255000	0.290
Coefficient of variation	0.0320	0.0234	0.0422	0.1093	0.0121	0.0077	0.0084	0.0261

Tensile Coupon Test Results For Specimen P6-2 Steel Plate Table 4.3

Coupon number or statistical parameter	б <sub>у</sub> (мра)	E (MPa)	(3π)	ε <sub>st</sub> (με)	ε <sub>υ</sub>	G <sub>u</sub> (MPa)	ε <sub>τ</sub> (με)	>
6T1-2	286.3	208300	1370	21800	227000	453.1	313000	0.287
6T2-2	ı	209300	1	ı	1	i	ι	0.279
6T3-2	295.2	209600	1410	23700	205000	455.9	320000	ı
6T4-2	296.1	207300	1430	23400	213000	451.9	313000	0.281
Mean	292.5	208600	1400	23000	215000	453.4	315000	0.282
Coefficient of variation	0.0151	0.0043	0.0214	0.0214	0.0423	0.9037	0.0105	0.0121
6P6-2	296.1	207000	1430	23600	244000	453.2	347000	0.282
628-2	293.2	208100	1410	22300	253000	453.5	333000	0.278
6P10-2	386.5	208200	1380	23700	233000	453.6	347000	0.280
Mean	291.9	207800	1410	23200	243000	453,3	342000	0.280
Coefficient of variation	0.0188	0.0026	0.0148	6.0275	0.0337	0.0004	0.0193	0.0058

Tensile Coupon Test Results For Specimen P12 Steel Plate Table 4.4

12T1 315.7 20 12T2 316.0 20 12T3 317.0 20 12T4 325.1 20 12T5 320.0 20 Mean 318.9 20 Coefficient 0.0105 0.12P5 317.4 20 12P5 317.4 20	26. 205 2048 208600 205800	1560 1560 1550 1540	17400 17100 17000 19600 16600	193000 184000 193000 189000	520.8	253000	0.273
316.0 317.0 325.1 320.0 318.9 0.0105 317.4	205 2048 208600 205800	1550 1550 1550	17100 17000 19600 16600	184000 193000 189000 193000	524.9	258000	
317.0 325.1 320.0 318.9 0.0105 317.4	2048 208600 205800	1560 1560 1550 1540	17000 19600 16600	193000 189000 193000	530.7		1
325.1 320.0 318.9 0.0105 317.4 324.5	208600 205800 206100	1560 1550 <u>1540</u>	19600	189000	•	269000	,
320.0 318.9 0.0105 317.4 324.5	205800 <u>206100</u>	1550 1 <u>540</u>	16600	193000	513.0	269000	0.276
317.4	206100	1540	17500		519.6	267000	1
0.0105 317.4 324.5				190000	521.8	263000	0.274
317.4	0.0083	0.0088	0.0605	0.0187	0.0112	0.0105	0.0055
324.5	206100	1540	27306	186000	528.9	306000	0.283
	210000	1550	26000	196000	519.7	299000	0.278
12F8 307.0 20	204600	1500	16800	179000	524.3	306090	1
12P9 326.0 20	203700	1600	20000	200000	526.1	300000	0.281
12P10 321.7 20	204100	1580	19200	190000	52.5	297000	0.284
Mean S.S.S.	205700	1550	20700	190000	525.7	302000	0.282
Coefficient 0.0216 0.	0.0112	0.0221	0.1701	0.0388	0.0067	6.0123	0.6083

curvature exists at location A at that load. The average strain is always tensile indicative of membrane action. It increases relatively slowly up to a nondimensional load of about 9 when the average model strain equals the yield strain, 1400  $\mu\epsilon$ . Thereafter the average model strains increase rapidly up to a nondimensional load of about 15 where the strains increase less rapidly as discussed subsequently. The model strains increase slowly with load until yielding occurs, then more rapidly. The moderate increases in load over the greater portion of the yield plateau is due to the change in geometry of the membrane as it elongates due to yielding, making it stronger even though the maximum membrane stress has not changed.

The measured strains up to a load of about 6 agree reasonably well with the model strains. The rapid increase of the measured strain on the loaded side beyond this load is indicative of extensive yielding. The measured and model strains within the yield plateau of tensile straining indicate that the membrane is carrying the yield stress. The measured strains reach this value at a load of,  $q/q_C$ , of about 8 and the model strains at a load of about 10.

At location B, 81 mm from the fixed edge (F.g. 7.17), in Fig. 7.21 both the model and measured strains show a substantial difference between the strains on the loaded side and free side at a nondimensional load of 1 indicating substantial bending. The free side strains at this load are

only slightly in compression and therefore significant tension exists. At a load of about 4 the model strains on the two sides are equal indicating that no bending strains exist and that the section is in tension as the strains are positive. The same tendency for the bending strains to reduce is seen in the measured strains when both strains reach the yield value in tension. Beyond a load of 4 the free side model strains exceed those on the loaded side. This difference is attributed to the strain gradient due to the curvature of the plate. At higher loads,  $q/q_C = 16$ , the rate of increasing strain decreases with load.

The measured strains at low loads closely follow the model strains and when yielding occurs increase rapidly although not at the same load as the model strains.

Location C (Fig. 7.17) is 131 mm from the edge. At this location, as shown in Fig. 7.20, essentially no negative (compressive) strains exist on the free side indicating that on the average the section is in tension. Whatever bending strains exist in the model disappear at a nondimensional load of about 2.5. Thereafter the free side strains exceed the loaded side strains substantially as is consistent with significant membrane curvature. At low loads the measured strains are in good agreement with the model strains and although they exhibit the same general behaviour as the model strains the break in the curves reflect that yielding occurs at different loads. The december destrains, after the load corresponding to zero curvature at this section is reached,

exceed the loaded side strains with the difference representing the membrane curvature.

Locations D, E, F and G (Fig. 7.17), with the latter at the mid-width, are all remote from the edge. The corresponding strains are shown on Figs. 7.23 through 7.26. The free side strains always exceed the loaded side strains consistent with the membrane curvature. In no case is there a cross-over of strains with first one side exhibiting the maximum strain and then the other. The average strain is always a tensile strain. Again, these strains first increase slowly and then more rapidly with yielding. At both locations F and G compressive strains exist on the loaded side at low loads. These are consistent with initial bending of the plate before significant membrane action has developed.

# 7.3.1.1 Summary

Several distinct trends or patterns can be determined from the strain variation with load. Near the edge (Locations X, A, B and C), very distinct bending strains exist at low loads. At location A specifically, these strains increase up to the plastic collapse load ( $q/q_C$  = 1.0). The bending strains gradually reduce and become zero as the point of zero moment migrates towards the edge from point C at a nondimensional load of 2.5 to point B at a load of 4 and to point A at a load of 18.

The model strains increase slowly with load until yielding occurs, then more rapidly and finally, more slowly

again as discussed subsequently. The maximum measured strains did not reach the strain hardening region.

At or near mid-width compressive strains existed at low loads on the loaded surface indicating initial bending of the plates here.

The measured strains are in reasonable agreement with the finite element strains up to a nondimensional load of about 6. At the fixed edge on the free side (location X) compressive strains of over 30000  $\mu\epsilon$  were predicted and observed at the maximum load (results from the finite element model, Fig. 6.9, give tensile strains on the opposite face at this load of 170000  $\mu\epsilon$ ).

From the change in strains at the various locations with load the development of elastic and inelastic membrane behaviour can be deduced.

In the inelastic range, because of the discontinuous yielding of the actual plate, the model strains are more useful than the measured strains to examine the extension of yielding, i.e., how yielding progresses from one location to another.

On the loaded surface, tensile yielding progresses from the edge towards the mid-width with increased loading. Yielding occurs on this surface at location A at a load of  $q/q_C=1$  and at location G only when the load has reached a value of 13.3.

On the free surface, tensile yielding progresses in the opposite direction moving from the mid-width, location G, at a load of  $q/q_{\rm C}=2.9$  to location A at a load of 9.5.

Within the zone acting as a membrane, the strains are greater on the free side than on the loaded side as would be expected because of the curvature of the membrane of finite thickness. Furthermore the difference between the strains on the two sides increases with increasing load as the membrane curvature must increase. Table 7.1 lists for locations A to G, the model strains on the loaded and unloaded surfaces at  $\alpha$ load,  $q/q_C = 17.7$  and their differences. It is seen that the differences vary by only about 8% from location B to location G. At location A, where the point of zero curvature exists, the strains are about the same. With increasing load the plate, originally deformed in flexure, approaches a portion of a circular arc as indicated by the radii of curvature computed from the strain differences on the two surfaces. the location of the point of zero curvature the radius is of course infinite.

maximum near the edge. And reduces gradually towards the midwidth. This is consistent with the fact that the portion of the plate near mid-width has to strain less as a membrane to attain a circular shape than the plate at the edge which initially was bent in the opposite direction. Thus, overall, as load is increased the membrane tends to approach a

circular shape, first near the mid-width and then progressing towards the edge.

The decrease in the rate of straining with increasing load for the model at high loads at Iccation A and to a lesser extent at locations B and C can be explained by examining a free body of a unit length of the plate extending from the point of zero curvature (point A when  $q/q_C = 17.7$ ) to its mid-width as shown in Fig. 7.27 where the corresponding strain diagram is also shown. For the model with an aspect ratio of zero, a strip of unit length must be in equilibrium without forces in the longitudinal direction. Therefore, the strains are consistent with the forces required to hold the free body, subject to uniform pressure, in equilibrium. At higher loads, with increasing pressure, the rate of straining at the edges is less than at mid-width because, at the edge, the increase in the horizontal component is less than at the mid-width and the increase in the vertical component is obtained by the changing angle of inclination, while at the mid-width the increase in the force can only be obtained with increased straining.

# 7.3.2 Test P12

The variation of strain with load for test P12 at locations X, A, B, C, D, E, F and G, as shown on Fig. 7.18, are given on Figs. 7.28 through 7.35, respectively. The behaviour, in general, is similar to that of plate P6-2 with quantitative differences due to the increased plate thickness.

At location X at the toe of the weld where the plate is severely bent, the strains on the free side reach large compressive values. Significant bending strains exist for considerable distance near the edges and reach a maximum value at location B at a nondimensional load of 1.0. The bending strains at location A, closer to the edge, continue to increase, as does the average (membrane) strain, throughout the test. Away from the edge the bending strains diminish with load and eventually become zero for the model strains at a load of less than 2 at location C and at a load of 4 at location B. Thus, as for plate P6-2 the point of zero curvature migrates towards the edge with increasing load, moving from point C, for the model, at a load of 2 to point B at a load of 4 but never reaching point A.

In the central portion of the plate, away from the edges, the difference in strains between the free and loaded sides due to curvature is again evident. At lower loads, however, because of bending of the thicker plate, compressive strains exist on the loaded side near mid-width. The average strain is always tensile and increases with increasing load as would be expected in this region for membrane behaviour.

The maximum tensile strains reach about 26000 µE for model 2 and 21000 µE for the test at location A. Therefore, the plate was loaded beyond the yield plateau region and strain hardening had begun. The reduction in measured strain on the loaded side at location A just prior to failure may be

indicative of the rupture of the plate at the toe of the weld.

The measured and finite element strains are generally in remarkably good agreement in the elastic region with some divergences in the yield plateau region as would be expected because of the discontinuous yielding of the test specimen. The strain variation in plate P12 is consistent with significant bending at the edges, a point of zero curvature that migrates towards the edges as the load is increased, and membrane behaviour extending over 90% of the width of the plate at maximum load.

As for specimen P6-2, from the change in model strains at the various locations with load, the development of the membrane behaviour can be deduced. On the loaded surface, yielding in tension progresses from location A near the edges at a load of  $q/q_C=1.0$  to location B at a load of  $q/q_C=4.0$  but never reaches location C or those beyond. On the free surface, yielding in tension progresses from location G at a load of  $q/q_C=1.0$  to location B at a load of  $q/q_C$  of 4.0 but never reaches location A where the free surface is always in compression.

Within the zone acting as a membrane the strains are greater on the free side than on the loaded side and the a fference between them increases with increasing load as would be expected. Table 7.2 lists for locations A to G the model strains on the loaded and unloaded (free) surfaces at a load  $q/q_C$  = 6.6 and the differences between these strains

(the specimen failed at about this load). As for model 1, it is seen that the differences vary only by 3% in the central region from location C to G. At location A the plate is still bent in the opposite direction whereas for model 1 at the maximum load no curvature existed at this point.

Location B exhibits a relatively large curvature as would be expected in this transition zone between the plate acting as a pure membrane and at the edges where bending in the opposite direction exists. Again the plate is tending of form a circular arc which develops from the mid-width towards the edges where bending exists and where as noted previously the tensile strains are the highest.

The strain variation through the thickness at any location can be considered to consist of two parts: that due to the curvature of the plate acting as an inelastic membrane subject to fluid pressure and, because of its stiffness, that due to bending. At any location where the radius of curvature approaches the minimum value the bending strains are relatively small. At location B where the radius of curvature is considerably larger than in the central near-circular portion the plate in fact is partially straightened-out because of bending in the reverse direction and, of course, at A the bending is very pronounced. Thus, a point of zero curvature, which in this case would lie between locations X and A (and which for model 1 was at location A) represents the case where curvature due to membrane action is balanced by bending curvature in the opposite direction. The

total strain at this location must be such that the force developed in tension here is sufficient to keep the portion of the plate between the points of zero curvature near the edges in equilibrium.

#### 7.3.3 Test P6-1

The variation of measured strain with load for test F6-1 at locations X,A,B,C,D,E,F and G, as shown on Fig. 7.36, are given on Figs. 7.38 through 5, respectively. behave or resembles closely t. . . . plate P6-2 which has the same nominal thickness. Near the edge (locations X, A and B in particular) very distinct bending strains exist at low loads. At location A, these strains increase up to a load,  $\mathrm{q}/\mathrm{q}_\mathrm{C}$  of about 1. The net bending strains gradually reduce and become zero as the point of zero curvature migrates towards the edges from point C at a load of about 1.8 to point B at a load of 3 and to point A, apparently approached at a load of about 7. Keeping in mind that the plotted strains are measured strains and that yielding is discontinuous and therefore that strains beyond yielding are more difficult to assess, on the loaded surface yielding is seen to progress as in the previous tests, from location A at a load of about 1 to location F at a load of about 10. med surement was made at location G. On the free surface the progress we yielding is again in the opposite direction moving from location G at a load of about 3.5 to location A at a load of 8. In both cases at the intermediate locations, the yielding progresses rapidly across the half width. As

before, the strains, within the zone acting as a membrane, are greater on the free side than on the loaded side though a quantitative assessment of these differences is not appropriate because of the discontinuous yielding phenomenon. In so far as the strain may be assessed qualitatively it appears that the membrane action spreads from the mid-width to the edges and, at the maximum load, inelastic membrane behaviour extends over 90% of the width of the plate.

### 7.3.4 Test P16

The variation of strain with load for test P16 at locations X,A,B,C,D,E,F and G, as shown on Fig. 7.37, are given on Figs. 7.46 through 7.53, respectively. Except for quantitative differences resulting due to the increased plate thickness the behaviour, in general, is similar to that of plates P6-1, P6-2 and P12.

The strain variation in plate P16 is consistent with significant bending at the edges, a point of zero curvature that migrates towards the edges as the load is increased and inelastic membrane behaviour extending over 75% of the width of the plate at maximum load.

As would be expected, bending is much more significant than for the thinner plates. The point of zero curvature moves from a point between locations D and C to location C at a load of 4.2 to location B at a load greater than 5 and never arrives at location A. On the loaded surface yielding occurs at location A at a nondimensional load,  $q/q_C = 0.6$  and moves to location E at a load of about 5.5 and never gets to

locations F or G. On the free side, yielding occurs at midwidth, location G, at a load of about 1.1 and progresses to location B at a load of about 5.2 but never reaches A. In the central portion where inelastic membrane action occurs, although at some locations not through the entire thickness, the strains on the free surface are greater than those on the loaded surface. The variation in the difference in strains between the two surfaces at different locations suggest that significant bending still exists, all the while recognizing the discontinuous yielding process.

## 7.4 Strains Across The Width At Mid-Length

In this section the strain distribution across the half width at mid-length at four different load levels is presented. Some information presented previously is reiterated.

The measured strain distribution is plotted for the four tests at four different load levels in Figs. 5.20 to 5.35. Strains, as deduced using the ADINA finite element wodel, are plotted in the figures for tests P6-2 and P12. In examining these strains, it is again emphasized that measured strains in the yield plateau because of discontinuous yielding may vary between the yield strain and the strain hardening strain and still represent the same stress level. Because the ADINA model requires a finite slope in the region between yielding and strain hardening, model strains vary continuously.

In Fig. 5.21 for test P6-2 at a nondimensional load of about 1.0 the behaviour is essentially elastic with bending strains at the fixed edges and a point of zero curvature at about 190 mm from the toe of the weld. The model strains are in good agreement with the measured strains. Close scrutiny reveals that the average strain is tensile, indicative of a limited amount of membrane behaviour. Figure 5.22 for test P12 at the same load level exhibits very similar behaviour with bending at the edge and a point of zero curvature again 190 mm from the toe of the weld. The experimental results are in good agreement with the model values and show on the average a small tensile strain. Some inelastic bending, as would be expected, occurs near the edge of the plate.

Figures 5.25 and 5.26 compare the ADINA mode! results to the measured strains at loads approaching failure,  $q/q_C = 17.9$  for test P6-2 and  $q/q_C = 6.6$  for test P1.2. The strains in the former case are of course considerably larger.

From Fig. 5.25, both the measured and model strains indicate significant bending at the edge of the plate. ADINA predicts the point of zero curvature to be at 40 mm from the edge. The point of zero curvature has therefore migrated substantially towards the edge from its position of 190 mm at a normalized load of 1.0. Beyond the point of zero curvature where the plate acts is a membrane both the model and measured strains are on the tensile yield plateau. The model strains are higher near the hinge and reduce towards the midwidth. As discussed previously, this variation is attributed

to the fact that significant bending is required at the edges to develop membrane action there but not at the mid-width.

For test P12 bending again is found to exist at the edge of the plate and the point of zero curvature at  $q/q_{\rm C}$  = 6.6 is only about 80 mm from the edge as compared to 190 mm at  $q/q_{\rm C}$  = 1.0. Both the test results and the ADINA values show that the vast central portion of the plate is acting as a membrane with the strains on the free surface in the inelastic region and strains on the loaded surface in the elastic region. The difference in strains on the two surfaces is a measure of the membrane curvature.

Figures 5.29 and 5.30 for test P6-2 at  $q/q_C=5.36$  and test P12 at  $q/q_C=3.03$  respectively were drawn for the case when the average membrane strain at the point of zero curvature was about equal to the yield strain. It is noted that points of zero curvature at 70 mm and 100 mm from the edge for tests P6-2 and P12, respectively, are intermediate to their previous positions for higher and lower load levels. In the elastic region the test results (associated with the loaded surface) are in good agreement with the ADINA model strains. On the free surface the model strains are just within the inelastic range with measured strains above and below these values.

Figures 5.33 and 5.34, for tests P6-2 at  $q/q_C=10.9$  and test P12 at  $q/q_C=5.1$  respectively, both represent the case when significant inelastic membrane action exists. Bending still exists at the edges and the points of zero

higher and lower loads, as should be the case. With some scatter, the experimental values are in reasonable agreement with the model results. In the central portion inelastic straining exists. For tests P6-2 this extends through the thickness of the plate except for a limited elastic zone on the loaded surface near the mid-width. For test specimen P12, Fig. 5.34, the free surface has inelastic strains while the loaded surface, where the model and measured strains are in good agreement, remains elastic.

The average (membrane) strains of the 6 mm plate are greater than those of the 12 mm plate. But on the other hand because of its greater thickness, the strain variation through the thickness of the 12 mm plate is greater than that for the 6 mm plate.

For both specimens, bending strains at the edge increase with increasing bads. The point of zero curvature migrates towards the edge with increasing loads. Tensile membrane strains, tending to be higher near the hinge than at mid-width are first relatively small, reach the yield strain and then progress into inelastic strain hardening. The free surface membrane strains are greater than the loaded surface membrane strains to accommodate the curvature of the membrane as discussed previously.

#### 7.5 Membrane Behaviour

Figures 5.20 to 5.35 give strains across the width at four different load levels for the four tests. The strains

obtained from the ADINA models are shown on figures drawn for tests P6-2 and P12. At lower load levels (see Figs. 5.20 to 5.23) the strains away from the edges and between points of zero curvature on both the loaded and free surfaces are in the elastic range of behaviour. Figures 5.24 to 5.35 that are drawn for higher load worels show that the points of zero curvature move toward the edges as the load is increased. Between points of zero curvature and with higher load levels the strains on both surfaces of the plate become tensile, increase in magnitude, and eventually become inelastic. The free surface strains increase at a higher rate than do the strains on the loaded surface as the curvature of the membrane increases with increasing load.

A plate acting as an inelastic membrane and subject to fluid pressure loading, has a mean tensile strain that, for any given load, is the same at all locations across its width. Hence, the average tensile stress and curvature of such a plate is constant from edge to edge and the deformed shape is circular. The load-deflection behaviour of a plate subjected to fluid pressure loading and acting as an inelastic membrane from edge to edge is given by Ratzlaff and Kennedy (1985, 1986), hereafter referred to as the Ratzlaff-Kennedy model. Curves Mi, N and N' in Figs. 7.1 to 7.4 are derived from their analysis. In these figures the test results follow curve N' closely as discussed previously in Section 7.2 and indicate that, at the higher loads, the plates act predominantly as inelastic membranes. The basic

reason for the slight differences between the load-deflection behaviour of the test plates and that given by Ratclaff-Kennedy model is that the test plates with significant thickness do not form line hinges at the edge (i.e. hinges of no lateral extent). As discussed in Section 7.3 the plates first of all, at very small deflections, behave as flexural members. With increasing deflections elastic membrane action asserts itself and eventually becomes inelastic in nature. The initial deflected shape is dissimilar to the final deflected shape which gradually approaches that of a circular arc with divergences from this shape representing bending at the edges. More straining is required to approach the circular shape near the edges where the initial curvature is in the opposite direction. The result, therefore, is that the final membrane shape approaches closely that of a circle over the central portion where inelastic straining occurs, but at the so-called points of zero curvature, which will be subject to significant straining and bending to maintain equilibrium, the resulting curvature is zero. From the edges to the point of zero curvature the plate acts primarily flexurally. It follows therefore that while in most of this central portion the curvature is sensibly constant, within a short distance of the point of zero curvature the curvature changes rapidly to zero. The deflected membrane shape is therefore not exactly circular.

Figures 5.24 for test P6-1 and 5.25 for test P6-2 show that the measured and model strains across the width from a

point within about 40 mm of the edge are in the inelastic region. The model strains show a variation from a maximum value near the point of zero curvature to the mid-width, as discussed previously, and show a strain gradient consistent with a membrane approaching constant curvature in this region. All of these strains therefore reflect stresses in the yield plateau. It follows that within this strain range any increase in load carrying ability of the membrane must arise simply trom increased curvature, increased deflections, and increased inclination of the member near its edges.

Figure 5.26 gives the measured and model strains for test P12 and Fig. 5.27 the measured strains for test P16, at their failure loads. For both tests the free surface strains are inelastic while the loaded surface strains tend to lie in the elastic region. Therefore for these thicker plates full inelastic membrane behaviour had not developed at failure. The ADINA strains for test P12 show a constant difference between the free and loaded surface strains over most of the width where membrane action is occurring indicating that the deflected shape of the plate is circular.

## 7.6 Longitudinal Strains

For a plate acting as a membrane in a state of plane strain, the longitudinal strains at any location would be expected to be zero. Figures 5.36 through 5.39 that give measured longitudinal strains across the width at mid-length for tests P6-1, P6-2, P12, and P16 respectively, at four different load levels, show that these strains approach zero

along the long sides. However, the strains, at loads near failure, approach yield levels (about 1500  $\mu\epsilon$ ) near mid-width for the two thinner plates and about one-third of the yield strain values for the two thicker plates. The greater longitudinal strains at mid-width for the thinner plates are attributed to the relatively larger deflections of the thinner plates as compared to the thicker plates as shown in Fig. 5.19 for plates P6-2 and P16.

The near-zero longitudinal strains along the long edges for all plates is simply due to the restraint provided by the stiff reaction plate to which the test plates were welded. The condition of straining therefore deviates from that approaching plane strain along the long sides to a triaxial strain condition that increased to a maximum at mid-width.

### 7.7 Behaviour At Edges

Sections through the thickness and across the width at mid-length of the plate specimens taken after failure indicated that considerable bending had occurred at edge locations. The geometry at the edges was similar to that obtained from the ADINA analysis as shown in Figs. 6.7 and 6.8 for Model 1 and Model 2 (specimens P6-2 and P12), respectively. A considerable amount of inelastic deformation occurred within the region along the edges as was evident from small samples taken there that retained their deformed geometry after being isolated from the remainder of the specimen.

As discussed in Chapter 5, failure for all four specimens was observed to initiate within the HAZ near the toe of the weld and along the edges at mid-length. After initiation, separation or rupture of the steel plate progressed through the plate thickness to the unloaded surface as shown in Figs. 5.9 and 5.10. In some cases, rupture was observed to have began but not propagated through the plate thickness at locations along the long edge opposite to the actual failure.

# 7.8 Failure Loads And Derlected Shapes

Ratzlaff and Kennedy (1986) proposed relationships for a membrane subject to fluid pressure, relating the membrane deflection, the maximum membrane stress and strain, and the membrane edge inclination to the fluid pressure and the material properties as follows.

[7.1] 
$$\frac{\sin\theta}{\theta} = \frac{\pi}{180(1+\epsilon_s)}$$

[7.2] 
$$q = \frac{\sigma_s h \sin \theta}{(L/2 - d \sin \theta) (1 - v_s^2)}$$

[7.3] 
$$Wm = (L/2 - d \sin\theta) \frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}}$$

where  $d \sin\theta$  is the distance from the edge of the plate to the point of zero curvature. These equations are derived in Appendix A. Also derived in Appendix A is the relationship relating the maximum membrane stress, the fluid pressure, and

the membrane edge inclination, based on the material properties and also based on full plastification of the cross section in tension and bending at the support. This relationship is

$$[7.4] \qquad \left[\frac{\sigma_{s}}{\sigma_{u}}\right] \left[\frac{4 \sin^{2}\theta \left(1+\epsilon_{p}\right) \left(1-\nu_{s}^{2}\right)}{3 \cos\theta \left(\epsilon_{u}-\epsilon_{p}\right) \left(1-\nu_{s}^{2}\right)}\right] + \left[\frac{\sigma_{s}}{\sigma_{u}}\right]^{2} \frac{\left(1-\nu_{p}^{2}\right)^{2}}{\left(1-\nu_{s}^{2}\right)^{2}} \cos^{2}\theta \qquad 1$$

In the latter derivation (See Appendix A) it is assumed that the strain distribution at the edge is linear with the maximum tensile strain on the concave side set equal to  $\sigma_{\rm u}$ .

From the test (measured) geometry at failure ( $W_m$  and d  $\sin\theta$ ) and the stress-strain curves for the tensile coupons, the loads corresponding to failure were predicted from the Ratzlaff-Kennedy equations [7.3], [7.1], and [7.2] (in this order) and are given in Table 7.3 together with the test failure loads. The test/predicted values are in excellent agreement ranging from 0.965 to 1.002. The maximum strain on the loaded side at the very edge of the plate,  $\epsilon_u$ , predicted from [7.4] is also given in the table.

At the test failure loads, the maximum deflection and other quantities were determined from the ADINA models 1 and 2 for tests P6-2 and P12, respectively, as given in Table 7.4. Equations [7.1] through [7.4] can again be used together with the stress-strain curves as modelled by ADINA and the ADINA geometry, to predict the failure loads (This

parallels the calculations given in Table 7.3 based on the measured geometry). In making these predictions the test values for the inelastic strain ratio were used. These calculations lead to test/predicted ratios of 1.022 and 0.954, respectively, as shown in Table 7.4. Had the inelastic strain ratio been taken equal to 0.50 (no volume change as used in the ADINA models) the predicted failure loads from equations [7.1] to [7.3] would have increased substantially resulting in test/predicted ratios of 0.86 and 0.78, respectively. These data therefore establish that the simple model for the circular deflected shape between points of the curvature is consistent with the loads required to deform the membrane into that shape. Implicit in this is the fact that inelastic strain ratio is significantly different from 0.50.

In Figs. 7.54 to 7.57 are plotted deflected shapes of the plate at mid-length across the half width for the failure load. Each figure has 3 curves: (i) the deflected (test) shape as measured using the surface level indicator, (ii) the shape as deduced from the Ratzlaff-Kennedy equations based on measured values of  $W_{\rm m}$ , the maximum deflection, and of d  $\sin\theta$ , the distance measured from the edge to the point of zero curvature (i.e.,the geometry at failure) and the stress-strain curves, by matching membrane deflections at mid-width to the test deflections, and (iii) the shape as deduced from the Ratzlaff-Kennedy equations based on the test (measured) failure loads and the stress-strain curves (matching failure

loads). The data for these latter calculations are summarized in Table 7.5. In this table, using the stress-strain curves to determine coincident values of  $\sigma_{\rm S}$ ,  $\epsilon_{\rm S}$ , and  $v_{\rm S}$ , as determined from the stress-strain data, values of  $v_{\rm S}$ , as determined from the stress-strain data, values of  $v_{\rm S}$ , and  $v_{\rm S}$ , are computed iteratively from [7.1] through [7.4] for the test failure loads. The maximum strains at the edge based on the ADINA models and the ratio of these strains to those predicted using equations [7.1] through [7.4], as discussed subsequently, are also given in Table 7.5.

Curves (ii), heated on measured values of the maximum deflection of the membrane portion of the plate ( $W_m$ ) at midwidth and the distance to the point of zero curvature ( $d \sin \theta$ ) have been plotted through these two points and extended backward towards the edge based on the Ratzlaff-Kennedy relationships. In the membrane region these curves and the test curves are in reasonable agreement. The difference between the two curves arises from the edge behaviour and the fact that the membrane is not circular over the entire range between points of contraflexure.

Curves (iii) have been computed from the RatzlaffKennedy relationships for the test failure load and have been
drawn with zero deflection at the edge. Again the behaviour
of the plate at the edge and the non-circular shape between
the points of contraflexure accounts for the difference in
the deflections between curve (iii) and the test curve. The

point of zero curvature is closer to the edge than for the measured (test) curve.

It is concluded that both sets of predicted curves, one matched in terms of deflections and the other in terms of failure load, are in substantial agreement with the observed (test) deflections.

#### 7.9 Failure Mechanisms

Four failure mechanisms have been suggested by various researchers with the first three of these based on previous work as discussed in Section 2.4.

Ratzlaff and Kennedy (1986) put forward the hypothesis of a maximum tensile strain criterion which was used in the ADINA analysis here as an upper bound. Failure is postulated to occur when the tensile strain at the edge (on the loaded surface in these tests) due to combined tension and bending reaches the true fracture strain.

Ratzlaff and Kennedy, as discussed in Section 2.3, also postulated a maximum shear stress criterion, based on failure observations, that failure occurs at the edge when the average shear stress reaches  $0.577\sigma_{\rm u}$ . This criterion applied to the one test they did that failed in shear gave a test/predicted ratio of 0.97.

Kennedy and Cheng (1992) proposed the criterion that failure occurs when, under a state of plane strain, by extending the von Mises-Huber-Hencky yield criterion to ultimate the relationship

$$[7.5] 0.75 \left(\frac{P}{\sigma_{0}A}\right)^{-1} + 3 \left(\frac{V}{\sigma_{0}A}\right)^{-1} = 1.0 ,$$

where P and V are the tension and shear acting on a unit length of the plate cross-section through its thickness at the toe of the weld, A is the cross-sectional area (1xh) and  $\sigma_{\rm u}$  is the uniaxial ultimate tensile stress. As the equation shows, no stresses due to bending are considered. Because of the significant bending at the edge where failure occurred in these tests this criterion is not further considered.

The fourth criterion developed as a result of these tests, applicable only when a heat affected zone exists in an area of large tensile strains, such as existed here at the edge of the plate, is that failure occurs when the maximum strain reaches that which causes rupture in the HAZ. data obtained in these tests are limited but the results of the two tests for which ADINA analyses were performed (see Section 6.2) suggest that failure occurs when the true strain, under conditions of plane strain that exist at the edge of the plate, reaches the range of 88000  $\mu\epsilon$  to 170000 µa. Even the least of these values exceeds that proposed by Matic and Jolles (1987) very considerably. least value even exceeds that used by Bond (1993) by 1.26 times. For this criterion of maximum strain in the HAZ the ADINA strain of 170000  $\mu\epsilon$  at the failure load in test P6-2 was used to predict the failure load for both tests P6-2 and P6-1 because the stress-strain curves for the two 6 mm plates are quite similar, as given in Table 7.6. Of course the

individual stress-strain data for the respective test plates were used. Similarly the ADINA strain of 88000  $\mu\epsilon$  at the toe of the weld for test P12 was also used for test P16, again because of the similarity of the stress-strain curves for these two steels (see Table 7.6). The Ratzlaff-Kennedy (1986) model was used to predict the failure load based on the maximum strain at the toe of the weld, taken to be 170000  $\mu\epsilon$  for the 6 mm plates and 88000  $\mu\epsilon$  for the 12 and 16 mm plates. The results of these calculations are summarized in Table 7.6 and discussed subsequently.

Table 7.7 summarizes the non-dimensional failure loads,  $q/q_C$ , for the four tests and the predicted failure loads based on the three critania considered to be valid as discussed previously. The smallest predicted failure loads are those due to the straining in the HAZ. These loads were used to determine the test/predicted ratios which vary from 0.69 to 0.80 indicating that the predicted failure loads even using the HAZ strain limits were too high. This is attributed to the fact that the strains at the toe of the weld as shown by the ADINA model (Figs. 6.5 and 6.6) are very much non-linear and a basic simplifying assumption of the Ratzlaff-Kennedy model is that the strains vary linearly. Non-linear straining means that less load can be carried for the same maximum strain. In particular, examining Table 7.6, based on linear straining it is seen that the maximum deflections are greater than those observed as given in Table 7.3 and those predicted by DINA as given in Table 7.4.

These tables also show that the distances from the edge to the point of zero curvature, as observed (Table 7.3) and calculated by ADINA (Table 7.4), are in good agreement but are much greater than when the maximum strain in the HAZ is used with the assumption that the strain distribution is linear (Table 7.6).

As discussed in Section 7.8 the failure loads in tables 7.3 and 7.4 were determined from measured or computed (ADINA) deflected shapes of the central portion that acted as a membrane and gave predicted failure loads in good correspondence with the test values (test/predicted ratios range from 0.954 to 1 022). The question still remains, however, what maximum trains does the Ratzlaff-Kennedy model predict based on the farlure load and how do these compare with the strains determined from ADINA? Table 7.5 summarizes the calculations for determining these strains and compares them to the ADINA strains. The ADINA strains range from 1.4to 1.8 times the linear strains, suggesting that, within the bounds of all the assumptions made, that the strain concentration factor at the toe of the weld was about 1.5 on the average. It is emphasized again that in this comparison the contributions from the various assumptions both in the ADINA program and in the Ratzlaff-Kennedy model (particularly the deformation and straining at the edge) contributed to this number. It is possible that the strain concentration factor may even be higher.

Taking the strain concentration factor to be 1.5 and using the maximum linear strain for the 6 mm plates as that determined by ADINA divided by this factor, i.e.  $170000~\mu\epsilon~/1.5~=~113000~\mu~$  and for the 12 mm and 16 mm plates,  $88000~\mu\epsilon~/1.5~=~58700~\mu\epsilon$ , the Ratzlaff-Kennedy model can be used to determine the failure loads. The resulting failure loads, given in Table 7.8, give a mean test/predicted value of 1.00 and a coefficient of variation of 0.09. The lowest of these values was determined for test P6-1 for which it is recalled a copper backing bar was used that resulted in contamination of the weld metal with copper in the area where fracture initiated.

In the four major tests conducted here it is pointed out that two factors contributed to the failure at loads less than expected based on the shear limit which otherwise would likely have governed. These are of course the reduced ductility in the HAZ and the strain concentration due to the abrupt change in cross-section at this location. Neither of these factors need be present in a real structure as discussed subsequently.

ADINA strain differences, model 1, q/qc = 17.7 rable 7.1

Location	Free surface strain (με)	Loaded surface strain $(\mu\epsilon)$	Difference in strain (με)	(mm) d
<b>o</b>	0099	2475	4125	1721
ĵz,	7303	3185	4116	1725
M	10439	6363	4076	1742
Д	16097	12086	4011	1770
υ	21433	17525	3908	1817
<b>M</b>	22193	18386	3807	1865
K	20845	20861	- 16	* *

\* .. \* Approximate point of zero curvature

9.9 = 2b/b ADINA strain differences, model 2, rable 7.2

Location	Free surface strain (με)	Loaded surface strain $(\mu\epsilon)$	Difference in strain $(\mu\epsilon)$	(mm) d
ŋ	6221	883	5338	2510
វែរ	6214	889	5325	2516
ធ	6214	905	5309	2524
А	6204	932	5272	2542
υ	6158	975	5183	2585
ф	4830	1229	3601	3721
<b>~</b>	- 1156	26761	- 27917	- 430

Failure loads based on Ratzlaff-Kennedy model Table 7.3

	Mea	sured	Predi	Predicted	(equations	[7.1]	to [	[7.4])		
H es t	Wm (mm)	d sinθ (mm)	Ө	(3Ħ) 83	cs (Mpa)	<b>8</b>	(311) n3	<pre>qpred. (kPa)</pre>	G <sub>test</sub> (kPa)	gtest g <sub>pred</sub> .
P6-1	60.8	44.3	14.5	10700	294.5	0.33	41000	1174	1152	0.981
P6-2	71.8	40.6	16.9	14700	. 292.5	0.33	55000	1407	1411	1.002
P12	39.5	84. 1.	10.5	5620	318.9	0.29	31100	യ ഗ ന	006T	0.965
9 1 9	37.1	O.40	10.2	5340	9.708	0.31	300 #	6 6 6 7	89 80 80	0.970

**G**test 1.022 0.954 qpred. q<sub>test</sub> (kPa) 1909 1411 qpred. (kPa) 1381 2001 [7.4]) 28000 64000 ε<sub>α</sub> (με) ţ [7.1]0.29 0.33 8 (equations os (Mpa) 294.0 320.8 14400 5450 ε**3** (με) Predicted 16.7 10.3 θ 170000 88090 ευ (με) d sine (mm) ADINA 34.3 93.1 va (mm) 71.9 38.1 ı ł Table 7.4 P6-2 P6-1 Test P12 P16

Failure loads based on Ratzlaff-Kennedy model and ADINA geometry

1.35

88000

64600

6100

10.9

41.0

45.1

0.32

321.0

1909

P12

년 년

88900

58200

5820

10.6

55.5

42.0

0.35

307.4

2383

P16

 $\epsilon_{u}$  (pred.) Eu (ADINA) 1.38 1.79 170000 170000 ADINA ε<sub>α</sub> (με) 123100 94900 ε<sub>υ</sub> (με) [7.4]) 1590¢ 11500 (3Ħ) to 15.0 17.6 [7.1] θ d sinθ (mm) (equations 18.5 19.5 Wn nn) 66.3 78.1 Predicted 0.33 0.34 8 Os (Mpa) 293.7 294.4 q<sub>test</sub> (kPa) 1152 1411 P6-1 P6-2 Test

on Ratzlaff-Kennedy with ADINA strains strains based and geometry, Comparison of maximum model using test load 7.5 Table

Failure loads based on Ratzlaff-Kennedy model and ADINA strains in HAZ at test failure load Table 7.6

Test (με)         εu (με)         ψm (με)         μm (με)         ψm (με)		ADINA		Predicted		(equations	[7.1]	] to [7.4])	.4])			
170000       23500       296.0       0.38       170000       95.4       15.7       21.3       1685         170000       25000       296.5       0.34       170000       98.4       17.0       22.0       1762         88000       10600       322.6       0.31       88000       60.3       40.1       14.5       2518         88000       8100       321.6       0.44       88000       50.7       43.0       12.4       3107	H e t	(311) n <sub>3</sub>	(3π) 83	ов (Mpa)	s >	ε <sub>α</sub>	Wm (mm)	d sinθ (mm)	θ	q <sub>pred.</sub> (кра)	Gtest (KPa)	qtest qpred.
170000       256.5       0.34       170000       98.4       17.0       22.0       1762         88000       10600       322.6       0.31       88000       60.3       40.1       14.5       2518         88000       8100       321.6       0.44       88000       50.7       43.0       12.4       3107	P6-1	170000	23500	296.0	0.38	•	95.4	15.7	21.3	1685	1152	0.30
88000 10600 322.6 0.31 88000 60.3 40.1 14.5 2518 88000 8100 321.6 0.44 88000 50.7 43.0 12.4 3107	96-2	170000	25000	296.5	0.34		98.4	17.0	22.0	1762	1411	0.69
88000 8100 321.6 0.44 88000 50.7 43.0 12.4 3107	P12	88000	10600	322.6	0.31		60.3	40.1	14.5	2518	1909	0.76
	P 1 6	88000	8100	321.6	0.44		50.7	43.0	12.4	3107	2383	0.77

Table 7.7 Actual and predicted failure loads

		Nondimensional	Nondimensional Loads at Failure $(q/q_C)$	(d/dc)	
Specimen			Predicted		Test/Predicted Ratio
	Test	Shear Limit	Strain Limit	Strain in HAE	
P6-1	17.4	53.8	68.7	25.4	0.69
P6 - 2	17.9	45.1	62.0	22.4	0.80
P12	6.6	27.1	37.3	89.8	97.0
P16	5.5	22.7	37.2	7.22	77.0

Table 7.8 Failure loads based on Ratzlaff-Kennedy model and linear strains in HA2

Specimen	Strain in HAZ (uc)	q (test) (kpa)	q (pred.)	q (test)
P6-1	113000	1152	1340	0.86
P6-2	113000	1411	1344	1.05
P12	58700	1909	1821	1.05
<b>P16</b>	58700	2383	2285	1.04

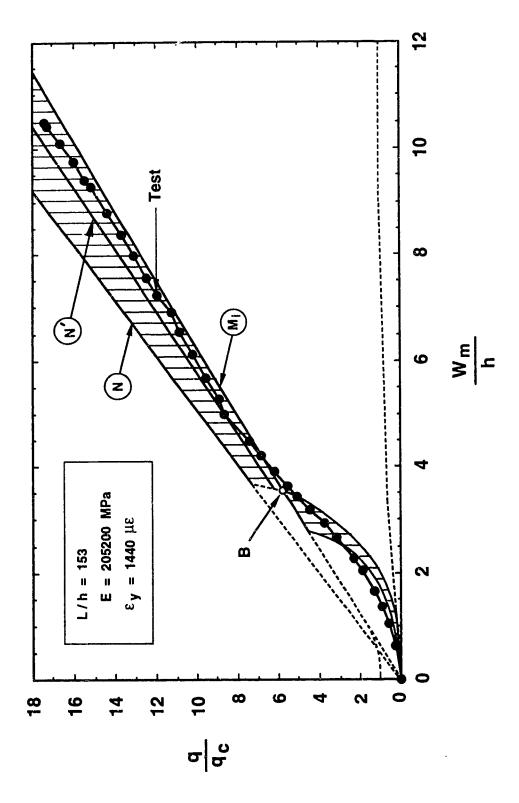


Figure 7.1 Load-deflection relationships, plate P6-1

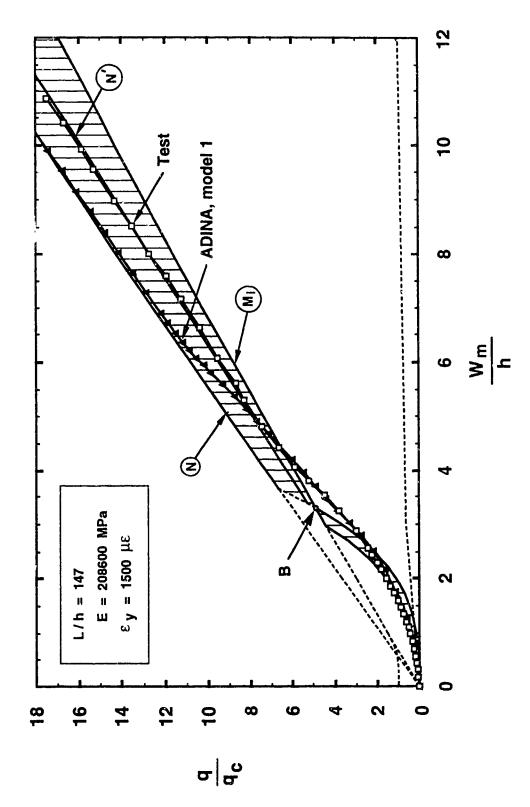


Figure 2 Load-deflection relationships, plate P6-2

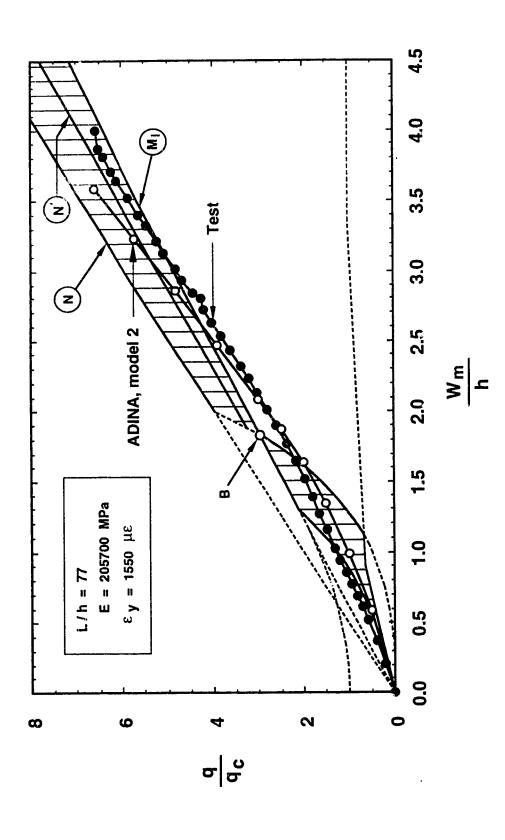


Figure 7.3 Load-deflection relationships, plate P12

<u>.</u>

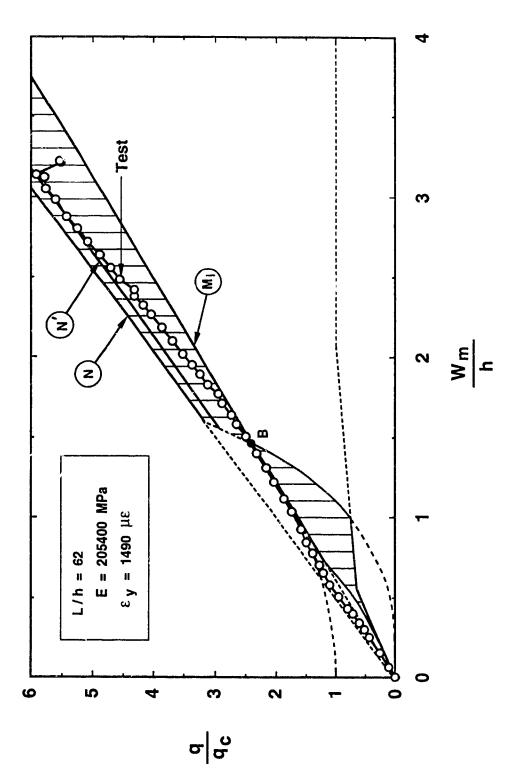
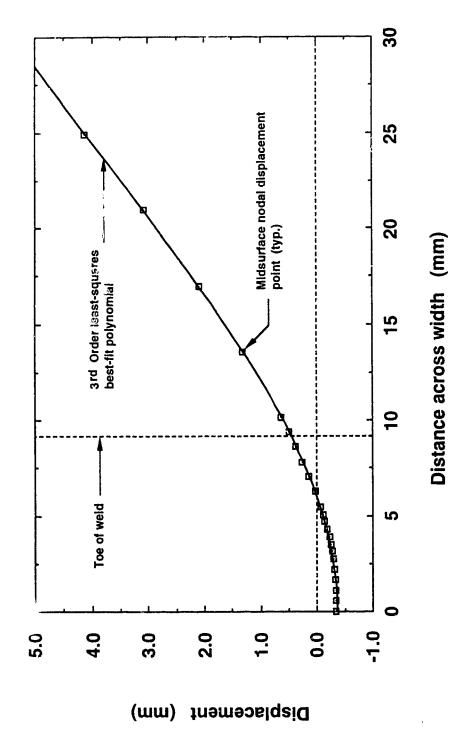
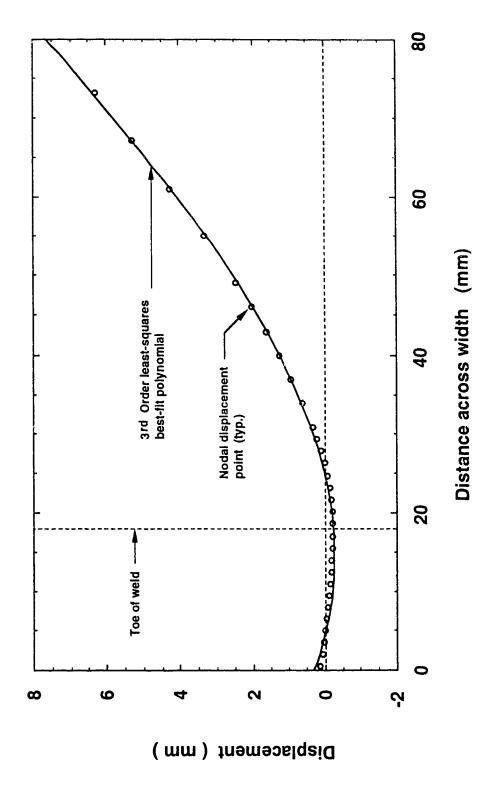


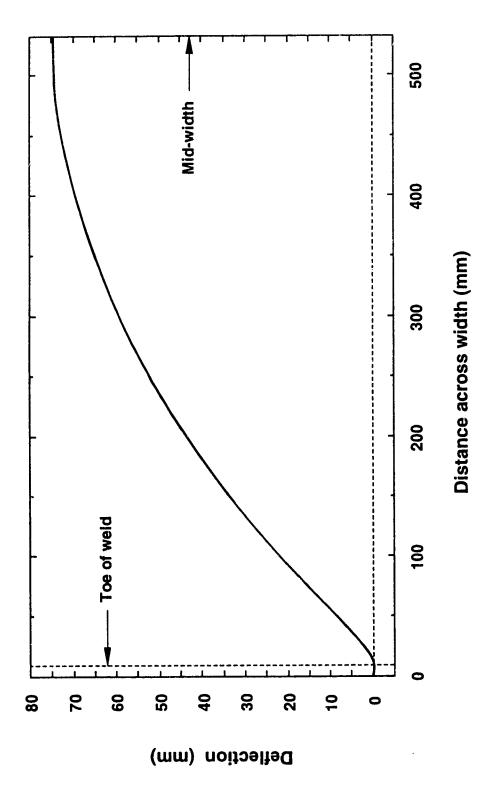
Figure 7.4 Load-deflection relationships, plate F16



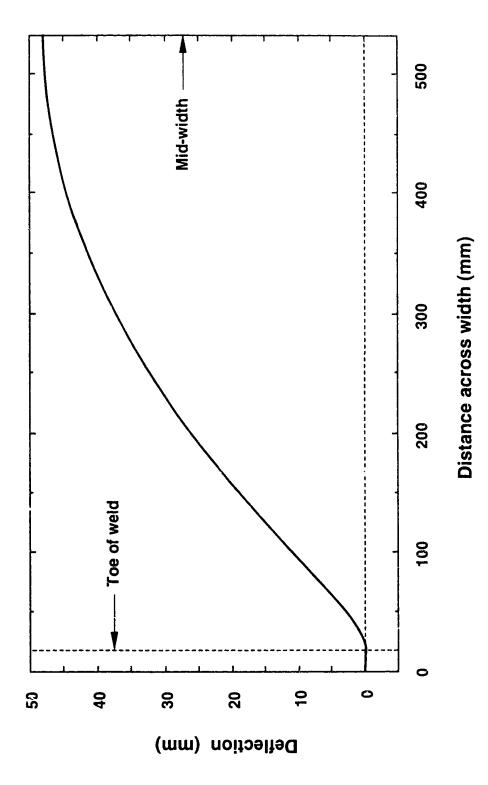
Model 1(test P6-2) midsurface nodal displacements, q/qc = 17.7 Figure 7.5



9.9 Model 2(test P12) midsurface nodal displacements, q/qc Figure 7.6



Model 1 (test P6-2) mid-surface deflections at test failure load Figure 7.7



Model 2 (test P12) mid-surface deflections at test failure load Figure 7.8

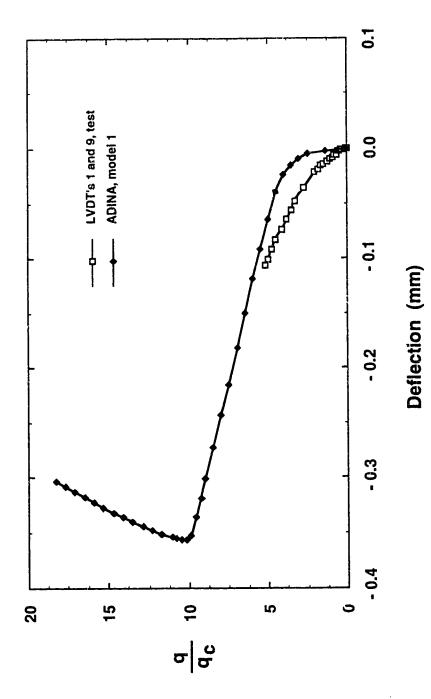
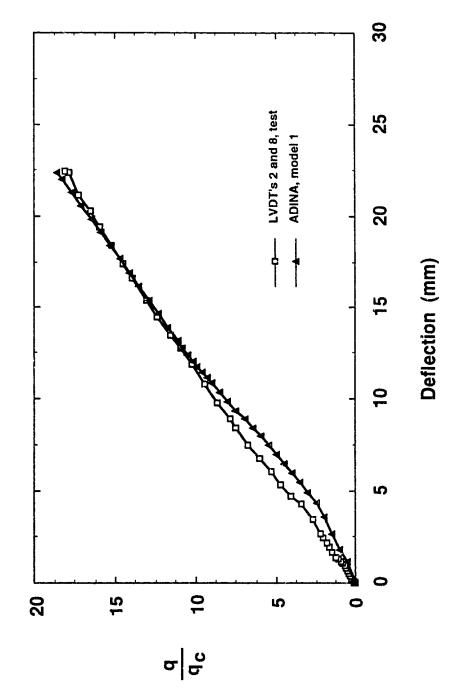
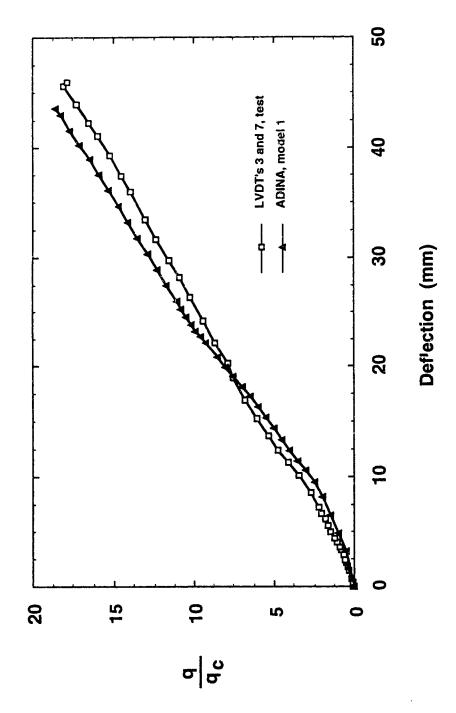


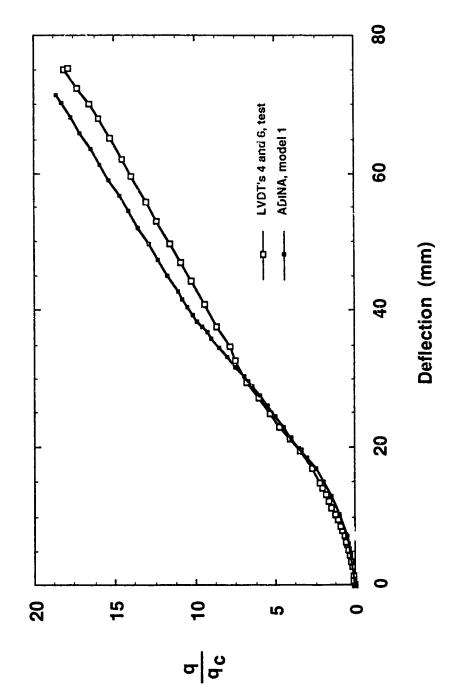
Figure 7.9 Deflection at toe of weld by LVDT's and ADINA model 1, test P6-2



Deflection at 87 mm from toe of weld by LVDT's and ADINA model 1, test P6-2



Deflection at 174 mm from toe of weld by  $\mathrm{LVDT}^+\mathrm{s}$  and ADINA model 1, test  $\mathrm{P6-2}$ Figure 7.11



Deflection at 348 mm from toe of weld by LVDT's and ADINA model 1, test P6-2 Figure 7.12

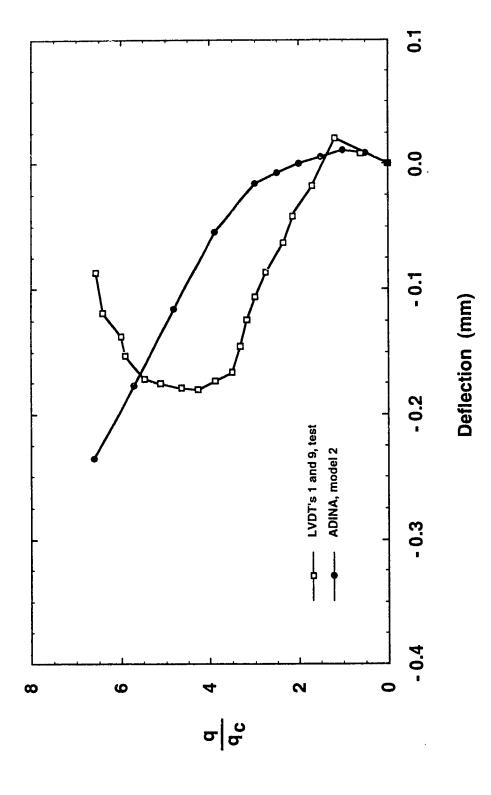
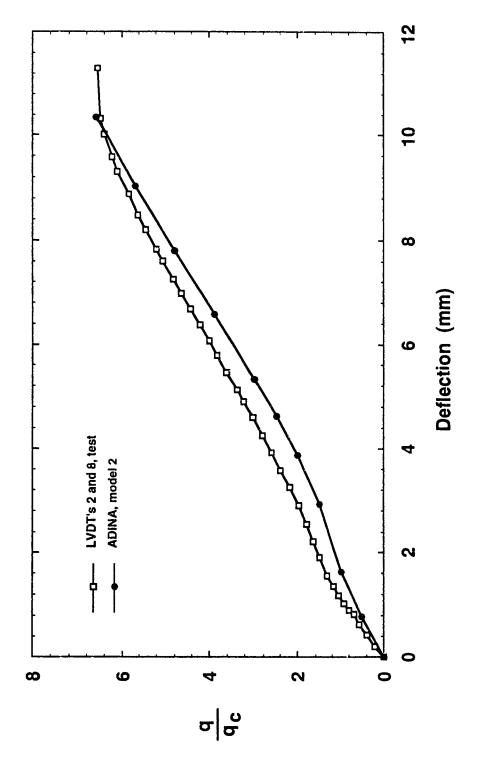
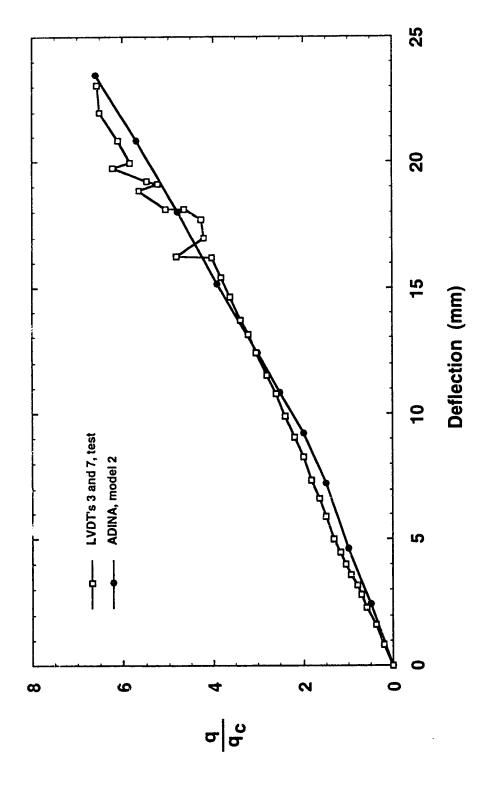


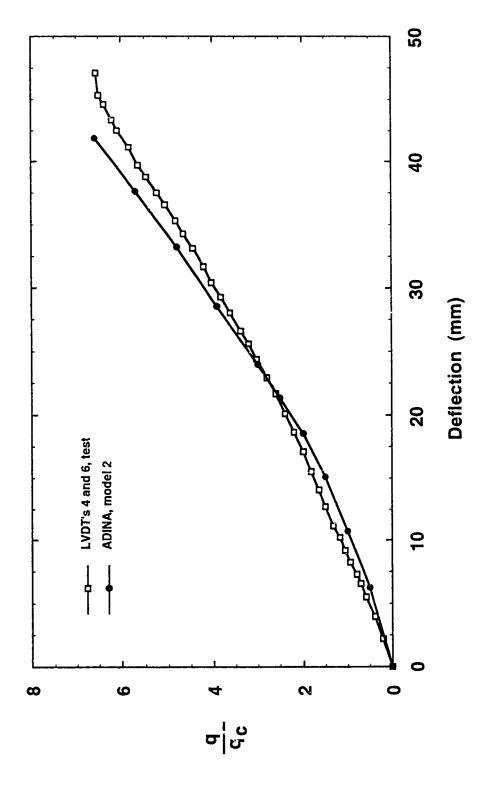
Figure 7.13 Deflection at toe of weld by LVDT's and ADINA model 2, test P12



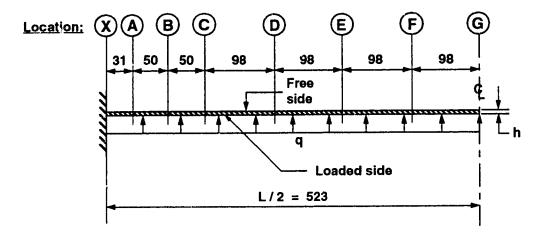
Deflection at 87 mm from toe of weld by LVDT's and ADINA model 2, test Pl2  $\,$ Figure 7.14



Deflection at 174 mm from toe of weld by LVDT's and ADINA model 2, test P12  $\,$ Figure 7.15

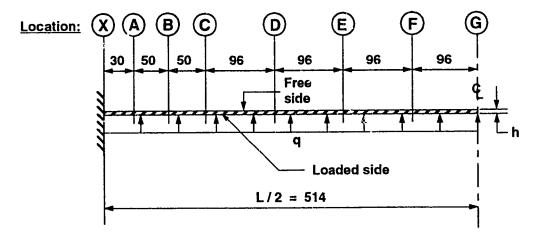


Deflection at 348 mm from toe of weld by LVDT's and ADINA model 2, test Pl2  $\,$ Figure 7.16



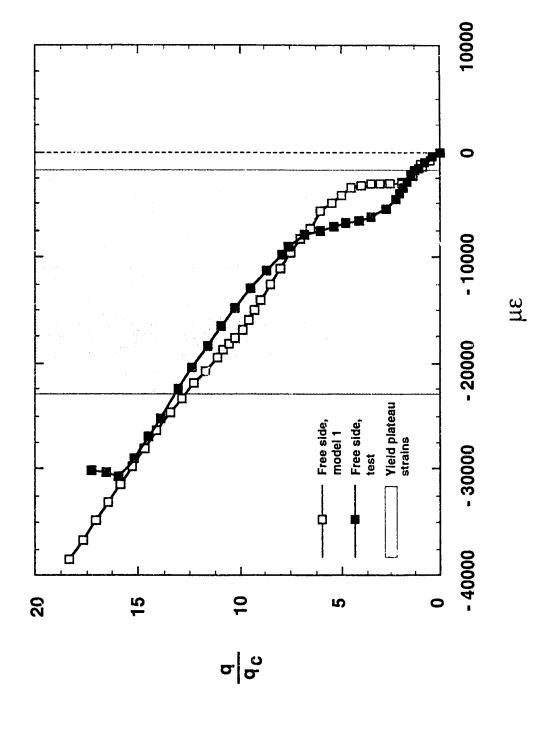
Note: All dimensions in mm

Figure 7.17 Locations for strain comparisons at mid-length between ADINA and test P6-2

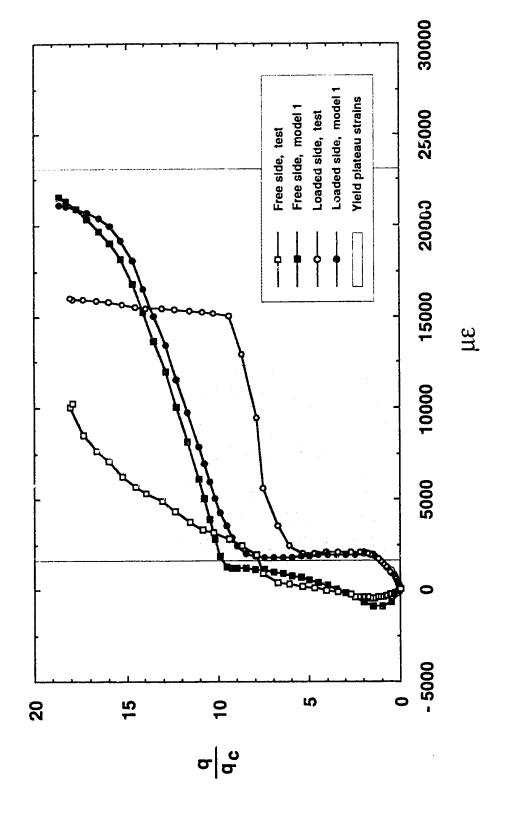


Note: All dimensions in mm

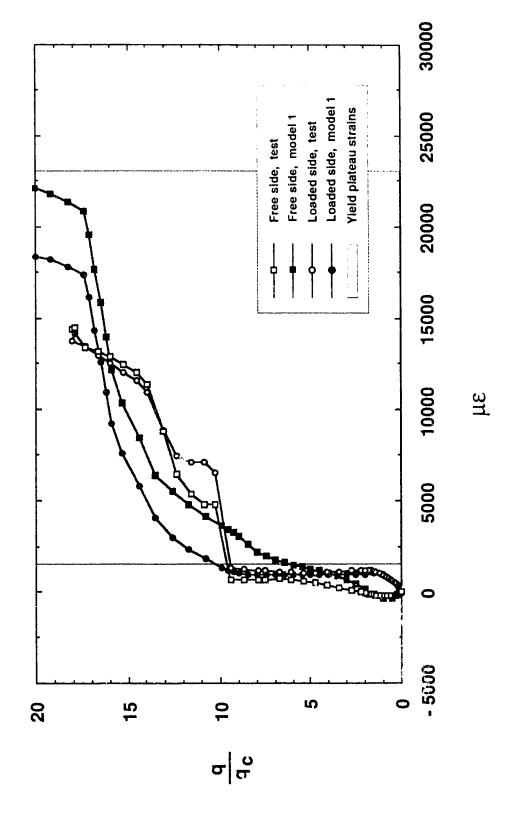
Figure 7.18 Locations for strain comparisons at mid-length between ADINA and test P12



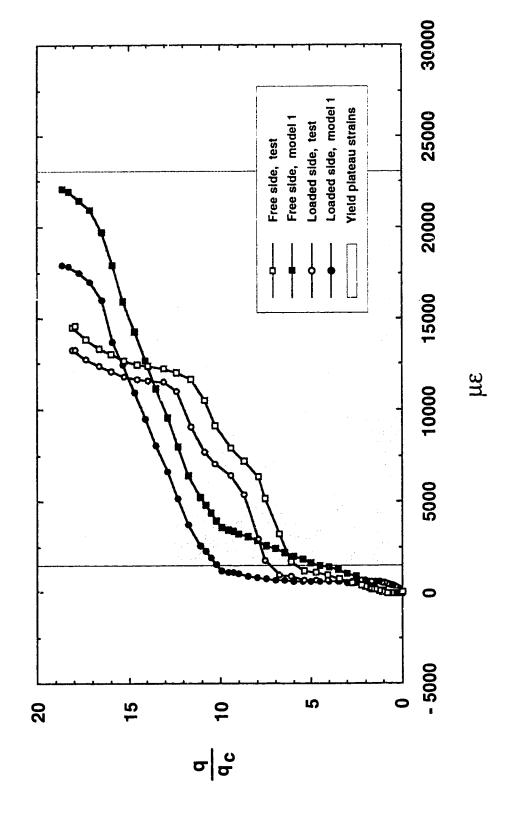
Strains at location  $\boldsymbol{x}$  given by ADINA and gauges on plate P6-2 Figure 7.19



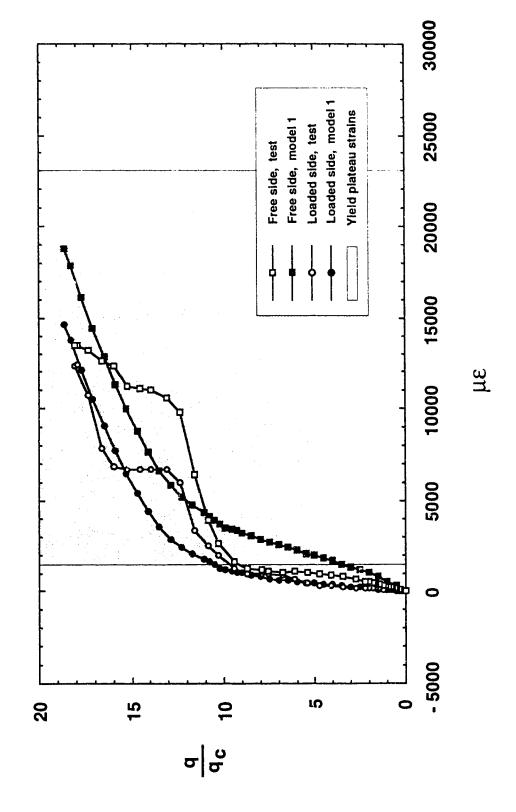
Strains at location  $\boldsymbol{A}$  given by ADINA and gauges on plate P6-2 Figure 7.20



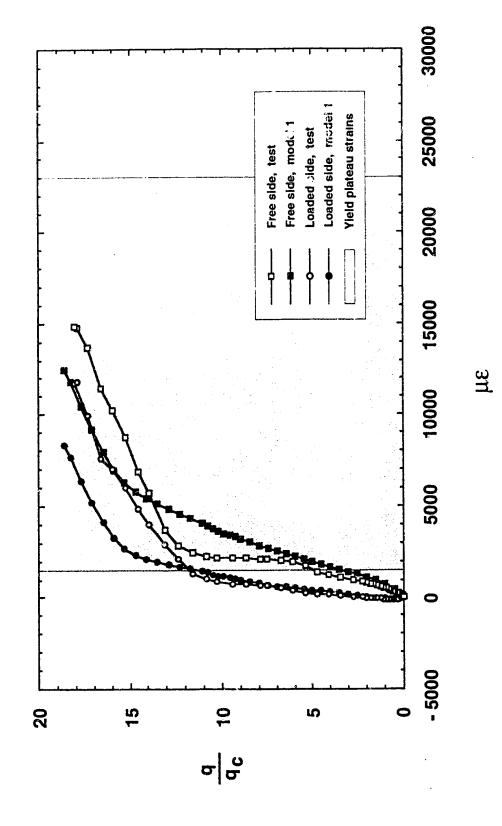
Strains at location  ${\bf B}$  given by ADINA and gauges on plate P6-2 gure 7.21



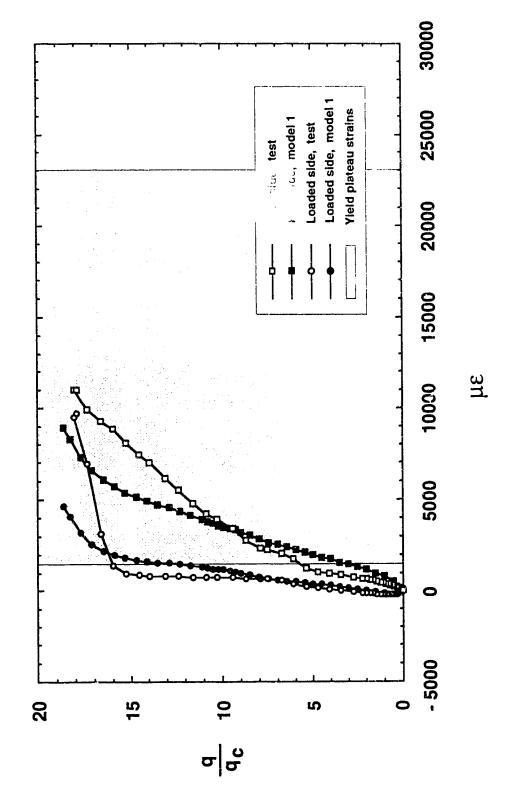
Strains at location C given by ADINA and gauges on plate P6-2 Figure 7.22



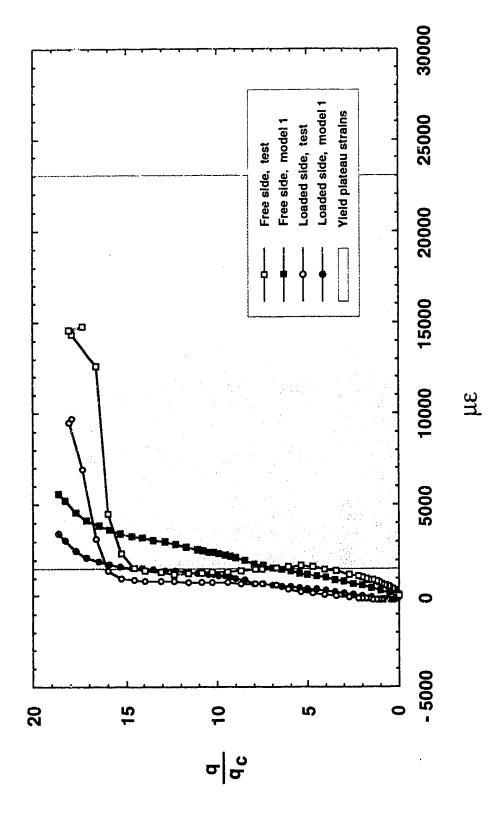
Strains at location  $\boldsymbol{D}$  given by ADINA and gauges on plate P6-2



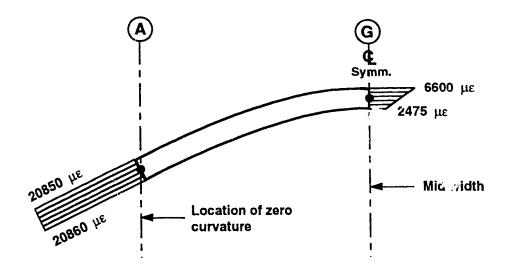
Strains at location  $\boldsymbol{E}$  given by ADINA and gauges on plate P6-2Figure 7.24



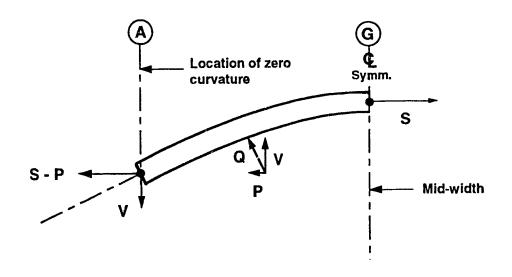
Strains at location F given by ADIMA and gauges on plate P6-2 Figure 7.25



Strains at location  ${\bf G}$  given by ADINA and gauges on plate P6-2 Figure 7.26

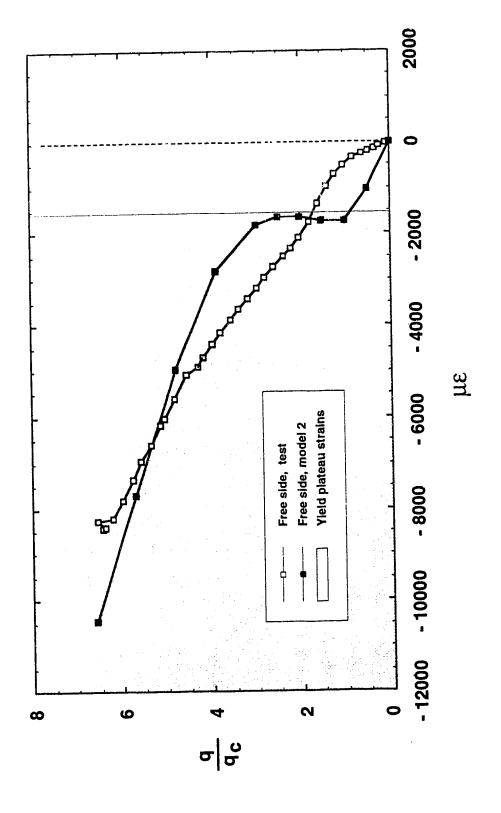


## (a) Strains

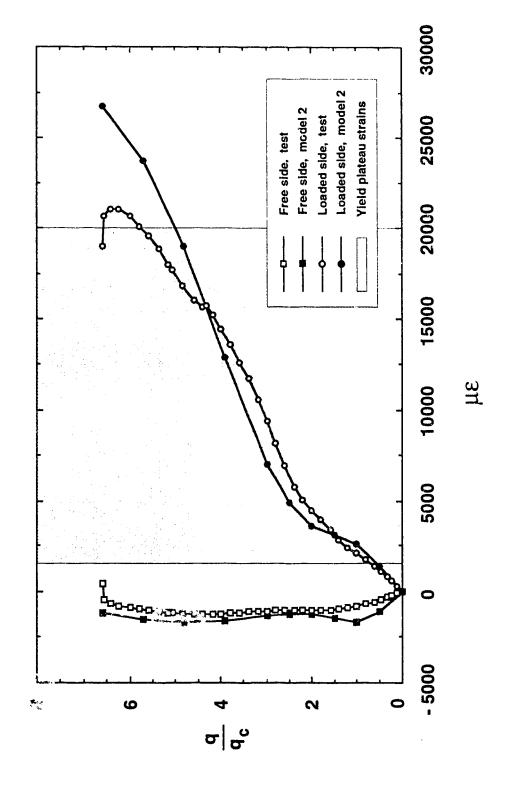


## (b) Free body diagram

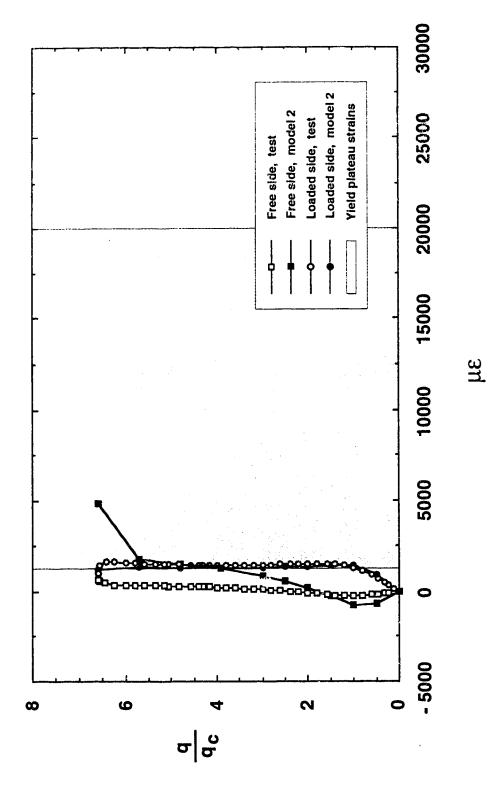
Figure 7.27 Strain and free body diagrams from location  $\bf A$  to location  $\bf G$ , model 1, q/qc = 17.7



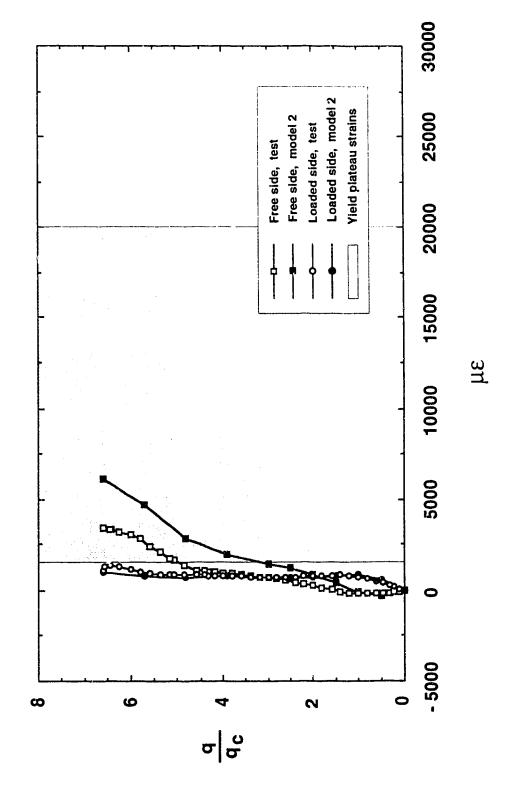
Strains at location  $\boldsymbol{x}$  given by ADINA and gauges on plate P12 Figure 7.28



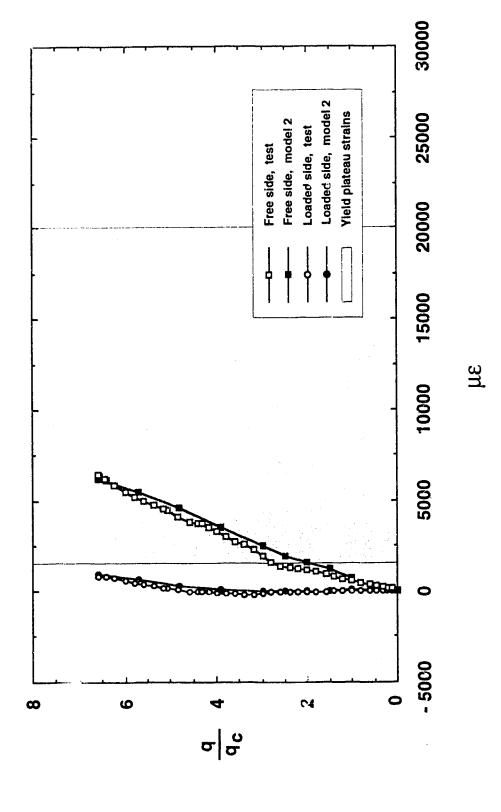
Strains at location A given by ADINA and gauges on plate P12 Figure 7.29



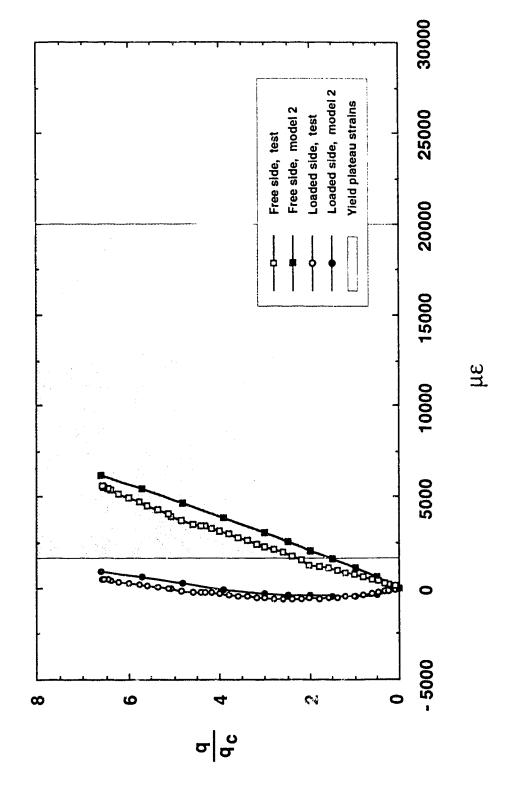
piate P12 Strains at location B given by ADINA and gauges o Figure 7.30



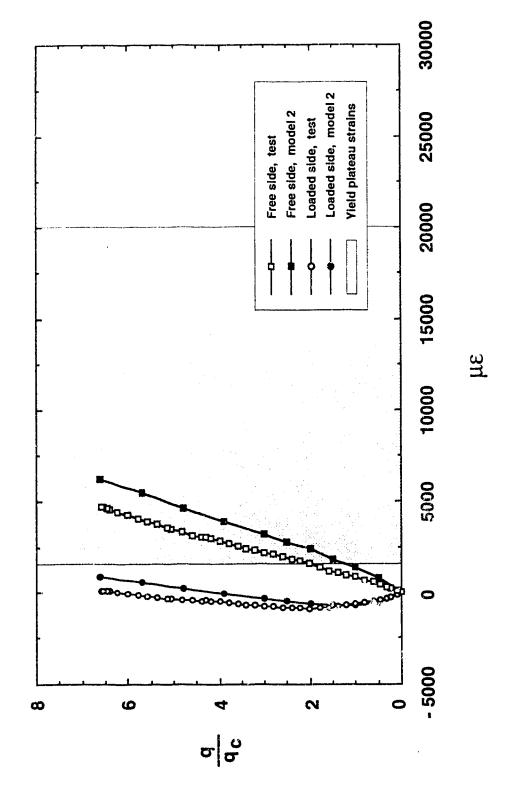
Strains at location C given by ADINA and gauges on platge F12 Figure 7.31



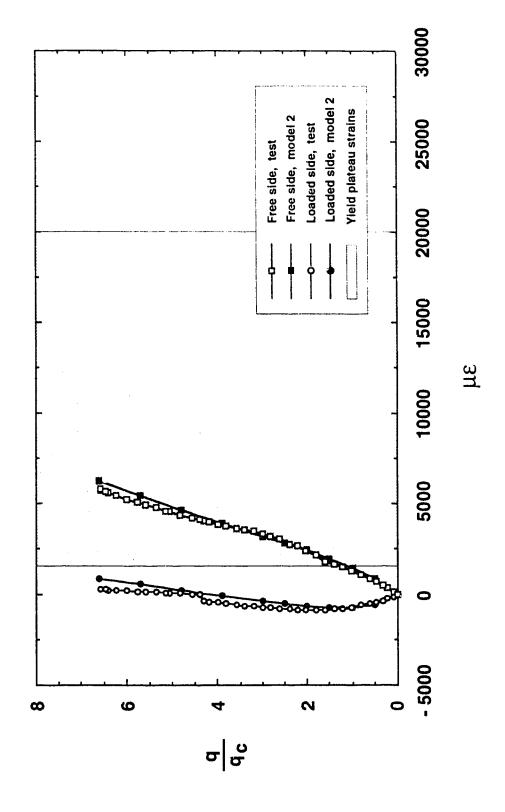
Strains at location  $\boldsymbol{D}$  given by ADINA and gauges on plate P12 Figure 7.32



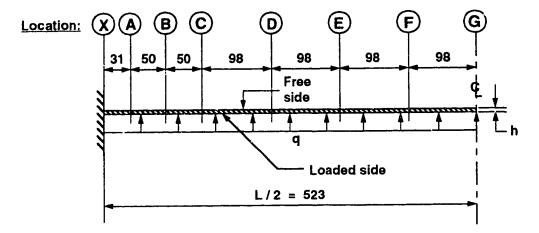
Strains at location **E** given by ADINA and gauges on plate P12 Figure 7.33



Strains at location **F** given by ADINA and gauges on plate P12 Figure 7.34

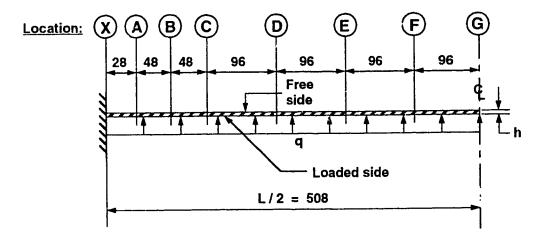


Strains at location **G** given by ADINA and gauges on plate P12 Figure 7.35



Note: All dimensions in mm

Figure 7.36 Locations of strain measurements, test P6-1



Note: All dimensions in mm

Figure 7.37 Locations of strain measurements, test P16

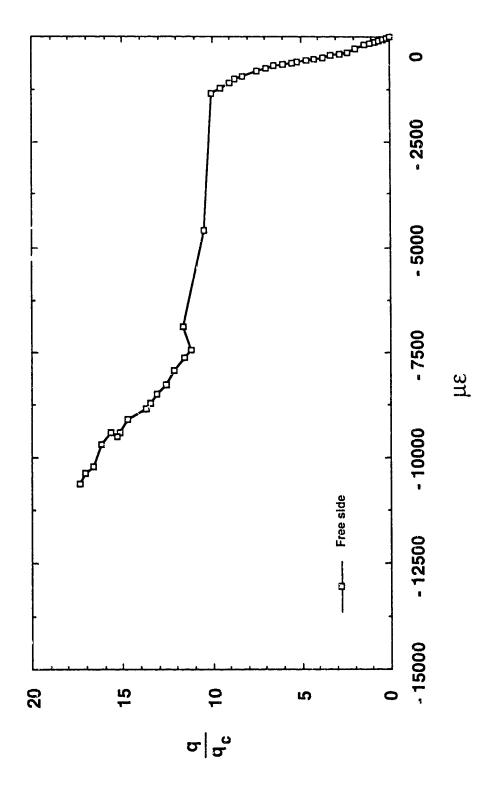


Figure 7.38 Strains at location X, test P6-1

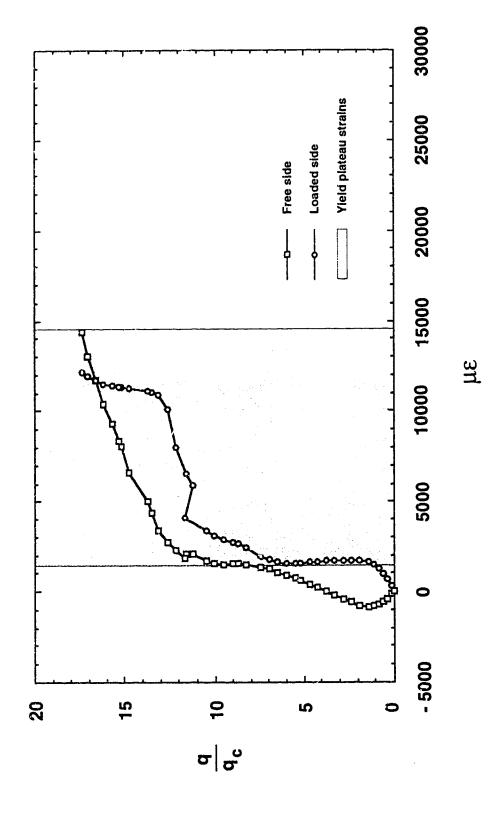


Figure 7.39 Strains at location A, test P6-1

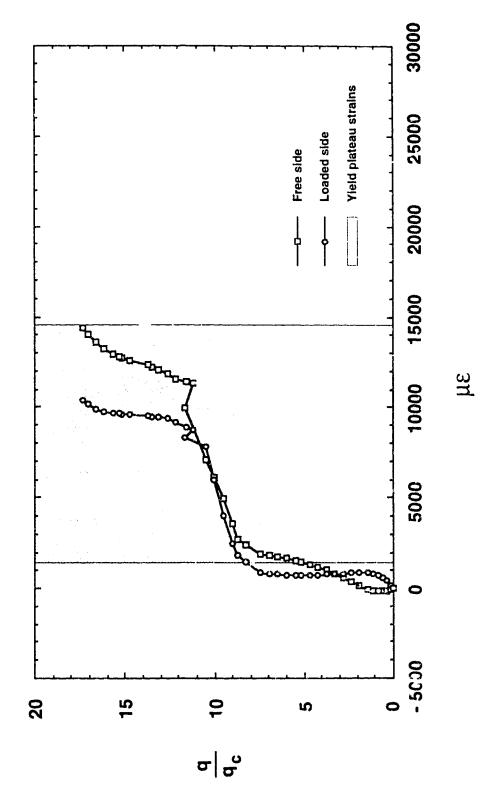


Figure 7.40 Strains at location B, test P6-1

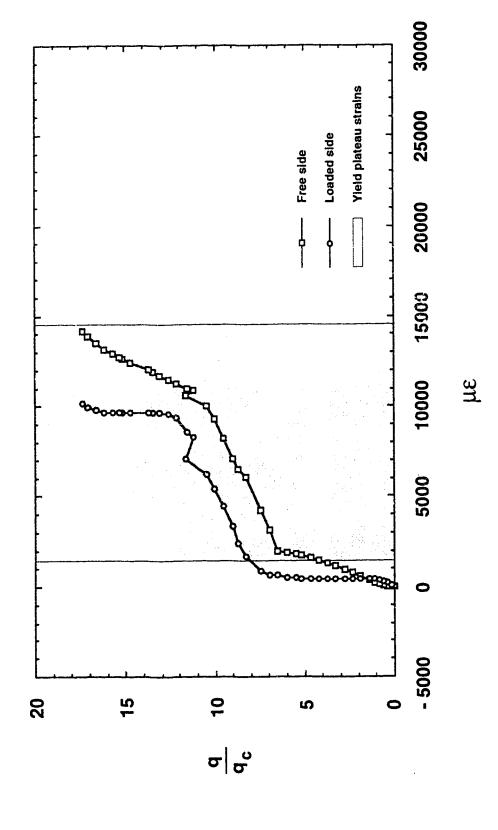


Figure 7.41 Strains at location C, test P6-1

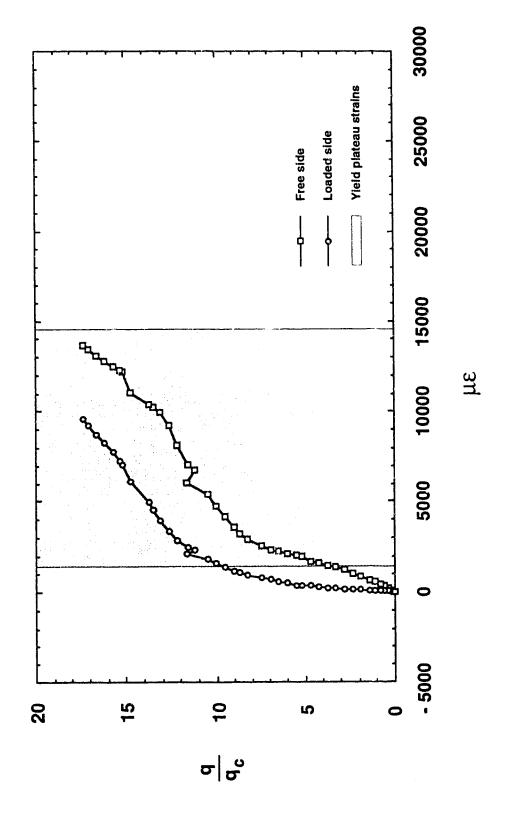


Figure 7.42 Strains at location  $\mathbf{D}$ , test P6-1

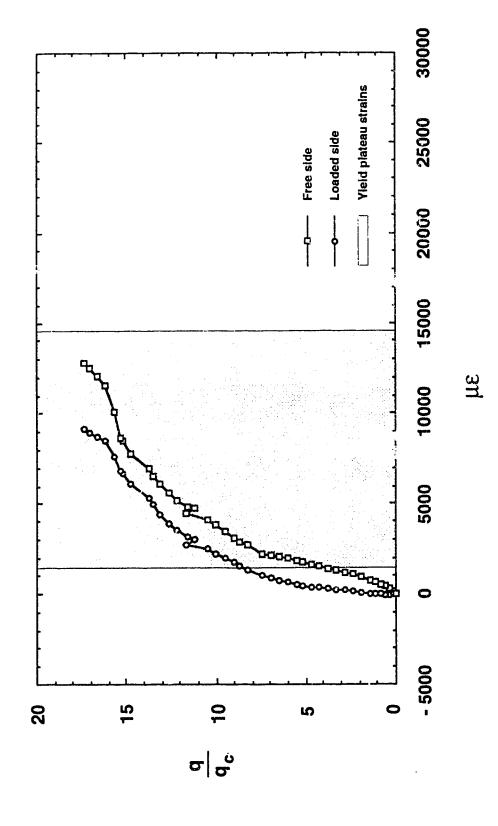


Figure 7.43 Strains at location E, test P6-1

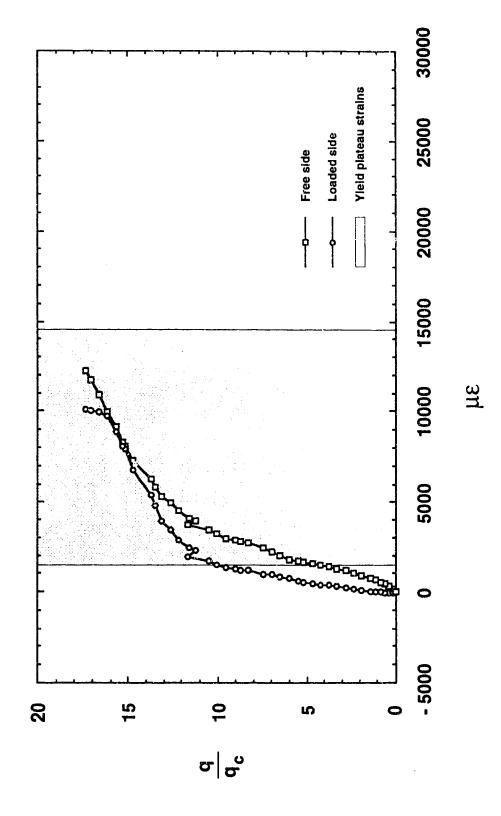


Figure 7.44 Strains at location F, test P6-1

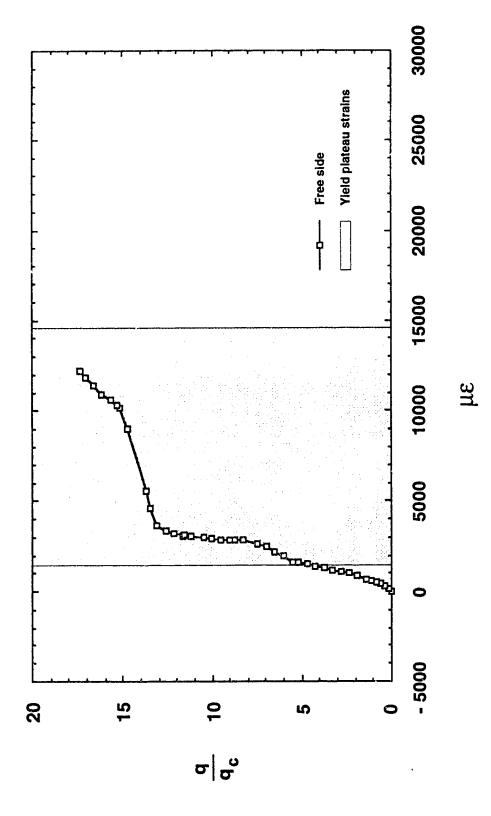


Figure 7.45 Strains at location G, test P6-1

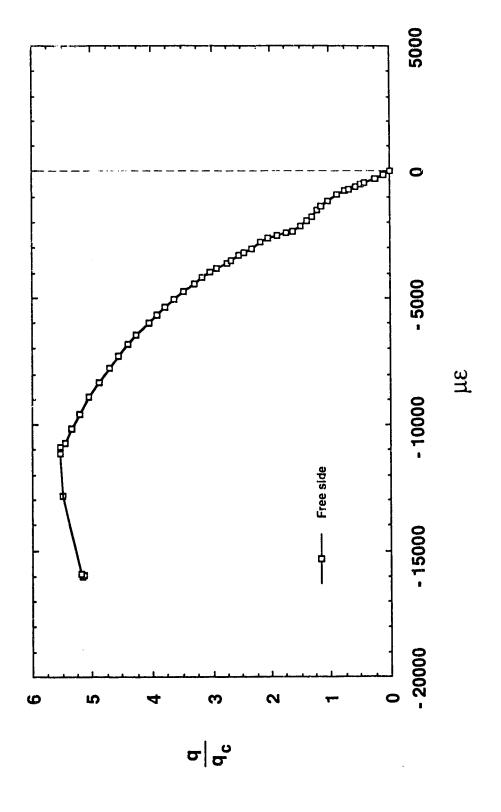


Figure 7.46 Strains at location X, test P16

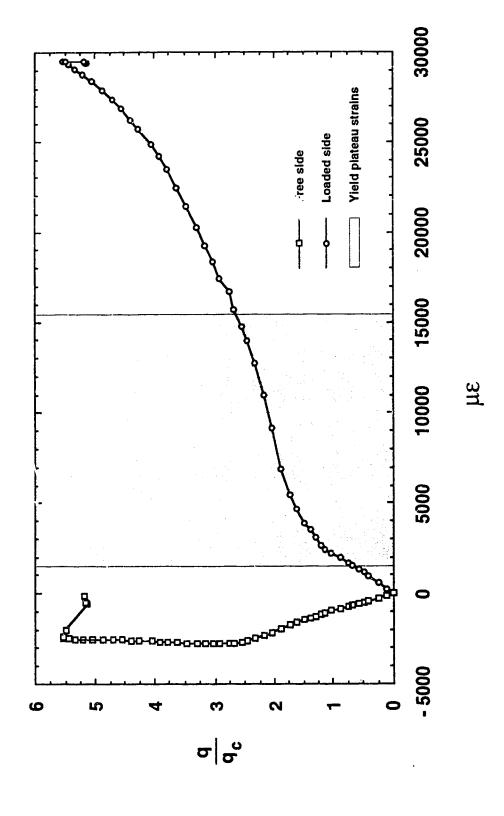


Figure 7.47 Strains at location A, test P16

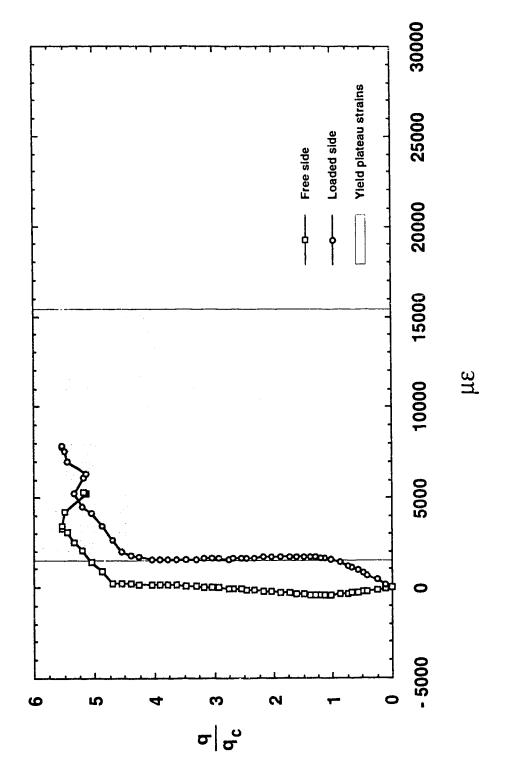


Figure 7.48 Strains at location B, test P16

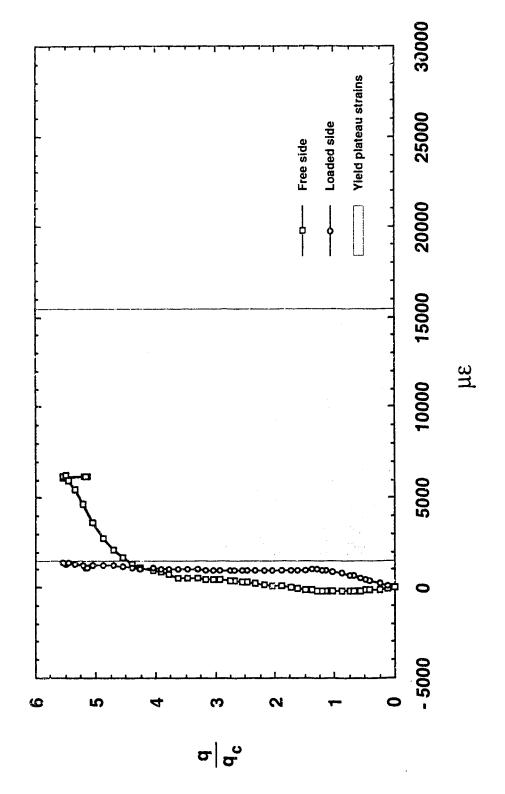


Figure 7.49 Strains at location C, test P16

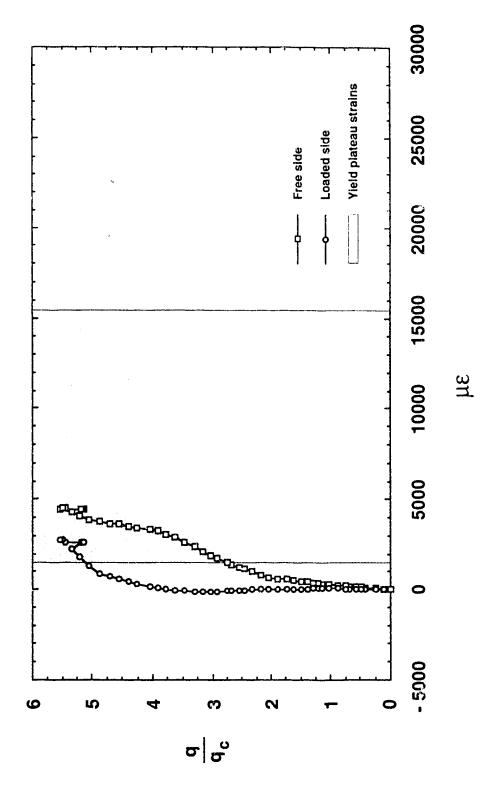


Figure 7.50 Strains at location D, test P16

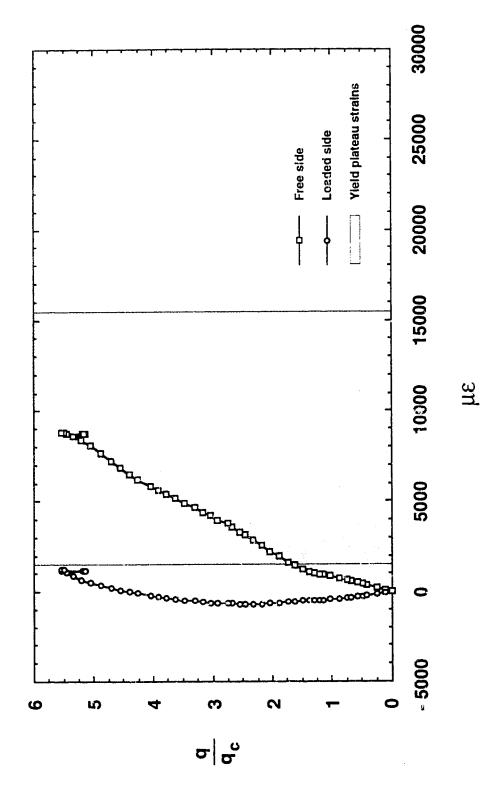


Figure 7.51 Strains at location E, test P16

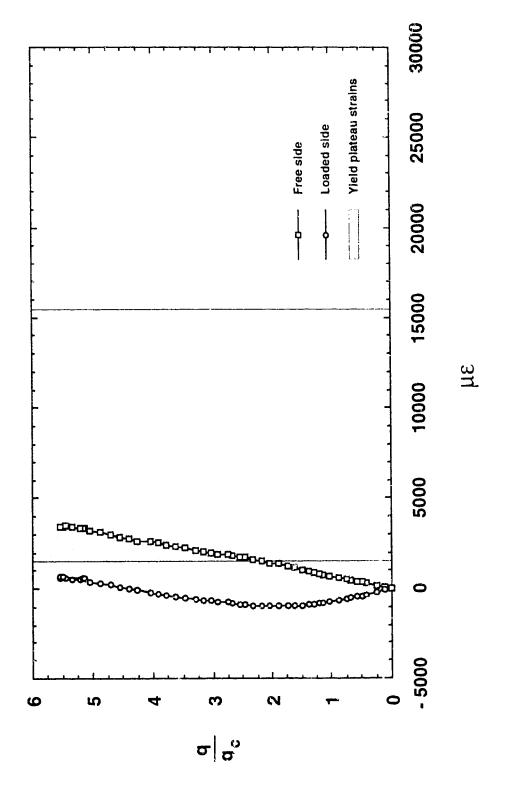


Figure 7.52 Strains at location F, test P16

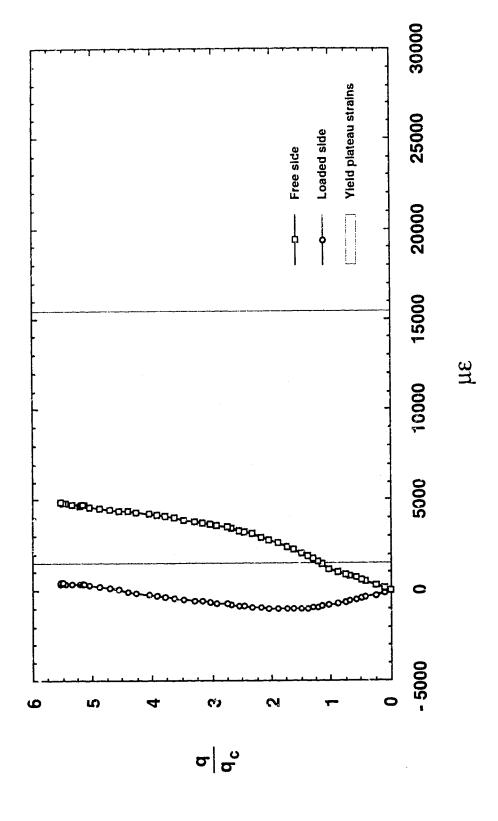


Figure 7.53 Strains at location **G**, test P16

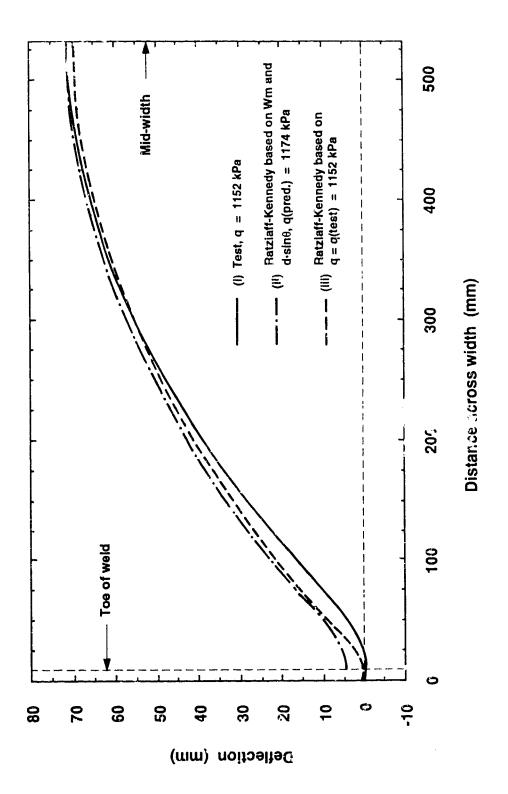


Figure 7.54 Deflected shapes, plate P6-1

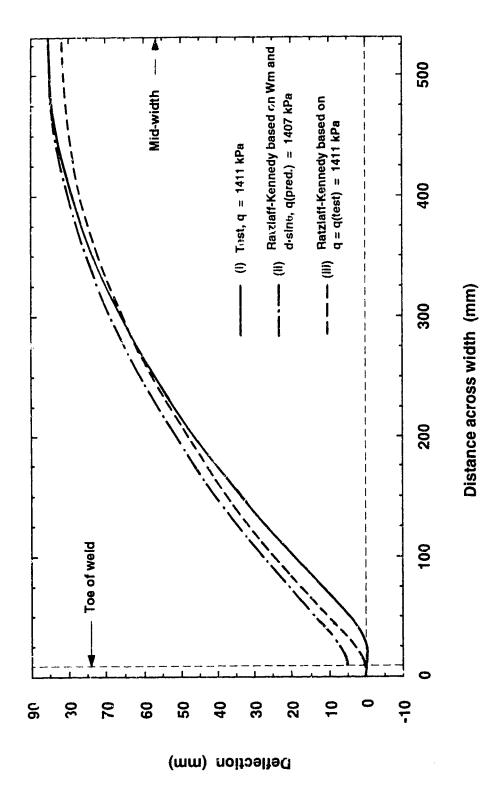


Figure 7.55 Deflected shapes, plate P6-2

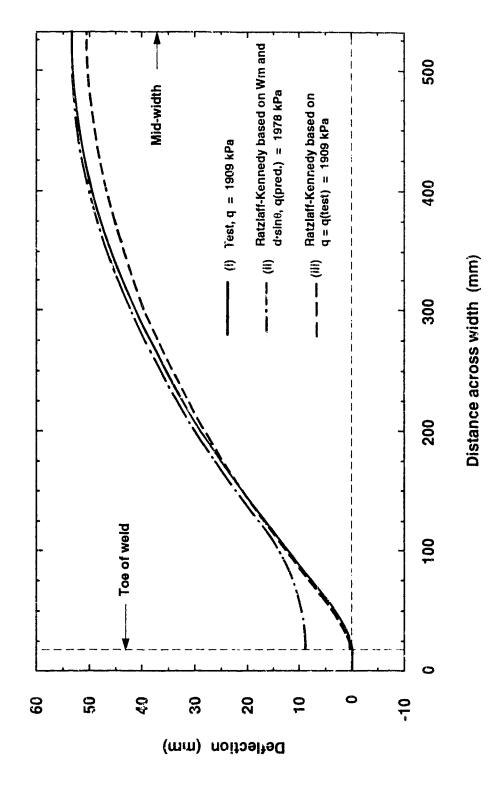


Figure 7.56 Deflected shapes, plate P12

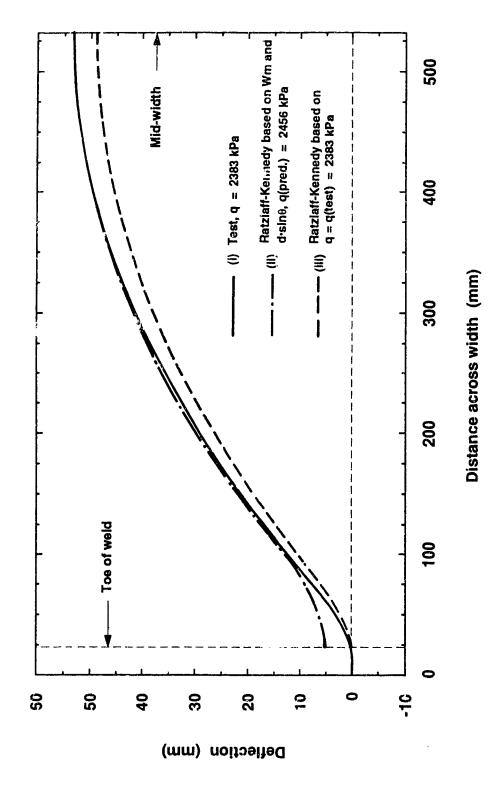


Figure 7.57 Deflected shapes, plate P16

# 8. SUMMARY, OBSERVATIONS, CONCLUSIONS AND APPLICATIONS

## 8.1 Summary

A series of four tests were conducted on transversely loaded rectangular mild steel plates, about 1000 mm wide by 3000 mm long with thicknesses of 6 mm to 16 mm, fixed against rotation and translation on all four edges. Therefore, the plates had an aspect ratio of the width divided by the length of about 0.34.and an effective width/thickness ratio of between 64 and 154.

The plates were subjected to fluid pressure loading until failure occurred. Deflection and strain measurements were taken at numerous locations across the width and along the length throughout the loading.

Two models (both based on an aspect ratio of zero, implying plane strain conditions) were used to predict the behaviour of the test plates. The Ratzlaff-Kennedy (1986) model taking into account elastic and inelastic material behaviour and large deformations and hence flexural action and elastic and inelastic membrane behaviour including bending at the edges, describes behavioural domains in which the response would be expected to lie. Three failure modes of excessive straining in the steel, excessive straining in the HAZ, or shear, all at the edge of the plate, have been identified.

The ADINA finite element model, used to predict the behaviour of two of the test plates, takes into account geometric nonlinearities due to large displacements and large strains and, as well, nonlinear material properties.

Strain measurements from uniaxial tensile coupon tests gave values of Poisson's ratio of about 0.28. Just beyond the yield plateau the inelastic strain ratio reaches about 0.43 and then decrease almost linearly to about 0.35 at an axial strain of 200000  $\mu\epsilon$ .

#### 8.2 Observations

In general, the load-deflection responses were consistent with analyses that describe a domain of behaviour. Initially, the plates exhibit elastic flexural membrane action, then elastic membrane behaviour with inelastic hinges near the supports and mid-width and finally, inelastic flexural membrane action was observed.

Significant bending of the plate exists in a relatively narrow region at the edges. With increased loading the curvature at the edge increases and the width of the region decreases, the curvature decreasing from a maximum at the edge to zero at the limit of this region. Between the bending regions at the edges and between the points of zero curvature, the plates under the action of fluid pressure nearly formed the circular arc that would be expected.

For the same load levels, the measured deflections are in excellent agreement with those predicted by the Ratzlaff-Kennedy (1986) model and those of the ADINA finite element

model.

The pressures at failure for the 16 mm, 12 mm, and two 6 mm plates were 2383 kPa, 1909 kPa, 1411 kPa, and 1152 kPa with corresponding deflections of about 53, 54, 85, and 71 mm, respectively. The pressures at failure are 5.5, 6.6, 17.9 and 17.6 times the pressure required to develop a three-hinge collapse mechanism assuming flexural action only. The maximum deflections, in the same order, are 3.2, 4.0, 12.0 and 10.5 times the respective plate thicknesses.

Failure of all four plates occurred along one edge near mid-length. Rupture, initiated in the HAZ at the toe of the restraining weld, and cracks propagated through the thickness and along the length of the plate.

Recognizing the discontinuous nature of yielding, the strains from the ADINA model are in reasonable agreement with the measured strains.

At the failure loads and within the membrane portion of the plate, i.e., between the points of zero curvature, the thinner plates were fully yielded throughout with larger tensile strains occurring on the convex (free) surface than on the concave (loaded) surface. Even at failure loads, the thicker 12 mm and 16 mm plates were strained only elastically on the loaded surface.

# 8.3 Conclusions

Because of the inelastic membrane action that develops as inelastic straining occurs, transversely loaded steel plates can carry many times the fully plastic flexural collapse mechanism load. The ratio of the inelastic membrane failure load to the collapse mechanism load increases with increasing width/thickness ratios or with thinner plates. Both the maximum deflection at ultimate failure and the deflection normalized by dividing by the plate thickness increase with decreasing plate thickness.

Three failure mechanisms associated with rupture at the edge of the plate have been identified that are considered to be valid when significant bending exists at that location or when the transverse shears are large. These are: (i) a maximum tensile strain criterion where the tensile strains arising from the tensile forces and bending moments reach the true ultimate tensile strain of the steel, (ii) a maximum tensile strain criterion where the tensile strains occurring from the tensile forces and bending moments at the edges reach the ultimate tensile strain of the HAZ (This condition may be further exacerbated by introducing a strain concentration at this same location), (iii) a maximum shear stress criterion when the average shear stress at the edge The criterion giving the lowest failure reaches  $0.577\sigma_{\rm u}$ , load in a given set of circumstances will govern. Real structures can be designed to eliminate or at least minimize the third criterion. With failure in the HAZ precluded

because of good design it is likely that the shear criterion would govern.

All four plates achieved at least partial inelastic flexural membrane behaviour but are considered to have failed premacurety due to rupture of the steel in the HAZ.

Test/predicted ratios for the 4 tests ranging from 0.69 to 0.80 were obtained based on the failure criterion of rupture in the HAZ when one Ratzlaff-Kennedy model was used to predict the strains and the test strains were taken as the maximum strains computed from the ADINA models (strains could not be measured on the loaded surface at the toe of the weld). These test/predicted ratios are considered to be low because the ADINA model shows that the strains vary non-linearly shrough the thickness while the Ratzlaff-Kennedy model assumes a linear distribution of strain there.

A comparison of the ADINA strains at the failure loads with those computed from the Ratzlaff-Kennedy model suggests that the strain concentration factor at the toe of the weld was in the order of 1.5. When this factor is applied to the maximum tensile strains in the Ratzlaff-Kennedy model the test/predicted ratios range from 0.86 to 1.05 with a mean value of 1.00.

## 8.4 Applications

The highly predictable inelastic membrane behaviour, demonstrated by these large-scale tests, that results in transverse load carrying resistances of steel plates many times that based on fully plastic flexural collapse

mechanisms, suggests a completely different limit states design approach for those structures in which the deformations associated with inelastic membrane action are of little consequence. Obviously, for membrane action to develop, the remote edges of the plate must be anchored against translation. For any postulated loading condition, whether distributed over the entire plate or a portion thereof, the inelastic membrane behaviour at the factored load level can be established either using the Ratzlaff-Kennedy model or an inelastic finite element analysis using a program such as ADINA. Points of maximum tensile straining and maximum shear, whether at supports or under load points, are identified. The values of the tensile strain and shear at these locations are compared to the limiting values to matisfy the ultimate limit states criteria. It is suggested that the nominal shear resistance for the plate material is  $0.577\sigma_{\rm u}$  and the nominal maximum strain is the ultimate true strain (this presumes that the failure mode associated with high tensile straining in the HAZ has been obviated by good detailing). Resistance factors and load factors are established consistent with the desired reliability of the structure.

Based on analyses to date it is expected that the shear criterion would govern and that the postulated failure loads would be in the order of 25 to 50 times the fully plastic flexural collapse mechanism loads and increase with decreasing plate thickness. The membrane deflections also,

of course, increase as the thickness is decreased.

Two classes or structures for which it is suggested that local deformations, so long as fracture does not occur, cause little more than cosmetic damage, are arctic caisson-type structures used for petroleum exploration and production and the hulls of ice-breakers. Both of these structures are likely to be subject to large ice loads and deformations are not a serviceability issue. Therefore such structures can be designed utilizing the membrane action to the fullest.

Figure 8.1 shows that the deflected shape of an individual plate panel is directly related to the distribution of loads to which it has been subjected. Ratzlaff and Kennedy (1986) state, therefore, that if a panel is subjected to a loading pattern different from that which first occurred, then a section deflected shape would be at al. (1974) provided the developed as shown h second load become targe. The panel simply as. The variation in adapts to the r i not to be a problem. A:: loading on a par ing deformation; that have a matter of fact, occurred can be used directly to determine the intensity of the forces to which plate panels of the structure have been subjected. The structure itself becomes a conglomeration of virtually indestructable load measuring devices. The deflected shapes could be recorded and would thereby provide a significant clue to the distribution of load. Yearly observations would enable the operator to catalogue the

forces to which the structure has been subjected, thus accumulating a body of statistical data.

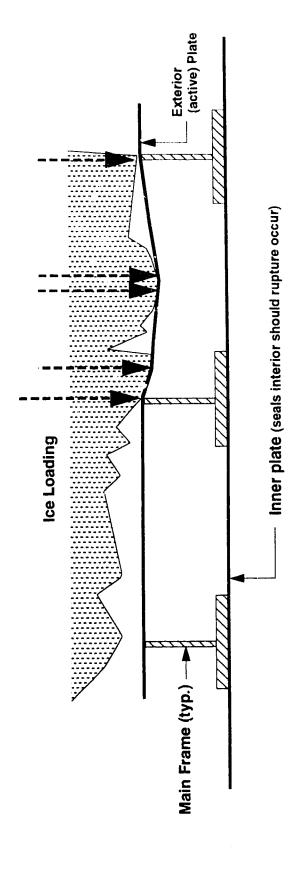
Consider the load-deformation response of plate P12 of Fig. 5.1. On unloading and reloading, the path is retraced. If a certain load has been reached and then removed, further cycles of loading to the same or a lesser level would simply cause elastic cycling. If on the other hand the maximum load on a subsequent cycle is greater, cycling would occur through this point on another line, elastically. It is anticipated that a significant number of cycles of loading dispraying this essentially elastic behaviour could be withstood before fatigue cracking would ensue.

For design, to minimize overall costs, parametric studies should be carried out to determine the best distribution of steel between the face plate subject to the heavy loads and the supporting members. It is envisaged that the optimum structure for a given plate thickness would have a minimum number of supporting bulkheads. In no case should a large number of intermediate stiffeners be used. These are unlikely to be able to accommodate the deformations associated with the development of inelastic membrane action without failing themselves. They therefore are ineffective. The volume of steel in such stiffeners, it is suggested, based on the behaviour observed here, is better placed in the membrane plate or its major supporting members. It is further suggested that satisfactory performance can be obtained when one-way membrane action is

developed. There is no real need for two-way action. The design concept is fundamentally to reduce the number of pieces incorporated in the structure. Of course, at the ends of the structure, provision must be made to anchor the membrane force in the original plane of the membrane.

A double-hulled approach appears to be desirable, with a second membrane behind the surface membrane at such a distance to allow full membrane action of the surface membrane to be developed without impinging on it. Should rupture of the surface membrane occur, fluids would be retained within the ruptured cell. Of course, once the surface membrane is deformed and unloaded, on reloading it deforms elastically until the previous maximum load is approached. Figure 8.1 shows a possible cross-section of the geometry for a structure designed in accordance with these principles.

The use of the full strength of continuous steel plates developed with inelastic membrane action has further applications in any ci. cumstance where deformations that develop are of little consequence. Other applications could include structures for storing materials such as tanks, bins, and hoppers.



Partial section through ice-resisting wall Figure 8.1

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#### APPENDIX A

The relationships proposed by Ratzlaff and Kennedy (1986) for a membrane subject to fluid pressure relating the membrane deflection, maximum stress, strain, the edge inclination, to the fluid pressure and the material properties are derived here.

Figure A.1(a) is a free body diagram of the half-width of a unit length of plate with zero aspect ratio subject to fluid pressure and assumed to have constant curvature from the point of zero curvature, point Z, to the mid-width. The angle  $\theta$  is the membrane edge inclination. Consider Fig. A.1(b), a free body diagram of the central circular portion from point Z to the mid-width, extending over a horizontal distance of L/2 - d sin $\theta$ . The strain arising from the extension of the plate under fluid pressure due to the membrane force, S, is

[A.1a] 
$$\epsilon_{\rm s} = \frac{L_1 - L_0}{L_0}, \text{ or }$$

$$[A.1b] \qquad \epsilon_{\rm s} = \frac{\frac{\theta\pi \, L_{\rm o}}{180 \, \sin\!\theta} - L_{\rm o}}{L_{\rm o}}, \text{ or }$$

[A.1c] 
$$\varepsilon_{\rm B} = \frac{\theta \pi}{180 \sin \theta} - 1, \text{ or }$$

[A.1d] 
$$\frac{\sin \theta}{\theta} = \frac{\pi}{180 (1 + \varepsilon_s)}$$

where q is expressed in degrees. Summing forces in the vertical direction gives

[A.2a] 
$$q\left(\frac{L}{2} - d\sin\theta\right)$$
  $V = S\sin\theta$ , or

[A.2b] 
$$q = \frac{\sigma_{h} \sin \theta}{(1 - v_{h})(\frac{L}{2} - d \sin \theta)}$$

when S is replaced by  $\sigma_{\text{s}} h / (1 - v_{\text{s}}^{-2})$  which takes into account the state

of plane strain (zero straining in the longitudinal direction). Summing moments about point Z and summing access in the horizontal direction give, respectively.

[A.3] 
$$Q \cdot \frac{\underline{L} - d \sin \theta}{2 \cos \frac{\theta}{2}} = S \cdot W_{p_0}, \text{ and}$$

$$[A.4] \qquad Q \cdot \sin\left(\frac{\theta}{2}\right) + S \cos\theta - S = 0$$

and combining [A.3] and [A.4] gives

[A.5] 
$$W_{m} = \frac{(\frac{L}{2} - d \sin\theta) (1 - \cos\theta)}{\sin\theta} = (\frac{L}{2} - d \sin\theta) \frac{\sqrt{1 - \cos\theta}}{\sqrt{1 + \cos\theta}}$$

Consider the free body diagram of Fig. A.1(c). The fluid pressure is indicated by its horizontal and vertical components. Assuming that the strain distribution at the cross section through point O and K is linear with an average axial strain,  $\epsilon_{\rm p}$ , equal to that due to the axial force  $\rm P_{\rm O}$  when the maximum strain due to bending and axial forces equals  $\epsilon_{\rm u}$  at point K, the radius of curvature is

$$[A, 6] \qquad \rho = \frac{h (1 + \varepsilon_t)}{2 (\varepsilon_t - \varepsilon_t)}$$

For simplicity, assuming that the centre line of the deformed plate shape can be described by a cubic function, the minimum radius of curvature corresponding to the maximum bending strains at the plate edge is found from geometry to be

$$[A.7] \rho_{\text{MM}} = \frac{d \cos \theta}{2}$$

McDermott et al. (1974) also used a cubic function to describe the geometry at the plate edge when analyzing hull damages from ship collisions. They suggested that it was a reasonable and simple way in which to approximate the change in curvature from zero to a maximum value at the edge.

The eccentricity, e, of the maximum membrane force, S, can be shown to be

$$[A.8] \qquad e = \frac{d \sin^2 \theta}{3}$$

Summing moments about point O and considering centre-line dimensions, the moment at the edge is

$$[A.9] \qquad M_{\odot} = S \cdot e$$

when the contribution to the moment of the fluid pressure is neglected (This moment is only of the order of 1% of the moment Se).

The axial force, Po, is

$$[A.10] \qquad \qquad P = S \cdot \cos \theta$$

when neglecting the horizontal component of the fluid pressure.

Setting  $\rho_{min}$  of [A.7] equal to  $\rho_{c}$  of [A.6] gives

[A.11] 
$$d = \frac{h(1 + \varepsilon_1)}{\cos\theta (\varepsilon_3 - \varepsilon_1)}$$

and substituting [A.11] in [A.8] gives

[A.12] 
$$e = \frac{h \sin \theta (1 + \varepsilon_1)}{3 \cos \theta (\varepsilon_0 - \varepsilon_1)}$$

By replacing the expression for d of [A.11] and for e of [A.12] in [A.9], the edge moment becomes

[A.13] 
$$M_0 = \frac{\text{Sh sin}^2 \theta (1 + \epsilon_{\text{p}})}{3 \cos \theta (\epsilon_{\text{p}} - \epsilon_{\text{p}})}$$

For plane strain conditions for a unit length of plate, the

maximum membrane force corresponding to uniaxial stress-strain conditions  $(\sigma_i \epsilon_s)$  is

$$[A.14] S = \frac{\sigma_s h}{(1 - v_s)}$$

Substituting [A.14] in [A.13] gives

[A.15] 
$$M_{\odot} = \frac{\sigma_{s} h \sin^{2}\theta (1 + \epsilon_{p})}{3 \cos\theta (\epsilon_{u} - \epsilon_{p}) (1 - v_{s}^{2})}$$

and similarly for [A10]

$$[A.16] P_o = \frac{\sigma_s h \cos \theta}{(1 - v_s^2)}$$

The coexisting axial load and moment at the edge are given by the

interaction equation,

$$[A.17] \qquad \frac{M}{M_{\odot}} + \left(\frac{P}{P_{\odot}}\right) = 1$$

based on full plasticity. Because the strains are exceptionally

harge,  $M_{\rm tr}$  and  $P_{\rm u}$  are determined using  $\sigma_{\rm u}$ , the ultimate tensile strength, and an inelastic strain ratio of 0.50.

Substituting  $M_o$  and  $P_o$  of [A.15] and [A.16], respectively, in [A.17] gives

$$\left[ \frac{\sigma_{i}}{\sigma_{i}} \right] \left[ \frac{4 \sin^{2}\theta \left(1 + \varepsilon_{i}\right) \left(1 - v_{i}^{2}\right)}{3 \cos\theta \left(\varepsilon_{ii} - \varepsilon_{i}\right) \left(1 - v_{ii}^{2}\right)} \right]$$

$$+ \left[ \frac{\sigma_{ii}}{\sigma_{ii}} \right]^{2} \frac{\left(1 - v_{ii}^{2}\right)^{2}}{\left(1 - v_{ii}^{2}\right)^{2}} \cos^{2}\theta = 1$$

This equation, describing the relationship among the maximum membrane stress and the edge inclination is based on the aterial properties of the plate,  $\sigma_s$ ,  $\sigma_u$ ,  $\epsilon_u$ ,  $\epsilon_p$ ,  $\nu_s$ , and  $\nu_p$ .

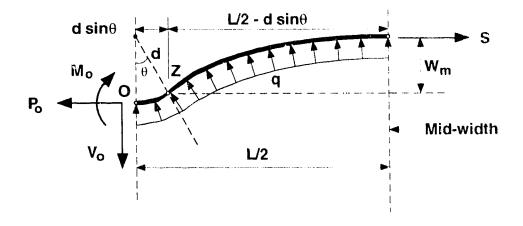


Figure A.1(a) Free body diagram of half-width of plate under fluid pressure

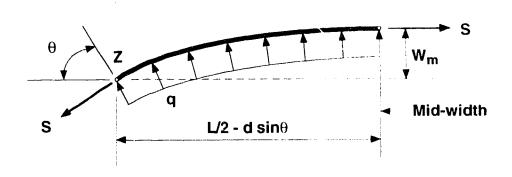


Figure A.1(b) Free body diagram of membrane under fluid pressure

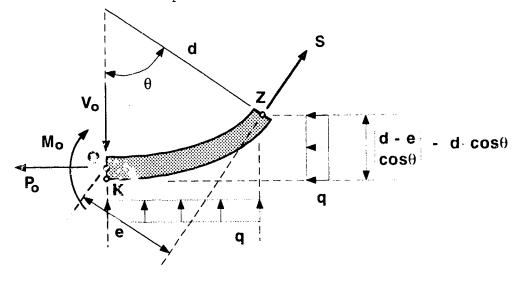


Figure A.1(c) Free body diagram of plate at edge