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THE UNIVERSITY OF ALBERTA

CHAOS AND THE ALPHA RHYTHM ELECTROENCEPHALOGRAM: A DYNAMICAL STUDY

BY

ANTHONY C. K. SOONG

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

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THE UNIVERSITY OF ALBERTA

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled *Chaos and the alpha rhythm electroencephalogram: a dynamical study* submitted by Anthony C. K. Soong in partial fulfillment of the requirements for the degree of Master of Science in Biomedical Physics.

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In memory of my father

Abstract

This thesis investigates the dynamics underlying the alpha-rhythm of the human electroencephalogram (EEG) and here reports evidence that the dynamics is chaotic as given by the modern theory of Chaos.

Within this theory, Takens' Theorem provided a means for multi-dimensional dynamical analysis of the system from a time series in only one variable. This is accomplished by reconstructing an attractor that has the same dynamical properties as the attractor underlying the original time series. Further dynamical understanding can be gained by classifying this reconstructed attractor with the correlation exponent and the largest Lyapunov exponent. These measures of dimensionality were calculated on a VAX 11/750 computer.

The calculated values of the largest Lyapunov exponents for both the filtered and unfiltered alpha-rhythm EEG were positive, thereby providing evidence that the underlying dynamics of the alpha-rhythm EEG is chaotic. Supporting evidence for chaotic dynamics is also provided by the saturation of correlation exponents with increasing phase-space dimensions. Moreover, Fourier analysis of the EEG showed that the alpha-rhythm EEG is not multiperiodic.

The results presented in this thesis suggest that the underlying mechanism cannot be as simple as supposed by the current hypothesis that the alpha-rhythm arises from the synchronization of the pyramidal cells by the thalamus. For, if we accept the synchronization hypothesis, it will be necessary to explain why these cells are being synchronized in a chaotic manner. The results also furnish an estimate of the lower bound of the system's degrees of freedom and for the dimensionality of the alpha attractor. Based upon these results, a new hypothesis on brain dynamics is presented.

Preface

Since all that is leaves us [illegible] handed,
The only return from all that is, loss and ruin,
It can be supposed that what the world has not, is,
And what it has, is not.

The Ruba'iyat of Omar Khayyam

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Chapter 1: Introduction

"...twentieth-century science will be remembered for just three things: relativity, quantum mechanics and chaos."[1]

"...and chaos is as common as daffodils in the spring"[2]

The human brain is singularly concerned with thought processes, memory, imagination, creativity, learning and consciousness (Carpenter, 1978). Besides these higher functions, all information about the surrounding environment is transmitted to the brain by sensory organs. This information is then processed, in a largely unknown manner, and actions are taken as a result. Thus the brain is a highly complex dynamical system; uniquely, it is the only system complex enough to ponder its complexity (Shipton, 1975) and it is widely thought that understanding the human brain is one of the most formidable scientific endeavors ever pursued (Gevins, 1984). Presumably, such understanding is likely to emerge only from a wide range of individual investigations. The present study investigates the dynamics underlying the alpha-rhythm of the human electroencephalogram (EEG); the results provide evidence that the dynamics is "chaotic" in the sense given by the mathematical Theory of Chaos.

As is well known, brain research has thus far been largely dominated by a reductionist approach which has been notably successful in elucidating the dynamics of the neuron. The reductionist approach, however, has not yielded insights into the dynamical behaviour of the brain as a whole because it is the interactions among the elementary dynamical units that lead to macroscopic brain behaviour. Typically these biological interactions differ from the random interactions studied in statistical mechanics where the dynamical elements are assumed to regain statistical independence after random collision-interactions. Moreover, biological systems typically show hierarchic organization and thus

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[1] J. Gleick, 1987. *Chaos: Making a New Science*, Viking, N.Y., p6.
[2] J. Ford, 1986. "Chaos: solving the unsolvable, predicting the unpredictable!", *Chaotic Dynamics and Fractals*, (M. F. Barnsley and S. G. Demko eds.), Academic Press, p 3.

have holistic properties. In other words, we cannot deduce the brain's behaviour as a total system from the dynamical properties of its elementary neural constituents (Stuart *et al.*, 1978; 1979). Instead, we must take into account the emergence of new modes of organization arising from nonlinear cooperative interactions among the dynamical constituents. Therefore the brain must be studied as a nonlinear dynamical system.

A natural variable for macroscopic investigation into the dynamical behaviour of the brain is the electroencephalogram (EEG). Introduced by Hans Berger in 1929, the EEG provides a means to quantify or describe ongoing brain activities (Shipton 1975). If all information transfers in the brain are mediated by electrical phenomena, then the EEG can be considered a measure of the activity of the brain. (A more detailed description of the EEG can be found in Chapter Two of this thesis.) The EEG, being a macrovariable, was at first expected to provide a window into the functions of the mind. Although there now exists a massive literature on the EEG, little or no insight has been gained into its underlying dynamical properties. Attempts, so far, to use the EEG as a method for understanding mental processes or as a tool in the fundamental behavioural sciences have failed (Shipton 1975). This lack of progress is due mainly to two reasons.

1) Much of the research has been phenomenological in character, directed towards the modelling of the EEG data with autoregressive and other probabilistic methods (Gevins, 1984). Though useful in clinical diagnosis, this line of research seems unlikely to illuminate the underlying dynamics of the brain. What is needed now is a move beyond phenomenological research, a move to dynamical studies of the brain.

2) Until recently, mathematical methods were not available for the study of nonlinear systems; the standard procedure was to approximate nonlinear systems by linear systems. Because biological systems are typically highly nonlinear (Stuart, 1987), the linear approximation did not give any new significant insights into the dynamical properties.

The last 15 years have seen an explosion in the field of nonlinear dynamics. The repercussions are being felt in such areas as hydrodynamics (Brandstater *et al.*, 1983), chemistry (Roux *et al.*, 1983) climatic variability (Lin and Lian, 1986; Nicolis and Nicolis, 1986; Nicolis and Nicolis, 1984), biochemistry (Markus *et al.*, 1985; 1984), ecology (Schaffer, 1985), cardiology (Glass *et al.*, 1986) and neurobiology (Babloyantz and Destexhe, 1986; Aihara and Matsumoto, 1986). The pinnacle of this explosion is a new theory that treats deterministic but 'random' systems macroscopically: the Theory of Chaos.

The main idea of this new theory is to investigate the behaviour of limit sets. In practice, only attracting limit sets ("attractors") are of interest since nonattracting limit sets cannot be observed in physical systems or simulations (Mandelbrot, 1977). An attractor is then the object on which the trajectories of a deterministic dynamical system accumulate. As we shall see later, if an attractor is sensitive to initial conditions then it is a "chaotic attractor". Two initial conditions that are arbitrarily close together on a chaotic attractor will, in time, diverge, but this implies that no matter how precisely the initial condition is known, the long-time behaviour of the system can never be predicted (Haken, 1983). This unpredictability is the hallmark of chaotic systems: deterministic 'randomness'.

At first glance, deterministic randomness appears to be a contradictory idea; a system is either deterministic or it is random. Mathematical ideas developed within this new theory, however, have now shown that there exists an important area of overlap between these two apparently polar ideas, where a system may have both random and deterministic properties. There are three possibilities how this may arise:

1) The system is originally deterministic, but with the passage of time this determinism weakens and the system eventually becomes a random system;
2) The system is originally random, but spontaneous interactions between elementary dynamical elements lead to stabilities having deterministic character.

3) Determinism and randomness coexists in the same system. i.e. the system is weakly deterministic with an element of chance embedded within it.

Intuitively, these possibilities pertain to the system's complexity. It may be hoped that chaos theory will eventually distinguish these possibilities in a systematic manner. This is an important desideratum since it would serve to clarify the notion of "complexity".

With respect to our current understanding of the human brain, one cannot assert that the brain is dynamically random, nor can one assert that it is dynamically deterministic. On the one hand, we have, within modern neural network theory, a notion of randomness associated with the brain; on the other hand, the consistent reaction of the brain to external stimuli betokens an element of determinism. It therefore seems reasonable to make the overall hypothesis that brain dynamics includes both random and deterministic features. Under such a hypothesis, the methodological appropriateness of chaos theory seems self-evident.

Clearly, we cannot expect to investigate the overall hypothesis, stated above, with respect to all brain activity. So we concern ourselves with a single phenomenon - the alpha-rhythm EEG. This phenomenon was selected because

i) it has a relatively "simple" waveform,

ii) it is a phenomenon familiar to all brain researchers,

iii) it is widely considered to reflect a "ground state" or "resting state" of the brain.

Methodologically, the study incorporates three mutually exclusive hypotheses:

1) The system is chaotic (i.e. deterministically random),
2) The system is random only, hence not chaotic,
3) The system is deterministic only, hence not chaotic,

of which only hypothesis 1 is in conformity with the overall hypothesis stated previously. The investigation should provide insights that will be useful for the construction of mathematical models, for example, the system's minimum number of degrees of freedom. This thesis, however, is in no way a thesis about mathematical models of the brain; no attempt will be made to construct a mathematical model of the brain or of the alpha-rhythm EEG. Indeed, given our present lack of knowledge as to the underlying dynamics, it would seem that the time is far from ripe enough for the construction of mathematical models capable of explaining the alpha rhythm.

Before we go on to study the dynamics of the alpha-rhythm, some account of the present theories on the physiological correlates of the EEG is necessary; this is given in Chapter Two, not as a detailed investigation into the physiology of the human brain, but rather as common ground for the discussion of the dynamical investigation that follows. It should be noted that, though the EEG is routinely used to infer and hence diagnose brain conditions, the mathematical analysis of this thesis applies strictly to the observed trace. While some biological inferences of its results are mentioned, the thesis does not pretend to reach authoritatively into the biological realm itself.

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Chapter 2: Physiological correlates of the EEG

2:1 Introduction

Most physiologists who have investigated the human brain have adopted a reductionist view of nature. Although it has led to a detailed understanding of the neuron, this method of investigation has intrinsic limitations of the kind mentioned in the introduction. This chapter will discuss current hypotheses concerning the human EEG.

2:2 Gross anatomy of the brain

Although dynamical understanding is unlikely to come from neuroanatomical considerations alone, a working knowledge of the brain's structure and interconnections is necessary for a comprehensive understanding of the EEG.

The elementary dynamical units of the human brain are the neurons. Morphologically, neurons are notable for their variety of form and size. Since the early studies of Ramón y Cajal, the three major regions of the neuron - dendrites, soma (cell body) and axons - have been regarded as standard equipment of fully developed neurons (Palay and Chan-Palay, 1977). The dendrites are the information reception regions of the cell. The information transfer is mediated by neurotransmitters at synapses (and in a more subtle manner at ephaptic junctions). Schematically, one may say that synapses, form between axons and dendrites, the former transferring information to the latter which, in turn, pass the information on to the cell body or soma. The cell body of the nerve cell consists of the neucleus surrounded by cytoplasm which contains organelles that sustain the cell: it is the metabolic and synthetic center of the cell. The axons of the neuron are the transmission lines of the cell. These spread forth from the cell body and form synapses with other neurons. Some axons are myelinated by oligodendrocytes in the central nervous system or Schwann cells in the peripheral nerves. Myelination increases the speed of transmission of the action potential but whether this is the only function of the oligodendrocytes and Schwann cells is unknown. It must be stressed that, upon closer inspection, the traditional morphological subdivision of the nerve cell into three distinct regions cannot be upheld. For example, there exist in the human brain neurons that lack a single characteristic process that can be called an axon (Palay and Chan-Palay, 1977). Nevertheless, for our purpose, the adaptation of the simpler view of three distinct regions is sufficient.

The human brain, which is constructed from neurons and other supporting cells, can be divided into three basic units: the cerebellum, the brain stem and the cerebral hemispheres. The cerebellum develops from the rhombic lip and is a derivative of the metencephalon. The brain stem can be further divided into four distinct regions: the diencephalon, the mesencephalon, the metencephalon and the myelencephalon. The diencephalon consists of the thalamus and the hypothalamus. It is the most rostral of the brain stem segments and is intimately related to the telencephalon (forebrain). The mesencephalon consists of the superior and inferior colliculli, midbrain reticular formation and cerebral peduncles. It is the smallest and least differentiated brain stem segment. The metencephalon consists of the tegmentum, and the pons. Together with the most caudal brain stem segment, the medulla oblongata (metencephalon), it forms the hind brain. The cerebral hemispheres are the massive paired lobes that developed from the telencephalon. The paired hemispheres are nearly mirror images of each other, and they consist of a highly convoluted gray cortex, the cerebral cortex, on top of a layer of massive white matter and deeply located basal ganglia.

2:3 The cerebral cortex

The neurons of the cerebral cortex are arranged in layers which are divided into three regions: archicortex, paleocortex and neocortex (Martin, 1985). The archicortex and the paleocortex contain only three cell-layers and are more primitive than the neocortex, which contains six cell-layers. The neocortex is a sheet of cells, with a surface area of about 2 200 cm^2 and a thickness of 1.3 to 4.5mm, that forms a cap over much of the cerebrum (Schmidt,1978). The estimated 10^{10} neurons within this sheet are wrinkled into a complex pattern of sulci and gyri (Carpenter, 1984).

When freshly cut, the neocortex has a striped appearance due to layers of somata alternating with layers containing mainly axons (Martin, 1985). Functionally, neurons that are related are aggregated together and interconnections within the aggregate are principally up and down along the columnar axis (Kuffer *et al.*, 1984). In layer III of the neocortex are associational neurons for intercolumn communications. Other interneurons are also present to mediate intercolumn communication but, unlike the associational neurons, they cross several layers of the neocortex. Although various methods have been used to study these interconnections, the resolutions of these studies are insufficient to determine the fine grain of neuronal interconnections.

The neocortex receives almost all of its subcortical afferents from the thalamocortical tracts (Schmidt, 1978). Inputs from other parts of the nervous system and the sense organs travel through a fine relay station at the thalamus before entering the neocortex. There is, however, one sensory pathway that does not pass through the thalamus: the olfactory tract (Guyton, 1981). Of all the afferent nerves that pass through the thalamus, most form synapses with pyramidal cells in the neocortex. Therefore, one can view the thalamus as a modifier to the output of the neocortex. But it is important to view the neocortex and the thalamus as one working unit and not to view the thalamus as merely a relay station.

The neurons forming the neocortex can be divided into two categories by their morphology: pyramidal and stellate cells (Martin, 1985). The pyramidal cells are the excitatory cells of the cortex; they project their axons to other areas of the brain and spinal cord. In particular, the main efferent outputs from pyramidal cells of layer V go to subcortical destinations (Carpenter, 1984). These cells have a resting potential of -50 to -70 mV and an action potential of approximately 100 mV. Because the hyperpolarization after an action potential is small, pyramidal cells can fire at frequencies up to 100 Hz. Their dendrites often cross several layers and are oriented mainly perpendicularly to the surface of the brain but they can also have horizontal projections. This organization enables inputs from many different layers to enter at different levels of [illegible]ndritic tree. Moreover, booster zones exist in the dendrites that enable remote e[illegible]y inputs to be actively conducted to the trigger zones of the axon.

The stellate cells are neurons with round-shaped cell-bodies. The axons of these cells terminate on nearby neurons and never leave the cortex (Martin, 1985). Thus, stellate cells form interneurons between cortical columns. Among the important types of stellate cells are the basket cells, which wrap around the soma of the postsynaptic neurons forming dense synaptic connections. Basket cells are inhibitory cells and they act to isolate one column of pyramidal cells from neighbouring columns. Another important type of stellate cells has its axons oriented vertically; they transmit information from the thalamus to interneurons or to the pyramidal cells.

Within the neocortex are cells of another type - the glia cells. Glia cells do not manifest any action potentials (Kuffler *et al.*, 1984) and are recognized as the metabolic supporting cells of the brain (although this may not be their only function). There are approximately 10 times more glia cells than there are neurons (Martin, 1985). The cellular membrane of the glia cell is sensitive to K^+; thus, they are postulated to take up the excess K^+ from the neurons during neuronal activity.

2:4 The electroencephalogram

Neuronal activities produces changes in the electric field which can be measured as electrical voltage fluctuations on the surface of the neocortex or on the surface of the scalp. The former is called the electrocortiogram, ECoG, and the latter is known as the electroencephalogram, EEG. The frequencies in the EEG range from 1 to 50 Hz, with amplitudes of 20 to 100 μV. The amplitudes of the recordings are attenuated by the scalp and the skull, as demonstrated by the amplitude of the ECoG being greater than the EEG by a factor of approximately 10 (Schmidt, 1978). The bulk of the potentials recorded from the scalp or the surface of the cortex are postulated to be the results of the extracellular current flows associated with the neuronal activities of numerous neurons (Martin, 1985). For example, it has been calculated that an electrode on the surface of the neocortex with a surface area of 1 mm^2 is in contact with 100 000 neurons down to a depth of 0.5 mm (Schmidt, 1978). With EEG recordings, the electrode is in contact with approximately ten times as many neurons.

Early authors considered that the EEG represented the summed action potentials of cortical neurons (Adrian and Yamagiva, 1935). However, by the early sixties, it was generally accepted that the EEG chiefly reflects slow changes in the membrane potentials of cortical neurons (Chang, 1951; Caspers, 1959), particularly the postsynaptic excitatory and inhibitory potentials of vertically oriented pyramidal cells (Martin, 1985). Unlike that of the motorneurons of the spinal cord, the postsynaptic potential of cortical cells lasts for a long time (Curtis and Eccles, 1959; Creutzfeldt *et al.*, 1966), but existence of activity in the deeper layers of the cortex during certain EEG waves emphasized that deeper structures were also important in the genesis of the EEG (Bishop and Clare, 1952).

There are many experiments that suggest a primary role for pyramidal cells in the genesis of the EEG. Scheibel and Scheibel (1964) demonstrated that, during ontogenesis, the EEG waves develop at a comparable time to the morphological differentiation of apical dendrites. In addition, most of the specific thalamocortical afferent fibres terminate at layers III and IV, where the pyramidal cells are located, further supporting the hypothesis that apical dendrites of the cortical pyramidal cells play an important role in EEG generation (Creutzfeldt and Houchin, 1974). The dendrites of pyramidal cells are also oriented perpendicular to the surface of the neocortex (Martin, 1985). Therefore, a synaptic potential on the dendrites is recorded with relatively little attenuation because the sources and sinks are perpendicular to the surface of the neocortex. Glia cells, on the other hand, have no specific orientation and their contributions to the EEG are probably insignificant (Martin, 1985). Further evidence that pyramidal cells are involved in the genesis of the EEG is seen through the correlation between the post-synaptic cellular depolarization and the surface negative potential. The correlation, however, is not absolute and a slight variation in the degree of correlation occurs because the thalamic relay nuclei are not absolutely synchronous (Verzeano and Negiski, 1960; Verzeano *et al.*, 1970; Andersen and Andersson 1968).

2:5 Single cell recordings

Before going on to the cellular origins of the EEG, the voltage recordings from a single neuron need to be discussed. The net ionic current generated by the cell can be recorded as a voltage across the resistance of the extracellular space. For a single excitatory post-synaptic potential, there is a current i flowing inward through the post-synaptic membrane and outward along a large area of the cellular membrane, completing the circuit. There are four resistors in the circuit: two representing the membrane resistance, R_m, one for the intracellular resistance, R_{in}, and one for the extracellular resistance, R_{ex}. For an intracellular recording, the voltage recorded, V_{in}, is given by

$$V_{in} = (R_m + R_{ex})i$$

(Martin, 1985). Since the membrane resistance is much larger than the extracellular resistance, the voltage measured is approximately the voltage drop across the membrane resistance. For an extracellular electrode, the voltage measured, V_{ex}, is simply

$$V_{ex} = i.R_{ex}$$

Because the membrane resistance is much less than that of the extracellular resistance, the voltage so measured is much smaller than that measured by the intracellular electrode. This small voltage poses a large hardware problem in extracellular recordings -- e.g. the EEG.

To record an EEG at least two electrodes are needed: one reference electrode and one or more active electrodes. The recorded voltage is then the voltage difference between an active electrode and the reference electrode. The reference electrode thus plays a key role in the voltages recorded in the EEG. Great care must be taken to choose the proper placement of the reference. There is, as yet, no recognized ideal placement of the reference electrode; any placement must be seen as interfering with the EEG. Hence a great deal of effort has been spent in the last ten years to estimate the activity of the reference electrode.

Methods such as removal of the global average and radial current density studies have their associated problems. So far little progress has been made (Nunez, 1981).

2:6 Physiological correlates of the EEG

The physiological correlates of the EEG are such that an excitatory postsynaptic potential located on the distal end of the pyramidal cell will cause a negative potential to be recorded (Martin, 1985; Creutzfeldt and Houchin, 1974). This excitatory potential will cause a site of inward current, a "sink", at the distal end of the dendrite and a flow of outward current along the large expanses of the extrasynaptic membrane. The extracellular space near the sink, at the surface of the neocortex, will then be negative and this will in turn cause a negative, upward deflection on the EEG recorder. (By convention, a positive voltage is a downward deflection.) For an excitatory postsynaptic potential located on the proximal end of the dendritic tree of the pyramidal cell, a sink will then be generated at the proximal end of the dendrite and a flow of outward current along the expanse of membrane at the distal end. This will then cause the extracellular space near the surface of the neocortex to be positive and a positive deflection will be recorded on the EEG. An inhibitory postsynaptic potential on the distal end of dendrite will cause a site of outward current, a "source", at the distal end and a flow of inward current along the large membrane expanse at the proximal end. The extracellular space near the surface of the neocortex will, therefore, be positive. Hence, the EEG recorded will be positive. On the other hand, an inhibitory postsynaptic potential at the proximal end of the dendrite causes the genesis of a source there and a flow of inward current along the expense of membrane near the distal end. The potential at the surface of the neocortex will therefore be positive and the EEG recording will show a positive deflection. It can now be seen that cortical synaptic events cannot be determined uniquely from the sign of the EEG alone (Martin, 1985). It must be stressed that so far only single cell activities have been discussed and that the EEG represents the activities of thousands of neurons.

The depth profile recordings further support this causal relationship between the EEG and synaptic activity. Depth profiles of cortical potential have shown that the polarity is reversed between the surface and deep layers of the cortex (Creutzfeldt and Houchin, 1974). This demonstrates that there is a source and sink configuration along the columnar axis of the cortex implying that the electrical activities of the pyramidal cells are involved.

Because of the importance of sources and sinks in the generation of the EEG, a large amount of effort has been spent within the last decade in locating neural sources and sinks. These enquiries appear to be of little significance since the summation of two or more spatially distinct generators gives arbitrary positions to the sources and sinks, depending on the relative strength of the generators (Creutzfeldt and Houchin, 1974).

In 1963 Jung postulated that the EEG is a result of an oscillating dipole. However, neither the dipole theory nor the dendritic potential theory fully explain the EEG phenomenon (Creutzfeldt and Houchin, 1974). The data relating cellular activity and gross activity is indirect and much more data is needed before a causal relationship can be considered.

2:7 Alpha-rhythm EEG

A specific pattern of EEG commonly observed is the alpha-rhythm. Alpha waves are associated with a state of relaxed wakefulness, and they are most predominant in the parietal and the occipital lobes of the neocortex (Martin, 1985). The main cellular correlates of the alpha-rhythm seem to be excitatory postsynaptic potentials (Creutzfeldt and Houchin, 1974). Inhibitory postsynaptic potentials appear not to be of significance during alpha episodes. This is most likely because the excitatory postsynaptic potentials during alpha often do not reach threshold and little inhibition is necessary.

Frequency analysis of the alpha-rhythm has shown that the power spectrum contains a predominantly broad peak between 8 and 13 Hz. Consequently, the alpha-rhythm is defined as the EEG recorded between 8 and 13 Hz from a relaxed but alert individual with eyes closed. This definition can be interpreted in two ways. One can interpret the alpha-rhythm as the EEG signal in which the frequency range of 8-13 Hz dominates (Figure 2.1), or one could consider the alpha-rhythm as strictly a signal of 8-13 Hz with any other frequencies present regarded as noise. With respect to the second definition, the power of the frequencies outside the 8 to 13 Hz range can be reduced significantly by band-pass filtering. A pictorial representation of the resulting signal can be found in Figure 2.2.

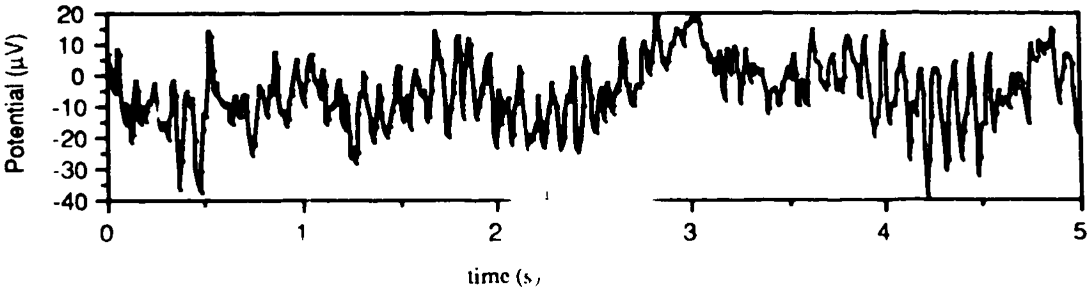

Figure 2.1: Alpha-rhythm EEG trace from a normal 45-years-old male.

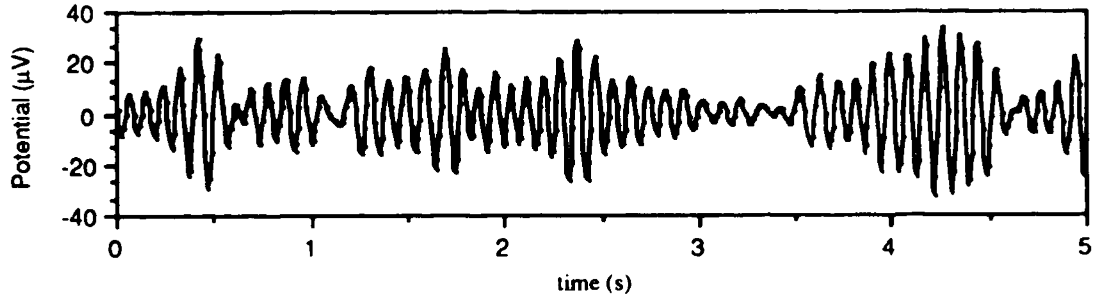

Figure 2.2: Raised cosine-filtered alpha-rhythm. The original signal used was the same as that in Figure 2.1

The synchronous appearance of the alpha-rhythm between 8 and 13 Hz has been postulated as the result of the synchronization of the neocortex by the thalamus (Guyton, 1981; Schmidt 1978; Andersen and Andersson, 1968). This postulation was based upon the experimental observation that the alpha-rhythm is destroyed with the removal of the thalamocortical pathway or the removal of the thalamus, while the rhythm of the thalamus survives the interruption of the thalamocortical pathway and the removal of the neocortex (Schmidt, 1978). Moreover, the signals from the thalamus are carried along by fibres that terminate around layers III and IV (Carpenter, 1981). When these thalamic connections are cut, the functions of the corresponding cortical area are almost entirely absent (Guyton, 1981, Creutzfeldt and Struck 1962) and the postsynaptic excitatory potential is virtually nonexistent (Watanabe and Creutzfeldt, 1966). The appearance of the alpha-rhythm thus seems highly dependent on the synchronous signals sent by the thalamus.

During the desynchronization of the alpha-rhythm (for example during periods of arousal), simultaneous recordings from nearby neurons show little or no correlation between their activities (Creutzfeldt and Houchin, 1974). Intracellular recordings show that, during desynchronization, sub-threshold postsynaptic activities still occur. Therefore, desynchronization must be due to neurons firing in a random manner and not to a lack of postsynaptic potentials. This reorganization of neuronal activity may or may not be accompanied by an increase in neuronal activity. This suggests that a possible mechanism for the disappearance of alpha and appearance of non-rhythmical slow waves under pathological conditions is the disturbance of the thalamus resulting in the loss of the alpha-rhythm (Mergenhagen *et al.*, 1968).

It is important to realize that the source of the alpha-rhythm has only been postulated to be the result of synchronization by the thalamus. The evidence today suggests that this is a possible mechanism but is by no means a proven fact. Nunez (1981) presented some reservations about this mechanism. His objection is based on six points:

1. The experimental evidence was gathered with anesthetized cats, and cats do not manifest an alpha-rhythm normally;
2. Quantitative correlation between cortical and thalamic potentials has not yet been shown;

3 Strong thalamic interactions do not necessarily imply thalamic pacemaking;

4 Isolated cortex experiments do not necessarily support the pacemaker idea because the EEG may require continuous random input;

5 The spatial EEG data do not appear to support the idea of a fixed spatial pattern;

6 Anatomical evidence shows that most of the input to a region of the cortex is from other cortical regions.

Even if the alpha-rhythm is synchronized by the thalamus, the reductionist investigations into its origins have not given a satisfactory reason for the synchronization process. Moreover, there would remain the question as to why there is a neural pacemaker and what purpose it may serve. In the case of the human heart, it is intuitively obvious that a pacemaker is necessary to coordinate the contraction of the cardiac muscle for the propulsion of the blood. In the case of the alpha rhythm, however, no such equivalent function is intuitively visible.

Lippold (1973) suggested another physiological mechanism behind the alpha-rhythm in his book, *The Origin of the Alpha-rhythm*. He postulated that the alpha-rhythm is not generated by the pyramidal cells but rather by the electrical activity generated in the extraocular muscles. This picture, however, is not consistent with the spatial location of the alpha-rhythm (Nunez, 1981). If the extraocular muscles were involved, the alpha-rhythm should be predominantly close to the orbital fissure. Instead, alpha is more prominent in the occipital regions of the brain. Furthermore, this idea contradicts volume conduction considerations of depth recordings of human alpha-rhythm (Nunez, 1981). Hogan and Fitzpatrick (1988) have shown convincing evidence that extraocular muscles activities are not the source of the alpha-rhythm. The persistence of the alpha-rhythm in isolated canine brain preparation during normoxia and its destruction during hypoxia have confirmed the neural origins of the alpha-rhythm. Moreover, EEG recordings from the isolated brain are free from extraneural influences and exclude the possibility that alpha activity is caused or influenced by the electrocardiogram, EKG, or the electromagnetogram, EMG. The hypothesis that the alpha-rhythm is caused by resonance of the intracranial tissue induced by the cerebral blood flow is also excluded because cerebral blood flow is non-pulsatile.

2:8 EEG's from other areas of the brain

Spontaneous EEG can also be recorded from the cerebellum, allocortical areas of the hippocampus and pyriform area. The rhythmic θ waves of the hippocampus are caused by rhythmical discharges of the septal nuclei (Brücke, 1959; Petsche *et al.*, 1962; 1965; Stumpf *et al.*, 1962; Apostol and Creutzfeldt, 1974). For the prepyriform cortex, it is assumed that it is driven by the olfactory bulb (Creutzfeldt and Houchin, 1974). For the cerebellar EEG, the main generators are probably the Purkinje cells.

2:9 EEG's from different species

EEG can be recorded from various species with distinctively varying patterns. The differences in the EEG across species can be attributed to the difference in the folding of the neocortex (Creutzfeldt and Houchin, 1974). The data, however, are inadequate for a comparative study.

2:10 Conclusion

From the preceding discussion, it is apparent that very little is known about the physiological origins of the EEG. The central stumbling block appears to be the transition from the microscopic picture to the macroscopic picture. With the reductionist method of investigation adopted in the past, the voltage recordings from a single neuron are well understood. But there has not been much advancement made in understanding the aggregate voltage recording from a large number of neurons (EEG). As stated in the Introduction, this lack of advancement seems to betoken an inherent limitation in reductionist methodology. A more holistic approach now seems to be possible throuth recent developments in nonlinear dynamics, but we should also consider the likelihood of success through classical dynamics.

2:12 Bibiography

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Chapter 3: Brain Dynamics

3:1 Introduction

In the nineteenth century, "all physicists ... [saw in classical dynamics] a firm and final foundation ... for all natural sciences"[1]. On that basis physicists, like biologists, adopted a reductionist view of nature. As a result, the classical method of investigation into dynamical systems is to determine the equation of motion for each elementary dynamical unit. The collected set of differential equations gives the behaviour of the dynamical system. Hence the problem of analyzing dynamical systems is reduced to the integration of these equations of motion. Since this set of equations is completely deterministic, the emphasis is on the study of exact, nonrandom, solutions. The possibility of obtaining random solutions from any deterministic system is considered absurd because randomness and determinism are contradictory ideas. Therefore, classical physics concentrates on exact solutions from integrable systems.

Unfortunately, real systems are typically non-linear, and analytically nonintegrable. Mathematics has not yet advanced sufficiently to solve many nonlinear systems analytically; the standard approach is that of linear approximation, but, with highly nonlinear systems, linear approximations is rarely effective. Therefore, many physicists in the past avoided nonlinear problems. With the advent of modern computers, numerical integration can now generate solutions as accurately as is needed to solve most practical problems. This computation, however, hinges on one assumption: that arbitrary accuracy of the solution can be calculated from arbitrarily accurate initial conditions (Tomita and Kai, 1978). In other words, states that are similar initially will be similar for all time, i.e., there is "orbital stability". As will be evident later, this assumption is not met by most nonlinear dynamical systems. The use of classical dynamics to understand complex systems, such as the human brain, is thus questionable.

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[1] A. Einstein, 1949. "Einstein's Autobiography", in *Albert Einstein: Philosopher-Scientist* (P. A. Schilpp ed.), The Library of Living Philosophers, Inc., p 21.

3:2 The beginnings of nonlinear dynamics

An indication that classical dynamics cannot be used to gain an understanding of complex dynamical systems can be found as early as 1889 in the work of Poincaré. Throughout the nineteenth century, the study of classical dynamics was dominated by a search for integrable systems. These are systems in which a canonical transformation replaces the Hamiltonian in the coordinate and the momentum variables by ones expressed solely in action-angle variables. Hamiltonians expressed in this form eliminate consideration of the potential energy. However, generalizing an earlier treatment of the 3-body problem by Hans Bruns, Poincaré showed in 1889 that the 3-body system and systems of greater complexity are analytically nonintegrable. This result was referred to as the "Poincaré catastrophe", being catastrophic in the sense that it virtually brought to a halt the further development of classical dynamics (Prigogine, 1980).

The Poincaré catastrophe demonstrated that even classically deterministic systems may exhibit extreme sensitivity to initial conditions, which contradicts the assumption of orbital stability. Hence such a system's behaviour becomes random, making prediction impossible, or, in the words of Poincaré, "we have fortuitous phenomena"[1]. In terms of perturbation theory, Poincaré's result showed that arbitrarily small deviations from a stable orbit in celestial mechanics can lead to drastically different orbits having unpredictable behaviour. This arises because resonances affect conditionally periodic behaviour, destroying the original simplicity of the motion by transferring large amounts of energy from one degree of freedom to another, ultimately leading to random behaviour.

These aberrant orbits are said to be "homoclinic" and the Poincaré catastrophe amounts to a demonstration that even Hamiltonian systems can exhibit homoclinic orbits. Poincaré was himself deeply disturbed by this finding and did not pursue the matter further.

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[1] Poincaré, H., 1903. "Science and Method", in *The Foundations of Science*, (trans. G. B. Halsted, 1946; J. McKeen Cattell, ed.), The Science Press, Lancaster, p397.

Even though nonlinear phenomena were known to be important in fluid dynamics, little progress was made in this field after the death of Poincaré. The Navier-Stokes Equation, although deterministic, is not, in general, solvable. Thus no connection was made between the onset of turbulence in fluid mechanics and the onset of randomness in celestial mechanics investigated by Poincaré. Interest in nonlinear problems then abated, to be rekindled in the 1950s in connection with the solitons, but general nonlinear methods were still undeveloped at that time.

3:3 Modern era of nonlinear dynamical analysis

The modern era of dynamical analysis began in 1963 when Lorenz observed that, in a completely deterministic system, all solutions were bounded but unstable. Ruelle and Takens (1971), without the knowledge of Lorenz's work, used the words "strange attractor" to describe this type of dynamical behaviour in turbulent flow. The field of dynamical studies continued in the 1970s with researchers working in isolated groups on different problems. Eventually, towards the latter half of the 1970s, their work began to merge together into a powerful new "Theory of Chaos".

This renewed interest in nonlinearity focussed attention on Poincaré's recognition of the close relationship between dynamics and topology. This led to the generalization of homoclinic orbits in terms of hyperbolic sets. These sets are defined on an n-dimensional phase space in which certain axes become stretched and others contracted. This situation captures Poincaré's notion of sensitivity to initial conditions because points initially close together on a hyperbolic set may become widely separated as a result of the topological distortion induced by this stretching and contraction. It is precisely this topological distortion of an attractor that makes its behaviour chaotic.

3:4 Conclusion

Attempts at understanding the human brain via methods of classical dynamics have not been very successful, and it is at least plausible that the lack of success arises from the classical requirement of understanding dynamical systems through analytical solutions of deterministic equations. It remains to be seen whether the new theory of chaos will prove more able to unlock some of the brain's secrets, but there are reasonable grounds for exploring this possibility.

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Chapter 4: Chaos Theory

4:1 Introduction

Apart from invasive procedures, the only measures of brain activity are the electroencephalogram (EEG) and the magnetoencephalogram (MEG). The MEG, however, is available in only a few centres and is technically very difficult to achieve. The EEG, on the other hand, is a very common clinical technique. But it is a time-series of only one variable - the electrical potential at the surface of the scalp - and very little is known about its underlying dynamics.

The alpha-rhythm EEG is considered to represent an alert but relatively quiescent mental state of the brain. Some general inferences about the alpha-rhythm can be made from phenomenological evidence. In the case of the well known phenomenon of alpha-blocking, there is the circumstance that the rhythm typically returns if external stimulation or purposeful mental activity is abated. Thus, so far as an alertly restful mental state is concerned, it seems reasonable to regard the alpha-rhythm as corresponding to an equilibrium condition to which the system returns following transitory perturbations. This seems to correspond to an attractor state in chaos theory. In other words, if the alpha state arises from a dynamical system with d degrees of freedom, then it can be modelled by a set of d differential equations and the dynamics of the system will stabilize on an attractor, which is a bounded subregion of the d-dimensional phase-space. For the alpha-rhythm, however, the degrees of freedom of the system are unknown. This chapter will explore how modern developments in nonlinear dynamical theory (as introduced in section 3:3) can be used to gain insights into the dynamical behaviour of the alpha-rhythm EEG, with particular emphasis on characterizing the attractor and determining the minimum number of degrees of freedom for the system.

4:2 Reconstruction of the attractor

Since the EEG is a time-series in only one variable, an obvious question is whether multi-dimensional dynamical understanding can be obtained from the EEG alone. Taken's theorem gives a method for the reconstruction of the multidimensional phase-space and the attractor from a single time-series (Takens, 1981). It is important to understand that this reconstruction does not recreate the underlying attractor; it constructs another attractor that has the same dynamical properties. Consider, for example, a digitized time-series $\{x(t), x(t+\tau), x(t+2\tau), \ldots x(t+n\tau)\}$. From this time-series and for integers f and m much less than n, vectors of f-dimension can be constructed:

$$\begin{aligned}
\mathbf{x}(t) &= \{x(t), x(t+m\tau), x(t+2m\tau), \ldots x(t+(f-1)m\tau)\},\\
\mathbf{x}(t+m\tau) &= \{x(t+m\tau), x(t+2m\tau), x(t+3m\tau), \ldots x(t+fm\tau)\},\\
\mathbf{x}(t+2m\tau) &= \{x(t+2m\tau), x(t+3m\tau), x(t+4m\tau), \ldots x(t+(f+1)m\tau)\},\\
&\ldots
\end{aligned} \tag{4.1}$$

m being referred to as the time-delay. Takens (1981) proved that there exists a deterministic dynamical system, F, such that a one-to-one correspondence exists between the dynamical properties of $\mathbf{x}(t) = F(\mathbf{x}(t + m\tau))$ and the dynamical properties of the system that generated the time-series, in particular, with invariant dimensionality and Lyapunov exponents. Theoretically, almost any time-delay (m) can be used, but there are three practical limitations (Parker and Chua, 1987; Froehling *et al.*, 1981). If m is too small, then the attractor will be restricted to the diagonal of the phase-space. If m is too large, then the structure of the attractor disappears. If m is too close to some period of the system, then the components of that period will be under-represented in the reconstruction.

When plotted on the f-dimensional phase-space, the set of vectors gives a digitized sample of the phase-space trajectory. Alternatively, these vectors can be the result of an f-dimensional difference equation. Thus, the dynamics of the EEG can be analyzed as a set of difference equations instead of a set of differential equations. For an example of a difference equation, consider a 1-dimensional nonlinear difference equation (or mapping) of the form

$$X_{t+1} = F(X_t), \tag{4.2}$$

the simplest of which is the logistic equation,

$$X_{t+1} = \alpha X_t(1 - X_t). \qquad (4.3)$$

For nontrivial dynamics, Equation 4.3 must be restricted to $0 < X < 1$ and $1 < \alpha < 4$. A natural question to ask at this point is the location of fixed points or equilibrium points (x). In other words, we wish to determine points of fixed dynamics, points where $X_{t+1} = X_t$. This can be done easily by solving the equation

$$x = \alpha x(1 - x). \qquad (4.4)$$

In this case, two such points exist: the trivial point 0 and the point $x_1 = 1 - 1/\alpha$. It is also of some interest to determine periodic points of period p. For example, periodic points of period 2 are points satisfying the following equation

$$x_2 = F o F(x_2). \qquad (4.5)$$

For convenience, we introduce the notations $F^{(2)}$ to denote the composite function FoF, $F^{(3)}$ to denote FoFoF, *et seq.*. Fixed points, we might note, are just a special case of periodic points.

The question arises as to the stability of the periodic points. Following Devaney (1986), a periodic point of period p, x_p, is stable (an attractor) if $|F^{(p)\prime}(x_p)| < 1$ (where $F^{(p)\prime}(x_p)$ is the derivative of $F^{(p)}(x)$ with respect to x evaluated at x_p), or is unstable (a repeller) if $|F^{(p)\prime}(x_p)| > 1$. It is now apparent that as α increases from 1 to 3, the equilibrium point, x_1, is stable. As α further increases beyond 3, that point becomes unstable but two new stable periodic points of period 2 are born. As the parameter continues to increase, the stable periodic points of period 2 become unstable and bifurcate to give four initially stable periodic points of period 4. These in turn will become unstable and give way to stable periodic points of period 8 which bifurcate to period 16, 32, ... 2^n upon further increases in α. However the increment of parameter α where any one cycle is stable become progressively smaller (May, 1976). Therefore, even though this bifurcation process is an infinite process, it will be bounded above by some critical parameter value. In other words, α will converge onto this upper bound with the rate of convergence and the quantitative location of the stability set given by universal constants - the Feigenbaum numbers

(Feigenbaum, 1979; Feigenbaum, 1977). For the case of equation 4.3 this upper bound occurs at the parameter value of 3.5700... (May, 1976). As the parameter α is increased beyond this point, there appears an infinite number of periodic points with periods of all value. There also exists here an uncountable number of initial conditions that never lead to periodic behaviour but are nevertheless bounded. This is the region where chaos can be found. Thus we have the following definition of chaos

> A map $F:V \rightarrow V$ is chaotic if F has sensitive dependence on initial conditions, F is topologically transitive and periodic points are dense in V (Devaney, 1986).

This transition from periodicity to chaos is called the period-doubling route to chaos. May (1976) gives an excellent introductory account of the finer structure of the chaotic regime.

One of the more powerful methods for studying chaotic dynamics is to determine whether the map is topologically conjugate to maps of known dynamical behaviour because mappings that are topologically conjugate are completely equivalent in terms of their dynamics. Two maps, $f:A \rightarrow A$ and $g:B \rightarrow B$, are then said to be topologically conjugate if there exists a homeomorphism h:A [illegible] such that $h \circ f = g \circ h$. It can be easily shown that for $\alpha = 4$, Equation 4.3 is topologically conjugate to

$$Y_{t+1} = 2Y_t \pmod 1), \tag{4.6}$$

where

$$X_t = \sin^2 \pi Y_t \tag{4.7}$$

is the topological conjugacy map. The proof of existence and uniqueness for equation 4.6 is trivial (Ford, 1986) and the solution is

$$Y_t = 2^t Y_0 \tag{4.8}$$

where Y_0 is the initial condition of the system. The initial condition can also be written as a binary digit string in sequence space, Σ_2, which is then the sequence space on the two symbols 0 and 1 if

$$\Sigma_2 = \{s = (s_0 s_1 s_2 s_3 \ldots) | \; s_j = 0,1\}. \tag{4.9}$$

It then becomes apparent that equation 4.6 is just a shift map: a map $\sigma:\Sigma_2 \rightarrow \Sigma_2$ given by $\sigma(s_0s_1s_2s_3...) = s_1s_2s_3...$. Hence, the shift map deletes the first entry of the sequence.

What happens to determinism (existence and uniqueness of solutions) for equation 4.6 after the transformation to symbol space? It is apparent that determinism takes on meaning only if we can specify the initial condition to arbitrary accuracy by some experimental or theoretical means. But is this possible? The binary sequence representing the initial condition could also be a representation of a semi-infinite coin toss for which 0 represents heads and 1 represents tails (Ford, 1986). The set of all initial conditions then represents the set of all possible random coin toss sequences. Since there does not exist a finite algorithm or experiment that can compute or measure random digit strings, the initial condition cannot be constructed with arbitrary accuracy. This implies that the solution to equation 4.3 cannot be constructed. Thus we arrive at a critical hallmark of chaos: deterministic randomness (hence the usage of the words 'deterministic chaos' to describe this phenomenon): deterministic because equation 4.6 satisfies an existence and uniqueness theorem, random because the future behaviors of the system are the results of random processes. This led Ford (1986) to equate chaos with randomness. But he stressed that the word random must not be assumed to imply only a uniform probability distribution. Chaos, for example, does not preclude loaded dice. The association of chaos with randomness, however, complicates the issue because of preconceived ideas about randomness and chaos will hamper understanding these deterministic but 'random' systems. It might be more appropriate to use a completely new word to describe random-looking solutions from deterministic systems.

From the preceding discussion, it might be supposed that, with a knowledge of the set of difference equations for the EEG, mathematical tools within the theory of chaos can be used to gain some understanding of the dynamical behaviour of the system. But in the case of the EEG this set of equations is still unknown. Nevertheless, insights into the dynamics might still be gained by characterizing the reconstructed attractor. As will be evident later, this characterization will also determine the number of degrees of freedom for the dynamical system underlying the EEG.

4:3 Classification of attractors

4:3.1 What is an attractor?

An attractor is a compact subregion of the phase-space for which phase-space trajectories that begin initially within some neighbourhood of the subregion will, in time, be within this subregion. An attractor therefore attracts nearby phase-space orbits. To define an attractor precisely, let S be a compact metric space, f a homeomorphism and A a subset of S which is positively invariant under f. A is then an attractor if

1. there exists a neighbourhood U of A such that

$$A = \bigcap_{t \geq 0} f^t(U)$$

and

2. $f|_A$ is topologically transitive

(Steeb and Louw, 1986). Different types of attractors are known, the simplest being a point attractor and the most complex a chaotic attractor. Attractors which have fractional (fractal) dimensionality are known as "strange attractors". Most strange attractors are chaotic. A chaotic attractor is one that shows sensitive dependence on initial conditions. In this way, chaotic behaviours of attractors then refer to the underlying dynamics and not to the geometry of the attractor. Chaotic attractors represent nonperiodic behaviour, and so, the phase-space trajectories never cross; but they do come arbitrarily close to each other. Geometrically the orbit is an infinitely long line confined in a finite area. The neighbourhood of an attractor in which orbits are attracted is known as the "basin of the attractor".

4:3.2 Classifying attractors

The question then arises as to what are the dynamical properties of the reconstructed EEG attractor? In order to answer this question, the reconstructed attractor needs to be classified. The central concept in classifying attractors is that of their complexity, which is usually discussed in terms of the attractor's dimensionality. Traditionally, dimensionality is identified with the number of degrees of freedom of a system. This traditional view, however, is not enough for attractor classification because an attractor's dimensionality also refers to the topology of the attractor and the complexity of the phase-space trajectories on the attractor. Thus, classification of the attractor gives insights into the complexity of the dynamics and the topology of the attractor. From another point of view, the dimension of an attractor is the amount of information necessary to specify the position of a point on the attractor to a given accuracy (Barnett and Chen, 1988). Methods for attractor classification are still at an early stage of development and so there exists, in literature, many different measures of dimensionality. These measures can be separated into two groups: those that require only a concept of distance (a metric) and those that require both a metric and a probability (natural) measure. The following definitions of dimensionality are as given by Farmer *et al.* (1983).

4:3.3 Metric dimensions

Two concepts of dimensionality can be defined for sets which have a metric: Hausdorff dimension and Kolmogorov capacity.

4.3.3a Hausdorff dimension

A familiar measure of dimensionality is the Hausdorff dimension, D_H, introduced in 1919. Unlike the dimensionality of regular geometric objects, the dimensions of chaotic sets can be noninteger. An intuitive idea of the Hausdorff dimension can be obtained by considering a regular geometric object such as a cube. If each side of the cube is linearly doubled, the volume of the cube will be eight times larger than the original one because $2^3 = 8$. More generally, if each spatial dimension of an arbitrary D-dimensional object is multiplied by j, its volume will be $k = j^D$ times that of the original. Reversing this, the dimension of an object can be taken as

$$D = \frac{\ln(k)}{\ln(j)} \tag{4.10}$$

In this way, D is no longer restricted to integer values.

To define the Hausdorff dimension precisely, cover the p-dimensional set (the attractor) with a collection of p-dimensional cubes of variable sides ε_i and define

$$l_d(\varepsilon) = \inf \sum_i \varepsilon_i^d , \tag{4.11}$$

and

$$l_d = \lim_{\varepsilon \to 0} l_d(\varepsilon) , \tag{4.12}$$

where the infimum is to be understood to extend over all possible coverings subject to the constraint that $\varepsilon_i \leq \varepsilon$. It can be shown that there exists a unique d^* such that

$$\begin{aligned} d < d^* &\Rightarrow l_d = \infty \\ d > d^* &\Rightarrow l_d = 0 . \end{aligned} \tag{4.13}$$

This unique d^* is called the Hausdorff dimension (i.e. $d^* = D_H$).

4:3.3b Kolmogorov capacity

The Kolmogorov capacity, D_C, is similar to the Hausdorff dimension. To calculate the Kolmogorov capacity, one employs a box-counting algorithm. The phase-space is divided into small cubes of size ε and one counts the number of cells, $N(\varepsilon)$ that contain at least one point of the attractor. The Kolmogorov capacity is then given by:

$$D_c = \lim_{\varepsilon \to 0} \frac{\log\left[N(\varepsilon)\right]}{\log\left[\frac{1}{\varepsilon}\right]} . \tag{4.14}$$

If the location of the $N(\varepsilon)$ cubes covering the attractor are specified, then the location of the attractor is specified to an accuracy ε (Farmer *et al.*, 1983)The Kolmogorov capacity thus measures the amount of information necessary to specify the location of the attractor to a given accuracy, ε, because, for small ε, $\log[N(\varepsilon)] = D_c \log[\varepsilon^{-1}]$.

4:3.3c Fractal dimension

Current evidence tends to support the conjecture that the Hausdorff dimensions and the Kolmogorov capacity take on a common value (Farmer *et al.*, 1983). Under such a conjecture, this common value is called the fractal dimension (D_F). This definition of the fractal dimension is different from that given by Mandelbrot (1982). In his book *The Fractal Geometry of Nature,* Mandelbrot defines the fractal dimension as the Hausdorff dimension regardless of any commonality with Kolmogorov capacity.

4:3.4 Probabilistic dimensions

In order to characterize the dynamical properties of an attractor, one must also take into account the density of trajectories within the attractor. To this end, a measure of the relative frequency with which an orbit visits different parts of the attractor, the "natural measure", ($\mu(c)$), must be introduced.

For a precise definition of the natural measure, let $\mu(x,C)$ be the fraction of time that the trajectories originating from an initial condition, x, in the basin of attraction spend in a cube, C, of the attractor. In particular,

$$\mu(x,C) = \lim_{\tau \to \infty} \mu_\tau(x,C) \tag{4.15}$$

where $\mu_\tau(x,C)$ is the fraction of some finite time τ that the trajectories spend in C. If almost every x gives the same $\mu(x,C)$, then the natural measure of the attractor is equal to $\mu(x,C)$. The natural measure thus measures the relative time-averaged probability of occupancy by the trajectories at different regions of the attractor.

If an attractor has a well defined metric and a natural measure, it can be characterized by four other measures of dimensionality: the information dimension, the θ-capacity, the θ-Hausdorff dimension, and the pointwise dimension.

4:3.4a Information dimension

As introduced by Balatoni and Renyi (1956), the information dimension, d_I, is given by

$$d_I = \lim_{\varepsilon \to 0} \frac{I(\varepsilon)}{\log\left[\frac{1}{\varepsilon}\right]} \tag{4.16}$$

where

$$I(\varepsilon) = \sum_{i=1}^{N(\varepsilon)} P_i \log\left[\frac{1}{P_i}\right] , \tag{4.17}$$

$$P_i = \mu(C_i) \tag{4.18}$$

and C_i is the $i^{\underline{th}}$ cube of side ε. If $I(\varepsilon) < \log|N(\varepsilon)|$, as is the case for unequal probabilities, then $D_C \geq d_I$.

In information theory, $I(\varepsilon)$ is the amount of information necessary to describe the state of a system to an accuracy ε (Shannon, 1948). In other words, it is the information gained in making a measurement that is uncertain by an amount ε. For small ε,

$$I(\varepsilon) \approx d_I \log\left[\frac{1}{\varepsilon}\right] . \tag{4.19}$$

Consequently, the information dimension quantifies the speed at which the information necessary to specify a point on the attractor increases as ε decreases.

4:3.4b θ-capacity

The θ-capacity is, essentially, the capacity of that part of the attractor which has the highest probability of being visited by a trajectory. Consequently, consider a quantity $N(\varepsilon;\theta)$ which is the minimum number of cubes of side ε needed to cover at least a fraction, θ, of the natural measure of the attractor. The θ-capacity, $d_c(\theta)$, is given by

$$d_c(\theta) = \lim_{\varepsilon \to 0} \frac{\log\left[N(\varepsilon;\theta)\right]}{\log\left[\frac{1}{\varepsilon}\right]} . \tag{4.20}$$

It is then obvious that the $N(\varepsilon;\theta)$ cubes must be chosen such that their combined natural measure is greater than or equal to θ.

4:3.4c θ-Hausdorff dimension

As the θ-capacity is related to the Kolmogorov capacity, the θ-Hausdorff dimension is related to the Hausdorff dimension. Let

$$I_d(\varepsilon;\theta) = \inf \sum_i \varepsilon_i^d \ , \tag{4.21}$$

where the infimum extends over all possible $\varepsilon_i < \varepsilon$ which covers a fraction θ of the total probability of the set. There then exists a value of d and a d_0 such that

$$\begin{aligned} I_d(\theta) &= \infty \qquad d < d_0 \\ I_d(\theta) &= 0 \qquad d > d_0 \end{aligned} \tag{4.22}$$

where

$$I_d(\theta) = \lim_{\varepsilon \to 0} I_d(\varepsilon;\theta) \ . \tag{4.23}$$

The value d_0 is then known as the θ-Hausdorff dimension ($d_H(\theta)$). In this way, the θ-Hausdorff dimension is a probability dimension that is based upon the Hausdorff dimension.

4:3.4d Pointwise dimension

Another probability dimension in use is the pointwise dimension, d_p. It measures the rate of decrease of the probability that a trajectory will visit a sphere as the radius of the sphere decreases. Expressly, the pointwise dimension is the exponent with which that probability decreases. To define the pointwise dimension, let

$$d_p(x) = \lim_{\varepsilon \to 0} \frac{\log\left[\mu(B_\varepsilon(x))\right]}{\log[\varepsilon]} \ , \tag{4.24}$$

where μ is the natural probability measure on the attractor and $B_\varepsilon(x)$ is a sphere with radius ε centered about x. If $d_p(x)$ is constant for all x with respect to the measure μ, then $d_p(x)$ is the pointwise dimension and $d_p = d_p(x)$.

4:3.4e Dimension of the natural measure

Current evidence is in agreement with the conjecture that all the dimensional measures that depend upon both metric and probability properties take on a common value (Farmer *et al.* 1983). If this conjecture is satisfied for a particular dynamical system, then this common value is called the dimension of the natural measure (D_μ). Typically, the measure $D_\mu \leq D_f$. A more detailed discussion and review of the relationships between these measures can be found in Farmer *et al.* (1983)

4:4 Practical measures of dimensionality

4:4.1 Correlation exponent

The calculation of most of the measures of dimensionality defined in section 4:3 are time consuming and almost impossible for phase-space dimensions larger than two. A more efficient method of classifying the reconstructed attractor from an experimental time-series in one variable is the correlation exponent, υ, as defined by Grassberger and Procaccia (1983a; 1983b; 1983c). It can be shown that the correlation exponent is a lower bound of the Hausdorff dimension (Grassberger and Procaccia; 1983c). The error associated with this calculation is discussed by Holzfuss and Mayer-Kress (1986).

To enable us to define the correlation exponent, consider a time-series $\{x(t), x(t+\tau), x(t+2\tau), \ldots x(t+n\tau)\}$. Let $\underline{x}_i(t)$ be a point in d-dimensional phase-space with coordinates $\{x(t_i), x(t_i+m\tau), \ldots x(t_i+(d-1)m\tau)\}$. The "correlation integral" is then given by

$$C(r) \equiv \lim_{n \to \infty} \frac{1}{n^2} \sum_{i,j=1}^{n} \Theta(r - |\underline{x}_i - \underline{x}_j|)$$

$$\equiv \int_0^r d^d r' \; E(\underline{r}') \qquad (4.25a)$$

where $\Theta(x)$ is the Heaviside function and $E(\underline{r}')$ is the "standard correlation function"

$$E(\underline{r}') \equiv \lim_{n \to \infty} \frac{1}{n^2} \sum_{i,j=1}^{n} \delta(\underline{r}' - \underline{x}_i - \underline{x}_j) \qquad (4.25b)$$

(Grassberger and Procaccia, 1983a; 1983b; 1983c). The correlation integral is said to measure the spatial correlation of the points on the attractor. For small values of r,

$$C(r) \propto r^{\upsilon} \tag{4.26}$$

where υ is called the correlation exponent. As shown, though not rigorously, by Grassberger and Procaccia (1983a; 1983b, 1983c), this correlation exponent is also a lower bound for the Hausdorff dimension and the information dimension.

However, the Grassberger and Procaccia definitions of C(r) as the correlation integral and E($\underline{r}'$) as the standard correlation function may be misleading. The double sum in equation 4.25a counts the number of pairs of points that are within a distance r from each other; n^2 represents the total number of possible pairs. Thus,

$$\frac{1}{n^2} \sum_{i,j=1}^{n} \Theta(r - |\underline{x}_i - \underline{x}_j|)$$

is just the probability that a pair of points will be within a distance r from each other. In other words, C(r) is a probability distribution function. We recall now the definition of the probability density function, f(x)

$$f(x) = \frac{d(F(x))}{dx} \quad \text{or} \quad F(x) = \int_{-\infty}^{x} f(u)\, du$$

where F(x) is the probability distribution function[1]. Then, by the integral relationship between C(r) and E($\underline{r}'$) in equation 4.25a and the definition of E($\underline{r}'$) in equation 4.25b, it is immediately apparent that E($\underline{r}'$) satisfies the three properties of probability densities: in particular

$$E(\underline{r}') \geq 0, \quad \text{for all } x,$$

$$\int_{-\infty}^{\infty} E(\underline{r}')\, d^d r' = 1,$$

---

[1] The term probability distribution function is used in mathematical literature. Statisticians currently use the term cumulative distribution function. An older term, the frequency function is a synonym for the density function (Feller, 1968).

and for any a,b ($a \le b$)

$$p[a < r \le b] = \int_a^b E(r')\,dr'.$$

Thus, C(r) is a probability distribution function and E(r') is the probability density function associated with that distribution.

The correlation exponent can distinguish between signals from a deterministic chaotic system and a stochastic system (Grassberger and Procaccia, 1983c). If the attractor is a chaotic attractor then υ saturates at some embedding phase-space dimension, while random noise induces no saturation. The phase-space dimension at which saturation occurs may be taken as defining the minimum number of variables, or the minimum number of differential equations, necessary to describe the dynamics of the system.

Moreover, any time-series such as an EEG, obtained from an experiment, will be contamina[illegible] some experimental noise, but the correlation exponent can disentangle the chaotic attractor from the random noise. The algorithm for determining υ from a noisy signal is as follows. First construct a plot of log[C(r)] versus log[r] for values of phase-space dimension larger than the saturation dimension, which will produce a fan-shaped plot. For length scales larger than the noise strength, all curves are linear with slope equal to υ, while the slope will be equal to d for length-scales less than the noise strength, the transition from a slope of d to a slope of υ occurring at the same length scale for all the curves. The position of this break is of the order of the noise strength. Thus, this method not only provides a method of characterizing the attractor but also gives an estimate of the level of random noise in the experimental system (Ben-Mizrachi, 1984).

4:4.2 Lyapunov exponents

Lyapunov exponents are the logarithms of Lyapunov numbers; the latter measure the average stability properties of an orbit on an attractor. This circumstance provides a quantitative basis for classifying attractors. In addition, there is a connection between average stability properties and dimension. The Lyapunov exponents, moreover, provide an especially effective method for calculating an attractor's dimensionality (Farmer *et al.*, 1983; Wolf *et al.*, 1985).

As regards the classification of attractors, the Lyapunov exponents measure the average exponential rates of divergence (positive exponent) or convergence (negative exponent) of nearby orbits in phase-space. In the presence of an attractor, phase-space trajectories will converge toward a subset of the phase-space but, on the attractor itself, neighbouring trajectories may diverge. Points close together in phase-space correspond to nearly identical states; therefore, if the trajectories of initially nearby points move apart, the corresponding states become different. The rate at which such separation grows therefore expresses the extent to which the system's dynamical behaviour is sensitive to small differences between the initial states. In this way, exponential divergence, manifested by a positive Lyapunov exponent, represents extreme sensitivity to initial conditions; a positive Lyapunov exponent, representing exponential divergence, thus serves as a functional definition of chaos (Farmer *et al.*, 1983; Wolf *et al.*, 1985). Therefore, any dynamical system containing at least one positive Lyapunov exponent is chaotic and any attractor containing one positive Lyapunov exponent is a chaotic attractor. Exponential expansion of even one principal axis, however, is not consistent with the idea of bounded attractor states. Hence, some sort of folding process must occur in the phase-space to bring widely separated trajectories close together on a chaotic attractor. It is this folding of the phase-space that makes chaotic systems unpredictable and their orbits homoclinic.

A good account of Lyapunov exponents is given by Wolf *et al.*, (1985). Given a continuous dynamical system in an n-dimensional phase-space, the time evolution of an infinitesimal n-sphere of initial conditions is monitored. The sphere, in time, will deform to an n-ellipsoid because the growth of the sphere is not, in general, the same for all dimensions. Hence if $P_i(t)$ be the length of the ellipsoidal principle axis at time t, the $i^{\underline{th}}$ dimensional Lyapunov exponent, λ_i, is given by

$$\lambda_i = \lim_{t \to \infty} \frac{1}{t} \log \left[\frac{P_i(t)}{P_i(t_0)} \right] . \tag{4.27}$$

It must be stressed that one cannot speak of a well-defined direction of the principal axis because which of the axes is principal changes with time.

The sum of the Lyapunov exponents is thus the time-average divergence or convergence of phase-space trajectories. Any dissipative system will have at least one negative exponent and the sum of the exponents will be negative (Wolf *et al.*, 1985). Since the sum of the Lyapunov exponents corresponds to the time-average divergence (or contraction) of phase-space velocity, a negative sum then corresponds to an average contraction of the phase-space. Thus the post-transient motion of the trajectories will occur on a zero-volume limit-set which is known as an attractor.

There is also a close link between Lyapunov numbers and information theory since the Lyapunov number measures the rate at which the system creates or destroys information. The negative exponents correspond to the loss of information and the positive exponents correspond to the growth of information.

4:5 Conclusion

The preceding sections have assembled all the necessary apparatus to analyze an experimental time-series. In particular, the correlation exponent and the Lyapunov exponent will be used in the next two chapters to characterize the reconstructed attractor from the alpha-rhythm time-series. Evidence will be provided showing that the underlying dynamics of the alpha-rhythm EEG is chaotic.

4.6 Bibliography

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Chapter 5: Chaos and the alpha-rhythm EEG

5:1 Introduction

Although nearly 60 years have passed since Berger introduced the EEG, little progress has been made in understanding the underlying brain dynamics (Gevins, 1984). For example, whereas a system's time evolution is traditionally studied within a phase-space having dimensions equal to the system's dynamical degrees of freedom, we do not know the degrees of freedom for the dynamics underlying the EEG. However, it is now well established that the time-delay transformation of an experimental time-series in one variable yields coordinates for the construction of an appropriate phase-space (Grebogi *et al.*, 1987; Babloyantz and Destexhe, 1986; Babloyantz *et al.* 1985; Wolf *et al.*, 1985; Grassberger and Procaccia, 1983a; 1983b; 1983c; Takens, 1981). In these developments, a central concern has been the characterization of attractor states. This approach was used here in a preliminary study of the human EEG alpha-rhythm, and the results provide preliminary evidence that the alpha-rhythm exhibits the properties of a chaotic attractor.

5:2 The chaotic approach

5:2.1 Method of time-delay reconstruction

In this method, the time-series (in, say, the electric potential, V) is expressed as a sequence of N points separated by a fixed sampling interval, τ. For example, a 1-dimensional phase-space can be constructed from the deterministic map, $V_{t+k\tau} = f\,(V_{t+(k-1)\tau})$, where τ is the fixed sampling interval, $k = 1,2,...,N-1$, and t is the selected initial time. The map f provides a representation in discrete time of a dynamical system which, in continuous time, is given by a first-order differential equation.

More generally, we can construct an n-component vector, $V_1 = [V(t), V(t+\tau), V(t+2\tau),..., V(t + (n-1)\tau)]$ by taking the first n terms of the discrete-time-series, where $n \ll N$. A second n-dimensional vector, V_2, is then constructed from the terms $[V(t+\tau), ..., V(t+n\tau)$. By continuing this process, we obtain a series of time-delayed vectors, V_i, $(i = 1, 2, ..., N-n)$, each a state-vector determining a point in n-dimensional phase-space, so that the series forms a dynamical, discrete time trajectory in n-dimensional space. If the system, in time, converges on an attractor (i.e. a stable subregion of phase-space), then the dynamics underlying the original time-series is deterministic in character. It is a remarkable fact, established by Takens' theorem (Takens, 1981), that the attractor reconstructed by the method of time-delayed coordinates is dynamically equivalent to that underlying the original time-series. If the EEG alpha-rhythm can be viewed as an attractor, we can therefore expect the time-delay method to be applicable to its investigation.

5:2.2 The validity of the chaotic approach

In order to investigate the validity of the chaotic approach, consider first the traditional approaches to EEG studies, though it must be emphasized that it would be beyond the scope of this thesis to enter into a detailed discussion or general review of this vast area; Nunez (1981), Gevins (1984; 1987) provide more detailed accounts.

5:2.2a Traditional approaches to EEG studies

From a historical point of view, one of the first types of analyses performed on the EEG was frequency analysis. At present, two distinctive types of analytical methods have been used to obtain information about the frequency content of the background EEG (Gevins, 1987). The first method measures the zero crossings of the EEG and sometimes also the zero crossings of its first and second derivatives. The number and properties of the wave, such as peak amplitude or area under the curve, are then tabulated with respect to times between zero crossings. By presenting the data in this manner, one can obtain a rough approximation of the power spectrum. The second and more commonly used method applies Fourier analysis to the EEG time-series. Mathematically, this process decomposes the EEG as a family of sines and cosines. It is, however, important not to interpret this decomposition to mean that the EEG reflects separately generated sines and cosines, but only that it can be mathematically represented this way. Nevertheless, this decomposition allows one to construct the power spectrum of the signal. Hence, the processes of computing the power spectrum are collectively known as spectral analyses. The use of spectral analysis has been shown to be useful in comparing short samples of data from patients against age-matched normative values (John *et al.*, 1987), the quantification of drug effects (Gevins and Cutillo, 1987), metabolic effects (Smith, 1987), and various disease states (Gotman, 1987).

With respect to the alpha-rhythm, the most popular method of power spectrum estimation is the Fast Fourier Transform (FFT). The broad peak between 8 and 13 Hz resulting from these analyses is known as the alpha power spectrum. The precise location of this peak differs from subject to subject and, for the same subject, it may change over time (for more detail see Nunez, 1981).

The FFT, however, produces a power spectrum with high variance. Whether this variance is actually part of the data or an artifact due to the application of the algorithm is unclear at the present time. As a result, calculation of the power spectrum from autoregressive (AR) models is becoming more popular because it tends to give a smoother spectrum. For stationary signals and long data segments, the FFT and AR methods give the same results. For shorter data segments, the AR method seems better since it does not attempt to estimate as many parameters as the FFT method (Gevins, 1987). We will discuss the AR model in more detail later.

As to the stationary nature of the EEG, the human brain is a dynamical system interacting with the environment, and so the EEG cannot be considered stationary for long periods of time. Even for short periods of time, the brain cannot be considered stationary because there is too much internal brain activity. Nevertheless, statistical tests of stationarity have shown the EEG being stationary for several seconds to several minutes (McEwen, 1975; Gasser, 1977; Sugimoto *et al.*, 1977). Therefore, the question as to the exact length of time the EEG is stationary is still an open question.

Recently, a large amount of work has been done in the spatial analysis of the EEG. In order to obtain spatial information about the EEG, multichannel recordings are necessary. Because the potential at one point on the scalp is only defined relative to the potential at another point, the electric potential at a given point on the scalp is ambiguous. Consequently, spatial analysis was introduced to provide a method of obtaining unambiguous statements of the electric field that are reference-independent (Lehmann, 1987). These statements can then be analyzed to give functional-physiological interpretations of the data. The simplest analysis that can be preformed on the spatial data is the construction of the equipotential maps of the EEG, which gives a clinically useful visual representation of the data set (Lehmann, 1971). Paranjape (in preparation), using the maximum entropy estimater, shows that clinically useful characterization of the EEG can be obtained via a 2-dimensional power spectrum. Another advantage of multichannel analysis is the possibilty of obtaining reference-free data sets by the calculation of the radial current density, which is the second spatial derivative. This computation, however, has received very little attention until recently and has not yet been fully explored (Lehmann 1987). Spatial analysis has also shown some promise in source localization (Nunez, 1981).

Another avenue of active research in the study of brain waves is feature extraction and data standardization. The main purpose of these studies is to characterize the signal properties most relevant to the hypothesis under consideration, by forming summary indices called features and thereby effecting a reduction in the amount of data needed for direct analysis. This can be accomplished by three methods: the heuristic, the statistical and the classifier-direct.

1) The heuristic method uses summary features that are quantitative analogs to those used in the traditional visual assessment of the EEG strip chart. The major disadvantage of this method is that validation studies are necessary to determine the correlation between the summary features and the strip-chart characteristic. Although this method has been useful in some studies, the tediousness of such analyses together with the lack of assurance that the maximum amount of information has been extracted from the data probably contribute to its scarcity (Gevins, 1987).

2) The most familiar statistical method is principal component factor analysis (PCA). This method seems to be quite successful since fewer than 10 basis images account for over 90% of the variance of the background EEG (Lazar, 1988). There are, however, several drawbacks to this method (Gevins, 1987). During PCA, a number of arbitrary decisions have to be made, such as the segment of time-series to transform, whether to apply a digital low-pass filter, and what experimental conditions to establish. Moreover, the small amount of residual variance of the EEG may be crucial for the interpretation of the results. Finally, statistical methods of feature extraction often do not derive the measures that best distinguish the clinical categories or conditions of the experiment.

3) The classifier-direct method, via statistical pattern-recognition algorithms, has been developed and used successfully in choosing features that are "optimal" for hypothesis testing. These algorithms, however, can identify a large number of features and the optimal classifier is often some combination of these features. The only method available to determine which is the optimal combination is to try each combination. But this is impossible if the number of features is large because of "combinatoric explosion". Therefore, some heuristic strategy must be used to choose the optimal combination. The problem of determining the optimal combinations remains an unsolved problem (Gevins, 1987).

Mathematical modeling remains one of the most difficult and active areas of research associated with the study of the EEG. The main emphasis, so far, is in characterization of the signal in terms of a mathematical model determined by a set of parameters. The justifications for employing a particular method are generally sought in pragmatic arguments and not in theoretical arguments; a model is useful or valid if, for example, it can distinguish between different states of the brain. In other words, if the set of parameters for a particular model is different for each state of the brain, then the model has the potential of being clinically useful thus the justifies method. The main emphasis of these studies, therefore, is not to construct an explanatory biophysical model. Indeed, because of the pragmatic justifications, the link (if any) between the biophysical content and the mathematical model has been described as neither specific nor essential (Lopes Da Silva and Mars, 1987).

The most general parametric model is the Autoregressive Moving Average model (ARMA). The basic model assumes that the process underlying the generation of the EEG is filtered white noise and the signal is, in a wide sense, stationary. Because of the difficulties in determining the coefficients of the ARMA model, the standard method is to use the AR model as an approximation to the ARMA model. It is important to understand that AR modeling is not just an approximation of the ARMA model: for example, there exist processes that are strictly AR. Jansen *et al.* (1981), using the Akaike's final prediction error criterion, demonstrated that a fifth-order AR model can estimate EEG characteristics in 90 percent of the cases but better characteristic estimation can be obtained using a tenth-order AR model.

The main limitation of this method is that, whereas it is always possible to calculate the AR model parameters, the biophysical or physiological interpretation of the results is not always possible. The central issue here is that *the ARMA only models the time-series and not the dynamical system that generates the time-series. Since an infinite number of processes can generate the same time-series and ARMA models are not process-dependent, very little information is gained about the dynamical system.* Therefore, even if the model generates a time-series that gives a 100% correlation to the observed EEG, it does not imply that one has modelled the system, nor can one make any specific conclusion about the dynamical system.

Implicit to the foregoing paragraph is a distinction which, though important in principle, is not always easy to make in practice. The distinction is between mathematical models for empirical data (as e.g., for a biological time-series), and empirical theories about such data. In a narrow sense, a mathematical model may be considered as empirically valid if it reproduces or closely approximates the empirical phenomenon in question. In that regard, a given empirical data set might possibly be modeled in a variety of ways mathematically. Thus there is some degeneracy in going from the data set to mathematical models - each of which may be <u>mathematically valid</u> so far as its ability to account for the data is concerned. Such degeneracy can be viewed as a caveat against the obviously falacious assumption that processes generating the same output must be identical processes. The concept of "explanation" in natural science is a notoriously complex one, and so the concept of an "explanatory theory" can be no less complex. Nevertheless, it is probably fair to say that most scientists are able to maintain a sufficiently clear distinction between the idea of a phenomenological model and an explanatory theory. So far as biophysics is concerned, a biophysical theory is not just a mathematical construction, but rather an embedding of biophysical concepts into a mathematical structure.

The objective in this thesis is not to model the alpha-rhythm, but to ask whether we can obtain some characterization of the EEG signal, and its underlying dynamics, of a sort that might be useful in realistic theoretical studies of these systems. [illegible], the interest of empirical science involves our making a distinction between mathematically valid models and those having theoretical usefulness. One of the current hopes about chaos theory is that it may help us in that direction by revealing more about the dynamics underlying time-series like the EEG than the "classical" approaches have thus far achieved.

5:2.2b A plausibility argument

The question now concerns the validity of the chaotic approach in considering the EEG alpha-rhythm. This amounts to showing that the premise of Takens' theorem is satisfied.

The premise of Takens' theorem is that the time-series is produced by a deterministic dynamical system in which all sources of the system's dynamics are endogenized, i.e., the system itself produces its own dynamics. The central question is then:

Can the dynamical system underlying the alpha-rhythm EEG be considered an endogenous deterministic dynamical system?

It is clear that the absolute answer to this question is unknown at present. A definite proof that the dynamical system can or cannot be considered an endogenous deterministic dynamical system does not yet exist in literature. However, one can make a plausible argument that the dynamical system underlying the alpha-rhythm EEG can be considered an endogenous deterministic dynamical system.

The alpha-rhythm of the human EEG appears as an oscillatory pattern within the range 8-13 Hz. Typically, the alpha rhythm of a given individual has a stable frequency within the range mentioned, usually with spindle-like appearance due to waxing and waning of its amplitude. Its behavioral correlate is a state of alert relaxation with closed eyes. Upon opening one's eyes or other sensory stimulation, or the subject's engagement in purposeful mental activity, the alpha-rhythm gives way to a signal of lower amplitude and higher frequency (so-called "alpha-blocking").

The alpha-rhythm cannot be considered as representing an isolated dynamical system but, from the phenomenological observations given above, it does seem reasonable to regard the rhythm as representing a distinctive type of brain state, at least so far as accessibility through the EEG is concerned. The alpha-rhythm is thus operationally isolated as a quasi-stationary system, even though the total brain is simultaneously active in many other modes. The rhythm is considered the background EEG of the brain - as opposed to evoked potentials (which occur in response to external stimuli). We can thus consider external influences to the alpha-rhythm as being negligible, so that the system is in effect an endogenous one, i.e. one producing its own dynamics. As regards the system's determinism, one may argue as follows: With respect to our current understanding of the human brain, one cannot assert that the brain is dynamically random, nor can one assert that it is dynamically deterministic. On the one hand, we have, within modern neural network theory, a notion of randomness associated with the brain; on the other hand, the consistent reaction of the brain to external stimuli betokens an element of determinism. It therefore seems reasonable to make the overall hypothesis that brain dynamics may include both random and deterministic features. This hypothesis is not as far-fetched as it might at first seem. For the olfactory bulb, Skarda and Freeman (1987) constructed a mathematical model which yielded sustained chaotic activity that was statistically indistinguishable from the background EEG of resting animals. If we make the hypothesis stated above, the methodological appropriateness of chaos theory seems virtually self-evident.

The issue can be put another way. In the absence of existing knowledge as to the actual dynamics of the alpha-rhythm (or a generally accepted theory thereof), one may ask what precise evidence or arguments manifestly rule out the very possibility of the dynamics having deterministic character. One cannot say that the alpha-dynamics is indescribable, so it is reasonable to suppose there exists some set of variables of state, $\{\underline{x}_i\}$, that will describe the system. Since the argument above shows that the alpha-system generates its own dynamics, it now follows that there exists some function F such that

$$\underline{X}_t = F(\underline{X}_{t-1});$$

in which case we have a dynamical equation[1].

In summary then we have provided plausibility arguments for:

1. considering the dynamical system as an endogenous system;
2. the existence of the dynamical equation;
3. the dynamical equation to have deterministic properties.

Therefore, at least on plausibility grounds, the assumptions of Takens' theorem seem satisfied and the chaotic approach appears to be valid.

Also, in connection with the phenomenon of alpha-blocking, it is well known that

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[1] It is not always clear in the literature as to exactly what is meant by the term "dynamical system". Whereas in Physics the expression "dynamical system" refers to the time-varying system of interest and/or the kinetics and forces acting on the system, in Mathematics the expression refers to the system of differential or difference equations. To avoid confusion, I will use the expression "dynamical system" in the Physics sense, to refer to the system of interest, and the expression "dynamical equation" to refer to the system of differential or difference equations.

the rhythm typically returns, in quietly resting subjects, when external stimulation or purposeful mental activity is abated. This suggests that, within the behavioral constraints mentioned before, the alpha-rhythm arises from a dynamical state to which the system returns following transitory perturbations. It, therefore, seems reasonable to consider the alpha-rhythm as corresponding to an attractor state associated with a maintained condition of alert relaxation.

5:2.3 Classification of the alpha-attractor

A central concept in classifying attractors is that of their complexity, which is usually discussed in terms of dimensionality (Farmer *et al.*, 1983; section 4.3 of this thesis). The simplest kind is a point attractor having a dimension of zero; a limit cycle has a dimension of one. Those with fractional dimensionality are fractal objects known as strange attractors, and most strange attractors are deterministically chaotic.

To explain the latter concept, we note first that the existence of an attractor in the phase-space betokens a deterministic dynamics; we then note that phase-space trajectories may diverge within the attractor itself. Since points close together in phase-space correspond to nearly identical states, points with separating orbits become unlike each other. The rate at which such separation grows therefore expresses the extent to which the system's dynamical behaviour is sensitive to small differences in initial states. Exponential divergence of the orbits, measured by a positive Lyapunov exponent, serves to define chaos (Wolf *et al.*, 1985; Farmer *et al.*, 1983); i.e., any attractor with at least one positive Lyapunov exponent is a chaotic attractor. Thus, a chaotic dynamics is globally deterministic in that its trajectories remain on an attractor, but locally random in that there is an exponentially fast loss of predictability concerning the details of behaviour within the attractor. In that sense, chaos is a matter of deterministic randomness.

The correlation exponent, υ, as defined by Grassberger and Procaccia (1983a; 1983c), provides a further method for classifying attractors. This measure of the attractor was developed in direct response to the impossibilities of calculating the fractal dimension for high embedding-phase-space dimensions (see Chapter 4). The correlation exponent then provides a method of classifying attractors with phase-space dimension larger than two.

Apart from its re[illegible] computational ease, the correlation exponent has the advantage of disentangl[illegible]aotic behaviour from the random noise contaminating any experimental signal (Ben-Mizrachi *et al.*, 1983). If the attractor is deterministically chaotic, then υ reaches a stable value ("saturates") at some phase-space dimension, while random noise induces no saturation. The phase-space dimension at which saturation occurs, i.e., the embedding dimension, may be taken as defining the minimal dimensionality of the phase-space necessary for proper embedding of the reconstructed attractor. The embedding dimension therefore equals the minimal number of degrees of freedom for the original dynamics sampled by the experimental time-series.

5:3 Three experiments

We recall from Chapter 1 that, methodologically, this study incorporates three mutually exclusive hypotheses:

1) The system is chaotic (i.e. deterministically random),
2) The system is random only, not chaotic,
3) The system is deterministic only, not chaotic,

of which, only hypothesis 1 is effectively consistent with the overall hypothesis stated in Chapter 1. The first experiment tests the validity of hypothesis 1 in connection with the alpha-rhythm EEG. Since these three hypotheses are mutually exclusive, if the experimental findings support hypothesis 1, then they contradict hypotheses 2 and 3.

5:3.1 Experiment one

5:3.1a Introduction

The purpose of this experiment is to test the hypothesis that *the dynamics underlying the human alpha-rhythm correspond to a chaotic attractor.*

If the dynamics underlying the alpha-rhythm EEG correspond to an attractor, then the correlation exponent, as a function of embedding-phase-space dimension, will saturate. If the system has a chaotic dynamics, then the largest Lyapunov exponent will have positive value.

Both filtered and unfiltered alpha-rhythm data were analyzed in this experiment. These two different representations of data were chosen because there exist two ways in which the definition of the alpha-rhythm can be interpreted (see Chapter 2). The alpha-rhythm can be considered as the EEG signal in which the frequency range 8-13 Hz dominates, or as a signal of strictly 8-13 Hz with other frequencies present only as noise. The unfiltered EEG data in this experiment corresponds to the first interpretation while the filtered EEG data corresponds to the second interpretation.

5:3.1b Method

EEG data from two human subjects were obtained through an array of 32 channels with a reference electrode positioned on the left ear (subject A is a 45-year-old male, and subject B is a 32-year-old female). With respect to the very small number of subjects included in this study, we point out that we are not here describing a statistically designed experiment; our objective was the reconstruction and characterization of an alpha-attractor, which is achieved by completely deterministic methods.

A continuing problem in EEG analysis concerns the placement of the reference electrode. Since *a priori* knowledge of the location of the sources is not possible in EEG studies, there does not exist an optimal location for the placement of the reference electrode. Bipolar recordings, such as those analyzed in Mayer-Kress and Layne (1987a; 1987b) and Babloyantz *et al.* (1985; 1986) have many disadvantages (see arguments in Nunez, 1981). Although the left-ear reference does not eliminate the problems associated with the activity of the reference electrode, it does have some advantages (for example, it is less directionally selective).

Although the algorithms used in this experiment were relatively inexpensive computationally compared to other algorithms in this field, they still require a long time to run (and are therefore computationally expensive). As a result, purely on pragmatic grounds, a single channel was chosen for this preliminary study. Moreover, apart from the issue of economy, this study asks "is there evidence that the alpha-rhythm arises from a chaotic dynamics?". Clearly, the question as to whether the findings reported here apply to the spatial distribution of the alpha-rhythm is a separate question.

A single channel positioned over the left posterior parietal region was selected for analysis (Channel 25 in the array described in Koles *et al.*, in press). This position was selected because the alpha-rhythm is very robust in its area. The data were digitized to 12 bits with a sampling rate of 120 Hz. Analyses were based on: (a) unfiltered data, (b) the same data subjected to a 5-15 Hz raised cosine filter, thus emphasizing the 8-13 Hz frequency range.

The algorithm developed by Grassberger and Procaccia (1983a; 1983c) was used to calculate the correlation exponent, using 8,000 points (≅ 67 s of data) of the reconstructed attractor. The slope of the linear region of this plot was determined with a method essentially similar to that of Babloyantz and Destexhe (1986) and Mayer-Kress *et al.* (1987). The largest Lyapunov exponent was calculated from 4,000 points (by the algorithm given in Wolf *et al.* (1985), using a slightly modified version of the Fortran code in order to accommodate our method of data storage from 32 channels.

A time delay of 1/120 s (≅ 9 ms) was chosen, giving the maximum number of data points for our calculations. This value is just below the range of 10-40 ms used by Mayer-Kress and Layne (1987a; 1987b) but, in light of their statement that their "results were stable within fairly large ranges"[1], we feel that this slight deviation would cause insignificant differences in our calculations. Furthermore, our chosen value of delay time is typical for power-spectral analysis of the EEG; Froehling *et al.* (1981) recommend that the delay time should be of the order of "typical sampling times used in power spectral

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[1] G. Mayer-Kress and S.P. Layne, 1987. "Dimensionality of the human electroencephalogram", in *Ann. N. Y. Acad. Sci.*, Vol. 504, p71

analysis"[1]. All calculations reported here were performed on a VAX 11/750 computer with a VMS 4.6 Operating System.

5:3.1c Results

A relatively simple method of gaining some qualitative understanding of the dynamics is through the plotting of the phase portraits. The 2-dimensional phase-space trajectories obtained from the filtered and unfiltered alpha-rhythm EEG are shown in Figure 5.1 and 5.2 respectively. These portraits were constructed in the phase-space spanned by $X(t)$ and $X(t + \tau)$. Figure 5.1a shows that the flow of the phase-space trajectory is clockwise and that it spirals away from the centre of the ellipses. The trajectory reaches a local maximum and then spirals toward the centre and flows around the small ellipse for a while (Figure 5.1b). Figure 5.1c shows the trajectory expanding once more to a medium size ellipse. It then flows around this ellipse for a while (Figure 5.1d) and then spirals toward a very small ellipses. It must be noted that these shifts between smaller and larger ellipses are aperiodic; thus, the presence of a chaotic dynamics is already suggested even in these 2-dimensional portraits.

The phase-portraits of the unfiltered alpha-rhythm (Figure 5.2) have the same general properties as that of Figure 5.1. However, the path of the trajectory in Figure 5.2 is modulated by low frequency waves (< 3 Hz). Moreover, some high frequency oscillations of the path is present because of the roughness of the trajectory. Thus it can be readily seen that the phase portrait of the filtered alpha-rhythm shows more coherence than that of the unfiltered alpha rhythm. This is of course an expected characteristic of filtered signals. It must now by stress that we can only visualize these phase portraits at very low dimensions (< 3), and thus very little specific information is gained about the underlying dynamics. Nevertheless, they do allow for an intuitive feeling about the dynamics.

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[1] H. Froehling, J.P. Crutchfield, J.D. Farmer, N.H. Packard and R. Shaw, 1981. "On determining the dimension of chaotic flows", *Physica*, Vol. 3D, p611.

Figure 5.1a)

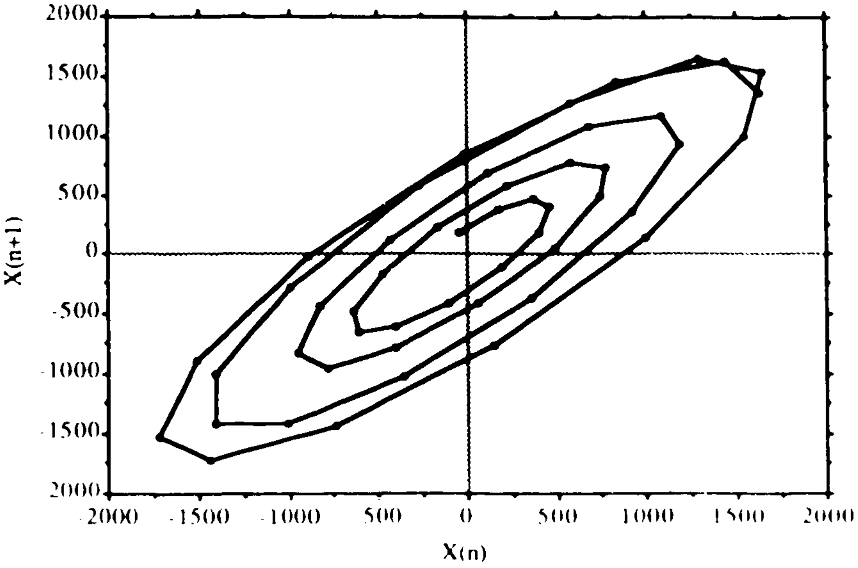

Figure 5.1b)

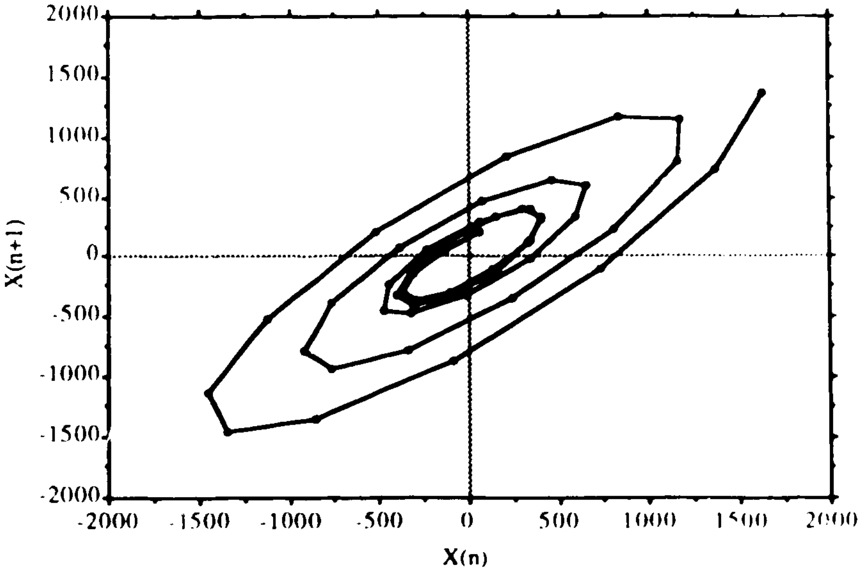

Figure 5.1c)

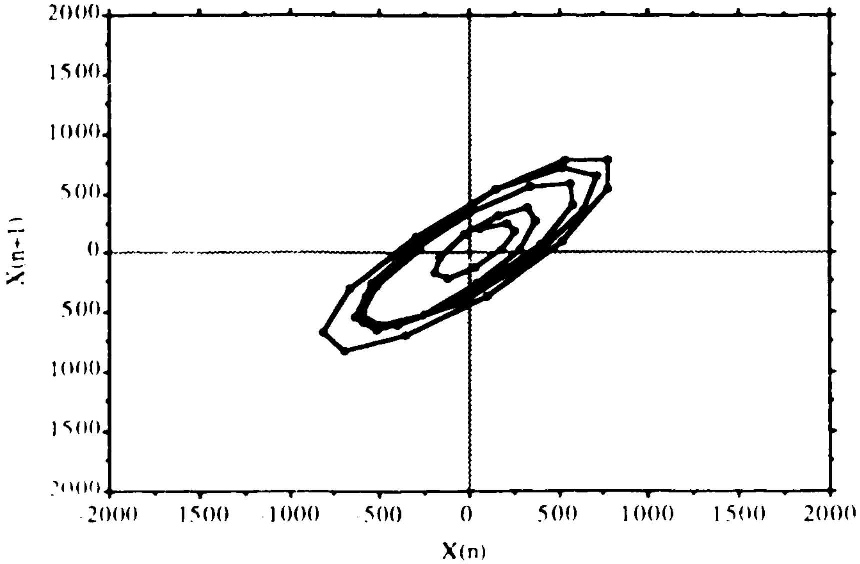

Figure 5.1d)

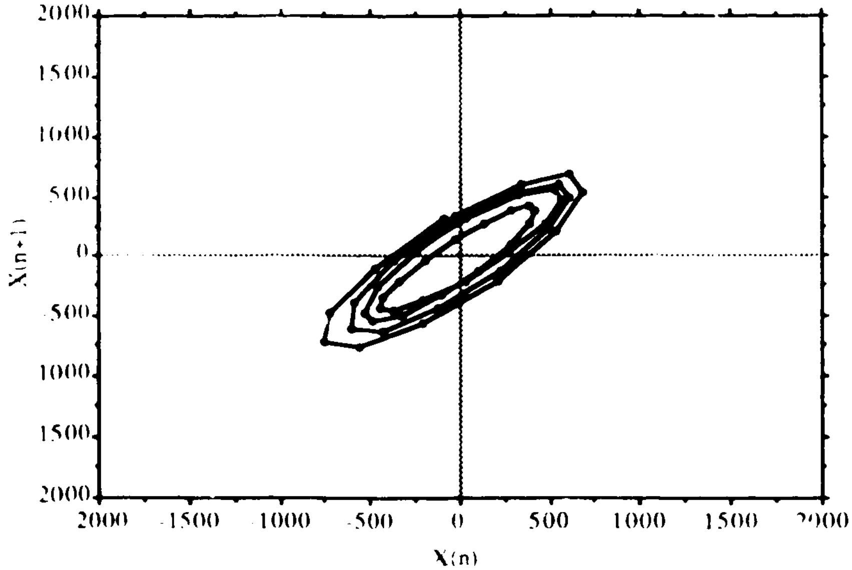

Figure 5.1e)

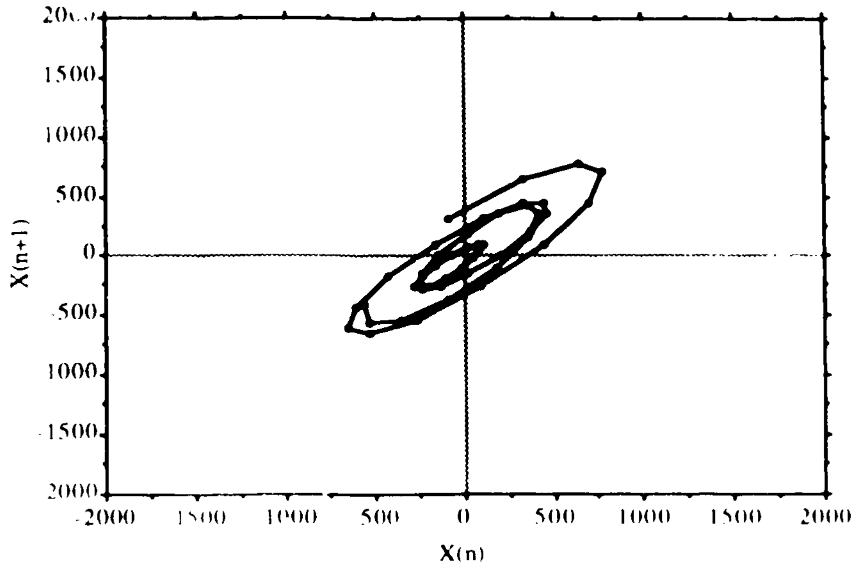

Figure 5.1: Two-dimensional phase portraits of the human alpha-rhythm electroencephalogram filtered by a raised cosine filter set at 5 to 15 Hz for a) 1 ≤ n ≤ 50 b) 50 ≤ n ≤ 100; c) 100 ≤ n ≤ 150; d) 150 ≤ n ≤ 200; e) 200 ≤ n ≤ 251. The portraits are drawn in computer units to facilitate later calculations.

Figure 5.2a)

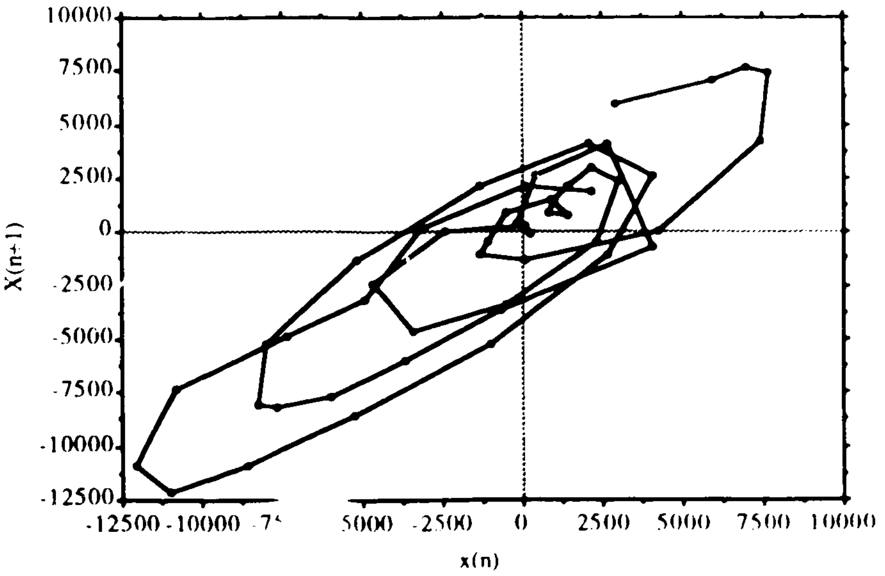

Figure 5.2b)

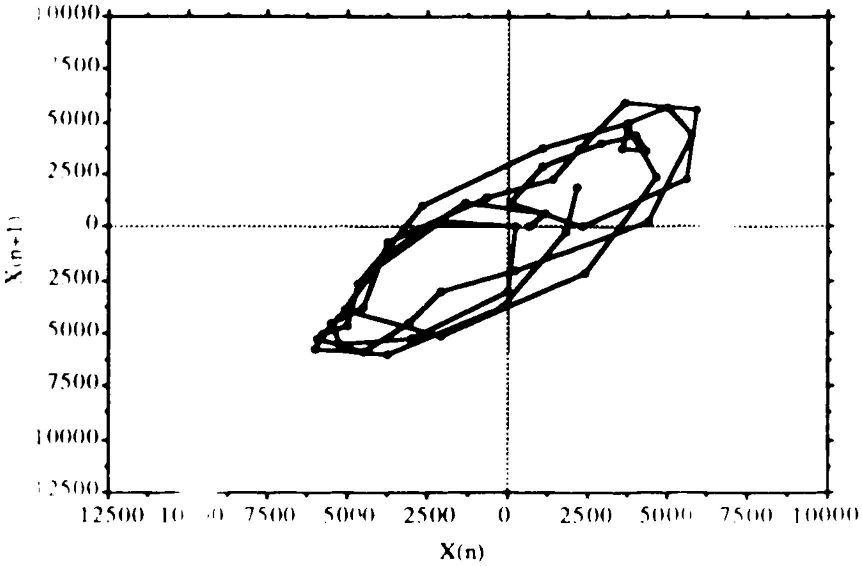

Figure 5.2c)

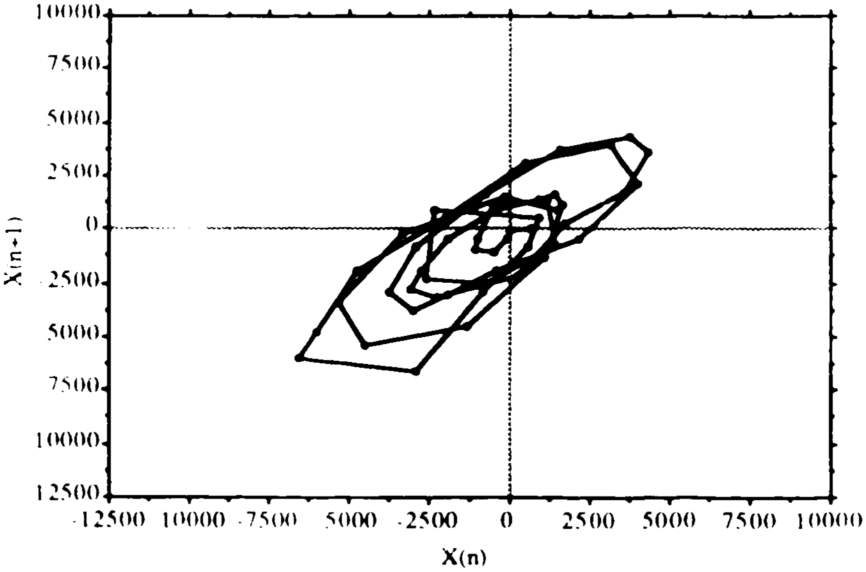

Figure 5.2d)

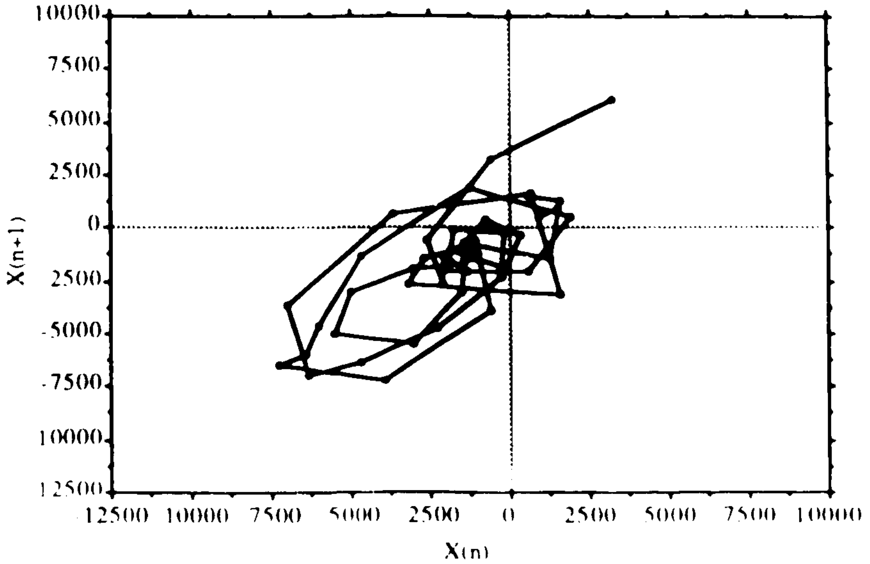

Figure 5.2e)

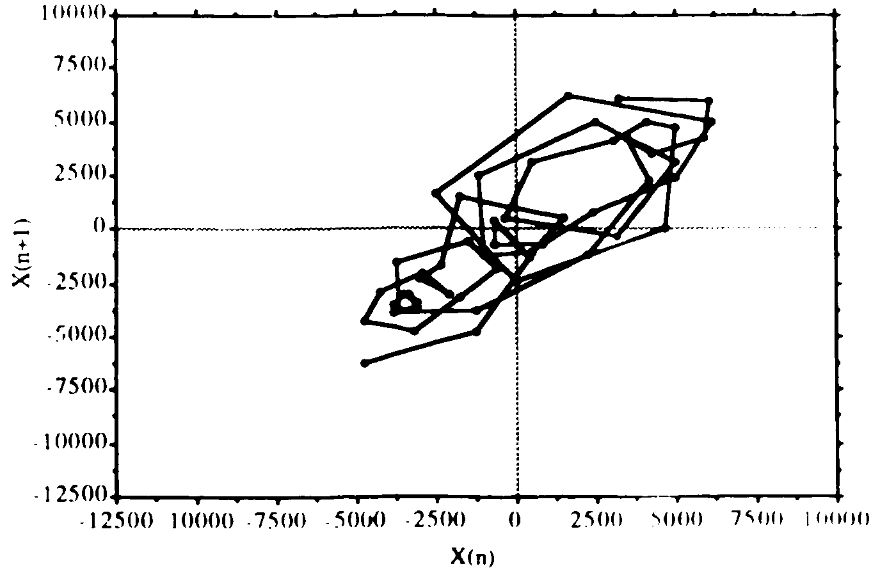

Figure 5.2: Phase portraits of human alpha-rhythm electroencephalogram in 2-dimensional phase-space for a) $1 \leq n \leq 50$; b) $50 \leq n \leq 100$; c) $100 \leq n \leq 150$; d) $150 \leq n \leq 200$; e) $200 \leq n \leq 251$. The portraits are drawn in computer units to facilitate later calculations.

Evidence of the presence of an attractor is provided by the log[C(r)] *vs* log[r] curve, shown in Figure 5.3. This curve contains three distinct regions: 1, a horizontal region for small log[r] where, because of digital sampling, the phase points do not have any neighbours within a circle of radius r_1 (where r_1 is very small); 2, a linear region for intermediate values of log[r] where the correlation exponent is calculated (as described above); 3, another horizontal region for large values of log[r] where all points are within a radius r_3 of each other ($r_3 >> r_1$). Region 3 is indicative that the dynamics correspond to an attractor since all points are confined to some subregion of the phase-space.

As mentioned earlier, an important property of an attractor is its dimensionality. In that regard, the results obtained in Experiment 1 are shown in Figure 5.4 and summarized in Table 5.1.The correlation exponents for the filtered data were found to saturate at

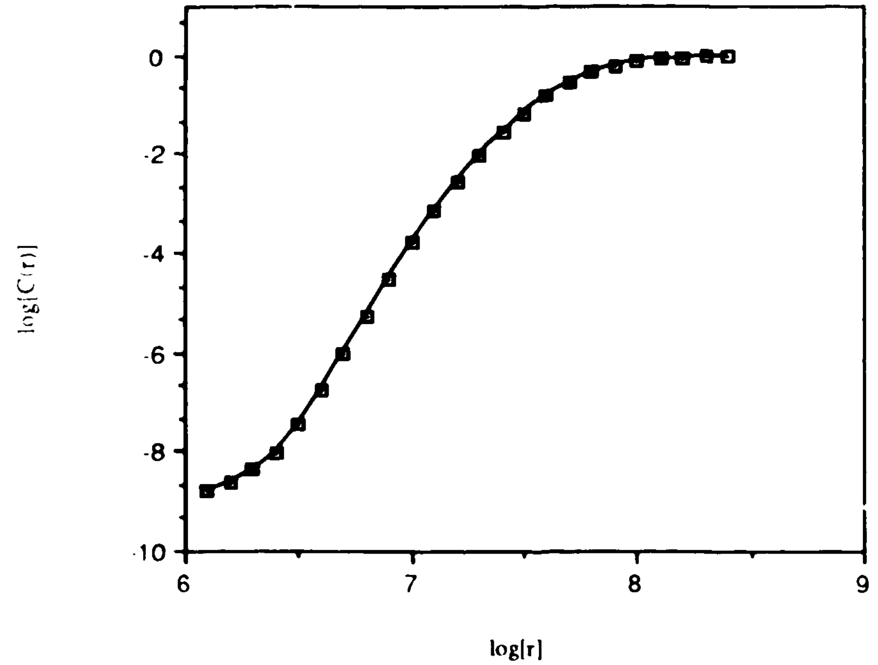

Figure 5.3: A graph of log[C(r)] as a function of log[r] from the unfiltered data of Subject B. The reconstruction phase-space dimension is 32. The shape of this curve is typical for all our alpha-rhythm data

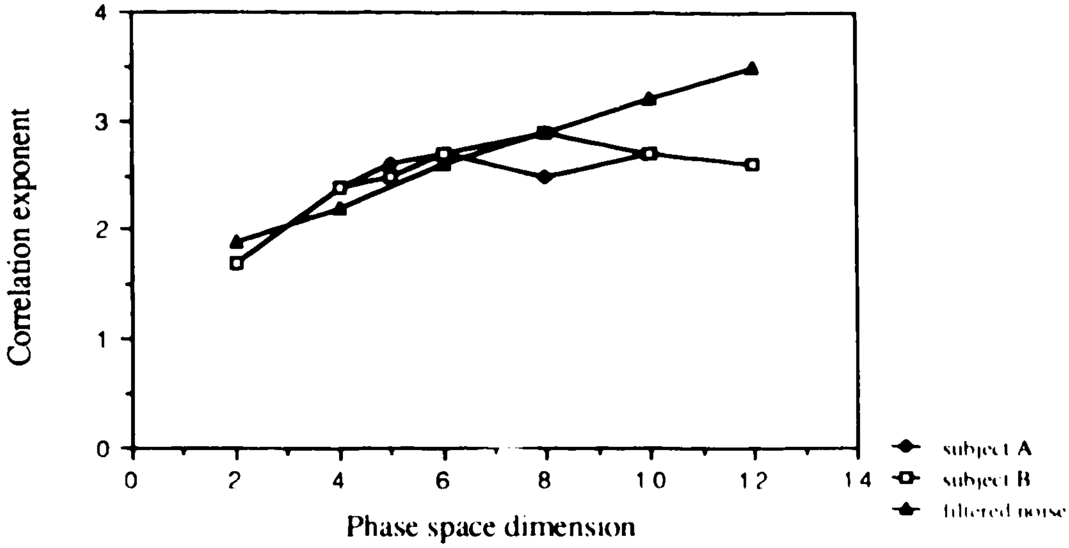

Figure 5.4: The correlation exponent for filtered white noise and for alpha-rhythm from a 45-year-old male (subject A) and a 32-year-old female (subject B) as a function of the embedding-phase-space dimension.

Table 5.1: The saturation dimension, correlation exponent, and Lyapunov exponent for filtered and non-filtered EEG alpha-rhythms obtained from two normal human subjects: a 45-year-old male (subject A) and a 32-year-old female (subject B):

Data	saturation dimension, d.	correlation exp., υ.	Lyapunov exp. λ.
A: filtered	5	2.6 ± 0.1	+ 2.5 ± 0.3
B: filtered	5	2.7 ± 0.1	+ 2.3 ± 0.2
A: unfiltered	27	7.8 ± 0.2	+(2.8 ± 0.6) × 10^{-2}
B: unfiltered	27	7.1 ± 0.1	+(5.7 ± 0.8) × 10^{-2}

(Values for d and υ were calculated from 8,000 points of the reconstructed attractor, those for λ from 4,000 points. The errors cited are in units of the sample standard deviation.)

embedding dimension 5 for both subjects. The value of the correlation exponent given in Table 5.1 gives a the lower bound to the Hausdorff dimension of the attractor. As previously mentioned, such a saturation result means that the alpha episodes in this preliminary study correspond to attractor states arising from a deterministic dynamics. Table 5.1 also shows that the filtered data were found to have a positive Lyapunov exponent for each subject, so that the filtered alpha-rhythm data corresponds to a chaotic attractor.

With respect to the unfiltered alpha-rhythm, the correlation exponent saturated at embedding dimension 27 for both subjects (Table 5.1). Moreover, the unfiltered data were found to have a small but positive Lyapunov exponent (Table 5.1). As stated before, such results provide evidence that the dynamics of the unfiltered alpha-rhythm also corresponds to a chaotic attractor.

5:3.2 Experiment 2

5:3.2a Introduction

This experiment was designed to test the hypothesis that *the filtered alpha-rhythm has the same dynamics as a filtered white-noise signal havingthe same power spectrum as the filtered alpha-rhythm*.

As is well known, sources of a widely different character may yield one and the same power spectrum. Because the filtered alpha-rhythm has a broadband power spectrum, the possibility arises that the alpha-rhythm has a random dynamical source. Therefor, if a signal having a known white-noise source is shaped to yield the same power spectrum as the filtered alpha-rhythm, then subjected to the same measures as in Experiment 1, it will provide a control experiment in testing the above hypothesis.

5:3.2b Method

A random white-noise signal generated by the VAX 11/750 random number generator was filtered by a fifth-order autoregressive (AR) filter. A fifth-order filter was chosen because it has been demonstrated that a fifth-order AR model is a good power-spectrum estimator of the EEG (Jansen *et al.*, 1981). The AR filter parameters were calculated from 30 seconds of EEG data using the Burg's method (Kay, 1988). The correlation exponent as a function of embedding-phase-space dimension was calculated by the same methods as stated in section 5:3.1b.

5:3.2c Results

The power spectrum of the AR-filtered noise as compared to the filtered alpha-rhythm is shown in Figure 5.5. The correlation exponent did not saturate for phase-space dimension less than 18 (Figure 5.6), whereas filtered alpha saturated for a phase-space dimension of 5 (Figure 5.4). This implies that the number of degrees of freedom for the AR-filtered noise is at least three times greater than that of filtered alpha-rhythm. But that would then imply that the dynamics underlying these data is vastly different from that of the filtered alpha-rhythm. In other words, the dynamics underlying the filtered alpha-rhythm is <u>not the same</u> as a filtered random white-noise signal with the same power spectrum.

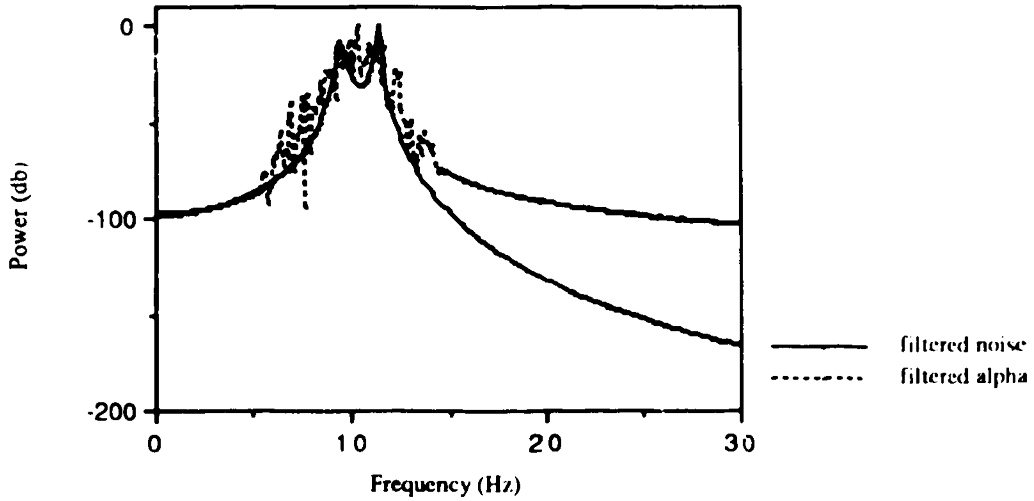

Figure 5.5: The power spectrum of filtered alpha-rhythm as estimated by the FFT and that as estimated by a fifth-order AR model from 30 seconds of data. The spectrum is calculated from 0 to 60 Hz., of which only the 0 to 30 Hz. range is shown.

Since the correlation exponent did not saturate, the Lyapunov exponent could not be calculated because the embedding phase-space dimension could not be determined. However, as can be seen from Figure 5.6, the curve of the correlation exponent as a function of phase-space dimension shows early signs of saturation. This is not a surprising result since the random number generator in the VAX 11/750 computer is topologically conjugate to the Baker map. Because the Baker map is a chaotic map (Devaney, 1986), the random number generator is then a chaotic dynamical system. In this light, one would expect the correlation exponent to saturate at very high phase-space dimensions - high dimensionality because the VAX random number generator is a very good random number generator.

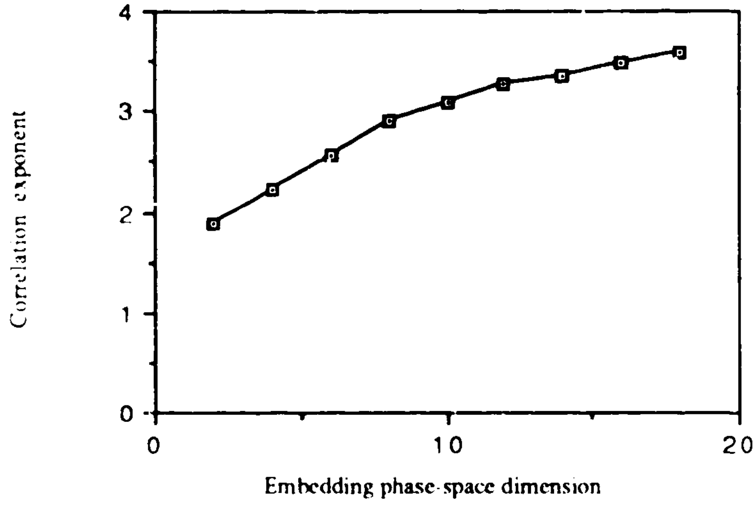

Figure 5.6: The correlation exponent as a function of embedding phase-space dimension for AR filtered white noise.

Freeman (1987) interpreted the saturation induced by "pseudo-random" signals as a measure of the "curse of dimensionality": the lowest phase-space dimension for which the value of the correlation exponent is no longer equal to the embedding dimension defines the upper limit of resolution for that sample size. But this cannot be the correct interpretation since the random number generator is a chaotic dynamical system, and so saturation is expected: the curve in Figure 5.6 will not be linear as would be expected for a purely random source.

From the preceding discussion, it will be recognized that Experiment 2 does not constitute an ideal control experiment, since the control signal is not itself derived from a purely random source. But the results do serve to emphasize an important feature. Namely, that in chaotic systems - where the element of randomness is always present - the *degree* of randomness present is related to the complexity of the underlying dynamics and, hence, to the saturation dimension of the attractor. Thus, the results obtained strongly suggest that the degree of randomness underlying the AR filtered white noise is markedly greater than that underlying the filtered alpha-rhythm. In turn, this serves to emphasize the point, made earlier, that similarity of power spectra does not provide evidence for similarity of the dynamics underlying those spectra.

5.4.3 Experiment 3

5:3.3a Introduction

The purpose of this experiment was to test the hypothesis that *the filtering process induced the chaotic dynamics.*

In terms of the experimental logic, this experiment, like Experiment 2, may be considered as a control experiment. We are here concerned with the possibility that the experimental methodology adopted in Experiment 1 created the observed chaotic dynamics. Thus, if a random white-noise signal is filtered with the same filter used in Experiment 1, and is then subjected to the same measures as in Experiment 1, this experiment will then provide a control against the hypothesis stated above. For, if the calculated quantities are closely similar for the two sets of data - EEG and control signals - then we cannot reject the hypothesis stated above.

5:3.3b Method

A white noise signal was generated using the random number generator of the VAX 11/750 computer. This signal was then digitally filtered with the same raised cosine filter as in Experiment 1; the correlation exponent as a function of embedding-phase-space dimension was calculated via the same method as that outline in section 5:3.1b.

5:3.3c Results

White noise filtered by the same procedure as used on the EEG data did not show saturation of the correlation exponent for phase-space dimension less than 18 (Figure 5.7). As with Experiment 2, this again implies that the number of degrees of freedom of the filtered white noise signal is at least 3 times greater than that of the filtered alpha-rhythm. These results tend to reject the hypothesis stated in 5:3.3a and to support the hypothesis that *the dynamics underlying the alpha-rhythm is __not__ similar to that of band-passed white noise*.

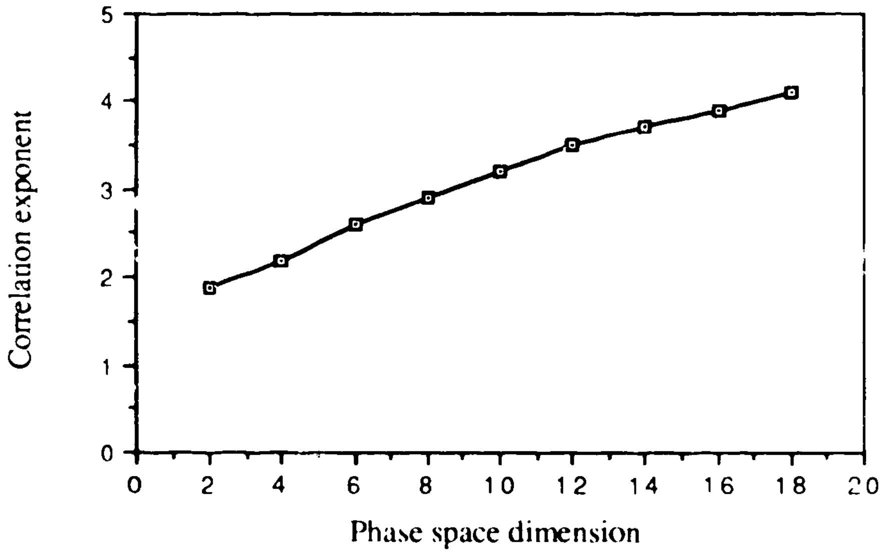

Figure 5.7: The correlation exponent for filtered white noise at various phase-space embedding dimensions.

However, the correlation exponent for band-passed filtered [illegible] much less sign of saturation (Figure 5.5) than that of the AR filtered white noise (Figure 5.4). As stated in section 5:3.2c, saturation of the correlation exponent is expected for "pseudo-random" signal. Moreover, since the AR filter is a much more restrictive filter, the phase-space dimension for which the correlation exponent saturates will be lower than that for the band-pass-filtered noise.

As in Experiment 2, the Lyapunov exponent could not be calculated because the dimension of the embedding phase-space could not be determined.

5:4 Discussion

The chaotic property of the alpha-rhythm is not attributable to noise since random noise does not yield saturation of the correlation exponent (Babloyantz and Destexhe, 1986; Grassberger and Procaccia, 1983a; 1983b; 1983c). Nor is the chaos induced by the filtering process since the correlation exponent for the unfiltered EEG data did saturate (see section 5:3.1c), thus exhibiting the presence of chaos in the absence of filtering procedures, whereas white noise filtered by the same procedure as the EEG did not show saturation of the correlation exponent for phase-space dimension less than 18 (see section 5:3.3c). Moreover, there is evidence that the dynamics underlying the alpha-rhythm is not that of filtered white noise since AR-filtered white noise with the same power spectrum as the alpha-rhythm did not saturate at phase-space dimension less than 18 (see section 5:3.2c). Fourier analysis revealed a broadband power spectrum for the filtered alpha-rhythm (see section 5:3.2c and Figure 5.5), so that the dynamics does not arise from a multiperiodic system (Grassberger and Procaccia, 1983c; Parker and Chua, 1987).

Babloyantz *et al.* (1985), using a time-series of 4000 points sampled at 100 Hz (40 s of data) and a time delay of 100 ms, did not find saturation of the correlation exponent up to embedding dimension 10. This choice of delay-time, however, is the same as the period of the 10 Hz alpha-rhythm. Therefore, the components of that period will be under-represented in the reconstruction (see discussion of the reconstruction time in Chapter 4). Thus, the conclusions they draw may be questioned. Mayer-Kress and Layne

(1987a; 1987b), using a time-series of 10 - 30 s sampled at 500 Hz and a delay time of 10 - 40 ms, also did not observe saturation of the correlation exponent up to dimension 20. All these studies were done with bipolar recording and, hence, are not directly comparable to the results presented here. However, we also obtained no saturation for the unfiltered EEG at dimensions ≤ 20.

At this point, we should say something about the correlation exponent itself. For a circle, of radius r, set on an attractor, the Grassberger-Procaccia correlation integral, C(r), measures the spatial correlation of points on the attractor obtained from a time-series. For small r, it is known that C(r) behaves according to a power law. In a system with F degrees of freedom, C(r) scales as r^F for a signal arising from noise, but as r^υ (with $\upsilon < F$) when the signal arises from deterministic chaos (Grassberger and Procaccia, 1983c).

Although our unfiltered EEG's had insufficient data points to obtain a reliable error term for the saturation of υ, saturation was nevertheless found to occur at embedding dimension of 27. To ensure that we did observe saturation, values of υ were calculated up to dimension 34. The saturation of the unfiltered signals provides evidence that contributions from noise to the high dimensionality do not mask the effects due to chaotic dynamics of high complexity.

So far as external noise is concerned, it's presence has been shown to yield two characteristic regions in a plot of log [C(r)] against log [r], whereas noise-free data do not show this effect (Ben-Mizrachi *et al.*, 1983; Grassberger and Procaccia, 1983c) (see discussion in Chapter 4). This external noise effect was absent in both the filtered and the unfiltered data in our study; hence, the high dimensionality of the unfiltered EEG cannot be attributed to the presence of external noise. The small but positive Lyapunov exponents calculated for the unfiltered data indicate that, whatever the reason for the high embedding dimension for the correlation exponent, the chaotic dynamics is still discernible. These findings thus suggest that the EEG is effectively a much less noisy signal than is often supposed.

We have already explained why the alpha-rhythm may be anticipated to correspond to an attractor state but, in view of the structural regularity of the alpha signal, the question naturally arises as to why the underlying dynamics should be chaotic. Interestingly, the EEG of petit mal raises the converse question as to why an epileptic seizure, traditionally viewed as a "chaotic" breakdown of organized brain dynamics, should yield the well known regularity of the "wave and spike" EEG.

In the one case we are asking why a regular signal should arise from chaotic dynamics and, in the other case, why a "chaotic" dynamics should yield a coherent signal. Clearly, this is not simply a symmetric restatement of a single question since the reference is to different brain states, norm [illegible] pathological, respectively. However, the latter question is sharpened by the fact that [illegible]oyantz and Destexhe (1986) found a chaotic attractor [illegible]ave and spike" EEG of petit mal seizure. The embedding dimension in this case [illegible] correlation exponent 2.05 ± 0.09, and the largest Lyapunov exponent positive w[illegible]gnitude 2.9 ± 0.6. These values are close to those shown in Table 5.1 for the filtered alpha-rhythm in our study, although what constitutes a significant difference in these values is not yet clear. Meanwhile, the circumstance that positive Lyapunov exponents are found for both petit mal and alpha EEG's serves to symmetrize the questions raised above in the sense that both situations appear to involve chaotic dynamics in the technical sense.

As regards petit mal, a chaotic dynamics mean that the random element traditionally associated with epileptic seizure is also deterministic; this suggests that some mode of cooperative dynamics underlies the coherent pattern of the petit mal EEG. Similarly, cooperative dynamics seem to underlie the alpha-rhythm. This can be seen from the sigmoidal curve in Figure 5.3. In the first place, the curve reveals that the alpha rhythm corresponds to an attractor state.

More generally, just as a linear curve on these axes would betoken a purely random or noisy source, the sigmoidal curve is diagnostic of a chaotic attractor in that it reveals local randomness for middle-range values of log[r] with global determinism for large log[r], where by global determinism we mean that the dynamical trajectories are bounded within a subregion of the phase-space, so that the curve saturates for large log[r].

The plot is also recognizable as a typically "cooperative curve", corresponding to the spontaneous emergence of macroscopic order from nonlinear interactions among elementary dynamical constituents of the system. Thus, a possible explanation is that the chaos arises from unpredictable changes in the number and spatial distribution of elementary units contributing to the alpha-rhythm and in the dynamical patterns of their coupling interactions. This possibility arises because it is known that coupled limit-cycle oscillators may lead to a chaotic regime (Yamada and Fujisaka, 1978). Hence, the dynamics underlying the quasi-sinusoidal alpha-rhythm may correspond to N coupled limit cycles. In any case, our findings suggest that the dynamics cannot be a simple synchronization effect driven by (e.g. thalamic) pacemaker cells.

It seems that, for the alpha-rhythm and petit mal EEG alike, it is necessary to take the role of the extracellular space into account in order to accommodate such cooperative modes. In that respect, we draw attention to [illegible] circumstance that, in its dynamical aspect, the neuron has no definite spatial boundary (Smart *et al.*, 1978; 1979), the unbounded neuron can then be regarded as operating in an exchange field which mediates cooperative modes of behaviour.

From a physiological point of view, the findings presented here suggest the coupling of stability and instability, i.e., a stable attractor-state of alert relaxation coupled with an instability evinced by positive Lyapunov exponents. Such a coupling would correspond to a state of great sensitivity to new sensory inputs that would throw the dynamics from the alpha attractor to a new regime; this would be consonant with the ideas proposed by Freeman and Viana di Prisco (1986) in their studies of mechanisms in the olfactory bulb of the rabbit. It might be anticipated that the dynamics underlying such processes may be studied in humans through the method of event-related potentials.

5:5 Conclusion

In summary, the method used in this study has shed some light on dynamical aspects of the human EEG alpha-rhythm. The hypothesis of section 5:3.1, that the alpha state corresponds to an attractor is supported. In addition, we have an estimate for the lower bound of the system's degrees of freedom and for the dimensionality of the alpha attractor. From the results in section 5:3.2, one can conclude that the dynamics underlying the alpha rhythm is different from that of band-pass filtered white noise, and that the filtering process did not create the observed chaotic dynamics. In section 5:3.3, it was shown that filtered white noise with the same power spectrum did not have the same underlying dynamics as that of the filtered alpha rhythm. The three experiments together, therefore, seem to provide evidence that the dynamics is indeed nonlinear and chaotic in the technical sense.

It must, however, be emphasized that this is a preliminary study based on only two subjects. While it is true that this number compares favourably with existing reports in the literature concerning applications of chaos theory to the human EEG, it nevertheless remains an open question as to how far the results reported here - and elsewhere - will turn out to be statistically representative.

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Chapter 6 Conclusion

The main motivation for undertaking this study was to ask whether the application of modern nonlinear dynamical theory to the EEG alpha rhythm might teach us something unrevealed by more traditional studies of brain dynamics. The rationale adopted here was described in Chapter 1.

It was seen in Chapter 2 that a central stumbling block against the advancement of our understanding of brain dynamics was in the transition from the microscopic to the macroscopic picture. This transition was discussed in connection with the reductionist views that have dominated brain research over the past 60 years. The current conception is that the alpha-rhythm is the result of the synchronization of the neocortex by the thalamus. This hypothesis, however, does not address the question as to why there should be a neural pacemaker and what purpose it could serve. In order to gain a better understanding of the underlying dynamics of the EEG, it was concluded that the brain must be studied holistically.

There are at present two methods for studying dynamical systems: classical dynamics and nonlinear dynamics. In Chapter 3, it was argued that classical dynamics is inappropriate for analyzing the human brain because biological systems are typically nonlinear and analytically nonintegrable. The standard procedure for studying such systems is that of numerical integration, but this hinges on the assumption of orbital stability. However, as can be seen from the work of Poincaré, even classically deterministic systems may exhibit extreme sensitivity to initial conditions, which contradicts the assumption of orbital stability.

Within the last 20 years, significant advances have been made in the field of nonlinear dynamics, culminating in the development of the theory of chaos. In Chapter 4 it was shown that Takens' theorem provides a means for multi-dimensional dynamical analysis of the system from a time-series in only one variable. This is accomplished by reconstructing an attractor that has the same dynamical properties as the attractor underlying the original time-series. Further dynamical understanding can be gained by classifying this reconstructed attractor via the correlation exponent and the largest Lyapunov exponent. These measures of dimensionality were chosen because calculations of other measures of dimensionality are impractical for phase-space dimensions larger than two.

In Chapter 5, the calculated values of the largest Lyapunov exponents for both the filtered and unfiltered alpha rhythm EEG were shown to be positive, thereby providing evidence that the underlying dynamics of the alpha-rhythm EEG are chaotic. Supporting evidence for chaotic dynamics is also provided by the saturation of correlation exponents with increasing phase-space dimensions. Moreover, Fourier analysis of the EEG showed that the alpha-rhythm EEG is not multiperiodic. Hence the underlying mechanism cannot be as simple as synchronization of the pyramidal cells by the thalamus, for, if we accept the synchronization hypothesis, it will be necessary to explain why these cells are being synchronized in a chaotic manner.

A more reasonable hypothesis is based upon the well known fact that a system of n limit cycles can exhibit chaotic behaviour. The state of alert relaxation (the background state of the cortex) may correspond to such a mixed state, with many limit cycles. Exactly how many limit cycles there are is presently unknown and may be regarded as a new avenue of research. For the sake of argument let us assume that each brain process is represented by just one limit cycle. The dynamics of the brain then stabilizes onto one of these limit cycles upon processing information. Which limit cycle it stabilizes on will then depend on the process. The dynamics of these limit cycles can be studied through the method of event-related potentials.

While working on the rabbit, Freeman and Viana di Prisco (1986) postulated a similar mechanism for the olfactory bulb. They suggested that the bulbar background state is that of a chaotic attractor. This chaotic background state corresponds to that of great sensitivity to new sensory inputs because it allows for easy transition to other attractor states. With each inhalation, the rabbit makes a test of the environment that may result in the convergence of the bulbar mechanism to a limit cycle. The hypothesis presented here for the alpha-rhythm differs from that presented by Freeman and Viana di Prisco in that the chaotic dynamics underlying the alpha-rhythm is the result of n coupled limit cycles. However, the two hypotheses are similar in that, when processing information, the dynamics will diverge from the chaotic attractor and converge on a limit cycle. In any case, we have shown that the alpha-rhythm of the human EEG reveals the presence of a cooperative dynamics.

Although some insights into the dynamics underlying the alpha-rhythm EEG were gained in this work, much more is still required for any significant understanding of the human brain. The results presented here have, nevertheless, weakened current ideas about the origins of the alpha-rhythm, and furnished an estimate of the lower bound of the system's degrees of freedom and for the dimensionality of the alpha attractor. Moreover, the methods have provided evidence that the dynamics is indeed nonlinear and chaotic in the technical sense. Based upon these results, a new hypothesis on brain dynamics was presented. Further studies can now be made to test this hypothesis.

More generally, two issues are brought into special focus by this study, both suggesting avenues for further research. The first concerns the neurological interpretation to be placed on the idea of sensitivity to initial conditions. The second concerns the critical parameters underlying the cooperative dynamics shown to be present.

The theory of chaotic dynamics is still unfolding, and it would be naive to suppose that the results obtained in the present study can carry the interpretational assurance yielded by long-established traditional methods. On the other hand, chaos theory has already revealed a number of interpretational complexities that went unrecognized in traditional physics. We therefore have, on the one hand, the possibility that the results reported here reveal a merely coincidental fit between EEG data and the evolving concepts of chaos theory and, on the other hand, the possibility that these results show actual properties of a sort not apparent through the use of traditional methods. In this situation it seems best to adopt the view that, at the very worst, the use of chaos theory has served to generate new kinds of questions about brain dynamics by drawing attention to possibilities not revealed by traditional approaches.

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