Column Out-of-Plumbs: $\frac{\Delta_0}{h} \times 10^3$ Rad.

Column No.	Storey No. 27		26		25		24	
	x Axis	y Axis	x Axis	y Axis	x Axis	y Axis	x Axis	y Axis
1	0.00	-1.04	0.87	-0.87	4.34	2.60	-4.34	-2.60
2	-0.50	-0.52	-0.92	-0.87	-1.00	0.87	0.00	0.00
3	0.25	-0.52	-0.04	0.87	-0.50	-3.47	0.50	-0.87
4	0.25	0.52	-0.54	-1.74	-1.50	-0.87	-1.00	2.60
5	-0.25	-1.04	0.04	-1.74	0.50	1.74	-1.50	0.00
6	0.52	0.52	2.60	-0.87	-0.87	-1.74	-2.60	3.47
7	0.00	1.00	1.74	1.08	0.87	0.50	-0.87	0.50
8	-1.04	-0.50	0.00	-0.92	-1.74	-1.00	0.00	0.00
9	0.00	1.04	0.87	0.87	-2.60	-0.87	-0.87	3.47
10	-1.50	0.00	-2.00	-1.74	-1.50	2.60	-1.00	0.00
11	-1.00	1.56	-1.33	0.00	-1.00	-1.74	-1.00	1.74
12	-1.00	1.04	-1.33	-0.87	-1.00	-0.87	0.00	-0.87
13	-1.00	-0.52	-1.33	0.00	-1.00	0.87	-0.50	1.74
14	1.04	-0.52	-0.87	-0.87	0.00	0.87	-1.74	1.74
15	0.00	0.00	0.87	0.00	-3.47	0.00	-0.87	2.00
16	2.60	-0.50	0.00	0.58	-2.60	2.00	-0.87	1.50
17	2.00	-1.50	2.67	-0.75	2.00	1.00	-1.50	0.00
18	0.00	-1.00	1.00	-1.33	2.00	-1.00	-2.00	-1.00
Mean	0.02	-0.11	0.13	-0.51	-0.50	0.08	-1.12	0.75
Stand. Dev.*	1.03	0.85	1.31	0.89	1.85	1.60	1.08	1.59

* Defined in Appendix A

TABLE 6.4 MEASURED COLUMN OUT-OF-PLUMBS IN BUILDING A

Class Delimitations ($\times 10^3$ Rad.)	f_i x Axis	f_i y Axis	f_i Total	f_i/nw Total
-7.5 to -6.5	1	1	2	2.2
-6.5 to -5.5	0	1	1	1.1
-5.5 to -4.5	2	1	3	3.3
-4.5 to -3.5	2	3	5	5.5
-3.5 to -2.5	18	24	42	45.9
-2.5 to -1.5	50	40	90	98.3
-1.5 to -0.5	98	100	198	216.2
-0.5 to 0.5	144	144	288	314.4
0.5 to 1.5	77	77	154	168.1
1.5 to 2.5	35	30	65	71.0
2.5 to 3.5	19	31	50	54.6
3.5 to 4.5	8	5	13	14.2
4.5 to 5.5	2	0	2	2.2
5.5 to 6.5	2	1	3	3.3
6.5 to 7.5	0	0	0	0.0
Total	458	458	n=916	

f_i = frequency

w = class width = 0.001 Rad.

n = sample dimension (total number of measurements)

TABLE 6.5 FREQUENCY FUNCTION OF COLUMN OUT-OF-PLUMBS FOR BUILDING A

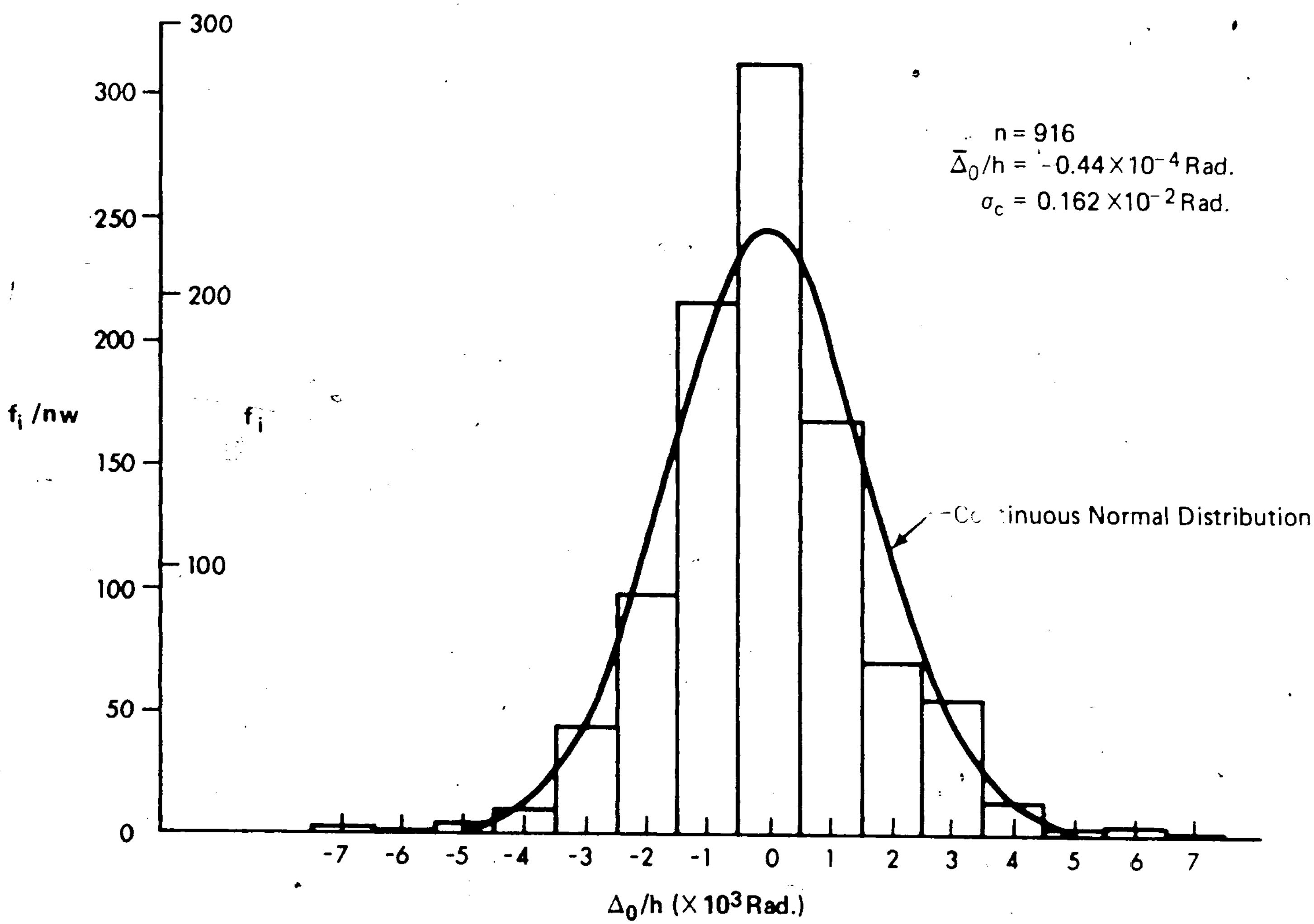

Figure 6.5 Column out-of-plumb distribution for building A

prescribed tolerance of 1/500. The result was expected. Superimposed on the histogram is the normal distribution* calculated from the given mean and standard deviation. The area under both the histogram and the normal curve is unity. A comparison of the two graphs indicates that the out-of-plumb population can effectively be assumed normally distributed.

Two more quantities characterize a distribution. As described in Appendix A, the skewness and the peakedness (kurtosis) of a distribution are defined by non-dimensional quantities. For the distribution of Fig. 6.5, the skewness factor is +0.14 indicating that the distribution is slightly skewed to the right. The kurtosis factor is 4.9, indicating a distribution more peaked than a perfect normal distribution with a kurtosis factor of 3.0.

Similar histograms are given in Figs. 6.6 through 6.8 showing the distributions of column out-of-plumbs measured in buildings B, A and B combined, and C. All populations are approximately normally distributed. The characteristics are quite similar in all cases and are summarized in Table 6.6. The standard deviations given in Table 6.6 are comparable with those of Tables 6.1 and 6.2.

The probability densities of the absolute values of the measurements for buildings A, B, A and B combined, and C are given in Figs. 6.9 through 6.12. The negative part of the normal distributions of Figs. 6.5 to 6.8 is literally folded over and added to the positive part. The distribution that results when the mean is zero or relatively close to zero is called a "half-normal" distribution. A description of the half-normal distribution is given in Appendix A.

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* Definition given in Appendix A.

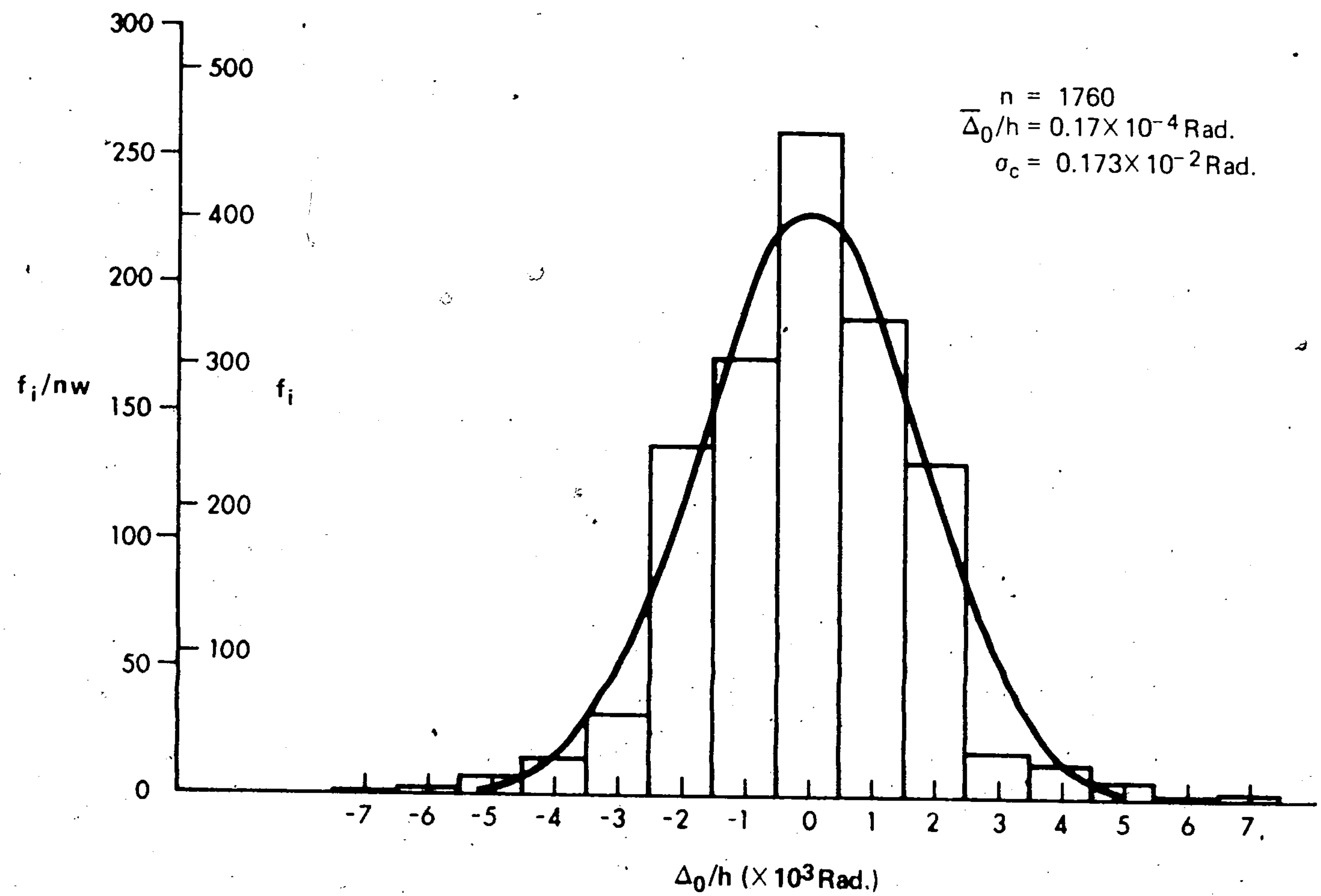


Figure 6.6 Column out-of-plumb distribution for building B

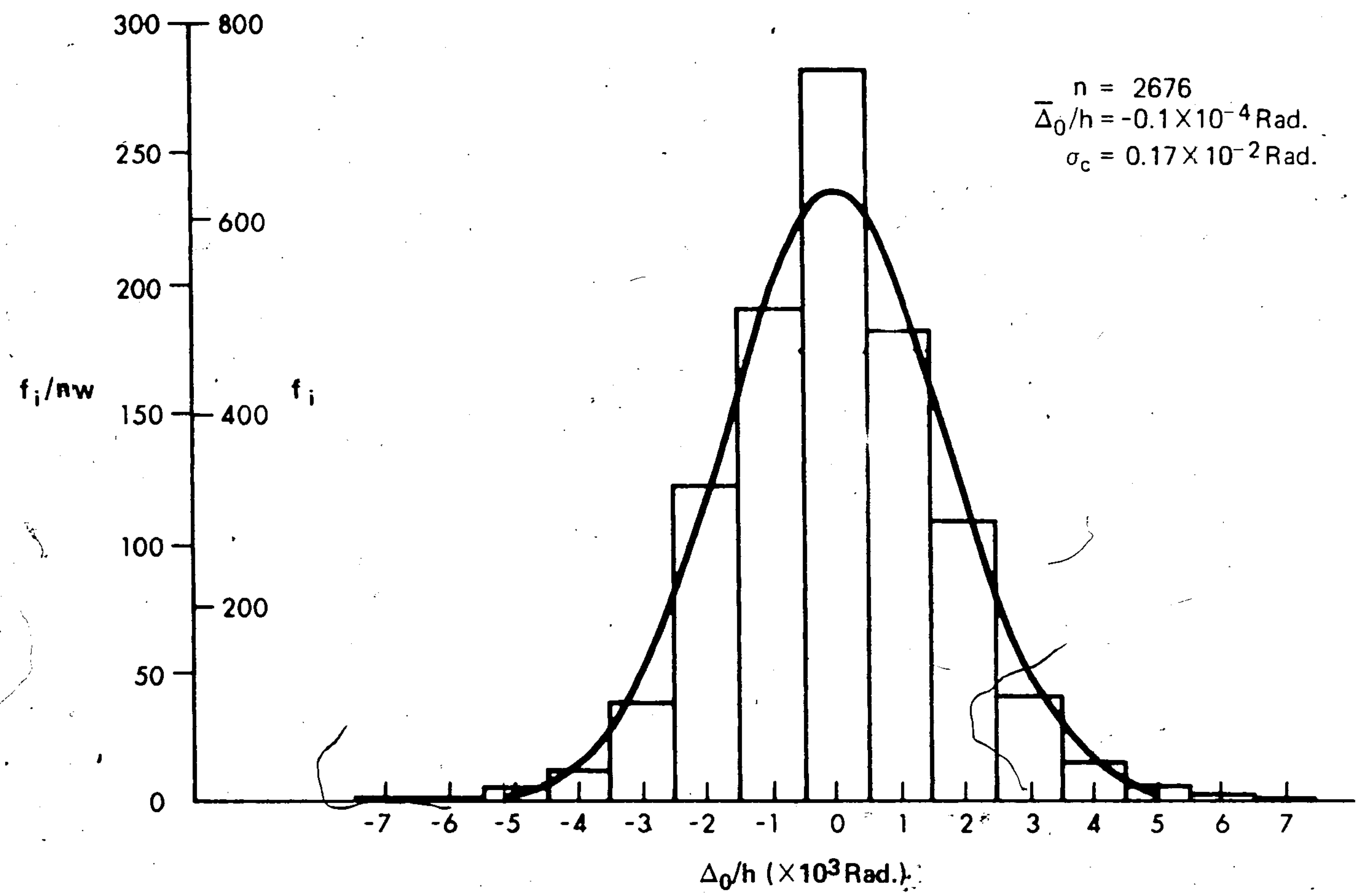


Figure 6.7 Column out-of-plumb distribution for buildings A and B

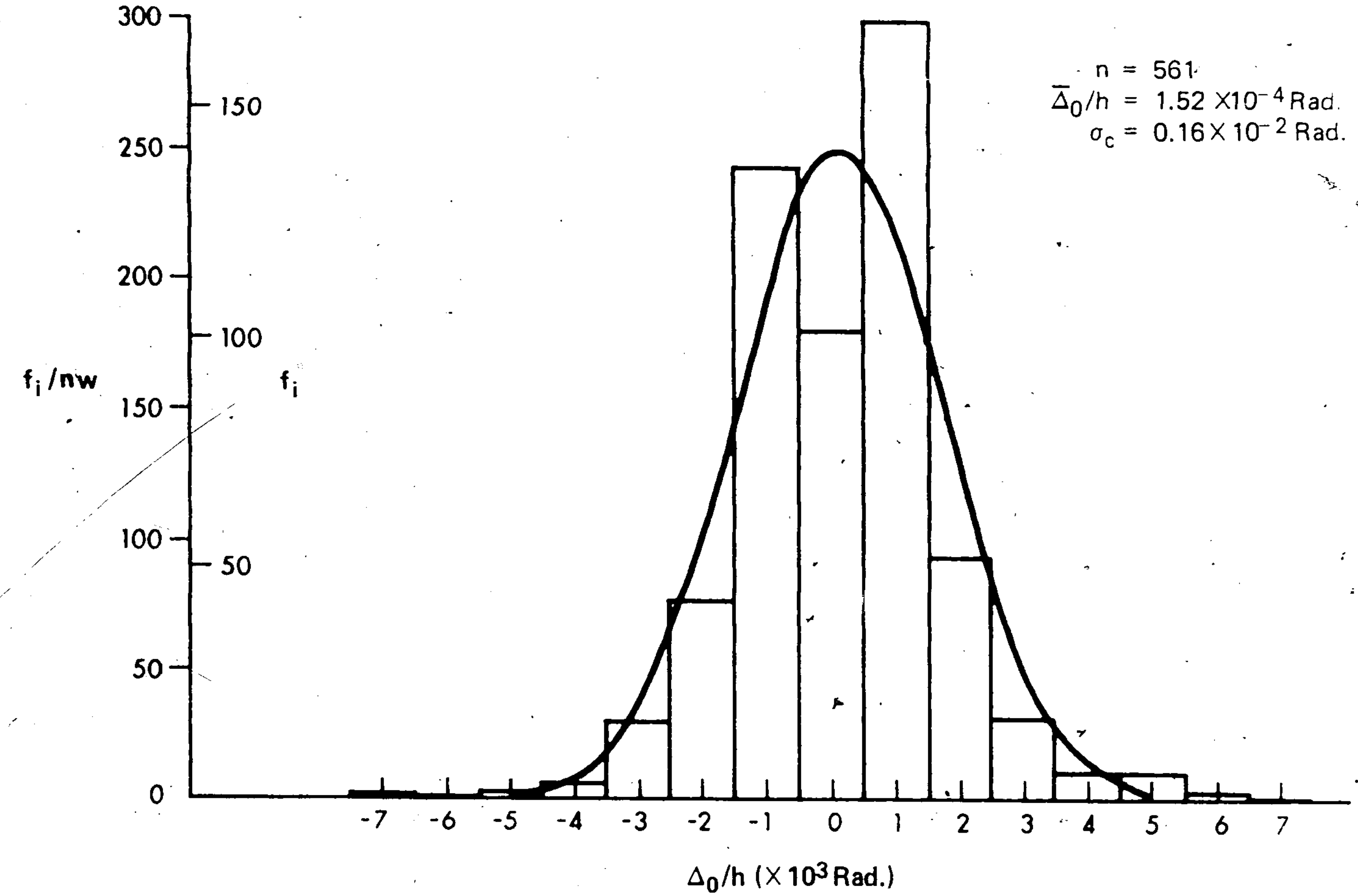


Figure 6.8 Column out-of-plumb distribution for building C

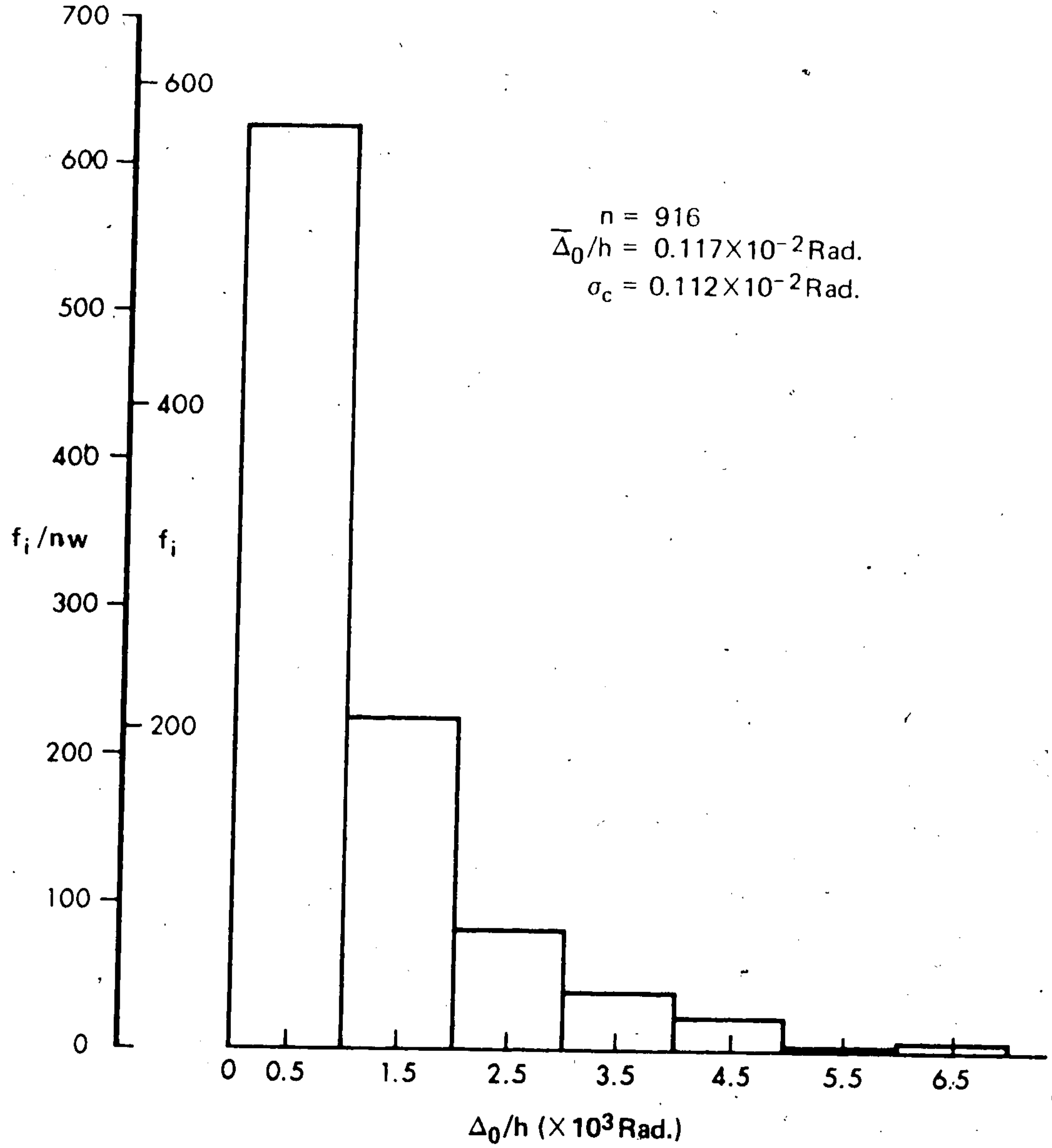


Figure 6.9 **Distribution of absolute values of column out-of-plumbs for building A**

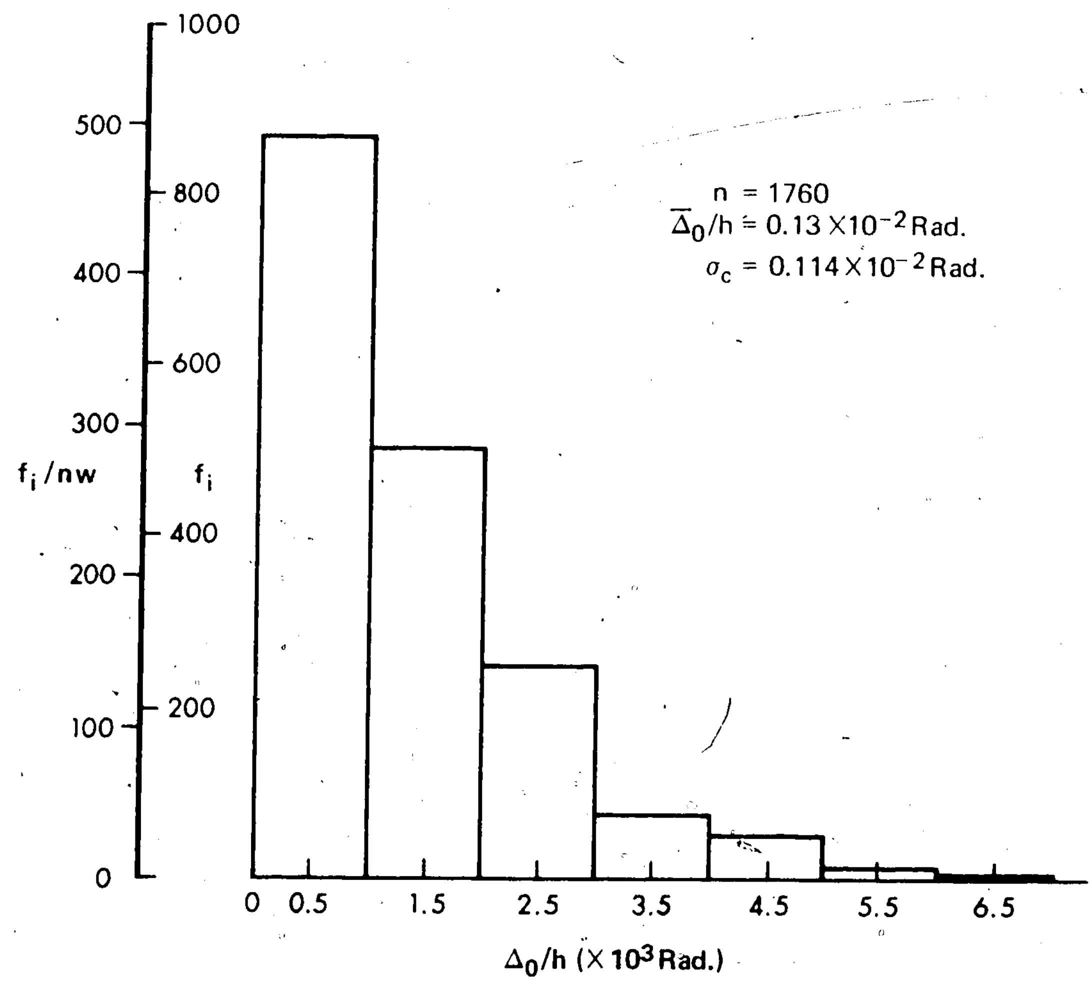


Figure 6.10 Distribution of absolute values of column out-of-plumbs for building B

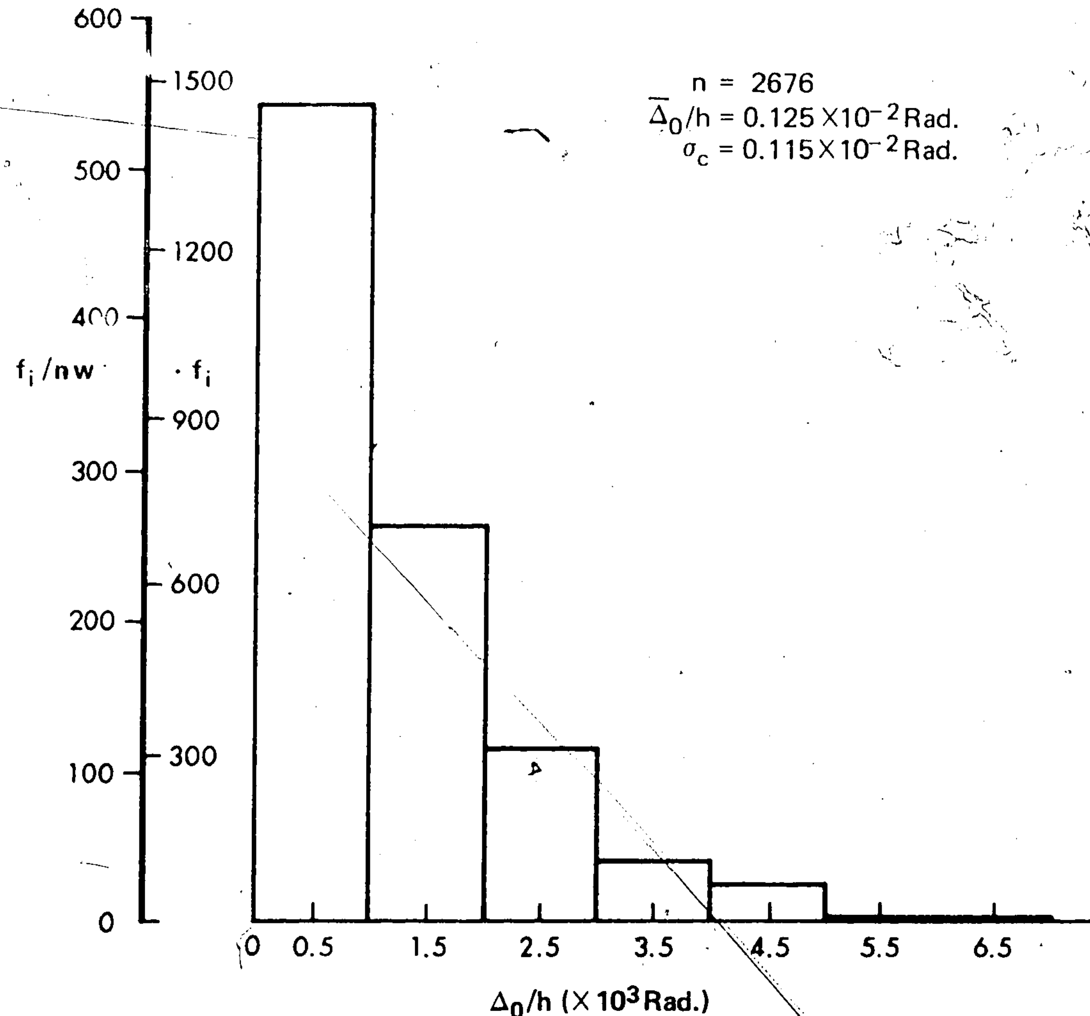


Figure 6.11 Distribution of absolute values of column out-of-plumbs for buildings A and B

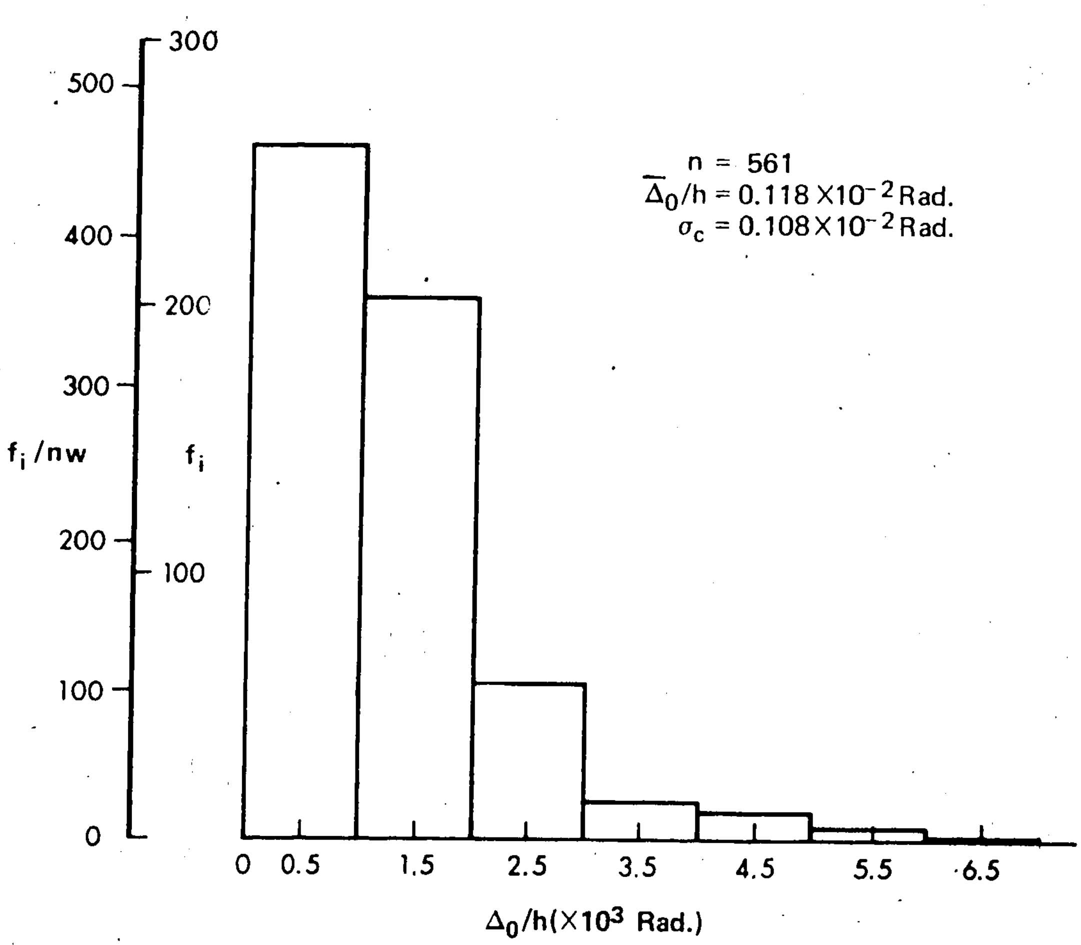


Figure 6.12 Distribution of absolute values of column out-of-plumbs for building C

Building	Type	Sample Dimension n	Mean (Eqn. A-14)* (x 10^2 Rad.)	Stand. Dev. (Eqn. A-16)* (x 10^2 Rad.)	Skewness (Eqn. A-20)*	Kurtosis (Eqn. A-22)*
ALGEBRAIC VALUES						
A	x Axis	458	-0.004	0.164	0.27	5.1
	y Axis	458	-0.005	0.160	0.00	4.7
	Total	916	-0.004	0.162	0.14	4.9
B	x Axis	880	-0.009	0.167	-0.13	3.8
	y Axis	880	0.012	0.178	0.17	4.2
	Total	1760	0.002	0.173	0.05	4.1
A + B	Total	2676	-0.001	0.170	0.09	4.5
C	Total	561	0.015	0.160	0.05	4.7
ABSOLUTE VALUES						
A	x Axis	458	0.119	0.113	1.74	7.5
	y Axis	458	0.115	0.112	1.51	6.1
	Total	916	0.117	0.112	1.62	6.8
B	x Axis	880	0.126	0.110	1.24	5.2
	y Axis	880	0.134	0.119	1.43	6.1
	Total	1760	0.130	0.114	1.35	5.8
A + B	Total	2676	0.125	0.115	1.50	6.4
C	Total	561	0.118	0.108	1.68	6.9

* See Appendix A

TABLE 6.6 STATISTICAL CHARACTERISTICS OF COLUMN OUT-OF-PLUMBS

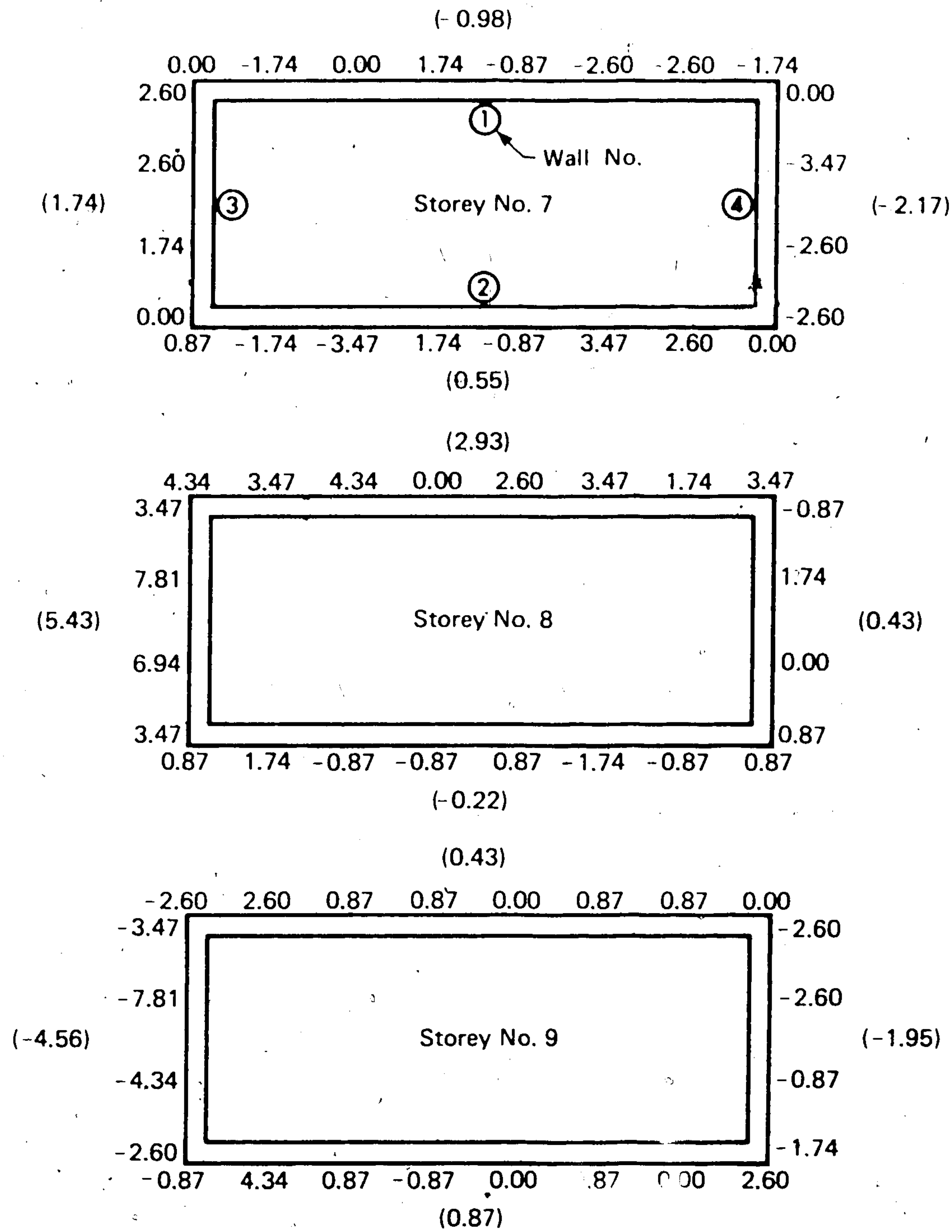
The corresponding statistical characteristics are listed in Table 6.6. In this table, the values of the mean and standard deviation are always positive and of the same order of magnitude, with a resulting coefficient of variation (standard deviation/mean) slightly lower than unity. The standard deviations of the half-normal distributions are 47 percent lower than the standard deviations of the corresponding normal distributions. The measure of kurtosis, in the order of 6.5, indicates that the half-normal distribution approaches the exponential distribution characterized by a factor of 9.0.

6.4.2 Wall Out-of-Plumbs

In a manner similar to the column deviations, the wall out-of-plumbs are conveniently expressed in the non-dimensional form Δ_0/h, where Δ_0 is the horizontal deviation of the top of the wall from a plumb line passing through the base of the wall and h is the height of the wall.

Several measurements are needed to define the out-of-plumb of a wall. A minimum of four measurements were taken at regular intervals along the walls. In some cases, up to 15 measurements were necessary to define the out-of-plumbs of long walls. As an example, the values and locations of measurements taken at three adjacent storeys in building A are given in Fig. 6.13. The sign convention adopted is the same as that used for columns, that is, a value is positive when the top of a wall leans in the positive direction of the axis.

The measurements taken on a cast-in-place reinforced concrete wall are not totally independent of each other. This observation is based on the fact that a wall is being cast in a continuous form and that the chance of measuring large out of plumb variations



Wall Out-of-Plumbs: Δ_0/h ($\times 10^3$ Rad.)

Figure 6.13 Typical core-wall measurements on building A

within a short distance along the wall is remote. The average out-of-plumb value of each individual wall therefore is used in statistical manipulations. This, in turn, implies that the deviations of the individual walls forming a core are independent of each other, which is a reasonable assumption.

Two out-of-plumb values are used to characterize a wall. One is measured in the direction "perpendicular" to the plane of the wall and is the average out-of-plumb discussed above. The second is measured in the direction "parallel" to the plane of the wall and is the average of the measurements taken at the two extremities. According to this definition, the parallel out-of-plumb for wall No. 1 at storey 7 in Fig. 6.13 is (0.0026 + 0)/2 = 0.0013 Rad.

The average perpendicular and parallel wall out-of-plumbs, as defined above, are given in Tables 6.7 through 6.10 for buildings A and B. The measurements for building B were taken by the research team but those for building A were supplied by the surveyor on the job site. The results for the 5 storeys above level 22 in the latter were not available.

The perpendicular and parallel out-of-plumbs are plotted separately on Figs. 6.14 through 6.19 for building A, B, and A and B combined. The type of graph used is the same as for the column data. Once again, each discrete distribution can be fitted by a normal distribution.

The characteristics of each statistical distribution are summarized in Table 6.11. The standard deviation can be taken as 0.0028 Rad. for both parallel and perpendicular out-of-plumbs. For all practical purposes, a common mean value of 0.00028 Rad. can be

Out-of-Plumbs : x 10^3 Rad.

Storey	Wall No.*			
	1	2	3	4
22	1.95	1.02	12.37	-4.20
21	2.86	1.48	1.56	-1.54
20	-0.87	-0.87	0.69	0.72
19	2.60	-1.74	-1.74	5.79
18	0.12	2.11	5.64	0.00
17	-5.82	-6.57	-0.87	-0.93
16	2.93	-0.11	8.25	0.87
15	0.00	-0.22	3.91	-3.69
14	5.32	3.26	-4.12	-1.09
13	-1.20	-0.98	-0.22	1.95
12	-2.50	-1.52	-2.60	-2.60
11	0.87	2.17	3.47	0.87
10	-1.85	-0.98	2.17	0.22
9	0.43	0.87	-4.56	-1.95
8	2.93	-0.22	5.43	0.43
7	-0.98	0.55	1.74	-2.17
6	0.14	-0.43	0.68	-0.65
5	2.92	0.36	-1.76	-0.54
4	-4.17	0.55	1.91	6.95
3	0.31	-0.28	1.74	-2.08
2	3.27	0.40	2.17	-3.76
1	-4.40	1.45	3.76	2.89

* Wall numbering given in Fig. 6.2.

TABLE 6.7 PERPENDICULAR WALL OUT-OF-PLUMBS FOR BUILDING A

Out-of-Plumbs : x 10^3 Rad.

Storey	Wall No.*			
	1	2	3	4
22	-11.28	-10.42	0.00	-1.74
21	-1.30	0.00	0.00	1.74
20	0.00	1.74	0.43	-0.43
19	-0.43	-1.74	-0.43	0.43
18	4.34	4.34	0.87	3.04
17	-3.47	0.87	-1.74	-9.55
16	1.30	-0.87	-2.17	-0.87
15	-3.04	-0.87	1.74	2.60
14	0.43	0.00	1.74	-3.91
13	-2.60	-2.60	2.17	-2.60
12	3.04	3.04	-3.47	2.17
11	-0.87	-0.43	0.43	-1.74
10	-3.04	-2.17	-1.74	1.30
9	1.30	2.17	1.74	2.17
8	1.30	-1.30	0.43	-0.87
7	6.51	0.87	1.30	3.47
6	-1.74	0.00	-0.43	-0.87
5	1.08	-2.15	-0.54	2.15
4	7.29	0.00	0.00	0.00
3	3.13	0.00	2.60	-0.52
2	-0.87	-0.43	-0.87	1.30
1	-1.74	4.34	0.00	-1.74

* Wall numbering given in Fig. 6.2.

TABLE 6.8 PARALLEL WALL OUT-OF-PLUMBS FOR BUILDING A

Out-of-Plumbs : x 10^3 Rad.

Storey	Wall No.*								
	1	2	3	4	5	6	7	8	9
34	-0.97	-0.83	-	2.76	-2.19	1.46	1.98	1.48	-0.42
33	2.19	0.00	-	2.71	-0.83	1.56	-1.67	-2.37	-0.31
32	0.73	2.08	-	5.73	3.33	-3.54	-1.39	-2.37	-3.33
31	-0.26	1.67	-	1.70	1.67	0.42	0.52	-1.22	-0.42
30	1.72	-1.46	-	1.04	-4.04	-0.91	-0.73	-3.31	0.10
29	1.88	1.41	-	-0.31	0.21	-2.50	-0.42	1.47	-0.20
28	2.29	0.52	-	1.17	0.63	-3.44	1.98	-0.59	-1.46
27	0.05	-1.25	-	0.68	-1.15	-2.50	-0.52	-0.26	-0.83
26	1.25	0.10	-	0.21	-0.63	-2.08	-3.44	1.87	1.04
25	0.05	-0.21	-	-2.71	-0.52	1.04	4.17	2.24	-1.67
24	0.89	0.96	-	-0.39	-2.92	1.35	7.29	1.82	6.94
23	0.63	1.82	-	2.08	-3.13	-1.88	0.31	-1.30	-0.21
22	2.08	0.21	-	-0.20	1.04	3.65	2.92	-0.63	1.94
21	0.94	0.28	-	0.76	-2.08	-1.87	1.98	-2.27	2.71
20	0.68	0.26	-	4.69	-1.88	-1.46	1.25	5.00	-1.04
19	-1.72	2.50	5.52	-1.35	-0.83	2.50	1.25	2.08	-0.47
18	0.98	3.52	3.52	1.95	4.04	1.82	5.73	0.00	-1.39
7	1.09	0.47	1.98	3.02	-1.87	0.73	-1.56	0.42	1.15
[illegible]	0.37	0.94	3.75	-5.42	-2.92	-1.46	-2.92	8.75	-3.85
[illegible]	-0.99	1.20	2.50	-0.73	0.00	1.65	-2.92	-4.17	1.04
14	2.71	1.46	-0.52	-2.78	-2.08	0.83	-2.19	0.00	-0.83
13	2.73	-0.59	0.91	4.43	0.39	-1.25	2.73	0.52	1.04
12	-2.05	0.43	-2.41	0.52	3.52	-0.52	-0.91	-5.21	-2.02
11	1.69	9.33	-1.17	-3.13	-9.51	-2.47	-2.60	-1.82	-4.56
10	1.99	-3.52	0.26	-2.29	2.08	1.30	0.78	-4.69	0.46
9	3.52	1.86	0.26	1.30	0.65	0.76	0.00	-0.52	2.08
8	2.15	-1.60	-0.98	0.91	1.82	2.73	0.13	1.04	2.21
7	0.49	-0.91	0.78	0.70	-1.59	4.77	-4.43	-1.56	2.73
6	-1.63	0.33	2.60	-1.56	0.26	0.00	-1.30	-0.42	3.13
5	2.15	0.52	-0.17	2.99	0.13	-0.15	-3.52	5.28	-1.74
4	-0.62	0.62	2.40	-0.91	-2.21	0.91	1.17	-0.91	2.41
3	-4.86	-0.89	-1.09	-1.04	0.00	0.52	-0.13	0.13	0.70
2	3.99	-0.07	-0.35	-2.74	-0.65	-1.17	-0.52	-0.78	-1.85
1	-1.39	-0.05	-1.27	-1.04	-1.31	-1.56	-0.52	-1.69	1.62

*Wall numbering given in Fig. 6.3

TABLE 6.9 PERPENDICULAR WALL OUT-OF-PLUMBS FOR BUILDING B

Out-of-Plumbs : x 10^3 Rad.

Storey	Wall No.*								
	1	2	3	4	5	6	7	8	9
34	-2.08	-0.83	-	0.27	-0.83	-0.83	0.42	-3.73	-0.42
33	4.17	-0.20	-	4.17	-2.92	-3.75	-0.42	2.29	3.33
32	-0.42	0.00	-	2.50	0.42	0.83	5.00	3.75	0.21
31	0.79	-0.26	-	2.08	1.25	0.00	-1.67	1.25	0.83
30	-1.04	1.46	-	0.21	2.29	0.21	-0.63	-1.04	1.88
29	-3.33	-2.50	-	5.83	2.08	1.04	0.63	-0.21	0.21
28	-0.73	1.25	-	1.67	1.67	-0.42	2.50	2.08	3.33
27	0.42	-0.42	-	-1.67	-1.25	0.00	-3.13	0.63	2.71
26	-2.50	0.42	-	2.92	3.75	-0.21	0.00	-1.67	0.42
25	-0.83	-0.63	-	0.83	-0.42	0.42	0.00	0.21	0.00
24	-2.50	2.08	-	-0.83	0.00	0.42	2.08	2.50	4.17
23	1.67	-0.42	-	2.08	1.67	-1.46	0.83	2.08	0.83
22	0.00	-0.63	-	0.00	-0.21	6.67	1.04	0.21	1.67
21	0.94	0.28	-	0.76	-2.08	-1.88	1.98	0.42	1.88
20	-4.17	4.17	-	-0.83	1.25	0.42	1.04	1.67	1.46
19	-3.75	-2.50	-1.67	-0.83	2.50	0.00	-3.33	-1.25	-2.29
18	-0.42	2.50	5.42	3.13	1.82	-0.52	1.82	1.82	4.58
17	0.21	0.00	-4.58	1.25	-0.63	1.67	1.25	2.50	1.46
16	-2.92	-5.83	0.63	0.42	3.13	1.46	1.25	-1.67	0.83
15	0.63	2.08	-1.67	2.92	0.00	-2.50	3.75	-3.33	4.38
14	-0.83	-0.21	1.67	2.92	0.42	2.92	2.50	2.92	0.83
13	2.08	-1.56	0.78	-0.52	-0.52	3.80	3.38	0.00	2.08
12	-0.52	-1.95	1.95	-2.47	-0.52	-2.60	-2.21	-0.65	1.43
11	0.00	-2.60	3.65	-0.26	1.04	4.17	4.43	3.65	5.73
10	-1.04	4.43	-1.95	-2.21	-1.56	0.65	1.69	0.26	2.08
9	-2.34	3.91	5.08	1.30	1.04	2.21	1.82	0.52	5.73
8	0.26	-0.13	-0.52	-0.78	0.52	2.08	2.34	1.69	-3.39
7	1.04	2.60	-1.04	-2.08	0.26	0.26	2.60	-1.95	0.78
6	0.26	1.56	-0.26	-1.56	2.08	0.13	0.52	-3.13	-1.56
5	-1.78	-5.73	0.26	4.69	0.78	1.30	-0.78	-1.56	0.78
4	2.08	0.71	2.08	1.41	-0.31	2.19	0.91	-2.40	1.04
3	-4.34	-0.69	2.08	-0.69	0.00	-1.52	0.43	-1.30	-2.60
2	-0.17	-0.35	3.47	0.69	0.17	-2.08	1.39	2.43	2.78
1	-1.56	0.52	-0.28	0.35	-1.39	-1.39	-0.35	0.52	-0.69

*Wall numbering given in Fig. 6.3

TABLE 6.10 PARALLEL WALL OUT-OF-PLUMBS FOR BUILDING B

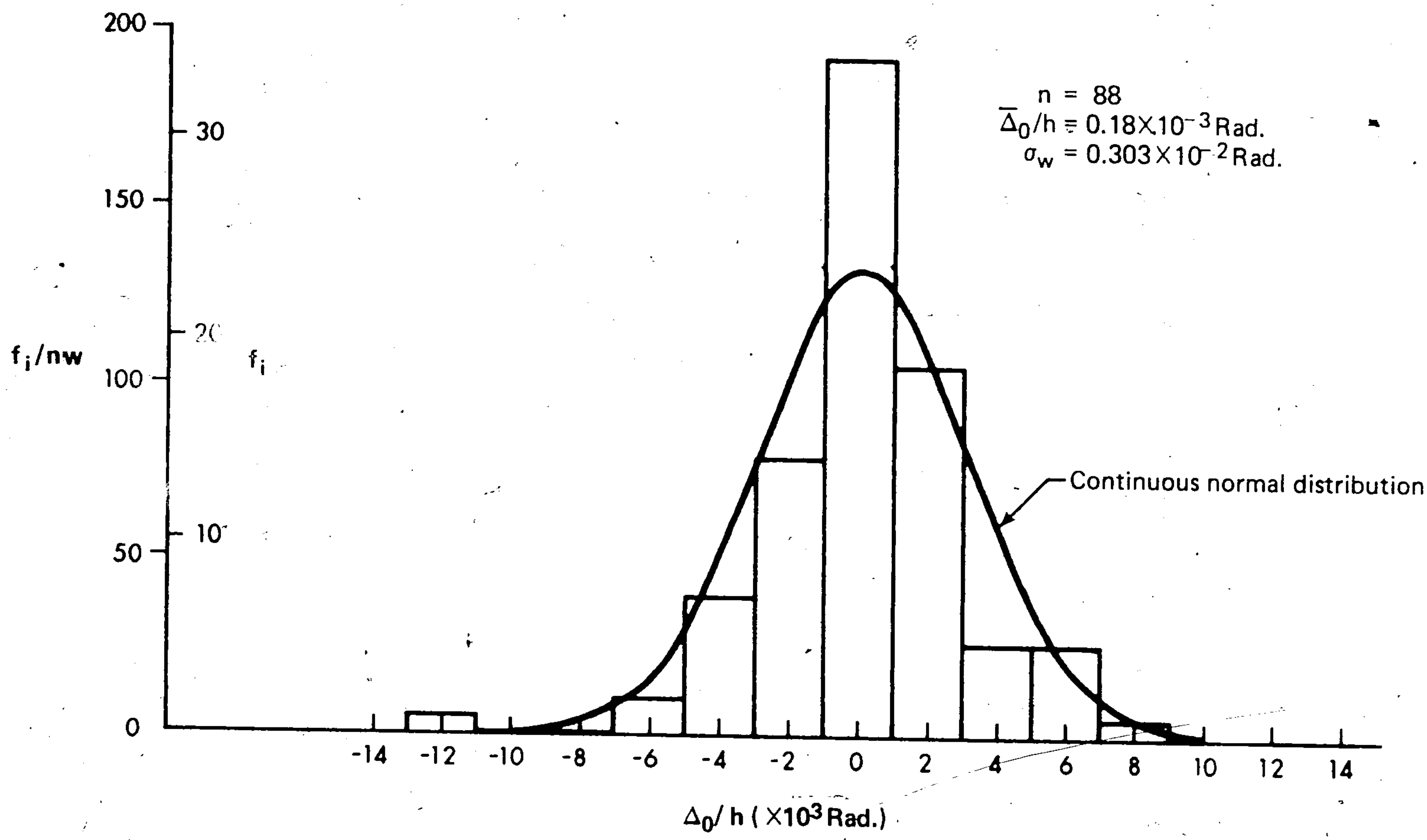


Figure 6.14 Distribution of perpendicular wall out-of-plumbs for building A

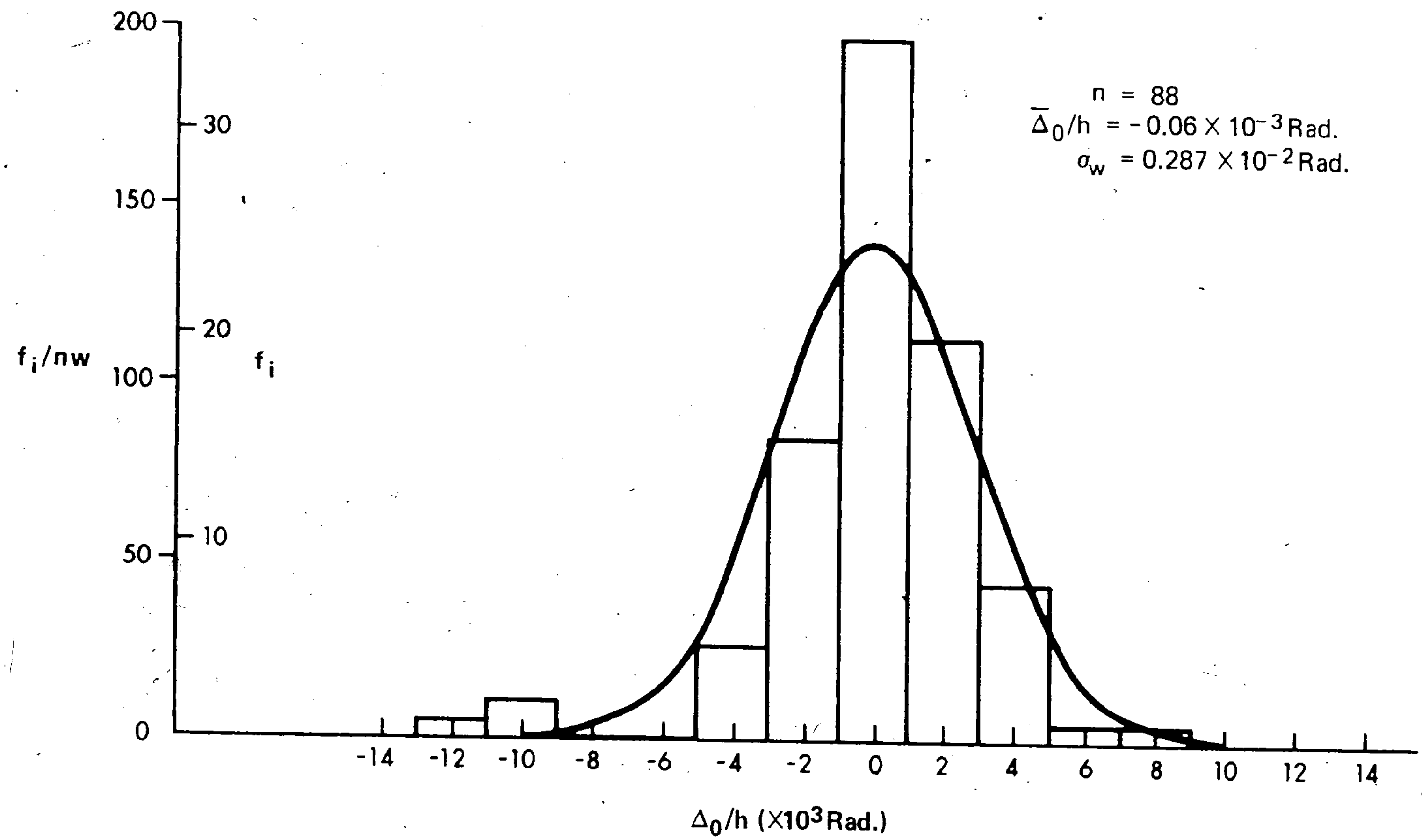


Figure 6.15 Distribution of parallel wall out-of-plumbs for building A.

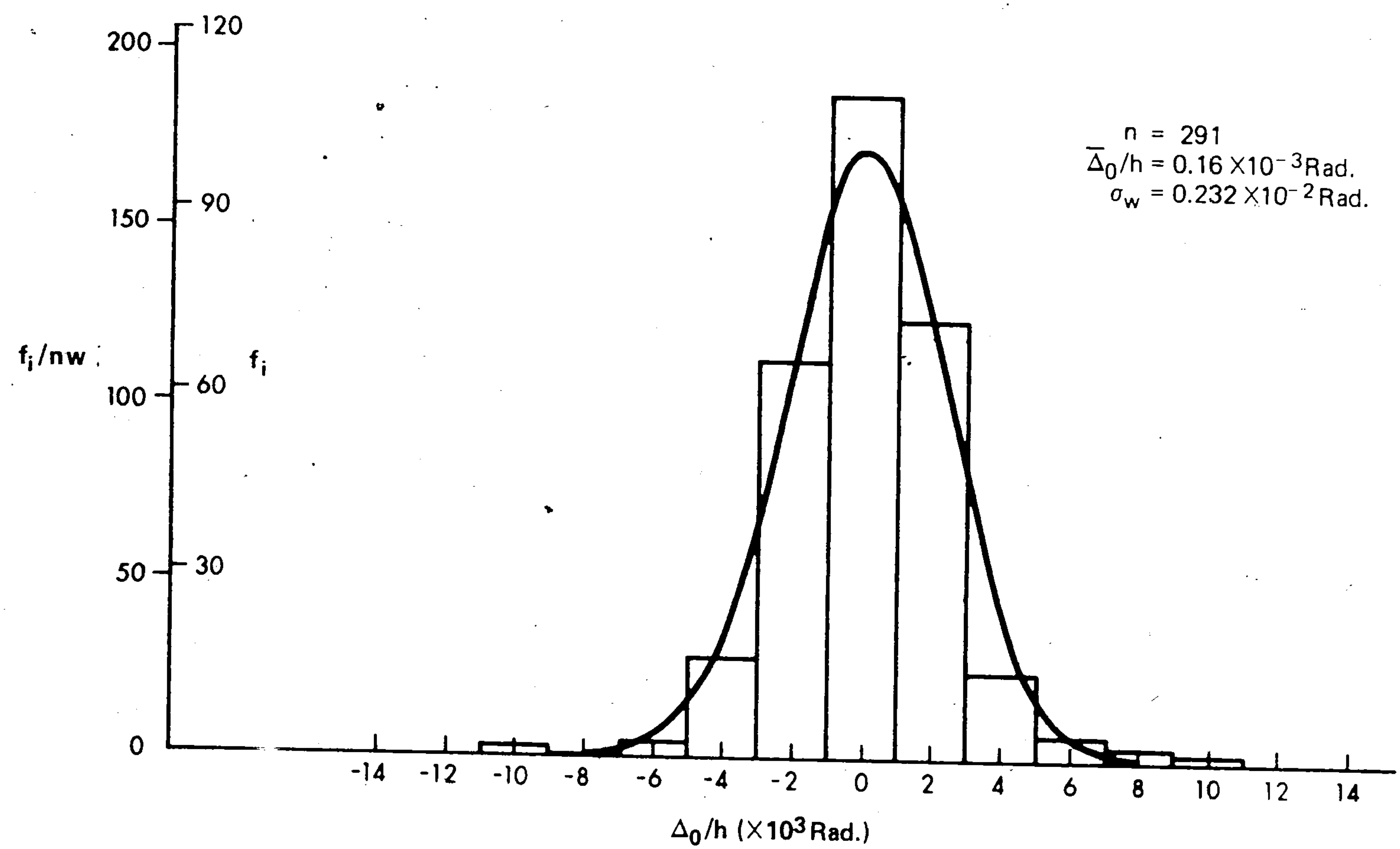


Figure 6.16 Distribution of perpendicular wall out-of-plumbs for building B

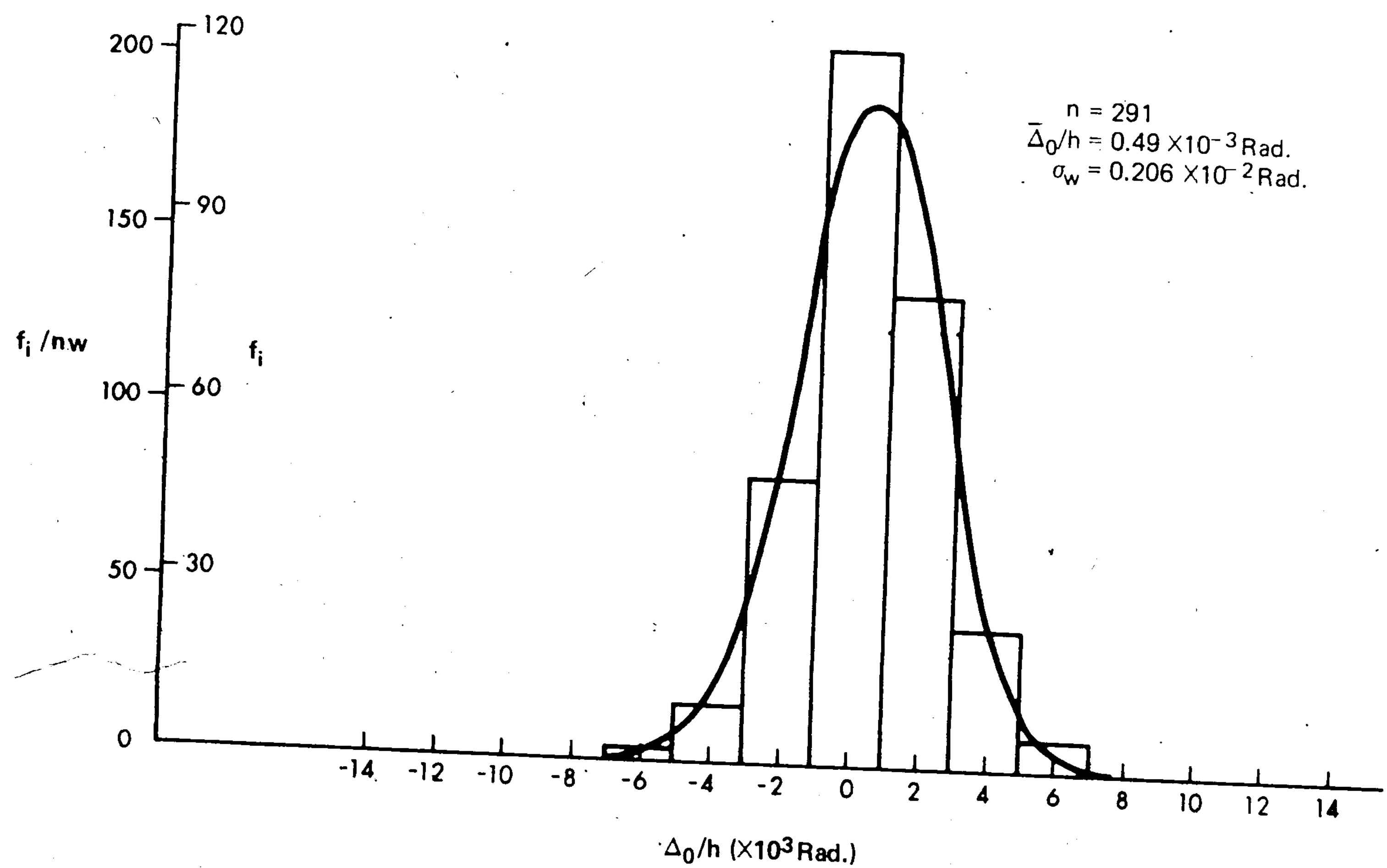


Figure 6.17 Distribution of parallel wall out-of-plumbs for building B

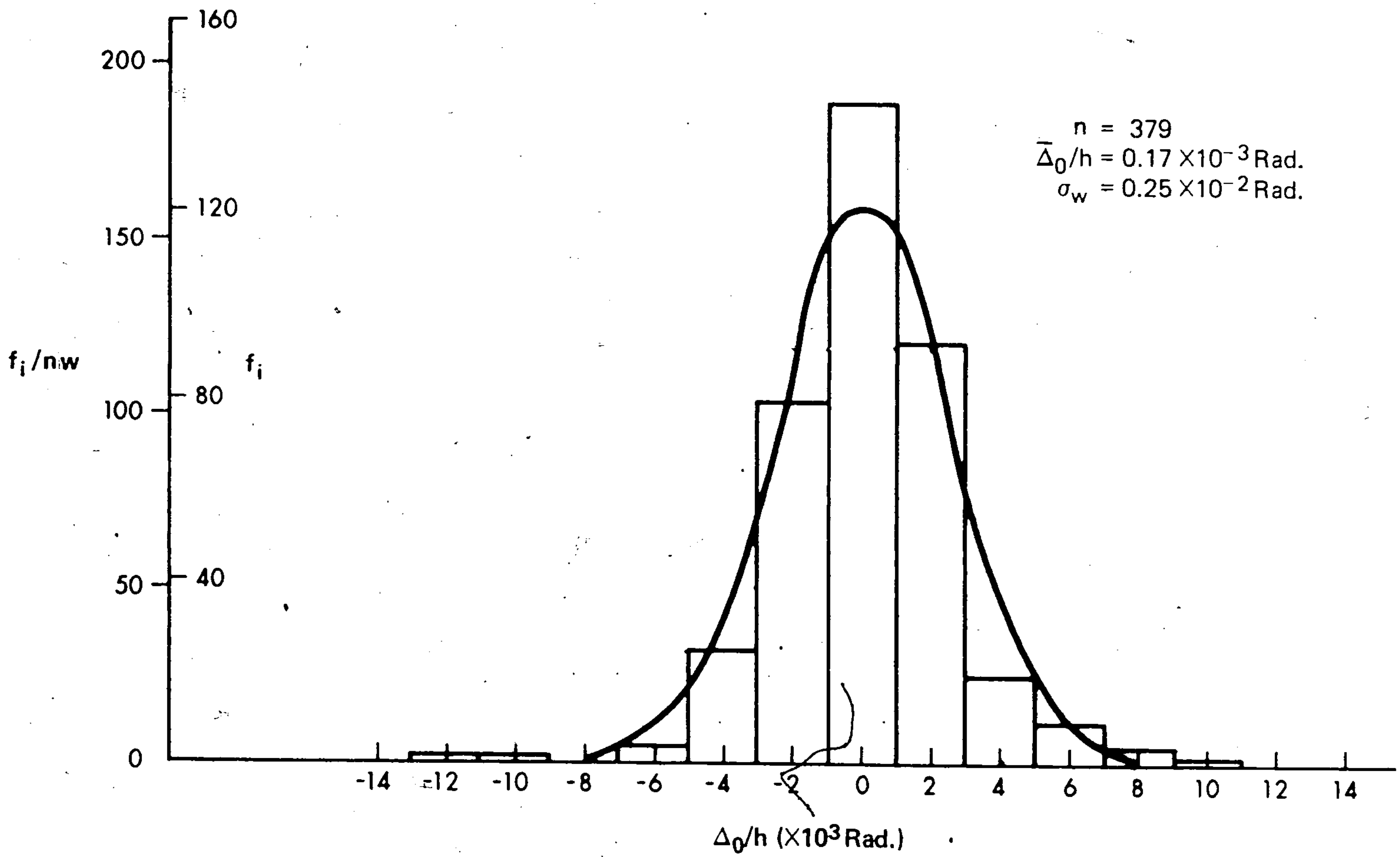


Figure 6.18 Distribution of perpendicular wall out-of-plumbs for buildings A and B

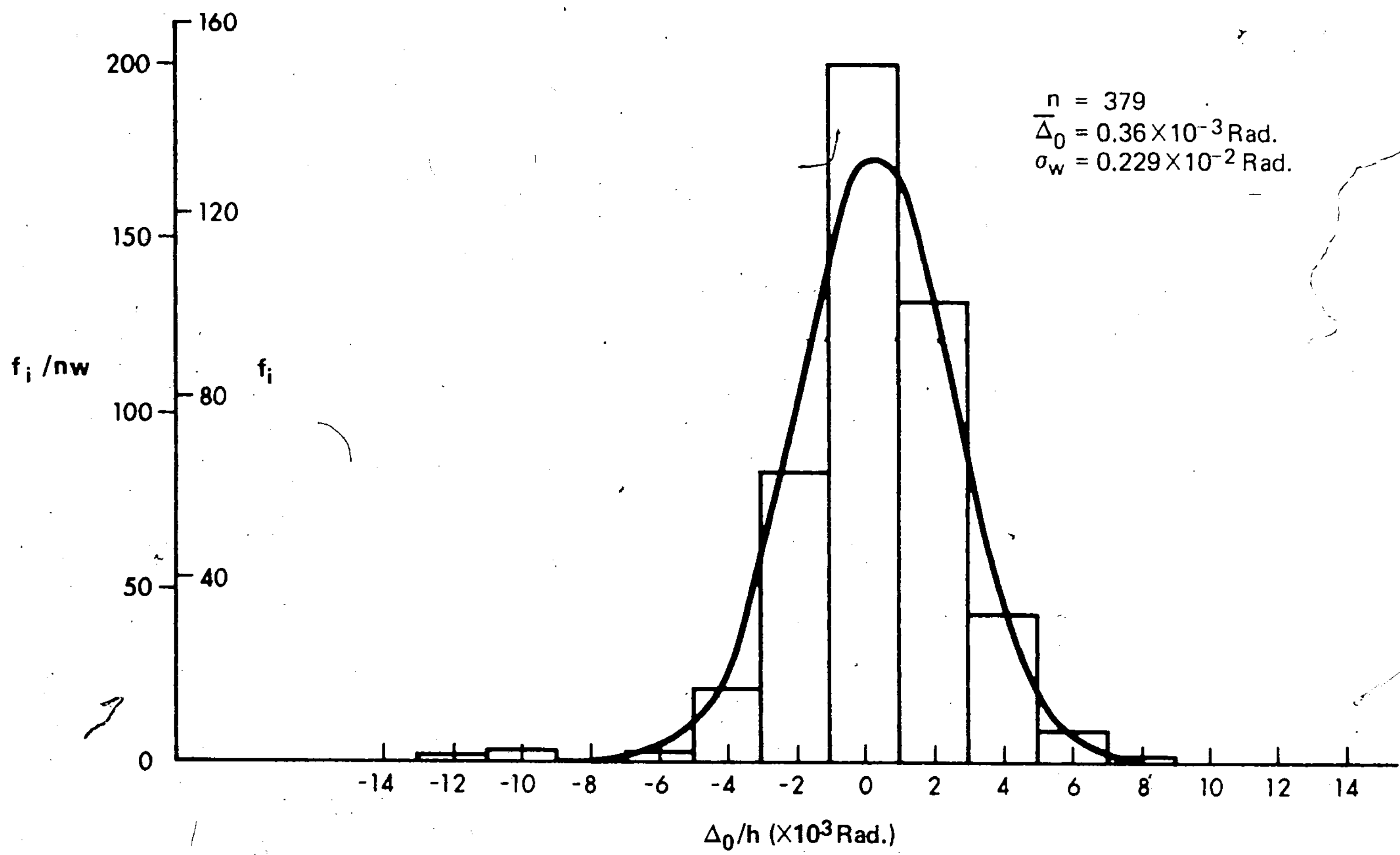


Figure 6.19 Distribution of parallel wall out-of-plumbs for buildings A and B

Building	Type	Sample Dimension n	Mean (Eqn. A-14)* (x 10^2 Rad.)	Stand. Dev. (Eqn. A-16)* (x 10^2 Rad.)	Skewness (Eqn. A-20)*	Kurtosis (Eqn. A-22)*
A	Perpen.	88	0.018	0.303	-0.57	5.6
	Parallel	88	-0.006	0.287	-1.18	7.3
B	Perpen.	291	0.016	0.232	0.29	4.9
	Parallel	291	0.049	0.206	0.06	3.3
A + B	Perpen.	379	0.017	0.250	-0.05	5.63
	Parallel	379	0.036	0.229	-0.60	6.37

* See Appendix A

TABLE 6.11 STATISTICAL CHARACTERISTICS OF WALL OUT-OF-PLUMBS assumed for both types of out-of-plumbs, therefore eliminating the need to differentiate between perpendicular and parallel deviations. The values listed in Table 6.11 also demonstrate that the distributions are slightly skewed and somewhat more peaked than a normal distribution.

CHAPTER VII

STATISTICAL ANALYSIS

7.1 Effects of Column Out-of-Plumbs

Since the column out-of-plumb population is normally distributed, the variate Δ_0/h can be standardized as described in Appendix A. A design value for column deviations from plumb is obtained by rearranging the terms in Eq. (A-44).

$$\frac{\Delta_d}{h} = \frac{\overline{\Delta}_0}{h} + \beta\,\sigma_c \tag{7.1}$$

In this expression Δ_d/h is the distributed random variable, β is the standardized Δ_d/h value, $\overline{\Delta}_0/h$ is the arithmetic mean of the discrete population, and σ_c is the standard deviation.

In the present study, the mean, valued at -1.0×10^{-5} Rad. in Fig. 6.7, may clearly be neglected in Eq. (7.1). In other words, the population is assumed normally distributed about a mean of zero. The expression is then reduced to

$$= \beta\,\sigma_c \tag{7.2}$$

The quantity β is found from the "Tables of the Standard Cumulative Normal Distribution" (Table A-1) for a prescribed cumulative probability of occurrence. For example, the probability of having a value falling within the limits $\pm\, 2\sigma_c$ is 0.9544. The selection of an appropriate β

in the present case is very arbitrary. A study described in Ref. 58 has shown that the probability of failure of a building under normal conditions should not be higher than 3×10^{-4} during the 30-year life of the structure. This corresponds to a β factor of approximately 3.5. A factor of 3.0 has been used in Ref. 61 in the derivation of design criteria based on limit states. The New Canadian Standard CSA-S16.1, "Steel Structures for Buildings - Limit States Design", has used β factors ranging between 2.9 and 4.0(59). The selected factor, β, commonly called the "safety index", should fall within these limits. A conservative β of 3.5 corresponding to a probability of being exceeded of 4.6×10^{-4} will be used in this thesis. This choice will be subject to further discussions in Appendices B and C.

7.1.1 Horizontal Force at Connection Point

The horizontal forces shown in Fig. 2.2 result from the fact that the column is out-of-plumb. The force $P\Delta_x/h$, for instance, is transmitted by the connection to the beam or floor diaphragm and then to the core. A safe estimate of this additional force in the connection is:

$$F_d = \beta\sigma_c P = 3.5 \times 0.0017\ P \simeq 0.006\ P \qquad (7.3)$$

where F_d is the absolute value of the force, P is the factored axial load in the column obtained for a specific load combination, $\beta = 3.5$, and $\sigma_c = 0.0017$ from Table 6.6. Equation (7.3) indicates that a connection between one column and the adjacent beam should be designed for 0.6 percent of the factored axial load to resist the force created by the out-of-plumb of the column. The force F_d has a probability of not being exceeded, defined by the safety index β, of 99.954 percent or, in other words, a probability of being exceeded of 4.6×10^{-4}, if P is assumed deterministic.*

In the common case of two column segments connected at a floor level and having different axial loads, different heights, and different out-of-plumbs, the extra force, F, at the beam-to-column connection is an algebraic summation of the type shown in Fig. 7.1.

$$F = \left(\frac{P\Delta}{h}\right)_1 + \left(\frac{P\Delta}{h}\right)_2 \qquad (7.4)$$

If $\quad F = P_1X + P_2Y$

and $\quad X \sim> N(\mu_x, \sigma_x), \quad Y \sim> N(\mu_y, \sigma_y)$

Then $\quad F \sim> N(P_1 \mu_x + P_2 \mu_y, \sqrt{P_1^2 \sigma_x^2 + P_2^2 \sigma_y^2})$,

if independence is satisfied.†

For $\mu_x = \mu_y = 0$ and $\sigma_x = \sigma_y = \sigma$,

$$F \sim> N(0, \sigma\sqrt{P_1^2 + P_2^2})$$

Thus, in the case of two column segments, the force F is still normally distributed and has a new standard deviation defined as above.

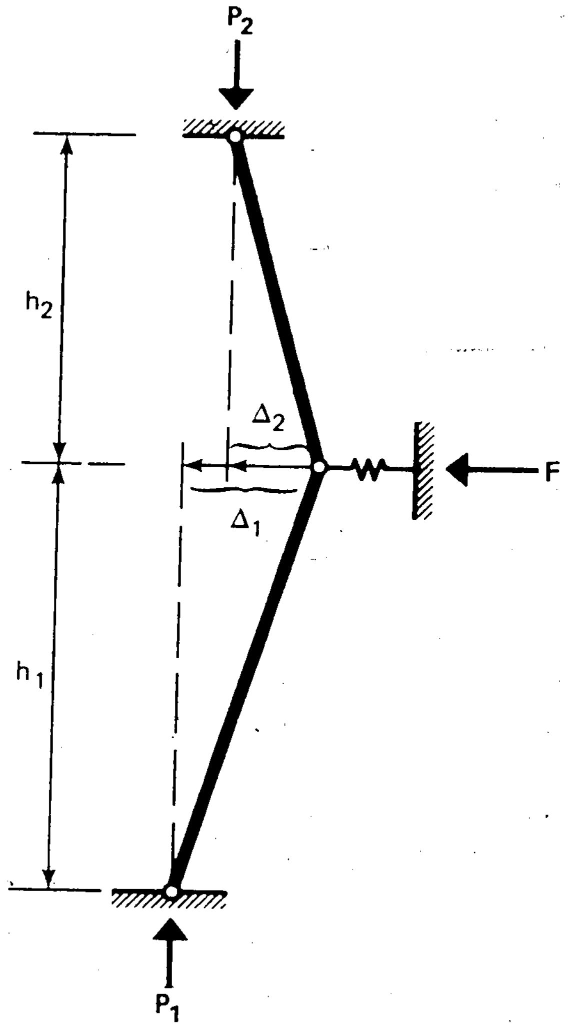
Equations (7.2) and (7.4) are combined to give:

$$F_d \doteq \beta\sigma_c\sqrt{P_1^2 + P_2^2} \qquad (7.5)$$

where F_d is the absolute value of the extra force used in the design of a beam-to-column connection when two columns are present. When $P_1 = P_2$, $F_d = 3.5 \times 0.0017 \times \sqrt{2}\ P = 0.0084\ P$. The extra force to be resisted by the connection as given by expression (7.5) is 0.84 percent of the

* See Appendix B

† See Appendix C



$F = (P\Delta/h)_1 + (P\Delta/h)_2$

Figure 7.1 **Horizontal force required to stabilize two out-of-plumb columns**

average axial load in the columns, which is significantly lower than the 2 percent presently used[43].

7.1.2 Horizontal Shear in the Plane of the Floor

The out-of-plumbs of the columns above and below a given floor produce extra shears in the plane of that floor. For example, the floor system of Fig. 6.2 must transmit to the core an extra shear resulting from the 16 out-of-plumb columns on lines 8 to 15. These columns have random inclinations and the resulting shear (for example in the x-direction) is:

$$F = \sum_{j=1}^{n} \left[\frac{P\Delta_0}{h}\right]_j \tag{7.6}$$

where $n = 16$, the number of columns considered in the example.

As in the preceding case, an expression for F may be obtained from the statistical sum of the standard deviations corresponding to the 16 columns.

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \tag{7.7}$$

F_d is the absolute value of the extra horizontal shear due to n out-of-plumb columns and P_j is the factored column axial load for the load combination considered. The other terms have been defined previously. Equation (7.7) is general and includes Eqs. (7.3) and (7.5), applied previously to connection design, for n = 1 and 2.

7.1.3 Moment in Floor

Moments in any portion of a floor due to a group of out-of-plumb columns can also be determined. For example, the moment in the plane of the floor at point 0 in Fig. 6.2 is produced by the x and y out-of-plumbs of the columns on lines 8 to 15, thus:

$$M = \sum_{j=1}^{16} \left[\frac{P\Delta_x}{h} L_y + \frac{P\Delta_y}{h} L_x\right]_j \qquad (7.8)$$

In Eq. (7.8), L_x and L_y are the lever arms in the x and y directions from the column to the point at which the moment is calculated. Since L_x and L_y are also coefficients (similar to P), the same summation rule applies.

$$M_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} [P^2(L_x^2 + L_y^2)]_j} \qquad (7.9)$$

M_d is the absolute value of the design moment in the plane of the floor due to a group of n out-of-plumb columns.

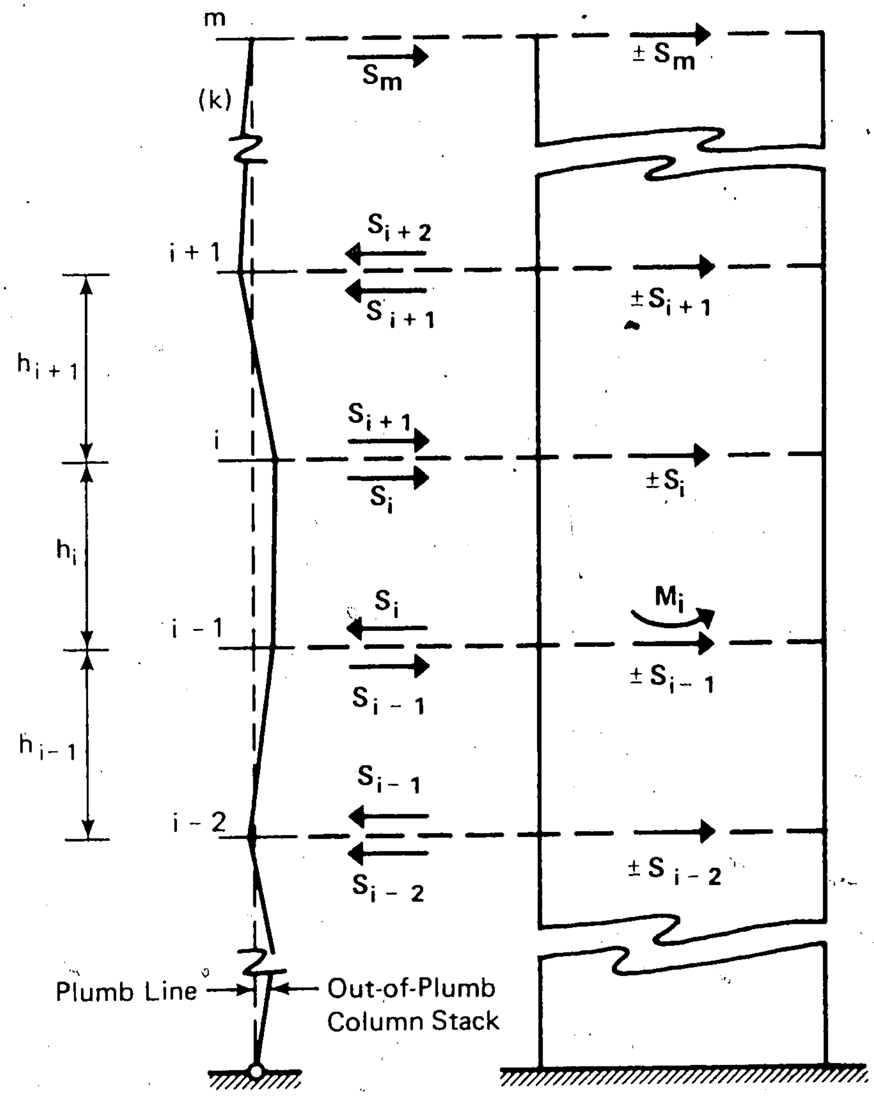
7.1.4 Shear in Core

The core (or any other bracing system) must stabilize the columns by resisting the forces induced by the vertical loads acting on the columns in their deformed positions. Fig. 7.2 shows that the absolute value of the out-of-plumb shear resisted by the core between floors i-1 and i is S_i and depends only on the out-of-plumbs of the columns at storey i (storey below floor level i).

$$S_i = \sum_{j=1}^{n_i} \left[\frac{P\Delta_0}{h}\right]_j \qquad (7.10)$$

where n_i is the number of columns at storey i. Then,

$$S_{di} = \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} P_j^2} \qquad (7.11)$$



$\overline{m}$ = Total number of storeys

i, k = Storey indices $i + 1 \leqslant k \leqslant m$

Figure 7.2 Shears and moments in core due to column out-of-plumbs

where S_{di} is the absolute value of the shear in the core at storey i caused by the out-of-plumbs of the columns and P_j is the factored axial load in column j. The summation extends over the n_i columns in storey i.

7.1.5 Moment in Core

Fig. 7.2 shows that the moment at floor level i due to the out-of-plumb columns is:

$$M_i = \sum_{k=i+1}^{m} S_k h_k \tag{7.12}$$

where k is a storey index used for storeys above level i. Substituting S_k as given by Eq. (7.10) in the expression for the moment, gives:

$$M_i = \sum_{k=i+1}^{m} \left[\sum_{j=1}^{n_k} \left(\frac{P\Delta_0}{h}\right)_j \right]_k h_k$$

Transforming the summation inside the brackets results in:

$$M_i = \sum_{k=i+1}^{m} \left(\beta\sigma_c \sqrt{\sum_{j=1}^{n_k} P_j^2}\right)_k h_k$$

Transforming again for the other summation yields:

$$M_{di} = \beta\sigma_c \sqrt{\sum_{k=i+1}^{m} \left[\sqrt{\sum_{j=1}^{n_k} P_j^2}\right]_k^2 h_k^2}$$

which can be written as:

$$M_{di} = \sqrt{\sum_{k=i+1}^{m} (S_d h)_k^2} \tag{7.13}$$

where M_{di} is the moment in the core at floor level i caused by the out-of-plumbs of the columns, S_{dk} is the shear at storey k given by Eq. (7.11),

h_k is the height of storey k, and m is the total number of storeys in the building.

7.1.6 Torque in Core

The torque due to out-of-plumb columns, at a specific storey of the core, depends only on the columns at that storey in a manner similar to the shear. The torque at each storey is obtained by combining the expressions developed for the moments in the floors (7.9) and the shears in the core (7.11).

$$T_i = \sum_{j=1}^{n_i} \left[\frac{P\Delta_x}{h} L_y + \frac{P\Delta_y}{h} L_x\right]_j \tag{7.14}$$

or

$$T_{di} = \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} [P^2(L_x^2 + L_y^2)]_j} \tag{7.15}$$

where L_x and L_y are the distances (lever arms) along the x and y axes between a particular column and the center of resistance of the core.

7.1.7 Lateral Deflections

An equivalent column inclination, Δ_d/h, constant for a specified number of columns may be obtained from Eqs. (7.6) and (7.7). For $F = F_d$,

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2}}{\sum_{j=1}^{n} P_j} \tag{7.16}$$

This equation would be considerably simplified if expressed only in terms of β, σ_c and n, the total number of columns in the structure. Assuming that

P_j is constant for all the columns gives:

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c}{\sqrt{n}} \tag{7.17}$$

Generally, the column axial loads differ greatly in a structure. As demonstrated in Appendix D, the formulation (7.17) is always unconservative with respect to the "exact" expression (7.16). It is also demonstrated in the Appendix that

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c}{\sqrt[2.2]{n}} \tag{7.18}$$

gives a safe estimate when n is reasonably large. For structures of one and two storeys, Eq. (7.16) is recommended.

A set of horizontal forces is obtained from the structural configuration shown in Fig. 5.1(a) where the constant slope is defined by either one of the equations above. These forces can be added to the wind forces and used to calculate the lateral deflections of a structure. Applications of Eqs. (7.16) and (7.18) are given in the next chapter.

7.2 Effects of Wall Out-of-Plumbs

The results summarized in Table 6.11 show that the perpendicular and parallel wall out-of-plumb populations can be described by a normal distribution and statistical characteristics common to both. Conservatively, the mean, $\overline{\Delta}_0/h$, is taken as 0.00028 Rad. and the standard deviation, σ_w, as 0.0028 Rad.

Since the core depends entirely on itself for stability (the frame is assumed pinned at each floor level and the core cantilevered from the foundation), only moments, torques, and extra lateral deflections induced in the core by the wall out-of-plumbs must be calculated.

The deviations measured on the walls are affected in some ways by the presence of variations in wall thickness. The problem is treated in Appendix E.

7.2.1 Moment in Core

The expression used to describe a standardized normal variable (A-44) can also be used to describe the wall out-of-plumbs:

$$\frac{\Delta_d}{h} = \frac{\bar{\Delta}_0}{h} + \beta\sigma_w \tag{7.19}$$

where Δ_d/h is the design wall out-of-plumb and β is the safety index introduced in section 7.1. The moment due to out-of-plumbs at the base of the one-storey wall shown in Fig. 7.3(a) is:

$$M = P\Delta_0 \tag{7.20}$$

where P is the total factored load carried by the wall and Δ_0 is the actual averaged out-of-plumb of the wall. Similarly, the moment in either the x or y direction at the base of the one-storey core section of Fig. 7.3(b) is:

$$M = \sum_{j=1}^{n} (P\Delta_0)_j \tag{7.21}$$

where P_j is the factored axial load carried by one of the n individual wall segments and Δ_{0j} is the actual perpendicular or parallel out-of-plumb of the wall, depending on the direction considered.

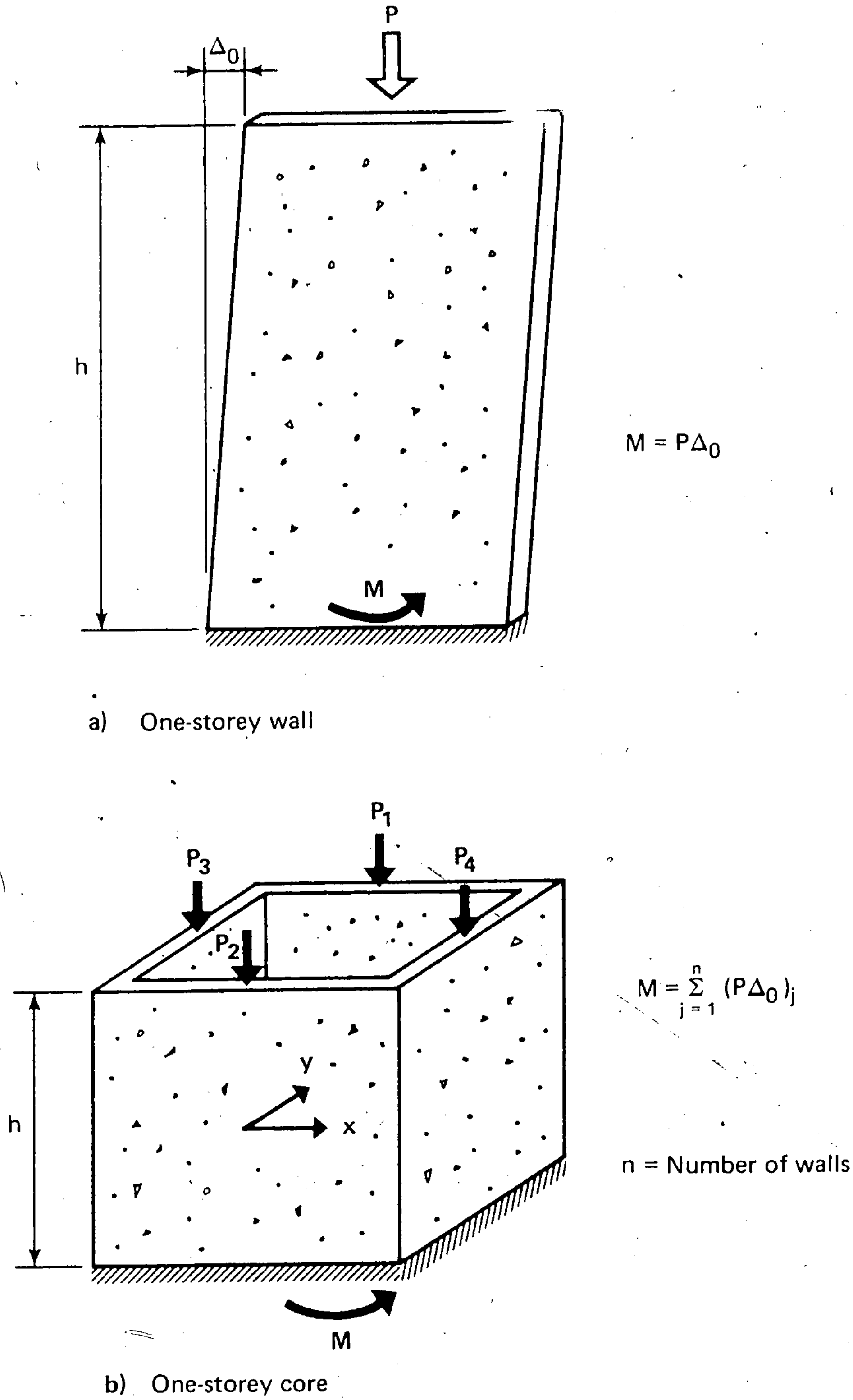


Figure 7.3 Moments in one-storey walls

Equations (7.20) and (7.21) together with (7.19) may be adapted for design by the transformation of section 7.1.1.

$$M_d = \frac{\overline{\Delta}_0}{h} \sum_{j=1}^{n} (Ph)_j + \beta\sigma_w \sqrt{\sum_{j=1}^{n} (Ph)_j^2} \qquad (7.22)$$

The terms in the above expression have been defined previously in this chapter.

Equation (7.22) would be simplified if the mean were neglected. The actual mean could be zero, as in the case of column out-of-plumbs, due to the similarities between these two variables. It is possible that the measured mean is different from zero by reason of the relative small sample size (379). The question, at this point, is to find the percentage of the total moment that is contributed by the mean in practical situations.

Assuming constant axial loads, Eq. (7.22) becomes $M_d = 0.00028\ n\ Ph + 3.5 \times 0.0028 \sqrt{n}\ Ph$. The ratio of the first term to the total expression for M_d gives

$$\frac{0.00028 \sqrt{n}}{0.00028 \sqrt{n} + 0.0098}$$

which is the percentage contribution of the mean to the total moment in terms of n. For n = 1, the mean accounts for less than 3 percent of the total but for n = 5 and 20, the contributions are 6 and 11 percent respectively.

Assuming, for the reasons listed above, that the mean could be negligible, Eq. (7.22) would become:

$$M_d = \beta\sigma_w \sqrt{\sum_{j=1}^{n} (Ph)_j^2} \qquad (7.23)$$

and σ_w is taken as 0.0028. The validity of Eq. (7.23) will be discussed in the next chapter.

The design equation for the moments due to initial wall deviations in a multi-storey core is obtained in a manner similar to Eq. (7.13) for column out-of-plumbs. Using the notation adopted in Fig. 7.2, the moment is calculated as:

$$M_i = \sum_{k=i+1}^{m} M_k \tag{7.24}$$

where M_i, the moment at level i, is the algebraic summation of the individual storey-moments above level i. When the contribution of the mean is accounted for, the corresponding design equation becomes:

$$M_{di} = \frac{\overline{\Delta}_0}{h} \sum_{k=i+1}^{m} \left[\sum_{j=1}^{n_k} (Ph)_j \right]_k + \beta\sigma_w \sqrt{\sum_{k=i+1}^{m} \left[\sum_{j=1}^{n_k} (Ph)_j^2 \right]_k} \tag{7.25}$$

When the contribution of the mean is neglected, this expression is reduced to:

$$M_{di} = \sqrt{\sum_{k=i+1}^{m} (M_d^2)_k} \tag{7.26}$$

where M_{dk} is given by Eq. (7.23) for each level k above level i. The variable n in Eq. (7.23) is then replaced by n_k, the number of walls at storey k.

7.2.2 Torque in Core

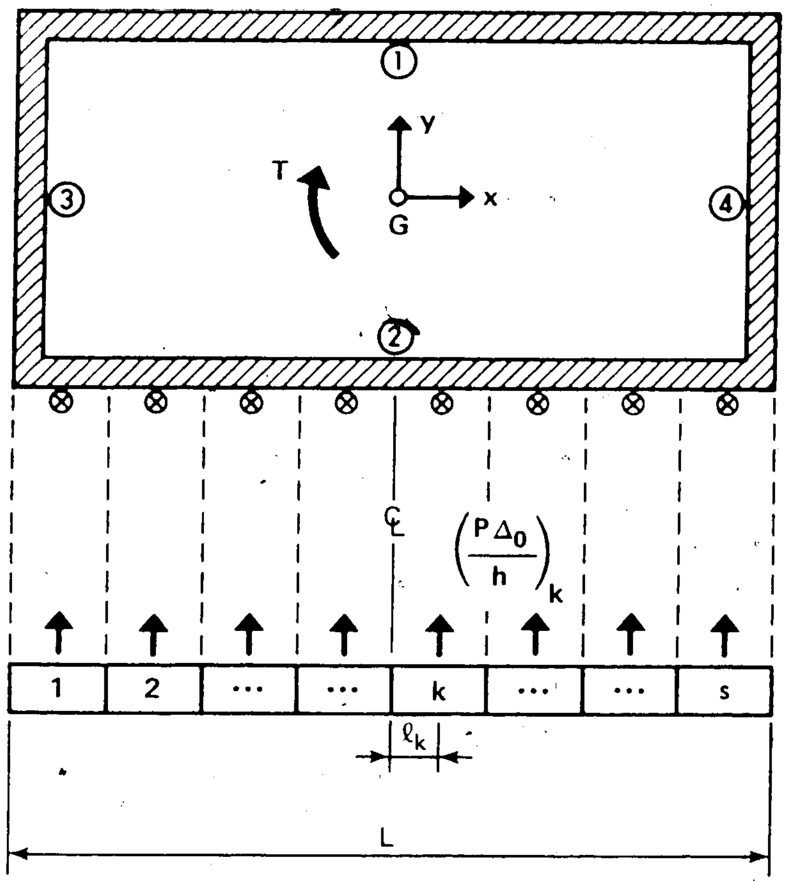
The cantilevered wall shown in Fig. 7.3(a) is stabilized against the in-plane out-of-plumb by a moment at the base. However, due to the relatively small thickness of the wall, the stability against an out-of-plumb in the orthogonal direction must be ensured by some means other than the wall itself.

As an example, the support required to stabilize wall No. 2 in the y-direction, as shown in Fig. 7.3(b), is provided by the adjacent walls, Nos. 3 and 4, spanning at right angles. Assuming conservatively that the base of wall No. 2 is pinned in the y-direction and that the supports of the adjacent walls are only effective at the top of the wall, a total horizontal force of value $P\Delta_y/h$ is induced to stabilize wall No. 2. The out-of-plumb value for wall No. 2 in the y-direction is Δ_y.

This force, however, is not distributed equally to the stabilizing walls (Nos. 3, 4) since the wall to be stabilized (No. 2) has variable out-of-plumbs at different sections. When several walls are assembled orthogonally to form a core, the presence of these unbalanced forces could result in the formation of a significant torque in the core.

A set of unequal horizontal forces on a wall can be visualized as a corresponding force applied eccentrically with respect to the center of the wall. The eccentricity of this force is a function of the actual state of plumbness of the wall and consequently characterizes that wall. The eccentricity can be considered as a variable with a particular distribution and can be evaluated from the out-of-plumb measurements taken on the individual walls by the procedure described below.

As shown in Fig. 7.4, a wall may be subdivided into as many segments as there are measurements taken on that wall. The contribution of the wall to the total torque can be approximated by:



T: Torque (positive clockwise)

G: Center of resistance of the core

ℓ_k: Distance from segment k to the center of the wall

L: Total length of the wall

⊗: Location of measurements

Figure 7.4 Evaluation of ratio e/L

$$T = Pe = \sum_{k=1}^{s} (P \frac{\Delta_0}{h} \ell)_k$$

where

T = resulting torque

P = total factored axial load on the wall

e = equivalent "eccentricity"

s = number of segments in a wall

P_k = axial load carried by segment k

Δ_{0k} = measured out-of-plumb of segment k

ℓ_k = distance from the center of segment k to the center of the wall

h = height of the wall

Making the assumption that each segment carries an equal share of the total load on the wall and solving for e, gives:

$$e = \sum_{k=1}^{s} (\frac{\Delta_0}{h} \ell)_k \frac{P_k}{P}$$

The ratio P_k/P is equal to $1/s$. Dividing both sides by the length, L, of the wall gives:

$$\frac{e}{L} = \frac{1}{s} \sum_{k=1}^{s} (\frac{\Delta_0}{h} \frac{\ell}{L})_k \qquad (7.27)$$

The measured out-of-plumb, Δ_0/h, of each segment k is multiplied by the ratio ℓ_k/L pertaining to that segment. The sum of the s individual products is then multiplied by a constant, $1/s$, to result in a dimensionless equivalent "eccentricity", e/L, which characterizes the wall.

These quantities are listed in Tables 7.1 and 7.2 for the walls of buildings A and B respectively. The values are then plotted separately for each building in Figs. 7.5 and 7.6 and combined in Fig. 7.7. The characteristics of each distribution are listed in Table 7.3.

The distributions are approximately normal. They are reasonably peaked and slightly skewed. The absolute value of the mean $\bar{e}/L$, can be taken as 0.5×10^{-4} and the standard deviation, σ_e, as 4.0×10^{-4}.

The contribution of a wall to the total torque in a core is then calculated as:

$$T = P \frac{e}{L} L \tag{7.28}$$

where e/L, since normally distributed, can be approximated by:

$$\frac{e}{L} = \frac{\bar{e}}{L} + \beta\sigma_e \tag{7.29}$$

The safety index β is 3.5.

The design torque in the core at any storey i is obtained from a statistical formulation combining the two expressions above.

$$T_{di} = \frac{\bar{e}}{L} \sum_{j=1}^{n_i} (PL)_j + \beta\sigma_e \sqrt{\sum_{j=1}^{n_i} (PL)_j^2} \tag{7.30}$$

The torque at storey i depends only on the n_i out-of-plumb walls at that storey. By assuming an eventual mean of zero for a population with a larger sample dimension, Eq. (7.30) would be reduced to:

e/L x 10^4

Storey No.	Wall No.*			
	1	2	3	4
22	1.95	1.24	-0.81	-3.94
21	-0.09	-2.21	0.54	-2.49
20	0.18	3.54	-0.27	5.36
19	-2.30	-4.25	-4.61	4.85
18	-0.09	-3.54	3.80	0.85
17	6.20	9.92	-4.34	4.98
16	-4.68	3.32	1.09	9.22
15	2.44	-3.12	5.97	-1.36
14	-4.68	1.09	-2.44	-1.36
13	4.14	5.49	2.44	-0.81
12	5.49	-2.17	-0.54	-0.54
11	-2.58	-1.09	-2.17	0.54
10	-0.07	0.61	0.00	1.36
9	-0.67	-0.27	-1.90	-1.36
8	1.15	0.14	0.27	-1.09
7	2.10	-2.78	2.71	2.17
6	1.03	1.42	-3.98	-1.36
5	1.89	-2.69	0.74	0.18
4	-5.06	-2.48	3.04	8.68
3	-3.20	8.48	5.21	0.29
2	8.56	-1.53	1.57	-0.97
1	-6.78	1.24	0.49	-3.86

* Wall numbering given in Fig. 6.2

TABLE 7.1 VARIABLE e/L FOR BUILDING A

$e/L \times 10^4$

Storey No.	Wall No.*								
	1	2	3	4	5	6	7	8	9
34	-0.80	2.33	-	-3.92	-5.08	2.87	8.46	0.87	-4.69
33	-8.27	1.17	-	3.91	4.17	-2.21	-8.33	1.11	5.60
32	-7.49	6.06	-	0.91	-3.65	7.55	-1.30	1.42	1.04
31	4.46	-3.65	-	-0.14	-8.33	14.58	-6.38	2.72	0.00
30	0.94	-0.52	-	-4.69	4.40	-6.35	11.26	-1.43	-0.91
29	0.65	8.43	-	0.78	-2.87	2.08	0.13	-1.53	-1.69
28	-3.26	-2.80	-	-5.05	-3.39	4.82	-2.47	2.97	-2.87
27	-0.81	-5.34	-	-0.33	-3.78	1.30	-1.17	-2.35	2.08
26	4.30	3.39	-	-2.34	-1.82	1.04	3.00	-4.45	-4.95
25	0.29	1.04	-	1.95	1.69	-4.18	-0.52	2.23	-2.08
24	5.37	-1.69	-	-4.53	-11.46	3.26	-8.07	-1.87	-1.82
23	0.39	-1.86	-	3.52	12.24	2.87	0.65	-3.14	2.34
22	-2.28	1.95	-	0.39	-3.39	3.52	2.08	1.56	0.78
21	4.23	-3.26	-	1.20	1.04	6.51	-3.00	-5.11	0.78
20	-2.12	-8.04	-	-0.39	-9.64	0.26	2.60	0.80	-2.87
19	-2.51	3.94	-0.39	-6.38	-2.08	-1.04	-9.64	0.93	-4.17
18	3.78	-3.99	-1.79	-6.35	-7.65	-1.95	-7.49	5.97	-1.63
17	-0.29	-4.33	-1.43	-0.91	3.91	-0.91	1.69	5.34	1.95
16	2.31	0.72	0.52	-3.13	-1.04	1.30	2.08	-1.54	3.78
15	-4.07	-1.07	-1.17	5.08	2.08	-1.17	-2.08	-5.38	-1.30
14	0.91	-0.37	-4.82	-5.47	-14.06	3.13	-4.56	3.37	1.04
13	-4.23	3.38	2.12	-1.30	-1.14	-4.75	-8.30	2.18	5.86
12	1.16	-1.57	-3.01	0.65	1.79	-1.89	-7.00	4.46	4.31
11	-6.19	-2.28	1.79	1.63	1.79	-0.10	-7.16	3.01	-0.16
10	1.24	-4.07	-6.51	-2.87	-6.25	-2.93	-11.07	5.41	-6.10
9	-1.99	-1.95	-5.86	-2.93	-8.14	-1.20	-8.14	-1.75	-5.21
8	1.18	0.55	-1.55	-7.32	-11.39	-5.83	-7.98	-7.30	1.47
7	-2.54	0.41	-2.28	-2.85	2.66	1.07	-2.93	5.89	-1.14
6	2.48	1.22	-4.49	0.00	4.88	0.00	1.63	3.93	-3.91
5	4.27	2.22	-0.94	-1.47	-10.25	1.97	-0.49	4.39	1.47
4	-3.00	10.51	0.77	1.14	6.51	-8.63	-6.02	-3.74	6.59
3	-0.52	2.50	-4.50	-6.50	-2.28	-1.30	-1.79	-3.09	0.41
2	-0.75	2.31	-1.30	-1.14	3.74	-0.81	2.60	-0.65	-0.43
1	-1.68	-2.22	-3.36	-3.91	-2.60	-0.65	6.51	-1.14	1.52

* Wall numbering given in Fig. 6.3

TABLE 7.2 VARIABLE e/L FOR BUILDING B

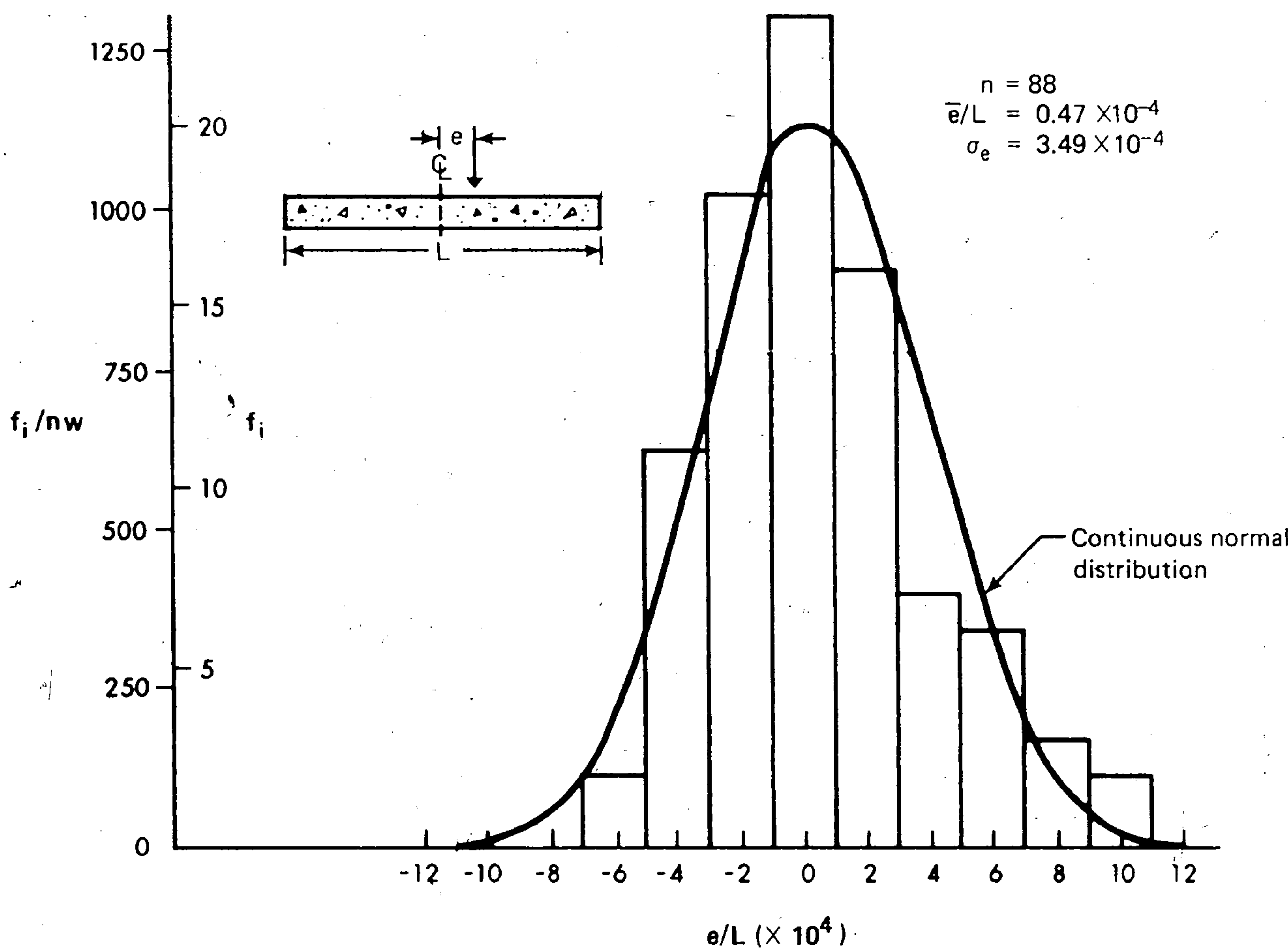


Figure 7.5 Distribution of e/L for building A

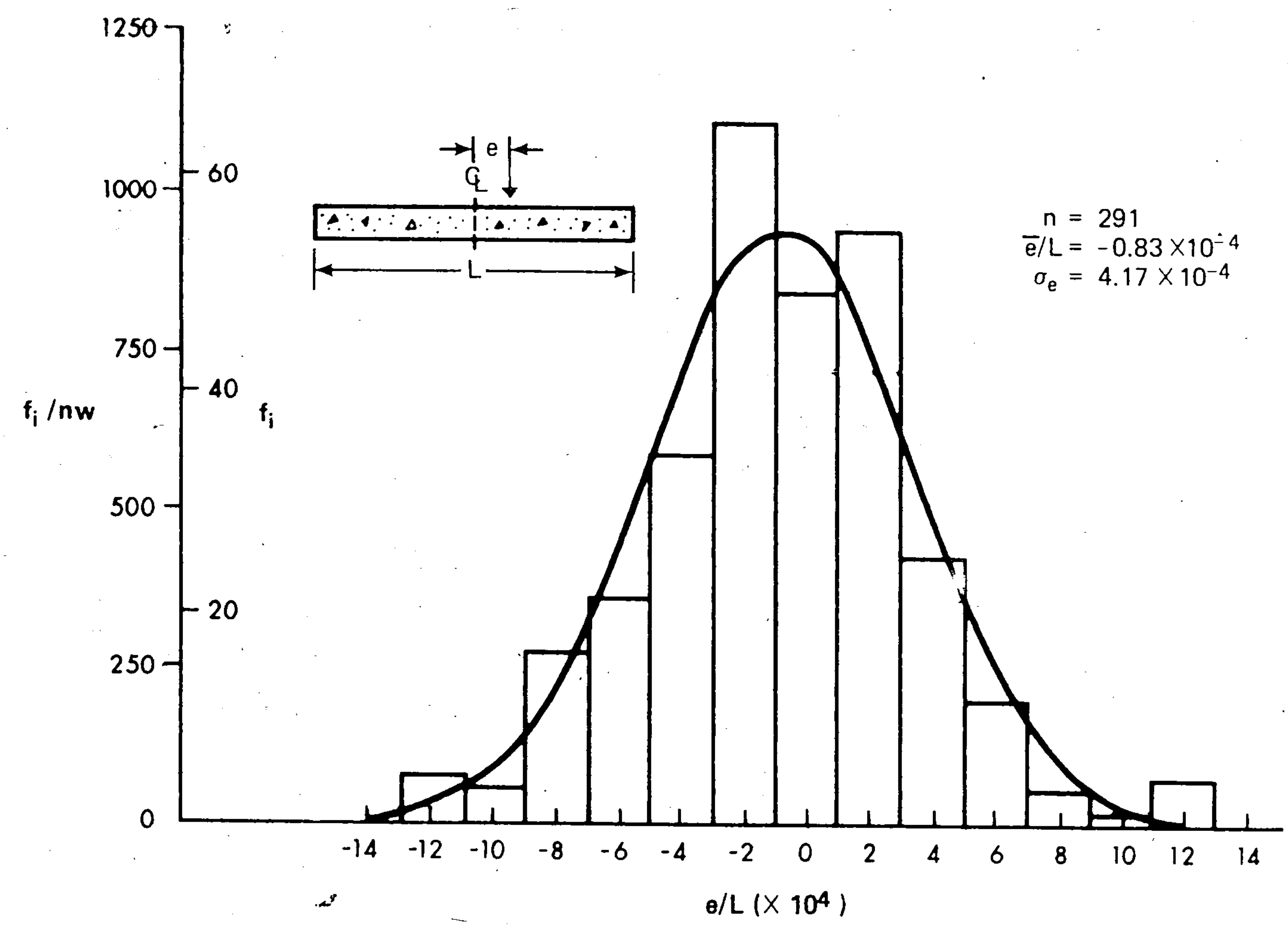


Figure 7.6 Distribution of e/L for building B

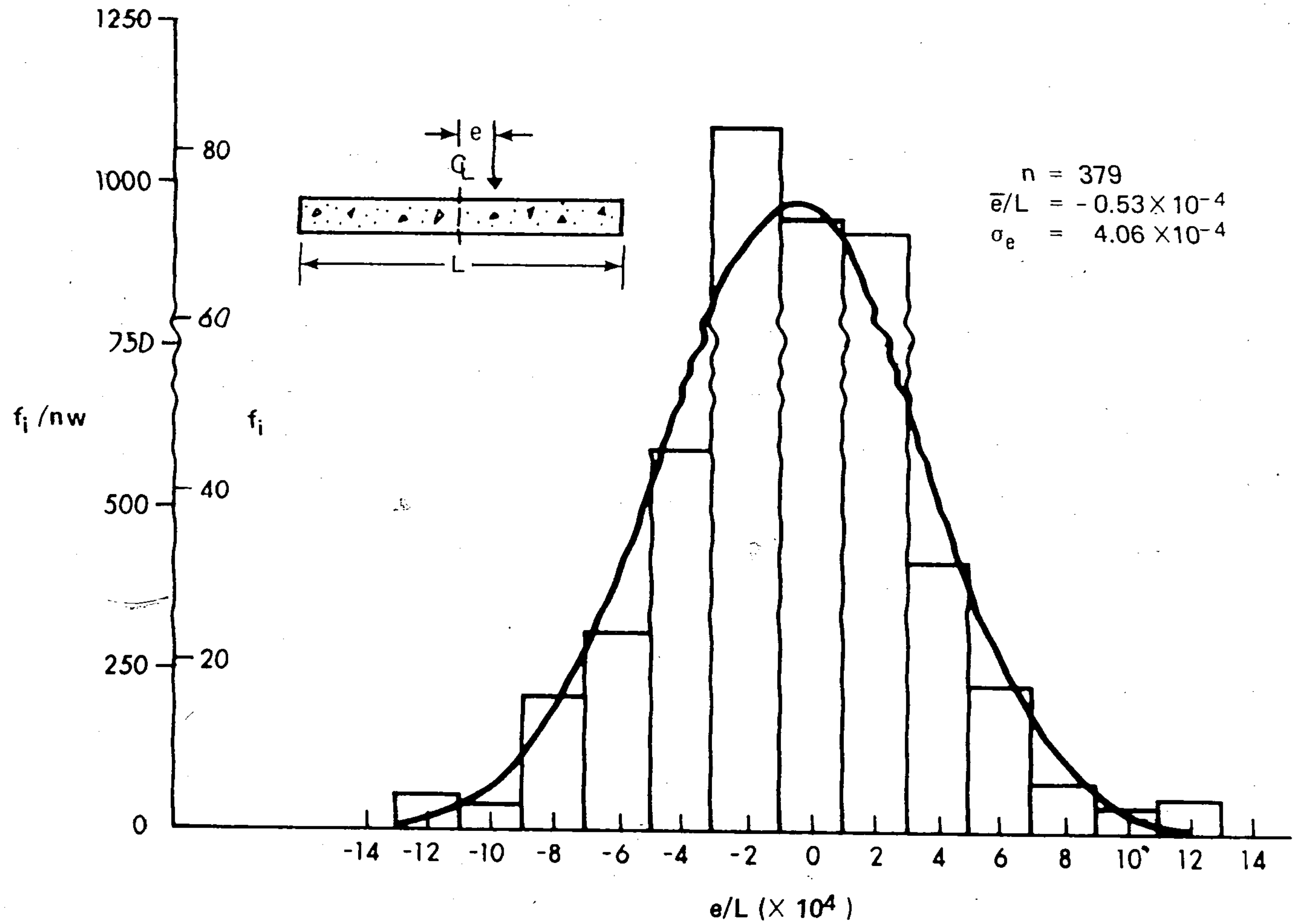


Figure 7.7 Distribution of e/L for buildings A and B

Building	n	Mean ($\times 10^4$)	Stand. Dev. ($\times 10^4$)	Skewness	Kurtosis
A	88	0.47	3.49	0.65	3.21
B	291	-0.83	4.17	0.06	3.79
A + B	379	-0.53	4.06	0.10	3.81

TABLE 7.3 STATISTICAL CHARACTERISTICS FOR e/L

$$T_{di} = \beta\sigma_e \sqrt{\sum_{j=1}^{n_i} (PL)_j^2} \qquad (7.31)$$

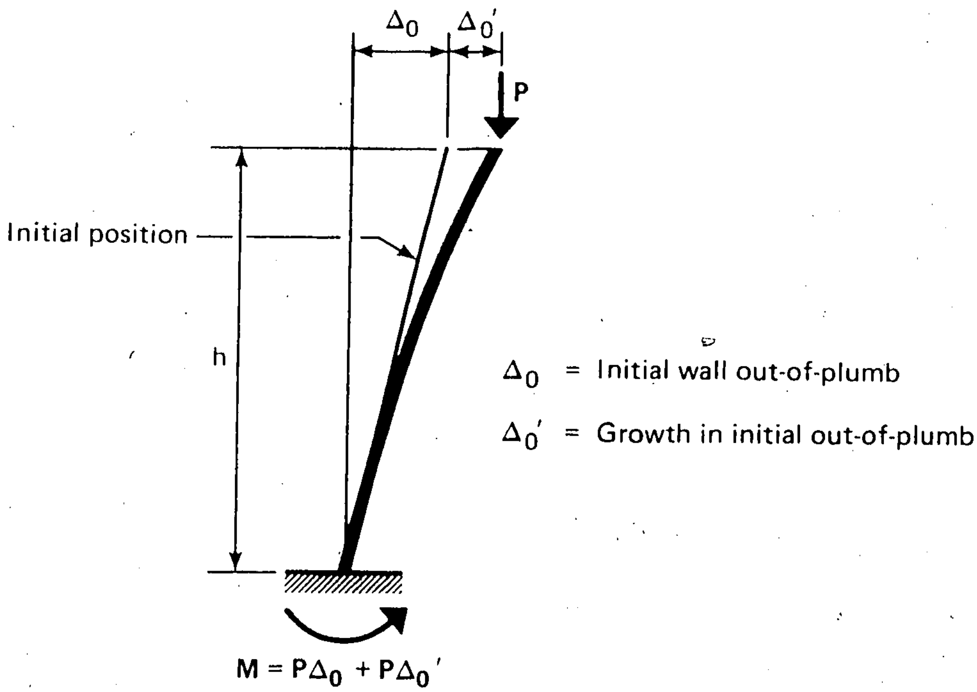
The two equations above are only applicable to a reinforced concrete structure consisting of an orthogonal assembly of cast-in situ walls.

7.2.3 Lateral Deflections

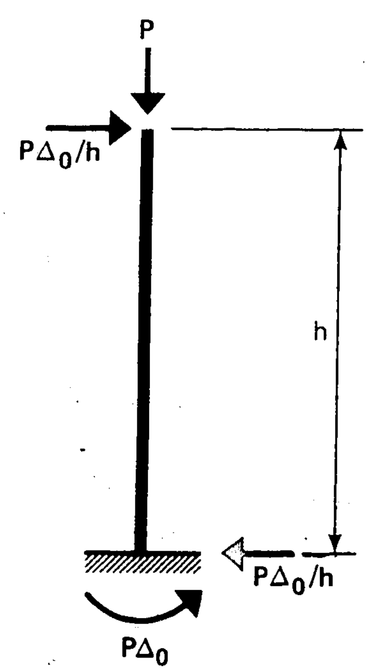
It has been demonstrated in section 7.1.7 that a structure deflects laterally as the result of the axial loads acting on the out-of-plumb columns. In a similar manner, the axial loads acting on the out-of-plumb walls forming the core force the structure to deflect an additional amount. As shown in Fig. 7.8(a), a vertical load P applied to an out-of-plumb wall section induces an additional lateral deflection Δ_0'. The moment at the base of the one-storey wall is then the sum of the moment $P\Delta_0$ defined in section 7.2.1 and a smaller moment $P\Delta_0'$.

An estimation of the component $P\Delta_0'$ of the total moment can be obtained from the equivalent model shown in Fig. 7.8(b). A fictitious horizontal force $P\Delta_0/h$ is applied at the top of the perfectly vertical wall section. In a manner similar to the iterative procedure described in Chapter III, the structure is analyzed to determine the converged moment $P(\Delta_0 + \Delta_0')$ and the corresponding deflection Δ_0'.

The model shown in Fig. 7.8(b) can be used to evaluate an equivalent wall out-of-plumb value, Δ_d/h, for the general case of a combination of n walls. The derivation is similar to that of section 7.1.7 for columns. The fictitious force at the top of the wall section is:



a) Cantilevered out-of-plumb wall section



b) Equivalent Model (First order values shown)

Figure 3 Effect of wall out-of-plumbs on the lateral deflection of a core

$$F = \frac{P\Delta_0}{h} \tag{7.32}$$

Eq. (7.32) combined with Eq. (7.19) becomes:

$$F_d = \frac{\overline{\Delta}_0}{h} P + \beta\sigma_w P \tag{7.33}$$

For n walls,

$$F = \sum_{j=1}^{n} \left(\frac{P\Delta_0}{h}\right)_j \tag{7.34}$$

or

$$F_d = \frac{\overline{\Delta}_0}{h} \sum_{j=1}^{n} P_j + \beta\sigma_w \sqrt{\sum_{j=1}^{n} P_j^2} \tag{7.35}$$

An equivalent out-of-plumb, Δ_d/h, constant for a specified number of walls may be obtained from Eqs. (7.34) and (7.35). For $F = F_d$,

$$\frac{\Delta_d}{h} = \frac{\overline{\Delta}_0}{h} + \beta\sigma_w \frac{\sqrt{\sum_{j=1}^{n} P_j^2}}{\sum_{j=1}^{n} P_j} \tag{7.36}$$

According to Appendix D, this expression can be written as:

$$\frac{\Delta_d}{h} = \frac{\overline{\Delta}_0}{h} + \frac{\beta\sigma_w}{2.2\sqrt{n}} \tag{7.37}$$

Assuming that every wall is out-of-plumb in the same direction by the amount Δ_d/h given in Eq. (7.37), a set of horizontal forces, at each floor level can be calculated.

The total sway of a structure is obtained by a second order analysis where the applied forces are those caused by the wind loads together with the lateral forces due to column and wall out-of-plumbs. A statistical combination is required to account for the fact that the walls and the columns may induce deflections in opposite directions. The total lateral load at a specific floor level is then:

$$H = H_{wind} + \sqrt{H_c^2 + H_w^2} \qquad (7.38)$$

where H_c and H_w are the lateral loads representing the effects of the column and wall out-of-plumbs respectively.

Eq. (7.38) is not exact if the mean in expression (7.37) is to be included. The exact expression can be easily derived but is more complex. However, the difference in the results is not significant and Eq. (7.38) can be adopted.

7.3 Summary

The various statistical characteristics that have been recommended for use in design in this chapter and in the previous one have been summarized in Table 7.4.

	Function	Mean (Rad.)	Standard Deviation (Rad.)
Column Out-of-Plumbs	All Purposes	$\frac{\overline{\Delta}_0}{h} = 0.0$	$\sigma_c = 0.0017$
Wall Out-of-Plumbs	Moment & Deflection	$\frac{\overline{\Delta}_0}{h} = 0.00028$	$\sigma_w = 0.0028$
	Torque	$\frac{\overline{e}}{L} = 0.00005$	$\sigma_e = 0.0004$

Safety Index	$\beta = 3.5$

TABLE 7.4 DESIGN VALUES

CHAPTER VIII

APPLICATIONS

Several equations serving different purposes have been presented in the previous chapter but no examples of applications have yet been presented. In this chapter the applicability of these equations will be checked against the corresponding results obtained from the measurements taken on buildings A and B.

8.1 Column Out-of-Plumbs

8.1.1 Force at Connection Point

A connection between one column and a beam must be designed to resist the extra horizontal force due to the eventual out-of-plumb of the column. This force was estimated as 0.6 percent of the factored axial load in the column. The force is increased to 0.84 percent of the average axial load in the more common case of two column segments connected at a floor level, as shown in Fig. 7.1.

Two cases must be considered in the transfer of these forces in a braced structure:

1. The bent to be designed is stabilized by a stiffer structure outside the plane of the bent. This could be the case, for instance, for column stacks 1 to 6 in the structure shown in Fig. 6.? [illegible] extra forces originating from each column stack a[illegible] dire[illegible] transmitted to the core by the floor diaphragm [illegible] individual [illegible]nections must be designed for

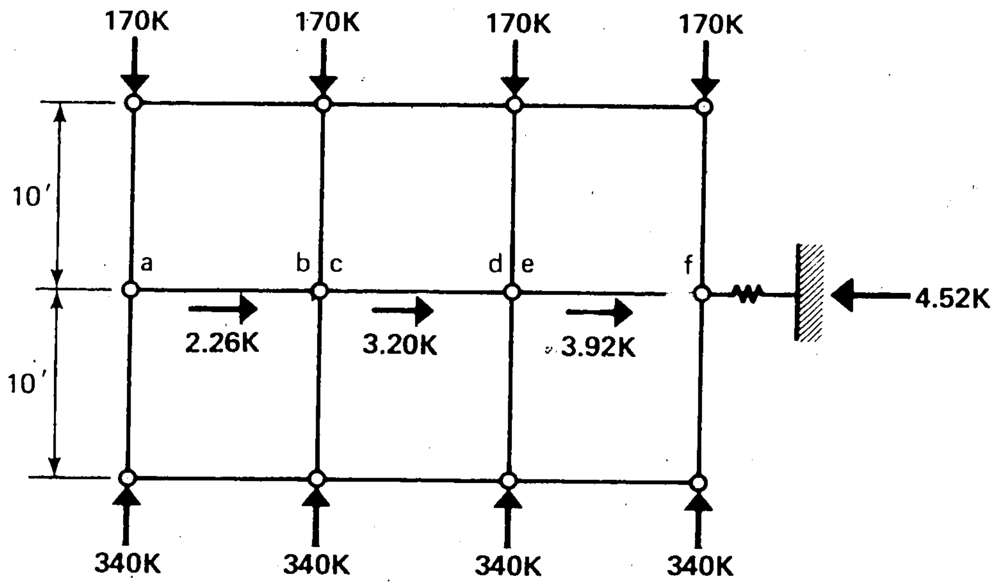
horizontal shears equal to 0.6 or 0.84 percent of the column axial loads, depending on the case (see section 7.1.1). The floor diaphragms, in turn, must be designed to resist the appropriate horizontal shears given by Eq. (7.7).

2. The bent to be designed is braced in the plane of the bent. The extra shears due to column out-of-plumbs are transferred from bay to bay and the connections must be designed accordingly.

An example showing the gradual increase in the horizontal force, when transmitted to the bracing system, is given in Fig. 8.1. The columns in the upper and lower storeys of the frame carry individual axial loads of 170 and 340 kips respectively. According to Eq. (7.7), the force in the girder a-b is 2.26 kips and originates from the two left hand columns. The force in girder c-d is 3.2 kips and is produced by the axial loads acting on four out-of-plumb columns; the shears being transmitted from left to right. The force in girder e-f is 3.92 kips and the bracing system finally resists a total force of 4.52 kips. The gradual increase in shear is non-linear and the connections in the vicinity of the bracing structure have to resist the larger shears.

8.1.2 Shear in the Plane of the Floor

The horizontal shears in the plane of the floor, estimated by Eq. (7.7), are compared to the values calculated from the measured column out-of-plumbs in buildings A and B. The actual forces in the x and y directions, calculated for all the columns at each storey, are listed in columns 3 and 4 of Tables 8.1 and 8.3. The corresponding



$$F_d = \beta \sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \qquad (7.7)$$

$\beta = 3.5$

$\sigma_c = 0.0017$ Rad.

Figure 8.1 Transfer of shear in beam-to-column connections

1	2	3	4	5	6	7	8	9	10
Floor No.	Storey Height (Ft.)	Storey Force (Kips)		Storey Moment (Ft.-K.)	Shear in Core (Kips)		Moment in Core (Ft.-K.)		Torque in Core (Ft.-K.)
		x Axis	y Axis		x Axis	y Axis	x Axis	y Axis	
27	20	0.04	- 0.16	10.74	0.04	- 0.16	0.00	0.00	10.74
26	12	0.36	- 1.36	85.20	0.40	- 1.52	0.80	- 3.20	95.94
25	12	- 2.35	1.84	57.79	- 1.95	0.32	5.60	-21.44	153.73
24	12	- 3.40	2.92	-205.57	- 5.35	3.24	-17.80	-17.60	- 51.84
23	12	4.50	0.16	-182.13	- 0.85	3.40	-82.00	21.28	-233.97
22	12	7.19	- 8.85	497.56	- 6.34	- 5.45	-92.20	62.08	263.58
21	12	- 6.88	6.77	104.97	- 0.54	1.32	-16.12	- 3.32	368.55
20	12	- 5.13	- 5.99	-131.15	- 5.67	- 4.67	-22.60	12.52	237.40
19	12	2.68	5.98	-157.84	- 2.99	1.31	-90.64	-43.52	5.33
18	12	4.44	- 7.93	-294.82	1.45	- 6.62	-126.52	-27.80	-215.27
17	12	- 3.72	- 0.10	154.05	- 2.27	- 6.72	-109.12	-107.24	-61.22
16	12	2.79	1.72	-123.43	0.52	- 5.00	-136.36	-187.88	-184.65
15	12	- 7.27	12.29	-317.10	- 6.75	7.29	-130.12	-247.88	-501.93
14	12	5.87	- 0.66	684.63	- 0.88	6.63	-211.12	-160.40	182.83
13	12	15.41	-18.61	647.17	14.53	-11.99	-221.68	-80.84	830.05
12	12	-26.74	9.72	-672.88	-12.21	- 2.26	-47.32	-224.60	157.17
11	12	14.23	0.58	-534.81	2.02	- 1.68	-193.84	-251.72	-377.64
10	12	5.08	-25.66	512.65	7.10	-27.34	-169.60	-271.88	134.00
9	12	-16.33	37.07	-190.02	- 9.24	9.73	-84.40	-599.96	-55.01
8	12	4.80	- 9.66	145.26	- 4.44	0.07	-195.28	-483.20	90.17
7	12	- 9.73	23.43	988.50	-14.16	23.50	-248.56	-482.36	1078.75
6	12	15.80	1.47	-830.72	1.64	24.97	-418.48	-200.36	248.03
5	12	5.46	- 8.98	95.87	7.10	15.99	-398.80	99.28	343.89
4	12	14.30	-26.84	140.90	21.40	-10.85	313.60	291.16	484.79
3	12	3.02	- 2.31	-398.58	24.42	-13.16	-56.80	160.96	86.21
2	12	-45.97	24.17	730.38	-21.55	11.01	236.24	3.04	816.59
1	20	- 3.03	-15.38	-1486.87	-24.58	- 4.37	-22.36	135.16	-670.28
							-513.96	47.76	

TABLE 8.1 . FORCES IN BUILDING A FROM ACTUAL COLUMN OUT-OF-PLUMBS

1	2	3	4	5	6	7
Floor No.	Storey Height (Ft.)	$\left[\sqrt{\sum_{j=1}^{n_i} P_j^2}\right]_i$ (Kips)	Storey Force (Eq. 7.7) (Kips)	Shear in Core (Eq. 7.11) (Kips)	Moment in Core (Eq. 7.13) (Ft.-K.)	Torque in Core (Eq. 7.15) (Ft.-K.)
27	20	215	1.28	1.28	0.0	79.6
26	12	691	4.31	4.12	25.6	259.6
25	12	1000	7.23	5.95	55.7	377.0
24	12	1265	9.59	7.52	90.5	477.3
23	12	1523	11.77	9.06	127.8	575.1
22	12	1776	13.92	10.56	167.8	670.7
21	12	2029	16.05	12.08	210.3	766.7
20	12	2280	18.16	13.57	255.4	861.4
19	12	2528	20.26	15.05	302.9	955.7
18	12	2777	22.35	16.52	352.6	1049.9
17	12	3024	24.43	17.99	404.6	1143.2
16	12	3271	26.50	19.47	458.5	1236.7
15	12	3517	28.58	20.92	514.6	1329.7
14	12	3337	28.85	19.86	572.6	1393.2
13	12	3556	29.02	21.15	620.2	1439.7
12	12	3773	30.84	22.45	670.1	1527.6
11	12	4003	32.73	23.79	722.2	1615.5
10	12	4207	35.77	25.04	776.7	1703.4
9	12	4422	36.32	26.32	832.8	1786.3
8	12	4640	38.14	27.60	890.7	1878.4
7	12	4855	39.96	28.90	950.3	1965.9
6	12	5071	41.78	30.18	1011.5	2052.9
5	12	5287	43.60	31.46	1074.4	2140.4
4	12	5502	45.41	32.74	1138.8	2227.5
3	12	5718	47.21	34.02	1204.7	2315.0
2	12	5933	49.02	35.29	1272.0	2401.7
1	20	6149	50.84	36.59	1340.7	2489.2
					1527.4	

TABLE 8.2. FORCES IN BUILDING A FROM STATISTICAL CALCULATIONS

1	2	3	4	5	6	7	8	9	10
Floor No.	Storey Height (Ft.)	Storey Force (Kips)		Storey Moment (Ft.-K.)	Shear in Core (Kips)		Moment in Core (Ft.-K.)		Torque in Core (Ft.-K.)
		x Axis	y Axis		x Axis	y Axis	x Axis	y Axis	
34	24	- 2.12	- 0.60	6.31	- 2.12	- 0.60	0.00	0.00	6.31
33	12	0.21	0.96	64.58	- 1.91	0.36	-50.88	-14.40	70.89
32	12	1.48	1.25	-338.27	- 0.43	1.61	-73.80	-10.08	-267.39
31	12	0.68	- 1.70	409.35	0.25	- 0.09	-78.96	9.24	141.96
30	12	- 5.93	0.06	-498.94	- 5.68	- 0.03	-75.96	8.16	-356.98
29	12	7.16	2.83	40.35	1.48	2.81	-144.12	7.80	-316.63
28	12	- 0.60	2.93	125.64	0.88	5.74	-126.36	41.52	-191.00
27	12	-10.56	9.82	321.64	- 9.68	15.56	-115.80	110.40	130.65
26	12	2.01	- 7.48	-398.29	- 7.67	8.08	-231.96	297.12	-267.65
25	12	10.66	- 4.85	709.03	2.99	3.24	-324.00	394.08	441.38
24	12	-15.50	11.14	-828.74	-12.51	14.38	-288.12	432.96	-387.36
23	12	8.17	- 4.72	1528.07	- 4.34	9.66	-438.24	605.52	1140.71
22	12	0.94	-20.54	-726.16	- 3.40	-10.87	-4[illegible]0.[illegible]2	721.44	414.55
21	12	4.01	22.97	222.34	0.61	12.10	-531.12	591.00	636.88
20	12	1.72	- 2.18	-1987.66	2.34	9.92	-523.80	736.20	-1350.77
19	12	- 9.99	-35.03	1016.84	- 7.65	-25.11	-495.72	855.24	-333.93
18	12	19.37	9.83	841.27	11.71	-15.29	-587.52	553.92	507.33
17	12	- 3.16	30.03	-708.78	8.55	14.74	-447.00	370.44	-201.45
16	12	-17.88	- 6.47	-318.22	- 9.33	8.28	-344.40	547.32	-519.66
15	12	19.00	-14.21	596.11	9.66	- 5.94	-456.36	646.68	76.45
14	12	15.85	4.70	-530.72	25.51	- 1.24	-340.44	575.40	-454.28
13	12	-14.40	2.92	853.66	11.11	1.68	- 34.32	560.52	399.39
12	12	-29.54	-21.74	151.78	-18.43	-20.06	99.00	580.68	551.17
11	12	13.11	28.85	-2289.57	- 5.31	8.79	-122.16	339.96	-1738.40
10	12	5.16	-27.09	1920.94	- 0.15	-18.30	-185.88	445.44	182.54
9	12	- 6.33	11.38	-327.14	- 6.48	- 6.91	-187.68	225.84	-144.60
8	12	7.12	35.16	-585.46	0.64	28.25	-265.44	142.92	-730.05
7	12	-20.56	-36.81	36.68	-19.93	- 8.56	-257.76	481.92	-693.37
6	12	45.88	-12.41	2901.36	25.95	-20.97	-496.92	379.20	2207.99
5	24	- 8.14	21.83	832.05	17.80	0.86	-185.52	127.56	1375.94
4	27	-31.81	15.34	444.57	-14.00	16.20	241.68	148.20	1820.50
3	16	- 3.12	4.27	2528.19	-17.12	20.46	-136.32	585.60	-707.69
2	15	17.49	6.17	994.50	0.36	26.63	-410.24	912.96	286.82
1	15	5.64	3.75	1748.31	6.00	30.38	-404.84	1312.41	-1461.50
							-314.84	1768.11	

TABLE 8.3 FORCES IN BUILDING B FROM ACTUAL COLUMN OUT-OF-PLUMBS

1	2	3	4	5	6	7
Floor No.	Storey Height (Ft.)	$\left[\sqrt{\sum_{j=1}^{n_i} P_j^2}\right]_i$ (Kips)	Storey Force (Eq. 7.7) (Kips)	Shear in Core (Eq. 7.11) (Kips)	Moment in Core (Eq. 7.13) (Ft.-K.)	Torque in Core (Eq. 7.15) (Ft.-K.)
34	24	398	2.37	2.37	0.0	160.6
33	12	689	4.73	4.10	56.9	279.7
32	12	998	7.22	5.94	75.2	407.2
31	12	1303	9.77	7.75	103.6	537.1
30	12	1628	12.41	9.69	139.2	671.5
29	12	1945	15.09	11.57	181.4	801.8
28	12	2263	17.75	13.46	228.4	932.8
27	12	2580	20.42	15.35	279.8	1063.8
26	12	2899	23.09	17.25	335.0	1195.4
25	12	3216	25.76	19.14	393.8	1326.4
24	12	3594	28.70	21.38	455.9	1481.6
23	12	3855	31.36	22.94	523.1	1589.7
22	12	4173	33.80	24.83	591.1	1720.7
21	12	4491	36.48	26.72	662.0	1852.2
20	12	4809	39.15	28.61	735.5	1984.0
19	12	5129	41.83	30.52	811.7	2115.0
18	12	5447	44.52	32.41	890.5	2246.0
17	12	5766	47.20	34.31	971.7	2377.7
16	12	6084	49.87	36.20	1055.4	2508.7
15	12	6404	52.56	38.10	1141.3	2640.3
14	12	6723	55.25	40.00	1229.4	2772.0
13	12	7040	57.92	41.89	1319.8	2903.0
12	12	7361	60.60	43.80	1412.3	3035.3
11	12	7679	63.29	45.69	1506.9	3166.3
10	12	7998	65.97	47.59	1603.6	3298.0
9	12	8316	68.65	49.48	1702.2	3429.0
8	12	8636	71.33	51.38	1802.8	3560.6
7	12	8955	74.02	53.28	1905.3	3692.3
6	12	9274	76.71	55.18	2009.7	3824.0
5	24	9592	79.39	57.07	2116.0	3963.3
4	27	9911	82.07	58.97	2520.6	4095.4
3	16	10234	84.77	60.89	2981.4	4163.0
2	15	10567	87.53	62.87	3136.5	4365.4
1	15	10952	90.55	65.16	3275.2	4516.2
					3418.0	

TABLE 8.4 FORCES IN BUILDING B FROM STATISTICAL CALCULATIONS

storey forces given by Eq. (7.7) are listed in column 4 of Tables 8.2 and 8.4. The absolute values of the measured and predicted forces described above are compared in Figs. 8.2 and 8.3. The forces obtained from the measurements are represented by the solid circles. The direction of the forces is not relevant since the only purpose of these figures is to compare the observed and predicted magnitudes. Equation (7.7), with $\beta = 3.5$ and $\sigma_c = 0.0017$ Rad., appears to be an upper bound on the predicted forces in the floor diaphragms.

If more measurements were available from several similar structures, the computed values shown by the solid circles would eventually fill the area corresponding to the predicted values with a density corresponding to that of a normal distribution. Depending on the probability chosen for design (β factor), a few points may be found outside the limits. In the absence of measurements, such a situation can be artificially created by a Monte Carlo simulation[54]. In this method, applied to the present case, out-of-plumbs of known distribution and characteristics are randomly generated by a computer for every column segment in a fictitious structure.

8.1.3 Moment in the Plane of the Floor

The adequacy of Eq. (7.9) in predicting moments in floor diaphragms due to column out-of-plumbs can be checked as in the previous section. The results obtained from Eq. (7.9) are compared in Figs. 8.4 and 8.5 with those taken from column 5 of Tables 8.1 and 8.3. These figures and others to come in this chapter have the characteristics of Figs. 8.2 and 8.3.

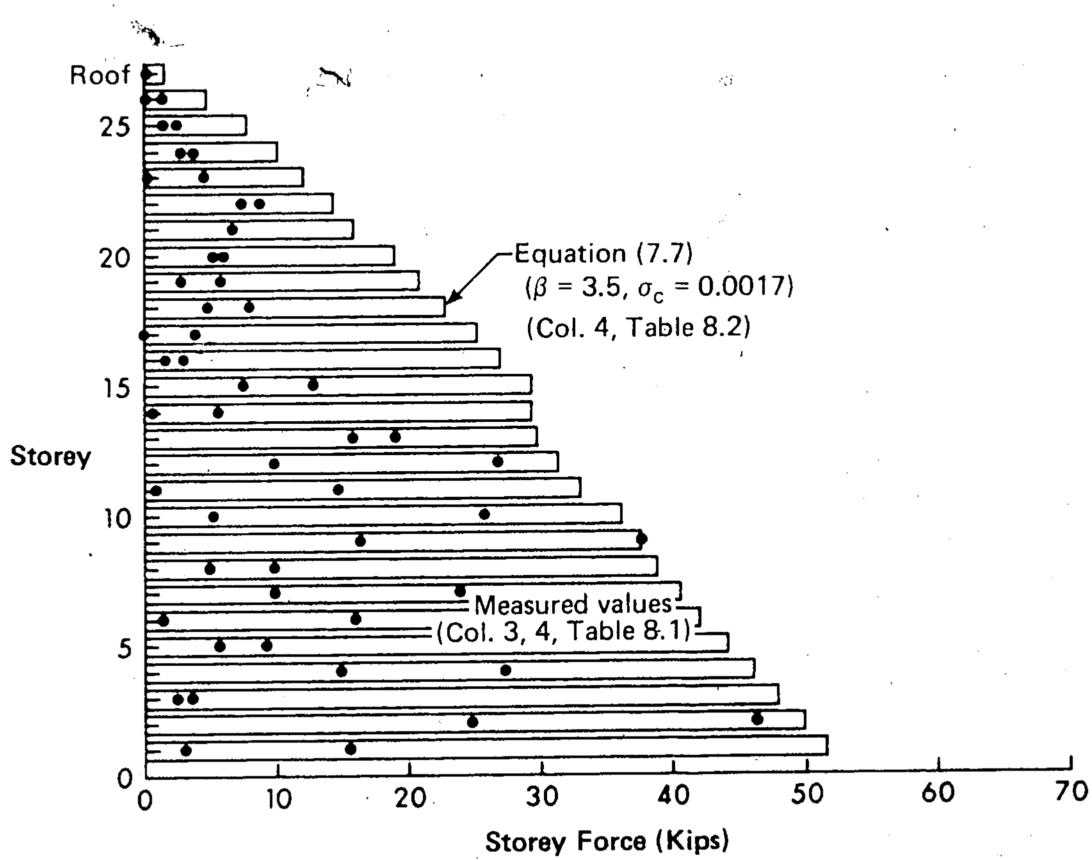


Figure 8.2 Force in the plane of the floor at each level of building A

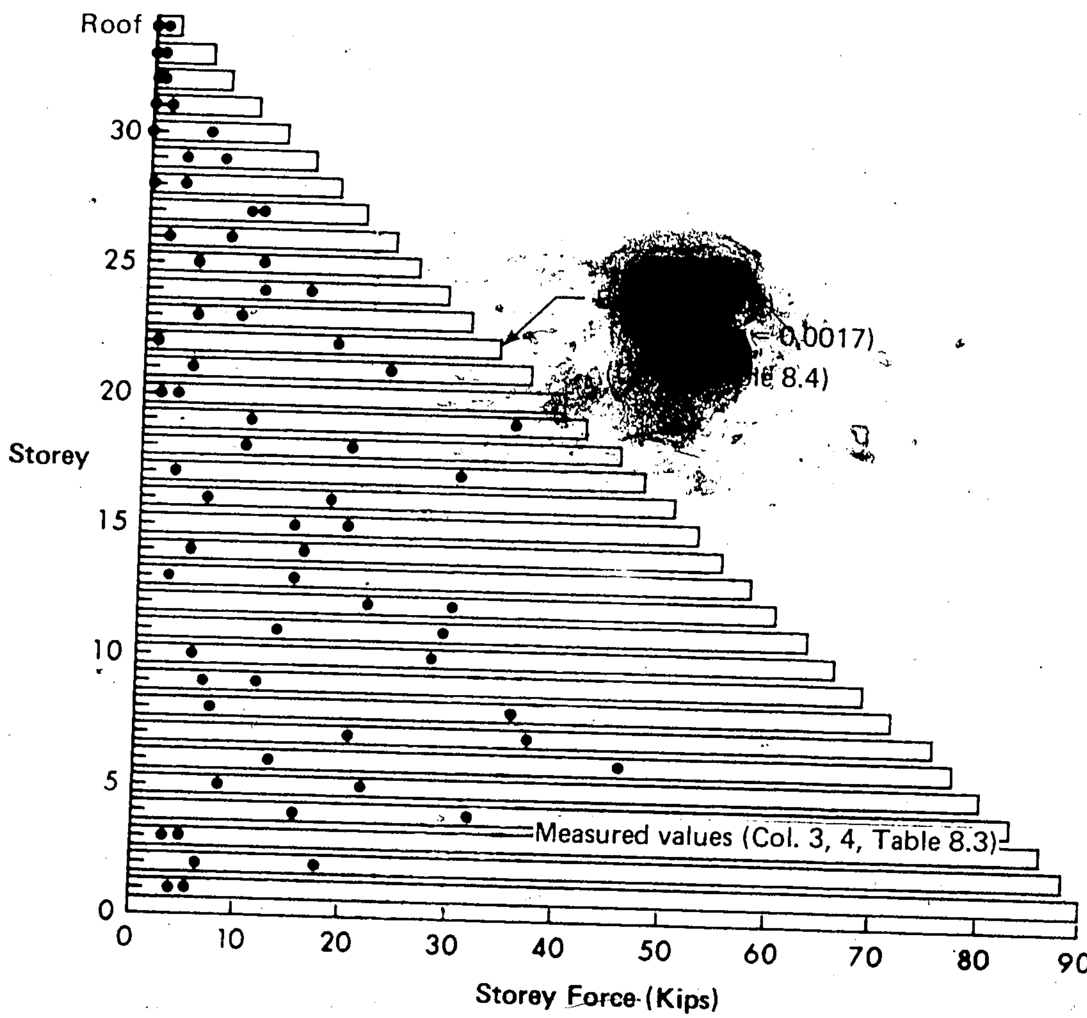


Figure 8.3 **Force in the plane of the floor at each level of building B**

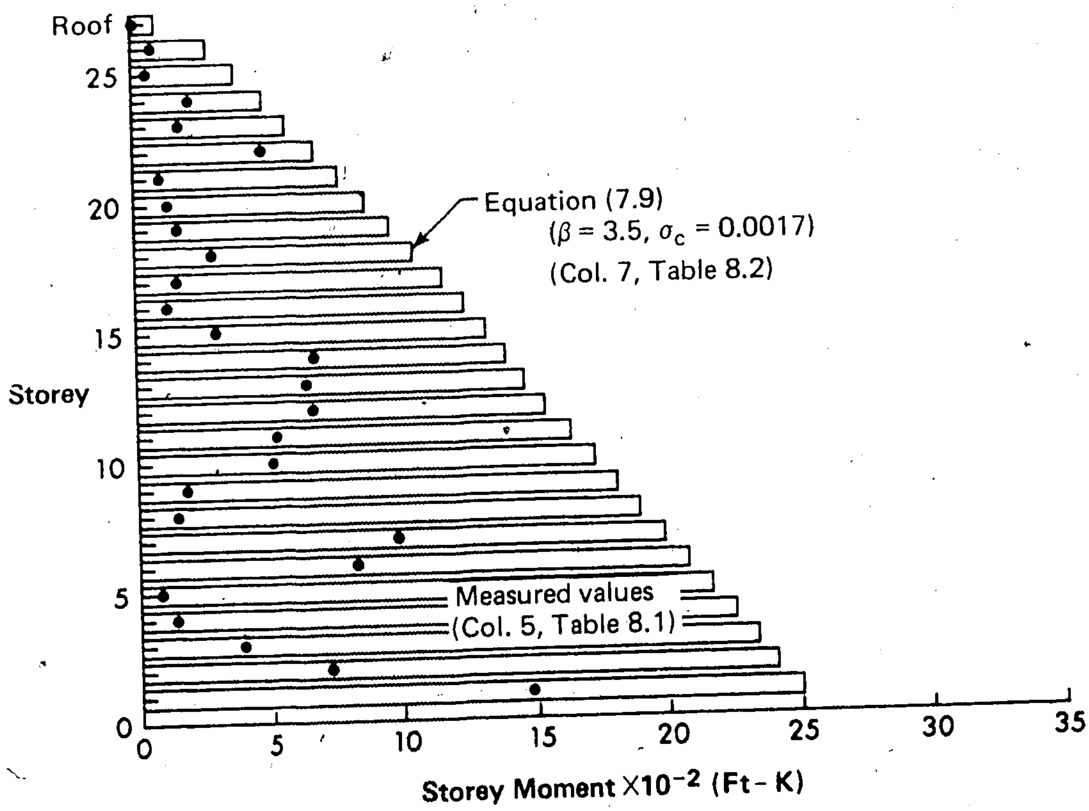


Figure 8.4 Moment in the plane of the floor at each level of building A

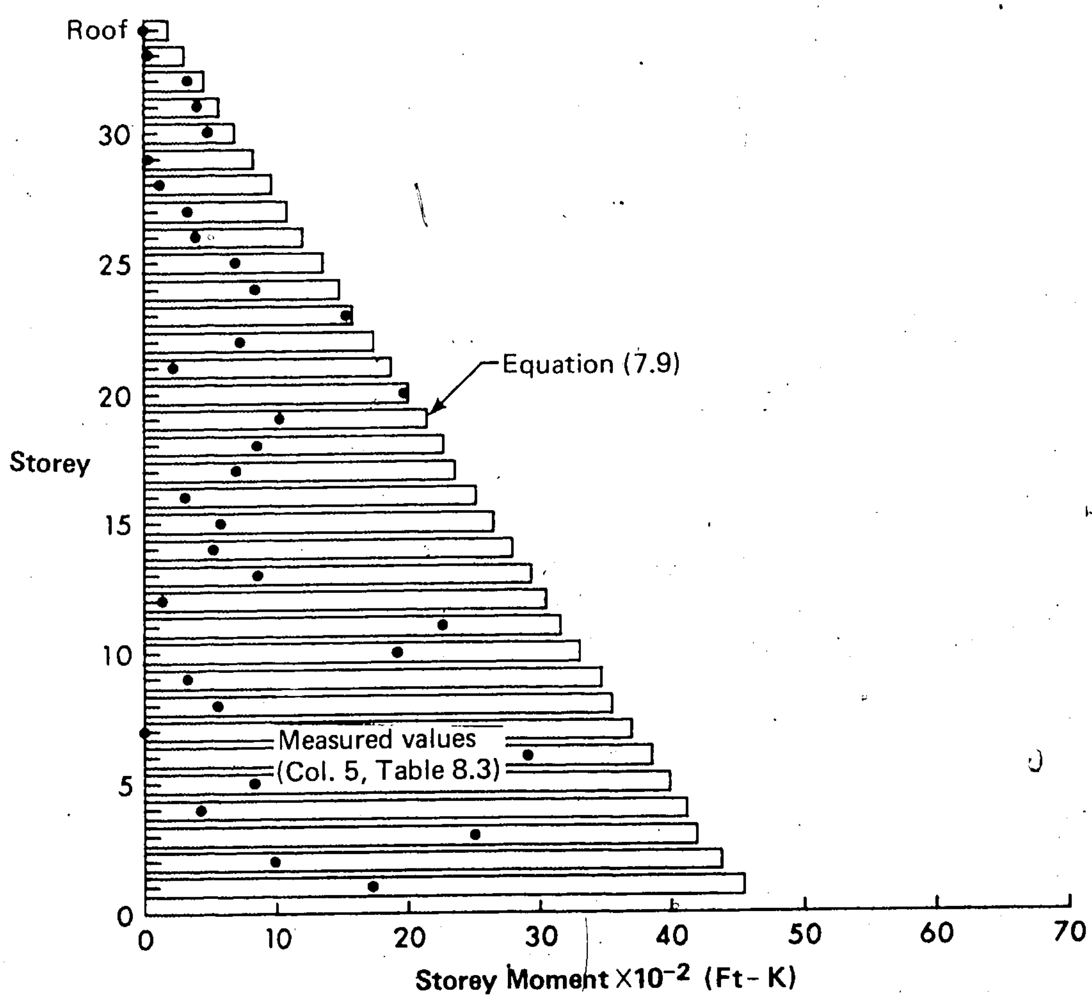


Figure 8.5 **Moment in the plane of the floor at each level of building B**

The moments shown are calculated, for convenience, by considering every column present in the structure at each storey. In this particular case, n becomes n_i and Eq. (7.9) takes the form of Eq. (7.15) derived to calculate the torques in the core. These results are listed in column 7 of Tables 8.2 and 8.4. The figures show that Eq. (7.9) provides a good estimate of the moments in the floor diaphragms.

8.1.4 Shear, Moment, and Torque in Core

Calculations and figures are provided in this section to verify the application of Eqs. (7.11), (7.13), and (7.15), all three expressions giving estimates of forces to be resisted by the core. The measured and predicted shears resisted by the core at each storey of buildings A and B are compared in Figs. 8.6 and 8.7. Similarly, the moments are compared in Figs. 8.8 and 8.9 and the torques in Figs. 8.10 and 8.11. The plotted quantities are taken from Tables 8.1 to 8.4 and their respective origins are indicated on the figures. In each case, the proposed equation seems adequate.

8.1.5 Lateral Deflections

The presence of out-of-plumb columns in a structure forces the structure to sway laterally. All the columns participate in this action. In the case of building A, the lateral deflection curves obtained from the measurements in the x and y directions have been plotted in Fig. 8.12 against the results given by the equations derived in section 7.1.7. Curves 1 and 2 show the results of Eqs. (7.16) and (7.18) while curves 3 and 4 present the actual deflections obtained from the measurements. The values shown in abcissa have no units since they only serve the purpose of indicating the relative deflections.

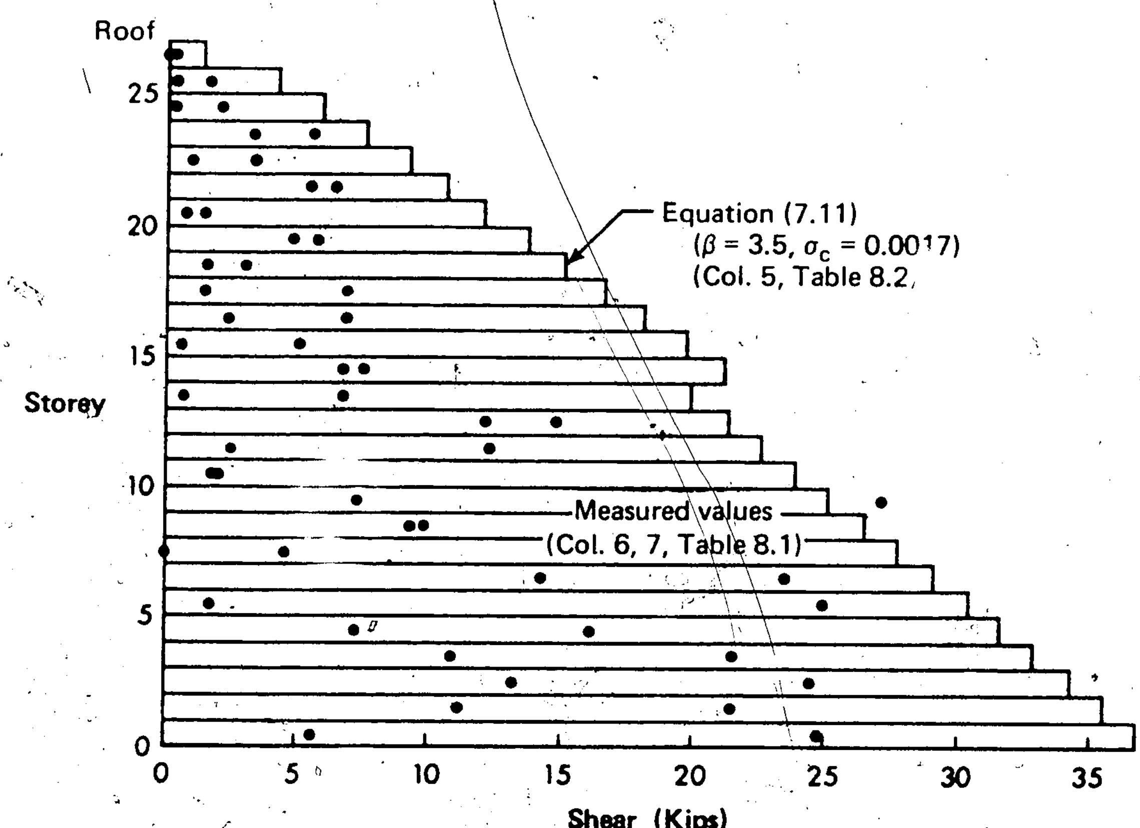


Figure 8.6 Shear due to column out-of-plumbs in the core of building A

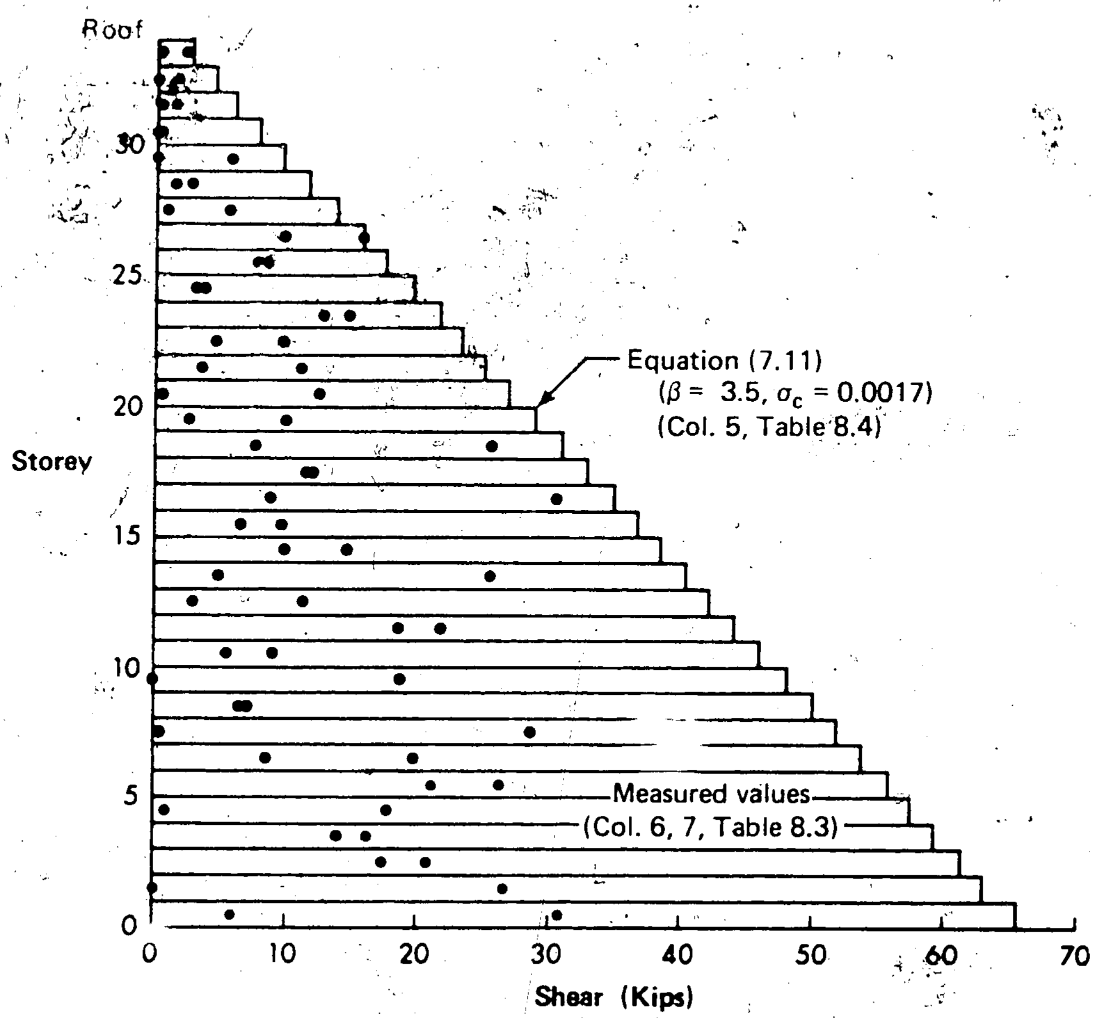


Figure 8.7 Shear due to column out-of-plumbs in the core of building B

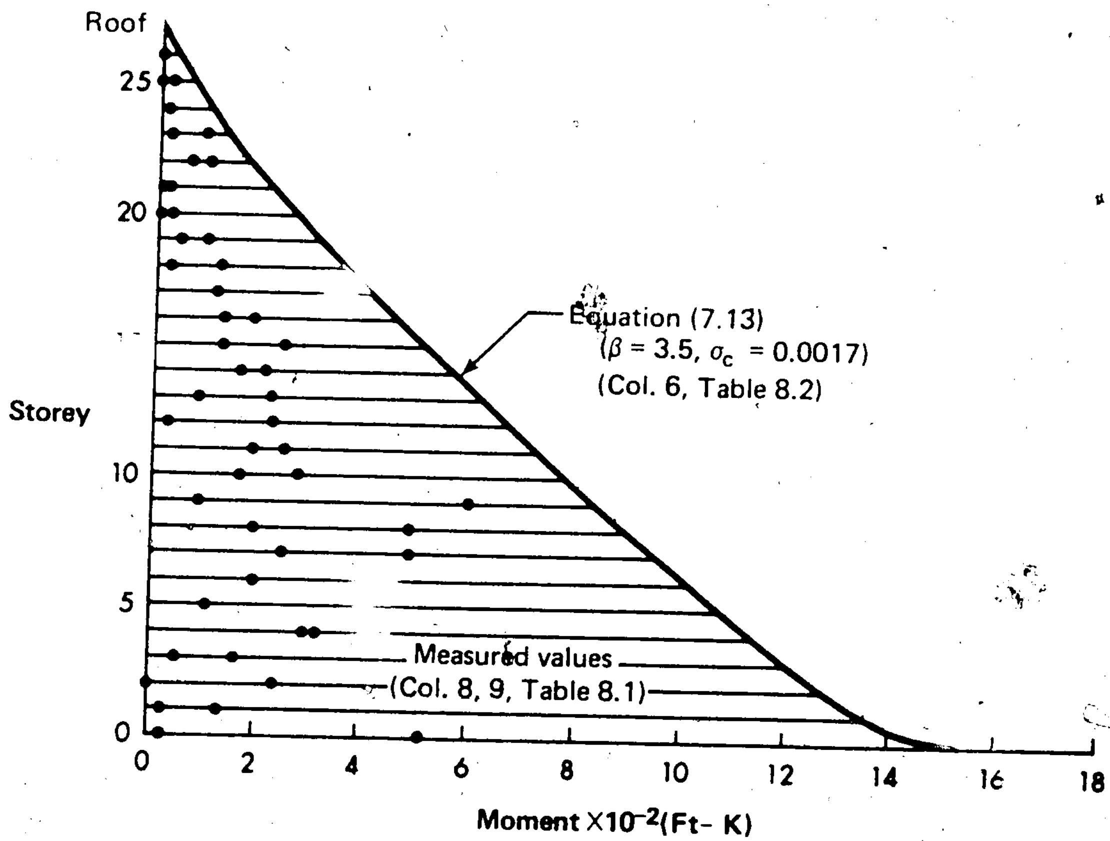


Figure 8.8 Moment due to column out-of-plumbs in the core of building A

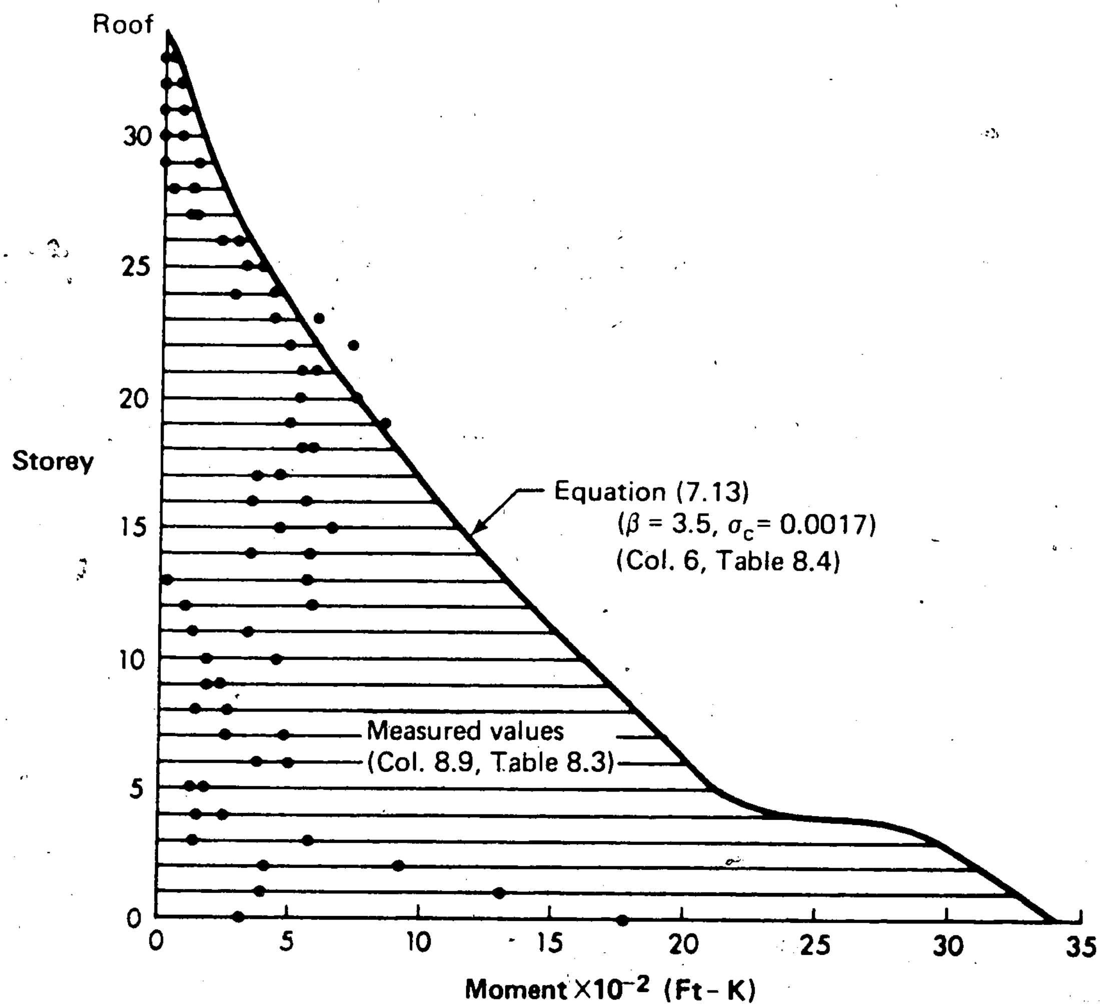


Figure 8.9 Moment due to column out-of-plumbs in the core of building B

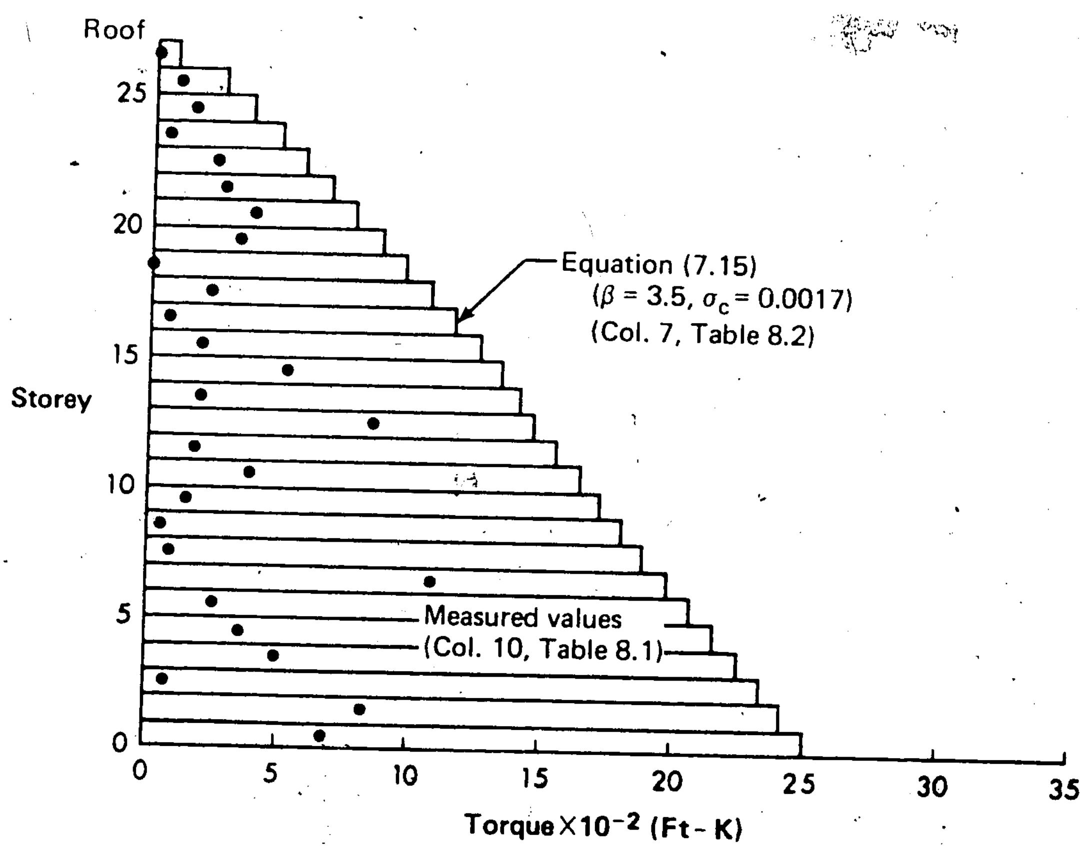


Figure 8.10 Torque due to column out-of-plumbs in the core of building A

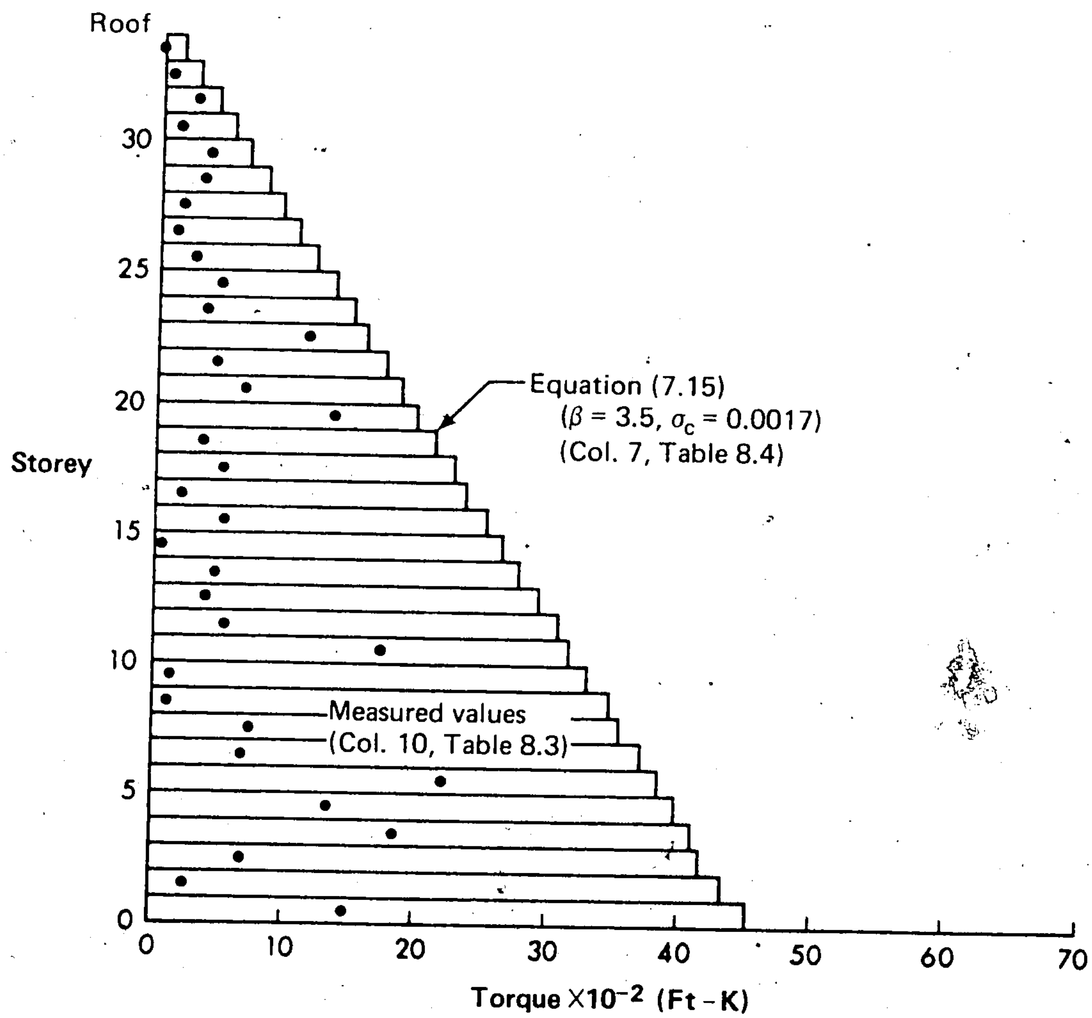


Figure 8.11 Torque due to column out-of-plumbs in the core of building B

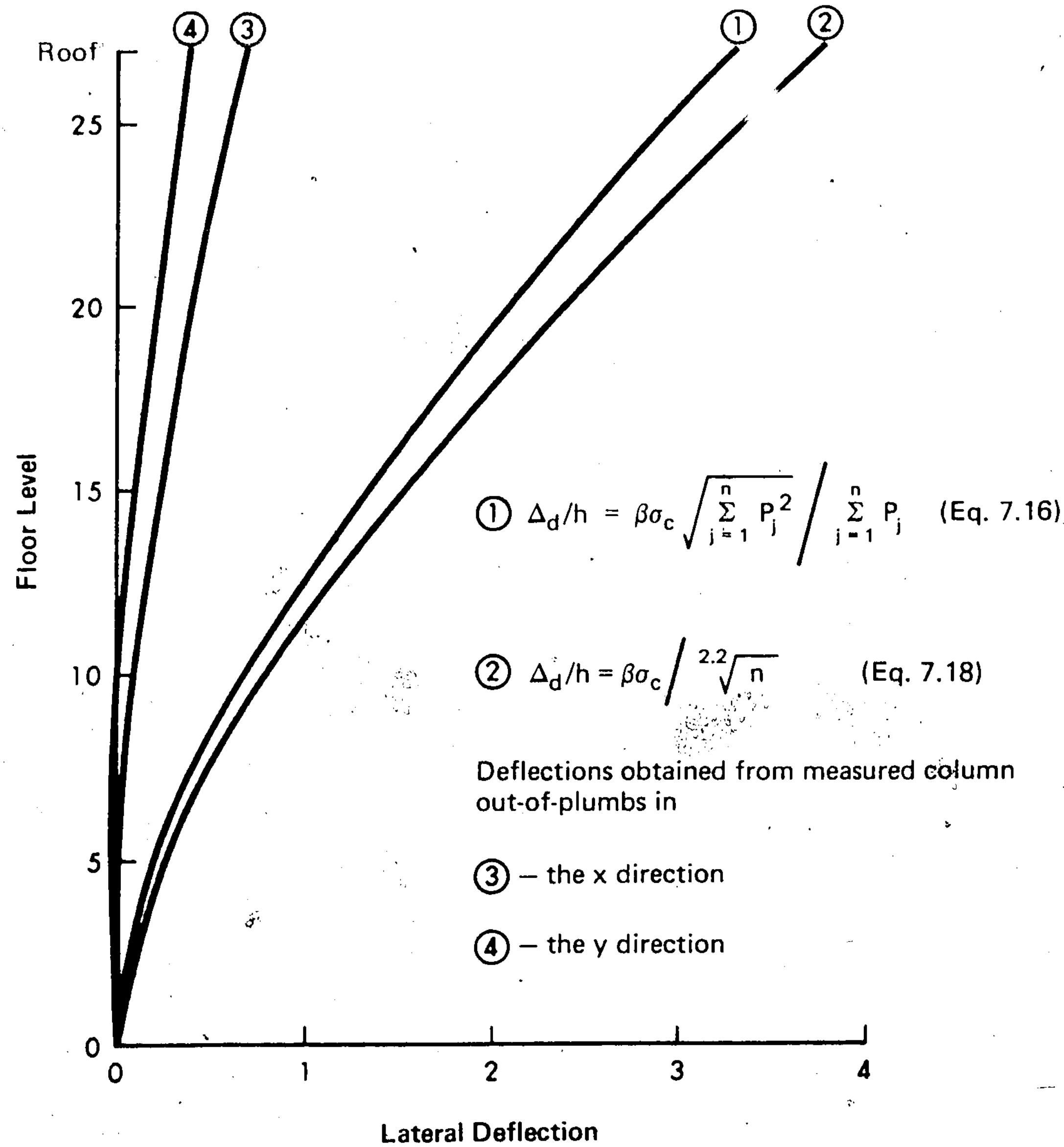


Figure 8.12 Lateral deflections caused by initial column out-of-plumbs for building A

It is seen that Eq. (7.18) gives a conservative estimate of the more exact expression (7.16). The equivalent slope Δ_d/h given by Eq. (7.16) is 3.5 x 0.0017 x 19755/367682 = 0.00032 Rad., compared to 0.00037 Rad. given by Eq. (7.18) with n = 458. The storey forces used in the determination of the sway of building A were obtained by using these values and the actual column axial loads in the structure.

To demonstrate that Eqs. (7.16) and (7.18) are general, the study must be extended to structures of different size. Three simple structures denoted as E.1, E.2, and E.3 have been selected and are shown in the insets to Figs. 8.13 to 8.15. E.1 and E.2 are 5-storey buildings with 20 and 50 columns respectively, while E.3 is a 10-storey building containing 50 columns.

A Monte Carlo simulation has been used to generate random out-of-plumbs for the columns of frame E.1. The population generated was normally distributed and had a mean of zero and a standard deviation of 0.0017 Rad. Every column of the frame has been allocated one of these values. A set of storey forces was obtained and was applied to the shear-wall to calculate the lateral deflections. The process was repeated 50 times with different values and the results were plotted in Fig. 8.13. Also plotted is the curve resulting from Eq. (7.16), where Δ_d/h = 3.5 x 0.0017 x 1173/4500 = 0.00155 Rad. (in this case, the same result is obtained from Eq. (7.18)). It is observed that none of the curves exceeds the limit given by Eq. (7.16) or (7.18).

Similar computations were made for structures E.2 and E.3 and the results are presented in Figs. 8.14 and 8.15. Uniform slopes in the order of 0.001 Rad. were calculated from both Eqs. (7.16) and (7.18). The prescribed curves were again not exceeded. It can be

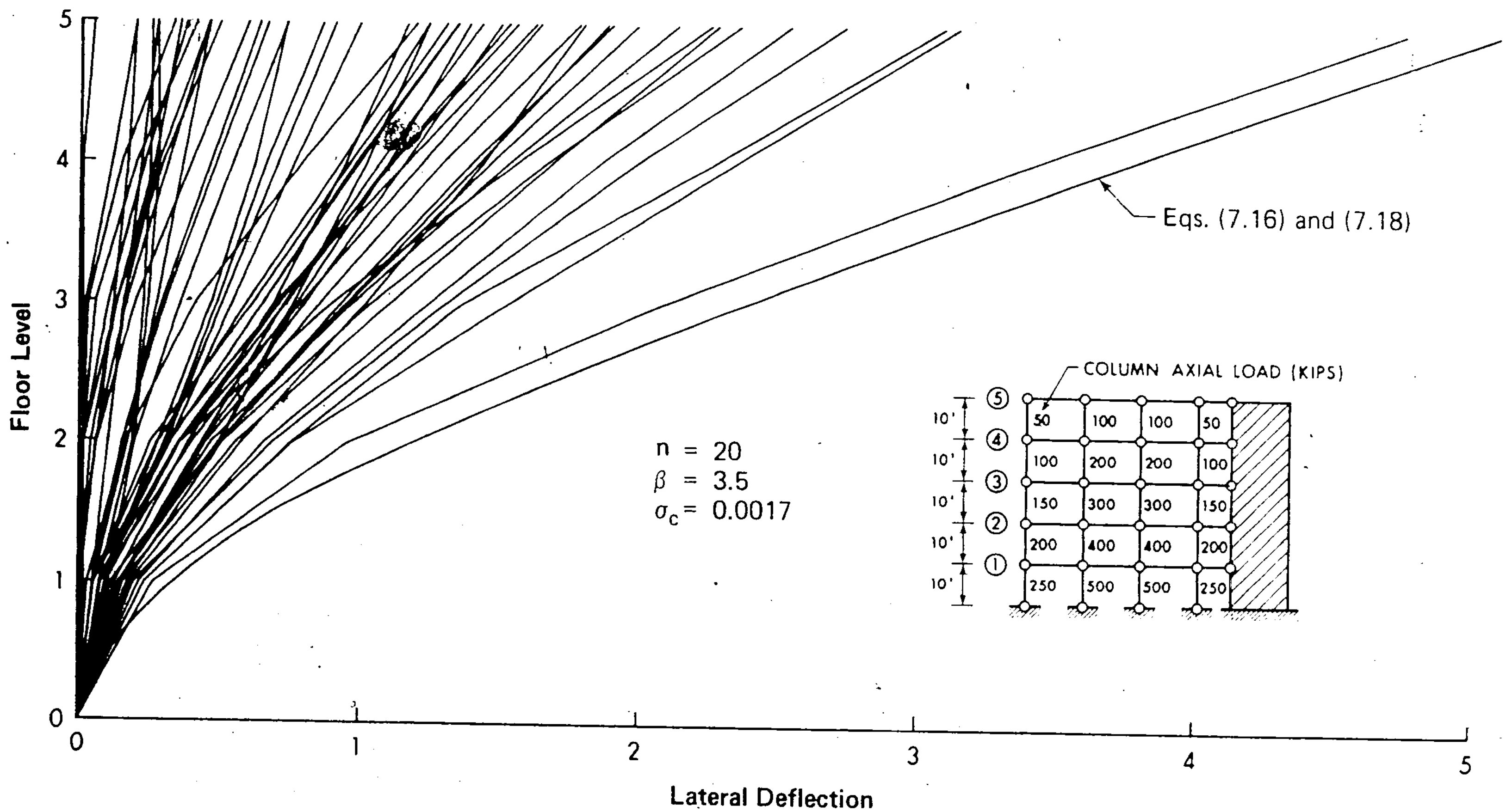


Figure 8.13 Lateral deflections caused by initial column out-of-plumbs for building E.1

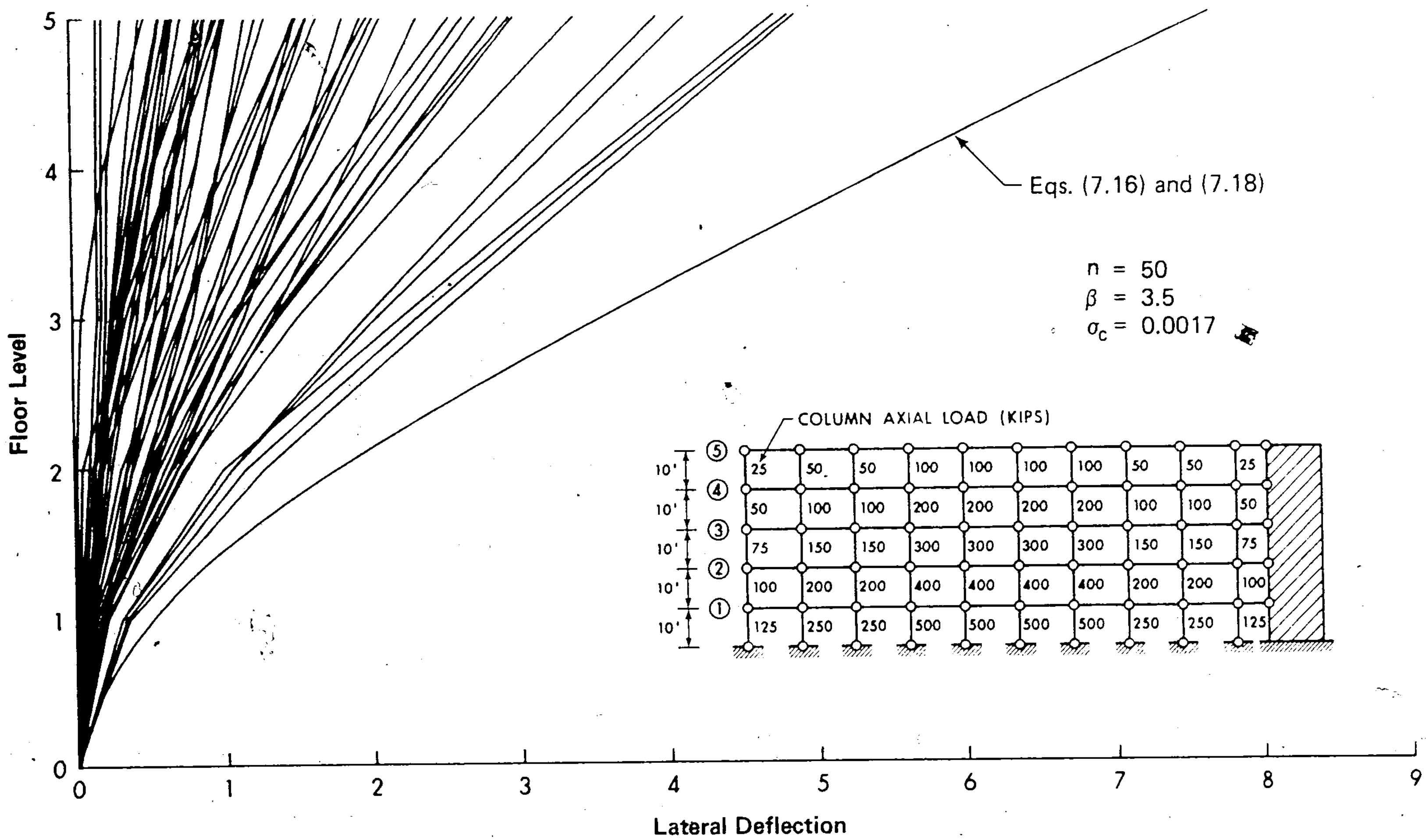


Figure 8.14 Lateral deflections caused by initial column out-of-plumbs for building E.2

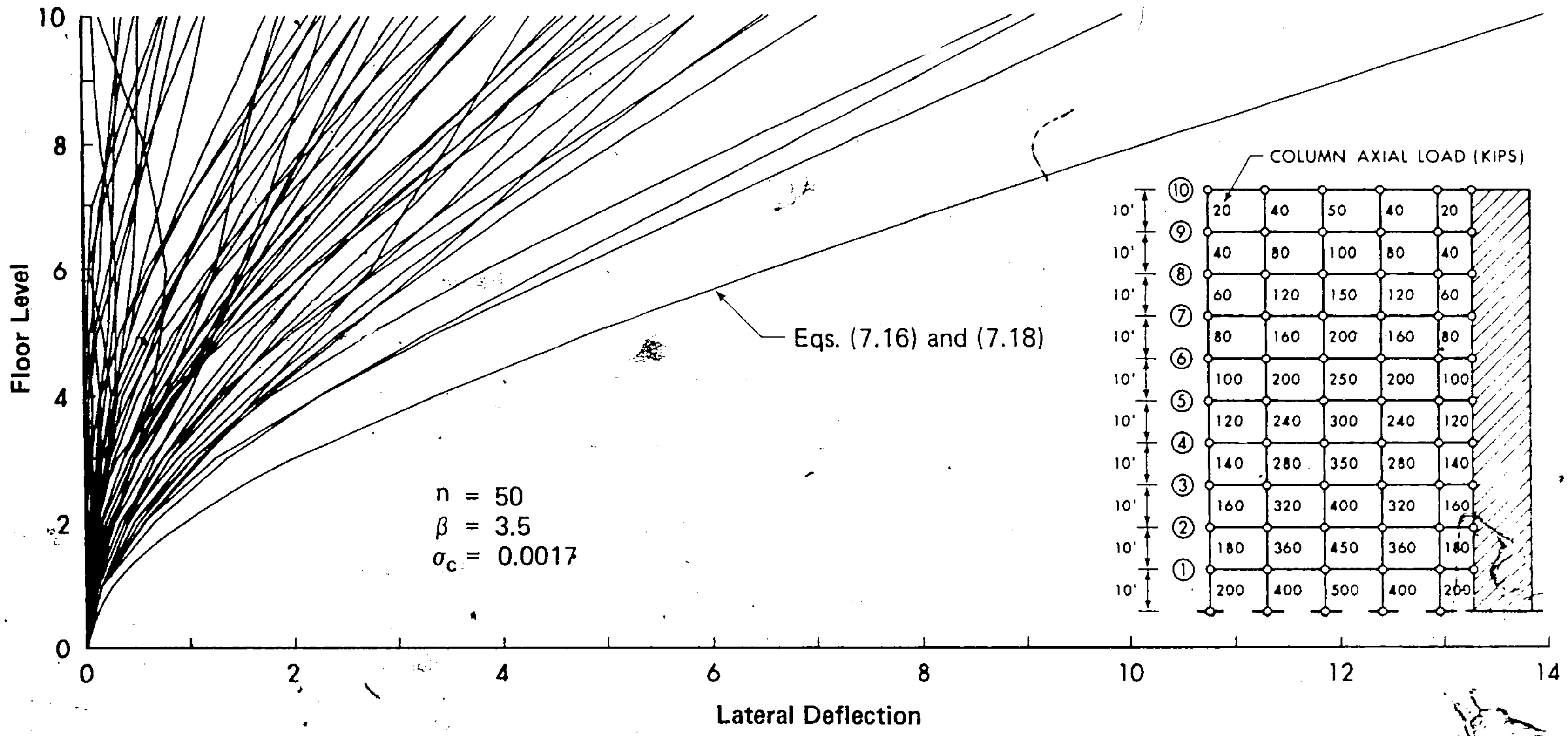


Figure 8.15 Lateral deflections caused by initial column out-of-plumbs for building E.3

concluded that Eq. (7.18) gives a good upper bound for estimating the lateral deflections induced by initial column out-of-plumbs for the types of structures studied.

8.2 Wall Out-of-Plumbs

The actual cores of buildings A and B are formed of eight and nine walls per storey, respectively. However, only the four exterior walls will be considered in the following applications, using the information given by the measurements. It will be assumed that the four walls carry the total axial loads in the actual cores. The moments, torques, and lateral deflections calculated in this manner will be larger than in the actual cases where the same loads are carried by more than four walls. These effects will be discussed in the next chapter.

The wall dimensions used in the computations are given in Figs. 6.2 and 6.3. In building A, the core dimensions are 87' x 29' up to level 14 and are reduced to 67' x 29' in the upper section. In building B, the dimensions are assumed to be 66' x 38' for the total height of the building.

The share of the total axial load carried by the individual walls is assumed proportional to the length of the walls. These values are listed in Tables 8.5 and 8.7. The axial loads in the upper 22 storeys of the 27-storey building A are used in the calculations since results were obtained for only 22 storeys.

8.2.1 Moment in Core

The moments in the cores of buildings A and B have been calculated at each storey from the information given by the measurements on the walls and have been listed in columns 6 and 7 of Tables 8.5 and 8.7. The moments prescribed by Eq. (7.25), which considers the contribution of the mean, and Eq. (7.26), which neglects it, are listed in columns 3 and 4 of Tables 8.6 and 8.8.

The measured and predicted moments described above are plotted in Figs. 8.16 and 8.17 for direct comparison. Fig. 8.17 proves that the apparent conservativeness of the proposed equations in Fig. 8.1 is due to the actual out-of-plumb arrangement in building A. This arrangement resulted in the calculation of comparatively small moments in the core. Moreover, the contribution of the mean, which accounts for 15 and 25 percent of the contribution of the standard deviation at the base of buildings A and B, is apparently a significant factor.

8.2.2 Torque in Core

The torques at each core level, as predicted by Eqs. (7.30) and (7.31), are listed in Tables 8.6 and 8.8 for buildings A and B. The torques calculated from the actual measurements are listed in Tables 8.5 and 8.7 and are plotted against the proposed design values in Figs. 8.18 and 8.19. Since all the measured values lie well within the limits prescribed by Eq. (7.31) and since the contribution of the mean accounts for less than 6 percent of the contribution of the standard deviation in both cases, Eq. (7.31) seems appropriate for use in design.

1	2	3	4	5	6	7	8
Storey No.	Storey Height (Ft.)	Axial Load (Kips)			Moment (Ft.-K.)		Torque (Ft.-K.)
		Total	Walls #1,2*	Walls #3,4*	x Axis	y Axis	
22	12	1382	484	207	0.00	0.00	7.49
21	12	2726	954	409	-167.12	12.93	-17.01
20	12	4140	1449	621	-181.90	71.14	45.28
19	12	5520	1932	828	-141.17	40.95	84.21
18	12	6894	2413	1034	-151.20	61.08	- 44.74
17	12	8274	2896	1241	170.16	174.15	315.08
16	12	9652	3378	1448	52.90	-424.68	12.51
15	12	11022	3858	1653	288.87	-363.11	4.52
14	12	12406	4652	1551	52.33	-287.06	-162.39
13	12	13782	5168	1723	- 20.38	151.32	441.12
12	12	15154	5683	1894	-307.48	7.22	158.22
11	12	16406	6197	2006	- 11.47	-296.46	-207.35
10	12	17898	6712	2237	- 3.82	-101.87	40.36
9	12	19306	7240	2413	-359.24	-341.45	- 82.02
8	12	20650	7744	2581	-246.09	-115.22	80.77
7	12	22022	8258	2753	- 64.61	123.82	- 9.89
6	12	23398	8774	2925	652.23	238.53	141.72
5	12	24766	9287	3096	288.78	85.89	- 56.38
4	12	26146	9805	3268	123.35	428.35	-532.11
3	12	27526	10322	3441	1127.65	73.94	529.04
2	12	28910	10841	3614	1438.38	148.85	669.33
1	20	30286	11357	3786	1200.10	645.35	-584.38
					1857.10	322.00	

* Wall numbering given in Fig. 6.2.

TABLE 8.5 FORCES IN CORE OF BUILDING A FROM ACTUAL WALL OUT-OF-PLUMBS

1	2	3	4	5	6
Storey No.	Storey Height (ft.)	Moment (Ft.-K.)		Torque (Ft.-K.)	
		Eq. (7.25)*	Eq. (7.26)*	Eq. (7.30)**	Eq. (7.31)**
22	12	0.0	0.0	69.1	65.3
21	12	92.3	87.5	136.3	128.7
20	12	207.9	193.5	207.0	195.5
19	12	354.7	325.9	276.0	260.7
18	12	524.7	477.9	344.7	325.6
17	12	716.9	647.3	413.7	390.7
16	12	930.0	832.8	482.6	455.8
15	12	1162.6	1033.0	551.1	520.5
14	12	1413.5	1246.7	851.2	806.2
13	12	1698.6	1489.8	945.6	895.7
12	12	1999.2	1743.6	1039.9	984.9
11	12	2314.2	2008.2	1133.4	1073.6
10	12	2643.0	2281.8	1228.1	1163.3
9	12	2988.5	2567.3	1324.8	1254.8
8	12	3361.9	2863.9	1417.0	1342.1
7	12	3736.9	3169.3	1511.0	1431.
6	12	4126.3	3484.3	1605.5	1520.
5	12	4530.0	3808.8	1699.3	1609.
4	12	4946.1	4142.1	1794.1	1699.3
3	12	5376.2	4484.6	1888.7	1788.9
2	12	5819.9	4835.9	1983.7	1878.9
1	20	6277.2	5196.0	2078.1	1968.3
		7416.4	6165.2		

* with $\beta = 3.5$, $\bar{\Delta}_0/h = 2.8 \times 10^{-4}$, and $\sigma_w = 2.8 \times 10^{-3}$.

** with $\beta = 3.5$, $\bar{e}/L = 0.5 \times 10^{-4}$, and $\sigma_e = 4.0 \times 10^{-4}$.

TABLE 8.6 FORCES IN CORE OF BUILDING A FROM STATISTICAL CALCULATIONS

1	2	3	4	5	6	7	8
Storey No.	Storey Height (Ft.)	Axial Load (Kips)			Moment (Ft.-K.)		Torque (Ft.-K.)
		Total	Walls #1,2*	Walls #3,9*	x Axis	y Axis	
34	24	312	100	56	0.00	0.00	- 0.82
33	12	1962	628	353	- 3.85	- 4.53	- 16.67
32	12	3208	1027	577	42.79	51.77	- 5.42
31	12	4326	1376	787	54.24	105.18	6.94
30	12	5440	1730	990	75.12	155.94	- 16.27
29	12	6560	2086	1194	97.38	186.10	120.88
28	12	7672	2446	1396	- 81.47	354.80	-139.60
27	12	8790	2795	1600	- 71.02	520.91	-102.81
26	12	9896	3147	1801	- 74.02	500.73	109.83
25	12	11012	3502	2004	-125.68	623.91	29.75
24	12	12114	3852	2205	-292.17	637.38	-232.69
23	12	13226	4206	2407	-138.02	810.64	12.79
22	12	14342	[illegible]	2610	- 20.77	1018.44	1.67
21	12	15456	[illegible]	2813	- 0.60	1196.07	52.63
20	12	16550	5263	3012	220.76	1387.30	-390.23
19	12	17664	5617	3215	352.93	1469.10	- 2.70
18	12	18922	6017	3444	126.16	1369.05	- 53.10
17	12	20186	6419	3674	364.48	2107.00	-188.47
16	12	21446	6820	3903	518.30	2088.91	20[illegible].16
15	12	22708	7221	4133	-202.68	2263.77	-283.76
14	12	23938	7612	4357	207.65	2416.14	- 35[illegible]
13	12	25194	8012	4585	41.70	2927.45	- 94.09
12	12	26456	8413	4815	199.24	3291.20	1.02
11	12	27716	8814	5044	-306.82	3322.93	-461.48
10	12	28974	9214	5273	-929.03	5056.49	-424.77
9	12	30600	9615	5685	-509.12	489[illegible]	-489.17
8	12	31496	10016	5732	-168.95	6252.96	112.62
7	12	32758	10417	5962	- 68.51	6051.05	-223.92
6	12	34016	10817	6191	638.75	5979.25	66.53
5		35548	11304	6470	836.68	5674.81	497.23
4		36176	11504	6584	-1496.94	6560.72	754.35
3		40886	13002	7441	225.71	7113.85	54.38
2		42470	13505	7730	-867.73	5854.57	88.23
1	15	43978	13985	8004	-1228.49	7374.[illegible]	-415.94
					-1405.32	6955.80	

* Wall numbering given in Fig. 6.3.

TABLE 8.7 FORCES IN CORE OF BUILDING B FROM ACTUAL WALL OUT-OF-PLUMBS

1	2	3	4	5	6
Storey No.	Storey Height (ft.)	Moment (Ft.-K.)		Torque (Ft.-K.)	
		Eq. (7.25)*	Eq. (7.26)*	Eq. (7.30)**	Eq. (7.31)**
34	24	0.0	0.0	14.6	13.7
33	12	40.2	38.1	91.7	86.3
32	12	134.4	125.7	150.0	141.0
31	12	252.3	232.8	201.4	189.3
30	12	385.7	351.7	253.2	238.0
29	12	535.6	483.3	305.3	287.0
28	12	701.5	627.2	357.1	335.7
27	12	882.4	782.3	409.1	384.6
26	12	1077.7	948.1	460.6	433.0
25	12	1286.5	1123.6	512.5	481.8
24	12	1508.6	1308.7	563.8	530.0
23	12	1743.1	1502.6	615.6	578.7
22	12	1990.1	1705.1	667.5	627.5
21	12	2249.2	1916.0	719.4	676.2
20	12	2520.1	2135.0	770.[illegible]	724.1
19	12	2801.9	2361.2	822.[illegible]	772.8
18	12	3095.1	2594.9	88[illegible]	827.9
17	12	3403.3	2839.6	93[illegible].5	883.2
16	12	3726.1	3094.5	998.2	938.3
15	12	4062.8	3359.2	1056.9	993.5
14	12	4412.9	3633.0	1114.1	1047.3
13	12	4775.2	3914.9	1172.6	1102.3
12	12	5150.1	4205.2	1231.3	1157.5
11	12	5537.5	4503.6	1290.0	1212.7
10	12	5936.9	4809.9	1348.5	1267.7
9	12	6348.0	5123.7	1412.3	1327.2
8	12	6777.2	5450.1	1465.9	1378.0
7	12	7210.8	5777.9	1524.6	1433.2
6	12	7656.0	6113.0	1583.2	1488.3
5	24	8112.1	6454.9	1654.5	1555.3
4	27	9670.1	7773.9	1683.7	1582.8
3	16	11391.1	9221.5	1903.0	1788.9
2	15	12154.4	9801.6	1976.6	1858.1
1	15	12852.8	10321.6	2046.8	1924.1
		13567.5	10851.6		

* with $\beta = 3.5$, $\overline{\Delta}_0/h = 2.8 \times 10^{-4}$, and $\sigma_w = 2.8 \times 10^{-3}$.

** with $\beta = 3.5$, $\overline{e}/L = 0.5 \times 10^{-4}$, and $\sigma_e = 4.0 \times 10^{-4}$.

TABLE 8.8 FORCES IN CORE OF BUILDING B

FROM STATISTICAL CALCULATIONS

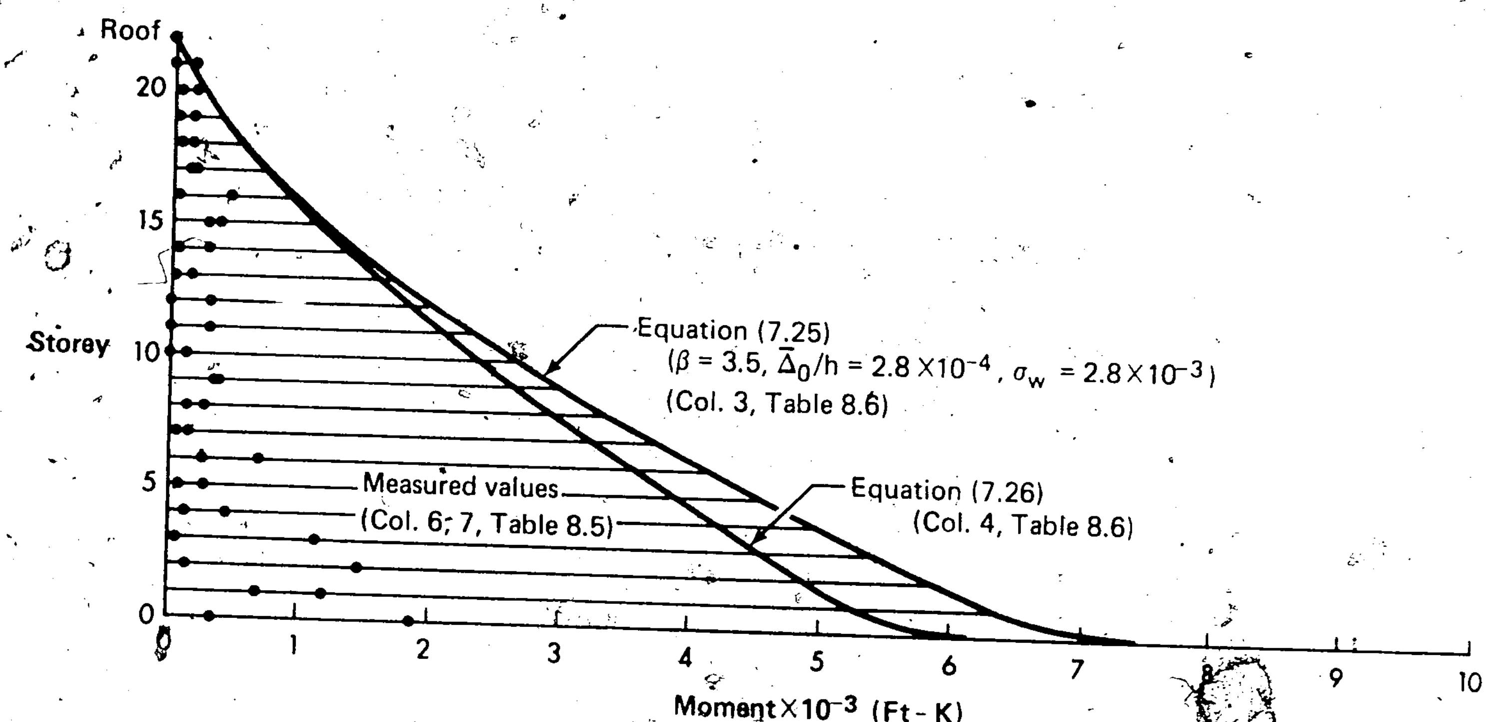


Figure 8.16 Moment due to wall out-of-plumbs in the core of building A

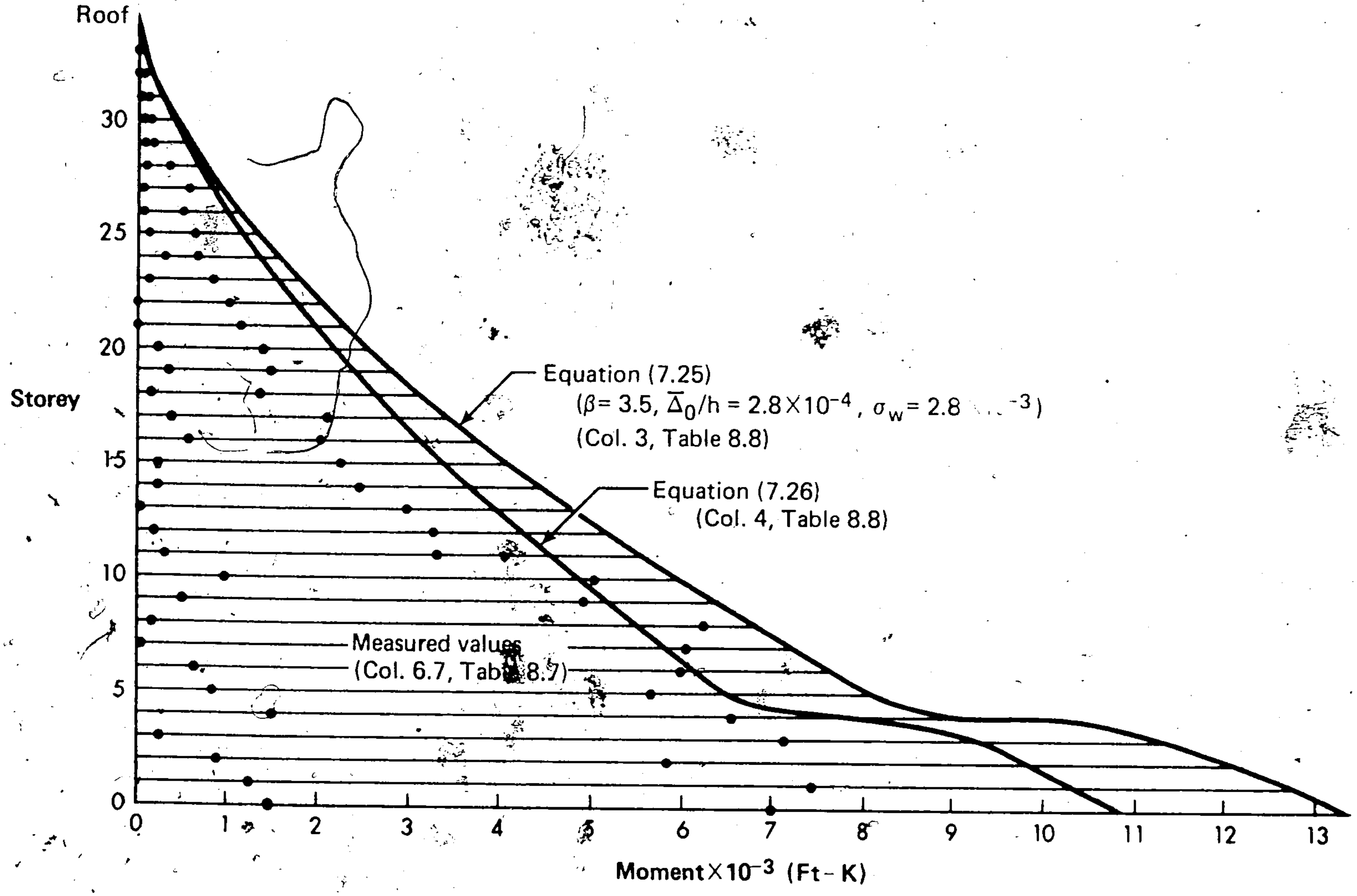


Figure 8.17 Moment due to wall out-of-plumbs in the core of building B

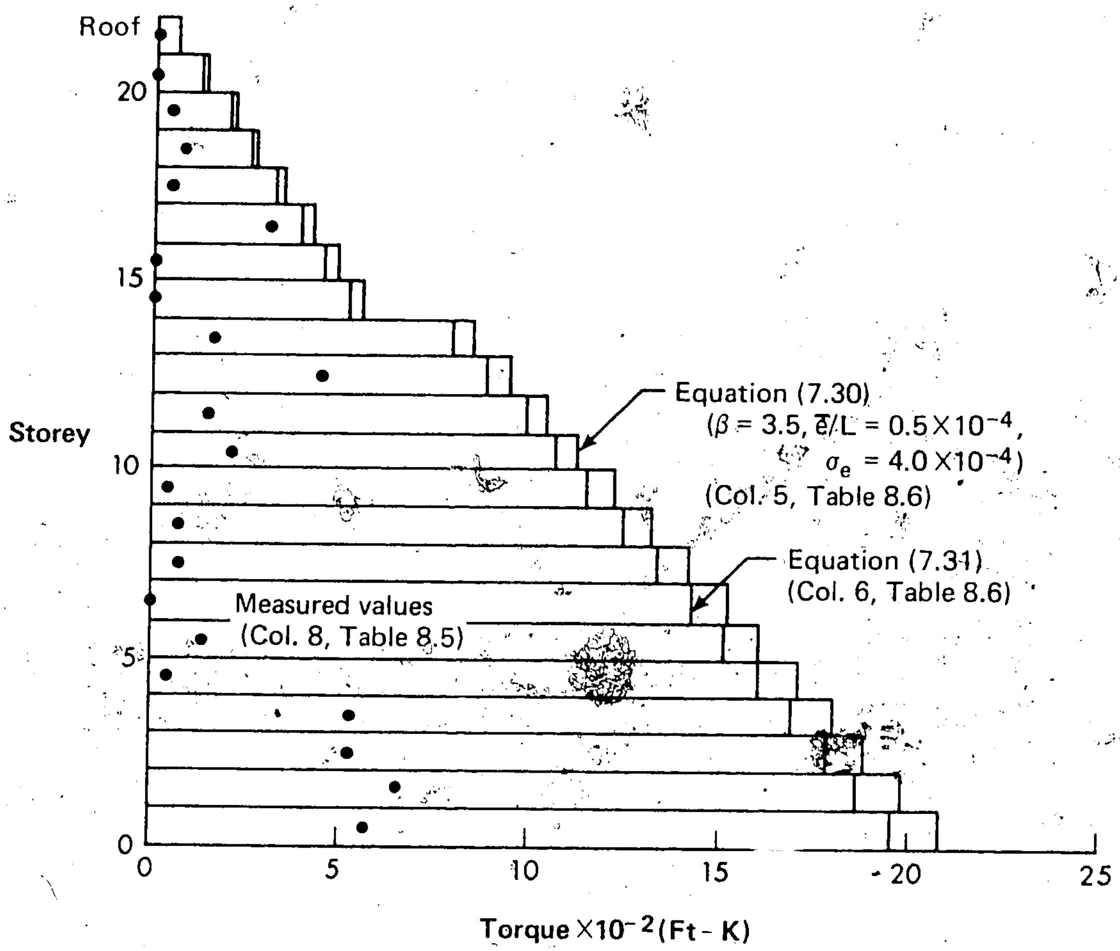


Figure 8.18 Torque due to wall out-of-plumbs in the core of building A

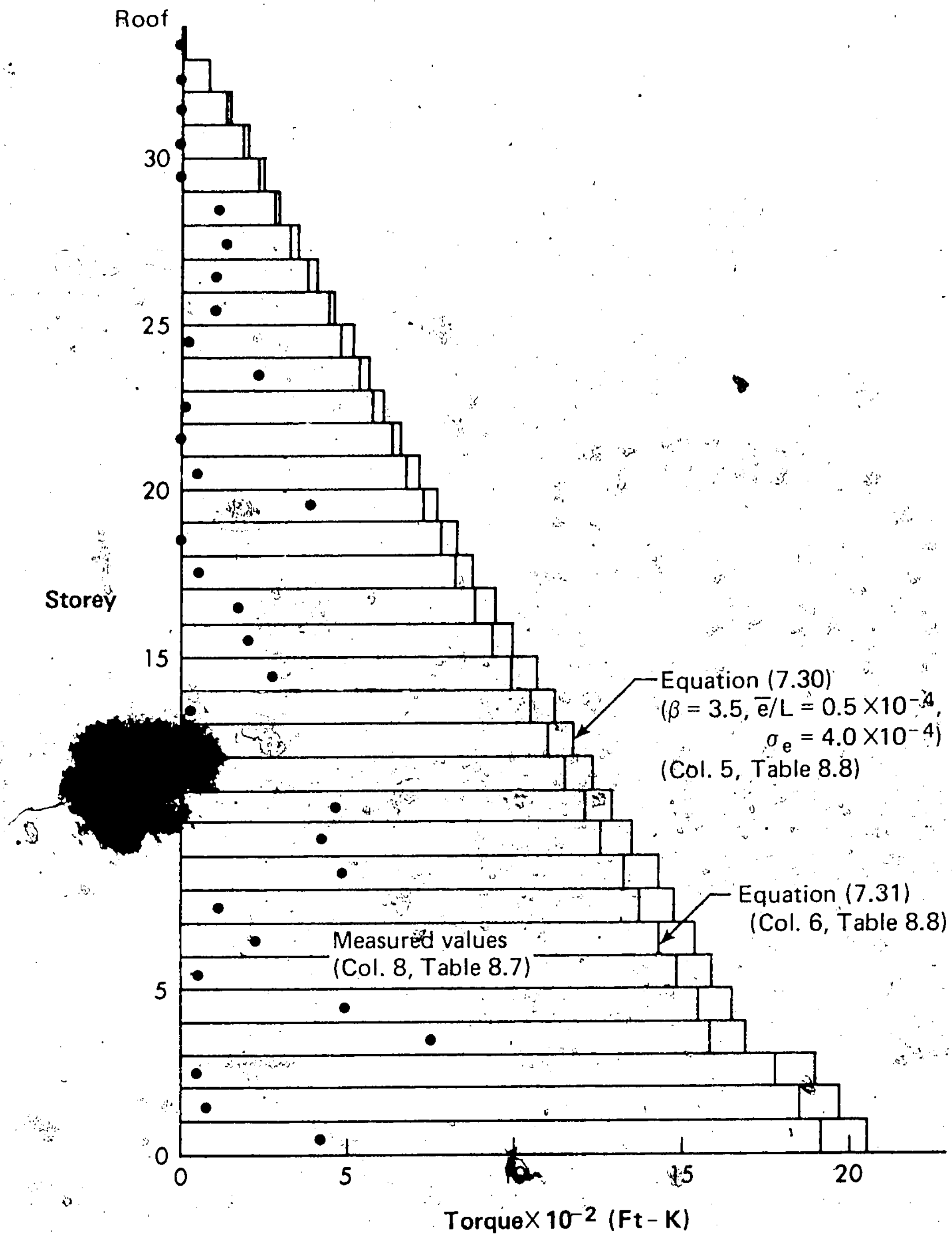


Figure 8.19 Torque due to wall out-of-plumbs in the core of building B

8.2.3 Lateral Deflections

Equation (7.37), which gives an equivalent wall out-of-plumb for use in an overall stability analysis of a structure, has the same form and consequently the same characteristics as Eq. (7.18) derived for column out-of-plumbs. Therefore, the results obtained in section 8.1.5 can be extended to the present case.

As applied to the 22-storey building A, the equivalent slope of the walls forming the core is $\Delta_d/h = 0.00028 + 3.5 \times 0.0028/\sqrt[2.2]{88} =$ 0.00156 Rad. and the slope calculated for building B (where n = 136) is 0.00133 Rad.

CHAPTER IX

COMPARATIVE STUDY

Design equations have been derived and their applicability has been confirmed in the previous two chapters, but the significance of the effects in terms of the overall structural design has not been determined. The importance of the out-of-plumb effects on design can be evaluated in terms of corresponding wind effects.

The study in this chapter is limited in scope but gives interesting results when applied to core-braced structures. Where appropriate, the design recommendations given in section 5.4 are compared with the results of the techniques developed in this report. A certain consistency is achieved by using the specified loads in the calculations. No attempt is made to include the wind effects on the column axial loads, which would have resulted in increased axial loads on the leeward side and reduced axial loads on the windward side, with no net difference in the total gravity load.

9.1 Horizontal Forces at Connection Point

Wind pressures of 35.0 psf in the upper sections of buildings A and B and 22.0 psf at ground level were obtained from the simple procedure prescribed by the National Building Code of Canada[60]. Of these pressures, a proportion 8/13 is applied directly to the windward side of the building and a proportion 5/13 acts as a suction on the leeward side. The basic wind pressure with a return period of 30 years is 8.5 psf for these buildings.

The shears caused by wind and transmitted to the core were calculated at specific beam-to-column connections in both buildings. The values presented in column 4 of Table 9.1 were obtained from the windward pressures given above applied to the particular connection tributary areas (see Figs. 6.2 and 6.3). The horizontal forces created in these connections by the out-of-plumbs of the columns are presented in the same table. A sample calculation using Eq. (7.5) is given below the table. The out-of-plumb to wind shear ratios given in column 6 indicate that, in a tall braced building, the out-of-plumb shears generally govern in the design of the connections, while in the top storeys they still account for an important fraction of the wind shears.

The out-of-plumb shears become even more critical in a frame of the type shown in Fig. 8.1 where the forces are transferred to a bracing system in the plane of the frame. The shears due to column out-of-plumbs are increased from bent to bent during the transfer but the wind forces remain constant. It is understood that other significant forces, such as the P-Δ forces described in Chapter IV, are also present.

The model shown in Fig. 9.1(a) was used as an example in Ref. 23 to estimate the out-of-plumb forces in girder-to-column connections. All columns are assumed to be erected with initial out-of-plumbs of 0.002 Rad.[36] (see section 5.4.2). The constant forces of value 1.02 kips created at each connection are added algebraically when transferred to the bracing system.

1	2	3	4	5	6
Building	Floor Level	Column Location	Wind Shear (kips)	Out-of-Plumb Shear (Eq. 7.5) (kips)	Out-of-Plumb Shear / Wind Shear
A	1*	Exterior	6.2	13.6†	2.19
		Corner	3.3	8.2	2.48
	23**	Exterior	7.5	2.8	0.37
		Corner	3.9	1.7	0.44
B	1*	Exterior	3.9	16.6	4.26
		Corner	2.0	11.8	5.90
	31**	Exterior	5.0	2.1	0.42
		Corner	2.6	1.0	0.38

* Design wind pressure = 8/13 x 22 = 13.54 psf.

** Design wind pressure = 8/13 x 35 = 21.54 psf.

† Sample calculation:

$$F_d = 3.5 \times .0017\sqrt{1643^2 + 1585^2} = 13.6 \text{ kips.}$$

TABLE 9.1 RATIO OF OUT-OF-PLUMB SHEAR TO WIND SHEAR AT BEAM-TO-COLUMN CONNECTIONS

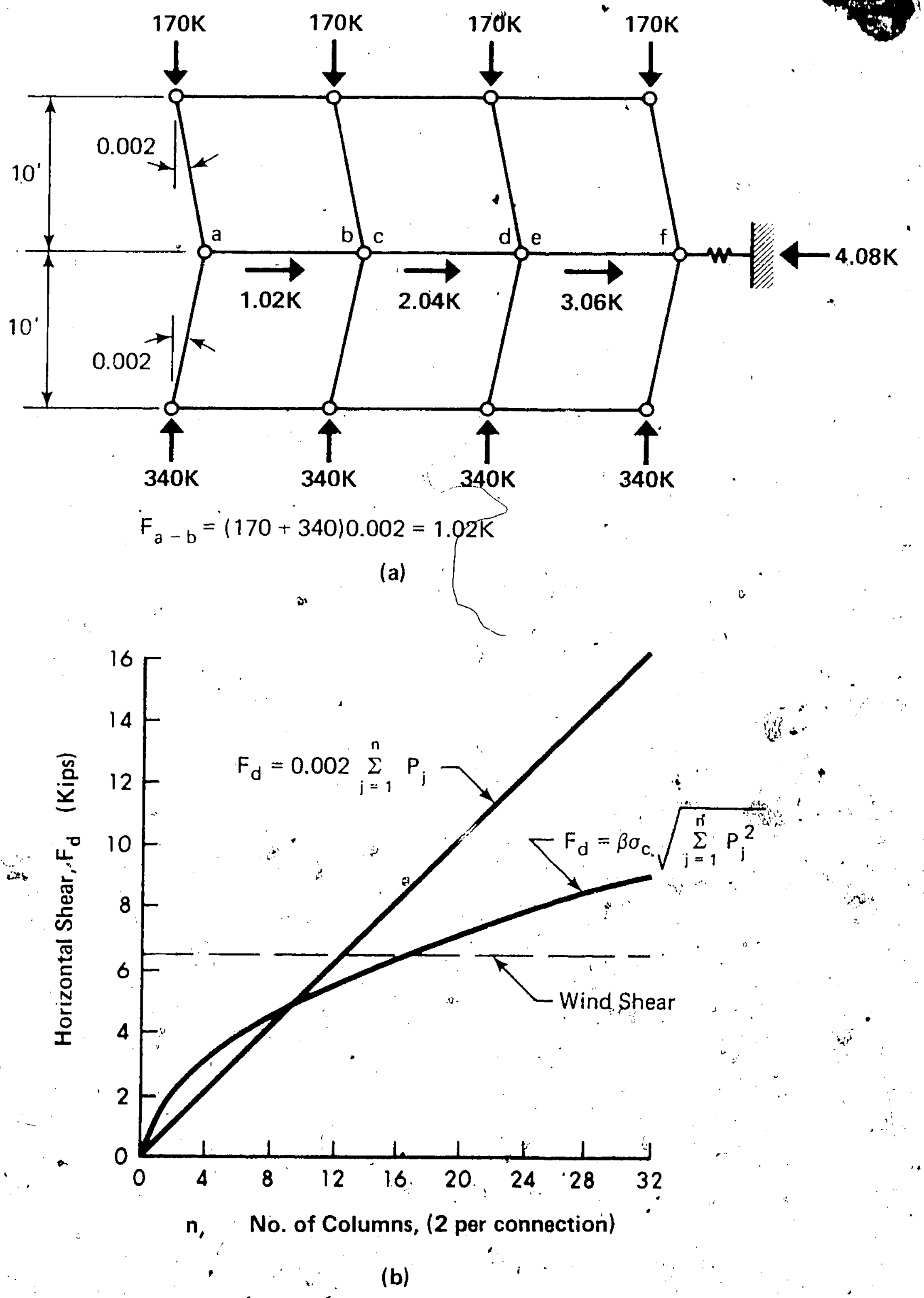


Figure 9.1 Distribution of horizontal shear in a braced bent

The same frame has been used in the example of Fig. 8.1 for an eventual comparison. The horizontal shears from Figs. 8.1 and 9.1(a) are plotted in Fig. 9.1(b) as a function of the number of columns (2 per connection). The figure shows that the shears predicted by a constant column out-of-plumb of 0.002 are unconservative for a small number of columns but become excessively large as the number of columns involved increases. An arbitrary wind shear, calculated for a pressure of 21.54 psf and a span perpendicular to the plane of the frame of 30 ft., has been plotted on the graph to point out that the wind shear is independent of the number of columns in the bent. The relative importance of the wind shear with respect to the out-of-plumb shear is given by the gravity-to-wind load ratio at the section under study.

9.2 Shear and Moment in the Plane of the Floor

The portion of the floor delimited by column lines No. 8 to 15 in Fig. 6.2 must be designed for shears and moments in the plane of the floor. Assuming a wind force in the x direction, the shear transmitted to the core at a specific floor level by the portion of the floor described above, is roughly equal to the sum of the lateral forces concentrated at connections No. 8, 9, 14 and 15. A shear equal to 17.6 kips is calculated at floor level 1 for a wind pressure of 22 psf and a tributary area of 800 ft^2. This value is compared in Table 9.2 with the shear calculated at the same storey from the 16 out-of-plumb columns on lines 8 to 15. The out-of-plumb shear is about twice as large as the wind shear at that level. The values obtained at floor level 23 show that the wind controls in the upper storeys.

1	2	3	4	5
	Floor Level	Wind Effect	Out-of-Plumb Effect	$\frac{\text{Out-of-Plumb Effect}}{\text{Wind Effect}}$
Shear In Floor	1* 23**	17.6 k. 21.0 k.	36.0 k.† 7.4 k.	2.0 0.35
Moment In Floor	1* 23**	222 ft.-k. 265 ft.-k.	2206 ft.-k.†† 452 ft.-k.	9.9 1.71

* Design wind pressure = 22.0 psf

** Design wind pressure = 35.0 psf

† $F_d = 3.5 \times .0017 \sqrt{\sum_{j=1}^{16} P_j^2}$ (Eq. 7.7)

†† $M_d = 3.5 \times .0017 \sqrt{\sum_{j=1}^{16} [P^2 (L_x^2 + L_y^2)]_j}$ (Eq. 7.9)

TABLE 9.2 COMPARISON OF OUT-OF-PLUMB AND WIND EFFECTS IN FLOOR SYSTEMS

The moments calculated at the same floor sections, under the same conditions, are listed in the second half of the table. The out-of-plumb moments obtained from Eq. (7.9) far exceed the wind moments at every storey of building A.

9.3 Shear, Moment, and Torque in Core

The simplified method of Ref. 60 was used to calculate the moments and shears caused by a basic wind pressure of 8.5 psf at each storey of buildings A and B. The results are presented in Tables 9.3 to 9.5. The tabulated values have been calculated for a wind applied perpendicular to the short face of the buildings. The wind loads (windward and leeward components combined) and resulting shears and moments in the orthogonal direction can be obtained by factoring the tabulated values by the appropriate building length-to-width ratio given below the tables.

A comparison of the shears given in column 4 of Tables 9.3 and 9.5 with those given in column 5 of Tables 8.2 and 8.4 reveals that the column out-of-plumbs create shears in the cores of buildings A and B which do not exceed 4.5 and 3.3 percent of the wind shears (depending on the direction of the wind).

The out-of-plumb moments given in column 6 of Table 8.2 are compared in Fig. 9.2 with the corresponding wind moments for building A. Although still small, the fraction of moment due to column out-of-plumbs is relatively larger in the upper section of the structure than at the base. This [illegible] part due to the gravity-to-wind load ratio but is largely a reflection of the fact that wind

1	2	3	4	5
Storey No.	Storey Height (Ft.)	Wind Load (Short Span) (kips)	Shear (Kips)	Moment (Ft.-K.)
27	20	35.0	35.0	0.0
26	12	56.0	91.0	700
25	12	42.0	133.0	1792
24	12	42.0	175.0	3388
23	12	42.0	217.0	5488
22	12	39.4	256.4	8092
21	12	39.4	295.8	11169
20	12	39.4	335.2	147[illegible]
19	12	39.4	374.6	187[illegible]
18	12	39.4	414.0	232[illegible]
17	12	39.4	453.4	2820[illegible]
16	12	39.4	492.8	33645
15	12	36.8	529.6	39558
14	12	36.8	566.4	45914
13	12	36.8	603.2	52710
12	12	36.8	640.0	59949
11	12	36.8	676.8	67629
10	12	34.1	710.9	75750
9	12	34.1	745.0	84281
8	12	34.1	779.1	93221
7	12	31.5	810.6	102570
6	12	31.5	842.1	112298
5	12	28.9	871.0	122403
4	12	28.9	899.9	132855
3	12	26.3	926.2	143654
2	12	26.3	952.5	154768
1	20	35.0	987.5	166198
				185948

Wind pressure, $q(\frac{1}{30})$ = 8.5 psf.

Building dimensions: Long span, 147' Short span, 99' Ratio = 1.485

TABLE 9.3 SHEAR AND MOMENT DUE TO WIND IN CORE OF (27-STOREY) BUILDING A

1	2	3	4	5
Storey No.	Storey Height (Ft.)	Wind Load (Short Span) (kips)	Shear (kips)	Moment (Ft.-K.)
22	12	19.7	19.7	0.0
21	12	39.4	59.1	236
20	12	39.4	98.5	946
19	12	39.4	137.9	2128
18	12	39.4	177.3	3782
17	[illegible]	39.4	216.7	5910
16	12	39.4	256.1	8510
15	12	36.8	292.9	11584
14	12	36.8	329.7	15098
13	12	36.8	366.5	19055
12	12	36.8	403.3	23453
11	12	36.8	440.1	28292
10	12	34.1	474.2	33574
9	12	34.1	508.3	39264
8	12	34.1	542.4	45364
7	12	31.5	573.9	51872
6	12	31.5	605.4	58759
5	12	28.9	634.3	66024
4	12	28.9	663.2	73636
3	12	26.3	689.5	81594
2	12	26.3	715.8	90184
1	20	35.0	750.8	98773
				113789

Wind pressure, $q\ (\frac{1}{30}) = 8.5$ psf

Building Dimensions:

- Long span, 147' Ratio = 1.485
- Short span, 99'

TABLE 9.4 SHEAR AND MOMENT DUE TO WIND IN CORE OF (22-STOREY) BUILDING A

1	2	3	4	5
Storey No.	Storey Height (Ft.)	Wind Load (Short Span) (kips)	Shear (kips)	Moment (Ft.-K.)
34	24	54.4	54.4	0.0
33	12	81.6	136.0	1306
32	12	54.4	190.4	2938
31	12	48.4	238.8	5222
30	12	48.4	287.2	8088
29	12	48.4	335.6	11534
28	12	48.4	384.0	15562
27	12	48.4	432.4	20170
26	12	48.4	480.8	25358
25	12	48.4	529.2	31128
24	12	48.4	577.6	37478
23	12	48.4	626.0	44410
22	12	48.4	674.4	51922
21	12	48.4	722.8	60014
20	12	48.4	771.2	68688
19	12	45.3	816.5	77942
18	12	45.3	861.8	87740
17	12	45.3	907.1	98082
16	12	45.3	952.4	108967
15	12	45.3	997.7	120396
14	12	45.3	1043.0	132368
13	12	45.3	1088.3	144884
12	12	42.3	1130.6	157944
11	12	42.3	1172.9	171511
10	12	42.3	1215.2	185586
9	12	42.3	1257.5	200168
8	12	42.3	1299.8	215258
7	12	39.3	1339.1	230856
6	12	39.3	1378.4	246925
5	24	59.0	1437.4	263466
4	27	77.1	1514.5	297964
3	16	59.6	1574.1	338855
2	15	39.1	1613.2	364041
1	15	37.8	1651.0	388239
				413004

Wind pressure, $q(\frac{1}{30}) = 8.5$ psf.

Building Dimensions : Long Span, 152' Ratio = 1.333
Short Span, 114'

TABLE 9.5 SHEAR AND MOMENT DUE TO WIND IN CORE OF BUILDING B

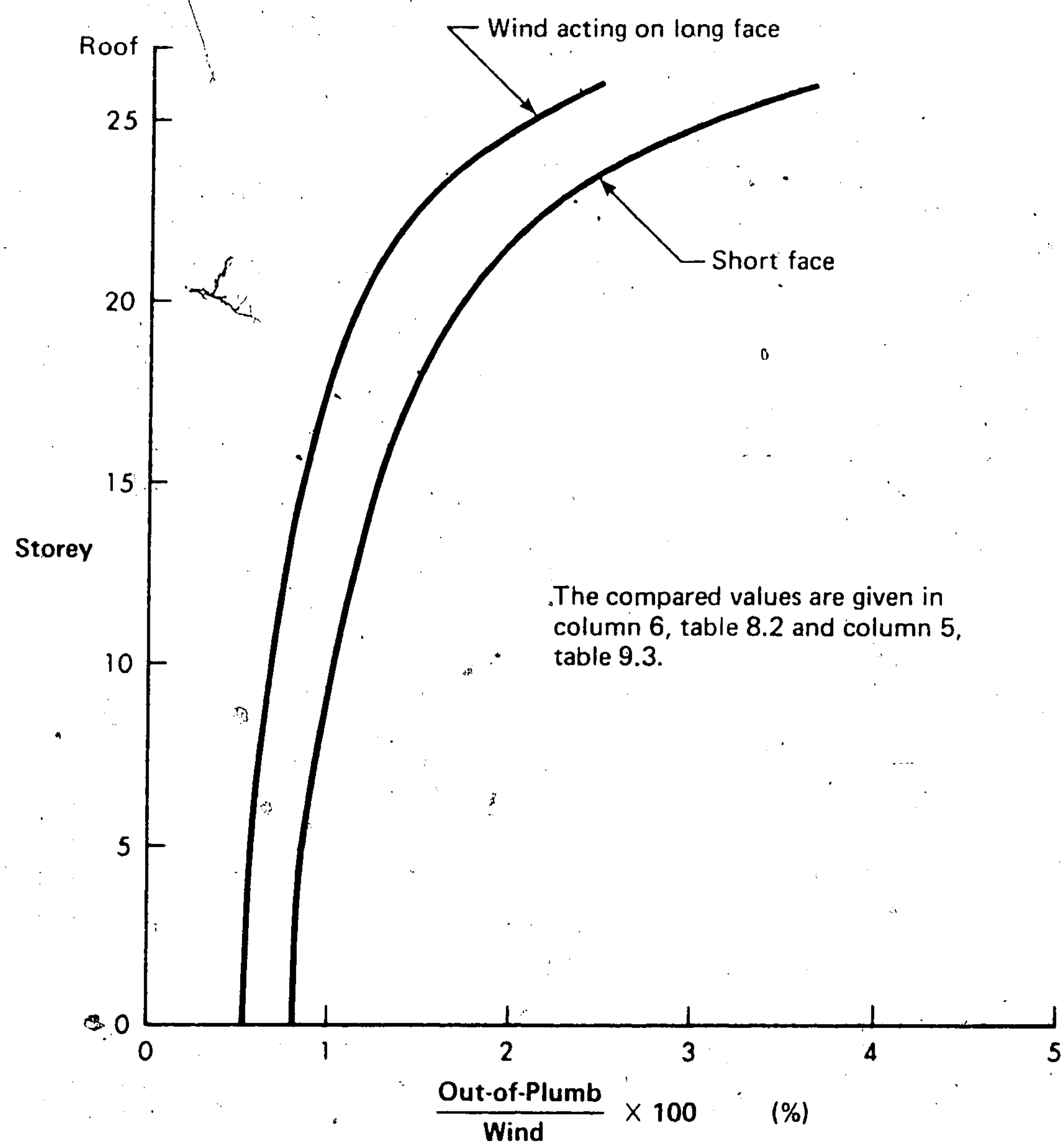


Figure 9.2 **Comparision of moments caused by column out-of-plumbs and moments due to wind in building A**

moments are added algebraically from storey to storey while out-of-plumb moments are combined statistically according to Eq. (7.13).

A similar comparison is presented in Fig. 9.3 for the moments caused by the wall out-of-plumbs of building A. The figure indicates that the out-of-plumb moments can be proportionally quite large in the upper storeys of a tall building. Fortunately, these results are not significant. A core is designed for the forces at its base and in general for the forces at one or two other locations along the core. Therefore, the forces present in the upper section of the core are resisted by a stiffer and stronger core than required.

The actual buildings, A and B, had cores formed of eight and nine orthogonal walls respectively. The moments, torques, and lateral deflections calculated for only four walls are consequently larger than in the real case. Assuming that the actual eight walls of building A carry an equal share of the total vertical load, it can be shown that the moments predicted by Eq. (7.26) are reduced by 55 percent. Thus, in the case of the wind acting on the short face of the building as shown in Fig. 9.3, the moment is reduced from 39 to about 19 percent at the top of the building and from 6.5 to about 3.5 percent at the base.

The results presented in Fig. 9.4 show that the moments in the core due to column and wall out-of-plumbs are negligible in the case of building B. A reduction of 55 percent also exists when the calculations are based on the actual nine walls.

There are no recommendations related to wind in the Canadian National Building Code[60] which allow a calibration of the torques

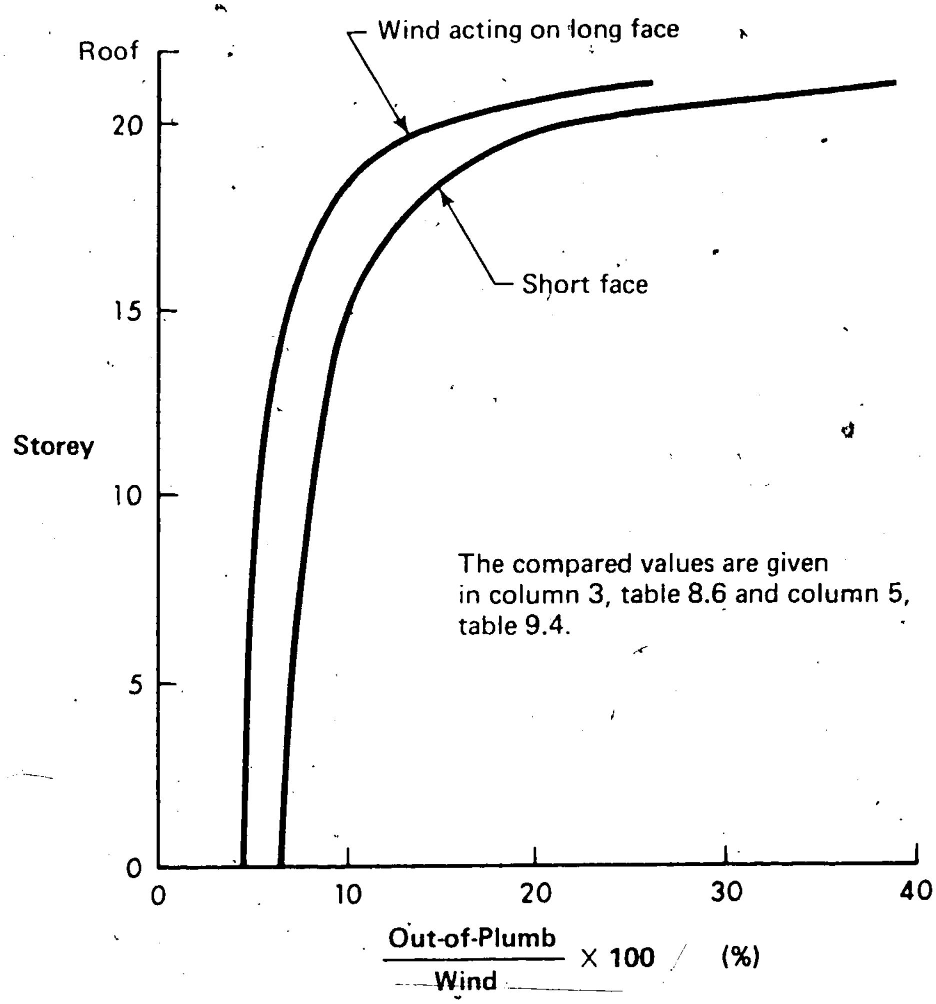


Figure 9.3 **Comparison of moments caused by wall out-of-plumbs and moments due to wind in building A**

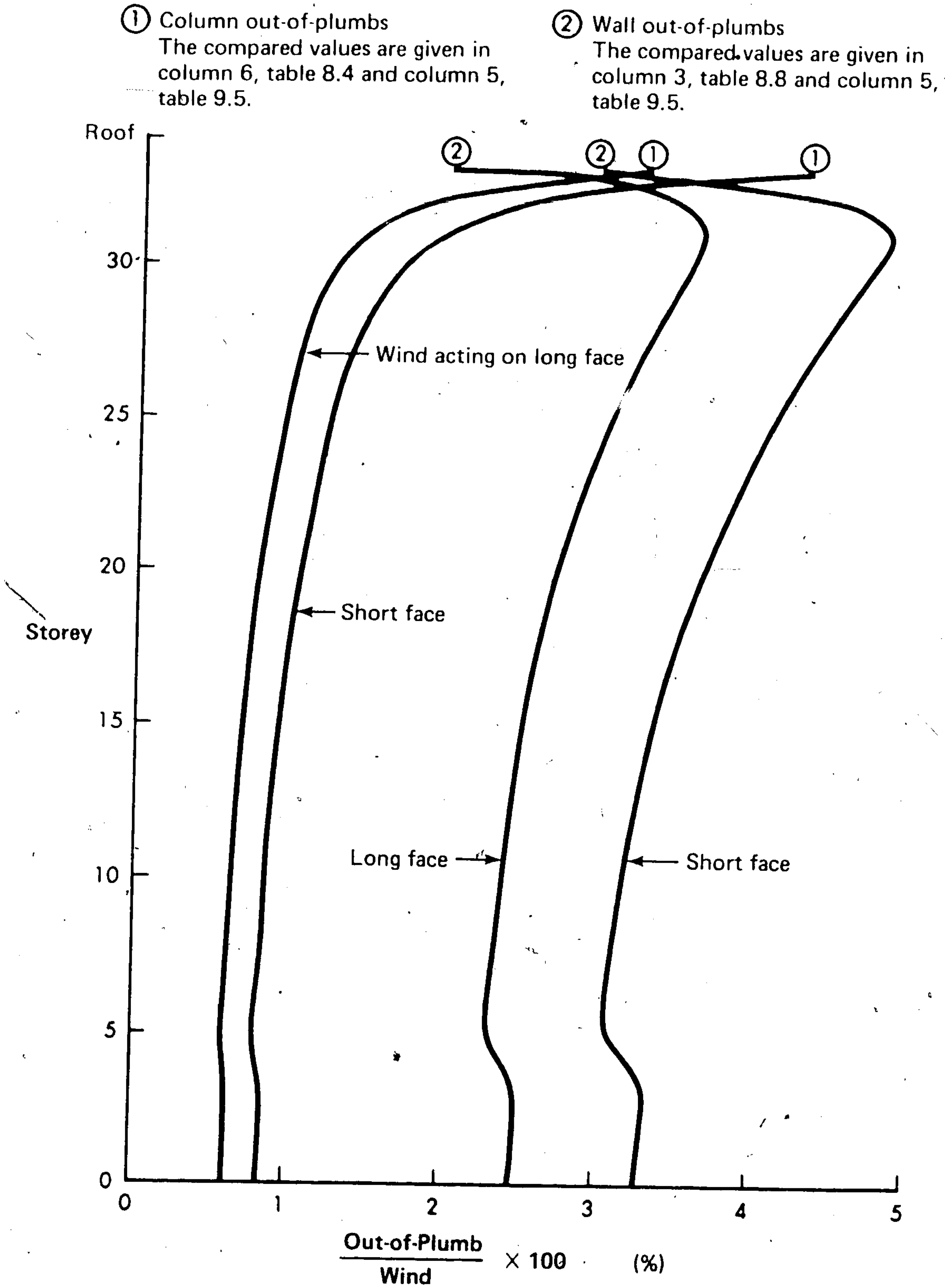


Figure 9.4 Comparision of moments caused by column and wall out-of-plumbs and moments due to wind in building B

given in column 7 of Tables 8.2 and 8.4 and in columns 5 and 6 of Tables 8.6 and 8.8. However, the simple static analysis described in the commentary on the effects of earthquakes in Supplement No. 4 to the National Building Code of Canada can be used to calibrate the out-of-plumb torques. The total lateral forces for an earthquake occurring in a seismic zone 2 are calculated as 1032 kips and 1560 kips, respectively, for buildings A and B. The estimated total weights of both buildings are approximately 61450 and 100000 kips. The calculated lateral forces acting through the design eccentricity recommended by the code(60) produce torsional moments in the order of 7585 and 28236 ft.-k., respectively, at the base of the buildings.

The code recommends a design eccentricity equal to 1.5 times the distance between the calculated center of mass and the center of resistance of the structure, plus an accidental eccentricity equal to 0.05 times the plan dimension in the direction of the computed eccentricity. By reason of its symmetry, building A has an accidental eccentricity equal to 7.35 ft. The calculated eccentricity for building B is 7.0 ft. and the accidental eccentricity is equal to 7.6 ft.

The torques created by the out-of-plumbs in these buildings account for 33 and 38 percent of the accidental torque in the case of the columns and 27 and 17 percent, respectively, in the case of the walls. When the calculated eccentricity is also accounted for in building B, the proportions are reduced to 16 percent for the columns and 7 percent for the walls. The proportions related to the walls in both buildings are further reduced by a factor of 2.5 of the calculations based on the actual number of walls forming the cores. The out-of-plumbs could therefore induce torsional effects that are not necessarily negligible.

9.4 Lateral Deflections

It has been shown in sections 5.1 and 5.4 that the most common technique in an overall stability analysis for initial out-of-plumbs consists in assuming that columns and walls all lean in the same direction, as in Fig. 5.1(a). The standards described in section 5.4, which refer to that model, were applied to building A and compared with the results of Eq. (7.16) and (7.18) in Fig. 9.5. It was shown in section 8.1.5 that curves 1 and 2 in Fig. 9.5 were upper bounds on the actual lateral deflections for this structure. In fact, the actual deflection curves obtained from the measured column out-of-plumbs in the x and y directions were shown in Fig. 8.12 to be well within these limits. By superimposing Fig. 8.12 on Fig. 9.5, the conservative nature of the various code recommendations is evident.

The West German expression (No. 5)[47], which is a function of the building height, gives the closest estimate. The lateral deflections given by curves 4 and 6 are about 13 times larger than the limit given by Eq. (7.18) at the top of the building. Curve No. 6, suggested by the Swedish Concrete Regulations (B7-1968)[46], has been obtained for 6 columns with a slope of 0.007 Rad. and 10 columns with a slope of 0.0035 Rad., resulting in a slope of 0.0048 Rad. at each storey.

By extending the study, it can be ascertained whether some of these recommendations still have the same relationship when applied to structures of different heights. Figs. 9.6 and 9.7 present curves obtained for a 10-storey building (E.3) and a 5-storey building (E.1) respectively. It is observed that the different code requirements are

$$① \quad \Delta_d/h = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \Big/ \sum_{j=1}^{n} P_j \quad \text{......... (Eq. 7.16)}$$

$$② \quad \Delta_d/h = \beta\sigma_c \Big/ \sqrt[2.2]{n} \quad \text{.................. (Eq. 7.18)}$$

$$③ \quad \Delta_d/h = 0.002 \quad \text{..... [CSA-S16.1] ... (Section 5.4.2)}$$

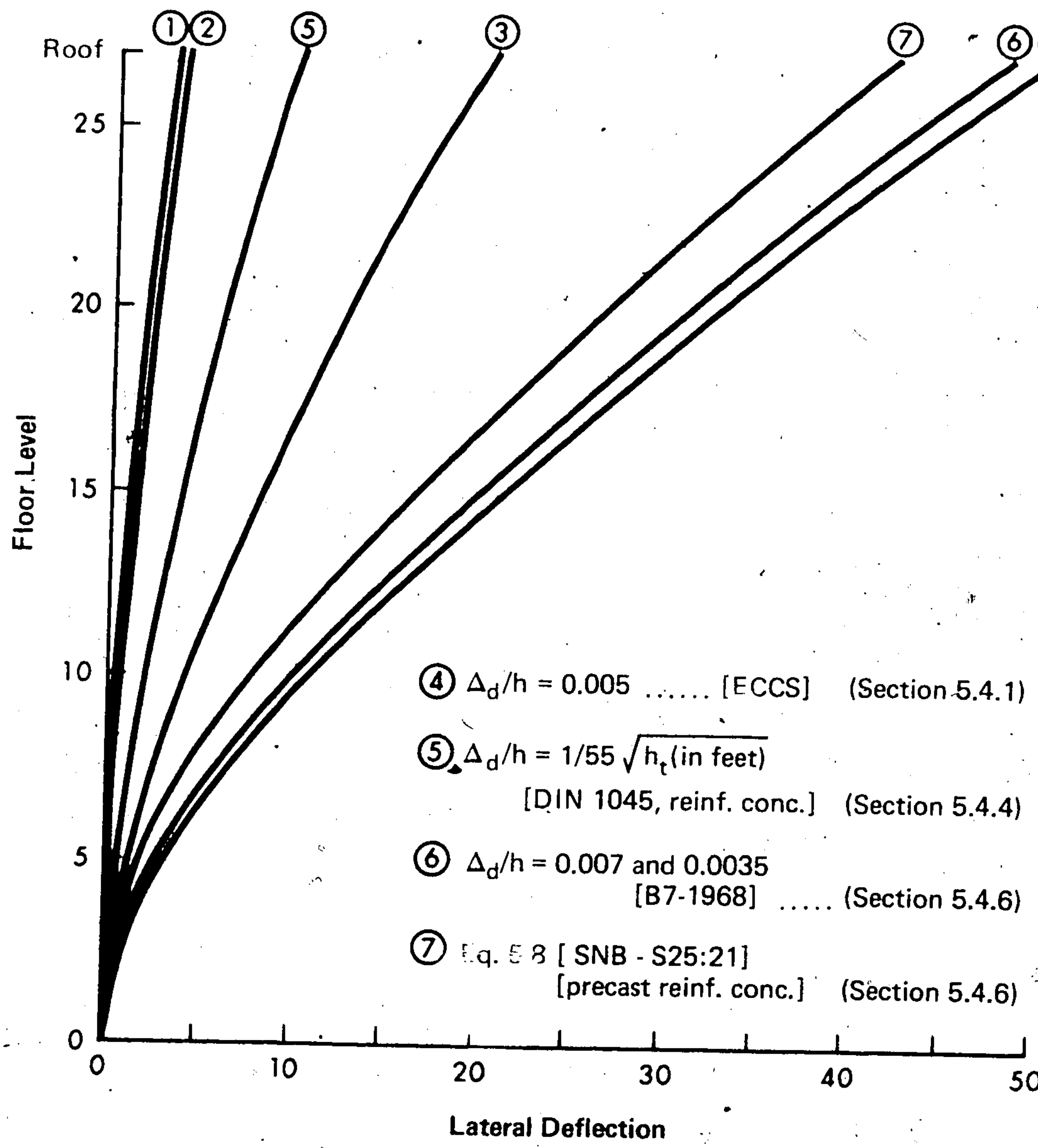


Figure 9.5 **Comparison of lateral deflections derived from different code specifications for building A**

① $\Delta_d/h = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \Big/ \sum_{j=1}^{n} P_j$ .... (Eq. 7.16)

② $\Delta_d/h = \beta\sigma_c \Big/ \sqrt[2.2]{n}$ .... (Eq. 7.18)

③ $\Delta_d/h = 0.002$ .... [CSA-S16.1] .... (Section 5.4.2)

④ $\Delta_d/h = 0.005$ .... [ECCS] .... (Section 5.4.1)

⑤ $\Delta_d/h = 1/55 \sqrt{h_t \text{ (in feet)}}$

[DIN 1045, reinf. concrete] (Section 5.4.4)

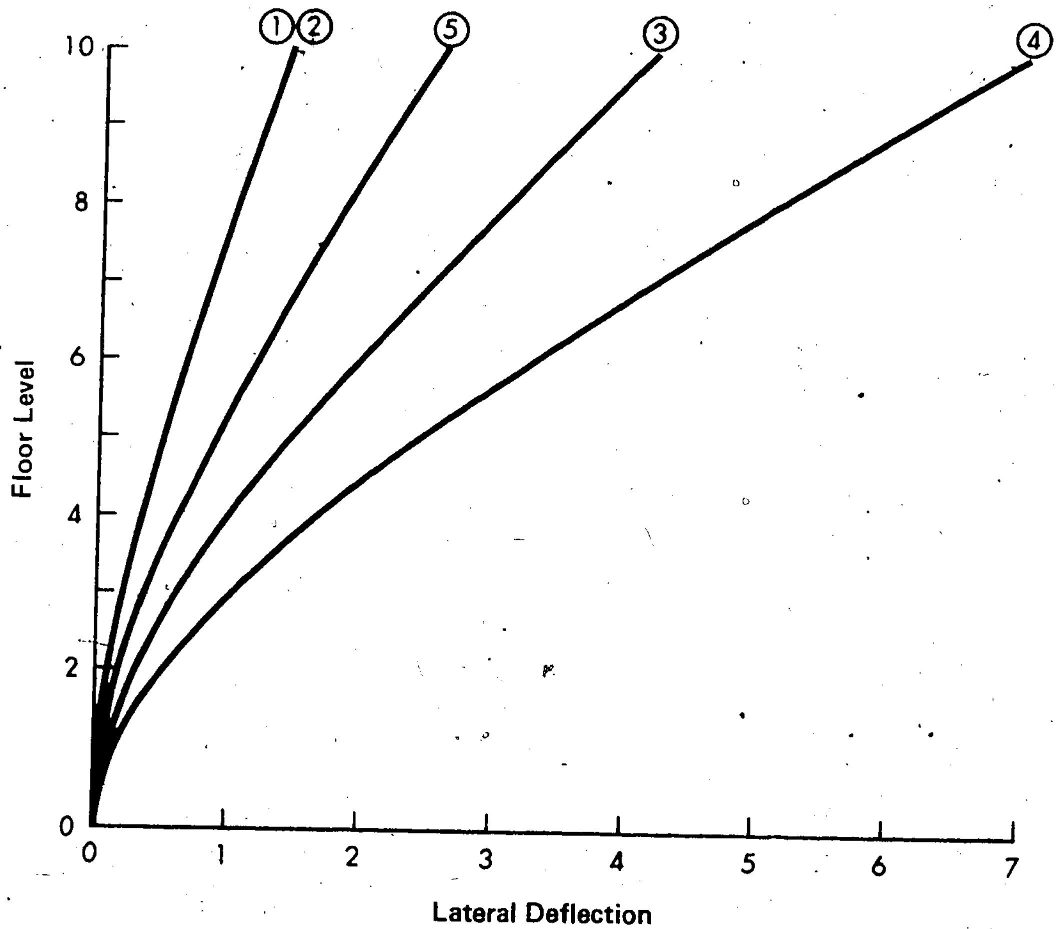


Figure 9.6 Comparison of lateral deflections derived from different code specifications for building E.3

① $\Delta_d/h = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \Big/ \sum_{j=1}^{n} P_j$ .... (Eq. 7.16)

② $\Delta_d/h = \beta\sigma_c \Big/ \sqrt[2.2]{n}$ .... (Eq. 7.18)

③ $\Delta_d/h = 0.002$ .... [CSA - S16.1] .... (Section 5.4.2)

④ $\Delta_d/h = 0.005$ .... [ECCS] .... (Section 5.4.1)

⑤ $\Delta_d/h = 1/55\sqrt{h_t \text{ (in feet)}}$ [DIN 1045, reinf. concrete] (Section 5.4.4)

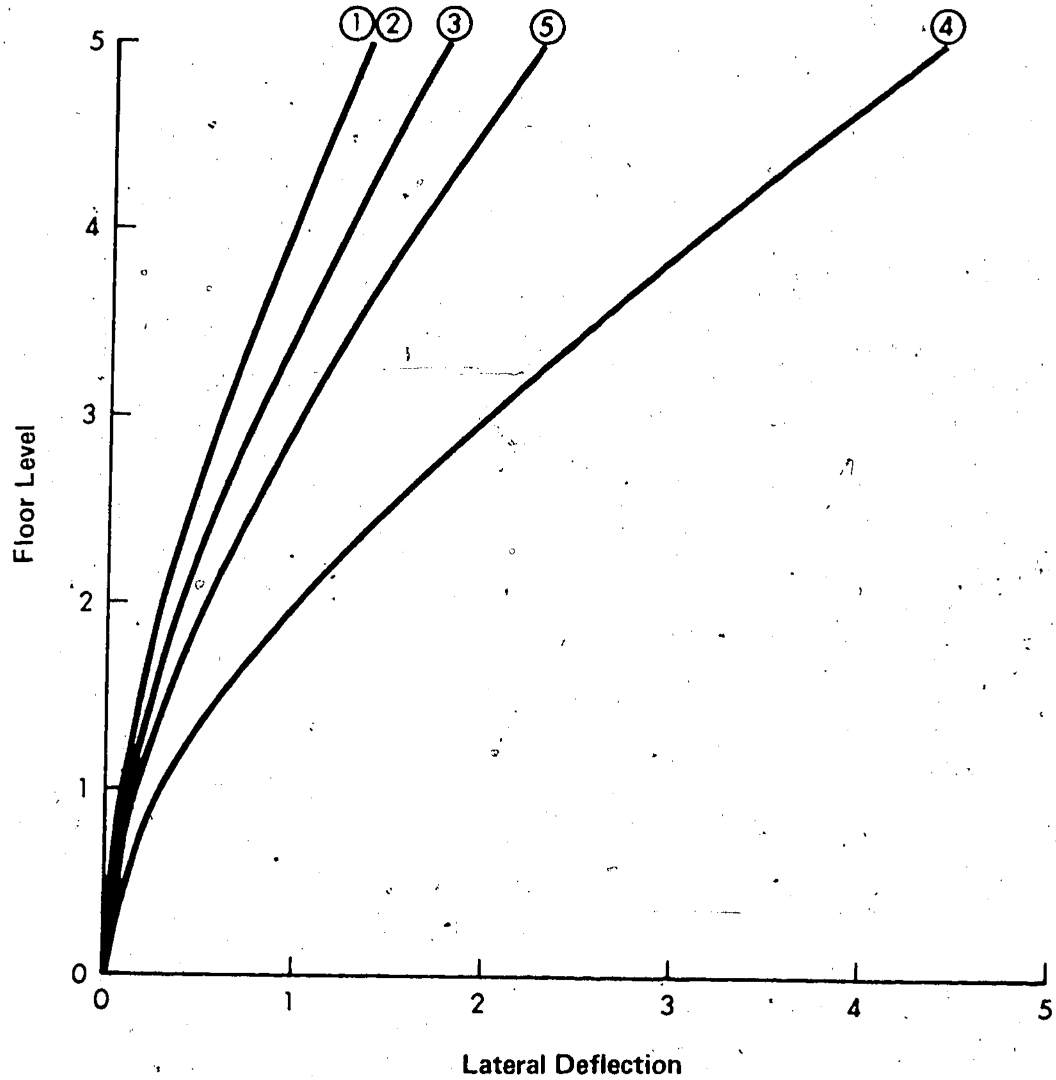


Figure 9.7 Comparison of lateral deflections derived from different code specifications for building E.1

more appropriate to low structures in the sense that the disparities between their predictions and the results of Eq. (7.18) are significantly reduced. The West German recommendation[47] in general estimates the deflections reasonably well while the European recommendation[41] still remains excessively conservative. Any comparison between the different code recommendations, however, must be carefully interpreted for the reasons given in section 5.1.

For the buildings under study, the deflections predicted by Eqs. (7.18) and (7.37) account for a small percentage of the deflections due to wind, as shown in Table 9.6. The column out-of-plumbs have apparently no significant effect on the overall stability of the structures. The sway induced by the wall out-of-plumbs accounts for less than 5 percent of the sway due to wind when the actual number of walls is used in the calculations. Although the lateral deflections are small for these buildings of 20 storeys and over, it cannot immediately be concluded that the overall stability of other buildings is not affected by out-of-plumbs. The gravity-to-wind load ratio of a building, the type of building, and the number of columns and/or walls present in the building are all significant factors to be taken into account.

An example is given in Fig. 9.8 where a one-storey braced structure is analyzed for t[illegible]ferent combinations of columns and walls. Eqs. (7.16) and (7.[illegible]6) are used to calculate the horizontal forces due to out-of-plumbs [illegible] the given axial loads. The forces H_c and H_w obtained for column and wall out-of-plumbs, respectively, are combined according to Eq. (7.38). Since the deflections are directly related to the applied lateral loads in this case, the ratios given in Fig. 9.8 are a measure of the sway induced by out-of-plumbs to the sway

Column Out-of-Plumbs:

Building	No. of Columns n	Δ_d/h* (x 10^4 Rad.)	$\frac{\text{Out-of-Plumb}}{\text{Wind}}$ x 100 (%)
A	458	3.67	1.0
B	880	2.73	1.0

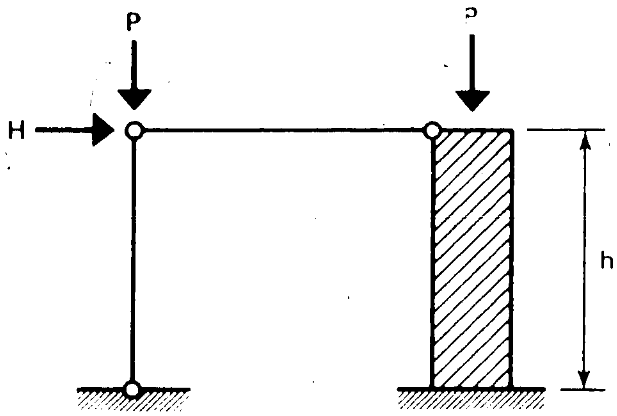
* $\Delta_d/h = 3.5 \times .0017/\sqrt[2.2]{n}$ (Eq. 7.18)

Wall Out-of-Plumbs:

Building	No. of Walls n	Δ_d/h** (x 10^3 Rad.)	$\frac{\text{Out-of-Plumb}}{\text{Wind}}$ x 100 (%)
A	(4 walls/storey) 88	1.56	6.0
	(8 walls/storey) 176	1.20	4.6
B	(4 walls/storey) 136	1.33	3.5
	(8 walls/storey) 306	1.00	2.6

** $\Delta_d/h = 0.00028 + 3.5 \times .0028/\sqrt[2.2]{n}$ (Eq. 7.37)

TABLE 9.6 LATERAL DEFLECTIONS CAUSED BY OUT-OF-PLUMBS AS A PERCENTAGE OF DEFLECTIONS DUE TO WIND



1 Column:

$$H_c = \frac{P\,\Delta_d}{h} = 80\left[3.5 \times 0.0017\,\frac{\sqrt{80^2}}{80}\right] = 0.48\text{ k}$$

1 Wall:

$$H_w = \frac{P\,\Delta_0}{h} = 80\left[0.00028 + 3.5 \times 0.0028\,\frac{\sqrt{80^2}}{80}\right] = 0.8\text{ k}$$

$$\frac{\sqrt{H_c^2 + H_w^2}}{H} = 0.052$$

4 Columns:

$$H_c = \frac{\Sigma\,P\,\Delta_0}{h} = 80\left[3.5 \times 0.0017\,\frac{\sqrt{4 \times 20^2}}{80}\right] = 0.24\text{ k}$$

2 Walls:

$$H_w = \frac{\Sigma\,P\,\Delta_0}{h} = 80\left[0.00028 + 3.5 \times 0.0028\frac{\sqrt{2 \times 40^2}}{80}\right] = 0.58\text{ k}$$

$$\frac{\sqrt{H_c^2 + H_w^2}}{H} = 0.035$$

Figure 9.8 Example—proportion of sway

caused by the lateral loads. The 5.2 percent out-of-plumb sway obtained for the one-column, one-wall structure is reduced to 3.5 percent for the four-column, two-wall structure carrying the same total load.

CHAPTER X

DISCUSSION

The previous chapters have been devoted to the presentation and discussion of aspects of structural stability related to P-Δ and out-of-plumb effects. The general discussion presented in this chapter should establish a logical link between these different sections and emphasize the major characteristics of the study.

10.1 P-Δ Effects

The P-Δ effects were briefly discussed in the first sections of the thesis with emphasis placed on the creation and transfer of additional horizontal forces in structures. Practical techniques for including second order effects in analysis were presented. It was demonstrated that when an approximate second order analysis is performed, the horizontal forces in the structure are not distributed in a proper manner. Generally, the errors in these forces do not affect the design of structural members but do affect the design of connections and floor diaphragms when the second order effects are important. Such a situation may occur in a pin-connected frame which relies on a separate lateral support system for stability. The transfer of horizontal forces in this type of structure is more critical than in other types of structures.

As a general rule, the correct horizontal force distribution should be evaluated at specific storeys in any type of structure when the storey shears given in Fig. 3.3 are significant compared to the applied wind forces. A redistribution of the forces based on the free-body diagram method described in section 4.3 is then recommended, since the concept of the free-body diagram is familiar to most designers. When the sway forces are not significant, the horizontal forces given by a first order analysis can be used in the design of the connections and floor systems.

10.2 Out-of-Plumb Effects

The study of the effects of out-of-plumbs on the stability of structures was primarily based on core-braced buildings in which a pin-connected steel frame is supported by a central reinforced concrete core. Although the transfer of the horizontal forces is more critical in this type of structure, the results obtained on core-braced buildings can be extrapolated to other types of structures including moment resisting frames. The sway of a continuous structure creates extra moments in the members and the additional forces produced are reduced to a minimum. The out-of-plumb effects in moment resisting frames are not investigated as such in this thesis.

Statistical methods are essential to describe the nature of the out-of-plumb forces. A certain probability of occurrence can then be associated with the selected factor of safety. A safety index of 3.5, corresponding to a probability of failure of 4.6×10^{-4}, has been adopted, based on discussions presented in section 7.1 and in Appendices B and C. The adequacy of the selected safety index and the applicability of the equations derived in Chapter VII have been confirmed in Chapter VIII.

The out-of-plumb measurements taken as a part of this research project and presented in Chapter VI compare quite well with the published measurements listed in section 6.1. The statistical populations obtained are normally distributed, resulting in greatly simplified calculations. The calculated means are generally small enough to be neglected when the sample dimensions are sufficiently large to present a realistic estimate of the distribution characteristics. Unfortunately, the standard deviations obtained in this thesis are specific to the building measured. They are apparently typical for steel columns and cast-in situ reinforced concrete walls, as observed by comparing the results of the three different buildings A, B, and C. The standard deviation for the reinforced concrete walls is almost double that measured for the steel columns, as shown in Table 7.4.

Data from other structures are required to estimate the effects on the standard deviation of variables like:

- the structural material,
- the type of structure,
- the erection and plumbing techniques,
- the skill and experience of the constructor.

The population mean may also be large in some cases and the possibility of systematic variations due to erection techniques or errors caused by the use of a faulty instrument, for instance, should be evaluated. While the mean of the column population is almost zero, the correspondin[illegible] for the walls is not negligible.

10.2.1 Connection and Flo[illegible]gn in Braced Buildings

It was demonstr[illegible]ections 9.1 and 9.2 that the action of column out-of-plumbs in a [illegible]aced building generally controls the design of beam-to-column connections and of floor diaphragms for horizontal shears and moments. The forces and moments given by Eqs. (7.7) and (7.9) should be computed for the loading cases of Ref. 36, using the appropriate load combination factors. In tall buildings with uniformly varying gravity loads, the shears and moments can be calculated at specific levels and the remaining values obtained by interpolation.

The model shown in Fig. 5.1(b) was incorrect for the assessment of forces in the plane of the floors. With a suitable uniform slope equal to $\beta\sigma_c$, as given by Eq. (7.2), the model would produce an upper bound on the forces, which would be equivalent to totally neglecting the random nature of column out-of-plumbs (See Appendix C). The application shown in Fig. 9.1(b) provides a graphical representation of the problem. The straight line is the result of the algebraic summation of the individual column forces suggested by the model of Fig. 5.1(b). The curve, obtained from a statistical summation, represents more exactly the actual forces in the structure. In this particular example, since $\Delta_d/h = 0.002$ is lower than $\beta\sigma_c = 0.006$, the actual forces are underestimated by the simple model for frames having fewer than 10 columns. Beyond this limit, a significant reduction takes place.

Each beam-to-column connection in a moment resisting frame can be conservatively designed for an extra horizontal force equal to 0.85 percent of the largest axial load in the two columns above and below the floor (see Eq. 7.5). This assumes that no significant transfer of force exists in the structure and that a minimum bracing force is required to stabilize the columns. A slight reduction of this value might be expected in future from an appropriate study on the effects of column out-of-plumbs in continuous frames.

The basic equation used in Russia for the calculation of out-of-plumb horizontal forces follows the pattern observed in the present study[50]. Eq. (5.4) should be compared to Eq. (7.18), considering that ε_1 is equal to three times the standard deviation obtained from measurements taken on concrete structures. The design value $\varepsilon_1 = 0.012$, defined as the total change in slope between two columns (at their intersection), is twice as large as $\beta\sigma_c = 0.006$ for one column in this thesis and has been described as too large in Ref. 48.

The need for a variable safety index, which has been observed in the study presented in Fig. 5.4, is most likely to compensate for the neglect of the variable axial loads P_J combined as in Eq. (7.16) or (7.7). It has been shown in Appendix D that an expression of the form of Eq. (7.18) underestimates the "exact" force given by Eq. (7.16) for a small number of columns. The Russians observed this trend in their study and compensated by imposing a larger factor of safety.

The Swedish Building Regulations[51] described in section 5.4.6 also present the results of a comprehensive statistical approach. The regulations are, in some respects, in good agreement with the findings of this thesis. However, the force in a connection, given by Eq. (5.6), is equal to about 3.5 percent of the average axial load in the load bearing elements; which is large when compared to the 0.84 percent found in this study.

10.2.2 Core Design

Shears and moments in a core from both column and wall out-of-plumbs can be neglected, according to the calculations presented in Chapter IX. However, as shown in section 9.3, the torques account for a significant percentage of the minimum torsional moments prescribed by the Canadian National Building Code[60]. The empirical minimum eccentricity prescribed by the code has been intended to account for possible additional torsion arising from various sources listed in Supplement No. 4 to the Canadian National Building Code. It is not mentioned, however, that the minimum eccentricity is also intended to account for the possibility of a torque caused by out-of-plumbs but there is no apparent reason why it should not.

In view of the significance of the out-of-plumb torques it might be more reasonable to include the torsional moments given by Eqs. (7.15) and (7.31) specifically. In a three-dimensional analysis of a structure under earthquake loading, the out-of-plumb torques would be added to the prescribed calculated and accidental torsional moments. In an analysis for wind loads, the out-of-plumb torques would be considered alone since, in this case, there is no provision for a minimum eccentricity of load application. In mixed construction (core-braced building), the torques would be evaluated at specific storeys for the factored axial loads and combined according to the expression $\sqrt{T_c^2 + T_w^2}$ to account for their random nature.

It is common practice, however, to neglect the three-dimensional effects in most buildings, based on a recognition of the fact that the torques are comparatively small and that a structure is generally much stiffer and stronger in torsion than necessary.

10.2.3 Overall Stability

When the overall stability of a structure under combined lateral and gravity loads is to be assessed, the extra lateral deflections caused by column and wall out-of-plumbs can be disregarded. The deflections caused by the out-of-plumbs account for a very small percentage of the deflections due to wind, as demonstrated in section 9.4 (Table 9.6).

The uniform slopes given by Eqs. (7.18) and (7.37) are suitable for the assessment of the overall stability of a structure when the loading case considered is associated with the vertical loads acting alone. The presence of initial imperfections in a structure gives rise to an initial sway of the structure. Lateral forces are computed and applied to the structure to produce additional deflections and corresponding P-Δ shears and forces. The concept has been developed in Ref. 23 and has been adapted for the Canadian Standards S16.1[36] for a constant slope of 0.002. A discussion of the standard is given in section 5.4.2.

In a core-braced building or an equivalent composite structure, the forces obtained from Eqs. (7.18) and (7.37) should be combined together according to the expression $\sqrt{H_c^2 + H_w^2}$ to account for their random nature. As before, the forces can be evaluated at specific floors and the intermediate values obtained by interpolation. The factored axial loads from the gravity load case are used in the equations[36].

The fictitious horizontal load principle discussed in section 3.2 and applied to a cantilevered member in section 7.2.3 can be used to justify the application of Eq. (7.18) to moment resisting frames.

The extra moments created in the structure in this manner resist the sway induced by column out-of-plumbs.

Taking for granted that Eqs. (7.16) and (7.18) provide an upper bound on the lateral deflections, the different code recommendations described in section 5.4 were shown to be conservative in Figs. 9.5 to 9.7. The study demonstrated that a fixed slope, such as 0.002 or 0.005, is not an appropriate model for general application. The West German expression given by equation No. 5 in Fig. 9.5 provided a close estimate in every case. The variable h_t suggested by the Germans is in some ways equivalent to the variable n in Eq. (7.18).

The prescription of the Swedish Building Code for reinforced concrete[51], Eq. (5.8), conforms in principle to the views of the present study. However, the expression assumes that 20 percent of the maximum inclination obtained from field measurements is a systematic variation and 80 percent is random. The maximum inclination of 0.015 used in the equation seems slightly high, as demonstrated in Fig. 9.5. The reduction factor γ, which accounts for the tolerance requirements and the degree of control, has no real significance, in view of the discussion presented in section 5.3.

The entire provision for out-of-plumbs summarized in Fig. 5.5 is, in general, acceptable. The most curious statement is that the three types of forces given by Eqs. (5.6) to (5.8) cannot be combined.

10.3 Concluding Remarks

The stability of a structure cannot be properly ensured unless all the major destabilizing forces in the structure are properly resisted. For this reason, P-Δ as well as out-of-plumb forces should be given consideration in design.

The statistical method presented herein for steel frames and cast-in situ reinforced concrete walls can also be applied to cast-in situ or precast concrete frames,as long as the characteristics of the respective member out-of-plumb populations are known or estimated. A similar type of approach can be adopted for the evaluation of forces in the bracing members which provide lateral support to the compression flange of initially crooked beams and girders. An assembly of bracing members and beams in a horizontal plane can be treated as an assembly of beams and out-of-plumb columns in the vertical. The required statistical characteristics would be obtained from a survey of beam deviations in buildings under construction.

CHAPTER XI

SUMMARY AND CONCLUSIONS

11.1 Summary

Different forces which are likely to affect the strength and stability of buildings and their components were investigated in this thesis. The forces were classified in three categories: the first order forces, the P-Δ forces, and the forces due to initial out-of-plumbs.

The P-Δ forces were discussed briefly and approximate methods for their determination were presented.

The thesis essentially concentrated on the investigation of out-of-plumb effects and the development of suitable design procedures. Statistical methods provided an appropriate means of defining the problems of stability and strength related to structural out-of-plumbs.

Measurements were made on steel columns and concrete walls in two tall core-braced buildings and one large industrial building under construction to determine actual characteristics of out-of-plumbs for use in the statistical calculations.

Equations were derived for the design of connections, floor diaphragms, and vertical bracing systems affected by the out-of-plumb forces and methods were suggested for the evaluation of the building sway movements.

Comparisons with corresponding first order effects have demonstrated that while some out-of-plumb effects are negligible, others may be very significant. Moreover, when compared to the results based on present standards, the effects were generally found to be either under- or overestimated.

The investigation resulted ultimately in the creation of more rational clauses for design standards which are related to the stability of structures and individual members. The proposed clauses are listed in the following section.

11.2 Conclusions

The investigation is concluded by presenting the results of the research in the form of proposed clauses for design standards. The actual sections of the Canadian Standard S16.1 "Steel Structures for Buildings - Limit States Design"[36] which relate to the overall stability of structures and of individual members are rewritten in view of the present findings. Some recommendations which relate to concrete structures are presented separately for consideration for the appropriate concrete standards.

The section numbering adopted below corresponds to that of the Canadian Standard but the nomenclature used is that of this thesis.

12.2.1 Proposed Clauses for CSA-S16.1 "Steel Structures for Buildings"

8.6 Stability Effects

8. .1 The analyses referred to in Clauses 8.4 and 8.5 shall include the sway effects produced by the vertical loads acting on the structure in its displaced configuration, unless the structure is designed in accordance with the provisions of Clause 8.6.3.

For certain types of structures where the vertical loads are small, where the structure is relatively stiff and where the lateral load resisting elements are well distributed, the sway effects may not have a significant influence on the design of the structure (See clause 9.3.2(b)).

8.6.2 For structures in which the sway effects have been included in the analysis to determine the design moments and forces (see Appendix J), the effective length factors for members shall be based on the sidesway prevented condition (See clause 9.3.2(a)).

(a) Where a loading combination produces significant relative lateral displacements of the column ends, the sway effects shall include the effect of the vertical loads acting on the displaced structure but need not include the sway effects produced by initial column out-of-plumbs.

(b) However, in a steel frame the sway effects shall not be less than those produced by the vertical loads acting on the structure assumed displaced an amount equal to $0.006/\sqrt[2.2]{n}$, where n is the total number of columns in the structure (see section J-3 of Appendix J).

(c) In mixed construction, composed of steel columns and cast-in situ reinforced concrete walls arranged orthogonally, the sway effects shall not be less than those produced by the vertical loads acting on

(i) the columns assumed out-of-plumb by the amount given in clause 8.6.2(b) and

(ii) the walls assumed out-of-plumb by an amount equal to $0.0003 + 0.01/\sqrt[2.2]{n}$, where n is the total number of one-storey walls in the structure.

The storey forces obtained in this manner shall be combined according to $\sqrt{H_c^2 + H_w^2}$ and applied to the structure as in Appendix J. In this expression H_c and H_w are the forces obtained from the column and wall out-of-plumbs respectively.

8.6.3 For structures in which the sway effects have not been included in the analysis, the use of effective length factors greater than 1.0 (sidesway permitted case) for the design of columns, provides an approximate method of accounting for the sway effects in moment resisting frames (see clause 9.3.3). This provision shall not be used for structures analyzed in accordance with Clause 8.5.

19. Stability of Structures and Individual Members

19.1 General

19.1.1 In the design of a steel structure, care shall be taken to ensure that the structural system is adequate to resist the forces caused by the factored loads and to ensure that a complete structural system is provided to transfer the factored loads to the foundations, particularly when there is a dependence on walls, floors, and roofs acting as shear resisting elements or diaphragms. (See also Clause 8.6).

19.1.2 Design drawings shall indicate all load resisting elements essential to the integrity of the completed structure and shall show details necessary to ensure the effectiveness of the load resisting system. Design drawings shall also indicate the requirements for roofs and floors used as diaphragms.

19.1.3 Erection drawings shall indicate all load resisting elements essential to the integrity of the completed structure. Permanent and temporary load resisting elements essential to the integrity of the partially completed structure shall be clearly specified on the erection drawings.

19.1.4 Where the portion of the structure under consideration does not provide adequate resistance to applied lateral forces and other destabilizing forces, provision shall be made for transferring the forces to adjacent lateral load-resisting elements.

(a) Beam-to-column connections and floor diaphragms shall be designed to resist horizontal forces due to column out-of-plumbs given by

$$F_d = 0.006 \sqrt{\sum_{j=1}^{n} P_j^2}$$

where n = number of participating columns above and below floor level.

P_j = factored axial loads in the individual columns.

(b) Individual sections of floor diaphragms shall also be designed for in-plane moments given by

$$M_d = 0.006 \sqrt{\sum_{j=1}^{n} [P^2(L_x^2 + L_y^2)]_j}$$

where n and P are defined as in clause 19.1.4(a) and L_x and L_y are lever arms, taken in two orthogonal directions, from the column to the point at which the moment is calculated.

19.1.5 The structure shall be analysed to ensure that adequate resistance to torsional deformations has been provided. As a minimum, the bracing system in a structure shall be capable of resisting a torsional moment, T_c, at each storey, given by the expression of Clause 19.1.4(b), with the summation applied to the total number of columns in the storey. The torsional moment is calculated with respect to the center of resistance of the structure and shall account for the effects of the gravity loads acting on the out-of-plumbs of the columns.

(a) A steel structure shall be designed for the above torsional moment.

(b) Mixed construction, composed of steel columns and cast-in situ reinforced concrete walls arranged orthogonally shall be able to resist a torsional moment at a specific storey given by:

(i) the expression defined in Clause 19.1.5(a) as applied to the columns and,

(ii) the expression below as applied to the walls:

$$T_w = 0.0015 \sqrt{\sum_{j=1}^{n} (PL)^2_j}$$

where n = total number of walls in the storey

P = factored axial load in each individual wall

L = length of the wall.

The torques obtained in (b) shall be combined according to $\sqrt{T_c^2 + T_w^2}$ at each storey.

19.2 Stability of Columns

19.2.1 Beam-to-column connections shall have adequate strength to transfer the applied forces, the sway forces (see Appendix J), plus the forces calculated as follows:

(a) In a simple braced frame, the forces described in Clause 19.1.4(a).

(b) In a continuous frame, 0.85 percent of the largest factored axial load in the two columns above and below the floor at a specific connection.

These forces shall be computed for the loading cases of Clause 7.2.4 using the appropriate load combination factors.

APPENDIX J - Guide To Calculation Of Stability Effects

J.1 This guide provides one approach to the calculation of the additional bending moments and forces generated by the vertical loads acting through the deflected shape of the structure. By this approach, the above moments and forces are incorporated into the results of the analysis of the structure. However, due to the approximate nature of the method, the horizontal forces to be used in the design of floor diaphragms and beam-to-column connections are inexact but can be easily corrected when required.

Alternatively, a second order analysis, which formulates equilibrium on the deformed structure, may be used to include the stability effects.

J.2 Combined Loading Case

Step 1 - Apply the factored load combination to the structure (Clause 7.2.2).

Step 2 - Compute the lateral deflections at each floor level (Δ_i) by first order elastic analysis.

Step 3 - Compute the artificial storey shears V'_i due to the sway forces.

$$V'_i = \frac{\Sigma P_i}{h_i} (\Delta_{i+1} - \Delta_i)$$

Step 4 - Compute the artificial lateral loads H'_i.

$$H'_i = V'_{i-1} - V'_i$$

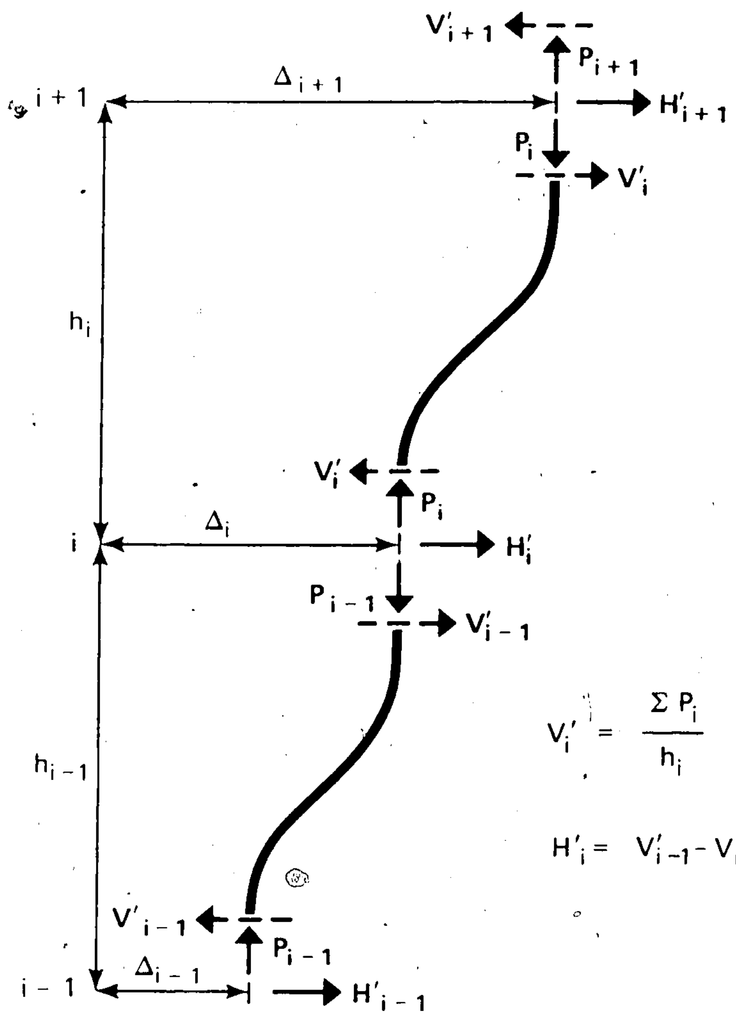
Step 5 - Repeat Step 1 applying the artificial lateral loads H'_i in addition to the factored load combination.

Step 6 - Repeat Steps 2 through 5 until satisfactory convergence is achieved. Lack of convergence within 5 cycles may indicate an excessively flexible structure. In no case shall the building sway exceed the recommended maximum values for deflections given in Appendix I.

Convergence can be achieved in one cycle by using the expression given below instead of the equation given in step 3 in the calculation of the artificial storey shears.

$$V'_i = \frac{1}{\dfrac{1}{\dfrac{\Sigma P_i(\Delta_{i+1} - \Delta_i)}{h_i}} - \dfrac{1}{\Sigma V_i}}$$

where ΣV_i = total first order shear at storey i; the other terms are defined in Figure 11.1.



$$V_i' = \frac{\Sigma P_i}{h_i} (\Delta_{i+1} - \Delta_i)$$

$$H'_i = V'_{i-1} - V'_i$$

where:

V'_i = Artificial shear at storey i due to sway forces,

ΣP_i = Sum of the column axial loads at storey i,

h_i = Height of storey i, and

Δ_{i+1}, Δ_i = Displacements of levels i + 1 and i respectively.

Figure 11.1 Sway forces due to vertical loads

J.3 Vertical Loads Only

Since vertical loads do not normally produce significant sway deflections of the structure, the initial sway forces are computed on the basis of the sway displacements in each storey produced by initial column out-of-plumbs.

$$\Delta_i = \frac{0.006\ h_i}{\sqrt[2.2]{n}}$$

where, Δ_i = net sway displacement at storey i (equivalent to $\Delta_{i+1} - \Delta_i$).

h_i = height of storey i

n = total number of columns in the building.

Using these deflections, the calculations are started at step 3 of the procedure described in J.2.

J.4 Horizontal Force Distribution

The procedure described in section J.2 produces horizontal forces slightly in error. The correct forces should be evaluated at specific storeys when the artificial storey shears, V_i', are significant compared to the applied lateral loads at these storeys. The individual column shears are calculated from equilibrium of each column with the moments, axial loads, and lateral deflections obtained from the second order analysis. The correctly distributed horizontal forces are then determined by equilibrium of these shears and applied forces at floor levels. In all other cases, the horizontal forces given by a first order analysis should be used.

12.2.2 Proposed Clauses for Concrete Buildings

(a) A structure composed exclusively of load bearing cast-in situ concrete walls should be designed for an extra sway produced by the vertical loads acting on the walls assumed out-of-plumb an amount equal to $0.0003 + 0.01/{}^{2.2}\sqrt{n}$, where n is the total number of one-storey walls in the structure.

(b) A structure composed of load bearing cast-in situ concrete walls arranged orthogonally should be able to resist a torsional moment at a specific storey given by:

$$T_w = 0.0015 \sqrt{\sum_{j=1}^{n} (PL)^2_j}$$

where, n = total number of walls in the storey

P = factored axial load in each individual wall

L = length of the wall

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APPENDIX A

PROBABILISTIC AND STATISTICAL CONCEPTS

The following appendix presents a summary of the probabilistic and statistical concepts introduced in the report. Although there are many excellent texts available to fulfill this purpose, it is thought that a condensed summary of the essential concepts will greatly facilitate comprehension of the material. For more detailed discussion with illustrations from civil engineering practice, Refs. 54 and 55 are recommended.

A-1 Probability

If an experiment is conducted N times, and a particular attribute A occurs n times, then the limit of n/N as N becomes large is defined as the probability of the event A, denoted Pr(A).

However, a more general definition is needed to cover the case in which an estimate of the outcome of an event is principally intuitive. In this case: "The probability Pr(A) is a measure of the degree of belief held in a specified proposition A". This interpretation of probability is a broader concept and includes the first definition.

A-2 Probability Rules

1. If $Pr(A)$ and $Pr(\bar{A})$ represent respectively the probabilities of the event A occurring and not occurring, then

$$Pr(\bar{A}) = 1 - Pr(A) \tag{A-1}$$

2. If A and B are two independent events, then the probability that both A and B will happen, known as the "joint probability" denoted by "and", is the product of the respective individual probabilities - that is,

$$Pr(A \text{ and } B) = Pr(AB) = Pr(A)\,Pr(B) \tag{A-2}$$

3. If A and B are two mutually exclusive events - that is $Pr(AB) = 0$ - then the probability denoted by "or" that one of these two events will take place is given by the sum of their individual probabilities:

$$Pr(A \text{ or } B) = Pr(A+B) = Pr(A) + Pr(B) \tag{A-3}$$

4. The probability of an event A is a number greater than or equal to zero but less than or equal to unity. The probability of a certain (absolute) event B is unity.

$$0 \leq Pr(A) \leq 1 \tag{A-4}$$

$$Pr(B) = 1 \tag{A-5}$$

A-3 Random Variables

A random variable is a function defined on a sample space. For example, in the toss of two dice, the sample space consists of the

36 possible pairs of outcomes. The sum or the average or even the square of the summed value for each pair of tosses is a random variable, because it is a function defined for every point in the sample space.

A sample space involving either a finite number or a countable infinity of elements is said to be "discrete". A discrete random variable is one that can take on only a countable number of values. A second type of random variable is a "continuous" variable. A continuous random variable may take on any value in one or more intervals and results from measured, rather than counted data.

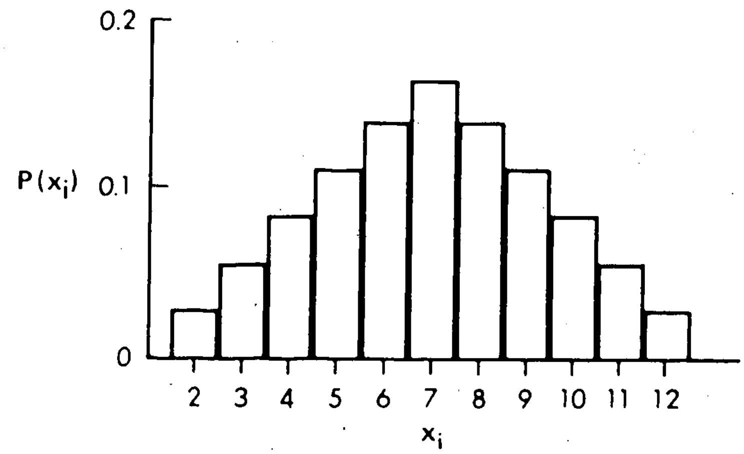
A-4 Probability Function and Cumulative Distribution

A-4.1 Discrete Random Variable

In the two dice example, the probability of each value of the random variable x representing the sum of the results of the two tosses is obtained by adding the probabilities of appropriate points in the sample space. Because each of the 36 points is equally likely and their total probability must add to 1, each point has associated with it a probability of 1/36. The "probability function", $P(x_i)$, obtained is sketched in Fig. A-1(a).

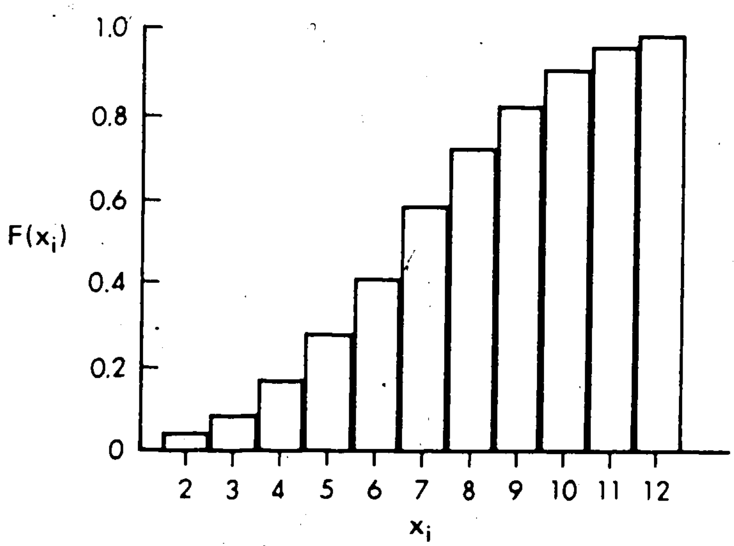
The function $F(x_i)$ plotted in Fig. A-1(b) gives the probability of obtaining a value smaller than or equal to some value x_i of the discrete random variable x and is known as the "cumulative distribution function" of that random variable. $F(x_i)$ can be obtained by summing the values of the probability function over those points in the sample space for which the random variable takes on a value less than or equal to x_i - that is,

x_i	2	3	4	5	6	7	8	9	10	11	12
$P(x_i)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36



a) Probability function

x_i	< 2	2	3	4	5	6	7	8	9	10	11	≥ 12
$F(x_i)$	0	1/36	3/36	6/36	10/36	15/36	21/36	26/36	30/36	33/36	35/36	1



b) Cumulative distribution function

Figure A.1 Statistical distribution for sum of values in tossing two dice

$$Pr(x \leq x_i) = F(x_i) = \sum_{x \leq x_i} P(x_i) \tag{A-6}$$

Clearly,

$$0 \leq F(x_i) \leq 1 \quad \text{for all } x_i \tag{A-7}$$

$$F(x_i) \geq F(x_j) \quad \text{for } x_i \geq x_j \tag{A-8}$$

The complement of the distribution function gives the probability that the random variable exceeds a specified value - that is,

$$P_r(x > x_i) = 1 - F(x_i) \tag{A-9}$$

Also,

$$\sum P(x_i) = 1 \tag{A-10}$$

A-4.2 Continuous Random Variable

The case of a continuous random variable is treated in a manner similar to the discrete variable. Here, the distributions are represented by smooth continuous curves and the discrete summations are replaced by integrations.

If $F(x_i)$ is the cumulative distribution of a continuous random variable x, then

$$\lim_{x_i \to -\infty} F(x_i) = F(-\infty) = 0$$

$$\lim_{x_i \to \infty} F(x_i) = F(\infty) = 1 \tag{A-11}$$

For a discrete random variable the probability function $P(x_i)$ was defined as the probability associated with the value x_i. Such a direct definition is clearly no longer meaningful for a continuous random variable. Instead, the definition of the cumulative distribution function is used to define the "probability density function" f(x) of a continuous variate x as follows:

$$f(x) = \lim_{\Delta_x \to 0} \frac{Pr(x_i \leq x \leq x_i + \Delta_x)}{\Delta_x} = \frac{d}{dx}[F(x)] \quad \text{(A-12)}$$

Probability for a continuous random variable may thus be interpreted in terms of relative area under the curve defined by the probability density function. As an example, different probabilities are represented by the shaded areas on Fig. A-2 for a continuous random variable x with probability density function f(x).

A-5 Expected Value or Mean

The best known measure of central tendency is the "expected value", more frequently called the "arithmetic mean", or sometimes "the mean".

When the mathematical form of the distribution is known, the expected value is defined as

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx \qquad x = \text{continuous random variable}$$

(A-13)

$$E(x) = \sum_i x_i p(x_i) \qquad x = \text{discrete random variable}$$

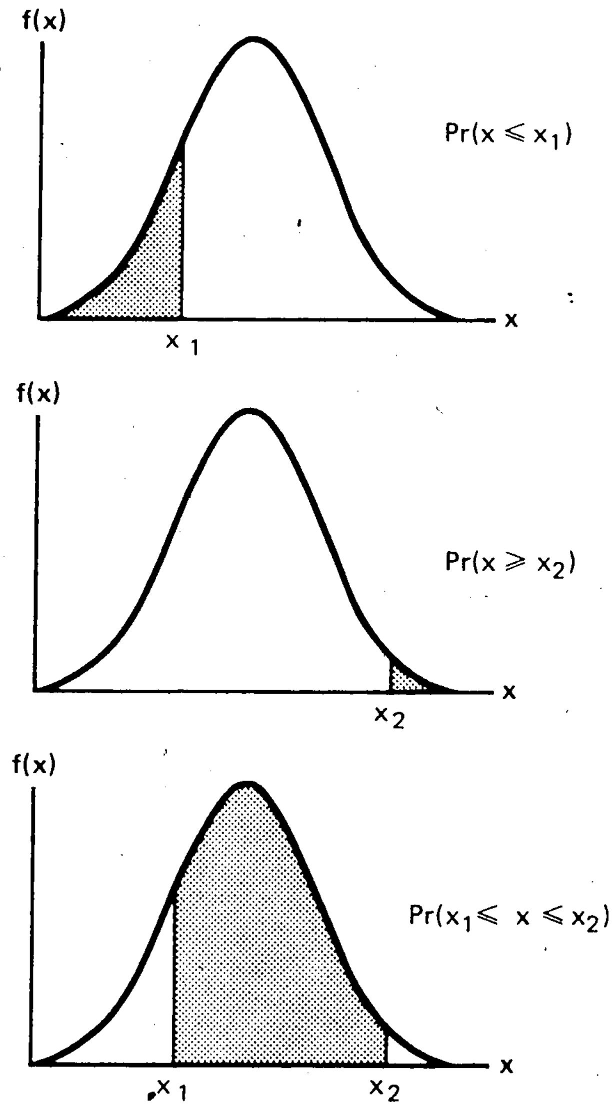


Figure A.2 Probability density function, f(x)

The mean is more frequently estimated from the values of n observations. The "data mean", denoted by $\bar{x}$ or μ, is calculated as

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{A-14}$$

where x_i, i = 1, 2, ..., n, are the values for the n data points. Other measures of central tendency, such as mode and median are described in Refs. 54 and 55.

A-6 Moments of a Distribution

In addition to the mean, other characteristics are frequently used to describe the distribution spread, symmetry, and peakedness. These characteristics may be summarized by the moments of the distribution. For the purpose of simplicity only the expressions defining the moments from data will be presented.

A distribution is completely specified once all its moments are known. However, many distributions can be adequately described by the first four moments, and discussion will be limited to these moments.

The first central moment is always zero

$$m_1 = 0 \tag{A-15}$$

and is the difference between the mean and itself.

A-6.1 Variance and Standard Deviation

The second moment about the mean is a measure of dispersion. It is known as the "variance" m_2, var(x) or σ_x^2.

$$\sigma_x^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \frac{1}{n^2} \left(\sum_{i=1}^{n} x_i \right)^2 \tag{A-16}$$

Eq. (A-16) leads to what statisticians call a "biased estimate". The corresponding unbiased formula is

$$s^2 = \frac{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2}{n(n-1)} \tag{A-17}$$

Eq. (A-17) is generally used instead of (A-16) as an estimate of the variance where only a small number of observations is available.

The square root of the variance is known as the "standard deviation" and is denoted by the symbol σ_x. A non-dimensional characteristic called "coefficient of variation" is of special importance and is defined as,

$$v = \frac{\sigma_x}{\bar{x}} \tag{A-18}$$

A-6.2 Skewness

The third moment about the mean is related to the asymmetry or "skewness" of a distribution.

$$m_3 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^3}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i^3}{n} - 3 \frac{\sum_{i=1}^{n} x_i^2}{n} \frac{\sum_{i=1}^{n} x_i}{n} + 2 \left(\frac{\sum_{i=1}^{n} x_i}{n} \right)^3 \tag{A-19}$$

The third moment is generally standardized in order to compare the symmetry of two distributions where the scales of measurement differ.

$$\alpha_3 = \frac{m_3}{\sigma^3} \tag{A-20}$$

A single peaked distribution with $\alpha_3 < 0$ is said to be skewed to the left, that is, it has a left "tail" as shown in Fig. A-3(a). If $\alpha_3 > 0$, the distribution is skewed to the right. For symmetric distribution, $\alpha_3 = 0$.

A-6.3 Kurtosis

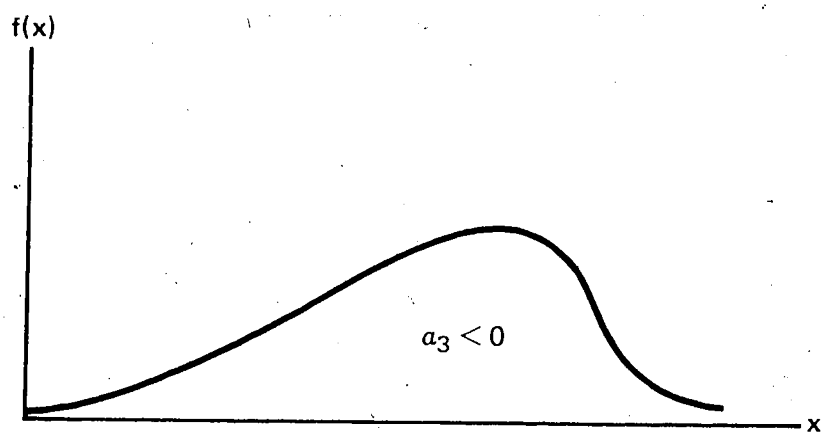
The fourth moment about the mean is related to the peakedness, also called "kurtosis" of the distribution, and is defined as

$$\begin{aligned} m_4 &= \frac{\sum_{i=1}^{n} (x_i - \bar{x})^4}{n} \\ &= \frac{\sum_{i=1}^{n} x_i^4}{n} - 4\,\frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i^3}{n^2} + 6\left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2 \frac{\sum_{i=1}^{n} x_i^2}{n} - \\ &\quad - 3\left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^4 \end{aligned} \tag{A-21}$$

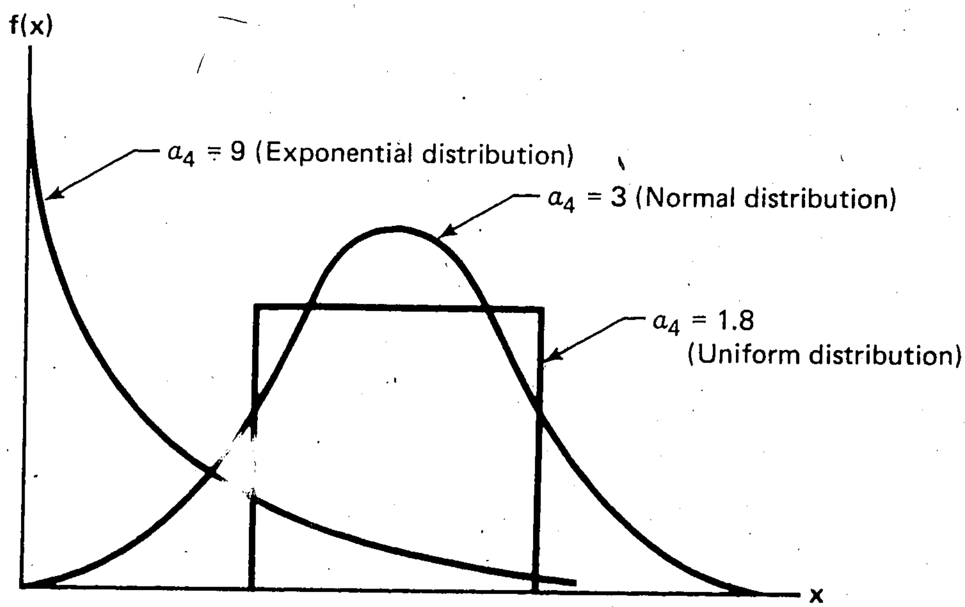
The quantity

$$\alpha_4 = \frac{m_4}{\sigma^4} \tag{A-22}$$

is a relative measure of kurtosis. As shown in Fig. A-3(b), α_4 is 1.8 for a uniform distribution, 3.0 for a perfectly normal distribution, and 9.0 for an exponential distribution.



(a) Distribution skewed to the left



(b) Relative measure of kurtosis

Figure A.3 Moments of a distribution

A-7 Covariance and Coefficient of Correlation

The joint behavior of two random variables x and y is usually summarized by the "covariance", $\sigma_{x,y}$.

$$\sigma_{x,y} = E(xy) - E(x)E(y) \tag{A-23}$$

If x and y are independent, $\sigma_{x,y} = 0$.

The standardized measure of the linear relationship between two variates is the "coefficient of correlation", ρ,

$$\rho = \frac{\sigma_{x,y}}{\sigma_x \sigma_y}$$

$$= \frac{\sum_{i=1}^{n} x_i y_i - \dfrac{\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n}}{\sqrt{\left[\sum_{i=1}^{n} x_i^2 - \dfrac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}\right]\left[\sum_{i=1}^{n} y_i^2 - \dfrac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right]}} \tag{A-24}$$

ρ lies between and includes -1 and 1. If ρ = 0, the variates are said to be uncorrelated. The correlation coefficient only gives a measure of the linear relationship between two variables.

A-8 First Order Probabilistic Approach

In a first order probabilistic approach, the first two moments are used to characterize a random variable. The mean, standard deviation, and correlation coefficient concisely describe the best predictions, the uncertainty, and the joint behavior of the variables.

A-9 Moment Algebra

Properties of Expectation

$$E(c) = c \tag{A-25}$$

$$E(cx) = c\bar{x} \tag{A-26}$$

where c is a deterministic constant.

$$\text{var}\ (c) = 0 \tag{A-27}$$

$$\text{var}\ (cx) = c^2\ \sigma_x^2 \tag{A-28}$$

Sum of Random Variables

$$\text{Let } z = x + y \tag{A-29}$$

$$\text{then } \bar{z} = \bar{x} + \bar{y} \tag{A-30}$$

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_{x,y} \tag{A-31}$$

If x and y are uncorrelated

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 \tag{A-32}$$

Difference of Random Variables

$$\text{Let } z = x - y \tag{A-33}$$

$$\text{Then } \bar{z} = \bar{x} - \bar{y} \tag{A-34}$$

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 - 2\sigma_{x,y} \tag{A-35}$$

If x and y are uncorrelated

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 \tag{A-36}$$

If x and y are uncorrelated, whether z is the sum or the difference of x and y, the variances σ_x^2 and σ_y^2 always add to give σ_z^2.

Product of Random Variables

$$\text{Let } z = xy \tag{A-37}$$

$$\text{Then } \bar{z} = \bar{x}\ \bar{y} + \sigma_{x,y} \tag{A-38}$$

If x and y are uncorrelated

$$\bar{z} = \bar{x}\,\bar{y} \qquad \text{(A-39)}$$

$$\sigma_z^2 = \bar{x}^2\sigma_y^2 + \bar{y}^2\sigma_x^2 + \sigma_x^2\sigma_y^2 \qquad \text{(A-40)}$$

which is simplified to

$$v_z^2 = v_x^2 + v_y^2 + v_x^2 v_y^2 \qquad \text{(A-41)}$$

A-10 Normal Distribution

The "normal (or Gaussian) distribution" is the most widely used model in applied probability theory. Its probability density function as shown in Fig. A-4(a) is,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] \qquad \text{(A-42)}$$

$$-\infty < x < \infty, \; -\infty < \mu < \infty, \; \sigma > 0$$

The mean, μ, and the variance, σ^2, of the normal distribution are estimated by Eqs. (A-14) and (A-16) respectively. The cumulative normal distribution is

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(z-\mu)^2}{2\sigma^2}\right] dz \qquad \text{(A-43)}$$

This expression gives the probability of a randomly selected value from a normal distribution. Most text-books provide a table of the cumulative distribution function of a "standardized" normal random variable, which is defined as,

$$\frac{x-\mu}{\sigma} \qquad \text{(A-44)}$$

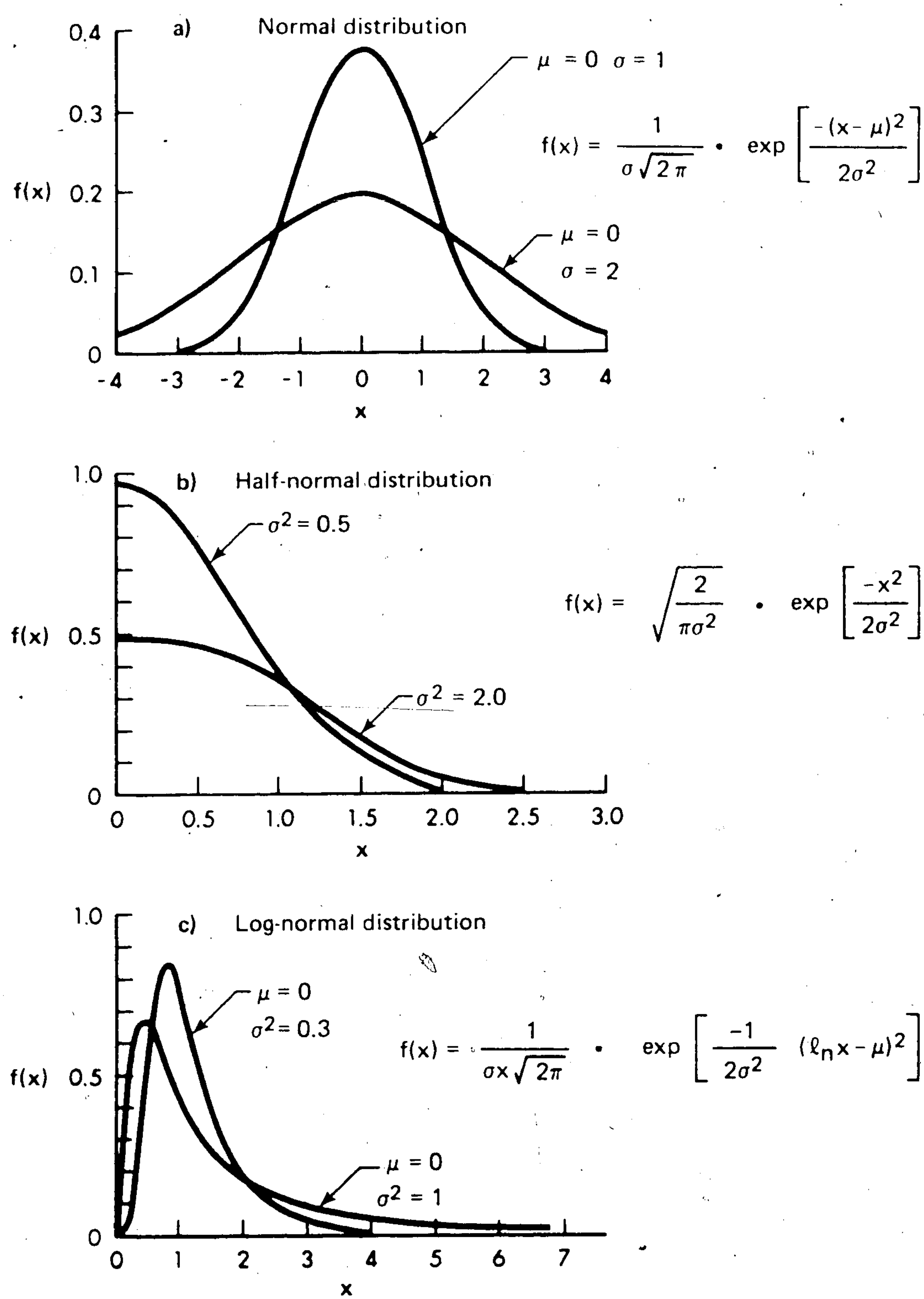


Figure A.4 Continuous statistical distributions

and has a mean 0 and a standard deviation 1.
The cumulative probabilities of the standardized normal distribution are given in Table A-1. For a random variable following a normal distribution, 68.3 percent of the probabilities are within $\pm 1\sigma$ around the mean and 95.5 and 99.7 percent of the probabilities are within the $\mu \pm 2\sigma$ and $\mu \pm 3\sigma$ ranges respectively.

A-11 Central Limit Theorem

The "central limit theorem" is a justification of the wide use of the normal distribution. This theorem states tha' under very general conditions, as the number of variables in the sum becomes large, the distribution of the sum of random variables will approach the normal distribution.

Even if the number of variables involved is only moderately arge, as long as no one variable dominates and as long as the variables e not highly dependent, the distribution of their sum will be nearly normal[55].

A-12 Half-Normal Distribution

The "half-normal distribution" is used to describe normally distributed variates in which only the absolute deviations a ... ' the mean are known.

The probability density function is

$$f(x) = \sqrt{\frac{2}{\pi\sigma^2}} \exp\left[-\frac{x^2}{2\sigma^2}\right] \qquad \text{(A-45)}$$

$$F(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{z^2}{2}} \, dz$$

y	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot9^{2}0097$	$\cdot9^{2}0358$	$\cdot9^{2}0613$	$\cdot9^{2}0863$	$\cdot9^{2}1106$	$\cdot9^{2}1344$	$\cdot9^{2}1576$
2·4	$\cdot9^{2}1802$	$\cdot9^{2}2024$	$\cdot9^{2}2240$	$\cdot9^{2}2451$	$\cdot9^{2}2656$	$\cdot9^{2}2857$	$\cdot9^{2}3053$	$\cdot9^{2}3244$	$\cdot9^{2}3431$	$\cdot9^{2}3613$
2·5	$\cdot9^{2}3790$	$\cdot9^{2}3963$	$\cdot9^{2}4132$	$\cdot9^{2}4297$	$\cdot9^{2}4457$	$\cdot9^{2}4614$	$\cdot9^{2}4766$	$\cdot9^{2}4915$	$\cdot9^{2}5060$	$\cdot9^{2}5201$
2·6	$\cdot9^{2}5339$	$\cdot9^{2}5473$	$\cdot9^{2}5604$	$\cdot9^{2}5731$	$\cdot9^{2}5855$	$\cdot9^{2}5975$	$\cdot9^{2}6093$	$\cdot9^{2}6207$	$\cdot9^{2}6319$	$\cdot9^{2}6427$
2·7	$\cdot9^{2}6533$	$\cdot9^{2}6636$	$\cdot9^{2}6736$	$\cdot9^{2}6833$	$\cdot9^{2}6928$	$\cdot9^{2}7020$	$\cdot9^{2}7110$	$\cdot9^{2}7197$	$\cdot9^{2}7282$	$\cdot9^{2}7365$
2·8	$\cdot9^{2}7445$	$\cdot9^{2}7523$	$\cdot9^{2}7599$	$\cdot9^{2}7673$	$\cdot9^{2}7744$	$\cdot9^{2}7814$	$\cdot9^{2}7882$	$\cdot9^{2}7948$	$\cdot9^{2}8012$	$\cdot9^{2}8074$
2·9	$\cdot9^{2}8134$	$\cdot9^{2}8193$	$\cdot9^{2}8250$	$\cdot9^{2}8305$	$\cdot9^{2}8359$	$\cdot9^{2}8411$	$\cdot9^{2}8462$	$\cdot9^{2}8511$	$\cdot9^{2}8559$	$\cdot9^{2}8605$
3·0	$\cdot9^{2}8650$	$\cdot9^{2}8694$	$\cdot9^{2}8736$	$\cdot9^{2}8777$	$\cdot9^{2}8817$	$\cdot9^{2}8856$	$\cdot9^{2}8893$	$\cdot9^{2}8930$	$\cdot9^{2}8965$	$\cdot9^{2}8999$
3·1	$\cdot9^{3}0324$	$\cdot9^{3}0646$	$\cdot9^{3}0957$	$\cdot9^{3}1260$	$\cdot9^{3}1553$	$\cdot9^{3}1836$	$\cdot9^{3}2112$	$\cdot9^{3}2378$	$\cdot9^{3}2636$	$\cdot9^{3}2886$
3·2	$\cdot9^{3}3129$	$\cdot9^{3}3363$	$\cdot9^{3}3590$	$\cdot9^{3}3810$	$\cdot9^{3}4024$	$\cdot9^{3}4230$	$\cdot9^{3}4429$	$\cdot9^{3}4623$	$\cdot9^{3}4810$	$\cdot9^{3}4991$
3·3	$\cdot9^{3}5166$	$\cdot9^{3}5335$	$\cdot9^{3}5499$	$\cdot9^{3}5658$	$\cdot9^{3}5811$	$\cdot9^{3}5959$	$\cdot9^{3}6103$	$\cdot9^{3}6242$	$\cdot9^{3}6376$	$\cdot9^{3}6505$
3·4	$\cdot9^{3}6631$	$\cdot9^{3}6752$	$\cdot9^{3}6869$	$\cdot9^{3}6982$	$\cdot9^{3}7091$	$\cdot9^{3}7197$	$\cdot9^{3}7299$	$\cdot9^{3}7398$	$\cdot9^{3}7493$	$\cdot9^{3}7585$
3·5	$\cdot9^{3}7674$	$\cdot9^{3}7759$	$\cdot9^{3}7842$	$\cdot9^{3}7922$	$\cdot9^{3}7999$	$\cdot9^{3}8074$	$\cdot9^{3}8146$	$\cdot9^{3}8215$	$\cdot9^{3}8282$	$\cdot9^{3}8347$
3·6	$\cdot9^{3}8409$	$\cdot9^{3}8469$	$\cdot9^{3}8527$	$\cdot9^{3}8583$	$\cdot9^{3}8637$	$\cdot9^{3}8689$	$\cdot9^{3}8739$	$\cdot9^{3}8787$	$\cdot9^{3}8834$	$\cdot9^{3}8879$
3·7	$\cdot9^{3}8922$	$\cdot9^{3}8964$	$\cdot9^{4}0039$	$\cdot9^{4}0426$	$\cdot9^{4}0799$	$\cdot9^{4}1158$	$\cdot9^{4}1504$	$\cdot9^{4}1838$	$\cdot9^{4}2159$	$\cdot9^{4}2468$
3·8	$\cdot9^{4}2765$	$\cdot9^{4}3052$	$\cdot9^{4}3327$	$\cdot9^{4}3593$	$\cdot9^{4}3848$	$\cdot9^{4}4094$	$\cdot9^{4}4331$	$\cdot9^{4}4558$	$\cdot9^{4}4777$	$\cdot9^{4}4988$
3·9	$\cdot9^{4}5190$	$\cdot9^{4}5385$	$\cdot9^{4}5573$	$\cdot9^{4}5753$	$\cdot9^{4}5926$	$\cdot9^{4}6092$	$\cdot9^{4}6253$	$\cdot9^{4}6406$	$\cdot9^{4}6554$	$\cdot9^{4}6696$
4·0	$\cdot9^{4}6833$	$\cdot9^{4}6964$	$\cdot9^{4}7090$	$\cdot9^{4}7211$	$\cdot9^{4}7327$	$\cdot9^{4}7439$	$\cdot9^{4}7546$	$\cdot9^{4}7649$	$\cdot9^{4}7748$	$\cdot9^{4}7843$
4·1	$\cdot9^{4}7934$	$\cdot9^{4}8022$	$\cdot9^{4}8106$	$\cdot9^{4}8186$	$\cdot9^{4}8263$	$\cdot9^{4}8338$	$\cdot9^{4}8409$	$\cdot9^{4}8477$	$\cdot9^{4}8542$	$\cdot9^{4}8605$
4·2	$\cdot9^{4}8665$	$\cdot9^{4}8723$	$\cdot9^{4}8778$	$\cdot9^{4}8832$	$\cdot9^{4}8882$	$\cdot9^{4}8931$	$\cdot9^{4}8978$	$\cdot9^{5}0226$	$\cdot9^{5}0655$	$\cdot9^{5}1066$
4·3	$\cdot9^{5}1460$	$\cdot9^{5}1837$	$\cdot9^{5}2199$	$\cdot9^{5}2545$	$\cdot9^{5}2876$	$\cdot9^{5}3193$	$\cdot9^{5}3497$	$\cdot9^{5}3788$	$\cdot9^{5}4066$	$\cdot9^{5}4332$
4·4	$\cdot9^{5}4587$	$\cdot9^{5}4831$	$\cdot9^{5}5065$	$\cdot9^{5}5288$	$\cdot9^{5}5502$	$\cdot9^{5}5706$	$\cdot9^{5}5902$	$\cdot9^{5}6089$	$\cdot9^{5}6268$	$\cdot9^{5}6439$
4·5	$\cdot9^{5}6602$	$\cdot9^{5}6759$	$\cdot9^{5}6908$	$\cdot9^{5}7051$	$\cdot9^{5}7187$	$\cdot9^{5}7318$	$\cdot9^{5}7442$	$\cdot9^{5}7561$	$\cdot9^{5}7675$	$\cdot9^{5}7784$
4·6	$\cdot9^{5}7888$	$\cdot9^{5}7987$	$\cdot9^{5}8081$	$\cdot9^{5}8172$	$\cdot9^{5}8258$	$\cdot9^{5}8340$	$\cdot9^{5}8419$	$\cdot9^{5}8494$	$\cdot9^{5}8566$	$\cdot9^{5}8634$
4·7	$\cdot9^{5}8699$	$\cdot9^{5}8761$	$\cdot9^{5}8821$	$\cdot9^{5}8877$	$\cdot9^{5}8931$	$\cdot9^{5}8983$	$\cdot9^{6}0320$	$\cdot9^{6}0789$	$\cdot9^{6}1235$	$\cdot9^{6}1661$
4·8	$\cdot9^{6}2067$	$\cdot9^{6}2453$	$\cdot9^{6}2822$	$\cdot9^{6}3173$	$\cdot9^{6}3508$	$\cdot9^{6}3827$	$\cdot9^{6}4131$	$\cdot9^{6}4420$	$\cdot9^{6}4696$	$\cdot9^{6}4958$
4·9	$\cdot9^{6}5208$	$\cdot9^{6}5446$	$\cdot9^{6}5673$	$\cdot9^{6}5889$	$\cdot9^{6}6094$	$\cdot9^{6}6289$	$\cdot9^{6}6475$	$\cdot9^{6}6652$	$\cdot9^{6}6821$	$\cdot9^{6}6981$

TABLE A-1 STANDARD CUMULATIVE NORMAL DISTRIBUTION where $x \geq 0$ and $\sigma > 0$ is a scale parameter which does not equal the standard deviation of the distribution. A plot of a half-normal distribution is given in Fig. A-4(b).

A-13 Log-Normal Distribution

The "log-normal distribution" is the model for a random variable having a logarithm which follows the normal distribution with parameters μ and σ. Thus the probability density function for x is

$$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}(\ell n\, x - \mu)^2\right] \qquad \text{(A-46)}$$

where $x > 0$, $-\infty < \mu < \infty$, $\sigma > 0$

The log-normal distribution as shown in Fig. A-4(c) is skewed to the right, the degree of skewness increasing with increasing values of σ. Note that μ and σ are scale and shape parameters respectively and not location and scale parameters as in the normal distribution.

By the central limit theorem, it can be shown that the distribution of the product of n independent positive variates approaches a log-normal distribution.

The cumulative values for $y = \ell n\, x$ can be obtained from the tabulation of the standardized normal distribution and the corresponding values of x are found by taking antilogs.

APPENDIX B

EFFECTS OF NON-DETERMINISTIC GRAVITY LOADS

B-1 Deterministic Gravity Loads

The horizontal force created by the gravity loads acting on the out-of-plumb of one column is,

$$F = P \frac{\Delta_0}{h} \tag{B-1}$$

where P is a deterministic axial load and Δ_0/h is a normally distributed out-of-plumb variable with a mean, μ_c, equal to zero and a standard deviation, σ_c, equal to 0.0017 Rad. (see Fig. 6.7).

$$\frac{\Delta_0}{h} \sim> N(0,0.0017) \tag{B-2}$$

For this case, the resulting design force is given by Eq. (7.3) in section 7.1.1:

$$F_d = \beta \, \sigma_c \, P \tag{B-3}$$

The safety index β has been selected as 3.5. The absolute value of the force F_d then has a probability of not being exceeded (given by Table A-1) of 0.99954, assuming that P is not a variable.

B-2 Non-Deterministic Gravity Loads

A random gravity load with a known distribution would produce the horizontal force given by Eq. (B-4) when acting on an out-of-plumb column.

$$F = P_0 \frac{\Delta_0}{h} \tag{B-4}$$

The force in Eq. (B-4) has a distribution with a mean μ_f and a variance σ_f^2 given by expressions (A-38) and (A-40) in Appendix A.

$$\mu_f = \mu_c \mu_p + \sigma_{c,p} \tag{B-5}$$

$$\sigma_f = \sqrt{\mu_c^2 \sigma_p^2 + \mu_p^2 \sigma_c^2 + \sigma_c^2 \sigma_p^2} \tag{B-6}$$

where, μ_c = 0.0

μ_p = mean of gravity load population

$\sigma_{c,p}$ = covariance = 0.0

σ_c = 0.0017

σ_p = standard deviation of gravity load population

The covariance $\sigma_{c,p}$ is zero since obviously there is no correlation between the axial load and the out-of-plumb of a column. Then,

$$\mu_f = 0.0$$

$$\sigma_f = \sigma_c \sqrt{\mu_p^2 + \sigma_p^2} \tag{B-8}$$

The gravity load is the sum of dead and live loads. Since the dead and live loads are not correlated[59], Eqs. (A-30) and (A-32) are used to define μ_p and σ_p:

$$\mu_p = \mu_d + \mu_\ell \tag{B-9}$$

$$\sigma_p = \sqrt{\sigma_d^2 + \sigma_\ell^2} \tag{B-10}$$

In these expressions, the subscripts d and ℓ define the dead and live load distribution characteristics respectively. It can be assumed that the standard deviation of the dead load is not very significant compared to the standard deviation of the live load in Eq. (B-10)[58,59]. The dispersion of the gravity load distribution will be very close to the dispersion of the live load distribution.

The sustained live load distributions for the instantaneous load, the lifetime maximum load, and the loads for two intermediate time periods are given in Fig. B-1[61]. Of these, only the instantaneous distribution is known from live load surveys[62]. Various theoretical models have been proposed to derive the lifetime maximum live load distribution from load survey data[63,64]. Using the live load model of Ref. 63 and the live load data of Ref. 62, the statistics of lifetime maximum live load have been derived through simulation in Ref. 64.

What is needed in the present case is the distribution of the lifetime maximum live load intensities given by the dashed curve in Fig. B-1 and not the arbitrary point-in-time loads obtained from the live load surveys. For all practical purposes, the lifetime maximum live load distribution can be reasonably approximated by a normal distribution. A computer simulation has shown that the product of two normal variables is also very close to a normal. The design horizontal force for the case of variable gravity loads can then be defined by Eq. (A-44):

$$F_d = \mu_f + \lambda \sigma_f \qquad \text{(B-11)}$$

The safety index λ in this expression is not necessarily 3.5 as for β. Since $\mu_f = 0$,

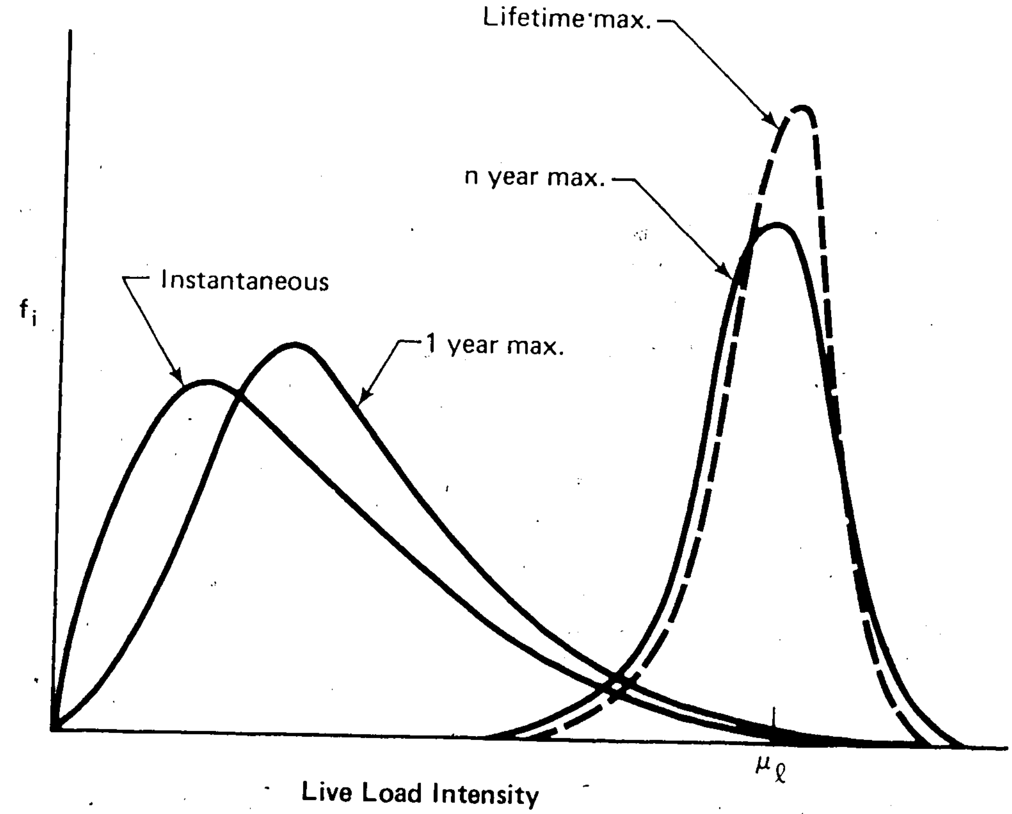


Figure B.1 Probability distributions of live load intensities

$$F_d = \lambda \sigma_c \sqrt{\mu_p^2 + \sigma_p^2} \qquad \text{(B-12)}$$

Eq. (B-12), derived for a non-deterministic load, should be compared to Eq. (B-3) to determine the value of the factor λ that would make the horizontal forces given by the two equations equal.

B-3 Estimation of Dead and Live Load Parameters

The four parameters μ_d, μ_ℓ, σ_d, and σ_ℓ must be estimated in order to determine the safety [illegible]

Approximations giv[illegible] 59 are listed in Table B-1. The lifetime maximum dead load [illegible]med to have an average value equal to the design value with a coe[illegible]ent of variation of 0.07[58,59]. This implies that the maximum dead load will be within ±14 percent of the design value in 95 percent of all structures.

For an office building floor designed for 50 psf, the expected maximum live load (not including partitions) over a 30-year life would be reduced to about 35 psf[59]. It has been indicated in Ref. 64 that the Canadian National Building Code formula[60] for reduction of floor load with tributary area, $0.3 + 10/\sqrt{A}$, is fairly consistent with calculated maximum lifetime loads based on measurements. Therefore, the ratio of expected 30-year load to National Building Code design load will be assumed to be 0.7, independent of tributary area. The results of load surveys[62] indicate that the coefficient of variation for maximum floor loads is about 0.3 and is unchanged with increasing area.

For office and residential buildings, the expected load at a given time is approximately equal to the 30-year load for an infinite area. For office buildings, this corresponds to $0.7(0.3 + 10/\sqrt{\infty})$ 50 psf =

	Mean μ / Specified Load	Coeff. of Variation σ/μ
Dead Load	1.0	0.07
Live Load		
- Maximum 30 years	0.7	0.3
- At any time	$0.21/(0.3 + 10/\sqrt{A})$	$0.3 + 0.4/\sqrt{A}$

A is the Tributary Area in sq. ft.

TABLE B-1 PROBABILISTIC ASSUMPTIONS FOR GRAVITY LOADS

10.5 psf, a value confirmed by survey results(59). On the other hand the coefficient of variation of a load at any time increases with a decrease in area. The equation given is based on Table 7 of Ref. 62.

Approximate formulas are given in Ref. 64 for the mean and standard deviation of the maximum sustained live load during a structure's life plus the largest extraordinary event which occurs during the random duration of this maximum sustained load. The approximations fit the actual distributions in the upper fractiles of these loads.

$$\mu_\ell = 14.9 + 763/\sqrt{A_1} \text{ psf} \qquad \text{(B-13)}$$

$$\sigma_\ell = \sqrt{11.3 + 15000/A_1} \text{ psf} \qquad \text{(B-14)}$$

where A_1 is the "influence area" which corresponds to four times the more common "tributary area", A, in the case of single-storey column loads.

These results have shown that for columns, the prescribed design loads as a function of area for the Canadian National Building Code(60) correspond approximately to the 0.9 fractile of the maximum total load. As shown in Fig. B-2, the NBC prescribed load is 50 psf for office buildings and for $A > 200$ ft.^{2}, this value is reduced by a factor $0.3 + 10/\sqrt{A}$. The prescribed live load P in a column is then estimated by Eq. (B-15) for a column tributary area larger than 200 ft.^{2}.

$$P = \mu_\ell + \psi\sigma_\ell \qquad \text{(B-15)}$$

The value for the safety index ψ which corresponds to the 0.9 fractile is 1.3 and μ_ℓ and σ_ℓ are given by Eqs. (B-13) and (B-14) or are taken from Table B-1.

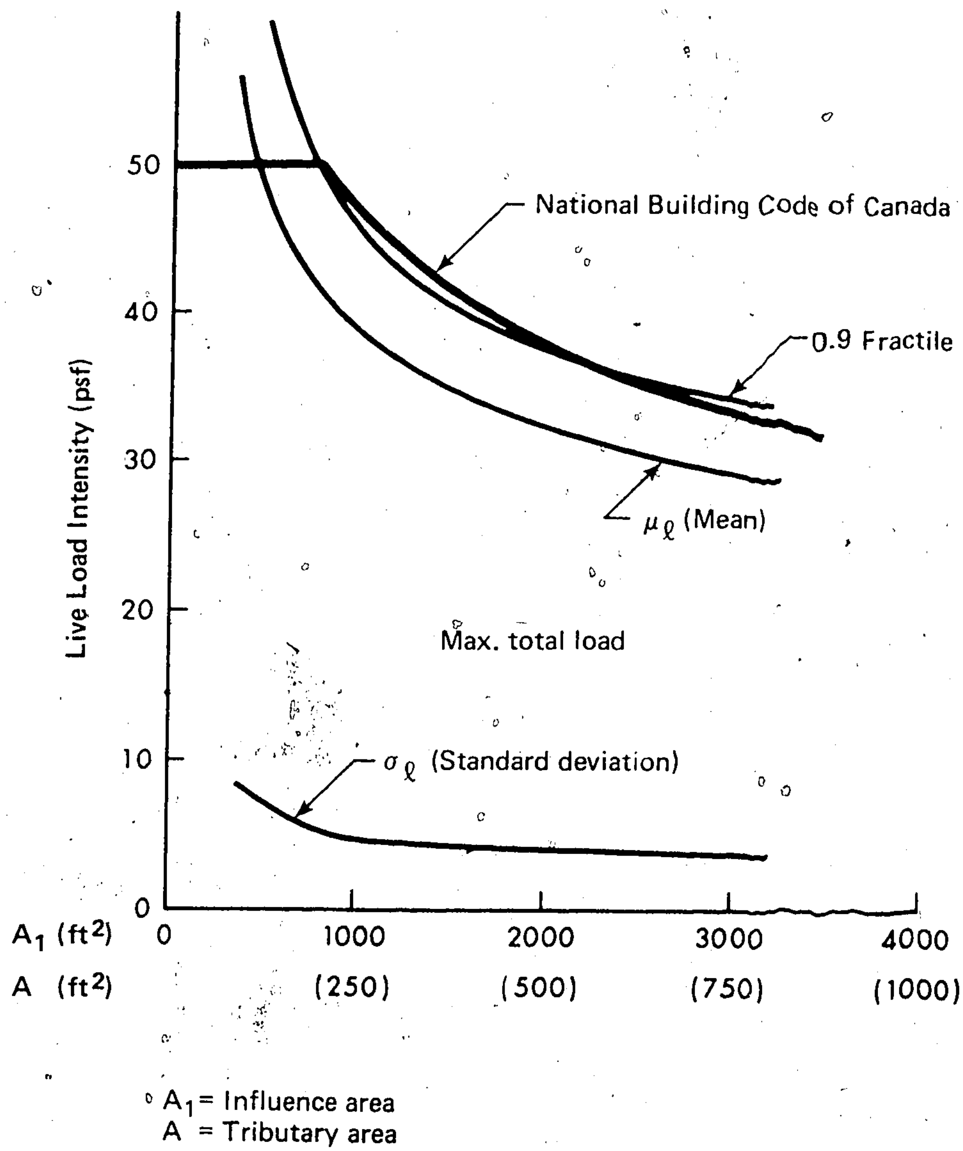


Figure B.2 Prescribed live loads for column design

Example:

$$A = 200 \text{ ft.}^2, \; A_1 = 800 \text{ ft.}^2$$

Prescribed load = 50 x 200 = 10000 lbs.

$$\mu_\ell = (14.9 + 763/\sqrt{800})\ 200 = 8375 \text{ lbs.}$$

$$\sigma_\ell = \sqrt{11.3 + 15000/800} \times 200 = 1096 \text{ lbs.}$$

$$P = 8375 + 1.3 \times 1096 = 9800 \text{ lbs.}$$

$\therefore$ 2 percent difference.

Using the values of Table B-1,

$$\mu_\ell = 0.7 \times 50 \times 200 = 7000 \text{ lbs.}$$

$$\sigma_\ell = 0.3 \times 7000 = 2100 \text{ lbs.}$$

$$P = 7000 + 1.3 \times 2100 = 9730 \text{ lbs.}$$

$\therefore$ 2.8 percent difference.

B-4 Probability Calculations

The distributions of the two random variables in Eq. (B-4) are given in Fig. B-3(a,b) with their corresponding probabilities. The dead load is not included in the load distribution shown in (b) in order to simplify the calculations. The results should not be changed significantly. The distribution represented by the continuous curve in Fig. B-3(c) is the distribution of the variable horizontal force, $P\Delta_0/h$, for a deterministic axial load P. The shape of the distribution is the same as in (a) but the scale is different. The variance in this case is $P^2\sigma_c^2$ according to Eq. (A-28). The horizontal

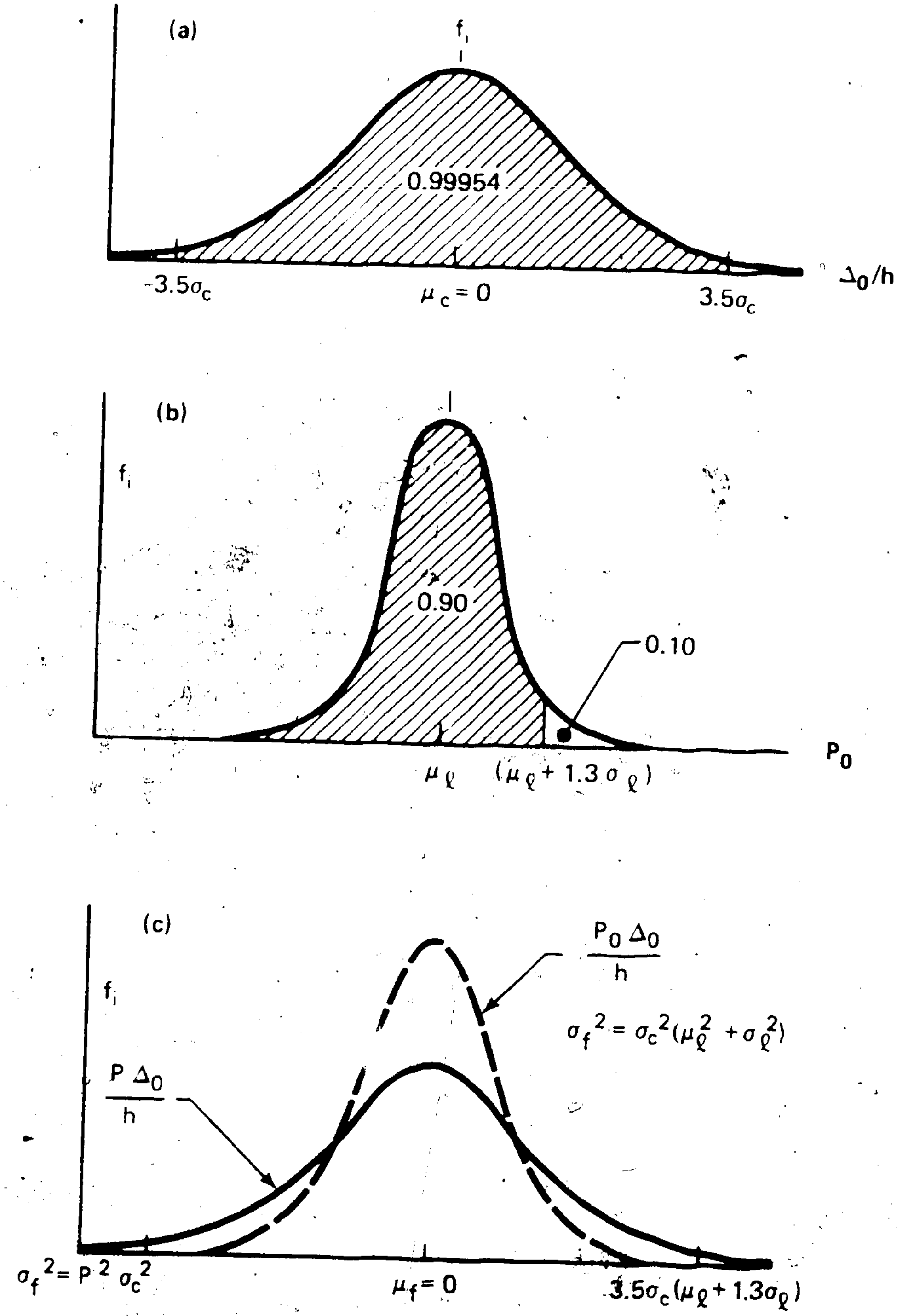


Figure B.3 Distributions

shear distribution for a variable axial load P_0 is represented by the dotted curve on the same figure. The variance is now equal to $\sigma_c^2(\mu_\ell^2 + \sigma_\ell^2)$ and is always smaller than the variance in the case of the deterministic load. In terms of standard deviations,

$$\sigma_c(\mu_\ell + 1.3\,\sigma_\ell) > \sigma_c\sqrt{\mu_\ell^2 + \sigma_\ell^2} \tag{B-16}$$

The variance of the di[illegible]on assuming P non-deterministic is reduced from the variance assuming P deterministic. The probability is then greater under the dotted curve that the shear force will be less than $3.5\sigma_c(\mu_\ell + 1.3\sigma_\ell)$. By assuming P deterministic, the absolute value of the shear has a 99.954 percent chance of being less than this limit.

$$\Pr\left[\left|\frac{P\Delta_0}{h}\right| < 3.5\sigma_c\,(\mu_\ell + 1.3\sigma_\ell)\right] = 0.99954 \tag{B-17}$$

When th[illegible]xial load is random,

$$\Pr\left[P_0\,\frac{\Delta_0}{h} < 3.5\,\sigma_c\,(\mu_\ell + 1.3\sigma_\ell)\right] = \;?$$

Dividing both sides of the inequality by the standard deviation of the population, gives

$$\Pr\left[\frac{P_0\Delta_0/h}{\sigma_c\sqrt{\mu_\ell^2+\sigma_\ell^2}} < \frac{3.5\,(\mu_\ell + 1.3\sigma_\ell)}{\sqrt{\mu_\ell^2+\sigma_\ell^2}}\right] = \;? \tag{B-18}$$

As shown in section A-10 of Appendix A, when a normally distributed variable with a mean equal to zero is divided by the standard deviation of the population, the variable is said to be standardized. The expression on the left hand side of the inequality is then the standardized horizontal shear for a non-deterministic load and can be called F_{st}. The new safety index λ for the force obtained in Eq. (B-12) is given by the expression on the right hand side. The factor λ can be evaluated from the values of Table B-1 or from Eqs. (B-13) and (B-14). From Table B-1,

$$\mu_\ell = 0.7 \text{ P}$$

$$\sigma_\ell = 0.3\ \mu_\ell = 0.3 \times 0.7 \text{ P} = 0.21 \text{ P}$$

$$\lambda = \frac{3.5\ (0.7 + 1.3 \times 0.21)\ \text{P}}{\sqrt{0.7^2 + 0.21^2}\ \text{P}} = 4.66$$

The probability of F_{st} being lower than this value is obtained from the table of the Standard Cumulative Normal Distribution (Table A-1).

$$\Pr\ [F_{st} < 4.66] = 0.9999984$$

$$\Pr\ [|F_{st}| < 4.66] = 0.9999968$$

Using Eqs. (B-13) and (B-14) with A = 200 ft.^{2} and A_1 = 800 ft.^{2}, μ_ℓ = 8375 lbs. and σ_ℓ = 1096 lbs.

$$\lambda = \frac{3.5\ (8375 + 1.3 \times 1096)}{\sqrt{8375^2 + 1096^2}} = 4.06$$

$$\Pr\ [F_{st} < 4.06] = 0.999975$$

$$\Pr\ [|F_{st}| < 4.06] = 0.999951$$

Due to the random nature of the gravity loads, the horizontal shear given by Eq. (B-3) has a real probability which corresponds to $\beta \simeq 4.2$ when the loads prescribed by the Canadian National Building Code are used in combination with $\beta = 3.5$ in Eq. (B-3).

It is possible to calculate the value of ψ in Eq. (B-15) which would have held the probability at 0.99954 (β = 3.5) in Eq. (B-3). This occurs when $(\mu_\ell + \psi\sigma_\ell) = \sqrt{\mu_\ell^2 + \sigma_\ell^2}$ in Eq. (B-18). Then,

$$\Pr\,[F_{st} < 3.5] = 0.99954$$

The quadratic equation obtained has a root equal to

$$\psi = \frac{\sqrt{\mu_\ell^2 + \sigma_\ell^2} - \mu_\ell}{\sigma_\ell} \qquad \text{(B-19)}$$

The index ψ will be close but never equal to zero according to this equation. An indeterminate result is obtained when the variation σ_ℓ^2 is zero. The values in Table B-1 for the maximum live load give ψ = 0.15, while Eqs. (B-13) and (B-14), for the example presented previously, yield ψ = 0.065. Thus, if a gravity load given by the mean shown in Fig. B-2 was used, the resulting horizontal shear calculated by Eq. (B-3) would have a 99.95 percent chance of not being exceeded.

APPENDIX C

DEGREE OF DEPENDENCE OF COLUMN OUT-OF-PLUMBS

It seems likely that a certain correlation exists between the out-of-plumbs of the columns in a structure. Whether it significantly affects the results of the theory developed in Chapter VII has yet to be verified.

Assuming that z is the sum of two random variables, x and y, the variance of the new variable z is given by Eq. (A-31) in Appendix A:

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 + 2\sigma_{x,y} \tag{C-1}$$

The variance σ^2 is defined in section A-6.1 and the covariance $\sigma_{x,y}$ in section A-7. The horizontal force at a connection point, as given by Eq. (7.4) in the case of two out-of-plumb columns, is now a normally distributed variable of the form:

$$F \sim> N(P_1\mu_x + P_2\mu_y,\ \sqrt{P_1^2\sigma_x^2 + P_2^2\sigma_y^2 + 2P_1P_2\sigma_{x,y}})$$

For $\mu_x = \mu_y = 0$ and $\sigma_x = \sigma_y = \sigma$,

$$F \sim> N(0,\ \sigma\sqrt{P_1^2 + P_2^2 + 2\rho\, P_1P_2}\ )$$

where $\rho = \sigma_{x,y}/\sigma^2$ is defined in section A-7 of Appendix A as the coefficient of correlation, which is the standardized measure of the joint behavior of two random variables. When $\rho = 1.0$, the variates are positively perfectly correlated and when $\rho = -1.0$, they are perfectly negatively correlated. On a graph, these conditions are represented by straight lines of slope +1 and -1 respectively. If $\rho = 0.0$, the variates are said to be uncorrelated or perfectly independent.

The horizontal force caused by n out-of-plumb columns, assuming a certain degree of correlation between the columns, is then given by:

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2 + 2\rho \sum_{j=1}^{n-1} P_j \, P_{j+1}} \qquad \text{(C-2)}$$

The second term under the root sign is the summation of all the possible independent combinations of pairs of adjacent columns. An upper bound is found when $\rho = 1$, i.e. when there exists a positive linear dependence between the variables. Then,

$$F \sim> N(0, \sigma\sqrt{(P_1 + P_2)^2})$$

$$F \sim> N(0, \sigma\,(P_1 + P_2))$$

The resulting design horizontal force for the general case of n columns is:

$$F_d = \beta\sigma_c \sum_{j=1}^{n} P_j \qquad \text{(C-3)}$$

This is equivalent to the model shown in Fig. 5.1(b) with all the columns out-of-plumb by $\beta\sigma_c$. The lower bound is obtained by assuming perfect independence between the variables ($\rho = 0$).

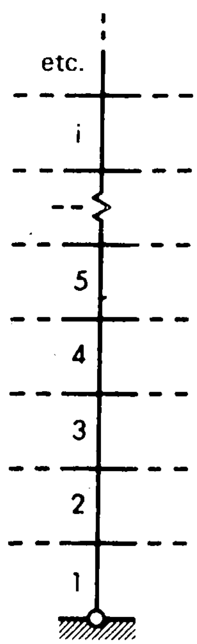
$$F \sim> N(0, \sigma\sqrt{P_1^2 + P_2^2})$$

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2} \qquad \text{(C-4)}$$

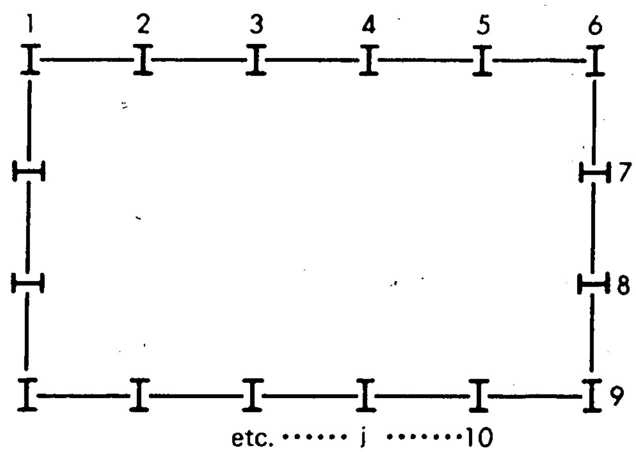
This formulation is used in Chapter VII, sections 7.1.1 and 7.1.2.

Different combinations of out-of-plumb columns taken in a vertical line and in a storey are shown in Fig. C-1. The correlations corresponding to the different combinations, denoted as a, b, c, and d, can be calculated and can be visualized graphically. An estimation of the actual coefficient of correlation for each case can be obtained from Eq. (A-24). The results obtained for building B are listed in Table C-1 and the graphical representations of the two different correlations pertaining to group 'a' are given in Figs. C-2 and C-3. Fig. C-2 shows the correlation between columns adjacent in vertical lines and Fig. C-3 shows the correlation between adjacent columns at each storey. The values plotted on each figure represent a sample of the total number of observations. The out-of-plumbs in the x and y directions are considered together.

The results shown in column 2 of Table C-1 for groups a and b indicate a slight but non-significant dependence between columns in vertical lines. The scatter of the points shown in Fig. C-2 confirms these results. The coefficients of correlation obtained in cases c and d where the pairs are one and two steps apart are even closer to zero. This result was expected. It indicates a decreased dependence of the variables as the compared columns are taken farther apart. The usual practice of erecting tier columns does not seem to induce a significant degree of correlation between the column out-of-plumbs from one floor to another. The correlation that could have existed initially from floor to floor is apparently wiped out during



Elevation



Plan View

Combinations

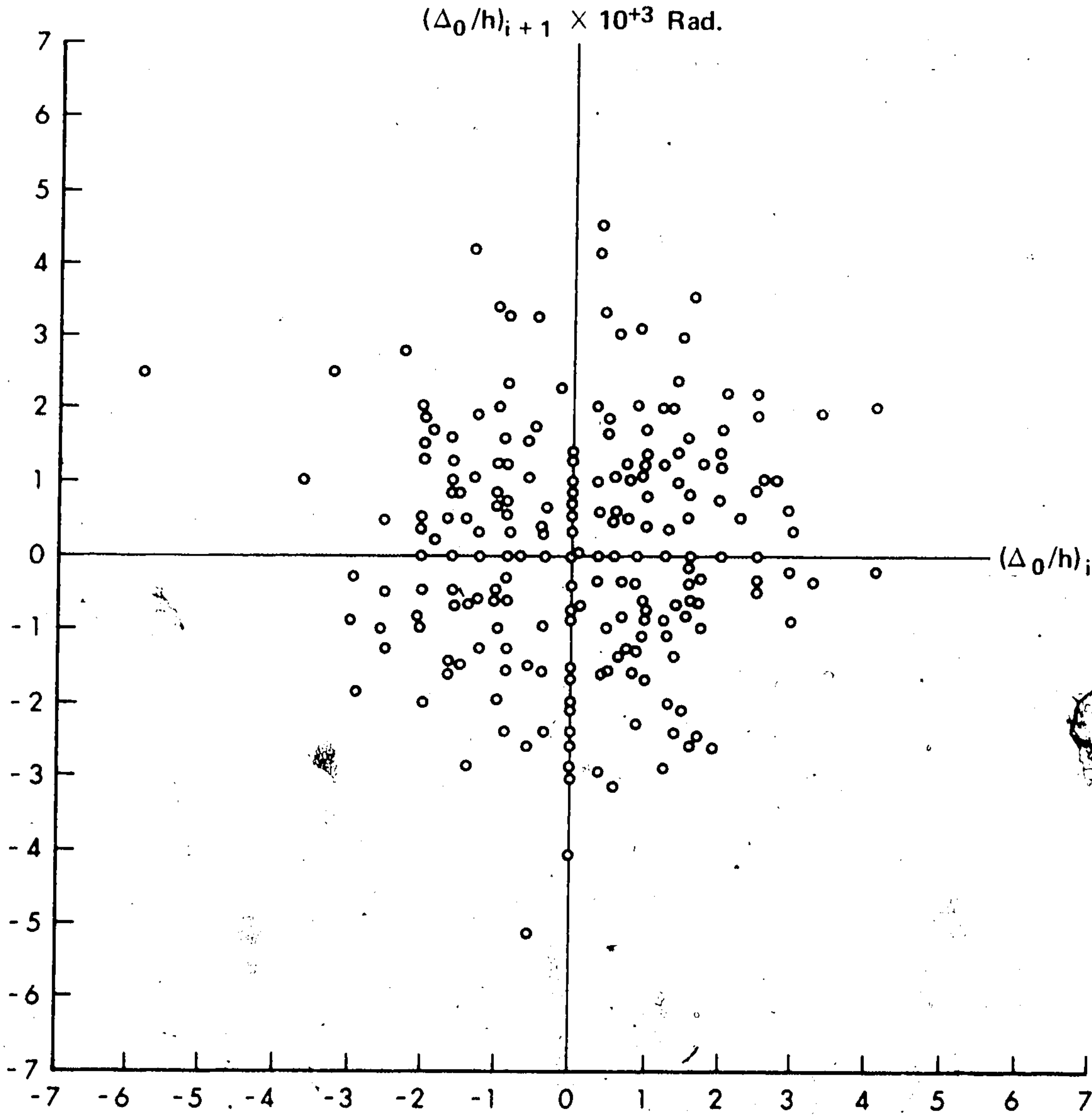
a — 1-2, 3-4, 5-6, etc . . . (adjacent columns)
b — 2-3, 4-5, 6-7, etc . . . (adjacent columns)
c — 1-3, 4-6, 7-9, etc . . . (1 step apart)
d — 1-4, 5-8, 9-12, etc . . . (2 steps apart)

Figure C.1 Column combinations for the evaluation of the degree of correlation between out-of-plumbs

1	2	3
Combination Type*	Coefficient of Correlation, ρ	
	In A Vertical Line	In A Storey
a - Adjacent Columns	0.072	0.133
b - Adjacent Columns	0.063	0.238
c - One Step Apart	-0.008	0.068
d - Two Steps Apart	0.024	-0.022

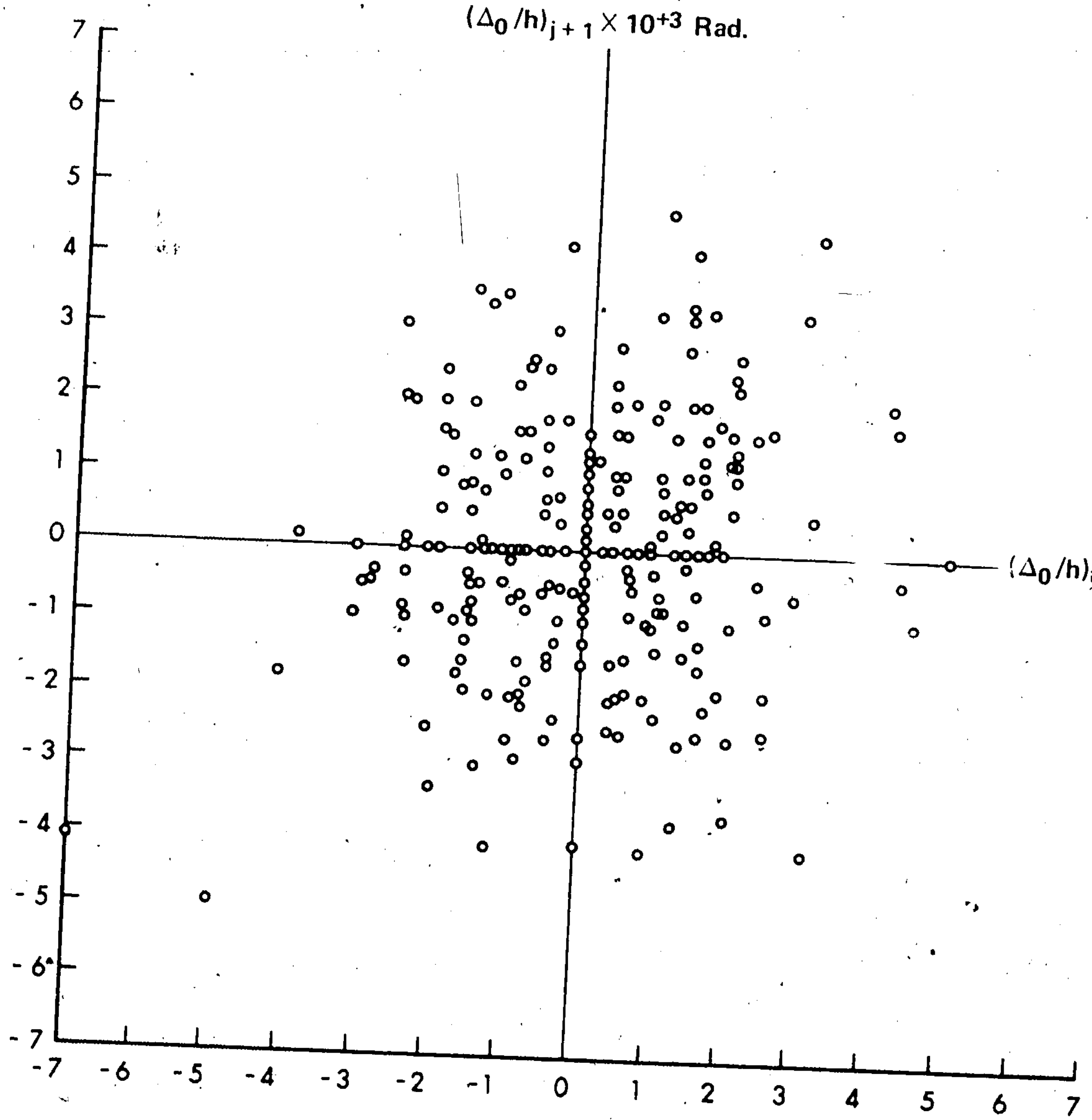
* Defined in Fig. B-1

TABLE C-1 COEFFICIENTS OF CORRELATION FOR BUILDING B



Coefficient of correlation = 0.072 (from data)

Figure C.2 **Correlation between two out-of-plumb columns adjacent in a vertical line in building B**



Coefficient of correlation = 0.133 (from data)

Figure C.3 Correlation between two out-of-plumb columns adjacent in a storey in building B

the construction of the building by the effects described earlier in section 5.3.

Coefficients of correlation of 0.133 and 0.238 for adjacent columns in a storey were obtained for building B. The plot corresponding to $\rho = 0.133$ is given in Fig. C-3 and shows considerable scatter. A positive coefficient of correlation in this case indicates that two adjacent columns in a storey lean in the same positive or negative direction more often than predicted by the theory developed in Chapter VII based on total independence. Since an unsafe situation could result, an evaluation of this effect is mandatory.

The results obtained for combination types c and d in column 3 of Table C-1 confirm earlier observations that the correlation decreases rapidly as the columns forming the pairs are taken farther apart. It is believed that the correlation within a storey is in great part tied in to fabrication errors. When girders for a specific storey are cut slightly shorter or longer, the adjustment of these girders between the columns in a bent might force the columns to lean in the same direction in the plane of the bent.

In summary, a correlation does exist between column-of-plumbs. Although it is negligible from storey to storey, it is significant within a storey. For all practical purposes, $\rho = 0.0$ between columns in vertical lines, $\rho = 0.2$ between adjacent columns in a storey, $\rho = 0.1$ for pairs one step apart in a storey, and $\rho = 0.0$ for pairs more than one step apart.

Considering the case of two adjacent columns at a same storey Eq. (C-2) becomes:

$$F_d = \beta\sigma_c \sqrt{P_1^2 + P_2^2 + 2(0.2)\, P_1 P_2}$$

Assuming equal axial load in the columns,

$$F_d = \sqrt{2.4}\ \beta\sigma_c P$$

The lower bound given by Eq. (C-4) is

$$F_d = \sqrt{2}\ \beta\sigma_c P$$

Then, the correlation existing between two adjacent columns increases the horizontal force predicted by Eq. (C-4) by 10 percent.

For the general case of n columns at the same storey, Eq. (C-2) yields:

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2 + 2[0.2 \sum_{j=1}^{n-1} P_j P_{j+1} + 0.1 \sum_{j=1}^{n-2} P_j P_{j+2} + 0.0]}$$

Assuming that

$$\sum_{j=1}^{n-1} P_j P_{j+1} \simeq \sum_{j=1}^{n} P_j^2$$

and

$$\sum_{j=1}^{n-2} P_j P_{j+2} \simeq \sum_{j=1}^{n} P_j^2,$$

$$F_d = \beta\sigma_c \sqrt{1.6 \sum_{j=1}^{n} P_j^2} = 1.26\ \beta\sigma_c \sqrt{\sum_{j=1}^{n} P_j^2}$$

which constitutes an increase of 26 percent from the lower bound given by Eq. (C-4). The calculations show that the 26 percent limit is attained at 15 columns.

It remains to check whether this effect is reduced or increased when columns from different storeys are combined. By summing separately within each storey, Eq. (C-2) becomes:

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} P_j^2 + \sum_{j=1}^{n_{i+1}} P_j^2 + 2[0.2(\sum_{j=1}^{n_i-1} P_j P_{j+1} + \sum_{j=1}^{n_{i+1}-1} P_j P_{j+1}) + 0.1(\sum_{j=1}^{n_i-2} P_j P_{j+2} + \sum_{j=1}^{n_{i+1}-2} P_j P_{j+2})]}$$

where n_i is the number of columns at storey i.

This equation can be simplified within each storey as before:

$$F_d = \beta\sigma_c \sqrt{1.6 \sum_{j=1}^{n_i} P_j^2 + 1.6 \sum_{j=1}^{n_{i+1}} P_j^2}$$

$$F_d = 1.26\ \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} P_j^2 + \sum_{j=1}^{n_{i+1}} P_j^2}$$

Eq. (C-4), applied to the same case, becomes:

$$F_d = \beta\sigma_c \sqrt{\sum_{j=1}^{n_i} P_j^2 + \sum_{j=1}^{n_{i+1}} P_j^2}$$

This shows that the increase of 26 percent from the lower bound remains when columns from different storeys are com' ned.

Eq. (C-2), however, is not practical in a design for the horizontal out-of-plumb forces. It seems more logical to use the expression giving the lower bound with an increased safety index which would account for the correlation effects and other factors. When a factor β of 3.5 is used in Eq. (C-4), the real probability of being exceeded is not 4.6×10^{-4} but 5×10^{-3} corresponding to a β value of 2.8. In other words, in five out of a thousand times, the horizontal forces calculated for $\beta = 3.5$ would exceed the predicted values.

The applications given later in Chapter VIII, where several measured and predicted quantities are directly compared, will justify the choice of a safety index equal to 3.5 under the above conditions. More important is the fact that the safety index β is actually increased from 3.5 to 4.2, as shown in Appendix B, because of the random nature of the gravity loads. The combination of the effects observed in Appendices B and C results in an average safety index of 3.5 with a corresponding probability of 0.99954.

APPENDIX D

LATERAL DEFLECTIONS DUE TO COLUMN OUT-OF-PLUMBS

Expression (7.16) reproduced below as (D-1) requires excessive computational efforts for structures with a large number of columns. An investigation is needed to determine whether this equation can be simplified.

$$\frac{\Delta_d}{h} = \beta\sigma_c \frac{\sqrt{\sum_{j=1}^{n} P_j^2}}{\sum_{j=1}^{n} P_j} \qquad \text{(D-1)}$$

When the column axial loads are assumed to be constant, the variable P_j disappears and Eq. (D-1) is reduced to:

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c}{\sqrt{n}} \qquad \text{(D-2)}$$

In practical structures the axial loads differ and cause Eq. (D-2) to be unconservative. A more general formulation would be,

$$\frac{\Delta_d}{h} = \frac{\beta\sigma_c}{\sqrt[x]{n}} \qquad \text{(D-3)}$$

where the variable x is a function of the number of columns in a structure and the variations in column axial loads. The influence of these two factors on the variable x can be evaluated.

Equating (D-1) and (D-3) yields:

$$\sqrt[x]{n} = \frac{\sum_{j=1}^{n} P_j}{\sqrt{\sum_{j=1}^{n} P_j^2}}$$

which in turn becomes:

$$x = \frac{\ell n\ n}{\ell n\ R} \qquad \text{(D-4)}$$

where

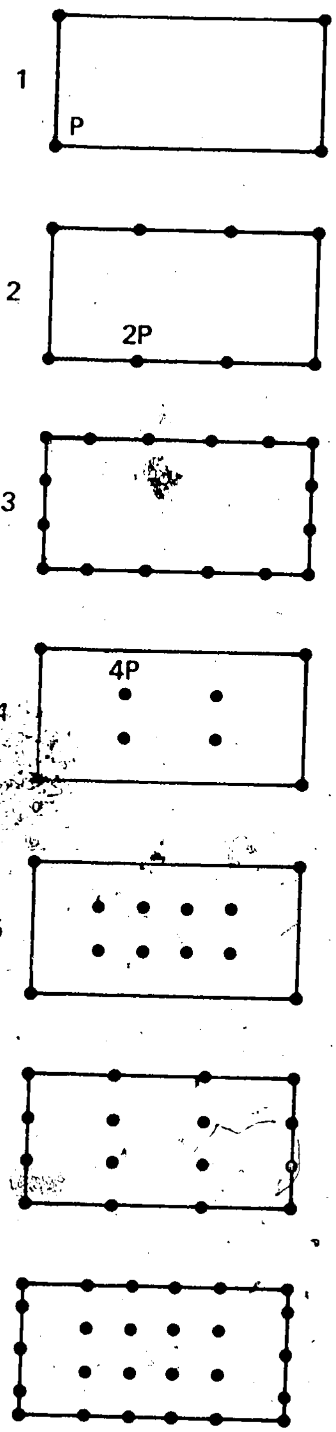
$$R = \frac{\sum_{j=1}^{n} P_j}{\sqrt{\sum_{j=1}^{n} P_j^2}} \qquad \text{(D-5)}$$

The minimum x value is 2.0 and is obtained when the axial loads in Eq. (D-4) are constant.

$$x = \frac{\ell n\ n}{\ell n\left(\frac{nP}{\sqrt{nP^2}}\right)} = \frac{\ell n\ n}{\ell n\ \sqrt{n}} = \frac{\ell n\ n}{1/2\ \ell n\ n} = 2 \qquad \text{(D-6)}$$

The upper limit is not defined but is in the order of 2.5 in practical structures. Larger values are obtained only in very unusual cases.

Fig. D-1 shows the column layouts of seven different building cross-sections. The variable x is calculated for typical 1, 2, 6, and 10-storey buildings by assigning the relative axial loads P, 2P, and 4P to the corner, exterior, and interior columns respectively. The column axial loads are increased uniformly from floor to floor. For instance, if the top column of a 3-storey column stack carries 2P, the middle one and the lower one carry 4P and 6P respectively.



Layout	1 Storey n/R	1 Storey X	2 Storeys n/R	2 Storeys X	6 Storeys n/R	6 Storeys X	10 Storeys n/R	10 Storeys X
1	4/2.00	2.00	8/2.68	2.11	24/4.4	2.14	40/5.61	2.14
2	8/2.68	2.11	16/3.60	2.16	48/5.91	2.18	80/7.52	2.17
3	16/4.13	2.04	32/5.21	2.10	96/8.55	2.13	160/10.88	2.13
4	8/2.43	2.34	16/3.25	2.35	48/5.34	2.31	80/6.80	2.29
5	12/3.13	2.18	24/4.20	2.21	72/6.90	2.21	120/8.78	2.20
6	16/3.6	2.16	32/4.83	2.20	96/7.93	2.20	160/10.09	2.20
7	26/4.67	2.11	52/6.26	2.15	156/10.28	2.17	260/13.08	2.16

Figure D.1 X values for different column layouts

In all cases, except in the very unusual case No. 4, x does not exceed 2.2. A similar pattern is observed in all arrangements: The factor x increases slightly when passing from a one-storey to a two-storey building; the value remains fairly constant as other storeys are added and finally decreases when n becomes large. This behavior is explained by the fact that when n becomes large, the difference between the axial loads becomes less significant and R tends towards $\sqrt{n}$. At the limit, x is 2.0 as shown in Eq. (D-6). The notable increase of the variable x in the 2-storey buildings of Fig. D-1 reflects the factor of 2 between the axial loads at each storey. The number of columns is then too small to hide the effect of the axial load variations.

As applied to the actual axial loads in the 27-storey building A (Fig. 6.2), $x = \ln 18.62 = 2.10$. The column layout of building A is given in example No. 3 in Fig. D-1.

In view of these considerations, Eq. (D-3) with $x = 2.2$ is recommended for use in design. However, the "exact" expression (D-1) should be used in the case of one or two-storey structures for a more accurate evaluation of Δ_d/h.

APPENDIX E

EFFECT OF WALL THICKNESS VARIATIONS ON MEASUREMENTS OF OUT-OF-PLUMBS

The exact deviation from plumb at a specific section of a wall is obtained by using the average of two measurements, one taken on either side of the wall. Measurements taken on one side only do not account for the unavoidable thickness variations of the wall. However, it is physically impossible to take double measurements at each wall section.

It is possible to determine to what extent the measurements are affected by estimating the distribution of the wall thickness variations and combining the resulting variance with the variance of the wall out-of-plumb population. The variance is defined as the squared value of the standard deviation.

Thickness measurements were taken wherever possible with a measuring tape on the core of building B. At least two measurements were taken per vertical section of the wall.

The variables that must be distributed and used in the calculations are the deviations from the mean at each individual section. The wall section shown in Fig. E-1(a) is thicker at the bottom and the measurement taken as shown results in an out-of-plumb value smaller than the actual. In (b) the recorded deviation is larger than it should be while in (c) the actual out-of-plumb is recorded. In other cases, as in (d), the thickness variation does

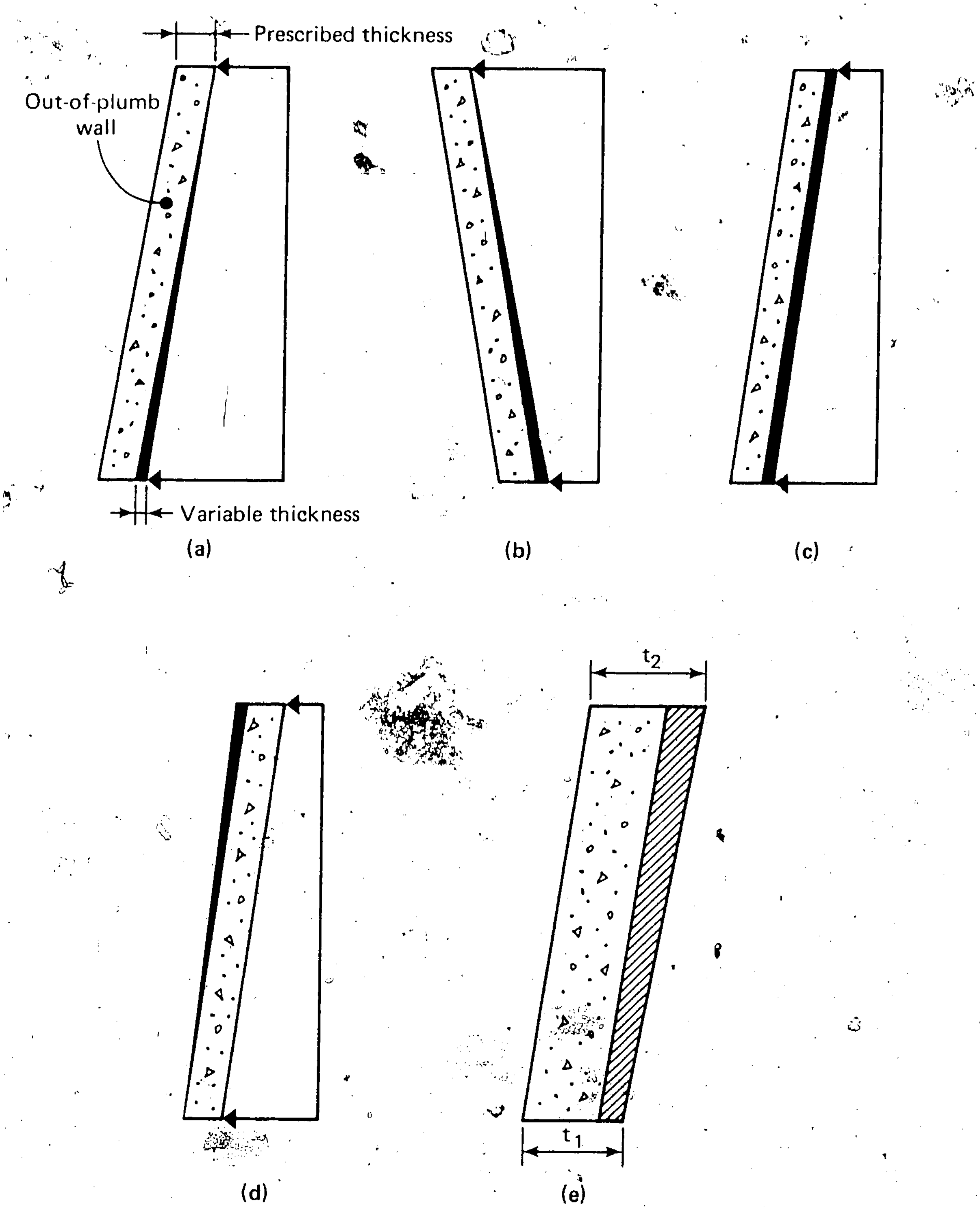


Figure E.1 Wall thickness variations

not affect the measurements. For a set of two measurements at the section shown in Fig. E-1(e), the mean thickness is $(t_i + t_2)/2$ and the values to be distributed are $t_1' = t_1$ - mean and $t_2' = t_2$ - mean.

The distribution obtained is given in Fig. E-2 together with the mean, the standard deviation, and other characteristics. The distribution is close to normal. The mean, of course, is zero and the standard deviation is 0.16". The variance of the measured out-of-plumbs is the sum of the variance of the actual out-of-plumbs and the variance of the deviations of the wall thickness from the mean at specific sections.

$$\text{var}\ (\Delta_0)_m \quad = \quad \text{var}\ (\Delta_0)_{act} + \text{var}\ (\delta_t)$$

or

$$\text{var}\ (\Delta_0)_{act} \quad = \quad \text{var}\ (\Delta_0)_m - \text{var}\ (\delta_t) \qquad \text{(E-1)}$$

The standard deviation of the measured out-of-plumbs for building B is given in Table 6.11 and is approximately 0.0023. A representative value in units of inches is obtained by multiplying the standard deviation by the standard storey height in practical structures, say 144". The variance is then $(144" \times 0.0023)^2 = (0.33")^2$ and

$$\text{var}\ (\Delta_0)_{act} \quad = \quad (0.33)^2 - (0.16)^2 \quad = \quad 0.08\ \text{in.}^2$$

The actual standard deviation should therefore be

$$\sigma \quad = \quad \sqrt{0.08} \quad = \quad 0.29"$$

which constitute an insignificant reduction from 0.33". Since the effect is slightly on the conservative side when neglected, a reduction will not be applied to the measured standard deviation in this report.

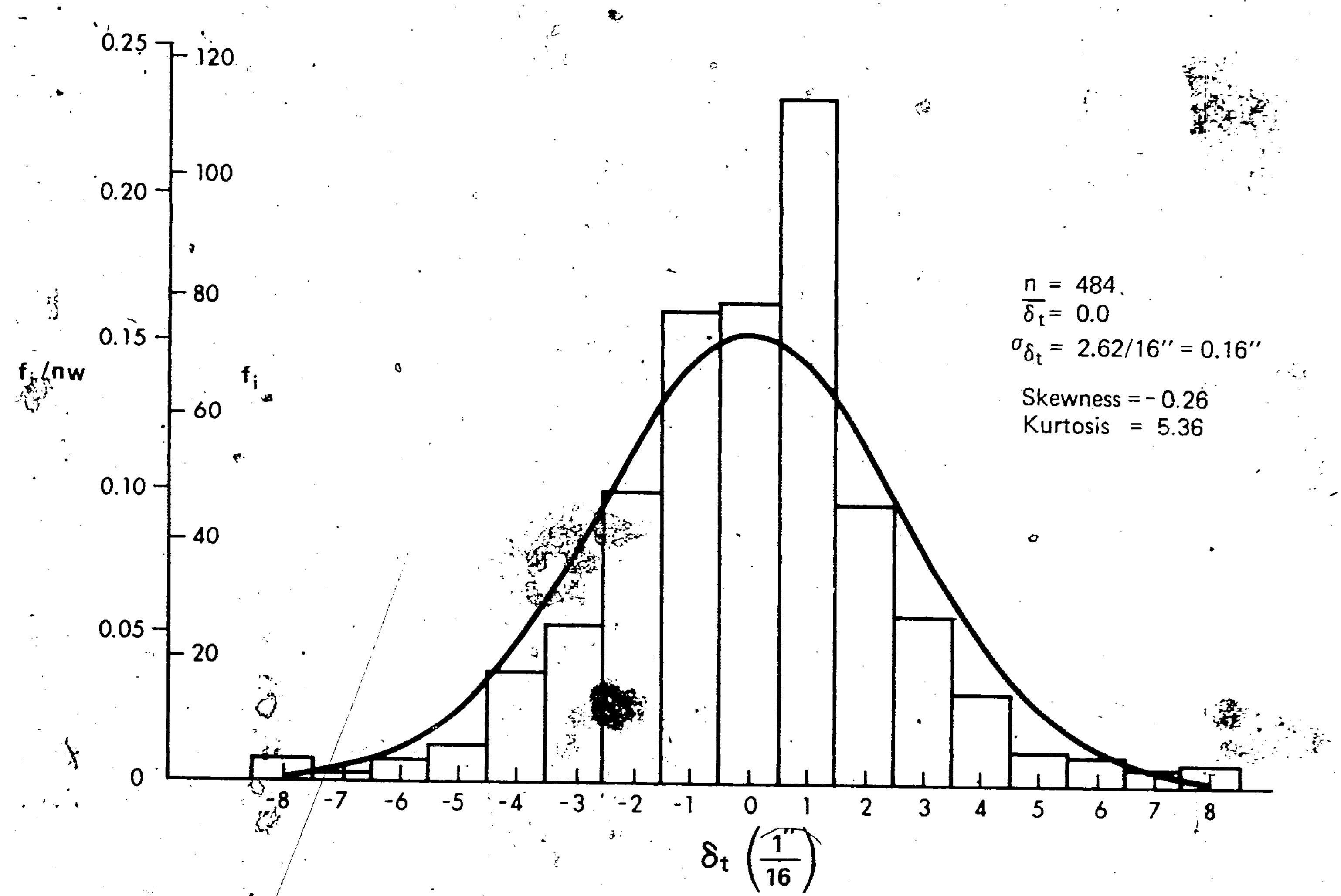


Figure E.2 Distribution of wall thickness deviations from the mean at a cross section