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LATERALLY LOADED PILES

by

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A REPORT

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a report entitled "Laterally Loaded Piles " as submitted by Brynja Gudmundsdottir in partial fulfilment of the requirements for the Degree of Master of Engineering.

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LATERALLY LOADED PILES

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Table of Contents

List of Tables

List of Figures

Figure		Page

NOTATION

Symbol	Unit	Definition
A	-	Coefficient for ultimate soil resistance (p-y curve :sand)
A_m	-	Coefficient for moment (Matlock-Reese)
A_s	-	Coefficient for slope (Matlock-Reese)
A_y	-	Coefficient for deflection due to shear (Matlock-Reese)
B	-	Coefficient for soil resistance (p-y curve:sand)
B_m	-	Coefficient for moment (Matlock-Reese)
B_s	-	Coefficient for slope (Matlock-Reese)
B_y	-	Coefficient for deflection due to moment (Matlock-Reese)
C	-	Coefficient in the parabolic section (p-y curve:sand)
C_u	kPa	Undrained shear strength
C_s	-	Shape factors for steel piles (Broms)
C_y	-	Coefficient for deflection (Matlock-Reese)
D	m	Diameter or width of pile
E	kPa	Modulus of elasticity - Youngs modulus
E_p	kPa	Modulus of elasticity of pile
E_s	kPa	Soil modulus (Matlock-Reese)
e	m	Eccentricity of load
f	m	Distance from ground surface or 1.5 pile diameter below ground surface to location of maximum bending moment (Broms)

f_y	kPa	Yield stress of pile material (Broms)
I	m^4	Moment of inertia
I_p	m^4	Moment of inertia of pile section
I_{yP}	-	Displacement influence factor for applied horizontal load (Poulos)
I_{yM}	-	Displacement influence factor for applied moment(Poulos)
I_{yF}	-	Displacement influence factor for fixed head pile (Poulos)
$I_{\theta P}$	-	Rotation influence factor for applied horizontal load (Poulos)
$I_{\theta M}$	-	Rotation influence factor for applied moment (Poulos)
J	-	Empirical adjustment factor (p-y curve soft clay)
K	kPa	Coefficient fo earth pressure
K_a	kPa	Coefficient of active earth pressure
K_o	kPa	Coefficient of earth pressure at rest
K_p	kPa	Coefficient of passive earth pressure
K_R	-	Pile flexibility factor
k_h	kN/m^3	Coefficient of horizontal subgrade reaction
k	kN/m^3	Coefficient of soil modulus (Matlock -Reese)
L	m	Embedded length of pile
M	kNm	Moment

M_{yield}	kNm	Yield or ultimate moment resistance of the pile section
m_h	kN/m⁴	ratio between coefficient of horizontal subgrade reaction and depth below surface
m	-	slope of line between points m and u (p-y curve:sand)
N_p	-	Coefficient of ultimate resistance (p-y curves)
n	-	Power of the parabolic section (p-y curve sand)
n_1	-	Coefficient, function of the unconfined compressive strength (Broms)
n_2	-	Coefficient, function of the pile material (Broms)
P	kN	Lateral load
P_a	kN	Working load for single pile (Broms)
P_m	kN	Maximum allowable working load (Broms)
P_u	kN	Ultimate lateral load
p	kN/m	Load per unit length of pile
p_u	kN/m	Ultimate resistance from theory (p-y curve:sand)
p_{cd}	kN/m	Ultimate resistance well below ground surface (p-y curve:sand)
p_{ct}	kN/m	Ultimate resistance near ground surface (p-y curve:sand)
p_0	kN/m²	Contact pressure corresponding to earth pressure at rest

p_o'	kN/m^2	Contact pressure on vertical face for horizontal displacement y_o
p_a	kN/m_2	Contact pressure on area acted upon by active earth pressure
p_p	kN/m_2	Contact pressure on area acted upon by passive earth pressure (= p_o' + p)
Q	kN	Axial load
q_u	kPa	Unconfined compressive strength
S	-	Slope of the pile
S	m^3	Section modulus about an axis perpendicular to the load plane
T	m	Relative stiffness factor (Matlock-Reese)
V	kN	Shear force
x	m	Depth below ground surface
x_r	m	Depth below ground surface to transition in coefficient of ultimate resistance equation (p-y curve:soft clay)
x_t	m	Depth below ground surface to transition in ultimate soil resistance equation (p-y curve:sand)
y	m	Displacement of pile
y_o	m	Initial displacement, required for increasing the coefficient of earth pressure on a vertical wall from K_o to K_o
y_m	m	Deflection at point m (p-y curve: sand)
y_k	m	Deflection of point k (p-y curve: sand)

y_{50}	m	Deflection at one half the ultimate soil resistance
z	-	Depth coefficient (Matlock-Reese)
γ	kN/m^3	Unit weight of the soil
γ_s	-	Poisson's ratio
β	1/m	Factor for cohesive soil (Broms)
η	1/m	Factor for cohesionless soil (Broms)
βL	-	Dimensionless length factor for cohesive soil
ηL	m	Dimensionless length factor for cohesionless soil
ε_{50}	-	Strain corresponding to one half the maximum principal stress difference
ϕ	°	Angle of internal friction
σ_x	kN/m^2	Overburden pressure
Θ	-	Pile rotation

1. INTRODUCTION

Large lateral loads and moments on superstructures caused by waves, winds, seismic forces, surcharges etc. are transferred to the desired soil strata by means of a single pile or a pile group.

In designing piles for lateral load, the designer should avail himself of more than one method whenever possible.

In this project a single active pile will be investigated. The deflection at the ground surface and the maximum moment in the pile will be calculated. This will be done for three different methods and the results will be compared for different types of cohesive soil (overconsolidated clay and normally consolidated clay), and for cohesionless soil (medium dense sand). However comparison with test result for calculated values is not possible because the examples are calculated by taking representative values for the properties of each type of soil .

Many different methods for calculating the lateral capacity of piles are presented in the literature. Those selected here are the ones developed by Broms (1964a) and (1964b), Matlock-Reese (1961) and Poulos (1971). All these methods are well known and are widely used.

In chapter two the methods are introduced and described. The theory on which they are based is reviewed, togerther with how the parameters are used in each method.

In chapter three the example design problem is introduced and used are given. The deflection at the ground surface and the maximum moment in the pile are calculated for these methods.

In chapter four the results are discussed and compared and explanation is sought when they do not agree.

2. DESCRIPTION OF METHODS

2.1 Introduction

In this chapter the methods used are reviewed. First the subgrade reaction coefficient is explained, based mainly on the work of Terzaghi (1955) and McClelland and Focht (1958). The differential equation which the subgrade reaction method has as its basis is presented, togerther with how it is used for both the Broms method and the Matlock-Reese method.

Broms (1964a and b) method is outlined both for cohesive and cohesionless soil. The Matlock-Reese (1961) hand solution is reviewed, and also the construction of p-y (soil reaction-pile deflection) curves which is necessary to use that method. Construction of p-y curves is based on Reese et.al.(1975) for overconsolidated clay or stiff clay, on Matlock (1970) for normally consolidated clay or soft clay, and on Reese et.al. (1974) for cohesionless soil. The last method that is reviewed is the one by Poulos (1971) which is based on an elastic solution. The elastic analysis is discussed, followed by a description of how calculation is done by this method.

2.2 Subgrade reaction method

The coeffient of horizontal subgrade reaction k_h is defined as the ratio between a horizontal pressure per unit area of vertical surface and the corresponding horizontal displacement. Thus it is a measurement of the ability of the soil to resist horizontal deformation. The value of k_h depends on the elastic properties of the subgrade and on the dimension of the area acted upon by the subgrade pressure.

Consider a pile which has been driven into or is buried in subgrade. Before any horizontal force has been applied to the pile, the surface of contact between the pile and the subgrade is acted upon at any depth x below the surface by a pressure p_0 which is equal to or greater than the earth pressure at rest. If the pile is moved to the right the pressure at the left side will drop to very small value and on the right side will increase from p_0 to p_0'. The lateral displacement y_0 required to produce this change is very small and can be neglected. After the pile has moved a distance y_1 the pressure at each side will be:

$$\text{left side (active state) } p = 0 \tag{2.1}$$

$$\text{right side (passive state) } p = p_0' + k_h y_1 \tag{2.2}$$

The subgrade modulus of a stiff clay, k_h, is generally considered to have the same value at every point of the subgrade contact, independent of depth. Therefore, at any time, the subgrade reaction p is almost uniformly distributed over the right hand face of the pile. However, due to progressive consolidation of clay under constant load, y_1 increases and k decreases with time. Both quantities approach an ultimate value, which is the value that should be used in design.

For cohesionless subgrade material the values of k_h and y_1 are independent of time. However, the elastic modulus of sand increases with depth, therefore the coefficient of subgrade reaction is determined by $k_h = m_h x$ where x is the depth below subgrade and m is the ratio between coefficient of horizontal subgrade reaction and depth below surface. The value m_h is assumed to be the same for every point of the surface contact (Figure 2.1).

The width D of the pile also influences the horizontal displacement. For piles of diameter D_1 and nD_1 the lengths of the bulbs of pressure measured in the direction of movement of the pile are L and nL respectively (see Figure 2.1) Furthermore in both clay and sand the modulus of elasticity is constant in the horizontal direction. Hence in clay as well as in sand the horizontal displacement y increases in direct proportion to the width D.

2.3 Analytical design methods by subgrade reaction

For solving laterally loaded piles the pile-soil system is treated as analogous to a beam on an elastic foundation. These analyses have as their basis the Winkler model, which assumes that a medium can be approximated by an infinite series of closely-spaced, independent springs.

If we look at the elementary theory of bending, it is found that stresses and deflections in beams are directly proportional to applied loads. Looking at one element of the beam in Figure 2.2 and taking equilibrium, summing forces gives:

$$\sum F_y = 0 \Rightarrow p = -\frac{dV}{dx} \tag{2.3}$$

which is the rate at which the shearing force changes with the distance x from the midpoint of the length of the beam. Summing moments about point n gives:

$$\sum M_n = 0 \Rightarrow V = \frac{dM}{dx} - Q\frac{dy}{dx} \tag{2.4}$$

If the effects of shearing deformation and shortening of the beam axis are neglected, the expression for the curvature of the axis of the beam is:

$$EI \frac{d^2y}{dx^2} = -M \tag{2.5}$$

Combining these equations by substituion and differentiation yields:

$$EI \frac{d^4y}{dx^4} + Q \frac{d^2y}{dx^2} = P \tag{2.6}$$

which is the basic differential equation for bending of beam columns. It is also used for laterally loaded piles where the shear force is corrected by subgrade reaction theory, and y increases approximately in proportion to the applied load.

For cohesive soil the shear force is the horizontal pressure and is equal to:

$$p = k_h y \qquad (2.7)$$

For cohesionless soil it becomes:

$$p = m_h x y \qquad (2.7a)$$

because modulus of elasticity increases approximately in direct proportion to depth. Therefore it is assumed without serious error that the pressure p required to produce a given horizontal displacement y increases in direct proportion to the depth as shown. Equation 2.6 then becomes:

$$EI \frac{d^4y}{dx^4} + Q \frac{d^2y}{dx^2} = k_h y \quad \text{or} \quad m_h x y \qquad (2.6a)$$

To solve this equation one must make assumptions concerning the end conditions and then determine the constants.

Both Broms and Matlock-Reese use this equation for their solution. Broms assumes that the axial load is negligible compared to the buckling load, and he uses the shear force as previously described. Matlock-Reese use an iterative procedure to account for the non-linear behaviour relationship between pile deflection and soil resistance, until satisfactory compatibility is obtained between the predicted behaviour of the soil and the load-deflection relationship required by an elastic pile.

2.4 Broms' theoretical-empirical method

For the coefficient of horizontal subgrade reaction, Broms uses Terzaghi's values for cohesionless soils. For cohesive soil he established the following expression:

$$k = \frac{n_1 n_2}{D}(40q_u \text{ to } 160q_u) \tag{2.8}$$

The use of $80q_u$ gives good agreement with Terzaghi's values The value n_1 is a function of the unconfined compressive strength q_u,n_2 is a function of the pile material, and D is the pile diameter. The values of n_1 and n_2 are given in Table 2.1. The values of k_h for cohesionless soil are given in Table 2.2 for both Terzaghi's and Reese's recommendations.

For design Broms developed two basic design conditions, which are discussed below.

2.4.1 Allowable lateral deflection at working loads

Broms made the simplifying assumption that the axial load is negligible compared to the buckling load, and equation (2.6) thus reduces to

$$EI \frac{d^4y}{dx^4} = -k_h y \tag{2.6b}$$

He solved this equation for three pile conditions:

1) Fixity: free-headed or restrained,
2) Length: short, intermediate or long,
3) Soil type: cohesive or cohesionless.

He incorporated the criteria for various pile conditions in the corresponding equations and developed graphical relationships. From these curves it is possible to determine either the theoretical lateral deflection or the theoretical applied load if the other is known. Allowable lateral deflection or allowable load can then be determined by applying the appropriate adjustments.

2.4.2 Ultimate or failure load

This criterion assumes two modes of failure:
1) shear in the soil in the case of short stiff piles,
2) bending of the pile in the case of long piles (as governed by the plastic yield resistance of the pile section).

In the case of long piles Broms assumed that the pile developed plastic hinges permitting sufficient rotation to mobilize the bending moments along the pile length. Based on these assumptions he treated the pile as a statically determinant beam with a distributed load corresponding to the particular condition. Below is a summary of the design procedure for Broms' method. Figures that are referred to are at the end of the chapter.

Summary of Broms design method.

STEP 1:

Determine the general soil type (cohesive or cohesionless) within the critical depth below the surface (about four to five pile diameters D)

STEP 2:

Determine the horizontal coefficient of subgrade reaction k within the critical depth from equation (2.8) for cohesive soil, or by selecting an appropriate value from Table 2.2 for cohesionless soil.

STEP 3:

Adjust k_h for loading and soil conditions:

a. Cyclic loading in cohesionless soil:

1. $k_h = 1/2k_h$ from Step 2 for medium to dense sand
2. $k_h = 1/4k_h$ from Step 2 for loose soil

b. Static loads resulting in soil creep:

1. Soft and very soft normally consolidated clays
   $k_h = (1/3 \text{ to } 1/6)\ k_h$ from Step 2
2. Stiff to very clays
   $k_h = (1/4 \text{ to } 1/2)\ k_h$ from Step 2

STEP 4:

Determine pile parameters:

a. Modulus of elasticity E

b. Moment of inertia I

c. Section modulus S about an axis perpendicular to the load plane

d. Yield stress of pile material f_y

e. Embedded pile length L

f. Diameter or width D

g. Eccentricity of applied load e for free-headed piles -- i.e., vertival distance between ground surface and lateral load

h. Dimensionless shape factor C_s (for steel piles only):

1. Use 1.3 for piles with circular cross-section
2. Use 1.1 for H-section when the applied load is in the direction of the pile's maximum resisting moment (normal to pile flanges).
3. Use 1.5 for H-section piles when the applied load is in the direction of the pile's minimum resisting moment (parallel to pile flanges)

i. M , the resisting moment of the pile = $C_s f_y S$

STEP 5:

Determine factor β or η:

a. $\beta = \sqrt[4]{k_h D/4EI}$ for cohesive soil, or

b. $\eta = \sqrt[5]{k_h/EI}$ for cohesionless soil

STEP 6:

Determine the dimensionless length factor:

a. βL for cohesive soil, or

b. ηL for cohesionless soil.

STEP 7:

Determine if the pile is long or short:

a. Cohesive soil

1. $\beta L > 2.5$ (2.25) (long pile)

2. $\beta L < 2.0$ (2.25) (short pile)

3. $2.0 < \beta L < 2.5$ (intermediate pile)

b. Cohesionless soil

1. $\eta L > 4.0$ (long pile)

2. $\eta L < 2.0$ (short pile)

3. $2.0 < \eta L < 4.0$ (intermediate pile)

<u>STEP 8:</u>

Determine other soil parameters:

a. Rankine passive pressure coefficient for cohesionless soil $K_p = \tan^2(45 + \phi/2)$, where ϕ = angle of internal friction.

b. Average effective soil unit weight γ over embedded length.

c. Cohesion C_u = one-half the unconfined compressive strength $q_u/2$.

<u>STEP 9:</u>

Determine the ultimate (failure) load P_u for a single pile:

a. Short Free- or Fixed-Headed Pile in Cohesive Soil
Using L/D (and e/D for free-headed case), enter Figure 2.5a, select the corresponding value of $P_u/C_u D^2$, and solve for P_u.

b. Long Free- or Fixed-Headed pile in Cohesive Soil
Using $M_{yield}/C_u D^3$ (and e/D for free-headed case), enter Figure 2.5b, select the corresponding value of $P_u/C_u D^2$, and solve for P_u.

c. Short Free- or Fixed-Headed Pile in Cohesionless Soil

Using L/D (and e/L for the free-headed case), enter Figure 2.6a, select the corresponding value of $P_u/ K_p D^3 \gamma$, and solve for P_u.

d. Long Free- or Fixed-Headed Pile in Cohesionless Soil

Using $M_{yield}/D^4 \gamma K_p$, (and e/D for the free-headed case), enter Figure 2.6b select the corresponding value of $P_u/ K_p D^3 \gamma$, and solve for P_u.

e. Intermediate Free- or Fixed-Headed Pile in Cohesionless Soil

Calculate P for both a short pile (Step 9c) and a long pile (Step 9d) and use smaller value.

STEP 10:

Calculate the maximum allowable working load for a single pile P_m from the ultimate load P_u determined in Step 9:

$$P_m = P_u/2.5$$

STEP 11:

Calculate the working load for a single pile P_a corresponding to a given design deflection y at the ground surface, or the deflection corresponding to a given design load. If P_a and y are not given, substitute the value of P_m from Step 10 for P_a in the following cases and solve for y_m:

a. Free- or Fixed-Headed Pile in Cohesive Soil

Using βL (and e/L for the free-headed case), enter Figure 2.3, selecting the corresponding value of $y k_h DL/P_a$, and solve for P_a or y.

b. Free- of Fixed-Headed Pile in Cohesionless Soil

Using nL (and e/L for the free-headed case), enter Figure 2.4, select the corresponding value of $y(EI)^{3/5}k^{2/5}/P_a L$, and solve for P_a or y.

STEP 12:

If $P_a \geq P_m$, use P_m and calculate y_m (Step 11)

If $P_a < P_m$, use P_a and y.

If P_a and y are not given,use P_m and y_m.

STEP 13:

Reduce the allowable load selected in Step 12 to account for method of installation: for driven piles use no reduction, and for jetted piles use 0.75 of the value.

2.4.3 Moments by Broms method

The mode of failure of laterally loaded pile depends on the depth of embedment and on the degree of end restraint.

For short piles failure takes place when the soil yields along the total length of the pile, and the pile rotates as a unit around a point located at some depth below ground surface (in the case of a free head pile) Restrained failure takes place when the applied load is equal to the ultimate lateral resistance of the soil, and the pile moves as a unit through the soil.

For long piles the mechanism of failure is when a plastic hinge forms at the location of the maximum bending moment. Failure takes place when the bending moment is equal to the moment resistance of the pile section (for free-head piles). Restrained pile failure takes place when two plastic hinges form along the pile. The two plastic hinges form when the maximum positive bending moment at depth f or (1.5D + f) below ground surface, and the maximum negative bending moment at the bottom of the pile cap or lateral bracing system, both reach the yield resistance of the pile section.

For the case of a restrained pile, an intermediate length of pile has also to be taken into account. In this case failure takes place when the restraining moment at the head of the pile is equal to the ultimate moment resistance of the pile section, and the pile rotates around a point located at some depth below the ground surface.

The maximum moment occurs at the depth below surface where the shear force in the pile is equal to zero, at depth (f + 1.5D) for cohesive soil and f for cohesionless soil.

Cohesive soil:

The distance f and the maximum bending moment M_{max} can be calculated from the two equations

$$f = \frac{P}{9C_u D} \quad \text{and} \quad M_{max}^{pos} = P(e + 1.5D + 0.5f) \qquad (2.9)$$

Cohesionless soil:

Here it has been assumed that lateral deflections are sufficiently large at failure to develop the full passive resistance (equal to three times the passive Rankine earth pressure), from the ground surface to the location of maximum bending moment. Thus we have for free-head piles:

$$f = 0.82\sqrt{P/\gamma DK_p} \quad \text{and} \quad M_{max}^{pos} = P(e + 0.67f) + Qa \quad (2.10)$$

2.5 Matlock-Reese hand solution

2.5.1 Deflections

When using this method a set of p-y curves are needed. Here they will be constructed semi-empirically for a static loading condition and will be described in next section.

The primary solution will consist of finding the set of elastic deflections of the pile (including the short jacket leg extension) which will simultaneously satisfy:

1) the non-linear resistrance deformation relations which are predicted for the soil,
2) the elastic bending properties of the piles,
3) the angular stiffness of the upper structure at the pile to structure connection.

The force-deformation characteristics of the soil are described by a set of predicted p-y curves. In Figure 2.7 there is a graphical definition of p-y curves and in Figure 2.8 there are typical resistance curves for soil at various depths. Since solution for the interaction problem relies on repeated applications of elastic theory, a secant modulus of soil reaction E_s is required, which is defined as $E_s = -p/y$.

This is only a computation device which is generally independent of pile size and depends primarily on soil properties (not a unique soil property).

The differential equation for a beam is:

$$EI \frac{d^4y}{dx^4} = p \qquad \frac{d^4y}{dx^4} + \frac{E_s}{EI} y = 0 \tag{2.6c}$$

This is the same equation as used for Broms' method. Here this equation is solved by trial and error by estimating the value of T, relative stiffness factor, until $T_{obtained}$ is equal to T_{tried}. Then correct set of E_s are found and deflection and bending moments are computed.

In this solution the deflection y of the pile at any depth x is:

$$y = A_y \frac{P_t T^3}{EI} + B_y \frac{M_t T^2}{EI} \tag{2.11}$$

where A_y, B_y are functions of $z = x/T$, relative stiffness factor T is $T^5 = EI/k$, and subscript t refers to the pile at the top.

It is convenient to define an additional set of nondimensional deflection coefficients by rearranging equation (2.11) as:

$$y = C_y \frac{P_t T^3}{EI} \qquad \text{where } C_y = A_y + \frac{M_t}{P_t T} B_y \tag{2.12}$$

To begin the solution of the example problem it is necessary to assume temporarily that the form of soil modulus variation $E_s = kx$ will be a satisfactory approximation of the actual final E_s variation. Available non-dimensional solutions are limited to a pile of constant bending stiffness.

The slope at the top of the pile is:

$$S_y = A_{st}\frac{P_t T^2}{EI} + B_{st}\frac{M_t T}{EI} \tag{2.13}$$

where subscript s stands for shear and subscript t for at top.

The relation between M_t and S_t is:

$$S_t = \frac{e}{3.5EI} M_t \tag{2.14}$$

Combining these two equations yields:

$$\frac{M_t}{P_t T} = \frac{A_{st} T}{\frac{e}{3.5} - B_{st} T} \tag{2.15}$$

Since the relative stiffness factor T depends on the coefficient of soil modulus variation k and this quantity in turn depends on non-linear resistance characteristics, the solution must proceed by a repeated trial and adjustments of the values of T (or k) until the deflection and resistance patterns of the pile agree as closely as possible with the resistance-deflection (p-y) relations previously estimated for the soil. Even though the final set of secant modulii ($E_s = -p/y$) may not vary in a perfectly linear fashion with depth, proper fitting of $E_s = kx$ will usually produce satisfactory solutions.

The steps in the Matlock-Reese method are as follows:

1) Calculations are made for certain depths x
2) Estimate the first value of T (trial value)
3) Calculate $z = x/T$
4) Find the coefficients A_y and B_y given in Table 2.3 (or find C_y)
5) Calculate y from equation (2.11)
6) Find p for this y from p-y curves
7) Calculate $E_s = -p/y$
8) Set up a graph E_s vs x and find k from this graph (see Figure 2.9), and giving more weight to points at depth less than $x = 0.5T$.
9) Calculate $T_{obtained} = \sqrt[5]{EI/k}$
10) Compare $T_{obtained}$ and T_{trial} and when they are the same the trial and error process is completed. Use a graph of T_{trial} vs $T_{obtained}$ (see Figure 2.10.) The final set of computations for the E value is made as a check.

2.5.2 Moments

Computations of values of bending moments along the pile are made by application of the equation:

$$M = A_M P_t T + B_M M_t \tag{2.16}$$

The non-dimensional coefficients A and B are found in Table 2.2. Subscript m refers to moments.

2.6 Construction of p-y curves.

2.6.1 Overconsolidated clay (Reese and Welch 1975)

The step by step procedure for this material is:

1) Obtain the best possible estimate of the variation of shear strength and effective unit weight with depth, and of the value of ε_{50}, the strain corresponding to one-half the maximum principal stress difference. If no value of ε_{50} is available use a value of 0.005 or 0.010, the larger value being more conservative. Here the lower value will be used, ε_{50}= 0.005 (Reese and Welch).
2) The ultimate soil resistance p is computed according to:

$$p = \left(3 + \frac{\gamma x}{C_u} + 0.5\ \frac{x}{D}\right) C_u D \tag{2.17a}$$

$$p = 9 C_u D \tag{2.17b}$$

and the smaller value is used for each depth.

3) Compute displacement y_{50} at one-half the ultimate soil resistance.

$$y_{50} = 2.5D\varepsilon_{50} \qquad (2.18)$$

4) The points on the curve are now computed by:

$$\frac{p}{p_u} = 0.5\left(\frac{y}{y_{50}}\right)^{1/4} \qquad (2.19)$$

(Beyond $y = 16y_{50}$, $p = p_u$ for all values of y)

2.6.2 Normally consolidated clay (Matlock 1970)

The steps are the same as for overconsolitated clay but the values and equations are different. The procedure given here is for submerged clay soils, which are normally consolidated or slightly overconsolidated.

1) Here the value of ε_{50} may be assumed to be between 0.005 and 0.020, the smaller value being more applicable to brittle or sensitive clay and the larger value to disturbed or remolded soils or unconsolidated sediments. An intermediate value of ε_{50} = 0.010 is probably satisfactory for most purposes.

2) Ultimate resistance.

If soft clay soil is confined so that plastic flow around a pile occurs in horizontal planes, the ultimate resistance per unit length of pile may be expressed as:

$$p_u = N_p C_u D \qquad (2.20)$$

where

$$N = \begin{cases} 9 \text{ at a considerable depth below the surface} \\ 3 + \dfrac{\sigma_x}{C_u} + J\,\dfrac{x}{D} \text{ (between the free soil surface and depth the variation is described by this equation)} \\ 2\text{-}4 \text{ very near the surface in front of the pile} \\ 3 \text{ (for a cylindrical pile, this is believed to be appropriate)} \end{cases}$$

The value of J should to be determined empirically. Here the value of J = 0.5 will be used as it is considered to be appropriate for this type of clay.

3) Compute y_{50} using Skempton's approach

$$y_{50} = 2.5\varepsilon_{50} D \qquad (2.21)$$

4) The points on the curve are now computed by

$$\frac{p}{p_u} = 0.5 \left(\frac{y}{y_{50}}\right)^{1/3} \qquad (2.22)$$

Beyond $y = 8y_{50}$, $p = p_u$ for all values of y. The final shape of the p-y curve of clay is shown in Figure 2.11 for normally consolidated clay. The shape for overconsolidated clay is almost the same but the power in the equation is different.

2.6.3 Cohesionless soil (Reese, Cox and Koop 1974)

For construction of p-y curves for sand, the outline from Reese et.al.(1974) is used. The method is based on theory as well as empiricism.

Recommended procedure:

1) Obtain soil properties and pile dimensions ϕ, γ, D
2) Use the following parameters for computing soil resistance:

   $\alpha = \frac{\phi}{2}$, $\beta = 45 + \frac{\phi}{2}$, $K_o = 0.4$, $K_a = \tan^2(45 - \frac{\phi}{2})$
3) The following equation are used to calculate soil resistance:

   a) Ultimate resistance near ground surface:

$$p_{ct} = \gamma H\left(\frac{K_o H \tan\phi \sin\beta}{\tan(\beta-\phi)\cos\alpha} + \frac{\tan\beta}{\tan(\beta-\phi)}(D + H\tan\beta\tan\alpha) + K_o H\tan\beta(\tan\phi\sin\beta - \tan\alpha) - K_a D\right) \quad (2.23)$$

   b) Ultimate resistance well below the ground surface.

$$p_{cd} = K_a D\gamma H(\tan^8\beta - 1) + K_o D\gamma H\tan\phi\tan^4\beta \quad (2.24)$$

4) Find the intersection x_t of two above equations $p_{ct} = p_{cd}$

5) Select depths at which p-y curves are desired.

6) Establish $y_u = 3b/80$ and $p_u = Ap_c$, where A is an empirical adjustment factor given in Figure (2.10a)

7) Establish $y_m = b/60$ and $p_m = Bp_c$, where B is an empirical adjustment factor given in Figure (2.10b).

8) Establish the slope of the initial portion of the p-y curve by selecting the appropriate value of k_h.

9) Parabola to be fitted between points k and m

$$p = Cy^{1/n} \qquad (2.25)$$

10) Fit the parabola between these points :

a) Slope of line between u and m: $m = \frac{p_u - p_m}{y_u - y_m}$

b) The power of the parabola n: $n = \frac{p_m}{m y_m}$

c) Obtain C as follows: $C = \frac{p_m}{y^{1/n}}$

d) Determine point k as: $y_k = (\frac{c}{kx})^{n/n-1}$

e) Use the equation in step 9 to compute the points.

The final shape of the p-y curve is given in Figure 2.12.

2.7 Poulos method

2.7.1 Elastic analysis

The methods previously discussed are based on the Winkler model or spring medium, and the continuity of the soil is not taken into account. In this method it is assumed that the soil is an elastic mass and an ideal homogeneous, isotropic, semi-infinite elastic material, having a Young's modulus E and Poisson's ratio γ_s which are unaffected by the presence of the pile.

In this analysis the pile is assumed to be a thin rectangular strip of width D (or if circular the diameter is used instead of the width), length L and having constant flexibility EI_p.

For an elastic condition the horizontal shear stress developed between the soil and the sides of the pile is not taken into account. The pile is divided into elements all of equal length except the top and bottom elements which are of half length. Each element is acted upon by a uniform horizontal stress p which is assumed to be constant across the width of the pile.

For this condition the horizontal displacement of the soil and of the pile are equal along the pile (for purely elastic conditions). In this analysis the soil and pile displacements are evaluated and equated at the element center, except for the top and bottom elements where displacements are calculated by equating soil and pile displacements at these points. Using the appropriate equilibrium conditions, sufficient equations are qbtained to solve for the unknown horizontal displacement at each element.

Two conditions of practical interest at the pile head are considered:

1) a free-head pile, free rotation occurs

2) a fixed-head pile, no rotation occurs

Two major variables influencing pile behaviour are the length to diameter ratio L/D and a factor K_R, herein known as the pile flexibility factor $K_R = \frac{E_p I_p}{E L^4}$, a dimensionless measure of the flexibility of the pile relative to the soil.

2.7.2 Calculation of displacement

Calculations by this method are performed by using charts. Displacement and rotation are expressed in terms of dimensionless influence factors which are function of the pile flexibility factor K . The length to diameter ratio is relatively small so $\gamma_s = 0.5$ may be used for all conditions.

The horizontal displacement is expressed as:

for a free-head pile $$y = I_{yP}\frac{P}{EL} + I_{yM}\frac{M}{EL} \qquad (2.26)$$

for a fixed-head pile $$y = I_{yF}\frac{P}{EL} \qquad (2.27)$$

The rotation θ for a free-head pile is:

$$\theta = I_{\theta P}\frac{P}{EL^2} + I_{\theta M}\frac{M}{EL^2} \qquad (2.28)$$

Values of $I_{y,\theta}$ are given in Figures 2.14 to 2.17. From theory I_{yM} and $I_{\theta M}$ should be the same but this is not quite true. The difference is usually 5% except for very flexible piles where it is 10 to 15%. This discrepancy indicates the order of accuracy of the corresponding values of $I_{\theta M}$.

2.7.3 Calculation of moments

Moments are calculated by charts of K and L/D. From these charts the value M/PL is obtained from Figure 2.18 for free-head piles and Figure 2.19 for fixed-head piles. For free-head piles the maximum moment is at depth of 0.1L to 0.4L below the surface, the lower value being associated with stiffer piles. For fixed-head piles the maximum moment is at the head of the pile except if the pile is very flexible.

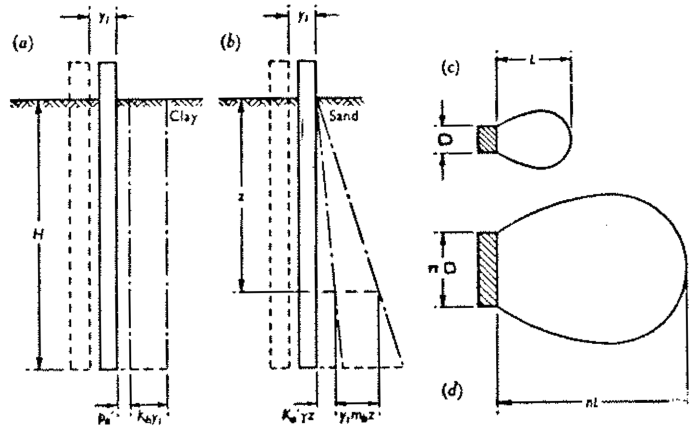


Figure 2.1 - Distribution of lateral pressure a) in stiff, clay b) in sand, c) influence of width of beam on dimensions of bulb of pressure (Terzaghi 1955).

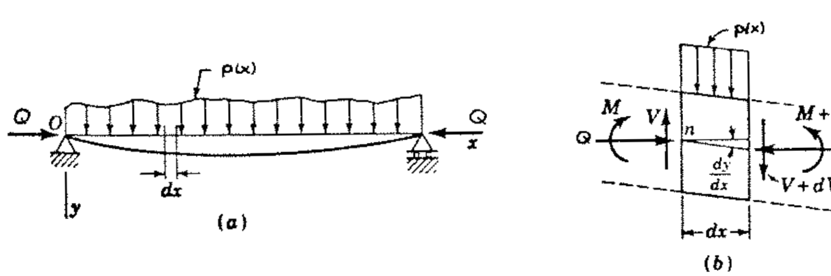


Figure 2.2 - a) Beam coulumn under lateral load b) force on one element of the beam coulumn (Terzaghi 1955).

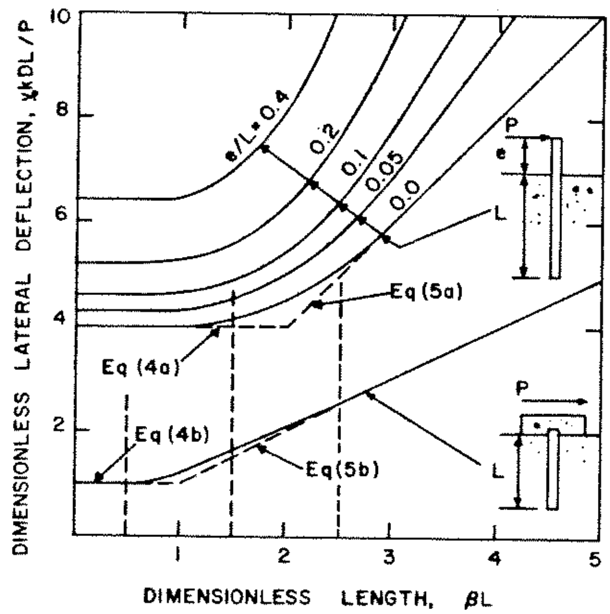


Figure 2.3 - Cohesive soil - Lateral deflections at ground surface (Broms 1964a)

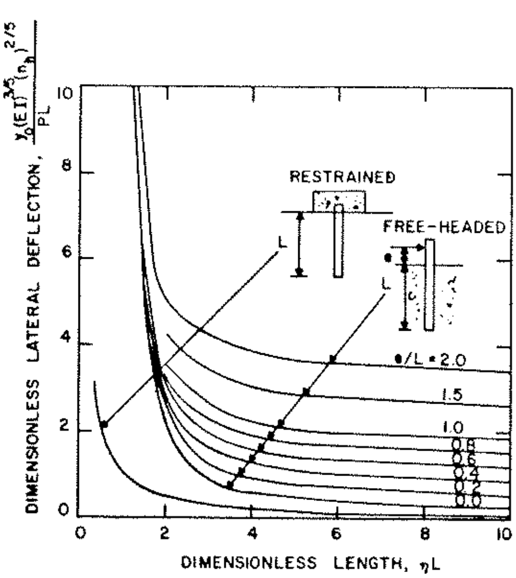
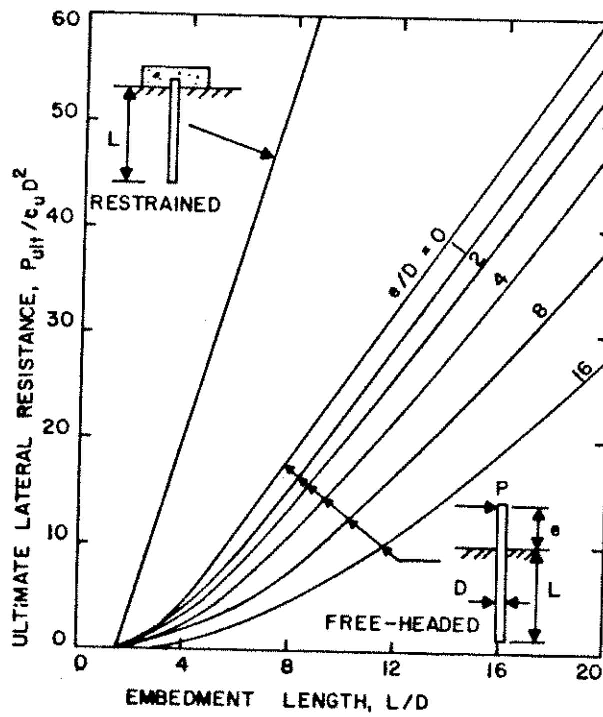


Figure 2.4 - Cohesionless soil - Lateral deflections at ground surface (Broms 1964b).



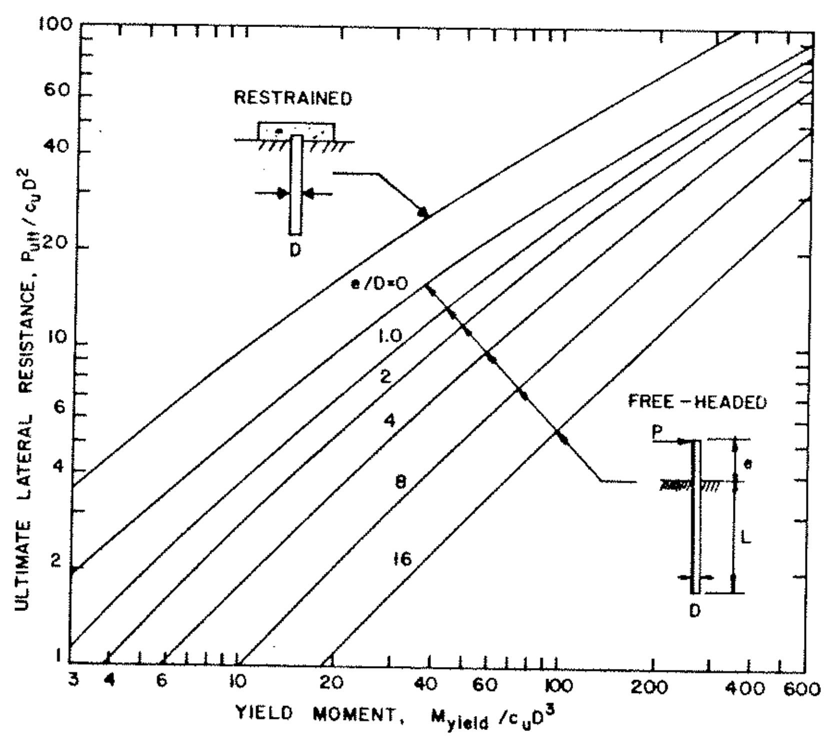
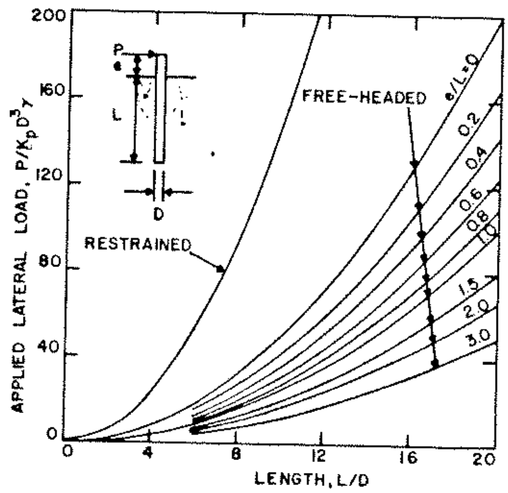


Figure 2.5 - Cohesive soil - Ultimate lateral resistance
a) short piles, b) long piles (Broms 1964a)



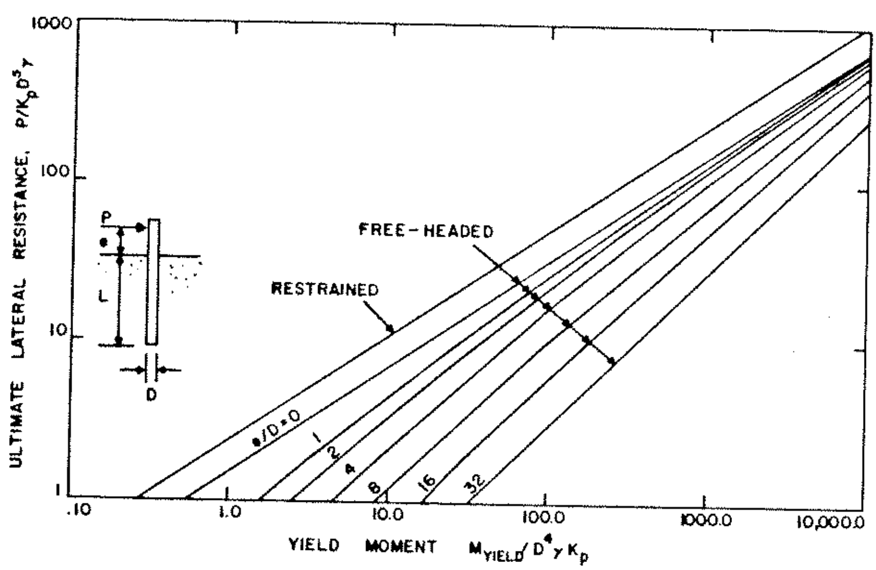


Figure 2.6 - Cohesionless soil - Ultimate lateral resistance
a) short piles, b) long piles (Broms 1964b)

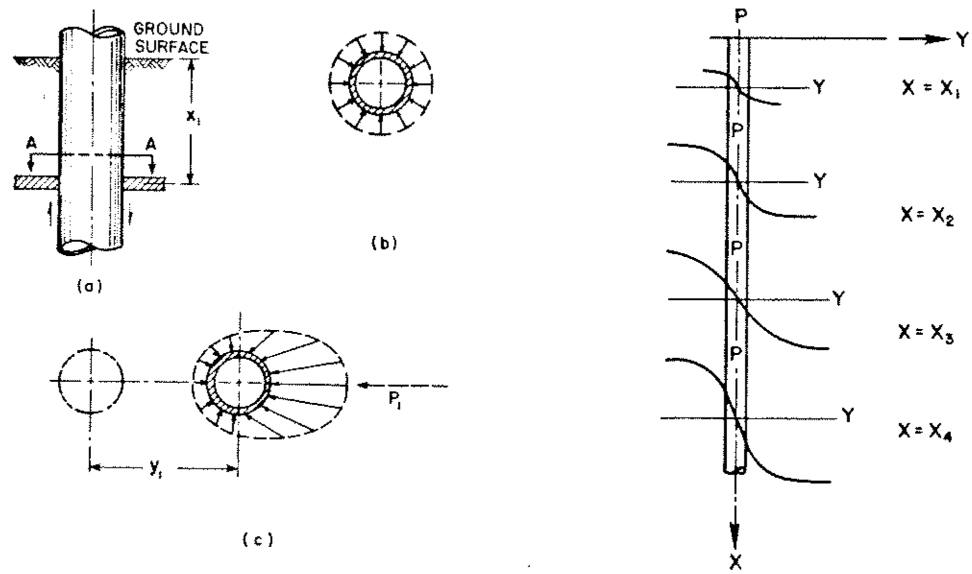


Figure 2.7 - Graphical definition of p-y: a) pile elevation b) view AA - earth pressure distribution prior to lateral loading c) view AA - earth pressure distribution after lateral loading d) p-y curves, (Reese et.al. 1974).

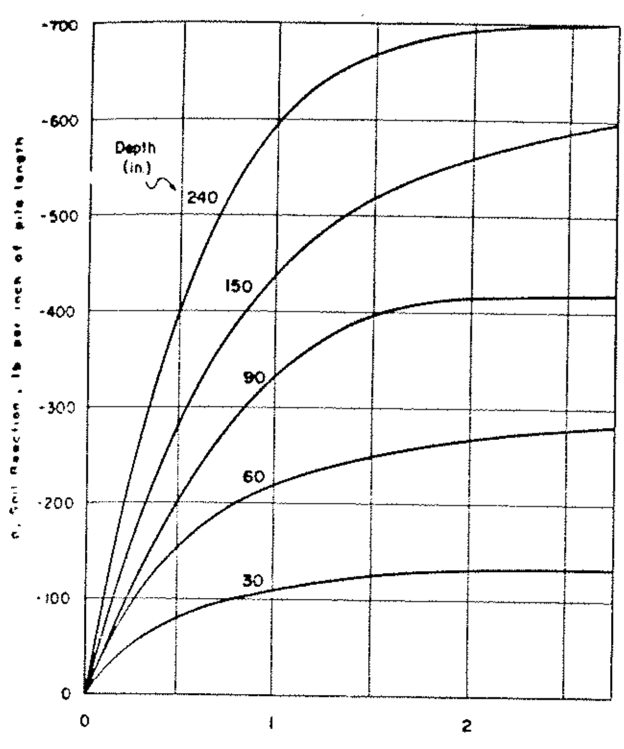


Figure 2.8 - Typical resistance-deflection curves predicted for soil at various depth, (Matlock-Reese 1961)

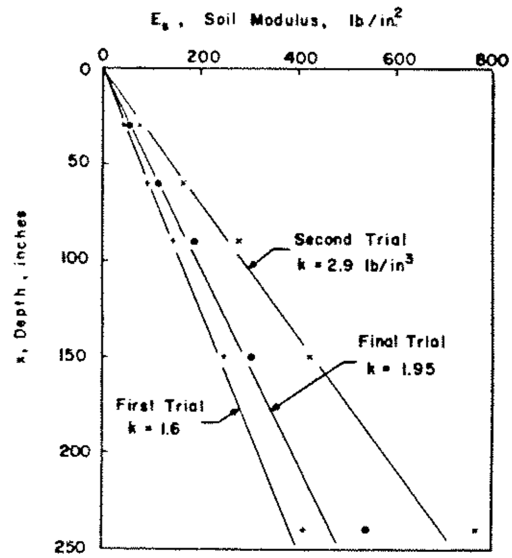


Figure 2.9 - Trial plots of soil modulus values, (Matlock-Reese 1961).

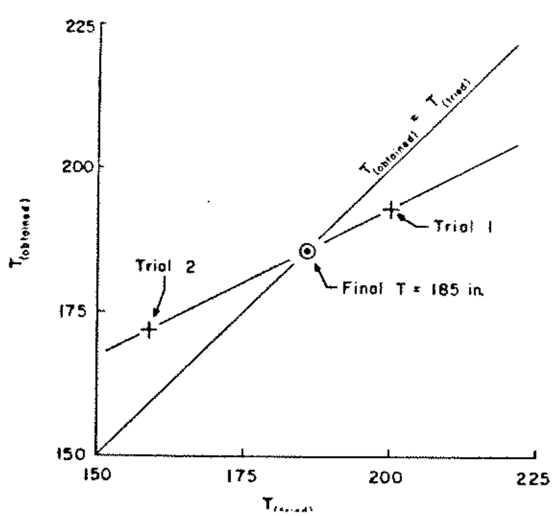


Figure 2.10 - Interpolation for final value of relative stiffness factor T, (Matlock-Reese 1961).

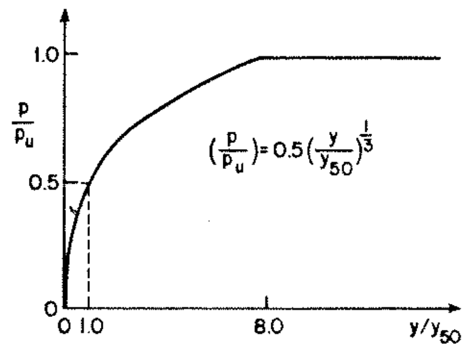


Figure 2.11 - Characterstic shapes of p-y curves for soft clay short term static loading, (Matlock 1970).

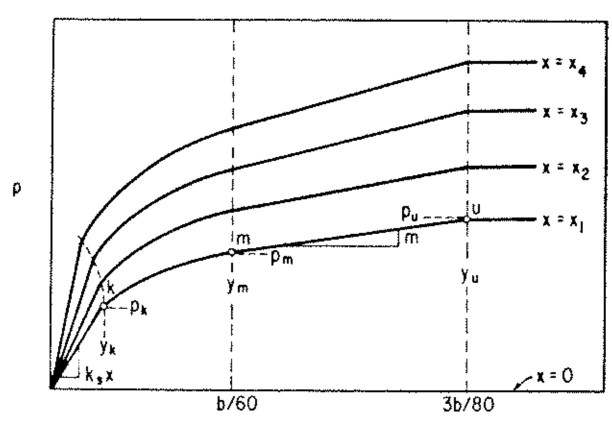


Figure 2.12 - Typical family of p-y curves for proposed criteria in cohesionless sand (Reese et.al. 1974).

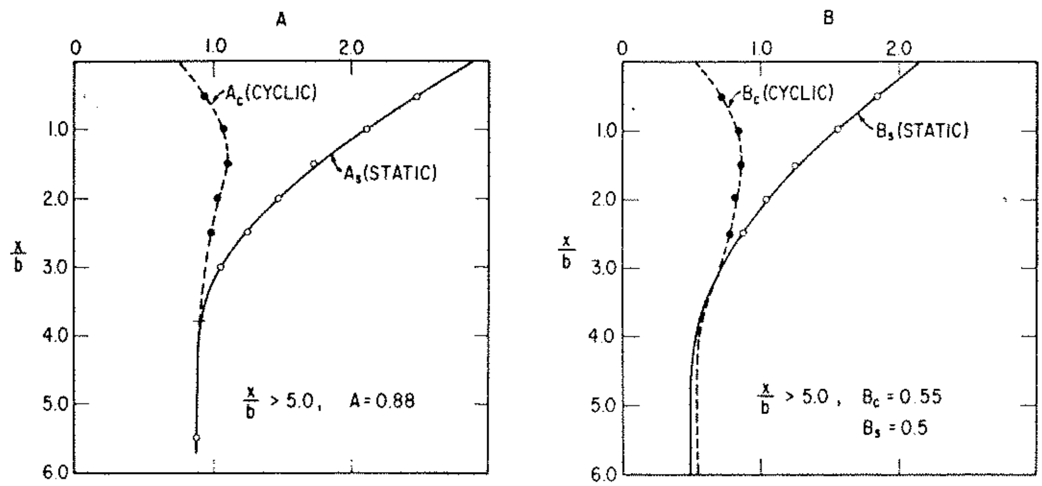


Figure 2.13 - Non-dimensional coefficients for ultimate soil resistance vs depth for sand a) coefficient A, b) coefficient B, (Reese et.al. 1974).

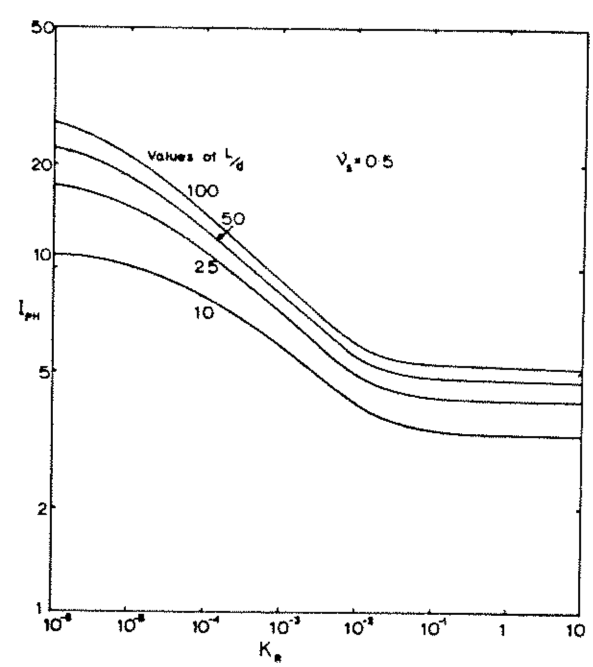


Figure 2.14 - Influence factor I_{yP} - Free-head pile, (Poulos 1971).

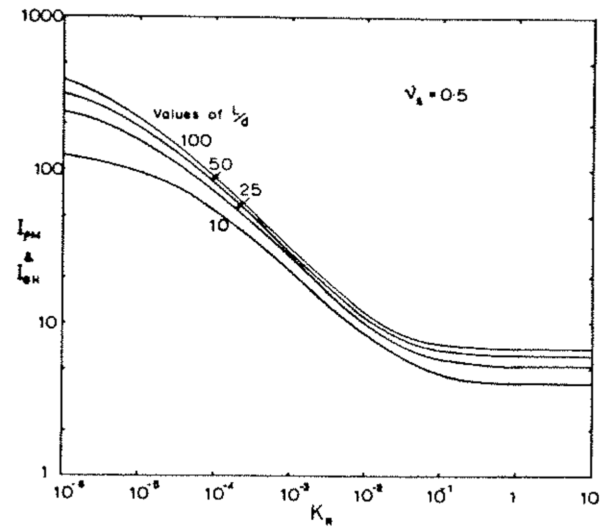


Figure 2.15 - Influence factor I_{yM} and $I_{\theta P}$ - Free-head pile, (Poulos 1971).

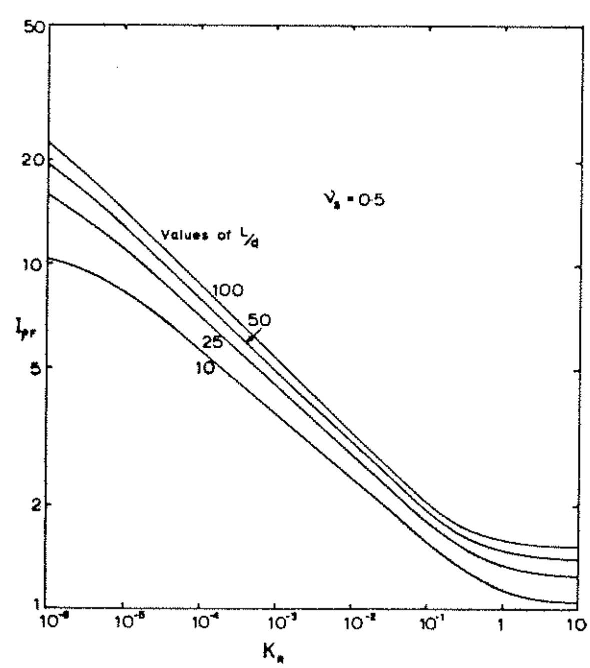


Figure 2.16 - Influence factor I_{yF} - Fixed-head pile, (Poulos 1971).

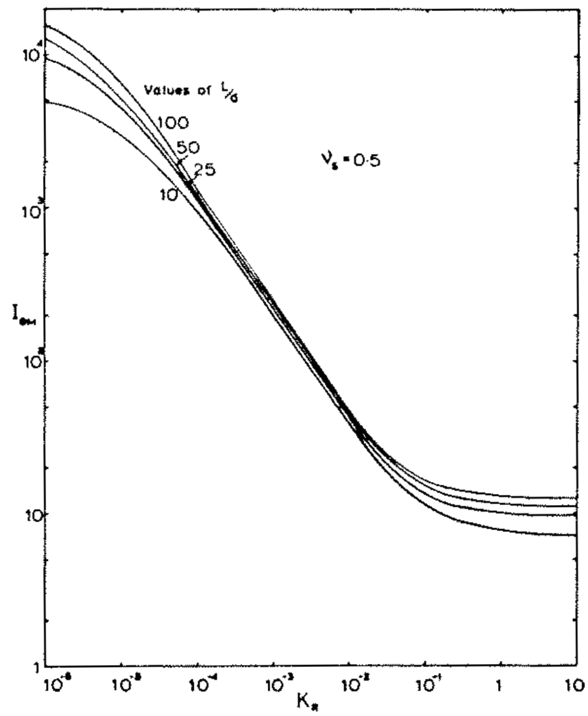


Figure 2.17 - Influence factor $I_{\theta M}$ - Free-head pile, (Poulos 1971).

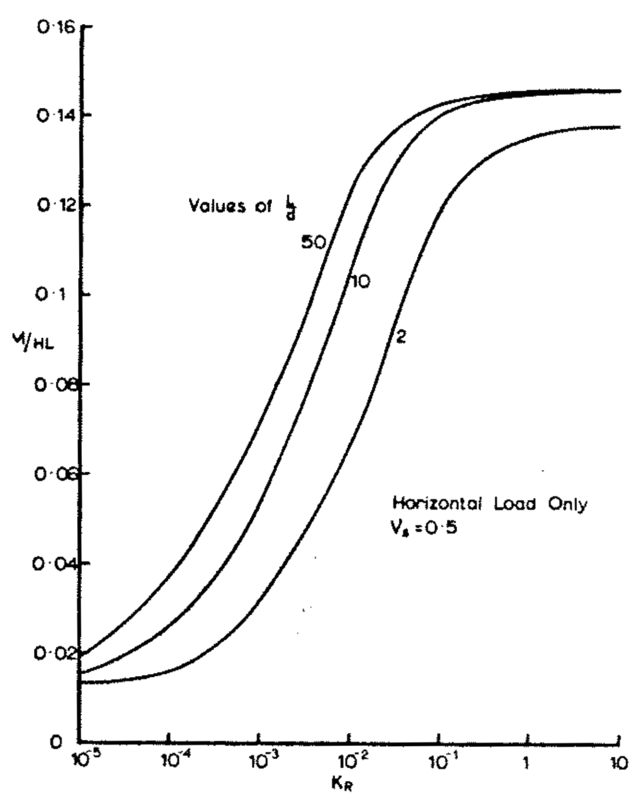


Figure 2.18 - Maximum moment in free-head pile, (Poulos 1971).

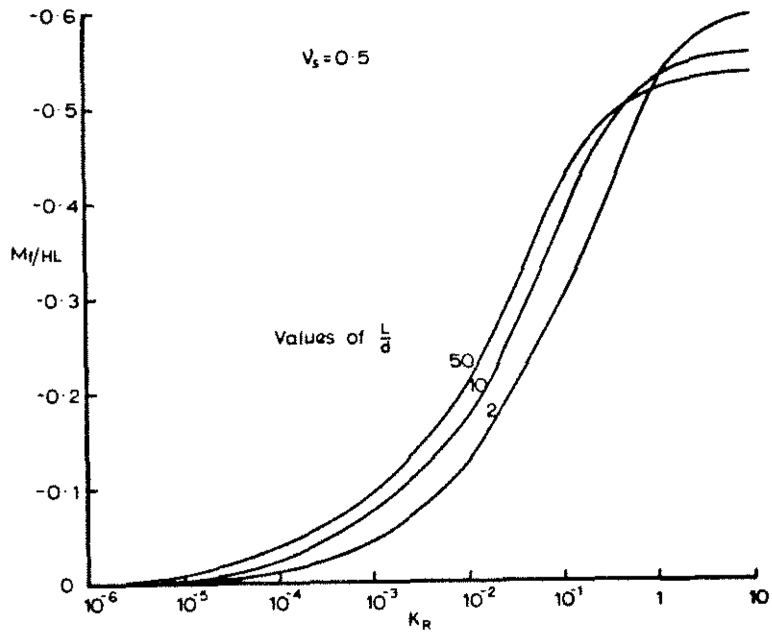


Figure 2.19 - Fixing moment at head of fixed-head pile, (Poulos 1971),

Table 2.1: Evaluation of the coefficient n_1 and n_2, (Broms 1964a).

Unconfined Compressive Strength q_u, tons per square foot	Coefficient n_1
Less than 0.5	0.32
0.5 to 2.0	0.36
Larger than 2.0	0.40

Pile Material	Coefficient n_2
Steel	1.00
Concrete	1.15
Wood	1.30

Table 2.2: Coefficient of horizontal subgrade reaction k, (Reese et.al 1974).

TERZAGHI'S VALUES OF k FOR SUBMERGED SAND

Relative Density	Loose	Medium	Dense
Range of Values of k (lbs/in^3)	2.6 - 7.7	7.7 - 26	26 - 51

RECOMMENDED VALUES OF k FOR SUBMERGED SAND

(Static and Cyclic Loading)

Relative Density	Loose	Medium	Dense
Recommended k (lbs/in^3)	20	60	125

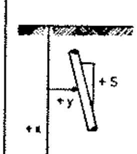
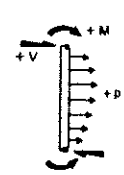
Table 2.3: Coefficients and equations for Matlock-Reese hand solution, (Matlock-Reese 1961).

Table 1

Coefficients and Equations for Long Piles, $E_s = kx$

Z	A_y	A_S	A_M	A_V	A_p
0.0	2.435	-1.623	0.000	1.000	0.000
0.1	2.273	-1.618	0.100	0.989	-0.227
0.2	2.112	-1.603	0.198	0.956	-0.422
0.3	1.952	-1.578	0.291	0.906	-0.586
0.4	1.796	-1.545	0.379	0.840	-0.718
0.5	1.644	-1.503	0.459	0.764	-0.822
0.6	1.496	-1.454	0.532	0.677	-0.897
0.7	1.353	-1.397	0.595	0.585	-0.947
0.8	1.216	-1.335	0.649	0.489	-0.973
0.9	1.086	-1.268	0.693	0.392	-0.977
1.0	0.962	-1.197	0.727	0.295	-0.962
1.2	0.738	-1.047	0.767	0.109	-0.885
1.4	0.544	-0.893	0.772	-0.056	-0.761
1.6	0.381	-0.741	0.746	-0.193	-0.609
1.8	0.247	-0.596	0.696	-0.298	-0.445
2.0	0.142	-0.464	0.628	-0.371	-0.283
3.0	-0.075	-0.040	0.225	-0.349	0.226
4.0	-0.050	0.052	0.000	-0.106	0.201
5.0	-0.009	0.025	-0.033	0.013	0.046

Z	B_y	B_S	B_M	B_V	B_p
0.0	1.623	-1.750	1.000	0.000	0.000
0.1	1.453	-1.650	1.000	-0.007	-0.145
0.2	1.293	-1.550	0.999	-0.028	-0.259
0.3	1.143	-1.450	0.994	-0.058	-0.343
0.4	1.003	-1.351	0.987	-0.095	-0.401
0.5	0.873	-1.253	0.976	-0.137	-0.436
0.6	0.752	-1.156	0.960	-0.181	-0.451
0.7	0.642	-1.061	0.939	-0.226	-0.449
0.8	0.540	-0.968	0.914	-0.270	-0.432
0.9	0.448	-0.878	0.885	-0.312	-0.403
1.0	0.364	-0.792	0.852	-0.350	-0.364
1.2	0.223	-0.629	0.775	-0.414	-0.268
1.4	0.112	-0.482	0.688	-0.456	-0.157
1.6	0.029	-0.354	0.594	-0.477	-0.047
1.8	-0.030	-0.245	0.498	-0.476	0.054
2.0	-0.070	-0.155	0.404	-0.456	0.140
3.0	-0.089	0.057	0.059	-0.213	0.268
4.0	-0.028	0.049	-0.042	0.017	0.112
5.0	0.000	0.011	-0.026	0.029	-0.002

Term	Equation	Sign Convention
Depth	$x = ZT$	
Deflection	$y = A_y \frac{P_t T^3}{EI} + B_y \frac{M_t T^2}{EI}$	
Slope	$S = A_s \frac{P_t T^2}{EI} + B_s \frac{M_t T}{EI}$	
Moment	$M = A_M P_t T + B_M M_t$	
Shear	$V = A_V P_t + B_V \frac{M_t}{T}$	
Soil Reaction	$p = A_p \frac{P_t}{T} + B_p \frac{M_t}{T^2}$	

3. CALCULATIONS

3.1 Introduction

In this chapter the deflection at ground surface and maximum moment for a certain pile will be calculated by the previously described methods. First the parameters for both the soil and pile are given and the results of the calculations for each method are given. Finally a summary of the calculations is given at the end of the chapter.

3.2 Dimensions of pile and properties of soil

For the three different types of soil the dimensions and properties of the soil are assumed the same. The pile is assumed to be of reinforced concrete, having a circular cross section.

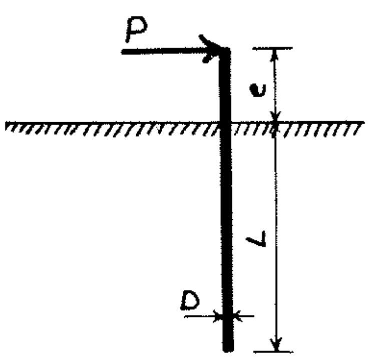


Figure 3.1
Example calculated.

$P = 200kN$

$L = 16m$

$e = 0.8m$

$D = 0.8m$

$E = 4x10^6 psi = 2.82x10^7 kN/m^2$

$I = --- = 0.0201m^4$

$EI = 5.6682\ 10^5 kNm^2$

The values of the soil parameters are given in Table 3.1.

Table 3.1 -- Soil parameters used in example

properties \ soil	over-consolidated clay	normally consolidated clay	cohesion-less soil
unconfined compressive strength q_u	100kPa	30kPa	-
unit weight γ	20kN/m³	20kN/m³	19kN/m³
cohesion C_u	50kPa	15kPa	-
angle of internal friction ϕ	-	-	30°
modulus of elasticity E	3500kN/m²	940 kN/m²	3447 kN/m²

3.3 Broms method

This method is described in section 2.4. Here calculations are made for a certain force so steps 9 and 10 are omitted. The results from the calculations are summarized in Table 3.2 and all equations and figures used are found in Chapter 2.

3.4 Calculation of p-y curves

To be able to use the Matlock-Reese hand solution a set of p-y curves is needed, so they will be constructed first. For this, calculations are done for certains depths: here the selected depths are x = 0, 0.5, 1.0, 1.5, 2.0, 2.5 and 5.0 m.

Table 3.2 -- Calculation of y and M by Broms Method.

parameter \ soil	over-consolidated clay	normally consolidated clay	cohesion-less soil
$k_h' = \dfrac{n_1 n_2 80 q_u}{D}$	n_1 = 0.36 n_2 = 1.15 q = 100kPa	n_1 = 0.32 n_2 = 1.15 q = 30kPa	Use Reese recom.
k_h' (kN/m³)	4140	1104	16290
$k_h = (1-1/3)k'$	1380	368	16290
β, η	0.1485	0.1067	0.4917
βL, ηL (m)	2.38	1.71	7.87
length	interm(long)	short	long
e/L	0.05	0.05	0.05
$y k_h DL/P$ $y(EI)^{3/5} k_h^{2/5}/PL$	5.7 -	4.8 -	- 0.4
y (m)	0.0645	0.2038	0.00943
f	0.556	1.85	1.717
M (kNm)	455.6	585.2	390.0

3.4.1 Cohesive soil

For overconsolidated soil the procedure is described in section 2.6.1. First the value of P is calculated according to equations (2.17a) or (2.17b) and the lower of the two values is used in the analysis. The value of y_{50} is computed according to equation (2.18) where ε_{50} = 0.005.

The curves are generated using equation (2.19). Beyond y =

$16y_{50}$, $p = p_u$ for all values of y.

The values of p versus y are given in Table 3.3 for overconsolidated soil and plotted in Figure 3.2.

Table 3.3 -- Calculation of p for each depth and deflection. Overconsolidated clay (unit kN/m)

y	x = 0m	x = 0.5m	x = 1.0m	x = 1.5m	x = 2.0m	x = 2.5m	x = 5.0m
0.16	120.0	140.5	161.0	181.5	202.0	222.5	325.0
0.12	111.7	130.7	149.8	168.9	188.0	207.1	302.4
0.10	106.7	124.9	143.2	161.4	179.6	197.8	289.0
0.08	100.9	118.1	135.4	152.6	169.6	187.1	273.3
0.06	93.9	109.9	126.0	142.0	158.1	174.1	254.3
0.04	84.9	99.3	113.8	128.3	142.8	157.3	229.8
0.02	71.4	83.5	95.7	107.9	120.1	132.3	193.2
0.015	66.4	77.7	89.0	100.4	111.8	123.1	179.8
0.009	58.4	68.4	78.4	88.4	98.4	108.4	158.3
0.006	52.8	61.8	70.9	79.9	88.9	97.9	143.0
0.004	47.7	55.9	64.0	72.2	80.3	88.5	129.2
0.002	40.1	47.0	53.8	60.7	67.5	74.4	108.7
0.001	33.7	39.5	45.3	51.0	56.8	62.6	91.0

For normally consolidated clay the procedure is given in section 2.6.2 and equation (2.20) is used for calculation of p_u where:

$$N_p = \begin{cases} 3 \text{ very near the surface} \\ 3 + \dfrac{\sigma_x}{C_u} + J\dfrac{X}{D} \text{ from surface fo depth } x_r \\ 9 \text{ below depth } x_r \end{cases}$$

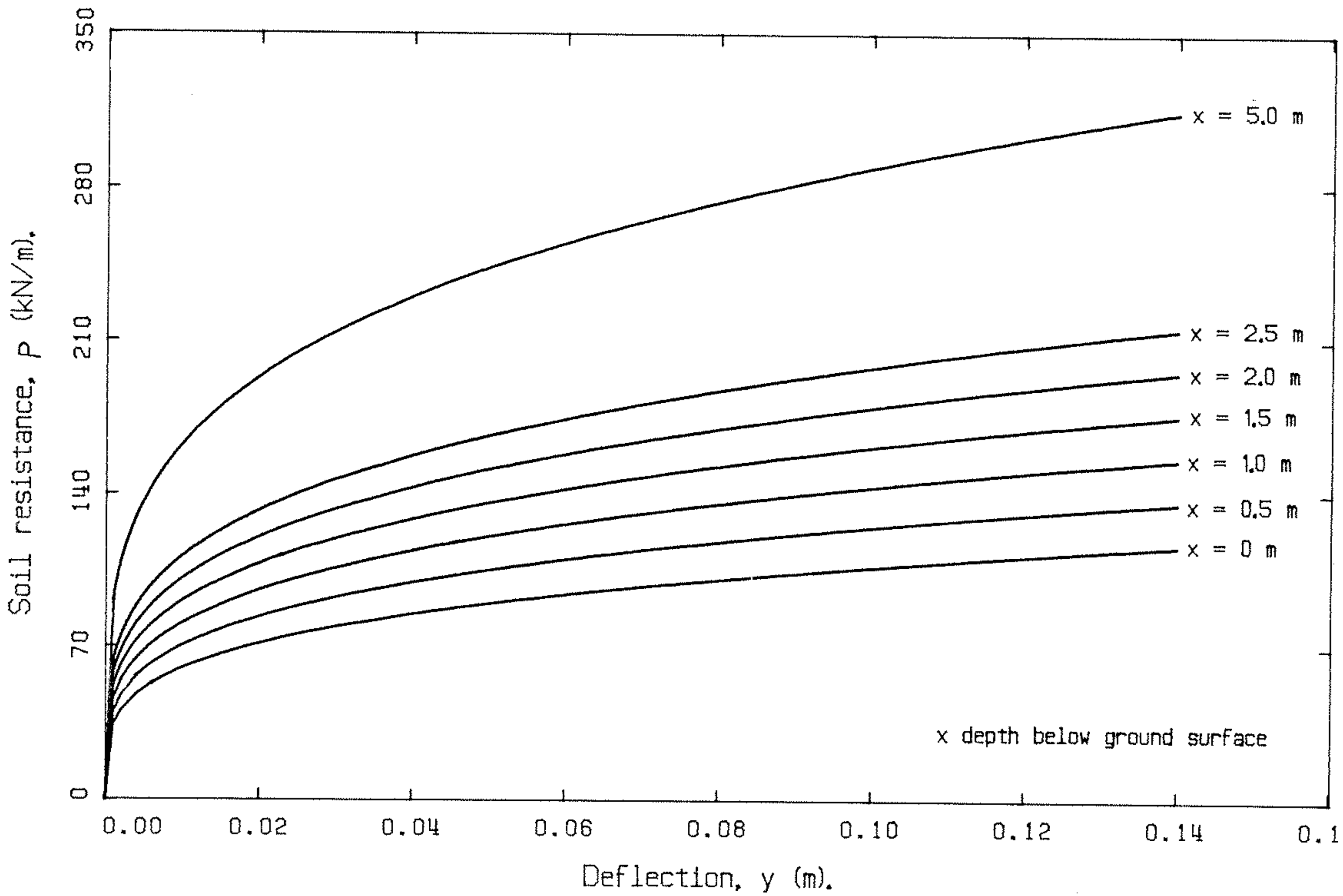


Figure 3.2 - P-y curves for overconsolidated clay.

where depth x_r is found by equating the two lasts parts for N that is:

$$x_r = \frac{6D}{\frac{\gamma D}{C_u} + J} = 3.06\text{m}$$

The value of y_{50} is computed by using equation (2.18) and here the value of ε_{50} used is 0.010.

Equation (2.21) is used to generate the curves beyond $y = 8y_{50}$ and $p = p_u$.

The values of p versus y for normally consolidated soil are given in Table 3.4 and the plotted in Figure 3.3.

3.4.2 Cohesionless soil

The step by step procedure for calculating the p-y curve for sand is given in section 2.6.3 and the calculated values for each step are given here:

1) From Table 3.1 the soil parameters and pile dimensions are: $\phi = 30°$, $\gamma = 19\text{kN/m}^2$, $D = 0.8\text{m}$.
2) $\alpha = 15°$, $\beta = 60°$, $K_0 = 0.4$ and $K_a = \tan^2 30$
3-4) Find x_t - found by equating equation (2.22) and equation (2.23); x = 10.913m and p_c is calculated for selected depths in Table 3.6.
5) The selected depths are the same as before x = 0, 0.5, 1.0, 1.5, 2.0, 2.5 and 5.0m.
6) $y_u = \frac{3D}{80} = 0.03\text{m}$ and $p_u = Ap_c$

Table 3.4 -- Calculation of p for each depth and deflection. Normally consolidated clay (unit kN/m)

y	x = 0m	x = 0.5m	x = 1.0m	x = 1.5m	x = 2.0m	x = 2.5m	x = 3.0m	x = 5.0m
0.16	36.0	47.7	59.5	71.2	83.0	94.7	106.5	108.0
0.14	43.4	45.7	56.9	68.1	79.4	90.6	101.7	103.3
0.12	32.7	43.4	54.1	64.7	75.4	86.1	96.8	98.1
0.10	30.8	40.8	50.9	60.9	71.0	81.0	91.1	92.3
0.08	28.6	37.9	47.2	56.6	65.9	75.2	84.5	85.7
0.06	26.0	34.4	42.9	51.4	59.9	68.3	76.8	77.9
0.04	22.7	30.1	37.5	44.9	52.3	59.7	67.1	68.0
0.02	18.0	23.9	29.8	35.6	41.5	47.4	53.3	54.0
0.015	16.4	21.7	27.0	32.4	37.7	43.0	48.4	49.1
0.009	13.8	18.3	22.8	27.3	31.8	36.3	40.8	41.4
0.006	12.1	16.0	19.9	23.8	27.8	31.7	35.6	36.1
0.004	10.5	14.0	17.4	20.8	24.3	27.7	31.1	31.6
0.002	8.4	11.1	13.8	16.5	19.3	22.0	24.7	25.1
0.001	6.6	8.8	11.0	13.1	15.3	17.5	19.6	19.9

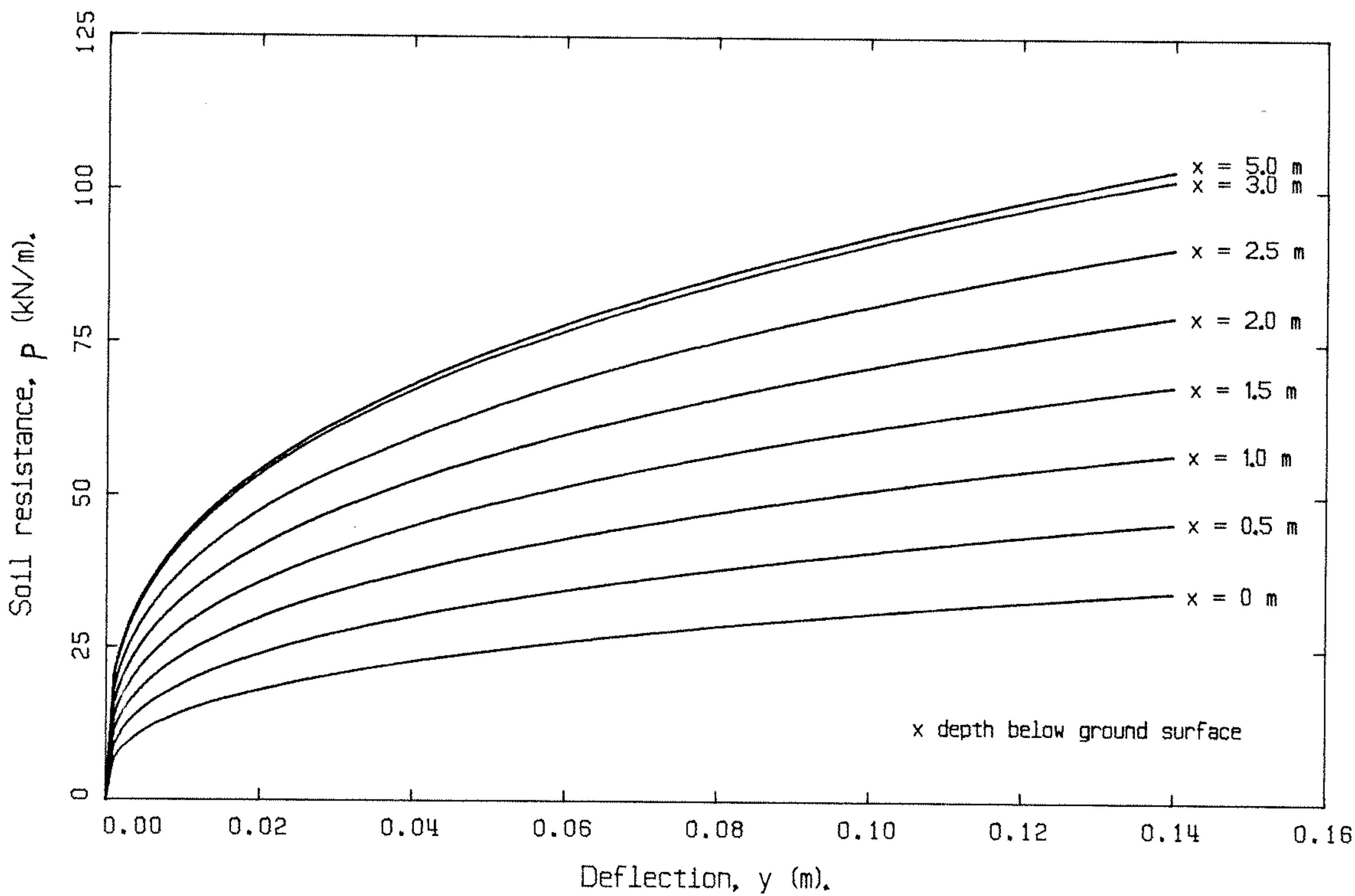


Figure 3.3 - P-y curves for normally consolidated clay.

7) $y_m = \frac{D}{60} = 0.0133m$ and $p_m = Bp_c$

where A and B are taken from Figure 2.10 and given in Table 3.5.

8) The slope of the initial portion of the p-y curve is $k_h x$ where $k_h = 16290 kN/m^3$.

9) Between points k and m equation (2.25) is used to generate the points (see Figure 2.12).

10) Constants m, n, C, y are calculated for each depth and they are given in Table 3.6.

The p-y curves for cohesionless soil are given in Figure 3.4.

Table 3.5 -- Coefficient A and B from Figure 2.13

depth x	x/D	A	B
0.0	0.000	2.87	2.14
0.5	0.625	2.38	1.78
1.0	1.250	1.95	1.38
1.5	1.875	1.57	1.11
2.0	2.500	1.22	0.84
2.5	3.125	1.00	0.68
5.0	6.250	0.88	0.50

Table 3.6 -- Calculation of p-y curve for cohesionless soil

x	p kN/m	p kN/m	p kN/m	m kN/m²	n -	C -	k x kN/m²	y m	p kN/m
0.5	29.36	69.85	52.24	1036.60	3.7083	167.366	8145	0.0049	39.88
1.0	76.86	149.88	106.07	2628.61	3.0263	441.740	16290	0.0046	74.62
1.5	142.53	223.77	158.21	3933.83	3.0163	662.020	24435	0.0045	110.37
2.0	226.40	276.21	190.18	5161.92	2.7632	907.300	32580	0.0037	119.59
2.5	328.35	328.35	223.28	6304.32	2.6563	1134.326	40.726	0.0032	130.47
5.0	1116.73	977.44	555.37	25324.63	1.6447	7666.828	81450	0.0024	195.77

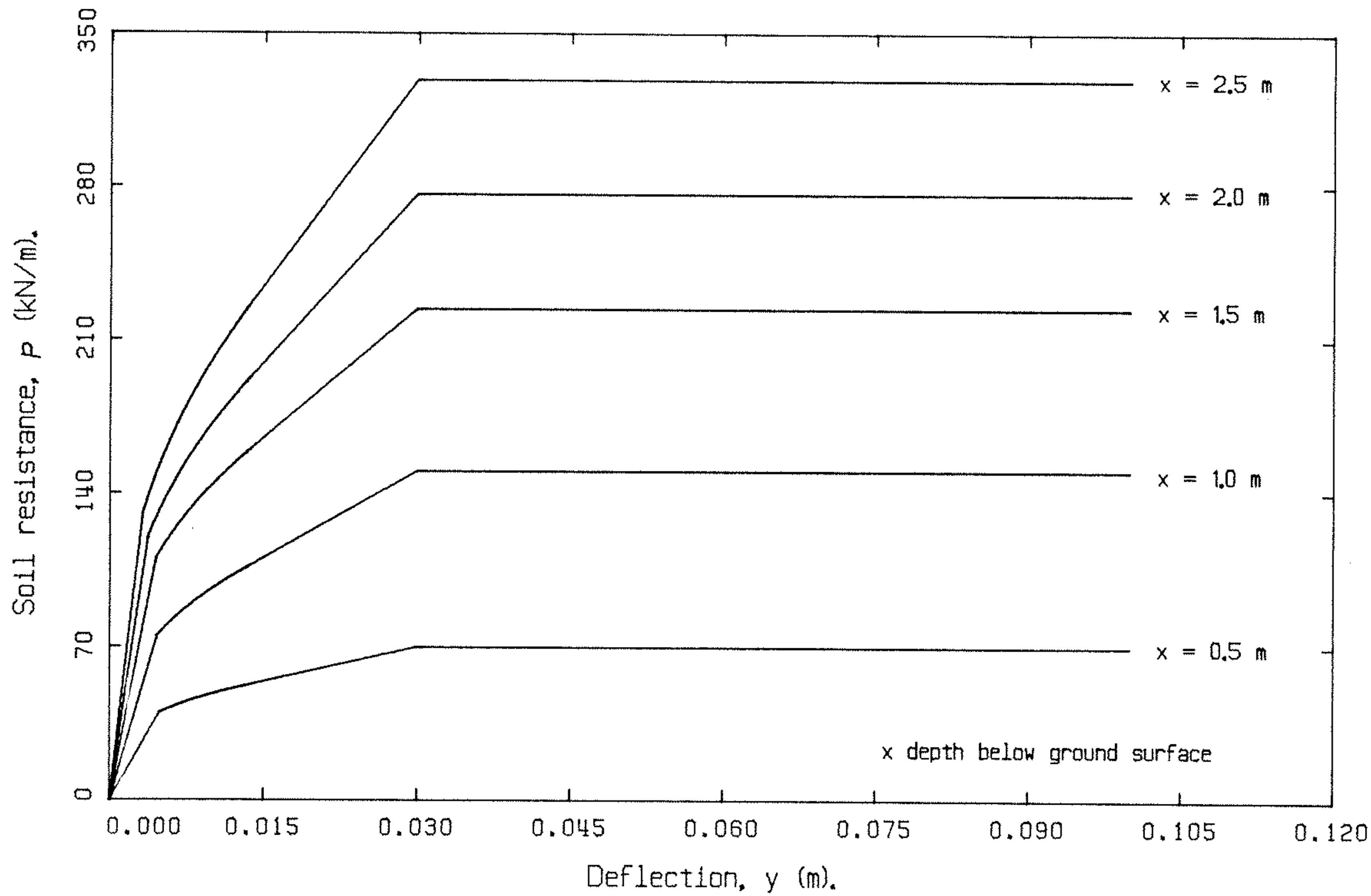


Figure 3.4 - P-y curves for cohesionless soil.

3.5 Matlock-Reese hand solution

3.5.1 Deflections

The procedure for this method is given in section 2.5.1 and the results of calculations are given in Tables 3.7 to 3.9 for overconsolidated clay, normally consolidated clay and cohesionless soil respectively. This method is an iterative procedure. The results of the iterations are presented in Figures 3.5 to 3.8 and in Tables 3.7 to 3.9.

3.5.2 Moments

Here the moment at the top of the pile is M = 0. The maximum moment in the pile thus occurs when A_m is maximum, which is A_m=0.772 at z = 1.4. The maximum moment for the three soil types are given below.

Overconsolidated clay:

$M_{max} = A_m P_t T$ = 0.772 200 3.10 = 478.6kNm

at depth x = 1.4T = 4.34m

Normally consolidated clay:

M_{max} = 0.772 200 4.25 = 656.2kNm

at depth x = 5.95m

Cohesionless soil:

M_{max} = 0.772 200 2.15 = 332.0 kNm

at depth x = 3.01m

Table 3.7 -- Calculation of y by Matlock-Reese method. Overconsolidated clay.

T_{trial}	x	Z=x/T	A_y	y	-p	E	comments
5	0.0	0.0	2.435	0.1074	108	1005	
	0.5	0.1	2.273	0.1003	125	1246	k_{obt} =
	1.0	0.2	2.122	0.0936	140	1486	562.5
	1.5	0.3	1.952	0.0861	156	1812	
	2.0	0.4	1.796	0.0792	170	2147	T_{obt} = 4
	2.5	0.5	1.664	0.0725	183	2524	
	5.0	1.0	0.962	0.0424	233	5495	
2.5	0.0	0.0	2.435	0.0143	63	4701	
	0.5	0.2	2.112	0.0116	73	6271	
	1.0	0.4	1.796	0.0099	79	7980	k_{obt}=
	1.5	0.6	1.496	0.0083	86	10424	3409
	2.0	0.8	1.216	0.0067	92	13731	
	2.5	1.0	0.962	0.0053	97	18302	T_{obt} =2.78
	5.0	2.0	0.142	0.00078	75	95785	
3.0	0.0	0.000	2.435	0.0232	73	3147	
	0.5	0.167	2.171	0.0207	84	4058	k_{obt} =
	1.0	0.333	1.901	0.0181	94	5193	2184.2
	1.5	0.500	1.664	0.0159	102	6415	
	2.0	0.667	1.400	0.0133	108	8120	T_{obt} =3.04
	2.5	0.833	1.173	0.0112	113	10089	o.k.
	5.0	1.667	0.336	0.0032	121	37813	Z_{max}=5.33

Table 3.8 -- Calculation of y by Matlock-Reese method. Normally consolidated clay.

T_{trial}	x	z = x/T	A_y	y	$-p$	E	comments
5	0.0	0.0	2.435	0.1074	31	288	
	0.5	0.1	2.273	0.1003	41	409	k_{obt} =
	1.0	0.2	2.112	0.0932	49	526	267.4
	1.5	0.3	1.952	0.0861	58	674	
	2.0	0.4	1.796	0.0792	66	833	T_{obt} =
	2.5	0.5	1.664	0.0725	73	1007	4.6
	3.0	0.6	1.496	0.0660	79	1197	
	5.0	1.0	0.962	0.0424	69	1627	
2.5	0.0	0.0	2.435	0.0134	16	1194	
	0.5	0.2	2.112	0.0116	20	1724	k_{obt} =
	1.0	0.4	1.796	0.0099	24	2374	1313.4
	1.5	0.6	1.496	0.0083	26	3152	
	2.0	0.8	1.216	0.0067	29	4328	T_{obt} =
	2.5	1.0	0.962	0.0053	31	5849	3.4
	3.0	1.2	0.738	0.0047	33	7036	
	5.0	2.0	0.142	0.00078	19	24358	

Table 3.8 -- continued

T_{trial}	x	z = x/T	A_y	y	$-p$	E	comments
4.25	0.0	0.0	2.435	0.0660	27	409	
	0.5	0.1176	2.245	0.0608	34	559	k_{obt} =
	1.0	0.2353	2.056	0.0557	42	754	419.4
	1.5	0.3529	1.869	0.0506	48	949	
	2.0	0.4706	1.689	0.0457	55	1204	T_{obt} =
	2.5	0.5882	1.513	0.0410	60	1463	4.23 ok
	3.0	0.7059	1.345	0.0365	64	1758	z_{max}=
	5.0	1.1765	0.764	0.0207	54	2609	3.76

Table 3.9 -- Calculation of y by Matlock-Reese method. Cohesionless soil.

T_{trial}	x	z = x/T	A_y	y	-p	E	comments
5	0.0	0.0	2.435	0.1074	0	0	
	0.5	0.1	2.273	0.1003	70	697	k_{obt} =
	1.0	0.2	2.112	0.0932	150	1608	1692
	1.5	0.3	1.952	0.0861	224	2599	
	2.0	0.4	1.796	0.0792	276	3487	T_{obt} =
	2.5	0.5	1.644	0.0725	328	4530	3.20
2.5	0.0	0.0	2.435	0.0134	0	0	
	0.5	0.2	2.112	0.0116	50	4310	k_{obt} =
	1.0	0.4	1.796	0.0099	95	9596	9230
	1.5	0.6	1.496	0.0083	135	16364	
	2.0	0.8	1.216	0.0067	147	21940	T_{obt}=
	2.5	1.0	0.962	0.0053	155	29245	2.28
2.15	0.0	0.0	2.435	0.0085	0	0	k_{obt} =
	0.5	0.2326	2.0598	0.0072	44	6111	11875
	1.0	0.4651	1.6970	0.0060	80	13445	
	1.5	0.6977	1.3563	0.0048	111	23319	T_{obt} =2.17
	2.0	0.9302	1.0486	0.0037	123		z_{max} =
	2.5	1.1628	0.7797	0.0027	130		7.44

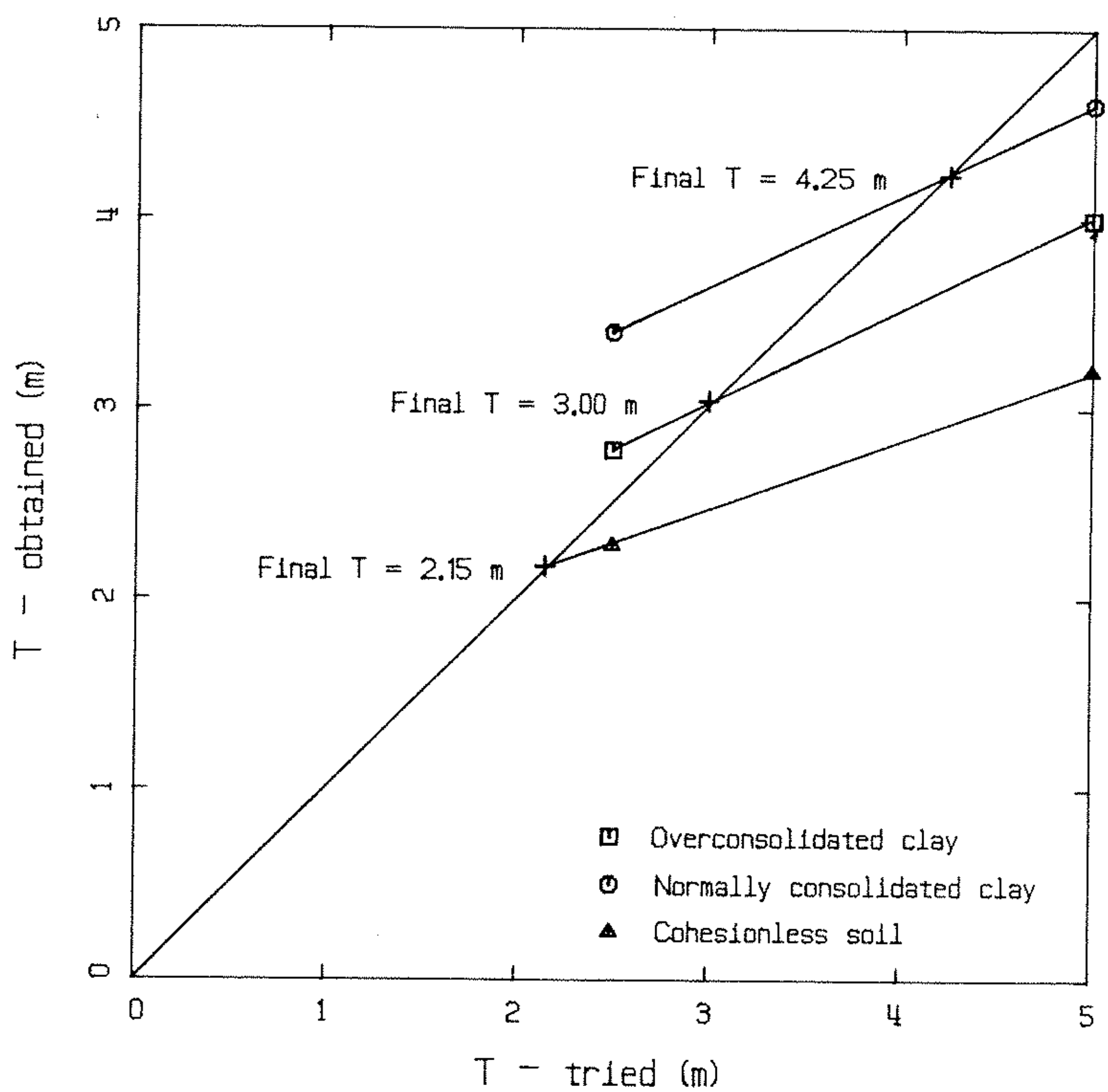


Figure 3.5 - Interpolation for final value of relative stiffness factor.

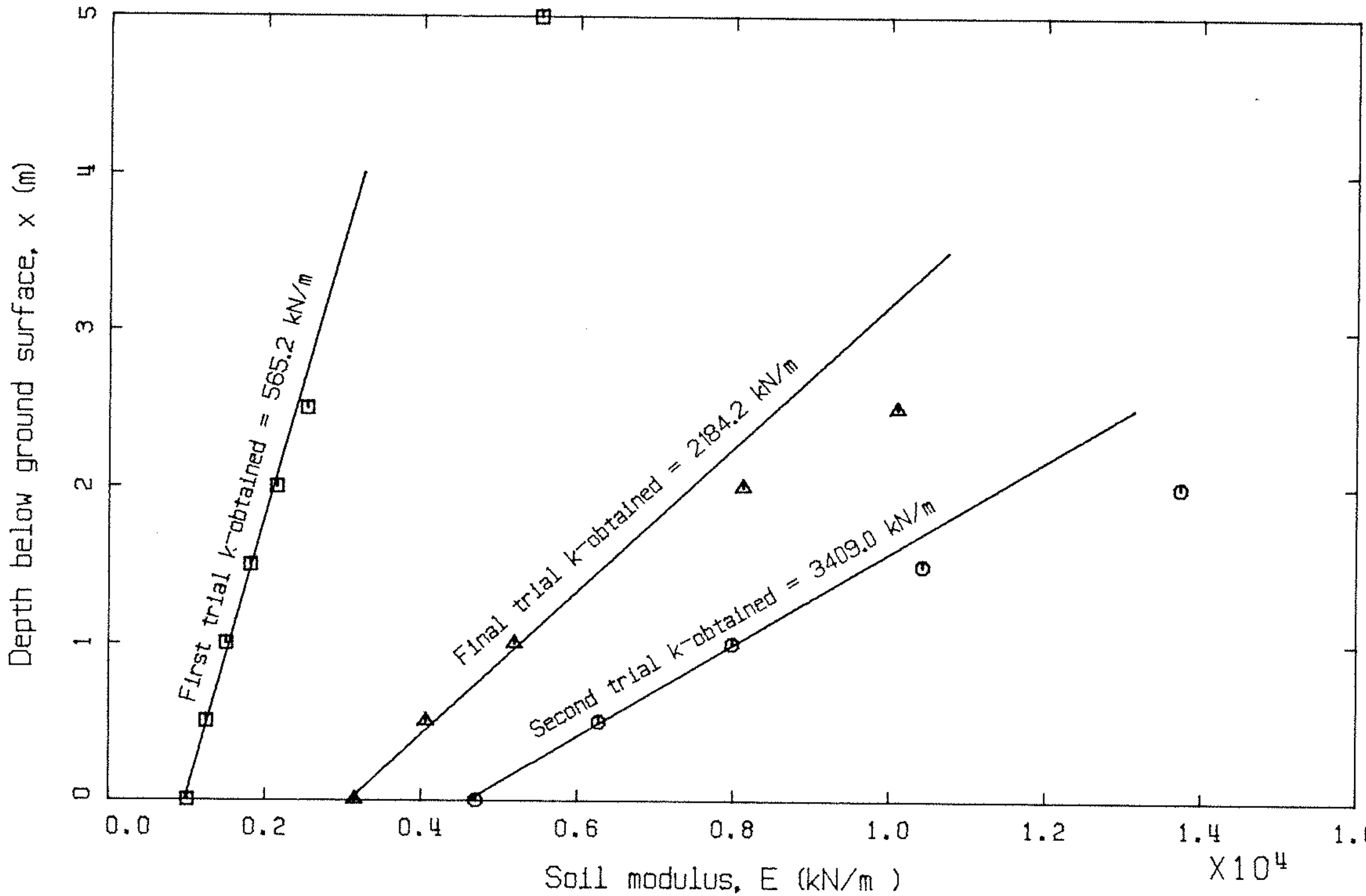


Figure 3.6 - Trial plots of soil modulus values - overconsolidated clay.

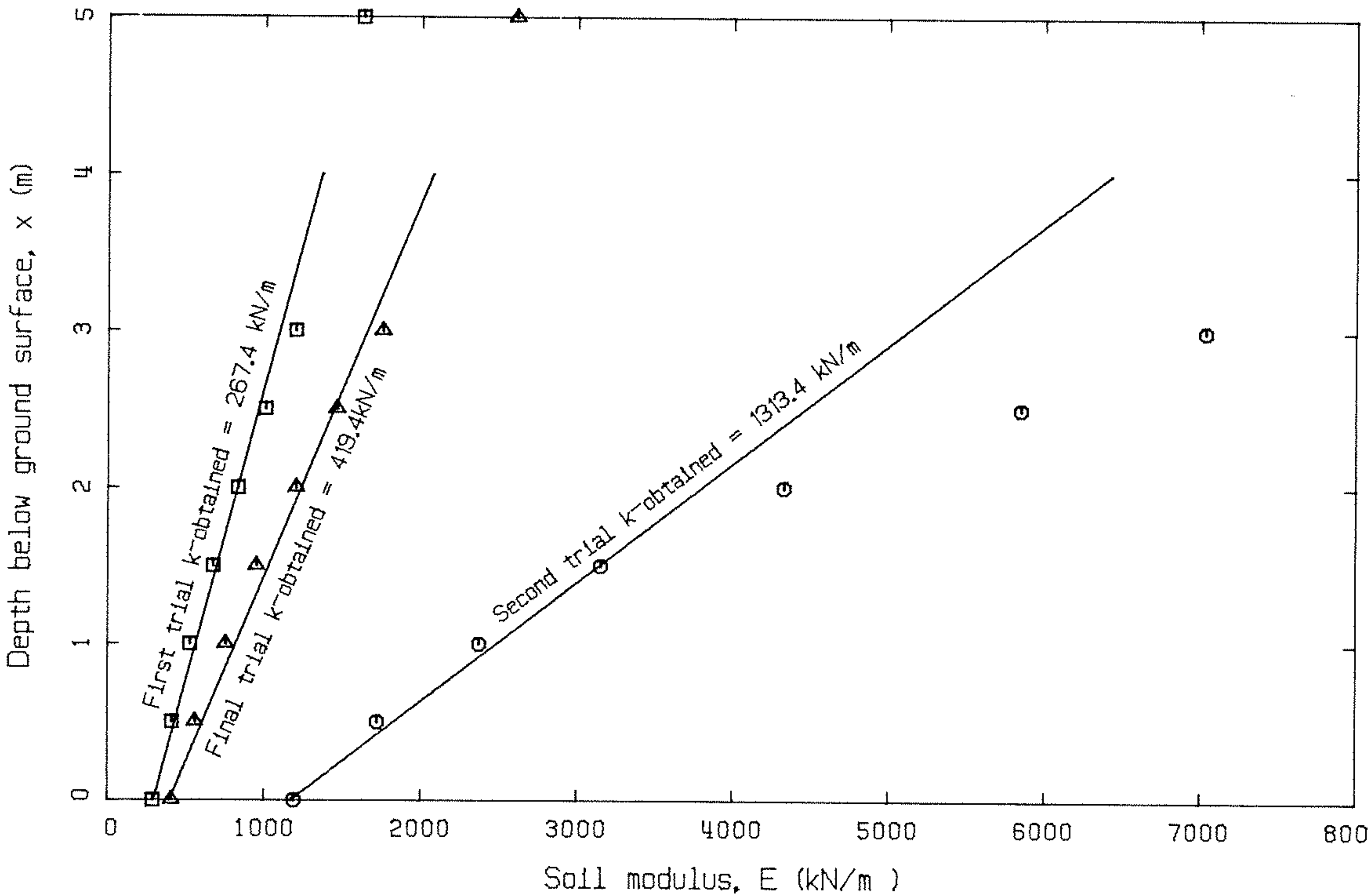


Figure 3.7 - Trial plots of soil modulus values - normally consolidated clay.

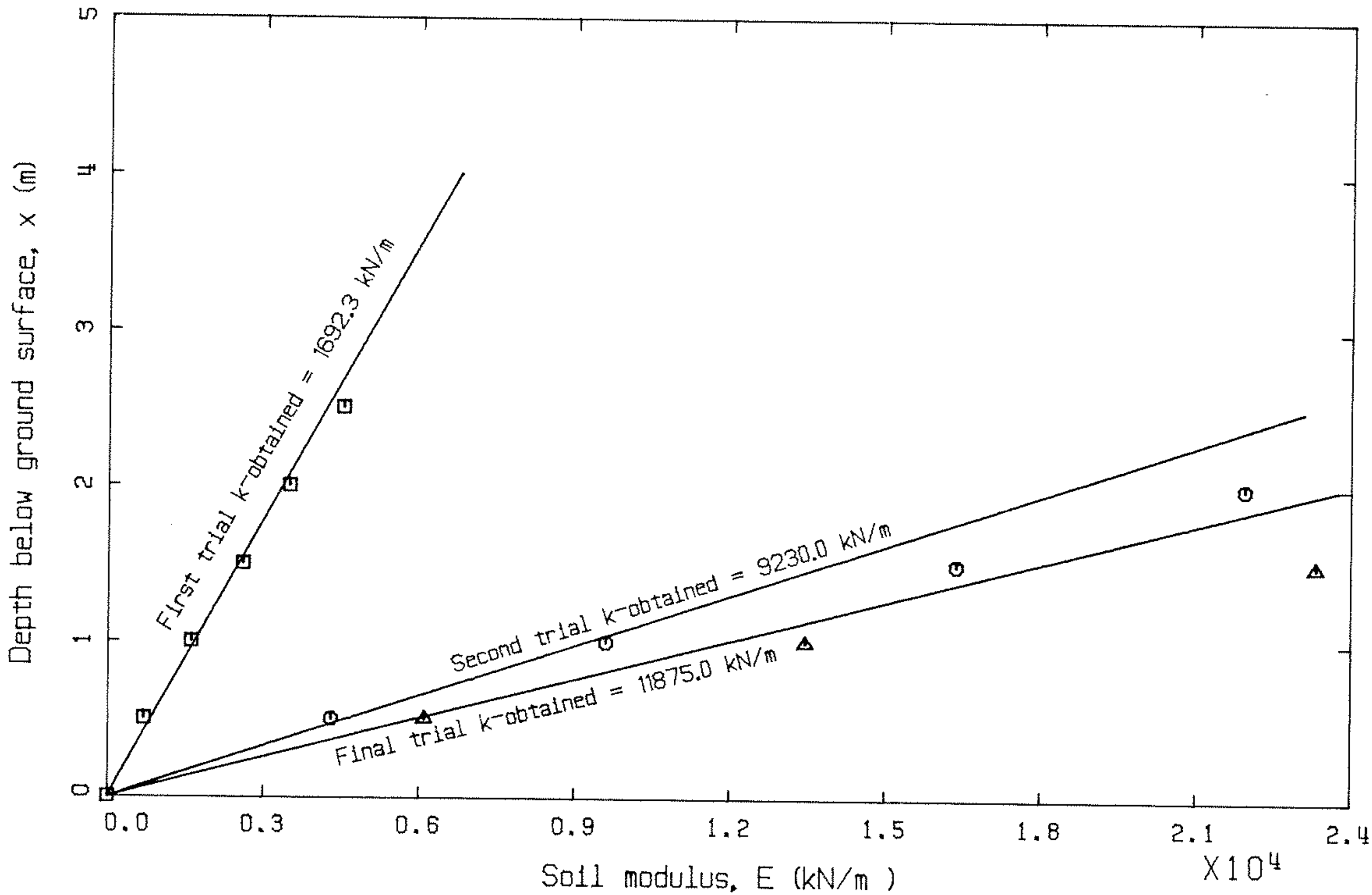


Figure 3.8 - Trial plots of soil modulus values - cohesionless soil.

3.6 Poulos method

This method is described in section 2.7 and all graphs needed for this calculations are given there. The results of the calculations are given in Table 3.10.

Table 3.10 - Calculation of y and M by Poulos method.

soil / parameter	over-consolidated clay	normally consolidated clay	cohesion-less soil
$E_p I_p$ (kNm²)	5.66 10^5	5.66 10^5	5.66 10^5
E (kN/m²)	3500	940	3447
L (m)	16	16	16
K_R	0.00247	0.00920	0.00251
L/D	20	20	20
I_{yP}	6.8	5.0	6.8
y (m)	0.0242	0.0665	0.0247
M/PL	0.063	0.103	0.064
M (kNm)	201.6	330.7	204.8

3.7 Summary of results.

Table 3.11 is a summary of results for all methods for deflections at ground surface y and the maximum moment M that occurs in the pile.

Table 3.11 -- Summary of results using different methods.

A. Deflection (unit m)

soil / method	over-consolidated clay	normally consolidated clay	cohesion-less soil
Broms	0.0645	0.2038	0.0093
Matlock-Reese	0.0233	0.0660	0.0088
Poulos	0.0242	0.0652	0.0247

B. Moment (unit kNm)

soil / method	over-consolidated clay	normally consolidated clay	cohesion-less soil
Broms	455.6	585.2	390.0
Matlock-Reese	478.2	656.2	332.0
Poulos	201.6	330.7	204.8

4. DISCUSSION OF RESULTS

4.1 Introduction

In the two previous chapters three different methods were introduced for calculating deflection and maximum moment for laterally loaded piles, and example calculations were presented in Chapter 3. A summary of the results is given in Table 3.11. In this chapter an explanation of the results will be sought.

4.2 Cohesionless soil

For cohesionless soil the deflection calculated by the Broms method and Matlock-Reese method compare fairly well with each other but Poulos method gives over 2 times greater values. The moment calculated by Poulos method is lower than calculated by the other methods. The explanation lies in that for the Poulos method a constant value of Youngs modulus E is used which is not valid for cohesionless soil. When constant value of Youngs modulus is used the subgrade reaction method agrees better with test results, for example Gleser result, than does the elastic solution with constant E (Pise 1972). This is true for both deflections and moments.

Using an increasing modulus E of the soil with depth is more realistic than using a constant value. But an elastic solution with variable E equivalent to the Mindlin solution, which Poulos method is based on, for constant E is not available so an approximate analysis must be used, E = $N_h x$, where E and k_h have the same rate of increase with depth. Solutions based on varying E also give better agreement with the results of Gleser than do solutions for constant E . The fact that, as shown by Pise, the subgrade reaction solution gives better agreement with Gleser's results than elastic solution for constant E, stems from the use of varying k rather from the superiority of the subgrade reaction approach. (Poulos 1972).

Uncertainities in determining E remain the same as in determining the modulus of subgrade reaction (Pise 1972).

4.3 Cohesive soil

For cohesive soil the deflections calculated by the Matlock-Reese method and the Poulos method compare fairly well. On the other hand the moments calculated by the Broms method and the Matlock-Reese method compare fairly well, and those calculated by Poulos are much lower. The deflections calculated by the Broms method are 2.5 to 3 greater higher than calculated by the other two methods. In the following paragraphs some points are mentioned that might explain this difference.

Broms (1964) says that deflection depends primarily on the dimensionless length factor βL. He gives two equations to calculate deflections at ground surface for an unrestrained pile, one for βL less than 1.5 and another for βL greater than 2.5, that is:

$$\beta L < 1.5 \qquad y_o = \frac{4P(1+1.5\frac{e}{L})}{KDL} \tag{4.1}$$

$$\beta L > 2.5 \qquad y_o = \frac{2P\beta(\beta+1)}{K_\infty D} \tag{4.2}$$

He then shows that the lateral deflection y_o at the ground surface can be expressed as a function of the dimensionless quantity $y_o kD \frac{L}{D}$ versus dimensionless length βL. It seems that for βL between 1.5 and 2.5 an extrapolation is made between these equations (see Figure 2.5). The value of the dimensionless length βL for the cases calculated here are in this range.

Also, for βL less than 1.5 the stiffness of the pile is not taken into account for calculation of deflection except for selecting the equation.

When Broms compared this method to case histories for various types of soil and degrees of end restraint he found that the measured lateral deflections at the ground surface varied from between 0.5 to 3.0 times the calculated deflections. For short piles the lateral deflections are inversely proportional to the assumed coefficient of horizontal subgrade reaction, k_h, and thus also to the measured average unconfined compressive strength of the supporting soil. Thus small variations in q will have large effects on the calculated lateral deflections. Also, agreement between calculated and measured lateral deflections improves with decreasing shear strength of the soil.

Broms does not describe any case histories for unrestrained (free-head) concrete piles driven into the soil.

When comparing elastic solution and subgrade reaction method, relationship between the Young's modulus and the coefficient of horizontal subgrade reaction has to be established. Poulos does this by equating the elastic and subgrade reaction for displacement of a stiff clay and states that this is the most accurate way. And then compares his solution with the one by Hetenyi (1946). There all the values from the subgrade reaction are greater than from the elastic theory. The difference becomes increasingly marked as the stiffness of the pile descreases. Comparisons between the corresponding solutions for moments give that the largest difference between the two solutions again occurs for relativly flexible piles, for which the subgrade reaction overestimates the moments. However, the two methods are in reasonable agreement for stiff piles and, in general, the agreement is better than for displacement.

Kosics points out that Vesic states that the subgrade reaction method underestimates the deflections, while Poulos states that it overestimates them. The difference lies in the different basis of relating k_h and E values. Also, the disadvantages of the subgrade reaction method is that k_h depends upon the pile properties as well as the soil properties. And that could mean that the same k_h should not be used for piles of different stiffnesses. Consequently, the results from a lateral load test on a particular pile cannot be directly applied to the analysis of other piles or piles groups with different conditions of end restraints although the soil conditions are the same. In order to obtain agreement between elastic solution and subgrade reaction solution, different k should be used for piles of different stiffnesses. The elastic solution also has its limitations. The method is limited to constant E value. The term E is not only going to vary from point to point in the soil mass, but also at a given point it will vary with stress conditions at that point. Also, the Mindlin solution is used and that includes the assumption that the soil is capable of resisting tensile stresses on one side of the pile. This assumption would not be valid in the critical zone near the ground surface.

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