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A Study of Small-Signal
Stability of Multi-Machine Power Systems

by

Wenzhi Chen ©

A thesis submitted to the Faculty of Graduate St dies and Research in partial fulfillment of the requirements for the degree of Master of Science

Department of Electrical Engineering

Edmonton, Alberta

Fall 1995

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The undersigned certify that they read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **A Study of Small-Signal Stability of Multi-Machine Power Systems** submitted by **Wenzhi Chen** in partial fulfillment of the requirements for the degree of **Master of Science**.

K.E. Bollinger

Supervisor: K.E. Bollinger

Dr. H.J.J Seguin

Dr. D.G. Fisher

Date: June 14, 1995

Dedicated to my wife Zian

and to the memory of my father

Abstract

This research project includes the development of power system models and the design of control strategies to enhance power system stability. The research areas of this thesis include power system models, control and small-signal stability study. A major concern to electric power engineers is the small-signal stability of a power system, particularly the inter-area oscillation problem.

A small-signal stability program was developed in this thesis as one of basic analytical tools for investigating low frequency inter-area oscillations. The basic concepts, mathematical models and system state space model formulation that underlie this program are described, along with an illustration of its application to a small-scale hypothetical system. The computer program was used to do an eigenvalue analysis and to identify the various oscillation modes of the study system. The power system is characterized by a local mode and an inter-area mode. The results of the study shows that Power System Stabilizers (PSSs) can increase the damping of inter-area oscillations. PSSs based on pole placement techniques and other control techniques are also designed in the thesis to enhance the overall system stability. Comprehensive simulation studies were conducted to evaluate the performance of PSSs under various system conditions and disturbances. The small-signal stability program was applied to ensure there are no adverse effects on the other modes. Results are presented to show a comparison between the various design approaches.

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I am also indebted to my brother Wenhao and his family for their support and encouragement during the past five years. To them I own a permanent debt of gratitude.

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List of Symbols

x_d = d-axis synchronous reatance

L_d = d-axis stator winding self-inductance

x_q = q-axis synchronous reatance

L_q = q-axis stator winding self-inductance

x_d' = d-axis transient reactance

L_d' = d-axis stator transient inductance

x_d'' = d-axis subtransient reactance

x_l = leakage reactance

L_l = stator winding leakage inductance

x_e = line reactance

ω_o = synchronous speed

x_q' = q-axis transient reactance

L_q' = q-axis stator transient inductance

x_q'' = q-axis subtransient reactance

r_a = stator winding resistance

H = rotor mechanical inertia constant

K_D = rotor mechnical damping constant

K_s = synchronizing torque coefficient

A_{sta} = saturation curve coefficients

B_{stat} = saturation curve coefficients

Y = nodal admittance matrix

δ = rotor angle

ω = angular speed

ψ = flux linkage

P = real electric power

Q = reactive electric power

v_t = generator terminal voltage

L_{ad} = d-axis mutual inductance

L_{aq} = q-axis mutual inductance

L_{fd} = d-axis stator/field winding mutual inductance

R_{fd} = field d-axis winding resistance

L_{1q} = q-axis damper winding inductance

R_{1q} = q-axis damper winding resistance

i_t = generator terminal current

L_{ad}' = d-axis mutual flux linage

v_d = d-axis component of terminal voltage

v_q = q-axis component of terminal voltage

i_d = d-axis component of terminal current

i_q = q-axis component of terminal current

P_e = electric power at generator terminals

P_m = applied mechanical power

A_d = device state matrix

B_d = control bus input matrix

K_{stab} = gain of a power system stabilizer

K_e = gain of a exciter

Y_d = device admittance matrix

T_e = electric power torque

T_m = mechnical power torque

Δ = small change

Σ = summation

s = Laplace operator

p = differential operator d/dt

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Chapter 1

Introduction

1.1 Power System Stability

Power system stability was first recognized as an important problem in the 1920s. Since then, power system stability has obtained a much broader perspective and become an independent discipline. Power system stability may be described as that property of a power system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable operating state following perturbations [4,35,54]. Since electrical power is generated exclusively by synchronous generators, maintenance of synchronism among the generating units is one of the basic conditions for a stable electrical power supply [45]. Under disturbed conditions, the change in electrical torque of a synchronous machine can be composed of two components:

$$\nabla T_e = T_s \nabla \delta + T_D \nabla \omega \tag{1.1}$$

where,

T_s is the synchronizing torque coefficient,

$T_s \nabla \delta$ is called the synchronizing torque component,

T_D is the damping torque coefficient,

$T_D \nabla\omega$ represents the damping torque component.

It can be seen that system stability is determined by both components of torque for each of the synchronous machines. Lack of sufficient synchronizing torque results in instability through an aperiodic drift in rotor angle, while lack of sufficient damping torque results in oscillatory instability [35].

Note that in a more complex system involving DC line or asynchronous operation, loss of synchronism by slipping of a pole by a machine may or may not be an acceptable state depending on the circumstances and operating criteria [41, 56]. The system may also become unstable because of the collapse of load voltage.

1.2 Non-linear Characteristics of Power System

An electrical power system is inherently nonlinear due to saturation, limits in excitation and governor-prime mover systems, and power-angle relation. During a major disturbance, such as a severe fault, and up to about 3 seconds following the disturbance, the power system behaves in a nonlinear way and a nonlinear model for simulation is required. After the first 3 seconds or in response to a small disturbance the system behaves in a linear or quasi-linear way and a linearized model of the power system for analysis is valid. When in the linear state, the response of the system is composed of the natural modes of the system, which are defined by the dynamic characteristics of the system. These modes can have an exponential or sinusoidal form. Their magnitude may decay or increase. Any system which exhibits a non-decaying mode is deemed unstable [35].

The stability of a linear system is completely independent of the input. In contrast, the stability of a nonlinear system depends on the type and magnitude of input, and the initial state. These factors have to be taken into account in defining the stability of a nonlinear system. For a nonlinear model, solutions cannot be obtained in analytical form. Instead, time is broken down into steps and the difference equations solved "step-by-step." A numerical method has to be utilized which is essentially a digital simulation technique.

1.3 Transient Stability and Small Signal Stability

It has been common practice to classify power system stability into two categories: small signal stability (or small disturbance stability) and transient stability (or large disturbance stability). A detailed classification of power system stability is listed by Kundur in reference 35. Some good historical summaries of power system stability are contained in the references 35, 54 and 56. Small-signal stability is the ability of a power system to maintain synchronism under small disturbances and is a fundamental requirement for normal operation of a power system. Such disturbances occur continually in power systems due to small variations in loads and generation [24,36]. Under a small disturbance, nonlinear equations describing the dynamics of a power system can be linearized. The solution can be obtained either in the time domain or in the frequency domain. Transient stability is the ability of a power system to maintain synchronism under a severe disturbance and is related to the transient period. Stability is determined by both the initial operating state of the system and the severity of the disturbance. Linearization is not valid in this case and hence, the mathematical model is a set of nonlinear differential equations with or without algebraic constraints [56]. A solution to the nonlinear problem may be obtained in the time domain.

1.3.1 Statement of the Problem

Until now, much of the effort and interest related to power system stability has been focused on transient stability, and the system is designed and operated so as to meet a set of reliability criteria concerning transient stability. The analysis of small-signal stability is not as widespread as transient stability analysis. There are no standard study procedures or commonly accepted performance criteria with regard to the small signal stability. The small-signal stability has been revealed largely by transient programs. This is mainly because, in the past, a system that remained stable for the first few seconds after a severe disturbance was sure to remain stable for small perturbations. However, it may not be true for present systems [24,38]. The complexity of a power system with long EHV transmission lines, large generating units and fast excitation capabilities has made stability and control problems more difficult than ever [38]. In some cases small-signal stability is more limiting than transient stability and power transfers across major interferences are being curtailed owing to the stability of inter-area oscillations. The deterioration in the stability of the inter-area modes may be a result of the power system being operated at higher stress levels due to an increase in inter-utility power transfer to compensate for delays in the addition of new transmission and generation facilities. High-response exciters, while improving transient stability, adversely effect small-signal stability associated with local plant modes of oscillations by introducing negative damping.

It has been recognized that normal operation of a bulk electric power system is characterized, in part, by generator speed variations, or oscillations. Most oscillations between generator rotors are positively damped, but in some circumstances, these oscillations may persist, or increase in magnitude, with time. Unstable oscillations may represent interaction among a few generators and have mainly local effects, or they may represent interaction among large groups of generators and have widespread effects. The latter are usually hard to analyze and remedy. These potentially unstable oscillations may lead to a serious disruption of service. In today's practical power systems, small-signal stability is largely a problem of insufficient damping of oscillations. These oscillations can be divided into the following four modes:

(1) **Local modes** are associated with the swinging of units at a generating station with respect to the rest of the power system. The term local is used because

the oscillations are localized at one station or a small part of the power system.

(2) **Inter-area modes** are the swinging of many machines in one part of the system against machines in other parts. They occur when two or more groups of closely coupled machines are interconnected by weak ties.

(3) **Control modes** are related to the generating unit and other control devices. They may be caused by poorly tuned exciters, speed governors, HVDC converters or static var. compensators.

(4) **Torsional modes** are related to the turbine-generator shaft system rotational components. These modes are often caused by interaction with excitation controls, speed governors, HVDC controls and series-capacitor-compensated lines [24,35].

1.3.2 Approaches to Small Signal Stability

Theoretically, any methods applicable to linear systems may be utilized in the study of the small signal stability of power systems. Methods used in conventional linear control theory, such as Routh and Nyquist criteria, have been adopted for assessing dynamic stability of power systems. These methods are restricted to the analysis of small systems, such as a single machine connected to an infinite system. They are also of limited value in the analysis of system having a wide range in frequencies of oscillations [9, 40]. However, eigenvalue analysis has been shown to be very effective for analysis of multi-machine systems in particular. It illuminates the behavior of the system for any mode of oscillation.

The eigenvalues of the system are indicative of system performance. The real part is a measure of the amount of damping: negative parts are damped, but positive ones represent unstable conditions that may lead to catastrophic consequences. The imaginary part is related to the damped natural frequency of oscillation of the corresponding mode. In general, system eigenvalues are functions of all control and design parameters. They may be sensitive to changes in the system design or operation. An eigenvector includes the relative magnitudes and phases of all generator speed deviations for that mode of oscillation. This close relationship between mathematical results and the physical system is very useful in determining how to improve inadequate damping [11, 12, 59].

Evaluation of the small signal stability of a power system requires the computation of the eigenvalues of a very large unsymmetrical and sparse matrix. Though conventional eigenvalue programs are robust and converge fast, for example, the MASS (Multi-Area Small signal Stability) program uses QR methods [23], such programs do not exploit sparsity of the system matrix and demand a large amount of computational memory storage and CPU time. As a result, their application is limited to relatively small power systems [36]. On the other hand, for a bulk power system with thousands of state variables, often only a specific set of eigenvalues with certain features of interest, such as local mechanical models, and their corresponding inter-area modes need to be calculated [74].

To overcome the limitation of conventional eigenvalue programs, a number of special methods have been developed with three basic properties:

(1) Sparsity oriented techniques can be utilized,

(2) A specific set of eigenvalues can be handled efficiently,

(3) Good convergence and numerical stability [71].

These methods focus on evaluating a selected subset of eigenvalues associated with complete system responses. The AESOPS (Analysis of Essentially Spontaneous Oscillations in Power Systems) program is one such technique [24]. Two sparsity-based eigenvalue techniques: simultaneous iterations and modified Arnoldi method, and their application to the small signal stability analysis of large power systems, are described by Wang and Semlyen [71]. The PEALS (Program for Eigenvalue Analysis of Large Systems) incorporates these techniques [24,36]. This is an efficient program capable of simulating systems having up to 12,000 buses and 1,000 machines. It is mainly aimed at computation of slow inter-area oscillatory modes. The S-method introduced by Udida and Nagao is more efficient for finding the unstable modes [66]. The SMA (Selective Modal Analysis) calculates eigenvalues associated with selected modes of interest by using sensitivity based techniques [69]. It first identifies variables that are significant to the selected modes, and then forms a reduced order model that involves only the significant variables.

Each method mentioned above is developed only for a particular type of application. However, none of these techniques satisfy all the requirements of analyzing the small signal stability of power systems. The best approach, therefore, is to apply several techniques in a complementary manner [36]. The following summarizes the measures taken to improve damping and enhance stability.

- Readjustment of generator control,
- Addition of stabilizing controls to generating units,
- Modulation of DC links,
- Imposition of operating restrictions,
- Reinforcement of the transmission system,
- Control of reactive power [12].

1.3.3 Inter-Area Oscillations

When electric power systems are interconnected through ties of relatively small capacity, low frequency inter-area oscillations can cause problems to normal operating conditions. There also exist local, lightly damped, higher frequency mode of oscillations. These oscillations must be considered in planning, designing and operating a large power system. Unstable inter-area electromechanical oscillations have been encountered in the last 30 years by electric power utilities throughout the world [12]. For example, in December 1959, unstable oscillations of approximately 0.25 Hz occurred between the interconnected power system of Michigan, Ontario and Quebec. More recently, between the period of August 1987 to May 1988, the power systems of south and southeast Brazil experienced low frequency inter-area oscillations. In recent years, the stability of inter-area oscillations has become a concern for many power pools in North America, and a number of research projects have been conducted [12]. The following briefly summarizes the results and observations by researchers.

Inter-area oscillations are inherent to interconnected power systems. These oscillations are associated with weak transmission links and heavy power transfer. Their frequencies range from 0.2 to 0.8 Hz. Inter-area oscillations can be spontaneous, that is, initiated by a small disturbance such as a change in load or generation. These types of disturbances occur continually in a power system, and thus a power system with an unstable inter-area mode is impossible to operate. Inter-area oscillations can also be excited by a large disturbance, and if they are unstable or poorly damped, they dominate the response of the system starting from 2 to 3 seconds following the disturbance. In addition, because inter-area oscillations are due to a natural mode of the system, they cannot be eliminated. However, their damping and frequency magnitude can be modified. The factors that effect the damping of low frequency oscillations are complex. In many cases low damping is due to a combination of many factors. Load characteristics have some major effects on the stability of inter-area modes. Other factors include increased stress of the weak interconnections between oscillating groups of generators, the type and location of system loads, the strength of voltage regulation in the system, and the type of generator excitation control [12].

1.3.4 Tools for Inter-Area Oscillations

The commonly used tools for analysis of inter-area oscillation problems include a load-flow program, a transient stability program and a small-signal stability program. Load-flow is used to initialize the transient and small signal stability programs. It must be capable of producing good post-fault conditions for use with the small signal stability program. The transient stability program is the workhorse of stability analysis. Its nonlinear models and step-by-step time simulation accurately analyze the system behavior following severe faults. Small-signal stability programs use a linearized power system model and apply modal analysis techniques. Using these techniques, generators that may experience an oscillation, or may be candidates for stability controls can be identified. The small signal stability program can be used as the fundamental tool for designing power system damping controls.

Controls for damping inter-area oscillations may be accomplished by Power System Stabilizers (PSS), Static Var Compensator (SVC) modulation, HVDC link modulation, or Flexible AC Transmission System (FACTS) modulation. Of these controls, the most cost-effective one is the power system stabilizer, which is an auxiliary control applied through the exciter of a generator. To be effective, power system stabilizers must be placed on a generator with a strong participation in the mode to be controlled [12].

1.4 Objectives of Project

In this study, the major concern is the small-signal stability problem, particularly the inter-area oscillation problem. The objectives of this research are to study the fundamentals of low frequency inter-area oscillations, to study methods for designing controls to remedy the local and inter-area low-frequency oscillations, and to define methods and demonstrate analytical tools for studying these oscillations. The project will formulate models for the study of the small-signal stability of multi-machine power systems and compute the eigenvalues, eigenvectors and sensitivities of the system. In addition, Power System Stabilizers will be designed by using pole placement and sensitivity techniques. The goal of this project is to provide quantitative information on system oscillatory behaviours.

1.5 Outline of Thesis

A basic review of the fundamental theory for small-signal stability is given in Chapter 2. Various basic concepts, such as state space model and representation, eigenvalues and eigenvectors, participation factor, controllability and observability as well as linearization are covered. Modal analysis, eigenvalue analysis and sensitivity analysis are briefly discussed. Lyapunov's stability theorem, one of most important stability theorems, is presented. Thus, this chapter provides the basic analytic tools to be used in the following chapters.

Chapter 3 presents the formulation of the mathematical model of a multi-machine power system. A computer package for small-signal stability study is developed. The basic ideas and mathematical models that underlie this program are described, such as the principle steps of modeling, the model structure, the reference frame transformation, generator models, turbine/governor models, excitation system models, load models, network models and the system model formulation of an interconnected system.

Chapter 4 deals with Power System Stabilizers design techniques. The basic functions of a PSS and general design procedures are discussed. A review of various PSSs based on different control theories is presented. Pole placement and linear optimal control techniques re described in some detail. A case study is also presented in Chapter 4. With a small-signal stability program developed for this thesis, an eigenvalue analysis was carried out to identify the various oscillation modes of a small hypothetical power system. The system is characterized by a local and an inter-area oscillations. Pole placement techniques are used in design PSSs to enhance the damping of overall system. Comprehensive simulation studies were conducted to evaluate the performance of these PSSs. Though the study system is very simple compared to a modern power system, it does help to understand how to implement a small-signal stability analysis and how to design a PSS to remedy the local and inter-area oscillations.

Chapter 5 gives the main results of the study and lists some topics for further investigation.

Chapter 2

Fundamental Theory for Small-Signal Stability Study

2.1 State Space Concepts

The state of a dynamic system is the smallest set of system variables that, together with the inputs, totally determines the system behavior. By this definition, the state of the system contains sufficient information about the processes occurring in this system prior to some initial instant, such that all processes following this instant could be computed from them. State variables are those variables that determine the system state. All the state variables of a dynamic system are elements of a vector called a state vector. If a system can be completely described by **n** state variables, the **n**-dimensional space whose **n** coordinate axes represent the **n** state variables is referred to as a state space [56, 57, 58].

2.2 Linear State Space Model Representation

A multi-input and multi-output (MIMO) linear dynamic system, such as a power system, may be represented by the following forms,

$$\dot{x}=Ax+Bu \tag{2.1}$$

$$y=Cx+Du \tag{2.2}$$

where,

x is a state vector of **n**-dimension,

y is an output vector of **m**-dimension,

u is an input vector of **r**-dimension,

A, **B**, **C**, **D** are compatible matrices.

Eq. (2.1) is referred to as the state equation, while (2.2) is called the output equation. Eqs. (2.1) and (2.2) together compose a state space representation in the time domain. A particular set {**A**, **B**, **C**, **D**} is designated as a system representation. If one or more of these parameters vary with time, the system is defined as a time-varying state model. By contrast, if these parameters remain unchanged during system operation, the system is called a time invariant system. So far, time invariant models have been used in most cases for small signal stability of a power system. In this study, we assume the systems under study are time invariant systems. The state equations can be represented by a block diagram as shown in Figure 2.1.

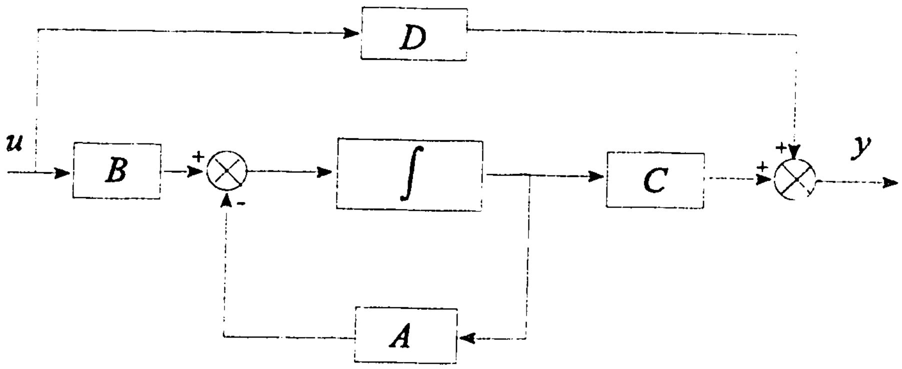


Figure 2.1 Block diagram of state space representation

Note that, the system representation is not unique. A system described by {**A**, **B**, **C**, **D**}, i.e., Eqs. (2.1) and (2.2), can assume a transformation given by,

$$x = Tz \tag{2.3}$$

where,

T is a non-singular square matrix.

Eq.(2.1) can be transformed as,

$$\dot{x} = T\dot{z} = ATz + Bu \tag{2.4}$$

Rewriting Eq. (2.4) yields,

$$\dot{z} = T^{-1}ATz + T^{-1}Bu \tag{2.5}$$

Similarly, Eq.(2.2) can be transformed as,

$$y = Cx + Du = CTz + Du \tag{2.6}$$

If Eqs. (2.1) and (2.2) are replaced by Eqs. (2.5) and (2.6), i.e., to choose a state vector **z** instead of **x** to represent the system, the input output relationship of the system does not change. Thus, the state variables can be chosen freely. For instance, in modeling a synchronous machine, state variables may be chosen from voltages, currents and flux linkage. However, the number of state variables will be the same in all cases, and is equal to the order of the model, and the eigenvalues will be independent of the choice made [36].

2.3 Transfer Function

A s-domain transfer function is an algebraic representation of the relationship between the input and output variables of a system. It is typically obtained by taking the Laplace transform of the system differential equation with zero initial conditions. A transfer function totally determines both dynamic and steady state behavior of the process output when the input signal is specified. Given state space equations of a system, the transfer function of the system can be easily obtained and vice versa. Taking the Laplace transform of Eqs. (2.1) and (2.2), with zero initial conditions gives,

$$sX(s)=AX(s)+Bu(s) \tag{2.7}$$

and,

$$Y(s)=CX(s)+Du(s) \tag{2.8}$$

Solving for Y(s) yields,

$$Y(s)=\left[C(sI-A)^{-1}B+D\right]u(s) \tag{2.9}$$

By definition, the transfer function is then,

$$G(s)=\frac{Y(s)}{u(s)}=C(sI-A)^{-1}B+D \tag{2.10}$$

where,

G(s) is referred to as a transfer function matrix.

2.4 Solution of the State Space Equation

The solution of the state equations can be obtained in different ways. Taking the Laplace transforms of Eqs. (2.1) and (2.2), yields the state equations in the frequency domain. The equations are given by,

$$sX(s)-X(0)=AX(s)+Bu(s) \tag{2.11}$$

and,

$$Y(s)=CX(s)+Du(s) \tag{2.12}$$

X(s) can be solved as,

$$\begin{aligned} X(s)&=(sI-A)^{-1}[X(0)+Bu(s)] \\ &=\frac{adj(sI-A)}{\det(sI-A)}[X(0)+Bu(s)] \end{aligned} \tag{2.13}$$

and **Y**(s) as,

$$Y(s)=C\frac{adj(sI-A)}{\det(sI-A)}[X(0)+Bu(s)]+Du(s) \tag{2.14}$$

The Laplace transforms of **X**(s) and **Y**(s) are seen to have two components. One is dependent on the initial conditions and the other on the inputs. These are the Laplace transforms of free and zero-state components of the state and output vectors. By taking the inverse of Laplace transform of **X**(s) and **Y**(s), solutions in the time domain can be obtained as,

$$x(t)=e^{A(t-t_0)}x(t_0)+\int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau \tag{2.15}$$

and,

$$y(t)=Ce^{A(t-t_0)}x(t_0)+C\int_{t_0}^{t}e^{A(t-\tau)}Bu(\tau)d\tau+Du(t) \qquad (2.16)$$

2.5 Solution of Transfer Function

The transfer function of a system having **n** distinct eigenvalues may be written as,

$$\frac{y}{u}=k\frac{(s-z_1)(s-z_2)\ \dots\ (s-z_m)}{(s-p_1)(s-p_2)\ \dots\ (s-p_n)} \qquad (2.17)$$

where,

k is the transfer function gain,

z_i is the system zero,

p_j is the system pole,

with **m≤n.**

Using the partial fraction expansion, the transfer function can also be written in terms of the residues. The general form is given by,

$$\frac{y}{u}=\frac{c_1}{(s-p_1)}+\frac{c_2}{(s-p_2)}+\ \dots\ +\frac{c_n}{(s-p_n)} \qquad (2.18)$$

where,

c_i is called the residue of **G(s)** at pole p_i .

The transfer function residues can also be computed by left and right eigenvectors [35]. The residue is given by,

$$c_i = \psi_i B C \phi_i \tag{2.19}$$

The frequency response of the transfer function may be obtained by setting $s=j\omega$ in the transfer function and varying ω, the frequency in rad/s, over the frequency range of interest. The solution of **G(s)** in the time domain can be obtained by inverting the Laplace transform of Eq.(2.18) as the sum of modes,

$$g(t) = c_1 e^{p_1 t} + c_2 e^{p_2 t} + \ \dots\ + c_n e^{p_n t} \tag{2.20}$$

2.6 Eigenvalue and Eigenvector

The values of s that satisfy the characteristic equation of matrix **A** are known as eigenvalues and are given by,

$$\det(sI - A) = 0 \tag{2.21}$$

The general eigenvalue can be expressed as,

$$\lambda = \alpha + j\omega \tag{2.22}$$

Associated with each eigenvalue (λ_i) of **A** are the right eigenvector and left eigenvectors. The right eigenvector is defined by,

$$A\phi_i=\lambda_i\phi_i \qquad i=1,2,\cdots,n \tag{2.23}$$

where,

$$\phi_i=\left(\phi_{1i} \quad \phi_{2i} \quad . \quad . \quad . \quad \phi_{ni}\right)^T \tag{2.24}$$

Similarly, the left eigenvector is determined by,

$$\psi_i A=\lambda_i\psi_i \qquad i=1,2,\cdots,n \tag{2.25}$$

where,

$$\psi_i=\left(\psi_{1i} \quad \psi_{2i} \quad . \quad . \quad . \quad \psi_{ni}\right)^T \tag{2.26}$$

From the above definition, it follows that if the eigenvalues are distinct, the left and right eigenvectors corresponding to different eigenvalues are orthogonal. The results can be represented by,

$$\psi_j\phi_i=0 \tag{2.27}$$

and,

$$\psi_i\phi_i=C_i \tag{2.28}$$

Usually the eigenvectors are normalized to one, i.e.,

$$\psi_i \phi_i = 1 \tag{2.29}$$

It follows that,

$$\psi\phi = I, \qquad \psi = \phi^{-1} \tag{2.30}$$

where,

$$\phi = [\phi_1, \phi_2, \ldots, \phi_n] \tag{2.31a}$$

$$\psi = [\psi_1^T, \psi_2^T, \ldots, \psi_n^T]^T \tag{2.31b}$$

Therefore,

$$A\phi = \phi\Lambda \tag{2.32}$$

Or rewriting the above equation as,

$$\Lambda = \phi^{-1} A \phi \tag{2.33}$$

Obviously, Λ is a diagonal matrix, with the eigenvalues as diagonal elements.

2.7 Modal and Eigenvalue Analysis

Introducing a new state vector **z**, related to the original state vector **x** by the transformation, $x=\phi z$, the Eqs. (2.1) and (2.2) can be expressed as

$$\dot{z}=\phi^{-1}A\phi z+\phi^{-1}Bu \tag{2.34}$$

and,

$$y=C\phi z+Du \tag{2.35}$$

Substituting Eq.(2.33) into Eq.(2.34) yields,

$$\dot{z}=\Lambda z+\phi^{-1}Bu \tag{2.36}$$

The dynamic equations are decoupled since Λ is diagonal and the resulting equations may be solved separately. The solution is of the form.

$$z_i(t)=c_i e^{\lambda_i t}+o_i(t) \tag{2.37}$$

For simplicity, 0_i represents zero-state components of the state, the same as the second term of Eq.(2.16). Since,

$$x=\phi z, \qquad x=\sum_{i=1}^{n}\phi_i z_i \tag{2.38}$$

then substituting for z_i using Eq.(2.37) results in,

$$x(t)=\sum_{i}^{n} \phi_i[c_i e^{\lambda_i t}+o_i(t)] \tag{2.39}$$

where,

n is the number of the states,

$c_i e^{\lambda_i t}$ is called a mode of the system.

Every system variable can be expressed as a linear combination of dynamic modes. The value of c_i may be determined by using the orthogonal property of the left and right eigenvectors. Pre-multiplying Eq.(2.38) by ψ_i,

$$\psi_i x=\sum_{1}^{n} \psi_i \phi_i z_i=z_i \tag{2.40}$$

If the initial condition of the state vector is given by $x(0)$, then,

$$z_i(0)=\psi_i x(0) \tag{2.41}$$

and for simplicity, assuming that zero-state component in Eq. (2.37) is zero yields,

$$z_i(0)=c_i \tag{2.42}$$

therefore,

$$c_i = \psi_i x(o) \tag{2.43}$$

The scalar product (c_i) represents the magnitude of excitation of the **ith** mode resulting from the initial condition. The time dependent characteristicof a mode corresponding to an eigenvalue λ_i is given by,

$$e^{\lambda_i t} \tag{2.44}$$

Therefore, the stability of the system is determined by the eigenvalues. Analysis of the eigen-properties of matrix **A** provides complete information regarding the stability characteristics of the system. If matrix **A** is real, complex eigenvalues always occur in conjugate pairs, and each pair corresponds to an oscillatory mode, which has the form,

$$e^{\alpha t}\sin(\omega t+\theta) \tag{2.45}$$

Eq.(2.45) represents a damped sinusoid for negative α. The imaginary component of the eigenvalue determines the frequency of oscillation. The frequency in Hz can be computed by,

$$f = \frac{\omega}{2\pi} \tag{2.46}$$

A real eigenvalue corresponds to a non-oscillatory mode. A negative real eigenvalue represents a decaying mode. The amplitude of mode z_i decays with time, the larger its magnitude, the faster the decay. This type of mode is said to be asymptotically stable. When the real part is positive, the amplitude of the mode increases with time. Obviously, the system is aperiodic unstable. Therefore, the real component of the eigenvalue provides the damping contribution. From the above analysis, it can be seen that every solution representing a free response depends on three quantities:

(1) an eigenvalue determines the decay/growth rate of the response,

(2) an eigenvector determines the shape of the response,

(3) initial condition determines the degree to which each mode will participate in the free response [5].

The damping ratio is calculated by,

$$\zeta=-\frac{\alpha}{\sqrt{\alpha^2+\omega^2}} \tag{2.47}$$

Since the damping ratio determines the rate of decay of the amplitude of the oscillation, it is a quantitative measure of the degree of system damping. An absolute stability limit ($\zeta=0$) is of theoretical interest only. For satisfactory operation of the system each mode must have a certain minimum value of damping ratio. The minimum acceptable value of damping ratio is not clear. However, situations with damping ratio of less than 0.03 must be accepted with caution [38].

2.8 Lyapunov's Stability Theory

The most general concepts and theorems of stability still used today are due to Lyapunov, who in 1892 set forth the general framework for the solution of such problems [54]. Lyapunov's stability theory can be applied to both linear and nonlinear systems. He outlined two approaches to the problem of stability, popularly known as "Lyapunov's first method" and the "Lyapunov's second method" or the "direct" method. The distinction is based on the fact that the "first method" depends on finding an approximate solution to the differential equations, thereas in the "second method," no such knowledge is necessary. The stability criterion of the "first method" is given by the explicit solution of a nonlinear differential equation, i.e., by eigenvalues of matrix A, for small-signal stability. A negative real part represents a damped oscillation, and the system is stable, whereas, a positive real part represents an oscillation of increasing amplitude, and the system is unstable. If the real part is zero, the mode will oscillate with a constant amplitude. The effect of eigenvalues on system behavior is summarized in the Table 2.1.

Table 2.1 Effect of eigenvalues on system behavior

$\lambda = \alpha \pm j\omega$	Real part (α)		imaginary part (ω)	
Effect on system	$\alpha \geq 0$	unstable	$\omega = 0$	not oscillatory
	$\alpha < 0$	stable	$\omega \neq 0$	oscillatory

2.9 Lyapunov's Second Method

Lyapunov's second method can be best explained by using an energy concept. The energy of an object is always positive. If the derivative of the energy is always negative, the energy of the object will become zero. In other words, the object will go into a static state. For a purely mathematical system, a fiction energy function, called the Lyapunov's function, denoted by **V(x)**, or **V(** $x_1, x_2 \ldots x_n$**)** is constructed. If **V(x)** is always positive, and its derivative is always negative, then the system state must approach zero, which means that the system is asymptotically stable. Construction of Lyapunov's function is very important, and is sometime very difficult. Assuming a linear time-invariant system represented by,

$$\dot{x}=Ax \tag{2.48}$$

If the Lyapunov function **V(x)** is chosen as the following quadratic form,

$$V(x)=x^{T}Px \tag{2.49}$$

where,

P is a real symmetrical positive definite matrix.

then its derivative is,

$$\dot{V}(x) = \dot{x}^T P x + x^T P \dot{x}$$

$$= x^T A^T P x + x^T P A x \qquad (2.50)$$

$$= x^T (A^T P + P A) x$$

Eq.(2.50) can be simplified as,

$$\dot{V}(x) = -x^T Q x \qquad (2.51)$$

Asymptotic stability requires that **Q** must be positively definite, or,

$$-Q = A^T P + P A \qquad (2.52)$$

The implication of this equation must be clearly understood. For example, if **Q** is a positive definite matrix, **Q=I**, then if the equation has no solution or more than one solution, the origin is not asymptotically stable. On the other hand, if the unique solution is such that **P** is positive definite, then the origin is asymptotically stable in the large [54].

2.10 Eigenvalue Sensitivity and Participation Factor

To predict the system performance under different parameter settings, eigenvalues can be recalculated for every parameter selection with the aid of sensitivities to determine a suitable step size. However, for a large system, recalculating the eigenvalues can be time consuming and costly [77]. An alternative approach is to use eigenvalue sensitivities and eigenvalue tracking. The advantages of using sensitivities have been shown by many researchers [24,77]. The sensitivity analysis has been used to identify different system modes, to choose suitable model precision, such as PSSs setting and tuning, and to estimate the required accuracy of field measurement for simulation studies [77].

Left and right eigenvectors may be used to determine the eigenvalue sensitivity of changes in the system state matrix. Consider a change in $\mathbf{A}_{ki,}$

$$(A+\Delta A)(\phi_i+\Delta\phi_i)=(\lambda_i+\Delta\lambda_i)(\phi_i+\Delta\phi_i) \qquad (2.53)$$

Neglecting products of small quantities yields,

$$A\phi_i+A\Delta\phi_i+\Delta A\phi_i=\lambda_i\phi_i+\Delta\lambda_i\phi_i+\lambda_i\Delta\phi_i \qquad (2.54)$$

or,

$$(A-\lambda_i I)\Delta\phi_i+\Delta A\phi_i=\Delta\lambda_i\phi_i \qquad (2.55)$$

Recall that,

$$A\phi_i=\lambda_i\phi_i \qquad (2.56)$$

and hence multiplying Eq.(2.55) by ψ_i gives,

$$\psi_i(A-\lambda_i I)\Delta\phi_i+\psi_i\Delta A\phi_i=\psi_i\Delta\lambda_i\phi_i=\Delta\lambda_i\psi_i\phi_i \quad (2.57)$$

Since,

$$\psi_i(A-\lambda_i I)=0 \quad (2.58)$$

and,

$$\psi_i\phi_i=1 \quad (2.59)$$

the result is,

$$\Delta\lambda_i=\psi_i\Delta A\phi_i \quad (2.60)$$

and it follows that,

$$\frac{\Delta\lambda_i}{\Delta a_{kj}}=\psi_{ki}\phi_{ij} \quad (2.61)$$

ΔA is chosen to be zero except for a change in A_{ki} (as a_{ki}), thus, the sensitivity of the eigenvalue λ_i to the element a_{ki} of the state matrix is equal to the product of the left eigenvector element Ψ_{ik} and the right eigenvector element Φ_{ik}. The participation factor is defined as,

$$p_{ki} = \Phi_{ki} \Psi_{ik} \qquad (2.62)$$

with,

$$\sum_{i=1}^{n} p_{ki} = 1 \qquad (2.63)$$

By comparing Eq.(2.61) with (2.62), we see that the participation factor $\mathbf{p}_{ki}$ is actually equal to the sensitivity of the eigenvalue λ_i to the diagonal element $\mathbf{a}_{kk}$ of the state matrix **A**, i.e.,

$$p_{ki} = \frac{\partial \lambda_i}{\partial a_{kk}} \qquad (2.64)$$

The speed participation, corresponding to a generator rotor, provides the sensitivity of the eigenvalue to a change in the direct mechanical damping at the shaft of that generator. A positive real part of the speed participation will provide damping for a torque proportional to the negative of the speed of the generator. Conversely, a negative real part of the participation will reduce the damping of the mode for a torque proportional to the negative of the speed. When the speed participation is zero, the eigenvalue is not influenced by the generator. The speed participation is thus a good indicator of the effect that a power system stabilizer, fitted to the generator, will have on damping a particular mode. The imaginary part of the participation vector indicates the effect of changes in the state matrix diagonal entries on the frequency of the mode.

2.11 Controllability and Observability

Controllability and observability are important indicators for the assessment of system output signals to be used for damping oscillations and the location of the controller input. The coupling between the input and the state determines whether any state can be controlled by a particular input? This is a controllability problem. The relationship between the state and the output provides the information about the state that can be observed from the output. This is the observability problem.

If the eigenvalues of the system matrix **A** in Eq.(2.1) are distinct, the system of Eqs. (2.1) and (2.2) can be diagonalized by $\mathbf{x}=\phi\mathbf{z}$ (see section 2.7). Rewriting Eq.(2.35) and (2.36), results in,

$$\dot{z}=\Lambda z+\psi Bu \qquad (2.65)$$

and,

$$y=C\phi z+Du \qquad (2.66)$$

Apparently if one row in $\psi\mathbf{B}$ is zero, the corresponding state variable cannot be changed by the input. In other words, for a linear system, with distinct eigenvalues, the system is completely controllable if and only if no row of $\psi\mathbf{B}$ has all zero elements. Similarly, if any column in $\mathbf{C}\phi$ is zero, then no information about a corresponding state variable can be provided from the output This means, for a linear system with distinct eigenvalues, the system is completely observable if and only if no column of $\mathbf{C}\phi$ has all zero elements. These results can be simply stated as,

(1) the system described by Eqs. (2.1) and (2.2) is controllable if and only if,

$$rank\ co = rank\begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} = n$$

(2) the system represented by Eqs. (2.1) and (2.2) is observable if and only if,

$$rank\ ob = rank\begin{bmatrix} C^T & A^TC^T & \cdots & (A^T)^{n-1}C^T \end{bmatrix} = n$$

2.12 Linearization of State Equations

It has been mentioned that a power system is inherently nonlinear. Thus, it may be inappropriate to apply the above mentioned linear state space concepts for a small-signal stability study of power system. Fortunately, within the limits of normal engineering accuracy requirements, under a small disturbance, nonlinear equations describing the dynamics of a power system may be linearized by applying smallperturbation theory. Such a system is supposed to be linear over a reasonably wide range of operation, i.e., the following superposition properties hold,

$$\textit{if} \quad y=f(x), \quad \textit{then} \quad cy=f(cx) \tag{2.67}$$

and,

$$y=f(x_1+x_2)=f(x_1)+f(x_2) \tag{2.68}$$

For example, given a time-invariant system described by the following state equation,

$$\dot{x}=f(x,u) \tag{2.69}$$

and an output equation,

$$y=g(x,u) \tag{2.70}$$

where,

x is a state variable vector,

u is an input vector,

$f(.,.)$ and $g(.,.)$ are assumed to have continuous partial derivatives of all orders.

An expansion of Eq.(2.69) into a Taylor series about ($\mathbf{x}_0$, $\mathbf{u}_0$) yields the small-signal, the equilibrium point (operational point) is then done. For small perturbations Δx and Δu, it follows that,

$$x = x_0 + \Delta x, \qquad u \simeq u_0 + \Delta u \tag{2.71}$$

and,

$$\dot{x}_0 = f(x_0, u_0), \qquad \dot{x} = f[(x_0 + \Delta x), (u_0 + \Delta u)] \tag{2.72}$$

The state equation can then be written as,

$$f(x,u) = f(x_0,u_0) + \frac{\partial f}{\partial x}\Big|_{x_0,u_0} \Delta x + \frac{\partial f}{\partial u}\Big|_{x_0,u_0} \Delta u + higher \quad order \quad term \tag{2.73}$$

Since ($\mathbf{x}$-$\mathbf{x}_0$) is very small, all terms of order two and higher can be neglected. This yields,

$$\Delta \dot{x} = x - x_0 = \frac{\partial f}{\partial x}\Big|_{x_0,u_0} \Delta x + \frac{\partial f}{\partial u}\Big|_{x_0,u_0} \Delta u \tag{2.74}$$

and similarly,

$$\Delta y = y - y_0 = \frac{\partial g}{\partial x}\Big|_{x_0,u_0} \Delta x + \frac{\partial g}{\partial u}\Big|_{x_0,u_0} \Delta u \tag{2.75}$$

Therefore, the state equation and output equation become,

$$\Delta\dot{x}=A\Delta x+B\Delta u \tag{2.76}$$

$$\Delta y=C\Delta x+D\Delta u \tag{2.77}$$

Where, **A**, **B**, **C** and D matrices are determined by,

$$A=\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}, \quad B=\begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_r} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_r} \end{pmatrix} \tag{2.78}$$

and,

$$C=\begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_n}{\partial x_1} & \cdots & \frac{\partial g_n}{\partial x_n} \end{pmatrix}, \quad D=\begin{pmatrix} \frac{\partial g_1}{\partial u_1} & \cdots & \frac{\partial g_1}{\partial u_r} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_n}{\partial u_1} & \cdots & \frac{\partial g_n}{\partial u_r} \end{pmatrix} \tag{2.79}$$

Chapter 3

Mathematical Models of Multi-Machine Power Systems

3.1 Introduction

A preliminary step for the analysis of the small-signal stability of a power system is to represent the power system by a state space model. Formulation of a state space model of a multi-machine power system is quite detailed and complex, even though the problem may be considered to have been solved at some levels, since the models of all major devices in power systems have been studied extensively and standard models for these devices have been proposed [12, 55]. Power systems are composed of many kinds of devices, such as transmission lines, transformers, static and dynamic loads, steam or hydraulic turbines, boilers, generators, exciters, governors and power system stabilizers. The respective models of each of these devices are represented by a set of differential and algebraic equations. Usually, these differential equations are nonlinear and may be linearized at the system operating point of interest by Taylor series expansion. These equations are then integrated into a single system state model. An understanding of each device's characteristics and accurate modeling of its dynamic performance are very important in the study of power system stability. This chapter will introduce some of the models used in this thesis without involving excessive detail.

Power system models are often conveniently defined in terms of the major subsystems of equipment that are active in determining the system performance, such as synchronous generators, interconnecting transmission network, loads and control systems. Figure 3.1 shows a structure of the complete power system for small-signal stability analysis.

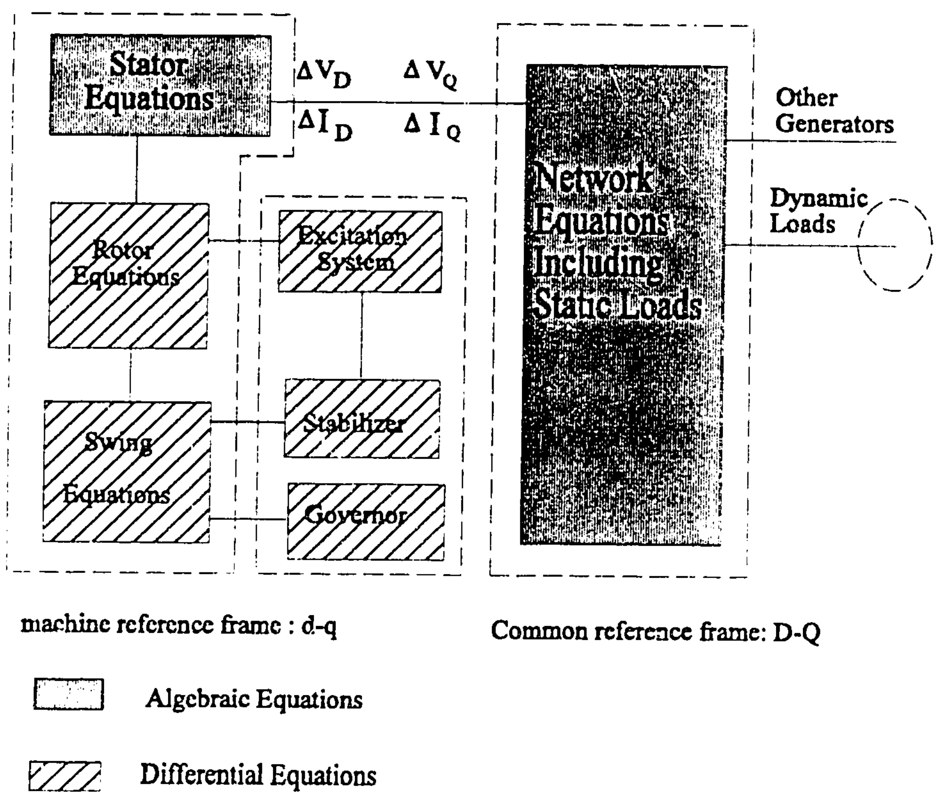


Figure 3.1 Structure of power system model adapted from [35] and [3]

3.2 Basic Considerations for Modeling a Power System

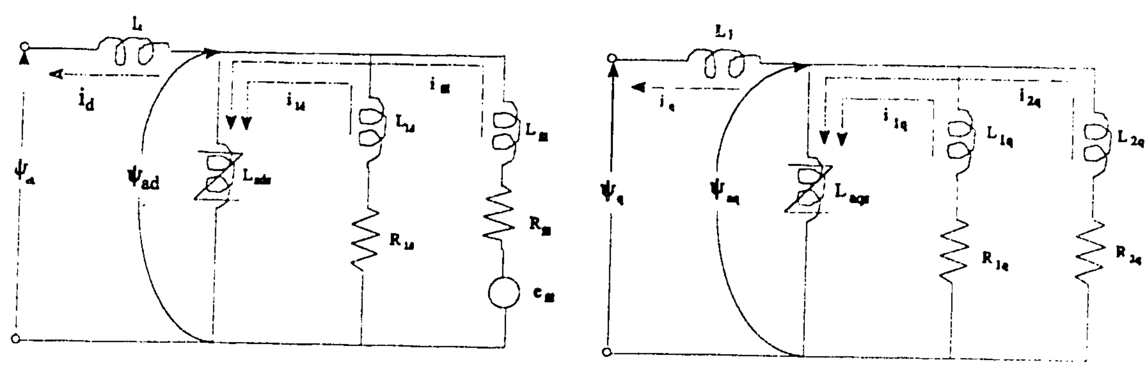
The basic requirements for a properly designed and operated power system are that the system must be able to meet the continually changing load demands and that the power supply must maintain both a constant frequency and voltage with high reliability. Three factors should be considered when formulating a model: accuracy, practical implementation and computation difficulties [16]. A bulk power system may consist of hundreds of generators and thousands of buses. A complete description of the power system may need a very large number of equations. For example, modeling a typical modern power system would require more than several thousand differential and algebraic equations. If the full power system model were to be utilized for design purposes then the controller would require hundreds or even thousands of variables from across the power system to be measured and fed back for a single stabilizer signal. This approach is totally unrealistic. The dynamic model is required to represent all the important system dynamics for the design, while remaining as simple as possible. Generally, only reduced order models are used in the design, such as a single machine infinite bus model using output feedback control model [73].

3.3 Synchronous Generator Model

Synchronous machines may be modeled in varying degrees of complexity, depending on the purpose of the model usage and the needed accuracy of results. In a small-signal stability study, usually a detailed model is used for the generators under investigation, and the classical models are used for rest of the system. Generator models are exclusively represented by Park's equations developed by Park in the 1920s. In Park's equations the terminal voltages and currents are transformed, from the actual voltage and currents, to a reference frame fixed in the rotor, i.e., each generator model is expressed in its own direct axis-quadrature (d-q) reference frame. Park's equations for detailed and classical models are included. References 35,23 and 24 contain commonly used models and details of derivation.

3.3.1 Detailed Model

The detailed model which is commonly used in power system stability analysis includes one d-axis amortisseur and two q-axis amortisseurs. The equivalent circuits are shown in Figure 3.2.



(a) d-axis (b) q-axis

Figure 3.2 Synchronous generator equivalent circuits

3.3.1.1 Stator Equations

The d-axis stator voltage is given by,

$$v_d = -r_a i_d + x_q'' i_q + E_d'' \tag{3.1}$$

where,

$$E_d'' = -\omega L_{aq}'' \left(\frac{\psi_{1q}}{L_{1q}} + \frac{\psi_{2q}}{L_{2q}} \right) \tag{3.2}$$

ψ = flux linkages

i_d = d-axis stator current

i_q = q-axis stator current

r_a = stator winding resistance

x_q'' = q-axis subtransient reactance

The q-axis stator voltage is represented by,

$$v_q = -r_a i_q - x_d'' i_d + E_q'' \tag{3.3}$$

where,

$$E_q'' = \omega L_{ad}'' \left(\frac{\psi_{fd}}{L_{fd}} + \frac{\psi_{1d}}{L_{1d}} \right) \tag{3.4}$$

3.3.1.2 Rotor Equations

The field flux linkage equation is,

$$p\psi_{fd}=\frac{\omega_0 R_{fd}}{L_{adu}}E_{fd}-\omega_0 R_{fd} i_{fd} \tag{3.5}$$

and the d-axis damper winding flux linage equation is,

$$p\psi_{1d}=-\omega_0 R_{1d} i_{1d} \tag{3.6}$$

while the q-axis damper winding flux linkage equations are,

$$p\psi_{1q}=-\omega_0 R_{1q} i_{1q} \tag{3.7}$$

$$p\psi_{2q}=-\omega_0 R_{2q} i_{2q} \tag{3.8}$$

where, the rotor currents are given by,

$$i_{fd}=\frac{1}{L_{fd}}(\psi_{fd}-\psi_{ad}) \tag{3.9}$$

$$i_{1d}=\frac{1}{L_{1d}}(\psi_{1d}-\psi_{ad}) \tag{3.10}$$

$$i_{1q}=\frac{1}{L_{1q}}(\psi_{1q}-\psi_{ad}) \tag{3.11}$$

$$i_{2q}=\frac{1}{L_{2q}}(\psi_{2q}-\psi_{ad}) \tag{3.12}$$

The **d**-and **q**-axis mutual flux linkages are given by,

$$\psi_{ad}=L''_{ads}(-i_d+\frac{\psi_{fd}}{L_{fd}}+\frac{\psi_{1d}}{L_{1d}}) \tag{3.13}$$

$$\psi_{aq}=L''_{aqs}(-i_q+\frac{\psi_{1q}}{L_{1q}}+\frac{\psi_{2q}}{L_{2q}}) \tag{3.14}$$

where,

$$L''_{ads}=\frac{1}{\frac{1}{L_{ads}}+\frac{1}{L_{fd}}+\frac{1}{L_{1d}}} \tag{3.15}$$

$$L''_{aqs}=\frac{1}{\frac{1}{L_{aqs}}+\frac{1}{L_{1q}}+\frac{1}{L_{2q}}} \tag{3.16}$$

3.3.1.3 Saturation Effects

Both x_{ad} and x_{aq} are assumed to vary due to magnetic path saturation. The effects of d-axis magnetic saturation may be considered by the following equations,

$$L_{ads}=K_{sd}L_{adu} \tag{3.17}$$

and,

$$L_{ads(incr)}=K_{sd(incr)}L_{adu} \tag{3.18}$$

where,

K_{sd} is the d-axis total saturation factor;

$K_{sd(incr)}$ is the d-axis incremental saturation factor, which is associated with the perturbed value of flux linkages and currents and can be calculated by,

$$K_{sd(incr)}=\frac{1}{1+B_{sat}A_{sat}e^{B_{sat}(\psi_{ato}-\psi_{T1})}} \tag{3.19}$$

The distinction between incremental and total saturation can be shown by Figure 3.3.

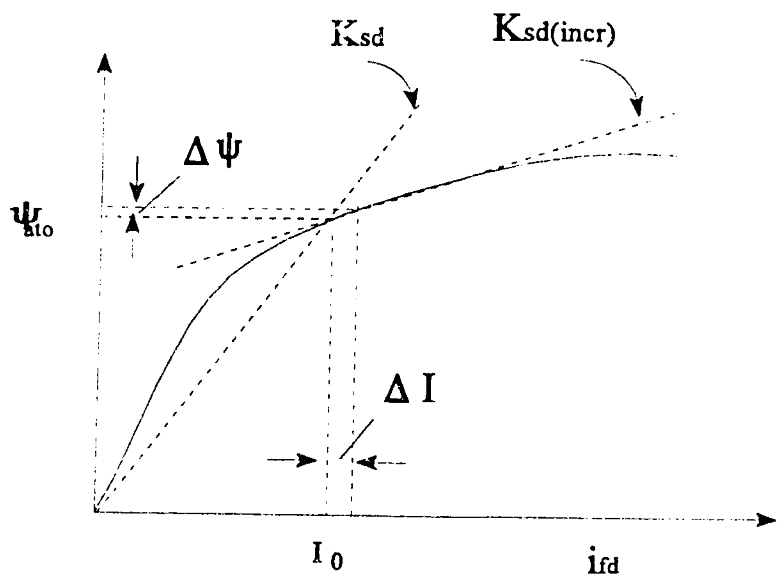


Figure 3.3 Incremental and total saturation

A similar treatment applies to q-axis saturation.

3.3.1.4 Electro-Mechanical Rotor Equations

The acceleration equations are,

$$p\Delta\omega=\frac{1}{2H}(T_m-T_e-K_D\Delta\omega) \tag{3.20}$$

$$p\delta=\omega_0\Delta\omega \tag{3.21}$$

The air-gap torque is determined by,

$$T_e=\psi_{ad}i_q-\psi_{aq}i_d \tag{3.22}$$

where,

K_D is the damping factor,

H is the inertia constant.

The above differential-algebraic equations constitute the detailed model of the synchronous machine. For small signal stability studies these equations are linearized about a steady state operating point. State variables are usually chosen to be the change in the fluxes, ψ_{fd}, ψ_{1q}, ψ_{2q} and ψ_{1d}, in speed, ω, and load angle, δ, of the generator.

3.3.1.5 Reference Frame Transformation

For the solution of interconnecting network equations, all voltages and currents must be expressed in a common (**D-Q**) reference frame. Usually a reference frame rotating at synchronous speed is used as the common reference. Axis transformation equations are used to transform between the individual machine (**d-q**) and network (**D-Q**) reference frames given by Eq. (3.23),

$$\begin{pmatrix} v_D \\ v_Q \end{pmatrix} = \begin{pmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} v_d \\ v_q \end{pmatrix} \tag{3.23}$$

Where δ is defined as the angle, by which the generator **q**-axis leads the **D**-axis as shown in Figure 3.4. The relationship between generator(**d-q**) and network (**D-Q**) reference can also be illustrated by figure 3.4.

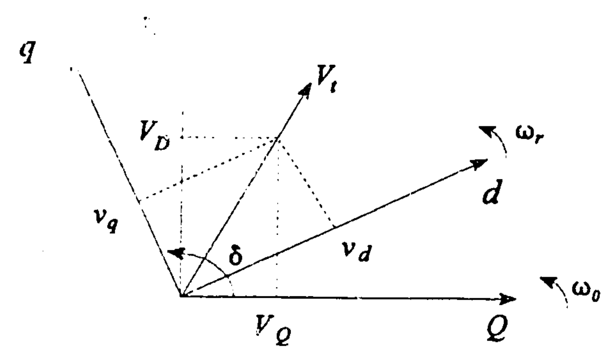


Figure 3.4 Reference frame transformation

After linearization, the operating points are obtained from a load flow program, and after eliminating changes in stator flux linkage and rotor currents by application of the incremental versions of equations, Park's equations may be represented in the state space form, as,

$$\dot{x}_d = A_d x_d + B_d \Delta v \tag{3.24}$$

and,

$$\Delta i_s = C_d x_d - Y_d \Delta v \tag{3.25}$$

where,

x_d = the perturbed value of the individual device state variables,

A_d = devices state matrix,

Δi_s = represents the change in terminal current at the device bus injected into the AC network,

B_d = control bus input matrix,

Δv = represents the change in the terminal voltage at the device bus,

Y_d = device admittance matrix.

3.3.2 Classical Generator and Infinite Bus

The classical generator model assumes that a generator with constant voltage behind x_d' is connected through a reactance x_e to an infinite bus as shown in Figure 3.5, where δ is the angle by which **E'** leads the infinite bus voltage V_B.

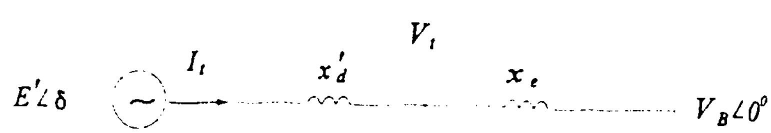
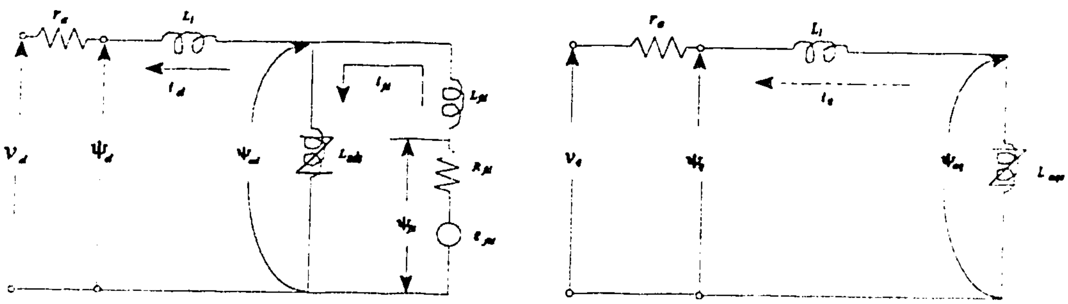


Figure 3.5 Classical generator and infinite bus

For this reduced model, amortisseurs are neglected. There is only one field winding in the model. The equivalent circuit is represented in Figure 3.6.



d - axis *q - axis*

Figure 3.6 Equivalent circuit of a single machine infinite bus

3.3.2.1 Stator Equations

For the corresponding circuits shown in Figures 3.5 and 3.6, the d-q axis stator voltages are given by,

$$v_d = -r_a i_d - \psi_q = -r_a i_d + (L_l i_q - \psi_{aq}) \tag{3.26}$$

$$v_q = -r_a i_q + \psi_d = -r_a i_q - (L_l - \psi_{ad}) \tag{3.27}$$

where,

$$\psi_{ad} = L_{ads} i_d + L_{ads} i_{fd} = L_{ads} i_d + \frac{L_{ads}}{L_{fd}} (\psi_{fd} - \psi_{ad}) \tag{3.28}$$

$$\psi_{aq} = -L_{aqs} i_q \tag{3.29}$$

3.3.2.2 Rotor Equations

Since there is only one field winding in the rotor, the rotor equation becomes very simple as shown in Eq. (3.5),

$$p\psi_{fd} = \frac{\omega_0 R_{fd}}{L_{adu}} E_{fd} - \omega_0 R_{fd} i_{fd} \qquad (3.30)$$

3.3.2.3 Swing Equations

The electro-mechanical rotor equations are the same as Eqs. (3.20) and (3.21). The corresponding equations may be represented by the block diagram shown in Figure 3.7.

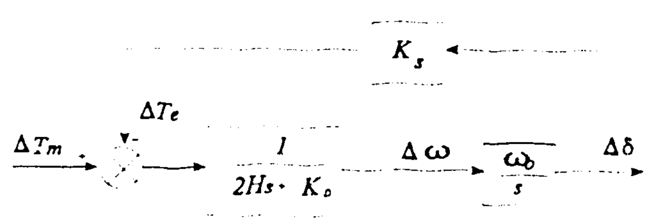


Figure 3.7 Block diagram of swing equations

where,

K_D is the damping factor,

H is inertia constant,

ω_0 is the rated speed,

K_s is the synchronizing torque coefficient given by,

$$K_s = \left(\frac{E'V_B}{x_d' + x_e} \right) \cos\delta_0 \tag{3.31}$$

3.3.2.4 System Block Diagram

A complete block diagram for a single machine infinite bus (SMIB) with excitation system, turbine/governor and power system stabilizer is shown in Figure 3.8.

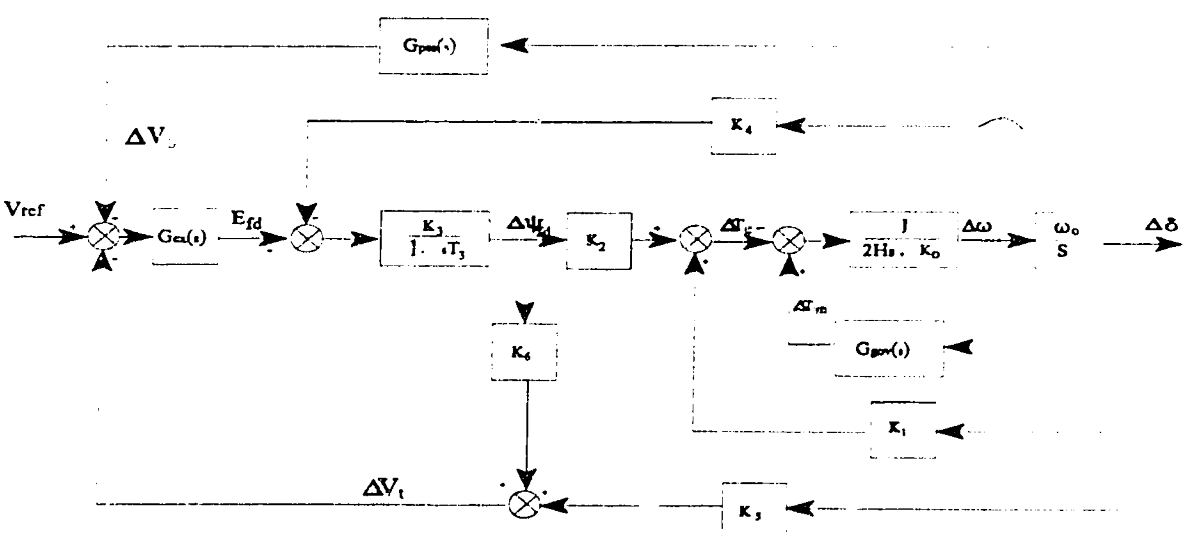


Figure 3. 8 Block diagram of complete system

3.4 Turbine/Governor Model

The basic function of a governor is to control speed and/or generator loading. The primary speed/load control function involves feeding back speed error to control the gate position of a hydro unit or steam valves of a fossil-fired unit. A block diagram for turbine/governor is depicted in Figure 3.9. Other types of governors can found in [24, 32].

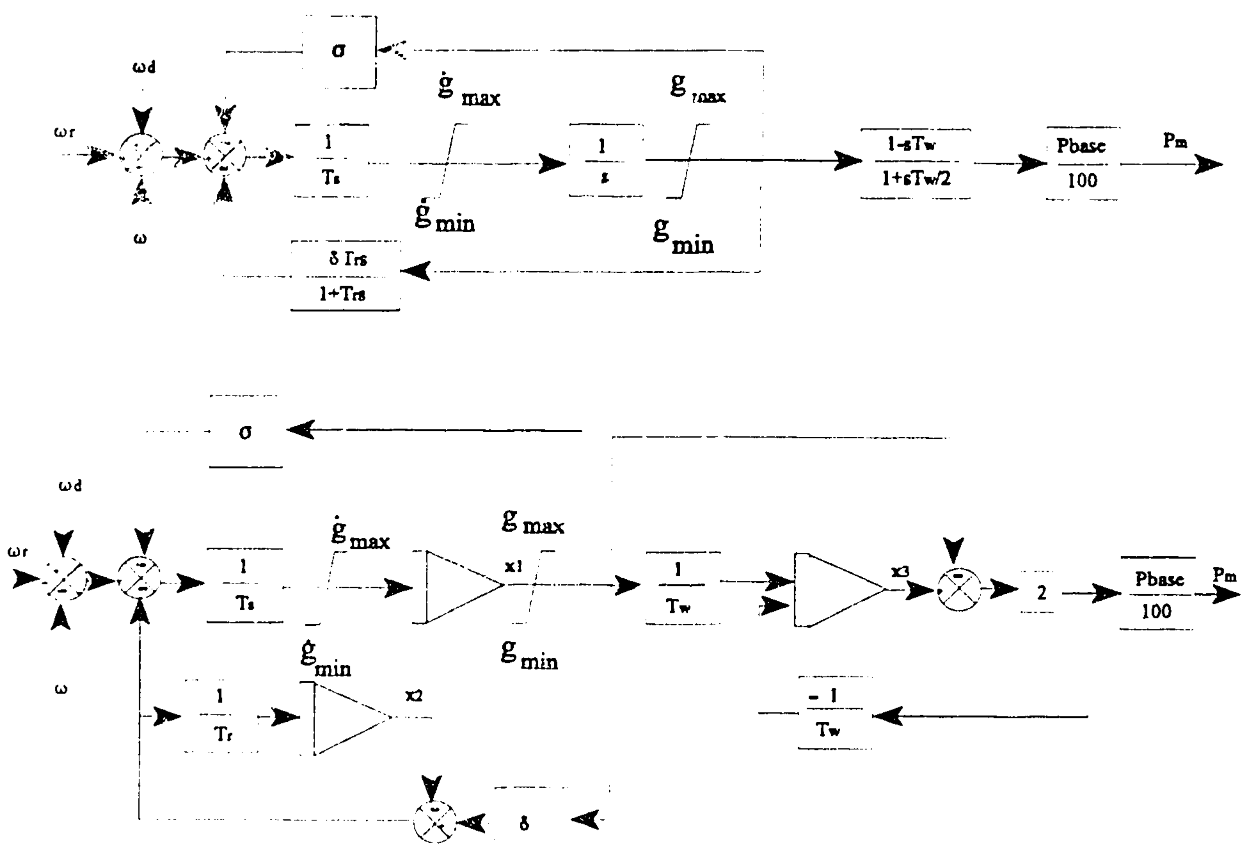


Figure 3.9 Block diagram and analog wiring diagram of hydraulic turbine and mechanical hydraulic governor [79]

The differential equations for the governor block diagram are given by,

$$\dot{x}_1 = -\frac{1}{T_s}(\sigma+\delta)x_1 + \frac{1}{T_s}x_2 + \frac{1}{T_s}u_g \qquad (3.32)$$

$$\dot{x}_2 = \frac{\delta}{T_r}x_1 - \frac{1}{T_r}x_2 \qquad (3.33)$$

$$\dot{x}_3 = \frac{3}{T_w}x_1 - \frac{2}{T_w}x_3 \qquad (3.34)$$

and,

$$u_g = \omega_r + \omega_d - \omega \qquad (3.35)$$

3.5 Excitation Model

The basic function of an excitation system is to provide direct current to the synchronous generator field winding. In addition, the excitation system performs control and protective functions essential to the satisfactory performance of a power system by controlling the field voltage and thereby the field currents. The performance requirements of excitation systems are determined by considering the synchronous generator as well as the power system. Excitation systems are of many different designs, and there is no single mathematical model that is adequate for all types. Based on performance, they can be classified into linear and nonlinear. Based on the excitation power system used, they can be

divided into **DC**, **AC** and Static types. Models for stability studies are given in [33].
Consider a simple static exciter represented by the block diagram in Figure 3.10.

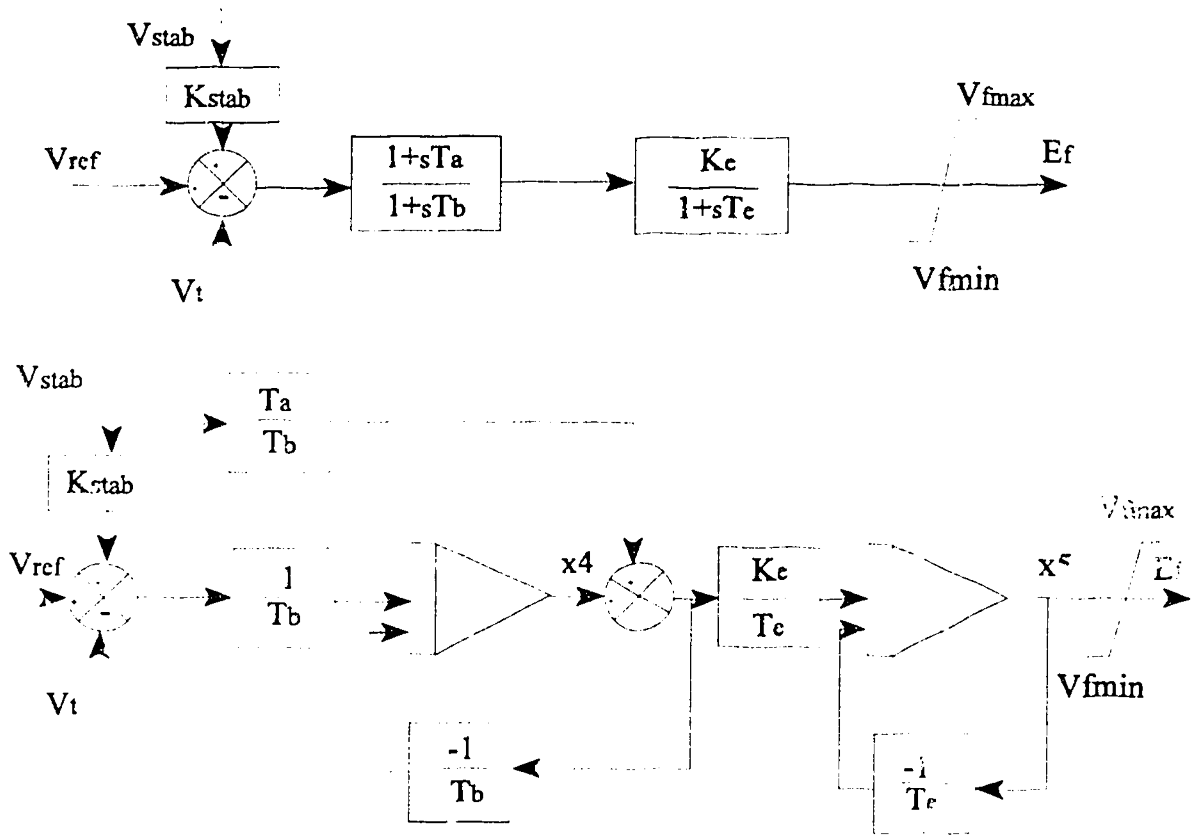


Figure 3.10 Block diagram of a simple exciter [79]

Differential equations for this exciter can be written directly from the analog wiring diagram.
These are given by,

$$\dot{x}_4 = -\frac{1}{T_b}x_4 + \frac{1}{T_b}(1 - \frac{T_a}{T_b})u_e \qquad (3.36)$$

$$\dot{x}_5 = \frac{K_e}{T_e}x_4 - \frac{1}{T_e}x_5 + \frac{K_e T_a}{T_e T_b}u_e \tag{3.37}$$

and,

$$u_e = V_{ref} + K_{stab} V_{stab} - V_t \tag{3.38}$$

3.6 Power System Stabilizer Model

The simple power system stabilizer used in this thesis is represented by the block diagram as shown in Figure 3.11. Models of PSSs are contained in [32]. The application of stabilizers to control generator excitation systems is the most cost-effective method of enhancing the small-signal stability of power systems. The basic function of a PSS is to add damping to the generator rotor oscillations by modulating the generator excitation to develop a component of electrical torque in phase with rotor speed deviations. The design techniques of a PSS are described in chapter 4 of this thesis.

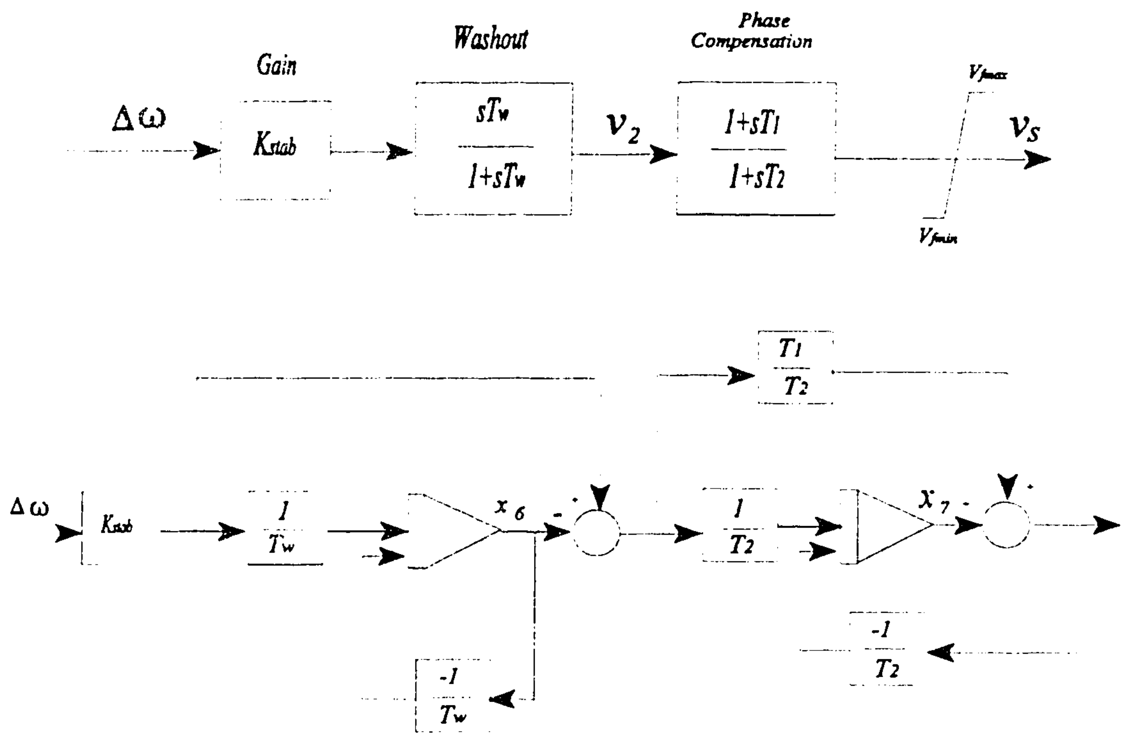


Figure 3.11 Block diagram of simple PSS

The corresponding differential equations as deduced from the PSS block diagram are given by,

$$\dot{x}_6 = -\frac{1}{T_w} x_6 + \frac{1}{T_w} u_p \tag{3.39}$$

$$\dot{x}_7 = \frac{1}{T_2}\left(\frac{T_1}{T_2} - 1\right) x_6 - \frac{1}{T_2} x_7 - \frac{1}{T_2}\left(\frac{T_1}{T_2} - 1\right) u_p \tag{3.40}$$

with,

$$u_p = K_{stab} \Delta \omega \tag{3.41}$$

3.7 Load Model

The electric power consumed by different types of loads is determined by the terminal voltage and frequency. The dependence of active and reactive consumption on voltage and/or frequency is generally described in the following mathematical forms:

$$\frac{P_L}{P_{L0}} = \left(\frac{V}{V_0}\right)^{m_v} \left(\frac{f}{f_o}\right)^{m_f} \tag{3.42}$$

$$\frac{Q_L}{Q_{L0}} = \left(\frac{V}{V_0}\right)^{n_v} \left(\frac{f}{f_o}\right)^{n_f} \tag{3.43}$$

where

P_{L0} = initial value of the active component of load,

Q_{L0} = initial value of the reactive component of load,

V_0 = initial value of the bus voltage magnitude,

m_v, m_f, n_v and n_f = parameters determined by the load.

Load models are traditionally classified into two broad categories: static models and dynamic models. Static loads are relatively unaffected by frequency changes [6,18]. They can be classified as linear (constant impedance) and nonlinear loads. Loads are usually assumed to be linear (constant impedance). In this study, linear loads are represented by,

$$B_L=-\frac{Q_{L0}}{v_0^2} \tag{3.44}$$

$$G_L=\frac{P_{L0}}{v_0^2} \tag{3.45}$$

while nonlinear loads are described as,

$$\frac{P_L}{P_{L0}}=\left(\frac{V}{V_0}\right)^m \tag{3.46}$$

$$\frac{Q_L}{Q_{L0}}=\left(\frac{V}{V_0}\right)^n \tag{3.47}$$

These equations must be linearized. Linearization of Eqs.(3.46) and (3.47) yields,

$$\begin{pmatrix} \Delta i_D \\ \Delta i_Q \end{pmatrix}=\begin{pmatrix} G_{DD} & B_{DQ} \\ -B_{QD} & G_{QQ} \end{pmatrix}\begin{pmatrix} \Delta v_D \\ \Delta v_Q \end{pmatrix} \tag{3.48}$$

where,

$$G_{DD}=\frac{P_{L0}}{v_0^2}\left[(m-2)\frac{v_{D0}^2}{v_0^2}+1\right]+\frac{Q_{L0}}{v_0^2}\left[(n-2)\frac{v_{D0}v_{Q0}}{v_0^2}\right] \tag{3.49}$$

$$B_{DQ}=\frac{Q_{L0}}{v_0^2}\left[(n-2)\frac{v_{Q0}^2}{v_0^2}+1\right]+\frac{P_{L0}}{v_0^2}\left[(m-2)\frac{v_{D0}v_{Q0}}{v_0^2}\right] \tag{3.50}$$

$$B_{QD}=\frac{Q_{L0}}{v_0^2}\left[(n-2)\frac{v_{D0}^2}{v_0^2}+1\right]-\frac{P_{L0}}{v_0^2}\left[(m-2)\frac{v_{D0}v_{Q0}}{v_0^2}\right] \tag{3.51}$$

$$B_{QQ}=\frac{P_{L0}}{v_0^2}\left[(m-2)\frac{v_{Q0}^2}{v_0^2}+1\right]-\frac{Q_{L0}}{v_0^2}\left[(n-2)\frac{v_{D0}v_{Q0}}{v_0^2}\right] \tag{3.52}$$

The effect of terminal voltage has always been taken into account in stability studies, but the effect of frequency change has only been considered in recent years. More and more attention has been given to adequate modeling of consumer behavior because it may have a marked influence on the accuracy and reliability of stability investigations [56].

3.8 Network Model

In power system stability analysis, the electromagnetic transient processes of network elements are generally neglected [56]. The transmission, and/or interconnection lines, are assumed to be represented by linear lumped parameter systems Similarly, the transformers are represented by their steady equivalent circuit in algebraic equations. As a result, the network is described by algebraic equations. The network equations in terms of node admittance matrix can be written as,

$$I=YU \tag{3.53}$$

where,

Y is the nodal admittance matrix.

This equation may be written in partitioned form as,

$$\begin{pmatrix} I_N \\ I_L \end{pmatrix} = \begin{pmatrix} Y_{NN} & Y_{NL} \\ Y_{LN} & Y_{LL} \end{pmatrix} \begin{pmatrix} V_N \\ V_L \end{pmatrix} \tag{3.54}$$

Eq.(3.54) can be written as,

$$I_N=Y_{NN}V_N+Y_{NL}V_L \tag{3.55}$$

$$I_L=Y_{LN}V_N+Y_{LL}V_L \tag{3.56}$$

Rewriting Eq.(3.56) as,

$$V_L=Y_{LL}^{-1}I_L-Y_{LL}^{-1}Y_{LN}V_N \tag{3.57}$$

and substituting it into Eq.(3.55), results in,

$$I_N=(Y_{NN}-Y_{NL}Y_{LL}^{-1}Y_{LN})V_N+Y_{NL}Y_{LL}^{-1}I_L \tag{3.58}$$

If loads are represented by constant impedances, $i_L=0$, then Eq.(3.58) becomes,

$$I_N=(Y_{NN}-Y_{NL}Y_{LL}^{-1}Y_{LN})V_N \tag{3.59}$$

which can be simplified as,

$$I_N=Y_{NN}^{*}V_N \tag{3.60}$$

3.9 Formulation of System State Space Matrix

The state equations for each generator described by Eqs.(3.24) and (3.25) may be put into matrix equation form,

$$\dot{x}=A_Dx+B_D\Delta V \tag{3.61}$$

$$\Delta I=C_Dx-Y_D\Delta V \tag{3.62}$$

For the interconnecting transmission network, with the load represented by a constant impedance, the node admittance matrix may be represented by,

$$\begin{pmatrix} \Delta I \\ 0 \end{pmatrix}=\begin{pmatrix} Y_{NN} & Y_{NL} \\ Y_{LN} & Y_{LL} \end{pmatrix}\begin{pmatrix} \Delta V \\ \Delta V_L \end{pmatrix} \tag{3.63}$$

where,

$\Delta I=[\Delta i_1,\Delta i_2, \cdots \Delta i_n]^T$ is the collection of all changes in the current injected by the devices into the network,

$\Delta V=[\Delta v_1,\Delta v_2 \cdots \Delta v_n]^T$ is the collection of all the device terminal voltage changes,

$\Delta V_L=[\Delta v_1,\Delta v_2 \cdots \Delta v_L]^T$ is the collection of all the load bus voltage changes.

By eliminating the load bus voltage changes, Eq.(3.63) may be reduced to,

$$\Delta I=Y^{*}_{NN}\Delta V \tag{3.64}$$

It follows that,

$$Y^{*}_{NN}=(Y_{NN}-Y_{NL}Y^{-1}_{LL}Y_{LN}) \tag{3.65}$$

Substituting Eq.(3.64) into Eq.(3.62) yields,

$$\Delta V=(Y^{*}_{NN}+Y_D)^{-1}C_D x \tag{3.66}$$

The state equation for the overall system representation may be obtained by substituting Eqs.(3.66) and (3.64) into Eq.(3.61). The result is,

$$\dot{x}=(A_D+B_D(Y^{*}_{NN}+Y_D)^{-1}C_D)x \tag{3.67}$$

or,

$$\dot{x}=Ax \qquad (3.68)$$

where,

A is the state matrix of the complete system and is given by,

$$A=(A_D+B_D(Y_{NN}^{*}+Y_D)^{-1}C_D) \qquad (3.69)$$

The program developed in this thesis was based on the above models and computing methods and is illustrated by Figure 3.12. The program was written as a Matlab M-file. The program was first tested on a benchmark model, which has been used to study the fundamental nature of interarea oscillations by other reseachers [34]. The system consists of two similar areas connected by a weak tie. Each area consists of two coupled units, each having of 900 MVA and 20 KV. Generators are modeled by detailed models.

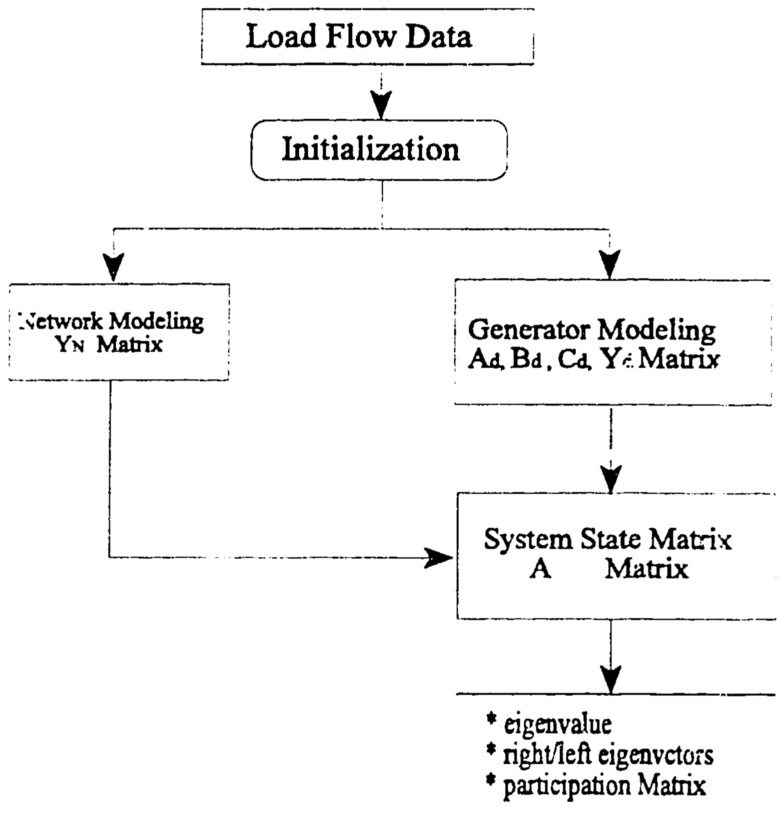


Figure 3.12 Conceptual overview of a small signal program structure

Chapter 4

Design of Power System Stabilizers

4.1 Introduction

The science of applying power system stabilizers has developed rapidly since the 1960s. The use of PSS is one of the common remedies for inadequate damping where oscillations have been particularly troublesome. The basic function of a stabilizer is to extend stability limits by modulating generator excitation to enhance damping of the electric power system during low frequency oscillations. The controller contributes positive damping to an electromechanical oscillation by producing a component of electrical torque on the rotor that is in phase with the speed variation. A comprehensive summary on the theory and applications of a PSS can be found in [40].

A power system stabilizer is basically a linear controller [12]. It consists of a gain, a washout, and two or three stages of compensation blocks, normally in the form of first order lead/lag blocks. Rotor speed, bus frequency, electrical power, or a combination of electrical power and speed or frequency are most commonly used as input signals to PSSs, and the output of a stabilizer is added to the AVR input summing junction. The design of a PSS controller includes the determination of appropriate settings for the adjustable parameters, i.e., gain and time constants in the washout and lead/lag stages' blocks. The principle steps of the design process are .. follows.

(1) D[illegible]ine the PSS location using participation factors calculated by a sm[illegible] [illegible]nal stability program;

(2) Design the compensation by applying linear design techniques;

(3) Evaluate the performance of the PSS under various system conditions and disturbances by utilizing load flow and transient programs;

(4) Ensure that there are no adverse effect[illegible] on the other modes by using [illegible]igenvalue analysis [12].

In this chapter, basic design theories for pole placement techniques are presented. These include pole placement via conventional control, pole placement via state feedback and linear optimal control. A case study is carried out, which utilized modeling, stability and control techniques introduced in this thesis.

4.2 Pole Placement Technique via Conventional Control Theory

For a single-input single-output time invariant system, pole placement techniques can be used to shift a pair of dominant poles to a newly assigned location in the s-plane. The proximity to instability can be defined by phase and gain margins between the plant open loop response and the (-1+j0) point. These are used to design a compensator that causes the response specifications to be met. Note that in phase compensation techniques, the adverse effects on other modes can be taken into account right at the beginning. whereas, in the case of pole-placement techniques such effects are checked after the initial design, and possibly reduced by trial and error. Moreover, the former is more suitable when two or more modes are to be controlled by the same compensator. Consider a simple feedback control system as shown in Figure 4.1.

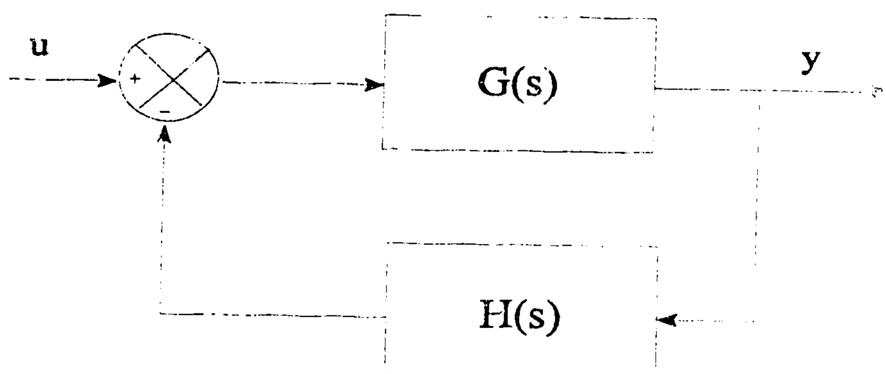


Figure 4.1 A feedback control system

where,

G(s) is the transfer function of open-loop plant,

H(s) is the transfer function of the controller.

The closed-loop transfer function of the compensated plant may be represented by,

$$G_c(s)=\frac{G(s)}{1+G(s)H(s)} \tag{4.1}$$

where 1+G(s)H(s) is called the characteristic equation of the closed-loop system. Suppose that a system eigenvalue is to be moved to a new location in the s-plane denoted by λ_0. λ_0 is subjected to the characteristic equation of the closed-loop system, i.e.,

$$H(\lambda_0)=\frac{-1}{G(\lambda_0)} \tag{4.2}$$

This can be written in terms of magnitude and phase as,

$$|H(\lambda_0)|=\frac{1}{|G(\lambda_0)|} \tag{4.3}$$

and,

$$arg(H(\lambda_0))=180-arg(G(\lambda_0)) \tag{4.4}$$

Obviously, the magnitude and phase of the compensator at λ_0 can be computed from the magnitude and phase of the plant at λ_0. λ_0 is chosen to satisfy the specified damping ratio. The compensator generally consists of a washout and a series of lead/lag functions as shown below.

$$K\frac{sT_w}{1+sT_w}\frac{1+sT_1}{1+sT_2}\cdots\cdots\frac{1+sT_{2n-1}}{1+sT_{2n}} \tag{4.5}$$

The washout, $\frac{sT_w}{1+sT_w}$, is intended to eliminate the DC and to attenuate very low frequency components of the measured signal. Each lead or lag block compensation $\frac{1+sT_{2i-1}}{1+sT_{2i}}$ is used to improve the system performance. "The decision to use phase-lead rather than phase-lag is arbitrary. In general, phase-lead increases the bandwidth of the closed-loop transfer function and hence the system will respond faster to set-point adjustments. Phase-lag tends to 'slow' the system down. Phase-lead tends to increase the noise input to the process and this limits the amount of phase-lead that can be contributed by the compensator [8]." Phase-lead is limited to a maximum of 60^0 for practical purpose related to noise amplification and phase-lag can provide more than 40^0 for practical reasons. The angle, θ_m, that the i^{th} block can provide is determined by,

$$\sin\theta_m = \frac{1-\alpha_i}{1+\alpha_i} \tag{4.6}$$

where,

$$\alpha_i = \frac{T_{2i}}{T_{2i-1}} \tag{4.7}$$

The frequency at which this maximum angle occurs is given by,

$$\omega_m = \frac{1}{\sqrt{\alpha_i}T_{2i-1}} \tag{4.8}$$

which is usually chosen to be near the frequency of λ_0. Finally, the gain K is chosen to meet the magnitude equation. This technique guarantees an eigenvalue at λ_0, but does not guarantee that the poorly damped eigenvalue will shift to λ_0.

4.3 Pole Placement via State Space Feedback

For a system with state space representation, state space feedback is commonly used to obtain a suitable closed-loop pole location. A number of pole placement techniques based on this theory have been applied in the design of PSS. These include full state feedback [58], a full order observer [28], output feedback [16], a low order dynamic compensator designed by the projective method [15] as well as eigenstructure assignment by decentralized feedback control [44]. Of these five methods somewhat more attention is paid to the last three methods with emphasis on the usage of the techniques rather than on rigorous theory. Suppose that a controllable linear time invariant system defined by,

$$\dot{X}=AX+Bu \tag{4.9}$$

$$Y=CX \tag{4.10}$$

where,

X= state vector n×1,

u=control signal m×1,

Y=output vector r×1,

A, **B**, **C**= constant matrices

Wonham [28,58] showed that a controllable system is always pole-assignable by an appropriate state feedback. By substituting the control law into Eq.(4.9), the closed loop system matrix is given by,

$$A\text{-}BK \tag{4.11}$$

The poles of the closed loop system are determined by the characteristic equation

$$|sI\text{-}(A\text{-}BK)| \tag{4.12}$$

A block diagram of full state feedback control system is shown in Figure 4.2.

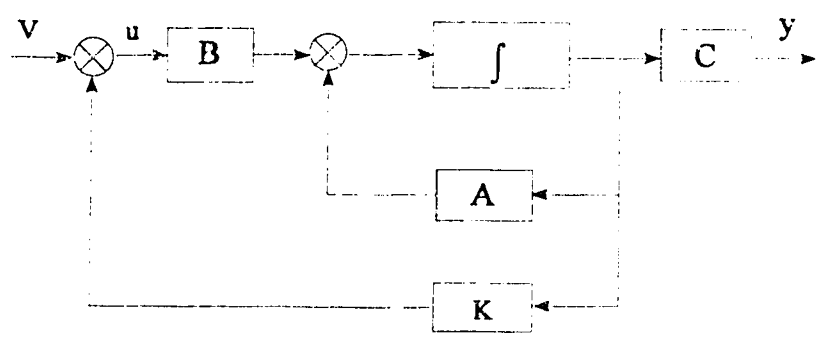


Figure 4.2 Full state feedback system

Design steps for multi-input cases described below is taken from references [39, 52, 58].

Step 1. Check whether the pair (**A**,**B**) is controllable by

$$rank\ Q = rank\left[B \;\; AB \;\; \cdots \;\; A^{n-1}B\right] = n \tag{4.13}$$

If it is not, then convert to the Kalman canonical form.

Step 2. Randomly choose suitable dimensions $\mathbf{F}_0$. If **A** has repeated eigenvalues, then define

$$A_0 = A + BF_0 \tag{4.14}$$

until $\mathbf{A}_0$ has distinct eigenvalues.

Step 3. Randomly pick **u**, which has suitable dimensions, then define

$$B_0 = Bu \tag{4.15}$$

until $(\mathbf{A}_0, \mathbf{B}_0)$ is controllable.

Step 4. Assign the desired poles Λ^d by a suitable choice of state feedback $\hat{F}$ such that

$$\sigma(A_0 - B_0\hat{F}) = \Lambda^d \tag{4.16}$$

Step 5. The feedback control law **F** is found by,

$$F = F_0 + u\hat{F} \tag{4.17}$$

Then check whether the closed loop matrix has the required poles.

$$A\text{-}BF \tag{4.18}$$

From a practical view, the necessity of full state feedback is undesirable. This is obvious when considering large order systems and the cost of measuring and feeding back each state variable. Output feedback is more attractive because it requires fewer measured variables. Davison [16] indicated that if the system is controllable and observable and if rank [B]=m and rank [C] = r, then max(m, r) eigenvalues are assignable almost arbitrarily. The eigenvalues of the closed loop system are determined by,

$$|sI-(A-BFC)| \tag{4.19}$$

A block diagram for the output feedback control system is shown in 4.3.

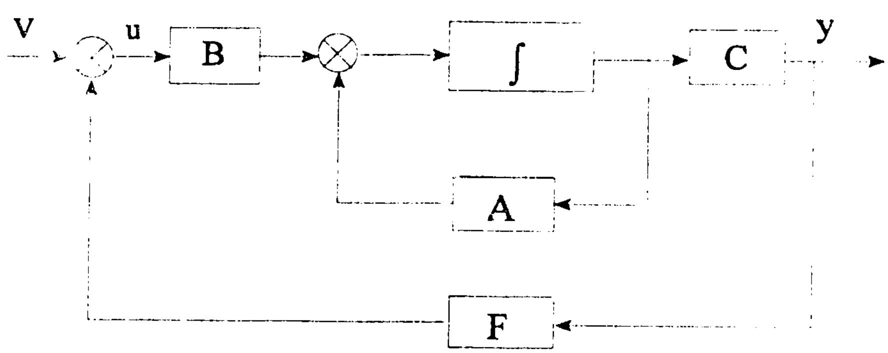


Figure 4.3 Output feedback system

4.4 Linear Optimal Designs

The design of a linear optimal stabilizer is commonly formulated as a linear optimal (time invariant) regulator problem [75,76]. The design theory for the linear optimal stabilizer is very simple. Consider a linearized power system model described by,

$$\dot{X} = AX + Bu \tag{4.20}$$

where,

A and **B** are system matrices,

X and **u** are the state variable vector and control signal vector, respectively.

An optimal control signal **u** can be found from minimization of a chosen quadratic performance index:

$$J=\frac{1}{2}\int_0^\infty (X^TQX+u^TRu)dt \tag{4.21}$$

subject to the system dynamics constraint (4.20). The optimal control is a linear function in terms of the system state variables X as

$$u=-KX=-R^{-1}B^TPX \tag{4.22}$$

where,

Q and **R** are the weighting matrices,

K is the feedback gain matrix,

P is the solution of Riccati equation.

The Riccati equation is the key to the design of the linear optimal control system and is given by,

$$A^{T}P+PA-PBR^{-1}B^{T}P+Q=0 \qquad (4.23)$$

Once **Q** and **R** are known, the matrix **P** can be obtained by solving equation (4.23). Then the optimal control signal **u** can be calculated by equation (4.22). With **u** determined, the closed-loop system equation becomes

$$\dot{X}=Ax-BKx=(A-BK)x=GX \qquad (4.24)$$

where,

$$G=A-BR^{-1}B^{T}P \qquad (4.25)$$

Therefore, the eigenvalues of the closed-loop system **G** depend upon the selection **Q** and **R** for the cost function.

A very important and difficult problem that challenges the designer is how to select the weighting matrices **Q** and **R**. Different matrices pairs ($\mathbf{Q}_i$, $\mathbf{R}_i$) determine a different feedback gain $\mathbf{K}_i$, which results in different dynamic performance for the closed-loop system. Large portions of the literature have been written for this problem. The traditional way to select (**Q**,**R**) is to apply a trial and error method.

Many simulation studies have to be done in the time domain with different weighting matrices to choose the one that provides the desired performance [12].

Yu et al [75,76] have proposed a new method to determine **Q** and **R** in conjunction with the dominant eigenvalue shift of the closed-loop system to the left of the complex plane as far as the practical controller permits. However, Vaahedi [68] has pointed out that this approach may not give the "best" response as may be judged by inspection of load angle swing following a fault in the simulation studies, because the generator is essentially nonlinear and some of the variables and controls have reached maximum values.

It is more "realistic" if only $\Delta\delta$ and $\Delta\omega$ are used as feedback. This has been referred to as **suboptimal control** in the literature.

$$K_s = \left[k_1, k_2, 0, \cdots, 0\right] \qquad (4.26)$$

where $\mathbf{k}_1$, $\mathbf{k}_2$ are obtained from optimal controller gain matrix.

Comments

The design procedure for PSS via conventional control is based on a trial-and-error approach, while the design of PSS by state space method enables engineers to design a controller having the desired closed loop roots. State space methods enable engineers to avoid the burden of tedious computations by using computers.

The application of conventional control is readily applied to linear time invariant system having a single input and a single output [51]. Conventional PSSs are generally designed with the parameter settings fixed to their optimal values under a specific operating condition [43]. Usually only one stabilizing signal, rotor speed, AC bus frequency or electrical power, is injected into each excitation control loop. As a result, the interactions between generators cannot be considered easily and this may result in one or more modes of oscillation being left undamped.

In the 1980s attention was directed towards a multi-input multi-output (MIMO) solution with the constraint that local control is applied [31, 75]. Various PSSs based on different control theories, such as state space feedback, linear optimal control theory, adaptive control theory, artificial neural network (ANN), have been proposed for the multi-machine power systems with multi-mode oscillations [78]. The reasons are that modern control theory enables engineers to design a control system having the desired closed-loop poles or optimal control system with respect to given performance indexes. Also, modern control theory enables the designer to include initial conditions. State-space methods are particularly well suited for digital computer computations because of their time-domain representation. This enables a designer to avoid the burden of tedious manual computations. Furthermore, it is not necessary that state variables represent physical quantities of the actual system. Those variables that are neither measurable nor observable may be chosen as state variables. However, design by modern control theory requires an accurate mathematical description of system dynamics [39,52].

Pole placement methods select closed-loop poles without regard to the control effort required to achieve that response. On the other hand, optimal control can select poles that result in some balance between system errors and control effort. These two methods require that every controller have access to all the measurements taken from the system. For large scale systems, like a power system, this requirement presents a need for long-distance communication that in most cases is impractical and uneconomical.

4.5 Case Study

Consider a two-machine infinite-bus power system as shown in Figure 4.4. The generators are mutually coupled and radially connected to an infinite bus. Each plant can be described by a fourth-order synchronous model equipped with a second order exciter. Each generator unit has six state variables, i.e., $\Delta\delta$, $\Delta\omega$, $\Delta\psi_{fd}$, $\Delta\psi_{1q}$, x_1, x_2. Loads are modeled as constant impedance. The system parameters are listed in appendix B.

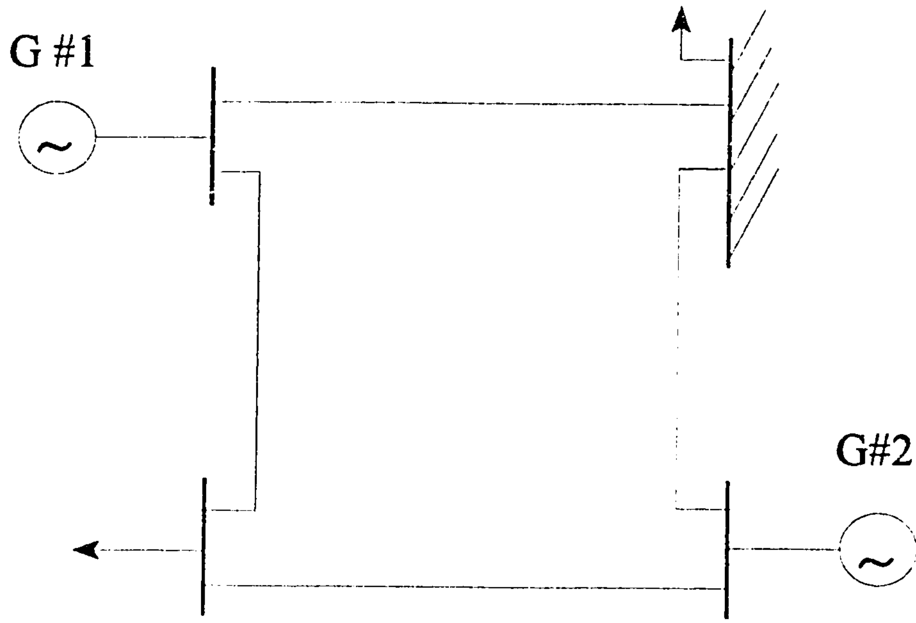


Figure 4.4 Two-machine infinite-bus power system

4.5.1 Eigenvalue Analysis

The system state **A** matrix, eigenvalues, participation factors, damped frequencies and damping ratios can be calculated by the computer program developed in this thesis. The load flow program initializes the small-signal stability program and computes the operating points of the system. The initial operating point is given in Table 4.1.

Table 4.1 Initial operating points

Bus No.	MW (P)	MVR(Q)	Voltage (V)	Angle (δ)
G#1	180.00	200.00	0.998	38.21
G#2	850.00	40.00	1.01	39.51
G#3 (Infinite bus)	-646.00	611.00	1.00	0.00
4	0.00	0.00	1.00	38.86

The system matrix A (no PSS) at this operating point is calculated by the computer program and listed in Eq.(4.27). The program structure is illustrated in Figure 3.12. State space model formulation described in this thesis is easy to implement as compared to the sub-matrix approach by Undrill [67]. Once the state space matrix A is formulated, an eigenvalue program is applied to calculate the eigenvalues, damping ratios, damped frequencies, eigenvectors and participation factors.

$$A=\begin{bmatrix} -0.0011 & -0.1714 & 4.28697 & -2.2722 & 0 & 0 & 0 & 0.0055 & 0.0127 & -0.0004 & 0 & 0 \\ 377 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.3392 & -141.46 & 9.41660 & 0.2405 & 0 & 0.0001 & 0.0177 & -0.0001 & 0 & 0 & 0 \\ 0 & -0.1756 & -6.002 & -19794 & 0 & 0 & 0 & 0.0001 & 0.0126 & -0.0005 & 0 & 0 \\ 0 & 0.0025 & -1.9150 & -2.239 & -0.8333 & 0 & 0 & 0.0000 & 0.0011 & 0.0000 & 0 & 0 \\ 0 & -3.0561 & 2297.9 & 2687.3 & 200.0 & -20.0 & 0 & -0.0168 & -1.2761 & -0.0199 & 0 & 0 \\ 0 & 0.0000 & 0.0109 & -0.0006 & 0 & 0 & -1.875 & -0.1292 & -14.512 & -0.3558 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 377 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0000 & 0.0189 & -0.0014 & 0 & 0 & 0 & -03859 & 141.09 & 6.7328 & 0 & 0.2498 \\ 0 & 0.0001 & 0.0148 & -0.0008 & 0 & 0 & 0 & -0.1539 & 4.5717 & -2.0353 & 0 & 0 \\ 0 & 0.0000 & 0.0016 & -0.0000 & 0 & 0 & 0 & 0.0009 & -0.8743 & -0.9249 & -0.1000 & 0 \\ 0 & 0.0239 & 2.4461 & -0.0759 & 0 & 0 & 0 & 1.4760 & -1360.0 & -1438.8 & 1400.0 & -20.0 \end{bmatrix} \quad (4.27)$$

Eigenvalues are calculated and shown in Table 4.2. It can be seen that the system is stable with two oscillatory modes and eight non-oscillatory modes. The frequencies of these two oscillatory modes are 1.254 and 0.875 Hz. They are lightly damped with damping ratios 0.098 and 0.0626.

Table 4.2 Eigenvalues, damped frequency and damping ratio of the system

Modes No.	Eigenvalue (λ)	Damped frequencies (f_d)	Damping ratio (ζ)
1	-145.70		
2	-137.79		
3	-23.36		
4	-11.84		
5, 6	-0.78±7.88i	1.254	0.098
7	-5.59		
8, 9	-0.34 ±5.50i	0.875	0.0626
10	-0.66		
11	-1.77		
12	-1.51		

The participation factors are listed in Table 4.3. Only the magnitudes of the participation factors are shown in the table since the angle does not provide any useful information [35]. It can be shown that the oscillatory mode of 1.254 Hz is associated primarily with the rotor angle and speed of generator #1, and the 0.875 Hz mode is primarily determined by those of generator #2. The eight non-oscillatory modes are associated with the field and damping winding and AVR. The participation factors indicated that each generator should have a PSS to enhance the system damping for this case.

Table 4.3 The system participation factor matrix

states	Eigenvalues											
	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}
$\Delta\omega_1$	0.0002	0.0000	0.0000	0.0376	0.5144	0.5144	0.0130	0.0000	0.0000	0.0011	0.0000	0.0000
$\Delta\delta_1$	0.0002	0.0000	0.0000	0.0341	0.5123	0.5123	0.0104	0.0000	0.0000	0.0000	0.0007	0.0000
$\Delta\psi_{fd1}$	0.9654	0.0000	0.0000	0.1064	0.0087	0.0087	0.0886	0.0000	0.0000	0.0000	0.0004	0.0000
$\Delta\psi_{1q1}$	0.0012	0.0000	0.0000	0.6448	0.0737	0.0737	1.7279	0.0000	0.0000	0.0634	0.0000	0.0000
Δx_{11}	0.0000	0.0000	0.0000	0.0180	0.0025	0.0025	0.0755	0.0000	0.0000	0.9376	0.0000	0.0000
Δx_{12}	0.0355	0.0000	0.0000	1.6280	0.0442	0.0442	0.7382	0.0000	0.0000	0.0002	0.0000	0.0000
$\Delta\omega_2$	0.0000	0.0008	0.0038	0.0000	0.0002	0.0002	0.0000	0.4975	0.4975	0.0000	0.0627	0.1207
$\Delta\delta_2$	0.0000	0.0008	0.0035	0.0000	0.0001	0.0001	0.0000	0.5154	0.5154	0.0000	0.0039	0.0301
$\Delta\psi_{fd2}$	0.0000	1.0299	0.0304	0.0000	0.0000	0.0000	0.0000	0.0111	0.0111	0.0000	0.0092	0.0053
$\Delta\psi_{1q1}$	0.0000	0.0025	0.0222	0.0000	0.0000	0.0000	0.0000	0.1210	0.1210	0.0000	6.2001	5.1739
Δx_{21}	0.0000	0.0001	0.0059	0.0000	0.0000	0.0000	0.0000	0.0387	0.0387	0.0000	4.7623	5.8424
Δx_{22}	0.0000	0.0259	1.0215	0.0000	0.0000	0.0000	0.0000	0.0642	0.0642	0.0000	0.3698	0.2462

Note that a high participation factor is a necessary, but not a sufficient condition for a PSS at the generator unit to effectively damp interarea oscillations. For a large power system, following the initial screening based on participation factors, evaluations using frequency response methods should be applied to decide appropriate locations for the stabilizers [35]. By adjusting the gain of the PSS of one generator, while setting the gain of PSS of the other generator at zero, it can be seen that the damping of the generator with adjusted gain changes primarily as shown in Table 4.4.

Table 4.4 System modes and K_{stab}

K_{stab1}	K_{stab2}	oscillation modes	K_{stab1}	K_{stab2}	oscillatory modes
0	0	-0.78±7.8i -0.34±5.5i	0	0	-0.78±7.8i -0.34±5.5i
1	0	-0.86±7.73i -0.34±5.5i	0	1	-22.9±2.66 -0.78±7.88i -0.43±5.45i
5	0	-1.32±6.95i -6.23±1.87i -0.34±5.5i	0	5	-22.3±6.06i -0.78±7.88i -0.77±5.26i -1.89±0.14i
15	0	-0.34±5.5i -0.34±5.5i -0.73±0.16i	0	15	-20.41±12.71i -0.78±7.86i -1.7±3.86i -1.99±0.35i
-5	0	-0.34±5.5i -0.7±8.5i -0.57±0.33i	0	-5	-0.78±7.8i 0.1±5.7i

The eigenvalues, damping ratio, damped frequency and dominant states obtained from the participation factor of the whole system (generator, AVR, turbine/governor and PSS) are listed in Table 4.5. After including the turbine/governor, the system order increases to 22. There are two oscillatory modes with damped frequencies of 1.3 and 0.9 Hz. These two oscillatory modes are well-damped with damping ratios 0.16 and 0.30. Table 4.5 also shows the dominant and significant states of every eigenvalue, which provides the useful information for the poles.

Table 4.5 Eigenvalues, participation factor of the whole system

Modes	Eigenvalues	Damped Frequency	Damping Ratio	Dominant States	Significant states
1	-141.31			$\Delta\psi_{fd}$ of G#1	
2	-140.75			$\Delta\psi_{fd}$ of G#2	
3	-28.61			PSS of G#1	
4	-19.63			AVR of G#1	
5	-22.73			PSS of G#2	
6	-20.03			AVR of G#2	
7, 8	-1.35±8.16i	1.298	0.1633	$\Delta\omega, \Delta\delta$ of G#1	$\Delta\psi_{1q}$ of G#1
9	-10.08			GOV of G#1	
10, 11	-1.8±5.67i	0.902	0.3032	$\Delta\omega, \Delta\delta$ of G#2	$\Delta\psi_{1q}$ of G#2
12	-5.01			PSS of G#1	
13	-1.97			GOV of G#2	
14	-0.61			$\Delta\psi_{1q}$ of G#1	AVR of G#1
15	-0.14			AVR of G#2	
16	-0.47			$\Delta\psi_{1q}$ of G#2	$\Delta\delta$ of G#2
17	-0.85			GOV of G#1	AVR of G#1
18	-1.01			GOV of G#2	$\Delta\psi_{1q}$ of G#2
19	-1			$\Delta\psi_{1q}$ of G#1	AVR,GOV of G#1
20	-1			PSS of G#2	
21, 22	-0.4			GOV of G#1	

4.5.2 Pole Placement via Conventional Control

Conventional design methods described in section 4.2 are applied to determine the gain and time constant of the PSS. One set of parameters for the PSSs, using conventional methods, are listed below:

Generator #1: T_w= 1.1, T_1=0.27, T_2=0.035, K_{stab}=0.5;

Generator #2: T_w= 11, T_1=0.115, T_2=0.044, K_{stab}=0.1.

With the addition of the PSSs (delta-omega type), the damping of the oscillatory modes has been enhanced as the real parts of the eigenvalues increased significantly. The damped frequencies changed slightly, but the damping ratios changed considerably. The eigenvalues, damped frequencies and damping ratios are listed in the Table 4.6.

Table 4.6 Eigenvalues of the system with AVR and PSS

Eigenvalue/(Frequency in Hz, Damping Ratio)					
-141.22	-140.76	-1.41±8.18i f_d = 1.30, ζ=0.1695	-19.78	-22.73	-20.01
-1.84±5.75i ω_d=0.92, ξ=0.3049	-0.61	-0.91±0.09i f_d=0.01, ζ =0.9951	-0.11	-0.55	-0.09

The system performance with PSS included can be analyzed by observing the transient responses following a step disturbance either in load or the reference voltage. Only disturbances applied in the reference voltage are used in the following system performance evaluations. The transient responses of systems with and without

PSSs are shown in Figure 4.5 and 4.6 respectively. It can be seen that the dynamic characteristic of the system with PSSs is much better than that of the system without PSSs.

4.5.3 Full State Feedback

Since the system is completely controllable, without considering the control effort and measurement problem, poles of the system can be assigned arbitrarily. If the goal is to achieve high damping, the desired poles can be assigned as shown in Table 4.7.

Table 4.7 Desired eigenvalues

Desired eigenvalues				
-145	-7.8±7.8i	-137	-23	-12
-5.5±5.5i	-5.6	-1.77	-1.51	-0.66

The gain matrix K can be found as shown in Eq.(4.28). It can be seen that the value of each component of the gain matrix is very high. This implies that a great effort is required. The step simulation results are shown in Figure 4.7. The dynamic characteristic is better than that of conventional PSS as it take less time to steady state and less overshoot.

$$K_f \begin{bmatrix} 66.9 & 8.8 & 5.0 & 25.1 & -4.8 & -0.1 & -1001.4 & 6.7 & 101.8 & 8.9 & 98.4 & 1.3 \\ 14.4 & 1.9 & 0.1 & 5.4 & -1.0 & -0.0 & -215.2 & 1.4 & 21.9 & 1.9 & 21.1 & 0.3 \end{bmatrix} \qquad (4.28)$$

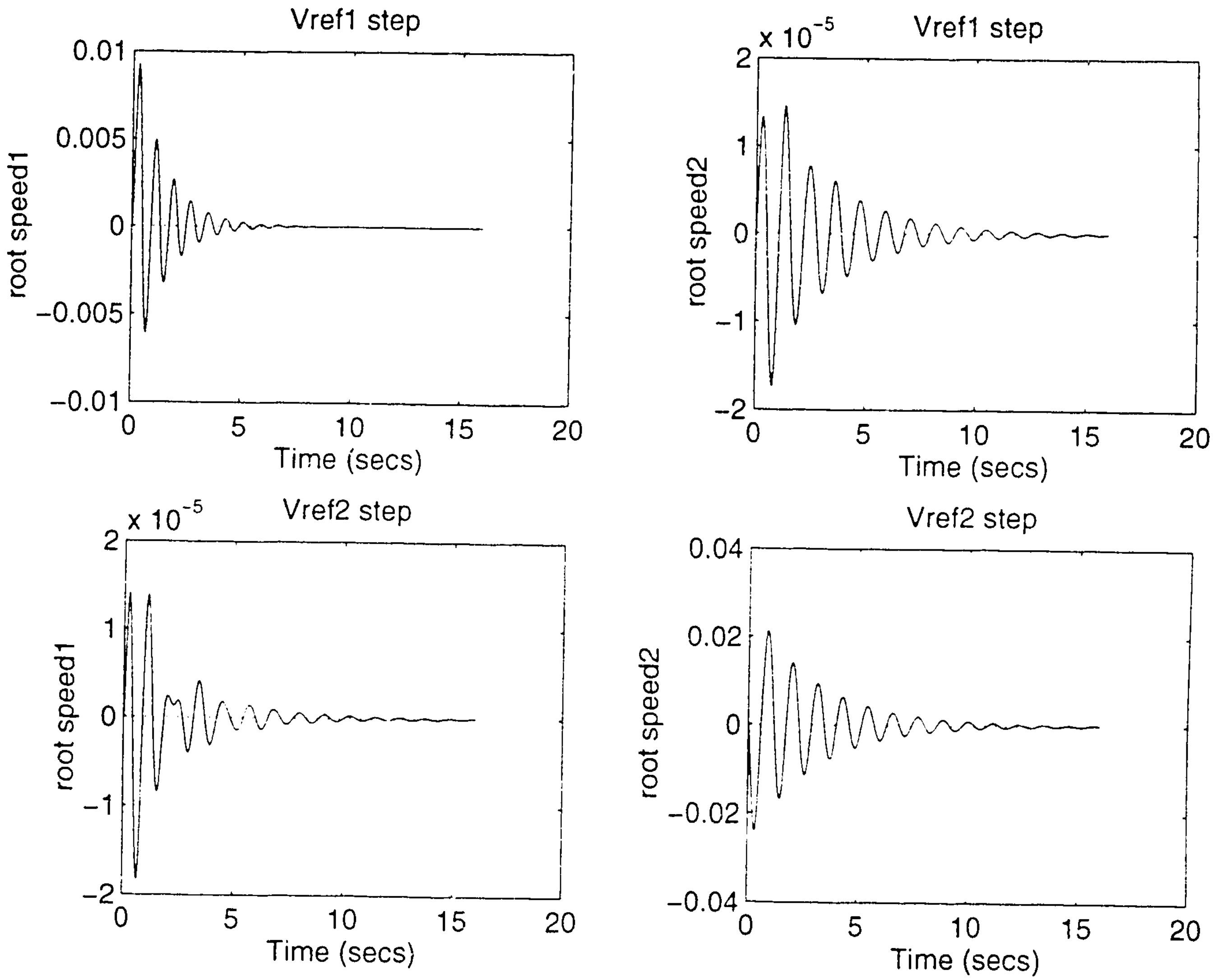


Figure 4.5 Step simulation without PSSs

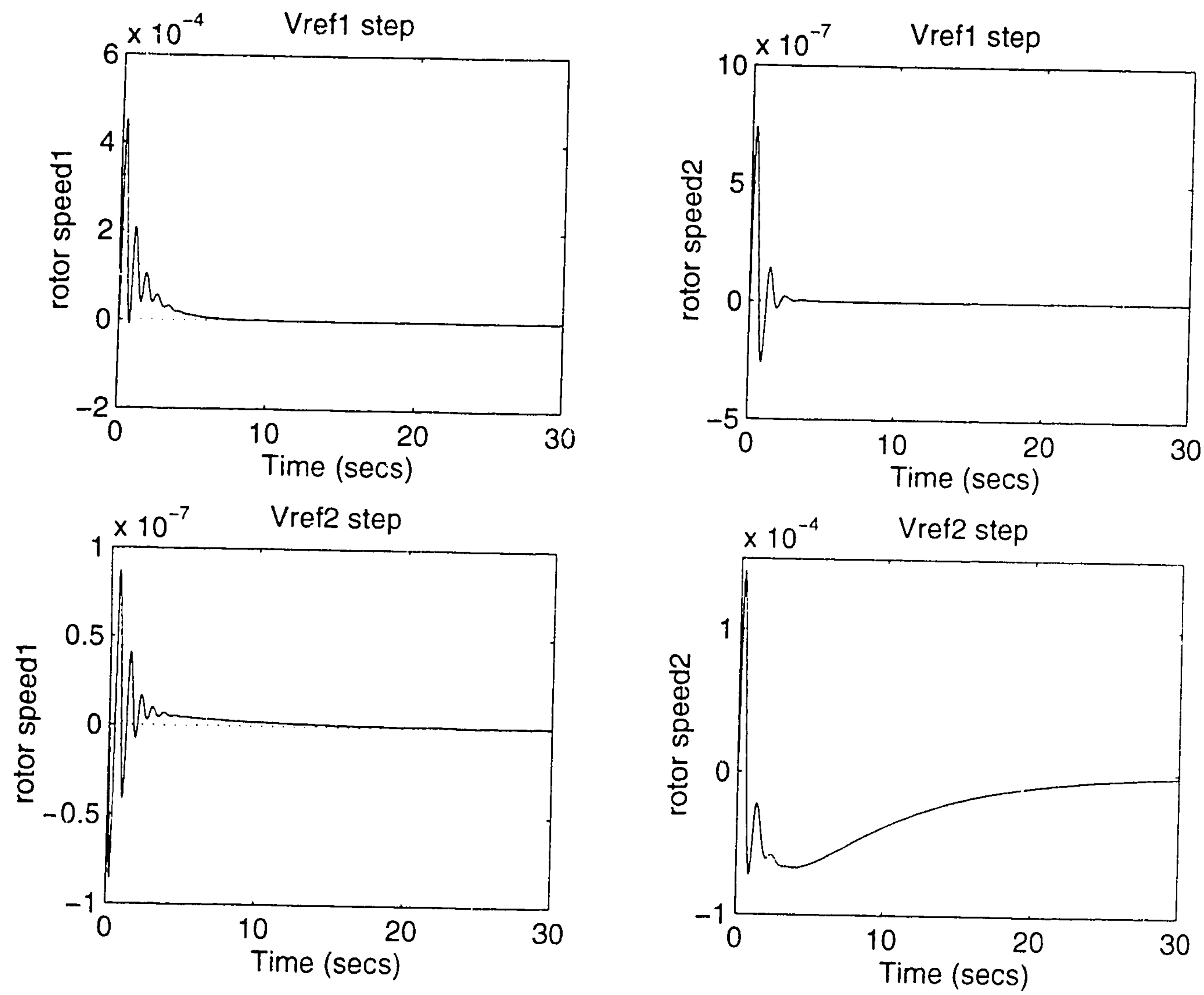


Figure 4.6 Step simulation with PSSs

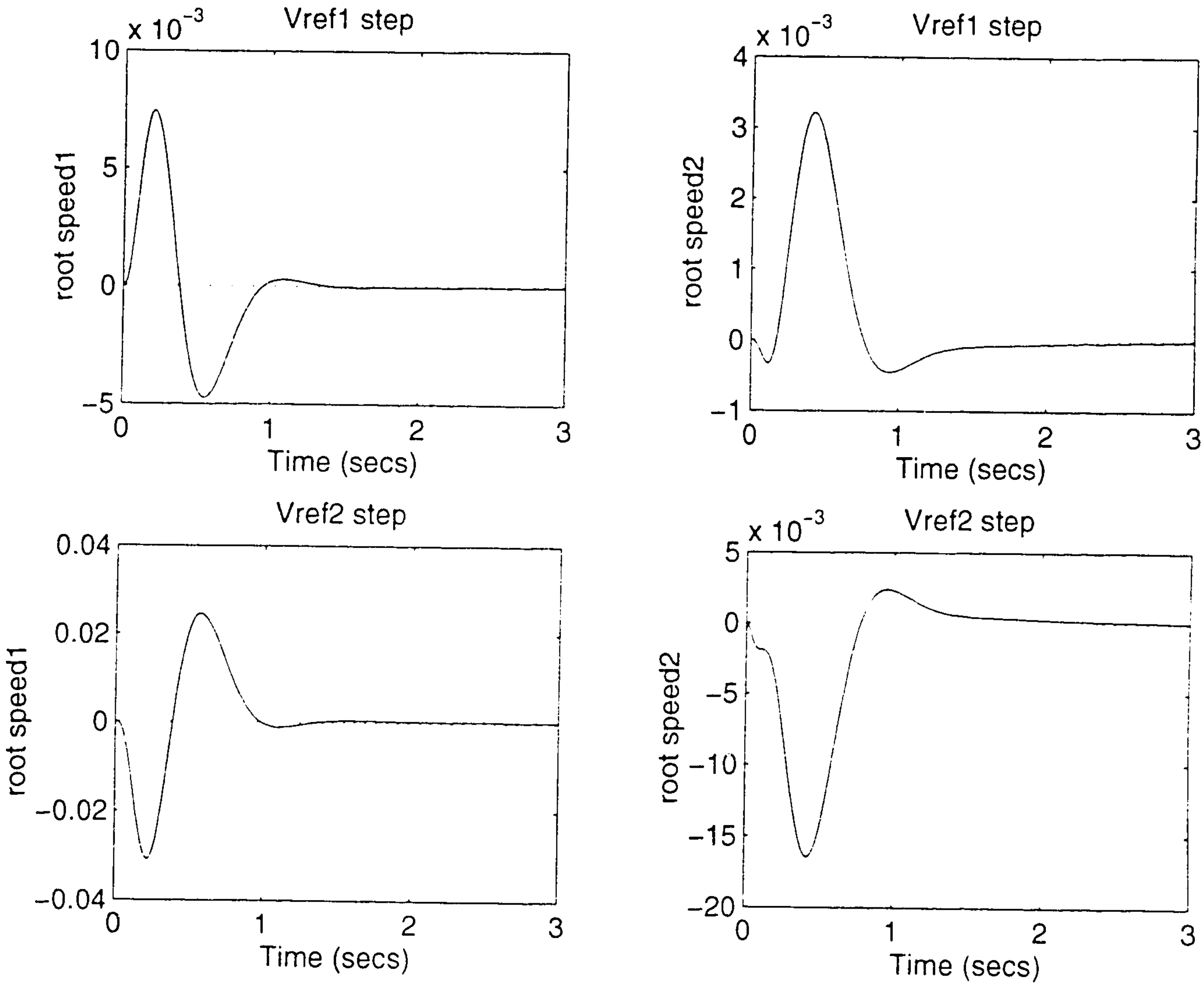


Figure 4.7 Step simulation with full state feedback

4.5.4 Linear Optimal Control

Suppose all the state variables are measurable, so that optimal control can be applied to enhance the damping of the system. For simplicity, assume the weighting matrices Q and R are unit diagonal matrices. Following the method described in this chapter, the optimal controller gain matrix K is obtained as,

$$K_o = \begin{bmatrix} 7.3376 & -0.0485 & 6.8161 & 10.1943 & 0.9559 & 0.8971 & 0.0114 & 0.0000 & -0.0036 & -0.0018 & 0.0002 & 0.0000 \\ 0.0063 & -0.0002 & 0.0059 & 0.0004 & -0.0000 & 0.0000 & -24.2347 & -0.0943 & -1.7273 & -3.2775 & 7.5860 & 0.8656 \end{bmatrix} \quad (4.29)$$

It can be seen that the value of each component of gain matrix is smaller than that of pole placement. If R decreases, the value of each component of the gain matrix, K may be increased because it needs more control effort. The closed-loop eigenvalues are listed in Table 4.8. The system has 4 pairs of complex eigenvalues. These modes are stable and three of them are well damped. The simulation results are shown in Figure 4. 8. It can be seen that the dynamic characteristic is better than that of conventional PSS because it takes less time to steady state and has less overshoot.

Table 4. 8 Eigenvalues for system with optimal controller

Eigenvalues/ damping ratios			
-177.58	-140.6±14.19i ζ=0.324	-132.86	-19.9±5.81i ζ=0.1714
-142±8.19i ζ=0.999	-1.06	-0.89±0.04i ζ=0.995	-0.66

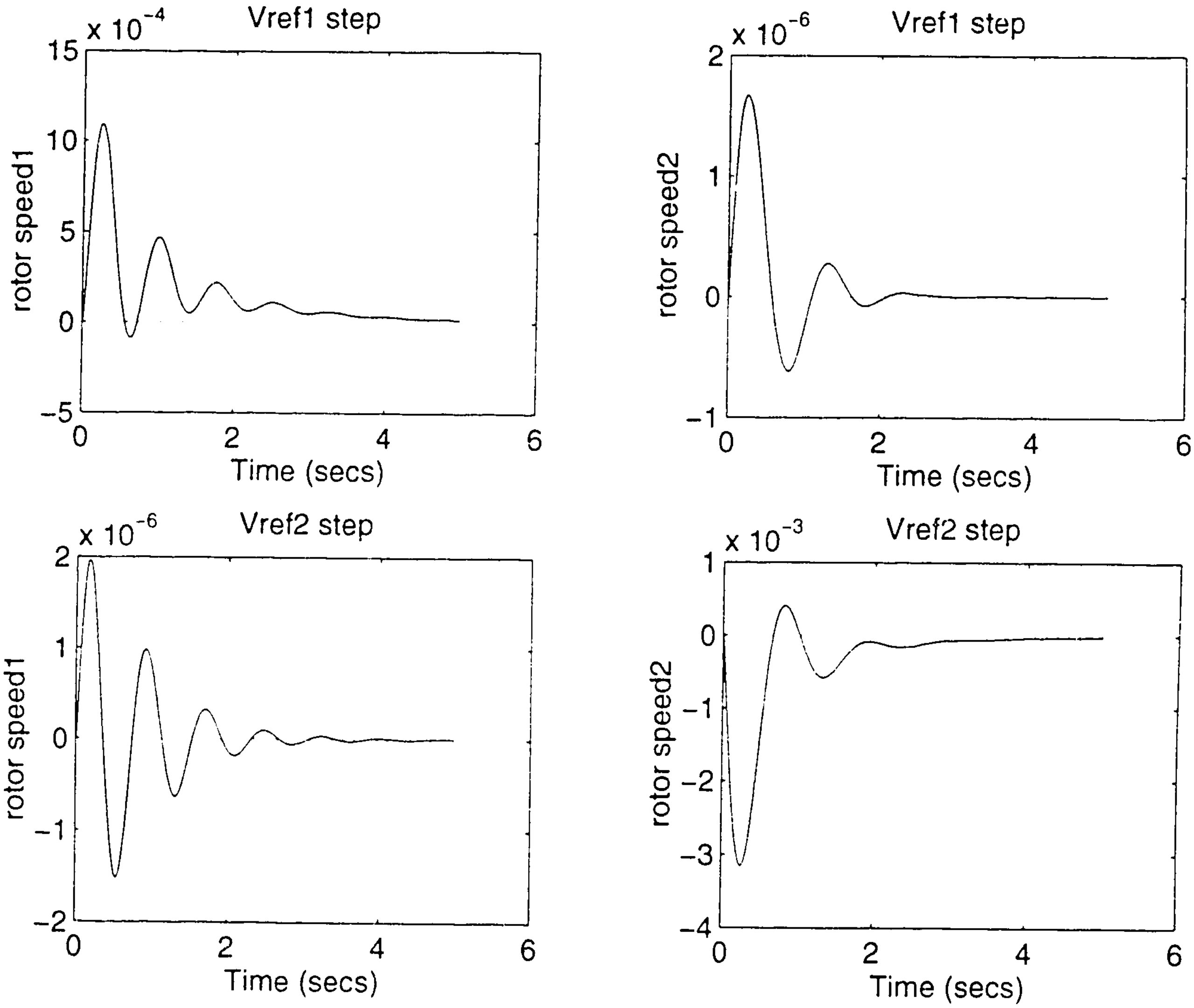


Figure 4.8 Step simulation with optimal control

4.5.5 Suboptimal Control

Rotor speed and angle of a generator are generally considered as measurable. Assume only these two variables can be taken from the system. In this case, it is better to use suboptimal control as described in this chapter. The suboptimal controller gain matrix can be found easily from the optimal controller gain matrix,

$$K_{so}=\begin{bmatrix} 7.3376 & -0.0485 & 0 & 0 & 0 & 0 & 0.0114 & -0.000 & 0 & 0 & 0 & 0 \\ 0.0063 & -0.0002 & 0 & 0 & 0 & 0 & -24.2347 & -0.0943 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.30)$$

The eigenvalues of the closed-loop system are listed in the Table 4.9. The system has two oscillatory modes. The damping ratios of these two modes are 0.138 and 0.37. The step simulation results are better than that of conventional PSS as shown in figure 4.9 as it takes less time to steady state.

Table 4.9 Eigenvalues for the system with suboptimal controller

Eigenvalues/Damping ratios				
-145.77	-138.56	-18.36	-0.1105	-1.111±7.98i ζ=0.138
-2.47 ±6.15 ζ=0.37	-5.66	-0.65	-1.75	-1.5

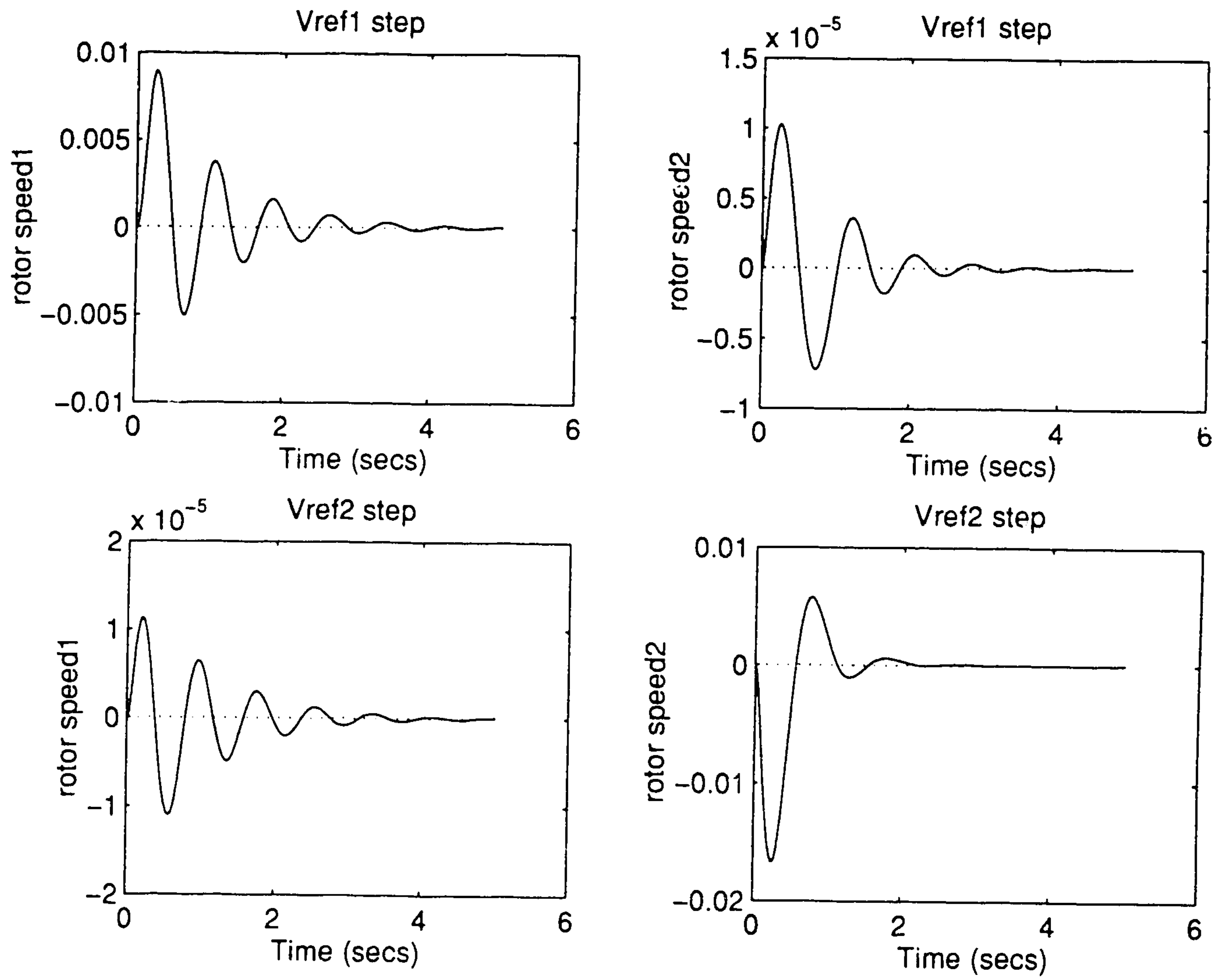


Figure 4.9 Step simulation with suboptimal control

Chapter 5

Conclusions and Future Research

5.1 Conclusions

Under certain conditions, large electric power systems show a tendency toward spontaneous low-frequency oscillations. These slow oscillations disrupt the normal, smooth operation of the system and may lead to a serious operating problem [24]. These oscillations must be considered in planning, designing and operating a large power system. Low-frequency oscillations are an inherent phenomena of interconnected AC power systems and one source is related to a relatively weak interconnection between groups of strongly coupled generators [12].

To investigate low frequency oscillations, several tools are required, such as a load flow program, a transient stability program and a small-signal stability program. The load flow program is used to initialize the transient and small-signal stability programs. The transient stability program simulates the system behavior following severe faults, while the small-signal stability program is used to identify the generators that may experience an oscillation, or may be candidates for stability control. A small-signal stability program was developed in this thesis as a basic analytical tool for studying low frequency inter-area oscillations. The program can be used to determine the dynamic stability or instability of power systems under different operating conditions. It can be used to calculate the associated eigenvalues and eigenvectors, participation factor, natural and damped frequencies, and damping ratio. The program was first tested on a benchmark model developed by other researchers, then used to study the small signal stability of a two-machine infinite bus power system. The main results of the investigation have been presented in Chapter 4.

The state space model formulation described in this thesis is easier to implement than the sub-matrix approach by Undrill [67]. The procedure is illustrated in Figure 3.12. The synchronous generators, controllers, loads, and network are represented by differential or algebraic equations. These equations are then linearized at an operating point. Through a suitable reference transformation, these linearized equations are integrated as a system state-space model. The dynamic model is required to represent all the important dynamics of the system accurately, while remaining as simple as possible. The Park's equation provides a mathematical model for synchronous generators and has high practical value particularly in the analysis of generator dynamic characteristics. A synchronous machine can be modeled in varying degrees of complexity depending on the purpose of the model and the required accuracy of results. A detailed sixth order model and a simple classic model are described in this thesis. The detailed model is used for the small-signal stability study, while the classical model is used to represent a remote generator to keep the whole system model simple. A fourth-order generator model is used in the case study in chapter 4.

Eigenvalue techniques are well suited for dynamic stability analysis of multi-machine power systems. Modes of generator rotor oscillations, which tend to be mixed with each other, may be separated and identified by eigenvalue analysis. The eigenvalues associated with each mode give a clear and precise indication of oscillation frequency and damping. Factors associated with each eigenvalue are revealed by participation factors (from a sensitivity study). This provides valuable information to enhance damping of a system and to determine the location of a PSS. In the case study, it is shown that the two-machine infinite bus power system has two oscillation modes, with frequencies at 1.25 and 0.875 Hz, slightly damped with damping ratios 0.098 and 0.063. The oscillation modes are primarily associated with the rotor speed and angle of generators. Each generator requires a PSS to enhance the damping of the overall system according to the participation factors of the system.

A power system stabilizer is a very cost effective supplementary control device to enhance the damping of the electric power system during low frequency oscillations. The conventional PSSs usually use only one stabilizing signal, rotor speed, AC bus frequency or electrical power, which is injected into each excitation control loop. As a result, the interactions among generators cannot be considered easily and this results in one or more modes of oscillation being left undamped. The design of conventional PSSs is usually conducted using transfer functions, together with graphical techniques, such as root-locus and bode plots. The design of the control system is based on a trial-and-error procedure that in general, will not yield optimal control of the system. The conventional design method was carried out in the base case study to determine the gain and time constants of a given PSS. This is described in Chapter 4. After the newly-designed PSSs are installed, the damping ratio was increased from 0.063 and 0.098 to 0.17 and 0.30 respectively.

Much effort has been made to develop a multi-machine stabilizer using multi-input multi-output concepts. One approach is to construct an asymptotically stable closed-loop system by specifying the desired locations for the closed-loop poles, then, determine the feedback gain matrix K such that the system will have the desired closed loop characteristic equation. This is called pole placement or eigenvalue assignment. Another approach is to design a controller K such that a quadratic performance index is minimized. This formulation to determine the optimal control law is often termed the quadratic optimal control problem. The pole placement, optimal and sub-optimal are used to design PSSs in this thesis.

The pole placement methods select closed-loop poles without regard to the effect on the control effort required to achieve the pre-specified response. On the other hand, optimal control can select poles that result in some balance between system errors and control effort. These two methods assume that the complete state x(t) of the plant can be accurately measured at all times and is available for feedback. This is an unrealistic assumption for a large scale power system because it presents a need for long distance communication that can be impractical and uneconomical. As shown in chapter 4, the closed loop eigenvalues may be assigned to desired locations but may require unrealistic effort and time to realize.

Linear optimal PSS design problems are commonly considered as a linear-quadratic design problem, The design theory is simple. However, how to choose a cost function (weighting matrices Q and R) is still a problem for designers. Considerable research has been done on this problem. The main results can be found in references [13, 47, 48, 75]. The prevalent idea is to move dominant eigenvalues of the closed loop system to pre-specified locations. These controls are based on state space models, which enable the engineer to design a control system having desired closed-loop poles or optimal control system with respect to a given performance index. Also, modern control theory enables the designer to include initial conditions. The state space methods are particularly suited for digital computer computations because of their time-domain approach. This enables a designer to avoid the burden of tedious computations. The elements of the gain matrix K obtained from optimal control are much smaller than those obtained from full state feedback as shown in Chapter 4.

Most of the dynamic performance evaluation has been carried out by transient response in the literature. In this thesis both transient response and eigenvalues are used to evaluate the system performance.

5.2 Future Research

(1) The power system under study in this thesis is relatively simple. In order to further explore the factors influencing the performance of a PSS, an extension to a multi-area power system is necessary.

(2) Application of new design techniques for PSS design, such as H_∞ methods offer considerable promise. The latest development in feedback control design is H_∞ optimization, which was developed by Zames in 1982 [12]. Reference 12 provides a detailed design procedure. H_∞ approach is a frequency domain technique. Thus, most design specification used in phase compensation technique can be easily adopted in the new design method. The technique is used to design a feedback controller with two objectives. One is to reduce the effect of a disturbance on the plant output, subject to a closed-loop stability constraint. The other is to design a controller that performs satisfactorily under a wide range of system operating conditions.

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Appendix A

One Damper Winding Model

Reference Frame Transformation

From the reference transformation equation:

$$\begin{pmatrix} v_d \\ v_q \end{pmatrix} = \begin{pmatrix} -\cos\delta & \sin\delta \\ \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} v_D \\ v_Q \end{pmatrix} \tag{1}$$

Incremental form,

$$\begin{pmatrix} \Delta v_d \\ \Delta v_q \end{pmatrix} = \begin{pmatrix} -\cos\delta & \sin\delta \\ \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \Delta v_D \\ \Delta v_Q \end{pmatrix} + \begin{pmatrix} \sin\delta & \cos\delta \\ \cos\delta & \sin\delta \end{pmatrix} \begin{pmatrix} v_{D_0} \\ v_{Q_0} \end{pmatrix} \Delta\delta \tag{2}$$

or,

$$\begin{pmatrix} \Delta v_d \\ \Delta v_q \end{pmatrix} = \begin{pmatrix} -\cos\delta & \sin\delta \\ \sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} \Delta v_D \\ \Delta v_Q \end{pmatrix} + \begin{pmatrix} v_{q_0} \\ -v_{d_0} \end{pmatrix} \Delta\delta \tag{3}$$

Rewriting,

$$\Delta v_d = -\cos\delta\,\Delta v_D + \sin\delta\,\Delta v_Q + v_{q0}\Delta\delta \tag{4}$$

$$\Delta v_q = -\sin\delta\,\Delta v_D + \cos\delta\,\Delta v_Q - v_{d0}\Delta\delta \tag{5}$$

Similarly,

$$\begin{pmatrix}\Delta i_d\\ \Delta i_q\end{pmatrix}=\begin{pmatrix}-\cos\delta & \sin\delta\\ \sin\delta & \cos\delta\end{pmatrix}\begin{pmatrix}\Delta i_D\\ \Delta i_Q\end{pmatrix}+\begin{pmatrix}\sin\delta & \cos\delta\\ \cos\delta & \sin\delta\end{pmatrix}\begin{pmatrix}i_{D_0}\\ i_{Q_0}\end{pmatrix}\Delta\delta \tag{6}$$

or,

$$\begin{pmatrix}\Delta i_d\\ \Delta i_q\end{pmatrix}=\begin{pmatrix}-\cos\delta & \sin\delta\\ \sin\delta & \cos\delta\end{pmatrix}\begin{pmatrix}\Delta i_D\\ \Delta i_Q\end{pmatrix}+\begin{pmatrix}i_{q_0}\\ -i_{d_0}\end{pmatrix}\Delta\delta \tag{7}$$

Stator Equations

$$v_d = -r_a i_d + x_q' i_q + E_d' \tag{8}$$

$$v_q = -r_a i_q - x_d' i_d + E_q' \tag{9}$$

where,

$$E_d' = -\omega L_{aq}' \frac{\psi_{1q}}{L_{1q}} \quad , \quad E_q' = \omega L_{ad}' \frac{\psi_{fd}}{L_{fd}}$$

$$L_{ad}' = \frac{1}{\dfrac{1}{L_{ad}}+\dfrac{1}{L_{fd}}} \quad , \quad L_{aq}' = \frac{1}{\dfrac{1}{L_{aq}}+\dfrac{1}{L_{1q}}}$$

Therefore,

$$v_d = -r_a i_d + x_q' i_q - \omega L_{aq}' \frac{\psi_{1q}}{L_{1q}} \tag{10}$$

$$v_q = -r_a i_q - x_d' i_d + \omega L_{ad}' \frac{\psi_{fd}}{L_{fd}} \tag{11}$$

Rewriting these two equations as

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} -r_a & x_q' \\ -x_d' & -r_a \end{pmatrix}^{-1} \left[\begin{pmatrix} v_d \\ v_q \end{pmatrix} + \begin{pmatrix} \omega L_{aq}' \dfrac{\psi_{1q}}{L_{1q}} \\ -\omega L_{ad}' \left(\dfrac{\psi_{fd}}{L_{fd}} \right) \end{pmatrix} \right] \tag{12}$$

Incremental form,

$$\begin{pmatrix} \Delta i_d \\ \Delta i_q \end{pmatrix} = \begin{pmatrix} -r_a & x_q' \\ -x_d' & -r_a \end{pmatrix}^{-1} \left[\begin{pmatrix} \Delta v_d \\ \Delta v_q \end{pmatrix} + \begin{pmatrix} \omega L_{aq}' \dfrac{\Delta \psi_{1q}}{L_{1q}} \\ -\omega L_{ad}' \dfrac{\Delta \psi_{fd}}{L_{fd}} \end{pmatrix} \right] \tag{13}$$

Note that,

$$\begin{pmatrix} -r_a & x_q' \\ -x_d' & -r_a \end{pmatrix}^{-1} = \frac{1}{r_a^2 + x_d' x_q'} \begin{pmatrix} -r_a & -x_q' \\ x_d' & -r_a \end{pmatrix} \tag{14}$$

Rewriting Eq.(13) as,

$$\Delta i_d = \frac{1}{r_a^2 + x_d' x_q'} \left(-r_a \Delta v_d - x_q' \Delta v_q - \omega r_a L_{aq}' \frac{\Delta \psi_{1q}}{L_{1q}} + \omega x_q' L_{ad}' \frac{\Delta \psi_{fd}}{L_{fd}} \right) \tag{15}$$

and,

$$\Delta i_q = \frac{1}{r_a^2 + x_d' x_q'}\left(x_d' \Delta v_d - r_a \Delta v_q + \omega x_d' L_{aq}' \frac{\Delta \psi_{1q}}{L_{1q}} + \omega r_a L_{ad}' \frac{\Delta \psi_{fd}}{L_{fd}} \right) \tag{16}$$

Substituting Δv_d and Δv_q into Eqs.(15) and (16) and rearranging,

$$\Delta i_d = m_1 \Delta v_D + m_2 \Delta v_Q + m_3 \Delta \delta + m_4 \Delta \psi_{fd} + m_5 \Delta \psi_{1q} \tag{17}$$

$$\Delta i_q = n_1 \Delta v_D + n_2 \Delta v_Q + n_3 \Delta \delta + n_4 \Delta \psi_{fd} + n_5 \Delta \psi_{1q} \tag{18}$$

where,

$$m_1 = \frac{1}{r_a^2 + x_d' x_q'}(r_a \cos \delta - x_q' \sin \delta) \tag{19}$$

$$m_2 = \frac{-1}{r_a^2 + x_d' x_q'}(r_a \sin \delta + x_q' \cos \delta) \tag{20}$$

$$m_3 = \frac{1}{r_a^2 + x_d' x_q'}(-r_a v_{q0} + x_q' v_{d0}) \tag{21}$$

$$m_4 = \frac{1}{(r_a^2 + x_d' x_q') L_{fd}} x_q' L_{ad}' \tag{22}$$

$$m_5 = \frac{-1}{(r_a^2 + x_d' x_q') L_{1q}} r_a L_{aq}' \tag{23}$$

$$n_1 = \frac{-1}{r_a^2 + x_d' x_q'}(x_d' \cos\delta + r_a \sin\delta) \tag{24}$$

$$n_2 = \frac{1}{r_a^2 + x_d' x_q'}(x_d' \sin\delta - r_a \cos\delta) \tag{25}$$

$$n_3 = \frac{1}{r_a^2 + x_d' x_q'}(x_d' v_{q0} + r_a v_{d0}) \tag{26}$$

$$n_4 = \frac{1}{(r_a^2 + x_d' x_q')L_{fd}} r_a L_{ad}' \tag{27}$$

$$n_5 = \frac{1}{(r_a^2 + x_d' x_q')L_{1q}} x_d' L_{aq}' \tag{28}$$

Eqs.(17) and Eq(18) and (7) yield,

$$\begin{pmatrix} \Delta i_D \\ \Delta i_Q \end{pmatrix} = \begin{pmatrix} \cos\delta & -\sin\delta \\ -\sin\delta & -\cos\delta \end{pmatrix} \begin{pmatrix} 0 & m_3 - i_{q0} & m_4 & m_5 \\ 0 & n_3 + i_{d0} & n_4 & n_5 \end{pmatrix} \begin{pmatrix} \Delta\omega \\ \Delta\delta \\ \Delta\psi_{fd} \\ \Delta\psi_{1q} \end{pmatrix} + \begin{pmatrix} \cos\delta & -\sin\delta \\ -\sin\delta & -\cos\delta \end{pmatrix} \begin{pmatrix} m_1 & m_2 \\ n_1 & n_2 \end{pmatrix} \begin{pmatrix} \Delta v_D \\ \Delta v_Q \end{pmatrix} \tag{29}$$

The d-and q-axis mutual flux linkages are given by,

$$\psi_{ad} = L'_{ads}\left(-i_d + \frac{\psi_{fd}}{L_{fd}} \right) \tag{30}$$

where,

$$L'_{ads} = \frac{1}{\frac{1}{L_{ads}} + \frac{1}{L_{fd}}}$$

$$\Delta\psi_{ad} = L'_{ads}\left(-\Delta i_d + \frac{\Delta\psi_{fd}}{L_{fd}} \right) \tag{31}$$

substituting Δi_d into Eq(31),

$$\Delta\psi_{ad} = L'_{ads}\left(-(m_1\Delta v_D + m_2\Delta v_Q + m_3\Delta\delta + m_4\Delta\psi_{fd} + m_5\Delta\psi_{1q}) + \frac{\Delta\psi_{fd}}{L_{fd}} \right) \tag{32}$$

$$\Delta\psi_{ad} = km_1\Delta v_D + km_2\Delta v_Q + km_3\Delta\delta + km_4\Delta\psi_{fd} + km_5\Delta\psi_{1q} \tag{33}$$

where,

$$km_1 = -m_1 L'_{ads} \quad , \quad km_2 = -m_2 L'_{ads} \quad , \quad km_3 = -m_3 L'_{ads}$$

$$km_4 = -L'_{ads}(m_4 - \frac{1}{L_{fd}}) \quad , \quad km_5 = -m_5 L'_{ads}$$

$$\psi_{aq} = L'_{aqs}\left(-i_q + \frac{\psi_{1q}}{L_{1q}} \right) \tag{34}$$

where,

$$L'_{aqs}=\frac{1}{\frac{1}{L_{aqs}}+\frac{1}{L_{1q}}}$$

$$\Delta\psi_{aq}=L'_{aqs}\left(-\Delta i_q+\frac{\Delta\psi_{1q}}{L_{1q}}\right) \tag{35}$$

substituting Δi_q into Eq.(35),

$$\Delta\psi_{aq}=L'_{aqs}\left(-(n_1\Delta v_D+n_2\Delta v_Q+n_3\Delta\delta+n_4\Delta\psi_{fd}+n_5\Delta\psi_{1q})+\frac{\Delta\psi_{1q}}{L_{1q}}\right) \tag{36}$$

$$\Delta\psi_{aq}=kn_1\Delta v_D+kn_2\Delta v_Q+kn_3\Delta\delta+kn_4\Delta\psi_{fd}+kn_5\Delta\psi_{1q} \tag{37}$$

where,

$$kn_1=-n_1L'_{aqs}\ ,\ kn_2=-n_2L'_{aqs}\ ,\ kn_3=-n_3L'_{aqs}$$

$$kn_4=-n_4L'_{aqs}\ ,\ kn_5=-L'_{aqs}(n_5-\frac{1}{L_{1q}})$$

The air-gap torque is determined by

$$T_e=\psi_{ad}i_q-\psi_{aq}i_d \tag{38}$$

$$\Delta T_e=\psi_{ad0}\Delta i_q+i_{q0}\Delta\psi_{ad}-\psi_{aq0}\Delta i_d-i_{d0}\Delta\psi_{aq} \tag{39}$$

Substituting Δi_d, Δi_q, $\Delta\psi_{ad}$, $\Delta\psi_{aq}$ into Eq.(39),

$$\Delta T_e = k_1 \Delta v_D + k_2 \Delta v_Q + k_3 \Delta \delta + k_4 \Delta \psi_{fd} + k_5 \Delta \psi_{1q} \tag{40}$$

where,

$$k_1 = \psi_{ad0} n_1 + i_{q0} k m_1 - \psi_{aq0} m_1 - i_{d0} k n_1 \tag{41}$$

$$k_2 = \psi_{ad0} n_2 + i_{q0} k m_2 - \psi_{aq0} m_2 - i_{d0} k n_2 \tag{42}$$

$$k_3 = \psi_{ad0} n_3 + i_{q0} k m_3 - \psi_{aq0} m_3 - i_{d0} k n_3 \tag{43}$$

$$k_4 = \psi_{ad0} n_4 + i_{q0} k m_4 - \psi_{aq0} m_4 - i_{d0} k n_4 \tag{44}$$

$$k_5 = \psi_{ad0} n_5 + i_{q0} k m_5 - \psi_{aq0} m_5 - i_{d0} k n_5 \tag{45}$$

State space matrix formulation

Electro-Mechanical Rotor Equations

$$p\omega = \frac{1}{2H}(T_m - T_e - K_D \omega) \tag{46}$$

Where,

K_D is the damping factor or coefficient in pu.
H is inertia constant.

$$p\Delta\omega = \frac{1}{2H}(\Delta T_m - \Delta T_e - K_D \Delta\omega) \tag{47}$$

Substituting ΔT_e into, and $\Delta T_m=0$, yields,

$$a_{11}=\frac{-K_D}{2H}\ ,\quad a_{12}=\frac{-K_3}{2H}\ ,\quad a_{13}=\frac{-K_4}{2H}\ ,$$

$$a_{14}=\frac{-K_5}{2H}\ ,\quad b_{11}=\frac{-K_1}{2H}\ ,\quad b_{12}=\frac{-K_2}{2H}\ .$$

If $\Delta T_m \neq 0$, then,

$$b_{13}=\frac{\Delta T_m}{2H}$$

For including turbine/governor

$$a_{15}=-\frac{2}{2H}\frac{P_{base}}{100}\ ,\quad a_{17}=\frac{2}{2H}\frac{P_{base}}{100}\ .$$

$$p\Delta\delta=\omega_0\Delta\omega \tag{48}$$

where,

$$\omega_0=2\Pi f_0 \tag{49}$$

$$a_{21}=\omega_0=2\Pi f_0\ ,\quad b_{21}=b_{22}=0$$

Rotor Equations

$$p\psi_{fd}=\frac{\omega_0 R_{fd}}{L_{adu}}E_{fd}-\omega_0 R_{fd} i_{fd} \tag{50}$$

where,

$$i_{fd}=\frac{1}{L_{fd}}(\psi_{fd}-\psi_{ad}) \tag{51}$$

therefore,

$$p\Delta\psi_{fd}=\frac{\omega_0 R_{fd}}{L_{adu}}\Delta E_{fd}-\omega_0 R_{fd}\frac{1}{L_{fd}}(\Delta\psi_{fd}-\Delta\psi_{ad}) \tag{52}$$

substituting $\Delta\psi_{ad}$ into the Eq.(52), assume $\Delta E_{fd}=0$,

$$a_{31}=0\ ,\quad a_{32}=\frac{\omega_0 R_{fd}}{L_{fd}}km_3\ ,\quad a_{33}=\frac{\omega_0 R_{fd}}{L_{fd}}(km_4-1)\ ,\quad a_{34}=\frac{\omega_0 R_{fd}}{L_{fd}}km_5\ ,$$

$$b_{31}=\frac{\omega_0 R_{fd}}{L_{fd}}km_1\ ,\quad b_{32}=\frac{\omega_0 R_{fd}}{L_{fd}}km_2$$

if $\Delta E_{fd}\neq 0$, then,

$$b_{34}=\frac{\omega_0 R_{fd}}{L_{adu}}\ ,\quad a_{3,9}\ or\ a_{3,6}=\frac{\omega_0 R_{fd}}{L_{adu}}$$

$$p\psi_{1q}=-\omega_0 R_{1q} i_{1q} \tag{53}$$

$$i_{1q}=\frac{1}{L_{1q}}(\psi_{1q}-\psi_{ad}) \tag{54}$$

Therefore,

$$p\Delta\psi_{1q}=-\omega_0 R_{1q}\frac{1}{L_{1q}}(\Delta\psi_{1q}-\Delta\psi_{ad}) \tag{55}$$

where,

$$a_{41}=0\ ,\quad a_{42}=\frac{\omega_0 R_{1q}}{L_{1q}}kn_3\ ,\quad a_{43}=\frac{\omega_0 R_{1q}}{L_{1q}}kn_4$$

$$a_{44} = \frac{\omega_0 R_{1q}}{L_{1q}}(kn_5 - 1) \quad ,$$

$$b_{41} = \frac{\omega_0 R_{1q}}{L_{1q}} kn_1 \quad , \quad b_{42} = \frac{\omega_0 R_{1q}}{L_{1q}} kn_2$$

Therefore,

$$\begin{pmatrix} \Delta \dot{\omega} \\ \Delta \dot{\delta} \\ \Delta \psi_{fd} \\ \Delta \psi_{1q} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} \Delta \omega \\ \Delta \delta \\ \Delta \psi_{fd} \\ \Delta \psi_{1q} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{pmatrix} \begin{pmatrix} \Delta v_D \\ \Delta v_Q \end{pmatrix} + \begin{pmatrix} b_{13} & 0 \\ 0 & 0 \\ 0 & b_{34} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta T_m \\ \Delta E_{fd} \end{pmatrix} \tag{56}$$

Note that,

$$\Delta E_{fd} = \Delta x_5 \quad , \quad \Delta T_m = 2(\Delta x_3 - \Delta x_1)\frac{P_{base}}{100}$$

Turbine/Governor Model

$$\dot{x}_1 = -\frac{1}{T_s}(\sigma + \delta)x_1 + \frac{1}{T_s}x_2 + \frac{1}{T_s}u_g \tag{57}$$

with,

$$u_g = \omega_r + \omega_d - \omega \tag{58}$$

Assuming $\omega_r, \omega_d = 0$, incremental form of Eq.(57),

$$\Delta \dot{x}_1 = -\frac{1}{T_s}\Delta \omega - \frac{1}{T_s}(\sigma+\delta)\Delta x_1 + \frac{1}{T_s}\Delta x_2 \tag{59}$$

where,

$$a_{51}=-\frac{1}{T_s} \quad , \quad a_{55}=-\frac{\sigma+\delta}{T_s} \quad , \quad a_{56}=\frac{1}{T_s}$$

$$\dot{x}_2 = \frac{\delta}{T_r}x_1 - \frac{1}{T_r}x_2 \tag{60}$$

Incremental form,

$$\Delta \dot{x}_2 = \frac{\delta}{T_r}\Delta x_1 - \frac{1}{T_r}\Delta x_2 \tag{61}$$

where,

$$a_{65}=-\frac{\delta}{T_r} \quad , \quad a_{66}=\frac{1}{T_r}$$

$$\dot{x}_3 = \frac{3}{T_w}x_1 - \frac{2}{T_w}x_3 \tag{62}$$

$$\Delta \dot{x}_3 = \frac{3}{T_w}\Delta x_1 - \frac{2}{T_w}\Delta x_3 \tag{63}$$

where,

$$a_{75}=-\frac{3}{T_W} \quad , \quad a_{77}=-\frac{2}{T_W}$$

Excitation Model

$$\dot{x}_4 = -\frac{1}{T_b}x_4 + \frac{1}{T_b}(1 - \frac{T_a}{T_b})u_e \tag{64}$$

$$\dot{x}_5 = \frac{K_e}{T_e}x_4 - \frac{1}{T_e}x_5 + \frac{K_e T_a}{T_e T_b}u_e \tag{65}$$

with,

$$u_e = V_{ref} + V_s - V_t \tag{66}$$

assuming ΔV_{ref}, $\Delta V_s = 0$, $u_e = -\Delta V_t$, then,

$$\Delta v_t = \frac{v_{d0}}{v_{t0}}\Delta v_d + \frac{v_{q0}}{v_{t0}}\Delta v_q \tag{67}$$

since,

$$\Delta v_d = -r_a \Delta i_d + L_l \Delta i_q - \Delta \psi_{ad} \tag{68}$$

and,

$$\Delta v_q = -r_a \Delta i_d - L_l \Delta i_d + \Delta \psi_{aq} \tag{69}$$

Substituting Δi_d, Δi_q, $\Delta \psi_{ad}$, $\Delta \psi_{aq}$ into Eq.(67) and rearranging results in,

$$\Delta v_t = ke_1 \Delta v_D + ke_2 \Delta v_Q + ke_3 \Delta \delta + ke_4 \Delta \psi_{fd} + ke_5 \Delta \psi_{1q} \tag{70}$$

where,

$$ke_1 = \frac{v_{d0}}{v_{t0}}(-r_a m_1 + L_l n_1 - kn_1) + \frac{v_{q0}}{v_{t0}}(-r_a n_1 - L_l m_1 + km_1) \tag{71}$$

$$ke_2 = \frac{v_{d0}}{v_{t0}}(-r_a m_2 + L_l n_2 - kn_2) + \frac{v_{q0}}{v_{t0}}(-r_a n_2 - L_l m_2 + km_2) \tag{72}$$

$$ke_3 = \frac{v_{d0}}{v_{t0}}(-r_a m_3 + L_l n_3 - kn_3) + \frac{v_{q0}}{v_{t0}}(-r_a n_3 - L_l m_3 + km_3) \tag{73}$$

$$ke_4 = \frac{v_{d0}}{v_{t0}}(-r_a m_4 + L_l n_4 - kn_4) + \frac{v_{q0}}{v_{t0}}(-r_a n_4 - L_l m_4 + km_4) \tag{74}$$

$$ke_5 = \frac{v_{d0}}{v_{t0}}(-r_a m_5 + L_l n_5 - kn_5) + \frac{v_{q0}}{v_{t0}}(-r_a n_5 - L_l m_5 + km_5) \tag{75}$$

$$\dot{x}_4 = -\frac{1}{T_b}x_4 - \frac{1}{T_b}\left(1 - \frac{T_a}{T_b}\right)\Big[(ke_1 \Delta v_D + ke_2 \Delta v_Q + ke_3 \Delta \delta + ke_4 \Delta \psi_{fd} + ke_5 \Delta \psi_{1q}) - v_s\Big] \tag{76}$$

where,

$$a_{8,1} \ or \ a_{5,1} = 0 \ , \quad a_{8,2} \ or \ a_{5,2} = \frac{-1}{T_b}\left(1 - \frac{T_a}{T_b}\right)ke_3$$

$$a_{8,3} \ or \ a_{5,3} = \frac{-1}{T_b}\left(1 - \frac{T_a}{T_b}\right)ke_4 \ , \quad a_{8,4} \ or \ a_{5,4} = \frac{-1}{T_b}\left(1 - \frac{T_a}{T_b}\right)ke_5 \ ,$$

$$a_{8,8} \ or \ a_{5,5} = \frac{-1}{T_b} \ ,$$

$$b_{8,1} \ or \ b_{5,1} = \frac{-1}{T_b}\left(1 - \frac{T_a}{T_b}\right)ke_1 \ , \quad b_{8,2} \ or \ b_{5,2} = \frac{-1}{T_b}\left(1 - \frac{T_a}{T_b}\right)ke_2$$

$$b_{8,3} \; or \; b_{5,3} = \frac{-1}{T_b}(1-\frac{T_a}{T_b})$$

if including v_s, then,

$$a_{8,1} \; or \; a_{5,1} = \frac{1}{T_b}(1-\frac{T_a}{T_b})\frac{T_1}{T_2}K_{stab} \; , \; a_{8,10} \; or \; a_{5,7} = \frac{-1}{T_b}(1-\frac{T_a}{T_b})\frac{T_1}{T_2} \; ,$$

$$a_{8,11} \; or \; a_{5,8} = \frac{1}{T_b}(1-\frac{T_a}{T_b})$$

where,

$$\dot{x}_5 = \frac{K_e}{T_e}x_4 - \frac{1}{T_e}x_5 - \frac{K_e T_a}{T_e T_b}[(ke_1\Delta v_D + ke_2\Delta v_Q + ke_3\Delta\delta + ke_4\Delta\psi_{fd} + ke_5\Delta\psi_{1q}) - v_S] \tag{77}$$

$$v_s = -\frac{T_1}{T_2}K_{stab}\Delta\omega - \frac{T_1}{T_2}\Delta x_6 + \Delta x_7 \tag{78}$$

$$a_{9,2} \; or \; a_{6,2} = -\frac{k_e T_a}{T_e T_b}ke_3 \; , \; a_{9,3} \; or \; a_{6,3} = -\frac{k_e T_a}{T_e T_b}ke_4$$

$$a_{9,3} \; or \; a_{6,3} = -\frac{k_e T_a}{T_e T_b}ke_4 \; , \; a_{9,4} \; or \; a_{6,4} = -\frac{k_e T_a}{T_e T_b}ke_5$$

$$a_{9,8} \; or \; a_{6,5} = \frac{k_e}{T_e} \; , \; a_{9,9} \; or \; a_{6,6} = \frac{-1}{T_e} \; ,$$

$$b_{9,3} \; or \; b_{6,3} = \frac{k_e T_a}{T_e T_b}$$

If including v_s, then,

$$a_{9,1} \text{ or } a_{6,1} = \frac{k_e T_a}{T_e T_b}\frac{T_1}{T_2}K_{stab} \;,\; a_{9,10} \text{ or } a_{6,7} = -\frac{k_e T_a}{T_e T_b}\frac{T_1}{T_2} \;,\; a_{9,11} \text{ or } a_{6,8} = \frac{k_e T_a}{T_e T_b}$$

$$b_{9,1} \text{ or } b_{6,1} = -\frac{k_e T_a}{T_e T_b}ke_1 \;,\; b_{9,2} \text{ or } b_{6,2} = -\frac{k_e T_a}{T_e T_b}ke_2$$

Power System Stabilizer

The corresponding differential equations are

$$\dot{x}_6 = -\frac{1}{T_w}x_6 + \frac{1}{T_w}u_p \tag{79}$$

$$\dot{x}_7 = \frac{1}{T_2}\left(\frac{T_1}{T_2}-1\right)x_6 - \frac{1}{T_2}x_7 - \frac{1}{T_2}\left(\frac{T_1}{T_2}-1\right)u_p \tag{80}$$

with $u_p = K_{stab}\Delta\omega$

$$\Delta\dot{x}_6 = -\frac{1}{T_w}\Delta x_6 + \frac{1}{T_w}K_{stab}\Delta\omega \tag{81}$$

where,

$$a_{10,1} \text{ or } a_{7,1} = \frac{1}{T_W}K_{Stab} \;,\; a_{10,10} \text{ or } a_{7,7} = -\frac{1}{T_W}$$

For Pe feedback, since,

$$P_e = T_e\omega \tag{82}$$

therefore,

$$\Delta P_e = \Delta\omega T_{e0} + \Delta T_e \omega_0 \tag{83}$$

Substituting Eq.(40) into yields,

$$\Delta P_e = \Delta \omega T_{e0} + \omega_0 (k_1 \Delta v_D + k_2 \Delta v_Q + k_3 \Delta \delta + k_4 \Delta \psi_{fd} + k_5 \Delta \psi_{1q}) \tag{84}$$

$a_{10,1}$ or $a_{7,1} = \frac{1}{T_W} K_{Stab} \omega_0 T_{e0}$, $a_{10,2}$ or $a_{7,2} = \frac{1}{T_W} K_{Stab} \omega_0 k_3$

$a_{10,3}$ or $a_{7,3} = \frac{1}{T_W} K_{Stab} \omega_0 k_4$, $a_{10,4}$ or $a_{7,4} = \frac{1}{T_W} K_{Stab} \omega_0 k_5$

$a_{10,10}$ or $a_{7,7} = -\frac{1}{T_W}$,

$b_{10,1}$ or $b_{7,1} = \frac{1}{T_W} K_{Stab} \omega_0 k_1$, $b_{10,2}$ or $b_{7,2} = \frac{1}{T_W} K_{Stab} \omega_0 k_2$

$$\Delta \dot{x}_7 = \frac{1}{T_2}(\frac{T_1}{T_2} - 1)\Delta x_6 - \frac{1}{T_2}\Delta x_7 - \frac{1}{T_2}(\frac{T_1}{T_2} - 1) K_{stab} \Delta \omega \tag{85}$$

where,

$a_{11,1}$ or $a_{8,1} = -\frac{1}{T_2}(\frac{T_1}{T_2} - 1) K_{Stab}$, $a_{11,10}$ or $a_{8,7} = \frac{1}{T_2}(\frac{T_1}{T_2} - 1)$

$a_{11,11}$ or $a_{8,8} = -\frac{1}{T_2}$

For Pe feedback, since,

$$P_e = T_e \omega \tag{86}$$

therefore,

$$\Delta P_e = \Delta \omega T_{e0} + \Delta T_e \omega_0 \tag{87}$$

Substituting Eq.(40) into yields,

$$\Delta P_e = \Delta \omega T_{e0} + \omega_0 (k_1 \Delta v_D + k_2 \Delta v_Q + k_3 \Delta \delta + k_4 \Delta \psi_{fd} + k_5 \Delta \psi_{1q}) \tag{88}$$

$$a_{11,1} \text{ or } a_{8,1} = -\frac{1}{T_2}\left(\frac{T_1}{T_2}-1\right)T_{e0} \qquad a_{11,4} \text{ or } a_{8,4} = -\frac{1}{T_2}\left(\frac{T_1}{T_2}-1\right)K_{Stab}\omega_0 k_5 \ ,$$

$$a_{11,10} \text{ or } a_{8,7} = \frac{1}{T_2}\left(\frac{T_1}{T_2}-1\right) \ , \qquad a_{11,11} \text{ or } a_{8,8} = -\frac{1}{T_2}$$

$$b_{11,1} \text{ or } b_{8,1} = -\frac{1}{T_2}\left(\frac{T_1}{T_2}-1\right)K_{Stab}\omega_0 k_1 \ , \ b_{11,2} \text{ or } b_{8,2} = -\frac{1}{T_2}\left(\frac{T_1}{T_2}-1\right)K_{Stab}\omega_0 k_2$$

Appendix B

System Parameters

Table B.1 Generator parameters

	Generator #1	Generator #2
MVA	175	900
r_a	0.0032	0.0032
x_l	0.22	0.24
x_d	0.74	0.74
x_d'	0.27	0.27
x_q	0.45	0.48
x_q'	0.40	0.44
H	3.91	4
K_D	8.5	15
A_{stab}	0.0003	0.0003
B_{stab}	6.0	6.0

Table B.2 Excitation system parameters

T_a	1	1
T_b	1.2	10
T_e	0.05	0.05
K_e	10	70

Table B.3 Turbine/generator parameters

T_W	0.2	1
T_s	0.3	0.3
T_r	10.3	10.3
P_{do}	0.1	0.05
T_{dr}	0.4	0.4

Table B.4 Power system stabilizer parameters

T_1	0.27	0.115
T_2	0.035	0.044
T_w	1.1	11
K_{stab}	1	0.5

Table B.5 Initial generator terminal condition

Generator	MW (P)	MVR(Q)	Voltage (V)	Angle (δ)
G#1	180	200	0.981	38.94
G#2	850	40	0.992	40.43
G#3 (Infinite bus)	-646	611	1	0