

LIMITATIONS OF STABILITY ANALYSIS IN GEOTECHNICAL PRACTICE

by

Norbert R. Morgenstern*

ABSTRACT — Case histories are used to illustrate the empirical fracture that influence the selection of the design Factor of Safety as calculated by limit equilibrium methods of stability analysis. These case histories are concerned with: 1) the behaviour of a plastic clay fill placed wet of optimum moisture content, 2) the behaviour of a structured, contractant alluvial clay subjected to embankment loading, 3) progressive failure along a weak bentonite layer during an excavation, 4) progressive failure along a brittle clay of high plasticity beneath a dam, 5) cumulative deformations in a weak dam foundation as a result of cyclic reservoir loads.

RESUMO — Recorre-se a casos de obra para ilustrar os factores empíricos que influenciam a escolha do coeficiente de segurança de projecto, no contexto dos métodos de análise de estabilidade por equilíbrio limite. Os casos de obra referem-se a: 1) comportamento de um aterro de argila plástica colocada com teor de humidade superior ao óptimo, 2) comportamento de uma argila aluvionar estruturada e contráctil submetida ao carregamento de um aterro, 3) rotura progressiva ao longo de uma camada branda de bentonite, durante uma escavação, 4) rotura progressiva ao longo de uma camada de argila frágil de alta plasticidade, debaixo de uma barragem, 5) deformações cumulativas numa fundação branda de barragem, em resultado do carregamento cíclico devido à albufeira.

INTRODUCTION

It is a great honour to have been invited to deliver the 6th Manuel Rocha Memorial Lecture.

The professional career of Manuel Rocha touched all of us who work in Geotechnical Engineering; firstly by his own contributions as a scholar and a consulting engineer, and secondly through the impact of the great National Laboratory that he did so much to create. It is a fitting memorial that a lecture series commemorates the achievements of such a distinguished engineer.

The theme of this presentation is concerned with the selection of the appropriate Factor of Safety when concerned with problems of stability of slopes, excavations or dam foundations. Case histories will illustrate that there are many difficulties involved in selecting the correct Factor of Safety and that there are many problems where it is extremely difficult, if not impossible, to converge on the optimum design by means of limit equilibrium analyses alone. The spirit of this theme was already anticipated in Manuel Rocha's General Report to the International Society for Rock Mechanics Symposium on Rock Mechanics Related to Dam Foundations (Rocha, 1978) where he stressed that:

"it should be emphasized that fixing the values of the coefficients of safety to be used is done in a predominantly empirical way, by successive adjustments, in accordance with the behaviour of similar works under operation".

* University Professor of Civil Engineering, University of Alberta, Edmonton, Canada

PRINCIPLES OF LIMIT EQUILIBRIUM ANALYSIS

Limit equilibrium methods for stability analysis remain a basic tool for design and evaluation in Geotechnical Engineering. Morgenstern and Sangrey (1978) discuss their foundation and application at length and point out that regardless of the specific procedure for carrying out the computations, the following principles are common to all methods of analysis:

- 1) A slip mechanism is postulated.
- 2) The shearing resistance required to equilibrate the assumed slip mechanism is calculated by means of statics.
- 3) The calculated shearing resistance required for equilibrium is compared with the available shear strength in terms of the Factor of Safety.
- 4) The mechanism with the lowest Factor of Safety is found by iteration.

It is important to define the Factor of Safety clearly and to understand its role. The favoured definition is as follows:

The Factor of Safety is that factor by which the shear strength parameters may be reduced in order to bring the slope into a state of limiting equilibrium along a given slip surface.

Morgenstern and Sangrey (op. cit.) expand on the merit of this definition, compare it with others that have been used and note some limitations associated with it.

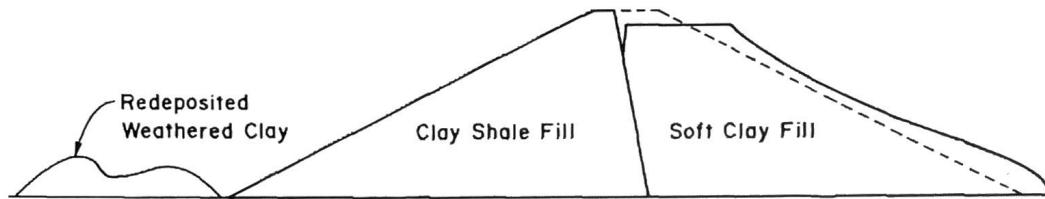
One well-recognized role of the Factor of Safety is to account for uncertainty and to act as a factor of ignorance with regard to the reliability of factors that are input to the analysis. These include strength parameters, pore pressure distribution and stratigraphy. However, an additional major role of the Factor of Safety is that it constitutes the empirical tool whereby deformations are limited to tolerable amounts within economic restraints. In this way, the choice of the Factor of Safety is greatly influenced by the accumulated experience with a particular soil or rock mass. Since the degree of risk that can be taken is also much influenced by experience, the actual magnitude of the Factor of Safety used in design will vary with material type and performance requirements.

The case histories that follow are intended to illustrate these points.

CLAY FILL EMBANKMENT

Figure 1 illustrates schematically a long embankment about 15 m high constructed as part of an industrial waste retention scheme. The site was an abandoned clay pit that had been utilized in the manufacture of bricks. As the clay pit was developed, weathered clay near the surface, which was unsuitable in brick manufacture, was stripped and deposited on the previously uncovered floor of the pit where it subsequently absorbed water and softened. The economics of the project made it desirable to use this softened clay for part of the fill while freshly excavated hard clay was available for the remainder.

As shown in Figure 1, fresh hard clay was used for fill in the upstream portion of the embankment while the softened clay was used in the downstream section. This softened clay was substantially wet of Proctor optimum moisture content and design of the embankment for stability seemed a relatively straight forward undertaking.



In-Situ Clay Shale

Fig. 1 — Section of clay fill embankment

The downstream embankment was to be constructed with a homogeneous remoulded saturated clay, on a relatively strong foundation. Slip circle analysis in terms of the undrained strength (C_u , $\phi = 0$) was appropriate and the downstream slope was selected to have a Factor of Safety of about 1.8. This appeared to be a conservative design based on Factors of Safety in conventional use and on the view that the clay fill material behaviour was well understood.

When the embankment was completed, substantial bulging developed in the downstream direction and a crack opened between the stiff upstream fill and the ductile downstream material. These features are also illustrated in Figure 1. Although the embankment had not collapsed, it would continue to deteriorate and it was not performing as intended. The Factor of Safety was inadequate to ensure the serviceability of the embankment.

What had been ignored in this simple case was the extreme ductility of the clay. Axial strains of 30–40% are mobilized at the peak undrained strength of this material and hence the design Factor of Safety implied strains of 15–20%, far in excess of the levels of strain usually accommodated in conventional earthworks. There had been limited experience in utilizing such wet fill for the shoulders of a berm; it is usually wasted on most jobs. Hence insufficient attention was paid to the level of strain developed at the design Factor of Safety. In order to restrict the deformation to levels commonly accepted, a Factor of Safety of about 4 would be required.

Earth moving at this project was economical and the stability was augmented without difficulty by constructing an additional low level berm in the downstream direction. The cracks were repaired and the embankment behaved in an acceptable manner.

The case illustrates that when clays wet of optimum are used as structural fill in embankments, Factors of Safety higher than usual are necessary to restrict the deformations and ensure serviceability. In modern practice, a deformation analysis by means of finite element methods would be appropriate to address this point in more detail.

EMBANKMENT ON ALLUVIAL FOUNDATION

As part of a water supply and fly ash retention scheme, in conjunction with a large coal-fired generating station, it was required to construct several kilometers of embankment on alluvial soils adjacent to the Mersey River in the United Kingdom. Details of the foundation and its behaviour during construction have been reported by Al-Dhahir et al. (1970).

Economic design required the utilization of adjacent materials in embankment fill, which influenced fill zonation. The properties of the fill are not relevant to the behaviour under discussion here.

Initial design was based on a combination of in-situ vane testing and laboratory testing. The embankment was sectioned initially using undrained strengths and it was evident that stage construction was necessary to achieve stability. There was ample time to proceed with stage construction, even without the installation of sand drains. Most of the project was completed successfully in two years using field observations of pore pressures and deformations to control construction.

One portion of the embankment had been located over somewhat weaker clays and a longer construction period was anticipated. A control section through this portion of the embankment was also instrumented to guide construction. On-going evaluation of stability was being undertaken in terms of effective stress. It is this section that revealed behaviour of special interest.

The embankment and instrumented section are shown in Figure 2. On-going stability assessment relied on pore pressure measurements, surface displacement monitoring, and effective stress analyses. The clays were only slightly sensitive and did not appear to have characteristics significantly different from those commonly found in alluvial environments.

The construction history of the embankment and the response of selected piezometers are given in Figure 3. As anticipated, high construction pore pressures developed in response to the applied embankment load. Significant pore pressure dissipation occurred during the shut-down seasons, but the rate of pore pressure dissipation decreased with increasing effective stress. As shown by Al-Dhahir et al. (1970) this was due to the reduction in coefficient of consolidation with increasing effective stress, caused primarily by the reduction in permeability in the alluvial soil as cracks, fissures and root holes close readily in response to small increases in effective stress.

As the embankment reached full height, it became apparent that further dissipation of pore pressure during construction would not contribute much to stability. Stability analyses in terms of effective stress indicated that the Factor of Safety would drop below target values of about 1.3–1.4 but would still remain significantly above unity.

In August of 1966, the calculated Factor of Safety based on observed pore pressures was about 1.25 and only about 2 m of fill remained to be placed. A report came from the field that some pore pressures were going up suddenly, grossly disproportionate to the amount of fill that had been placed in the past few days (see P 2.5). Instructions to the field were to monitor for displacements carefully. None were observed until after the pore pressure peaked, when cracks

formed in the embankment and a slip occurred. The spike in the pore pressures rapidly dissipated back to their original trend line. The slip was of limited consequence and a stabilizing berm was readily added to the section of embankment constructed over the weaker clays.

In the parlance of the 1960's, the behaviour was attributed to structural breakdown of the clay at some critical strain, followed by reconsolidation to its pre-breakdown effective stress. In more modern terms one might say that undrained yielding occurred within the foundation and the pore pressures that developed were in accordance with the type of yield surface often displayed by natural soft clays. Crooks et al. (1984) have attempted to analyze the yielding of this foundation, together with many other cases, in a quantitative manner.

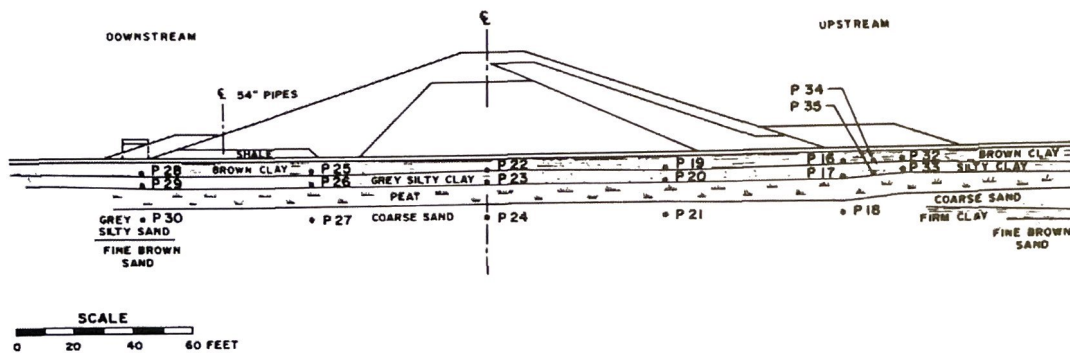


Fig. 2 — Fiddler's Ferry embankment (Al-Dhahir et al., 1970)

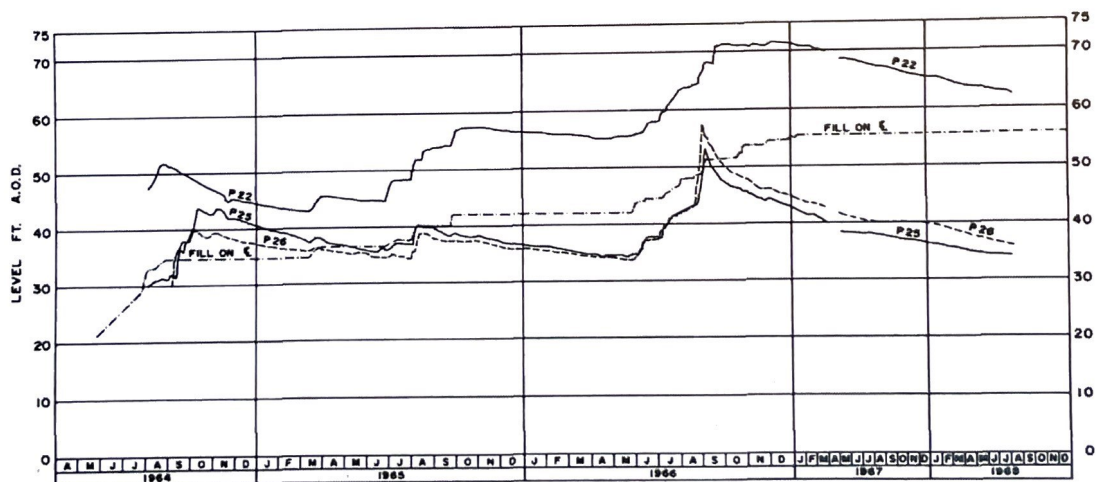


Fig. 3 — Pore pressure history at Fiddler's Ferry embankment

From the perspective of limit equilibrium analyses, one notes that when dealing with contractant alluvial clays, a Factor of Safety of about 1.25, based upon effective stress analyses, may be insufficient to preclude the development of undesirable deformations if the foundation can be subjected to increments of undrained loading. It is also of interest to note that the sudden onset of the destabilizing process limits the application of the observational method. There was little opportunity to take remedial action once undrained yielding began.

PROGRESSIVE FAILURE IN AN EXCAVATION

Progressive failure occurs in a material with distinct peak and residual strengths such that the mobilization of shear strength parameters at failure is nonuniform along the slip surfaces. As will be illustrated below, the occurrence of progressive failure constitutes a serious impediment to assuring stability based on limit equilibrium methods of analysis alone.

The Edmonton Convention Centre is situated in a 20 m deep excavation on the north bank of the North Saskatchewan River in the centre of the city of Edmonton, Canada. Construction began in 1980. The excavation was supported by tangent pile walls with six levels of prestressed anchors embedded into the adjacent soil and bedrock. The performance of the excavation was carefully monitored to assure that ground movements remained within tolerable limits. Site investigation had indicated the presence of several locations of practically horizontal bentonite within the bedrock. One of these was known by experience to control adjacent slope stability. Observations from the slope indicator measurements during the excavation revealed that substantial movement occurred at this bentonite layer. This bentonite layer had been sampled during the site investigation stage and carefully inspected. It was not sheared *in-situ* at the location of the slope indicator measurement prior to excavation. Therefore the large localized movements that occurred were a result of the excavation process. Balanko et al. (1982) have summarized the details of the construction of the tied-back wall as well as its performance during excavation.

The stratigraphic sequence consists of approximately 3.1 m of undifferentiated sand, silt and clay fill with organic inclusions underlain by 1.7 m of lacustrine clayey silt with low to medium plasticity. Beneath the silt is a highly plastic, stiff to very stiff clay to a depth of 8.8 m. Sandy clay (till) of low to medium plasticity is found beneath the lacustrine clay to a depth of 15.2 m. This till often contains water-bearing sand lenses. Pre-glacial dense sands and gravels are found beneath the till and above the Cretaceous clay shale bedrock.

The bedrock is interbedded shale and sandstone starting at an average depth of 18.3 m. Coal seams and bentonite layers are found throughout the bedrock to a depth of over 60 m. The extent of the bentonite layers is not known, but there are several layers that, from local experience, are believed to be continuous. The properties and continuity of these bentonite layers control the stability at depth. Discontinuous slip surfaces and till were found within the top few metres of the bedrock indicating local weakening by glacial drag. This was also a consideration in project design but it proved not to be as significant as the dominant bentonite layer.

The final section for purposes of overall stability evaluation is shown in Figure 4, together with the earth pressure loading diagram that was adopted in design. This pressure diagram arose from consideration of the earth pressure required to minimize adjacent ground relaxation and it is more restrictive than if it arose from the requirements to limit movements of adjacent buildings.

As illustrated in Figure 4, several overall stability mechanisms had to be investigated, but only the deepest, which had the lowest Factor of Safety, need be considered here. The overall

design Factor of Safety against sliding on this slip surface was 1.8. In this analysis, peak shear strength was attributed to the deep bentonite layer based upon careful sample inspection or samples.

To assess whether the peak strength of the bentonite could be relied upon as excavation proceeded, a finite element analysis was conducted. This analysis employed hyperbolic stress-strain relations and modelled the excavation and anchor installation in steps. Yielding in the potential shear zone was investigated by characterizing this material as transversely anisotropic. If the stress mobilized in the horizontal direction were to exceed the peak strength, the horizontal modulus of rigidity would be reduced, together with the available strength, in order to promote slip and strength reduction. This technique is described in more detail by Morgenstern and Simmons (1982), who have used it to evaluate movements in the shear zone beneath a dam.

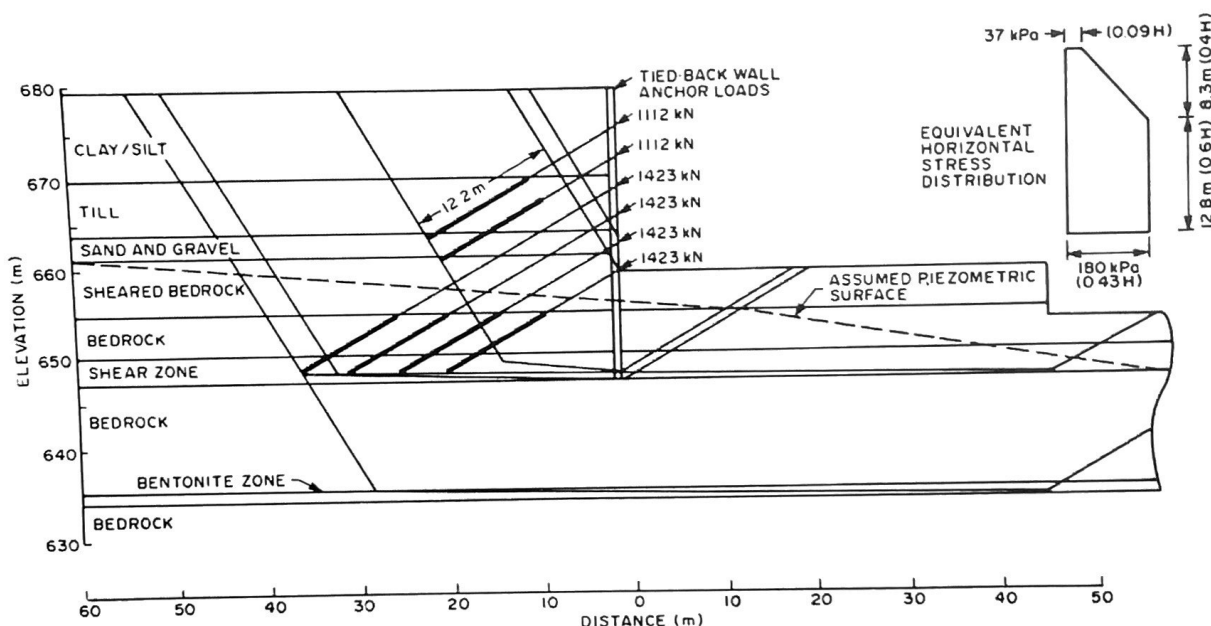


Fig. 4 — Stability of Edmonton Convention Centre (Chan and Morgenstern, 1987)

The finite element analysis conducted for purposes of design indicated that localized movements of about 13 mm could be expected but that the shear strength of the bentonite would not be exceeded.

All aspects of the excavation and support system were instrumented. Of particular interest here are the observations from inclinometer SI-80-4, located at the mid-point, just behind the major tangent pile wall. It is clear that intensive shearing occurred at a depth of around 46.3 m which coincides with the lowest zone of weakness identified from the site investigation. Slip of the order of 30 mm was observed when the excavation was completed. There was a direct correlation between the movements and the amount of material being excavated as indicated in Figure 5.

The bentonite seams underneath the convention centre have a peak friction angle of 14° and a residual friction angle of about 7° . The transition from peak to residual is abrupt and it

is likely that the slip movements that developed reduced the strength locally to close the residual. If residual strength is used along the bentonite zone, the Factor of Safety drops to about 1.4, a reduction of about 22%. Practical deformation analyses available in the early 1980's were not able to forecast these effects in a reliable manner.

Progressive failure phenomena create a fundamental limitation to the validity of stability analyses based on limit equilibrium analyses alone. When strain-weakening materials exist, and strains arise from either loading or unloading, it cannot be assumed that the peak strength parameters identified at the site investigation stage will be mobilized at the time of collapse. Strain-weakening can result in a substantial reduction in available strength parameters and this process can only be investigated by undertaking a deformation analysis.

Fortunately there have been major advances in conducting deformation analyses with strain-weakening materials by means of finite element methods. Figure 5 also compares the results of such a simulation with the observations behind the tangent pile wall and excellent agreement is found. Moreover, the analysis reveals the migration of progressive failure as excavation proceeds which facilitates the evaluation of the Factor of Safety. Details of these calculations have been given by Chan and Morgenstern (1987).

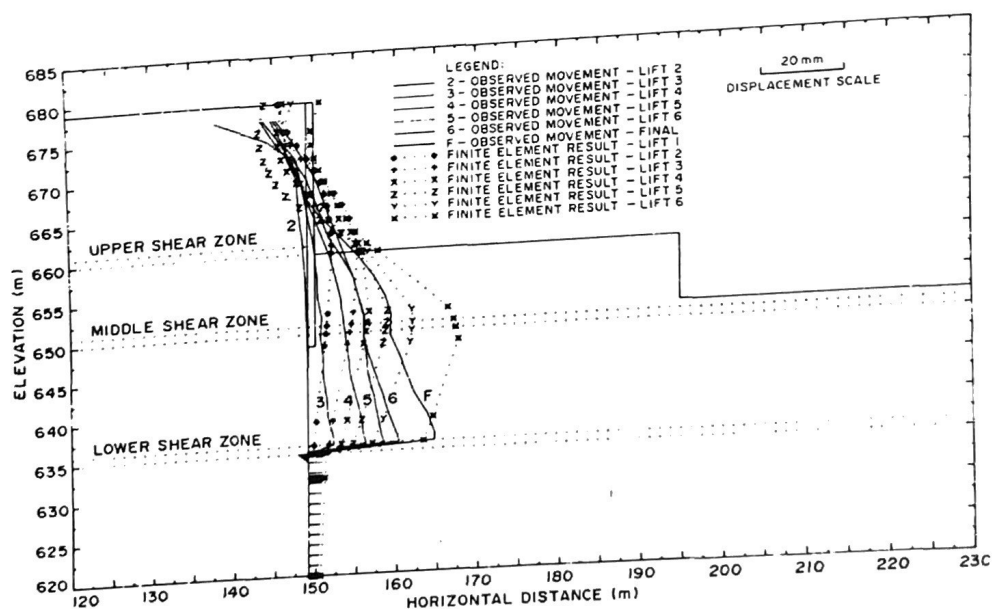


Fig. 5 — Inclinator response at Edmonton Convention Centre
(Chan and Morgenstern, 1987)

PROGRESSIVE FAILURE IN A DAM FOUNDATION

The Paddle River dam is located approximately 140 km northwest of Edmonton, Canada. The embankment is situated in the Paddle River valley which is approximately 600 m wide and

35 m deep at the dam site. Construction of the dam commenced in September 1981 and was completed in the summer of 1985. The service spillway is a concrete chute located on the right abutment. The low level conduit is provided with an automatic gate control designed to regulate the reservoir elevation for the purposes of providing flood attenuation on the Paddle River. Additional description of the project concept is provided by Thiessen and Ramage (1986).

The earthfill dam has been constructed with a wide central clay core, gravel shells, upstream impervious blanket, berms on both upstream and downstream slopes, and downstream relief wells. A cross-section through the dam is shown in Figure 6. The extensive stabilising berms are evident. A staged construction approach was used to facilitate design modifications for the embankment as the project developed.

The embankment is constructed on a stratified alluvial and glacial deposit. The general sequence of materials is up to 3 m of alluvial clay, silty sand and gravel overlying a stiff sandy clay till unit 10 m thick. Below the till are the sub-till sediments, a 10 m thick sequence of silt and slickensided high plastic clay grading in the downward direction to a medium grained sand. Below the sand is the sandstone bedrock with minor occurrences of clay shale. The surficial deposit has been removed from the base of the central core and upstream impervious blanket.

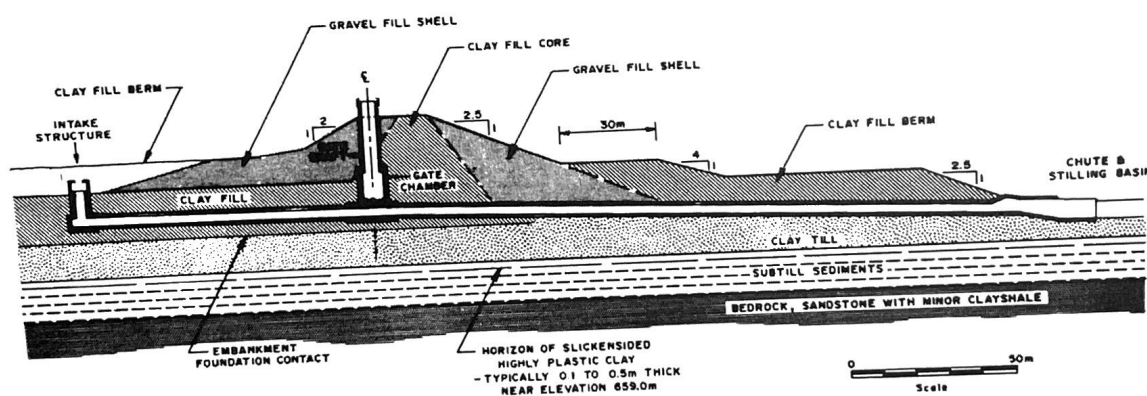


Fig. 6 — Paddle River dam section

The sub-till clay was a pro-glacial lake deposit that was subsequently disturbed by the deposition of the overlying till. As a result, it possesses a sequence of discontinuous, sub-horizontal, slickensides. The peak frictional strength in this direction is about $20^{\circ} - 22^{\circ}$. But with only a small deformation beyond peak, the strength falls rapidly to its residual value of about 9° . This is an example of strain-weakening and it was recognized in design that strains induced by construction would lead to a reduction in available shearing resistance and therefore the maximum strength could not be relied upon everywhere along a potential slip surface. As discussed in the previous case, a deformation analysis would be required to calculate the extent of the shear band that developed during construction. At the time of the design of this dam, it was not possible to calculate the development of the shear band with any accuracy.

Therefore it was necessary to estimate the location of the residual strength zone by means of judgement, guided by simple elastic stress calculations.

In the intervening years, there has been considerable progress in the numerical modelling of shear band propagation. The following illustration, taken from Chan (1986), was prompted by the circumstances encountered at the Paddle River dam.

An embankment 12.5 m high is constructed incrementally as shown in Figure 7. A strain weakening layer of clay, 5 cm thick, is present 5 m below foundation level. The clay is taken to fail in an undrained manner with a peak strength of 100% at a plastic strain of 2%. The strength then falls off with increasing plastic strain following a stress-strain relation advocated by Prevost and Hoeg (1975). The analysis is not restricted to this relation and other formulations of strain-weakening, in either total or effective stresses can be adopted.

As the height of the embankment increases, the peak shear strength is mobilized in the clay layer and with further heightening, a shear band with reduced strength, migrates along the clay layer. A cusped distribution of available shear strength develops and, as illustrated in Figure 7, when the embankment attains a height of 12.5 m, approximately 50% of the clay layer beneath the embankment has been reduced in strength from the peak value. The details of the distribution will depend upon the constitutive relation used to model strain-weakening.

More recent advances in modelling progressive failure have been made by Potts et al. (1990) and Chen (1990).

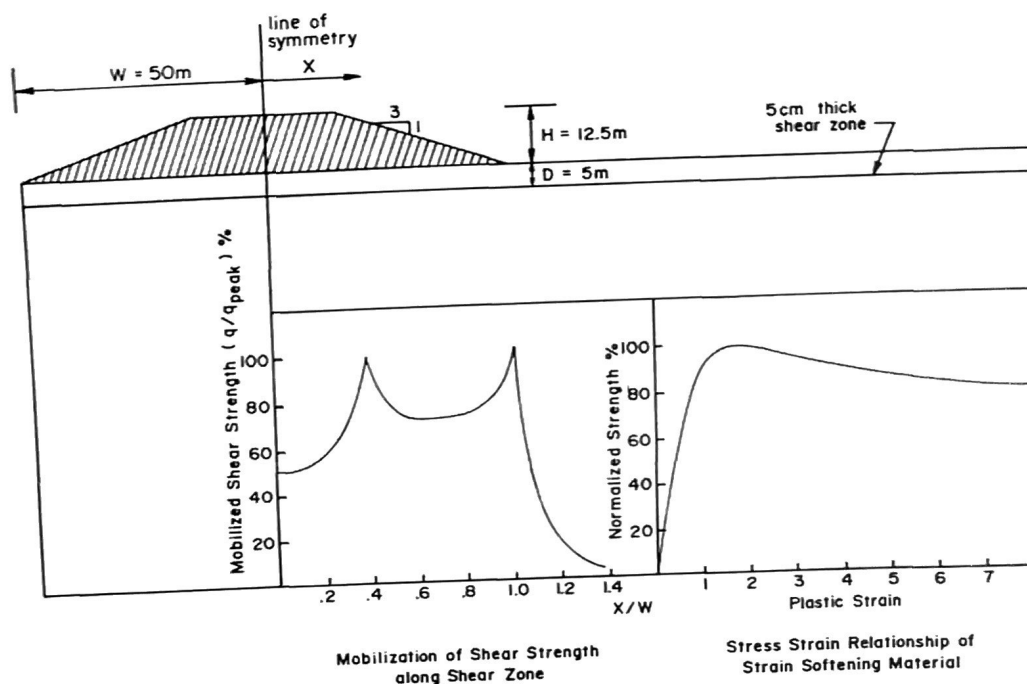


Fig. 7 — Progressive failure in a dam foundation (Chan, 1986)

CUMULATIVE MOVEMENT IN A DAM FOUNDATION

Gardiner Dam was constructed on the South Saskatchewan River in south-central Saskatchewan, Canada between 1958 and 1968. Bedrock consists of overconsolidated bentonitic clay shale and sandstone which were known to pose unusual and difficult foundation and stability problems. Geotechnical performance of the dam and associated works has been described in detail by Jaspar and Peters (1979).

Continuing deformations of the embankment and foundation have caused some concern, particularly as movements have been concentrated along foundation shear zones with a history of previous movements. An analysis of these deformations presented by Morgenstern and Simmons (1982) clarified the nature of the load-deformation mechanism and highlighted certain limitations associated with the use of conventional limit equilibrium methods in practice.

The project consists of a main river valley embankment and a smaller subsidiary embankment across Coteau Creek, as well as a spillway, power tunnels, and outlet works. After construction commenced, it became necessary to revise original designs because of stability problems encountered with soft weathered shale. Embankment slopes had to be substantially flattened and this proved to be satisfactory until construction was well advanced. Foundation displacements were then experienced in weak shale zones. Additional slope flattening, closely controlled fill placement, and more extensive berming were required to complete the embankments. As a consequence of the foundation conditions, much reliance was placed upon field observation of performance and a very comprehensive suite of monitoring instrumentation was installed. Piezometers measuring pore pressures in the shale foundation, particularly in the shear zones, showed a very high level of response and virtually no dissipation since construction was completed. In addition, a pattern of yearly incremental displacements was detected. These were non-recoverable shear zone slips related to the increase of reservoir level as runoff accumulated each spring. The slips were definitely related to the water thrust changes; creep movements at more or less constant reservoir level were small in comparison with the slips associated with the change in reservoir level. The Coteau Creek embankment has a somewhat simpler configuration than the main dam and therefore it was chosen for detailed analysis. Figure 8 summarizes its performance. The average slip under the embankment centreline, for the period reported, was 0.027 m/year.

As part of a dam safety review, Morgenstern and Kaiser (1978) assessed the available strength, pore pressures and movement data which was extremely comprehensive and they carried out a stability assessment using limit equilibrium methods of analysis. The resulting Factor of Safety was less than commonly accepted for major structures under full reservoir and, in view of the cumulative movements, consideration was given to increasing the Factor of Safety by excavation of a portion of the crest and upstream fill, as indicated in Figure 9.

It is common practice to associate increases in Factor of Safety with decreased movements. However, given the sensitivity associated with undertaking major modifications of a large water-retaining dam, it was judged prudent to study the deformation mechanism in more detail by constructing a finite element model of the dam that more or less simulated its deformation history and that would allow assessment of any proposed modifications to the section. To be

realistic, the model had to account for highly localized slip and had to match sensibly the following stages: 1) foundation preparation, 2) embankment construction, 3) first filling of reservoir, and 4) cyclic reservoir operation.

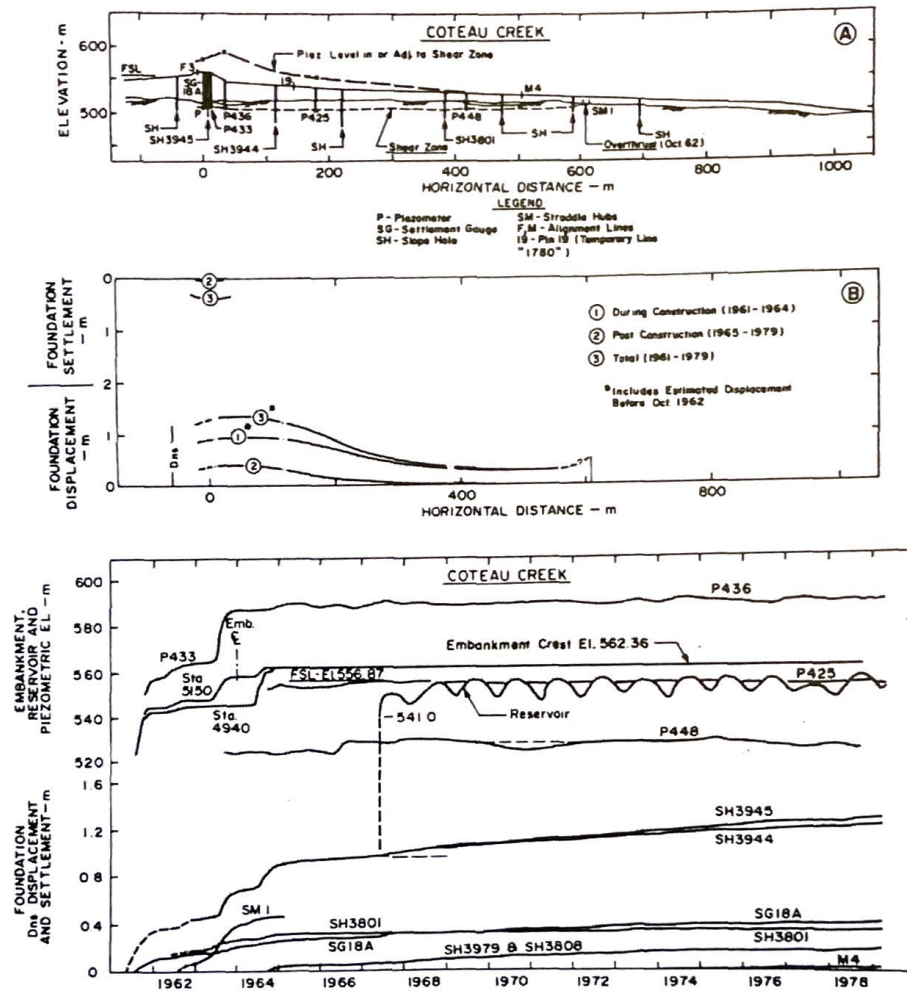


Fig. 8 — Movements of Coteau Creek embankment (Jaspar and Peters, 1979)

As shown by Morgenstern and Simmons (1982), it was possible to develop a history-matched model that embraced reasonably the behaviour of the dam to the end of first filling.

In order to simulate the behaviour during reservoir operation, it was postulated that the upstream random fill, which had not been particularly well compacted, displayed a cyclic reduction in stiffness, as illustrated schematically in Figure 10. The unloading behaviour on a given cycle was stiffer than the loading behaviour, which is in general terms equivalent to a reduction in modulus. As the reservoir cycled, slip accumulated in the shear zone in response to the reduction in stiffness of the shell material. The computed average slip per cycle of reservoir loading was close to that observed which increased confidence in the model.

It was then possible to interrogate the model with regard to the proposed excavation. The slip per cycle of reservoir loading was reduced, but only by about 30%. Clearly any broad empirical correlation between Factor of Safety and anticipated movement does not reckon with

the implication of cyclic reduction in stiffness of an embankment seated on a sheared shale foundation, already at a low Factor of Safety. In general, when the overall Factor of Safety is low and critical elements of the soil mass have yielded, it is not advisable to relate deformation patterns to Factor of Safety on an empirical basis. The deformation mechanism should be evaluated on its own.

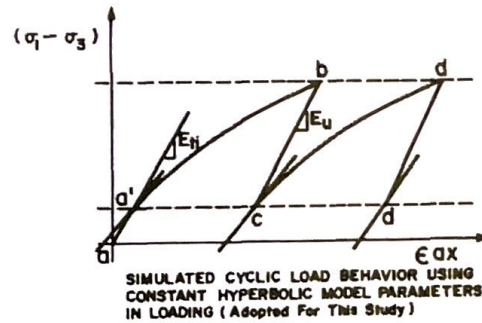


Fig. 9 — Cyclic stiffness reduction (Morgenstern and Simmons, 1982)

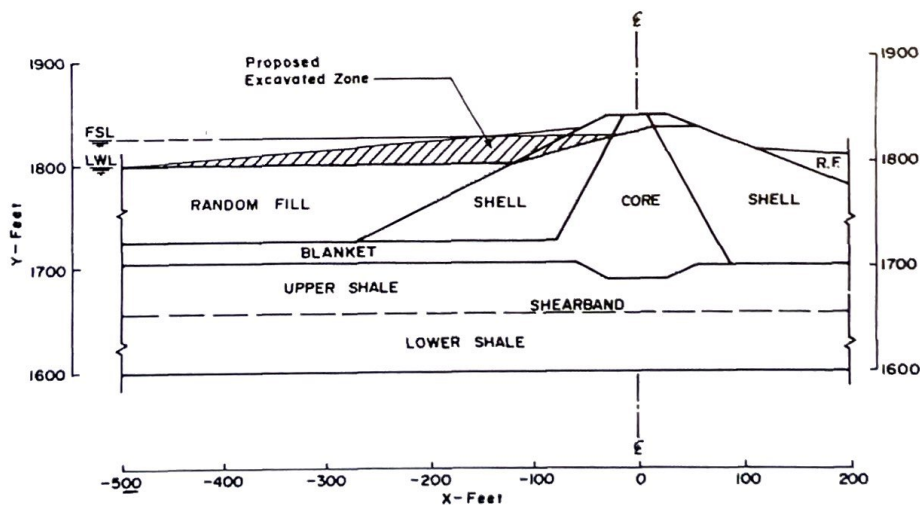


Fig. 10 — Modified section (Morgenstern and Simmons, 1982)

CONCLUSIONS

Five case histories associated with slope problems have been summarized. They involve the following:

- i) the behaviour of a plastic clay fill placed wet of optimum moisture content,
- ii) the behaviour of a structured, contractant alluvial clay supporting an embankment at a Factor of Safety of about 1.25,
- iii) the progressive failure along a weak bentonite layer during an excavation,

- iv) the progressive failure along a brittle clay of high plasticity, subjected to embankment loading,
- v) the cumulative deformations in a weak dam foundation as a result of cyclic reservoir loads.

In each case, traditional analyses with sometimes traditional values of Factor of Safety were not capable of inhibiting undesirable behaviour. It is not that the methods of analysis themselves are wrong; but that their application is bounded by the experience within which they have been developed and applied in the past. The Factor of Safety appropriate for design must reflect this fact.

As the profession makes greater demands in terms of more efficient, more daring use of geotechnical material, to ensure proper performance, it will be increasingly essential to pay attention to the implications of the deformations that develop within the earth structure in its various phases of construction and operation. This clearly cannot be addressed by means of limit equilibrium methods of analysis alone.

REFERENCES

- AL-DHAHIR, Z., KENNARD, M.F. and MORGENSTERN, N.R., 1970 — *Observations on pore pressures beneath the ash lagoon embankments at Fiddler's Ferry power station*. In-Situ Investigations in Soils and Rocks, British Geotechnical Society, p. 265–276.
- BALANKO, L.A., MORGENSTERN, N.R. and YACHYSHYN, R., 1982 — *Tangent pile wall*, Edmonton Convention Centre. Proc. Symp. Application of Walls to Landslide Control Problems, ed. by R. Reeves, ASCE, New York, p. 108–123.
- CHAN, D.H., 1986 — *Finite Element Analysis of Strain-Softening Material*. Ph.D. Thesis, University of Alberta.
- CHAN, D.H. and MORGENSTERN, N.R., 1987 — *Analysis of progressive deformation of the Edmonton Convention Centre excavation*. Canadian Geotechnical Journal, Vol. 24, p. 430–440.
- CHEN, Z., 1990 — *Analysis of Progressive Failure of Carsington Dam*. Ph.D. Thesis, University of Alberta.
- CROOKS, J.H., BECKER, D.E., JEFFERIES, M.G. and MCKENZIE, K., 1984 — *Yield behaviour and consolidation, I: Pore pressure response*. Sedimentation and Consolidation Models, ASCE, New York, p. 356–381.
- JASPAR, J.L. and PETERS, N., 1979 — *Foundation performance of Gardiner Dam*, Canadian Geotechnical Journal. Vol. 16, p. 758–788.
- MORGENSTERN, N.R. and KAISER, P.K., 1978 — *Analyses of the stability of Gardiner Dam, South Saskatchewan River Project*. Report to Chief Engineer, Prairie Farm Rehabilitation Administration, Regina, Saskatchewan.
- MORGENSTERN, N.R. and SANGREY, D.A., 1978 — *Methods of stability analysis*. In Landslides: Analysis and Control, ed. by R.L. Schuster and R.J. Krizek, Special Report No. 176, Transportation Research Board, National Academy of Sciences, Washington, D.C., p. 155–171.
- MORGENSTERN, N.R. and SIMMONS, J. V., 1982 — *Analysis of the movements of the Gardiner Dam*. Proc. 4th Int. Conf. Numerical Methods in Geomechanics, Edmonton, Vol. 3, p. 1003–1028.
- POTTS, D.M., DOUNIAS, G.T. and VAUGHAN, P.R., 1990 — *Finite element analysis of progressive failure of Carsington embankment*. Geotechnique. Vol. 40, p. 79–102.
- PREVOST, J.H. and HOEG, K., 1975 — *Soil mechanics and plasticity analysis of strain softening*. Geotechnique. Vol. 25, p. 279–297.

- ROCHA, M., 1978 — *General Report: Analysis and design of the foundations of concrete dams*. Proc. International Symposium on Rock Mechanics Related to Dam Foundation, Brazilian Committee on Rock Mechanics, Rio de Janeiro, Vol. 2, p. III. 11-III.70.
- THIESSEN, J.W. and RAMAGE, R.G., 1986 — *Paddle River Dam, a project manager's review of dam safety concerns*. Proc. Dam Safety Seminar, Edmonton, BiTech Publishers Ltd., p. 559-568.

VOTO DE AGRADECIMENTO

Após o final da Lição, o Inv. E. Maranha das Neves, do Laboratório Nacional de Engenharia Civil, propôs um voto de agradecimento ao Prof. Morgenstern:

Foi com grande satisfação que acedi ao convite da organização da 6.^a Lição Manuel Rocha para dizer algumas palavras após a interessante palestra que o Professor Norbert Morgenstern acaba de pronunciar.

Tratando-se fundamentalmente de um agradecimento pareceu-me adequado que o dirigisse em inglês ao nosso conferencista de hoje.

Professor Morgenstern,

The lesson that you have just presented to us deals with a geotechnical subject that can only be conveniently tackled when scientific fundaments, well founded empiricism, technical knowledge and large experience are available. Even though and depending on the real case, a well balanced combination of these ingredients is of a paramount importance.

There is no doubt, and this lesson proves it, that you are one of the few persons able to fulfill these conditions at the highest level and we had the fortunate possibility of listening today to a brilliant and thoroughly stimulating lecture.

The adopted procedures were always cast by a clear engineering objective and this fact constitutes an interesting link with Manuel Rocha, the patron of these lectures.

It is then with the very great pleasure that I propose a vote of thanks to Professor Norbert Morgenstern for a most memorable sixth Manuel Rocha Lecture.