University of Alberta

An Application of Traditional and Flow Capturing Location-Allocation Models to Edmonton, Alberta.

BY

Anne Leontien Gerding

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of Master of Science.

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The undersigned certify that they have read, and recommended to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **An Application of Traditional and Flow Capturing Location-Allocation Models to Edmonton, Alberta** here submitted by **Anne Leontien Gerding** in partial fulfilment of the requirements for the degree of **Master of Science**.

M.J. Hodgson

E. Erkut

M. Bhargava

December 10, 1993

This Thesis is Dedicated to

my Grandfathers

Professor Harm Gerding

Mr. W.T.G.A. Binnendijk

Thankyou

ABSTRACT

This thesis applies [illegible] allocation (LA) models, specifically the *p*-median model and the [illegible] Flow Capture Location Model (FCLM), to a real world network. Most [illegible] based on small test problems, however, testing using larger realistic problems is important for several reasons. These reasons include the ability to test solution methods, to demonstrate to potential users the availability and applicability of techniques usually introduced in technical journals; and most importantly allow investigation into aspects of LA models which are not observable using small contrived problem settings. Cities have structure, reflected by population distributions, travel time/distance relationships and traffic flows.

Using Edmonton data I assess the useability of simple Cartesian distances as compared to travel times to locate facilities. Initial results are encouraging, although using the Cartesian separation measure does result in a decreased ability to conveniently provide services to the demand population. I then demonstrate the FCLM on the network, assessing the importance of taking a systematic approach to locating traffic based facilities. Results confirm the need for such an approach, as using a 'naive' solution approach results in a poor service strategy as compared to the greedy and best solution approaches. The cannibalization by facilities of the demand in the network is certainly shown to be a serious problem. Lastly, I illustrate the importance of using the correct location model to suit consumer behaviour. The conclusions reached suggest that using the incorrect service assumption to serve demand centred in nodes and arising from flows has a strong impact on the service conveniently available to them. As a contribution to the LA literature, this thesis stands as a demonstration, comparison, and evaluation of two important location models as applied to a medium sized Canadian city.

Acknowledgements

I would like to take this opportunity to acknowledge those individuals who enabled me to complete this Thesis successfully. First of all I would like to thank my supervisor Professor John Hodgson, and the oral examination committee. Secondly I would like to thank the City of Edmonton Transportation Department, particularly Allen Brownlee and Zig Dumpich from the Forecasting and Assessment Branch for providing me with the necessary data. Thirdly I would like to thank my family for supporting me through this sometimes trying time - especially my brother Harm Gerding who eased the pain of recalcitrant computers along the way. Last, but not least I would like to thank my husband for putting up with me - thanks for sticking it out. I couldn't have done it without you!

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and the effect of turning penalties, add further complexity and effort to the data collection process.

It is reasonable to ask whether it is actually necessary to onerously collect more realistic distance data rather than using a simple Cartesian distance matrix. Love and Morris (1979), for example, make the comment that in their urban samples the Cartesian distance metric proved superior to the rectangular distance function. If the Cartesian distance measure is shown to result in an acceptable location pattern, and does not significantly detract from the objective, it may be very useful to planners. Using a Cartesian distance matrix would significantly decrease the time required to collect more representative distance data. **The first objective of this study is to explore, in the context of the *p*-median model, the potential of using Cartesian distance data in LA modelling versus using network travel time values.**

A change in the way the demand population is perceived to act will also result in different location patterns. In traditional LA problems, demand is usually expressed as a weight at a node, with a value in proportion to the target population represented by the node. The traditional view assumes that consumers of a service travel between home or work to the facility and return directly. This does not consider the realistic possibility that consumers may stop for other optional services between the planned destination and home or work origin (Hodgson and Rosing, 1990; Goodchild and Noronha, 1987).

Goodchild and Noronha (1987), Berman, Larson and Fouska (1992), Hodgson and Rosing (1992) and Hodgson (1990) have suggested that some facilities, (for example dry cleaning outlets, billboards or banking services), may be better located if demand is modelled as being exerted along a link in the network. In response Berman, Larson and Fouska (1992) and Hodgson (1990) formulated the Flow Capturing Location Model (FCLM). The objective of this model is to locate *p* facilities so that the number of customers travelling past at least one facility is maximized. There has been little effort to evaluate this model's abilities in a real world situation, however. **The second objective of this study is to augment the knowledge base of the characteristics and capacity of the FCLM by applying it to locating facilities on the City of Edmonton's transportation network. The third objective of the thesis ties the first two objectives together by investigating the trade off between serving demands arising from flows and demands centred in nodes.**

To accomplish these objectives, the study first analyses the effects of using Cartesian distances in the *p*-median model. This analysis is performed by comparing the results of the *p*-median model runs using the Cartesian distances to the results obtained using a network travel time matrix obtained from the City of Edmonton.

Since the FCLM model has only been tested on small theoretical networks by Hodgson (1990), and Berman, Larson and Fouska (1992), little is known of its ability to locate facilities on a real world network. I add to this knowledge by testing the FCLM on the transportation network of the City of Edmonton, comparing the results of a globally optimal and two heuristic solution procedures.

Both flow based and resident based demands in a network require services.

However, no one has investigated the trade off between serving both these types of demands in a real world network. Therefore, the *p*-median model is evaluated using the locations determined by the FCLM; and vice versa, the FCLM is evaluated using the locations determined using the *p*-median model. This last objective follows up on Hodgson and Rosing's (1992) suggestion that their work be applied to a real-world network, and thus results in an original contribution to the LA literature.

I provide some background to the study of location-allocation, as applicable to this thesis, in Chapter 2. This includes a review of some the traditional LA models used, and a discussion of a number of the solution methods which have been developed to solve them. Special emphasis is placed on the *p*-median model, as this model is used as the basis for comparison in the study. I compare the performances of the Cartesian distance matrix created by the author and the network travel time matrix obtained from the City of Edmonton in Chapter 3. In Chapter 4, I review the FCLM, including a discussion of its origins and its mathematical formulation. I then present the approach taken to test the suitability of the FCLM model in fulfilling its proclaimed objective, and analyze the results of using this model on Edmonton's transportation network. In Chapter 4 I also consider the trade-off between the *p*-median and FCLM objectives. Lastly, I bring forward conclusions and recommendations in Chapter 5.

Chapter 2

The Location-Allocation Problem

"Location theory developed as a discipline that addressed questions related to the spatial organization of activities." (Ghosh and Rushton, 1987, pg. 1). In facilities location problems we are interested in locating optimally one or more new facilities, while simultaneously allocating the demand for the service provided by these facilities to them. Technical progress in the determination of solution methods for solving a variety of practical location problems has sparked interest in the application of facility location concepts to planning endeavours (Ghosh and Rushton, 1987). It is now becoming more accepted that the theory of facility location provides a logical basis for the effective planning of the locations of various facilities, both in real world applications and in the private and public sectors.

Location theory has its roots in Weber's (1909) plant location problem, in which he tried to find the production location between a market place and two raw material sources resulting in the least transportation cost. Prior to the 1960's, before the ready availability of digital computers, however, study in facilities location was minimal. This was primarily because solution methods were restricted to geometric and physical analog methods, which are limited to solving simple facility or transport-cost functions (Kuhn and Kuenne, 1962). There were no mathematical solution methodologies which could deal with the more complex single facility or multi-facility problems, where locations and allocations must be simultaneously determined.

Since the 1960's serious attention has been focused on the study of facilities location due to the development of solution algorithms, and access to computers which can more easily deal with them. Of particular importance are Kuhn and Kuenne's (1962) solution algorithm for the single facility problem, and Cooper's (1963) solution algorithm for the multi-facility Weber problem. These solution methods allow for the simultaneous location of facilities and allocation of demand to them; hence the term location-allocation (LA) (Ghosh and Rushton, 1987). Initially, LA solution methods dealt with the location of facilities in continuous space; where we solve for any location in the identified space. In the 1960's and early 1970's location theorists such as Maranzana (1964), Tietz and Bart (1968), and Revelle and Swain (1970), developed solution algorithms for the discrete space LA problem; where we choose optimal locations from a predetermined set of possible ones.

In early work, the primary objective underlying the solution approaches for the facility location problem was to minimize the transportation cost incurred in the system. The optimal solution, therefore, was the set of locations which resulted in the least transportation cost. This objective is closely related to the *p*-median model which solves for facility locations which minimize the aggregate weighted distance from the demand points to the nearest facility.

The 1970's saw the recognition of alternative objective functions, in particular the *Set* and *Maximal Covering problems* (Toregas and ReVelle, 1972; Church and ReVelle, 1974, 1976). The term *covering* describes the demand population which is served by the facilities being located. The set covering model determines the "minimal number and the location of facilities that ensures that no demand point will be farther than [a preset]...maximal service distance from a facility" (Church and ReVelle, 1976, pg. 408). Although the model minimizes the number of facilities which are located, it does not consider the feasibility of establishing the number of facilities required to serve the population within this maximal service distance. An alternative approach is to maximize the demand covered within a desired service distance with the location of a predetermined number of facilities (ReVelle and Swain, 1970). This is the objective of the maximal covering location model (MCLM). More recently, work has expanded to include more complex spatial interaction based objectives (O'Kelly, 1987).

The facilities to be located may be either noxious or salutary in nature. Examples of noxious facilities include landfills or nuclear sites, and it is typically preferable to locate them far from the demand population. Salutary facilities are beneficial in nature, for example day cares or medical centres, and it is preferable to locate them close to the demand population. In this thesis I limit my concern to the location of salutary facilities, and consequently to objective functions which locate facilities close to the demand. The following sections of this chapter, therefore, review the development of the traditional *p*-median model from Weber's work to the development of discrete space solution formulations. Accordingly, after laying out the nomenclature used in this work, this chapter reviews the genesis of the single facility problem followed by an investigation of the multi-facility or multi-Weber problem, in particular the *p*-median problem.

Definitions

Within the space being considered in the LA problem there exist N demand points located at (x_i, y_i). The demand points are locations at which some population exists which needs the services provided by the p facilities located at (x_j, y_j); p representing the number of facilities to be located. The weight factor W_i represents the quantity of demand population which exists at each demand point.

The distance between points in the LA problem is represented by d_{ij}. Although it is possible to define d_{ij} as a measure of travel cost, travel time, linear distance, or other characteristics, within this thesis I only consider the Cartesian and travel time separation measures. In the Cartesian plane, distance is measured in a straight line from point to point and is represented as:

$$d_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2]^{\frac{1}{2}} \qquad 2.1$$

The travel time values used were obtained from the City of Edmonton's Forecasting and Assessment Branch (FAB).

In continuous space the p facilities may be placed anywhere in the space. In discrete space, however, the locations of the facilities are limited to a set of predetermined potential locations. In the case of networks, Hakimi proved that for distance minimizing LA approaches "to find a *p*-median, one must only examine all subsets of [the vertices of the weighted graph] containing *p* vertices" (Hakimi, 1965, pg. 465). Therefore, because the *p*-median definition of optimality is used here, an optimum solution to the location of the facility can be found at the vertices of the transportation network used in our problem.

The Single Facility Problem

The single facility location problem, as Wesolowsky (1993) notes, is known by many names; including, for example, the generalized Weber problem, general Fermat problem, Steiner problem, median problem, weighted median problem, and Euclidean Minisum Distance Location Problem. The single facility problem on a plane was addressed in the 17th century by Fermat, who attempted to solve for the location of a point whose sum of distances to three other points in a plane was a minimum. Alfred Weber, who pioneered location theory, incorporated the Fermat problem into his treatise on industrial location (1909). Weber used the weight of raw material and products to be transported as factors in determining the optimal location of a manufacturing industry on a plane; see Figure 2.1. His analysis was limited to considering raw materials from two sources and one market for finished goods.

The single facility problem using unweighted demand points, also called the median problem, may be solved by:

$$\textit{Minimizing} \quad Z = \sum_{i=1}^{N} d_{ij} \qquad 2.2$$

where as noted previously,

$$d_{ij} = [\,(x_i - x_j)^2 + (y_i - y_j)^2\,]^{\frac{1}{2}} \qquad 2.3$$

To solve this problem Torricelli used a system of equilateral triangles constructed on the sides of the triangle made by the three points in Fermat's problem. He showed that the circles circumscribing these triangles intersect at the optimal point (Love, Morris, Wesolowsky, 1988).

A number of mechanical and geometrical solutions were devised to solve the single facility problem where the weights at the demand points are unequal; the weighted median problem. In this case it is necessary to:

$$\textit{Minimize} \quad Z = \sum_{i=1}^{N} W_i\, d_{ij} \qquad 2.4$$

The mathematician Varignon developed the Varignon frame (Figure 2.2), a mechanical analog solution method in which the relative locations of the demand points are placed on a surface, and the weights and distances associated with each of these points are simulated by scaled weights and wire lengths respectively. The wires are connected to each other, and at each demand point have a weight W_i corresponding to the quantity of demand at the location attached to them. The final resting point of the intersection of the connected wires is the optimal facility location (Kennelly, 1968). This solution method was the first method which could deal with more than three demand points.

Launhardt (1882) developed a geometrical procedure similar to Torricelli's, but in order to deal with weighted nodes, the triangles on the sides of the triangle formed by the three points of the simple single facility problem are not equilateral. Rather, they are proportional to the weights of the demand points, and the distances between them; see Figure 2.3.

This method was used by Weber, who also developed a method for solving the single facility LA problem, called the method of isodopanes. Isodopanes are lines of equal total transportation cost, the sum of the costs of getting the raw materials from the supply point to the production facility and the product to the market. The resulting cost surface is useful for showing the structure of the single facility problem. Lloyd and Dicken (1977) demonstrated that isodopanes provide a powerful representation of a space cost surface, which is flexible in dealing with various location applications.

These cumbersome methods were used to solve single facility problems until the 1960's in the belief that the problem could not be solved analytically (Ghosh and Rushton, 1987). Kuhn and Kuenne (1962) then rediscovered Weiszfeld's (1937) solution algorithm to the single facility problem; a method which may be used to solve this problem for more than three demand points. This solution algorithm represents the first widely known mathematical solution to the single facility problem; a solution approach made feasible because of the availability of computers. The solution method as presented independently by Kuhn and Kuenne (1962) and Weiszfeld (1937) is to find the location (x_j, y_j) which minimizes:

$$Z = \sum_{i=1}^{N} W_i \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{\frac{1}{2}} \qquad 2.5$$

It is a principle of differential calculus that when the first partial derivatives of a function, with respect to each of the variables, are equated to zero a maximum, minimum, or saddlepoint is obtained. Ostresh (1978a, 1978b) showed that the following derivation converges to a minimum. Setting the partial derivatives:

$$\frac{dZ}{dx_j}, \frac{dZ}{dy_j} \qquad 2.6$$

to zero we obtain the optimizing conditions:

$$x_j = \frac{\sum_{i=1}^{N} \frac{W_i\, x_i}{[\,(x_i - x_j)^2 + (y_i - y_j)^2\,]^{\frac{1}{2}}}}{\sum_{i=1}^{N} \frac{W_i}{[\,(x_i - x_j)^2 + (y_i - y_j)^2\,]^{\frac{1}{2}}}} \qquad 2.7$$

$$y_j = \frac{\sum_{i=1}^{N} \frac{W_i\, y_i}{[\,(x_i - x_j)^2 + (y_i - y_j)^2\,]^{\frac{1}{2}}}}{\sum_{i=1}^{N} \frac{W_i}{[\,(x_i - x_j)^2 + (y_i - y_j)^2\,]^{\frac{1}{2}}}} \qquad 2.8$$

However, at this point x_j and y_j, whose values we are seeking, also exist on the right side of the equations. Kuhn and Kuenne (1962) recommended that the initial values used in this procedure be the weighted mean of the demand points (x,y):

$$\bar{x} = \frac{\sum_{i=1}^{N} W_i\, x_i}{\sum_{i=1}^{N} W_i} \qquad 2.9$$

$$\bar{y} = \frac{\sum_{i=1}^{N} W_i\, y_i}{\sum_{i=1}^{N} W_i} \qquad 2.10$$

The resulting values for x_j or y_j are then iteratively used as starting solutions until they converge within a satisfactory tolerance.

Ostresh (1978a) provided the convergence proof for this algorithm and determined that "the algorithm converges to the unique minimum point, *unless* it happens to "land" on a non-optimal given point" (Ostresh, 1978a, pg. 154). Ostresh (1978b) further developed a method which overcame this problem, and presented a method which avoids landing on a non-optimal demand point during solution.

The Multi-Facility Problem

The location problem can be extended to the location of an interacting system of facilities. This problem is called the multi-facility problem, the multi-Weber problem, Cooper problem or location-allocation problem. Scott (1970; 1971) notes that there are in the basic LA problem two underlying problems that must be solved; the locations of the facilities and the allocation of demand to them. If the locations of the facilities were known, determining the allocations or flows between the demand points and the facilities would be trivial as each demand point would simply be allocated to the nearest facility. Alternatively, if it were known what groups of demand points are allocated to facilities, then the multiple location problem would be separable into p independent location problems; each of which could be solved using the Kuhn and Kuenne (1962) algorithm. In realistic situations, however, neither the locations or allocations are known, and an LA solution algorithm which locates facilities and allocates demand simultaneously is called for.

The LA Problem in Continuous Space

Cooper (1964) presented an important heuristic solution method, called the *alternate location and allocation algorithm*, for solving the multi-facility location problem in continuous space. This algorithm, commonly known as the Cooper algorithm, clearly shows the interdependencies of the facility locations and the demand allocations. His algorithm works by first dividing the problem space into p approximately equal subsets. Using the Kuhn and Kuenne (1962) procedure the optimal single location within each subset is determined; the locate step. Each demand point is then reviewed to determine whether it could be allocated to a facility location closer than the one it was initially allocated to; the allocate step. If there are any changes in the allocations of the demand points, new locations for the facilities are determined, again using the exact location method. The algorithm terminates when no further changes occur. This alternating locate-allocate mechanism of the solution algorithm gave rise to the term location-allocation. This algorithm is heuristic and may fail to find an optimal solution; several different starting solutions should therefore be attempted.

The LA Problem in Discrete Space

Maranzana (1964) developed a heuristic solution algorithm for the multi-facility problem. It is the Cooper algorithm in discrete space; and operates by alternately forcing each node to assign to its nearest facility and then locating the facilities so that they minimize the aggregate distance between them and the demand assigned to them. Rosing *et al.* (1979) comment that the quality of solutions generated by this algorithm decreases rapidly as p increases.

The Tietz and Bart (1968) heuristic is a robust solution method used to solve the multi-facility LA problem in discrete space. This solution method has a good trade off between efficiency and robustness - how quickly a solution is produced, and how close this solution is to optimality, respectively; see Rosing *et al.* (1979) for an excellent review of this algorithm. It solves the multi-facility LA problem by choosing p facility locations from the N possible facility locations in the space. Each vacant location is then substituted as one of the chosen facility locations, and the one resulting in the most improvement in the objective function is kept. This process continues until no improvement is possible. Because the initial set of facilities may not result in the optimal solution, the algorithm is run with various random starting solutions; the more starting solutions the better the chance that the final solution is optimal.

ReVelle and Swain (1970) formulated a globally optimum, discrete space, linear programming solution to the p-median problem. Formally they stated the problem:

$$\textit{Minimize} \quad Z = \sum_{i=1}^{N} \sum_{j=1}^{N} W_i d_{ij} x_{ij} \qquad 2.11$$

Where d_{ij} is the distance between nodes i and j; and x_{ij} represents the proportion of node i's demand served at j. In some cases, because a node may represent all of the demand within an area, it is assigned a distance value to represent the distance travelled between it and the dispersed demand it represents. The Revelle and Swain (1970) formulation requires that the distance between node i and itself be less than the distance between it and any other node:

$$d_{ii} < d_{ij} \quad \forall j \neq i \qquad 2.12$$

This ensures that if a node has a facility its demand will be allocated to itself, i.e.: $x_{ii} >= 1.0$. Because all W_i and d_{ij} are non-negative, the trivial minimum for this solution is to set all $x_{ij} = 0$ for all i,j. To force the p-median solution a number of constraints are placed on equation 2.11.

$$\sum_{j=1}^{N} x_{ij} \geq 1 \qquad \forall i \qquad 2.13$$

This ensures all of the demand at i is served. Optimality forces this constraint to the equality condition.

$$x_{jj} - x_{ij} \geq 0 \qquad \forall i, j \quad (i \neq j) \qquad 2.14$$

This ensures that only locations with a facility have demand allocated to them. Because a location with a facility allocates all of its demand to that facility, the condition $x_{jj} = 1$ signals the existence of a facility. If $x_{jj} = 1$, x_{ij} may be as high as 1.0, but if $x_{jj} = 0$ all x_{ij} must equal 0.

$$\sum_{j=1}^{N} x_{ij} = p \qquad 2.15$$

ensures that exactly p locations are chosen to which demand may be allocated.

The linear programming form of the p-median model only makes sense if all x_{ij} = 1 or 0. Fractional x_{ij} are meaningless because they represent the existence of fractional facilities. ReVelle and Swain (1970) determined that as a linear programming algorithm their model does tend to solve to a 1 or 0 condition for x_{ij}. In the case that x_{ij} does not solve to either 0 or 1, binary linear programming is used which forces x_{ij} equal to 0 or 1, by allowing only 0 or 1 as a solution.

ReVelle and Swain's (1970) formulation provides a simple and understandable method for defining the p-median location-allocation problem. The authors suggest that their formulation of the problem may be used to determine the locations of the "demand points to which the populations of communities are assigned ... [or] with no change in theory be used for the location of the supply points from which goods are to emanate" (ReVelle and Swain, 1970, pg. 31).

Conclusion

Location-allocation problems optimize the location of a facility or facilities and the allocation of demand to these facilities. The origins of the problem lie in Weber's (1909) plant location problem, which sought the optimal location for a processing plant between two sources of materials and a market place (Ghosh and Rushton, 1987). Since the 1960's, iterative solution algorithms have been available to solve both the single and multi-facility LA problems.

Heuristic algorithms are important in LA modelling as globally optimal solutions are expensive to determine; and although their ability to find optimal solutions is not guaranteed, their efficiency in determining solution sets presently makes them a viable solution for large LA problems. Because of the combined robustness and efficiency characteristics displayed by the Tietz and Bart heuristic algorithm, I shall use it as the solution method throughout this thesis.

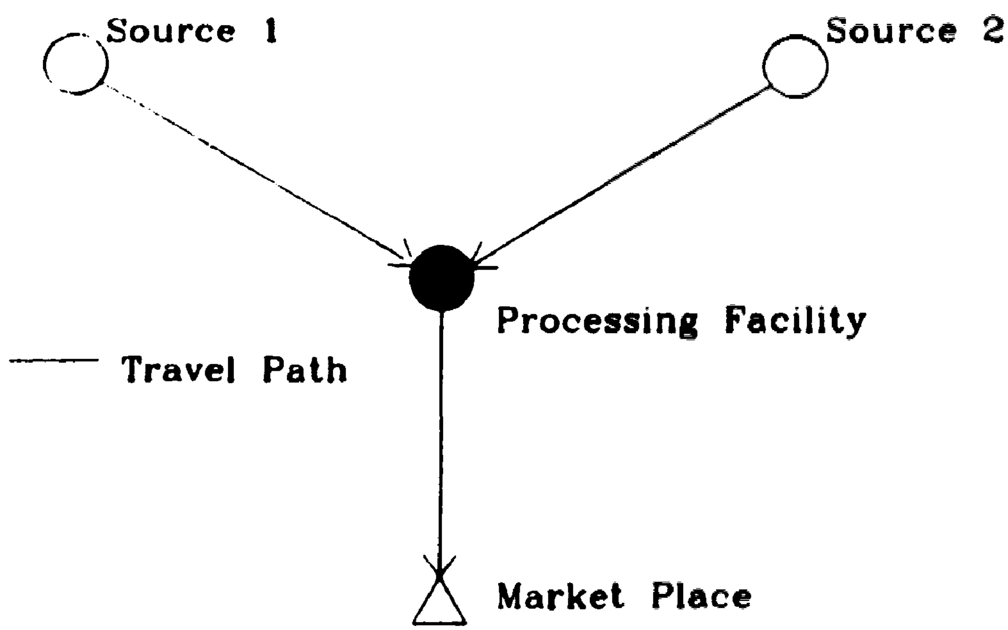


Figure 2.1: Weber's Problem

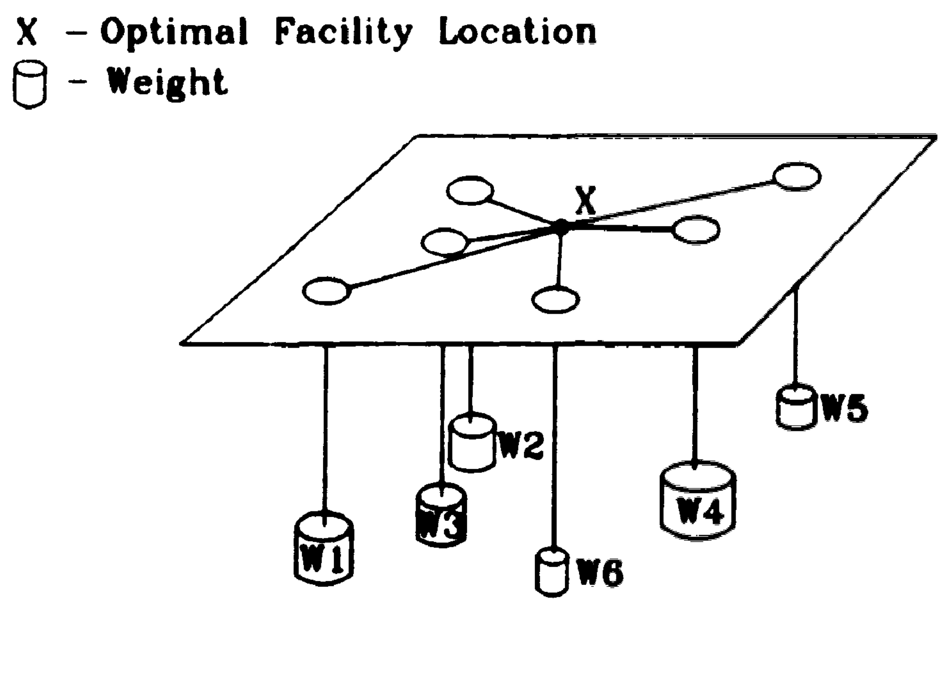


Figure 2.2: The Varignon Frame

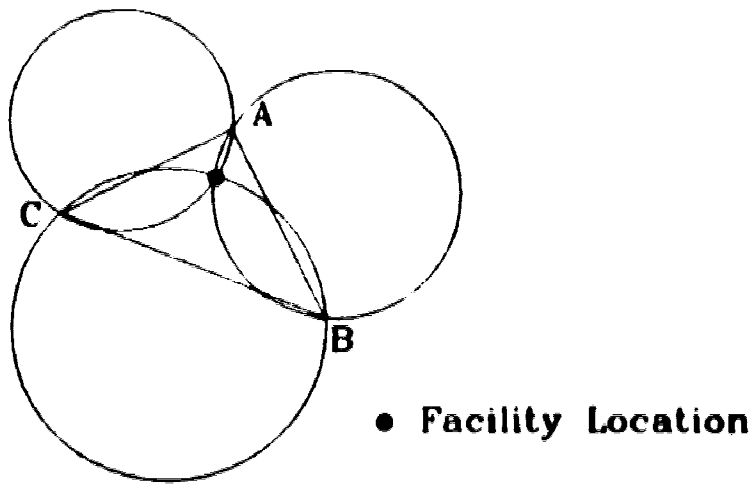


Figure 2.3: Launhardt's Method

Chapter 3

Representing Travel Disutility with Cartesian Distances and Travel Time

In the previous chapter I introduced the *p*-median model, which minimizes aggregate weighted distance. The formal model specifies the use of distance, but since the fundamental purpose of LA is to minimize travel *disutility*, it has been suggested that other measures of spatial separation, such as travel time, be used (Hodgson and Doyle, 1978). Two issues arise in the choice of a distance measure. One is the effort or cost of obtaining data, and the other is the error that may result from using an inappropriate measure. I compare the use of a simple measure, *Cartesian distance*, and a complex one, *travel time*.

Obtaining and preparing travel data may, if not readily available, be the most time consuming activity in LA modelling. The task of calculating travel times in a system of freeways, arterials and collector streets is enormous. Conversely, calculating Cartesian distances is easily and quickly done. The accuracy of distance data, however, affects the accuracy of locational results (Hodgson, 1991). The absolute differences of these separation measures are not of concern; distance and travel time are measured in different units. The relative differences in these values, however, is critical. Cartesian distances are not strictly proportional to travel times in a transportation network; although Kolesar (1979) showed that they are strongly proportional. Travel times take into account the effects of different types of roads, (i.e.: arterials, collectors, freeways), which carry different traffic volumes at different speeds. Cartesian distance values simply assume that traffic is able to traverse the distance between points under the same conditions regardless of the location of these points. Such relative differences could be much more variable and perhaps more significant in heavily urbanized areas. Consequently, we would expect that the relative differences in separation measures affect the facility locations chosen.

In this chapter I test the potential usefulness of Cartesian distances in the context of the *p*-median model. Results obtained from using a.m. peak hour, 07:00 - 08:14 a.m., travel time data for Edmonton, Alberta, are used as a basis for comparison under the assumption that these values more accurately represent the travel disutility experienced by the demand population. This is an attempt to determine whether the data collection process may be simplified by using Cartesian distances, rather than arduously obtained travel times, in LA applications without sacrificing locational accuracy excessively. I use the term *distance* to represent Cartesian distances and the term *time* to represent travel times. The experiment examines the degree to which the relative differences between separation measures affect LA; i.e.: how much poorer off people are when the less sophisticated separation measure is used. This analysis could result in tangible advantages and benefits in the planning processes undertaken in the real world by reducing the time spent on the collection of distance data.

The Experiment

I use the *p*-median model for this experiment because it is the traditional LA problem, and much work has been based on it. Other important reasons include the ability to easily present this solution graphically, and the simplicity of the model which enables the effective isolation of the effects of error in a single variable (Hodgson and Storrier, 1993).

For the purposes of this experiment I assume that the time measurements provided by the City of Edmonton Forecasting and Assessment Branch (FAB) are correct (,i.e.: represent the actual *travel disutility* incurred in the space;) and that the Cartesian distance measurements are incorrect. The experiment is straightforward. I let:

Λ_T = The optimal set of facilities found using the time data
Λ_D = the optimal set of facilities found using the distance data
Z_{TT} = Objective function evaluated using Λ_T and time data
Z_{DD} = Objective function evaluated using Λ_D and distance data
Z_{DT} = Objective function evaluated using Λ_D and time data

The excess travel time incurred through using Cartesian distance is:

$$Z_{DT} - Z_{TT} \qquad 3.1$$

By measuring the aggregate weighted distance using the correct separation measure for both solutions, the difference in objective functions (equation 3.1) arises strictly from differences in location. This is the excess aggregate weighted travel time arising from the incorrect locations and is hence termed location error. This argument is provided in greater detail by Hodgson and Neuman (1993) who use Casillas' term optimality error and suggest the use of the term locational error for its greater generality.

The Study Area

Edmonton, a Canadian prairie city, had in 1990 a population of 605,538 and covered an approximate area of 700 km^2 (Status Report, 1991). The city is primarily planned around the traditional grid system. A large river, the North Saskatchewan, flows from the west to the east of the city, and is spanned by 9 bridges, two of which only support one-way traffic. There are a total of 2,927 roadway kilometres: freeways/ expressways 103km, arterials 729km, collectors 451km, and local roads 1644km (Figure 3.1). A large number of streets in the downtown core are designated as one way for normal vehicle traffic. There is, furthermore, a public transit system, including 12.3 km of Light Rail Transit (LRT) line. A truck route system is in effect, limiting truck traffic to designated routes; and a 143km Bicycle Route system is also available, allowing bicycle traffic to traverse the city reasonably efficiently, particularly in the river valley areas (Status Report, 1991).

These transportation systems for trucks and bicycles impact capacity and congestion and, therefore, the travel time experienced by drivers. The freeway and minutes) for all links except feeder links:

$$Time = (D_L/S)x(1+(0.15*(V/0.75*L*1750)^4))*60 \qquad 3.2$$

Where D_L - Length of the Link
L - Number of lanes in one direction
V - Volume assigned to the link
1750 - Number of vehicles handled by each lane at capacity
S - The free flow speed of the link. (Based on the posted speed limit adjusted to reflect actual average speeds attained in light traffic conditions).

Feeder links are treated differently, because they are artificial links connecting the zone centroid to the transportation network. They represent a capacity of 10,000 vehicles per hour, and a speed of 37.5 km/hr (Working paper, 1990). Although capacity, and other travel limitations such as one-way streets, bridge crossings, and other characteristics related to the network are considered by EMME\2, travel penalties such as turning are not. Thus even this separation measure, which we consider correct, is subject to some error.

To deal with trip assignment (determining along which links the demand from various zones travels) the EMME/2 model uses a built-in iterative equilibrium assignment method. This algorithm assigns trips to different routes after each assignment using the times obtained in the previous iteration until an optimum assignment is determined (Working Paper, 1990). As noted in the City's Working Paper (1990) on the recalibration of the regional travel model "The final link flows are a linear combination of the individual flows from each iteration, weighted to produce an 'optimum' result that minimises (sic) overall time in the network" (pg. 34). In the case of intrazonal times, "where the assignment model gave zero time, a value of one half of the shortest time from each zone to any of the other zones was used... [and] no additional terminal times were added to the time estimated in the assignment program" (Working paper, 1990, pg. 19). In this experiment, however, intrazonal times were set to zero, to make the comparison with the Cartesian matrix fair; the Cartesian distance matrix assumes a distance of 0.0 between a centroid and the associated demand in the zone.

A major recalibration of the City's transportation model is undertaken every 5 years, the last being 1989, to update information such as direction flows (to within 10% of observed flows), travel survey results; and completed road construction projects. In 1989, a.m. peak hour flows used in the model were within 5% of observed flows. Each year information available on a yearly basis, such as zonal populations, is updated.

A 1990 population data file and a 8.5" x 11" map of the transportation zones were also obtained from FAB. The population file is based on the 1990 municipal census. I digitized the zonal map and the zonal centroids, for the purposes of cartographic displays.

Preparing The Data

As is often the case with data obtained from various sources, the data had to be prepared to ensure compatibility with the software used. FAB provided the time matrix in ASCII format, in minutes, and the file had to be edited to remove extraneous information such as colons, and zone identifiers. As the data came with a minimum of two decimal places, it could be converted from minutes to seconds. This was necessary to avoid a loss of precision, since the LA program used converts the values to integers by truncation.

City population data, expressing the existing demand, was obtained in Lotus[RM] 123[RM] format and converted to ASCII. The file was organized such that each zone had its own population value. FAB determined these population values by combining the populations from each enumeration area within the zone, and then dividing the populations of those enumeration areas which were covered by more than one zone. The branch carefully divided populations according to address, and added these values to the population value of the zone including those addresses (Von Leiningen, 1992).

To be compatible with the LA program, the digitized values for the Cartesian nodes and graphics database also had to be manipulated. For the graphics database I digitized the whole zone map, and organized the input file such that each set of lines in the file identified the X and Y coordinates of a line on the map. To graph the facility point location in each transportation zone I used the node coordinate file. This file uses the same X,Y coordinate system as the zone map. The coordinate system used is based on the 3TM projection, and reference coordinates could be determined from the digital map files provided by FAB. Using the same dimensions ensures that the map is to scale in both the X and Y directions, and gives an accurate graphic representation of Edmonton.

Results

For this experiment I used the Teitz and Bart (1968) heuristic with 30 random starts. Although there are optimal algorithms which can solve a problem of this size, they were unavailable to me. The software used in this experiment takes into account the bi-directionality of the data, provides a user-friendly interface and graphics, and, as explained in Chapter 2, I expect this algorithm to be robust. These important criteria are among those outlined by Zanakis and Evans (1981) as making the use of a heuristic desirable and advantageous. Each data matrix was processed separately for 1 - 20, 25, 30, 35, 40, 45, and 50 facilities. The resultant aggregate weighted time, the location error, and the percent location error values are recorded in Table 3.1.

Cost-effectiveness graphs illustrating the variation in objective function values as the number of facilities increases are a common way to evaluate the results of LA models. Figure 3.2 compares the Z_{TT} and Z_{DT} values within such a graph; clearly showing the differences between the performance of the *p*-median using the time and distance separation matrices. The Z_{DT} values appear good compared to the Z_{TT} values. Both curves show a decrease in the average weighted time values, and in the benefit derived from adding another facility, as the number of facilities increases. There is no clear 'break' point visible indicating the point at which the benefit of investing in an additional facility would drop significantly; rather the curves are smooth and drop gradually throughout.

In Figure 3.3 I plotted the Z_{TT} and Z_{DD} values against the number of facilities, scaling them so that the largest value of each solution is set to one-hundred percent. This graph exhibits the behaviour of the distance and time solutions without the distortions introduced by evaluating the distance solutions using the time matrix. Both curves are smooth, gradually decreasing, and concave. It is clear that although the Z_{DD} values are higher for $p < =5$ than the Z_{TT} values, they are consistently lower for $p > 5$. A possible reason for this result may be related to the characteristics of the separation measures used. For example, using the distance data implies that the demand population can travel uninhibited in every direction along a straight line; whereas using the time data means that the demand population is constricted to the links in the network and is affected by the volume/delay characteristics of those links. As the number of facilities increases the demand travelling according to the time scenario may find itself increasingly travelling over more and more links in the network which are slower. i.e.: If only a few facilities are located based on the *p*-median objective the demand can take advantage of all the fast network links in the city. However, if many facilities are located, they will more likely be placed in locations which can only be reached using slower links in the network. Travel according to the distance metric would not be affected in this manner, as travel could still proceed in the same way regardless of the locations of the facilities. As a result the aggregate weighted time does not decrease as quickly, or in proportion to, the aggregate weighted distance.

I also plotted the percent location error values against p to see if this difference is related to the number of facilities. If a pattern exists it could provide valuable information about the 'closeness' in results we could expect between the two separation measures as the number of facilities increases. The percent location error was calculated as:

$$\left(\frac{Z_{DT}}{Z_{TT}} - 1\right) * 100 \qquad 3.3$$

This provides a value which shows what percentage more travel time the demand incurs if facilities are located using the Cartesian separation matrix than if the facilities were located using the travel time matrix. For example, at $p = 10$, the location error is 4.8%. The results are graphed in Figure 3.4, which shows that there is no apparent trend in the location error as the number of facilities increases. The apparently lower error when 25 facilities or more are located may not be born out for the missing values of p.

Dividing the location error by the total population gives the extra time the average individual must travel to access a facility located using the Cartesian rather than the travel time separation measure. The results of this calculation (recorded in Table 3.1) show that the individual is not subjected to excessively greater travel times; especially when $p > 3$. The individual typically incurs no more than 20 extra seconds of travel time in order to access a facility. Planners may consider this finding as strengthening the argument for using the Cartesian rather than the travel time separation measure when locating facilities; particularly when a data source such as FAB is unavailable.

Directly comparing the Z_{TT} and Z_{DT} values obtained, however, shows that it is in some cases possible to operate fewer facilities with equally good travel times when using the travel time rather than the Cartesian distance matrix. For example, the Z_{TT} value determined at $p = 12$, is lower than the Z_{DT} value determined at $p = 14$; using the Cartesian distance matrix results in having to add two facilities to the network to obtain a similar aggregate weighted travel time value. This finding suggests that it may be worthwhile to incur the expense of collecting travel time data, especially if its use results in having to invest in fewer facilities.

To more critically judge how good the solutions obtained are requires further investigation. This investigation takes the form of a simulation similar to the one employed by Ebert and Neuman (1991) in their efforts to explain the success of a heuristic in finding a good solution. Using Hodgson's LA program 10 000 random solutions were enumerated for each value of p and, using the following formula, plotted in a frequency histogram using SPSS/PC+[RM] :

$$\frac{Z_{RT}}{Z_{R_{min}}} \times 100 \qquad \forall R \qquad 3.4$$

Where Z_{RT} is the objective function value determined for the random solution (randomly generated set of facilities) using travel time; R is the set of all random solutions and R_{min} is the smallest value so determined. This index is the excess travel for a particular random solution vis a vis using the best random solution obtained. For example, for $p = 2$, there are 80 random solutions with excess travel between 10 and 15 percent (See Figure 3.5). These histograms allow us to assess the Z_{TT} and Z_{DT} values to see how good a solution was obtained. See Table 3.2 and Figure 3.6 for the R_{min} value found for each value of p, and the percent aggregate weighted time determined compared to this value using A_T and A_D.

The Z_{TT} solution values from the original experiment are lower than the minimum random solution, except for $p = 1, 2$: locating the facilities using the heuristic is better than the best random solution enumerated. The fact that the Z_{TT} solutions obtained using the heuristic are superior to the solutions obtained using the random methodology substantiates the claim that the heuristic is robust.

The Z_{DT} values obtained in the experiment were, except for $p = 1$-3, also better than the minimum random solution; representing a savings of as much as 24.8% at $p = 50$. The Z_{DT} values for $p = 1$-3 are at worst better than 80% of the random values (see Figure 3.5). These findings may provide some comfort to those users forced to use the Cartesian separation measure elsewhere; where resources have not been brought to bear on extensive network travel time data collection.

To further analyze location error on the network, I mapped Λ_T and Λ_D for p = 1 - 50, and their associated allocation patterns. A comparison of Figure 3.7, representing Λ_T, and Figure 3.8, representing Λ_D, for p = 15, shows that the river and locations of bridges have a stronger impact on the allocation of demand when using the time data than when using the distance data. The time solutions tend to avoid river crossings, particularly in areas not served by bridges; see for example the allocation pattern to facility 'a' Figure 3.7. The distance solutions, however, do result in the allocation of demand to facilities across the river, regardless of the convenient accessibility or existence of a bridge in the area; see for example the allocation pattern to facility 'a' Figure 3.8.

The inefficiency of the distance solution, and the associated misallocation of demand to facilities due to the river and location of bridges, is clearly observable in Figures 3.7 to 3.9. In Figure 3.8, facility 'a' thinks that it covers the demand in the dot shaded area on the west side of the river. However, as may be seen on Figure 3.9 facility 'a' does not actually cover this demand, resulting in very long allocations of the demand to facility 'b.' In the time solution (Figure 3.7) there is a realization that facility 'a' cannot cover the demand in the southwest area of the river better, and facility 'b' is located closer to this demand to improve the travel times in this area.

Another interesting example of the locational inefficiency resulting from the use of the distance matrix vs. the time matrix is shown in Figure's 3.10 and 3.11. From the diagrams it is clear that if the city is to be served by a single facility that the use of an accurate representation of the travel separation is needed, because the difference in locations and travel time (see also Figure 3.2) is so great. Using the distance matrix results in the facility being placed in the downtown area whereas the use of the time matrix results in the facility being placed away from the downtown core towards the east. This obvious difference in facility location is most likely due to congestion in the city core during the a.m. peak hour, which negatively affects the aggregate weighted travel time incurred by the demand population travelling there.

Based on the above analysis the concept of *equity* becomes an important factor in the determination of what constitutes a better facility location. Penalties created by barriers, such as the river, and other factors such as capacity and delay have no effect on the location of facilities using the distance separation matrix. However, because the demand population is affected by these impediments the equity of locating facilities using the distance separation matrix may be compromised. The degree to which the effects of location error impact on different locations is an extension of this research which might be pursued.

Conclusions

I have used the *p*-median model to test how well facilities would be located in the City of Edmonton using Cartesian distance data. The results obtained show that there are trade offs which must be considered in the ultimate decision as to what separation measure should be used. In particular the trade off between the expense of collecting more representative data and the excess travel incurred in the system.

Although this excess travel may be acceptable to the individual, it may also result in the need to add more facilities to the system. Therefore, to avoid incurring the expense of adding facilities, it may be advantageous to incur the expense of collecting better data. Comparison with the randomly generated solutions indicates the robustness of the Tietz and Bart heuristic in determining facility locations. The difference between the results obtained using the two separation measures is small in comparison with the variation among randomly generated solutions. We might conclude from this that locating facilities using the Cartesian separation measure is preferable to using a random solution generating method.

Table 3.1: Results of the Experiment

p	Aggregate Weighted Time (Seconds) Z_{TT}	Aggregate Weighted Time (Seconds) Z_{DT}	Location Error (Seconds) $Z_{DT}-Z_{TT}$	Percent Location Error $(\frac{Z_{DT}}{Z_{TT}}-1)*100$	Z_{TT} as Percent of Z_{TT} at $p = 1$	Z_{DD} as Percent of Z_{DD} at $p = 1$	Excess Time per individual (Seconds per individual) $\frac{Z_{DT} - Z_{TT}}{Population}$
1	631399700	681294200	49894500	7.9	100	100	82
2	405361900	528147500	122785600	30.3	64.2	72.92	203
3	333118500	351336300	18217800	5.5	52.76	57.56	30
4	293533400	305349700	11816300	3.9	46.49	48.7	19
5	265732800	275196800	9464000	3.4	42.09	43.04	15
6	243297300	256095100	12797800	5.2	38.53	38.12	20
7	222997700	229433000	6435300	3.0	35.32	34.97	10
8	210570400	222018000	11447600	5.5	33.35	32.55	18
9	198581900	211677700	13095800	6.7	31.45	30.37	21
10	190520200	199909300	9389100	4.8	30.17	28.54	15
11	183530000	196402400	12872400	6.9	29.08	26.92	21

Table 3.2: Results of Random Solution Simulation

p	Random Solution Minimum Value (Seconds) R_{min}	Percent Aggregate Weighted Time $\frac{Z_{TT}}{R_{min}}*100$	Percent Aggregate Weighted Time $\frac{Z_{DT}}{R_{min}}*100$
1	631399684	100	107.9
2	405361818	100	130.3
3	336692718	98.9	104.3
4	307035217	95.6	99.5
5	290537355	91.5	94.7
6	278456459	87.4	92.0
7	258909668	86.1	88.6
8	251128329	83.8	88.4
9	238512216	83.2	88.7
10	236822464	80.4	84.4
11	227277351	80.7	86.4
12	225161117	78.9	83.4
13	211484785	81.4	86.2
14	203450234	81.9	87.7
15	203631433	79.2	85.0
16	194377423	80.41	84.0
17	195450879	77.5	84.1
18	190472925	77.1	83.5
19	189310491	75.4	81.9
20	186141802	75.2	80.3

25	166083353	77.0	79.8
30	158521345	75.3	76.9
35	149853918	74.3	76.0
40	140961693	74.7	76.4
45	132706244	75.4	76.4
50	129123821	73.7	75.2

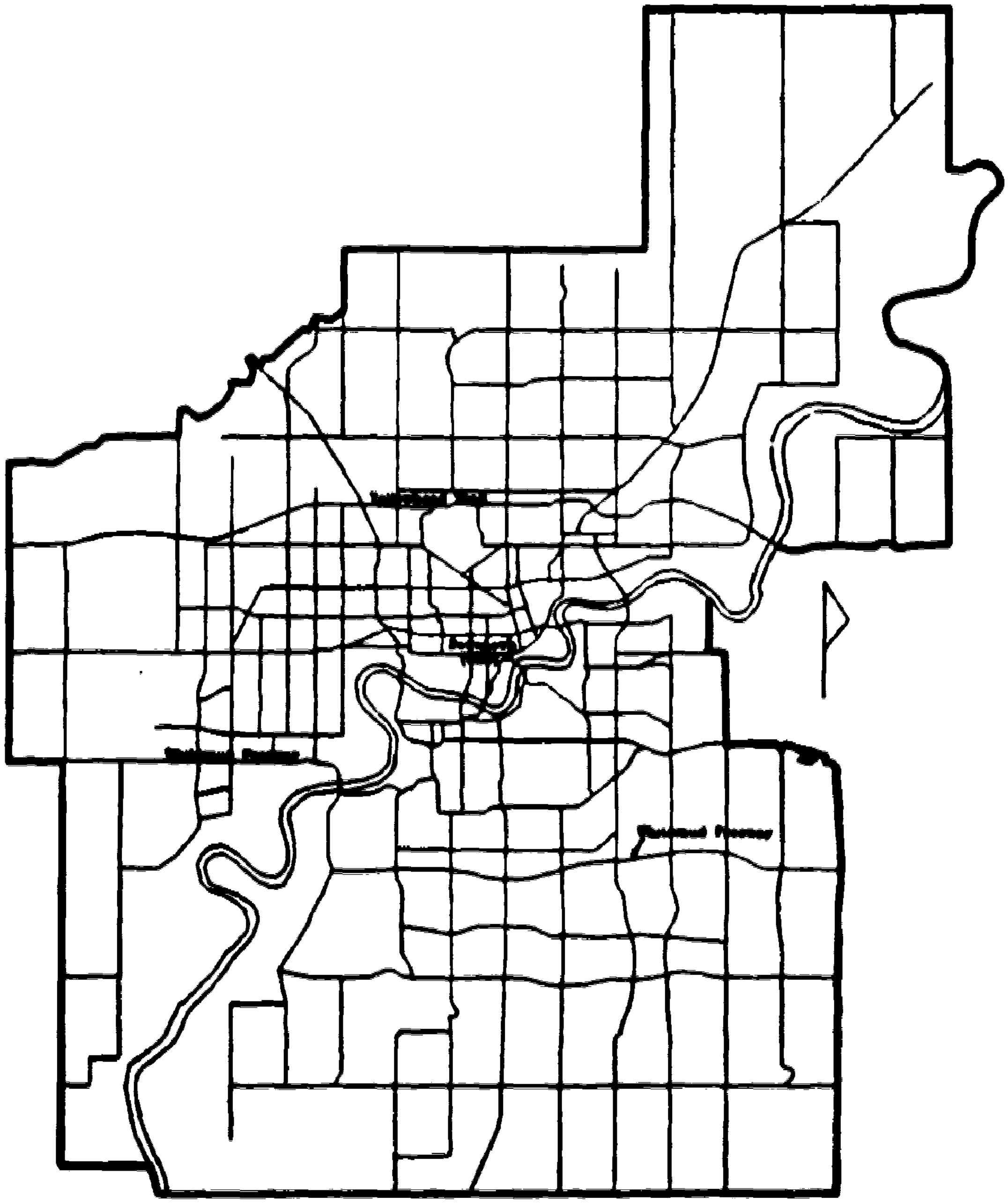
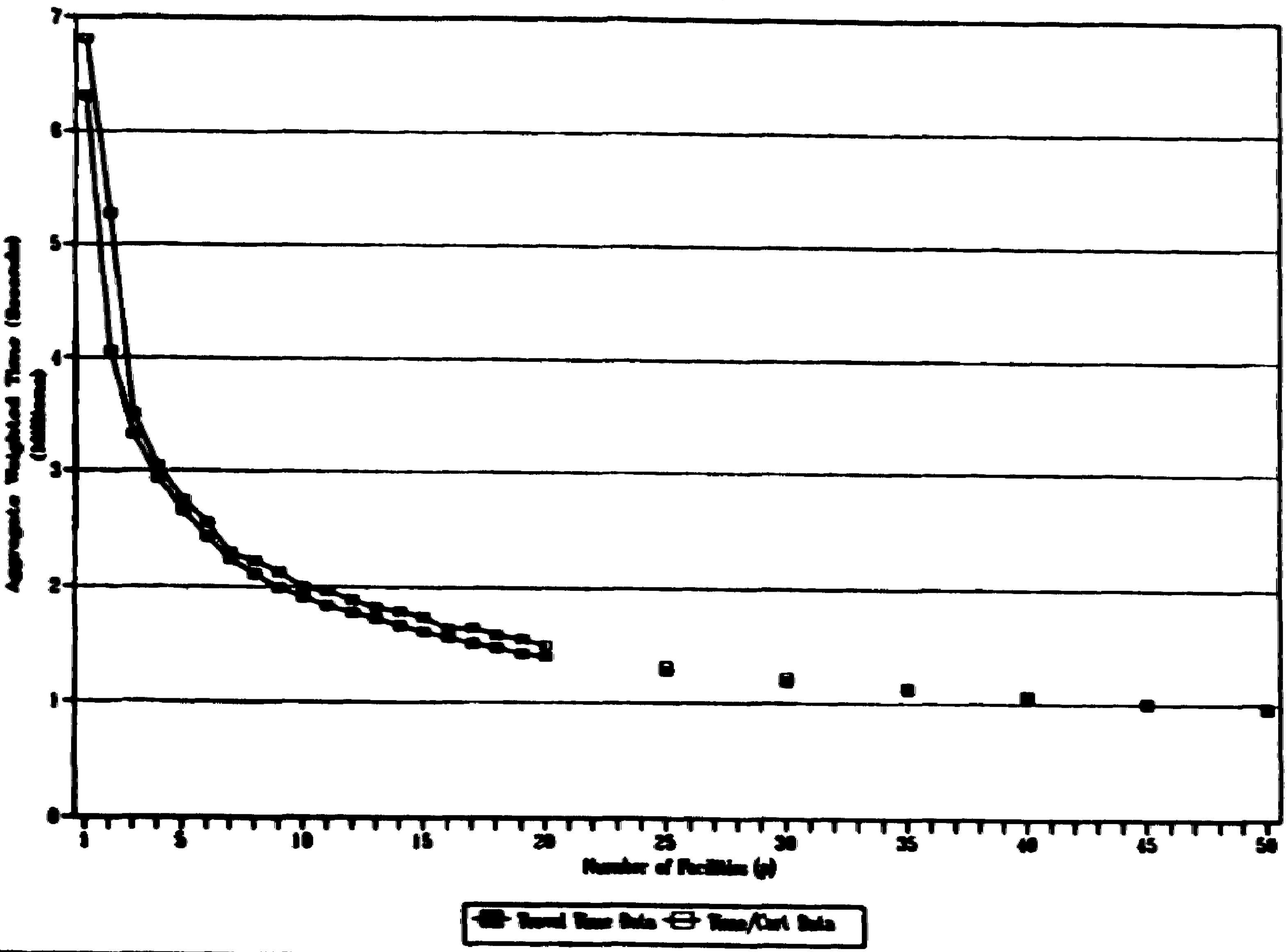


Figure 3.1: The City of Edmonton Transportation Network

Figure 3.2: Comparison of Travel Data:
Travel Time and Time/Cart Data



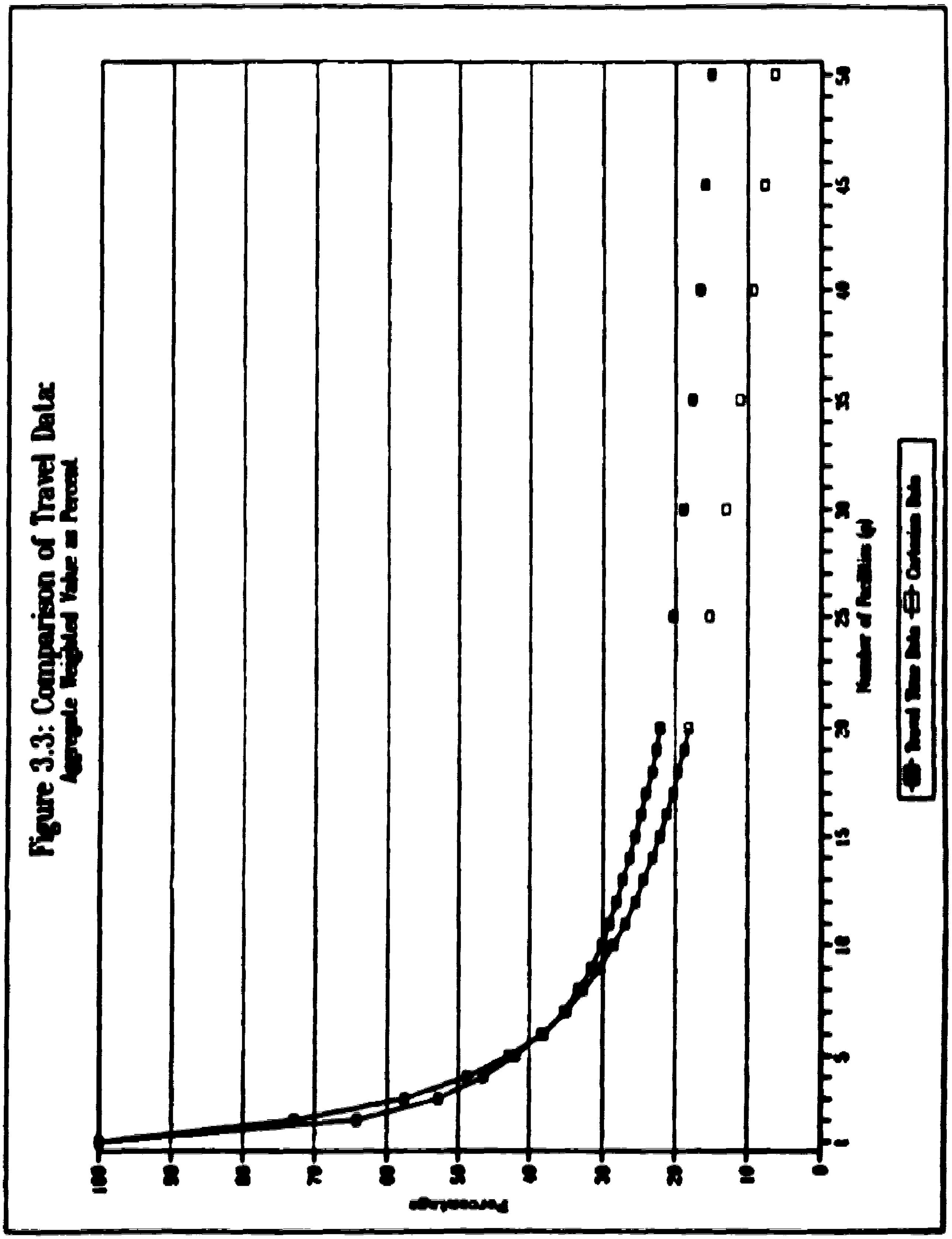


Figure 3.3: Comparison of Travel Data:
Aggregate Weighted Value as Percent

Figure 3.4: Percent Location Error:
Excess Travel Time Incurred

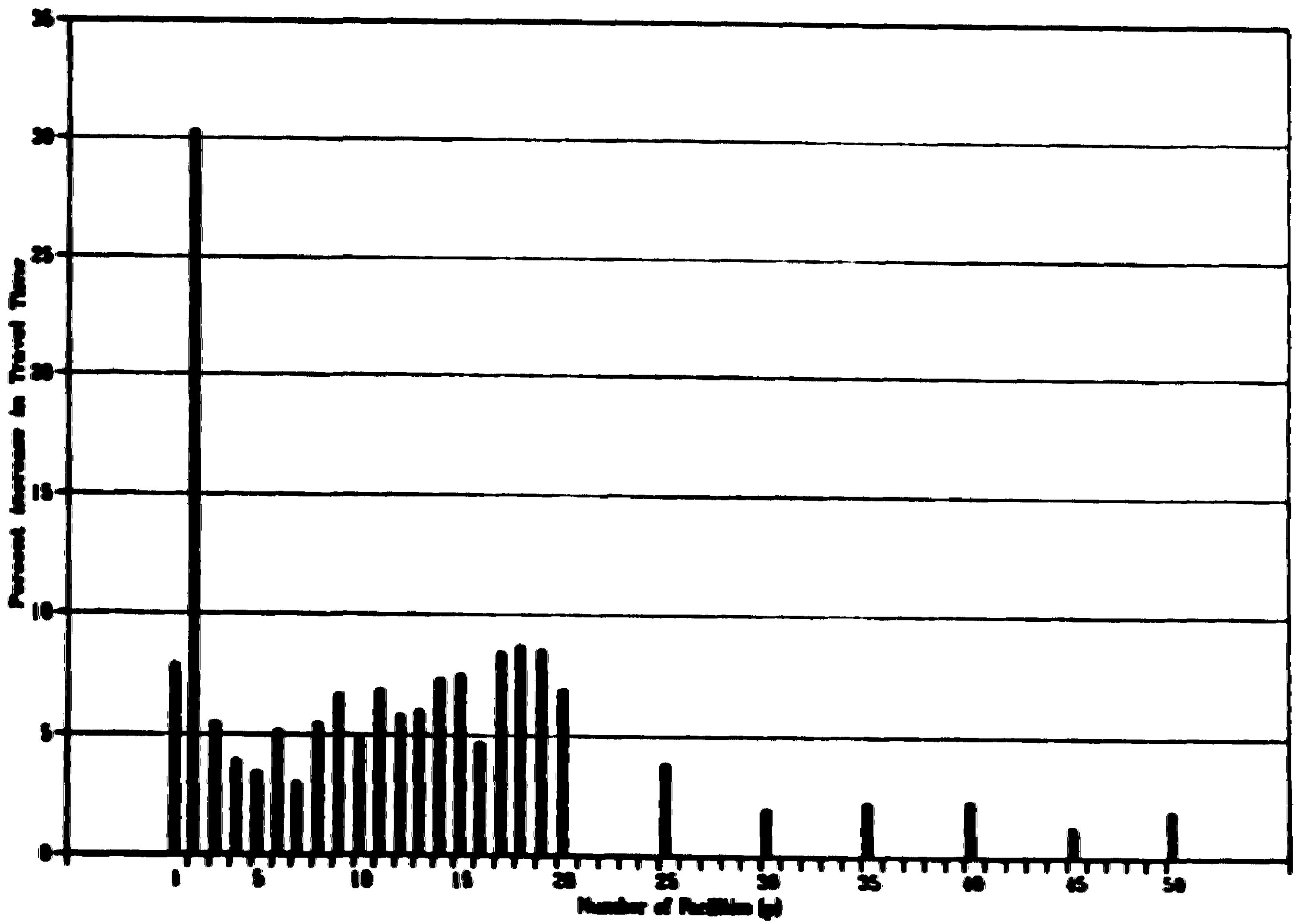
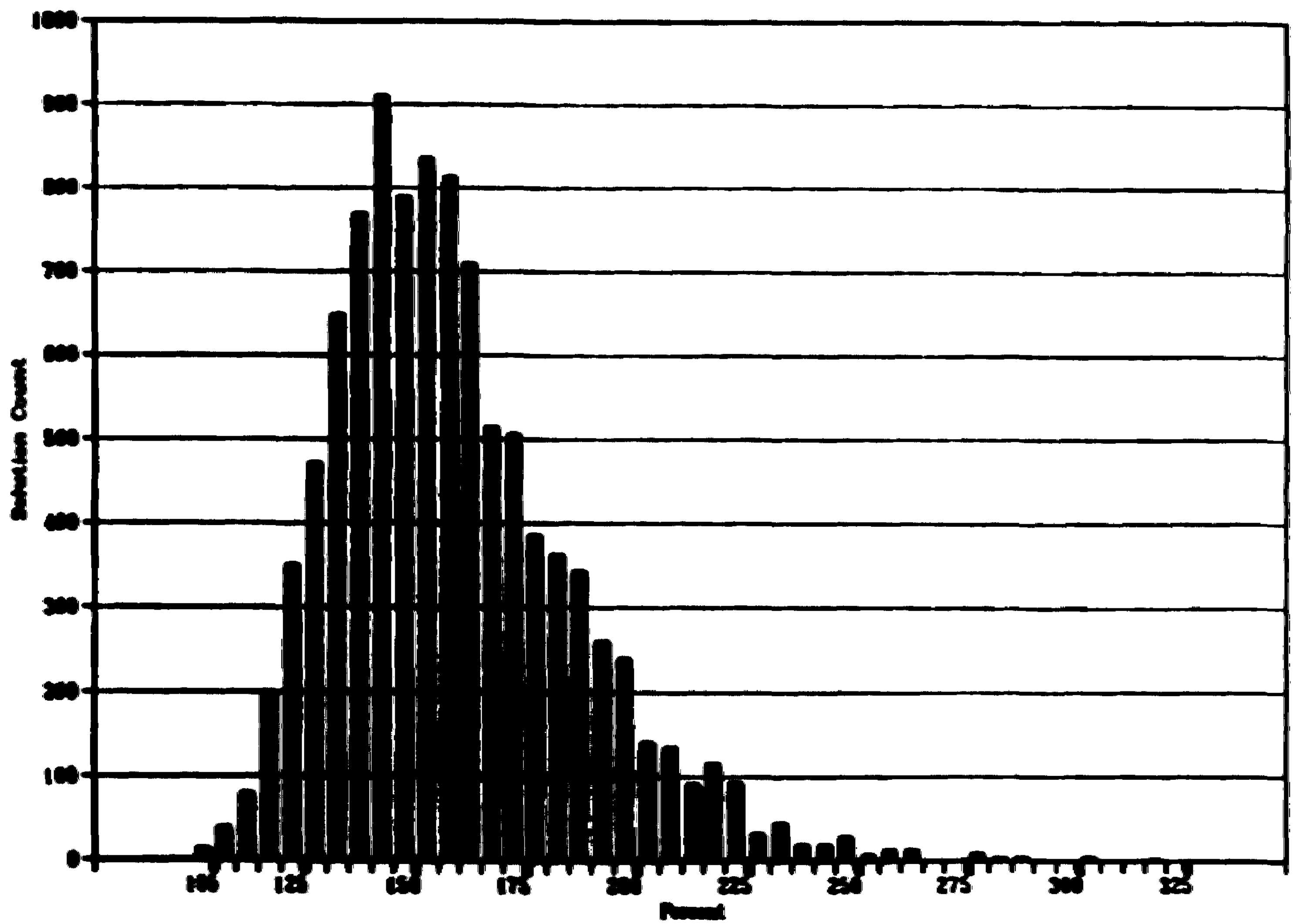


Figure 3.5: Random Solutions, p = 2

As Percent of Travel Time Solution



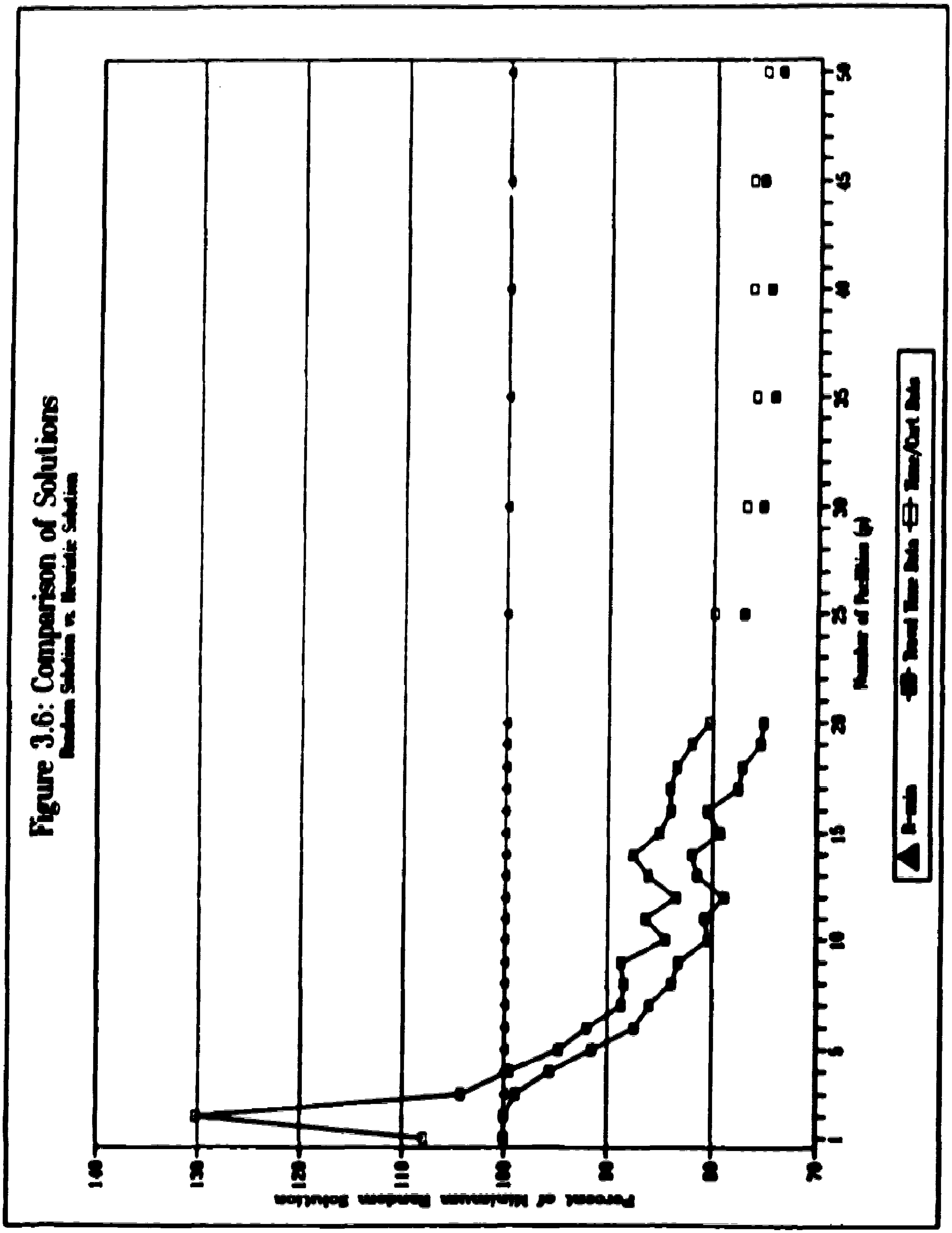


Figure 3.6: Comparison of Solutions
Random Solution vs. Heuristic Solution
Percent of Minimum Random Solution
140
130
120
110
100
90
80
70
Number of Facilities (n)
5
10
15
20
25
30
35
40
45
50
Travel Time Rule
Time/Cost Rule

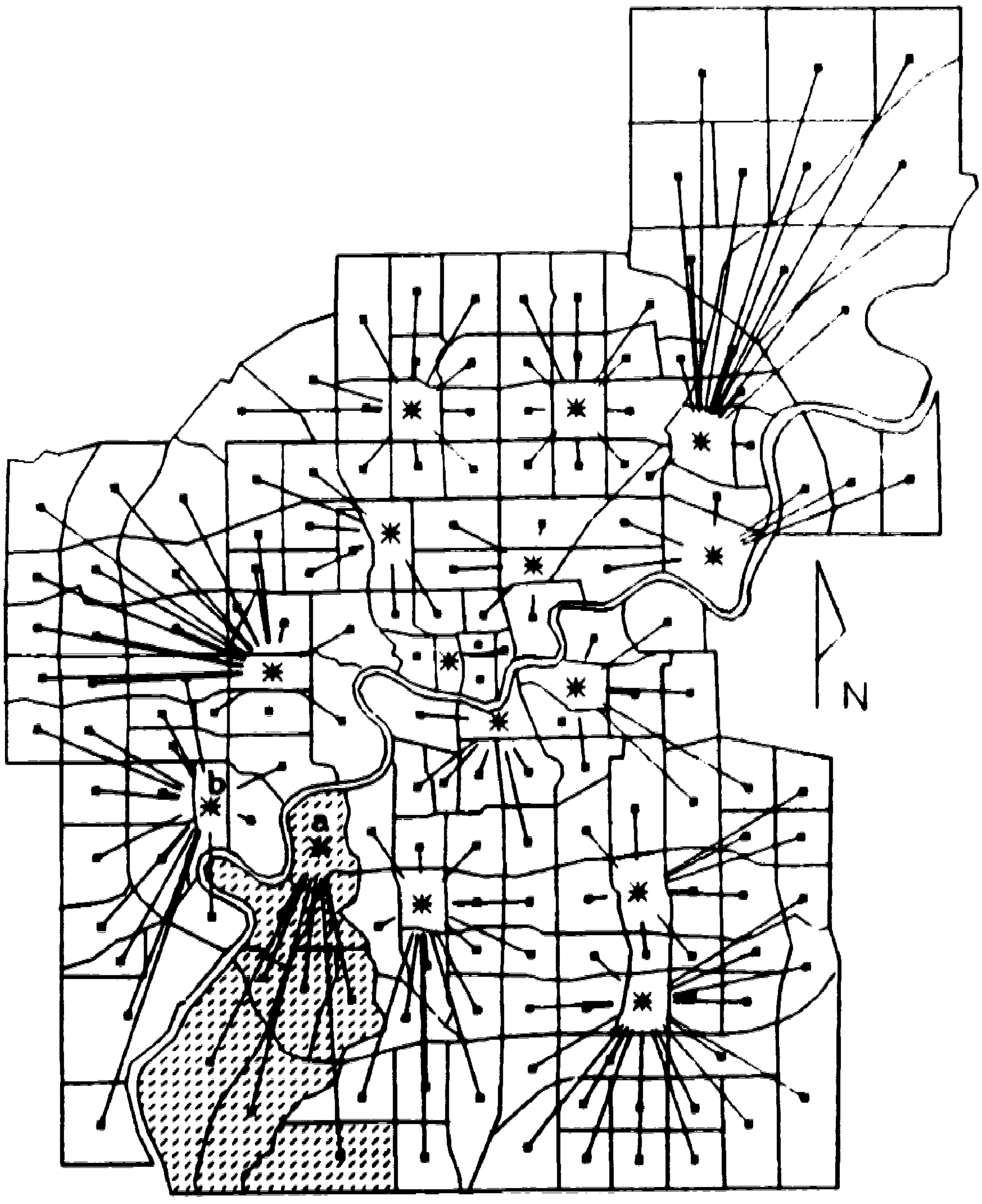


Figure 3.7: Time Solution, p = 15

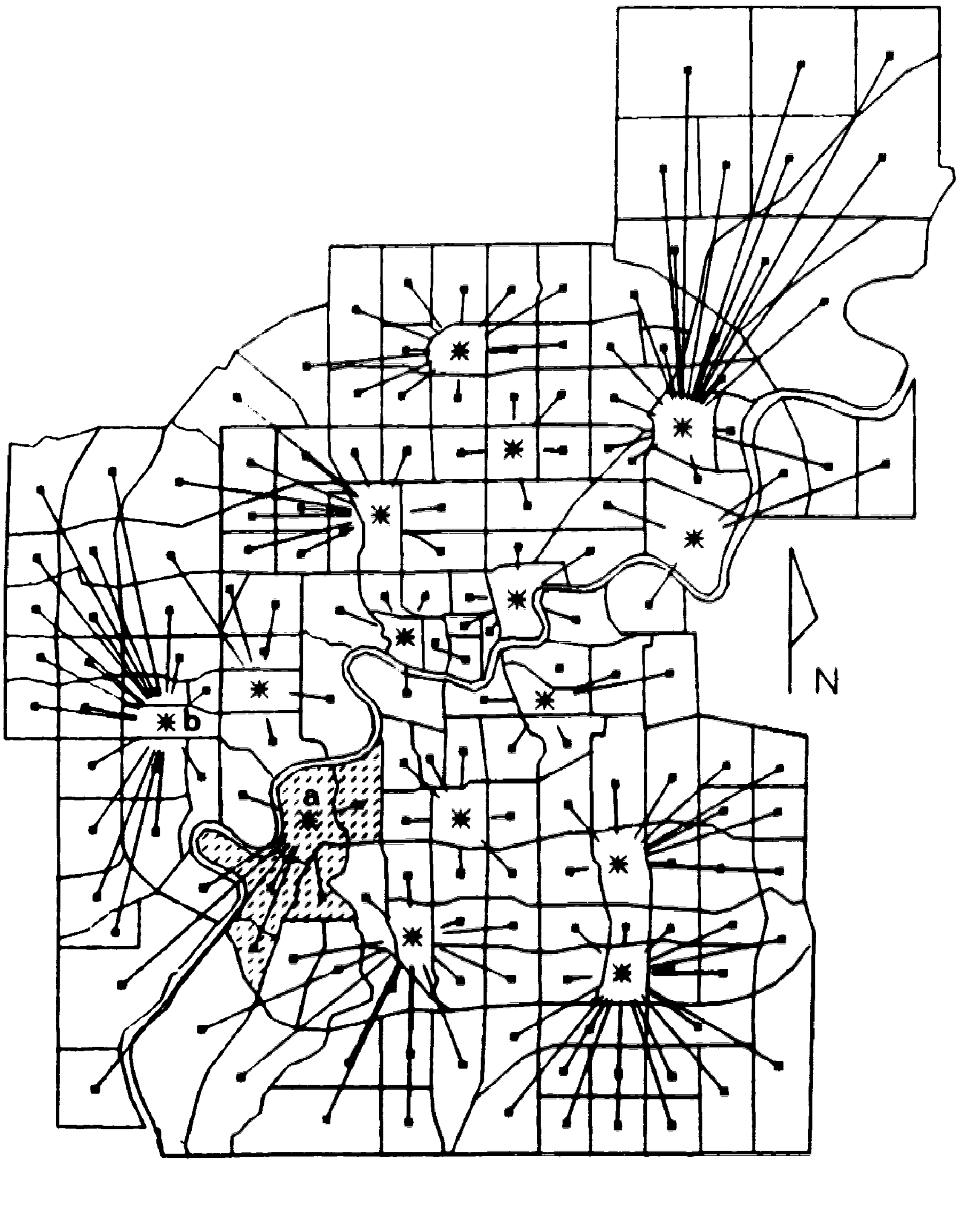


Figure 3.8: Distance Solution, p = 15

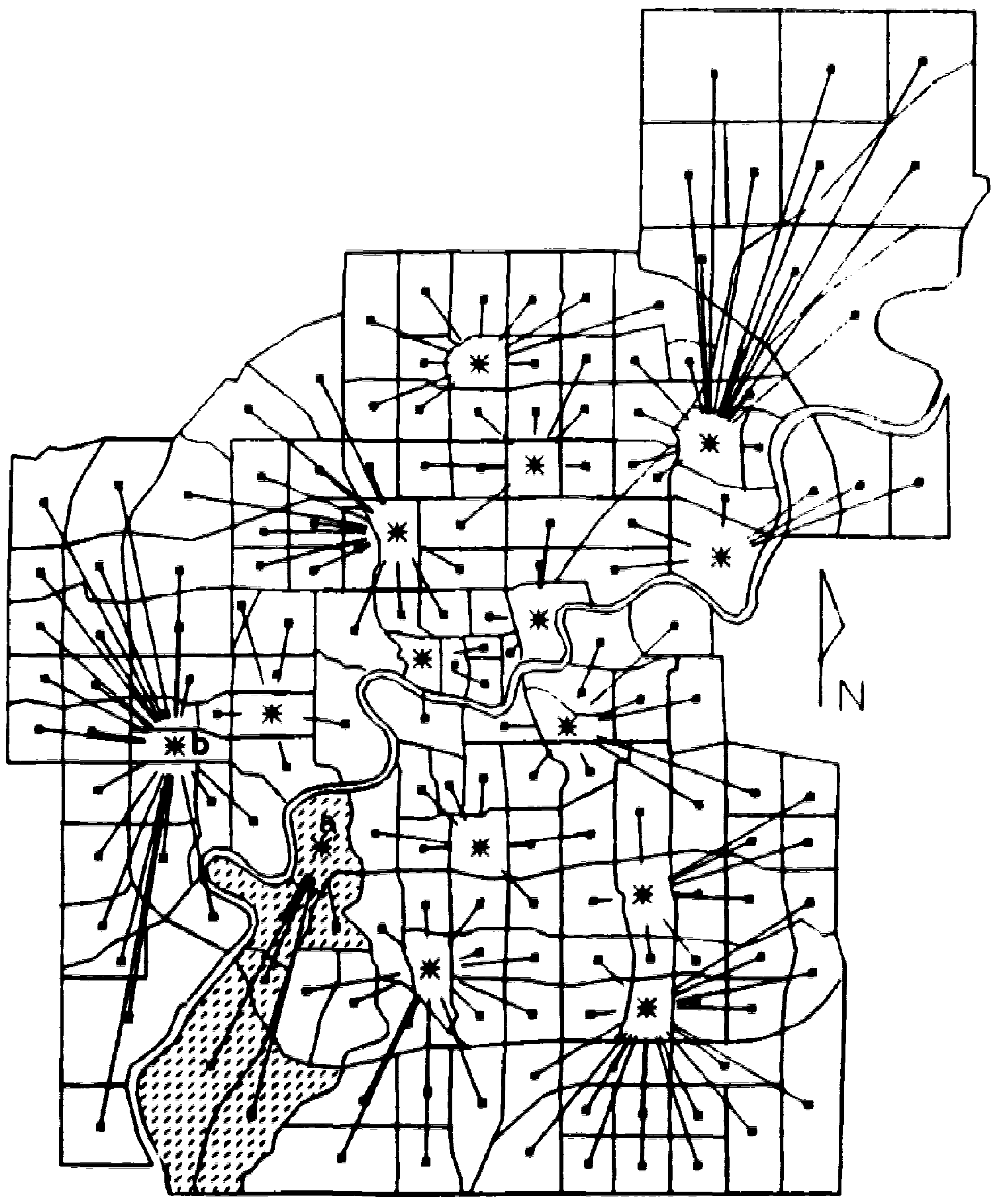


Figure 3.9: Time Matrix, Cartesian Facility Locations
p = 15

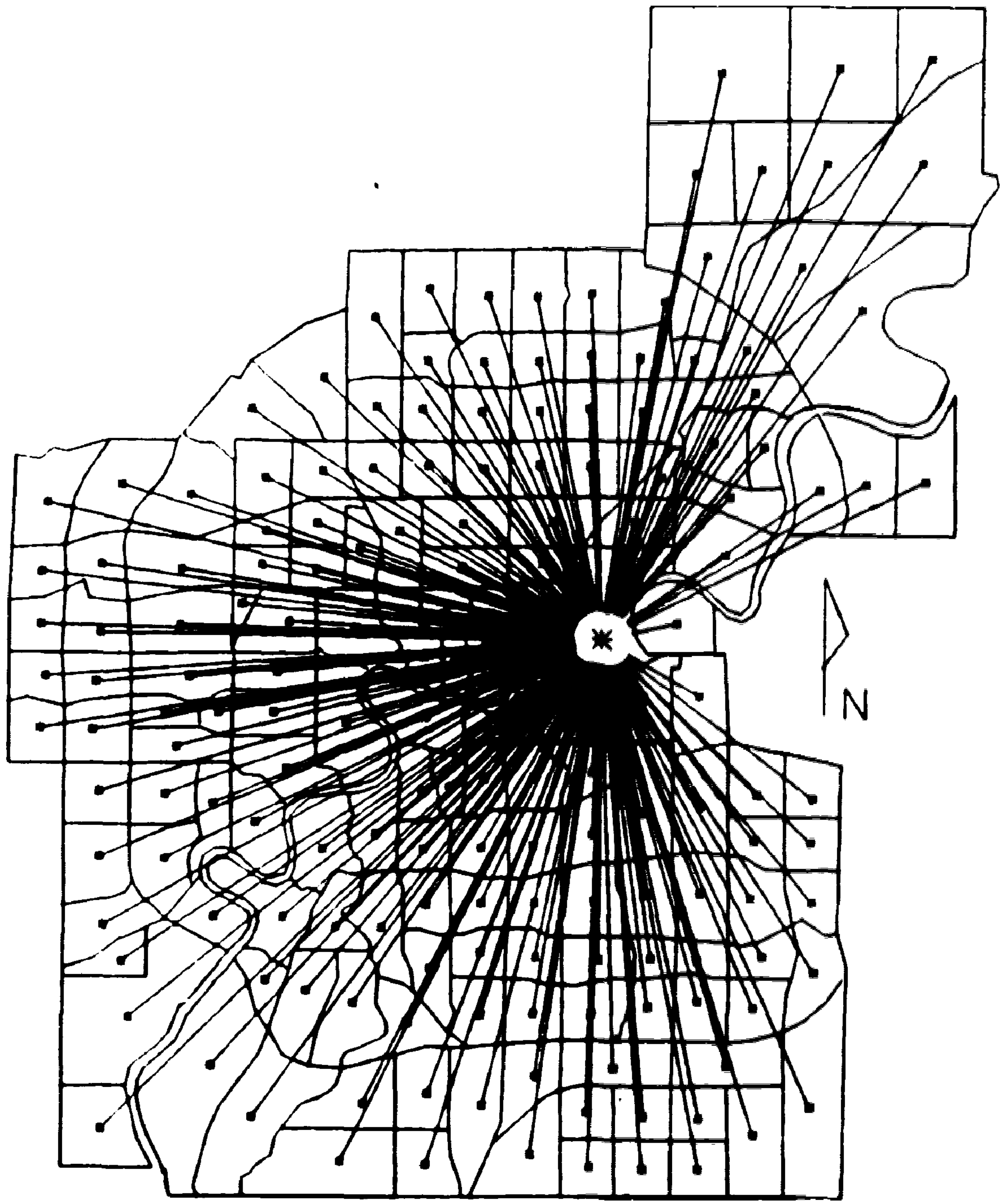


Figure 3.10: Time Solution, p = 1

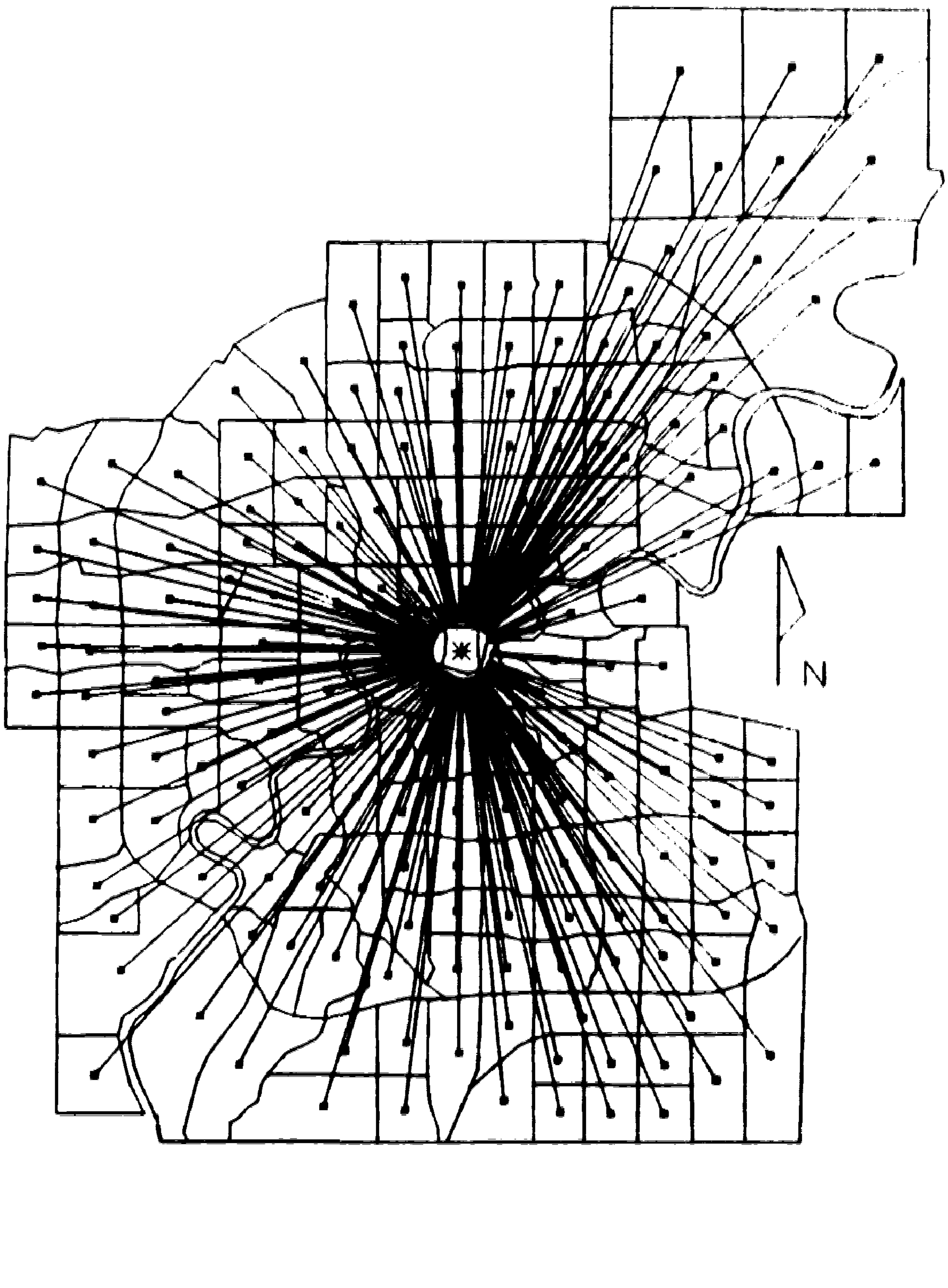


Figure 3.11: Distance Solution, p = 1

Chapter 4

An Application of Flow Capturing LA Models to Edmonton, Alberta

The *p*-median model, considered in Chapters 2 and 3, and most other location-allocation (LA) models, deal with demand for facilities as expressed as a weight at a node; the weight a value in proportion to the amount of target population represented by the node. This point oriented approach to demand implies that the population demanding the service either remains at the node location to be served (if a facility is located there) or travels to the closest facility located by the LA model and then returns.

In contrast, for some types of services, demand may be expressed during a trip. In such cases, demand would be better represented as being exerted by flow between origins and destinations in the transportation network. For example, demand for convenience stores, dry cleaning outfits or banking machines may be generated by people travelling past the service and impulsively stopping to use them. These types of service facilities have been termed *discretionary services* by Berman, Larson and Fouska (1992). Clearly then, modelling demand as point based is a very simplistic, unrealistic, way of modelling some types of trip activity.

Very little has been done in terms of modelling demand as a flow on a network; particularly since traditional LA models cannot interpret this different expression of demand. Hodgson (1990) and Berman, Larson and Fouska (1992) have, however, independently developed a model to deal with demand as a flow. This model, called the Flow Capturing Location Model (FCLM) by Hodgson (1990), determines the optimal locations of *p* discretionary service facilities so that demand in the system is maximally served. Hodgson (1990) tested the FCLM on a small theoretical network and demonstrated the advantages of using an LA approach to avoid damaging self competition, which he termed flow *cannibalization*. Berman, Larson and Fouska (1992) proposed a greedy heuristic solution method for the Flow Capturing Location Problem (FCLP); and a method for determining the number of facilities necessary to capture a predefined percentage of the available flow in the network. Hodgson and Rosing (1992) considered the trade off between serving point and flow based demand using a dual objective function; a hybrid of the *p*-median and FCLM models.

Other than their work, there is presently no literature available which directly deals with the FCLP. Furthermore, none of the papers deals with a real world situation and they are, therefore, incomplete in their ability to analyze the location-allocation characteristics of this modelling approach. Transportation systems are complex and planned structures, not the small contrived networks used in the above mentioned literature. The travel distributions used are also critical to portraying the activity and movement of the demand on the network, and only Hodgson (1990) used a more realistic, though simple, flow structure, estimated using the gravity model. In this chapter, I expand on the work done by Hodgson (1990) and Hodgson and Rosing (1992), and test the FCLM on the actual transportation network of the City of

Edmonton, Alberta, using morning peak hour traffic data. I look specifically at a number of solution algorithms for the FCLM, and analyze the trade off of serving a point based demand using a flow based approach (the FCLM), and a flow based demand using a node based approach (the *p*-median model).

Mathematical Definition of the FCLM

The objective of the Flow Capturing Location Model is to maximize the amount of *flow* (number of customers or demand) captured by the *p* facilities being located. Hodgson (1990) and Berman, Larson and Fouska (1992) showed that these *p* optimal locations occur at nodes. Flows are *captured* by passing a facility. We:

$$Maximize\quad Z = \sum_{q \in Q} f_q y_q \tag{4.1}$$

Where q indicates a particular origin-destination pair;

Q is the set of all OD pairs;

f_q is the flow between OD pair q; and

For simplicity it is assumed that all of the flows for an OD pair take the same path.

y_q is a binary variable which is 1 if f_q is captured, 0 if not.

We can use the single variable q to represent the $_{ij}$ pair because the unit of demand is the <u>flow</u> between the origin and destination pair; and the $_{ij}$ pair is never separated. Since both $_i$ and $_j$ range from 1 to N, because there are no flows between places and themselves, and since the flow data are bi-directional, q ranges from 1 to (N^2 - N.)

Constraints are:

$$\sum_{k \in C_q} x_k \geq y_q \qquad \forall\, q \in Q \tag{4.2}$$

Where k indicates a potential facility location;

C_q is the set of nodes capable of capturing f_q, ie: the nodes on path q between the origin and destination pair;

x_k is a binary variable which is:

1 if there is a facility at location k, and 0 if not.

By this constraint, if a facility exists at location k, $x_k = 1$, it may either capture the flow f_q or not (y_q may take the value 0 or 1). If no facility exists at k, $x_k = 0$, then that node cannot capture the passing flow f_q (y_q must be 0).

$$\sum_{k \in K} x_k = p \tag{4.3}$$

Where K is the set of potential facility locations; and *p* is the predefined number of facilities to be located.

This ensures that the total number of facilities located equals *p*.

Hodgson (1990) comments that this formulation is structurally identical to the Maximal Covering Location Model (MCLM). The only difference is that, in terms of the MCLM, C_q is defined as the set of nodes capable of covering the demand q within a predefined range. In the FCLP, C_q is defined as the set of nodes capable of capturing the flow on path q (Hodgson, 1990).

Flow Cannibalization and Solution Procedures

Cannibalization is a term applied to describing the potential for self competition which may exist if facilities are located solely on the basis of flow (Hodgson, 1990). This occurs when facilities occur on the same flow paths, thus covering some flows multiple times; potentially at the cost of not covering flows elsewhere in the system. In the case of transportation networks, flows travel between origins and destinations. The paths they take pass through two or more nodes which are potential facility locations. If a large flow exists between an O-D pair, and multiple covering is ignored, more than one facility would quite likely be located at nodes on the path between them. Flows may thus be covered several times, a waste of facilities, since the goal is to cover as many flows as possible, with no benefit accruing to covering any flow more than once. Where a facility is one of several facilities covering flows several times, it is likely that it would be able to capture more new flows if it were located elsewhere. Hodgson (1990) provides an extensive discussion and examples of flow cannibalization.

To demonstrate the potentially damaging effects of flow cannibalization I compare the results of two heuristic solution algorithms. Hodgson's "naive" algorithm simply locates the *p* facilities at the *p* locations which are passed by the greatest amount of flow. This approach pays no heed to the potential for cannibalization, and it is likely that facilities are placed along paths of high flow volumes which may already have been served. Such capturing lies at the heart of the cannibalization process.

Hodgson's "greedy" algorithm also locates facilities in an incremental way, but heeds the potential for flow cannibalization. Its first choice will always be at the location where it will capture the most flow. Then, recognizing that these flows have been captured, it removes them from further consideration. The next facility is located to capture the most yet uncaptured flows and those flows are in turn removed from further consideration. This continues until all *p* facilities are located. This solution method makes it more likely that flows along other paths are served. Hodgson (1990) observed this approach to be very robust in his test problem.

To further assess the robustness of the greedy approach I compare the results of the greedy algorithm with the results of a binary integer programming solution algorithm, and a vertex substitution algorithm. For solving actual location problems of small size the binary integer programming method provides optimal solutions (Hodgson, 1990); Professor Rosing, Erasmus Universiteit Rotterdam, kindly used this method to solve for $p = 1 - 15$. For larger problems a vertex substitution heuristic similar to the Tietz and Bart (1968) has been developed by Rosing (Hodgson, Rosing, Storrier, 1993); who also used this method to solve my problem for $p = 16 - 50$.

There is an onus on planners to ensure that facilities are located so that the associated demand has convenient and easy access to the services provided there. To do so the planner must make assumptions as to the expected behaviour of the demand population; including, for example, whether the demand arises from flows or is centred in nodes. Clearly, the LA model used to place facilities in the system must reflect this behaviour. What is unclear, however, is what the impact of making the incorrect service assumption is on the facility locations. In the second part of this chapter, I address the third objective of the thesis and investigate the trade off between serving demands arising from flows and demands centred in nodes. Consequently, the *FCLM* is evaluated under the assumption that the demand is point based, and the *p*-median model is evaluated under the assumption that the demand is flow based.

Hodgson and Rosing (1992) performed this experiment on a small theoretical network, and one of their major findings was that the *p*-median model is more susceptible to impairment by flow capture behaviour than is the FCLM by *p*-median behaviour. They commented that this finding is counter-intuitive, as the *p*-median objective is typically fairly tolerant to shifts in locations; and they had expected that the FCLM would suffer significantly if facilities were moved from major arteries because of a *p*-median emphasis. They go on to suggest that these findings are quite possibly a result of their use of a theoretical network and the way the flow data was generated. They propose that further work focus on replicating their experiment on a real world network; this work represents that contribution. Unlike Hodgson and Rosing (1992), however, I look only at the two extreme objectives, and do not consider intermediate values resulting from their mix.

To compare the two models fairly required solving a 703 node *p*-median problem. Since Hodgson's program does not deal with such large problems I used the LADSS software developed by Densham (1992), also using the Tietz and Bart (1968) algorithm, to locate facilities on Edmonton's transportation network. This software was also used to evaluate the effect of locating facilities at locations determined using the FCLM. The results of the optimal and vertex substitution algorithms are used as the FCLM locations, and a short program written by Hodgson is used to evaluate the flow captured using the *p*-median facility locations.

Data

The design of the following experiment is very similar to the one used in Chapter 3. Data was obtained through (FAB), but it differs because the FCLM uses flow data rather than the point based data used in Chapter 3 to analyze the impact of differing separation measures on facility location.

To perform this experiment a list of the 3TM coordinates of the nodes, and their connections (links) to other nodes in the city's transportation network, was obtained from FAB. These nodes represent the intersections of the freeway and arterial road network in the city. The network topology is critical to the FCLP, because it deals with flows between origin and destination pairs travelling along links and through intermediate nodes. The trip origin/destinations of each zone are represented by a point at the centroid of the zone, from which one or more feeder links radiate onto the appropriate road link. These centroids were provided by FAB and were calculated using their data and EMME/2 modelling procedures. The length of each link was also made available, to enable calculation of the shortest path between each origin and destination pair.

The number of morning peak hour trips between each pair of zones was made available in the form of a trip table. These values were calculated by the City of Edmonton using their EMME/2 transportation model. Factors representing the trips made for purposes other than going to work or post secondary education were applied to the trips to determine the volume of all-purpose trips in the network. These factors were based on data collected in the 1984 travel survey, and were manipulated to maintain consistency between districts for the 1989 recalibration (Working Paper, 1990). Initially it was expected that there would be a total of 31 152 (N^2 - N) flows in the network, however some OD pairs did not have any flows; in actuality a total of 23 350 OD flows exist in the network.

Ford and Fulkerson's (1962) shortest path algorithm was used to determine the travel links used between OD pairs in the transportation network for the experiment. Once these were determined it was possible for Hodgson to remove from the network all those nodes and links which were not along the shortest paths determined for the set of OD pairs. Having no flow, these nodes would never be selected as facility locations. By removing them from the system the remaining data was easier to manipulate and process through the FCLM. A total of 703 nodes remained in the network, from an original total of 1722 nodes; Figure 4.1 shows this reduced network. FAB also provided a number of AutoCAD™ map files which show Edmonton and the surrounding region. A road map of freeways, arterials, collectors, and associated text was provided, as well as the locations of each node, zone centroid, and feeder link in the network; Figure 3.1 shows the basic transportation network of Edmonton. Additional layers outlining the transportation zones and districts were also provided. Each set of information was provided on a different layer, and it was, therefore, easy to look at various combinations of information by overlaying the layers.

Demonstration

The first objective of this demonstration is to show, in a real world context, the degree to which cannibalization can detract from flow capturing and to investigate the robustness of the greedy heuristic algorithm. The cannibalizing issue is investigated by comparing results of the naive and greedy algorithms. This, and the investigation of the greedy model's robustness, is further accomplished through a comparison of these results with those obtained using the globally optimum and vertex substitution solution methods.

For p = 1-15, optimal solutions were determined by Professor K.E. Rosing using the LAMPS software package. This solution method solves the problem as a continuous linear program, resolving non-binary solution variables through branch and bound (Hodgson and Rosing, 1992). For p = 16-50, the best of 50 randomly started (200 for p = 20, 25, ... 50) vertex substitution solutions, also calculated by Professor Rosing, are used to investigate the greedy heuristic's performance. This situation is not ideal, of course, because the vertex substitution algorithm itself is subject to local minimization problems. The term *best solution* is, therefore, used throughout. For p = 1-15 it refers to the optimal solution and for p = 16-50 it refers to the vertex substitution solution.

The second objective of this demonstration is to show, in a real world context, the effect of serving point based demand using a flow capturing approach, and serving flow based demand using a point based approach. Both models used are first run on the network, shown in Figure 4.1, to determine where they would locate facilities. The resulting sets of facility locations are then evaluated in the other model. Both problems are run using the 703 node network, with each node eligible to be a facility location.

The LADSS software was used to deal with this problem, and works by processing the data through a number of program modules. The first module, *Shortest Path Algorithm*, reads in the network point and link data and creates a candidate and demand string file. The point data file holds, for each node in the network, the node name, the region (used if the problem space is partitioned), the weight of the demand at the node, the candidacy of the node (1 if the node is a potential candidate, 0 if not), and the X and Y coordinates of the node. The link file holds, for each OD pair in the network, the name of the origin node, the destination node, and the travel time incurred along the link. The candidate string file produced holds for each candidate a string of all the demand nodes it can serve; whereas the demand string produced holds for each demand point a string of all the candidates it can be served by. Densham and Rushton (1992) comment that a large saving in processing time is possible when both a candidate and demand string are used because each string optimizes the retrieval of demand and candidate information.

The next module, used to solve the p-median model, *Find locations (solve model)*, reads the candidate and demand strings and enables the user to process them using the Teitz and Bart (1968) heuristic. The last module employed, *Evaluate a location set*, produces a statistical analysis of the solution, using the candidate and demand strings and the solution obtained using the previous module. This output includes the location-allocation pattern of the solution, the total amount of demand served, and the total time travelled.

There are a number of significant limitations to the LADSS software. Links, unlike Hodgson's LA program which can deal with bi-directional data, are treated as being symmetric. i.e.: the length from i to j is the same as the length from j to i, and one way links are treated as symmetric two way links. Hodgson and Storrier (1993) demonstrate that this limitation results in a minor degradation of results. To compensate, the average of the two time values was used. Another significant limitation to the software made it possible to only obtain values for p=5-15. This limitation is a result of the inability of the program to deal with long candidate or demand strings, which result when the number of demand and candidate nodes in the network is large. LADSS does make it possible to limit the time travelled between nodes, so that not every interaction between the nodes in the network is recorded. This is very useful for large values of *p* because the allocation area of each facility is small. However, when *p* is small the time value, called the Z-Limit by Densham (1992), must be large to ensure that all demand in the network is served. As the network used in this thesis is very big, the candidate and demand strings are very large for $p < 5$, causing the program to crash. A third limitation is the time required to solve for large values of *p*, this has also limited the results obtained. In fact only three random starting solutions were used to determine the best solutions found because of time limitations; for example, to obtain a solution for $p = 15$ using one random start took 1.08 hours. The solutions obtained may therefore not be as good as solutions obtained previously in the thesis. However, the Tietz and Bart (1968) algorithm has been shown to be robust.

Results

Comparing the results of the greedy, naive, and optimal solution algorithms clearly shows the problem of cannibalization. When considering this problem, I use the term "expected flow" for the amount of flow captured if redundancy were not considered, i.e.: if flows captured for a second or greater time could be counted as new flows captured. I use the term "actual flow" for the amount of flow captured at least once; covering a flow more than once is not credited. Figure 4.2 shows that of all the solution approaches the naive algorithm *expects* to cumulatively capture the most flow; followed by the optimal, and then the greedy, solution algorithms. However, the curve representing the *actual* flow captured using the naive algorithm shows that it captures the least amount of flow. Figure 4.3 also shows that the naive heuristic expects to capture a lot of flow at each facility it locates; more so than the optimal and greedy algorithms. However, Figure 4.3 clearly shows that the amount of flow actually captured using the naive solution varies drastically; some facilities capture much new flow whereas others, such as the sixth, eighth, and fifteenth facility capture no new flow at all. Furthermore, instead of each consecutive facility location capturing less new flow than the previous facility location, as is the case with the greedy and best solutions, a facility can capture a varying amount of flow which can be more or less than that captured by the previous facility.

Insight into this may be gained by looking at Figures 4.4, 4.5, 4.6 and 4.7, which show the facility locations, and expected and actual flow captured, in a system of 15 facilities using the naive, greedy and optimal solution approaches. It is clear on Figure 4.4 that the naive algorithm chooses facility locations along travel paths with a high percentage of the total available flow. Unlike the facility locations on Figures 4.6 and 4.7 which are widely distributed, facility locations on Figure 4.4 are concentrated in two areas; bundled on paths as seen on the Whitemud Freeway, and clustered in the downtown core area. As may be seen on Figure 4.4, box A and Figure 4.5, box B, the expected value of captured flow is much higher than the actual flow captured.

i.e., in reality, facility I captures the largest proportion of flow in box B, and the other facilities each capture a little of the remaining *uncaptured* flow. The facilities are located so that they are competing for the same flow; rather than being located to capture previously uncaptured flow.

As explained previously the naive algorithm does not remove captured flows from the system before locating the next facility. As a result, when the algorithm looks for the next best facility location it is likely that it will locate it beside or along the same path as the previous facility; it actually appears that there is a lot of flow available for capture in the area. As shown in the example, most or all of the flow in the area has been captured, and the new facility has no customer base left to serve: cannibalization has occurred. This may be clearly seen on Figure 4.8, which shows the difference between the actual and expected flow captured; the cannibalized flow. This shortfall is clearly very significant, and the problem of cannibalization may be seen in various degrees at almost every facility location. Cannibalization is shown to be a serious problem as it results in a significant reduction in the amount of flow captured in the network; and therefore a reduction in the service provided conveniently to the demand population.

Figures 4.6 and 4.7 show that the pattern of facility locations determined using the greedy and optimal solution algorithms is much more widely distributed. Multiple facilities are not placed next to each other, or as close together as they were by the naive algorithm. A look at the area enclosed by box A on Figure 4.4 shows that four facilities were located there along a straight line; the same area on Figures 4.6 and 4.7 shows only two facilities being located, and not along the same straight line. The locations do seem to reflect the movement of flows to the centre of the City in the a.m. peak hour. It is not, however, possible to use these figures to explain why the optimal algorithm performs better than the greedy algorithm, because the facility location patterns are actually very similar.

Also apparent in the above analysis is the closeness in the results of the greedy and optimal solution algorithms. To provide more information about this 'closeness' the results of the greedy and best algorithms for p = 1 to 50 were also plotted (Figure 4.9). In Figures 4.2 and 4.9 the best solution is only slightly better overall than the greedy solution. The results presented in Figure 4.9 assert that the greedy heuristic is robust in determining facility locations in the transportation network. To better show the differences between these two solution approaches the percentage of flow captured by the greedy algorithm compared to the best solution was calculated for p = 1 to 50:

$$\frac{\textit{Greedy solution}}{\textit{Best solution}} \times 100 \qquad 4.4$$

The results are tabulated in Table 4.1. The greedy solutions range from capturing 99.1 % of the flow captured by the best solution to 100 %. These results suggest that it is acceptable to use the greedy heuristic, as it is clearly very robust.

This knowledge is especially valuable as it can be very time consuming, even impossible, to obtain globally optimal solutions; particularly as p increases. A comparison of the run times for the greedy and best solution algorithms used here further supports the use of the greedy algorithm when a better solution algorithm is not available. On a Convex 210 mainframe computer, the greedy algorithm required a total of 29.1 CPU seconds for $p=1$-50. To obtain the optimal solution, however, required a total of 12.3 days CPU time for $p=2$-15; and the time required to solve the Vertex Substitution Heuristic for $p=16$-50 rose linearly with p. For the 200 runs performed for $p = 50$, 6.7 days of CPU time were required.

Figure 4.2 also shows the impact of the p-median locations on the FCLM. These values were plotted to see how effective the use of facility locations determined using the p-median objective, are compared to the those determ[illegible] using the optimal, greedy and naive solution algorithms. It is clear in Figure 4.2 that this trade-off results in a very poor level of service, significantly worse than what the optimal, greedy, and even naive solution algorithms are able to provide. Generally the amount of flow increases as p increases, similar to the other curves. However, in some cases, because for each value of p the complete set of locations may change - unlike the greedy and naive heuristics, the total flow captured may actually drop.

To analyze the trade off between serving point based demand using a flow capture approach, and serving flow based demand using a point based approach, I evaluated the solutions obtained using the p-median model in the FCLM model, and evaluated the solutions obtained using the FCLM in the p-median model. As noted previously, solutions for the p-median model, in the 703 node network, were determined using the LADSS software. By using the same network to solve for both p-median and FCLM solutions, I could compare them fairly. The terms used in this comparison are:

Λ_p = The facility locations determined using the p-median model
Λ_φ = The facility locations determined using the FCLM
Z_{pp} = The objective function values obtained by evaluating the p-median model using Λ_p
$Z_{\varphi\varphi}$ = The objective function values obtained by evaluating the FCLM using Λ_φ
$Z_{p\varphi}$ = The objective function values obtained by evaluating the p-median model using Λ_φ
$Z_{\varphi p}$ = The objective function values obtained by evaluating the FCLM using Λ_p

The underlying rationale for the comparison of the two models is to assess the negative effects of either using a p-median model where demand is actually flow based or the FCLM where demand is actually point based. Because these effects are negative, I use the term *damage* for the comparison measure. The damage resulting from using the FCLM when the p-median model is appropriate is:

$$(\frac{Z_{pf}}{Z_{pp}} - 1) \times 100 \qquad 4.5$$

(The percent excess aggregate weighted distance).

Alternately, the damage resulting from using the *p*-median model when the FCLM is appropriate is:

$$\frac{Z_{fp}}{Z_{ff}} \times 100 \qquad 4.6$$

(The percentage of flow captured). The results are recorded in Table 4.2, and plotted in Figures 4.12 and 4.13 respectively.

Comparing the Z_{pp} and Z_{pf} (Figure 4.10), and Z_{ff} and Z_{fp} (Figure 4.11) values obtained clearly shows the damage in service provision when using the incorrect service assumption. In each case the FCLM locations result in an increase in the distance which must be traversed by the demand population, and the *p*-median locations result in a reduction of the flow captured in the network. This is not unexpected as A_p and A_f do not necessarily contain any of the same facility locations, and were obviously not determined based on the same optimization criterion.

In Figure 4.10, the pattern exhibited by the Z_{pp} and Z_{pf} values as *p* increases shows a monotonic decrease in the aggregate weighted time (in optimal values we must observe this). In each case, the addition of a facility results in a decrease in the aggregate weighted time. As may be observed in Figure 4.11, however, the Z_{fp} values, unlike the Z_{ff} values, do not exhibit monotonicity. In fact, at $p = 8$ and 12 less flow is captured than at $p = 7$ and 11 respectively.

Table 4.2 and Figure 4.12 show that the damage incurred, or the percent excess in the aggregate weighted time, ranges from 32.4 at p = 6 to 69.3% at $p = 15$. As *p* increases the percent difference in aggregate weighted time also increases with some exceptions. Clearly, the damage to the *p*-median objective is positively associated with the number of facilities, and the convenient availability of service provided to flow based demand is significantly compromised.

Table 4.2, in conjunction with Figure 4.13, presents the damage to the FCLM objective; the percent of optimal flow captured when facilities are located to serve point based demand. The flow captured using A_p ranges from capturing as little as 38.0% to as much as 47.4% of the flow captured using A_f. Thus, the change in facility locations significantly affects the ability of the FCLM to serve the flow based demand in the network. The pattern exhibited for $p = 5 - 15$ suggests that the damage to the FCLM varies little in response to *p*, as additional facilities make little or no impact on the percent of optimal flow captured.

Conclusions :

In the first experiment undertaken in this chapter, a non-cannibalizing, a globally optimum, and a cannibalizing solution approach were compared to see how serious the effects of cannibalization are on a real world network. Hodgson (1990) showed the greedy algorithm to be very robust, and in this real world example it is actually more so than in his example. Similarly, this experiment shows that the effects of cannibalization can be severe; a large percentage of the demand is not served because of the location pattern of the facilities. In some cases the location of facilities appears unnecessary as they capture little or no flow. This suggests that Hodgson's (1990) results are not just an artifact of using a small artificial network, as on a real world network similar results are obtained.

In the second experiment, the trade off between a flow capturing and *p*-median objective function was evaluated. In both cases, running the *p*-median model using the FCLM solutions and running the FCLM using the *p*-median solutions, the impact of making an incorrect service assumption appears significant. Evaluating the models under the incorrect service assumption results in a poor facility location pattern and the two types of demand population, *flow* and *point based*, are significantly inconvenienced. These results suggest that it is vital to the success of the planning process that the type of demand requiring services in the transportation system is correctly identified.

Table 4.1: Flow Captured by Solution Algorithms

	Flow Captured				Cumulative Flow Captured						
p	1 Naive Exp	2 Naive Act	3 Greedy	4 Optimal	5 Naive Exp	6 Naive Act	7 Greedy	8 Optimal	9 VSH	10 *p*-median	11 Greedy to Best Solution ratio
1	7446	7446	7446	7446	7446	7446	7446	7446	7446		100
2	7363	6722	6722	6722	14809	14168	14168	14168	14168		100
3	6573	6522	6522	6522	21382	20690	20690	20690	20690		100
4	6443	2107	4827	4827	27825	22797	25517	25517	25517		100
5	6108	1895	3958	3958	33933	24693	29475	29475	29475	13933	100
6	6056	0	3834	3834	39989	24693	33309	33309	33309	14048	100
7	5991	3218	3076	3076	45980	27911	36385	36385	36385	16866	100
8	5756	0	2838	2838	51736	27911	39223	39223	39223	14918	100
9	5706	1153	2724	2724	57442	29064	41947	41947	41947	18235	100
10	5587	2667	2240	2240	63029	31730	44187	44187	44187	19689	100
11	5584	2289	2132	2132	68613	34019	46319	46319	46319	21747	100
12	5462	197	2091	2091	74075	34216	48410	48410	48410	21317	100
13	5419	1397	1660	1660	79494	35613	50070	50070	50070	22915	100
14	5389	138	1550	1776	84883	35751	51620	51846	51846	23901	99.6

15	5354	0	1405	1396	90237	35751	53025	53242	53242	25243	99.6
16							54364		54704		99.4
17							55595		56011		99.3
18							56716		57211		99.1
19							57798		58342		99.1
20							58868		59426		99.1
21							59856		60425		99.1
22							60779		61323		99.1
23							61676		62030		99.4
24							62529		62947		99.3
25							63292		63672		99.4
26							64047		64380		99.5
27							64757		65027		99.6
28							65408		65728		99.5
29							66029		66326		99.6
30							66650		66953		99.5
31							67208		67522		99.5
32							67711		68022		99.5
33							68182		68491		99.5

34							68642		68975		99.5
35							69071		69407		99.5
36							69472		69800		99.5
37							69868		70244		99.5
38							70229		70617		99.5
39							70548		70982		99.4
40							70866		71358		99.3
41							71174		71665		99.3
42							71465		71954		99.3
43							71743		72243		99.3
44							72020		72514		99.3
45							72280		72794		99.3
46							72528		73050		99.3
47							72754		73279		99.3
48							72972		73495		99.3
49							73175		73718		99.3
50							73365		73918		99.3

Legend : Description of Data in Table 4.1	
Column	Description of Data
1	Flow expected to be captured using Naive algorithm.
2	Flow actually captured using Naive algorithm.
3	Flow captured using Greedy algorithm.
4	Flow captured using Optimal solution algorithm.
5	Expected cumulative flow captured using Naive algorithm.
6	Actual cumulative flow captured using Naive algorithm.
7	Cumulative flow captured using Greedy algorithm.
8	Cumulative flow captured using Optimal solution algorithm.
9	Cumulative flow captured using Vertex Substitution Heuristic.
10	Cumulative flow captured by FCLM where facility locations determined using p-median objective.
11	Percent flow captured using Greedy algorithm compared to Best solution (Equation 4.4).

Table 4.2: Results of Trading off FCLM and p-Median objectives

p	1 Z_{pp} (Seconds)	2 Z_{pf} (Seconds)	3 $(\frac{Z_{pf}}{Z_{pp}}-1)*100$ % Excess Aggregate Weighted Time	4 Z_{ff} (Flow Captured)	5 Z_{fp} (Flow Captured)	6 $\frac{Z_{fp}}{Z_{ff}}*100$ % Flow Captured
5	400750759	553514742	38.12	29475	13933	47.27
6	361525004	478707207	32.41	33309	14048	42.17
7	325237804	446595556	37.31	36385	16866	46.35
8	305763767	431345522	41.07	39223	14918	38.03
9	289102887	415081154	43.58	41947	18235	43.47
10	276001712	407452899	47.63	44187	19689	44.56
11	261398130	399897648	52.98	46319	21747	46.95
12	249879850	383838733	53.61	48410	21317	44.03
13	238753645	379698425	59.03	50239	22915	45.61
14	228496673	377584745	65.25	51846	23901	46.10
15	218944684	370568213	69.25	53242	25243	47.41

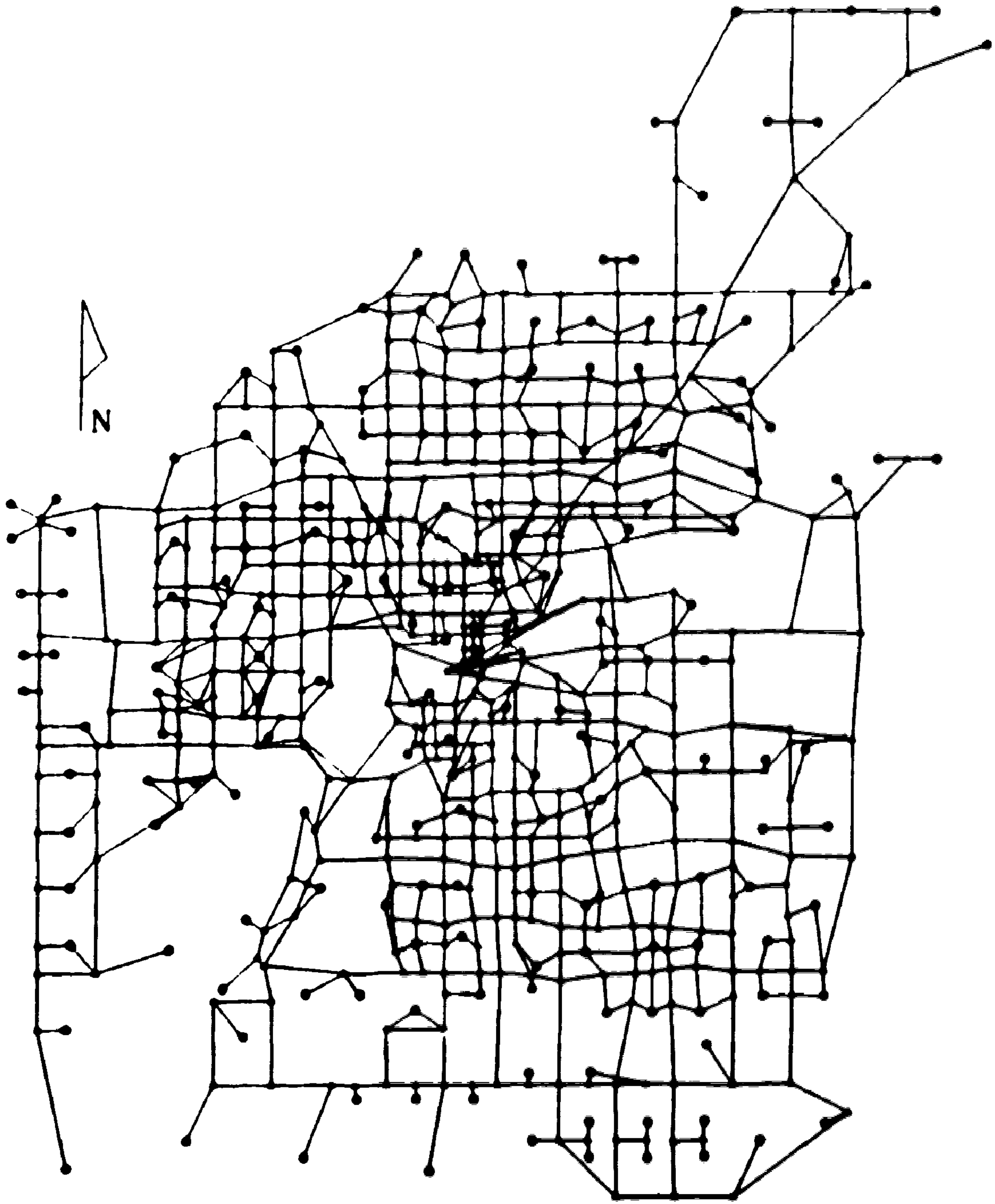


Figure 4.1: City of Edmonton Transportation Network Extraneous Links and Nodes Removed

Figure 4.2: Cumulative Flow Captured
p = 1 to 15

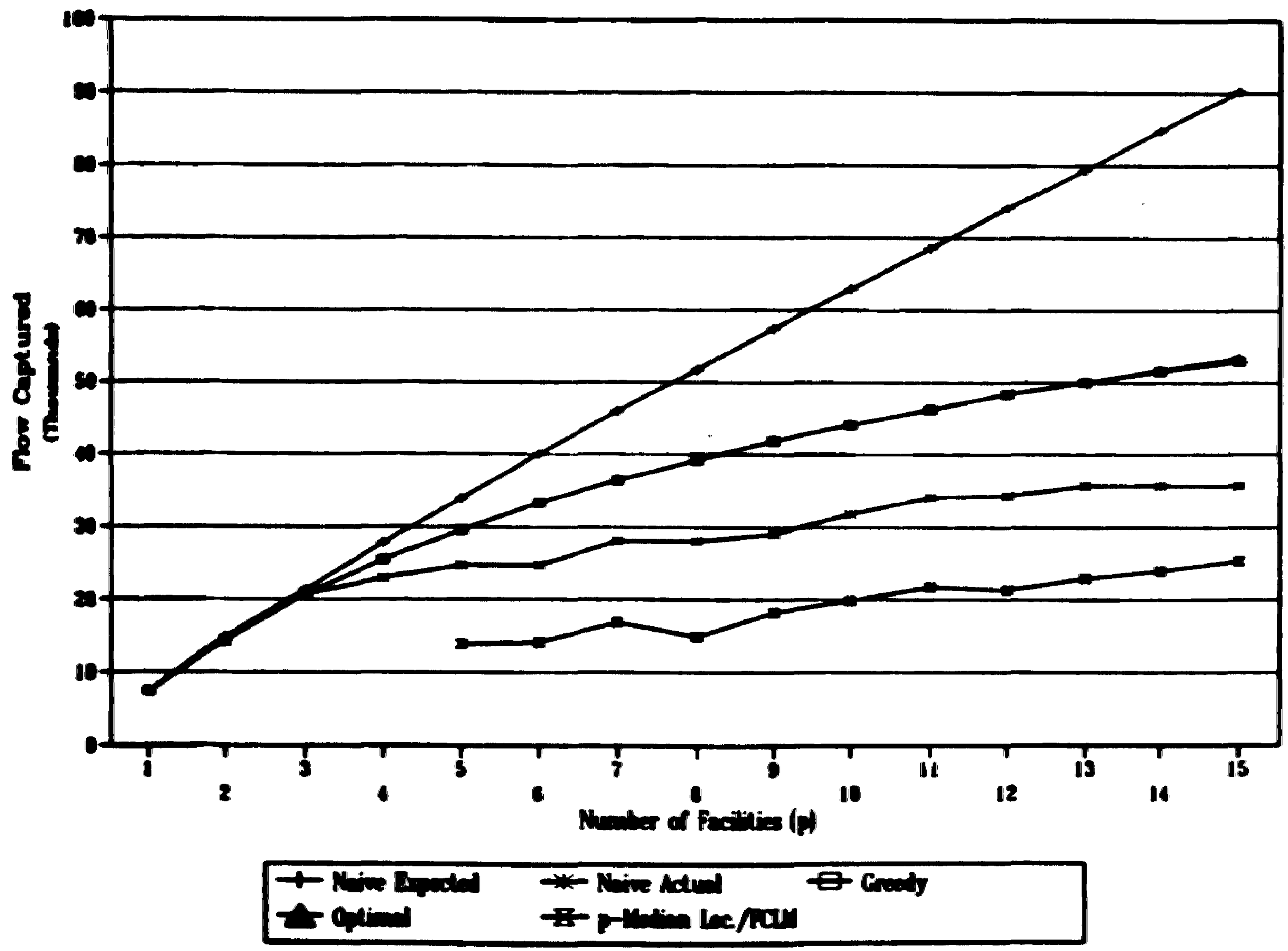
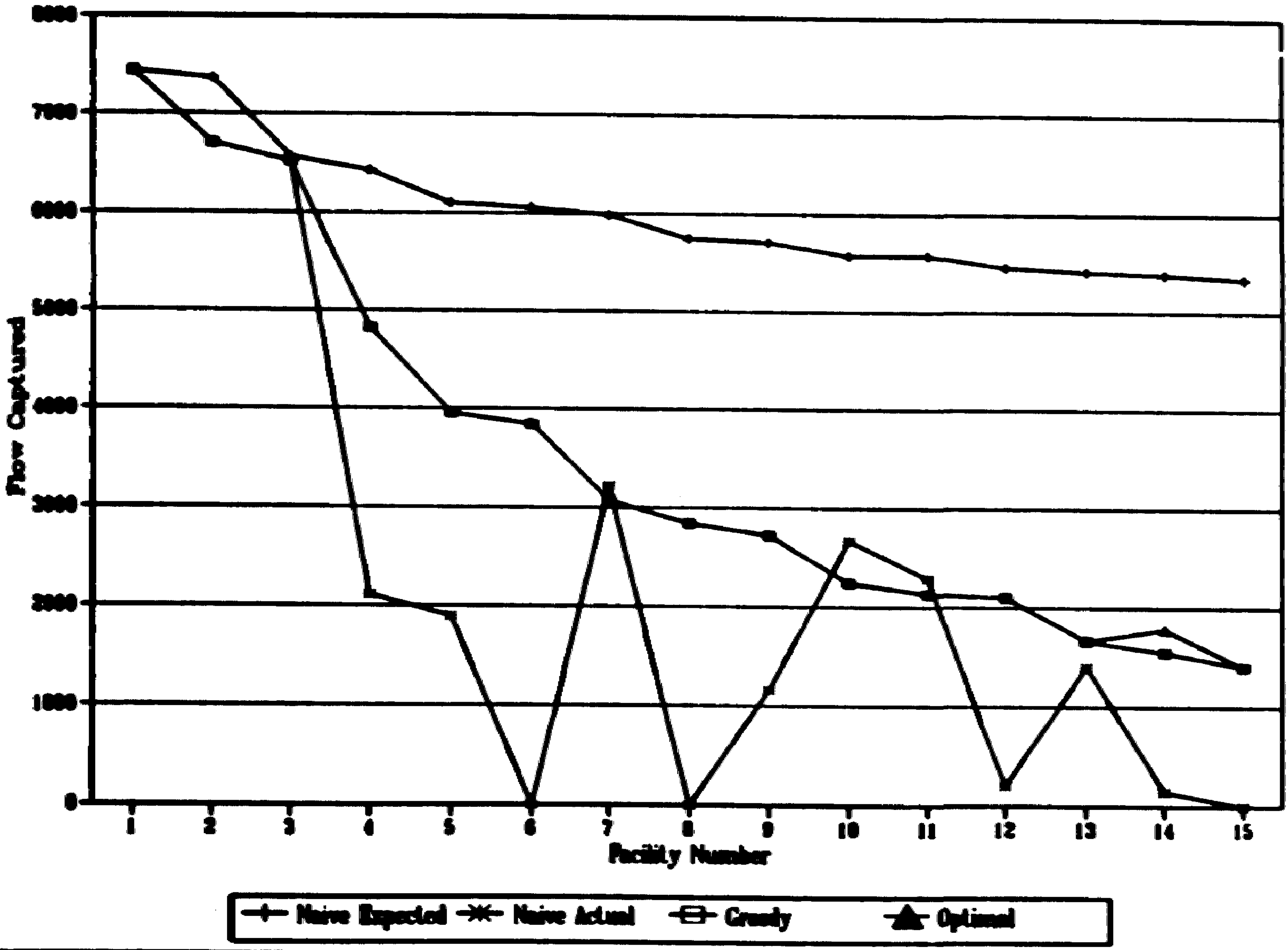


Figure 4.3: Flow Added by each Facility

p = 15



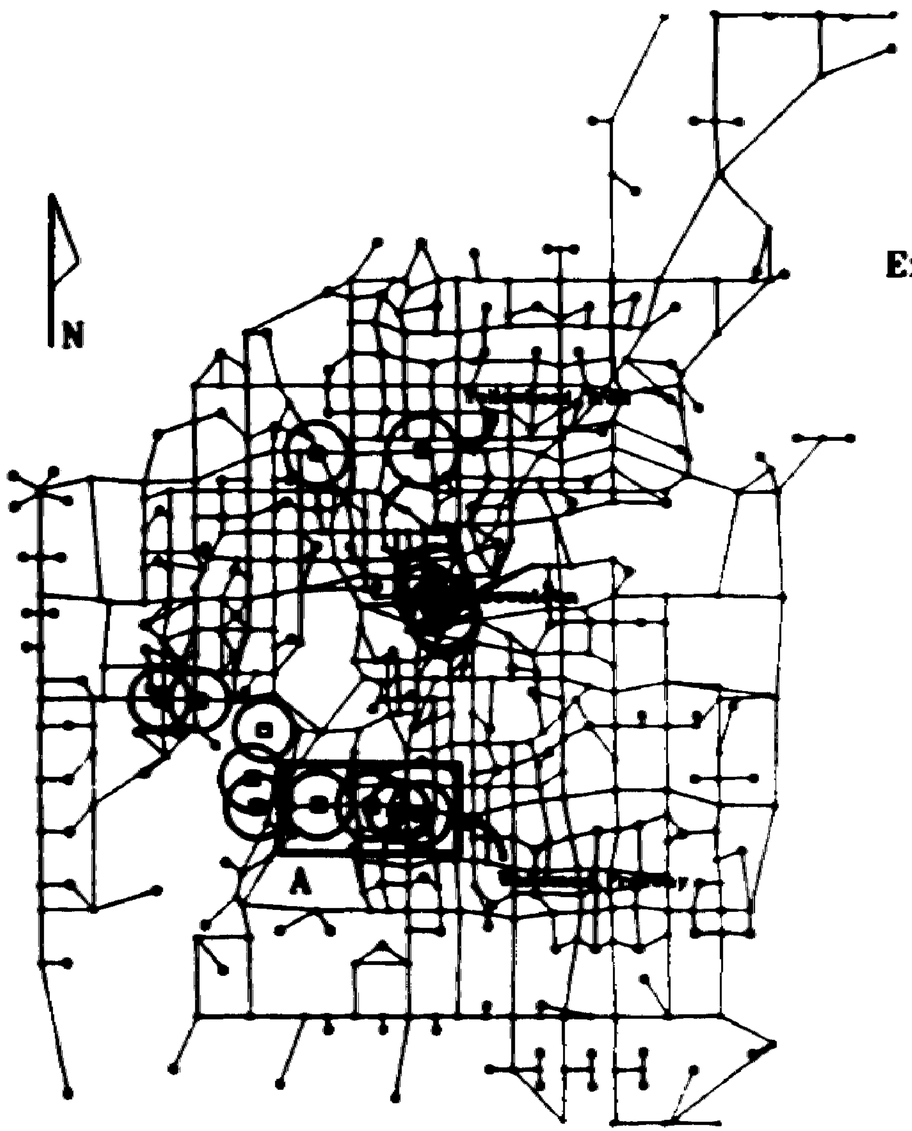


Figure 4.4: Naive Solution
Expected Flow Captured, p = 15

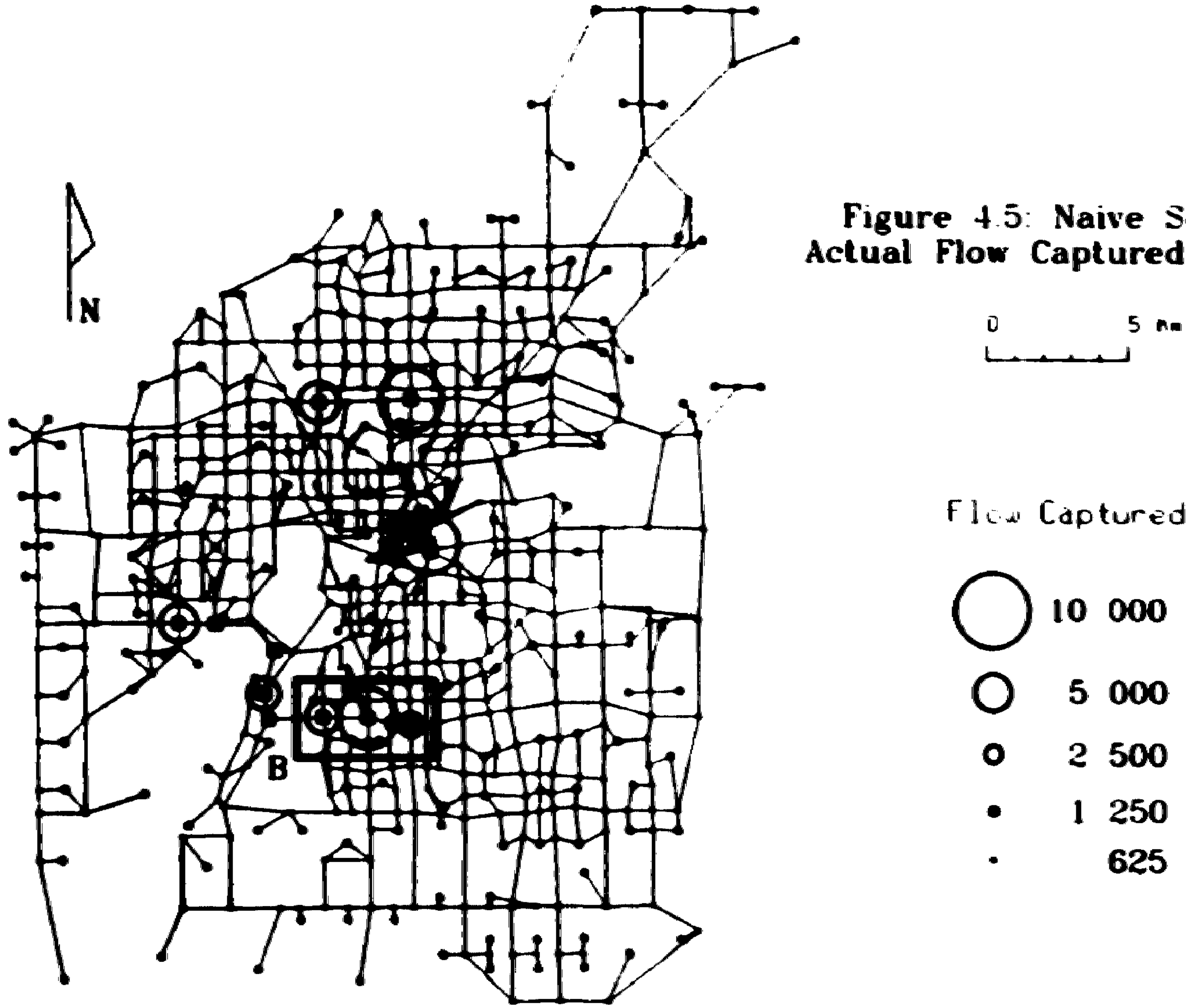


Figure 4.5: Naive Solution
Actual Flow Captured, p = 15

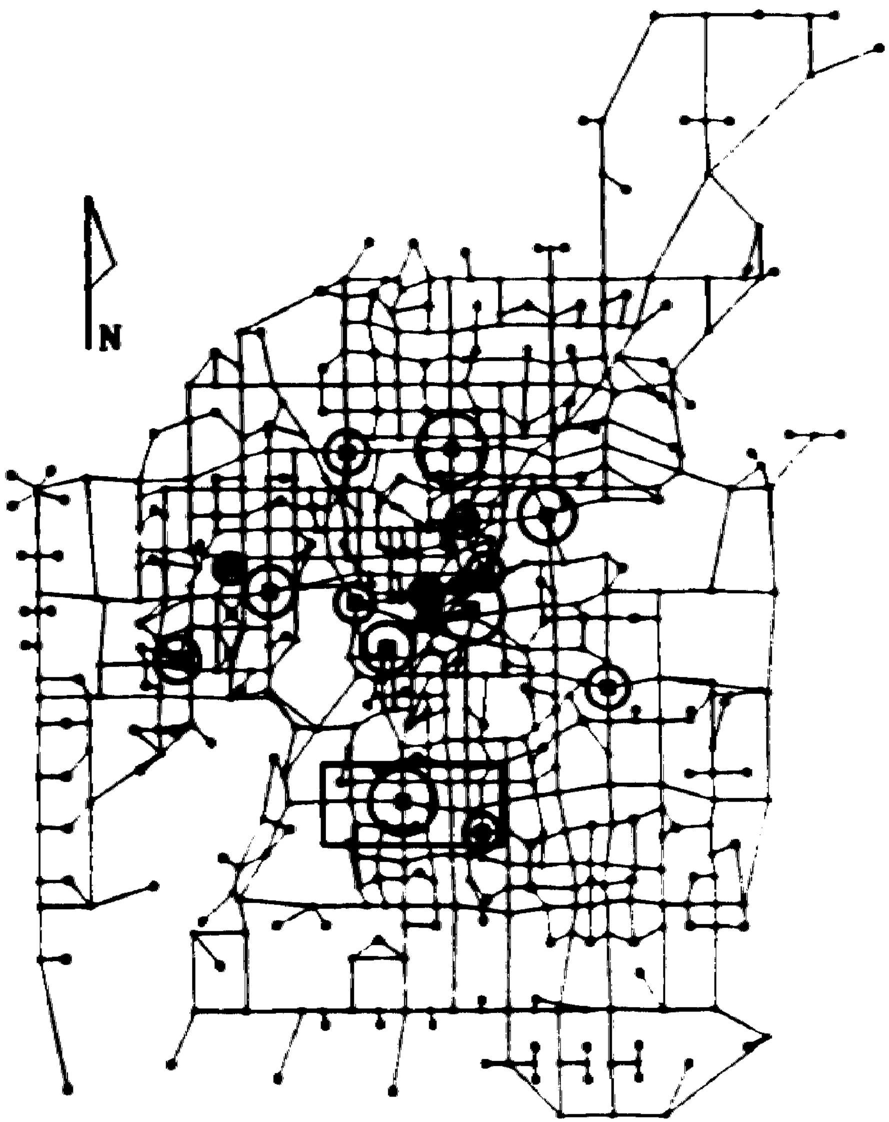


Figure 4.6: Greedy Solution
Flow Captured, p = 15

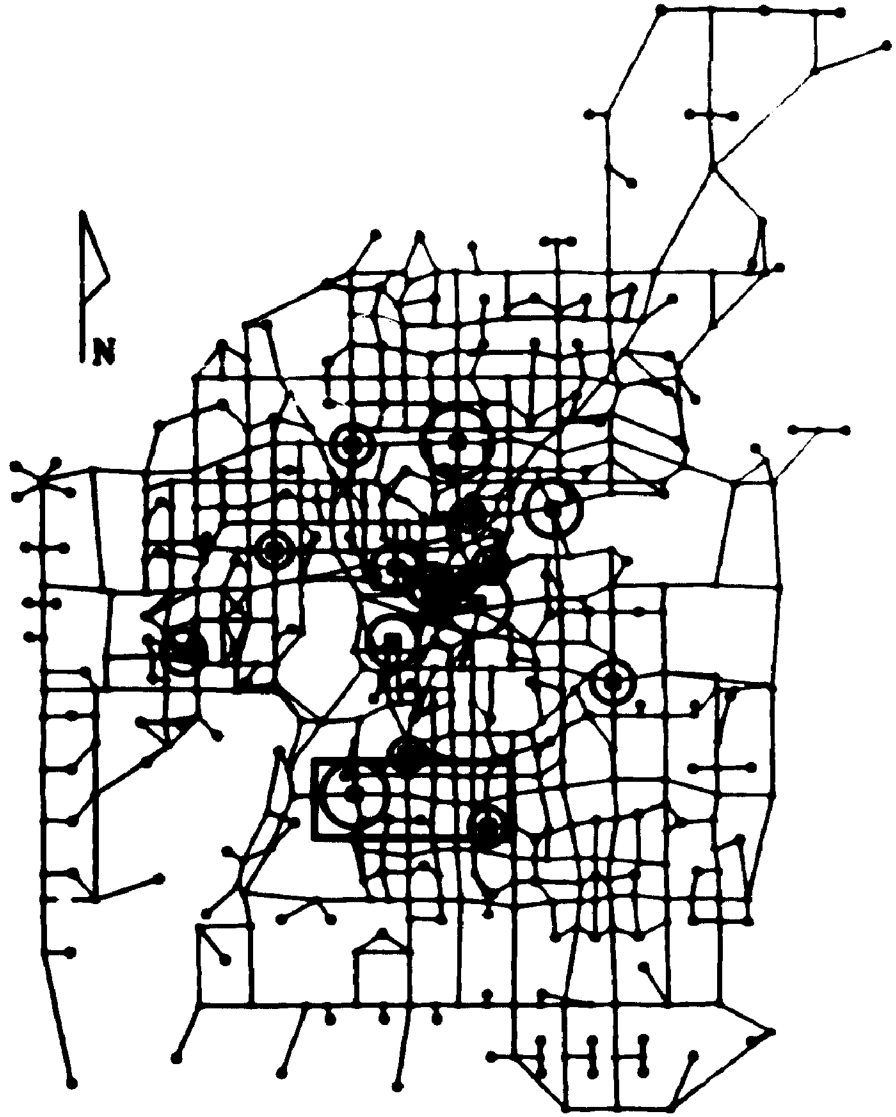


Figure 4.7: Optimal Solution
Flow Captured, p = 15

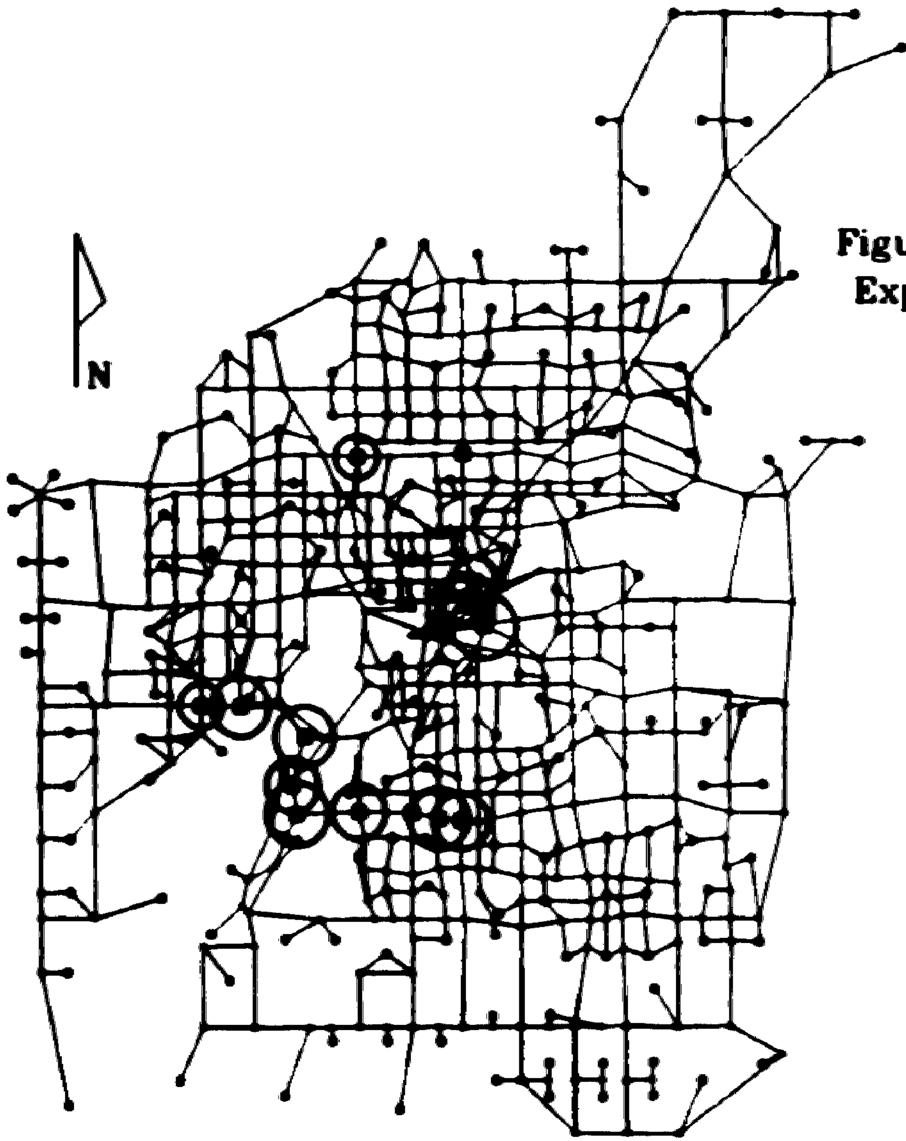


Figure 4.8: Naive Solution Shortfall
Expected - Actual Flow Captured
p = 15

Figure 4.9: Cumulative Flow Captured

Greedy vs. Best Solution, p = 1-50

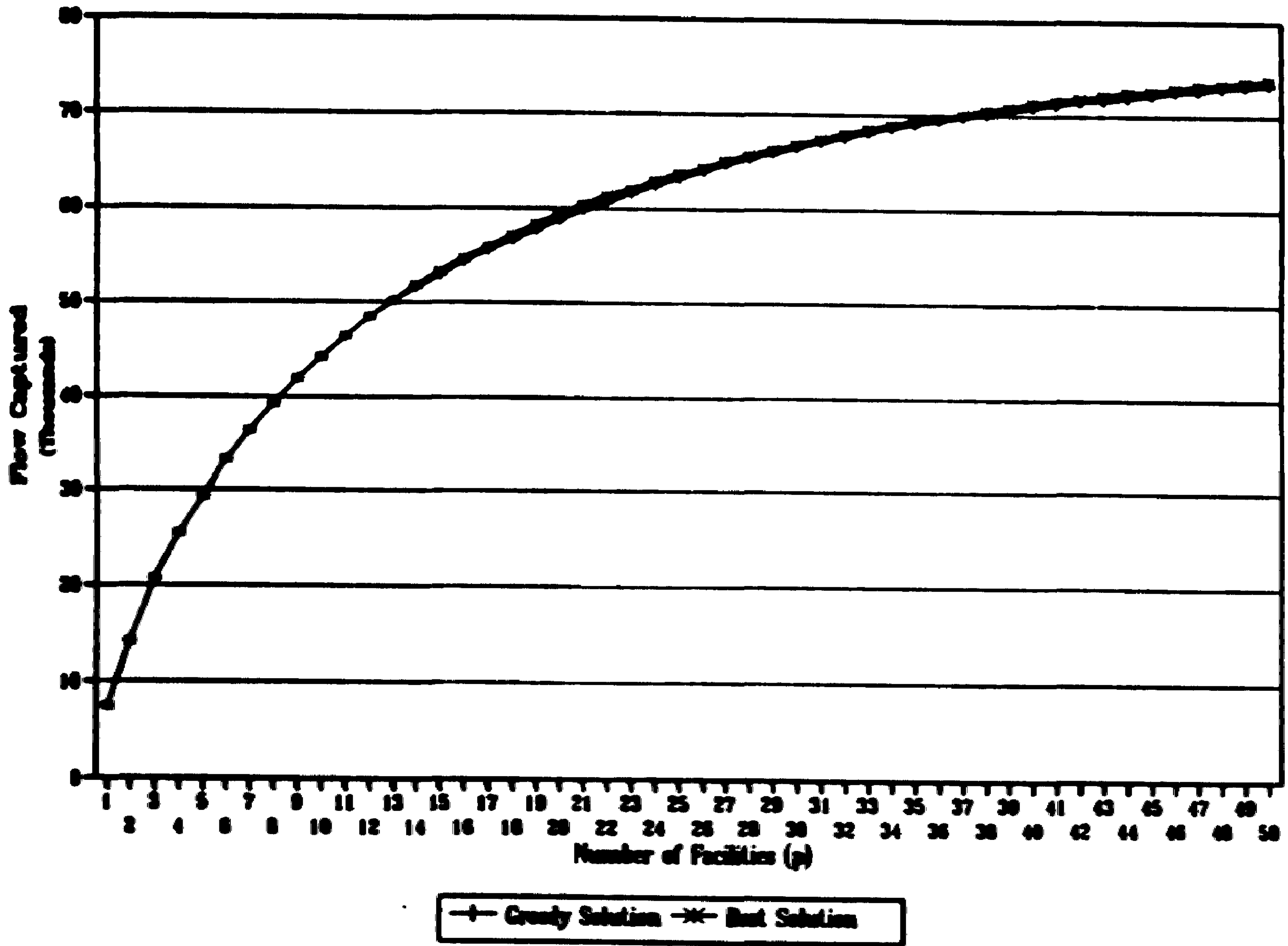


Figure 4.10: Impact on p–Median Model

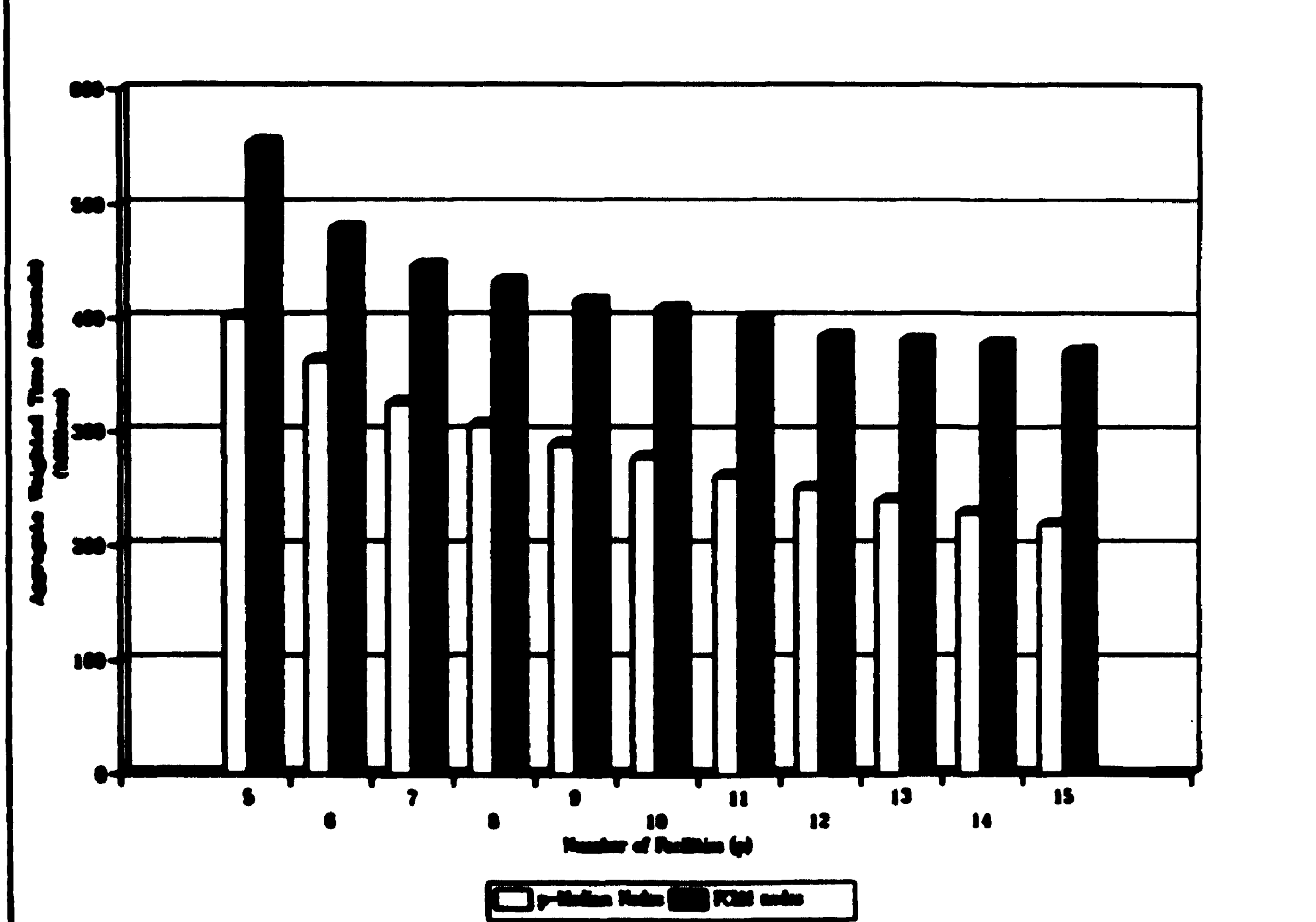


Figure 4.11: Impact on FCLM

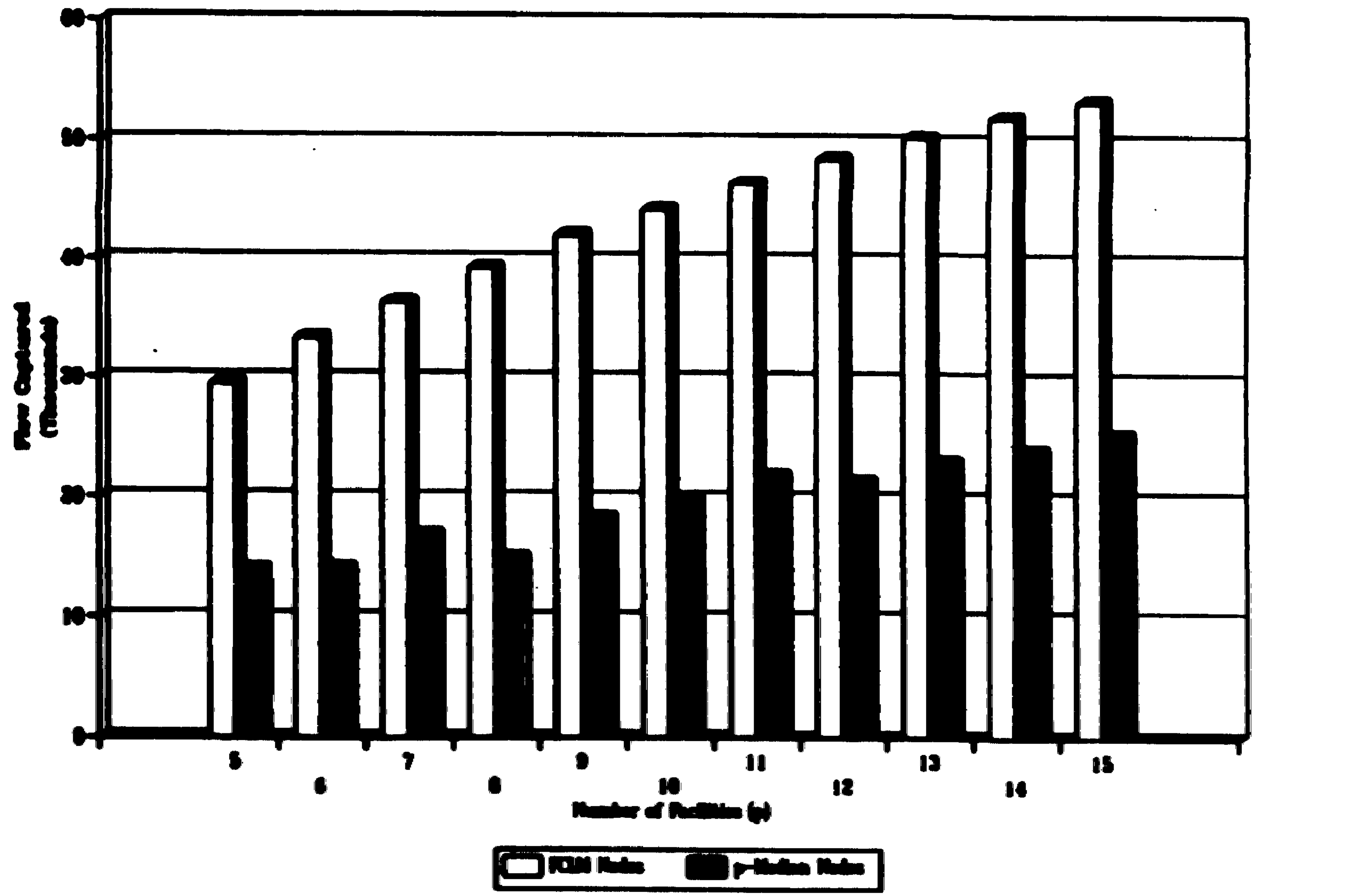


Figure 4.12: Impact on Service

p-Median Model using Flow Based Demand

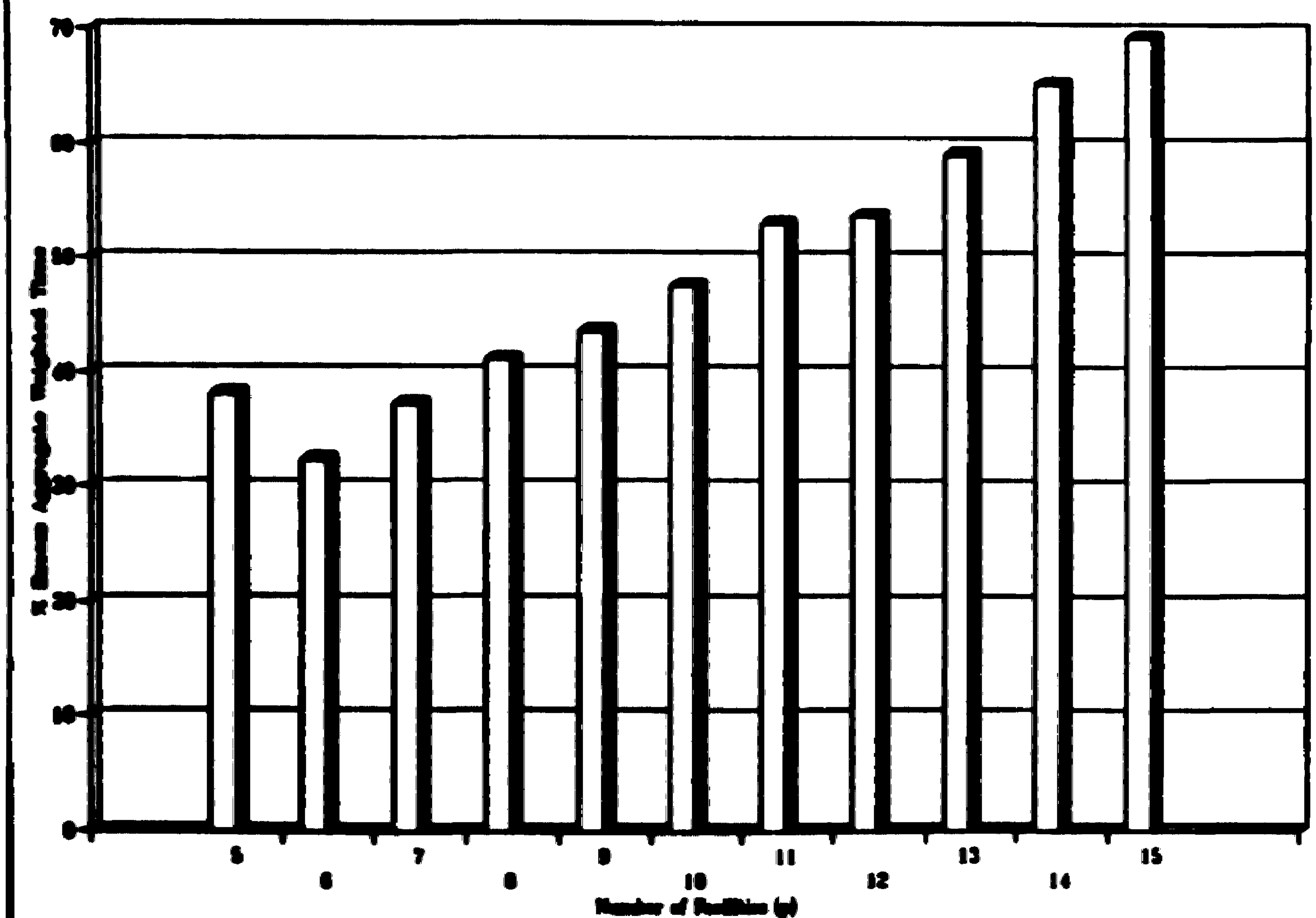
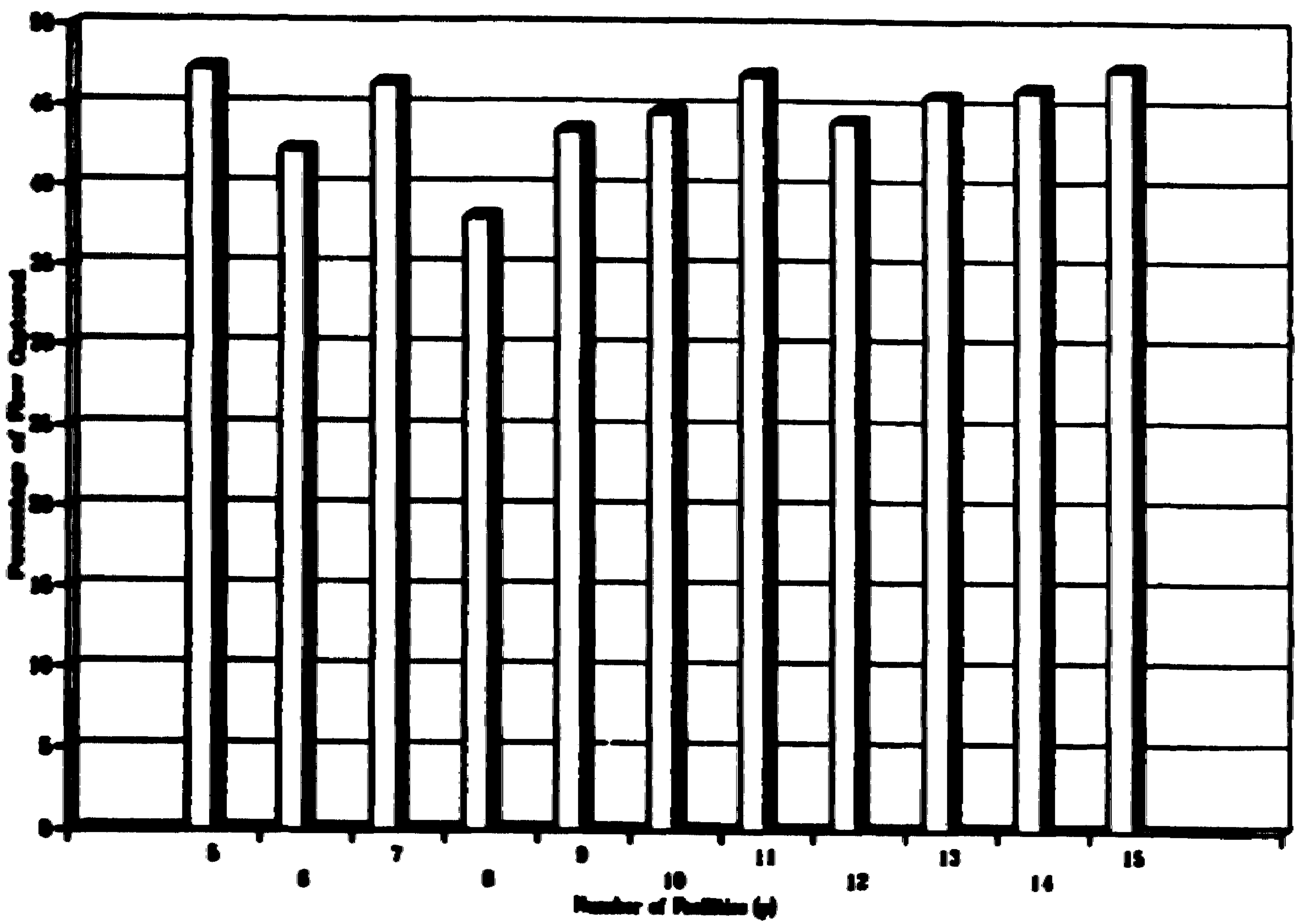


Figure 4.13: Impact on Service



Chapter 5

Conclusion

A number of choices face a practitioner or researcher about to embark on a location-allocation project. Central are the choice of data type and source and the choice of model. The theoretical LA literature can guide these choices, but the real world situations being confronted must inform the final choice. In this thesis, using data for Edmonton, Alberta, I consider the trade offs involved between two sources of spatial separation data and the consequences of choosing the incorrect one of the two types of LA model. I first explored the potential of using easily and cheaply obtainable Cartesian distance measurements, rather than the more complex and expensive to collect travel time data, in the *p*-median model. I then compared the results of a cannibalizing vs. non cannibalizing solution approach to the FCLP, and considered the ability of the non-cannibalizing, greedy, solution algorithm to locate facilities on a real world network as opposed to a simple theoretical network. Lastly, I looked at the impact of using an incorrect service assumption on facility locations; in the context of the *p*-median and FCLM LA models.

The results obtained in the distance experiment, using the *p*-median model, in Chapter 3 show that Cartesian distance data performs well compared to actual travel time data. However, there is a trade off to be considered in deciding what separation measure should be used. In particular the trade off between incurring the expense of collecting more representative separation data, and the reduced ability to provide services when the Cartesian separation measure is used. This investigation also confirms the robustness of the use of a vertex substitution heuristic solution algorithm for the *p*-median model.

The results obtained, however, are specific to Edmonton's transportation network, which, as any other network is unique. The results are, therefore, open for discussion and comparison with further work. It is suggested here that further investigation of the concept of equity, by analyzing the impact that barriers, and other travel penalties have on the benefits received by individuals in location-allocation solutions, would be interesting. It would be naive to suggest that further testing of the use of Cartesian distances is not required, as on the basis of one set of correct separation values it is impossible to generalize to other systems and conditions. A regression analysis, studying the relationship between the travel times and Cartesian distances, may also provide further insight into the effects of barriers on Cartesian distance data. Further evidence from different cities would be required before we can put sufficient confidence in the use of Cartesian distance measurements to use them routinely for LA modelling applications. The results are definitely encouraging enough to make further experimentation worthwhile, and to determine whether the costs involved in obtaining 'better' data are justified.

It is clear from the results obtained in the FCLM experiment in Chapter 4 that locating facilities without taking into account the effects of cannibalization can result in a significant waste of resources. In the experiment, placing facilities along the same busy stretches of road resulted in some flows being captured at least once, whereas *other flows in the network were not captured at all; in fact cannibalization is shown to* be a real problem. Thus the use of the FCLM, which avoids cannibalization while locating facilities, is vital to the optimal provision of services in the transportation network. A comparison of the greedy and globally optimal solution approaches confirms earlier work, with small contrived test problems, that the greedy heuristic used provides excellent results. This is very encouraging, because if these results are general, the use of such a heuristic may be considered a viable alternative for large facility location problems which cannot be solved using globally optimal solution methods.

I would suggest that it would further be worthwhile to investigate how often facilities are placed along certain types of road categories versus others. It would be interesting to determine whether these locations would actually be able to capture the flows associated with them in real life. For example, it may be inconvenient for travellers to access facilities placed close to bridge structures even though theoretically they may capture large amounts of flow. The effects of data aggregation on the FCLM could also be considered, as well as the effect of different topographic features.

Investigating the trade off between serving point based demand using a flow capturing approach and serving flow based demand using a point based approach provides an opportunity to observe what effect making an incorrect service assumption has on the facility locations in the network. This investigation is very valuable from the standpoint that if this effect seriously compromises the ability to provide convenient access to services, that planners using LA models must be sensitive to the type of demand being considered and identify it correctly. From the experiment, it is clear that when point based demand is served by facilities located using the FCLM, the ability to conveniently serve the demand is significantly compromised. For each value of p the amount of extra distance which must be traversed, using the FCLM rather than the p-median model, is significant. In fact, it appears that as p increases, the extra aggregate weighted distance to be travelled also increases. Evaluating the impact that using a p-median model service assumption has on flow based demand also shows that the ability of the resulting facility locations to provide convenient service is compromised. In the experiment, the sets of facility locations found are consistently unable to capture more than half of the flow captured using the FCLM solution. Furthermore, an increase in p does not result in any improvement in the service which is provided.

These results strongly underline the need for the planner to be aware of the type of demand for which facilities are being located. The impact of locating facilities using the incorrect service assumption is clearly too significant to ignore. These results lead me to suggest that research is needed to develop ways of correctly identifying the type of demand associated with the service being considered. Delving further into the trade off between the two objective functions, as Hodgson and Rosing (1992) did, is also an important research priority if we are to best model the correct service assumption for facility location models.

In the exploration of the objectives laid out in the introduction to this thesis, a number of important observations could be made. These observations add to the general knowledge of LA modelling, in specific our understanding of the usefulness of Cartesian distances in the context of the *p*-median model, the ability of the FCLM to locate facilities on a real world transportation network, and the impact that an incorrect service assumption has on the location of facilities. The knowledge consolidated here, therefore, provides further insight into how populations can be better served; the ultimate goal of the LA modeller. As a contribution to the LA literature, this thesis stands as a demonstration, comparison, and evaluation of two important location models as applied to a medium sized Canadian City; as well as providing a solid basis for future research projects.

BIBLIOGRAPHY

Berman, O., and D. Simchi-Levi (1988). "A heuristic algorithm for the travelling salesman location problem on networks." *Operations Research*, 36:478-84.

Berman, Oded, Richard C. Larson, and Nikoletta Fouska (1992). "Optimal location of discretionary service facilities." *Transportation Science*, 26:201-211.

Church, Richard L., and Charles S. ReVelle (1974). "The maximal covering location problem." *Papers of the Regional Science Association*, 32:101-118.

Church, Richard L., and Charles S. ReVelle (1976). "Theoretical and computational links between the *p*-median, location set-covering, and the maximal covering location problem." *Geographical Analysis*, 8:406-415.

Cooper, L. (1963). "Location-Allocation Problems." *Operations Research*, 11:331-343.

Cooper, L. (1964). "Heuristic methods for location-allocation problems." *SIAM Review*, 6:37-53.

Densham, Paul. (1990). LADSS, distributed by the National Center for Geographic Information and Analysis, State University of New York at Buffalo, Buffalo, New York 14261, U.S.A.

Densham, P.J., and G. Rushton (1992). " Strategies for solving large location-allocation problems by heuristic methods." *Environment and Planning A*, 24:289-304.

Erkut, Erhan, and Susan Neuman (1991). "Comparison of four models for dispersing facilities." *INFOR*, 2:[illegible].

Ford, L.R., and Fulkerson, D.R. (1962). *Flows in Networks*, Princeton: Princeton University.

Ghosh, A. (1979). "Changes in spatial efficiency of periodic markets." *Tijdschrift Voor Economische en Sociale Geografie*, 70:293-299.

Ghosh, A., and Gerard Rushton (1987). "Introduction." *Spatial Analysis and Location-Allocation Models*, New York: Van Nostrand Reinhold Company, pp 1-18.

Goodchild, M.F. (1979). "The aggregation problem in location-allocation." *Geographical Analysis*, 11:240-255.

Goodchild, M.F., and Valerian T. Noronha (1987). "Location-allocation and

Canada." (Unpublished).

Hodgson, M. John, and A. Leontien G. Storrier (1993). "Functional error in geographical analysis: The case of spatial separation in location-allocation." (Unpublished).

Kennelly, Robert A. (1968). "The location of the Mexican steel industry." In *Readings in Economic Geography : The Location of Economic Activity*, Ed. Robert H.T. Smith, Edward J. Taaffe, and Leslie J. King. Chicago: Rand McNally & Company, pp 126-157.

Kolesar, P.J. (1979). "Travel time and travel distance models." In *Fire Department Deployment Analysis*, Ed. W.E. Walker, J.M. Chaiken, and E.J. Ignall. New York: North Holland, pp 157-201.

Kuhn, Harold W., and Robert E. Kuenne (1962). "An efficient algorithm for the numerical solution of the generalized Weber problem in spatial economics." *Journal of Regional Science*, 4:21-33

Launhardt, W. (1882). "Die bestinmmuns des eweckmaessigsten standortes einer siewerblichen anlage." *Zeitschrift des Vereines Deutscher Ingenieure*, 26:106-115.

Lloyd, P.E., and P. Dicken (1977). *Location in Space: A Theoretical Approach to Economic Geography*. New York: Harper and Row, 3rd Ed.

Love, R.F., and J.G. Morris (1972). "Modelling inter-city road distances by mathematical functions." *Operational Research Quarterly*, 23:61-71.

Love, R.F., and J.G. Morris (1979). "Mathematical models of road travel distances." *Management Science*, 25:130-139.

Love, Robert F., James G. Morris, and George O. Wesolowsky (1988). *Facilities location: Models and methods*. New York: Elsevier Science Publishing Company Inc.

Maranzana, F.E. (1964). "On the location of supply points to minimize transport costs." *Operational Research Quarterly*, 15:261-270.

O'Kelly, Morton E. (1987). "Spatial interaction and location-allocation models." In *Spatial Analysis and Location-Allocation Models*, Ed. A. Ghosh and G. Rushton. New York: Van Nostrand Reinhold Company, pp 302-326.

Ostresh, Lawrence M. Jr. (1978a). "Convergence and descent in the Fermat location problem." *Transportation Science*, 12:153-164.

Ostresh, Lawrence M. Jr. (1978b). "On the Convergence of a Class of Iterative Methods for Solving the Weber Location Problem." *Operations Research*, **26**:597-609.

ReVelle C., and R. Swain (1970). "Central facilities location." *Geographical Analysis*, **2**:30-42.

Rosing, K.E., E.L. Hillsman, and Hester Rosing-Vogelaar (1979). "The robustness of two common heuristics for the *p*-median problem." *Environment and Planning A*, **11**:373-380.

Scott, Allen J. (1970). "Location-allocation systems: A review." *Geographical Analysis*, **2**:95-119.

Scott, Allen J. (1971). *Combinatorial Programming, Spatial Analysis and Planning*. London: Methuen & Co Ltd.

Tietz, Michael B., and Polly Bart (1968). "Heuristic methods for estimating the generalized vertex median of a weighted graph." *Operations Research*, **16**:955-961.

Toregas, C., and C. ReVelle (1972). "Optimal location under time or distance constraints." *Paper of the Regional Science Association*, **28**:133-143.

Transportation Department (1989). *1989 travel survey working paper: survey design, organisation and analysis*. Edmonton: City of Edmonton.

Transportation Department (1990). *Working Paper: Recalibration of the City of Edmonton regional travel model to 1989 base conditions and development of travel demand forecasts 1989 - long term*. Edmonton: City of Edmonton.

Transportation Department (1991). *1990 Transportation System Status Report*. Edmonton: City of Edmonton.

Weber, A. (1909). *Uber den Standort der Industrie*. Tubingen: Mohr.

Weiszfeld E. (1937). "Sur le point lequel la somme des distances de n points donnes est minimum." *Tohoku Mathematical Journal*, **43**:355-386.

Wesolowsky, George O. (1993). "The Weber problem: history and perspectives." *Location Science*, **1**:5-23.

Zanakis, Stelios H., and James R. Evans (1981). " Heuristic 'optimization': why, when, and how to use it." *Interfaces*, **3**:84-91.

Conversations :

Bakker, J., Professor of Transportation Engineering, Department of Civil Engineering, University of Alberta. January 26, 1992.

Brownlee, A., Supervisor, Forecasting and Transportation Planning Branch. Transportation Department, City of Edmonton. October 23, 1991.

Von Leiningen, S., Transportation Technician, Forecasting and Transportation Planning Branch. Transportation Department, City of Edmonton. January 23, 1992.

Location-allocation Software :

Location-allocation decision support system (LADSS). Developed by Paul J. Densham, and M.P. Armstrong (1990).

Other Location-Allocation software developed by M.J. Hodgson, University of Alberta.

Data :

The travel time data used in this thesis were kindly provided to me by the City of Edmonton Forecasting and Assessment Branch.

Chapter 1

Introduction

Services are for people. Depending on the services desired, people can either travel to a facility to access the service(s) or receive the benefit of that service at their location. For example, consumers travel to shopping and banking services, whereas fire and carpet cleaning services are delivered to the consumer. Planners should locate these facilities so that the services offered are accessible but respect budget, space, and political constraints. Both the economy and public (political) pressure play major roles in the decision making process at all levels of government and in the private sector. Added to the complexity of these constraints is the need to consider the location of a new facility or facilities within a system or network of existing or proposed facilities.

Location-allocation (LA) analysis deals with the location of systems of facilities. LA models optimally locate facilities, based on some objective such as reducing travel time on a transportation network, and allocate the demand population to these facility locations. The demand population is the clients requiring the service(s) offered by the facility: senior citizens, single parent families, school children, for example. A variety of LA models have been developed to meet the needs of consumers. Examples include the *p*-median model, which minimizes the aggregate weighted distance between services and customers; and the maximal covering location model (MCLM) which locates a pre-determined number of facilities to maximize the number of clients served. The LA model used by the planner depends on the objectives set out in the location problem. Regardless of the application for which LA modelling is used it is the overall intent that services are effectively delivered and that the client is satisfied. The interested reader is referred to Hodgson, Rosing and Shmulevitz (1993) for an excellent review of practical LA applications.

As may be expected, changes in the inputs of the LA model result in changes in the facility locations. The accuracy of the separation measure, or distance value, used to represent the amount of travel required by demand populations to access facility locations, has an important impact on the accuracy of the facility locations determined (Hodgson, 1991). A diversity of separation measures are available and used in LA modelling; for example: Cartesian or Manhattan metric, travel time, or network distance. Choosing which will best represent the actual travel requirements encountered by consumers is not always simple. Love and Morris (1972, 1979) specifically considered which distance functions most accurately portray travel distances between points in the transportation network.

Regardless of whether the correct or best separation measure is determined, it may not be easy or possible to collect this distance data. Collecting distance data is probably one of the most time consuming activities in LA modelling. This is particularly true if a non-Cartesian distance measure is desired to represent real world geographical regions where one-way streets, congestion and route capacities exist. Additional criteria such as shortest path travel time or distance, intermediate stops, arterial system also impacts the travel time experienced by the motorist, by providing high speed limit zones, more lanes, and rapid access to other areas in the city. An effort in transportation planning called the UNI project, which turned a number of roads and bridges into one-ways, and a recent overhaul of the signal timing system, have further improved travel within the city (Bakker, 1992).

Data

FAB divided Edmonton into a transportation zone system, most recently updated in 1983. Arterial roadways, major transit corridors, neighbourhood boundaries, and natural boundaries delineate a total of 177 transportation zones. The principle was to make the zones as homogeneous, in terms of the activities taking place in them, as possible (Brownlee, 1991). FAB also tried to ensure that these zones encompass the federal census tract enumeration areas as much as possible (Brownlee, 1991). Using simple visual estimation, I positioned a zonal centroid in the approximate centre of each transportation zone. I furthermore used these centroids to determine the distances between zones, thus, the location data is subjective, but provides a simple method of determining node locations.

Understanding the derivation of the time data is crucial to accepting its correctness, and hence the appropriateness of its use in this experiment. Time data was obtained from FAB, which has expended much effort in obtaining accurate a.m. peak hour time information to assist in their short and long range travel demand forecasting. The a.m. peak hour extends from 7:00 - 8:14 a.m., and reflects the peaks in tripmaking from outlying and 'close in' areas; the existence of peaking patterns is supported by actual traffic counts. As of 1990, their data is processed using the EMME/2 transportation system; software developed by INRO Consultants.

Understand that using a.m. peak hour data is not necessarily the most suitable for the location of many types of facilities. However, there is an advantage to using this data rather than off peak data. As noted in the introduction, I am concerned with the relative differences between the separation measures used. These differences are likely to be more significant in a.m. peak hour data due to the greater congestion and capacity problems encountered during these times. By comparing the two extreme cases then, Cartesian distance vs. a.m. peak hour travel time, the location error determined in this experiment is probably greater than it would be using off peak travel times. Therefore, the experiment shows more clearly the limitations of the distance data used here than if off peak time data were used.

The FAB calculated travel times by summing the times along each link on the shortest path between each origin and destination. Note that the time data provided is bi-directional, and reflects the time disutility incurred in each direction along the link. This better reflects the reality of travel experienced by the demand population than if the travel time had been assumed to be symmetrical. Specifically, the EMME/2 model uses link capacity and free-flow speed to calculate the link travel times, subject to specified volume/delay functions and the traffic volume assigned to the link (Working Paper, 1990). The FAB used the following formula to calculate the travel time (in

12	177730100	187824700	10094600	5.8	28.15	25.53	17
13	172082800	182263800	10171000	6.0	27.26	24.36	17
14	166531100	178516300	11985200	7.3	26.37	23.22	19
15	161375500	173038900	11663400	7.5	25.56	22.06	20
16	156300600	163203600	6903000	4.7	24.75	21.18	11
17	151483800	164405900	12922100	8.4	24	20.3	21
18	146834800	159069500	12234700	8.7	23.26	19.56	20
19	142697000	155067000	12370700	8.5	22.6	18.81	20
20	139945800	149443200	9497400	6.9	22.16	18.13	16
25	127901400	132590400	1325904	3.8	20.25	15.3	8
30	119291700	121864500	1218645	2.0	18.89	13.07	4
35	111385100	113941200	1139412	2.2	17.64	11.05	4
40	105354000	107639800	1076398	2.3	16.69	9.24	4
45	100010800	101355200	1013552	1.2	15.84	7.7	2
50	95146200	97048200	970482	1.9	15.06	6.35	3

impulsive shopping: The case of gasoline retailing." In *Spatial Analysis and Location-Allocation Models*, Ed. A. Ghosh and G. Rushton. New York: Van Nostrand Reinhold Company, pp 121-36.

Hakimi, S.L. (1964). "Optimum locations of switching centres and the absolute centers and median of a graph." *Operations Research*, 12:450-459.

Hakimi, S.L. (1965). "Optimum distribution of switching centres in a communication network and some related graph theoretic problems." *Operations Research*, 13:463-475.

Hillsman E., and R. Rhoda (1978). "Errors in measuring distances from populations to service centers." *Annals of the Regional Science Association*, 12:74-88.

Hodgson, M.J. (1990). "A flow capturing location-allocation model." *Geographical Analysis*, 22:270-279.

Hodgson, M.J. (1991). "Stability of solutions to the *p*-median problem under induced data error." *INFOR*, 29:167-183.

Hodgson, M.J., and P. Bayne (1990). "Accessibility implications of the multiservice centre concept for Edmonton." In *A World of Real Places*, Ed. P.J. Smith and E.L. Jackson. Edmonton: University of Alberta, pp 23-33.

Hodgson, M.J., and P. Doyle (1978). "The location of public services considering the mode of travel." *Socio-Economic Planning Sciences*, 12:49-54.

Hodgson, M.John, and Susan Neuman (1993). "A GIS approach to eliminating source C aggregation error in *p*-median models." *Location Science*, 2:155-170.

Hodgson M.J., and K. Rosing (1990). "A dual objective approach to the study of solution stability: Flow capturing and *p*-median." *EGI Discussiestukken*, Nr 90-4.

Hodgson M. John, and Kenneth E. Rosing (1992). "A network location-allocation model trading off flow capturing and *p*-median objectives." *Annals of Operations Research*, 40:247-260.

Hodgson, M. John, Kenneth E. Rosing, and Felix Shmulevitz (1993). "A review of location-allocation applications literature." *ISOLDE VI Survey Papers: Studies in Locational Analysis*,
Issue 5, pp 3-30.

Hodgson, M. John, Kenneth E. Rosing, and A. Leontien G. Storrier (1993). "An illustration of the flow capturing location model with network traffic in Edmonton,