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## **Web Slenderness Limits for Non-Compact Beams**

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## A B S T R A C T

Current specifications used for the design of steel structures in Canada place no limit on the web slenderness of sections used as non-compact beams. Designers customarily apply the rule for compact shapes (which, in turn, is also used for plastic design sections), a procedure which is unnecessarily conservative.

This report gives the results of an experimental study established to determine a suitable limit for the web slenderness of non-compact shapes used as beams. The results of a previous theoretical study on beam-columns are used to substantiate the experimental results. The limit suggested is significantly higher than that now being used.

## A C K N O W L E D G E M E N T S

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## INTRODUCTION

Depending upon its behavior at ultimate load, a steel flexural member is classified as non-compact, compact, or suitable for plastic design. Members suitable for plastic design must be able to reach their plastic moment capacity and, in addition, must be able to undergo considerable inelastic rotation before the onset of local buckling. Sections classified as compact must be able to reach their plastic moment before local buckling but there is no similar requirement with respect to rotation. Sections termed non-compact are those which need only to reach their yield moment before unloading occurs as a result of local buckling of the elements. The three categories of behavior are illustrated by the diagrammatic moment-rotation curves given in Fig. 1.1.

The compression elements of the cross-section under bending are those parts which will eventually suffer local buckling and cause the member to unload. Even though loaded by means of flexure, these parts are essentially under axial compression. The buckling strength of an axially loaded plate is largely an inverse function of its width-to-thickness ratio. This parameter is therefore a convenient means of defining whether a cross-section is non-compact, compact, or suitable for plastic design. Specifications and standards should therefore stipulate the upper limits of the width-to-thickness ratios for the flange and web elements corresponding to the three categories of beam behavior. Current Canadian Standards Association<sup>1</sup> requirements for webs and flanges are given in Fig. 1.2. No limiting ratio is specified for

the webs of non-compact beams. Designers would presumably apply the rule for compact sections (which also applies to plastic design sections).

As the requirements for each of the three categories (plastic design, compact, non-compact) are successively less severe, it seems unnecessarily conservative to apply the same web slenderness criterion throughout. A recent study has considered the problem of the limiting web slenderness for compact beams<sup>2</sup> and concluded that the value shown in Fig. 1.2 can be safely raised to  $520/\sqrt{F_y}$ . It was the purpose of the investigation presented herein to examine the limiting web slenderness for cross-sections classified as non-compact.

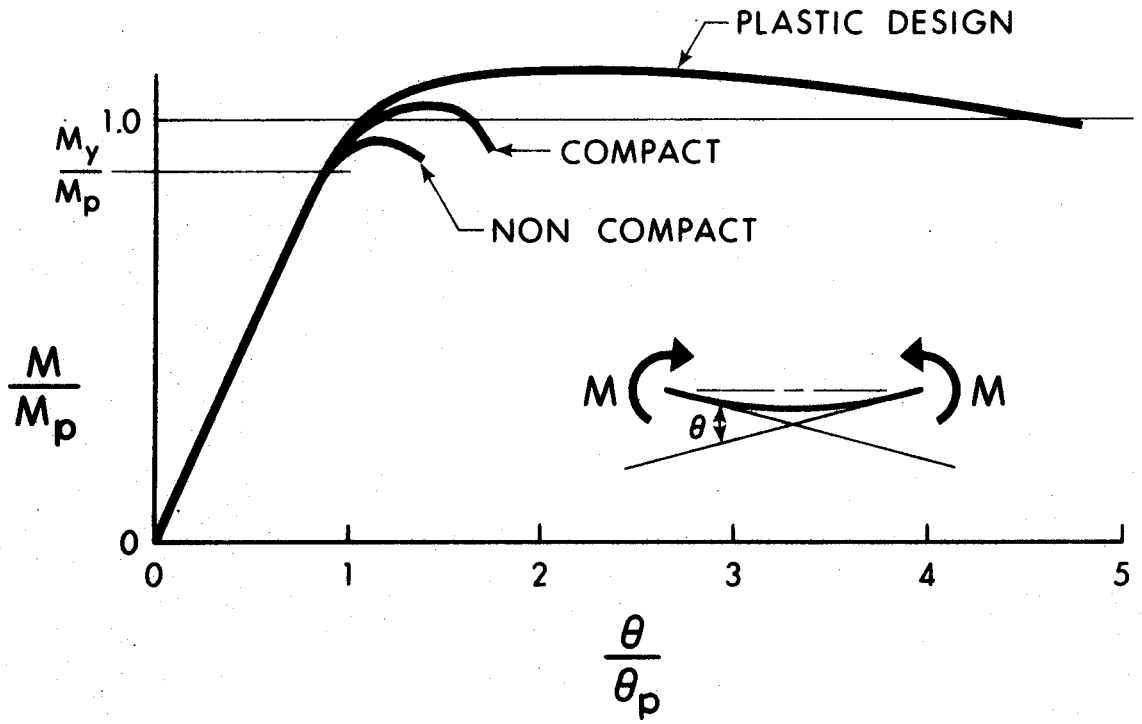


FIG. 1.1 TYPES OF MOMENT ROTATION BEHAVIOR

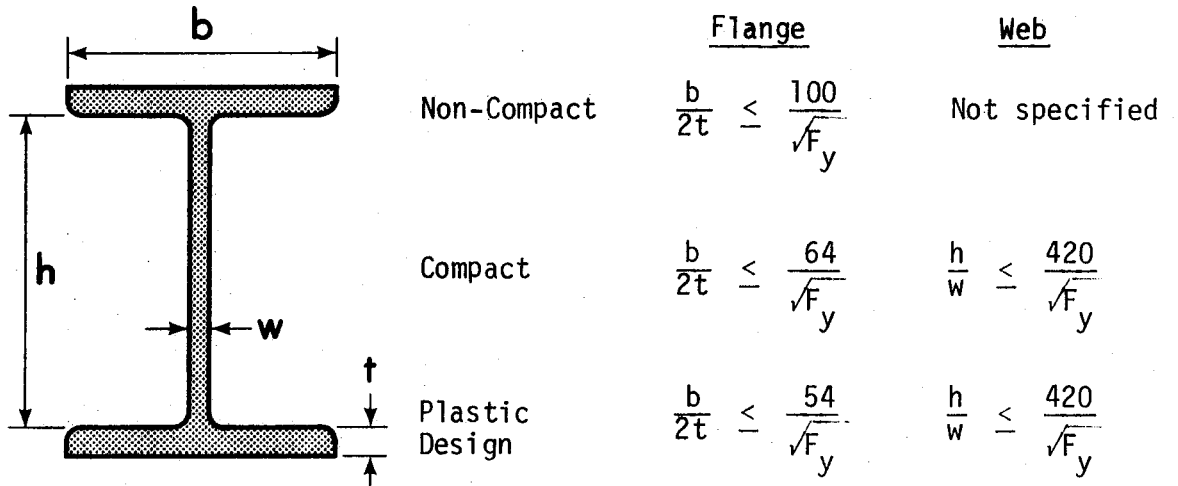


FIG. 1.2 PRESENT CSA S16 WIDTH-THICKNESS LIMITS

## 2. PREVIOUS INVESTIGATIONS

The first significant study on unstiffened plate girders was conducted at Lehigh University in 1935.<sup>3</sup> It was concluded from the results of tests on unstiffened plate girders that an upper limit of 80 on web slenderness ( $d/w$ ) was required if the shear capacity of the beam was to be reached. This is somewhat more liberal than present specifications would permit for the grade of steel used in the Lehigh tests.

Haaiker and Thurlimann reported in 1960 on an extensive theoretical and experimental study on cross-sections expected to have large plastic deformations without buckling<sup>4</sup>. They concluded that an appropriate limit was

$$\frac{d}{w} \leq \frac{370}{\sqrt{F_y}}$$

where  $d$  is the depth between flange centroids,  $w$  is the web thickness and  $F_y$  is the specified minimum yield stress (ksi).

In 1961, Basler conducted an extensive study on plate girders<sup>5</sup>. From the results of two tests in the series, it was suggested that the limiting web slenderness ratio for unstiffened girders might be raised to 100 ( $F_y = 33$  ksi). In terms of current specification nomenclature, this is equivalent to a limit of  $574/\sqrt{F_y}$ .

Tests on unstiffened, hybrid beams having web slenderness ratios of from 70 to 145 were reported by Carscaddan in 1968<sup>6</sup>. He



concluded that, in order for a beam to reach shear yield before local web buckling, the web slenderness ratio must be less than 67 ( $F_y = 36$  ksi). This is equivalent to  $h/w \leq 402/\sqrt{F_y}$ .

Research at the University of Texas<sup>7</sup> on unstiffened, continuous plate girders suggests that plastic design can be used for beams with web slenderness ratios as high as  $750/\sqrt{F_y}$ , providing that the shear stress is lower than the critical web buckling stress calculated from elastic theory.

Recently, research at the University of Alberta has been conducted on web slenderness limits for compact beams and for compact beam-columns. The first study<sup>2</sup>, by Holtz and Kulak, concluded on the basis of test results that the web slenderness limit for compact beams could be safely raised from the present value of  $420/\sqrt{F_y}$  to  $520/\sqrt{F_y}$ . The second study<sup>8</sup>, by Perlynn and Kulak, used both theoretical and experimental approaches to examine the problem of web buckling of beam-columns. The theoretical method verified the value for  $520/\sqrt{F_y}$  chosen in the earlier experimental study for compact beams. This method will be also used in connection with this report on non-compact beams.

### 3. EXPERIMENTAL PROGRAM

#### 3.1 Scope

Two specimens were used to provide experimental evidence of the behavior of non-compact beams. The flanges of these beams were proportioned so as to just meet the CSA S16 requirement as shown in Fig. 1.2. The webs were proportioned well above both the present limit and the proposed limit for compact webs.

In addition to these two tests, the results of certain previous tests can be included in the examination. These are beams which were tested under the program which developed rules for web slenderness limits for compact sections. They differ from the primary tests in this series in that their flanges were proportioned to meet the rules proscribed for compact sections, that is, their flanges are stockier. The test results which have been chosen for inclusion here include only specimens which failed initially as a result of web rather than flange buckling. Thus, their behavior can be considered as an adjunct to the present tests; caution must be used, however, as web and flange buckling cannot be considered to act independently. (An examination of the results of the work described in Ref. 2 shows that three specimens reported therein failed by web buckling. Only the results of two specimens, WS-3 and WS-4, will be used here, however. The third specimen, WS-6 showed lateral movement of the compression flange between brace points; it is likely that the bracing mechanism permitted distortion of the cross-section to occur<sup>9</sup>.)

All beams were simply supported and were subjected to a symmetric, two-point loading. The compression flange of each beam was braced in order to preclude the possibility of premature local buckling.

The loading arrangement required that stiffeners be placed on the beam webs at the load and reaction points. In the region of high moment, these stiffeners extended less than half the depth of the beam and were located only on the tension side of the web. It was felt that the influence of the stiffener on the buckling behavior of the web would therefore be minimal and that the member could be considered effectly unstiffened.

### 3.2 Description of Specimens

Only the two specimens directly concerned with the present investigation will be described here. There were a few very minor differences for the beams tested in the earlier program<sup>2</sup>. These differences, however, are not considered to have any effect on the results.

The beams were fabricated from CSA G40.12 plate<sup>10</sup>, which has a specified minimum yield strength of 44 ksi for plates up to 1-1/2 in. thick. The flanges of the specimens were 11 in. by 3/8 in., resulting in a width to thickness of 14.67. This meets the non-compact limit of 15.1 as specified by CSA S-16 for steel with a yield point of 44 ksi.

The webs of the two specimens were 27 and 29 inches deep by 1/4 in. thick, giving nominal width-to-thickness ratios of 108 and 116, respectively. These were chosen as likely non-compact limits from the results of the earlier series of tests on compact beams. For comparison,

the present allowable web slenderness for steel with a yield point of 44 ksi is 63.

The beams were proportioned so that the shear stress developed when the beam reached the yield moment would be less than the inelastic buckling shear stress.

Horizontal bracing was provided at the load and reaction points. At the load points, a threaded rod was welded to the compression flanges in the plane of the web and a bracing arrangement braced on Watt's straightline mechanism<sup>9</sup> was attached. This arrangement effectively prevents any lateral movement of the compression flange but does not restrict vertical deflections in the plane of loading, nor does it restrict torsional rotations or local buckling of flange or web. At the reaction points, lateral movement of the tension flange was prevented by means of threaded rods running from the specimen to the test frame. It was felt that the reaction hangers and loading jacks, because they were stressed in tension and were attached to the compression flange and tension flange respectively, also provided some measure of torsional restraint to the specimen.

### 3.3 Test Set-up and Procedure

Load was applied by means of two hydraulic rams acting in tension and suspended from a testing frame. The jacks were pin-connected to lugs which were welded to the tension flange in the central portion of the beam. The beam reactions were taken by hanger rods passing through the floor of the laboratory. These were also pin-connected to lugs welded to the compression flange at the ends of the beam.

Load was measured by means of calibrated electric resistance load cells attached to each jack. Vertical deflections were measured at the load and reaction points by means of dial gages mounted on pedestals and with the plungers bearing against the compression flange. Mechanical rotation meters were used to measure rotation and these readings were taken at the load and reaction points. The meters were attached to the stiffeners on the web.

Out-of-plane deflections of the web were measured by means of an apparatus consisting of three dial gages mounted on a light but rigid frame. The frame consisted of two legs and a mounting bracket that held the gages in position relative to the legs. The dial gages measured deflections relative to a chord joining the tips of the legs. After initially calibrating the instrument by placing it on a surface known to be flat, out-of-plane deflections of the web could be obtained.

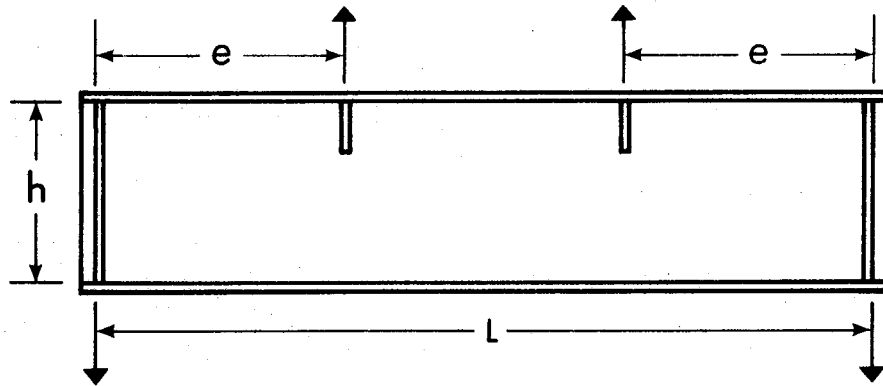
The deflection of the flange tips relative to each other was also measured. This was done by dial gages mounted on magnetic stands on the outside tip of the compression flange with a thin wire running from the plunger to the outside tip of the tension flange.

Web and flange deflections were taken at the same longitudinal locations along the beam. The locations were roughly at the midpoint between load and reaction and approximately one foot on each side of each load. This resulted in readings being taken at locations of high shear and low moment, at high shear and high moment and at high moment and zero shear.

Load was initially applied in increments of one fifteenth of the expected yield load until the plot of load versus vertical deflection

that was maintained during the test indicated that yielding had begun. After that point, the load was controlled by enforcing increments of equal deflection. At each increment, load was allowed to stabilize as were all deflections before any readings were taken.

The general configuration of the specimens is shown in Fig. 2.1. Also shown are the principal dimensions of the two test specimens specifically established for this program as well as the two specimens from the program on compact beams.



SPECIMEN	WEB DEPTH $h$ (in)	$L$ (in)	$e$ (in)	$h/w$	$(h/w)\sqrt{F_y}$
WS-12-N	26.9	192	78	104.1	675
WS-13-N	29.0	216	90	113.0	732
*WS-3	30.0	240	99	115.3	744
*WS-4	35.0	288	126	137.8	890

\* FROM REFERENCE 2

ALL WEBS FABRICATED FROM 1/4 in. THICK PLATE

FIG. 3.1 CONFIGURATION OF SPECIMENS

## 4. TEST RESULTS

### 4.1 General

Both of the non-compact beam specimens fabricated for this program had flanges and webs made from the same rollings. Using standard tensile coupons, it was established that the plate used for the flanges had a static yield strength of 34.7 ksi. The plate used for the webs showed that the static yield stress for this material was 42.0 ksi.

The ultimate bending capacities of the two non-compact beams as well as the two specimens tested earlier are listed in Table 4.1. These results are also shown in Fig. 4.1 where the ratio of actual ultimate bending capacity to yield moment is plotted as a function of web slenderness as measured by  $(h/w)/\sqrt{F_y}$ . It can be seen that a reasonably regular progression occurs between web slenderness and beam strength.

Non-dimensionalized relationships between moment capacity and rotation are shown in Fig. 4.2. These beams displayed the expected behavior of non-compact sections, that is, they reached the yield moment and then the capacity decreased. The rotations plotted are those relative rotations occurring between the load points. Both the moments and rotations have been non-dimensionalized so that comparisons can easily be made.

Fig. 4.3 is a non-dimensional plot of the measured out-of-plane deflection of the web versus the vertical deflection of the load point. The locations at which the deflections were measured are shown in the insets to the figure. The deflection of the flanges, measured



at the same locations as the web deflections, are shown in Fig. 4.4. The flange deflection is simply the movement of the compression flange tip relative to the tension flange tip on the same side of the beam.

Comparing the data in Figs. 4.3 and 4.4, it is seen that, in each case, the web started to deflect at an increased rate slightly before the flange did. This indicates that the web began to buckle before the flange, a conclusion supported by visual observations during the test.

In the previous investigation which involved compact beams<sup>2</sup>, it was concluded that the amount of shear present did not have a significant influence on the moment capacity but that initial web deflections apparently were of some consequence. Obviously, the two non-compact specimens tested here will not provide enough information to state whether or not initial web deflections were influential. The initial web deflections for the two specimens were 0.0061 and 0.0056 in. Taken over the corresponding depths, these are well below the permissible values for initial out-of-flatness given by CSA W59.1<sup>11</sup>.

## 4.2 Results and Conclusions

If only the results of the two non-compact sections are considered, a straight-line projection as shown on Fig. 4.1 indicates that a web slenderness limit,  $(h/w)\sqrt{F_y}$ , of 773 would be satisfactory for non-compact beams. Drawing a curve through all four points on the plot gives a value of about 770.

The work on beam-columns done by Perlynn and Kulak<sup>8</sup> includes a theoretical development for predicting the strength of compact members under both axial load and moment. This proved to be in good agreement with the experimental results and it can be adapted for use here. In its general form, the relationship developed was

$$\alpha = \frac{h}{100w} \sqrt{F_y} \sqrt{\frac{0.01241}{1 - 0.695 \left(\frac{P}{P_y}\right)^{0.3846}}} \quad 4.1$$

where  $\alpha = \frac{\sigma_{cr}}{\sigma_y}$

$\sigma_{cr}$  = stress at time of buckling

$\sigma_y$  = yield stress of material  $\equiv F_y$

P = applied axial load

$P_y$  = yield load

A value of  $\alpha = 0.77$  was determined (on the basis of one test) by Haaiker and Thurlimann in the work from which Eqn. 4.1 was derived<sup>4</sup>. Using this value, and setting the term  $P/P_y = 0$ ;

$$\frac{h}{w} \sqrt{F_y} = 690 \quad 4.2$$

Eqn. 4.2 represents the theoretical limit at which a beam will just reach the yield point of the material before buckling occurs in the web. Reference 8 develops another expression, similar to Eqn. 4.1, which represents the limit for attaining the yield moment before buckling in either the web or flange occurs. This is

$$\alpha = \frac{h}{100w} \sqrt{F_y} \sqrt{\frac{0.01225}{1 - 0.619 \left(\frac{P}{P_y}\right)^2} 0.3846} \quad 4.3$$

Again using  $\alpha = 0.77$  and setting the applied axial load equal to zero, the solution becomes

$$\frac{h}{w} \sqrt{F_y} = 695 \quad 4.4$$

If the web slenderness limits obtained theoretically are considered the "correct" values, then the results obtained experimentally verify the theoretical prediction to within about 12%. This level of agreement is reasonable when it is considered that the number of tests in the experimental work is small and that the theoretical development itself uses certain quantities derived on the basis of tests.

TABLE 4.1 TEST RESULTS

Specimen	$\frac{h}{w} \sqrt{F_y}$	$M_u$ in.-kips	$\frac{M_u}{M_y}$	Buckled Element
WS-12-N	675	5367	1.038	Web
WS-13-N	732	5696	1.016	Web
*WS-3	744	5740	1.003	Web
*WS-4	890	6820	0.977	Web

\*From Ref. 2

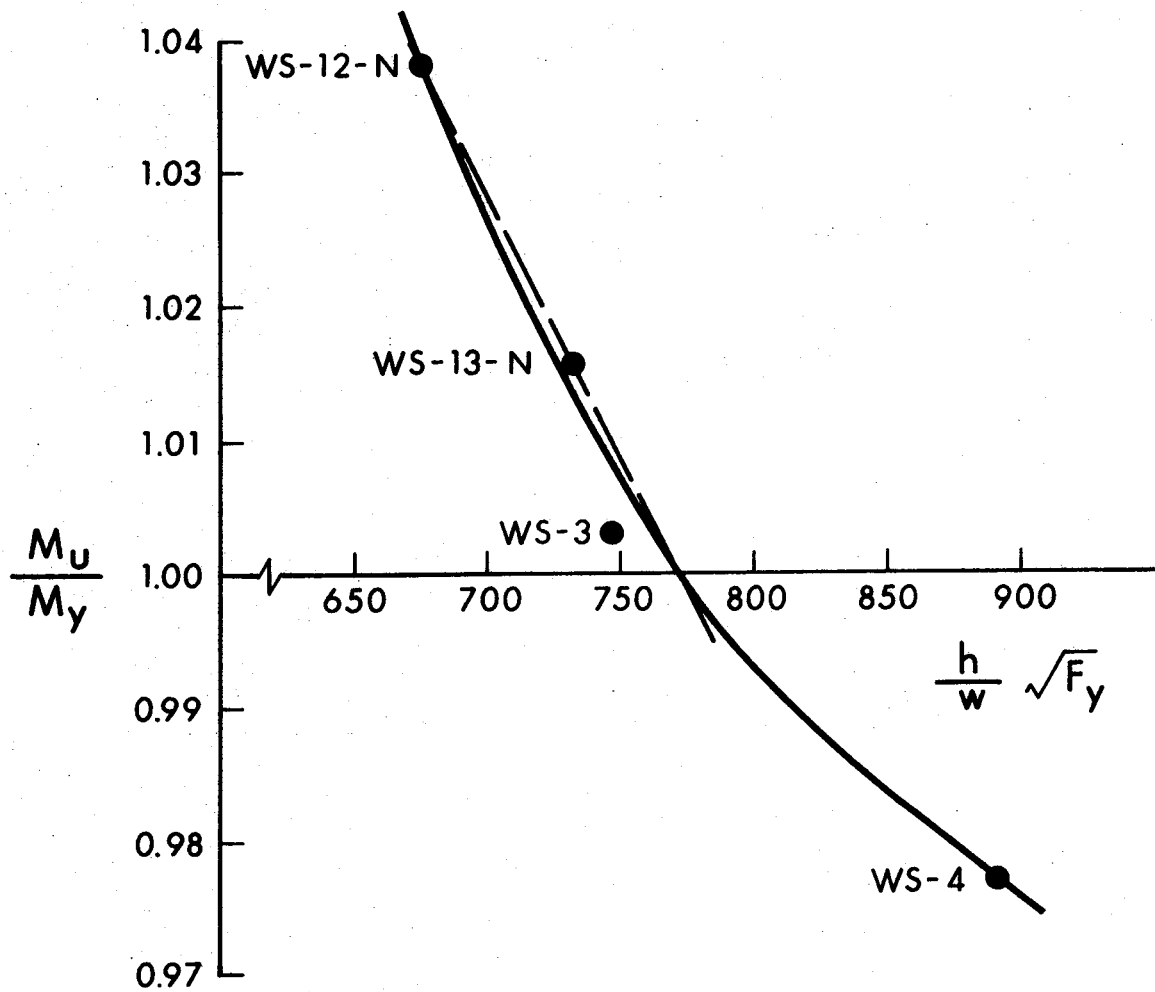


FIGURE 4.1 EFFECTS OF WEB SLENDERNESS ON MOMENT CAPACITY

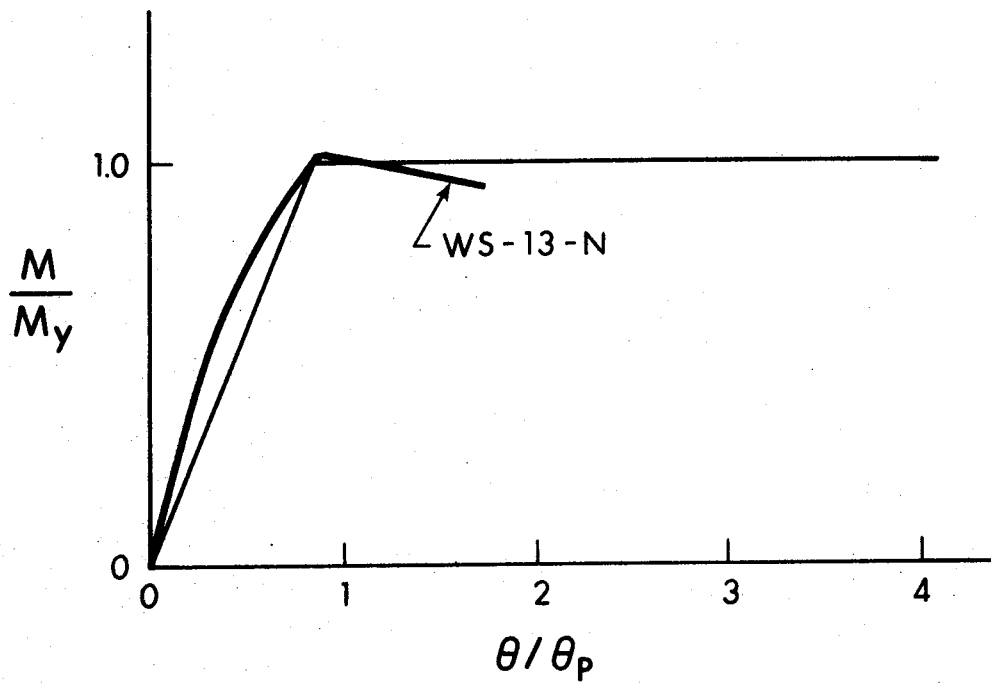
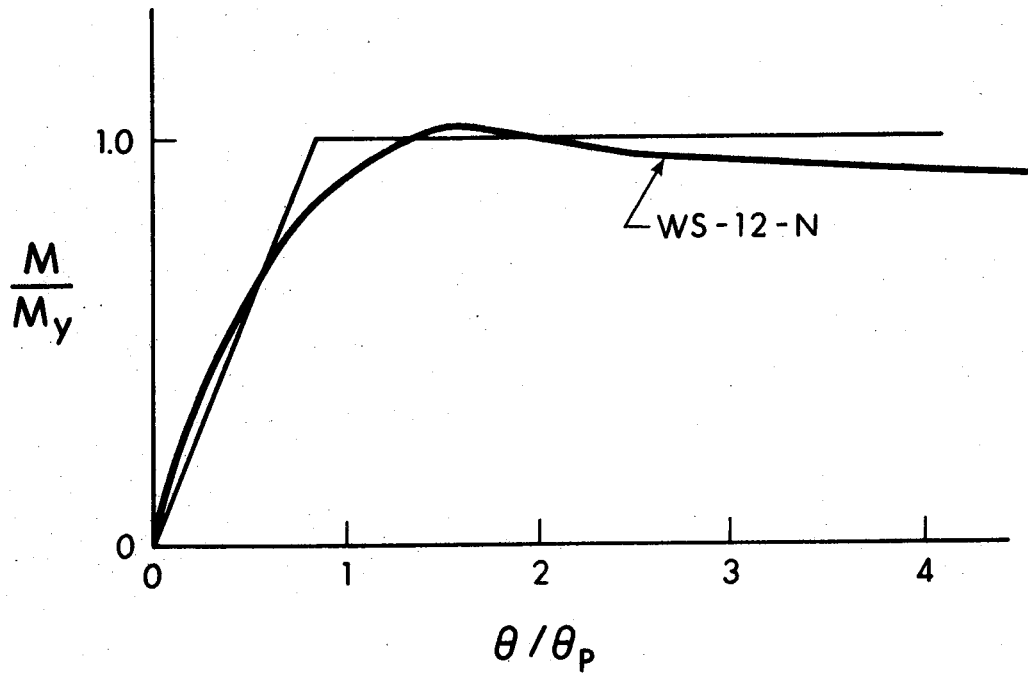


FIG. 4.2 MOMENT VS ROTATION BEHAVIOR

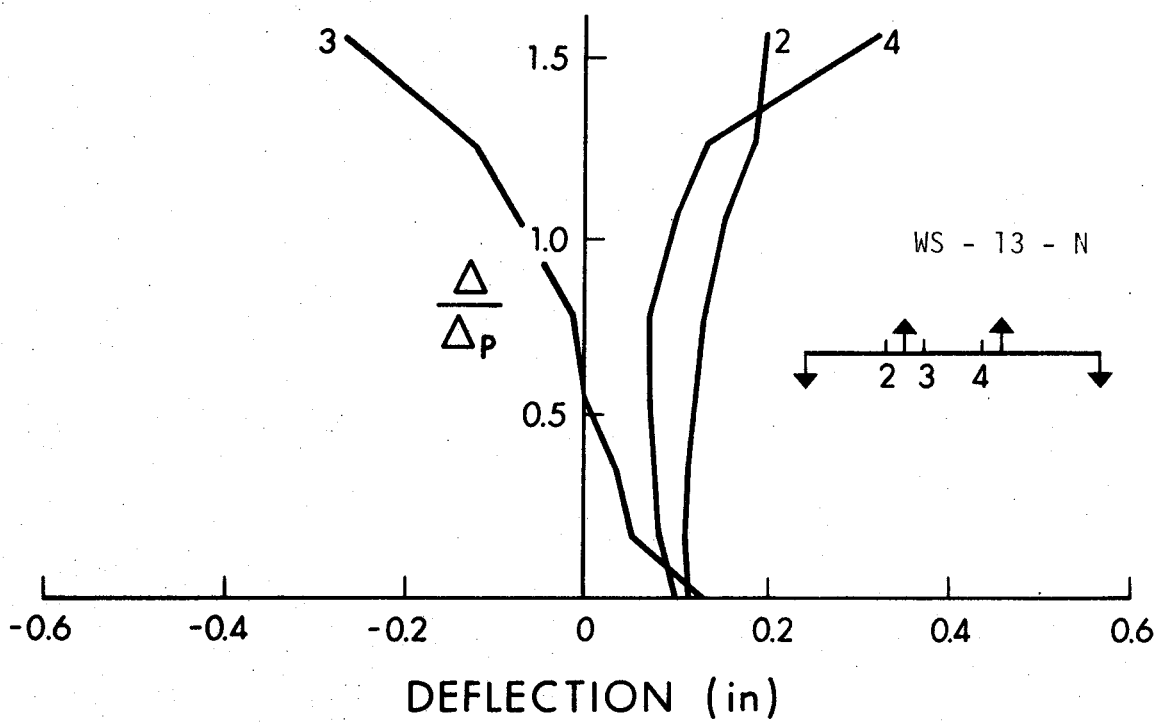
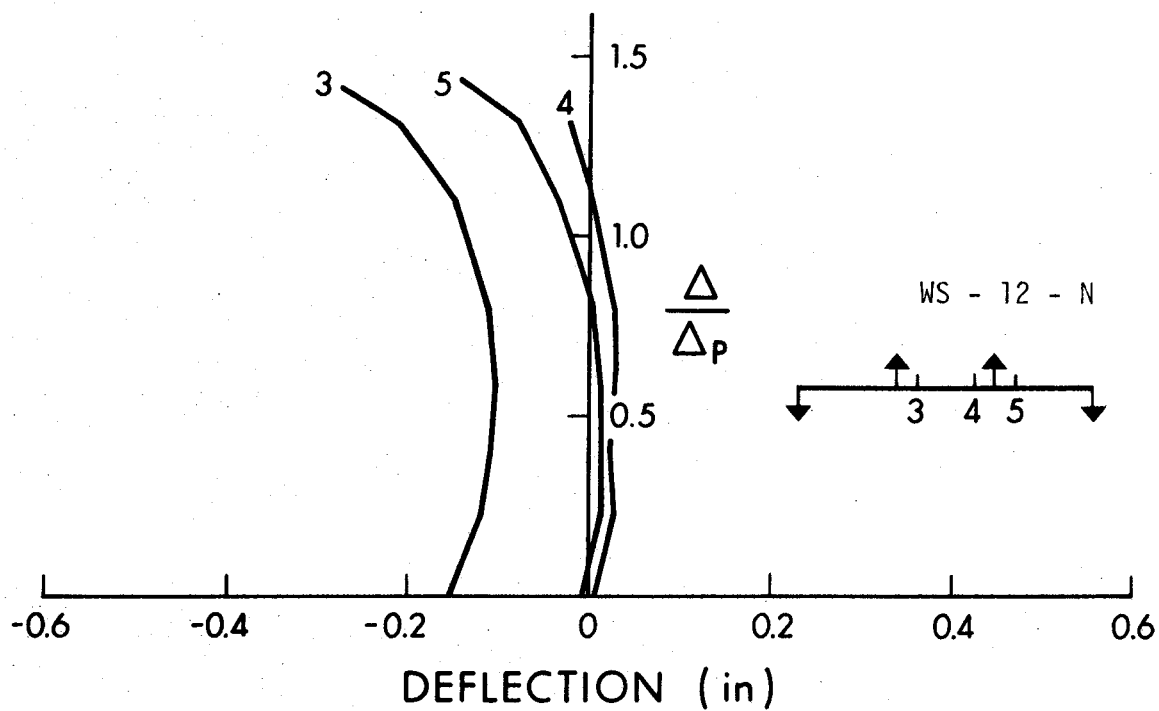


FIG. 4.3 WEB DEFLECTIONS

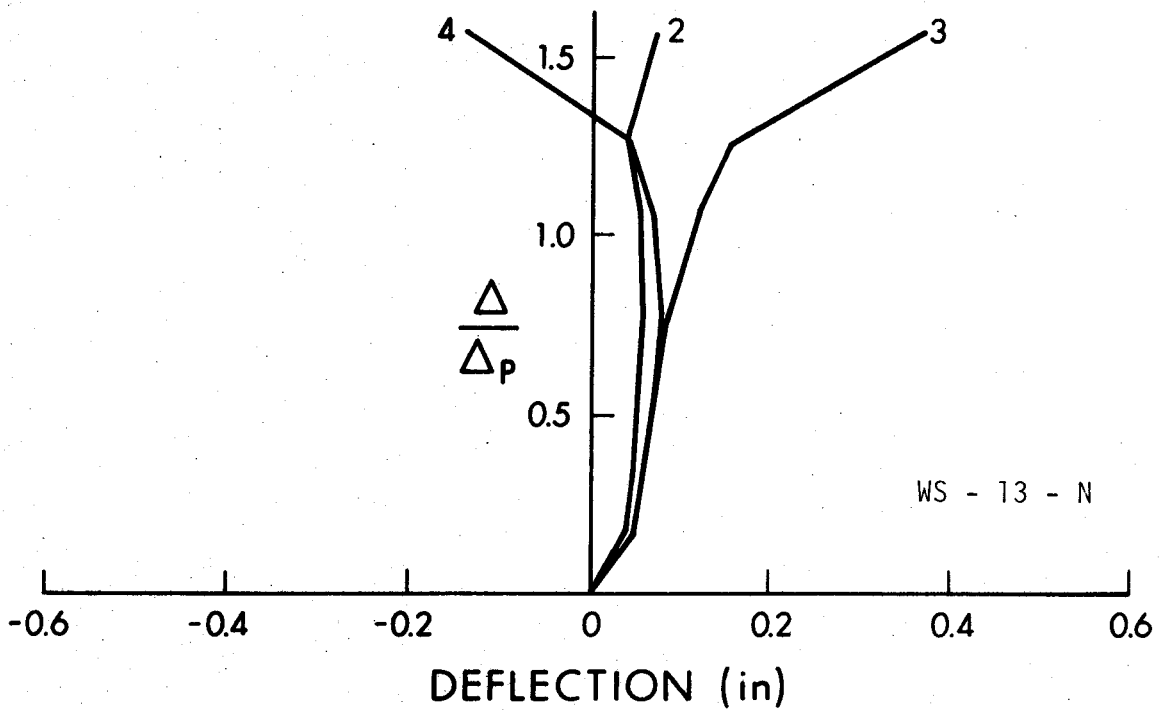
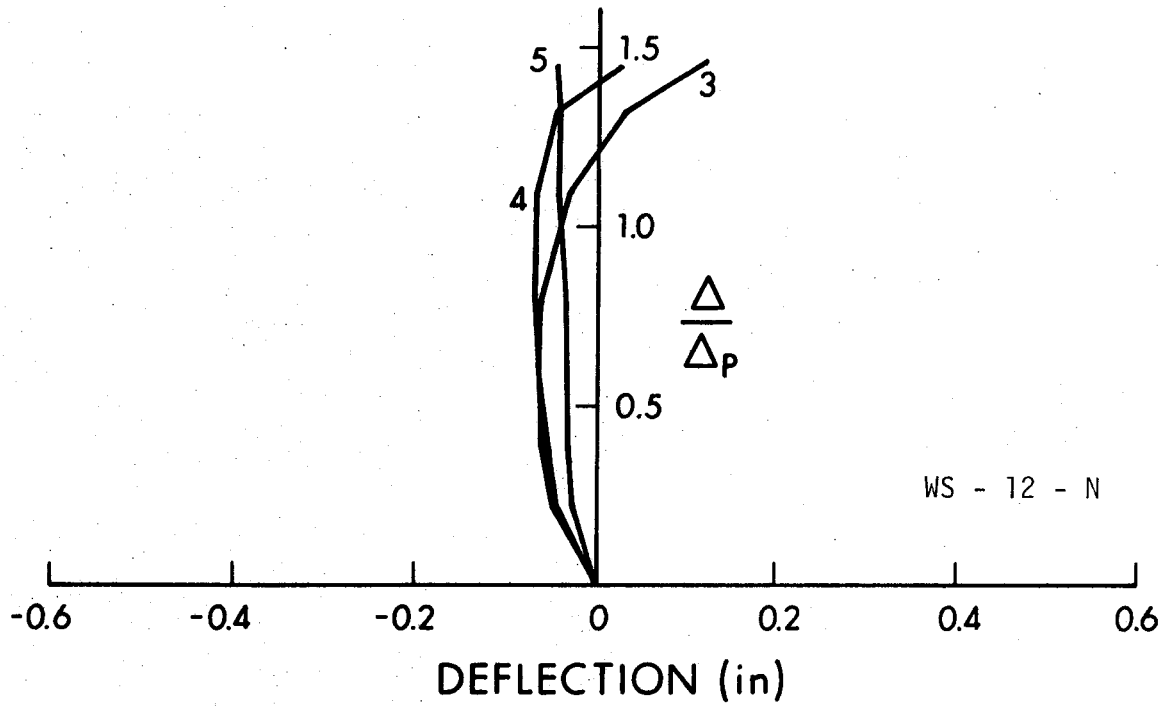


FIG. 4.4 FLANGE DEFLECTIONS



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