University of Alberta

Hierarchical Location-Allocation Modeling Based on Spatial Interaction

BY

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Table of Contents

List of Tables

Table 5.7: Benefit provided by facilities of different levels in the actual system and the spatial choice-based solutions 69

List of Figures

List of Abbreviations

AWD	Average Weighted Distance
BDD	Benefit/Demand Disproportion
IIA	Independence from Irrelevant Alternatives
LA	Location-Allocation
SI	Spatial Interaction

Chapter 1. Introduction.

1.1. Hierarchical systems and location-allocation models

People use **services**. Some services are provided by **facilities**, so that people **(patrons)** must travel to a facility to obtain a service. Some services have a hierarchical (multi-level) structure, i.e. services may be conceived of as being of different levels: some services may be more time-consuming, costly and valuable than others. **The level of service** characterizes a service's cost, demand and utility. Low-level services cost little to provide and are frequently required. High-level services are more costly to provide and are less frequently required. Examples of low-level services are first aid in medicine and cash withdrawal in banking. Examples of high-level services are heart surgery in medicine and large-scale investing in banking. The higher the level of service, the farther people will travel to obtain it. We must also distinguish **the level of facility**, which indicates the highest level of service offered by a particular facility. Because there are different levels of services and facilities it is often useful to organize them into hierarchical systems. A **hierarchical system** consists of several levels of facilities, which collectively provide goods and/or services (Narula, 1984).

Spatial hierarchies may be divided into those which are successively inclusive and those which are successively exclusive (Narula, 1984). In a **successively inclusive** hierarchy a higher-level (level k) facility offers services unique to it as well as services available at the next lower-level (level $k-1$) facility (Eitan, Narula, and Tien, 1991). An example of a successively inclusive hierarchy is a health care system consisting of three levels.

- Level 1. Local/village health centre: first aid and preventive services (a general practitioner and assistant).
- Level 2. Community health centre: first aid with preventive services *plus* some therapeutic procedures.
- Level 3. Medical centre: first aid with preventive services, some therapeutic procedures, *plus* specialized and curative services.

Planners and businessmen often face the task of locating facilities so that the services offered are accessible, but also respect various constraints (budget, space availability, policy). The complexity of the constraints, the large number of combinations of possible facility locations and the necessity to consider the level of facility make this task complicated. Location-allocation (LA) analysis helps to accomplish this task. It deals with the location of facility systems and allocation of the population's demand to them while providing the highest possible convenience. Various LA models have been developed in recent decades, some of them dealing with hierarchical systems[1].

1.2. Study objectives

The p-median LA model minimizes the aggregate weighted distance from patrons to facilities. It was modified for spatial hierarchies by, *inter alia*, Calvo and Marks (1973), Harvey, Hung, and Brown (1974), Banerji and Fisher (1974), Narula and Ogbu (1979), Tien and El-Tell (1984), Narula and Ogbu (1985), Serra and ReVelle (1994) and Galvao, Espejo, and Boffey (2002)[2]. Dealing with spatial hierarchies increases the complexity of the model because solutions involve determining both the locations and

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[1] In this work I consider successively inclusive spatial hierarchies. For brevity, I will skip the words "successively inclusive".
[2] Hereafter, I organize the reference enumeration in a chronological order.

hierarchical levels of facilities. In the works mentioned above, the hierarchical p-median model either was solved for relatively small data sets or some heuristic (approximation) procedures were applied (the latter do not guarantee the optimality of the solution). In recent years, computer hardware has been much improved and mathematical programming software (solvers) have been developed. **The first objective of this study is to apply a modern solver (CPLEX 6.5.1) to the hierarchical p-median model for problems of relatively large size (three levels, 150 demand nodes and the same number of potential facility sites).**

The p-median model is widely used, in spite of several shortcomings that are undesirable in the context of LA analysis. One of these is the least-cost allocation rule – the model allocates patrons to the nearest facility. Real-world observations, *inter alia* Hodgson (1981), Kloos (1990) and Oppong (1992), have shown that patrons frequently bypass the nearest facilities. Consequently, the p-median model based on the unrealistic least-cost allocation rule does not provide the highest possible convenience to the patrons.

To overcome this problem, Hodgson (1978, 1981), O'Kelly and Storbeck (1984), Hodgson (1984, 1986, 1988), Oppong (1992) and Oppong and Hodgson (1998) incorporated spatial interaction (SI) models for LA analysis. The authors used heuristic procedures to solve their models. **The second objective of this study is to formulate a previously developed interaction-based LA model in mathematical optimization terms and to solve it CPLEX 6.5.1.**

In the 1980-90s, spatial interaction theory experienced a new stage of development in response to evidence that previously developed SI models were underspecified. **The third objective of this study is to employ a recently developed SI model (the spatial choice model) in LA analysis, and to compare its performance with previously developed ones.**

A hierarchical system is characterized by two features: its hierarchical structure and its spatial configuration. **Hierarchical structure** implies the composition of the number of levels of facilities; **spatial configuration** refers to the locational patterns of facilities in a region (Okabe, Okunuki, and Suzuki, 1997). Most of the previous LA models concentrated on the spatial configuration of a system, assuming the number of facilities at each level to be given. **The fourth objective of this study is to develop an LA model which optimizes both the spatial configuration and structure of the hierarchy.**

1.3. Study area

The LA models will be demonstrated with the data set presented by Oppong (1992). This is the data covering health care facilities location in the Suhum district, the Eastern region, Ghana (Fig. 2.1). The actual health care system was three-level, consisting of twenty-three low-level (village medical rooms), six middle-level (community health centres) and one high-level (district hospital) facilities (Fig. 2.2). The population of Suhum was 102,481 (1984); the area was 877.5 square kilometres. The district capital, Suhum, had 19,298 people; it was the hub of most activities in the district. This was a predominantly rural area with 150 populated places; the population of each place is used as the proxy for demand. Walking was the most popular mode of travel in the district; few people could afford buying a motor-bike or even a bicycle. I use

Euclidean metrics based on the projected coordinates[3] to measure the distance between populated places. The full list of the populated places, their X, Y coordinates and population value is given (Appendix I).

1.4. Thesis structure

The work is organized in the following way. In Chapter 2, I show the traditional approach to hierarchical LA analysis. Special emphasis is placed on hierarchical *p*-median models. A brief literature review is followed by a demonstration of the hierarchical *p*-median model and its optimal solution for the study area. In Chapter 3, I review previously developed interaction-based LA models and formulate one of them (the Batty-based model) as a mathematical optimization. In Chapter 4, a recently developed SI model of spatial choice is employed in the LA framework and I perform a sensitivity analysis to demonstrate the flexibility of the new LA model. Hierarchical structure optimization issues are considered in Chapter 5. Lastly, I bring forward conclusions and recommendations in Chapter 6.

[3] The Transverse Mercator projection was used to get X and Y coordinates.

Chapter 2. Traditional hierarchical LA approaches

2.1. Literature review

Traditional hierarchical LA models can be divided into two classes depending on their objective: covering and minisum (Narula, 1984). Covering models locate facilities to maximize the number of clients served. These were extensively applied for services which *are delivered* to patrons, such as emergency services (for a review of LA model applications see Hodgson, Rosing, and Shmulevitz, 1993). Minisum models were extensively applied to services assuming that *patrons travel to a facility*, for example shopping or non-emergency health care. These minimize the distance travelled by patrons. In this work I assume that people travel to get their services, so I do not discuss hierarchical covering LA models. The interested reader is referred to articles by Schilling *et al.* (1979), Charnes and Storbeck (1980), Moore and ReVelle (1982), Church and Eaton (1987), Serra, Marianov, and ReVelle (1992), Gerrard and Church (1994), Marianov and Serra (2000) and Church and Gerrard (2003).

The first studies of hierarchical spatial systems arose in the field of Economic Geography: Christaller (1933) and Losch (1956) developed Central Place Theory. The appearance of single-level LA models with the minisum objective, for example, ReVelle and Swain (1970), gave an impetus to developing LA models for hierarchies. Schultz (1970) and Calvo and Marks (1973) formulated the first hierarchical LA models, but did not provide solution methods. Later, piecemeal level-by-level solution techniques were introduced and used by Dokmeci (1973), Banerji and Fisher (1974), Harvey *et al.* (1974), Dokmeci (1977, 1979). Fisher and Rushton (1979) concluded that solution can be approached in three ways:

- The highest-level facilities are located first and subsequent lower level places are constrained to include them; the *top-down method.*
- The lowest level facilities are located first and subsequent higher facilities are selected from them; the *bottom-up method.*
- The middle level facilities are located first and then higher level facilities are selected from them and lower level ones constrained to include them; the *middle-out method.*

Hodgson (1984) demonstrated that piecemeal approaches are inferior to the simultaneous location of facilities of all hierarchical levels. He adapted the Teitz and Bart (1968) vertex substitution heuristic to a three-level hierarchy. Narula and Ogbu (1979) tested five different heuristic procedures to obtain simultaneous solutions for a two-level hierarchical system with referral[4]. Narula and Ogbu (1985) proposed a solution procedure that resulted in a lower bound for the same problem and in some instances provided an optimal solution. Weaver and Church (1991) provided a solution technique for the non-referral uncapacitated hierarchical LA model, tested it for a three-level thirty-node data set, and obtained optimal or near-optimal solutions. Eitan *et al.* (1991) proposed a general hierarchical LA model considering referrals, capacity constraints, and variable and fixed costs, but the proposed model was too complex and no solution technique was provided. Okabe *et al.* (1997) presented a model which optimizes both the spatial organization and the hierarchical structure, solving the model using heuristics based on Voronoi polygon construction. The authors assumed that the solution space is infinite, i.e. the model was

[4] In the referral type of hierarchy some part of the demand can be referred from one level to another. For example, a patron with a special need can travel to an inappropriate level of facility (a local health centre) and then be advised to travel to a higher-level facility (hospital).

continuous. Galvao *et al.* (2002) developed a three-level LA model with referral and solved it optimally with CPLEX 7.5. However, the model was optimally solved only for small data sets (10 and 15 demand nodes); for larger problems heuristic procedures were developed and applied.

2.2. The hierarchical *p*-median model

Assuming that patrons are concentrated in populated places (demand points, or nodes) and that there is a definite number of possible facility sites, the hierarchical successively inclusive *k*-level *p*-median model can be formulated as:

$$Minimize \sum\sum\sum U_k W_i d_{ij} X_{ij}^k$$

$$Minimize \sum_{k\in\mathbf{K}}\sum_{i\in\mathbf{I}}\sum_{j\in\mathbf{J}} U_k W_i d_{ij} X_{ij}^k \qquad (1)$$

Subject to :

$$\sum_{j\in\mathbf{J}} X_{ij}^k = 1, i \in \mathbf{I},\ k \in \mathbf{K} \qquad (2)$$

$$\sum_{\substack{k\geq k^* \\ (k\in\mathbf{K})}} Y_j^k \geq X_{ij}^{k^*}, i \in \mathbf{I}, j \in \mathbf{J}, k^* \in \mathbf{K} \qquad (3)$$

$$\sum_{j\in\mathbf{J}} Y_j^k = p_k, k \in \mathbf{K} \qquad (4)$$

$$\sum_{k\in\mathbf{K}} Y_j^k \leq 1,\ j \in \mathbf{J} \qquad (5)$$

$$X_{ij}^k \in \{0,1\}\ i \in \mathbf{I}, j \in \mathbf{J},\ k \in \mathbf{K} \qquad (6)$$

$$Y_j^k \in \{0,1\}, j \in \mathbf{J}, k \in \mathbf{K} \qquad (7)$$

The *decision variables* are X_{ij}^k, which equals 1 if the patrons from node *i* are allocated to a level *k* facility at location *j,* and 0 otherwise; and Y_j^k, which equals 1 if a level *k* facility is located at location *j,* and 0 otherwise. The *parameters* of the model are p_k, which is the number of level *k* facilities; **J**, which is the set of potential facility sites;

I, which is the set of demand nodes; **K**, which is the set of hierarchy levels[5]; W_i , which is the demand (the number of patrons) concentrated in node *i*; d_{ij} , which is the distance between locations *i*, and *j* and U_k , which is the proportional usage of facilities at level *k*.

The objective function (1) minimizes the aggregate weighted travel distance from demand points to the nearest facility sites at each level. Constraint (2) ensures that all demand for all services at all *i*'s is served; constraint (3) states that service can only be obtained at points where appropriate level facilities are located (the service of k^* level can be obtained at a k level facility, such that $k \geq k^*$); constraint (4) specifies the number of facilities of each level to be located; constraint (5) states that each location can have at most one facility; constraints (6) and (7) make all decision variables binary.

The constraint set (3) enforces the successive inclusiveness property and makes the model more difficult to solve than the single-level one. For the study area (150 demand nodes and the same number of potential facility sites) I used CPLEX 6.5.1 to obtain the optimal solution of the model[6]. The number of facilities at each level was taken to be the same as in the actual system, i.e. one high-level, six middle-level and twenty-three low-level facilities. The proportional usage parameter (U_k) was estimated by Oppong (1992) and equals 0.188, 0.203 and 0.609 for high-, middle- and low-level services respectively.

A *p*-median solution can be evaluated by the average weighted distance (AWD) value, i.e. the average distance a patron travels to get service of the appropriate level. The lower this value the better facilities are located according to the *p*-median principle;

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[5] Hereafter, I assume that the higher level of service the higher its ordinal number, for example low-level service is 1st level service, middle-level is 2nd level service, etc.

[6] The execution time is 21 sec. All calculations for this thesis were done on a personal computer with Intel ® Pentium 4 CPU 2.80 GHz, 1GB RAM.

correspondingly the optimal p-median solution gives the minimum AWD. The optimal solution was compared to the actual system (Table 2.1)[7]; it is also shown how the proposed solution could improve the actual system in terms of the AWD. In the actual system, facilities are located sub-optimally in terms of the p-median criterion. Low-level services provision could be improved by more than 20% in terms of AWD, which means in the actual system patrons must travel almost one-fifth farther than they could do in the optimal p-median system. To reach optimality, the p-median model (Fig. 2.3 and 2.4) locates four middle-level facilities and twelve low-level facilities differently than the actual system does. Using ArcGIS 8.3, I constructed Voronoi (or Thiessen) polygons, which delineate the nearest surrounding area for each facility. For each demand node the link demonstrating its demand allocation is shown.

Note that the allocation links do not cross the Voronoi polygon borders; the facility service areas coincide with their Voronoi polygons. This demonstrates the *least-cost allocation rule*. Minimizing the weighted travel distance the p-median model allocates patrons to the nearest facility of the appropriate level. As a result, the high-level facility in Suhum has fewer nodes allocated than the neighbouring low-level facilities (Fig. 2.3). The low-level demand is concentrated around the district capital; the optimal p-median solution locates several low-level facilities around Suhum to serve this low-level demand. The least-cost allocation rule is also evident for the middle-level demand allocation (Fig. 2.4).

[7] Unfortunately it is impossible to compare the optimal p-median solution to that presented by Oppong (1992). The data he used for calculating AWD differs from those presented in Appendix I, which makes the comparison meaningless.

2.3. The least-cost allocation and hierarchical LA modeling

The least-cost allocation is unrealistic and undesirable for hierarchical LA modeling. As observed in the real world (*inter alia,* Hodgson (1981), Kloos (1990), Oppong (1992)), facilities of different levels have different attractiveness for people. Patrons who need a lower-level service may bypass the nearest lower-level facility and attend a higher-level facility at a greater distance for one or several of the following reasons:

- They perceive that low-level services can be performed better at a higher-level facility. For example, the thought might be that a surgeon might treat a broken finger better.
- They perceive the level of service required to be higher than it actually is; also they wish to avoid possible referral.
- Multi-purpose trips: higher-level facilities are usually located in larger towns where a greater variety of other goods and services are available, so a visit to a doctor can be combined with a visit to a non-grocery shop, for example.
- Uncertainty about travel times and facility locations.

As a result, the allocation pattern in Fig. 2.3 is unrealistic. For example, the district hospital in Suhum would likely have more allocation links than the neighbouring village medical rooms. Patrons would rather travel several hundred meters farther and bypass the nearest low-level facilities. Therefore, locating facilities based on the least-cost allocation (Fig. 2.3 and 2.4) can lead to underutilized low-level facilities and overutilized middle- and high-level facilities. The former can be considered as a non-optimal use of available resources in providing health care services. The latter is evidence of non-convenience of the proposed facility system for patrons.

2.4. Conclusion

I have demonstrated the traditional (p-median) approach to hierarchical LA modeling. The p-median model for the study area was solved optimally with CPLEX 6.5.1. It was shown that the actual system is inefficient in terms of the p-median criterion. At the same time, the latter hardly can be used in a hierarchical LA context because of the unrealistic least-cost allocation rule, leading to non-convenience of the proposed facility location.

Table 2.1: The *p*-median solution and the actual system.

Level of service	Actual system, AWD in km	Optimal solution	
		AWD, km	Improvement, %[8]
Low level (1st)	1.38	1.06	23.2
Middle level (2nd)	3.75	3.34	10.9
High level (3rd)	9.93	9.93	0.0
Overall	3.47	3.19	8.1

[8] Calculated as $(AWD_{actual} - AWD_{solution})/AWD_{actual} * 100\%$

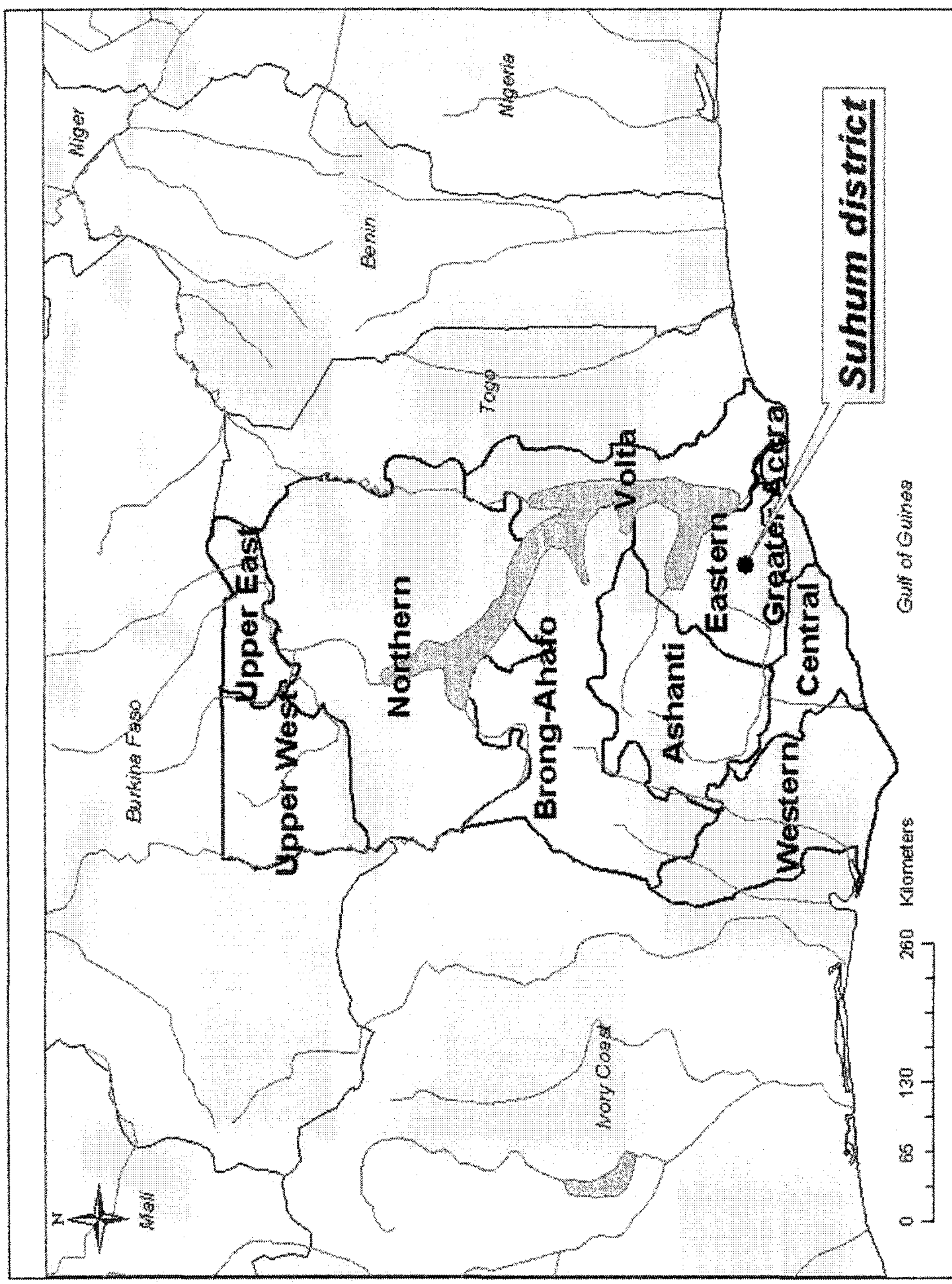


Figure 2.1: Ghana regions

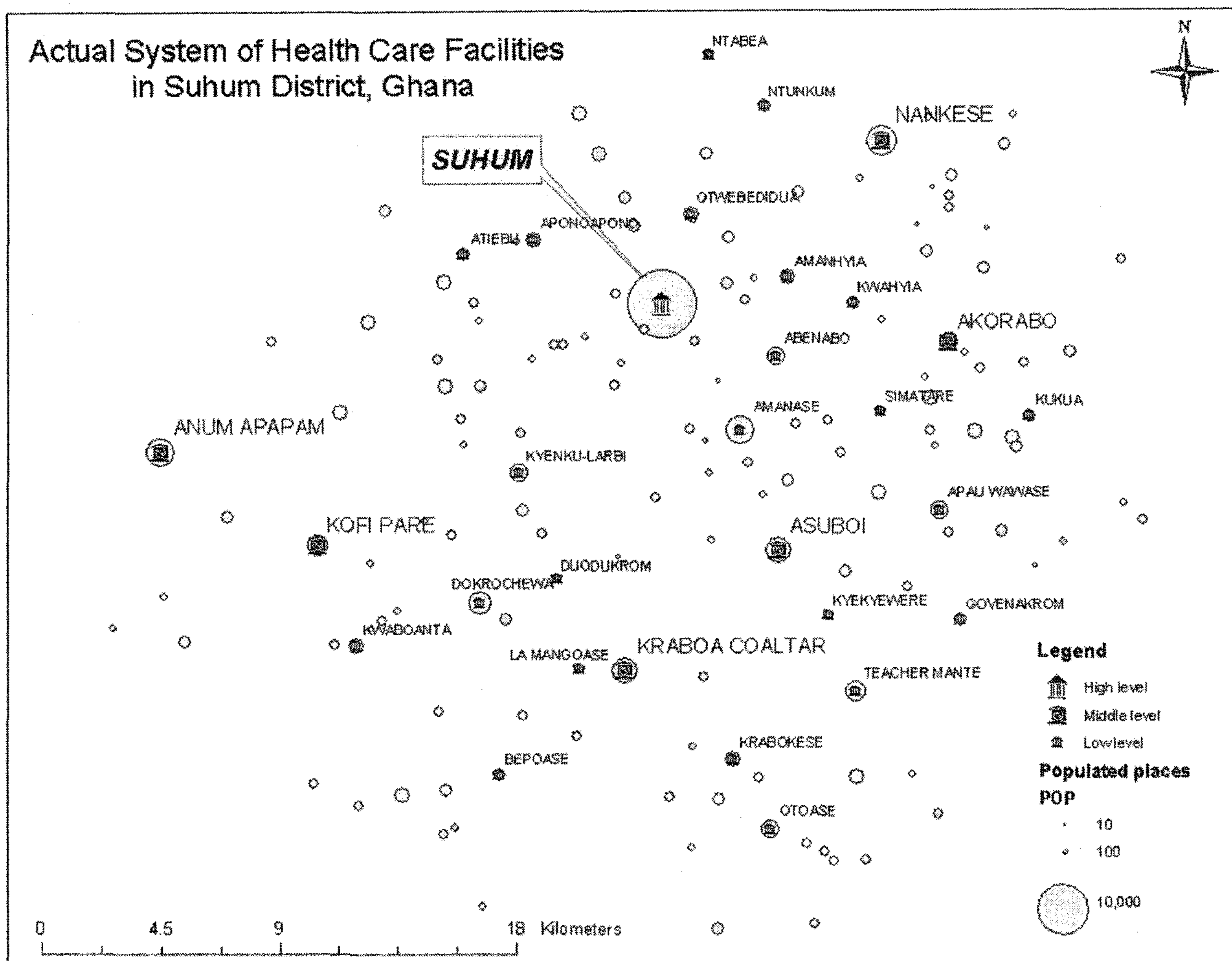


Figure 2.2. Source: Oppong (1992)

P-median optimal solution (1,6,23).
Low-level demand allocation

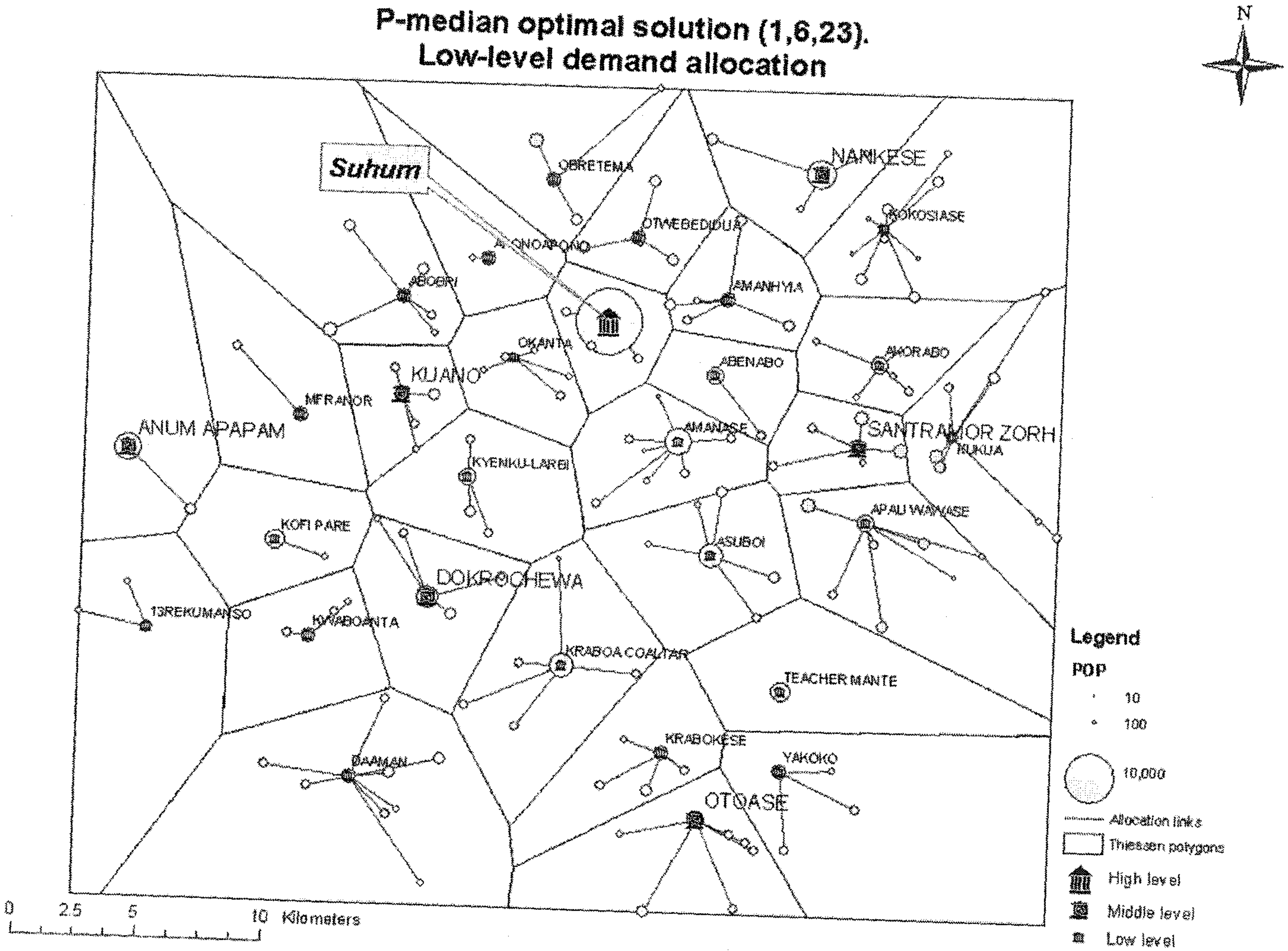


Fig. 2.3

P-median optimal solution (1,6,23).
Middle-level demand allocation.

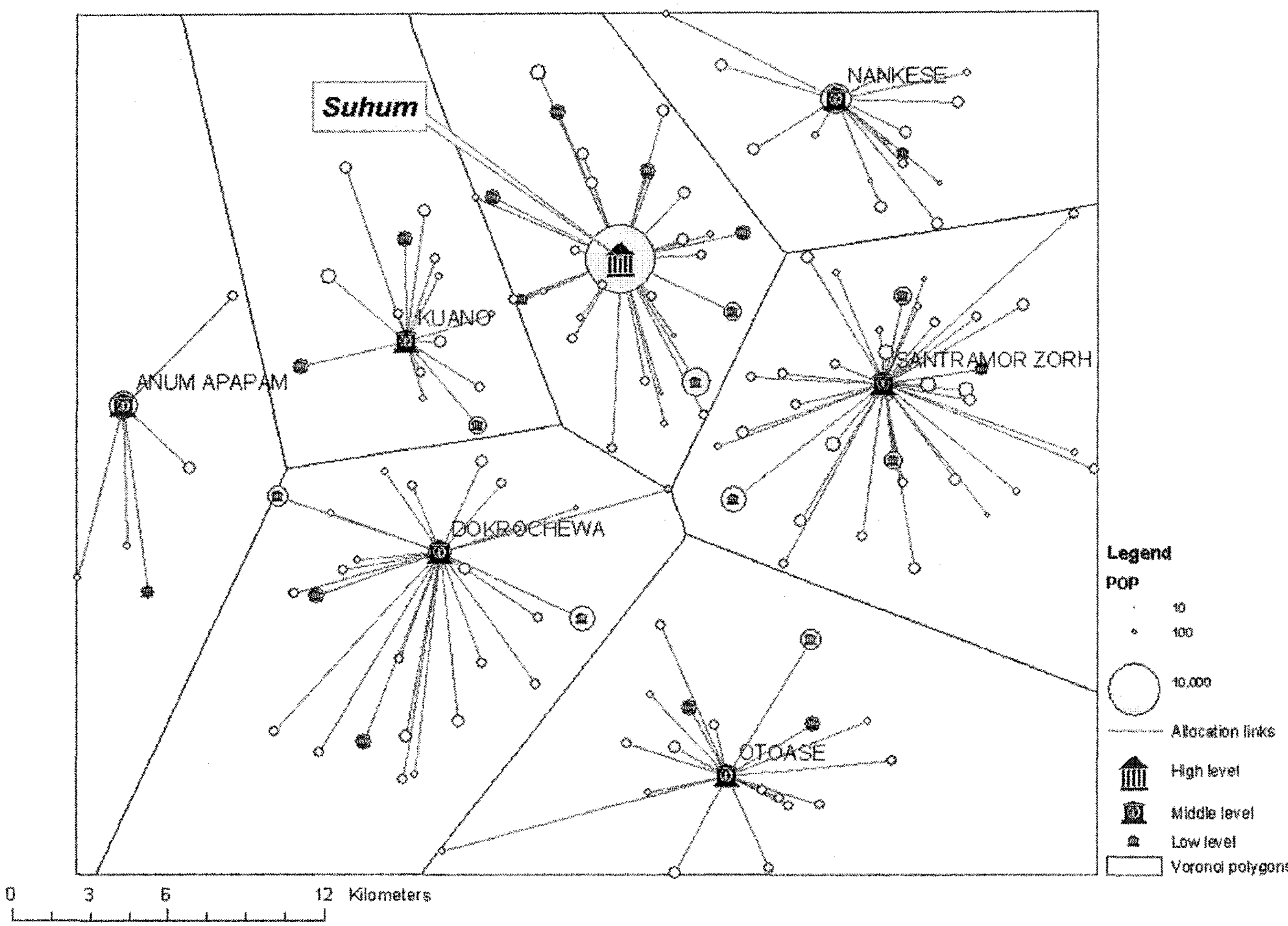


Fig. 2.4

Chapter 3. Interaction-based hierarchical LA approaches

3.1. Previous research

The least-cost allocation results in non-convenient facility location and non-optimal spending of available resources. To overcome these consequences and to render LA models more realistic, spatial interaction methods have been applied to location-allocation. The rationale for spatial interaction-based LA models is the fact that the patronage of facilities depends on the *benefit* accruing to patrons from getting services there. These models state that such patronage is directly proportional to facility attractiveness and inversely proportional to the disutility of travelling there.

The first attempt to incorporate SI models into on LA framework was by Hodgson (1978), but the model had the drawback of a travel-cost minimization objective. Later Hodgson (1981) used the consumer welfare maximization criterion advanced by Wilson (1976); he considered facilities of two different sizes and probabilistic allocations, but did not consider clearly distinguished hierarchical levels. In both cases, he applied an adaptation of the Teitz and Bart (1968) heuristics to obtain the solution. O'Kelly and Storbeck (1984) formulated an interaction-based LA model based on the Wilson (1976) criterion and probabilistic allocation. They solved it for a 2-level, 16-demand point problem using lagrangean relaxation and subgradient search techniques. For a 3-level hierarchical LA model Hodgson (1984, 1986, 1988) employed the Batty (1978) model allocating demand to maximize consumers' benefit:

$$S_j^{\alpha} / \exp(\beta d_{ij})$$

(8)

where S_j is some measure of the attractiveness facility at j [9], d_{ij} is the disutility of travelling from demand point i to facility at j expressed by the distance, and α (attractiveness exponent value) and β (distance impedance parameter) are empirically determined parameters. Oppong (1992) and Oppong and Hodgson (1998) applied this model to the Suhum study area. An adaptation of the Teitz and Bart (1968) heuristic substitution algorithm was used to get the solution.

3.2. The Batty-based hierarchical LA model.

In the works mentioned above, the Batty-based model was not fully formulated as a mathematical optimization model; only the objective function was provided. I formulate the k-level Batty-based model as:

$$Maximize \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{c \in \mathbf{C}} \sum_{\substack{k \in \mathbf{K} \\ (k \geq c)}} W_i U_c B_{ij}^{ck} X_{ij}^{ck} \tag{9}$$

$$\text{where } \mathrm{B}_{ij}^{ck} = S_j^k / \exp(\beta^c d_{ij}) \tag{10}$$

Subject to :

$$\sum_{\substack{k \in \mathbf{K} \\ (k \geq c)}} \sum_{j \in \mathbf{J}} X_{ij}^{ck} = 1, i \in \mathbf{I}, c \in \mathbf{C} \tag{11}$$

$$Y_j^k \geq X_{ij}^{ck}, i \in \mathbf{I}, \mathrm{j} \in \mathbf{J}, k \in \mathbf{K}, c \in \mathbf{C}(c \leq k) \tag{12}$$

$$\sum_{k \in \mathbf{K}} Y_j^k \leq 1, \ \mathrm{j} \in \mathbf{J} \tag{13}$$

$$\sum_{j \in \mathbf{J}} Y_j^k = p_k, \mathrm{k} \in \mathbf{K} \tag{14}$$

$$X_{ij}^{ck} = \{0,1\} \ \mathrm{i} \in \mathbf{I}, \mathrm{j} \in \mathbf{J}, \mathrm{c} \in \mathbf{C}, \mathrm{k} \in \mathbf{K} \tag{15}$$

$$Y_j^k \in \{0,1\}, \mathrm{j} \in \mathbf{J}, \mathrm{k} \in \mathbf{K} \tag{16}$$

---

[9] The common way to measure the attractiveness of a facility is to use its size.

The objective function (9) maximizes overall patrons' benefit. The benefit (10) depends on the level of facility where the demand is served. Therefore, we must distinguish the set of *levels of facilities* (**K**) from that of *levels of services* (**C**)[10]. X_{ij}^{ck} is the allocation variable which equals 1 if the service of level c at point i is served at a facility of level k at site j, or 0 otherwise. Consequently, $\mathrm{B}_{\mathrm{ij}}^{\mathrm{ck}}$ is the parameter of the patrons' benefit derived from this allocation. It is directly proportional to the attractiveness of the facility of level k at site j (S_j^k) and inversely proportional to the exponential function of the distance between i and j (d_{ij})[11]. Constraint (11) assures that all demand at all levels is served; constraint (12) ensures that demand of level c can be served only at a facility of level k such that $c \leq k$. Constraint (13) assures that one location can have at most one facility and constraint (14) is the budget constraint. Constraints (15) and (16) make all decision variables binary.

The Batty-based hierarchical model is more complex than the hierarchical *p*-median model because:

❖ One set of level-specific allocation variables $\mathrm{X}_{\mathrm{ij}}^{\mathrm{k}}$ is divided into c sets of $\mathrm{X}_{\mathrm{ij}}^{\mathrm{ck}}$ variables to distinguish the level of service demanded (c) and the level of facility where this demand is allocated (k).

❖ The set of constraints (3) is replaced by the more extensive set of constraints (12) to specify where patrons can be allocated. Now each combination of c and k $(c \leq k)$ has its own set of constraints.

---

[10] Recall, that "the level of service" and "the level of facility" are not synonyms (see p.1)

[11] The distance impedance parameter (β^c) is specific to the level of service c

The model parameters taken from Oppong (1992) are shown in Table 3.1. All values were calculated based on the actual travel patterns observed in the study area. Note that the distance impedance value (β^c) is the highest for middle-level services, which contravenes notions of Central Place Theory by Christaller (1933)[12]. Oppong (1992) explained this away as a result of the relative importance of particular low-level facilities (Kukua and Apau Wawase).

First, I evaluated the actual facility system against the Batty-based model; then the optimal Batty-based solution was obtained. I used CPLEX 6.5.1; the total execution time for each calculation did not exceed 35 sec[13]. In examining the actual system I used the LA model presented in (9-16); the only difference is that the facility locations (Y_j^k) were given by the actual system. In the low-level demand allocation (Fig. 3.1) many places are assigned, not to the closest facilities but to more distant middle- and high-level facilities. The allocation links of Suhum, for instance, extend far beyond its Voronoi polygon. The Batty model thus can capture the bypassing effect observed in real-world situations. The effect is so strong that the demand from eight locations with low-level facilities is assigned to higher-level facilities[14]. Such a situation was termed by Oppong (1992) as "direct bypassing"[15]. That makes the low-level facilities superfluous, serving no demand. The presence of such facilities, which do not contribute to the overall system benefit, is evidence that the actual system is not optimal.

[12] People travel shorter distances for getting low-level services than for getting higher-level ones. Therefore, in spatial hierarchies the distance impedance value is larger for lower levels than for higher levels.
[13] Oppong (1992) reported that in his calculations one run of the heuristic required 11.4 hours. A computer with a 386 processor was used.
[14] Such places are indicated by the pushpin symbol in Fig. 3.1.
[15] The "indirect bypassing" refers to the situation when there is no facility in the place of origin but the user ignores closer facilities in favor of a more distant one.

The optimal Batty-based system (Fig. 3.2) does not produce direct bypassing. The model changed the locations of fourteen low-level facilities to maximize their contribution to overall benefit[16]. Many low-level facilities are, however, self-served -- they serve only the patrons from the places where they are located. The larger attractiveness of high- and middle-level facilities still compensates for the greater distance to travel there, so their allocation links cross the Voronoi polygon borders.

The numerical results of the actual facility system and the Batty-based model are summarized in Tables 3.2 and 3.4. In these tables, and in the following chapters, the patrons' benefit analysis is performed from the two points of view. In a hierarchical system the overall benefit can be broken down in two ways:

- Benefit by service level, which measures the patrons' convenience in getting services of an appropriate level.
- Benefit by facility type (level)[17], which demonstrates the contribution of each facility type to overall system benefit. It also serves as a proxy for the relative importance of the facility type in a hierarchy.

The Batty-based optimal system improves the consumer benefit by 4.2% (Table 3.2). The service level-by-level benefit ratio is almost the same in the actual and optimal systems. Low-level demand is the largest in the system, so it produces the highest benefit; correspondingly the high-level benefit is the lowest. The Batty-based model improves patrons' convenience in obtaining low- and middle-level services by optimally locating facilities of the appropriate levels; the location of the high-level facility remains the same.

[16] In addition, one middle-level facility has been relocated; three low-level facilities have been promoted to middle-level ones and, correspondingly, three middle-level facilities have been reduced to low-level ones without changing locations.
[17] Hereafter, I assume that the term "type of facility" is synonymous with "level of facility", i.e. the actual system has facilities of three types: high-, middle and low-level ones.

It is also interesting to look at the contribution to the overall benefit by the facility type (Table 3.3). Low-level and middle-level facilities are less attractive than the high-level one, so the total contribution of the 29 facilities to the system benefit is less than that of the high-level one -- in the actual system it is only 35%. The Batty-based model increases the role of the low- and middle-level facilities by optimally locating them; their contribution increases to 40%, but it is still less than a half. Note also that even in the optimal system, the 23 low-level facilities make very little contribution to overall benefit (8%).

3.3. The shortcomings of the Batty-based model

In my discussion of bypassing (p. 11) I mentioned four reasons making the least-cost allocation unrealistic and affecting the patrons' choice of the facility: the higher quality of services provided at higher level facilities, uncertainty about the level of service required, multi-purpose trips and uncertainty about travel times. The Batty-based model deals only with the first two, combining them into the attractiveness value. However, facility attractiveness alone does not consider multipurpose trips, for example.

The Batty-based model relaxes the least-cost allocation rule, but this relaxation has an effect only for the lower-level demand allocation (Fig. 3.2). Most of the middle-level demand is still allocated to the nearest facility (Fig. 3.3); the facility allocation links do not extend beyond their Voronoi polygons.

The Batty benefit criterion (8) reflects the patrons' convenience. The Batty-based LA model maximizes the overall system benefit increasing the patrons' convenience. We might expect that the facilities providing higher benefit would attract and serve more patrons. However, we observe disproportions between the percentage of benefit provided by a facility type and the percentage of demand allocated to it (Table 3.4). The high-level facility provides the largest share of benefit, but has less demand allocated than the middle-level facilities. Further, serving low-level demand, one high-level facility provides four times (400 %) as much benefit as twenty-three low-level ones. At the same time, the high-level facility serves only 5% more demand than the low-level ones. Patrons' level-by-level facility allocation pattern does not coincide with benefit contribution. These disproportions[18] are a shortcoming of the model – in a system-wide context some patrons are inconveniently allocated to less beneficial low-level facilities.

The BDD problem is likely to be the consequence of the Batty (1978) model property termed in the consumer choice literature as Independence from Irrelevant Alternatives (IIA). To demonstrate this property, consider two alternative destinations (facilities), n and k, to travel to from the demand point i. According to the Batty (1978) model the probabilities that an individual from i will travel to n and k will be equal to, respectively:

$$p_{in} = S_n^{\alpha} \exp(-\beta d_{in}) / \sum_{j \in \mathbf{J}} \mathrm{S}_{\mathrm{j}}^{\alpha} \exp(-\beta \mathrm{d}_{\mathrm{ij}}) \text{ and } p_{ik} = S_k^{\alpha} \exp(-\beta d_{ik}) / \sum_{j \in \mathbf{J}} \mathrm{S}_{\mathrm{j}}^{\alpha} \exp(-\beta \mathrm{d}_{\mathrm{ij}})$$

(17)

where **J** is the whole set of destinations to which to travel. In other words, the probability of traveling to any given facility is directly proportional to the consumer's benefit provided by this facility (numerator) and inversely proportional to the consumer's benefit provided by all other facilities (denominator) in the system. As the denominator in (17) is constant for the given set of facilities (**J**) the ratio of probabilities of choosing n over k is

[18] Hereafter, I shall call them benefit/demand disproportions (BDD).

$$p_{in} / p_{ik} = S_n^{\alpha} \exp(-\beta d_{in}) / S_k^{\alpha} \exp(-\beta d_{ik})$$

(18)

Now assume that a new destination, *m*, is added to the system. It can change the spatial flows to *n* and *k* because of the denominator term in (17), and as a result, the ratio of probabilities. However the formula (18) does not reflect these possible changes. The IIA property can be formulated as, "the ratio of the probabilities of an individual selecting two alternatives is unaffected by the addition of a third alternative" (Fotheringham, Brunsdon, and Charlton, 2000). This property is undesirable and may be erroneous in the context of SI and LA modelling.

3.4. Conclusion

SI models were incorporated into an LA framework to consider bypassing. The Batty-based model was solved heuristically for the study area by Oppong (1992). I proposed the full formulation of the Batty-based model and solved it optimally with CPLEX 6.5.1. The optimally solved Batty-based model takes into account bypassing at the low-level demand allocation, but middle-level demand is still allocated to the closest facility with some rare exceptions. The Batty-based model deals with only two (out of four) reasons, which make the least-cost allocation unrealistic in spatial hierarchy. Benefit/demand disproportions (BDD) are observed – the high-level facility providing the largest benefit share gets less demand allocated than the middle-level ones. The BDD problem is likely to be a consequence of the Independence from Irrelevant Alternative (IIA) property, which is undesirable and may be erroneous in LA modeling.

Table 3.1: The Batty-based model parameters for the study area, by Oppong (1992).

Hierarchy level	Attractiveness value (S_j)	Distance impedance β^c	Usage of facilities (U_k)
High (3^{rd})	4.2	0.148	0.609
Middle (2^{nd})	2.7	0.264	0.203
Low (1^{st})	1.0	0.254	0.188

Table 3.2: Benefit broken down by service level and demand allocation in the Batty-based model.

Service	Actual system		Batty-based model		
	Benefit	%	Benefit	%	Improvement[19], %
Low-level	113,467.0	63.3	119,603.0	64.1	3.8
Middle-level	35,830.2	19.9	37,178.3	19.9	5.4
High-level	29,934.5	16.7	29,934.5	16.0	0.0
Overall	179,231.84	99.9	186,715.5	100.0	4.2

Table 3.3: Benefit broken down by facility type (level) in the actual and the Batty-based facility systems.

Facilities	Actual system		Batty-based system		Improvement[20], %
	Benefit	%	Benefit	%	
Low-level (23)	12,067.7	6.7	15,026.7	8.0	24.5
Middle-level (6)	52,165.9	29.1	60,873.7	32.6	16.7
High-level (1)	114,998.0	64.2	110,815.0	59.3	-3.6
Overall	179,231.8	100.0	186,715.5	99.9	4.2

[19] Improvement columns for both p-median models are calculated as $(B_{Batty} - B_{actual})/B_{actual}*100\%$
[20] Improvement columns for both p-median models are calculated as $(B_{Batty} - B_{actual})/B_{actual}*100\%$

Table 3.4: Benefit broken down by facility level and demand allocation in the Batty-based model.

Facilities	Low-level demand			Middle-level demand		
	Benefit		Demand allocation, %	Benefit		Demand allocation, %
	Value	%		Value	%	
Low level (23)	15026.7	12.6	25.9	-	-	-
Middle level (6)	44061.7	36.8	43.6	16812.0	45.2	64.4
High level (1)	60514.2	50.6	30.5	20366.3	54.8	35.6
Overall	119602.6	100.0	100.1	37178.3	100.0	100.0

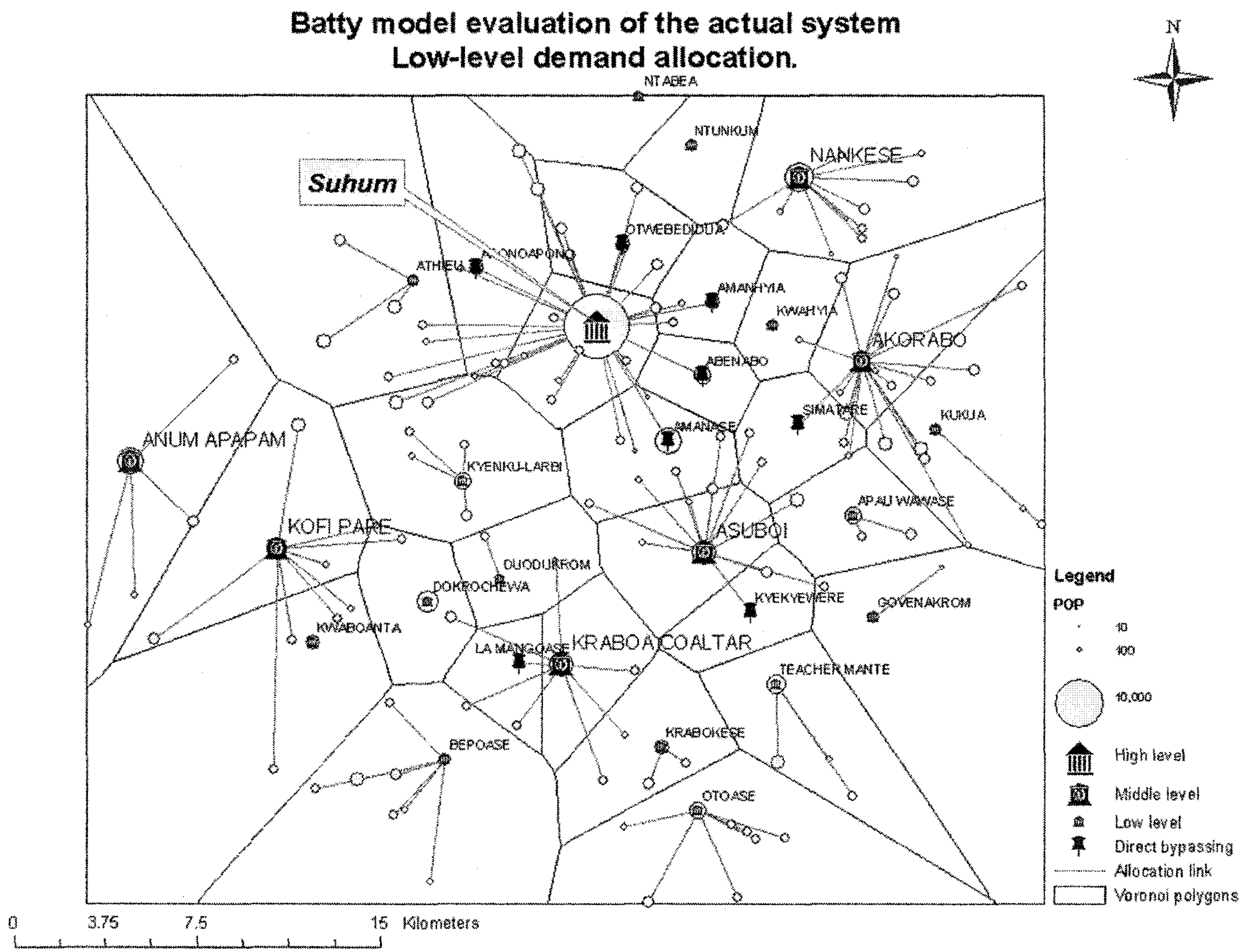

Batty model evaluation of the actual system
Low-level demand allocation.
N
NTABEA
NTUNKUM
NANKESE
Suhum
OTWEBEDIDUA
ATHEIA
AMANHYIA
KWAHYIA
AKORABO
ABENABO
ANUM APAPAM
AMANASE
SIMATARE
KUKUA
KYENKU-LARBI
APAU WAWASE
KOFI PARE
ASUBOI
DUODURROM
DOKROCHEWA
KYEKYEWERE
GOVENAKROM
KWABOANTA
LA MANGOASE
KRABOA COALTAR
TEACHER MANTE
KRABOKESE
BEPOASE
OTOASE
Legend
POP
10
100
10,000
High level
Middle level
Low level
Direct bypassing
Allocation link
Voronoi polygons
0
3.75
7.5
15
Kilometers

Fig. 3.1

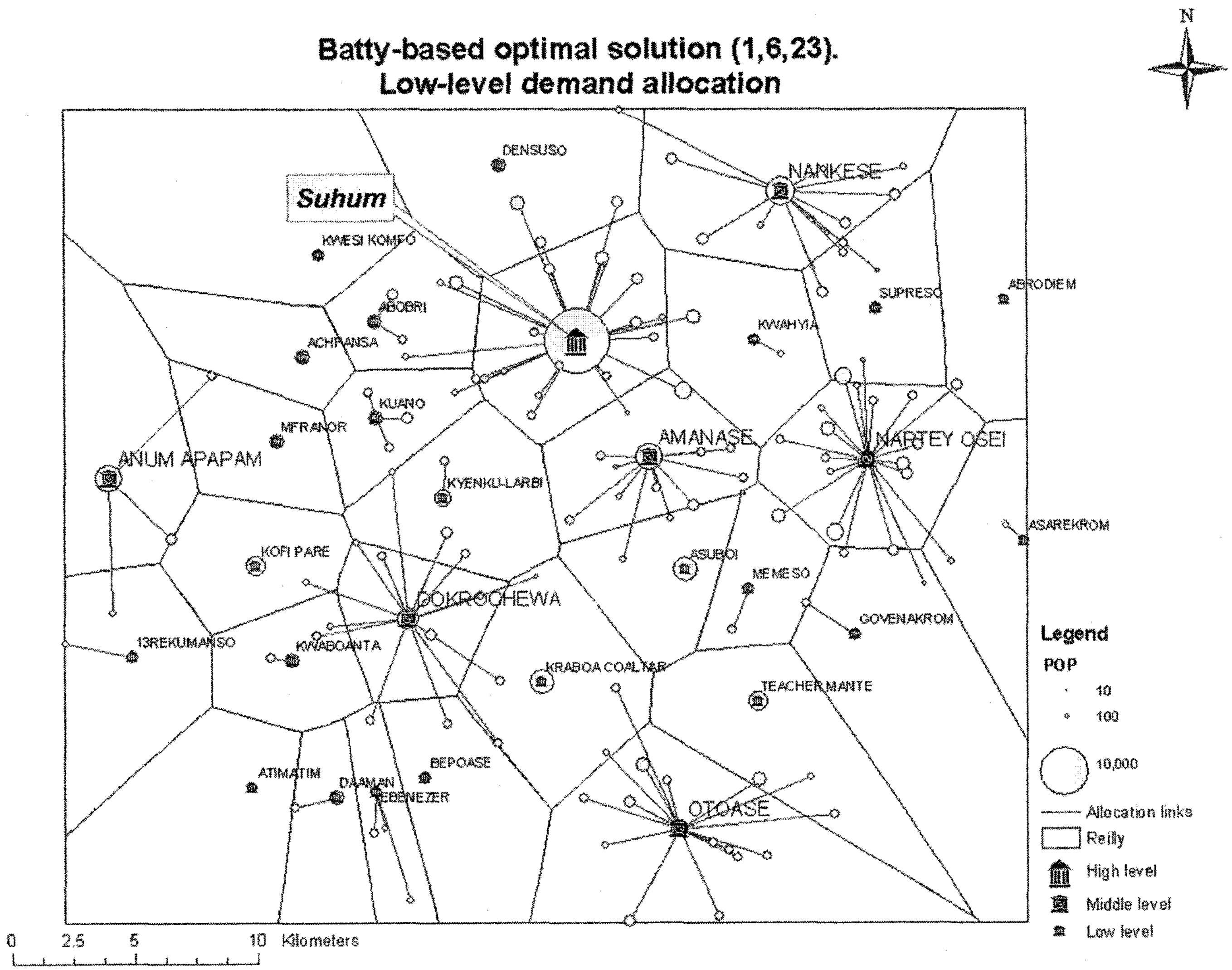

Batty-based optimal solution (1,6,23).
Low-level demand allocation
N
Suhum
DENSUSO
NANKESE
KWESI KOMFO
ABOBRI
SUPRESO
ABRODIEM
KWAHYIA
KUANO
MFRANOR
ANUM APAPAM
AMANASE
NARTEY OSEI
KYENKU-LARBI
ASAREKROM
KOFI PARE
ASUBOI
MEMESO
DOKROCHEWA
GOVENAKROM
13REKUMANSO
KWABOANTA
KRABOA COALTAR
TEACHER MANTE
ATIMATIM
DAAMAN
EBENEZER
BEPOASE
OTOASE
Legend
POP
10
100
10,000
Allocation links
Reilly
High level
Middle level
Low level
0 2.5 5 10 Kilometers

Fig. 3.2

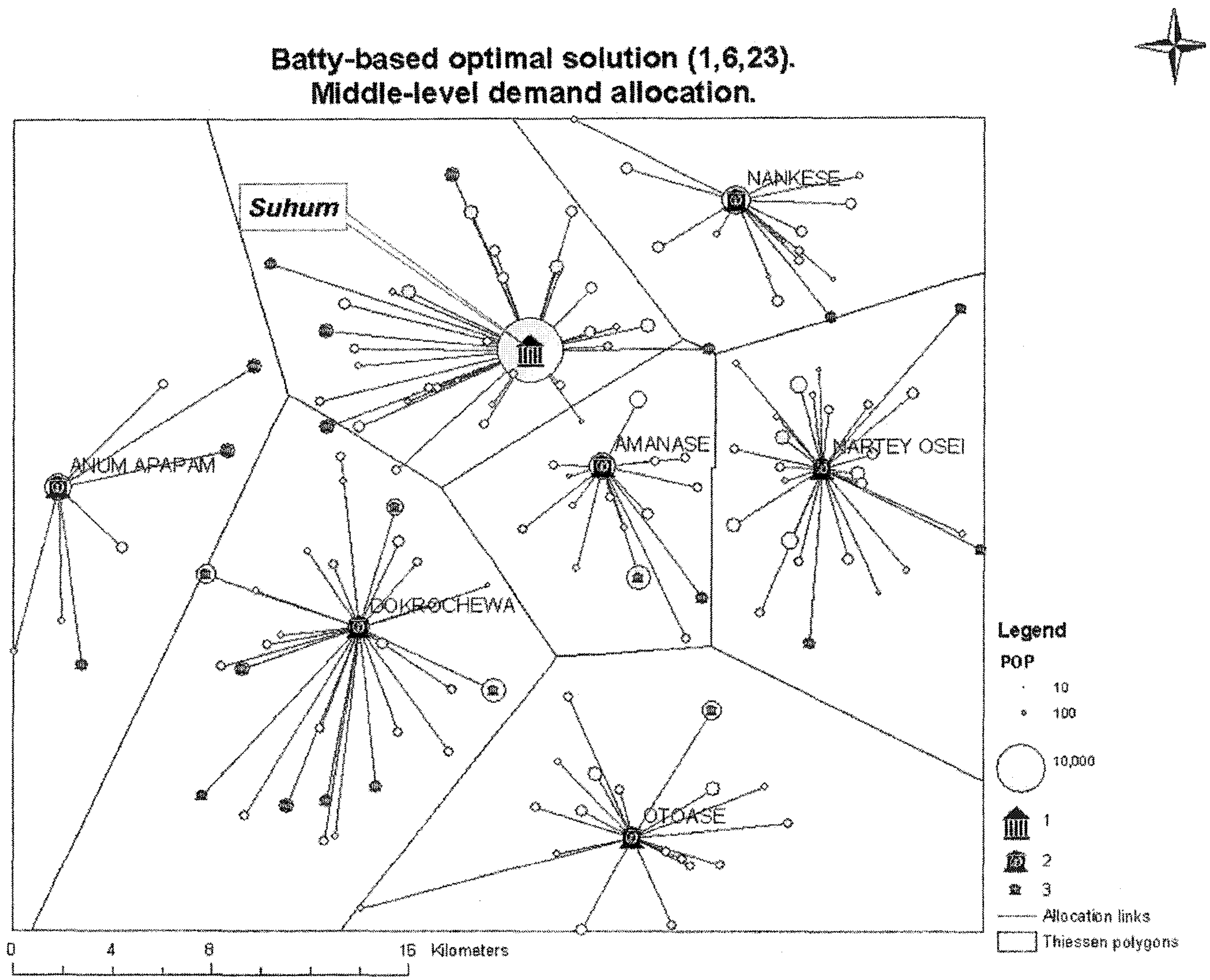

Batty-based optimal solution (1,6,23).
Middle-level demand allocation.
Suhum
NANKESE
ANUM APAPAM
AMANASE
NARTEY OSEI
DOKROCHEWA
OTOASE
Legend
POP
10
100
10,000
1
2
3
Allocation links
Thiessen polygons
0
4
8
16
Kilometers

Fig. 3.3

Chapter 4. A Spatial Choice Interaction Model in an LA framework

4.1. The Spatial Choice Theory

To overcome the IIA property, extensive research was done in the field of spatial interaction theory. A new form of spatial interaction model was developed and presented by Fotheringham (1983a, 1983b, 1986), Fotheringham and O'Kelly (1989), Fotheringham (1991) and Fotheringham *et al.* (2000). The models are alternatively referred to as *spatial choice models*, *competing destination models*, and *spatial information processing models*. For simplicity I shall use the term *spatial choice model(ing)*. Its rationale is the following: a patron's spatial choice in deciding where to travel to depends on:

- The destination (facility) attributes, e.g. its attractiveness and the disutility of travelling to it.
- How she/he processes the information about the destination attributes.

The first factor was thoroughly considered and calibrated in the previously developed SI models, including the Batty (1978) model, whereas the second factor was left out of consideration mostly because the task of expressing the manner of processing the information in an SI model in quantitative terms is difficult.

It is generally recognized that individuals have a limited capacity to process information[21]. In particular, Bettman (1979) argues that an individual can process information about no more than seven alternative destinations at once. However, in most spatial choice situations, the number of possible destinations is much larger. For example, there are thirty facilities providing the low-level health care services in the study area. To

[21] For the full list of references supporting this statement the interested reader is referred to (Fotheringham *et al.*, 2000), p. 226

simplify the choice process, individuals employ some information-processing strategy where clusters of facilities are *initially* considered and only *then* is a particular destination chosen within a selected cluster. Therefore (17) should be modified to

$$p_{ik} = S_k^{\alpha} \exp(-\beta d_{ik}) * L_i(k \in \mathbf{M}) / \sum_{j \in \mathbf{J}} S_j^{\alpha} \exp(-\beta d_{ij}) * L_i(j \in \mathbf{M})$$

(19)

where $L_i(k \in \mathbf{M})$ is the likelihood that the considered destination *k* is in the individual's (from origin *i*) chosen cluster **M** and $L_i(j \in \mathbf{M})$ is the likelihood that some destination *j* is in the individual's chosen cluster **M**.

Unfortunately, it is very difficult or even impossible to know how people cluster alternative destinations in space. To overcome this problem, researchers use some destination attributes associated with the processing of spatial information that affect the likelihood of a destination being considered. Different approaches to calculating the likelihood function have been presented, *inter alia*, by Meyer and Eagle (1982), Borgers and Timmermans (1987), Fik and Mulligan (1990), and Lo (1990, 1991).

Fotheringham (1983a, 1986) proposed using an accessibility function as a proxy for such a likelihood function. The accessibility function measures the proximity of a destination to all other possible destinations. Fotheringham and Trew (1983) and Pellegrini, Fotheringham, and Lin (1997) applied this accessibility function to store choice modelling. Pellegrini and Fotheringham (2002) noted that, frequently, in the store selection process (especially in non-grocery), an *agglomeration effect* exists -- individuals are attracted to large clusters of destinations in order to minimize costs of comparison shopping. Thus people perceive additional benefit from the facility's spatial neighbourhood.

4.2. The spatial choice interaction approach in an LA framework

In health care, the agglomeration effect results in facilities being located in more populated places; clusters attract more patrons, because of multipurpose trips, for instance. Assuming that the population of a place is a proxy of its "centrality" and that there is a non-linear relationship between the likelihood (L_i) and accessibility, we can define:

$$L_i(k \in \mathbf{M}) = A_k^{\delta} = (\sum_j W_j^{\alpha} / \exp(\beta d_{jk}))^{\delta}$$

(20)

From (19) and (20) we can conclude that an individual from *i* will travel to that destination *k* for which the expression

$$S_k^{\alpha} * \exp(-\beta d_{ik}) * \mathrm{A}_{\mathrm{k}}^{\delta}$$

(21)

is greatest.

If parameter δ equals zero, no additional benefit from spatial surroundings is perceived and (21) converts to the conventional Batty (1978) model. If δ is positive, then the agglomeration effect exists -- the model assumes that the more central a place is, the more patrons are attracted there, so the more central sites have an advantage for locating facilities and allocating demand to them. The model takes into account the additional patrons' benefit accrued from attending the more centrally located place. If δ is negative, then 'the competition effect' exists -- the facilities within large clusters are unattractive for patrons[22], so isolated places have an advantage for locating facilities.

The next question is how to calculate the accessibility of a potential site (A_k) from (20) and (21) to capture the spatial neighbourhood effect in the best way. This question can be divided into several topics:

- Size attractiveness value (α) and distance impedance parameter (β). Fotheringham *et al.* (2000) suggested that they usually are set to 1 and –1 respectively[23]. Obviously, the best values can be calculated through a model calibration procedures based on real-world observations. It is impossible to get these values from the Suhum district dataset. In (20) I use the exponential function of the distance impedance. For simplicity I assumed that $\alpha = 1$ and $\beta = 0.254$ [24] for A_k calculation.
- Distance. The accessibility term expresses the likelihood of a destination being considered by an individual. The distance used for calculating the accessibility can be different from the physical distance that will be traveled by an individual. Ideally we would include psychological, or mental, distance: the distance based on the patron's perception. Again new field studies would be required to get the values of psychological distance, so I will use Euclidean distance.
- The accessibility exponent value δ. There are several studies in which this parameter was calibrated. For example, Fotheringham (1986) and Pellegrini and Fotheringham (1999) used US data for small- and large-scale migrations; Fotheringham and Trew

[22] The competition effect is unlikely to be observed in health care or non-grocery shopping patterns. However, it is observed in the other spatial interaction patterns, such as migration or choosing place of residence. In the current work I shall consider it only for purposes of sensitivity analysis.
[23] Fotheringham *et al.* (2000) used the power function for distance impedance.
[24] The distance impedance parameter value for low-level service demand which produces the most trips in the system.

(1983), Thill (1995) and Pellegrini *et al.* (1997) used shopping attendance data. Obviously, the decision process for migration or shopping differs from that for seeking medical services. It is beyond the scope of this work to determine the particular δ value for the study area. Rather, I perform a sensitivity analysis to investigate how the model behaves under different values of δ.

Based on (20) and (21), a new hierarchical LA model can be formulated. The objective function maximizes the consumers' benefit considering the accessibility of a potential facility site:

$$Maximize \sum\sum\sum\sum W_i U_c B_{ij}^{ck} X_{ij}^{ck} \tag{22}$$

$$\text{where } \mathrm{B}_{\mathrm{ij}}^{\mathrm{ck}} = S_j^k * A_j^\delta / \exp(\beta^c d_{ij});\ A_j = \sum_{n \in \mathbf{J}} W_n / \exp(0.254 * d_{jn}) \tag{23}$$

In combination with the constraints (11)-(16), the objective function forms a spatial choice-based LA model. All parameters are taken from Table 3.1. In this model patrons' benefit depends on:

- The distance patrons must travel (d_{ij})
- The level of facility from which patrons obtain services (S_j^k)
- The place where a facility is located, its spatial neighbourhood (A_j).

4.3. The spatial choice-based model results

As with the models considered above, I solved the spatial choice-based model optimally with CPLEX 6.5.1. In order to investigate the behaviour of the spatial choice-based LA model, I performed a sensitivity analysis. The accessibility exponent value (δ) is crucial to the model's performance; I varied this parameter from -1 to $+1$ in increments of 0.2. Execution time depended on the δ value and varied from 20 seconds ($\delta = 1$) to 95 seconds ($\delta = -1$). The model output results at the upper and lower bounds of δ (+1 and −1) are illustrated in maps. Then the spatial choice-based model behaviour will be shown – the mean accessibility of the facility locations and the level-by-level benefit distribution and patron allocation will be plotted against the δ value[25].

If $\delta = 1$ (Fig. 4.1, 4.2) then a strong spatial agglomeration effect exists. Patrons get much benefit from being served by the high-level facility located in the most accessible place (Suhum); looking for low-level services (Fig. 4.1) they are ready to bypass not only the nearest low-level facilities but also the middle-level ones. Note that the north-central and the north-eastern parts of the district have no low-level facilities at all. The presence of the high- and middle-level facilities, which are so "beneficial", makes the low-level facilities unnecessary in the surrounding areas. The low-level facilities, mostly located in the western and south-eastern parts of the district, in the relatively isolated places, serve only their own places. Middle-level demand allocation (Fig. 4.2) does not have the least-cost pattern. The allocation links are affected by strong, evident centripetal force. The benefit perceived from being served in the capital is so high that patrons bypass the nearest middle-level facilities.

If $\delta = -1$ (Figs. 4.3, 4.4) a strong spatial competition effect exists. Patrons perceive additional benefit from being served by high- and middle-level facilities, but the latter tend to be located at isolated sites, in the periphery of the study area (Fig. 4.3). The

[25] Recall, that low- and middle-level demand produces the most trips in spatial hierarchy. Therefore, our attention will be concentrated on the benefit derived from getting low- and middle-level services.

high-level facility is located at the isolated place (Asarekrom), which has the third lowest accessibility value. Five out of six middle-level facilities are located in corners of the study area, close to its borders. Most of the low-level demand is allocated to the middle- and high-level facilities. The middle-level demand allocation (Fig. 4.4) as in the agglomeration case does not have the least-cost pattern. However, in this case allocation links have a strong centrifugal pattern. The facility isolation provides more benefit, so the allocation links of the isolated facilities, located in the corners of the study area extend beyond their Voronoi polygons.

The δ values (+1 and –1) considered above are the upper and lower bounds of the sensitivity analysis. Figs. 4.5-4.9 demonstrate the model's behaviour between these values. The mean accessibility values[26] were calculated and plotted against the appropriate δ values (Fig. 4.5)[27]. We might expect that as δ grows, patrons perceive more benefit from attending more accessible places and that, correspondingly, the model would locate facilities at sites with high accessibility. However, the behaviour of three plotted accessibility measures does not completely confirm those expectations. The mean accessibility of middle-level facility sites increases as we increase the δ value; the only exception is where δ changes from –1 to –0.8 when accessibility decreases due to replacing a middle-level facility in Suhum (the highest accessibility value) with a high-level one. The changes in overall mean accessibility and those of the low-level facilities

[26] The accessibility for each facility site was calculated by (20). Then:

- Accessibilities of all facility sites were summed and divided by 30 (mean accessibility).
- Accessibilities of the sites with middle-level facilities were summed and divided by 6 (mean accessibility of middle-level facilities).
- Accessibilities of the sites with low-level facilities were summed and divided by 23 (mean accessibility of low-level facilities).

[27] The high-level facility was placed in Suhum during all tests except one when δ was equal to –1 (as observed in Figs. 4.3, 4.4), so the accessibility of the high-level facility site for all other tests remains the same and was not plotted.

are not so straightforward. When δ is negative, accessibilities decrease up to $\delta = -0.6$ and then start increasing. They reach their maximum at $\delta = 0.2$ and then again start decreasing. This somewhat erratic pattern may be explained by the relationship between the accessibility value and demand allocation, which is also affected by δ (Fig. 4.6). The mean accessibility of the low-level facilities corresponds closely to the proportion of low-level demand served by them. The higher the accessibility exponent, the less demand is served by low-level facilities, the less is their contribution to system benefit, and, consequently, they are located in less accessible places. When δ is negative, isolation becomes an asset; middle-level facilities tend to be located in less accessible places, so lower-level facilities become more and more important for serving low-level demand.

Figs. 4.5 and 4.6 indicate that the higher the accessibility exponent value, the more demand is served by higher-level facilities and the less it is served by low-level ones. Fig. 4.7 demonstrates the same relationship between the value of δ and the amount of the middle-level demand served at the high-level facility. The higher the agglomeration effect (the higher the value of δ), the more middle-level demand is allocated to the high-level facility.

These changes in the demand allocation may be explained by the following feature of hierarchically structured services. On the one hand, patrons bypass the nearest low-level facilities, tending to get low-level services at a higher level. On the other hand, there is some countervailing force to this bypassing, for instance, if a high-level facility is too far, a patron will not travel there. At one extreme, all facilities would be located in one place (the district capital in the given study area); at the other extreme all facilities would be located according to the distance minimization criterion (the p-median model). The spatial choice-based model is flexible enough to find "an ideal balance" between these two forces. Having the number of facilities at each level predefined and selecting the appropriate accessibility exponent value (greater or equal to 0 for the health care spatial interaction pattern) will allow the location of high- and middle-level facilities with respect to increased low-level demand, but at the same time will take into account the maximum distance patrons are willing to travel to get services at the higher-level facilities. Thus the model locates low-level facilities in the less accessible areas to serve this "isolated" demand.

In Chapter 3 we observed the BDD problem, which is evidence of the Batty-based model shortcoming. In the spatial choice-based model the benefit distribution and demand allocation depend on the accessibility exponent value (Fig. 4.6, 4.8). In the positive range of δ, as this value grows the role of the high-level facility rises; it has more and more low-level demand allocated. Finally, at $\delta = 0.7$ the two curves cross and the high-level facility has more low-level demand allocated (Fig. 4.6). Beyond that point the BDD is not observed – the high-level facility has the largest benefit contribution and the maximal demand allocation; the middle-level facilities have the second largest values; low-level ones have the minimal values. In the negative range of δ no BDD is observed after the –0.2 point. The results in the agglomeration and competition cases are more balanced than those of the Batty-based model (Table 4.1). The greater the contribution to the overall benefit of each facility type, the more demand is allocated to it. The BDD is observed in the middle-level services (Fig. 4.7, 4.9). Under positive values of δ, the high-level facility has the largest benefit contribution, but less demand allocated.

However, the gap becomes smaller as δ grows. Middle-level service demand allocation has more inertia than does that of the low-level, which may be explained by its higher distance impedance parameter (β). The BDD is not observed for the competition case (Table 4.2) – the high-level facility contributes little benefit and has little demand allocated.

The accessibility exponent value is the crucial parameter for the model – it reflects the degree of bypassing. For the given study area conditions (developing world, patrons, mostly walking, high distance impedance), bypassing resulting from the spatial neighbourhood may be small. However, in the developed countries with cars and mass transit it might be greater. The spatial choice-based LA model is flexible enough to reflect these changes by selecting the appropriate value of δ.

4.4. The spatial choice-based model vs. the Batty-based model

To compare the spatial choice model to the Batty-based model, the actual facility system and the optimal Batty-based solution were evaluated against the spatial choice model (agglomeration case). In the actual system, the allocation links directed to Suhum cover almost half of the district (Fig. 4.10), more than half the low-level demand is served there. Thirteen low-level facilities experience direct bypassing to the high-level facility. Comparison with the Batty model solution (Fig. 4.11) gives a similar picture. The number of direct bypassing points is slightly fewer (10), but they still exist. In the agglomeration case, patrons perceive such strong additional benefit from the facility neighborhood that the Batty model solution, obtained without considering it, is sub-optimal.

The role of different types of facilities in providing low-level benefit is shown in Table 4.3. Spatial choice theory takes into account additional bypassing (agglomeration effect), so it locates low- and middle-level facilities optimally, increasing their contribution to the objective function. However, their contribution is very small; 23 low-level and 6 middle-level facilities provide, respectively, only 3.0% and 18.3% of the low-level benefit (Table 4.1). At the same time one high-level facility contributes almost 80% of the low-level benefit.

Note also the very small difference (2.1%) in the overall benefit value between the actual system and the spatial choice-based solution (Table 4.3). A similar picture is observed in the middle-level benefit (the improvement is 1.7%). By optimizing spatial configuration only the effectiveness of the system cannot be improved significantly. That leads us to the other approach of improving the effectiveness of the hierarchical facility systems – optimizing their hierarchical structure.

. Oppong (1992) noted that the facility system with one additional middle-level facility, but without low-level ones, could provide more benefit than the actual system. In both Batty-based and spatial choice-based models the contribution of the 23 low-level facilities to the overall system benefit is very small (8% as a maximum). Real-world observations, as well as the actual system evaluation by SI models, show that patrons bypass low-level facilities – they do not use them. All these are evidence of facility underutilization, which in turn constitutes non-optimal spending of available resources. How to improve patrons' convenience without using additional resources, but instead by optimizing hierarchical structure, is discussed in the next chapter.

4.5. Conclusion

The Batty-based LA model addresses only two (out of four) of the reasons, which affect patron's choice of facility. The BDD problem is observed in the Batty-based model results. The Batty model has the IIA property, which is undesirable for SI and LA modeling. Fotheringham (1983a, 1983b, 1986), Fotheringham and O'Kelly (1989), Fotheringham (1991) and Fotheringham *et al.* (2000) presented a new spatial choice model in an SI framework, and I have implemented this model for LA modeling. The new spatial choice-based LA model is flexible – by selecting the appropriate parameter (accessibility exponent value, δ) it is possible to take into account the spatial agglomeration/competition effect which was observed in various spatial interaction situations. This effect causes additional bypassing, which the Batty-based model failed to consider. Sensitivity analysis shows that the δ value affects both the location of the facilities and the allocation of demand to them. The BDD problem can be resolved by choosing the appropriate δ value. For health care this value should be positive (the agglomeration effect). However, under this effect the role of low-level facilities in providing benefit to the patrons is shown to be incredibly small. Prespecifying the number of facilities to be provided at each level limits the model's ability to improve patrons' benefit. The LA models, simultaneously optimizing spatial configuration and hierarchical structure, are discussed in the next chapter.

Table 4.1: Benefit derived from getting low-level services and low-level demand allocation

by facility type, %

Facilities	Batty-based		Agglomeration case		Competition case	
	Benefit	Allocation	Benefit	Allocation	Benefit	Allocation
Low level (23)	12.6	25.9	3.0	20.4	22.4	31.3
Middle level (6)	36.8	43.6	18.3	36.6	59.5	43.2
High level (1)	50.6	30.5	78.7	43.0	18.1	25.5
Overall	100.0	100.1	100.0	100.0	100.0	100.0

Table 4.2: Benefit derived from getting middle-level services and middle-level demand allocation by facility type, %

Facilities	Batty-based		Agglomeration case		Competition case	
	Benefit	Allocation	Benefit	Allocation	Benefit	Allocation
Middle level (6)	45.2	64.4	20.2	55.7	79.0	67.7
High level (1)	54.8	35.6	79.8	44.3	21.0	32.3
Overall	100	100.0	100.0	100.0	100.0	100.0

Table 4.3: Low-level benefit broken down by facility type (level) in the actual system and the optimal Batty solution examined by the spatial choice model.

Facilities	Actual system, benefit, mln	Batty-based system		Spatial choice- based	
		Benefit, mln	Improvement[28] %	Benefit, mln	Improvement[29]
Low-level (23)	58.6	47.8	-18.4	69.0	17.7
Middle-level (6)	310.5	410.0	32.0	418.7	34.8
High-level (1)	1868.7	1801.3	-3.6	1798.1	-3.8
Overall	2237.9	2259.2	1.0	2285.8	2.1

[28] Improvement calculated as $(B_{Batty} - B_{actual})/B_{actual}*100\%$
[29] Improvement calculated as $(B_{spatial\ choice} - B_{actual})/B_{actual}*100\%$

Spatial choice-based solution (1,6,23). Agglomeration case.
Low-level demand allocation

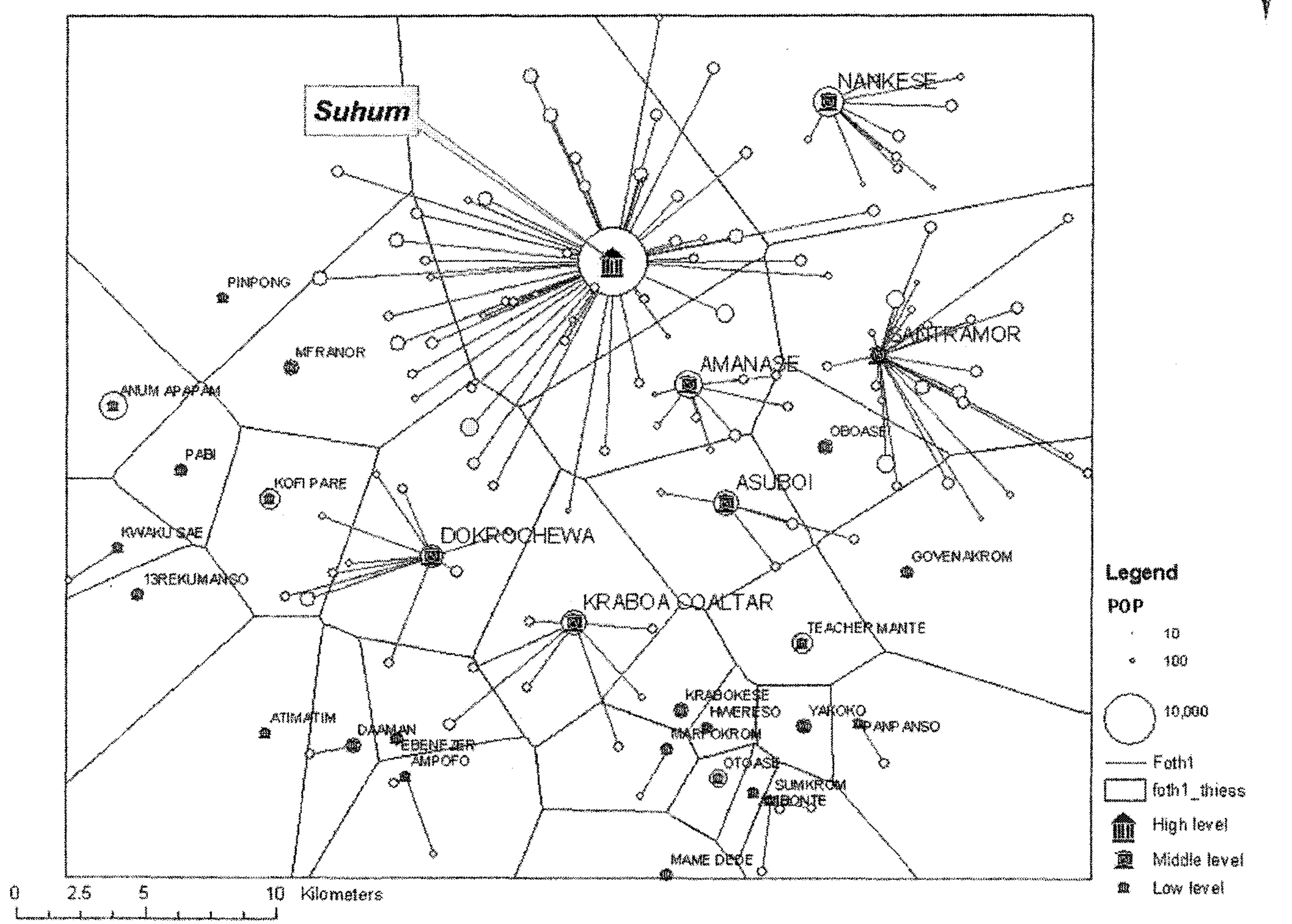


Fig. 4.1

**Spatial choice-based solution (1,6,23). Aggregation case.
Middle-level demand allocation.**

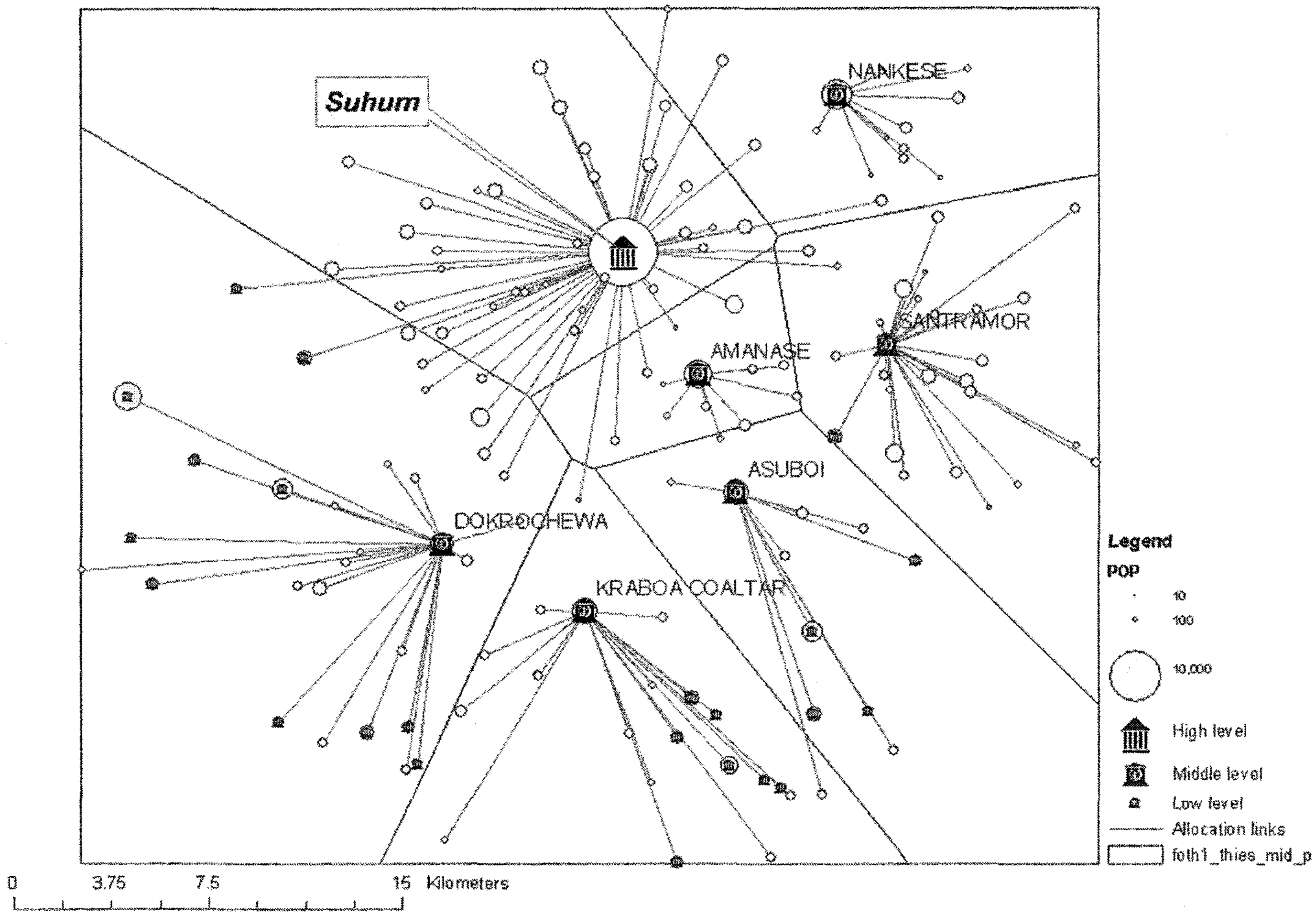


Fig. 4.2

Spatial choice-based solution (1,6,23). Competition case.
Low-level demand allocation

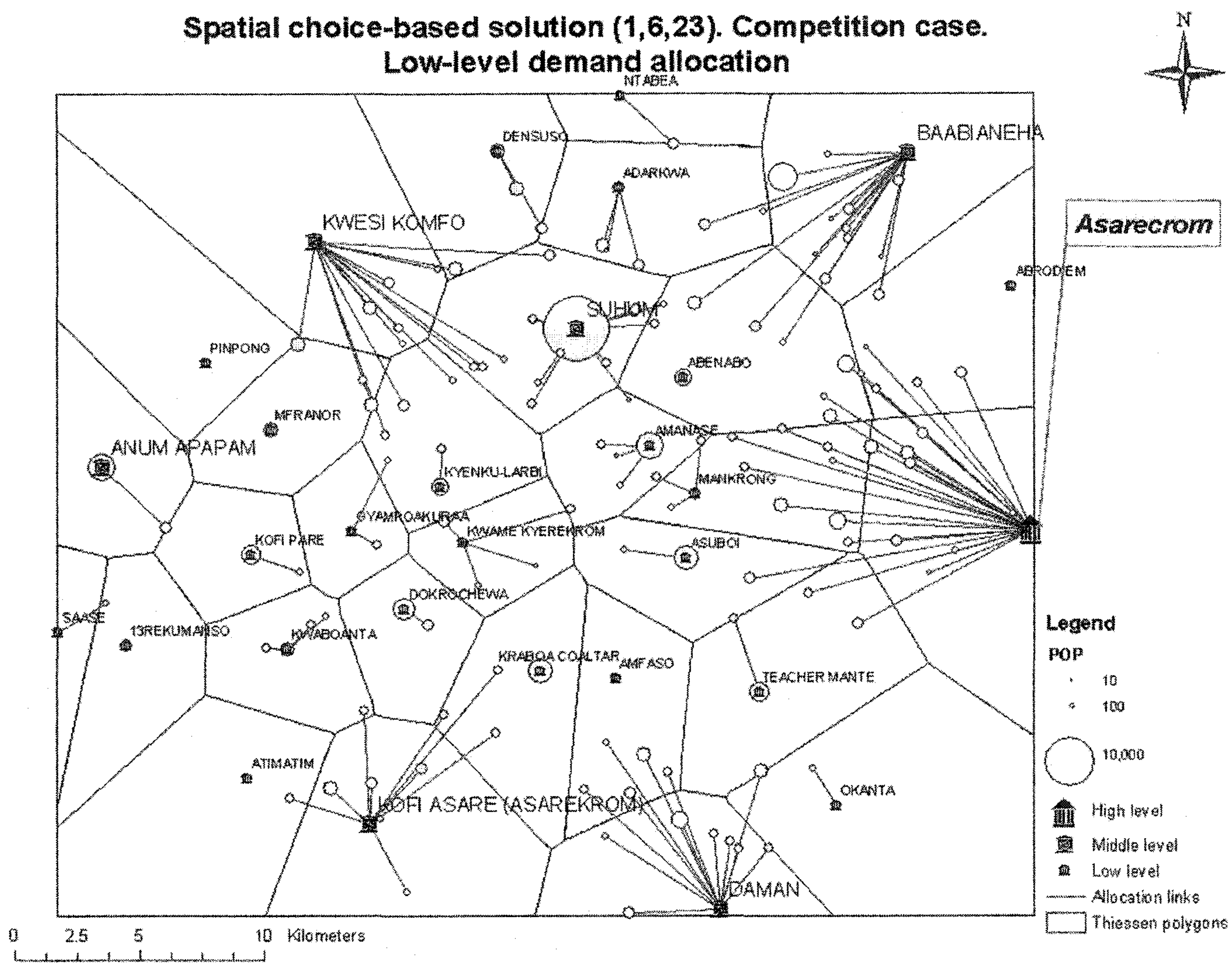


Fig.4.3

Spatial choice-based solution (1,6,23). Competition case.
Middle-level demand allocation.

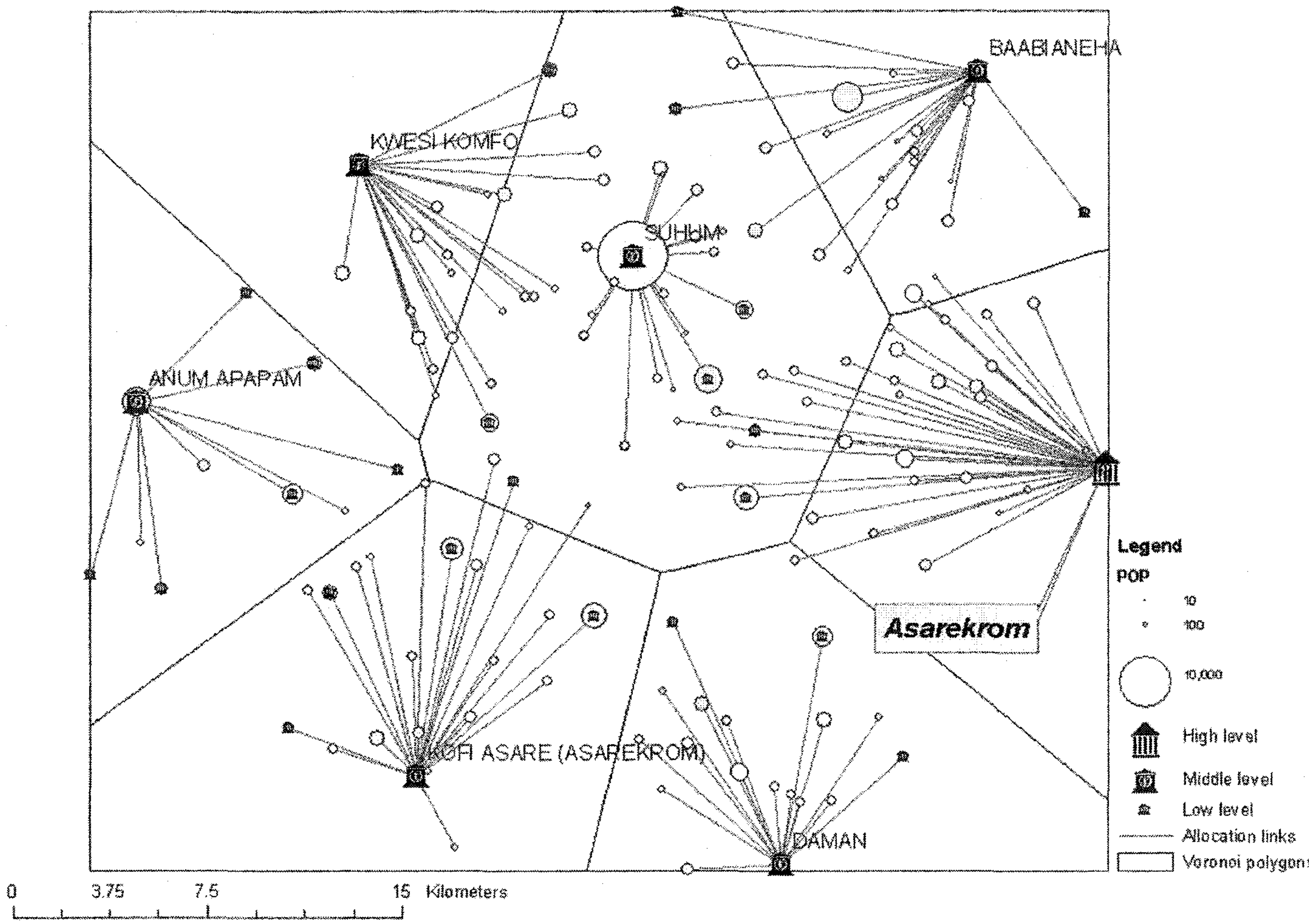


Fig. 4.4

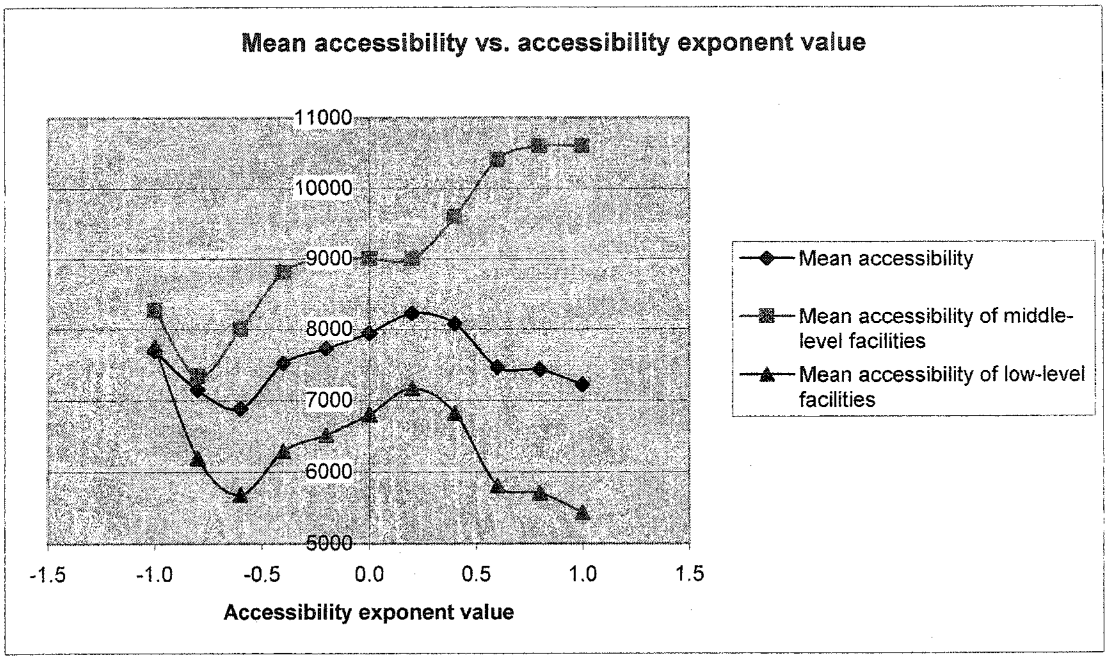

Mean accessibility vs. accessibility exponent value
11000
10000
9000
8000
7000
6000
5000
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Accessibility exponent value
Mean accessibility
Mean accessibility of middle-level facilities
Mean accessibility of low-level facilities

Fig. 4.5

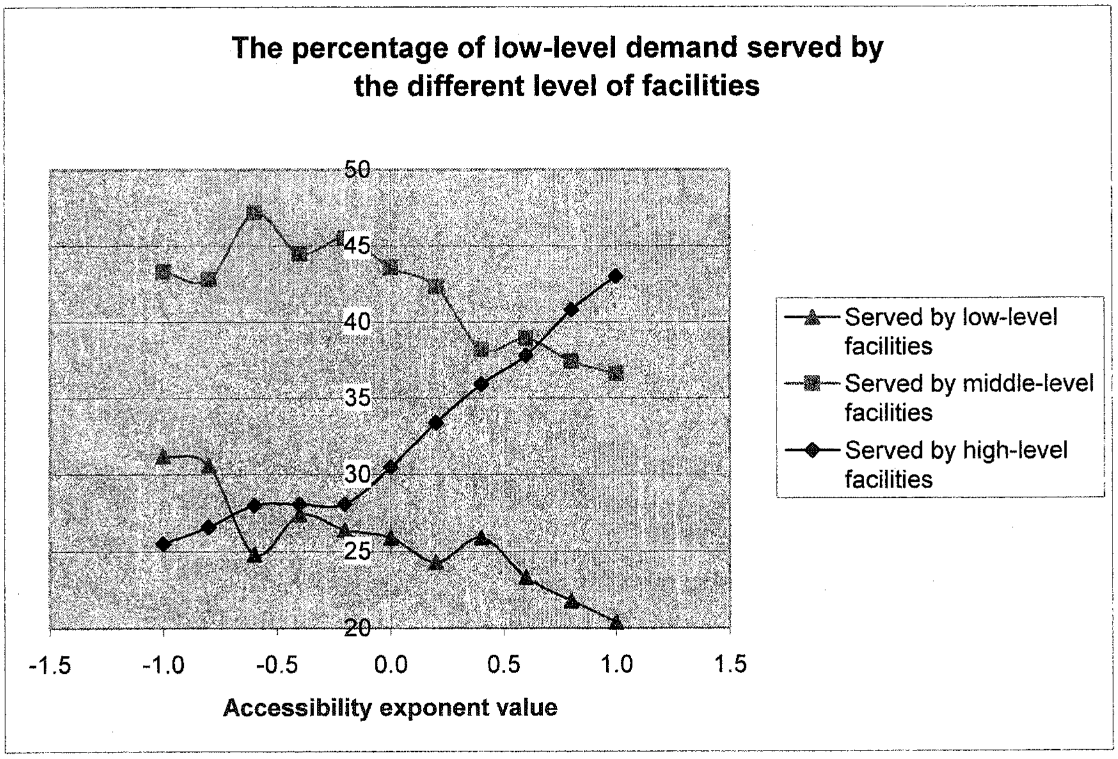

The percentage of low-level demand served by the different level of facilities
50
45
40
35
30
25
20
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Accessibility exponent value
Served by low-level facilities
Served by middle-level facilities
Served by high-level facilities

Fig. 4.6.

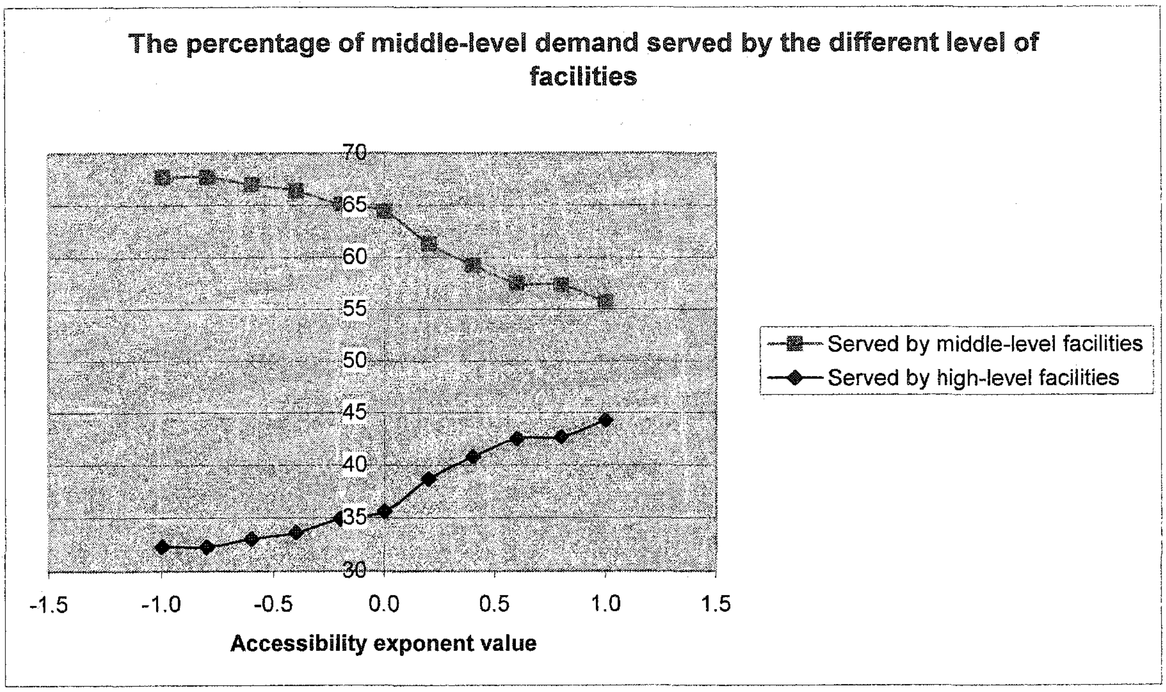

The percentage of middle-level demand served by the different level of facilities
70
65
60
55
50
45
40
35
30
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Accessibility exponent value
Served by middle-level facilities
Served by high-level facilities

Fig. 4.7

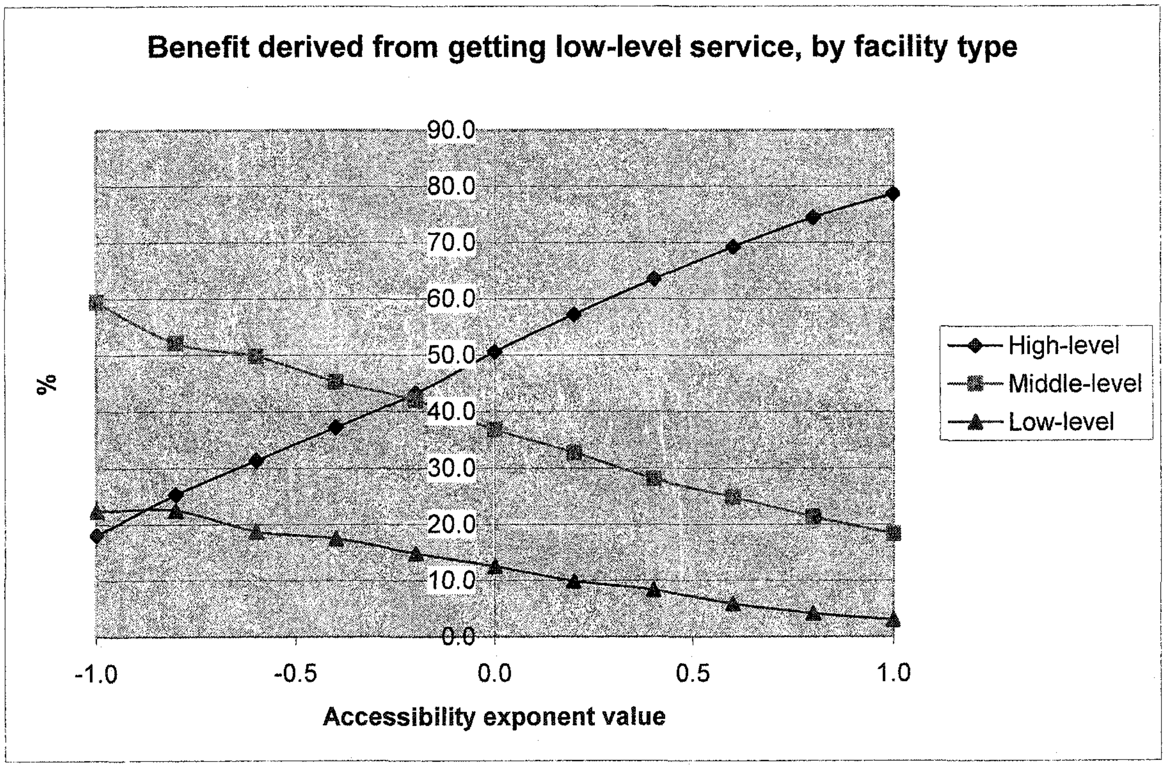
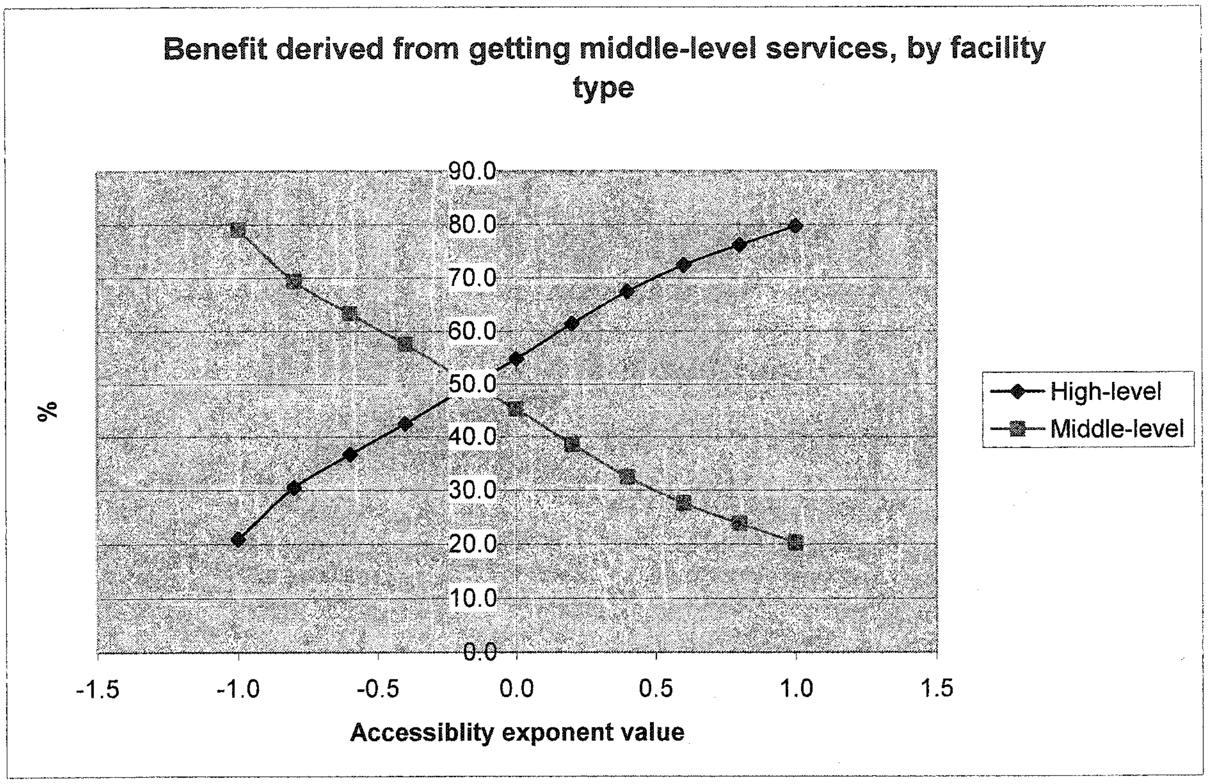

Benefit derived from getting low-level service, by facility type
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
%
-1.0
-0.5
0.0
0.5
1.0
Accessibility exponent value
High-level
Middle-level
Low-level

Fig. 4.8



Benefit derived from getting middle-level services, by facility type
%
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Accessiblity exponent value
High-level
Middle-level

Fig. 4.9

Spatial choice-based model evaluation of the Actual system.
Low-level demand allocation.

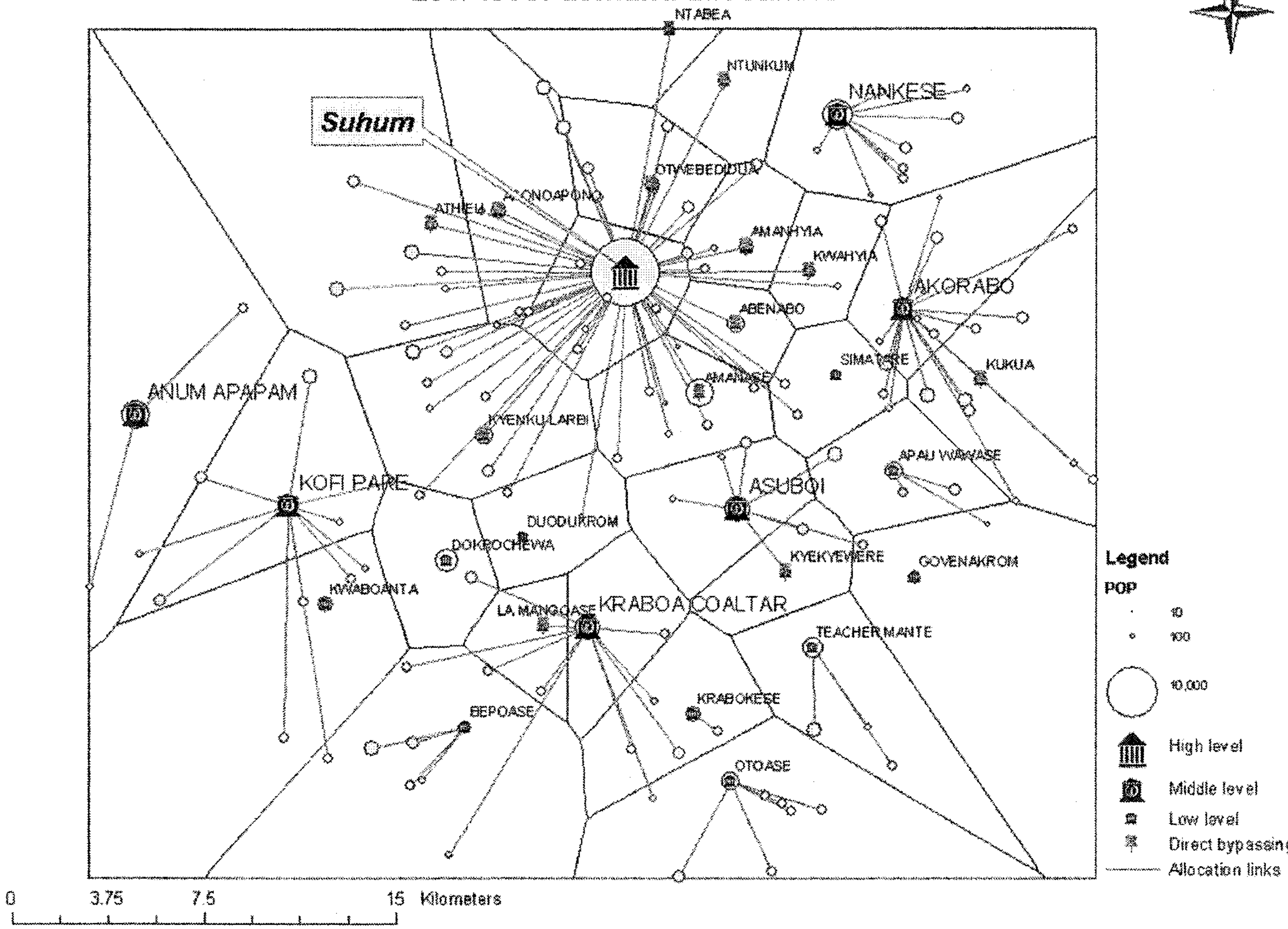


Fig. 4.10

**Spatial choice-based model evaluation of the Batty-based system.
Low-level demand allocation.**

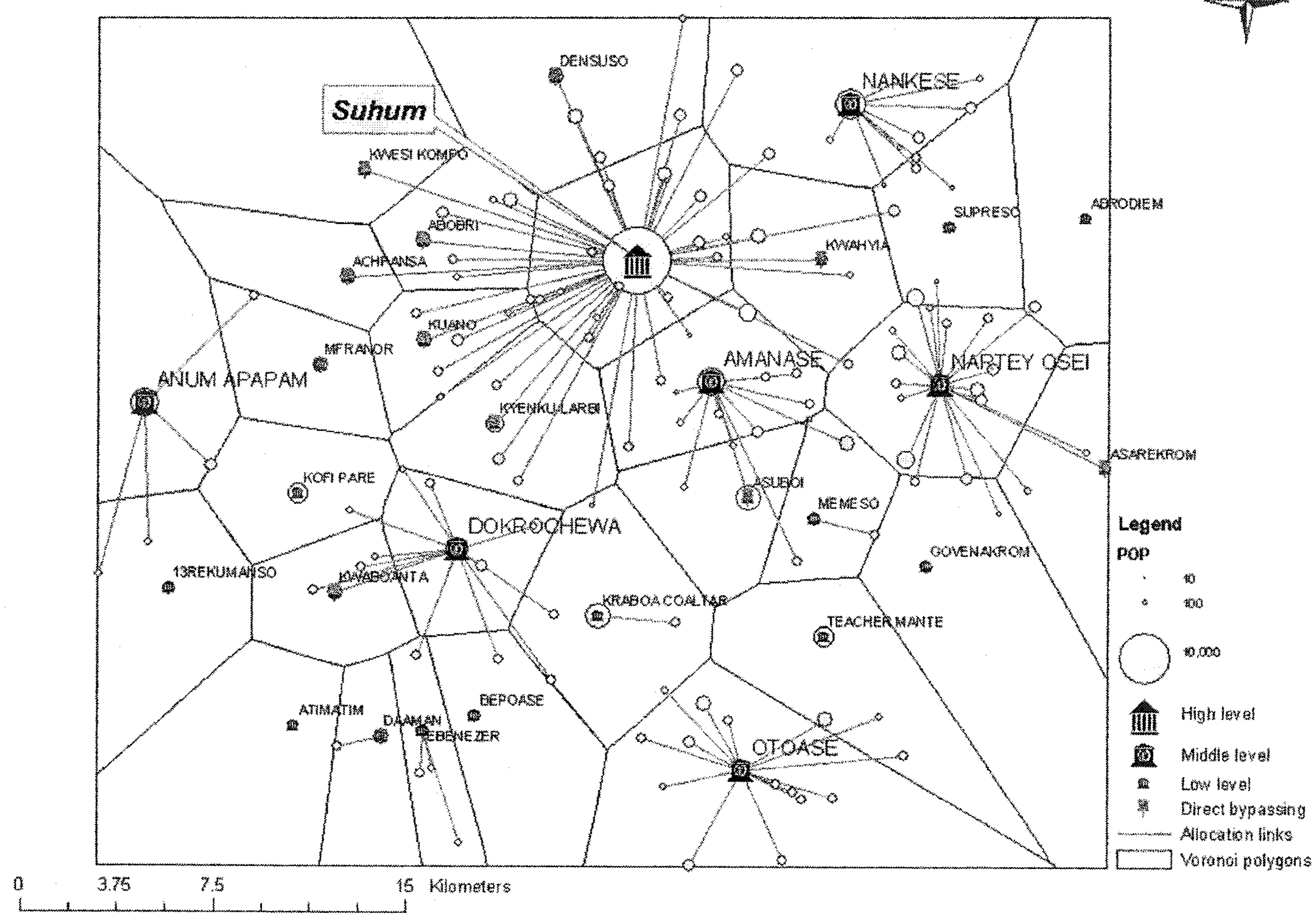


Fig. 4.11

Chapter 5. Optimizing hierarchical structure

5.1. Previous attempts at optimizing hierarchical structure

Most of the previously developed LA models optimized the spatial configuration of the hierarchical system. In other words, they recommended *where* to locate facilities assuming that the number of facilities at each level to be given. In particular, the budget constraints (4) and (14) specified how many facilities of each level must be located. However, the analysis of bypassing in Chapters 3 and 4 demonstrates that some patrons do not use facilities of one (low) level, preferring being served at another (higher) level. It was also shown that the low-level facilities provide a very small contribution to the objective function of the interaction-based LA models, revealing them to be unimportant in terms of the overall system benefit. Optimization would be greater if the LA models simultaneously answer the question "*how many* to locate" along with "*where to* locate". In this chapter I present such models and compare their results to those that optimize spatial configuration only.

The simultaneous optimization of spatial configuration and hierarchical structure was considered by Dokmeci (1973, 1977, 1979). She considered a three-level, 12-node problem. The number of high-level facilities was fixed at one; the number of low-level and middle-level facilities was varied correspondingly from one to twelve and from one to five. The spatial configuration of facilities was optimized by a bottom-up, Cooper (1963)-based heuristic. The best hierarchical structure was found by enumeration. Later, the solution for the extended four-level, 18-node problem was found. These works had two shortcomings:

- Mentioned in Chapter 2, the bottom-up approach is inferior to the simultaneous location of facilities of all levels.
- Explicit enumeration makes the LA model very computationally expensive and unsolvable for problems like the study area because of their significant size.

Okabe *et al.* (1997) proposed a model solved by a heuristic algorithm based on Voronoi diagrams tessellations; they considered the cases of two-, three- and four-level hierarchies. The possible number of facilities at each level was not initially bounded, but defined by the model. However, as they noted, the algorithm could be applied only by assuming that the demand is uniformly distributed over a region, unacceptable for most real-world problems, such as the study area.

Least-cost allocation was assumed both by Dokmeci (1973, 1977, 1979) and Okabe *et al.* (1997). They also assumed that facilities will be located in a continuous plane -- there is an infinite number of possible facility locations. Usually in real world situations planners and businessmen must select the location of a facility from a definite number of sites, e.g. to decide at which communities facilities should be located. The works used the heuristic procedures, which did not guarantee optimal solutions.

5.2. Hierarchical structure optimization in the LA models. The *p*-median model

The hierarchical LA models considered in the previous chapters (*p*-median and interaction-based) can be transformed to simultaneously optimize spatial configuration and hierarchical structure. The optimal combination of facilities of different levels (hierarchical structure) can be modelled by changing constraints (4) and (14):

$$\sum_{j \in \mathbf{J}} \sum_{k \in \mathbf{K}} C^k * Y_j^k = P$$

(24)

where C^k is the cost of establishing one facility of level k and P is the overall budget available. Introducing the level-specific cost parameter (C^k) associated with 1/0 location variable (Y_j^k) allows the LA model to optimize the hierarchical structure. In particular, for the considered three-level hierarchy we equate the cost of a high-level facility to several times that of a middle-level facility and to many times that of a low-level facility. Given the cost parameters and the overall budget limit, the hierarchical LA models find the best combination of facilities at different levels -- that combination which gives the best objective function value.

Level-by-level cost parameters were not provided for the study area, but I estimate them from the existing hierarchical structure. Assuming that establishing one low-level facility demands 1 cost unit, I assume that one middle-level facility costs 4 units and one high-level facility costs 23 units.[30] Manipulating these cost values estimates the overall budget in the actual system to be 23*1+4*6+1*23=70 cost units.

I solved the *p*-median model with optimal hierarchical structure ((1)-(3), (5)-(7), (24)) with CPLEX 6.5.1 (Figs. 5.1 and 5.2). The execution time is 585 sec., significantly more than the time required by the predefined *p*-median model[31] (21 sec.), but entirely manageable. The optimal hierarchical structure proposed by the *p*-median model differs from the actual system. Middle-level facilities are relatively cheap and provide both low-

[30] The actual system has 23 low-level facilities per 6 middle-level and per 1 high-level ones. Correspondingly, 23/6=3.8(3), rounded to 4 and 23/1=23.

[31] For conciseness I shall call the p-median model with the predefined hierarchical structure (considered in Chapter 2) as "*p*-median predefined", whereas the *p*-median with the optimal hierarchical structure will be called as "*p*-median optimally structured".

and middle-level services. Thus the optimal structure includes seven middle-level and nineteen low-level facilities (Fig. 5.1.). In terms of the objective function (AWD) it is better to replace four low-level facilities with a single middle-level one. Correspondingly, the AWD for middle-level services has been decreased (Table 5.1). However, the fewer number of the low-level facilities was reflected by low-level AWD increasing. As a result, the improvement in the overall AWD reached by the simultaneous optimization is almost the same as that reached by optimizing spatial configuration only. At the same time, the optimally structured p-median model provides quite different spatial configuration. Comparing Figs. 3.3 and 5.1 it is seen that besides replacing four low-level facilities, the optimally structured p-median model changes the locations of three low-level and one middle-level facilities. One low-level facility (Amanase) is promoted to a middle-level one (Fig. 5.2). The larger number of middle-level facilities results in their more significant role in serving demand (Table 5.2).

It is also of particular interest how both the hierarchical structure and spatial configuration reflect the changes in the overall budget available (P). I varied this parameter from 40 to 100 in increments of 1. Total execution time was around 9 hours. The AWD and the number of facilities are plotted against the budget available: Fig 5.3 summarizes the information about the spatial configuration of the hierarchy, whereas Fig. 5.4 illustrates its structure. The pattern of changes in the total AWD curve is usual for the p-median model (Fig. 5.3). It is concave, monotonically decreasing as the budget increases. The diminishing returns rule is evident – the rate of the total AWD decrease resulting from adding another cost unit becomes smaller and smaller. There is no clear break point indicating the point at which further AWD decrease resulting from an additional cost unit would drop significantly.

However, separately, the AWD for the services of different levels do not decrease monotonically; the curves are not concave; they are rather some complex functions of the budget with numerous local minima. The middle-level AWD *increases* as the available budget changes from 52 to 53 and from 55 to 56 but this increase is compensated for by a corresponding decrease in low-level AWD.

Hierarchical structure is subject to dramatic changes under certain values of P. Starting from a budget of 76 cost units available, the optimal hierarchical structure has two high-level facilities, which results in decreasing the number of low-level facilities from 24 to 14 and the number of middle-level ones from 7 to 4. Fig. 5.3 shows the corresponding "jump up" of the low- and middle-level AWD's. However, because one additional high-level facility is added the AWD for high-level services decreases 26% (from 9.9 km to 7.33 km), so this significant improvement more than compensates for increasing of the lower-level AWD's. Changes in the hierarchical structure are reflected by those in the spatial configuration. The low- and middle-level demand allocation curves for the optimally structured p-median solution with 76 units available (Figs. 5.5, 5.6) look completely different from those with 70 units available. In the 70-unit case, 27 facilities provide low-level services, so 27 Voronoi polygons delineate their service areas. When P=76, only 18 facilities can serve low-level needs. The similar changes are observed in the middle-level service provision. Only 6 facilities provide middle-level services in the 76-unit case instead of 7 ones in the 70-unit case.

The actual hierarchical structure appeared to be close to the optimal one offered by the p-median model; the changes do not provide much improvement in terms of AWD. However, as seen in Figs. 5.3 and 5.4 the optimal structure can change dramatically as a result of adding only one budget unit. The changes in hierarchical structure are reflected by those in spatial configuration as we deal with the simultaneous optimization. There is no straightforward way to predict at which breakpoint these radical changes will occur. The analysis should take into account not only the exact amount of resources available (X cost units for instance) but also a range of values surrounding it ($X \pm 1, X \pm 2$, etc.).

5.3. Hierarchical structure optimization in the LA models. The Batty-based model

I also relaxed the assumption concerning the given number of facilities at each level for the interaction-based models (the Batty-based and the agglomeration case of the spatial choice-based models). Constraints (14) were replaced by (23), the same cost values were applied; the models were solved with CPLEX 6.5.1. For the Batty-based model the execution time was 30 seconds. The Batty-based model offers more significant changes in the hierarchical structure (Fig. 5.7 and 5.8) than the p-median model does. Patrons perceive more benefit from being served by higher-level facilities. Middle-level facilities are relatively cheap and provide more benefit than the low-level ones. Consequently, the model locates as many middle-level centres as possible (11 centres), one high-level facility as high-level demand must be met, and the remaining 3 cost units are spent on the low-level facilities.

The radical changes in hierarchical structure result in significant improvement of the benefit derived by patrons from getting low- and middle-level services (Table 5.3). The predefined Batty-based model improves the middle-level benefit value slightly (4%), whereas the optimally structured model does so by almost 20%. A similar picture occurs for low-level benefit: 15% more is provided by the optimally structured model. Breaking down the overall benefit value by the level of facility highlights the importance of middle-level facilities (Table 5.4). Due to the very small number of low-level facilities in the optimal system their role in providing the overall benefit decreases by 80% compared with the actual system. The contribution of the high-level facility is also slightly decreased. However, the large number of middle-level facilities and their optimal location provide so much additional benefit that the overall benefit is improved. The optimally structured Batty-based model provides three times as much convenience improvement as the Batty-based model with the predefined number of facilities does.

The BDD problem is resolved in the optimally structured hierarchy (Table 5.5). Having the number of low-level facilities predefined forces the LA model to allocate some demand to them even though they are unattractive and provide little benefit. That results in overall benefit losses. In the optimal hierarchical structure, the low attractiveness of the local medical rooms results in their low number (3) and the low share of demand allocation. By contrast, middle-level facilities are attractive, provide more benefit and have the most low-level demand allocated.

A sensitivity analysis was also performed for the Batty-based model. The total consumers' benefit was broken down by the service level and plotted as a function of a budget available (Fig. 5.9). All benefit measures increase smoothly as the budget grows.

Some fluctuations in this trend appear for *P* of 86, 90 and 96 cost units. Under these values the hierarchical structure is changed by adding one more high-level facility (Fig. 5.10). Unlike the *p*-median model these changes are not "one-time", but rather "periodical" – when *P*=86 the second high-level facility has been added in the first time, but at *P*=87 it disappears. The second time the optimal structure includes two high-level facilities at *P*=90, but at *P*=91 it disappears again. Only after *P*=98 does the structure "consistently" have two high-level facilities. Without having a clear explanation of such strange function behaviour I suggest that it may be accounted for by a feature of local spatial structure and/or the high distance impedance parameter of the middle-level services.

The other conclusion which can be drawn from the sensitivity analysis is that the Batty-based model establishes as many middle-level facilities as possible. It can be seen from Fig. 5.10 that low-level facilities are expendable; as soon as the budget available for low- and middle-level facilities becomes divisible by 4 (the cost of one middle-level facility) the model locates no low-level facilities. It indicates that the actual 3-level hierarchical structure in the study area is far from optimal. A two-level hierarchy with a larger number of middle-level facilities provides more benefit. This finding corresponds to the conclusion drawn by Oppong and Hodgson (1998)[32].

[32] The actual system provides less benefit than the system with optimally located high- and middle-level facilities only.

5.4. Hierarchical structure optimization in the LA models. The spatial choice-based model

The spatial choice-based model was also subjected to hierarchical structure optimization. Because the agglomeration effect is likely to be observed in the health care situation, I fixed the accessibility exponent value at 1 (agglomeration case). The execution time was 135 seconds. In the agglomeration case, patrons perceive benefit both from the facility level and from its spatial neighbourhood expressed in the accessibility of the place where it is located. It is the most "beneficial" to locate two high-level facilities in the most accessible places (Fig. 5.11). The remainder of the budget is divisible by 4, so low-level facilities, which provide so little benefit, are substituted for middle-level ones. Patrons do not need low-level facilities; they are ready to travel farther to get served at higher levels. Two high-level facilities are located at accessible sites; they have the highest attractiveness values; only their allocation links extend beyond their Voronoi polygons. Note also that the low-level demand allocation coincides with the middle-level one.

The optimally structured spatial choice-based model improves the benefit provided by the services of all levels (Table 5.6). Note that high-level benefit, which was not affected by the predefined model, is improved as well. The optimally structured model provides five times as much improvement as the predefined model does. This improvement is the result of eliminating low-level facilities and increasing the role of high-level facilities (Table 5.7). The low-level facilities are not necessary for service provision. In the predefined model they provide only 3% of the low-level service benefit, but have 20% demand allocated, thus decreasing the overall system benefit (Table 5.8).

As for the Batty-based model the optimally structured model resolves the BDD problem by minimizing the number of low-level facilities and allocating demand to more beneficial middle- and high-levels.

The relationship between the service benefit/number of facilities and the budget available for the spatial choice-based model has the same pattern as for the Batty-based model. Total patrons' benefit smoothly grows as the budget available rises (Fig. 5.12). Fluctuations in this growth are observed only once (for the Batty-based model it was three times). The change in structure caused by adding the second high-level facility happens when P=66, a lower value than for the Batty-based model (Fig 5.13). They have the same "periodical" pattern: only after P=70 the second high-level facility is consistently included in the system.

Both interaction-based LA models propose optimal hierarchical structures, which are quite different from the existing one. The low-level facilities are unattractive and provide so little benefit that they are replaced by middle-level ones. The optimal spending of the budget according to the interaction-based models is to spend only the residue of the division $(P-C^{l})/4$ for low-level facilities. At some values of P the optimal hierarchical structure contains no low-level facilities at all. That means that according to the interaction-based criteria the actual system is sub-optimal and should be changed to a two-level system with only high- and middle-level facilities.

In a successively inclusive hierarchy, simultaneous optimization of both the hierarchical structure and spatial configuration is necessary. For all models, relaxing the requirement for a specified number of facilities at each level leads to the improvement of the objective function. For all models, the diminishing returns rule is seen. However, level-by-level service benefit values, as well as the hierarchical structure, can differ dramatically by adding a single budget unit.

Letting a model decide how many facilities of each level must be located leads us to the "level-by-level convenience" problem. The overall patrons' convenience in the k-level system can be divided into k level-specific measures, which show us the patrons' convenience in getting services of a particular level. For example, the optimal solution can provide very high convenience for getting middle-level services and very low convenience for getting low-level ones. An ability to avoid such an undesirable "conflict of interest" would be a good asset for a model. In this sense the solutions (hierarchical structure) provided by the interaction-based models seem to be more balanced than those given by the p-median model. For the latter, the gradual changes in the total budget available result in frequent dramatic level-to-level "conflicts of interest". Adding one more middle- or high-level facility leads to removing several low-level ones, increasing AWD for low-level services.

Interaction-based models result in hierarchical structures in which middle-level facilities are dominant. The level-specific convenience indices (Figs. 5.9 and 5.12) are much more consistent in reflecting changes in the budget value. For these models, adding a new middle-level facility, in most cases, leads to patrons' benefit improvement for both low-level and middle-level services.

Optimal hierarchical structure depends on the costs of establishing facilities of different types. In the actual system, middle-level facilities are relatively cheap, around one-fifth the cost of high-level facilities. However, if the middle-level facility costs were half the costs of high-level one, the optimal hierarchical structure would have been changed rapidly. For example, the optimal p-median solution would include no middle-level facilities at all (Fig. 5.14). The cost/efficiency ratio of each facility type is crucial for defining the optimal hierarchical structure.

5.5. Conclusions

Optimization of multi-level facility systems implies finding not only the optimal facility locations but also the optimal hierarchical structure – the best combination of facilities of different levels. The latter depends on both the specific benefit value provided by a facility of a given level and the establishing costs. As is the case with the optimal spatial configuration, the optimal hierarchical structure varies from model to model. The hierarchical p-median model proposes a combination of facilities close to the actual one. The interaction-based LA models provide optimal structures which are significantly different from the actual. As would be expected, for all models, optimizing the hierarchical structure results in the improvement of the objective function value. However, for the p-median model this improvement is negligible, whereas for the interaction-based models it can be as high as 10%. Such improvement in patrons' convenience is significant, especially taking into account that no additional resources are needed. Simultaneously optimizing hierarchical structure and facility location seems to be crucially important in generating a new hierarchical service system; suggestions about how many levels it should contain will therefore likely be valuable for planners.

Table 5.1: AWD in the actual system and in the *p*-median solutions

Level of service	Actual system, AWD, km	*P*-median predefined		*P*-median optimally structured	
		AWD, km	Improvement[33], %	AWD, km	Improvement, %
Low-level	1.380	1.060	23.2	1.170	15.2
Middle-level	3.750	3.340	10.9	3.000	20.0
Overall	3.47	3.190	8.1	3.188	8.1

Table 5.2: Level-by-level demand allocation in the *p*-median solutions

Model	# of demand points served[34] by facilities, by level of facility			Percentage of low-level demand served at facilities, by level of facility			Percentage of middle-level demand served, by level of facility	
	Low	Middle	High	Low	Middle	High	Middle	High
P-median predefined	117	148	35	60.4	19.7	20.0	64.8	35.2
P-median optimally structured	103	170	27	52.2	27.8	20.0	70.3	29.7

[33] Both *Improvement* columns for p-median models are calculated as $(Z_{actual} - Z_{pmed})/Z_{actual}*100\%$

[34] Only low- or middle-level demand is considered. Note that each point has the both types of demand, so each point is counted twice.

Table 5.3: Benefit provided by service level in the actual system and the Batty-based solutions

Service	Actual system, benefit	Batty-based predefined		Batty-based optimally structured	
		Benefit	**Improvement[35], %**	**Benefit**	**Improvement, %**
Low-level	113,467.0	119,603.0	5.4	130,603.0	15.1
Middle-level	35,830.2	37,178.3	3.8	42,680.3	19.1
High-level	29,934.6	29,934.6	0.0	29,934.6	0.0
Overall	179,231.8	186,715.5	4.2	203,218.0	13.4

Table 5.4: Benefit provided by facilities of different levels in the actual system and the Batty-based solutions

Facilities	Actual system, benefit	Batty-based predefined		Batty-based optimally structured	
		Benefit	Improvement[36], %	Benefit	Improvement, %
Low-level	12,067.7	15,026.7	24.5	2,397.0	-80.1
Middle-level	52,165.9	60,873.7	16.7	90,910.0	74.2
High-level	114,998.0	110,815.0	-3.6	109,911.0	-4.4
Overall	179,231.84	186,715.5	4.2	203,218.0	13.4

[35]The improvement values provided by the Batty-based models are calculated as $(B_{Batty} - B_{actual})/B_{actual}*100\%$

[36]The improvement values provided by the Batty-based models are calculated as $(B_{Batty} - B_{actual})/B_{actual}*100\%$

Table 5.3: Benefit provided by service level in the actual system and the Batty-based solutions

Service	Actual system, benefit	Batty-based predefined		Batty-based optimally structured	
		Benefit	Improvement[35], %	Benefit	Improvement, %
Low-level	113,467.0	119,603.0	5.4	130,603.0	15.1
Middle-level	35,830.2	37,178.3	3.8	42,680.3	19.1
High-level	29,934.6	29,934.6	0.0	29,934.6	0.0
Overall	179,231.8	186,715.5	4.2	203,218.0	13.4

Table 5.4: Benefit provided by facilities of different levels in the actual system and the Batty-based solutions

Facilities	Actual system, benefit	Batty-based predefined		Batty-based optimally structured	
		Benefit	Improvement[36], %	Benefit	Improvement, %
Low-level	12,067.7	15,026.7	24.5	2,397.0	-80.1
Middle-level	52,165.9	60,873.7	16.7	90,910.0	74.2
High-level	114,998.0	110,815.0	-3.6	109,911.0	-4.4
Overall	179,231.84	186,715.5	4.2	203,218.0	13.4

[35]The improvement values provided by the Batty-based models are calculated as $(B_{Batty} - B_{actual})/B_{actual}*100\%$

[36]The improvement values provided by the Batty-based models are calculated as $(B_{Batty} - B_{actual})/B_{actual}*100\%$

Table 5.5: Benefit derived from getting low-level services and low-level demand allocation in the Batty-based solutions by facility type, %

Facilities	Batty-based predefined		Batty-based optimally structured	
	Benefit	Allocation	Benefit	Allocation
Low level	12.6	25.9	1.8	3.8
Middle level	36.8	43.6	52.0	66.1
High level	50.6	30.5	46.1	30.0
Overall	100.0	100.1	100.0	100.0

Table 5.6: Benefit provided by service level in the actual system and the spatial choice-based solutions

Service	Actual system, benefit, mln	Spatial choice predefined		Spatial choice optimally structured	
		Benefit, mln	Improvement[37], %	Benefit, mln	Improvement, %
Low-level	2237.9	2285.8	2.1	2362.7	5.6
Middle-level	730.6	743.3	1.7	777.5	6.4
High-level	834.8	834.8	0.0	857.3	2.7
Overall	3803.3	3864.0	1.6	3997.5	5.1

[37] The improvement values provided by the spatial choice-based models are calculated as $(B_{spatial\ choice} - B_{actual})/B_{actual}*100\%$

Table 5.7: Benefit provided by facilities of different levels in the actual system and the spatial choice-based solutions

Facilities	Actual system, benefit, mln	Spatial choice predefined		Spatial choice optimally structured	
		Benefit, mln	Improvement[38], %	Benefit, mln	Improvement, %
Low-level	58.6	69.0	17.7	0.0	-100.0
Middle-level	425.3	569.1	33.8	443.6	4.3
High-level	3319.4	3225.9	-2.8	3554.0	7.1
Overall	3803.3	3864.0	1.6	3997.5	5.1

Table 5.8: Benefit derived from getting low-level services and low-level demand allocation in the spatial choice-based solutions by facility type, %

Facilities	Spatial choice predefined		Spatial choice optimally structured	
	Benefit	Allocation	Benefit	Allocation
Low level	3.0	20.4	0.0	0.0
Middle level	18.3	36.6	14.1	45.4
High level	78.7	43.0	85.9	54.6
Overall	100.0	100.1	100.0	100.0

[38] The improvement values provided by the spatial choice-based models are calculated as $(B_{spatial\ choice} - B_{actual})/B_{actual} * 100\%$

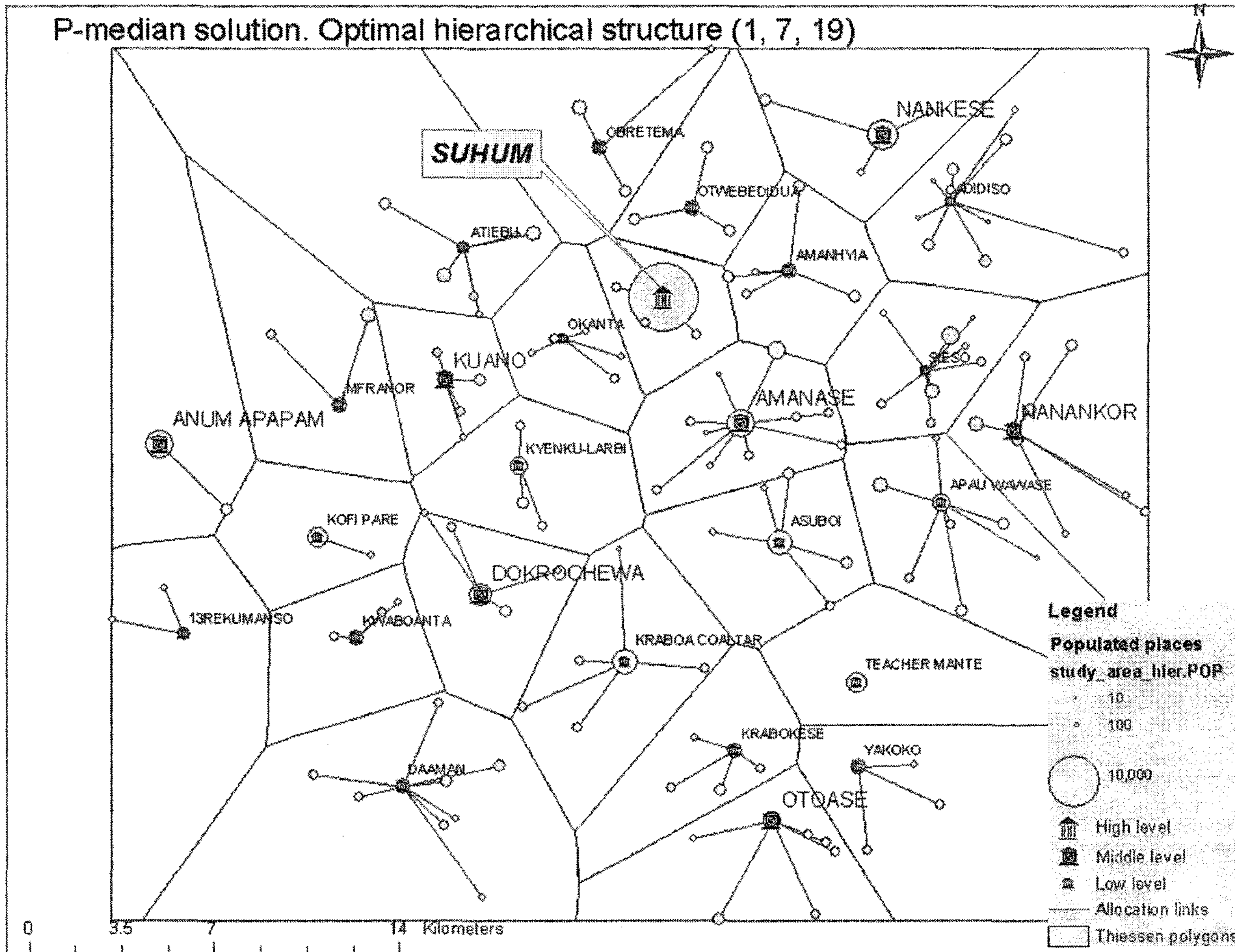

P-median solution. Optimal hierarchical structure (1, 7, 19)
N
SUHUM
NANKESE
OBRETEMA
OTWEBEDIDUA
ATIEBU
AMANHYIA
OKANTA
KUANO
MFRANOR
ANUM APAPAM
AMANASE
KYENKU-LARBI
APAU WAWASE
ASUBOI
KOFI PARE
DOKROCHEWA
13REKUMANSO
KWABOANTA
KRABOA COALTAR
TEACHER MANTE
KRABOKESE
YAKOKO
DAAMAN
OTOASE
0
3.5
7
14 Kilometers
Legend
Populated places
study_area_hier.POP
10
100
10,000
High level
Middle level
Low level
Allocation links
Thiessen polygons

Fig. 5.1

P-median model. Optimal hierarchical structure (1,7,19).
Middle-level demand allocation.

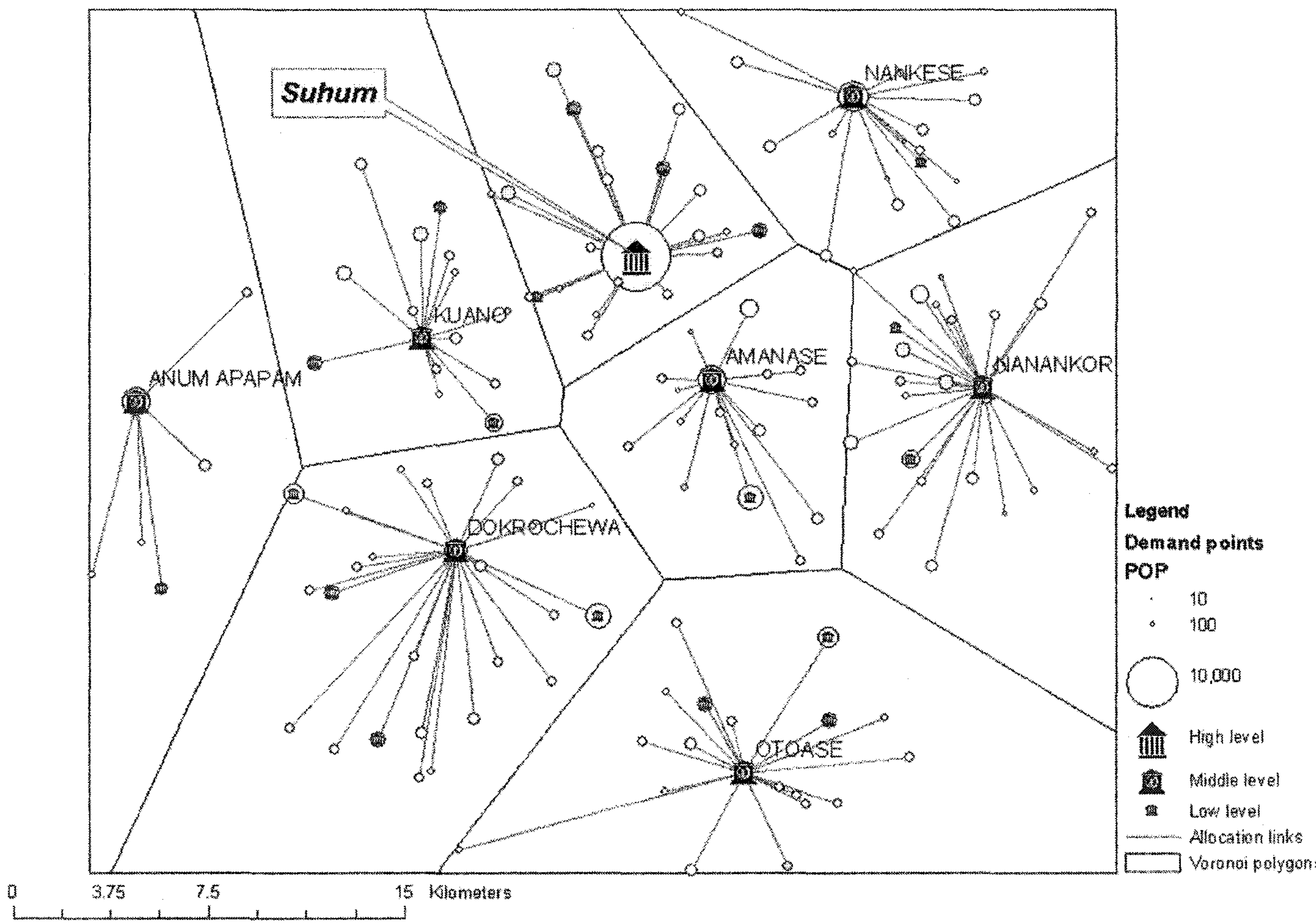


Fig. 5.2

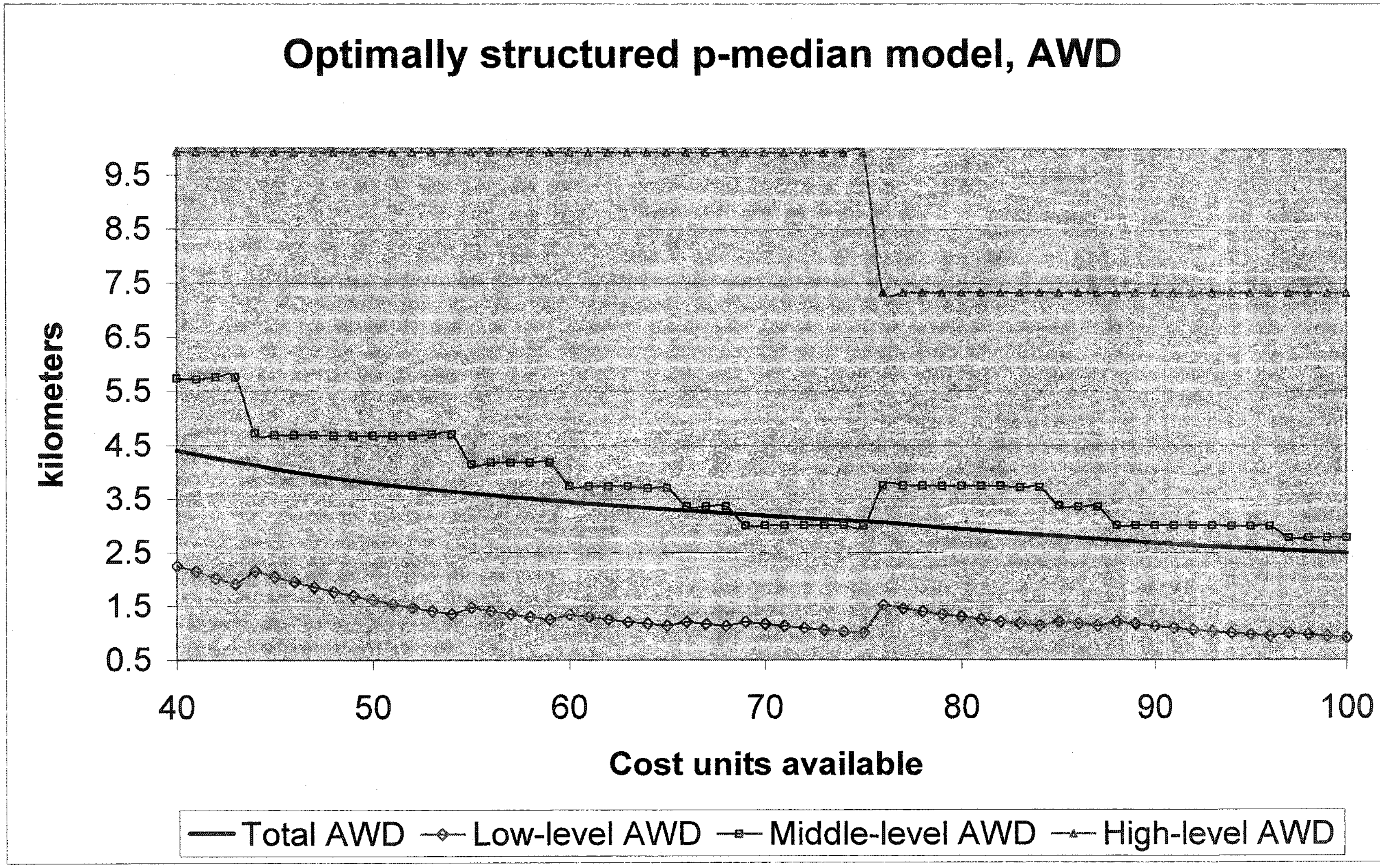

Optimally structured p-median model, AWD
kilometers
9.5
8.5
7.5
6.5
5.5
4.5
3.5
2.5
1.5
0.5
40
50
60
70
80
90
100
Cost units available
Total AWD
Low-level AWD
Middle-level AWD
High-level AWD

Fig. 5.3.

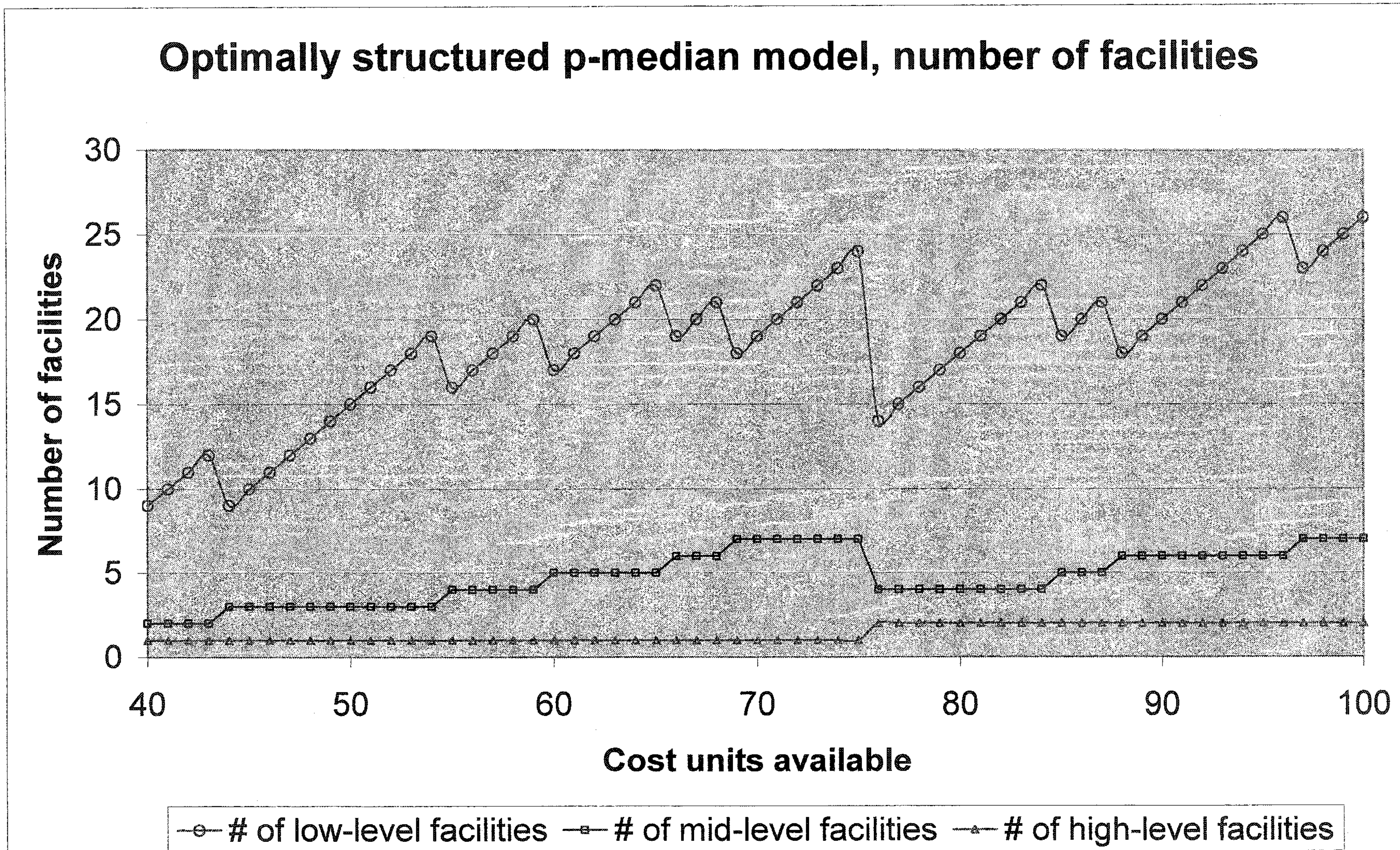

Optimally structured p-median model, number of facilities
Number of facilities
0
5
10
15
20
25
30
40
50
60
70
80
90
100
Cost units available
# of low-level facilities
# of mid-level facilities
# of high-level facilities

Fig. 5.4

P-median model solution. Optimal hierarchical structure (2,4,12).
Low-level demand allocation.

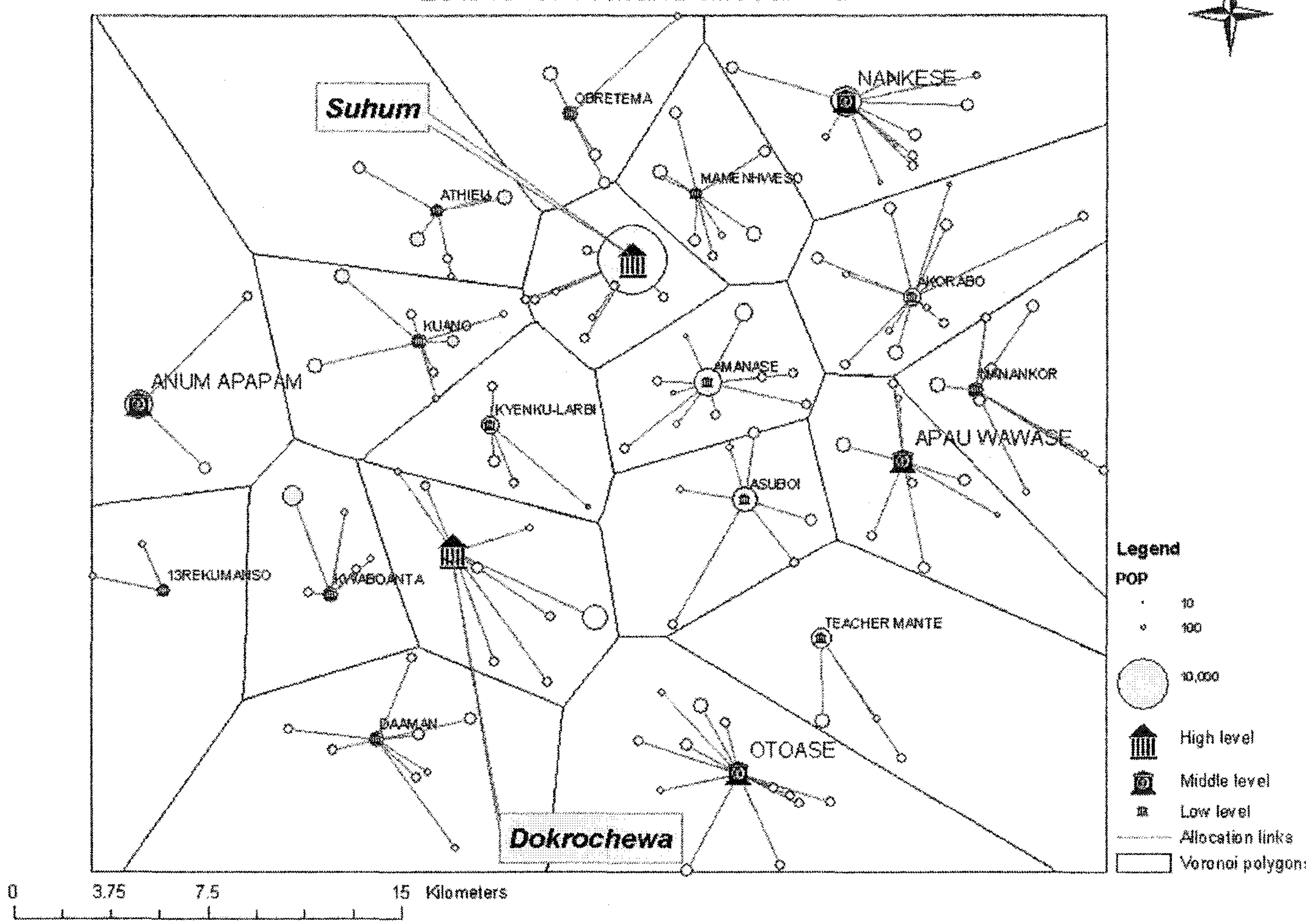


Fig. 5.5

P-median model solution. Optimal hierarchical structure (2,4,12). Middle-level demand allocation.

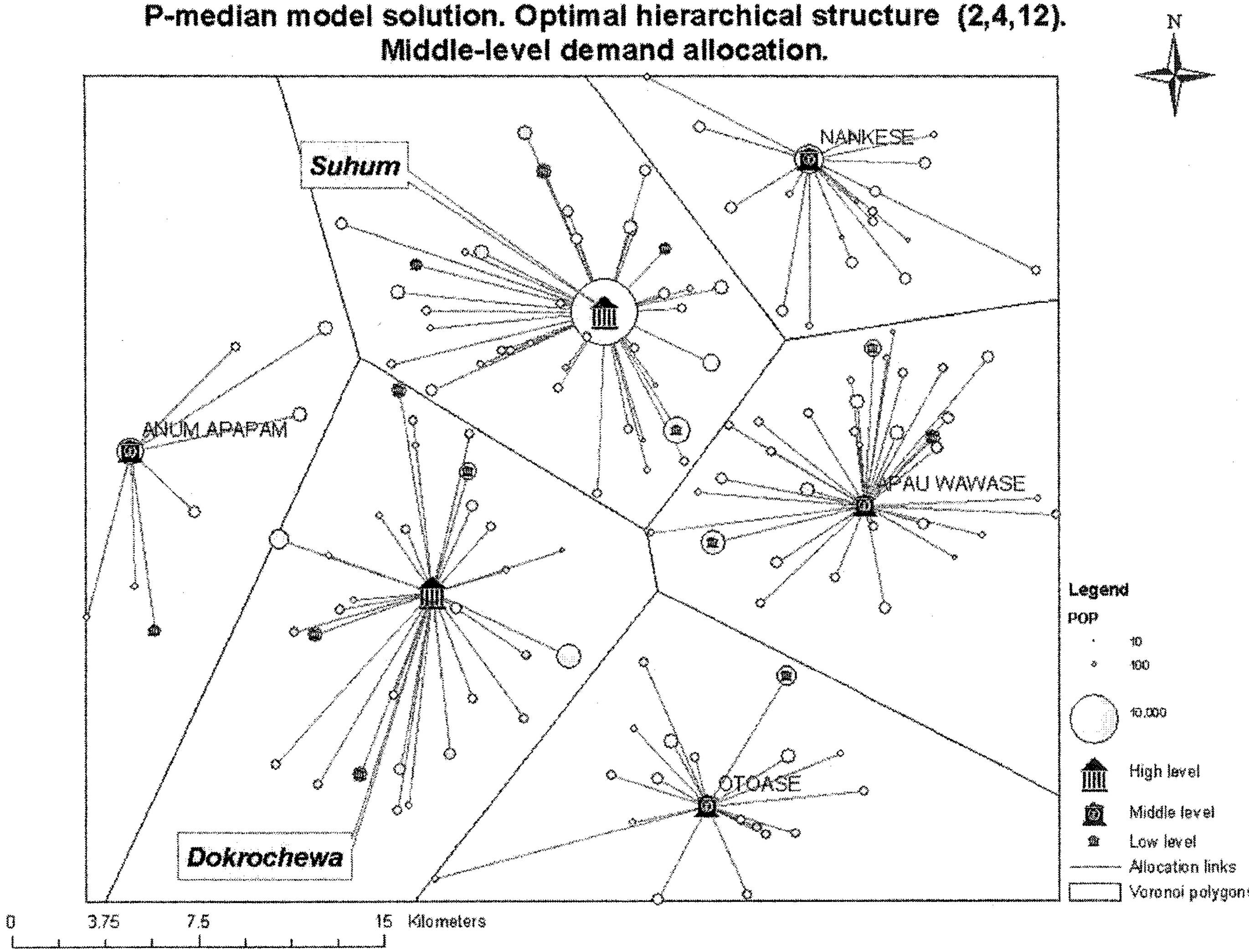


Fig. 5.6

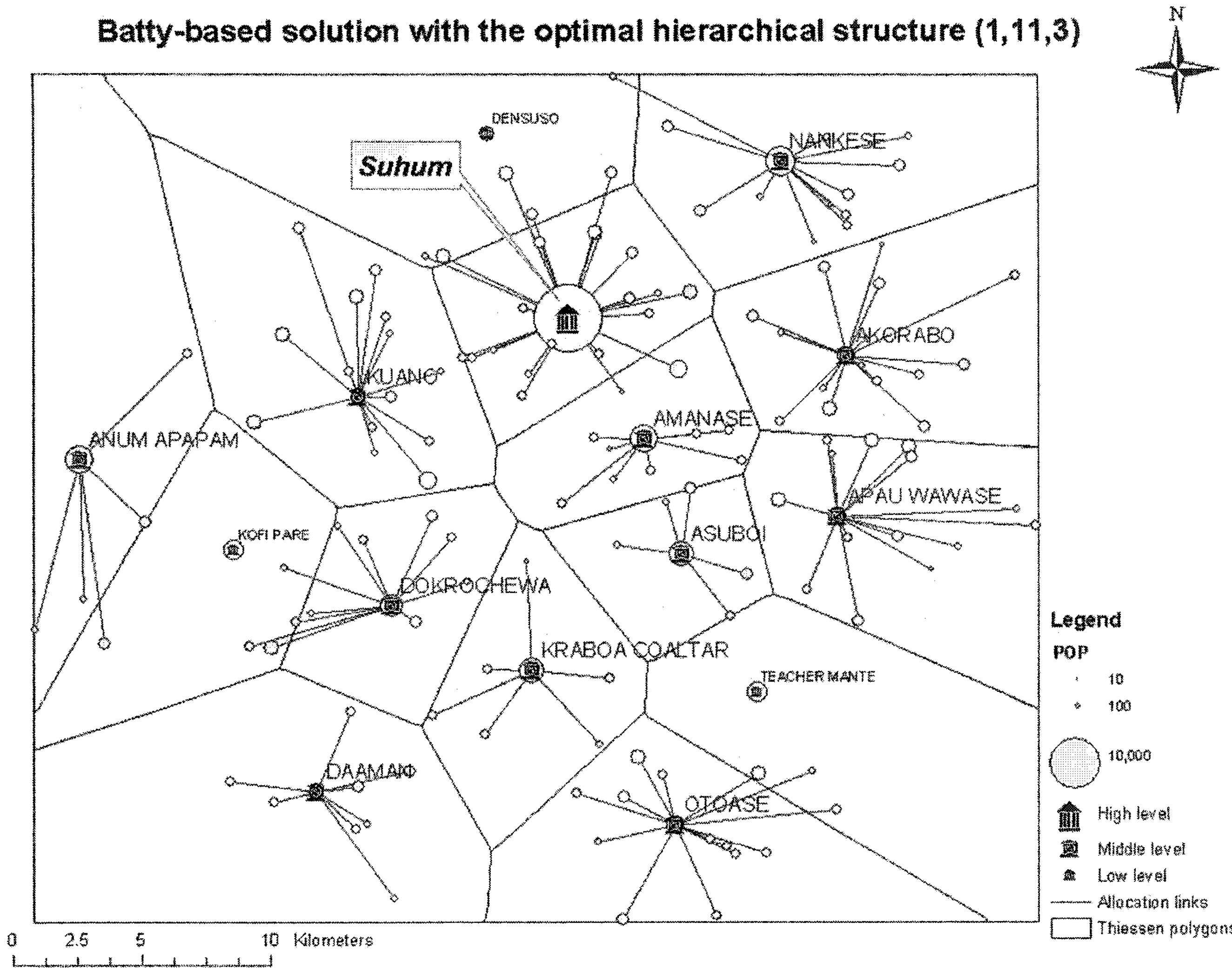

Batty-based solution with the optimal hierarchical structure (1,11,3)
N
DENSUSO
Suhum
NANKESE
AKORABO
ANUM APAPAM
AMANASE
APAU WAWASE
KOFI PARE
ASUBOI
DOKROCHEWA
KRABOA COALTAR
TEACHER MANTE
OTOASE
Legend
POP
10
100
10,000
High level
Middle level
Low level
Allocation links
Thiessen polygons
0 2.5 5 10 Kilometers

Fig. 5.7

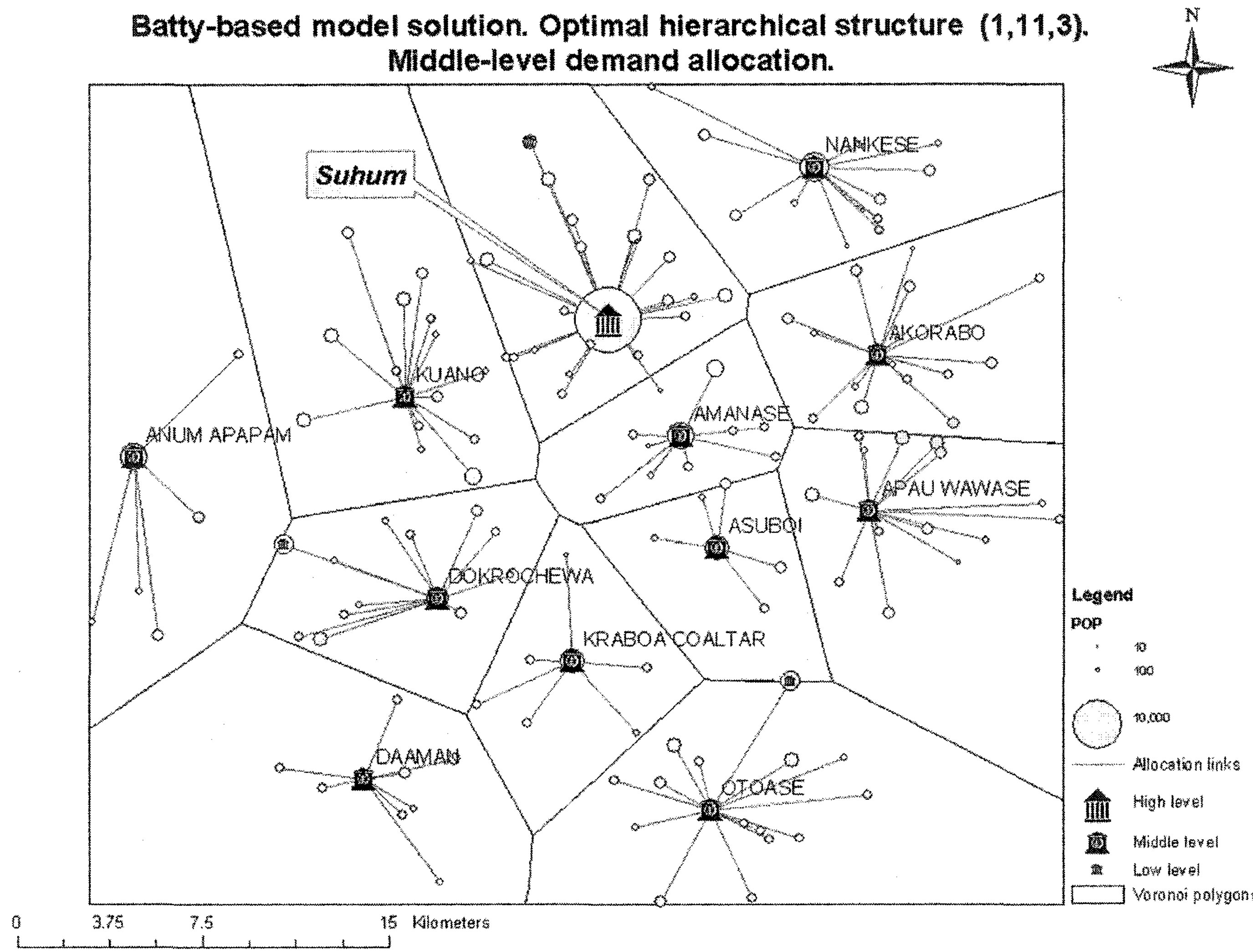

Batty-based model solution. Optimal hierarchical structure (1,11,3).
Middle-level demand allocation.
N
Suhum
NANKESE
AKORABO
KUANO
AMANASE
ANUM APAPAM
APAU WAWASE
ASUBOI
DOKROCHEWA
KRABOA COALTAR
DAAMAN
OTOASE
Legend
POP
10
100
10,000
Allocation links
High level
Middle level
Low level
Voronoi polygons
0
3.75
7.5
15
Kilometers

Fig. 5.8

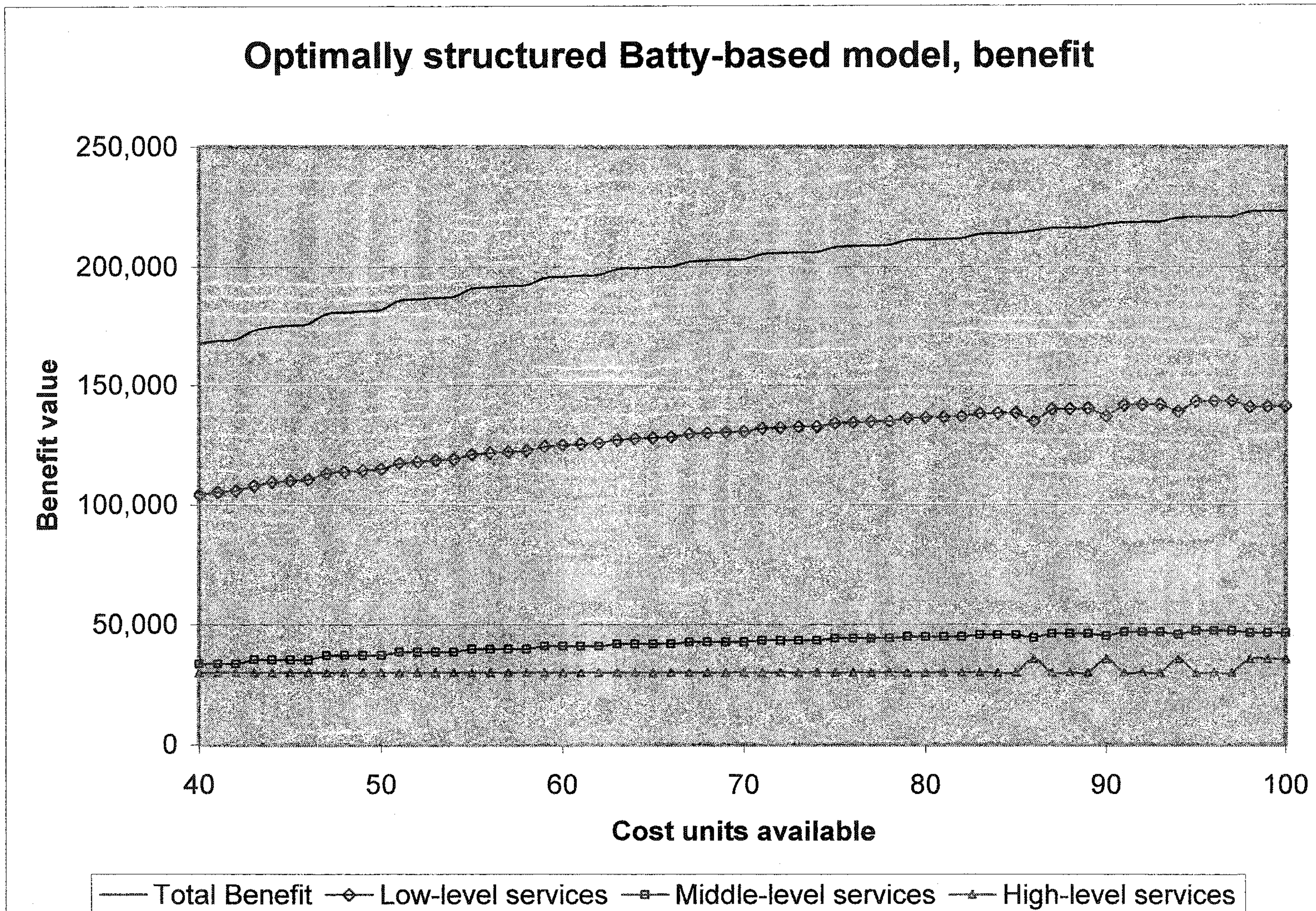

Optimally structured Batty-based model, benefit
Benefit value
250,000
200,000
150,000
100,000
50,000
0
40
50
60
70
80
90
100
Cost units available
Total Benefit
Low-level services
Middle-level services
High-level services

Fig. 5.9

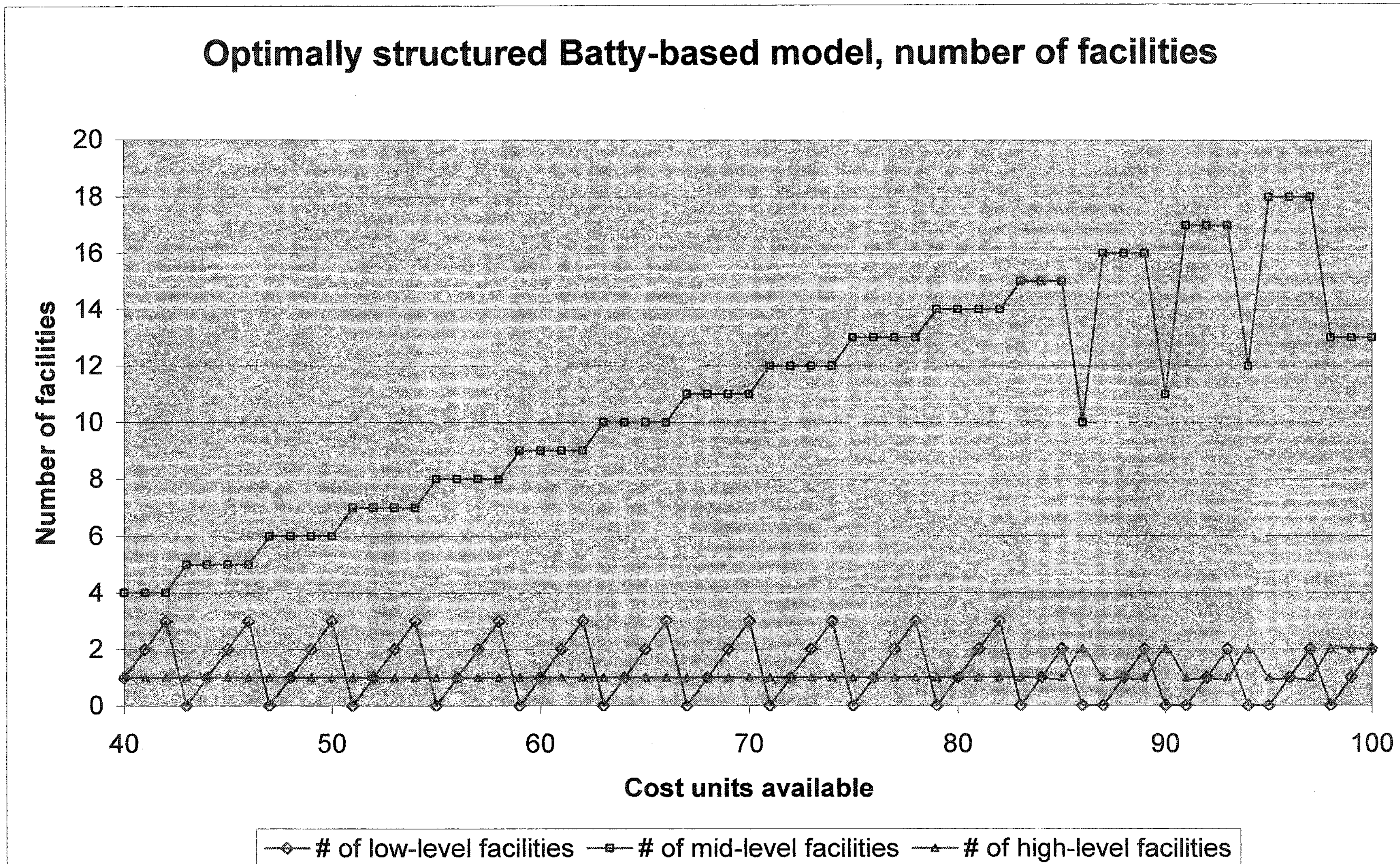

Optimally structured Batty-based model, number of facilities
Number of facilities
0
2
4
6
8
10
12
14
16
18
20
Cost units available
40
50
60
70
80
90
100
# of low-level facilities
# of mid-level facilities
# of high-level facilities

Fig. 5.10

Optimal spatial choice-based solution (2,6,0). Agglomeration case.
Low-level demand allocation

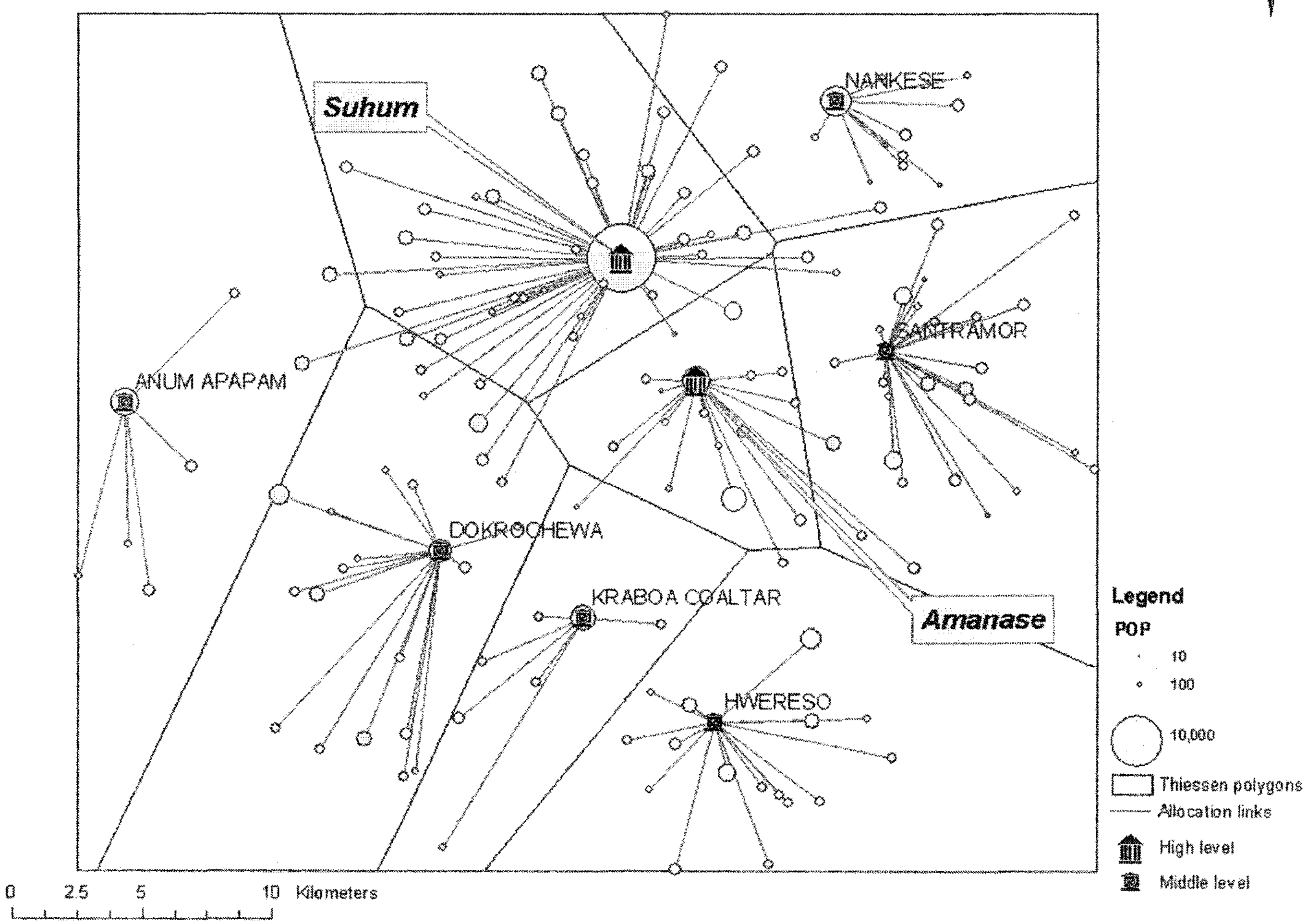


Fig. 5.11

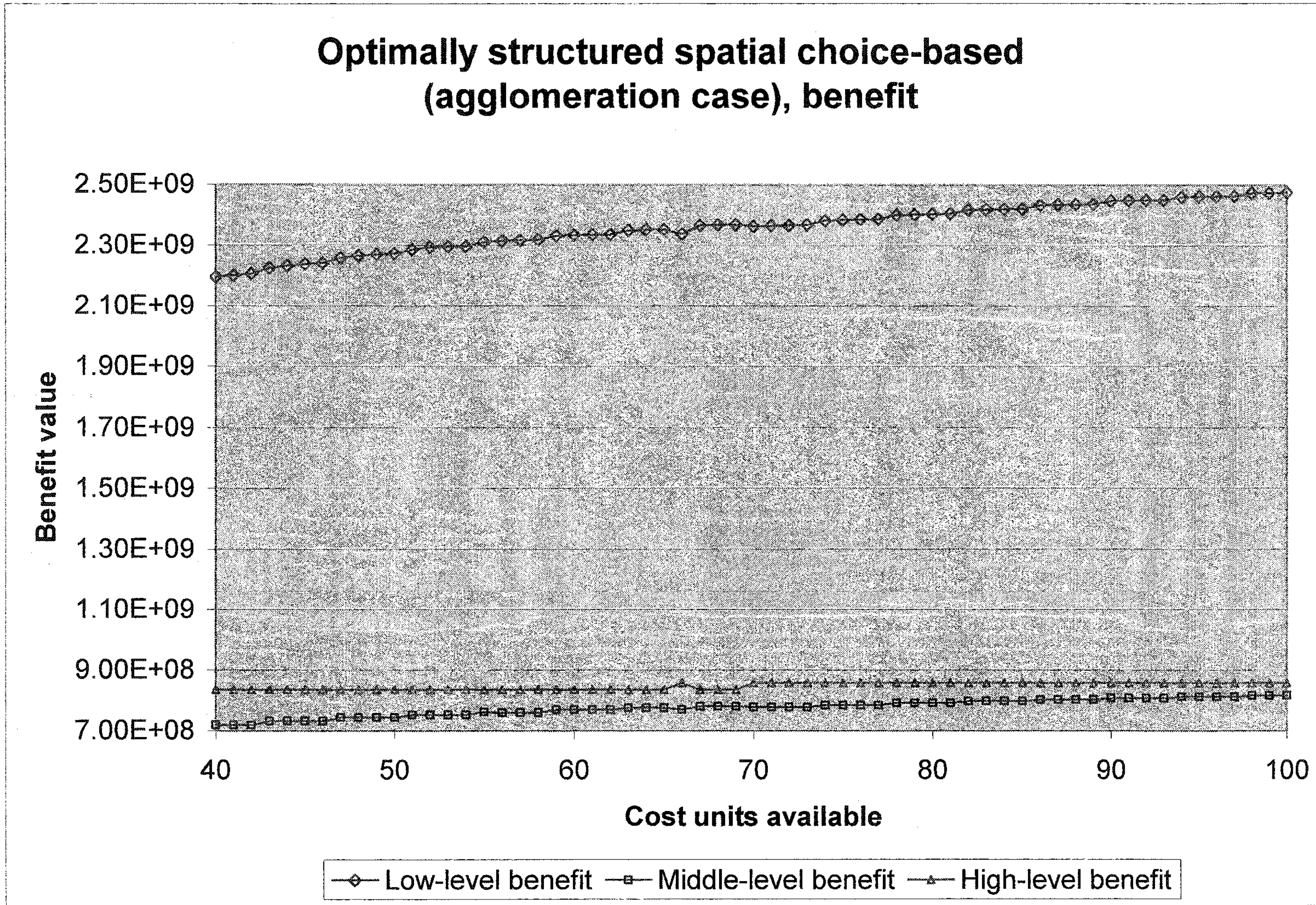

Optimally structured spatial choice-based
(agglomeration case), benefit
2.50E+09
2.30E+09
2.10E+09
1.90E+09
1.70E+09
1.50E+09
1.30E+09
1.10E+09
9.00E+08
7.00E+08
Benefit value
40
50
60
70
80
90
100
Cost units available
Low-level benefit
Middle-level benefit
High-level benefit

Fig. 5.12

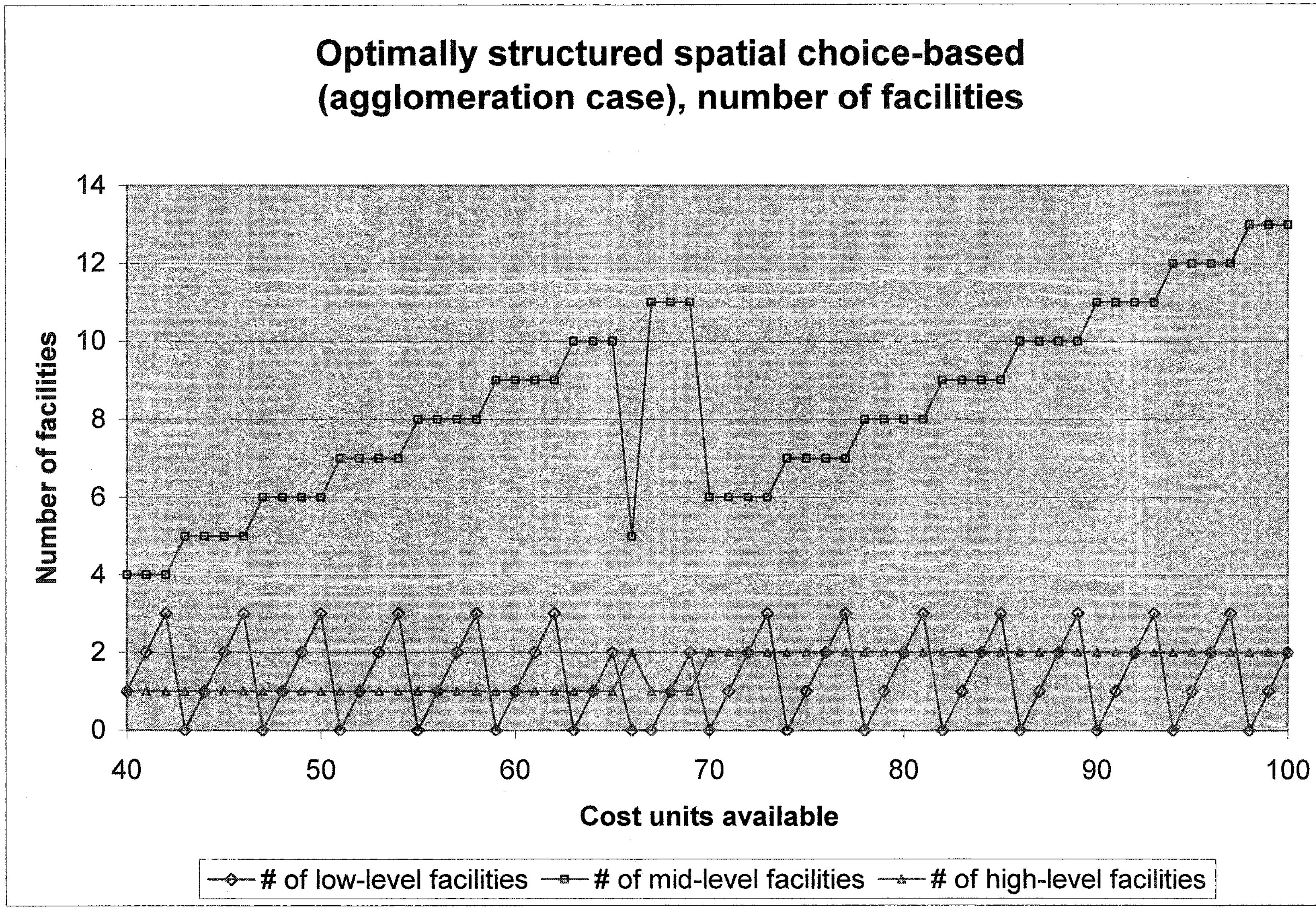

Optimally structured spatial choice-based
(agglomeration case), number of facilities
Number of facilities
0
2
4
6
8
10
12
14
Cost units available
40
50
60
70
80
90
100
# of low-level facilities
# of mid-level facilities
# of high-level facilities

Fig. 5.13

P-median solution (2,0,24). Changed cost value for the middle-level facilities. Low-level demand allocation

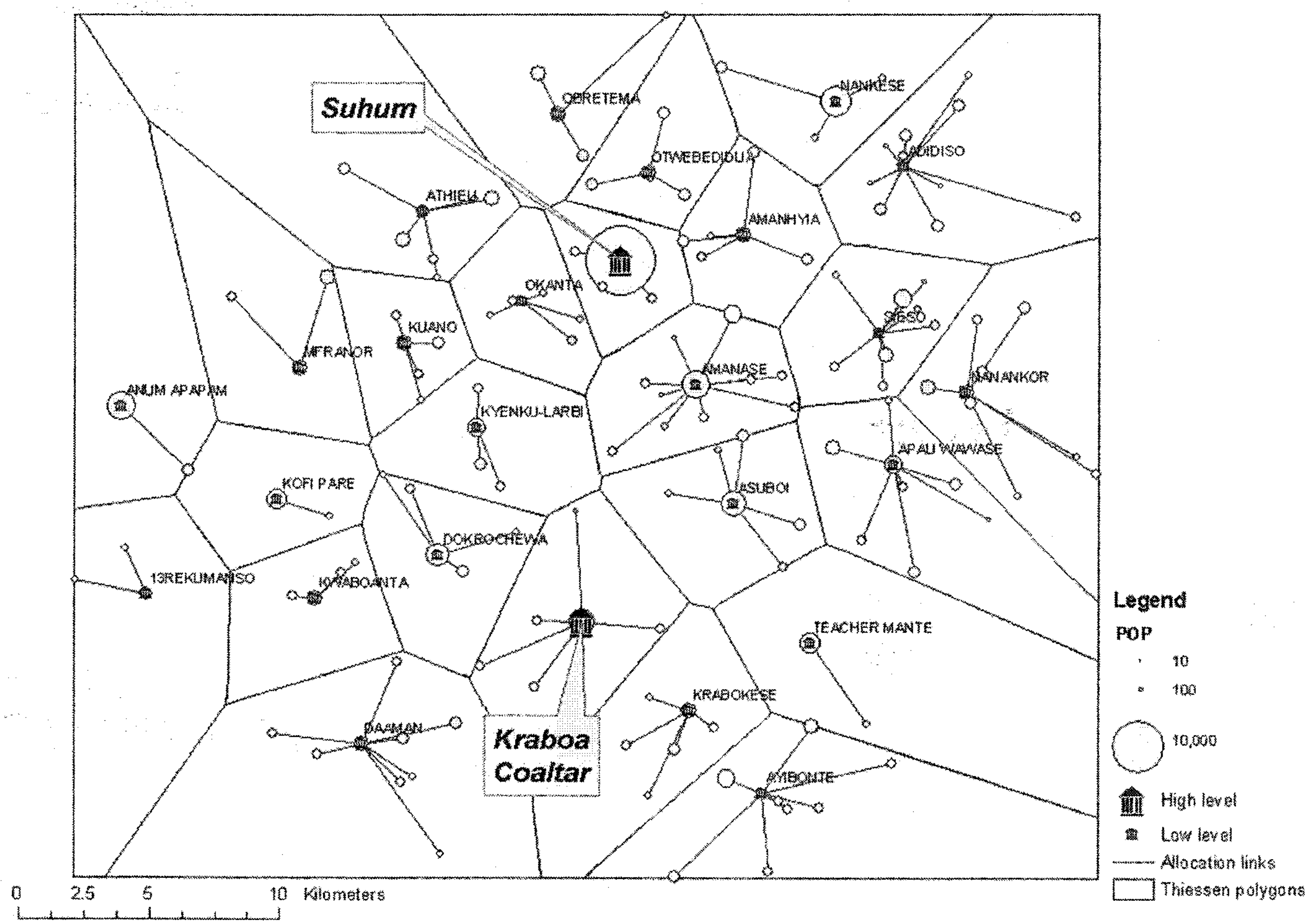


Fig. 5.14

Chapter 6. Summary and recommendations for further research

6.1. Research summary

The purpose of LA analysis and modeling is to locate facility systems that provide the highest possible convenience for patrons. LA analysis is more complex to perform in hierarchically structured services than in single-level ones. **First,** the need to determine both the location and the type (level) of facility results in additional computational costs. That is why heuristic methods were used for most of the previous hierarchical LA models. They were reasonable in terms of computing time, but did not guarantee the optimality of the solution. Fortunately, improvement of hardware and software in recent years allows us to get the optimal solutions for hierarchical LA problems of real world size. In the work reported here, the CPLEX 6.5.1 solver software was used to solve 150-node, 3-level *p*-median problems.

Second, it was observed that the distance minimization objective does not provide the highest convenience for patrons for multi-level systems. A spatial interaction approach was introduced to the LA framework to overcome this problem. Its underlying principle is that patrons face trade-offs between minimizing travel costs and increasing benefit from being served by a more attractive (higher-level) facility. The best combination of these factors provides the highest patrons' benefit, i.e. convenience. I formulated the Batty-based LA model as a mathematical program and solved it optimally with CPLEX 6.5.1.

Third, the development of SI theory has shown that facility attendance depends not only on facility attractiveness but also on its spatial neighborhood. A recently developed LA model based on the SI spatial choice theory considers the additional patron's benefit perceived from being served by a facility located at a more accessible place (the spatial agglomeration effect). It is also shown that the model can be applied in the case in which the isolation of a facility would be an asset (spatial competition),[39] -- the model is flexible with regard to patron perceptions.

Fourth, the highest possible convenience in the spatial hierarchy can be achieved only by simultaneous optimizing its spatial configuration and its hierarchical structure. The latter has not received significant attention in previous LA research. This simultaneous approach makes LA models more complex and demands more computing resources, but at the same time allows one to make recommendations about the structure of the hierarchical system, for example how many levels would be optimal. Simultaneous optimization could easily be applied to other hierarchical location models as well (set-covering, maximal covering).

6.2. Suggestions for the future research

The LA models presented in this thesis can be improved. Here are some suggestions for future research:

❖ Relax the "unlimited capacity" assumption. The size of a facility is included in the interaction-based models, but only as a parameter of patrons' attraction. The concentration of demand served by the high-level facility can lead to congestion, service delays, etc. Obviously, modelling capacity limitations will make the model more realistic.

[39] It does not seem to be appropriate with respect to health care, but could be applied in disposal sites or poison manufactures location, for example.

❖ Non-Cartesian distances. Road network distances might affect the model results. It will be of particular interest to investigate if road network distances or other measures, such as travel time, have a significant effect on the spatial choice-based model.

The presented spatial choice-based LA model looks very promising for real-world application once its key parameters are calibrated. It is necessary to estimate three parameters (size, distance impedance values and accessibility exponent value), which might introduce additional difficulties. However, Fotheringham and Trew (1983), Pellegrini *et al.* (1997), Pellegrini and Fotheringham (1999) demonstrated the methodology of spatial choice model calibration and in particular agglomeration effect estimation based on local-scale spatial interaction data. The appropriate data can be obtained from either real-world observations or health care organizations.

In real world situations, planners and businessmen are faced with many factors in their location decisions. Some of them can be quantitatively expressed, others cannot. Therefore both qualitative and quantitative methods should be used in decision making support. The beauty of Geography is its ability to wed both approaches into one methodological framework. Understanding that the various solutions are suggestions or recommendations rather than final decisions, I tried to demonstrate the potential of quantitative methods to generate facility locations within complex service systems. The models were tested on Oppong (1992)'s health care example, but also could be applied to other hierarchical systems.

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Appendix I

ID	NAME	X	Y	Population
1	SUHUM	32.482	31.194	19298
2	KOFI PARE	19.367	22.015	1403
3	ASUBOI	36.857	21.85	2341
4	NANKESE	40.786	37.415	3414
5	KRABOA COALTAR	31.024	17.235	2492
6	ANUM APAPAM	13.368	25.589	2921
7	KUKUA	46.391	26.924	705
8	KWAHYIA	39.703	31.267	578
9	KYEKYEWERE	38.787	19.408	370
10	TEACHER MANTE	39.827	16.422	1507
11	OTOASE	36.604	11.187	1348
12	KRABOKESE	35.164	13.829	991
13	LA MANGOASE	29.322	17.285	363
14	DUODUKROM	28.516	20.745	220
15	DOKROCHEWA	25.56	19.821	1820
16	AMANHYIA	37.233	32.229	779
17	KOFIGYA	34.897	31.984	485
18	TETTEH NKWANTA	35.955	32.173	245
19	NIIFIO	31.386	34.191	530
20	OMENAKO	31.049	35.281	678
21	NANKESE NINGO	39.989	35.992	265
22	OBRETEMA	30.072	36.92	887
23	DENSUSO	29.329	38.442	1026
24	TRAYO	37.607	35.462	651
25	MAMENG DONYA	44.16	30.419	94
26	AKORABO	43.346	29.752	1199
27	ASIEDU	42.729	35.686	77
28	OKORASE	33.681	29.765	396
29	APEATU	34.578	28.266	119
30	GYATO	40.785	30.64	169
31	AKWADUM	26.529	19.177	502
32	AMANASE	35.425	26.419	2853
33	BUDU (KONKONURU)	35.691	25.187	359
34	MANKRONG	37.226	24.471	699
35	PANPANSO	41.988	13.298	249
36	OKANTA	42.938	11.806	420
37	DZATSUI	40.198	10.076	289
38	KOFISAH	38.995	10.021	354
39	SUMKROM	38.63	10.357	339
40	AYIBONTE	37.964	10.653	326

ID	NAME	X	Y	Population
41	AMFASO	34.051	16.972	353
42	BREKUMANSO	14.328	18.308	685
43	PABI	15.939	23.103	671
44	MFRANOR	20.196	27.082	991
45	KOFI ASARE (ASAREK	24.156	11.02	310
46	EBENEZER	24.256	12.692	559
47	WURUDUWURUDU	23.973	15.668	283
48	MARFOKROM	34.612	12.314	615
49	AWORESO	32.75	12.43	412
50	MAME DEDE	34.584	7.422	649
51	ANOM	34.35	22.237	142
52	DEDEWA	47.725	22.177	206
53	ASAREKROM	50.75	23.028	282
54	BETEMANO	50	23.677	178
55	SIESO	42.435	28.429	176
56	KONKUNURU	34.077	26.03	95
57	TETEKASOM	30.642	28.14	427
58	NSUTA	26.93	33.621	143
59	OKONAM	27.512	29.077	276
60	KWABENA KUMI	28.355	29.652	437
61	SOWATEY	25.562	28.063	572
62	KUANO	24.244	28.057	895
63	OKANTA	28.695	29.651	420
64	KWAO NARTEY	44.548	28.734	308
65	AMEDE	43.934	29.34	159
66	MFRANTA	24.811	26.857	461
67	AMANFORO	30.92	28.957	151
68	BOKOR	29.545	29.954	139
69	ALIKROM	31.795	30.208	349
70	ADUMASA TRAYO	33.647	34.395	47
71	NANKESE AYISA	42.576	38.33	139
72	YAKOKO	39.877	13.204	842
73	MANKRONG KETEWA	36.28	23.942	227
74	NTUNKUM	36.352	38.731	499
75	OTWEBEDIDUA	33.534	34.618	947
76	BEPOASE	26.265	13.282	578
77	APAU WAWASE	42.995	23.357	1307
78	SIMATARE	40.728	27.14	472
79	APONOAPONO	27.583	33.653	952
80	ATEIBU	24.932	33.111	638
81	ABENABO	36.795	29.176	1263
82	GOVENAKROM	43.811	19.198	568
83	KYENKU-LARBI	27.003	24.771	1157

ID	NAME	X	Y	Population
84	NTABEA	34.266	40.719	273
85	KOKOSIASE	43.337	35.305	366
86	ADIDISO	43.372	34.869	332
87	NTOWKROM	29.211	14.729	377
88	ARKUKROM	22.391	19.534	262
89	OGBOLU	21.807	19.151	432
90	KWABOANTA	20.822	18.158	1031
91	DAAMAN	22.613	12.478	814
92	KWABOANTA ADA	19.99	18.232	423
93	SAASE	11.631	18.85	258
94	KWAKU SAE	13.531	20.082	246
95	ATIMATIM	19.221	12.919	434
96	ODUMKYERE	20.937	12.101	294
97	AMPOFO	24.598	11.26	204
98	ODOMPONINASE	25.665	8.262	218
99	OPAREKROM	24.476	22.43	396
100	GYAMPOAKURAA	23.451	22.948	251
101	AKOTUAKROM	33.607	10.528	229
102	DAMAN	38.249	7.609	338
103	AYEH KOKUOSO	32.197	23.853	360
104	NSUANTA	30.804	21.573	71
105	OWAWASE	43.371	22.506	319
106	AWISAM	45.364	25.59	536
107	AWISAM	45.91	25.749	536
108	KWAKYE	42.802	25.807	193
109	SANTRAMOR	42.682	27.608	738
110	NARTEY OSEI	44.325	26.356	751
111	ABOABO SONKOR	27.053	26.302	408
112	ABOABO (AMANASE)	37.5	26.65	459
113	AGBEMEHIA	34.214	24.803	260
114	HWANABENYA	33.473	26.462	304
115	BEKOEKROM	33.659	14.327	227
116	HWERESO	36.135	13.171	444
117	KWAME KYEREKROM	27.907	22.499	384
118	DOME	23.924	29.083	466
119	ACHEANSA	21.288	30.536	789
120	SRA (PRAPRABABIDA)	25.515	30.582	246
121	FAWOTRIKOSIE	27.172	23.348	549
122	OBUOTUMPAN	43.436	36.117	492
123	OBOMOFO DENSUA	42.504	33.232	482
124	NKATEKWAN	45.45	37.27	519
125	BAABIANEHA	45.821	38.393	236
126	ABESIM	30.71	31.575	411

ID	NAME	X	Y	Population
127	ADARKWA	34.153	36.949	674
128	MAMENHWESO	34.963	33.773	632
129	KORANSANG	47.975	29.393	557
130	OBOASE	40.692	24.024	956
131	OBOASE ABOABO	39.243	25.563	339
132	KUAHO	41.776	20.452	289
133	KORADASO	46.194	28.968	380
134	KWESI KOMFO	21.936	34.764	605
135	KWADJO HUM	24.892	25.855	200
136	SANTRAMOR ZORH	42.595	26.395	398
137	MEMESO	39.443	21.049	482
138	ABOBRI	24.2	32.037	779
139	BETENASE	21.365	21.335	266
140	ABUNABUN	35.625	31.366	371
141	PINPONG	17.585	29.804	405
142	YAW DONKORKROM	27.174	15.505	285
143	ABRODIEM	49.942	32.929	299
144	ABOABO ODUMASE	38.748	26.773	459
145	ASAREKROM	44.757	34.12	97
146	SUPRESO	44.668	32.578	521
147	TRAGO	42.109	34.234	100
148	AFRANSU DEDEWA	46.627	21.251	105
149	NANANKOR	45.774	26.12	751
150	PRAPRABABIDA	25.329	31.259	432