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THE UNIVERSITY OF ALBERTA

THE DESIGN OF LIGHT INDUSTRIAL BUILDINGS

by

CHARLES DONALD NIXON

(C)

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH  
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THE UNIVERSITY OF ALBERTA  
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## ABSTRACT

Research information relevant to the design of single storey light industrial buildings of structural steel is summarized. Information that is not directly applicable, or that is overly conservative for this type of structure is modified so that it can be used directly, and results from areas not previously investigated are reported. Topics covered include loads, cost of different framing arrangements, design of flexural and compression members, resistance to lateral loads, cost of steel construction, and expansion joint requirements. Limit States Design is used throughout. Areas requiring additional research are noted.

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## LIST OF SYMBOLS

A	Area
$A_b$	Brace area
$A_f$	Flange area
$A_m$	Area of fusion face
$A_t$	Area of tension chord
$A_w$	Web area; throat area of weld
a	Girt spacing; ground acceleration
B	width of base plate
$B_r$	Factored bearing resistance
b	Exterior column spacing; flange width
C	Length of base plate; connection flexibility factor; compression force
$C_e$	Exposure factor; buckling load; external pressure coefficient
$C_f$	Compressive force
$C_{fb}$	Compressive resistance of a "fully braced" column
$C_g$	Girder flexibility coefficient; gust factor
$C_j$	Joist flexibility coefficient
$C_{MAX}$	Maximum carrying capacity of a column
$C_{n\infty}$	Force coefficient for an infinitely long member
$C_p$	External pressure coefficient
$C_{pi}$	Internal pressure coefficient
$C_r$	Factored compressive resistance
$C_s$	Snow load coefficient
$C_t$	Tangent buckling load

$C_w$	Warping torsion constant
$C_y$	Axial compressive load at yield stress
$C_\phi$	Torsional buckling load
$c$	Distance between centroid of cantilever girder and link beam reaction
$D$	Specified dead load
$D_N$	Building dimension normal to earthquake motion
$d$	Depth; diameter
$E$	Young's modulus of steel
$E_C$	Young's modulus of concrete
$e$	Edge distance
$e_x$	Eccentricity
$F$	Force; foundation factor; flexibility factor
$F'_C$	Unconfined compressive strength of concrete
$F_U$	Ultimate capacity of connection; ultimate tensile strength
$F_y$	Specified minimum yield stress
$G$	Shearing modulus
$G'$	Diaphragm rigidity per unit width
$G_L$	Restraint parameter at lower end of column
$G_U$	Restraint parameter at upper end of column
$g$	Ground snow load; acceleration due to gravity; footing width
$H$	Horizontal force
$h$	Depth of 24-hour rainfall; clear depth of web between flanges; height
$I$	Importance factor
$I_C$	Column moment of inertia

$I_g$	Girder moment of inertia
$I_{gr}$	Reduced girder moment of inertia
$I_j$	Joist moment of inertia
$I_y$	Weak axis moment of inertia of a wide flange section
$J$	St. Venant torsion constant
$K$	Coefficient reflecting type of construction
$k$	Reduction factor for a member of finite length; stiffness; distance from outer face of flange to web toe of fillet; effective length factor
$k_{id}$	Ideal stiffness
$k_x$	Shielding factor
$L$	Specified live load; length
$L_g$	Girder length
$L_j$	Joist length
$L_v$	Deck span
$M$	Bending moment
$M_f$	Factored bending moment
$M_{f1}$	Smaller factored moment
$M_{f2}$	Larger factored moment
$M_o$	Bending moment due to dead load; critical buckling moment of a simply supported beam subjected to uniform bending
$M_p$	Plastic moment
$M_r$	Factored moment resistance
$M_u$	Elastic buckling moment
$N$	Length of bearing
$n$	Number of bolts
$n_s$	Number of seam fasteners per side lap

$n_{sh}$	Number of sheets of cladding per wall panel
$P$	Load
$P_{ult}$	Ultimate load
$p$	Specified external wind pressure
$p_i$	Specified internal wind pressure
$Q$	Specified lateral load; diaphragm rigidity
$q$	Reference velocity pressure; shear flow; modulus of subgrade reaction
$R$	Resistance of a member; reaction
$r$	Radius of gyration
$r_t$	Radius of gyration about its axis of symmetry of a tee section comprising the compression flange and 1/6 of the web
$S$	Seismic response factor
$S_x$	Section modulus
$s$	Joist spacing; roof snow load
$s_c$	Flexibility of edge fasteners
$s_s$	Flexibility of seam fasteners
$T$	Effect of specified temperature change; period of vibration
$t$	Thickness
$V$	Base shear; total potential
$V_r$	Factored shear resistance
$W$	Seismic weight
$w$	Uniformly distributed load; width of web
$w_j$	Self weight of a joist
$X_u$	Ultimate tensile strength of electrode

$Z_x$	Plastic section modulus
$\alpha$	Load factor; ponding coefficient; overhang length divided by column spacing
$\alpha_T$	Coefficient of thermal expansion
$\beta$	Safety index
$\gamma$	Importance factor; density of water; shear strain
$\Delta$	Deflection; elongation
$\delta$	Initial crookedness; deflection
$\epsilon$	Strain
$\epsilon_u$	Ultimate strain
$\theta$	End slope; rotation
$\lambda$	Non-dimensional slenderness ratio
$\mu$	Coefficient of friction
$\sigma$	Standard deviation
$\sigma_{rc}$	Maximum residual compressive stress
$\sigma_{rt}$	Maximum residual tension stress
$\phi$	Performance factor
$\psi$	Load combination factor
$\omega$	Equivalent uniform moment coefficient

## CHAPTER I

### INTRODUCTION

In the past decade structural research has concentrated on problems related to the design of tall buildings, and has resulted in more rational and economical design procedures for such structures. In contrast, however, research related to the design of low buildings (industrial buildings, warehouses, shopping centres, etc.) has lagged significantly, even though such buildings form a large portion of the market for the steel industry. Although it is not possible to determine the tonnage of steel used each year in Canada for construction of low buildings, it is likely that it is as high, or higher, than that used for tall buildings.

A surprisingly large number of situations exist in the design of this class of building for which either little design information exists, or for which the design information is overly conservative. For example, the effectiveness of girts and corrugated metal wall cladding in bracing exterior columns has been an open question for some time. Another problem is that current design procedures (1,2) may not be completely satisfactory when applied to the design of the overhanging portions of girders in cantilever roof framing schemes.



The purpose of this dissertation is to summarize information relevant to the design of low buildings, to modify information that is not directly applicable or that is overly conservative when applied to low buildings, and to develop new information. Since the subject matter is very broad, the research is limited primarily to the design of single storey light industrial buildings, such as that shown in Figure 1.1. The roof girders may be either simply connected to the supporting columns, or they may be cantilevered over the column tops and separated by "link" beams. Open web steel joists span in a direction perpendicular to the main framing. Lateral load resistance may be provided through a direct acting bracing system, diaphragm action of the wall cladding, masonry block, or some combination of these. Not included are problems related to the design of rigid frames, or to structures in which the effect of a travelling overhead crane is a dominant design feature. Design of these structures is considered elsewhere (3).

The design process for a building structure can be outlined as follows:

1. the over-all size and shape of the building is established, and bay sizes are chosen, based on the intended use of the building,
2. the loads (dead load and loads caused by snow, rain, wind and earthquake) are estimated,

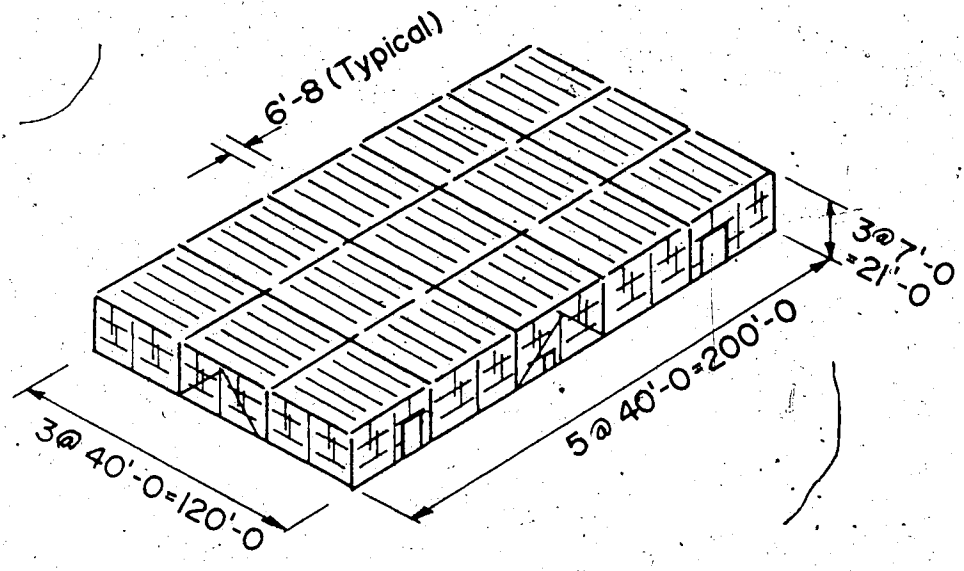


FIGURE I.1 · A LIGHT INDUSTRIAL BUILDING STRUCTURE

3. choices of the most suitable construction materials are made, considering cost, availability and construction time,
4. an economical framing scheme is selected,
5. a preliminary member selection is made,
6. the structure is analyzed to determine if the members chosen are reasonable,
7. re-design is performed where required,
8. the drawings, specifications, and other contract documents are prepared, and contracts let,
9. the fabricator's shop and erection drawings are approved, construction is initiated, and
10. during construction, periodic inspection of the job-site is undertaken to ensure that construction is in accordance with the drawings and specifications.

Although most of these steps will involve the designer to some extent, only steps 2, 4, 5 and 6 will be discussed further here.

CHAPTER II of this dissertation contains a summary of information on the loading conditions appropriate to the design of low buildings. Typical dead loads are summarized, and background information is given for the treatment of snow, wind and earthquake loads. A detailed treatment of rain ponding is presented. In some locations in Canada, ponding of rainwater can cause higher moments and deflections than those caused by snow.

CHAPTER III contains a comparison of the relative

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costs, of various bay sizes, roof framing schemes, and methods of resisting lateral load. Use is made of cost information provided by several steel fabricators, so that cost comparisons are not based on weight savings alone, but rather on the cost of unfabricated steel plus the costs of fabrication and erection.

The design of flexural members is treated in CHAPTER IV. Extensive use is made of a finite element computer program<sup>(4)</sup> that accounts for the effects of residual stresses and progressive yielding in the members to determine design procedures for cantilever girders. Since deflection can sometimes govern the design of a flexural member, a section in this Chapter is devoted to the derivation of a live load deflection limitation for a girder supporting an asphaltic roof membrane.

The design of compression members is discussed in CHAPTER V. Bracing requirements for exterior compression members braced by diaphragm-girt bracing are discussed.

Methods of resisting lateral load are discussed in CHAPTER VI. The mechanism for the transfer of load from the walls to the roof, diaphragm action in the plane of the roof, and the transfer of load from the roof to the foundation are described. Results of a series of tests to determine the strengths of column bases subjected to combined shears and axial forces are presented.

CHAPTER VII contains a detailed description of a method of estimating the cost of steel construction. The method forms the basis for comparing framing schemes and layouts described in CHAPTER III. A summary of the investigation is presented in CHAPTER VIII.

In each chapter worked problems illustrate the design process, using the small rectangular structure shown in Figure 1.1 as an "example building". Thus, in CHAPTER II the loads on the building are computed, in CHAPTER IV various flexural members are designed, and so on. Although an irregular structure such as a "T" shaped building or a building with skewed girder to column connections could have been selected, the structure shown in Figure 1.1 has the advantage of simplicity, and yet at the same time can be used to illustrate many of the design situations encountered in practice. The exterior dimensions of the building, and the bay sizes, are assumed to be based on functional requirements (step 1, above). The structure is located in Ottawa, Ontario.

## CHAPTER II

### LOADS

#### 2.1 Introduction

Before the member selection process can begin the loads on the building must be estimated. The loads considered in this chapter are dead loads, and those live loads due to snow, rain, wind and earthquake. For a single storey building it is the effects of the vertical loads on the roof that most strongly effect the building cost - the effects of lateral loads are usually accommodated rather easily. This is in contrast to a tall building or tower, where the effects of the lateral loads can become very severe. Before discussing loads, however, the Limit State Design method will be briefly reviewed<sup>(1)</sup>.

#### 2.2 Limit States Design

To fulfil its intended purpose a building must provide an acceptable margin of safety against collapse at the factored load levels and be serviceable at the specified load levels. When a structure or a member in the structure fails to fulfil its intended purpose a limit state is said to have been reached. Limit states involving collapse, such as gross yielding or instability, are called ultimate limit states.

In Figure 2.1 frequency curves are plotted for the effect of the loads on a structural member (for example, the maximum bending moment) and the resistance of the member to this load effect (for example, the plastic moment). In the shaded area the effect of the loads is greater than the resistance of the member and failure occurs.

In Limit States Design the probability of reaching an ultimate limit state is kept sufficiently low by requiring that the factored member resistance (ie., the resistance  $R$  multiplied by a performance factor  $\phi$ ) be greater than, or at least equal to, the effect of the factored loads. The performance factor is taken as 0.9 for steel design in general, but 0.67 for bolts and the plates in bearing type connections.

The effect of the factored loads is the bending moment, column load or other structural effect due to the specified loads multiplied by the load factors  $\alpha$ , a load combination factor  $\psi$ , and an importance factor  $\gamma$ :

$$\alpha_D D + \gamma \psi (\alpha_L L + \alpha_Q Q + \alpha_T T)$$

where

- D = specified dead load,
- L = specified live load,
- Q = specified lateral load (from an earthquake, for example),
- T = influences resulting from temperature changes or from differential settlement,

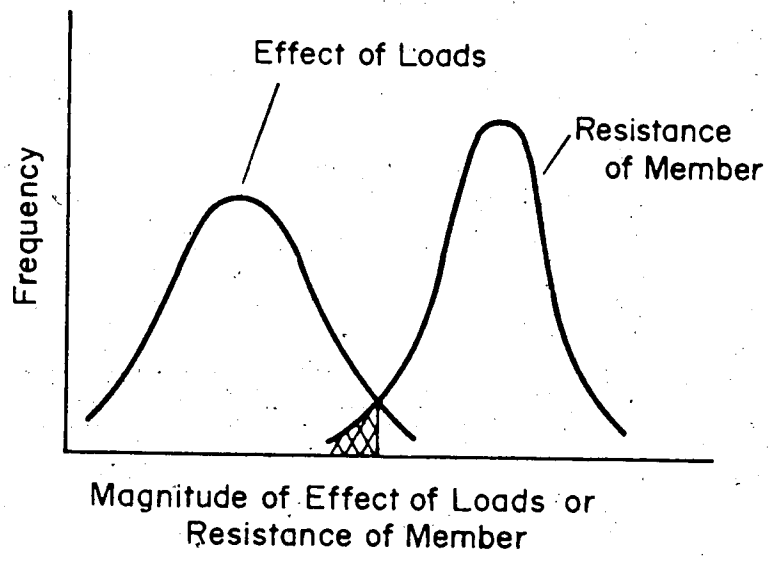


FIGURE 2.1 FREQUENCY DISTRIBUTION CURVES



$\alpha_D$ ,  $\alpha_L$ ,  $\alpha_Q$  and  $\alpha_T$  = load factors equal to 1.25  
(or, in cases of overturning,  
uplift or stress reversal, 0.85),  
1.5, 1.5 and 1.25 respectively,

$\gamma$  = an importance factor equal to 1.0 for most build-  
ings (but 0.80 for buildings where the rate of  
human occupancy is low), and

$\psi$  = a load combination factor equal to 1.0 when only  
one of L, Q or T acts, 0.7 when any two of L, Q or  
T acts, and 0.6 when all three act together.

Limit states that effect the serviceability of the  
structure, such as excessive deflection or vibration, are called  
serviceability limit states. Serviceability limit states are  
checked at the specified load levels, using the appropriate load  
combination factors.

Because of the greater rationality of Limit States  
Design, and because Canadian Structural Standards all appear to be  
moving towards the limit states format, only CSA Standard  
S16.1-1974 "Steel Structures for Buildings - Limit States  
Design"<sup>(1)</sup> will be used for member selection in this report.

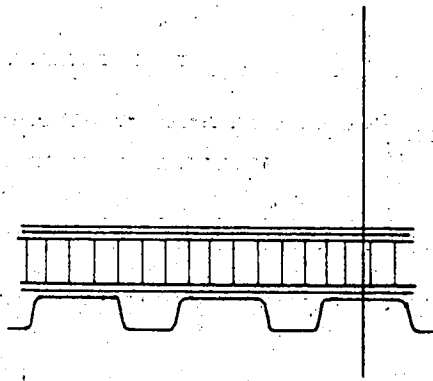
## 2.3 Loads

### 2.3.1 Dead Load

The dead load supported by a roof girder includes the weights of materials comprising the roof, the weights of mechanical units on the roof, ductwork carried through the joists, lighting fixtures, sprinkler laterals, fans, etc., the weight of the joists, and the weight of the steel girder itself. Typical weights of materials commonly used in roof construction and weights of mechanical units are given in the Handbook of Steel Construction<sup>(5)</sup> as well as in various manufacturers catalogues.

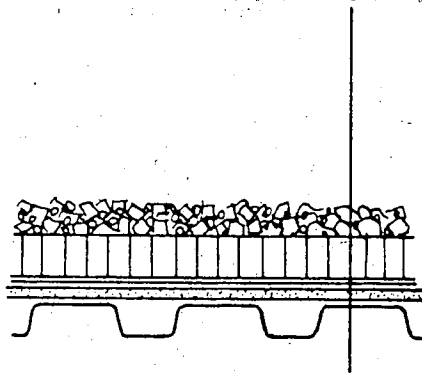
Two different roof systems are in common use at the present time: conventional roof systems and inverted, sometimes called "upside down", roof systems<sup>(6)</sup>. Typical weights of materials for both systems are given in Figure 2.2.

The weights of light mechanical units, ductwork, lighting fixtures, etc. are often assessed as a uniformly distributed dead load of approximately 5 psf. Since mechanical and electrical services may be suspended rather indiscriminately from the lower chords and webs of the joists, it may be more appropriate to treat these loads as live loads instead of dead loads and to consider the effects of different load patterns. In some cases particularly heavy mechanical units may be supported by the top chords of some of the joists. Their effects should be assessed separately.



Gravel.....	0-10 Psf.
3-Ply Roofing Membrane.....	3
2 In. Rigid Insulation.....	3
Vapor Barrier.....	-
1.5 In. Steel Deck.....	2
(0.030 In. Thick)	

(a) Conventional Roof System



Gravel.....	10-20 Psf.
2 In. (Waterproof) Rigid Insulation.....	3
3- Ply Roofing Membrane.....	3
0.5 In. Gypsum Board.....	2.5
1.5 In. Steel Deck.....	2
(0.030 In. Thick)	

(b) Inverted Roof System

FIGURE 2.2 TWO COMMON ROOF SYSTEMS

Although the dead weights of joists and girders can usually be estimated closely by experienced designers, rules of thumb are often useful for those without this experience. One such rule can be derived for joists subjected to a uniformly distributed load by first estimating the tension chord area  $A_t$  from:

$$\phi A_t F_y = \frac{swL_j^2}{8d} \times \frac{12}{1000} \quad (2.1)$$

where

- $F_y$  = specified minimum yield stress, ksi,
- $w$  = factored load carried by the joists, psf,
- $s$  = joist spacing, ft.,
- $L_j$  = joist span, ft.,
- $d$  = joist depth, in.

Assuming that the total cross sectional area is three times that of the tension chord area, a steel density of 490 lbs/ft.<sup>3</sup>, a span to depth ratio of 24 and a yield stress of 55 ksi, the joist weight  $w_j$  in lbs/ft.<sup>2</sup> of roof area is:

$$w_j = 0.0007 wL_j \quad (2.2)$$

The assumption that the total area is three times the tension chord area is clearly an approximation, but it is felt that it is accurate enough for use in the derivation of this rule of thumb. Note that the number 0.007 in this equation has the units of feet.

---

Example 2.1

Given

Estimate the specified dead load supported by an interior girder in the structure of Figure 1.1. Assume a specified snow load of 48 psf and an inverted roof system with 10 psf of gravel.

Solution

From Figure 2.2, the weights of the materials in the roof are estimated as:

gravel .....	10 psf
2 in. insulation .....	3
3-ply roof membrane .....	3
0.5 in. gypsum board .....	2.5
1.5 in. steel deck .....	<u>2</u>
total .....	20.5 psf

As discussed above, it may be more appropriate to consider the various mechanical and electrical services as live loads. However, for simplicity a uniform dead load of 5 psf will be assumed.

The self-weight of the 40 foot long joists can be estimated as:

$$\begin{aligned}
 w_j &= 0.0007 wL_j && \text{(Eq. 2.2)} \\
 &= 0.0007 \times (1.25 \times 25.5 + 1.5 \times 48) \times 40 \\
 &= 2.9 \text{ psf}
 \end{aligned}$$

This will be rounded up to 3.0 psf. The self weight of the girder will also be estimated as 3.0 psf, subject to later verification.

Thus the specified dead load supported by the girder is:

$$\begin{aligned} D &= 20.5 + 5 + 3.0 + 3.0 \\ &= 31.5 \text{ psf.} \end{aligned}$$

### 2.3.2 Snow Load

The live load that governs the design of roof systems for most locations in Canada is snow load. As shown in References (7), (8) and (9), snow load on the flat roof of a building is determined by multiplying the ground snow load by a snow load coefficient that accounts for building exposure:

$$s = C_s g \quad (2.3)$$

where  $s$  = specified roof load in psf

$C_s$  = snow load coefficient

$g$  = ground snow load in psf

Since the minimum recommended live load is 20 psf, (7) the specified roof snow load should not be less than this value.

Ground snow loads are based on measurements of snow depths taken at over 200 stations across Canada for periods ranging from 10 to 18 years (8). From these records the depths that would be equalled or exceeded on the average of once in 30 years were found and converted to loads by assuming that 1 inch of snow

weighed 1 psf. To this load was added the weight of a 1-day rain-fall at the time of year when the snow depths were the greatest. Ground snow loads are tabulated for many locations in Canada in Reference (8).

The basic snow load coefficient, used to convert the ground snow load to a roof snow load, is 0.8 except that for roofs exposed to wind this may be reduced to 0.6. These coefficients are based on results of a continuing snow load survey carried out by the Division of Building Research of the National Research Council of Canada (9). As illustrated in Figure 2.3(a), an "exposed" roof must be in generally open, level terrain containing only scattered buildings and trees and not likely to become shielded in the future by taller buildings closer than 15 feet or  $10h$ , where  $h$  is the difference in height between the two buildings in feet. In addition, parapets or other projections on the roof must not be higher than  $g/25$  feet, where  $g$  is defined in Equation (2.3). When taller buildings are closer than  $10h$ , but more than 15 feet away, as shown in Figure 2.3(b), 0.8 should be used over that portion of the roof within  $10h$  feet from the taller building.

---

#### Example 2.2

#### Given

Estimate the snow load on the roof of the structure shown in Figure 1.1. The structure is located in Ottawa, Ontario ( $g = 60$  psf) and is not situated in an exposed location.

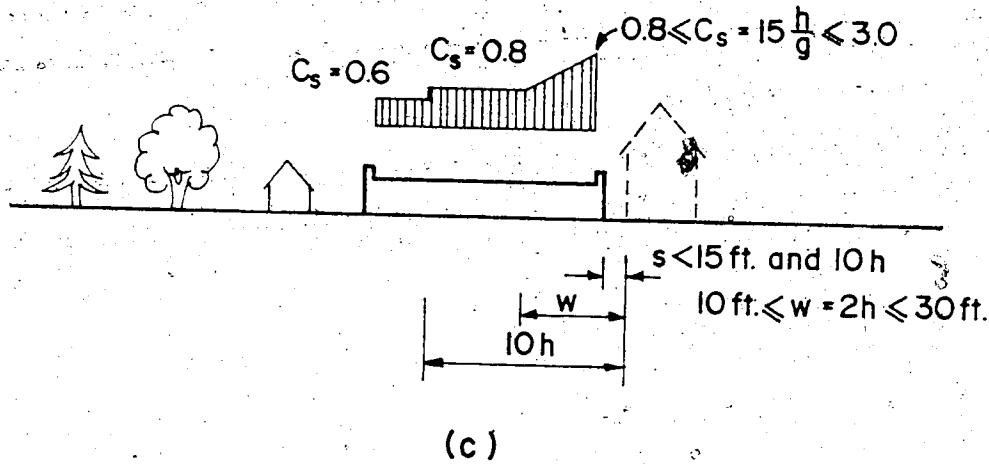
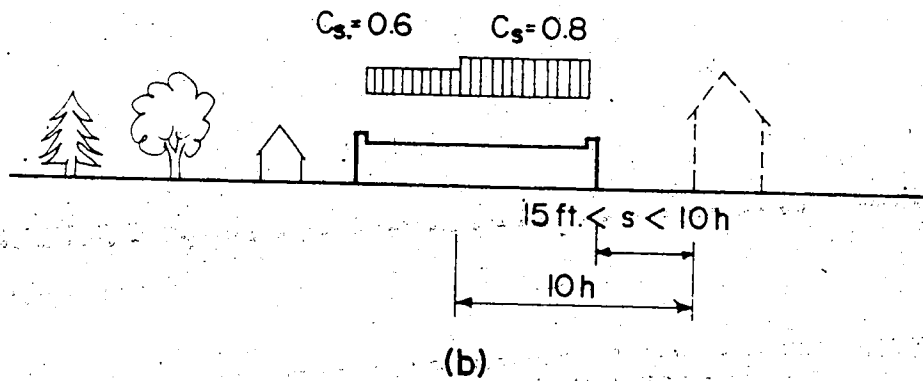
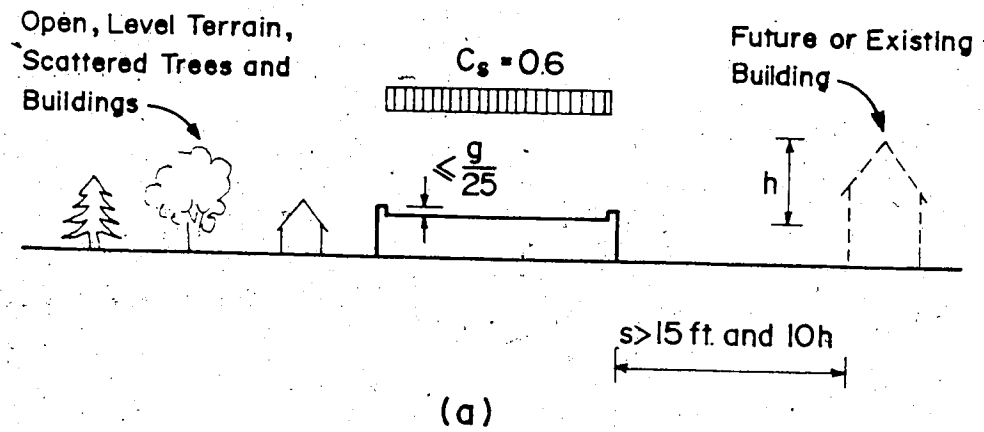


FIGURE 2.3 EFFECT OF SHELTER ON SNOW LOAD COEFFICIENTS



Solution

Since the roof of the structure is not exposed to the wind, a snow load coefficient of .8 should be used:

$$\begin{aligned}
 s &= C_s g && \text{(Eq. 2.3)} \\
 &= 0.8 \times 60 \\
 &= 48 \text{ psf.}
 \end{aligned}$$

---

Drifting of snow and subsequent pile-up on the roof can occur in the vicinity of an upper roof or other projection. It may also occur if a taller building is less than 15 feet and 10 h. away, as illustrated in Figure 2.3(c).

Wind flows in streamlines which can be taken as relatively laminar over smooth terrain. These streamlines are deflected by the building, crowding over the top and around the sides. Where the streamline flow is broken turbulence occurs, resulting in an area of "aerodynamic shade", in which a strong wind velocity does not exist in any direction. Falling snow transported horizontally by the wind, and snow carried along the roof surface, is deposited in these areas in drifts.

For large upper roofs it can be assumed that (for the worst case) the snow drift extends to the top of the upper roof, creating a drift of height h. With an assumed density of 15 pcf the weight of snow at the base of the drift is 15 h psf, so that the snow load coefficient becomes 15 h/g. The difference between

the density of 15 pcf used here and the density of 1 psf per inch depth used earlier approximately accounts for the effect of the 1-day rainfall. For projections and small roof obstructions of height  $h$  it can be assumed that the drift only extends two thirds of the height because the large reservoir of snow on the upper roof of the preceding case is not present. Thus, the snow load coefficient is  $10 h/g$ . It has been observed that these values of  $C_s$  rarely exceed 3.0 and 2.0, respectively, and these two values are therefore taken as upper limits<sup>(9)</sup>. Approximate lengths of these drifts are  $2 h$ , but not less than 10 feet or more than 30 feet<sup>(9)</sup>.

For relatively short upper roofs of length  $L$  less than 50 feet the snow pile-up may be less than that given by  $15 h/g$ , and a smaller value may be judged appropriate by the designer<sup>(9)</sup>. One method of determining this is to use a linear reduction between  $15 h/g$  for a roof 50 feet long and  $10 h/g$  for a projection

$$C_s = (10 + \frac{L}{10}) \frac{h}{g} \quad (2.4)$$

but not greater than an upper limit of  $2 + L/50$ . A comparison of Equation (2.4) with snow load measurements<sup>(10,11)</sup> is shown in Figure 2.4.

Because snow can drift, and thereby accumulate unevenly, consideration should also be given to the effects of partial loading by designing for the more severe of<sup>(9)</sup>:

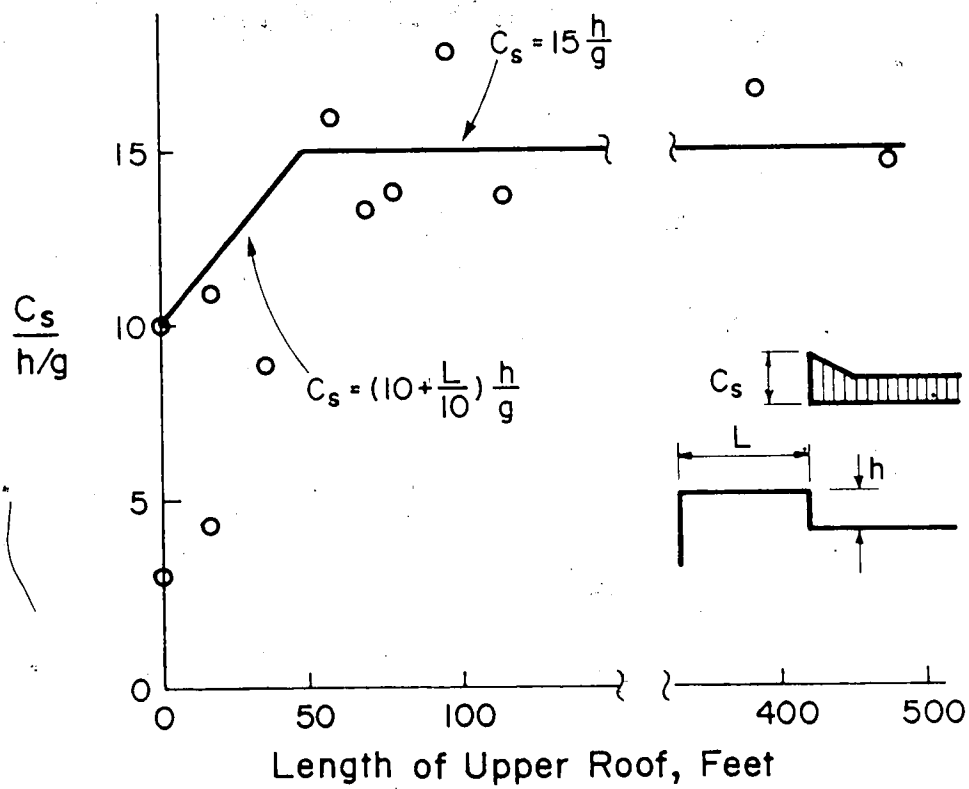


FIGURE 2.4 SNOW PILE-UP AGAINST A RELATIVELY SHORT UPPER ROOF

1. dead load plus snow load over the total roof area, and
2. dead load plus snow load over the total roof area except that one half of the snow load is removed from one particular portion of the roof area.

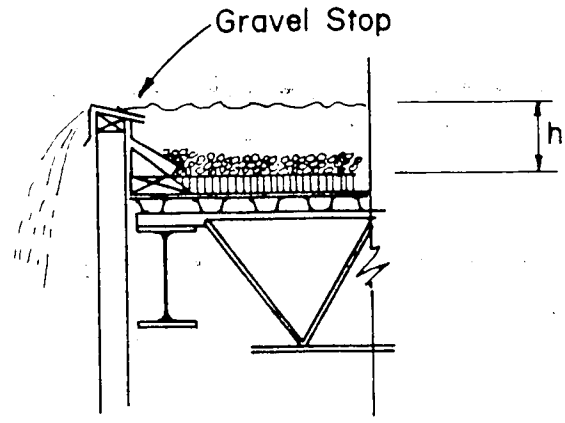
Loading case 1 usually governs the design of columns and simply supported girders and joists. As discussed in Section 4.4, however, loading case 2 governs the design of cantilever girders.

### 2.3.3 Rain Load

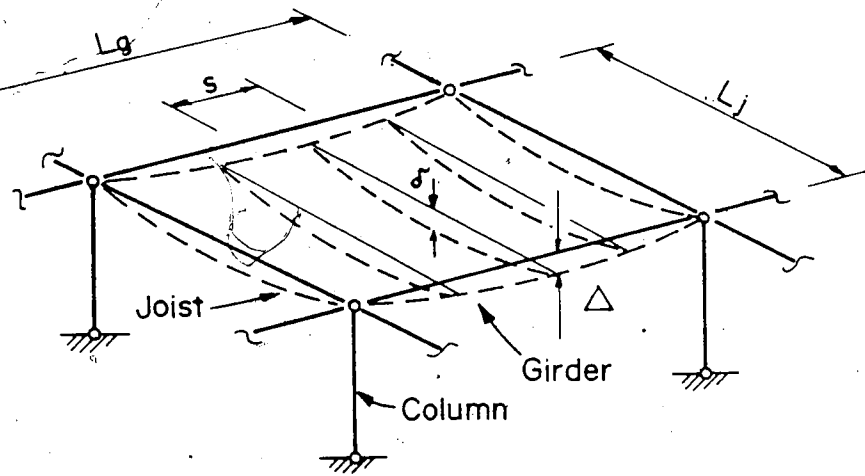
The roof of a structure should be capable of resisting the more severe of dead load plus snow load or dead load plus the load resulting from the 24-hour rainfall at that location(7).

Rainfall depths for many locations in Canada are tabulated in Reference (8). Since the heaviest rainfalls usually occur in the spring or early summer, the effects of snow and the 24-hour rainfall need not be considered to act together.

The amount of water that is retained by a roof depends on whether or not a gravel stop (Figure 2.5(a)) extends around the perimeter of the roof, as well as on the number and location of roof drains and on the amount of maintenance given to these



(a)



(b)

FIGURE 2.5 PONDING OF RAINWATER ON A SIMPLY SUPPORTED ROOF SYSTEM

drains. In keeping with the provisions for rain load given Reference (7), however, it will be assumed in this report that the roof drains are blocked and are not functioning. Thus if the height of the gravel stop is  $h$ , it is possible for the roof to be flooded to a depth  $h$  above the undeflected roof position as illustrated Figure 2.5(a). If gravel stops are not used water can spill out over the roof edges, and in this case water will be retained only in the deflected shapes of the roof members.

Ponding of rainwater on the flat roof of an industrial building is not considered to be a problem for most locations in Canada. However, in locations where the snow load is light, roof loads can be close to those used for design in the southern United States, and ponding of rainwater can cause higher moments and deflections than those caused by snow.

Consider as an example a location where the specified roof snow load is 20 psf and the 24-hour rainfall is 4 inches. If a gravel stop at least 4 inches high extends around the perimeter of the building, water is prevented from spilling out. Therefore the roof must be capable of carrying this depth of rain. If the roof system is infinitely stiff, ponding will not occur and the rain will act as a uniformly distributed load. Since the weight

of water is approximately 5 psf for each inch of depth, the dead load plus 4 inches of rain will cause exactly the same moments and deflections as dead load plus 20 psf of snow. For a flexible roof system, however, water will flow (pond) towards the mid-spans of the roof members so that the moments and deflections caused by rain will be greater than those caused by snow. In the case of a very flexible roof system, ponding of rainwater may lead to collapse.

Most of the research on ponding reported in the literature (12,13,14,15,16) is based on an analysis of a simply supported girder with infinitely stiff joists - ie., the joist deflection  $\delta$  in Figure 2.5(b) is equal to zero. If a half sine wave deflected shape is assumed for a particular girder and the external work done by the loads is equated to the internal strain energy stored in the girder, it is shown in these references that:

$$M = M_0 \left( \frac{1}{1 - C_g} \right) \tag{2.5}$$

$$\Delta = \Delta_0 \left( \frac{1}{1 - C_g} \right) \tag{2.6}$$

- where
- M = maximum moment due to dead load and water in deflected shape,
  - M<sub>0</sub> = maximum moment due to dead load plus the water surcharge  $\gamma h$ ,
  - $\Delta$  = maximum deflection due to dead load and water in deflected shape,
  - $\Delta_0$  = maximum deflection due to dead load plus the water surcharge  $\gamma h$ ,
  - $\gamma$  = density of water.

The parameter  $C_g$  is a measure of the girder flexibility, given by:

$$C_g = \frac{\gamma L_j L_g^4}{\pi^4 E I_g} \quad (2.7)$$

where  $L_j$  = girder spacing = joist length,

$L_g$  = girder length,

$I_g$  = girder moment of inertia,

$E$  = Young's modulus.

For design,  $h$  can be conservatively taken as the depth of the 24-hour rainfall or the height of the gravel stop - whichever is less. This essentially assumes that there is an infinite reservoir of water available for ponding on the girder being designed. This condition is approximated if the girder in question is erected at a slightly lower elevation than the remaining girders in the roof system, thereby allowing water to flow from remote areas of the roof.

If  $C_g$  is greater than or equal to unity, all of the water will collect in the deflected shape and deflections will increase either as long as the rain continues or until the member fails. This situation is analagous to an initially crooked, pin-ended column loaded with an axial force equal to the Euler buckling load. If  $C_g$  is less than unity, an equilibrium position will eventually be reached.



Roof joists, however, are not infinitely stiff, and in an actual roof system both the girders and joists will deflect and retain water. The effect of the interaction between girders and joists can be considerable, and should not be ignored. (17, 18,19) Solutions to the equilibrium equations for this case are presented in chart form in Reference (17).

For the type of building under consideration here the spans involved are usually not excessively long, and the joists and girders are normally not cambered. If the roof members are cambered, however, the charts in Reference (18) may be used to determine the effects of ponding.

A method is presented in Appendix A that gives the same results as Reference (17), except that charts are not required. To determine girder moments and deflections the quantity

$$\alpha = 1 + \frac{8}{\pi^2} \left( \frac{C_j}{1 - C_j} \right) \quad (2.8)$$

in which 
$$C_j = \frac{\gamma s L_j^4}{\pi^4 E I_j} \quad (2.9)$$

where  $s$  = joist spacing, and

$I_j$  = joist moment of inertia

is first computed. The moments and deflections can then be determined from:

$$M = \alpha M_o \left( \frac{1}{1 - \alpha C_g} \right) \quad (2.10)$$

$$\Delta = \alpha \Delta_o \left( \frac{1}{1 - \alpha C_g} \right) \quad (2.11)$$

If Equations (2.10) and (2.11) are compared with Equations (2.5) and (2.6), it can be seen that the equations derived for the case of infinitely stiff joists can still be used providing the dead load and the water density are first multiplied by  $\alpha$ .

After the maximum girder deflection  $\Delta$  is determined from Equation (2.11), the load  $w$  per unit length of the most heavily loaded joist can be determined by referring to Figure 2.5(b). Assuming first that the joists do not deflect:

$$w = (D + \gamma\{h + \Delta\})s \quad (2.12)$$

To account for the water in the deflected shape of the joist, use is made of Equation (2.5), substituting the subscript "j" for "g":

$$w = (D + \gamma\{h + \Delta\})s \left( \frac{1}{1 - C_j} \right) \quad (2.13)$$

For convenience in design, when length is in feet and moment of inertia is in inches to the fourth power, the parameters  $C_g$  and  $C_j$  can be determined from:

$$C_g = \frac{L_j L_g^4}{325000 I_g} \quad (2.14)$$

and

$$C_j = \frac{s L_j^4}{325000 I_j} \quad (2.15)$$

The amplification factors shown in brackets in Equations (2.10), (2.11) and (2.13) are appropriate only as long as the members remain elastic. Since yielding can be expected to occur at the factored load level, moments and deflections will be greater than those that would be determined by using these equation.

Ideally it would be desirable to derive amplification factors that are a function of bending moment and that include the effects of possible yielding in the girders and joists. Design for strength would then consist of computing the moments due to the factored loads, and ensuring that the factored member resistances are not exceeded. The derivation of these amplification factors, however, is considered to be beyond the scope of this dissertation.

In the meantime, the following conservative approach is suggested. The design is considered to be satisfactory providing the roof members do not yield under dead load plus rain load at the specified load level, and that the live load deflections do not exceed maximum recommended values. (20)

Adequate strength can be ensured for the girders by requiring that:

$$M < (F_y - \sigma_{rc}) S_x \quad (2.16)$$

where

$\sigma_{rc}$  = maximum residual compressive stress, and

$S_x$  = section modulus.

For joists, the tension chord will begin to yield before the heavier compression chord, and inelastic action can be avoided by ensuring that:

$$w < 8 (F_y - \sigma_{rt}) \frac{A_t d}{L_j^2} \quad (2.17)$$

where  $\sigma_{rt}$  = the maximum residual tension stress.

For rolled wide flange girders  $\sigma_{rc}$  can be taken as 13 ksi (2) and for hot rolled hat section chords  $\sigma_{rt}$  can be taken as 20 ksi (21).

---

### Example 2.3

#### Given

As will be shown later in this section, Ottawa is not a location in Canada in which ponding is a problem. To illustrate the application of the theory presented above, however, consider a flat roofed industrial building located in Vancouver, a location where ponding generally governs the design of roof members. Bay sizes are 30 x 35 feet, with simply supported girders spanning in the 30 foot direction. The joists are evenly spaced at 5 foot centers.

Based on a specified dead load of 20 psf (ie., a conventional roof system), a ground snow load of 34 psf and a snow load coefficient of 0.8, W18x45 girders ( $F_y = 44$  ksi) and 20 inch deep open web joists ( $F_y = 55$  ksi) were selected for the structure. The design is to be checked for dead load plus the load resulting from a 24-hour rainfall of  $h = 4.5$  inches.

#### Solution

The cross section properties for the girder are given in Reference (5) and for the joist in the manufacturers catalogue:

$$I_g = 706 \text{ in}^4$$

$$S_x = 79.1 \text{ in}^3$$

$$I_j = 151 \text{ in}^4$$

$$A_t = 0.757 \text{ in}^2$$

$$d = 19.0 \text{ in}$$

The following quantities are first computed:

$$\begin{aligned}
 C_g &= \frac{L_j L_g^4}{325000 I_g} && \text{(Eq. 2.14)} \\
 &= \frac{35 \times (30)^4}{325000 \times 706} \\
 &= 0.124
 \end{aligned}$$

$$\begin{aligned}
 C_j &= \frac{s L_j^4}{325000 I_j} && \text{(Eq. 2.15)} \\
 &= \frac{5 \times (35)^4}{325000 \times 151} \\
 &= 0.153
 \end{aligned}$$

$$\begin{aligned}
 \alpha &= 1 + \frac{8}{\pi^2} \left( \frac{C_j}{1 - C_j} \right) && \text{(Eq. 2.8)} \\
 &= 1 + \frac{8}{\pi^2} \left( \frac{0.153}{1 - 0.153} \right) \\
 &= 1.15
 \end{aligned}$$

Assuming, for simplicity, that the girder loading can be idealized as a uniformly distributed load, the unamplified center-line moment and deflection under 20 psf dead load plus

$$\gamma_h = 62.4 \frac{\text{lbs}}{\text{ft}^3} \times \frac{4.5 \text{ in}}{12 \text{ in/ft}} = 23.4 \text{ psf}$$

is  $M_o = 171 \text{ ft-k}$  and  $\Delta_o = 1.35 \text{ in}$ .

The amplified moment is:

$$\begin{aligned}
 M &= \alpha M_o \left( \frac{1}{1 - \alpha C_g} \right) && \text{(Eq. 2.10)} \\
 &= 1.15 \times 171 \left( \frac{1}{1 - 1.15 \times 0.124} \right) \\
 &= 229 \text{ ft-k}
 \end{aligned}$$

the amplified girder deflection is

$$\begin{aligned}\Delta &= \Delta_0 \left( \frac{1}{1 - \alpha C_g} \right) && \text{(Eq. 2.11)} \\ &= 1.15 \times 1.35 \left( \frac{1}{1 - 1.15 \times 0.124} \right) \\ &= 1.81 \text{ in.}\end{aligned}$$

and the load per unit length of the most heavily loaded joist is

$$\begin{aligned}w &= (D + \gamma \{h + \Delta\}) \left( \frac{1}{1 - C_j} \right) s && \text{(Eq. 2.13)} \\ &= \left( 20 + \frac{62.4}{12} \times \{4.5 + 1.81\} \right) \times 5 \\ &= 312 \text{ plf.}\end{aligned}$$

Yielding does not occur in the girders providing

$$\begin{aligned}M &< (F_y - \sigma_{rc}) S_x && \text{(Eq. 2.16)} \\ &= (44 - 13) \times \frac{79.1}{12} \\ &= 204 \text{ ft-k}\end{aligned}$$

and in the joists providing

$$\begin{aligned}w &< 8 (F_y - \sigma_{rt}) \frac{A_t d}{L_j^2} && \text{(Eq. 2.17)} \\ &= 8 (55 - 20) \times \frac{0.757}{(35)^2} \times 19 \times \frac{1000}{12} \\ &= 274 \text{ plf}\end{aligned}$$

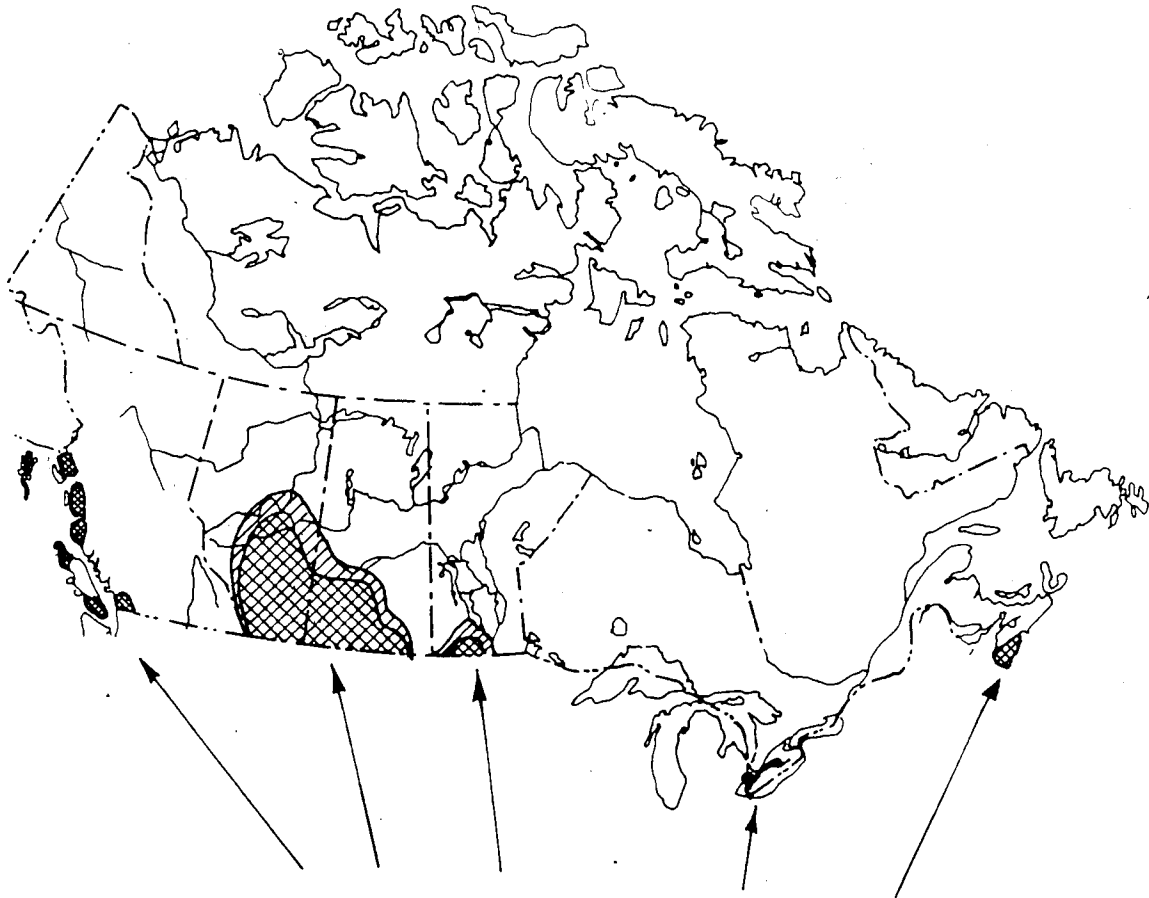
Thus yielding occurs in both the beams and the joists so that the design, although satisfactory for snow load, is not satisfactory for rain load.

---

In order to determine where, in Canada, ponding could be a problem the map shown in Figure 2.6 was prepared. For each location listed in Reference (8) a series of conventional roof systems were designed for a specified dead load of 20 psf plus the appropriate ground snow load at that location using a snow load coefficient of 0.8. Square bays were assumed, with rolled wide flange girders available from Canadian mills ( $F_y = 44$  ksi), and short span steel joists (with 55 ksi hat section chords). Girders were simply supported. Spans were varied from 25 to 40 feet in 5 foot increments, and the span at which rain controlled instead of snow was used in preparing the map.

The white areas indicate those locations where rain did not control. Ottawa is one such location. The cross-hatched areas indicate those locations where rain always controlled. The single-hatched areas indicate those locations where either snow or rain could control, depending on the spans. Thus ponding governed on the west coast of British Columbia, in central and southern Alberta and Saskatchewan, in southern Manitoba, in southern Ontario in the vicinity of the Great Lakes, and in southern Nova Scotia. It should be noted, however, that had this





Locations Where Rain Load May Be More Severe Than Snow Load

FIGURE 2.6 PONDING MAP OF CANADA

study assumed inverted rather than conventional roof systems the joists and girders would have been stiffer because of the increased dead loads. Figure 2.6 therefore overestimates the severity of ponding action for inverted roof systems.

The treatment of ponding of rainwater thus far in this report has considered only simply supported roof systems. Consider next the behaviour of a cantilever roof system, in which girders in alternate bays cantilever over the column tops and are separated by simply supported link beams. In Reference (18) it is shown that when the rain starts to fall it is deposited in a uniform layer which, initially, acts as a uniform load. As the roof members deflect the link beams are lifted up by the hinge points and the water on these spans flows towards the cantilever spans. The "shoreline" will always lie between the supports and the hinge points. Therefore ignoring the small end moments produced by the water lying outside the supports, the cantilever spans act essentially as if they were simply supported, and the equations derived above can also be used here. In computing  $C_g$  the girder length should be taken as the length between columns, and not between hinge points.

#### 2.3.4 Wind Load

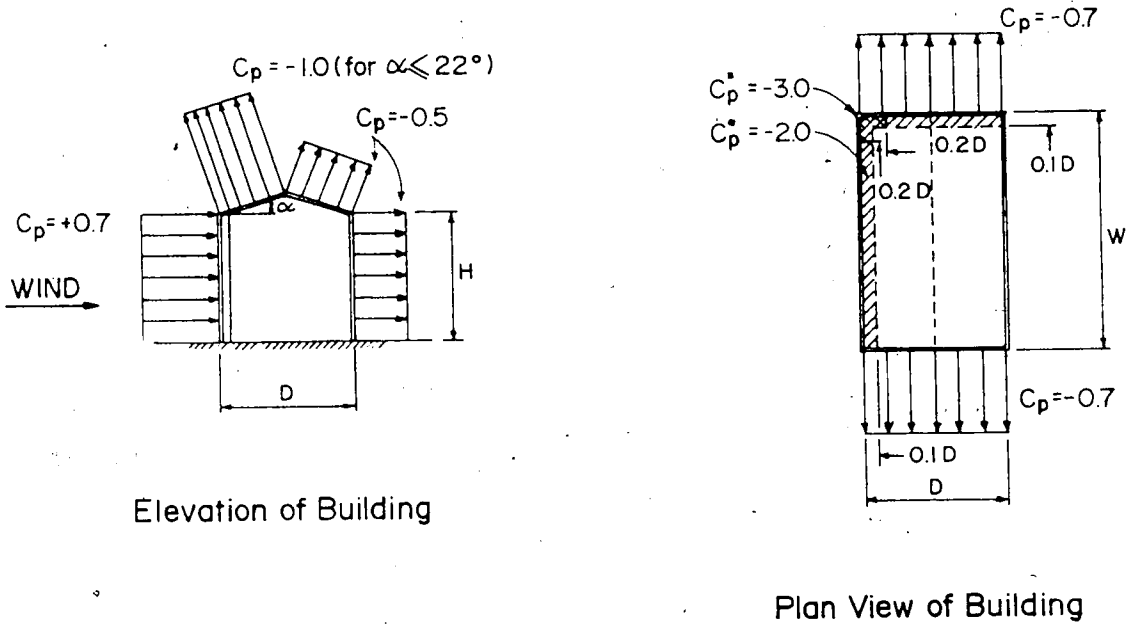
Three different approaches to assessing the effects of

wind on buildings are outlined in References (7) and (9):

1. use of wind tunnel tests,
2. a detailed procedure, and
3. a simplified procedure.

Wind tunnel tests would normally not be considered economically justifiable for an individual single storey building; however, many tests have been carried out on this class of structure to determine pressure coefficients, such as those presented in Reference (9). When the detailed procedure is applied to low-rise buildings wind loads can be half those given by the simple procedure, depending on the shelter conditions. However, using this approach with low-rise structures violates several basic assumptions made in its development(22). In the derivation, the building in question was assumed to be taller than the surrounding buildings and was assumed to be slender enough so as to not change the character of the approaching turbulent flow. Wind flow around a low building can be strongly affected by the surrounding terrain and neighbouring structures. In addition, gusts are further altered by the building itself if it is wide in proportion to its height. Thus the simple procedure is usually used to determine wind loads on these types of buildings.

Wind forces on single storey, flat roof buildings act as pressure on the windward wall, suction on the leeward and side walls, and suction over the roof, as shown in Figure 2.7.



INTERIOR PRESSURES	$C_{pi}$
1. Openings mainly in windward wall.	+0.7
2. Openings mainly in leeward wall.	-0.5
3. Openings mainly in walls parallel to wind direction.	-0.7
4. Openings uniformly distributed in all 4 walls.	-0.3

FIGURE 2.7 PRESSURE COEFFICIENTS FOR BUILDINGS<sup>(9)</sup>

With the simple procedure the external pressures and suction at the specified load level for wind at right angles to the building are given by (7)

$$p = C_e C_g C_p q \quad (2.18)$$

where

- $p$  = the specified external pressure or suction acting normal to the surface, psf,
- $C_e$  = the exposure factor, equal to 1.0 for buildings less than 30 feet high,
- $C_g$  = the gust effect factor equal to 2.0 when designing structural members and 2.5 when designing cladding or connections between the cladding and the frame,
- $C_p$  = the external pressure coefficient shown in Figure 2.7 for the location being considered, and
- $q$  = the reference velocity pressure, psf.

Reference velocity pressures are tabulated for many locations in Canada in Reference (8). The reference velocity pressures for the design of cladding and the design of structural members for deflection and vibration is that based on a probability of 1 in 10 of being exceeded in any one year. The reference velocity pressure for the design of structural members for strength is that based on a probability of 1 in 30 of being exceeded in any one year.

Although wind blowing at right angles to a building usually causes the largest uplift for the roof as a whole, wind blowing diagonally across the roof can produce more severe local suctions near the corners and along the edges. Values of  $C_p^*$  to be used in determining these local effects are also given in Figure (2.7).

In addition to pressures and suctions on the outside of the building, air leakage around doors and windows results in an internal pressure,  $p_i$ , (or suction, depending on the locations of the openings) given by

$$p_i = C_e C_{pi} q \quad (2.19)$$

when the effect of gusts inside the building is not considered to be important, and

$$p_i = C_e C_g C_{pi} q \quad (2.20)$$

when the effect of gusts is considered to be important. Values of the internal pressure coefficient,  $C_{pi}$ , are also given in Figure 2.7. It is left to the designer to determine whether the effect of gusts is significant inside the building. Use of Equation (2.20) might be appropriate, for example, in an industrial building where use of the building requires that large doors be left open during the working day.

One shortcoming is that although it is conservative, it is not rational to add the internal pressure to the external suction when investigating the effects of local suction on the roof since they are both based on different wind directions. The values of  $C_p^*$  are based on wind blowing along the roof diagonal, while the values of  $C_{pi}$  are based on wind blowing perpendicular to the walls.

In some situations, such as in design of the roof deck diaphragm, only the combined effects of pressure on the windward wall and suction on the leeward wall need to be considered. In this case it is the shape factor for the building as a whole, also denoted  $C_p$  and defined as the difference between the pressure coefficients for these walls:

$$C_p = 0.7 - (-0.5) = 1.2 \quad (2.21)$$

that is used in Equation (2.18) to determine the specified loads.

---

#### Example 2.4

##### Given

The door and window openings in the structure shown in Figure 1.1 are located mainly in one of the 200 foot long walls. Determine the specified wind loads for:

1. strength design of the girts,
2. strength design of the roof deck diaphragm and the bracing members in the end walls, and
3. calculation of building sway. e

The structure is located in Ottawa, Ontario ( $q_{1/10} = 6.2$  psf,  $q_{1/30} = 7.8$  psf). Assume that gusts do not occur inside the building.

### Solution

#### 1. Strength Design of the Girts

A study of the external and internal pressure coefficients given in Figure 2.7 indicates that the largest loads occur on the walls for the following two wind directions:

1. on the side walls when the wind is blowing against the wall with the openings, and
2. on the windward wall when the wind is blowing parallel to the wall with the openings.

In the first case the net effect is a suction directed outward, while in the second case it is a pressure directed inward.

Although both wind directions produce loads that are equal in magnitude, the first case, wind blowing against the wall with the



openings, is the more severe for the design of the girts. In this situation the compression (inside) flanges of the girts in the side walls are laterally unsupported over a longer span.

The external wind suction is given by:

$$\begin{aligned}
 p &= C_e C_g C_p q^{1/30} && \text{(Eq. 2.18)} \\
 &= 1.0 \times 2.0 \times 0.7 \times 7.8 \\
 &= 10.9 \text{ psf}
 \end{aligned}$$

and the internal wind pressure is given by:

$$\begin{aligned}
 p_i &= C_e C_{pi} q && \text{(Eq. 2.19)} \\
 &= 1.0 \times 0.7 \times 7.8 \\
 &= 5.5 \text{ psf}
 \end{aligned}$$

giving a net inward pressure of  $10.9 + 5.5 = 16.4$  psf.

## 2. Strength Design of the Roof Deck Diaphragm and the Bracing Members in the End Walls

In this situation only the combined effects of the outside pressures need to be considered. As discussed earlier, the shape factor for the building as a whole is  $C_p = 1.2$ . Thus:

$$\begin{aligned}
 p &= C_e C_g C_p q^{1/30} && \text{(Eq. 2.18)} \\
 &= 1.0 \times 2.0 \times 1.2 \times 7.8 \\
 &= 18.7 \text{ psf}
 \end{aligned}$$

### 3. Calculation of Building Sway

Since sway calculations are based on a 1:10 reference velocity pressure, the specified load for calculation of building sway can be computed from the results above as:

$$\begin{aligned} p &= \frac{q_{1/10}}{q_{1/30}} \times 18.7 \\ &= \frac{6.2}{7.8} \times 18.7 \\ &= 14.9 \text{ psf} \end{aligned}$$

---

Preliminary results of a research project that is still in progress at the time of writing<sup>(23)</sup> indicate that some changes may be appropriate for the pressure coefficients given above. Specifically, it is suggested that:

1. the leeward wall coefficient be increased from -0.5 to -0.7,
2. the local corner coefficient be reduced from -3.0 to -2.0,
3. the edge coefficient be reduced from -2.0 to -1.5,
4. the leeward roof coefficient be reduced from -0.5 to -0.2,  
and
5. the windward roof coefficient be reduced from -1.0 to -0.8.

However, these results should be considered tentative until the project is completed, since it appears that other aspects of the effects of wind load, such as fatigue on the cladding connectors, may become important if the wind loads are reduced<sup>(23)</sup>.

Since the shape of a building changes during erection, wind loads can be higher during this time than after completion of the building (9). Consider a series of bents consisting of columns and either girders or joists. The force  $F_1$  on a particular member in the windward bent is (9)

$$F_1 = C_e C_g (C_{n\infty} k) A \quad (2.22)$$

where  $C_{n\infty}$  = force coefficient for an infinitely long member (see Figure 2.8),  
 $k$  = reduction factor for a member of finite length (see Figure 2.9), and  
 $A$  =  $hL$  = area normal to wind direction.

For members that are located in the remaining bents the effect of shielding should be taken into account. As illustrated in Figure 2.10 for the case of two bents, the force  $F_2$  on the leeward member is (9):

$$F_2 = k_x F_1 \quad (2.23)$$

where  $k_x$  is the shielding factor. The circles in Figure 2.10 represent the numerical values given in Reference (9). Note that for solid shapes and realistic separation ratios,  $x/h$ , it is necessary to extrapolate these points as indicated by the broken line. Tests are required to determine if the extrapolation shown is valid.

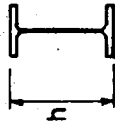




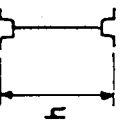
Beams	Columns			Girts	Joists
					
2.05	1.6	1.9	2.0	0.6	0.4

FIGURE 2.8 FORCE COEFFICIENT  $C_{n\infty}$  FOR AN INFINITELY LONG MEMBER<sup>(9)</sup>

$\frac{L}{h}$	5	10	20	35	50	100	$\infty$
Solid Shapes	0.60	0.65	0.75	0.85	0.90	0.95	1.0
Joists	0.96		0.98		0.99		1.0

FIGURE 2.9 REDUCTION FACTOR  $k$  FOR MEMBERS OF FINITE LENGTH<sup>(9)</sup>

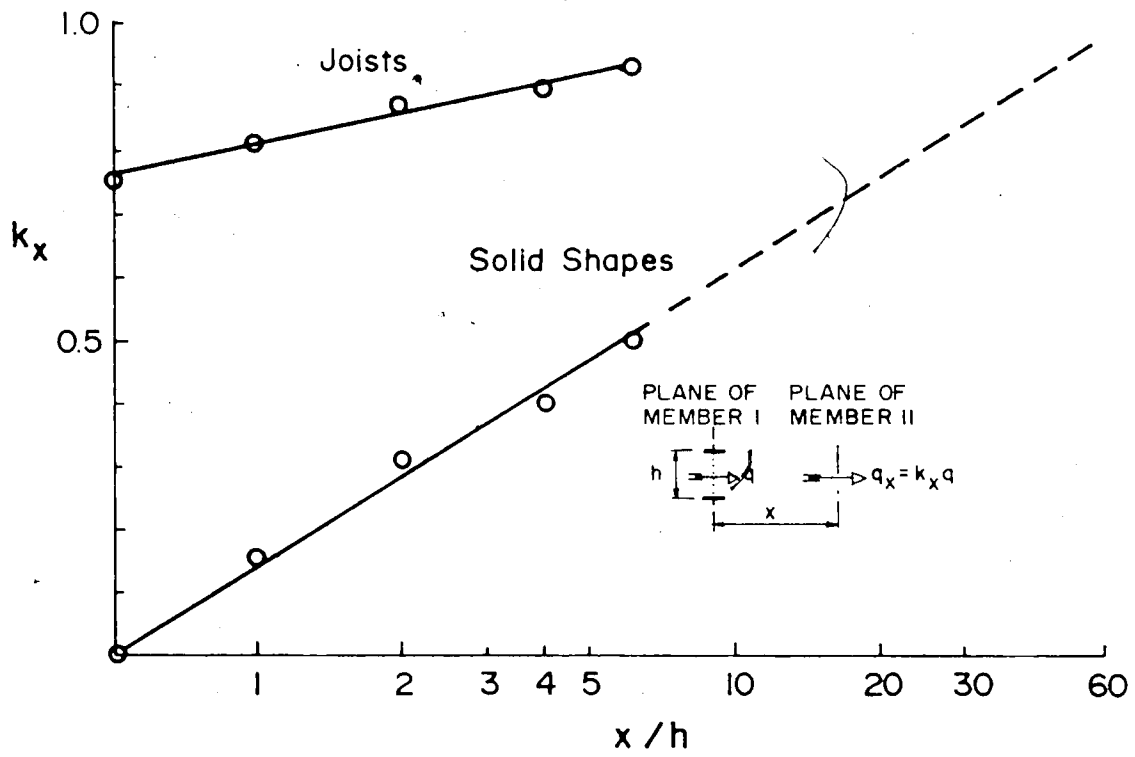


FIGURE 2.10 SHIELDING FACTOR  $k_x$  <sup>(9)</sup>

For a series of  $n$  bents it is assumed that the net wind force can be estimated by successive applications of Equation (2.23):

$$\begin{aligned}
 \sum_{i=1}^n F_i &= F_1 + F_2 + F_3 + \dots + F_n \\
 &= F_1 + k_x F_1 + k_x (k_x F_1) + \dots + k_x^{n-2} (k_x F_1) \\
 &= F_1 (1 + k_x + k_x^2 + \dots + k_x^{n-1}) \\
 &= F_1 \left( \frac{1 - k_x^n}{1 - k_x} \right) \qquad (2.24)
 \end{aligned}$$

### Example 2.5

#### Given

In order to clearly illustrate the basic concepts involved in computing wind loads during erection, the structure shown in Figure 1.1 is not a very good choice because of the large number of different members involved. Consider instead a relatively simple structure consisting of ten bents. As illustrated in Figure 2.11, each bent is 200 ft. long and consists of 18 in. deep girders 40 ft. long connecting into the webs of 8 in. columns 21 ft. high. Compute the specified wind force acting at roof level. Assume a reference velocity pressure of 7.8 psf.

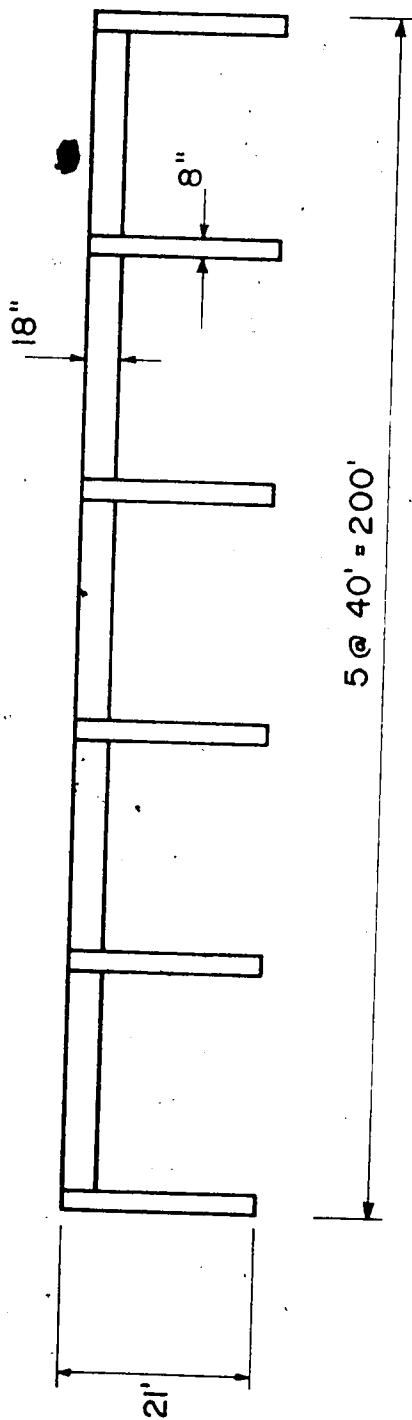


FIGURE 2.11 BENT EXPOSED TO WIND DURING ERECTION



Solution

The force on the girders in the windward bent is:

$$F_1 = C_e C_g (C_{n\infty} k) qA \quad (\text{Eq. 2.22})$$

From Figure 2.8 the force coefficient for an infinitely long member is 2.05. To account for the actual girder slenderness of:

$$\frac{L}{h} = \frac{200 \times 12}{18} = 133$$

a reduction factor of 0.96 will be used (see Figure 2.9). Thus

$$\begin{aligned} F_1 &= 1.0 \times 2.0 \times (2.05 \times 0.96) \times \frac{7.8}{1000} \times 200 \times \frac{18}{12} \\ &= 9.21 \text{ kips} \end{aligned}$$

To determine the shielding factor the separation ratio is

$$\frac{x}{h} = \frac{40 \times 12}{18} = 26.7$$

so that, using Figure 2.10,  $k_x = 0.8$ . Thus

$$\sum_{i=1}^{10} F_i = F_1 \left( \frac{1 - k_x^{10}}{1 - k_x} \right) \quad (\text{Eq. 2.24})$$

$$= 9.21 \left( \frac{1 - (0.8)^{10}}{1 - 0.8} \right)$$

$$= 41.1 \text{ kips}$$

Similarly, the force on the windward set of columns is:

$$\begin{aligned}
 F_1 &= C_e C_g (C_n^\infty k) qA && \text{(Eq. 2.22)} \\
 &= 1.0 \times 2.0 \times (1.9 \times 0.8) \times \frac{7.8}{1000} \times 6 \times 21 \times \frac{8}{12} \\
 &= 2.0 \text{ kips}
 \end{aligned}$$

and the shielding factor is:

$$\frac{x}{h} = \frac{40 \times 12}{8} = 60$$

so that  $k_x = 0.98$ , and

$$\begin{aligned}
 \sum_{i=1}^{10} F_i &= F_1 \left( \frac{1 - k_x^{10}}{1 - k_x} \right) && \text{(Eq. 2.24)} \\
 &= 2.0 \cdot \left( \frac{1 - (0.98)^{10}}{1 - 0.98} \right) \\
 &= 18.3 \text{ kips}
 \end{aligned}$$

It will be assumed that all of the girder force and half of the column force acts at roof level. The remaining half of the columns force is assumed to be transferred directly to the foundation. Thus the total force at roof level is  $41.1 + \frac{18.3}{2} = 50.3$  kips. This force must be resisted by the bracing members in the side walls.

---

One shortcoming in the above procedure for computing wind

forces during erection is in using the same reference velocity pressure for the short erection period that is used for the lifetime of the structure. A statistical analysis of wind data is required to determine a wind pressure appropriate for this shorter period. However, this is considered to be beyond the scope of this dissertation. Another shortcoming is the lack of consideration of the wind loading on a single storey structural frame before the wall cladding is installed. This research, which would involve wind tunnel testing, is also beyond the scope of this dissertation.

#### 2.3.5 Earthquake Loads

A quasi-static procedure to determine the design loads on a building due to earthquake motion is described in References (7) and (9). A dynamic approach is also outlined. In comparing the two procedures it was found that larger base shears were predicted for single storey buildings with the dynamic approach. However, because both methods contain major simplifications and assumptions it is not really possible to designate one procedure as being more correct than the other. For simplicity, only the static procedure will be discussed further.

With this approach the specified horizontal shear acting at the base of a single storey building is given by:

$$V = ASKIFW \quad (2.25)$$

where  $A$  = horizontal design ground acceleration expressed as a decimal part of  $g$  the acceleration due to gravity,

$S$  = a seismic response factor,

$K$  = a coefficient that reflects the type of construction and the energy absorptive capacity of the structure,

$I$  = an importance factor, equal to 1.0 for light industrial buildings,

$F$  = a foundation factor, and

$W$  = roof dead load plus 25 percent of the roof snow load.

Since ground motion during an earthquake is multi-directional the motion of the building will consist of a simultaneous translation about two horizontal axes, plus vertical and torsional motions.

It is considered adequate, however, to design only for independent motion about each of the horizontal axes, providing torsional effects are also considered<sup>(9)</sup>. For a single storey building the specified horizontal earthquake force acting at roof level is equal in magnitude, but opposite in direction, to the base shear  $V$ .

An equation of the same form as Equation (2.25) can be derived by assuming that the structure is infinitely stiff, and moves as a rigid body. The maximum base shear is therefore equal to the product of the mass of the building and the maximum ground acceleration,  $a$ :

$$\begin{aligned} V &= \frac{W}{g} a \\ &= CW \end{aligned} \quad (2.26)$$

where C can be described as an earthquake coefficient. The corresponding value of C in Equation (2.25) is the product ASKIF.

Chart 12 in Reference (8) divides Canada into four seismic zones of differing earthquake risk. In each region the peak horizontal ground acceleration A that has a probability of 1/100 of being exceeded in any one year has been determined and is also shown in this chart. Thus in Ottawa (zone 2) the acceleration A is 0.04, while Edmonton is not considered to be in a zone of earthquake risk.

The simplified analysis leading to Equation (2.26), in which the building is assumed to move as a rigid body, is valid only for an infinitely stiff structure. The actual motion of the building depends on both its mass and stiffness, characterized by its fundamental period of vibration, T. The seismic coefficient in Equation (2.25), defined as (7):

$$S = \frac{0.5}{T} \quad (2.27)$$

but not greater than 1.0

is therefore the term that accounts for the dynamic properties of the building. Measured values of fundamental periods for single storey buildings vary from 0.05 seconds for a stiff building with large areas of masonry in-filled walls to 0.15 seconds for a flexible rigid frame with light cladding (24). The empirical equation (7):

$$T = \frac{0.05 h}{\sqrt{D}} \quad (2.28)$$

where  $h$  = building height, in feet, and  
 $D$  = the dimension of the building in the direction of  
the applied earthquake force, in feet

is in reasonable agreement with these measurements. From Equation (2.27), since the seismic coefficient is greater than unity for all periods less than 0.125 seconds, the dynamic properties of many single storey buildings will not influence the value of base shear, and these buildings will attract the same base shear as if they were infinitely stiff.

The factor  $K$  reflects the ability of the structure to undergo plastic deformation when subjected to cyclic loading, and appears to be based primarily on observed damage to buildings, although some analytical work has been done in this area. (25) For the types of structures considered here,  $K$  should be taken as 1.0 for structures with "X" or "K" bracing designed to resist both tensile and compressive forces, and 1.3 for structures with "X" or "K" bracing designed for tensile forces only, or for structures with one or more infilled bays (7). Research is required to determine if standard connections satisfy the requirements of a "moment resisting space frame", discussed in Reference (7). If so,  $K$  could be reduced to 0.8.

Foundation conditions play a significant role in determining structural response. Experience has shown that

buildings founded on unconsolidated soil suffer more damage than those founded on well consolidated soil or rock. For this reason recommended values for the foundation factor  $F$  vary from 1.0 for rock to 1.5 for loose coarse-grained soils. The product  $FS$ , however, need not exceed 1.0<sup>(7)</sup>.

---

### Example 2.6

#### Given

Compute the specified earthquake forces acting in the 120 foot direction on the structure shown in Figure 1.1. The specified roof dead load is shown in Example 2.1 to be 31.5 psf, and the specified roof snow load is shown in Example 2.2 to be 48 psf. The structure is located in Ottawa, Ontario ( $A = 0.04$ ), has a direct acting bracing system designed to resist tension only ( $K = 1.3$ ), and is founded on compact coarse-grained soil ( $F = 1.3$ ). The dimensions of the structure are 200 ft. x 120 ft. x 20 ft..

#### Solution

The period of vibration of the structure in the 120 ft. direction is

$$\begin{aligned}
 T &= \frac{0.05 h}{\sqrt{D}} && \text{(Eq. 2.28)} \\
 &= \frac{0.05 \times 21}{\sqrt{120}} \\
 &= 0.10 \text{ seconds}
 \end{aligned}$$

therefore the seismic coefficient for this direction is

$$\begin{aligned}
 S &= \frac{0.5}{\sqrt[3]{T}} && \text{(Eq. 2.27)} \\
 &= \frac{0.5}{\sqrt[3]{0.10}} \\
 &= 1.1
 \end{aligned}$$

however this need not exceed 1.0. Also, since the product of the seismic coefficient and the foundation factor need not exceed 1.0, the "effective" value of F is also 1.0.

The weight W is defined as the roof dead load plus 25 percent of the snow load, or:

$$\begin{aligned}
 W &= \left(31.5 + \frac{48}{4}\right) \times \frac{200 \times 120}{1000} \\
 &= 1040 \text{ kips}
 \end{aligned}$$

Thus the base shear is

$$\begin{aligned}
 V &= ASKIFW && \text{(Eq. 2.25)} \\
 &= 0.04 \times 1.0 \times 1.3 \times 1.0 \times 1.0 \times 1040 \\
 &= 54.1 \text{ kips}
 \end{aligned}$$

The specified horizontal force acting at roof level is equal in magnitude, but opposite in direction, to the base shear.

---



One aspect of earthquake forces on buildings that deserves attention occurs in the design of irregular buildings, for example with "L" or "T" shaped lay-outs. Torsional effects may become important when, as in these cases, the centers of mass and rigidity do not coincide. This subject is considered further in Reference (26). However, even with a rectangular lay-out accidental torsion can be introduced. To account for uncertainties in estimating dead and live loads, possible future additions of wall panels, inelastic action, variation in estimating rigidities, and simplifications inherent in the approximate earthquake analysis it is recommended that the calculated eccentricity  $e$  between the centers of mass and rigidity be replaced by

$$e_x = 1.5 e + 0.05 D_N \quad (2.29)$$

or

$$e_x = 0.5 e - 0.05 D_N \quad (2.30)$$

where  $D_N$  is the building dimension at right angles to the direction of the earthquake loading, whichever causes the greater effect in the member concerned<sup>(9)</sup>. If  $e_x$  exceeds  $0.25 D_N$  either a dynamic analysis should be performed or the effects of torsion in the static procedure should be doubled<sup>(7)</sup>.

### Example 2.7

#### Given

Compute the effects of accidental torsion on the structure shown in Figure 2.12. The specified base shear is 54.1 kips.

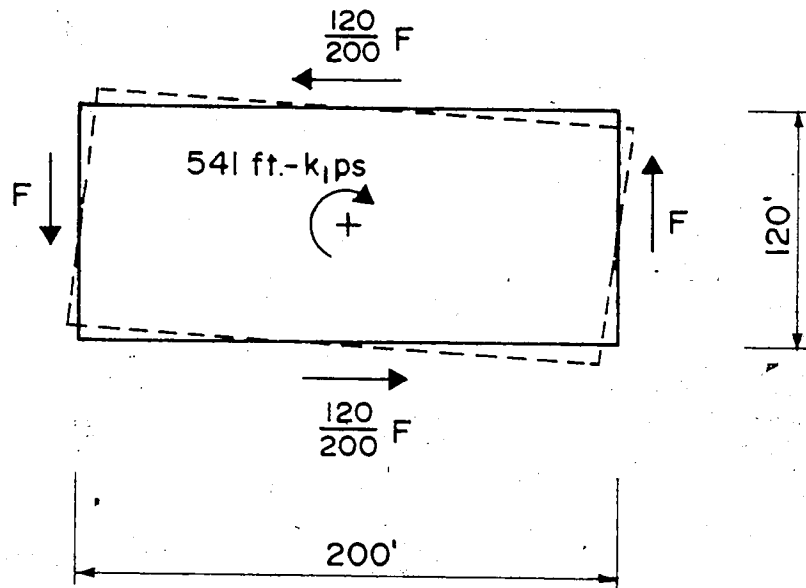


FIGURE 2.12 EFFECT OF TORSION

Solution

The accidental eccentricity is

$$\begin{aligned} e_x &= 0.05 D_N && \text{(Eq. 2.29)} \\ &= 0.05 \times 200 \\ &= 10 \text{ ft.} \end{aligned}$$

therefore the accidental torsion that must be accounted for is

$$T = 54.1 \times 10 = 541 \text{ ft-kips}$$

As illustrated in Figure 2.12, this torque is resisted by the bracing members in the end and side walls.

Assuming that the roof deck diaphragm rotates as a rigid body and that the bracing members are all equally stiff, the largest forces will be developed in the walls furthest away from the center of rigidity. Denoting this force as  $F$ ,

$$F \times 200 + \frac{120}{200} F \times 120 = 541$$

or

$$F = 2.0 \text{ kips}$$

The bracing members must be designed to resist this force, in addition to the bracing forces computed assuming no torsion.

---


In order to prevent damage to non-structural elements it is recommended in Reference (7) that the building sway at the specified load level not exceed  $1/200$  of the building height. To prevent collision between adjacent buildings it is recommended that either the separation must be at least twice the sum of the individual deflections or the buildings must be tied together to act as a unit (7). In the case of a long building separated into two closely spaced shorter ones with an expansion joint, the expansion joint width may be governed by this separation requirement.

#### 2.3.6 Other Loadings

The roof of a structure should also be capable of supporting loads incidental to construction and maintenance. To cover these occasional short time loads a specified load of 20 psf is recommended in Reference (7), either uniformly distributed over the total roof area or on any portions of the roof area, whichever produces the most critical effects in the members concerned. This load is not additive to loads due to snow or rain. To simulate concentrated loads it is recommended in Reference (7) that a 300 lb. force be applied over a 2-1/2 ft. by 2-1/2 ft. area of the roof, located so as to cause maximum effects.

The roof deck should also have the strength and stiffness necessary for successful application and satisfactory performance of the roofing membrane. To insure this, it is recommended in Reference (27) that for design of the roof deck only the minimum uniform factored dead plus live load be 72.5 psf, and that the maximum deflection under a minimum uniform live load of 40 psf not exceed  $1/240$  of the span.

#### 2.4 Summary



In this chapter the design process is outlined and loads that are used in design of single storey buildings are discussed. Examples illustrate procedures for estimating dead load and loads caused by snow, wind and earthquake. A detailed treatment of ponding of rainwater is presented. It is shown that in some locations in Canada moments and deflections caused by ponding of rainwater can exceed those caused by snow.

CHAPTER III  
ECONOMICS OF FRAMING ARRANGEMENTS

3.1 Introduction

In light industrial buildings, where interior architectural features are often minor, the best structural arrangement is usually a compromise between functional requirements and economic considerations. Although primary importance must be attached to function, cost is also a basic factor.

In CHAPTER I over-all dimensions for an example building 200 x 120 feet in plan and 21 feet high, having square bays measuring 40 x 40 feet, were assumed to be established from functional requirements. This lay-out is shown in Figure 3.1. In this chapter a framing scheme is selected and the cost of structural steel in this building is compared with the cost of structural steel in buildings with other bay sizes, bay shapes and framing schemes in order to determine if the proposed lay-out is reasonably economical. The cost studies presented are based on a method developed with the assistance of several major fabricators, and is described in detail in CHAPTER VII. In CHAPTERS IV, V, and VI, where specific design procedures are discussed, various members in this building are selected and designed as illustrative examples.

No attempt is made in this chapter to determine the

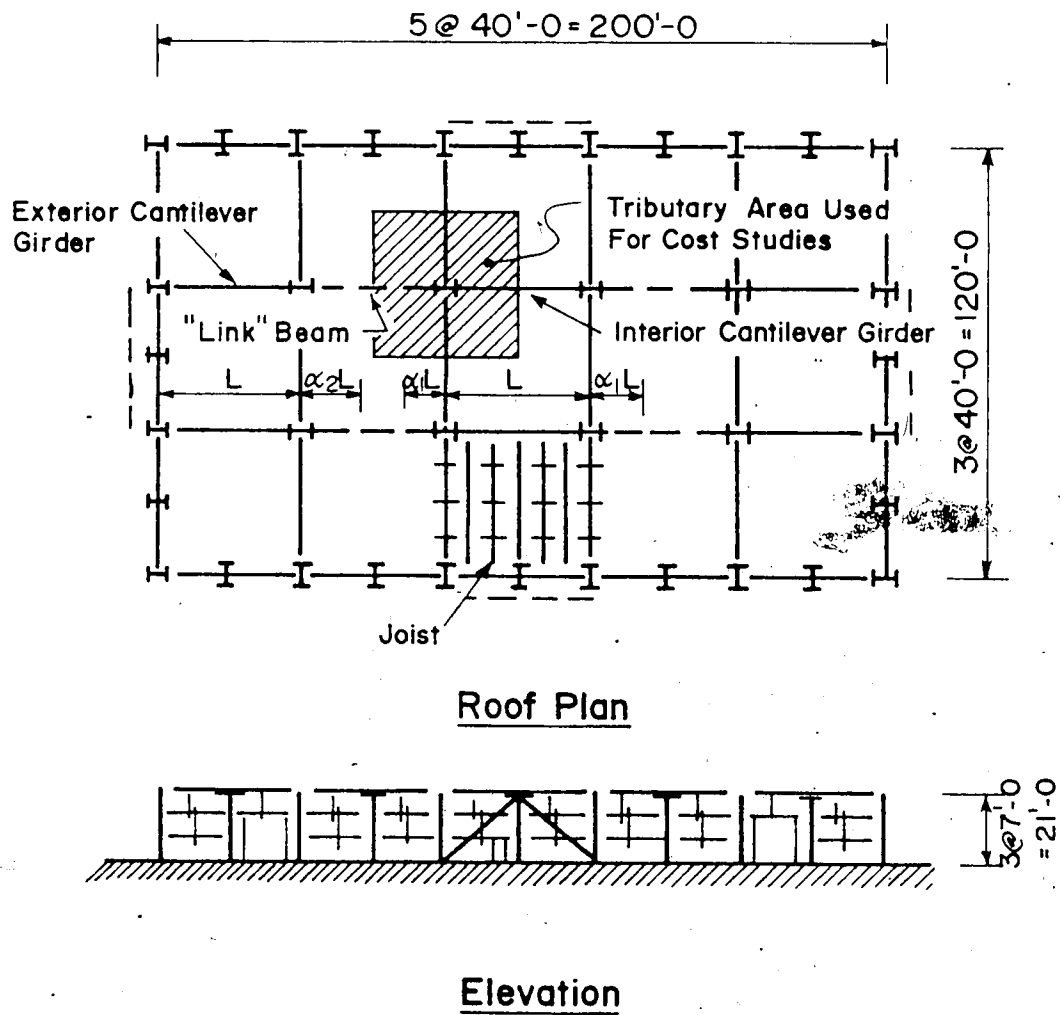


FIGURE 3.1 PROPOSED LAY-OUT OF EXAMPLE BUILDING

"optimum" lay-out and framing scheme - ie., those that result in minimum building cost - since this also depends on costs associated with other trades. Cost of extra wall cladding required to maintain a constant clear height when large bay areas are used, for example, is not included. Effect of cost of mechanical ductwork on building cost is also not considered. The cost study indicates only the effect of different lay-outs and framing schemes on cost of structural steel in the building.

Briefly, the cost of structural steel is assumed to be composed of a material cost, that depends on the weight of steel involved (including connection weight), a fabrication cost, cost of outside purchases such as open web steel joists and roof deck, and an erection cost. The fabrication cost of a beam or a column is assumed to depend on the number and complexity of the connections that must be made. Open web steel joists, on the other hand, are often purchased from another fabricator who specializes in making them, usually at a fixed price per ton. Roof deck is usually purchased at a fixed price per square foot. The erection cost is assumed to depend closely on the number of members to be handled. Also included is the cost of high strength bolts and anchor bolts, cost of drafting, receiving and shipping, profit, overhead and freight. The unit prices and wages used are generally representative of those at the time of writing (July, 1978) in the Ottawa area.

The cost comparisons in this chapter are meaningful only if it can be assumed that the structural steel will be



supplied by a fabricator who evaluates the labor cost separately for each job. If the fabricator's bid assumes that cost is simply proportional to weight, then the weight comparisons in Reference (28) will better reflect the effect of change of bay size and shape on cost.

Unless noted otherwise, the cost comparisons are based on a typical interior bay of a building with a cantilever roof framing scheme, such as that shown in Figure 3.1, and wide flange columns. Bays are assumed to be square.

Short span open web steel joists span in a direction perpendicular to the main frame, spaced evenly at not more than 6'-8" on center. Girders, beams and columns are rolled wide flange shapes available from Canadian mills, in G40.21 44W steel. Joist chords are assumed to have a minimum yield strength of 55 ksi, while web material is assumed to have a minimum yield strength of 36 ksi. The specified roof dead load (including the weight of the steel) is 31.5 psf and the specified roof snow load is 48 psf.

### 3.2 Cantilever Overhang Distances

In a cantilever roof framing scheme the girders cantilever over the supporting columns and are separated by simply supported "link" beams, as illustrated in Figure 3.1. The overhang distance divided by the distance between columns is defined as the overhang ratio for the girder. The overhang ratio

for an interior cantilever girder is denoted  $\alpha_1$  and the overhang ratio for an exterior cantilever girder is denoted  $\alpha_2$ . The most economical combination of overhang ratios are those that result in the least cost of the total girder system - in Figure 3.1 an interior cantilever girder, two exterior cantilever girders and two "link" beams. Although it is not usually true, in this case least cost and least weight are synonymous since the fabrication and erection costs are independent of the overhang distances.

As discussed in Section 4.4, for short overhang ratios the girders behave as if they were completely supported along their compression flanges, and the weights decrease as the overhang ratios increase. For long overhang ratios, however, the girder carrying capacities are limited by lateral-torsional instability, involving both the cantilevers and the main spans, and the girder sizes must be increased over what would normally be required if the compression flanges were braced. Since the compression flanges of the "link" beams are laterally braced by the roof deck, their weights always decrease as the overhang ratios increase.

Using the design procedures developed in Section 4.4, girder systems were designed for various combinations of interior and exterior overhang ratios. The results are shown in Figure 3.2, where the weight of the girder system is plotted against the overhang ratios. It is seen that the least weight occurs for an interior overhang ratio of 0.14 and for an exterior overhang ratio of 0.18. These values are used for the girders in the example building, and in all other cost studies in this chapter.

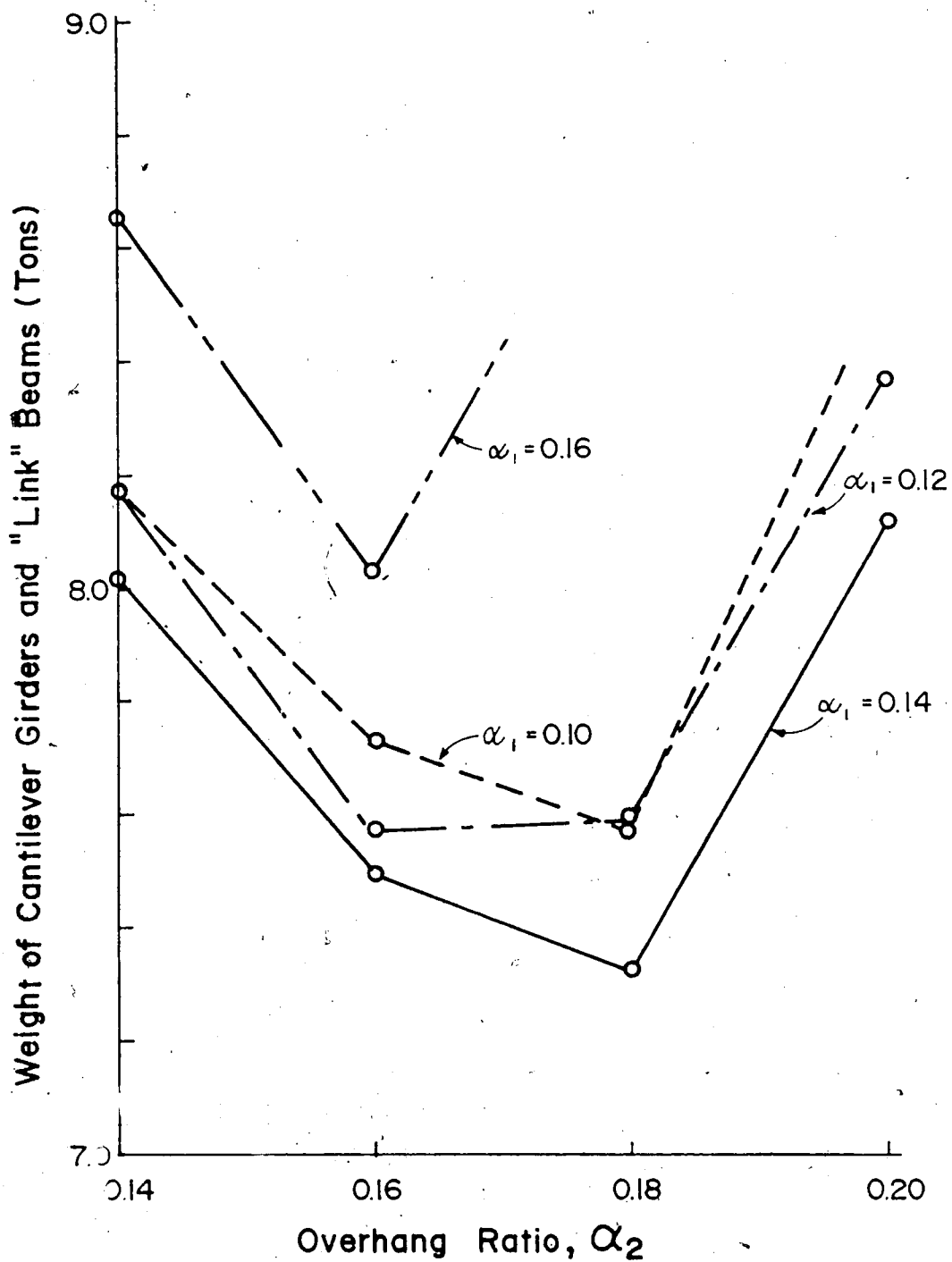


FIGURE 3.2 EFFECT OF OVERHANG RATIOS ON WEIGHT OF STEEL

### 3.3 Effect of Clear Span on Cost

Figure 3.3 illustrates the effect of clear span on cost per square foot of tributary roof area of footings, structural steel f.o.b. job-site, roof deck, and erection. The sum of the individual costs is also shown. To illustrate the estimating procedure, typical calculations for a 40 ft. x 40 ft. bay size are summarized in the following example. For complete details the reader is referred to CHAPTER VII:

---

#### Example 3.1

##### Given

Compute the components of the total cost for the 40 ft. x 40 ft. shaded area shown in Figure 3.1. The cantilever girder is a W21x73 and the "link" beam is a W21x55. The column is a W8x40. The open web steel joists are 24 inches deep and weigh 15.3 lbs./ft. The footing dimensions are 4'-2" x 4'-2" x 1'-2" reinforced with 6 - #5 bars each way. Use the procedure described in CHAPTER VII.

##### Solution

#### 1. Footing Cost

The footing cost is assumed to be comprised of:

- the cost of excavating and back filling at \$10.00 per cubic yard of soil removed,

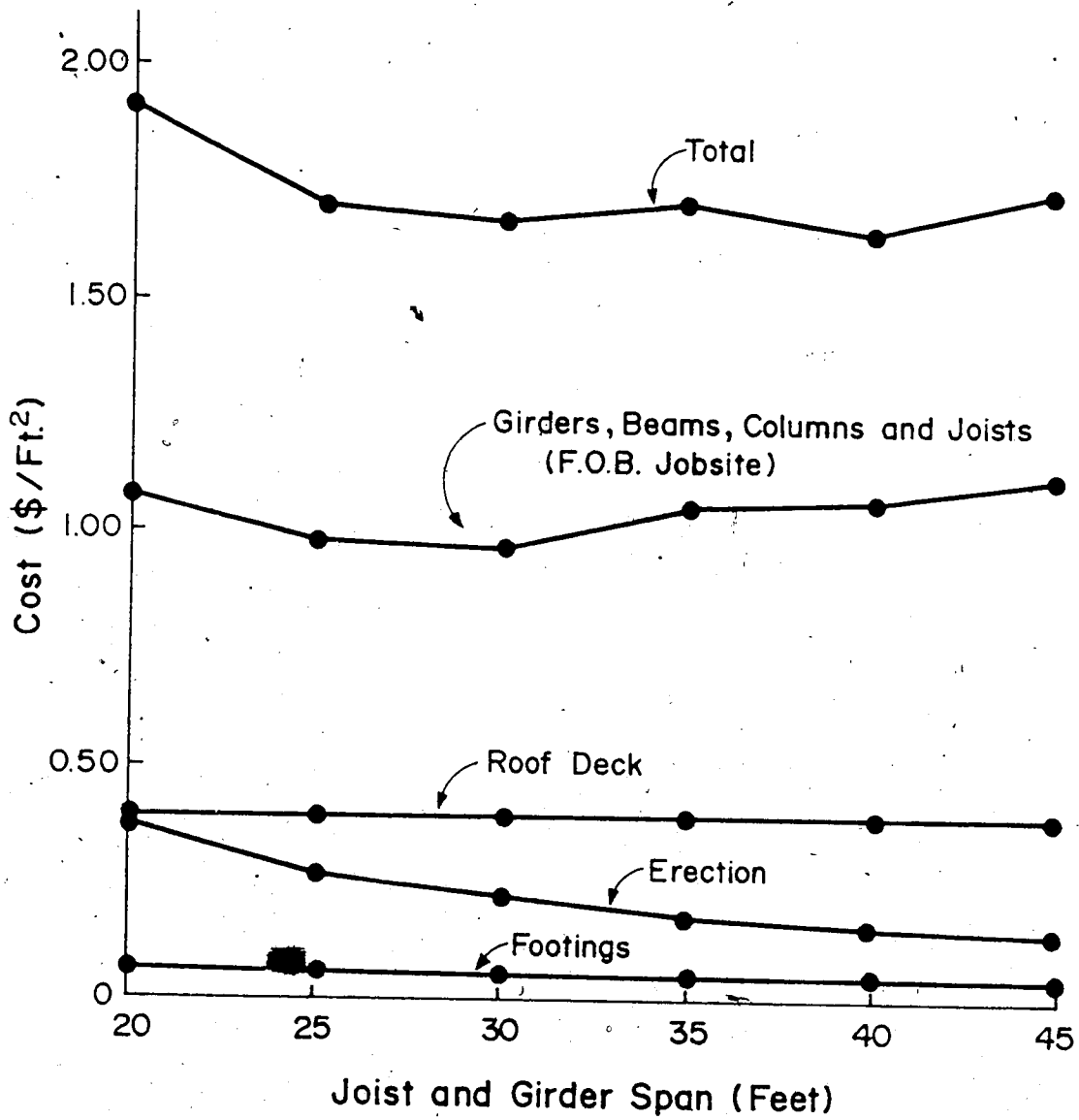


FIGURE 3.3 EFFECT OF BAY SIZE ON COST (Cantilever Roof Framing)

- the cost of formwork for the vertical sides of the footing at \$1.50 per square foot,
- the cost of reinforcing steel at \$0.28 per pound, and
- the cost of concrete at \$30.00 per cubic yard.

Summing up these costs and dividing by 1600 square feet, the result is \$0.05 per square foot of tributary roof area.

## 2. Cost of Girders, Beams and Columns

- cost of steel:		
3440 lbs. x \$0.15/lb.	=	\$516.07
- connections and scrap (5%)	=	\$25.80
- drafting:		
4 man-hrs. x \$14.00/man-hr.	=	\$56.00
- fabrication:		
3.63 man-hrs. x \$20.00/man-hr.	=	\$72.60
- receiving, shipping and freight:		
1.72 tons x \$35.00/ton	=	<u>\$60.20</u>
	sub-total	= \$730.67
- sales and administration overhead (10%)	=	<u>\$73.07</u>
	total	= \$803.74

Dividing this cost by 1600 results in \$0.50 per square foot.

### 3. Cost of Outside Purchases

- joists:

3672 lbs. x \$0.25/lb. = \$918.00

or \$0.57 per square foot

- roof deck is assumed to cost \$0.40  
per square foot.

### 4. Cost of Erection

- unloading and sorting	=	0.43 man-hrs
- erecting	=	1.60
- plumbing	=	0.60
- bolting	=	2.00
- joists	=	2.67
- deck	=	<u>4.27</u>
total	=	11.6 man-hrs.

At a labor rate of \$20.00 per man-hr., this is \$232.00. Adding to this the cost of 0.4 hrs. of crane rental at \$30.00 per hr., and dividing by 1600 square feet, the cost per square foot is \$0.15.

The total cost is found by adding up the component costs:

$$\begin{aligned} \text{total cost} &= 0.04 + 0.50 + 0.57 + 0.40 + 0.15 \\ &= \$1.66 \text{ per square foot.} \end{aligned}$$

as illustrated in Figure 3.3. Note that these costs do not include profit or sales taxes.

The minimum cost of structural steel occurs for bay sizes of approximately 25 ft. x 25 ft. to 40 ft. x 40 ft. Cost increases rapidly for smaller clear spans, but much more slowly for larger clear spans. Although it is not considered in this study, the cost of joists for the longer clear spans can be reduced by using lighter intermediate span joists instead of the short span joists assumed. In this case the fabricated cost of the joists decrease, while the erection cost increases.

Since the cost of the roof deck is assumed to be constant per square foot of roof area, it will not be considered in any of the following comparisons. Similarly, since the footing cost is small and almost constant, it will also not be considered further.

#### 3.4 Effect of Girder Framing and Column Type on Cost

Curves are shown in Figure 3.4 for the fabricated cost of simply supported roof girders and for the fabricated cost of cantilever girders with simply supported "link" beams. In all cases the cantilever roof system results in a lower cost by approximately 5 to 15 cents per square foot, depending on the column spacing. For this reason it was selected for the example building. Similar comparisons could also be made for continuous or plastically designed girder systems, however this was considered to be beyond the scope of this dissertation.

The curves shown in Figure 3.5 illustrate the effect of choice of either wide flange or hollow structural steel sections



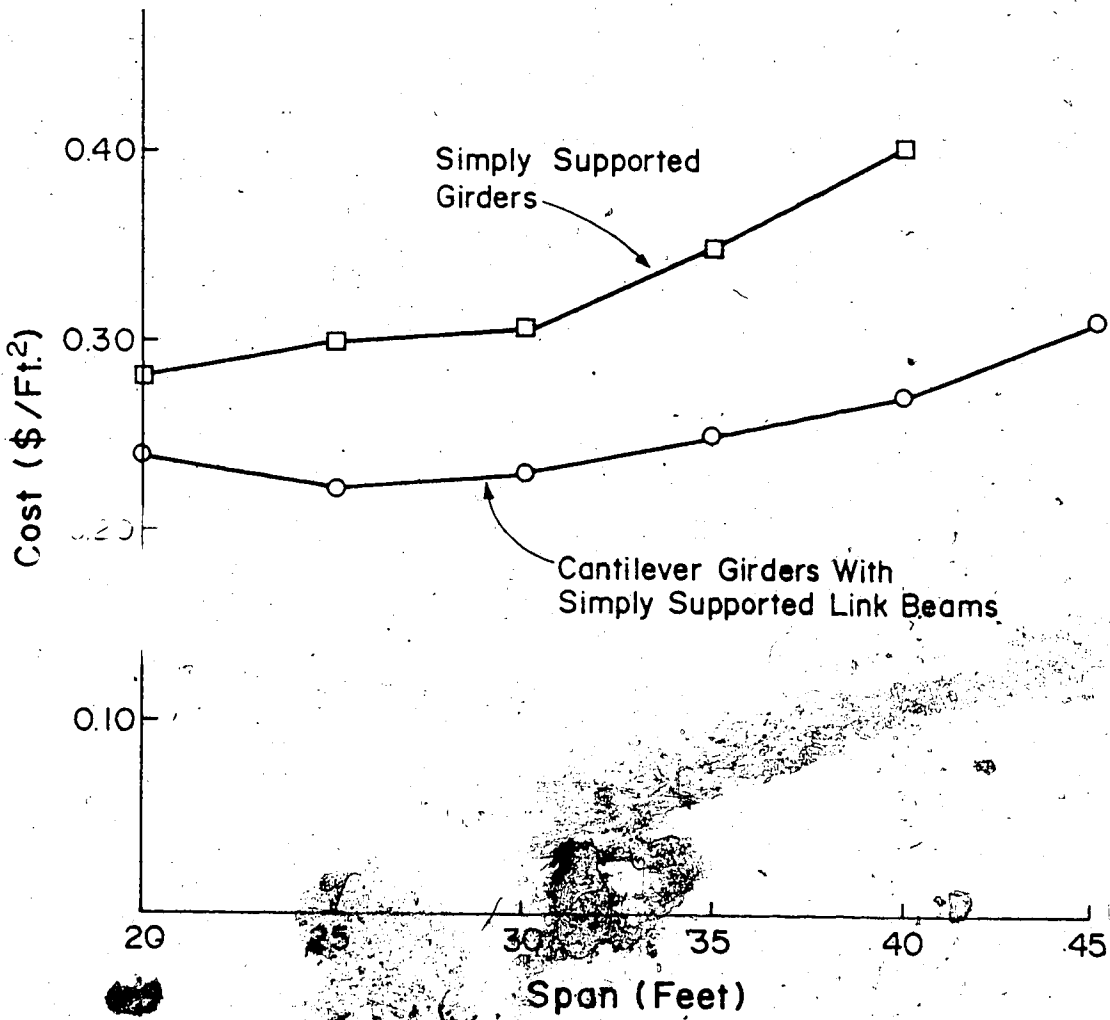


FIGURE 3.4 FABRICATED COST OF GIRDER SYSTEMS

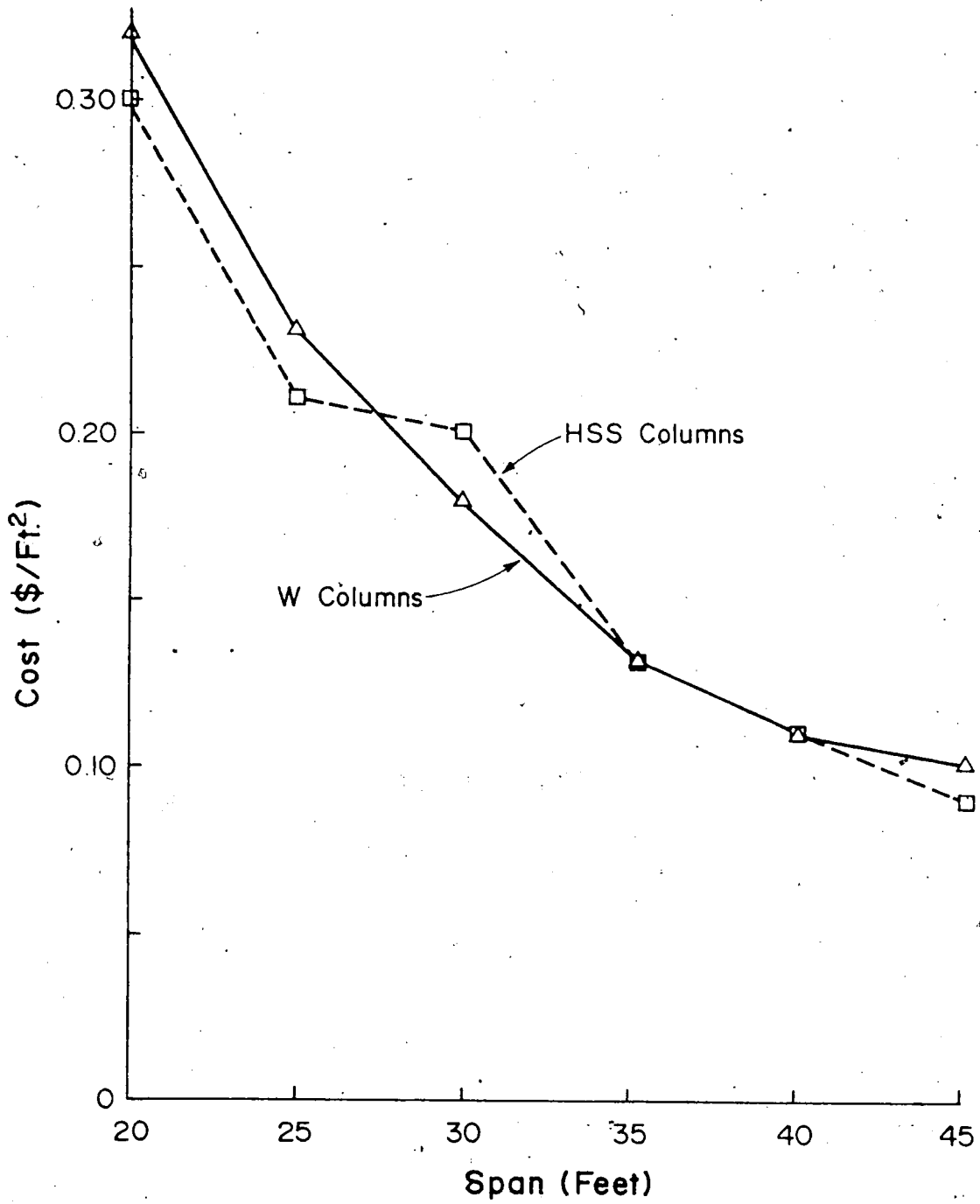


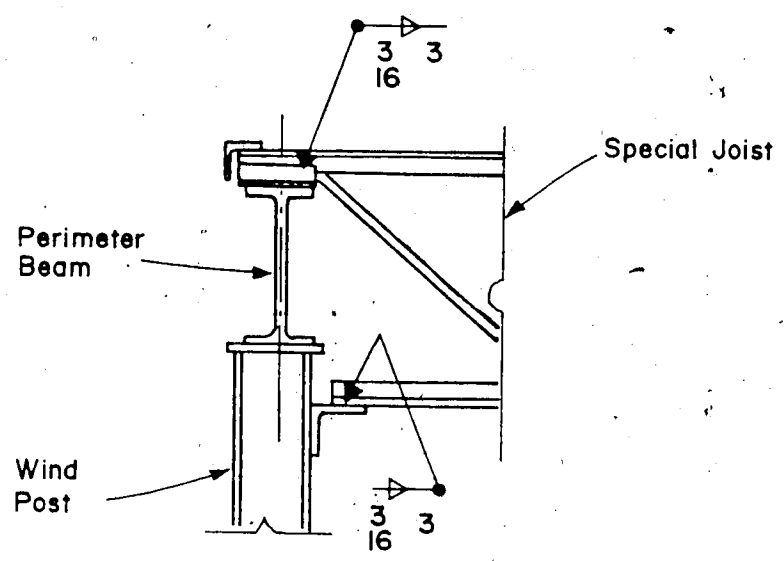
FIGURE 3.5 FABRICATED COST OF COLUMNS

on cost. Based on these curves there does not appear to be a strong argument for one type of column over the other, particularly for 40 ft. spans. One disadvantage with hollow structural steel columns is that with simple construction, where cap plates are not provided, rain water can collect inside the columns before the roof deck is erected. Thus drainage holes must be punched through one of the side walls at the lower ends of the columns. On the other hand, lateral-torsional instability problems with the exterior compression members, (discussed in Section 5.3.2) do not occur. Wide flange columns will be selected for the example building, not for reasons of economy, but rather so that these stability problems can be illustrated.

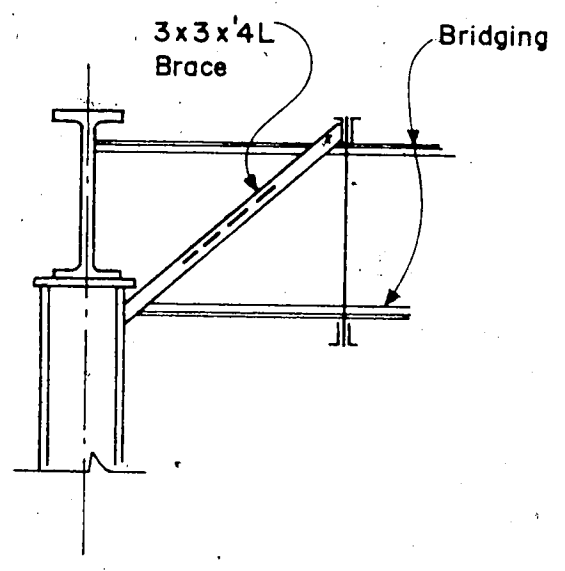
### 3.5 Joist Spacing

Since erection costs decrease as the number of joists decreases, joists should be spaced as far apart as possible without overloading the roof deck. Wherever possible the joist spacing should also be selected so that the roof deck is continuous over three or more spans. Roof decks can be produced economically in lengths up to 36 feet, however the most convenient length for erection is about 21 to 24 feet. Longer lengths become more difficult to handle.

Another consideration that can affect joist spacing occurs if the perimeter roof beams are continuous over intermediate "wind posts", as illustrated in Figures 3.1 and 3.6(a). In this case special joists at these locations can be designed to



(a)



(b)

FIGURE 3.6 SUPPORT CONDITIONS AT UPPER END OF WIND POSTS

transfer the reactions from the tops of the wind posts into the roof deck. Where this is not possible either braces must be designed to transfer these forces, as shown in Figure 3.6(b), or the perimeter beams must be designed to resist combined weak axis bending and torsion.

The joists in the example building in this report will be evenly spaced at 6'-8" on centers. This is the maximum allowable joist spacing for strength design of the 0.030 inch thick, 1.5 inch deep roof deck, and at the same time ensures that joists are available to brace the tops of the wind posts in the 200 ft. long walls. The roof deck will be ordered from the supplier in lengths of 20'-6", allowing continuity over three spans and a 6 inch overlap between adjacent sheets.

Although there are not any obstructions on the roof for snow to drift and pile-up against, this situation often occurs with other low-rise buildings. In this case the joist spacing in the vicinity of the pile-up should be reduced so that:

1. the same joists can be used as in other areas of the roof, and
2. the roof deck is not overloaded.

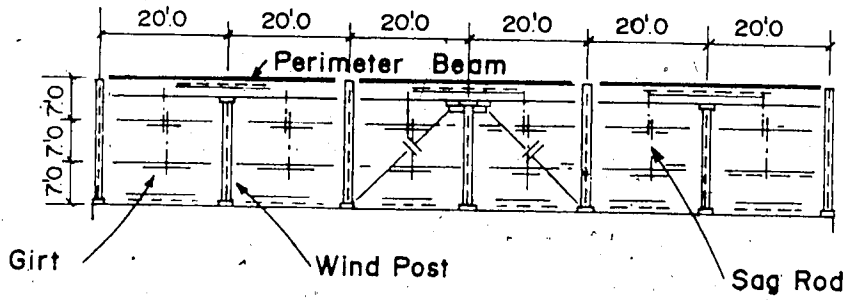
In some cases consideration should also be given to use of a heavier roof deck in these areas.

### 3.6 Wall Framing Schemes

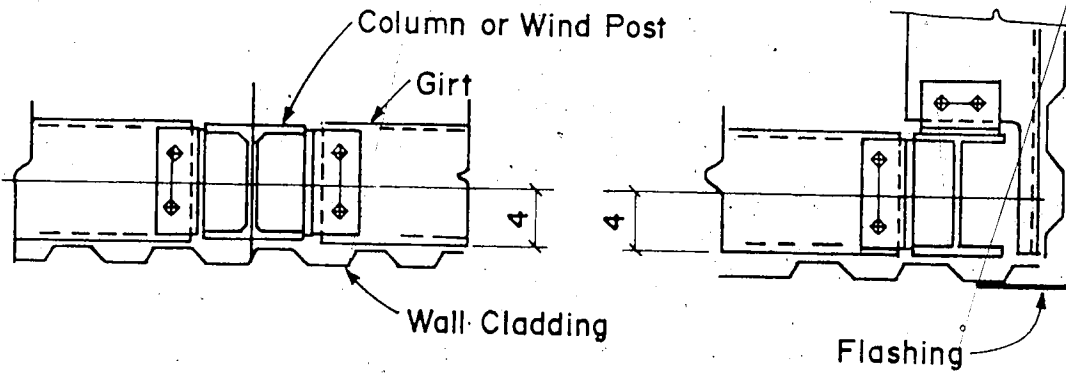
The exterior walls of the example building consist of girts and metal wall cladding. Many different framing schemes for these walls are possible. One solution is to design both the perimeter beams and the girts as simply supported members spanning between exterior columns, however this is impractical for the 40 foot spans involved here. Many girt load tables do not tabulate loads for spans in excess of approximately 25 feet, perhaps the upper limit on column spacing for this type of wall construction.

Another solution, the one chosen here, is illustrated in Figure 3.7(a). In this case intermediate "wind posts" are provided to reduce the girt spans. As illustrated in Figure 3.7(b), the girts span between the columns and wind posts, and are designed as simply supported members 20 feet long. Sag rods are provided at mid-spans to reduce the unsupported lengths of the compression flanges. In order to have the fewest number of girts (thereby reducing fabrication and erection cost) the girt spacing should be established to meet the loading requirements of the cladding, with the first girt 7'-0" minimum from the finished floor level to allow for any man-doors and windows which are normally not framed out. The perimeter roof beams are designed as two-span continuous members, using the wind posts as intermediate supports.

Other solutions are also possible. For example, the girts could be supplied in 40 foot lengths and connected to the



(a) Exterior Wall



(b) Details

FIGURE 3.7 WALL FRAMING SCHEME SELECTED FOR EXAMPLE BUILDING

outside flanges of the columns and wind posts, as illustrated in Figure 3.8(a). Since the girts are continuous over two spans, lighter sections can usually be used. However, as indicated in Section 5.3.4, diaphragm action of the wall cladding cannot be guaranteed with this framing scheme to resist lateral loads or to brace the exterior compression members. This scheme was not selected. Another solution is to use slotted connections, as illustrated in Figure 3.8(b), so that the wind posts do not carry axial loads. In this case the perimeter beams would have to be designed to span 40 feet instead of 20 feet. No attempt has been made to perform a cost comparison of the various alternatives, however it is expected that the arrangement shown in Figure 3.7 is reasonably economical.

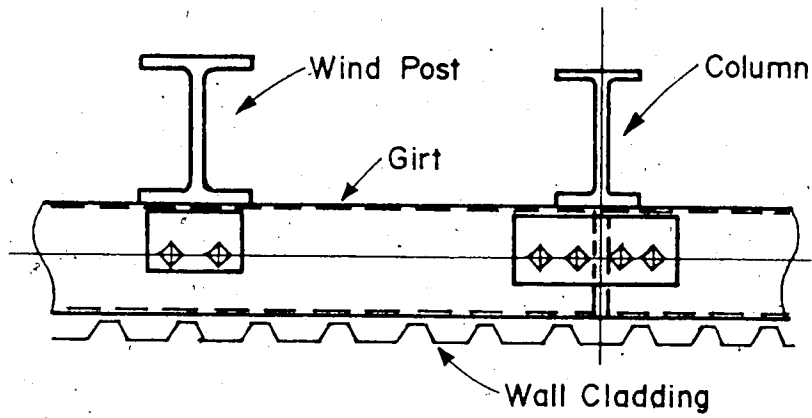
### 3.7 Methods of Resisting Lateral Load

#### 3.7.1 Lateral Load Transfer in the Plane of the Roof

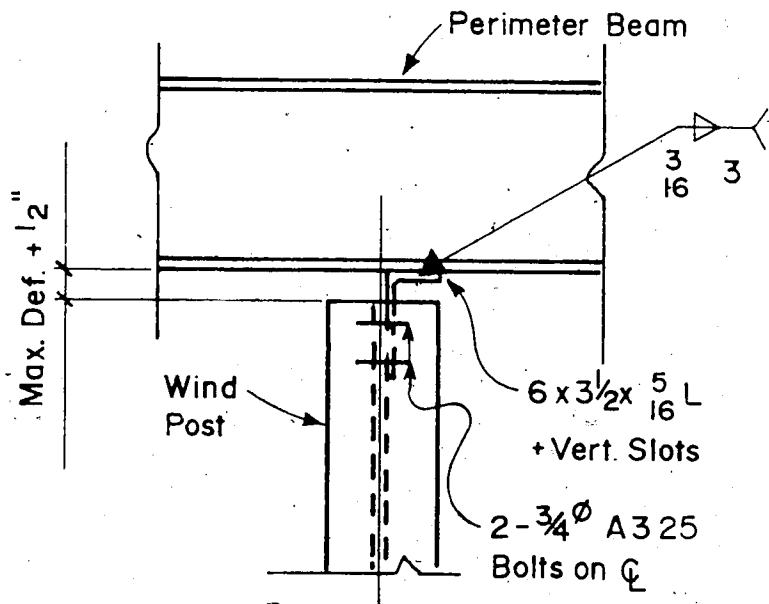
Lateral loads can be transferred in the plane of the roof to the end walls through an arrangement of bracing members provided for this purpose, or through diaphragm action of the roof deck. Figure 3.9 illustrates one possible arrangement of bracing members for the example building.

If diaphragm action is used to resist the lateral loads instead, current design practice<sup>(29)</sup> requires shear transfer elements to be welded on the top flange on all interior girders, between joists, and a trimmer angle to be connected to the beams





(a) Alternate Girt Framing Scheme



(b) Alternate Perimeter Beam Detail

FIGURE 3.8 ALTERNATE WALL FRAMING SCHEMES

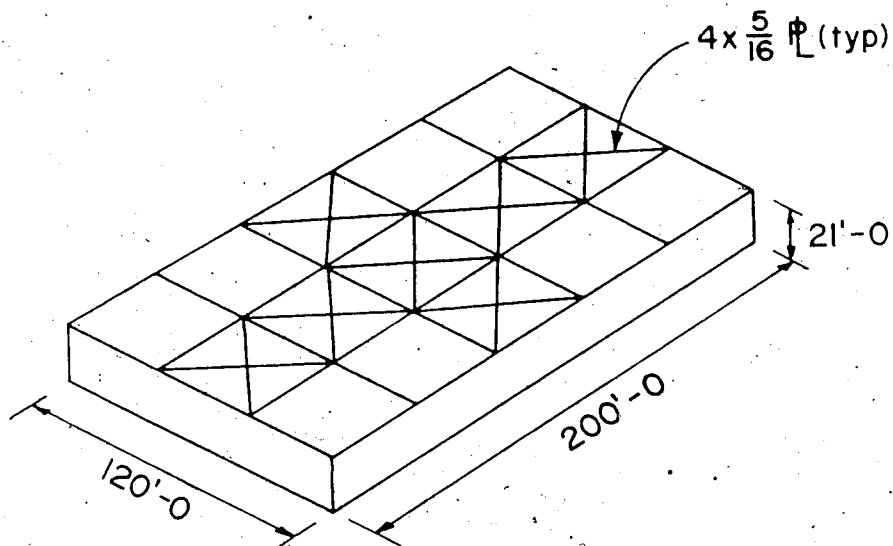


FIGURE 3.9 DIRECT ACTING BRACING SYSTEM

and joists around the perimeter of the building, as illustrated in Figure 3.10. When the walls consist of girts and cladding, however, the trimmer angle will be required in any event, in order to attach the cladding. As illustrated in the following example, diaphragm action is usually the most economical way to transfer loads in the plane of the roof to the end walls.

### Example 3.2

#### Given

Compare the cost of the direct acting bracing system shown in Figure 3.9 with the cost of ensuring diaphragm action of the roof deck by providing the shear transfer elements shown in Figure 3.10. Assume that 10 percent more time is required to connect the roof deck when diaphragm action is being considered, but that the trimmer angle will be required with both schemes to connect the wall cladding at roof level.

#### Solution

##### 1. Cost of the Direct Acting Bracing System

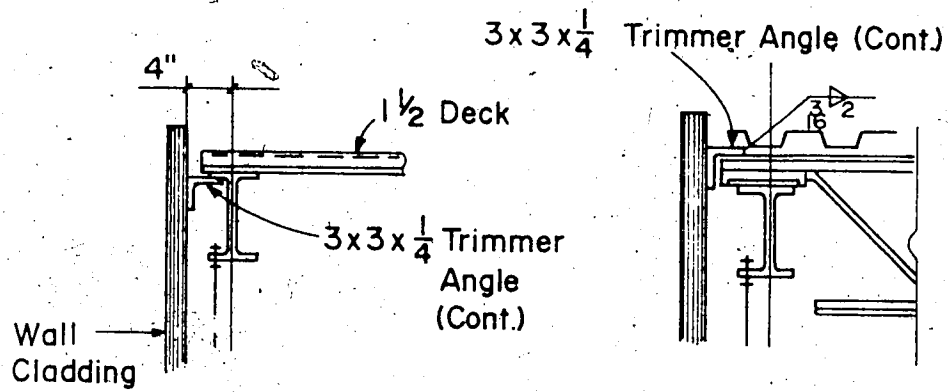
- cost of plate material

3440 lbs. x \$0.16 lb. = \$550.00

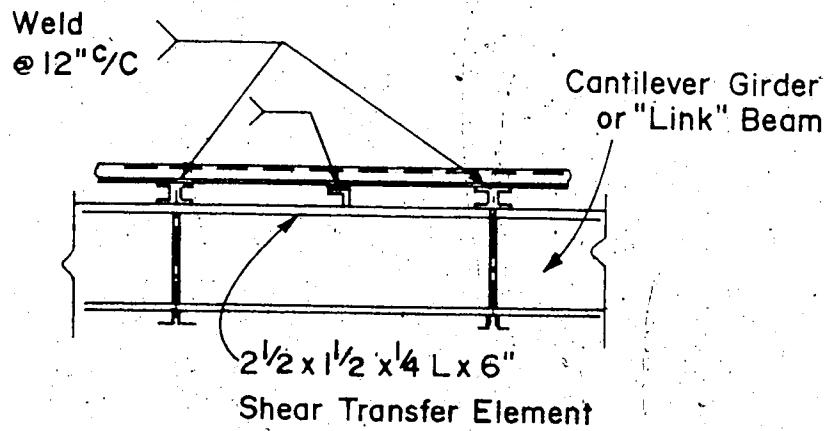
- connections, scrap, drafting and

fabrication

Nil



(a) Trimmer Angle



(b) Shear Transfer Element

FIGURE 3.10 REQUIREMENTS FOR DIAPHRAGM ACTION

- receiving, shipping and freight		
1.72 tons x \$35.00/ton	=	<u>\$60.20</u>
	sub-total	= \$610.20
- sales and administration overhead (10%)	=	\$61.00
- erection		
7 bays x 4 m.h./bay x \$20.00/m.h.	=	<u>\$560.00</u>
	total	= \$1231.20

## 2. Cost of Ensuring Diaphragm Action

- cost of 30 feet of L 2-1/2 x 2-1/2 x 1/4 for shear transfer elements:		
123 lbs. x \$0.15/lb.	=	\$18.45
- connections, scrap, drafting		Nil
- fabrication (10 man-minutes each)		
60 elements x 10 m.m. x 1/60 m.h./m.m. x \$20.00/m.h.	=	\$200.00
- receiving, shipping and freight		<u>Nil</u>
	sub-total	= \$218.45
- sales and administration overhead (10%)	=	\$21.85
- erection (connecting roof deck)		
$\frac{200 \times 120 \text{ ft.}^2}{12000 \text{ ft.}^2/\text{day}}$ x 32 m.h./day		
x \$20.00/m.h. x 0.1	=	<u>\$128.00</u>
	total	= \$368.30

Comparing this with the cost of the direct acting bracing system (\$1231.20) it can be seen that the cost savings is substantial.

In some situations, however, it may not be advisable to rely on diaphragm action, and a direct acting bracing system should be used even though it is more expensive. This might be the case, for example, if the roof deck contains a large number of openings for mechanical units.

---

### 3.7.2 Lateral Load Transfer in the Plane of the End Walls

Lateral load can be transferred from the roof to the foundation through a direct acting bracing scheme, concrete or masonry shear walls, cladding, semi-rigid frame action, or some combination of these. A cost comparison has not been made to determine which method will result in overall building economy, however the bracing scheme shown in Figure 3.7(a) is expected to be among the least costly, and will be selected for the example building. This will be used in combination with diaphragm action of the wall cladding to resist the lateral loads.

### 3.8 Summary

Although the most important consideration in determining the lay-out of an industrial building is that it not interfere with the intended use of the building, cost is also a basic factor. It is shown that when the labor cost is evaluated separately in the cost estimate the cost of structural steel per square foot of roof area is relatively constant over a large range of clear spans. This is not to say that the building cost is independent

of clear span, since costs associated with other trades have not been considered. This study concentrates only on the cost of structural steel in the building.

CHAPTER IV  
FLEXURAL MEMBERS

4.1 Introduction

Flexural members are those members in a structure that are required to resist loads acting perpendicular to their longitudinal axes. As discussed in CHAPTER II, the governing load combination for flexural members in the roof is usually dead load plus snow load. In locations where the snow load is light, however, dead load plus rain load can sometimes govern the design. Another load combination, dead load plus wind suction, should also be investigated in designing members near the perimeter of the roof if the snow load is light.

The names given to the various types of flexural members depend on their location in the building and on their specific functions. In the structure shown in Figure 4.1, for example, interior cantilever girders extend over the supporting columns. Exterior cantilever girders, located in the end bays, extend over the columns at one end only. These girders are connected by simply supported "link" beams. Open web steel joists span in a direction perpendicular to the main framing scheme and are welded to the top flange of the girder system. Special open web steel joists, spanning between columns, prevent lateral displacement and twisting of the girders at the column lines, and also brace the tops of the columns. Perimeter beams span between exterior



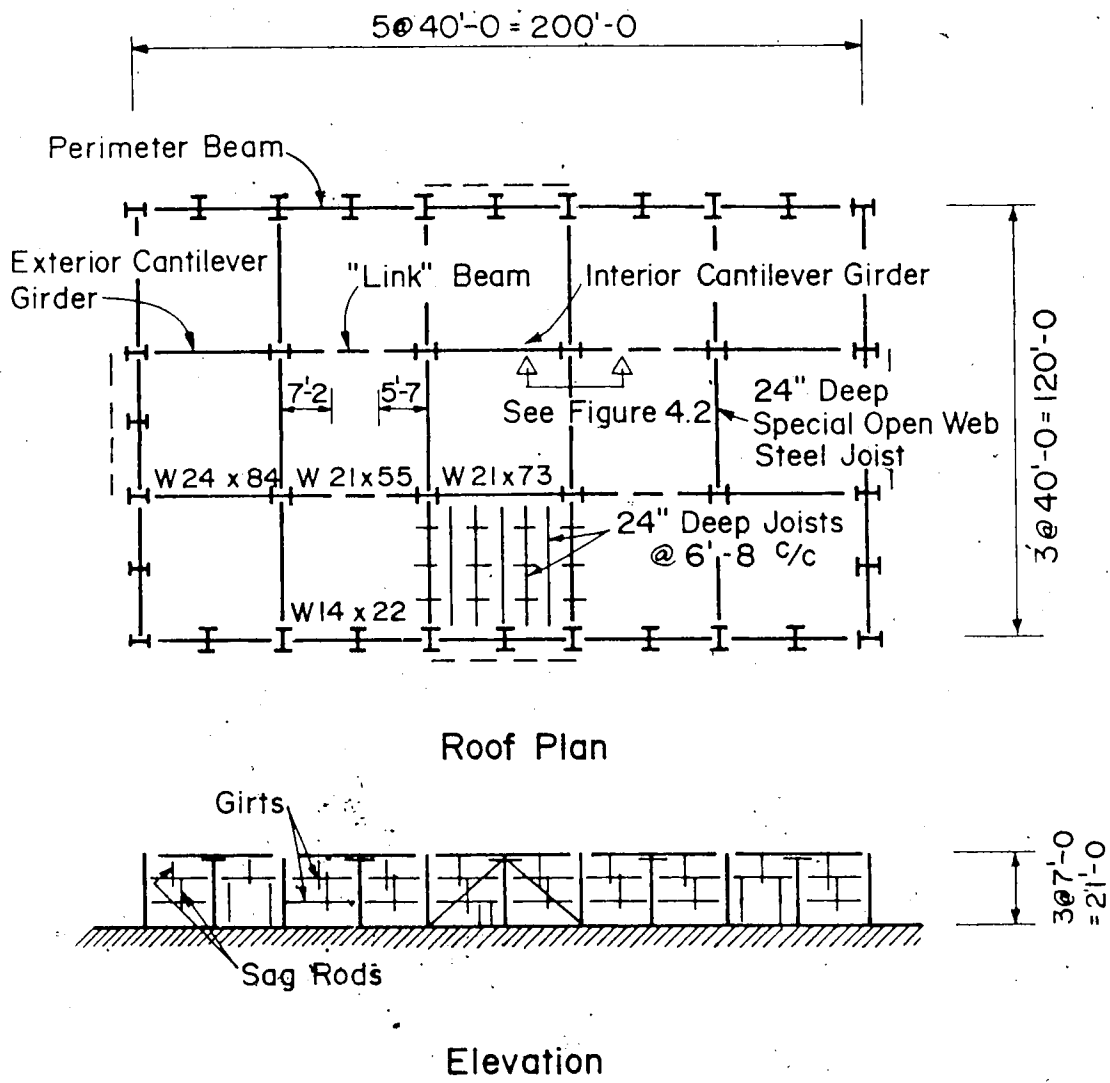


FIGURE 4.1 FLEXURAL MEMBERS

columns and are continuous over wind posts located at the mid-span points. Roof deck is welded to the top flanges of the joists. On the sides of the building, girts and corrugated metal wall cladding resist wind loads. If the wind post-to-perimeter beam connections are detailed so as to not transmit axial loads (Figure 3.8(b)), the wind posts behave as vertical flexural members.

In some situations flexural members, such as lintels (members supporting a wall above a window or door opening), may also be subjected to torsional moments caused by eccentric loads. If the tops of the wind posts are not braced (see Figure 3.6) the perimeter beams are also subjected to torsion. The design of members subject to combined bending and torsion is not considered in this dissertation<sup>(30)</sup>. The design of plate girders<sup>(31)</sup>, or girders requiring large web openings for the passage of pipes or ductwork<sup>(32)</sup>, plastically designed continuous girder systems<sup>(33)</sup>, and various problems associated with design of trusses and joists are also not considered.

In the first part of the chapter the behaviour of laterally supported and laterally unsupported girders is reviewed. Design procedures are then presented for girders used in cantilever roof framing schemes for which specific design information is not available. The design of several other types of flexural members is then considered. Following this, lateral bracing, deflection and camber requirements are reviewed and examined. Finally, the design of wide flange girder webs to prevent shear failures and the design of bearing stiffeners is discussed.

#### 4.2 Behaviour of Laterally Supported Members

If the compression flange of the girder is braced so that only in-plane deflections can occur, failure is initiated by local buckling of one of the plate elements comprising the cross section. If both the flange and web plates are stocky enough,  $M_p$ , the plastic moment capacity, can be reached and maintained at least until strain-hardening occurs. This section is designated a Class 1 section. A Class 2 section can also reach the plastic moment, but local buckling occurs shortly thereafter. A Class 3 section is only capable of reaching  $M_y$ , the yield moment, before local buckling occurs, while a Class 4 section fails by local buckling before reaching the yield moment. The limiting width-to-thickness ratios for the flange and web plates for all four classes of sections are given in Reference (1).

Most rolled wide flange sections used as beams and girders in light industrial buildings are usually either Class 1 or 2 sections for which the factored moment resistance is computed from:

$$M_r = \phi M_p = \phi Z F_y \quad (4.1)$$

where  $\phi = 0.9$  is the performance factor discussed in Section 2.2 and  $Z$  is the plastic section modulus. Occasionally a Class 3 section is used for which:

$$M_r = \phi M_y = \phi S F_y \quad (4.2)$$

where  $S$  is the elastic section modulus. Light sections used as girts are usually Class 4 sections. Although behaviour of girts will be discussed in a later section, detailed design of these members will not be considered.

#### 4.3 Behaviour of Laterally Unsupported Members

If a member is not braced at sufficiently close intervals along its length, the possibility of out-of-plane, or lateral-torsional, instability exists. Certain types of flexural members are more likely to fail in this way than other types. A wide flange member used as a girder is usually not torsionally strong, and the member has a tendency to buckle by twisting and deflecting laterally before the in-plane strength described in section 4.2 can be reached.

The elastic buckling strength of a girder is controlled by a number of factors, including shape of the cross section, member length, end restraint, restraint along the length of the member, load pattern, point of load application, and modulus of elasticity, but not on the yield stress of the material. In Reference (34) a general equation is derived for the elastic buckling moment  $M_u$  of a laterally unsupported wide flange section or channel subjected to end moment or transverse forces in the plane of the web:

$$M_u = \frac{\pi}{\omega L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} \quad (4.3)$$

where

$$\omega = \frac{K \sqrt{1 + \frac{\pi^2 EC_w}{GJL^2}}}{C_1 \left( \sqrt{1 + \frac{\pi^2 EC_w}{GJ(KL)^2}} \left( \frac{C_2}{2} + 1 \right) \pm \pi C_2 \sqrt{\frac{EC_w}{GJ(KL)^2}} \right)} \quad (4.4)$$

is an equivalent uniform moment coefficient,  $G$  is the shear modulus,  $I_y$  is the moment of inertia of the cross section about the weak axis,  $K$  is an "effective length" coefficient,  $L$  is the length of the girder between points of support,  $C_w$  is the torsion warping constant, and  $J$  is the St. Venant torsion constant. The coefficients  $C_1$ ,  $C_2$  and  $K$  depend primarily on the loading and support conditions. The "+" sign is positive for bottom flange loading, and negative for top flange loading. For centroidal loading, or for end moments only,  $C_2$  is equal to zero.

For the case of a simply supported member subjected to uniform bending by equal and opposite end moments,  $C_1$  and  $K$  are equal to unity,  $C_2$  is equal to zero, so that  $\omega$  is also equal to unity. Numerical values of these coefficients for several other loading and support conditions are given in Reference (34), and in a slightly different form, in References (35), (36) and (37).

For I-shaped members the elastic buckling moment given by Equation (4.3) can be conservatively taken as (1,38):

$$M_u = \frac{S}{\omega} \sqrt{\sigma_1^2 + \sigma_2^2} \quad (4.5)$$

where

$$\sigma_1 = \frac{20,000}{Ld/A_f} \quad (4.6)$$

$$\sigma_2 = \frac{250,000}{L/r_t)^2} \quad (4.7)$$

in which

$d$  = overall depth of the section,

$A_f$  = compression flange area, and

$r_t$  = radius of gyration about its axis of symmetry of a tee section comprising the compression flange and one sixth of the web.

Assuming that the member fails while it is still elastic, the factored moment resistance is(1):

$$M_r = \phi M_u \quad (4.8)$$

However, if the elastic buckling moment exceeds  $2/3 M_p$  for Class 1 and 2 sections, or  $2/3 M_y$  for Class 3 sections, the assumption of elastic action is no longer valid(2). When failure occurs through inelastic lateral-torsional instability an estimate of the factored moment resistance can be obtained from the empirical equations(1):

$$M_r = 1.15 \phi M_p \left(1 - \frac{0.28 M_p}{M_u}\right) \quad (4.9)$$

but not greater than  $\phi M_p$

for Class 1 and Class 2 sections, and

$$M_y = 1.15 \phi M_y \left( 1 - \frac{0.28 M_y}{M_u} \right) \quad (4.10)$$

but not greater than  $\phi M_y$

for Class 3 sections.

#### 4.4 Lateral Buckling in Cantilever Construction

The main framing scheme in light industrial buildings often consists of girders cantilevered over the supporting columns and separated by simply supported "link beams" as shown in Figures 4.1 and 4.2. Open web steel joists are welded to the top flanges of the girders. In turn, steel roof deck is welded to the joists. In this situation the (lower) compression flanges of the girders, adjacent to the columns, are unbraced except at the columns and premature failure can occur as a result of lateral-torsional instability. Two questions of concern to the designer are: (a) what should the overhang lengths be, and (b) for these overhangs how can the carrying capacities of the girders be determined?

Overhang lengths considered optimum by different designers can vary greatly. Overhang lengths recommended for interior girders<sup>(28)</sup> are approximately 14 to 15 percent of the distances between column lines, however 17 percent appears to be



FIGURE 4.2    DETAIL OF INTERIOR CANTILEVER GIRDER OVERHANG



more usual while longer overhangs (up to 25 percent) are not uncommon. Often the overhang lengths are selected so that the cantilever girder to link beam splices are located where the inflection points would be in a continuous girder system subjected to a uniformly distributed load.

After the overhang lengths have been selected, the question of design must be considered. Formulas are available to determine the lateral buckling moment of a cantilever beam fixed at the support and subjected to a point force at the free end. However, in a light industrial building the supported end of the cantilever is not completely restrained by the remaining portion of the girder. An approximate but conservative procedure recommended in Reference (34) is to consider the warping constant  $C_w$  to be zero in the buckling formula for a cantilever beam. If the reaction from the link beam to the cantilever girder is located a distance "c" above the centroid of the cantilever girder, the relevant buckling formula is (39):

$$M_u = 4.0 \sqrt{\frac{EI_y GJ}{\alpha L}} \left( 1 - \frac{c}{\alpha L} \sqrt{\frac{EI_y}{GJ}} \right) \quad (4.11)$$

where

$M_u$  = elastic buckling moment at the base of the cantilever,

$\alpha$  = cantilever overhang length divided by the distance  $L$  between columns.

In this section the results of a series of finite element computer studies to determine the overhang lengths corresponding to maximum load carrying capacity are presented for both interior and exterior cantilever girders. The load carrying capacity of a particular wide flange member depends on the girder span, the overhang length, the number of joists along the span, the location of the link beam reaction, the loading pattern (ratio of live load to dead load), the specified yield stress of the steel, the magnitude and distribution of residual stress, the beneficial effects of strain hardening, and the strength and stiffness of the roof deck. For short overhangs the load carrying capacity is equal to the maximum capacity of the member cross section (as if it were completely braced), providing that the roof deck possesses adequate strength and stiffness. For longer overhangs the carrying capacity is governed by lateral-torsional instability involving both the overhanging portions of the girder and the main span.

The computer program used in the analysis is a modified version of an eigenvalue (buckling) program described in Reference (4). The computer program accounts for all of the variables listed above with the exception of roof deck strength since it is a buckling, not a large deformation, computer program, and strain hardening. The roof deck diaphragm is assumed to be semi-flexible, with a flexibility factor,  $F$ , of  $90 \times 10^{-6}$  in./lb. (29). The link beam reactions are assumed to act at points located halfway between the centroid of the cantilever girder cross section and the top flange (ie.,  $c = d/4$ ). The specified yield stress of the steel is assumed to be 44 ksi.

#### 4.4.1 Interior Cantilever Girders

An upper limit on the overhang ratio  $\alpha$  of an interior cantilever girder can be determined if it is assumed that the full capacity of the cross section can be reached. As discussed in Section 2.3.2, the roof must be capable of supporting the more severe of either (9):

1. dead load plus snow load over the total roof area or
2. dead load plus snow load over the total roof area except that one half of the snow load is removed from one particular portion of the roof area.

The overhang ratio can then be selected so that the maximum positive bending moment in the member under the most severe loading condition is equal to the maximum negative bending moment under the most severe loading condition for this case. For a cantilever girder, full dead load plus snow load never produces the most severe effects. The longest negative moment region occurs when half of the snow load is removed from the main span, and the largest positive moment occurs when half of the snow load is removed from one overhang.

Consider as a simple example a series of interior cantilever girders of lengths  $L$  with overhang lengths  $\alpha L$ , separated by simply supported link beams. Open web steel joists are evenly

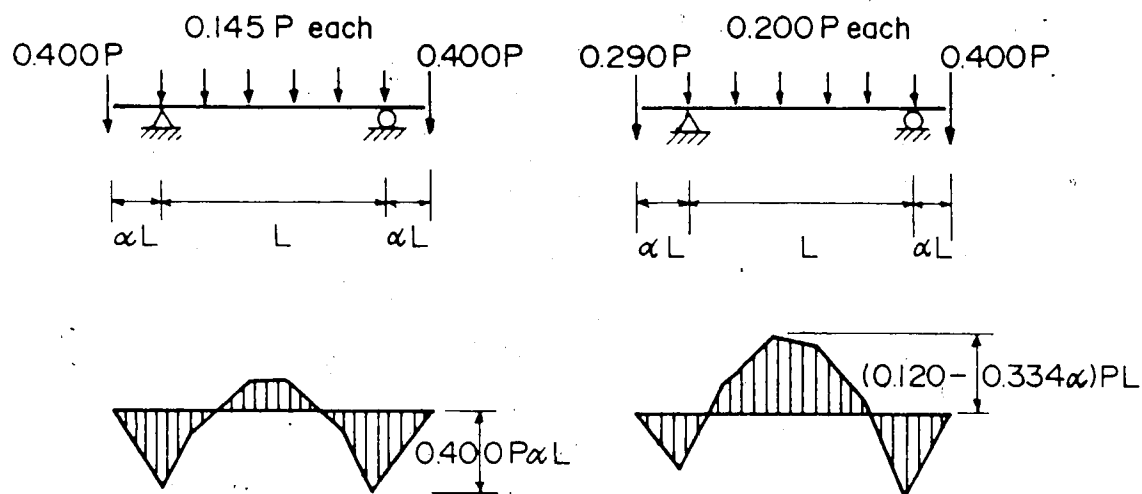
spaced at  $0.20 L$  center-to-center. The structure is assumed to be located in a region of relatively light snowfall, where the specified snow load is equal to the specified dead load. Denoting the factored dead load plus the full factored snow load multiplied by the bay area as  $P$ , the maximum factored moments are those shown in Figures 4.3(a) and (b). Equating the maximum negative moment to the maximum positive moment:

$$0.400 P_{\alpha}L = (0.120 - 0.334\alpha) PL \quad (4.12)$$

or  $\alpha = 0.163$ .

Proceeding in a similar fashion<sup>(40)</sup>  $\alpha$  can be found for a different number of joists along the span, and for different live load to dead load ratios. However, it appears to be relatively insensitive to changes in these variables and to be constant at approximately 0.16 in most cases.

However, it is not reasonable to assume that the full cross section strength is available unless the member is laterally and torsionally braced at sufficiently close intervals. Results of a series of preliminary studies<sup>(40,41)</sup> indicate that for a short overhang ratio the girder carrying capacity is unaffected by lateral-torsional instability, and that the carrying capacity of the girder increases as the overhang length increases. For a longer overhang ratio the girder capacity is reduced as a result of member instability, and can be less than that of a member with the same cross section used as a simply supported girder. The



(a) Longest Negative Moment Region

(b) Largest Positive Moment

FIGURE 4.3 DESIGN CONDITIONS FOR AN INTERIOR CANTILEVER GIRDER ( $L=D$ )

overhang ratio corresponding to the maximum load carrying capacity is defined as the optimum overhang length.

The results of the preliminary studies also indicate that, except for a very short overhang ratio, lateral-torsional instability of an interior cantilever girder can be expected to occur while the member is elastic; that is, while the maximum negative moment given in Figure 4.3(a) is less than  $2/3 M_p$  for a Class 1 or a Class 2 section. Thus Equation (4.3) can be used to determine the member carrying capacity. Substitution of the negative moment  $0.4 P\alpha L$  into Equation (4.3) and re-arranging leads to:

$$\sqrt{\frac{PL^2}{EI_y GJ}} = 2.5 \frac{\pi}{\omega\alpha} \sqrt{1 + \frac{\pi^2 EC_w}{GJL^2}} \quad (4.13)$$

This function, determined from a finite element analysis, is shown by the solid lines in Figure 4.4.

Failure occurs under the loading condition shown in Figure 4.3(b), which causes the maximum positive moment, when the plastic moment capacity of the cross section is reached - that is, when

$$(0.120 - 0.334\alpha) PL = M_p \quad (4.14)$$

Substituting

$$M_p = fSF_y \quad (4.15)$$

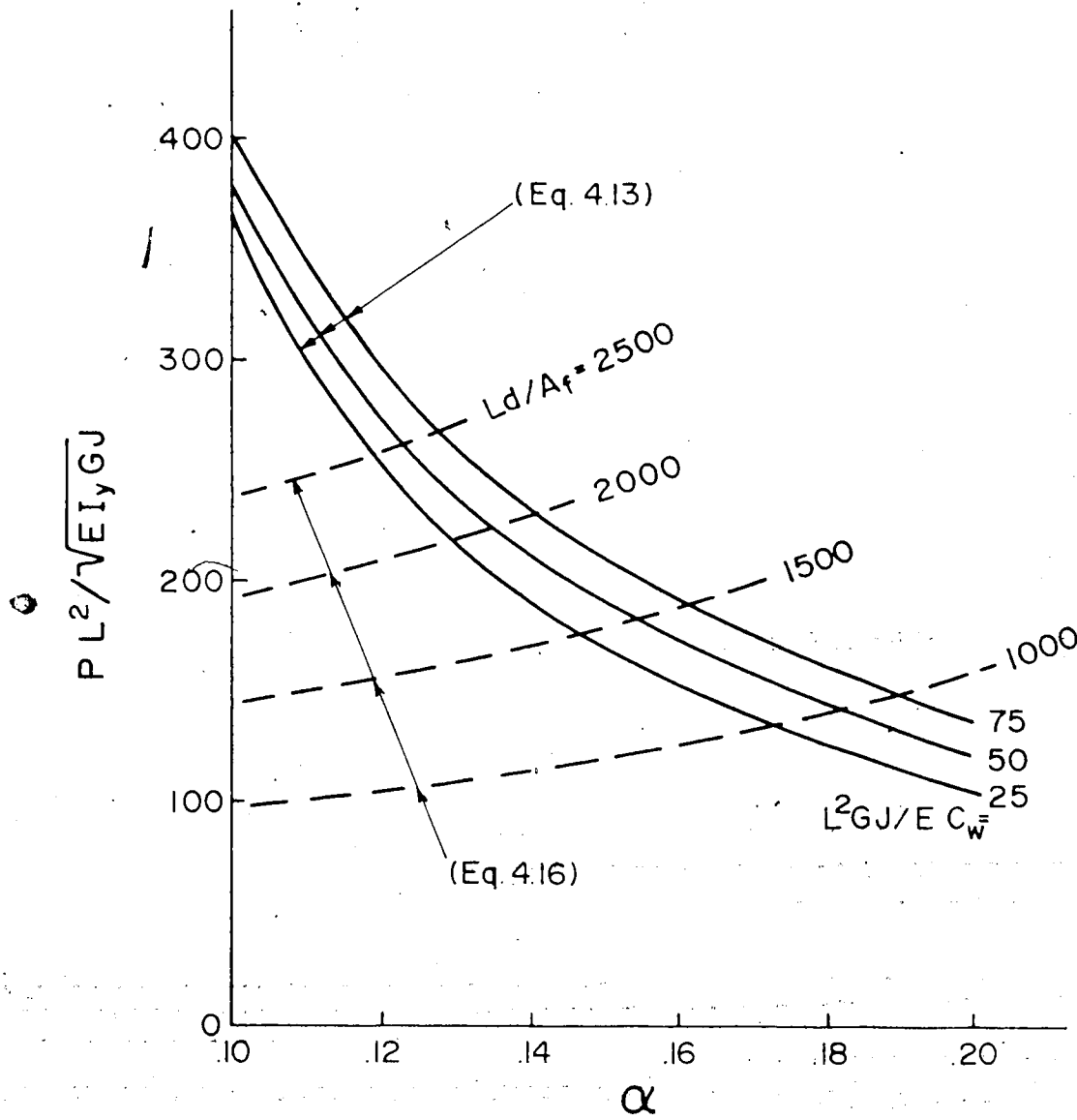


FIGURE 4.4 PLOTS OF EQUATIONS (4.13) AND (4.16) FOR THE INTERIOR CANTILEVER GIRDER SHOWN IN FIGURE 4.3

where  $f \approx 1.10$  is the shape factor for a wide flange cross section<sup>(38)</sup> into Equation (4.14), multiplying both sides by  $L/\sqrt{EI_y GJ}$ , using the approximate expressions for the cross section properties given in Reference (38), and substituting  $F_y = 44$  ksi leads to:

$$\frac{PL^2}{\sqrt{EI_y GJ}} = \frac{0.008 Ld/A_f}{0.120 - 0.334\alpha} \quad (4.16)$$

Equation (4.16) is shown by the broken lines in Figure 4.4.

For a given girder and column spacing,  $Ld/A_f$ ,  $L^2 GJ/EC_w$ , and  $L^2 \sqrt{EI_y GJ}$  are constant. Under these conditions Figure 4.4 can be interpreted as a plot of roof load  $P$  versus the overhang ratio  $\alpha$ . The girder is used most efficiently when the overhang length is such that  $P$  is a maximum. Thus the optimum value of  $\alpha$  corresponds to the intersection point of the strength curve (broken line) with the stability curve (solid line), and is therefore a function of the non-dimensional parameters  $Ld/A_f$  and  $L^2 GJ/EC_w$ . Figure 4.5, constructed from Figure 4.4, plots  $\alpha$  versus the  $Ld/A_f$  ratio for three different values of  $L^2 GJ/EC_w$ . Also shown in Figure 4.5 is the equation

$$\alpha = 0.20 - \frac{Ld/A_f}{31000} \quad (4.17)$$

but less than or equal to 0.16



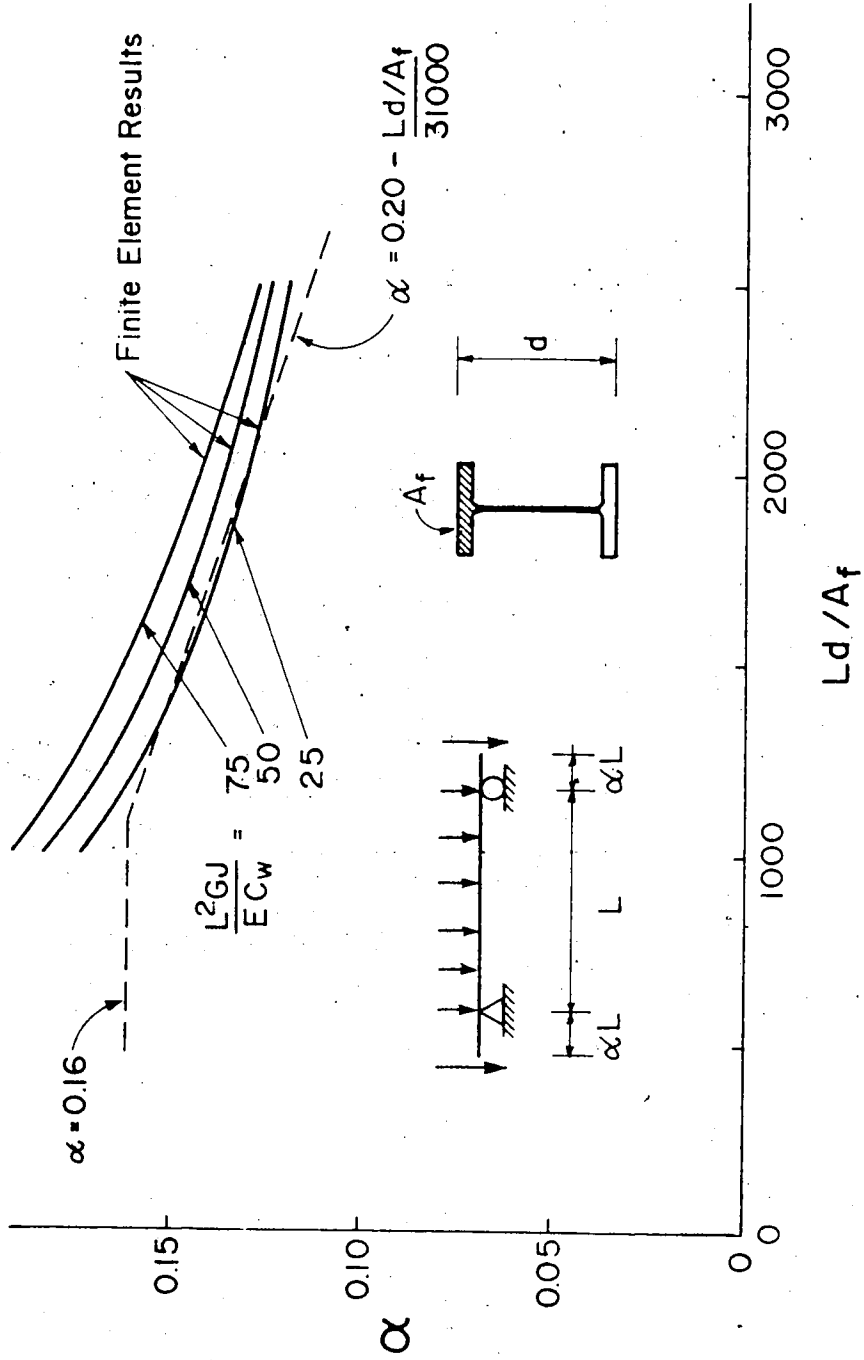


FIGURE 4.5 OPTIMUM OVERHANG RATIOS FOR INTERIOR CANTILEVER GIRDERS

which provides a conservative estimate of the predicted overhang values. An upper limit of  $\alpha = 0.16$  is selected to be consistent with the analysis at the beginning of this section, which assumed that the full cross section strength was available to resist negative bending. Similar analyses have been performed to determine the effect of the number of joists along the span, the live load to dead load ratio, and the class of section. These results are summarized in Figure 4.6 for an  $L^2GJ/EC_w$  ratio of 50. Also shown is Equation (4.17). The effects of these variables appear to be minor.

#### 4.4.2 Exterior Cantilever Girders

A girder located in an end bay overhangs at one end only, as shown in Figure 4.1. It can be expected, therefore, that the overhang ratio corresponding to the maximum load carrying capacity of an exterior girder is greater than that for an interior girder, which overhangs on both ends. Using a procedure similar to that described above, finite element studies were done for the particular girder system shown in the insert to Figure 4.7. Also as shown in Figure 4.7, the empirical equation

$$\alpha = 0.22 - \frac{Ld/A_f}{50000} \quad (4.18)$$

but less than or equal to 0.21

provides a close fit to the finite element results.

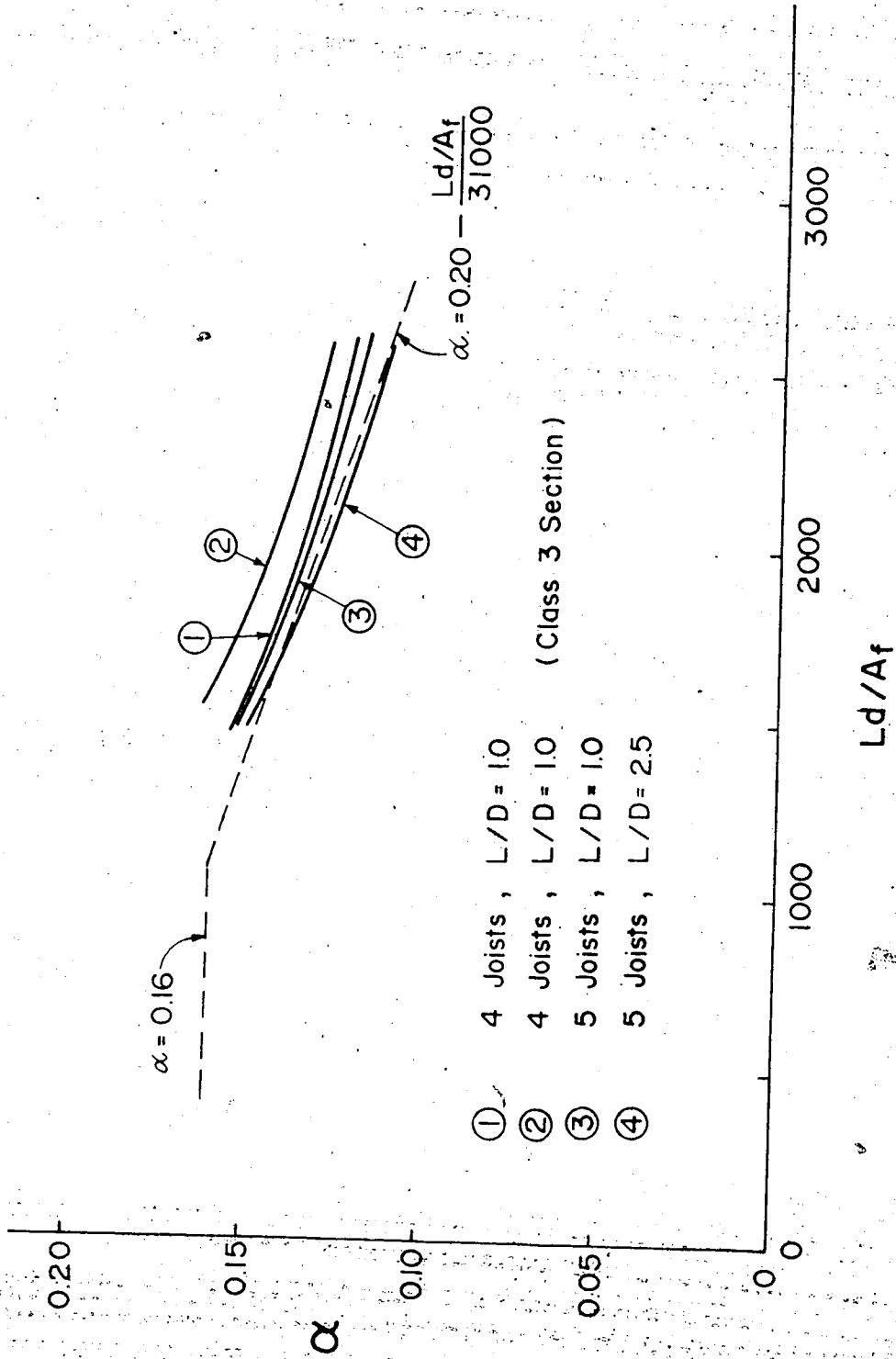


FIGURE 4.6 EFFECT OF OTHER VARIABLES ON OPTIMUM OVERHANG RATIOS

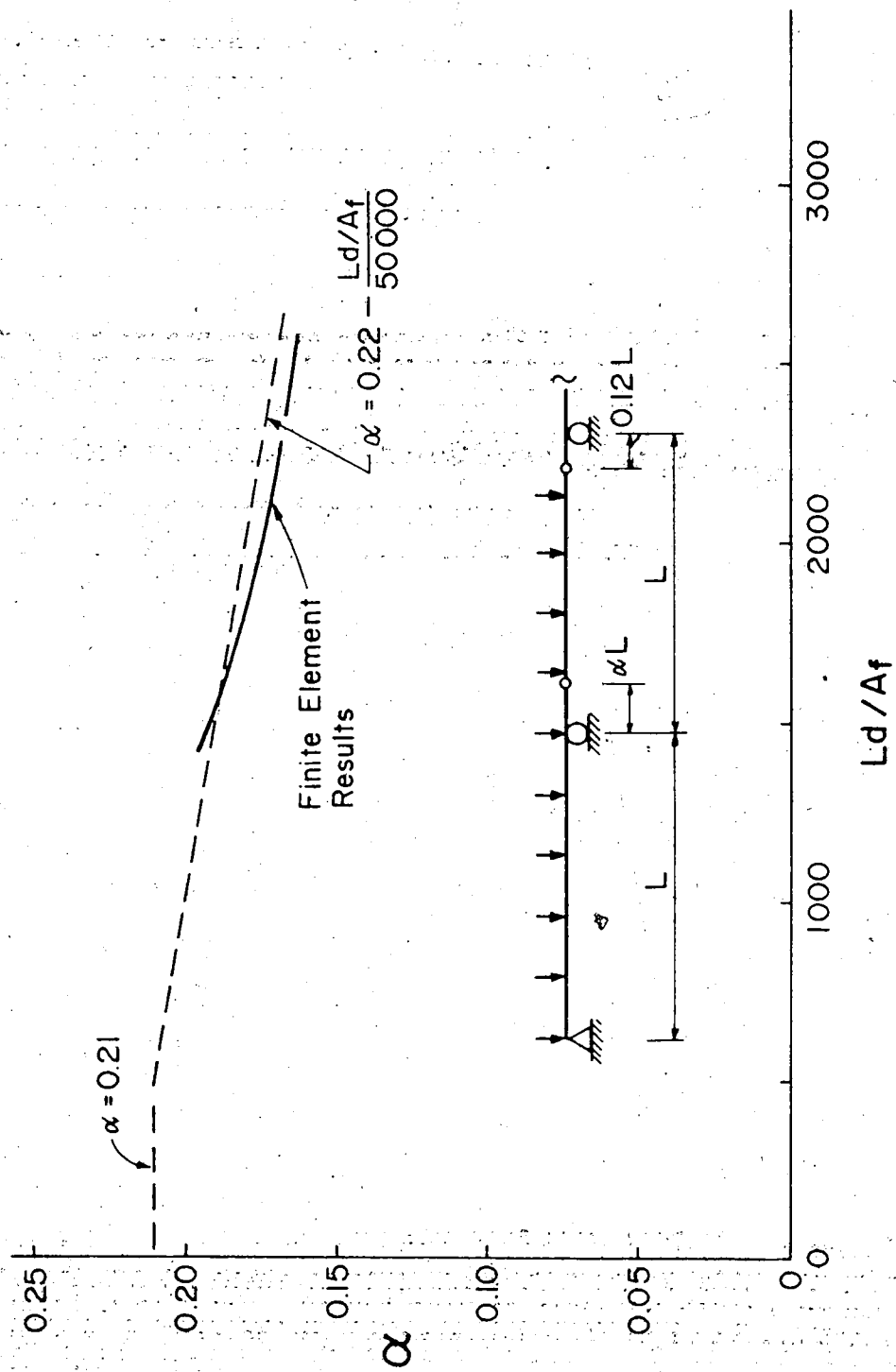


FIGURE 4.7 OPTIMUM OVERHANG RATIOS FOR EXTERIOR CANTILEVER GIRDERS

The upper limit of 0.21 is based on the assumption that lateral-torsional instability does not occur, and that the full cross section strength can be reached.

---

#### Example 4.1

##### Given

The roof dead load and snow load for the structure shown in Figure 4.1 were determined in Examples 2.1 and 2.2 as 31.5 psf and 48 psf, respectively. Select a wide flange section for the interior cantilever girder. Use G40.21 44W steel ( $F_y = 44$  ksi).

##### Solution

As discussed earlier in this section, if the overhang ratio  $\alpha$  is less than or equal to the "optimum value" given by Equation (4.17), the load pattern governing design of the girder consists of full dead load and full snow load, except that half of the snow load is removed from one overhang. This produces the largest positive moment in the cantilever girder. In this case an overhang ratio of  $\alpha = 0.14$  was selected on the basis of preliminary calculations (see Section 3.2). Equation (4.17) must therefore be used in a slightly different form. The overhang is optimum providing:

$$\frac{d}{A_f} \leq (0.20 - \alpha) \frac{31000}{L} \quad (\text{Eq. 4.17})$$

$$= (0.20 - 0.14) \times \frac{31000}{40 \times 12}$$

$$= 3.88 \text{ in.}^{-1}$$

Try a W21x73

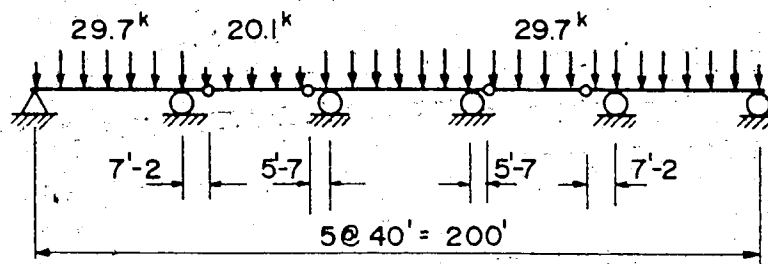
Relevant cross section properties and dimensions of a W21x73 section are listed in Reference (5):

$$d/A_f = 3.46 \text{ in.}^{-1}$$

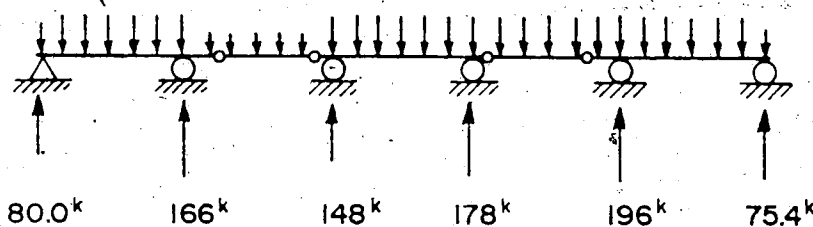
$$Z_x = 172 \text{ in.}^3$$

Since the  $d/A_f$  ratio provided is less than that required, only the load pattern producing the maximum positive moment need be investigated - that is, failure by lateral-torsional instability of the overhang is not a relevant failure mode. Note that the purpose of limiting the  $d/A_f$  ratio in this way is to ensure that the section is sufficiently stocky so that lateral-torsional instability cannot occur.

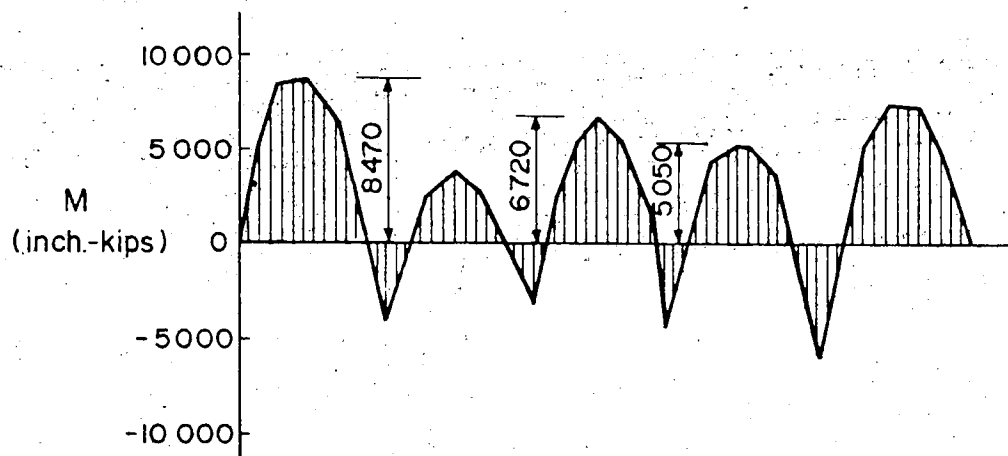
The distributed roof loads are transmitted to the girder as a series of point forces at the joist locations, as shown in Figure 4.8. Each point force is equal to the factored roof load multiplied by the tributary area of the joist. In the region of full snow load these forces are:



(a) Geometry and Loads



(b) Reactions



(c) Moments

FIGURE 4.8 LOADS ACTING ON GIRDER SYSTEM

$$\begin{aligned} & \alpha_D D + \gamma \psi \alpha_L L \\ & = [1.25 \times 31.5 + 1.0 \times 1.0 \times 1.5 \times 48] \times \frac{40 \times 6.67}{1000} \\ & = 29.7 \text{ kips} \end{aligned}$$

and in the region of half snow load they are

$$\begin{aligned} & \alpha_D D + \gamma \psi \alpha_L \frac{L}{2} \\ & = [1.25 \times 31.5 + 1.0 \times 1.0 \times 1.5 \times \frac{48}{2}] \times \frac{40 \times 6.67}{1000} \\ & = 20.1 \text{ kips.} \end{aligned}$$

The corresponding bending moments are also shown in Figure 4.8.

Results of calculations (not shown) indicate that a W21x73 section is a Class 1 section in 44W steel, therefore the factored moment resistance, assuming full lateral support by the roof deck is:

$$\begin{aligned} M_r &= \phi M_p = \phi Z F_y && \text{(Eq. 4.1)} \\ &= 0.9 \times 172 \times 44 \\ &= 6811 \text{ inch-kips} \end{aligned}$$

Since this exceeds the 6720 inch-kips required, the section is satisfactory. It is also the lightest section that can be selected.

Use a W21x73



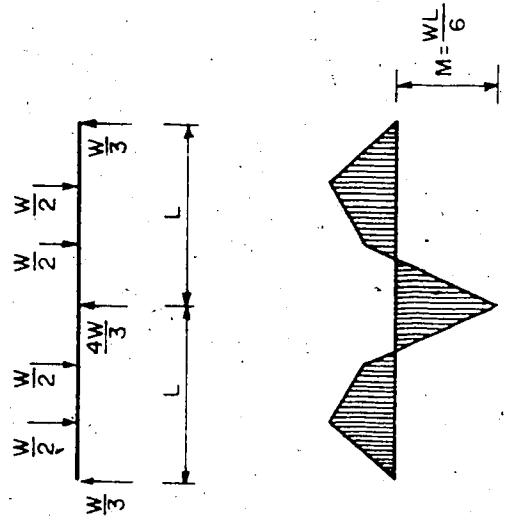
Although the design calculations are not shown for the exterior cantilever girder, the procedure followed is the same. The W24x84 section indicated is the lightest wide flange section that can be selected.

#### 4.5 Perimeter Roof Beams

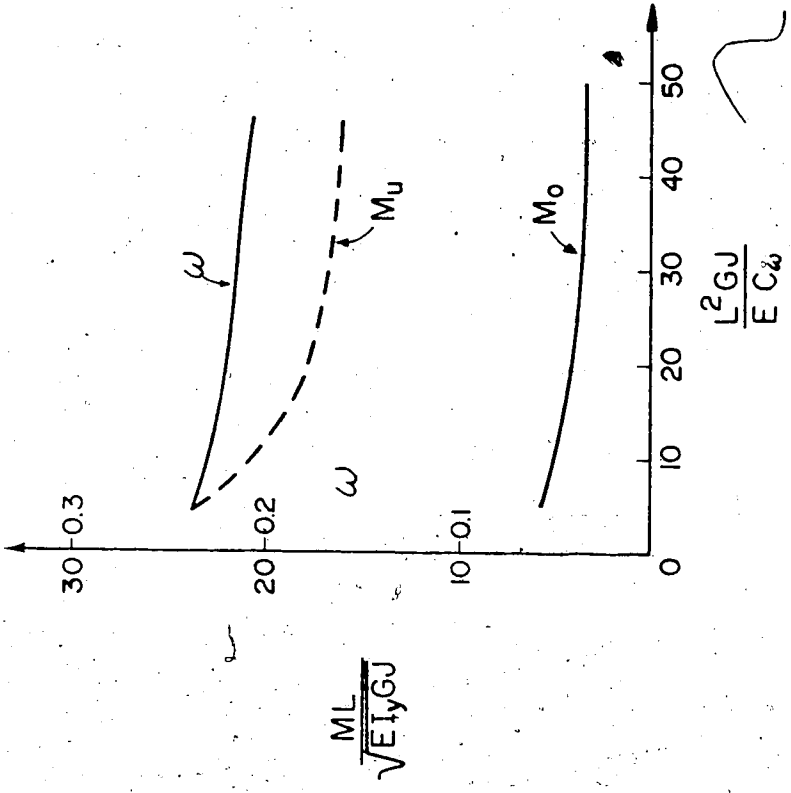
When a perimeter roof beam is simply supported at the ends and continuous over an intermediate wind post, as shown in Figure 4.1, it is subjected to negative bending moments in the vicinity of the interior support. The beam is braced along the top flange by the roof deck diaphragm and lateral deflections and twisting rotations are prevented at the ends and at the wind post. However, since the member is not completely braced along the compression flange in the vicinity of the interior support, failure can occur through lateral-torsional instability.

The problem analyzed is that shown in Figure 4.9(a). The member is subject to top flange loading, and is partially braced as described above. The elastic lateral-torsional buckling moment  $M_U$  can be determined from Equation (4.3) providing the equivalent uniform moment coefficient  $\omega$  is known. The factored moment resistance can then be determined from Equations (4.8) to (4.10).

From an examination of Equation (4.4) it can be seen that for a given loading and support condition the equivalent uniform moment coefficient depends only on the non-dimensional



(a) Loading Pattern



(b) Finite Element Analysis Results

FIGURE 4.9 PERIMETER ROOF BEAM ANALYSIS

slenderness parameter  $L^2GJ/EC_w$ . This relationship can be conveniently determined by using an elastic finite element computer program.

For a given slenderness parameter the equivalent uniform moment coefficient is equal to the buckling moment  $M_O$  for a simply supported member subjected to uniform bending divided by the maximum moment  $M_u$  in the perimeter beam when the applied loads are such that buckling occurs. This study was performed for slenderness parameters varying from 5 to 50, and the results are summarized in Figure 4.9(b). As indicated in this Figure, the equivalent uniform moment coefficient varies from approximately 0.21 to 0.24. For design, a value of 0.2 is therefore conservative.

It is expected that the range of the slenderness parameter selected will cover most situations encountered in practice. For example, for a W14x22 section with a span  $L$  of 20 feet, the parameter is 15.1, while for a W10x21 section it is 19.0.

---

Example 4.2Given

Select a wide flange section for the perimeter beam shown in Figures 4.1 and 4.9. The maximum factored negative bending moment can be shown to be 1180 inch-kips. Use G40.21 44W steel ( $F_y = 44$  ksi).

SolutionTry a W14x22

Relevant cross section properties and dimensions of a W14x22 section are listed in Reference (5):

$$Z = 33.1 \text{ in.}^3$$

$$J = .22 \text{ in.}^4$$

$$I_y = 7.0 \text{ in.}^4$$

$$C_w = 314 \text{ in.}^6$$

Results of preliminary calculations indicated that this is a Class 1 section in 44W steel. The factored moment resistance is:

$$\begin{aligned}
 M_r &= \phi M_p = \phi Z F_y && \text{(Eq. 4.1)} \\
 &= 0.9 \times 33.1 \times 44 \\
 &= 1310 \text{ inch-kips}
 \end{aligned}$$

However, since the compression flange in the vicinity of the wind post is laterally unsupported, the possibility of lateral-torsional buckling should also be investigated. The elastic lateral-torsional buckling moment is:

$$\begin{aligned}
 M_u &= \frac{\pi}{\omega L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} && \text{(Eq. 4.3)} \\
 &= \frac{\pi}{0.2 \times 20 \times 12} \sqrt{30000 \times 7.0 \times 11500 \times .22 + \left(\frac{\pi \times 30000}{20 \times 12}\right)^2 \times 7.0 \times 314} \\
 &= 1930 \text{ inch-kips}
 \end{aligned}$$

Since this exceeds 2/3 of  $M_p$ , inelastic action occurs and:

$$\begin{aligned}
 M_r &= 1.15 \phi M_p \left(1 - \frac{0.28 M_p}{M_u}\right) && \text{(Eq. 4.9)} \\
 &= 1.15 \times 1310 \left(1 - \frac{0.28 \times 33.1 \times 44}{1930}\right) \\
 &= 1190 \text{ inch-kips}
 \end{aligned}$$

Since this exceeds the factored moment of 1180 inch-kips the section is adequate. It is also the lightest section that can be selected.

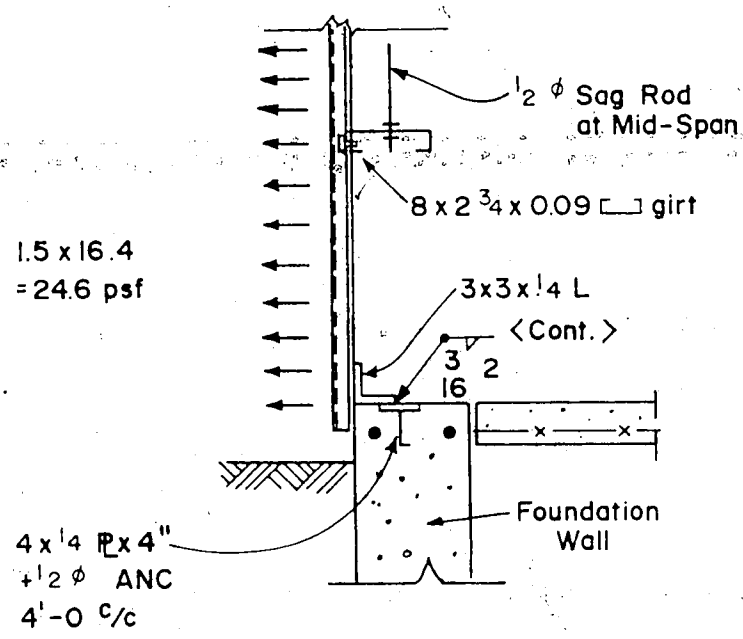
Use a W14x22

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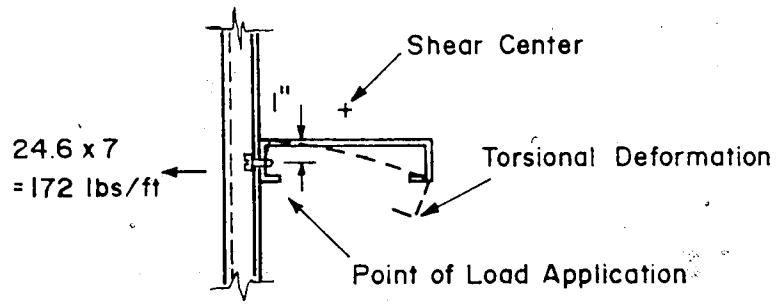
#### 4.6 Girts

Girts are subjected to the effects of wind action, their self weight, and depending on the method of attachment, may also have to support the weight of the wall cladding. For the detail shown in Figure 4.10(a), however, it can be assumed that the weight of the cladding is transferred directly by bearing of the trimmer angle against the foundation. It can also be assumed that the effect of the self weight of the girts is small since they are partially supported by the wall cladding and the sag rods. Thus the primary loading is wind loading. Wind loading on girts was discussed in Example 2.4, where it was shown that the most severe loading is wind suction, illustrated in Figure 4.10(a).

As illustrated in Figure 4.10(b), the point of load application and the shear center do not coincide. Thus the possibility of lateral-torsional buckling (that is, a sudden snap from a perfectly straight configuration into a buckled configuration) is precluded since torsional deformations occur from the onset of loading. Nevertheless, current design procedures are based on buckling as the failure mode (42).



(a)



(b)

FIGURE 4.10 GIRT LOADING

Sections used as girts are usually cold-formed sections having a specified yield stress of approximately 50 to 55 ksi. Design generally consists of first computing the loads on the girts and then selecting appropriate sections from a manufacturer's catalogue. The 8" x 2.75" x 0.090" girt shown in Figure 4.10 was chosen in this way, using a ten foot unsupported length since it has been shown that the sag rod is effective in acting as a brace (43).

Recently, however, research has aimed at treating the problem as one of combined bending and torsion instead of buckling. (43,44,45,46) These studies have included the bracing effect of the wall cladding as well as the restraint provided by the sag rods.

The bracing effect of the wall cladding is characterized by its shear rigidity  $Q$ , its rotational restraint  $F$ , and its shear strength  $q$ . The quantities  $Q$  and  $q$  can be determined from the results of a shear test on a typical wall panel, as shown in Figure 4.11, and using the relationships:

$$P = G'by = Q\gamma \quad (4.19)$$

and

$$q = \frac{P_{ult}}{b} \quad (4.20)$$

where  $G'$  = rigidity per unit width  
 $Q$  = rigidity per panel width



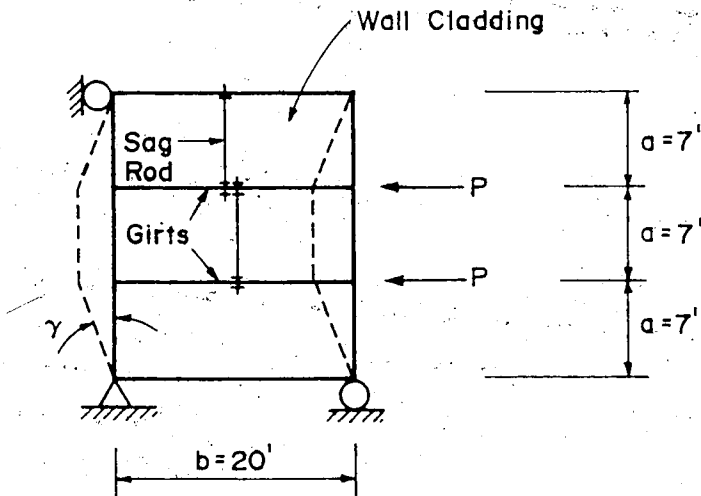


FIGURE 4.11 SHEAR TEST ON WALL PANEL

and all of the other variables are as shown in Figure 4.11.

Alternatively, as shown in Section 5.3.4, these quantities can be determined analytically(47,48). The quantity  $F$  is a function of the cross bending rigidity of the diaphragm and the local rigidity of the diaphragm to resist twisting of the girt and must be determined experimentally(44).

The maximum load that a girt can carry may be limited by one of the following:

1. excessive normal stresses,
2. premature failure of the wall cladding as a shear diaphragm, and, possibly,
3. a serviceability requirement.

Computer programs have been written from which load-deflection curves, and hence failure, loads can be obtained(44,45).

To illustrate girt behaviour when the first of the above failure modes governs, a series of girts having the cross section shown in Figure 4.10 were analyzed using the computer program described in Reference (45). The lengths  $L$  of the girts varied from 12 to 24 feet, and all had sag rod braces at mid-spans. To prevent possible premature failure in the wall cladding due to excessive bracing forces the stiffness  $Q$  was set equal to infinity. The stiffness  $F$  was set equal to zero. Failure was assumed to occur when the maximum normal stress was

equal to 1.15 multiplied by the yield stress  $F_y$  (55 ksi) since the maximum stresses only occurred at the corners. This approach is that recommended in Reference (46). The failure moments non-dimensionalized by the yield moment  $M_y$  are shown in Figure 4.12. For the particular girts studied, the effect of torsion appears to be small.

Additional work is required to determine the strengths of different girt sections, including different arrangements of sag rods, and the effect of premature failure in the wall cladding. An acceptable maximum angle of twist at the specified load level must also be determined. These studies are considered to be beyond the scope of this dissertation.

#### 4.7 Bracing Requirements

##### 4.7.1 Diaphragm Braced Beams

The most frequently used bracing in a light industrial building is the roof deck, connected to the top chord of the joists which, in turn, are connected to the girders. If the roof deck has adequate shear strength and stiffness it acts as a shear diaphragm, and the joists and girders then can be assumed to be laterally braced and to be capable of attaining their full in-plane strengths.

In Reference (1) it is recommended that if bracing is

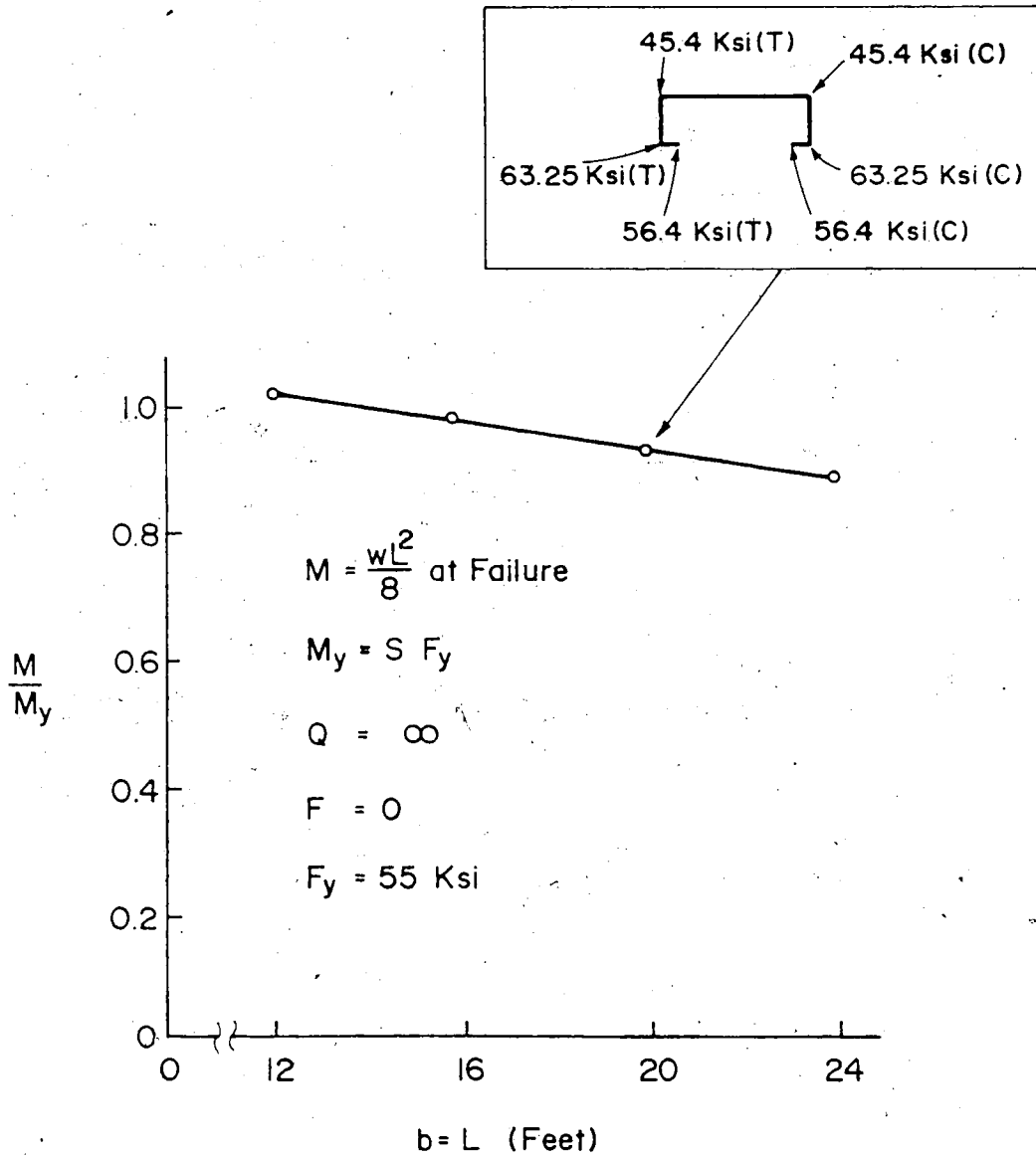


FIGURE 4.12 BENDING STRENGTH OF GIRT SHOWN IN FIGURES 4.10 AND 4.11.

provided by the roof deck, the deck and the connections should be designed to resist forces uniformly distributed along the compression flanges or chords, equal to 5 percent of the maximum forces in the flanges or chords, unless smaller values can be justified by analysis. The analysis presented here, a modification of those presented in References (49) and (50), indicates that a reduction may be appropriate.

For the case of a simply supported I-shaped beam of depth  $d$ , length  $L$ , braced along its top flange by a shear diaphragm of stiffness  $Q$  (see Section 4.6), and subjected to uniform bending, a conservative estimate of the elastic buckling moment is<sup>(49)</sup>:

$$M_u = Qd \quad (4.21)$$

Assuming for illustration purposes either a Class 1 or a Class 2 section, the member is capable of reaching its full in-plane strength if  $M_r$ , determined from Equation (4.9), is equal to  $\phi M_p$ . For this to occur,  $M_u$  must be at least equal to  $2.15 M_p$ . Substituting this value of  $M_u$  into Equation (4.21), the required diaphragm stiffness for full bracing is:

$$Q = \frac{2.15 M_p}{d} \quad (4.22)$$

The stiffness provided depends on the roof deck profile, the method of fastening, the length of the roof deck units, and the span between joists, and is characterized by its flexibility factor  $F$ <sup>(29)</sup>. If the width of the diaphragm tributary to one beam is denoted  $b$ , the relationship between  $Q$  and  $F$  is easily determined by comparing Equation (4.19) with a similar Equation in Reference (29):

$$Q = \frac{10^6 b}{F} \quad (4.23)$$

In addition, the diaphragm must be adequately strong.

The maximum shear strain  $\gamma$  in the roof diaphragm can be shown to be (49):

$$\gamma = \left( \frac{1}{\frac{M_u}{M_p} - 1} \right) \frac{\pi \delta}{L} \quad (4.24)$$

where  $\delta$  is an "equivalent" initial imperfection including both initial crookedness and twist, approximately  $L/400$ . Using the relationship<sup>(29)</sup>:

$$q = \frac{10^6}{F} \gamma \quad (4.25)$$

the corresponding shear flow  $q$  can be obtained. Allowable shears are published in Reference (29).

---

Example 4.3Given

Use the above analysis to determine if a simply supported W21x55 girder ( $M_p = 5544$  inch-kips,  $d = 20$  in.) 40 feet long is capable of reaching its full in-plane strength if the top flange is braced by roof deck with the following properties:

$$q = 200 \text{ lbs./ft.}$$

$$F = 70 \text{ micro-inches/lb.}$$

Assume that the width of the roof deck tributary to the girder is 35 feet. Compare with the 5 percent rule recommended in Reference (1).

Solution

The stiffness  $Q$  is determined first:

$$\begin{aligned} Q &= \frac{10^6 b}{F} && \text{(Eq. 4.23)} \\ &= \frac{10^6 \times 35}{70} \times \frac{12}{1000} \\ &= 6000 \text{ kips} \end{aligned}$$

Since this exceeds the right-hand side of Equation (4.22):

$$\frac{2.15 M_p}{d} = \frac{2.15 \times 5544}{20} = 596 \text{ kips}$$

the diaphragm is adequately stiff.

The elastic buckling moment is:

$$\begin{aligned}
 M_u &= Qd && \text{(Eq. 4.21)} \\
 &= 6000 \times 20 \\
 &= 120000 \text{ inch-kips}
 \end{aligned}$$

The maximum shear strain in the diaphragm is:

$$\begin{aligned}
 \gamma &= \left( \frac{1}{\frac{M_u}{M_p} - 1} \right) \frac{\pi \delta}{L} && \text{(Eq. 4.24)} \\
 &= \left( \frac{1}{\frac{120000}{5544} - 1} \right) \frac{\pi}{400} \\
 &= 0.000380 \text{ radians}
 \end{aligned}$$

and the shear flow is

$$\begin{aligned}
 q &= \frac{10^6}{F} \gamma && \text{(Eq. 4.25)} \\
 &= \frac{10^6}{70} \times 0.000380 \times 12 \\
 &= 65 \text{ lbs./ft.}
 \end{aligned}$$

This is substantially less than the allowable shear of 200 lbs./ft., therefore the full in-plane strength can be reached.



Next, the shear flow is determined using the 5 percent rule discussed above. The force in the compression flange is approximately equal to the plastic moment divided by the lever arm:

$$F = \frac{5544}{20} = 277 \text{ kips}$$

The bracing force per unit length of girder is:

$$w = \frac{0.05 \times 277}{40} \times 1000 = 345 \text{ lbs./ft.}$$

and the resulting shear in the diaphragm is:

$$q = \frac{345 \times 40}{2 \times 35} = 197 \text{ lbs./ft.}$$

approximately three times greater than that computed using the detailed analysis.

---

Several deficiencies in the above analysis should be studied, however, before it can replace the 5 percent rule:

1. The analysis is valid only for simply supported girders subjected to uniform bending. While this is conservative for

simply supported girders with other loading patterns, it may not be for different support conditions (eg., girders in cantilever roof framing schemes).

2. The shear values tabulated in Reference (29) are allowable values - ultimate values are not given. Since the bracing requirements are based on the ultimate load level, ultimate shear values should also be determined.
3. If the roof deck is being used to brace more than one girder (the usual case) the implicit assumption made above is that is the same for all girders. Research is required to determine the statistical distribution of girder out-of-straightness before a less conservative approach can be determined.

All of these studies are considered to be beyond the scope of this dissertation.

#### 4.7.2 Special Open Web Steel Joists in Cantilever Framing Schemes

An example of an interior column in a building with a cantilever roof framing scheme is shown in Figure 4.13(a). The building is assumed to be "sidesway prevented". If the column base plate and cap plate connections behave as pins, and the special open web steel joists are both stiff enough and strong enough, the column buckles in the half sine wave shown by the broken lines.

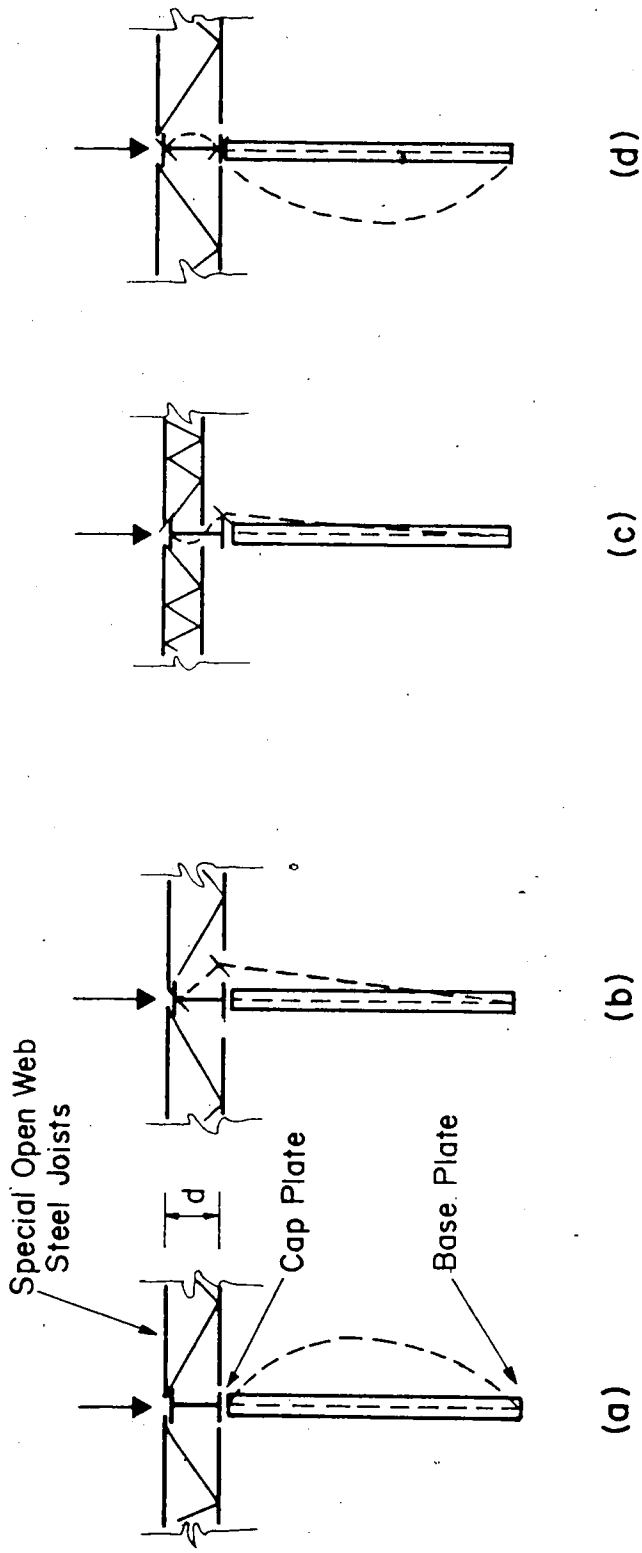


FIGURE 4.13 VARIOUS POSSIBLE FAILURE MODES FOR COLUMNS

However, if the bottom chords are not connected to the column top, as shown in Figure 4.13(b), the column behaviour approaches that of a pin-ended, "sideways permitted" column, for which the effective length factor is infinity. Expressions for the required stiffness and strength of the special open web steel joists required to prevent translation of the top of the column are derived below. The analysis parallels that described in Reference (51) to determine bracing requirements for continuous columns.

The joists used to brace the top of the column in Figure 4.13(a) must be classified as "special open web steel joists", as opposed to "tie joists". A tie joist is defined in Reference (1) as a joist designed to resist gravity load only, and has an extended bottom chord to facilitate erection. Frequently the bottom chord extension of a tie joist is a lighter section than the remainder of the bottom chord, and is connected to the column with a slotted connection. A tie joist is not designed to develop a fixed-end moment, and under vertical loads either the bottom chord extension will buckle, or the slotted connection will slip. In this sense a tie joist connection must be regarded as a non-structural connection.

It is tacitly assumed that the web buckling failures shown in Figures 4.13(c) and (d) do not occur before failure of the column as a pin-ended member. It is also assumed that a web crippling failure (discussed in Section 4.9) does not occur in the girder.

The web buckling failure shown in Figure 4.13(c) is not analyzed in any of the usual references on stability. However, it is unlikely to occur if the depth of the joist is approximately equal to the depth of the girder since the bottom flange of the girder cannot move sideways. This is usually the case in light industrial buildings with square, or nearly square, bays. The web buckling failure shown in Figure 4.13(d) is associated mainly with a deep plate girder, and usually does not occur in the web of a rolled wide flange shape. In a doubtful situation, however, the bearing resistance of the girder web based on buckling can be checked using the approach described in Reference (1).

To simplify the analysis the restraint provided by the bottom chord of each special open web steel joist is represented by a linear elastic spring with a spring constant  $k$ . The spring constant can be determined by noting that the stiffness of the special open web steel joist in Figure 4.14(a) is defined by:

$$M = \frac{2EI}{L_j} \theta \quad (4.26)$$

where

$M$  = restraining moment

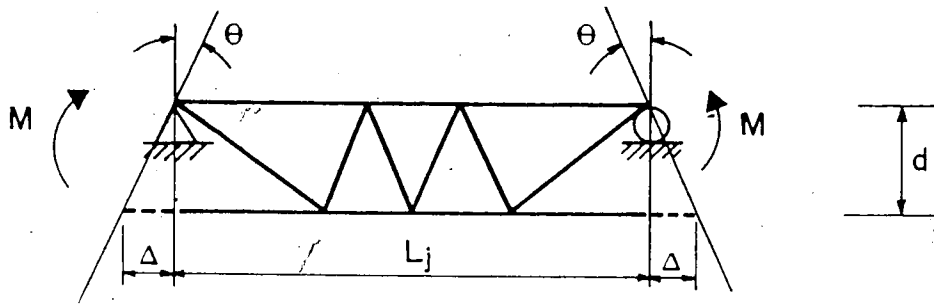
$\theta$  = end rotation

$I$  = moment of inertia, and

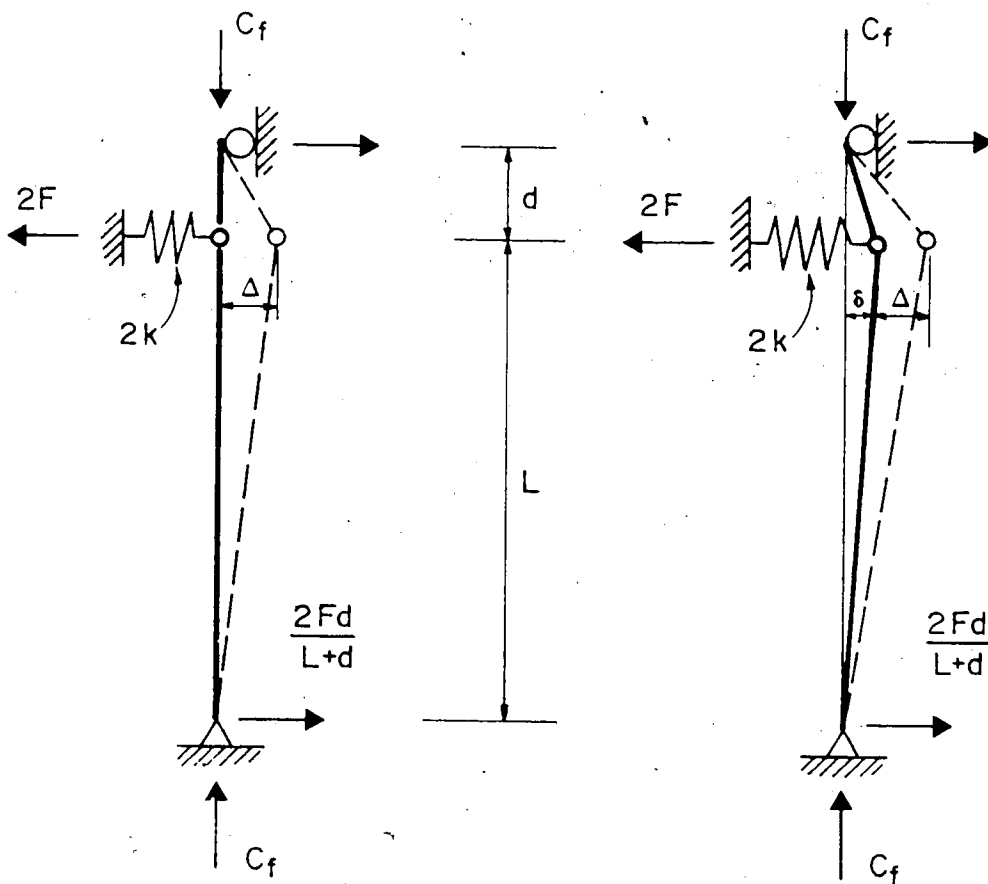
$L_j$  = length.

Using the relationships

$$F = \frac{M}{d} \quad (4.27)$$



(a)



(b)

(c)

FIGURE 4.14 ANALYSIS OF BRACING REQUIREMENTS FOR SPECIAL OPEN WEB STEEL JOISTS

$$\text{and } \theta = \frac{\Delta}{d} \quad (4.28)$$

where

$F$  = axial force in top and bottom chords

$d$  = joist depth

$\Delta$  = extension of end of bottom chord,

the following equation can be derived:

$$F = k \Delta \quad (4.29)$$

where

$$k = \frac{2EI}{L_j d^2} \quad (4.30)$$

is the spring constant for one special joist.

As illustrated in Figure 4.13(a), the special open web steel joists and the girder are assumed to be of equal depth  $d$ . The column is of length  $L$  measured from the top of the base plate to the underside of the cap plate, and is subjected to a factored compression force  $C_f$  based on "sidesway prevented" buckling. Note that the above definition of  $L$  differs from that used in Reference (1), where  $L$  is taken as the length from the top of the base plate to the centroid of the restraining member. This value of  $L$  is used since it is assumed that the cap plate connection is ineffective in restraining the column for weak axis buckling.

Consider first a perfectly straight and plumb column. The idealized column model is shown in Figure 4.14(b). The girder web and column are represented by pin-ended bars of lengths  $d$  and

L respectively, supported by a spring. Since this spring represents the stiffnesses of the bottom chords of the special open web steel joists on both sides of the column, its stiffness is  $2k$ , where  $k$  is defined by Equation (4.30):

The column is subjected to a small lateral displacement  $\Delta$ , resulting in the horizontal forces shown. The buckling condition can be found by summing moments about the cap plate:

$$\left(\frac{2Fd}{L+d}\right)L = C_f\Delta \quad (4.31)$$

If  $k_{id}$  is defined as the spring stiffness required to brace the ideal column, substitution of

$$F = k_{id}\Delta \quad (4.32)$$

into Equation (4.31) yields:

$$k_{id} = \left(\frac{L+d}{d}\right) \frac{C_f}{2L} \quad (4.33)$$

Thus if  $k$  determined from Equation (4.30) is equal to or greater than  $k_{id}$  determined from Equation (4.33), the column does not buckle in the "sidesway permitted" mode, and the top of the column can be assumed to be braced.



This analysis, however, does not give any indication of the bottom chord force  $F$  that must be resisted. In order to obtain some guidance in this respect, the effect of column out-of-plumb and eccentricity of loading must be taken into account.

As shown in Figure 4.14(c), it is assumed that these imperfections can be represented by a fictitious out-of-plumb  $\delta/L$ . The maximum out-of-plumb permitted by Reference (1) is  $L/500$ . Multiplying by a factor of two to approximate the effect of eccentricity of loading, (51)

$$\frac{\delta}{L} = \frac{1}{250} \quad (4.34)$$

After the compression force is applied, the top of the column is assumed to deflect laterally an additional amount  $\Delta$ , producing the horizontal forces shown. In addition, the column must undergo bending deflections between its ends caused by the initial crookedness of the member. Only the sideways deflections are indicated by the broken line in Figure 4.14(c), however, since the results of the analysis are unaffected by the magnitude of the deflections between the ends of the column.

The growth in deflection  $\Delta$  is determined by summing moments about the cap plate:

$$\left(\frac{2Fd}{L+d}\right)L = C_F (\delta + \Delta) \quad (4.35)$$

Substituting Equations (4.29) and (4.33) into Equation (4.35) and solving for  $\Delta$ ,

$$\Delta = \frac{\delta}{\frac{k}{k_{id}} - 1} \quad (4.36)$$

If the additional deflection  $\Delta$  at incipient buckling is limited to the initial deflection  $\delta$ , then the actual spring stiffness must be at least twice the ideal spring stiffness:

$$k = 2 k_{id} \quad (4.37)$$

and the force that must be resisted by each bottom chord is

$$F = k\Delta = (2 k_{id}) \frac{L}{250} \quad (4.38)$$

Using Equations (4.27), (4.33) and (4.30), Equations (4.37) and (4.38) become:

$$\frac{2EI}{L_j d^2} = \left(\frac{L+d}{d}\right) \frac{C_f}{L} \quad (4.39)$$

$$M = 0.004 (L+d) C_f \quad (4.40)$$

#### Example 4.4

##### Given

In the structure shown in Figure 4.1, 24 in. deep special open web steel joists 40 ft. long are used to brace the

cantilever girders and columns. As illustrated in Figure 4.8, the maximum factored axial force in the columns is 196 kips. The column length, measured from the top of the base plate to the cap plate is 19 ft. Determine the required moment of inertia and the end moments that must be resisted.

### Solution

The required moment of inertia is:

$$\begin{aligned} I &= \frac{L_j d^2}{2E} \left( \frac{L + d}{d} \right) \frac{C_f}{L} && \text{(Eq. 4.39)} \\ &= \frac{40 \times (24)^2}{2 \times 30000} \left( \frac{19 + 2}{2} \right) \frac{196}{19} \\ &= 42 \text{ in.}^4 \end{aligned}$$

and the end moments are

$$\begin{aligned} M &= 0.004 (L + d) C_f && \text{(Eq. 4.40)} \\ &= 0.004 (19 + 2) \times 196 \\ &= 16.5 \text{ ft.-kips.} \end{aligned}$$

The end moments  $M$  may be such as to produce either a concave up or a concave down deflected shape, the worst case governing design of the special open web steel joists. In addition, the special open web steel joists must be capable of resisting the negative end moments resulting from the various loading cases. Usually the end moments that must be developed to brace the columns are small relative to these other moments.

---

An alternative solution is to brace the top of the column with braces extending from the cap plate at an angle of approximately 45 degrees to the level of the roof deck. The results of the above analysis can be applied to this case also. Denoting  $A_b$  as the area of either brace, and noting that each brace has a stiffness of  $A_b E / (2\sqrt{2}L)$ , the required bracing area to satisfy the stiffness requirements, from Equations (4.33) and (4.37):

$$A_b = 2\sqrt{2} \left(1 + \frac{d}{L}\right) \frac{C_f}{E} \quad (4.41)$$

Similarly, dividing Equation (4.40) by the depth  $d$ , and multiplying by  $\sqrt{2}$ , the force that must be resisted by each brace is

$$F = 0.004 \left(1 + \frac{L}{d}\right) \sqrt{2} C_f \quad (4.42)$$

Note that one brace is in tension, while the other is in compression.

## 4.8 Deflection and Camber Requirements

### 4.8.1 Deflection Limitations

A building must be designed to be safe from collapse at the factored load level, and to be serviceable at the specified load level. Excessive deflections can result in a number of serviceability problems.

An asphalt roof becomes unserviceable if cracking occurs, allowing penetration of rainwater. In simply supported roof systems, cracks can occur along the column lines if the relative rotation of adjacent panels becomes excessive. In cantilever roof framing schemes, cracks can occur in the vicinity of the connections between the main girders and the simply supported "link" beams, caused by the sharp changes in curvatures at these locations. Excessive deflections of the perimeter roof beams can cause distress in the exterior walls. Overhead cranes cannot operate smoothly if the crane runway beams are too flexible. Other difficulties could be listed; however, the above examples indicate the importance of limiting deflections to acceptable amounts. Since the dead loads are acting during the construction stage, it is primarily the live load deflections, occurring after construction, that are of interest.

Calculation of the deflection of a structural member should not present any unusual difficulties for the designer. Formulas to calculate the deflections of simply supported and continuous girders, subjected to a large number of loading conditions, are provided in Reference (5). Since snow can drift on a flat roof and thereby accumulate unevenly, it is recommended in Reference (7) that, in computing deflections caused by snow loadings, the effects of drifting be approximated by removing half of the snow load from any one portion of the roof, if that loading condition causes increased deflections in the member concerned.

The computed deflections should be such that they are not visually objectionable and that the various serviceability problems discussed above do not occur. In the absence of a more detailed evaluation, recommended values of maximum specified live load deflections for roof girders in industrial buildings are (20):

Simple span members supporting a bituminous membrane or other inelastic roof coverings:

$$\frac{\Delta}{L} \leq \frac{1}{240} \quad (4.43)$$

Simple span members supporting metal or other elastic roof coverings:

$$\frac{\Delta}{L} \leq \frac{1}{180} \quad (4.44)$$

The practice of limiting the deflection in this way appears to have originated with Tredgold in the early 19<sup>th</sup> century, who recommended a limit of 1/480 (52). This was increased by American engineers later in the same century to 1/360 for houses. In the intervening one hundred years these values have been gradually modified to the limitations in use at the present time.

While deflections can generally be computed rather closely, the deflection limits are much less precise. In general deflection limits are "rules of thumb" that have worked well in the past, but there is very little analytical or experimental information reported in the literature to indicate that they are appropriate for today's materials and construction practices.

An analysis is presented below to derive a deflection requirement for simply supported girders supporting a conventional roof system, as illustrated in Figure 4.15. The objective of the analysis is not to recommend that the 1/240 limitation for this type of roof system be changed. Rather, the objective is to indicate by the assumptions that must be made the areas requiring further research before such a recommendation could be made.

The deflection due to the specified live load is denoted as  $\Delta$ . While continuity is not required between adjacent girders, it is required in the roof if the cracking of the roof membrane over the columns is to be prevented.

A cross section through the roof is shown in section A-A. A continuous and uniform coating of a fire retardant adhesive is applied to the top surface of 1.5 inch steel deck. While the adhesive is still tacky, the vapor barrier is embedded. Adhesive is then applied to the surface of the vapor barrier and the 2 inch rigid insulation board is embedded in the adhesive. Finally a bituminous roof membrane consisting of coal tar rein-

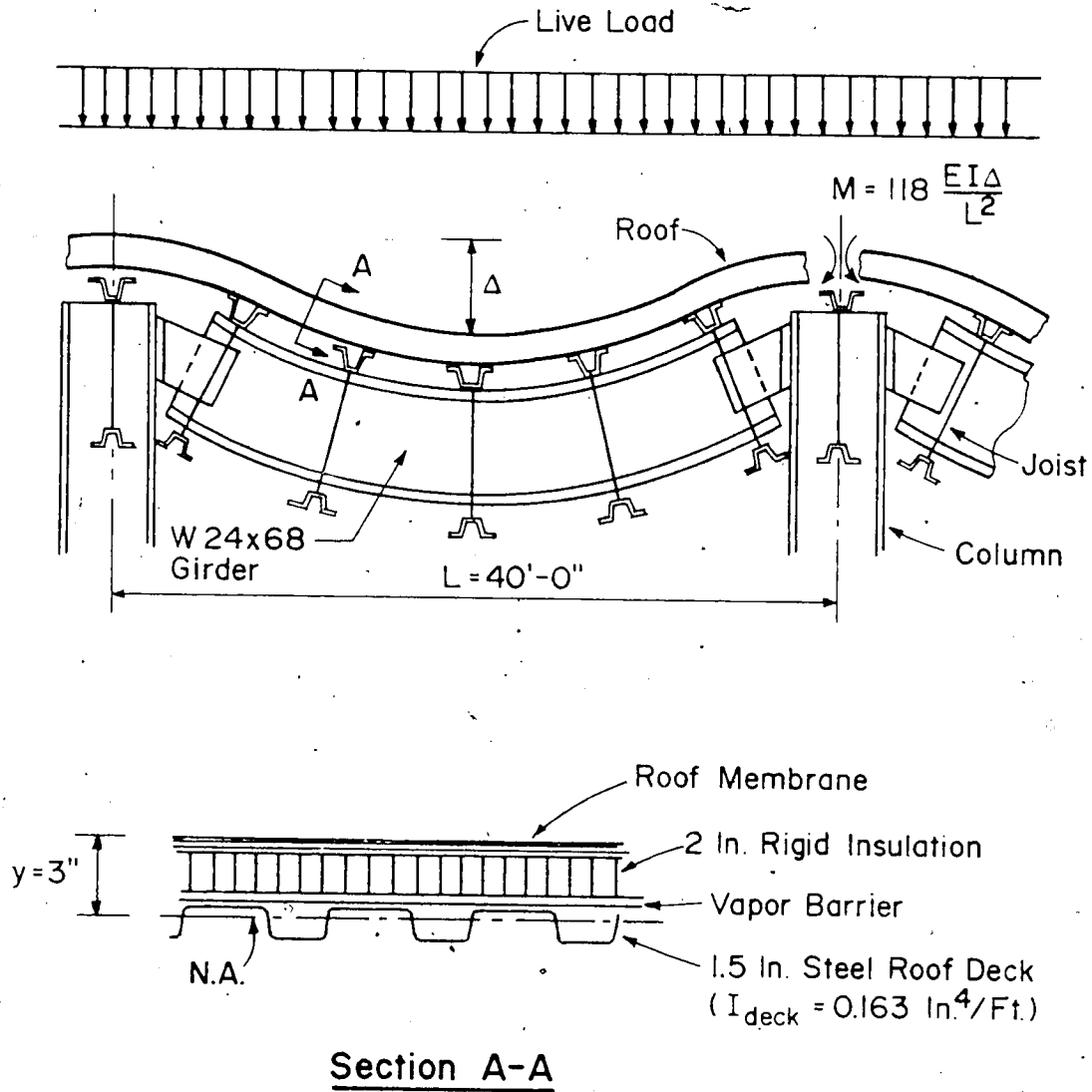


FIGURE 4.15 DEFLECTION OF A SIMPLY SUPPORTED ROOF SYSTEM



forced with organic fibre felts is applied to the top surface of the insulation.

The roof shown in section A-A is assumed to behave as a composite beam. Thus, plane sections before bending are assumed to remain plane after bending. This assumption is conservative since any slip along the glued joints, or shearing deformation in the rigid insulation, will reduce the strain in the roof membrane. Because the modulus of elasticity of the non-structural components is much smaller than the modulus of elasticity of the roof deck, the neutral axis of the composite beam is assumed to be unchanged from the neutral axis of the steel deck alone. The distance  $y$ , measured from the neutral axis to the top of the membrane, is assumed to equal 3 inches.

The negative bending moment  $M$ , in the roof over the columns, depends on the relative stiffness of the roof deck and the girders. The results of a computer analysis, for the design shown in Figure 4.15, indicates that:

$$M = 118 \frac{EI\Delta}{L^2} \quad (4.45)$$

where  $I$  is the moment of inertia of the roof deck per unit width multiplied by the bay width, assumed to equal the girder span  $L$ .

The strain  $\epsilon$  in the roof membrane is:

$$\epsilon = \frac{My}{EI} \quad (4.46)$$

which, after substitution of Equation (4.45) becomes:

$$\epsilon = 118 \left(\frac{Y}{L}\right) \left(\frac{\Delta}{L}\right) \quad (4.47)$$

For  $y$  equal to 3 inches and  $L$  equal to 40 feet,

$$\epsilon = 0.738 \left(\frac{\Delta}{L}\right) \quad (4.48)$$

The ultimate strain  $\epsilon_u$  and the coefficient of thermal expansion  $\alpha_t$  are (53,54):

$$\epsilon_u = 0.007 \quad (4.49)$$

$$\alpha_t = 50 \times 10^{-6}/^{\circ}\text{C} \quad (4.50)$$

The strain  $\epsilon_u$  varies with temperature and the value given above is based on laboratory tests conducted on a membrane at  $-40^{\circ}\text{C}$ . The coefficient of thermal expansion also varies with temperature. The value reported above is valid for temperatures between approximately  $0^{\circ}\text{C}$  and  $-35^{\circ}\text{C}$ .

Assuming a sudden drop in temperature  $\Delta T$  of  $30^{\circ}\text{C}$ , and using a factor of 0.8 with  $\epsilon_u$  to approximate the scatter that can be expected from varying field conditions, cracking of the roof membrane occurs when

$$\alpha_T \Delta T + \epsilon = 0.8 \epsilon_u \quad (4.51)$$

or, substituting numerical values for  $\alpha_T$  and  $\epsilon_u$ , and Equation (4.48) for  $\epsilon$

$$50 \times 10^{-6} \times 30 + 0.738 \left(\frac{\Delta}{L}\right) = 0.8 \times 0.007 \quad (4.52)$$

which, after solving for  $\Delta/L$  gives

$$\frac{\Delta}{L} = \frac{1}{180} \quad (4.53)$$

Equation (4.53) can be compared to the 1/240 recommendation for this type of roof system.

Several of the assumptions must be examined further. It has been implied that the thermal strains that occur slowly from summer to winter can be ignored, and that cracking is associated only with a sudden drop in temperature. This assumption is valid if the strains are either lost through relaxation of the roof membrane or if the average coefficient of thermal expansion of the membrane during the warmer temperatures is small.

The factor of 0.8 used with  $\epsilon_u$  to account for the quality of workmanship, effects of weather, etc. is an estimate only, and some other number could have been used equally well. A limited test program described in Reference (54) indicates that the effect of surface irregularities (stress raisers) may be small.

---

Example 4.5Given

Calculate the live load deflection of the W21x73 cantilever girder shown in Figure 4.1, and compare it with the limiting value of 1/240 recommended in Reference (20). The specified snow load is 48 psf.

Solution

The most severe snow load pattern consists of full snow load over the main span and one overhang, with half snow load on the other overhang. Since the loading is unsymmetrical the maximum deflection does not occur at mid-span. For simplicity, however, it will be assumed that it is sufficiently accurate to take the mid-span deflection as the maximum deflection. Also, this deflection will be computed on the basis of a distributed load, rather than on the basis of a series of point forces at the joist locations.

The maximum deflection due to the load along the main span is (5):

$$\Delta_1 = \frac{5wL^4}{384 EI} \quad (4.54)$$

where

$w$  = the uniformly distributed load

$L$  = the distance between columns

$E$  = Young's modulus, and

$I$  = the girder moment of inertia.

The negative moment  $M$  at the column due to the fully loaded overhang is

$$M = \alpha_1 (1 - \alpha_2) \frac{wL^2}{2} \quad (4.55)$$

where

$\alpha_1$  = the overhang ratio for the interior cantilever girder and

$\alpha_2$  = the overhang ratio for the exterior cantilever girder.

which produces an upward deflection at mid-span of <sup>(5)</sup>:

$$\Delta_2 = \frac{ML^2}{8EI} \quad (4.56)$$

Thus the net downward deflection, using Equations (4.54) to (4.56), is:

$$\begin{aligned} \Delta &= \Delta_1 - \frac{3}{2} \Delta_2 \\ &= \frac{5wL^4}{384 EI} \left( 1 - \frac{36}{5} \alpha_1 \{ 1 - \alpha_2 \} \right) \end{aligned} \quad (4.57)$$

The moment of inertia of the W21x73 section is given in Reference (5):

$$I = 1600 \text{ in.}^4$$

Noting from Figure 4.1 that

$$L = 40 \text{ ft.}$$

$$\alpha_1 = 0.14$$

$$\alpha_2 = 0.18$$

and that

$$w = \frac{48 \times 40}{1000} = 1.92 \text{ k/ft.}$$

the deflection is:

$$\Delta = \frac{5 \times 1.92 \times (40)^4 \times 1728}{384 \times 29000 \times 1600} \left(1 - \frac{36}{5} \times 0.14 \times \{1 - 0.18\}\right) \quad (\text{Eq. 4.57})$$

$$= 0.413 \text{ in.}$$

Since this is less than the limiting value of:

$$\frac{L}{240} = \frac{40 \times 12}{240} = 2.0 \text{ in.}$$

the section is satisfactory.

---

#### 4.8.2 Camber Requirements

Damage to the roofing membrane can also occur at mid-span if the total deflections are such that rainwater drainage cannot take place and ponding of water occurs. In this case the membrane is subjected to an accelerated rate of deterioration, resulting in a decrease in adhesive and flow properties<sup>(53)</sup>. Partly to reduce the possibility of this type of damage, and partly for appearance, Reference (1) recommends that long span joists and trusses be cambered upwards so as to be approximately flat under dead loads.

The extra fabrication costs associated with cambering of joists and trusses are usually small since a jig must be made for fabrication whether or not camber is required. Cambering of girders can become expensive, however, since an extra step is required during the fabrication process. Cambering is usually not necessary for girders in light industrial buildings where the spans are moderate.

#### 4.9 Web Crippling

Large vertical compressive stresses occur in the web of a girder in the vicinity of concentrated loads and reactions. If these forces are too large they can cause local yielding to occur, followed by web crippling. Web crippling should not be confused with web buckling. While over-all web buckling is also a possibility in some situations, it is usually associated with the slender web of a deep plate girder rather than with a rolled wide flange girder.

The following formulas are recommended in Reference (1) to determine the factored bearing resistance,  $B_r$ , of an unstiffened girder web:

1. for interior loads,

$$B_r = 1.25 \phi_w (N + 2k) F_y \quad (4.58)$$

2. for end reactions,

$$B_r = 1.25 \phi_w (N + k) F_y \quad (4.59)$$

where  $N$  = length of bearing, but not less than  $k$  for end reactions  
and

$k$  = distance from outer face of flange to web toe of  
flange-to-web fillet.

Equations (4.58) and (4.59) are based on a series of



six tests reported in Reference (55). In these tests the bearing lengths  $N$  varied from approximately 3-1/2 to 11 inches. A similar series of tests are reported in Reference (59), however in these tests a much smaller bearing length was used (1/2 inch). In this latter reference it is concluded that the load is resisted by a length of web equal to  $N + 5k$  instead of  $N + 2k$ . This has caused some designers to feel that the  $N + 2k$  recommendation may be overly conservative.

In fact, the  $N + 2k$  recommendation appears to be conservative only because it does not account for the bearing length-to-web slenderness ratio. Therefore, it is not valid for all values of this ratio.

To illustrate this, consider the following two equations:

1. for interior loads,

$$B_r = \phi \left( 15 + 3.25 \sqrt{\frac{N + 2k}{w}} \right) F_y w^2 \quad (4.60)$$

2. for end reactions,

$$B_r = \phi \left( 15 + 3.25 \sqrt{\frac{N + k}{w}} \right) F_y w^2 \quad (4.61)$$

Equation (4.60) is used with a  $k$  distance of zero in the design of cold formed steel girders to prevent web crippling<sup>(42)</sup>. A com-

parison of Equation (4.60) without the effect of the performance factor and the test results of References (55) and (56) is given in Figure 4.16. Agreement is generally good, with both sets of test results falling on the same straight line. Equation (4.48) plots as a parabola on this Figure and therefore cannot predict both sets of test results well.

#### 4.10 Shear in Rolled Wide Flange Girder Webs

Shear in the web of a rolled wide flange section used as a girder does not normally control design. It should be checked, however, especially for short spans and heavy loads. Assuming elastic behaviour, the shear stress distribution in the web is parabolic(57). The maximum shear stress,  $f_v$ , occurs at mid-depth, and can be approximated closely by:

$$f_v = \frac{V_r}{A_w} \quad (4.62)$$

where  $V_r$  = factored shear resistance

$A_w$  = web area,  $d \times w$

Since the bending stress is zero at this location, the maximum octahedral shear stress theory indicates that yielding occurs when(57):

$$f_v = \frac{F_y}{\sqrt{3}} \quad (4.63)$$

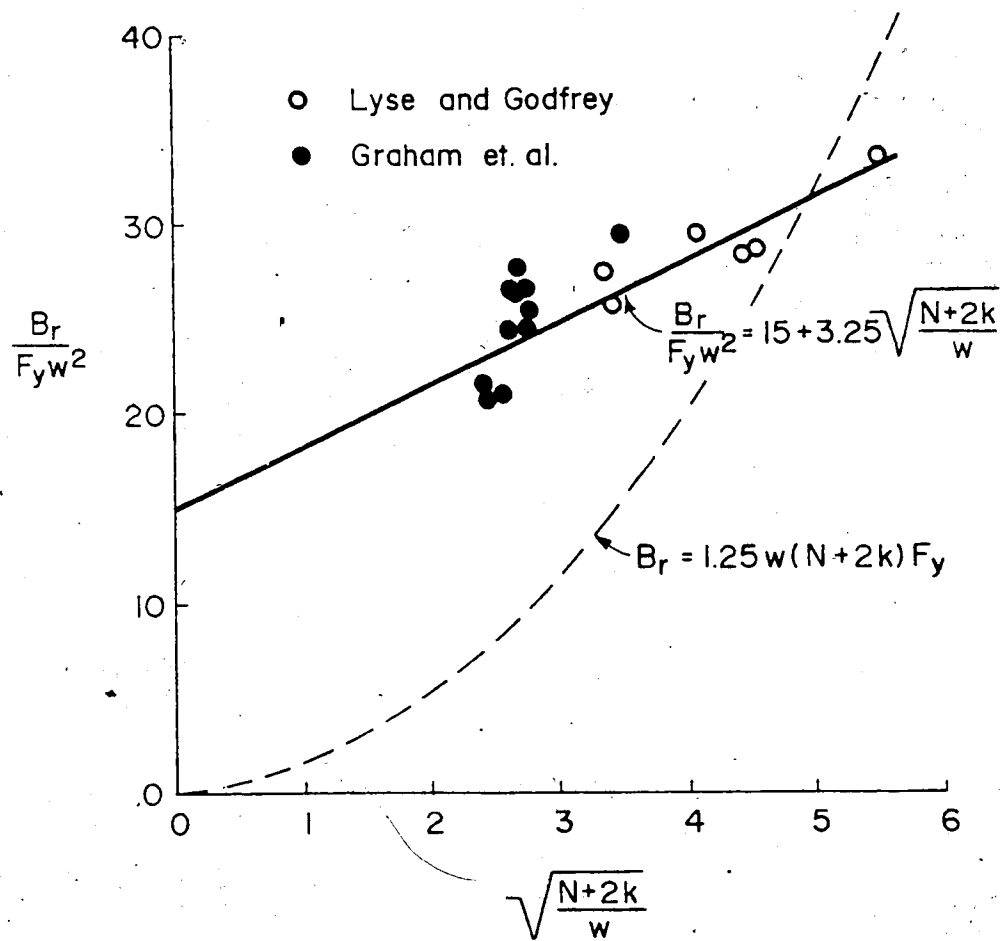


FIGURE 4.16 SUMMARY OF WEB CRIPPLING TEST RESULTS

Equating the right hand sides of Equations (4.62) and (4.63), and incorporating a performance factor  $\phi$  leads to:

$$\begin{aligned} V_r &= \phi A_w \frac{F_y}{\sqrt{3}} \\ &= 0.58 \phi A_w F_y \end{aligned} \quad (4.64)$$

Equation (4.64) indicates the shear force at which plastic flow begins, not that causing collapse of the girder. In Reference (1) this is assumed to be:

$$V_r = 0.66 \phi A_w F_y \quad (4.65)$$

to account for the beneficial effects of strain-hardening.

Equation (4.65) is based on the assumption that failure does not occur first by web buckling. Buckling of an unstiffened girder web does not occur before it has completely yielded providing (1):

$$\frac{h}{w} < \frac{386}{\sqrt{F_y}} \quad (4.66)$$

The web slenderness ratios of most rolled wide flange shapes are less than the limiting value given by Equation (4.66).

#### Example 4.6

##### Given

The maximum shear force caused by the factored loads

acting on the W21x55 link beam in Figure 4.1 is 70.5 kips. Check this member for shear. Use G40.21 44W steel ( $F_y = 44$  ksi).

Solution

The cross section dimensions of a W21x55 section are listed in Reference (5):

$$d = 20.8 \text{ in.}$$

$$t = 0.522 \text{ in.}$$

$$w = 0.375 \text{ in.}$$

First the possibility of web buckling is investigated. The clear web depth-to-thickness ratio is:

$$\frac{h}{w} = \frac{d - 2t}{w} = \frac{20.8 - 2 \times 0.522}{0.375} = 52.7$$

Since this is less than the limiting value given by:

$$\frac{h}{w} = \frac{386}{\sqrt{F_y}} = \frac{386}{\sqrt{44}} = 58.2 \quad (\text{Eq. 4.66})$$

premature web buckling does not occur. Therefore the factored shear resistance is:

$$\begin{aligned} V_r &= 0.66 \phi A_w F_y \\ &= 0.66 \times 0.9 \times 20.8 \times 0.375 \times 44 \\ &= 204 \text{ kips.} \end{aligned} \quad (\text{Eq. 4.65})$$

Since the actual shear force is only 70.5 kips the member is adequate in shear.

---

#### 4.11 Summary

In this chapter the design of laterally braced and unbraced girders is reviewed. Next the various stability problems encountered in the design of light industrial buildings are considered, followed by a discussion of bracing and deflection requirements for flexural members. Finally, web crippling and shear resistance of girder webs are examined.

## CHAPTER V

### COMPRESSION MEMBERS

#### 5.1 Introduction

Compression members can be broadly classified as either interior compression members or exterior compression members. These in turn can be classified as either columns or beam-columns. Columns are those members in a structure that are required to resist only compressive forces. Beam-columns, in addition, must resist bending moments.

Interior compression members must resist axial forces, and with some framing schemes, end moments, but are generally not subjected to transverse forces. (An exception to this is if these members support a travelling overhead crane.) Interior compression members are classified as columns if the connections at the upper ends and the base plate connections behave as pins. Otherwise, these members are capable of developing moments from frame action and should be designed as beam-columns. An example of an interior column is shown in Figure 5.1.

Exterior compression members may be subjected to a large number of different load combinations. Although the framing

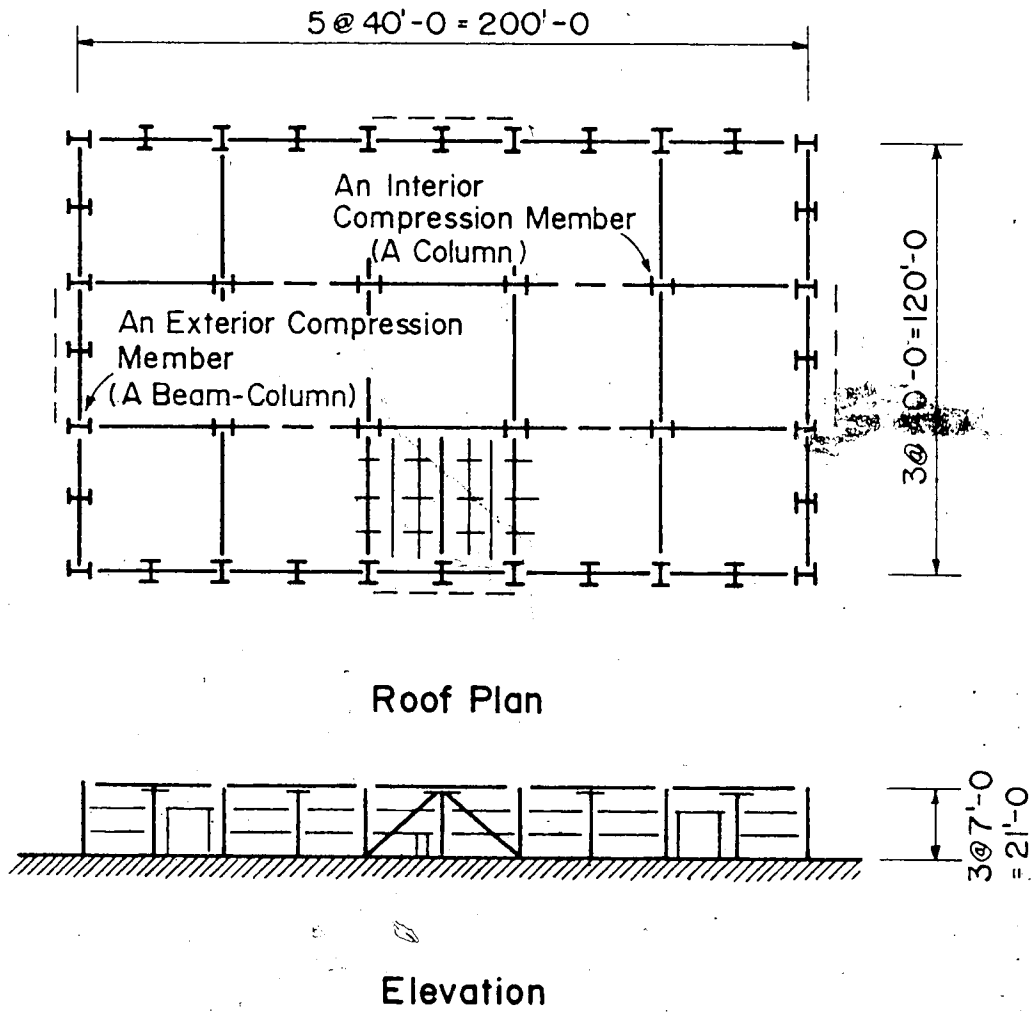


FIGURE 5.1 COMPRESSION MEMBERS

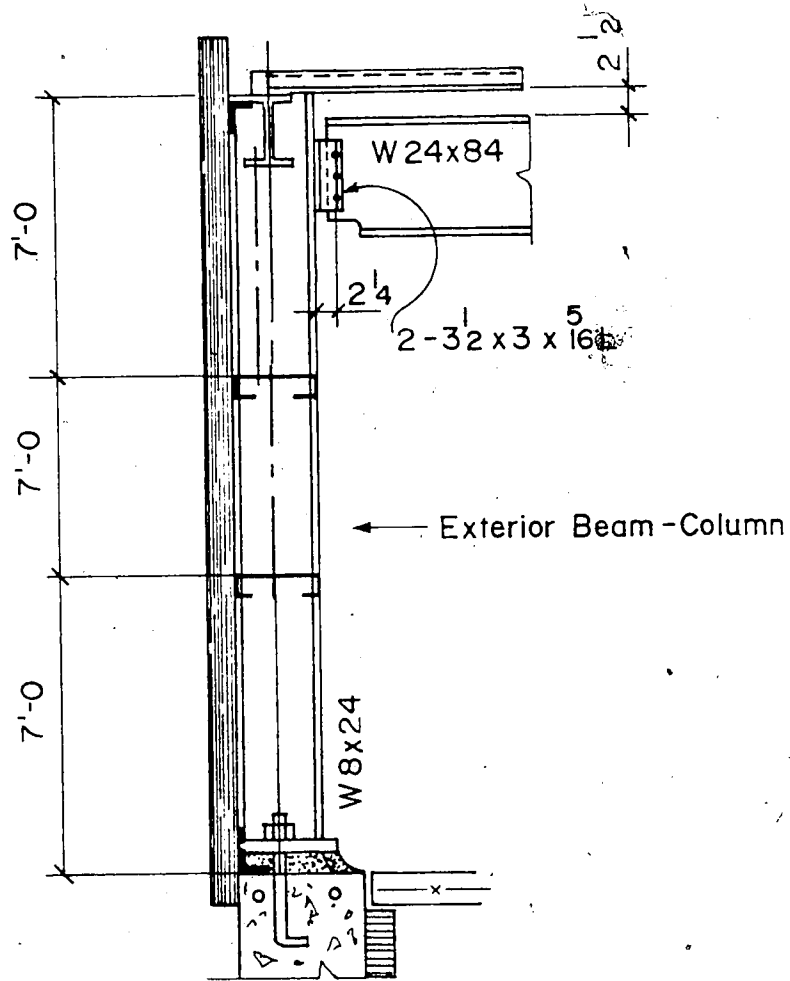


scheme and method of wall construction may be such that some of these members behave as columns, usually they must also resist bending moments.

If the flexural members are connected to the flanges of the compression members they are subjected to end moments due to the load eccentricities. An example of this is the exterior beam-column shown in Figure 5.1 and again in Figure 5.2. If these members participate in rigid or semi-rigid frame action they are also subjected to end moments from lateral loads and sway effects.

If the siding consists of girts and metal wall cladding, as illustrated in Figure 5.2, wind loads are transferred to the exterior beam-columns as a series of transverse point forces at the girt locations. If precast concrete wall panels are used as siding, the exterior compression members do not resist transverse forces. In this situation it is usually assumed that the panels act as one-way slabs, transferring wind loads directly to the roof and foundation. If the siding consists of masonry in-filled panels, the force distribution along the lengths of the beam-columns can be approximated on the basis of the areas tributary to the members considered.

Interior compression members are usually unsupported along their lengths. Their effective lengths depend on the fixity of the bases and on the restraint provided at their tops by the flexural members.



**FIGURE 5.2** · EXAMPLE OF AN EXTERIOR  
**COMPRESSION MEMBER**

Exterior compression members may be restrained (or partially restrained) along their lengths by the wall cladding. Their effective lengths depend on the type of siding and on the method of connection. For example, if the siding consists of girts connected into the webs of the compression members, and metal wall cladding, failures may be either by instability between girts or by strong axis bending. On the other hand, masonry connected directly into the webs prevents weak axis bending, and failures must be accompanied by strong axis bending only.

In this chapter the design of interior pin-ended columns is considered first. After a discussion of pin-ended columns, the effect of end restraint is then considered, followed by a discussion of the design of interior beam-columns. Design procedures for exterior columns and beam-columns are considered next, followed by a discussion of bracing requirements. Finally, design procedures for base plates and cap plates are reviewed.

## 5.2 Interior Columns

### 5.2.1 Axially Loaded Columns

The strength of a steel column depends on its length, cross-sectional properties, the amount of initial crookedness, the yield stress of the steel and the magnitude and distribution of residual stresses. A load deflection curve for a perfectly straight, pin-ended and concentrically loaded column is shown in

Figure 5.3. Initially the column compresses axially. Lateral deflections do not occur until the tangent buckling load  $C_T$  is reached, at which point the column suddenly snaps into a deflected (buckled) configuration. The column continues to bend with increasing load until a maximum value,  $C_{MAX}$ , is reached, after which unloading occurs<sup>(2)</sup>,  $C_T$ , however, is a sufficiently accurate prediction of the strength  $C_{MAX}$ , and can be easily determined from buckling theory.

Behaviour of a real column is more difficult to determine. Figure 5.3 also illustrates the behaviour of a column with an initial crookedness  $\delta/L$ . Lateral deflections are present throughout the loading history so that the possibility of buckling - that is, a sudden snap from a perfectly straight to a deflected configuration - is precluded. The maximum load,  $C_{MAX}$ , must be determined by sophisticated computational methods, often requiring the use of a digital computer. The column strength found in this way is close to  $C_T$  if  $\delta/L$  is small, but can be significantly less for large values of  $\delta/L$ .

Residual stresses, caused by uneven cooling of the member after rolling, can also have a major influence on the strength of a steel column. Figure 5.4 shows measurements of average residual stresses across the plate elements for W shapes and hot rolled HSS sections<sup>(58)</sup>. After rolling, those portions of the cross section that have higher surface area to mass ratios (flange tips and center portion of the web of W shapes and outside

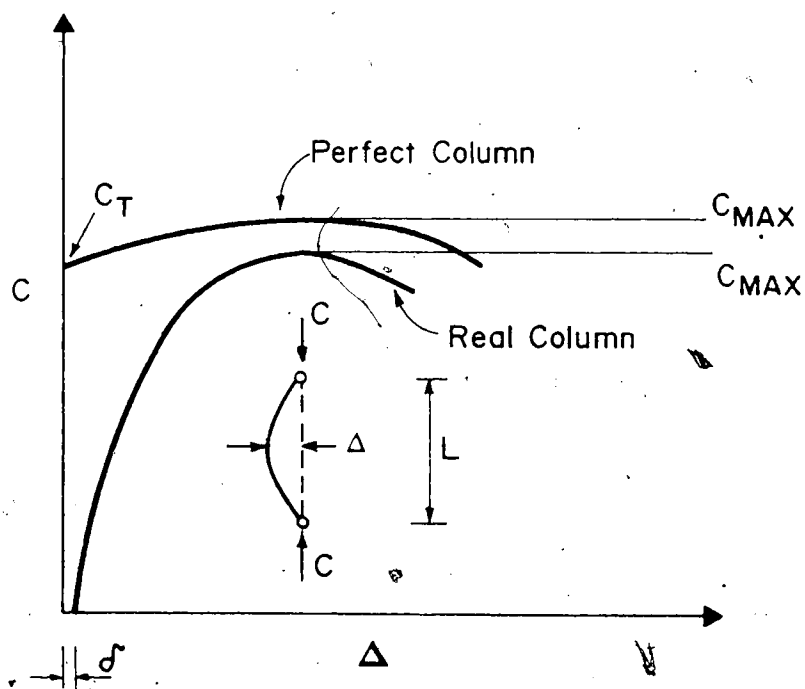


FIGURE 5.3 COLUMN LOAD-DEFLECTION CURVES

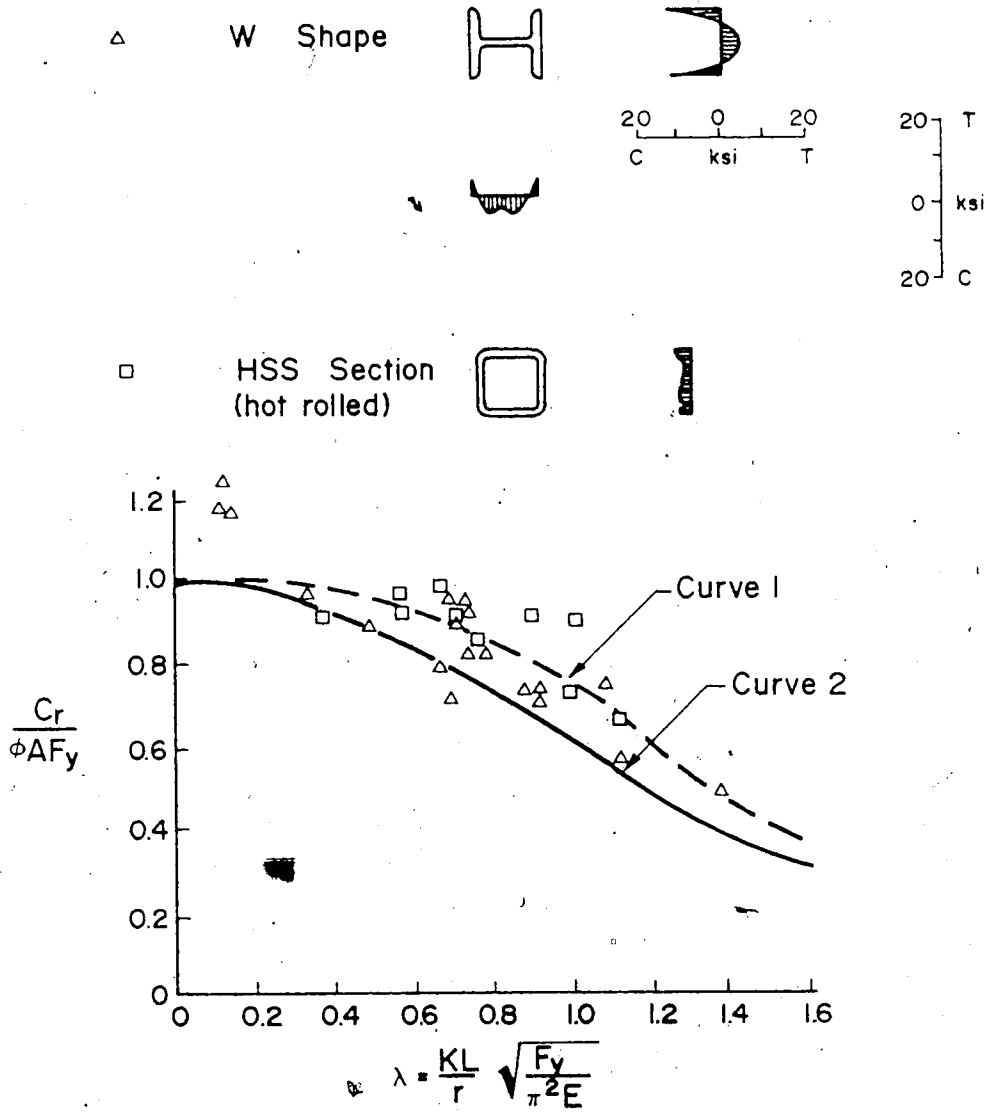


FIGURE 5.4 RESIDUAL STRESS DISTRIBUTION<sup>(58)</sup> AND COLUMN STRENGTH CURVES<sup>(2)</sup>

faces of HSS sections) cool and contract more rapidly, and solidify first. These portions tend to resist further contraction of the still plastic portions (flange to web junctions of W shapes, inside face of HSS sections) and therefore develop compressive residual stresses. Those portions of the cross section that cool last develop tensile residual stresses. Thus the maximum compressive residual stress, averaged across the thickness of a HSS section (approximately 5 ksi in Figure 5.4), is much smaller than the maximum compressive residual stress averaged across the flange tip of a W section (approximately 15 ksi in Figure 5.4). The magnitude and distribution of the residual stresses depend primarily on the shape of the cross section, the rolling temperature and the cooling conditions. The magnitude and distribution are relatively independent of the yield stress level<sup>(58)</sup>.

The effects of both residual stresses and initial crookedness have been considered in developing the three column strength curves proposed in Reference (2). Curve 2 has been adopted as the basic design curve for Limit States design<sup>(1)</sup>. This curve is divided into four parts:

$$0 \leq \lambda < 1.0 \quad C_r = \phi A F_y (1.035 - 0.202\lambda - 0.222\lambda^2) \quad (5.1a)$$

$$1.0 \leq \lambda < 2.0 \quad C_r = \phi A F_y (-0.111 + 0.636\lambda^{-1} + 0.087\lambda^{-2}) \quad (5.1b)$$

$$2.0 \leq \lambda < 3.6 \quad C_r = \phi A F_y (0.009 + 0.877\lambda^{-2}) \quad (5.1c)$$

$$3.6 \leq \lambda \quad C_r = \phi A F_y \lambda^{-2} = \phi A \frac{286000}{\left(\frac{kL}{r}\right)^2} \quad (5.1d)$$

where

$$\lambda = \frac{kL}{r} \sqrt{\frac{F_y}{\pi^2 E}} \quad (5.2)$$

$C_r$  = factored compression resistance,

$\phi$  = 0.9 = performance factor discussed in Section 2.2

$A$  = cross section area

$\lambda$  = non-dimensional slenderness ratio

$L$  = column length

$r$  = radius of gyration of the cross section about the axis of bending, and

$k$  = the effective length factor.

Equation (5.1) is appropriate for the design of W shape columns of the type normally rolled and fabricated in Canada, and for other doubly symmetric Class 1, 2 and 3 sections (1). Although it is recommended in Reference (1) that it not be used for design of cold formed non stress-relieved hollow structural steel sections, recent research reported in Reference (59) indicates that this restriction may not be appropriate.

The effective length factor of a column is equal to the ratio of the effective length,  $kL$ , to the actual length,  $L$ . The effective length, in turn, is defined as the length of a fictitious pin-ended column with the same buckling load as the actual column - that is,



$$C_e = \frac{\pi^2 EI}{(kL)^2} \quad (5.3)$$

where

$C_e$  = the buckling load and

$I$  = the moment of inertia about the axis of buckling.

As illustrated in Figure (5.5), the effective length factor depends on whether or not buckling is accompanied by sidesway, and on the conditions of restraint at the ends of the member (2).

The restraint parameter  $G_u$  at the upper end of the column is

$$G_u = \frac{\sum I_c/L_c}{\sum I_g/L_g} \quad (5.4)$$

where the subscripts "c" and "g" are for column and girder, respectively, and the summation signs extend over the members framing into the joints. For single storey structures the restraint parameter  $G_L$  at the lower end of the column depends on the base plate and footing dimensions and on the soil conditions, as discussed in the next section. Use of these nomographs will be illustrated in Example 5.1.

Although it is conservative to use Equation (5.1) for the design of columns of square or circular hollow structural steel sections in CSA G40.21 Class H steel, and for all stress

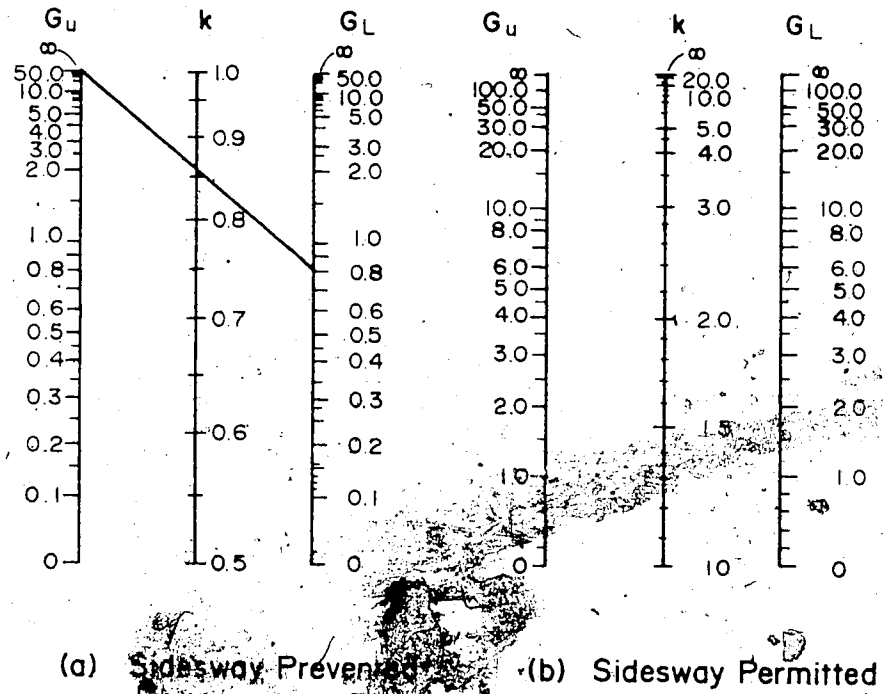


FIGURE 5.5 EFFECTIVE LENGTH NOMOGRAPHS

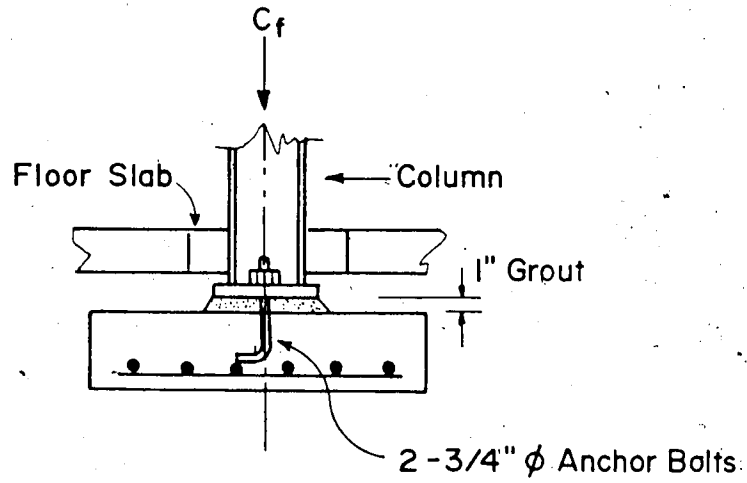
relieved bisymmetric shapes regardless of steel grade, Curve 1 in Reference (2) may be more appropriate. This curve, which reflects the increased carrying capacity of these columns, is defined by:

$$\begin{aligned}
 0 \leq \lambda < 0.15 & C_r = \phi A F_y \\
 0.15 \leq \lambda < 1.2 & C_r = \phi A F_y (0.990 + 0.122\lambda - 0.367\lambda^2) \\
 1.2 \leq \lambda < 1.8 & C_r = \phi A F_y (0.051 + 0.0801\lambda^{-2}) \\
 1.8 \leq \lambda < 2.8 & C_r = \phi A F_y (0.008 + 0.942\lambda^{-2}) \\
 2.8 \leq \lambda & C_r = \phi A F_y \lambda^{-2} = \phi A \frac{286000}{\left(\frac{kL}{r}\right)^2}
 \end{aligned}$$

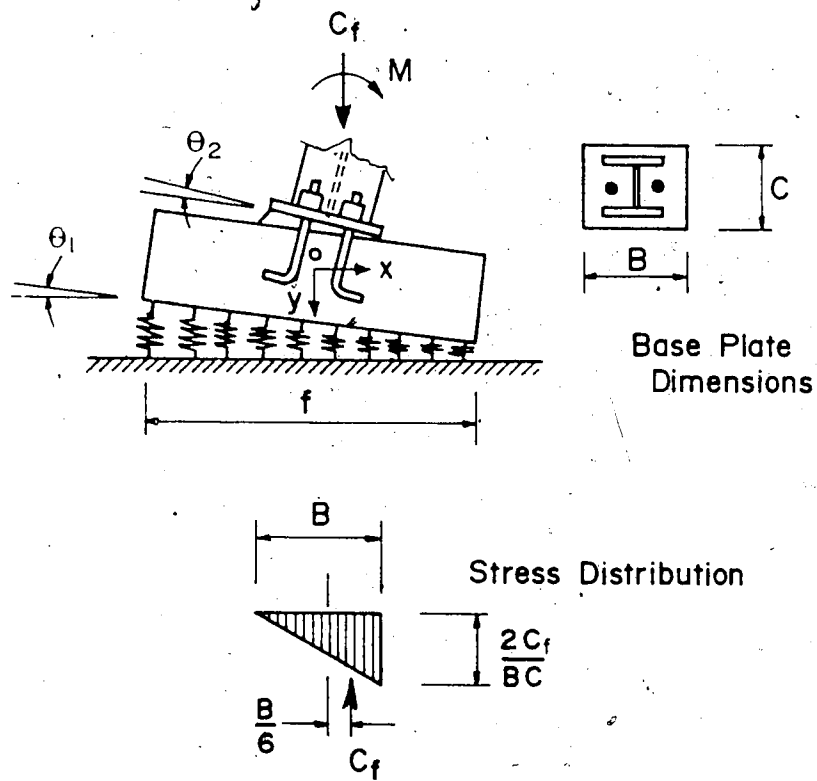
Since the width of the band of column curves that forms the basis for Curve 1 is smaller than that for Curve 2, a performance factor of 0.9 is conservative when applied to these columns (2). To indicate the difference in the load carrying capability of W and hot-rolled HSS sections a comparison is made in Figure 5.4 of the predicted strengths and observed test results (58).

### 5.2.2 Effect of Foundation Restraint

An example of a commonly accepted "pinned" column base connection is shown in Figure 5.6(a). Theoretically, the restraint parameter  $G_L$  is infinity. To account for the effect of partial restraint offered by the foundation, however, it is recommended in Reference (2) that  $G_L$  be taken as 10.0. Similarly, the restraint parameter for a fixed base column is theoretically



(d) The Actual Foundation



(b) The Idealized Model

FIGURE 5.6 ANALYSIS OF FOUNDATION STIFFNESS

zero, but to allow for a small amount of foundation rotation it is recommended that  $G_L$  be taken as 1.0.

The results of research reported in Reference (60) indicate that use of  $G_L$  equal to 10.0 for the base plate detail of Figure 5.6(a) may be far too conservative. In this reference it is shown that the rotational restraint offered by a pinned connection is often sufficient to increase the buckling load of the column to approximately that of the fixed-end case. The structure under consideration is assumed to be sideway prevented.

The restraint parameter for the foundation is defined by (31);

$$M = \frac{2EI}{G_L L} \theta \quad (5.6)$$

where  $M$  is the restraining moment resulting from a small rotation  $\theta$  of the lower end of the column. This rotation is assumed to be composed of an angle  $\theta_1$ , the slope of the footing relative to the undisturbed soil, and  $\theta_2$ , the slope of the base plate relative to the footing:

$$\theta = \theta_1 + \theta_2 \quad (5.7)$$

The footing is treated as a rigid beam on an elastic foundation, as shown in Figure 5.6(b). The reaction of the soil

on an elemental area  $gdx$  of the footing is  $gqydx$ , where  $g$  is the footing width,  $q$  is the "modulus of subgrade reaction", and  $x$  and  $y$  are as defined in Figure 5.6(b). Values of  $q$  for several types of soils are given in Reference (65). Summing moments about point  $o$  by integrating over the length  $f$  of the footing and noting that  $y = \theta_1 x$  leads to:

$$\theta_1 = \frac{12}{gqf^3} M \quad (5.8)$$

The rotation  $\theta_2$  is found from the initial slope to the moment rotation curve for the base plate connection. As illustrated in Figure 5.6(b), the base plate is in full contact with the underlying concrete and the anchor bolts are unloaded for applied moments up to:

$$M = \frac{C_f B^3}{6} \quad (5.9)$$

where  $B$  is the base plate width. The angle  $\theta_2$  can be estimated by assuming that the depth to which the strains are transmitted is equal to the base plate length  $B$ :

$$\theta_2 = \frac{2C_f}{E_c B C} \quad (5.10)$$

where  $E_c$  is Young's modulus for the concrete. Thus

$$\theta_2 = \frac{12M}{B^2 E_C C} \quad (5.11)$$

The total angle change,  $\theta_L$ , is found by substituting Equations (5.8) and (5.11) into Equation (5.7):

$$\theta = 12 \left( \frac{1}{9qf^3} + \frac{1}{B^2 C E_C} \right) M \quad (5.12)$$

and  $G_L$  is found by substituting Equation (5.12) into Equation (5.6):

$$G_L = \left( \frac{24 E_C I}{L} \left( \frac{1}{9qf^3} + \frac{1}{B^2 C E_C} \right) \right) \quad (5.13)$$

Since this is a buckling analysis, the base plate rotational restraint is based on small values of rotation. Thus the base plate is assumed to be in full contact with the concrete footing. The number and location of the anchor bolts, and the thickness of the base plate do not appear in the above derivation. As illustrated in the following example,  $G_L$  determined from Equation (5.13) is usually much less than the value of 10.0 recommended in Reference (2).

#### Example 5.1

#### Given

Select a column to resist the 196 kip force shown in Figure 4.8. Assume base plate dimensions B and C equal 11 inches

and that the length  $f$  of the footing is 4 feet. Assume that Young's modulus for the concrete is 3000 ksi and that the soil is relatively poor, with a subgrade modulus of only 30 lbs./in.<sup>2</sup>. The length of the column measured from the underside of the base plate to the top of the cap plate is 19'-5".

Solution

Try a W8x40

The section properties for the W8x40 section are listed in Reference (5):

$$A = 11.8 \text{ in.}^2$$

$$r = 2.04 \text{ in.}$$

$$I = 49.1 \text{ in.}^4$$

The restraint parameter is:

$$\begin{aligned} G_L &= 24 \frac{EI}{L} \left( \frac{1}{9qf^3} + \frac{1}{BC^2E_C} \right) \\ &= \frac{24 \times 29 \times 10^6 \times 49.1}{233} \left( \frac{1}{50 \times 30 \times (50)^3} + \frac{1}{11 \times (11)^2 \times 3 \times 10^6} \right) \\ &= 0.82 \end{aligned}$$

Assuming  $G_u$  is equal to infinity, from Figure 5.5(a) the effective



length factor is  $K = 0.86$ . Thus:

$$\begin{aligned} \lambda &= \frac{KL}{r} \sqrt{\frac{F_y}{\pi^2 E}} && \text{(Eq. 5.2)} \\ &= \frac{0.86 \times 233}{2.04} \sqrt{\frac{44}{\pi^2 \times 29000}} \\ &= 1.22 \end{aligned}$$

and

$$\begin{aligned} C_r &= \phi A F_y (-0.111 + 0.636 \lambda^{-1} + 0.087 \lambda^{-2}) && \text{(Eq. 5.1)} \\ &= 0.9 \times 11.8 \times 44 (-0.111 + 0.636 \times (1.22)^{-1} + 0.087 \times (1.22)^{-2}) \\ &= 219 \text{ kips} \end{aligned}$$

Since this exceeds the factored load of 196 kips the W8x40 section is adequate. It is also the lightest section that can be selected.

The analysis described above does not give any information on the required strength (ie., moment capacity) of the base plate connection. Although a stiff base plate connection usually also has inherent strength, the two terms are not interchangeable. For the column base to be fixed or partially fixed, it must be both adequately stiff and strong.

Since the analysis is based on buckling considerations, the column is assumed to remain perfectly straight until the

buckling load is reached. This implies that the base moment is zero for loads less than the buckling load and undefined at the buckling load. An upper bound estimate equal to the plastic moment capacity of the cross section is recommended in Reference (61), however this is considered to be unnecessarily high for design purposes by some engineers (62). In fact, the base moment cannot exceed the reduced plastic moment,  $M_{PC}$  (31).

A rigorous analysis to determine a strength requirement must be based on an initially imperfect column, and should also include the effects of residual stress and progressive yielding of the column cross section, as well as the effects of non-linear behaviour in the concrete footing and in the soil underneath the footing. Such an analysis is considered to be beyond the scope of this report. In order to indicate the relationship between some of the more important quantities involved, however, an approximate analysis is presented below.

As illustrated in Figure 5.7, the column is assumed to be initially crooked an amount  $\delta = 0.001 L(2)$ . The connection at the top of the column is assumed to act as a pin. The restraint provided by the base plate and footing is represented by a rotational spring with a restraint parameter  $G_L$ . The column is therefore statically indeterminate. Selecting the restraining moment  $M$  at the column base as the redundancy, from Equation (5.6) the angle change  $\theta$  is:

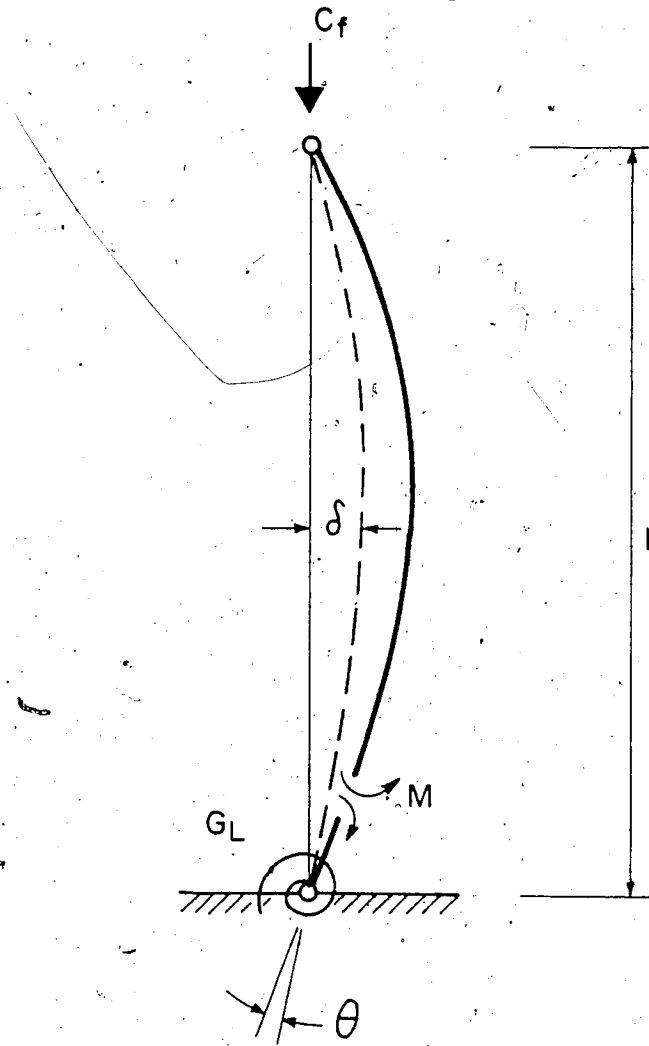


FIGURE 5.7 DERIVATION OF A STRENGTH REQUIREMENT

$$\theta = \frac{G_L L}{2EI} M \quad (5.14)$$

To ensure compatibility the lower end of the column must also undergo this angle change. In Reference (39) it is shown that the slope deflection equation for the member, modified to include the effect of axial force, is:

$$\theta = -\frac{ML}{2EIU} \left( \frac{1}{2U} - \frac{1}{\tan 2U} \right) + \left( \frac{\pi}{2U} \right) \frac{\pi \delta}{L}$$

where

$$2U = \pi \sqrt{\frac{C_f}{C_e}} \quad (5.16)$$

in which  $C_e$  is the Euler buckling load given by Equation (5.3) with  $k = 1.0$ .

Equating the right hand sides of Equations (5.14) and (5.15) and solving for  $M$  leads to:

$$\frac{M}{C_e \delta} = \frac{\frac{1}{\pi^2} \left( \frac{1}{(\frac{\pi}{2U})^2 - 1} \right)}{\frac{1}{2U} \left( \frac{1}{2U} - \frac{1}{\tan 2U} \right) + \frac{G_L}{2}} \quad (5.17)$$

### Example 5.2

Given

Compute the moment that must be resisted by the base

plate and foundation described in Example 5.1. The restraint parameter for the lower end of the column is 0.82.

Solution

The moment of inertia of the W8x40 section is listed in

Reference (5):

$$I = 49.1 \text{ in.}^4$$

The Euler buckling load is:

$$\begin{aligned} C_e &= \frac{\pi^2 EI}{L^2} && \text{(Eq. 5.3)} \\ &= \frac{\pi^2 \times 29000 \times 49.1}{(233)^2} \\ &= 259 \text{ kips} \end{aligned}$$

thus

$$\begin{aligned} 2U &= \pi \sqrt{\frac{C_f}{C_e}} && \text{(Eq. 5.16)} \\ &= \pi \sqrt{\frac{196}{259}} \\ &= 2.73 \end{aligned}$$

Substitution of this value of  $2U$  and  $G_L = 0.82$  into the right hand side of Equation (5.17) gives:

$$\frac{M}{C_e \delta} = 0.715 \quad (\text{Eq. 5.17})$$

or

$$\begin{aligned} M &= 0.715 \times 259 \times (0.001 \times 233) \\ &= 43.2 \text{ inch-kips.} \end{aligned}$$

This is a very small moment and can be resisted without inducing calculable axial forces in the anchor bolts since they are unloaded for applied moments up to:

$$\begin{aligned} M &= \frac{C_f B}{6} \quad (\text{Eq. 5.9}) \\ &= \frac{196 \times 11}{6} \\ &= 359 \text{ inch-kips.} \end{aligned}$$

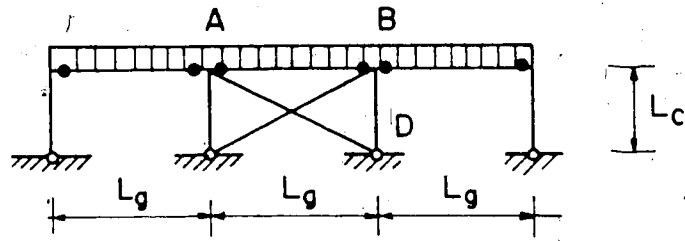
To summarize, it appears that the detail shown in Figure 5.6, usually thought of as a pinned base connection, is in fact a fixed base connection, even if the soil is a relatively low strength soil. For the specific case considered in Examples 5.1 and 5.2 the restraint parameter  $G_L$  was found to be lower than the value of unity usually used for fixed bases<sup>(2)</sup>. At the same time the moment that must be resisted by the base is very small, and in fact can be resisted without straining the anchor bolts. Additional analytical and experimental research is required, however, to determine the effect of non-linear behaviour in the steel,

concrete and the soil, as well as the effect of different soil types. Specific recommendations cannot be made at this time.

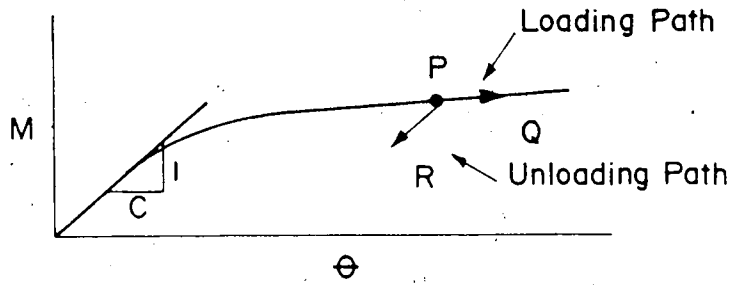
### 5.2.3 Restraint Provided By Girders With Semi-Rigid Connections

When resistance to lateral loads and sway effects is provided by a suitable system of bracing, advantage can be taken of the restraining effect of semi-rigid girder-to-column connections to reduce the effective lengths of the columns to values less than the actual lengths. If, instead, the structure is designed to resist gravity loads on the basis of simple construction and is proportioned to resist lateral loads on the basis of rigid frame action, the flexibility of the connections must be considered in assessing the stability of the structure(1). Procedures for accounting for the restraint provided by semi-rigid connections are presented below.

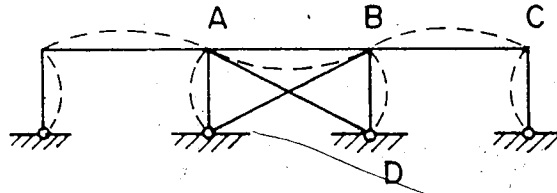
Figure 5.8(a) shows a "sidesway prevented" pinned base frame in which the girders are semi-rigidly connected to the columns. It may be assumed that under gravity loads, before application of lateral loads, the connections are loaded well into the inelastic ranges of the moment-rotation curves, indicated in Figure 5.8(b) by point p(63). Plastic hinges are therefore shown schematically in the girders at each connection. As the vertical loads are increased, the frame will suddenly snap into a deflected (buckled) configuration. If the joints are rigid, these additional deflections are as shown in Figure 5.8(c).



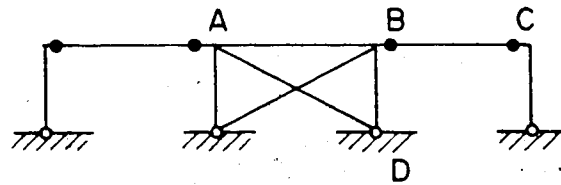
(a)



(b)



(c)



(d)

FIGURE 5.8 RESTRAINT PROVIDED BY GIRDERS IN SIMPLE CONSTRUCTION



The connections on girders such as BC, therefore, tend to follow the loading path PQ in Figure 5.8(b), and the plastic hinges act as real hinges during the buckling motion. The connections on girders such as AB unload during the buckling motion, and behave as elastic, semi-rigid connections. Thus column BD is restrained only by girder AB, as illustrated in Figure 5.8(d).

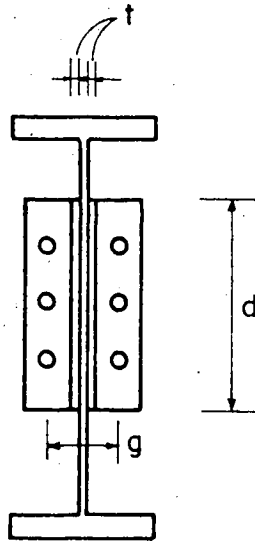
The restraint parameter  $G_u$  can be determined from the slope-deflection equation for a girder of length  $L_g$  and moment of inertia  $I_g$  (64):

$$\theta = CM + \frac{ML_g}{2EI_g} \quad (5.18)$$

where  $C$  is the flexibility of the connection, defined as the inverse of the slope of the moment-rotation curve as shown in Figure 5.8(b). Values of connection flexibilities are tabulated in References (64) and (65). For example, the flexibility of a double web angle connection is given in Figure 5.9, adapted from Reference (64).

Solving Equation (5.18) for  $M$ ,

$$M = \left( \frac{1}{1 + \frac{2EI_g C}{L_g}} \right) \frac{2EI_g}{L_g} \theta \quad (5.19)$$



$$C = 3.66 \times 10^{-4} d^{-24} f^{-1.81} g^{0.15}$$

FIGURE 5.9 EXPRESSION FOR THE FLEXIBILITY  
"C" OF A DOUBLE ANGLE CONNECTION

For a rigid connection  $C = 0$  and the term in brackets is unity. Therefore it is seen that the effect of the semi-rigid connection can be accounted for in assessing the girder restraint by using a reduced girder moment of inertia  $I_{gr}$ :

$$I_{gr} = \left( \frac{1}{1 + \frac{2EI_g C}{L_g}} \right) I_g \quad (5.20)$$

Thus the restraint parameter for the upper end of the column is:

$$G_u = \frac{\sum I_c / L_c}{\sum I_{gr} / L_g} \quad (5.21)$$

Since only one girder frames into the joint the summation sign in the denominator can be omitted.

### Example 5.3

#### Given

In simply supported roof framing schemes the restraint provided to the columns by the nominally pin-ended girders is often sufficient to almost completely fix them at their upper ends. As an example, compute the restraint parameter  $G_u$  for weak axis bending of a 20 ft. long W8x28 column if W21x62 girders 40

ft. long frame into the column web. The connection detail is the double clip angle connection shown in Figure 5.9 in which:

$$d = 12 \text{ in.}$$

$$t = 0.3125 \text{ in.}$$

$$g = 3.75 \text{ in.}$$

The frame is braced.

### Solution

The moments of inertia of the column and girder are listed in Reference (5):

$$I_C = 21.6 \text{ in.}^4$$

$$I_g = 1330 \text{ in.}^4$$

The connection flexibility constant  $C$  is, from Figure 5.9:

$$\begin{aligned} C &= 3.66 \times 10^{-4} d^{-2.4} t^{-1.8} g^{0.15} \\ &= 3.66 \times 10^{-4} 12^{-2.4} 0.3125^{-1.8} 3.75^{0.15} \\ &= 9.42 \times 10^{-6} \text{ radians/inch-kip.} \end{aligned}$$

Thus:

$$I_{gr} = \left( \frac{1}{1 + \frac{2EI_g C}{L_g}} \right) I_g \quad (\text{Eq. 5.20})$$

$$= \left( \frac{1}{1 + \frac{2 \times 29000 \times 1330 \times 9.42 \times 10^{-6}}{40 \times 12}} \right) \times 1330$$

$$= 529 \text{ in.}^4$$

and:

$$G_u = \frac{\sum I_c / L_c}{\sum I_{gr} / L_g} \quad (\text{Eq. 5.21})$$

$$= \frac{\frac{21.6}{20}}{\frac{529}{40}}$$

$$= 0.082$$

From Figure 5.5(a) it is seen that the restraint provided is almost sufficient to completely fix the column at its upper end.

---

#### 5.2.4 Interior Beam-Columns

A beam-column is a member subjected to both axial force and bending moments. Beam-columns occur in the interior of a braced structure if the vertical forces are eccentric, and in an unbraced structure if lateral loads and sway effects are resisted by rigid or semi-rigid frame action.

The carrying capacity of an interior beam-column depends

on the cross section of the member, the member length, degree of end restraint, grade of steel, and the ratio of the end moments. Assuming that premature plate buckling is avoided by keeping the width-thickness ratios of the plate elements less than the limiting values specified in Reference (1), the member capacity is limited either by a strength failure at one of the member ends, or by a stability failure.

Exact methods for the analysis of beam-columns of wide flange or square hollow structural steel sections are available (2), however these approaches are too involved for the design office and use is made instead of empirical interaction equations. A detailed discussion of the behaviour of beam-columns and the development of interaction equations may be found in Reference (2).

For the purposes of this report it will be assumed that the beam-columns are subjected only to compression and strong axis bending. The interaction equations recommended in Reference (1) for either Class 1 or Class 2 I-shaped members become:

$$\frac{M_f}{\phi M_p} \leq 1.0 \quad (5.22)$$

$$\frac{C_f}{\phi C_y} + 0.85 \frac{M_f}{\phi M_p} \leq 1.0 \quad (5.23)$$

and

$$\frac{C_f}{C_r} + \frac{\omega M_f}{M_r \left(1 - \frac{C_f}{C_{ex}}\right)} \leq 1.0 \quad (5.24)$$

Equations (5.22) and (5.23) are strength equations; their purpose is to prevent the occurrence of a plastic hinge at either the upper or lower end of the beam-column. In these equations:

$M_f$  = maximum moment resulting from the factored loads,  
and

$M_p$  = plastic moment of the cross section about its strong axis.

Equation (5.24) is a stability equation; its purpose is to prevent failure between the ends of the beam-column. In this equation,

$\omega$  = a coefficient used to determine an equivalent uniform bending effect when the end moments are not equal and opposite,

$C_{ex}$  = Euler buckling load about the X axis,

$M_r$  = factored moment resistance of the member about the X axis in the absence of an axial force, but considering the possibility of lateral-torsional instability.

For Class 3 I-shaped members it is recommended that the strength equations be replaced by<sup>(1)</sup>:

$$\frac{C_f}{\phi C_y} + \frac{M_f}{\phi M_y} \leq 1.0 \quad (5.25)$$

where  $M_y$  is the yield moment of the cross section about its strong axis but that the stability equation remain unchanged.

For Class 1 or Class 2 square hollow structural steel sections either hot rolled, or cold rolled and stress relieved such that the residual stresses do not exceed  $0.3 F_y$ , Equations (5.22) to (5.24) may be used directly<sup>(66)</sup>.

Until recently, frames were either classified as sidesway prevented (braced) or sidesway permitted (unbraced) and the effective length factors were determined from the appropriate nomograph. However the lateral stiffnesses of both braced and unbraced frames are approximately the same, since they are designed to the same deflection limitation for lateral loads. Sidesway is therefore not prevented in a braced frame any more than it is in an unbraced frame.

This consideration led to a newer approach to design<sup>(67)</sup>. With this approach both braced and unbraced structures are analyzed using a method that includes the sway effects, a "second order" analysis. Then, at the design stage, the effective length factors are determined from the nomograph in Figure 5.5(a).



Since the sway effects are included in the analysis, they are not considered again during the design by using effective column length factors greater than unity. Procedures for performing a second order analysis are discussed in Section 6.3.

If the analysis includes the sway effects, the equivalent uniform bending coefficients can be computed from (67):

$$\omega = 0.6 + 0.4 M_{f1}/M_{f2} \text{ for members bent in single curvature} \quad (5.26)$$

$$\omega = 0.6 - 0.4 M_{f1}/M_{f2} \text{ for members bent in double curvature, but not less than 0.4} \quad (5.27)$$

where  $M_{f1}/M_{f2}$  = ratio of the smaller moment to the larger moment at opposite ends of the unbraced length, in the plane of bending considered;

Otherwise,

$$\omega = 0.85 \text{ for members bent in double curvature or subject to moment at one end} \quad (5.28)$$

$$\omega = 1.0 \text{ for members bent in single curvature due to moments at both ends} \quad (5.29)$$

There are no interior beam-columns in the building selected for the illustrative examples in this report. However

the interaction equations discussed in this section are also used for designing exterior beam-columns, therefore examples illustrating their use will be given after design of these members is discussed (Section 5.3.3).

### 5.3 Exterior Compression Members

#### 5.3.1 Full Bracing

Design of exterior compression members differs from the design of interior compression members in that they are usually braced, or partially braced, along their lengths by the wall cladding. Full bracing of a compression member is defined as bracing such that any increase in stiffness or strength does not significantly increase the carrying capacity of the member. For example, an eight inch thick masonry wall connected into the web of an exterior column is sufficient to prevent weak axis instability, and to force failure to occur through strong axis instability instead. Increasing the thickness of the masonry wall to twelve inches does not cause a proportionate increase in the column strength. The column is fully braced by the eight inch thick wall.

For simplicity, and because it has been found that it is usually not economical to provide anything less than full bracing<sup>(68)</sup>, only the design of fully braced compression members are considered in this report. After the design procedures are

presented for these members, bracing requirements will be discussed.

Only exterior compression members in "sideways prevented" frames are examined. If the frame under consideration is unbraced, it is assumed that the  $P\Delta$  effects are included at the analysis, rather than at the design stage<sup>(67)</sup>.

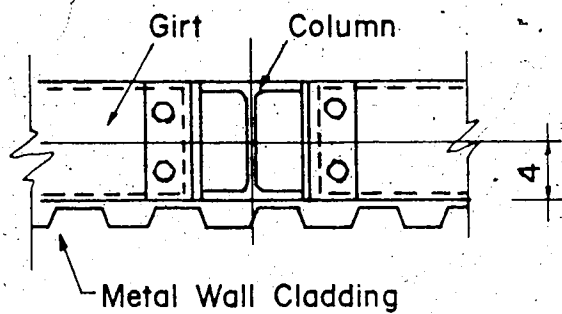
### 5.3.2 Exterior Column

The strength of an exterior column can also be determined from Equation (5.1), discussed in connection with behaviour of interior columns. Procedures for determining the non-dimensional slenderness ratio,  $\lambda$ , for four different types of wall construction are given below.

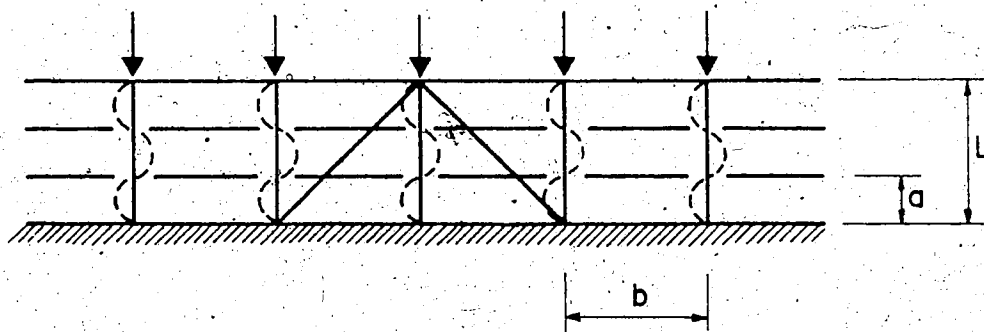
#### 1. Columns Braced in the Weak Direction By Metal Siding

If the wall consists of girts with metal wall cladding, connected so as to brace the columns in the weak direction, as shown in Figure 5.10(a), column failure may occur either by strong axis instability, or by weak axis instability between girts, as shown in Figure 5.10(b).

If failure occurs through strong axis instability the effective length is determined, as for an interior column, by assessing the conditions of restraint at the ends of the member. It is usually conservatively assumed that the flexural stiffness



(a)



(b)

FIGURE 5.10 COLUMNS BRACED IN THE WEAK DIRECTION BY METAL SIDING

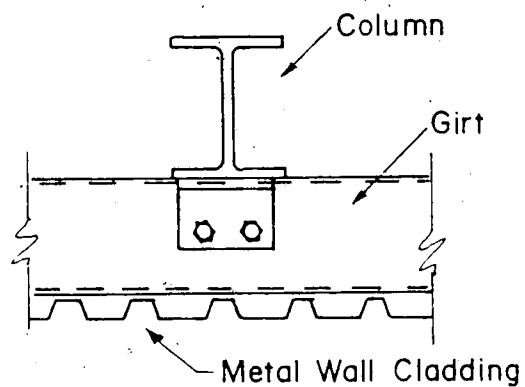
of the wall cladding does not inhibit strong axis bending. In this respect it should be noted that, as illustrated in Figure 5.8(d), an exterior column is not restrained at its upper end by a girder with a semi-rigid connection. In this situation the connection, which was shown to act as a plastic hinge under vertical loads, acts as a real hinge during the buckling motion. A procedure for assessing the restraint at the base of the column is discussed in Section 5.2.3.

If failure occurs through weak axis instability between girts the effective length depends on the conditions of end restraint and on the number of girts. Rather than attempting a rigorous analysis, it is assumed for simplicity that the effective length is equal to the girt spacing.

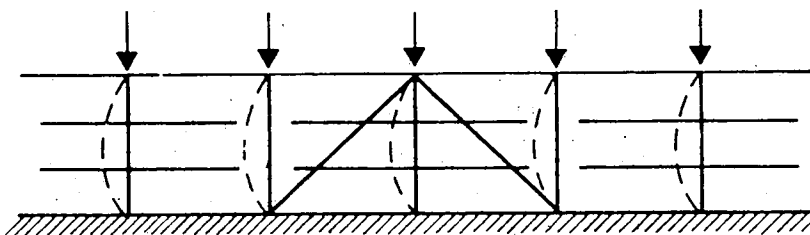
After the effective lengths for both failure modes are determined, the non-dimensional slenderness ratios are computed using Equation (5.2). The largest slenderness ratio so computed governs the design. The factored compressive resistance of the column is then determined from Equation (5.1).

## 2. Columns Braced About Their Outside Flanges By Metal Siding

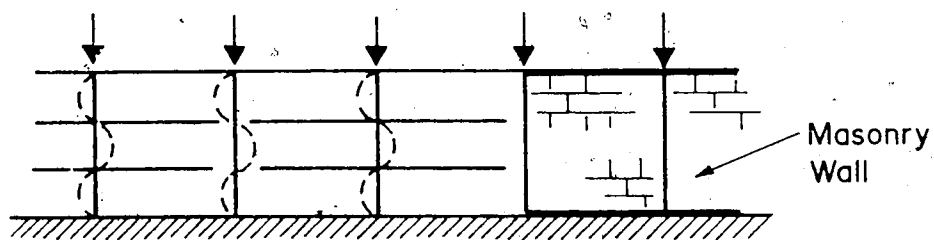
If the wall consists of girts connected to the flanges of the columns, as shown in Figure 5.11(a), the wall cladding is unable to function as a shear resistant diaphragm (see Section 5.3.4). The columns are unsupported along their lengths and



(a)



(b)



(c)

**FIGURE 5.11 COLUMNS BRACED ABOUT THEIR OUTSIDE FLANGES BY METAL SIDING**

failure occurs by weak axis instability of all the columns as a unit, as illustrated in Figure 5.11(b). It should be noted that in this situation the axial stiffnesses of the girts alone cannot be developed to brace the columns.

The effective length factor of a particular column is determined by assessing the conditions of end restraint. The column is restrained at the top by the perimeter roof beams connecting into it. The amount of restraint depends on the flexibility of the semi-rigid connection, and can be determined as described in Section 5.2.4. The column is restrained at its base by the foundation, as discussed in Section 5.2.3. The factored compressive resistance of the column is then determined from Equations (5.1) and (5.2).

However, if one or more wall panels consist of masonry in-fill, as shown in Figure 5.11(c), the axial stiffnesses of the girts can be developed, and failure of the column may occur either by strong axis instability, or by weak axis instability between girts. The longest effective length is used to compute the factored compressive resistance.

### 3. Columns Braced in the Weak Direction by Masonry

If the wall consists of masonry connected into the webs of the columns, as shown in Figure 5.12, failure can occur only through strong axis instability. As discussed for the case of





girts connected into the column web, the effective length of a particular column can be determined by assessing the conditions of restraint at the ends of the member, assuming that the wall does not inhibit strong axis bending. The factored compressive resistance is then determined using Equations (5.1) and (5.2). Tests are required to determine if it is overly conservative to neglect the flexural stiffness of the masonry wall.

#### 4. Columns Braced About Their Outside Flanges By Masonry

If the masonry is connected into the outside flange of the column instead, failure may occur either through strong axis instability, as discussed above, or by a twisting motion about the restrained outside flange as shown in Figure 5.13. The effective lengths for both failure modes should be computed, and the longer used for design with Equations (5.1) and (5.2).

The critical load for the torsion buckling mode is (68,69,70):

$$C_{\theta} = \frac{\pi^2 E Y_y}{L^2} + \frac{2GJ}{d^2} \quad (5.30)$$

where

G = shearing modulus,

J = St. Venant's torsion constant, and

d = depth of wide flange section.

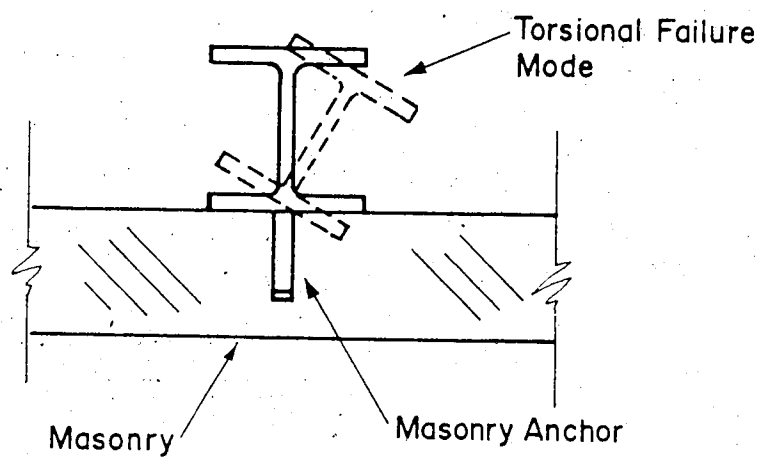


FIGURE 5.13 COLUMNS BRACED ABOUT THEIR OUTSIDE FLANGES BY MASONRY

The first term in Equation (5.30) is the Euler buckling load for the column for weak axis instability. The second term is the additional load that can be resisted as a result of the restraint along the length of the member. The effective length is determined by equating the right hand sides of Equations (5.3) and (5.30):

$$\frac{\pi^2 EI_y}{(kL)^2} = \frac{\pi^2 EI_y}{L^2} + \frac{2GJ}{d^2} \quad (5.31)$$

or

$$kL = L \sqrt{\frac{1}{1 + \frac{2GJL^2}{\pi^2 EI_y d^2}}} \quad (5.32)$$

The effective lengths for both failure modes should be computed, and the longer used for design with Equations (5.1) and (5.2).

#### Example 5.4

##### Given

Although the exterior compression member shown in Figure 5.2 does not act as an axially loaded column, the factored compressive resistance is still required for use with the more general beam-column interaction equation. Compute the factored compressive resistance of this member using a W8x24 shape as a trial section. Use G40.21 44W steel ( $F_y = 44$  ksi).

Solution

The section properties for the W8x24 section are listed in Reference (5):

$$r_x = 3.42 \text{ in.}$$

$$r_y = 1.61 \text{ in.}$$

$$A = 7.06 \text{ in.}^2$$

Results of preliminary calculations (not shown) indicate that the W8x24 section is a Class 2 section in 44W steel, therefore the factored compressive resistance can be computed using Equation (5.1). As discussed in the text, the governing non-dimensional slenderness ratio is the largest of those corresponding to strong axis instability over the length of the column, and weak axis instability between girts.

The non-dimensional slenderness ratio for strong axis instability can be determined by assessing the conditions of restraint at the ends of the member. Assuming a restraint parameter of 10 at the lower end of the column (a "pinned" base) and a restraint parameter of infinity at the upper end, the effective length factor can be determined from Figure 5.5(a) as 0.97.

Assuming that the actual column length, measured from the base plate to the centroidal axis of the bolt group in the girder-to-column header connection is 20 feet, the non-dimensional slenderness ratio is:

$$\begin{aligned}
 \lambda &= \frac{KL}{r} \sqrt{\frac{F_y}{\pi^2 E}} && \text{(Eq. 5.2)} \\
 &= \frac{0.97 \times 20 \times 12}{3.42} \sqrt{\frac{44}{\pi^2 \times 29000}} \\
 &= 0.842
 \end{aligned}$$

Also as discussed in the text, it will be assumed that the effective length for weak axis instability between girts is equal to the girt spacing of 7 feet. The non-dimensional slenderness ratio is, therefore,

$$\begin{aligned}
 \lambda &= \frac{KL}{r} \sqrt{\frac{F_y}{\pi^2 E}} && \text{(Eq. 5.2)} \\
 &= \frac{7 \times 12}{1.61} \sqrt{\frac{44}{\pi^2 \times 29000}} \\
 &= 0.648
 \end{aligned}$$

Using the governing slenderness ratio of 0.842, the factored compressive resistance is:

$$\begin{aligned}
 C_r &= \phi A F_y (1.035 - 0.202\lambda - 0.222\lambda^2) && \text{(Eq. 5.1)} \\
 &= 0.9 \times 7.06 \times 44 (1.035 - 0.202 \times 0.648 - 0.222 \times (0.648)^2) \\
 &= 198 \text{ kips.}
 \end{aligned}$$

This result will be used in Example 5.5 where the member shown in Figure 5.2 is analyzed as a beam-column.

### 5.3.3 Exterior Beam-Columns

The interaction equations discussed in Section 5.2.5 for an interior beam-column can also be used for the design of an exterior beam-column. However, since this member may be subjected to transverse forces in addition to end moments, the maximum factored bending moment does not necessarily occur at one of the ends of the member.

The maximum factored bending moment is multiplied by an equivalent uniform bending coefficient,  $w$ , to obtain a uniform bending effect<sup>(1)</sup>. If the beam-column is subjected to a compressive force and end moments, the equivalent uniform bending coefficient can be determined as for an interior beam-column. If the beam-column is subjected to a compressive force and transverse forces, but not to end moments, the equivalent uniform bending coefficient is 0.85 for one transverse force and unity for two or more<sup>(1)</sup>. If the beam-column is subjected to a compressive force, end moments and to any number of transverse forces, the equivalent uniform bending coefficient is unity<sup>(1)</sup>.

As discussed in Section 5.3.2, the factored compressive resistance depends on the type of wall construction. The factored moment resistance also depends on the type of wall construction. This will be explained below for the four types of wall construction considered earlier.

## 1. Columns Braced in the Weak Direction By Metal Siding

An approximate procedure to determine the factored moment resistance of the member is to assume that each segment of the beam-column between girts behaves as a simply supported member subjected to end moments. The factored moment resistance is then that of the most heavily loaded segment, computed using Equations (4.3) and either (4.9) or (4.10).

In computing the elastic lateral-torsional buckling moment  $M_u$ , the equivalent uniform bending coefficient is determined using the moments at the end of the segment, not at the ends of the member. This will usually not be equal to the equivalent uniform bending coefficient used in the numerator of the stability interaction equation to obtain a uniform bending effect.

## 2. Columns Braced About Their Outside Flanges By Metal Siding

If one or more panels of the wall consist of masonry infill, as shown in Figure 5.11(c), it will be assumed that the factored moment resistance can be determined by analysing the most heavily loaded segment between girts, as discussed above. Otherwise, it will be assumed that the member is completely unbraced along its length.

This is obviously an approximation since some rotational restraint will be provided by resistance of the girts to

bending. The amount, however, will depend on the flexibility of the girt-to-column connections and on the unsupported lengths of the girts. An exact analysis of this is considered to be beyond the scope of this report.

### 3. Columns Braced In The Weak Direction By Masonry

Since the member is completely laterally supported along its length the factored moment resistance can be based on the full in-plane strength of the cross section.

### 4. Columns Braced About Their Outside Flanges By Masonry

As discussed in Section 4.3, the equivalent uniform bending coefficient used to determine the elastic lateral-torsional buckling moment depends on the loading and support conditions, and also on the non-dimensional parameter  $L^2GJ/EC_w$ . In the following studies it is assumed that the member is pin-ended, and that the tension flange is restrained in such a way that translation, but not rotation, is prevented. Three different loading cases will be examined: end moments only, transverse forces only, and end moments with transverse forces.

#### (a) Member Subjected to End Moments Only

Consider a pin-ended member subjected to end moments  $M_{f1}$  and  $M_{f2}$ , where  $M_{f2}$  is numerically larger than  $M_{f1}$ . For



convenience in later algebraic manipulation, the equivalent uniform bending coefficient for end moments is denoted  $\omega_1$ .

The relationship between the equivalent uniform bending coefficient and the end moment ratio, determined with an elastic finite element computer program is shown by the solid lines in Figure 5.14 for two different  $L^2GJ/EC_w$  ratios. The  $L^2GJ/EC_w$  ratios of 10 and 35 correspond to the extremes likely to be encountered in light industrial buildings. For example, for a W8x24 member 21 feet long this ratio is 33.

The procedure used to construct these curves is as follows. For a member with given  $L^2GJ/EC_w$  and  $M_{f1}/M_{f2}$  ratios, the critical value of  $M_{f2}$  is determined using the finite element computer program. The elastic lateral-torsional buckling moment  $M_u$  of a member with the same cross section, unrestrained along its length, and subjected to uniform bending is then computed from Equation (4.13). The equivalent uniform bending coefficient is, then, by definition,

$$\omega_1 = \frac{M_u}{M_{f2}} \quad (5.33)$$

As indicated by the broken lines in Figure 5.14, a close estimate of the equivalent uniform bending coefficient is:

$$\omega_1 = 0.5 + 0.4 M_{f1}/M_{f2} \quad (5.34)$$

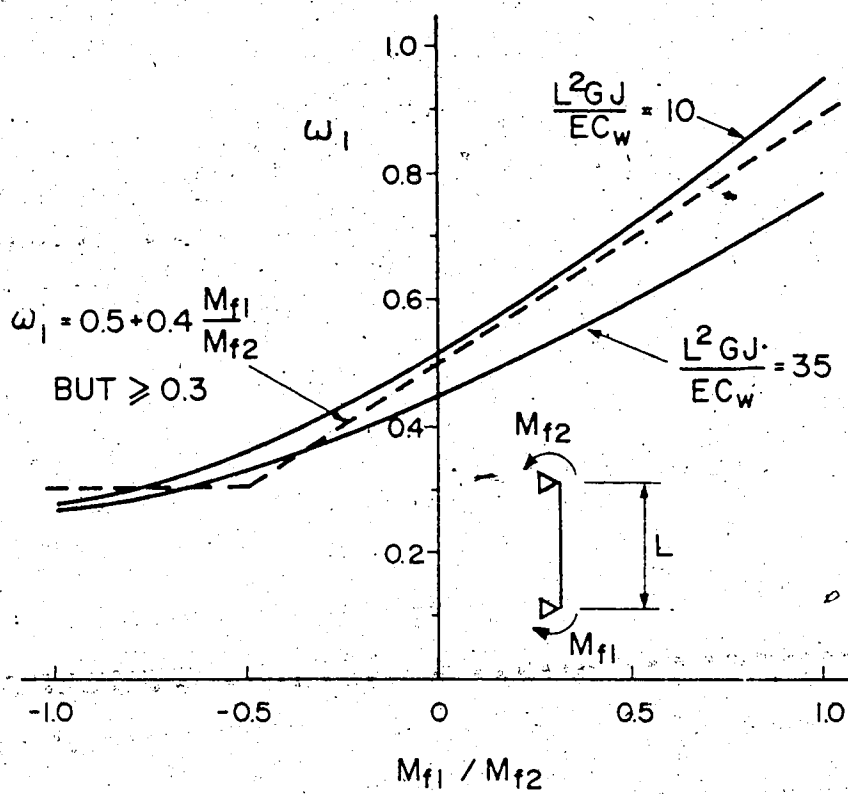


FIGURE 5.14 MEMBER SUBJECTED TO END MOMENTS ONLY

for members bent in single curvature, and

$$\omega_1 = 0.5 - 0.4 M_{F1}/M_{F2} \quad (5.35)$$

for members bent in double curvature, but not less than 0.3. These equations may be compared with the equations recommended in Reference (1), for laterally unbraced members. The difference reflects the effect of the restraint along the column. If this restraint is present, the lower value of omega given above may be used.

(b) Members Subjected to Transverse Forces Only

Similar studies were done for the loading cases shown in Figure 5.15. Denoting the equivalent uniform bending coefficient for transverse forces as  $\omega_2$ , it can be seen that a close and generally conservative approximation is  $\omega_2 = 0.4$ . As discussed above, the difference between this value and the value of 1.0 recommended in Reference (1) reflects the effect of the restraint.

(c) Members Subjected to End Moments and Transverse Forces

Consider next a member subjected to both end moments and transverse forces, as illustrated in Figure 5.16. The maximum moment, which may either occur at one of the ends of the member, or at some point along the member length, is denoted  $M_3$ . The maximum moment caused by end moment alone is denoted  $M_1$ , and the maximum moment caused by the transverse forces alone is denoted  $M_2$ . The elastic lateral-torsional buckling moment of a simply supported unbraced beam with the same length and cross sectional properties is denoted  $M_0$ .

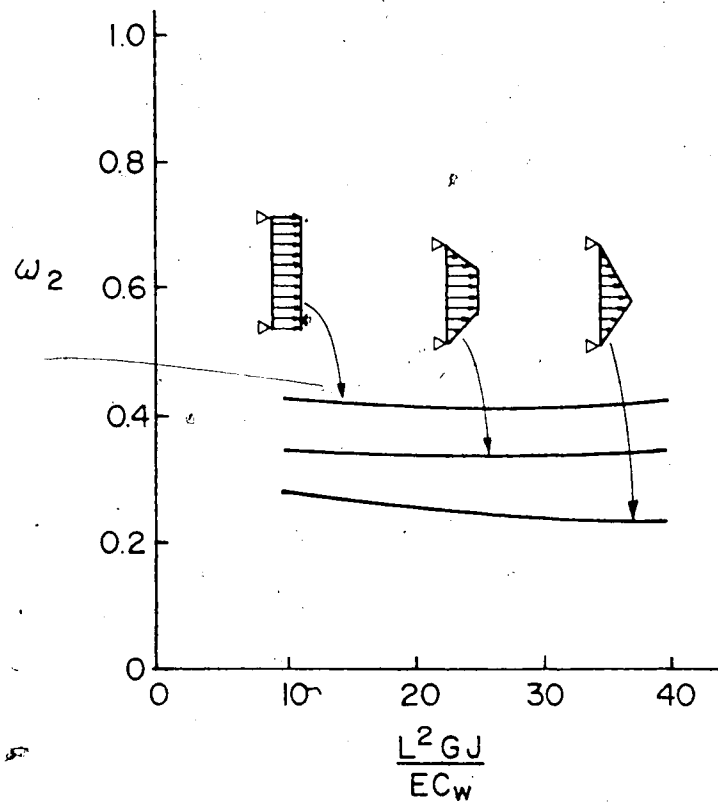


FIGURE 5.15 MEMBER SUBJECTED TO TRANSVERSE FORCES ONLY

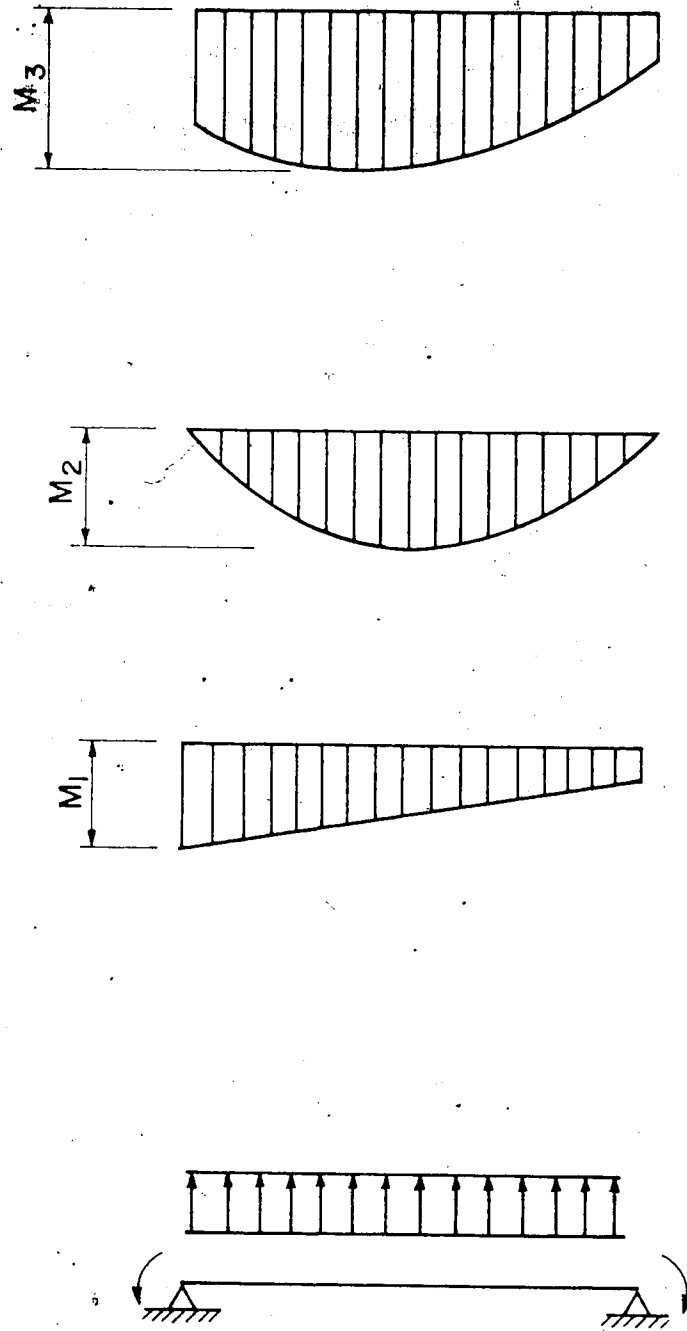


FIGURE 5.16 MEMBERS SUBJECTED TO BOTH END MOMENTS AND TRANSVERSE FORCES

In Reference (71) it is shown that if the buckled shapes for both loadings acting separately are identical, then the buckling condition when both loading occur at the same time is:

$$\omega_1 M_1 + \omega_2 M_2 = M_0 \quad (5.36)$$

If the buckled shapes are similar, but not identical, this relationship is approximate.

Denoting  $\omega_3$  as the equivalent uniform bending coefficient for the combined loading case,

$$\frac{(\omega_1 M_1 + \omega_2 M_2)}{M_3} M_3 = M_0 \quad (5.37)$$

or

$$\omega_3 M_3 = M_0 \quad (5.38)$$

where

$$\omega_3 = \frac{\omega_1 M_1 + \omega_2 M_2}{M_3} \quad (5.39)$$

### Example 5.5

#### Given

Select a wide flange section for the exterior beam-

column shown in Figures 5.1 and 5.2. The length of the member, measured from the underside of the base plate to the centroid of the girder connection is 20 feet. Use G40.21 44W steel ( $F_y = 44$  ksi).

Solution

Try a W8x28

The section properties for the W8x28 section are listed in Reference (5):

$$\begin{array}{ll} I_x = 82.5 \text{ in.}^4 & C_w = 258 \text{ in.}^6 \\ I_y = 18.2 \text{ in.}^4 & Z = 23.1 \text{ in.}^3 \\ J = 0.343 \text{ in.}^4 & A = 7.06 \text{ in.}^2 \end{array}$$

The factored compressive resistance for this member was shown in Example 5.4 to be 198 kips.

Two different load combinations must be investigated:

1. dead load plus snow load, and
2. dead load plus snow load and wind load.

With the first load combination the member is subjected to an eccentric load equal to the 80 kip reaction from the exterior cantilever girder (see Figure 4.8). Assuming, as illustrated in

Figure 5.2, that the distance from the centroid of the girder connection to the column face is 2-1/4 inches, the eccentricity is equal to this amount plus the half depth of the section, or 6-1/4 inches. Thus the member can be designed for a concentric axial load of 80 kips and an end moment of  $80 \times 6.25 = 500$  inch-kips, as illustrated in Figure 5.17(a).

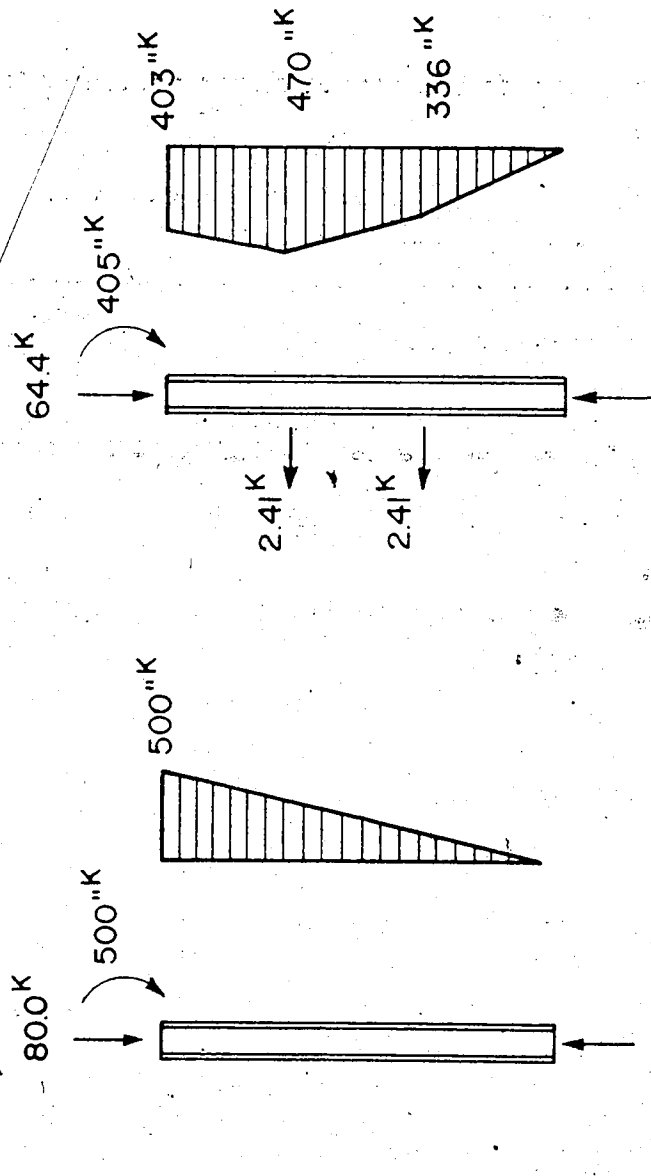
The second load combination is illustrated in Figure 5.17(b). In this Figure the axial load and end moment from the first load case are multiplied by the ratio of the factored dead load and reduced snow load to the factored dead load and full snow load:

$$\frac{1.25 \times 31.5 + 0.7 \times 1.5 \times 48}{1.25 \times 31.5 + 1.5 \times 48} = 0.806$$

where 31.5 psf is the specified dead load (Example 2.1), 48 psf is the specified snow load, 0.7 is the load combination factor for snow load and wind load acting together, and 1.25 and 1.5 are the dead and live load factors, respectively. The wind forces are equal to the specified wind load of 16.4 psf (Example 2.4) multiplied by the live load combination factor of 0.7, the live load factor of 1.5 and the tributary area of one girt, 7 x 20 square feet.

The Euler buckling load for strong axis instability, required for use with the stability interaction equation is:





(a)  $1.25 D + 1.5 L$

(b)  $1.25 D + 0.7 (1.5 L + 1.5 Q)$

FIGURE 5.17 DESIGN CONDITIONS FOR THE EXTERIOR BEAM-COLUMN

$$C_e = \frac{\pi^2 EI}{L^2} \quad (\text{Eq. 5.3})$$

$$= \frac{\pi^2 \times 29000 \times 82.5}{(20 \times 12)^2}$$

$$= 410 \text{ kips.}$$

The equivalent uniform bending coefficient, used to reduce the maximum bending moment, is:

$$\omega = 0.6 + 0.4 \frac{M_{f1}}{M_{f2}} = 0.6 \quad (\text{Eq. 5.26})$$

for loading case 1, and 1.0 for loading case 2.

#### Loading Case 1

The factored moment resistance is computed by assuming that the most heavily loaded beam-column segment (ie., the upper third) acts as a simply supported, laterally unbraced member subjected to the end moments

$$M_{f2} = 500 \text{ inch-kips}$$

$$M_{f1} = \frac{2}{3} \times 500 \text{ inch-kips}$$

Therefore the equivalent uniform bending coefficient used to compute the elastic lateral-torsional buckling moment of the segment is:

$$\begin{aligned}
 \omega &= 0.6 + 0.4 \frac{M_{f1}}{M_{f2}} && \text{(Eq. 5.26)} \\
 &= 0.6 + 0.4 \times \frac{2}{3} \\
 &= 0.87
 \end{aligned}$$

The buckling moment is

$$\begin{aligned}
 M_u &= \frac{\pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} && \text{(Eq. 4.3)} \\
 &= \frac{\pi}{0.87 \times 84} \sqrt{29000 \times 18.2 \times 11500 \times 0.343 + \left(\frac{\pi \times 29000}{84}\right)^2 \times 18.2 \times 258} \\
 &= 3745 \text{ inch-kips}
 \end{aligned}$$

Results of calculations (not shown) indicate that the flange width-to-thickness ratio and (for the 80 kip axial load) that the web height to thickness ratio meet the requirements of a Class 2 Section (1). Thus the factored moment resistance can be computed using Equations (4.8) and (4.9).

The plastic moment capacity is:

$$\begin{aligned}
 M_p &= ZF_y && \text{(Eq. 4.1)} \\
 &= 23.1 \times 44 \\
 &= 1016 \text{ inch-kips}
 \end{aligned}$$

The factored moment resistance is then determined as:

$$M_r = 1.15 \phi M_p \left(1 - 0.28 \frac{M_p}{M_u}\right) \quad (\text{Eq. 4.9})$$

$$= 1.15 \times 0.9 \times 1016 \left(1 - 0.28 \times \frac{1016}{3745}\right)$$

$$= 972 \text{ inch-kips}$$

however this must not exceed

$$M_r = \phi M_p$$

$$= 0.9 \times 1016$$

$$= 914 \text{ inch-kips (governs)}$$

The strength and stability equations can now be checked:

$$\frac{M_f}{\phi M_p} \leq 1.0 \quad (\text{Eq. 5.22})$$

$$\frac{500}{0.9 \times 1016} \leq 1.0$$

$$0.55 \leq 1.0 \quad (\text{OK})$$

$$\frac{C_f}{\phi C_y} + 0.85 \frac{M_f}{\phi M_p} \leq 1.0 \quad (\text{Eq. 5.23})$$

$$\frac{80}{0.9 \times 7.06 \times 44} + \frac{0.85 \times 500}{0.9 \times 1016} \leq 1.0$$

$$0.29 + 0.46 \leq 1.0$$

$$0.75 \leq 1.0 \quad (\text{OK})$$

$$\frac{C_f}{C_r} + \frac{\omega M_f}{M_r \left(1 - \frac{C_f}{C_{ex}}\right)} \leq 1.0 \quad (\text{Eq. 5.24})$$

$$\frac{80}{198} = \frac{0.6 \times 500}{914 \left(1 - \frac{80}{410}\right)} \leq 1.0$$

$$0.40 + 0.41 \leq 1.0$$

$$0.81 \leq 1.0 \quad (\text{OK})$$

therefore the member can satisfactorily resist loading case 1.

### Loading Case 2

Proceeding in a manner similar to that described above, it can be easily shown that the factored moment resistance for this loading case is also 914 inch-kips. Checking the strength and stability interaction equations:

$$\frac{M_f}{\phi M_p} \leq 1.0 \quad (\text{Eq. 5.22})$$

$$\frac{470}{0.9 \times 1016} \leq 1.0$$

$$0.51 \leq 1.0 \quad (\text{OK})$$

$$\frac{C_f}{\phi C_y} + 0.85 \frac{M_f}{\phi M_p} \leq 1.0 \quad (\text{Eq. 5.23})$$

$$\frac{64}{0.9 \times 7.06 \times 44} + \frac{0.85 \times 470}{0.9 \times 1016} \leq 1.0$$

$$0.23 + 0.44 \leq 1.0$$

$$0.67 \leq 1.0 \quad (\text{OK})$$

$$\frac{C_f}{C_r} + \frac{\omega M_f}{M_r \left(1 - \frac{C_f}{C_{ex}}\right)} \leq 1.0 \quad (\text{Eq. 5.24})$$

$$\frac{64}{198} + \frac{1.0 \times 470}{914 \left(1 - \frac{60}{410}\right)} \leq 1.0$$

$$0.32 + 0.60 \leq 1.0$$

$$0.92 \leq 1.0 \quad (\text{OK})$$

Therefore the member is also satisfactory under this loading condition. Similar calculations for other sections indicate that the W8x24 section is the lightest section that can be selected.

Use a W8x24

---

#### 5.3.4 Bracing Requirements For Exterior Columns

In the last two sections procedures were developed for the design of "fully braced" columns and beam-columns. In this section bracing requirements are discussed.

Only bracing requirements for columns will be considered, even though most exterior compression members act as beam-columns. It is assumed that the requirements for columns are conservative when applied to beam-columns since both flanges of the columns are loaded in compression, whereas for beam-columns the outside flanges are usually less heavily loaded in compression than the inside flanges.

### 1. Columns Braced In The Weak Direction By Metal Siding

As discussed in Section 4.6, a metal wall diaphragm is characterized by its stiffness  $Q$  and its strength  $q$ . An expression for the ideal stiffness  $Q_{id}$  required for "full bracing" is given in Reference (68), however it is somewhat lengthy and will not be repeated here. A simplification of this expression is possible for a column in a light industrial building if it is noted that the bending stiffnesses of the girts are usually much greater than the torsional stiffness of the column:

$$Q_{id} = \frac{C_{fb}}{K_2} \quad (5.40)$$

where

$C_{fb}$  = "fully braced" factored compressive resistance,  
and

$K_2$  = a constant given in Table 5.1 that depends upon the number of girts along the length of the column. (68)

The strength requirement given in Reference (68) can also be simplified if the bending stiffnesses are much greater than the torsional stiffness:

$$q = \frac{2K_4Q}{ab} \left( \frac{1}{\frac{K_2Q}{C_{fb}} - 1} \right) \quad (5.41)$$

where

$K_4$  = a constant given in Table 5.1,

$\delta$  = initial crookedness of column, approximately  $L/480$  (68), and

$a, b$  = girt spacing and span as shown in Figure 4.11.

No. of Girts	$f_1$	$K_2$		$K_4$
		Eqn. (5.40)	Eqn. (5.41)	
1	1	0.810	0.810	1.0
2	0.97	0.684	0.912	0.866
3	0.89	0.810	0.950	0.707

TABLE 5.1 CONSTANTS  $f_1$ ,  $K_2$  AND  $K_4$ 

Corrugation	Cladding Fastened in Every		
	Alternate Corrugation	Third Corrugation	Fourth Corrugation
0.99	13.4	17.5	19.6

TABLE 5.2 SHEETING CONSTANT K



Although the stiffness and strength provided could be determined from the results of a shear test on the assembly shown in Figure 4.11, ensuring that the cladding profile and the method of attachment used in the test are the same as those used in the structure, it is usually not economically feasible to do this. Another option is to use empirical expressions for the stiffness and strength(72). However, the expressions given in Reference (72) are based on only six test results, and therefore are not generally applicable. A third possibility, perhaps the most favourable, is to compute these quantities analytically(47,48).

In Reference (47) a detailed analysis is given to do this. Consideration is given to the effects of:

1. warping at the panel ends,
2. shear strain in the panel,
3. axial strain in the girts
4. deformation of the cladding to girt fasteners,
5. deformation of the seam fasteners, and
6. deformation of the edge fasteners.

Of these, only items 1, 5 and 6 appear to have a major effect, the other items being of lesser importance. Based on this analysis, and considering only the three major items, the stiffness provided is:

$$Q = G'b = \frac{\frac{b^2}{L}}{\frac{0.144 bd^4 f_1 K}{Et^3 L^3} + \frac{(n_{sh} - 1)s_s}{n_s} + \frac{2s_s}{n_c}} \quad (5.42)$$

where

$d$  = pitch of corrugations,

$t$  = cladding thickness,

$n_{sh}$  = number of sheets per panel,

$n_s$  = number of seam fasteners per side lap,

$s_s$  = slip per seam fastener per unit load,

$n_c$  = number of edge fasteners per column,

$s_c$  = slip per edge fastener per unit load,

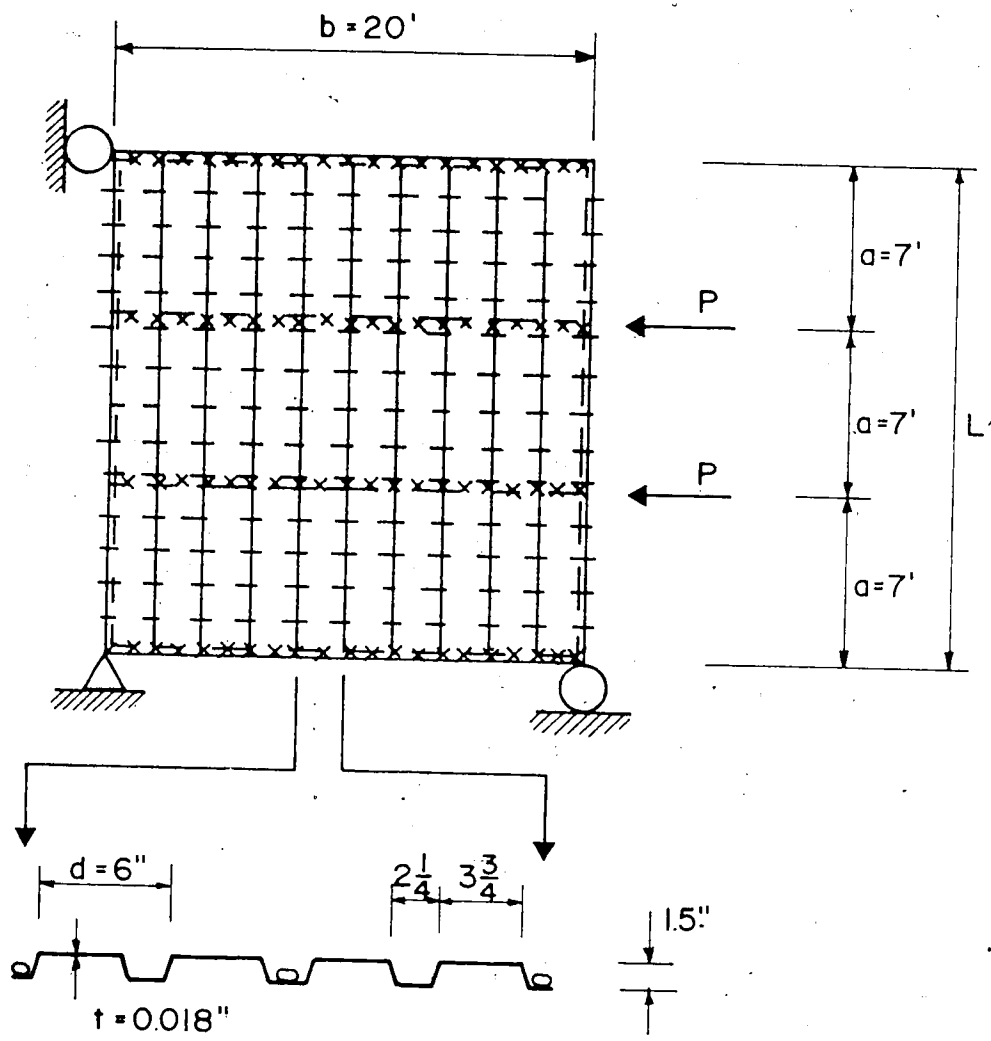
$f_1$  = a constant given in Table 5.1, and

$K$  = a sheeting constant given in Table 5.2 for the specific profile shown in Figure 5.18.

Many of these quantities are illustrated in Figure 5.18. The quantities  $s_s$  and  $s_c$  depend on the types of fasteners and on the thickness of the cladding. For 1/4 inch diameter Barber Colman Teks self-drilling/tapping screws and for 3/16 inch diameter Monel pop rivets with cladding thicknesses from 0.0180 inches to 0.050 inches:

$$s_s = s_c = 0.060 \text{ inches/kip.}$$

In order to determine the cladding strength, three different failure modes are considered<sup>(47)</sup>:



x = Tek self drilling /tapping screws  
 - = Monel pop rivets

FIGURE 5.18 SCREW CONNECTED DIAPHRAGM

1. failure of the seam fasteners,
2. failure of the sheet to girt fasteners, and
3. failure of the edge fasteners.

A fourth failure mode, sheet buckling, is considered in Reference (73), however it appears that this mode is not likely for panels with usual cladding thicknesses and girt spacings. For these diaphragms the first of the three failure modes above generally governs, and (47):

$$q = \phi \frac{n_s F_u}{L} \quad (5.43)$$

where  $F_u$  is the ultimate strength of one fastener. For 1/4 inch diameter Barber Colman Teks self-drilling/tapping screws (47):

$$F_u = 0.34 \text{ kips for each } 0.01 \text{ inch} \quad (5.44)$$

thickness of cladding,

and for 3/16 inch diameter Monel pop rivets (47):

$$F_u = 0.14 \text{ kips for each } 0.01 \text{ inch} \quad (5.45)$$

thickness of cladding.

The quantity  $\phi$  is a performance factor which, for illustration purposes, will be assumed to equal 0.75. Additional research is required to determine a more rational performance factor. This is considered to be beyond the scope of this dissertation.

---

Example 5.6Given

Determine whether the W8x24 section selected in Example 5.5 is "fully braced" by the wall cladding. The factored compressive resistance was shown to be 198 kips.

The exterior wall construction is shown in Figure 5.10. The cladding, selected to resist wind load acting normal to the wall, is 0.018 inches thick with the profile shown in Figure 5.18. The sheets are fastened to the girts with 1/4 inch diameter Barber Colman Tek's self-drilling/tapping screws in alternate corrugations and to each other along the vertical seams with 3/16 inch diameter Monel pop rivets spaced 18 inches apart.

Solution

Notation:

L = panel length = 252 in.

a = girt spacing = 84 in.

b = panel width = 240 in.

d = pitch of corrugations = 6 in.

t = cladding thickness = 0.018 in.

$n_{sh}$  = number of sheets per panel =  $\frac{240}{24} = 10$

$n_s$  = number of seam fasteners per side lap:

4 self-drilling/tapping screws and

$$\frac{252}{18} + 1 = 15 \text{ pop rivets}$$

$s_s$  = slip per seam fastener per unit load = 0.060 in./kip

$n_c$  = number of edge fasteners per column =  $n_s$

$s_c$  = slip per edge fastener per unit load =  $s_s$

$f_1$  = a constant = 0.97 (Table 5.1)

$K_2$  = constants (Table 5.1):

0.684 for Equation (5.40)

0.912 for Equation (5.41)

$K_4$  = a constant = 0.866 (Table 5.1)

$K$  = sheeting constant = 13.4 (Table 5.2)

$F_u$  = ultimate fastener strengths in shear:

For the screws,

$$F_u = 0.34 \times \frac{0.018}{0.010} = 0.61 \text{ kips} \quad (\text{Eq. 5.44})$$

and for the pop rivets

$$F_u = 0.14 \times \frac{0.018}{0.010} = 0.25 \text{ kips} \quad (\text{Eq. 5.45})$$

The stiffness provided by the cladding is:

$$Q = \frac{\frac{b^2}{L}}{\frac{0.144 bd^4 f_1 K}{Et^3 L^3} + \frac{(n_{sh} - 1) s_s}{n_s} + \frac{2s_c}{n_c}} \quad (\text{Eq. 5.42})$$

$$= \frac{\frac{(240)^2}{252}}{\frac{0.144 \times 240 \times (6)^4 \times 0.97 \times 13.4}{29000 \times (0.018)^3 \times (252)^3} + \frac{9 \times 0.060}{19} + \frac{2 \times 0.060}{19}}$$

$$= 896 \text{ kips}$$

The cladding strength is:

$$\begin{aligned}
 q &= \phi \frac{n_s F_u}{L} && \text{(Eq. 5.43)} \\
 &= 0.75 \times \frac{(4 \times 0.61 + 15 \times 0.25)}{21} \times 1000 \\
 &= 221 \text{ lbs./ft.}
 \end{aligned}$$

The required stiffness is:

$$\begin{aligned}
 Q_{id} &= \frac{C_{fb}}{K_2} && \text{(Eq. 5.40)} \\
 &= \frac{198}{0.684} \\
 &= 289 \text{ kips.}
 \end{aligned}$$

Since the stiffness provided, 896 kips, exceeds this, the wall cladding may be satisfactory.

The strength required is:

$$\begin{aligned}
 q &= \frac{2K_4Q}{ab} \left( \frac{1}{\frac{K_2Q}{C_{fb}} - 1} \right) && \text{(Eq. 5.41)} \\
 &= \frac{2 \times 0.866 \times 896 \times \frac{21 \times 12}{480}}{7 \times 21 \times 144} \left( \frac{1}{\frac{0.912 \times 896}{198} - 1} \right) \times 12000 \\
 &= 148 \text{ lbs./ft.}
 \end{aligned}$$

Since the strength provided, 221 lbs./ft., exceeds this, the wall cladding is satisfactory to provide "full bracing". It should be noted, however, that the performance factor of 0.75 is an assumed value, not a rationally derived value. This example should therefore be regarded only as an example to illustrate the basic theory and general conclusions about diaphragm requirements should not be made.

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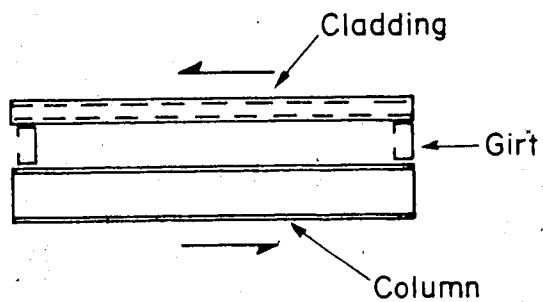
## 2. Columns Braced About Their Outside Flanges By Metal Siding

In the previous case the diaphragm and the flanges of the columns and girts were in the same plane. In this case, as shown in Figure 5.11(a), this is not true.

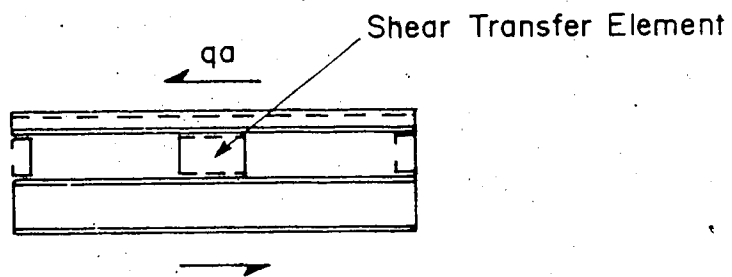
For the wall cladding-girt-column assembly to be able to function as a diaphragm, a path must be provided for transfer of the shear flow  $q$ , acting along the edges of the panel parallel to the columns as shown in Figure 5.19(a), down to the columns. Usually the girt-to-column connections are too flexible and weak to do this, and it is assumed for the purposes of this dissertation that the panel stiffness and strength cannot be developed. Tests are required on typical connections before a less conservative approach can be suggested.

In Reference (47) it is recommended that in designing new structures shear transfer elements be used between girts to





(a) A Flexible Diaphragm



(b) Improved Behaviour

FIGURE 5.19 INDIRECT SHEAR TRANSFER

provide this load path, as illustrated in Figure 5.19(b). These connectors may also serve another purpose; to prevent premature failure of the wall cladding by buckling. If connectors are used the stiffness and strength can be computed as if the girts were connected into the web instead of the flange<sup>(47)</sup>. It should be noted that an identical situation occurs in design of roof deck diaphragms<sup>(29)</sup>.

If the walls of the structure are insulated, it is usual practice to attach the cladding to sub-girts, which in turn are fastened to the structural girts as shown in Figure 5.20. The insulation is then positioned as shown. Additional research is required before it can be known if this wall system is capable of functioning as a diaphragm, even if the structural girts are connected into the webs of the columns, unless shear connectors are used.

If one or more bays are infilled, as shown in Figure 5.11(c), bracing is provided by the axial stiffnesses and strengths of the girts. In this situation the girts and the connection to the masonry infilled bay should be designed for one percent of the sum of the forces in the columns<sup>(1)</sup>.

### 3. Columns Braced In The Weak Direction By Masonry

With this type of wall construction (see Figure 5.12) it is assumed that either the column bears directly against the

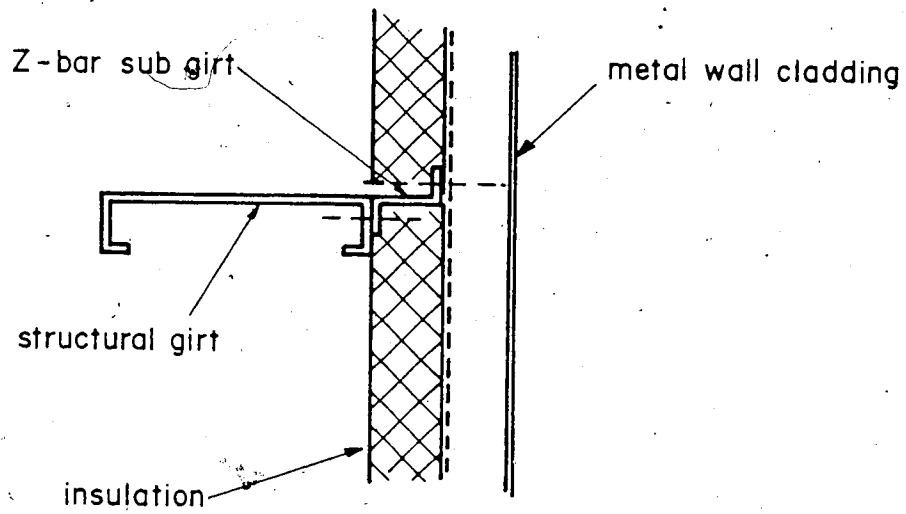


FIGURE 5.20 INSULATED WALL CLADDING

masonry wall from the onset of loading, or that it bends about its weak axis a small amount until contact is established. Bracing is therefore provided by bearing, and masonry anchors are not required for this purpose.

Masonry anchors are, however, required to resist wind load acting normal to the wall. Reference (7) is very specific about the type of anchorage that should be used. They must be fabricated from corrosion-resistant steel straps at least 1-1/2 inches wide and 1/4 inches thick, and must not be spaced vertically more than 1'4" on centers. The masonry anchors must be of a length at least twice the thickness of the masonry, and have 2 inch right angle bend at one end completely embedded in mortar. The other end, also with a 2 inch right angle bend, is welded to the column. As discussed in Reference (74), however, these empirical standards represent only guesses rather than rigorous rational performance criteria; a search of the literature reveals almost no supporting test evidence.

#### 4. Columns Braced About Their Outside Flanges

By Masonry

This type of wall construction is illustrated in Figure 5.13. The masonry anchors must be capable of resisting the wind forces acting normal to the wall, and must be capable of resisting the shear forces needed to prevent translation of the outside flange of the column. This shear is approximately one percent of the axial force in the column.

## 5.4 Base Plates

Steel columns or beam-columns supported by concrete footings or foundations are provided with base plates to avoid overloading the supporting material. If lateral loads and sway effects are resisted by a direct acting bracing system, the base plates of interior columns are subjected to compression forces and shear forces due to column out-of-plumb and building sway. The base plate of an exterior compression member in a braced bay may be subjected to shear and to compression or tension, depending on whether or not the vertical component of the force in the bracing system exceeds the compression force caused by column action. Base plates of exterior compression members may also be subjected to shear caused by transverse forces acting along the lengths of these members. In this section the design of base plates subjected to compression or tension and to shear is discussed.

### 5.4.1 Base Plates Subjected to Compression

A satisfactory bearing surface is defined as one for which 75 percent of the entire contact area is in full bearing, and separation of the remaining portion does not exceed 0.01 inches, except adjacent to toes of flanges where a localized separation of 0.25 inches is permissible<sup>(1)</sup>. In order to obtain a satisfactory bearing surface both the column end and the base plate must be adequately flat. These flatness requirements can be met if the column is sawn to length, but usually will not be if it is

torch cut without milling. A base plate cut from a rolled steel plate less than two inches thick (the usual case for light industrial buildings) is sufficiently flat to meet these requirements, and may be used without milling<sup>(1)</sup>.

When the column end is sufficiently flat the compression force in the column is transferred directly through bearing to the base plate. The column to base plate connection must therefore be designed only to resist the shear force caused by the column out-of-plumb and building sway. In the case of a lightly loaded column some fabricators may consider it more economical to torch cut instead of sawing the column to length and weld the base plate directly to the unprepared column end. In this case the column to base plate connection must also be designed to resist the compression force in the column.

The compression force is transferred from the underside of the base plate to the concrete foundation by bearing. The factored bearing resistance per unit of bearing area,  $B_r$ , is<sup>(75)</sup>:

$$B_r = 0.85 \phi_c F'_c \quad (5.46)$$

where  $\phi_c = 0.63$  is the performance factor for concrete in bearing, and  $F'_c$  is the unconfined compressive strength of the concrete. Thus the base plate area  $A$  must be such that:

$$B_r A \geq C_f \quad (5.47)$$

Under certain conditions, if the concrete in the immediate vicinity of the base plate is confined by the unloaded concrete further away from the base plate, the factored bearing resistance may be increased. However, when the column base is grouted, as shown in Figure 5.21(a), it is doubtful that advantage can be taken of this confining action since the bearing resistance of the grout alone is given by Equation (5.46).

---

#### Example 5.7

##### Given

The interior W8x40 column of Example 5.1 is subjected to a factored axial load of 196 kips, caused by dead load and full snow load. The unconfined compressive strength of the grout is 3000 psi. Determine the base plate dimensions required.

##### Solution

The dimensions of the W8x40 section are listed in Reference (5):

$$b = 8.08 \text{ in.}$$

$$d = 8.25 \text{ in.}$$

The minimum size base plate, based on fabrication

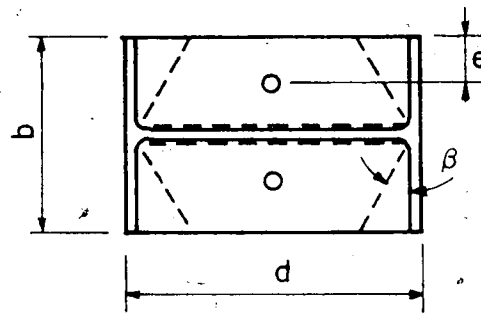
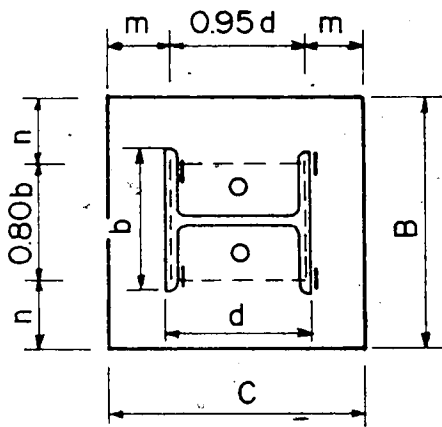
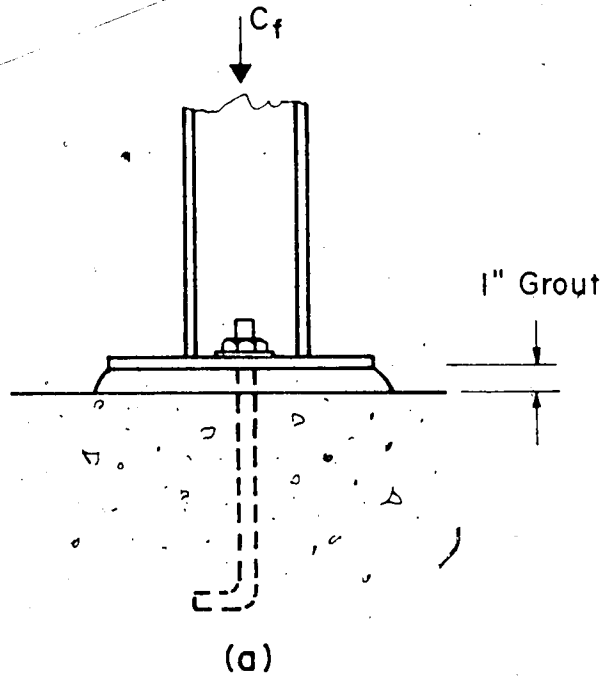


FIGURE 5.21 AXIALLY LOADED COLUMNS



considerations, is assumed to equal the actual  $b \times d$  of the column plus one half inch, rounded up to the nearest one half inch. Thus trial base plate dimensions of 9 in. x 9 in. are selected.

The factored bearing resistance of the group per unit of bearing area is:

$$\begin{aligned} B_r &= 0.85 \phi_c F'_c && \text{(Eq. 5.46)} \\ &= 0.85 \times 0.63 \times 3000 \\ &= 1600 \text{ psi} \end{aligned}$$

The minimum base plate dimensions selected are adequate providing:

$$B_r A \geq C_f \quad \text{(Eq. 5.47)}$$

or

$$\frac{1600 \times 9 \times 9}{1000} \geq 196$$

or

$$130 \geq 196$$

A larger base plate is therefore required.

Try an 11 in. x 11 in. base plate

As before, this base plate is adequate providing:

$$B_r A \geq C_f \quad \text{(Eq. 5.47)}$$

or

$$\frac{1600 \times 11 \times 11}{1000} \geq 196$$

or

$$194 \bullet 196$$

This is a very small amount of underdesign and will be considered to be acceptable.

Use a 11 in. x 11 in. base plate

---

The following method is recommended in Reference (75) to determine the thickness,  $t$ , of a base plate that is not subjected to bending moment, shear, or tension forces. It is assumed that (see Figure 5.21(b)):

1. the factored axial force is uniformly distributed over the base plate of dimensions  $B \times C$ ,
2. the base plate exerts a uniform pressure upon the foundation, and
3. the base plate strip projecting beyond the assumed dotted rectangle shown in Figure 5.21(b) acts as a cantilever subjected to the uniform pressure.

The required base plate thickness is determined by equating the factored moment acting on that portion of the plate taken as a

cantilever to the factored moment resistance of the plate and solving for the plate thickness:

$$t = \sqrt{\frac{2C_f m^2}{BC \phi F_y}} \text{ or } \sqrt{\frac{2C_f n^2}{BC \phi F_y}} \quad (5.48)$$

whichever is greater.

Both  $m$  and  $n$  and therefore  $t$  approach zero if the column base plate is approximately the same size as the column, as shown in Figure 5.21(c). In this case the base plate thickness can be determined by yield line analysis<sup>(76)</sup>. It is assumed that:

1. the base plate exerts a uniform pressure upon the foundation,
2. the edges of the base plate under the column flanges are simply supported, and
3. yield lines form along the broken lines shown in Figure 5.21(c).

The required base plate thickness is:

$$t = 0.43 b \sqrt{\frac{C_f}{\phi F_y (1 - \beta^2) BC}} \quad (5.49)$$

where

$$\beta = \sqrt{0.75 + \frac{1}{4\lambda^2} - \frac{1}{2\lambda}}$$

$$\lambda = 2d/b$$

However, the failure mode shown in Figure 5.21(c) is also possible for the column base plate shown in Figure 5.21(b). Therefore the

more severe of Equations (5.48) and (5.49) should govern design.

---

### Example 5.8

#### Given

Determine the required thickness of the 11 in. x 11 in. base plate selected in Example 5.7. The factored axial force in the column is 196 kips. Use G40.21 44W steel ( $F_y = 44$  ksi).

#### Solution

The dimensions of the W8x40 section are listed in Reference (5):

$$b = 8.08 \text{ in.}$$

$$d = 8.25 \text{ in.}$$

The failure mode shown in Figure 5.21(b) will be considered first. From the base plate geometry shown in this figure,

$$m = \frac{C - 0.95d}{2} = \frac{11 - 0.95 \times 8.25}{2} = 1.58 \text{ in.}$$

and

$$n = \frac{B - 0.80b}{2} = \frac{11 - 0.80 \times 8.08}{2} = 2.27 \text{ in.}$$

Therefore

$$\begin{aligned}
 t &= \sqrt{\frac{2C_f n^2}{BC \phi F_Y}} && \text{(Eq. 5.48)} \\
 &= \sqrt{\frac{2 \times 196 \times (2.27)^2}{11 \times 11 \times 0.9 \times 44}} \\
 &= 0.650 \text{ in.}
 \end{aligned}$$

Next, the failure mode shown in Figure 5.21(c) will be investigated:

$$\begin{aligned}
 \lambda &= \frac{2d}{b} = 2 \times \frac{8.25}{8.08} = 2.04 && \text{(Eq. 5.49)} \\
 \beta &= \sqrt{0.75 + \frac{1}{4\lambda^2} - \frac{1}{2\lambda}} \\
 &= \sqrt{0.75 + \frac{1}{4 \times (2.04)^2} - \frac{1}{2 \times 2.04}} \\
 &= 0.655
 \end{aligned}$$

therefore

$$\begin{aligned}
 t &= 0.43 b \beta \sqrt{\frac{C_f}{\phi F_Y (1 - \beta^2) BC}} \\
 &= 0.43 \times 8.08 \times 0.655 \sqrt{\frac{196}{0.9 \times 44 \times (1 - (0.655)^2) \times 11 \times 11}} \\
 &= 0.610
 \end{aligned}$$

Since this does not exceed the 0.650 inch thickness computed earlier, it does not govern. The actual base plate

thickness will be this computed value, rounded up to the nearest one quarter of an inch. Although the thickness could have been rounded up to one sixteenth or one eighth of an inch instead, one quarter of an inch was chosen since fabricators are more likely to have this plate in stock. However, this is strictly a matter of judgement.

Use an 11 in. x 11 in. x 3/4 in. plate

---

#### 5.4.2 Base Plates Subjected to Shear

The base plate of an interior column is subjected to a shear force caused by possible column out-of-plumbness and building sway. The base plate of an exterior compression member, in addition, is subjected to shear caused by wind forces acting along the member length. If the member is at a braced bay the base plate is also subjected to a shear equal to the horizontal component of the force in the bracing member.

These shear forces are transferred from the end of the compression member to the base plate through the welds, and from the base plate to the anchor bolts by bearing. The welds must be able to transfer these shears, and the base plate thickness must be such that a bearing failure does not occur.

The shear resistance of a fillet weld is taken as the smaller of that corresponding to failure of the base metal (1):

$$V_r = \phi A_m \times (0.66 F_y) \quad (5.50)$$

where

$$A_m = \text{area of the fusion face}$$

$$= \text{nominal weld size} \times \text{weld length}$$

$$0.66 F_y = \text{shear yield stress, approximately } F_y / \sqrt{3}$$

or that corresponding to failure of the weld itself (1):

$$V_r = 0.50 \phi A_w X_u \quad (5.51)$$

where

$$A_w = \text{throat area of weld}$$

$$= 0.707 \times \text{nominal weld size} \times \text{weld length}$$

$$X_u = \text{ultimate tensile strength of the electrode, given}$$

by the electrode classification number.

The base shear on an interior column is usually very small and the column to base plate connection often consists of the four minimum size fillet welds shown in Figure 5.21(b). With this weld lay-out the base plate can be connected to the end of the column using down-hand welding, without turning the column, thereby resulting in economies in fabrication. Minimum fillet weld sizes are 3/16 inch for base plate thicknesses up to 1/2 inch, inclusive, 1/4 inch for base plate thicknesses over 1/2 inch and up to 3/4 inch, inclusive (77). The over-all length of each fillet weld should not be less than four times the nominal size (77).

The bearing resistance of the base plate is (1):

$$B_r = \phi t n e F_u \leq 3 \phi t d n F_u \quad (5.52)$$

where

- $n$  = number of anchor bolts
- $e$  = edge distance measured from the center of the bolt hole to the edge of the plate in the direction of the applied load,
- $F_u$  = ultimate tensile strength of the plate  
= 65 ksi for G40.21 44W steel, and
- $d$  = anchor bolt diameter.

As discussed earlier, the base plate of an interior column is subjected to a shear force caused by the column out-of-plumb and building sway. In order to determine the magnitude of this shear, consider the free-body diagram of an interior column of height  $h$  in the middle of the structure, as shown in Figure 5.22. Summing moments about the top of the column, the base shear  $H$  is

$$H = C_f \frac{\Delta}{h} \quad (5.53)$$

where  $\Delta$  is the column sway, equal to the sum of the column out-of-plumbness, which must not exceed  $h/500$  (1), and the maximum building sway due to lateral loads and second order effects. One way to determine the building sway would be to compute it



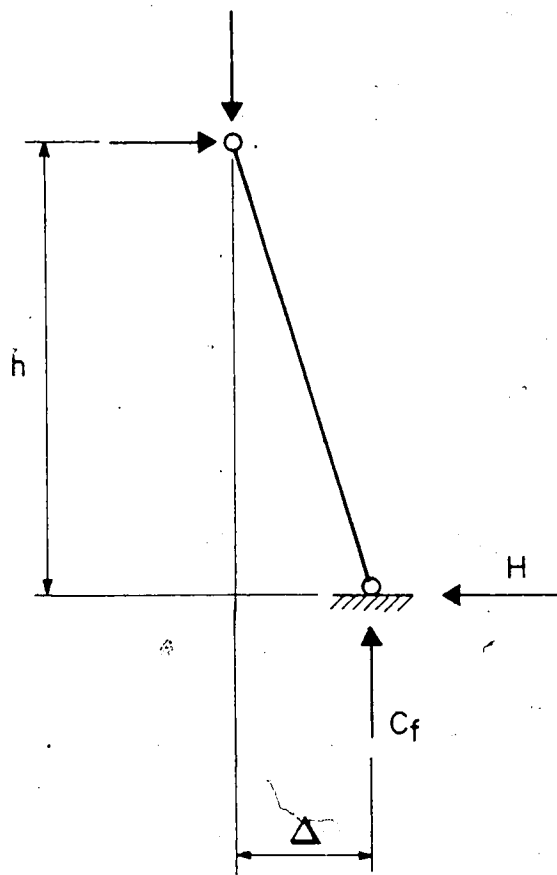


FIGURE 5.22 SHEAR CAUSED BY COLUMN SWAY

directly, using the procedures developed in CHAPTER VI. As an approximation, however, it can be estimated by multiplying the maximum recommended sway of  $h/200$  for industrial buildings(20) by the ratio of the reference wind pressures used for strength and deflection calculations, approximately 1.3, by the live load factor of 1.5, by the load combination factor of 0.7, and by an assumed factor of 1.2 to account for second order effects. Thus an estimate of the shear is:

$$\begin{aligned} H &= C_f \left( \frac{1}{500} = \frac{1}{200} \times 1.3 \times 1.5 \times 0.7 \times 1.2 \right) \\ &= 0.01 C_f \end{aligned} \quad (5.54)$$

or 1 percent of the axial force in the column.

---

#### Example 5.9

##### Given

As shown in Figure 5.23, the base plate of a compression member at a braced bay is subjected to a factored shear of 26.3 kips acting in the plane of the wall. In addition, it is subjected to a factored shear of 3.45 kips due to the transverse wind forces at the girt locations (see Figure 5.17(b)). The base plate is 1/2 inch thick and the edge distance is 1-1/2 inches. Determine the weld requirements and check the base plate for

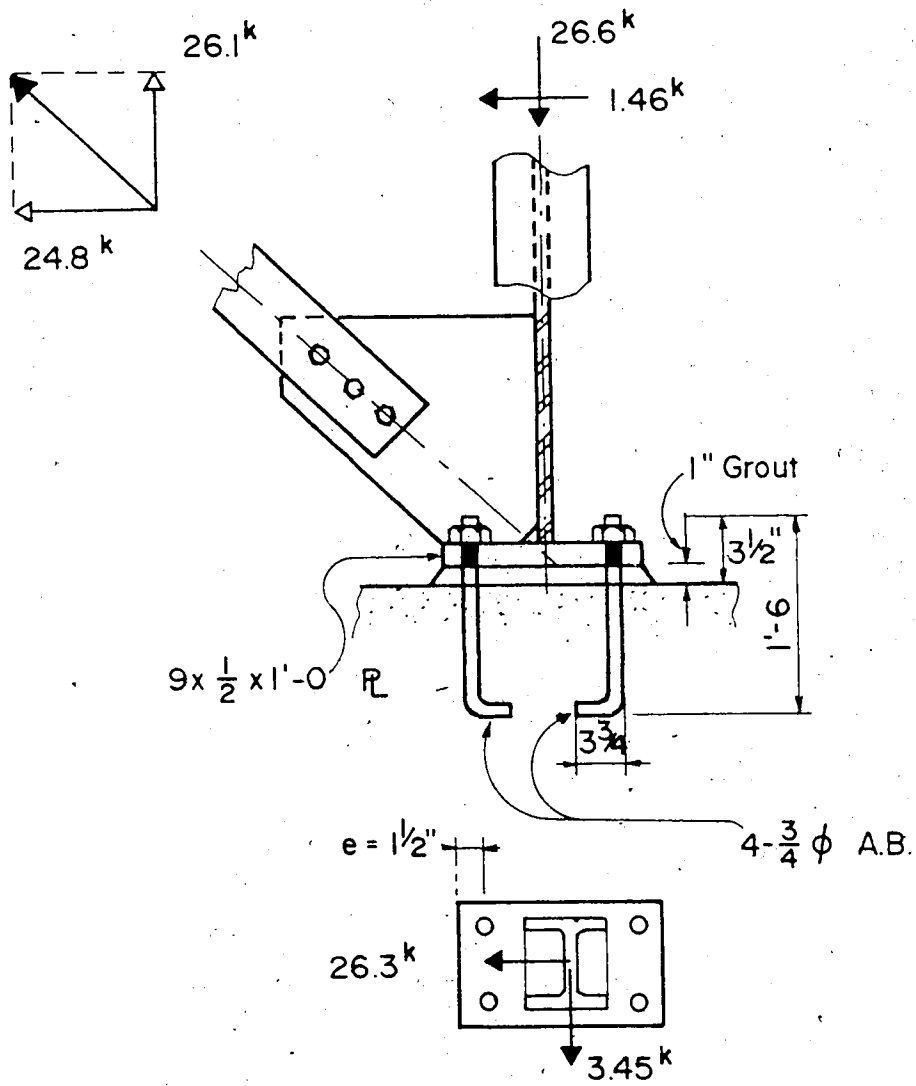


FIGURE 5.23 DETAIL OF BRACED BAY

adequacy in bearing. Use G40.21 44W steel ( $F_y = 44$  ksi,  $F_u = 65$  ksi) and E70 electrodes ( $X_u = 70$  ksi).

### Solution

The total factored shear that must be resisted is:

$$\begin{aligned} V &= \sqrt{(26.3)^2 + (3.45)^2} \\ &= 26.5 \text{ kips} \end{aligned}$$

A weld thickness of 1/4 inch will be selected. Based on failure of the base metal the factored shear resistance is:

$$\begin{aligned} V_R &= \phi A_m (0.66 F_y) && \text{(Eq. 5.50)} \\ &= 0.9 \times \frac{1}{4} \times 1 \times 0.66 \times 44 \\ &= 6.53 \text{ kips/inch} \end{aligned}$$

and based on failure in the weld material it is:

$$\begin{aligned} V_R &= 0.50 \phi A_w X_u && \text{(Eq. 5.51)} \\ &= 0.50 \times 0.9 \times 0.707 \times \frac{1}{4} \times 1 \times 70 \\ &= 5.57 \text{ kips/inch (governs)} \end{aligned}$$

Therefore the required weld length is  $25.7/5.57 = 4.61$  inches.

Use four welds 1-1/2 inches long as shown in Figure 5.27(b).

The factored bearing resistance of the base plate, is:

$$\begin{aligned}
 B_r &= \phi t n e F_u && \text{(Eq. 5.52)} \\
 &= 0.67 \times \frac{1}{2} \times 4 \times 1.5 \times 65 \\
 &= 130 \text{ kips (governs)}
 \end{aligned}$$

However, this must not exceed

$$\begin{aligned}
 B_r &= 3 \phi t d n F_u && \text{(Eq. 5.53)} \\
 &= 3 \times 0.67 \times \frac{1}{2} \times \frac{3}{4} \times 4 \times 65 \\
 &= 196 \text{ kips}
 \end{aligned}$$

Since 130 kips exceeds the factored shear of 26.5 kips the base plate is adequate in bearing.

#### 5.4.3 Base Plates Subjected to Tension

A compression member, by definition, is a member subjected to an axial compressive force. Nevertheless, the base plate of an exterior compression member at a braced bay will be subjected to tension if the vertical component of the force in the bracing member exceeds the compression force. This is most likely

to occur in the summer when there is no snow load on the roof.

Denoting the net tension force per anchor bolt as  $F$ , the required base plate thickness  $t$  can be determined by yield line theory. Assuming the yield line pattern shown in Figure 5.21(c) the internal work  $D$  is shown in Reference (76) to be:

$$D = 4\phi M_p \left( \frac{1}{\beta} + \beta + \frac{\lambda}{2} \right) \quad (5.55)$$

The external work  $W$ , assuming two anchor bolts arranged as shown, is:

$$W = 2F \left( 1 - \frac{2e}{b} \right) \quad (5.56)$$

where  $e$  is the edge distance parallel to the flanges. Setting  $D = W$  and solving for  $M_p$ ,

$$M_p = \frac{F \left( 1 - \frac{2e}{b} \right)}{2\phi \left( \frac{1}{\beta} + \beta + \frac{\lambda}{2} \right)} \quad (5.57)$$

Differentiating with respect to  $\beta$ , and setting  $dM_p/d\beta = 0$  leads to the requirement that  $\beta$  equal unity. Thus the assumed yield line pattern shown in Figure 5.21(c) is valid providing  $d$  is greater

than or equal to  $b$ , which is always true for wide flange sections used as columns. Substituting  $\beta$  equal to unity in Equation 5.57 and setting the result equal to  $M_p = F_y t^2/4$  leads to:

$$t = \sqrt{\frac{2F (1 - \frac{2e}{b})}{\phi F_y (2 + \frac{\lambda}{2})}} \quad (5.58)$$

### 5.5 Cap Plates

A detail of a cap plate connection for an interior column in a building with a cantilever roof framing scheme is shown in Figure 5.24. The main purpose of the cap plate is to facilitate erection. The minimum cap plate length is approximately equal to the column depth plus a 2-3/4 inch overhang on either side to provide sufficient space for bolting. The minimum cap plate width is equal to the flange width of the column, plus 1 inch, and the minimum cap plate thickness is approximately 1/2 inch. These dimensions are shown in Figure 5.24.

The horizontal force to be transferred at the top of the column is equal to that at the bottom of the column, approximately 1 percent of the factored axial load in the column. Since this force is small, the column to cap plate welds are nominal, as for the base plate connection discussed in Section 5.4.2, and the bolts are selected to be compatible with the thickness of the cap plate and the girder flange.

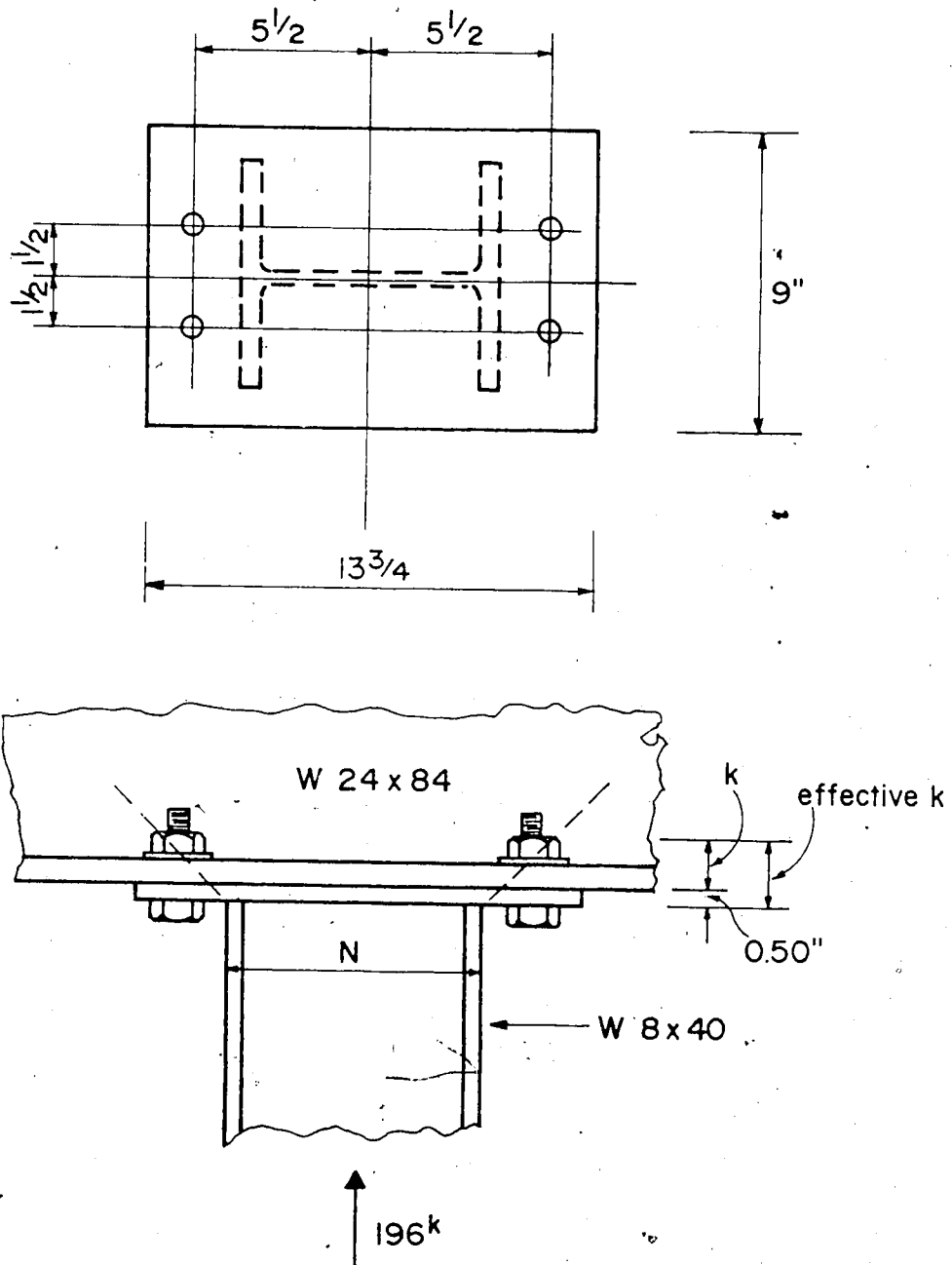


FIGURE 5.24 CAP PLATE DETAIL



## 5.6 Design of Bearing Stiffeners

Bearing stiffeners are required to prevent web crippling wherever the concentrated loads or reactions exceed the factored bearing resistance of the girder web. As discussed in Section 4.9, the factored bearing resistance of the girder web shown in Figure 5.32 is given by

$$B_r = 1.25 \phi w (N + 2k) F_y \quad (\text{Eq. 5.58})$$

where  $k$  can now be interpreted as the "effective"  $k$  distance shown in Figure 5.24, which includes the cap plate thickness. The bearing length  $N$  is the column depth rather than the cap plate length, since the cap plate is not designed to distribute the load over the overhanging portions.

If the column is oriented at ninety degrees to the position shown in Figure 5.24, so that the column and girder webs are at right angles to each other, the length  $N$  should be taken as the thickness of the column web. If the column is fabricated from a hollow structural steel section instead of a wide flange section, the length  $N$  should be taken as twice the wall thickness. In both these situations, since the bearing lengths are short, Equation (4.58) will be overly conservative. As discussed in Section 4.9, use of a bearing length of  $N + 5k$  may be more appropriate in these cases.

---

Example 5.10Given

Check to see if the factored bearing resistance of the W24x84 girder web shown in Figure 5.24 is exceeded by the 196 kip column load. The cap plate thickness  $t$  is 0.50 inches and the bearing length  $N$  is 8.25 inches, the depth of the W8x40 column. Use G40.21 44W steel ( $F_y = 44$  ksi).

Solution

The relevant properties of the W24x86 section are listed in Reference (5):

$$w = 0.470 \text{ in.}$$

$$k = 1.56 \text{ in.}$$

The "effective"  $k$  distance is:

$$\begin{aligned} k_e &= k + t \\ &= 1.56 + 0.50 \\ &= 2.06 \text{ in.} \end{aligned}$$

therefore the factored bearing resistance is:

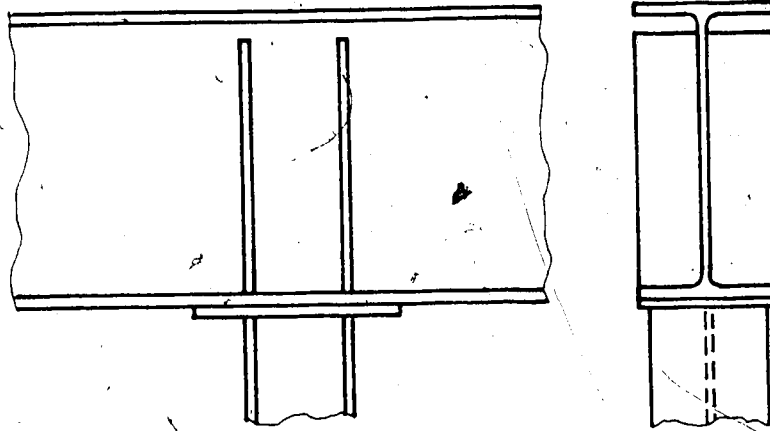
$$\begin{aligned}
 B_r &= 1.25 \phi w (N + 2k) F_y && \text{(Eq. 5.58)} \\
 &= 1.25 \times 0.9 \times 0.470 (8.25 + 2 \times 2.06) \times 44 \\
 &= 288 \text{ kips}
 \end{aligned}$$

Since this exceeds the factored load of 196 kips web crippling is not a problem.

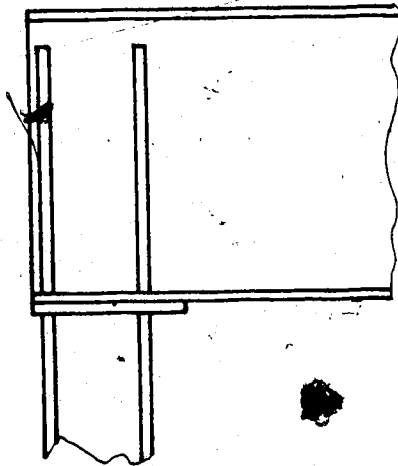
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If the girder web had been slender, bearing stiffeners would have been used as shown in Figure 5.25(a) to "extend" the column flanges thereby allowing the load to be distributed into the girder web less abruptly, eliminating the possibility of web crippling. Bearing stiffeners are also required at unframed ends of single-web girders (Figure 5.25(b)) having web slenderness ratios greater than  $402/\sqrt{F_y}(l)$ .

Bearing stiffeners must bear against the flanges through which they receive their loads. They extend approximately to the edges of the flange plates. Sufficient contact area must be provided between the stiffeners and the flanges so that the loads can be transferred by bearing. The stiffeners must not fail prematurely either by column-type buckling or by local buckling. In addition, the connections between the stiffeners and the girder web must be adequate to transfer the loads.



(a) At Interior Column



(b) At Exterior Column

FIGURE 5.25 BEARING STIFFENERS

In order to clear the web to flange fillets, the corners of the stiffeners are cut, so that the complete stiffener area is not effective in bearing. The factored bearing resistance,  $B_r$ , on the contact area  $A$  is<sup>(1)</sup>:

$$B_r = 1.50 \phi A F_y \quad (5.59)$$

Since the load is transferred from the stiffeners to the girder web through the welds, stiffeners need not bear against the unloaded flanges.

In order to ensure that premature failure of the bearing stiffeners by local buckling does not occur, the stiffener plates should be stocky enough so that they are capable of reaching the yield load. The required width-thickness limitation is<sup>(1)</sup>

$$\frac{b}{t} \leq \sqrt{\frac{100}{F_y}} \quad (5.60)$$

The stiffeners and a portion of the girder web are assumed to act as a column in resisting the compressive loads or reactions. Because the ends of a stiffener are not completely free to rotate and because the loads are not applied at opposite ends of the stiffener, as in a column, the assumption is made that the effective column length is three-quarters of the stiffener length<sup>(1)</sup>. In addition, a portion of the web plate equal to 25 times the web thickness, at interior locations, or 12 times the

web thickness, at the end of the girder, is assumed to act with the stiffeners in resisting the loads or reactions by column action.

#### 5.7 Summary

In this Chapter the design of interior, pin-ended columns is considered first, followed by analyses of various forms of end restraints. The design of interior beam-columns is then reviewed briefly. The design of exterior columns and beam-columns and bracing requirements for metal wall cladding are then discussed. Finally, the design of base plates, cap plates, and bearing stiffeners are considered.

## CHAPTER VI

### RESISTANCE TO LATERAL LOADS

#### 6.1 Introduction

Lateral loads on a structure may be caused by wind, earthquake or other agents. In a tall building or tower the lateral loads are usually dominant in the design. In a single storey structure, however, these effects are of lesser importance, and it is the vertical loads that tend to govern the weight of steel in the structure. Nevertheless, it is important to ensure that paths exist for lateral loads to be transferred from the walls to the roof, in the plane of the roof to the end walls, and finally down to the foundation. In this section various methods of transferring lateral loads are examined.

#### 6.2 Transfer of Wind Loads From the Walls to the Roof

As discussed in Section 2.3.4, wind acts as a pressure on the windward wall and a suction on the leeward wall. If the walls consist of girts and metal wall cladding, these loads are transferred from the cladding to the girts, from the girts to the columns, and from the columns to the roof as a series of point forces. Instead of computing the effects of pressure and suction separately, it is convenient in designing the lateral load resisting system to consider only their combined effect.

If the walls of the building consist of precast concrete wall panels, the sheathing functions as a series of uniformly loaded slabs, simply supported at the foundation and at the roof level. Under this condition the roof edge is subjected to a uniformly distributed load instead of point forces.

If the walls consist of masonry block, the roof edge is subjected to both point forces and a uniformly distributed load. The portion of the load transferred directly to the roof edge and the portion transferred indirectly through the columns can be determined using the tributary area concept(7).

### 6.3 Sway Effects

#### 6.3.1 Detailed Derivation

After the lateral loads on the structure are computed, an estimate of the building deflection can be made by performing a first order analysis - i.e., one that does not consider the effect of the vertical load. In fact, as the structure sways laterally the vertical load acting through this sidesway causes an overturning moment that is not predicted by a first order analysis. In a braced frame this is responsible for an increase in the roof deflection, an increase in the shears in the roof deck, and an increase in the axial forces in the bracing members. In the case of a rigid frame it is responsible for an increase in the axial loads and moments in the beam-columns.



Consider the braced frame of length  $L$  depth  $d$  and height  $h$  shown in Figure 6.1. Lateral loads and sway effects are assumed to be resisted by a direct acting bracing system. The structure is subjected to a uniformly distributed load  $w$  acting at roof level, and to a uniform roof load of intensity  $p$ . The roof is assumed to be capable of functioning as a shear diaphragm.

The deflected shape of the roof of the structure in Figure 6.1 approaches a straight line only if the stiffness of the roof diaphragm is very large relative to the stiffnesses of the end walls. If, on the other hand, the end walls are very stiff relative to the roof, the deflected shape is approximately sinusoidal.

Due to the effect of lateral load alone the maximum roof deflection  $\Delta$  is assumed to be given by (29):

$$\Delta = \Delta_D + \Delta_F + \Delta_W \quad (6.1)$$

where

$\Delta_D$  = the component of deflection due to the deformation of the bracing members in the end bays,

$\Delta_F$  = the maximum flexural deflection of the roof due to deformation of the perimeter members, and

$\Delta_W$  = the maximum shear deflection of the roof due to deformation of the roof deck diaphragm.

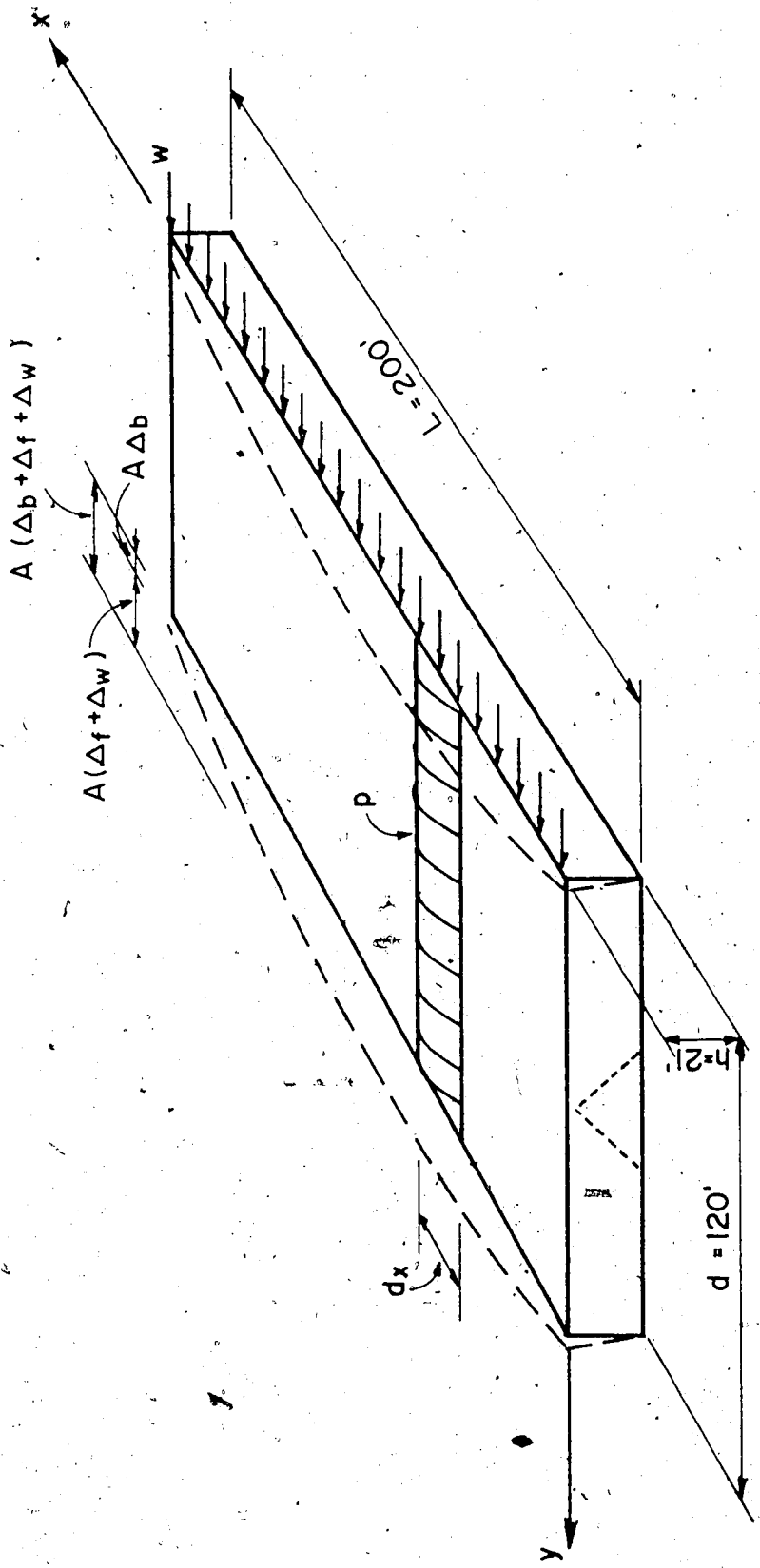


FIGURE 6.1 ANALYSIS OF PA EFFECT IN BRACED FRAMES

Under the combined effects of vertical load and lateral load it is assumed that the above deflections are amplified by an amplification "A" so that the maximum deflection  $\delta$  is:

$$\delta = A (\Delta_D + \Delta_F + \Delta_w) \quad (6.2)$$

An expression for this amplification factor is derived below.

The XY coordinate system is selected as shown in Figure 6.1. The roof deflection  $y$  at any distance  $x$  less than  $\frac{L}{2}$  is assumed to given by:

$$y = A (\Delta_D + \Delta_F \sin \frac{\pi x}{L} + \Delta_w \frac{2x}{L}) \quad (6.3)$$

Defining  $k$  as the stiffness of the bracing system in either end wall:

$$\Delta_D = \frac{wL}{k} \quad (6.4)$$

Also, (29):

$$\Delta_F = \frac{5wL^4}{384EI} \quad (6.5)$$

$$\Delta_w = \frac{wL^2 F}{8 \times 10^6 d} \quad (6.6)$$

where  $I$  is the moment of inertia of the diaphragm flange members about the centroidal axis of the diaphragm and  $F$  is the "flexibility factor" for the roof deck in units of micro-inches per pound.

Methods of computing the flexibility factor are discussed in Section 6.4.

The amplification factor  $A$  can be determined by formulating the expression for the total potential  $V$ , differentiating it with respect to  $A$ , and setting the result equal to zero. From symmetry, only half of the structure need be analysed. Thus:

$$\begin{aligned}
 V = & \frac{1}{2}k (A \Delta_b)^2 + \frac{1}{2} \int_0^{L/2} EI (A \Delta_f \sin \frac{\pi x}{L})^2 dx \\
 & + \frac{1}{2} \frac{2 x 10^6 d}{FL} (A \Delta_w)^2 \\
 & - \int_0^{L/2} \frac{pd}{2h} A^2 (\Delta_b + \Delta_f \sin \frac{\pi x}{L} + \Delta_w \frac{2x}{L})^2 dx \\
 & - \int_0^{L/2} wA (\Delta_b + \Delta_f \sin \frac{\pi x}{L} + \Delta_w \frac{2x}{L}) dx \quad (6.7)
 \end{aligned}$$

The first term in Equation (6.7) is the strain energy stored in the bracing system. The second term is the strain energy stored in the diaphragm flanges. The third term is the strain energy stored in the diaphragm web. The fourth and fifth terms are the losses in potential due to the vertical and lateral loads, respectively.

Performing the required integration and setting the derivative of V with respect to A equal to zero leads to:

$$\begin{aligned} & \left( \frac{\Delta_b L}{4} + \frac{5\pi^4 \Delta_f L}{8 \times 384} + \frac{\Delta_w L}{8} \right. \\ & \left. - \frac{pdL}{wh} \left\{ \frac{\Delta_b^2}{4} + \frac{\Delta_f^2}{8} + \frac{\Delta_w^2}{12} + \frac{\Delta_b \Delta_f}{\pi} + \frac{\Delta_b \Delta_w}{4} + \frac{2\Delta_f \Delta_w}{\pi^2} \right\} \right) A \\ & = \frac{\Delta_b L}{4} + \frac{\Delta_f L}{2\pi} + \frac{\Delta_w L}{8} \end{aligned} \quad (6.8)$$

Noting in Equation (6.8) that

$$\frac{5\pi^4}{8 \times 384} \approx \frac{1}{2\pi} \quad (6.9)$$

the sum of the three terms in the outside set of brackets ( ) on the left hand side is approximately equal to the sum of the three terms on the right hand side. Thus dividing both sides of Equation (6.8) by this amount, multiplying and dividing the second term in brackets by:

$$\Delta = \Delta_b + \Delta_f + \Delta_w \quad (\text{Eq. 6.1})$$

and then solving for A gives:

$$A = \frac{1}{1 - \alpha \frac{pdL\Delta}{wLh}} \quad (6.10)$$

where

$$\alpha = \frac{\frac{1}{4} + \frac{1}{8} \left(\frac{\Delta_f}{\Delta_b}\right)^2 + \frac{1}{12} \left(\frac{\Delta_w}{\Delta_b}\right)^2 + \frac{1}{\pi} \left(\frac{\Delta_f}{\Delta_b}\right) + \frac{1}{4} \frac{\Delta_w}{\Delta_b} + \frac{2}{\pi^2} \frac{\Delta_f \Delta_w}{\Delta_b^2}}{\left(\frac{1}{4} + \frac{1}{2\pi} \frac{\Delta_f}{\Delta_b} + \frac{1}{8} \frac{\Delta_w}{\Delta_b}\right) \left(1 + \frac{\Delta_f}{\Delta_b} + \frac{\Delta_w}{\Delta_b}\right)} \quad (6.11)$$

Since the total vertical load P is

$$P = pdL \quad (6.12)$$

and the total lateral load H is

$$H = wL \quad (6.13)$$

Equation (6.10) can be simplified to

$$A = \frac{1}{1 - \alpha \frac{P\Delta}{Hh}} \quad (6.14)$$

Sway effects are also referred to as  $P\Delta$  effects, because of the  $P\Delta$  term appearing in the denominator of Equation (6.14).

Limits on  $\alpha$  can be set. If the end walls are very stiff and the roof deflection is caused mainly by shear deformation in the deck

$$\alpha = \frac{\frac{1}{12}}{\frac{1}{8}} = \frac{2}{3} = 0.67 \quad (6.15)$$

If the end walls are very stiff and the roof deflection is caused mainly by flexural deformation of the perimeter roof members:

$$\alpha = \frac{\frac{1}{8}}{\frac{1}{2}} = 0.78 \quad (6.16)$$

If the end walls are flexible and the roof deck is rigid:

$$\alpha = \frac{\frac{1}{4}}{\frac{1}{4}} = 1.0 \quad (6.17)$$

Thus  $\alpha$  must always be between the limiting values of 0.67 and 1.0.

Since the shear in the diaphragm web, the axial forces in the diaphragm flanges, and the axial forces in the end wall bracing members are all proportional to the roof deflection, which in turn is proportional to the lateral load, sway effects can be conveniently accounted for in design by multiplying the lateral load by the amplification factor  $A$ . This approach, which can also be shown to be valid for single storey rigid or semi-rigid frames, is followed in the illustrative examples in this chapter.

### 6.3.2 Simplification For Preliminary Design

The amplification factor cannot be computed from Equation (6.14) until after the bracing members and the roof diaphragm are designed and their properties are known. Therefore for preliminary design it is necessary to be able to account for the sway effects approximately.

As discussed above, the amplification factor is used to multiply the lateral loads, which may be caused by either wind or earthquake. The first assumption that will be made is that the roof diaphragm is rigid so that the quantity  $\alpha$  used in computing the amplification factor is 1.0. This assumption is conservative since  $\alpha$  must always be between 0.67 and 1.0 as shown earlier.

The quantity  $\Delta$  is the sway due to the lateral load  $H$ , and is usually not likely to exceed approximately  $h/200$  at the specified load level for wind or earthquake. At the factored load level, when snow load is not considered to act in combination with lateral load,

$$\Delta = 1.5 \frac{h}{200} \quad (6.18)$$

If snow load is considered,

$$\Delta = 0.7 \times 1.5 \frac{h}{200} \quad (6.19)$$


---



Example 6.1Given

Estimate the factored, amplified lateral loads on the structure shown in Figure 6.1 due to:

1. wind in the summer, and
2. wind in the winter.

The specified loads were shown to be:

$$D = 31.5 \text{ psf (Example 2.1)}$$

$$L = 48 \text{ psf (Example 2.2)}$$

$$Q = 18.7 \text{ psf (Example 2.4)}$$

Solution

1. Wind in the Summer

The factored vertical load is:

$$P = 1.25 D = 1.25 \times 31.5 \times \frac{200 \times 120}{1000} = 945 \text{ kips}$$

The factored, but unamplified lateral load is:

$$H = 1.5 Q = 1.5 \times 18.7 \times \frac{200 \times 21}{2 \times 1000} = 58.9 \text{ kips}$$

The estimated deflection at the factored load level is:

$$\Delta = 1.5 \frac{h}{200} = 1.5 \times \frac{21 \times 12}{200} = 1.89 \text{ in.} \quad (\text{Eq. 6.18})$$

The amplification factor is:

$$A = \frac{1}{1 - \frac{P\Delta}{Hh}} = \frac{1}{1 - \frac{1.0 \times 945 \times 1.89}{58.9 \times 21 \times 12}} \quad (\text{Eq. 6.14})$$

$$= 1.14$$

Therefore the factored, amplified lateral load is:

$$H = 58.9 \times 1.14 = 67.2 \text{ kips}$$

## 2. Wind in the Winter

The factored vertical load is:

$$P = 1.25 D + 0.7 \times 1.5L$$

$$= (1.25 \times 31.5 + 0.7 \times 1.5 \times 48) \times \frac{200 \times 120}{1000}$$

$$= 2155 \text{ kips.}$$

The factored but unamplified lateral load is:

$$\begin{aligned} H &= 0.7 \times 1.5 Q \\ &= 0.7 \times 1.5 \times 18.7 \times \frac{200 \times 120}{2 \times 1000} \\ &= 41.2 \text{ kips} \end{aligned}$$

The estimated deflection at the factored load level is:

$$\begin{aligned} \Delta &= 0.7 \times 1.5 \frac{h}{200} && \text{(Eq. 6.19)} \\ &= 0.7 \times 1.5 \times \frac{21 \times 12}{200} \\ &= 1.32 \text{ in.} \end{aligned}$$

The amplification factor is:

$$\begin{aligned} A &= \frac{1}{1 - \alpha \frac{P\Delta}{Hh}} = \frac{1}{1 - \frac{1.0 \times 2155 \times 1.32}{41.2 \times 21 \times 12}} && \text{(Eq. 6.14)} \\ &= 1.38 \end{aligned}$$

Therefore the factored, amplified lateral load is:

$$H = 41.2 \times 1.38 = 56.9 \text{ kips.}$$

The most severe lateral load occurs during the summer.

Although the sway effect is smaller than in the winter (an amplification factor of 1.14 versus one of 1.38) the lateral load is not subject to a reduction by the load combination factor.

Similar calculations (not shown) indicate that the design of the lateral load resisting system is not governed by earthquake loading.

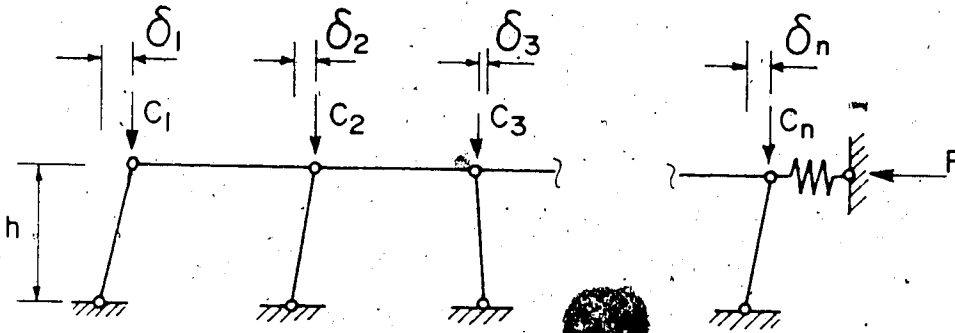
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### 6.3.2 Effect of Column Out-of-Plumbs

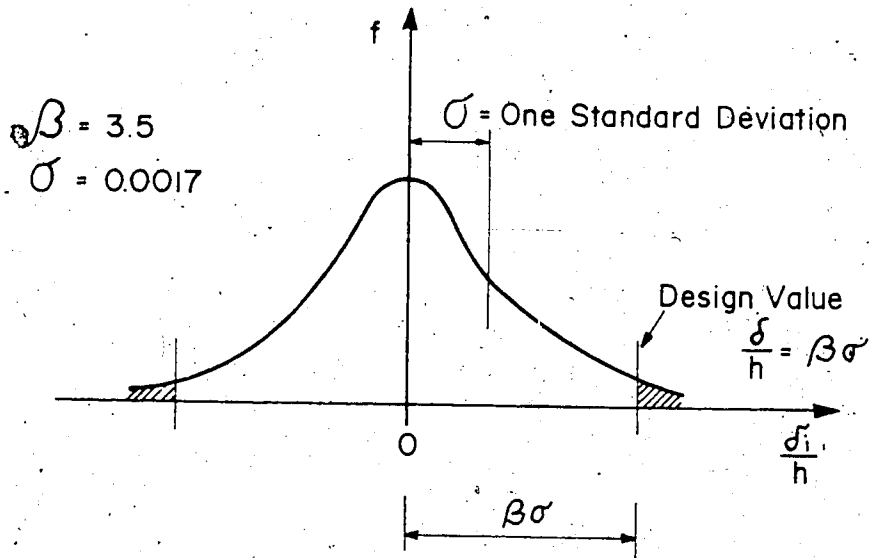
In Example 6.1 only the effects of dead load with full lateral load and the effects of dead load with reduced snow load and lateral load were considered. A third combination, dead load with full snow load and a small lateral load to simulate column out-of-plumb $\bar{s}$  should also be considered.

The maximum out-of-plumb permitted in Reference (1) for a column is 1/500 of the column height. If the distribution of out-of-plumbness of columns in a building is measured, it would be found that some columns may be out-of-plumb in varying amounts in one direction, while others would be out-of-plumb by varying amounts in the other direction, as illustrated in Figure 6.2(a). Although the actual out-of-plumbs should in most cases be less than the maximum permitted value given above, it is possible that some columns may be out-of-plumb by more than this.

Based on a study of column out-of-plumbs in buildings<sup>(78)</sup> it may realistically be assumed that after erection there is an equal probability of a particular out-of-plumbness  $\delta_i$  occurring in one direction or the other direction.



(a) A series of n out-of-plumb columns



(b) Frequency diagram of out-of-plumbs

FIGURE 6.2 BRACING FORCE REQUIRED TO RESIST THE EFFECT OF COLUMN OUT-OF-PLUMB

The probability density function of  $\delta_i/h$  thus will be symmetric, as illustrated in Figure 6.2(b), with a mean close to zero, and in the analysis in Reference (78) it is therefore assumed that  $\delta_i/h$  follows a normal probability law. Since  $\delta_i$  is a random variable, the out-of-straightness  $\delta_i/h$  also will be random. Since  $h$  is a deterministic parameter, the statistical characteristics of  $(\delta_i/h)$  thus will be directly proportional to those of  $\delta_i$ .

Based on these assumptions it can be shown that the effect of "n" out-of-plumb columns can be simulated with a fictitious lateral force given by(78):

$$F = \beta \sigma \sqrt{\sum_{i=1}^n C_i^2} \quad (6.20)$$

where

$C_i$  = factored axial load in column "i",

$\delta$  = 0.0017 = standard deviation of measured out-of-plumbs (see Figure 6.2 (b)),

$\beta$  = 3.5 = a "safety index" such that there is an acceptably small probability of a given  $\delta_i/h$  exceeding  $\beta\sigma$

Use of Equation (6.20) is illustrated in the following example.

### Example 6.2

#### Given

Estimate the fictitious lateral load to simulate the

effect of out-of-plumb columns in the structure shown in Figure 6.1. A plan view of the column lay-out is shown in Figure 5.1. The specified dead load is 31.5 psf and the specified snow load is 48 psf.

### Solution

The factored roof load is first determined:

$$1.25 D + 1.5 L = 1.25 \times 31.5 + 1.5 \times 48 = 111 \text{ psf.}$$

By actual count, the number of columns and their associated tributary areas and loads are:

<u>No. of Columns</u>	<u>Trib. Area, (ft.<sup>2</sup>)</u>	<u>C (kips)</u>
8	1600	178
20	400	44
8	350	39
4	100	11

therefore the lateral force used to simulate column out-of-plumbs

is:

$$H = 8\sigma \sqrt{\sum_{i=1}^n C_i^2} \quad (\text{Eq. 6.20})$$

$$= 3.5 \times 0.0017 \times \sqrt{8 \times (178)^2 + 20 \times (44)^2 + 8 \times (39)^2 + 4 \times (11)^2}$$

$$= 3.3 \text{ kips}$$

Since this is much smaller than the lateral loads computed in Example 6.1 the effects of this loading case are negligible. A systematic study is required, using the approach in this example, to determine if this is generally true - that is, to determine if the effect of column out-of-plumbs can be ignored for single storey buildings.

---

#### 6.4 Load Transfer in the Plane of the Roof

In Section 6.2 methods of lateral load transfer from the walls to the roof were discussed, and in Section 6.3 it was shown how sway effects can be accounted for by amplifying these loads. In this section transfer of these (amplified) lateral loads in the plane of the roof to the braced end walls is discussed.

Lateral loads can be transferred in the plane of the roof by either diaphragm action of the roof deck or by a direct acting bracing system designed for this purpose. As shown in CHAPTER III, significant cost savings are often possible if diaphragm action is used since the roof deck is required in any event to resist the vertical loads. Nevertheless, in some situations the engineer may wish to employ a direct acting bracing scheme. Both mechanisms of load transfer are discussed in this section.



#### 6.4.1 Diaphragm Action

When diaphragm action is used, the roof is designed to act as a large horizontal plate girder of length  $L$  and depth  $d$ , as shown in Figure 6.3(a). As illustrated in Figure 6.3(b), the flanges of the girder consist of trimmer angles, required on all four sides of the diaphragm. The web of the girder consists of the roof deck. The supports are the braced end walls.

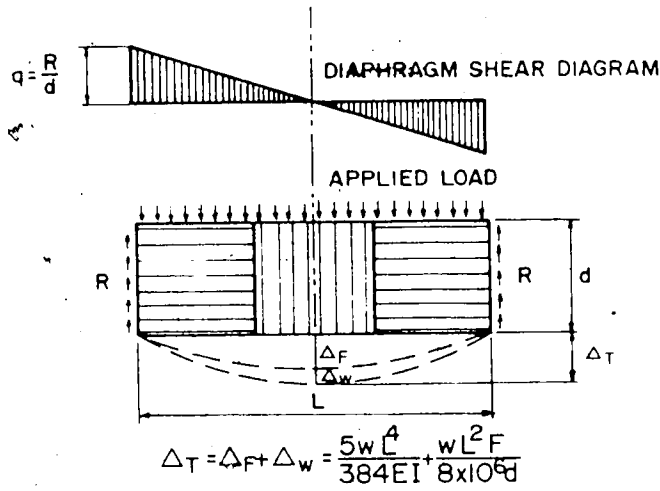
To simplify the design it is assumed that the flanges resist only flexural forces and that the web resists only shear forces (29). Thus the maximum axial force  $C$  in either the top or bottom flange is:

$$C = \frac{M}{d} = \frac{wL^2}{8d} \quad (6.21)$$

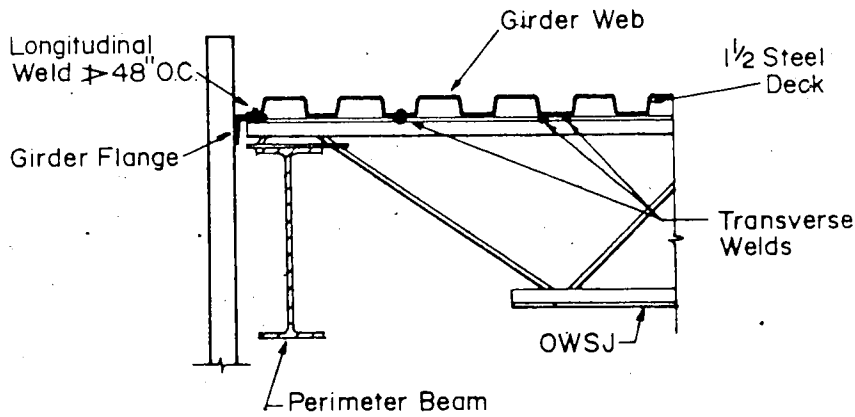
where  $M$  is the mid-span bending moment due to the lateral load  $w$ . The maximum shear flow,  $q$ , in the web is:

$$q = \frac{R}{d} = \frac{wL}{2d} \quad (6.22)$$

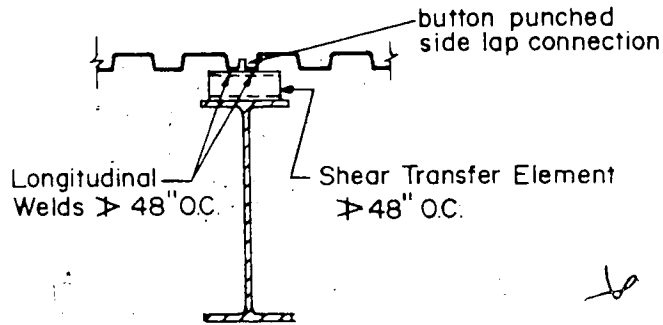
where  $R$  is the girder reaction. The maximum roof deflection  $\Delta_T$  is determined as the sum of a flexural component and a shear component:



(a) Plate Girder Analogy



(b) Girder Flange and Web



(c) Shear Connector

FIGURE 6.3 DIAPHRAGM ACTION

$$\Delta_T = \frac{5wL^4}{384 EI} + \frac{wL^2F}{8 \times 10^6 d} \quad (6.23)$$

where  $I$  is the moment of inertia of the trimmer angles about the centroidal axis of the girder, and  $F$  is the "flexibility factor" of the roof deck, in units of micro-inches per pound. This is illustrated schematically in Figure 6.3(a)

In Equations (6.21) to (6.23) it is assumed that the lateral load acts as a uniformly distributed load. Although this is valid for an earthquake load when the roof is subjected to full snow load, since the earthquake load is proportional to the roof load, it is clearly a simplification for wind load if the walls consist of girts and cladding. For this latter case a more detailed analysis could be made by assuming the lateral load acts as a series of point forces at each column location, as discussed in Section 6.2, however, for simplicity this will not be done here.

As illustrated in Figure 6.3(b), trimmer angles are usually required along the roof perimeter if the walls consist of girts and cladding; these members can also function as the girder flanges, thus resulting in a cost saving. If the wall construction is such that trimmer angles are not ordinarily required, for example with masonry block, a cost saving may be possible if the perimeter roof beams and joists are designed to act as the flanges instead. In this case, however, tests are required to determine if the joist seats are able to transfer the shear adequately.

The ability of the roof deck to resist web shear depends on a number of factors including deck thickness, deck profile, fastener pattern, span, and length of deck unit supplied, but, as illustrated in Figure 6.3(a), not on the orientation of the units relative to the direction of the applied loads(29).

Diaphragm strengths and flexibility factors for a common roof deck profile and various fastener patterns have been published in Reference (29), and are reproduced in Table 6.1. Other deck profiles have been included in the literature of different companies supplying these products, and in technical reports. (79,80,81) Unfortunately much of the test information summarized in the literature of the roof deck suppliers is proprietary and is not readily available.

The values listed in Table 6.1 are allowable values, extrapolated from various large scale test results in which a factor of safety of 3.0 has been incorporated. At the time of writing they have not been converted to a limit states format, therefore in the example that follows the diaphragm shear computed using the factored loads is divided by the live load factor of 1.5 and is then compared with the allowable shear. Additional research is required before an appropriate performance factor can be determined.

---

No. of Transverse Welds/Unit = 3 per support													
		Side Lap			Button Punch 12 in. o.c.			Button Punch 18 in. o.c.			Button Punch 24 in. o.c.		
SPAN (ft.)	Deck th.	.030	.036	.048	.030	.036	.048	.030	.036	.048	.030	.036	.048
5	q	336	495	817	300	453	766	282	431	740	20+22R	15+128R	10+54R
	F	17+22R	13+128R	9+54R	19+22R	14+128R	9+54R	20+22R	15+128R	10+54R	20+22R	15+128R	10+54R
6	q	295	433	709	259	390	657	240	368	630	240	368	630
	F	19.5+185R	15.7+107R	10+45.2R	22+185R	17.2+107R	10.4+45.2R	23.2+185R	18+107R	11+45.2R	23.2+185R	18+107R	11+45.2R
7	q	266	386	633	230	343	580	212	321	554	212	321	554
	F	21+158R	17+91R	12+38R	24+158R	19+91R	12+38R	26+158R	20+91R	13+38R	26+158R	20+91R	13+38R
8	q	224	350	577	208	307	520	190	286	496	190	286	496
	F	24.2+138R	19.3+80.6R	13.4+33.9R	27.2+138R	21.6+80.6R	14.5+33.9R	29.4+138R	23.1+80.6R	15.3+33.9R	29.4+138R	23.1+80.6R	15.3+33.9R

No. of Transverse Welds/Unit = 5 per support													
		Side Lap			Button Punch 12 in. o.c.			Button Punch 18 in. o.c.			Button Punch 24 in. o.c.		
SPAN (ft.)	Deck th.	.030	.036	.048	.030	.036	.048	.030	.036	.048	.030	.036	.048
5	q	418	624	1074	382	580	1019	364	559	992	364	559	992
	F	16+55R	12+32R	8+13R	17+55R	13+32R	8+13R	17+55R	13+32R	8+13R	17+55R	13+32R	8+13R
6	q	360	533	926	326	490	870	308	469	842	308	469	842
	F	18+46.3R	14.2+25.7R	9.5+11.3R	20+46.3R	15.2+25.7R	10+11.3R	21+46.3R	15+25.7R	10.2+11.3R	21+46.3R	15+25.7R	10.2+11.3R
7	q	322	469	821	286	426	764	268	404	735	268	404	735
	F	20+39R	16+22R	11+9R	22+39R	17+22R	11+9R	24+39R	18+22R	12+9R	24+39R	18+22R	12+9R
8	q	290	421	733	256	378	675	238	357	546	238	357	546
	F	22.2+34.7R	18+20R	12.2+8.5R	25+34.7R	19.6+20R	13.1+8.5R	27+34.7R	21+20R	13.6+8.5R	27+34.7R	21+20R	13.6+8.5R

Notes:

R = ratio of span of deck unit to average length of deck unit supplied.

TABLE 6.1 DIAPHRAGM STRENGTH AND FLEXIBILITY

Example 6.3Given

In Example 6.1 the factored, amplified lateral load on the structure of Figure 6.1 was shown to be 67.2 kips, i.e.,  $w = 67.2/200 = 0.336$  kips/ft. This load is to be transferred in the plane of the roof to the end walls by diaphragm action. Select:

1. the girder flanges (trimmer angles) using an angle section in G40.21 44W steel ( $F_y = 44$  ksi.),
2. the number of transverse arc spot welds per deck unit and the spacing of the side lap button punch connections, and
3. compute the maximum roof deflection at service load level using a specified wind pressure of 14.9 psf, as computed in Example 2.4 for a probability of occurrence of once in ten years.

Assume that the roof deck units are 1.5 inches deep, 0.030 inches thick, 2 feet wide, 21 feet long, and with flutes at 6 inches on centers, as shown in the inserts to Table 6.1. The building dimensions  $L$ ,  $d$  and  $h$  are 200, 120 and 20 feet, respectively, and the roof joists are evenly spaced on 6'-8" centers.

Solution1. Design of the Trimmer Angles

As illustrated in Figure 6.3(b), the roof deck is

welded to the trimmer angle. The weld spacing must not exceed 48 inches, as discussed later. It is assumed, therefore, that wind loads acting on the cladding are transferred directly into the roof deck, and that this angle does not have to resist a local bending moment. The size of the angle will be determined by assuming that it behaves as a column with a length equal to the distance between roof joists. Conservatively, load transfer through the joist seats to the perimeter roof beams can be ignored. Instability will be assumed to occur about the weak (Z-Z) axis of the angle.

Try a 3 x 3 x 5/16 angle

The cross section properties for a 3 x 3 x 5/16 angle are given in Reference (5):

$$r_z = 0.59 \text{ in.}$$

$$A = 1.78 \text{ in.}^2$$

$$b = 3 \text{ in.}$$

$$t = 0.3125 \text{ in.}$$

The non-dimensional slenderness ratio is

$$\begin{aligned}
 &= \frac{KL}{r} \sqrt{\frac{F_y}{\pi^2 E}} && \text{(Eq. 5.2)} \\
 &= \frac{1.0 \times 8 \times 12}{0.59} \sqrt{\frac{44}{\pi^2 \times 29000}} \\
 &= 2.02
 \end{aligned}$$

Since  $2.0 < \lambda < 3.6$  the factored compressive resistance is given by

$$\begin{aligned} C_r &= \phi A F_y (0.009 + 0.877 \lambda^{-2}) && \text{(Eq. 5.1)} \\ &= 0.9 \times 1.78 \times 44 (0.009 + 0.877 \times (2.02)^{-2}) \\ &= 15.8 \text{ kips} \end{aligned}$$

The maximum compressive force is

$$\begin{aligned} C &= \frac{wL^2}{8d} \\ &= \frac{0.336 \times (200)^2}{8 \times 120} && \text{(Eq. 6.21)} \\ &= 14.0 \text{ kips} \end{aligned}$$

Since this is less than  $C_r = 15.8$  kips, the  $3 \times 3 \times 5/16$  angle is satisfactory. Similar calculations also indicate that it is also the lightest member that can be selected.

## 2. Design of Shear Diaphragm

As indicated above, the diaphragm strengths in Table 6.1 are allowable values. Therefore, in computing the maximum diaphragm shear the specified, not factored, lateral load must be used. This is done by dividing the factored lateral load by the live load factor of 1.5

$$w = \frac{0.336}{1.5} = 0.224 \text{ kips/ft.}$$



The maximum diaphragm shear is

$$\begin{aligned}
 q &= \frac{wL}{2d} \\
 &= \frac{0.224 \times 200}{2 \times 120} \times 1000 \quad (\text{Eq. 6.22}) \\
 &= 187 \text{ lbs./ft.}
 \end{aligned}$$

which, from Table 6.1, can be safely resisted if there are 3 transverse welds per unit and the side lap connections are at 24 inches center-to-center. Interpolating linearly between the tabulated values of 240 and 212 lbs./ft. for deck spans of 6 and 7 feet, respectively, the allowable diaphragm shear for a span of 6'-8" is 221 lbs./ft.

### 3. Calculation of Roof Deflection

The lateral load is found by multiplying the wind pressure by one half of the height of the building

$$w = 14.9 \times \frac{21}{2} = 156 \text{ lbs./ft.}$$

The moment of inertia of the trimmer angles about the centroidal axis of the roof diaphragm is

$$I = \sum A \left(\frac{d}{2}\right)^2$$

$$= 2 \times 1.78 \times \left( \frac{120 \times 12}{2} \right)^2$$

$$= 1.85 \times 10^6 \text{ in.}^4$$

The flexibility factor must also be obtained by interpolation from Table 6.1

$$F = 25.1 + 167 R$$

where R is the ratio of the deck span to the length of unit supplied, so that

$$F = 25.1 + 167 \times \frac{6.67}{21}$$

$$= 78.1 \text{ micro inches per pound.}$$

The roof deflection can now be computed as

$$\Delta_T = \frac{5wL^4}{384 EI} + \frac{wL^2F}{8 \times 10^6 d} \quad (\text{Eq. 6.23})$$

$$= \frac{5 \times 156 \times (200)^4 \times 1728}{384 \times 29 \times 10^6 \times 1.85 \times 10^6} + \frac{156 \times (200)^2 \times 78.1}{8 \times 10^6 \times 120}$$

$$= 0.11 + 0.51$$

$$= 0.62 \text{ inches.}$$

Note that this is the roof deflection relative to the end walls, and is not the maximum column sway. In Example 6.6 this roof deflection will be added to the end wall deflection and the sum will be compared to recommended values of maximum column sway.

---

The strengths and flexibility factors given in Table 6.1 appear to be derived, in part at least, from empirical equations given in Reference (82). For 1.5 inch deep galvanized deck units 2 feet wide, flutes of 6 inches center-to-center, and button punched side lap connections some simplification of these equations is possible. The strength equation can be written:

$$q = (q_1 + q_2) \frac{q_3}{q_3} \quad (6.24)$$

but not greater than  $q_1 + q_2$  or  $\frac{3.25 \times 10^6 t}{2L_v^2}$

where

$$q_1 = \frac{92 tK}{L_v} \quad (6.25)$$

$$q_2 = \frac{L_v}{a_s} \sqrt{q_1 \left( 0.31 + \frac{1}{C_1 L_v t} \right)} \quad (6.26)$$

$$q_3 = \frac{3600 t}{a_s} \quad (6.27)$$

$$K = \frac{1000}{\sqrt{1 + \left( \frac{C_2}{t} \right)^4}} \quad (6.28)$$

$t$  = deck thickness in inches,

$L_v$  = deck span in feet,

$C_1$  = 4 for 3 transverse welds/unit

= 5 for 5 transverse welds/unit

$C_2$  = 0.043 for 3 transverse welds/unit,

= 0.038 for 5 transverse welds/unit, and

$a_s$  = spacing of the button punch connections, in feet.

Similarly, the equation for the flexibility factor can be written:

$$F = F_1 + F_2 + F_3 \quad (6.29)$$

where

$$F_1 = \frac{1}{12t} \quad (6.30)$$

$$F_2 = \frac{L_v^2}{80} \left( 0.31 + \frac{1}{C_1 L_v t} \right) \frac{q_1}{q_1 + q_2} \quad (6.31)$$

$$F_3 = \frac{R}{C_3 L_v t^3} \quad (6.32)$$

$C_3 = 33$  for 3 transverse welds/unit  
 $133$  for 5 transverse welds/unit.

Substitution of numerical values into these equations produces values of  $q$  and  $F$  that are almost identical to those given in Table 6.1. Unfortunately the theoretical background for the general form of these equations is not given in Reference (82), nor are these expressions compared with test results.

It is standard practice in Canada to fasten individual deck units to the supporting framework with arc spot ("puddle") welds. Depending on their locations, arc spot welds are divided into two groups; transverse welds and longitudinal welds. As shown in Figures 6.3(b) and (c), transverse welds are perpendicular to the deck flutes while longitudinal welds are parallel to the deck flutes. In Reference (29) it is recommended that the longitudinal weld spacing,  $a_w$ , not exceed the maximum distances given in Table

6.2, or 4 feet. Since roof joists are spaced, typically, 5 or more feet apart, special shear transfer elements are therefore usually required on all interior girders between joists. This is illustrated in Figure 6.3(c).

For lightly loaded roofs it may be more economical to omit the shear connectors. However, additional research is required before appropriate diaphragm strengths and flexibility factors can be determined. Although some work has been done in this area<sup>(47,48)</sup> it is more applicable to screw connected metal wall diaphragms (see Section 5.3.4) since the fasteners are assumed to be elastic - perfectly plastic. This is not a valid assumption for arc spot welds.

#### 6.4.2 Direct Acting Bracing Schemes

It is usually more economical to transfer loads in the plane of the roof by diaphragm action of the roof deck rather than through a direct acting bracing system. However in some situations, for example when mechanical requirements dictate a large number of roof openings, it may be inadvisable to rely on diaphragm action. Only a small amount of research has focussed on the behaviour of diaphragms with openings<sup>(83)</sup>.

Bracing members are usually flat plates, angles or rods. Use of thin flat plates may at first glance appear to be particularly attractive since they can be field welded to the top

Deck Gauge	Nominal Core Thickness	Factor*
22	.030 in.	1380
20	.036	1650
18	.048	2200
16	.060	2750
20-20	.036/.036	2750
20-18	.036/.048	3120

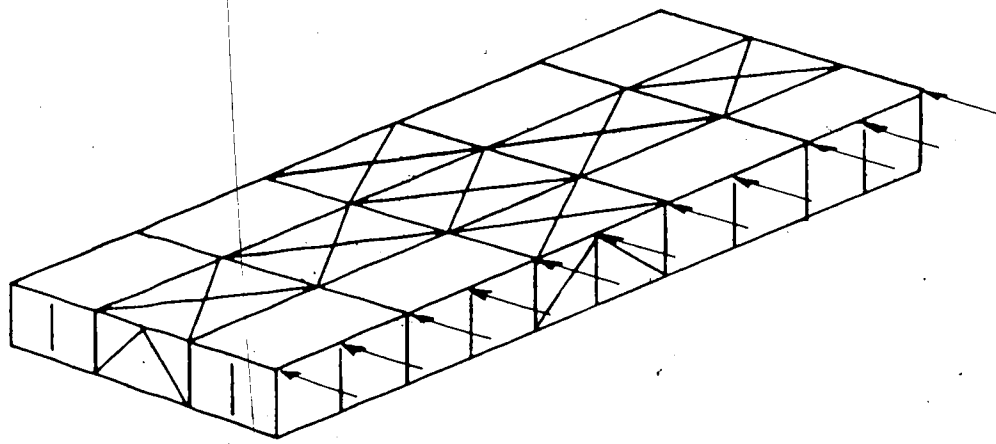
\*Divide factor in this Table by the specified diaphragm shear to get spacing of welds in feet to framing members parallel to flutes. The spacing in no case should exceed 4 ft.

TABLE 6.2 LONGITUDINAL WELD SPACING FACTORS

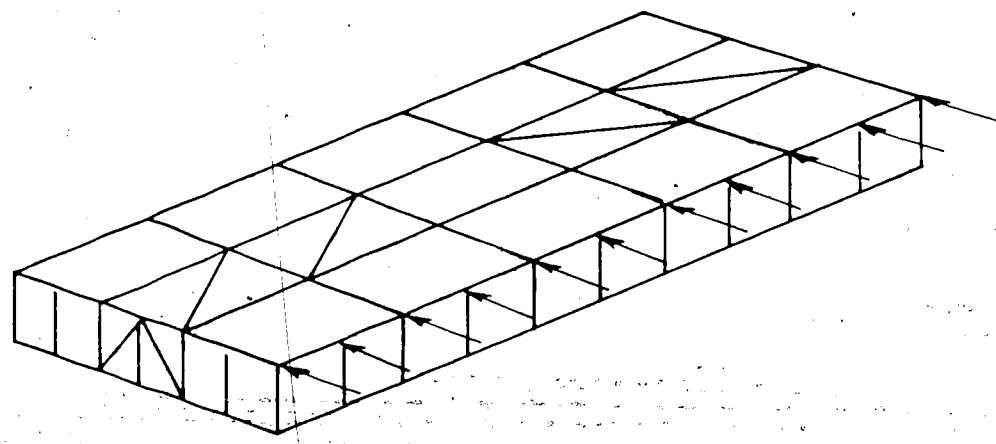
surfaces of the top chords of the roof joists while angles or rods are usually attached to the underside of the top chords in order to allow proper connection of the roof deck. Erection is less costly since there is no interference with the top chord bridging. However, flat plates are susceptible to bending or kinking during shipment to and storage at the job-site. Their use, therefore, may not be advisable unless precautions are taken to avoid this type of damage.

A typical roof bracing scheme is illustrated in Figure 6.4(a). It is apparent that bracing schemes can be highly indeterminate, even with a simple building lay-out such as this one. However, the bracing members are usually so slender that they are ineffective in resisting compression and the simplifying assumption can be made that all compression diagonals are buckled and only the tension diagonals are effective. Thus the statically indeterminate bracing scheme shown in Figure 6.4(a) can be reduced to the statically determinate system shown in Figure 6.4(b).

Often calculated bracing forces may be so small that the design of the bracing members is governed by an arbitrary decision on the maximum slenderness ratio that should be permitted for these members. In Reference (1) a maximum value of 200 is recommended for members designed to resist compression, and 300 for tension members. It is suggested however that this latter requirement could be waived if it is shown that flexibility, sag, vibration or slack will not be detrimental to the performance of the structure. For light



(a) Actual Lay-out



(b) Simplified Statically Determinant Lay-out

FIGURE 6.4 DIRECT ACTING BRACING SYSTEM



industrial buildings without overhead travelling cranes the slenderness requirement for tension members can usually be waived.

#### 6.5 Load Transfer in the Planes of the End Walls

Lateral loads are transmitted from the roof to the foundation through the end walls. These loads may be transferred in the plane of the end walls by diaphragm action, a direct acting bracing system, or a combination of these mechanisms.

Diaphragm action of the wall cladding was discussed in Section 5.3.4. As discussed in that section, it is assumed that the girts are connected into the column webs. If the girts are connected into the column flanges, or if the girts are connected into the webs but the walls contain a large number of door and window openings, it may not be possible to develop diaphragm action and a direct acting bracing scheme may have to be used instead.

Several different types of bracing systems are shown in Figure 6.5. Although the "right-left" and "X" bracing systems are perhaps more common, "K" and "A" bracing systems are often used for framing around wall openings. These bracing systems may be designed as either "tension only" or "tension-compression" bracing systems.

One question that sometimes arises with the tension-

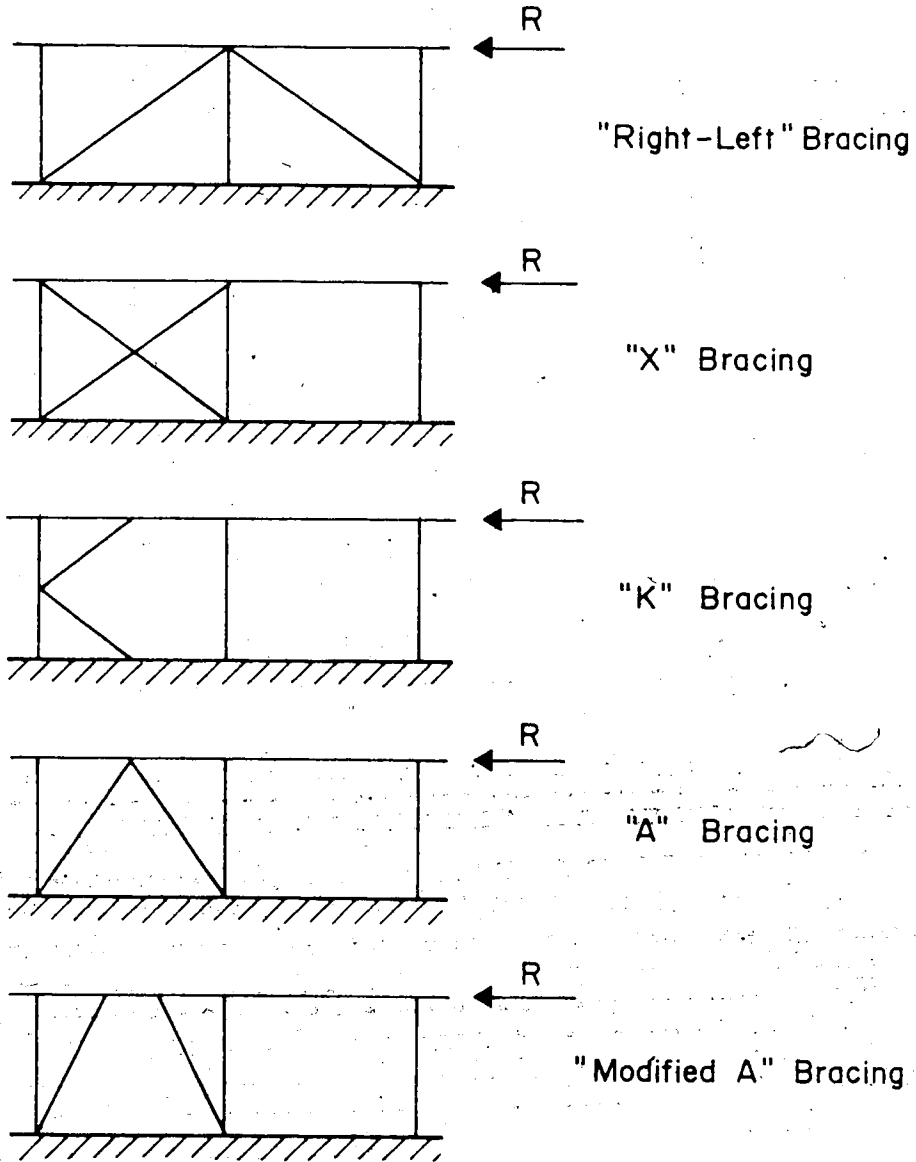


FIGURE 6.5 VARIOUS TYPES OF BRACING SYSTEMS

compression "X" bracing system concerns the effective length of the compressive brace if it is connected to the tension brace at its mid-point. To analyze this problem, consider the braced bent shown in Figure 6.6(a). The applied shear is denoted  $R$ , and the resulting forces in each brace are denoted  $C$ , one of which is a compression force and the other a tension force. The length of each brace is denoted  $l$ . The buckled shape of the system is as shown.

The tension brace acts as a spring. The spring stiffness  $k$  includes the bending stiffness of the brace, plus a stiffness term due to the tension force<sup>(39)</sup>:

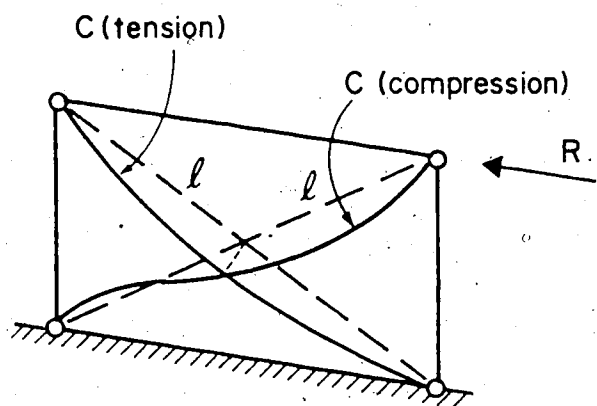
$$k = \frac{48EI}{l^3} + \frac{4C}{l} \quad (6.33)$$

where  $I$  is the moment of inertia for out-of-plane bending. The compression brace can therefore be idealized as shown in Figure 6.6(b), for which the critical elastic load is<sup>(39)</sup>:

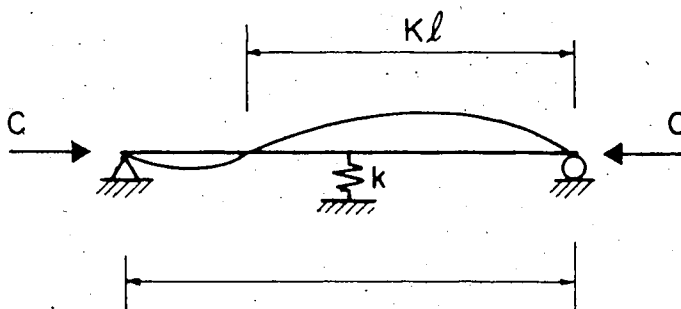
$$C_{cr} = \left(1 + \frac{3}{16} \frac{k l}{C_e}\right) C_e \quad (6.34)$$

where  $C_e$  is the Euler buckling load

$$C_e = \frac{\pi^2 EI}{l^2} \quad (\text{Eq. 5.5})$$



(a) BUCKLED SHAPE



(b) EFFECTIVE LENGTH OF COMPRESSION BRACE

FIGURE 6.6 ANALYSIS OF "X" BRACE

where

$$K = \frac{1}{\sqrt{1.9 + \frac{3C}{4C_e}}} \quad (6.36)$$

is the effective length factor. Using Equation (6.36) it is seen that  $K$  must always be less than 0.725, thus use of  $K$  equal to unity may be overly conservative. After  $K$  is computed the member can be designed as a column using Equations (5.1) and (5.2). The discussion of maximum slenderness ratios in Section 6.4.2 for bracing members in the plane of the roof applies equally well for bracing members in the planes of the end walls.

If a "tension only" direct acting bracing system is coupled with diaphragm action, the ultimate shear load  $R$  that can be resisted is equal to the shear capacity of the wall cladding added to the horizontal component of the factored yield load of the tension brace, providing the brace is fabricated from hot rolled steel. This approach is possible because it has been demonstrated that both have sufficient ductility for the complete system to develop its ultimate strength(84). Initial tightness of the brace between 0 and 20 percent of the yield load does not significantly reduce this ultimate strength(84).

#### Example 6.4

##### Given

In Example 6.3 diaphragm action of the roof deck was used to transfer the factored lateral load of 0.336 kips/ft.

acting at roof level on the structure shown in Figure 6.1 to the end walls. As indicated, the structure is 200 ft. long, 120 ft. wide and 21 ft. high. The exterior columns are spaced 20 ft. apart. In this example a combination of diaphragm action of the wall cladding and a "tension-only right-left" direct acting bracing system will be used to transfer the fictitious "plate girder" reactions of  $R = 0.336 \times 100 = 33.6$  kips down to the foundations.

In Example 5.6 the factored diaphragm strength was shown to be 221 lbs./ft., however of this available strength 148 lbs./ft. is required to brace the exterior columns, leaving  $221 - 148 = 73$  lbs./ft. to resist the lateral loads. The braces will be fabricated from G40.21 44W steel ( $F_y = 44$  ksi).

### Solution

The portion of the reaction that can be resisted by diaphragm action, denoted  $R_1$ , is:

$$\begin{aligned} R_1 &= qd \\ &= \frac{73}{1000} \times 120 \\ &= 8.76 \text{ kips} \end{aligned}$$

therefore the portion  $R_2$  that must be resisted by the bracing system is:

$$\begin{aligned}
 R_2 &= R - R_1 \\
 &= 33.6 - 8.76 \\
 &= 24.8 \text{ kips.}
 \end{aligned}$$

The axial force in the tension brace is:

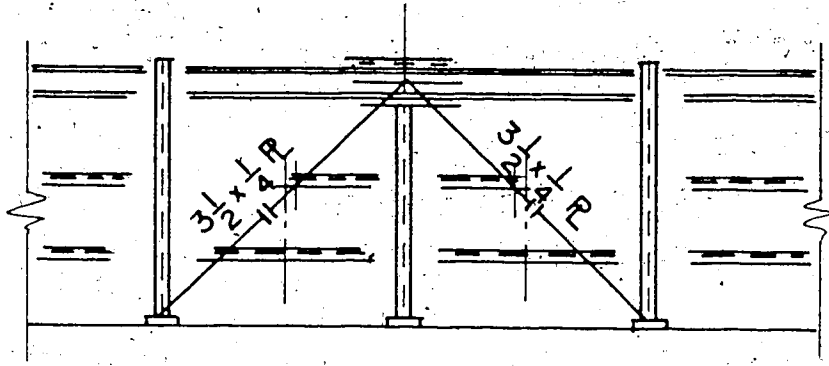
$$\begin{aligned}
 T &= \frac{\sqrt{(21)^2 + (20)^2}}{20} \times 24.8 \\
 &= 36.0 \text{ kips,}
 \end{aligned}$$

and the required area of steel is:

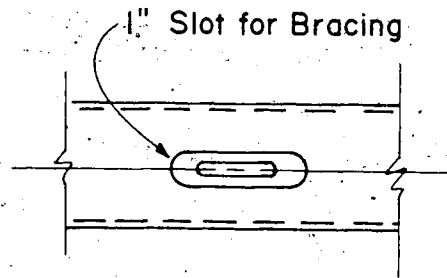
$$\begin{aligned}
 A &= \frac{T}{\phi F_y} \\
 &= \frac{36.0}{0.9 \times 44} \\
 &= 0.91 \text{ in.}^2
 \end{aligned}$$

This will be provided by using 3-1/2 in. x 1/4 in. flat plates, as illustrated in Figure 6.7(a). Two additional points should be noted, however:

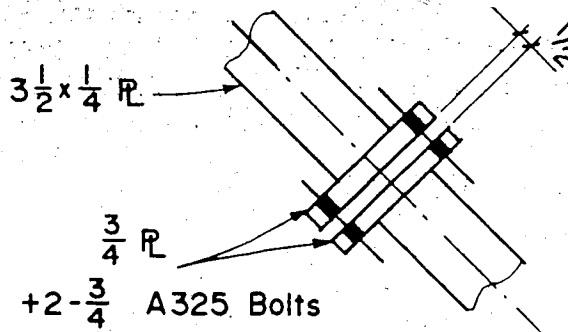
1. as illustrated in Figure 6.7(b), the girts in the braced bays must be slotted to provide clearance for the bracing members, and
  2. since flat plates are susceptible to bending and kinking a pre-tension detail must be provided, such as that shown in Figure 6.7(c).
-



(a) BRACING LAY-OUT



(b) DETAIL THROUGH GIRTS



(c) PRE-TENSION DETAIL

FIGURE 6.7 DETAILS OF BRACING SCHEME SELECTED



## 6.6 Load Transfer From the End Walls to the Foundation

A typical detail at the lower end of an exterior beam-column is shown in Figure 5.23. The horizontal component of the axial force in the bracing member and the shear resisted by diaphragm action of the wall cladding is transferred from the lower end of the column into the foundation through the anchor bolts. Although the column, by definition, is subjected to a compression force, the column base will be subjected to a tensile force if the vertical component of the force in the brace exceeds the compressive force in the column.

Anchor bolts used in light industrial buildings are usually fabricated from 3/4 or 7/8 inch diameter steel rods with a minimum specified yield stress of 36 ksi. The bolts are threaded at one end and bent at the other to form a right angle, as shown in Figure 5.23. The dimensions shown are representative of those used in practice; however actual dimensions may vary from those indicated.

In heavy industrial buildings anchor bolts may consist instead of 1-1/4 to 3 inch diameter rods with a nut and washer anchorage instead of a right angle bend. The strength of this type of connection is considered elsewhere (85).

Very little information is available to assist the designer in determining the number of anchor bolts required, which

depends on a number of variables, including:

1. the type of loading,
2. anchor bolt configuration and dimensions,
3. strength of anchor bolt steel,
4. dimensions of the foundation,
5. method of reinforcement,
6. distance from the anchor bolts to the edge of the concrete,
7. concrete strength,
8. grout strength, and
9. quality of the grouting operation.

In Appendix B a series of test results is summarized to determine the effect of the first of the variables listed above. The remaining variables were held fixed at values typical of those encountered in practice. For example, the foundation simulated was a ten inch thick wall using concrete with a compressive strength  $F'_c$  of 3000 psi, and the anchor bolts were fabricated from steel with a specified minimum yield strength of 36 ksi. The anchor bolt diameter  $d$  was  $3/4$  inch. Most anchor bolts used in this type of construction are, in fact,  $3/4$  inch in diameter. Four additional tests were then performed to determine the effects of oversize holes in the base plates and the effects of incomplete grouting.

Since the effects of the remaining variables were not studied, the results are strictly speaking of very limited use. In particular, the anchor bolt diameter must be 3/4 inch and the concrete strength 3000 psi. Nevertheless, the results are non-dimensionalized so that these two variables appear in the strength equation. In the absence of any other experimental data, this equation should provide at least some guidance for the designer. However, the effect of these and the other variables should be studied. This is considered to be beyond the scope of the dissertation.

Tests were performed on column stubs for various ratios of  $P/F$ , where  $P$  is the shear force per anchor bolt and  $F$  is the net vertical force per anchor bolt. For experimental convenience the column stubs were connected to the foundation walls using only two anchor bolts per connection - it is implied that the strength of a four bolt connection would be twice as great. Typical results are shown in Figure 6.8 in which  $P$  is plotted against the shear slip movement.

In all of the tests involving shear the ultimate strengths were reached only after large shear slips had occurred, caused by crushing of the concrete in front of the anchor bolts. Connection failures are therefore defined as the loads corresponding to bearing failures in the concrete rather than failure of the anchor bolts themselves. In general, these loads corresponded to slips of approximately 3/8 inch. For the pure

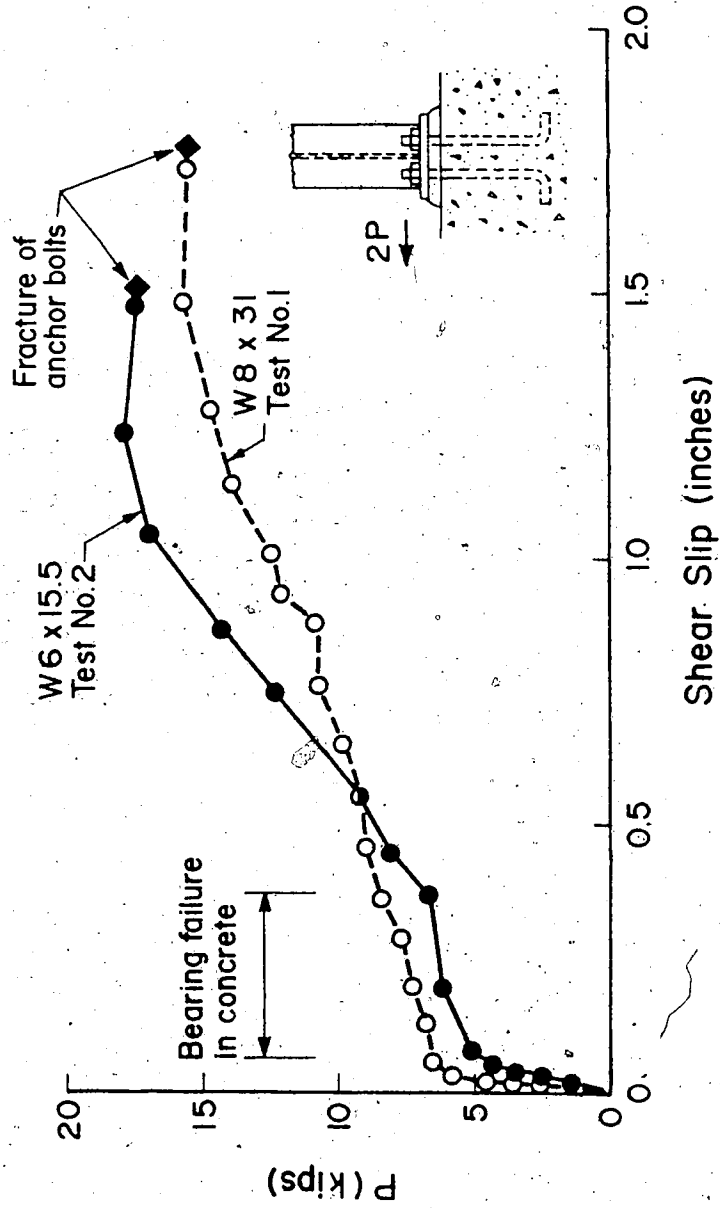


FIGURE 6.8 TEST RESULTS FOR PURE SHEAR CASE

tension tests connection failures are defined as the ultimate strengths of the bolts. The test results are summarized in Figure 6.9. Also shown in Figure 6.9 is the empirical equation

$$P = 0.85 F'_c A + \mu F \quad (6.37)$$

for combined shear and compression, where  $A$  is an assumed rectangular bearing area of concrete in front of each anchor bolt equal in width to  $d$  and in length to  $5d$ , and  $\mu$  is the coefficient of friction between the underside of the steel base plate and the grout, assumed to be 0.65. For combined shear and tension, the magnitude of the tension force  $F$  does not appear to affect the bearing strength, so that

$$P = 0.85 F'_c A \quad (6.38)$$

For the purpose of this report a performance factor of 0.9 will be used with Equations (6.37) and (6.38) since, as indicated in Figure 6.8, there is a considerable reserve strength over the load causing bearing failure in the concrete. A different approach, which will not be pursued further here, would be to base the connection strength on fracture of the anchor bolts, rather than on a bearing failure, as is currently done with headed studs used in composite beam design<sup>(31)</sup>. In this case an appropriate performance factor would have to be determined.

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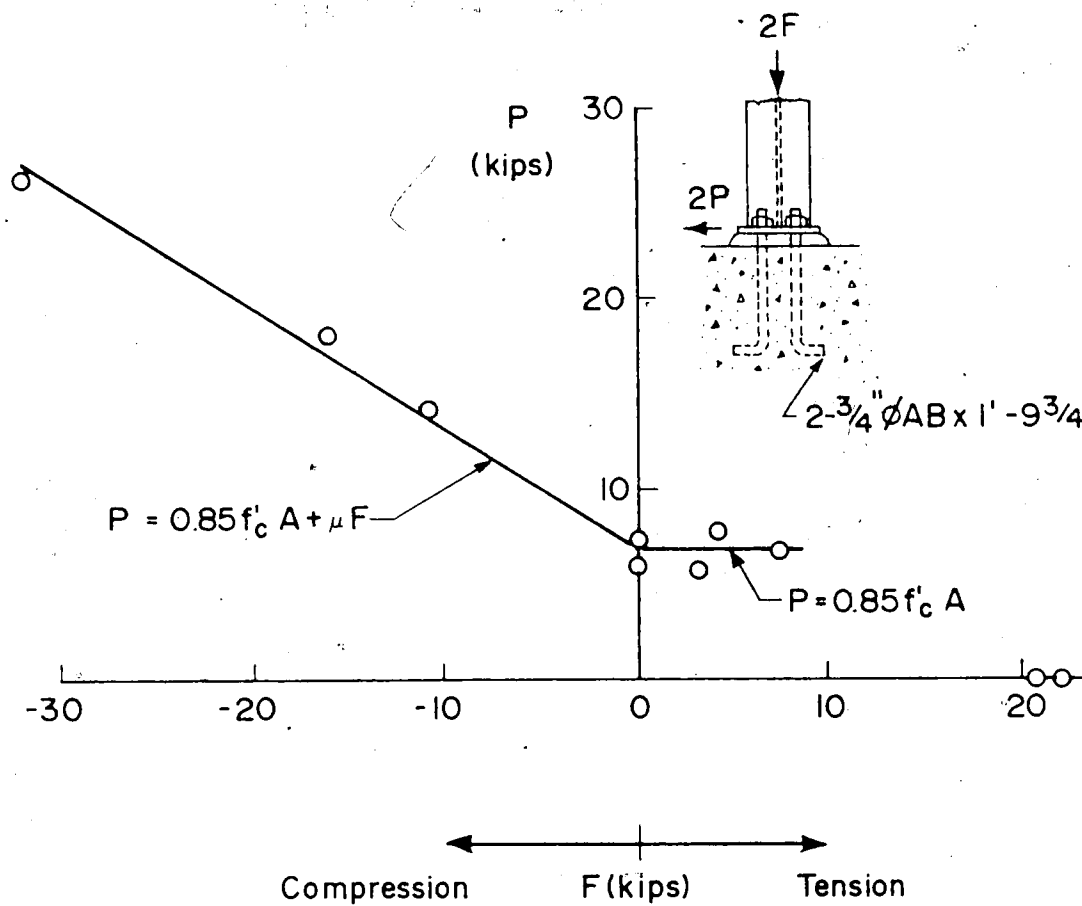


FIGURE 6.9 INTERACTION DIAGRAM

Example 6.5Given

As shown in Example 6.4, the lateral loads are transferred into the foundation partly by diaphragm action and partly by a direct acting bracing system. The resulting forces acting on the base of a column at a braced bay are shown in Figure 5.23. The compression force shown in the column is that resulting from factored dead load only. Determine the number of 3/4 inch diameter anchor bolts required. Assume a specified concrete strength of 3000 psi.

Solution

Assume 4 anchor bolts are required

Since the vertical component of the force in the brace does not exceed the compressive force in the column the base plate connection is subjected to combined shear and compression. The shear capacity of one anchor bolt is therefore given by:

$$P = 0.85 F'_c A + \mu F \quad (\text{Eq. 6.37})$$

where

$$\begin{aligned} A &= (5d) \times d \\ &= 5 \times 0.75 \times 0.75 \\ &= 2.81 \text{ in.}^2 \end{aligned}$$

Thus

$$P = 0.85 \times 3 \times 2.81 + \frac{0.65}{4} (26.6 - 22.4)$$
$$= 7.88 \text{ kips.}$$

This is reduced by the performance factor of 0.9 to 7.09 kips.

The number of anchor bolts required is found by dividing the total shear by the capacity of one bolt:

$$n = \frac{24.8 + 1.46}{7.09} = 3.70$$

Use 4 anchor bolts

---

#### 6.7 Column Sidesway

If the exterior walls of a structure consist of masonry, the maximum column sway is equal to the maximum roof deflection relative to the end walls. In this case it is assumed that the end walls are so stiff that they do not deform appreciably. If the walls consist of girts and cladding and/or a direct acting bracing system, the deflections of the end walls should be computed and added to the maximum roof deflection to obtain the maximum column sway.



Consider the end wall shown in Figure 6.7(a) in which the lateral load is resisted by a combination of diaphragm action of the wall cladding and a direct acting bracing system. As discussed in Section 5.3.4, the stiffness per unit width of a metal diaphragm is denoted  $G'$ . Re-writing Equation (4.19), and allowing for a change in notation, the stiffness  $k_1$  of the diaphragm-clad wall is defined by:

$$F = k_1 \Delta_b = Q \frac{\Delta}{h} = \frac{G'd}{h} \Delta_b \quad (6.39)$$

thus

$$k_1 = \frac{G'd}{h} \quad (6.40)$$

The stiffness  $k_2$  of the tension brace of area  $A$ , length  $l$  and angle of inclination  $\theta$  is easily determined as:

$$k_2 = \frac{AE}{l} \cos^2 \theta \quad (6.41)$$

It is assumed that since the bracing is "tension-only" bracing the compression brace has buckled and is ineffective in resisting lateral loads. The force-deformation relationship for the end wall is, therefore,

$$R = \left( \frac{G'd}{h} + \frac{AE}{l} \cos^2 \theta \right) \Delta_b \quad (6.42)$$

A limitation must be imposed on the sway of an industrial structure subjected to wind loads for several reasons: (1) aesthetic or visual considerations, (2) serviceability of the structure, and (3) damage to non-structural components. For industrial structures without travelling overhead cranes, recommended values of maximum column sway at specified load levels and a 1:10 wind pressure vary from 1/400 to 1/200 of the column heights, depending on the type of wall construction (20).

Aesthetic or visual limitations are largely a matter of personal opinion, however there is some evidence that building sway can be easily seen if it amounts to about 1/250 of the height (86).

Deflection limitations based on serviceability requirements are also difficult to quantify, and have not been analyzed in a systematic manner. In order to prevent doors, folding partitions, etc. from jamming, a deflection limitation of 1/240 has been suggested (87).

Deflection limitations related to damage of non-structural components such as exterior walls are more easily determined. For example, consider a masonry wall of dimensions  $L \times h \times t$ , as shown in Figure 6.10, and wind load acting normal to the side  $L$ . If the maximum column sway at midspan is too large, damage may occur at the base of the wall due to out-of-plane bending.

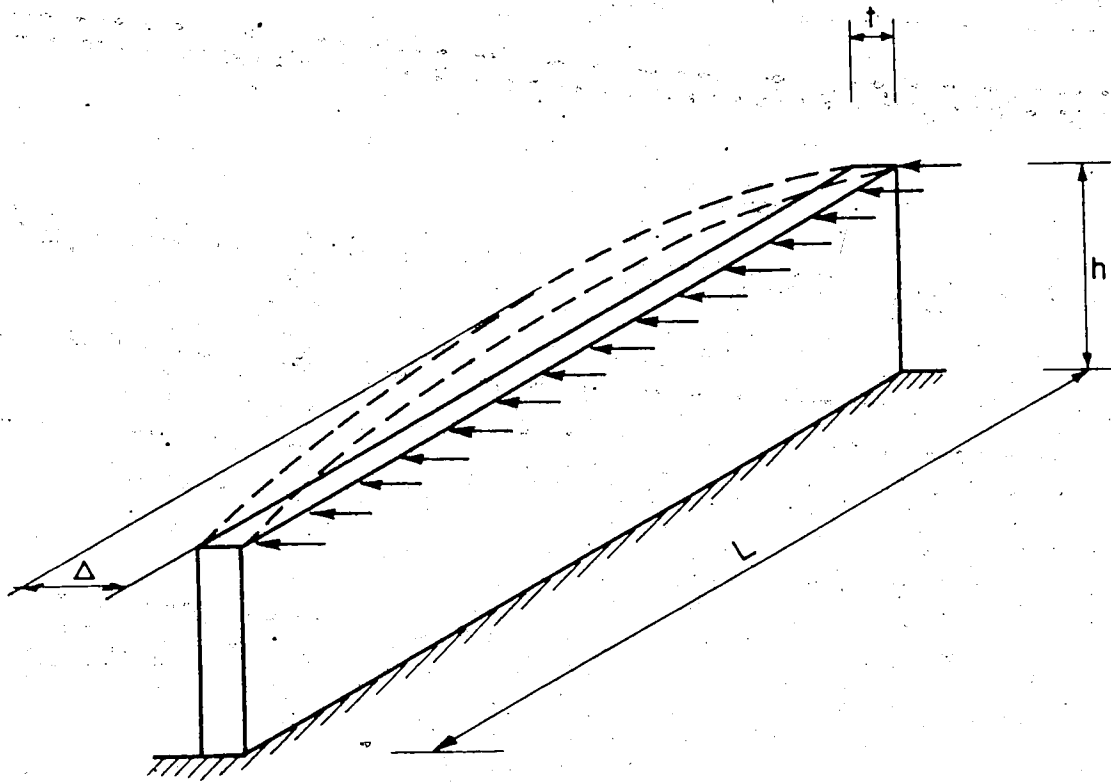


FIGURE 6.10 DEFORMED SHAPE OF MASONRY WALL

To analyze this, a unit width of the wall at midspan of thickness  $t$  (the thickness of the masonry units) and height is isolated and treated as a vertical cantilever beam subjected to a point force  $P$  at its end. The tip deflection is given by(5):

$$\Delta = \frac{Ph^3}{3EI} \quad (6.43)$$

where  $I$  is the moment of inertia given by:

$$I = \frac{t^3}{12} \quad (6.44)$$

and  $E$  is Young's modulus, given by(7):

$$E = 1000 f'_m \quad (6.45)$$

in which  $f'_m$  is the compressive strength of the masonry. In this analysis the compressive strength is assumed to be less than 3000 psi, and the mortar is assumed to be either type M or S. However, the approach can be used for other strengths and mortar types equally well. Limiting the bending stress at the base of the cantilever to  $0.32 f'_m$ (7) Equation (6.43) can be re-written as:

$$\frac{\Delta}{h} = \frac{1}{4690} \frac{h}{t} \quad (6.46)$$

For a wall 21 feet high with 10 inch thick masonry units, for example, the maximum column sway becomes  $1/186$ . This is larger than the largest value recommended in Reference (20).

A similar analysis can be performed for a building with walls consisting of girts and metal cladding. However, substitution of typical cross section properties and material strengths indicates that these walls are usually flexible enough so that wall failure caused by excessive out-of-plane deflection cannot be used to establish a realistic deflection limitation. A deflection limitation of  $1/200$  applied to this type of structure would appear to be based purely on aesthetic considerations.

---

#### Example 6.6

#### Given

In Example 2.4 the specified wind load on the building shown in Figure 6.1, based on a 1:10 wind pressure, was shown to be 14.9 psf. The maximum roof deflection, relative to the end walls, was shown in Example 6.3 to be 0.62 inches. As described in Example 5.8 the end walls consist of 1.5 inch deep metal wall cladding units for which the stiffness  $G'$  is 3.75 kips/inch. In addition, a 3-1/2 in. x 1/4 in. flat, selected in Example 6.4, is used to assist in resisting lateral load. The angle this bracing member makes with the horizontal is 46.4 degrees. As indicated in

Figure 6.1 the building dimensions  $h \times d \times L$  are 21 ft.  $\times$  120 ft.  $\times$  200 ft. Compute the maximum column sway and compare it with the maximum recommended value of  $1/200$  of the building height for this class of structure.

### Solution

The reaction  $R$  from the roof deck diaphragm is computed first by multiplying the wind pressure by the tributary area of the windward wall:

$$R = \frac{14.9}{1000} \times \frac{21}{2} \times \frac{200}{2} = 15.7 \text{ kips.}$$

The end wall deflection is, therefore:

$$\Delta_D = \frac{R}{\frac{G'd}{h} + \frac{AE}{L} \cos^2 \theta} \quad (\text{Eq. 6.42})$$

$$= \frac{15.7}{3.75 \times \frac{120}{21} + \frac{3.5 \times 0.25 \times 29000}{20 \times 12} \cos^3 \theta} \quad (46.4)$$

$$= \frac{15.7}{21.4 + 34.7}$$

$$= 0.280 \text{ in.}$$

and the maximum column sway is:

$$\Delta = 0.62 + 0.28 = 0.90 \text{ inches.}$$

Since this is less than the maximum recommended value of

$$\Delta = \frac{h}{200} = \frac{21 \times 12}{200} = 1.26 \text{ inches}$$

it is considered to be acceptable. One could at this point re-compute the amplification factor A using Equation(6.14) with a deflection  $\Delta = 1.5 \times 0.90$  and iterate, however this will not be done here.

---

## 6.8 Summary

In this Chapter, the response of the structure to lateral loads was examined. Following a brief discussion of load transfer from the wall to the roof a detailed analysis of sway effects in single storey structures was presented. Load transfer in the plane of the roof to the end walls, and from the end walls to ground level were then considered. Next, a series of tests to determine strengths of anchor bolts subjected to combined shear and axial load was summarized. Finally, lateral deflection limitations were discussed.

## CHAPTER VII

### ESTIMATING THE COST OF STEEL CONSTRUCTION

#### 7.1 Introduction

The methods of cost estimating used by steel fabricators can vary from simple to comparatively complex and detailed procedures. Small to intermediate size fabricators may simply multiply the weights of beams, columns, joists and other members by unit costs that have been developed by trial and error and found to be appropriate. Erection costs may or may not be considered separately. This method may be quite satisfactory for many structures. It can, however be very unreliable when extended to structures with unusual framing or with a large number of light members. Larger fabricators, therefore, usually analyze in detail the five major items that determine the cost of the structure: material, drafting, fabrication labor, outside purchases such as joists, bridging and roof deck, and erection labor.

Various methods of either estimating(88,89,90) or reducing the cost of steel construction(91,92,93) are available in published form. In this chapter a method of estimating that closely follows the procedure described in Reference (88) is presented. All five major cost items are included. The total cost so determined represents the cost to the general contractor. The cost to the owner will be higher, depending on the general contractor's overhead and profit. The method has been used in

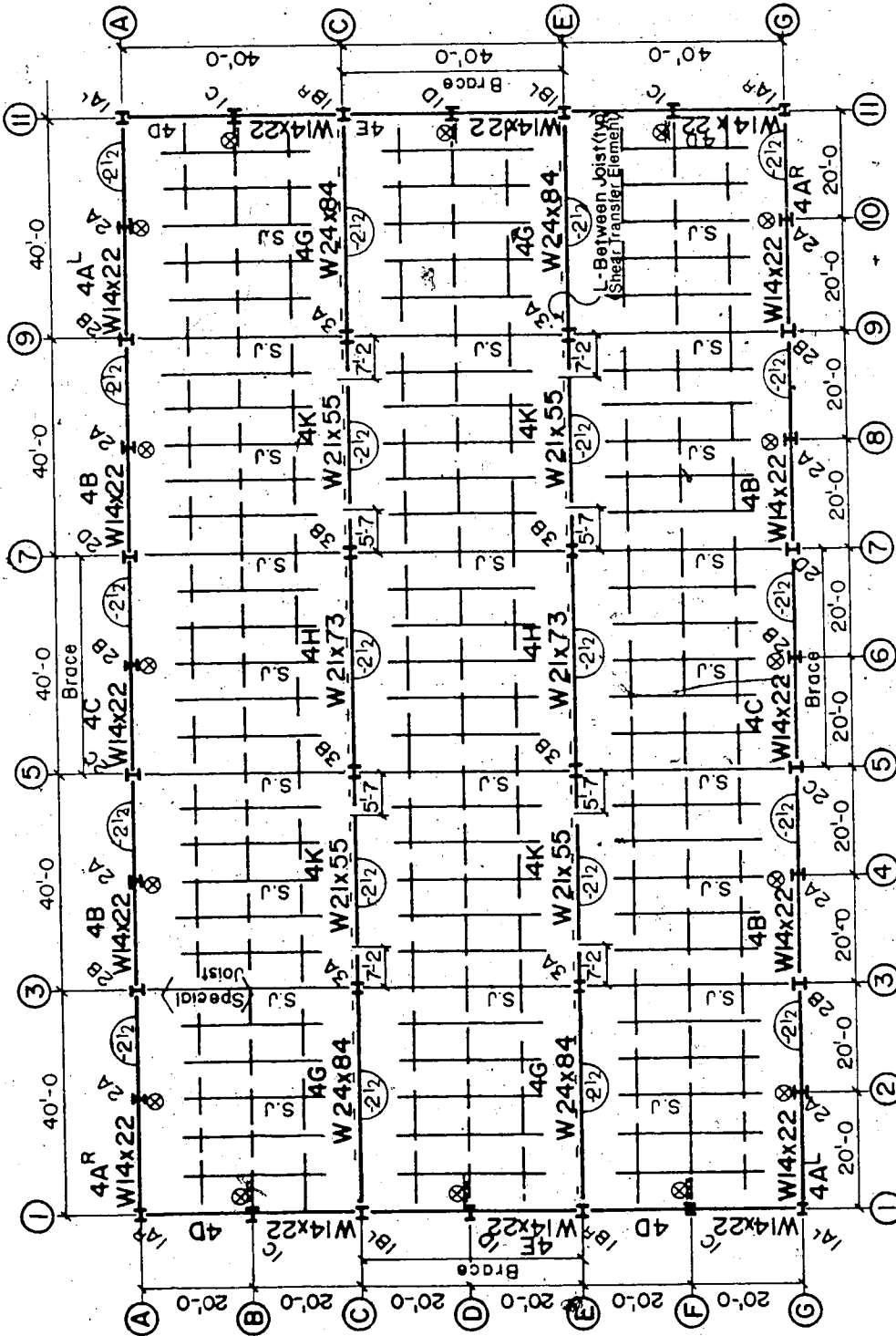


**NOTES**

Material - G40-21-44W  
 Paint - One Shop  
 Coat GP40C  
 Conn: 3/4" A325 Bolts  
 or Welded  
 All Joists to be  
 24" SS Joist Spaced  
 at 6'-8" c/c  
 All Bridging to be  
 1 1/2" x 1 1/2" x 1/8" L  
 Top & Bottom Chord  
 All Deck to be  
 1/2" - 0.030" L.Z.C.

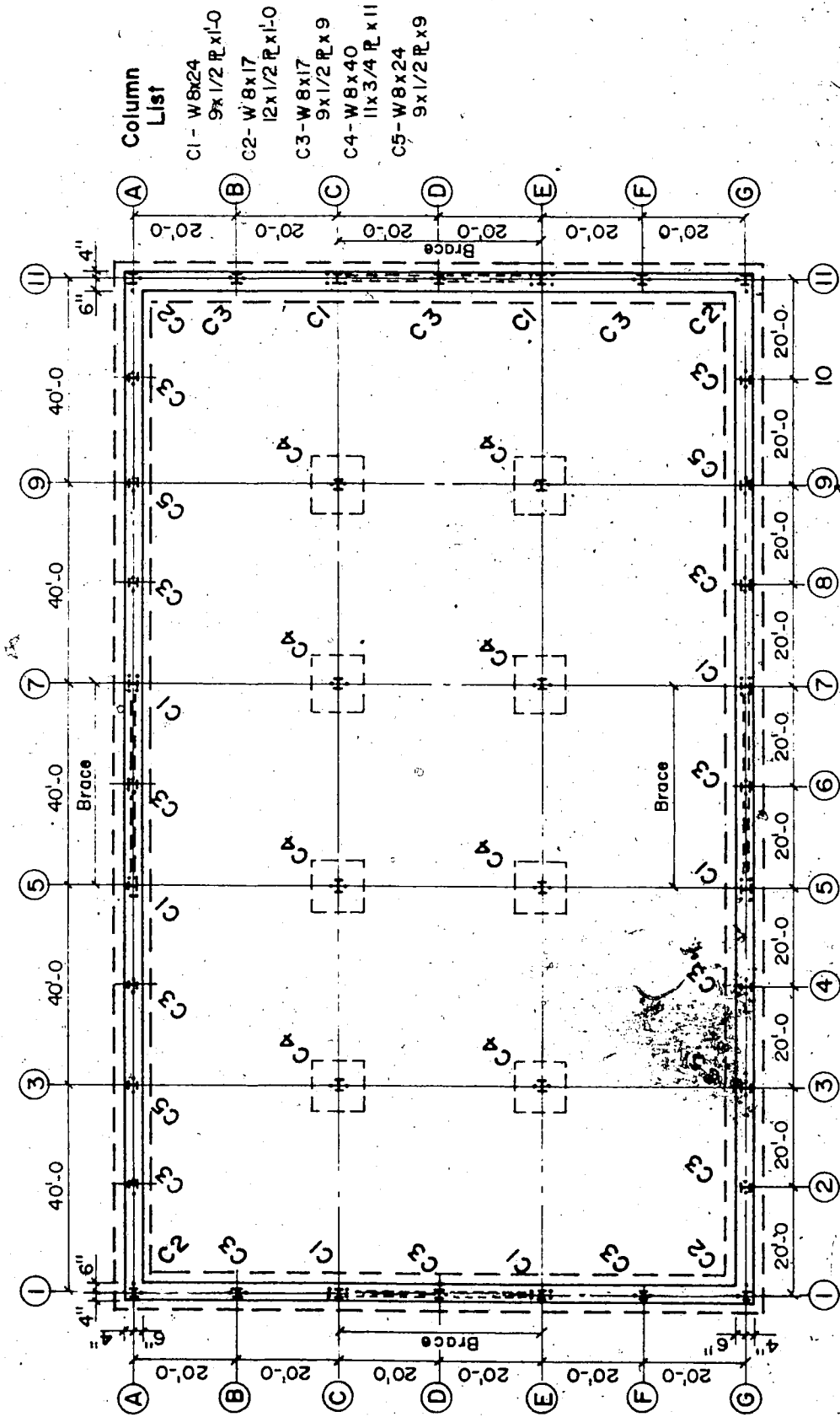
**ROOF LOADING**

SPECIFIED DEAD  
 LOAD: 25.5 PS.F.  
 SPECIFIED LIVE  
 LOAD: 48 PS.F.



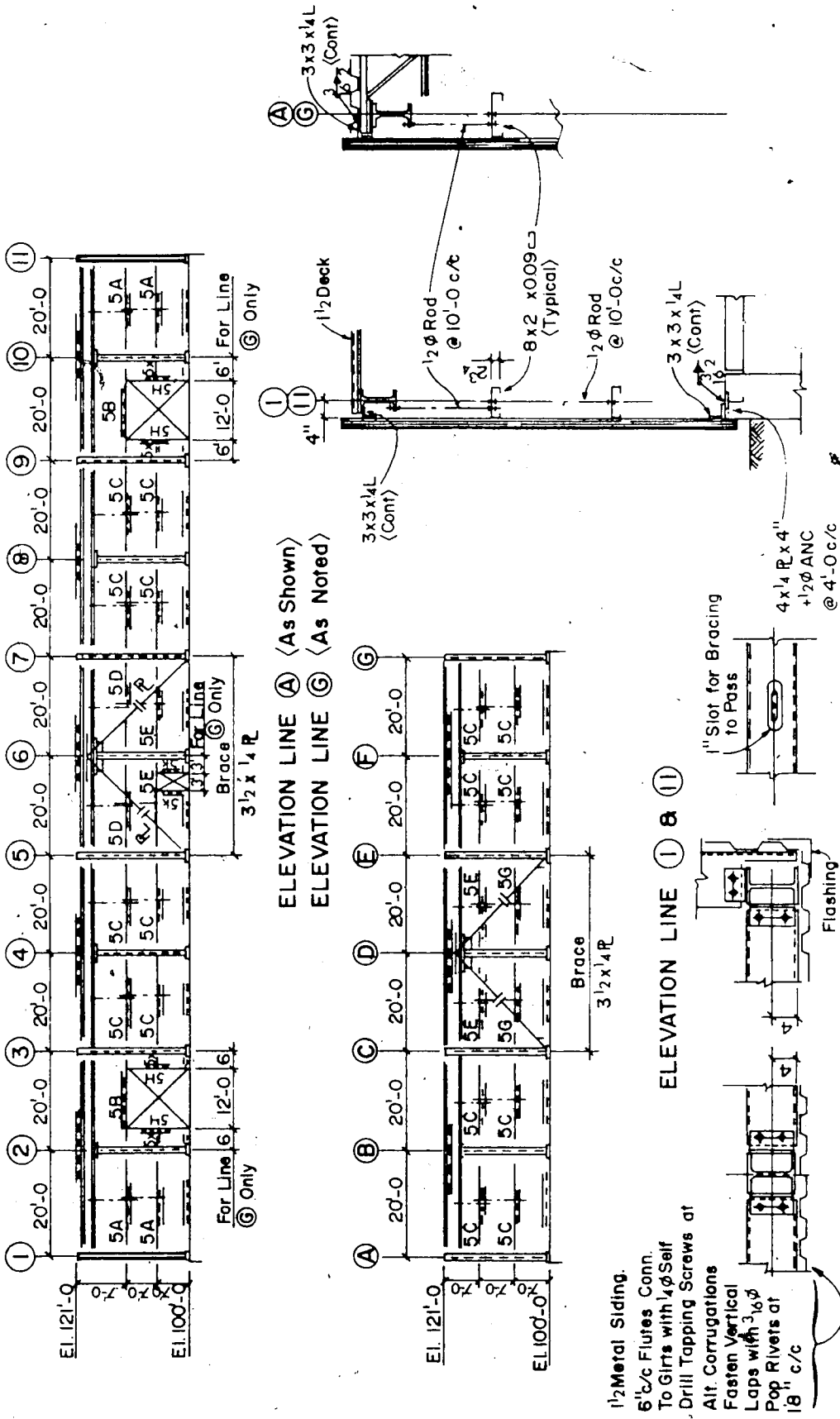
**ROOF PLAN**  
 Top of Steel (u/s Deck) El 121'-0" u/N

**FIGURE 7.1(a) ERECTION DRAWINGS FOR EXAMPLE BUILDING**



Foundation & Anchor Bolt Plan  
Fin Floor Slab El. 100'-0"

FIGURE 7.1(b)



TYPICAL GIRT & SIDING DETAILS

TYPICAL GIRT CONN. DETAILS

FIGURE 71(c)

1/2" Metal Siding.  
 6 1/2" Corrugated Metal Flutes Conn.  
 To Girts with 1/4" Self  
 Drill Tapping Screws at  
 Alt. Corrugations  
 Fasten Vertical  
 Laps with 3/16" φ  
 Pop Rivets at  
 18" c/c

ELEVATION LINE A (As Shown)  
 ELEVATION LINE G (As Noted)

ELEVATION LINE 1 & 11

1" Slot for Bracing  
 to Pass

Flashing

4x1/4 R x 4"  
 + 1/2" φ ANC  
 @ 4'-0" c/c

3x3x1/4 L  
 (Cont)

1/2" Deck

1/2" φ Rod  
 @ 10'-0" c/c

8x2 x 0.09" (Typical)

1/2" φ Rod  
 @ 10'-0" c/c

3x3x1/4 L  
 (Cont)

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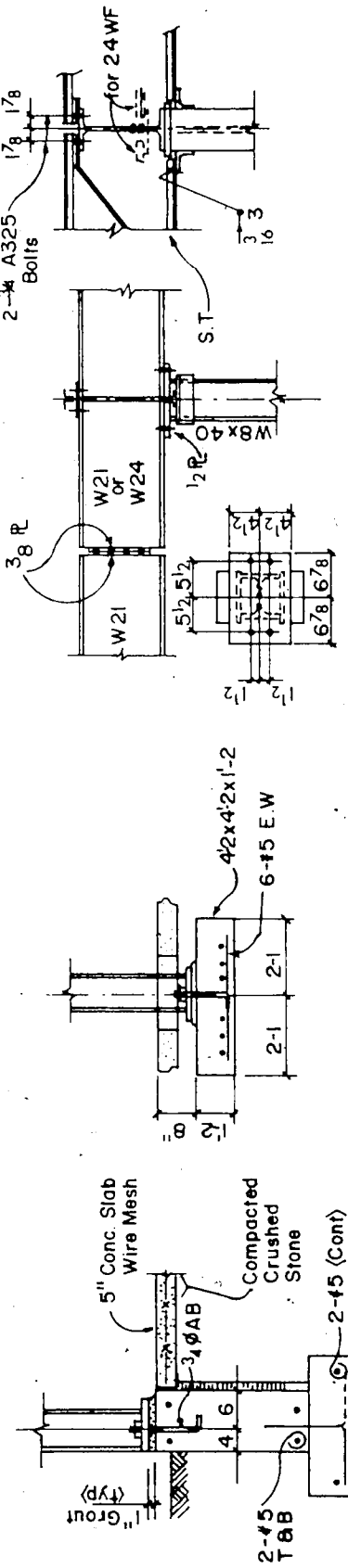
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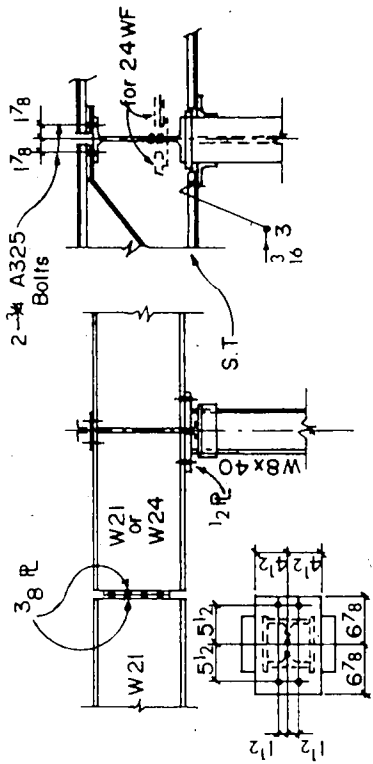
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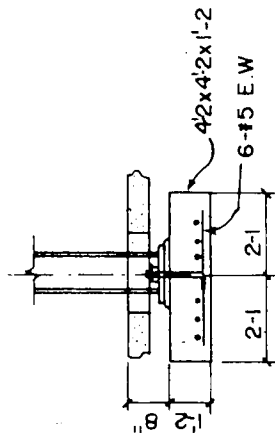
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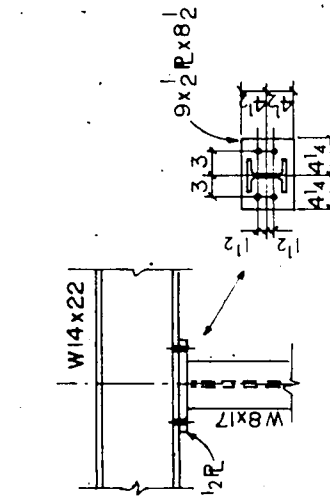
Typical Footing & Foundation Details



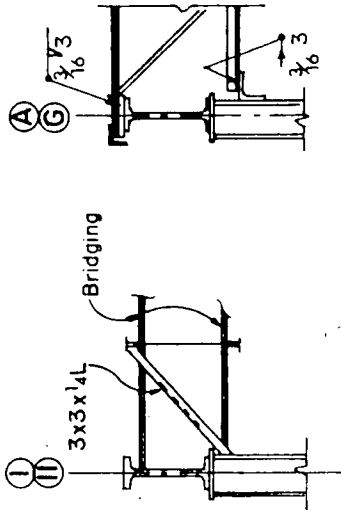
Typical Interior Beam Conn Detail



Typical Wind Post-Detail

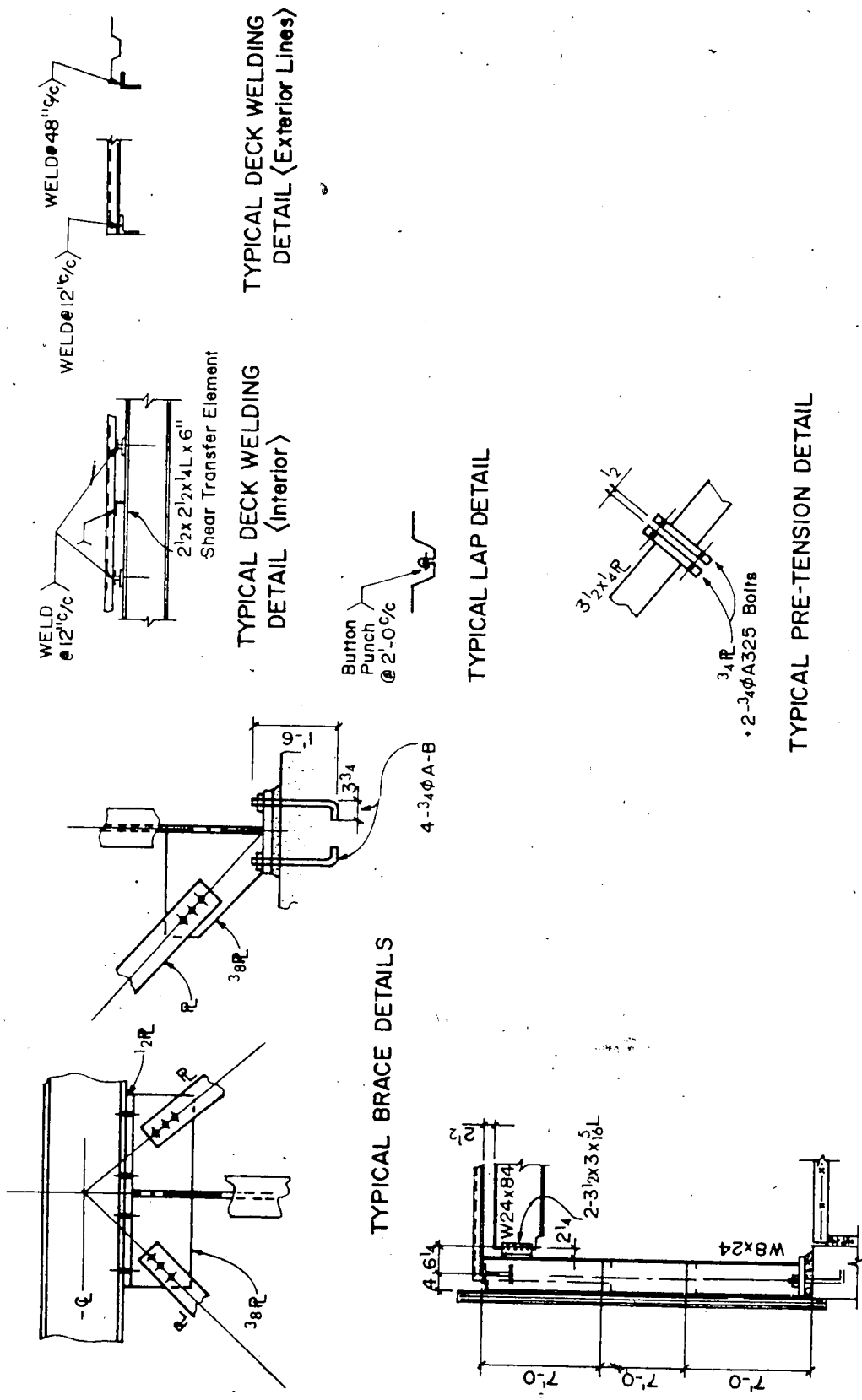


Typical Special Joist Detail Exterior Lines



Typical Brace Details at Locations

FIGURE 7.1(d)



TYPICAL DETAIL LINE I & II

FIGURE 7.1(e)

CHAPTER III to determine the relative costs of different framing schemes, and will be used in this chapter to estimate the cost of structural steel in the building shown in Figure 7.1.

## 7.2 Material

In order to determine this cost the fabricator must obtain drawings of the structure and from these determine the numbers, sizes, lengths and weights of different sections required, as shown in Table 7.1.

The unit prices used depend on a number of factors, including the following:

1. The size of section (unit prices are higher for lighter sections than for heavier ones). For simplicity, however, some fabricators may use a weighted average for wide flange columns and girders.
2. The grade of steel (grade 55W is more expensive than grade 44W, for example).
3. The specific lengths required. Most sections are stocked in either 40 or 60 foot lengths, and if the lengths required do not fit evenly into these stock lengths the amount of waste will affect the unit cost used.

Qty.	Item	Wt. (lbs/ft)	Length	Weight (lbs)	Fab. Est. (m.h.)
4	W24x84	84	47'-2	15848	7.2
2	W21x73	73	51'-1	7470	4.0
4	W21x55	55	27'-3	5995	6.0
16	W14x22	22	40'-0	14080	24.0
8	W8x40	40	19'-5	6213	16.0
12	W8x24	24	21'-0	6048	28.8
4	W8x17	17	21'-0	1428	9.6
10	W8x17	17	19'-8	3343	24.0
6	W8x17	17	19'-10	2023	14.4
8	3-1/2x1/4 P	2.8	29'-0	650	4.0
32	L3x3x1/4	4.9	40'-0	6272	8.0
60	L2-1/2x2-1/2x1/4	4.1	0'-6	123	10.0
64	C8x2-3/4x0.09	4.75	20'-0	6080	32.0
64	1/2" $\phi$	0.67	7'-0	300	16.0
	sub-totals...			75823	204
87	24" OWSJ	15.3	40'-0	53244	
180	L1-1/2x1-1/2x1/8	1.23	20'-0	4428	

TABLE 7.1 MATERIALS REQUIRED

4. The delivery requirements. Most fabricators stock commonly used sections, however if a certain section is not available and if the delivery requirement is such that it is not possible to wait for delivery from a steel mill, it may be necessary for the fabricator to purchase this section from a large steel warehouse at a higher cost.
5. The amount of repetition of member sizes. The lowest mill prices are for relatively large orders, therefore small quantities of a large number of different member sizes may not warrant buying from the mill, but rather from a warehouse.

Approximate costs in the Ottawa area at the time of writing (July, 1978) are shown in Table 7.2. Costs for different geographic regions can usually be obtained through local steel fabricators.

### 7.3 Drafting

Two different types of drawings are required by the steel supplier: (1) erection drawings, which are drawn first, contain mark numbers for each member in the structure and are used at the job site to help position these members; (2) shop drawings, which are drawn next, contain the detailed information required by the shop to fabricate these members.

Erection drawings for the example building are shown in



Description	Cost (\$/lb)
W Sections	0.15
HSS Sections	0.22
Girts	0.18
Plate	0.15
Angles	0.15
Rods	0.17

TABLE 7.2 UNIT COSTS OF MATERIALS

Figure 7.1. As indicated here, erection drawings show the member framing at roof level, typical or unusual details, elevations, etc.. Mark numbers are given to all members, however identical members receive the same mark number. For example, the two interior cantilever girders are each given the same mark number, 4H.

The drafting time required for the erection drawings depends on the complexity and size of the structure. Skewed buildings, buildings with varying bay sizes, buildings with column bases at different elevations all require more drafting time than buildings with simpler lay-outs. Approximately 8 man-hours were required for the erection drawings shown in Figure 7.1, including checking.

Shop drawings are made for each different mark number. The time required to prepare these drawings depends on the complexity of the connections. For the structure shown in Figure 7.1, approximately one man-hour would be required for each beam, column, and girt mark number (including checking). By actual count, 28 different mark numbers have to be assigned, therefore the drafting time is 28 man-hours. Adding to this the time required for the erection drawings, an estimate of the total drafting time is:

$$8 + 28 = 36 \text{ man-hours}$$

#### 7.4 Fabrication

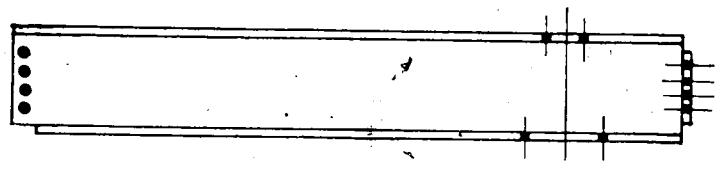
The main items considered in estimating the cost of

fabrication are:

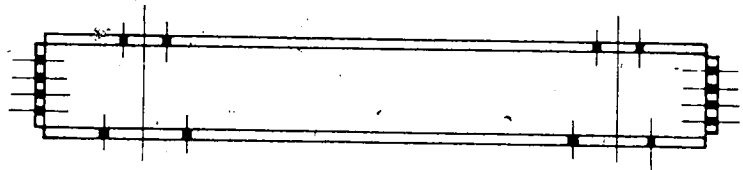
1. handling of steel for various operations in the fabrication process;
2. cutting to length;
3. drilling and punching of holes;
4. preparation of base plates, cap plates, header plate connections and other details;
5. assembly of details and welding;
6. cleaning and painting.

Most fabricators develop a number of tables for estimating man-hour requirements for different classes of work. These tables are initially verified by time studies and are thereafter updated by compiling historical records of completed projects. Each member in the structure is examined for the man-hour content associated with each of the above items and a total shop time required to complete the job is determined.

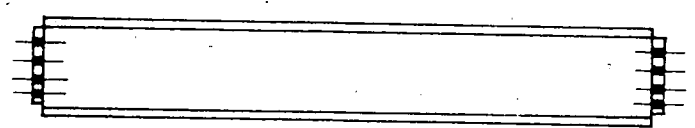
Approximate shop times for flexural and compression members are shown in Figures 7.2 and 7.3, determined after consultation with several major fabricators. Shop time can vary greatly from one shop to another; indeed this was found to be the case for the various fabricators consulted. For example, the shop time required for the interior cantilever girder shown in Figure 7.2 may vary from as little as 1.5 man-hours for a large, well equipped



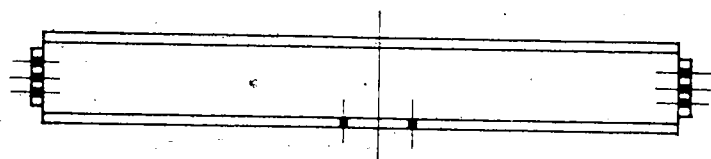
Exterior Cantilever Girder 1.8 m.h.  
(add 0.4 m.h. per stiffener)



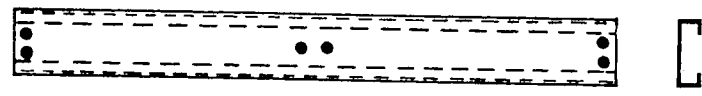
Interior Cantilever Girder 2.0 m.h.  
(add 0.4 m.h. per stiffener)



Simply Supported Beam 1.5 m.h.

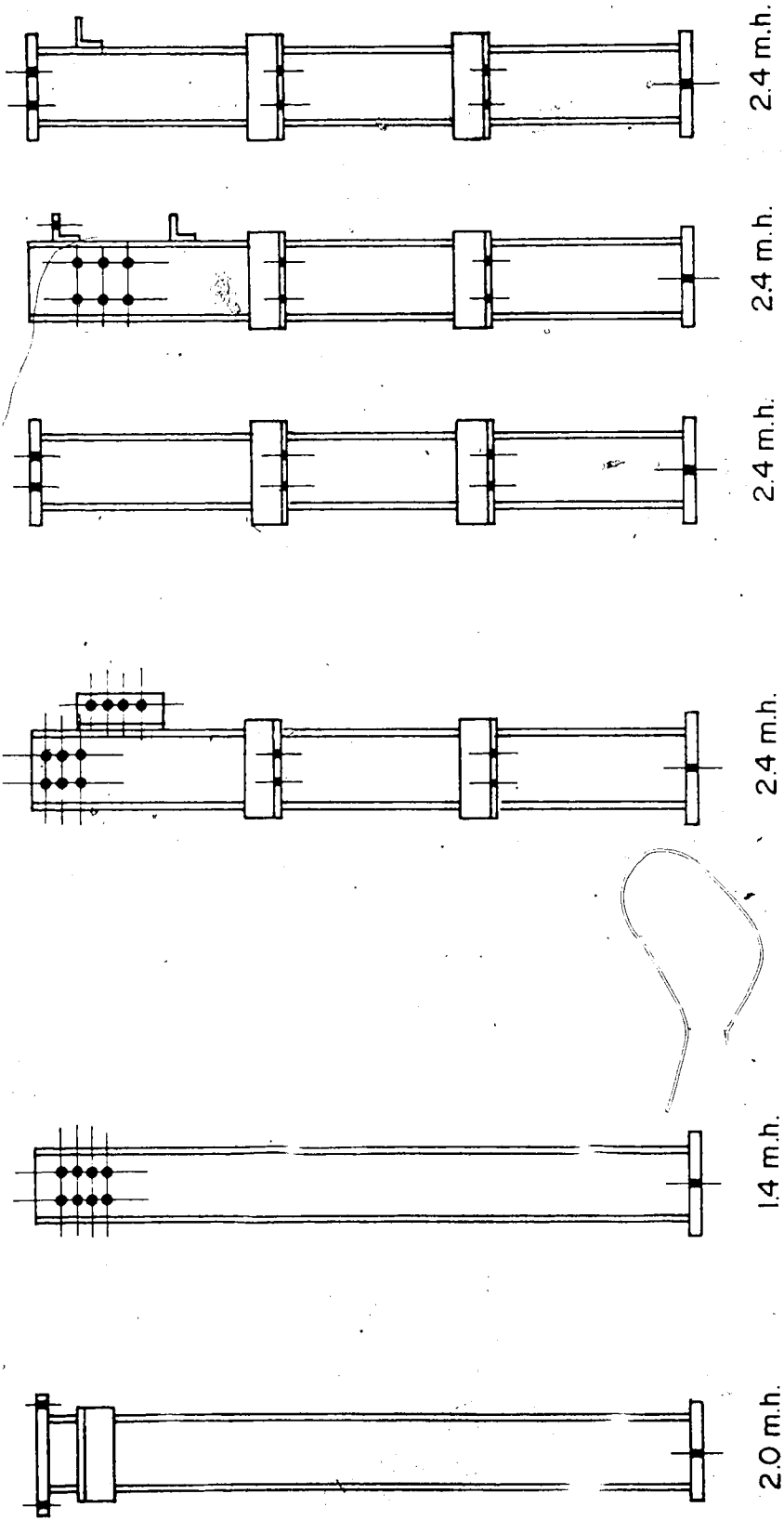


Perimeter Beam 1.5 m.h.



Girt 0.5 m.h.

FIGURE 7.2 FABRICATION ESTIMATES FOR FLEXURAL MEMBERS



Interior Compression Members

Exterior Compression Members

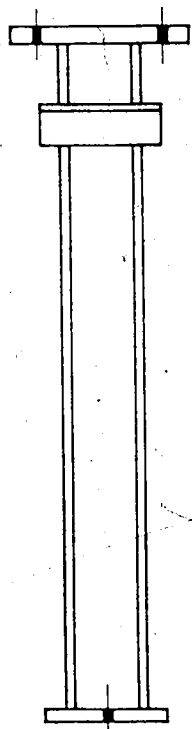
FIGURE 7.3 FABRICATION ESTIMATES FOR COMPRESSION MEMBERS

fabricating shop to as much as 2.5 man-hours for a small shop.

The man-hour estimates are based on the assumption that the time required for fabrication of flexural and compression members depends primarily on the complexity of the work at the connections, and not on the lengths or depths of the members. As indicated in Figure 7.4 this is an approximation since the time required for sawing, handling, fabrication of base plates and cap plates, and cleaning and painting all depend on the lengths and sizes of the members. An estimate of the fabrication time for the structure shown in Figure 7.1 is given in Table 7.1.

#### 7.5 Outside Purchases

Outside purchases consist of items normally erected, but not made by the fabricator, such as joists, bridging and steel roof deck. Some fabricators may be equipped to make their own joists. It is assumed here however, that they are purchased by another fabricator who specializes in making them. Joists and bridging are often sold to the fabricator on a weight basis. For fabricators in the Ottawa area this cost at the time of writing is approximately \$500/ton. Roof deck is always sold on a square footage basis. For 0.030 inch galvanized steel deck 1-1/2 inches thick this cost is approximately \$0.40 per square foot.



	W8 x 40 20'-0 800 lb	W6 x 15.5 10'-0 155 lb
sawing to length	0.20 man-hours	0.15 man-hours
handling in shop	0.20	0.15
fab. base plate	0.45	0.30
weld base plate	0.10	0.10
fab. cap plate	0.50	0.40
weld cap plate	0.10	0.10
fab. angles	0.10	0.10
weld angles	0.25	0.25
clean and paint	0.20	0.10
<b>total</b>	<b>2.10</b>	<b>1.65</b>

FIGURE 7.4 DETAILED EVALUATION FOR INTERIOR COLUMNS

## 7.6 Erection

It is usually more difficult to estimate the erection cost since the actual time taken to do the work often depends on factors that are beyond the estimator's control, such as inclement weather and accuracy of placement of the anchor bolts. Approximate labor rates are shown in Table 7.3 for erection of single storey industrial buildings. In preparing this table it was assumed that the structure is to be erected during the summer, that the job site is located in an open area with good access roads, that the anchor bolts are placed accurately, and that the job site is close to the city. As indicated the erection time is assumed to depend on the number of members in the structure, rather than on their weights. Use of Table 7.3 is illustrated in the following example.

---

### Example 7.1

#### Given

Estimate the total number of man-hours required for erection of structural steel in the building shown in Figure 7.1. Assume a five man crew consisting of one supervisor and four men, and a crane with a rated capacity of 15 tons. Include the roof deck, but not the wall cladding.



Description	Capacity per 8 Hr. Day
1. Unloading and sorting	150 lifts with crane
2. Erecting beams, columns and special joists (1 or 2 bolts per connection)	60 lifts with crane
3. Plumbing	40 columns
4. Bolting	250 bolts
5. Welding joists and bridging	60 joists
6. Girts Sag rods	20 girts 100 sag rods
7. Trimmer angles (40 ft. lengths)	25 lengths
8. Bracing	8 braced bays
9. Roof deck	12000 sq.ft.

Note: This table is based on a crew consisting of  
1 supervisor and 4 men.

TABLE 7.3 ERECTION ESTIMATES

Solution

The numbers of different members in the structure are:

beams - 26

columns - 40

special open web steel joists - 22

joists - 65

girts - 64

sag rods - 64

high strength bolts - 350 (approximately)

lengths of 40 ft. long trimmer angles - 32

braced bays - 4

area of roof deck -  $120 \times 200 = 24000 \text{ ft.}^2$

To determine the days required for unloading and sorting of steel it is assumed that the joists are shipped to the job site in twelve bundles of six joists each, and that each bundle of joists and each beam and column is unloaded separately:

$$\text{days} = \frac{26 + 40 + 15}{150} = 0.54$$

Next, the beams, columns and special open web steel joists are erected, using only one or two bolts per connection:

$$\text{days} = \frac{26 + 40 + 22}{60} = 1.47$$

After this, the remaining joists, bridging and roof deck are lifted up to roof level. When this is completed the crane is no longer required at the job-site. For computation of crane rental costs the number of days that it must be at the site is  $0.54 + 1.47 = 2.0$  days.

The man-hour requirement for plumbing of a particular structure is strongly dependent on the geometry of the structure, since plumbing of one column in a bent affects the plumbness of other columns in the same bent. Using Table 7.3, however, an estimate of the time required for plumbing is:

$$\text{days} = \frac{40}{40} = 1.0$$

The time required for installing the remaining bolts and tightening is:

$$\text{days} = \frac{350}{250} = 1.40$$

The time required to weld the joists and connect the bridging is:

$$\text{days} = \frac{65}{60} = 1.08$$

The time required for the girts is:

$$\text{days} = \frac{64}{20} = 3.20$$

and for the sag rods is:

$$\text{days} = \frac{64}{100} = 0.64$$

The time for the trimmer angles is:

$$\text{days} = \frac{32}{25} = 1.28$$

The time required for the bracing is:

$$\text{days} = \frac{4}{8} = 0.5$$

The time required to spread and connect the roof deck is:

$$\text{days} = \frac{24000}{12000} = 2.0$$

The total number of days is therefore:

$$\begin{aligned} \text{days} &= 0.54 + 1.47 + 1.0 + 1.4 + 1.08 + 3.20 + 0.64 \\ &\quad + 1.28 + 0.5 + 2.0 \\ &= 13.1 \end{aligned}$$

This is rounded up to 14 days to include other work not explicitly considered, such as touch-up painting and clean-up of the construction site.

---

#### 7.7 Cost of the Erected Steel Frame

After an examination of these four major cost items, the individual costs are assembled on a final estimate sheet. An example of a final estimate sheet for the structure of Figure 7.1 is shown in Table 7.4. Although federal and provincial sales taxes would normally be included, they are omitted in this example since they vary from province to province.

The unit costs shown for the drawing office and the shop include both direct wage rates and operating overhead. Operating overhead accounts for various costs associated with running the fabricating shop, including:

1. Utilities (electricity, water and oil);
2. Depreciation on shop and machinery;

Item	Unit	Unit Cost	Cost
1. Material:			
- W shapes	62448 lbs.	\$0.15/lb.	\$9367.
- Angles and Plates	7045 lbs.	\$0.15/lb.	\$1057.
- Girts	6080 lbs.	\$0.18/lb.	\$1094.
- Rods	300 lbs.	\$0.17/lb.	\$51.
Sub-Total .....			\$11569.
2. Connections (5% of above)			\$578.
3. Scrap (2% of above)			\$231.
4. Stores:			
- A325 bolts	350	\$0.40	\$140.
- Anchor bolts	80	\$1.50	\$120.
5. Drafting	36 man-hours	\$14./man-hour	\$504.
6. Shop	204 man-hours	\$20./man-hour	\$4080.
7. Receiving and Shipping	37.9 tons	\$25./ton	\$948.
Sub-Total .....			\$18170.
8. Sales and Administration Overhead (10%)			\$1817.
9. Freight	37.9 tons	\$10./ton	\$379.
10. Outside Purchases:			
- Joists and Bridging	28.8 tons	\$500./ton	\$14400.
- 0.030 inch roof deck	24000 sq.ft.	\$0.40/sq.ft.	\$9600.
Sub-Total .....			\$44366.
11. Erection:			
- Labor	506 man-hours	\$20./man-hour	\$11200.
- Crane	2 days	\$250./day	\$500.
Total .....			\$56066.
Profit (10%) .....			\$5607.
			\$61673.
		Quote .....	\$62000.

TABLE 7.4 FINAL SUMMARY SHEET

3. Shop maintenance;
4. Cost of welding rods, paint, etc.;
5. Employee benefits.

Receiving and shipping as well as sales and administration overhead costs are usually evaluated separately. Receiving and shipping costs account for unloading of material delivered from the steel mill, handling, storage, and re-loading of the fabricated steel for shipment to the job site. Sales and administration overhead covers those costs that cannot be attributed directly to a particular job, such as executive and office salaries, accounting services, and advertising. The unit cost used for erection is also intended to cover the cost of erection overhead and rental of equipment, such as air compressors, welding machines, etc. Since a crane is only at the construction site for part of the erection time, its cost is evaluated separately.

#### 7.8 Summary

In this chapter a method of estimating the cost of structural steel was presented. Rather than simply assuming that cost is proportional to weight, a breakdown into material, drafting, fabrication and erection costs is made.

## CHAPTER VIII

### SUMMARY

Much of the research in steel construction during the past ten or fifteen years has been directed towards the behaviour of tall structures, while single storey structures have been somewhat neglected by comparison. Various aspects of the design of single storey structures where more information might be useful have been investigated.

This dissertation has as its aim the collection of research information that is directly applicable to the design of light industrial buildings, the modification (wherever possible) of existing information so that it is directly applicable, the development of new design procedures for commonly occurring situations that cannot be covered in this manner, and the identification of areas requiring future research. The term "light industrial building" is used to designate a single storey building in structural steel. The roof systems usually consist of open web steel joists and cantilever girders separated by simply supported "link" beams. Simply supported girders are also commonly used. Resistance to lateral loads is usually provided by diaphragm action or a direct acting bracing system. Rigid frame structures are not considered.

In the course of this investigation the following general areas were considered:



1. Effect of loads,
2. Effect building layout,
3. Design of flexural members,
4. Design of compression members,
5. Design of members to resist lateral loads, and
6. Estimation of cost of fabrication and erection.

Within each area the approach has been to survey the existing literature and design practices and then to either extend the state-of-the art or to define those areas requiring future work. The result of the investigation can be summarized as follows:

#### Loads

1. Loads produced by drifting snow around an upper roof less than 50 feet long can be determined by using a linear reduction between a snow load coefficient of  $15 h/g$  for roofs greater than fifty feet long, and a snow load coefficient of  $10 h/g$  for parapets (see Figure 3.4).
2. Ponding of rainwater is the governing live load (not snow) for the western coast of British Columbia, central and southern Alberta, areas in the vicinity of the Great Lakes, and some of the eastern provinces (see Figure 2.6).

3. Moments and deflections produced by ponding of rainwater on a flexible roof system can be determined by using an amplification factor, as described in Section 2.3.3.
4. There is a lack of experimental data concerning the effects of wind on single storey structures under construction. For example, there is very little information available on the effect of wind on a structural frame on which only the roof deck has been placed, or the "shielding effect" when several bents are in series.
5. Use of the same wind pressure  $q$  in determining loads during the short erection period as that used in determining loads on the completed structure does not appear to be reasonable, but further research is required before recommendations can be made.

Economics of Different Lay-Outs For Structures in the Ottawa Area  
(June, 1978):

6. For cantilever roof framing schemes the overhang ratios resulting in least cost are approximately 0.18 and 0.14 of the column spacing for exterior and interior cantilever girders, respectively (see Figure 3.2).

7. For a given roof framing scheme (for example a cantilever roof framing scheme) a minimum cost of structural steel occurs for spans of approximately 25 to 40 feet (see Figure 3.3). Costs of structures with larger clear spans, for example structures with long span trusses, were not determined since the cost data used was not considered valid for such structures. Additional work should be done in this area.
8. Cantilever roof framing schemes are less expensive than simple roof framing schemes regardless of bay size (see Figure 3.4).
9. For small bay sizes the use of hollow structural steel section columns can result in a cost savings. As the bay sizes increase the cost per square foot for both types of columns is approximately equal however (see Figure 3.5).
10. It is usually more economical to resist lateral loads in the plane of the roof by the diaphragm action of the roof deck than by a direct acting bracing system designed for this purpose.
11. Although the studies were performed for a location with a heavy roof snow load, the general conclusions are also valid for a location with a light roof snow load. (41)

Flexural Members

12. For a given column spacing and overhang ratios the slenderness ratios of the girders (characterized by the  $L_d/A_f$  ratios) must be less than those determined from Equations (4.17) and (4.18) for efficient use of the sections. These girders may be designed as if they were completely braced along their lengths.
13. Girders with large  $L_d/A_f$  ratios fail by lateral-torsional instability, involving both the overhangs and the main spans (Equation 4.11).
14. Additional research is required to properly determine the behaviour of girders subjected to suction. These members fail by combined bending and torsion, not lateral-torsional instability, since the loads do not pass through the shear centers of the cross sections (Figure 4.10).
15. Although research has been done to determine the bracing requirements for a simply supported beam subjected to uniform bending and braced by roof deck along its top flange additional research is required to determine the effect of different loading and support conditions and the effect of more than one beam.

16. Bracing requirements for special open web steel joists in cantilever roof framing schemes are summarized in Section 4.7.2.
17. A live load deflection limitation of  $1/180$  of the span is derived for a simple span girder supporting an asphaltic roof membrane, instead of the  $1/240$  recommendation in Reference (20). Experimental verification of several of the assumptions is necessary.
18. A search of the literature reveals almost no available test information concerning the capacity of bearing stiffeners. Experimental work is required to determine the margin of safety inherent with the conventional design procedure (1).

#### Compression Members

19. Commonly accepted details for "pinned" column bases may be both stiff enough and strong enough to act as "fixed" bases, however experimental verification is required.
20. Tests are required to determine minimum base plate thickness requirements to ensure that the assumption of a uniform bearing stress is reasonable.
21. The stiffnesses of "simple" girder-to-column connections are often large enough to reduce the restraint parameters  $G_u$  at

the tops of the columns to values approaching the fixed-ended case.

22. Bracing requirements, based on a simplification of the results of Reference (68) are suggested for exterior columns braced along their lengths by girts and a diaphragm.
23. Tests are required to determine procedures for cap plate design if the length of the cap plate is to be selected to prevent crippling of the girder web.

#### Resistance to Lateral Loads

24. The  $P$  effect is responsible for increases in bending moments in beam-columns, increase in diaphragm shears, increases in axial forces in bracing members, and increases in column shears. The effects can be conveniently accounted for in design if the lateral loads are multiplied by an amplification factor, as discussed in Section 6.3.
25. Tests should be performed on diaphragm assemblies with and without shear transfer elements between joists, to determine the effect of these elements on diaphragm strength and stiffness.
26. The effect of roof openings on diaphragm action should be investigated.

27. The results of a series of tests to determine the strengths of anchor bolt connections are presented. Additional tests are required to determine the effects of those variables not yet considered.
28. Additional research is required to quantify the reasons for sway limitations in industrial buildings.

#### Fabrication and Erection

29. The estimating information presented in CHAPTER VII should be extended to include structures with long clear spans so that cost studies can be done for these cases.

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APPENDIX A  
ANALYSIS FOR PONDING

A.1 Introduction

In Section 2.3.3 equations were presented for the computation of moments and deflections in roof members subjected to ponding loads. Derivations of these equations for roof systems with infinitely stiff joists can be found in References (16) to (19). In this Appendix a method is presented for extending these results to include the effect of joist flexibility. The results are identical to the chart solutions presented in Reference (17).

A.2 Effect of Joist Flexibility

Consider the roof system shown in Figure 2.5. The effect of joist flexibility on the girders can be accounted for by adding a point force at each joist location equal to the additional weight of water contained in the deflected shape of either joist at the location. This force is determined by multiplying the area in the deflected joist shape by the joist spacing and by the density of water.

If the effect of deck flexibility is considered to be negligible, the joist deflection  $\delta$  can be determined from Equation (2.6) after substituting the subscript "j" for "g" and  $\delta$  for  $\Delta$



$$\delta = \delta_0 \left( \frac{1}{1 - C_j} \right) \quad (\text{A.1})$$

where

$$\delta_0 = \frac{5wL_j^4}{384EI_j} \quad (\text{A.2})$$

$$w = (D + \gamma h + \gamma \Delta) s \quad (\text{A.3})$$

and all other terms are defined in Section 2.3.3.

Assuming a sinusoidal deflected joist shape with a maximum amplitude  $\delta$  the weight of water in the deflect shape,  $w_1$ , is

$$w_1 = \int_0^{L_j} \gamma s \delta \sin \frac{\pi x}{L_j} dx \quad (\text{A.4})$$

Substituting Equations (A.1) to (A.3) into (A.4) and performing the required integration leads to

$$w_1 = (\alpha - 1) (D + \gamma (h + \Delta)) s L_j \quad (\text{A.5})$$

where

$$\alpha = 1 + \frac{8}{\pi^2} \frac{C_j}{1 - C_j} \quad (\text{A.6})$$

If the joists were rigid the point forces  $w_2$  acting at each joist location would be, from Figure 2.5,

$$w_2 = (D + \gamma (h + \Delta)) sL_j \quad (A.7)$$

Adding Equations (A.5) and (A.7), it can be seen that if joist flexibility is considered the point forces acting at each joist location are given by  $\alpha w_2$  instead of  $w_2$ . This is equivalent to assuming rigid joist behaviour and multiplying both the dead load and water density by  $\alpha$ .

### A.3 Summary

The equations presented in Section 2.3.3 for ponding of rainwater on the flat roof of a building are derived in this Appendix.

APPENDIX B

## ANCHOR BOLT TESTS

B.1 Introduction

A review of the literature indicates that almost no research has been performed to determine the strengths of column base details in light industrial buildings. As opposed to tall buildings, large sign posts, lamp standards, etc., where the overturning moments are large, column bases in single storey industrial buildings are usually not subjected to appreciable tension forces. Instead, it is the behaviour of the connections under the action of shear forces or combined shear and moderate amounts of tension or compression that is important. For this reason a series of tests were undertaken to determine the strengths of anchor bolt connections subjected primarily to shear forces.

The basic variables that can affect the strength of an anchor bolt connection are listed in Section 6.6. In this Appendix details of the test set-up and the testing procedure are outlined, and the results are analyzed.

B.2 Test Specimens

In order to simulate the behaviour of a column base connection in an actual structure, the test specimen shown in Figure B.1 was developed. The exterior foundation walls of the

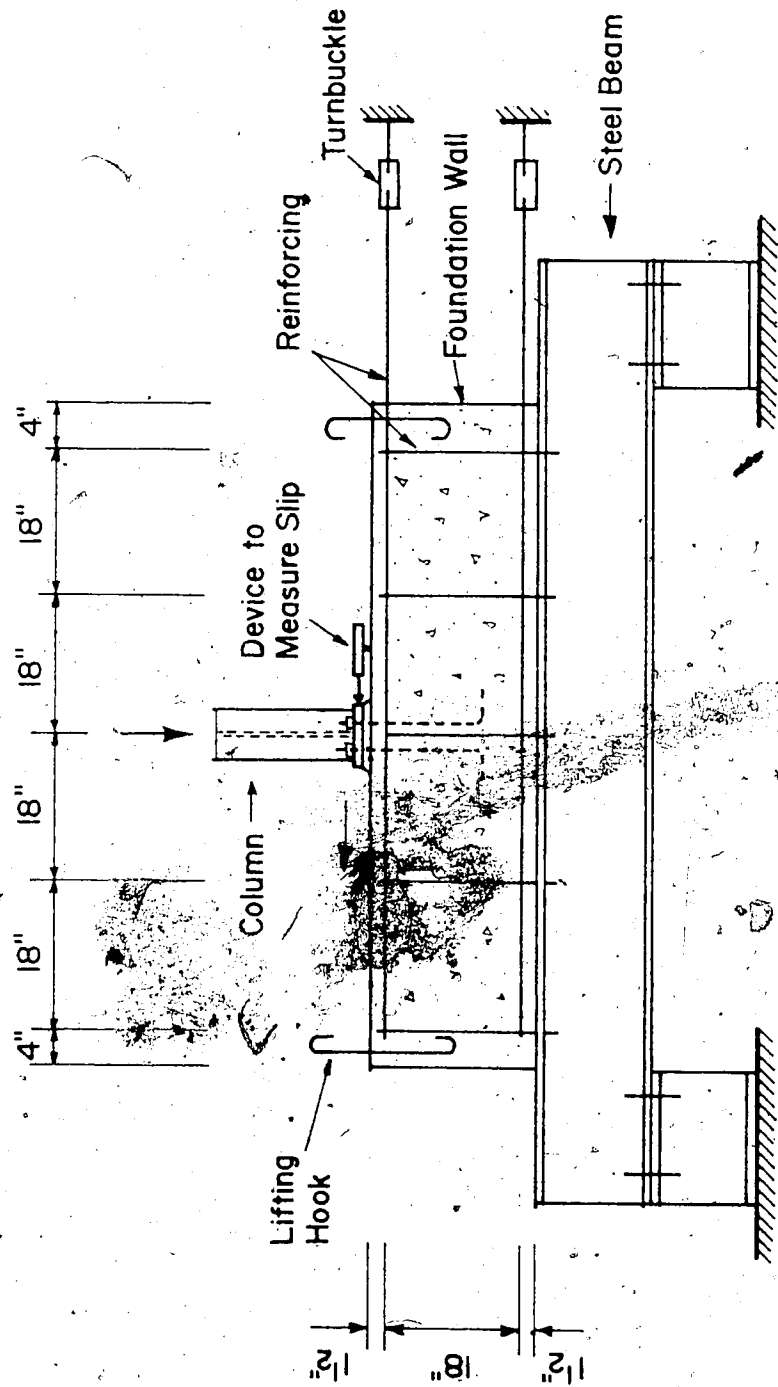


FIGURE B.1 TEST SPECIMEN

hypothetical structure were assumed to be 10 inches thick and the concrete was assumed to have a 28 day compressive strength of 3000 psi. The walls were assumed to be reinforced with #4 reinforcing bars (60 ksi. yield) spaced evenly at 18 inch centers on each face, both horizontally and vertically, with 1-1/2 inches of cover. The test specimen shown in Figure B.1 represents a portion of this wall.

The height of the specimen was selected so that at least two lines of longitudinal reinforcement would be present. The vertical reinforcing bars were connected to the top flange of the supporting steel beam with nuts and washers to simulate the bond that would be provided by the remainder of the wall. The specimen length was selected by assuming that the stresses caused by the applied loads would spread into the concrete along 45 degree lines emanating from the edges of the base plate and then doubling the length so obtained. The longitudinal reinforcing bars were connected by turn buckles to rigid supports.

Both W8x31 and W6x15.5 stubs columns were used in the tests. The base plates for the W8x31 columns measured 10" x 10" x 3/4" and the base plates for the W6x15.5 columns measured 8" x 8" x 3/4". Holes 5/16 of an inch larger than the 3/4 inch diameter of the anchor bolts were drilled into the base plates in accordance with standard practice<sup>(1)</sup>. Relatively thick base plates were used to ensure that failures occurred either in the walls or in the anchor bolts, rather than in the base plates. For simplicity, only two anchor bolts were used for each connection, as shown in Figure B.2. It is tacitly assumed that a four bolt connection would have twice the failure load of a two bolt connection.

### B.3 Concrete and Grout

In order to simulate construction practice 3000 psi concrete was ordered directly from a commercial ready mix plant. The actual 28 day compressive strength was found to be 3020 psi. The grout used was a high early strength grout with a strength after one week of 3570 psi.

### B.4 Anchor Bolts

The anchor bolts were also ordered commercially. All of the anchor bolts used in the tests were fabricated from the same heat to the dimensions shown in Figure 5.23. The steel conformed to the ASTM A36 Specification with a minimum specified yield strength of 36 ksi. Tension tests on unthreaded steel stock indicated that the actual yield stress was 49.3 ksi, and that the actual ultimate strength was 73.5 ksi. The steel was ductile, and exhibited a sharply defined yield point.

### B.5 Reinforcing Steel

The reinforcing steel conformed to CSA Standard A23.1, with a minimum specified yield point of 60 ksi.

### B.6 Casting and Curing

All fourteen specimens were cast at one time in order to

minimize the effect of variation of concrete strength. The concrete was vibrated into place with an internal vibrator, and the anchor bolts were positioned along the wall center-lines so that 3-1/2 inches projected above the top surface of the concrete. The right angle bends of the anchor bolts were pointed along the lengths of the walls. The specimens were trowelled and left to moist cure for one week. At the end of the week the formwork was removed and the specimens were left for a minimum of three more weeks before testing.

#### B.7 Grouting

One week prior to testing of a wall a one inch thick pad of grout was placed on the specimen in the vicinity of the projecting anchor bolts. The column stub was then lifted into place and supported by shims under the base plate at each of the four corners. The grout was then trowelled flush with the edges of the base plate, and the nuts on the anchor bolts were finger tightened. After several days the nuts were snug tightened with a hand wrench.

#### B.8 Testing

As indicated in Figure B.1, two jacks were required for each test. The horizontal force was applied through one jack, and the vertical force through the other. The loads were applied in increments, and slip of the column base plate relative to the top surface of the concrete was measured for each load increment, using

the device indicated symbolically in Figure B.1. Loading continued until either failure occurred or one of the jacks ran out of travel. In the two tests involving pure tension the deflection measuring device was mounted vertically in order to measure the upward movement of the base plate.

### B.9 Results

The loads causing large slips and the ultimate loads are tabulated for each of the fourteen tests in Table B.1. In this report the load causing a "large" slip is arbitrarily defined as that corresponding to a slip of  $3/8$  inch. For the pure tension tests the load causing "large" slip and the ultimate load are taken to be synonymous. In most cases slips of  $3/8$  of an inch correspond to points on the flat portions of the load-slip curves, indicating bearing failures in the concrete. Although the connections are capable of resisting larger shear forces, they can do so only at the expense of greatly increased slips. Therefore these loads are taken as the failure loads.

The ultimate failure mode was in all cases (except those in which jack travel requirements were exceeded) fracture of one or both of the anchor bolts, accompanied by extensive splitting of the grout as shown in Figure B.2. The specimens were dismantled at the ends of the tests and the cracked pieces of grout removed. The concrete was crushed in the vicinity of the anchor bolts, and in a few cases was split longitudinally, caused by the anchor bolts acting as wedges.



Test	Description	Load Per Anchor Bolt, Kips	
		Shear Load at Large Slip (3/8")	Ultimate Load
1	Pure Shear	8.5	16.0
2		6.5	18.3
3	Pure Tension	-	21.0
4		-	21.5
5	Shear + Tension	7.8	18.4
6		6.0	12.0
7		7.0	11.0
8	Shear + Compression	14.5	-
9		18.5	-
10		26.5	-
11	Pure Shear, Slotted Base plates	4.3	-
12		7.0	15.6
13	Incomplete Grouting	6.0	19.0
14		5.0	18.8

TABLE B.1 SUMMARY OF TEST RESULTS



FIGURE B.2 ULTIMATE FAILURE MODE OF TEST NO.2

### B.9.1 Effect of Type of Loading

Test results for pure shear, pure tension, combined shear and tension, and combined shear and compression are plotted in Figures B.3 to B.6, respectively. The types of loading and the column sizes are indicated on the Figures.

As discussed above, the failure loads are taken as those corresponding to slips of  $3/8$  inch, indicated by the broken vertical lines in Figures B.3, B.5 and B.6. These failure loads are plotted to form an interaction diagram in Figure B.7, in which the vertical axis corresponds to the shear force  $P$ , in kips, per anchor bolt, and the horizontal axis corresponds to the axial force  $F$ , in kips, per anchor bolt. Also shown are empirical relationships which provide a close fit to the test results.

In the absence of axial load the strength of the connection is approximately 7.2 kips per anchor bolt, caused by a combination of bearing, adhesion, and friction resulting from the small (but unmeasured) normal force caused by tightening the nuts on the anchor bolts. As indicated in Figure B.7, the bearing strength does not appear to be affected by the presence of a tension force on the connection. Assuming a compressive force  $F$  acting on the area of base plate tributary to each anchor bolt, and a coefficient of friction between grout and steel of 0.65, the strength of the connection is increased by  $0.65 F$  per bolt.

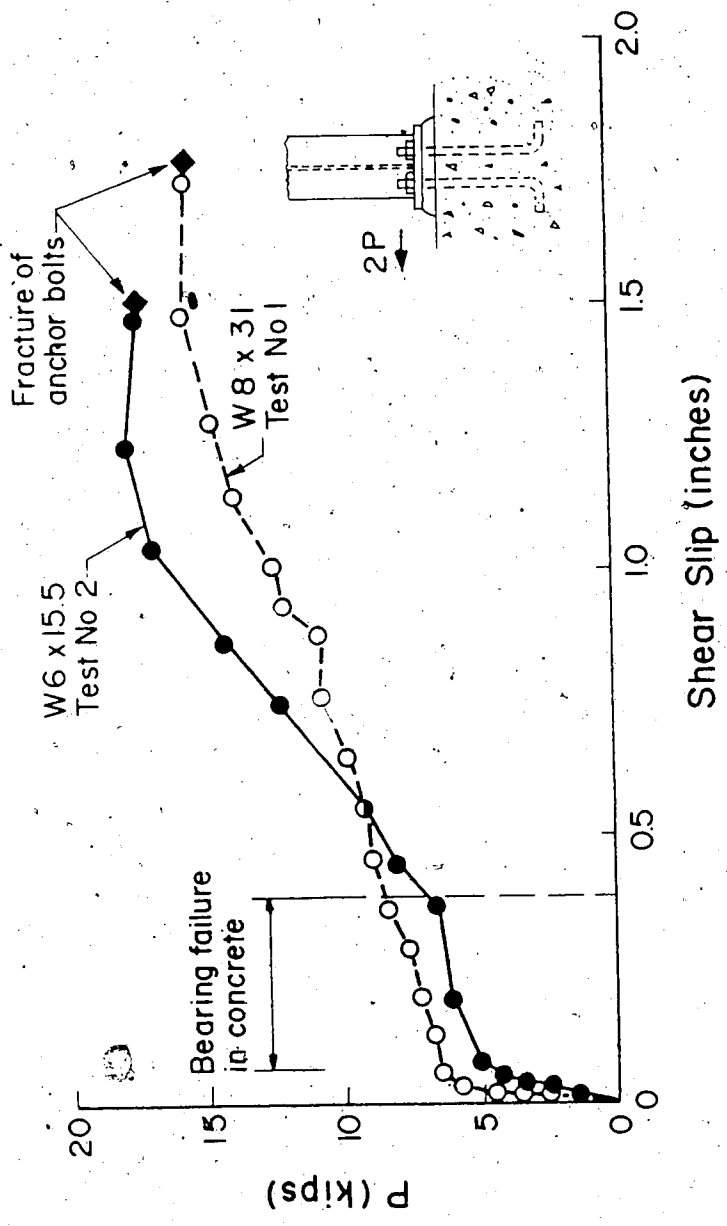


FIGURE B.3 TEST RESULTS FOR PURE SHEAR CASE

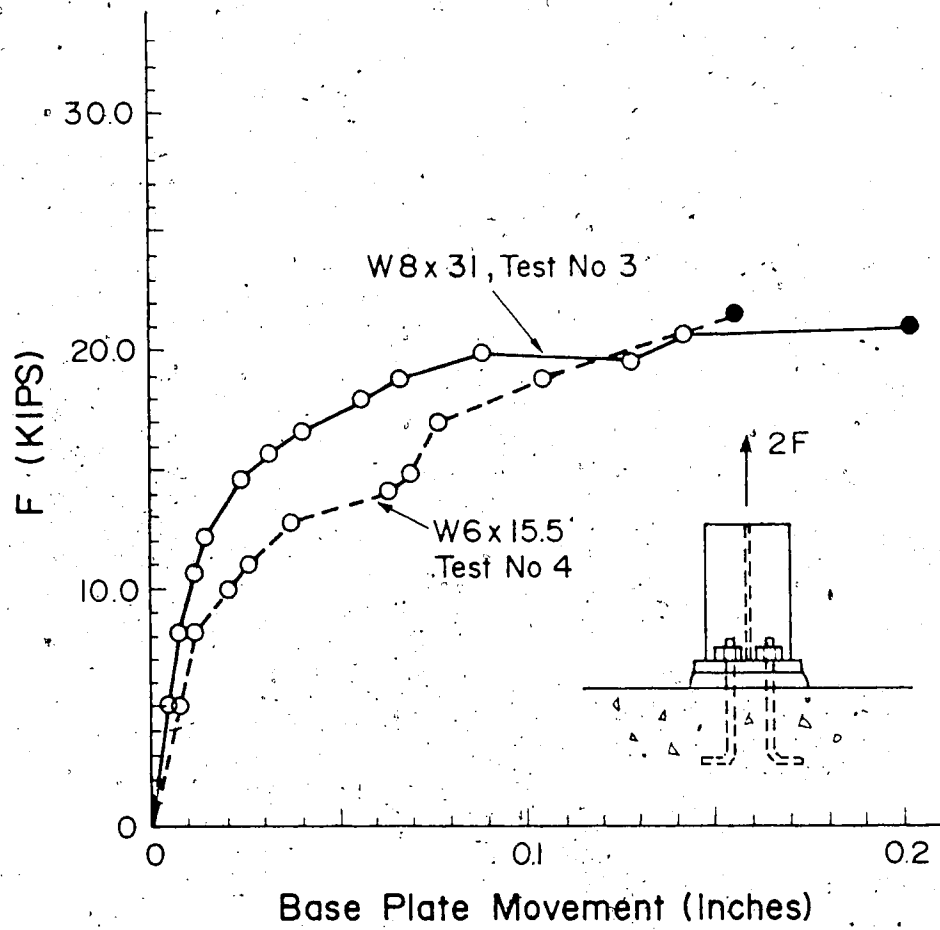


FIGURE B.4 PURE TENSION

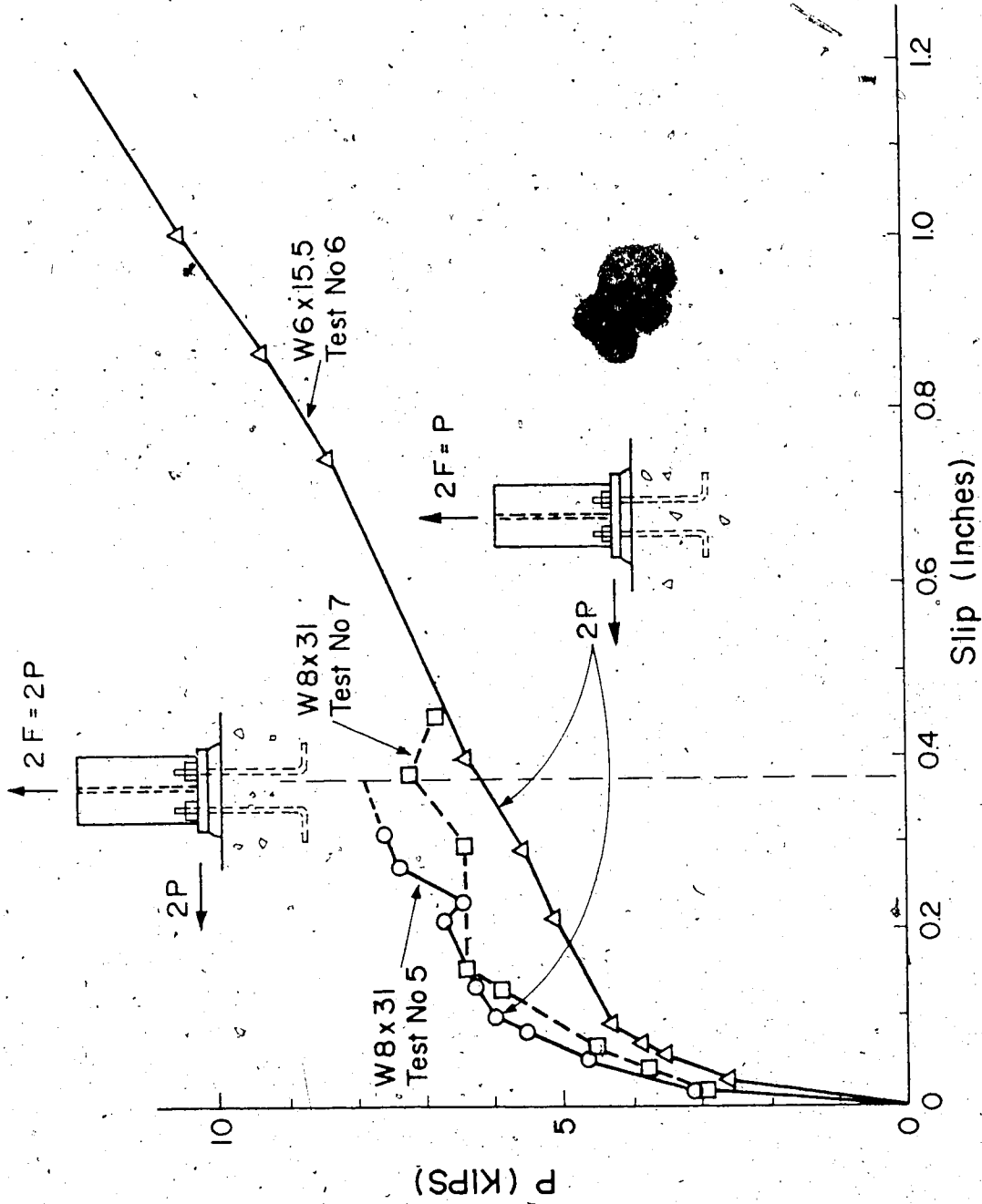


FIGURE B.5 COMBINED SHEAR AND TENSION

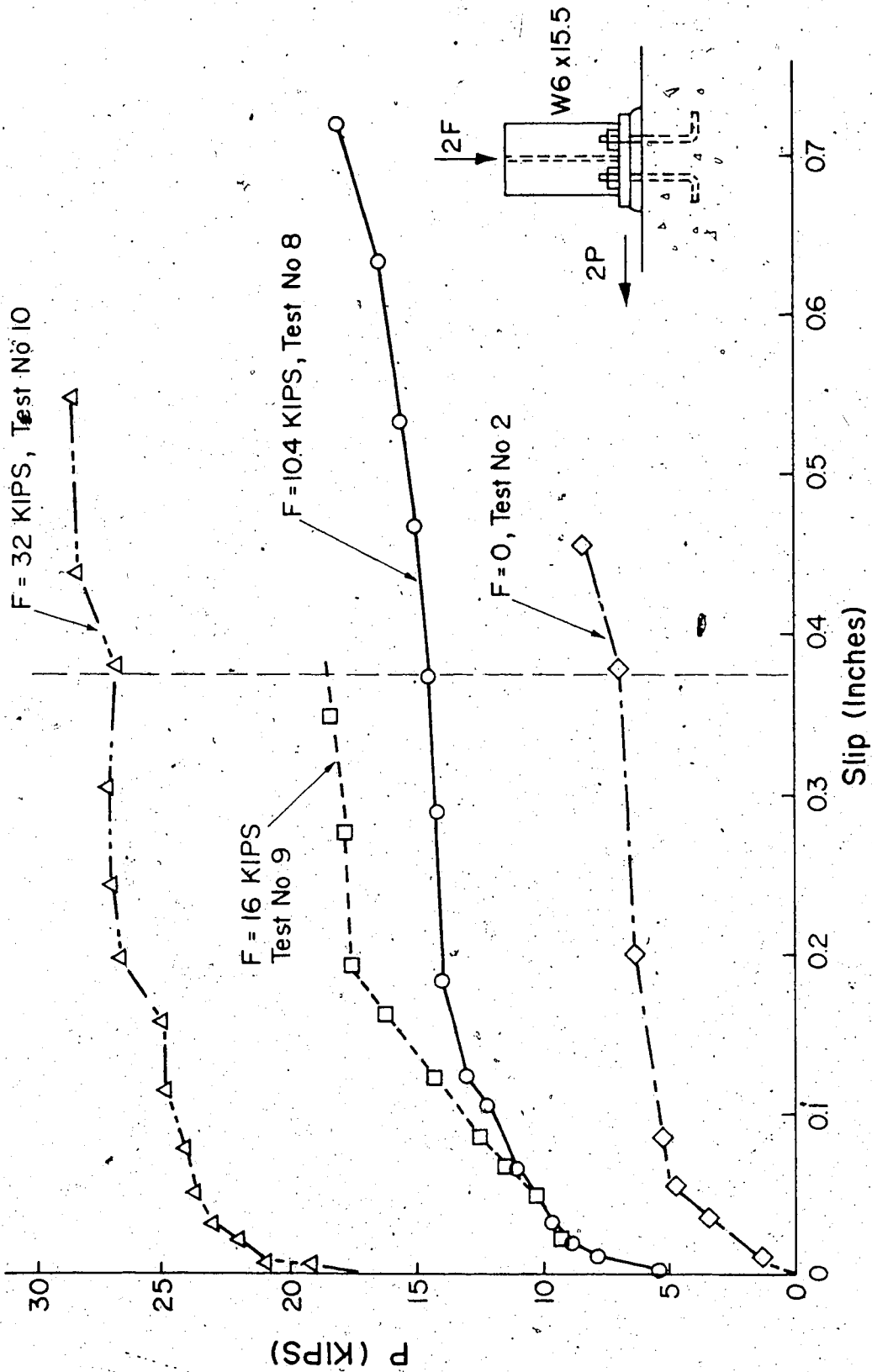


FIGURE B.6 COMBINED SHEAR AND COMPRESSION

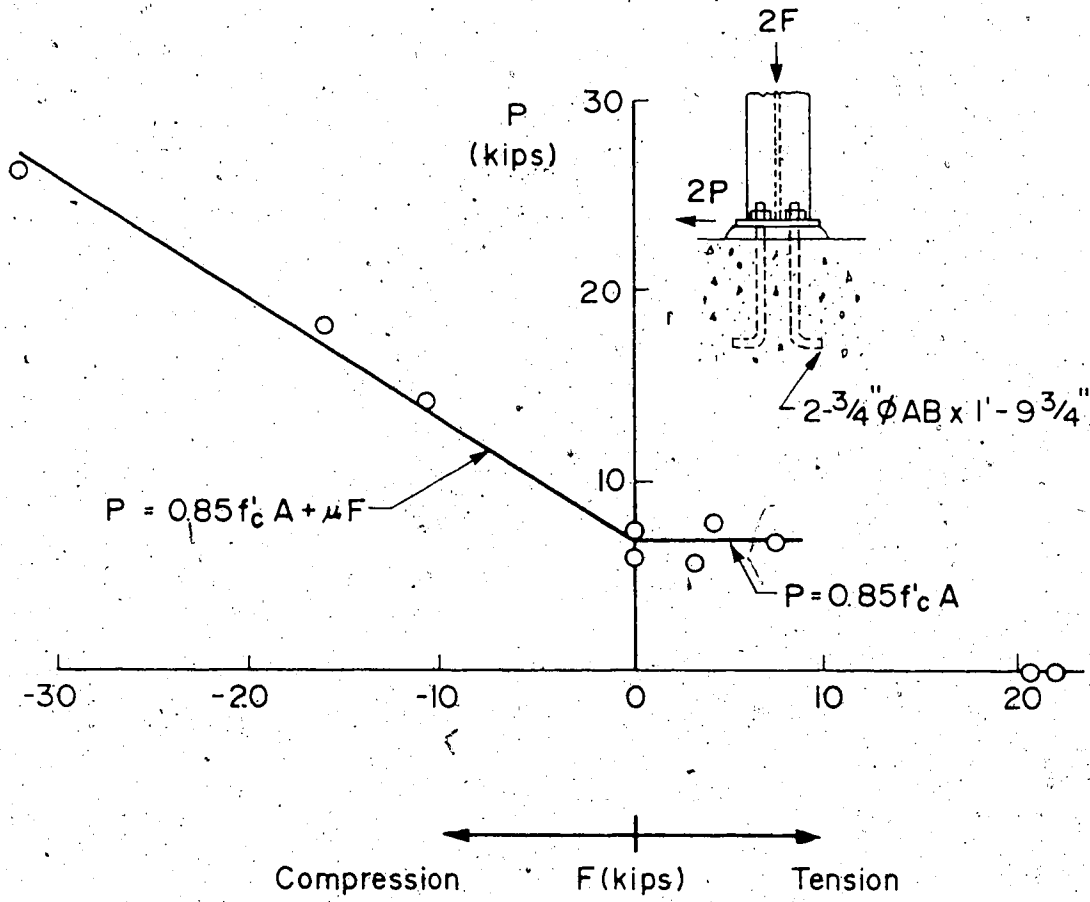


FIGURE B.7 INTERACTION DIAGRAM



### B.9.2 Effect of Oversize Holes

Slotting of the base plate may be required, for example, if the anchor bolts are improperly located at the construction site. If the base plate is slotted, the washer is often welded to the base plate in order to assist in the force transfer.

Test results for a base plate with slotted holes are compared in Figure B.8 with tests results for a base plate with slotted holes but with the washers welded to the nuts and base plate. Also shown are the test results for an unslotted base plate.

It is seen that in both cases the shear forces causing large slips are approximately one half to two-thirds of that for the unslotted base plate, although the ultimate strengths do not appear to be reduced significantly. Considerable connection deformation is required before the behaviour of the slotted base plate with welded washers is similar to that of the unslotted base plate. Prior to this, the effect of welding the washer is small.

### B.9.3 Effect of Incomplete Grouting

As indicated in the insert to Figure B.9, pure shear tests were performed on column bases with ungrouted regions near the middle portions of the base plates of 25 and 50 percent of the total base plate areas. The ungrouted areas were formed by positioning one inch thick square pieces of styrofoam over the anchor bolts

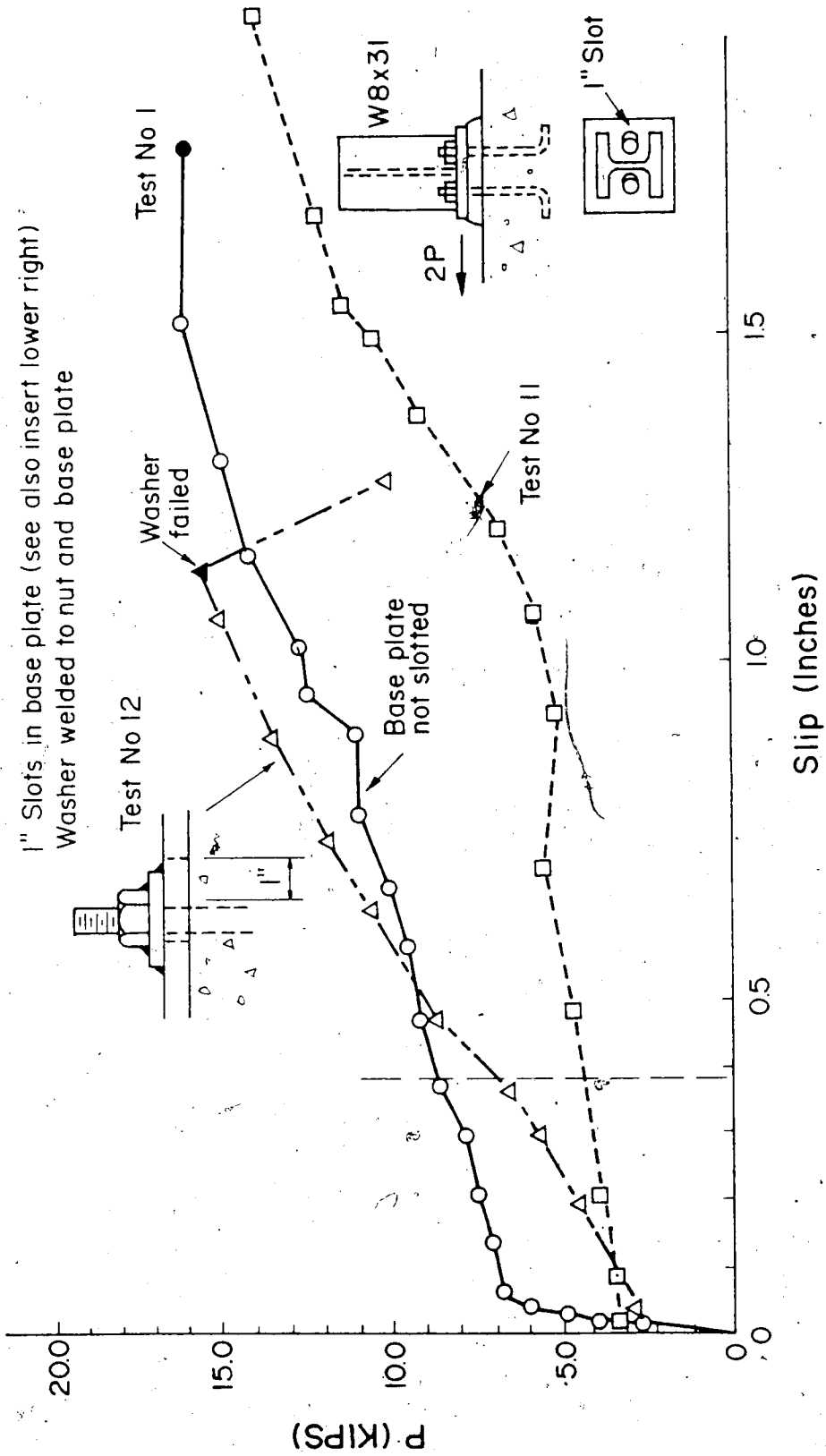


FIGURE B.8 EFFECT OF SLOTTING

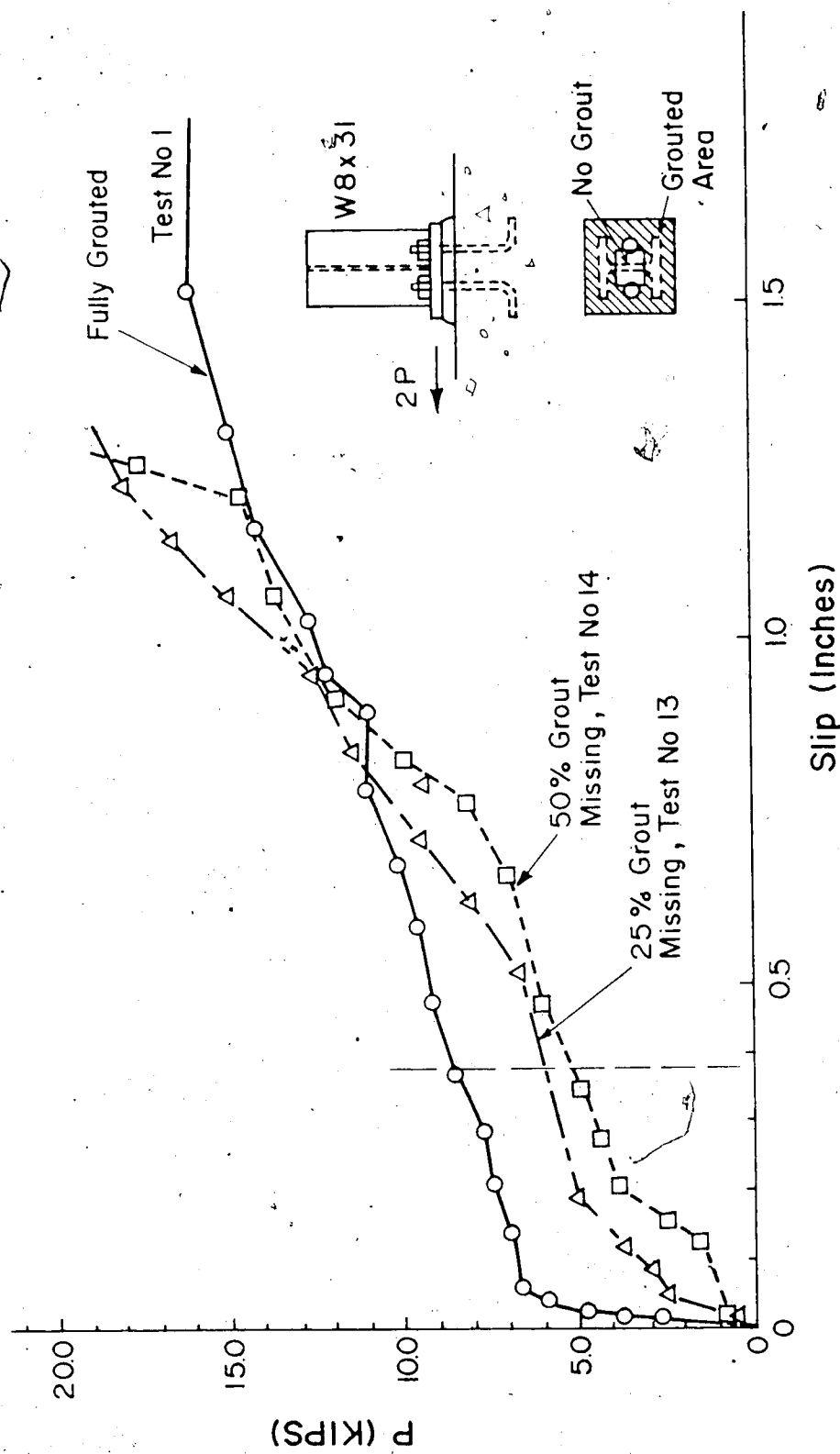


FIGURE B.9 EFFECT OF GROUTING

prior to the grouting operation.

It is seen from Figure B.9 that the strengths corresponding to large slips are reduced by approximately one half to two-thirds of the strength of the fully grouted case. Ultimate strengths, however, are not reduced.

#### B.10 Summary

A series of tests to determine the behaviour of anchor bolt connections are summarized in this Appendix.

APPENDIX C

## EXPANSION JOINT REQUIREMENTS

C.1 Introduction

In other portions of this report the effects of vertical loads and horizontal loads due to winds and earthquakes were studied. In this Appendix the effects of movements caused by temperature changes are considered.

Consider as an example the rectangular one storey industrial building shown in Figure C.1. The structure is braced in one direction by direct acting bracing systems located in the end bays. It is braced in the other direction by masonry shear walls. The building is assumed to be longer than it is wide.

The foundation of the structure, below the frost line, remains at a relatively constant temperature while the walls and roof do not. Thus an increase in temperature causes the structure to deform approximately as shown in Figure C.1. This movement can cause a number of problems if the structure is too long:

- (a) yielding of the end tension diagonals;
- (b) cracking and crushing of the masonry walls along their bases;
- (c) various serviceability problems related to maximum column sways at the ends of the building.

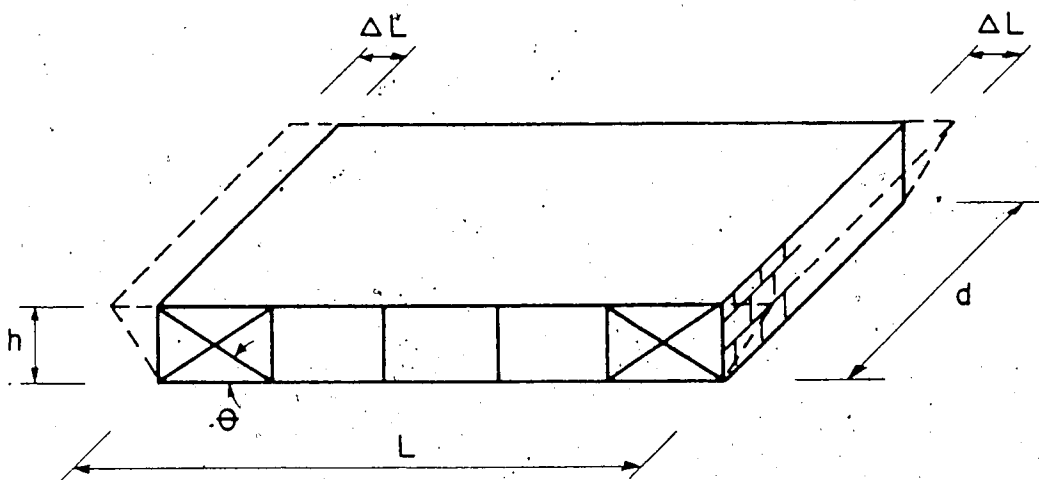


FIGURE C.1 EXAMPLE BUILDING

Similar problems can occur if the temperature drops. If the building is "L", "T", or "H" shaped the different segments may push or pull against each other, possibly damaging the walls or joints.

The usual solution to these problems is to provide one or more expansion joints. Maximum distances between expansion joints are often determined by "rules of thumb" based on past experience (Figures C.2 to C.4). Expansion joints (also called "isolation" or "separation" joints) extend from the top of the foundation completely through the walls and roof of the building, dividing the structure into two or more closely spaced but separate, smaller structures (Figure C.5(a)).

As illustrated in Figure C.5(b) the portions on either side of the expansion joint are designed to act independently of each other, requiring double lines of beams, columns and bracing members. If diaphragm action is used to resist lateral loads in the plane of the roof, separate trimmer angles must be provided along both sides of the expansion joint. An expansion joint detail is therefore an expensive detail, and in some situations the designer may wish to exceed the empirical limits given in Figures C.2 to C.4 by evaluating the effects of temperature changes analytically.

The maximum thermal movement,  $\Delta L$ , depends on a number of variables, including:

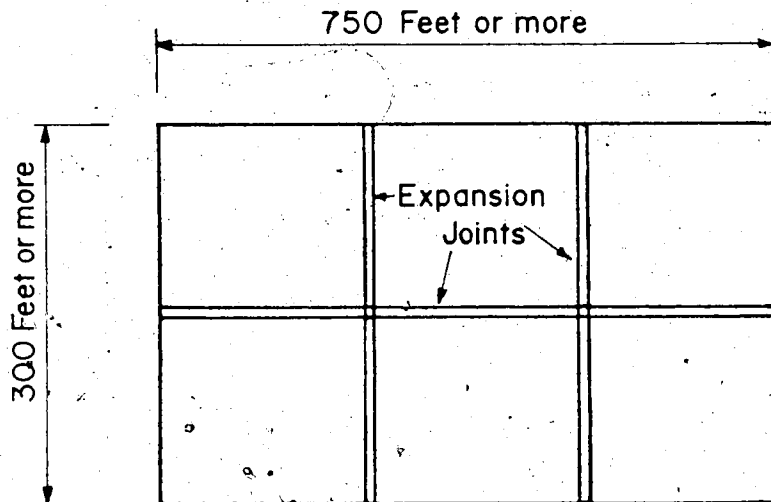
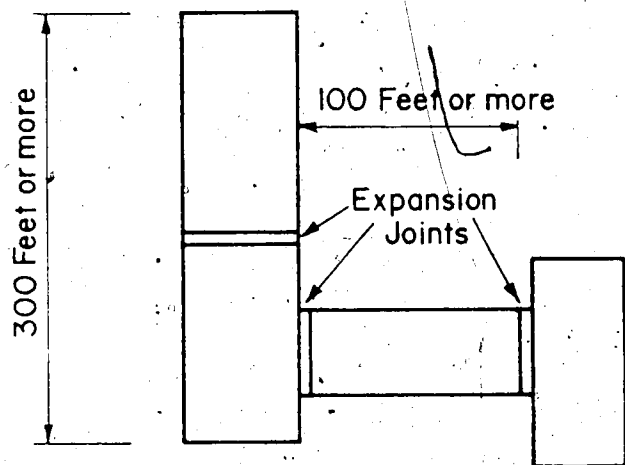


FIGURE C.2 LENGTHS BETWEEN EXPANSION JOINTS<sup>(94)</sup>



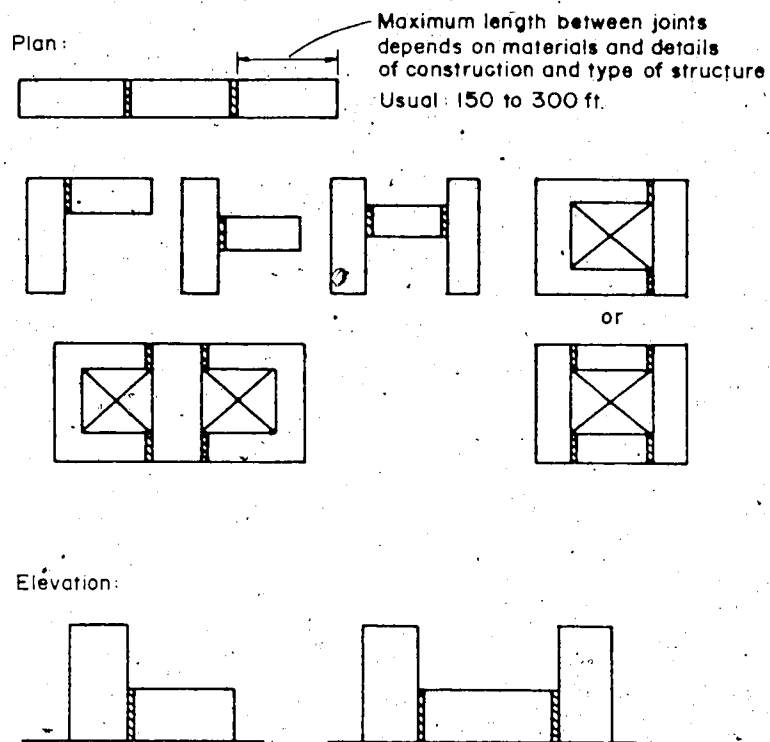


FIGURE C.3 LENGTHS BETWEEN EXPANSION JOINTS (95)

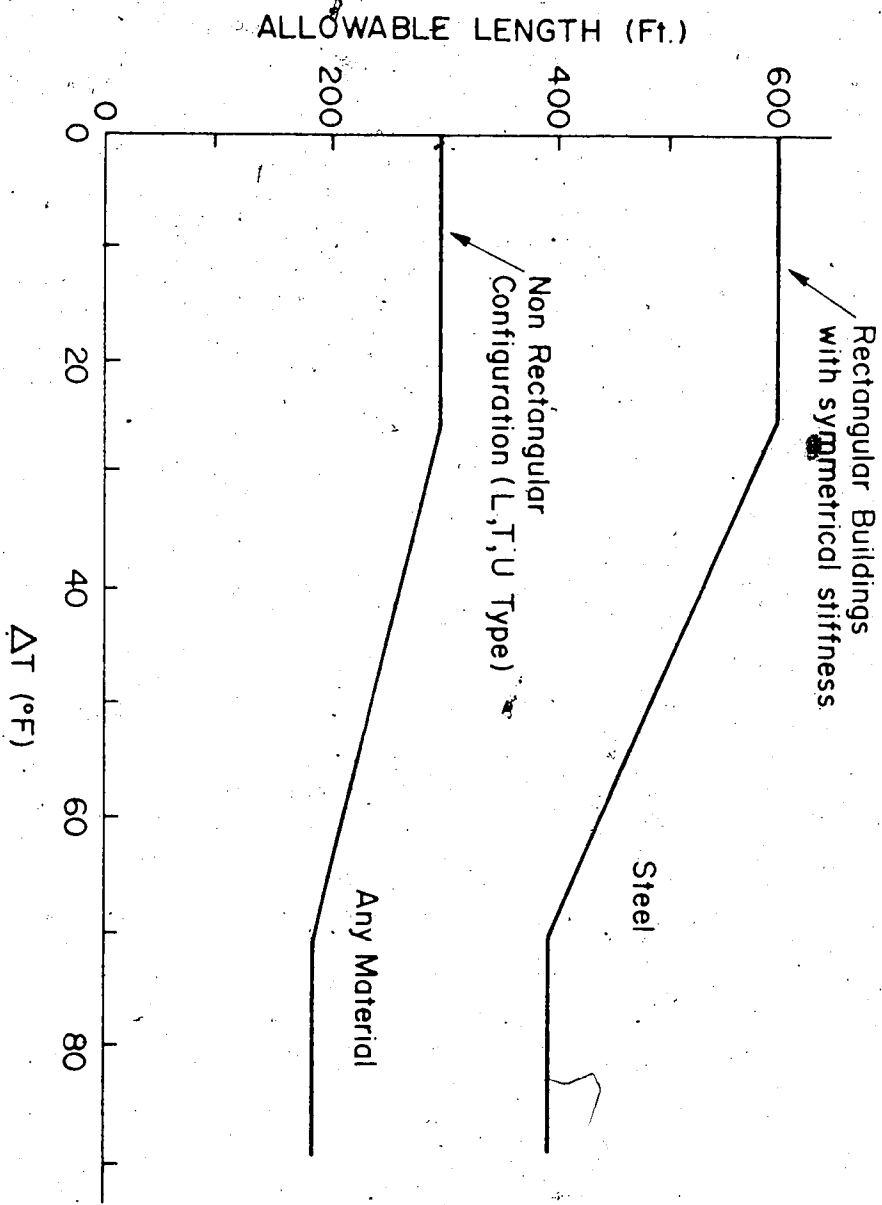
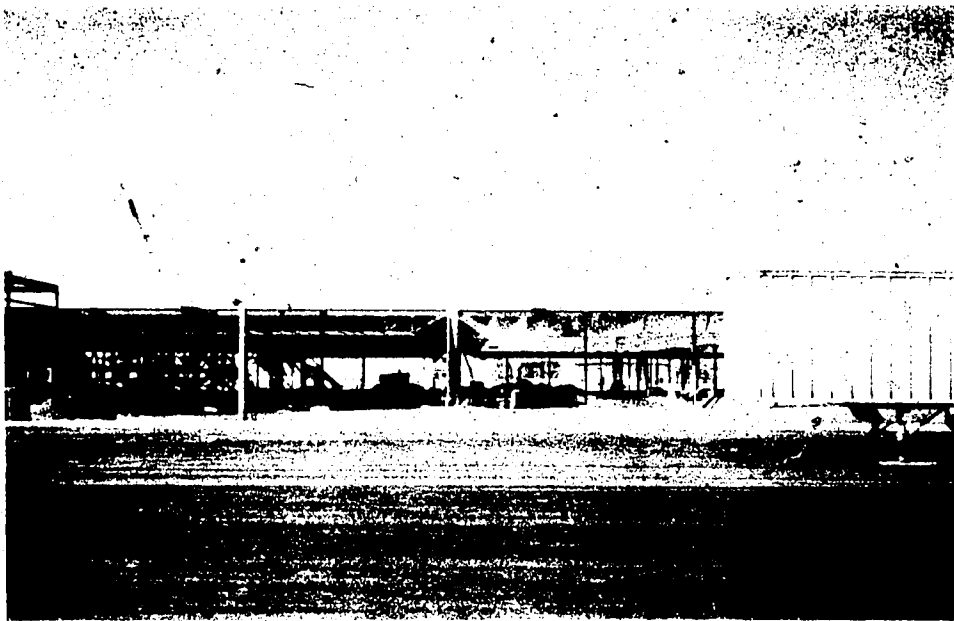
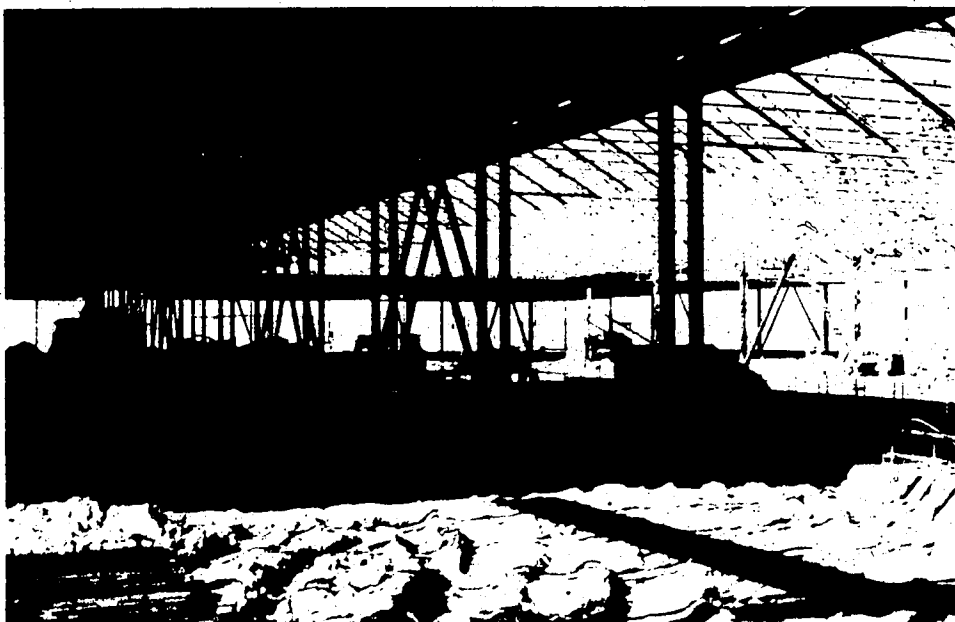


FIGURE C.4 LENGTHS BETWEEN EXPANSION JOINTS (96)



(a)



(b)

FIGURE C.5 CONSTRUCTION DETAILS OF AN EXPANSION JOINT

1. the range of outside temperatures,
2. provisions made for inside temperature control,
3. time of construction,
4. effectiveness of the wall and roof insulation,
5. locations and relative stiffnesses of the braced bays,
6. the building shape,
7. the cross section areas of the roof members, and
8. the flexibility factor of the roof deck diaphragm, or
9. the arrangement and sizes of bracing members in the plane of the roof.

A simple closed form solution that includes all of these variables is considered to be beyond the scope of this report. In this report the factored thermal movement is assumed to be given by an equation of the form<sup>(96)</sup>:

$$\Delta L = 1.25 L_{\text{eff}} \alpha \cdot T \quad (C.1)$$

in which 1.25 is the load factor for thermal effects as described in Section 2.2,  $L_{\text{eff}}$  is an "effective" building length,  $\alpha$  is the coefficient of thermal expansion for steel ( $6.5 \times 10^{-6}/^{\circ}\text{F}$ ) and  $\Delta T$  is the specified temperature change.

### C.2 Effective Building Length, $L_{\text{eff}}$

Structures with a symmetrically arranged bracing system experience temperature displacements of a relatively simple pattern,

as illustrated in Figure C.6(a). Since the wall bracing is symmetrical the roof members can be assumed to elongate horizontally relative to a fixed point at the center of the wall. In this case the effective length is equal to one half the actual length.

$$L_{\text{eff}} = \frac{L}{2} \quad (\text{C.1})$$

If, on the other hand, the bracing is not symmetrical and one end is stiffer than the other as illustrated in Figure C.6(b), the deflection at the stiff end,  $\Delta_1$ , is less than that at the other end,  $\Delta_2$ . It is recommended in Reference (96) that for computing  $\Delta L_1$

$$L_{\text{eff}} = \frac{1}{3} L \quad (\text{C.2})$$

and for computing  $\Delta L_2$

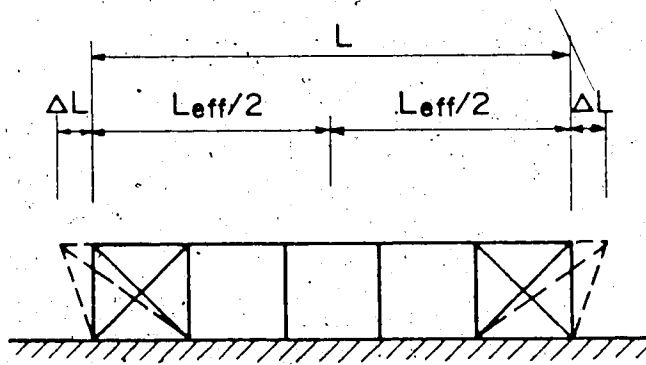
$$L_{\text{eff}} = \frac{3}{4} L \quad (\text{C.3})$$

In the limit, for an infinitely stiff end, for computing  $\Delta L_1$

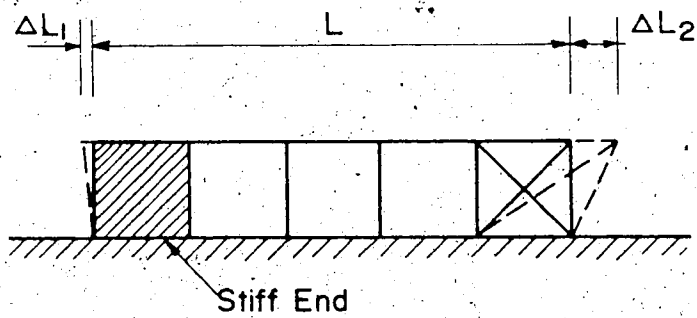
$$L_{\text{eff}} = 0 \quad (\text{C.4})$$

and for computing  $\Delta L_2$

$$L_{\text{eff}} = L \quad (\text{C.5})$$



(a) Symmetrically Arranged Bracing System



(b) Structure with Unsymmetrically Arranged Bracing System

FIGURE C.6 EFFECT OF BRACING LAY-OUT ON THERMAL MOVEMENTS

### C.3 Specified Temperature Change, T

Assume, initially, that the structure is erected and plumbed during the winter when the outside temperature is  $-40^{\circ}\text{F}$ . The temperature of the steel frame is therefore also equal to this value. If the structural steel is exposed to view from the inside of the structure after it is completed (the usual case for the type of building being considered here) the temperature of the steel must be approximately  $70^{\circ}\text{F}$  (room temperature). In some type of industrial buildings, for example fabricating shops which may not be well insulated and in which a large amount of welding is done, the inside temperature at roof level may be as high as  $100^{\circ}\text{F}$ . The temperature range for winter construction is therefore approximately  $110^{\circ}\text{F}$  to  $140^{\circ}\text{F}$ .

Assume next that the structure is erected and plumbed during the summer when the average outside temperature is  $70^{\circ}\text{F}$ . If, after completion, the inside temperature is maintained at  $70^{\circ}\text{F}$ , the temperature range is zero. Such a structure would, theoretically, not require any expansion joints regardless of length. This assumes, however, that the structure will not be vacated at some time in the future. If this would happen during the winter and the furnace is shut off, the inside temperature will, after a period of time, approach the outside temperature. The temperature range is therefore approximately equal to that for the first case considered.

While it is always conservative to assume the worst cases,

discussed above, it may not always be economical to do so. Although summer and winter temperatures are published in Reference (8) for various locations in Canada, a design temperature change for purposes of calculating expansion joint spacing is not given. Therefore this quantity must, at present, be based on engineering judgement.

In Reference (96) an empirical method is presented for computing suggested temperature changes for various locations in the United States. For locations along the United States - Canada border, the outside temperature changes are in the order of 100°F to 130°F, however it is suggested that for purposes of expansion joint calculations that the design temperature changes for buildings heated but not air conditioned be taken as approximately 50°F. In this report a specified temperature change of 60°F will be used.

#### C.4 Maximum Length of a Structure Without Expansion Joints

The structure shown in Figure C.1 will be used to illustrate the calculations involved. This building can be constructed without an expansion joint, providing (1):

$$\text{Factored Resistance} \geq \text{Effect of Factored Loads} \quad (\text{C.6})$$

and that at the specified load level serviceability problems do not occur.

Yielding of the end diagonals will occur if the building



is too long. If the bracing members are inclined at an angle  $\theta$  to the horizontal, as illustrated in Figure C.7(a), a temperature rise  $1.25 \Delta T$  results in a strain  $\epsilon$  in the tension bracing members of

$$\epsilon = \frac{\Delta L}{h} \sin\theta \cos\theta - 1.25 \alpha \Delta T \quad (C.7)$$

In Equation (C.7) the  $\alpha \Delta T$  term accounts for the beneficial effect of the temperature rise in the bracing member itself. Using Equation (C.1) Equation (C.7) becomes:

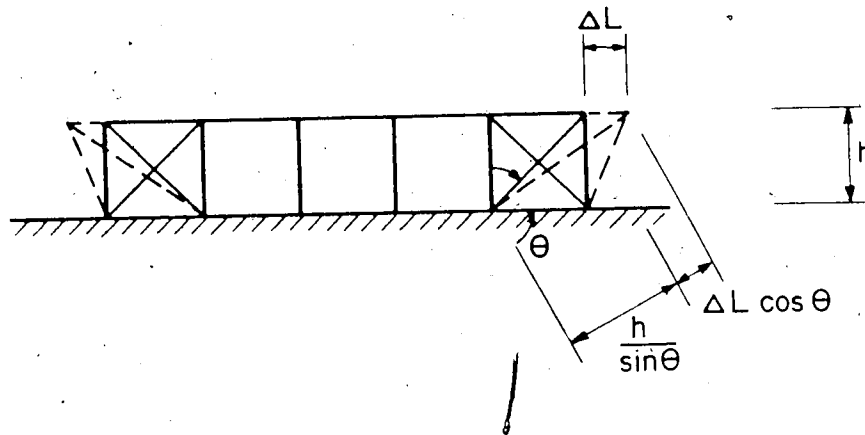
$$\epsilon = \frac{1.25 L_{eff}}{h} \alpha \Delta T \sin\theta \cos\theta - 1.25 \alpha \Delta T \quad (C.8)$$

Although the bracing members tend to resist this movement the area of roof framing is usually so large that the bracing will have to distort by approximately this amount. The force  $F$  in the bracing members due to temperature is equal to the strain multiplied by the bracing area  $A_b$  and Young's Modulus  $E$ :

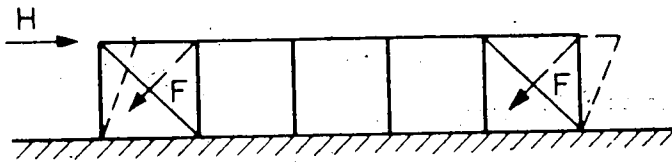
$$F = 1.25 A_b E \alpha \Delta T \left( \frac{L_{eff}}{h} \sin\theta \cos\theta - 1 \right) \quad (C.9)$$

Therefore Equation (C.6) becomes

$$\phi A_b F_y > F \quad (C.6a)$$



(a)



(b)

FIGURE C.7 EFFECTS OF TEMPERATURE AND LATERAL LOAD

where  $\phi = 0.9$  is the performance factor and  $F_y$  is the specified minimum yield stress of the steel.

---

### Example C.1

#### Given

The building shown in Figure C.1 has the following dimensions:  $L = 300$  ft., and  $h = 20$  ft. The bracing members in the side walls are inclined at  $45^\circ$  to the horizontal and are fabricated from steel rods with a cross sectional area of  $1 \text{ in.}^2$ . Assume that the steel is CSA G40.21 grade 44W ( $F_y = 44$  ksi). Determine if an expansion joint is required using yielding of the bracing members as a criteria.

#### Solution

Since the structure is symmetrically braced the effective building length is:

$$L_{\text{eff}} = \frac{L}{2} = \frac{300}{2} = 150 \text{ ft.} \quad (\text{Eq. C.1})$$

The tensile force in each bracing member due to temperature is:

$$F = 1.25 A_b E \alpha \Delta T \left( \frac{L_{\text{eff}}}{h} \sin\theta \cos\theta - 1 \right) \quad (\text{Eq. C.9})$$

$$= 1.25 \times 1.0 \times 30 \times 10^3 \times 6.5 \times 10^{-6} \times 60 \left( \frac{150}{120} \times \frac{1}{2} - 1 \right)$$

$$= 40.2 \text{ kips}$$

The bracing members do not undergo gross yielding providing

$$\phi A_b F_y \geq F_T \quad (\text{Eq. C.6a})$$

or

$$0.9 \times 1.0 \times 44 \geq 40.2$$

or

$$39.6 \geq 40.2$$

Although this indicates that yielding does occur, the proposed lay-out may still be judged to be acceptable in view of the various uncertainties involved in computing  $\Delta T$ , and since the left hand side of Equation C.6(a) is approximately equal to the right hand side.

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In Section 6.7 it is shown that if the masonry walls at the ends of the structure are idealized as vertical cantilever members subjected to horizontal point forces at their ends, allowable compressive stresses at their bases are exceeded if the maximum roof deflection exceeds

$$\Delta_{\max} = \frac{1}{4690} \left(\frac{h}{t}\right) h \quad (\text{C.10})$$

where  $t$  is the thickness of the masonry. Although it would be preferable to be able to express the following equation in a limit states format, design of reinforced masonry still (1977) uses allowable stress procedures<sup>(8)</sup>. Failure therefore is assumed to occur if:

$$L_{\text{eff}} \alpha \Delta T > \frac{1}{4690} \left(\frac{h}{t}\right) h \quad (\text{C.11})$$

### Example C.2

#### Given

The end walls of the structure shown in Figure C.1 are constructed of masonry units 10 inches thick. Additional details are described in Example C.1. Determine if an expansion joint is required through the structure using failure of the masonry walls as a criteria.

#### Solution

Failure of the end walls occurs if:

$$L_{\text{eff}} \alpha \Delta T \geq \frac{1}{4690} \left( \frac{h}{t} \right) h \quad (\text{Eq. C.11})$$

As shown in Example C.1, the effective building length is 150 feet.

The left hand side of Equation (C.11) is:

$$L_{\text{eff}} \alpha \Delta T = 150 \times 6.5 \times 10^{-6} \times 60 = 0.0585 \text{ ft.}$$

Since this is less than the right hand side

$$\frac{1}{4690} \left( \frac{h}{t} \right) h = \frac{1}{4690} \times \frac{20 \times 12}{10} \times 20 = 0.102 \text{ ft.}$$

an expansion joint is not required.

In some situations, for example if an overhead travelling crane is located in the end bays, it may be desirable to limit the maximum column sway at the specified load level. For example, if the maximum permissible column sway is taken as  $h/400$ , then an expansion joint is not required providing:

$$L_{\text{eff}} \alpha \Delta T \geq \frac{h}{400} \quad (\text{C.12})$$

Example C.3Given:

Determine if an expansion joint is required in the building shown in Figure C.1 using a maximum column sway of  $h/400$  as a criteria.

Solution

As shown in Example C.1, the effective building length is 150 feet. The left hand side of Equation (C.12) is:

$$L_{eff}\alpha\Delta T = 150 \times 6.5 \times 10^{-6} \times 60 = 0.0585 \text{ ft.}$$

Since this exceeds the right hand side

$$\frac{h}{400} = \frac{20}{400} = 0.050 \text{ ft.}$$

an expansion joint is required. It should be noted, however, that it is often very difficult to justify numerically a particular value of a permissible column sway, for example the  $h/400$  value selected here.

C.5 Expansion Joint Width

The theoretical expansion joint width  $w$  between two

building segments of lengths  $L_1$  and  $L_2$  is:

$$w = \alpha_T (L_{\text{eff}1} + L_{\text{eff}2}) \alpha \Delta T \quad (\text{C.13})$$

where  $L_{\text{eff}1}$  and  $L_{\text{eff}2}$  are the effective lengths of the two segments. Usually widths computed using Equation (C.17) vary from one to two inches. As discussed in Section 2.3.5, expansion joint widths may be governed by recommended separation distances to prevent the building segments from colliding during an earthquake.

#### C.6 Summary

Various empirical and analytical procedures are discussed in this chapter for evaluating the need for "through building" expansion joints. The analytical procedures are illustrated with examples.