H-Infinity Filtering Based Fault Detection, Estimation and Fault Tolerant Control

by

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Abstract

This thesis investigates the integrated fault detection, estimation and fault tolerant control problem for linear systems and Lipschitz nonlinear systems. Faults and disturbances are taken into consideration in a unified formulation. A H-infinity observer-based fault detection filter (FDF) is applied to generate residual signals. The FDF is designed to minimize the influence of disturbances and maximize the sensitivity of faults at the same time. Then a new online fault estimation scheme is designed by applying H-infinity filtering to residual signals instead of system outputs. Compared with existing literature, in which system outputs are commonly adopted for fault estimation, the proposed fault estimation scheme based on residual signals can achieve more accurate fault estimation results due to the fact that the influence of disturbances is minimized in the residual signal produced by the H-infinity FDF. Finally a fault tolerant controller is designed to retain the system stability and performance by compensating for the faults. The integrated scheme consists of three essential steps that are centered around the residual signal, namely, generating the residual signal for fault detection, and filtering the residual signal for fault estimation and fault compensation. In this framework, fault diagnosis and fault compensation are designed simultaneously in a closed-loop system. This integrated scheme is considered to be one of the most important contributions of this thesis. To demonstrate the effectiveness of the proposed method, two examples are given and simulation results are presented.

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Chapter 1

Introduction

1.1 Historical Development

Due to the increasing demands on system safety and reliability nowadays, fault detection, estimation and fault tolerant control have attracted great attention from worldwide research communities. Components of an industrial system have their own special capabilities to achieve certain functions. Obviously, the system can achieve full-functional performance only if all the components are healthy and functional. A single small component malfunction may cause a damaging impact on the overall system. Traditionally, enhancing the quality and reliability of individual system components can improve the system's robustness and dependability. But a fault-free system operation still cannot be guaranteed [5].

To satisfy the increasing requirements of high reliability and survivability of complex control systems, it is important to handle unexpected faults. Generally speaking, faults may change the system's dynamics, in another word, the system may not behave like what it is supposed to be when a fault occurs.. Most faults occur inside a system, which may interrupt the power supply, break a signal link in a communication channel, or create a leakage in a pipe [2]. Usually, unexpected faults cause losses from an economical perspective. Therefore, fault diagnosis is of vital importance to ensure the system safety and maintain reliable system operation. Fault detection can prevent the system from further failure by generating alarms to operators so essential protective actions can be executed.

The Fault Diagnosis problem is also very important in the fault-tolerant control system, which is designed to retain some portion of its control integrity in the event of possible component faults. This is achieved by incorporating an element of automatic reconfiguration, once a malfunction has been detected [25]. Fault tolerant control takes system faults into consideration and reduces the negative influence of the internal faults. A dynamic fault tolerant controller should automatically adjust its control law by analysing the faults to guarantee the closed-loop system achieves an acceptable performance.

The research of model-based fault diagnosis can be traced back to the beginning of 1970s. Since then, extensive research has been done and many fault diagnosis frameworks have been developed. The term "model-based" is used to characterize the application of the powerful techniques of mathematical modelling, such as state estimation and system identification for FDI [25]. Model-based fault diagnosis is based on analytical redundancy generated from the system model. From 1970s to 1990s, plenty of theoretical results were established in this area. The objectives of a model-based FDI algorithm has been widely accepted as monitoring of the plant during its normal working conditions so as to detect the occurrence of faults (Fault Detection), recognize their location (Fault Isolation) and, if possible, their time evolution (Fault Identification) [25]. Some mathematical or theoretical models are used to analyse if faults exist in the system. The model-based FDI techniques include observer-based approach, parity-space approach, and parameter identification based methods [5]. A number of methods have been developed to handle this problem. For example, Patton et. al proposed an application of disturbance principles for robust fault diagnosis [14, 23, 22, 29]. Ding et. al developed a robust fault detection filter to diagnose the faults [5, 10, 31, 39].

Automatic control systems and algorithms are becoming more and more complicated and sophisticated. Consequently, there is a growing demand for fault tolerance approaches, which can be achieved not only by improving the individual reliabilities of the functional units but also by an efficient fault detection, isolation and accommodation concept. A fault is understood as any kind of malfunction in the actual dynamic system, the plant, that leads to an unacceptable anomaly in the overall system performance [7]. Generally speaking, fault tolerant control can be classified into two types: passive fault tolerant control (PFTC) and active fault tolerant control (AFTC) [38]. There are some important results for PFTC, for example, the LQ reliable control by Hsieh et.al [15], the reliable control systems possessing actuator redundancies by Jiang et.al [16], the Reliable control of nonlinear systems by Liang et.al [17] and the reliable robust flight tracking control by Liao et.al [18] etc. In the literature, PFTC is also named as reliable control systems or control systems with integrity. In contrast to PFTC, AFTC reacts to the system component failures actively by reconfiguring control actions so that the stability and acceptable performance of the entire system can be maintained [38].

There are several reconfigurable control design methods proposed by researchers, such as: the linear quadratic [20, 21], the pseudo-inverse [35, 1, 12], the eigenstructure assignment [36, 37], the H_∞ robust control [33, 35, 34] and the linear matrix inequality [4, 8] etc.

1.2 Performance Requirements of System Subject to Faults

Researchers have analysed the faults' impacts on industrial systems for many years, since the faults may cause enormous damage and even risk operators' life. Basically, we define four notions to assess the properties of the system: safety, reliability, availability and dependability.

Safety is defined as the absence of danger. A safety system is designed to protect the current equipments from permanent damage when the faults happen unexpectively. It can be achieved by reducing or even shutting down the inputs from the controller, for example, reducing the voltage or shutting down the power. Thus, the safety system needs the information to scale the faults and make judgement by system automatically or by operator manually. A safety system is proposed to shut down the future operation of the overall system to reduce and avoid potential dangers of the system and its surroundings. The safety operation mechanism will be triggered if the system performance exceeds the outlier of degraded performance region. The safety system and fault tolerant controller work in separate regions of the signal space and satisfy complementary aims [2].

Reliability is used to describe the probability that system can run fully functionally under normal conditions. Fault diagnosis and fault tolerant control cannot change the reliability of the plant, but they can enhance the reliability of the overall system [2]. The reason is that the fault tolerant control can guarantee the system still work functionally even with some bounded faults.

A fault tolerant control system can reduce the possibilities that faults develop into a failure. The closed loop system will remain fully functional while the performance might degrade within the certain acceptable range.

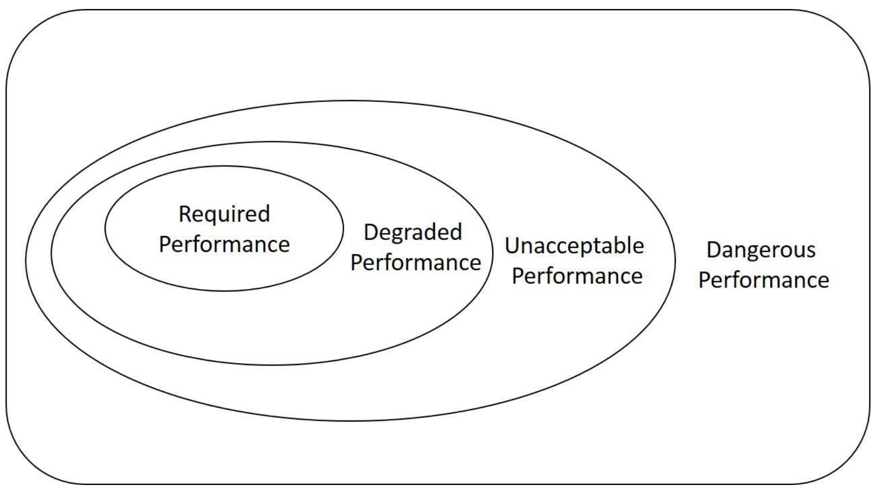

Figure 1.1: Regions of system performance

The system can satisfy its pre-defined performance in the region of required performance. The system should keep working in this range during its running time. Although the uncertainties and disturbances exist in the system, the robust controller should keep the system running in this region all the time. Furthermore, the controller may still maintain the system performance in this region even with some small faults, but at the same time it makes the faults harder to diagnose.

The region of degraded performance shows the region where the systems are still allowed to work even not satisfying the required performance. Faults will cause the system degradation from the required region of performance. Fault tolerant controller should start activating the recover actions to keep the system away from unacceptable performance region, and it should try to lead the system back to required performance region.

The unacceptable performance region is able to be avoided by applying the fault tolerant control. This region is between the degraded performance region and dangerous performance region, where the system should never reach.

1.3 Summary of Contributions

In this thesis, we focus on observer-based fault diagnosis and fault tolerant control problem. Although observer-based techniques have been developed for many years, there still remains an open problem of simultaneous fault detection and fault tolerant control. By using the new framework provided in this thesis, we can diagnose system faults and compensate for the faults at the same time. We develop a new integrated H_∞ filtering based fault diagnosis and fault tolerant control scheme to improve system performance. The main contributions of this thesis are summarized as follows:

1. A H_∞ observer-based fault detection filter is applied to generate residual signals. Then an online fault estimation scheme is designed by applying H_∞ filtering to the residual signals instead of the system outputs.

2. The fault detector and fault estimator are co-designed while the fault tolerant controller is designed separately. The closed-loop system stability and performance are analysed.

3. The fault tolerant controller is designed to stabilize the system and compensate for the fault effects simultaneously.

4. For linear systems, faults and disturbances are taken into consideration in a unified formulation and the robustness of fault detection and fault estimation is achieved. .

5. The framework is also extended to Lipschitz nonlinear systems.

1.4 Outline of the Thesis

The thesis is organized as follows:

- Chapter 2: In this chapter, we survey the techniques currently used for model-based fault diagnosis. we introduce the typical observer design method and residual generation, which leads us to design a new fault diagnosis framework. Then we talk about the classic H_∞ synthesis method, which will be used in the following designs.

- Chapter 3: In this chapter, we consider the problem of simultaneous fault detection and tolerant control for linear system. Using the observer-based fault detection filter, we can obtain the purified faults by reducing the impact from system disturbances. Then we estimate the faults by using H_∞ synthesis. Furthermore, we design a fault tolerant controller which can take effect in both fault-free case and fault-occurred case. A quadrotor example is provided to show the effectiveness.

- Chapter 4: In this chapter, we apply the framework to Lipschitz nonlinear problem. We obtain the residual signals by using the Lipschitz nonlinear observer, which can be further used to estimate the faults. Then a fault tolerant controller has been designed for the nonlinear system. A satellite model is simulated to illustrate the framework design.

- Chapter 5: Conclusions and future work are presented in this chapter.

Chapter 2

Background and Framework

In this chapter we list the classic methods and techniques which will be used in the following chapters, including observer-based residual generation fault diagnosis and H_∞ synthesis. The mathematical framework and notation used throughout the thesis are introduced.

2.1 Fault Diagnosis

Engineers enhance the equality and robustness of system components to improve the reliability and dependability of a system. These components include actuators, sensors, controllers and even the supervisor computer. However, faults might still exist in the system and system cannot guarantee to be fault free during the operation periods. Therefore, it is vital important to monitor the process and diagnose the faults. These two methods have become an essential ingredient in modern control systems.

2.1.1 Faults

There are many kinds of malfunction in the automatic control system. Faults, disturbances and model uncertainties all may change the performance of the system. Actually, they are three different kinds of malfunctions. Disturbance is a kind of perturbation that cannot be eliminated, which is always caused by the environment, such as air resistance. Model uncertainties is mostly caused by the inaccurate parameter realization of the system model, which can be reduced but cannot be excluded. Model uncertainties can change the model parameters of the system.

The faults are those elements which should be detected and whose effects should be removed by remedial actions. Disturbances and model uncertainties are nuisances, but whose effects on the system performance are handled by appropriate measures like filtering or robust design [2]. Faults may happen in the plants, actuators and sensors, which are depicted in the Figure 2.1.

Since the faults of system may lead to substantial damage on the industrial equipments and even cause risk of people life, the researchers have developed a set of theorems to analyse and classify the faults as the following three cases:

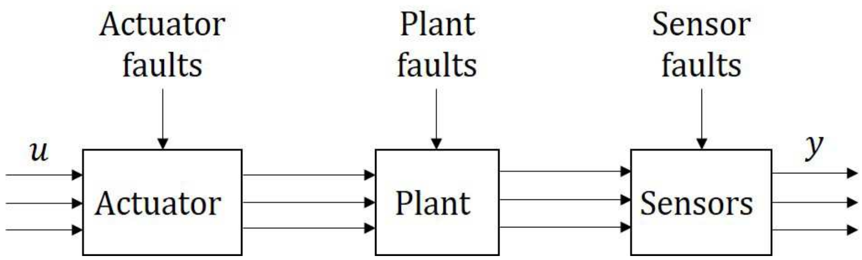

Figure 2.1: Schematic description of the systems with faults

1. **Actuator faults:** The faults happened in the actuators may influence the control input of the system, such as thrusts and currents. The plant properties will not be affected, however the controller of the automatic system may be varied or even be shut down.

2. **Plant faults:** The system's dynamic input and output properties can be affected when faults happen in the plant.

3. **Sensor faults:** The plant properties will not be affected, however the readings from sensors will not be accurate and may have a large bias.

There are different methods to detect the faults from different locations and faults can be categorized by their sizes and temporal behaviours. Abrupt faults occur, for example, in a breakdown of the power supply whereas steadily increasing faults are brought about by wear, and intermittent faults by an intermitted electrical contact [2].

2.1.2 Diagnostic Problem

Before implementing the fault tolerant control, the first task is to analyse the faults in the system. Generally, for a given continuous model-based system, fault diagnosis is always achieved by fault detection and fault estimation. The diagnostic problem has to be solved under real-time constraints by exploitation of the information included in a dynamical model and in the time evolution of the signals [2]. Figure 2.2 illustrates the diagnostic problem: for a given input u and output y of the system, analyses the fault f by using the designed diagnostic algorithm. Fault diagnosis is always composed by the following three essential tasks.

1. **Fault detection**: Detect the occurrence of faults in the functional units of the process, which lead to undesired or intolerable behavior of the whole system [5]. One main purpose of fault detection is to trigger alarm when faults occur in the system.

2. **Fault isolation**: Classify the different faults to generate different kinds of alarm signals to show where and which the faults happened.

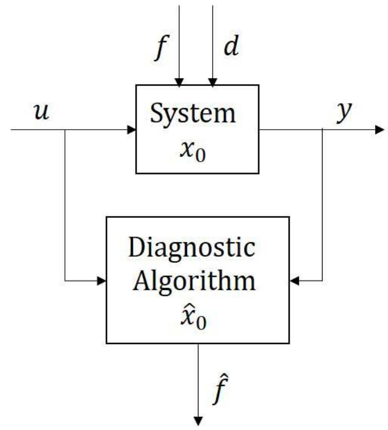

Figure 2.2: Fault diagnosis

3. **Fault analysis or identification**: Determine the types, magnitudes and causes of the faults. Give the details and the scales of the faults.

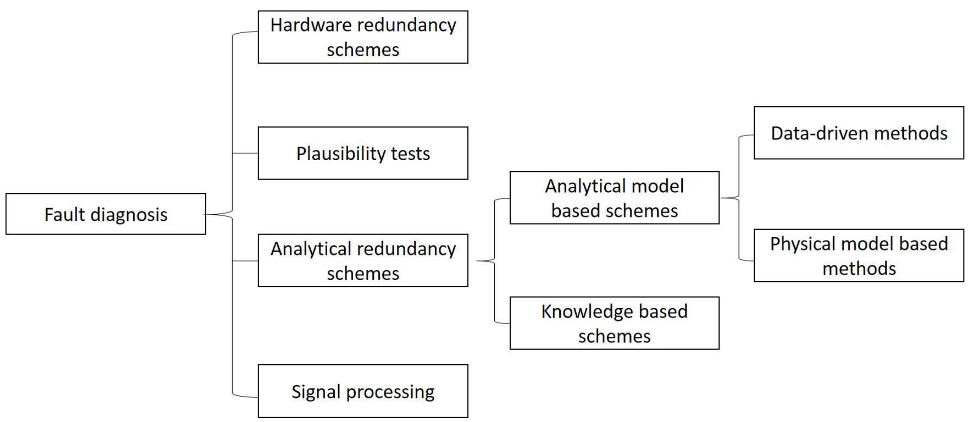

Figure 2.3: Classification of fault diagnosis methods

Fault diagnosis technique is currently receiving more and more attention. Figure 2.3 depicts the basic classification of the fault diagnosis methods.

1. **Hardware redundancy schemes**: The most essential components in this scheme is the reconstruction of the process by applying the redundant hardware. Then if the output of the hardware system is different from the one of its redundant component, the faults occurred in the process component can be detected. The hardware redundancy based fault diagnosis is highly reliable but costs much more than other schemes, for example, triple modular redundancy.

2. **Plausibility tests**: The plausibility test is used to check the simple physical laws while the system is running. Faults may cause the system losing the plausibility. Therefore, checking the plausibility will give us a simple way to analyse the faults

occurred in the system. However, the limitation is that the plausibility test may not work effectively in some complex systems.

3. **Analytical model based schemes**: In model-based fault diagnosis techniques, a process model is adopted to analyse the faults. The process model is often designed as a software which can run in the supervisor computer. By using the process modelling technique, we can get the model which can describe the system dynamic behaviours. So that we can reconstruct the process behaviour online.

4. **Signal processing based fault diagnosis**: The faults can be diagnosed by a suitable signal processing scheme, since signals usually carry information of faults, for example, fault locations and magnitudes. The signal processing based schemes are mainly used for those processes in the steady state, and their efficiency for the detection of faults in dynamic systems, which are of a wide operating range due to the possible variation of input signals, is considerably limited [5].

2.2 Fault Tolerant Control

A safety system interrupts the operation of the overall system to avoid dangers for the system and its environment. It is invoked if the outer border of the region of unacceptable performance is exceed [2]. The architecture of fault tolerant control is showed in Figure 2.4. There are major steps of designing the fault tolerant control for the system.

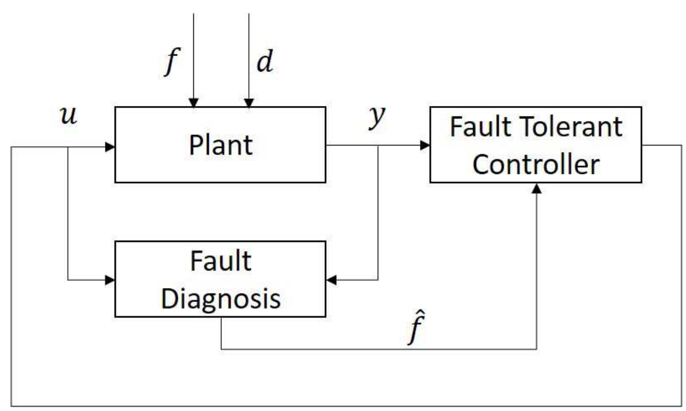

Figure 2.4: Architecture of fault tolerant control

1. The fault diagnosis part uses the measured input and output from the system and tests their consistency with the plant model. Its results is a characterisation of the faults with sufficient accuracy for the controller re-design [2].

2. The fault tolerant controller uses the fault information from fault diagnosis block, and adjusts the controller to the faulty situation.

There are two methods often used to design the active fault tolerant controller.

1. Multiple controllers are designed especially for switching based control, which means the system has multiple controllers: a nominal controller and fault tolerant controllers. The overall control modes are independent. The system supervisor level will switch the controller to fault tolerant control mode when faults are detected and switch back when no fault exists any more, e.g. Markovian jump linear system [19].

2. The other one is the all-in-one controller, which can handle both the nominal case and faulty case by one controller. Note that in this method fault tolerant control is always composed of two parts: nominal controller and faulty compensation controller. Basically, the nominal controller can be any proper controller that can satisfy the system performance. Then the faulty compensation controller is considered to be an add-on to the nominal controller, which is designed to handle the faults and guarantee the system performance under the faulty case.

2.3 The Integrated Framework for Fault Detection, Estimation and Fault Tolerant Control

In this section, we will introduce the classic observer-based fault diagnosis techniques, which will be used in this thesis to both the robust linear fault diagnosis and Lipschitz nonlinear fault diagnosis. Generally, The observer-based approach consists of two steps: observer design and residual generation.

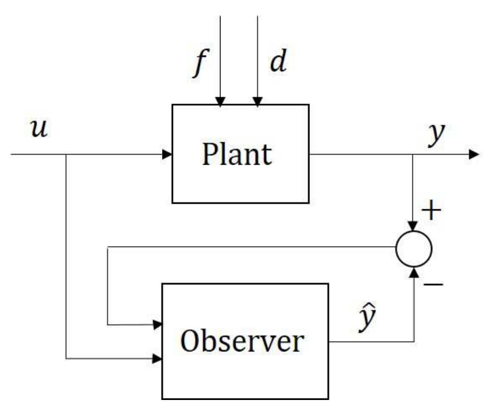

Figure 2.5: Observer-based approach

In this thesis, a fault detection observer and a fault estimation scheme are designed in a modified H_∞ framework, and then a H_∞ fault compensation control law is designed. The proposed schemes realize an active fault tolerant control system, designed in a unified H_∞ problem setting. Unlike several existing fault tolerant control schemes, where only a fault estimation scheme is designed for fault compensation, an integrated design of fault detection, estimation and fault compensation control is investigated in this thesis.

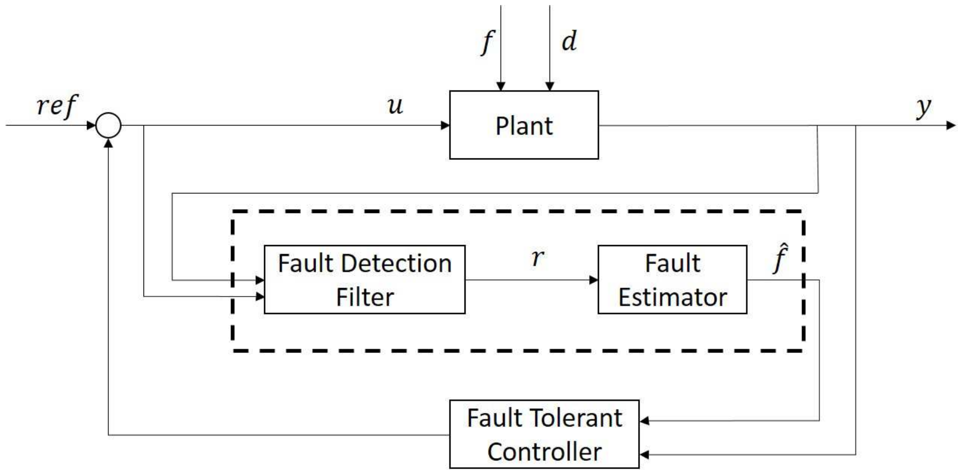

Figure 2.6: The integrated framework

The design methodology presented in this paper is shown in Figure 2.6. It is noted that the fault tolerant controller only utilizes the information of estimated faults $\hat{f}(t)$ and the system output $y(t)$. Due to the fact that the residual signals (with disturbances minimized) are used for fault estimation, the influences of control feedback on fault detection and fault estimation is further reduced, which makes it possible to design the fault compensation law and fault estimation law separately. The following 3-step procedure consists of:

- An observer-based robust fault detection filter (FDF) is used to detect faults, and handle disturbances. The residual signal $r(t)$ from the FDF will be generated by solving an H_∞/H_- optimization problem, which will minimize the sensitivity of $r(t)$ to $d(t)$ and maximum the sensitivity of $r(t)$ to $f(t)$ [25].

- Since the disturbances from residual signal have been filtered, we can estimate the faults $f(t)$ in a more accurate fashion using directly the residual signal $r(t)$. A simple solution is proposed by designing a filter $K_{est}(s)$, to minimize the error $\tilde{f}(t)$ between the estimated faults $\hat{f}(t)$ and the actual faults $f(t)$.

- Finally a fault tolerant control law $u(t)$ is to be designed to stabilize the system and compensate for the faults simultaneously. A dynamic stabilizing controller and a compensating controller are designed for this purposed.

2.4 H_∞ Synthesis

The H_∞ synthesis is a classical method and has been widely used in controller design. For ease of reference in the following chapters, first we list some necessary lemmas and theorems in this section. Note that all lemmas and theorems in this section are collected from Dullerud's book *course in robust control theory: a convex approach* [6]. The proofs have been omitted in this section and can be referred to the book.

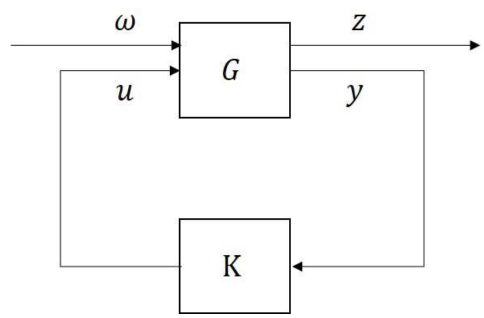

Figure 2.7: Standard setup

Consider the system described by the following block diagram, where the plant G and K are assumed to be real rational and proper; w is a vector signal including noises, disturbances, and reference signals, z is a vector signal including all controlled signals and tracking errors, u is the control signal, and y is the measurement. It will be assumed that state-space models of G and K are available and their realizations are assumed to be stabilizable and detectable [40]. Describe the system $G(s)$ as the following matrix:

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right] \tag{2.1}$$

where $A \in R^{n \times n}$. Realize the dynamic controller $K(s)$ as:

$$K(s) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \tag{2.2}$$

where $A_K \in R^{n_K \times n_K}$. Combine these two state space realizations into one which describes the map from ω to z [6]. We obtain

$$S(G,K) = \left[\begin{array}{c|c} A_L & B_L \\ \hline C_L & D_L \end{array}\right] = \left[\begin{array}{cc|c} A + B_2 D_K C_2 & B_2 C_K & B_1 + B_2 D_K D_{21} \\ B_K C_2 & A_K & B_K D_{21} \\ \hline C_1 + D_{12} D_K C_2 & D_{12} C_K & D_{11} + D_{12} D_K D_{21} \end{array}\right] \tag{2.3}$$

Now define a matrix J such that

$$J = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \tag{2.4}$$

which collects the representation for $K(s)$ into one matrix. We can parametrize the closed-loop relation in terms of the controller realization as follows [6]. At the very beginning we can define the following matrices.

$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C_1 & 0 \end{bmatrix}, \quad \underline{C} = \begin{bmatrix} 0 & I \\ C_2 & 0 \end{bmatrix}$$
$$\underline{B} = \begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix}, \quad \underline{D}_{12} = \begin{bmatrix} 0 & D_{12} \end{bmatrix}, \quad \underline{D}_{21} = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix}$$

where all the parameters above can be calculated directly from the information of $G(s)$. Then we define

$$\begin{aligned} A_L &= \bar{A} + \underline{B}J\underline{C} \\ B_L &= \bar{B} + \underline{B}J\underline{D}_{21} \\ C_L &= \bar{C} + \underline{D}_{12}J\underline{C} \\ D_L &= D_{11} + \underline{D}_{21}J\underline{D}_{21} \end{aligned}$$

Now we can design for a controller $K(s)$ such that the closed loop is contractive and internally stable [6].

Lemma 2.1 *[6] Suppose*
(a) P_{X_L}, H_{X_L} and Q are matrices and that H_{X_L} is symmetric;
(b) The matrices W_P and W_Q are full rank matrices satisfying $ImW_P = kerP_{X_L}$ and $ImW_Q = kerQ$, where ImW_P and ImW_Q are the image spaces of W_P and W_Q, $kerP_{X_L}$ and $kerQ$ are the kernel spaces of P_{X_L} and Q.
Then there exists a matrix J such that

$$H_{X_L} + P_{X_L}^T J^T Q + Q^T J P_{X_L} < 0 \tag{2.5}$$

if and only if, the inequalities $W_P^T H_{X_L} W_P < 0$ and $W_Q^T H_{X_L} W_Q < 0$ both hold.

Lemma 2.2 *[6] Suppose $M_L(s) = C_L(Is - A_L)^{-1}B_L + D_L$. Then the followings are equivalent conditions.*
(a) The matrix A_L is Hurwitz and $||M_L(s)||_\infty < \gamma$, where $\gamma > 0$;
(b) There exists a symmetric positive definite matrix X_L such that

$$\begin{bmatrix} A_L^T X_L + X_L A_L & X_L B_L & C_L^T \\ * & -\gamma I & D_L^T \\ * & * & -\gamma I \end{bmatrix} < 0 \tag{2.6}$$

Now define the matrices

$$P_{X_L} = [\underline{B}^T \;\; 0 \;\; \underline{D}_{12}^T] \tag{2.7}$$

$$Q = [\underline{C} \;\; \underline{D}_{21} \;\; 0] \tag{2.8}$$

and further

$$H_{X_L} = \begin{bmatrix} \bar{A}^T X_L + X_L \bar{A} & X_L \bar{B} & \bar{C}^T \\ * & -\gamma I & D_{11}^T \\ * & * & -\gamma I \end{bmatrix} \tag{2.9}$$

It follows that the inequality in (2.6) is exactly

$$H_{X_L} + Q^T J^T P_{X_L} + P_{X_L} J Q < 0 \tag{2.10}$$

By using the lemma above, There exists a matrix J if and only if $W_P^T H_{X_L} W_P < 0$ and $W_Q^T H_{X_L} W_Q < 0$ both hold.

Lemma 2.3 *[6] Suppose X and Y are symmetric, positive definite matrices in $R^{n\times n}$; and n_K is a positive integer. Then there exists matrices $X_2, Y_2 \in R_{n\times n_K}$ and symmetric matrices $X_3, Y_3 \in R^{n_K\times n_K}$, satisfying*

$$\begin{bmatrix} X & X_2 \\ X_2^T & X_3 \end{bmatrix} > 0 \tag{2.11}$$

and

$$\begin{bmatrix} X & X_2 \\ X_2^T & X_3 \end{bmatrix}^{-1} = \begin{bmatrix} Y & Y_2 \\ Y_2^T & Y_3 \end{bmatrix} \tag{2.12}$$

if and only if

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \text{ and } rank \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \leq n + n_K \tag{2.13}$$

Theorem 2.1 *[6] There exists a $K(s)$ if and only if there exists symmetric matrices $X > 0$ and $Y > 0$ such that*
(a)

$$\begin{bmatrix} N_X & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A^TX + XA & XB_1 & C_1^T \\ * & -\gamma I & D_{11}^T \\ * & * & \gamma I \end{bmatrix} \begin{bmatrix} N_X & 0 \\ 0 & I \end{bmatrix} < 0 \tag{2.14}$$

(b)

$$\begin{bmatrix} N_Y & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} YA^T + AY & YC_1^T & B_1 \\ * & \gamma I & D_{11} \\ * & * & -\gamma I \end{bmatrix} \begin{bmatrix} N_Y & 0 \\ 0 & I \end{bmatrix} < 0 \tag{2.15}$$

(c)

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0, \tag{2.16}$$

where N_X and N_Y are full-rank matrices whose images satisfy

$$ImN_X = ker[C_2 \quad D_{21}] \tag{2.17}$$
$$ImN_Y = ker[B_2^T \quad D_{12}^T] \tag{2.18}$$

We now outline this procedure. Suppose X and Y have been found satisfying Theorem 2.1. Note that Y always has its inverse Y^{-1} since Y is a symmetric matrix. Then by Lemma 2.1 we can construct the matrix $X_L = \begin{bmatrix} X & X_2^T \\ X_2 & I \end{bmatrix}$ satisfying

$$X - Y^{-1} = X_2X_2^T \tag{2.19}$$

where the order n_K should be smaller than n. By Lemma 2.2 we know that there exists a solution to

$$H_{X_L} + Q^TJ^TP_{X_L} + P_{X_L}JQ < 0 \tag{2.20}$$

and that any such $J = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix}$ provides the state space realization for a feasible controller $K(s)$.

2.5 Conclusion

The chapter reviews the typical model-based fault diagnosis technique at the beginning, discusses the different types of faults that might occur in the system; then propose a integrated H_∞ filtering based fault diagnosis framework, which is the most significant contribution in this thesis. The framework can be used to detect, estimate and compensate for the faults simultaneously. At the end of the chapter, we provide a brief review of classical H_∞ synthesis, which will be applied in the following chapters.

Chapter 3

H_∞ Filtering Based Fault Diagnosis and Fault Tolerant Control for Linear Systems

In this chapter, we study the H_∞ filtering based fault diagnosis problem and fault tolerant control for linear systems. We introduce a new dynamic framework which can detect, estimate and compensate for the faults simultaneously. In the linear system model, both faults and disturbances are included. We can obtain the residual signals after we properly filter the disturbances with H_∞/H_- fault detection filter, to generate the "cleaner" residual signals (i.e. those unwanted disturbances are attenuated in the residual signals). It is observed that for linear systems subject to additive faults, e.g. actuator and sensor biases, and oscillatory cases, the residual signals will manifest similar signal profiles as the faults. This feature can be used to not only detect the faults but also estimate the sizes of the faults. In this work, instead of using the output, we filter the residual signals for fault estimation. Then a fault tolerant controller can been designed based on the estimated faults. Finally, we present a quadrotor control example which shows the effectiveness of the method.

3.1 System Description

Consider the following linear time-invariant system.

$$\Sigma_f : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_f f(t) + B_d d(t) \\ y(t) = Cx(t) + D_f f(t) + D_d d(t) \end{cases} \tag{3.1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^p$ the control input vector, $y(t) \in R^q$ the output vector, $d(t) \in R^m$ the disturbance vector; and $f(t) \in R^l$ is an unknown vector that represents possible faults and will equal to zero when no faults exist in the system. Without loss of generality, we assume that $d(t), f(t)$ are L_2-norm bounded in the paper. $A, B, C, B_d, B_f, D_f, D_d$ are known matrices with appropriated dimensions. Assume that (A, B) is stabilizable, (A, C) is detectable and (A, B_d) is controllable.

3.2 H_∞/H_- Fault Detection Filter Design

In this section, we design an observer-based fault detection filter (FDF). Since (A, C) is assumed to be detectable, an observer is guaranteed to exist. Given the following Luenberger observer,

$$\begin{aligned}\dot{\hat{x}}(t) =& A\hat{x}(t) + Bu(t) + H(y(t) - \hat{y}(t))\\ \hat{y}(t) =& C\hat{x}(t)\\ r(t) =& y(t) - \hat{y}(t)\end{aligned} \tag{3.2}$$

where the observer gain H should be chosen to guarantee the stability of the observer. $\hat{x}(t) \in R^n$ and $\hat{y}(t) \in R^p$ represent the state and the output estimation vectors, respectively. $r(t)$ is the output residual signal generated by the observer. We can define $e(t) = x(t) - \hat{x}(t)$, then the error dynamics of states can be described as:

$$\begin{aligned}\dot{e}(t) =& (A - HC)e(t) + (B_f - HD_f)f(t)\\ &+ (B_d - HD_d)d(t)\\ r(t) =& Ce(t) + D_f f(t) + D_d d(t)\end{aligned} \tag{3.3}$$

The main objective of this work is to design observer (3.2) such that the following conditions are satisfied:

1. $(A - HC)$ is Hurwitz;
2. $||G_{rd}(j\omega)||_\infty := \sigma_{max}(G_{rd}(j\omega)) < \gamma$
3. $||G_{rf}(j\omega)||_- := \sigma_{min}(G_{rf}(j\omega)) > \beta$

where

$$G_{rd}(s) = C(sI - A + HC)^{-1}(B_d - HD_d) + D_d \tag{3.4}$$

$$G_{rf}(s) = C(sI - A + HC)^{-1}(B_f - HD_f) + D_f \tag{3.5}$$

Conditions 1) and 2) refer to the general requirements for H_∞ estimation, where condition 2) represents the worst-case criterion for the effect of disturbances on the residual signal r. Condition 3) stands for the worst-case criterion for the sensitivity of $r(t)$ to $f(t)$. Therefore, these three conditions represent the most important performance of the fault detection filter [24]. Then the problem can be depicted as: given $\gamma > 0$, the observer gain H can be determined by solving the following optimization problem:

$$\max_{H, Y>0, Y_1 = Y_1^T} \beta \text{ subject to} \tag{3.6}$$

$$\begin{bmatrix} \Phi & Y(B_d - HD_d) & C^T \\ * & -\gamma I & D_d^T \\ * & * & -\gamma I \end{bmatrix} < 0$$

$$\begin{bmatrix} D_f^T D_f - \beta^2 & (B_f - HD_f)^T Y_1 + D_f^T C \\ * & \Gamma \end{bmatrix} > 0$$

where $\Phi = (A - HC)^T Y + Y^T (A - HC)$ and $\Gamma = Y_1(A - HC) + (A - HC)^T Y_1 + C^T C$.

It is important to highlight the work by Wang and Yang [30], in which the H_∞/H_- design considerably increasing the design efficiency in comparison with the standard robust fault detection design approaches given in [5]. The detailed derivations of the above problem are omitted here.

By solving this optimization problem, we can obtain the available H to ensure the performance of fault detection filter. Then the so-called residual evaluation function $||r||_{2,T}$ is determined by

$$||r||_{2,T} = \left[\int_{t_1}^{t_2} r^T(t) r(t) dt \right]^{\frac{1}{2}}, T = t_2 - t_1 \tag{3.7}$$

$t \in (t_1, t_2]$ is the finite-time window and the adaptive threshold $J_{th} = \underset{t>0}{sup}||r(t)||$ is calculated by the method from M.Zhong's paper [39]. When $||r||_{2,T} > J_{th}$, the system will alarm for the faults. Since an evaluation of residual signals over the whole time range is impractical, it is desired that the faults will be detected as early as possible [39].

3.3 H_∞ Fault estimation

From (3.4), we can obtain the transfer matrix $G_{rf}(s)$. First we define $A_r = A - HC$, $B_r = B_f - HD_f$, $C_r = C$ and $D_r = D_f$. Note that all these variables can be calculated after obtaining H by solving the optimization problem in (3.6) for the fault detection filter.

Then the transfer matrix above can be rewritten as:

$$G_{rf}(s) = C_r(sI - A_r)^{-1} B_r + D_r \tag{3.8}$$

$$= \left[\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array} \right] \tag{3.9}$$

Therefore, the residual signal can be obtained as $r(s) = (C_r(sI - A_r)^{-1} B_r + D_r) f(s)$. To measure the accuracy of fault estimation, we introduce $\tilde{f}(t) = \hat{f}(t) - f(t)$ to evaluate the estimation quality. Since the residual signal $r(t)$ is generated from the output $y(t)$ and the observer-filtered output $\hat{y}(t)$, it contains certain information about faults, which leads us to design a dynamic estimation law $\hat{f}(t) = K_{est}(s) r(t)$ to estimate the faults directly by the residual signal. We call $\hat{f}(t)$ the fault estimate if the estimation error $\tilde{f}(t)$ converges to zero. The formulation of fault estimation is illustrated in the Figure 3.1.

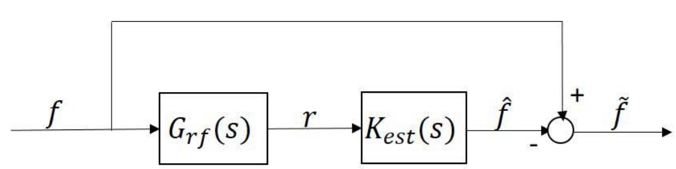

Figure 3.1: Formulation of fault estimation

The system illustrated by Figure 3.1 can be reformulated into a standard H_∞ control problem as given in Figure 3.2 [24].

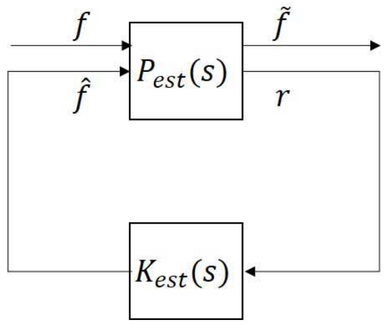

Figure 3.2: Standard setup

$$\begin{bmatrix} \tilde{f} \\ r \end{bmatrix} = P_{est}(s) \begin{bmatrix} f \\ \hat{f} \end{bmatrix} \tag{3.10}$$

The transfer matrix $P_{est}(s)$ is given as follows.

$$P_{est}(s) = \begin{bmatrix} I & -I \\ G_{rf}(s) & 0 \end{bmatrix} = \left[\begin{array}{c|cc} A_r & B_r & 0 \\ \hline 0 & I & -I \\ C_r & D_r & 0 \end{array}\right] \tag{3.11}$$

The transfer matrix for this H_∞ estimation problem is described by

$$T_{\tilde{f}f}(s) = I - K_{est}(s)G_{rf}(s) \tag{3.12}$$

Then the H_∞ fault estimation problem is transferred to finding an admissible $K_{est}(s) \in RH_\infty$ such that

$$||T_{\tilde{f}f}(s)||_\infty < \gamma_k \tag{3.13}$$

where γ_k is a given positive constant. Note that A_r is Hurwitz since we have designed a proper observer gain H to guarantee $A_r = A - HC$ is Hurwitz. Thus the fault estimation problem has been reformulated into a general H_∞ control problem, which could be solved by the standard method.

Theorem 3.1 *[5] There exists a H_∞ filtering fault estimator $K_{est}(s)$ for the system (3.1), if and only if there exists symmetric matrices $X > 0$ and $Y > 0$ such that*

$$\begin{bmatrix} N_X & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A_r^T X + XA_r & XB_r & 0 \\ * & -\gamma_k I & I \\ * & * & -\gamma_k I \end{bmatrix} \begin{bmatrix} N_X & 0 \\ 0 & I \end{bmatrix} < 0 \tag{3.14}$$

$$\begin{bmatrix} N_Y & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} A_r Y + YA_r & 0 & B_r \\ * & -\gamma_k I & I \\ * & * & -\gamma_k I \end{bmatrix} \begin{bmatrix} N_Y & 0 \\ 0 & I \end{bmatrix} < 0 \tag{3.15}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \tag{3.16}$$

where N_X and N_Y are full-rank matrices whose images satisfy

$$ImN_X = ker[C_r \quad D_r] \tag{3.17}$$

$$ImN_Y = ker[0 \quad -I] \tag{3.18}$$

Proof. It can be directly proven by Theorem 2.1. ■

By Lemma 2.3, define a X_2, which is calculated by $X - Y^{-1} = X_2X_2^T$. Combine these two state space realizations into one which describes the mapping from f to $\tilde{f}$. We obtain

$$S(P_{est}(s), K_{est}(s)) = \left[\begin{array}{c|c} A_L & B_L \\ \hline C_L & D_L \end{array}\right] = \left[\begin{array}{cc|c} A_r & 0 & B_r \\ B_KC_r & A_K & B_KD_r \\ \hline D_KC_r & C_K & -I + D_KD_r \end{array}\right] \tag{3.19}$$

Now define the matrix

$$J = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \tag{3.20}$$

which collects the matrices for $K_{est}(s)$ into one matrix. We can parametrize the closed-loop relation in terms of the controller realization as follows. First we provide the following definitions.

$$\bar{A} = \begin{bmatrix} A_r & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_r \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \quad \underline{C} = \begin{bmatrix} 0 & I \\ C_r & 0 \end{bmatrix}$$
$$\underline{B} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, \quad \underline{D}_{12} = \begin{bmatrix} 0 & I \end{bmatrix}, \quad \underline{D}_{21} = \begin{bmatrix} 0 \\ D_r \end{bmatrix}$$

which are all given in terms of the state space matrices of $P_{est}(s)$. Then we have

$$\begin{aligned} A_L &= \bar{A} + \underline{B}J\underline{C} \\ B_L &= \bar{B} + \underline{B}J\underline{D}_{21} \\ C_L &= \underline{D}_{12}J\underline{C} \\ D_L &= D_{11} + \underline{D}_{21}J\underline{D}_{21} \end{aligned}$$

Now we look for a estimator $K_{est}(s)$ such that the closed loop is contractive and internally stable.

Lemma 3.1 *[6] Suppose $M_L(s) = C_L(Is - A_L)^{-1}B_L + D_L$. Then the followings are equivalent conditions.*
(a) The matrix A_L is Hurwitz and $||M_L(s)||_\infty < \gamma_k$;
(b) There exists a symmetric positive definite matrix X_L such that

$$\begin{bmatrix} A_L^TX_L + X_LA_L & X_LB_L & C_L^T \\ * & -\gamma_kI & D_L^T \\ * & * & -\gamma_kI \end{bmatrix} < 0 \tag{3.21}$$

Now we are ready to present the main theorem for the fault estimation design.

Theorem 3.2 *For the system (3.1), $K_{est}(s) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$ is a H_∞ filtering based fault estimator if there exist symmetric matrices A_K, B_K, C_K and D_K such that*

$$\begin{bmatrix} \Pi & A_r X_2^T + C_r B_K^T + X_2 A_K & X B_r + X_2 B_r D_r & -C_r D_K^T \\ * & A_K^T + A_K & X_2 B_r + B_K D_r & -C_K^T \\ * & * & -\gamma_k I & I - D_r D_K^T \\ * & * & * & -\gamma_k I \end{bmatrix} < 0 \tag{3.22}$$

where $\Pi = A_r^T X + X A_r + C_r B_K^T X_2 + X_2 B_K C_r$, X and X_2 satisfy with $X - Y^{-1} = X_2 X_2^T$.

Proof. By substituting the parameter in A_L, B_L, C_L and D_L, we can obtain:

$$A_L = \begin{bmatrix} A_r & 0 \\ B_K C_r & A_K \end{bmatrix}, \quad B_L = \begin{bmatrix} B_r \\ B_K D_r \end{bmatrix}$$
$$C_L = \begin{bmatrix} -D_K C_r & -C_K \end{bmatrix}, \quad D_L = \begin{bmatrix} I - D_K D_r \end{bmatrix}$$

Define the matrices

$$P_{X_L} = [\underline{B}^T \;\; 0 \;\; \underline{D}_{12}^T] \tag{3.23}$$
$$Q = [\underline{C} \;\; \underline{D}_{21} \;\; 0] \tag{3.24}$$

and further

$$H_{X_L} = \begin{bmatrix} \bar{A}^T X_L + X_L \bar{A} & X_L \bar{B} & \bar{C}^T \\ * & -\gamma_k I & D_{11}^T \\ * & * & -\gamma_k I \end{bmatrix} \tag{3.25}$$

It follows that the inequality in Lemma 3.1 is exactly

$$H_{X_L} + Q^T J^T P_{X_L} + P_{X_L} J Q < 0 \tag{3.26}$$

By using the Lemma 3.1 above, there exists a matrix J satisfying (3.26) if and only if $W_P^T H_{X_L} W_P < 0$ and $W_Q^T H_{X_L} W_Q < 0$ both hold, where

$$P_{X_L} = \begin{bmatrix} X_2 & I & 0 & 0 \\ 0 & 0 & 0 & -I \end{bmatrix} \tag{3.27}$$

$$Q = \begin{bmatrix} 0 & I & 0 & 0 \\ C_r & 0 & D_r & 0 \end{bmatrix} \tag{3.28}$$

$$H_{X_L} = \begin{bmatrix} A_r^T X + X A_r & A_r X_2^T & X B_r & 0 \\ * & 0 & X_2 B_r & 0 \\ * & * & -\gamma_k I & I \\ * & * & * & -\gamma_k I \end{bmatrix} \tag{3.29}$$

Replacing the matrices H_{X_L}, P_{X_L} and Q in the inequality, we can easily proof the theorem. ■

3.4 Output Feedback Fault Tolerant Controller Design

In this section, firstly, a controller $K_c(s)$ is designed for the fault-free model. Then an control compensation law V is determined, which is to be added to the normal control law in order to reduce the faults' effects on the system.

3.4.1 Fault-Free System

The LTI model (3.1) can be rewritten as the following equations when there is no fault occur in the system.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_d d(t) + Bu(t) \\ z(t) &= x(t) \\ y(t) &= Cx(t) + D_d d(t) \end{aligned} \tag{3.30}$$

where $z(t)$ is the controlled signal. So that the transfer matrix $P_c(s)$ can be expressed as:

$$P_c(s) = \left[\begin{array}{c|cc} A & B_d & B \\ \hline I & 0 & 0 \\ C & D_d & 0 \end{array}\right] \tag{3.31}$$

The following assumptions have been made in the previous section, which are listed below:
(1) (A, B) is stabilizable and (C, A) is detectable.
(2) (A, B_d) is controllable

Define the following two Hamiltonian matrices associated with the model above:

$$N_\infty = \begin{bmatrix} A & \gamma_N^{-2} B_d B_d^T - BB^T \\ -I & -A^T \end{bmatrix}, \quad J_\infty = \begin{bmatrix} A^T & \gamma_N^{-2} I - C^T C \\ -B_d B_d^T & -A \end{bmatrix} \tag{3.32}$$

where $\gamma_N > 0$ is given.

Theorem 3.3 *[40] There exists an admissible controller such that $||T_{zw}||_\infty < \gamma_N$ if and only if the following three conditions hold:*
(i) $H_\infty \in dom(Ric)$ and $X_\infty := Ric(H_\infty) > 0$
(ii) $J_\infty \in dom(Ric)$ and $Y_\infty := Ric(J_\infty) > 0$
(iii) $\rho(X_\infty Y_\infty) < \gamma_N^2$
where $dom(*)$ is domain of Riccati, $Ric(*)$ refers to the stabilizing solution of an ARE. Moreover, when these conditions hold, one such controller is

$$K_c(s) := \left[\begin{array}{c|c} \hat{A}_\infty & (I - \gamma_N^{-2} Y_\infty X_\infty)^{-1} Y_\infty C^T \\ \hline -B^T X_\infty & 0 \end{array}\right] \tag{3.33}$$

where $\hat{A}_\infty := A + (\gamma_N^{-2} B_d B_d^T - BB^T) X_\infty + (I - \gamma_N^{-2} Y_\infty X_\infty)^{-1} Y_\infty C^T C$.
Furthermore, the set of all admissible controllers such that $||T_{zd}||_\infty < \gamma_N$ equals the set of all transfer matrices from y to u in where $Q \in RH_\infty$, $||Q||_\infty < \gamma_N$.

$$M_\infty(s) = \left[\begin{array}{c|cc} \hat{A}_\infty & (I - \gamma^{-2} Y_\infty X_\infty)^{-1} Y_\infty C^T & (I - \gamma^{-2} Y_\infty X_\infty)^{-1} B \\ \hline -B^T X_\infty & 0 & I \\ -C & I & 0 \end{array}\right] \tag{3.34}$$

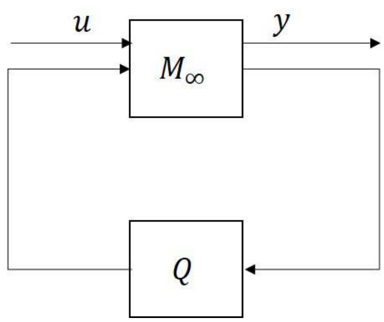

Figure 3.3: Standard setup

By using the theorem, we can obtain a controller $K_c(s)$ which can guarantee the stability of the system with disturbances. Suppose that the minimal state space realization of dynamic controller $K_c(s)$ is given by (3.35).

$$\begin{aligned} \dot{x}_k(t) &= A_k x_k(t) + B_k y(t) \\ u_N(t) &= C_k x_k(t) \end{aligned} \tag{3.35}$$

where $A_k = \hat{A}_\infty$, $B_k = (I - \gamma_N^{-2} Y_\infty X_\infty)^{-1} Y_\infty C^T$, $C_k = -B^T X_\infty$.

3.4.2 Fault Compensation

In this subsection, a fault compensation control law $u_{ad}(t)$ will be designed. By adding $u_{ad}(t)$ to the normal control law $u_N(t) = K_c y(t)$, we aim to reduce the impact of the faults on the system, which leads to

$$\begin{aligned} u(t) &= u_N(t) + u_{ad}(t) \\ &= K_c y(t) + V\hat{f}(t) \end{aligned} \tag{3.36}$$

The compensation control law, $u_{ad}(t) = 0$ in the normal case and $u_{ad}(t) \neq 0$ in the faulty cases. Since a robust control law $K_c(s)$ is designed to handle the disturbances of the system, hereby we focus on compensating for the faults by ignoring $d(t)$ in the following analysis. Replace the fault estimate $\hat{f}(t)$ by $f(t) - \tilde{f}(t)$ and use (3.35), then the closed-loop system can be written as:

$$\Sigma_{cl} : \begin{cases} \dot{x}(t) &= Ax(t) + BC_K x_k(t) + (BV + B_f)f(t) - BV\tilde{f}(t) \\ \dot{x}_k(t) &= A_K x_k(t) + B_K C x(t) + B_K D_f f(t) \\ y(t) &= Cx(t) + D_f f(t) \end{cases} \tag{3.37}$$

Define $\xi(t) = [x(t) \; x_k(t)]^T$ and $\eta(t) = [f(t) \; \tilde{f}(t)]^T$, we can rewrite the augmented state space model as follows,

$$\begin{aligned} \dot{\xi}(t) &= A_c \xi(t) + B_c \eta(t) \\ y(t) &= C_c \xi(t) + D_c \eta(t) \end{aligned} \tag{3.38}$$

where,

$$A_c = \begin{bmatrix} A & -BC_k \\ B_kC & A_k \end{bmatrix}$$
$$B_c = \begin{bmatrix} BV + B_f & -BV \\ B_kD_f & 0 \end{bmatrix}$$
$$C_c = \begin{bmatrix} C & 0 \end{bmatrix}, \quad D_c = \begin{bmatrix} D_f & 0 \end{bmatrix}$$

It is clear that the fault compensation problem can also be handled in a H_∞ control framework, for which the objective is to minimize $||G_{y\eta}(s)||_\infty$ to guarantee that faults $f(t)$ and estimation errors $\tilde{f}(t)$ cause minimal negative influence to the system.

Theorem 3.4 *Given $\gamma_c > 0$, then $||G_{y\eta}(s)||_\infty < \gamma_c$ if there exists symmetric matrices $P_1 > 0$, $P_2 > 0$, and a matrix V, such that the following matrix inequality holds:*

$$\begin{bmatrix} AP_1 + P_1A^T & P_1C^TB_k^T + BC_kP_2 & BV + B_f & -BV & P_1C^T \\ * & A_kP_2 + P_2A_k & B_kD_f & 0 & 0 \\ * & * & -\gamma I & 0 & D_f^T \\ * & * & * & -\gamma I & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{3.39}$$

Proof. By applying the Lemma 3.1 and substituting the parameters A_L, B_L, C_L, D_L by A_c, B_c, C_c, D_c, the theorem is proven. ■

3.5 Simulation

In this section, we apply the proposed scheme to a quadrotor system example given in [3] to verify its effectiveness. The quadrotor helicopter platform is driven by four dc-motors and has three encoders to measure the yaw, pitch and roll angles. Each propeller generates a lift force to control the pitch and roll axes. The total torque generated by the propellers causes the body to move around the yaw axis. The voltage signals going to the motors and the pitch and yaw encoder signals are transmitted to the controller. For this system, several assumptions are given in order to simplify the modeling process without loss of generality [3][11].

(1) The structure is supposed to be rigid and strictly symmetrical.

(2) The center of mass and the body fixed frame origin are assumed to coincide.

(3) The moment is proportional to the dc-motor voltage.

(4) The change of attitude angle range is limited to $(-10°, 10°)$.

(5) The air resistance can be ignored at low speed.

Remark 3.1 *The above assumptions are made to eliminate the nonlinear factors and simplify the modeling. Generally, the torque for a dc-motor with respect to speed shows quadratic relationship and there are varieties of quadrotor research work assuming that the torque is proportional to the square of voltage, such as [13, 32]. But since the change of attitude angle is limited within a small range, as given in assumption (3) in the above, the torque is proportional to the dc-motor voltage. We adopt the linear quadrotor model to show the effectiveness of the proposed fault estimation and fault tolerant control scheme.*

Based on the assumptions, the dynamic system of the quadrotor can be described as

$$\ddot{\psi}(t) = \frac{K_f}{J_p} l(V_f(t) - V_b(t)) \tag{3.40}$$

$$\ddot{\theta}(t) = \frac{K_f}{J_r} l(V_r(t) - V_l(t)) \tag{3.41}$$

$$\ddot{\phi}(t) = \frac{K_{fc}}{J_y}(V_f(t) + V_b(t)) + \frac{K_{fn}}{J_y}(V_r(t) + V_l(t)) \tag{3.42}$$

where $\psi(t)$, $\theta(t)$, $\phi(t)$ denote the pitch, roll, and yaw angles, respectively. K_{fc}, K_{fn} are the counter rotation propeller torque-thrust constant and the normal rotation propeller torque-thrust constant, respectively; K_f is the thrust constant; l is the distance between the motors and the encoder pivot; J_p is the equivalent moment of inertia about the pitch axis; J_r is the equivalent moment of inertia about the roll axis; J_y is the equivalent moment of inertia about the yaw axis; and V_f, V_b, V_r, V_l represent the front, back, right, and left motor voltages of the helicopter system, respectively [3]. Main parameters associated with the quadrotor model are given by Table 3.1.

Table 3.1: Values of model parameters

Symbol	Value	Unit
K_{fn}	0.0036	$N.m/V$
K_{fc}	-0.0036	$N.m/V$
K_f	0.1188	N/V
l	0.197	m
J_y	0.110	$kg.m^2$
J_p	0.0552	$kg.m^2$
J_r	0.0552	$kg.m^2$

System can be rewritten as the state-space model

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{3.43}$$

where $x(t) = [\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}]^T \in R^6$ is the state vector, and $y(t) = [\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}]^T \in R^6$ is the output of the system. The control input vector $u(t)$ can be described as $[V_f, V_b, V_r, V_l]^T \in$

R^4. The system matrices of the state space model are:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{lK_f}{J_p} & -\frac{lK_f}{J_p} & 0 & 0 \\ 0 & 0 & \frac{lK_f}{J_r} & -\frac{lK_f}{J_r} \\ \frac{K_{fc}}{J_y} & \frac{K_{fc}}{J_y} & \frac{K_{fn}}{J_y} & \frac{K_{fn}}{J_y} \end{bmatrix}$$

$$C = I_{6\times 6}$$

The angles can be directly measured by encoders. The corresponding angle velocity is computed by taking the derivative of the position and filtering the result using a second-order lowpass filter. Clearly, the above model is stabilizable and observable. Then we suppose faults and disturbances may exist in the system. Define $B_f = B$, $B_d = I_6$ and $D_d = I_6$. The main parameters of the system are provided in the Table 3.1. The initial state $x_0 = [1, 0, 0, 0, 0, 0]^T$. Then the state-space model (3.43) can be generalized for the disturbances and faults as following:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_f f(t) + B_d d(t) \\ y(t) &= Cx(t) + D_d d(t) \end{aligned} \tag{3.44}$$

3.5.1 Step Faults Case

There are 4 motors in the quadrotor system, hereby we assume that the front motor has a fault, which is manifested as a step change (e.g. a bias), given as follows,

$$\begin{aligned} f_1(t) &= \begin{cases} 1, & t \geq 10 \text{ (sec)} \\ 0, & 0 \leq t < 10 \text{ (sec)} \end{cases} \\ f_2(t) &= f_3(t) = f_4(t) = 0 \end{aligned} \tag{3.45}$$

By using the above H_∞ fault estimation method, we can estimate the faults $\hat{f}(t)$, which is shown in Figure 3.4. To show the accuracy and effectiveness of the estimation law, we also provide the Figure 3.5, which represent the estimation error $\tilde{f}(t)$ between estimated faults and actual faults.

One can notice that the estimation error contains only small noise in our simulation for quadrotor model, and the estimated faults $\hat{f}(t)$ is considered to be accurate. The performance of the proposed fault estimation law is satisfactory.

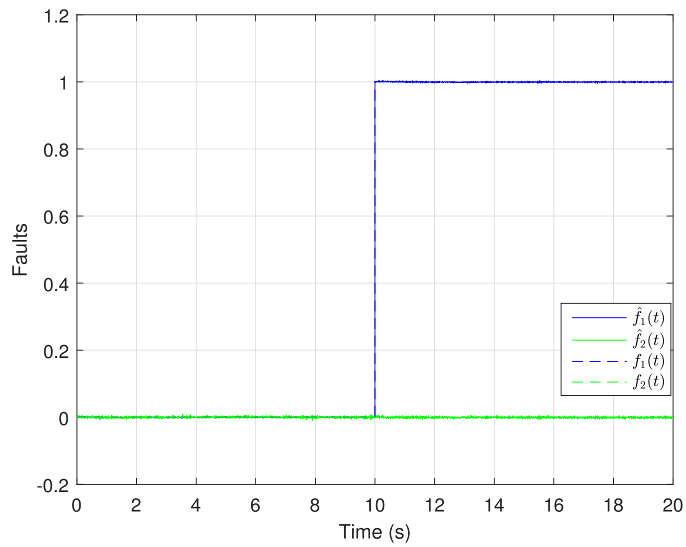

Figure 3.4: Estimation of faults $\hat{f}(t)$ and actual faults $f(t)$

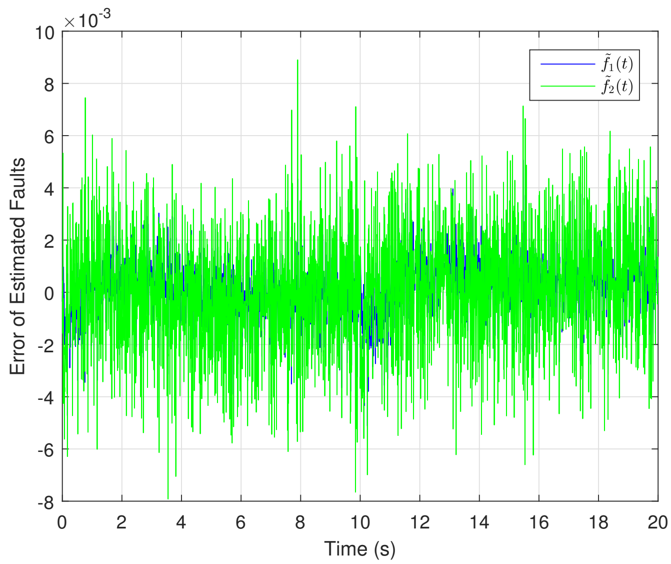

Figure 3.5: Error of estimated faults $\tilde{f}(t)$

Figure 3.6 shows the output $y(t)$ of the faulty system. From the figure, we can clearly see that the fault induced transient response at ($t = 10s$) decays to zero after a few seconds, which demonstrates that the fault tolerant controller can guarantee the stability and reduce the effects of faults.

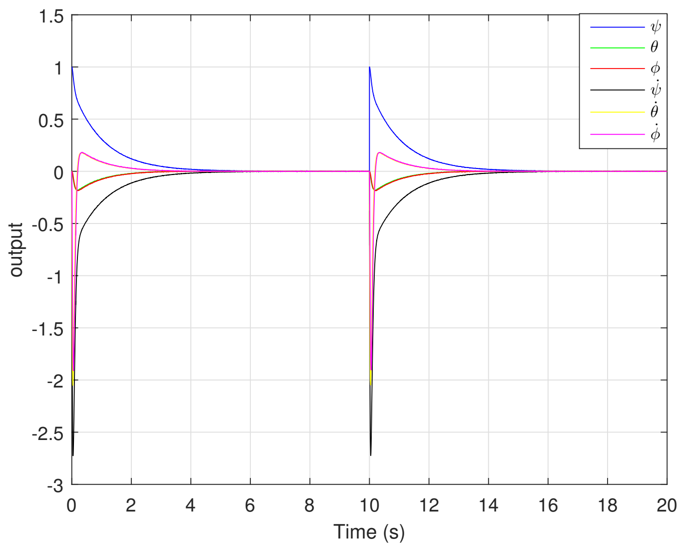

Figure 3.6: Output $y(t)$

3.5.2 Sinusoidal Faults Case

In this subsection, we simulate the sinusoidal faults as:

$$f_1(t) = \begin{cases} 1 + 0.1cos(\pi t), & t \geq 10 \text{ (sec)} \\ 0 & 0 \leq t < 10 \text{ (sec)} \end{cases}$$
$$f_2(t) = f_3(t) = f_4(t) = 0 \tag{3.46}$$

By using the above H_∞ fault estimation method, we can estimate the faults $\hat{f}(t)$, which has been showed in Figure 3.7.

One can notice that the estimated faults $\hat{f}(t)$ are accurate compared to the actual faults $f(t)$. Figure 3.8 shows the output $y(t)$ of faulty system. From the figure, we can clearly see that after the transient response for fault occurring ($t = 10s$), the system responses are recovered from the faults, which shows that the fault tolerant controller can retain the stability and reduce the effects of faults. Based on the simulation results, we conclude that the proposed fault tolerant control law is effective in handling the faults.

3.6 Conclusion

In this chapter, a H_∞ fault estimation law is studied based on the residual generation from fault detection filter. An output feedback fault tolerant controller $u(t)$ is developed based on the online estimation of faults, where the fault compensation control law is designed to reduce the fault effects on system.

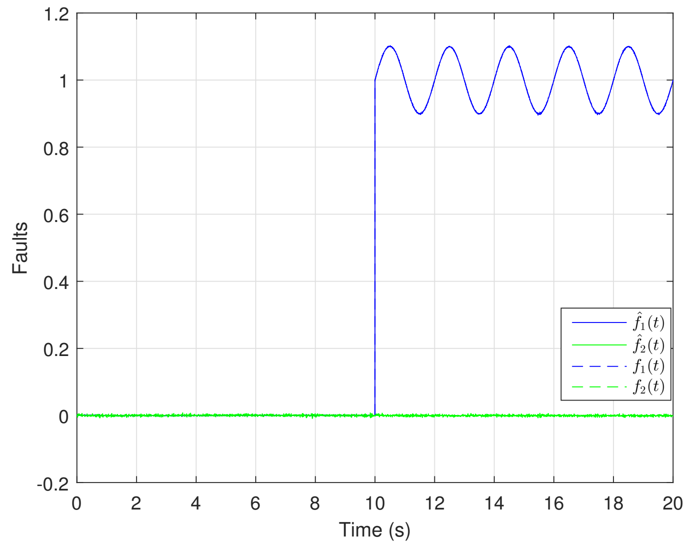

Figure 3.7: Estimation of faults $\hat{f}(t)$ and actual faults $f(t)$

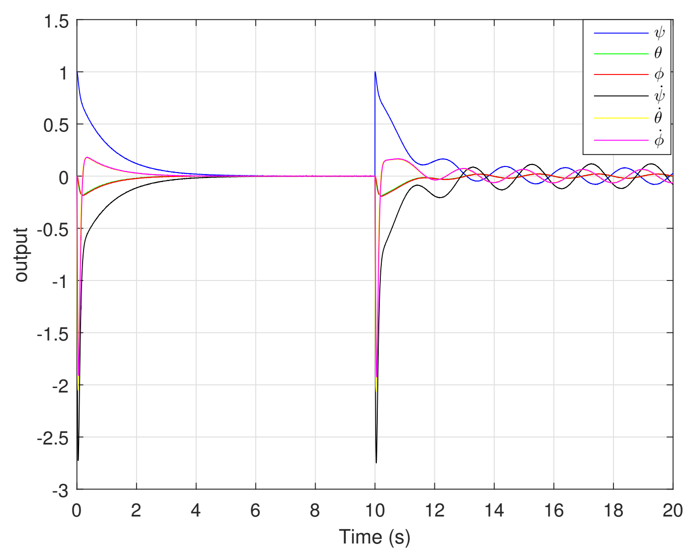

Figure 3.8: Output $y(t)$

Chapter 4

H_∞ Filtering Based Fault Diagnosis and Fault Tolerant Control for Lipschitz Nonlinear Systems

4.1 The Lipschitz Observer Design

A class of nonlinear systems that has been paid much attention in the literature is the Lipschitz nonlinear systems of the following form. Many Lyapunov-like design techniques have been proposed for such systems.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \Phi(x,t) \\ y(t) &= Cx(t), \quad A \in R^{n\times n}, \quad C \in R^{p\times n} \end{aligned} \tag{4.1}$$

where (A, B) is stabilizable, (A, C) is detectable and the function $\Phi(x, t)$ satisfies a uniform Lipschitz condition globally in x, i.e,

$$||\Phi(x_1, t) - \Phi(x_2, t)|| \leq \alpha||x_1 - x_2|| \tag{4.2}$$

$\forall u \in R^m$, $t \in R$ and $\forall x_1(t), x_2(t) \in R^n$. Here $\alpha \in R^+$ is referred to as the Lipschitz constant and is independent of x and t. Lipschitz systems constitute a very important class. Any nonlinear system $\dot{x} = f(x, u, t)$ can be expressed in the form of (4.1) if $f(x, u, t)$ is continuously differentiable with respect to x. Many nonlinear systems are at least locally Lipschitz, such as trigonometric nonlinearities occuring in robotics, nonlinearities which are square or cubic in nature, etc. The function $\Phi(x, t)$ can also be considered as a perturbation affecting the system [26].

In existing results on observer design for systems of the form (4.1)-(4.2), the observer described as the class of Luenberger-like observers as following,

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + \Phi(\hat{x}, t) + H(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{4.3}$$

The observer error dynamics are then given by

$$\begin{aligned}\dot{e}(t) &= (A - HC)e(t) + \Phi(x,t) - \Phi(\hat{x},t) \\ r(t) &= Ce(t)\end{aligned} \tag{4.4}$$

where $e(t) = x(t) - \hat{x}(t)$.

Lemma 4.1 *[25] If a gain H is chosen s.t $\alpha < \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}$ and it satisfies the Lyapunov equation $(A - HC)^T P + P(A - HC) = -Q$, the estimation error in (4.4) is asymptotically stable.*

Lemma 4.1 provides a very important sufficient condition for the existence of an observer, but does not provide insights for the design problem. Raghavan proposed an algorithm based on the following lemma,

Lemma 4.2 *[27] If there exists an $\epsilon > 0$ such that the Algebraic Riccati Equation (ARE) in (4.5) has a symmetric positive definite solution P, then the observer gain $H = \frac{1}{2\epsilon}PC^T$ stabilizes the error dynamics in (4.4) for all $\Phi(x,t)$ with a Lipschitz constant α.*

$$AP + PA^T + P(\alpha^2 I - \frac{1}{\epsilon}C^T C)P + I + \epsilon I = 0 \tag{4.5}$$

Based on this result, Raghavan proposed an iterative binary search procedure over ϵ, to obtain the observer gain. However, given a particular system of the form (4.1) with a specific Lipschitz constant α, this procedure may fail even if the pair of matrices (A, C) is observable. Moreover, Lemma 4.2 provides no insight into what conditions the matrix $(A - HC)$ must satisfy to ensure observer stability. The answer to this puzzle was provided by Rajamani in the following lemma,

Lemma 4.3 *[28] The observer gain H stabilizes the error dynamics in (4.4) for all Φ with a Lipschitz constant α if H is chosen so as to ensure that $(A - HC)$ is stable and such that*

$$\min_{\omega \in R^+} \sigma_{min}(A - HC - j\omega I) > \alpha \tag{4.6}$$

The detailed proof can be found in the Rajamani's paper [28]. Basically, the proof of this theorem is done in the following three parts, Part 1: If $\min_{\omega \in R^+} \sigma_{min}(A - HC - j\omega I) > \alpha$, then there exists $\epsilon > 0$ such that the Hamiltonian matrix

$$\bar{H} = \begin{bmatrix} A - HC & \alpha^2 I \\ -I - \epsilon I & -(A - HC)^T \end{bmatrix} \tag{4.7}$$

has no imaginary axis eigenvalues.
Part 2: If the matrix $\bar{H}$ has no imaginary axis eigenvalues and if $(A - HC)$ is stable, then there exists a symmetric positive definite solution $P = P^T$ to the Riccati equation

$$(A - HC)^T P + P(A - HC) + \alpha^2 PP + I + \epsilon I = 0 \tag{4.8}$$

Part 3: The existence of a positive definite matrix P satisfying (4.8) ensures that the observer (4.3) for the system (4.1) is asymtotically stable.

From the design perspective, it can be related to the H_∞ theory by rewriting (4.5) as:

$$||[sI - (A - HC)]^{-1}||_\infty < \frac{1}{\alpha} \tag{4.9}$$

where the left side of (4.9) is equivalent to the H_∞ norm of the transfer function $T_{z\Delta\Phi}(s)$ between z and $\Delta\Phi$ in the following standard form:

$$\begin{aligned} \dot{e}(t) &= (A - HC)e(t) + \Delta\Phi \\ z(t) &= e(t) \end{aligned} \tag{4.10}$$

where $\Delta\Phi = \Phi(x,t) - \Phi(\hat{x},t)$. Then if we can find a feasible H to satisfy $||T_{z\Delta\Phi}(s)||_\infty < \frac{1}{\alpha}$, the static observer gain H will satisfy the Lemma 4.3. By using the bounded real lemma, we have the following theorem,

Theorem 4.1 *[28] The followings are equivalent conditions.*
(a) The matrix $A - HC$ is Hurwitz and $||T_{z\Delta\Phi}(s)||_\infty < \frac{1}{\alpha}$;
(b) There exists a symmetric positive definite matrix X such that

$$\begin{bmatrix} (A - HC)^T X + X(A - HC) & X & I \\ * & -\frac{1}{\alpha}I & 0 \\ * & * & -\frac{1}{\alpha}I \end{bmatrix} < 0 \tag{4.11}$$

The theorem is directly reached by applying bounded real lemma, so the proof is omitted here. Therefore, we can design a static observer gain H by Theorem 4.1.

4.2 Fault Detection

Consider a nonlinear system with actuator faults as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \Phi(x,t) + B_f f(t) \\ y(t) &= Cx(t) + D_f f(t) \end{aligned} \tag{4.12}$$

where $x(t) \in R^n$ is the state vector; $u(t) \in R^m$ and $y(t) \in R^p$ are, respectively, the control input and output vectors; $f(t) \in R^k$ is the unknown fault vector; A, B, B_f, C, D_f are constant real matrices of appropriate dimensions; the function $\Phi(x,t)$ satisfies a uniform Lipschitz condition globally in x,

$$||\Phi(x_1,t) - \Phi(x_2,t)|| \leq \alpha||x_1 - x_2|| \tag{4.13}$$

$\forall u(t) \in R^m$, $t \in R$ and $\forall x_1(t), x_2(t) \in R^n$. Here $\alpha \in R^+$ is referred to as the Lipschitz constant and is independent of x and t. Note that the proposed approach to fault estimation could address a control-dependent nonlinearity $\Phi(x,t)$, but fault accommodation would be much more complex. Finally, although they could be taken into consideration, uncertainties and unknown inputs are not introduced since they call for robust adaptive techniques.

Faults are described by the vector $f(t) \in R^q$, assumed to be zero prior to the failure time, non-zero and differentiable after the fault occurrence (note that continuity at the fault occurrence time is not required). It is assumed that the fault vector and its time derivative are bounded. This assumption is quite general in the literature. It typically says that FDI and FTC are designed for situations where the system is not "exploding", which is not really a restriction. Note that as a consequence, the Lipschitz condition is indeed satisfied in the bounded region of the state space that is practically considered. At the case, general faults are assumed, where no assumptions are made on the time-domain or frequency-domain properties of the faults.

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + \Phi(\hat{x}, t) + H(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{4.14}$$

The observer in (4.14) with the residual signal $r(t) = y(t) - \hat{y}(t)$ is used as the residual generator, and the object is to develop conditions on the observer gain H that guarantee robustness of fault detection. To this end, it can be seen that the residual dynamics are given by:

$$\begin{aligned} \dot{e}(t) &= (A - HC)e(t) + (B_f - HD_f)f(t) + \Phi(x, t) - \Phi(\hat{x}, t) \\ r(t) &= Ce(t) + D_f f(t) \end{aligned} \tag{4.15}$$

where $e(t) = x(t) - \hat{x}(t)$.

Definition 4.1 *Given the i_{th} fault $f_i(t)$ ($f_i(t) = 0, t < t_f$, and $f_i(t) \neq 0, t \geq t_f$) is detectable if there exists a residual generator and $t_1 > 0$, such that the produced residual signal $r(t)$ satisfies that $r(t) = 0, t < t_f$ and $r(t) \neq 0, t \geq t_f + t_1$.*

Theorem 4.2 *Given the system described in (4.12) and (4.13), the residual generator in (4.15) achieves fault detectability (according to Definition 4.1) if there exists a symmetric positive definite matrix X such that*

$$\begin{bmatrix} (A - HC)^T X + X(A - HC) & X & I \\ * & -\frac{1}{\alpha}I & 0 \\ * & * & -\frac{1}{\alpha}I \end{bmatrix} < 0 \tag{4.16}$$

Proof. Based on the Theorem 4.1, if (4.16) is satisfied, then the observer provides an accurate estimation of $x(t)$, and $\hat{y}(t) = C\hat{x}(t)$ is the accurate estimate of $y(t)$ for fault-free case. In this case, the residual signal $r(t)$ is approximately zero. In the faulty case, based on (4.15) the fault vector $f(t)$ has directly reflected in the residual signal $r(t)$. Therefore, the fault detection according the Definition 4.1 is achievable by this structure. ■

Then the residual evaluation function $||r||_{2,T}$ is determined by

$$||r||_{2,T} = \left[\int_{t_1}^{t_2} r^T(t)r(t)dt\right]^{\frac{1}{2}}, T = t_2 - t_1 \tag{4.17}$$

where $t \in (t_1, t_2]$ is the finite-time window and the adaptive threshold $J_{th} = \underset{t>0}{sup}||r(t)||$ is calculated by the method from Zhong's paper [39]. When $||r||_{2,T} > J_{th}$, the system will alarm for the faults.

4.3 H_∞ Fault Estimation

Define $A_r = A - HC$, $B_r = B_f - HD_f$, $C_r = C$, $D_r = D_f$. Since we have already got the observer gain H from Theorem 4.2, A_r and B_r are known constant matrix. Then the residual dynamics can be expressed as:

$$\begin{aligned} \dot{e}(t) &= A_r e(t) + B_r f(t) + \Delta\Phi \\ r(t) &= C_r e(t) + D_r f(t) \end{aligned} \tag{4.18}$$

where $\Delta\Phi = \Phi(x,t) - \Phi(\hat{x},t)$. By the Lipschitz nonlinear constraints (4.13), it is easy to show that $||\Delta\Phi|| \leq \alpha||e(t)||$. Note that $f = 0$ when no fault exists in the system, where the residual dynamics will be exactly same with (4.10), for which the stability has been proved by the Lemma 4.3. By defining the variables: $\tau = [\tau_1 \quad \tau_2]^T = [\Delta\Phi \quad f(t)]^T$, the error dynamics can be represented as:

$$\begin{aligned} \dot{e}(t) &= A_r e(t) + [I \quad B_r]\tau \\ r(t) &= C_r e(t) + [0 \quad D_r]\tau \end{aligned} \tag{4.19}$$

To measure the accuracy of fault estimation, we introduce $\tilde{f}(t) = f(t) - \hat{f}(t)$ to evaluate the estimation quality. Since the residual signal $r(t)$ is generated from output $y(t)$ and observer-filtered output $\hat{y}(t)$, it should contains certain information about faults, which leads us to design a dynamical estimation law $\hat{f}(t) = K_{est}(s)r(t)$ to estimate the faults directly from residual signal. Since $\tilde{f}(t) = f(t) - \hat{f}(t)$, we can say $\hat{f}(t)$ can be a fault estimator if $\tilde{f}(t)$ is bounded within a small range.

This can also be represented by Figure 4.1 where the plant $P_{est}(s)$ has the state space representation in (4.20) and where the controller $K_{est}(s)$ is the dynamic estimator of the faults.

The transfer matrix $P_{est}(s)$ can be written as:

$$P_{est}(s) = \left[\begin{array}{c|cc} A_r & \begin{bmatrix} I & B_r \end{bmatrix} & 0 \\ \hline 0 & \begin{bmatrix} 0 & -I \end{bmatrix} & I \\ C_r & \begin{bmatrix} 0 & D_r \end{bmatrix} & 0 \end{array}\right] \tag{4.20}$$

Theorem 4.3 *Given system (4.12) and (4.13), the estimated faults $\hat{f}(t)$ achieves fault estimation if $K_{est}(s)$ is chosen such that $||T_{f\tilde{f}}(s)||_\infty < \gamma$, where $\gamma > 0$ is given.*

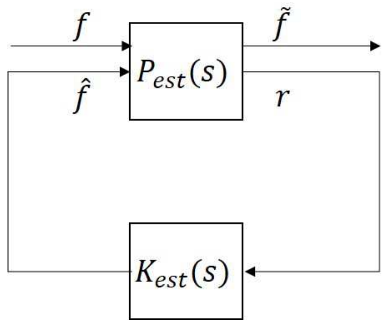

Figure 4.1: Standard setup

Proof. It is easily seen that if the H_∞ norm of the closed-loop transfer function is bounded by a given positive constant γ, then estimated error $||\tilde{f}(t)|| < \gamma||f(t)||$. Therefore, we say $\hat{f}(t)$ is an estimated faults since it can estimate the magnitude of the faults $f(t)$. ■

Define

$$\begin{aligned} A_p = A_r, \quad B_{p1} &= \begin{bmatrix} I & B_r \end{bmatrix}, \quad B_{p2} = 0, \\ C_{p1} = 0, \quad C_{p2} = C_r, \quad D_{p11} &= \begin{bmatrix} 0 & -I \end{bmatrix}, \\ D_{p12} = I, \quad D_{p21} &= \begin{bmatrix} 0 & D_r \end{bmatrix}. \end{aligned}$$

Then the transfer matrix $P_{est}(s)$ can be rewritten as:

$$P_{est}(s) = \left[\begin{array}{c|cc} A_p & B_{p1} & 0 \\ \hline 0 & D_{p11} & D_{p12} \\ C_{p2} & D_{p21} & 0 \end{array}\right] \tag{4.21}$$

Then the estimator $K_{est}(s)$ can be realized as:

$$K_{est}(s) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right] \tag{4.22}$$

Combine these two state space realizations into one which describes the map from f to $\tilde{f}$. We obtain

$$\begin{aligned} M_L(s) &= \left[\begin{array}{c|c} A_L & B_L \\ \hline C_L & D_L \end{array}\right] \\ &= \left[\begin{array}{cc|c} A_p & 0 & B_{p1} \\ B_K C_{p2} & A_K & B_K D_{p21} \\ \hline D_K C_{p2} & C_K & D_{p11} + D_K D_{p21} \end{array}\right] \end{aligned} \tag{4.23}$$

Now define the matrix

$$J = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \tag{4.24}$$

which collects the representation for $K_{est}(s)$ into one matrix. We can parametrize the closed-loop relation in terms of the controller realization as follows. First make the following definitions.

$$\bar{A} = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_{p1} \\ 0 \end{bmatrix}, \quad \bar{C} = 0, \quad \underline{C} = \begin{bmatrix} 0 & I \\ C_{p2} & 0 \end{bmatrix},$$
$$\underline{B} = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}, \quad \underline{D}_{12} = \begin{bmatrix} 0 & I \end{bmatrix}, \quad \underline{D}_{21} = \begin{bmatrix} 0 \\ D_{p21} \end{bmatrix}.$$

which are entirely in terms of the state space matrices for $P_{est}(s)$. Then we have

$$A_L = \bar{A} + \underline{B}J\underline{C}, \quad B_L = \bar{B} + \underline{B}J\underline{D}_{21},$$
$$C_L = \underline{D}_{12}J\underline{C}, \quad D_L = D_{p11} + \underline{D}_{12}J\underline{D}_{21}.$$

Obviously, we can obtain the following equations by replacing with the A_K, B_K, C_K and D_K.

$$A_L = \begin{bmatrix} A_r & 0 \\ B_K C_r & A_K \end{bmatrix}$$
$$B_L = \begin{bmatrix} I & B_r \\ 0 & B_K D_r \end{bmatrix}$$
$$C_L = \begin{bmatrix} D_K C_r & C_K \end{bmatrix}$$
$$D_L = \begin{bmatrix} 0 & I + D_K D_r \end{bmatrix}$$

Now we look for a dynamic estimator $K_{est}(s)$ such that the closed-loop transfer function $M_L(s)$ is contractive and internally stable.

Lemma 4.4 *[6] Suppose*
(a) P, Q and H are matrices and that H is symmetric;
(b) The matrices W_P and W_Q are full rank matrices satisfying $I_m W_P = kerP$ and $I_m W_Q = kerQ$.
Then there exists a matrix J such that

$$H + P^* J^* Q + Q^* J P < 0 \tag{4.25}$$

if and only if, the inequalities

$$W_P^* H W_P < 0 \text{ and } W_Q^* H W_Q < 0 \tag{4.26}$$

both hold.

Theorem 4.4 *[6] The matrix A_L is Hurwitz and $||M_L(s)||_\infty < \gamma$ if and only if there exists a symmetric positive definite matrix X_L such that*

$$\begin{bmatrix} A_L^* X_L + X_L A_L & X_L B_L & C_L^* \\ * & -\gamma I & D_L^* \\ * & * & -\gamma I \end{bmatrix} < 0 \tag{4.27}$$

Proof. Theorem 4.4 can be proven readily by using bounded real lemma. ■

Now define the matrices

$$P_{X_L} = \begin{bmatrix} \underline{B}^* X_L & 0 & \underline{D}_{12}^* \end{bmatrix}$$
$$Q = \begin{bmatrix} \underline{C} & \underline{D}_{21} & 0 \end{bmatrix}$$
$$H_{X_L} = \begin{bmatrix} \bar{A}^* X_L + X_L \bar{A} & X_L \bar{B} & \bar{C}^* \\ * & -\gamma I & D_L^* \\ * & * & -\gamma I \end{bmatrix}$$

It follows that ineuqality in (4.27) is exactly

$$H_{X_L} + Q^* J^* P_{X_L} + Q_{X_L}^* J Q < 0 \tag{4.28}$$

Lemma 4.5 *[6] Suppose X and Y are symmetric positive definite matrices in $R^{n\times n}$; and n_K is a positive integer. Then there exist matrices X_2, $Y_2 \in R^{n\times n}$ and symmetric matrices X_3, $Y_3 \in R^{n_K \times n_K}$, satisfying*

$$\begin{bmatrix} X & X_2 \\ X_2^* & X_3 \end{bmatrix} > 0 \text{ and } \begin{bmatrix} X & X_2 \\ X_2^* & X_3 \end{bmatrix}^{-1} = \begin{bmatrix} Y & Y_2 \\ Y_2^* & Y_3 \end{bmatrix}$$

if and only if

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \text{ and rank} \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \leq n + n_K$$

Theorem 4.5 *There exists a H_∞ filtering based fault estimator $K_{est}(s)$ for the system (4.12), if and only if there exists symmetric matrices $X > 0$ and $Y > 0$ such that*
(a)

$$\begin{bmatrix} N_X & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} A_r^* X + X A_r & 0 & X B_r & 0 \\ * & -\gamma I & 0 & 0 \\ * & * & -\gamma I & D_r^* \\ * & * & * & -\gamma I \end{bmatrix} \begin{bmatrix} N_X & 0 \\ 0 & I \end{bmatrix} < 0 \tag{4.29}$$

(b)

$$\begin{bmatrix} N_Y & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} A^* Y + Y A & 0 & I & B_r \\ * & -\gamma I & 0 & -I \\ * & * & -\gamma I & 0 \\ * & * & * & \gamma I \end{bmatrix} \begin{bmatrix} N_Y & 0 \\ 0 & I \end{bmatrix} < 0 \tag{4.30}$$

(c)

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0, \tag{4.31}$$

where N_X and N_Y are full-rank matrices whose images satisfy

$$Im N_X = ker[C_r \quad 0 \quad D_r] \tag{4.32}$$
$$Im N_Y = ker[0 \quad I] \tag{4.33}$$

Suppose X and Y have been found satisfying Theorem 4.5, we can find a matrix $X_2 \in R^{n \times n}$ such that $X - Y^{-1} = X_2 X_2^*$ by Lemma 4.5. Then

$$X_L = \begin{bmatrix} X & X_2^* \\ X_2 & I \end{bmatrix} \tag{4.34}$$

has the properties desired above. As seen before, the order n_K need to be no larger than n, and in general can be chosen to be the rank of $X - Y^{-1}$. Next by Lemma 4.4 we know that there exists a solution to

$$H_{X_L} + Q^* J^* P_{X_L} + P_{X_L}^* J Q < 0 \tag{4.35}$$

and that any such solution J provides the state space realization for a feasible estimator K_{est}.

4.4 Fault Tolerant Controller Design

In this section, firstly, a controller K_N is designed for the fault-free model. Then a control compensation law V is determined, which is to be added to the normal control law in order to reduce the faults' effects on the system.

4.4.1 Fault-Free Case

Now we design the fault-free part of control at first. For the fault-free system, $f(t)$ should always be zero. That is to say the system (4.12) can be rewritten as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu_N(t) + \Phi(x,t) \\ y(t) &= Cx(t) \end{aligned} \tag{4.36}$$

Furthermore, the error dynamic of the fault-free system can be derived directly from (4.15), such that

$$\dot{e}(t) = (A - HC)e(t) + \Phi(x,t) - \Phi(\hat{x},t) \tag{4.37}$$

Assuming that a Moore-Penrose pseudo-inverse exists, the observer-based state feedback controller can be proposed as:

$$u_N(t) = K_N \hat{x}(t) - B^\dagger \Phi(\hat{x},t) \tag{4.38}$$

where $B^\dagger$ is the pseudo-inverse of B, K_N is the control gain to be determined for the fault-free case. Then (4.36) can be rewritten as:

$$\dot{x}(t) = (A + BK_N)\hat{x}(t) + \Phi(x,t) - \Phi(\hat{x},t) \tag{4.39}$$

By augmenting the state $x(t)$ and the dynamic error $e(t)$ in a vector $\eta(t)$, we can reach the following equations.

$$\begin{aligned}\dot{\eta}(t) &= \begin{bmatrix} A+BK_N & -BK_N \\ 0 & A-HC \end{bmatrix}\eta(t) + \begin{bmatrix} I \\ I \end{bmatrix}\Delta\Phi \\ y(t) &= \begin{bmatrix} C & 0 \end{bmatrix}\eta(t)\end{aligned} \tag{4.40}$$

where $\eta(t) = [x(t) \quad e(t)]^T$ and $\Delta\Phi = \Phi(x,t) - \Phi(\hat{x},t)$. Then it is easy to see that K_N in the above equations can be calculated by the classic H_∞ techniques.

Theorem 4.6 *Given $\gamma > 0$, then $||G_{y\Delta\Phi}(s)||_\infty < \gamma$ if there exists symmetric matrices $P_1 > 0$, $P_2 > 0$ and a matrix Y such that the following matrix inequality holds:*

$$\begin{bmatrix} AP_1 + P_1A^T + Y + Y^T & -Y & P_1 & C^T \\ * & P_2(A-HC) + (A-HC)^TP_2 & P_2 & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -\gamma I \end{bmatrix} < 0 \tag{4.41}$$

where $Y = P_1BK_N$.

Proof. Define symmetric matrices P_1 and P_2 such that $P = diag(P_1, P_2)$ is also symmetric. By using the Lemma 2.2, we can easily know that $||G_{y\Delta\Phi}(s)||_\infty < \gamma$ is equivalent to the following inequity.

$$\begin{bmatrix} P_1(A+BK_N) + (A+BK_N)^TP_1 & -P_1BK_N & P_1 & C^T \\ * & P_2(A-HC) + (A-HC)^TP_2 & P_2 & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -\gamma I \end{bmatrix} < 0 \tag{4.42}$$

However, the (4.42) is not a solvable LMI. Luckily we can use some standard notations to transfer the inequality to a solvable one. By left and right multiplying with the symmetric matrix $diag(P_1, P_2)$, the inequality (4.42) can be rewritten as (4.41), which can be directly solved by LMI technique. ■

4.4.2 Fault Compensation

In this subsection, a fault compensation control law $u_{ad}(t)$ will be designed. By adding $u_{ad}(t)$ to the normal control law $u_N(t)$, we can reduce the impact of the faults on the system, which leads to

$$\begin{aligned} u(t) &= u_N(t) + u_{ad}(t) \\ &= K_c(s)\hat{x}(t) - B^\dagger\Phi(\hat{x},t) + V\hat{f}(t) \end{aligned} \tag{4.43}$$

The compensation control law $u_{ad}(t) = 0$ in the normal case and $u_{ad}(t) \neq 0$ in the faulty cases.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \Phi(x,t) + B_f f(t) \\ y(t) &= Cx(t) + D_f f(t) \end{aligned} \tag{4.44}$$

By replacing estimation $\hat{f}(t)$ by $f(t) - \tilde{f}(t)$, the system can be rewritten as:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BK_N\hat{x}(t) + \Delta\Phi + B_f f(t) + BV\hat{f}(t) \\ \dot{e}(t) &= A_r e(t) + B_r f(t) + \Delta\Phi \end{aligned} \tag{4.45}$$

where $\Delta\Phi = \Phi(x,t) - \Phi(\hat{x},t)$. By augmenting the state x and the error dynamics e, we can reach the following equations.

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} &= \begin{bmatrix} A + BK_N & -BK_N \\ 0 & A_r \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B_f + BV & -BV & I \\ B_r & 0 & I \end{bmatrix} \begin{bmatrix} f(t) \\ \tilde{f}(t) \\ \Delta\Phi \end{bmatrix} \\ y(t) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} D_f & 0 & 0 \end{bmatrix} \begin{bmatrix} f(t) \\ \tilde{f}(t) \\ \Delta\Phi \end{bmatrix} \end{aligned} \tag{4.46}$$

Then we define $\eta(t) = [f(t) \ \ \tilde{f}(t) \ \ \Delta\Phi]^T$ to make the expression simple.

Theorem 4.7 *Given $\gamma > 0$, then $||G_{y\eta}(s)||_\infty < \gamma$ if there exists symmetric matrices $P_a > 0$, $P_b > 0$ and a matrix V such that the following matrix inequality holds:*

$$\begin{bmatrix} (A+BK_N)P_a + P_a(A+BK_N)^T & -BK_N & B_f + BV & -BV & I & P_aC^T \\ * & A_rP_b + P_bA_r^T & B_r & 0 & I & 0 \\ * & * & -\gamma I & 0 & 0 & D_f^T \\ * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & -\gamma I & 0 \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{4.47}$$

Proof. Define symmetric matrices P_a and P_b such that $P = diag(P_a, P_b)$ is also symmetric. We know that $||G_{y\eta}(s)||_\infty < \gamma$ is equivalent to the following inequity (4.48) by applying the Lemma 2.2.

$$\begin{bmatrix} \Gamma & -P_aBK_N & P_a(B_f + BV) & -P_aBV & P_a & C^T \\ * & P_bA_r + A_r^TP_b & P_bB_r & 0 & P_b & 0 \\ * & * & -\gamma I & 0 & 0 & D_f^T \\ * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & -\gamma I & 0 \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{4.48}$$

where $\Gamma = P_a(A+BK_N)+(A+BK_N)^T P_a$. But we have to use some notations to transfer the inequality (4.48) to a solvable one. The inequality (4.48) can be rewritten as (4.47) in the Theorem 4.7 by left and right multiplying with the symmetric matrix $diag(P_a, P_b)$. Note that the parameters K_N, A_r and B_r can all be calculated by the previous lemmas or theorems. That's to say, we only need to solve P_a, P_b and V from this inequality. Then the inequality can be solved by LMI technique. ■

4.5 Simulation

In this section, a system model for a rigid body satellite in a circular orbit is adopted [9]. The three coordinate frames for the dynamics of the satellite are shown in Figure 4.5.

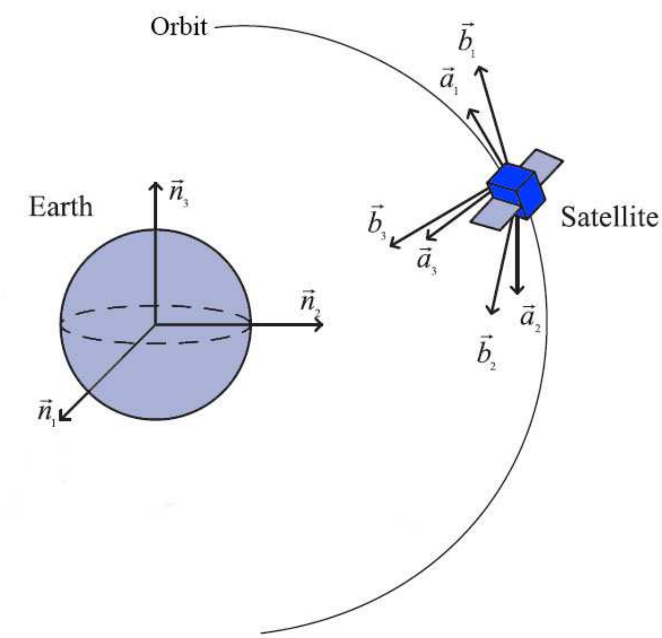

Figure 4.2: Rigid body satellite in a circular orbit

The local vertical and local horizontal reference frame A with its origin at the center of mass of the satellite, and a set unit vectors $\vec{a_1}, \vec{a_2}, \vec{a_3}$; a body-fixed reference frame B with basis vectors $\vec{b_1}, \vec{b_2}, \vec{b_3}$; and a Newtonian inertial reference frame N with a set unit vectors $\vec{n_1}, \vec{n_2}, \vec{n_3}$. The dynamics of a rigid-body satellite in a circular orbit is displayed with the well-known Euler's moment equation. The nonlinear motion equation is represented as:

$$J\dot{\omega} + \omega \times J\omega = 3\omega_0^2 \xi \times J\xi + u + T_d \tag{4.49}$$

where $\omega = [\omega_x\ \omega_y\ \omega_z]^T$ is the angular velocity of the satellite in a body-fixed reference frame; $J = diag\{J_1, J_2, J_3\}$ is the symmetric inertia matrix of the rigid satellite, and $J_i(i = 1, 2, 3)$

are the principle axis moments of inertia of the satellite; ω_0 is the constant orbit rate; $u = [u_1 \ u_2 \ u_3]^T$ is the control torque vector and T_d represents the external disturbance torques. The nonlinear term ξ is

$$\xi = \begin{bmatrix} -sin\theta & sin\phi & cos\theta & cos\phi & cos\theta \end{bmatrix}^T \tag{4.50}$$

The notation $a^\times$ for a vector $a = [a_1 \ a_2 \ a_3]^T$ is used to represent the skew-symmetric matrix

$$a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

To describe the orientation of the body-fixed reference frame B with respect to A, in terms of three Euler angles ψ, θ and ψ, which are roll, pitch and yaw attitude angles respectively, a particular sequence of three successive body-axis rotations is symbolically denoted by

$$\psi\vec{a_3} \to \theta\vec{a_2'} \to \phi\vec{a_1''}$$

in which $\psi\vec{a_3}$ represents a rotation about the $\vec{a_3}$-axis of the frame $\vec{a_1}, \vec{a_2}, \vec{a_3}$ with an angle ψ to the frame $\vec{a_1}', \vec{a_2}', \vec{a_3}'$,$\theta\vec{a_2}'$ a rotation about the $\vec{a_2}'$-axis of the frame $\vec{a_1}', \vec{a_2}', \vec{a_3}'$ with an angle θ to the frame $\vec{a_1}'', \vec{a_2}'', \vec{a_3}''$ with an angle ϕ to the frame $\vec{b_1}, \vec{b_2}, \vec{b_3}$. The kinematic differential equation of an orbiting rigid body can be described as

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \frac{1}{cos\theta} \begin{bmatrix} cos\theta & sin\phi sin\theta & cos\phi sin\theta \\ 0 & cos\phi & -sin\phi cos\theta \\ 0 & sin\phi & cos\phi \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \frac{\omega_0}{cos\theta} \begin{bmatrix} sin\psi \\ cos\theta cos\psi \\ sin\theta cos\psi \end{bmatrix} \tag{4.51}$$

For small attitude angles, the dynamic equation becomes

$$\begin{aligned} J_1\dot{\omega}_x - (J_2 - J_3)\omega_y\omega_z + 3\omega_0^2(J_2 - J_3)\psi &= u_1 + T_{d1} \\ J_2\dot{\omega}_y - (J_3 - J_1)\omega_z\omega_x + 3\omega_0^2(J_1 - J_3)\theta &= u_2 + T_{d2} \\ J_3\dot{\omega}_z - (J_1 - J_2)\omega_x\omega_y &= u_3 + T_{d3} \end{aligned}$$

and the kinematic equation can be linearized as

$$\begin{aligned} \dot{\phi} &= \omega_x + \omega_0\psi \\ \dot{\theta} &= \omega_y + \omega_0 \\ \dot{\psi} &= \omega_z - \omega_0\phi \end{aligned}$$

choose state variable x as

$$x(t) = \begin{bmatrix} \phi(t) & \theta(t) & \psi(t) & \omega_x(t) & \omega_y(t) & \omega_z(t) \end{bmatrix}^T$$

the satellite attitude control system model with actuator faults can be obtained as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + \Phi(x,t) + B_f f(t) \\ y(t) &= Cx(t) + D_f f(t) \end{aligned} \tag{4.52}$$

where

$$A = \begin{bmatrix} 0 & 0 & \omega_0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\omega_0 & 0 & 0 & 0 & 0 & 1 \\ -3\omega_0^2 J_1^{-1}(J_2 - J_3) & 0 & 0 & 0 & 0 & 0 \\ 0 & -3\omega_0^2 J_2^{-1}(J_1 - J_3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi(x,t) = \begin{bmatrix} 0 & \omega_0 & 0 & J_1^{-1}(J_2 - J_3)\omega_y\omega_z & J_2^{-1}(J_3 - J_1)\omega_z\omega_x & J_3^{-1}(J_1 - J_2)\omega_x\omega_y \end{bmatrix}^T$$

$$B = \begin{bmatrix} 0 & 0 & 0 & J_1^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & J_2^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & J_3^{-1} \end{bmatrix}, C = 0.2 \times I_{6\times 6}$$

$$B_f = D_f = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

The satellite is assumed to be operating in a small angle range with small angular velocity, therefore the nonlinear function $\Phi(x,t)$ is locally Lipschitz with a Lipschitz constant α.

$$||\Phi(x,t) - \Phi(\hat{x},t)|| \leq \alpha ||x(t) - \hat{x}(t)||$$

Note that α can be obtained by calculating the upper bound of the changing rate of the function $\Phi(x,t)$ with respect to the angular velocity in the operation range.

The inertia matrix is assumed as $J = diag\{18, 21, 24\}$. The satellite is equipped with three actuators dispatched at each principle axis, and each actuator is with limited control torque $u_{max} = 2N \cdot m$. The orbital angular velocity $\omega_0 = 0.0012\ rad/s$. The Lipschitz constant is calculated as $\alpha = 0.33$. By applying the Theorem 4.2, the static observer gain H can be designed as

$$H = \begin{bmatrix} 14.4371 & 0 & -0.0021 & -18.8657 & 0 & 0.0062 \\ 0 & 14.4371 & 0 & 0 & -3.9193 & 0 \\ 0.0021 & 0 & 14.4371 & 0.0052 & 0 & -15.2908 \\ 23.8657 & 0 & -0.0052 & 14.4371 & 0 & 0 \\ 0 & 8.9193 & 0 & 0 & 14.4371 & 0 \\ -0.0062 & 0 & 20.2908 & 0 & 0 & 14.4371 \end{bmatrix}$$

Furthermore, by using the Theorem 4.3 and Theorem 4.4, the dynamic fault estimator

$K_{est}(s) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$ can be obtained as:

$$A_K = \begin{bmatrix} -202.5410 & 0 & 0.0020 & 1.0000 & 0 & 0 \\ 0 & -202.5140 & 0 & 0 & 1.0000 & 0 \\ -0.0020 & 0 & -202.5300 & 0 & 0 & 1.0000 \\ -501.7960 & 0 & 0.0050 & -0.3000 & 0 & 0 \\ 0 & -500.2330 & 0 & 0 & -0.3000 & 0 \\ -0.0030 & 0 & -501.2850 & 0 & 0 & -0.3000 \end{bmatrix},$$

$$B_K = \begin{bmatrix} 3165.444 & 0 & 0 & 59.83 & 0 & -0.02 \\ 0 & 3165.027 & 0 & 0 & 12.436 & 0 \\ 0 & 0 & 3165.27 & -0.017 & 0 & 48.494 \\ 7880.153 & 0 & -0.061 & -41.015 & 0 & 0 \\ 0 & 7902.777 & 0 & 0 & -41.021 & 0 \\ 0.071 & 0 & 7883.388 & 0 & 0 & -41.017 \end{bmatrix},$$

$$C_K = \begin{bmatrix} 0 & 0 & 0 & -0.063 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.063 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.063 \end{bmatrix},$$

$$D_K = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Then we go further to design the fault tolerant controller, which can be calculated by the H_∞ synthesis technique. We obtain the normal controller gain K_N as following:

$$K_N = \begin{bmatrix} -12 & 0 & -0.0048 & -26 & 0 & 0 \\ 0 & -14 & 0 & 0 & -30.3333 & 0 \\ 0.0064 & 0 & -16 & 0 & 0 & -34.6667 \end{bmatrix}$$

After that, we design the fault compensator by applying the Theorem 4.7, then obtain the gain of V.

$$V = \begin{bmatrix} -6.0477 & 0.0000 & 0.0004 \\ -0.0000 & -6.1211 & -0.0000 \\ -0.0014 & 0.0000 & -8.1326 \end{bmatrix}$$

4.5.1 Oscillated Faults Case

In this case, it is assumed that the system has a biased attenuated oscillated fault happened in the first channel of faults which is given as follows,

$$f_1(t) = \begin{cases} 1 + 0.1cos(\pi t)e^{-0.1t}, & t \geq 10 \text{ (sec)} \\ 0 & 0 \leq t < 10 \text{ (sec)} \end{cases} \tag{4.53}$$

The Figure 4.3 shows the residual signal $r(t)$ generated from the observer. Obviously, the residual signal jumps a big step during the faults happen ($t \geq 10$), which means the fault detection has be achieved.

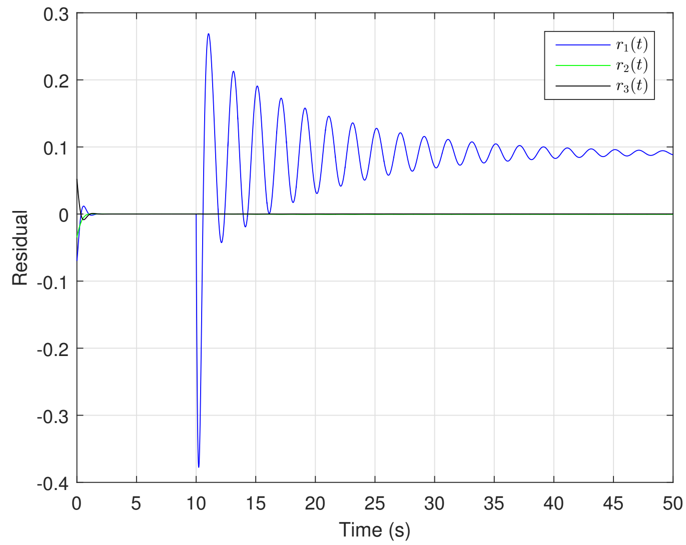

Figure 4.3: Residual generation

The relationship between real faults $f(t)$ and estimated faults $\hat{f}(t)$ (dotted-line) is illustrated in Figure 4.4.

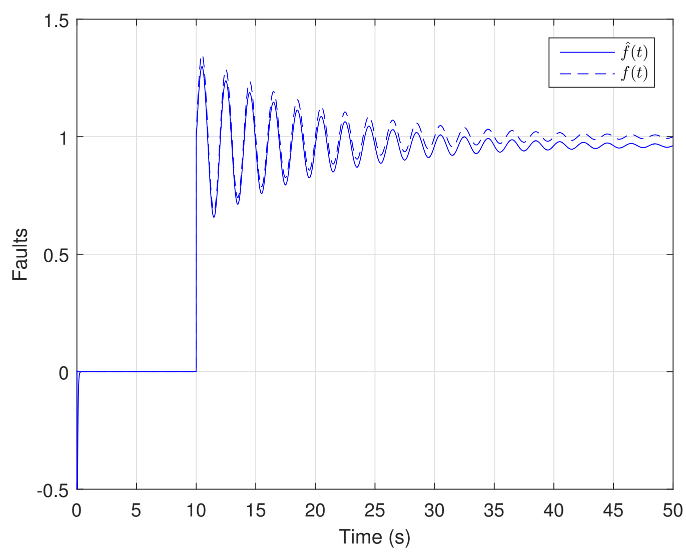

Figure 4.4: Estimation of faults

One can notice that the estimated faults $\hat{f}(t)$ are accurate compared to the actual faults $f(t)$. Figure 4.5 shows the state $x(t)$ of faulty system. From the figure, we can clearly see that after the transient response for faults' occurrence ($t = 10s$), the system responses are recovered from the faults, which shows that the fault tolerant controller can retain the stability and reduce the effects of faults.

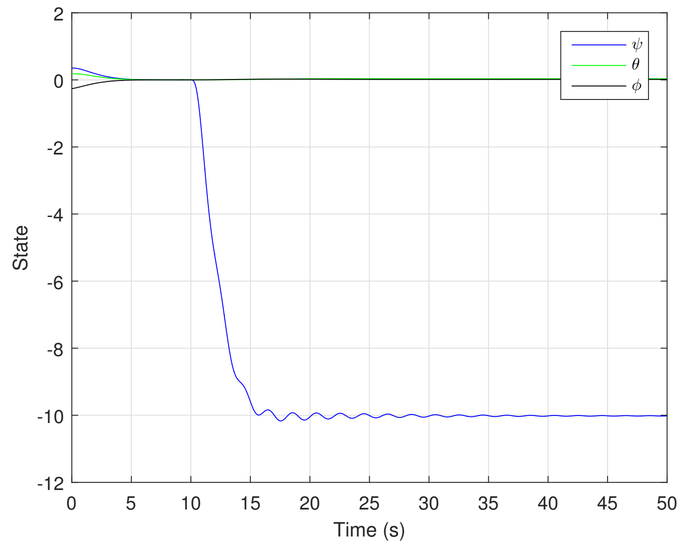

Figure 4.5: System state under faults occurrence

4.5.2 Multiple Faults Case

In the second case, we assume that the system has three different kinds of biased attenuated oscillated faults happened in all these three fault channels in different time, which can be described by equations (4.54).

$$
\begin{aligned}
f_1(t) &= \begin{cases} 1 + cos(\pi t)e^{-0.1t}, & t \geq 10 \text{ (sec)} \\ 0 & 0 \leq t < 10 \text{ (sec)} \end{cases} \\
f_2(t) &= \begin{cases} 0.8 + 1.5cos(\pi t)e^{-0.15t}, & t \geq 20 \text{ (sec)} \\ 0 & 0 \leq t < 20 \text{ (sec)} \end{cases} \\
f_3(t) &= \begin{cases} 0.5 + cos(\pi t)e^{-0.12t}, & t \geq 30 \text{ (sec)} \\ 0 & 0 \leq t < 30 \text{ (sec)} \end{cases}
\end{aligned} \tag{4.54}
$$

The Figure 4.6 shows the residual signal $r(t)$ generated from the observer. The residual signals vary at three occasions: 10s, 20s and 30s, which are exactly the faults' occurrence time. That's to say the system can detect all these three faults together.

The relationship between real faults $f(t)$ and estimated faults $\hat{f}(t)$ (dotted-line) is described in Figure 4.7. One can notice that the estimated faults $\hat{f}(t)$ are accurate compared to the actual faults $f(t)$.

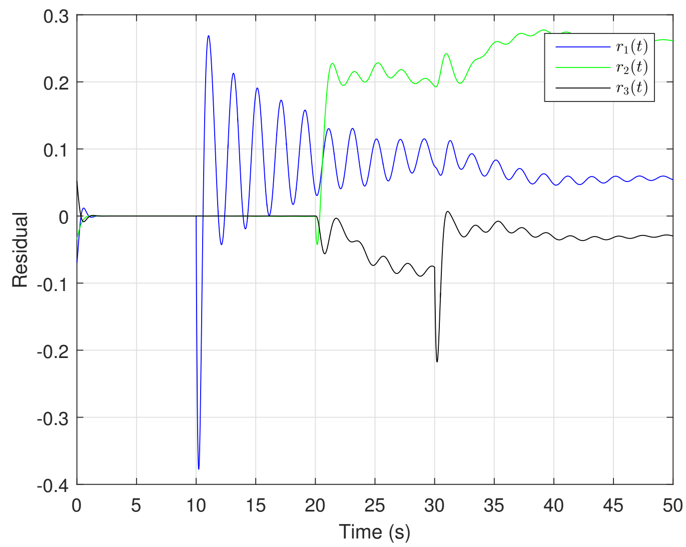

Figure 4.6: Residual generation

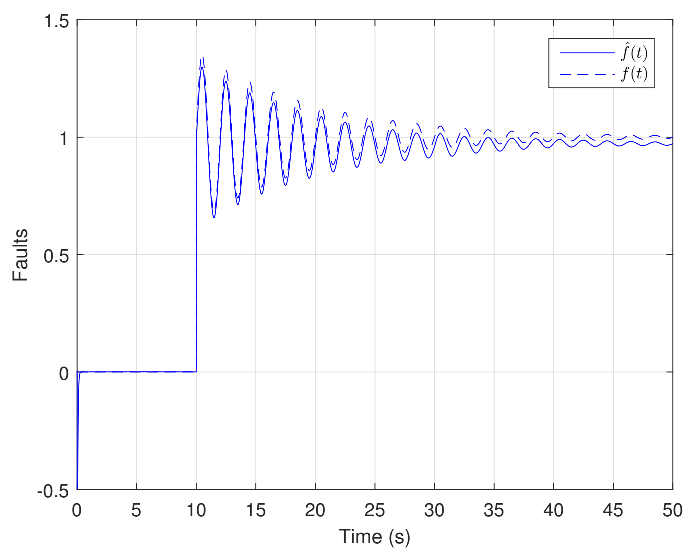

Figure 4.7: Estimation of faults

4.5.3 Piecewise Step Faults Case

In the third case, we simulate that the system has a piecewise step fault happened in the first fault channel as equations (4.55).

$$f_1(t) = \begin{cases} 0.5, & 10 \leq t < 30 \text{ (sec)} \\ 0 & \end{cases} \tag{4.55}$$

The Figure 4.8 shows the residual signal $r(t)$ generated from the observer. It is seen that the residual jumps a big step during the faults happen at $t \geq 10$ and drops back to the original states when the faults disappear at $t \geq 30$. Then the faults can be detected by this method.

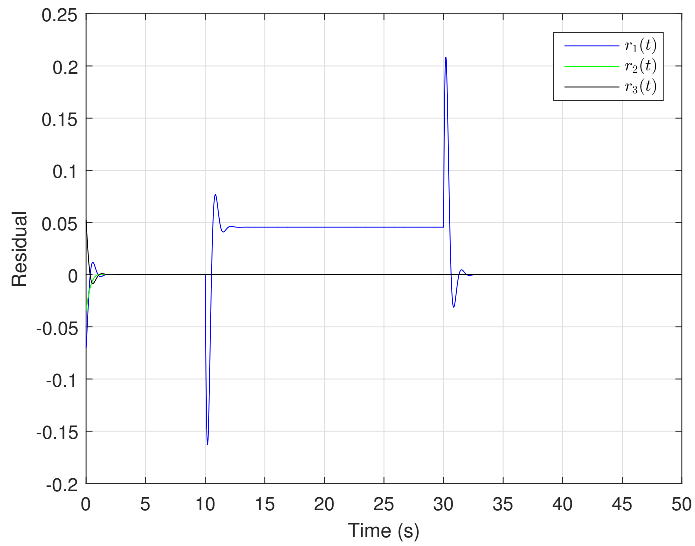

Figure 4.8: Residual generation

The relationship between real faults $f(t)$ and estimated faults $\hat{f}(t)$ (dotted-line) is illustrated in Figure 4.9.

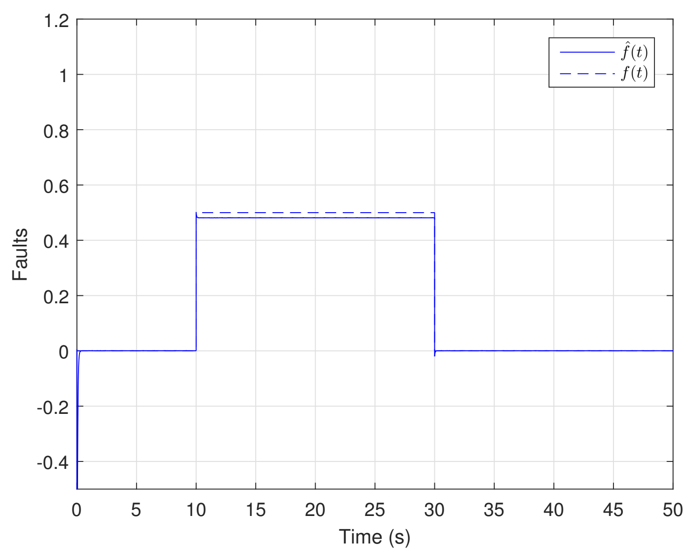

Figure 4.9: Estimation of faults

One can notice that the estimated faults $\hat{f}(t)$ are accurate compared to the actual faults $f(t)$. Figure 4.10 shows the state $x(t)$ of faulty system. From the figure, we can clearly see that after the transient response for faults' occurrence ($t = 10s$), the system responses

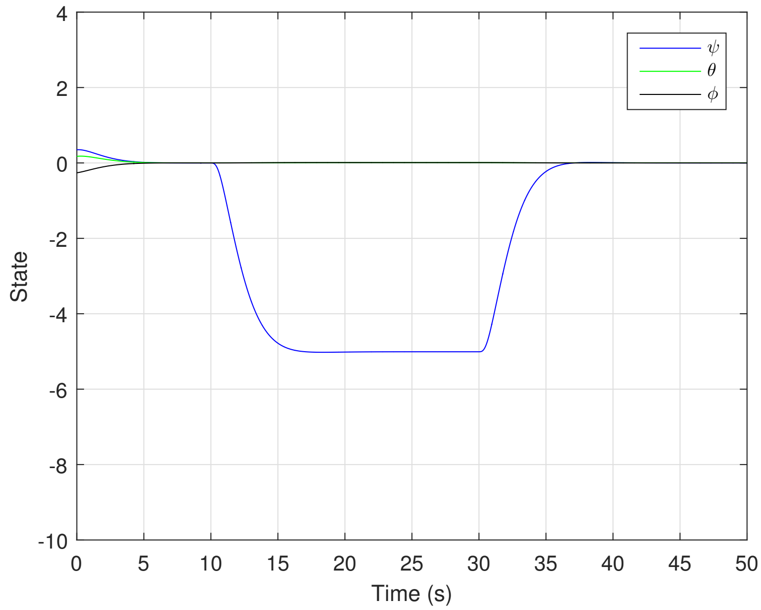

Figure 4.10: System state under faults occurrence

are recovered from the faults, which shows that the fault tolerant controller can retain the stability and reduce the effects of faults. After the faults disappear at $t = 30s$, the system will be back on origin state by the effects of the fault tolerant controller.

4.6 Conclusion

This chapter gives an idea of extending the H_∞ filtering based fault diagnosis framework from linear system to Lipschitz nonlinear system. Based on the simulation results, we conclude that the proposed fault tolerant control law is effective in handling the faults.

Chapter 5

Conclusion and Future Work

In this thesis, we focus on the model-based fault diagnosis problem. Although observer-based techniques have been developed for many years, there still remains an open problem of conducting fault detection and fault tolerant control simultaneously. By using the new framework proposed in this paper, we can diagnose the faults and compensate the faults at the same time.

We give a brief review of the previous techniques about model-based fault diagnosis and develop a new H_∞ filtering based fault diagnosis framework at the beginning of the thesis. We use the H_∞ filtering observer design technique to obtain the filtered residual signals and separate the useful information of the residual signals from disturbances. Then we apply the classic H_∞ synthesis method to both the linear system and the Lipschitz nonlinear system.

For the linear system, we use the integrated framework to detect the faults and apply the fault tolerant control simultaneously. We obtain the purified faults information by filtering the residual signals generated from the observer. Then we estimate the faults by using H_∞ synthesis. After that, we design a fault tolerant controller which can handle both fault-free case and fault-occurred case. At the end of the discussion, a quadrotor example is illustrated to show the effectiveness of the framework.

For the Lipschitz nonlinear system, we obtain the residual signals by using a classic Lipschitz nonlinear observer. Then the faults can be detected and estimated by the H_∞ filtering based fault diagnosis framework. Then a fault tolerant controller has been designed for the Lipschitz nonlinear system. Finally, we introduce a satellite model to validate the framework.

In the future work, we may take uncertainties into consideration for the linear system. Also, we can apply the scheme to the more realistic quadrotor control problem, for example, how to maintain the stability if one of the motors has lost its control efficiency. Then for the Lipschitz nonlinear system, external disturbances could be an interesting topic to be discussed in the future work. Besides, the nominal controller u_N in Lipschitz nonlinear system is just a simplification, which can be improved in the continuing research.

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