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MUSIC ANALYSIS/SYNTHESIS BY OPTIMIZED MULTIPLE WAVETABLE INTERPOLATION:
RESULTS IN DETAIL

by

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Optimized multiple wavetable interpolation is a new music analysis/synthesis method introduced in the author's doctoral thesis [2]. This technical report serves as a supplement to the thesis, providing additional tables of results.

1 Overview

Music analysis/synthesis is a general process in which a recorded sound is analyzed in such a way that a musician can modify the analysis data and synthesize an altered sound from the modified data.

Multiple wavetable interpolation is a form of music analysis/synthesis introduced by Horner [1], based on a previous method by M.-H. Serra et al. [3]. The algorithm involves three basic steps: 1) The recorded sound is reduced to a set of breakpoints by piecewise linear approximation of the spectral envelopes of its harmonics; 2) the spectrum at each breakpoint is matched by determining weightings for a small number of wavetables; and 3) the sound is resynthesized using multiple wavetable additive synthesis by interpolating between the weightings for each wavetable at consecutive breakpoints.

Optimized multiple wavetable interpolation generalizes and optimizes multiple wavetable interpolation. The method uses a clustering algorithm to select a bank of wavetables such that the wavetables will be useful in matching the breakpoint spectra of a wide variety of harmonic tones played by various instruments. The breakpoint-matching algorithm selects subsets of the wavetables in the wavetable bank which best match each breakpoint spectrum, subject to the constraint that a wavetable which ceases to be used at a given breakpoint must be faded out by the next breakpoint and one which comes into use must be faded in. The algorithm introduces the use of the single-source acyclic weighted shortest path algorithm to choose breakpoint matches in a globally optimal way. The output of the algorithm is a sequence of n -tuples of pairs of wavetable indices and weights which can serve as a control stream for a hardware or software synthesizer.

The method was tested on 198 tones played by sixteen different instruments at pitches which spanned the range from A1 to B6 at intervals of a minor third. The tones were grouped by pitch into five subranges so that a greater number of harmonics could be retained for the lower tones to provide greater fidelity on resynthesis without introducing artefacts by exceeding the limit of the Nyquist frequency. A separate wavetable bank was selected for each group of tones.

2 Multi-Level Search Results

In the first stage of the breakpoint-matching algorithm, an initial match is found for each breakpoint. A match consists of a subset of the wavetables (*basis spectra*) in the wavetable bank, along with a weighting factor for each wavetable, chosen to minimize the matching error in a least-squares sense.

One of the methods used to find initial matches to each breakpoint spectrum was a multi-level pruned search. At the first level, matches consisting of only a few wavetables were found by exhaustive search; the search tree was then pruned by conducting a second-level exhaustive search to augment those matches found by the first-level search to the desired size.

The tables in Appendix A present the results of the multi-level search in detail for all five groups of tones. Tables 1–5 list the times (in seconds) and the root-mean-square (RMS) matching errors for single-level exhaustive searches to depths 1, 2, 3, and 4 for each tone in Groups 1–5, respectively. Tables 6–15 indicate, for each tone, the time (in seconds) required to augment an initial match by one or two tables (T_{aug}), the total search time of the two-level search (T_{srch}), and the matching error of the augmented match.

3 Oscillator Assignment Optimization Results

In the final stage of the breakpoint-matching algorithm, a weighted wavetable is assigned to each available oscillator at each breakpoint. The oscillator assignments are constrained by the requirement that a wavetable which begins to be used at a particular breakpoint must be faded in from a weighting of zero at the previous breakpoint; similarly, a wavetable which is used at one breakpoint but not at the next must be faded out to zero amplitude between the two breakpoints.

Due to this constraint, it is not possible, in general, to use all the wavetables of each initial match since, at a particular breakpoint, the number of wavetables in the match plus the number of wavetables which must be faded in and/or out may exceed the number of oscillators available for use in synthesis.

The optimization algorithm thus treats the wavetables of an initial match at a given breakpoint as candidates which may or may not be used in the final match at that breakpoint. In order to allow a wavetable to come into use earlier than it is actually needed (according to an initial match) or to be retained for use at subsequent breakpoints, the initial matches are overlapped by one or two breakpoint; that is, the wavetables of an initial match at a particular breakpoint are added to the sets of wavetables eligible for use at the preceding and following breakpoints.

A globally optimal set of oscillator assignments (relative to some defined error measure) is then found by modeling the problem as a vertex-weighted directed acyclic graph (DAG) and using the single-source acyclic weighted shortest path algorithm to select the final match to the spectrum at each breakpoint, taking into account the requirement to fade wavetables in and out as they begin and cease to be used with non-zero weightings.

The tables in Appendix B present the results of oscillator assignment optimization in detail for all five groups of tones. Each table indicates, for each tone in the relevant group, the time (in seconds) required to optimize the breakpoint matches (T_{opt}), the total time for both the initial search and optimization (T_{total}), and the RMS matching error of the final matches. Tables 16–25 show the results of match optimization for three oscillators; Tables 26–35, for four oscillators; and Tables 36–45, for five oscillators.

To prevent unreasonably large optimization graphs, a size limit of 15 was specified for the wavetable sets produced by overlapping in the 5-oscillator case, and the results in Tables 36–45 reflect this choice.

4 Comparison with Horner's Method

Horner [1] used a simple form of oscillator assignment for his experimentation with multiple wavetable interpolation: a match of size $N_{\text{osc}} - 1$ was found by exhaustive search for the breakpoint spectrum with the largest RMS amplitude, and the other breakpoint spectra were matched by working forward and backward from the peak breakpoint, changing at most one wavetable from one breakpoint to the next by exhaustive search. This method might be called *constrained matching*, since all matches are constrained to consist of $N_{\text{osc}} - 1$ wavetables.

Tables 46 to 50 in Appendix C presents the results of implementing and testing Horner's constrained matching on the 198 instrumental tones selected for this research, using the same wavetable bank for each group of tones as was selected for testing the method of optimized multiple wavetable interpolation proposed in the accompanying thesis [2].

It should be clearly noted that this comparison with Horner's method is restricted to the breakpoint spectrum matching and oscillator assignment components only. Horner did not use large, general-purpose wavetable banks, but used a genetic algorithm to select small sets of basis spectra (two to ten wavetables in size) which were particular to each instrument being matched.

Figures 1–5 compare the results of Horner's constrained matching method with those of optimized matching using 3, 4, and 5 oscillators for the tones of Groups 1–5, respectively.

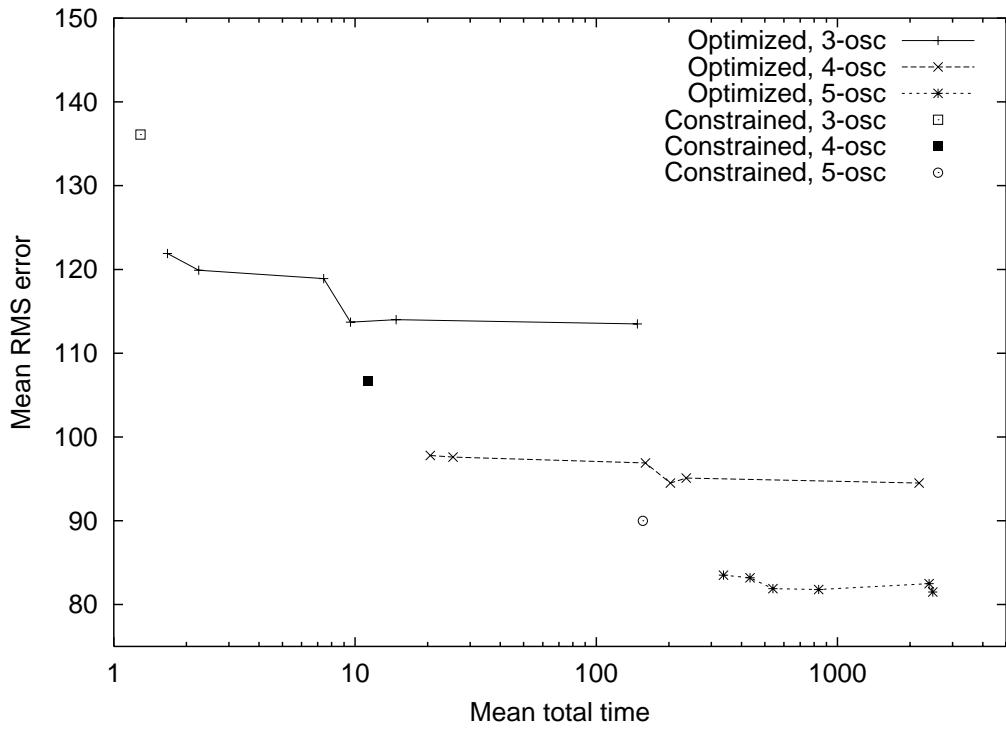


Figure 1: Comparison of Horner's constrained matching with optimized multi-level exhaustive search results for Group 1.

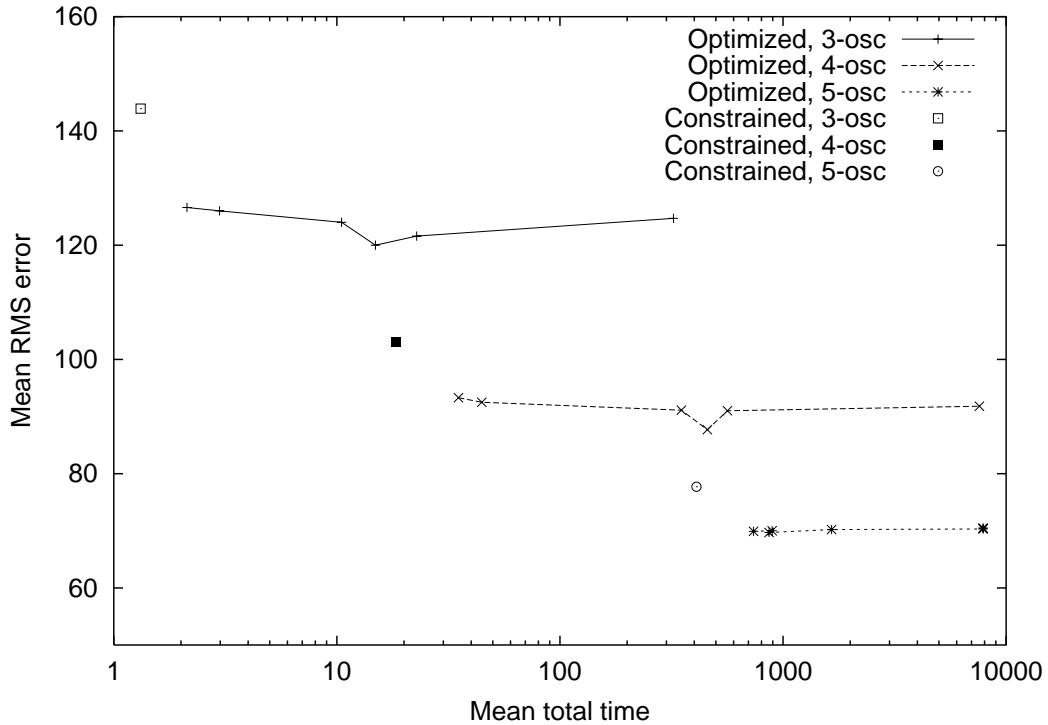


Figure 2: Comparison of Horner's constrained matching with optimized multi-level exhaustive search results for Group 2.

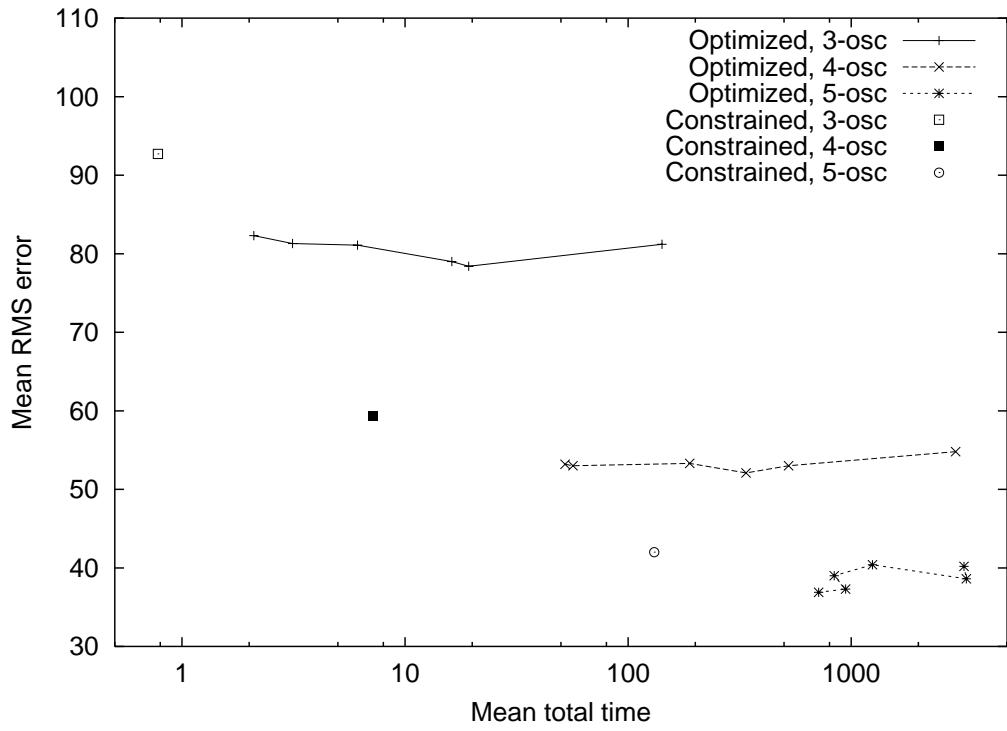


Figure 3: Comparison of Horner's constrained matching with optimized multi-level exhaustive search results for Group 3.

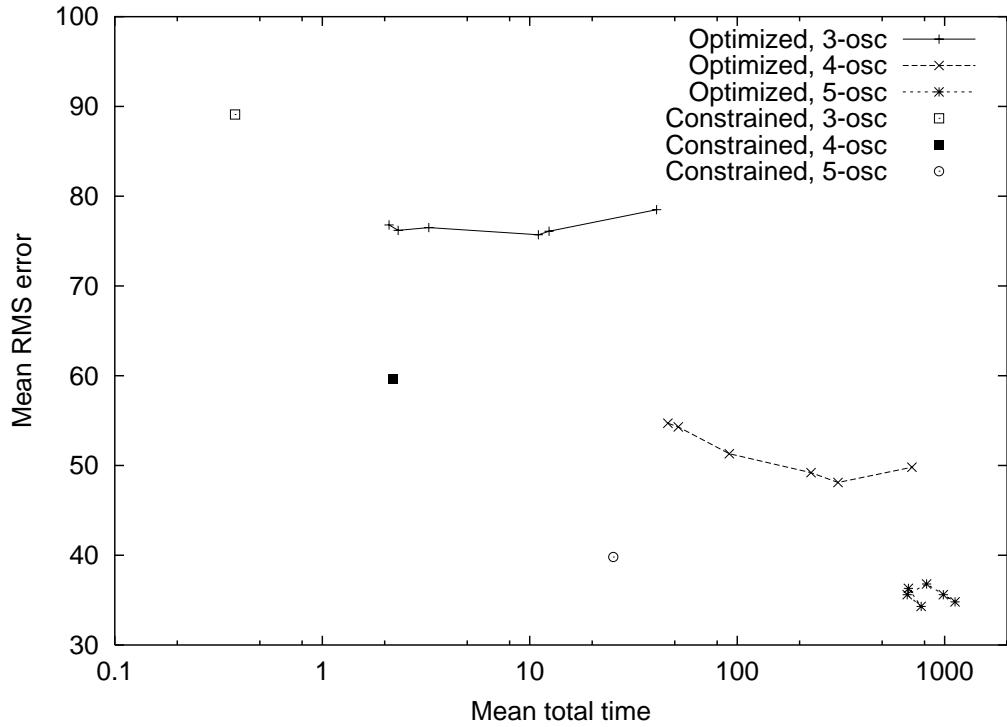


Figure 4: Comparison of Horner's constrained matching with optimized multi-level exhaustive search results for Group 4.

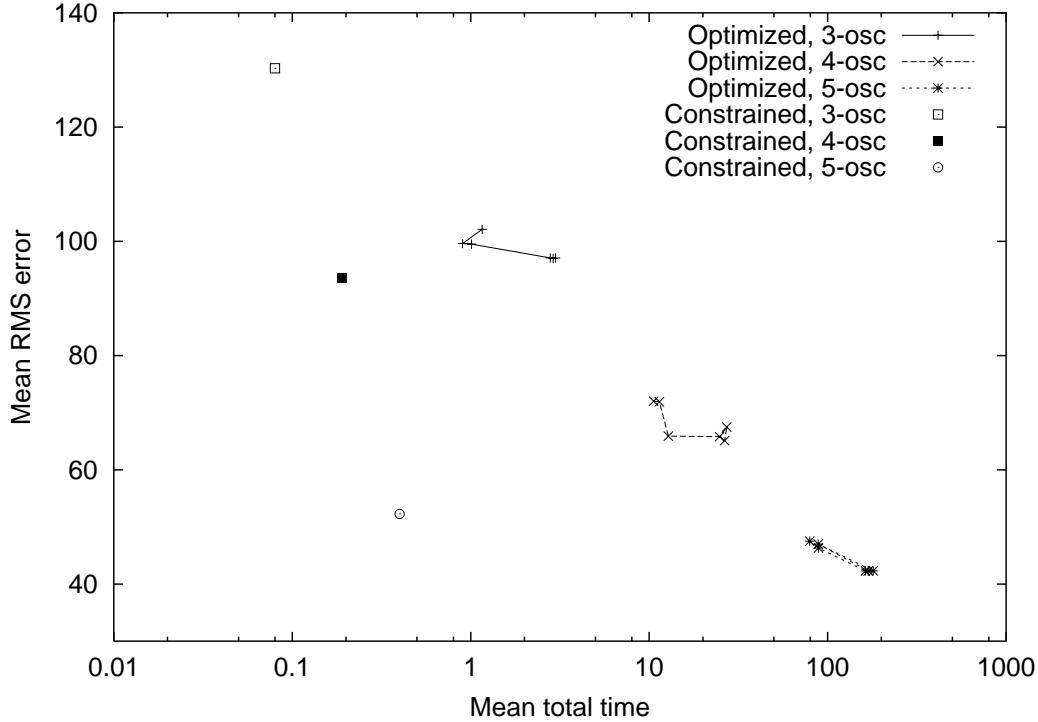


Figure 5: Comparison of Horner’s constrained matching with optimized multi-level exhaustive search results for Group 5.

While Horner’s constrained matching method is faster than any of the types of multi-level exhaustive search optimization for a given number of oscillators, the error levels produced by Horner’s method are significantly higher than those of the optimized matches, and in some cases are closer to those achieved by optimization with one fewer oscillators.

5 Conclusion

Optimized multiple wavetable interpolation analysis/synthesis provides a useful model for the compact representation and rapid synthesis of a wide variety of instrumental tones. Using only a small collection of data—the wavetable banks—and a sparse control stream—for each breakpoint, a time index, a frequency differential, and a wavetable index and weight for each oscillator—the method enables the synthesis of dynamically changing harmonic spectra with only a few oscillators. The technique of grouping tones by pitch range allows the use of different numbers of partials for tones of different pitch without creating synthesis artefacts due to exceeding the Nyquist frequency.

The method would be very suitable for incorporation in a hardware synthesizer, where it would offer both high fidelity and low cost. The model is also useful to electronic music composers in a software synthesis context, since its control stream is quite intuitive due to the use of common breakpoints and spectral ramps. The introduction of an oscillator assignment optimizer provides assurance to hardware designers and composers alike that the resources available, whether hardware oscillators or computational cycles, are used as fully as possible during synthesis.

The thesis [2] for which this report serves as a supplement should be consulted for a complete discussion of the algorithm and its implementation.

References

- [1] HORNER, A. Computation and memory tradeoffs with multiple wavetable interpolation. *Journal of the Audio Engineering Society* 44, 6 (June 1996), 481–496.
- [2] MOHR, J. *Music Analysis/Synthesis by Optimized Multiple Wavetable Interpolation*. PhD thesis, University of Alberta, Edmonton, Alberta, Canada, 2002.
- [3] SERRA, M.-H., RUBINE, D., AND DANNENBERG, R. Analysis and synthesis of tones by spectral interpolation. *Journal of the Audio Engineering Society* 38, 3 (Mar. 1990), 111–128.

A Search Results in Detail

GROUP 1		One-level Exhaustive Search							
		Depth 1		Depth 2		Depth 3		Depth 4	
Instr	Pitch	Time	Error	Time	Error	Time	Error	Time	Error
bsn	A#1	0.11	136.0	4.96	125.0	112	106.4	1646	85.7
	C#2	0.12	244.8	5.03	190.6	112	152.8	1650	129.1
	E2	0.15	177.3	4.94	140.9	112	112.1	1647	87.7
	G2	0.13	252.6	5.00	121.6	112	88.8	1643	73.9
	A#2	0.11	143.6	4.93	113.4	112	74.3	1643	55.4
	C#3	0.11	182.4	5.09	101.1	112	56.2	1643	37.5
clb	C#2	0.12	257.3	4.93	231.8	112	198.4	1644	170.4
	E2	0.12	159.8	4.92	141.9	112	129.4	1653	121.6
	G2	0.12	228.6	4.92	175.0	112	147.1	1643	129.4
	A#2	0.12	214.8	4.95	152.6	112	120.2	1643	106.4
	C#3	0.12	168.0	4.93	138.8	112	104.9	1643	90.4
hrn	E2	0.12	312.6	4.96	187.0	112	108.6	1643	87.1
	G2	0.12	112.0	4.92	81.5	112	56.1	1642	40.3
	A#2	0.11	255.6	4.93	107.8	112	73.7	1642	53.4
	C#3	0.11	94.1	4.93	47.2	112	30.0	1643	21.6
pno	A#1	0.11	133.6	4.92	91.7	112	65.1	1643	51.5
	C#2	0.11	152.1	4.92	110.5	112	98.5	1643	90.0
	E2	0.11	291.0	4.93	204.4	112	155.7	1642	112.9
	G2	0.12	273.7	4.94	166.2	112	95.4	1644	75.4
	A#2	0.12	229.7	4.96	162.2	112	134.1	1643	105.8
	C#3	0.12	126.7	4.93	60.1	112	35.7	1644	27.5
sax	A#1	0.11	107.5	4.93	85.5	112	71.6	1639	58.4
	C#2	0.12	135.9	4.92	119.6	112	97.2	1634	78.4
	E2	0.12	157.3	4.97	138.8	112	122.2	1636	108.0
	G2	0.12	138.1	4.96	109.9	112	82.7	1636	65.7
	A#2	0.11	113.3	4.97	89.7	112	72.0	1635	58.5
	C#3	0.11	178.6	4.95	131.4	112	104.1	1658	84.0
trb	E2	0.12	147.4	4.93	96.1	112	71.6	1671	58.4
	G2	0.11	150.5	4.94	104.5	112	72.5	1633	55.7
	A#2	0.11	126.6	4.93	80.0	112	54.6	1633	40.3
	C#3	0.11	296.4	4.93	164.8	112	124.5	1640	79.5
vla	C#3	0.27	366.1	11.11	297.5	251	239.6	3642	200.9
vlb	A#1	0.23	181.8	9.71	124.0	220	86.2	3184	63.7
	C#2	0.28	186.8	11.11	119.2	252	85.6	3634	72.4
	E2	0.23	169.6	9.73	128.9	220	106.9	3183	90.2
	G2	0.23	100.6	9.74	70.6	221	54.7	3184	46.0
	A#2	0.25	325.6	10.52	165.0	240	113.9	3443	85.6
	C#3	0.23	365.8	9.80	252.0	219	131.5	3183	97.7
vlc	C#2	0.24	171.8	10.42	118.3	231	87.4	3376	71.2
	E2	0.23	110.5	9.73	74.7	217	60.7	3182	52.6
	G2	0.23	204.4	9.74	103.2	217	74.8	3181	55.3
	A#2	0.23	362.3	9.73	174.0	217	104.2	3182	74.3
	C#3	0.25	200.9	10.98	116.3	244	78.2	3584	62.7
Mean		0.15	196.4	6.41	132.9	145	98.6	2114	79.4
Std. Dev.		0.06	76.3	2.40	50.9	54	40.4	772	35.2

Table 1: Group 1, one-level exhaustive search.

GROUP 2		One-level Exhaustive Search							
		Depth 1		Depth 2		Depth 3		Depth 4	
Instr	Pitch	Time	Error	Time	Error	Time	Error	Time	Error
bsn	E3	0.07	209.5	5.21	109.5	181	60.6	4271	33.4
	G3	0.07	817.7	5.16	331.3	181	172.6	4280	78.2
	A♯3	0.07	1150.4	5.15	327.7	180	170.9	4270	58.1
	C♯4	0.07	141.0	5.17	58.2	180	23.8	4271	12.5
	E4	0.08	328.5	5.14	56.2	180	26.6	4270	15.1
cla	E3	0.08	246.2	5.15	191.6	180	144.1	4272	112.9
	G3	0.08	228.7	5.18	176.4	180	96.2	4271	65.2
	A♯3	0.07	156.9	5.33	123.1	180	89.9	4285	66.0
	C♯4	0.08	402.2	5.19	240.1	180	138.5	4274	100.2
	E4	0.08	113.0	5.14	84.2	180	55.6	4254	38.7
clb	E3	0.08	177.2	5.16	95.5	180	77.2	4215	60.9
	G3	0.08	150.8	5.14	114.8	180	72.2	4215	57.0
	A♯3	0.08	391.0	5.14	266.0	180	175.6	4216	113.4
	C♯4	0.07	439.6	5.16	195.4	181	125.8	4216	75.6
eng	E3	0.15	112.8	10.14	71.3	352	48.1	8268	33.1
	G3	0.16	158.3	10.12	84.4	354	60.2	8212	44.6
	A♯3	0.16	137.5	10.04	88.1	354	56.5	8301	42.3
	C♯4	0.15	124.1	10.04	77.5	354	53.8	8477	39.5
	E4	0.16	163.6	10.11	120.7	352	84.6	8201	56.3
flt	E3	0.15	185.0	10.04	77.8	348	46.1	8123	23.7
	G3	0.20	231.8	13.37	110.4	462	42.0	10812	19.8
hrn	E3	0.08	116.6	5.17	47.2	180	20.5	4262	10.7
	G3	0.08	214.5	5.17	94.5	180	46.0	4322	23.8
	A♯3	0.08	293.5	5.35	148.5	180	53.3	4263	24.3
	C♯4	0.08	471.8	5.30	72.9	180	38.1	4236	21.7
	E4	0.08	203.8	5.17	47.8	180	18.0	4216	9.9
obo	A♯3	0.15	213.2	10.12	157.4	352	115.2	8195	83.1
	C♯4	0.16	163.6	10.05	91.9	352	54.6	8198	33.7
	E4	0.15	163.7	10.12	69.7	352	37.9	8236	24.9
pno	E3	0.08	124.7	5.25	70.9	180	48.6	4236	32.0
	G3	0.07	124.7	5.26	69.4	180	47.2	4228	36.2
	A♯3	0.08	156.3	5.46	86.4	180	54.8	4228	34.7
	C♯4	0.07	112.7	5.19	69.4	180	45.1	4218	29.6
	E4	0.08	138.8	5.21	68.3	180	40.5	4219	24.8
sax	E3	0.08	132.3	5.16	80.6	180	58.6	4218	39.9
	G3	0.08	251.5	5.16	190.4	180	131.9	4219	95.9
	A♯3	0.08	187.9	5.16	115.6	180	73.8	4220	50.7
	C♯4	0.08	173.5	5.16	111.6	180	71.6	4218	45.9
	E4	0.07	519.1	5.17	329.0	180	208.5	4219	137.5

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GROUP 2		One-level Exhaustive Search							
		Depth 1		Depth 2		Depth 3		Depth 4	
Instr	Pitch	Time	Error	Time	Error	Time	Error	Time	Error
tpt	G3	0.08	78.8	5.17	64.3	180	49.4	4217	38.4
	A♯3	0.08	207.4	5.17	121.3	180	72.5	4220	47.5
	C♯4	0.08	81.0	5.17	60.4	180	38.7	4252	24.4
	E4	0.08	101.8	5.16	57.3	180	32.5	4243	19.5
trb	E3	0.08	387.3	5.17	231.5	180	99.1	4245	50.4
	G3	0.07	132.8	5.16	72.9	180	38.3	4244	19.7
	A♯3	0.08	184.6	5.16	81.7	180	38.5	4244	22.9
	C♯4	0.08	353.1	5.17	187.4	180	74.9	4246	37.6
	E4	0.08	308.4	5.16	83.1	180	21.6	4239	9.3
vla	E3	0.34	400.8	22.66	340.4	785	256.1	18346	196.3
	G3	0.44	382.4	29.40	305.0	1015	210.0	23698	156.3
	A♯3	0.35	764.8	23.69	414.9	818	306.1	19152	218.0
	C♯4	0.31	325.3	20.34	232.8	702	174.0	16454	124.6
	E4	0.42	536.5	27.38	356.0	937	235.2	21960	164.2
vlb	E3	0.16	187.1	10.15	112.0	350	73.7	8215	53.4
	G3	0.16	148.5	11.18	98.5	386	77.5	9043	57.5
	A♯3	0.15	142.6	10.15	80.1	350	43.7	8208	29.9
	C♯4	0.18	143.9	12.01	80.1	414	54.9	9702	36.7
	E4	0.22	195.4	14.52	97.6	500	60.4	11718	37.1
vlc	E3	0.21	137.3	14.30	95.8	493	70.7	11534	52.6
	G3	0.16	144.6	10.15	89.3	350	58.4	8200	39.6
	A♯3	0.15	193.8	10.16	119.3	350	77.1	8202	50.4
	C♯4	0.18	211.6	12.02	149.6	415	90.8	9698	61.0
	E4	0.21	152.4	13.68	90.3	472	58.0	11035	38.9
vln	G3	0.15	261.4	10.15	191.5	350	139.7	8203	104.7
	A♯3	0.31	302.1	20.76	217.6	714	118.9	16776	77.1
	C♯4	0.38	330.7	25.80	170.7	887	110.9	21040	76.6
	E4	0.21	242.5	14.30	149.0	493	98.0	11664	59.9
Mean		0.14	251.7	9.17	138.4	317	86.0	7450	56.9
Std. Dev.		0.09	183.0	6.06	87.9	208	60.3	4880	43.4

Table 2: Group 2, one-level exhaustive search.

GROUP 4		One-level Exhaustive Search							
		Depth 1		Depth 2		Depth 3		Depth 4	
Instr	Pitch	Time	Error	Time	Error	Time	Error	Time	Error
cla	G5	0.02	459.3	0.63	150.8	14.2	53.3	208	23.4
	A♯5	0.02	103.4	0.62	33.7	14.2	10.8	208	6.1
	C♯6	0.02	267.4	0.62	63.0	14.0	19.9	208	7.9
eng	G5	0.03	91.7	1.21	40.2	27.4	18.1	399	7.0
flt	G5	0.04	77.9	1.77	20.2	41.0	11.2	584	8.4
	A♯5	0.05	60.3	1.81	13.5	41.0	5.4	600	3.1
	C♯6	0.04	47.2	1.56	11.4	34.5	6.3	524	3.4
	E6	0.05	114.5	1.70	19.0	38.6	7.5	567	4.3
glk	G5	0.02	637.0	0.63	424.4	14.1	346.7	208	237.1
	A♯5	0.02	394.4	0.62	365.9	14.0	272.6	208	181.9
	C♯6	0.01	212.1	0.62	154.0	14.0	104.2	208	65.8
	E6	0.02	890.3	0.63	163.4	14.0	129.1	209	62.4
obo	G5	0.03	108.3	1.23	38.2	27.5	12.8	411	4.8
	A♯5	0.03	117.5	1.22	35.5	27.6	13.4	414	6.5
	C♯6	0.03	159.6	1.21	36.7	27.1	12.4	412	4.8
	E6	0.03	61.7	1.25	20.9	27.1	5.5	402	3.1
pno	G5	0.02	69.1	0.62	25.7	14.0	12.6	208	8.6
	A♯5	0.02	21.6	0.62	5.9	13.9	2.9	208	1.7
	C♯6	0.01	21.5	0.63	6.8	14.1	2.8	208	1.5
	E6	0.02	38.0	0.62	8.4	14.1	3.8	208	2.0
sax	G5	0.02	114.6	0.62	54.9	14.9	29.0	208	12.8
	A♯5	0.01	103.9	0.62	31.6	14.0	10.3	208	4.8
	C♯6	0.02	64.0	0.62	21.8	14.1	6.7	208	4.2
tpt	G5	0.01	79.2	0.62	31.3	14.1	9.1	208	4.4
	A♯5	0.02	90.7	0.62	22.9	14.1	6.2	208	3.5
	C♯6	0.02	86.7	0.62	15.6	14.4	5.6	208	2.8
vla	G5	0.12	403.3	4.28	225.4	97.6	109.7	1405	61.9
	A♯5	0.07	173.5	2.60	106.8	58.9	41.4	856	21.8
	C♯6	0.06	97.4	2.20	45.7	48.5	18.5	721	9.6
vlc	G5	0.08	282.0	3.00	123.1	68.1	61.1	984	38.3
vln	G5	0.07	339.0	2.58	139.9	60.8	60.2	848	37.6
	A♯5	0.08	159.8	3.04	83.3	69.1	48.9	992	29.8
	C♯6	0.11	176.4	4.39	80.1	99.9	39.8	1451	25.2
	E6	0.06	151.1	2.68	67.6	60.8	38.9	881	26.4
Mean		0.04	184.5	1.42	79.0	32.2	45.2	470	27.3
Std. Dev.		0.03	187.2	1.08	97.1	24.6	74.9	354	50.1

Table 4: Group 4, one-level exhaustive search.

GROUP 5		One-level Exhaustive Search							
		Depth 1		Depth 2		Depth 3		Depth 4	
Instr	Pitch	Time	Error	Time	Error	Time	Error	Time	Error
flt	G6	0.01	50.3	0.06	24.5	0.30	10.9	0.89	8.0
	A♯6	0.02	55.7	0.09	20.2	0.47	10.5	1.40	7.0
glk	G6	0.01	610.5	0.03	406.6	0.14	247.9	0.42	142.5
	A♯6	0.00	528.0	0.03	424.6	0.15	316.9	0.42	172.6
pno	G6	0.00	19.2	0.03	4.5	0.14	3.5	0.42	2.5
	A♯6	0.00	24.8	0.03	5.8	0.14	2.8	0.42	2.0
vln	G6	0.01	198.4	0.10	59.6	0.48	33.7	1.42	23.0
	A♯6	0.01	161.3	0.11	75.6	0.52	45.1	1.55	31.5
Mean		0.01	206.0	0.06	127.7	0.29	83.9	0.87	48.6
Std. Dev.		0.01	234.2	0.04	179.5	0.17	124.8	0.51	68.5

Table 5: Group 5, one-level exhaustive search.

GROUP 2		Augmented Search								
		2+1			3+1			4+1		
Instr	Pitch	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error
bsn	E3	1.6	6.8	66.2	4.7	185	38.5	6.8	4278	26.1
	G3	1.4	6.6	214.3	2.6	184	110.9	4.2	4284	48.3
	A#3	0.8	6.0	171.1	1.6	182	62.6	2.5	4273	44.4
	C#4	3.2	8.3	25.2	5.8	186	13.1	6.6	4278	8.7
	E4	1.6	6.8	26.6	2.6	183	15.4	6.5	4277	11.4
cla	E3	1.4	6.6	144.1	2.6	183	125.8	4.6	4277	92.5
	G3	1.6	6.8	109.1	2.9	183	73.4	6.2	4277	51.5
	A#3	2.6	7.8	94.8	5.5	186	69.5	6.8	4292	54.0
	C#4	1.6	6.8	195.6	2.6	183	107.7	4.6	4279	78.9
	E4	2.6	7.8	58.1	5.2	185	44.1	7.8	4262	31.8
clb	E3	2.2	7.4	83.0	4.7	185	64.1	6.8	4222	52.5
	G3	2.6	7.8	83.8	5.0	185	57.9	7.2	4222	51.2
	A#3	3.0	8.2	182.1	4.7	185	125.1	6.8	4223	77.8
	C#4	1.6	6.8	128.8	3.9	185	88.1	5.5	4222	54.1
eng	E3	6.3	16.5	49.0	13.7	366	38.0	20.7	8289	27.3
	G3	5.8	16.0	60.6	15.8	370	46.4	26.4	8238	37.6
	A#3	10.5	20.6	58.8	19.4	373	46.3	28.3	8329	37.0
	C#4	7.1	17.3	58.7	18.7	372	42.0	26.4	8503	32.9
	E4	7.9	18.0	87.3	15.7	368	61.0	23.9	8225	46.5
flt	E3	10.1	20.2	47.7	20.4	369	26.0	28.1	8151	16.6
	G3	9.3	22.8	47.6	29.7	492	20.6	51.3	10863	12.3
hrn	E3	3.2	8.3	23.4	5.8	186	13.5	7.8	4270	7.8
	G3	2.8	8.0	50.8	5.2	185	27.6	7.2	4329	15.5
	A#3	2.4	7.5	68.9	3.4	184	35.3	6.2	4269	16.9
	C#4	1.6	6.8	40.2	3.9	184	22.4	5.5	4242	16.1
	E4	1.4	6.5	23.4	3.7	184	10.6	6.0	4222	6.1
obo	A#3	7.1	17.3	118.7	17.2	370	88.7	23.2	8218	69.1
	C#4	8.9	19.1	64.1	19.8	372	40.2	27.1	8225	26.1
	E4	6.3	16.5	39.6	15.2	367	28.0	26.1	8262	18.6
pno	E3	4.0	9.2	50.8	5.5	186	36.7	6.8	4243	26.4
	G3	2.6	7.8	49.0	5.5	186	36.9	6.8	4235	29.7
	A#3	3.2	8.4	58.3	5.5	186	41.0	7.5	4235	28.8
	C#4	2.0	7.2	51.8	3.4	184	32.2	6.5	4225	22.1
	E4	3.0	8.2	42.6	4.0	184	26.4	6.2	4225	21.3
sax	E3	3.6	8.8	61.7	5.8	186	44.4	7.5	4226	33.2
	G3	3.2	8.3	137.5	5.5	186	105.3	6.5	4226	77.5
	A#3	3.2	8.3	76.1	5.8	186	53.1	7.5	4227	41.4
	C#4	3.2	8.3	78.8	5.8	186	54.1	7.5	4225	37.4
	E4	2.0	7.2	229.8	4.7	185	153.7	5.9	4225	110.6

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GROUP 2		Augmented Search								
Depth + 1		2+1			3+1			4+1		
Instr	Pitch	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error
tpt	G3	2.2	7.3	52.7	5.5	186	40.7	6.2	4223	32.6
	A♯3	1.8	7.0	76.4	3.2	183	50.0	4.6	4225	33.6
	C♯4	2.0	7.2	44.7	5.0	185	28.7	6.5	4259	17.5
	E4	2.0	7.2	32.6	5.2	185	22.3	7.5	4251	13.7
trb	E3	2.4	7.6	123.6	4.2	184	65.7	6.2	4251	33.1
	G3	2.6	7.8	42.4	5.2	185	22.7	7.5	4251	12.2
	A♯3	3.0	8.2	45.0	4.2	184	25.4	6.8	4251	14.4
	C♯4	2.2	7.4	91.0	3.7	184	38.1	6.5	4253	17.0
	E4	1.8	7.0	23.2	2.9	183	10.2	4.9	4244	5.2
vla	E3	20.7	43.4	258.7	81.8	866	202.5	127.6	18474	167.8
	G3	48.1	77.5	219.3	160.8	1175	167.8	242.2	23940	131.0
	A♯3	30.2	53.9	313.6	86.2	904	232.6	149.6	19302	178.9
	C♯4	28.7	49.1	176.3	70.2	772	137.4	113.6	16568	100.8
	E4	67.8	95.3	237.9	139.3	1076	176.5	208.3	22168	132.8
vlb	E3	9.3	19.5	79.5	20.6	371	60.8	29.1	8244	46.6
	G3	12.0	23.2	78.1	25.0	411	64.2	33.2	9076	50.4
	A♯3	8.6	18.7	45.4	19.7	370	31.4	28.5	8237	25.9
	C♯4	19.8	31.8	57.9	32.7	447	42.3	42.3	9744	31.4
	E4	26.7	41.2	63.8	46.6	547	42.8	61.3	11779	29.4
vlc	E3	16.4	30.7	74.2	43.1	536	57.1	52.0	11586	42.6
	G3	11.3	21.4	64.5	23.2	373	46.1	29.6	8230	31.2
	A♯3	11.3	21.4	82.0	19.1	369	55.9	28.5	8231	40.7
	C♯4	13.8	25.8	98.9	25.0	440	69.1	39.4	9737	48.2
	E4	16.2	29.9	61.2	37.2	509	43.8	51.5	11086	30.0
vln	G3	8.9	19.1	144.4	20.1	370	113.6	30.2	8233	87.3
	A♯3	30.1	50.9	132.3	70.5	785	84.3	114.5	16891	60.3
	C♯4	52.2	78.0	115.2	131.1	1018	83.2	189.3	21229	61.2
	E4	16.4	30.7	101.0	39.6	532	68.6	54.6	11719	45.7
Mean		9.2	18.4	92.4	21.6	339	62.8	32.1	7482	45.1
Std. Dev.		12.7	18.5	63.4	32.8	240	46.5	49.9	4928	36.1

Table 8: Group 2, two-level exhaustive search with 1-table augmentation.

GROUP 2		Augmented Search								
Depth + 2		1+2			2+2			3+2		
Instr	Pitch	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error
bsn	E3	1.8	1.8	66.2	76	81	36.6	207	388	25.9
	G3	2.0	2.1	218.6	67	72	97.7	115	296	53.5
	A♯3	1.1	1.2	171.1	38	43	76.3	69	249	44.6
	C♯4	3.7	3.8	25.2	151	156	12.5	253	433	8.3
	E4	1.6	1.7	27.5	75	81	15.2	115	295	11.0
cla	E3	1.8	1.9	144.1	66	71	113.7	115	295	95.0
	G3	1.6	1.7	125.7	75	81	74.6	126	306	55.2
	A♯3	2.9	2.9	96.5	123	129	69.2	241	421	54.4
	C♯4	2.0	2.1	196.9	75	80	123.3	115	295	76.7
	E4	3.0	3.1	58.1	122	127	39.3	230	410	32.4
clb	E3	2.7	2.8	83.3	103	108	60.9	207	387	51.9
	G3	2.7	2.8	83.8	122	127	61.7	218	398	49.9
	A♯3	2.9	2.9	184.9	141	146	124.6	207	387	74.9
	C♯4	1.8	1.9	152.9	75	80	82.3	172	353	58.9
eng	E3	7.5	7.7	49.0	294	304	33.7	598	950	26.9
	G3	7.0	7.2	60.6	273	283	45.3	686	1040	37.1
	A♯3	12.0	12.2	58.5	495	505	43.8	847	1201	36.6
	C♯4	8.4	8.6	58.5	330	339	41.9	801	1155	32.1
	E4	8.6	8.7	88.7	364	374	57.3	685	1037	44.4
flt	E3	12.7	12.9	49.7	478	488	24.4	890	1238	15.3
	G3	11.4	11.6	48.6	442	455	20.6	1294	1756	11.6
hrn	E3	3.9	4.0	23.4	153	158	11.1	253	433	7.6
	G3	3.8	3.9	50.8	133	138	26.5	230	410	14.7
	A♯3	3.2	3.3	70.6	114	120	25.1	149	329	18.8
	C♯4	2.5	2.5	40.2	77	82	28.0	172	352	13.8
	E4	1.9	2.0	23.1	67	72	11.3	161	341	6.0
obo	A♯3	7.9	8.1	119.3	331	341	85.1	756	1108	68.3
	C♯4	11.8	11.9	64.1	423	433	39.4	871	1223	25.5
	E4	7.1	7.3	39.6	298	308	25.6	671	1023	18.0
pno	E3	5.1	5.2	51.3	191	197	35.8	242	422	26.6
	G3	3.4	3.5	49.0	126	131	37.0	242	422	28.9
	A♯3	4.1	4.2	65.8	152	157	37.6	241	421	29.8
	C♯4	2.6	2.7	51.8	95	100	35.6	149	329	22.1
	E4	4.4	4.5	44.3	143	149	26.2	172	352	19.8
sax	E3	4.5	4.5	61.6	170	175	40.7	254	434	30.8
	G3	4.1	4.1	137.3	151	156	103.1	241	421	75.9
	A♯3	4.2	4.3	76.1	151	156	51.9	252	432	39.6
	C♯4	4.1	4.2	79.9	151	156	50.6	252	432	37.9
	E4	2.9	2.9	243.7	95	100	158.2	207	387	107.8

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GROUP 2		Augmented Search								
Depth + 2		1+2			2+2			3+2		
Instr	Pitch	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error
tpt	G3	2.8	2.9	52.6	104	109	40.5	242	422	33.9
	A♯3	2.0	2.1	112.0	85	90	54.1	138	318	32.4
	C♯4	2.7	2.8	44.7	95	100	29.8	218	398	18.7
	E4	2.4	2.5	32.6	94	100	20.2	230	410	14.4
trb	E3	3.1	3.2	126.5	113	118	60.2	184	364	35.9
	G3	3.5	3.5	42.4	123	128	22.3	231	411	10.9
	A♯3	3.9	3.9	45.0	141	146	24.7	185	365	12.1
	C♯4	2.8	2.9	90.5	104	109	43.9	162	342	17.0
	E4	2.5	2.6	23.2	85	90	10.0	126	306	5.1
vla	E3	22.0	22.3	258.7	970	993	197.3	3722	4507	161.3
	G3	53.9	54.3	219.3	2260	2290	159.2	7053	8068	128.4
	A♯3	33.1	33.5	313.7	1393	1417	220.4	3769	4587	170.2
	C♯4	30.8	31.1	176.9	1344	1365	125.9	3066	3768	96.9
	E4	78.9	79.3	238.5	3156	3184	167.7	6104	7041	129.3
vlb	E3	11.6	11.8	79.6	438	448	55.8	906	1256	46.0
	G3	15.2	15.3	78.1	562	574	60.5	1094	1480	52.0
	A♯3	9.9	10.0	50.1	402	412	30.4	862	1212	25.0
	C♯4	21.1	21.2	58.2	927	939	39.3	1434	1848	30.5
	E4	28.9	29.1	63.9	1248	1262	39.8	2041	2541	28.2
vlc	E3	18.0	18.2	74.7	769	783	54.7	1896	2389	42.6
	G3	15.8	15.9	64.4	530	540	42.1	1018	1368	32.7
	A♯3	12.8	13.0	83.8	529	540	53.2	843	1193	39.0
	C♯4	16.0	16.1	101.4	647	659	66.3	1102	1517	47.3
	E4	18.3	18.5	61.3	762	775	40.9	1642	2114	29.6
vln	G3	10.3	10.4	144.9	420	430	107.2	922	1272	84.2
	A♯3	33.7	34.0	133.0	1411	1431	80.5	3106	3820	58.9
	C♯4	57.5	57.9	115.7	2434	2459	78.8	5721	6608	58.3
	E4	19.3	19.5	100.0	769	783	60.7	1733	2226	43.0
Mean		10.6	10.8	94.4	432	441	60.7	947	1264	44.4
Std. Dev.		14.2	14.3	64.3	592	598	44.9	1442	1644	35.0

Table 9: Group 2, two-level exhaustive search with 2-table augmentation.

GROUP 4 Depth + 1		Augmented Search								
		2+1			3+1			4+1		
Instr	Pitch	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error	T _{aug}	T _{srch}	Error
cla	G5	0.67	1.30	86.2	0.83	15.1	30.3	1.4	209	17.0
	A♯5	0.49	1.12	12.9	0.94	15.2	7.4	1.4	209	3.8
	C♯6	0.71	1.34	26.7	1.33	15.3	11.1	1.4	209	5.4
eng	G5	1.66	2.87	20.2	3.68	31.1	8.8	5.4	404	4.7
flt	G5	3.56	5.33	11.4	8.30	49.3	8.8	10.9	595	6.7
	A♯5	4.10	5.92	5.6	9.47	50.5	3.6	12.6	613	2.5
	C♯6	2.57	4.13	6.7	6.43	41.0	4.0	8.9	533	2.6
	E6	2.12	3.82	7.8	6.25	44.9	4.8	10.6	578	2.8
glk	G5	0.24	0.88	349.8	0.70	14.8	276.6	1.2	209	196.9
	A♯5	0.41	1.04	321.7	0.80	14.8	245.8	1.2	209	146.3
	C♯6	0.56	1.18	124.9	0.89	14.9	86.4	1.4	209	43.1
	E6	0.26	0.89	129.1	0.85	14.8	83.8	1.4	210	43.5
obo	G5	2.16	3.37	15.2	4.36	31.9	5.9	5.5	417	3.3
	A♯5	2.66	3.91	15.3	4.34	31.9	8.1	5.6	420	5.0
	C♯6	2.51	3.72	12.8	4.24	31.4	5.4	5.5	418	3.1
	E6	1.21	2.42	5.8	3.43	30.6	3.5	5.5	407	2.1
pno	G5	0.83	1.46	17.6	1.18	15.2	11.1	1.5	209	6.9
	A♯5	0.72	1.34	3.4	1.15	15.1	2.2	1.5	209	1.3
	C♯6	0.79	1.43	3.5	1.14	15.2	1.8	1.4	209	1.3
	E6	0.86	1.48	4.1	1.19	15.2	2.9	1.5	209	1.5
sax	G5	0.60	1.22	31.5	0.93	15.8	15.7	1.4	209	9.6
	A♯5	0.61	1.23	11.4	1.08	15.1	6.1	1.4	209	3.7
	C♯6	0.60	1.23	6.8	1.04	15.1	5.1	1.4	209	3.5
tpt	G5	0.61	1.23	10.1	1.13	15.2	5.6	1.5	209	3.4
	A♯5	0.79	1.41	7.1	1.19	15.2	4.1	1.5	209	2.9
	C♯6	0.79	1.41	5.9	1.30	15.7	3.6	1.5	209	2.1
vla	G5	17.38	21.67	121.9	51.71	149.3	71.2	66.8	1472	47.4
	A♯5	3.69	6.31	48.8	15.86	74.7	24.1	24.7	881	16.1
	C♯6	5.50	7.71	20.2	14.26	62.8	11.8	18.1	739	7.2
vlc	G5	13.77	16.78	66.6	26.71	94.8	42.9	34.1	1018	30.9
vln	G5	6.37	8.94	64.3	15.96	76.8	41.3	24.4	872	29.0
	A♯5	12.04	15.05	50.8	24.62	93.8	34.2	35.0	1027	22.9
	C♯6	22.02	26.40	46.4	55.01	154.9	28.5	75.5	1526	20.3
	E6	10.52	13.18	40.7	21.46	82.2	29.5	27.5	908	22.4
Mean		3.66	5.08	50.4	8.64	40.9	33.4	11.8	482	21.2
Std. Dev.		5.31	6.34	80.9	13.47	37.7	62.2	18.1	371	40.9

Table 12: Group 4, two-level exhaustive search with 1-table augmentation.

GROUP 4 Depth + 2		Augmented Search								
		1+2			2+2			3+2		
Instr	Pitch	T_{aug}	T_{srch}	Error	T_{aug}	T_{srch}	Error	T_{aug}	T_{srch}	Error
cla	G5	0.88	0.90	85.2	20	21	25.9	28	42	17.0
	A♯5	0.68	0.70	12.9	15	15	6.9	27	41	4.0
	C♯6	0.92	0.94	27.7	21	22	9.0	34	48	4.8
eng	G5	2.19	2.22	20.4	50	51	7.2	98	125	4.3
flt	G5	4.03	4.08	11.4	107	108	8.6	222	263	6.5
	A♯5	4.43	4.48	5.6	122	124	3.1	268	309	2.2
	C♯6	3.07	3.11	6.7	77	78	3.6	168	202	2.3
	E6	2.03	2.07	8.0	63	65	4.4	173	212	2.7
glk	G5	0.39	0.41	349.8	7	7	237.5	20	34	199.6
	A♯5	0.56	0.58	322.6	12	13	205.9	22	36	151.9
	C♯6	0.66	0.68	139.7	17	18	88.7	27	41	62.9
	E6	0.38	0.39	129.1	8	9	62.8	24	38	42.8
obo	G5	2.66	2.70	15.1	65	66	5.2	122	150	2.9
	A♯5	2.86	2.89	15.8	80	81	7.1	122	150	4.6
	C♯6	2.85	2.88	12.8	76	78	4.8	119	146	3.0
	E6	1.44	1.48	5.8	36	38	3.2	96	123	2.0
pno	G5	1.23	1.25	16.9	24	24	9.7	34	48	7.4
	A♯5	0.88	0.90	3.4	21	22	1.8	32	46	1.5
	C♯6	1.12	1.14	3.6	24	24	1.8	32	46	1.1
	E6	1.17	1.17	4.1	26	27	2.4	34	48	1.3
sax	G5	0.73	0.74	31.5	18	19	13.9	27	42	8.6
	A♯5	0.81	0.82	11.3	18	19	4.9	31	45	3.6
	C♯6	0.82	0.83	6.8	18	19	4.2	29	43	3.3
tpt	G5	0.73	0.75	9.9	18	19	4.9	32	46	3.2
	A♯5	0.95	0.97	7.1	24	25	3.6	34	48	2.6
	C♯6	0.95	0.97	6.1	24	24	2.9	34	48	2.1
vla	G5	18.55	18.60	121.7	517	521	67.0	1390	1488	43.4
	A♯5	4.17	4.25	49.1	111	113	23.5	432	491	14.8
	C♯6	5.36	5.41	21.2	166	168	10.3	378	426	6.3
vlc	G5	15.30	15.38	67.6	413	416	40.1	735	803	28.9
vln	G5	6.11	6.17	64.9	192	194	38.7	439	500	27.2
	A♯5	14.18	14.28	50.7	362	365	30.7	681	750	21.3
	C♯6	22.65	22.77	48.4	665	669	26.2	1514	1614	18.7
	E6	12.62	12.69	40.7	318	321	27.3	587	648	21.2
Mean		4.07	4.11	51.0	110	111	29.3	237	269	21.5
Std. Dev.		5.68	5.70	81.4	160	161	53.2	367	391	42.1

Table 13: Group 4, two-level exhaustive search with 2-table augmentation.

GROUP 5 Depth + 1		Augmented Search								
		2+1			3+1			4+1		
Instr	Pitch	T_{aug}	T_{srch}	Error	T_{aug}	T_{srch}	Error	T_{aug}	T_{srch}	Error
flt	G6	0.22	0.28	11.0	0.44	0.74	8.0	0.78	1.67	5.9
	A♯6	0.69	0.78	10.5	1.39	1.86	7.0	1.88	3.28	5.0
glk	G6	0.07	0.10	248.0	0.09	0.23	193.1	0.21	0.63	97.5
	A♯6	0.06	0.09	317.1	0.11	0.26	172.6	0.15	0.57	86.9
pno	G6	0.10	0.13	3.5	0.20	0.34	2.9	0.23	0.65	2.2
	A♯6	0.10	0.13	3.0	0.18	0.32	2.1	0.23	0.65	1.7
vln	G6	0.48	0.58	34.2	1.36	1.84	23.5	2.12	3.55	18.1
	A♯6	0.55	0.66	45.1	1.19	1.71	31.9	1.92	3.47	24.7
Mean		0.28	0.34	84.1	0.62	0.91	55.1	0.94	1.81	30.3
Std. Dev.		0.25	0.28	124.8	0.59	0.76	79.7	0.88	1.39	39.2

Table 14: Group 5, two-level exhaustive search with 1-table augmentation.

GROUP 5 Depth + 2		Augmented Search								
		1+2			2+2			3+2		
Instr	Pitch	T_{aug}	T_{srch}	Error	T_{aug}	T_{srch}	Error	T_{aug}	T_{srch}	Error
flt	G6	0.23	0.24	11.1	1.29	1.35	8.0	2.36	2.66	5.7
	A♯6	0.76	0.78	10.6	4.11	4.21	7.0	7.22	7.69	4.9
glk	G6	0.09	0.09	248.3	0.43	0.46	151.8	0.51	0.65	98.0
	A♯6	0.08	0.09	317.1	0.34	0.37	198.3	0.62	0.77	86.9
pno	G6	0.17	0.17	3.5	0.65	0.68	2.5	1.09	1.23	2.1
	A♯6	0.19	0.19	3.0	0.65	0.68	2.0	0.99	1.13	1.6
vln	G6	0.54	0.55	35.8	2.58	2.68	23.2	7.13	7.61	18.0
	A♯6	0.67	0.68	45.1	3.38	3.49	31.5	6.13	6.65	24.5
Mean		0.34	0.35	84.3	1.68	1.74	53.0	3.26	3.55	30.2
Std. Dev.		0.27	0.28	124.7	1.48	1.51	77.0	3.03	3.19	39.4

Table 15: Group 5, two-level exhaustive search with 2-table augmentation.

B Optimization Results in Detail

GROUP 1 3 Oscillators		Three-Table Match Optimization								
		3+0			2+1			1+2		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
bsn	A#1	2.90	115	114.5	2.61	11.7	116.6	2.49	7.39	116.8
	C#2	1.74	114	171.7	1.80	9.7	171.7	1.79	5.49	171.7
	E2	1.31	113	118.3	1.50	8.8	123.1	1.37	4.27	123.1
	G2	2.25	115	103.6	2.32	11.6	106.8	2.32	7.32	106.8
	A#2	2.50	114	90.1	2.60	11.1	90.4	2.59	7.09	90.4
	C#3	1.96	114	69.7	1.69	9.6	69.7	1.68	5.38	69.7
clb	C#2	1.31	113	206.9	0.49	7.5	214.2	0.50	2.80	214.2
	E2	3.25	115	136.6	2.31	11.3	136.2	1.68	6.08	134.9
	G2	1.95	114	161.3	1.57	9.8	160.2	0.57	4.17	158.1
	A#2	1.52	114	135.3	1.23	9.7	135.3	0.95	5.15	135.3
	C#3	1.77	114	121.9	1.29	8.9	118.4	1.44	4.34	119.7
hrn	E2	1.19	113	121.4	1.08	9.0	121.3	1.05	5.15	121.2
	G2	2.02	114	65.7	1.55	9.2	65.6	1.61	5.01	65.6
	A#2	0.89	113	86.6	0.89	8.5	87.8	0.96	4.96	86.6
	C#3	2.52	115	39.1	2.17	11.0	39.1	2.29	7.09	39.1
pno	A#1	1.49	113	74.2	1.20	9.1	75.0	1.18	4.88	75.0
	C#2	4.03	116	109.0	2.81	11.8	107.8	2.20	7.00	107.9
	E2	1.99	114	188.3	1.06	9.6	188.3	1.29	6.79	181.3
	G2	2.75	115	115.6	1.81	10.3	137.5	1.26	5.96	128.1
	A#2	1.68	114	155.8	1.39	9.9	152.5	1.88	7.38	152.5
	C#3	2.22	114	49.3	2.01	11.3	51.2	2.02	7.42	51.2
sax	A#1	2.32	114	82.4	1.76	11.1	83.2	1.76	7.16	83.2
	C#2	2.39	114	108.1	1.54	10.6	107.5	1.41	6.31	107.5
	E2	1.91	114	129.0	1.39	10.0	129.0	1.28	5.68	129.0
	G2	2.30	114	95.4	2.16	11.0	95.4	1.72	6.12	95.4
	A#2	1.77	114	84.2	1.53	11.0	82.3	1.54	7.14	82.3
	C#3	2.06	114	120.7	2.11	11.9	118.4	2.06	8.16	118.2
trb	E2	1.04	113	79.2	0.96	8.6	79.2	0.91	4.31	79.2
	G2	1.96	114	83.7	1.60	10.1	83.7	1.60	6.30	83.7
	A#2	1.64	114	65.0	1.59	10.9	65.0	1.58	7.38	65.0
	C#3	2.21	114	141.7	1.33	8.6	141.4	1.40	4.60	138.0
vla	C#3	4.84	256	270.6	4.75	30.2	269.8	4.94	22.24	268.8
vlb	A#1	4.85	225	107.7	4.36	27.3	108.7	4.35	20.45	108.7
	C#2	7.49	259	103.1	6.72	33.0	103.0	6.87	24.67	103.1
	E2	6.68	227	123.5	4.24	29.0	121.7	4.26	23.36	121.7
	G2	6.58	228	69.5	6.44	34.2	69.5	6.07	26.87	69.5
	A#2	2.83	243	127.2	2.78	24.5	127.2	2.85	16.05	127.2
	C#3	2.29	221	162.6	1.86	17.0	162.6	1.88	8.38	162.6
vlc	C#2	4.70	236	105.0	4.66	24.5	105.7	4.11	14.91	105.7
	E2	7.01	224	71.2	6.87	28.3	71.2	6.93	20.23	71.2
	G2	5.14	222	96.4	5.33	29.7	96.2	5.11	21.71	96.2
	A#2	2.06	219	122.6	1.68	16.7	117.3	2.07	10.17	126.5
	C#3	5.93	250	95.9	5.64	27.0	95.9	5.08	18.48	96.4
Mean		2.87	148	113.5	2.48	14.8	114.0	2.39	9.58	113.7
Std. Dev.		1.75	55	43.5	1.68	8.1	43.7	1.66	6.67	43.2

Table 16: Group 1, 3-oscillator, 3-table match optimization.

GROUP 1 3 Oscillators		One- and Two-Table Match Optimization								
		2+0			1+1			1+0		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
bsn	A#1	0.49	5.45	124.0	0.49	1.23	124.2	0.70	0.81	124.1
	C#2	0.34	5.37	180.2	0.34	1.08	180.2	0.77	0.89	180.2
	E2	0.24	5.18	137.6	0.21	0.74	137.6	0.77	0.92	137.6
	G2	0.44	5.44	117.5	0.39	0.92	117.5	0.79	0.92	117.5
	A#2	1.55	6.48	94.9	1.55	2.50	94.9	2.30	2.41	94.9
	C#3	1.46	6.55	72.1	1.45	2.19	72.1	1.77	1.88	72.1
clb	C#2	0.74	5.67	215.0	0.56	1.09	214.2	0.98	1.10	214.2
	E2	1.18	6.10	137.3	1.04	1.58	137.2	1.45	1.57	137.2
	G2	0.96	5.88	160.9	0.64	1.59	161.0	1.46	1.58	161.0
	A#2	0.89	5.84	135.3	1.26	2.21	135.3	1.75	1.87	135.3
	C#3	1.00	5.93	133.4	0.90	1.43	133.4	1.35	1.47	133.4
hrn	E2	1.15	6.11	121.6	1.13	2.28	121.5	1.23	1.35	162.3
	G2	0.90	5.82	70.4	0.90	1.63	70.4	1.01	1.13	70.4
	A#2	1.38	6.31	82.9	1.23	2.59	82.9	0.85	0.96	88.1
	C#3	2.45	7.38	39.3	1.91	2.86	39.3	2.37	2.48	39.3
pno	A#1	0.25	5.17	81.0	0.21	0.95	81.0	0.69	0.80	81.0
	C#2	0.55	5.47	108.2	0.41	1.36	108.3	0.88	0.99	108.3
	E2	1.68	6.61	191.0	1.17	3.37	189.7	1.80	1.91	200.2
	G2	0.28	5.22	148.8	0.22	1.58	150.3	1.08	1.20	152.6
	A#2	1.45	6.41	156.7	1.48	3.05	172.5	1.70	1.82	172.8
	C#3	0.48	5.41	53.5	0.49	1.43	53.5	1.34	1.46	53.5
sax	A#1	0.46	5.39	83.2	0.45	1.40	85.7	0.59	0.70	86.3
	C#2	0.32	5.24	115.5	0.33	1.07	115.5	0.50	0.62	115.5
	E2	0.22	5.19	136.5	0.22	0.76	136.6	0.64	0.76	136.6
	G2	1.24	6.20	97.9	1.18	1.72	97.9	1.63	1.75	97.8
	A#2	2.19	7.16	84.3	2.38	3.53	84.3	2.21	2.32	84.4
	C#3	0.65	5.60	124.9	0.31	1.75	126.9	0.90	1.01	126.9
trb	E2	0.21	5.14	89.1	0.25	1.16	84.9	0.69	0.81	84.9
	G2	0.26	5.20	90.8	0.26	1.42	90.8	0.70	0.81	96.9
	A#2	0.38	5.31	73.3	0.39	1.85	73.3	0.60	0.71	82.4
	C#3	0.87	5.80	137.6	0.84	2.00	137.6	1.63	1.74	137.6
vla	C#3	0.87	11.98	286.8	0.77	3.40	286.7	2.16	2.43	287.0
vlb	A#1	0.73	10.44	117.4	0.74	3.44	117.4	1.22	1.45	118.5
	C#2	1.02	12.13	106.5	1.10	4.19	116.5	2.35	2.63	116.4
	E2	1.05	10.78	125.8	1.01	4.91	125.8	3.28	3.51	127.2
	G2	1.19	10.93	69.7	1.21	3.49	69.7	2.36	2.59	69.7
	A#2	2.57	13.09	125.6	2.46	4.04	125.6	3.87	4.12	125.6
	C#3	2.13	11.93	159.1	2.15	3.20	159.1	2.85	3.08	159.1
vlc	C#2	0.83	11.25	107.9	0.70	2.68	107.9	1.96	2.20	107.9
	E2	0.80	10.53	71.2	0.75	2.12	71.2	1.22	1.45	71.2
	G2	2.79	12.53	96.8	2.83	4.85	96.8	2.37	2.60	98.9
	A#2	0.82	10.55	144.5	0.72	3.01	155.7	2.07	2.30	162.5
	C#3	1.34	12.32	106.0	1.20	3.17	110.9	2.51	2.76	110.9
Mean		1.00	7.41	118.9	0.94	2.25	119.9	1.52	1.67	121.9
Std. Dev.		0.66	2.69	45.2	0.65	1.11	45.6	0.80	0.84	46.2

Table 17: Group 1, 3-oscillator, 2- and 1-table match optimization.

GROUP 2 3 Oscillators		Three-Table Match Optimization								
		3+0			2+1			1+2		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
bsn	E3	1.89	183	89.8	1.30	8.1	90.0	1.30	3.1	90.0
	G3	1.83	183	289.2	1.94	8.5	300.1	1.35	3.5	271.8
	A♯3	0.35	180	189.6	0.29	6.3	185.8	0.29	1.5	185.8
	C♯4	2.60	183	47.1	2.14	10.4	46.9	1.95	5.8	46.9
	E4	0.73	181	38.9	0.51	7.3	38.9	0.35	2.0	38.9
cla	E3	1.03	181	151.1	0.69	7.3	151.0	0.62	2.5	151.0
	G3	1.00	181	130.6	0.46	7.3	143.0	0.54	2.2	137.4
	A♯3	2.80	183	112.6	2.03	9.8	114.0	1.73	4.6	110.2
	C♯4	1.14	181	295.8	1.33	8.1	227.7	0.90	3.0	228.3
	E4	2.84	183	68.4	1.48	9.3	67.7	1.47	4.6	67.7
clb	E3	1.98	182	91.7	1.90	9.3	86.1	1.35	4.2	86.1
	G3	1.86	182	124.4	1.34	9.1	117.0	1.31	4.1	117.0
	A♯3	2.38	182	234.4	1.50	9.7	229.2	1.26	4.2	223.9
	C♯4	1.39	182	236.2	1.22	8.0	235.1	0.60	2.5	163.4
	E4	1.98	182	91.7	1.90	9.3	86.1	1.35	4.2	86.1
eng	E3	2.84	355	57.1	2.26	18.8	57.1	2.19	9.9	57.1
	G3	3.78	358	77.0	3.69	19.7	76.5	3.68	10.9	76.5
	A♯3	4.92	359	75.8	3.49	24.1	74.2	3.70	15.9	74.2
	C♯4	4.37	358	70.2	2.60	19.9	68.9	2.68	11.3	70.4
	E4	3.45	355	113.3	3.28	21.3	108.4	3.45	12.1	106.2
flt	E3	7.75	356	77.8	5.61	25.8	78.3	5.88	18.8	77.2
	G3	8.28	470	104.2	6.76	29.6	102.1	6.89	18.5	99.6
hrn	E3	2.89	183	45.1	2.68	11.0	40.6	2.66	6.7	40.6
	G3	2.95	183	87.0	2.36	10.4	80.3	2.36	6.3	80.3
	A♯3	1.35	181	119.5	1.37	8.9	149.7	1.45	4.8	116.4
	C♯4	1.42	181	66.6	0.71	7.5	43.6	0.63	3.1	42.9
	E4	1.02	181	38.7	0.63	7.1	36.6	0.57	2.6	36.6
obo	A♯3	4.43	356	135.5	2.82	20.1	133.6	2.76	10.9	134.4
	C♯4	5.74	358	91.8	4.26	23.4	81.5	3.78	15.7	81.5
	E4	2.91	355	54.5	2.63	19.1	52.6	2.64	9.9	52.6
pno	E3	4.39	184	73.5	3.28	12.5	67.4	3.21	8.4	68.1
	G3	2.24	182	60.8	1.44	9.2	57.8	1.50	5.0	57.8
	A♯3	4.05	184	88.9	3.38	11.8	77.3	2.39	6.6	82.9
	C♯4	1.68	182	87.5	0.95	8.2	66.1	0.95	3.7	66.1
	E4	1.34	181	51.9	1.17	9.4	51.9	1.31	5.8	51.9
sax	E3	3.31	183	79.3	2.69	11.5	76.3	2.23	6.7	76.2
	G3	3.43	183	172.1	2.52	10.8	168.3	2.04	6.1	169.5
	A♯3	2.35	182	94.8	1.93	10.2	95.4	1.94	6.2	95.4
	C♯4	3.56	184	104.0	2.27	10.6	105.2	2.04	6.2	106.7
	E4	2.94	183	295.1	1.31	8.5	274.3	1.29	4.2	291.4

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GROUP 2 3 Oscillators		Three-Table Match Optimization								
		3+0			2+1			1+2		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
tpt	G3	2.62	183	59.1	1.88	9.2	59.0	1.74	4.6	59.3
	A♯3	1.66	182	94.7	0.86	7.9	93.4	0.44	2.5	119.6
	C♯4	3.17	183	52.8	1.59	8.8	51.9	1.38	4.2	51.9
	E4	2.13	182	38.2	1.24	8.4	38.2	1.19	3.7	38.2
trb	E3	1.69	182	164.8	1.74	9.3	163.2	0.94	4.1	156.4
	G3	2.43	182	66.6	2.17	10.0	62.2	2.14	5.6	62.2
	A♯3	1.42	181	62.5	1.07	9.3	62.5	1.05	5.0	62.5
	C♯4	1.54	182	123.1	1.07	8.5	113.9	1.10	4.0	122.5
	E4	1.03	181	25.6	0.69	7.7	42.3	0.65	3.2	42.3
vla	E3	8.79	794	296.2	7.50	50.9	296.5	7.54	29.8	296.3
	G3	17.69	1033	262.7	16.77	94.3	267.0	16.37	70.7	267.0
	A♯3	11.92	830	384.6	10.55	64.5	383.4	9.49	43.0	383.4
	C♯4	12.03	714	220.5	10.22	59.3	218.7	9.32	40.4	218.3
	E4	22.06	959	325.4	21.81	117.1	325.1	21.21	100.5	325.8
vlb	E3	7.32	357	111.7	5.53	25.0	106.5	5.56	17.4	104.4
	G3	6.55	393	93.4	5.46	28.7	93.0	5.29	20.6	92.9
	A♯3	4.95	355	62.9	4.23	22.9	62.2	3.44	13.4	63.8
	C♯4	13.72	428	94.0	11.69	43.5	89.5	11.80	33.0	89.0
	E4	12.93	513	109.3	13.68	54.9	107.0	12.50	41.6	106.4
vlc	E3	11.60	505	101.6	8.76	39.5	100.7	9.40	27.6	98.3
	G3	8.72	359	91.3	6.80	28.2	84.4	6.21	22.1	84.4
	A♯3	9.55	360	122.8	7.80	29.2	119.0	7.00	20.0	116.6
	C♯4	12.92	428	144.1	8.97	34.8	143.3	7.74	23.8	143.4
	E4	13.38	485	102.9	11.00	40.9	101.2	10.84	29.3	100.5
vln	G3	5.05	355	177.2	3.95	23.1	173.7	3.36	13.8	173.4
	A♯3	17.19	731	214.6	12.68	63.6	207.9	12.78	46.8	207.9
	C♯4	21.35	908	179.4	20.69	98.7	177.3	19.65	77.5	177.8
	E4	9.12	502	149.5	7.95	38.6	146.7	7.25	26.8	145.7
Mean		5.31	323	124.7	4.37	22.8	121.6	4.13	14.9	120.0
Std. Dev.		5.12	213	79.7	4.73	23.1	78.2	4.60	18.8	76.4

Table 18: Group 2, 3-oscillator, 3-table match optimization.

GROUP 2 3 Oscillators		One- and Two-Table Match Optimization								
		2+0			1+1			1+0		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
bsn	E3	0.16	5.4	98.4	0.17	0.39	98.4	0.31	0.38	98.4
	G3	1.30	6.5	275.7	0.36	1.01	362.4	0.89	0.96	281.0
	A♯3	0.75	6.0	185.8	0.76	1.12	185.8	1.10	1.17	185.8
	C♯4	0.36	5.6	52.0	0.36	1.15	51.5	0.69	0.76	53.5
	E4	1.17	6.4	38.9	0.76	1.26	38.9	0.55	0.63	49.3
cla	E3	0.80	6.0	157.6	0.75	1.26	157.6	1.11	1.19	157.6
	G3	2.06	7.3	134.8	1.26	1.77	137.4	1.49	1.57	137.4
	A♯3	0.35	5.5	115.5	0.23	0.74	117.3	0.66	0.73	116.7
	C♯4	1.13	6.3	216.7	0.96	1.62	216.4	1.03	1.11	216.4
	E4	0.27	5.5	77.6	0.24	0.89	77.0	0.51	0.59	78.1
clb	E3	0.95	6.2	86.1	0.73	1.09	86.1	0.99	1.07	86.1
	G3	1.05	6.2	102.7	0.89	1.25	102.7	1.18	1.26	102.7
	A♯3	0.48	5.7	241.1	1.08	1.60	237.5	1.50	1.58	237.5
	C♯4	1.87	7.1	163.4	0.86	1.36	163.1	0.62	0.69	177.4
	E4	0.61	10.7	114.7	0.45	1.71	115.0	1.32	1.48	115.0
flt	E3	1.66	11.9	79.9	1.46	4.11	82.1	2.63	2.78	79.8
	G3	1.33	14.8	104.7	1.21	3.97	105.1	3.08	3.28	101.4
hrn	E3	0.39	5.6	43.3	0.39	1.17	43.3	0.60	0.68	44.4
	G3	1.79	7.0	80.0	1.80	2.86	80.0	2.27	2.35	80.2
	A♯3	1.81	7.0	114.4	1.59	2.23	114.1	1.44	1.52	119.6
	C♯4	0.96	6.2	42.9	0.88	1.80	42.9	1.64	1.72	42.9
	E4	1.49	6.7	30.5	1.63	2.41	30.5	1.78	1.86	30.6
obo	A♯3	1.52	11.6	138.9	1.71	2.69	142.2	2.24	2.39	142.2
	C♯4	0.80	11.0	89.2	0.76	3.67	89.2	0.68	0.84	99.0
	E4	0.38	10.6	58.0	0.37	1.36	58.2	1.19	1.34	58.2
pno	E3	1.06	6.3	70.6	0.74	2.51	72.8	1.79	1.87	76.2
	G3	0.27	5.5	59.3	0.28	0.93	59.3	0.88	0.95	59.3
	A♯3	0.67	5.9	93.8	0.64	1.43	91.1	1.25	1.33	91.6
	C♯4	1.19	6.4	67.0	1.19	1.85	67.0	0.93	1.00	67.2
	E4	0.37	5.6	65.9	1.12	3.03	65.3	1.30	1.38	88.4
sax	E3	0.53	5.7	79.2	0.54	1.46	78.7	0.64	0.72	82.3
	G3	0.62	5.8	182.3	0.48	1.40	183.3	1.10	1.18	184.0
	A♯3	0.42	5.6	108.7	0.42	1.48	108.7	0.49	0.57	108.7
	C♯4	1.33	6.5	105.9	1.19	2.11	105.9	1.85	1.93	107.5
	E4	1.13	6.3	286.1	1.08	2.01	286.3	1.42	1.49	286.3

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GROUP 2 3 Oscillators		One- and Two-Table Match Optimization								
		2+0			1+1			1+0		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
tpt	G3	0.30	5.5	63.2	0.27	0.92	63.7	0.51	0.59	63.7
	A♯3	1.13	6.3	102.6	0.84	1.48	133.9	1.23	1.31	133.9
	C♯4	0.19	5.4	54.1	0.22	1.01	53.9	0.48	0.56	53.9
	E4	0.15	5.4	54.4	0.15	0.66	54.5	0.57	0.65	54.4
trb	E3	1.38	6.6	179.5	1.07	1.86	179.4	0.93	1.01	194.5
	G3	0.40	5.6	71.1	0.39	1.32	71.2	0.44	0.51	72.7
	A♯3	0.58	5.8	76.3	0.62	1.54	76.3	0.69	0.77	75.7
	C♯4	1.25	6.5	113.8	0.98	2.04	113.8	1.11	1.19	113.8
	E4	0.77	6.0	42.9	0.76	1.55	42.9	0.52	0.60	81.7
vla	E3	5.01	27.6	315.0	4.85	7.04	314.7	6.63	6.97	314.7
	G3	2.22	31.6	288.1	2.23	9.07	288.1	3.77	4.21	288.5
	A♯3	1.68	25.4	394.7	1.40	6.27	395.0	4.01	4.36	394.5
	C♯4	1.94	22.2	224.5	4.73	8.37	223.8	5.78	6.09	223.9
	E4	3.75	31.2	337.0	3.85	13.17	336.8	6.45	6.87	338.2
vlb	E3	3.16	13.4	106.7	0.93	2.47	109.4	2.32	2.48	109.4
	G3	2.24	13.4	95.5	1.91	5.11	95.5	1.59	1.75	96.0
	A♯3	2.77	13.0	66.4	3.34	4.33	79.2	4.22	4.37	79.2
	C♯4	2.77	14.8	88.0	2.33	5.45	87.7	4.70	4.88	87.2
	E4	3.40	17.9	105.5	3.02	8.74	106.0	5.33	5.55	106.5
vlc	E3	1.68	16.0	97.2	1.71	3.48	97.2	3.44	3.65	97.2
	G3	1.28	11.4	89.8	1.29	5.57	89.9	2.97	3.13	87.2
	A♯3	1.32	11.5	118.1	1.16	3.51	117.3	2.22	2.37	117.3
	C♯4	2.36	14.4	148.4	1.95	5.07	146.5	2.45	2.63	149.5
	E4	2.50	16.2	100.4	2.12	5.68	100.2	4.67	4.88	99.7
vln	G3	0.90	11.1	188.2	0.64	2.46	190.5	1.91	2.06	188.3
	A♯3	2.01	22.8	209.8	1.78	4.92	210.2	3.31	3.62	210.4
	C♯4	3.72	29.5	177.6	3.49	9.50	176.8	7.86	8.24	176.5
	E4	1.47	15.8	145.2	1.45	4.38	143.8	2.22	2.43	144.0
Mean		1.34	10.5	124.0	1.25	2.98	126.0	2.00	2.13	126.6
Std. Dev.		1.00	6.8	78.2	1.03	2.51	81.2	1.68	1.76	77.4

Table 19: Group 2, 3-oscillator, 2- and 1-table match optimization.

GROUP 4 3 Oscillators		Three-Table Match Optimization								
		3+0			2+1			1+2		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
cla	G5	2.92	16.9	232.5	2.22	3.5	160.8	2.27	3.2	163.1
	A \sharp 5	2.05	16.1	22.4	1.67	2.8	23.0	1.68	2.4	23.0
	C \sharp 6	3.44	17.4	65.9	1.98	3.3	63.3	1.75	2.6	58.2
eng	G5	4.32	31.3	38.2	3.96	6.9	37.6	3.89	6.1	38.4
flt	G5	8.32	49.3	19.2	6.80	12.1	18.5	6.88	11.0	18.4
	A \sharp 5	13.00	54.0	11.6	11.32	17.2	11.4	10.17	14.7	11.6
	C \sharp 6	9.20	44.2	11.4	7.44	11.5	11.4	7.53	10.6	11.4
	E6	6.76	45.8	14.3	6.40	10.2	13.7	5.85	7.9	12.8
glk	G5	1.48	15.5	365.0	0.58	1.5	362.2	0.51	0.9	362.2
	A \sharp 5	1.83	15.8	306.8	1.08	2.1	338.3	0.30	0.9	338.3
	C \sharp 6	3.01	17.0	130.7	1.94	3.1	147.2	1.36	2.1	151.6
	E6	1.51	15.5	145.9	1.08	2.0	145.9	0.96	1.4	145.9
obo	G5	6.76	34.8	43.9	4.54	7.9	38.8	4.96	7.7	38.8
	A \sharp 5	8.82	36.8	43.4	6.39	10.3	40.9	6.11	9.0	41.5
	C \sharp 6	5.27	32.3	44.4	4.02	7.7	44.8	4.08	7.0	44.9
	E6	3.29	30.3	11.6	3.02	5.4	12.1	3.08	4.6	12.1
pno	G5	4.05	18.1	28.7	1.38	2.9	23.8	2.57	3.8	23.6
	A \sharp 5	3.14	17.1	6.1	2.00	3.3	5.4	1.94	2.8	5.4
	C \sharp 6	3.95	17.9	7.4	3.17	4.6	6.8	3.59	4.7	6.3
	E6	4.06	18.1	11.2	2.41	3.9	10.0	2.41	3.6	10.0
sax	G5	2.26	17.3	49.6	1.05	2.2	46.9	1.29	2.0	46.7
	A \sharp 5	2.25	16.2	23.4	1.70	2.9	28.4	2.21	3.0	23.8
	C \sharp 6	2.06	16.1	14.6	1.79	3.0	14.8	1.74	2.5	14.8
tpt	G5	2.63	16.6	29.5	1.81	3.0	25.2	2.06	2.9	25.2
	A \sharp 5	3.20	17.2	25.1	2.17	3.6	25.0	2.10	3.1	24.9
	C \sharp 6	3.29	17.3	18.3	2.59	4.0	17.5	2.10	3.1	17.7
vla	G5	27.85	125.8	216.6	24.95	46.6	206.0	23.43	42.0	206.0
	A \sharp 5	15.07	74.1	102.1	12.95	19.2	94.6	10.29	14.5	90.8
	C \sharp 6	14.82	63.8	43.6	12.35	20.1	42.6	9.93	15.3	41.3
vlc	G5	26.19	94.2	151.6	25.67	42.5	153.2	24.76	40.2	150.2
vln	G5	13.36	74.4	150.4	13.66	22.6	149.2	12.40	18.6	147.8
	A \sharp 5	25.58	94.6	100.9	21.56	36.7	98.6	21.70	36.0	98.7
	C \sharp 6	38.10	138.1	108.0	36.32	62.7	95.9	31.41	54.2	93.4
	E6	19.84	80.8	76.3	16.70	29.9	74.4	16.48	29.2	74.7
Mean		8.64	40.9	78.5	7.31	12.4	76.1	6.88	11.0	75.7
Std. Dev.		9.11	33.6	89.0	8.65	14.9	88.5	7.89	13.5	88.7

Table 22: Group 4, 3-oscillator, 3-table match optimization.

GROUP 4 3 Oscillators		One- and Two-Table Match Optimization								
		2+0			1+1			1+0		
Instr	Pitch	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error
cla	G5	0.48	1.08	158.9	0.59	0.82	161.5	0.70	0.72	149.1
	A♯5	1.07	1.67	27.5	1.04	1.17	27.5	1.32	1.34	25.7
	C♯6	0.44	1.04	62.7	0.41	0.67	65.8	1.07	1.09	87.9
eng	G5	0.68	1.88	38.0	0.63	1.08	38.7	0.84	0.87	46.2
flt	G5	2.18	3.98	19.4	2.06	2.64	19.3	2.87	2.91	17.4
	A♯5	2.39	4.19	11.8	2.13	2.58	12.2	3.02	3.07	11.8
	C♯6	1.20	2.80	10.3	1.20	1.65	10.3	1.99	2.03	10.1
	E6	1.01	2.71	14.0	0.53	0.85	14.1	0.79	0.84	14.1
glk	G5	0.82	1.42	362.2	0.65	0.80	362.2	0.62	0.64	364.9
	A♯5	0.42	1.02	343.2	1.45	1.66	333.2	0.56	0.58	333.3
	C♯6	0.69	1.29	152.4	1.01	1.24	162.0	0.50	0.51	162.0
	E6	0.50	1.10	142.1	0.38	0.48	142.1	0.46	0.48	142.1
obo	G5	0.90	2.10	33.8	0.83	1.39	33.8	1.28	1.31	33.5
	A♯5	1.32	2.52	39.6	1.21	1.55	39.6	1.72	1.75	39.6
	C♯6	1.40	2.60	48.0	1.44	1.88	48.0	1.58	1.61	47.0
	E6	2.44	3.64	13.8	2.23	2.58	13.8	0.64	0.67	15.5
pno	G5	0.58	1.18	25.9	0.66	1.03	25.8	0.79	0.81	27.3
	A♯5	0.70	1.30	5.9	0.71	0.96	6.2	0.96	0.98	6.2
	C♯6	0.83	1.43	7.8	0.89	1.20	7.8	1.21	1.22	6.6
	E6	1.00	1.60	9.1	1.05	1.48	9.1	0.86	0.88	12.1
sax	G5	1.11	1.71	46.7	1.15	1.30	47.1	1.12	1.14	49.2
	A♯5	0.46	1.06	27.7	0.32	0.48	26.8	0.48	0.49	26.5
	C♯6	0.41	1.01	16.2	0.27	0.42	15.7	0.24	0.26	16.2
tpt	G5	0.49	1.09	25.2	0.37	0.48	25.2	0.48	0.49	25.2
	A♯5	0.61	1.21	26.7	0.47	0.68	20.1	0.81	0.83	21.1
	C♯6	0.65	1.25	18.8	0.54	0.74	17.4	0.91	0.93	17.4
vla	G5	4.39	8.69	217.9	4.11	6.17	217.7	4.68	4.80	218.1
	A♯5	1.30	3.90	103.4	1.41	2.17	102.4	1.73	1.80	102.3
	C♯6	2.49	4.69	42.2	1.82	2.53	41.8	2.04	2.10	41.8
vlc	G5	7.52	10.52	146.1	7.61	9.84	141.7	8.86	8.94	143.0
vln	G5	2.63	5.23	139.4	2.00	3.06	140.9	2.40	2.47	136.5
	A♯5	5.42	8.42	97.6	5.27	7.51	97.9	6.80	6.88	99.3
	C♯6	9.77	14.17	92.2	6.99	9.18	91.3	6.79	6.90	90.7
	E6	4.67	7.37	72.8	4.24	6.48	72.0	9.04	9.10	72.5
Mean		1.85	3.26	76.5	1.70	2.32	76.2	2.06	2.10	76.8
Std. Dev.		2.15	3.14	88.9	1.84	2.49	88.4	2.36	2.38	88.2

Table 23: Group 4, 3-oscillator, 2- and 1-table match optimization.

GROUP 5 3 Oscillators		Three-Table Match Optimization								
		3+0			2+1			1+2		
Instr	Pitch	<i>T</i> _{opt}	<i>T</i> _{total}	Error	<i>T</i> _{opt}	<i>T</i> _{total}	Error	<i>T</i> _{opt}	<i>T</i> _{total}	Error
flt	G6	3.30	3.60	16.6	3.11	3.39	16.6	2.52	2.76	16.6
	A [#] 6	4.85	5.32	18.0	4.58	5.36	18.0	5.04	5.82	18.0
glk	G6	0.61	0.75	249.8	0.38	0.48	249.1	0.28	0.37	249.4
	A [#] 6	0.56	0.71	363.0	0.34	0.43	363.4	0.36	0.45	363.4
pno	G6	1.18	1.32	4.6	1.06	1.19	4.6	1.10	1.27	4.6
	A [#] 6	1.08	1.22	4.6	0.98	1.11	4.6	0.97	1.16	4.6
vln	G6	5.57	6.05	52.6	5.58	6.16	52.2	4.81	5.36	52.7
	A [#] 6	4.42	4.94	67.2	4.40	5.06	67.2	4.43	5.11	67.2
Mean		2.70	2.99	97.1	2.55	2.90	97.0	2.44	2.79	97.1
Std. Dev.		2.07	2.24	134.6	2.12	2.38	134.6	2.05	2.31	134.6

Table 24: Group 5, 3-oscillator, 3-table match optimization.

GROUP 5 3 Oscillators		One- and Two-Table Match Optimization								
		2+0			1+1			1+0		
Instr	Pitch	<i>T</i> _{opt}	<i>T</i> _{total}	Error	<i>T</i> _{opt}	<i>T</i> _{total}	Error	<i>T</i> _{opt}	<i>T</i> _{total}	Error
flt	G6	0.56	0.62	18.8	0.51	0.56	18.9	0.55	0.56	18.8
	A [#] 6	1.93	2.02	18.6	1.94	2.06	18.6	2.49	2.51	18.5
glk	G6	0.28	0.31	254.8	0.23	0.25	254.8	0.27	0.28	254.8
	A [#] 6	0.55	0.58	367.2	0.55	0.58	367.2	0.63	0.63	384.4
pno	G6	0.27	0.30	4.6	0.27	0.33	4.6	0.47	0.47	4.7
	A [#] 6	0.34	0.37	4.8	0.34	0.40	4.8	0.55	0.55	5.3
vln	G6	1.93	2.03	54.7	1.13	1.24	55.2	2.21	2.22	57.7
	A [#] 6	1.72	1.83	72.3	1.65	1.76	72.3	2.03	2.04	72.2
Mean		0.95	1.01	99.5	0.83	0.90	99.6	1.15	1.16	102.1
Std. Dev.		0.77	0.80	136.0	0.66	0.70	136.0	0.92	0.92	140.7

Table 25: Group 5, 3-oscillator, 2- and 1-table match optimization.

GROUP 2 4 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
bsn	E3	48	4319	59.4	61	142	57.9	30	215	59.6
	G3	18	4298	177.5	29	101	189.2	34	218	152.9
	A♯3	10	4280	117.6	3	46	177.8	1	183	68.0
	C♯4	49	4320	25.1	49	205	25.1	40	226	23.5
	E4	21	4291	24.2	18	99	23.5	10	193	23.5
cla	E3	45	4317	131.9	27	98	128.2	17	200	129.9
	G3	65	4336	104.3	36	117	92.7	8	191	85.2
	A♯3	70	4355	91.3	65	194	92.4	39	225	91.5
	C♯4	71	4345	173.9	27	107	139.9	30	213	159.4
	E4	77	4331	55.5	51	178	55.2	53	238	52.1
clb	E3	47	4262	73.8	36	144	73.8	48	233	74.0
	G3	74	4289	72.9	33	160	80.5	48	233	73.3
	A♯3	55	4271	166.1	29	175	152.8	38	223	155.1
	C♯4	61	4277	118.6	44	124	132.6	35	220	110.5
eng	E3	57	8325	46.4	50	354	45.3	48	414	47.6
	G3	80	8292	59.7	83	366	59.9	42	412	61.8
	A♯3	142	8443	62.3	104	609	61.7	116	489	61.8
	C♯4	102	8579	54.7	86	425	56.3	80	452	54.5
	E4	82	8283	85.2	71	445	84.4	54	422	85.0
flt	E3	176	8299	48.2	150	638	47.7	156	526	48.3
	G3	221	11033	55.5	177	632	55.7	165	657	57.8
hrn	E3	69	4331	23.9	60	218	23.7	71	257	24.5
	G3	64	4386	50.8	82	220	51.0	50	235	45.2
	A♯3	29	4292	68.4	23	143	69.6	33	217	61.3
	C♯4	26	4262	28.5	27	109	34.5	17	201	29.7
	E4	29	4245	21.0	12	84	19.4	13	197	19.8
obo	A♯3	84	8279	118.4	64	405	115.4	73	443	112.5
	C♯4	170	8368	60.1	90	523	60.3	84	456	60.3
	E4	91	8327	42.7	98	406	43.1	78	445	40.3
pno	E3	88	4324	53.0	87	284	46.9	59	245	48.1
	G3	36	4264	47.5	21	152	47.0	23	209	46.8
	A♯3	93	4321	59.2	61	218	58.3	72	258	64.7
	C♯4	41	4259	53.4	26	126	50.1	18	202	52.4
	E4	38	4257	35.1	33	182	34.6	37	221	32.4
sax	E3	81	4299	60.2	87	262	62.0	48	234	59.5
	G3	83	4302	136.6	57	213	130.6	78	264	132.0
	A♯3	61	4281	75.1	55	211	72.9	50	236	71.4
	C♯4	95	4313	83.3	78	234	82.5	55	241	86.1
	E4	64	4283	251.3	31	131	215.7	37	222	197.5

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GROUP 2 4 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
tpt	G3	62	4279	50.1	54	163	49.8	51	237	49.6
	A♯3	25	4245	64.8	38	128	68.8	22	205	57.9
	C♯4	57	4309	40.6	49	149	40.2	52	237	36.5
	E4	77	4320	32.2	47	147	31.9	68	253	30.5
trb	E3	40	4285	107.5	28	146	112.5	38	222	99.1
	G3	69	4313	40.6	46	174	36.8	55	240	36.7
	A♯3	43	4287	37.1	41	187	37.5	36	220	37.3
	C♯4	38	4284	61.2	31	140	72.5	28	212	61.1
	E4	27	4266	16.2	19	109	15.8	16	199	16.1
vla	E3	272	18618	258.2	259	1252	254.7	207	1073	254.4
	G3	594	24292	225.8	507	2797	224.5	445	1620	225.7
	A♯3	347	19499	316.5	334	1751	316.2	310	1214	317.1
	C♯4	367	16821	185.8	329	1694	184.0	263	1035	182.4
	E4	646	22606	259.9	536	3720	257.9	533	1609	255.9
vlb	E3	162	8377	79.9	160	608	80.5	174	545	81.1
	G3	217	9260	84.6	158	732	82.2	159	570	82.2
	A♯3	95	8303	47.7	97	509	46.8	100	470	46.2
	C♯4	327	10029	72.9	334	1273	71.3	339	786	71.6
	E4	380	12098	78.0	307	1569	76.0	344	891	77.6
vlc	E3	300	11834	80.3	271	1054	79.5	281	816	79.5
	G3	195	8395	72.2	152	692	64.1	205	578	67.3
	A♯3	300	8502	93.7	224	764	90.2	238	607	94.5
	C♯4	353	10051	122.0	224	883	116.5	312	752	114.9
	E4	406	11441	84.8	314	1089	78.4	358	867	78.7
vln	G3	144	8347	143.8	134	564	143.2	107	477	144.8
	A♯3	439	17215	157.2	385	1816	156.7	400	1185	153.5
	C♯4	697	21737	143.2	629	3088	140.8	596	1614	141.3
	E4	279	11943	118.8	245	1028	116.5	221	753	119.6
Mean		146	7595	91.8	122	563	91.0	119	458	87.7
Std. Dev.		155	5023	64.0	135	728	62.8	134	365	61.1

Table 28: Group 2, 4-oscillator, 4-table match optimization.

GROUP 2 4 Oscillators		Three-Table Match Optimization									
		3+0			2+1			1+2			
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	
bsn	E3	8.4	189	59.5	5.0	11.8	62.2	5.1	6.9	62.2	
	G3	9.7	191	181.7	9.9	16.5	195.9	5.8	7.9	190.3	
	A♯3	3.5	184	135.3	2.4	8.4	135.3	2.4	3.6	135.3	
	C♯4	13.3	193	27.0	10.0	18.3	26.8	9.0	12.8	26.8	
	E4	2.2	182	24.4	1.3	8.1	24.4	5.9	7.6	23.1	
cla	E3	18.5	199	133.3	14.7	21.3	133.3	15.8	17.7	133.3	
	G3	13.8	194	87.2	11.2	18.0	97.7	1.7	3.4	124.1	
	A♯3	15.3	195	88.9	9.8	17.6	95.2	7.7	10.6	93.8	
	C♯4	15.0	195	141.5	22.4	29.2	181.6	13.0	15.1	179.2	
	E4	15.7	196	57.9	6.4	14.2	56.8	6.4	9.5	56.8	
clb	E3	9.9	190	74.9	9.2	16.6	78.7	5.8	8.6	77.7	
	G3	7.8	188	76.2	5.2	13.0	80.6	4.9	7.7	80.5	
	A♯3	13.5	194	158.3	6.8	15.0	161.2	5.2	8.1	170.6	
	C♯4	5.7	187	130.8	25.9	32.7	124.2	8.8	10.7	128.4	
	E4	12.4	364	49.0	9.0	25.5	49.4	8.5	16.2	49.4	
eng	G3	17.1	371	62.8	16.7	32.7	62.8	16.7	23.9	62.8	
	A♯3	24.9	379	61.7	15.9	36.5	60.2	17.0	29.2	61.0	
	C♯4	22.1	376	55.9	26.6	43.9	53.4	26.8	35.4	54.0	
	E4	15.0	367	89.6	14.0	32.0	89.6	14.6	23.3	89.9	
	flt	E3	48.2	396	50.1	30.5	50.7	47.8	33.5	46.4	50.5
	G3	45.4	507	63.3	34.5	57.3	62.7	35.7	47.3	58.1	
hrn	E3	13.6	194	27.0	14.0	22.3	25.0	14.0	18.0	25.0	
	G3	13.5	194	46.3	12.4	20.4	49.8	12.5	16.4	49.8	
	A♯3	15.4	195	55.7	15.2	22.7	67.3	13.3	16.6	66.3	
	C♯4	24.7	205	28.2	14.8	21.6	33.8	14.2	16.7	33.7	
	E4	3.1	183	21.8	4.9	11.4	20.3	4.8	6.8	20.3	
obo	A♯3	21.5	374	117.1	9.9	27.2	116.5	11.4	19.5	117.0	
	C♯4	32.8	385	62.3	23.1	42.2	65.6	19.1	31.0	65.6	
	E4	12.1	364	40.7	11.2	27.7	40.3	11.2	18.5	40.3	
pno	E3	28.4	208	51.4	20.5	29.7	49.3	20.6	25.8	49.7	
	G3	12.0	192	48.0	5.2	13.0	48.7	6.1	9.6	48.7	
	A♯3	26.4	206	63.2	20.4	28.8	62.2	12.4	16.6	68.3	
	C♯4	6.9	187	54.8	8.3	15.5	52.3	8.2	10.9	52.3	
	E4	19.5	199	32.4	19.3	27.5	37.3	21.6	26.1	33.0	
sax	E3	17.9	198	60.4	15.5	24.3	59.7	11.1	15.6	59.8	
	G3	21.0	201	133.2	15.9	24.2	134.0	11.5	15.6	135.6	
	A♯3	12.9	193	75.2	10.1	18.4	76.0	10.2	14.5	76.0	
	C♯4	21.3	201	85.3	11.8	20.1	85.5	9.8	14.0	86.4	
	E4	15.9	196	227.8	14.0	21.2	218.8	6.4	9.3	239.8	

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GROUP 2 4 Oscillators		Three-Table Match Optimization								
		3+0			2+1			1+2		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
tpt	G3	13.3	193	52.3	8.5	15.8	53.0	7.4	10.3	53.5
	A♯3	9.1	189	72.5	2.9	9.9	78.3	2.9	5.0	78.0
	C♯4	18.2	198	41.5	6.6	13.8	42.4	5.7	8.5	42.2
	E4	10.7	191	33.9	4.5	11.7	33.9	4.3	6.8	34.1
trb	E3	32.9	213	98.8	28.1	35.7	96.5	9.2	12.4	95.5
	G3	12.2	192	45.8	9.9	17.7	42.0	10.3	13.8	42.0
	A♯3	16.9	197	35.6	13.6	21.8	36.5	13.6	17.5	36.5
	C♯4	7.0	187	70.0	4.0	11.4	88.7	4.5	7.4	89.8
	E4	4.3	184	19.6	11.3	18.3	16.1	11.3	13.9	16.1
vla	E3	39.7	825	257.8	31.6	75.0	259.8	32.5	54.8	259.9
	G3	98.8	1114	225.8	88.3	165.8	228.3	88.3	142.6	228.7
	A♯3	60.1	878	315.9	51.2	105.1	313.8	43.2	76.7	313.9
	C♯4	65.9	768	183.3	52.3	101.4	182.5	45.0	76.1	182.0
	E4	140.3	1077	258.2	130.6	225.9	258.0	132.8	212.1	257.2
vlb	E3	55.2	405	84.4	37.1	56.6	86.0	38.4	50.2	85.9
	G3	39.3	425	81.8	26.9	50.1	81.0	27.0	42.3	81.0
	A♯3	59.5	409	47.4	43.1	61.8	47.0	44.2	54.2	47.9
	C♯4	98.4	512	70.4	73.0	104.8	67.9	87.5	108.7	67.9
	E4	83.2	583	76.3	90.0	131.2	78.2	81.0	110.1	77.4
vlc	E3	67.0	560	79.0	48.2	78.9	80.1	49.5	67.7	79.4
	G3	57.2	407	71.3	40.0	61.4	68.6	36.1	52.0	68.6
	A♯3	62.8	413	95.6	46.6	68.0	94.1	40.5	53.5	92.9
	C♯4	85.8	501	116.5	53.1	78.9	115.7	44.8	60.9	115.1
	E4	87.1	559	77.4	66.6	96.5	77.9	65.4	83.9	77.7
vln	G3	25.5	376	144.3	18.6	37.7	142.1	14.2	24.6	143.2
	A♯3	99.0	813	150.4	69.5	120.4	152.2	69.6	103.6	153.2
	C♯4	133.3	1020	140.7	126.7	204.7	139.5	115.4	173.3	140.2
	E4	49.3	542	115.0	40.0	70.7	115.8	37.4	56.9	113.4
Mean		32.7	350	91.1	26.2	44.6	92.5	24.3	35.1	93.3
Std. Dev.		31.8	235	62.1	27.4	44.9	62.7	27.2	40.9	63.5

Table 29: Group 2, 4-oscillator, 3-table match optimization.

GROUP 4 4 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
cla	G5	48	256	78.1	53	74	77.2	62	76	69.4
	A♯5	54	262	13.8	41	56	12.9	41	56	13.9
	C♯6	45	253	30.3	34	56	25.8	60	75	23.4
eng	G5	95	494	24.9	93	144	25.0	100	131	22.8
flt	G5	249	833	12.7	212	320	12.9	194	243	12.8
	A♯5	347	947	6.5	283	407	6.6	246	296	6.3
	C♯6	280	804	7.7	230	308	7.6	207	248	7.6
	E6	261	828	9.3	257	322	9.0	182	227	8.9
glk	G5	45	253	276.7	30	37	278.3	21	36	289.1
	A♯5	47	255	266.6	14	27	206.3	26	41	268.9
	C♯6	92	300	111.9	87	105	124.2	58	73	100.9
	E6	34	243	83.3	32	41	83.9	33	48	112.2
obo	G5	138	549	19.3	127	193	18.5	98	130	17.1
	A♯5	205	619	23.1	175	256	22.7	204	236	24.0
	C♯6	155	567	16.4	141	219	16.0	95	126	15.3
	E6	80	482	5.7	86	124	5.7	75	106	5.6
pno	G5	104	312	17.0	54	78	15.4	75	90	15.1
	A♯5	83	291	3.2	65	87	3.2	56	71	3.4
	C♯6	80	288	4.9	52	76	4.8	64	79	4.2
	E6	70	278	5.6	56	83	5.3	62	77	5.9
sax	G5	41	249	31.6	31	50	29.8	30	46	27.4
	A♯5	59	267	13.6	58	77	13.2	32	46	13.4
	C♯6	67	275	7.8	35	54	8.0	28	44	7.4
tpt	G5	69	277	11.9	52	71	11.1	58	73	12.0
	A♯5	48	256	7.3	44	69	7.6	41	56	7.2
	C♯6	57	265	7.5	44	68	7.5	62	78	7.5
vla	G5	814	2219	150.8	676	1197	151.9	627	776	139.8
	A♯5	394	1250	62.2	302	415	62.5	350	425	59.9
	C♯6	369	1090	27.7	319	487	28.1	348	411	27.4
vlc	G5	625	1609	89.4	675	1091	87.8	556	651	89.8
vln	G5	396	1244	80.6	395	589	79.8	347	424	71.3
	A♯5	629	1621	69.6	570	935	68.5	567	661	68.7
	C♯6	1094	2545	63.5	918	1587	63.4	884	1039	63.7
	E6	459	1340	54.1	395	716	53.7	448	530	51.2
Mean		225	695	49.8	195	306	48.1	186	227	49.2
Std. Dev.		252	602	66.9	224	382	62.3	212	249	68.3

Table 32: Group 4, 4-oscillator, 4-table match optimization.

GROUP 4 4 Oscillators		Three-Table Match Optimization								
		3+0			2+1			1+2		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
cla	G5	18.4	32.4	77.6	12.0	13.3	93.5	12.3	13.2	95.4
	A \sharp 5	10.5	24.5	12.2	8.0	9.1	13.7	8.0	8.7	13.7
	C \sharp 6	23.2	37.2	21.1	10.3	11.6	29.1	8.7	9.6	29.4
eng	G5	22.8	49.8	21.7	20.4	23.3	22.5	19.1	21.3	23.5
flt	G5	48.6	89.6	12.5	37.3	42.6	12.5	35.2	39.3	12.4
	A \sharp 5	90.7	131.7	6.5	75.0	80.9	6.5	63.5	68.0	6.5
	C \sharp 6	57.6	92.6	7.5	44.9	49.0	7.5	44.5	47.6	7.5
	E6	36.9	75.9	8.8	34.4	38.2	8.8	29.6	31.7	8.9
glk	G5	21.1	35.1	328.0	9.2	10.1	331.5	10.9	11.3	331.5
	A \sharp 5	10.4	24.4	289.7	4.9	5.9	328.4	5.8	6.4	328.4
	C \sharp 6	53.6	67.6	100.8	10.0	11.2	128.9	6.2	6.9	136.8
	E6	8.5	22.5	120.6	4.5	5.4	120.6	4.1	4.5	120.7
obo	G5	45.6	73.6	17.0	25.2	28.6	16.4	26.7	29.4	16.3
	A \sharp 5	63.1	91.1	24.2	39.9	43.8	23.7	37.5	40.4	23.5
	C \sharp 6	29.7	56.7	17.1	20.6	24.3	17.3	20.7	23.6	17.3
	E6	14.3	41.3	5.4	13.9	16.3	5.7	14.0	15.5	5.7
pno	G5	26.7	40.7	13.5	6.3	7.8	15.9	15.0	16.2	15.5
	A \sharp 5	19.7	33.7	3.4	11.1	12.4	3.2	9.0	9.9	3.2
	C \sharp 6	27.1	41.1	4.2	20.6	22.0	3.9	24.9	26.0	4.0
	E6	28.3	42.3	4.8	13.9	15.4	4.3	14.0	15.2	4.3
sax	G5	12.6	27.6	29.3	4.2	5.4	29.6	5.0	5.7	29.3
	A \sharp 5	13.4	27.4	11.5	8.3	9.5	13.1	12.1	12.9	13.0
	C \sharp 6	11.4	25.4	7.1	8.9	10.1	7.5	8.5	9.3	7.5
tpt	G5	15.1	29.1	12.0	8.6	9.8	11.4	8.8	9.6	11.4
	A \sharp 5	20.6	34.6	8.3	11.9	13.3	8.6	11.4	12.4	8.6
	C \sharp 6	21.2	35.2	7.2	15.5	16.9	6.5	12.4	13.4	6.6
vla	G5	186.9	284.9	143.7	161.5	183.2	145.0	141.9	160.5	145.0
	A \sharp 5	96.7	155.7	61.9	76.5	82.8	61.0	55.2	59.4	62.3
	C \sharp 6	104.1	153.1	26.5	79.0	86.7	26.9	57.7	63.1	26.4
vlc	G5	194.4	262.4	85.8	191.5	208.3	86.5	178.2	193.6	87.5
vln	G5	81.2	142.2	74.4	82.8	91.7	77.6	69.9	76.1	77.6
	A \sharp 5	188.7	257.7	68.8	147.7	162.8	67.3	146.5	160.8	67.7
	C \sharp 6	276.5	376.5	62.0	267.4	293.8	61.4	211.2	234.0	61.9
	E6	137.9	198.9	50.2	108.4	121.6	50.6	105.5	118.2	50.1
Mean		59.3	91.6	51.3	46.9	52.0	54.3	42.2	46.3	54.7
Std. Dev.		65.7	89.2	75.1	62.1	68.4	80.1	53.2	58.8	80.4

Table 33: Group 4, 4-oscillator, 3-table match optimization.

GROUP 5 4 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
flt	G6	32.1	33.0	11.4	31.3	32.6	11.4	28.9	29.6	11.8
	A♯6	36.7	38.1	10.8	36.8	41.0	10.8	36.1	38.0	10.8
glk	G6	7.2	7.6	209.8	7.3	7.8	197.5	9.0	9.2	215.6
	A♯6	4.3	4.7	203.9	3.5	3.9	234.8	4.3	4.6	203.9
pno	G6	14.8	15.2	3.3	13.1	13.8	3.3	8.8	9.1	3.4
	A♯6	12.1	12.5	3.2	14.4	15.1	3.2	13.9	14.2	2.9
vln	G6	56.9	58.3	35.6	57.1	59.8	36.3	56.2	58.0	35.5
	A♯6	40.8	42.3	42.5	39.9	43.4	42.5	34.0	35.7	42.6
Mean		25.6	26.5	65.1	25.4	27.2	67.5	23.9	24.8	65.8
Std. Dev.		18.8	19.2	88.7	18.7	20.0	93.4	17.9	18.6	90.0

Table 34: Group 5, 4-oscillator, 4-table match optimization.

GROUP 5 4 Oscillators		Three-Table Match Optimization								
		3+0			2+1			1+2		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
flt	G6	15.8	16.1	11.1	15.2	15.5	11.3	10.6	10.8	11.3
	A♯6	21.6	22.1	10.9	19.8	20.6	10.9	21.4	22.2	10.9
glk	G6	5.3	5.4	216.4	2.0	2.1	216.7	1.9	2.0	216.4
	A♯6	4.3	4.5	203.6	2.3	2.4	251.1	2.3	2.4	251.1
pno	G6	4.9	5.0	3.4	4.3	4.4	3.4	4.3	4.5	3.4
	A♯6	4.4	4.5	3.2	3.7	3.8	3.2	3.7	3.9	3.2
vln	G6	26.0	26.5	35.7	23.8	24.4	35.6	20.3	20.9	36.1
	A♯6	17.5	18.0	43.1	17.6	18.3	43.3	17.4	18.1	43.3
Mean		12.5	12.8	65.9	11.1	11.4	71.9	10.2	10.6	72.0
Std. Dev.		8.8	9.0	90.2	8.9	9.2	101.4	8.3	8.6	101.3

Table 35: Group 5, 4-oscillator, 3-table match optimization.

GROUP 1 5 Oscillators		Five-Table Match Optimization								
		4+1			3+2			2+2+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
bsn	A#1	240	1895	89.2	255	547	79.9	258	398	83.3
	C#2	221	1880	124.2	239	542	125.5	184	289	124.1
	E2	347	2002	89.5	292	541	87.3	237	321	88.7
	G2	224	1878	79.9	289	624	78.7	213	363	79.9
	A#2	257	1910	63.3	214	537	63.8	290	412	63.0
	C#3	336	1990	40.1	318	631	40.2	377	482	39.7
clb	C#2	110	1759	166.4	101	320	167.4	88	161	168.5
	E2	210	1873	123.6	198	511	122.3	212	351	122.7
	G2	258	1909	129.5	306	566	130.5	292	404	130.8
	A#2	222	1875	110.9	281	551	109.0	225	347	109.3
	C#3	322	1974	93.2	373	664	92.2	370	466	93.1
hrn	E2	223	1874	82.0	107	345	81.5	215	317	81.3
	G2	173	1825	44.5	276	567	45.3	261	356	43.7
	A#2	227	1877	54.4	140	389	53.2	187	279	54.1
	C#3	162	1815	19.0	282	585	20.3	207	337	19.0
pno	A#1	206	1858	52.1	287	569	50.1	213	317	51.4
	C#2	365	2019	91.3	305	661	92.3	196	336	91.8
	E2	179	1829	105.6	216	487	118.2	266	385	119.0
	G2	302	1955	77.0	287	568	77.2	117	238	71.7
	A#2	226	1879	100.8	274	566	94.0	327	448	109.7
	C#3	281	1934	28.5	326	673	28.4	223	373	28.5
sax	A#1	269	1918	63.3	263	566	64.6	365	512	65.9
	C#2	344	1988	80.0	253	545	78.7	272	410	79.7
	E2	235	1878	106.6	236	496	101.8	218	336	106.4
	G2	320	1965	73.0	305	607	72.0	307	437	71.1
	A#2	390	2036	63.1	214	526	63.3	363	512	64.4
	C#3	312	1981	88.3	318	609	88.1	292	448	88.0
trb	E2	295	1975	60.0	314	584	60.5	296	389	60.0
	G2	203	1844	53.6	244	536	54.2	235	354	53.6
	A#2	248	1892	41.8	250	564	44.0	266	415	41.8
	C#3	208	1854	72.6	187	457	78.4	169	250	77.8
vla	C#3	638	4333	217.2	643	1759	217.6	624	1119	216.3
vlb	A#1	601	3826	71.7	649	1662	70.9	708	1154	71.7
	C#2	592	4280	80.0	631	2020	79.7	577	1094	80.3
	E2	569	3788	94.4	526	1624	95.4	628	1121	94.4
	G2	418	3644	53.3	505	1585	53.1	470	1055	52.7
	A#2	446	3924	82.8	654	1531	80.4	476	856	82.7
	C#3	455	3663	95.4	738	1345	103.3	392	581	95.5
vlc	C#2	810	4225	76.0	739	1573	76.8	727	1051	76.4
	E2	487	3710	58.4	360	1433	58.5	458	867	58.4
	G2	645	3868	60.8	589	1598	60.9	657	1136	60.4
	A#2	731	3939	78.7	593	1033	85.7	460	648	79.3
	C#3	777	4410	70.1	807	1803	71.1	722	1089	70.0
Mean		351	2483	81.5	358	835	81.8	340	540	81.9
Std. Dev.		178	947	35.2	184	491	35.4	170	303	35.7

Table 36: Group 1, 5-oscillator, 5-table match optimization.

GROUP 1 5 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error
bsn	A#1	154	1800	90.1	134	264	90.4	111	230	92.6
	C#2	185	1835	129.2	184	279	134.1	103	222	134.8
	E2	133	1780	90.5	83	160	90.5	92	209	94.4
	G2	246	1889	79.8	221	360	79.8	156	277	79.1
	A#2	241	1884	62.6	230	342	62.5	232	352	63.0
	C#3	192	1835	39.3	160	254	39.3	95	214	39.2
clb	C#2	141	1785	166.0	108	175	171.2	183	299	166.9
	E2	192	1845	123.3	309	438	123.3	179	299	124.4
	G2	112	1755	129.6	104	208	131.4	59	177	134.1
	A#2	153	1796	111.5	149	261	109.6	63	180	109.6
	C#3	153	1796	94.2	118	205	95.0	82	201	93.9
hrn	E2	131	1774	86.1	87	181	85.8	42	159	89.2
	G2	246	1888	45.9	177	262	45.4	131	250	45.4
	A#2	107	1749	54.6	64	149	55.1	340	457	54.3
	C#3	252	1895	22.5	281	401	22.3	198	317	22.4
pno	A#1	190	1833	52.0	159	253	52.0	87	205	51.9
	C#2	298	1941	93.2	256	385	91.7	269	390	94.3
	E2	130	1772	117.2	135	246	124.9	185	303	119.8
	G2	178	1822	78.0	136	248	80.6	102	220	75.6
	A#2	241	1884	104.3	90	203	112.4	83	202	105.8
	C#3	159	1803	29.0	120	261	29.0	165	286	29.1
sax	A#1	248	1887	62.4	152	289	65.8	177	296	62.8
	C#2	191	1825	81.1	121	250	83.4	124	243	86.3
	E2	111	1747	108.9	124	235	109.3	88	206	110.1
	G2	171	1807	71.0	152	272	70.6	139	258	72.2
	A#2	321	1956	63.7	210	348	62.7	233	352	63.1
	C#3	172	1830	89.9	130	276	89.3	119	238	90.0
trb	E2	80	1751	58.5	61	145	58.5	78	196	58.5
	G2	110	1743	55.6	125	236	55.5	136	255	54.8
	A#2	298	1931	42.1	302	440	41.8	142	262	44.9
	C#3	229	1869	81.8	347	423	75.3	222	340	81.6
vla	C#3	667	4309	218.0	622	1065	217.5	747	1030	219.9
vlb	A#1	600	3784	73.0	478	883	72.5	372	621	72.0
	C#2	690	4324	78.0	635	1098	78.1	414	708	77.9
	E2	354	3537	96.4	350	806	96.3	423	675	96.9
	G2	818	4002	52.3	787	1330	52.3	610	864	52.1
	A#2	660	4103	80.0	614	960	80.0	438	701	79.8
	C#3	149	3332	96.5	151	316	96.5	132	365	100.4
vlc	C#2	402	3778	76.4	408	693	77.3	315	568	77.5
	E2	564	3746	57.5	530	883	57.6	520	768	58.7
	G2	510	3691	60.4	447	886	60.7	323	569	61.1
	A#2	354	3536	74.9	283	448	80.7	190	415	78.9
	C#3	686	4270	70.7	540	860	70.9	541	813	70.6
Mean		284	2398	82.5	253	434	83.2	220	377	83.5
Std. Dev.		194	942	35.7	183	306	36.4	164	218	36.3

Table 37: Group 1, 5-oscillator, 4-table match optimization.

GROUP 2 5 Oscillators		Five-Table Match Optimization								
		4+1			3+2			2+2+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
bsn	E3	279	4557	45.5	226	614	47.0	255	343	45.5
	G3	171	4455	76.8	227	523	85.2	234	310	72.4
	A♯3	137	4410	49.5	115	364	54.2	44	89	55.4
	C♯4	242	4520	16.5	246	679	13.7	274	437	14.6
	E4	317	4594	16.1	285	580	15.7	288	375	16.1
cla	E3	157	4434	103.1	118	413	101.1	118	193	103.2
	G3	239	4516	77.8	221	527	66.1	168	254	76.2
	A♯3	241	4533	75.5	353	774	72.3	261	397	75.6
	C♯4	199	4478	125.3	162	457	122.5	146	230	112.0
	E4	249	4511	44.7	184	594	45.8	293	427	43.7
clb	E3	280	4502	63.6	192	579	61.9	253	368	63.5
	G3	305	4527	65.6	195	593	62.1	202	336	65.6
	A♯3	227	4450	110.8	178	565	104.1	139	290	109.4
	C♯4	235	4457	98.3	277	630	85.1	269	354	90.4
eng	E3	381	8670	37.1	527	1477	37.9	375	700	38.0
	G3	517	8755	49.5	529	1569	49.0	499	807	48.9
	A♯3	483	8812	52.8	368	1569	50.7	495	1028	52.3
	C♯4	412	8915	45.2	509	1664	45.0	389	753	46.3
	E4	524	8749	68.9	360	1397	73.1	489	884	69.1
flt	E3	507	8658	33.5	577	1815	34.7	528	1045	32.8
	G3	649	11512	31.1	529	2285	32.1	708	1212	32.3
hrn	E3	258	4528	15.4	188	621	16.5	297	463	15.6
	G3	313	4642	33.2	245	655	30.5	281	425	38.0
	A♯3	276	4545	35.2	184	513	40.6	295	421	32.2
	C♯4	189	4431	21.3	232	584	22.9	153	241	22.6
	E4	184	4406	14.0	194	535	11.0	136	214	12.0
obo	A♯3	490	8708	99.4	561	1669	98.4	696	1058	98.4
	C♯4	540	8765	47.1	639	1862	45.6	517	977	47.7
	E4	588	8850	31.6	412	1435	33.1	568	902	32.5
pno	E3	235	4478	38.4	179	601	37.7	232	436	37.3
	G3	153	4388	37.2	295	717	38.3	206	344	37.2
	A♯3	229	4464	45.8	241	662	48.0	269	433	43.2
	C♯4	313	4538	39.5	196	525	36.8	214	319	35.3
	E4	250	4475	26.3	215	567	26.4	255	410	26.6
sax	E3	282	4508	51.5	172	606	51.2	285	468	51.5
	G3	222	4448	114.7	259	680	112.2	272	436	105.3
	A♯3	343	4570	58.4	247	679	59.9	290	454	59.1
	C♯4	178	4403	63.8	240	672	66.6	252	416	67.7
	E4	285	4510	158.9	244	631	149.6	174	280	171.8

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GROUP 2 5 Oscillators		Five-Table Match Optimization								
		4+1			3+2			2+2+1		
Instr	Pitch	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error
tpt	G3	206	4429	44.7	270	692	44.4	243	358	45.3
	A♯3	183	4408	47.7	225	543	47.4	160	255	53.6
	C♯4	181	4440	30.9	223	621	30.4	182	290	31.0
	E4	196	4447	22.6	209	619	24.5	204	311	22.7
trb	E3	197	4448	60.1	183	547	67.4	172	296	65.6
	G3	180	4431	24.9	192	603	24.2	138	273	25.0
	A♯3	320	4571	30.7	265	630	25.9	242	395	24.4
	C♯4	122	4375	25.5	59	401	25.2	146	261	26.1
	E4	208	4452	10.0	168	474	9.7	139	234	9.5
vla	E3	928	19402	225.6	966	5473	229.0	852	1971	224.2
	G3	1059	24999	196.1	1189	9257	197.3	1164	3691	195.9
	A♯3	855	20157	266.4	924	5511	272.2	1033	2598	267.5
	C♯4	908	17476	164.6	837	4605	164.8	875	2351	161.0
	E4	1070	23238	219.8	1107	8148	220.7	1066	4455	217.9
vlb	E3	448	8692	69.3	374	1630	73.3	527	1003	69.4
	G3	435	9511	75.7	415	1895	75.4	368	975	71.3
	A♯3	350	8587	38.6	404	1616	40.3	315	755	38.1
	C♯4	561	10305	60.3	535	2383	58.0	567	1548	59.7
	E4	811	12590	60.1	623	3164	62.6	599	1921	59.6
vlc	E3	635	12221	68.6	576	2965	68.8	690	1527	68.3
	G3	475	8705	59.5	475	1843	57.8	445	1014	54.6
	A♯3	514	8745	77.4	494	1687	80.0	366	932	75.7
	C♯4	541	10278	101.1	481	1998	92.6	611	1303	95.7
	E4	647	11733	65.9	621	2735	64.1	704	1530	61.4
vln	G3	453	8686	123.7	411	1683	123.1	596	1055	118.8
	A♯3	813	17704	124.6	840	4660	123.0	881	2423	123.2
	C♯4	936	22165	120.3	946	7554	119.7	860	3510	119.3
	E4	416	12135	90.4	558	2784	96.5	441	1278	89.2
Mean		399	7881	70.5	386	1651	70.2	394	866	69.7
Std. Dev.		240	5158	52.9	252	1881	53.0	257	877	52.6

Table 38: Group 2, 5-oscillator, 5-table match optimization.

GROUP 2 5 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
bsn	E3	246	4517	44.2	279	360	46.3	98	284	42.9
	G3	78	4358	90.4	122	194	119.2	167	351	115.7
	A♯3	131	4401	49.5	77	120	55.4	13	195	49.5
	C♯4	252	4523	13.6	258	414	13.5	192	378	13.5
	E4	82	4352	15.6	69	150	15.4	33	216	15.7
cla	E3	159	4431	117.6	205	276	97.8	76	259	124.3
	G3	200	4471	78.0	218	299	76.9	56	239	68.4
	A♯3	196	4481	73.7	263	392	74.6	148	334	74.9
	C♯4	228	4502	122.2	162	242	110.0	190	373	112.2
	E4	252	4506	44.4	199	326	43.3	239	424	45.4
clb	E3	183	4398	63.7	118	226	63.7	123	308	66.7
	G3	189	4404	65.9	157	284	65.3	177	362	63.3
	A♯3	134	4350	115.6	135	281	118.0	108	293	117.6
	C♯4	123	4339	93.4	530	610	97.4	165	350	89.0
eng	E3	270	8538	35.4	221	525	35.4	220	586	37.9
	G3	312	8524	49.2	325	608	50.1	171	541	50.1
	A♯3	779	9080	50.6	532	1037	50.2	453	826	51.0
	C♯4	370	8847	45.4	383	722	45.6	344	716	45.8
	E4	352	8553	70.1	298	672	67.8	207	575	70.2
flt	E3	624	8747	32.7	640	1128	31.9	697	1066	33.2
	G3	1006	11818	31.6	883	1338	30.9	647	1139	31.6
hrn	E3	333	4595	15.6	310	468	15.6	405	591	15.7
	G3	346	4668	30.6	235	373	33.2	249	434	30.2
	A♯3	139	4402	32.0	97	217	32.0	160	344	39.1
	C♯4	121	4357	20.6	117	199	26.9	74	258	21.1
	E4	135	4351	12.5	38	110	11.8	119	303	11.1
obo	A♯3	399	8594	99.0	286	627	98.0	309	679	98.1
	C♯4	407	8605	47.9	435	868	44.9	298	670	46.3
	E4	447	8683	32.7	453	761	32.9	375	742	31.3
pno	E3	170	4406	38.1	232	429	36.2	162	348	36.6
	G3	133	4361	40.0	83	214	38.1	99	285	38.0
	A♯3	231	4459	44.9	208	365	44.2	231	417	50.8
	C♯4	207	4425	37.8	113	213	40.4	80	264	35.5
	E4	201	4420	25.4	82	231	25.7	69	253	25.9
sax	E3	173	4391	51.7	163	338	52.7	189	375	50.4
	G3	206	4425	111.0	202	358	106.6	228	414	114.6
	A♯3	269	4489	58.3	196	352	58.7	177	363	58.7
	C♯4	237	4455	61.5	264	420	66.3	166	352	64.8
	E4	187	4406	166.6	136	236	171.5	136	321	153.1

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GROUP 2 5 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error
tpt	G3	293	4510	45.2	237	346	45.4	225	411	44.6
	A♯3	120	4340	52.1	184	274	55.7	159	342	46.2
	C♯4	251	4503	28.7	243	343	31.7	177	362	29.8
	E4	286	4529	23.2	229	329	22.6	259	444	26.1
trb	E3	103	4348	63.9	94	212	74.5	107	291	74.9
	G3	211	4455	25.7	225	353	25.2	145	330	24.7
	A♯3	212	4456	23.8	202	348	24.0	178	362	24.0
	C♯4	119	4365	41.7	137	246	40.5	124	308	37.2
	E4	86	4325	9.9	82	172	9.9	63	246	10.0
vla	E3	1370	19716	222.4	1308	2301	219.4	961	1827	220.2
	G3	1757	25455	196.5	1960	4250	193.0	1796	2971	191.7
	A♯3	1336	20488	258.8	1406	2823	258.3	1218	2122	264.1
	C♯4	1202	17656	156.4	1466	2831	154.1	1188	1960	155.7
	E4	1314	23274	215.2	1511	4695	212.7	1437	2513	210.8
vlb	E3	476	8691	69.4	451	899	69.8	638	1009	70.0
	G3	527	9570	73.7	600	1174	68.6	517	928	71.2
	A♯3	484	8692	36.5	451	863	36.8	447	817	37.0
	C♯4	604	10306	60.3	397	1336	59.1	468	915	58.1
	E4	651	12369	60.6	1053	2315	56.1	713	1260	58.8
vlc	E3	976	12510	67.5	1003	1786	67.2	858	1394	67.0
	G3	433	8633	56.6	512	1052	52.9	545	918	56.2
	A♯3	398	8600	78.4	510	1050	74.8	624	993	77.4
	C♯4	402	10100	101.7	802	1461	91.6	545	985	88.5
	E4	577	11612	64.4	853	1628	61.0	1113	1622	59.2
vln	G3	552	8755	120.7	540	970	120.5	365	735	120.4
	A♯3	1554	18330	121.4	1731	3162	117.5	1314	2099	118.5
	C♯4	1266	22306	117.3	1470	3929	116.9	1542	2560	114.6
	E4	948	12612	88.9	988	1771	87.0	933	1465	88.1
Mean		433	7883	70.3	453	894	70.0	399	738	69.9
Std. Dev.		397	5260	52.3	451	1021	51.6	406	639	51.8

Table 39: Group 2, 5-oscillator, 4-table match optimization.

GROUP 3 5 Oscillators		Five-Table Match Optimization								
		4+1			3+2			2+2+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
cla	G4	245	1503	45.0	315	474	47.0	293	332	46.8
	A♯4	254	1512	28.3	267	407	28.4	289	329	27.8
	C♯5	294	1553	74.3	232	386	74.3	250	309	70.3
	E5	225	1483	74.6	169	337	77.2	290	359	65.2
eng	G4	574	3018	54.9	405	880	52.9	582	766	50.4
	A♯4	415	2859	25.1	393	850	29.5	496	711	24.9
	C♯5	398	2842	13.8	433	899	14.1	532	747	13.5
	E5	421	2868	17.3	361	828	18.1	482	689	15.4
flt	G4	632	3833	22.7	614	1448	20.6	684	1053	21.9
	A♯4	932	5601	21.8	966	2391	21.4	964	1711	21.8
	C♯5	771	5069	17.5	792	2096	17.1	878	1377	16.4
	E5	458	3776	10.8	602	1440	11.5	509	809	10.5
hrn	G4	106	1369	3.6	225	370	4.0	98	136	3.6
	A♯4	167	1430	2.7	168	294	3.8	202	243	2.7
	C♯5	186	1449	2.7	163	299	2.8	183	220	2.7
obo	G4	540	2984	43.4	508	920	50.9	436	628	48.4
	A♯4	400	2845	56.5	474	983	49.2	475	664	51.8
	C♯5	408	2862	30.7	466	948	32.4	462	664	29.3
	E5	545	2990	22.7	507	883	21.3	550	721	22.9
pno	G4	179	1438	13.6	265	433	10.9	206	299	9.8
	A♯4	167	1426	16.1	164	333	15.4	178	266	13.6
	C♯5	200	1459	11.7	172	345	12.1	184	282	12.2
	E5	209	1468	30.0	301	469	26.5	214	306	20.9
sax	G4	220	1479	28.0	211	379	28.8	252	314	24.0
	A♯4	239	1498	30.1	293	461	30.5	153	235	35.6
	C♯5	220	1479	34.9	240	403	34.3	267	340	35.7
	E5	308	1566	10.9	172	340	12.9	243	311	12.8
tpt	G4	207	1466	15.7	160	328	18.0	215	278	15.5
	A♯4	209	1468	25.4	267	417	23.6	240	303	24.8
	C♯5	255	1514	16.6	290	453	15.6	254	320	17.5
	E5	215	1474	7.0	188	360	7.1	191	276	7.0
trb	G4	266	1525	8.8	221	370	9.7	218	265	8.4
	A♯4	183	1443	8.8	175	301	8.6	193	244	8.8
	C♯5	199	1458	8.5	224	378	9.4	270	329	8.5
vla	G4	917	5862	176.8	660	2438	168.3	900	1708	157.0
	A♯4	937	6834	144.4	883	3218	150.1	967	1855	145.8
	C♯5	817	5763	107.5	760	2296	106.4	804	1549	109.9
	E5	1455	10338	109.3	1344	6795	111.0	1488	4318	109.2
vlc	G4	574	4690	69.7	631	1848	70.3	566	1116	71.6
	A♯4	803	4769	58.2	893	2038	56.1	816	1381	55.8
	C♯5	823	5447	59.4	937	2613	56.8	911	1941	54.9
	E5	928	6671	56.4	868	3090	58.3	915	1851	57.3
vln	G4	698	5147	80.5	657	2176	82.5	747	1502	78.0
	A♯4	483	3837	43.4	529	1460	44.8	592	948	43.2
	C♯5	949	7180	55.5	719	3280	60.8	815	1791	56.6
	E5	1049	6732	55.4	1025	3183	50.8	947	1789	53.0
Mean		471	3202	40.2	463	1246	40.4	487	839	39.0
Std. Dev.		313	2151	37.3	293	1265	37.0	313	779	35.8

Table 40: Group 3, 5-oscillator, 5-table match optimization.

GROUP 3 5 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error
cla	G4	250	1505	44.1	189	225	45.0	251	316	45.8
	A♯4	147	1402	27.8	196	232	25.6	90	153	28.8
	C♯5	253	1508	72.7	184	239	70.7	171	235	67.7
	E5	226	1481	68.9	170	236	65.4	149	214	66.4
eng	G4	599	3030	51.4	534	705	49.0	475	606	49.9
	A♯4	512	2944	23.7	639	841	23.8	589	720	23.5
	C♯5	607	3038	13.3	520	722	12.9	625	757	12.8
	E5	540	2974	16.8	589	782	16.7	511	643	16.2
flt	G4	802	3980	20.5	827	1173	20.4	815	991	19.3
	A♯4	1157	5778	18.7	1210	1909	19.1	1024	1282	17.2
	C♯5	1024	5280	17.2	1108	1565	16.0	1175	1414	16.2
	E5	1089	4382	10.2	902	1167	10.1	779	958	9.9
hrn	G4	50	1310	4.0	26	61	3.9	132	196	6.0
	A♯4	96	1357	3.1	95	134	3.1	210	274	3.3
	C♯5	238	1499	2.6	235	270	2.6	186	250	2.6
obo	G4	467	2898	48.9	508	687	52.9	288	416	48.7
	A♯4	572	3003	46.5	530	705	47.8	427	558	47.6
	C♯5	671	3112	29.1	696	885	28.1	536	666	28.9
	E5	381	2813	21.6	351	509	22.8	223	351	21.8
pno	G4	118	1373	11.3	342	431	10.1	211	276	10.5
	A♯4	199	1454	17.1	207	292	14.3	141	206	15.6
	C♯5	125	1380	11.5	384	478	11.7	289	354	11.7
	E5	113	1368	26.3	249	338	21.7	174	239	22.4
sax	G4	289	1544	26.2	223	281	26.1	359	424	22.2
	A♯4	216	1471	27.5	268	346	26.8	240	305	28.5
	C♯5	294	1549	35.1	350	420	32.9	184	249	31.5
	E5	221	1476	11.8	273	337	11.7	210	275	12.0
tpt	G4	287	1542	12.5	338	397	13.4	248	313	13.8
	A♯4	378	1633	18.6	357	416	20.6	201	266	23.1
	C♯5	251	1506	15.6	122	184	17.0	204	269	14.9
	E5	314	1569	6.9	278	359	6.2	279	344	6.6
trb	G4	249	1504	11.0	89	133	10.6	154	218	10.2
	A♯4	137	1393	9.6	103	150	8.9	32	96	9.5
	C♯5	200	1455	8.5	207	262	8.4	247	311	8.3
vla	G4	1131	6020	169.3	1241	1994	149.1	898	1182	150.7
	A♯4	1737	7558	139.5	1774	2587	137.0	1560	1904	137.5
	C♯5	1337	6228	103.4	1467	2159	101.0	1280	1558	97.8
	E5	1252	9957	107.3	1867	4523	102.2	1967	2532	98.6
vlc	G4	1143	5221	68.7	1112	1624	66.5	1291	1519	64.8
	A♯4	531	4461	58.3	740	1271	55.8	513	732	54.6
	C♯5	641	5216	57.9	761	1742	54.6	593	858	53.7
	E5	1382	7053	54.9	1586	2452	53.9	1628	1959	52.9
vln	G4	1050	5454	80.4	1006	1717	80.1	908	1161	74.6
	A♯4	762	4090	41.3	765	1095	41.0	719	904	40.9
	C♯5	1362	7507	53.7	1589	2479	53.1	1514	1876	51.6
	E5	1125	6736	51.7	1185	1955	44.3	1214	1541	46.9
Mean		577	3283	38.6	617	945	37.3	563	715	36.9
Std. Dev.		443	2224	36.1	497	911	33.9	487	595	33.4

Table 41: Group 3, 5-oscillator, 4-table match optimization.

GROUP 4 5 Oscillators		Five-Table Match Optimization								
		4+1			3+2			2+2+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
cla	G5	281	490	37.1	325	367	34.4	262	284	33.3
	A♯5	236	445	8.7	293	334	9.3	209	225	8.5
	C♯6	237	446	13.2	259	307	16.1	216	240	11.8
eng	G5	482	886	12.3	572	697	11.3	531	587	11.4
flt	G5	729	1324	9.8	683	946	9.8	824	946	9.8
	A♯5	795	1408	4.8	623	932	5.1	785	922	4.8
	C♯6	521	1054	5.9	540	742	5.9	525	612	5.6
	E6	670	1248	6.8	734	946	6.6	649	724	6.6
glk	G5	166	375	211.7	184	218	262.3	190	198	236.0
	A♯5	248	457	206.9	222	258	185.3	139	153	172.7
	C♯6	268	477	71.5	262	303	95.0	269	288	108.8
	E6	139	349	56.6	204	242	58.0	233	243	55.7
obo	G5	439	856	10.8	658	808	10.2	474	546	10.4
	A♯5	472	892	14.8	366	516	15.4	431	518	14.6
	C♯6	418	836	9.2	541	687	8.6	541	625	9.1
	E6	607	1014	3.8	582	705	4.0	699	742	3.7
pno	G5	224	433	15.9	332	380	11.8	287	312	10.5
	A♯5	246	455	2.5	290	336	2.4	321	344	2.3
	C♯6	294	503	3.3	299	345	3.0	232	257	3.0
	E6	208	417	3.8	275	323	3.7	236	264	4.1
sax	G5	292	501	18.5	204	246	16.8	300	320	15.3
	A♯5	205	414	7.7	231	276	7.9	164	184	7.6
	C♯6	174	383	6.0	277	320	5.6	217	237	5.6
tpt	G5	248	457	8.7	290	336	6.7	213	233	7.8
	A♯5	162	371	5.4	263	311	5.1	237	263	5.1
	C♯6	238	447	4.7	229	277	4.3	174	199	4.5
vla	G5	1388	2860	117.1	1492	2980	116.4	1628	2216	110.3
	A♯5	886	1767	41.7	870	1361	39.7	1038	1176	40.5
	C♯6	708	1447	20.6	820	1246	20.6	824	1010	20.1
vlc	G5	969	1987	68.1	1221	2024	69.8	1180	1629	68.3
vln	G5	746	1618	58.6	893	1393	59.0	711	930	60.4
	A♯5	955	1982	52.5	1047	1797	53.4	928	1327	53.5
	C♯6	1594	3120	48.8	1671	3285	47.3	1639	2382	47.0
	E6	861	1769	42.9	926	1574	41.5	961	1310	41.4
Mean		503	985	35.6	549	818	36.8	537	660	35.6
Std. Dev.		360	728	51.4	382	764	55.9	403	571	52.0

Table 42: Group 4, 5-oscillator, 5-table match optimization.

GROUP 4 5 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error	T _{opt}	T _{total}	Error
cla	G5	170	378	36.6	245	266	33.0	179	194	39.3
	A♯5	302	510	8.6	210	225	8.6	164	180	8.7
	C♯6	238	446	10.7	166	188	13.0	247	262	12.5
eng	G5	501	900	12.0	490	541	11.0	377	408	11.0
flt	G5	926	1510	10.1	809	917	9.9	755	805	9.7
	A♯5	967	1567	4.8	981	1105	4.7	929	980	4.6
	C♯6	659	1183	5.7	787	865	5.3	867	908	5.4
	E6	818	1385	6.5	955	1020	6.4	934	979	6.2
glk	G5	217	425	240.4	150	157	240.8	97	112	256.4
	A♯5	334	542	192.2	61	74	172.7	91	106	237.5
	C♯6	262	470	85.0	105	123	98.8	229	244	91.4
	E6	171	380	63.6	154	163	62.7	129	144	62.3
obo	G5	600	1011	9.9	623	689	10.1	433	464	9.6
	A♯5	689	1103	14.4	658	739	12.5	670	702	12.3
	C♯6	666	1078	7.5	694	772	7.3	431	462	7.2
	E6	392	794	3.7	421	459	3.7	361	392	3.8
pno	G5	378	586	11.3	299	323	10.8	152	167	11.6
	A♯5	258	466	2.6	290	312	2.2	253	268	2.6
	C♯6	251	459	2.8	242	266	2.9	253	268	2.7
	E6	261	469	3.5	306	333	3.4	222	237	3.3
sax	G5	121	329	15.2	145	164	15.1	101	117	15.2
	A♯5	268	476	6.6	321	340	6.7	153	168	6.6
	C♯6	332	540	5.8	159	178	5.2	131	146	5.5
tpt	G5	305	513	7.2	273	292	6.6	180	195	6.8
	A♯5	255	463	4.8	224	249	4.7	209	224	4.7
	C♯6	278	486	4.4	231	255	4.1	262	278	4.5
vla	G5	1880	3285	99.4	1897	2418	100.1	2404	2553	96.5
	A♯5	1359	2215	36.3	1433	1546	35.4	1419	1494	35.4
	C♯6	1110	1831	18.7	1050	1218	18.0	1119	1181	18.1
vlc	G5	1253	2237	62.6	1053	1469	63.8	1507	1602	58.7
vln	G5	1542	2390	55.1	1440	1634	54.8	1370	1447	52.0
	A♯5	1438	2430	50.9	1742	2107	49.0	1219	1313	50.6
	C♯6	1489	2940	46.1	2095	2764	45.9	2163	2318	42.6
	E6	1489	2370	39.7	1690	2011	38.6	1331	1414	38.2
Mean		652	1123	34.8	659	770	34.3	628	669	36.3
Std. Dev.		504	848	52.9	582	728	51.8	615	652	59.1

Table 43: Group 4, 5-oscillator, 4-table match optimization.

GROUP 5 5 Oscillators		Five-Table Match Optimization								
		4+1			3+2			2+2+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
flt	G6	114	116	7.9	141	144	7.8	113	115	7.9
	A♯6	240	243	7.6	295	303	7.7	232	238	7.6
glk	G6	52	53	145.0	49	50	145.0	29	30	145.0
	A♯6	43	44	114.5	39	40	114.5	45	46	114.5
pno	G6	78	79	2.7	58	59	2.7	68	69	2.7
	A♯6	75	76	2.2	94	95	2.1	89	90	2.1
vln	G6	347	351	25.9	341	349	25.9	343	348	26.1
	A♯6	386	389	32.3	393	400	32.4	365	371	32.2
Mean		167	169	42.3	176	180	42.3	161	163	42.3
Std. Dev.		138	139	55.7	144	147	55.7	135	137	55.7

Table 44: Group 5, 5-oscillator, 5-table match optimization.

GROUP 5 5 Oscillators		Four-Table Match Optimization								
		4+0			2+2			3+1		
Instr	Pitch	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error	T_{opt}	T_{total}	Error
flt	G6	113.9	114.8	8.2	112.1	113.4	8.2	99.8	100.5	8.3
	A♯6	119.5	120.9	7.7	119.4	123.6	7.7	117.1	119.0	7.7
glk	G6	22.1	22.5	150.2	22.9	23.4	145.0	27.8	28.0	154.0
	A♯6	10.7	11.1	146.2	7.7	8.1	146.1	10.6	10.9	146.2
pno	G6	55.3	55.7	2.7	46.8	47.5	2.7	26.1	26.4	2.8
	A♯6	42.0	42.4	2.2	52.8	53.5	2.2	47.3	47.6	2.3
vln	G6	206.0	207.4	26.3	208.4	211.1	26.1	199.3	201.1	26.1
	A♯6	131.2	132.8	32.5	127.2	130.7	32.6	99.3	101.0	32.4
Mean		87.6	88.5	47.0	87.2	88.9	46.3	78.4	79.3	47.5
Std. Dev.		66.5	67.0	63.4	66.8	67.9	62.2	63.1	63.7	64.3

Table 45: Group 5, 5-oscillator, 4-table match optimization.

C Comparison with Horner's Method

GROUP 1		Horner's Constrained Matching					
		3 Oscillators		4 Oscillators		5 Oscillators	
Instr	Pitch	Time	Error	Time	Error	Time	Error
bsn	A♯1	1.06	125.0	10.8	113.3	158	94.2
	C♯2	1.06	191.3	10.7	161.2	155	137.8
	E2	1.06	140.9	10.7	126.4	155	111.2
	G2	1.06	122.2	10.7	98.0	157	91.5
	A♯2	1.06	113.5	10.7	82.8	156	70.7
	C♯3	1.06	101.2	10.7	64.4	154	52.5
clb	C♯2	1.06	232.5	10.7	202.1	154	174.6
	E2	1.07	144.1	10.7	132.7	154	125.8
	G2	1.01	182.4	10.7	158.8	154	142.5
	A♯2	1.17	152.7	10.7	127.6	154	119.1
	C♯3	1.09	146.9	10.7	125.6	154	113.3
hrn	E2	1.08	187.8	10.7	112.2	154	96.2
	G2	1.13	81.6	10.7	60.5	154	48.0
	A♯2	1.14	109.8	10.7	76.5	154	62.1
	C♯3	1.06	48.6	10.7	31.8	154	25.3
pno	A♯1	1.05	92.0	10.7	71.0	154	59.1
	C♯2	1.06	110.9	10.7	102.0	154	97.2
	E2	1.06	217.3	10.7	170.2	154	121.4
	G2	1.06	169.1	10.7	99.9	154	87.7
	A♯2	1.15	176.8	10.7	145.5	154	109.9
	C♯3	1.03	62.8	10.7	42.7	154	33.6
sax	A♯1	1.09	86.6	10.7	77.1	154	66.7
	C♯2	1.05	119.9	10.7	106.4	154	93.3
	E2	1.06	138.9	10.7	128.8	154	121.9
	G2	1.07	110.5	10.7	89.5	154	81.1
	A♯2	1.06	90.0	10.7	78.5	154	69.5
	C♯3	1.06	135.6	10.7	112.4	154	100.1
trb	E2	1.06	96.1	10.7	73.8	154	60.8
	G2	1.06	106.7	10.7	80.3	154	64.9
	A♯2	1.06	90.2	10.7	66.1	154	51.6
	C♯3	1.07	164.9	10.7	135.2	154	96.1
vla	C♯3	2.02	299.8	13.1	254.9	159	230.9
vlb	A♯1	1.78	126.5	12.5	96.1	159	76.9
	C♯2	2.03	127.5	13.1	101.4	161	81.7
	E2	1.80	131.1	12.5	113.7	160	105.4
	G2	1.80	72.3	12.5	61.5	160	54.9
	A♯2	1.87	166.0	12.8	117.5	158	92.7
	C♯3	1.78	252.0	12.5	136.5	159	111.3
vlc	C♯2	1.84	119.0	12.7	94.8	159	82.1
	E2	1.79	74.7	12.5	67.2	159	61.0
	G2	1.78	104.3	12.5	84.8	159	65.2
	A♯2	1.77	184.5	12.5	110.6	158	87.8
	C♯3	1.97	147.7	13.0	94.2	163	80.4
Mean		1.29	136.1	11.3	106.7	156	90.9
Std. Dev.		0.36	51.7	0.9	41.7	3	37.2

Table 46: Group 1, Horner's constrained matching results.

GROUP 2		Horner's Constrained Matching					
		3 Oscillators		4 Oscillators		5 Oscillators	
Instr	Pitch	Time	Error	Time	Error	Time	Error
bsn	E3	0.90	109.5	16.8	77.1	408	47.5
	G3	0.89	336.0	16.8	192.9	410	86.1
	A♯3	0.89	327.7	16.6	171.9	411	62.7
	C♯4	0.89	58.3	17.2	30.7	425	16.7
	E4	0.80	57.8	16.6	27.2	421	16.9
cla	E3	0.95	191.7	16.6	144.7	413	120.5
	G3	0.85	178.9	16.6	104.0	409	73.3
	A♯3	0.93	125.5	16.7	97.8	409	82.1
	C♯4	0.89	286.5	16.8	173.2	410	138.5
	E4	0.92	84.8	16.9	67.0	407	48.1
clb	E3	0.96	95.6	16.8	83.1	406	69.0
	G3	0.87	114.8	16.8	94.7	405	63.2
	A♯3	0.92	276.4	16.8	199.6	405	139.1
	C♯4	0.91	196.3	17.0	167.9	406	109.4
eng	E3	1.43	71.4	18.2	55.7	408	46.9
	G3	1.41	84.4	18.0	66.3	408	57.8
	A♯3	1.42	91.3	18.1	64.3	408	58.3
	C♯4	1.43	77.9	18.0	60.9	409	48.9
	E4	1.45	125.7	18.1	96.4	409	78.9
flt	E3	1.40	96.4	18.2	59.8	409	39.6
	G3	1.79	113.8	19.0	64.0	415	36.4
hrn	E3	0.90	54.4	17.2	32.5	406	17.5
	G3	0.90	106.3	16.8	54.4	405	41.5
	A♯3	0.90	159.2	16.8	67.2	405	38.7
	C♯4	0.90	73.3	16.8	43.8	405	23.2
	E4	0.90	50.1	16.8	25.4	405	12.9
obo	A♯3	1.46	157.5	18.1	127.4	408	102.7
	C♯4	1.42	96.1	18.3	73.6	408	62.1
	E4	1.41	69.9	18.1	43.2	408	34.5
pno	E3	0.89	75.3	16.8	53.0	405	38.8
	G3	0.90	69.4	16.9	53.1	405	43.0
	A♯3	0.90	97.2	17.3	72.6	405	48.3
	C♯4	0.90	72.8	16.7	53.7	405	38.3
	E4	0.90	68.3	16.8	45.5	405	26.6
sax	E3	0.89	80.8	17.4	74.1	405	67.5
	G3	0.89	194.7	18.1	163.7	405	121.8
	A♯3	0.90	115.6	17.9	89.9	405	74.8
	C♯4	0.98	113.4	17.7	87.6	405	75.3
	E4	0.90	349.7	17.7	249.6	405	161.3

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GROUP 2		Horner's Constrained Matching					
		3 Oscillators		4 Oscillators		5 Oscillators	
Instr	Pitch	Time	Error	Time	Error	Time	Error
tpt	G3	0.90	64.4	17.7	55.6	405	50.1
	A♯3	0.96	128.4	17.6	80.2	405	60.0
	C♯4	0.92	60.5	17.5	47.0	405	32.7
	E4	0.90	57.5	17.8	36.0	405	25.9
trb	E3	0.90	232.8	17.7	140.5	405	70.4
	G3	0.93	78.9	17.7	51.2	405	36.9
	A♯3	0.85	82.5	17.8	47.4	405	29.7
	C♯4	0.90	188.0	18.0	98.9	405	44.0
	E4	0.90	87.2	17.7	23.9	405	12.9
vla	E3	2.72	341.2	22.7	275.6	414	242.1
	G3	3.44	307.3	24.8	243.3	418	207.9
	A♯3	2.86	415.1	23.1	341.8	417	286.0
	C♯4	2.50	237.5	22.5	202.6	414	183.8
	E4	3.20	377.1	23.7	290.3	419	241.6
vlb	E3	1.42	121.6	19.3	97.6	415	81.9
	G3	1.54	98.8	19.7	86.6	413	78.0
	A♯3	1.42	83.7	19.6	52.5	410	38.0
	C♯4	1.61	91.8	19.5	75.5	412	64.1
	E4	1.88	114.2	20.2	86.7	413	65.8
vlc	E3	1.86	102.6	20.5	84.5	415	72.6
	G3	1.41	98.5	19.0	73.7	411	61.1
	A♯3	1.42	122.2	19.3	100.4	411	84.3
	C♯4	1.61	159.4	19.5	120.8	411	101.7
	E4	1.78	110.8	20.3	84.3	411	69.2
vln	G3	1.42	201.7	19.1	158.6	408	128.6
	A♯3	2.52	226.7	21.9	174.9	417	141.2
	C♯4	3.05	197.0	23.2	145.7	417	127.5
	E4	1.86	151.2	20.2	122.5	410	97.1
Mean		1.32	143.9	18.4	103.1	409	77.7
Std. Dev.		0.63	90.0	2.0	68.5	5	56.7

Table 47: Group 2, Horner's constrained matching results.

GROUP 3		Horner's Constrained Matching					
		3 Oscillators		4 Oscillators		5 Oscillators	
Instr	Pitch	Time	Error	Time	Error	Time	Error
cla	G4	0.43	80.4	6.37	53.0	130	47.8
	A♯4	0.44	78.4	6.39	49.9	131	32.0
	C♯5	0.50	172.1	6.37	109.2	130	80.4
	E5	0.40	232.1	6.37	105.6	130	60.2
eng	G4	0.72	107.9	7.02	72.2	131	57.6
	A♯4	0.68	58.1	7.03	42.2	131	29.1
	C♯5	0.71	27.9	7.02	21.7	131	15.7
	E5	0.71	37.5	7.02	26.0	131	21.6
flt	G4	0.88	79.0	7.44	50.4	131	25.5
	A♯4	1.21	113.5	8.25	55.4	134	25.8
	C♯5	1.14	72.8	8.07	39.4	133	19.0
	E5	0.91	33.0	7.52	20.2	132	10.6
hrn	G4	0.44	42.8	6.38	13.6	130	5.4
	A♯4	0.43	37.9	6.37	9.1	130	3.5
	C♯5	0.43	34.7	6.36	10.9	130	4.9
obo	G4	0.70	101.5	7.03	72.2	131	53.4
	A♯4	0.70	109.4	7.02	79.0	131	60.0
	C♯5	0.71	92.0	7.02	46.6	131	33.2
	E5	0.70	76.8	7.02	47.2	131	27.9
pno	G4	0.43	23.4	6.36	14.8	129	10.5
	A♯4	0.45	41.0	6.37	27.0	130	22.7
	C♯5	0.43	28.6	6.37	17.3	129	15.1
	E5	0.43	62.3	6.37	43.2	129	35.0
sax	G4	0.43	57.9	6.37	38.8	129	26.8
	A♯4	0.44	68.7	6.37	50.0	130	29.3
	C♯5	0.43	60.7	6.37	44.0	129	32.1
	E5	0.44	48.1	6.37	19.7	129	11.9
tpt	G4	0.43	44.6	6.37	25.0	130	14.5
	A♯4	0.44	66.6	6.37	33.8	130	22.7
	C♯5	0.43	57.2	6.37	32.1	129	21.8
	E5	0.45	24.1	6.36	11.5	129	7.7
trb	G4	0.46	58.2	6.37	25.9	129	12.7
	A♯4	0.44	62.7	6.36	26.4	130	10.7
	C♯5	0.43	28.7	6.37	11.4	130	8.5
vla	G4	1.31	336.9	8.41	237.8	134	160.6
	A♯4	1.56	232.8	8.89	184.1	135	156.3
	C♯5	1.35	177.9	8.41	140.0	134	113.6
	E5	2.16	212.5	10.44	147.2	138	112.6
vlc	G4	1.08	146.6	7.92	95.4	133	74.2
	A♯4	1.05	107.1	7.90	82.3	133	56.2
	C♯5	1.19	115.5	8.19	81.2	133	58.0
	E5	1.43	136.4	8.79	82.7	134	61.8
vln	G4	1.19	158.2	8.10	115.6	133	82.1
	A♯4	0.96	96.8	7.52	63.7	132	43.3
	C♯5	1.55	105.9	9.04	80.2	135	63.7
	E5	1.50	118.0	8.75	79.5	134	54.4
Mean		0.78	92.7	7.17	59.4	131	42.0
Std. Dev.		0.43	65.5	1.00	47.8	2	36.8

Table 48: Group 3, Horner's constrained matching results.

GROUP 4		Horner's Constrained Matching					
		3 Oscillators		4 Oscillators		5 Oscillators	
Instr	Pitch	Time	Error	Time	Error	Time	Error
cla	G5	0.18	179.7	1.77	89.4	24.4	45.4
	A♯5	0.19	34.8	1.76	15.5	24.4	10.6
	C♯6	0.19	73.3	1.76	34.6	24.4	14.6
eng	G5	0.32	44.9	2.08	28.7	25.1	15.7
flt	G5	0.37	27.1	2.38	13.3	25.6	11.1
	A♯5	0.58	20.6	2.41	7.7	25.7	5.3
	C♯6	0.41	12.5	2.25	8.8	25.4	6.2
	E6	0.37	22.3	2.34	10.0	25.5	7.3
glk	G5	0.19	424.4	1.76	365.8	24.4	280.2
	A♯5	0.19	368.1	1.76	304.5	24.5	218.0
	C♯6	0.19	177.5	1.77	122.2	24.4	93.7
	E6	0.19	163.4	1.77	131.2	24.4	72.2
obo	G5	0.32	47.0	2.08	22.9	25.1	11.8
	A♯5	0.33	45.4	2.08	25.5	25.0	13.4
	C♯6	0.32	51.7	2.08	25.3	25.1	11.6
	E6	0.32	20.9	2.08	7.0	25.1	4.3
pno	G5	0.19	28.7	1.76	14.4	24.4	10.9
	A♯5	0.19	8.5	1.77	4.9	24.4	3.1
	C♯6	0.19	9.1	1.76	4.6	24.4	3.0
	E6	0.19	12.1	1.77	4.5	24.4	4.0
sax	G5	0.22	58.6	1.77	43.2	24.4	21.3
	A♯5	0.23	35.3	1.77	13.6	24.4	6.9
	C♯6	0.11	23.9	1.76	9.1	24.4	5.8
tpt	G5	0.20	32.8	1.76	13.0	24.4	7.9
	A♯5	0.23	36.1	1.77	9.4	24.4	6.2
	C♯6	0.21	21.4	1.77	10.2	24.4	4.8
vla	G5	1.03	249.5	3.73	170.3	28.2	112.5
	A♯5	0.64	109.0	2.82	69.8	26.5	44.9
	C♯6	0.54	50.3	2.59	30.1	26.0	19.7
vlc	G5	0.86	181.7	3.05	120.9	26.9	67.9
vln	G5	0.63	162.7	2.80	93.4	26.5	65.5
	A♯5	0.73	107.3	3.04	77.3	26.9	58.1
	C♯6	1.06	104.3	3.78	67.3	28.4	46.4
	E6	0.65	83.2	2.86	57.3	26.8	41.3
Mean		0.38	89.1	2.19	59.6	25.3	39.8
Std. Dev.		0.25	99.8	0.58	82.6	1.1	60.7

Table 49: Group 4, Horner's constrained matching results.

GROUP 5		Horner's Constrained Matching					
		3 Oscillators		4 Oscillators		5 Oscillators	
Instr	Pitch	Time	Error	Time	Error	Time	Error
flt	G6	0.09	25.9	0.20	15.6	0.41	10.1
	A♯6	0.13	23.5	0.31	14.6	0.61	9.4
glk	G6	0.04	406.6	0.10	248.9	0.20	153.6
	A♯6	0.04	425.1	0.10	366.1	0.21	172.7
pno	G6	0.04	5.6	0.11	4.5	0.21	3.0
	A♯6	0.04	5.9	0.11	3.7	0.21	2.4
vln	G6	0.13	66.9	0.28	41.9	0.64	29.9
	A♯6	0.14	82.5	0.31	53.5	0.68	37.1
Mean		0.08	130.3	0.19	93.6	0.40	52.3
Std. Dev.		0.05	178.4	0.10	136.8	0.22	69.7

Table 50: Group 5, Horner's constrained matching results.