# ANN Based Voltage Stability Margin Prediction

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Abstract: This paper presents ANN based model for predicting stability margin for a power system prone to voltage instability. Such a model may be employed either for direct prediction of the stability margin based on the existing loading conditions or for forecasting the loading conditions for a future time period and then providing an estimate of the stability margin. The neural networks employed are the Multi Layer Perceptron (MLP) with a second order learning rule and the Radial Basis Function (RBF) network. The simulation results for an sample 5-bus system indicate that the ANN models provide a fairly accurate and fast prediction of the stability margin making them suitable for application in an on-line energy management system.

Keywords: Voltage Stability, Energy Margin, ANN.

### I. INTRODUCTION

Problems related to voltage stability have recently been considered as the major concerns in the planning and operation of power systems. Voltage stability is concerned with the ability of a power systems to maintain acceptable voltages at all buses in the system under normal conditions and after being subjected to a disturbance. A system enters a state of voltage instability when a disturbance, increase in load demand, or change in system condition causes a progressive and uncontrollable decline in voltage and the process may result in *voltage collapse*. Voltage instability has been attributed to the lack of adequate reactive support and the difficulty in the flow of required reactive power on the transmission network.

Analysis of voltage instability [1] for a given system state involves the examination of the static and dynamic aspects of the problem. The static approach is concerned with the determination of the proximity of the system state to the voltage instability boundary while the dynamic approach examines the actual mechanism that leads to a state of voltage instability.

A knowledge of the voltage stability margin is of vital importance to utilities in order to operate their system with maximum security and reliability. The system operator must be provided with an accurate and fast method to predict the voltage stability margin so as to initiate the necessary control actions.

Several methods have been reported in the literature [2], [3], [4], [5] to determine the proximity to voltage instability. These methods, although fairly accurate, are not computationally effective for real-time application in the energy management system. Hence, for on-line application, there is a need to develop a method which can quickly predict the voltage stability margin.

Recently there has been considerable interest in Artificial Neural Networks (ANN) to power system problems [7], S. C. Srivastava, Senior Member, IEEE Department of Electrical Engineering Indian Institute of Technology Kanpur 208 016, India. Email: scs@iitk.ac.in

[8], [9], [10], [11] because of various advantages offered by them including their high computational rates.

This paper presents an ANN based model for voltage stability margin prediction. Section II presents derivation of a voltage stability proximity indicator based on an energy function that reflects the system loading conditions. Section III describes an algorithm to calculate low-voltage load-flow solutions necessary to determine the energy margin. Section IV presents the ANN based model for margin prediction. The margin may be predicted using existing loading conditions on a network or loading conditions that have been forecasted by a different ANN. Section V presents the results obtained by applying the ANN based models to an sample 5-bus system and section VI gives the main conclusions.

# II. ENERGY MARGIN AS A PROXIMITY INDICATOR TO VOLTAGE INSTABILITY

In this section the energy function based voltage stability indicator is discussed. The expression of energy margin as a function of the system state is derived first for a simple radial system. This expression is generalized for an n bus system.

Consider a two-bus system shown in Fig. 1 with a single series transmission line connecting Buses 1 and 2. Bus 1 is assumed to be a slack bus with voltage magnitude fixed at 1.0 p.u., while a constant P-Q type of load is delivered at Bus 2. The governing algebraic equations for the real and reactive power flows on the line are given as

$$f(\alpha, V) = P_L + B_{12}V\sin\alpha = 0 \tag{1}$$

$$g(\alpha, V) = Q_L - B_{22}V^2 - B_{12}V\cos\alpha = 0$$
 (2)

where V= voltage magnitude at Bus 2 and  $\alpha = \delta_2 - \delta_1$  is the phase angle difference from Bus 2 to Bus 1. Multiplying both sides of (2) by  $V^{-1}$  we get,

$$g(\alpha, V) = \frac{Q_L}{V} - B_{22}V - B_{12}\cos\alpha = 0$$
(3)

The energy based stability margin to indicate vulnerability to voltage instability is obtained by integrating the



Fig. 1. Sample two bus sytem

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function composed of  $f(\alpha, V)$  and  $g(\alpha, V)$  from a lower limit corresponding to a Stable Operating Point(SEP)  $x^s = (\alpha^s, V^s)$  to an upper limit corresponding to a particular low-voltage power flow solution  $x^u = (\alpha^u, V^u)$  as

$$E(x^{u}, x^{s}) = \int_{x^{s}=(\alpha^{u}, V^{u})}^{x^{u}=(\alpha^{u}, V^{u})} [f(\alpha, V) g(\alpha, V)] \bullet [d\alpha \, dV]^{T}$$
  

$$= \int_{\alpha^{s}}^{\alpha^{u}} (P_{L} + B_{12}V\sin\alpha) \, d\alpha$$
  

$$+ \int_{V^{s}}^{V^{u}} \left(\frac{Q_{L}}{V} - B_{22}V - B_{12}\cos\alpha\right) \, dV$$
  

$$= \frac{1}{2}B_{22}(V^{u})^{2} + \frac{1}{2}B_{22}(V^{s})^{2} - B_{12}V^{u}\cos\alpha^{u}$$
  

$$+ B_{12}V^{s}\cos\alpha^{s} + Q_{L}\ln\left(\frac{V^{u}}{V^{s}}\right) + P_{L}(\alpha^{u} - \alpha^{s})$$
  
(4)

As the loading on the system increases the two solutions  $x^s$  and  $x^u$  approach each other and the energy margin E steadily decreases. The threshold loading level at which  $x^s$  and  $x^u$  coalesce and the energy margin reduces to zero represents the *Voltage Instability* point.

For a general n bus system, (1) and (2) at the  $i^{\text{th}}$  bus can be written as

$$f_{i}(\boldsymbol{\alpha}, \mathbf{V}) = P_{i} - \sum_{j=1}^{n} B_{ij} |V_{i}|| V_{j} |\sin(\alpha_{i} - \alpha_{j})$$

$$- \sum_{j=1}^{n} G_{ij} |V_{i}^{s}|| V_{j}^{s} |\cos(\alpha_{i} - \alpha_{j})$$

$$g_{i}(\boldsymbol{\alpha}, \mathbf{V}) = (V_{i})^{-1} \left[ Q_{i} + \sum_{j=1}^{n} B_{ij} |V_{i}|| V_{j} |\cos(\alpha_{i} - \alpha_{j}) \right]$$

$$- (V_{i}^{s})^{-1} \left[ \sum_{j=1}^{n} G_{ij} |V_{i}^{s}|| V_{j}^{s} |\sin(\alpha_{i}^{s} - \alpha_{j}^{s}) \right]$$
(6)

The constant terms in (55) and (6) are included so that f and g are identically zero at the SEP even when network transfer conductances are included. The energy function  $\mathcal{E}(\mathbf{x}^s, \mathbf{x}^u)$  is defined as in (7) given below. This is a scalar quantity dependent on system state (bus voltage magnitudes and phase angles) with the property that the current operating state defines a local minimum of this energy. Normally, the small random variations in the system which disturb its state from the SEP and add a small amount of energy are compensated by the system damping. At a secure operating point, where the energy well is deep, these random effects are negligible. But as the system moves towards a state vulnerable to voltage collapse, the depth of the energy well decreases and the system states (particularly the voltage magnitudes) becomes highly sensitive to load changes. Under such conditions there is a possibility that these random variations could push the state out of the potential well that defines its stable equilibrium point. A necessary condition for the state to escape this well is that it receives energy greater than the energy value of the closest Unstable Equilibrium Point (UEP) on the boundary of the well. The UEPs or the saddle points correspond to alternative solutions of the load flow equations, referred to as the *low-voltage solutions*.

$$\begin{aligned} \mathcal{E}(\mathbf{x}^{s}, \mathbf{x}^{u}) &= \int_{(\boldsymbol{\alpha}^{s}, \mathbf{V}^{s})}^{(\boldsymbol{\alpha}^{u}, \mathbf{V}^{u})} \left[ \mathbf{f}^{T}(\boldsymbol{\alpha}, \mathbf{V}) \, \mathbf{g}^{T}(\boldsymbol{\alpha}, \mathbf{V}) \right]^{T} \left[ d\boldsymbol{\alpha} \, d\mathbf{V} \right]^{T} \\ &= -\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} |V_{i}^{u}| |V_{j}^{u}| \cos(\alpha_{i}^{u} - \alpha_{j}^{u}) \\ &+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} |V_{i}^{s}| V_{j}^{s}| \cos(\alpha_{i}^{s} - \alpha_{j}^{s}) \\ &- \left[ \sum_{i=1}^{n} \int_{V_{i}^{s}}^{V_{i}^{u}} \frac{Q_{i}(x)}{x} \, dx \right] - \mathbf{P}^{T}(\boldsymbol{\alpha}^{u} - \boldsymbol{\alpha}^{s}) \\ &- \sum_{i=1}^{n} \left[ \sum_{j=1}^{n} \left( G_{ij} |V_{i}^{s}| |V_{j}^{s}| \cos(\alpha_{i}^{s} - \alpha_{j}^{s}) \right) \right. \\ &\left. \left( \alpha_{i}^{u} - \alpha_{j}^{s} \right) \right] + \sum_{i=1}^{n} \left[ \left( V_{i}^{s} \right)^{-1} \sum_{j=1}^{n} \left( G_{ij} |V_{i}^{s}| |V_{j}^{s}| \sin(\alpha_{i}^{s} - \alpha_{j}^{s}) \right) \right] \\ &\left. \sin(\alpha_{i}^{s} - \alpha_{j}^{s}) \right) \left( V_{i}^{u} - V_{i}^{s} \right) \right] \end{aligned}$$

Therefore, the energy function  $\mathcal{E}$  shows the height of the *Potential Barrier* between the operable solution and a low-voltage solution. As the system parameters ( loads and generation) move towards the point of voltage instability, the low-voltage solutions decrease in number. Immediately before collapse, only the operable solution and a single low voltage solution exist. These two solutions eventually coalesce and the steady-state equilibrium point is lost. In [6] Dobson et al., have shown that a system always loses the steady state stability by a saddle node bifurcation between the operable solution and a Type-1 low-voltage solution. A Type-1 solution has the property that the linearized system about that equilibrium point has a single positive eigen value. Therefore, for the calculation of energy based voltage stability margin the set of low voltage solutions to be determined may be restricted to Type-1 solutions, thus reducing the computational requirements.

# III. Algorithm for Calculation of Type-1 Low Voltage Solutions

In order to calculate the energy based stability measure, it is important to first find the appropriate unstable equilibrium (or low-voltage) solutions of a system.

For an *n* bus system, there can be upto  $2^{n-1}$  load flow solutions. In [2], Tamura et al. presented an algorithm which determines all the low voltage solutions using all the  $2^{n-1}$  combinations of initial guess vectors. This method

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was not computationally efficient. A simplified method was also presented by the same authors where only (n-1)combinations of initial voltage guess vectors are used to run the load flow. Furthermore, the solutions obtained by the simplified method correspond to the Type-1 solutions [5]. Based on this the main steps in the algorithm to calculate Type-1 low-voltage solutions and their associated energy measures are given below :

1. Obtain the stable operating solution  $V_i^s$  for the system from the flat start.

2. For each bus i (except slack) in the system calculate an initial low voltage guess  $V_i^u$  using a closed form equation (18).

3. Form the initial voltage guesses with  $V_j^s$  for  $j \neq i$  and  $V_j^u$  for j = i.

4. Perform the load flow solution for 'Bus i' using the Newton-Raphson method in rectangular coordinates.

5. If a solution exists, calculate the associated energy measure.

This energy measure is an indication of the voltage security in the area of Bus i.

# A. Calculation of $V_i^u$

The load flow equations at bus i in an n bus system can be expressed in rectangular coordinates as follows:

$$P_{i} = \sum_{j=1}^{n} \left\{ e_{i} \left( e_{j} G_{ij} + f_{j} B_{ij} \right) + f_{i} \left( f_{j} G_{ij} - e_{j} B_{ij} \right) \right\}$$
(8)

$$Q_{i} = \sum_{j=1}^{n} \left\{ f_{i} \left( e_{j} G_{ij} + f_{j} B_{ij} \right) - e_{i} \left( f_{j} G_{ij} - e_{j} B_{ij} \right) \right\}$$
(9)

where  $Y_{ij} = (G_{ij} - jB_{ij})$  is the  $ij^{\text{th}}$  element of  $\mathbf{Y}_{\text{BUS}}$ ,  $S_i = (P_i + jQ_i)$  is the complex power at Bus *i* and  $V_i = (e_i + jf_i)$  is the complex voltage at Bus *i*. (8) and (9) can be rewritten as

$$P_{i} = \left(e_{i}^{2} + f_{i}^{2}\right)G_{ii} + e_{i}A_{i} + f_{i}B_{i}$$
(10)

$$Q_{i} = (e_{i}^{2} + f_{i}^{2}) B_{ii} + A_{i}f_{i} - B_{i}e_{i}$$
(11)

where,

$$A_{i} = \sum_{j=1}^{n} (e_{i}G_{ij} + f_{j}B_{ij})$$
(12)

$$B_{i} = \sum_{j=1}^{n} (f_{j}G_{ij} + e_{j}B_{ij})$$
(13)

From (10) and (11),  $f_i$  can be derived as

$$f_i = \alpha \, e_i + \beta \tag{14}$$

where

$$\alpha = \left(\frac{A_i B_{ii} + B_i G_{ii}}{A_i G_{ii} - B_i B_{ii}}\right) \tag{15}$$

$$\beta = \left(\frac{P_i B_{ii} - Q_i G_{ii}}{B_i B_{ii} - G_{ii} A_i}\right) \tag{16}$$

Substituting (14) into (10) and eliminating  $f_i$  we get

$$ae_i^2 + be_i + c = 0 (17)$$

where  $a = (1 + \alpha^2)G_{ii}$ ,  $b = (2\alpha\beta G_{ii} + A_i + \alpha\beta_i)$  and  $c = (G_{ii}\beta^2 + B_i\beta - P_i)$ . The solutions of (17) are given as

$$e_i = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{18}$$

Thereon,  $f_i$  can be obtained by substituting  $e_i$  into (10). Thus, we have two solutions  $V_i^s = (e_i^s, f_i^s)$  and  $V_i^u = (e_i^u, f_i^u)$ .

## IV. ANN BASED MODEL FOR VOLTAGE STABILITY MARGIN PREDICTION

The proposed ANN based model for the determination of the energy margin is shown in Fig. 2. The input to the neural network consists of real and reactive power injections at all load buses in the system for a particular loading condition. The output of the network is the energy margin.

For implementing this model the Multi Layer Perceptron (MLP) structure with a second-order learning rule and the Radial Basis Network (RBF) were used. The network was trained with different sets of loading conditions and energy margins. The range of energy margins should cover the entire range of its variation. After training the network will be able to determine the energy margin which serves as an indicator of the systems proximity to the voltage instability boundary. The complete block diagram for the ANN based voltage stability margin predictor is given in Fig. 3. It contains the ANN models for load forecasting at each node utilizing the load history of the previous N time period and that for the energy margin determination described above. The load power factor at each node  $(pf_1, \ldots, pf_n)$  is assumed to remain constant.

# V. System Studies and Results

The effectiveness of the energy based voltage stability margin and the proposed ANN model was studied using the Stagg and El-Abiad five bus system given in the appendix. The results are given below.

## A. Calculation of energy margin

For the 5-bus system, all the real and reactive loads were assumed to be a linear function of the parameter  $\lambda$ ( $\lambda = 0$  for base case loading). As the value of  $\lambda$  is varied, the number of type-one low-voltage solutions changes.



Fig. 2. Block diagram for ANN based voltage stability margin determination

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Fig. 3. Block diagram for ANN based load forecasting and voltage stability margin prediction

Table I shows the base case ( $\lambda = 0$ ) solution  $V^s$  of the system and the initial low voltage vector  $V^u$  calculated using 17. These two solutions were combined to produce 4 initial low voltage guess vectors. The Newton-Raphson load flow in rectangular coordinates was run using each of these initial low voltage guess vectors. Two load flows corresponding to  $V_o^{(2)}$  and  $V_o^{(4)}$  converged giving the two low voltage solutions  $V^{(2)}$  and  $V^{(4)}$  shown in Table II. The energy margins calculated, using (7) corresponding to these two solutions are 6.79 and 6.74.

As the loading parameter  $\lambda$  was increased the number of type-one low voltage solutions decreased and so did the energy margin. For a point close to voltage instability  $(\lambda = 2.3)$  there was only one low voltage solution shown in Table III corresponding to an initial low voltage guess  $V_o^{(5)}$  at Bus 5 and the energy margin was 1.7341 at this loading condition. At  $\lambda = 2.366$  no low-voltage solution exists.

The parameter  $\lambda$  was varied in the range of 0 to 2.37. For each loading case, the lowest energy margin at a particular load bus is denoted as the "system energy margin". The variation of the system energy margin as a function of  $\lambda$  is shown in Fig. 4. As this figure shows, the system energy margin remains fairly constant up to a threshold of  $\lambda = 1.5$  beyond which it goes down rapidly indicating a vulnerability to voltage collapse.

Bus	$V^s$		$V^u$		
no.	е	f	е	f	
2	1.0461	-0.0512	-0.0069	0.0039	
3	1.0202	-0.0892	0.0062	-0.0095	
4	1.0191	-0.095	0.0035	-0.009	
5	1.012	-0.109	0.0227	-0.0452	

TABLE I Base Case Solution and Initial low-voltage vector  $V^u$ 

TABLE II Type-1 low-voltage solutions and energy margins ( $\lambda=0.0$ )

$\operatorname{Bus}$	Initial		Converged		Energy		
no.	value $V_0^{(2)}$		value $V^{(2)}$		Margin		
	е	f	e	f			
2	-0.0069	0.0039	0.567	-0.049			
3	1.02	-0.089	0.107	-0.078	6.79		
4	1.019	-0.095	0.1928	-0.06			
5	1.012	-0.109	0.188	-0.144			
Bus	Initial		Conv	erged	Energy		
no.	value $V_0^{(4)}$		value	$V^{(4)}$	Margin		
	e	f	е	f			
2	1.046	-0.051	0.57	-0.05			
3	1.02	-0.089	0.107	-0.078	6.74		
4	0.0035	-0.009	0.188	-0.059			
5	1.012	-0.019	0.204	-0.145			

	TABL	E III				
LOW-VOLTAGE	SOLUTIONS	AND E	NERGY	MARGIN	$(\lambda =$	2.3)

Bus	Initial		Converged		Energy
no.	value $V_0^{(2)}$		value $V^{(2)}$		Margin
	е	f	e	f	
2	1.046	-0.051	0.785	-0.158	
3	1.02	-0.089	0.563	-0.282	1.734
4	1.019	-0.095	0.534	-0.302	
5	0.023	-0.045	0.404	-0.332	

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## B. Prediction of energy margin for forecasted loading conditions of the system

A RBF network was employed for forecasting the real power loads of the 5-bus system for the first day (Monday) of the week. For energy margin calculation for the forecasted real loading, it was assumed that all the loads in the system have constant power factor. Furthermore, the generation in the system was varied in proportion to the load variation.

A MLP network with 30 hidden *log* sigmoid neurons was trained for a sum squared error goal of 0.001. The inputs to the network consisted of the real and reactive power injections (Fig 3) at all the load buses, the generation and output of the network was the energy margin.



Fig. 4. Variation of system energy margin  $\mathcal{E}$  with loading

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Fig. 5. Loading and Predicted Energy Margin

During the training phase, about 200 loading conditions were generated randomly varying between 50% and 200% of the base case loading and for each the corresponding energy margin was calculated as described above. This data consisting of power system loading conditions and energy margin was used to train the neural network.

The predicted and calculated energy margins for the 23 hourly loading conditions forecasted were found to be in close agreement. The maximum error between the predicted and the calculated values was 0.8728 p.u.

Fig. 5(A) shows the forecasted loading while Fig. 5(B)shows the variation of energy margin predicted for the 23 hours forecasted data. The energy margin prediction was consistent with the loading conditions on the system. Comparing two curves it can be seen that the predicted margin is lowest at peak loading conditions (e.g., at hour 10 and 20) and high at the valley load condition (e.g., at hour 4).

# VI. CONCLUSIONS

In this paper, a fast method for predicting the voltage stability margin using ANN has been proposed. An energy function based indicator was used to define the proximity of a system to voltage instability. The proposed ANN model was used to predict the voltage stability margin for forecasted loading conditions of a 5-Bus system. The results reveal the following :

1. The variation of the energy function based stability indicator with respect to changes in system load is smooth. so that the system security can be checked by periodically calculating the type-one solutions and their associated energy measures.

2. The results of energy margin predicted with the proposed ANN model are close to the actual value calculated. The model can provide fairly accurate estimate of system margin to the operator.

3. The response of the ANN model was extremely fast. The testing time for the prediction of energy margin for a given loading condition was less than 5ms.

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#### APPENDIX



Fig. 6. Stagg and El-Abiad System

### TABLE IV GENERATOR BUS DATA

Bus	$P_G$	$V_{spec.}$	Load	
no.	(MW)	(p.u.)	P (MW)	Q (MVAR)
1(slack)		1.06	0.0	0.0
2	40.0	1.0	20.0	10.0

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## TABLE V Load Bus Data

Bus	Load		
no.	P (MW)	Q (MVAR)	
3	45.0	15.0	
4	40.0	5.0	
5	60.0	10.0	

## TABLE VI Line Data

From	То	Series Impedance		Shunt Susceptance
Bus	Bus	R p.u.	<i>X</i> p.u.	$\frac{1}{2}B$ p.u.
1	2	0.02	0.06	0.06
1	3	0.08	0.24	0.05
2	3	0.06	0.18	0.04
2	4	0.06	0.18	0.04
2	5	0.04	0.12	0.03
3	4	0.01	0.03	0.02
4	5	0.08	0.24	0.05

# BIOGRAPHIES

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