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Assessment of Students' Declarative, Procedural, and Strategic Knowledge in the Area of Mathematical Word Problem Solving

by

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ABSTRACT

This exploratory study investigated the existing delcarative, procedural and strategic mathematical word problem solving knowledge of one hundred and seventy nine grade 4, 5 and 6 students. A model of mathematical word problem solving based on the work of Kilpatrick (1975), Dillon (1966) and Messick (1984) was proposed and the Mathematical Word Problem Solving Test (MWPST) incorporating an indivdiualized think-aloud procedure devised. The main purpose of the study was to determine whether the MWPST could assess students' declarative, procedural and strategic knowledge as outlined in the model.

Results indicated that while students' performance improved in succeedng grades, within each grade students' level of declarative knowledge differed markedly, supporting a continuity model of development (Dean, 1988; Seigler, 1986).

Relative to task variables, students had difficulty with problems involving more than one step (Quintere, 1982), extraneous information (Muth, 1984) and the drawing of logical conclusions (Caldwell and Golden, 1979). Consistent with previous findings by Kloosterman (1988), students' attitudes toward mathematics were significantly correlated with test performance.

Most students used four procedures when solving problems - examining the problem, choosing a plan, representing and carrying out the plan. Above average students also identified a goal, supporting the view by Andre (1986) and Solman (1988)

that domain specific and procedural knowledge are interdependent.

Cognitive and metacognitive strategy use also depended on student's declarative and procedural knowledge (Nickerson, 1985) with higher scoring students generating more strategies. Below average problem solvers tended to "react to" tasks (Schmidt, 1973). While average problem solvers employed a variety of cognitive strategies, metacognitive strategy use was isolated and inconsistant. Above average problem solvers appeared to actively monitor their progress.

Overall, the results supported the model and the MWPST as a way of measuring student word problem solving knowledge as well as the interrelatioship among declarative, procedural and strategic knowledge outlined in the model and postulated by Brown (1981), Carey (1985), Glaser (1984) and Sternberg (1985). Further research is recommended to expand this beginning knowledge base.

Table of Contents

List of Tables

List of Figures

I. INTRODUCTION

A. Rationale

Results of a recent National Assessment of Educational Progress (NAEP) in the United States indicated that the ability of nine to thirteen year olds to solve word problems has declined significantly since the first assessment in 1972 (Carpenter, Kepner, Corbitt, Linquist and Reys, 1980a; 1980b). This drop is unfortunate because learning to solve word problems should help prepare students to use mathematics in the real world. Understanding the reasons for this drop in performance would be useful for instruction. However, many existing instruments provide information not on how a student performed but on whether or not the student completed the problem correctly. Messick (1984) and Wiggins (1989) suggested that more information was needed on students' performance and that tests to assess both product and process information would be valuable.

Ebel (1979), McAloon (1984), and McKillop (1979) also noted that traditional tests of mathematical ability have tended to focus on whether students obtain correct answers rather than investigating their ability to apply procedures and strategies to solve mathematical word problems. Kantowski (1977) and Ekenstam and Greger (1983) suggested that there was a need to develop test instruments that assess the students' ability to solve word problems correctly as well as their use of processes and strategies when solving problems. Unfortunately, such conceptual processes are not easily observed or measured by techniques currently in use (Greenwood and Anderson, 1983).

If a process oriented test was available it should have a strong theoretical base in order to account for a wide range of student knowledge and information in the relevant domain. Wide applicability is needed to avoid responding to one area such as content knowledge at the expense of other critical and potentially overlapping areas such as how the person approached the task and what strategies are employed to complete the word problem. Previous studies (Lee, 1982; Muth, 1986; Quintere, 1982) have examined specific aspects of declarative, procedural, and strategic knowledge. In developing a model of word problem solving to account for a holistic view of the interaction of declarative, procedural, and strategic knowledge, the present study provided an integrated view of ways and means to measure and address the interaction of all three aspects of knowledge. This investigation began with conceptualizing the factors important in mathematical word problem solving. Based on this conceptualization, a model of mathematical word problem solving was developed along with a test to assess the declarative, procedural and strategic knowledge of elementary students.

B. Purpose of the Study

The primary purpose of this study was to develop a prototype of a test to assess declarative, procedural and strategic knowledge of students in the area of mathematical word problem solving at the Grade 4, 5, and 6 levels. In order to arrive at a series of problems tapping declarative, procedural and strategic knowledge in mathematical word problem solving, a model of word problem solving that recognized and accounted for the interaction among declarative, procedural and strategic knowledge when a person is involved in a problem solving task was necessary and was developed as a preliminary step to the development of the test itself. As well, the ability of the word problem solving test to tap the declarative, procedural and strategic knowledge of elementary students using a think-aloud procedure provided initial support for the proposed model as well as the problem solving test.

A second purpose of the study was to evaluate the effectiveness of the test format and administration procedures in assessing declarative, procedural and strategic knowledge in word problem solving. It was expected that this total approach would provide insight into students' strategies, processes, and content knowledge.

C. Significance of the Study

This study has practical and theoretical implications. With regard to the practical implications, the information gained from assessing students' declarative, procedural and strategic knowledge can be used to develop and implement instructional programs that address all three areas of knowledge. Specific problem areas also can be isolated for later intervention. This type of assessment focuses attention on more than the student's correct answer. Data gained from the present study will serve as a precursor to the development of additional assessment devices and have implications for cognitive instructional programs that facilitate the discovery and acquisition of declarative, procedural and strategic knowledge in word problem solving.

Theoretically, the test and its administration procedure deviate from traditional ways of measuring word problem solving performance. Many traditional tests, such as the Canadian Test of Basic Skills (King, 1984), view word problem solving skill as a unidimensional attribute measuring only solution correctness. The test developed for this study affords an opportunity to assess students' word problem solving abilities in many interrelated facets of knowledge. Meichenbaum, Burland, Gruson, & Cameron (1985) and Wiggins (1989) stated that assessment instruments such as the one proposed seem more valid for evaluating cognitive instructional programs than the product-oriented measures used by some researchers to assess process outcomes. Thus, the resulting instrument and model used in the present study will enhance the knowledge base necessary to evaluate the effectiveness of process oriented programs in a more meaningful way.

D. Definition of Terms

Word Problem - refers to a written statement or statements that contain numerals and pose a question that requires using familiar mathematical operations and strategies to obtain a solution.

Mathematical Word Problem Solving - The building of a representation of the words of a problem and finding the solution to the problem using the rules of arithmetic and algebra (Mayer, 1986).

Declarative Knowledge - Knowledge of the content and facts of a discipline (Dillon, 1986).

Procedural Knowledge - The specific or general processes undertaken in the actual encoding, transforming and sorting of information (Kirby, 1984). Cross and Paris (1988) explained that procedural knowledge reflects an appreciation of how skill operate or are applied.

Strategic Knowledge - Strategic knowledge according to

Messick (1984) refers to knowledge individuals have about alternatives for goal setting and planning.

Strategies - Strategies are defined as techniques, principles or rules that will facilitate the acquisition, manipulation, integration, storage and retrieval of information across situations and settings (Alley and Deshler, 1979).

Construct - "Some postulated attribute of people, assumed to be reflected in test performance. In test validation the attribute about which we make statements in interpreting a test is a construct." (Cronbach and Meehl, 1955, p. 59)

II. REVIEW OF LITERATURE

A. Introduction

Attempts to assess acquisition, maintenance, and transfer of generic problem solving strategies may be providing us with much information about "what" students learn, but little on "how" students apply that knowledge (Glaser, 1984). Traditional achievement test scores tend to assess products, outcomes, and content, not "how" a student learns (McAloon, 1984). Kirby (1984) noted that "how" a student learns concerns the use of strategies to regulate information processing. To assess and learn more about how students regulate the processing of information requires, as Kirby stated, that researchers bridge the gap that appears to exist between cognitive theory and educational applications. One educational application of cognitive theory and the regulation of information processing is the assessment of mathematical problem solving strategies.

While the assessment of problem solving strategies has been problematic, recent advances have begun to provide researchers with methods to use to gain insight into how individuals solve problems. For example, use of verbal reporting techniques found in recent research to examine cognitive processes (Romberg and Collis, 1985; Uprichard, Phillips and Soriano, 1986) have provided additional information on the nature of strategies used by students to regulate information processing in mathematical word problem solving. The use of verbal reporting techniques, coupled with other testing procedures such as the inclusion of written work would contribute to bridging the gap noted by Kirby (1984) between cognitive theory and educational applications. In addition, such testing procedures could provide insight into internal cognitive processes rather than having to infer processes based on differential performance on multiple choice tests of mathematical problem solving or reviewing procedures used to solve computer presented problems (Ginsburg, 1984; Nickerson, 1985).

However, confusion still exists within the problem solving literature regarding such concepts as cognitive strategies, processes, and heuristics (Best, 1986; Kirby, 1984). As well, other discrepant issues such as the distinction between the constructs cognition and metacognition and the roles of knowledge and development in processing information need to be clarified. These issues will be addressed before examining problem solving within a theoretical and educational format.

B. Cognition: A Conceptual Framework

The meaning of the construct "strategy" and the similar terms "process" and "heuristic" can be vague and confusing.

Processes usually refer to a class of cognitive functioning that is involved in the actual encoding, transforming, and sorting of information, while strategies are responsible for controlling or planning the use of these processes (Kirby, 1984). The basic problem is that strategies have process aspects and vice versa. When a person performs a realistic task, the distinction between process and strategy is blurred, making it difficult to impose a strict dichotomy between the two.

The literature on mathematical problem solving also refers to a heuristic in much the same way that persons in other academic areas refer to a strategy. Heuristics have been defined as rules of thumb that problem solvers find useful (Andre, 1986; Best, 1986) and as general strategies "that help problem solvers approach and understand a problem and efficiently marshal their resources to solve it" (p. 9) (Schoenfeld, 1985a). Schoenfeld also drew a distinction between heuristic strategies and managerial or control strategies which are reasonable decisions about which strategies to try and when to try them. Strategies have also been defined as techniques that assist in the acquisition, manipulation, integration, storage and retrieval of information across situations and settings (Alley and Deshler, 1979). Pressley, Goodchild, Fleet, Zajchowksi and Evans (1987), on the other hand, stated that strategies were processes that when matched to the requirements of the task, facilitate performance.

It would appear that the terms process, strategy, and heuristic have been used interchangeably by some writers. However, for the purposes of this study, processes will refer to procedures used in problem solving. This reference is in keeping with the distinction made by Kirby (1984) between process and strategy. The terms heuristic and strategy appear to denote the same construct and could be used interchangeably to refer to techniques that facilitate problem solving by helping the problem solver approach, understand, and solve a problem. In addition, the term managerial strategies (Schoenfeld, 1979) will be used to refer to knowledge of which strategies to use as well as when, where, and how to use them. Both heuristic strategies and managerial strategies are cognitive strategies but managerial strategies may be considered higher-order strategies.

In effect, the difficulty in separating processes from strategies exists due to the difficulty differentiating cognition from metacognition. Schmidt (1973) defined cognition as "any activity of becoming or being aware of something or having an object of consciousness" (p. 106). For example, during a conversation a person can not only listen and respond but also actively monitor responses, selectively choose input, as well as note emotional tone and nonverbal cues. This notion of cognition is inclusive, covering more than the traditional narrow intellectual processes such as problem-solving, and extending to include affective and emotional awareness. Flavell (1979) has a similar notion of cognition. He stated that cognitive processes habitually intrude themselves into virtually every human psychological process and activity.

Flavell (1979) also noted that individuals play an active role in cognitive environmental interchanges. According to Flavell and Wellman (1977) this active role on the part of the individual involved two aspects. One aspect involved being a participant in a cognitive event. The second aspect involved being an observer or monitor of that cognitive event. Flavell called the active monitoring and consequent regulation and arrangement of the processes involved in a cognitive event metacognition. Specifically, Flavell (1977) stated that metacognition included "the active monitoring and consequent regulation and orchestration of these (cognitive) processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective" (p. 232).

While the term metacognition is a relatively new term, referents to the label "metacognition have been inconsistent, especially regarding an operational definition of this construct (Lawson, 1984). For example, Sternberg (1986) divided thinking into three kinds of skills: executive processes called metacomponents, and two types of nonexecutive learning processes called performance components and knowledge-acquisition components. The executive processes are used to plan, monitor and evaluate thinking. Performance components are used to carry out the thinking while the knowledge-acquisition components are used in learning new information. Sternberg went on to state that being a good thinker involves four elements including access to workable strategies for solving problems, the ability to represent information mentally, the motivation to use thinking skills and a knowledge base.

Brown (1981) distinguished two types of metacognition: 1) knowledge about cognition and 2) regulation of cognition. Knowledge about one's own cognition includes such things as knowledge about one's own cognitive resources, and knowledge about how compatible the demands of learning situations are with one's own resources. Knowledge about cognition is stable over time, can be stated by the learner, may not be accurate and is late developing. The second type of metacognition, regulation of cognition, "consists of the self-regulatory mechanisms used by an active learner during an ongoing attempt to solve problems" (Brown, 1981 p. 21). These activities are thought to be relatively unstable, rarely statable, and relatively independent of the learner's age. While these two types of metacognition may be distinguished conceptually, Brown (1981) argued that both types of metacognition are closely related and should not be separated if one is to understand metacognition.

Schoenfeld (1985a) also pointed out the limitations of viewing metacognitive acts narrowly as active, intentional, and within the individual's awareness. This description characterizes idealized behaviour and tends to isolate metacognitive behaviour too much from other levels of cognition. Schoenfeld suggested that a broad theoretical approach to human cognitive performance was needed to deal with all issues related to performance. He noted that a great deal of competent human performance can be attributed to the almost automatic, often unconscious, accessing of what might be called "situational representations and associated sets of responses" (p. 365).

Earlier, Vygotsky (1962) described two phases in the development of knowledge. The first phase was the automatic unconscious acquisition of knowledge which was followed by gradual increases in active conscious control over the knowledge. It may be that this is essentially the difference between what has been termed cognition and metacognition. That is, the metacognitive components of monitoring and regulating are similar to the gradual increases in active conscious control over knowledge. The unconscious acquisition of knowledge and the knowledge itself is cognition. Indeed, Brown (1980) has made a similar statement concerning the distinction between cognition and metacognition.

The issue of conscious control raises another interesting dimension to the continuing debate over cognition and metacognition. Lawson (1984) stated that knowledge about metacognition has been limited to conscious reportable knowledge described by Brown (1981) as "stable, statable, fallible, and late developing" (p. 211). Yet, not all executive processes are consciously controlled processes. Kirby (1984) stated a distinction should be drawn between established automatized strategies and the "to-be-constructed" strategies that are not yet automatized. These "to-be-constructed" strategies are most open to conscious control and reporting. Those strategies which have become automatized may not be open to conscious control and not readily reported (Shiffrin and Dumais, 1981). Nickerson, Perkins and Smith (1985) also noted that metacognitive knowledge may be accessed intentionally or automatically and may be influential with or without entering consciousness.

Brown (1978) pointed out that the concept of knowing about knowing cannot be separated from the concept of knowledge itself. The implication is that one needs to have not only the domain-specific knowledge that is essential to skilled performance, but the knowledge of when and how to apply the knowledge in specific contexts (Glaser, 1984). Several writers have pointed out that possession of a body of knowledge that is relevant to a particular domain does not guarantee that such knowledge will be effectively applied in that domain (Carey, 1985; Glaser, 1988; Nickerson et. al., 1985; Walker, 1987). For example, Barrows and Tamblyn (1980) claimed that the possession of a large body of knowledge in medicine provides no assurance that the possessor knows when or how to apply that knowledge in the care of patients. However, as Brown (1978) has noted, this type of reasoning illustrates the interdependence of metacognition with its content area. Despite the earlier illustration, the debate over whether problem solving and reasoning are independent of or interdependent with knowledge continues (Perkins, 1985; Walker, 1987).

In order to rationalize the distinction between cognition and metacognition with the definition of cognition cited earlier by Schmidt (1973), it is important to note that Schmidt's notion of cognition appeared to entail a continuum of development involving the individual from "reacting to" to "knowing about". As such, the metacognitive component would appear to come at the "knowing about" end of the continuum. It might be argued however, that the "knowing about" in Schmidt's notion of cognition is really the executive or metacognitive component referred to by Brown, Flavell, and Sternberg. Therefore, the notion of metacognition may not really be separate from cognition but at one end of the continuum of cognitive experience / development within a dynamic and broad definition of cognition. Using Flavell's (1979) concept of the variables of person, task and strategy involved in a cognitive event makes it reasonable to conclude that the same person may exhibit greater use of metacognitive strategies under one circumstance rather than another as a result of task demands. This has particular implications for the research at hand in that the nature of the items may elicit different degrees of active conscious monitoring on the basis of the tasks rather than the person alone. In other words, a dynamic relationship may exist.

Acceptance of a definition of cognition such as proposed by Schmidt (1973) may be expedient and ecologically valid. Such a definition accounts for individuals "reacting to" situations which may result from passivity, task demands or automatized unconscious responses. As well, the definition accounts for the "knowing about" or the conscious knowledge and control component which some term "meta".

C. Development and Cognitive Information Processing

There is no one detailed and comprehensive view of the development of metacognitive knowledge and executive processing (Lawson, 1984). In general, the evidence from developmental studies seems to support a view of both dimensions of cognition outlined by Brown (1981) as developing quite early in the child and in a fashion more compatible with a continuity model of development (Siegler, 1986). Lunzer (1986) noted that the idea of relatively discontinuous stages has little support from research, nor is there much evidence for phases of accelerated intellectual growth which are bound by longer periods of equilibrium marked by a slower rate of development. What evidence there is would suggest that the acquisition of cognitive schemes is a gradual business and new competencies always build on old. Kurdek and Burt (1982) also supported the view that children's cognitive abilities were better conceptualized as developing along a continuum than in a step-like fashion.

Several recent studies (Bingham, Rembold & Yussen, 1986; Dean, 1987; Liebling, 1988) supported the notion of a developmental continuum in the area of cognition. Liebling (1988) examined elementary school-age children's (Grades 1, 3, 5) knowledge, comprehension, production of, and reasoning about directives. Her findings indicated that children's knowledge of directives continued to develop during the elementary school years. Between the ages of 6 and 11, students improved in their ability to articulate their understanding of the appropriate use of directive in specified social circumstances.

Dean (1987) tested ninety-seven children in grades kindergarten, 2, 4, 6, and 8 on ten probability problems based on Piaget and Inhelder's (1975) research. This study confirmed a developmental trend in children's probability concepts. Children younger than 6 or 7 knew that the number of desired objects was directly related to the desired outcome. Somewhat older children also knew that numbers of undesired objects were inversely related to the desired outcome. By 10 or 11 children knew that probability depended on the relation between numbers of desired and undesired objects within a set. Very few children in this study constructed quantitative ratios (reflecting formal operational level according to Piaget and Inhelder) suggesting that even 14 and 15-year-old adolescents rarely compute quantitative ratios on two-set problems. The implications of these findings for the proposed study are that few of the subjects may have reached formal operational level and that most may not have access to or employ managerial strategies.

The development of skills used in identifying the form and content of main idea was researched by Bingham, Rembold & Yussen (1986). Subjects included college students as well as 2nd, 5th, and 8th grade students. It was found that with age, children improved in their identification of the best main idea statement and developed in their ability to distinguish between important and unimportant story elements.

On the other hand, Pressley, Levin, Ghatala, and Ahmad (1987) investigated whether there were developmental increases in metacognitive benefits solely from taking a test. Seventy-two children, thirty-six from ages 6 to 8 and thirty-six from ages 9 to 11, took part in the study. Using forty test items (20 easy; 20 advanced) from the Peabody Picture Vocabulary Test, subjects were asked to estimate their overall performance at the end of the test. Results indicated that the older children were more accurate and less variable than younger children.

The results of the Pressley et al (1987) study supported the findings of Bingham et al (1986) indicating that improved monitoring and accuracy occurs with increases in age. However, the asking of questions after test performance in the Pressley et al (1987) study and the multiple-choice answer format in the Bingham et al (1986) study provided limited information on how children processed these tasks.

Yeotis and Hosticka (1980) also questioned the notion of an automatic step from concrete thought to formal thought due to age and general environment. They cited Herron's (1975) work to indicate that fewer than 25% of college freshmen are functioning fully in a formal operational mode in their thought processes. More recently, Nickerson (1985) reported evidence that students go through years of formal education without acquiring a sufficiently deep understanding of the fundamental concepts they have studied and are not able to apply these concepts effectively in new contexts.

Others, like Brown (1981) saw a relationship between the development of content knowledge and the development of reasoning skills. Recent studies by Carey (1985) and Bullock, Gelman, and Baillargeon (1982) have demonstrated that young children's competencies are more like older children's than once was assumed. Carey (1985) and Bullock et al (1982) provided further evidence that there were classification structures available to the young and that these structures were used under some circumstances. For example, in Carey's study a 4 1/2-year-old dinosaur expert was able to organize his knowledge within a class-inclusion structure. This should not be possible according to a theory that holds that a preschooler lacks concrete operations and therefore cannot deal with such structures.

It is generally assumed that metacognition is gradually acquired throughout development as learners experience new and varied demands on their cognitive skills (Duell, 1986). Hagen (1971) suggested that through learning experiences, learners come to realize that they can control how much they learn by the activities or strategies they use. This view is also held by others (Brown, 1981; Glaser, 1988; Neimark and Lewis, 1983; Yoetis and Hosticka, 1980). Brown, for example, noted that the regulation of cognition was late developing and that many young learners were unaware of their own capabilities and limitations as learners. Such learners were not always able to take the most appropriate actions in any given learning situation. Although Brown (1981) acknowledged incidences of both metacognitive awareness and self-regulating behaviors in quite young children involved in simple tasks, she regarded both of these dimensions as late developing. She supported this view by arguing that children, even child experts, were limited in the degree to which their learning and processing could be extended across domains.

Messick (1984) noted that cognitive abilities were still developing well into adulthood. Whereas the young child might reveal a given capacity on one carefully crafted task, the older child will reveal it on many tasks. The implication is that there are major differences in the range of situations to which young children can apply competence. It would seem that the competence of the young child is fragile and that of the older child is fluid and generalizable. While the young child needs to be tested with a particular set of stimuli in a particular setting with a particular task, the older child can transfer his or her knowledge across a variety of domains. Therefore, the current research will utilize familiar mathematical content in a recognized format and be conducted at the child's school, a familiar setting.

The emergence of higher order executive strategies depends on the continuous development of a broad base of information about lower level strategies. Without entrenched metacognitive knowledge - that contains specific information about a variety of lower level strategies such as rehearsal and organization - it is unlikely that sophisticated, higher level executive strategies will develop (Kurtz and Borkowski, 1987; Pressley et al, 1987).

Researchers (Brown, 1981; Dean, 1987; Leibling, 1988) have noted that a child's cognitive development is not bound by age or stage, rather it appears to progress at an individual rate. While metacognitions may appear as a function of development, and while there are some exceptions, they do not seem to be available to young children (Brown, 1981; Gelman, 1986). It would appear that cognitive development is restricted by the child's competence to access, generalize and transfer knowledge (Gelman, 1986).

D. Knowledge and Cognitive Information Processing

A number of authors have noted the interdependence of content and reasoning skills (Chi, 1985; Glaser, 1984; Kantowski, 1977; Webb, 1977). Many of these writers find the position - that there are general overarching formal operations - to be wanting, and see instead development of skills within each performance domain (Nickerson et al, 1985).

Messick (1984) noted that a person's structure of knowledge in a subject area included not only declarative knowledge about substance (information about what) but also procedural knowledge about methods (information about how) and strategic knowledge about alternatives for goal setting and planning (information about which, when and possibly why). Although the acquisition of declarative and procedural knowledge is an explicit goal of typical instruction in most subject areas, strategic knowledge is rarely so and must be acquired by induction (Greeno, 1984).

Dillon (1986) also addressed the distinction between the different types of knowledge underlying successful problem solving within disciplines. She defined declarative knowledge as the content and facts of a discipline and procedural knowledge as the specific or general processes being explicated as well as the level of the processes. In addition, Dillon mentioned a third type of knowledge and used Nickerson's (1985) term of self-knowledge to delineate it from the other types of knowledge. Self-knowledge is defined as knowledge about our own strengths or limitations. Self-knowledge seems comparable to Brown's (1981) notion of metacognition.

Similar categories of knowledge were mentioned in an article by Cross and Paris (1988) when discussing strategic reading. Declarative knowledge about reading included an understanding of what factors influenced reading. Procedural knowledge reflected an appreciation for how skills operate or are applied and conditional knowledge was seen as an understanding of the occasions when particular strategies are required and why they effect reading.

The distinction between declarative versus procedural knowledge is usually described as the distinction between knowing what and knowing how. Knowing what refers to being able to talk about something; knowing how refers to being able to do that something. For most school-acquired intellectual skills, the objectives of instruction involve both declarative and procedural knowledge about a skill (Andre, 1986; Soloman, 1988). Acquiring procedural knowledge does not insure that declarative knowledge will be acquired and vice versa. Schoenfeld (1979) and Perkins (1985) also noted the relationship between procedural and declarative knowledge. Indeed, Schoenfeld emphasized the limitations of merely having a strategy as opposed to knowing when and how to apply it.

Many strategies appear to be context-bound. This does not mean that general strategies do not exist (Perkins, 1986; Perkins and Soloman, 1989). However, general strategies provide only limited power for complex intellectual tasks. This dependence on the interaction of knowledge and cognitive processes was evident in studies of novice and expert chess players (Chi, 1978; Chase and Simon, 1973). Young expert chess players exhibited superior recall for chess positions when compared with low-knowledge adults who knew little about chess. This superiority is attributed to the influence of knowledge in the content area rather than the exercise of memory strategies. The hypothesis is that change in the knowledge base can produce sophisticated cognitive performance (Glaser, 1984). If novices do not have the resources in a particular domain, they do not gain much from a general strategy.

One way to characterize the difference between an expert's understanding of a problem and that of a novice is with respect to the richness of the knowledge base from which each derives understanding. While both novices and experts must make use of the memory's stored knowledge to solve problems, findings indicating that experts tend to obtain solutions faster and typically verbalize a smaller number of steps in the solution process suggest that the relevant knowledge they possess may be stored in larger chunks (Nickerson, 1985; Perkins and Soloman, 1989; Sweller, 1988).

Carey (1985) also suggested that reasoning and problem solving are greatly influenced by experience with new information. For example, Carey analyzed young children's (ages 4 to 10) understanding of biological concepts. Her findings indicated that older children's (ten year olds) acquisition of domain-specific information resulted in structured knowledge that is reflected in the ability to think about properties and to reason appropriately. Her research suggested that concepts develop with increasing knowledge in a domain.

Similar results were reported by Hegarty, Just, and Morrison (1988) who noted the relationship between knowledge and reasoning in their study of mechanical reasoning with subjects who had different levels of mechanical ability. Subjects with a high-knowledge of mechanical systems used both qualitative and quantitative means of dealing with problems while those with low mechanical knowledge used only qualitative methods. These results suggested that high-knowledge individuals were more flexible problem solvers who could vary their solution process depending on the demands of the problem.

Perkins (1985) discussed the notion of general and domain specific strategies. He noted that as performance in a domain improved, quite likely it became attuned to the specific requirements of the domain; that is, the higher the level of competence concerned, the fewer general cognitive-control strategies there were. Improving cognitive awareness may require instruction in several specific subject matters rather than in overarching cognitive skills.

Studies by Carey (1985), Chi (1978), and Siegler and Richards (1982) have pointed out the significance for development of teaching reasoning along with information. As Siegler and Richards (1982) stated:

> "Developmental psychologists until recently devoted almost no attention to changes in children's knowledge of specific content....Recently, however, researchers have suggested that knowledge of specific content domains is a crucial dimension of development in its own right and that changes in such knowledge may underlie other changes previously attributed to the growth of capabilities and strategies." (p. 930)

While experts and novices in a domain typically do not differ with respect to general strategies, they differ markedly in both the quantity and the quality of domain-specific knowledge they possess (Walker, 1987). Walker (1987) investigated whether domain-specific expertise could compensate for low overall aptitude level when people were learning new domain related material. Participants were low-aptitude and high-aptitude enlisted Army personnel, grouped on the basis of their Army aptitude test of general/technical ability and their level of baseball knowledge. Subjects were presented with a taped and written fictional account of an inning of a baseball game. After listening to the tape, subjects wrote down what they could remember and completed a multiple-choice test on the story. Results demonstrated that individuals with high baseball knowledge outperformed low baseball knowledge individuals, regardless of overall aptitude level. Effective comprehension seems to depend on the learner having sufficient knowledge of the topic so that the words from which the text is built evoke the concepts necessary for understanding (Adams and Bruce, 1980). This finding provides further evidence of the importance of domain knowledge to effective cognitive processing.

It would appear that the acquisition of specific content knowledge is seen as a factor in acquiring increasingly sophisticated problem-solving ability (Siegler & Richards, 1982). Indeed, results of recent studies (Adams, Kasserman, Yearwood, Perfetto, Bransford, and

Franks, 1988; Gelman, 1988) have pointed to the interdependence of domain specific knowledge and processes. The emergence of new domains seems to lead to more powerful and more highly differentiated inductive thinking.

Research has also indicated that strategies are often a relatively automatic consequence of previously acquired knowledge (Bransford, Sherwood, Vye, and Riesser, 1986). Competencies in a domain and the ability to think about the domain seem to develop hand in hand. The important factor appears to be knowing when to activate the knowledge that is needed. Thus, the possession of a body of knowledge relevant to a particular task domain does not guarantee that knowledge will be effectively applied in that domain (Nickerson et al, 1985; Pressley et al, 1987). The implication is that one needs not only the domain specific knowledge but also the knowledge of when and how to apply and transfer that knowledge in specific contexts (Perkins and Soloman, 1989).

Given the interdependence of "meta" with content area and with cognitive processing, Forrest-Pressley and Walker (1984) and Glaser (1988) have argued for more research in defining the role of metacognition in academic content areas. In addition, they stress the need to explore the relationship between the cognitive and metacognitive aspects of academic areas. One specific content area receiving increased attention is that of mathematical word problem solving.

E. MATHEMATICAL PROBLEM SOLVING: Definitions and Models

Lester (1983; 1985) noted the lack of agreement among theorists and researchers within mathematics education on the nature of word problem solving. Some researchers view word problem solving as applying previous experience, knowledge, skills, and understanding to new or unfamiliar situations (Alberta Education, 1985; Best, 1986; Krulick and Rudnick, 1980; Linquist, 1984). While agreeing, in part, with this definition, Lester (1980; 1985), Nickerson et. al. (1985) and Foxman, Joffe, and Ruddock (1984) stated that the procedure for determining the solution to a mathematical problem may be unknown or not obtained by mere application of routine or rote learning techniques. Such definitions suggest that if a technique or procedure is known and easily applied, then a problem solving situation does not exist.

Definitions of mathematical word problem solving put forth by Crowley and Miller (1986), Mayer (1986), Kantowski (1977), and Swing and Peterson (1988) encompass both product and process components and seem more applicable generally and to elementary school mathematics in particular. For example, Mayer (1986) interpreted mathematical word problem solving as building a representation of the words of the problem and finding the solution of the problem using the rules of arithmetic and algebra. Similarly, Crowley and Miller (1986) defined mathematical word problem solving to be the "interpretation of information and the analysis of data to arrive at a single acceptable response or to provide the basis for one or more acceptable alternatives" (p. 36).

The definitions put forth by Mayer (1986) and Crowley and Miller (1986), stated above, will be applied in this study as they do not demand that word problem solving be directed toward performance on some intellectually demanding task but rather recognize that problem solving is relative and at times automatic. Crowley and Miller's definition is also broad enough to include problem solving at all mathematical content levels. There is a recognition that many mathematical skills are still being acquired, therefore, deciding what process to use and how to use it creates a problem for many children (Campbell, 1984).

Models of Mathematical Word Problem Solving

The complexity of the word problem solving process is reflected in the number of models reported in the literature (i.e. Briars and Larkin, 1984; Dunlap and McKnight, 1980; Gagne, 1983; Kintsch and Greeno, 1985; Lester, 1985; Polya, 1962; Webb, 1977). Many of these models are classified as either descriptive or prescriptive. Descriptive models attempt to describe or explain the experiences of the students as they solve a problem. For example, Klausmeir and Goodwin (1966) identified five phases: 1) setting a goal, 2) appraising the situation, 3) attempting to reach the goal, 4) verifying the solution, and 5) obtaining the goal. These phases consider only one level of processing - what Sternberg (1986) refers to as nonexecutive processes.

Prescriptive models, such as Polya's (1962) are more a proposal for teaching students how problem solvers ought to think. The four phases necessary to be successful at problem sclving according to Polya are 1) understanding the problem, 2) devising a plan, 3) following the plan, and 4) looking back. While these phases include general cognitive processes and imply metacognitive processing, this is not explicitly stated. Indeed, Polya's model focuses on instruction in problem solving and equipping students with a set of heuristics under the assumption that this is sufficient to make them good word problem solvers. Lester (1985) and Schoenfeld (1985a) criticized such approaches for their overemphasis on developing general heuristic skills and their ignoring of the managerial skills necessary to regulate one's own activity.

Other researchers presented models which proposed a synthesis of the various prescriptive and descriptive models (i.e. Gagne, 1983; Webb, 1977; Uprichard, Phillips, and Soriano, 1986). Uprichard et al (1986) outlined an extensive model that described the actual behaviour exhibited while engaged in problem solving and prescribed the thought processes that could be followed in order to be a successful problem solver. They hypothesised that solving mathematical word problems required prerequisite knowledge in mathematics and language, as well as utilization of six dimensional strategies - reading, analyzing, estimating, translating, computing and verifying. They claimed their schema could be utilized to design viable tools for diagnosis and instructional sequences that facilitate the acquisition of abilities needed to solve mathematical word problems successfully (Uprichard et al, 1986).

However, Uprichard et al (1986) limited diagnosis and instruction to the areas of prerequisite knowledge (declarative knowledge) and the six dimensions of the problem solving process (procedural knowledge). The existence of cognitive and/or metacognitive strategies was implied based on students' performance on a limited number of problems and adherence to the six procedural steps. In addition, the inclusion of estimating as a necessary component in problem solving is questionable. While this is a desirable strategy, even Uprichard et al (1986) admitted that students rarely use this strategy and they provided no evidence to suggest that inclusion of this strategy improved success when solving problems.

Recently, information processing models (Briars and Larkin, 1984; Kintsch and Greeno, 1985) have attempted to explain how children solve word problems. Problems dealt with in both models are limited to one-step addition and subtraction word problems at the grade 1 and 2 mathematics level. The subject's performance on computer presented problems is observed and related to the sequence of steps the computer model produced to obtain a solution. It is assumed that the steps executed by the model can be compared with strategies executed by expert problem solvers. While problem representation and procedure are important, they do not assure that the subject understands the problem nor do they provide insight into how and why subjects choose certain representations and/or procedures.

Information processing (IP) models have come under heavy criticism from Schoenfeld (1985b) and Simon and Simon (1979). These authors, noting the literature on expert problem solvers' automatic responses and lack of overt strategies, questioned the ability of IP models to delineate strategies particularly when novices show more evidence of strategic processing when solving problems. Nickerson (1985) criticized the emphasis placed on features of the problem and finding appropriate ways to represent the psychological processes involved in problem solving in the form of a computer step. Schoenfeld (1985a) also questioned the assumption that important strategies for computer problem solving are important strategies for human problem solving. This aspect needs to be researched further and checked against reality. Even Kintsch (1988) presented some concerns with his 1984 IP model of problem solving. Kintsch noted that IP models were not always sensitive to situational information. As well, he stated that models such as Kintsch and Greeno's (1985) and Briars and Larkin's (1984) have no way to cope with subtle context demands. Within these models the student either used the right step or not.

A more comprehensive model of problem solving was presented by Lester (1985). It combines a cognitive component based on Polya's (1962) model and a metacognitive component that considers the three classes of variables (person, task, strategy) outlined by Flavell and Wellman (1977). The model purports to describe the categories of the cognitive component in terms of points during problem solving where metacognition actions might occur. Lester saw metacognitive decisions serving as a guide to cognitive actions. Cognitive decisions may influence or result in metacognitive actions. However, cognitive actions are not always guided by metacognitive decisions as implied by Lester's model. This one way reaction negates the interaction reported by others between cognition and metacognition and ignores research suggesting limited use of metacognitive strategies, especially in young children (Brown, 1978; Nickerson, 1985; Pressley et. al., 1987; Schmidt, 1973; Sternberg, 1986).

In summary, while each model looked at specific and/or general aspects of problem solving, no one model considered all the variables entailed when studying problem solving. Kilpatrick (1975) has outlined the range of variables involved in problem solving. These included three independent variables - subject, task, and situation, which are derived from the necessary components of a problem solving event including the problem solver (subject) solving a problem (task) under a set of conditions (situation). Four dependent variables also were identified by Kilpatrick and include: product variables (solution correctness, time), process variables (procedures, heuristics), evaluation variables (personal view of process and correctness), and concomitant variables (trait variables) which are derived from the subjects' responses to the problem task. Any problem solving event involves a complex interaction among these seven variables as described by Kilpatrick. The models cited above, except Lester's (1985), ignored the metacognitive aspects of problem solving. None have included situational variables and few take into account the continuous interaction of the person with the task.

The model proposed in this paper attempts to incorporate 1) the variables of person, task and situation delineated by Kilpatrick (1975) and Flavell and Wellman (1977); 2) a set of procedures for successful problem solving; and 3) the cognitive/metacognitive strategies utilized by students when solving problems. Points two and three (procedures and strategies) listed above include, but are not limited to, Kilpatrick's dependent variables. The Model recognizes the interaction among all three components as well as the importance of affective variables in performance at any point during the problem solving process. (See Figure 1)

Proposed Model of Problem Solving

The circle in the center contains Kilpatrick's (1975) independent variables: person, task, and situation. Person variables are those quantities which describe or measure specific attributes of the person. They include such attributes as age, sex, attitude, persistence, and mathematical topics studied or background knowledge. The category of task variables includes the semantic content or mathematical meaning of a word problem, the mathematical structure of a word problem, and the manners in which a problem may be presented. Situation variables describe the physical, psychological, or social environment in which the problem event takes place. This circle would incorporate what is referred to as declarative knowledge (Dillon, 1986; Messick, 1984) since it should include the domain content and facts - the 'what' - to which a person has been exposed.

The middle circle outlines six steps believed to be utilized by a person when solving mathematical word problems. These include:

1. Examine Problem - involves reading and/or examining the printed information. "Understands" was not used as the first step as suggested by Polya (1962) because many students read, attempt and complete problems without understanding them (Mayer, 1986; Nickerson, 1985).
2. Identifies Goal - involves recognizing the end state, what has to be resolved to arrive at a solution to the problem.
3. Chooses Plan - involves deciding what mathematical operation(s), diagrams, and/or formulas are needed to obtain a response.
4. Represents Plan - involves transforming the plan into usable mathematical language or symbolism.

Model of Mathematical Word Problem Solving

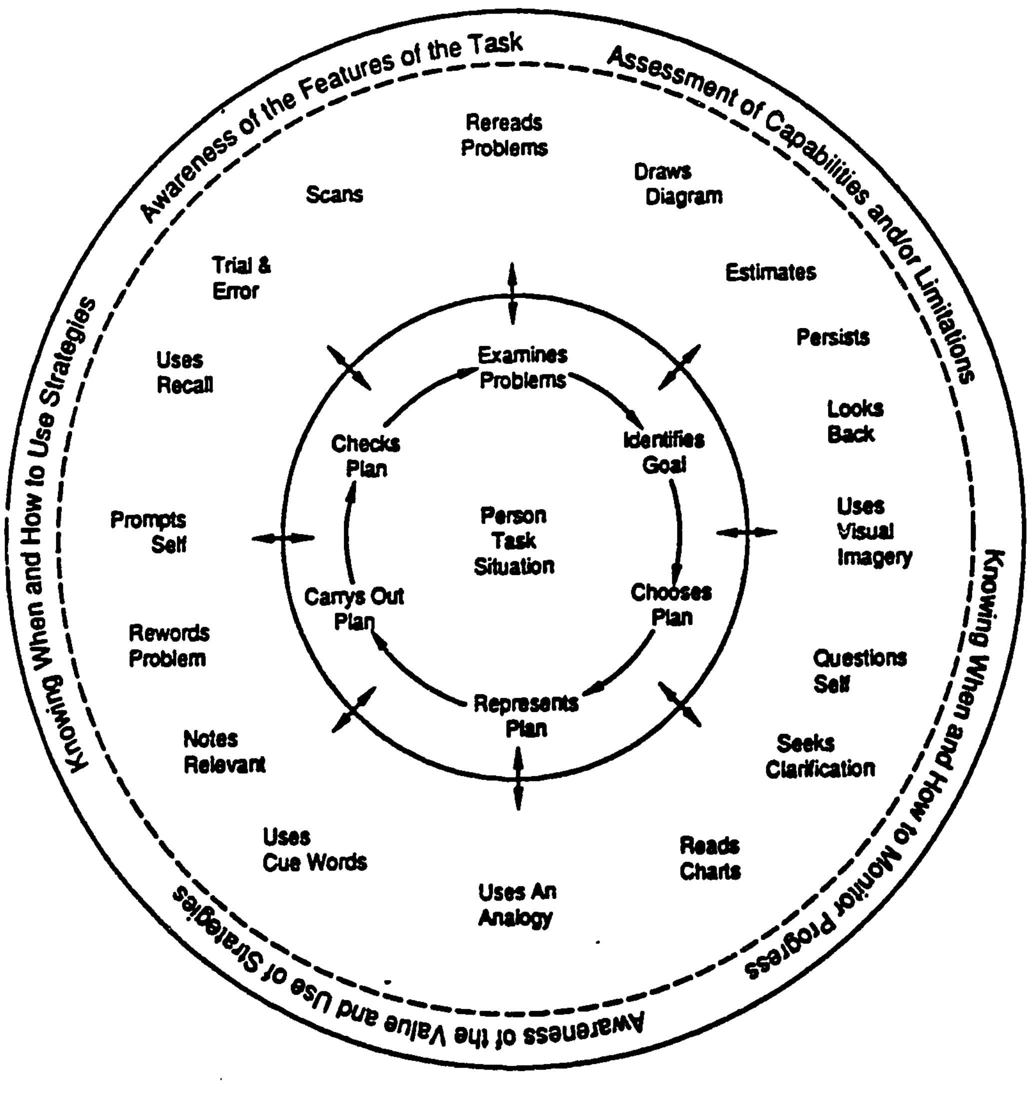


Figure 1

5. Carrys out Plan - involves actually implementing the chosen plan.

6. Checks Plan - involves looking back to check computation and/or to verify that the response is in line with the identified goal.

These six steps are based on the results of two pilot studies that noted that good problem solvers used similar steps when solving mathematical word problems (French, submitted; 1988). Information gained while the person was going through these steps to solve a problem includes what Messick (1984) referred to as procedural knowledge - information about 'how' a person solves problems. While the steps are somewhat sequential, they are not mutually exclusive. The circular presentation of the steps is a recognition that problem solvers are individuals with unique approaches to specific tasks at different times and under specific conditions. A person may use all six steps in the suggested sequence when solving a problem or as few as three or four steps in a somewhat random order. For example, a person may see the words "all together", assume addition is required and add the given numerals and then go on to the next problem or go back and read the problem.

The outer circle notes some of the strategies (both cognitive and metacognitive) students use when solving problems. While certain tasks evoke specific strategies, most students have a repertoire of strategies such as those given in Figure 1 which are often cited in the literature (i.e. Alberta Education, 1983; Lester, 1985; Schoenfeld, 1982). Examples of cognitive strategies include:

- Recognizing cue words in the problem statement
- Noting all the relevant information in the problem statement
- Looking back at the information given to verify data as copied correctly
- Seeking clarification of uncertainties
- Rereading the problem
- Drawing a diagram to visually represent the problem
- Restating the problem in his/her own words.

Examples of metacognitive strategies are:

- Verbalizing an awareness of the value of using specific strategies for certain problems
- Knowing when and how to monitor progress
- Recognition of ones capabilities and limitations
- Having an awareness of the how and when to use strategies.

The outer circle, in the model, includes both heuristic (cognitive) strategies and managerial (metacognitive) strategies, as suggested by Schoenfeld (1985), with a broken line separating them. This broken line signifies that both heuristic and managerial strategies are cognitive strategies but managerial strategies could be considered higher-order strategies in the cognitive continuum. This section of the model considers strategic knowledge as defined by Messick (1984) in that it recognizes the person's ability to know when and which strategies to apply as well as their understanding of their own knowledge, strengths and limitations.

The two-way arrows between the circle point to the dynamic interaction between all components of the problem solving process. Each aspect serves a purpose that should be considered when evaluating problem solving ability or devising programs to teach mathematical problem solving.

The model is fairly broad, serving as an assessment and instructional tool. By incorporating declarative, procedural and strategic knowledge with person, task and situation variables it recognizes the continued interaction between development and knowledge throughout life.

F. Factors Affecting Mathematical Word Problems

Conceptual models for solving mathematical word problems also contain common traits that good problem solvers possess According to Meiring (1979) a number of factors characterize good problem solvers. These include the ability to estimate and analyze, to understand mathematical concepts and terms, and to interpret quantitative and linguistic data.

Mayer (1982, 1983) considered certain types of knowledge to be relevant in mathematical word problem solving. These included: 1) linguistic, factual, and algorithmic knowledge, 2) schema knowledge, and 3) strategic knowledge. Mayer (1982) stated that these types of knowledge contributed to the understanding of the problem and helped the problem solver know how and when to apply operators until the goal was achieved.

The types of knowledge outlined by Mayer (1982) are essential components in the model proposed by the author. Each type of knowledge taps an area that needs to be addressed to ascertain its influence on problem solving at the elementary school level. Areas pertaining to these types of knowledge accentuated in the research literature on word problem solving include computational skills, reading ability, problem tasks, attitude toward mathematics, and process and strategy use. Each of these variables will be dealt with in the context of declarative, procedural and strategic knowledge relative to mathematical word problem solving. This in no way negates the complex interaction among the three types of knowledge.

Variables Influencing Declarative Knowledge in Mathematical Word Problem Solving

The importance of person, task and situation variables has been noted by Kilpatrick (1975). Lester (1985) and Uprichard et. al. (1986) also included person and task variables within their problem solving models. While it is recognized that many components interact within and among each of the three variables, a limited number, relevant to the current research study, will be reviewed to determine their effect on word problem solving. These include variables such as reading and computational ability, attitude toward math and task specifics.

<u>Reading</u> - Some researchers (Balow, 1964; Lees, 1976; Lyda and Duncan, 1967; Vanderlinde, 1964) contended that reading ability was significantly associated with word problem solving ability. However, results of more recent studies (Jerman, 1973; Knifong and Holtan, 1977) questioned this hypothesis as well as the validity of past studies equating word problem solving with reading ability.

Results of studies conducted by Vanderlinde (1964) and Lyda and Duncan (1967) suggested that teaching quantitative vocabulary produced significantly better results on word problem tasks. Balow (1964) also noted that reading and computational ability effect word problem solving ability. However, Vanderlinde (1964) and Lyda and Duncan (1967) did not control or account for the effects of time and treatment in their data analysis. Likewise, an analysis of covariance, controlling for reading, significantly diminished the apparent effects of reading on word problems in the Balow study.

More recently Carpenter et. al. (1980a) questioned the impact of reading ability on word problem solving performance. They noted that during the NAEP mathematical assessment, all word problems were presented in both a written and oral format. Consequently, inability to read should not have been a factor in students' word problem solving. But results indicated that students (ages 9-13) exhibited a decline in word problem solving since a prior assessment.

Conflicting results were obtained by Ballew and Cunningham (1982) who examined the effects of several variables on word problem solving. Two hundred and forty-four sixth grade students were administered three tests made up of randomly chosen word problems from the grade 3 to 8 basal textbooks of one mathematics series. Students completed a.) a computation test containing equations from the word problems; b.) a word problems test in which problems were read aloud and presented in written format and c.) a word problem test requiring silent reading by the student. Results suggested that individual students vary in their problem solving abilities across different components. For example, 26% of students had computational difficulties. More relevant was the fact that students performed better on problems when they were presented in oral and written format. Based on these results, Ballew and Cunningham suggested that inability to read word problems is a major obstacle for grade six students.

Yet, Ballew and Cunningham (1982) did not note if tests were parallel nor did they obtain a measure of students' reading ability. They also noted that many students had difficulty setting up a word problem after they read it. Therefore, Ballew and Cunningham postulated that differences in word problem solving ability may go beyond basic reading skills. Indeed, ability to solve word problems may be related to processing differences or a combination of variables.

Support for word problem difficulties going beyond basic reading ability was substantiated in studies by Jarman (1973) and Moyer, Moyer, Sowder and Threadwill-Sowder (1984). Both studies looked at the impact of varying reading demands on word problem tasks. Jarman presented grade four to six students with word problems in three different reading format lengths while keeping other variables constant. On the other hand, Moyer et. al. (1984) administered word problems to grade 3 to 7 students in the usual textbook format and a reduced verbal format. Results of both studies indicated that reduced reading demands in a word problem statement did not influence performance. Similarly, Paul, Nibbelink and Hoover (1986) found that controlling for sentence length or vocabulary level did not effect elementary student's ability to solve word problems.

Knifong and Holtan (1977) also looked at whether poor reading skills contribute to failure on mathematical word problem solving tasks. Using information from a previous study the authors interviewed thirty-five students whose errors were attributed to reading factors. Students 1) read the problem, 2) described the problem situation, 3) stated what had to be ascertained and 4) how to work the problem. Results indicated that students were proficient in all areas (92%-98% correct) except on stating how to work the problem (37% correct). These results add support to Ballew and Cunningham's (1982) hypothesis that word problem solving ability involves more than basic reading skills.

Research dealing with the impact of reading ability on mathematical problem solving varied in results and point of view. Knifong and Holtan (1977) found poor reading ability to account for approximately ten percent of erred problems. In contrast, Ballew and Cunningham (1982) considered an inability to read problems a major obstacle in solving word problems. Generally, results indicated that the impact of reading ability on word problem solving has been overrated. Equating word problem solving difficulty with reading difficulty appears to be too great a simplification. However, knowing how students interpret and process words in a problem is a major concern (Suydam, 1982).

Computation - A further factor proposed as impacting on student's mathematical word problem solving ability is computational skill. Knifong and Holtan (1976) analyzed students' responses on the word problem solving section of a section of the Metropolitan Achievement Test. Frequent clerical and computational errors were positively identified as the main source of difficulty when solving word problems. However, studies by Bidwell (1983), Caldwell and Goldin (1979), and results of the NAEP assessment (Carpenter et. al., 1980 a, b) demonstrated that students performed better on rote calculations than simple word problems.

Carpenter et. al. (1980b) in a follow-up paper on the NAEP mathematics assessment studied the relationship between student's computational and word problem solving abilities. They noted that solution correctness for multiplication and division problem was significantly lower than for addition and subtraction problems due to computational errors. In addition, for most single operation word problems, performance was closely related to computational skills.

These findings were inconsistent with those reported by Bidwell (1983). Bidwell (1983) attempted to measure and contrast the ability of 11 and 12 year olds to do simple calculation exercises and relational problems. Five hundred and twenty-three subjects completed a twenty-four item test. Results showed that the students tested were better able to perform rote calculations than solve simple word problems.

While researchers such as DeVault (1981) have noted that competence in basic computation skills was necessary to solve many mathematical word problems correctly, Lichtenberg (1984) and Zweng (1984) cautioned against focusing too much attention on computation. Indeed, Lesh (1981) reported that computational ability does not ensure a student will know when to use an operation or how to use the answer once obtained.

Lesh's (1981) comment suggested a need to look beyond solution correctness. While students may be proficient at computation they may not understand word problems. For example, a student drawing an appropriate diagram and choosing the correct mathematical operation but making a computation error is better at solving this specific word problem than a student whose choice of mathematical operation indicates a lack of understanding but executes the computation correctly. In many cases the actual problem task determines word problem solving capabilities.

Task Variables - Researchers, however, disagree on what they consider appropriate problem solving tasks. According to Goldin (1983) "a task is a problem when steps or processes are detected between the posing of the task and the answer". Such a definition covers a wide variety of problems including textbook word problems. In fact, problems used by both Webb (1979) and Quintere (1981) in their studies were classified as typical textbook word problems.

In other studies (Ekenstam and Greger, 1983; Sherrill, 1983; Zweng, 1979) researchers also used typical textbook word problems. These researchers found that elementary school students experienced difficulties solving all word problems except those requiring only one step with no extraneous information. These researchers noted that school math texts do not provide enough examples of multi-step problems that require the use of various heuristics.

Dolan (1983), Kilpatrick (1975) and Lesh (1981) reported similar complaints with textbook word problems. They stated that textbooks were limited in the problem types presented and that sets of problems were usually located immediately after the material that provided practice in the operation required to solve the problem. Dolan (1983) also agreed with Sherrill's (1983) comment that textbook problems do not require the use of many heuristics and usually involved reading and writing an equation and occasionally checking the results. Indeed, Dolan noted that many textbook problems eliminate the need for students to determine the operations necessary to solve the problem, and in some instances, even eliminate the need to read the problem.

Despite these difficulties other authors (Ball, 1986; Charles, 1981; Springer, 1977) suggested using typical textbook word problems to develop concepts, to motivate, to provide practice for algorithms and as stepping stones to extend the student's skills level in solving everyday problems. Steffe and Black (1983) noted that the tasks and methods they used may be routine to adults but may not be routine to young students. Indeed, Havel (1985) pointed out that even grade six students often depend on help with word problems adults consider routine.

Results of studies dealing with elementary school students' ability to solve typical textbook word problems demonstrated that students had difficulties with different types of word problems. For example, in the elementary grades, word problems that can not be solved by a routine application of a single arithmetic operation cause a great deal of difficulty (Quintere, 1983; Silver and Thompson, 1984. Several studies (Jerman and Mirman, 1974; Nesher, 1976; Quintere, 1982, 1983; Zweng, 1979) have looked at students' difficulty with problems with more than one step.

In her study, Quintere (1982) individually observed thirty-six 9-13 year old students solving one and two-step word problems. She found that while over eighty percent of students could correctly solve one-step problems, the percentage solving two-step problems was significantly lower. Both Jarman and Mirman (1974) and Nesher (1976) evaluated large groups of students solving one-step and multi-step word problems. They found that the number of steps needed to solve the problem had a significant effect on student performance.

Zweng (1979) also attempted to identify the ability of elementary school children to solve different types of word problems. Twenty-four students in grades three to six were interviewed solving 15 typical textbook word problems. Problem types were those involving only one computational step, two steps or containing extraneous information. Reading and computational demands were controlled at each grade level. Results indicated a gradual increase in ability to solve one-step problems - 79% correct at grade three to 100% correct at grade six by various subjects. Performance on problems with extraneous information was significantly lower than on one-step problems, but improved with age. However, performance on multi-step problems was considerably lower and did not increase across grade levels.

Student difficulty with word problems containing extraneous information was noted on the NAEP assessment by

Carpenter et. al. (1980) and Muth (1986). Muth's (1986) survey of six popular mathematics textbooks resulted in few examples of extraneous information in mathematical word problems. She contended that since learners are confronted with extraneous information problems in testing situations and real life, it was to their advantage to have exposure to such word problems in classroom situations.

Other studies such as those by Nesher (1976) and Cohen and Stover (1981) showed that students (grades five and six) scored significantly lower on problems containing extraneous information than those without extraneous information. However, Nesher's use of only four division problems in his study and Cohen and Stover's manipulation of other variables such as vocabulary and order of numerical data presentation limited the conclusions that can be drawn from these studies.

More recently Muth (1984) and Englert, Culatta and Horn (1987) have investigated the effects of irrelevant linguistic and numerical information embedded within word problems. Subjects in the Englert et. al. study were forty-eight grade two and four students (24 each grade). Twenty-four of these subjects were classified as learning disabled by their school district and 24 average (12 at each grade for each category). Students were individually administered sixteen addition problems with and without extraneous information. Results indicated that while learning disabled subjects were slower and less accurate than their average peers, both groups in each grade had decrements in word problem solving when irrelevant information was presented in the problem statement. Similar results were noted by Muth (1984) in his study of sixth graders (n=200) ability to solve fifteen word problems with and without extraneous information.

Other studies have looked at the effect of various task variables on word problem solving ability. For example, Caldwell and Goldin (1979) examined the importance of problem content. They presented students in grades four to six with different types of abstract and concrete word problems. Results indicated that students solved significantly more concrete and factual problems than abstract problems.

Burns and Yonally (1964) on the other hand found order of presentation of numerical data had an effect on solving one and two step problems. They found that fifth grade students had difficulties solving textbook word problems in which numerical data were presented in an order other than the required order to solve the problem. Given students' reported difficulties with typical textbook word problems, it would appear that textbook word problems constitute a problem solving task for elementary students.

<u>Situational Variables</u> - However, students are not only confronted with unique word problem solving tasks but often expected to perform them under unfamiliar conditions. For example, reports in the United States (Carpenter et al, 1980a) and Canada (Robitaille, 1981) indicated that students at all grade levels were routinely administered group achievement tests. These tasks require responses to multiple-choice items in several curriculum areas, including mathematical word problem solving. This method deviates from the students' routine presentation and response format. Indeed, the multiple-choice format might alter students' approach to the task and/or induce uncharacteristic behaviors such as guessing.

Several of these group administered tests also have time restrictions. For example, the Canadian Test of Basic Skills (King, 1984) is used in several Canadian provinces and requires students to complete each subtest in a set time frame. This time restriction may have affective and performance effects on students' word problem solving ability.

Vakali (1984-85) and Sowder, Threadgill-Sowder, Moyer and Moyer (1986) studied different response models on word ›blems as well as other variables. While Sowder et. al. 9t.6) required only the mathematical operation symbol for a response, Vakali (1985) considered the effects of asking students to solve word problems mentally. In Vakali's study, ninety-two grade three students were individually interviewed solving 24 addition and subtraction word problems (with and without carrying and borrowing). Subjects were asked to read the problem aloud, solve it "in your head" and give their answer orally. Latency and accuracy of responses were recorded. Results demonstrated that subjects were slower and less accurate on problems requiring carrying and borrowing. Given the response method, these results are not surprising. At the grade three level, students rarely are expected or encouraged to complete complex computation mentally.

Afferbach and Johnson (1984) and Ericson and Simon (1984) also noted that the use of "think-aloud" procedures may effect a subject's performance on a task. For example, having a subject verbalize his thoughts while completing a word problem and/or respond to queries about the process after the task is completed may effect his performance. (Think-aloud procedures are discussed in greater detail in the assessment section of the literature review.) Unfortunately, researchers are limited in methods to gain insight into subjects' strategic behaviour while solving word problems and must currently rely on "think-aloud" procedures. However, use of familiar tasks, requiring routine procedures and response methods, as in the current study, may minimize the effect of the "think-aloud" procedure on the subjects' word problem solving performance.

<u>Affective Variables</u> - Silver (1985) commented on the dearth of information on non-cognitive aspects of word problem solving. He stated that when students approach word problem solving tasks, especially challenging tasks, they do not enter such situations as purely cognitive beings. For example, Silver noted that a student's attitude toward mathematics and word problem solving in particular can have a powerful influence on the nature and quality of his/her performance on a word problem solving task.

Marvin (1982) also looked at students' attitudes toward mathematics. Marvin had teachers administer the Attitudes Toward Arithmetic Scale (Kramer, 1970) to 850 grade four to six students. Her findings indicated that while students considered mathematics fun and important, they also cited fear of word problems as their main reason for disliking mathematics. These results suggested that a student's progress in problem solving may be hindered by his/her attitude towards word problems.

More recently, Kloosterman (1988) attempted to demonstrate how attitudes towards mathematics (including word problem solving) could be explained in terms of motivational variables. Four hundred and eighty-six grade seven students were administered a series of instruments that tapped self-confidence in mathematics, attribution style, mathematical ability and thoughts about success and failure in mathematics. Correlations between the variables indicated that self-confidence and success in mathematics were positively related to effort in mathematics. While Kloosterman's study confirmed a positive correlation between success, effort and self-confidence based on student reports, it did not look at the relationship between affective factors and actual word problem solving tasks.

Given increased demands for more emphasis on word problem solving in the math curriculum (Thompson and Rathmell, 1988), McLeod (1988) expressed concern regarding the lack of research on the affective characteristics of problem solvers. McLeod noted that many students have intense reactions to word problem solving. He went on to state that information on these emotional reactions should be helpful in planning instructional techniques that consider affective issues.

In addition, Silver (1985) noted that the current emphasis on computer models of problem solving has resulted in a movement away from the affective issues of word problem solving performance. He suggested a renewed effort to uncover the affective/cognitive link in problem solving. While such an effort is not the focus of this study, the author recognizes the influence of affective variables on word problem solving. Administering the Attitudes Toward Arithmetic Scale and seeking responses to questions on attitudes toward problem solving after the testing is completed, in a small way addresses affective issues in the present study.

Summary

The impact of declarative knowledge on word problem solving skills has often been over-simplified. The articles mentioned in this brief review point to the complex interaction of numerous variables. Each person brings to a problem solving situation unique personal characteristics that interact and react to various word problem tasks with their own set of demands.

Variables such as reading ability and computational ability appear to have limited effect on performance on grade level word problem solving (Bidwell, 1983; Carpenter et. al., 1980; Moyer et. al., 1984). However, other variables such as the type of word problem given, required response method and attitude towards word problems seem to have an impact on the person's overall performance on familiar word problem tasks (Kramer, 1982; Muth, 1984; Quintere, 1982; Valaki, 1984-85).

While teachers and researchers can discern some of the impact of these variables on students' word problem solving performance by recording and observing behavioral manifestations and written and oral work in relevant academic areas, other influences are not as easily identified. How and why students solve word problems in particular ways are not always evident.

G. Variables Influencing Procedural and Strategic Knowledge in Mathematical Word Problem Solving

The influences of procedural and strategic knowledge on word problem solving will be discussed in this section. As Kirby (1984) noted, these two concepts are not easily separated as each has aspects of the other. However, in this study procedural knowledge will refer to the procedures/methods involved in the actual encoding, transforming and sorting of information. Strategic knowledge is seen as being responsible for the controlling and planning of these procedures or processes (Kirby, 1984; Messick, 1984).

According to Uprichard et. al. (1986) students utilized a six-dimensional strategy to successfully solve word problems. These six "strategies" were also the procedures students went through in the problem solving process. As previously stated, the six dimensions are: reading, analyzing, estimating, translating, computing and verifying. Uprichard et. al. studied teachers' ability to diagnose these six-dimensions of the problem solving process.

Uprichard et. al. (1986) trained ten teachers to conduct structured and unstructured interviews of fourth and fifth grade children individually and in group settings. Four students from each of the ten classes received individual interviews as well as being involved in the class interview. Of the forty students, complete data during both interview situations were obtained on 32 subjects. During each interview students solved two problems, one single-operation word problem and one multi-operation word problem. Students were then rated as having competency or no competency on each of the six dimensions by three raters. Ratings were based on student responses to questions and written protocols during each interview situation. Rater agreements for the two single-operation problems ranged from 66% (translating) to 91% (computing) and from 66% (verifying) to 97% (translating) for the two multi-operational word problems. The authors concluded that structured interviews can be used successfully for group classroom diagnosis of word problem procedures/strategies possessed by individual students.

The conclusion reached by Uprichard et. al. (1986) seemed unwarranted for a number of reasons. First, the authors saw no reason to calculate inter-rater reliability, a questionable decision when student responses were judged in only two categories - competency/no competency. Second, the number of problems (2) in each interview situation was too limited to provide an indication of the subjects' word problem solving ability. A third concern is a low percentage of agreement by raters on whether the subjects did or did not exhibit competency in the interview situations. Given the structured nature of both interview situations and the availability of written protocols such low percentages of agreement are unacceptable. Finally, the actual interviews were structured to elicit responses on the six dimensions, not to determine the students current procedure for solving word problems. For example, questions asked in the "non-structured interview" included: "What do we want to find out?" and "What do you think the answer will be close to?" However, results of this study did indicate that students in grade four and five rarely estimate or check their responses. Lee (1982) also noted that students rarely check their responses.

The Uprichard et. al. study was discussed in detail because it examined several procedures used when solving word problems. Previous research (Knifong and Holtan, 1977; Sowder et. al., 1986) was limited to looking at one or two procedures.

The Knifong and Holtan (1977) study, previously discussed, noted that while students could read problems they had difficulty choosing a plan to solve the problem. Sowder et. al. (1986) investigated the ability of 167 sixth graders to choose the mathematical operations required to solve twelve word problems. No actual computation was required. Results indicated that students had difficulty choosing correct operations, especially multiplication (25% accuracy). Quintere (1982) also concluded that some of the students' difficulties with two-step problems stemmed from their inability to choose a plan and understand the requirements of the task. It would appear that students do not realize that particular contexts call for particular operations, especially with multi-step word problems (Sadowski and McLlveen, 1984).

Other procedures, such as representing a plan and carrying out a plan do not appear to be problematic. Students seemed to accomplish these steps, whether right or wrong, with few difficulties. Carpenter, et. al. (1980a) in their discussion of the National Council of Teachers of Mathematics (NCTM) Standards noted the need for increased emphasis of representational modes. The Standards pointed to the importance of focusing students' attention on various representation modes (i.e. diagrams, mental images, oral language, written symbols) and asking students to translate from one representation to another.

Some authors (Mayer, 1983; Silver, 1988) noted that students not only differed in their approach to solving a problem but also in the problem solving strategies they employed. Hiebert (1984) and Greenwood and Anderson (1983) discussed two kinds of knowledge students must acquire about mathematics in order to solve problems. The first form included knowledge about symbols such as numerals, signs, rules and procedures while the other form dealt with understanding which accounted for intuitions and ideas about how mathematics works.

In a similar manner, Schoenfeld (1982) stated what while an adequate knowledge of basic facts and principles was necessary in problem solving, students also needed to master basic problem solving techniques and strategies. Strategies help one to select appropriate approaches to problems and to terminate fruitless approaches. Schoenfeld went on to note that managerial strategies were not possessed by all students, especially those at the elementary school level.

Indeed, Romberg and Collis (1985) found that children differ in their cognitive capacity to deal with mathematical word problems and in the strategies they use to solve these problems. Subjects for the Romberg and Collis study were eleven children beginning grade three. Seven subjects were classified as having high ability and four low ability based on several previously administered developmental tests. Three times throughout the year (February, April, May) subjects were individually interviewed solving twelve addition and subtraction word problems with and without carrying and borrowing. The high ability group, in all three interviews, had a greater percentage of word problems correct (79% to 95%) than the low ability group (12% to 70%). Subjects in the high ability group also had a greater repertoire of appropriate strategies that they used more frequently than the lower ability group.

For example, Romberg and Collis noted that the high ability group represented the model concretely, wrote a sentence with the numerical response, counted and used routine procedures. The low ability group occasionally used these strategies but used a significantly higher number of inappropriate strategies such as incorrect sentences and choosing the wrong operation. These results suggest that children who differ in their cognitive-processing capacity also differ in their access to correct answers and strategy usage. Romberg and Collis (1985) made no claims of generalizability noting that the study only described the problem solving behaviour of a small group of subjects.

However, Romberg and Collis suggested that there was too much emphasis on paper and pencil algorithmetic procedures at the early stages of development and not enough emphasis on applications and understanding. Avital (1983), Burkhardt (1983), Gagne (1983) and Wachsmush (1983) have encouraged the practice of algorithmetics at a younger school age. Such practice, they stated, might help make these skills automatic and allow more time to develop flexibility in the use of processes and strategies. Results of several studies suggest a need to develop both algorithmetic and strategic abilities together (Lee, 1982; Romberg and Collis, 1985; Webb, 1979).

Lee (1982) examined the ability of 16 fourth graders to acquire and use specific strategies. Eight of these subjects were classified as being at the early concrete operational stage and eight at the late concrete operational stage on the basis of their performance on two Piagetian tasks. Half of the subjects in each group were then assigned to an instruction or no-instruction group. Subjects in the instruction group received 20 word problem solving sessions in a nine week period. Preinstruction and postinstruction measures of subjects solving ten challenging textbook word problems, similar to those presented in the instruction sessions, were obtained in individual interviews. Results of the preinstruction interviews indicated that both groups of subjects exhibited few strategies and could not solve any of the problems. Following instruction, the instruction groups made significant gains in solution correctness. The two instruction groups had 58% and 88% of the word problems correct compared with 4% and 8% correct in the no-instruction group.

More significant was the increased use of appropriate strategies in the instruction groups. These students frequently used strategies such as drawing a picture, making a chart, finding a pattern and recognizing similar problems. Students in the no-instruction group rarely used these strategies and their limited use was often inappropriate.

For example, they included irrelevant information in their charts. Students in the no-instruction group were also more inclined to impulsively perform an operation on the numbers in the problem statement. These findings demonstrated that students can learn to acquire and use specific strategies.

As noted previously, Silver (1985) encouraged further investigation of the affective/cognitive link in word problem solving performance. Garofalo and Lester (1985) agreed that affective reactions may influence different types of cognitive processes and strategies. They noted that decisions about whether to persevere or not might be influenced by student confidence or anxiety. Kloosterman (1988) also felt that self confidence in mathematics was probably related to students' thoughts about their success of failure in this academic area. For example, students who attribute their success to help from others may not feel capable of making decisions on which path to pursue or to trying alternative procedures.

Thompson and Rathmell (1988) in their discussion of the proposed NTCM Standards for school mathematics commented on the relative emphasis that should be afforded to conceptual development, mathematical reasoning and word problem solving in future mathematics curricula. This increased emphasis is intended to propose a balanced curriculum that focuses on both mathematical ideas and processes. Similar positions have been advocated by Alberta Education (1983; 1985) and

Callahan and Garofalo (1988). Callahan and Garofalo further suggested designing instruction to help develop students' metacognitive knowledge.

Summary

Elementary school aged students appear capable of acquiring and using procedural and strategic knowledge. While research indicated that students used the procedures of reading, choosing and representing a plan, and carrying out the plan regularly, their plans, representations and responses are not always accurate. Few students used procedures such as identifying a goal, estimating and checking responses (Carpenter et. al., 1980; Lee, 1982; Uprichard et. al., 1986). Students' views of word problem solving appear to be limited to carrying out one of the four basic mathematical operations on some or all of the numerals in the problem statement. Flanders (1987) and Silver (1988) noted that this rote model of word problem solving is often reinforced in textbooks and mathematics classes.

However, elementary school students also demonstrated that they are capable of acquiring and using processes and strategies. Appropriate use of strategies has been noted to have a positive effect on problem solving ability (Lee, 1982; French, submitted; Romberg and Collis, 1986). Research also suggested that strategy use may be influenced by affective variables. Indeed, it seems reasonable to presume that students with a weak knowledge of mathematical concepts and/or lack of confidence or liking for the subject area would experience greater difficulties in word problem solving and vice versa.

The increased emphasis on word problem solving in mathematics curricula that has been advocated in the United States and parts of Canada can have an impact on how word problem solving skills will be taught and assessed at the elementary school level. The intent is to move students away from static routine procedures. Instead, students are encouraged to use flexible representation models and to understand and apply strategies and processes to varied word problem solving tasks and situations (Alberta Education, 1983; Thompson and Rathmell, 1988).

With the changing focus of mathematics in the schools and research findings on word problem solving ability comes the need to reconsider current means of assessing word problem solving skills. Researchers have noted the impact of declarative, procedural and strategic knowledge on word problem solving ability. Therefore, instruments and/or techniques developed to assess word problem solving skills must attempt to gain some insight into the student's declarative, procedural and strategic knowledge in this area.

H. Mathematical Problem Solving and Assessment

Holowinskiy (1980) pointed to the need to depart from the traditional quantitative assessment approaches of measuring cognitive skills. He noted that such approaches are imprecise, emphasize product not process, make unwarranted assumptions regarding underlying cognitive skills, and are more like achievement tests than cognitive tests. Vygotsky (1978) also claimed that intelligence tests have limited utility, since they tap only mental functions that have been developed and provide no information on the child's ability to learn. Similarly, Anastasi (1984) noted that current cognitive and achievement tests frequently assess developed abilities (skills acquired through years of training and practice) and not why individuals perform as they do.

More recently, Neill and Medina (1989) noted that standardized test results were inaccurate, inconsistent and biased against minorities, females and students from low-income families. They went on to state that standardized intelligence and achievement tests contributed to the reification and ranking of the construct intelligence. Standardized tests are constructed with the assumption that the skill being measured is one-dimensional and static and that all individuals perceive information and solve problems the same way (Campione and Brown, 1987; Neill and Medina, 1989; Menick, 1987).

Glaser (1984) stated that while it is important to know what knowledge a child has in mathematics, it is also necessary to know his/her reasoning skills when solving word problems . Such information helps assess students' initial concepts so that misconceptions can be corrected. In addition, information about ways students currently process mathematical information and the strategies they employ could provide insight into the reason for their success or difficulty with problem solving.

Allwood (1976) suggested three factors to take into account when analyzing problem solving skills: 1) the type of problem, 2) the individual's knowledge base, and 3) the strategies characteristically used by the individual when interacting with the problem type. In other words, assessment techniques should not only look at solutions and procedures but also strategies used when solving different word problem tasks.

In addition, Walker (1987) and Campione and Brown (1987) noted the need to assess student's cognitive ability level in familiar domains to provide more specific and more accurate indications of the person's capabilities. Using standardized achievement and intelligence tests that assess an individual's cognitive abilities in unfamiliar domains can underestimate the individual's potential as an information processor.

Nickerson (1985) found that results on current assessment devices often imply that students understand certain concepts based on their performance on a limited number of items that tap domain specific knowledge. For example, a student pointing to a picture representing a word is supposed to represent a deep understanding of the meaning of the word; or, using a mathematical operation to solve a problem is supposed to constitute evidence that the student understands the problem in more than a superficial way. According to Nickerson (1985), when one really understands a concept he/she can usually demonstrate it in a variety of ways - i.e. communicate it effectively, apply the concept consistently and correctly; use it in a variety of contexts; and/or draw analogies. However, as Lester (1983) noted, traditional tests provide insufficient data about such cognitive behaviour.

According to Wiggins (1989) authentic assessment

> 'is most accurate and equitable when it entails human judgement and dialogue, so that the person tested can ask for clarification of questions and explain his or her answers.'
>
> (p. 704)

Wiggins (1989) contended that such a testing procedure allows for observation of the learner while he/she tackles and solves problems, marshals evidence, arranges arguments and takes actions to address the problem.

To compensate for the limited information and subjective assumptions of traditional tests, Meichenbaum, Burland, Gruson, and Cameron (1985) highlighted the value of using multiple assessment approaches to study the relationship between cognition, metacognition, and performance. These approaches included interviews, think-aloud assessments, actual performance on tasks, and observations. Neill and Medina (1989) also concluded that high quality methods of assessment would ensure the use of a variety of forms of measurement, resulting in more valid and useful measures of competence, achievement and ability. Therefore, the present study will employ a variety of measures of mathematical problem solving including think-aloud methods, teacher ratings, and an instrument format that allows sufficient space to include all the student's workings.

Ginsburg (1981) suggested clinical interviews (a think aloud procedure) as a method to assess strategy use as standardized tests preclude exploration and explanations of responses. Further, naturalistic observations are not practical and tend to be subjective (Ginsburg, 1981). The clinical interview entails a subject verbalizing while completing a task or responding to open-ended questions upon the completion of a task or both. During the interview, the examiner and the subject can query, observe, or clarify actions or verbalizations. According to Ginsburg (1981) these interviews serve three purposes: 1) the discovery of cognitive activity, 2) the identification of cognitive activity and 3) the evaluation of levels of competency.

The clinical interview method facilitates rich verbalizations which may indicate underlying processes, clarify ambiguous statements, and test alternative hypotheses. Finally, the clinical interview attempts to ascertain the student's highest level of competence in a particular area. The individual format and flexibility of an interview provides latitude to clarify, persist, motivate and challenge (Ginsburg, 1984; Schoenfield, 1985a). However, to be successful the clinical interview should employ tasks that channel the subject's activity into one particular area and demand reflection on the part of the subject. Questions should also be contingent on the subject's responses. During interviews, researchers usually use introspective (verbalizations of thought processes while completing the task) and/or retrospective (verbalizations of thought processes after completion of the task) think-aloud procedures to assess the subject's knowledge and understanding of specific concepts.

Despite the increased use of verbal data for research it is not without controversy (Nickerson, 1985; Schoenfeld, 1985a). Swanson, Schwartz, Ginsburg, and Kossan (1981) and Cavanaugh and Perlmutter (1982) questioned the reliability and validity of subject's verbal reflections. These authors noted that the information gained through verbal reports was constrained by the subject's knowledge and access to his/her own inner thoughts and automatized unconscious responses resulting in incomplete verbal reports. Another cause of concern with verbal reports concerns verbal reporting by subjects with limited linguistic skills such as the very young or the mentally handicapped.

On the other hand, think aloud procedures using introspective methods of data collection provide the researcher with information about facts and processes the subject employed as well as descriptions of some of their cognitive strategies (Afferback and Johnson, 1984; Ericson and Simon, 1984; Swanson et al, 1981). However, Ericson and Simon (1984) also noted that the actual method of having subjects verbalise while performing a task may interfere with their routine way of thinking and working and thus effect time on task, strategy selection and actual output.

While retrospective methods may compensate for some of the apparent difficulties with introspective methods, they appear to have more drawbacks. Greenwood and Anderson 1983) noted that asking for verbal reports after the task was completed could result in incomplete and inaccurate data due to memory demands, interference, and subjects making up accounts of what they thought happened. In addition, subjects already have some knowledge and past experiences which they tend to express rather than their actual thought processes (Swanson et al, 1981).

However, while many researchers recognize the problems associated with think aloud procedures, they consider interviews ecologically valid (Afferback and Johnson, 1984; Ericson and Simon, 1984; Schoenfeld, 1985b; Swanson et al, 1981). Massey and Gelman (1988) found that interviews employing think-aloud procedures provided insight into the nature of thought processes which would be otherwise unknown.

Ericson and Simon (1980, 1984), Ginsburg (1981) and Swanson et al (1981) suggested using other sources of data and nonspecific, noncued probing to increase the reliability of think-aloud procedures. Afferback and Johnson (1984) also offered suggestions to improve the reliability and validity of verbal reports. They noted that: 1) subjects could receive training in the think-aloud technique prior to the actual interview; 2) subjects should be older than eight; 3) interviewers receive training regarding the interview process and their role within this process; 4) analysis of the data gained during the interview should be systematic but not rigid; 5) persons rating the data receive training; and 6) other sources of data be gathered for verification of the verbal data.

Glaser (1984) suggested that using multiple approaches should help in assessing all aspects of problem solving in order to learn the student's current state of knowledge as well as how to help the student move to new levels of reasoning. However, no one source of data is ever complete, especially, as Schoenfeld (1985) noted, when attempting to assess such complex cognitive phenomena as mathematical problem solving strategies. Indeed, verbal reports are currently our only avenue to accessing reasoning processes underlying higher level cognitive activity (Afferback and Johnson, 1984).

Summary

The apparent dynamic interaction between cognition, knowledge and development (Brown, 1981; Gelman, 1986; Pressley et al, 1987) necessitates the development of assessment devices that go beyond scoring correct answers. Instead, assessment devices tapping mathematical problem solving skills must incorporate ways and means of discovering processes and strategies used when solving problems (Ekenstam and Greger, 1983; Lester, 1983; Threadwill-Sowder, 1985). In addition, Walker (1987) suggested using familiar content to bridge the gap between cognitive theory and educational applicability.

The model of mathematical problem solving proposed by the author (Figure I) served as a guide to develop a prototype of an assessment devise to tap the knowledge, processes and cognitive strategies of elementary students in the relevant area. Because the model takes into account declarative, procedural and strategic knowledge it is relatively inclusive. However, the ability of the instrument developed to measure existing word problem solving skills using a think-aloud procedure has not been addressed. Therefore, the present study has been designed to explore the validity of the instrument designed to tap mathematical word problem solving and the theory underlying the test. Chapter III provides a detailed overview of the development of the test.

III. Method

A. Introduction

Crocker and Algina (1986) noted that the validity of test scores on an instrument and the validity of the theory about the construct being studied are inseparably linked. This study addressed the psychometric and psychological foundations of the instrument developed to assess mathematical word problem solving at an elementary school level. The instrument used in this study was based on the model of mathematical word problem solving presented in Chapter II, Figure I. The purpose of this study was to explore the relationship between the model and test.

This chapter contains a description of the process used to develop the instrument and procedure used to assess mathematical word problem solving. In addition, the sample, published instruments used in this study, and the data collection procedure are described.

B. Author Designed Instrument

Mathematical Word Problem Solving Test (MWPST) :

The development of an instrument to tap mathematical word problem solving ability as hypothesized in the author's model began with a review of current elementary mathematics textbooks. Testing children in familiar domains in order to obtain more accurate estimates of individuals' potential and current knowledge is advocated by Campione and Brown (1987) and Walker (1987). One way of addressing familiar content is to examine textbook content. This review provided insights into the mathematical concepts, procedures and strategies, and the word problem types introduced in math textbooks. Based on the author's model of mathematical word problem solving which addressed the interrelationship among declarative, procedural, and strategic knowledge, the review delineated content in the textbooks representing these three knowledge areas.

Following a review of three current mathematics textbooks used in Canadian schools (Addison-Wesley; Copp Clark Pittman; Holt Rhinehart & Winston), a list of the types of strategies, mathematical operations, and word problem types introduced in grades four, five and six was outlined. (See Appendix A) While definite procedures were not listed in the textbooks apart from reminders to read carefully or to check responses, some Canadian provinces such as Alberta distribute documents on problem solving advocating procedures similar to Polya's model (Alberta Government, 1984).

The information contained in Appendix A, was used to create a pool of forty three word problems using a similar format to that in the textbooks and tapping similar strategies, problem types and operations. Computation and reading demands were minimized as the purpose of the test was to tap problem solving ability not computation or reading comprehension skills. Reading level was verified through the use of the Fry Readability Graph (1977) to be at the 3.6 to 3.9 grade level. Operations and concepts on the test had been introduced at least one year prior to grade four. Indeed, Flanders (1987) in his review of several mathematics textbooks, found that less than 50% of the material introduced in each elementary grade was new. Problems chosen for the test reflected important aspects of the curriculum that had been consistently emphasized for a number of years as well as specific strategies outlined in current textbooks and the literature on mathematical problem solving.

The pool of items was reviewed both formally and informally. Three teachers, one at each of the grade four, five and six levels, evaluated the items to determine the appropriateness of their content, strategic demands and structure. Items were reviewed as well for grammar, spelling, and readability as suggested by Crocker and Algina (1986).

Eighteen items were eliminated because they were viewed as being too easy, too difficult, unclear, or duplicating others. Twenty-five of forty-three items were retained. Four of the twenty-five received minor revisions.

The twenty-five items were pilot tested on sixty-four grade four and five students in the Lunenberg School District of Nova Scotia (French,1988). Following an analysis of test results, five items were eliminated as they were too difficult and/or their discrimination power inadequate. (See Appendix B for a listing of the difficulty and discrimination indexes.) A sixth item that was difficult for students was retained because it was felt that it might result in the use of interesting strategic behaviour. Three items received minor changes in wording due to student comments. These revisions resulted in a test of twenty items

The final version of the test, with its twenty word problems, was reviewed by two experts in the field of mathematics education. One individual was involved in mathematics education at the university level and the other was a consultant in mathematics education at the school board level. These two people, along with the author, agreed on a few minor revisions .

A list of the final word problems, noting their major strategy types, problem types and mathematical operations, is contained in Appendix C.

Format of the Mathematical Word Problem Solving Test

Word problems were presented in a typed, orderly format. Each problem was arranged in a manner to allow ample space for subjects to include their written work with no more than four problems per page. Protocols contained all written actions taken by the problem solver to reach a solution (see Appendix D). A cover sheet containing instructions, two sample problems and space for the student to record his/her name, age, grade and date of birth was also included.

The preceding method of presentation was chosen because it is an efficient, informative way to administer individual (or group) tests. As well, French (1988) noted that students in the pilot study commented on the test format noting that it was less confusing than a multiple choice format. Students also stated that they felt comfortable providing their written workings as it was similar to classroom experiences. Students' written work was used to note correct and incorrect responses, error types and the use of specific strategies or procedures. For example, raters could note if a student checked his/her response, underlined relevant information, corrected computation errors, drew a diagram, or tried other procedures through access to the students' written work.

In addition to providing their written workings, students were encouraged to "think aloud" while solving problems. These verbalizations were audio-taped. Students' verbalizations provided indications and/or verifications of the procedures and strategies they utilized in the problem solving situation which could not be gleaned from written protocols. Use of only the written work of the students to surmise problem solving strategies and procedures is limited and subjective. For example, an examiner, with only the written protocols of a group of students, has no way of knowing if students actually read the problem, reread it, prompted themselves, estimated or skipped the problem and returned to it later. On the other hand, an examiner may attribute specific strategies to students based on written work which are not part of their repertoire. For example, an examiner may assume a student can distinguish between relevant and nonessential information when in fact the student randomly choose certain numbers. Having students "think aloud" while solving problems eliminated some of these concerns.

Audio-taping the students verbalizations was efficient and provided more complete data for analysis. Students could proceed at their own pace as the examiner monitored progress and probed or encouraged when necessary. Attempting to note strategies and procedures, as the student completes word problems, without audio-taping, can be a demanding, impersonal technique that is distracting to both the student and the examiner. Students may be uncomfortable knowing someone is noting their behaviour which may lead to a focus on the examiner, an attempt to keep pace with the examiner, anxiety, or a disruption in normal problem solving behaviour. The process of noting students' procedures and strategies as they complete word problems also places excessive memory demands and time restraints on the examiner. In addition, while noting one strategy an examiner may miss seeing the student employ another. While the presence of a tape recorder may initially be disconcerting, it should prevent some of the more serious difficulties encountered by examiners attempting to monitor and record students' strategies and procedures as they complete word problems.

The combined use of oral reports and written calculations provided insights into the student's declarative, procedural, and strategic knowledge in mathematical word problems. Specifically, this format gave an indication of students' current knowledge in mathematical concepts, their ability to carry out mathematical operations, the types of errors they made, the procedures they used to obtain a solution, and the strategies they employed during the entire process.

Scoring System for Mathematical Word Problem Solving Test:

The scoring system developed by the researcher considered declarative, procedural, and strategic knowledge. Several authors have advocated the importance of using a scoring system that goes beyond noting only the correctness of the response (Kantowski, 1977; Ibe, 1984; Schoenfeld, 1982). These authors suggested employing a scoring system that considers both the problem solving process and the correctness of the results. Scoring only the final response, they noted, could result in students' problem solving ability being overestimated or unrecognized.

A scoring system taking into account the product, processes, and strategies involved in solving each problem was developed. Both the students' written work and verbalizations were needed to score the MWPST in order to obtain an indication of students' declarative, procedural, and strategic knowledge in mathematical problem solving. The scoring system has three component parts which are completed for individual subjects. These include:

1.) Noting of declarative knowledge by dichotomously scoring each item and recording the type of error(s) made per problem.

2.) Noting of procedural knowledge by listing the steps used in the problem solving process for each word problem. While some steps were evident from the written protocol others were not and could only be verified by the subject's verbal report.

3.) Noting strategic knowledge by recognizing which strategies were used throughout the problem solving process for each word problem. Again, while some strategies may be inferred from the written protocol, subjects' verbal reports appear to provide greater insight into their strategic knowledge.

Each component of the scoring system encompassed an aspect of the model of mathematical word problem solving presented in Figure I. While each component was scored separately all three components must be considered when evaluating a student's current word problem solving ability. The first component of the scoring system dealt with the student's declarative knowledge in the area of mathematical word problem solving. It provided an indication of the student's facility with familiar mathematical tasks and operations. Chi (1985), Messick (1984) and Pressley et al (1987) have noted the interdependence of domain specific knowledge and procedural and strategic knowledge. Scoring students' responses and noting error types afforded some indication of the students' declarative knowledge. Comparisons could then be made among the students declarative, procedural, and strategic knowledge in the area of mathematical word problem solving.

Reviewing the student's written protocol while listening to his/her verbalizations indicated which items were answered correctly and the types of errors students made. While the correctness of most responses was evident from the written protocols, occasionally students worked problems out mentally and verbalized their answers. As all transcribed interviews were reviewed, students' responses could be checked and confirmed. In incidents where answers were incorrect, examiners noted the type of error(s) made. Errors were then coded depending on the type(s) made and recognizing that students could make more than one error per problem. Error

Table III.I

Error Types and Code Numbers

Code		Error Type
1	Computation	- any error in calculations
2	Operation	- incorrect mathematical operation used
3	Clerical	- information copied or represented incorrectly
4	Concept	- response indicates difficulty with mathematical concept (eg perimeter, subtraction with zero)
5	Extraneous	- irrelevant information included in the problem solving process
6	Steps	- failure to complete all steps
7	No Understanding	- response indicates a lack of understanding of the requirements of the problem
8	No Attempt	- skips problem / no solution process
9	Other	- e.g.: does not draw logical conclusion or reads, then write down a number in problem statement

categories (See Table III.1) were based on student responses during the pilot testing of the MWPST and research on errors in mathematical word problem solving (Carpenter et. al., 1980; Knifong and Holtan, 1976; Quintere, 1982).

The model of mathematical word problem solving presented in Figure I lists six steps or procedures that could be used during the problem solving process. As noted previously, these steps were chosen based on pilot studies (French, 1985; 1988) and research on procedures used by students when solving word problems (Lester, 1985; Uprichard et. al., 1986). Table III.2 provides the codes for different combinations of steps that students could and did employ during the problem solving process.

Strategies were coded from 1 to 49 (Table III.3), representing strategies employed by elementary students in grades four to six when solving the given word problems. These strategies are ones often cited in the literature (i.e. Alberta Education, 1983; Lee, 1982; Lester, 1985; Romberg and Collis, 1985; Schoenfeld, 1982) and were used by students in a previous study of mathematical word problem solving strategies by French (1985). While the strategies listed in Table III.3 are all cognitive strategies, those listed below the dotted line (coded 45 to 49) are considered higher order cognitive strategies or metacognitive strategies (Flavell, 1977). This list is intended to give an overview of strategies students may employ while solving word problems. It is not

Table III.2

Steps Used to Solve Problems and Codes for Various Step Sequences

Steps when Solving Word Problems

1. Examines Problem
2. Identifies Goal
3. Chooses Plan
4. Represents Plan
5. Carries Out Plan
6. Checks Plan

Code	Steps
1	1 3 4 5
2	1 2 3 4 5
3	1 3 4 5 6
4	1 2 3 4 5 6
5	1 3 5
6	1 (no attempt)
7	1 2 3 5
8	other: reads, gives number in problem statement

intended as an exhaustive list and students may use strategies that have not been included in the list.

Information gained about students' declarative, procedural, and strategic knowledge by analyzing their written work and verbalizations was coded and recorded on a summary sheet. Appendix E contains a copy of the summary sheet. Summarizing the information provided a visual profile of students' performance on the word problem test in the different knowledge areas proposed in the model.

Teacher Rating Scale:

Teachers of the students completing the MWPST were asked to rate students in their class on mathematical word problem solving and computation skills. A five point rating scale was used that included the following categories:

5 - Very Good (grade usually > or = to 85)

4 - Good (grade usually between 75 and 84)

3 - Average (grade usually between 65 and 74)

2 - Slightly Below Average (grade usually between 50 and 64)

1 - Below Average (grade usually < 50).

Teachers were asked to assign one of the above numerical values (1 -5) to each student in each of the categories of computation and word problem solving. These values were to

Table III.3
Cognitive and Metacognitive Strategies with Codes

Codes	Strategies
1	- reads problem
2	- underlines/marks relevant information
3	- rereads the problem
4	- copies numeric data/words correctly
5	- organizes written format
6	- chooses correct operation(s) or method
7	- sequences steps of problem correctly
8	- search behaviour -looks back/looks ahead
9	- notes nonessential information
10	- notes all relevant information
11	- pauses to consider options
12	- notes/verbalizes goal
13	- supplies missing or implied information
14	- estimates answer
15	- picks up cues from context
16	- draws a diagram or chart
17	- attempts other procedures
18	- uses an analogy
19	- uses visual imagery
20	- uses trial and error
21	- restates problem
22	- draws upon past experience
23	- questions self
24	- verbalizes answer
25	- persists
26	- checks answer
27	- judges whether an answer is reasonable
28	- notes errors and self corrects
29	- verbally prompts self
30	- draws logical conclusion
31	- seeks clarification
32	- repeats key points
33	- reads chart
34	- works computation out mentally
35	- works backward
36	- skips, then returns to problem later
37	- rules out options verbally
38	- checks back to problem when finished
39	- verbalizes plan before starting calculations
40	- guesses
41	- states an acceptable alternate method
45	- comments on own ability
46	- monitors progress
47	- aware of strategy use and value
48	- knows when to use strategies
49	- awareness of the features of the task

represent the teacher's estimate of the student's general level of performance in these two mathematics areas.

C. Description of Published Instruments

The first aspect of the model presented in Figure I considered the students declarative knowledge. Declarative knowledge incorporates person, task, and situation variables. Students performance on different tasks completed using specific procedures was evaluated as was the impact of specific characteristics the person brought to the problem solving situation. Past research has noted the impact of variables such as reading (Ballew and Cunningham, 1982), computation (Zweng, 1984), and attitude (Silver, 1985) on mathematical word problem solving.

Results of four subtests on the Canadian Test of Basic Skills (CTBS) (King, 1984), the scores on the three major areas of the Cognitive Abilities Test (CAT) (Thorndike and Hagan, 1983) and The Attitude Toward Arithmetic Scale (ATAS) (Kramer, 1970) were used in this study. These results provided an indication of the subjects' current status in reading, computation, and word problem solving as well as an indication of their attitude toward mathematics. Since these variables may influence declarative knowledge in word problem solving, their impact as measured on the CTBS, ATAS and CAT, was examined in relation to word problem solving performance. As well, results on the problem solving subtest of the CTBS were used as measure of concurrent validity for the test developed for this study. This is in keeping with Smith and Glass's (1987) suggestion that a connection be established between the test being developed and other indicators of the same construct that use different measurement methods. While the CTBS uses a multiple choice format and the MWPST uses a combination verbal and written format, both are intended to measure mathematical problem solving.

The Canadian Test of Basic Skills (CTBS) (King, 1984) had been administered to all grade five and six students one to two months prior to collecting data for this study. The grade four classes were administered four subtests of the CTBS before being assessed on the MWPST. Results of the CTBS were used to give an estimate of the subjects' current levels of vocabulary, reading comprehension, computational skills, and problem solving abilities from a more traditional perspective.

The CTBS was normed on 30,137 grade one to twelve students in all Canadian provinces and territories. Its internal consistency reliability coefficients range from .87 to .96 for the five main areas assessed, with a composite reliability coefficient of .97 for all grades. Content validity is based on over forty years of continuous research, consideration of courses of study and recommendations of national curriculum groups. Considering the content validity, reliability, and local norming this instrument should provide a reasonable suitable indication of the students' current levels of knowledge in the areas of mathematics and reading.

The Attitude Toward Arithmetic Scale (Kramer, 1970) is often used to assess students' attitudes toward mathematical problem solving and computation. Marvin (1982) normed this instrument on 850 grade four , five, and six students. Test/retest reliability was .86. The scale has fifteen items. Each item has a scale value, ranging from 1 to 10.5, assigned by Marvin. Higher values are assigned to positive statements. Therefore the higher a person's score the more positive his attitude toward arithmetic. Appendix F lists the items and provides a table indicating their assigned values.

Teachers at one of the schools in the research study administered the Cognitive Abilities Test (CAT) (Thorndike and Hagen, 1983) to all students every year. These results were available and used in this study. The CAT has four subtests of verbal ability ; three subtests of quantitative ability; and three subtest measuring nonverbal ability. Results in these three major areas are suppose to provide an indication of academic achievement and abstract reasoning.

Reliability estimates (K-R 20) are quite high, ranging from .89 to .96. Test -retest reliabilities for 4000 grade 5, 7, and 9 students ranged from .76 to .96. Correlations of the CAT with the Standford-Binet (3rd edition) for 550 persons ranged from .65 to .75. While correlations among the verbal, quantitative and nonverbal batteries indicate a substantial amount of overlap (.72 to .78), the authors claim that each battery provides special information to assist teachers.

D. The Sample

A total of one hundred and seventy-nine students enrolled in either grade four, five or six in one of three Dartmouth District Schools composed the sample. There were sixty-five grade six students, sixty-five grade five students and forty-nine grade four students. Nine teachers worked with the 179 students. All nine teachers completed the teacher rating form.

E. Collection of Data

After obtaining initial approval from the Dartmouth District School Board, participating schools were contacted by telephone, to arrange a meeting to explain the research project. During these meetings, the purpose of the research was explained, the type of data to be collected and the intended data collection method outlined, questions and concerns answered, time lines discussed, and co-operation enlisted. Teachers were also asked to distribute parent consent forms to the children in their class to take home to their parents. The consent forms contained a brief description of the research project, the child's role, a place to give written consent and assurances of confidentiality.

This method of distribution was chosen so schools would not have to disclose information some parents might consider personal.

Having obtained parent permission the researcher and two graduate students, familiar with and trained in conducting research using think aloud procedures served as the examiners. Involving more than one examiner in the research process is considered good practice (Mathison, 1988). However, in this case it was also necessary in order to collect the data within the given time line provided by the schools. While both graduate students had assisted in prior research projects involving think aloud procedures, their projects were in the area of reading. Therefore the researcher met with the two graduate students hired as examiners, outlined the proposed research and data collection procedures and reviewed introspective and retrospective think aloud procedures. Examiners also had an opportunity to practise the techniques with each other and several volunteer school age children. These two examiners were responsible for collecting data on 68 (37.9%) subjects.

As part of the training session, the researcher provided each examiner with a summary sheet outlining the procedure to employ during individual testing situations (Appendix G). Examiners were then given the names of the teachers and students with whom they would work, sufficient copies of the Attitude Toward Arithmetic Scale and the appropriate Teacher

Rating Scales. One of the hired examiners and the researcher were also responsible for administering four subtests of the CTBS to the grade four students. Both the examiner and the researcher had used the CTBS on previous occasions.

One examiner met with each class of students to explain the purpose of the project and answer their questions. Students then completed the Attitude Toward Arithmetic Scale. Items were read aloud and explanations given to student queries. The two classes of grade four students were administered the CTBS. Following completion of the attitude scale and the CTBS, examiners met with students individually.

During the individual sessions students first received training in the "think aloud" procedure using a grade four reading level cloze passage. The training acquainted the student with the concept of verbalizing his/her thoughts on a task while actually completing the task. In addition to obtaining an unbiased first hand look at the student's reading and verbalization abilities, training using a reading passage did not contaminate the collection of data for the think aloud procedure involved in mathematical word problems.

The MWPST designed for this study was presented to the students who filled in their age, grade and date of birth and read the instructions. Students were instructed to read each word problem aloud and to verbalize their thoughts, aloud, while they completed the problem. A reminder to keep talking as they worked on the problems and to feel free to say anything that came to mind was given periodically where required.

The cover sheet also contained two sample problems. Students read the sample problems aloud, verbalizing their thoughts as they completed them. These sample problems were completed to ensure that students understood the requirements of the testing situation. The tape recorder was turned on when students were ready to begin word problem number one.

All students read each problem aloud. While students completed the word problems in many unique ways, past experience (French, submitted; 1988) demonstrated that three main response styles were used by students. First, the majority of students read the problem and immediately began to write down the numbers and carry out the mathematical operation, verbalizing the numbers and mathematical procedure. A second group of students read the problem, talked about what they were going to do, and on occasion explained why they were doing it. These students completed the written work quietly. A final group of students read the problem, paused and/or reread the problem, talked about what had to be found to solve the problem, wrote and described their response process, and verbally gave their response in the form of a sentence.

While completing the word problem test, students were encouraged to verbalized all their thoughts while engaged in solving the problem. Introspective and retrospective think-aloud procedures were used. Unknown vocabulary was identified and unfamiliar concepts explained if students sought clarification. For example, some students were informed that a hectare was a unit of measurement.

Examiners used their own judgement according to established procedures regarding when to probe or query, being careful not to provide hints or mislead. As a general rule examiners queried or probed when students completed written work but were hesitant to verbalize or when students read the problem and some time elapsed (approximately one minute) without written or verbal actions. Probes were also used to ensure and/or clarify a students reasoning behind a written or verbal response. Open ended probes such as 1) "Tell me what you are thinking about now?" 2) "I noticed your eyes moving, are you looking anywhere special?" and 3) "You seem to be taking longer here, what is the reason?" were used. However, examiners recognized that students differed in their ability and willingness to "talk aloud". Therefore, students were encouraged but not pressured to verbalize. Periodic review of the tapes during the data collection period followed by discussion with the examiners was undertaken by the researcher.

Upon completion of the test items, the examiners asked the student a few questions in an informal manner. Questions dealt with things such as attitude toward math, ways to become a better problem solver, and strategies to use when solving a difficult problem. Students were thanked for their co-operation and returned to class. The examiner labeled the tape and noted relevant information on the back of the student's test protocol. For example, examiners recorded any of the student's actions (i.e. looking back at the problem statement) or reactions (ie rapport) that might not be evident on written protocols or heard on the tapes. Audio-tapes of students solving the word problems were then transcribed.

F. Data Analysis

Data analysis focused on whether the MWPST, developed for this study, provided a measure of the student's declarative, procedural, and strategic knowledge as suggested by the proposed model of problem solving. To accomplish this verification of the test and the model, data were analyzed both qualitatively and quantitatively.

The summary sheets of each student's written and verbal input on the MWPST were completed and the information on these sheets used in the data analysis. To complete these summary sheets the researcher noted solution correctness and the codes for error type(s) per problem, the steps used in the problem solving process and the different strategies employed. Responses to informal questions at the end of the MWPST were also noted and summarized. Appendix H contains six examples of these transcripts, two at each grade level.

In addition to the review of strategies, errors, and steps in the problem solving process by the researcher, two graduate students who were familiar with strategy assessment procedures were employed to review a randomly selected portion of the tapes. Raters were given summary sheets, a copy of the coding guides, clean xeroxed copies of subjects' transcripts and protocols, and an explanation of the scoring procedure. Following the independent review by raters, the researcher met with raters to mediate any discrepancies that existed regarding steps and strategies used or error types. In a few cases where a consensus could not be reached a majority decision stood.

The purpose of rater review was to ensure the consistency of rater judgements and to assess the practicality and clarity of the coded guidelines. Of the 179 taped interviews, a random selection of 24 tapes was reviewed by one rater. The second examiner reviewed a random selection of 54 different tapes. Inter-rater reliability was calculated by dividing the number of agreements by the number of agreements plus disagreements.

It is important to note that this study employed a number of data collection methods and sources as suggested by Miechenbaum et al (1986) and Ericsson and Simon (1984). As well, suggestions by Afferback and Johnson (1984) to improve the reliability and validity of verbal reports were incorporated into the study. For example, subjects were older than eight and received training in think aloud techniques and examiners were familiar with think aloud methods.

Firestone (1987) suggested using triangulation of data to gain greater confidence in research findings. Mathison (1988) described triangulation of data as the use of "multiple methods, data sources, and researchers to enhance the validity of research findings" (p. 13). In this study, use of specific strategies and steps were confirmed by students' introspective/retrospective verbal reports rather than assumed from their written work. In addition, other researchers assisted in data collection and corroboration of test data was obtained from other sources such as teacher ratings. Perhaps, as Mathison (1988) postulated, the different methods tapped different domains of knowledge. In this study students' written work on their test protocols and teacher ratings gave an indication of their declarative knowledge while the think aloud procedure provided insight into students' procedural and strategic knowledge in the area of mathematical word problem solving.

G. Research Questions

To summarize, the study was designed to address the following questions:

1. Is the MWPST a reliable and valid instrument instrument to measure mathematical word problem solving skills?

2. Does the MWPST provide insight into the subjects' declarative knowledge as outlined in the proposed model of word problem solving?

3. Does the MWPST provide insight into the subjects' procedural knowledge as outlined in the proposed model of word problem solving?

4. Does the MWPST provide insight into the subjects' strategic knowledge as outlined in the proposed model of word problem solving?

5. Does the MWPST provide evidence of a relationship among the three types of knowledge (declarative, procedural, and strategic) included in the proposed model of word problem solving?

IV. RESULTS AND DISCUSSION

A. Introduction

This chapter focuses on the results of the assessment of grade 4,5, and 6 students obtained through the use of the Mathematical Word Problem Solving Test (MWPST) as well as the emerging traits the test purports to measure. As Loevinger (1957) noted, test data are a manifestation of the traits being measured just as the traits are representations of our current understanding of the construct being measured. In this case the MWPST was administered using specific procedures to determine whether it measured students' declarative, procedural, and strategic knowledge in the area of mathematical problem solving as outlined in the model presented in Figure I. Cronbach and Meehl (1955) noted that many types of evidence are relevant to construct validation, including content validity, criterion validity and item characteristics. The results of the qualitative and quantitative data analysis focused on the ability of the MWPST to support and verify the hypothesized model of word problem solving.

B. Sample

One hundred and seventy-nine students in grades four, five and six were individually interviewed while being asked to solve word problems. Students in grade 4 ranged in age from

9-06 to 11-11 with a mean age of 10-05. Grade 5 students had ages ranging from 10-05 to 13-02 with a mean of 11-04 while the ages of grade six students ranged from 11-06 to 14-01 with a mean of 12-03. The high age means may be due to data being collected at the end of the school year and the early cut-off date for school entrance in Nova Scotia. (Grade one students in N.S. must be six by the end of September as compared with six by the end of December in a number of other provinces.) There was almost an even distribution of subjects by sex overall and by each grade. Of the students 51.6% were females and 48.6% were males. Table IV.1 contains a summary of age and sex characteristics of students by grade.

Table IV.1
Percentage and Number of Students by Sex and Age at Each Grade Level

	Grade 4	Grade 5	Grade 6	Total
Ages				
Mean Ages	10.05	11.04	12.03	11.04
Age Ranges	9.06 - 11.11	10.05 - 13.02	11.06 - 14.01	9.06- 14.01
Sex				
Females	26	33	33	92(51.4%)
Males	23	32	32	87(48.6%)
Total	49	65	65	179(100%)

C. Research Question 1: Is the MWPST a reliable and valid instrument to measure mathematical word problem solving skills?

Classical true-score test procedures (Gulliksen, 1950) were applied to look at the psychometric properties of the instrument itself to see if the test was valid and reliable. Loevinger (1957) considered item responses as signs and samples of behaviour. As such they can indicate and represent the presence of traits. In order to make inferences from test behaviour to behaviour outside the testing situation, the items themselves have to be sensitive reliable signs of the behaviour being measured. This necessitated the calculation of certain item characteristics such as item difficulties and item discrimination indexes. In addition, the internal consistency of the test was obtained and a factor analysis of the items completed. Different types of validity - content, concurrent, and construct - were also addressed.

Item number 19 required students to use a visual imagery strategy to obtain the correct solution. Since no student had the correct answer or reported using visual imagery strategies on this item it was dropped from all data analysis. Students' written protocols and verbalizations indicated that they saw item number 19 as a straight forward perimeter problem.

Table IV.2

Item Difficulties, Item Discriminations, Item/Total Test Score Correlations on the MWPST

Item Number	Diff.	Dis.	Item/ Total Score (a)
1	.72	.44	.30
2	.60	.59	.39
3	.73	.38	.25
4	.79	.44	.35
5	.60	.59	.39
6	.40	.60	.45
7	.60	.46	.32
8	.83	.26	.19
9	.10	.23	.35
10	.48	.86	.61
11	.89	.32	.39
12	.48	.79	.52
13	.03	.07	.16
14	.83	.43	.43
15	.90	.24	.22
16	.57	.67	.44
17	.48	.75	.49
18	.50	.71	.53
20	.77	.47	.38

a Diff = Item Difficulty; Dis = Item Discrimination; Item/Total Score = Item Total-Score Correlation

Table IV.2 lists the various item statistics for the items on the MWPST. Item difficulty indicated the proportion of students who had the item correct. For this test, item difficulties ranged from .03 to .90 with most of the items falling within the .38 to .73 range. Allen and Yen (1979) noted that item difficulties of .3 to .7 maximized the information provided by the test about differences among subjects.

Some items (8,11,14,15) had difficulty levels over .8 indicating that they were completed correctly by most students. Three of these items were simple one-step problems (two of the three also contained extraneous information) and one (8) required the drawing of a diagram for a correct response. While the difficulty indexes suggest that all four of these items should be dropped, consideration must be given to other factors such as discrimination indexes, strategies generated by these items, and concepts assessed before such a decision can be made.

Conversely, two items proved to be very difficult for the majority of students. Only ten percent of the students responded correctly to number 9 and three percent to number 13. Both of these items demanded that the student understand the problem statement and be able to go beyond the information given to obtain the correct answer. Many students understood certain aspects of the problems but did not draw on all the given information to reach a logical conclusion. For example, problem 9 required students to find out how many book shelves were needed for 44 books if 8 fit on each shelf. Most students applied the correct mathematical operation and reported their answer without recognizing that the remaining books needed an extra shelf. Many students actually said "5 shelves and 4 books left over."

Item discrimination indexes are also presented in Table IV.2. These were obtained by subtracting the proportion of students with the correct answer in the top 30% of the sample from the proportion of subjects with the correct answer in the lowest 30% of the sample. For most items, discrimination indexes ranged from .30 to .75 within acceptable levels for discriminating among students (Crocker and Algina, 1986).

The low discrimination index for item number 13 is likely due to its difficulty. This item was answered correctly by few students in either the upper or lower 30% of the entire sample. Other items with relatively low discrimination indexes included problems number 8,9,and 15. Problem number 9 was the other word problem which the majority of students had difficulty obtaining the correct answer. Problems number 8 and 15, on the other hand, were fairly simplistic, resulting in most students obtaining the correct response and lowering the discrimination index. Item-reliability estimates are a measure of the item/total-test score point-biserial correlations. These correlations indicate the degree to which responses on one item are related to the total test score. The item/test correlations range from .16 to .61 with the majority in the

.30 to .52 range. Items 8 and 13 had the lowest correlations with the total test scores. As noted previously, these items had weak discrimination indexes and were either too easy (# 8) or too difficult (# 13) for most students.

Finally, the internal consistency coefficient was obtained to provide an index of both item content homogeneity and item quality (Crocker and Algina, 1986). The Kuder Richardson 20 (KR 20), for dichotomously scored items, was calculated. The KR 20 for the MWPST was .80, indicating that the different items on the MWPST measured the same underlying trait or highly correlated traits (Sax,1989).

Factor analysis was used to help determine whether the set of item intercorrelations was homogeneous. Using the nineteen item scores on the Mathematical Word Problem Solving Test, a principal component analysis with orthogonal rotations (varimax) was performed on the one hundred and seventy-nine student sample. Using a criterion of eigenvalue greater than 1, a six factor solution was obtained. (Eigenvalues were: 4.41, 1.28, 1.25, 1.17, 1.07, 1.01.) The loadings of the item scores on the six orthogonal factors are presented in Table IV.3.

Loadings greater than .30 are considered salient different from zero and therefore contribute to the solution. Main loadings of items are defined as the largest loading of that item while secondary loadings are defined as loadings less than the largest or main loading, but still salient. The six factors that emerged were:

Factor I - Practical, One-step Problems - Items 4, 11, 12, and 16 - These four problems dealt with practical everyday concerns such as spending money or measuring a person's height. Subtraction was required or implied (#12) to answer each of these problems correctly. While two of these problems were one-step problems (#4, #11) and two were two-step problems (#12, #16) many students treated the two-step problems as one-step problems. However, problems loading on Factor I were relatively straight forward apart from number 12 which loaded almost equally on another factor (Factor V). Secondary loadings from problems 5,10,18, and 20 also contributed to this first factor. These items also involved a give and take relationship. Three problems (#'s 5,10,18) required division to obtain the correct solution, an inverse operation of subtraction, but appeared to be less straight forward than those loading mainly on Factor I. Again several subjects treated the two-step problems as one-step problems.

Factor II - Essential Information/Two-step - Items 3,5,10, and 17. These problems required careful reading of the problem statement to select the essential information and/or to eliminate nonessential information to obtain a correct solution. All of these problems had three or more numerals in the problem statement. These were nonroutine problems that should have caused students to pause and think about how to solve the problem. Indeed, students had difficulty choosing the correct operation and information needed to solve these problem. Items 12 and 14 had secondary loadings on this factor. These items also contained three or more numerals in their problem statements and required the choosing of relevant information and the appropriate mathematical operation to obtain a solution. Problems with secondary loadings on this factor seemed to be more obvious than those loading on the primary factor. However, problem number 18 loaded equally on Factors II and III. This is also a nonroutine problem requiring care in choosing the relevant information and deciding which mathematical operation to employ. Most of the problems that had main and secondary loadings on this factor were two-step problems or were often solved using more than one step.

Factor III - Understanding/Wording - Items 6 and 7 - While these problems appeared to be simplistic, they actually necessitated careful reading of the problem statement to discern what was being asked. Indeed, more students used only three procedural steps to solve these two problems

Table IV.3

Rotated Factor Matrix of Items on MWPST

Item Number	Factors I	II	III	IV	V	VI	C a
4	.63			.32			.53
11	.64						.53
16	.69						.58
12	.39	.30			.37		.53
3		.69					.50
5	.35	.67					.60
10	.31	.55	.40				.58
17		.41	.37				.42
18	.30	.36	.37				.44
6			.61				.50
7			.76				.64
1				.78			.65
2				.63			.50
14		.30		.34	.47		.49
15					.66		.53
20	.37				.40		.37
8			.40		-.50		.48
9						.61	.51
13						.82	.70

a Amount of Common Variance - Commonality than for other problems which on average took four procedural steps. (The procedural step omitted by students was "represents plan". Students worked out the computation mentally rather than writing the equation out.) Four items had secondary loadings on Factor III- items 10,17,8, and 18. To answer these items correctly students had to read the problem statement carefully as these problems appear to be more complicated than the two loading on this factor.

Factor IV - One-step/ Wording - Items 1 and 2 - Both of these problems required the completion of only one mathematical operation to arrive at the correct solution. Both could also be solved using multiplication. However, these problems had to be read carefully to recognize what was being asked. Items 4 and 14 loaded on this factor. Numbers 4 and 14 are one-step word problems and multiplication can be used to solve number 14. Items 4 and 14 were solved correctly by more students and presented fewer difficulties for students than problems 1 and 2. For example, most students appeared to understand number 4 but made computational errors. Conversely, errors on number 1 were mainly due to choosing the wrong mathematical operation and on number 2 because students misunderstood the problem.

Factor V - Obvious Problems - Items 8,14,15, and 20 - These four problems were solved correctly by 77% or more of all the students completing the test. Three of the problems (14,15, and 20) were traditional, simple problems with the information in the problem statement presented in order of usage and with miminal computational demands. Two of the problems were one-step problems (#14, #15) and #20 was a two-step problem. Errors made by students on #20 were due primarily to completing only one step of the problem, thereby treating it as a one-step problem. Number 8 also loaded mainly, but negatively, on this factor. Most students (23 out of 29) who had #8 wrong had a raw score of eleven or less on the MWPST. However, the nontraditional problem statement without numerals coupled with the nontraditional response format and the need to comprehend the requirements of the task created difficulties for these students on #8 but not on the straight forward problems (14,15,20) loading on this factor. As noted previously, item 12 loaded almost equally on this factor and was often treated by students as a simple one-step problem.

Factor VI - Logic Problems - Items 9 and 13 - These problems required students to go beyond the information given to arrive at a logical solution. Students appeared to find these problems difficult as they were answered correctly by only 10% (#9) or fewer (#13) students. Errors were the result of lack of understanding of the requirements of the problem or a failure to draw a logical conclusion.

The factor analysis provided a means of determining how the items on the word problem test may represent fewer variables than the items themselves. As well factor analysis acted as a form of construct validity as it provided an indication of which item types clustered together. These item clusters could than be compared with those proposed originally when the test was being developed (Appendix C). Most of the items fell into the expected categories. For example, problems requiring more than one step and careful reading loaded on the same factors.

Crocker and Algina (1986) noted that variations in responses to item clusters can be attributed to variations among subjects on a common underlying factor. In this case, the Mathematical Word Problem Solving Test used problems that were designed to tap subjects' declarative, procedural, and strategic knowledge as outlined in the proposed model of word problem solving. The factor analysis revealed that several underlying factors determined a persons success on word problems.

Another approach to construct validation is to determine if the test differentiates between groups. In this study it was proposed that subjects with high declarative knowledge would also possess high procedural and strategic knowledge and thus be better word problem solvers and vice versa. Crocker and Algina (1986) and Cronbach and Meehl (1955) suggested that finding such expected differences adds support to the theory underlying the construct and the instrument for measuring the construct. Examination of total item correct/incorrect responses offered little insight into group differences in mathematical word problem solving as outlined in the proposed model (Figure I). In the sections that follow, differences in students' declarative, procedural, and strategic knowledge in mathematical problem solving are discussed in response to the remaining research questions. Following this examination, a statement can be made regarding the ability of the MWPST to measure the theoretical construct that it was designed to measure.

The review of current elementary mathematical textbooks to ascertain the type of word problems and the suggested strategies introduced, helped establish items which sampled the content being measured. Further review of the items by elementary school teachers, experts in the field of elementary mathematics, pilot testing and a final revision of items all helped contribute to the content validity of the word problem solving test. As noted by Loevinger (1954) the content of items should account for the trait(s) believed to be measured and the context of measurement.

Concurrent validity refers to the relationship between a test score and a measure of a related construct. In Nova Scotia all children are administered the CTBS at certain grade levels and several schools also administer the CAT on a regular basis to obtain estimates of students' current mathematical abilities. The correlation between the raw scores on the CTBS problem solving subtest and MWPST was .51 and between the raw scores on the CAT quantitative and MWPST .48 across the three grades. These findings indicated that while there was some overlap, the MWPST was tapping some different aspects of word problem solving. Given the different response format and administrative procedures these findings were not unexpected. The correlation between teachers' ratings of word problem solving ability in school and students' performance on the MWPST was .65 with a range of .63 for grade sixes to .71 for grade fours. There appears to be a moderately strong relationship between word problem solving on the test and in the classroom as noted by teachers, a further indication of the concurrent validity of the MWPST.

It would appear that the MWPST developed for this study, while in need of minor revisions, has content validity and internal consistency reliability. While construct validity has been addressed somewhat through the factor analysis, it needs to be examined in greater detail. The proposed model suggests an interrelationship between declarative, procedural, and strategic knowledge in the area of mathematical word problem solving. Each of these areas (declarative, procedural, strategic) has to be examined separately. The interaction between the three types of knowledge must also be examined to determine whether the test developed in conjunction with the model supports the hypothesized interrelationship.

D. Research Question 2: Does the MWPST provide insight into the subjects' declarative knowledge as outlined in the proposed model of word problem solving?

The proposed model of mathematical word problem solving (Figure I) addressed the interrelationship among declarative, procedural, and strategic knowledge. The ability of the MWPST to tap students' declarative knowledge will be discussed first. As noted previously, students' declarative knowledge encompassed person, task and situational variables. Person variables included specific attributes of the person (Kilpatrick, 1975) which they brought to the problem solving situation. Dillon (1986) and Messick (1984) noted that declarative knowledge is influenced by what the student brings to the problem solving situation. Therefore, some of the variables often cited as having an impact on students' problem solving ability, such as reading, computational abilities and attitude toward mathematics, were considered in response to this question. Problem solving tasks were also discussed.

Quantitative analysis techniques were used mainly to assess subjects' current level of declarative knowledge in mathematics. As item responses are considered signs and samples of behaviour (Loevinger, 1957) that represent the presence of other traits, the number of word problems answered correctly and the types of errors made were considered an indication of students' declarative knowledge. Measures of reading ability, computation, attitude toward mathematics, and teachers' ratings of computation ability were also considered in the analysis.

To answer this question, the correctness of item responses was calculated and error types (see Table III.1) per word problem and grade were noted. Several one-way analyses of variance (ANOVA) were carried out to test the differences among mean problems correct per grade. The Scheffe procedure was used to test the significance of these comparisons. Correlations of some of the variables cited in the literature review as having an impact on word problem solving with total number of problems correct on the MWPST are presented. These variables included the relationship of word problem solving to reading, computation, attitude toward math, and for some students (n=75) verbal, quantitative and nonverbal reasoning. Students' results on the word problem test were also compared with their teachers' rating of computational ability. Students' performances on different types of word problems were also discussed.

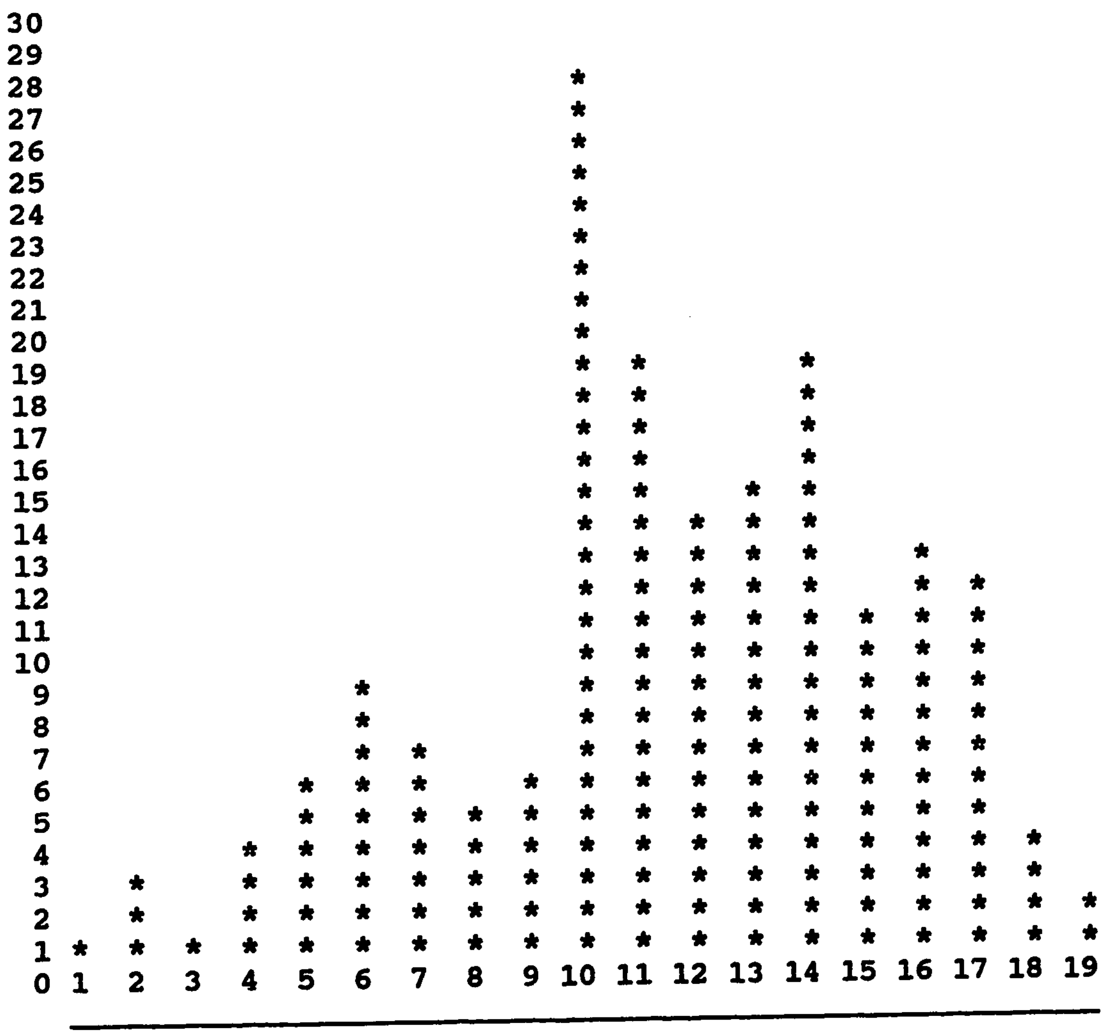
In addition, students were assigned to problem solving groups based on their raw score (number correct) on the MWPST. Students with a raw score equal to or greater than 16 (84% - 100%) were considered above average word problem solvers. Students whose raw scores ranged from 10 to 15 (53% - 79%) were considered average problem solvers and those whose raw score was equal to of less than 9 (0% - 47%) were considered below average problem solvers. Problem correctness and error types were noted for students in each raw score group. Again the ANOVA and Scheffe procedures were used to determine significant differences among the three groups.

By grouping students in such a manner the researcher was able to compare the strategies, procedural steps used, and solution correctness of above average, average and below average problem solvers. Such comparisons among the three raw score groups added insight into the ability of the MWPST to differentiate among students - a form of construct validity. As noted previously, differences were expected. Failure to find expected differences would raise doubts about the researcher's proposed model of problem solving ability, the adequacy of the MWPST, or both. In addition, such groupings provided an opportunity to look at the relationship between the content knowledge and strategies employed by students with different levels of problem solving ability.

Correctness of Item Response and Error Types

The histogram in Figure II provides an indication of the distribution of raw scores for all students. The distribution is negatively skewed indicating that students tended to answer most word problems correctly. The majority of students (106) had scores that fell within the range of 10 to 15 correct. Forty-two students had fewer than 50% of the word problem right, and 31 students had raw scores of 16 or greater. Overall, females scored slightly higher on average than males.

Frequencies of Raw Scores on MWPST



Raw Score on Word Problem Solving Test

Figure II

Students in Grade 4 averaged 10.02 correct responses on the word problem solving test, grade 5's averaged 11.31, and grade 6's averaged 12.80 (See Table IV.4). The overall average number correct was 11.50 with a range of 1 to 19 word problems correct. A one-way analysis of variance (ANOVA) ($F = 7.75$, $df = 2/176$, $p < .05$) indicated significant differences in problems correct by grade. Results of the Scheffe test demonstrated that there was a significant difference ($p < .05$) between the average scores of grade 4 and grade 6 students. Appendix I contains the ANOVA tables.

Data analysis revealed a small increase in average raw scores on the MWPST among the three grades. As the test was developed to ensure that reading and computational demands were not beyond those introduced at any of the grade levels being assessed, the gradual increase in scores in each grade could have been due to a number of factors. One possible explanation is the amount of exposure to and practice on word problems by the subjects in grade six particulary since textbook material is often considered repetitive and overlapping (Dolen, 1983; Flanders, 1987). A second possibility is that there appears to be a developmental factor operating. While this was a cross sectional study the gradual increase in scores by grade provided some evidence to support Dean (1987), Pressely et al (1987), Seilger (1986) and others who advocated a continuity model of cognitive development.

Table IV.4

Distribution of Students by Raw Score Group per Grade

Grade	Above Average	Average	Below Average	Total
4	7 (14)	28 (57)	14 (28)a	49
5	10 (15)	39 (60)	16 (25)	65
6	14 (21)	40 (61)	12 (18)	65
Total	31 (17)	106 (59)	42 (23)	179

a Percent of total number of students in grade and overall.

As noted earlier in this chapter, students were divided into three groups based on their performance on the word problem test. Table IV.4 lists the number of students in each raw score group by grade. The mean raw score for students in the below average group was 6.02. Students in the average group had a mean score of 12.10 correct, while those in the above average group had a mean of 16.84 correct (See Table IV.5). As expected, the one-way ANOVA ($F = 362.91$, $df = 2/176$, $p < .05$) revealed significant differences among raw score groups. Scheffe results indicated that students in the above average word problem solving group did significantly better ($p < .05$) than subjects in the other two groups. Average problem solvers also solved significantly more problems correctly than below average word problem solvers.

While there was a gradual increase in scores by grade, within each grade subjects differed markedly in their word problem solving performance on the MWPST. Word problem solving skills seem to develop differently for individuals even within the same age groups. Such findings are in accord with Yoetis and Hosticka's (1980) observation that it is difficult to represent content knowledge and the development of reasoning skills in terms of age and general environment. In this study similar ages and school environments still produced differences in word problem solving performance, offering further confirmation of Yoetis and Hosticka's findings.

In addition to looking at the number of word problems correct, the number and types of errors were calculated. The type(s) of errors a student made also provided insight into his/her problem solving ability. For example, a student who had a problem wrong because of an error in calculation probably had a better understanding of the task than a student who had the problem wrong due to completing only one step or using the wrong mathematical operation. As students could make more than one error per problem, the number of errors and word problems correct per student do not total nineteen. ANOVAs followed by Scheffes indicated that grade fours had significantly more errors than sixes ($F=6.32$, $df=2/176$, $p < .05$) but the differences between 4's and 5's, and 5's and 6's were not significant. The mean number of errors per student in grades 4, 5, and 6 was 11.49, 9.31, and 7.33 respectively.

Table IV.5

Mean Number of Word Problems Correct by Grade and Raw Score Group

	Mean	SD
Grade 4	10.02	3.89
Grade 5	11.31	3.79
Grade 6	12.80	3.54
Below Average	6.02	2.12
Average	12.10	1.75
Above Average	16.84	.89
Total	11.50	3.90

Errors were also classified by type (Table III.1). Inter-rater reliability coefficients averaged .97 with a range of .94 to .99 for classification of errors by type. Table IV.6 lists the proportion and type of errors made by students in each grade. Error types were similar in the three grades. A series of 2x2 Chi Square analyses were performed to investigate whether significant differences in proportion of errors by type between grades and raw score groups existed.

Students in grade 4, on average, made significantly more computational errors than grade 5's (Chi Square = 5.06, $p < .05$). Results presented in Table IV.6 indicated that, on average, grade four students made 1.3 computational errors while students in grades 5 and 6 made less than one computational error each. Grade fours also had significantly more concept errors than grade sixes (Chi Square = 10.56, $p < .01$) and were more likely not to attempt a word problem than students in either grade 5 or 6 (Chi Square $\geq$ 17.64, $p < .001$).

However, students made relatively few errors in these categories. The main sources of errors for all three grades were similar and included errors choosing the correct mathematical operation to solve the problem, completing all the required steps, and lack of understanding of the requirements of the problem. Grade 6's had significantly more errors in the "other" type than students in grades 4 and 5 (Chi Square $\geq$ 7.78, $p < .05$). Students in grade six appeared to have a better grasp of the requirements of specific word problem but didn't always draw a logical conclusion.

The types of errors made by subjects were fairly consistent across grades. Computation errors were relatively small in proportion when compared to other types of errors. This is consistent with Carpenter et al (1980) and Bidwell (1983) who noted that inability to do computation is not a

Table IV.6

Proportion of Errors by Type per Grade

Error Types [c]	Grade 4 (49) P	N	E/S	Grade 5 (65) P	N	E/S	Grade 6 (65)[a] P	N	E/S [b]
1	.11	63	1.3	.07	46	.7	.09	44	.6
2	.25	138	2.8	.26	154	2.4	.22	107	1.7
3	.01	5	.1	.02	9	.1	.02	7	.1
4	.03	18	.4	.02	14	.2	.01	3	<.1
5	.07	38	.8	.07	45	.7	.08	37	.6
6	.23	130	2.7	.25	148	2.3	.22	105	1.6
7	.19	105	2.1	.23	140	2.2	.22	106	1.6
8	.05	28	.6	.00	0	.0	.01	4	<.1
9	.07	38	.8	.08	49	.8	.13	64	.9

a Number of subjects in each grade.
b P = Proportion; N = Actual number of errors by type per grade; E/S = Number of errors divided by number of students per grade.
c 1 = computation; 2 = operation; 3 = clerical; 4 = concept; 5 = extraneous; 6 = steps; 7 = no understanding; 8 = no attempt; 9 = other

major source of difficulty in word problem solving for elementary students. These results were also in line with those of Quintere (1982) and Sowder (1986) who noted that elementary students have difficulties understanding problems and in choosing the correct operation to solve a problem.

The mean number of errors per student for the above average raw score group was 2.67. Students in the average raw score group had a mean of 8.16 errors while the mean number of errors for those in the below average group was 16.60. Results presented in Table IV.7 also demonstrate that the proportion of errors by type differ within the three raw score groups. Errors by students in the below average group indicated difficulties in almost all areas, especially in choosing the correct operation, completing all steps, understanding the problem and computation. Results in Table IV.7 indicated that, on average, each student in the below average group made 4.8 errors when choosing the correct operation to complete the problem, 4.3 errors due to completing only one step in a two-step problem, and 2.8 errors because they did not understand the requirements of the task. Students in the average group had similar difficulties. On the other hand, errors of subjects in the above average word problem solving group were mainly due to lack of understanding of the problem or not drawing a logical conclusion.

It would appear that below average problem solvers have not grasped the content knowledge needed to be successful mathematical word problem solvers. While many of their errors indicated a lack of understanding of the task requirements, the majority demonstrated a lack of domain specific knowledge. Walker (1987) and Seigler and Richards (1982) have pointed out the importance of specific content knowledge as a factor in acquiring increasingly sophisticated problem solving ability.

Above average problem solvers seemed to complete most problems automatically, having few specific content errors. Their "no understanding" errors were due to their lack of success on mainly one problem - item number 13 - solved correctly by only five of the one hundred and seventy nine students in the study. Errors in the "other" category appear to be the result of not drawing a logical conclusion on item number 9. Some of the students in this group understood the problem and used the correct mathematical operation but did not go that one step further. However, students in this group demonstrated they had grasped the relevant domain specific knowledge. As Perkins (1985) noted competence in a domain is often accompanied by improved performance and a realization of the specific requirements of the domain. The performance of students with above average problem solving skills on the MWPST indicated the importance of declarative knowledge in successful word problem solving.

Table IV.7

Proportion of Errors by Type per Raw Score Group

Error Types c	Below Average 0 - 9 (42) P	N	E/S	Average 10 - 15 (106) P	N	E/S	Above Average 16 - 19 (31)a P	N	E/S b
1	.09	59	1.4	.10	87	.8	.08	7	.2
2	.29	203	4.8	.22	189	1.8	.08	7	.2
3	.01	8	.2	.02	13	.1	.00	0	.0
4	.03	20	.5	.02	15	.1	.00	0	.0
5	.07	51	1.2	.08	66	.6	.04	3	.1
6	.26	180	4.3	.22	189	1.8	.17	14	.5
7	.17	119	2.8	.23	199	1.9	.40	33	1.1
8	.03	24	.6	.01	8	<.1	.00	0	.0
9	.05	33	.8	.11	99	.9	.23	19	.6

a Number of subjects in each raw score group.
b P = Proportion; N = Number of errors by type per raw score group; E/S = # of errors divided by # of subjects.
c 1 = computation; 2 = operation; 3 = clerical; 4 = concept; 5 = extraneous; 6 = steps; 7 = no understanding; 8 = no attempt; 9 = other

Problem Solving Tasks

The individual word problems yielded a variety of error types as can be noted in Table IV.8. Some specific error trends also were evident. For example, word problem number 16 required subjects to extract information from a chart. (Appendix E) The proportion of clerical errors for problem 16 was higher than for any other item. The correct response for word problem number 9 necessitated drawing a logical conclusion and an inability to do this was the main source of error.

In general, the types of errors made by students were expected due to the demands of specific problems. For example, the greatest proportions of errors on one step problems were due to computation errors and not choosing the correct mathematical operation, while errors on two step problems were due mainly to failure to complete all the required steps. Extraneous information in one of the word problems (number 5) created difficulties for students but for the other two word problems it did not. A proportion of the errors for problems 11 and 14 were due to the inclusion of extraneous information in the solution process. However, the extraneous information in word problem 11 and 14 may have been too obvious in the problem statement.

The actual type of problem also had an effect on students' performance. Students performed better on simple one-step problems, with and without extraneous information. Completing two-step problems resulted in a number of errors for students at all grade levels. Other areas of concern included word problem types that deviated from the traditional format of: choosing an operation and carrying out the operation. Such problems would include numbers 9 and 13 that required the student to go beyond the information given to

Table IV.8

Percentage of Students Responding Correctly by Word Problem Type and the Proportion of Errors by Type per Problem

Prob. No.	Prob. Type	% Correct	Error Type by Code 1	2	3	4	5	6	7	8	9a
1	one step	72	.42	.56						.02	
4	one step	79	.66	.03	.03	.28					
7	one step	60	.08	.87	.01	.01				.01	
15	one step	90	.18	.64	.12				.06		
3	two step	73	.19	.36	.01		.03	.41			
6	two step	40	.01	.14			.08	.73	.01	.02	.01
10	two step	48	.07	.38	.02			.49	<1	.03	<1
12	two step	48	.05	.19	<1b		.02	.69	.02	<1	.02
18	two step	50	.03	.22			.15	.48	.06	.05	<1
20	two step	77	.20	.11		.02		.66			
5	extraneous	60	.10	.28	.02		.57		.02	.01	
11	extraneous	89	.26	.42			.26			.05	
14	extraneous	83	.10	.43			.33		.07		.07
8	draw	83					.77				.23
2	logic	65	.03	.24			.02		.80	.05	.03
9	logic	10	.03	.24		<1			.23	.02	.48
13	logic	3							.97	.03	
16	chart	57	.30	.04	.13	.15	.01	.35			.01
17	trial/ error	48	.01	.33		<1			.42	.02	.21

a 1 - computation; 2 - operation; 3 - clerical; 4 - concept; 5 - extraneous; 6 - steps; 7 - understanding; 8 - no attempt; 9 - other; b <1 = less than 1%

reach a satisfactory conclusion. Most students treated these as simple one-step problems needing only a correct choice of operation. Indeed, only 10% of all students had number 9 correct and only 3% had number 13 right.

Student difficulty with different problem types had been documented in past research. Previous studies found that elementary students had difficulties with word problems with more than one step (Quintere, 1982; Zweng, 1979), extraneous information (Englert et al 1987; Muth, 1984) or problems that demand logical conclusions beyond the facts given (Caldwell and Goldin, 1979). The large number of errors on the MWPST due to failure to complete both steps of the problem, including extraneous information in the solution process or an inability to draw a logical conclusion are in line with past findings. These findings also demonstrate the importance of the actual task in word problem solving performance (Kilpatrick, 1975) and support inclusion of this variable in the proposed model of word problem solving.

Variables Impacting on Word Problem Solving

To determine how well the students' performance on the word problem test compared with their current performance in school, teachers were asked to rank each student on a 5 point scale. Teachers could rank students from 1-below average to 5-above average in each of the areas of computation and word problem solving. The correlations between computation and word problem solving on the MWPST ranged from .514 for grade sixes to .583 for grade fours. Correlations between problem solving performance on the MWPST and teachers' ratings have already been mentioned in response to research question 1. The relationship between word problem solving and computation was significant ($p < .001$) for grade fours. Grade four students have had less exposure and practice in computation than students in grades five or six. While grade four students had more computation errors than students in grades five or six, errors in computation were not the main source of difficulty for these students.

Correlations between the number of word problems correct and measures of vocabulary, reading comprehension, and computation on the Canadian Test of Basic Skills (CTBS) were in the moderate range (See Table IV.9). Similarly, the correlations between verbal, quantitative, and nonverbal ability on the Cognitive Abilities Test (CAT) and the word problem solving test were moderate. While all correlations were significant ($p < .001$),they seemed to indicated only a moderate relationship between these variables and word problem solving.

Suydam (1982) stated that success at word problem solving seemed to be related to how students interpret and process what they read. Results of previous research on the impact of reading on word problem solving have been inconclusive (e.g. Balow, 1964; Moyer et al, 1984). However, most of these studies considered reading skills rather than reading comprehension. In this study an inability to read the problems could not be considered a source of error as a review of the audio tapes indicated that all students except one could orally read the word problems. The one student who had poor oral reading skills still had average problem solving performance on the test. In addition, examiners told students words they did not know or misread. The results in this study support those of Knifong and Holtan (1977), Moyer et al (1984), and Paul et al (1987) who found that more than basic oral reading ability is involved in word problem solving.

Table IV.9

Intercorrelations Between CTBS and CAT Measures and Word Problem Correctness on the MWPST

Variables	1	2	3	4	5	6	7a
1.CTBS - Voc.	1.0	.747	.561	.693	.535	.447	.447
2.CTBS - R.C.		1.0	.499	.689	.582	.393	.490
3.CTBS - Comp.			1.0	.458	.503	.546	.400
4.CAT - Ver.				1.0	.541	.612	.505
5.CAT - Quan.					1.0	.647	.480
6.CAT - NonV.						1.0	.515
7.Problems C.							1.0

a Variables: CTBS - VOC = Vocabulary; CTBS - R.C. = Reading Comprehension; CTBS - Comp. = Computation; CAT - Ver = Verbal; CAT - Quan. = Quantitative; CAT - NonV. = Nonverbal; Problems C. = Problems Correct on MWPST

Equating word problem solving ability with oral reading skills is indeed an over simplification of a very complex process (Lesh, 1981).

Correlations between the Attitude Toward Arithmetic Scale and word problem correctness overall, by grade and by raw score group were in the moderate to low range. Overall, the correlation between solution correctness and attitude was .528 ($p < .001$). Correlations between average problems correct per grade and attitude were all significant ($p < .001$), .416 for sixes, .524 for fours and .676 for fives.

A review of the raw scores on both the arithmetic attitude scale and the word problem solving test indicated a positive relationship between the two measures. Indeed, raw scores on the attitude scale for the above average group ranged from 28 to 75 with all but two students having scores above 58. Similar results were noted for the below average group whose raw scores on the attitude scale ranged from 12 to 60 with all but 4 students scoring below 42.

Results indicated that students who liked mathematics tended to be better at word problem solving and vice versa. This liking may translate into more effort on the part of individual students as suggested by Kloosterman (1988). This was evident on some occasions during data collection for this study as students in the below average group sometimes made no attempt to solve a word problem. The results of the attitude scale and performance on the word problem solving test implied a link between affective and performance variables. This could be likened to McLeod's (1988) work on actively confronting the material as well as the issues of self-concept and self acceptance of responsibility for outcomes.

In summary, the Mathematical Word Problem Solving Test developed for this study appears to provide an indication of students' declarative knowledge in the area of word problem solving. Students' raw scores on the MWPST were generally in the average range with smaller proportions of subjects in the below and above average groups. Students in grades 4, 5, and 6 exhibited different proportions of errors by type. Specific word problems evoked specific error types. While word problem type (task) and attitude toward arithmetic appear to be related to solution correctness, the relationship between other variables, such as reading and computation, to word problem solving is not as clear.

Comparisons among students in the three raw score groups (above average, 16-19; average, 10-15; below average 0-9 correct) provided insight into the types of errors made by good and weak problem solvers as well as which types of problems created difficulties for students in each group. Results of comparisons among the three raw score groups also demonstrated the impact of attitude on problem solving. Students with positive attitudes toward arithmetic as measured on the arithmetic attitude scale tended to perform better on the MWPST than those who expressed negative attitudes. Attitudes were one aspect of person variables that seemed to influence not only declarative knowledge but, as will be noted later, also procedural and strategic knowledge. Further discussion of results will indicate that students who had a more positive attitude toward arithmetic tended to have more word problems correct, used more procedures and employed more strategies than students with a negative attitude, indicating the interrelation among person, task, and procedural, declarative and strategic knowledge.

The MWPST, administered using a think aloud procedure, appeared to tap a range of declarative knowledge and offered insights into the declarative knowledge of the participating students in the area of word problem solving.

E. Research Question 3: Does the MWPST provide insight into the students' procedural knowledge as outlined in the proposed model of word problem solving?

The proposed model of word problem solving (Figure I) outlines six steps that students could use when solving problems. Evaluation of the use of these steps or some combination of steps provided an indication of students' procedural knowledge when solving mathematical word problems.

While students' written protocols give some indication of the procedures used when solving word problems, other procedures are often assumed. Therefore, to provide a more complete measure of subjects' procedural knowledge all audio tape transcriptions were reviewed to identify procedures used by subjects on each problem. This review provided an indication of the different combinations of problem solving steps used to solve each word problem. As well, the average number of steps used to solve each problem and the average number of steps used by subjects in each grade and raw score group were calculated. The ANOVA and Scheffe procedures were again used to test the signifance of these means.

All transcripts were reviewed by the researcher. A random selection of 78 tapes was also reviewed by two graduate students familiar with think-aloud methods. The inter-rater reliability coefficient for step combinations used was .93 with a range of .91 to .96.

The mean number of steps used to solve word problems was 3.96 for fourth graders, 4.11 for fifth graders, and 4.30 for sixth graders (See Table IV.10). The overall mean number of steps used to solve problems was 4.13. The results of a one-way ANOVA ($F = 7.36$, df 2/176, $p < .05$) of the average number of steps used to solve word problems by subjects in grades 4, 5, and 6 indicated significant differences. Application of the Scheffe procedure resulted in significant differences between the mean number of step used by grade 4's and 6's.

The majority of students used four steps - examining the problem, choosing a plan, representing the plan, and carrying out the plan - to solve word problems. While different problems occasionally prompted more or fewer steps, this was very rare. Silver (1988) and Flanders (1987) noted that most students are exposed to and learn ritual ways of dealing with problems. Consistent use of the same procedures when solving word problems on the MWPST, regardless of task, appeared to confirm this point. For example, students seemed to have no difficulty choosing an operation to solve a problem. However, the number of errors due to choosing an incorrect operation provided further evidence of the application of a rote procedure when solving word problems. It would appear that while students have procedures they can apply to solve a problem, they are not always using them appropriately. Indeed, Schoefeld (1984) made the same point. He stated that having a set of procedures can be limiting unless one knows when and how to use them.

Students in the below average raw score group used 3.79 steps on average while those in the average and above average groups used 4.08 and 4.81 steps respectively. Results of the one-way ANOVA ($F = 73.09$, $df = 2/176$, $p > .05$) revealed significant differences in the average number of steps used to solve word problem by subjects in the raw score groups. Use of the Scheffe test indicated students considered above average word problem solvers used significantly ($p < .05$) more steps to solve word problems than those considered average and

Table IV.10

Mean Number of Steps Used to Solve Word Problems by Grade and Raw Score Group

	Mean	SD
Grade 4	3.96	.45
Grade 5	4.11	.44
Grade 6	4.30	.49
Below Average	3.79	.42
Average	4.08	.33
Above Average	4.81	.40

below average problem solvers. Students in the average group also used significantly more steps than those in the below average group. These results indicate a relationship exists between solution correctness and steps used.

Few students, except those in the above average raw score group, identified a goal and/or checked their plan. Knifong and Holtan (1977), Lee (1982) and Uprichard et al (1986) noted similar findings. The fact that students in the below average and average groups used fewer steps than those in the above average group contradicts comments by Nickerson (1985) and Perkins and Sweller (1989). These authors found experts verbalized fewer steps when completing problems. However, the subjects in these studies were experts in the content area and could make decisions regarding the consideration of several options at one time. The extent of verbalizations by the students on most of the tasks in the present study could indicate that they have not reached the "expert" level. These students may still be developing their word problem solving skills, making this process less automatized. As well, the tasks themselves may have made it easier for poorer word problem solvers to compute mentally without having to identify a goal or represent a plan, both areas of difficulty noted by Knifong and Holten (1977).

A calculation of the mean number of steps used to solve each word problem provided little insight into the impact of step usage. On average four steps were used to solve each problem. Students used the fewest number of steps for word problem number 2 (3.63) and the greatest number for number 17 (4.36). Given that 133 students used an average of 4 steps per problem, while 11 students used 3 steps on average and 35 students used 5 steps, a mean of four steps is not surprising.

A review of different step combinations (Table IV.11) per word problem indicates that for all problems most subjects used the following combination of steps:

1. examines problem
3. chooses plan
4. represents plan
5. carries out plan.

The second most popular combination included the above four steps plus step number 2 - identifies goal. The majority of students using this combination were in the above average problem solving group. Specific word problems also elicited different step combinations. For example, word problems with minimal computational demands were often completed without representing the plan. However, a review of test protocols and transcripts indicated that in many cases using only three steps resulted in errors due to an over simplification of the task. Problem number 17 necessitated checks of different trials to see if they met all conditions in the word problem statement. Step 6, checks plan, was used more for problem # 17 than for other problems. Several students only used one step for problem 2. They read the word problem statement, and immediately wrote down a number in the problem statement, suggesting a lack of understanding of the requirements of this task.

Nickerson et al (1986) and Glaser (1988) noted that the possession of a body of knowledge did not guarantee that the knowledge would be used effectively. In this case, a review of students protocols indicated that they knew the basic mathematics needed to solve word problems, yet their application of that knowledge varied significantly. Perhaps, as Carey (1985), Greeno (1984) and Seigler and Richards (1982) suggested, there is a need to reconsider ways and means of teaching word problem solving so that knowledge of how and

Table IV.11

Different Step Combinations Used by 10% or More of the Students when Solving Word Problems

Problem Number	Codes for Step Combinations 1 (1345)	2 (12345)	4 (123456)	5 (135)	6 (1)a
1	147	23			
2	79	50			41
3	150	23			
4	139	27			
5	132	40			
6	114	23		30	
7	121			40	
8	157	18			
9	146	20			
10	127	38			
11	140	28			
12	120	24		28	
13	130			18	
14	142	28			
15	124			36	
16	135	42			
17	104		42		
18	102	31		28	
20	138	24			

a Steps: 1 - examines problem; 2 - identifies plan; 3 - chooses plan; 4 - represents plan; 5 - carries out plan; 6 - checks plan when to use procedures and strategies is developed along with content information (declarative knowledge).

It would appear that the recommendation in the NCTM Standards discussed by Thompson and Rathmell (1988) to emphasize different representational modes is well founded. Most students in this study assumed that finding the right mathematical equation was the means to solving all the problems. This practice, often reinforced in current mathematics textbooks and teaching methods, places too much emphasis on set ways to complete word problems and not enough on developing and using different representational modes to solve a task.

In summary, the MWPST developed for this study provided an indication of which procedures noted in the proposed model (Figure I) were used most often to solve problems. Students generally used four steps to solve all problems. The use of more steps by successful word problem solvers seemed to indicate a positive relationship between solution correctness and steps used. The number and combination of steps used also appeared to be related to the perceived demands of the word problem task. Based on the range of information obtained relative to procedural knowledge, it would appear that using a think-aloud procedure when administering the MWPST enables an understanding of students' procedural knowledge when solving word problems.

F. Research Question 4: Does the MWPST provide insight into subjects' strategic knowledge as noted in the proposed model of word problem solving?

The model of mathematical word problem solving presented in Figure I noted some of the cognitive and metacognitive strategies students used while solving word problems. The written protocols and audio taped transcripts of students involved in this study were reviewed to ascertain their use of different strategies when completing word problems on the MWPST.

Because of the inherent difficulties in diagnosing strategy use based only on written protocols, transcripts of the audio tapes of students solving the word problems on the test were also reviewed. During this review raters noted which of the 41 cognitive strategies and 5 metacognitive strategies listed in Table III.3 were utilized by the students to solve each word problem. From this, the researcher had an indication of specific strategies employed by individual students as well as specific strategies used when solving each word problem. The mean number of strategies used by students in each grade and raw score group was also calculated. A series of one-way ANOVA 's and Scheffe comparisons determined the significances among these means. Finally, student responses to informal questions dealing with strategy application were listed and discussed.

As noted previously, all transcripts were reviewed by the author and a random selection of 78 tapes reviewed by two graduate students familiar with think aloud methods. Fifty four tapes were reviewed by one graduate student and 24 tapes were reviewed by the other student. Interrater reliability coefficients for the strategy data ranged from .87 to .94 with a mean of .91.

The mean number of different cognitive strategies exhibited by grade 4's while solving all word problems on the test was 19.02. The mean number of strategies for grade 5's was 19.75 and for grade 6's the mean was 21.60 (See Table IV.12). A one-way ANOVA ($F = 4.87$, $df = 2/176$, $p < .05$) indicated that there was a significant difference between cognitive strategy use by students in different grades. Scheffe results revealed a significant differences ($p < .05$) between grade four and six students. (These strategies were generated at different times during the testing process in response to task demands. The numbers do not reflect mean number of strategies used to solve each problem but the mean number of different strategies subjects used throughout the testing situation.)

Calculation of the mean number of metacognitive strategies employed at each grade level resulted in means of .37, .54, and .80 for grades four, five and six respectively. The mean differences among grades on metacognitive strategy use failed to reach significance ($F = 2.36$, $df = 2/176$).

Combining all types of strategies (those considered to be cognitive and those seen as being metacognitive) used by students produced means of 19.36 for 4's, 20.26 for 5's, and 22.42 for 6's. Again a one-way ANOVA followed by the Scheffe test indicated significant differences ($F = 4.99$, $df = 2/176$, $p < .05$) only between grade 4 and grade 6 students' strategy use.

The differences in mean number of cognitive or metacognitive strategies used between one grade to the next were not significant. This finding is in line with Lunzer's (1986) contention that development of cognitive strategies is a gradual undertaking. The gradual increases most likely reflected the fact that new competencies are developing as students are reviewing old content and being introduced to new material. Results of recent studies by Dean (1987) and Liebling (1988) are also in line with those of the present study and support the notion of a developmental continuum for cognition strategies. Likewise, it is generally assumed that metacognitive strategies are gradually acquired (Brown, 1981; Duell, 1986; Glaser, 1988) and dependent on the continuous development of a broad base of content knowledge and lower level strategies (Messick, 1984; Pressely et al, 1987). In this study students in grade six had significantly more problems correct and used significantly more cognitive strategies than students in grade four.

Table IV.12

Mean Number of Different Cognitive and Metacognitive Strategies Used by Students in Each Grade

	Mean	SD
Cognitive Strategies		
Grade 4	19.02	4.60
Grade 5	19.75	4.57
Grade 6	21.60	4.67
Metacognitive Strategies		
Grade 4	.37	.86
Grade 5	.54	1.11
Grade 6	.80	1.19
Cognitive + Metacognitive Strategies		
Grade 4	19.36	5.25
Grade 5	20.26	5.26
Grade 6	22.42	5.57

The mean number of cognitive strategies used by students in the different raw score groups indicated vast discrepancies. The mean number of strategies employed by below average word problem solvers (0-9 correct) to solve all word problems on the test was 15.11. For the average group (10-15 correct) the mean was 20.74 while the mean for the above average group (16-19 correct) was 25.32 (See Table IV.13). A one-way ANOVA and Scheffe test produced significant differences among all three groups ($F = 83.89$, $df = 2/176$, $p < .05$).

The mean number of metacognitive strategies employed by students in the various raw score groups to solve all word problems on the test also demonstrated significant differences among the three groups. Students in the below average group showed no evidence of using any metacognitive strategies. Word problem solvers in the average group, on average, used .31 metacognitive strategies. Above average problem solvers had a mean of 2.32 metacognitive strategies. Results of a one-way ANOVA and the Scheffe test indicated significant differences among all three groups ($F = 109.08$, $df = 2/176$, $p <.05$).

Calculations of the mean number of cognitive and metacogntive strategies combined for each raw score group revealed results consistent with separate calculations. The mean number of combined strategies were 15.11 for the below average raw score group, 21.04 for the average group to 27.68

Table IV.13

Mean Number of Different Cognitive and Metacognitive Strategies Used by Students in Raw Score Groups

	Mean	SD
Cognitive Strategies		
Below Average	15.11	2.80
Average	20.74	3.49
Above Average	25.32	3.78
Metacognitive Strategies		
Below average	.00	.00
Average	.31	.59
Above Average	2.32	1.38
Cognitive + Metacognitive Strategies		
Below Average	15.11	2.80
Average	21.05	3.79
Above Average	27.68	4.74

for the above average word problem solving group. Again a one-way ANOVA and use of the Scheffe test resulted in significant differences among all three groups ($F = 101.08$, $df = 2/176$, $p < .05$).

Results indicated that students in the above average raw score group used significantly more cognitive and metacognitive strategies than students in the average and below average groups, suggesting a direct relationship between declarative knowledge and strategies employed. Students with more word problems correct used more steps and strategies than students with fewer problems correct. These findings support those of other researchers including Walker (1987), Presseley et al (1987) and Brown (1981). Chi (1978), Perkins (1986) and Glaser (1984) also noted that above average problem solvers have sufficient knowledge of the topic to allow for flexible cognitive strategy use whereas weak problem solvers may still be concentrating on acquiring content mastery.

Strategy Use by Subjects

Appendix J contains a listing of the different types of strategies used by individual students. Students are listed in order of their raw score on the word problem test developed for this study. The strategies they employed throughout the test are noted by an "x" under the strategy code number. While strategies noted were not used for each problem, this listing provides an indication of the students' repertoire of strategies.

In order to make further comparisons of strategy use by students achieving different raw scores on the word problem solving test, students were further divided into six groups based on their raw score on the word problem solving test developed for this study. These groupings and the number of students in each grouping included: Group 1 - 31 students with scores in the range of 16 to 19; Group 2 - 30 students with scores of 14 or 15; Group 3 - 29 students with scores of 12 or 13; Group 4 - 47 students with scores of 10 or 11; Group 5 - 18 students with scores between 7 and 9; and Group 6) - 24 students with scores between 0 and 6.

Some strategies were used by 80% or more of all students on at least one of the problems. These strategies along with their code number from Table III.3 included:

1 - reads problem

4 - copies numeric data/words correctly

5 - organizes written work

6 - chooses correct operation (s) or response method

10 - notes all relevant information

15 - picks up on clues from context/statement

33 - reads a chart correctly

These strategies are typically used in all word problem solving situations, with the exception of number 33. Many students appeared to use these strategies automatically as they worked out problems, even if the task demands suggested other alternatives could be used.

Almost all students, regardless of word problem solving performance on the MWPST, demonstrated they had a repertoire of general strategies such as picking up on cues in the problem statement and noting relevant information. Walker (1987) also noted that most students have general strategies which they apply to several situation and tasks.

Other strategies were exhibited by fewer than 10% of all subjects. These strategies and their codes from Table III.3 were:

14 - estimates (< 5%)

18 - uses an analogy (< 5%)

19 - uses visual imagery (< 5%)

22 - draws upon past experience

35 - works backward

36 - skips, then returns to problem later

41 - states an acceptable alternate method

47 - aware of strategy use and value (< 5%)

48 - knows when to use strategies (< 5%)

These strategies seem to require a greater understanding of the word problem solving task as well as an ability to see the problem from different perspectives. Use of such strategies also assumes that the individual is aware of ways to approach the task and can be selective in his/her methods.

While assessing students' strategy use, raters looked for two types of strategies - cognitive and metacognitive. Both types were considered evidence of strategic behaviour but at different levels on a developmental continuum. Use of metacognitive strategies implied an active, conscious control over knowledge. Few students in this study exhibited evidence of metacognitive strategy use. Yet the problems on the MWPST often required active conscious control over the material as recommended by Kirby (1984). Indeed, students who did activate metacognitive behaviour were mainly in the above average raw score group (16 - 19 correct) A few students in the average groups (10 -15 correct) used metacognitive strategies but their use of such strategies was isolated and inconsistent.

Several strategies were used by some subjects in the six raw score groups in each grade. While percentage of use varied, there was 20% or less discrepancy between the use of these strategies from above average problem solvers (16 - 19 correct) to below average problem solvers (0 - 6 correct). Strategies that fall within this category include:

3 - rereads problem (60% -72%)

7 - sequences steps of problem correctly (72% -85% excluding students with raw scores > or = to 16)

23 - questions self (32% - 45%)

25 - persists (4% - 20%)

31 - seeks clarification (33% - 44%)

34 - works computation out mentally (32% - 5[illegible]%)

Again some of the strategies (3, 7) noted in this group seemed familiar to most elementary school students as they are usually introduced in math textbooks. Others, such as 23, 25, and 31 depend on the students' recognizing that the demands of the task are not obvious.

For a number of strategies, students in the six raw score groupings exhibited marked differences in the degree to which they were used. Table IV.14 lists these strategies and the percentage of subjects within each raw score group who used them. Chi square analysis revealed that a twenty percentage point difference was significant at $p < .01$.

Table IV.14 indicates that for a number of strategies there is a gradual decline in their being generated which is in line with raw scores on the word problem solving test. More word problem solvers with a greater number of problems correct tended to use these strategies, except for strategy number 40 which involved guessing. Examples of such strategies include:

2 - marks/underlines relevant information (32% - 4%)

8 - search behaviour - looks back (93% - 29%)

9 - notes extraneous information (100% - 54%)

11 - pauses to consider options (84% - 46%)

16 - draws a diagram or chart (45% - 0%)

17 - attempts other procedures (32% - 4%)

21 - restates problem (48% - 17%)

28 - notes errors and self corrects (97% - 61%)

29 - verbally prompts self (97% - 42%)

32 - repeats key points (90% - 61%)

37 - rules out options verbally (31% - 4%)

38 - goes back to problem when finished (41% - 4%)

Table IV.14

Percentage of Students Using Specific Strategies by Raw Score

Strategy Code	Raw Score >/= 16 (29)	14/15 (29)	12/13 (27)	10/11 (47)	7/8/9 (21)	</= 6 (25)a
2-marks info	31.3	10.3	11.1	8.5	19.0	4.0
8-looks back	93.1	72.4	70.4	70.2	38.1	28.0
9-extraneous	100.0	100.0	100.0	100.0	85.7	56.0
11-pauses	82.8	82.8	62.9	72.3	71.4	48.0
12-notes goal	100.0	65.5	33.3	17.0	4.7	4.0
13-supplies info	100.0	100.0	85.2	74.5	66.7	8.0
16-draws diagram	44.8	17.2	18.5	10.6	0.0	8.0b
17-other ways	31.0	27.5	3.7	14.9	9.5	8.0
20-trial/error	93.1	72.4	70.4	44.7	19.0	4.0
21-restates	48.3	55.1	55.5	21.3	19.0	16.0
24-oral answer	96.6	87.5	66.7	57.4	19.0	24.0
26-checks	62.1	24.1	18.5	6.3	0.0	8.0
27-answer ok	89.6	55.1	51.9	29.8	18.5	8.0
28-self corrects	96.6	82.8	81.4	78.7	61.9	60.0
29-prompts self	96.6	93.1	77.8	63.8	42.9	40.0
30-logical outcome	100.0	48.2	33.3	4.2	0.0	0.0
32-key points	89.7	82.8	81.4	78.7	61.9	60.0
37-rules out ways	31.0	10.3	18.5	23.4	9.5	4.0
38-rechecks info	41.4	17.2	22.2	8.5	4.7	4.0
39-verbal plan	89.6	75.9	66.7	46.8	33.3	24.0
40-guesses	0.0	6.8	14.8	8.5	19.0	20.0
45-ability	34.5	6.8	0.0	0.0	0.0	0.0
46-monitors	82.8	31.0	7.4	4.2	0.0	0.0
49-task awareness	82.8	6.8	11.1	0.0	0.0	0.0

a Number of students in each category.

b Percentage of students who used this strategy other than for word problem number 8, which required drawing a diagram for the answer.

These results seemed to indicate that better word problem solvers were cautious, careful with calculations, used different ways of solving the problem and talked to themselves about the problem.

Students differed significantly in their domain specific strategy use. Students in the above average group generated a variety of strategies that indicated they understood the requirements of the task and ways to achieve the correct response. While students in the average groups (10/11, 12/13, 14/15) employed more strategies than those in the below average groups (7/8/9, 0-6), with few exceptions, their strategy use was narrow and not always goal directed. For example, some students in the average groups underlined relevant information but used it inappropriately in the problem solution. Such behaviour again addressed the relationship between declarative knowledge and strategic knowledge and added further support to Schoenfeld's (1979) contention that having a strategy without knowing when and how to use it can be very limiting.

There was a marked decline or sudden drop in strategy use by students in the 6 raw score groups for some strategies, such as:

12 - notes / verbalizes goal (100% - 4%)

13 - supplies missing or implied information (100% - 8%)

20 - uses trial and error (94% - 4%)

26 - checks answer (62% - 0%)

27 - judges whether an answer is reasonable (87% - 8%)

30 - draws logical conclusion (100% - 0%)

45 - awareness of abilities (34% - 0%)

46 - monitors progress (68% - 0%)

49 - awareness of the features of the task (55% - 0%)

The overwhelming use of the above strategies by subjects with scores above 16 on the word problem test seems to demonstrate that these subjects understood the word problem tasks. Such strategy use requires going beyond rote processes to consider the features of the task, the monitoring of progress and becoming involved in the solving of word problems.

Again the relationship between declarative knowledge and strategy use, as suggested in the proposed model (Figure I), was evident since above average word problem solvers had more problems correct and consistently used more strategies than problem solvers in the other two groups and vice-versa. This finding supported those reported by Adams et al (1988) and Gelman (1988) who noted that acquisition of specific domain content can lead to more highly differentiated use of strategies. In addition, the fact that metacognitive strategies were used at all grade levels in this study provided further evidence of Brown's (1980) hypothesis that metacognitive strategies were not age dependent. The results of this study are similar to those of Romberg and Collis (1985) who found that high ability students used more strategies and on a more consistent basis than low ability problem solvers.

Overall, the types of metacognitive strategies used were related mainly to awareness and monitoring of the task (19 and 27 respectively out of 179 students) and an awareness of one's own abilities (12 of 179 students). Only two of the 179 students indicated an awareness of when to use strategies or the value of strategy use. Other students may also be using metacognitive strategies but as suggested by Ericson and Simon (1984) some students have trouble verbalizing the use of metacognitive strategies. Kirby (1984) and Shiffin and Deumais (1981) also noted that the use of specific strategies may become so automatized that they may not be open to conscious control or be readily reported by students. While the issue of verbalizations remains a concern, the MWPST attempted to control for the issue of automatization and conscious control by varying the difficulty of the problems.

The limited use of metacognitive strategies at these grade levels adds support for their separation from cognitive oriented strategies in the model in Figure I. However, since some students did report or give evidence of metacognitive use, their inclusion within the strategy aspect of the proposed model is necessary. Several authors (Brown, 1980; Carey, 1985; Dean, 1987; Glaser, 1988; Messick, 1984) have noted that metacognition is gradually acquired and restricted by the person's competence to access, generalize and transfer knowledge. For the most part, elementary students appear to be still acquiring basic competencies.

One strategy, number 40 - guessing, showed a gradual increase rather than decline with lowering grades. No subjects in the 16 or above category exhibited this strategy, yet 21% of subjects with scores less than or equal to 9 used it. On some occasions guessing resulted in a correct response, but, in the majority of cases it did not.

Students in the below average raw score groups (0 - 9 correct) tended to be very passive problem solvers. They often read the problem and immediately performed a mathematical operation. These problem solvers may be at the "reacting to" level of cognitive development (Schmidt, 1973) in that they are going through the motions of completing word problems without considering how or why they perform as they do. Responses to informal questions at the end of the word problem solving test as well as their use of the "guess" strategy seems to confirm this hypothesis.

<u>Strategy Types Used per Word Problem</u>

The different word problems on the test designed for this study elicited specific strategies, yet had certain strategies in common. For example, for all problems the majority of subjects in each grade:

1 - read the problem

4 - copied numeric data/words correctly

5 - organized written work

6 - chose the correct operation (s) or response method

10 - noted all relevant information.

However, strategy type and usage varied between and within grades. Table IV.15 lists the number of different strategies used by subjects in grades 4, 5, and 6 to solve each word problem as well as the average number of strategies employed overall to solve the problem. The average number of strategies is rounded off to the nearest whole number. While a number of word problems elicited a high number of different strategies, often specific strategies were exhibited by only a few subjects. This fact is evident when one compares the average number of strategies used with the number of different strategy types.

Different word problems resulted in the use of several common strategies as well as strategies specific to the problem performance is influenced by the way the task is represented and the demands of that task. The task demands on the MWPST coupled with the ways individuals completed the word problems resulted in unique performances among students. For example, word problems number 9 and 13 produced 34 and 35 different strategies each, yet, on average, only six and five strategies were used, respectively, to arrive at an answer. These two problems also had the fewest correct responses. This apparent discrepancy occurred because most students gave little attention to the demands of the task and assumed it required applying the correct operation. A few students, however, recognized that the solutions to these word problems were not obvious and contemplated several goals, responses and options in an attempt to reach a satisfactory conclusion.

These results point to the interaction between task, person and strategies throughout the problem solving process. Students who viewed the tasks as routine reacted to them in a routine manner. Students who recognized the demands of the problems became actively involved in the problem solving process, employing different strategies and procedures to obtain solutions. Similar findings were reported by Nickerson (1985) and Kilpaterick (1975) who note the relationship between task and person.

Word problems eliciting 30 or more different strategies were numbers 5, 6, 9, 10, 12, 13, 17, and 18. Four of these problems (6,10,12,18) required the completion of more than one step to obtain a solution. Two (9,13) problems demanded that the subject draw a logical conclusion or draw a diagram to help in the solution process. The remaining two necessitated using a trial and error technique (17) or recognizing extraneous information (5).

Interestingly, many of the problems that elicited the greatest number of strategies also loaded on the same factors (See Table IV.3). For example, problems #9 and #13 were the only ones loading on Factor VI (Logic Problems) while problems number 5, 10, 12, 17, and 18 all had loadings on Factor II (Choosing Essential Information). Conversely, a number of the simple one step problems that produced a reduced number of strategies loaded on the same factor - Factor V (Obvious One Step Problems). These findings point to a relationship among problem type, solution correctness, and strategy use. This finding also added support to the hypothesized model of Mathematical Word Problem Solving (Figure I) and the MWPST developed to measure the validity of the proposed model. The model suggested that successful problem solving was the result of the interrelationship among several variables. One of the variables included under declarative knowledge was the actual problem solving task. For many students in this study, especially those in the above average group (16 or more problems correct), word actual problem tasks evoked an increase in procedure and strategy use, indicating an interrelationship among declarative, procedural, and strategic knowledge.

In addition, students at the various raw score levels within the different grades exhibited different strategy use when solving problems. As has been noted, significant differences occurred among the three raw score groups regarding the use of strategies.

Appendix K contains coded listings of the strategies used to solve individual word problems by raw score group in each grade. The strategy codes were listed in Table III.3 and used in reviewing transcripts. As stated, most students employed a small number of strategies when solving specific word problems. Therefore, the number of strategy types noted for each problem in Appendix K may present a distorted picture if they are viewed as being consistently applied by all students within a particular raw score group. A more accurate way of viewing these strategies per problem would be to consider that for each problem, specific strategies were employed by individual students in each raw score group per grade. However, it should be remembered that individual students exhibited these strategies while solving word problems, even if it was not on a consistent basis.

Overall, grade four students used some strategies less frequently than students in grades five and six. Strategies used less frequently included : 2 - rereading; 8 - looking back; 11- pausing; and 23 - questioning self. Both fourth and fifth graders were less likely than sixth graders to employ the strategies of 26 -checking answers and 27 - judging response reasonableness.

Fourth graders exhibited fewer metacognitive strategies than fifth and sixth graders. As well they used a narrower range of types of metacognitive strategies (see Appendix J). Some of the grade four students monitored their progress, showed awareness of the features of tasks, and recognized their strengths and limitations. None indicated an awareness of strategy use and/or value. For seven problems (1,4,7,8,13,14,15) fourth graders at all raw score levels did not use any metacognitive strategies. This may, in part, be due the simplicity of the particular word problem or to an automatized response to this type of problem.

A number of fifth graders employed metacognitive strategies to solve specific problems, but usage was not consistent. Appendix K also contains the metacognitive strategy types exhibited by a limited number of individual students in grade 5. No fifth grader used metacognitive strategies for word problems number 1, 4, and 15 - all one-step problems.

While grade 6 students employed more metacognitive strategies to solve problems than grade 4's or 5's, their use was inconsistent. All types of metacognitive strategies were exhibited on a limited bases. Only word problem number 4 produced no metacognitive strategies for the grade six students. As this was a simple one-step subtraction problem it may have been a rote task allowing a more automatized response for this group of students.

The number and variety of strategies generated for problems on the MWPST supports the contention of Havel (1985) and Steffe and Black (1983) that textbook word problems do constitute problem solving tasks for elementary students. Dolan's (1983) and Sherrill's (1983) comments that textbook word problems do not require the use of many strategies were not supported when the results of this study are considered. While some students at this level applied routine procedures and strategies, many others generated a variety of strategies to solve each problem task. Indeed, up to thirty five different strategies were used by different students when solving some word problems on the MWPST.

Evidence of Strategy Use on Responses to Informal Questions

Students' responses to their choice of a favorite subject and whether they liked math were interesting. While over 85% stated they liked math, 65% cited other subjects as favorites or could not make a clear decision. Students were reluctant to classify themselves as better word problem solvers than their classmates, even those with high raw scores. Less than 5% noted that they were better at word problem solving than others in their class.

Responses to the question dealing with things to do to become a better word problem solver evoked interesting responses. Summaries of the responses are presented in Tables IV.16, IV.17, and IV.18 for students in each grade by raw score group. Students in the below average problem solving groups in all three grades gave responses that indicated they felt more effort would make them better word problem solvers.

Students in the average groups in all grades frequently verbalized responses that indicated a need for more effort on their part to be a better problem solver. In addition, they noted other strategies such as looking for clues and checking their work more often. One student in the average group noted

Table IV.16
Grade 4 Students Responses to How to Be a Better Word Problem Solver

Raw Score 0 - 9	Raw Score 10 - 15	Raw Score 16 - 19
-study harder	-study harder	-study harder
-practice	-practice	-practice
-know x's tables	-know x's tables	-know x's tables
	-look for clues	-look for clues
	-ask for help	-ask for help
	-improve reading (a)	-do it for fun
	-learn things of by heart	-work on estimating (a)
		-do it for the challenge
		-by getting older because as you older, you get better (a)

a Responses given by only one student.

Table IV.17
Grade 5 Student Responses to How to Be a Better Word Problem Solver

Raw Score 0 - 9	Raw Score 10 - 15	Raw Score 16 -19
-check work	-check work	-check work
-know x's tables	-know x's tables	-know x's tables
-pay attention	-pay attention	-pay attention
-get a new teacher	-practice	-practice
-use a calculator	-reread	-reread
	-read slowly	-concentrate
	-study	-study
	-read out loud	-be careful
	-look for clues	-look for clues
	-make up own word problems	-make up own word problems
	-teacher give you extra work (a)	-teacher give you extra work
	-work on problems at home	-work on problems at home
		-get the full picture (a)
		-do it for fun
		-parents give extra work
		-problems in (a) old math books
		-stay after school for help
		-don't rush
		-check errors
		-read more (a)

a Responses given by only one student.

Table IV.18

Grade 6 Student Responses to How to Be a Better Word Problem Solver

Raw Score 0 - 9	Raw Score 10 -15	Raw Score 16 - 19
-reread	-reread	-reread
-study	-study	-study
-practice	-practice	-practice
-try different operation	-try different operations	-try different operations
	-look for clues	-look for clues
	-don't rush	-take time
	-more effort	-more effort
		-pay attention
		-say it to yourself
		-write info down
		-more experience
		-don't get anxious
		-think about why problem solving is important (a)

a Response given by only one student.

that improved reading ability also improved word problem solving but could not explain why.

Responses from students in the above average category in all three grades revealed a variety of responses, especially in grade 5. Students gave the standard responses of the need for more effort, attention, and care, but then went beyond to include more insightful suggestions. While some responses indicated good strategies to use such as estimating and writing relevant information down, others dealt with the interaction of performance and affective variables. For example, some of these students noted that performance is better when you are doing something for the fun of it or because it is challenging. Others noted the importance of remaining calm and not getting anxious when success is not forthcoming.

Three different students in the above average group each gave an unique yet thoughtful and appropriate response. One student pointed out the relationship between reading and word problem solving. She noted that reading more books improved reading ability and that good reading was important when solving word problems. Another student indicated that a relationship between age and ability existed. He stated that as you get older you have more experience with word problems and therefore your ability to solve them improves. Another student noted the relevance of word problem solving to everyday situations throughout life.

Responses to a question addressing what should be done when one does not know how to solve a word problem produced varied and interesting responses (See Tables IV.19, IV.20, and IV.21 for a summary of these strategies.). Students in the below average groups in each grade gave responses that indicated in general that they would ask someone for help, skip the problem, or just guess at a response. A few students responded that they would use concrete aids (fingers, calculators), or consider different operations. Most of these strategies are passive, requiring limited active involvement by the subject.

While students in the average group for all grades relied on help from others, rereading, guessing, and waiting; a number of different responses also were stated. Alternative responses included strategies such as reading aloud, doing one step at a time, understanding the problem, drawing a diagram and looking for details. In contrast to students in the below average group, students in the average group noted they often skipped a problem and went to the next one but subsequently returned to the skipped problem to complete it.

Again students in the above average problem solving group in all grades gave a greater number of different responses. Standard responses (such as rereading, checking, and drawing) were given by a number of students. As well, several students mentioned alternative strategies such as picking the answer which sounds most reasonable, thinking of a similar problem,

Table IV.19

Grade 4 Responses to How to Solve a Difficult Word Problem

Raw Score 0 - 9	Raw Score 10 - 15	Raw Score 16 19
-ask for help	-ask for help	-ask for help
-skip	-skip	-leave,come back
-look at it	-look at it	-look for hints
-try different operations	-try different operations	-try different operations
-write all numbers down	-decide what it is telling me	-see which answer sounds best
-underline info(a)	-put down any answer	-draw a picture
	-reread	-reread
	-sit there	-think about it happening to me

a Response given by only one student.

Table IV.20

Grade 5 Responses to How to Solve Difficult Problems

Raw Score 0 - 9	Raw Score 10 - 15	Raw Score 16 - 19
-ask for help	-ask for help	-ask for help
-ship	-skip,come back	-skip,come back
-try different operations	-try different operations	-try different operations
-reread	-reread	-reread
-look for clues	-look for clues	-look for clues
-sit & think	-sit & think	-think till it clicks
-guess	-guess	-concentrate
-trial/error	-estimate	-estimate
	-draw a diagram	-draw a diagram
	-work out on scrap paper	-think of combinations
	-use concrete things	-check for errors in statement
	-look for details	-read 3 times for main idea/tidbits understanding (a)
	-play with numbers	-split it up then bring together (a)
	-think out in my head (a)	-explain to myself
	-say it aloud	-imagine self in(a) similar situation
		-ask myself what I going to do
		-do each sentence separately (a)
		-realize sometimes you can't figure it out (a)

Table IV.21

Grade 6 Responses to How to Solve a Difficult Word Problem

Raw Score 0 - 9	Raw Score 10 - 15	Raw Score 16 - 19
-reread	-reread	-reread
-ask for help	-ask for help	-ask for help
-try different operations	-try different operations	-try different operations
-skip	-skip, come back	-skip, come back
-count on fingers	-draw	-draw
-guess	-guess	-check answer
-use a calculator	-use scrap paper	-write all info
	-persist	-persist
	-sit & wait	-consider possibilities & pick best one
	-go through steps [a]	-check for understanding
	-look at question	-find another way to look at it
	-look at someone else's work	-think of a similar problem
		-ask for explanation
		-listen for cues from teacher
		-look for key words

[a] Response given by only one student.

making sure you understand the problem, and clarifying some aspects of the word problem statement.

Unique response were verbalized by four individual students. These included: 1) "Read the problem over three times. First to get the main idea, then to get the tidbits and last for understanding." 2) "Split the problem up into sections and then bring the pieces together." 3) "Image yourself in a similar situation." and 4) "Realize that sometimes you may not know enough to be able to figure out a problem". Such responses indicate an awareness of ones own abilities, strategy use and task demands.

Responses to the informal questions are in line with strategy use by students in each grade at the different raw score levels when completing the word problem test. Students in the above average group demonstrated a repertoire of more strategy types than students in the remaining two groups. Some strategies verbalized by a few above average problem solvers indicated an understanding of their capabilities and an awareness of task demands. A limited number of students within the average and below average groups provided strategies indicating an attempt to understand the problem.

Comments by students in reply to informal questions at the completion of the MWPST proved informative. Students at all grade levels who had scores in the below average range indicated passive means of improving their word problem solving skills or to solve a difficult problem. Suggestions like: "I'd seek help", "I have to work harder", and "You need to know your times tables" demonstrated a belief that all that was needed to become a better problem solver was to learn more facts and put extra effort into the task. Schoenfeld (1979) and Perkins (1985) both noted that learning more facts and increasing work effort were no guarantee that strategic or procedural knowledge would improve or be applied appropriately. However, this is very individual. Results of this study and others by researchers such as Adams et al (1988), Walker (1987) and Gelman (1988) have demonstrated that acquisition of specific content knowledge is one factor in acquiring increasingly sophisticated problem solving skills.

Average problem solvers at all grade levels voiced strategies similar to those given by below average subjects in response to the informal questions. While average word problem solvers provided more strategies, their strategies still centered around the need for more practice, effort and attention.

Above average word problem solvers in all three grades presented unique as well as standard strategies in response to informal questions. Suggestions focused on the need to monitor progress, understand the problem, become more experienced, and enjoy the problem solving process. These strategies imply active participation in the word problem solving event or the "knowing about" end of the cognitive development continuum (Schmidt, 1973). Comments by some above average word problem solvers also denoted an understanding of the interaction between affective and cognitive variables. But as Silver (1985) and McLeod (1988) both advocated, the influence of affective variables needs to be addressed further to determine their actual impact on word problem solving.

G. Research Question 5: Does the MWPST provide evidence of a relationship among the three types of knowledge (declarative, procedural and strategic) included in the proposed model of word problem solving?

To date, each type of knowledge considered in the proposed model of word problem solving has been examined separately. During this process the interrelationship among the three types of knowledge has become evident. However, while this relationship may be implied it must be proven. To address this relationship correlations between a number of variables were calculated. These included comparing solution correctness (declarative knowledge) with the average number of steps used (procedural knowledge) and the average number of cognitive and metacognitive strategies used (strategic knowledge) per subject. Information gleaned from reviewing students' audio taped transcripts and written protocols must also be included to support the statistical data.

Table IV.22 shows the Pearson product-moment correlations among the four variables of raw score, mean steps used, cognitive strategies, and metacognitive strategies. All correlations were significant at the $p < .001$ level. The results indicated a high to moderate relationship among all variables.

Table IV.22

Intercorrelations Among the Variables of Raw Score on MWPST, Steps, Cognitive Strategies, and Metacognitive Strategies

Variables	1	2	3	4
1. raw score	1.00	.590*	.755*	.602*
2. mean steps		1.000	.634*	.601*
3. cognitive strategies			1.000	.750*
4. metacognitive strategies				1.000

* $p < .001$

In addition, information gleaned from the review of transcripts indicated that students in the above average group had more problems correct and used more steps and strategies than students in the average and below average groups. These findings seemed to indicate that a relationship among declarative, procedural and strategic knowledge in the area of word problem solving exists. In addition, the results on the MWPST supported similar findings by Bransford et.al. (1986), Perkins and Soloman (1989) and Nickerson et.al. (1986) indicating that knowledge in a domain and knowledge of when and how to apply and transfer that knowledge seem to be interrelated.

As noted throughout this chapter, students with the greatest number of word problems correct, regardless of grade level, tended to use more steps and exhibited more strategies than subjects with an average or below average number of word problems correct. The results presented in the correlation matrix, coupled with descriptive information from students' transcripts presented in this chapter seemed to indicate that the MWPST can provide an indication of subjects' declarative, procedural, and strategic knowledge in the area of mathematical word problem solving and that these aspects form a dynamic interactive relationship.

Discussion of the results throughout this chapter has indicated a positive relationship among the three types of knowledge outlined in the proposed model of word problem solving and measured on the MWPST when it is administered using a think aloud procedure.

The gradual increase in solution correctness, steps used, and strategies employed within successive grades indicated developing ability in the area of mathematics word problem solving within the limitations that this is cross sectional rather than longitudinal data. In general, as students became more proficient with the content needed to solve mathematical word problems their success increased. The repetition of content material throughout grades (Flanders, 1987) and the extra exposure and practice should ensure some improvement in performance on word problem solving tasks. As differences among students at different grade levels were miminal, results suggested that exposure and repetition were not enough to develop problem solving skills for all students.

The model (Figure I) on which the MWPST was based considered word problem solving performance to be multi-dimensional, involving a constant interaction among declarative, procedural, and strategic knowledge. The interrelationship among declarative, procedural, and strategic knowledge has been postulated by several researchers including Brown (1981), Messick (1984), Nickerson et al (1985), and Sternberg (1985). Research findings by others such as Carey (1985), Chi (1985), Glaser (1984) and Perkins and Soloman (1989) also concur with the findings of this study and indicate the interdependence of content and reasoning skill.

More importantly, the results of this study support the use of the MWPST developed for this study to measure mathematical word problem solving ability as outlined in the model presented in Figure I. The ability of the test and testing procedure to tap and distinguish among students' declarative, procedural and strategic knowledge when solving word problems provides verification of the proposed model and indicates that the MWPST has construct validity. Mathematical word problem solving is too complex a skill to be judged by subjects obtaining a right or wrong answer. Many factors interact when students solve word problems. However, the information gleaned from the MWPST developed to support the proposed model of Mathematical Word Problem Solving provided an indication of elementary students current word problem solving skills in three interrelated knowledge areas.

H. Limitations

As in all studies that are carried out in education this one had some limitations. The main ones are listed below.

1. Information gained from verbal reports is often constrained by the subjects' linguistic skills or because the strategy is so well known and utilized that its use may have become automatized and not within conscious control of the subject (Cavanaugh and Perlmutter, 1982; Nickerson, 1985). However, it is hoped that the verbal reports coupled with the subjects' written responses and teacher ratings enhanced confidence in the findings.

2. Usually students do not verbalize their thoughts while solving word problems. Attempts to minimize the effects of this procedure, such as using familiar material (Wiggins, 1989) and practicing the think-aloud procedure on another task (Afferback and Johson, 1984), were employed. However,the actual think-aloud procedure may have influenced the students' performance on the word problem test.

3.The fact that the word problem solving sessions were audio-taped may have influenced some students' performance. Students may have become anxious, careless, or overly conscious of the taping. While there was little overt evidence of these behaviors and students' performance on the MWPST was generally in line with classroom performance on similar tasks, an affective factor may have influenced some students' performance. However, taping of sessions was necessary in order to evaluate students' procedures and strategy usage in a complete manner.

4. The actual use of verbal reports as data is questionable. However, the pros (Ginsburg, 1981) and cons (Cavanaugh and Perlmutter, 1982) of verbal reports as data have been discussed. However, Afferback and Johnson (1984) and Erison and Simon (1984) concluded that verbal reports are a valuable and reliable source of data when they are elicited with care and interpreted with full understanding of the circumstances under which they were obtained, particularly if they are collected concurrently with other measures of behaviour.

V. SUMMARY AND RECOMMENDATIONS

The model of Mathematical Word Problem Solving presented in Figure I evolved from a review of literature in several interrelated areas including cognition, knowledge, development and mathematical word problem solving, as well as two pilot studies (French submitted; 1988). Word problem solving is hypothesised to involve the dynamic interaction among the three components of declarative, procedural, and strategic knowledge. As well, the model emphasized the interaction between knowledge and reasoning in word problem solving (Brown, 1981; Gelman, 1986; Pressley et al, 1987).

From this model the Mathematical Word Problem Solving Test (MWPST) was developed to reflect "word problem solving" as defined in the model. The MWPST, administered using a think-aloud procedure, was intended to tap declarative knowledge in mathematical word problem solving as well as the procedures and strategies used by elementary students in grades four, five, and six when solving mathematical problems. The focus of this study was to determine whether performance on the MWPST could in some way be demonstrated to be related to word problem solving as outlined in the proposed model. Cronbach and Meehl (1955) and Loevinger (1957) noted that such a finding would add validity to the theory underlying the test.

It is difficult, if not impossible, to measure or observe "word problem solving" as defined in the proposed model when using tests with a timed, multiple choice format. Therefore, as Loevinger (1957) suggested test developers should define the traits that can be manifested in the test data and interpreted as measures of the construct. Loevinger (1957) also noted that these traits should be observable. As the MWPST with its think-aloud procedure sought to measure students' declarative, procedural, and strategic knowledge in mathematical word problem solving as outlined in Figure I, each type of knowledge had to be discerned as students' solved problems.

To assist in the delineation of declarative, procedural, and strategic knowledge the conditions under which possession of the different knowledge types could be observed were identified. These included:

a) Declarative - A review of students' written protocols and transcripts was used to note solution correctness and error types provided an indication of students' current declarative knowledge in the area of mathematical word problem solving. Performance on different problem types and students' attitude toward mathematics were also considered.

b) Procedural - A review of the transcripts of students' verbalizations and their written work was used to ascertain which, if any, of the procedures and their sequence as outlined in Figure I were used while solving word problems.

c) Strategic - A review of the transcripts of verbalizations and written work was used to note which, if any, of the strategies listed in Table III.3 students were using throughout the problem solving process.

Following the paper/pencil/verbal format procedure, test data was collected and analyzed. Results revealed a number of interesting findings regarding the test itself and the underlying theory. Discussion of these results will focus, in part, on the reliability of the format, procedure and scoring system of the MWPST. The ability of the MWPST to measure declarative, procedural, and strategic knowledge will also be discussed in light of whether the observed behaviour can be interpreted as measuring problem solving as hypothesized in Figure I.

Mathematical Word Problem Solving Test

All subjects completed the word problem test by reading the problems aloud and verbalizing their thoughts as they worked out the problem. Subjects' verbalizations were taped. In this study, the impact of the think-aloud procedure was reduced by using familiar content, a practice advocated by Campione and Brown (1987), Walker (1987) and Wiggins (1989). The think-aloud procedure did not appear to interfere with subjects' performance on the word problem tasks. Indeed, the positive distribution of raw scores on the test and the significant correlations between students performance on the test and in class indicated relatively consistent performance.

However, it should be remembered that once a non familiar person and/or a non familiar situation is introduced into any setting, a change in an individual from his/her natural day to day functioning is possible. This is a particular problem for the person attempting to assess cognitive performance since the very framing of even an open ended question provided a previously untapped referent for the participant. In attempting to control for this, the research and the hired examiners tried to be as unobtrusive as possible, a practice advocated by Baron (1987).

Solving word problems using a think-aloud procedure is a departure from the traditional method of assessing word problem solving skills. The pros and cons of think-aloud procedures have been discussed previously (Afferback and Johnson, 1984; Ericson and Simon, 1984; Greenwood and Anderson, 1983). While think-aloud methods are not without flaws, a more viable or less subjective method of strategy assessment has yet to be found (Afferback and Johnson, 1984). Think-aloud testing procedures not only provided insight into a students' understanding of a problem but, as Nickerson (1985) noted, such procedures help avoid the problem of assumptions being made about students' understanding of a problem based only on the correctness or incorrectness of their responses and an error analysis.

In the current study, subjects' verbalizations revealed a number of procedures and strategies that might have been missed altogether or assumed if relying only on written work. For example, completion of multiple-choice tests or paper/pencil tests that discourage verbal output give no indication that students reread the problem, pause, rule out some options, or monitor their progress. Massey and Gelman (1988) reached a similar conclusion, noting that think-aloud procedures provided insight into the nature of thought processes that might be otherwise unknown.

Test Format:

Another important aspect of a test is its format. The format in the MWPST allowed space for actual written workings. The majority of students included all their written work except in a few cases where the computational demands made it easy to work the word problem out mentally. A careful analysis of subjects' written work went beyond just noting if the response was correct to include looking for clues on procedures and strategies utilized in the problem solving process. For example, raters noted if students checked their responses, underlined relevant information, corrected errors, or tried other procedures. Anastasi, (1985), Ebel, (1985),and McAloon, (1983) noted the importance of students including their written workings. They contended that test protocols that contained students' written work provided additional insight into their declarative, procedural, and strategic knowledge that could not be gained on traditional multiple-choice tests.

The inclusion of written work also enhanced the validity of the research findings as it provided one indicator of the students' word problem solving abilities that could be used in conjunction with their verbal reports. The use of various data sources is advocated by Firestone (1987) and Mathison (1988) as a way of triangulating data. As well, Meichenbaum et al (1986) and Neill and Medina (1989) also encouraged the use of multiple

approaches to assessment, including actual performance on task. In this study, students' written work and verbalizations while solving word problems as well as teacher ratings of their problems solving ability were used.

Scoring System

Raters hired to review transcripts had no difficulty following the scoring system. This may be due to the fact they were familiar with think-aloud research procedures. However, the researcher found having a summative page, containing solution correctness and coded error types, steps used, and strategies employed, to be useful when analyzing results and summarizing individual performance for feedback to teachers. As suggested by Ekenstam and Gregor (1983) the scoring system went beyond noting only declarative knowledge and addressed the students' procedural and strategic knowledge. The present study offered evidence to support the feasibility of such a procedure in day to day practice. Teachers reported feeling very positive on the nature of the data contained in the scoring system, and, as noted, the system was not problematic nor overly time consuming for the raters.

Psychometric Properties of the MWPST

Cronbach and Meehl (1955) advocated the use of many types of evidence to support the construct validity of a proposed test, including content validity, concurrent validity, and item characteristics. Loevinger (1957) also noted the importance of choosing items which were sensitive signs of the traits that were considered significant to the construct being measured. With these points in mind, item were analysized to determine their reliability and validity.

One of the first things that must be considered is the content of the items. Smith and Glass (1987) stated that there should be a logical consistency between the content of the test and the proposed construct. The procedure used in developing the MWPST included pilot testing and reviews by experts in the field of mathematics education and teachers in elementary schools. Coupled with the use of familiar content these reviews indicated that the MWPST had content validity.

Smith and Glass (1987) went on to note the importance of establishing an empirical connection between performance on the proposed test and some other measure that purports to measure the same construct. A correlation between the total test score on the MWPST and the problem solving subtest of the Canadian Test of Basic Skills (CTBS) was in the moderate range (.51) while correlations between total score on the MWPST and teachers' rating of problem solving ability were moderately high (.64 to .72). These correlations indicated that the MWPST had concurrent validity. The CTBS, the MWPST, and teachers' ratings were all assumed to be measures of word problem solving. However, only the MWPST overtly considered students' declarative, procedural, and strategic knowledge in mathematical word problem solving. The correlations between these measures and only the total test scores on the MWPST were more realistically an indication of a relationship among measures of students' declarative knowledge in mathematical problem solving.

Students' performance on items administered using a think aloud procedure was also addressed. Their performance was reflected in the item difficulty and discrimination indexes. Along with providing an indication of how many students responded correctly to an item, item difficulty indexes indicated which items students found easy or difficult. Items seen as too easy or too difficult may cause students to generate few strategies, and result in people overestimating or underestimating students' word problem solving ability.

Either is not a true indication of students' current word problem solving ability. Likewise, items that do not discriminate among students fail to recognize and account for individual differences in mathematical word problem solving. Poorly constructed items that neither discriminate nor differentiate among students raise questions about the validity of the test, the underlying construct, or both.

Application of Classical true-score testing procedures (Gulliksen, 1950), indicated that the test, while needing minor revisions, appeared to be a reliable instrument. The majority of items had difficulty and discrimination indexes that were within acceptable levels according to measurement theorists such as Crocker and Algina (1986).

With regard to the revisions, the reliability and validity of the test could be enhanced by eliminating items that did not discriminate among subjects. However, the ability of the item to elicit strategies must be considered concurrently with item statistics since strategy generation was one of the purposes of the test items. Consideration of the ability of an item to generate students' strategies and processes is a departure from normal item selection procedures as outlined by Crocker and Algina (1986). Typically, statistical procedures are the base for deciding whether an item will be included on a test or not.

However, the purpose of this test was to go beyond gaining only declarative information in the form of right/wrong response patterns and attempt to assess procedural and strategic knowledge. Applying traditional item selection procedures, for example, would mean that item number 8 would be eliminated. However, item 8, on average, elicited more strategies than most items with excellent difficulty and discrimination indexes. While it could be revised it should not be dropped.

This raises the interesting question of whether or not a test can tap qualitative data within the rigour of traditional measurement based test design. Results of the present study appear to offer a positive answer to this question. Applying suggested rigour to item development, constructing items within a theoretical framework and calculating item statistics using Classical true-score test procedures are traditional guidelines for test developers (Crocker and Algina, 1986). Administrating items using a think-aloud method was a departure from standardized administration procedures advocated by traditionalists. However, the item statistics in this study were calculated on items administered using a think-aloud procedure to avoid using a product based assessment instrument to measure process oriented behaviors. While the emphasis was on the ability of the items to tap declarative, procedural and strategic knowledge, gaining additional statistical information on the item allowed the test developer to make more informed decisions on the quality and performance of the items as a measure of the proposed model of word problem solving.

Relationship Between the MWPST and the Model of Mathematical Word Problem Solving in Figure I

Results on the MWPST, administered using a think aloud procedure, revealed a number of findings with regard to students' knowledge of the content area and how they approached and solved word problems. What was observed and how these observed behaviors can be interpreted with regard to their ability to measure declarative, procedural, and strategic knowledge in the proposed model of mathematical word problem solving will now be discussed. Each knowledge area will be considered separately and then their interrelationship will be addressed.

Declarative Knowledge as Measured on the MWPST - Results on the MWPST demonstrated a gradual increase in the number of word problems solved correctly by grade but students made similar types of errors in each grade. However, within each grade there were vast discrepancies in students' ability to solve word problems. When students were grouped according to their raw scores on the MWPST there were obviously significant differences among their performances on the MWPST and among the types of errors they made. Results also indicated that the actual task and the students' attitude toward mathematics had an impact on the students' word problem solving ability.

Reading and computation were controlled in this study and appeared to have little impact on word problem solving for this group of students.

Declarative Knowledge as used in the proposed model of word problem solving is defined by Dillon (1986) and Messick (1984) as knowledge of the content and facts of a discipline It takes into account how a person will respond to a particular task in a certain situation (Kilpatrick, 1975).

Item content in the MWPST was typical of word problems introduced in elementary mathematics textbooks to which students had been exposed and the majority of item difficulty and discrimination indexes were adequate. As well, teacher ratings of word problem solving was in line with performance on the MWPST, Thus, it would appear that the MWPST provided an indication of students' current declarative knowledge in mathematical word problem solving. In addition, the majority of students with high scores on the Attitude Toward Arithmetic Scale also had high scores on the MWPST, indicating, as Silver (1985) suggested, a positive relationship between students' attitudes and performance on word problem solving tasks.

Students' declarative knowledge was influenced by the actual task, a finding often noted in the word problem solving literature (Burns and Yonally, 1964; Muth, 1984; and Quintere, 1982). How students perceived a task also had an impact on their performance. For example, problem number 13 was seen as a simple perimeter problem by the majority of students. This perception resulted in a number of incorrect responses. Brown (1987) and Menick (1987) both noted that problem solvers perceive information in unique ways that influence their performance on tasks.

While rapport had been established with most students prior to beginning the taping of the word problem solving session, the procedure was a departure from their normal way of completing word problems. In the classroom situations, students tended to complete all written requirements for a problem silently. Solving the problems using a think-aloud procedure could have caused students to take more care with their responses, to check their responses, or in some cases to actual read the problem. The think-aloud procedure could also have caused children to be more focused on the task than if they were working silently by themselves. In some cases it may have inhibited performance. However, the actual impact of the situation on students' declarative knowledge may be minimal, as students could not use knowledge they did not possess. The moderately high correlations between teacher ratings of problem solving and results on the MWPST indicated that students' performance was fairly consistent in class and in the testing situation.

Overall, the MWPST appeared to provide an indication of students' current level of declarative knowledge in mathematical word problem solving. Individual performance on the MWPST appeared to be the result of personal characteristics coupled with the word problem tasks being completed using a think-aloud procedure. For example, students with few problems correct (<9) on the MWPST had no difficulty reading items but tended to be passive problem solvers. They completed problems in a routine manner regardless of the task demands. Most of these students also had low scores on the Attitude Toward Arithmetic Scale.

Procedural Knowledge as Measured on the MWPST - A review of students' audio-taped transcripts and written protocols revealed that students at all grade levels generally used four procedures on a consistent bases. While students examined the problem, chose a plan, represented the plan, and carried out the plan, they rarely identified a goal or checked their response. Students with above average raw scores (16 or more correct) on the MWPST tended to identify a goal and some checked their responses, steps rarely used by average (10 - 15 correct) or below average (9 or less correct) problem solvers. The word problem task also had an impact on the number of steps used.

Procedural knowledge as used in the model refer to what Messick (1984) called information about "how" a person solved a problem. Students' verbalizations as they solved the problems coupled with their written work provided an indication of how they proceeded to solve the problem. In this study, students were required to read the problem aloud, ensuring that the first procedural step noted in the model

(Figure I) was always completed whether it was part of the students usual behaviour or not.

Results were similar to those of past studies on how students solved word problems conducted by Lee (1982), Sadowski and McLlveen (1984) and Uprichard et. al. (1986) in that students in their studies also had difficulty choosing a plan to complete a problem correctly. While students may have had difficulty choosing the correct plan, they all had access to a number of plans. However, Soloman (1988) and Schoenfeld (1979) pointed out the limitations of having a plan as opposed to knowing when and how to apply it.

The actual tasks also resulted in differences in how many steps students used to complete word problems. Tasks with minimal computational demands were sometimes completed using only three steps while items designed to tap specific strategies such as trial and error tended to require five steps. Again a relationship between the task, an aspect of declarative knowledge, and the students' procedural knowledge on the MWPST was noted.

Four of the procedures (reads problem, chooses plan, represents plan, caries out plan) noted in the model of word problem solving (Figure I) usually require overt behaviour that can be observed easily, while two of the procedures can be completed mentally. The use of a think-aloud format when administering the MWPST was therefore necessary to obtain more complete information on students' procedural knowledge.

Viewing only written protocols gave no indication of whether students actual identified a goal or used internal verbalizations to check their response. It may be that students used these procedures automatically as Swanson et.al. (1981) suggested. However, students' overt behaviour and the number of steps observed indicated that while students may complete word problems in an automatic, set way they rarely identified a goal or checked their response. Indeed, students, especially those in the average and below average groups, read the problem, immediately began to write down a mathematical equation, and then went on to the next problem when the calculations were complete. The number of errors due to lack of understanding and choosing the wrong operation by students in these two raw score groups also added support to this finding of limited goal identification and checking behaviour when solving problems on the MWPST. Conversely, students who exhibited above average problem solving ability tended to state what had to be found and often checked their responses.

The ability of the MWPST with its pencil/paper/verbal format to identify differences in procedures used pointed to its ability to tap students' procedural knowledge as outlined in the proposed model of mathematical word problem solving.

Strategic Knowledge as Measured on the MWPST - Results on the MWPST indicated a gradual increase in the number and type of strategies used by students by grade. All students had some general strategies that they applied to most word problems. However, there was a significant difference in the number of specific strategies used by students with above average word problem solving ability when compared to those with average and below average word problem solving ability on the MWPST. Few students used metacognitive strategies when solving problems. Finally, the word problem solving task itself effected the number and type of strategies students employed.

Strategic knowledge, as used in the model and defined by Messick (1984), refers to the persons ability to know when and which strategies to apply as well as an understanding of ones own knowledge, strengths and limitations. The model of word problem solving (Figure I) distinguished between two types of strategies - heuristic (cognitive) strategies and managerial (metacognitive) strategies. While both were considered cognitive strategies, managerial strategies were considered higher order strategies in the cognitive continuum.

Again the format of the MWPST allowed for a more complete indication of students' strategy use than the traditional multiple-choice test or test requiring only written work. Strategy use was not assumed, rather students' verbalizations or written implementation of strategies were noted. While students may possess other strategies that were not verbalized, the think-aloud format of the MWPST afforded an opportunity to glean information about current strategy use.

Sternberg (1985) questioned whether or not all thinking was available to conscious awareness due to the role of automaticity. Kirby (1984) suggested that automaticity played a role only in strategies that were well established as opposed to strategies that were to be constructed. In other words, where an activity involved newer challenges building on an existing knowledge base, the issue of automaticity need not rule out students access to strategies. With this in mind, items were constructed to challenge students in familiar content and strategy areas in mathematics yet minimize the impact of automaticity.

The MWPST administered using a think-aloud procedure did provide an indication of students' strategic knowledge in mathematical word problem solving. Students verbalizations and written protocols indicated the use of a variety of strategies throughout the word problem solving test. Students with above average problem solving skills generated a greater number and variety of strategies than those with average or below average problem solving ability. This ability to generate strategies was also evident on their responses to informal questions regarding problem solving. Above average word problem solvers also tended to employ metacognitive strategies. This is in line with Brown (1981) and Gelman's (1986) finding that there is a relationship between the development of content and the development of reasoning about that content.

A few students whose performance on the MWPST was above average used no metacognitive strategies. In fact, a number of students solved problems correctly without the apparent use of metacognitive strategies, indicating that these strategies may not be necessary for solving the word problems on the MWPST. Indeed, Nickerson (1985) and Herron (1975) reported that often adults in college do not demonstrate the use of metacognitive reasoning. However, the MWPST was designed to provide insight into students' current strategies, be they cognitive or metacognitive. Nevertheless, the significant difference in the use of metacognitive strategies by above average problem solvers and their lack of use by below average problems solvers indicated that such strategies do have an impact on problem solving ability. As well as proving insight into individual differences among students, use of metacognitive strategies demonstrated that some students regardless of age or grade had access to higher order strategies, supporting the findings of Dean (1987), Gelman (1986) and Leibling (1988) that cognitive development in not bound by age or stage.

The MWPST did provide an indication of students' strategic knowledge as outlined in the proposed model of word problem solving.

Interrelationship Among Declarative, Procedural, and Strategic Knowledge as Measured on the MWPST - Throughout the analysis of data and from the administration of the MWPST using a think-aloud procedure the interrelationship between students' declarative, procedural, and strategic knowledge in mathematical word problem solving was evident. Students who had more word problems correct tended to use more steps and generated a greater number and variety of strategies than those who had fewer problems correct. A gradual increase in word problem solving ability by grade was noted. The moderately high intercorrelations among problems correct, steps used, and strategies employed also provided support for an interrelationship among students' declarative, procedural, and strategic knowledge as proposed in the model and evidenced on the MWPST.

Evidence of each type of knowledge provided insight into a different but related aspect of the students word problem solving ability. If only one level of knowledge had been assessed, it would have provided a partial picture of the students' word problem solving ability. For example, knowing a student had received a low score on a word problem test did not provide insights into how or why he performed as he did or things that need to be addressed to improve his ability in this area.

The model presented in Figure I emphasized the interaction between the three types of knowledge when solving word problems, implying that each was necessary for successful problem solving. Results on the MWPST appeared to verify that successful word problem solving involved an interaction among a student's declarative, procedural, strategic knowledge in the relevant area. Carey (1985), Gelman (1988), Siegler and

Richards (1982) and Walker (1987) also reached a similar conclusion, noting the significance of acquiring content knowledge along with knowledge of when and how to apply and transfer that knowledge.

The results on the MWPST provided support for the construct of "mathematical word problem solving" outlined in Figure I. Results also indicated that the MWPST administered using a think-aloud procedure could provide an indication of students current level of word problem solving ability as proposed in the model in Figure I. Successful problem does appear to involve all three levels of knowledge.

Summary

The model of mathematical word problem solving developed for this study considered the impact of declarative, procedural, and strategic knowledge on elementary students word problem solving ability. This model was fairly comprehensive, and as previously discussed, incorporated several variables hypothesized to influence word problem solving but only partially addressed in other models of mathematically word problem solving. attempting to verify the model in Figure I through the development of a test and testing procedure is also a departure from previous practices outlined in the problem solving literature.

The assessment instrument, the Mathematical Word Problem Solving Test (MWPST), developed for this study provided an indication of students' current declarative, procedural, and strategic knowledge and support for the underlying model of word problem solving. The ability of the MWPST, administered using a think-aloud procedure, to assess students' declarative, procedural, and strategic knowledge suggested that instruments can be developed to assess cognitive reasoning. Such instruments are needed, as Pressely et. al. (1985) and Meichenbaum et.al. (1986) suggested, to provide a more process orientated approach to the measurement and evaluation of cognitive processing and the effects of cognitive intervention than traditional product oriented tests.

While further research is necessary with the MWPST it did appear to tap declarative, procedural and strategic knowledge in the present sample. While many researchers attempt to get at procedural and strategic knowledge using informal measures (Romberg and Collis, 1985; Uprichard et al, 1986) or taking existing measures (Knifong and Holtan, 1977) and in effect changing the discrimination power of the items and the administration procedure, the MWPST represented an integrated approach to tapping declarative, procedural, and strategic.

Recommendations for Future Research

A number of issues emerged during this study which require further examination. These issues are presented below.

1. The Mathematical Word Problem Solving Test developed for the study is in need of minor revisions. Few students generated a number of different strategies while completing items number 9 and 13. In addition, similar logic problems are introduced in elementary textbooks. Eliminating both of these items because of their difficult level and limited strategy use by the majority of students would eliminate one type of problem. One or both of these problem could be retained but changed to avoid being misinterpreted. Two items - (# 8) requiring drawing a diagram and item (# 14) containing extraneous information - were answered correctly by the majority of students but appeared to tap a number of strategies. These two items should be retained and revised to make them more challenging. Perhaps a better example of a problem tapping visual imagery strategy could also be developed and included. Following these revisions the test should be readministered and item statistics recalculated. Such revisions should increase the reliability and validity of the test.

2. While use of a think-aloud procedure did not appear to effect subjects' performance on the word problem solving test, its impact is not fully known. Future research could address this concern by gathering observational data in actual classroom settings. The additional data would provide another data source for verification of results, a practice advocated by Meichenbaum et al (1986) and Neill and Medina (1989). In addition, an alternate form of the MWPST could be developed. This alternate form of the MWPST could be administered without the think-aloud procedure so that the impact of the think-aloud procedure can be ascertained.

3. The Mathematical Word Problem Solving Test was administered individually to subjects. Teachers may find individual assessment time consuming and inefficient. Future research could address ways of administering process oriented tests to small groups. Attempts by Uprichard et al (1986) to assess strategy/procedure use in groups were so structured that findings were questionable. As well, group assessment is often contaminated by modelling of strategies, during the assessment, that are not part of the students' present repertoire of strategies. Nevertheless, group assessments would be more efficient. Whether group assessments are as effective and informative as an individual assessment is yet to be determined.

4. The test used in this study was designed for students in grades 4, 5, and 6. The viability of the proposed model of word problem solving to develop prototypes of tests at other grade levels requires further study.

5. In the current study examiners and raters hired to assist in the collection and scoring of data were familiar with think-aloud procedures. A more detailed scoring and administration guide may be needed to assist those who are not familiar with process oriented assessment techniques.

6. A sixth issue for future research may be the video taping of students completing the word problem solving test. Video tapes show subjects overt actions, latency times, and reactions to specific word problems. While video taping is even a greater departure from subjects routine way of solving problems, situations could be structured so taping would not be obvious.

7. The present study explained the effectiveness of the proposed model of word problem solving as a guide for developing an assessment device. While further research on the model is necessary, future research should also consider whether the model can also be used as a guide to develop cognitive instructional programs in the area of word problem solving. Assessing subjects' declarative, procedural, and

strategic knowledge in word problem solving is informative. However, information gained from this assessment should be used to enhance future performance or to initiate intervention, if needed, in all the knowledge areas related to word problem solving. Campione and Brown (1987) and Menick (1987) noted the direct link between assessment and instruction, stating that each supported and enriched the other.

8. Coupled with the need to develop a cognitive instructional program based on the model of word problem solving is the need to evaluate the effectiveness of such cognitive instructional program. Too often researchers have relied on product oriented assessment devises to evaluate process oriented programs (Wiggins, 1989). Development of a parallel test of word problem solving performance so that pre or post measures are available to denote improved problem solving performance without having to contend with practice effects when employing the same test for both measures, would be beneficial. Such a test tapping procedural, declarative and strategic knowledge could offer greater insight into the effectiveness of cognitive instructional programs.

9. Data obtained in this study seemed to suggest developmental trends in the acquisition of word problem solving skills. However, longitudinal research would provide insight into the development of word problem solving skills at different age

and grade levels since the present study used cross-sectional methodology making direct year to year comparisons impossible.

10. The reported dearth of knowledge on the affective influence on word problem solving performance needs to addressed (MacLeod, 1988; Silver, 1985). Results of the present study indicated a relationship between students' attitudes toward mathematics and word problem solving performance. However, this only tapped the surface rather than uncover the affective/cognitive link in word problem solving. Affective areas that could be addressed include motivation, self-esteem as it relates to word problem solving performance, and perceived level of control over the problem solving situation.

By way of a conclusion, this study offered a number of insights into the declarative, procedural, and strategic knowledge of elementary students in the area of word problem solving as well as a "ways and means" of assessing procedure and strategy use. Future research in these areas is required to expand the knowledge base in pursuit of more effective teaching and assessment procedures.

VI. BIBLIOGRAPHY

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Appendix A

Strategies, Operations, and Problem Types Introduced in Grade 4, 5, and 6 Textbooks

Strategies emphasized in Grade 4 textbooks:

choosing the correct operation
understanding the problem
organizing information
using cue words
drawing a diagram
reading a chart
using a simple example
trial and error
drawing conclusions

Strategies emphasized in Grade 5 textbooks:

all of the above mentioned, plus

estimating
choosing essential information

Strategies emphasixed in Grade 6 textbook:

all those mentioned under grades four and five, plus

using visual imagery
drawing a graph

Summary of Strategies used in Grades 4,5, and 6:

choosing the correct operation
understanding the problem
organizing information
using cue words
drawing a diagram
reading a chart
using a simple example
trial and error
drawing conclusions
choosing essential information
estimating
using visual imagery
drawing a graph

In all grades and textbooks more attention was given to the first four strategies noted in the summary list. Other strategies received periodic notice at intervals throughout textbooks.

Types of Word Problems in Grade 4,5, and 6 Textbooks:

Grade Four

one-step - problems requiring the use of one operation and one set of calculations, no extraneous information

two-step - problems requiring two sets of calculations and one or more mathamatical operations

reading charts - problems requring the extraction of pertinent information from a given chart

drawing a diagram - problems requiring the student to draw a diagram for a response or to assist in the solution process

logic - problems requiring the drawing of a logical conclusion based on the information given or obtained in the response

Grade Five

All types mentioned under grade four, plus

one-step with extraneous information added to the problem statement

estimating - one-step problems that encouraged the practice of estimating responses

trial and error - problems that require trying different number combinations to obtain a response

Grade Six

All types mentioned under grades four and five, plus

drawing a graph - problems requiring the organization of given information into a graph for the solution

visual imagery - problems that require going beyond the obvious given information and mentally visualizing some aspect of the problem

The majority of word problems in textbooks at all grade levels were one and two step problems with and without extraneous information. The ratio of one-step to two-step problems ranged from 1:3 to 1:5 in favor of one-step problems. Other types received periodic attention.

Mathamatical Operations in Word Problems in Grades 4,5, & 6:

Operation	4	5	6
addition	x	x	x
subtraction	x	x	x
multiplication	x	x	x
division	x	x	x
charting	x	x	x
percentage			x
decimals			x
fractions	x	x	x
graphing			x

In grade four textbooks most of the word problems require application of addition or subtraction operations to arrive at the correct solution.

Grade five textbooks provide more practice in the application of multiplication or division operations to obtain solutions to word problems.

However, grade six textbooks appear to have an even distribution of problems using the four different types of operations (+,x,-, divide).

While charting problems are given limited but equal attention at all three grade levels, other operations receive differential treatment in textbooks at the different grade levels.

Appendix B

Item Discrimination and Difficulty Indexes for the Pilot Test

Item Statistics for Items in the Pilot Study

Item No. Pilot	Item No. MWPST	Difficulty	Discrimination
1	18	.50	.73
2	-	.92	.06 a
3	14	.69	.40
4	2	.46	.54
5	11	.80	.40
6	16	.63	.74
7	6	.37	.62
8	19	.00	.00 b
9	-	.90	.20 a
10	17	.49	.74
11	9	.23	.60 c
12	12	.57	.87
13	-	.76	.20 a
14	3	.63	.40
15	5	.38	.60 c
16	4	.69	.40
17	-	.82	.20 a
18	15	.67	.66
19	10	.46	.94
20	7	.57	.66
21	20	.57	.66
22	8	.86	.40
23	-	.89	.20 a
24	1	.42	.54
25	13	.13	.24 c

a = Items dropped ; b = Item retained ; c = Items revised

Appendix C

Word Problems on MWPST, Strategies, Mathematical Operations

Problem	Strategy a	Operation
1. Jan played 8 records one Saturday. Each record lasted 18 minutes. How long did Jan play her recorder player?	choosing operation	x
2. Ron's grandmother came for a visit. She said her home was 3925 kilometers away. How far must she travel on the complete trip?	drawing conclusion	+ or x
3. Mary had 77 cents. She spent 28 cents, then she found 15 cents. How much money does Mary have?	organizing information	- &+
4. Sam spent $8.67 on video games. What change did he receive from $10.00 ?	choosing operation	-
5. 72 cookies were baked. There were 48 chocolate chip cookies and 24 peanut butter cookies. Each child got 3 cookies. How many children were there?	extraneous information choosing essential information	-
6. One rabbit eats 2 pounds of food each week. How much food will 5 rabbits eat in 2 weeks?	choosing essential info	x&x
7. Jeff has 13 gold fish. He has 5 gold fish less than Jay. How many fish does Jay have?	choosing operation	+
8. Sally is taller than Joanne. Joanne is taller than Lorna. Lorna is shorter than Sally. Draw and label three lines to represent each girl's height.	draw a diagram/ draw a gragh	
9. Eight books fit on each shelf in the bookcase. How many shelves will be needed to place 44 books in the bookcase?	draw a conclusion / logic	-

10. Mr.Power divided 126 hectares of his land equally among his three sons. One son, John, sold 15 hectares of his land. How much land does John have left.	choosing operations context clue	divide & -
11. Ms. Cooper measured the heights of sam and David. Sam was 137 cm. tall, David was 152 cm. tall and Ms. Cooper was 181 cm. tall. How much taller is David than Sam?	extraneous information	-
12. Sid bought 3 packages of peanuts at 50 cents each. He had 12 cents left. How much money did Sid have at first?	choosing operations choosing essential information	x&+
13. Mr. Jones is taking out a wall to make 2 bedrooms that join each other into one large bedroom. The rooms are 4 meters by 3 meters and 4 meters by 4 meters. What will be the perimeter of the new bedroom?	draw a diagram notes essential information	+
14. Tim bought 8 apples at 21 cents each. Oranges are 25 cents each. How much did Tim pay for the eight apples?	extraneous information	x
15. Allen had some cars. He gave 6 cars to Stephen and now he has 12 cars. How many cars did Allen have in the first place?	choosing information choosing operation	+
16. (Chart of store prices) Mark bought his dog a rubber toy, a dog bed, and a dog collar. How much change will he receive from $20.00?	reading a chart choosing operation	+&-
17.Mary has 7 coins. The total value of the coins is 80 cents. What are the 7 coins?	trial and error choosing essential information	

18. Patti had 24 extra crayons. She gave an equal number of her extra crayons to Joan, Bill and Erin. Joan than gave half of these crayons to another person. How many crayons does Joan have left?	choosing operations divide organizing information & -
19. A rectangular tent requires 8 tent pegs along each side and 6 tent pegs on each end. How many tent pegs are required?	using visual imagery +
20. Pete has 7 boy scout badges and Matt has 12 badges. Jay has twice as many as Pete and Matt together. How many badges does Jay have?	choosing operations +&+ organizing information +&x

---

a While all of the problems required choosing the correct operation, understanding the problem, and organizing information, others were designed to tap specific strategies

Word Problem Types on the MWPST:

7 - one step problems - 3 with extraneous information (Number of problem on the MWPST 1,4,5,7,11,14,15)

6 - two-step problem (# on MWPST 3,6,10,12,18,20)

1 - charting - two-steps (# on MWPST 16)

1 - drawing a diagram/graph (#8, and helpful for #13 on MWPST)

1 - trial and error (#17 on MWPST)

3 - logic (#2,9,13 on the MWPST)

1 - visual imagery (# on MWPST 19)

--
20

Appendix D

Mathematical Word Problem Solving Test

Mathamatical Word Problem Solving Test

Name____
Grade_______ Age_______ Date of Birth____________________

Instructions:

1. Listen to the examiners instructions.
2. Complete sample problems on this page.
3. Turn page and begin the test.
4. Read each word problem carefully.
5. Include all your written work in the space provided.
6. Attempt all the word problems.

Sample Word Problems:

-Example 1:

Paul has 234 stamps in his stamp book. He buys 54 more from a dealer. How many stamps does he have now?

-Example 2:

Mary placed 18 running shoes into boxes. Two shoes were placed in each box. How Many boxes were used?

1. Jan played 8 records one Saturdey. Each record lasted 18 minutes. How long did Jan play her record player?

Answer ____________

---

2. Ron's grangmother came for a visit. She said her home was 3925 kilometers away. How far must she travel on the complete trip?

Answer ____________

---

3. Mary had 77 cents. She spent 28 cents, then she found 15 cents. How much money does Mary have?

Answer ____________

---

4. Sam spent $8.67 on video games. What change did he receive from $10.00 ?

Answer ____________

5. 72 cookies were baked. There were 48 chocolate chip cookies and 24 peanut butter cookies. Each child got three cookies. How many children were there?

Answer ____________

---

6. One rabbit eats 2 pounds of food each week. How much food will 5 rabbits eat in 2 weeks?

Answer ____________

---

7. Jeff has 13 gold fish. He has 5 gold fish less than Jay.How many fish does Jay have?

Answer ____________

---

8. Sally is taller than Joanne. Joanne is taller than Lorna. Lorna is shorter than Sally. Draw and label three lines to represent each girl's height.

9. Eight books fit on each shelf in the bookcase. How many shelves will be needed to place 44 books in the bookcase?

Answer ______________

10. Mr. Power divided 126 hectares of his land equally among his three sons. One son, John, sold 15 hectares of his land. How many hectares does John have left?

Answer ______________

11. Ms. Cooper measured the heights of Sam and David. Sam was 137 cm. tall, David was 152 cm. tall and Ms. Cooper was 181 cm. tall. How much taller is David than Sam?

Answer ______________

12. Sid bought 3 packages of peanuts at 50 cents each. He had 12 cents left. How much money did Sid have at first?

Answer ______________

13. Mr. Jones is taking out a wall to make 2 bedrooms that join each other into one large bedroom. The rooms are 4 meters by 3 meters and 4 meters by 4 meters. What will be the perimeter of the new bedroom?

Answer ____________

14. Tim bought 8 apples at 21 cents each. Oranges are 25 cents each. How much did Tim pay for the 8 apples?

Answer ____________

15. Allen had some cars. He gave 6 cars to Stephan and now he has 12 cars. How many cars did Allan have in the beginning?

Answer ____________

16. PET SHOP SUPPLIES

Rawhide Bones ___	$1.29	Dog Bed _________	$8.09
Dog Collar ______	$4.78	Rubber Toys _____	$1.25
Flea Collar _____	$2.99	Dog Dish ________	$3.50

Mark bought his dog one rubber toy, a dog bed , and a dog collar. How much change will he receive from $20.00 ?

Answer ____________

17. Mary has 7 coins. The total value of the coins is 80 cents. What are the 7 coins?

Answer ______________

---

18. Patti had 24 extra crayons. She gave an equal number of her extra crayons to Joan, Bill and Erin. Joan than gave half of these crayons to another person. How many crayons does Joan have left?

Answer ______________

---

19. A rectangular tent requires 8 tent pegs along each side and 6 tent pegs on each end. How many tent pegs are required

Answer ______________

---

20. Pete has 7 boy scout badges and Matt has 12 badges. Jay has twice as many as Pete and Matt together. How many badges does Jay have ?

Answer ______________

Appendix E

Summary Sheet and Example of Scored Protocol

SUMMARY SHEET

Name ______________________ Grade _______ Age ______

Problem	R/W	Steps	Strategies	Error(s)
- 1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				

Scoring Procedure:

Using student's test protocol and transcript of their verbalizations:

1. Look at each problems separately.

2. Note if response is correct or not and indicate under R/W column on the Summary Sheet

3. If wrong, note code for type of error under Error on the Summary Sheet. Codes for errors are in Table III.1.

4. Review written work and transcripts for of different steps and the sequence in which they were used. Record the code number for the step sequence under Steps on the Summary Sheet. Codes for step sequences are in Table III.2

5. Review written work and transcripts for evidence of different strategies employed to answer each problem. Record the code number under Strategies on the Summary Sheet. Codes for strategies are listed in Table III.3.

SUMMARY SHEET

Name 095 Grade 5 Age 10-11

Problem	R/W	Steps	Strategies	Error(s)
1	x	1	1,4,5,24,39,24	2
2		1	1,3,4,5,6,13,29,39,24,10	
3	x	1	1,3,4,5,39,21, 24,	2 & 6
4		1	1,4,5,6,10,39,15,24,	
5	x	1	1,3,4,5,11,28, 24,	2 & 7
6	x	1	1,3,4,5,11,24,38,23,15,24	2 & 6
7		1	1,4,5,6,10,11,39,15,24	
8		1	1,5,19,8,16,10	
9	x	1	1,4,5,6,39,24	9
10	x	1	1,4,5,39,24,15,11,28,24	2 & 6
11		1	1,4,5,6,10,9,39,24,15,32	
12	x	1	1,4,5,15,39,	2 & 6
13	x	1	1,4,5,39,15,	7
14		1	1,4,5,6,9,10,39,24,15,37	
15		1	1,4,5,6,10,39,24,15	
16		1	1,4,5,6,7,8,9,10,33,39,28,15,24	
17		1	1,4,5,10,11,24	
18	x	1	1,3,4,5,11,39,23,24	2 & 6
19	x	1	1,3,4,5,39,24,15	7
20	x	1	1,4,5,39,24,15	6

Student Number 095

1. Jan played 8 records one Saturdey. Each record lasted 18 minutes. How long did Jan play her record player?

```
 1
 18
  8
 26
```

Answer 26

---

2. Ron's grangmother came for a visit. She said her home was 3925 kilometers away. How far must she travel on the complete trip?

```
1 1
3925
3925
7850
```

Answer 7850

---

3. Mary had 77 cents. She spent 28 cents, then she found 15 cents. How much money does Mary have?

```
   1
   28
 + 15
   43
```

Answer 43

---

4. Sam spent $8.67 on video games. What change did he receive from $10.00 ?

```
 09 9
$10.00
$ 8.67
$ 1.33
```

Answer $1.33

5. 72 cookies were baked. There were 48 chocolate chip cookies and 24 peanut butter cookies. Each child got three cookies. How many children were there?

```
  72
+ 3
  75
```

Answer 75

---

6. One rabbit eats 2 pounds of food each week. How much food will 5 rabbits eat in 2 weeks?

```
  2
X 2
  4
```

Answer 4

---

7. Jeff has 13 gold fish. He has 5 gold fish less than Jay. How many fish does Jay have?

```
  13
+ 5
  18
```

Answer 18

---

8. Sally is taller than Joanne. Joanne is taller than Lorna. Lorna is shorter than Sally. Draw and label three lines to represent each girl's height.

```
____________________ S

________________ J

_____________
```

9. Eight books fit on each shelf in the bookcase. How many shelves will be needed to place 44 books in the bookcase?

```
      0 5 R4
  8 ) 44
      0
       44
       40
        4
```

Answer 5 R4

---

10. Mr. Power divided 126 hectares of his land equally among his three sons. One son, John, sold 15 hectares of his land. How many hectares does John have left?

```
   1
  126
   15
  141
```

Answer 141

---

11. Ms. Cooper measured the heights of Sam and David. Sam was 137 cm. tall, David was 152 cm. tall and Ms. Cooper was 181 cm. tall. How much taller is David than Sam?

```
   4
  152
  137
   15
```

Answer 15

---

12. Sid bought 3 packages of peanuts at 50 cents each. He had 12 cents left. How much money did Sid have at first?

```
  50
  12
  62
```

Answer 62

13. Mr. Jones is taking out a wall to make 2 bedrooms that join each other into one large bedroom. The rooms are 4 meters by 3 meters and 4 meters by 4 meters. What will be the perimeter of the new bedroom?

```
 4
 4
 4
 3
15
```

Answer 15 m

---

14. Tim bought 8 apples at 21 cents each. Oranges are 25 cents each. How much did Tim pay for the 8 apples?

```
   21
   x8
$1.68
```

Answer $1.68

---

15. Allen had some cars. He gave 6 cars to Stephan and now he has 12 cars. How many cars did Allan have in the beginning?

```
12
+6
18
```

Answer 18

---

16. PET SHOP SUPPLIES

Rawhide Bones	$1.29	Dog Bed	$8.09
Dog Collar	$4.78	Rubber Toys	$1.25
Flea Collar	$2.99	Dog Dish	$3.50

Mark bought his dog one rubber toy, a dog bed , and a dog collar. How much change will he receive from $20.00 ?

```
   1 2            19 9
$ 4.78         $ 20.00
  8.09         $ 14.12
  1.25         $  5.88
$14.12
```

Answer $5.88

17. Mary has 7 coins. The total value of the coins is 80 cents. What are the 7 coins?

1 quarter
5 dimes = 50 cents
1 nickel 75
5
80

Answer ________

18. Patti had 24 extra crayons. She gave an equal number of her extra crayons to Joan, Bill and Erin. Joan than gave half of these crayons to another person. How many crayons does Joan have left?

1
24
08
16
8
8

Answer 8

19. A rectangular tent requires 8 tent pegs along each side and 6 tent pegs on each end. How many tent pegs are required

8
6
14

Answer 14

20. Pete has 7 boy scout badges and Matt has 12 badges. Jay has twice as many as Pete and Matt together. How many badges does Jay have ?

12
7
19

Answer 19

Appendix F

Attitude Toward Arithmetic Scale

Attitudes Towards Arithmetic

Name________________________
Date____________________

Check (X) only the statements which express your feeling toward arithmetic.

1. I feel arithmetic is an important subject. ____
2. Arithmetic is something you have to do even though it is not enjoyable. ______
3. Working with numbers is fun. ______
4. I have never liked arithmetic. ______
5. I like arithmetic better than any other subject. ______
6. I get no satisfaction from studying arithmetic. ______
7. I like arithmetic because step2 follows step 1. ______
8. I am unsuccessful at working word or story problems.______
9. I like working all types of arithmetic problems. ______
10. I dislike arithmetic and never use it outside school. ______
11. I like arithmetic better than I did because I see how I can use it in every day life. ______
12. I have no feelings toward arithmetic one way or another. ______
13. I like arithmetic because it makes me think. ______
14. I like arithmetic but I like other subjects just as well. ______
15. Getting an answer and proving it in arithmetic makes me feel good. ______

Statements 1-15 appeared in: Damer, Dlass (1970). Readings in Mathematics . Boston: Allyn and Bacon.

Assigned Values for Items on the
Attitude Toward Arithmetic Scale

Item Number	Assigned Value
1	7.2
2	3.3
3	8.7
4	1.5
5	10.5
6	2.6
7	7.9
8	2.0
9	9.6
10	1.0
11	8.2
12	5.2
13	9.5
14	5.6
15	9.0

Higher values are assigned to positive statements.

Appendix G

Testing Procedure

PROCEDURE DURING INDIVIDUAL TESTING

1. Be relaxed and friendly with the student.

2. See that the student is comfortable and at ease.

3. Present the cloze reading passage to the student.

4. Explain that you want him/her to read the passage aloud and to verbalize what he/she is thinking about as he/she fills in the responses. Use coaching and probing where necessary.

5. Present MWPST to student.

6. Have student fill in name, age, grade, date of birth and read the instructions.

7. Student completes sample problems on cover page using think aloud procedure.

8. Turn on tape recorder and begin at problem 1. During testing, tell students any words they do not know. Encourage students to verbalize or continue working but avoid statements that offer insight into problems or interfere with their verbalizations. Respond to querries for help or confirmation indirectly: Ex. "What do you think?", "Do what you think is best!"

9. When the student is finished, ask him/her the following questions, in an informal manner.
A. What is your favorite subject? Why?
B. Do you like math?
C. Do you think you are better at problem solving than other people in your class? Why?
D. What kinds of things might make you a better problem solver?
E. What do you do if you are given a word problem and don't know how to solve it? What are ways to figure out the answer?

10. Thank the student.

11. Label each tape with the student's name, grade and school.

Appendix H

Samples of Students' Transcripts

Evidence of strategies such as 2,4,5,6,7,9,10,16,26 was gleaned from written protocals.

S = Student E = Examiner

S - 147 - Gr 4

S (Reads #1).(1) I have to multiply. (39)
E (Prompts to verbalize).
S I'm multiplying cause that's what I have to do. Cause if I did something else it wouldn't work out right. (37)

E Why do you have to multiply?
S To find out what the answer is. Why do you have to multiply! That's a hard question.

E What are you going to multiply?
S Going to multiply how long did Jan listen on her record player.

E That's what you want to find out?
S Yah.
E What are you actually going to multiply?
S The number of records times ... multiply by how long each record took. (39)

E OK. Go ahead and do that if you think that's what you should do.
S The answer is 144 minutes. (24)
E Finished? #2

S (Reads #2).(1) I have to find out the answer by doubling so (13) that would be 3925 multiplied by 2 (39) to find out the correct answer. This is a hard one! The answer is 7850. (24)

E O.K. Finished? #3

S (Reads #3). First I subtract, then I add. (39)
E Why?
S Because first it tells me she spent, spent means she spent, that's the key words (15) to subtract. How much (15) is the key words to add. So I have 77 subtract 28 which is ... 49. Then I have to add 15 ... that's 64. My answer is 64 cents. (24)

S (Reads #4). Subtract (39) because it said what change did he receive and that means what change from 10 dollars, so that would mean to subtract from the actual $8.67.(21) I subtract ... (whispers).

E You're subtracting now?
S Yes. Move my decimal ... the answer is $1.33. (24)

E #5
S (Reads #5). (Repeats "Each child gets 3 cookies). (32) I don't know what I have to do in this! Oh, I have to divide 72 into 3, 72 into 3. (39) Why? (23) Because each child got 3 cookies, and there were 73 cookies and the key word were there were "how many were there".(15) So that tells me in my mind to divide.

E How many what were there?
S How many children were there, (12) so that tells me to divide. So, how many 3's are in 7, 2 so that means 2 down here. So that means 20 up here. So I multiply 3 times 20 and that gives me 60, then subtract that would give me 12. How many 3's in 12? 4 3's in 12. Subtract 12 from 12 and I have 0. So I put 24 here. That would be 24 children. (24)

E #6
S (Reads #6) So, oh oh! That's a hard one.

E (Prompts to verbalize).
S I'm thinking that 2 pounds each week. (32) I'll have to multiply and multiply again.(39) There is 2 lbs. each week. 2 lbs. in 2 weeks is 2 x 2, so that equals 4, and there's 5 rabbits. So I have to multiply 4 by 5 is 20. My answer is 20. (24)

S (Reads #7) Tells me to add (39) because "how many" fish does Jay have. (12) So "how many" (15) is the key word in my mind that tells me to add. 13 add 5, I'm adding the # of Jeff's goldfish then I add the 5 extra ones Jeff has. (39) My answer will be 18. (24)

S (Reads #8) Rereads "Sally is taller than Jo". (32) (8) So this is Sally. This is Jo, or Joanne. Lorna is shorter than Sally,(32) (8) so this is Lorna. There is no answer there because I drew a picture for the answer.

E #9
S (Reads #9)
S Tells me to divide (39) because the key words (15) are "how many shelves will be needed". (32) 8 divide by 44. I can't divide 8 into 4 - so I have to figure out the closest number to 44, in the 8 times table, so I go over it. (29) 8 x 1 is 8, 8 x 2 is 16, 8 x 3 is 24, 8 x 4 is 32, 8 x 5 is 40 and I will stop there. That means there's 5 8's.

Subtract 40 from 44 and I am left with 4. Can I divide it again? (23) No! So my answer is 5 with a remainder of 4. (24)

S (Reads #10 and says "What's a hectare" (31) to which E explains). I have to divide then I have to subtract. (39) I have to divide the 3 sons, it says equally (15) among his 3 sons, so that means I have to divide equally among the 3 sons. So that means 3 divided by 126.
This is going to be very hard (laughs). How many 3's are in 126? How many 3's are in just 12? (23) There are 4 three's in 12, 4 threes. So I subtract 120 from 126 and am left with 6. Two more things, 2 more hectares can be given to each son, so that will answer 0.
Now why I have to subtract is (reads) "One son, John, sold 15 hectares of his land. How many hectares does John have left?" (32)

S Those are the key words, (15) in my mind, to tell me I have to subtract, "have left". That means each son gets 42 hectares. So I have to subtract 42 from 15, and I am left with, just wait a second, (29) 27. The answer is 27. (24) I am finished.

S (Reads #11). I have to subtract (39) because the key words (15) are how much taller is David than Sam. That tells me in my mind I have to divide ... (11) subtract. I'll subtract 152 from 137 and my mind tells me that I have left ... 15 cms. So David is 15 cms taller. (24)

E O.K. #12.
S (Reads #12). First I have to multiply, then I have to subtract. (39) Why? (23) Because Sid bought 3 packages, 50 cents each. (32) That means 3 packages of peanuts at 50 cents. (21) Those are my key words (15) to multiply. So I multiply 50 multiplied by 3 and I get 150 or $1.50. Then I have to subtract $1.50 subtract by 12.
E Why do you have to subtract?

S Why I have to subtract? "How much money did he have at first" (32) means I have to subtract. (38) Oh! I don't have to subtract. I just have to multiply but I'll subtract anyway it tells me he'll have $1.38 left.

E You didn't tell me why you're subtracting.
S But I had to subtract because I didn't have to subtract in the first place. I thought I have to subtract to find out how much money he had now and how much money he had at first. (12) But it just says how much he had at first. So the answer is $1.50 and I'm finished. (24)

S (Reads #13). Perimeter will be (11)... I have to add. (39) Why I have to add? (23) Because it says "What will be the perimeter" (32) and the perimeter, I know, is the distance around. So I have to find out what is it, what will be, (12) so I have to add. The numbers are 4, 4, 4, 4, and 3 ... that will give me 15. So the answer is 15. (24)

S (Reads #14). I have to multiply. (39) Why? (23) Because it says "How much did Tim pay for the 8 apples?" (32) I know each apple equals 21 cents. So how much did Tim pay tells me I have to multiply 8 times 21 or 21 times 8 (12)... turn it around ... he paid $1.68. (24)

S (Reads #15). I have to subtract then I have to add.(39) Why? (23) It says "How many cars did Allen have in the beginning?" (32) and I know he gave 6 cars to Steven and now he has 12 cars. (21) So what I have to do is subtract 12 from 6 and that'll give me 6. Then I have to add 6 onto 12 to give me 18. My answer is 18. (24)

E Why did you add?

S I had to add because it said "How many cars" and that gave me the key word, (15) "how many cars" and "in the beginning" were the key words to add.

S (Reads #16), (reads $20.00 as $2.00 with subsequent correction by E).

I will add then I will subtract. (39) Why? (23) Well, I have to see how much a dog bed, a rubber toy and dog collar cost altogether then subtract that amount from $20.00. (39)

So the dog bed is $8.09. (8) The dog collar is $4.78, (8) the rubber toy is worth $1.25. (33) I add these up to give me 22 carry the 2. Add 2 onto the 7 which is 9, then add 2 again which is 11. Put down my decimal, carry the 1, 1 add 8 is 9. 9 add 5 is 14. The total is $14.12.

I subtract $20.00 from the total $14.12 and I get ... 5 dollars and 88 cents change.

S (Reads #17). I have to divide.(39) Why? (23) Tells me how many coins I have and I have to divide the number of coins. The amount of the money divided into the number of coins.

7 into 80, will be 10, 70 subtract 10. Then I have to ... one more is 7 cents (whispers) and ...(11)

E (Prompt to verbalize).
S I'm thinking that I did this question right so I have 7 dimes then I have to take away one 5 cents to give me 5, then take away 2 five cents to give me 10.
That would be 7 coins, 7 tens. If I had 6 coins that would be 60. Oh Oh! I did this question wrong, don't have to subtract.(28) I don't have to subtract, multiply, divide or add. I don't have to do anything. (49)

All I have to do is figure out what are the 7 coins. (12) I do have to do something, I have to add. Why? I have to add the 7 coins to see if they total 80. What I have to do is make a graph of 7 coins (16) ... 1, 2, 3, 4, 5, 6, 7, then add them. (39) (49)

This could be a 5 cent, a 25 cent which is 30, then 4 dimes to total 70 cents. I added the coins in my head. 5 add 25 is 30, add 40 is 70. (20) Oh! My total is 80. Oh my goodness. (27)

Let's see, let's see. (29) What if I added on extra ten? (20) No, I have to add 2 25's, give me 50. 10 and 20, no.

This is a really hard question and I don't think I can do it. We did something like this before and it took me a long time to do it.

E Would you like to come back to it? (36)
S Yes.
E Try #18.
S (Reads #18). I have to divide then subtract. (39) Why? (23) I have to divide 24 extra crayons among Joan, Bill and Erin, the 3 kids. (13)

So I have to divide 3 into 24 to give me ... 8. Why did it give me 8? Because I can't divide 3 into 2 so there has to be an equal. (15) equal. equal amount. So I use my 3 x table, I worked my way up from 3 times 7 because I know off by heart it's 21.

So I add 3 more to get 3 x 8 = 24. So I put 3 x 8 = 24 subtract it gives 0. Each child gets 8 crayons. Then Joan gave half her crayons to another person so I have to subtract, 8 crayons for each children, 8 crayons for Joan. Half of 8 is 4, I knew that in my mind. My answer is 4. (24)

S (Reads #19). So I think I'll draw a picture to get my mind going. (16) A rectangular tent needs 8 pegs. (32) Sometimes I draw a picture when my mind is boggled. (48)

8 tent pegs along each side, and 6 tent pegs. There's 9 pegs here 1, 2, 3, 4, 5, 6, 7, 8. 8 pegs here, and 6 pegs here 1, 2, 3, 4, 5, 6 and 1, 2, 3, 4, 5, 6.

I have to multiply (39) I think, (23) multiply to take a short cut instead of adding 8 add 8 add 6 add 6. No, I won't multiply, I won't. (17) I'll add. Why?

Because add 8 add 8 add 6 I want to find out how many pegs are required. How many (15) are the key words to tell me to add. That's what I'll do. (29) Add 8 add 8 add 6 add 6.

That gives me 16 and 12. In my mind I'm adding the total of 8 + 8 is 16, so I add 6 + 6 is 12. The total is 20 ... 32. My answer is 32. (24)

E Are you finished #19? Go on to #20.

S (Reads #20). I have to add then multiply. (39) Why? (23) Because I have to add how many badges does Jay have, that tells me to multiply. The key words (15) that tell me to add are "Pete and Matt together". (32) I know Pete has 7, Matt has 12. So I have to add to get 19. Then I have to multiply 19 by 2 to find out how many badges does Jay have. (12)

My answer is 38 badges. (24)

E Are you finished? Why don't you go back to #17.
S I'll draw a picture, (16) 7 coins. The total I'll just leave over here, 80 cents. (10) What if 3 of them, (20) no 5 of them were dimes. Put 10, 10, 10, 10, 10 to give me 50 cents.

So I have to subtract now 80 from 50 to leave me 30, so I need 30 cents left in 2 coins. (46) What could they be? (23) I know, a quarter and a nickel.

My answer is 5 dimes, one quarter, and one nickel. Thank-you. (24)

E Thank-you. I want to ask you a few questions. Tell me what your favorite subject is.

S Math.
E Oh, it is. Why?

S Because it's in my father's family. My pere used to be a mathematician and he used to make mint coins.

E OK. Why do you <u>like</u> math?

S It's fun, keeps your mind going .. what's that word I'm trying to think of. It keeps your mind in the world.

E I assume you find math easy.

S Sometimes. If it's something we did before like to review a page of multiplication then it's fun and easy.

If it's something new I find it not fun and easy but more complicated and challenging.
E Do you think you are a better problem solver than any other person in your class?

S Sometimes, at something I'm good at I might think of myself as pretty good.

If it's something they can do really good and I can do a little bit worse than them, then I don't think I'm pretty good at that. But I am still good. One day we had a hard math page and I got everyone wrong. I just broke apart, that never happened in my life.

E That happens sometimes. What kinds of things might make you a better problem solver. (Repeats)

S The teacher does it the way I like. I don't like expanded or stacked. We just did like estimating. Maybe if I worked more on my estimations I might be better.

E Word problem question.
S I would either do 1 or 2 things. If it was a word problem I'd read it over and over and over and over again until I had it in my mind. That's what I would do to solve it. Or I would draw a picture (like #17) to make me understand and solve it.

S = Student E = Examiner

S - 151 - Gr 4

S (Reads #1). Well it looks like a multiplication question because it's 8 x 18. 8 x 8 = 64. I put down ... put down 4, carry 6, 8 x 1 = 8, 9, 10, 11, 12, 13, 14. The answer is 144 minutes.

S (Reads #2). You're gonna have to add 2 times because you have to go and come back. Add 3925 plus 3925. 5 + 5 = 10 carry the 1, 2 + 2 = 4, plus 1 is 5, 9 + 9 = 18 carry the 1, 3 + 3 = 6 and 1 is 7. 7850.

S (Reads #3). First you have to subtract, 77 take away 28. From that answer we'll add 15. So 7, borrow from ten's space, 17 - 8, 9. 6 - 2 = 4.

E Why subtract.
S They are taking away 28 cents to buy candy. She had 77 cents and she took away 28 so there will be 49 left so she has to add 49 + 15. 49 + 15, 9 + 5 = 14 carry the 1, 4 is 5 and 1 is 6.

E Why add 49 to 15?
S Because she had 49 cents left then found 15 so she had 64 cents left.

S (Reads #4). Since 8.67 on video games and he had $10.00 so it's gonna be subtracting because he had $10.00 and he spent $8.67, so 10 dollars take away 8.67. You have to borrow from, one from this question because there's only 0 and you can't subtract it. Take away 1, that's a zero. That becomes a 10, cross it out and make it a 9. That becomes a 10, cross out that 10 and make it a 9, and that becomes a 10. 9 - 7 = 3, 9 - 6 = 3, 9 - 8 = 1 and that's the answer. 131 change, 133 change.

S (Reads #5). There was 72 cookies, 48 choc. chip and 24 p. butter. It's gonna have to be a divided by question because 72 divided out of 3 because there's 3 children who each split up 72 cookies, so ...

No, I'm gonna find out how many chidren were there and each child <u>got</u> 3 cookies. So, um I'm gong to divide it by 72 out of 3.

I'll do one way, how many 3's in 7, 2. Put that up there. 20 x 3 = 60. Add that. 2 + 0 = 2. 7 + 6 = 13. Can't do this. 7 - 6 = 1, that = 12. How many 3's in 2 12. Well, 3 times something = 12, that = 4.

Put the 4 here, no. Put 12 there and the 4 up here. Subtract to 0. There's 24 children.
S (Reads #6). That'll be a multiplication sentence because its 2 x 5 because there are 5 rabbits and each does 2 lbs. 2 x 5 = , oh 2 weeks! There will be 5 rabbits in 2 weeks. That'll be 5 rabbits x 2 = 10.

Then, um. 10 x 2 because that's how many lbs. 10 rabbits eat in 2 weeks. 10 x 2 = ... 2 x 1 = and 2 so 20 lbs. for each 5 rabbits in 2 weeks.

S (Reads #7). This is a subtracting sentence because they are comparing. So Jeff has 13 take away 5 = 12, 11 ... 8. 8 fish Jay has so Jeff has 8 fish.

S (Reads #8). So, Sally will be tallest (repeats) and Joanne middle, Lorna is smallest.

S (Reads #9). This is a divided by sentence because you take how many 8's are in 44. How many 8's in 4 ... 5. 5 and a remainder of 4. The answer is 5 with a remainder of 4.

S (Reads #10). First divided by 3 out of 126. Because um we have to find out how many each son has of hectares. So 3 get 126. How many 3's in 126. 40 ...

E (Verbal prompt)
S I'm trying to think how many 3's are in 126. I know there are 4 in 12. Six, so that's 42. Each child has 42 hectares. Since each has 42 we have to take away 15 because John sold 15.

42 - 15, cross out 4 makes it 3, 12 - 5 = 7. 3 - 1 = 2. John has 27 hectares.

S (Reads #11). It looks like David is taller than Sam so and David is 152 cm, put down that because he is taller. Sam was 137 cm, put down that. This is a subtraction question because we're trying to find out how much David is taller.

So take 1 out of 5 = 4. 12 - 7 = 5. 4 - 3 = 1, 1 - 1 = 0. So David is 15 cm taller than Sam. The answer is 15 cms.

S (Reads #12). There is 3 packages and 15 cents. So ... divided 3 out of. How many 3's in 5 = 1. Take away 30, 5 - 3 = 2, 0 - 0 = 0. How many 3's in 20? 3, 6, 9, 12, 15, 18. 18, remainder 2. How many 3's in 18? 6.
E Why are you dividing?
S Because I'm trying to find out how much money Sid has. 16 with a remainder of 2. So, he had ...
E (Verbal Prompt)
S I'm thinking oh! I just add up 16 and 2. 16 right here,

I'll add the remainder of 2. He had $1.62.

E Why add 16 + 2?
S Because (pause) (sigh)
E I'm not saying it's right or wrong I just want to know why you add the 16 and the 2.

S Well, that was a dollar and that was a six. I thought in my mind that was a 60, that was 100 and that was a 2, so I just said $1.62 so that was the answer. $1.62.

S (Reads #13). I'll just add 4, 3 and 4, 4 because those are the meters in the room. 4 and 4 is 8 = 3 is 11 and 4 is 15. So 15. There is usually 100, well ... oh. So ... 15 ...

E 15 what?
S Meters. So the answer is (writes 15)
E Finished?
S (Reads #14). Multiply 8 x 21 because you're sort of adding 8 21's so 8 x 21. 8 x 21. 8 x 1 = 8 and ... 1 x .. uh, 2 x 8 = 16 and he spent $1.68 for the apples.

S (Reads #15). I'm gonna add 12 + 6 because he gave he had 6 and 12 cars so I just add. 6 + 2 is 8 and 1 is 18 so altogether he had 18 cars in the beginning.

S (Reads #16). Up here he has all kinds of things how much they cost. So I'll add the prices of 1 dog bed that's &8.09, 1 rubber toy that's $1.25 and a dog collar that's $4.78.

Once I add all these up I'm just gonna subtract $20.00 out of this answer. 9 + 5 = 4 and 8 is 22, put the 2 there carry the 2. 7 + 2 is 9 and 2 is 11, carry the 1. 8 + 1 is 9 and 1 is 10 and 4 is 14.

Now I'm gonna subtract $20.00 out of $14.12 to find out how much change he'll have. Take one out of the 2, that's becomes a 10 cross it out and make a 9. That becomes a 10 cross it out and make that a 9, and that becomes a 10. 10 -2 = 8, 9 - 1 = 8, 9 - 4 = 5, and 1 - 1 = 0. So how much change he had is $5.88, dollars and 88 cents.

S (Reads #17). Well, it can't be 7 dimes because that would be 70 cents, one is gonna have to be a quarter. I'll put down 25 cents right there. And he'll have maybe another ... now ... dime so that's ten cents. If I add 6 more dimes it would be 85 cents so that can't be right so I'll put a 5 cent here and 5 dimes. Now, let's try to add all these up to make 80 cents.
5 + 5 is 10, put down 0 carry 1. 2 and 1 is 3 and 1 is 4, 1 is 5, 1 is 6, 1 is 7 so I did something wrong ... no I didn't I didn't put another 10 cents there, sorry, cause I only had

4 dimes. That is 1, 2, 3, 4, 5, 6, 7, 8 so that makes 80 cents. The 7 coins are a quarter, a dime no 1, 2, 3, 4, 5 dimes and 1 nickel. That makes 7 coins.

S (Reads #18). Well, Patti had 24 extra crayons nad there's 3 people so we'll divide it by 24 outa 3. So 3 outa 24. How many 3's in 24, that's 12, 15, 18, ... 3, 6, 9, 12, 15, 18, 21. 8 three's in 24, so the answer is they gave each child 8 crayons. Joan then gave half of her crayons which is 8, take away 4 cause that's half her crayons, to another person. So 8 take away 4 = 4. That's how many Joan had left, 4 crayons.

S (Reads #19). Well, there is ... each side so there's, in a rectangular shape there's 4 sides, in each side there's 8, so 2 sides so there's 2 8's, 8 + 8 = 16. Now add 2 sixes to 16. 6, 6 and 6 is 8, carry the 1, 28. There's 28 pegs required. The answer is 28.

S (Reads #20). Well, you have to add Pete and Matt's badges to see how much Jay has because he has more than both of them required ... both of them together I mean. So 7 + 12, 7 and 2 is 9 carry the 1, and 1 that's 19. So ... he has twice as many as Pete and Matt so we have to add another 19 to find out. 19 + 19 is 9 + 9 = 18 put down 8 carry 1, 1, 1 and 1 = 3. So the answer has to be 39 badges because Jay has more, 38 badges. Jay has 38 badges.

E Why 38?
S Because he twice more than Pete and Matt together so you had to add 2 19's because that's how much Matt and Pete had together. The answer is 38 because that's how much 19 and 19 is.

E Favorite subject?
S Math because I like it a lot. I like working with numbers so I like it more than any other subject.

E Why?
S I just find it that way.

E Better problem solver than others?
S Sometimes when I'm doing good and really whizing through math problems but no altogether ... some people are better than me.

E Things to improve?
S I could study all the numbers and multiplication facts more.
E Problems you couldn't solve?
S I would go over it a couple of times then ask the teacher and she could do a few things to help me understand it.

E Anything else?
S No.

S = Student E = Examiner

S - 074 - Gr 5
S. whispers a bit when solving by operations, does a lot in her head.

S (Reads #1). Each record lasted 18 minutes, that's 18 times 8. 8 x 8 = 64, 8, 9, 10, 11, 12, 13, 14 ... 144.

S (Reads #2). She has to go back so you count going back too? 3925 x 2 ... 10 ... see I'm just figuring what to put there.

S (Reads #3). 77 take away 28. (Pause) 64. She has 64 cents left.

S (Reads #4).

S (Reads #5). 72 ... 72, 48, 2, 3, 4, 8 ... the answer is 20 ... 48 children ... 48 children.

S (Reads #6).

E (Prompts to verbalize)
S I'm thinking ... is this hard or is this going to be easy. 2 lbs each week, how much food will 5 rabbits eat in 2 weeks. 2 x 2 ... 4 lbs, for 2 weeks ... 4 x 5 = 20, 5 rabbits eat 2 lbs a week. This is hard! (erases)

O.K. 2 lbs, 2 weeks, 4 lbs. Now there's 5 rabbits and they eat 2 lbs a week, so 5 rabbits and 2 lbs would would be 10, 10 lbs. The answer is 10 lbs.
E Why are you scratching your head?

S I'm thinking am I gonna get this right! There's 2 lbs of food a week, so for 2 weeks there'd be 4. 5 rabbits would eat 10 in a week. I'm thinking it's going to be hard. 1 rabbit eats 2 lbs of food each week. 5 rabbits 2 lbs a piece, 20 lbs.

E How did you get the 20?
S There's 5 rabbits, they each eat 2 lbs a week, so that's 10 lbs for a week, and another 10 lbs, so it's 20 lbs.

S (Reads #7) That's easy ... 18.

S (Reads #8). (Rereads most aloud, rest to self).

E You can just make lines, don't need full drawings.

S Sally is taller than Joanne, Joanne is taller than Lorna, Lorna is shorter than Sally. There!

S (Reads #9). 44 divided by 8 ... 44 divided 8 (whispers) ... 4 left and you ... 44 books, there's 8 on each shelf, 8 and 8 and 8 ... is 16, 16, 16, 16. 64 but it says 44 ... 12, 13, 14, 15, 16, 17, 18, 19, 20, that's 40 but there's 4 left. 4 books left. You're gonna need an extra bookcase!

So, there's 40 books. That'd be 8 x 5 = 40 so you need 6.

E Explain.
S 8 x 5 is 40 so you need 5 bookcases, then you have 4 left over so you need 6.

S (Reads #10). O.K. There's 3 into 126. I'm just getting used to 2 digit division. 12 ... 6 ... remainder nothing. He has 42, he sold 15 so that'd be ... 2 ... 12 and he has 27 hectares ...

S (Reads #11). David than Sam ... she has nothing to do with this ... they're tricking me!

S I'm thinking how to take it away.
E Do what you think is best.
S 15. (Reads #12). $1.63.
S (Reads #13). 8.7 meters.

E How did you get that?
S I added 4.3 m and 4.4 m. The simple way of doing it.
S (Reads #14). $1.68
S (Reads #15). 18

E This one, start reading it here.
S (Reads #16). He bought him this (S is circling items in price list). $8.09, he bought him a rubber toy and a dog collar ... plus 1.25. He will have $14.12 left.

S (Reads #17). 25 ... 25 ... 5 ... that's 50. 10 and 10. 25, 10, 10, 10 ... 25, 35, 45, 55 ... 65, 70, 80. 80 cents. I made it.

E Do you do those kind in class anytime?
S No I'm just ... my mind is swirling.

S (Reads #18). She has 4 crayons left because among 3 people what times 3 goes into 24? 8 x 3 is 24. We have 24 so if they are all divided equally she gave half which is 4 crayons.

S (Reads #19). 28 tent pegs.

S (Reads #20).

E Are you finished? Like math?
S Yes, sometimes I find it boring because it can get on your nerves if you want to do art or DPA.

E Do you think you're good at it?
S Yes.
E You did very well. How solve problem stuck on?

S Sit there and concentrate or ask for help. Sometimes [illegible] up like in enrichment they are really hard. I give up [illegible] trick questions but they're not math questions.

E How could you improve?
S Study, concentrate ... go over the times tables every night.

E Can you think of anything else?

S Study this test!

S = Student E = Examiner

S - 095 - Gr 5

S (Reads #1) I have to add 18 and 8. (39) 8 plus 8 is sixteen, carry 1, 1 and 1 , 2 . 28 minutes (24)
E Why did you add?
S Because it "how long" and that means to add.

S (Reads #2). She ... I have to ...(11) (Rereads #2).(3) Add 3925 to ...(11)

E (Prompt to verbalize)
S Trying to find another thing to add.(23) (Rereads #2). (3) I have to add 3925 to 3925. (13)(39) O.K. (29) 5 + 5 is 10 carry 1. 1 +.2 = 3 and 2 is 5. 9 + 9 are 18 carry the 1. 1 and 3 is 4, 4 + 3 are 7. She must travel 7850 km on the complete trip. (24)

E Why add?
S It said how far must she travel so I knew I had to add. (39)

S (Reads #3). I should add 28 + 15 (39) because it says how much money does she have, after she spent 77 cents.(21) She had 77 cents so I should add 28 and 15. 8 and 5 are 13, 1 + 2 are 3 and 1 is 4. She has 43 cents now. (24)

S (Reads #4). I subtract $8.67 from $10.00. (39) 10 subtract 7 is 3, 6 from 9 is 3, 8 from 9 is 1. He received 1.33 from a 10 dollar bill (24) and want to subtract it from say what change did he receive so you know you have to subtract. (15)

S (Reads #5). (Rereads #5). (3) You should ...(11) add 48 no (28) you should add 72 and 3. 2 and 3 is 5 and 7 and nothing is 7, so there were 75 children. (24) Why I added was it said how many children were there.

S (Reads #6). I have to add (39) ... (11) (rereads #6).(3) I have to add 2 lbs and ... no 1 rabbit and 5 rabbits. Total is 6. (24) (rereads #6). (38)

E (Prompt to verbalize)
S Trying to figure out what to add.(23) (Rereads #6). (3) I need to add ...(11) 2 lbs and 2 weeks, and I get 4. So, 5 rabbits eat 4 lbs in 2 weeks. (24) Why I had to add is because it said how much. (15)

S (Reads #7). I have to subtract (39) .. (11) I'm looking at 13 and 5 and I have to add (28) 5 to 13 (39) because it says how many. (15) Jay has 18 goldfish. (24)

S (Reads #8). So, Sally is tallest (8) and then there's Joanne and then Sally. Why I should draw lines is because it says to. (16)

S (Reads #9). I should divide 8 into 44. (39) 8 into 4 is 0, 4 ... 4 is 4 ... times 5 is 40, closest to 44. There's um 5 shelves with a remainder of 4. 5 shelves will be needed to place 44 books and there will be remainder of 4 shelves left over. (24)

S (Reads #10). I have to take 15 away from 126. (39) 5 from 6 is 1, 1 from 2 is 1, 0 from 1 is 1. He has 101 hectares left. (24) Why I know to subtract is because it says how many (15) ...(11) (28) which means I should add (39) (erases). I should add. 6 + 5 is 11, 1 + 2 is 3 and 1 is 4, 0 + 1 is 1. He has 141 hectares left. (24) I did that because it says how many. (15)

S (Reads #11). Subtract 137 from 152. (39) Can't take 7 from 2, change 5 to 4 and 2 to 12 and 7 from 12 is 5. 3 from 4 is 1 and 1 from that is 0. So David is 15 cms taller than Sam. (24) I knew to subtract because it says how much taller is David. (32) (15)

S (Reads #12). I need to add 50 and 12 (39) which gives me 62 cents. I knew to add because it says how much money did he have. (15)

S (Reads #13). I need to add 4 and 3 and 4 and 4. (39) That becomes 2. 15 m so the perimeter of the new bedroom is 15 m. (39) It said what will be so I knew I had to add.

S (Reads #14). I need to multiply 8 by 21, (39) which is $1.68 so Tim paid $1.68 for the 3 apples. (24) I knew to multiply because it said how much. (15) If I added it would be too low, if I subtracted it would be too low and divided .. way too low. (37)

S (Reads #15). I need to add 12 and 6, (39) 18. He had 18 cars at first. (24) How I knew to add is because it said how many which means to add. (15)

S (Reads #16). A dog collar is $4.78, (8, 16) a dog bed is $8.09 (8) (33) and 1 rubber toy is $1.25. (8, 16) I'm adding 4.78, 8.09 and 1.25 to get the answer and subtract it from a 20 dollar bill. (39)

8 and 9 is 17 and 5 is 22, and 2 + 2 is 4 and 7 is 11. 1 + 1 is 2, 2 and 4 are 6, 6 and 8 are 14. Now subtract 14.12 from 20 dollars. Change 2 to 1 ... 0 outa 10 oh 9. 2 from 10 is 8. 1 from 9 is 8, 4 from 9 is 5, 1 from 9 is 8. He gets 5.88, his change is 5.88 from his 10 dollar bill (28)

er 20 dollar bill. First I added then subtracted because how <u>much</u> change will be. (15)

S (Reads #17). I need 1 quarter and (11) ... 5 dimes ... that's 50 and 25 ... 75 ... and I need 1 nickel. That comes to 80 cents. Her 7 coins are 1 quarter, 5 dimes, 1 nickel. (24)

S (Reads #18). So I need (11) ... to ... (Rereads #18). (3) I need to subtract (39) um (11) ... I'm trying to figure out what number goes into 24 evenly, and I can't do it. (23) Subtract 8 from 24 ... 14 from 8 is 6 so ... I just subtracted 8 from 24 and got 16 and now I'm going to figure out how many crayons Joan had left. So, she has 8 crayons left. (24) How I got that is because I subtract 8 from 24 to get 16 and 8 goes into 16 evenly 8 times so I subtracted 8 from 16 .. 8.

S (Reads #19). I need to add 8 and 6. (39) (Rereads #19). (3) 8 and 6 are 14, so 14 pegs are required. (24) How I got that and why I know to add is because it said how many. (15)

S (Reads #20). I need to add 7 + 12. (39) 2 and 7 are 9, 1 and 0 is 1. So Jay has 19 badges. (24) I know to add because it says how many. (15)

E Favourite subject?
S Art and language arts because I like to write and draw, even though I can't do it very good.

E Like math?
S No.
E How be better problem solver?
S If I read the statement more clearly and read it over more times to see what I need to add, subtract, multiply or divide.

E Anything else?
S No.
E Better problem solver than others?
S No because I am usually one of the last people to do the problem when we have our fundamentals ... just reading it over and over and over and I still can't get it and they usually are done before me.

E How do a problem stuck on?
S I'd read it a lot of times, first quickly for main idea, second to pick up tidbits I need, the third time if I have to keep going until I understand it more.

E Anything else?
S No

S = Student E = Examiner

S - 057 - Gr 6

S (Reads #1). 18 minutes for each record is ... 18 multiplied by 8.

E Why multiply?
S To find out how long she played her records. 18 ... 18 minutes x 8 records = 8 x 8 = 64 ... 8 x 1 is 8 + 6 is 14. 144.

S (Reads #2). To find out how far to travel she has to multiply or add ... multiply 3925 by 2. 3925 x 2 = 5 x 2 = 10 carry 1, 2 x 2 is 4 + 1 = 5, 9 x 2 is 18 carry 1, 3 x 2 = 6 plus 1 is 7. 7850.

S (Reads #3). To find out how much money Mary has you put down 77 cents - 28 cents that's . Cross out 7, put down 6, carry 10, put down 10. 17 - 8 = 9, 6 - 2 = 4.

E Why subtract?
S To find out how much money Mary has left. She's 49 cents. She found 15 and you have to add 15 cents to 49. 9 + 5 is 14 carry 1, 12 + 1 is 5 and 1 is 6 ... 64 cents left.

S (Reads #4). You have to subtract \$8.64 from \$10.00. \$10.00 - \$8.64 is ... You can't take 7 from 0 so you go over the 1 from the 10 and make it 0 and put 9 ... put 9 down beside on top of 8 and 0 and put 9 on top of the 6 and 0 and add 1. 9 from 6 is 3, put in your decimal, take 8 from 9 leaves a 1 the answer is \$1.33.

S (Reads #5). 72 cookies divided by 3, 72 divided by 3 to find out how many children. 3 into 7 2 times. 3 x 2 is 6, 7 - 6 is 1, bring down the 2, 3 goes into 12 4 times, 4 x 3 is 12, 12 take away 12 is 0. So there were 24 children.

S (Reads #6). To find out how much food 5 rabbits eat in 2 weeks you have to multiply 2 ... one rabbit eats 2 lbs in one week, how much can 5 eat in 2 weeks ... multiply 5 x 2 ... 5 rabbits x 2 ... O.K. 2 x 2 is 4. First you have to mulitply 2 lbs by 2 weeks, 2 x 2 = 4. Then multiply 4 lbs of food by 5 to find how many lbs 5 rabbits eat.

4 x 5 = 20. The answer is 20. 20 lbs of food.

S (Reads #7). To find out how many fish Jay has you have to subtract 5 from 13. 13 - 5 is 7 so Jay must have 7 fish.

S (Reads #8). Sally is taller than Joanne, but Joanne is taller than Lorna (repeats the same). Lorna is shorter than Sally, O.K. O.K.

S (Reads #9). You have to multiply ... 8 books on each shelf ... you have to multiply 44 x 8. To find out how many shelves are needed. 8 x 4 is 32, carry 3, 4 x 8 = 32 plus 3 is 35 so 352 sshelves will be needed to place 44 books on the bookcase.

S (Reads #10). 126 divided by 6 ... 126 divided by, you have to divide 3 into 126 ... you have to divide 126 into 3 ... hold it ... 3 into 126. Divide 3 into 126 to see how much each son got. 3 doesn't go into 1 so you have to go to the 2, you have to go to the 1 and 2. 3 into 12 is 4, 3 x 4 is 12 ... 12 - 12 is 0, bring down 6. 3 goes into 6 2 times. 3 x 2 is 6. 6 - 6 is 0. So, John got 42 hectares. If he sold 15, 42 - 15 would leave you. You can't subtract 5 from 2. Borrow from the 4. 40 - 10 is 30, give the ten put it in the ones place. 12 - 5 is ... 12 - 5 is 7.

3 - 1 is 2. So, John has 27 hectares left.

S (Reads #11). O.K. You don't need to know how tall Ms. Cooper was because it says how much taller was David than Sam. You have to subtract 150 ... 137 from 152 to find out how much taller David was than Sam.

Can't take 7 from 2 so you borrow from the tens place. Cross out 5 put down 4 bring over 10. 12 - 7 is 5. 4 - 3 is 1. 1 - 1 = 0. So, David was 15 cms taller than Sam.

S (Reads #12). First divide 50 into 3 to find out how much money he had at first. 3 goes into 5 once. 3 x 1 = 3, 5 - 3 = 2 bring down 0. 3 goes into 20 6 times 3 x 6 is 18. 20 - 18 = 2. Do I put a decimal?

E Do whatever you think is right.
S O.K. Sid bought 3 packages of peanuts at 50 cents each. Oh! For 50 cents each. Multiply 50 by 3 to find out how much he had at the beginning.

You can't subtract 3 from 0 so you go to the 10's place. 5 cross out put down 4 bring 10 to ones place. Subtract 3 from 10 = 7, 4 - 0 = 4. Is 4. I did something wrong I think.

E Why?
S Because he bought 3 packs each at 50 cents ... 50 x 3 .. I didn't .. I did something wrong. 50 3 times is 150 so .. Why it's this is complicated because I can't go over to the 5 because ... I thought I was subtracting! I don't have to go over!

3 x 0 = 0, 5 x 3 = 15, put decimal in 2 places. That's $1.00 so the peanuts cost him $1.50 then he had 12 cents left, add it on.
2 + 0 = 2, 5 + 1 = 6, 1 + 0 is 1. Put your decimal in. Answer is $1.62.

I got confused.

S (Reads #13). O.K. (Rereads #13). To find out perimeter you have to multiply 4 by 3, I think. Or add 4 and 4 and 3 and 4. 2 4's = 8 and 4 are 12, and 3 are 15. So the perimeter was 15 m.

E Why multiply then add?
S This would be shorter instead of writing it all down. I added 4 + 4 and 4 + 3 to find the perimeter of the new bedroom.

S (Reads #14). You don't need to know how much the oranges were because he bought apples not oranges. Multiply 21 x 8, 8 x 1 = 8, 2 x 8 = 16. He paid $1.68 for the apples.

S (Reads #15). (Rereads). He had 6 cars .. he has 12 cars now .. he had 6 cars also. You add 12 + 6 to see how many he had. He had 18 cars at the beginning.

S (Reads #16). A dog bed is $8.09, a rubber toy is $1.25, a dog collar is $4.78. Add these together. 8 + 9 = 17 + 5 is 23, carry 2. 7 + 2 = 9 + 2 is 11, carry 1. 8 + 1 = 9 + 1 is 10 + 4 is 14. It cost him $14.13. Subtract $14.13 from $20.00. You can't subtract 3 from 0 so you .. cross out 2, put down 9 .. The 9 above the first 0, a 9 above the second 0. Put down the 1, the 10. 10 - 3 = 7, 9 - 1 = 8. 9 - 4 = 5, 1 - 1 = 0. Put in decimal. He would receive $5.87 from $20.00.

S (Reads #17). (Rereads #17). Let's see. .25 cents. Forget about the 25 cents. First let's divide 7 coins into 80 cents ... 7 goes into 8 7 times ... O.K. Divide 7 into 80 just to see if it goes evenly to find what the 7 coins are. 7 goes into 8 one time, 7 x 1 = 7. 8 - 7 is 1, bring down 0. 7 goes into 10 1 time, 7 x 1 = 7. 10 - 7 = 3. Remainder of 3 so you couldn't have ... your answer is 11 remainder 3. Let's see ... 7 coins ... 7 ... O.K.

It couldn't be so ... 7 coins ... 7 dimes would be 70 cents so that couldn't be right because there's 7 coins and ... I did something wrong.

E What are you thinking?
S I don't know!
E Want to come back to it?

S (Reads #18). (Rereads half question #18 - first 2 sentences) So 1, 2, 3 ... 3 people. 24 divided by 3 to find out how many crayons each person got. 3 doesn't go into 2, 3 goes into 24. 3 into 24 is 8. 8 x 3 is 24. 24 -24 = 0. Each person, Joan, Bill and Erin got 8 crayons. Joan gave half her crayons to another person. Half of 8 is 4 so 8 - 4 = 4. Joan has 4 crayons left.

S (Reads #19). (Rereads #19). There's 2 sides of the tent so you add 8 + 8 to see how many tent pegs you need for the sides. 8 + 8 = 16 and there's 2 ends so add 6 + 6 to see how many pegs the ends need. 6 + 6 = 12. You add 16 + 12 to see how many pegs all the way around.

16 + 12 = 6 - 2 = 4, 1 - ... oh add! 6 + 2 = 8, 1 + 1 = 2, so you need 28 pegs for the whole tent.

S (Reads #20). Jay has twice as many ... Pete has 7 and Matt has 12 badges. Jay has twice as many as Pete and Matt together. How many badges does Jay have? Twice as many as Pete and Matt together. You have to add 12 + 7 to see how many Jay has.

12 + 7 = 7 + 2 = 9 + 1, 1 + 0 =1.

E O.K.
S The answer is 19. Jay has 19 badges.

E Back to #17.
S I'll start all over again. (Reads #17). I'll write down 7 coins and 80 cents.

There's 80 cents and 7 coins. 25 cents .. 25 cents. 25 cents + 25 cents is 50 cents. There's 2 coins and 50 cents plus a dime is 60 cents. There's 3 and there's 80 cents. O.K. 3 coins .. 60 .. um 60 cents = 3 .. + 4 nickels is 60 cents + 4 nickels. 4 nickels is 5 cents + 5 cents + 5 cents + 5 cents or 5 x 4 is 20 cents so 20 cents to 60 cents gives you 80 cents and there's 7 coins. The answer is 2 quarters, 1 dime and 4 nickels.

E Favourite subject?

S Probably art because you get to talk and I like to do murals ...
E Draw and do things with your hands.
Do you like math?

S No but I usually get good marks. It's one of my best subjects.

E Better problem solvers in your class?
S Probably because some people like to do problems. They usually do good on them. I'd rather do other things than problems so they probably do better than me.

E Where do you fit?
S Probably in the middle.
E How be better problem solver?

S Practicing probably. But I usually do good at problems. Putting more effort might make me better. My Dad was a math whiz too, but more studying might help me.

E Problem can't solve?
S Tell my teacher I don't understand it and she helps me understand it without giving me the answer. I might go ahead and come back to it and read it over and over until I understand it. I'd write everything on a paper and go from there.

S = Student E = Examiner

S - 060 - Gr 6

S (Reads #1). 18 records, each lasts 18 minutes (32) so you multiply. (39) (laughs) Because there's 18 records lasts 18 minutes so you multiply because you can't subtract, you can't add, you can't multipy or divide.

S (Reads #2). (Rereads #2). (3) (Rereads #2). (3) (After verbal prompt from E). There's not enough information! (Rereads #2). (3) There's not enough information. (23) There's only 1 number .. it doesn't say how far Ron's house is.

E What do you want to do?

S Come back to it I guess. (36)

S (Reads #3). You add 77 cents for the 15 and 28 cents (39) because you can't divide, subtract, or multiply because she has <u>more</u>. (15)

S (Reads #4). Subtract 8 from, subtract \$10.00 from \$8.76 because you take some away, (39) he spends \$8.67 from \$10.00.

S (Reads #5). You divide, (28) no multiply 72 times 24 times 3. (39) Must have to add. (28) (Rereads #5). (3) Divide 3 into 72 then multiply 48 times 24. Can I skip this one? (36)

E If you'd like to move on, sure.

S (Reads #6). You multiply (28) no add. (39)
E Tell me why you added.
S (Rereads #6). I don't know! (40) You can't multiply ... you can't divide because there's 3 numbers.

E I'm not saying it's right or wrong, I just want to know why you added.

S Well you can't multiply because there's 3 numbers, you only need 2 for multiplying. You can't divide because it says how <u>much</u>, you can't divide because there's 3 numbers, you can't multiply because there's 3 numbers. (37)

S (Reads #7). You subtract (39) because it says how many fish so you subtract 13 from 7 (28) 5.

S (Reads #8 as Jay is taller than ...)
Sally is taller than Joanne. (8) Joanne is taller than Lorna. (8) Lorna is shorter than Sally. (32) There.

S (Reads #9). (Rereads #9). (3) Multiply (39) because there's 8 books and you need 44 books in the bookcase.

S (Reads #10). (Asks for explanation of hectares. (31) You subtract 126 subtract 15 (39) because it says how many hectares does John have left, (32) that means subtract. (15)

S (Reads #11). Subtract 173 from 152 (39) because it says how much (15) no (28) you add. How much taller. No, you multiply (40) er! you subtract. (39) Yah.

E Why subtract?
S Because it says how much taller so you subtract. (15)

S (Reads #12). Add 50 cents + 12, (39) it says how much. (15) That's wrong. (28) So, you add. (39) (Rereads #12). (3) You multiply 50 x 12. (39)

S (Reads #13). (Rereads #13). (3) Add 4 + 4 + 4 + 3. (39)

S (Reads #14). (Rereads #14). (3) You divide (40) er multiply

E Why?
S Because it says (reads #14 again) (3) so you multiply because it says how much. (15) You can't add because it wouldn't make sense. You can't subtract, because it wouldn't make sense. You can't divide because it wouldn't make sense. (37)

S (Reads #15). Well you add 12 + 6 (39) because he had so much and he gave 12 away. And now he has 12 left er 6 to Steven and now he has 12 left. (32) So you have to add 12 + 6. (39)

S (Reads #16). One rubber, a dog bed, $8.09 (8, 33) and a dog collar .. rubber toy $1.25 (8,33) then he bought a dog collar .. $4.78.

E What are you doing?
S Adding or subtracting (40) because it says how much change because he already bought a dog bed and you can't really add because you can't get more money from buying something. Oh! I was adding. I was supposed to subtract (28) no add.
E Tell me what you're doing.
S I'm getting confused. You can't add them or subtract them. How much change will he receive from $20.00 (32) so you have to subtract them. (39) How much change will he receive from $20.00? (15) So you add 8.09 and 1.25 then $4.78, add those up. (adds) Then subtract $14.12 from $20.00. (39)

S (Reads #17). (Rereads #17). (3) 7 coins so that's

E (Verbal prompt).
S I'm just thinking what could it be? (23) 10 cents , 70 then 5 or 2. 5 cents, 10

S (Reads #18). (Rereads #18). (3) Equal numbers of 24 is 4 because she gave 4 to Joan, 4 to Bill, 4 to Erin. Joan gave half her crayons to another person. (32) How many does Joan have left? Well, 24 then 4 .. 4 x 2 or 4 - 2 = 2.

S (Reads #19 with help pronouncing rectangular)
(Rereads #19). (3)
Add 8 + 6. 8 + 10 on each side. 2 eights is 16, add it. Then add 6 + 6 is 12, so add 16 and 12 together because it says how many tent pegs. (39)

S (Reads #20). (Rereads #20). (3) (Rereads). (3) Add 12 + 7 then times 2. (39)
E Why add?
S Add 12 + 7 to see how much they had together.

E Why multiply?
S You multiply the answer of 18 times 2, it says twice. (15) Jay has twice as many so you take 19 x 2.

E Let's go back to #2.
S (Reads #2). There's no other number so she has to travel 3925 miles .. kms away. (24)

E #5
S (Reads #5). Add 72 + 48 + 24. (39) Then times 3 .. to see how many cookies there are .. or how many children. (40) I added because you couldn't do anything unless you got the number for how many cookies there were. Then times to answer to how many children times 3 because each child got 3 cookies.

E Favourite subject?
S English because it's easy.
E Like math?
S No.

I just hate it if I don't get it and don't really understand it. Some people really understand math but I don't.
E Better problem solver than classmates.
S No. I'm not good at it. I know I'm not.
E How improve?
S Try harder. Do some at home. Get help at school.
E If couldn't solve one?
S Probably guess at it or reread it.

Appendix I

ANOVA Tables

Table 1

ANOVA for Mean Number of Word Problems Correct by Grade

Source	df	SS	MS	F
Between	2	219.52	109.76	7.75*
Within	176	2491.23	14.15	
Total	178	2710.75		

* p < .05

Table 2

ANOVA of Mean Word Problems Correct by Raw Score Group

Source	df	SS	MS	F
Between	2	2181.72	1090.86	362.91*
Within	176	528.03	3.01	
Total	178	2710.75		

* p < .05

Table 3

ANOVA of Mean Number of Steps Used by Grades 4, 5, & 6 Students

Source	df	SS	MS	F
Between	2	3.24	1.62	7.36*
Within	176	39.52	.22	
Total	178	42.76		

* P < .05

Table 4

ANOVA of the Mean Number of Steps Used by Raw Score Groups

Source	df	SS	MS	F
Between	2	19.46	9.73	73.09*
Within	176	23.52	.13	
Total	178	42.76		

Table 5

ANOVA of the Mean Number of Cognitive, Metacognitive, and Total Number of Strategies for Students in Grades 4, 5, & 6

Source	df	SS	MS	F
Cognitive Strategies				
Between	2	208.42	104.21	4.87*
Within	176	3746.64	21.28	
Total	178	3955.06		
Metacognitive Strategies				
Between	2	5.47	2.73	2.36
Within	176	203.94	1.15	
Total	178	209.41		
Cognitive and Metacognitive Strategies				
Between	2	285.66	142.83	4.99*
Within	176	5029.19	28.57	
Total	178	5314.85		

* $p < .05$

Table 6

ANOVA of the Mean Number of Cognitive, Metacognitive, and Total Number of Strategies by Raw Score Groups

Source	df	SS	MS	F
Cognitive Strategies				
Between	2	1930.26	965.13	83.89*
Within	176	2024.80	11.50	
Total	178	3955.06		
Metacognitive Strategies				
Between	2	115.91	57.95	109.08*
Within	176	93.50	.53	
Total	178	209.41		
Cognitive and Metacognitive Strategies				
Between	2	2840.34	1420.17	99.86*
Within	176	2474.51	14.01	
Total	178	5314.85		

* $p < .05$

Appendix J

Strategy Types by Code Used by Individual Students

STRATEGY TYPES USED BY INDIVIDUAL SUBJECTS

Strategy Types by Code

ID 123	G 4	RS 56	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	45	46	47	48	49
011	6	19	x		x	x	x	x	x	x	x	x	x	x	x		x	x				x				x		x	x	x	x	x	x	x	x	x				x	x				x			
050	6	19	x		x	x	x	x	x	x	x	x	x	x	x		x	x	x			x	x	x	x	x	x	x	x	x	x	x		x	x	x		x		x	x			x	x		x	x
012	6	18	x		x	x	x	x	x	x	x	x	x	x	x	x	x					x				x		x	x	x	x	x	x	x	x					x	x							
071	6	18	x		x	x	x	x	x	x	x	x	x	x	x		x	x				x	x			x		x		x	x	x	x	x	x	x					x			x	x			x
184	4	18	x	x	x	x	x	x	x	x	x	x	x	x	x		x		x			x	x			x		x	x	x	x	x		x	x	x			x	x	x			x	x			
146	4	18	x		x	x	x	x	x	x	x	x	x	x	x		x	x				x	x			x		x		x		x		x	x	x			x	x	x							
002	6	17	x		x	x	x	x	x	x	x	x		x	x		x					x				x		x	x	x	x	x	x	x	x						x				x			
005	6	17	x	x	x	x	x	x	x	x	x	x	x	x	x		x					x				x		x		x	x	x		x	x	x					x				x			
007	6	17	x	x	x	x	x	x	x	x	x	x		x	x		x		x			x				x		x		x	x	x		x	x	x					x				x			x
009	6	17	x		x	x	x	x	x		x	x	x	x	x		x					x				x			x	x	x	x	x	x	x						x			x	x			x
021	6	17	x		x	x	x	x	x	x	x	x	x	x	x		x					x			x	x			x	x	x	x	x		x	x					x			x				
055	6	17	x			x	x	x	x	x	x	x		x	x		x	x				x			x	x		x	x	x	x	x	x	x	x										x			x
073	5	17	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x			x	x			x	x	x	x	x	x	x		x	x	x				x					x			x
094	5	17	x			x	x	x	x	x	x	x	x	x	x		x					x	x			x		x	x	x	x	x		x	x			x	x		x				x		x	x
096	5	17	x	x	x	x	x	x	x	x	x	x	x	x	x		x						x			x			x	x	x	x		x	x						x			x				
105	5	17	x			x	x	x	x		x	x	x	x	x		x					x	x			x		x	x		x	x		x	x	x			x	x	x				x			
129	5	17	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x				x			x	x	x		x	x	x	x	x	x	x	x			x		x			x	x			x
036	6	17	x	x	x	x	x	x	x	x	x	x	x	x	x		x		x			x			x		x		x		x	x		x	x	x			x	x	x							
019	6	16	x		x	x	x	x	x	x	x	x	x	x	x		x	x				x			x	x		x	x	x	x	x	x	x	x	x			x	x					x			x
040	6	16	x		x	x	x	x	x	x	x	x	x	x	x		x	x				x	x						x	x		x			x	x					x							
053	6	16	x		x	x	x	x	x	x	x	x	x	x	x		x	x	x			x	x		x	x	x	x	x	x	x	x		x	x	x		x		x	x				x			x
056	6	16	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x			x	x		x	x	x	x	x	x	x	x	x	x	x			x		x	x			x	x			x
059	6	16	x		x	x	x	x	x	x	x	x	x	x	x		x		x						x	x		x	x	x	x	x	x	x	x				x		x			x				x
069	6	16	x	x	x	x	x	x	x	x	x	x	x	x	x		x					x				x		x	x	x	x	x	x	x	x						x							
074	5	16	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x			x	x		x	x	x	x	x	x	x	x		x	x	x				x	x			x	x			x
075	5	16	x			x	x	x	x	x	x	x	x	x	x		x					x			x	x		x	x	x	x	x	x	x	x	x			x						x			x
091	5	16	x			x	x	x	x	x	x	x		x	x		x					x	x			x			x	x	x	x		x	x						x							
101	5	16	x		x	x	x	x	x	x	x	x	x	x	x		x	x	x			x			x	x			x	x	x	x			x						x				x		x	x
150	4	16	x		x	x	x	x	x	x	x	x	x	x	x		x	x				x	x		x	x		x	x	x	x	x		x	x				x	x	x			x	x			x
154	4	16	x			x	x	x	x	x	x	x		x	x		x					x	x		x	x		x	x	x	x	x		x	x				x	x	x		x					
183	4	16	x		x	x	x	x	x	x	x	x	x	x	x		x	x				x			x	x		x	x	x	x	x		x	x						x		x		x			x
013	6	15	x		x	x	x	x	x		x	x	x	x	x		x					x				x			x	x	x	x	x	x	x						x			x				
015	6	15	x		x	x	x	x	x		x	x	x	x	x		x									x			x	x	x	x			x	x												
016	6	15	x		x	x	x	x	x	x	x	x	x	x	x		x								x				x	x	x	x	x	x	x	x								x				
044	6	15	x		x	x	x	x	x	x	x	x	x		x		x					x							x	x	x	x		x	x						x							
045	6	15	x		x	x	x	x	x	x	x	x	x	x	x		x	x	x			x			x			x				x		x	x						x							
067	6	15	x		x	x	x	x	x	x	x	x	x	x	x		x		x			x	x			x				x	x	x		x	x					x	x							
029	6	15	x		x	x	x	x	x	x	x	x	x		x		x					x	x		x	x					x	x		x	x						x							
097	5	15	x	x	x	x	x	x	x	x	x	x		x	x		x					x	x			x			x	x	x			x	x			x			x							
104	5	15	x		x	x	x	x	x		x	x	x	x	x		x	x								x				x	x	x	x	x	x					x	x							

STRATEGY TYPES USED BY INDIVIDUAL SUBJECTS

Strategy Types by Code

ID 123	G 4	RS 56	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	45	46	47	48	49
109	5	15	x			x	x	x	x	x	x	x	x	x	x		x					x	x		x	x			x	x	x				x	x			x		x	x						
119	5	15	x		x	x	x	x	x		x	x	x		x		x		x										x	x	x	x			x	x					x							
014	6	14	x		x	x	x	x	x	x	x	x	x	x	x		x					x				x		x	x	x	x			x	x						x							
038	6	14	x		x	x	x	x	x		x	x	x	x	x		x	x	x			x		x		x		x	x	x	x		x	x	x	x									x			
048	6	14	x		x	x	x	x	x	x	x	x	x		x		x					x							x						x													
057	6	14	x	x	x	x	x	x	x	x	x	x	x	x	x		x		x			x	x		x	x		x	x	x	x	x	x	x	x			x			x				x			
061	6	14	x			x	x	x	x		x	x	x		x		x					x	x			x			x	x	x		x	x	x													
062	6	14	x		x	x	x	x	x	x	x	x	x	x	x		x	x	x				x	x	x	x			x	x	x			x	x						x							
027	6	14	x		x	x	x	x	x		x	x	x		x		x						x		x	x					x			x	x	x					x							
090	5	14	x		x	x	x	x	x	x	x	x		x	x		x					x	x	x	x	x					x		x	x	x						x							
092	5	14	x			x	x	x	x	x	x	x	x	x	x		x		x			x		x	x			x		x	x	x		x	x	x			x		x							
093	5	14	x		x	x	x	x	x	x	x	x	x	x	x		x					x	x			x	x		x	x	x			x	x			x			x							
102	5	14	x		x	x	x	x	x	x	x	x	x	x	x		x					x	x			x				x	x	x			x						x							
103	5	14	x	x	x	x	x	x	x	x	x	x	x	x	x		x						x		x	x				x	x		x	x	x	x					x	x						
126	5	14	x		x	x	x	x	x	x	x	x	x		x		x					x		x	x	x				x	x		x	x	x				x		x							
142	4	14	x			x	x	x	x	x	x	x	x	x	x		x	x	x	x		x	x		x	x		x	x	x	x	x	x	x	x	x				x	x				x			
145	4	14	x		x	x	x	x	x	x	x	x	x		x		x					x	x			x				x	x	x		x	x													
158	4	14	x		x	x	x	x	x		x	x		x	x		x					x	x		x	x			x	x	x			x	x					x	x							x
163	4	14	x		x	x	x	x	x		x	x	x		x		x						x			x				x	x			x	x					x	x			x					
173	4	14	x			x	x	x	x	x	x	x	x		x		x					x				x		x		x	x			x	x					x	x							
136	5	14	x		x	x	x	x	x	x	x	x	x		x		x					x	x			x				x	x			x	x					x								
006	6	13	x		x	x	x	x	x		x	x					x		x			x	x					x	x	x	x		x	x	x	x												
063	6	13	x		x	x	x	x	x	x	x	x		x	x		x	x				x	x		x	x	x		x	x	x			x	x			x										
064	6	13	x		x	x	x	x	x	x	x	x		x	x		x					x	x		x	x	x		x	x	x	x		x	x			x			x							
066	6	13	x			x	x	x	x	x	x	x		x	x		x					x	x		x	x			x	x	x		x	x	x	x					x							
025	6	13	x		x	x	x	x	x		x	x	x		x		x						x		x	x				x	x				x						x							
077	5	13	x			x	x	x	x	x	x	x	x	x	x		x					x	x		x				x	x	x	x	x								x							
080	5	13	x		x	x	x	x	x	x	x	x	x	x	x		x	x				x			x			x		x	x	x	x	x	x			x	x		x							x
106	5	13	x		x	x	x	x	x		x	x					x									x	x			x				x	x						x							
118	5	13	x		x	x	x	x	x	x	x	x	x		x		x					x			x	x			x	x	x	x		x	x				x		x							
130	5	13	x		x	x	x	x	x	x	x	x	x		x		x					x							x	x	x	x	x	x	x	x												
131	5	13	x			x	x	x	x	x	x	x	x				x						x							x	x		x	x	x	x					x							
151	4	13	x			x	x	x	x	x	x	x					x					x	x	x		x				x	x	x		x	x	x				x	x							
161	4	13	x		x	x	x	x	x	x	x	x	x	x	x		x					x	x		x	x			x		x		x	x	x				x	x	x	x						
135	5	13	x		x	x	x	x	x	x	x	x					x					x	x	x		x					x			x	x						x							
052	5	13	x		x	x	x	x	x		x	x					x									x	x			x				x	x			x	x		x							
008	6	12	x		x	x	x	x	x		x	x			x		x	x				x	x						x	x	x			x	x								x					
010	6	12	x		x	x	x	x	x	x	x	x	x		x		x					x			x	x		x	x	x	x	x	x	x	x	x												
022	6	12	x		x	x	x	x	x	x	x	x	x		x		x								x										x	x												
047	6	12	x		x	x	x	x	x	x	x	x	x		x		x					x								x	x					x												

STRATEGY TYPES USED BY INDIVIDUAL SUBJECTS

Strategy Types by Code

ID 123	G 4	RS 56	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	45	46	47	48	49
068	6	12	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x				x	x		x	x		x	x	x	x			x	x	x		x	x	x	x				x			
098	5	12	x		x	x	x	x	x		x	x		x	x		x	x				x	x		x	x			x	x	x	x		x	x				x		x							
100	5	12	x		x	x	x	x	x		x	x	x		x		x								x	x					x		x	x	x						x							
111	5	12	x	x	x	x	x	x	x		x	x			x		x					x	x							x				x	x													
132	5	12	x		x	x	x	x	x	x	x	x	x		x		x																		x	x					x	x						
134	5	12	x		x	x	x	x	x		x	x	x		x		x					x			x	x			x	x	x	x	x	x	x	x				x	x		x					
152	4	12	x	x	x	x	x	x	x	x	x	x		x	x		x						x			x					x			x	x						x							
181	4	12	x		x	x	x	x	x	x	x	x	x		x		x					x	x			x				x	x			x	x	x				x	x	x						
185	4	12	x			x	x	x	x	x	x	x	x		x		x					x				x				x					x	x												
186	4	12	x		x	x	x	x	x	x	x	x	x		x		x								x				x	x					x	x												
018	6	11	x		x	x	x	x	x	x	x	x	x		x		x		x			x				x			x	x			x	x	x	x												
024	6	11	x			x	x	x	x		x	x	x				x					x			x					x			x	x	x													
042	6	11	x		x	x	x	x	x	x	x	x	x		x		x		x										x		x				x						x							
051	6	11	x	x	x	x	x	x	x	x	x	x			x		x	x	x							x				x			x	x	x													
054	6	11	x		x	x	x	x	x	x	x	x	x		x		x	x				x			x	x			x	x	x		x	x	x	x												
065	6	11	x		x	x	x	x	x	x	x	x		x	x		x								x	x			x	x	x		x	x	x													
070	6	11	x			x	x	x	x	x	x	x	x	x	x		x					x	x							x	x			x	x						x							
026	6	11	x			x	x	x	x	x	x	x			x		x						x			x					x				x	x					x							
028	6	11	x		x	x	x	x	x	x	x	x	x		x		x								x				x	x	x			x	x	x					x							
076	5	11	x		x	x	x	x	x	x	x	x	x		x		x		x			x			x			x	x		x		x	x			x		x	x	x							
078	5	11	x		x	x	x	x	x		x	x	x		x		x													x	x										x							
099	5	11	x		x	x	x	x	x	x	x	x					x					x	x			x			x	x	x			x	x				x									
112	5	11	x			x	x	x	x		x	x	x		x		x							x	x	x	x							x	x	x				x	x							
155	4	11	x		x	x	x	x	x	x	x	x	x		x		x								x	x				x	x			x	x				x	x	x							
156	4	11	x		x	x	x	x	x	x	x	x	x				x					x	x		x	x				x	x		x	x	x				x		x							
146	4	11	x		x	x	x	x	x	x	x	x			x		x									x								x	x													
160	4	11	x	x	x	x	x	x	x	x	x	x	x	x	x		x									x				x	x			x	x			x			x							
176	4	11	x		x	x	x	x	x	x	x	x	x		x		x					x	x		x	x			x		x		x	x	x	x					x							
182	4	11	x		x	x	x	x	x	x	x	x	x		x		x					x				x			x	x	x		x	x	x	x					x							
003	6	10	x		x	x	x	x	x	x	x	x					x	x							x					x	x		x	x	x							x						
004	6	10	x		x	x	x	x	x		x	x	x		x		x											x		x	x	x	x	x	x	x												
017	6	10	x		x	x	x	x	x		x	x	x				x					x	x		x					x			x	x	x	x												
046	6	10	x		x	x	x	x	x		x	x	x		x		x													x					x													
049	6	10	x	x	x	x	x	x	x	x	x	x	x		x		x	x											x		x		x	x	x						x							
058	6	10	x		x	x	x	x	x	x	x	x	x		x		x	x				x	x		x	x	x		x	x	x		x	x	x	x			x									
079	5	10	x			x	x	x	x	x	x	x		x	x		x					x								x					x						x							
085	5	10	x	x	x	x	x	x	x		x	x		x			x	x					x		x	x				x	x		x	x														
088	5	10	x		x	x	x	x	x	x	x	x					x					x								x				x	x													
095	5	10	x		x	x	x	x	x	x	x	x	x	x	x		x									x				x	x			x	x				x	x	x							

STRATEGY TYPES USED BY INDIVIDUAL SUBJECTS

Strategy Types by Code

ID 123	G 4	RS 56	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	45	46	47	48	49
107	5	10	x		x	x	x	x	x	x	x	x	x				x									x	x	x	x	x	x			x	x	x					x							
108	5	10	x		x	x	x	x	x	x	x	x	x		x		x						x							x	x		x	x	x			x			x							
113	5	10	x			x	x	x	x		x	x					x	x					x		x					x			x	x	x	x												
114	5	10	x			x	x	x	x	x	x	x	x				x					x			x	x				x			x	x	x	x			x			x						
117	5	10	x			x	x	x	x	x	x	x	x				x								x										x	x												
124	5	10	x			x	x	x	x		x	x	x		x		x							x			x		x	x				x	x				x		x							
137	5	10	x			x	x	x	x		x	x	x	x	x		x									x									x	x			x		x							
140	4	10	x		x	x	x	x	x		x	x	x		x		x					x				x			x	x					x	x												
147	4	10	x		x	x	x	x	x	x	x	x			x		x					x								x	x				x	x												
148	4	10	x		x	x	x	x	x	x	x	x	x		x		x	x				x				x				x	x			x	x	x			x									
157	4	10	x			x	x	x	x		x	x		x	x		x					x				x				x				x	x				x									
162	4	10	x			x	x	x	x		x	x	x		x		x					x			x	x	x			x	x		x		x			x	x		x							
164	4	10	x		x	x	x	x	x		x	x			x		x									x	x			x	x			x	x						x							
167	4	10	x			x	x	x	x	x	x	x	x		x		x		x											x	x				x	x												
168	4	10	x		x	x	x	x	x	x	x	x	x		x		x					x					x			x	x		x	x	x						x							
169	4	10	x			x	x	x	x		x	x					x										x			x	x		x	x	x	x												
171	4	20	x			x	x	x	x	x	x	x	x		x		x											x		x					x	x												
181	4	10	x		x	x	x	x	x	x	x	x	x		x		x					x	x				x	x			x				x													
020	6	09	x		x	x	x	x	x	x	x	x	x		x		x																x		x	x												
023	6	09	x		x	x	x	x	x		x	x	x		x		x										x							x														
037	6	09	x		x	x	x	x	x		x	x	x		x		x							x	x	x			x	x	x		x															
120	5	09	x		x	x	x	x	x		x	x			x										x					x	x		x	x														
141	4	09	x	x	x	x	x	x			x	x	x				x	x												x				x														
172	4	09	x		x	x	x	x				x	x					x												x				x	x													
041	6	08	x		x	x	x	x	x		x	x	x		x		x					x	x												x													
081	5	08	x	x		x	x	x	x			x	x				x								x	x				x	x		x			x					x							
121	5	08	x			x	x	x	x		x	x					x								x					x	x			x	x					x		x						
139	4	08	x		x	x	x	x	x		x	x					x								x					x	x			x	x	x					x	x						
174	4	08	x			x	x	x	x		x	x			x		x								x	x				x	x		x	x	x	x					x							
039	6	07	x		x	x	x	x	x		x	x	x		x		x					x								x	x			x	x	x												
082	5	07	x		x	x	x	x	x	x	x	x	x		x		x												x	x			x	x	x							x						
083	5	07	x		x	x	x	x	x	x	x	x	x		x		x												x	x	x		x	x								x						
086	5	07	x		x	x	x	x	x	x	x	x			x		x								x						x		x	x	x						x							
115	5	07	x	x		x	x	x			x	x	x				x								x	x				x				x	x	x			x									
144	4	07	x	x	x	x	x	x	x	x	x	x	x		x		x					x			x				x	x									x		x							
175	4	07	x			x	x	x	x		x	x	x		x		x						x												x	x												
001	6	06	x		x	x	x	x		x	x	x					x								x						x		x	x	x													

STRATEGY TYPES USED BY INDIVIDUAL SUBJECTS

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ID  G RS                        Strategy Types by Code
123 4 56 1234567891011121314151617181920212223242526272829303132333435363738394041 4546474849

043 6 06 x xxxxx x x         x           x   x                 x x x
087 5 06 x xxxxxxx x     x   x x       x                 x x                     x
122 5 06 x  xxxx x x x       x             x x         x       x x
125 5 06 x  xxxx   x x       x                         x x         x
133 5 06 x xxxxx x x x       x                                   x
149 4 06 x xxxxx x x   x     x               x x   x   x x   x x x x       x
165 4 06 x  xxxx x x         x                         x         x
178 4 06 x xxxxx x x x       x x               x                 x x     x
072 6 05 x xxxxxx  x x       x           x   x         x x     x             x
084 5 05 x xxxxx x x         x                 x       x x   x x   x
089 5 05 x xxxxxxx x         x                         x           x
116 5 05 x  xxxx x x x       x                               x x x
143 4 05 x xxxxx x x x       x x         x                   x   x             x
170 4 05 x  xxxx   x         x                 x     x x     x x
128 5 04 x xxxx    x x   x   x               x         x x   x               x
153 4 04 x xxxxx x x x       x x               x x x   x x   x x x
166 4 04 x  xxx x  x         x                         x         x             x
177 4 04 x xxxxx   x x       x                           x   x x   x
179 4 03 x  xxxxxx x x       x                                 x x
060 6 02 x xxxxxx  x         x           x  x          x x     x x     x
138 4 02 x  xxx    x x                    x                    x x             x
158 4 02 x xxxx      x       x                  x            x     x x           x x
123 5 01 x  xxx    x                                   x           x         x
```

Appendix K

Strategies Coded for Each Word Problem by Raw Score Group in Each Grade

Strategies Used to Solve Each Problem by Groups in Grade 4

Problem Number	Strategies A= scores 1-9; B= scores 10-15; C= scores 16-19
1	A - 1,3,4,5,6,10,11,28,31,34,39 B - 1,3,4,5,6,10,11,21,37,39,40 C - 1,3,4,5,6,10,11,12,24,29,32,39,49
2	A - 1,3,4,5,6,10,11,13,15,29,30,39 B - 1,3,4,5,6,8,10,11,13,15,21,23,24,29,30,37,39,41 C - 1,33,4,5,6,10,13,15,21,24,29,30,32,39
3	A - 1,4,5,6,7,10,24,28,29,32,38 B - 1,3,4,5,6,7,10,12,21,24,28,29,31,41, C - 1,4,5,6,7,10,12,24,29,32,39,46,49
4	A - 1,4,5,6,10,12,28,34,39 B - 1,4,5,6,10,24,39 C - 1,4,5,6,10,12,15,21,24,29,39,49
5	A - 1,3,4,5,9,10,11,26,28,29,31,34,39 B - 1,3,4,5,6,9,10,11,12,22,23,24,28,29,31,32,37, 38,39,49 C - 1,3,4,5,6,8,9,10,12,21,23,24,28,29,32,39,46,49
6	A - 1,4,5,6,11,34,39 B - 1,3,4,5,6,7,10,11,21,24,28,29,32,34,38,39 C - 1,3,4,5,6,7,8,10,11,12,16,17,21,23,24,26,28, 32,38,39,46,49,
7	A - 1,3,4,5,6,10,21,24,34 B - 1,3,4,5,6,10,15,24,28,34,39, C - 1,4,5,6,10,11,12,15,24,29,39

-

-

8 A - 1,3,4,5,6,10,15,16,21,23,29,31,32,38,39
B - 1,3,4,5,6,8,10,11,12,16,21,23,29,31,32,38
C - 1,3,4,5,8,10,11,12,15,16,24,29,32

-

9 A - 1,3,4,5,6,11,23,25,27,28,34,39
B - 1,3,4,5,6,11,24,28,29,31,32,34,39
C - 1,3,4,5,6,10,11,12,13,21,22,24,29,30,32,38,39,46,49

-

10 A - 1,3,4,5,11,24,31,32,39
B - 1,3,4,5,6,7,10,13,15,21,24,28,29,32,39,49
C - 1,3,4,5,6,7,10,11,12,13,15,23,24,26,28,29,32,
39,46,49

-

11 A - 1,4,5,6,9,10,24,28,32,34,39
B - 1,4,5,6,9,10,24,29,32,39
C - 1,4,5,6,9,10,12,21,24,29,39,46

-

12 A - 1,3,4,5,6,7,10,12,28,32,24,39
B - 1,3,4,5,6,7,10,11,15,23,24,28,29,32,34,39
C - 1,4,5,6,7,10,11,12,15,21,23,24,29,32,38,39,46

-

13 A - 1,4,5,12,32,34,39
B - 1,3,4,5,8,16,23,24,29,38,39,
C - 1,3,4,5,11,21,24,23,29,31,32,39

-

14 A - 1,3,4,5,6,9,10,28,29,32,34,39
B - 1,3,4,5,6,9,10,11,24,32,39
C - 1,4,5,6,9,10,12,15,21,24,28,29,32,39,41

-

15 A - 1,4,5,6,10,15,27,28,34,39
B - 1,4,5,6,10,11,12,21,24,29,31,34,39
C - 1,4,5,6,10,12,15,28,32,34,39

-

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16 A - 1,2,3,4,5,6,7,8,9,10,24,28,28,32,33,34
B - 1,4,5,6,7,8,9,10,24,29,32,33
C - 1,2,3,4,5,6,7,8,9,10,12,15,24,28,29,32,33,39,46,49

-

17 A - 1,3,4,5,6,8,10,12,20,23,24,25,26,28,29,32
B - 1,3,4,5,6,8,10,11,12,16,17,20,23,26,27,28,
29,31,37,39
C - 1,3,4,5,6,8,10,11,12,16,17,20,23,24,25,
26,27,28,29,31,32,37,38,45,46,49

-

18 A - 1,3,4,5,6,7,10,13,15,28,29,31,32,34,39
B - 1,3,4,5,6,7,10,13,15,24,25,28,29,31,32,37,38,39,46
C - 1,3,4,5,6,7,10,11,12,13,15,21,23,24,28,29,32,
39,46,49

-

20 A - 1,3,4,5,6,7,10,15,24,39
B - 1,3,4,5,6,7,10,15,21,28,29,31,32,34,38,39
C - 1,4,5,6,7,10,12,15,24,27,28,29,34,39,46

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Strategies Used to Solve Each Problem by Groups in Grade 5

Problem Number	Strategies A= scores 1-9; B= scores 10-15; C= scores 16-19
1	A - 1,4,5,6,10,24,28,29,31,32,34 B - 1,3,4,5,6,10,11,12,21,23,24,27,28,29,32,39 C - 1,3,4,5,6,10,11,12,21,29,32,39
2	A - 1,2,4,5,6,8,10,11,13,15,23,24,28,30,31,32,39 B - 1,3,4,5,6,8,10,11,12,13,15,21,24,25,29,30, 32,36,38,39,40, C - 1,3,4,5,6,10,11,12,13,15,21,23,24,29,30,39
3	A - 1,2,4,5,6,7,8,10,11,24,28,29,39 B - 1,4,5,6,7,8,10,12,21,24,28,29,32,39 C - 1,4,5,6,7,8,10,12,24,28,29,38,39,46,49
4	A - 1,2,3,4,5,6,8,10,11,12,28,29,34 B - 1,4,5,6,10,12,21,23,24,28,29,37,39 C - 1,4,5,6,10,12,21,24,29,39,49
5	A - 1,3,4,5,6,9,10,11,22,23,24,27,28,29,32,34,40 B - 1,3,4,5,6,8,9,10,11,12,17,21,23,24,28,29,32,39,40,49 C - 1,3,4,5,6,8,9,10,12,24,28,29,32,39,45,46,49
6	A - 1,2,4,5,6,7,10,11,17,21,23,24,28,28,31,32, 34,35,36,37 B - 1,2,3,4,5,6,7,8,10,11,12,21,23,27,28,29, 32,34,36,37,38,39,46,49 C - 1,2,3,4,5,6,7,8,10,11,12,14,16,17,19,21,23, 25,26,29,32,34,37,38,39,45,46,48,49
7	A - 1,3,4,5,6,10,11,15,32,34,39 B - 1,3,4,5,6,10,15,24,25,28,34,37,39 C - 1,4,5,6,10,12,15,21,25,34,39

---

8 A - 1,3,4,5,6,8,10,11,16,24,29,31,32
B - 1,4,5,6,8,9,10,12,15,16,21,23,28,29,31,32,39
C - 1,3,4,5,6,8,10,12,15,16,21,24,26,28,29,31,32,
37,38,39,46,49

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9 A - 1,3,4,5,6,17,24,28,29,31,32,39
B - 1,4,5,6,10,11,12,21,23,24,27,28,29,30,32,37,39
C - 1,3,4,5,6,10,11,12,13,16,17,21,23,24,25,26,27,
28,29,30,31,32,36,37,38,39,45,46,48,49

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10 A - 1,3,4,5,6,11,24,29,31,40
B - 1,3,4,5,6,7,8,10,12,15,23,24,28,29,32,39,46,49
C - 1,3,4,5,6,10,12,13,15,24,26,27,28,29,32,39,
45,46,48,49

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11 A - 1,4,5,6,9,10,11,24,28,32,40
B - 1,2,3,4,5,6,9,10,12,21,24,28,29,39
C - 1,4,5,6,10,12,24,32,39,46,49

---

12 A - 1,4,5,6,8,11,24,27,28,29,31,32,39
B - 1,2,3,4,5,6,7,10,11,12,21,23,25,27,28,29,
31,32,34,39
C - 1,4,5,6,7,8,10,12,13,15,21,24,29,32,38,39,48,49

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13 A - 1,3,4,5,11,16,24,29,31
B - 1,2,3,4,5,6,8,11,12,23,24,27,28,29,32,34,37,39,40
C - 1,3,4,5,6,11,12,17,23,24,27,29,31,32,39,46

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14 A - 1,4,5,6,9,10,11,24,39
B - 1,3,4,5,6,8,9,10,12,17,21,24,27,28,32,37,39
C - 1,4,5,6,9,10,12,15,21,24,28,29,32,39,49

---

15 A - 1,4,5,6,10,11,15,24,31

B - 1,2,3,4,5,6,10,15,21,24,29,34,39
C - 1,4,5,6,10,12,32,34,39,49

---

16 A - 1,2,4,5,6,7,8,9,10,11,15,23,24,28,29,32,33,37,39
B - 1,2,4,5,7,8,9,10,12,22,24,29,32,33,39,46
C - 1,2,4,5,6,7,8,9,10,12,21,24,29,32,33,39,46,49

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17 A - 1,3,4,5,6,11,20,23,25,28,29,31,36,37
B - 1,3,4,5,6,8,10,17,20,21,22,23,25,26,27,28,29,
31,32,36,37,39
C - 1,2,3,4,5,6,8,10,11,12,16,17,20,22,23,24,25,26,
27,28,29,31,32,37,39,46,49

---

18 A - 1,4,5,6,11,15,23,27,28,29,31,32,39
B - 1,3,4,5,6,7,10,13,15,21,24,26,27,28,29,32,
34,35,39,49
C - 1,3,4,5,6,7,10,11,12,13,15,21,24,26,29,32,34,
37,39,46,49

---

20 A - 1,3,4,5,6,7,8,10,11,15,24,27,28,29,32,38,39
B - 1,3,4,5,6,7,8,10,24,27,28,39
C - 1,4,5,6,7,10,12,15,23,24,28,28,34,39,46,49

Strategies Used to Solve Each Problem by Group in Grade 6

-Problem
Number
19

Strategies
A= score 1-9; B= score 10-15; C= score 16-

---

1 A - 1,3,4,5,6,8,10,11
B - 1,2,3,4,5,6,10,11,17,21,24,28,29,32,32,34
C - 1,3,4,5,6,10,11,12,17,21,24,25,26,27,28,29,32,34,36,
38,40,49

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2 A - 1,3,4,5,6,7,10,13,15,22,23,29,30
B - 1,3,4,5,6,10,11,12,13,15,23,24,28,29,30,
31,32,34,39,40,
C - 1,3,4,5,6,8,10,11,12,13,15,21,23,24,27,28,29,
30,31,32,34,46

---

3 A - 1,3,4,5,6,7,10,11,27,28,29,32
B - 1,4,5,6,7,8,10,21,24,28,29,32,38,39
C - 1,4,5,6,7,8,10,12,21,22,23,24,26,27,28,29,
31,32,36,39,45

---

4 A - 1,3,4,5,6,8,10,29,32,39
B - 1,4,5,6,10,17,21,24,26,27,28,29,31,32,34,36,39
C - 1,4,5,6,10,12,24,26,29,32,34,45

5, A - 1,3,4,5,6,8,9,10,11,21,23,28,32,37,39
B - 1,2,3,4,5,6,8,9,10,11,12,17,21,22,23,27,28,29,
32,36,37,39,45,46
C - 1,3,4,5,6,8,9,10,11,12,15,21,23,24,27,28,29,
32,34,36,38,39,45,46,49

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6 A - 1,3,4,5,6,7,10,15,23,28,29,31,32
B - 1,2,3,4,5,6,7,8,10,11,15,21,23,27,28,29,32,34,
36,38,39,45
C - 1,3,4,5,6,7,8,10,11,12,15,21,23,24,25,26,27,28,29,
32,34,36,38,39,46

-

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7 A - 1,4,5,6,10,11,15,21,29,31,32
B - 1,3,4,5,6,8,10,12,15,17,21,24,27,27,29,32,34,39
C - 1,3,4,5,6,10,15,17,24,27,28,29,32,34,39,49

-

8 A - 1,4,5,6,8,10,11,15,16,21,28,29,31,32
B - 1,3,4,5,6,8,10,11,15,16,21,24,27,29,31,32,46
C - 1,3,4,5,6,8,10,11,12,16,21,23,24,26,27,28,29,31,
32,38,48,49

-

9 A - 1,3,4,5,6,11,28,32
B - 1,3,4,5,6,10,11,13,17,21,23,24,25,27,28,29,30,31,
32,34,36,38,39
C - 1,2,3,4,5,6,8,10,11,12,13,14,15,17,21,23,24,25,26,
27,28,29,30,31,32,38,39,45,46,49

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10 A - 1,4,5,6,10,11,15,32,34,
B - 1,4,5,6,7,8,10,11,12,15,21,23,24,26,27,28,29,32,
34,46
C - 1,2,4,5,6,7,8,10,11,12,15,23,24,28,29,32,46

-

11 A - 1,3,4,5,6,9,10,11,23,29,32,34,37
B - 1,2,4,5,6,8,9,10,12,21,24,26,28,29,32,34,39
C - 1,4,5,6,9,10,12,24,26,28,29,32,46

-

12 A - 1,3,4,5,6,8,11,23,29,31,34
B - 1,3,4,5,6,7,8,10,11,12,15,17,21,23,24,27,28,29,
31,32,34,36,37,38,39,46,49
C - 1,2,3,4,5,6,7,8,10,11,13,15,23,24,25,27,28,29,
31,32,36,39,46,49

-

13 A - 1,2,4,5,6,11,23,31,32,34,40
B - 1,3,4,5,6,8,16,23,25,28,29,31,32,34,36,39
C - 1,3,4,5,6,7,10,11,12,15,16,17,21,23,24,25,26,
27,28,29,30,31,32,37,39,45,46,49